

**QUANTUM EFFECTS ON CONFORMALLY COUPLED SCALARS
DURING INFLATION**



Ph.D. THESIS

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Department of Physics Engineering

Physics Engineering Programme

JULY 2018

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**ENFLASYON SÜRESİNCE
KONFORMAL BAĞLI SKALERLERE OLAN KUANTUM ETKİLERİ**

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Date of Submission : **14 May 2018**

Date of Defense : **03 July 2018**



FOREWORD

I would like to present my gratitude to my advisor Prof. Dr. Emre Onur KÂHYA for giving me the opportunity to work with him and for sharing his time with me throughout last five years of my Ph.D..

I would like to thank Dr. Sohyun PARK for her meritorious contributions to me and especially deeply to thank Prof. Dr. Richard Paul WOODARD for his helpful comments, discussions and supports to me during my Ph.D. thesis.

I thank jury members of my Ph.D. thesis; Prof. Dr. Neşe ÖZDEMİR, Assoc. Prof. Dr. Abdurrahman Savaş ARAPOĞLU, Prof. Dr. Tekin DERELİ and Prof. Dr. Osman Teoman TURGUT.

I thank Prof. Dr. Shantanu DESAI and Prof. Dr. Kâzım Yavuz EKŞİ for their precious supports, Assoc. Prof. Dr. Özgür AKARSU, Assoc. Prof. Dr. Tolga BİRKANDAN, Assoc. Prof. Dr. Mehmet ÖZKAN, Assoc. Prof. Dr. James SERVANTIE and Assoc. Prof. Dr. Ergün YARANERİ for their valuable helps to me.

I should remark that the work titled by One loop corrected conformally coupled scalar mode equations during inflation, Phys. Rev. D 96, 105003, (2017), in this thesis was supported by T.C. İSTANBUL TECHNICAL UNIVERSITY (İTÜ), The Scientific Research Project (BAP) coordination unity and the works titled by Quantum gravity corrections to the conformally coupled scalar self-mass-squared on de Sitter background. II. Kinetic conformal cross terms, Phys. Rev. D 96, 025001, (2017) and Quantum gravity corrections to the conformally coupled scalar self-mass-squared on de Sitter background, Phys. Rev. D 90, 124054, (2014) in this thesis were supported by The Scientific and Technological Research Council of Turkey (TUBITAK) 1001 Project. I thank T.C. İTÜ BAP coordination unity and TUBITAK.

July 2018

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TABLE OF CONTENTS

	<u>Page</u>
FOREWORD.....	vii
TABLE OF CONTENTS.....	ix
ABBREVIATIONS	xi
SYMBOLS.....	xiii
LIST OF TABLES	xv
LIST OF FIGURES	xvii
SUMMARY	xix
ÖZET	xxiii
1. INTRODUCTION	1
1.1 Purpose of Thesis	1
1.2 Literature Review	2
1.3 Hypothesis	4
2. QUANTUM GRAVITY CORRECTIONS TO THE CONFORMALLY COUPLED SCALAR SELF-MASS-SQUARED ON DE SITTER BACK- GROUND.....	7
2.1 Introduction	7
2.2 The self-mass-squared.....	9
2.2.1 Formal expressions for the one loop self-mass-squared.....	12
2.2.1.1 4-point contributions.....	12
2.2.1.2 3-point contributions.....	13
2.2.2 Propagators	14
2.3 Computation of the one loop self-mass-squared	17
2.3.1 Contribution from the 4-point vertices	17
2.3.2 3-point interaction.....	18
2.4 Renormalization	23
2.4.1 Construction of counterterms	23
2.4.2 Localization of divergences and a finite result	26
2.5 Discussion.....	34
3. QUANTUM GRAVITY CORRECTIONS TO THE CONFORMALLY COUPLED SCALAR SELF-MASS-SQUARED ON DE SITTER II: KINETIC-CONFORMAL CROSS TERMS.....	39
3.1 Introduction	39
3.2 The Self-Mass-Squared	41
3.2.1 Formal expressions for the one loop self-mass-squared.....	44
3.2.1.1 4-point contributions.....	44
3.2.1.2 3-point contributions.....	44
3.2.2 Propagators	46

3.3 Computation of the 4-point interactions.....	49
3.4 Computation of the 3-point interactions.....	50
3.4.1 3-point kinetic interactions	51
3.4.1.1 Local contributions for 3-point kinetic part.....	52
3.4.1.2 Logarithm contributions for 3-point kinetic part.....	53
3.4.1.3 Normal contributions for 3-point kinetic part.....	56
3.4.2 3-point cross interactions.....	59
3.4.2.1 Local contributions for 3-point cross part	60
3.4.2.2 Logarithm contributions for 3-point cross part.....	60
3.4.2.3 Normal contributions for 3-point cross part	62
3.5 Renormalization	66
3.5.1 Allowed counterterms and the associated vertices	67
3.5.2 Localization of divergences	68
3.5.3 Regularization.....	71
3.5.3.1 For 4-point part and 3-point kinetic part	71
3.5.3.2 For 3-point cross part.....	75
3.5.4 Renormalization.....	80
3.6 Discussion.....	82
4. ONE LOOP CORRECTED CONFORMALLY COUPLED SCALAR	
MODE EQUATIONS DURING INFLATION	85
4.1 Introduction	85
4.2 Our Previous Calculation.....	86
4.3 Effective Mode Equation	88
4.3.1 Schwinger-Keldysh formalism	88
4.3.2 Perturbative Solution	90
4.3.2.1 Local Corrections	93
4.3.2.2 Nonlocal Corrections.....	93
4.4 Discussion.....	95
5. LOOP CORRECTIONS TO PRIMORDIAL NON-GAUSSIANITY	97
5.1 Introduction	97
5.2 Slow-roll inflation and time-dependent zeta-zeta correlator	99
5.3 Bispectrum at tree-level.....	102
5.4 One-loop correction to bispectrum: a particular example	104
5.5 Discussion.....	106
6. CONCLUSIONS AND RECOMMENDATIONS	111
6.1 Conclusions	111
6.2 Recommendations	112
REFERENCES.....	113
APPENDICES	125
APPENDICES	127
APPENDIX A.1	127
Tables for Identities to extract derivatives	127
Tables for the coefficient functions of external operators.....	129
APPENDIX A.2	137

Tables for the coefficient functions of external operators for 3-point kinetic part	137
Tables for the coefficient functions of external operators for 3-point cross part	142
APPENDIX A.3	154
The Tables For All Finite Nonlocal Contributions From [1, 2].....	154
Key Integral Tables	157
The Examples For Calculating Key Integrals From Table A.42.....	159
Result Tables.....	160
The Examples From The Table A.45	163
CURRICULUM VITAE	166





ABBREVIATIONS

BICEP2	: Background Imaging of Cosmic Extragalactic Polarization2
BPHZ	: Bogoliubov Parasiuk Hepp Zimmermann
CC	: Conformally Coupled
CMB	: Cosmic Microwave Background
MMC	: Massless Minimally Coupled
SMS	: Self-Mass-Squared
1PI	: One Particle Irreducible





SYMBOLS

$-iM^2(x; x')$	: Self-Mass-Squared
$i\Delta(x; x')$	: Propagator
ϕ	: Scalar Field





LIST OF TABLES

	<u>Page</u>
Table 2.1 : Contributions to counterterms. All terms are multiplied by $\frac{i\tilde{\kappa}^2 H^{D-4}}{(4\pi)^{D/2}} \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)}$	29
Table 2.2 : Contributions to counterterms. All terms are multiplied by $\frac{i\tilde{\kappa}^2 H^{D-4}}{(4\pi)^{D/2}} \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)}$	30
Table 2.3 : Other contributions to counterterms. All terms are multiplied by $\frac{i\tilde{\kappa}^2 H^{D-4}}{(4\pi)^{D/2}} \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)}$	31
Table 2.4 : All Finite Nonlocal Contributions with $x \equiv \frac{y}{4}$, where $y(x; x')$ is defined in the equation (2.33).	32
Table 3.1 : Nonlocal logarithm contributions from the relation (3.72) with $x \equiv \frac{y}{4}$	56
Table 3.2 : Nonlocal logarithm contributions from the relation (3.111) with $x \equiv \frac{y}{4}$	61
Table 3.3 : Contributions to 3-point kinetic counterterms. All terms are multiplied by $\frac{i\kappa^2 H^{D-4}}{(4\pi)^{D/2}} \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)}$	72
Table 3.4 : Contributions to 3-point kinetic counterterms. All terms are multiplied by $\frac{i\kappa^2 H^{D-4}}{(4\pi)^{D/2}} \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)}$	72
Table 3.5 : Contributions to 3-point kinetic counterterms. All terms are multiplied by $\frac{i\kappa^2 H^{D-4}}{(4\pi)^{D/2}} \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)}$	73
Table 3.6 : All finite nonlocal 3-point kinetic contributions with $x \equiv \frac{y}{4}$, where $y(x; x')$ is defined in the equation (3.31).	74
Table 3.7 : Contributions to 3-point cross counterterms. All terms are multiplied by $\frac{i\tilde{\kappa}^2 H^{D-4}}{(4\pi)^{D/2}} \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)}$	77
Table 3.8 : Contributions to 3-point cross counterterms. All terms are multiplied by $\frac{i\tilde{\kappa}^2 H^{D-4}}{(4\pi)^{D/2}} \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)}$	77
Table 3.9 : Contributions to 3-point cross counterterms. All terms are multiplied by $\frac{i\tilde{\kappa}^2 H^{D-4}}{(4\pi)^{D/2}} \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)}$	78
Table 3.10 : All finite nonlocal 3-point cross contributions with $x \equiv \frac{y}{4}$, where $y(x; x')$ is defined in the equation (3.31).	79
Table A.1 : For $F(y)\nabla^4 A(y)$	127
Table A.2 : For $F(y)\partial_0^2 \partial_0'^2 A(y)$	128
Table A.3 : For $F(y)\nabla^2(\partial_0^2 + \partial_0'^2)A(y)$	128
Table A.4 : For $F(y)\nabla^2(\partial_0 \partial_0'^2)B(y)$	129
Table A.5 : For $\alpha \equiv (aa')^2 \square^2$ type terms.	129
Table A.6 : For $\beta_1 \equiv (aa')^2 H^2 \square$ type terms.	130
Table A.7 : For $\beta_2 \equiv aa'(a^2 + a'^2)H^2 \square$ type terms.	130

Table A.8 :	For $\gamma_1 \equiv (aa')^2 H^4$ type terms.	131
Table A.9 :	For $\gamma_2 \equiv aa'(a^2 + a'^2) H^4$ type terms.	132
Table A.10:	For $\gamma_3 \equiv aa'(a + a')^2 H^4$ type terms.	133
Table A.11:	For $\delta \equiv (a^2 + a'^2) H^2 \nabla^2 \square$ type terms.	133
Table A.12:	For $\varepsilon_1 \equiv aa' H^2 \nabla^2$ type terms.	134
Table A.13:	For $\varepsilon_2 \equiv (a^2 + a'^2) H^2 \nabla^2$ type terms.	135
Table A.14:	For $\varepsilon_3 \equiv (a + a')^2 H^2 \nabla^2$ type terms.	136
Table A.15:	For $\zeta \equiv \nabla^4$ type terms.	136
Table A.16:	For $\alpha \equiv (aa')^{\frac{D}{2}+1} \square^2$ type terms.	137
Table A.17:	For $\beta_2 \equiv (aa')^{\frac{D}{2}} (a^2 + a'^2) H^2 \square$ type terms.	137
Table A.18:	For $\gamma_1 \equiv (aa')^{\frac{D}{2}+1} H^4$ type terms.	138
Table A.19:	For $\gamma_2 \equiv (aa')^{\frac{D}{2}} (a^2 + a'^2) H^4$ type terms.	138
Table A.20:	For $\gamma_3 \equiv (aa')^{\frac{D}{2}} (a + a')^2 H^4$ type terms.	139
Table A.21:	For $\delta \equiv (aa')^{\frac{D}{2}-1} (a^2 + a'^2) \nabla^2 \square$ type terms.	139
Table A.22:	For $\varepsilon_1 \equiv (aa')^{\frac{D}{2}} H^2 \nabla^2$ type terms.	140
Table A.23:	For $\varepsilon_2 \equiv (aa')^{\frac{D}{2}-1} (a^2 + a'^2) H^2 \nabla^2$ type terms.	140
Table A.24:	For $\varepsilon_3 \equiv (aa')^{\frac{D}{2}-1} (a + a')^2 H^2 \nabla^2$ type terms.	141
Table A.25:	For $\zeta \equiv (aa')^{\frac{D}{2}-1} \nabla^4$ type terms.	141
Table A.26:	For $\alpha \equiv (aa')^{\frac{D}{2}+1} \square^2$ type terms.	142
Table A.27:	For $\beta_1 \equiv (aa')^{\frac{D}{2}+1} H^2 \square$ type terms.	142
Table A.28:	For $\beta_2 \equiv (aa')^{\frac{D}{2}} (a^2 + a'^2) H^2 \square$ type terms.	143
Table A.29:	For $\gamma_1 \equiv (aa')^{\frac{D}{2}+1} H^4$ type terms.	144
Table A.30:	For $\gamma_2 \equiv (aa')^{\frac{D}{2}} (a^2 + a'^2) H^4$ type terms.	146
Table A.31:	For $\gamma_3 \equiv (aa')^{\frac{D}{2}} (a + a')^2 H^4$ type terms.	147
Table A.32:	For $\delta \equiv (aa')^{\frac{D}{2}-1} (a^2 + a'^2) H^2 \nabla^2 \square$ type terms.	148
Table A.33:	For $\varepsilon_1 \equiv (aa')^{\frac{D}{2}} H^2 \nabla^2$ type terms.	149
Table A.34:	For $\varepsilon_2 \equiv (aa')^{\frac{D}{2}-1} (a^2 + a'^2) H^2 \nabla^2$ type terms.	151
Table A.35:	For $\varepsilon_3 \equiv (aa')^{\frac{D}{2}-1} (a + a')^2 H^2 \nabla^2$ type terms.	152
Table A.36:	For $\zeta \equiv (aa')^{\frac{D}{2}-1} \nabla^4$ type terms.	153
Table A.37:	All Finite Nonlocal “conformal” contributions with $x \equiv \frac{y}{4}$, where $y(x; x')$ is defined in the equation (4.13) [1].	154
Table A.38:	All Finite Nonlocal “kinetic” contributions with $x \equiv \frac{y}{4}$, where $y(x; x')$ is defined in the equation (4.13) [2].	155
Table A.39:	All Finite Nonlocal “cross” contributions with $x \equiv \frac{y}{4}$, where $y(x; x')$ is defined in the equation (4.13) [2].	156
Table A.40:	Integrals with a'	157
Table A.41:	Integrals with a'^2	157
Table A.42:	Integrals with a'^3	158
Table A.43:	Integrals with a'^4	158
Table A.44:	$-\frac{\tilde{u}(0,k)}{a^2} \frac{iH^8}{(4\pi)^2} \int d^4 x' (\text{Ext. opr}) \times [f(\frac{y_{++}}{4}) - f(\frac{y_{+-}}{4})]$ from conformal nonlocal terms.	160
Table A.45:	$-\frac{\tilde{u}(0,k)}{a^2} \frac{iH^8}{(4\pi)^4} \int d^4 x' (\text{Ext. opr}) \times [f(\frac{y_{++}}{4}) - f(\frac{y_{+-}}{4})]$ from kinetic nonlocal terms.	161

Table A.46:	$-\frac{\tilde{u}(0,k)}{a^2} \frac{iH^8}{(4\pi)^2} \int d^4x' (\text{Ext. opr}) \times [f(\frac{y_{++}}{4}) - f(\frac{y_{+-}}{4})]$ from cross non-local terms.....	162
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LIST OF FIGURES

	<u>Page</u>
Figure 2.1: The one-loop self-mass-squared from massless conformally coupled scalar. The first two diagrams represent the 3-point and 4-point interactions, respectively. The last diagram corresponds to the counterterms required to absorb ultraviolet divergences from the two primitive diagrams.	11
Figure 2.2: Contribution from the 4-point vertices.....	17
Figure 2.3: Contribution from the 3-point vertices.....	18
Figure 2.4: Contribution from counterterms.	23
Figure 5.1: One-loop correction is sourced by cubic and quartic self-interactions of ζ	101
Figure 5.2: Bispectrum: Tree-level and corrections to external legs.....	104
Figure 5.3: The remaining diagram at one-loop order.....	108



QUANTUM EFFECTS ON CONFORMALLY COUPLED SCALARS DURING INFLATION

SUMMARY

The effects arising from the small (quantum) scale in the early universe help us to understand large (cosmological) scale structure formation in today's universe. Throughout this thesis, we are interested in these small scale effects in the early universe.

First of all, we search to find the answer to the fundamental question “*What is the source of these quantum effects?*” in this thesis. The answer to this question is given by the particular example, the interactions between the “massless conformally coupled scalars ($\tilde{\phi}$)” and “gravitons ($h_{\mu\nu}$)” in the early universe which lead to quantum effects.

The reason for the particularly choice of these two particles can be summarized as: The quantum effects stem from the particles created in vacuum. In flat space, virtual particles affect on these particles created in vacuum. These effects of virtual particles diminish and finally disappear at the Planck time. But the expansion of curved space-time alters ‘Heisenberg’s energy-time uncertainty principle’ ($\Delta E \Delta t > \hbar/2$), allowing virtual particles to survive longer than in flat space. The reaction rate of interacted virtual particles increase in the expanding space-time if these particles are massless and conformally noninvariant.

In inflation scenarios, the expansion of curved space-time can lead to particle creation by increasing the existence time of virtual particle pairs in vacuum. If the expansion is accelerated, the effect is strongest and the virtual particles are massless and do not possess conformal symmetry. Therefore, the well known examples of the kind of these particles are massless, minimally coupled (MMC) scalars and gravitons in the literature. They can mediate large quantum effects during inflation.

Also, the named as cosmological perturbations as firstly quantum gravitational observables that were predicted by Mukhanov and Chibisov at 1981 for the scalar part and by Starobinsky at 1979 for the tensor part. The MMC scalars and gravitons are the source of scalar (WMAP9, Planck) and tensor perturbations (B-mode polarization), respectively, predicted by inflationary theories. The observables from these cosmological perturbation came into prominence via Cosmic Microwave Background (CMB) power spectrum.

In this thesis, the interaction between particularly “massless conformally coupled scalars” and “graviton” is taken as the source of the quantum effects during inflation and these effects are calculated as the corrections coming from the first order level. In order to analyze these effects, the effective field equation are solved.

In **Chapters 2.** and **3.**, the calculations of leading order quantum gravity corrections from these interactions are detailly given in de Sitter inflation time. *Firstly*, the corresponding only the lowest order Feynman diagrams, i.e., one-loop order

Feynman diagrams, are calculated applying the perturbation method in terms of the loop-counting parameter ($\kappa^2 \equiv 16\pi G$). *Second*, fully renormalized self-mass-squared (SMS) ($iM^2(x; x')$) for conformally coupled scalars (CCS) is obtained by using special calculation technique, dimensional regularization. In this technique, the counterterms help to extract the finite terms from the divergences. The fully renormalized self-mass-squared is obtained for every interaction (conformal-conformal interaction, kinetic-kinetic interaction, cross interaction), separately. Then, the total of them is given as

$$-iM^2(x; x') = -iM_C^2(x; x') + -iM_K^2(x; x') + -iM_{\text{cross}}^2(x; x') ,$$

here C and K denote conformal and kinetic interactions, respectively.

The total of them $-iM^2(x; x')$ is needed for the next step of the calculations, as

$$\partial_\mu \left(\sqrt{-\tilde{g}} \tilde{g}^{\mu\nu} \partial_\nu \right) \tilde{\phi}(x) - \frac{1}{6} \tilde{\phi}(x) \tilde{R} - \int d^4 x' M^2(x; x') \tilde{\phi}(x') = 0 .$$

These difficult calculations and their outcomes are published as two works.

In **Chapter 4.**, the Schwinger-Keldysh linearized effective field equation

$$\partial^2 \tilde{\Phi}_1(x; \vec{k}) - \int_{\eta_i}^0 d\eta' \int d^3 x' \mathcal{M}_1^2(x; x') \tilde{\Phi}_0(x'; \vec{k}) = 0 .$$

is solved by applying the perturbation method in terms of loop-counting parameter to the obtained self-mass-squared outcomes and conformally coupling scalar as

$$M^2(x; x') = \sum_{\ell=1}^{\infty} \kappa^{2\ell} \mathcal{M}_\ell^2(x; x') ,$$

$$\tilde{\Phi}(x; \vec{k}) = \sum_{\ell=0}^{\infty} \kappa^{2\ell} \tilde{\Phi}_\ell(x; \vec{k}) = \sum_{\ell=0}^{\infty} \kappa^{2\ell} \tilde{u}_\ell(\eta, k) \times e^{i\vec{k} \cdot \vec{x}} ,$$

and using some special integral techniques. It is seen that the found quantum corrected scalar mode functions ($\tilde{u}_\ell(\eta, k)$) have a secular growth effect behaviour, proportional to a logarithm of the scale factor at late times with physical time. The all calculations' details and their outcomes are published in the third work.

In **Chapter 5.**, the effect of loop corrections to primordial non-Gaussianity of Cosmic Microwave Background is investigated and the findings from it can be sorted as: A huge value for the non-Gaussianity which would exceed the observed value by at least 30 orders of magnitude may be obtained for single scalar inflaton. It is shown that the non-Gaussianity is a better place for quantum gravitational corrections: While power spectrum is related to the magnitude of mode function, non-Gaussianity is related to the time derivative of the mode function. All details are included in the published last work.

All outcomes are summed and the recommended future works are presented in **Chapter 6.**

In this way, the expected outcomes coming from quantum gravity effects for particular example from the interaction between a “massless conformally coupled scalars” and a “graviton” during inflation time are investigated in this thesis.





ENFLASYON SÜRESİNCE KONFORMAL BAĞLI SKALERLERE OLAN KUANTUM ETKİLERİ

ÖZET

“Erken evrende küçük ölçekteki (kuantum) etkiler, bugünkü evrenimizi oluşturan büyük ölçek (kozmozik) yapı oluşumunun temelini oluşturur”, kozmozik savından hareketle; bu tezde, evrenimizin başlangıç zamanındaki bu küçük ölçekli etkilerin evren ivmelenerek genişledikçe evrenin dinamiğine olan zamanla artan katkıları ile ilgilenildi.

O halde, akla gelen ilk ifade, *“enflasyonun başlangıç zamanındaki bu kuantum etkilerinin kaynağının ne olabileceği hakkındadır”*. Birbirleriyle etkileşmekte olan bazı parçacıklar bu kuantum etkilerinin nedeni olarak gösterilebilir. Bahsedilen “bazı parçacıklar” ifadesi ise, bu tezde sunulan özel örnek için, “kütlesiz konformal bağlanan skalerler (ϕ)” ve “gravitonlar ($h_{\mu\nu}$)” olarak atıflandırıldı.

Özel olarak bu parçacıkların neden tercih edildiğinden bahsedilmesi gerekirse, şu kısa açıklamanın yapılması yerinde olacaktır:

Düz uzayda, sanal parçacıklar vakumda parçacık oluşumları üzerinde etkiye sahiptir. Planck zamanında bu etkinin yok olması gerektiği için, etkiler uzun sürmez, kısıtlanır. İvmelenerek genişleyen evren senaryolarından da bilineceği üzere, enflasyon zamanında vakumda sanal parçacıkların varolma zamanı kısıtsızdır. Uzay-zamanın bu genişlemesinin hızı, artarak ivmelenmeye sahip şekilde ne denli gerçekleşiyorsa, etkisi de o denli kuvvetli olacaktır ve bu sanal parçacıklar konformal simetri özelliğine sahip olmak zorunda değildir. Uzay-zamanının genişlemesi ile ‘Heisenberg’ in enerji-zaman belirsizlik ilkesinin’ ($\Delta E \Delta t > \hbar/2$) değişimi, sanal parçacıkların düz uzayda bulunduğundan daha uzun süre varolmasına imkân sağlar.

Buna ek olarak, sanal parçacıkların diğer parçacıkları etkilemeleri için, vakumda sadece uzun süreli varolmaları yeterli olmaz, sıklıkla da varolmaları gerekir. Bu ise sanal parçacıkların birbirleriyle reaksiyona girme oranları ile belirlenir.

Böylece, varolma sıklıklarının artması, sanal parçacıkların diğer parçacıklara olan etkisini artırır. Konformal bağlanan parçacıklar için reaksiyon hızı, parçacıkların oluşma hızını azaltıcı etkiye sahiptir.

Tüm bunların sonucu olarak, kütlesiz ve konformal etkileşmeyen parçacıklarla ilgilenilir. Bu özelliğe sahip çalışılan iki özel parçacık ise kütlesiz minimal bağlanan skalerler ve gravitonlardır. Enflasyon teorileri tarafından, bu parçacıklar sırasıyla, skaler pertürbasyonların ve tensör pertürbasyonların kaynakları olarak kabul edilirler.

Yakın geçmişte, Kozmik Arka Plan (CMB) güç tayfı ile kozmozik pertürbasyonların gözlenmesinin gerçekleşebilme düşüncesi önem kazanmıştır. Bu gözlemlerin, kuantum kütle çekimsel gözlemlerin kaynağı olabileceğine dair ilk öngörüler ise, skaler pertürbasyonlar için Mukhanov ve Chibisov tarafından 1981 yılında ve tensörler için Starobinsky tarafından 1979 yılında ortaya atılmıştır.

Her ne kadar kozmolojik tensörel pertürbasyon bileşenleri şimdiye dek gözlemsel olarak haritalanmış olmasa da, kozmolojik skaler pertürbasyon bileşenleri WMAP9 ve Planck ile haritalanmıştır. Fakat BICEP2’den gelen B-mod polarizasyon sonuçları, tensörel bileşenlerin görüntüsünün yakın gelecekte eldesi hakkında umut verici olmaktadır. Bu gözlemlenebilirler Kozmik Arka Plan (CMB) güç tayfı ile ön plana çıkmıştır.

Bu tezde, yukarıda bahsedilen gravitonun kütesiz konformal bağlı skalerlerle olan etkileşimi araştırıldı. Eğer enflasyonu sürdüren skaler parçacık (inflaton), gravitona konformal bağlı skaler parçacıksa, bunun fiziksel sonuçlarının da olması beklenir.

Buraya dek kısaca değinildiği gibi, bu tez için, kuantum kütle çekimsel etkilerin kaynağı olarak özel olarak “gravitonun, konformal skaler bağlı parçacıklara” olan etkisi incelendi.

Sırada ise, “*bu iki parçacık arasındaki etkileşimlerin kuantum seviyedeki katkılarının nasıl olabileceği*” ile ilgilendi ve bu kuantum kütle çekimsel etkileşimlerin katkılarına, efektif alan denkleminin çözüme kavuşturulması ile ulaşılabileceği gösterildi.

Bu tezde, eğri uzayda kuantum alan teorilerine özel hesaplama teknikleri uygulanıp oldukça uzun bir yol izlenerek efektif alan denkleminin çözümüne ulaşıldığı detaylı şekilde gösterildi. Sadece ilk seviye ilmek (tek-ilmek) Feynman diyagramları düzeltmeleri hesaplanarak kuantum kütle çekimsel katkı hesabı yapıldı.

Hesaplamalarda *ilk adımda*, de Sitter enflasyon süresince konformal bağlanan skalerlerin, bu parçacıkların tamamıyla renormalize edilmiş kendi karesi alınmış kütleleri ($-iM^2(x;x')$) her bir etkileşim (konformal-konformal etkileşim, kinetik-kinetik etkileşim, konformal-kinetik ve kinetik-konformal etkileşim) için ayrı ayrı elde edildi. Bunun için, *önce* teoriyi veren lagrangian yoğunluğundaki her terime ilmek saydırma parametresi ($\kappa^2 \equiv 16\pi G$) cinsinden pertürbasyon açılımı uygulandı. *Sonra* ise tamamıyla renormalize edilmiş kendi karesi alınmış kütle hesaplanmasının yapılması için, tekniksel yöntem uygulandı. Kullanılan bu yöntem boyutsal regularizasyondur; burada, karşıt terimler yardımıyla hesaplarda yer alan sonlu terimler diverjanslıktan kurtarılır. Böylece elde edilen sonlu terimler, hesaplamaların gelecek adımında kullanılmak üzere hazırlanmış olunur. Konformal-konformal etkileşimler ile ilgili olan detaylı hesaplamalar ve elde edilen sonuçlar **2. Bölümde**, kinetik-kinetik ve çapraz (konformal-kinetik ve kinetik-konformal) etkileşimler ile ilgili yapılan detaylı hesaplamalar ve elde edilen sonuçlar ise **3. Bölümde** sunulmaktadır. Tüm etkileşimlerin toplam kendi karesi alınmış kütleleri

$$-iM^2(x;x') = -iM_C^2(x;x') + -iM_K^2(x;x') + -iM_{\text{capraz}}^2(x;x') ,$$

şeklinde ifade edilir. Burada C ve K sırasıyla konformal ve kinetik etkileşimleri simgeler. Bu iki bölüm, bu tezde yer alan ilk iki sıralı makale haline getirilmiş ve yayınlanmıştır.

İkinci adımda ise, konformal bağlı skalerlerin kendi karesi alınmış kütleleri ile gravitonların etkileşimleri için efektif alan denklemleri elde edildi.

$$\partial_\mu \left(\sqrt{-\tilde{g}} \tilde{g}^{\mu\nu} \partial_\nu \right) \tilde{\phi}(x) - \frac{1}{6} \tilde{\phi}(x) \tilde{R} - \int d^4x' M^2(x; x') \tilde{\phi}(x') = 0 .$$

Bu tezde özel olarak aşağıda da verilen Schwinger-Keldysh doğrusal efektif alan denklemi kullanıldı.

$$\partial^2 \tilde{\Phi}_1(x; \vec{k}) - \int_{\eta_i}^0 d\eta' \int d^3x' \mathcal{M}_1^2(x; x') \tilde{\Phi}_0(x'; \vec{k}) = 0 .$$

Burada ilk olarak, elde edilmiş olan kendi karesi alınmış kütle ifadesi ve konformal bağlı scalar alan aşağıdaki gibi yine ilmek saydırma parametresi cinsinden pertürbe edildi,

$$\begin{aligned} M^2(x; x') &= \sum_{\ell=1}^{\infty} \kappa^{2\ell} \mathcal{M}_\ell^2(x; x') , \\ \tilde{\Phi}(x; \vec{k}) &= \sum_{\ell=0}^{\infty} \kappa^{2\ell} \tilde{\Phi}_\ell(x; \vec{k}) = \sum_{\ell=0}^{\infty} \kappa^{2\ell} \tilde{u}_\ell(\eta, k) \times e^{i\vec{k} \cdot \vec{x}} , \end{aligned}$$

ve ardından çeşitli integral çözme teknikleri uygulanarak, denklem fiziksel zamanda çözümü olan mod fonksiyonlarına ($\tilde{u}_\ell(\eta, k)$) konformal bağlı skalerler için kavuşturuldu.

Görüldü ki, kuantum düzeltilmiş mod fonksiyonları fiziksel zamana bağlı olarak büyüme davranışı sergiliyor ve evrenin geç zaman diliminde ölçek faktörünün logaritmik olarak büyümesi ile orantılı olan bir davranışa sahip. Sonuç olarak, bu tezde tek-ilmek düzeltme hesabının çözümünde pertürbatif yöntemle gerçekleşen hesaplamaların yetersiz kaldığı ve pertürbatif olmayan yeni hesaplama tekniklerine ihtiyaç duyulduğu belirtildi. Bununla ilgili gereken detaylı hesaplamalar ve elde edilen sonuçlar **4. Bölümde** sunulmaktadır. Bu bölüm, bu tezde sıralamadaki üçüncü makale olarak yer alır ve yayınlanmıştır.

Son olarak, ilmek düzeltmelerinin ilkel Gaussyen olmayan Kozmik Mikrodalga Arkaplan'a (CMB) etkileri **5. Bölümde** incelendi. Tek-ilmek düzeltmesi için özetle gösterildi ki, kuantum düzeltmelerin Kozmik Mikrodalga Arkaplan da olabileceği muhtemel en iyi yeri olarak, Gaussyen olmayan yerin olması gerektiği ve bu yerin mod fonksiyonunun zamana bağlı türevi ile ilişkili olarak elde edilebileceği bölge olduğu sunuldu.

Sıfırncı mertebede kuantum düzeltmelerinin katkıları neredeyse kaybolurken, birinci ile birlikte diğer üst mertebelerden gelen katkıların, sıfırncı mertebe ile karşılaştırıldığında hiç de küçük olmadığı görüldü.

Dahası Gaussyen olmayan gözlemlerin büyük değerlerinin kaynağının tek skaler alan parçacık örneği olan inflaton olabileceği gösterildi. Gaussyen olmayan gözlemlerin bu büyüklük değerleri için ölçüt olarak, en azından otuz mertebesindeki büyüklüğe sahip Gaussyen olmayan büyüklük sınır aşımının alınması gerektiği sunuldu. Bu bölüm, bu tezde dördüncü makale haline getirilmiş ve yayınlanmıştır.

Tüm elde edilen sonuçların özeti ve gelecek çalışmalar hakkındaki öneriler **6. Bölümde** verilmiştir.

Böylece, bu tezde hedeflenen, enflasyon süresince kuantum kütle çekimi etkilerinin detaylı şekilde görülmesi; bunlar için yapılan hesaplamalar ve bu hesapların gözlemlenebilir parametrelere olan katkıları sunulmuştur.



1. INTRODUCTION

The effects arising from the small (quantum) scale in the early universe help us to understand large (cosmological) scale structure formation in today's universe. During this thesis, we investigate these effects in inflation era of the universe. We present the solutions to three fundamental questions for this era. In a nutshell, these questions and their solutions are given as the following:

The first one is about “*the source of the quantum effects during primordial inflation time*”. For this, we will give the details about the interactions leading to the quantum effects. These interactions are between two particles: graviton and massless conformally scalar particle. In this stage, we need advanced technical tools for our calculations.

The second one consists of the investigation about “*the quantum gravity corrections to conformally coupled scalar interacting with graviton*” by using the mentioned effects above. We will construct these corrections by using our calculation techniques coming from the first solution.

Final fundamental problem is “*What is the quantum correction to non-Gaussianity?*” In the solution to this problem, our goal is to calculate the loop corrections to 3-point function which is proportional non-Gaussianity parameter.

calumake the explanation about non-Gaussian cases coming from spectrum analyses.

Thus, we aim to present the obtained outcome coming from the solutions of these problems.

1.1 Purpose of Thesis

The main purpose of thesis is to see the leading quantum effects during inflation. These quantum effects will be seen via one particular example [1–3] by solving the effective field equation for a conformally coupled scalar (CCS) self-mass-squared (SMS) $-iM^2(x;x')$ interacting with a graviton during de Sitter inflation. For early

universe, in the inflation scenarios, the expansion of space-time can lead to particles creation by delaying the virtual pairs in vacuum [4]. If the expansion is accelerated, the effect is strongest [5–7]. And the virtual particles are massless and do not possess conformal symmetry [8]. The well known examples of the kind of these particles are massless, minimally coupled (MMC) scalars and gravitons. These particles are the source of scalar (map [9, 10]) and tensor perturbations (B-mode polarization [11]), predicted by inflationary theories.

Finally, the effect of loop corrections to primordial non-Gaussianity of Cosmic Microwave Background (CMB) will be investigated as will be seen in [12].

1.2 Literature Review

One-loop quantum gravity corrections to the CCSSMS on a locally de Sitter background were evaluated in the works [1, 2]. These effects were detaily illustrated and interpreted by solving effective mode equation in the work [3]. For this thesis, the conformal-conformal, kinetic-kinetic and cross interaction parts of SMS were worked in these works. These works complement the minimal-minimal part worked out in the old papers [13, 14] in the literature.

In late 1960's Leonard Parker's discovery about the creation of particle out of the vacuum in an expanding universe takes the place in the literature via his work [5–7], also one can reach the References [15, 16] for the historical notes on the discovery. The particle creation in an exponentially expanding universe had not been emphasized much until the idea of inflation [17–19] was proposed in early 1980's [16]. However, the inflationary paradigm drastically changed the situation and particle production during inflation has been attracting a great deal of attention since then [1, 13, 14, 20–90]. The unique particles having no conformal invariance are the massless, minimally coupled (MMC) scalars and gravitons, and these particles comprise the scalar [91] and tensor [92] perturbations predicted by inflationary theories.

As was listed in the work [3], a number of computations involving either scalar-mediated effects or graviton-mediated effects during inflation have been made over the past decades in the literature:

- For MMC scalars Yukawa coupled to massless fermions, a growing mass is induced to fermions [93–95], but there is no large correction on scalars [96].
- In MMC scalar quantum electrodynamics, assuming the scalars are charged, photons acquire a growing mass [97–100], but scalars receive no significant effect [35, 36].
- For MMC scalars with a quartic self-interaction, scalars gain an increasing mass (though it remains perturbatively small) at one and two loop orders [21, 101].
- MMC scalars have no significant effect on dynamical gravitons at one loop order [37, 38], but they induce a secular effect on the gravitational potentials of a point mass (specifically a secular decrease in the gravitational coupling G) [41]. One loop corrections from conformal fields to the gravitational potentials of a point mass were also found [40, 47, 49, 51].
- For quantum gravity plus massless fermions, the fermion field strength gets a secular growth effect from inflationary gravitons [29–31, 95].
- For quantum gravity plus MMC scalars, inflationary gravitons induce no significant corrections on the scalar mode functions at one loop [13, 14].
- For quantum gravity plus electrodynamics, inflationary gravitons induce secular effects on the dynamical photons and alter the electric field of a point charge and the magnetic field of a point magnetic dipole [70–72, 74, 75, 103, 104].

These secular effects are typically in the form of the logarithm of the scale factor $\ln(a)$ times the minuscule loop counting parameter GH^2 [105, 106].

As was given in the work [12], for the past fifteen years, there have been many efforts towards understanding loops in cosmology. Among this relatively large literature, the most influential works were those of Weinberg [105, 106]. In one of these works, Weinberg asserted a theorem [106] related to quantum loop effects in cosmology: in N -th order perturbation theory, quantum corrections can at most be of order $(\alpha \ln a(t))^N$, where α is the loop counting parameter and $a(t)$ is the scale factor. There were plenty of discussions [107–111] not about the existence but the type of “infrared logarithm” that arise from the quantum contributions in cosmological

correlations. Two types of infrared logarithm factors that appeared are time-dependent and time-independent.

1.3 Hypothesis

The most well-known examples of massless particles with no conformal invariance are the massless, minimally coupled (MMC) scalars and gravitons. These particles are the sources of the scalar and tensor perturbations predicted by inflationary theories [91,92]. The scalar component of which has been mapped with great precision [9, 10]. The tensor components have not been mapped yet, but with the recent announcement for the detection of the *B*-mode polarization by the BICEP2 collaboration [11], it is not hopeless to image them in the near future.

Furthermore, most probably, the founders of quantum gravity did not have high hopes that what they were doing would some day be tested or even have observational consequences. But, the CMB power spectrum opened that window to us and now we can name cosmological perturbations as first quantum gravitational observables that were predicted by Mukhanov and Chibisov [91] for the scalar part and by Starobinsky [92] for the tensor part.

One might expect strong quantum effects for the interaction between the MMC scalars and gravitons because these two particles (being massless and without conformal invariance) are the ones produced enormously during inflation [8]. However, many studies [13, 14, 20–27, 29–33, 35–38, 72, 112] suggest that the strongest effects are caused by non-derivative interactions. The reason why it happens is the differentiated scalar, so its kinetic energy is redshifted away during inflation so they interact very weakly with virtual gravitons. So one loop corrections from gravitons to the self-mass-squared of MMC scalar result in zero [13, 14]. The same phenomenon happens to the mirror case, namely the differentiated gravitons interacting with virtual scalars [37, 38]. In fact, these null results motivated the authors to consider the conformally coupled scalar interacting with virtual gravitons by noting that in the conformal coupling term the scalar is not differentiated. Moreover, the mode functions of conformally coupled scalar redshift to zero at tree order. Therefore any loop corrections to them would be dominant over the zero tree order result.

Thus, the obtained outcomes from this thesis contribute to literature and continue hopes of the observations to test them.





2. QUANTUM GRAVITY CORRECTIONS TO THE CONFORMALLY COUPLED SCALAR SELF-MASS-SQUARED ON DE SITTER BACKGROUND

We evaluate one loop quantum gravity corrections to the conformally coupled (CC) scalar self-mass-squared on a locally de Sitter background. In this paper¹ we consider only the conformal-conformal interaction part of the self-mass-squared. This complements the minimal-minimal part worked out in the previous paper [13] and we will add the minimal-conformal part in a follow-up paper to complete the full self-mass-squared at one loop order. The computation is performed using dimensional regularization and the results are fully renormalized by absorbing divergences with counterterms. The finite results can give rise to quantum corrections to the CC scalar mode functions and therefore to their power spectra.

2.1 Introduction

An intuitive way of understanding quantum effects is to examine the classical response to virtual particles. If there are not many virtual particles or they interact with the particles in question only weakly there will not be much quantum loop effect. In this respect it will be interesting to investigate the cases in which the number of virtual particles increases and their interactions with other particles. It has been known for a long time that the expansion of spacetime can lead to particle creation by delaying the annihilation of virtual pairs ripped out of the vacuum since Schrödinger first realized it [4]. In late 1960's Parker carried out explicit computations regarding particle production in expanding universe [5–7] and the results are summarized as follows: the effect is the strongest if the expansion is accelerated, and the virtual particles are massless and do not possess conformal symmetry [8].

The locally de Sitter background we consider is the most highly accelerated expansion with classical stability. The unique examples of massless particles with no conformal

¹This chapter is based on the paper Quantum gravity corrections to the conformally coupled scalar self-mass-squared on de Sitter background, *Phys. Rev. D* 90, 124054, (2017) by **Boran, S.**, Kâhya, E. O. and Park, S. .

invariance are the massless, minimally coupled (MMC) scalars and gravitons. These particles are the sources of the scalar and tensor perturbations predicted by inflationary theories [91, 92]. The scalar component of which has been mapped with great precision [9, 10]. The tensor components have not been mapped yet, but with the recent announcement for the detection of the B -mode polarization by the BICEP2 collaboration [11], it is not hopeless to image them in the near future.

The approximate tree order results for the scalar and tensor power spectra are

$$\begin{aligned}\Delta_{\mathcal{R}}^2(k) &= \lim_{t \gg t_k} \frac{k^3}{2\pi^2} \times 4\pi G \times |v(t, k)|^2 \simeq \frac{GH^2}{\pi\epsilon} , \\ \Delta_h^2(k) &= \lim_{t \gg t_k} \frac{k^3}{2\pi^2} \times 64\pi G \times |u(t, k)|^2 \simeq \frac{16GH^2}{\pi} .\end{aligned}\quad (2.1)$$

Here $v(t, k)$ and $u(t, k)$ are the mode functions of scalar and tensor perturbations, G is Newton's constant, $H(t)$ is the Hubble parameter and $\epsilon(t)$ is the slow roll parameter. The time t is taken to be much later than t_k , the time of first horizon crossing for the mode of wave number k . One can study loop corrections to this lowest order effect using a mathematical object called the one-particle-irreducible (1PI) 2-point function. The procedure is first to compute the 1PI 2-point function, for the case of scalar it is the self-mass squared, $-iM^2(x; x')$. Second, to use it to quantum-correct the linearized effective field equation, for the CC scalar it is

$$\partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \phi(x) \right) - \frac{1}{6} R \phi(x) - \int d^4 x' M^2(x; x') \phi(x') = 0 . \quad (2.2)$$

By solving this equation one can get loop corrections to the mode functions for the inflaton, where the conformal coupling term can be thought effectively as an inflaton potential (similar to the $m^2 \phi^2$ model) in the context of scalar-driven inflation models at the lowest order.

A number of computations in this regard have been made over the last decade [13, 14, 20–27, 29–33, 35–38, 43–48, 60–69, 72, 112–115]. (A simple worked-out example specifically for loop corrections to the power spectra can be found in [116].) In this paper we carry out the first part of this procedure, that is we compute the self-mass-squared of CC scalar interacting with inflationary gravitons at one loop order and renormalize it so to make the integral in (2.2) finite. In a follow-up paper we will solve the effective field equation to study one loop corrections to the tree order mode functions of the CC scalar field.

One might expect strong quantum effects for the interaction between the MMC scalars and gravitons because these two particles (being massless and without conformal invariance) are the ones produced enormously during inflation [8]. However, many studies [13, 14, 20–27, 29–33, 35–38, 72, 112] suggest that the strongest effects are caused by non-derivative interactions. The reason why it happens is the differentiated scalar, so its kinetic energy is redshifted away during inflation so they interact very weakly with virtual gravitons. So one loop corrections from gravitons to the self-mass-squared of MMC scalar result in zero [13, 14]. The same phenomenon happens to the mirror case, namely the differentiated gravitons interacting with virtual scalars [37, 38]. In fact, these null results motivated the authors to consider the conformally coupled scalar interacting with virtual gravitons by noting that in the conformal coupling term the scalar is not differentiated. Moreover, the mode functions of conformally coupled scalar redshift to zero at tree order. Therefore any loop corrections to them would be dominant over the zero tree order result.

In Section 2.2, we derive a formal expression for the CC scalar self-mass-squared. At one-loop order it consists of two parts, namely 4-point and 3-point interactions. Explicit computations of the two parts are performed in Section 2.3. The results are fully renormalized using appropriate counterterms in Section 2.4. Our discussions comprise Section 2.5.

2.2 The self-mass-squared

The Lagrangian which describes pure gravity and the interaction between gravitons and the conformally coupled scalar (in D spacetime dimensions to facilitate dimensional regularization) is

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}\sqrt{-g} - \frac{D-2}{8(D-1)}\phi^2 R\sqrt{-g} + \frac{1}{16\pi G}(R - (D-2)\Lambda)\sqrt{-g}, \quad (2.3)$$

where R is the Ricci scalar and $\Lambda = (D-1)H^2$ is the cosmological constant. Varying the 1PI effective action corresponding to the Lagrangian (2.3) with respect to the scalar field we have the linearized effective field equation (2.2),

$$\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi(x)\right) - \frac{1}{6}R\phi(x) - \int d^4x' M^2(x;x')\phi(x') = 0.$$

We work on the open conformal coordinate patch of de Sitter space

$$ds^2 = \hat{g}_{\mu\nu} dx^\mu dx^\nu = a^2(\eta) \left[-d\eta^2 + d\vec{x} \cdot d\vec{x} \right], \quad \text{where} \quad a(\eta) = -\frac{1}{H\eta}, \quad (2.4)$$

with the coordinate ranges

$$-\infty < x^0 \equiv \eta < 0, \quad -\infty < x^i < +\infty, \quad i = 1, 2, \dots, D-1. \quad (2.5)$$

Perturbation theory is expressed in terms of the background metric $\hat{g}_{\mu\nu}$ and conformally rescaled graviton field $h_{\mu\nu}$

$$g_{\mu\nu}(x) \equiv a^2 \left[\eta_{\mu\nu} + \kappa h_{\mu\nu}(x) \right], \quad \text{where} \quad \kappa^2 \equiv 16\pi G, \quad (2.6)$$

Due to the conformal coupling term (the second term) the matter sector (the first and second term) in the Lagrangian (2.3) is invariant under the conformal rescaling,

$$g_{\mu\nu} \equiv \Omega^2 \tilde{g}_{\mu\nu}, \quad \phi \equiv \Omega^{\frac{2-D}{2}} \tilde{\phi}. \quad (2.7)$$

$$\begin{aligned} \Rightarrow \mathcal{L}_{\text{Mat}} &= -\frac{1}{2} \partial_\mu \phi \partial_\nu \phi g^{\mu\nu} \sqrt{-g} - \frac{D-2}{8(D-1)} \phi^2 R \sqrt{-g}, \\ &= -\frac{1}{2} \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} \tilde{g}^{\mu\nu} \sqrt{-\tilde{g}} - \frac{D-2}{8(D-1)} \tilde{\phi}^2 \tilde{R} \sqrt{-\tilde{g}}. \end{aligned} \quad (2.8)$$

Taking $\Omega = a$ we can simply work with the conformally rescaled metric

$$\tilde{g}_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}. \quad (2.9)$$

of which the inverse and the volume element are expanded as

$$\tilde{g}^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^\mu{}_\rho h^{\rho\nu} + \mathcal{O}(\kappa^3), \quad (2.10)$$

$$\sqrt{-\tilde{g}} = 1 + \frac{1}{2} \kappa h + \frac{1}{8} \kappa^2 h^2 - \frac{1}{4} \kappa^2 h^{\rho\sigma} h_{\rho\sigma} + \mathcal{O}(\kappa^3). \quad (2.11)$$

The next step is using the above expressions for the metric in order to expand the Ricci scalar at second order in κ . The expression for $\tilde{R}$ is,

$$\begin{aligned} \tilde{R} &= \kappa \left(-h^\mu{}_\mu + h^{\mu\nu}{}_{,\mu\nu} \right) + \kappa^2 \left(-2h^{\mu\nu} h_{\lambda\nu,\mu}{}^\lambda + h^{\mu\nu} h_{\mu\nu,\lambda}{}^\lambda + h^{\mu\nu} h_{,\mu\nu} \right. \\ &\quad \left. + \frac{3}{4} h^{\mu\nu}{}_{,\lambda} h_{\mu\nu}{}^{,\lambda} + h^{\mu\nu}{}_{,\mu} h_{,\nu} - h^{\mu\nu}{}_{,\mu} h_{\lambda\nu}{}^{,\lambda} - \frac{1}{2} h^{\mu\nu}{}_{,\lambda} h^\lambda{}_{\nu,\mu} - \frac{1}{4} h_{,\mu} h^{,\mu} \right. \\ &\quad \left. - \frac{1}{2} h h^\mu{}_\mu + \frac{1}{2} h h^{\mu\nu}{}_{,\mu\nu} \right) + \mathcal{O}(\kappa^3). \end{aligned} \quad (2.12)$$

Using the perturbative expansion, the self-mass-squared $-iM^2(x; x')$ can be computed at any desired loop order. Our aim is to derive it at one loop order, which consists of the three Feynman diagrams depicted in Figure 2.1.

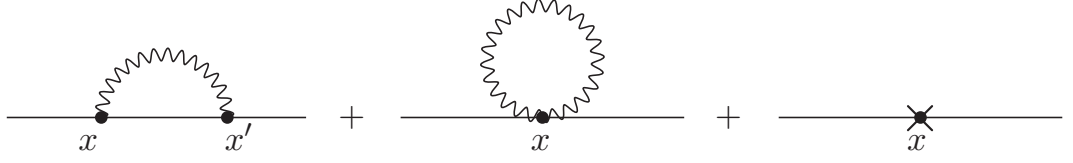


Figure 2.1: The one-loop self-mass-squared from massless conformally coupled scalar. The first two diagrams represent the 3-point and 4-point interactions, respectively. The last diagram corresponds to the counterterms required to absorb ultraviolet divergences from the two primitive diagrams.

The first two diagrams which represent the 3-point and 4-point interactions, respectively have the following analytic expressions.

$$\text{1st diagram: } -iM_{3\text{pt}}^2(x; x') \equiv \left\langle i \frac{\delta S[\phi, h]}{\delta \phi(x)} i \frac{\delta S[\phi, h]}{\delta \phi(x')} \right\rangle_0, \quad (2.13)$$

$$\text{2nd diagram: } -iM_{4\text{pt}}^2(x; x') \equiv \left\langle i \frac{\delta^2 S[\phi, h]}{\delta \phi(x) \delta \phi(x')} \right\rangle_0, \quad (2.14)$$

where the subscript 0 on the expectation value indicates that it is to be taken in the free theory. Once the action $S = \int d^D x \mathcal{L}$ is expanded computing these two expressions is straightforward. The 3-point and 4-point diagrams correspond to the $\tilde{\phi}^2 h$ and $\tilde{\phi}^2 h^2$ interactions which derive from expanding the matter part of the Lagrangian (2.8). The expansion of the scalar kinetic term is given in the previous work [13] and therefore we focus on the conformal coupling term in (2.8).

$$\begin{aligned} \mathcal{L}_{\text{CC}} &\equiv -\frac{D-2}{8(D-1)} \tilde{\phi}^2 \tilde{R} \sqrt{-\tilde{g}}, \\ &= -\frac{D-2}{8(D-1)} \kappa \tilde{\phi}^2 \left(-h_{\mu}^{\mu} + h^{\mu\nu}_{,\mu\nu} \right) - \frac{D-2}{8(D-1)} \kappa^2 \tilde{\phi}^2 \left(-2h^{\mu\nu} h_{\lambda\nu,\mu}^{\lambda} \right. \\ &\quad \left. + h^{\mu\nu} h_{\mu\nu,\lambda}^{\lambda} + h^{\mu\nu} h_{,\mu\nu} + \frac{3}{4} h^{\mu\nu}_{,\lambda} h_{\mu\nu}^{\lambda} + h^{\mu\nu}_{,\mu} h_{,\nu} - h^{\mu\nu}_{,\mu} h_{\lambda\nu}^{\lambda} \right. \\ &\quad \left. - \frac{1}{2} h^{\mu\nu}_{,\lambda} h^{\lambda}_{\nu,\mu} - \frac{1}{4} h_{,\mu} h^{\mu} - \frac{1}{2} h h_{\mu}^{\mu} + \frac{1}{2} h h^{\mu\nu}_{,\mu\nu} \right). \end{aligned} \quad (2.15)$$

In the following subsection we derive the formal expressions for the self-mass-squared from the 3-point and 4-point interactions in the conformal coupling term. For notational simplicity we drop $\sim$ from now on, but remember that our metric, the scalar field ϕ and graviton field $h_{\mu\nu}$ are all conformally rescaled ones:

$$\tilde{g}_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \quad \tilde{\phi} \equiv \phi. \quad (2.16)$$

2.2.1 Formal expressions for the one loop self-mass-squared

2.2.1.1 4-point contributions

The 4-point contributions to the self-mass-squared come from $\phi^2 h^2$ terms in (2.15).

Let us illustrate the derivation by calculating it from the first term,

$$\mathcal{L}_{4a} \equiv -\frac{D-2}{8(D-1)} \kappa^2 \phi^2 (-2h^{\mu\nu} h_{\lambda\nu, \mu}{}^\lambda). \quad (2.17)$$

From the defining expression (2.14), the first step is to vary the action with respect to the scalar field,

$$\begin{aligned} \frac{\delta^2 S_{4a}}{\delta\phi(x')\delta\phi(x)} &= -\frac{D-2}{8(D-1)} \frac{\delta}{\delta\phi(x')} \int d^D y \left[-2\kappa^2 \frac{\delta\phi^2(y)}{\delta\phi(x)} h^{\mu\nu}(y) \partial^\lambda \partial_\mu h_{\mu\nu}(y) \right], \\ &= -\frac{D-2}{8(D-1)} \left[-4\kappa^2 \delta^D(x-x') h^{\mu\nu}(x) \partial^\lambda \partial_\mu h_{\mu\nu}(x) \right], \\ &= -\frac{D-2}{8(D-1)} \left[-4\kappa^2 \delta^D(x-x') h^{\mu\nu}(x') \partial^\lambda \partial_\mu h_{\mu\nu}(x) \right]. \end{aligned} \quad (2.18)$$

In the last line we used the delta function to convert $h^{\mu\nu}(x)$ to $h^{\mu\nu}(x')$ in order to distinguish them from one another. It also is useful to see the tensorial structure of the graviton propagator from x to x' . The self-mass-squared is the functional integral of i times (2.18) over the relevant fields,

$$\begin{aligned} -iM_{4a}^2(x; x') &\equiv \left\langle i \frac{\delta^2 S_{4a}}{\delta\phi(x')\delta\phi(x)} \right\rangle_0, \\ &= \frac{D-2}{2(D-1)} \kappa^2 i \delta^D(x-x') \partial^\lambda \partial_\mu \left\langle h^{\mu\nu}(x) h_{\lambda\nu}(x') \right\rangle_0, \\ &= \frac{32(D-1)}{(D-2)} \tilde{\kappa}^2 i \delta^D(x-x') \partial^\lambda \partial_\mu \left\{ i[\mu\nu \Delta_{\lambda\nu}](x; x') \right\}. \end{aligned} \quad (2.19)$$

Here we define $\tilde{\kappa} \equiv \frac{D-2}{8(D-1)} \times \kappa$ for future convenience. Also note that an expression for the graviton propagator emerges in the last step:

$$i[\mu\nu \Delta_{\lambda\nu}](x; x') = \left\langle h^{\mu\nu}(x) h_{\lambda\nu}(x') \right\rangle_0. \quad (2.20)$$

Using the same procedure for the remaining terms in (2.15) we have the contributions from the 4-point vertices as

$$\begin{aligned}
-iM_{4\text{pt}}^2(x; x') = & \frac{32(D-1)}{(D-2)} \tilde{\kappa}^2 i \delta^D(x-x') \left\{ \partial^\lambda \partial'_\mu i^{[\mu\nu} \Delta_{\lambda\nu]}(x; x') - \frac{1}{2} \partial^\lambda \partial'_\lambda i^{[\mu\nu} \Delta_{\mu\nu]}(x; x') \right. \\
& - \frac{1}{2} \partial_\mu \partial'_\nu i^{[\mu\nu} \Delta^\alpha{}_\alpha](x; x') - \frac{3}{8} \partial^\lambda \partial'_\lambda i^{[\mu\nu} \Delta_{\mu\nu]}(x; x') - \frac{1}{2} \partial_\mu \partial'_\nu i^{[\alpha} \Delta^{\mu\nu]}_\alpha(x; x') \\
& + \frac{1}{2} \partial^\lambda \partial'_\mu i^{[\mu\nu} \Delta_{\lambda\nu]}(x; x') + \frac{1}{4} \partial^\mu \partial'_\lambda i^{[\lambda\nu} \Delta_{\mu\nu]}(x; x') + \frac{1}{8} \partial_\mu \partial'^\mu i^{[\alpha} \Delta^\beta{}_\beta](x; x') \\
& \left. + \frac{1}{4} \partial^\lambda \partial'_\lambda i^{[\alpha} \Delta^\beta{}_\beta](x; x') - \frac{1}{4} \partial_\mu \partial'_\nu i^{[\alpha} \Delta^{\mu\nu]}_\alpha(x; x') \right\}. \quad (2.21)
\end{aligned}$$

2.2.1.2 3-point contributions

The 3-point contributions derive from $\phi^2 h$ terms in (2.15):

$$\begin{aligned}
\mathcal{L}_{3\text{pt}} &= -\frac{D-2}{8(D-1)} \kappa \phi^2 (-h^\mu{}_\mu + h^{\mu\nu}{}_{,\mu\nu}) = -\tilde{\kappa} \phi^2 (-h^\mu{}_\mu + h^{\mu\nu}{}_{,\mu\nu}) \\
&\equiv \mathcal{L}_{3a} + \mathcal{L}_{3b}. \quad (2.22)
\end{aligned}$$

The 3-point self-mass-squared defined in (2.13) then can be written as

$$\begin{aligned}
-iM_{3\text{pt}}^2(x; x') &= \left\langle i \frac{\delta S_{3a}}{\delta \phi(x)} i \frac{\delta S_{3a}}{\delta \phi(x')} + i \frac{\delta S_{3a}}{\delta \phi(x)} i \frac{\delta S_{3b}}{\delta \phi(x')} \right. \\
&\quad \left. + i \frac{\delta S_{3b}}{\delta \phi(x)} i \frac{\delta S_{3a}}{\delta \phi(x')} + i \frac{\delta S_{3b}}{\delta \phi(x)} i \frac{\delta S_{3b}}{\delta \phi(x')} \right\rangle_0. \quad (2.23)
\end{aligned}$$

Here the variations of the action with respect to the scalar are

$$\frac{\delta S_{3a}}{\delta \phi(x)} = -\tilde{\kappa} \frac{\delta}{\delta \phi(x)} \int d^D y \left[-\phi^2(y) \partial_\mu \partial^\mu h(y) \right] = -\tilde{\kappa} \left[-2\phi(x) \partial_\mu \partial^\mu h(x) \right], \quad (2.24)$$

$$\frac{\delta S_{3b}}{\delta \phi(x)} = -\tilde{\kappa} \frac{\delta}{\delta \phi(x)} \int d^D y \left[\phi^2(y) \partial_\mu \partial_\nu h^{\mu\nu}(y) \right] = -\tilde{\kappa} \left[2\phi(x) \partial_\mu \partial_\nu h^{\mu\nu}(x) \right]. \quad (2.25)$$

By substituting (2.24), (2.25) into (2.23), we obtain

$$\begin{aligned}
-iM_{3\text{pt}}^2(x; x') &= -\tilde{\kappa}^2 i \Delta_{\text{cf}} \left\{ \partial^2 \partial'^2 i^{[\alpha} \Delta^\beta{}_\beta](x; x') - [\partial_\mu \partial_\nu \partial'^2 + \partial'_\mu \partial'_\nu \partial^2] i^{[\alpha} \Delta^{\mu\nu]}_\alpha(x; x') \right. \\
&\quad \left. + \partial_\mu \partial_\nu \partial'_\alpha \partial'_\beta i^{[\mu\nu} \Delta^{\alpha\beta]}(x; x') \right\}. \quad (2.26)
\end{aligned}$$

Here note that the appearance of the propagator for a massless, conformally coupled scalar

$$i\Delta_{\text{cf}}(x; x') = \left\langle \phi(x) \phi(x') \right\rangle_0. \quad (2.27)$$

In fact, there exists another kind of 3-point contributions taking one variation from the kinetic term and the other from the conformal coupling in the Lagrangian (2.8). The 3-point interactions from the scalar kinetic term are

$$\mathcal{L}_{K3pt} = -\frac{1}{2}\kappa\partial_\mu\phi\partial_\nu\phi\left(-h^{\mu\nu} + \frac{1}{2}\eta^{\mu\nu}h\right) \equiv \mathcal{L}_{K3c} + \mathcal{L}_{K3d}, \quad (2.28)$$

and the 3-point self-mass-squared from the ‘cross terms’ can be computed as

$$\begin{aligned} -iM_{3ptcross}^2(x; x') &= \left\langle i\frac{\delta S_{3a}}{\delta\phi(x)}i\frac{\delta S_{K3c}}{\delta\phi(x')} + i\frac{\delta S_{3a}}{\delta\phi(x)}i\frac{\delta S_{K3d}}{\delta\phi(x')} \right. \\ &\quad \left. + i\frac{\delta S_{3b}}{\delta\phi(x)}i\frac{\delta S_{K3c}}{\delta\phi(x')} + i\frac{\delta S_{3b}}{\delta\phi(x)}i\frac{\delta S_{K3d}}{\delta\phi(x')} \right\rangle_0. \end{aligned} \quad (2.29)$$

Including (2.26), (2.29) and the contributions from the kinetic terms given in [13] (which can be formally written as)

$$\begin{aligned} -iM_{3ptK}^2(x; x') &= \left\langle i\frac{\delta S_{K3c}}{\delta\phi(x)}i\frac{\delta S_{K3c}}{\delta\phi(x')} + i\frac{\delta S_{K3c}}{\delta\phi(x)}i\frac{\delta S_{K3d}}{\delta\phi(x')} \right. \\ &\quad \left. + i\frac{\delta S_{K3d}}{\delta\phi(x)}i\frac{\delta S_{K3c}}{\delta\phi(x')} + i\frac{\delta S_{K3d}}{\delta\phi(x)}i\frac{\delta S_{K3d}}{\delta\phi(x')} \right\rangle_0. \end{aligned} \quad (2.30)$$

completes the 3-point contributions. However, with the experience from [13] and the present work, each of the three expressions (2.26), (2.29) and (2.30) takes a number of heavy computations and can be treated separately. Thus we leave calculating the cross contributions as a follow-up project.

2.2.2 Propagators

From the 4-point and 3-point contributions (2.21) and (2.26), we see that the propagators are the main ingredients of the self-mass-squared. This subsection therefore provides with the scalar and graviton propagators. We follow three notational conventions employed in the previous work [13] for continuity and comparison with the current results. First, the background geometry is denoted with a hat, which was already introduced in (2.4),

$$\hat{g}_{\mu\nu} = a^2\eta_{\mu\nu} \quad \text{accordingly} \quad \hat{R} = D(D-1)H^2. \quad (2.31)$$

Second, considering the gauge choice for the graviton propagator in which time and space are treated differently, it is convenient to differentiate the spatial parts of the Lorentz metric and the Kronecker delta with a bar,

$$\bar{\eta}_{\mu\nu} \equiv \eta_{\mu\nu} + \delta_\mu^0\delta_\nu^0 \quad \text{and} \quad \bar{\delta}_\nu^\mu \equiv \delta_\nu^\mu - \delta_0^\mu\delta_\nu^0. \quad (2.32)$$

Third, it is useful to define the de Sitter length function $y(x; x')$ in terms of the de Sitter invariant length $\ell(x; x')$ from x^μ to x'^μ :

$$y(x; x') = 4 \sin^2 \left(\frac{1}{2} H \ell(x; x') \right) = a a' H^2 \left\{ \|\vec{x} - \vec{x}'\|^2 - \left(|\eta - \eta'| - i\delta \right)^2 \right\}, \quad (2.33)$$

where $a \equiv a(\eta)$ and $a' \equiv a(\eta')$. The propagator for a massless conformally coupled scalar has long been known [153],

$$i\Delta_{\text{cf}}(x; x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \Gamma\left(\frac{D}{2} - 1\right) \left(\frac{4}{y}\right)^{\frac{D}{2}-1}. \quad (2.34)$$

The graviton propagator was derived by adding the gauge fixing term to the Lagrangian [118],

$$\begin{aligned} \mathcal{L}_{\text{GF}} &= -\frac{1}{2} a^{D-2} \eta^{\mu\nu} F_\mu F_\nu, \\ F_\mu &\equiv \eta^{\rho\sigma} \left(h_{\mu\rho, \sigma} - \frac{1}{2} h_{\rho\sigma, \mu} + (D-2) H a h_{\mu\rho} \delta_\sigma^0 \right). \end{aligned} \quad (2.35)$$

The quadratic part of the gauge fixed Lagrangian can be partially integrated to extract the kinetic operator $D_{\mu\nu}^{\rho\sigma}$ as

$$\frac{1}{2} h^{\mu\nu} D_{\mu\nu}^{\rho\sigma} h_{\rho\sigma}, \quad (2.36)$$

where

$$\begin{aligned} D_{\mu\nu}^{\rho\sigma} &\equiv \left\{ \frac{1}{2} \bar{\delta}_\mu^{(\rho} \bar{\delta}_\nu^{\sigma)} - \frac{1}{4} \eta_{\mu\nu} \eta^{\rho\sigma} - \frac{1}{2(D-3)} \delta_\mu^0 \delta_\nu^0 \delta_0^\rho \delta_0^\sigma \right\} D_A \\ &\quad + \delta_{(\mu}^0 \bar{\delta}_{\nu)}^{(\rho} \delta_0^{\sigma)} D_B + \frac{1}{2} \left(\frac{D-2}{D-3} \right) \delta_\mu^0 \delta_\nu^0 \delta_0^\rho \delta_0^\sigma D_C. \end{aligned} \quad (2.37)$$

The three scalar differential operators are defined as

$$D_A \equiv \partial_\mu \left(\sqrt{-\hat{g}} \hat{g}^{\mu\nu} \partial_\nu \right), \quad (2.38)$$

$$D_B \equiv \partial_\mu \left(\sqrt{-\hat{g}} \hat{g}^{\mu\nu} \partial_\nu \right) - \frac{1}{D} \left(\frac{D-2}{D-1} \right) \hat{R} \sqrt{-\hat{g}}, \quad (2.39)$$

$$D_C \equiv \partial_\mu \left(\sqrt{-\hat{g}} \hat{g}^{\mu\nu} \partial_\nu \right) - \frac{2}{D} \left(\frac{D-3}{D-1} \right) \hat{R} \sqrt{-\hat{g}}. \quad (2.40)$$

The graviton propagator should obey the following defining equation,

$$D_{\mu\nu}^{\rho\sigma} \times i \left[{}_{\rho\sigma} \Delta^{\alpha\beta} \right] (x; x') = \delta_\mu^{(\alpha} \delta_\nu^{\beta)} i \delta^D(x - x'), \quad (2.41)$$

and by inverting the kinetic operator one can get the graviton propagator. This inversion procedure can be done for each scalar kinetic operator by setting the graviton propagator as a sum of constant tensor factors times scalar propagators:

$$i[{}_{\mu\nu} \Delta^{\rho\sigma}] (x; x') = \sum_{I=A,B,C} [{}_{\mu\nu} T_{\rho\sigma}^I] i\Delta_I(x; x'), \quad (2.42)$$

where the scalar propagators satisfy

$$D_I \times i\Delta_I(x; x') = i\delta^D(x - x') \quad \text{for} \quad I = A, B, C. \quad (2.43)$$

Here the tensor factors are given as

$$\left[\mu\nu T_{\rho\sigma}^A \right] = 2\bar{\eta}_{\mu(\rho}\bar{\eta}_{\sigma)\nu} - \frac{2}{D-3}\bar{\eta}_{\mu\nu}\bar{\eta}_{\rho\sigma}, \quad (2.44)$$

$$\left[\mu\nu T_{\rho\sigma}^B \right] = -4\delta_{(\mu}^0\bar{\eta}_{\nu)(\rho}\delta_{\sigma)}^0, \quad (2.45)$$

$$\left[\mu\nu T_{\rho\sigma}^C \right] = \frac{2}{(D-3)(D-2)}[(D-3)\delta_{\mu}^0\delta_{\nu}^0 + \bar{\eta}_{\mu\nu}][(D-3)\delta_{\rho}^0\delta_{\sigma}^0 + \bar{\eta}_{\rho\sigma}]. \quad (2.46)$$

The A -type propagator which is the same as the one for the MMC scalar has de Sitter invariant (as a function of y) and breaking parts [119, 120].

$$i\Delta_A(x; x') = A(y) + k\ln(aa'), \quad (2.47)$$

where $k \equiv \frac{H^{D-2}}{(4\pi)^{D/2}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}$. The de Sitter invariant part $A(y)$ is [120],

$$\begin{aligned} A(y) &\equiv \frac{H^{D-2}}{(4\pi)^{D/2}} \left\{ \Gamma\left(\frac{D}{2}-1\right) \left(\frac{4}{y}\right)^{\frac{D}{2}-1} + \frac{\Gamma(\frac{D}{2}+1)}{\frac{D}{2}-2} \left(\frac{4}{y}\right)^{\frac{D}{2}-2} - \pi \cot\left(\frac{\pi D}{2}\right) \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \right. \\ &\quad \left. + \sum_{n=1}^{\infty} \left[\frac{1}{n} \frac{\Gamma(n+D-1)}{\Gamma(n+\frac{D}{2})} \left(\frac{y}{4}\right)^n - \frac{1}{n-\frac{D}{2}+2} \frac{\Gamma(n+\frac{D}{2}+1)}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right] \right\}. \quad (2.48) \end{aligned}$$

Note that this de Sitter breaking solution still preserves homogeneity and isotropy and it is a well-known issue that there is no de Sitter invariant solution for the MMC scalar propagator [121].

The B -type and C -type propagators have the following de Sitter invariant solutions

$$\begin{aligned} i\Delta_B(x; x') &= i\Delta_{\text{cf}}(x; x') \\ &\quad - \frac{H^{D-2}}{(4\pi)^{D/2}} \sum_{n=0}^{\infty} \left\{ \frac{\Gamma(n+D-2)}{\Gamma(n+\frac{D}{2})} \left(\frac{y}{4}\right)^n - \frac{\Gamma(n+\frac{D}{2})}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right\}, \quad (2.49) \\ i\Delta_C(x; x') &= i\Delta_{\text{cf}}(x; x') \\ &\quad + \frac{H^{D-2}}{(4\pi)^{D/2}} \sum_{n=0}^{\infty} \left\{ (n+1) \frac{\Gamma(n+D-3)}{\Gamma(n+\frac{D}{2})} \left(\frac{y}{4}\right)^n \right. \\ &\quad \left. - \left(n - \frac{D}{2} + 3\right) \frac{\Gamma(n+\frac{D}{2}-1)}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right\}. \quad (2.50) \end{aligned}$$

The infinite series terms of a positive power of y vanish for $D = 4$ so that one only need to retain them when multiplying a fixed divergence. This makes these propagators and loop calculations manageable.

2.3 Computation of the one loop self-mass-squared

In this section we evaluate the formal expressions for the 4-point and 3-point contributions to the self-mass-squared given in (2.21) and (2.26). The contribution from the 4-point vertices turns out to be finite. To manage the divergences occurring in the 3-point interactions we put them in the form of external operators acting on functions of y . This form is convenient for renormalization in the next section.

2.3.1 Contribution from the 4-point vertices

We start with the analytic expression for the 4-point contribution $-iM_{4\text{pt}}^2(x; x')$ in (2.21) corresponding to the following Feynman diagram, Fig. 2.2.

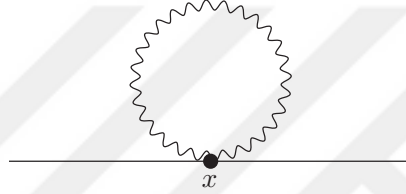


Figure 2.2: Contribution from the 4-point vertices.

We take the coincidence limit with the aid of the delta function. Again we work it out for the first term to demonstrate the procedure,

$$\delta^D(x - x') \partial^\lambda \partial_\mu i[\mu^\nu \Delta_{\lambda\nu}](x; x') = \lim_{x \rightarrow x'} \partial^\lambda \partial_\mu i[\mu^\nu \Delta_{\lambda\nu}](x; x') . \quad (2.51)$$

The constant tensor factors of this term are evaluated from (2.44), (2.45) and (2.46),

$$[\mu^\nu T_{\lambda\nu}^A] = \frac{D^2 - 3D - 2}{D - 3} \bar{\delta}_\lambda^\mu , \quad (2.52)$$

$$[\mu^\nu T_{\lambda\nu}^B] = \bar{\delta}_\lambda^\mu + (D - 1) \delta_0^\mu \delta_\lambda^0 , \quad (2.53)$$

$$[\mu^\nu T_{\lambda\nu}^C] = \frac{2}{(D - 3)(D - 2)} [(D - 3)^2 \delta_0^\mu \delta_\lambda^0 + \bar{\delta}_\lambda^\mu] . \quad (2.54)$$

and the differentiated scalar propagators become

$$\lim_{x \rightarrow x'} \partial^\lambda \partial_\mu i[\Delta_A](x; x') = \frac{H^D a^2}{(4\pi)^{D/2}} \frac{\Gamma(D - 1)}{\Gamma(\frac{D}{2})} \left\{ \frac{D - 1}{D} \delta_\mu^\lambda - \delta_0^\lambda \delta_\mu^0 \right\} , \quad (2.55)$$

$$\lim_{x \rightarrow x'} \partial^\lambda \partial_\mu i[\Delta_B](x; x') = \frac{H^D a^2}{(4\pi)^{D/2}} \frac{\Gamma(D - 1)}{\Gamma(\frac{D}{2})} \left\{ -\frac{1}{D} \delta_\mu^\lambda \right\} , \quad (2.56)$$

$$\lim_{x \rightarrow x'} \partial^\lambda \partial_\mu i[\Delta_C](x; x') = \frac{H^D a^2}{(4\pi)^{D/2}} \frac{\Gamma(D - 1)}{\Gamma(\frac{D}{2})} \left\{ \frac{2}{D(D - 2)} \delta_\mu^\lambda \right\} . \quad (2.57)$$

Therefore for the first term we have,

$$\begin{aligned} \lim_{x \rightarrow x'} \partial^\lambda \partial_\mu [\mu^\nu \Delta_{\lambda\nu}](x; x') &= \frac{H^D a^2}{(4\pi)^{D/2}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \frac{(D^4 - 6D^3 + 9D^2 + 4D - 16)}{(D-2)^2}, \\ &\rightarrow \frac{H^4 a^2}{16\pi^2} \times 8 \quad \text{in } D = 4. \end{aligned} \quad (2.58)$$

Using the same procedure for the remaining nine terms we obtain the 4-point contributions in $D = 4$ dimensions,

$$\begin{aligned} -iM_{4\text{pt}}^2(x; x') &= \frac{i\tilde{\kappa}^2}{3} \delta^4(x - x') \frac{H^4 a^2}{16\pi^2} \times \left\{ 1 \times (8) - \frac{1}{2} \times (20) - \frac{1}{2} \times (-16) - \frac{3}{8} \times (-32) \right. \\ &\quad \left. - \frac{1}{2} \times (16) + \frac{1}{2} \times (-8) + \frac{1}{4} \times (-8) + \frac{1}{8} \times (64) + \frac{1}{4} \times (-40) \right. \\ &\quad \left. - \frac{1}{4} \times (-16) \right\}, \\ &= 2i\kappa^2 \times \frac{H^4 a^2}{16\pi^2} \delta^4(x - x'). \end{aligned} \quad (2.59)$$

2.3.2 3-point interaction

In this subsection we evaluate the 3-point contribution $-iM_{3\text{pt}}^2(x; x')$ in (2.26) depicted in the following diagram, Fig. 2.3.

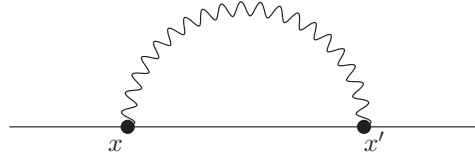


Figure 2.3: Contribution from the 3-point vertices.

Substituting the graviton propagator and contracting the tensor indices in (2.26) we have

$$\begin{aligned} &= -\tilde{\kappa}^2 i\Delta_{\text{cf}} \left\{ -\frac{4(D-1)}{D-3} \partial^2 \partial'^2 i\Delta_A + \frac{8}{(D-3)(D-2)} \partial^2 \partial'^2 i\Delta_C - \frac{4}{D-3} \left[-\nabla^2 \partial'^2 i\Delta_A \right. \right. \\ &\quad \left. \left. - \nabla^2 \partial^2 i\Delta_A + \frac{1}{D-2} \left[(D-3) \partial_0^2 + \nabla^2 \right] \partial'^2 i\Delta_C + \frac{1}{D-2} \left[(D-3) \partial_0'^2 + \nabla^2 \right] \partial^2 i\Delta_C \right] \right. \\ &\quad \left. + \frac{2(D-4)}{D-3} \nabla^4 i\Delta_A + 4\nabla^2 \partial_0 \partial_0' i\Delta_B + \frac{2}{(D-3)(D-2)} \left[(D-3)^2 \partial_0^2 \partial_0'^2 \right. \right. \\ &\quad \left. \left. + (D-3) \nabla^2 (\partial_0'^2 + \partial_0^2) + \nabla^4 \right] i\Delta_C \right\}. \end{aligned} \quad (2.60)$$

Here we applied

$$\partial'_i = -\partial_i, \quad \nabla'^2 = \nabla^2, \quad \partial^2 = -\partial_0^2 + \nabla^2. \quad (2.61)$$

Also, in the de Sitter background the covariant d'Alembertian operator is expressed as

$$\square \equiv \frac{1}{\sqrt{-\hat{g}}} \partial_\mu \left(\sqrt{-\hat{g}} \hat{g}^{\mu\nu} \partial_\nu \right) = -\frac{1}{a^2} \partial_0^2 - \frac{(D-2)H}{a} \partial_0 + \frac{1}{a^2} \nabla^2. \quad (2.62)$$

Let us remember that the expression for the conformal propagator was defined for de Sitter geometry in (2.34). Since we conformally rescaled our metric to flat space and the CC scalar is invariant under this rescaling we can work with the flat space limit of $i\Delta_{\text{cf}}$. In order to do the calculations more systematically we express all the propagators in terms of the de Sitter invariant function $y(x; x')$ in (2.33),

$$\begin{aligned} i\Delta_{\text{cf}} &= \frac{\Gamma\left(\frac{D}{2}-1\right)}{(4\pi)^{\frac{D}{2}}} \left(\frac{4}{\Delta x^2}\right)^{\frac{D}{2}-1} = (aa')^{\frac{D}{2}-1} \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \Gamma\left(\frac{D}{2}-1\right) \left(\frac{4}{y}\right)^{\frac{D}{2}-1} \\ &\equiv (aa')^{\frac{D}{2}-1} F(y), \end{aligned} \quad (2.63)$$

$$i\Delta_{A,B,C} \equiv A(y), B(y), C(y). \quad (2.64)$$

Obviously a similar flat space version for the graviton can not be used for this calculation since the graviton is not conformally invariant. Note also that the de Sitter breaking term in $i\Delta_A$ is dropped. Then the terms in the 3-point contribution can be classified as the following four types,

$$\begin{aligned} &F(y) \partial_0^2 \partial_0'^2 A(y), \quad F(y) \nabla^4 A(y), \\ &F(y) \nabla^2 [\partial_0^2 + \partial_0'^2] A(y) \quad \text{and} \quad F(y) \nabla^2 \partial_0 \partial_0' A(y). \end{aligned} \quad (2.65)$$

This allows to write the 3-point contribution as

$$\begin{aligned} -iM_{3\text{pt}}^2(x; x') &= -\tilde{\kappa}^2 (aa')^{\frac{D}{2}-1} \left\{ C_{1a} F(y) \partial_0^2 \partial_0'^2 A(y) + C_{2a} F(y) \nabla^4 A(y) \right. \\ &\quad + C_{3a} F(y) \nabla^2 [\partial_0^2 + \partial_0'^2] A(y) + C_b F(y) \nabla^2 \partial_0 \partial_0' B(y) + C_{1c} F(y) \partial_0^2 \partial_0'^2 C(y) \\ &\quad \left. + C_{2c} F(y) \nabla^4 C(y) + C_{3c} F(y) \nabla^2 [\partial_0^2 + \partial_0'^2] C(y) \right\}. \end{aligned} \quad (2.66)$$

Here the coefficients are

$$\begin{aligned} C_{1a} &\equiv -4\left(\frac{D-1}{D-3}\right), \quad C_{2a} \equiv -2\left(\frac{D-2}{D-3}\right), \quad C_{3a} \equiv 4\left(\frac{D-2}{D-3}\right), \quad C_b \equiv 4, \\ C_{1c} &\equiv \frac{2(D-2)^2}{(D-3)(D-2)}, \quad C_{2c} \equiv \frac{2}{(D-3)(D-2)} \quad \text{and} \quad C_{3c} \equiv \frac{(D-1)}{(D-3)(D-2)}. \end{aligned} \quad (2.67)$$

Recalling the self-mass-squared is eventually integrated over 4-dimensional spacetime in the effective field equation (2.2), we extract the derivatives outside the integral

to reduce the degree of divergence of the terms remaining inside the integral. This requires to convert all primed derivatives into unprimed ones (so that they can be freely moved outside the integral.) The final results are expressed in terms of eleven external operators acting on the functions of y . The procedure of extracting derivatives involves a number of indefinite integration. We denote this operation by

$$I[f](y) \equiv \int^y dy' f(y'). \quad (2.68)$$

For example, the following identity derived in [13]

$$f(y) \left\{ A''(y) \frac{\partial y}{\partial x^\rho} \frac{\partial y}{\partial x'^\sigma} + A'(y) \frac{\partial^2 y}{\partial x^\rho \partial x'^\sigma} \right\} = \partial_\rho \partial'_\sigma I^2[fA''](y) + \frac{\partial^2 y}{\partial x^\rho \partial x'^\sigma} I[f'A'](y). \quad (2.69)$$

describes the operation: the expression in the left-hand-side is converted to derivatives acting a function of y (as an indefinite integral function of y) plus an extra function of y in the right-hand-side. Similar identities can be found in [13].

$$f(y) \partial_0 \partial'_0 A(y) = \partial_0 \partial'_0 I^2[fA''](y) - \frac{1}{2} \nabla \cdot \nabla' I^3[f'A'](y) + H^2 a a' \left\{ (2-y) I[f'A'](y) - (D-1) I^2[f'A'](y) \right\}, \quad (2.70)$$

$$f(y) \partial_0 \partial'_j A(y) = \partial_0 \partial'_j I^2[fA''](y) + H a \partial'_j I^2[f'A'](y), \quad (2.71)$$

$$f(y) \partial_i \partial'_0 A(y) = \partial_i \partial'_0 I^2[fA''](y) + H a' \partial_i I^2[f'A'](y), \quad (2.72)$$

$$f(y) \partial_i \partial'_j A(y) = \partial_i \partial'_j I^2[fA''](y) - 2H^2 a a' \eta_{ij} I[f'A'](y). \quad (2.73)$$

Note that the derivatives are acting on functions of y externally in the above expressions. For our computation we need to derive these kind of expressions for a more complicated situation where there are four derivatives, spatial and temporal, acting on functions $A(y), B(y), C(y)$ that appear in equation (2.66),

$$\begin{aligned} F(y) \partial_\rho \partial_\sigma \partial'_\alpha \partial'_\beta A(y) &= \partial_\rho \partial_\sigma \partial'_\alpha \partial'_\beta \left\{ I^4[FA''''](y) \right\} \\ &+ \frac{\partial^4 y}{\partial x^\rho \partial x^\sigma \partial x'^\alpha \partial x'^\beta} \left\{ I[F'A'] + I^2[F'A''] + I^3[F'A'''] \right\} \\ &+ \left(2 \frac{\partial y}{\partial x^{(\rho}} \frac{\partial^3 y}{\partial x^\sigma) \partial x'^\alpha \partial x'^\beta} + 2 \frac{\partial y}{\partial x'^{(\alpha}} \frac{\partial^3 y}{\partial x'^\beta) \partial x^\rho \partial x^\sigma} + \frac{\partial^2 y}{\partial x^\rho \partial x^\sigma} \frac{\partial^2 y}{\partial x'^\alpha \partial x'^\beta} \right. \\ &+ \left. 2 \frac{\partial^2 y}{\partial x^\rho \partial x'^{(\alpha}} \frac{\partial^2 y}{\partial x'^\beta) \partial x^\sigma} \right) \left\{ I[F'A''] + I^2[F'A'''] \right\} + \left(4 \frac{\partial y}{\partial x^{(\rho}} \frac{\partial^2 y}{\partial x^\sigma) \partial x'^{(\alpha}} \frac{\partial y}{\partial x'^\beta)} \right. \\ &+ \left. \frac{\partial y}{\partial x^\rho} \frac{\partial y}{\partial x^\sigma} \frac{\partial^2 y}{\partial x'^\alpha \partial x'^\beta} + \frac{\partial y}{\partial x'^\alpha} \frac{\partial y}{\partial x'^\beta} \frac{\partial^2 y}{\partial x^\rho \partial x^\sigma} \right) \left\{ I[F'A'''] \right\}. \end{aligned} \quad (2.74)$$

Let us apply this method to the simplest cases, which is all of the derivatives are spatial, i.e. $F(y)\nabla^4 A(y)$ or $F(y)\nabla^4 C(y)$ terms in equation (2.66).

$$\begin{aligned}
F(y)\nabla^4 A(y) &= \partial_i \partial_i \partial_j' \partial_j' \left\{ I^4 [FA''''](y) \right\} + \frac{\partial^4 y}{\partial x^i \partial x^i \partial x'^j \partial x'^j} \left\{ I[F'A'] + I^2[F'A''] + I^3[F'A'''] \right\} \\
&+ \left(2 \frac{\partial y}{\partial x^{(i}} \frac{\partial^3 y}{\partial x^i) \partial x'^j \partial x'^j} + 2 \frac{\partial y}{\partial x'^{(j}} \frac{\partial^3 y}{\partial x'^j) \partial x^i \partial x^i} + \frac{\partial^2 y}{\partial x^i \partial x^i} \frac{\partial^2 y}{\partial x'^j \partial x'^j} + 2 \frac{\partial^2 y}{\partial x^i \partial x'^{(j}} \frac{\partial^2 y}{\partial x'^j) \partial x^i} \right) \\
&\times \left\{ I[F'A''] + I^2[F'A'''] \right\} + \left(4 \frac{\partial y}{\partial x^{(i}} \frac{\partial^2 y}{\partial x^i) \partial x'^{(j}} \frac{\partial y}{\partial x'^j)} + \frac{\partial y}{\partial x^i} \frac{\partial y}{\partial x^i} \frac{\partial^2 y}{\partial x'^j \partial x'^j} \right. \\
&\left. + \frac{\partial y}{\partial x'^j} \frac{\partial y}{\partial x'^j} \frac{\partial^2 y}{\partial x^i \partial x^i} \right) \left\{ I[F'A'''] \right\}. \tag{2.75}
\end{aligned}$$

As one can see from the above equation we need to take up to four derivatives of y with respect to spatial variable x^i or x'^j and make use of equation (2.73). Using the following derivative identities

$$\begin{aligned}
\frac{\partial y}{\partial x^i} &= 2H^2 aa' \Delta x_i, \quad \frac{\partial y}{\partial x'^j} = -2H^2 aa' \Delta x_j, \quad \frac{\partial^2 y}{\partial x^i \partial x^j} = 2H^2 aa' \eta_{ij}, \\
\frac{\partial^2 y}{\partial x^i \partial x'^j} &= -2H^2 aa' \eta_{ij}, \quad \frac{\partial^2 y}{\partial x^i \partial x^i} = \frac{\partial^2 y}{\partial x'^j \partial x'^j} = 2H^2 aa' (D-1), \\
\frac{\partial^3 y}{\partial x^i \partial x^i \partial x'^j} &= \frac{\partial^3 y}{\partial x^i \partial x'^j \partial x'^j} = \frac{\partial^4 y}{\partial x^i \partial x^i \partial x'^j \partial x'^j} = 0. \tag{2.76}
\end{aligned}$$

Eq. (2.75) will reduce into

$$\begin{aligned}
F(y)\nabla^4 A(y) &= \nabla^4 \left\{ I^4 [FA''''](y) \right\} + \left(\left[2H^2 aa' (D-1) \right]^2 + 2 \left[2H^2 aa' \right]^2 (D-1) \right) \\
&\times \left\{ I[F'A''] + I^2[F'A'''] \right\} + \left(4 \left[2H^2 aa' \right]^3 \Delta x_i \Delta x_j \eta_{ij} + \left[2H^2 aa' \right]^3 \Delta x_i \Delta x_i (D-1) \right. \\
&\left. + \left[2H^2 aa' \right]^3 \Delta x_i \Delta x_i (D-1) \right) \left\{ I[F'A'''] \right\} \\
&= \nabla^4 \left\{ I^4 [FA''''](y) \right\} + 8(D^2 - 1) \left[H^2 aa' \right]^2 \left\{ I[F'A''] + I^2[F'A'''] \right\} \\
&+ 16(D+1) \left[H^2 aa' \right]^3 \|\Delta \vec{x}\|^2 \left\{ I[F'A'''] \right\}. \tag{2.77}
\end{aligned}$$

The final form of (2.75) is:

$$\begin{aligned}
F(y)\nabla^4 A(y) &= \nabla^4 \left\{ I^4 [FA''''](y) \right\} + 4(D^2 - 1)H^4 (aa')^2 \left\{ I[F'A''] - I^2[F'A'''] \right\} \\
&+ 4(D+1)H^2 aa' \nabla^2 I^3[F'A'''], \tag{2.78}
\end{aligned}$$

where the following identity was used to get the above desired form:

$$aa' H^2 \|\Delta \vec{x}\|^2 f(y) = -\frac{1}{2}(D-1)I[f](y) - \frac{\nabla \cdot \nabla'}{4aa'H^2} I^2[f](y). \tag{2.79}$$

This result is tabulated in Table A.1. Similar, but much more tedious work should be done to extract derivatives out for pure temporal and temporal spatial mixed derivatives in equation (2.66). These results are all tabulated in Tables A.2, A.3, A.4, in the form of the following eleven external operators acting on functions of y .

$$\alpha \equiv (aa')^2 \square^2, \quad (2.80)$$

$$\beta_1 \equiv (aa')^2 H^2 \square, \quad (2.81)$$

$$\beta_2 \equiv aa'(a^2 + a'^2) H^2 \square, \quad (2.82)$$

$$\gamma_1 \equiv (aa')^2 H^4, \quad (2.83)$$

$$\gamma_2 \equiv aa'(a^2 + a'^2) H^4, \quad (2.84)$$

$$\gamma_3 \equiv aa'(a + a')^2 H^4 = 2\gamma_1 + \gamma_2, \quad (2.85)$$

$$\delta \equiv (a^2 + a'^2) \nabla^2 \square, \quad (2.86)$$

$$\varepsilon_1 \equiv aa' H^2 \nabla^2, \quad (2.87)$$

$$\varepsilon_2 \equiv (a^2 + a'^2) H^2 \nabla^2, \quad (2.88)$$

$$\varepsilon_3 \equiv (a + a')^2 H^2 \nabla^2 = 2\varepsilon_1 + \varepsilon_2, \quad (2.89)$$

$$\zeta \equiv \nabla^4. \quad (2.90)$$

The last step before completing this section is adding various components the 3-point contribution coming from $A(y)$, $B(y)$ and $C(y)$ in equation (2.66). The result can be written symbolically as

$$\begin{aligned} -iM_{3\text{pt}}^2(x; x') &= -\tilde{\kappa}^2 (aa')^{\frac{D}{2}-1} \left\{ \alpha f_\alpha(y) + \beta_1 f_{\beta_1}(y) + \beta_2 f_{\beta_2}(y) + \gamma_1 f_{\gamma_1}(y) + \gamma_2 f_{\gamma_2}(y) \right. \\ &\quad \left. + \gamma_3 f_{\gamma_3}(y) + \delta f_\delta(y) + \varepsilon_1 f_{\varepsilon_1}(y) + \varepsilon_2 f_{\varepsilon_2}(y) + \varepsilon_3 f_{\varepsilon_3}(y) + \zeta f_\zeta(y) \right\}. \end{aligned} \quad (2.91)$$

Here the functions, on which eleven external operators are acting, are given in the Tables A.5, A.6, A.7, A.8, A.9, A.10, A.11, A.12, A.13, A.14, and A.15 in Appendix A.1. This self-mass-squared will eventually be integrated over $d^4 x'$ in the effective field equation, (2.2). Thus, after extracting the derivative operators outside the integral we only need to retain D dimension for terms which diverge logarithmically² and higher at $x = x'$ for $D = 4$ in the coefficient functions, $f_i(y)$.

²Note that $y(x; x')$ vanishes like $(x - x')^2$ at coincidence and so $\int d^4 x' 1/y^2$ diverges logarithmically.

2.4 Renormalization

In this section we renormalize the scalar self-mass-squared by subtracting counterterms depicted in Fig. 2.4. First, we construct counterterms applying the Bogoliubov, Parasiuk, Hepp and Zimmermann (BPHZ) scheme [122–125]. The structure of our Lagrangian of a scalar, conformally coupled to gravity, allows us to determine three de Sitter invariant counterterms at one loop order. On the other hand, our gauge fixing condition (2.35) breaks de Sitter symmetry, which results in possibility of having de Sitter noninvariant counterterms. It turns out that there is only one noninvariant counterterm because the de Sitter breaking occurs in a particular way. Identifying these possible counterterms is of great importance for checking correctness of the calculation. Hundreds of terms arising from various places should all add up to terms which respect the symmetries not broken due to the gauge fixing term. We will soon show this occurs in a highly nontrivial way. The next step is to collect all the divergences occurred in the previous section and localize them (in the form of δ -function) so as to be absorbed by the local counterterms. Finally, we obtain a finite result which can be used to solve the effective field equation (2.2) at one loop order. This procedure is summarized in the following two subsections.

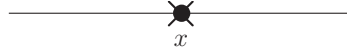


Figure 2.4: Contribution from counterterms.

2.4.1 Construction of counterterms

To construct possible counterterms we first note that the superficial degree of divergence for the scalar-graviton interaction in our Lagrangian (2.3) at one loop order is four. This means that in order to cancel those divergences, the corresponding counterterms should have a mass dimension of four. There are also two basic requirements for our case. One is, the counterterms must carry two scalar fields, each of which counts one dimension of mass. The other is, they should also contain one factor of the loop counting parameter κ^2 which has the dimension of $mass^{-2}$. We therefore require each counterterm to have an additional mass dimension of four. Because our scalar is massless, this can only be achieved by carrying four derivatives. There are three ways to form an invariant satisfying these requirements.

- All four derivatives act on scalars.
- Two derivatives act on scalars and the other two act on the metric.
- All four derivatives act on the metric.

Applying these three ways, we find five invariants listed as follows:

$$\begin{aligned} & \kappa^2 \varphi_{;\mu\nu} \varphi_{;\rho\sigma} g^{\mu\nu} g^{\rho\sigma} \sqrt{-g}, \quad \kappa^2 \partial_\mu \varphi \partial_\nu \varphi R g^{\mu\nu} \sqrt{-g}, \quad \kappa^2 \partial_\mu \varphi \partial_\nu \varphi R^{\mu\nu} \sqrt{-g}, \\ & \kappa^2 \varphi^2 R^2 \sqrt{-g} \quad \text{and} \quad \kappa^2 \varphi^2 R^{\mu\nu} R_{\mu\nu} \sqrt{-g}. \end{aligned} \quad (2.92)$$

Finally, specializing to the de Sitter background, i.e., $\hat{R}_{\mu\nu} = (D-1)H^2 \hat{g}_{\mu\nu}$, allows us to put the second and third of (2.92) together and the fourth and fifth together into one term. This results in three invariant counterterms using the rescaled field, $\tilde{\phi} \equiv \phi = a\varphi$, see Eq. (2.16),

$$\Delta\mathcal{L}_1 = \frac{1}{2}c_1 \kappa^2 \square \phi \square \phi a^2, \quad \Delta\mathcal{L}_2 = -\frac{1}{2}c_2 \kappa^2 H^2 \partial_\mu \phi \partial^\mu \phi, \quad \Delta\mathcal{L}_3 = \frac{1}{2}c_3 \kappa^2 H^4 \phi^2 a^2. \quad (2.93)$$

The vertices associated to these counterterms are,

$$\left. \frac{i\delta\Delta S_1}{\delta\phi(x)\delta\phi(x')} \right|_{\phi=0} = ic_1 \kappa^2 a^2 \square^2 \delta^D(x-x'), \quad (2.94)$$

$$\left. \frac{i\delta\Delta S_2}{\delta\phi(x)\delta\phi(x')} \right|_{\phi=0} = ic_2 \kappa^2 H^2 a^2 \square \delta^D(x-x'), \quad (2.95)$$

$$\left. \frac{i\delta\Delta S_3}{\delta\phi(x)\delta\phi(x')} \right|_{\phi=0} = ic_3 \kappa^2 H^4 a^2 \delta^D(x-x'). \quad (2.96)$$

Here the coefficients c_i will be determined by requirement of canceling the divergences.

The de Sitter noninvariant counterterm drawn to attention in the beginning of this section was constructed in Ref. [13] by carefully examining which symmetries are broken by our gauge choice (2.35). Here we review essential points of the construction strategy taken by [13] and finally give the unique noninvariant counterterm. The first point is that our gauge fixing term breaks only spatial special conformal transformations among the full $\frac{1}{2}D(D+1)$ de Sitter isometries:

- $(D-1)$ spatial translations :

$$\eta' = \eta, \quad x'^i = x^i + \varepsilon^i. \quad (2.97)$$

- $\frac{1}{2}(D-1)(D-2)$ rotations :

$$\eta' = \eta, \quad x'^i = R^{ij}x^j. \quad (2.98)$$

- 1 dilation :

$$\eta' = k\eta, \quad x'^i = kx^i. \quad (2.99)$$

- $(D-1)$ spatial special conformal transformations :

$$\eta' = \frac{\eta}{1 - 2\vec{\theta} \cdot \vec{x} + \|\vec{\theta}\|^2 x \cdot x}, \quad x'^i = \frac{x^i - \theta^i x \cdot x}{1 - 2\vec{\theta} \cdot \vec{x} + \|\vec{\theta}\|^2 x \cdot x}. \quad (2.100)$$

Hence we can use the residual symmetries respected by our gauge condition to restrict the form of noninvariant counterterms. Homogeneity (2.97) requires that the counterterm cannot depend on the spatial coordinates x^i . Isotropy (2.98) implies that spatial derivative operators ∂_i must be contracted into each another. Dilation symmetry (2.99) restricts that derivative operators and the conformal time η can only appear in the form $a^{-1}\partial_\mu$. These constraints and a number of partial integration lead us to three possible noninvariant counterterms,

$$\frac{1}{2}\kappa^2 a^{D-2} \square \phi \nabla^2 \phi, \quad \frac{1}{2}\kappa^2 a^{D-4} \nabla^2 \phi \nabla^2 \phi \quad \text{and} \quad -\frac{1}{2}\kappa^2 H^2 a^{D-2} \nabla \phi \cdot \nabla \phi. \quad (2.101)$$

Finally we note that our gauge fixing term (2.35) becomes Poincaré invariant in the flat space limit of $H \rightarrow 0$, where the conformal time $\eta = -e^{-Ht}/H$ with the comoving time t held fixed. Only the final term of (2.101) vanishes in the flat space limit and it can serve as a noninvariant counterterm. Using the rescaled field it becomes

$$\Delta \mathcal{L}_4 = -\frac{1}{2}\kappa^2 H^2 \nabla \phi \cdot \nabla \phi, \quad (2.102)$$

with the associated vertex,

$$\left. \frac{i\delta \Delta S_4}{\delta \phi(x) \delta \phi(x')} \right|_{\phi=0} = ic_4 \kappa^2 H^2 \nabla^2 \delta^D(x-x'). \quad (2.103)$$

To summarize, there are only four - three invariant and one noninvariant - counterterms allowed for the CC scalar self-mass-squared calculation at one loop order. Any occurrences of divergent terms which do not fit in one of these four counterterms would immediately imply errors in the calculation. Again we emphasize that this provides with a crucial check for perturbative quantum gravity computations like our current

work. It is amazing to observe that divergences of the “non-permitted” form occurring in various places add up to zero in the first column of TABLE 2.3. Note that the second column becomes finite for $D = 4$ by canceling the overall divergent factor $1/(D - 4)$.

2.4.2 Localization of divergences and a finite result

Now with the prepared counterterms, it is time to collect all the divergent terms and segregate them into a local function in the form of the counterterm vertices, that is, an operator acting on the $\delta^D(x - x')$. Recall that the contributions from 4-point interactions are already finite and the divergences from 3-point vertices involve powers of y that are not integrable for $D = 4$ dimensions,

$$\left(\frac{4}{y}\right)^D, \left(\frac{4}{y}\right)^{D-1} \text{ and } \left(\frac{4}{y}\right)^{D-2}. \quad (2.104)$$

Again we use the technique of extracting derivative operators to make the terms less singular. Specifically we use the following identity to extract d' Alembertian operators from these terms until they become integrable,

$$\begin{aligned} \square f(y) = & H^2 \left[(4y - y^2) f''(y) + D(2 - y) f'(y) \right] \\ & + \text{Res} \left[y^{\frac{D}{2}-2} f \right] \frac{4\pi^{D/2} H^{2-D}}{\Gamma(\frac{D}{2} - 1)} \frac{i}{\sqrt{-g}} \delta^D(x - x'). \end{aligned} \quad (2.105)$$

Here $\text{Res}[F]$ means the residue of $F(y)$. Applying this identity (2.105) each of the nonintegrable terms becomes,

$$\left(\frac{4}{y}\right)^D = \frac{2}{(D-1)D} \frac{\square}{H^2} \left(\frac{4}{y}\right)^{D-1}, \quad (2.106)$$

$$\left(\frac{4}{y}\right)^{D-1} = \frac{2}{(D-2)^2} \frac{\square}{H^2} \left(\frac{4}{y}\right)^{D-2} - \frac{2}{D-2} \left(\frac{4}{y}\right)^{D-2}, \quad (2.107)$$

$$\left(\frac{4}{y}\right)^{D-2} = \frac{2}{(D-4)(D-3)} \frac{\square}{H^2} \left(\frac{4}{y}\right)^{D-3} - \frac{4}{D-4} \left(\frac{4}{y}\right)^{D-3}. \quad (2.108)$$

Note that the logarithmically divergent power $1/y^{D-2}$ in (2.108) has reduced to the power $1/y^{D-3}$ which is integrable, however it has a divergent factor of $1/(D - 4)$.

Applying the key identity (2.105) to the power of $1/y^{D/2-1}$ we have zero in the form,

$$0 = \frac{\square}{H^2} \left(\frac{4}{y}\right)^{\frac{D}{2}-1} - \frac{D}{2} \left(\frac{D}{2} - 1\right) \left(\frac{4}{y}\right)^{\frac{D}{2}-1} - \frac{(4\pi)^{D/2} H^{-D}}{\Gamma(\frac{D}{2} - 1)} \frac{i}{a^D} \delta^D(x - x'). \quad (2.109)$$

Adding this expression of zero (2.109) to (2.108) allows us to segregate the divergence on the local term,

$$\begin{aligned} \left(\frac{4}{y}\right)^{D-2} &= \frac{2}{(D-4)(D-3)} \left\{ \frac{(4\pi)^{\frac{D}{2}} H^{-D}}{\Gamma(\frac{D}{2}-1)} \frac{i\delta^D(x-x')}{a^D} + \frac{\square}{H^2} \left[\left(\frac{4}{y}\right)^{D-3} - \left(\frac{4}{y}\right)^{\frac{D}{2}-1} \right] \right\} \\ &\quad - \frac{4}{D-4} \left\{ \left(\frac{4}{y}\right)^{D-3} - \frac{D(D-2)}{8(D-3)} \left(\frac{4}{y}\right)^{\frac{D}{2}-1} \right\}, \end{aligned} \quad (2.110)$$

$$\begin{aligned} &= \frac{iH^{-D}(4\pi)^{\frac{D}{2}}}{(D-4)(D-3)\Gamma(\frac{D}{2})} (D-2) \frac{\delta^D(x-x')}{a^D} \\ &\quad - \frac{\square}{H^2} \left\{ \frac{4}{y} \ln\left(\frac{y}{4}\right) \right\} + 2\left(\frac{4}{y}\right) \ln\left(\frac{y}{4}\right) - \left(\frac{4}{y}\right) + \mathcal{O}(D-4), \end{aligned} \quad (2.111)$$

or

$$\begin{aligned} \left(\frac{1}{y}\right)^{D-2} &= \frac{iH^{-D}\pi^{\frac{D}{2}}}{(D-4)(D-3)\Gamma(\frac{D}{2})4^{\frac{D}{2}}} 16(D-2) \frac{\delta^D(x-x')}{a^D} \\ &\quad - \frac{16\square}{4^D H^2} \left\{ \frac{4}{y} \ln\left(\frac{y}{4}\right) \right\} + \frac{32}{4^D} \left(\frac{4}{y}\right) \ln\left(\frac{y}{4}\right) - \frac{16}{4^D} \left(\frac{4}{y}\right) + \mathcal{O}(D-4). \end{aligned} \quad (2.112)$$

Substituting (2.111) into (2.107) the quadratically divergent power becomes,

$$\begin{aligned} \left(\frac{1}{y}\right)^{D-1} &= \frac{iH^{-D}\pi^{\frac{D}{2}}}{(D-4)(D-3)\Gamma(\frac{D}{2})4^{\frac{D}{2}}} \left\{ \frac{8\square}{D-2H^2} - 8 \right\} \frac{\delta^D(x-x')}{a^D} \\ &\quad - \frac{2\square^2}{4^D H^4} \left\{ \frac{4}{y} \ln\left(\frac{y}{4}\right) \right\} + \frac{\square}{H^2} \left\{ \frac{8}{4^D} \left(\frac{4}{y}\right) \ln\left(\frac{y}{4}\right) - \frac{2}{4^D} \left(\frac{4}{y}\right) \right\} \\ &\quad - \frac{8}{4^D} \left(\frac{4}{y}\right) \ln\left(\frac{y}{4}\right) + \frac{4}{4^D} \left(\frac{4}{y}\right) + \mathcal{O}(D-4). \end{aligned} \quad (2.113)$$

Similarly, the quartically divergent term becomes,

$$\begin{aligned} \left(\frac{1}{y}\right)^D &= \frac{iH^{-D}\pi^{\frac{D}{2}}}{(D-4)(D-3)\Gamma(\frac{D}{2})4^{\frac{D}{2}}} \left\{ \frac{4\square^2}{(D-2)(D-1)D H^4} - \frac{4\square}{(D-1)D H^2} \right\} \\ &\quad \times \frac{\delta^D(x-x')}{a^D} - \frac{1}{4^D} \left\{ \frac{1\square^3}{12 H^6} \left[\frac{4}{y} \ln\left(\frac{y}{4}\right) \right] + \frac{\square^2}{H^4} \left[\frac{1}{3} \left(\frac{4}{y}\right) \ln\left(\frac{y}{4}\right) - \frac{1}{12} \left(\frac{4}{y}\right) \right] \right. \\ &\quad \left. - \frac{1}{3} \left(\frac{4}{y}\right) \ln\left(\frac{y}{4}\right) + \frac{1}{6} \left(\frac{4}{y}\right) \right\} + \mathcal{O}(D-4). \end{aligned} \quad (2.114)$$

Plugging these expressions into (2.91) would make the divergent pieces almost look like the local counterterms except d'Alembertians acting on both powers of a and

$\delta^D(x-x')$. We employ the following identities to pass all factors of a to the left.

$$\square a^n \delta^D(x-x') = \left(a^n \square - [n^2 + n(D-1)] H^2 a^n - 2nH a^{n-1} \partial_0 \right) \delta^D(x-x'), \quad (2.115)$$

$$\begin{aligned} \square^2 a^n \delta^D(x-x') = & \left(a^n \square^2 - 2n[3n + (D-1)] H^2 a^n \square - 4nH a^{n-1} \partial_0 \square + 4n^3 H^3 a^{n-1} \partial_0 \right. \\ & \left. + 4n(n-1) H^2 a^{n-2} \nabla^2 + n^2 [n + (D-1)]^2 H^4 a^n \right) \delta^D(x-x') \end{aligned} \quad (2.116)$$

The following example for the term with the external operator α shows how we use these identities in order to put the divergent pieces in the form of the counterterms

$$\begin{aligned} (aa')^{\frac{D}{2}-1} \alpha \left\{ -\frac{(D+2)(D+4)H^{2D-4}\Gamma(\frac{D}{2}-1)^2}{128(D-2)(D+1)\pi^D} \left(\frac{1}{y}\right)^{D-2} \right\} \\ = -\frac{(D+2)(D+4)H^{2D-4}\Gamma(\frac{D}{2}-1)^2}{128(D-2)(D+1)\pi^D} (aa')^{\frac{D}{2}+1} \square^2 \left(\frac{1}{y}\right)^{D-2}, \end{aligned} \quad (2.117)$$

$$\begin{aligned} = -\frac{(D+2)(D+4)H^{2D-4}\Gamma(\frac{D}{2}-1)^2}{128(D-2)(D+1)\pi^D} \frac{iH^{-D}\pi^{\frac{D}{2}}}{(D-4)(D-3)\Gamma(\frac{D}{2})4^{\frac{D}{2}}} \\ \times a^{\frac{D}{2}+1} 16(D-2) \square^2 a^{\frac{D}{2}+1} \frac{\delta^D(x-x')}{a^D}, \end{aligned} \quad (2.118)$$

$$\begin{aligned} = -\frac{(D+2)(D+4)}{2^5(D-2)^3(D+1)} \frac{iH^{D-4}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)} \left\{ 16(D-2)a^2 \square^2 \right. \\ \left. - 8(D-2)^2(D-4)H^2 a^2 \square + 32(D-2)^2 H a \partial_0 \square - 8(D-2)^4 H^3 a \partial_0 \right. \\ \left. + 16(D-2)DH^2 \nabla^2 - (D-2)^2 D^2 H^4 a^2 \right\} \delta^D(x-x'). \end{aligned} \quad (2.119)$$

Here α is given in (2.80). Note that the first, second, fifth and sixth terms correspond to the counterterm vertices (2.94), (2.95), (2.103), and (2.96), respectively.

Using the same procedure for the terms with the remaining ten external operators, we can segregate all divergent terms into the form which derivatives acting on $\delta^D(x-x')$. The results are given in TABLE 2.1, TABLE 2.2 and TABLE 2.3. Note that we have another consistency check for the calculation: the contribution to the counterterm (2.94) vanishes as it must, otherwise this counterterm would not be zero in flat space and break Poincaré invariance.

Besides the local divergent terms, the remaining pieces are nonlocal and finite for $D = 4$ dimension. Again we illustrate how the nonlocal finite terms are identified

Table 2.1: Contributions to counterterms. All terms are multiplied by

$$\frac{i\tilde{\kappa}^2 H^{D-4}}{(4\pi)^{D/2}} \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)}$$

External operators	Coef. of $H^4 a^2 \delta^D(x-x')$
α	$\frac{D^2(D+2)(D+4)}{32(D-2)(D+1)}$
β_1	$\frac{(D-1)D(D+2)(D+4)}{8(D+1)}$
β_2	$-\frac{(D+2)(D+4)(D^2-2D-4)}{8(D+1)}$
γ_1	$\frac{D(D^5-35D^4+67D^3-2D^2-264D-64)}{32(D-2)(D+1)}$
γ_2	$\frac{2D^6+35D^5-108D^4-144D^3+384D+704}{8(D-2)^2(D+1)}$
Total	$\frac{D^7-29D^6+282D^5-540D^4-856D^3+224D^2+1664D+3328}{32(D-2)^2(D+1)}$

using the case with the α operator.

$$\begin{aligned}
& (aa')^{\frac{D}{2}-1} \alpha \left\{ -\frac{(D+2)(D+4)H^{2D-4}\Gamma(\frac{D}{2}-1)^2}{128(D-2)(D+1)\pi^D} \left(\frac{1}{y}\right)^{D-2} - \frac{3H^4}{32\pi^4} \frac{1}{y} \right\} \\
&= -\frac{(D+2)(D+4)H^{2D-4}\Gamma(\frac{D}{2}-1)^2}{128(D-2)(D+1)\pi^D} (aa')^{\frac{D}{2}+1} \square^2 \left(\frac{1}{y}\right)^{D-2} \\
&\quad - (aa')^{\frac{D}{2}+1} \square^2 \frac{3H^4}{32\pi^4} \frac{1}{y}, \tag{2.120}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{(D+2)(D+4)H^{2D-4}\Gamma(\frac{D}{2}-1)^2}{128(D-2)(D+1)\pi^D} (aa')^{\frac{D}{2}+1} \square^2 \\
&\quad \times \left\{ -\frac{16}{4^D} \frac{\square}{H^2} \left\{ \frac{4}{y} \ln\left(\frac{y}{4}\right) \right\} + \frac{32}{4^D} \left(\frac{4}{y}\right) \ln\left(\frac{y}{4}\right) - \frac{16}{4^D} \left(\frac{4}{y}\right) \right\} \\
&\quad - (aa')^{\frac{D}{2}+1} \square^2 \frac{3H^4}{32\pi^4} \left(\frac{1}{y}\right), \tag{2.121}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{3}{80} \frac{H^4}{\pi^4} (aa')^3 \square^2 \left\{ -\frac{16}{4^4} \frac{\square}{H^2} \left(\frac{\ln x}{x}\right) + \frac{32}{4^4} \left(\frac{\ln x}{x}\right) - \frac{16}{4^4} \left(\frac{1}{x}\right) \right\} \\
&\quad - (aa')^3 \square^2 \frac{3H^4}{32\pi^4} \left(\frac{1}{4x}\right), \\
&= \frac{3}{1280} \frac{H^2}{\pi^4} (aa')^3 \square^3 \left(\frac{\ln x}{x}\right) - \frac{3}{640} \frac{H^4}{\pi^4} (aa')^3 \square^2 \left(\frac{\ln x}{x} + \frac{9}{2x}\right). \tag{2.122}
\end{aligned}$$

This time we only write the nonlocal finite pieces without $\delta^D(x-x')$ from the power $(\frac{1}{y})^{D-2}$ in (2.112). In the final two lines, we re-define the function $y(x; x')$ in (2.33) as

Table 2.2: Contributions to counterterms. All terms are multiplied by

$$\frac{i\kappa^2 H^{D-4}}{(4\pi)^{D/2}} \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)}.$$

External operators	Coef. of $a^2 \square^2 \delta^D(x-x')$	Coef. of $H^2 a^2 \square \delta^D(x-x')$	Coef. of $H^2 \nabla^2 \delta^D(x-x')$
α	$-\frac{(D+2)(D+4)}{2(D-2)^2(D+1)}$	$\frac{(D-4)(D+2)(D+4)}{4(D-2)(D+1)}$	$-\frac{D(D+2)(D+4)}{2(D-2)(D+1)}$
β_1	0	$\frac{(D-1)(D+2)(D+4)}{2(D-2)(D+1)}$	0
β_2	0	$-\frac{(D+2)(D+4)}{2(D+1)}$	0
γ_1	$\frac{(D-20)(D+2)}{2(D-2)^2(D+1)}$	$\frac{D^5-37D^4+116D^3-172D^2-496D+576}{8(D-2)^2(D+1)}$	$\frac{(D-20)D(D+2)}{2(D-2)(D+1)}$
γ_2	$\frac{12(D+2)}{(D-2)^2(D+1)}$	$\frac{D^4+12D^3-5D^2-40D-184}{(D-2)^2(D+1)}$	$\frac{12(D^3+8)}{(D-2)^2(D+1)}$
ε_1	0	0	$\frac{D^4+12D^3-24D^2-20D+16}{(D-2)(D^2-1)}$
ε_2	0	0	$-\frac{13D^2+6D-16}{(D^2-1)}$
ε_3	0	0	$\frac{2(D-2)(D+2)(D+4)}{(D^2-1)}$
Total	0	$\frac{(D-4)(D^4-23D^3+124D^2+260D+208)}{8(D-2)^2(D+1)}$	$\frac{3D^5-3D^4-34D^3+96D^2+112D-192}{(D-2)^2(D^2-1)}$

Table 2.3: Other contributions to counterterms. All terms are multiplied by

$$\frac{i\tilde{\kappa}^2 H^{D-4}}{(4\pi)^{D/2}} \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)}.$$

External operators	Coef. of $Ha\partial_0\Box\delta^D(x-x')$	Coef. of $H^3a\partial_0\delta^D(x-x')$
α	$-\frac{(D+2)(D+4)}{(D-2)(D+1)}$	$\frac{(D-2)(D+2)(D+4)}{4(D+1)}$
β_1	0	$\frac{(D-1)(D+2)(D+4)}{2(D+1)}$
β_2	0	$-\frac{(D-2)(D+2)(D+4)}{2(D+1)}$
γ_1	$\frac{(D-20)(D+2)}{(D-2)(D+1)}$	$\frac{D^5-37D^4+112D^3-92D^2-480D+256}{8(D-2)(D+1)}$
γ_2	$\frac{24(D+2)}{(D-2)(D+1)}$	$\frac{D^4+12D^3-5D^2-64D-232}{(D-2)(D+1)}$
Total	0	$\frac{(D-4)(D^4-23D^3+124D^2+356D+400)}{8(D-2)(D+1)}$

$x \equiv \frac{y}{4}$. Also, note that we take the $D = 4$ limit to get the final result. Using the same method for the terms with the other external operators we obtain nonlocal finite terms for each case, i.e., each external operator. These newly found finite terms are added to the already found finite terms listed in TABLES A.5 - A.15. The summation of all the finite nonlocal terms from the 3-point interactions are given in TABLE 2.4.

Our final result for the regulated self-mass-squared consists of three finite parts:

- the local 4-point contributions given in (2.59),
- the local 3-point contributions coming from TABLES 2.1 - 2.3,
- the nonlocal pieces coming from 3-point interaction, i.e. TABLE 2.4,

It has the following form,

$$\begin{aligned}
 -iM_{\text{reg}}^2(x; x') = & -i\kappa^2 a^2 \left(d_1 \Box^2 + d_2 H^2 \Box + d_3 H^4 + d_4 H^2 \frac{\nabla^2}{a^2} \right) \delta^D(x - x') \\
 & + \text{Table 2.4} + 2i\kappa^2 \times \frac{H^4 a^2}{16\pi^2} \delta^4(x - x') + \frac{9i\kappa^2}{40} \times \frac{H^3 a}{16\pi^2} \partial_0 \delta^4(x - x') + \mathcal{O}(D - 4).
 \end{aligned}
 \tag{2.123}$$

Table 2.4: All Finite Nonlocal Contributions with $x \equiv \frac{y}{4}$, where $y(x; x')$ is defined in the equation (2.33).

External operators	Coef. of $\frac{\tilde{\kappa}^2 H^4}{(4\pi)^4}$
$(aa')^3 \square^3 / H^2$	$\frac{9 \ln x}{5x}$
$(aa')^3 \square^2$	$\frac{9 \ln x}{x} - \frac{21}{5x}$
$(aa')^3 H^2 \square$	$-\frac{267 \ln x}{5x} + \frac{51}{x}$
$(aa')^3 H^4$	$\frac{258 \ln x}{5x} - \frac{549}{5x}$
$(aa')^2 (a^2 + a'^2) \square^3 / H^2$	$-\frac{9 \ln x}{10x}$
$(aa')^2 (a^2 + a'^2) \square^2$	$-\frac{99 \ln x}{10x} - \frac{9}{10x}$
$(aa')^2 (a^2 + a'^2) H^2 \square$	$\frac{36 \ln x}{x} - \frac{51}{2x}$
$(aa')^2 (a^2 + a'^2) H^4$	$-\frac{108 \ln x}{5x} + \frac{174}{5x}$
$(aa')^2 H^2 \nabla^2$	$\frac{128 \ln x}{5x} + \frac{166}{5x}$
$(aa')^2 \nabla^2 \square$	$-\frac{64 \ln x}{5x}$
$(aa')(a^2 + a'^2) H^2 \nabla^2$	$-\frac{4 \ln x}{x} - \frac{2}{x}$
$(aa')(a^2 + a'^2) \nabla^2 \square$	$\frac{2 \ln x}{x} + \frac{2}{x}$
$(aa') \nabla^4$	0

The last term in the above equation comes from the second column of Table 2.3. Here the coefficients d_i are

$$d_1 = 0, \quad (2.124)$$

$$d_2 = \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left[\frac{7}{80} + \mathcal{O}(D-4) \right], \quad (2.125)$$

$$\begin{aligned} d_3 &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left[\frac{(D^7 - 29D^6 + 282D^5 - 540D^4 - 856D^3 + 224D^2)\Gamma(\frac{D}{2})}{2^{11}(D-4)(D-3)(D-1)^2(D+1)} \right. \\ &\quad \left. + \frac{(1664D + 3328)\Gamma(\frac{D}{2})}{2^{11}(D-4)(D-3)(D-1)^2(D+1)} \right], \\ &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left[\frac{3}{40(D-4)} + \frac{1021}{3600} - \frac{3\gamma}{80} + \mathcal{O}(D-4) \right], \end{aligned} \quad (2.126)$$

$$\begin{aligned} d_4 &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left[\frac{(3D^5 - 3D^4 - 34D^3 + 96D^2 + 112D - 192)\Gamma(\frac{D}{2})}{2^6(D-4)(D-3)(D-1)^2(D^2-1)} \right], \\ &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left[\frac{2}{9(D-4)} - \frac{59}{540} - \frac{\gamma}{9} + \mathcal{O}(D-4) \right]. \end{aligned} \quad (2.127)$$

Here γ is Euler's constant which is approximately equal to 0.577215. It appears here due to the expansion of the Gamma function. Following the BPHZ renormalization scheme, we will choose c_i 's to absorb the divergent terms:

$$c_i = -d_i + \Delta c_i \quad (2.128)$$

where Δc_i is the remaining arbitrary finite term for each of the four c_i . Taking the unregulated limit ($D=4$) provides us with the final renormalized result for the conformal-conformal interaction part of the full self-mass-squared at one loop order,

$$\begin{aligned} -iM_{\text{ren}}^2(x; x') &= -i\kappa^2 a^2 \left(\Delta c_1 \square^2 + \Delta c_2 H^2 \square + \Delta c_3 H^4 + \Delta c_4 H^2 \frac{\nabla^2}{a^2} \right) \delta^4(x - x') \\ &\quad + \text{Table 2.4} + 2i\kappa^2 \times \frac{H^4 a^2}{16\pi^2} \delta^4(x - x') + \frac{9i\kappa^2}{40} \times \frac{H^3 a}{16\pi^2} \partial_0 \delta^4(x - x') + \mathcal{O}(D-4). \end{aligned} \quad (2.129)$$

2.5 Discussion

We have calculated the self-mass-squared of a conformally coupled scalar interacting with a graviton at one loop order on a locally de Sitter geometry. A technical advantage of the scalar being conformally coupled to gravity is that we can do the computations around the flat space metric. Specifically, we can use the conformally rescaled scalar propagator for flat space. On the other hand, the graviton propagator stays the same because it is not conformally invariant. The computation was done using dimensional regularization and renormalized by absorbing divergences with BPHZ counterterms. The fully renormalized result is given in Eq. (2.129) with the finite nonlocal contributions in TABLE 2.4.

The purpose of this computation is to investigate quantum gravitational effects to the scalar mode function during Inflation. Despite the fact that our scalar is taken to be a spectator field during inflation driven by cosmological constant Λ , one might still apply the result to the inflaton of scalar-driven inflation because its potentials are considered to be nearly flat. The first step for checking whether or not gravitons give a significant correction to scalars is to compute and renormalize the scalar self-mass-squared $-iM^2(x; x')$, which concerns our present paper. The second step is to solve the quantum corrected effective field equation (2.2). At this point it should be noted that this field equation (2.2) is derived in the in-out formalism which one typically uses for scattering amplitude computations in flat space. However, in a time-dependent background such as de Sitter, the in-vacuum does not evolve to the out-vacuum and thus the in-out formalism is not applicable to our case. Therefore we will instead use the in-in formalism also called the Schwinger-Keldysh formalism [105, 126–128, 130–133] in which the effective field equation (2.2) is replaced by

$$a^4 \square \phi(x) - \frac{1}{6} R \phi(x) - \int_{\eta_i}^0 d\eta' \int d^3 x' \left\{ M_{++}^2(x; x') + M_{+-}^2(x; x') \right\} \phi(x') = 0. \quad (2.130)$$

Here in order to get $-iM_{++}^2(x; x')$ and $-iM_{+-}^2(x; x')$ (which are composed of $i\Delta_{++}(x; x')$ and $i\Delta_{+-}(x; x')$ respectively), we make the following substitutions for $y(x; x')$,

$$i\Delta_{++}(x; x') : \quad y \rightarrow y_{++}(x; x') \equiv a(\eta)a(\eta')H^2 \left[\|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| - i\delta)^2 \right], \quad (2.131)$$

$$i\Delta_{+-}(x; x') : \quad y \rightarrow y_{+-}(x; x') \equiv a(\eta)a(\eta')H^2 \left[\|\vec{x} - \vec{x}'\|^2 - (\eta - \eta' + i\delta)^2 \right]. \quad (2.132)$$

Note that $-iM_{++}^2(x; x')$ is the same as the usual in-out self-mass-squared $-iM^2(x; x')$.

Our aim is to determine if the tree order mode function of the CC scalar (see for example, [79])

$$\phi_0(t, \vec{x}) = u(t, k) e^{i\vec{k} \cdot \vec{x}}, \quad \text{where} \quad u(t, k) = \sqrt{\frac{\hbar}{2ck}} \frac{\exp \left[-ick \int_t^{t_i} \frac{dt'}{a(t')} \right]}{a(t)}, \quad (2.133)$$

gets corrections from the self-mass-squared at one loop order, which will be a future paper.

The conformally coupled scalar field interacts with gravity via a kinetic term as well as the conformal coupling term. The case with no conformal coupling, which is called minimal coupling, was studied in the previous work [13]. In this paper we examined the other one, which is conformal coupling. For the case of 4-point interactions depicted in Fig. 2.2, adding these two results will suffice. However, for the case of 3 point interactions graphically represented in Fig. 2.3, one has the possibility of mixing the two. One vertex can be a kinetic interaction and the other can come from the conformal coupling. In the current work, we have completed only the half of the first step, namely calculating the one loop scalar self-mass-squared from conformal coupling. In a subsequent paper we will include the purely kinetic terms [13] and the mixing of kinetic and conformal interactions in Eq. (2.29) and use these full results in order to solve the effective field equation (2.130).

We also would like to highlight a couple of points to corroborate correctness of our calculation, which is a question that comes into mind for any computation of this length. The first one is that the most divergent part of the graviton propagator (which consists of the A, B and C parts) is same as the conformally coupled scalar propagator as one can see from Eqs. (2.48)-(2.50). Adding the tensor factors of the A, B and C parts of the graviton propagator gives

$$\begin{aligned} & 2\bar{\eta}_{\mu(\rho}\bar{\eta}_{\sigma)\nu} - \frac{2\bar{\eta}_{\mu\nu}\bar{\eta}_{\rho\sigma}}{D-3} - 4\delta_{(\mu}^0\bar{\eta}_{\nu)(\rho}\delta_{\sigma)}^0 \\ & + \frac{2\left[(D-3)\delta_{\mu}^0\delta_{\nu}^0 + \bar{\eta}_{\mu\nu}\right]\left[(D-3)\delta_{\rho}^0\delta_{\sigma}^0 + \bar{\eta}_{\rho\sigma}\right]}{(D-3)(D-2)} = 2\eta_{\mu(\rho}\eta_{\sigma)\nu} - \frac{2}{D-2}\eta_{\mu\nu}\eta_{\rho\sigma}, \end{aligned} \quad (2.134)$$

which is the tensor factor of the graviton propagator in flat space. This means that the most singular terms in the full computation are identical to those of flat space. This is

why the sum of the coefficients for $a^2 \square^2 \delta^D(x-x')$ becomes zero in TABLE 2.2. The second, and much stronger, check comes from the analysis of the de Sitter noninvariant counterterms. The de Sitter symmetry is broken, due to the form of our gauge fixing term, in such a way only a subgroup of the full de Sitter group is preserved. Respecting the remaining symmetries turns out to make the divergent terms $Ha\partial_0 \square \delta^D(x-x')$ and $H^3 a \partial_0 \delta^D(x-x')$ become zero. This is exactly what happens in TABLE 2.3 in a highly nontrivial way. And finally the last check is getting zero for ∇^4 in TABLE 2.4. Since working with the most singular term is similar to working in flat space where the gauge fixing term vanishes, one can at most get quadratic divergence. Therefore the de Sitter breaking gauge fixing term can only occur at quadratically divergent level. This will suffice to give only ∇^2 which breaks de Sitter invariance but still respects the other symmetries that are not broken.

One point worth to note is that the result for the minimal coupling only. The massless, minimally coupled scalar gets no significant corrections from inflationary gravitons at one loop order [13, 14]. That reason why is that, even though inflation produces the vast ensemble of gravitons, they only interact with the MMC scalar via kinetic energy which redshifts to zero at late times. Adding conformal coupling engenders another interaction, which is non-derivative and so not redshifted. There is clearly one term that might give a big effect, which is the $H^4 a^2 \delta^4(x-x')$ term in Eq. (2.129). This term was not present in the MMC calculation studied in [13, 14]. But, unfortunately this can be removed by the conformal counterterm, $a^2 \Delta c_3 H^4 \delta^4(x-x')$ with an arbitrary constant Δc_3 in front. However the finite terms we got here may give some enhancement after we solve the effective field equation. In [14] the corresponding finite terms nontrivially added up to be zero but there is no a priori reason why they should add up to zero at the end in this calculation. Also including the MMC-CC cross terms might give an interesting effect and checking it explicitly is our goal for the series of this and subsequent papers.

Finally the most interesting and cosmologically relevant part would be the second step of solving the quantum corrected effective field equation for the scalar mode function. If the mode function gets quantum corrections, so does the power spectrum. Weinberg's analysis [105] suggests that the possibility of quantum loop corrections to the power spectra of primordial density perturbations. Our case, if we find

any correction, would serve as a specific example for his analysis. In the age of possibility of observing primordial gravitational waves with a detector like BICEP2 [11], resolving quantum corrections due to gravitons, even though it may still take a few decades more, does not sound utterly hopeless. We hope to provide a foreground for those future measurements.





3. QUANTUM GRAVITY CORRECTIONS TO THE CONFORMALLY COUPLED SCALAR SELF-MASS-SQUARED ON DE SITTER II: KINETIC-CONFORMAL CROSS TERMS

The present work¹ is the second part of a series of computations for the self-mass-squared of the conformally coupled (CC) scalar interacting with gravitons. This work includes the kinetic-kinetic and kinetic-conformal parts, and thus completes the full scalar self-mass squared at one loop order in de Sitter background when combined with the conformal-conformal part previously evaluated. We use dimensional regularization and renormalize the results by subtracting appropriate counterterms. The self-mass squared is finally ready to quantum-correct the CC scalar field equation so that one can study the effect of inflationary produced gravitons on the CC scalar and its observational consequences.

3.1 Introduction

It has been fifty years since Leonard Parker's breakthrough discovery in late 1960's that particles can be created out of the vacuum in an expanding universe [5–7]. See Refs. [15, 16] for the historical notes on the discovery. Parker also pointed out that the particle production is maximized if the expansion is exponential and the particles are not conformally invariant [5–7]. The Poincaré patch of de Sitter space we consider is such an exponentially expanding background. The particle creation in an exponentially expanding universe had not been emphasized much until the idea of inflation [17–19] was proposed in early 1980's [16]. However, the inflationary paradigm drastically changed the situation and particle production during inflation has been attracting a great deal of attention since then [1, 13, 14, 20–90]. The unique particles having no conformal invariance are the massless, minimally coupled (MMC) scalars and gravitons, and these particles comprise the scalar [91] and tensor [92] perturbations predicted by inflationary theories.

¹This chapter is based on the paper Quantum gravity corrections to the conformally coupled scalar self-mass-squared on de Sitter background. II. Kinetic conformal cross terms, *Phys. Rev. D* 96, 025001, (2017) by **Boran, S.**, Kâhya, E. O. and Park, S. .

The pioneering computations on the scalar and tensor perturbations were at tree order [91, 92], and it is a natural next step to include quantum loop corrections from MMC scalars and gravitons. A number of computations regarding quantum corrections in an inflationary background have been carried out [1, 13, 14, 20–90]. Besides the cause of inflation, which still has no consensus, studying quantum loop effects during inflation can be well estimated by taking the locally de Sitter background as an inflationary universe. A generic procedure to study loop effects is first to compute the one-particle-irreducible (1PI) 2-point function, and second to use it to quantum-correct the field equation of the particle in question. For the case of a scalar, the 1PI 2-point function is called the self-mass-squared and denoted by $-iM^2(x; x')$. In the present paper, we compute a part of $-iM^2(x; x')$ for the CC scalar, which is indeed the third and last part followed by the first done in Ref. [13] and the second in Ref. [1]. In a subsequent paper, we will use the full result of $-iM^2(x; x')$ to quantum-correct the CC scalar field equation

$$\partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \phi(x) \right) - \frac{1}{6} R \phi(x) - \int d^4 x' M^2(x; x') \phi(x') = 0, \quad (3.1)$$

and solve it to study how gravitons produced by inflation affect the dynamics of the CC scalar.

The motivation why we consider the CC scalar in particular is multifold. First, the tree order CC scalar mode functions

$$\phi_0(t, \vec{x}) = u_0(t, k) e^{i\vec{k} \cdot \vec{x}}, \quad \text{where} \quad u_0(t, k) = \sqrt{\frac{1}{2k}} \frac{\exp \left[-ik \int_{t_i}^t \frac{dt'}{a(t')} \right]}{a(t)}, \quad (3.2)$$

vanish as $a(t)$ exponentially grows. Hence, any finite result can be attributed to pure loop corrections. Second, we note that the scalar is not differentiated in the conformal coupling by recalling derivative interactions redshift away during inflation. In fact, the authors previously studied the interaction between MMC scalars and gravitons, which are the two non-conformally invariant particles vastly produced by inflation, and verified that both loop effects from gravitons [13, 14] and from MMC scalars [37, 38] redshift to zero. The reason for this is that their interactions are only through their kinetic energies, or in other words derivatives. Another observation is that the non-conformal and conformal combination seems to have more interesting effects than the combination of non-conformal particles. In MMC scalar quantum electrodynamics, a MMC scalar loop leads to photons having an increasing effective mass [22–24].

When a MMC scalar is Yukawa-coupled to a massless fermion, fermions acquire an effectively growing mass [26, 27]. Graviton loop corrections to a massless fermion cause the fermion field strength to grow with time [29–31]. Graviton loops also induce secular corrections to the electromagnetic field [70–76]. If our case does induce a secular loop effect, it will be added to this list of the non-conformal and conformal combinations having a significant effect. Furthermore, it might give an insight why this particular combination tends to give significant loop corrections.

The rest of this paper is organized as follows: In Section 3.2, we recast the definitions and notations employed in our previous paper [1]. We provide a formal expression for the CC scalar self-mass-squared which consists of 4-point and 3-point interactions at one-loop order. The 4-point interaction is computed in Section 3.3. The 3-point interaction is further divided by the kinetic-kinetic, conformal-conformal and kinetic-conformal parts. The first two parts were evaluated in Refs. [13] and [1], respectively and the kinetic-conformal part is computed in Section 3.4. We also compute the kinetic-kinetic part using the CC scalar propagator in 3.4. The results are fully renormalized and the unregulated limit is taken in Section 3.5. We summarize and discuss our results in Section 3.6.

3.2 The Self-Mass-Squared

We start with the Lagrangian describing pure gravity plus the interaction between gravitons and the massless CC scalar

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}\sqrt{-g} - \frac{D-2}{8(D-1)}\phi^2 R\sqrt{-g} + \frac{1}{16\pi G}(R - (D-2)\Lambda)\sqrt{-g}, \quad (3.3)$$

where G is Newton's constant, R is the Ricci scalar and $\Lambda = (D-1)H^2$ is the cosmological constant with the Hubble constant H . We work in D spacetime dimensions to facilitate dimensional regularization. Our background geometry is the open conformal coordinate patch of de Sitter space

$$ds^2 = \hat{g}_{\mu\nu}dx^\mu dx^\nu = a^2(\eta)\left[-d\eta^2 + d\vec{x}\cdot d\vec{x}\right], \quad \text{where } a(\eta) = -\frac{1}{H\eta}, \quad (3.4)$$

with the coordinate ranges

$$-\infty < x^0 \equiv \eta < 0, \quad -\infty < x^i < +\infty, \quad i = 1, 2, \dots, D-1. \quad (3.5)$$

The full metric is expressed as

$$g_{\mu\nu}(x) \equiv a^2 \left[\eta_{\mu\nu} + \kappa h_{\mu\nu}(x) \right], \quad \text{where} \quad \kappa^2 \equiv 16\pi G, \quad (3.6)$$

where $\hat{g}_{\mu\nu}$ is the background metric and $h_{\mu\nu}$ the conformally rescaled graviton field. The conformal coupling makes the matter sector of the Lagrangian conformally invariant:

$$g_{\mu\nu} \equiv \Omega^2 \tilde{g}_{\mu\nu}, \quad \phi \equiv \Omega^{\frac{2-D}{2}} \tilde{\phi}. \quad (3.7)$$

$$\begin{aligned} \Rightarrow \mathcal{L}_{\text{Mat}} &= -\frac{1}{2} \partial_\mu \phi \partial_\nu \phi g^{\mu\nu} \sqrt{-g} - \frac{D-2}{8(D-1)} \phi^2 R \sqrt{-g}, \\ &= -\frac{1}{2} \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} \tilde{g}^{\mu\nu} \sqrt{-\tilde{g}} - \frac{D-2}{8(D-1)} \tilde{\phi}^2 \tilde{R} \sqrt{-\tilde{g}}. \end{aligned} \quad (3.8)$$

We take $\Omega = a$ and work with the conformally rescaled metric

$$\tilde{g}_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}. \quad (3.9)$$

The expansions of the inverse and the volume element of the conformally rescaled metric $\tilde{g}_{\mu\nu}$ are

$$\tilde{g}^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^\mu{}_\rho h^{\rho\nu} + \mathcal{O}(\kappa^3), \quad (3.10)$$

$$\sqrt{-\tilde{g}} = 1 + \frac{1}{2} \kappa h + \frac{1}{8} \kappa^2 h^2 - \frac{1}{4} \kappa^2 h^{\rho\sigma} h_{\rho\sigma} + \mathcal{O}(\kappa^3). \quad (3.11)$$

Also the conformally rescaled Ricci scalar is

$$\begin{aligned} \tilde{R} &= \kappa \left(-h^\mu{}_\mu + h^{\mu\nu}{}_{,\mu\nu} \right) + \kappa^2 \left(-2h^{\mu\nu} h_{\lambda\nu,\mu}{}^\lambda + h^{\mu\nu} h_{\mu\nu,\lambda}{}^\lambda + h^{\mu\nu} h_{,\mu\nu} \right. \\ &\quad \left. + \frac{3}{4} h^{\mu\nu}{}_{,\lambda} h_{\mu\nu}{}^{,\lambda} + h^{\mu\nu}{}_{,\mu} h_{,\nu} - h^{\mu\nu}{}_{,\mu} h_{\lambda\nu}{}^{,\lambda} - \frac{1}{2} h^{\mu\nu}{}_{,\lambda} h^\lambda{}_{\nu,\mu} - \frac{1}{4} h_{,\mu} h^{,\mu} \right. \\ &\quad \left. - \frac{1}{2} h h^\mu{}_\mu + \frac{1}{2} h h^{\mu\nu}{}_{,\mu\nu} \right) + \mathcal{O}(\kappa^3). \end{aligned} \quad (3.12)$$

Using the perturbative expansion, we derive the self-mass-squared $-iM^2(x; x')$ at one loop order, which consists of the following three Feynman diagrams corresponding to the analytic expressions (written next to the diagrams, respectively):

$$\begin{aligned}
& \text{Diagram: a horizontal line with a wavy loop on top, labeled } x \text{ and } x' \text{ at the ends.} \\
& = -iM_{3\text{pt}}^2(x; x') \equiv \left\langle i \frac{\delta S[\phi, h]}{\delta \phi(x)} i \frac{\delta S[\phi, h]}{\delta \phi(x')} \right\rangle_0 + \\
& \text{Diagram: a horizontal line with a wavy loop on top, labeled } x \text{ at the end.} \\
& = -iM_{4\text{pt}}^2(x; x') \equiv \left\langle i \frac{\delta^2 S[\phi, h]}{\delta \phi(x) \delta \phi(x')} \right\rangle_0 + \\
& \text{Diagram: a horizontal line with a cross on top, labeled } x \text{ at the end.} \\
& = \text{counterterm} .
\end{aligned} \tag{3.13}$$

Here the subscript 0 means that the expectation value is taken in the free theory.

The 3-point and 4-point interactions (terms proportional to $\tilde{\phi}^2 h$ and $\tilde{\phi}^2 h^2$) can be read off from the expansion of the matter part of the Lagrangian (3.8). The kinetic term of the Lagrangian is expanded as [13]

$$\begin{aligned}
\mathcal{L}_K &\equiv -\frac{1}{2} \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} \tilde{g}^{\mu\nu} \sqrt{-\tilde{g}} , \\
&= -\frac{1}{2} \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} \left\{ \eta^{\mu\nu} + \kappa \left(\frac{1}{2} \eta^{\mu\nu} h - h^{\mu\nu} \right) + \kappa^2 \left(\frac{1}{8} \eta^{\mu\nu} h^2 - \frac{1}{4} \eta^{\mu\nu} h^{\rho\sigma} h_{\rho\sigma} \right. \right. \\
&\quad \left. \left. - \frac{1}{2} h h^{\mu\nu} + h^{\mu\rho} h_{\rho}{}^{\nu} \right) + \mathcal{O}(\kappa^3) \right\} .
\end{aligned} \tag{3.14}$$

and the conformal coupling term is expanded as [1]

$$\begin{aligned}
\mathcal{L}_{\text{CC}} &\equiv -\frac{D-2}{8(D-1)} \tilde{\phi}^2 \tilde{R} \sqrt{-\tilde{g}} , \\
&= -\frac{D-2}{8(D-1)} \kappa \tilde{\phi}^2 \left(-h_{\mu}{}^{\mu} + h^{\mu\nu}{}_{,\mu\nu} \right) - \frac{D-2}{8(D-1)} \kappa^2 \tilde{\phi}^2 \left(-2h^{\mu\nu} h_{\lambda\nu,\mu}{}^{\lambda} \right. \\
&\quad \left. + h^{\mu\nu} h_{\mu\nu,\lambda}{}^{\lambda} + h^{\mu\nu} h_{,\mu\nu} + \frac{3}{4} h^{\mu\nu}{}_{,\lambda} h_{\mu\nu}{}^{\lambda} + h^{\mu\nu}{}_{,\mu} h_{,\nu} - h^{\mu\nu}{}_{,\mu} h_{\lambda\nu}{}^{\lambda} \right. \\
&\quad \left. - \frac{1}{2} h^{\mu\nu}{}_{,\lambda} h^{\lambda}{}_{\nu,\mu} - \frac{1}{4} h_{,\mu} h^{\mu} - \frac{1}{2} h h_{\mu}{}^{\mu} + \frac{1}{2} h h^{\mu\nu}{}_{,\mu\nu} \right) .
\end{aligned} \tag{3.15}$$

In the following subsection we demonstrate the formal expressions of the 3-point and 4-point interactions for the self-mass-squared and drop tilde for notational simplicity. However, we re-emphasize here that our metric, the scalar field ϕ and graviton field $h_{\mu\nu}$ are conformally rescaled ones:

$$\tilde{g}_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} , \quad \tilde{\phi} \equiv \phi . \tag{3.16}$$

3.2.1 Formal expressions for the one loop self-mass-squared

We first recast the 4-point and 3-point contributions from Refs. [13] and [1] and derive the kinetic-conformal cross part of the 3-point interaction.

3.2.1.1 4-point contributions

The 4-point contribution from the kinetic term (3.14) is similar to [13] where the background metric for this case is flat. Therefore the expression becomes

$$\begin{aligned}
-iM_{\text{K4pt}}^2(x; x') &\equiv \left\langle i \frac{\delta^2 S_{\text{K4}}}{\delta \phi(x') \delta \phi(x)} \right\rangle_0, \\
&= \frac{i}{8} \kappa^2 \partial^\mu [i [\alpha_\Delta \Delta^\rho]_\rho(x; x') \partial_\mu \delta^D(x - x')] - \frac{i}{4} \kappa^2 \partial^\mu [i [\alpha^\beta \Delta_{\alpha\beta}](x; x') \partial_\sigma \delta^D(x - x')] \\
&\quad - \frac{i}{4} \kappa^2 \partial^\rho [i [\alpha_\Delta \Delta^{\rho\sigma}](x; x') \partial_\mu \delta^D(x - x')] + i \kappa^2 \partial^\mu [i [\alpha_{\beta\Delta} \Delta^{\rho\sigma}](x; x') \partial_\sigma \delta^D(x - x')] ,
\end{aligned} \tag{3.17}$$

where the expression

$$i[\mu^\nu \Delta_{\lambda\nu}](x; x') = \left\langle h^{\mu\nu}(x) h_{\lambda\nu}(x') \right\rangle_0, \tag{3.18}$$

is the graviton propagator which will be given in the next subsection. The 4-point contribution from the conformal coupling term (3.15) was calculated in [1].

3.2.1.2 3-point contributions

The first Feynman diagram that appears in (3.13) can be read off as two 3-point interaction vertices connected by a graviton propagator and a conformally coupled scalar propagator. There are two, conformal and kinetic, types of 3-point interactions for a scalar coupled to gravity conformally as one can see from (3.3).

First, we label the 3-point interactions from the conformal coupling term (3.15) as

$$\begin{aligned}
\mathcal{L}_{3\text{pt}} &= -\frac{D-2}{8(D-1)} \kappa \phi^2 (-h^\mu_\mu + h^{\mu\nu}_{,\mu\nu}) = -\tilde{\kappa} \phi^2 (-h^\mu_\mu + h^{\mu\nu}_{,\mu\nu}) \\
&\equiv \mathcal{L}_{3a} + \mathcal{L}_{3b},
\end{aligned} \tag{3.19}$$

where we defined $\tilde{\kappa} \equiv \frac{D-2}{8(D-1)} \times \kappa$. This leads to the *conformal-conformal* part of the 3-point interaction [1]

$$\begin{aligned}
& -iM_{3\text{pt}}^2(x; x') \\
& = \left\langle i \frac{\delta S_{3a}}{\delta \phi(x)} i \frac{\delta S_{3a}}{\delta \phi(x')} + i \frac{\delta S_{3a}}{\delta \phi(x)} i \frac{\delta S_{3b}}{\delta \phi(x')} + i \frac{\delta S_{3b}}{\delta \phi(x)} i \frac{\delta S_{3a}}{\delta \phi(x')} + i \frac{\delta S_{3b}}{\delta \phi(x)} i \frac{\delta S_{3b}}{\delta \phi(x')} \right\rangle_0, \\
& = -\tilde{\kappa}^2 i \Delta_{\text{cf}} \left\{ \partial^2 \partial'^2 i[\alpha_\alpha \Delta^\beta_\beta](x; x') - [\partial_\mu \partial_\nu \partial'^2 + \partial'_\mu \partial'_\nu \partial^2] i[\alpha_\alpha \Delta^{\mu\nu}](x; x') \right. \\
& \quad \left. + \partial_\mu \partial_\nu \partial'_\alpha \partial'_\beta i[\mu^\nu \Delta^{\alpha\beta}](x; x') \right\}. \tag{3.20}
\end{aligned}$$

This contribution was calculated in a previous work [1].

Second, we label the 3-point interactions from the kinetic term of the Lagrangian (3.14) as

$$\mathcal{L}_{K3\text{pt}} = -\frac{1}{2} \kappa \partial_\mu \phi \partial_\nu \phi \left(-h^{\mu\nu} + \frac{1}{2} \eta^{\mu\nu} h \right) \equiv \mathcal{L}_{K3c} + \mathcal{L}_{K3d}. \tag{3.21}$$

Taking c and d at each vertex point we obtain the *kinetic-kinetic* part of 3-point interaction of the self-mass-squared

$$\begin{aligned}
& -iM_{3\text{ptK}}^2(x; x') \\
& = \left\langle i \frac{\delta S_{K3c}}{\delta \phi(x)} i \frac{\delta S_{K3c}}{\delta \phi(x')} + i \frac{\delta S_{K3c}}{\delta \phi(x)} i \frac{\delta S_{K3d}}{\delta \phi(x')} + i \frac{\delta S_{K3d}}{\delta \phi(x)} i \frac{\delta S_{K3c}}{\delta \phi(x')} + i \frac{\delta S_{K3d}}{\delta \phi(x)} i \frac{\delta S_{K3d}}{\delta \phi(x')} \right\rangle_0, \\
& = \kappa^2 \left\{ \frac{1}{2} \partial_\alpha \partial'^\nu [i[\alpha^\beta \Delta^\rho_\rho](x; x')] \partial_\beta \partial'_\nu i \Delta_{\text{cf}}(x; x') - \partial_\alpha \partial'_\rho [i[\alpha^\beta \Delta^{\rho\sigma}](x; x')] \partial_\beta \partial'_\sigma i \Delta_{\text{cf}}(x; x') \right. \\
& \quad \left. - \frac{1}{4} \partial^\mu \partial'^\nu [i[\alpha_\alpha \Delta^\rho_\rho](x; x')] \partial_\mu \partial'_\nu i \Delta_{\text{cf}}(x; x') + \frac{1}{2} \partial^\mu \partial'_\rho [i[\alpha_\alpha \Delta^{\rho\sigma}](x; x')] \partial_\mu \partial'_\sigma i \Delta_{\text{cf}}(x; x') \right\}. \tag{3.22}
\end{aligned}$$

Note that this part can be read off from the 3-point contribution of Ref. [13] by replacing the minimally conformally coupled propagator term $i\Delta_A(x; x')$ by the conformal propagator $i\Delta_{\text{cf}}(x; x') \equiv \left\langle \phi(x) \phi(x') \right\rangle_0 = (aa')^{\frac{D}{2}-1} F(y)$ and dividing by $(aa')^{D-2}$ (extracting $(aa')^{\frac{D}{2}-1}$ from each vertex point). Although the structure of the kinetic part of our calculation is similar to Ref. [13] the powers of the scale factor gives quite a different form, and thus one has to redo all of the calculation using new identities.

Finally, the cross terms, which we name as *kinetic-conformal* part can be obtained by taking one kinetic term at one vertex and one conformal term at the other vertex:

$$\begin{aligned}
& \frac{\text{diagram with wavy line } a \text{ to } b \text{ (CC) and } c \text{ to } d \text{ (KC)}}{\text{CC}} + \frac{\text{diagram with wavy line } a \text{ to } b \text{ (KC) and } c \text{ to } d \text{ (CC)}}{\text{KC}} = -iM_{3\text{ptcross}}^2(x; x') \\
& = \left\langle i \frac{\delta S_{3a}}{\delta \phi(x)} i \frac{\delta S_{K3c}}{\delta \phi(x')} + i \frac{\delta S_{3a}}{\delta \phi(x)} i \frac{\delta S_{K3d}}{\delta \phi(x')} + i \frac{\delta S_{3b}}{\delta \phi(x)} i \frac{\delta S_{K3c}}{\delta \phi(x')} + i \frac{\delta S_{3b}}{\delta \phi(x)} i \frac{\delta S_{K3d}}{\delta \phi(x')} \right. \\
& \quad \left. + i \frac{\delta S_{3c}}{\delta \phi(x')} i \frac{\delta S_{K3a}}{\delta \phi(x)} + i \frac{\delta S_{3d}}{\delta \phi(x')} i \frac{\delta S_{K3a}}{\delta \phi(x)} + i \frac{\delta S_{3c}}{\delta \phi(x')} i \frac{\delta S_{K3b}}{\delta \phi(x)} + i \frac{\delta S_{3d}}{\delta \phi(x')} i \frac{\delta S_{K3b}}{\delta \phi(x)} \right\rangle_0. \tag{3.23}
\end{aligned}$$

Taking the variation of each term with respect to the scalar,

$$\frac{\delta S_{3a}}{\delta \phi(x)} = -\tilde{\kappa} \frac{\delta}{\delta \phi(x)} \int d^D y \left[-\phi^2(y) \partial_\mu \partial^\mu h(y) \right] = -\tilde{\kappa} \left[-2\phi(x) \partial_\mu \partial^\mu h(x) \right], \tag{3.24}$$

$$\frac{\delta S_{3b}}{\delta \phi(x)} = -\tilde{\kappa} \frac{\delta}{\delta \phi(x)} \int d^D y \left[\phi^2(y) \partial_\mu \partial_\nu h^{\mu\nu}(y) \right] = -\tilde{\kappa} \left[2\phi(x) \partial_\mu \partial_\nu h^{\mu\nu}(x) \right], \tag{3.25}$$

$$\frac{\delta S_{K3c}}{\delta \phi(x')} = \tilde{\kappa} \frac{\delta}{\delta \phi(x')} \int d^D y \left[\frac{1}{2} \partial_\mu \phi(y) \partial_\nu \phi(y) h^{\mu\nu}(y) \right] = -\tilde{\kappa} \partial'_\mu \left[\partial'_\nu \phi(x') h^{\mu\nu}(x') \right], \tag{3.26}$$

$$\frac{\delta S_{K3d}}{\delta \phi(x')} = \tilde{\kappa} \frac{\delta}{\delta \phi(x')} \int d^D y \left[-\frac{1}{4} \partial_\mu \phi(y) \partial_\nu \phi(y) \eta^{\mu\nu} h(y) \right] = \frac{\tilde{\kappa}}{2} \partial'_\mu \left[\partial'^\mu \phi(x') h(x') \right], \tag{3.27}$$

and plugging these into (3.23) leads to

$$\begin{aligned}
-iM_{3\text{ptcross}}^2(x; x') = & \tilde{\kappa}^2 \left\{ 2\partial'_\mu \left[\partial'_\nu i\Delta_{\text{cf}}(x; x') \partial_\rho \partial^\rho i[\mu^\nu \Delta^\gamma{}_\gamma](x; x') \right] \right. \\
& - \partial'_\mu \left[\partial'^\mu i\Delta_{\text{cf}}(x; x') \partial_\rho \partial^\rho i[\alpha{}_\alpha \Delta^\gamma{}_\gamma](x; x') \right] - 2\partial'_\mu \left[\partial'_\nu i\Delta_{\text{cf}}(x; x') \partial_\rho \partial_\sigma i[\mu^\nu \Delta^{\rho\sigma}](x; x') \right] \\
& + \partial'_\mu \left[\partial'^\mu i\Delta_{\text{cf}}(x; x') \partial_\rho \partial_\sigma i[\alpha{}_\alpha \Delta^{\rho\sigma}](x; x') \right] + 2\partial_\mu \left[\partial_\nu i\Delta_{\text{cf}}(x; x') \partial'_\rho \partial'^\rho i[\mu^\nu \Delta^\gamma{}_\gamma](x; x') \right] \\
& - \partial_\mu \left[\partial^\mu i\Delta_{\text{cf}}(x; x') \partial'_\rho \partial'^\rho i[\alpha{}_\alpha \Delta^\gamma{}_\gamma](x; x') \right] - 2\partial_\mu \left[\partial_\nu i\Delta_{\text{cf}}(x; x') \partial'_\rho \partial'_\sigma i[\mu^\nu \Delta^{\rho\sigma}](x; x') \right] \\
& \left. + \partial_\mu \left[\partial^\mu i\Delta_{\text{cf}}(x; x') \partial'_\rho \partial'_\sigma i[\alpha{}_\alpha \Delta^{\rho\sigma}](x; x') \right] \right\}. \tag{3.28}
\end{aligned}$$

3.2.2 Propagators

To express the propagators in the same notation as that of the previous work, we recall the three notational conventions employed in Refs. [1, 13]. First the background metric

is denoted with a hat,

$$\hat{g}_{\mu\nu} = \eta_{\mu\nu} \quad \text{accordingly} \quad \hat{R} = 0. \quad (3.29)$$

Second, the spatial parts of the Lorentz metric and the Kronecker delta are defined with a bar,

$$\bar{\eta}_{\mu\nu} \equiv \eta_{\mu\nu} + \delta_\mu^0 \delta_\nu^0 \quad \text{and} \quad \bar{\delta}_\nu^\mu \equiv \delta_\nu^\mu - \delta_0^\mu \delta_\nu^0. \quad (3.30)$$

Third, we define the de Sitter length function $y(x; x')$ which is related to the de Sitter invariant length $\ell(x; x')$ from x^μ to x'^μ as

$$y(x; x') = 4 \sin^2 \left(\frac{1}{2} H \ell(x; x') \right) = a a' H^2 \left\{ \|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| - i\delta)^2 \right\}, \quad (3.31)$$

where $a \equiv a(\eta)$ and $a' \equiv a(\eta')$.

The propagator for a massless conformally coupled scalar for the flat background is [153],

$$\begin{aligned} i\Delta_{\text{cf}}(x; x') &= \frac{\Gamma\left(\frac{D}{2}-1\right)}{(4\pi)^{\frac{D}{2}}} \left(\frac{4}{\Delta x^2}\right)^{\frac{D}{2}-1} = (a a')^{\frac{D}{2}-1} \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \Gamma\left(\frac{D}{2}-1\right) \\ \left(\frac{4}{y}\right)^{\frac{D}{2}-1} &\equiv (a a')^{\frac{D}{2}-1} F(y). \end{aligned} \quad (3.32)$$

Here we defined $F(y)$ above, in order to work with de Sitter invariant functions.

To obtain the graviton propagator, we add the gauge fixing term (first derived in Ref. [118]) to the Lagrangian,

$$\begin{aligned} \mathcal{L}_{\text{GF}} &= -\frac{1}{2} a^{D-2} \eta^{\mu\nu} F_\mu F_\nu, \\ F_\mu &\equiv \eta^{\rho\sigma} \left(h_{\mu\rho, \sigma} - \frac{1}{2} h_{\rho\sigma, \mu} + (D-2) H a h_{\mu\rho} \delta_\sigma^0 \right). \end{aligned} \quad (3.33)$$

We partially integrate the quadratic part of the gauge fixed Lagrangian to extract the kinetic operator $D_{\mu\nu}^{\rho\sigma}$ as follows

$$\frac{1}{2} h^{\mu\nu} D_{\mu\nu}^{\rho\sigma} h_{\rho\sigma}, \quad (3.34)$$

where

$$\begin{aligned} D_{\mu\nu}^{\rho\sigma} &\equiv \left\{ \frac{1}{2} \bar{\delta}_\mu^{(\rho} \bar{\delta}_\nu^{\sigma)} - \frac{1}{4} \eta_{\mu\nu} \eta^{\rho\sigma} - \frac{1}{2(D-3)} \delta_\mu^0 \delta_\nu^0 \delta_0^\rho \delta_0^\sigma \right\} D_A \\ &\quad + \delta_{(\mu}^0 \bar{\delta}_{\nu)}^{(\rho} \delta_0^{\sigma)} D_B + \frac{1}{2} \left(\frac{D-2}{D-3} \right) \delta_\mu^0 \delta_\nu^0 \delta_0^\rho \delta_0^\sigma D_C, \end{aligned} \quad (3.35)$$

and the three scalar differential operators are

$$D_A \equiv \partial_\mu \left(\sqrt{-\hat{g}} \hat{g}^{\mu\nu} \partial_\nu \right), \quad (3.36)$$

$$D_B \equiv \partial_\mu \left(\sqrt{-\hat{g}} \hat{g}^{\mu\nu} \partial_\nu \right) - \frac{1}{D} \left(\frac{D-2}{D-1} \right) \hat{R} \sqrt{-\hat{g}}, \quad (3.37)$$

$$D_C \equiv \partial_\mu \left(\sqrt{-\hat{g}} \hat{g}^{\mu\nu} \partial_\nu \right) - \frac{2}{D} \left(\frac{D-3}{D-1} \right) \hat{R} \sqrt{-\hat{g}}. \quad (3.38)$$

The graviton propagator obeys the defining equation,

$$D_{\mu\nu}^{\rho\sigma} \times i \left[\rho\sigma \Delta^{\alpha\beta} \right] (x; x') = \delta_\mu^{(\alpha} \delta_\nu^{\beta)} i \delta^D(x - x'), \quad (3.39)$$

and inverting the kinetic operator leads to the graviton propagator in the gauge of (3.33) being a sum of constant tensor factors times scalar propagators,

$$i[\mu\nu\Delta_{\rho\sigma}](x; x') = \sum_{I=A,B,C} [\mu\nu T_{\rho\sigma}^I] i\Delta_I(x; x'). \quad (3.40)$$

Here each of the scalar propagators obeys

$$D_I \times i\Delta_I(x; x') = i\delta^D(x - x') \quad \text{for} \quad I = A, B, C, \quad (3.41)$$

and the tensor factors are

$$[\mu\nu T_{\rho\sigma}^A] = 2\bar{\eta}_{\mu(\rho} \bar{\eta}_{\sigma)\nu} - \frac{2}{D-3} \bar{\eta}_{\mu\nu} \bar{\eta}_{\rho\sigma}, \quad (3.42)$$

$$[\mu\nu T_{\rho\sigma}^B] = -4\delta_{(\mu}^0 \bar{\eta}_{\nu)(\rho} \delta_{\sigma)}^0, \quad (3.43)$$

$$[\mu\nu T_{\rho\sigma}^C] = \frac{2}{(D-3)(D-2)} [(D-3)\delta_\mu^0 \delta_\nu^0 + \bar{\eta}_{\mu\nu}] [(D-3)\delta_\rho^0 \delta_\sigma^0 + \bar{\eta}_{\rho\sigma}]. \quad (3.44)$$

The A-type propagator is the same as the MMC scalar propagator, which consists of the de Sitter invariant and breaking parts [119, 120].

$$i\Delta_A(x; x') = A(y) + k \ln(aa'), \quad (3.45)$$

where $k \equiv \frac{H^{D-2}}{(4\pi)^{D/2}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}$ and the de Sitter invariant part $A(y)$ has the following solution [120],

$$\begin{aligned} A(y) \equiv & \frac{H^{D-2}}{(4\pi)^{D/2}} \left\{ \Gamma\left(\frac{D}{2} - 1\right) \left(\frac{4}{y}\right)^{\frac{D}{2}-1} + \frac{\Gamma(\frac{D}{2} + 1)}{\frac{D}{2} - 2} \left(\frac{4}{y}\right)^{\frac{D}{2}-2} - \pi \cot\left(\frac{\pi D}{2}\right) \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \right. \\ & \left. + \sum_{n=1}^{\infty} \left[\frac{1}{n} \frac{\Gamma(n+D-1)}{\Gamma(n+\frac{D}{2})} \left(\frac{y}{4}\right)^n - \frac{1}{n-\frac{D}{2}+2} \frac{\Gamma(n+\frac{D}{2}+1)}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right] \right\}. \quad (3.46) \end{aligned}$$

The B -type and C -type propagators are de Sitter invariant and have the following solutions

$$i\Delta_B(x; x') = i\Delta_{\text{cf}}(x; x') - \frac{H^{D-2}}{(4\pi)^{D/2}} \sum_{n=0}^{\infty} \left\{ \frac{\Gamma(n+D-2)}{\Gamma(n+\frac{D}{2})} \left(\frac{y}{4}\right)^n - \frac{\Gamma(n+\frac{D}{2})}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right\}, \quad (3.47)$$

$$i\Delta_C(x; x') = i\Delta_{\text{cf}}(x; x') + \frac{H^{D-2}}{(4\pi)^{D/2}} \sum_{n=0}^{\infty} \left\{ (n+1) \frac{\Gamma(n+D-3)}{\Gamma(n+\frac{D}{2})} \left(\frac{y}{4}\right)^n - \left(n-\frac{D}{2}+3\right) \frac{\Gamma(n+\frac{D}{2}-1)}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right\}. \quad (3.48)$$

The infinite sums vanish in $D = 4$ and each term is a positive power of y , therefore we only need to retain a small number of terms when multiplied by a potentially divergent quantity.

3.3 Computation of the 4-point interactions

In this section we evaluate the contribution from the kinetic 4-point interactions (3.17) and bring the conformal 4-point contribution from our previous work [1]. First note that in the coincidence limit the three scalar propagators become [13]

$$\lim_{x' \rightarrow x} i\Delta_A(x; x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left[-\pi \cot\left(\frac{\pi D}{2}\right) + 2\ln(a) \right], \quad (3.49)$$

$$\lim_{x' \rightarrow x} i\Delta_B(x; x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times -\frac{1}{D-2} \rightarrow -\frac{H^2}{(4\pi)^2}, \quad (3.50)$$

$$\lim_{x' \rightarrow x} i\Delta_C(x; x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times \frac{1}{(D-2)(D-3)} \rightarrow \frac{H^2}{(4\pi)^2}. \quad (3.51)$$

This leads the four contractions of the coincident limit graviton propagator in (3.17) to

$$\begin{aligned} i[\alpha_{\rho} \Delta^{\rho}{}_{\rho}](x; x') &\rightarrow -4\left(\frac{D-1}{D-3}\right) i\Delta_A(x; x') + \frac{H^2}{4\pi^2}, \\ i[\alpha^{\beta} \Delta_{\alpha\beta}](x; x') &\rightarrow \frac{(D-1)(D^2-3D-2)}{(D-3)} i\Delta_A(x; x') - \frac{H^2}{8\pi^2}, \\ i[\alpha_{\rho} \Delta^{\rho\sigma}](x; x') &\rightarrow -\frac{4}{(D-3)} i\Delta_A(x; x') + [2\delta_0^{\rho} \delta_0^{\sigma} + \bar{\eta}^{\rho\sigma}] \frac{H^2}{16\pi^2}, \\ i[\alpha_{\beta} \Delta^{\rho\sigma}](x; x') &\rightarrow \left(\frac{D^2-3D-2}{D-3}\right) \bar{\delta}_{\alpha}^{\sigma} i\Delta_A(x; x') - \delta_0^{\sigma} \delta_{\alpha}^0 \frac{H^2}{8\pi^2}. \end{aligned} \quad (3.52)$$

Next, substituting these relations into the expression (3.17), we obtain

$$\begin{aligned}
-iM_{\text{K4pt}}^2(x; x') &= i\kappa^2 \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \pi \cot\left(\frac{\pi D}{2}\right) \left\{ \left[\frac{1}{4} D(D-1) - D \right] \partial^\mu \partial_\mu \delta^D(x-x') \right\} \\
&\quad - i\kappa^2 \frac{H^2}{4\pi^{\frac{D}{2}}} \left\{ \left[\frac{1}{4} D(D-1) - D \right] \partial^\mu [\ln(a) \partial_\mu] \delta^D(x-x') + \frac{1}{4} \partial_0^2 \delta^D(x-x') \right\} \\
&\quad + \mathcal{O}(D-4). \tag{3.53}
\end{aligned}$$

To evaluate the action of derivatives on the Dirac delta function in (3.53), we note the following identities,

$$\begin{aligned}
\partial_\mu \partial^\mu \delta^D(x-x') &= \left[-\partial_0^2 + \nabla^2 \right] \delta^D(x-x'), \\
\partial^\mu \left[\ln(a) \partial_\mu \right] \delta^D(x-x') \\
&= - \left[H a \partial_0 + \frac{1}{2} \ln(aa') a^2 \square + \frac{(D-2)}{2} \ln(aa') H a \partial_0 - \ln(aa') \nabla^2 \right] \delta^D(x-x'). \tag{3.54}
\end{aligned}$$

The covariant d' Alembertian operator in our background is

$$\square \equiv \frac{1}{\sqrt{-\hat{g}}} \partial_\mu \left(\sqrt{-\hat{g}} \hat{g}^{\mu\nu} \partial_\nu \right) = -\frac{1}{a^2} \partial_0^2 - \frac{(D-2)H}{a} \partial_0 + \frac{1}{a^2} \nabla^2. \tag{3.55}$$

The following properties of the differential operators are also useful.

$$\partial'_i = -\partial_i, \quad \nabla'^2 = \nabla^2, \quad \partial^2 = -\partial_0^2 + \nabla^2. \tag{3.56}$$

By plugging (3.54), (3.55), and (3.56) into the expression (3.53), we obtain

$$\begin{aligned}
-iM_{\text{K4pt}}^2(x; x') &= i\kappa^2 \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left\{ \left[\pi \cot\left(\frac{\pi D}{2}\right) \frac{1}{4} D(D-5) - \frac{1}{4} \ln(aa') \right] H^2 a^{\frac{D}{2}} \square \right. \\
&\quad + \left[\pi \cot\left(\frac{\pi D}{2}\right) (D-2) - \frac{1}{2} - \frac{1}{2} \ln(aa') \right] H^3 a^{\frac{D}{2}-1} \partial_0 \\
&\quad \left. - \left[\pi \cot\left(\frac{\pi D}{2}\right) \frac{1}{4} D(D-5) + \frac{1}{2} \ln(aa') \right] H^2 \nabla^2 \right\} \delta^D(x-x') + \mathcal{O}(D-4). \tag{3.57}
\end{aligned}$$

This coincidence limit expression (3.57) will be used in Section 3.5.

3.4 Computation of the 3-point interactions

In this section we evaluate the contributions from the 3-point kinetic and cross (kinetic-conformal) interactions. Let us remember the form of all the propagators in

terms of the de Sitter invariant function $y(x; x')$ in (3.31),

$$i\Delta_{\text{cf}}(x; x') = \frac{\Gamma\left(\frac{D}{2}-1\right)}{(4\pi)^{\frac{D}{2}}} \left(\frac{4}{\Delta x^2}\right)^{\frac{D}{2}-1} = (aa')^{\frac{D}{2}-1} \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \Gamma\left(\frac{D}{2}-1\right) \left(\frac{4}{y}\right)^{\frac{D}{2}-1} \\ \equiv (aa')^{\frac{D}{2}-1} F(y), \quad (3.58)$$

$$i\Delta_{A,B,C}(x; x') \equiv A(y), B(y), C(y). \quad (3.59)$$

Following the organization of [13], we will split the 3-point interactions into three parts:

(i) *Local contributions* including the delta function;

$$\partial_\mu \partial'_\nu i\Delta_A(x; x') = \delta_\mu^0 \delta_\nu^0 \frac{i}{a^{D-2}} \delta^D(x-x') + A''(y) \partial_\mu y \partial'_\nu y + A'(y) \partial_\mu \partial'_\nu y \quad (3.60)$$

(ii) *logarithm contributions* originated from the factor of $k \ln(aa')$ in the A-type propagator

$$\kappa^2 \partial_i \partial'_i \left[i\Delta_A(x; x') \partial_i \partial'_i i\Delta_{\text{cf}}(x; x') \right], \quad (3.61)$$

(iii) *normal contributions* not having either delta function or $\ln(aa')$.

At this point, it should be noted that the 3-point conformal interactions (that we evaluated in Ref. [1]) do not contain the local and logarithm contributions.

3.4.1 3-point kinetic interactions

Contracting the tensor indices in the graviton propagator, the contribution from the kinetic 3-point interaction vertices (3.22) becomes

$$-iM_{3\text{ptK}}^2(x; x') = \kappa^2 \left\{ -\partial_i \partial'_i [i\Delta_A(x; x') \partial_i \partial'_i i\Delta_{\text{cf}}(x; x')] \right. \\ + C_A \partial_0 \partial'_0 [i\Delta_A(x; x') \partial_0 \partial'_0 i\Delta_{\text{cf}}(x; x')] - \partial_i \partial'_0 [i\Delta_A(x; x') \partial_i \partial'_0 i\Delta_{\text{cf}}(x; x')] \\ - \partial_0 \partial'_i [i\Delta_A(x; x') \partial_0 \partial'_i i\Delta_{\text{cf}}(x; x')] + \partial_0 \partial'_0 [i\Delta_B(x; x') \partial_i \partial'_i i\Delta_{\text{cf}}(x; x')] \\ + \partial_i \partial'_0 [i\Delta_B(x; x') \partial_0 \partial'_i i\Delta_{\text{cf}}(x; x')] + \partial_0 \partial'_i [i\Delta_B(x; x') \partial_i \partial'_0 i\Delta_{\text{cf}}(x; x')] \\ \left. + \partial_i \partial'_i [i\Delta_B(x; x') \partial_0 \partial'_0 i\Delta_{\text{cf}}(x; x')] - C_C \partial_0 \partial'_0 [i\Delta_C(x; x') \partial_0 \partial'_0 i\Delta_{\text{cf}}(x; x')] \right\}. \quad (3.62)$$

Here the coefficients C_A and C_C are

$$C_A \equiv \left(\frac{D-1}{D-3}\right) \quad \text{and} \quad C_C \equiv 2\left(\frac{D-2}{D-3}\right). \quad (3.63)$$

We will break up (3.62) into the three (local, logarithm and normal) parts and examine each of them in the next three subsections.

3.4.1.1 Local contributions for 3-point kinetic part

The local contributions in (3.62) come from the second, eighth, and ninth terms. We apply the relation (3.60) to these terms.

The second term becomes

$$\begin{aligned}
\kappa^2 C_A \partial_0 \partial'_0 [i\Delta_A(x; x') \partial_0 \partial'_0 i\Delta_{\text{cf}}(x; x')] &= \kappa^2 \left(\frac{D-1}{D-3} \right) \partial_0 \partial'_0 [i\Delta_A(x; x') \partial_0 \partial'_0 (aa')^{\frac{D}{2}-1} F(y)] , \\
&= \kappa^2 \left(\frac{D-1}{D-3} \right) \left\{ \partial_0 \partial'_0 [i\Delta_A(x; x') (\partial_0 \partial'_0 (aa')^{\frac{D}{2}-1} F(y))] + \partial_0 \partial'_0 [i\Delta_A(x; x') (\partial'_0 (aa')^{\frac{D}{2}-1} (\partial_0 F(y)))] \right. \\
&\quad \left. + \partial_0 \partial'_0 [i\Delta_A(x; x') (\partial_0 (aa')^{\frac{D}{2}-1} (\partial'_0 F(y)))] + \partial_0 \partial'_0 [(aa')^{\frac{D}{2}-1} i\Delta_A(x; x') (\partial_0 \partial'_0 F(y))] \right\} , \\
&= \kappa^2 \left(\frac{D-1}{D-3} \right) \left\{ \partial_0 \partial'_0 [i\Delta_A(x; x') (\frac{D}{2}-1)^2 H^2 (aa')^{\frac{D}{2}} F(y)] + \partial_0 \partial'_0 [i\Delta_A(x; x') (\frac{D}{2}-1) H (aa')^{\frac{D}{2}-1} \right. \\
&\quad \left. \times [a' \partial_0 + a \partial'_0] F(y)] + \partial_0 \partial'_0 [(aa')^{\frac{D}{2}-1} i\Delta_A(x; x') \partial_0 \partial'_0 F(y)] \right\} , \\
&\rightarrow -i\kappa^2 \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \pi \cot\left(\frac{\pi D}{2}\right) \left(\frac{D-1}{D-3} \right) \left\{ H^2 a^{\frac{D}{2}} \square + (D-2) \left[H^3 a^{\frac{D}{2}-1} \partial_0 \right. \right. \\
&\quad \left. \left. - \frac{(3D-8)}{4} H^4 a^{\frac{D}{2}} \right] - H^2 \nabla^2 \right\} \delta^D(x-x') + \mathcal{O}(D-4) . \tag{3.64}
\end{aligned}$$

Here we used the coincidence limit of the A-type propagator (3.49) and covariant d'Alembertian (3.55).

The eighth term becomes

$$\begin{aligned}
\kappa^2 \partial_i \partial'_i [i\Delta_B(x; x') \partial_0 \partial'_0 i\Delta_{\text{cf}}(x; x')] &= -\kappa^2 \nabla^2 [i\Delta_B(x; x') \partial_0 \partial'_0 (aa')^{\frac{D}{2}-1} F(y)] , \\
&= -\kappa^2 \left\{ \nabla^2 \left[i\Delta_B(x; x') (\partial_0 \partial'_0 (aa')^{\frac{D}{2}-1} F(y)) \right] + \nabla^2 \left[i\Delta_B(x; x') (\partial'_0 (aa')^{\frac{D}{2}-1} (\partial_0 F(y))) \right] \right. \\
&\quad \left. + \nabla^2 \left[i\Delta_B(x; x') (\partial_0 (aa')^{\frac{D}{2}-1} (\partial'_0 F(y))) \right] + \nabla^2 \left[(aa')^{\frac{D}{2}-1} i\Delta_B(x; x') (\partial_0 \partial'_0 F(y)) \right] \right\} , \\
&\rightarrow i\kappa^2 \frac{H^2}{(4\pi)^2} \nabla^2 \delta^4(x-x') + \mathcal{O}(D-4) . \tag{3.65}
\end{aligned}$$

The ninth term becomes

$$\begin{aligned}
& -\kappa^2 C_C \partial_0 \partial'_0 [i\Delta_C(x; x') \partial_0 \partial'_0 i\Delta_{cf}(x; x')] \\
& = -\kappa^2 2 \left(\frac{D-2}{D-3} \right) \left\{ \partial_0 \partial'_0 [i\Delta_C(x; x') (\partial_0 \partial'_0 (aa')^{\frac{D}{2}-1}) F(y)] \right. \\
& \quad + \partial_0 \partial'_0 [i\Delta_C(x; x') (\partial'_0 (aa')^{\frac{D}{2}-1}) (\partial_0 F(y))] + \partial_0 \partial'_0 [i\Delta_C(x; x') (\partial_0 (aa')^{\frac{D}{2}-1}) (\partial'_0 F(y))] \\
& \quad \left. + \partial_0 \partial'_0 [(aa')^{\frac{D}{2}-1} i\Delta_C(x; x') (\partial_0 \partial'_0 F(y))] \right\}, \\
& = -\kappa^2 2 \left(\frac{D-2}{D-3} \right) \left\{ \partial_0 \partial'_0 [i\Delta_C(x; x') \left(\frac{D}{2} - 1 \right)^2 H^2 (aa')^{\frac{D}{2}} F(y)] \right. \\
& \quad + \partial_0 \partial'_0 [i\Delta_C(x; x') \left(\frac{D}{2} - 1 \right) H (aa')^{\frac{D}{2}-1} [a' \partial_0 + a \partial'_0] F(y)] \\
& \quad \left. + \partial_0 \partial'_0 [(aa')^{\frac{D}{2}-1} i\Delta_C(x; x') \partial_0 \partial'_0 F(y)] \right\}, \\
& \rightarrow i\kappa^2 \frac{H^2}{(4\pi)^2} \left\{ -4H^2 a^2 \square - 8Ha\partial_0 + 8H^2 a^2 + 4\nabla^2 \right\} \delta^4(x-x') + \mathcal{O}(D-4). \quad (3.66)
\end{aligned}$$

Summing the three local contributions leads to

$$\begin{aligned}
-iM_{3\text{ptK}}^2(x; x') & = -i\kappa^2 \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left\{ \left[\pi \cot\left(\frac{\pi D}{2}\right) \left(\frac{D-1}{D-3}\right) + 2 \right] H^2 a^{\frac{D}{2}} \square \right. \\
& \quad + \left[\pi \cot\left(\frac{\pi D}{2}\right) \left(\frac{D-1}{D-3}\right) (D-2) + 4 \right] H^3 a^{\frac{D}{2}-1} \partial_0 \\
& \quad - \left[\pi \cot\left(\frac{\pi D}{2}\right) \left(\frac{D-1}{D-3}\right) (D-2) \frac{(3D-8)}{4} + 4 \right] H^4 a^{\frac{D}{2}} \\
& \quad \left. - \left[\pi \cot\left(\frac{\pi D}{2}\right) \left(\frac{D-1}{D-3}\right) + \frac{5}{2} \right] H^2 \nabla^2 \right\} \delta^D(x-x') + \mathcal{O}(D-4). \quad (3.67)
\end{aligned}$$

3.4.1.2 Logarithm contributions for 3-point kinetic part

The logarithm contributions come from the first to fourth terms in (3.62), which contain the $k \ln(aa')$ term in the A-type propagator (3.45). Because all the logarithm contributions are finite we can take the limit $D = 4$.

The first term is

$$\begin{aligned}
& -\kappa^2 \partial_i \partial'_i [i\Delta_A(x; x') \partial_i \partial'_i i\Delta_{cf}(x; x')] \rightarrow -\kappa^2 \partial_i \partial'_i [k \ln(aa') \partial_i \partial'_i (aa')^{\frac{D}{2}-1} F(y)] \\
& = -\kappa^2 k \ln(aa') (aa')^{\frac{D}{2}-1} \nabla^4 F(y), \\
& = -\kappa^2 \frac{H^2}{8\pi^2} \ln(aa') (aa') \nabla^4 F(y) + \mathcal{O}(D-4). \quad (3.68)
\end{aligned}$$

Here the $D = 4$ limit was taken for $k \equiv \frac{H^{D-2}}{(4\pi)^{D/2}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \rightarrow \frac{H^2}{8\pi^2}$.

The second term becomes

$$\begin{aligned}
& \kappa^2 C_A \partial_0 \partial'_0 [i \Delta_A(x; x') \partial_0 \partial'_0 i \Delta_{\text{cf}}(x; x')] \rightarrow \kappa^2 \left(\frac{D-1}{D-3} \right) \partial_0 \partial'_0 [k \ln(aa') \partial_0 \partial'_0 (aa')^{\frac{D}{2}-1} F(y)] \\
& = \kappa^2 \left(\frac{D-1}{D-3} \right) \left\{ \partial_0 \partial'_0 [k \ln(aa') (\partial_0 \partial'_0 (aa')^{\frac{D}{2}-1} F(y))] + \partial_0 \partial'_0 [k \ln(aa') (\partial'_0 (aa')^{\frac{D}{2}-1} (\partial_0 F(y)))] \right. \\
& \quad \left. + \partial_0 \partial'_0 [k \ln(aa') (\partial_0 (aa')^{\frac{D}{2}-1} (\partial'_0 F(y)))] + \partial_0 \partial'_0 [(aa')^{\frac{D}{2}-1} k \ln(aa') \partial_0 \partial'_0 F(y)] \right\}, \\
& = \kappa^2 \frac{3H^2}{8\pi^2} \left\{ H^2 \partial_0 \partial'_0 [\ln(aa') (aa')^2 F(y)] + H \partial_0 \partial'_0 [\ln(aa') (aa') [a' \partial_0 + a \partial'_0] F(y)] \right. \\
& \quad \left. + \partial_0 \partial'_0 [\ln(aa') i \delta^4(x-x')] \right\} + \mathcal{O}(D-4), \\
& = \kappa^2 \frac{H^2}{8\pi^2} \left\{ -[24 + 15 \ln(aa')] H^4 (aa')^3 F(y) + [6 + \frac{9}{2} \ln(aa')] H^2 (aa')^2 \nabla^2 I[F](y) \right\} \\
& \quad + i \kappa^2 \frac{H^2}{8\pi^2} \ln(aa') \left\{ 3 a^2 \square + 6 H a \partial_0 - 6 H^2 a^2 - 3 \nabla^2 \right\} \delta^4(x-x') + \mathcal{O}(D-4). \quad (3.69)
\end{aligned}$$

The third term gives

$$\begin{aligned}
& -\kappa^2 \partial_i \partial'_0 [i \Delta_A(x; x') \partial_i \partial'_0 i \Delta_{\text{cf}}(x; x')] \rightarrow -\kappa^2 \partial_i \partial'_0 [k \ln(aa') \partial_i \partial'_0 (aa')^{\frac{D}{2}-1} F(y)] \\
& = -\kappa^2 \left\{ k \ln(aa') \partial_i \partial'_0 [\partial_i \partial'_0 (aa')^{\frac{D}{2}-1} F(y)] + (\partial_i k \ln(aa')) \partial'_0 [\partial_i \partial'_0 (aa')^{\frac{D}{2}-1} F(y)] \right. \\
& \quad \left. + (\partial'_0 k \ln(aa')) \partial_i [\partial_i \partial'_0 (aa')^{\frac{D}{2}-1} F(y)] \right\}, \\
& = \kappa^2 \frac{H^2}{8\pi^2} \left\{ [1 - 2 \ln(aa')] [H^2 (aa') a'^2 \nabla^2 + H (aa') a' \partial'_0] \right\} F(y) \\
& \quad + i \kappa^2 \frac{H^2}{8\pi^2} \ln(aa') \nabla^2 \delta^4(x-x') + \mathcal{O}(D-4). \quad (3.70)
\end{aligned}$$

Similarly, the fourth term gives

$$\begin{aligned}
& -\kappa^2 \partial_0 \partial'_i [i \Delta_A(x; x') \partial_0 \partial'_i i \Delta_{\text{cf}}(x; x')] \rightarrow -\kappa^2 \partial_0 \partial'_i [k \ln(aa') \partial_0 \partial'_i (aa')^{\frac{D}{2}-1} F(y)] \\
& = -\kappa^2 \left\{ k \ln(aa') \partial_0 \partial'_i [\partial_0 \partial'_i (aa')^{\frac{D}{2}-1} F(y)] + (\partial_0 k \ln(aa')) \partial'_i [\partial_0 \partial'_i (aa')^{\frac{D}{2}-1} F(y)] \right. \\
& \quad \left. + (\partial'_i k \ln(aa')) \partial_0 [\partial_0 \partial'_i (aa')^{\frac{D}{2}-1} F(y)] \right\}, \\
& = \kappa^2 \frac{H^2}{8\pi^2} \left\{ [1 - 2 \ln(aa')] [H^2 (aa') a^2 \nabla^2 + H (aa') a \partial_0] \right\} F(y) \\
& \quad + i \kappa^2 \frac{H^2}{8\pi^2} \ln(aa') \nabla^2 \delta^4(x-x') + \mathcal{O}(D-4). \quad (3.71)
\end{aligned}$$

Collecting all the four logarithm contributions, we have

$$\begin{aligned}
-iM_{\log}^2(x; x') = & \kappa^2 \frac{H^2}{8\pi^2} \left\{ -15 \left[\frac{8}{5} + \ln(aa') \right] H^4 (aa')^3 F(y) \right. \\
& + \frac{9}{2} \left[\frac{4}{3} + \ln(aa') \right] H^2 (aa')^2 \nabla^2 I[F](y) - \left[-\frac{1}{2} + \ln(aa') \right] H^2 (aa')^2 \nabla^2 [6F(y) + 4yF'(y)] \\
& - \left[-\frac{1}{2} + \ln(aa') \right] H^2 (aa') (a^2 + a'^2) \nabla^2 [2F(y) + 2yF'(y)] \\
& + \left[-\frac{1}{2} + \ln(aa') \right] (aa') \nabla^4 I[F](y) - \ln(aa') (aa') \nabla^4 F(y) \\
& \left. + \ln(aa') [3a^2 \square + 6Ha\partial_0 - 6H^2 a^2 - \nabla^2] i\delta^4(x - x') \right\} + \mathcal{O}(D - 4) . \tag{3.72}
\end{aligned}$$

Here we used the following identities derived in Refs. [?, ?]

$$[a'\partial_0 + a\partial'_0]f(y) = -(D-1)Haa'f(y) + \frac{\nabla^2}{2H}I[f](y) , \tag{3.73}$$

$$[a\partial_0 + a'\partial'_0]f(y) = H(a+a')^2 y f'(y) + (D-1)Haa'f(y) - \frac{\nabla^2}{2H}I[f](y) . \tag{3.74}$$

The terms not multiplied by the delta function in Eq. (3.72) (namely, the nonlocal logarithm contributions) are summarized in TABLE 3.1. Later this will be added to TABLE 3.6. In the tables, we write the function $F(y)$ defined in Eq. (3.58) in terms of x , i.e., by rescaling $x \equiv \frac{y}{4}$ and take the $D = 4$ limit:

$$F(y) \rightarrow \frac{H^2}{(4\pi)^2} \frac{1}{x} . \tag{3.75}$$

The last term multiplied by the delta function will give finite local terms after renormalization.

Table 3.1: Nonlocal logarithm contributions from the relation (3.72) with $x \equiv \frac{y}{4}$.

External operators	Coef. of $\frac{\kappa^2 H^4}{(4\pi)^4}$
$(aa')^3 H^4$	$-\frac{48}{x} - \ln(aa') \frac{30}{x}$
$(aa')^2 H^2 \nabla^2$	$12 \ln(x) - \frac{10}{x} + \ln(aa') [9 \ln(x) + \frac{20}{x}]$
$(aa')(a^2 + a'^2) H^2 \nabla^2$	$-\frac{6}{x} + \ln(aa') \frac{12}{x}$
$(aa') \nabla^4$	$-\ln(x) + \ln(aa') [2 \ln(x) - \frac{2}{x}]$

3.4.1.3 Normal contributions for 3-point kinetic part

The remaining terms in the 3-point kinetic part (3.62) are normal contributions: The terms having the A -type propagator are

$$\begin{aligned}
 & -\kappa^2 \partial_i \partial'_i [i\Delta_A(x; x') \partial_i \partial'_i i\Delta_{\text{cf}}(x; x')] = -\kappa^2 (aa')^{\frac{D}{2}-1} \partial_i \partial'_i [A \partial_i \partial'_i F] , \\
 & \kappa^2 C_A \partial_0 \partial'_0 [i\Delta_A(x; x') \partial_0 \partial'_0 i\Delta_{\text{cf}}(x; x')] = \kappa^2 C_A \left\{ \left(\frac{D}{2} - 1\right)^2 H^2 \partial_0 \partial'_0 [(aa')^{\frac{D}{2}} [AF]] \right. \\
 & \quad \left. + \left(\frac{D}{2} - 1\right) H \partial_0 \partial'_0 [(aa')^{\frac{D}{2}-1} [A(a' \partial_0 + a \partial'_0) F]] + \partial_0 \partial'_0 [(aa')^{\frac{D}{2}-1} [A \partial_0 \partial'_0 F]] \right\} ,
 \end{aligned} \tag{3.76}$$

$$\begin{aligned}
 & -\kappa^2 \left\{ \partial_i \partial'_0 [i\Delta_A(x; x') \partial_i \partial'_0 i\Delta_{\text{cf}}(x; x')] + \partial_0 \partial'_i [i\Delta_A(x; x') \partial_0 \partial'_i i\Delta_{\text{cf}}(x; x')] \right\} \\
 & = -\kappa^2 \left\{ \left(\frac{D}{2} - 1\right) a^{\frac{D}{2}-1} H \partial_i \partial'_0 [A a'^{\frac{D}{2}} \partial_i F] + \partial_i \partial'_0 [(aa')^{\frac{D}{2}-1} A \partial_i \partial'_0 F] \right. \\
 & \quad \left. + \left(\frac{D}{2} - 1\right) a'^{\frac{D}{2}-1} H \partial'_i \partial_0 [A a^{\frac{D}{2}} \partial'_i F] + \partial'_i \partial_0 [(aa')^{\frac{D}{2}-1} A \partial'_i \partial_0 F] \right\} .
 \end{aligned} \tag{3.77}$$

The terms including the B -type propagator are

$$\kappa^2 \partial_0 \partial'_0 [i\Delta_B(x; x') \partial_i \partial'_i i\Delta_{cf}(x; x')] = -\kappa^2 \partial_0 \partial'_0 [(aa')^{\frac{D}{2}-1} B \partial_i^2 F] , \quad (3.78)$$

$$\begin{aligned} & \kappa^2 \left\{ \partial_i \partial'_0 [i\Delta_B(x; x') \partial_0 \partial'_i i\Delta_{cf}(x; x')] + \partial_0 \partial'_i [i\Delta_B(x; x') \partial_i \partial'_0 i\Delta_{cf}(x; x')] \right\} \\ &= \kappa^2 \left\{ \partial_i \partial'_0 [(aa')^{\frac{D}{2}-1} (\frac{D}{2} - 1) H a B \partial'_i F] + \partial_i \partial'_0 [(aa')^{\frac{D}{2}-1} B \partial_0 \partial'_i F] \right. \\ & \quad \left. + \partial'_i \partial_0 [(aa')^{\frac{D}{2}-1} (\frac{D}{2} - 1) H a' B \partial'_i F] + \partial'_i \partial_0 [(aa')^{\frac{D}{2}-1} B \partial'_0 \partial'_i F] \right\} , \end{aligned} \quad (3.79)$$

$$\kappa^2 \partial_i \partial'_i [i\Delta_B(x; x') \partial_0 \partial'_0 i\Delta_{cf}(x; x')] = -\kappa^2 \partial_i^2 [B \partial_0 \partial'_0 ((aa')^{\frac{D}{2}-1} F)] . \quad (3.80)$$

Finally the terms containing the C -type propagator are

$$\begin{aligned} & \kappa^2 C_C \partial_0 \partial'_0 [i\Delta_C(x; x') \partial_0 \partial'_0 i\Delta_{cf}(x; x')] = -\kappa^2 C_C \left\{ (\frac{D}{2} - 1)^2 H^2 \partial_0 \partial'_0 [(aa')^{\frac{D}{2}} [CF]] \right. \\ & \quad \left. + (\frac{D}{2} - 1) H \partial_0 \partial'_0 [(aa')^{\frac{D}{2}-1} [C(a' \partial_0 + a \partial'_0) F]] + \partial_0 \partial'_0 [(aa')^{\frac{D}{2}-1} [C \partial_0 \partial'_0 F]] \right\} . \end{aligned} \quad (3.81)$$

To reduce these normal contributions we will apply the following identity

$$\begin{aligned} \partial_0 \partial'_0 [(aa')^n f] &= (aa')^n \left\{ \frac{1}{2} (a + a')^2 [(D-1) y f' + y^2 f''] + \frac{1}{2} (a^2 + a'^2) [\square f + H^2 y f'] \right. \\ & \quad \left. + [n^2 - (D-1)[n - \frac{D-2}{2}]] H^2 a a' f + \frac{1}{2} [n - \frac{D-2}{2}] \nabla^2 I[f] - \nabla^2 f \right\} . \end{aligned} \quad (3.82)$$

Keeping in mind that the self-mass-squared will eventually be integrated in the quantum-corrected field equation (3.1), our goal is to make the integral finite by pulling the derivatives outside the integral. The strategy is to convert primed derivatives into unprimed ones so that we can freely pull them outside the integral. Once this procedure is completed, the self-mass-squared becomes in the form of eleven unprimed external operators acting on the functions of y . Extracting derivatives involves indefinite integrations and we denote this operation by

$$I[f](y) \equiv \int^y dy' f(y') . \quad (3.83)$$

The following example demonstrates the operation [?]:

$$f(y) \left\{ A''(y) \frac{\partial y}{\partial x^\rho} \frac{\partial y}{\partial x'^\sigma} + A'(y) \frac{\partial^2 y}{\partial x^\rho \partial x'^\sigma} \right\} = \partial_\rho \partial'_\sigma I^2[f A''](y) + \frac{\partial^2 y}{\partial x^\rho \partial x'^\sigma} I[f' A'](y) . \quad (3.84)$$

The terms in the left-hand-side are converted to the desired form of derivatives acting on a function of y plus an extra term as a function of y in the right-hand-side. Similar identities were also derived in Refs. [1, 13]

$$f(y)\nabla^2 A(y) = \nabla^2 I^2[fA''](y) + 2(D-1)H^2 aa' I[f'A'](y), \quad (3.85)$$

$$f(y)\partial'_0\partial'_0 A(y) = \partial'_0\partial'_0 I^2[fA''](y) + (\partial'_0\partial'_0 y) I[F'A'](y), \quad (3.86)$$

$$f(y)\partial'_i\partial'_0 A(y) = \partial'_i\partial'_0 I^2[fA''](y) + Ha' \partial'_i I^2[f'A'](y), \quad (3.87)$$

$$f(y)\partial'_i\partial'_j A(y) = \partial'_i\partial'_j I^2[fA''](y) + 2H^2 aa' \eta_{ij} I[f'A'](y). \quad (3.88)$$

Using these identities, all the remaining terms in (3.62) can be converted to functions of y which are acted upon by one of the following eleven external (unprimed) operators

$$\alpha \equiv (aa')^{\frac{D}{2}+1} \square^2, \quad (3.89)$$

$$\beta_1 \equiv (aa')^{\frac{D}{2}+1} H^2 \square, \quad (3.90)$$

$$\beta_2 \equiv (aa')^{\frac{D}{2}} (a^2 + a'^2) H^2 \square, \quad (3.91)$$

$$\gamma_1 \equiv (aa')^{\frac{D}{2}+1} H^4, \quad (3.92)$$

$$\gamma_2 \equiv (aa')^{\frac{D}{2}} (a^2 + a'^2) H^4, \quad (3.93)$$

$$\gamma_3 \equiv (aa')^{\frac{D}{2}} (a + a')^2 H^4 = 2\gamma_1 + \gamma_2, \quad (3.94)$$

$$\delta \equiv (aa')^{\frac{D}{2}-1} (a^2 + a'^2) \nabla^2 \square, \quad (3.95)$$

$$\varepsilon_1 \equiv (aa')^{\frac{D}{2}} H^2 \nabla^2, \quad (3.96)$$

$$\varepsilon_2 \equiv (aa')^{\frac{D}{2}-1} (a^2 + a'^2) H^2 \nabla^2, \quad (3.97)$$

$$\varepsilon_3 \equiv (aa')^{\frac{D}{2}-1} (a + a')^2 H^2 \nabla^2 = 2\varepsilon_1 + \varepsilon_2, \quad (3.98)$$

$$\zeta \equiv (aa')^{\frac{D}{2}-1} \nabla^4. \quad (3.99)$$

For instance, applying the identity (3.85) to the first term (which is the simplest) in (3.62) gives

$$\begin{aligned} -\partial_i\partial'_i[i\Delta_A(x;x') \partial_i\partial'_i i\Delta_{cf}(x;x')] &= -(aa')^{\frac{D}{2}-1} \partial_i\partial'_i[A \partial_i\partial'_i F] = -(aa')^{\frac{D}{2}-1} \nabla^2[A \nabla^2 F], \\ &= -(aa')^{\frac{D}{2}-1} \nabla^2[\nabla^2 I^2[AF''] + 2(D-1)H^2 aa' I[A'F']], \\ &= -(aa')^{\frac{D}{2}-1} [\nabla^4 I^2[AF''] + 2(D-1)H^2 aa' \nabla^2 I[A'F']], \\ &= -(\zeta I^2[AF''] + 2(D-1)\varepsilon_1 I[A'F']). \end{aligned} \quad (3.100)$$

To convert the fifth term (which is the most complicated) in (3.62) we use the identity (3.82),

$$\begin{aligned}
\kappa^2 \partial_0 \partial'_0 [i\Delta_B(x; x') \partial_i \partial'_i i\Delta_{\text{cf}}(x; x')] &= -\kappa^2 \partial_0 \partial'_0 [(aa')^{\frac{D}{2}-1} B \nabla^2 F] , \\
&= -\kappa^2 \partial_0 \partial'_0 [(aa')^{\frac{D}{2}-1} (\nabla^2 I^2 [BF''] + 2(D-1)H^2 aa' I[B'F'])] \\
&= -\kappa^2 \nabla^2 \partial_0 \partial'_0 [(aa')^{\frac{D}{2}-1} I^2 [BF'']] - \kappa^2 2(D-1)H^2 \partial_0 \partial'_0 [(aa')^{\frac{D}{2}} I[B'F']] , \\
&= -\kappa^2 \left\{ \frac{1}{2} \varepsilon_3 [(D-1)y I[BF''] + y^2 [BF'']] + \frac{1}{2} [\delta I^2 [BF''] + \varepsilon_2 y I[BF'']] + \left(\frac{D}{2} - 1\right)^2 \varepsilon_1 I^2 [BF''] \right. \\
&\quad - \zeta I^2 [BF''] + (D-1) \gamma_3 [(D-1)y [B'F'] + y^2 [B'F']'] + (D-1) [\beta_2 I[B'F'] + \gamma_2 y [B'F']'] \\
&\quad \left. + \frac{1}{2} (D-1)(D^2 - 4D + 4) \gamma_1 I[B'F'] + (D-1) \varepsilon_1 I^2 [B'F'] - 2(D-1) \varepsilon_1 I[B'F'] \right\} . \quad (3.101)
\end{aligned}$$

The result is simply written as

$$\begin{aligned}
-iM_{3\text{ptK}}^2(x; x') &= -\kappa^2 \left\{ \alpha f_\alpha(y) + \beta_2 f_{\beta_2}(y) + \gamma_1 f_{\gamma_1}(y) + \gamma_2 f_{\gamma_2}(y) + \gamma_3 f_{\gamma_3}(y) \right. \\
&\quad \left. + \delta f_\delta(y) + \varepsilon_1 f_{\varepsilon_1}(y) + \varepsilon_2 f_{\varepsilon_2}(y) + \varepsilon_3 f_{\varepsilon_3}(y) + \zeta f_\zeta(y) \right\} , \quad (3.102)
\end{aligned}$$

where we give the functions f_i (on which the external operators are acting) in TABLES A.16 through A.25.

3.4.2 3-point cross interactions

By contracting the tensor indices in (3.28), the contribution from the 3-point cross interactions (namely kinetic-conformal terms of the self-mass-squared) becomes

$$\begin{aligned}
-iM_{3\text{ptcross}}^2(x; x') &= \tilde{\kappa}^2 \left\{ C_1 \partial_i [\partial_i i\Delta_{\text{cf}}(x; x') (\nabla^2 i\Delta_A(x; x') + \nabla'^2 i\Delta_A(x; x'))] \right. \\
&\quad - C_1 \partial_i [\partial_i i\Delta_{\text{cf}}(x; x') (\partial_0^2 i\Delta_A(x; x') + \partial_0'^2 i\Delta_A(x; x'))] \\
&\quad + \partial_0 [\partial_0 i\Delta_{\text{cf}}(x; x') \nabla'^2 (-C_2 i\Delta_A(x; x') + C_3 i\Delta_C(x; x'))] \\
&\quad + \partial_0' [\partial_0' i\Delta_{\text{cf}}(x; x') \nabla^2 (-C_2 i\Delta_A(x; x') + C_3 i\Delta_C(x; x'))] \\
&\quad + C_4 \partial_0 [\partial_0 i\Delta_{\text{cf}}(x; x') \partial_0'^2 (i\Delta_A(x; x') - i\Delta_C(x; x'))] \\
&\quad + C_4 \partial_0' [\partial_0' i\Delta_{\text{cf}}(x; x') \partial_0^2 (i\Delta_A(x; x') - i\Delta_C(x; x'))] \\
&\quad + C_1 \partial_0 [\partial_i i\Delta_{\text{cf}}(x; x') \partial'_i \partial'_0 i\Delta_B(x; x')] + C_1 \partial_0' [\partial'_i i\Delta_{\text{cf}}(x; x') \partial_i \partial_0 i\Delta_B(x; x')] \\
&\quad + C_1 \partial_i [\partial_0 i\Delta_{\text{cf}}(x; x') \partial'_i \partial'_0 i\Delta_B(x; x')] + C_1 \partial'_i [\partial_0' i\Delta_{\text{cf}}(x; x') \partial_i \partial_0 i\Delta_B(x; x')] \\
&\quad \left. - C_1 \partial_i [\partial_j i\Delta_{\text{cf}}(x; x') \partial'_i \partial'_j i\Delta_A(x; x')] - C_1 \partial'_i [\partial'_j i\Delta_{\text{cf}}(x; x') \partial_i \partial_j i\Delta_A(x; x')] \right\} . \quad (3.103)
\end{aligned}$$

Here the coefficients are

$$C_1 \equiv 4, C_2 \equiv 4\left(\frac{D-2}{D-3}\right), C_3 \equiv \frac{4}{(D-3)}, C_4 \equiv 4\left(\frac{D-1}{D-3}\right). \quad (3.104)$$

In the following subsections we split this 3-point cross part into the local, logarithm, and normal contributions as we have done for the 3-point kinetic part.

3.4.2.1 Local contributions for 3-point cross part

The fifth and sixth terms of (3.103) produce the local contributions. We will again apply the equation (3.60). The fifth term becomes

$$\begin{aligned} & \tilde{\kappa}^2 C_4 \partial_0 [\partial_0 i\Delta_{\text{cf}}(x; x') \partial_0'^2 (i\Delta_A(x; x') - i\Delta_C(x; x'))] \\ &= \tilde{\kappa}^2 4 \left(\frac{D-1}{D-3}\right) \partial_0 [\partial_0 (aa')^{\frac{D}{2}-1} F(y) \partial_0'^2 (i\Delta_A(x; x') - i\Delta_C(x; x'))], \\ &= \tilde{\kappa}^2 4 \left(\frac{D-1}{D-3}\right) \partial_0 [\partial_0 (aa')^{\frac{D}{2}-1} F(y) \partial_0'^2(0)] = 0. \end{aligned} \quad (3.105)$$

The sixth term becomes

$$\tilde{\kappa}^2 C_4 \partial_0' [\partial_0' i\Delta_{\text{cf}}(x; x') \partial_0'^2 (i\Delta_A(x; x') - i\Delta_C(x; x'))] = 0. \quad (3.106)$$

That is, the local contribution coming from the 3-point cross part is zero.

$$-iM_{\text{3ptcross}}^2(x; x')_{\text{loc}} = 0. \quad (3.107)$$

3.4.2.2 Logarithm contributions for 3-point cross part

The logarithm contributions come from the second, fifth and sixth terms in (3.103). Since all the logarithm contributions are finite, we again take the limit $D = 4$. The second term is

$$\begin{aligned} & -\tilde{\kappa}^2 C_1 \partial_i [\partial_i i\Delta_{\text{cf}}(x; x') (\partial_0^2 i\Delta_A(x; x') + \partial_0'^2 i\Delta_A(x; x'))] \\ & \rightarrow -\tilde{\kappa}^2 4 \partial_i [\partial_i (aa')^{\frac{D}{2}-1} F(y) (\partial_0^2 k \ln(aa') + \partial_0'^2 k \ln(aa'))] \\ & -\tilde{\kappa}^2 4 (aa')^{\frac{D}{2}-1} k \partial_i [(\partial_i F(y)) [\partial_0 H a + \partial_0' H a']] = -\tilde{\kappa}^2 \frac{H^2}{8\pi^2} 4H^2(a^2 + a'^2)(aa') \nabla^2 F(y). \end{aligned} \quad (3.108)$$

The fifth term is

$$\begin{aligned}
& \tilde{\kappa}^2 C_4 \partial_0 [\partial_0 i\Delta_{\text{cf}}(x; x') \partial_0'^2 (i\Delta_A(x; x') - i\Delta_C(x; x'))] \\
& \rightarrow \tilde{\kappa}^2 4 \left(\frac{D-1}{D-3} \right) \partial_0 \left[\partial_0 (aa')^{\frac{D}{2}-1} F(y) \partial_0'^2 (i\Delta_A(x; x') - i\Delta_C(x; x')) \right] \\
& = \tilde{\kappa}^2 4 \left(\frac{D-1}{D-3} \right) \partial_0 \left[\left[\left(\frac{D}{2} - 1 \right) H a (aa')^{\frac{D}{2}-1} F(y) + (aa')^{\frac{D}{2}-1} \partial_0 F(y) \right] \partial_0'^2 (k \ln(aa')) \right], \\
& = \tilde{\kappa}^2 \frac{3H^2}{2\pi^2} [2H^4 (aa')^3 + 2(aa')^2 H^3 a \partial_0 + (aa') H^2 a^2] F(y). \tag{3.109}
\end{aligned}$$

Here note that only $i\Delta_A(x; x')$ has the logarithm piece, thus we can take only $i\Delta_A(x; x')$.

Similarly, the sixth term becomes

$$\begin{aligned}
& \tilde{\kappa}^2 C_4 \partial_0' [\partial_0' i\Delta_{\text{cf}}(x; x') \partial_0'^2 (i\Delta_A(x; x') - i\Delta_C(x; x'))] \\
& = \tilde{\kappa}^2 \frac{3H^2}{2\pi^2} [2H^4 (aa')^3 + 2(aa')^2 H^3 a' \partial_0' + (aa') H^2 a'^2] F(y). \tag{3.110}
\end{aligned}$$

Summing these three logarithm contributions results in

$$\begin{aligned}
-iM_{\text{ptcross}}^2(x; x')_{\log} &= \tilde{\kappa}^2 \frac{H^2}{8\pi^2} \left\{ 24H^4 (aa')^3 [5F(y) + 2yF'(y)] + 24H^4 (aa')^2 (a^2 + a'^2) yF'(y) \right. \\
&\quad \left. - 12H^2 (aa')^2 \nabla^2 I[F](y) - 4H^2 (aa') (a^2 + a'^2) \nabla^2 F(y) \right\}. \tag{3.111}
\end{aligned}$$

Note that this result includes only nonlocal (not having delta function) logarithm pieces. TABLE 3.2 summarizes the result and the nonlocal logarithm contributions will be added to TABLE 3.10 later.

Table 3.2: Nonlocal logarithm contributions from the relation (3.111) with $x \equiv \frac{y}{4}$.

External operators	Coef. of $\frac{\tilde{\kappa}^2 H^4}{(4\pi)^4}$
$(aa')^3 H^4$	$-\frac{144}{x}$
$(aa')^2 (a^2 + a'^2) H^4$	$-\frac{192}{x}$
$(aa')^2 H^2 \nabla^2$	$-24 \ln(x)$
$(aa') (a^2 + a'^2) H^2 \nabla^2$	$-\frac{8}{x}$

3.4.2.3 Normal contributions for 3-point cross part

Every term in the 3-point cross part in (3.28) generates normal contributions. The first term in (3.28) is

$$\begin{aligned}
& 2\tilde{\kappa}^2 \partial'_\mu \left[\partial'_\nu i\Delta_{\text{cf}}(x; x') \partial_\rho \partial^\rho i[\mu^\nu \Delta^\gamma_\gamma](x; x') \right] \\
&= \tilde{\kappa}^2 \left\{ -\frac{8}{(D-3)} \partial'_i [\partial'_i i\Delta_{\text{cf}}(x; x') \partial^2 i\Delta_A(x; x')] + \frac{8}{(D-2)} \partial'_0 [\partial'_0 i\Delta_{\text{cf}}(x; x') \partial^2 i\Delta_C(x; x')] \right. \\
&\quad \left. + \frac{8}{(D-3)(D-2)} \partial'_i [\partial'_i i\Delta_{\text{cf}}(x; x') \partial^2 i\Delta_C(x; x')] \right\}, \\
&= \tilde{\kappa}^2 \left\{ \frac{8}{(D-3)} \partial'_i [\partial'_i i\Delta_{\text{cf}}(x; x') \partial_0^2 i\Delta_A(x; x')] - \frac{8}{(D-3)} \partial'_i [\partial'_i i\Delta_{\text{cf}}(x; x') \nabla^2 i\Delta_A(x; x')] \right. \\
&\quad - \frac{8}{(D-2)} \partial'_0 [\partial'_0 i\Delta_{\text{cf}}(x; x') \partial_0^2 i\Delta_C(x; x')] + \frac{8}{(D-2)} \partial'_0 [\partial'_0 i\Delta_{\text{cf}}(x; x') \nabla^2 i\Delta_C(x; x')] \\
&\quad - \frac{8}{(D-3)(D-2)} \partial'_i [\partial'_i i\Delta_{\text{cf}}(x; x') \partial_0^2 i\Delta_C(x; x')] \\
&\quad \left. + \frac{8}{(D-3)(D-2)} \partial'_i [\partial'_i i\Delta_{\text{cf}}(x; x') \nabla^2 i\Delta_C(x; x')] \right\}. \tag{3.112}
\end{aligned}$$

The second term in (3.28) is

$$\begin{aligned}
& -\tilde{\kappa}^2 \partial'_\mu \left[\partial'^\mu i\Delta_{\text{cf}}(x; x') \partial_\rho \partial^\rho i[\alpha_\alpha \Delta^\gamma_\gamma](x; x') \right] \\
&= \tilde{\kappa}^2 \left\{ 4\left(\frac{D-1}{D-3}\right) \partial'_i [\partial'_i i\Delta_{\text{cf}}(x; x') \partial^2 i\Delta_A(x; x')] - 4\left(\frac{D-1}{D-3}\right) \partial'_0 [\partial'_0 i\Delta_{\text{cf}}(x; x') \partial^2 i\Delta_A(x; x')] \right. \\
&\quad - \left(\frac{8}{(D-3)(D-2)}\right) \partial'_i [\partial'_i i\Delta_{\text{cf}}(x; x') \partial^2 i\Delta_C(x; x')] \\
&\quad \left. + \left(\frac{8}{(D-3)(D-2)}\right) \partial'_0 [\partial'_0 i\Delta_{\text{cf}}(x; x') \partial^2 i\Delta_C(x; x')] \right\}, \\
&= \tilde{\kappa}^2 \left\{ -4\left(\frac{D-1}{D-3}\right) \partial'_i [\partial'_i i\Delta_{\text{cf}}(x; x') \partial_0^2 i\Delta_A(x; x')] + 4\left(\frac{D-1}{D-3}\right) \partial'_i [\partial'_i i\Delta_{\text{cf}}(x; x') \nabla^2 i\Delta_A(x; x')] \right. \\
&\quad + 4\left(\frac{D-1}{D-3}\right) \partial'_0 [\partial'_0 i\Delta_{\text{cf}}(x; x') \partial_0^2 i\Delta_A(x; x')] - 4\left(\frac{D-1}{D-3}\right) \partial'_0 [\partial'_0 i\Delta_{\text{cf}}(x; x') \nabla^2 i\Delta_A(x; x')] \\
&\quad + \left(\frac{8}{(D-3)(D-2)}\right) \partial'_i [\partial'_i i\Delta_{\text{cf}}(x; x') \partial_0^2 i\Delta_C(x; x')] \\
&\quad - \left(\frac{8}{(D-3)(D-2)}\right) \partial'_i [\partial'_i i\Delta_{\text{cf}}(x; x') \nabla^2 i\Delta_C(x; x')] \\
&\quad - \left(\frac{8}{(D-3)(D-2)}\right) \partial'_0 [\partial'_0 i\Delta_{\text{cf}}(x; x') \partial_0^2 i\Delta_C(x; x')] \\
&\quad \left. + \left(\frac{8}{(D-3)(D-2)}\right) \partial'_0 [\partial'_0 i\Delta_{\text{cf}}(x; x') \nabla^2 i\Delta_C(x; x')] \right\}. \tag{3.113}
\end{aligned}$$

The third term in (3.28) is

$$\begin{aligned}
& -2\tilde{\kappa}^2\partial'_\mu\left[\partial'_\nu i\Delta_{\text{cf}}(x;x')\partial_\rho\partial_\sigma i^{[\mu\nu}\Delta^{\rho\sigma]}(x;x')\right] \\
& = \tilde{\kappa}^2\left\{-4\partial'_i[\partial'_j i\Delta_{\text{cf}}(x;x')\partial_i\partial_j i\Delta_A(x;x')] + \frac{4}{(D-3)}\partial'_i[\partial'_i i\Delta_{\text{cf}}(x;x')\nabla^2 i\Delta_A(x;x')] \right. \\
& \quad + 4\partial'_i[\partial'_0 i\Delta_{\text{cf}}(x;x')\partial_0\partial_i i\Delta_B(x;x')] + 4\partial'_0[\partial'_i i\Delta_{\text{cf}}(x;x')\partial_0\partial_i i\Delta_B(x;x')] \\
& \quad - \frac{4}{(D-2)}\partial'_i[\partial'_i i\Delta_{\text{cf}}(x;x')\partial_0^2 i\Delta_C(x;x')] - \frac{4}{(D-3)(D-2)}\partial'_i[\partial'_i i\Delta_{\text{cf}}(x;x')\nabla^2 i\Delta_C(x;x')] \\
& \quad \left. - 4\left(\frac{D-3}{D-2}\right)\partial'_0[\partial'_0 i\Delta_{\text{cf}}(x;x')\partial_0^2 i\Delta_C(x;x')] - \frac{4}{(D-2)}\partial'_0[\partial'_0 i\Delta_{\text{cf}}(x;x')\nabla^2 i\Delta_C(x;x')] \right\}. \tag{3.114}
\end{aligned}$$

The fourth term in (3.28) is

$$\begin{aligned}
& \tilde{\kappa}^2\partial'_\mu\left[\partial'^\mu i\Delta_{\text{cf}}(x;x')\partial_\rho\partial_\sigma i^{[\alpha}{}_\alpha\Delta^{\rho\sigma]}(x;x')\right] \\
& = \tilde{\kappa}^2\left\{-\frac{4}{(D-3)}\partial'_i[\partial'_i i\Delta_{\text{cf}}(x;x')\nabla^2 i\Delta_A(x;x')] + \frac{4}{(D-3)}\partial'_0[\partial'_0 i\Delta_{\text{cf}}(x;x')\nabla^2 i\Delta_A(x;x')] \right. \\
& \quad + \frac{4}{(D-3)(D-2)}\partial'_i[\partial'_i i\Delta_{\text{cf}}(x;x')\nabla^2 i\Delta_C(x;x')] + \frac{4}{(D-2)}\partial'_i[\partial'_i i\Delta_{\text{cf}}(x;x')\partial_0^2 i\Delta_C(x;x')] \\
& \quad \left. - \frac{4}{(D-3)(D-2)}\partial'_0[\partial'_0 i\Delta_{\text{cf}}(x;x')\nabla^2 i\Delta_C(x;x')] - \frac{4}{(D-2)}\partial'_0[\partial'_0 i\Delta_{\text{cf}}(x;x')\partial_0^2 i\Delta_C(x;x')] \right\}. \tag{3.115}
\end{aligned}$$

The fifth term in (3.28) is,

$$\begin{aligned}
& 2\tilde{\kappa}^2\partial'_\mu\left[\partial'_\nu i\Delta_{\text{cf}}(x;x')\partial'_\rho\partial'^\rho i^{[\mu\nu}\Delta^\gamma{}_\gamma](x;x')\right] \\
& = \tilde{\kappa}^2\left\{\frac{8}{(D-3)}\partial_i[\partial_i i\Delta_{\text{cf}}(x;x')\partial_0'^2 i\Delta_A(x;x')] - \frac{8}{(D-3)}\partial_i[\partial_i i\Delta_{\text{cf}}(x;x')\nabla'^2 i\Delta_A(x;x')] \right. \\
& \quad - \frac{8}{(D-2)}\partial_0[\partial_0 i\Delta_{\text{cf}}(x;x')\partial_0'^2 i\Delta_C(x;x')] + \frac{8}{(D-2)}\partial_0[\partial_0 i\Delta_{\text{cf}}(x;x')\nabla'^2 i\Delta_C(x;x')] \\
& \quad - \frac{8}{(D-3)(D-2)}\partial_i[\partial_i i\Delta_{\text{cf}}(x;x')\partial_0'^2 i\Delta_C(x;x')] \\
& \quad \left. + \frac{8}{(D-3)(D-2)}\partial_i[\partial_i i\Delta_{\text{cf}}(x;x')\nabla'^2 i\Delta_C(x;x')] \right\}. \tag{3.116}
\end{aligned}$$

The sixth term in (3.28) is

$$\begin{aligned}
& -\tilde{\kappa}^2 \partial_\mu \left[\partial^\mu i\Delta_{\text{cf}}(x; x') \partial'_\rho \partial'^\rho i[\alpha_\alpha \Delta^\gamma_\gamma](x; x') \right] \\
& = \tilde{\kappa}^2 \left\{ -4 \left(\frac{D-1}{D-3} \right) \partial_i [\partial_i i\Delta_{\text{cf}}(x; x') \partial_0'^2 i\Delta_A(x; x')] + 4 \left(\frac{D-1}{D-3} \right) \partial_i [\partial_i i\Delta_{\text{cf}}(x; x') \nabla'^2 i\Delta_A(x; x')] \right. \\
& \quad + 4 \left(\frac{D-1}{D-3} \right) \partial_0 [\partial_0 i\Delta_{\text{cf}}(x; x') \partial_0'^2 i\Delta_A(x; x')] - 4 \left(\frac{D-1}{D-3} \right) \partial_0 [\partial_0 i\Delta_{\text{cf}}(x; x') \nabla'^2 i\Delta_A(x; x')] \\
& \quad + \left(\frac{8}{(D-3)(D-2)} \right) \partial_i [\partial_i i\Delta_{\text{cf}}(x; x') \partial_0'^2 i\Delta_C(x; x')] \\
& \quad - \left(\frac{8}{(D-3)(D-2)} \right) \partial_i [\partial_i i\Delta_{\text{cf}}(x; x') \nabla'^2 i\Delta_C(x; x')] \\
& \quad - \left(\frac{8}{(D-3)(D-2)} \right) \partial_0 [\partial_0 i\Delta_{\text{cf}}(x; x') \partial_0'^2 i\Delta_C(x; x')] \\
& \quad \left. + \left(\frac{8}{(D-3)(D-2)} \right) \partial_0 [\partial_0 i\Delta_{\text{cf}}(x; x') \nabla'^2 i\Delta_C(x; x')] \right\}. \tag{3.117}
\end{aligned}$$

The seventh term in (3.28) is

$$\begin{aligned}
& -2\tilde{\kappa}^2 \partial_\mu \left[\partial_\nu i\Delta_{\text{cf}}(x; x') \partial'_\rho \partial'_\sigma i[\mu^\nu \Delta^{\rho\sigma}](x; x') \right] \\
& = \tilde{\kappa}^2 \left\{ -4 \partial_i [\partial_j i\Delta_{\text{cf}}(x; x') \partial'_i \partial'_j i\Delta_A(x; x')] + \frac{4}{(D-3)} \partial_i [\partial_i i\Delta_{\text{cf}}(x; x') \nabla'^2 i\Delta_A(x; x')] \right. \\
& \quad + 4 \partial_i [\partial_0 i\Delta_{\text{cf}}(x; x') \partial'_0 \partial'_i i\Delta_B(x; x')] + 4 \partial_0 [\partial_i i\Delta_{\text{cf}}(x; x') \partial'_0 \partial'_i i\Delta_B(x; x')] \\
& \quad - \frac{4}{(D-2)} \partial_i [\partial_i i\Delta_{\text{cf}}(x; x') \partial_0'^2 i\Delta_C(x; x')] - \frac{4}{(D-3)(D-2)} \partial_i [\partial_i i\Delta_{\text{cf}}(x; x') \nabla'^2 i\Delta_C(x; x')] \\
& \quad \left. - 4 \left(\frac{D-3}{D-2} \right) \partial_0 [\partial_0 i\Delta_{\text{cf}}(x; x') \partial_0'^2 i\Delta_C(x; x')] - \frac{4}{(D-2)} \partial_0 [\partial_0 i\Delta_{\text{cf}}(x; x') \nabla'^2 i\Delta_C(x; x')] \right\}. \tag{3.118}
\end{aligned}$$

Finally, the eighth term in (3.28) is

$$\begin{aligned}
& \tilde{\kappa}^2 \partial_\mu \left[\partial^\mu i\Delta_{\text{cf}}(x; x') \partial'_\rho \partial'_\sigma i[\alpha_\alpha \Delta^{\rho\sigma}](x; x') \right] \\
& = \tilde{\kappa}^2 \left\{ -\frac{4}{(D-3)} \partial_i [\partial_i i\Delta_{\text{cf}}(x; x') \nabla'^2 i\Delta_A(x; x')] + \frac{4}{(D-3)} \partial_0 [\partial_0 i\Delta_{\text{cf}}(x; x') \nabla'^2 i\Delta_A(x; x')] \right. \\
& \quad + \frac{4}{(D-3)(D-2)} \partial_i [\partial_i i\Delta_{\text{cf}}(x; x') \nabla'^2 i\Delta_C(x; x')] + \frac{4}{(D-2)} \partial_i [\partial_i i\Delta_{\text{cf}}(x; x') \partial_0'^2 i\Delta_C(x; x')] \\
& \quad \left. - \frac{4}{(D-3)(D-2)} \partial_0 [\partial_0 i\Delta_{\text{cf}}(x; x') \nabla'^2 i\Delta_C(x; x')] - \frac{4}{(D-2)} \partial_0 [\partial_0 i\Delta_{\text{cf}}(x; x') \partial_0'^2 i\Delta_C(x; x')] \right\}. \tag{3.119}
\end{aligned}$$

We aim at putting these normal contributions in the form of unprimed derivatives acting on the functions of y . In our computation three derivatives, spatial and temporal, acting

on functions $A(y), B(y), C(y)$ arise, which requires an additional identity as follows,

$$F(y)\partial_\alpha\partial'_\rho\partial'_\sigma A(y) = \partial_\alpha\partial'_\rho\partial'_\sigma\left\{I^3[FA'''](y)\right\} + \frac{\partial^3 y}{\partial x^\alpha\partial x'^\rho\partial x'^\sigma}\left\{I[F'A'](y) + I^2[F'A''](y)\right\} \\ + \left(\frac{\partial y}{\partial x^\alpha}\frac{\partial^2 y}{\partial x'^\rho\partial x'^\sigma} + 2\frac{\partial y}{\partial x'^{(\rho}}\frac{\partial^2 y}{\partial x'^{\sigma)}\partial x^\alpha}\right)\left\{I[F'A''](y)\right\}. \quad (3.120)$$

Applying this identity to the first term of (3.103), which is the simplest because all of the derivatives are spatial, i.e., in the form of $F(y)\partial_i\nabla'^2 A(y)$,

$$2C_1 \tilde{\kappa}^2 \partial_i [\partial_i i\Delta_{cf}(x; x') \nabla^2 i\Delta_A(x; x')] = 2C_1 \tilde{\kappa}^2 \partial_i [\partial_i (aa')^{\frac{D}{2}-1} F(y) \nabla^2 A(y)] , \\ = 8 \tilde{\kappa}^2 (aa')^{\frac{D}{2}-1} \left(\partial_i \partial_i [F(y) \nabla^2 A(y)] - \partial_i [F(y) \partial_i \nabla^2 A(y)] \right). \quad (3.121)$$

Next, applying the identity (3.120) to the second term of the last expression (3.121) gives

$$F(y)\partial_i\nabla'^2 A(y) = \partial_i\nabla'^2\left\{I^3[FA'''](y)\right\} + \frac{\partial^3 y}{\partial x^i\partial x'^j\partial x'^j}\left\{I[F'A'](y) + I^2[F'A''](y)\right\} \\ + \left(\frac{\partial y}{\partial x^i}\frac{\partial^2 y}{\partial x'^j\partial x'^j} + 2\frac{\partial y}{\partial x'^{(j}}\frac{\partial^2 y}{\partial x'^{j)}\partial x^i}\right)\left\{I[F'A''](y)\right\}. \quad (3.122)$$

Further using the following derivatives of y with respect to x^i or x'^j ,

$$\frac{\partial y}{\partial x^i} = 2H^2 aa' \Delta x_i, \quad \frac{\partial y}{\partial x'^j} = -2H^2 aa' \Delta x_j, \quad \frac{\partial^2 y}{\partial x^i \partial x^j} = 2H^2 aa' \eta_{ij}, \\ \frac{\partial^2 y}{\partial x^i \partial x'^j} = -2H^2 aa' \eta_{ij}, \quad \frac{\partial^2 y}{\partial x^i \partial x^i} = \frac{\partial^2 y}{\partial x'^j \partial x'^j} = 2H^2 aa' (D-1), \\ \frac{\partial^3 y}{\partial x^i \partial x^i \partial x'^j} = \frac{\partial^3 y}{\partial x^i \partial x'^j \partial x'^j} = \frac{\partial^4 y}{\partial x^i \partial x^i \partial x'^j \partial x'^j} = 0, \quad (3.123)$$

leads Eq. (3.122) to

$$F(y)\partial_i\nabla'^2 A(y) = \partial_i\nabla'^2\left\{I^3[FA'''](y)\right\} \\ + \left(4H^4(aa')^2\Delta x_i(D-1) + 8H^4(aa')^2\Delta x_j\eta_{ij}\right)\left\{I[F'A''](y)\right\}. \quad (3.124)$$

Next, plugging the identity (3.85) and the expression (3.124) into Eq. (3.121) gives

$$\begin{aligned}
& 8 \tilde{\kappa}^2 \partial_i [\partial_i (aa')^{\frac{D}{2}-1} F(y) \nabla^2 A(y)] \\
&= 8 \tilde{\kappa}^2 (aa')^{\frac{D}{2}-1} \left(\partial_i \partial_i [\nabla^2 I^2 [FA'']](y) \right. \\
&\quad + 2(D-1)H^2 aa' I[F'A'](y) - \partial_i \left(\partial_i \nabla^2 \left\{ I^3 [FA'''](y) \right\} + \left(4H^4 (aa')^2 \Delta x_i (D-1) \right. \right. \\
&\quad \left. \left. + 8H^4 (aa')^2 \Delta x_j \eta_{ij} \right) \left\{ I[F'A''](y) \right\} \right) \Bigg), \\
&= 8 \tilde{\kappa}^2 (aa')^{\frac{D}{2}-1} \left(\nabla^4 I^2 [FA''](y) + 2(D-1)H^2 aa' \nabla^2 I[F'A'](y) - \nabla^4 I^3 [FA'''](y) \right. \\
&\quad \left. - 4(D-1)(D+1)H^4 (aa')^2 I[F'A''](y) - 8(D+1)H^6 (aa')^3 \|\Delta \vec{x}\|^2 [F'A''](y) \right). \tag{3.125}
\end{aligned}$$

We use the following identity

$$aa' H^2 \|\Delta \vec{x}\|^2 f(y) = -\frac{1}{2}(D-1)I[f](y) - \frac{\nabla \cdot \nabla'}{4aa'H^2} I^2[f](y). \tag{3.126}$$

to obtain

$$\begin{aligned}
& 8 \tilde{\kappa}^2 \partial_i [\partial_i (aa')^{\frac{D}{2}-1} F(y) \nabla^2 A(y)] = 8 \tilde{\kappa}^2 (aa')^{\frac{D}{2}-1} \\
&\quad \times \left(\nabla^4 I^2 [FA''](y) + 2(D-1)H^2 aa' \nabla^2 I[F'A'](y) - \nabla^4 I^3 [FA'''](y) \right. \\
&\quad \left. - 2(D+1)H^2 aa' \nabla^2 I^2 [F'A''](y) \right), \tag{3.127} \\
&= -\tilde{\kappa}^2 \left\{ \epsilon_1 16 \left((D+1)I^2 [F'A''](y) - (D-1)I[F'A'](y) \right) \right. \\
&\quad \left. + \zeta 8 \left(I^3 [FA'''](y) - I^2 [FA''](y) \right) \right\}. \tag{3.128}
\end{aligned}$$

The results including this pure spatial derivative, and also the pure temporal and temporal spatial mixed derivatives in equation (3.103) are tabulated in TABLES 3.7, 3.8, 3.9, 3.10, in the form of the eleven operators (3.89-3.99) acting on functions of y . Finally, by adding all the 3-point cross contributions coming from $A(y)$, $B(y)$ and $C(y)$ in (3.103), we have

$$\begin{aligned}
-iM_{3\text{ptcross}}^2(x; x') = & -\tilde{\kappa}^2 \left\{ \alpha f_\alpha(y) + \beta_1 f_{\beta_1}(y) + \beta_2 f_{\beta_2}(y) + \gamma_1 f_{\gamma_1}(y) + \gamma_2 f_{\gamma_2}(y) \right. \\
& \left. + \gamma_3 f_{\gamma_3}(y) + \delta f_\delta(y) + \epsilon_1 f_{\epsilon_1}(y) + \epsilon_2 f_{\epsilon_2}(y) + \epsilon_3 f_{\epsilon_3}(y) + \zeta f_\zeta(y) \right\}. \tag{3.129}
\end{aligned}$$

Here we give the functions f_i (on which the eleven external operators are acting) in TABLEs A.26 through A.36 in Appendix A.2.

3.5 Renormalization

In this section we absorb the divergences occurred in the primitive diagrams (the first two terms in Eq. (3.13)) by counterterms (the third term in Eq. (3.13)) and obtain a finite result for the scalar self-mass-squared. First we recall the appropriate counterterms for our Lagrangian (3.3), which we constructed by applying the theorem of Bogoliubov, Parasiuk, Hepp and Zimmermann (BPHZ) [122–125] in our previous work [1]. Since these BPHZ counterterm vertices are local (in the form of δ -function), the next task is to segregate all the divergent terms into a δ -function. The localized divergences can then be subtracted by the local counterterms. We perform this procedure of renormalization in the following subsections.

3.5.1 Allowed counterterms and the associated vertices

First of all, the structure of our Lagrangian (3.3) guides us to identify three de Sitter invariant counterterms at one loop order as

$$\Delta\mathcal{L}_1 = \frac{1}{2}c_1\kappa^2\Box\Box\phi a^2, \Delta\mathcal{L}_2 = -\frac{1}{2}c_2\kappa^2 H^2\partial_\mu\phi\partial^\mu\phi, \Delta\mathcal{L}_3 = \frac{1}{2}c_3\kappa^2 H^4\phi^2 a^2 \quad (3.130)$$

See Ref. [1] for the detailed derivation of these counterterms. The associated vertices for the counterterms are obtained by taking the same variation performed for the primitive terms:

$$\left. \frac{i\delta\Delta S_1}{\delta\phi(x)\delta\phi(x')} \right|_{\phi=0} = ic_1\kappa^2 \left[a^2\Box^2 + (D-2)H^2 a^2\Box + 2(D-2)Ha\partial_0\Box \right] \delta^D(x-x'), \quad (3.131)$$

$$\left. \frac{i\delta\Delta S_2}{\delta\phi(x)\delta\phi(x')} \right|_{\phi=0} = ic_2\kappa^2 H^2 a^2\Box \delta^D(x-x'), \quad (3.132)$$

$$\left. \frac{i\delta\Delta S_3}{\delta\phi(x)\delta\phi(x')} \right|_{\phi=0} = ic_3\kappa^2 H^4 a^2 \delta^D(x-x'). \quad (3.133)$$

In the first vertex (3.131) we correct the mistake in Eq. (94) of Ref. [1] (also in Eq. (130) of Ref. [13]), where the last two terms are missing. This mistake did not affect our previous results because the divergences in the forms of $a^2\Box^2\delta^D(x-x')$ and $Ha\partial_0\Box\delta^D(x-x')$ did not occur in the cases of Refs. [1, 13]. These two forms of

divergences only arise in the 3-point kinetic-conformal interactions, which is the main computation of the present paper. Moreover, as will be shown in the next subsection, these two divergences exactly match the relative coefficients of the first and third terms of Eq. (3.131) and thus get cancelled simultaneously, which serves as a nontrivial check for the correctness of our computation.

On the other hand, our gauge fixing Lagrangian (3.33) breaks the spatial special conformal symmetry among the full de Sitter symmetries and leads to one noninvariant counterterm [1]

$$\Delta\mathcal{L}_4 = -\frac{1}{2}\kappa^2 H^2 \nabla\phi \cdot \nabla\phi . \quad (3.134)$$

It should be noted that this noninvariant counterterm still respects the residual de Sitter symmetries (homogeneity, isotropy and dilation symmetry) and it also becomes Poincaré invariant in flat space [1]. These conditions indeed highly restrict the form of counterterms and leave only one possibility given in Eq. (3.134). The associated vertex to this noninvariant counterterm is

$$\left. \frac{i\delta\Delta S_4}{\delta\phi(x)\delta\phi(x')} \right|_{\phi=0} = ic_4 \kappa^2 H^2 \nabla^2 \delta^D(x-x') . \quad (3.135)$$

These four counterterms are the only possible ones for the CC scalar self-mass-squared at one loop order allowed by the symmetries of the bare Lagrangian for a scalar conformally coupled to gravity (3.3) and the gauge-fixing term (3.33). In fact, all the terms in the self-mass-squared must respect the same symmetries, which implies that any divergent term can be cancelled by one of these four counterterms. In other words, any divergence that cannot be cancelled by these counterterms means an error in our computation. Indeed this has served us as a crucial error-detecting method in this heavy calculation.

3.5.2 Localization of divergences

We now localize the divergent terms so as to put them in the form of the counterterm vertices. Keeping in mind that the terms in the self-mass-squared will eventually be integrated in the effective field equation (3.1), the first step is to make the powers of y ,

$$\left(\frac{4}{y}\right)^D, \left(\frac{4}{y}\right)^{D-1} \quad \text{and} \quad \left(\frac{4}{y}\right)^{D-2}, \quad (3.136)$$

integrable in $D = 4$ dimensions. This can be done by extracting d'Alembertian operators using the following identity,

$$\square f(y) = H^2 \left[(4y - y^2) f''(y) + D(2 - y) f'(y) \right] + \text{Res} \left[y^{\frac{D}{2}-2} f \right] \frac{4\pi^{D/2} H^{2-D}}{\Gamma(\frac{D}{2}-1)} \frac{i}{\sqrt{-g}} \delta^D(x - x'). \quad (3.137)$$

Here $\text{Res}[F]$ means the residue of $F(y)$. By applying this identity (3.137) and rearranging the terms, we can write these powers of y as

$$\left(\frac{4}{y}\right)^D = \frac{2}{(D-1)D H^2} \square \left(\frac{4}{y}\right)^{D-1}, \quad (3.138)$$

$$\left(\frac{4}{y}\right)^{D-1} = \frac{2}{(D-2)^2 H^2} \square \left(\frac{4}{y}\right)^{D-2} - \frac{2}{D-2} \left(\frac{4}{y}\right)^{D-2}, \quad (3.139)$$

$$\left(\frac{4}{y}\right)^{D-2} = \frac{2}{(D-4)(D-3) H^2} \square \left(\frac{4}{y}\right)^{D-3} - \frac{4}{D-4} \left(\frac{4}{y}\right)^{D-3}. \quad (3.140)$$

Note that extracting d'Alembertians subsequently transfers a quartically divergent power $1/y^D$ into a quadratically divergent $1/y^{D-1}$, a logarithmically divergent $1/y^{D-2}$ and finally into an integrable power of $1/y^{D-3}$. A new problem is that its coefficient has a divergent factor $1/(D-4)$. To take care of this divergent factor, we apply the identity (3.137) to the power of $1/y^{D/2-1}$ which leads to an expression of zero:

$$0 = \frac{\square}{H^2} \left(\frac{4}{y}\right)^{\frac{D}{2}-1} - \frac{D}{2} \left(\frac{D}{2}-1\right) \left(\frac{4}{y}\right)^{\frac{D}{2}-1} - \frac{(4\pi)^{D/2} H^{-D}}{\Gamma(\frac{D}{2}-1)} \frac{i}{a^D} \delta^D(x - x'). \quad (3.141)$$

We add this particular form of zero to (3.140) and segregate the divergent factor into the δ -function,

$$\begin{aligned} \left(\frac{4}{y}\right)^{D-2} &= \frac{2}{(D-4)(D-3)} \left\{ \frac{(4\pi)^{\frac{D}{2}} H^{-D}}{\Gamma(\frac{D}{2}-1)} \frac{i \delta^D(x - x')}{a^D} + \frac{\square}{H^2} \left[\left(\frac{4}{y}\right)^{D-3} - \left(\frac{4}{y}\right)^{\frac{D}{2}-1} \right] \right\} \\ &\quad - \frac{4}{D-4} \left\{ \left(\frac{4}{y}\right)^{D-3} - \frac{D(D-2)}{8(D-3)} \left(\frac{4}{y}\right)^{\frac{D}{2}-1} \right\}, \end{aligned} \quad (3.142)$$

$$\begin{aligned} &= \frac{i H^{-D} (4\pi)^{\frac{D}{2}}}{(D-4)(D-3) \Gamma(\frac{D}{2})} (D-2) \frac{\delta^D(x - x')}{a^D} \\ &\quad - \frac{\square}{H^2} \left\{ \frac{4}{y} \ln\left(\frac{y}{4}\right) \right\} + 2 \left(\frac{4}{y}\right) \ln\left(\frac{y}{4}\right) - \left(\frac{4}{y}\right) + \mathcal{O}(D-4). \end{aligned} \quad (3.143)$$

For the rest of computations we take the powers of $1/y$ instead of $4/y$:

$$\begin{aligned} \left(\frac{1}{y}\right)^{D-2} &= \frac{iH^{-D}\pi^{\frac{D}{2}}}{(D-4)(D-3)\Gamma(\frac{D}{2})4^{\frac{D}{2}}} 16(D-2) \frac{\delta^D(x-x')}{a^D} \\ &\quad - \frac{16}{4^D} \frac{\square}{H^2} \left\{ \frac{4}{y} \ln\left(\frac{y}{4}\right) \right\} + \frac{32}{4^D} \left(\frac{4}{y}\right) \ln\left(\frac{y}{4}\right) - \frac{16}{4^D} \left(\frac{4}{y}\right) + \mathcal{O}(D-4). \end{aligned} \quad (3.144)$$

Subsequently replacing $1/y^{D-2}$ in (3.139) by (3.144) makes the quadratically divergent power $1/y^{D-1}$ become

$$\begin{aligned} \left(\frac{1}{y}\right)^{D-1} &= \frac{iH^{-D}\pi^{\frac{D}{2}}}{(D-4)(D-3)\Gamma(\frac{D}{2})4^{\frac{D}{2}}} \left\{ \frac{8}{D-2} \frac{\square}{H^2} - 8 \right\} \frac{\delta^D(x-x')}{a^D} \\ &\quad - \frac{2}{4^D} \frac{\square^2}{H^4} \left\{ \frac{4}{y} \ln\left(\frac{y}{4}\right) \right\} + \frac{\square}{H^2} \left\{ \frac{8}{4^D} \left(\frac{4}{y}\right) \ln\left(\frac{y}{4}\right) - \frac{2}{4^D} \left(\frac{4}{y}\right) \right\} \\ &\quad - \frac{8}{4^D} \left(\frac{4}{y}\right) \ln\left(\frac{y}{4}\right) + \frac{4}{4^D} \left(\frac{4}{y}\right) + \mathcal{O}(D-4), \end{aligned} \quad (3.145)$$

and replacing $1/y^{D-1}$ in (3.138) by (3.145) makes the quartically divergent power $1/y^D$ become

$$\begin{aligned} \left(\frac{1}{y}\right)^D &= \frac{iH^{-D}\pi^{\frac{D}{2}}}{(D-4)(D-3)\Gamma(\frac{D}{2})4^{\frac{D}{2}}} \left\{ \frac{4}{(D-2)(D-1)D} \frac{\square^2}{H^4} - \frac{4}{(D-1)D} \frac{\square}{H^2} \right\} \\ &\quad \times \frac{\delta^D(x-x')}{a^D} - \frac{1}{4^D} \left\{ \frac{1}{12} \frac{\square^3}{H^6} \left[\frac{4}{y} \ln\left(\frac{y}{4}\right) \right] + \frac{\square^2}{H^4} \left[\frac{1}{3} \left(\frac{4}{y}\right) \ln\left(\frac{y}{4}\right) - \frac{1}{12} \left(\frac{4}{y}\right) \right] \right. \\ &\quad \left. - \frac{1}{3} \left(\frac{4}{y}\right) \ln\left(\frac{y}{4}\right) + \frac{1}{6} \left(\frac{4}{y}\right) \right\} + \mathcal{O}(D-4). \end{aligned} \quad (3.146)$$

Here we note that the d'Alembertian operator $\square$ acts on both a^D and $\delta^D(x-x')$ while it directly acts on $\delta^D(x-x')$ in the counterterm vertices. The following identities allow us to move the factors of a to the left of derivative operators:

$$\square a^n \delta^D(x-x') = \left(a^n \square - [n^2 + n(D-1)] H^2 a^n - 2nH a^{n-1} \partial_0 \right) \delta^D(x-x'), \quad (3.147)$$

$$\begin{aligned} \square^2 a^n \delta^D(x-x') &= \left(a^n \square^2 - 2n[3n + (D-1)] H^2 a^n \square - 4nH a^{n-1} \partial_0 \square + 4n^3 H^3 a^{n-1} \partial_0 \right. \\ &\quad \left. + 4n(n-1) H^2 a^{n-2} \nabla^2 + n^2 [n + (D-1)]^2 H^4 a^n \right) \delta^D(x-x'), \end{aligned} \quad (3.148)$$

$$\square a^n \partial_0 \delta^D(x-x') = a^n \left(\square - n[n + (D-1)] H^2 - 2nH a^{-1} \partial_0 \right) \partial_0 \delta^D(x-x'). \quad (3.149)$$

3.5.3 Regularization

3.5.3.1 For 4-point part and 3-point kinetic part

To illustrate the process of putting the divergences exactly in the forms of the counterterms, we take the term with the external operator α in Eq. (3.89) coming from the 3-point kinetic interactions as an example:

$$\begin{aligned}
\alpha \left\{ -\frac{DH^{2D-4}\Gamma(\frac{D}{2}-1)^2}{64(D-1)\pi^D} \left(\frac{1}{y}\right)^{D-2} \right\} &= -\frac{DH^{2D-4}\Gamma(\frac{D}{2}-1)^2}{64(D-1)\pi^D} (aa')^{\frac{D}{2}+1} \square^2 \left(\frac{1}{y}\right)^{D-2}, \\
&= -\frac{DH^{2D-4}\Gamma(\frac{D}{2}-1)^2}{64(D-1)\pi^D} \frac{iH^{-D}\pi^{\frac{D}{2}}}{(D-4)(D-3)\Gamma(\frac{D}{2})4^{\frac{D}{2}}} a^{\frac{D}{2}+1} 16(D-2)\square^2 a^{\frac{D}{2}+1} \frac{\delta^D(x-x')}{a^D} \\
&\quad -\frac{DH^{2D-4}\Gamma(\frac{D}{2}-1)^2}{64(D-1)\pi^D} (aa')^{\frac{D}{2}+1} \square^2 \left(-\frac{16}{4^D} \frac{\square}{H^2} \left\{ \frac{4}{y} \ln\left(\frac{y}{4}\right) \right\} + \frac{32}{4^D} \left(\frac{4}{y}\right) \ln\left(\frac{y}{4}\right) \right. \\
&\quad \left. -\frac{16}{4^D} \left(\frac{4}{y}\right) + \mathcal{O}(D-4) \right), \\
&= \frac{iH^{D-4}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)} \left\{ -\frac{D}{D^2-3D+2} H^2 a^2 \square^2 + \frac{D(D-4)}{2(D-1)} H^2 a^2 \square \right. \\
&\quad \left. -\frac{2D}{(D-1)} H a \partial_0 \square + \frac{D}{(D-1)} H^2 \nabla^2 - \frac{D(D-2)}{(D-1)} H^3 a \partial_0 + H^4 a^2 \right\} \delta^D(x-x') \\
&\quad + \frac{H^4}{(4\pi)^4} \left\{ \frac{1}{3} (aa')^3 \frac{\square^3 \ln(x)}{H^2 x} - \frac{1}{3} (aa')^3 \square^2 \left[\frac{2\ln(x)}{x} - \frac{1}{x} \right] \right\}. \tag{3.150}
\end{aligned}$$

Note that all the divergent pieces are exactly in the form of the counterterm vertices (3.131 - 3.133) and (3.135). Following the same process, the terms with the remaining ten external operators (3.89 - 3.99) can be localized into the desired form of external operators acting on $\delta^D(x-x')$. We tabulate the results from TABLE 3.3 to TABLE 3.5.

As we realized in section 3.5.1 the total coefficients for the contributions coming from the terms $\nabla^2 \partial_0 \delta^D(x-x')$ and $\nabla^2 \square \delta^D(x-x')$ to 3-point kinetic counterterms add up to zero as tabulated in TABLE 3.5. This is a very important check for the correctness of our calculation.

The terms in the last line of Eq. (3.150) are nonlocal and finite for $D = 4$ dimension. For notational simplicity, we define the function $y(x; x')$ as $x \equiv \frac{y}{4}$ and take the $D = 4$ limit for these finite nonlocal pieces. Again, applying the same procedure to the terms with the remaining ten external operators (3.89 - 3.99), we obtain nonlocal finite terms. We add these newly found finite terms to the finite terms tabulated in TABLES A.16

Table 3.3: Contributions to 3-point kinetic counterterms. All terms are multiplied by

$$\frac{i\kappa^2 H^{D-4}}{(4\pi)^{D/2}} \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)}.$$

External operators	Coef. of $H^4 a^2 \delta^D(x-x')$
α	$-\frac{D^3(D-2)}{16(D-1)}$
β_2	$\frac{D^6-9D^5+16D^4+36D^3-72D^2+16D+32}{16(D^2-3D+2)}$
γ_1	$\frac{15D^5-104D^4+284D^3-320D^2-32D+160}{16(D-1)}$
γ_2	$\frac{-4D^4+22D^3-11D^2-90D+108}{4}$
Total	$\frac{-15D^6+130D^5-406D^4+434D^3+372D^2-1112D+608}{8(D^2-3D+2)}$

Table 3.4: Contributions to 3-point kinetic counterterms. All terms are multiplied by

$$\frac{i\kappa^2 H^{D-4}}{(4\pi)^{D/2}} \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)}.$$

External operators	Coef. of $a^2 \square^2 \delta^D(x-x')$	Coef. of $H^2 a^2 \square \delta^D(x-x')$
α	$-\frac{D}{D^2-3D+2}$	$\frac{D(D-4)}{2(D-1)}$
β_2	$\frac{D}{D^2-3D+2}$	$\frac{D^5-6D^4+5D^3+34D^2-12D+8}{4(D-2)(D-1)^2}$
γ_1		$\frac{D^3+4D-8}{4(D-2)(D-1)}$
γ_2		-1
Total	0	$\frac{D^5-3D^4-14D^3+82D^2-60D+24}{4(D-2)(D-1)^2}$

Table 3.5: Contributions to 3-point kinetic counterterms. All terms are multiplied by

$$\frac{i\kappa^2 H^{D-4}}{(4\pi)^{D/2}} \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)}.$$

External operators	Coef. of $H^2 \nabla^2 \delta^D(x-x')$	Coef. of $Ha \partial_0 \square \delta^D(x-x')$	Coef. of $\nabla^2 \square \delta^D(x-x')$	Coef. of $Ha^{-1} \nabla^2 \partial_0 \delta^D(x-x')$
α	$\frac{D}{(D-1)}$	$-\frac{2D}{(D-1)}$		
β_2	$\frac{D(D^2-2D+4)}{(D^2-3D+2)}$	$\frac{2D}{(D-1)}$		
δ	$\frac{D(D^2-2D-4)}{4(D^2-3D+2)}$		$-\frac{D}{D^2-3D+2}$	$\frac{D^2}{D^2-3D+2}$
ε_1	$-\frac{4(D^2-7D+4)}{4(D-1)}$		$\frac{D}{D^2-3D+2}$	$-\frac{D^2}{D^2-3D+2}$
ε_2	$-\frac{D}{(D-1)}$			
Total	$-\frac{D(D^3-14D^2+28D-20)}{4(D^2-3D+2)}$	0	0	0

- A.25 and give the summation of the two (i.e., all the finite terms from the 3-point kinetic interactions) in TABLE 3.6.

In summary, the regulated self-mass-squared from the kinetic interactions consists of three finite parts:

- the local 4-point contributions given in Eq. (3.57),
- the local 3-point kinetic contributions arising from TABLES 3.3 - 3.5, and from Eqs. (3.67) and (3.72) (there are also the local 3-point kinetic contributions from our previous work, but they vanish),
- the nonlocal terms from the 3-point kinetic interactions listed in TABLE 3.6.

That is, we finally write it as

$$\begin{aligned}
-iM_{\text{regK}}^2(x; x') &= i\kappa^2 a^2 \left(d_1 \square^2 + d_2 H^2 \square + d_3 H^4 + d_4 H^2 \frac{\nabla^2}{a^2} + d_5 H \frac{\partial_0 \square}{a} \right) \delta^D(x-x') \\
&\quad + \text{Table 3.6} + i\kappa^2 \frac{1}{(4\pi)^2} a^2 \left(\frac{11}{4} \ln(aa') H^2 \square + \frac{11}{2} \ln(aa') H^3 \frac{\partial_0}{a} \right. \\
&\quad \left. - 6 \ln(aa') H^4 - \frac{3}{2} \ln(aa') H^2 \frac{\nabla^2}{a^2} \right) \delta^4(x-x') + \mathcal{O}(D-4), \quad (3.151)
\end{aligned}$$

Table 3.6: All finite nonlocal 3-point kinetic contributions with $x \equiv \frac{y}{4}$, where $y(x; x')$ is defined in the equation (3.31).

External operators	Coef. of $\frac{\kappa^2 H^4}{(4\pi)^4}$
$(aa')^3 \square^3 / H^2$	$\frac{\ln x}{3x}$
$(aa')^3 \square^2$	$-\frac{3 \ln x}{2x}$
$(aa')^3 H^2 \square$	$\frac{25 \ln x}{x} - \frac{3}{2x}$
$(aa')^3 H^4$	$-\frac{44 \ln x}{x} - \frac{26}{x} - \ln(aa') \frac{30}{x}$
$(aa')^2 (a^2 + a'^2) \square^3 / H^2$	$-\frac{\ln x}{6x}$
$(aa')^2 (a^2 + a'^2) \square^2$	$\frac{7 \ln x}{6x} - \frac{1}{6x}$
$(aa')^2 (a^2 + a'^2) H^2 \square$	$\frac{\ln x}{3x} + \frac{5}{6x}$
$(aa')^2 (a^2 + a'^2) H^4$	$-\frac{4 \ln x}{x} - \frac{7}{2x}$
$(aa')^2 H^2 \nabla^2$	$\frac{4 \ln x}{x} - \frac{14}{x} + \ln(aa') [9 \ln(x) + \frac{20}{x}]$
$(aa')^2 \nabla^2 \square$	$\frac{8 \ln x}{3x} + \frac{1}{3x}$
$(aa')^2 H^2 \nabla^2 \square^2$	$\frac{\ln x}{3x}$
$(aa') (a^2 + a'^2) H^2 \nabla^2$	$\frac{22 \ln x}{3x} - \frac{67}{6x} + \ln(aa') \frac{12}{x}$
$(aa') (a^2 + a'^2) \nabla^2 \square$	$-\frac{11 \ln x}{3x} + \frac{4}{6x}$
$(aa') \nabla^4$	$-\ln(x) + \frac{16}{3x} + \ln(aa') [2 \ln(x) - \frac{2}{x}]$

where the coefficients d_i are

$$d_1 = 0, \quad (3.152)$$

$$\begin{aligned} d_2 &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)} \left[\frac{(D^5 + 3D^4 - 14D^3 + 82D^2 - 60D + 24)}{2^2(D-2)(D-1)^3} \right] \right. \\ &\quad \left. + \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left[-2 + \left[\frac{1}{4}D(D-5) - \left(\frac{D-1}{D-3} \right) \right] \pi \cot\left(\frac{\pi D}{2}\right) \right] \right\}, \\ &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{4D^8 - 64D^7 + 396D^6 - 1239D^5 + 2111D^4 - 1886D^3}{2^3(D-4)(D-3)(D-1)^3} \right. \\ &\quad \left. + \frac{742D^2 + 44D - 72}{2^3(D-4)(D-3)(D-1)^3} - 4 + \frac{205}{18}\gamma + \mathcal{O}(D-4) \right\}, \quad (3.153) \end{aligned}$$

$$\begin{aligned} d_3 &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)} \left[\frac{(-15D^6 + 130D^5 - 406D^4 + 434D^3)}{2^3(D-2)(D-1)} \right] \right. \\ &\quad \left. + \frac{372D^2 - 1112D + 608}{2^3(D-2)(D-1)} \right] \\ &\quad \left. + \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left[-\frac{9}{2} + \left[\frac{1}{4}(D-2)(3D-8) \left(\frac{D-1}{D-3} \right) \right] \pi \cot\left(\frac{\pi D}{2}\right) \right] \right\}, \\ &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{9D^6 - 150D^5 + 914D^4 - 2774D^3 + 4596D^2 - 3960D - 1376}{2^4(D-4)(D-3)(D-1)} \right. \\ &\quad \left. - 9 + \frac{98}{3}\gamma + \mathcal{O}(D-4) \right\}, \quad (3.154) \end{aligned}$$

$$\begin{aligned} d_4 &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)} \left[-\frac{D(D^3 - 14D^2 + 28D - 20)}{2^2(D-2)(D-1)} \right] \right. \\ &\quad \left. + \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left[\frac{5}{2} - \left[\frac{1}{4}D(D-5) - \left(\frac{D-1}{D-3} \right) \right] \pi \cot\left(\frac{\pi D}{2}\right) \right] \right\}, \\ &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{-4D^6 + 56D^5 - 281D^4 + 638D^3 - 608D^2 + 108D + 96}{2^3(D-4)(D-3)(D-1)} \right. \\ &\quad \left. + 5 - \frac{65}{3}\gamma + \mathcal{O}(D-4) \right\}, \quad (3.155) \end{aligned}$$

$$d_5 = 0. \quad (3.156)$$

Here γ is the Euler's constant which arises from the expansion of the Gamma function.

3.5.3.2 For 3-point cross part

To illustrate how we put the divergences in the forms of the counterterms, we take the term with the external operator β_1 in Eq. (3.90) coming from the 3-point

kinetic-conformal cross interactions as an example:

$$\begin{aligned}
& \beta_1 \left\{ -\frac{(D-1)H^{2D-4}\Gamma(\frac{D}{2}-1)^2}{2\pi^D} \left(\frac{1}{y}\right)^{D-2} - \frac{3H^4}{2\pi^4} \left(\frac{1}{y}\right) \right\}, \\
& = -\frac{(D-1)H^{2D-4}\Gamma(\frac{D}{2}-1)^2}{2\pi^D} (aa')^{\frac{D}{2}+1} H^2 \square \left(\frac{1}{y}\right)^{D-2} - (aa')^{\frac{D}{2}+1} \square \frac{3H^4}{2\pi^4} \left(\frac{1}{y}\right), \\
& = -\frac{iH^{D-4}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2}-1)}{(D-4)(D-3)} \left\{ 16(D-1)H^2 a^2 \square + 4D(D-2)(D-1)H^4 a \right. \\
& \quad \left. + 16(D-2)(D-1)H^3 a \partial_0 \right\} \delta^D(x-x') + \frac{H^4}{(4\pi)^4} \left\{ 24(aa')^3 \square^2 \frac{\ln(x)}{x} \right. \\
& \quad \left. - 24(aa')^3 H^2 \square \left(\frac{2\ln(x)}{x} + \frac{3}{x} \right) \right\}. \tag{3.157}
\end{aligned}$$

Similarly, the terms with the remaining ten external operators can be localized into the desired form of external operators acting on $\delta^D(x-x')$. We give the results in TABLE 3.7, TABLE 3.8 and TABLE 3.9. The terms in the last line of Eq. (3.157) are nonlocal and finite for $D = 4$ dimension. We denote the function $y(x; x')$ by $x \equiv \frac{y}{4}$ and take the $D = 4$ limit for the finite nonlocal pieces. Applying the same procedure to the terms with the remaining ten external operators, we obtain the rest of nonlocal finite terms. We add these newly found finite terms to the finite terms in TABLES A.26 - A.36 and report the summation of the two (i.e., all the finite terms from the 3-point cross interactions) in TABLE 3.10.

Table 3.7: Contributions to 3-point cross counterterms. All terms are multiplied by

$$\frac{i\tilde{\kappa}^2 H^{D-4}}{(4\pi)^{D/2}} \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)}.$$

External operators	Coef. of $H^4 a^2 \delta^D(x-x')$
β_1	$-8D(D-1)$
β_2	$-\frac{D^5-6D^4+2D^3+16D^2+8D}{2(D-2)}$
γ_1	$-(D^4+6D^3-58D^2-173D+344)$
γ_2	$\frac{D^5-4D^4+10D^3-254D^2+764D-616}{2(D-2)}$
γ_3	$\frac{D^5+24D^4-253D^3+752D^2-852D+324}{(D-2)}$
Total	$\frac{20D^4-187D^3+696D^2-1180D+704}{(D-2)}$

Table 3.8: Contributions to 3-point cross counterterms. All terms are multiplied by

$$\frac{i\tilde{\kappa}^2 H^{D-4}}{(4\pi)^{D/2}} \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)}.$$

External operators	Coef. of $a^2 \square^2 \delta^D(x-x')$	Coef. of $H^2 a^2 \square \delta^D(x-x')$
β_1		$-\frac{32}{(D-2)}$
β_2	$-\frac{8}{(D-2)}$	$-\frac{2D^3-6D^2-36D+96}{(D-2)}$
γ_1		$-\frac{4(D^2+3D+1)}{(D-2)}$
γ_2		$\frac{2(D^2+4D+2)}{(D-2)}$
γ_3		$\frac{4(D^2+4D-10)}{(D-2)}$
Total	$-\frac{8}{(D-2)}$	$\frac{2D^3-8D^2-48D+168}{(D-2)}$

Table 3.9: Contributions to 3-point cross counterterms. All terms are multiplied by

$$\frac{i\tilde{\kappa}^2 H^{D-4}}{(4\pi)^{D/2}} \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)}.$$

External operators	Coef. of $H^2 \nabla^2 \delta^D(x-x')$	Coef. of $Ha\partial_0 \square \delta^D(x-x')$
β_2	$-\frac{8D(D-2)+32}{(D-2)}$	-16
ε_1	$-4(D-6)$	
ε_2	$-\frac{2D(D-8)}{(D-1)}$	
ε_3	$-\frac{4(D^3-3D^2+22D-16)}{(D-3)(D-1)}$	
Total	$-\frac{(D-2)(9D^4-71D^3+256D^2-400D+184)}{2(D-3)(D-1)}$	-16

The regulated self-mass-squared from the kinetic-conformal cross interactions consists of two finite parts:

- the local 3-point cross contributions from TABLES 3.7 - 3.9 and the equations (3.107) and (3.111),
- the nonlocal contributions from the 3-point cross interactions in TABLE 3.10,

and we can write it as

$$\begin{aligned}
 -iM_{\text{regcross}}^2(x; x') &= i\kappa^2 a^2 \left(d_1 \square^2 + d_2 H^2 \square + d_3 H^4 + d_4 H^2 \frac{\nabla^2}{a^2} + d_5 H \frac{\partial_0 \square}{a} \right) \delta^D(x-x') \\
 &\quad + \text{Table 3.10} + \mathcal{O}(D-4).
 \end{aligned} \tag{3.158}$$

Here $\tilde{\kappa} = \frac{D-2}{8(D-1)} \kappa$ is taken. The multiplier $(\frac{D-2}{8(D-1)})^2$ is inserted in the coefficients d_i .

Thus the coefficients d_i become

Table 3.10: All finite nonlocal 3-point cross contributions with $x \equiv \frac{y}{4}$, where $y(x; x')$ is defined in the equation (3.31).

External operators	Coef. of $\frac{\kappa^2 H^4}{(4\pi)^4}$
$(aa')^3 \square^3 / H^2$	$\frac{2 \ln x}{x}$
$(aa')^3 \square^2$	$-\frac{27 \ln x}{x}$
$(aa')^3 H^2 \square$	$-\frac{378 \ln x}{x} - \frac{59}{x}$
$(aa')^3 H^4$	$\frac{608 \ln x}{x} - \frac{436}{x}$
$(aa')^2 (a^2 + a'^2) \square^3 / H^2$	$\frac{\ln x}{x}$
$(aa')^2 (a^2 + a'^2) \square^2$	$-\frac{28 \ln x}{x} + \frac{1}{x}$
$(aa')^2 (a^2 + a'^2) H^2 \square$	$\frac{178 \ln x}{x} - \frac{26}{x}$
$(aa')^2 (a^2 + a'^2) H^4$	$-\frac{252 \ln x}{x} - \frac{66}{x}$
$(aa')^2 H^2 \nabla^2$	$-24 \ln(x) - \frac{142 \ln x}{3x} + \frac{508}{3x}$
$(aa')^2 \nabla^2 \square$	$\frac{76 \ln x}{3x}$
$(aa')(a^2 + a'^2) H^2 \nabla^2$	$-\frac{24 \ln x}{x} + \frac{58}{x}$
$(aa')(a^2 + a'^2) \nabla^2 \square$	$\frac{12 \ln x}{x} + \frac{4}{x}$
$(aa') \nabla^4$	$\frac{4}{x}$

$$\begin{aligned}
d_1 &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)} \left[-\frac{(D-2)}{2^3(D-1)^2} \right] \right\}, \\
&= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ -\frac{(D-2)^2}{2^4(D-4)(D-3)(D-1)^2} + \frac{1}{72}\gamma + \mathcal{O}(D-4) \right\}, \tag{3.159}
\end{aligned}$$

$$\begin{aligned}
d_2 &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)} \left[\frac{(D-2)(D^3-4D^2-24D+84)}{2^5(D-1)^2} \right] \right\}, \\
&= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{(D-2)^2(D^3-4D^2-24D+84)}{2^6(D-4)(D-3)(D-1)^2} + \frac{1}{24}\gamma + \mathcal{O}(D-4) \right\}, \tag{3.160}
\end{aligned}$$

$$\begin{aligned}
d_3 &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)} \left[\frac{(D-2)(20D^4-187D^3+696D^2-1180D+704)}{2^6(D-1)^2} \right] \right\}, \\
&= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{(D-2)^2(20D^4-187D^3+696D^2-1180D+704)}{2^7(D-4)(D-3)(D-1)^2} - \frac{17}{9}\gamma + \mathcal{O}(D-4) \right\}, \tag{3.161}
\end{aligned}$$

$$\begin{aligned}
d_4 &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)} \left[-\frac{(D-2)^3(9D^4-71D^3+256D^2-400D+184)}{2^7(D-1)^2} \right] \right\}, \\
&= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ -\frac{(D-2)^4(9D^4-71D^3+256D^2-400D+184)}{2^8(D-4)(D-3)(D-1)^2} + \frac{55}{108}\gamma + \mathcal{O}(D-4) \right\}, \tag{3.162}
\end{aligned}$$

$$\begin{aligned}
d_5 &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)} \left[-\frac{(D-2)^2}{2^2(D-1)^2} \right] \right\}, \\
&= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ -\frac{(D-2)^3}{2^3(D-4)(D-3)(D-1)^2} + \frac{1}{18}\gamma + \mathcal{O}(D-4) \right\}. \tag{3.163}
\end{aligned}$$

3.5.4 Renormalization

Finally, we sum the contributions from kinetic and cross interactions:

$$\begin{aligned}
-iM_{\text{reg}}^2(x; x') &= -iM_{\text{regK}}^2(x; x') + -iM_{\text{regcross}}^2(x; x'), \\
&= i\kappa^2 a^2 \left(d_1 \square^2 + d_2 H^2 \square + d_3 H^4 + d_4 H^2 \frac{\nabla^2}{a^2} + d_5 H \frac{\partial_0 \square}{a} \right) \delta^D(x - x') \\
&\quad + \text{Table 3.6} + \left(\frac{8(D-1)}{D-2} \right)^2 \text{Table 3.10} \\
&\quad + i\kappa^2 \frac{1}{(4\pi)^2} a^2 \left(\frac{11}{4} \ln(aa') H^2 \square + \frac{11}{2} \ln(aa') H^3 \frac{\partial_0}{a} - 6 \ln(aa') H^4 - \frac{3}{2} \ln(aa') H^2 \frac{\nabla^2}{a^2} \right) \\
&\quad \times \delta^4(x - x') + \mathcal{O}(D-4). \tag{3.164}
\end{aligned}$$

The resulting total coefficients d_i are

$$\begin{aligned} d_1 &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)} \left[-\frac{(D-2)}{2^3(D-1)^2} \right] \right\}, \\ &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ -\frac{(D-2)^2}{2^4(D-4)(D-3)(D-1)^2} + \frac{1}{72}\gamma + \mathcal{O}(D-4) \right\}, \end{aligned} \quad (3.165)$$

$$\begin{aligned} d_2 &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)} \left[\frac{D^6 - D^5 + 28D^4 + 56D^3 + 60D^2 + 288D - 144}{2^5(D-2)(D-1)^3} \right] \right. \\ &\quad \left. + \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left[-2 + \left[\frac{1}{4}D(D-5) - \left(\frac{D-1}{D-3} \right) \right] \pi \cot\left(\frac{\pi D}{2}\right) \right] \right\}, \\ &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{D^6 - D^5 + 28D^4 + 56D^3 + 60D^2 + 288D - 144}{2^6(D-4)(D-3)(D-1)^3} \right. \\ &\quad \left. - 4 + \frac{823}{72}\gamma + \mathcal{O}(D-4) \right\}, \end{aligned} \quad (3.166)$$

$$\begin{aligned} d_3 &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ -\frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)} \left[\frac{100D^7 - 853D^6 + 2230D^5 + 1040D^4}{2^6(D-2)(D-1)^2} \right. \right. \\ &\quad \left. \left. + \frac{-17136D^3 + 35824D^2 - 31648D + 10496}{2^6(D-2)(D-1)^2} \right] \right. \\ &\quad \left. + \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left[-\frac{9}{2} + \left[\frac{1}{4}(D-2)(3D-8) - \left(\frac{D-1}{D-3} \right) \right] \pi \cot\left(\frac{\pi D}{2}\right) \right] \right\}, \\ &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ -\frac{100D^7 - 853D^6 + 2230D^5 + 1040D^4 - 17136D^3 + 35824D^2}{2^7(D-4)(D-3)(D-1)^2} \right. \\ &\quad \left. + \frac{-31648D + 10496}{2^7(D-4)(D-3)(D-1)^2} - 9 + \frac{1159}{36}\gamma + \mathcal{O}(D-4) \right\}, \end{aligned} \quad (3.167)$$

$$\begin{aligned} d_4 &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)} \left[\frac{-9D^8 + 143D^7 - 1040D^6 + 4408D^5}{2^7(D-2)(D-1)^2} \right. \right. \\ &\quad \left. \left. + \frac{-11464D^4 + 19056D^3 - 19776D^2 + 11648D - 2944}{2^7(D-2)(D-1)^2} \right] \right. \\ &\quad \left. + \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left[\frac{5}{2} - \left[\frac{1}{4}D(D-5) - \left(\frac{D-1}{D-3} \right) \right] \pi \cot\left(\frac{\pi D}{2}\right) \right] \right\}, \\ &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{-9D^8 + 143D^7 - 1040D^6 + 4408D^5 - 11464D^4 + 19056D^3}{2^8(D-4)(D-3)(D-1)^2} \right. \\ &\quad \left. + \frac{-19776D^2 + 11648D - 2944}{2^8(D-4)(D-3)(D-1)^2} + 5 - \frac{2285}{108}\gamma + \mathcal{O}(D-4) \right\}, \end{aligned} \quad (3.168)$$

$$\begin{aligned} d_5 &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)} \left[-\frac{(D-2)^2}{2^2(D-1)^2} \right] \right\}, \\ &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ -\frac{(D-2)^3}{2^3(D-4)(D-3)(D-1)^2} + \frac{1}{18}\gamma + \mathcal{O}(D-4) \right\}. \end{aligned} \quad (3.169)$$

Now we choose the coefficients of the counterterm vertices (3.131 - 3.133) and (3.135) as

$$c_i = -d_i + \Delta c_i \quad \text{for } i = 1, 3, 4, \quad (3.170)$$

$$c_2 = -d_2 + (D-2)d_1 + \Delta c_2, \quad (3.171)$$

where Δc_i is the arbitrary finite terms which remain after the cancellation of the divergent parts. Note that the coefficients d_1 and d_5 have the relation

$$d_5 = 2(D-2) d_1, \quad (3.172)$$

which makes them simultaneously cancelled by subtracting the counterterm vertex (3.131). We finally obtain the final renormalized self-mass-squared from kinetic and kinetic-conformal cross parts at one loop order as

$$\begin{aligned} -iM_{\text{ren}}^2(x; x') = & i\kappa^2 a^2 \left(\Delta c_1 \square^2 + \Delta c_2 H^2 \square + \Delta c_3 H^4 + \Delta c_4 H^2 \frac{\nabla^2}{a^2} + 4\Delta c_1 H \frac{\partial_0 \square}{a} \right) \delta^4(x - x') \\ & + i\kappa^2 \frac{1}{(4\pi)^2} a^2 \left(\frac{11}{4} \ln(aa') H^2 \square + \frac{11}{2} \ln(aa') H^3 \frac{\partial_0}{a} - 6 \ln(aa') H^4 \right. \\ & \left. - \frac{3}{2} \ln(aa') H^2 \frac{\nabla^2}{a^2} \right) \delta^4(x - x') + \text{Table 3.6} + \left(\frac{8(D-1)}{D-2} \right)^2 \text{Table 3.10} + \mathcal{O}(D-4). \end{aligned} \quad (3.173)$$

3.6 Discussion

We have evaluated the kinetic and kinetic-conformal cross parts of the CC scalar self-mass-squared at one loop order in the cosmological patch of de Sitter space. The CC scalar has the technical advantage that its propagator with the conformally rescaled metric looks the same as the MMC scalar propagator in flat space,

$$i\Delta^{\text{flat}} = \frac{\Gamma(\frac{D}{2} - 1)}{4\pi^{D/2}} \frac{1}{\Delta x^{D-2}}, \quad (3.174)$$

where Δx^2 is the Poincaré length function defined as

$$\Delta x^2 \equiv \|\vec{x} - \vec{x}'\|^2 - \left(|\eta - \eta'| - i\delta \right)^2. \quad (3.175)$$

We have first dimensionally regulated the divergences, renormalized the result by subtracting the four possible BPHZ counterterms and then taken the unregulated limit of $D = 4$. The final result is given in Eq. (3.173) with the finite nonlocal contributions in TABLE 3.6 and TABLE 3.10. Adding our previous result from

the conformal-conformal part, Eq. (129) of Ref. [1] to Eq. (3.173) completes the full renormalized CC scalar self-mass-squared at one loop order.

The point of this computation is to examine the effects of inflationary produced gravitons on the CC scalar mode functions. In a subsequent paper [3] we will solve the quantum corrected CC scalar field equation (3.1) (with the full self-mass-squared) to obtain one loop corrections to the CC scalar mode functions. It is worthwhile at this point commenting that the Schwinger-Keldysh formalism [105, 126–128, 130–133] is necessary when studying quantum responses in a time-dependent background such as de Sitter. Applying the formalism amounts to replacing the in-out self-mass-squared by the ones from the Schwinger-Keldysh formalism,

$$a^4 \square \phi(x) - \frac{1}{6} R \phi(x) - \int_{\eta_i}^0 d\eta' \int d^3 x' \left\{ M_{++}^2(x; x') + M_{+-}^2(x; x') \right\} \phi(x') = 0. \quad (3.176)$$

At one loop order, $-iM_{++}^2(x; x')$ agrees with the in-out self-mass-squared $-iM^2(x; x')$. The $+ -$ self-mass-squared $-iM_{+-}^2(x; x')$ can be obtained by replacing $y(x; x')$ everywhere with

$$y(x; x') \rightarrow y_{+-}(x; x') \equiv a(\eta) a(\eta') H^2 \left[\|\vec{x} - \vec{x}'\|^2 - (\eta - \eta' + i\delta)^2 \right]. \quad (3.177)$$

It should be noted that the CC scalar field interacts with gravitons both through a kinetic term and the conformal coupling term. The case with a kinetic term only, also known as minimal coupling, was investigated in a previous work [13, 14]. When the conformal coupling is added, the 4-point interaction part simply adds the contributions from the kinetic and conformal coupling terms. However, for the 3-point interaction part which has two vertices, there are three possibilities: (i) both vertices are kinetic, (ii) both vertices are conformal (iii) one vertex is kinetic and the other vertex is conformal. The case (ii) was studied in a previous paper [1] and the cases (i) and (iii) were investigated in the present paper. The difference between the computations of the case (i) in Ref. [13] and in this paper is that in the former, the MMC scalar propagator was used (since there was no conformal coupling term); and in the latter we have used the CC scalar propagator.

We also would like to comment on a way to check the accuracy of our computation. We note that the most singular part of the graviton propagator in de Sitter background agrees with the conformally coupled scalar propagator and it can be compared with the

graviton propagator in flat space

$$i[\mu\nu\Delta_{\rho\sigma}^{\text{flat}}](x;x') = \left[2\eta_{\mu(\rho}\eta_{\sigma)\nu} - \frac{2}{D-2}\eta_{\mu\nu}\eta_{\rho\sigma} \right] \frac{\Gamma(\frac{D}{2}-1)}{4\pi^{D/2}} \frac{1}{\Delta x^{D-2}}, \quad (3.178)$$

here note that the scalar propagator multiplying the tensor factor is the flat space scalar propagator (3.174). As the counterterm analysis in Eq. (3.131) shows, the coefficients of the divergent terms d_1 of $a^2\Box^2\delta^D(x-x')$ and d_5 of $Ha\partial_0\Box\delta^D(x-x')$ should obey the relation $d_5 = 2(D-2)d_1$. And that is exactly what we get in Eqs. (3.165) and (3.169), so these divergent terms can be removed by the relevant counterterms. Also, as can be seen in TABLE 3.5, the total coefficients for the contributions coming from the terms $\nabla^2\partial_0\delta^D(x-x')$ and $\nabla^2\Box\delta^D(x-x')$ to 3-point kinetic counterterms add up to zero. There is no other reason why these relations are satisfied, but they turn out to satiate (through explicit computations) the conditions so that our counterterms respect the symmetries that are not broken by the gauge fixing term (3.33).

Finally, the CC scalar self-mass squared is ready to be employed in the CC scalar effective field equation so that one can study how the inflationary produced gravitons affect the CC scalar mode functions. In a subsequent paper [3] we will examine whether the mode functions get a secular growth effect in late times or not. If it does, it might leave an observational signature such as a correction to the scalar power spectrum. We will soon report whether this is the case or not.

4. ONE LOOP CORRECTED CONFORMALLY COUPLED SCALAR MODE EQUATIONS DURING INFLATION

We employ¹ a fully renormalized computation of the one loop contribution to the self-mass-squared of the conformally coupled (CC) scalar interacting with gravitons during inflation to study how inflationary produced gravitons affect the CC scalar evolution equation. The quantum corrected scalar mode functions turn out to get a secular growth effect, proportional to a logarithm of the scale factor at late times.

4.1 Introduction

Quantum fluctuations of infrared gravitons and massless, minimally coupled (MMC) scalars are vastly amplified during inflation, and eventually comprise primordial tensor and scalar perturbations [91, 92]. This is the lowest order effect directly derives from the inflationary produced virtual quanta [134]. The interactions of this ensemble of virtual gravitons and scalars among themselves and other particles can be quantified by the one-particle-irreducible (1PI) 2-point function of a particular kind of particle. A generic procedure for studying these interactions is first to compute the renormalized contribution of MMC scalars or gravitons to the 1PI function of a particle in question, then to use it to quantum-correct the field equation of the particle.

A number of computations involving either scalar-mediated effects or graviton-mediated effects during inflation have been made over the past decades:

- For MMC scalars Yukawa coupled to massless fermions, a growing mass is induced to fermions [29–31, 93, 94], but there is no large correction on scalars [96].
- In MMC scalar quantum electrodynamics, assuming the scalars are charged, photons acquire a growing mass [97–100], but scalars receive no significant effect [35, 36].

¹This chapter is based on the paper One loop corrected conformally coupled scalar mode equations during inflation, *Phys. Rev. D* 96, 105003, (2017) by **Boran, S.**, K  hya, E. O. and Park, S. .

- For MMC scalars with a quartic self-interaction, scalars gain an increasing mass (though it remains perturbatively small) at one and two loop orders [21, 101].
- MMC scalars have no significant effect on dynamical gravitons at one loop order [37, 38], but they induce a secular effect on the gravitational potentials of a point mass (specifically a secular decrease in the gravitational coupling G) [41]. One loop corrections from conformal fields to the gravitational potentials of a point mass were also found [40, 47, 49, 51].
- For quantum gravity plus massless fermions, the fermion field strength gets a secular growth effect from inflationary gravitons [29–31, 95].
- For quantum gravity plus MMC scalars, inflationary gravitons induce no significant corrections on the scalar mode functions at one loop [13, 14].
- For quantum gravity plus electrodynamics, inflationary gravitons induce secular effects on the dynamical photons and alter the electric field of a point charge and the magnetic field of a point magnetic dipole [70–72, 74, 75, 103, 104].

These secular effects are typically in the form of the logarithm of the scale factor $\ln(a)$ times the minuscule loop counting parameter GH^2 [105, 106].

The purpose of this paper is to work out leading loop corrections to the CC scalar mode functions from inflationary gravitons. We have already computed the fully renormalized, one loop contributions to the CC scalar self-mass-squared from gravitons [1, 2]. The result is summarized in Sec. 4.2. We solve the quantum corrected scalar field equation at one loop order in Sec. 4.3. We give our conclusions in Sec. 4.4.

4.2 Our Previous Calculation

We started with the bare Lagrangian of pure gravity plus the CC scalar

$$\mathcal{L} \equiv -\frac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}\sqrt{-g} - \frac{D-2}{8(D-1)}\phi^2 R\sqrt{-g} + \frac{1}{16\pi G}\left(R - (D-2)\Lambda\right)\sqrt{-g}. \quad (4.1)$$

Here R is the Ricci scalar, $\Lambda \equiv (D-1)H^2$ is the cosmological constant with the Hubble constant H , and G is Newton's constant. We worked on the open conformal coordinate

submanifold of de Sitter space

$$\hat{g}_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}, \quad \text{where } a(\eta) \equiv -\frac{1}{H\eta}. \quad (4.2)$$

The full metric consists of the background metric $\hat{g}_{\mu\nu}$ and the conformally rescaled graviton field $h_{\mu\nu}$

$$g_{\mu\nu}(\eta, \vec{x}) \equiv a^2(\eta) \left(\eta_{\mu\nu} + \kappa h_{\mu\nu}(\eta, \vec{x}) \right). \quad (4.3)$$

Here $\kappa^2 \equiv 16\pi G$ is the loop counting parameter of quantum gravity. The graviton propagator was obtained by adding a non-de Sitter invariant gauge fixing term [118, 135] to the invariant Lagrangian (4.1)

$$\mathcal{L}_{\text{GF}} \equiv -\frac{1}{2}\sqrt{-g}g^{\mu\nu}F_\mu F_\nu, \quad (4.4)$$

$$F_\mu \equiv \eta^{\rho\sigma} \left(h_{\mu\rho, \sigma} - \frac{1}{2}h_{\rho\sigma, \mu} + (D-2)aHh_{\mu\rho}\delta_\sigma^0 \right). \quad (4.5)$$

Owing to the conformal coupling term, the matter sector of the bare Lagrangian (4.1) is invariant

$$\begin{aligned} \tilde{\mathcal{L}}_{\text{Matter}} &= -\frac{1}{2}\partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} \tilde{g}^{\mu\nu} \sqrt{-\tilde{g}} - \frac{D-2}{8(D-1)} \tilde{\phi}^2 \tilde{R} \sqrt{-\tilde{g}} \\ &= -\frac{1}{2}\partial_\mu \phi \partial_\nu \phi g^{\mu\nu} \sqrt{-g} - \frac{D-2}{8(D-1)} \phi^2 R \sqrt{-g} = \mathcal{L}_{\text{Matter}}. \end{aligned} \quad (4.6)$$

under the conformal rescaling

$$\tilde{g}_{\mu\nu} \equiv \Omega^{-2}g_{\mu\nu} \quad \text{and} \quad \tilde{\phi} \equiv \Omega^{\frac{D-2}{2}}\phi. \quad (4.7)$$

Taking $\Omega = a$ gives the conformally rescaled metric as

$$\tilde{g}_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}. \quad (4.8)$$

We obtained the linearized effective CC scalar field equation with this conformally rescaled metric

$$\partial_\mu \left(\sqrt{-\tilde{g}} \tilde{g}^{\mu\nu} \partial_\nu \right) \tilde{\phi}(x) - \frac{1}{6} \tilde{\phi}(x) \tilde{R} - \int d^4x' M^2(x; x') \tilde{\phi}(x') = 0. \quad (4.9)$$

The scalar self-mass-squared $M^2(x; x')$ was computed and renormalized by subtracting off the one loop divergences using four counterterms [1, 2]

$$\Delta\tilde{\mathcal{L}} = \frac{1}{2}c_1\kappa^2\Box\tilde{\phi}\Box\tilde{\phi}a^2 - \frac{1}{2}c_2\kappa^2H^2\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} + \frac{1}{2}c_3\kappa^2H^4\tilde{\phi}^2a^2 - \frac{1}{2}c_4\kappa^2H^2\nabla\tilde{\phi}\cdot\nabla\tilde{\phi}. \quad (4.10)$$

Each coefficient c_i has two parts: a divergent part $-d_i$ which is chosen to exactly cancel the divergences occurring in the self-mass-squared, and a finite part Δc_i which remains arbitrary. The fully renormalized self-mass-squared takes the form [1, 2]

$$\begin{aligned}
-iM^2(x; x') = & i\kappa^2 a^2 \left(\Delta c_1 \square^2 + \Delta c_2 H^2 \square + \Delta c_3 H^4 + \Delta c_4 \frac{H^2}{a^2} \nabla^2 + 4\Delta c_1 \frac{H}{a} \partial_0 \square \right) \delta^4(x - x') \\
& + \text{Table A.37} + \text{Table A.38} + \text{Table A.39} \\
& + i\kappa^2 \frac{1}{(4\pi)^2} a^2 \left(\frac{11}{4} \ln(aa') H^2 \square + \left[\frac{9}{40} + \frac{11}{2} \ln(aa') \right] \frac{H^3}{a} \partial_0 \right. \\
& \left. + [2 - 6 \ln(aa')] H^4 - \frac{3}{2} \ln(aa') \frac{H^2}{a^2} \nabla^2 \right) \delta^4(x - x'). \tag{4.11}
\end{aligned}$$

Tables A.37, A.38 and A.39 are given in Appendix A.3. The Laplacian ∇^2 and the d'Alembertian $\square$ are

$$\nabla^2 \equiv \partial_i \partial_i \quad \text{and} \quad \square \equiv \frac{1}{a^2} [-\partial_0^2 - 2aH\partial_0 + \nabla^2]. \tag{4.12}$$

The de Sitter invariant length function is defined as

$$y(x; x') \equiv a(\eta)a(\eta') H^2 \left\{ \|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| - i\delta)^2 \right\}. \tag{4.13}$$

4.3 Effective Mode Equation

In this section we solve the effective field equation (4.9) for the scalar mode functions. We begin by explaining the necessity of the Schwinger-Keldysh formalism for cosmological settings, then we discuss how to solve the effective field equation perturbatively. Finally we work out the one loop contributions to the CC scalar mode functions from inflationary gravitons.

4.3.1 Schwinger-Keldysh formalism

The effective field equation (4.9) reveals two disturbing features if we were to interpret it in the sense of the *in-out* formalism:

- *Causality violation* - the in-out effective field equation at a point x^μ receives influence from points x'^μ which lie outside the past light-cone of x^μ ; and
- *Reality violation* - the quantum-induced scalar field acquires an imaginary part due to the nonzero imaginary part of the in-out self-mass-squared.

Neither of these features means the in-out formalism is wrong. It is the right answer to questions about matrix elements between scattering states if the system begins in free vacuum in the asymptotic past and ends up the same way in the far future. However it is not very relevant for cosmology in which we do not know how the universe begins and ends. The question of greater relevance for cosmology is how the system evolves if released at finite time in a prepared state. The Schwinger-Keldysh (or *in-in*) formalism [126–128, 136] answers this more relevant question and it is almost as simple to use as the Feynman diagrams of the in-out formalism. Therefore we would like to solve the linearized effective field equation for the plane wave mode solution $\tilde{\Phi}(x; \vec{k})$ which is given in terms of $\tilde{\phi}(x)$ as

$$\tilde{\Phi}(x; \vec{k}) = \left\langle \Omega_{\text{in}} \left| \left[\tilde{\phi}(x), \alpha^\dagger(\vec{k}) \right] \right| \Omega_{\text{in}} \right\rangle, \quad (4.14)$$

where $\alpha^\dagger(\vec{k})$ is the creation operator and $|\Omega_{\text{in}}\rangle$ is the in-vacuum state. Because excellent reviews on the Schwinger-Keldysh formalism exist [129–131, 133], we merely comment how to use it.

The distinct property of the Schwinger-Keldysh formalism compared to the in-out formalism is that each end of particle lines carries a $\pm$ polarity. Therefore, every in-out propagator gives rise to four Schwinger-Keldysh propagators. In our case, the in-out propagators depend on the de Sitter invariant length function $y(x; x')$ and the four propagators can be obtained by the following substitutions for $y(x; x')$

$$i\Delta_{++}(x; x') : y \rightarrow y_{++}(x; x) \equiv H^2 a a' \left[\|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| - i\delta)^2 \right], \quad (4.15)$$

$$i\Delta_{+-}(x; x') : y \rightarrow y_{+-}(x; x) \equiv H^2 a a' \left[\|\vec{x} - \vec{x}'\|^2 - (\eta - \eta' + i\delta)^2 \right], \quad (4.16)$$

$$i\Delta_{-+}(x; x') : y \rightarrow y_{-+}(x; x) \equiv H^2 a a' \left[\|\vec{x} - \vec{x}'\|^2 - (\eta - \eta' - i\delta)^2 \right], \quad (4.17)$$

$$i\Delta_{--}(x; x') : y \rightarrow y_{--}(x; x) \equiv H^2 a a' \left[\|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| + i\delta)^2 \right]. \quad (4.18)$$

Because external lines can be either $+$ or $-$, every 1PI N -point function of the in-out formalism gives rise to 2^N Schwinger-Keldysh 1PI N -point functions. The Schwinger-Keldysh effective action generates these 1PI functions. For our case of the 1PI 2-point function for the CC scalar field, it derives from

$$\begin{aligned} \Gamma[\tilde{\phi}_+, \tilde{\phi}_-] = & S[\tilde{\phi}_+] - S[\tilde{\phi}_-] - \frac{1}{2} \int d^4x \int d^4x' \left\{ \tilde{\phi}_+(x) M_{++}^2(x; x') \tilde{\phi}_+(x') \right. \\ & + \tilde{\phi}_+(x) M_{+-}^2(x; x') \tilde{\phi}_-(x') + \tilde{\phi}_-(x) M_{-+}^2(x; x') \tilde{\phi}_+(x') + \tilde{\phi}_-(x) M_{--}^2(x; x') \tilde{\phi}_-(x') \left. \right\} \\ & + \mathcal{O}(\tilde{\phi}_\pm^3). \end{aligned} \quad (4.19)$$

Here $\tilde{\phi}_+$ carries out forward time evolution from the prepared state while $\tilde{\phi}_-$ carries out backward evolution to the original state, and $S[\tilde{\phi}]$ is the classical action. Varying this action with respect to either $\tilde{\phi}_+$ or $\tilde{\phi}_-$ and setting them equal to $\tilde{\phi}$, we have the Schwinger-Keldysh effective field equation, which is the effective field equation given in (4.9) with $M^2(x; x')$ replaced by $M_{++}^2(x; x') + M_{+-}^2(x; x')$,

$$\partial_\mu \left(\sqrt{-\tilde{g}} \tilde{g}^{\mu\nu} \partial_\nu \right) \tilde{\Phi}(x; \vec{k}) - \frac{1}{6} \tilde{\Phi}(x; \vec{k}) \tilde{R} - \int_{\eta_i}^0 d\eta' \int d^3x' \left\{ M_{++}^2(x; x') + M_{+-}^2(x; x') \right\} \tilde{\Phi}(x'; \vec{k}) = 0. \quad (4.20)$$

Here $\eta_i = -1/H$ is the initial time at which the universe is assumed to be in free vacuum (i.e., in the Bunch-Davies vacuum). At one loop order $iM_{++}^2(x; x')$ is identical to the in-out self-mass-squared $iM^2(x; x')$. The $+-$ self-mass-squared $iM_{+-}^2(x; x')$ can be obtained by the substitution rule given in (4.16). One can check by examining the relations (4.15) and (4.16), the retarded self-mass-squared (the bracketed term in (4.20)) vanishes for $\eta' > \eta$ and it is real for $\eta > \eta'$. That is, the Schwinger-Keldysh effective field equation (4.20) is causal and real as desired.

4.3.2 Perturbative Solution

Because we only have one loop result for the scalar self-mass-squared, we must solve (4.20) perturbatively by expanding it in powers of the loop-counting parameter $\kappa^2 \equiv 16\pi G$:

$$M_{\text{ret}}^2(x; x') \equiv M_{++}^2(x; x') + M_{+-}^2(x; x') = \sum_{\ell=1}^{\infty} \kappa^{2\ell} \mathcal{M}_\ell^2(x; x'), \quad (4.21)$$

$$\tilde{\Phi}(x; \vec{k}) = \sum_{\ell=0}^{\infty} \kappa^{2\ell} \tilde{\Phi}_\ell(x; \vec{k}) = \sum_{\ell=0}^{\infty} \kappa^{2\ell} \tilde{u}_\ell(\eta, k) \times e^{i\vec{k} \cdot \vec{x}}, \quad (4.22)$$

where $\tilde{u}_\ell(\eta, k)$ are plane wave mode functions.

By substituting these expansions to the effective field equation (4.20), the zeroth order solution is²

$$\tilde{\Phi}_0(x; \vec{k}) = \tilde{u}_0(\eta, k) e^{i\vec{k} \cdot \vec{x}} = \frac{e^{-ik\eta}}{\sqrt{2k}} e^{i\vec{k} \cdot \vec{x}}. \quad (4.23)$$

²Since we are working with the conformally rescaled metric $\tilde{g}_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$, the zeroth order values are the same as in flat space, i.e., $\tilde{R}_0 = 0$.

The first order equation

$$\partial^2 \tilde{\Phi}_1(x; \vec{k}) - \int_{\eta_i}^0 d\eta' \int d^3x' \mathcal{M}_1^2(x; x') \tilde{\Phi}_0(x'; \vec{k}) = 0, \quad (4.24)$$

and its solution takes the same form

$$\tilde{\Phi}_1(x; \vec{k}) = \tilde{u}_1(\eta, k) e^{i\vec{k} \cdot \vec{x}}, \quad (4.25)$$

where the one loop corrected mode function $\tilde{u}_1(\eta, k)$ can be obtained by solving the following mode equation

$$(-\partial_\eta^2 - k^2) \tilde{u}_1(\eta, k) = \int_{\eta_i}^0 d\eta' \tilde{u}_0(\eta', k) \int d^3x' \mathcal{M}_1^2(x; x') e^{i\vec{k} \cdot (\vec{x}' - \vec{x})}. \quad (4.26)$$

The remaining task is then performing the integration of the right hand side of (4.26). One fact to keep in mind is that we have taken the initial state to be the Bunch-Davies vacuum, but we have not worked out loop corrections to the initial vacuum. This means that the first order solution $\tilde{u}_1(\eta, k)$ only makes sense at late times $\eta \rightarrow 0^-$. To see the leading late time behavior of $\tilde{u}_1$, it is illuminating to convert (4.26) from conformal time η to physical time $t \equiv -\ln(-H\eta)/H$,

$$\left[\frac{\partial^2}{\partial t^2} + 3H \frac{\partial}{\partial t} + \frac{k^2}{a^2} \right] \tilde{u}_1(t, k) = -\frac{1}{a^2} \int_{\eta_i}^0 d\eta' \tilde{u}_0(\eta', k) \int d^3x' \mathcal{M}_1^2(x; x') e^{i\vec{k} \cdot (\vec{x}' - \vec{x})}. \quad (4.27)$$

Given a presumed form for the late time limit of the right hand side (r.h.s.), the late time behavior of $\tilde{u}_1$ can be easily inferred as follows:

$$\text{r.h.s.} \rightarrow a \implies \tilde{u}_1 \rightarrow \text{Constant} + \frac{a}{4H^2}, \quad (4.28)$$

$$\text{r.h.s.} \rightarrow 1 \implies \tilde{u}_1 \rightarrow \text{Constant} + \frac{\ln(a)}{3H^2}, \quad (4.29)$$

$$\text{r.h.s.} \rightarrow \frac{1}{a} \implies \tilde{u}_1 \rightarrow \text{Constant} - \frac{1}{2H^2 a}, \quad (4.30)$$

$$\text{r.h.s.} \rightarrow a \ln(a) \implies \tilde{u}_1 \rightarrow \text{Constant} + \frac{a \ln(a)}{4H^2} - \frac{a}{4H^2}, \quad (4.31)$$

$$\text{r.h.s.} \rightarrow \ln(a) \implies \tilde{u}_1 \rightarrow \text{Constant} + \frac{\ln^2(a)}{6H^2} - \frac{\ln(a)}{9H^2}, \quad (4.32)$$

$$\text{r.h.s.} \rightarrow \frac{\ln(a)}{a} \implies \tilde{u}_1 \rightarrow \text{Constant} - \frac{\ln(a)}{2H^2 a} - \frac{1}{4H^2 a}. \quad (4.33)$$

Collecting all the contributions from conformal-conformal [1], kinetic-kinetic [2] and kinetic-conformal cross [2] parts, the regularized one loop self-mass-squared is

$$\begin{aligned}
-iM_{\text{reg}}^2(x; x') &= -iM_{\text{regconf}}^2(x; x') + -iM_{\text{regK}}^2(x; x') + -iM_{\text{regcross}}^2(x; x') , \\
&= i\kappa^2 a^2 \left(d_1 \square^2 + d_2 H^2 \square + d_3 H^4 + d_4 \frac{H^2}{a^2} \nabla^2 + d_5 \frac{H}{a} \partial_0 \square \right) \delta^D(x - x') \\
&\quad + \text{Table A.37} + \text{Table A.38} + \text{Table A.39} \\
&\quad + i\kappa^2 \frac{1}{(4\pi)^2} a^2 \left(\frac{11}{4} \ln(aa') H^2 \square + \left[\frac{9}{40} + \frac{11}{2} \ln(aa') \right] \frac{H^3}{a} \partial_0 \right. \\
&\quad \left. + [2 - 6 \ln(aa')] H^4 - \frac{3}{2} \ln(aa') \frac{H^2}{a^2} \nabla^2 \right) \delta^4(x - x') + \mathcal{O}(D - 4) ,
\end{aligned} \tag{4.34}$$

where Tables A.37, A.38 and A.39 are given in Appendix A.3 and the coefficients d_i are from combining results of [1, 2],

$$d_1 = \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ -\frac{(D-2)^2}{2^4(D-4)(D-3)(D-1)^2} + \frac{1}{72} \gamma + \mathcal{O}(D-4) \right\} , \tag{4.35}$$

$$\begin{aligned}
d_2 &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{D^6 - D^5 + 28D^4 + 56D^3 + 60D^2 + 288D - 144}{2^6(D-4)(D-3)(D-1)^3} \right. \\
&\quad \left. - \frac{327}{80} + \frac{823}{72} \gamma + \mathcal{O}(D-4) \right\} ,
\end{aligned} \tag{4.36}$$

$$\begin{aligned}
d_3 &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ -\frac{500D^7 - 4265D^6 + 11150D^5 + 5200D^4 - 85632D^3}{2^7 5(D-4)(D-3)(D-1)^2} \right. \\
&\quad \left. - \frac{178880D^2 - 157904D + 52336}{2^7 5(D-4)(D-3)(D-1)^2} - \frac{33421}{3600} + \frac{23207}{720} \gamma + \mathcal{O}(D-4) \right\} ,
\end{aligned} \tag{4.37}$$

$$\begin{aligned}
d_4 &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{27D^8 - 429D^7 + 3120D^6 - 13224D^5 + 34392D^4 - 57424D^3}{2^8 3(D-4)(D-3)(D-1)^2} \right. \\
&\quad \left. + \frac{60608D^2 - 36736D - 9600}{2^8 3(D-4)(D-3)(D-1)^2} + \frac{2759}{540} - \frac{2249}{108} \gamma + \mathcal{O}(D-4) \right\} ,
\end{aligned} \tag{4.38}$$

$$d_5 = \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ -\frac{(D-2)^3}{2^3(D-4)(D-3)(D-1)^2} + \frac{1}{18} \gamma + \mathcal{O}(D-4) \right\} . \tag{4.39}$$

Here $\gamma \approx 0.577215$ is Euler's constant.

Choosing the coefficients c_i of the counterterms

$$c_i = -d_i + \Delta c_i \quad \text{for } i = 1, 3, 4 , \tag{4.40}$$

$$c_2 = -d_2 + (D-2)d_1 + \Delta c_2 , \tag{4.41}$$

where Δc_i are arbitrary finite terms that remain after cancelling the divergent parts occurring in the primitive diagrams, and exploiting the relation between d_1 and d_5

$$d_5 = 2(D-2)d_1, \quad (4.42)$$

lead to the cancellation of all the divergent terms. Finally taking the unregulated limit ($D = 4$) gives

$$\begin{aligned} -iM_{\text{ren}}^2(x; x') &= i\kappa^2 a^2 \left(\Delta c_1 \square^2 + \Delta c_2 H^2 \square + \Delta c_3 H^4 + \Delta c_4 \frac{H^2}{a^2} \nabla^2 + 4\Delta c_1 \frac{H}{a} \partial_0 \square \right) \delta^4(x - x') \\ &\quad + \text{Table A.37} + \text{Table A.38} + \text{Table A.39} \\ &\quad + i\kappa^2 \frac{1}{(4\pi)^2} a^2 \left(\frac{11}{4} \ln(aa') H^2 \square + \left[\frac{9}{40} + \frac{11}{2} \ln(aa') \right] \frac{H^3}{a} \partial_0 \right. \\ &\quad \left. + [2 - 6 \ln(aa')] H^4 - \frac{3}{2} \ln(aa') \frac{H^2}{a^2} \nabla^2 \right) \delta^4(x - x'). \end{aligned} \quad (4.43)$$

4.3.2.1 Local Corrections

By inserting the local terms proportional to $\delta^4(x - x')$ into the right hand side of Eq. (4.27) and performing the integration gives local corrections to $\tilde{u}_1$ at late times ($\eta \rightarrow 0^-$ or equivalently $a \rightarrow \infty$),

$$\begin{aligned} \tilde{u}_{\text{local}} &= \tilde{u}_0 + \kappa^2 \tilde{u}_{1\text{local}} + \mathcal{O}(\kappa^4), \\ &\sim \frac{1}{\sqrt{2k}} \left\{ 1 - \frac{\kappa^2 H^2}{2^4 \pi^2} \left(\Delta c_4 - \frac{3}{4} \right) \ln(a) \right\}. \end{aligned} \quad (4.44)$$

While the zeroth order mode function $\tilde{u}_0$ goes to a constant, the first order local contribution to the mode function $\tilde{u}_{1\text{local}}$ grows like $\ln(a)$ though it is suppressed by a factor of $\kappa^2 H^2$.

4.3.2.2 Nonlocal Corrections

Nonlocal corrections come from inserting Tables A.37 - A.39 of nonlocal terms in Appendix A.3 into Eq. (4.27). We apply a seven-step procedure developed in [14] to evaluate (4.27):

- *Step 1:* Convert any factors of $1/y$ to $\ln(1/y)$ using the identities,

$$\frac{4}{y} = \frac{\square}{H^2} \left\{ \ln\left(\frac{y}{4}\right) \right\} + 3, \quad (4.45)$$

$$\frac{4}{y} \ln\left(\frac{y}{4}\right) = \frac{\square}{H^2} \left\{ \frac{1}{2} \ln^2\left(\frac{y}{4}\right) - \ln\left(\frac{y}{4}\right) \right\} + 3 \ln\left(\frac{y}{4}\right) - 2. \quad (4.46)$$

- *Step 2:* Factor out $\square$ and ∇^2 (which are w.r.t. x) outside the integration over x'^μ using the identities,

$$\frac{\square}{H^2} \longrightarrow -\left[a^2\partial_a^2 + 4a\partial_a + \frac{k^2}{a^2H^2}\right], \quad (4.47)$$

$$\nabla^2 \longrightarrow -k^2. \quad (4.48)$$

- *Step 3:* Combine the $++$ and $+-$ terms to extract a factor of i and a step function θ , which makes the effective mode equation (4.27) manifestly real and causal,

$$\ln\left(\frac{y_{++}}{4}\right) - \ln\left(\frac{y_{+-}}{4}\right) = 2\pi i\theta(\Delta\eta - \Delta x), \quad (4.49)$$

$$\ln^2\left(\frac{y_{++}}{4}\right) - \ln^2\left(\frac{y_{+-}}{4}\right) = 4\pi i\theta(\Delta\eta - \Delta x) \ln\left(\frac{1}{4}H^2aa'(\Delta\eta^2 - \Delta x^2)\right) \quad (4.50)$$

Here $a \equiv a(\eta)$, $a' \equiv a(\eta')$, $\Delta\eta \equiv \eta - \eta'$ and $\Delta x \equiv \|\vec{x} - \vec{x}'\|$. Note that the invariant length function y becomes

$$y_{\pm\pm} \longrightarrow -\frac{1}{4}H^2aa'(\Delta\eta^2 - \Delta x^2). \quad (4.51)$$

- *Step 4:* Make the change of variables $\vec{r} = \vec{x}' - \vec{x}$ to perform the angular integrations, and make the change of variables $r = \Delta\eta \cdot z$,

$$\begin{aligned} \int d^3x' \theta(\Delta\eta - \Delta x) F\left(\frac{1}{4}H^2aa'(\Delta\eta^2 - \Delta x^2)\right) e^{i\vec{k}\cdot(\vec{x}' - \vec{x})} \\ = 4\pi\theta(\Delta\eta) \int_0^{\Delta\eta} dr r^2 F\left(\frac{1}{4}H^2aa'(\Delta\eta^2 - r^2)\right) \frac{\sin(k\Delta x)}{k\Delta x}, \quad (4.52) \\ = 4\pi\theta(\Delta\eta) \Delta\eta^3 \int_0^1 dz z^2 F\left(aa'\left(\frac{1}{a'} - \frac{1}{a}\right)^2 \left(\frac{1-z^2}{4}\right)\right) \frac{\sin(k\Delta\eta z)}{k\Delta\eta z}. \end{aligned} \quad (4.53)$$

- *Step 5:* Perform the integration over z , which leads to a combination of elementary functions and sine and cosine integrals [96].
- *Step 6:* Make the last change of variables $a' = -1/H\eta'$ and perform the integration over a' .
- *Step 7:* Act external derivatives on the result of the integration with respect to a using (4.47).

Also, we adopt a simplification made in Ref. [14], which is to take the zeroth order solution $\tilde{u}_0(\eta, k)$ as its late time value $\tilde{u}_0(0, k) = 1/\sqrt{2k}$ and ignore the spatial phase factor $e^{i\vec{k}\cdot(\vec{x}'-\vec{x})}$:

$$\begin{aligned} & -\frac{1}{a^2} \int_{\eta_i}^0 d\eta' \int d^3x' \mathcal{M}_1^2(x; x') \tilde{u}_0(\eta', k) e^{i\vec{k}\cdot(\vec{x}'-\vec{x})} \\ & \longrightarrow -\frac{\tilde{u}_0(0, k)}{a^2} \int_{\eta_i}^0 d\eta' \int d^3x' \mathcal{M}_1^2(x; x'). \end{aligned} \quad (4.54)$$

This results in only elementary functions in the z integration (without sine and cosine integrals). Also, this simplification grants us neglecting any contribution from Tables A.37, A.38, and A.39 which contains a factor of ∇^2 . Then the integrals of the nonlocal terms take the following form,

$$-\tilde{u}_0(0, k) \frac{iH^8 a^{4-K}}{(4\pi)^4} \left(\frac{\square}{H^2}\right)^N \int_{\eta_i}^0 d\eta' a'^K \int d^3x' \left\{ f\left(\frac{y_{++}}{4}\right) - f\left(\frac{y_{+-}}{4}\right) \right\}, \quad (4.55)$$

where the constant K takes the values of 1, 2, 3, and 4 and the functions $f(x)$ are $1/x$, $\ln(x)/x$, $\ln x$ and $\ln^2 x$. The results of the integrations are given in Tables A.40-A.43 in Appendix A.3, and one example calculation using the seven-step procedure is provided in Appendix A.3. The results of (4.55) from conformal, kinetic and cross nonlocal parts are given in Tables A.44, A.45 and A.46 in Appendix A.3.

Finally, summing the results of Tables A.44, A.45 and A.46 (which are in the form of $\#_1 a \ln(a) + \#_2 a$) and using (4.28) and (4.31) gives

$$\begin{aligned} \tilde{u}_{\text{nonlocal}} &= \tilde{u}_0 + \kappa^2 \tilde{u}_{1\text{nonlocal}} + \mathcal{O}(\kappa^4), \\ &\sim \frac{1}{\sqrt{2k}} \left\{ 1 - \frac{\kappa^2 H^2}{\pi^2} \left[-\frac{42323}{2^9 \cdot 15} a \ln(a) + \frac{530953}{2^{10} \cdot 15} a \right] \right\}. \end{aligned} \quad (4.56)$$

The first order nonlocal contribution to the mode function $\tilde{u}_{1\text{nonlocal}}$ grows like $a \ln(a)$. It is dominant over the local contribution of $\ln(a)$ in (4.44) at late times ($a \rightarrow \infty$), thus the arbitrariness Δc_4 from the local counterterm is negligible in the late time limit.

4.4 Discussion

We have solved the one loop corrected, Schwinger-Keldysh effective field equation for a CC scalar interacting with a graviton during de Sitter inflation. It is instructive to take the inverse conformal transformation

$$\phi = \frac{\tilde{\phi}}{a} \quad \text{and} \quad u = \frac{\tilde{u}}{a}, \quad (4.57)$$

and express the result in physical time t ,

$$u_{\text{CC}} \sim \frac{1}{\sqrt{2k}} \left\{ \frac{1}{a} + GH^2 \left[\frac{42323}{2^5 \cdot 15\pi} \ln(a) - \frac{530953}{2^6 \cdot 15\pi} - \left(\Delta c_4 - \frac{3}{4} \right) \frac{\ln(a)}{a} \right] + \mathcal{O}(G^2 H^4) \right\}. \quad (4.58)$$

While the tree order CC scalar mode functions redshift to zero at late times, the one corrected mode functions gain a secular growth effect via the interaction with gravitons. This is in contrast to the case of the MMC scalar, for which the tree order mode functions become constant at late times, but they do not get significant corrections at one loop order [13, 14],

$$u_{\text{MMC}} \sim \frac{1}{\sqrt{2k}} \left\{ \frac{H}{k} + \mathcal{O}(G^2 H^4) \right\}. \quad (4.59)$$

It is interesting to see that the inflationary production of CC scalars is suppressed by $1/a$ due to the conformal invariance, but their interactions with gravitons turn out to be significant because the scalar is not differentiated (so not redshifted) in the conformal coupling. In the case of MMC scalar + graviton, although they both are copiously produced during inflation, they only interact through their kinetic energies, which redshift away at late times, hence they do not make significant loop corrections [13, 14, 37, 38].

Considering the undifferentiated scalar seems to be the key to have logarithmic corrections, this sort of secular loop effects might also present for a more general non-minimal coupling (such as $\xi R\phi^2$ with an arbitrary number ξ) in the context of Higgs inflation [137–139]. On the other hand, even though the loop counting parameter GH^2 is extremely small, the secular factor $\ln(a)$ would grow and eventually overcome it, then perturbation theory would break down. Hence it would be necessary to use a nonperturbative resummation method such as Starobinsky's stochastic technique [140]. There are nonperturbative techniques for various interactions in the literature [21, 29–31, 36, 140–149]. However, we leave these considerations for future work.

5. LOOP CORRECTIONS TO PRIMORDIAL NON-GAUSSIANITY

We discuss¹ quantum gravitational loop effects to observable quantities such as curvature power spectrum and primordial non-Gaussianity of Cosmic Microwave Background (CMB) radiation. We first review the previously shown case where one gets a time-dependence for zeta-zeta correlator due to loop corrections. Then we investigate the effect of loop corrections to primordial non-Gaussianity of CMB. We conclude that, even with a single scalar inflaton, one might get a huge value for non-Gaussianity which would exceed the observed value by at least 30 orders of magnitude. Finally we discuss the consequences of this result for scalar driven inflationary models.

5.1 Introduction

Most probably, the founders of quantum gravity did not have high hopes that what they were doing would some day be tested or even have observational consequences. The CMB power spectrum opened that window to us and now we can name cosmological perturbations as first quantum gravitational observables that were predicted by Mukhanov and Chibisov [91] for the scalar part and by Starobinsky [92] for the tensor part.

Since we already measured the lowest order effect in perturbation theory, the next logical step in any quantum field theory calculation is to go beyond this level, so-called the tree-level. This is mostly done in order to make precision tests of a particular model. Although one would expect only small corrections to already known physics (precision predictions) by calculating these higher order terms in perturbation theory, i.e. loops, new phenomena beyond our expectations can arise as was the case in the famous one-loop beta function calculation in quantum chromodynamics.

¹This chapter is based on the paper Loop Corrections to Primordial Non-Gaussianity, *Phys. Rev. D* 97, 043507, (2018) by **Boran, S.** and Kâhya, E. O. .

For the past fifteen years, there have been many efforts towards understanding loops in cosmology. Among this relatively large literature, the most influential works were those of Weinberg [105, 106]. In one of these works, Weinberg asserted a theorem [106] related to quantum loop effects in cosmology: in N -th order perturbation theory, quantum corrections can at most be of order $(\alpha \ln a(t))^N$, where α is the loop counting parameter and $a(t)$ is the scale factor.

There were plenty of discussions [107–111] not about the existence but the type of “infrared logarithm” that arise from the quantum contributions in cosmological correlations. Two types of infrared logarithm factors that appeared are time-dependent and time-independent. The obvious enticing aspect of the time-dependent infrared logarithm is that it grows with time. Having this case, the smallness of the loop counting parameter in quantum gravity gets counterbalanced by time-dependent infrared logarithms. If one assumes that we observe 50 e -folds of inflationary era, this time-dependent enhancement would only bring a factor of 50. Therefore there is not much hope of observing this effect any time soon.

Most of the discussions were around the quantum corrections to two-point correlation functions, i.e. power spectrum; since it is a quantity which is measured more accurately. The debate between the time-dependence and the time-independence camp went on and both parties published explicit calculations to support their claims [107–111]. The author of this work also contributed to this discussion claiming the time-dependence. In this work, we do not want to discuss the strengths and weaknesses of each point of view, but rather want to point out that the “small” time-dependent loop effects might not be that small; if one looks at higher orders of correlation functions such as three-point functions and even higher. The loop effects to the three-point function were discussed in two separate works. The first one is the work of Giddings and Sloth where they assumed the semiclassical approximation holds [58]. The second work is the work of Cogollo *et al.* [150] where there is an extra scalar field which induces the time-dependent effect. In this work, we will calculate the loop corrections to external legs of the three-point function using the Hartree approximation. We will show that the loop corrections have the potential to dominate the tree-level term by 30 orders of magnitude and therefore the perturbation theory will break down. If this effect survives after a nonperturbative analysis, this would immediately lead

to ruling out all single field inflationary models due to observational constraints on non-Gaussianity of the primordial curvature perturbation [151]. This claim might appear very odd, if one naively looks at the above theorem of Weinberg. But it turns out that the constancy of the tree-level mode function and time-dependency of loop corrections determine the faith of their contribution to the three-point function, bispectrum.

This paper is organized as follows. In Sec. 5.2 we review the case of the time-dependent zeta-zeta correlator arising from self-interaction of zeta at one-loop order. In Sec. 5.3 we use this one-loop corrected time-dependent mode function to calculate the one-loop corrected primordial bispectrum. We give our conclusions in Sec. 5.4.

5.2 Slow-roll inflation and time-dependent zeta-zeta correlator

The action of the model that we would like to consider consists of a scalar field ϕ , an Einstein-Hilbert part and a standard kinetic term

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} - \frac{1}{2} \phi_{,\mu} \phi_{,\nu} g^{\mu\nu} - V(\phi) \right]. \quad (5.1)$$

The metric is that of the Friedmann-Robertson-Walker one which our universe seems to prefer,

$$ds^2 = -dt^2 + a^2(t) d\vec{x} \cdot d\vec{x} \Rightarrow H \equiv \frac{\dot{a}}{a}. \quad (5.2)$$

We choose the background inflaton field to be constant at equal-time hypersurfaces as Maldacena [152] and Weinberg [105],

$$\phi(t, \vec{x}) - \phi_0(t) = 0. \quad (5.3)$$

The other $(D-1)$ conditions come from defining the unimodular part of the metric $\tilde{g}_{ij}$,

$$g_{ij} = a^2(t) e^{2\zeta(t, \vec{x})} \tilde{g}_{ij}(t, \vec{x}) \Rightarrow \sqrt{g} = a^{D-1} e^{(D-1)\zeta}. \quad (5.4)$$

By choosing the gauge in the above manner we switched from inflaton field, which is the dynamical variable ϕ of our theory, to ζ now parametrizing scalar fluctuations. Using Einstein's equations for the background scalar field ϕ , one can express its time-derivative in terms of the Hubble parameter as

$$\dot{\phi}^2 = -\frac{\dot{H}}{4\pi G}. \quad (5.5)$$

Another important quantity, called the first slow-roll parameter ε , is defined as

$$\varepsilon \equiv -\frac{\dot{H}}{H^2} \ll 1. \quad (5.6)$$

The next step is using perturbation theory for small fluctuations of the scalar and tensor fields. It became customary to use Arnowitt-Deser-Misner formalism to get the quadratic, cubic and even higher order parts of the action. The quadratic part of the action for ζ is [152]

$$S_{\zeta}^{(2)} = \frac{1}{8\pi G} \int d^4x \varepsilon [a^3 \dot{\zeta}^2 - a(\partial\zeta)^2] \quad (5.7)$$

and the cubic part of zeta is

$$S_{\zeta}^{(3)} = \frac{1}{2\pi G} \int d^4x \varepsilon^2 a^5 H \dot{\zeta}^2 \partial^{-2} \dot{\zeta} \equiv \int dt L_3(t). \quad (5.8)$$

One can vary the quadratic part of the action and equate to zero, to get the equation of motion for ζ as

$$-\partial_t(a^3 \varepsilon \dot{\zeta}) + a\varepsilon \partial^2 \zeta = 0. \quad (5.9)$$

The standard way of solving this equation for quantum fields is going into momentum space and expressing ζ as a mode sum

$$\zeta_{[k]}(t) = u_k(t)a + u_k^*(t)a_{-}^{\dagger}, \quad (5.10)$$

where $a_{-}^{\dagger}$ and a are creation and annihilation operators that obey canonical quantization conditions.

It is best to go to conformal time to write the expression for the mode function,

$$\begin{aligned} d\eta \equiv -adt \Rightarrow ds^2 &= -dt^2 + a^2(t)d\vec{x} \cdot d\vec{x} \\ &= a^2(t)(-d\eta^2 + d\vec{x} \cdot d\vec{x}), \end{aligned} \quad (5.11)$$

so that the geometry is conformally flat. Another motivation for choosing conformal time is related to the fact that there is not a unique choice of vacuum in curved space. One takes the expression (5.10) and uses that to solve equation (5.9) for $u_k(\eta)$,

$$u_k(\eta) = \frac{H}{\sqrt{2\varepsilon}} \frac{1}{\sqrt{2k^3}} (1 + ik\eta) e^{-ik\eta}, \quad (5.12)$$

which corresponds to the positive frequency modes. By choosing conformal time for coordinate system, it is easy to see that this solution for the mode function behaves

like Minkowski in the early time limit. This solution is called Bunch-Davies vacuum solution [153].

Let us define the curvature power spectrum:

$$\Delta_{\mathcal{R}}^2(k, t) \equiv \frac{k^3}{2\pi^2} \int d^3x e^{-i\vec{k}\cdot\vec{x}} \langle \Omega | \mathcal{R}(t, \vec{0}) \mathcal{R}(t, \vec{x}) | \Omega \rangle, \quad (5.13)$$

where $\mathcal{R}$ is related to the 3-curvature and is equal to ζ at the linearized order [154];

$$\mathcal{R}(t, \vec{x}) \equiv -\frac{a^2(t)}{4\nabla^2} R = \zeta(t, \vec{x}) + O(\zeta^2, \zeta h, h^2). \quad (5.14)$$

Therefore curvature power spectrum also goes by the name “zeta-zeta correlator” as well. The latest value of the curvature power spectrum constructed from measurements is [155]

$$\Delta_{\mathcal{R}}^2(k) = \left(2.198^{+0.076}_{-0.085}\right) \times 10^{-9} \left(\frac{k}{0.002 \text{ Mpc}^{-1}}\right)^{-0.0345 \pm 0.0062}. \quad (5.15)$$

The theoretical prediction at tree-level gives us

$$\left[\Delta_{\mathcal{R}}^2(k)\right]_{\text{tree}} \approx \frac{4Gk^3}{\pi} \times |u(t, k)|^2 \approx \frac{GH^2(t_k)}{\pi\epsilon}. \quad (5.16)$$

Although one would expect that tree-order quantum gravity calculations capture the full effect, it is natural to wonder what happens beyond that. Therefore we want to know if loop corrections to this measurable quantity make any difference. There have been many efforts to answer this questions in the past ten years [58, 156–166].

In a particular curious case [108] it was shown that one can get an enhanced time-dependent correction to ζ - ζ correlator at one-loop order coming from the Feynman diagrams in Fig. 5.1.

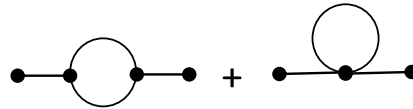


Figure 5.1: One-loop correction is sourced by cubic and quartic self-interactions of ζ .

The one-loop corrected curvature power spectrum gives

$$\left[\Delta_{\mathcal{R}}^2(k, t)\right]_{\zeta \text{ loops}} \approx \frac{GH^2}{\pi\epsilon} \left\{ 1 + \frac{27GH^2}{4\pi\epsilon} \ln(a) + O(G^2H^4) \right\}. \quad (5.17)$$

This corresponds to a correction to the tree-level scalar mode function as

$$\Rightarrow u(t, k) \approx \frac{H}{\sqrt{2\varepsilon}} \frac{1}{\sqrt{2k^3}} \left\{ 1 + \frac{27GH^2}{8\pi\varepsilon} \ln(a) \right\}, \quad (5.18)$$

if one uses Hartree approximation.

The ζ - ζ correlator becomes time-dependent if there is least one undifferentiated field in the action at the relevant order. These so-called infrared logarithms, as well as $1/\varepsilon$ term, enhance this one-loop effect by 3 orders of magnitude. But the smallness of $GH^2 \approx 10^{-10}$ overshadows this enhancement and makes the total one-loop correction to be at most at the order of 10^{-6} . The degree of the precision of the current experiments is well below the necessary level to untangle this one-loop effect. But still the effect is not hopelessly small.

For the last five years there has been some discussion about the time-dependence of the ζ - ζ correlator. It has even been claimed [111] that this quantity is constant at all loops, which we find to be highly dubious since even at tree-level it only asymptotes to a constant. The point of this work is not to argue the time-dependence of ζ - ζ correlator, but rather go towards another direction; which is loop corrections to three-point function and to see what the consequences of time dependence of ζ are. We believe that the real enhancement of time-dependent ζ - ζ correlator arises if one calculates three-point function for ζ . It turns out the one-loop correction to this quantity might totally dominate the tree-level result. Therefore when searching for enhanced quantum gravity corrections, the more interesting quantity to calculate is the three-point function, which is the subject of the next section.

5.3 Bispectrum at tree-level

One can write the primordial bispectrum in terms of the Fourier transformed three-point function as

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_\zeta(k_1, k_2, k_3). \quad (5.19)$$

Assuming a local form for the bispectrum where the non-Gaussian ζ field is produced from the Gaussian background ζ_g field as

$$\zeta([x]) = \zeta_g([x]) + (3/5) f_{\text{NL}} \zeta_g^2([x]) + \mathcal{O}(\zeta_g^3). \quad (5.20)$$

One can show that the bispectrum peaks at the so called “squeezed” triangle, for which one takes one wave number much smaller than the other two, i.e. $k_1 \approx k_2 \gg k_3$. For the case of squeezed limit bispectrum can be expressed in terms of power spectrum with the following equation

$$B_{\zeta}^{\text{local}}(k_1, k_2, k_3)_{k_1 \approx k_2 \gg k_3} \approx \frac{12}{5} f_{\text{NL}} P(k_1) P(k_3) , \quad (5.21)$$

where the late time limit of the power spectrum is

$$P(k) = |u_k|_{\eta \rightarrow 0}^2 = \frac{H^2}{2\epsilon} \frac{1}{2k^3} . \quad (5.22)$$

If Creminelli-Zaldarriaga consistency [167] condition for single field inflation models hold, the bispectrum in the local limit (or squeezed-limit) can be written as

$$B_{\zeta}(k_3 \ll k_1) = (1 - n_s) P(k_1) P(k_3) , \quad (5.23)$$

where $n_s(k)$ is called the spectral tilt index and is defined as

$$n_s - 1 \equiv \frac{d \ln(P(k))}{d \ln(k)} . \quad (5.24)$$

Instead of expressing the three-point function in terms of two-point functions, one can directly compute the in-in expectation value of ζ and use (5.8) for the interaction Hamiltonian and can get the following expression for the bispectrum [168]:

$$\begin{aligned} B_{\zeta}(k_1, k_2, k_3) &= 8i \frac{\epsilon^2}{H^2} \sum_{k_i} \left(\frac{1}{k_i^2} \right) u_{k_1}(\bar{\eta}) u_{k_2}(\bar{\eta}) u_{k_3}(\bar{\eta}) \\ &\times \int_{\eta_0}^{\bar{\eta}} d\eta \frac{1}{\eta^3} u'_{k_1*} u'_{k_2*} u'_{k_3*} + c.c. . \end{aligned} \quad (5.25)$$

The main point of this calculation is the integral that we have in the above expression for the bispectrum,

$$\int_{\eta_0}^{\bar{\eta}} d\eta \frac{1}{\eta^3} \tilde{u}'_{k_1*} \tilde{u}'_{k_2*} \tilde{u}'_{k_3*} . \quad (5.26)$$

If we take the tree-order mode function for the above expression it is obvious that we will get a small non-Gaussianity. This is due to the fact that the three-point function, therefore non-Gaussianity, is proportional to the change of the mode function for each wave number k_1, k_2, k_3 . Since for each mode the mode function itself goes to a constant after the horizon crossing, the change of those tree-level mode functions will be very small. On the other hand, the two-point function, therefore power spectrum,

is proportional to the magnitude of the mode function. Let us highlight this point by giving equations:

$$\begin{aligned}
\text{power spectrum} &\sim \langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \rangle \sim \delta^3(\mathbf{k}_1 + \mathbf{k}_2) |u_k|^2 \\
\text{non - Gaussianity} &\sim \langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \\
&\sim \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) u_{k_1}(\bar{\eta}) u_{k_2}(\bar{\eta}) u_{k_3}(\bar{\eta}) \\
&\times \int_{\eta_0}^{\bar{\eta}} d\eta \frac{1}{\eta^3} u'_{k_1*} u'_{k_2*} u'_{k_3*} + c.c.
\end{aligned} \tag{5.27}$$

5.4 One-loop correction to bispectrum: a particular example

We would like to find the answer to the following question: How big is the effect of loops to n-point functions of primordial curvature perturbation during Inflation? The answer of the above question for two-point and three-point functions is related to the magnitude and the time derivative of the scalar mode functions, respectively.

One should do the full computation to give a definitive answer of time dependence of bispectrum. In this work, our aim is to show that it is possible to get huge enhancements at one-loop order and for that we would like to consider the simplest case where the external legs are corrected

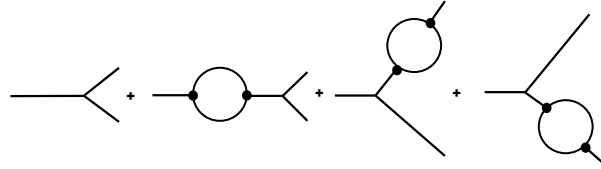


Figure 5.2: Bispectrum: Tree-level and corrections to external legs.

Here we would like to make a further simplification, namely use the form of the one-loop corrected mode function (5.27) where we applied a Hartree approximation.

The one-loop corrected mode function expression and its derivative with respect to conformal time and their long wavelength limits ($k\eta \ll 1$) are

$$\begin{aligned}
u_{tree} &= \frac{H}{\sqrt{2\varepsilon}} \frac{1}{\sqrt{2k^3}} (1 + ik\eta) e^{-ik\eta} \\
&\Rightarrow \frac{H}{\sqrt{2}} \frac{1}{\sqrt{2k^3}} \left\{ 1 + \frac{k^2 \eta^2}{2} + \dots \right\},
\end{aligned} \tag{5.28}$$

$$\begin{aligned}
u_{1\text{-loop}} &\Rightarrow \frac{H}{\sqrt{2\varepsilon}} \frac{1}{\sqrt{2k^3}} \left\{ 1 + O(1) GH^2 \ln(a) \right\} \\
&= u_{tree} \left\{ 1 + O(1) GH^2 \ln(a) \right\},
\end{aligned} \tag{5.29}$$

$$u'_{tree} \Rightarrow \frac{H}{\sqrt{2\varepsilon}} \frac{1}{\sqrt{2k^3}} \left\{ k^2 \eta + \dots \right\}, \quad (5.30)$$

$$\begin{aligned} u'_{1-loop} &\Rightarrow \frac{H}{\sqrt{2\varepsilon}} \frac{1}{\sqrt{2k^3}} k^2 \eta \left\{ 1 + O(1) GH^2 \frac{1}{k^2 \eta^2} \right\} \\ &= u'_{tree} \left\{ 1 + O(1) GH^2 \frac{1}{k^2 \eta^2} + \dots \right\}. \end{aligned} \quad (5.31)$$

The difference between the one-loop corrected mode function's and tree-level mode function's time derivative is of the order of $GH^2 \approx 10^{-10}$ as expected, but also multiplied with an additional factor of $1/k^2 \eta^2$. For the super-horizon modes ($k\eta \ll 1$) with the relevant 50 e -folds this brings an extra factor of 10^{20} which makes the one-loop correction to dwarf the tree-level part of the mode function.

The mathematical reasons for this huge effect is the following. The derivative with respect to conformal time brings an extra factor of $Ha(t)$ when it acts on a power of $a(t)$. Since the tree-level mode function is constant after horizon crossing, this time derivative does no good to boost the leading term, it simply annuls it. But for the case one-loop corrected mode function, the time derivative acts on the $\ln a$ and gives a chance to the constant leading term of the tree-level part of the mode function to survive. Not only does it survive, but also it gets boosted by the extra factor of $Ha(t)$.

Since the integral that appear in the the non-Gaussianity ($\zeta\zeta\zeta$ correlator) (5.27) has three factors of u'_{1-loop} the one-loop correction to the three-point function is 30 orders magnitude bigger than the tree-level term. Since the one-loop correction to $P(k)$ terms are very small, this extremely large value of bispectrum can only be achieved by having a huge f_{NL} parameter, if we look at equation (5.21). It also implies that Creminelli-Zaldarriaga consistency condition for single scalar field is not satisfied here, since equation (5.23) could not be satisfied with a bispectrum this big. However this does not mean the invalidation of the consistency condition, since the condition assumes time-independency a priori, although there are cases where the condition is violated [169, 170]. But most importantly an f_{NL} parameter of this magnitude results into ruling out all single field inflation models due to the observational limits on the non-Gaussianity parameter.

We would like to point out that we are not claiming Weinberg's theorem is incorrect. Mathematically what happens is that, the time derivative acting on the constant term,

which is the leading term in the long wavelength expansion of scalar mode function kills it. On the other hand, the quantum corrected time-dependent mode functions long wavelength expansion leave a room for the constant term to survive by letting the time derivative hit on the infrared logarithm. Due to this wondrous interaction, these one-loop terms dominate the tree-level term by 30 orders of magnitude, which results in breaking down of the perturbation theory. Therefore as Weinberg pointed out [106] we see the need of a non-perturbative method for cosmological correlations.

The integrals that appear in (5.27) can be evaluated [171] analytically in a general vacuum choice, called non-Bunch-Davies initial state which would lead to an increase of the non-Gaussianity parameter f_{NL} even at the tree-level [168, 172]. For the case of non-Bunch-Davies initial state the one-loop term again dominates the tree-level one and be ruled out as well.

5.5 Discussion

The success of inflationary cosmology is appalling. This simple idea solves homogeneity, flatness, horizon, isotropy and primordial monopole problems of standard cosmology with a single shot [17, 18, 173]. With inflation, linking quantum physics with cosmology, we can understand the origin of all matter from primordial quantum fluctuations. Getting first quantum gravitational data, such as curvature power spectrum with small error bars is a major success in itself.

It is therefore time to go beyond this tree-level effect and investigate possible consequences, which we can name as precision inflationary cosmology. Towards this direction, one possible thing to do is calculating loop corrections to cosmological correlations. At first look, one would naturally think that this is a futile effort due to the smallness of the loop counting parameter GH^2 . But still there was a lot of attention to loop corrections to power spectrum despite the smallness of them.

Cosmological loop corrections bring a typical infrared logarithm and are divided into three categories according to the form of the logarithmic factors: $\text{Log}(H\mu)$, $\text{Log}(kL)$ and $\text{Log}(a(t))$ [110]. The first case is claimed to be due to making an error in the implementing diffeomorphism invariant regularization and the second being a projection effect that will be removed if one computes observable quantities. The final

case is also dismissed in the mentioned work on the grounds of symmetry arguments as well as extrapolating this time-dependent effect to reheating and baryogenesis and claiming that predictivity of inflation will be lost. One can certainly reply to the above criticisms and perhaps one should. But in this work, we would like to bring a different viewpoint to this discussion that is more dramatic.

First of all, time-dependent zeta do occur even without loop corrections, such as multifield inflationary models and entropy perturbations. The time-dependence that we are interested in, that has the form of $\text{Log}(a(t))$, are originated from loop corrections. We investigated the minimal case where the only scalar field is the inflaton. We first reviewed time-dependent loop corrections to two-point functions, i.e. power spectrum, which arises due to ζ - ζ self interactions at one-loop order. In principle they are important; on the other hand, from an observational perspective they are irrelevant at the moment. We took the one-loop corrected scalar mode function and used that to calculate the loop corrections to external legs of the three-point function using Hartree approximation. We concluded that they grow with the square of the scale factor. We showed that the loop corrections have the potential of dominating the tree-level term by 30 orders of magnitude and lead to breaking down of the perturbation theory. If this effect survives after a nonperturbative analysis, that would result into an immediate ruling out all single-scalar driven models of inflation.

Therefore, non-Gaussianity is a better place, compared to power spectrum, to look for quantum gravitational corrections. The technical reason of this is:

1. Power spectrum is related to the magnitude of mode function.

Therefore it goes like : constant (tree-level) + a small correction (loops)

2. Non-Gaussianity is related to the time derivative of the mode function.

Therefore it goes like : almost zero (tree-level) + a not so small correction(loops)
compared to zero

Therefore the real treasure is hidden in the higher order correlation functions, not in the power spectrum. We also showed that this would imply a huge (10^{20} times bigger than tree-level prediction) non-Gaussianity f_{NL} parameter, leading to an immediate contradiction with the constraints on observed value of f_{NL} parameter.

At this point we would like to discuss the possible ways of avoiding huge f_{NL} parameter. Let us remember that there is another type of diagram that will contribute at one-loop order to three-point function. It might turn out that this vertex correction (Fig. 5.3) cancel the total contribution coming from Fig. 5.2.

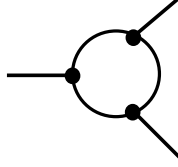


Figure 5.3: The remaining diagram at one-loop order.

For that end it is useful to look at the work of Cogolo *et al.* [150], where they show that this kind of diagram dominates the whole series of diagrams. They consider an extra scalar field and the effect arises due to that; but still a similar thing might happen for the single scalar field situation.

We want to conclude the discussion section by giving six points that should further be investigated in detail which might change the picture:

- i. Vertex correction
- ii. Hartree approximation
- iii. Single scalar field (inflaton), adiabatic perturbations
- iv. Time-dependent ζ - ζ correlator from loops
- v. Almost constant slow-roll parameter ε
- vi. A nonperturbative analysis.

The first possibility is the vertex correction exactly cancelling the effects given above. It might be that using Hartree approximation is the source of the mentioned effect. For that, one should do the full calculation and see if the effect is not there. One can imagine scenarios where a spectator field causing a similar effect which might cancel the ζ loops. This only happens for particular situations [150]. One could also try to incorporate the time-dependence of the ε parameter and investigate the consequences of that.

Our work highlights the need of a nonperturbative method to cosmological correlations. A nonperturbative method was found by Starobinsky and Yokoyama [141] for self-interacting scalar fields and Tsamis and Woodard for Scalar Quantum Electrodynamics [100], where they were able to resum the leading infrared log terms in the whole perturbative series. And it might turn out that, one can avoid a big f_{NL} parameter after calculating the quantum effects using a nonperturbative method. One can simply say that it is the fourth assumption that is wrong and maybe it is so. But no matter what the solution is, quantum loop corrections that result into time-dependent scalar mode functions have consequences that are so important and will be of such magnitude that they can not be swept under the rug.

We would like to end our discussion by pointing out to a curious work done by Pattison *et al.* [174], where the probability density function (PDF) of curvature perturbations were calculated by using stochastic δN formalism. During this period, due to quantum diffusion effects, stochastic force would determine the inflaton dynamics and PDF of ζ has the form elliptic theta functions. They claim that, in the limit where the potential is exactly flat and stochastic effects dominate, one gets highly non-Gaussian curvature perturbations. This claim, which might be an artefact of using stochastic δN formalism, should certainly be checked by making rigorous loop calculations. This work also shows the need for doing a one-loop calculation (fully renormalized with all the relevant interactions included) of the three-point function in particular and n-point functions of ζ in general.



6. CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

The purpose of this thesis was to illustrate the significance of these quantum gravity effects for the structure formation. It was demonstrated via our published works [1–3, 12].

As a particular example, we investigated the interaction of a conformally coupled scalar (CCS) with a graviton during de Sitter inflation. We calculated the quantity, SMS at one-loop order, in order to get the lowest order quantum correction to CCS by using particular calculation techniques.

In Chapter 2 (Ref. [1]), one-loop SMS contributions of solely conformal-conformal piece were constructed and they were calculated for a CCS interacting with a graviton. SMS contributions were fully renormalized by applying the dimensional regularization. The divergences were absorbed with counterterms for conformal-conformal pieces of SMS. The obtained finite terms were held to use in the next step, to solve the effective field equation.

In Chapter 3 (Ref. [2]), one-loop SMS contributions for the kinetic-kinetic and cross (kinetic-conformal plus conformal-kinetic) pieces were constructed and they were calculated for a CCS interacting with a graviton. Again, SMS contributions were fully renormalized by applying the dimensional regularization technique. But this time, the divergences were absorbed with counterterms for kinetic-kinetic and cross pieces of SMS. As was done in the same calculation of conformal-conformal piece, the obtained finite terms were held to use in the next step, to solve the effective field equation.

The finite results stemmed from the Chapters 2 and 3 were used in Chapter 4 (Ref. [3]). The one-loop corrected Schwinger-Keldysh effective field equation for a CCS interacting with a graviton during de Sitter inflation was solved. It was seen that the quantum corrected scalar mode functions grow secularly, proportional to a scale factor

times logarithm ($a \ln(a)$) of the scale factor at late times. Therefore, a nonperturbative calculation technique is needed.

In Chapter 5 (Ref. [12]), the effect of loop corrections to primordial non-Gaussianity of Cosmic Microwave Background (CMB) was investigated and the following results were reached:

- A huge value for the non-Gaussianity which would exceed the observed value by at least 30 orders of magnitude may be obtained for single scalar inflaton.
- It is shown that the non-Gaussianity is a better place to look for quantum gravitational corrections:
 1. Power spectrum is related to the magnitude of mode function.
Therefore it goes like : constant (tree-level) + a small correction (loops).
 2. Non-Gaussianity is related to the time derivative of the mode function.
Therefore it goes like : almost zero (tree-level) + a not so small correction (loops) compared to zero.

6.2 Recommendations

According to outcomes obtained from Chapters 2, 3 and 4, the perturbative calculation method for the loop corrections is not enough. Thus, a new nonperturbative calculation technique is needed as was done in those Refs. [21, 29–31, 36, 140–149]. A nonperturbative calculation method can be constructed for a this particular interaction in future work.

As a continuation of work done in Chapter 5, the remaining one-loop correction as was given by Fig. 5.3 in Chapter 5 can be calculated. For this calculation, one can also go beyond Hartree approximation and obtain the exact mode function expression in order to check secular effects.

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APPENDICES

APPENDIX A.1

Tables for Identities to extract derivatives

Tables for the coefficient functions of external operators

APPENDIX A.2

Tables for the coefficient functions of external operators for 3-point kinetic part

Tables for the coefficient functions of external operators for 3-point cross part

APPENDIX A.3

The Tables For All Finite Nonlocal Contributions From [1,2]

Key Integral Tables

The Examples For Calculating Key Integrals From Table A.42

Result Tables

The Examples From The Table A.45



APPENDICES

APPENDIX A.1

Tables for Identities to extract derivatives

Table A.1: For $F(y)\nabla^4 A(y)$.

External operators	The functions they act on
γ_1	$4(D^2 - 1)(I[F'A''] - I^2[F'A^{(3)}])$
ε_1	$4(D + 1)I^3[F'A^{(3)}]$
δ	$I^4[FA^{(4)}]$

Table A.2: For $F(y)\partial_0^2\partial_0'^2A(y)$.

External operators	The functions they act on
α	$I^4[FA^{(4)}]$
β_1	$\frac{1}{2}(D-2)^2I^4[FA^{(4)}]$
β_2	$-(D-2)(D-1)I^4[FA^{(4)}]$
γ_1	$\frac{1}{2}(D-2)^3(D-1)I^4[FA^{(4)}] - 2(D-1)(I^2[F'A'] + I^3[F'A''] + I^4[F'A^{(3)}])$ $+ (12 + 40y - 26y^2)(I[F'A''] + I^2[F'A^{(3)}]) + 8(D-1)(I^2[F'A''] + I^3[F'A^{(3)}])$ $+ (-48y + 64y^2 - 20y^3)I[F'A^{(3)}] + (D-1)(-24I^2[F'A^{(3)}] + 28I[yI[F'A^{(3)}]]$ $- 14I[y^2I[F'A^{(3)}]]) - 20(D-1)I[yI[F'A'']] + yI^2[FA^{(3)}]$
γ_2	$\frac{1}{2}(D-2)^2yI^3[FA^{(4)}] - 32y(I[F'A''] + I^2[FA^{(3)}])$ $- 12(D-1)(I^2[F'A''] + I^3[F'A^{(3)}])$ $+ (D-1)(24I^2[F'A^{(3)}] - 12I[yI[F'A^{(3)}]]) + (48y - 26y^2)I[F'A^{(3)}]$
γ_3	$\frac{1}{2}(D-2)^2(y^2I^2[FA^{(4)}] + yI^3[FA^{(4)}]) + \frac{1}{2}(D-2)^3yI^3[FA^{(4)}]$
δ	$-I^4[FA^{(4)}](y)$
ε_1	$\frac{1}{2}(D-2)2DI^4[FA^{(4)}] + 2(D-2)I[yI^2[FA^{(4)}]] - \frac{1}{2}(D-2)I[y^2I^2[FA^{(4)}]]$ $- \frac{1}{2}(D-2)DI[yI^3[FA^{(4)}]] - \frac{1}{4}(D-2)^3I^5[FA^{(4)}] - (D-2)^2I^4[FA^{(4)}]$ $- (D-2)(D-1)I^4[FA^{(4)}] + (I^3[F'A'] + I^4[F'A''] + I^5[F'A^{(3)}])$ $+ 10I^2[yI[F'A'']] + yI^2[F'A^{(3)}] - 4(I^3[F'A''] + I^4[F'A^{(3)}]) + 7I^2[y^2I[F'A^{(3)}]]$ $- 14I^2[yI[F'A^{(3)}]] + 12I^3[F'A^{(3)}]$
ε_2	$6(I^3[F'A''] + I^4[F'A^{(3)}]) + 6I^2[yI[F'A^{(3)}]] - 12I^3[F'A^{(3)}]$
ε_3	$-(D-2)yI^3[FA^{(4)}]$
ζ	$I^4[FA^{(4)}] + \frac{1}{2}(D-2)I^5[FA^{(4)}]$

Table A.3: For $F(y)\nabla^2(\partial_0^2 + \partial_0'^2)A(y)$.

External operators	The functions they act on
γ_1	$8(D-1)^2(I^2[F'A''] + I^3[F'A^{(3)}]) - 8(D-1)(I[F'A''] + I^2[F'A^{(3)}])$ $+ 16(D-1)y(I[F'A''] + I^2[F'A^{(3)}]) + (-16(D-1)y + 8(D-1)y^2)I[F'A^{(3)}]$ $- 8(D-1)(D+4)I^2[F'A^{(3)}] + 4(D-1)^2I[yI[F'A^{(3)}]]$
γ_2	$4(D-1)(I[F'A'] + I^2[F'A''] + I^3[F'A^{(3)}]) - 12(D-1)(I^2[F'A''] + I^3[F'A^{(3)}])$ $+ 8(D-1)y(I[F'A''] + I^2[F'A^{(3)}]) + 2(D-1)y^2I[F'A^{(3)}]$ $+ 24(D-1)I^2[F'A^{(3)}] - 12(D-1)I[yI[F'A^{(3)}]]$
δ	$-I^4[FA^{(4)}]$
ε_1	$-(D-2)(D-1)I^4[FA^{(4)}] + 4(D-1)(I^3[F'A''] + I^4[F'A^{(3)}])$ $+ 4(D+4)I^3[F'A^{(3)}] - 2(D-1)I^2[yI[F'A^{(3)}]]$
ε_2	$6(I^3[F'A''] + I^4[F'A^{(3)}]) + 6I^2[yI[F'A^{(3)}]] - 12I^3[F'A^{(3)}]$
ε_3	$-(D-2)yI^3[FA^{(4)}]$
ζ	$2I^4[FA^{(4)}] + \frac{1}{2}(D-2)I^5[FA^{(4)}]$

Table A.4: For $F(y)\nabla^2(\partial_0\partial_0'^2)B(y)$.

External operators	The functions they act on
γ_1	$2(D-1)(I[F'B'] + I^2[F'B''] + I^3[F'B^{(3)}]) - 4(D^2-1)(I^2[F'B''] + I^3[F'B^{(3)}])$ $+((4-2y)(D-1))(I[F'B''] + I^2[F'B^{(3)}]) + (8(D-1)y - 2(D-1)y^2)I[F'B^{(3)}]$ $+4((D-1)(D+4))I^2[F'B^{(3)}] - 2((D-1)(D+4))I[yI[F'B^{(3)}]]$
γ_2	$-12(D-1)I^2[F'B^{(3)}]$
δ	$\frac{1}{2}I^4[FB^{(4)}]$
ε_1	$\frac{1}{2}(D-2)(D-1)I^4[FB^{(4)}] + 2(D+1)(I^3[F'B''] + I^4[F'B^{(3)}])$ $+ (D+4)I^2[yI[F'B^{(3)}]] - 2(D+4)I^3[F'B^{(3)}]$
ε_2	$\frac{1}{2}yI^3[FB^{(4)}] + 6I^3[F'B^{(3)}]$
ε_3	$\frac{1}{2}y^2I^2[FB^{(4)}] + \frac{1}{2}(D-1)yI^3[FB^{(4)}]$
ζ	$-I^4[FB^{(4)}] - \frac{1}{4}(D-2)I^5[FB^{(4)}]$

Tables for the coefficient functions of external operators

Table A.5: For $\alpha \equiv (aa')^2\Box^2$ type terms.

$f_\alpha(y) = C_{1a} f_{\alpha(1a)}(y) + C_{1c} f_{\alpha(1c)}(y),$	
$f_{\alpha(1a)}(y)$	$I^4[FA^{(4)}]$
$f_{\alpha(1c)}(y)$	$I^4[FC^{(4)}]$
Total for $f_\alpha(y)$	
$-\frac{(D+2)(D+4)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{128(D-2)(D+1)}\left(\frac{1}{y}\right)^{D-2} - \frac{3H^4}{32\pi^4}\frac{1}{y}.$	

Table A.6: For $\beta_1 \equiv (aa')^2 H^2 \square$ type terms.

$f_{\beta_1}(y) = C_{1a} f_{\beta_1(1a)}(y) + C_{1c} f_{\beta_1(1c)}(y),$	
$f_{\beta_1(1a)}(y)$	$-(D-2)(D-1) I^4[FA^{(4)}]$
$f_{\beta_1(1c)}(y)$	$-(D-2)(D-1) I^4[FC^{(4)}]$
Total for $f_{\beta_1}(y)$	
$\frac{(D-1)(D+2)(D+4)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{128(D+1)} \left(\frac{1}{y}\right)^{D-2} + \frac{9H^4}{16\pi^4} \frac{1}{y}.$	

Table A.7: For $\beta_2 \equiv aa'(a^2 + a'^2)H^2 \square$ type terms.

$f_{\beta_2}(y) = C_{1a} f_{\beta_2(1A)}(y) + C_{1c} f_{\beta_2(1C)}(y),$	
$f_{\beta_2(1a)}(y)$	$\frac{1}{2}(D-2)^2 I^4[FA^{(4)}]$
$f_{\beta_2(1c)}(y)$	$\frac{1}{2}(D-2)^2 I^4[FC^{(4)}]$
Total for $f_{\beta_2}(y)$	
$-\frac{(D-2)(D+2)(D+4)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{256(D+1)} \left(\frac{1}{y}\right)^{D-2} - \frac{3H^4}{16\pi^4} \frac{1}{y}.$	

Table A.8: For $\gamma_1 \equiv (aa')^2 H^4$ type terms.

$f_{\gamma_1}(y) = C_{1a} f_{\gamma_1(1a)}(y) + C_{2a} f_{\gamma_1(2a)}(y) + C_{3a} f_{\gamma_1(3a)}(y) \\ + C_b f_{\gamma_1(b)}(y) \\ + C_{1c} f_{\gamma_1(1c)}(y) + C_{2c} f_{\gamma_1(2c)}(y) + C_{3c} f_{\gamma_1(3c)}(y),$	
$f_{\gamma_1(1a)}(y)$	$\frac{1}{2}(D-2)^3(D-1)I^4[FA^{(4)}] - 2(D-1)(I^2[F'A''] + I^3[F'A'''] + I^4[F'A^{(3)}]) \\ + (12 + 40y - 26y^2)(I[F'A''] + I^2[F'A^{(3)}]) + 8(D-1)(I^2[F'A''] + I^3[F'A^{(3)}]) \\ + (-48y + 64y^2 - 20y^3)I[F'A^{(3)}] + (D-1)(-24I^2[F'A^{(3)}] + 28I[yI[F'A^{(3)}]] \\ - 14I[y^2I[F'A^{(3)}]]) - 20(D-1)I[yI[F'A'']] + yI^2[FA^{(3)}]$
$f_{\gamma_1(2a)}(y)$	$4(D^2 - 1)(I[F'A''] - I^2[F'A^{(3)}])$
$f_{\gamma_1(3a)}(y)$	$8(D-1)^2(I^2[F'A''] + I^3[F'A^{(3)}]) - 8(D-1)(I[F'A''] + I^2[F'A^{(3)}]) \\ + 16(D-1)y(I[F'A''] + I^2[F'A^{(3)}]) + (-16(D-1)y + 8(D-1)y^2)I[F'A^{(3)}] \\ - 8(D-1)(D+4)I^2[F'A^{(3)}] + 4(D-1)^2I[yI[F'A^{(3)}]]$
$f_{\gamma_1(b)}(y)$	$2(D-1)(I[F'B'] + I^2[F'B''] + I^3[F'B^{(3)}]) - 4(D^2 - 1)(I^2[F'B''] + I^3[F'B^{(3)}]) \\ + ((4 - 2y)(D-1))(I[F'B''] + I^2[F'B^{(3)}]) + (8(D-1)y - 2(D-1)y^2)I[F'B^{(3)}] \\ + 4((D-1)(D+4))I^2[F'B^{(3)}] - 2((D-1)(D+4))I[yI[F'B^{(3)}]]$
$f_{\gamma_1(1c)}(y)$	$\frac{1}{2}(D-2)^3(D-1)I^4[FC^{(4)}] - 2(D-1)(I^2[F'C''] + I^3[F'C'''] + I^4[F'C^{(3)}]) \\ + (12 + 40y - 26y^2)(I[F'C''] + I^2[F'C^{(3)}]) + 8(D-1)(I^2[F'C''] + I^3[F'C^{(3)}]) \\ + (-48y + 64y^2 - 20y^3)I[F'C^{(3)}] + (D-1)(-24I^2[F'C^{(3)}] + 28I[yI[F'C^{(3)}]] \\ - 14I[y^2I[F'C^{(3)}]]) - 20(D-1)I[yI[F'C'']] + yI^2[FC^{(3)}]$
$f_{\gamma_1(2c)}(y)$	$4(D^2 - 1)(I[F'C''] - I^2[F'C^{(3)}])$
$f_{\gamma_1(3c)}(y)$	$8(D-1)^2(I^2[F'C''] + I^3[F'C^{(3)}]) - 8(D-1)(I[F'C''] + I^2[F'C^{(3)}]) \\ + 16(D-1)y(I[F'C''] + I^2[F'C^{(3)}]) + (-16(D-1)y + 8(D-1)y^2)I[F'C^{(3)}] \\ - 8(D-1)(D+4)I^2[F'C^{(3)}] + 4(D-1)^2I[yI[F'C^{(3)}]]$
Total for $f_{\gamma_1}(y)$	
$\frac{(D-20)(D-2)(D-1)D(D+2)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{32(D+1)} \left(\frac{1}{y}\right)^D \\ + \frac{(D-2)(D^5-35D^4+72D^3-100D^2-320D+256)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{256(D+1)} \left(\frac{1}{y}\right)^{D-1} \\ + \frac{(D^7-51D^6+240D^5+532D^4-5008D^3+7072D^2+1472D-5632)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2})^2}{1024(D-2)^2(D+1)} \left(\frac{1}{y}\right)^{D-2} - \frac{27H^4}{16\pi^4} \frac{1}{y}.$	

Table A.9: For $\gamma_2 \equiv aa'(a^2 + a'^2)H^4$ type terms.

$f_{\gamma_2}(y) = C_{1a} f_{\gamma_2(1a)}(y) + C_{3a} f_{\gamma_2(3a)}(y) \\ + C_b f_{\gamma_2(b)}(y) \\ + C_{1c} f_{\gamma_2(1c)}(y) + C_{3c} f_{\gamma_2(3c)}(y),$	
$f_{\gamma_2(1a)}(y)$	$\frac{1}{2}(D-2)^2 y I^3[FA^{(4)}] - 32y(I[F'A''] + I^2[FA^{(3)}]) \\ - 12(D-1)(I^2[F'A''] + I^3[F'A^{(3)}]) \\ + (D-1)(24I^2[F'A^{(3)}] - 12I[yI[F'A^{(3)}]]) + (48y - 26y^2)I[F'A^{(3)}]$
$f_{\gamma_2(3a)}(y)$	$4(D-1)(I[F'A'] + I^2[F'A''] + I^3[F'A^{(3)}]) \\ - 12(D-1)(I^2[F'A''] + I^3[F'A^{(3)}]) \\ + 8(D-1)y(I[F'A''] + I^2[F'A^{(3)}]) + 2(D-1)y^2 I[F'A^{(3)}] \\ + 24(D-1)I^2[F'A^{(3)}] - 12(D-1)I[yI[F'A^{(3)}]])$
$f_{\gamma_2(b)}(y)$	$-12(D-1)I^2[F'B^{(3)}]$
$f_{\gamma_2(1c)}(y)$	$\frac{1}{2}(D-2)^2 y I^3[FB^{(4)}] - 32y(I[F'C''] + I^2[FC^{(3)}]) \\ - 12(D-1)(I^2[F'C''] + I^3[F'C^{(3)}]) \\ + (D-1)(24I^2[F'C^{(3)}] - 12I[yI[F'C^{(3)}]]) + (48y - 26y^2)I[F'C^{(3)}]$
$f_{\gamma_2(3c)}(y)$	$4(D-1)(I[F'C'] + I^2[F'C''] + I^3[F'C^{(3)}]) \\ - 12(D-1)(I^2[F'C''] + I^3[F'C^{(3)}]) \\ + 8(D-1)y(I[F'C''] + I^2[F'C^{(3)}]) + 2(D-1)y^2 I[F'C^{(3)}] \\ + 24(D-1)I^2[F'C^{(3)}] - 12(D-1)I[yI[F'C^{(3)}]])$
Total for $f_{\gamma_2}(y)$	
$\frac{3(D-2)D(D^2+D-2)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{8(D+1)} \left(\frac{1}{y}\right)^D \\ + \frac{(D-2)(D^4+18D^3-17D^2+8D+8)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{64(D+1)} \left(\frac{1}{y}\right)^{D-1} \\ - \frac{(D^7-55D^6+536D^5-2424D^4+4916D^3-1840D^2-5552D+4544)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{1024(D-2)(D+1)} \left(\frac{1}{y}\right)^{D-2} \\ + \frac{3H^4}{16\pi^4} \frac{1}{y}.$	

Table A.10: For $\gamma_3 \equiv aa'(a+a)^2H^4$ type terms.

$f_{\gamma_3}(y) = C_{1a} f_{\gamma_3(1a)}(y) + C_{1c} f_{\gamma_3(1c)}(y),$	
$f_{\gamma_3(1a)}(y)$	$\frac{1}{2}(D-2)^2(y^2I^2[FA^{(4)}] + yI^3[FA^{(4)}]) + \frac{1}{2}(D-2)^3yI^3[FA^{(4)}]$
$f_{\gamma_3(1c)}(y)$	$\frac{1}{2}(D-2)^2(y^2I^2[FC^{(4)}] + yI^3[FC^{(4)}]) + \frac{1}{2}(D-2)^3yI^3[FC^{(4)}]$
Total for $f_{\gamma_3}(y)$	
$\frac{3H^4}{16\pi^4} \frac{1}{y}.$	

Table A.11: For $\delta \equiv (a^2 + a'^2)H^2\nabla^2\Box$ type terms.

$f_{\delta}(y) = C_{1a} f_{\delta(1a)}(y) + C_{3a} f_{\delta(3a)}(y) + C_b f_{\delta(b)}(y) + C_{1c} f_{\delta(1c)}(y) + C_{3c} f_{\delta(3c)}(y),$	
$f_{\delta(1a)}(y)$	$-I^4[FA^{(4)}]$
$f_{\delta(3a)}(y)$	$-I^4[FA^{(4)}]$
$f_{\delta(b)}(y)$	$\frac{1}{2}I^4[FB^{(4)}]$
$f_{\delta(1c)}(y)$	$-I^4[FC^{(4)}]$
$f_{\delta(3c)}(y)$	$-I^4[FC^{(4)}]$
Total for $f_{\delta}(y)$	
$\frac{H^4}{32\pi^4} \frac{1}{y}.$	

Table A.12: For $\varepsilon_1 \equiv aa'H^2\nabla^2$ type terms.

$f_{\varepsilon_1}(y) = C_{1a} f_{\varepsilon_1(1a)}(y) + C_{2a} f_{\varepsilon_1(2a)}(y) + C_{3a} f_{\varepsilon_1(3a)}(y) \\ + C_b f_{\varepsilon_1(b)}(y) \\ + C_{1c} f_{\varepsilon_1(1c)}(y) + C_{2c} f_{\varepsilon_1(2c)}(y) + C_{3c} f_{\varepsilon_1(3c)}(y),$	
$f_{\varepsilon_1(1a)}(y)$	$\frac{1}{2}(D-2)2DI^4[FA^{(4)}] + 2(D-2)I[yI^2[FA^{(4)}]] - \frac{1}{2}(D-2)I[y^2I^2[FA^{(4)}]] \\ - \frac{1}{2}(D-2)DI[yI^3[FA^{(4)}]] - \frac{1}{4}(D-2)^3I^5[FA^{(4)}] - (D-2)^2I^4[FA^{(4)}] \\ - (D-2)(D-1)I^4[FA^{(4)}] + (I^3[F'A'] + I^4[F'A''] + I^5[F'A^{(3)}]) \\ + 10I^2[yI[F'A''] + yI^2[F'A^{(3)}]] - 4(I^3[F'A''] + I^4[F'A^{(3)}]) \\ + 7I^2[y^2I[F'A^{(3)}]] - 14I^2[yI[F'A^{(3)}]] + 12I^3[F'A^{(3)}]$
$f_{\varepsilon_1(2a)}(y)$	$4(D+1)I^3[F'A^{(3)}]$
$f_{\varepsilon_1(3a)}(y)$	$-(D-2)(D-1)I^4[FA^{(4)}] + 4(D-1)(I^3[F'A''] + I^4[F'A^{(3)}]) \\ + 4(D+4)I^3[F'A^{(3)}] - 2(D-1)I^2[yI[F'A^{(3)}]]$
$f_{\varepsilon_1(b)}(y)$	$\frac{1}{2}(D-2)(D-1)I^4[FB^{(4)}] + 2(D+1)(I^3[F'B''] + I^4[F'B^{(3)}]) \\ + (D+4)I^2[yI[F'B^{(3)}]] - 2(D+4)I^3[F'B^{(3)}]$
$f_{\varepsilon_1(1c)}(y)$	$\frac{1}{2}(D-2)2DI^4[FC^{(4)}] + 2(D-2)I[yI^2[FC^{(4)}]] - \frac{1}{2}(D-2)I[y^2I^2[FC^{(4)}]] \\ - \frac{1}{2}(D-2)DI[yI^3[FC^{(4)}]] - \frac{1}{4}(D-2)^3I^5[FC^{(4)}] - (D-2)^2I^4[FC^{(4)}] \\ - (D-2)(D-1)I^4[FC^{(4)}] + (I^3[F'C'] + I^4[F'C''] + I^5[F'C^{(3)}]) \\ + 10I^2[yI[F'C''] + yI^2[F'C^{(3)}]] - 4(I^3[F'C''] + I^4[F'C^{(3)}]) \\ + 7I^2[y^2I[F'C^{(3)}]] - 14I^2[yI[F'C^{(3)}]] + 12I^3[F'C^{(3)}]$
$f_{\varepsilon_1(2c)}(y)$	$4(D+1)I^3[F'C^{(3)}]$
$f_{\varepsilon_1(3c)}(y)$	$-(D-2)(D-1)I^4[FC^{(4)}] + 4(D-1)(I^3[F'C''] + I^4[F'C^{(3)}]) \\ + 4(D+4)I^3[F'C^{(3)}] - 2(D-1)I^2[yI[F'C^{(3)}]]$
Total for $f_{\varepsilon_1}(y)$	
$\frac{(D^4+12D^3-24D^2-20D+16)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{64(D^2-1)} \left(\frac{1}{y}\right)^{D-2} + \frac{27H^4}{32\pi^4} \frac{1}{y}.$	

Table A.13: For $\varepsilon_2 \equiv (a^2 + a'^2)H^2\nabla^2$ type terms.

$f_{\varepsilon_2}(y) = C_{1a} f_{\varepsilon_2(1a)}(y) + C_{3a} f_{\varepsilon_2(3a)}(y) \\ + C_b f_{\varepsilon_2(b)}(y) \\ + C_{1c} f_{\varepsilon_2(1c)}(y) + C_{3c} f_{\varepsilon_2(3c)}(y),$	
$f_{\varepsilon_2(1a)}(y)$	$6(I^3[F'A''] + I^4[F'A^{(3)}]) + 6I^2[yI[F'A^{(3)}]] - 12I^3[F'A^{(3)}]$
$f_{\varepsilon_2(3a)}(y)$	$6(I^3[F'A''] + I^4[F'A^{(3)}]) + 6I^2[yI[F'A^{(3)}]] - 12I^3[F'A^{(3)}]$
$f_{\varepsilon_2(b)}(y)$	$\frac{1}{2}yI^3[FB^{(4)}] + 6I^3[F'B^{(3)}]$
$f_{\varepsilon_2(1c)}(y)$	$6(I^3[F'C''] + I^4[F'C^{(3)}]) + 6I^2[yI[F'C^{(3)}]] - 12I^3[F'C^{(3)}]$
$f_{\varepsilon_2(3c)}(y)$	$6(I^3[F'C''] + I^4[F'C^{(3)}]) + 6I^2[yI[F'C^{(3)}]] - 12I^3[F'C^{(3)}]$
Total for $f_{\varepsilon_2}(y)$	
$-\frac{(D-2)(13D^2+6D-16)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{128(D^2-1)}\left(\frac{1}{y}\right)^{D-2}.$	

Table A.14: For $\varepsilon_3 \equiv (a + a')^2 H^2 \nabla^2$ type terms.

$f_{\varepsilon_3}(y) = C_{1a} f_{\varepsilon_3(1a)}(y) + C_{3a} f_{\varepsilon_3(3a)}(y) \\ + C_b f_{\varepsilon_3(b)}(y) \\ + C_{1c} f_{\varepsilon_3(1c)}(y) + C_{3c} f_{\varepsilon_3(3c)}(y),$	
$f_{\varepsilon_3(1a)}(y)$	$-(D-2)yI^3[FA^{(4)}]$
$f_{\varepsilon_3(3a)}(y)$	$-(D-2)yI^3[FA^{(4)}]$
$f_{\varepsilon_3(b)}(y)$	$\frac{1}{2}y^2I^2[FB^{(4)}] + \frac{1}{2}(D-1)yI^3[FB^{(4)}]$
$f_{\varepsilon_3(1c)}(y)$	$-(D-2)yI^3[FC^{(4)}]$
$f_{\varepsilon_3(3c)}(y)$	$-(D-2)yI^3[FC^{(4)}]$
Total for $f_{\varepsilon_3}(y)$	
$\frac{(D+2)(D+4)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2})^2}{32(D^2-1)} \left(\frac{1}{y}\right)^{D-2} - \frac{H^4}{16\pi^4} \frac{1}{y}.$	

Table A.15: For $\zeta \equiv \nabla^4$ type terms.

$f_{\zeta}(y) = C_{1a} f_{\zeta(1a)}(y) + C_{2a} f_{\zeta(2a)}(y) + C_{3a} f_{\zeta(3a)}(y) \\ + C_b f_{\zeta(b)}(y) \\ + C_{1c} f_{\zeta(1c)}(y) + C_{2c} f_{\zeta(2c)}(y) + C_{3c} f_{\zeta(3c)}(y),$	
$f_{\zeta(1a)}(y)$	$I^4[FA^{(4)}] + \frac{1}{2}(D-2)I^5[FA^{(4)}]$
$f_{\zeta(2a)}(y)$	$I^4[FA^{(4)}]$
$f_{\zeta(3a)}(y)$	$2I^4[FA^{(4)}] + \frac{1}{2}(D-2)I^5[FA^{(4)}]$
$f_{\zeta(b)}(y)$	$-I^4[FB^{(4)}] - \frac{1}{4}(D-2)I^5[FB^{(4)}]$
$f_{\zeta(1c)}(y)$	$I^4[FC^{(4)}] + \frac{1}{2}(D-2)I^5[FC^{(4)}]$
$f_{\zeta(2c)}(y)$	$I^4[FC^{(4)}]$
$f_{\zeta(3c)}(y)$	$2I^4[FC^{(4)}] + \frac{1}{2}(D-2)I^5[FC^{(4)}]$
Total for $f_{\zeta}(y)$	
0.	

APPENDIX A.2

Tables for the coefficient functions of external operators for 3-point kinetic part

Table A.16: For $\alpha \equiv (aa')^{\frac{D}{2}+1} \square^2$ type terms.

$f_\alpha(y) = C_A f_{\alpha(A)}(y) + C_C f_{\alpha(C)}(y),$	
$f_{\alpha(A)}(y)$	$2I^2[AF'']$
$f_{\alpha(C)}(y)$	$2I^2[CF'']$
Total for $f_\alpha(y)$	
$-\frac{DH^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{64(D-1)}\left(\frac{1}{y}\right)^{D-2} + \mathcal{O}\left(\frac{1}{y}\right).$	

Table A.17: For $\beta_2 \equiv (aa')^{\frac{D}{2}}(a^2 + a'^2)H^2 \square$ type terms.

$f_{\beta_2}(y) = C_A f_{\beta_2(A)}(y) + f_{\beta_2(B)}(y) + C_C f_{\beta_2(C)}(y),$	
$f_{\beta_2(A)}(y)$	$-(\frac{D}{2}-1)\frac{D}{4}[AF] + (\frac{D}{2}-1)\frac{1}{2}(D-1)I[A'F] + I[A'F'] - \frac{1}{2}(D-1)I^2[A'F']$
$f_{\beta_2(B)}(y)$	$-(D-1)I[B'F']$
$f_{\beta_2(C)}(y)$	$-(\frac{D}{2}-1)\frac{D}{4}[CF] + (\frac{D}{2}-1)\frac{1}{2}(D-1)I[C'F] + I[C'F'] - \frac{1}{2}(D-1)I^2[C'F']$
Total for $f_{\beta_2}(y)$	
$\frac{(D-2)^2DH^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{64(D-1)}\left(\frac{1}{y}\right)^{D-1}$ $- \frac{(D-2)(D(D-4)-4)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{512}\left(\frac{1}{y}\right)^{D-2} + \mathcal{O}\left(\frac{1}{y}\right).$	

Table A.18: For $\gamma_1 \equiv (aa')^{\frac{D}{2}+1}H^4$ type terms.

$f_{\gamma_1}(y) = C_A f_{\gamma_1(A)}(y) + f_{\gamma_1(B)}(y) + C_C f_{\gamma_1(C)}(y),$	
$f_{\gamma_1(A)}(y)$	$-(\frac{D}{2}-1)(\frac{D}{2})(D^2-4D+4)[AF] + (\frac{D}{2}-1)(D-1)(D^2-4D+4)I[A'F]$ $+((\frac{D}{2}-1)^2-1)(2-y)-2(D-1)I[A'F'] + (\frac{D}{2}-1)(D-1)I^2[A'F']$ $+(D-1)yI[A'F'] - (D-1)\frac{1}{4}(D^2-4D+4)I^2[A'F']$
$f_{\gamma_1(B)}(y)$	$-(D-1)\frac{1}{4}(D^2-4D+4)I[B'F']$
$f_{\gamma_1(C)}(y)$	$-(\frac{D}{2}-1)(\frac{D}{2})(D^2-4D+4)[CF] + (\frac{D}{2}-1)(D-1)(D^2-4D+4)I[C'F]$ $+((\frac{D}{2}-1)^2-1)(2-y)-2(D-1)I[C'F'] + (\frac{D}{2}-1)(D-1)I^2[C'F']$ $+(D-1)yI[C'F'] - (D-1)\frac{1}{4}(D^2-4D+4)I^2[C'F']$
Total for $f_{\gamma_1}(y)$	
$\frac{(D-4)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{8(D-1)}\left(\frac{1}{y}\right)^{D-1}$ $\frac{(D-2)(D^5-9D^4+16D^3+84D^2-136D+32)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{1024(D-1)}\left(\frac{1}{y}\right)^{D-2} - \frac{3H^4}{128\pi^4}\frac{1}{y}.$	

Table A.19: For $\gamma_2 \equiv (aa')^{\frac{D}{2}}(a^2+a'^2)H^4$ type terms.

$f_{\gamma_2}(y) = C_A f_{\gamma_2(A)}(y) + f_{\gamma_2(B)}(y) + C_C f_{\gamma_2(C)}(y)$	
$f_{\gamma_2(A)}(y)$	$-(\frac{D}{2}-1)(\frac{D}{4})y[AF]' + (\frac{D}{2}-1)(D-1)\frac{1}{2}y[A'F]$ $-\frac{1}{2}y((4y-y^2)[A'F']' + D(2-y)[A'F'])$ $+(2-y)\frac{1}{2}y[A'F'] - (D-1)yI[A'F']$
$f_{\gamma_2(B)}(y)$	$-(D-1)y[B'F']$
$f_{\gamma_2(C)}(y)$	$-(\frac{D}{2}-1)(\frac{D}{4})y[CF]' + (\frac{D}{2}-1)(D-1)\frac{1}{2}y[C'F]$ $-\frac{1}{2}y((4y-y^2)[C'F']' + D(2-y)[C'F'])$ $+(2-y)\frac{1}{2}y[C'F'] - (D-1)yI[C'F']$
Total for $f_{\gamma_2}(y)$	
$-\frac{(D-2)^2DH^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{32}\left(\frac{1}{y}\right)^{D-1}$ $-\frac{(\frac{D}{2}-1)^2(D^2-17D+12)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{64}\left(\frac{1}{y}\right)^{D-2} + \frac{3H^4}{256\pi^4}\frac{1}{y}.$	

Table A.20: For $\gamma_3 \equiv (aa')^{\frac{D}{2}}(a+a)^2 H^4$ type terms.

$f_{\gamma_3}(y) = C_A f_{\gamma_3(A)}(y) + f_{\gamma_3(B)}(y) + C_C f_{\gamma_3(C)}(y),$	
$f_{\gamma_3(A)}(y)$	$ \begin{aligned} & -(\frac{D}{2}-1)(\frac{D}{4})((D-1)y[AF]' + y^2[AF]'') \\ & +(\frac{D}{2}-1)(D-1)\frac{1}{2}((D-1)y[A'F] + y^2[A'F]') - 2y[A'F'] \\ & +(2-y)\frac{1}{2}((D-1)y[A'F'] + y^2[A'F']') - (D-1)\frac{1}{2}((D-1)yI[A'F'] + y^2[A'F']) \end{aligned} $
$f_{\gamma_3(B)}(y)$	$-(D-1)((D-1)y[B'F'] + y^2[B'F']')$
$f_{\gamma_3(C)}(y)$	$ \begin{aligned} & -(\frac{D}{2}-1)(\frac{D}{4})((D-1)y[CF]' + y^2[CF]'') \\ & +(\frac{D}{2}-1)(D-1)\frac{1}{2}((D-1)y[C'F] + y^2[C'F]') - 2y[C'F'] \\ & +(2-y)\frac{1}{2}((D-1)y[C'F'] + y^2[C'F']') - (D-1)\frac{1}{2}((D-1)yI[C'F'] + y^2[C'F']) \end{aligned} $
Total for $f_{\gamma_3}(y)$	
$ \begin{aligned} & \frac{(D-2)^2(D+2)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{64}\left(\frac{1}{y}\right)^{D-1} \\ & - \frac{(\frac{D}{2}-1)^2(2D^3-8D^2+9D+14)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{64}\left(\frac{1}{y}\right)^{D-2} + \frac{3H^4}{256\pi^4}\frac{1}{y}. \end{aligned} $	

Table A.21: For $\delta \equiv (aa')^{\frac{D}{2}-1}(a^2+a'^2)\nabla^2\Box$ type terms.

$f_{\delta}(y) = C_A f_{\delta(A)}(y) + f_{\delta(B)}(y) + C_C f_{\delta(C)}(y),$	
$f_{\delta(A)}(y)$	$(\frac{D}{2}-1)\frac{1}{4}I^2[AF'] - I^2[AF''] + \frac{1}{4}I^3[A'F'] + C_A^{-1}I^2[AF'']$
$f_{\delta(B)}(y)$	$-\frac{1}{2}I^2[BF''] - I^2[BF'']$
$f_{\delta(C)}(y)$	$(\frac{D}{2}-1)\frac{1}{4}I^2[CF'] - I^2[CF''] + \frac{1}{4}I^3[C'F']$
Total for $f_{\delta}(y)$	
$ \frac{DH^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{128(D-1)}\left(\frac{1}{y}\right)^{D-2} + \frac{H^4}{96\pi^4}\frac{1}{y}. $	

Table A.22: For $\varepsilon_1 \equiv (aa')^{\frac{D}{2}} H^2 \nabla^2$ type terms.

$f_{\varepsilon_1}(y) = C_A f_{\varepsilon_1(A)}(y) + f_{\varepsilon_1(B)}(y) + C_C f_{\varepsilon_1(C)}(y),$	
$f_{\varepsilon_1(A)}(y)$	$ \begin{aligned} & -C_A^{-1} 2(D-1)I[A'F'] - \left(\frac{D}{2}-1\right) \frac{D}{4} \left(\frac{1}{2}I[AF] - [AF]\right) \\ & + \left(\frac{D}{2}-1\right) \frac{1}{8} (D^2 - 4D + 4) I^2[AF'] + \left(\frac{D}{2}-1\right) (D-1) \left(\frac{1}{2}I^2[A'F] - I[A'F]\right) \\ & + \left(\left(\frac{D}{2}-1\right)^2 - 1 - \left(\frac{D}{2}-1\right)\right) \frac{1}{2} I^3[A'F'] - \frac{1}{2} I^2[yA'F'] + 2I^2[A'F'] \\ & + (-2I[A'F'] + I^2[y[A'F']']) - (D-1) \left(\frac{1}{2}I^3[A'F'] - I^2[A'F']\right) \\ & - C_A^{-1} \left(\left(\frac{D}{2}-1\right) (D-1) I[AF'] + (D-1) I^2[A'F'] \right) \end{aligned} $
$f_{\varepsilon_1(B)}(y)$	$ \begin{aligned} & -\left(\frac{D}{2}-1\right)^2 I^2[BF''] - 2(D-1) \left(\frac{1}{2}I^2[B'F'] - I[B'F']\right) + \left(\frac{D}{2}-1\right) I[BF'] \\ & + \left(\frac{D}{2}\right) I^2[B'F'] - \left(\frac{D}{2}-1\right)^2 [BF] + 2(D-1) I[B'F'] \end{aligned} $
$f_{\varepsilon_1(C)}(y)$	$ \begin{aligned} & -\left(\frac{D}{2}-1\right) \frac{D}{4} \left(\frac{1}{2}I[CF] - [CF]\right) \\ & + \left(\frac{D}{2}-1\right) \frac{1}{8} (D^2 - 4D + 4) I^2[CF'] + \left(\frac{D}{2}-1\right) (D-1) \left(\frac{1}{2}I^2[C'F] - I[C'F]\right) \\ & + \left(\left(\frac{D}{2}-1\right)^2 - 1 - \left(\frac{D}{2}-1\right)\right) \frac{1}{2} I^3[C'F'] - \frac{1}{2} I^2[yC'F'] + 2I^2[C'F'] \\ & + (-2I[C'F'] + I^2[y[C'F']']) - (D-1) \left(\frac{1}{2}I^3[C'F'] - I^2[C'F']\right) \end{aligned} $
Total for $f_{\varepsilon_1}(y)$	
$ \begin{aligned} & -\frac{(D-2)^2 D H^{2D-4} \pi^{-D} \Gamma(\frac{D}{2}-1)^2}{32(D-1)} \left(\frac{1}{y}\right)^{D-1} \\ & -\frac{(D-2)(D^3-16D+16) H^{2D-4} \pi^{-D} \Gamma(\frac{D}{2}-1)^2}{256(D-1)} \left(\frac{1}{y}\right)^{D-2} - \frac{H^4}{192\pi^4} \frac{1}{y}. \end{aligned} $	

Table A.23: For $\varepsilon_2 \equiv (aa')^{\frac{D}{2}-1} (a^2 + a'^2) H^2 \nabla^2$ type terms.

$f_{\varepsilon_2}(y) = C_A f_{\varepsilon_2(A)}(y) + f_{\varepsilon_2(B)}(y) + C_C f_{\varepsilon_2(C)}(y),$	
$f_{\varepsilon_2(A)}(y)$	$\left(\frac{D}{2}-1\right) \frac{1}{4} y I[AF'] + \frac{1}{4} I^2[AF'] - C_A^{-1} \left(\frac{D}{2}\right) \left(\left(\frac{D}{2}-1\right) I[AF'] + I^2[A'F']\right)$
$f_{\varepsilon_2(B)}(y)$	$ \begin{aligned} & -\frac{1}{2} y I[BF''] - (I[BF'] + I[yBF'']) + \left(\frac{D}{2}\right) I^2[B'F'] - \frac{1}{2} (4y - y^2) [BF''] \\ & + D(2-y) [BF'] - \frac{1}{2} y [BF] \end{aligned} $
$f_{\varepsilon_2(C)}(y)$	$\left(\frac{D}{2}-1\right) \frac{1}{4} y I[CF'] + \frac{1}{4} I^2[CF']$
Total for $f_{\varepsilon_2}(y)$	
$ -\frac{(D-2)(D(2D-7)+4) H^{2D-4} \pi^{-D} \Gamma(\frac{D}{2}-1)^2}{128(D-1)} \left(\frac{1}{y}\right)^{D-2} - \frac{H^4}{96\pi^4} \frac{1}{y}. $	

Table A.24: For $\varepsilon_3 \equiv (aa')^{\frac{D}{2}-1}(a+a')^2 H^2 \nabla^2$ type terms.

$f_{\varepsilon_3}(y) = C_A f_{\varepsilon_3(A)}(y) + f_{\varepsilon_3(B)}(y) + C_C f_{\varepsilon_3(C)}(y),$	
$f_{\varepsilon_3(A)}(y)$	$(\frac{D}{2}-1)\frac{1}{2}(\frac{1}{2}(D-1)yI[AF'] + \frac{1}{2}y^2[AF'']) + \frac{1}{2}(\frac{1}{2}(D-1)yI[A'F'] + \frac{1}{2}y^2I[A'F'])$ $-C_A^{-1}((\frac{D}{2}-1)y[AF'] + yI[A'F'])$
$f_{\varepsilon_3(B)}(y)$	$-\frac{1}{2}((D-1)yI[BF''] + y^2[BF'']) - (\frac{D}{2}-1)(I[BF'] + I[yBF''])$ $-y[BF'] - y^2[BF''] + yI[B'F'] - \frac{1}{2}((D-1)y[BF'] + y^2[BF''])$
$f_{\varepsilon_3(C)}(y)$	$(\frac{D}{2}-1)\frac{1}{2}(\frac{1}{2}(D-1)yI[AF'] + \frac{1}{2}y^2[AF'']) + \frac{1}{2}(\frac{1}{2}(D-1)yI[A'F'] + \frac{1}{2}y^2I[A'F'])$
Total for $f_{\varepsilon_3}(y)$	
$-\frac{(D-2)^2 H^{2D-4} \pi^{-D} \Gamma(\frac{D}{2}-1)^2}{64} (\frac{1}{y})^{D-1} - \frac{5H^4}{384\pi^4} \frac{1}{y}.$	

Table A.25: For $\zeta \equiv (aa')^{\frac{D}{2}-1} \nabla^4$ type terms.

$f_{\zeta}(y) = C_A f_{\zeta(A)}(y) + f_{\zeta(B)}(y) + C_C f_{\zeta(C)}(y),$	
$f_{\zeta(A)}(y)$	$-C_A^{-1} I^2[AF''] + (\frac{D}{2}-1)\frac{1}{2}(\frac{1}{2}I^3[AF'] - I^2[AF']) + I^2[AF''] - \frac{1}{2}I^3[A'F']$ $+ (\frac{D}{2}-1)\frac{1}{2}I^2[AF'] + C_A^{-1} (-2I^2[AF''] + \frac{1}{2}I^3[A'F'])$
$f_{\zeta(B)}(y)$	$I^2[BF''] - (\frac{D}{2}-1)\frac{1}{2}I^2[BF'] + (2I^2[BF''] - \frac{1}{2}I^3[B'F']) + I^2[BF'']$
$f_{\zeta(C)}(y)$	$(\frac{D}{2}-1)\frac{1}{2}(\frac{1}{2}I^3[CF'] - I^2[CF']) + I^2[CF''] - \frac{1}{2}I^3[C'F'] + (\frac{D}{2}-1)\frac{1}{2}I^2[CF']$
Total for $f_{\zeta}(y)$	
$-\frac{H^4}{48\pi^4} \frac{1}{y}.$	

Tables for the coefficient functions of external operators for 3-point cross part

Table A.26: For $\alpha \equiv (aa')^{\frac{D}{2}+1}\square^2$ type terms.

$f_\alpha(y) = C_4 f_{\alpha(4A)}(y) - C_4 f_{\alpha(4C)}(y),$	
$f_{\alpha(4A)}(y)$	$2I^3[FA^{(3)}] - 2I^2[FA^{(2)}]$
$f_{\alpha(4C)}(y)$	$2I^3[FC^{(3)}] - 2I^2[FC^{(2)}]$
Total for $f_\alpha(y)$	
$0 \frac{1}{y}.$	

Table A.27: For $\beta_1 \equiv (aa')^{\frac{D}{2}+1}H^2\square$ type terms.

$f_{\beta_1}(y) = C_4 f_{\beta_1(4A)}(y) - C_4 f_{\beta_1(4C)}(y),$	
$f_{\beta_1(4A)}(y)$	$2(D-2)(D-1) (I^3[FA^{(3)}] - I^2[FA^{(2)}]) + 8 I[F'A']$
$f_{\beta_1(4C)}(y)$	$2(D-2)(D-1) (I^3[FC^{(3)}] - I^2[FC^{(2)}]) + 8 I[F'C']$
Total for $f_{\beta_1}(y)$	
$-\frac{(D-1)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{2} \left(\frac{1}{y}\right)^{D-2} - \frac{3H^4}{2\pi^4} \frac{1}{y}.$	

Table A.28: For $\beta_2 \equiv (aa')^{\frac{D}{2}}(a^2 + a'^2)H^2\Box$ type terms.

$f_{\beta_2}(y) = -C_2 f_{\beta_2(3A)}(y) + C_4 f_{\beta_2(4A)}(y) + C_3 f_{\beta_2(3C)}(y) - C_4 f_{\beta_2(4C)}(y),$	
$f_{\beta_2(3A)}(y)$	$-2(D-1) I[F'A']$
$f_{\beta_2(4A)}(y)$	$-(D-2)^2 (I^3[FA^{(3)}] - I^2[FA^{(2)}]) - 2 I[F'A'] - \frac{1}{2}(D-2)^2 [FA]$
$f_{\beta_2(3C)}(y)$	$-2(D-1) I[F'C']$
$f_{\beta_2(4C)}(y)$	$-(D-2)^2 (I^3[FC^{(3)}] - I^2[FC^{(2)}]) - 2 I[F'C'] - \frac{1}{2}(D-2)^2 [FC]$
Total for $f_{\beta_2}(y)$	
$-\frac{(D-2)^2 H^{2D-4} \pi^{-D} \Gamma(\frac{D}{2}-1)^2}{8} \left(\frac{1}{y}\right)^{D-1}$ $-\frac{(\frac{D}{2}-1) \frac{D}{2} (D-1) H^{2D-4} \pi^{-D} \Gamma(\frac{D}{2}-1)^2}{16} \left(\frac{1}{y}\right)^{D-2} + 0 \frac{1}{y}.$	

Table A.29: For $\gamma_1 \equiv (aa')^{\frac{D}{2}+1}H^4$ type terms.

$f_{\gamma_1}(y) = -C_1 f_{\gamma_1(2A)}(y) - C_2 f_{\gamma_1(3A)}(y) + C_4 f_{\gamma_1(4A)}(y) \\ + C_1 f_{\gamma_1(B)}(y) \\ + C_3 f_{\gamma_1(3C)}(y) - C_4 f_{\gamma_1(4C)}(y),$	
$f_{\gamma_1(2A)}(y)$	$-16(D-1)(yI[F'A'']) - 48(D-1)I^2[F'A''] + 16(D-1)I[yF'A'']$
$f_{\gamma_1(3A)}(y)$	$-(D-4)(D-1)^2I[F'A'] + 2(D-1)(2D-17)I^2[F'A''] \\ + (D-2)(D-1)^2I^2[F''A'] - 2(D-1)(D+5)(yI[F'A'']) \\ + 8D(D-1)I[F'A']$
$f_{\gamma_1(4A)}(y)$	$-\frac{D}{2}(D-2)[(4y-y^2)FA'' + D(2-y)FA'] \\ + \frac{D}{4}(D-2)^2(D-1)[FA] - \frac{D}{4}(D-2)^2(D-1)I[F'A] \\ + \frac{1}{2}(D-2)(D-1)[(4y-y^2)FA'' + D(2-y)FA'] \\ - \frac{1}{2}(D-2)^2(y[FA'] + yI[FA'' - F''A]) \\ - \frac{1}{4}(D-2)^3(D-1)([FA] + I^2[FA'' - F''A]) + (D-2)(D-1)^2I[F'A'] \\ + \frac{1}{2}(D-2)(D-1)I[(4y-y^2)F'A'' + D(2-y)F'A'] \\ - \frac{1}{2}(D-2)^2(y[FA]' - yI[FA'' - F''A]) \\ - \frac{1}{4}(D-2)^3(D-1)([FA] - I^2[FA'' - F''A]) \\ + (D-2)^2(2-y)I[F'A'] - (D-2)^2(D-1)I^2[F'A'] \\ + 6(D-2)(D-1)I^2[F'A''] + (D-2)(D-1)^2I^2[F''A'] \\ - (D-2)^3(D-1)(I^3[FA^{(3)}] - I^2[FA^{(2)}]) \\ + 2(D-2)^2(yI^2[FA^{(3)}] - yI[FA^{(2)}]) \\ - 2(D-1)(I^3[F'A''] - I^2[F'A']) + 8(yI[F'A''] - y[F'A']) \\ - 2(D-1)(1-y)(I^2[F'A''] - I[F'A']) \\ + (4y^3 - 16y^2)[F'A']' + D(4y-8)(y[F'A']) \\ + 2(D-2)(D-1)(yI[F'A']) - 2(D-2)(D-1)I^2[F'A'] \\ - 6(D-2)(D-1)I[F'A'] + 2(D-1)(D+3)I^3[F'A''] \\ - 24(y^2+1)I[F'A''] - 8(D-1)I[yI[F'A'']] + 8(y^2[F'A'']) \\ - 2(D-1)^2(yI^2[F'A'']) - (14D-86)(yI[F'A'']) \\ + (D-1)(12D-80)I^2[F'A''] + 24(D-1)(yI[F'A'']) \\ - 24(D-1)I[F'A''] - 24(D-1)I^2[F'A'']$
$f_{\gamma_1(B)}(y)$	$-60(D-1)I^2[F'B''] - 4(D-1)(yI[F'B''] + 4(D-1)I[yF'B'']) \\ - (D-1)(D(D-1)-4)I[F'B'] + 2(D-1)(2D-23)I^2[F'B''] \\ + (D-2)(D-1)^2I^2[F''B'] + 2(D^2-1)(yI[F'B'']) \\ + 8D(D-1)I[F'B''] + 2(D-2)(D-1)I[F'B']$

Table A.29 (continued): For $\gamma_1 \equiv (aa')^{\frac{D}{2}+1}H^4$ type terms.

$f_{\gamma_1(3C)}(y)$	$-(D-4)(D-1)^2 I[F'C'] + 2(D-1)(2D-17)I^2[F'C'']$ $+(D-2)(D-1)^2 I^2[F''C'] - 2(D-1)(D+5)(yI[F'C''])$ $+ 8D(D-1)I[F'C'']$
$f_{\gamma_1(4C)}(y)$	$-\frac{D}{2}(D-2)[(4y-y^2)FC'' + D(2-y)FC']$ $+\frac{D}{4}(D-2)^2(D-1)[FC] - \frac{D}{4}(D-2)^2(D-1)I[F'C']$ $+\frac{1}{2}(D-2)(D-1)[(4y-y^2)FC'' + D(2-y)FC']$ $-\frac{1}{2}(D-2)^2(y[FC'] + yI[FC'' - F''C])$ $-\frac{1}{4}(D-2)^3(D-1)([FC] + I^2[FC'' - F''C]) + (D-2)(D-1)^2 I[F'C']$ $+\frac{1}{2}(D-2)(D-1)I[(4y-y^2)F'C'' + D(2-y)F'C']$ $-\frac{1}{2}(D-2)^2(y[FC']' - yI[FC'' - F''C])$ $-\frac{1}{4}(D-2)^3(D-1)([FC] - I^2[FC'' - F''C])$ $+(D-2)^2(2-y)I[F'C'] - (D-2)^2(D-1)I^2[F'C']$ $+6(D-2)(D-1)I^2[F'C''] + (D-2)(D-1)^2 I^2[F''C']$ $-(D-2)^3(D-1)(I^3[FC^{(3)}] - I^2[FC^{(2)}])$ $+2(D-2)^2(yI^2[FC^{(3)}] - yI[FC^{(2)}])$ $-2(D-1)(I^3[F'C''] - I^2[F'C']) + 8(yI[F'C''] - y[F'C'])$ $-2(D-1)(1-y)(I^2[F'C''] - I[F'C'])$ $+(4y^3 - 16y^2)[F'C']' + D(4y-8)(y[F'C'])$ $+2(D-2)(D-1)(yI[F'C']) - 2(D-2)(D-1)I^2[F'C']$ $-6(D-2)(D-1)I[F'C'] + 2(D-1)(D+3)I^3[F'C'']$ $-24(y^2+1)I[F'C''] - 8(D-1)I[yI[F'C'']] + 8(y^2[F'C''])$ $-2(D-1)^2(yI^2[F'C'']) - (14D-86)(yI[F'C''])$ $+(D-1)(12D-80)I^2[F'C''] + 24(D-1)(yI[F'C''])$ $-24(D-1)I[F'C''] - 24(D-1)I^2[F'C'']$
Total for $f_{\gamma_1}(y)$	
$-\frac{(D^2+3D+1)(\frac{D}{2}-1)^2 H^{2D-4} \pi^{-D} \Gamma(\frac{D}{2}-1)^2}{2} \left(\frac{1}{y}\right)^{D-1}$ $-\frac{(D-2)(D^4+5D^3-57D^2-162D+348) H^{2D-4} \pi^{-D} \Gamma(\frac{D}{2}-1)^2}{64} \left(\frac{1}{y}\right)^{D-2} + \frac{3H^4}{32\pi^4} \frac{1}{y}.$	

Table A.30: For $\gamma_2 \equiv (aa')^{\frac{D}{2}}(a^2 + a'^2)H^4$ type terms.

$f_{\gamma_2}(y) = -C_1 f_{\gamma_2(2A)}(y) - C_2 f_{\gamma_2(3A)}(y) + C_4 f_{\gamma_2(4A)}(y) \\ + C_3 f_{\gamma_2(3C)}(y) - C_4 f_{\gamma_2(4C)}(y),$	
$f_{\gamma_2(2A)}(y)$	$-8(D-1)yI[F'A''] + 8(D-1)I^2[F'A''] + 8(D-1)I[yF'A'']$
$f_{\gamma_2(3A)}(y)$	$-6(D-1)yI[F'A''] + 8(D-1)I^2[F'A''] + \frac{1}{2}(D^2-2)(D-1)I[F'A']$
$f_{\gamma_2(4A)}(y)$	$\frac{D}{2}(D-2)(D-1)I[F'A'] - (D-2)^2y[FA]' + 4(yI[F'A''] - y[F'A']) \\ - 14(yI[F'A'']) + 16(D-1)I^2[F'A''] + 4(y^2[F'A'']) + 4(D-1)(yI[F'A''])$
$f_{\gamma_2(3C)}(y)$	$-6(D-1)yI[F'C''] + 8(D-1)I^2[F'C''] + \frac{1}{2}(D^2-2)(D-1)I[F'C']$
$f_{\gamma_2(4C)}(y)$	$\frac{D}{2}(D-2)(D-1)I[F'C'] - (D-2)^2y[FC]' + 4(yI[F'C''] - y[F'C']) \\ - 14(yI[F'C'']) + 16(D-1)I^2[F'C''] + 4(y^2[F'C'']) + 4(D-1)(yI[F'C''])$
Total for $f_{\gamma_2}(y)$	
$\frac{(D^2+6D-2)(\frac{D}{2}-1)^2 H^{2D-4} \pi^{-D} \Gamma(\frac{D}{2}-1)^2}{8} \left(\frac{1}{y}\right)^{D-1} \\ + \frac{(D-2)(D^4-3D^3+4D^2-212D-304) H^{2D-4} \pi^{-D} \Gamma(\frac{D}{2}-1)^2}{256} \left(\frac{1}{y}\right)^{D-2} + 0 \frac{1}{y}.$	

Table A.31: For $\gamma_3 \equiv (aa')^{\frac{D}{2}}(a+a)^2 H^4$ type terms.

$f_{\gamma_3}(y) = -C_2 f_{\gamma_3(3A)}(y) + C_4 f_{\gamma_3(4A)}(y) \\ + C_1 f_{\gamma_3(B)}(y) \\ + C_3 f_{\gamma_3(3C)}(y) - C_4 f_{\gamma_3(4C)}(y),$	
$f_{\gamma_3(3A)}(y)$	$-(D-4)(D-1)(yI[F'A']) + (D-2)(D-1)(I[F'A'] + I[yF''A']) \\ - 2(D-1)(yI[F'A''] + y^2I[F'A'']) + 4(D-1)I[yF'A'']$
$f_{\gamma_3(4A)}(y)$	$-\frac{1}{4}(D-2)^2(y^2[FA]'' + y^2[FA'' - F''A]) \\ - \frac{1}{4}(D-3)^2(y[FA]' + yI[FA'' - F''A]) + (D-2)(D-1)(y[F'A']) \\ + \frac{1}{4}(D-2)^2(D-1)(yI[FA'' - F''A] - y[FA]') \\ + (D-2)(D-1)I[(yF')'A'] \\ - D(D-2)^2(yI^2[FA^{(3)}] - yI[FA^{(2)}]) - (D-2)^2(y^2I[FA^{(3)}] - y^2[FA^{(2)}]) \\ - 2(yI[F'A''] - y[F'A']) - 2(D-1)(y[F'A']) + 4(D-1)I[yF'A''] \\ - 12(y[F'A'']) + 2(y^2[F'A''])$
$f_{\gamma_3(B)}(y)$	$4(D-1)(y[F'B''])$
$f_{\gamma_3(3C)}(y)$	$-(D-4)(D-1)(yI[F'C']) + (D-2)(D-1)(I[F'C'] + I[yF''C']) \\ - 2(D-1)(yI[F'C''] + y^2I[F'C'']) + 4(D-1)I[yF'C'']$
$f_{\gamma_3(4C)}(y)$	$-\frac{1}{4}(D-2)^2(y^2[FC]'' + y^2[FC'' - F''C]) \\ - \frac{1}{4}(D-3)^2(y[FC]' + yI[FC'' - F''C]) + (D-2)(D-1)(y[F'C']) \\ + \frac{1}{4}(D-2)^2(D-1)(yI[FC'' - F''C] - y[FC]') \\ + (D-2)(D-1)I[(yF')'C'] \\ - D(D-2)^2(yI^2[FC^{(3)}] - yI[FC^{(2)}]) - (D-2)^2(y^2I[FC^{(3)}] - y^2[FC^{(2)}]) \\ - 2(yI[F'C''] - y[F'C']) - 2(D-1)(y[F'C']) + 4(D-1)I[yF'C''] \\ - 12(y[F'C'']) + 2(y^2[F'C''])$
Total for $f_{\gamma_3}(y)$	
$\frac{(D^2+4D-10)(\frac{D}{2}-1)^2 H^{2D-4} \pi^{-D} \Gamma(\frac{D}{2}-1)^2}{8} \left(\frac{1}{y}\right)^{D-1} \\ + \frac{(D-2)(D^4+25D^3-204D^2+372D-192) H^{2D-4} \pi^{-D} \Gamma(\frac{D}{2}-1)^2}{256} \left(\frac{1}{y}\right)^{D-2} + 0 \frac{1}{y}.$	

Table A.32: For $\delta \equiv (aa')^{\frac{D}{2}-1}(a^2 + a'^2)H^2\nabla^2\Box$ type terms.

$f_\delta(y) = -C_1 f_{\delta(2A)}(y) - C_2 f_{\delta(3A)}(y) + C_4 f_{\delta(4A)}(y) \\ + C_1 f_{\delta(B)}(y) \\ + C_3 f_{\delta(3C)}(y) - C_4 f_{\delta(4C)}(y),$	
$f_{\delta(2A)}(y)$	$I^3[FA^{(3)}] - I^2[FA^{(2)}]$
$f_{\delta(3A)}(y)$	$I^3[FA^{(3)}] - I^2[FA^{(2)}]$
$f_{\delta(4A)}(y)$	$2(I^3[FA^{(3)}] - I^2[FA^{(2)}])$
$f_{\delta(B)}(y)$	$2(I^3[FB^{(3)}] - I^2[FB^{(2)}])$
$f_{\delta(3C)}(y)$	$I^3[FC^{(3)}] - I^2[FC^{(2)}]$
$f_{\delta(4C)}(y)$	$2(I^3[FC^{(3)}] - I^2[FC^{(2)}])$
Total for $f_\delta(y)$	
$-\frac{H^4}{16\pi^4} \frac{1}{y}.$	

Table A.33: For $\varepsilon_1 \equiv (aa')^{\frac{D}{2}} H^2 \nabla^2$ type terms.

$f_{\varepsilon_1}(y) = C_1 f_{\varepsilon_1(1A)}(y) - C_2 f_{\varepsilon_1(2A)}(y) \\ + C_2 f_{\varepsilon_1(3A)}(y) + C_4 f_{\varepsilon_1(4A)}(y) \\ - C_1 f_{\varepsilon_1(7A)}(y) \\ + C_1 f_{\varepsilon_1(B)}(y) \\ + C_3 f_{\varepsilon_1(3C)}(y) - C_4 f_{\varepsilon_1(4C)}(y),$	
$f_{\varepsilon_1(1A)}(y)$	$4(D-1)I[F'A'] - 4(D+1)I^2[F'A'']$
$f_{\varepsilon_1(2A)}(y)$	$4(yI[F'A']) + 2(D-1)I^2[F'A'] - 4I[F'A'] - 8I^2[yF'A''] \\ - 4(D-5)I^3[F'A''] + 4I^2[F'A''] \\ + (D-2)(D-1)(I^3[FA^{(3)}] - I^2[FA^{(2)}])$
$f_{\varepsilon_1(3A)}(y)$	$(\frac{D}{2}-1)(D-1)I^2[FA''] + \frac{1}{2}(D-4)(D-1)I^2[F'A'] \\ + \frac{1}{2}(D(D-7)+26)I^3[F'A''] \\ - \frac{1}{2}(D-2)(D-1)I^3[F''A'] + 4(D-1)I[F'A'] \\ + (D-1)I^2[yF'A''] - 4(D-1)I^2[F'A''] \\ + (D-2)(D-1)(I^3[FA^{(3)}] - I^2[FA^{(2)}])$
$f_{\varepsilon_1(4A)}(y)$	$-\frac{D}{8}(D-2)^2I^2[FA'] - \frac{1}{4}(D-2)I[(4y-y^2)[FA''] + D(2-y)[FA']] \\ + \frac{1}{4}(D-2)^3I[FA] + (D-2)^2[FA] \\ + \frac{1}{2}(D-2)(D-1)(I^2[FA''] - I^2[F'A']) \\ - \frac{1}{4}(D-2)I^2[(4y-y^2)F'A'' + D(2-y)F'A'] \\ - \frac{1}{2}(D-1)^2I^3[F'A'] + \frac{1}{2}(D-2)(D-5)I^3[F'A''] \\ - \frac{1}{2}(D-2)(D-1)I^3[F''A'] + 4(D-1)I[yFA'' - yI[FA'']] \\ - (D-2)I[y^2FA'' - y^2I[FA'']] - 2D(D-2)(I^3[FA^{(3)}] - I^2[FA^{(2)}]) \\ + D(D-2)I[yI^2[FA^{(3)}] - yI[FA^{(2)}]] \\ + 2(D-2)(2D-3)(I^3[FA^{(3)}] - I^2[FA^{(2)}]) \\ + \frac{1}{2}(D-3)^3(I^4[FA^{(3)}] - I^3[FA^{(2)}]) - I[yI[F'A''] - y[F'A']] \\ + (I^3[F'A''] - I^2[F'A']) - (D-2)I^2[yF'A'] \\ + 3(D-2)I^2[F'A'] + 4I[F'A'] - 2I^4[F'A''] + 7I^3[F'A''] \\ + 2I^2[y^2F'A''] + (D-1)I^2[yI[F'A'']] \\ - 12(D-3)I^3[F'A''] + I^2[yF'A''] - 12I^2[yF'A''] + 12I^2[F'A'']$
$f_{\varepsilon_1(7A)}(y)$	$4I[F'A'] - 4(D+1)I^2[F'A'']$
$f_{\varepsilon_1(B)}(y)$	$\frac{1}{2}(D-2)(D-1)I^2[FB''] + 3I^2[F'B'] - 4(D-7)I^3[F'B''] \\ - 2I^2[yF'B''] + 4I^2[F'B''] + \frac{1}{2}(D-1)(D+2)I^2[F'B'] \\ + \frac{1}{4}(2(D-2)(D-5)+64)I^3[F'B''] - \frac{1}{2}(D-2)(D-1)I^3[F''B'] \\ + (D-1)I^2[yF'B''] - 4(D-1)I^2[F'B''] + (D-2)I^2[F'B'] \\ + 4(yI^2[FB^{(3)}] - yI[FB^{(2)}]) + 2(D-2)(D-1)(I^3[FB^{(3)}] - I^2[FB^{(2)}])$

Table A.33 (continued): For $\varepsilon_1 \equiv (aa')^{\frac{D}{2}} H^2 \nabla^2$ type terms.

$f_{\varepsilon_1(3C)}(y)$	$ \begin{aligned} & \left(\frac{D}{2} - 1\right)(D-1)I^2[FC''] + \frac{1}{2}(D-4)(D-1)I^2[F'C'] \\ & + \frac{1}{2}(D(D-7) + 26)I^3[F'C''] \\ & - \frac{1}{2}(D-2)(D-1)I^3[F''C'] + 4(D-1)I[F'C'] \\ & + (D-1)I^2[yF'C''] - 4(D-1)I^2[F'C''] \\ & + (D-2)(D-1)(I^3[FC^{(3)}] - I^2[FC^{(2)}]) \end{aligned} $
$f_{\varepsilon_1(4C)}(y)$	$ \begin{aligned} & -\frac{D}{8}(D-2)^2I^2[FC'] - \frac{1}{4}(D-2)I[(4y-y^2)[FC''] + D(2-y)[FC']] \\ & + \frac{1}{4}(D-2)^3I[FC] + (D-2)^2[FC] \\ & + \frac{1}{2}(D-2)(D-1)(I^2[FC''] - I^2[F'C']) \\ & - \frac{1}{4}(D-2)I^2[(4y-y^2)F'C'' + D(2-y)F'C'] \\ & - \frac{1}{2}(D-1)^2I^3[F'C'] + \frac{1}{2}(D-2)(D-5)I^3[F'C''] \\ & - \frac{1}{2}(D-2)(D-1)I^3[F''C'] + 4(D-1)I[yFC'' - yI[FC'']] \\ & - (D-2)I[y^2FC'' - y^2I[FC'']] - 2D(D-2)(I^3[FC^{(3)}] - I^2[FC^{(2)}]) \\ & + D(D-2)I[yI^2[FC^{(3)}] - yI[FC^{(2)}]] \\ & + 2(D-2)(2D-3)(I^3[FC^{(3)}] - I^2[FC^{(2)}]) \\ & + \frac{1}{2}(D-3)^3(I^4[FC^{(3)}] - I^3[FC^{(2)}]) - I[yI[F'C''] - y[F'C']] \\ & + (I^3[F'C''] - I^2[F'C']) - (D-2)I^2[yF'C'] \\ & + 3(D-2)I^2[F'C'] + 4I[F'C'] - 2I^4[F'C''] + 7I^3[F'C''] \\ & + 2I^2[y^2F'C''] + (D-1)I^2[yI[F'C'']] \\ & - 12(D-3)I^3[F'C''] + I^2[yF'C''] - 12I^2[yF'C''] + 12I^2[F'C''] \end{aligned} $
Total for $f_{\varepsilon_1}(y)$	
$ -\frac{(D-2)(D-6)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{16}\left(\frac{1}{y}\right)^{D-2} + 0\frac{1}{y}. $	

Table A.34: For $\varepsilon_2 \equiv (aa')^{\frac{D}{2}-1}(a^2 + a'^2)H^2\nabla^2$ type terms.

$f_{\varepsilon_2}(y) = -C_1 f_{\varepsilon_2(2A)}(y) - C_2 f_{\varepsilon_2(3A)}(y) + C_4 f_{\varepsilon_2(4A)}(y) \\ + C_1 f_{\varepsilon_2(B)}(y) \\ + C_3 f_{\varepsilon_2(3C)}(y) - C_4 f_{\varepsilon_2(4C)}(y),$	
$f_{\varepsilon_2(2A)}(y)$	$2(yI[F'A']) - 2I^2[F'A'] - 6I^3[F'A''] - 4I^2[yF'A'']$
$f_{\varepsilon_2(3A)}(y)$	$\frac{D}{2}(\frac{D}{2} - 1)I^2[FA''] - 6I^3[F'A'']$
$f_{\varepsilon_2(4A)}(y)$	$\frac{D}{4}(D - 2)I^2[FA''] - 2(I^3[F'A''] - I^2[F'A']) + 2I^2[yF'A'' + yF''A'] \\ - 4I[F'A'] - 10I^2[F'A''] - 2I^2[yF'A'']$
$f_{\varepsilon_2(B)}(y)$	$4(yI^2[FB^{(3)}] - yI[FB^{(2)}])$
$f_{\varepsilon_2(3C)}(y)$	$\frac{D}{2}(\frac{D}{2} - 1)I^2[FC''] - 6I^3[F'C'']$
$f_{\varepsilon_2(4C)}(y)$	$\frac{D}{4}(D - 2)I^2[FC''] - 2(I^3[F'C''] - I^2[F'C']) + 2I^2[yF'C'' + yF''C'] \\ - 4I[F'C'] - 10I^2[F'C''] - 2I^2[yF'C'']$
Total for $f_{\varepsilon_2}(y)$	
$-\frac{D(D-8)(D-2)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{64(D-1)}\left(\frac{1}{y}\right)^{D-2} + \frac{3H^4}{4\pi^4}\frac{1}{y}.$	

Table A.35: For $\varepsilon_3 \equiv (aa')^{\frac{D}{2}-1}(a+a')^2 H^2 \nabla^2$ type terms.

$f_{\varepsilon_3}(y) = -C_1 f_{\varepsilon_3(2A)}(y) - C_2 f_{\varepsilon_3(3A)}(y) \\ + C_1 f_{\varepsilon_3(B)}(y) \\ + C_3 f_{\varepsilon_3(3C)}(y) - C_4 f_{\varepsilon_3(4C)}(y),$	
$f_{\varepsilon_3(2A)}(y)$	$(D-2)(yI^2[FA^{(3)}] - yI[FA^{(2)}])$
$f_{\varepsilon_3(3A)}(y)$	$(\frac{D}{2}-1)(yI[FA'']) + ((\frac{D}{2}-1)-2)I^2[yF'A''] \\ + (D-2)(yI^2[FA^{(3)}] - yI[FA^{(2)}])$
$f_{\varepsilon_3(4A)}(y)$	$\frac{1}{2}(D-2)(yI[FA'']) + \frac{1}{2}(D-2)I^2[yF'A''] \\ + 2(D-2)(yI^2[FA^{(3)}] - yI[FA^{(2)}]) - 2I^2[yF'A'']$
$f_{\varepsilon_3(B)}(y)$	$2(y^2I[FB^{(3)}] - y^2[FB^{(2)}]) + 2(D-2)(yI^2[FB^{(3)}] - yI[FB^{(2)}])$
$f_{\varepsilon_3(3C)}(y)$	$(\frac{D}{2}-1)(yI[FC'']) + ((\frac{D}{2}-1)-2)I^2[yF'C''] \\ + (D-2)(yI^2[FC^{(3)}] - yI[FC^{(2)}])$
$f_{\varepsilon_3(4C)}(y)$	$\frac{1}{2}(D-2)(yI[FC'']) + \frac{1}{2}(D-2)I^2[yF'C''] \\ + 2(D-2)(yI^2[FC^{(3)}] - yI[FC^{(2)}]) - 2I^2[yF'C'']$
Total for $f_{\varepsilon_3}(y)$	
$- \frac{(D-2)(D^3-3D^2+22D-16) H^{2D-4} \pi^{-D} \Gamma(\frac{D}{2}-1)^2}{64(D-3)(D-1)} \left(\frac{1}{y}\right)^{D-2} + \frac{9H^4}{8\pi^4} \frac{1}{y}.$	

Table A.36: For $\zeta \equiv (aa')^{\frac{D}{2}-1} \nabla^4$ type terms.

$f_\zeta(y) = C_1 f_{\zeta(1A)}(y) - C_1 f_{\zeta(2A)}(y) \\ - C_2 f_{\zeta(3A)}(y) + C_4 f_{\zeta(4A)}(y) - C_1 f_{\zeta(7A)}(y) \\ + C_1 f_{\zeta(B)}(y) \\ + C_3 f_{\zeta(3C)}(y) - C_4 f_{\zeta(4C)}(y),$	
$f_{\zeta(1A)}(y)$	$-2(I^3[FA^{(3)}] - I^2[FA^{(2)}])$
$f_{\zeta(2A)}(y)$	$-I^3[F'A'] + 2I^4[F'A''] - \frac{1}{2}(D-2)(I^4[FA^{(3)}] \\ - I^3[FA^{(2)}]) - 2(I^3[FA^{(3)}] - I^2[FA^{(2)}])$
$f_{\zeta(3A)}(y)$	$-\frac{1}{4}(D-2)(I^4[F'A''] + I^3[FA'']) + I^4[F'A''] \\ - \frac{1}{2}(D-2)(I^4[FA^{(3)}] - I^3[FA^{(2)}]) \\ - 2(I^3[FA^{(3)}] - I^2[FA^{(2)}])$
$f_{\zeta(4A)}(y)$	$-(D-2)(I^4[FA^{(3)}] - I^3[FA^{(2)}]) - 2(I^3[FA^{(2)}] - I^2[FA^{(2)}]) \\ + 3I^4[F'A''] - \frac{1}{4}(D-2)(I^4[F'A''] + I^3[FA''])$
$f_{\zeta(7A)}(y)$	$-2(I^3[FA^{(3)}] - I^2[FA^{(2)}])$
$f_{\zeta(B)}(y)$	$-\frac{1}{4}(D-2)I^3[FB''] - \frac{1}{2}I^3[F'B'] + 2I^4[F'B''] \\ - \frac{1}{4}(D-2)I^4[F'B''] - \frac{1}{2}I^3[F'B'] + I^4[F'B''] \\ - (D-2)(I^4[FB^{(3)}] - I^3[FB^{(2)}]) - 4(I^3[FB^{(3)}] - I^2[FB^{(2)}])$
$f_{\zeta(3C)}(y)$	$-\frac{1}{4}(D-2)(I^4[F'C''] + I^3[FC'']) + I^4[F'C''] \\ - \frac{1}{2}(D-2)(I^4[FC^{(3)}] - I^3[FC^{(2)}]) \\ - 2(I^3[FC^{(3)}] - I^2[FC^{(2)}])$
$f_{\zeta(4C)}(y)$	$-(D-2)(I^4[FC^{(3)}] - I^3[FC^{(2)}]) - 2(I^3[FC^{(2)}] - I^2[FC^{(2)}]) \\ + 3I^4[F'C''] - \frac{1}{4}(D-2)(I^4[F'C''] + I^3[FC''])$
Total for $f_\zeta(y)$	
$-\frac{H^4}{16\pi^4} \frac{1}{y}.$	

APPENDIX A.3

The Tables For All Finite Nonlocal Contributions From [1, 2]

For Tables A.37 and A.39, $\tilde{\kappa}^2 \equiv (\frac{D-2}{8(D-1)})^2 \kappa^2$.

Table A.37: All Finite Nonlocal “conformal” contributions with $x \equiv \frac{y}{4}$, where $y(x; x')$ is defined in the equation (4.13) [1].

External operators	Coeff. of $\frac{\tilde{\kappa}^2 H^4}{(4\pi)^4}$
$(aa')^3 \square^3 / H^2$	$\frac{9 \ln x}{5x}$
$(aa')^3 \square^2$	$\frac{9 \ln x}{x} - \frac{21}{5x}$
$(aa')^3 H^2 \square$	$-\frac{267 \ln x}{5x} + \frac{51}{x}$
$(aa')^3 H^4$	$\frac{258 \ln x}{5x} - \frac{549}{5x}$
$(aa')^2 (a^2 + a'^2) \square^3 / H^2$	$-\frac{9 \ln x}{10x}$
$(aa')^2 (a^2 + a'^2) \square^2$	$-\frac{99 \ln x}{10x} - \frac{9}{10x}$
$(aa')^2 (a^2 + a'^2) H^2 \square$	$\frac{36 \ln x}{x} - \frac{51}{2x}$
$(aa')^2 (a^2 + a'^2) H^4$	$-\frac{108 \ln x}{5x} + \frac{174}{5x}$
$(aa')^2 H^2 \nabla^2$	$\frac{128 \ln x}{5x} + \frac{166}{5x}$
$(aa')^2 \nabla^2 \square$	$-\frac{64 \ln x}{5x}$
$(aa')(a^2 + a'^2) H^2 \nabla^2$	$-\frac{4 \ln x}{x} - \frac{2}{x}$
$(aa')(a^2 + a'^2) \nabla^2 \square$	$\frac{2 \ln x}{x} + \frac{2}{x}$
$(aa') \nabla^4$	0

Table A.38: All Finite Nonlocal “kinetic” contributions with $x \equiv \frac{y}{4}$, where $y(x; x')$ is defined in the equation (4.13) [2].

External operators	Coeff. of $\frac{\kappa^2 H^4}{(4\pi)^4}$
$(aa')^3 \square^3 / H^2$	$\frac{\ln x}{3x}$
$(aa')^3 \square^2$	$-\frac{3 \ln x}{2x}$
$(aa')^3 H^2 \square$	$\frac{25 \ln x}{x} - \frac{3}{2x}$
$(aa')^3 H^4$	$-\frac{44 \ln x}{x} - \frac{26}{x} - \ln(aa') \frac{30}{x}$
$(aa')^2 (a^2 + a'^2) \square^3 / H^2$	$-\frac{\ln x}{6x}$
$(aa')^2 (a^2 + a'^2) \square^2$	$\frac{7 \ln x}{6x} - \frac{1}{6x}$
$(aa')^2 (a^2 + a'^2) H^2 \square$	$\frac{\ln x}{3x} + \frac{5}{6x}$
$(aa')^2 (a^2 + a'^2) H^4$	$-\frac{4 \ln x}{x} - \frac{7}{2x}$
$(aa')^2 H^2 \nabla^2$	$\frac{4 \ln x}{x} - \frac{14}{x} + \ln(aa') [9 \ln(x) + \frac{20}{x}]$
$(aa')^2 \nabla^2 \square$	$\frac{8 \ln x}{3x} + \frac{1}{3x}$
$(aa')^2 H^2 \nabla^2 \square^2$	$\frac{\ln x}{3x}$
$(aa')(a^2 + a'^2) H^2 \nabla^2$	$\frac{22 \ln x}{3x} - \frac{67}{6x} + \ln(aa') \frac{12}{x}$
$(aa')(a^2 + a'^2) \nabla^2 \square$	$-\frac{11 \ln x}{3x} + \frac{4}{6x}$
$(aa') \nabla^4$	$-\ln(x) + \frac{16}{3x} + \ln(aa') [2 \ln(x) - \frac{2}{x}]$

Table A.39: All Finite Nonlocal “cross” contributions with $x \equiv \frac{y}{4}$, where $y(x; x')$ is defined in the equation (4.13) [2].

External operators	Coeff. of $\frac{\tilde{\kappa}^2 H^4}{(4\pi)^4}$
$(aa')^3 \square^3 / H^2$	$\frac{2 \ln x}{x}$
$(aa')^3 \square^2$	$-\frac{27 \ln x}{x}$
$(aa')^3 H^2 \square$	$-\frac{378 \ln x}{x} - \frac{59}{x}$
$(aa')^3 H^4$	$\frac{608 \ln x}{x} - \frac{436}{x}$
$(aa')^2 (a^2 + a'^2) \square^3 / H^2$	$\frac{\ln x}{x}$
$(aa')^2 (a^2 + a'^2) \square^2$	$-\frac{28 \ln x}{x} + \frac{1}{x}$
$(aa')^2 (a^2 + a'^2) H^2 \square$	$\frac{178 \ln x}{x} - \frac{26}{x}$
$(aa')^2 (a^2 + a'^2) H^4$	$-\frac{252 \ln x}{x} - \frac{66}{x}$
$(aa')^2 H^2 \nabla^2$	$-24 \ln(x) - \frac{142 \ln x}{3x} + \frac{508}{3x}$
$(aa')^2 \nabla^2 \square$	$\frac{76 \ln x}{3x}$
$(aa')(a^2 + a'^2) H^2 \nabla^2$	$-\frac{24 \ln x}{x} + \frac{58}{x}$
$(aa')(a^2 + a'^2) \nabla^2 \square$	$\frac{12 \ln x}{x} + \frac{4}{x}$
$(aa') \nabla^4$	$\frac{4}{x}$

Key Integral Tables

Table A.40: Integrals with a' .

$f(x)$	$-\frac{iH^4}{16\pi^2} \times \int d^4x' a' \{f(\frac{y_{++}}{4}) - f(\frac{y_{+-}}{4})\}$
$\frac{1}{x}$	$-\frac{1}{2a} + \mathcal{O}(\frac{1}{a^2})$
$\frac{\ln(x)}{x}$	$-\frac{\ln(a)}{2a} + \frac{5}{4a} + \mathcal{O}(\frac{\ln(a)}{a^2})$
$\ln(x)$	$-\frac{1}{4a} + \frac{1}{18} + \mathcal{O}(\frac{1}{a^2})$
$\ln^2(x)$	$\frac{\ln(a)}{9} - \frac{1}{3} - \frac{\ln(a)}{2a} + \frac{5}{4a} + \mathcal{O}(\frac{1}{a^2})$

Table A.41: Integrals with a'^2 .

$f(x)$	$-\frac{iH^4}{16\pi^2} \times \int d^4x' a'^2 \{f(\frac{y_{++}}{4}) - f(\frac{y_{+-}}{4})\}$
$\frac{1}{x}$	$-\frac{1}{a} + \mathcal{O}(\frac{1}{a^2})$
$\frac{\ln(x)}{x}$	$-\frac{\ln(a)}{a} + \frac{3}{a} + \mathcal{O}(\frac{\ln(a)}{a^2})$
$\ln(x)$	$\frac{1}{12} - \frac{1}{2a} + \mathcal{O}(\frac{1}{a^2})$
$\ln^2(x)$	$\frac{\ln(a)}{6} - \frac{19}{36} - \frac{\ln(a)}{a} + \frac{3}{a} + \mathcal{O}(\frac{\ln(a)}{a^2})$

Table A.42: Integrals with a'^3 .

$f(x)$	$-\frac{iH^4}{16\pi^2} \times \int d^4x' a'^3 \{f(\frac{y_{++}}{4}) - f(\frac{y_{+-}}{4})\}$
$\frac{1}{x}$	$-\frac{\ln(a)}{a} + \frac{1}{a} + \mathcal{O}(\frac{1}{a^2})$
$\frac{\ln(x)}{x}$	$-\frac{\ln^2(a)}{2a} + \frac{2\ln(a)}{a} - \frac{3}{a} + \frac{\pi^2}{3a} + \mathcal{O}(\frac{\ln(a)}{a^2})$
$\ln(x)$	$\frac{1}{6} - \frac{\ln(a)}{2a} + \frac{1}{4a} + \mathcal{O}(\frac{1}{a^2})$
$\ln^2(x)$	$\frac{1}{3} \ln(a) - \frac{11}{9} - \frac{\ln^2(a)}{2a} + \frac{2\ln(a)}{a} - \frac{9}{4a} + \frac{\pi^2}{3a} + \mathcal{O}(\frac{\ln(a)}{a^2})$

Table A.43: Integrals with a'^4 .

$f(x)$	$-\frac{iH^4}{16\pi^2} \times \int d^4x' a'^4 \{f(\frac{y_{++}}{4}) - f(\frac{y_{+-}}{4})\}$
$\frac{1}{x}$	$-\frac{1}{2} + \mathcal{O}(\frac{1}{a})$
$\frac{\ln(x)}{x}$	$\frac{3}{4} + \mathcal{O}(\frac{1}{a})$
$\ln(x)$	$\frac{1}{6} \ln(a) - \frac{11}{36} + \mathcal{O}(\frac{1}{a})$
$\ln^2(x)$	$\frac{1}{6} \ln^2(a) - \frac{8}{9} \ln(a) + \frac{7}{4} - \frac{\pi^2}{9} + \mathcal{O}(\frac{\ln(a)}{a})$

The Examples For Calculating Key Integrals From Table A.42

As an example, let us choose the term $f(x) = \frac{1}{x}$ from the Table A.42 . The seven-step procedure is applied as follows:

$$\begin{aligned}
 \int d^4x' a'^3 f(x') &= \int d^4x' a'^3 \left\{ f\left(\frac{4}{y_{++}}\right) - f\left(\frac{4}{y_{+-}}\right) \right\}, \\
 &= \int d^4x' a'^3 \left\{ \left[\frac{\square}{H^2} \ln\left(\frac{y_{++}}{4}\right) + 3 \right] - \left[\frac{\square}{H^2} \ln\left(\frac{y_{+-}}{4}\right) + 3 \right] \right\}, \quad \leftarrow \text{Step 1} \\
 &= \frac{\square}{H^2} \int d^4x' a'^3 \left\{ \ln\left(\frac{y_{++}}{4}\right) - \ln\left(\frac{y_{+-}}{4}\right) \right\}, \quad \leftarrow \text{Step 2} \\
 &= \frac{\square}{H^2} \int d^4x' a'^3 2\pi i \theta(\Delta\eta - \Delta x), \quad \leftarrow \text{Step 3} \\
 &= 8\pi^2 i \frac{\square}{H^2} \int_{\eta_i}^{\eta} d^4\eta' a'^3 (\Delta\eta)^3 \int_0^1 dz z^2, \quad \leftarrow \text{Step 4 and 5} \\
 &= \frac{8\pi^2 i}{3H^4} \frac{\square}{H^2} \int_1^a d a' a' \left(\frac{1}{a'} - \frac{1}{a} \right)^3, \quad \leftarrow \text{Step 6} \\
 &= \frac{16\pi^2 i}{H^4} \left[-\frac{\ln(a)}{a} + \frac{1}{a} + \mathcal{O}\left(\frac{1}{a^2}\right) \right]. \quad \leftarrow \text{Step 7} \tag{A.1}
 \end{aligned}$$

The same procedure can be used for $f(x') = \ln(x)$ and $f(x') = \ln^2(x)$, but for the function $f(x') = \frac{\ln(x)}{x}$, a little more algebra is required. We choose our example again from Table A.42 to illustrate the algebra,

$$\begin{aligned}
 \int d^4x' a'^3 f(x') &= \int d^4x' a'^3 \left\{ f\left(\frac{\ln x_{++}}{x_{++}}\right) - f\left(\frac{\ln x_{+-}}{x_{+-}}\right) \right\}, \quad \text{taking } f(x') = \frac{\ln x}{x}, \\
 &= \int d^4x' a'^3 \left\{ \left(\frac{\square}{H^2} \left[\frac{1}{2} \ln^2\left(\frac{y_{++}}{4}\right) - \ln\left(\frac{y_{++}}{4}\right) \right] + 3 \ln\left(\frac{y_{++}}{4}\right) - 2 \right) \right. \right. \\
 &\quad \left. \left. - \left(\frac{\square}{H^2} \left[\frac{1}{2} \ln^2\left(\frac{y_{+-}}{4}\right) - \ln\left(\frac{y_{+-}}{4}\right) \right] + 3 \ln\left(\frac{y_{+-}}{4}\right) - 2 \right) \right\}, \\
 &= \frac{\square}{H^2} \int d^4x' a'^3 \left\{ \frac{1}{2} [\ln^2\left(\frac{y_{++}}{4}\right) - \ln^2\left(\frac{y_{+-}}{4}\right)] - [\ln\left(\frac{y_{++}}{4}\right) - \ln\left(\frac{y_{+-}}{4}\right)] \right\} \\
 &\quad + 3 \int d^4x' a'^3 \left\{ \ln\left(\frac{y_{++}}{4}\right) - \ln\left(\frac{y_{+-}}{4}\right) \right\},
 \end{aligned}$$

With this form, the rest steps (Step 2 - Step 7) can be applied in the same way as the above example. The result is

$$\int d^4x' a'^3 f(x') = \frac{16\pi^2 i}{H^4} \left[-\frac{\ln^2(a)}{2a} + \frac{2\ln(a)}{a} - \frac{3}{a} + \frac{\pi^2}{3a} + \mathcal{O}\left(\frac{\ln(a)}{a^2}\right) \right]. \tag{A.2}$$

Here the series representation of the Riemann zeta function is used.

$$\zeta(2) = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}. \tag{A.3}$$

Result Tables

Table A.44: $-\frac{\tilde{u}(0,k)}{a^2} \frac{iH^8}{(4\pi)^2} \int d^4x' (\text{Ext. opr}) \times [f(\frac{y_{++}}{4}) - f(\frac{y_{+-}}{4})]$ from conformal nonlocal terms.

External operators $\times f(x)$	Coefficient of $\tilde{u}(0,k) \times \frac{H^4}{(4\pi)^2}$
$(aa')^3 \square^3 / H^2 \times [\frac{9\ln x}{5x}]$	0
$(aa')^3 \square^2 \times [\frac{9\ln x}{x} - \frac{21}{5x}]$	0
$(aa')^3 H^2 \square \times [-\frac{267\ln x}{5x} + \frac{51}{x}]$	0
$(aa')^3 H^4 \times [\frac{258\ln x}{5x} - \frac{549}{5x}]$	0
$(aa')^2 (a^2 + a'^2) \square^3 / H^2 \times [-\frac{9\ln x}{10x}]$	$a [\frac{\ln(a)}{20} - \frac{9}{40}]$
$(aa')^2 (a^2 + a'^2) \square^2 \times [-\frac{99\ln x}{10x} - \frac{9}{10x}]$	$a [\frac{11\ln(a)}{40} - \frac{43}{40}]$
$(aa')^2 (a^2 + a'^2) H^2 \square \times [\frac{36\ln x}{x} - \frac{51}{2x}]$	$a [-\frac{101\ln(a)}{24} + \frac{101}{48}]$
$(aa')^2 (a^2 + a'^2) H^4 \times [-\frac{108\ln x}{5x} + \frac{174}{5x}]$	$a [\frac{3\ln(a)}{20} - \frac{83}{120}]$
Total	$a [-\frac{\ln(a)}{40} + \frac{9}{80}]$

Table A.45: $-\frac{\tilde{u}(0,k)}{a^2} \frac{iH^8}{(4\pi)^4} \int d^4x' (\text{Ext. opr}) \times [f(\frac{y_{++}}{4}) - f(\frac{y_{+-}}{4})]$ from kinetic nonlocal terms.

External operators $\times f(x)$	Coefficient of $\tilde{u}(0,k) \times \frac{H^4}{(4\pi)^2}$
$(aa')^3 \square^3 / H^2 \times [\frac{\ln x}{3x}]$	0
$(aa')^3 \square^2 \times [-\frac{3\ln x}{2x}]$	0
$(aa')^3 H^2 \square \times [\frac{25\ln x}{x} - \frac{3}{2x}]$	0
$(aa')^3 H^4 \times [-\frac{44\ln x}{x} - \frac{26}{x} - \ln(aa') \frac{30}{x}]$	0
$(aa')^2 (a^2 + a'^2) \square^3 / H^2 \times [-\frac{\ln x}{6x}]$	$a [\frac{4\ln(a)}{3} - 6]$
$(aa')^2 (a^2 + a'^2) \square^2 \times [\frac{7\ln x}{6x} - \frac{1}{6x}]$	$a [-\frac{14\ln(a)}{3} + \frac{58}{3}]$
$(aa')^2 (a^2 + a'^2) H^2 \square \times [\frac{\ln x}{3x} + \frac{5}{6x}]$	$a [-\frac{4\ln(a)}{3} + \frac{26}{3}]$
$(aa')^2 (a^2 + a'^2) H^4 \times [-\frac{4\ln x}{x} - \frac{7}{2x}]$	$a [4\ln(a) - \frac{34}{4}]$
Total	$a [-\frac{2\ln(a)}{3} + \frac{51}{2}]$

Table A.46: $-\frac{\tilde{u}(0,k)}{a^2} \frac{iH^8}{(4\pi)^2} \int d^4x' (\text{Ext. opr}) \times [f(\frac{y_{++}}{4}) - f(\frac{y_{+-}}{4})]$ from cross nonlocal terms.

External operators $\times f(x)$	Coefficient of $\tilde{u}(0,k) \times \frac{H^4}{(4\pi)^2}$
$(aa')^3 \square^3 / H^2 \times [\frac{2\ln x}{x}]$	0
$(aa')^3 \square^2 \times [-\frac{27\ln x}{x}]$	0
$(aa')^3 H^2 \square \times [-\frac{378\ln x}{x} - \frac{59}{x}]$	0
$(aa')^3 H^4 \times [\frac{608\ln x}{x} - \frac{436}{x}]$	0
$(aa')^2 (a^2 + a'^2) \square^3 / H^2 \times [\frac{\ln x}{x}]$	$a [-8\ln(a) + 36]$
$(aa')^2 (a^2 + a'^2) \square^2 \times [-\frac{28\ln x}{x} + \frac{1}{x}]$	$a [112\ln(a) - 452]$
$(aa')^2 (a^2 + a'^2) H^2 \square \times [\frac{178\ln x}{x} - \frac{26}{x}]$	$a [-712\ln(a) + 2952]$
$(aa')^2 (a^2 + a'^2) H^4 \times [-\frac{252\ln x}{x} - \frac{66}{x}]$	$a [128\ln(a) - 702]$
Total	$a [-352\ln(a) + 1834]$

In the Tables A.44, A.45 and A.46, the zeros are not exactly zero but sub-dominant to the leading terms. Below we give examples in Appendix A.3 for how the result Tables were made.

The Examples From The Table A.45

- We set zero for the following sub-dominant terms:

For example, the first term in Table A.45 is

$$\begin{aligned}
 (aa')^3 \frac{\square^3}{H^2} \times \left[\frac{\ln(x)}{3x} \right] &= a^3 \square^3 \int d^4 x' a'^3 \left[\frac{\ln(x)}{3x} \right], \\
 &= \frac{1}{3} a^3 H^4 \square^2 \left[-\frac{\ln(a)^2}{2a} + \frac{2\ln(a)}{a} - \frac{3}{a} + \frac{\pi^2}{3a} \right], \\
 &= a^2 H^4 \left[-\frac{2}{3} \ln(a)^2 + 4\ln(a) - \frac{17}{3} + \frac{4}{9} \pi^2 \right], \\
 \frac{1}{a^2} (aa')^3 \frac{\square^3}{H^2} \times \left[\frac{\ln(x)}{3x} \right] &= H^4 \left[-\frac{2}{3} \ln(a)^2 + 4\ln(a) - \frac{17}{3} + \frac{4}{9} \pi^2 \right] \rightarrow 0. \quad (\text{A.4})
 \end{aligned}$$

That is, we set zero for the terms of order $\mathcal{O}(\ln(a)^2)$ because they are sub-dominant to $a \ln(a)$ and a .

- Leading terms we keep:

For example, the fifth term in Table A.45 is

$$\begin{aligned}
 (aa')^2 (a^2 + a'^2) \frac{\square^3}{H^2} \times \left[-\frac{\ln(x)}{6x} \right] &= -\frac{1}{6} a^2 \frac{\square^3}{H^2} \int d^4 x' a'^4 \frac{\ln(x)}{x} - \frac{1}{6} a^4 \frac{\square^3}{H^2} \int d^4 x' a'^2 \frac{\ln(x)}{x}, \\
 &= -\frac{1}{6} a^2 \frac{\square^3}{H^2} \left(\frac{3}{4} \right) - \frac{1}{6} a^4 \frac{\square^3}{H^2} \left[-\frac{\ln(a)}{a} + \frac{3}{a} \right], \\
 \frac{1}{a^2} (aa')^2 (a^2 + a'^2) \frac{\square^3}{H^2} \times \left[-\frac{\ln(x)}{6x} \right] &= H^4 a \left[\frac{4}{3} \ln(a) - 6 \right]. \quad (\text{A.5})
 \end{aligned}$$



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PRESENTATIONS ON THE THESIS:

Boran, S., Kâhya, E. O. and Park, S. 2018: Quantum corrections to conformally coupled scalars during inflation. 16th QDIS WORKSHOP, April 21-23, 2018 MSGSU, İstanbul, Turkey (Invited speaker).

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