DOKUZ EYLÜL UNIVERSITY

GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

DESIGNING A ROBOT MANIPULATOR MECHANISM FOR HOLDING AND GRIPPING APPLICATIONS

by

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September, 2005

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M. Rıfat AKAL

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ABSTRACT

In this thesis; kinematic analysis of an n-dof serial manipulator are made. The determination of the link parameters, which are needed for the kinematic analysis of the robot manipulator and the extraction of the transformation matrices according to these parameters are explained.

The number of degrees of freedom (dof) of a mechanism depends on the number of links and joints and the types of joints used for construction of the mechanism. In this thesis I define links, joints, kinematic chains, mechanisms, and machines, then introduce the concepts of degrees of freedom and the loop mobility criterion. The kinematics analyses relations between the end-effectors location and the joint variables of serial and parallel manipulators. Both the direct and inverse kinematics has been analyzed.

The fourth chapter of the model is electronic circuits. Though it seems to be enough to control a 12 V DC motor, it is not possible to drive motors directly from the analog outputs because of the high operating and start up currents.

In the fifth chapter of the model a new gripper designed figures and production information about Laser Sintering System (SLS).

Keywords : Kinematics Analysis, Robot Manipulator, Link Parameters, Transformation Matrices.

TUTMA VE KAVRAMA UYGULAMALARI İÇİN ROBOT MANİPULATOR TASARIMI

ÖZ

Bu tez de, çok serbestlik dereceli bir robot manipülatörün kinematik incelemesi anlatılmıştır. Robot manipülatörün kinematik analizi için gerekli olan uzuv parametreleri belirlenmesi ve bu parametrelere göre transformasyon matrisleri oluşturulması açıklanmıştır.

Mekanizmanın serbestlik derecesi, eklem noktaların sayısına, türüne ve kullanılan eklem çeşidine bağlıdır. Bu tez de, bağlantıları, bağlantı kollarını, mekanizmaları ve mekanik bağlantıları tanıtacağım. Seri ve paralel manipülatör değişkenleri ile end-effector konumu arasında ki kinematik analiz ilişkisi anlatılmıştır. Direkt ve inverse kinematik için ayrı ayrı analizler verilmiştir.

Tezin dördüncü bölümünde, DC elektrik motoru ve elektronik kartı hakkında bilgi verilmiştir. DC motorun kontrolü için 12 V DC gerilim yeterli gibi görünmektedir. Motorun direkt olarak kontrolü için sadece bu yeterli değildir. Çünkü yüksek kalkış akımı ve yüksek işlem hızı gerekmektedir.

Tezin son bölümünde ise, dizayn edilmiş olan yeni end-effector'ün mekanik çizimleri ve laser sinterleme yöntemi hakkında teknik bilgiler bulunmaktadır.

Anahtar sözcükler : Kinematik Analiz , Robot Manipülatör , Link Parametreleri , Transformasyon Matrisi.

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CHAPTER ONE

DEGREES OF FREEDOM OF A MECHANISM

1. Introduction

A mechanism or mechanical manipulatar is made up of several links connected by joints. The number of degrees of freedom (**dof**) of a mechanism depends on the number of links and joints and the types of joints used for construction of the mechanism. in this section we define links, joints, kinematic chains, mechanisms, and machines, then introduce the concepts of degrees of freedom and the loop mobility criterion.

1.1 Mechanisms Parts

1.1.1 Links and Joints

The individual bodies that make up a mechanism are called the members or links. In this chapter, we treat all links as rigid bodies. The assumption of rigid bodies makes the study of mechanisms and robot manipulators a lot easier to understand. However, for high speed or highly loaded mechanisms, the elastic effects of a material body may become significant and should be taken into consideration. For convenience, certain nonrigid bodies, such as chains, cables, or belts, which momentarily serve the same function as rigid bodies, may also be considered as links. From the kinematic point of view, two or more members connected together such that no relative motion can occur between them will be considered as a single link.

Links in a mechanism or mechanical manipulator are connected in pairs. The connection between two links is called a joint. A joint provides some physical constraints on the relative motion between the two members. The kind of relative motion permitted by a joint is governed by the form of the contact surfaces between the members. The contact surface of a link is called a pair element. Two pair elements form a kinematic pair. Kinematic pairs can be classified into lower pairs and higher pairs, according to the type of contact. A kinematic pair is called a lower pair if the two elements contact each other with a substantial surface area. Typically, the forms of two mating lower-pair elements are geometrically identical, one being solid and the other hollow. A kinematic pair is called a higher pair if the pair elements are in contact at a point or along a line. There are six basic types of lower pairs and two of higher pairs that are frequently used in mechanisms and robot manipulators.

A revolute joint, R, permits two paired elements to rotate with respect to each other about an axis that is defined by the geometry of the joint. Hence the revolute joint imposes five constraints between the paired elements and is a l-dof joint. The revolute joint is sometimes called a tuming pair, ahinge, or a pin joint.

A prismatic joint, P, allows two paired elemeıııs to slide with respect to each other along an axis that is defined by the geometry of the joint. Hence the prismatic joint imposes five constraints between the paired elements and is a 1-dof joint. The prismatic joint is sometimes called a sliding pair.

A cylindrical joint, C, permits rotation about and independent translation along, an axis that is defined by the geometry of the joint. Hence the cylindrical joint imposes four constraints on the paired elemeıits and is a 2-dof joint.

A helical joint, H, allows two paired elements to rotate about, and translate along, an axis defined by the geometry of the joint. However, the translation is related to rotation by the pitch of a screw. Hence the helical joint imposes five constraints on the paired elements and is a 1-dof joint. The helical joint is sometimes called a screw pair.

A spherical joint, S, allows one element to rotate freely with respeel to the other about the center of the sphere in all possible orientations. No translation between the paired elements is permitted. Hence the spherical joint imposes three constraints on the paired elements and is a 3-dof joint.

A piane pair, E, permits two translational degrees of freedom along the plane of contact and a rotational degree of freedom about an axis normal to the plane of contact. Hence it imposes three constraints on the paired elements and is a 3-dof joint.

A gear pair, G, permits one gear to roll and slide with respect to the other at the point of contact between two meshing teeth in addition, the motion space of a spor, helical, or bevel gear is also assumed to be constrained on a plane. Hence a gear pair imposes four constraints on the paired elements and is a 2-dof joint.

A cam pair, Ep, is similar to the gear pair except that a spring is usually used to keep the two paired elements in contact. Hence the cam pair is a 2-dof joint.

Revolute, prismatic, cylindrical, helical, spherical, and plane pairs are lower pairs. Gear and cam pairs are higher pairs. Another frequently used joint is the universal joint. The universal joint is essentiall a combination of two interseering revolute joints, hence is a 2-dof joint.

1.1.2 Kinematic Chains, Mechanisms, and Machines

A kinematic chain is an assemblage of links that are connected by joints. When every link in a kinematic chain is connected to every other link by at least two distinct paths, the kinematic chain forms one or more closed loops and is called a closed-loop chain. On the other hand, if every link is connected to every other link by one and onlyone path, the kinematic chain is called an open-loop chain. It is also possible for a kinematic chain to be made up of both closed and open-loop chillus. We call such a chain a hybrid kinematic chain.

A kinematic chain is called a mechanism when one of its links is fixed to the ground. The fixed link is sometimes called the base. In a mechanism, one or more links may be assigned as the input links. As the input link(s) move with respect to the fixed link, all the other links will move according to the kinematic constraints imposed by the joints. Thus a mechanism is a device that transforms motion or torque from one or more input links to the others.

A machine is an assemblage of one or more mechanisms, along with other electrical and/or hydraulic components, used to transform external energy into useful work or other form of energy. For example, an electric drill is a machine. It consists of an electric motor, a speed reduction unit, a keyed chuck, and a trigger switch for the purpose of transforming electric energy into drilling work. However, the speed reduction unit itself is a mechanism and not a machine.

The terms mechanism and machine are sometimes used synonymously. According to the definitions above, however, there is a definite difference. An assemblage of parts is called a mechanism if it is used only for the transmission of motion, and it is called a machine if it is used to transform external energy into useful work. The mechanical manipulator of a robotic system is a mechanism. For the mechanism to become a machine, a microprocessor based controller, encoders and/or force sensors, and other accessories, such as a computer vision system, must be incorporated so that an external source of energy can be converted into useful work. Although a machine may consist of one or more mechanisms, a mechanism is not necessarily a machine since it does not necessarily have to do work but serves merely as a motion transformation device.

1.1.3 Degrees of Freedom of a Mechanism

Perhaps the first concem in a study of the kinematics of mechanisms is the number of degrees of freedom. The degrees of freedom of a mechanism are the number of independent parameters or inputs needed to specify the configuration of the mechanism completely. Except for some special cases, it is possible to derive a general expressian for the degrees of freedom of a mechanism in terms of the number of links, number of joints, and types of joints incorporated in the mechanism. The following notations are defined to facilitate the derivation of an equation:

C_i: number of constraints imposed by joint i.
f: degrees offreedom of a mechanism.
i: degrees of relative matian permitted by joint i.
j: number of joints in a mechanism, assuming that all the joints are binary.
j_i : number of joints with i degrees of freedom.
L: number of independent loops in a mechanism.
n: number of links in a mechanism, including the fixed link.
λ: degrees of freedom of the space in which a mechanism is intended to function.

Since we assume that all the joints are binary, a temary joint is counted as two binary joints, a quatenary joint is counted as three binary joints, and so on. We also assume that a single value of .i" applies to the motions of i all the maving links. That is, they all operate in the same working space: hence .i" = 6 for spatial mechanisms, and .i" = 3 for planar and spherical mechanisms.

Intuitively, the degree-of-freedom value of a mechanism is equal to the degrees of freedom associated with all the moving links minus the number of constraints imposed by the joints. Hence, if the links are all free of constraints, the degrees of freedom of an n-link mechanism, with one of its links fixed to the ground, would be equal to λ (n - 1). However, the total number of constraints imposed by the joints is equal to $\sum_{i=1}^{j} c_i$. Hence the degree of freedom value of a mechanism is generany given by

$$F = \lambda(n-1) - \sum_{i=1}^{j} c_i \tag{1.1}$$

The number of constraints imposed by a joint and the degrees of freedom permitted by the joint are equal to the motian parameter, λ ; that is,

$$\lambda = \mathrm{c_i} + f_i \tag{1.2}$$

Hence the total number of constraints imposed by the joints is

$$\sum_{i=1}^{j} c_i = \sum_{i=1}^{j} (\lambda - f_i) = j\lambda - \sum_{i=1}^{j} f_i \tag{1.3}$$

Substituting Eq. (1.3) into (1.1) yields

$$F = \lambda(n - j - 1) - \sum_{i} f_i \tag{1.4}$$

1.2 Position and Orientation for a Rigid Body

In this part, I explain to the kinematics of robot manipulators, I am constantly dealing with the locatian of several bodies in space. The body of interest include the links of a manipulator, the tools, the workpieces, and so on the identify the location of a body, a reference coordinate systems is established. We called this reference coordinate system the fixed frame, although in reality, it may not necessarily be fixed to the ground. In what follows we employ a Cartesian coordinate system to deseribe the locatiaon of a body, although other types of coordinate systems, such as the cylindrical coordinate system and spherical eaardinate system, may also be used.

The locatian of a body with respect to a reference coordinate system is known if the position of all the points of the body is known. If the body of interest is rigid, six independent parameters would be sufficient to deseribe its location in three-dimensional space. As shown in Fig. 1.1, we take the (x, y, z) coordinate system as the fixed frame. We also attach an (u, v, w) Cartesian coordinate system to the maving body and refer to it as the moving frame. Clearly, the pasitiofis of all the points of the rigid body can be determined when the location of the maving frame with respect to the fixed frame is known. This relative locatian can be considered as composed of the position of a point, say the origin Q, and the orientation of the maving frame with respect to the fixed frame. Further, if we assume that the maving frame coincides with the fixed frame initially, the locatian of the maving frame with respect to the fixed frame and the spatial displacement of a rigid body from the initially coincident position are equivalent.

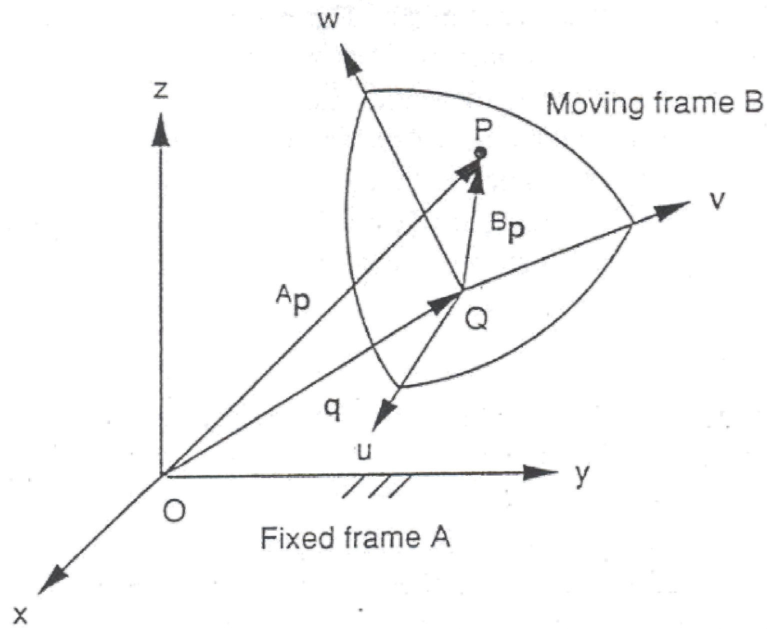

Figure 1.1 General space displacement

1.2.1 Description of the Position

The pasifian of any point with respect to the reference frame can be described by a 3 x 1 pasifian vector. For example, the pasition of apoint P in the referenee frame A as shown in Fig. 1.2 is written as

$$^{A}p = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} \tag{1.5}$$

where the subscripts x, y, and z represent the projections of the position vector only the three coordinate axes of the referenee frame. Since we are dealing with several coordinate systems, a leading superscript is used to indicate the coordinate system to which the vector is referred.

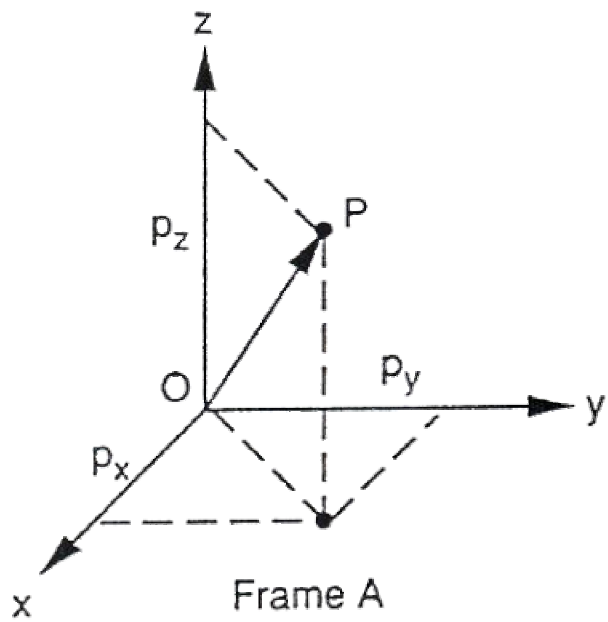

Figure 1.2 Point of P in space

1.2.2 Description of an Orientation

The orientation of a rigid body with respect to the fixed frame can be described in several different ways. In what follows we first describe the direction cosine representation followed by the screw axis representation and then the Euler angle representation. To deseribe the orientation of a rigid body, we consider the motion of a moving frame B with respeet to a fixed frame A with one point fixed. This is known as a rotation or a spherical motion. Without losing generaity, we assume that the origin of the moving frame is fixed to that of the fixed frame, as shown in Fig. 1.3.

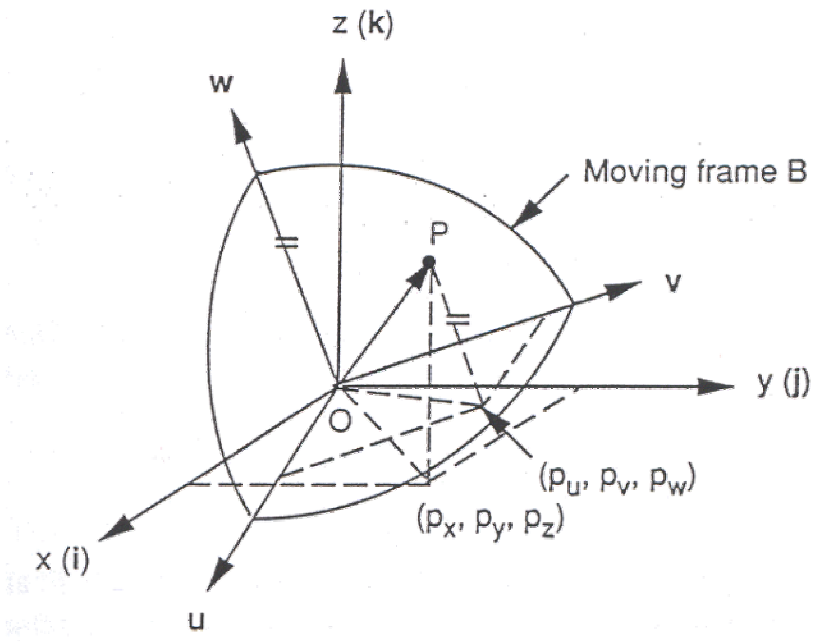

Figure 1.3 Orbicular displacement

1.2.3. Direction Cosine Representation

One convenient way of deseribing the orientation of a rigid body is by means of the directian cosines of the coordinate axes of the maving frame with respect to the fixed frame. Let **i**, **j**, and **k** denote three unit vectors pointing along the coordinate axes of the fixed frame A, and **u, v**, and **w** denote three unit veetors pointing along the coordinate axes of the moving frame B, respectively, as shown in Fig. 1.3. The three unit vectors **u, v**, and **w** can be expressed in the fixed frame A as follows:

$$^{A}u = u_x i + u_y j + u_z k$$

$$^{A}v = v_x i + v_y j + v_z k$$

$$^{A}w = w_x i + w_y j + w_z k \qquad (1.6)$$

The position vector of apoint P of the rigid body can be expressed either in the fixed frame A as

$$^{A}p = p_x i + p_y j + p_z k \tag{1.7}$$

or in the rotated frame B as

$$^{B}p = p_u u + p_y v + p_w w \tag{1.8}$$

Since P is apoint of the rigid body, ^{B}p is constant. However, ^{A}p depends on the orientation of B relative to A. Substituting Eg. (1.6) in (1.7), we obtain the vector p expressed in the fixed frame A as

$$^{A}p = (p_u u_x + p_v v_x + p_w w_x)i + (p_u u_y + p_v v_y + p_w w_y)j + (p_u u_z + p_v v_z + p_w w_z)k \tag{1.9}$$

Equating the x, y, and z components of ^{A}p in Eg. (1.9) to the corresponding components in (1.7) yields

$$\begin{aligned} p_x &= u_x p_u + v_x p_v + w_x p_w \\ p_y &= u_y p_u + v_y p_v + w_y p_w \\ p_z &= u_z p_u + v_z p_v + w_z p_w \end{aligned} \tag{1.10}$$

Writing Eq. (1.10) in a matrix form we obtain

$$^{A}p = {}^{A}R_B . {}^{B}p \tag{1.11}$$

where

$$^{A}R_B = \begin{bmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{1.12}$$

where the leading superscript A and the trajling subscript B indicate the order of transformation. In what follows we pass the leading superscript and trailing subscript from time to time whenever there are only two frames of reference and the order of transformation is clear.

We can the matrix ${}^{A}R_{B}$ the rotation matrix of the moving frame B with respect to the fixed frame A. The rotation matrix specifies the orientation of B completely with respect to A. It transforms the position vector of any point P from the moving frame B to the fixed frame A. From the definition above, we see that the columns of a rotation matrix represent three orthogonal unit vectors of the moving coordinate axes expressed in the fixed frame. Therefore, the rotation matrix is orthogonal. The orthogonality conditions can be stated as

$$u^2 = 1$$
$$v^2 = 1$$
$$w^2 = 1 \qquad (1.13)$$

and

$$u^T v = 0$$
$$v^T w = 0$$
$$w^T u = 0 \qquad (1.14)$$

Because of the orthogonality conditions above, only three of the line elements of ${}^{A}R_{B}$ are independent. Using the orthogonality conditions above, it can be shown that

$$u \text{ x } y = w,$$
$$y \text{ x } w = u,$$
$$w \text{ x } u = y. \qquad (1.15)$$

It can also be shown that

$$\det({}^{A}R_{B}) = 1 \tag{1.16}$$

Furthemore, due to the orthogonality conditions, the inverse transformation of a rotation matrix is equal to its transpose:

$${}^{B}R_{A} = {}^{A}R_{B}^{-1} = {}^{A}R_{B}^{T} \tag{1.17}$$

Since the columns of ${}^{A}R_{B}$ represent three unit vectors of the coordinate axes of frame B expressed in frame A, it follows that the rows of ${}^{A}R_{B}$ represent the three unit vectors defined along the coordinate axes of frame A and expressed in frame B. Therefore, the rotation matrix can be interpreted as a set of three column vectars or a set of three row vectors:

$${}^{A}R_{B} = \begin{bmatrix} {}^{A}u & {}^{A}v & {}^{A}w \end{bmatrix} = \begin{bmatrix} {}^{B}i^{T} \\ {}^{B}j^{T} \\ {}^{B}k^{T} \end{bmatrix} \tag{1.18}$$

1.3. Homogeneous Transformation

The first term on the right-hand side of the equation represents the contribution due to a rotation and the second term that due to a translation of the moving frame respect to the fixed frame. The equation is not in a compact form, because the 3 x 3 rotatian matrix does not provide for the translation. We can write a better-appearing form, we introduce the concepts of homogeneous coordinates and homogeneous transformatian matrix.

1.3.1 Homogeneous Coordinates

Let $p = [Px, Py, Pz]^{T}$ be the position vector of a point with respect to a reference frame A in three-dimensional space. We define the homogeneous coordinates of p as

$$\hat{p} \equiv [\rho p_x, \rho p_y, \rho p_z]^T \tag{1.19}$$

Thus the homogeneous coordinates of the point p in frame A are represented by a vector $\hat{p}$ in a four-dimensional space. The fourth coordinate ρ is a nonzero scaling factor. In general, an N-dimensionaI position vector becomes an (N+l) dimensional vector in a homogeneous coordinate system. The concept of homogeneous coordinates is useful in developing matrix transformations that include rotation, translation, scaling, and perspective transformation

From the definition above, we see that a three-dimensional vectar can be recovered from its four-dimensional homogeneous coordinates by dividing the first three homogeneous coordinates by the fourth coordinate; that is,

$$p_x = \frac{\hat{p}_x}{\rho} \quad p_y = \frac{\hat{p}_y}{\rho} \quad \text{and} \quad p_z = \frac{\hat{p}_z}{\rho} \tag{1.20}$$

We note that the homogeneous coordinates $\hat{p}$ are not unique, since any nonzero scaling factor ρ will yield the same three-dimensional vector p.

For example, $\hat{p}_1 \equiv [\rho_1 p_x, \rho_1 p_y, \rho_1 p_z, \rho_1]^T$ and $\hat{p}_2 \equiv [\rho_2 p_x, \rho_2 p_y, \rho_2 p_z, \rho_2]^T$ represent the same position vector $p \equiv [p_x, p_y, p_z]^T$ in a three-dimensional space. For the kinematics of mechanisms and robot manipulators, we choose a scaling factor of ρ=1 for convenience.

When the scaling factor is set to unity, the first three homogeneous coordinates represent the actual coordinates of a three-dimensional vector. Hence the position vector of the point is given simply as

$$\hat{p} \equiv [p_x, p_y, p_z, 1]^T \tag{1.21}$$

1.3.2 Homogeneous Transformation Matrix

The homogeneous transformation matrix is a 4 x 4 matrix that is defined for the purpose of mapping a homogeneous position vectür from one coordinate system into another. The matrix can be partitioned into four submatrices as follows:

$$^{A}T_{B}=\begin{bmatrix} ^{A}R_{B}(3*3) & . & ^{A}q(3*1) \\ \ldots\ldots\ldots\ldots & . & \ldots\ldots\ldots\ldots \\ \gamma(1*3) & . & \rho(1*1) \end{bmatrix} \quad (1.22)$$

The upper left 3 x 3 submatrix $^{A}R_{B}$ denotes the orientation of a moving frame B with respect to a reference frame A, the upper right 3 x 1 submatrix ^{A}q denotes the pasifian of the origin of the moving frame relative to the fixed frame, the lower left 1 x 3 submatrix y represents a perspecfive transformation, and the lower right element p is a scaling factor. For kinematics of mechanisms and robot manipulators, the scaling factor is set to unity and the 1 x 3 perspective transformation matrix is set to zero.

$$^{A}\hat{p}={}^{A}T_{B}*{}^{B}\hat{p} \quad (1.23)$$

where

$$^{A}T_{B}=\begin{bmatrix} ^{A}R_{B} & . & ^{A}q \\ \ldots\ldots\ldots\ldots & . & \ldots\ldots\ldots\ldots \\ 000 & . & 1 \end{bmatrix}$$

$$^{A}\hat{p}\equiv[p_x,p_y,p_z,1]^{T}$$

$$^{B}\hat{p}\equiv[p_x,p_y,p_z,1]^{T}$$

For example, the homogeneous transfonnation matrix for a simple rotation about the z-axis is given by

$$^{A}T_{B}(z,\theta)=\begin{bmatrix} c\theta & -s\theta & 0 & 0 \\ s\theta & c\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.24}$$

The transformation matrix for a pure translation is given by

$$^{A}T_{B}(q)=\begin{bmatrix} 1 & 0 & 0 & q_x \\ 0 & 1 & 0 & q_y \\ 0 & 0 & 1 & q_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.25}$$

Eq. (1.23) provides a transfonnation of homogeneous coordinates from one frame to another. Given the transformation matrix $^{A}T_{B}$, we can locate the moving frame B with respect to the fixed frame A; and gİven the location of a moving frame B with respect to a fixed frame A, we can find the transformation matrix $^{A}T_{B}$.

Although the transformation matrix $^{A}T_{B}$ is not orthogonal (i.e., $^{A}T_{B}^{-1} \neq {}^{A}T_{B}$), its inverse transformation does exist. We can obtain

$$^{B}p={}^{A}R_{B}^{T}\,{}^{A}p-{}^{A}R_{B}^{T}\,{}^{A}q \tag{1.26}$$

Using the homogeneous coordinates, Eq. (1.26) can be written as

$$^{B}\hat{p}={}^{A}T_{B}^{-1}*{}^{A}\hat{p} \tag{1.27}$$

Hence

$$ {}^{A}T_{B}^{-1} = {}^{B}T_{A} = \begin{bmatrix} {}^{A}R_{B}^{T} & . & -{}^{A}R_{B}^{T}\,{}^{A}q \\ \cdots\cdots\cdots & . & \cdots\cdots\cdots \\ 000 & . & 1 \end{bmatrix} \tag{1.28} $$

1.3.3 Composite Homogeneous Transformation

Homogeneous transfonnation matrices can be multiplied together to obtain a composice transformation matrix. However, special attention should be paid to the order of multiplication since finite rotations are not commutative. The problem is further complicated, since a rigid body can rotate about the coordinate axes of either a fixed reference frame or the body-attached moving frame. The following rules are helpful for the determination of a composite transformation matrix:

1. At the initial position, the moving frame B and the fixed frame A are coincident. Hence T=I is an identity matrix.

2. Rotation and translation about the coordinate axes of a fixed frame results in a premultiplication of the two matrices.

3. Rotation and translation about the coordinate axes of a moving frame results in a postmultiplication of the two matrices.

1.4 Mechanics of Robot Manipulators

The Robotic systems study involves both theoretical and applied research, which can be divided into manipulator design, basic mechanics, trajectory planning and control, programming and machine intelligence, and so on. Mechanics is a branch of science that deals with energy and forces and their effect on the motion of a mechanical system. We study involves three interrelated subjects: kinematics, statics, and dynamics.

1.4.1 Kinematics

Kiıematics deals with the aspects of motion wirhout regard to the forces and/or torques that cause it. The science of kinematics deals with the position, velocity, acceleration, and higher-order derivatives of the position variables with respect to time or other variables. Hence kinematics is concemed only with the geometrical and time properties of a motion. The joim variables of a robot manipulator are related to the position and orientation of the end effect for by the constraints imposed by the joints. These kinematic relations are the focal points of interest in a study of the kinematics of robot manipulators. The study can be approached from two different points of view: kinematic analysis and kinematic synthesis. However, the processes of kinematic analysis and kinematic synthesis are intertwined. A designer needs a skillful analysis ability to evaluate various design altematives under consideratian and to arrive at the best design. Hence a better understanding of the kinematics is the first concern in the design and control of robot manipulators.

Kinematic analysis deals with the derivation of relative motions among various links of a given manipulator. There are two types of kinematic analysis problems: direct kinematics and inverse kinematics in the programming of a robot manipulator, typically a set of desired positions and orientations, and perhaps the time derivatives of the positions and orientations of the end effector, are specified in space. The problem is to find all possible sets of actuated joint variable and orientation with the desired motion characteristics. This is known as inverse kinematics. On the other hand, sometimes the actuated joint variables and possibly their time derivatives are obtained from readings of sensors installed at the joints, from which we wish to find all possible sets of end effector positions and orientations and their corresponding time derivatives. This is called direct kinematics or forward kinematics. Both direct and inverse kinematics problems can be solved by various methods of analysis, such as geometric vector analysis, matrix algebra, screw algebra, and so on

Kinematic synthesis is the reverse process of kinematic analysis. In this case, a designer is challenged to devise a new manipulator or machine that will possess certain desired kinematic properties. Specifically, given a set of desirable positions and orientations of the end effector and possibly the time derivatives in space, the corresponding actuated joint variables and the type and geometry of a manipulator are to be defennined. Some researchers have focused on the synthesis of manipulators with the workspace as an optimality criterion. The kinematic synthesis problem can be further divided info three interrelated phases: type synthesis, number synthesis, and dimensional synthesis.

1.4.2 Statics

Statics deals with the relations of forees that produce equilibrium among the various members of a robot manipulator. A manipulator may be acted upon by forees that arise from various sources, such as forces of gravity, forces of applied loads, frictional forces, inertia forces, and so on. These forces must be considered carefully during the design stage of a robot manipulator so that its parts can be sized properly and the manipulator will function correctly. Obviously, inertia forces are excluded from the static force analysis. The forces of equilibrium depend on the configuration or posture of a robot manipulator and are not time dependent.

1.4.3 Dynamics

Dynamics deals with the forces and/or torques required to cause the motion of a system of bodies. The study includes inertia forces as one of the principal concems. The dynamics of a robot manipulator is a very complicated subject. Typically, the end effector is to be guided through a given path with certain prescribed motion characteristics. A set of torque and/or force functions must be applied at the actuated joints in order to produce that motion. These actuating torque and/or force functions depend not only on the spatial and temporal attributes of the given path but also on the mass properties of the links, the payload, the extemally applied forces, and so on.

The dynamics of robot manipulators can also be approached from two different points of view: dynamical analysis and dynamical synthesis.

Dynamical analysis deals with derivation of the equations of motion of a given manipulator. There are two types of dynamical analysis problems: direct dynamics and inverse dynamics. Direct dynamics can be defined as follows: Given a set of actuated joint torque and/or force functions, calculate the resulting motion of the end effector as a function of time; and inverse dynamics as: Given a trajectory of the end effector as a function of time, find a set actuated joint torque and/or force functions which will produce that motion. The computational efficiency of direct dynamics is not as critical since it is use d primarily for computer simulations of a robot manipulator. On the other hand, an efficient inverse dynamical model becomes extremely important for real-time, model-based control of a robot manipulator. Various methods of analysis are such as the Newton-Euler equations. The Lagrangian equations of motion and the principle of virtual work can be applied for the dynamical analysis of robot manipulators.

CHAPTER TWO

JACOBIAN ANALYSYS OF SERIAL MANIPULATORS

2. Introduction

I have studied the kinematics relations between the end-effectors location and the joint variables of serial and parallel manipulators on this chapter. Both the direct and inverse kinematics has been analyzed.

For some applications, it is necessary to move the end-effector of a manipulator along some desired paths with a prescribed speed. To achieve this goal, the motion of the individual joints of a manipulator must be carefully coordinated. There are two types of velocity coordination problems, called direct velocity and inverse velocity problems. For the direct velocity problem the input joint rates are given and the objective is to find the velocity state of the end effectors. For the inverse velocity problem the velocity state of the end effectors is given and the input joint rates required to produce the desired velocity are to be found.

We call the vector space spanned by the joint variables the joint space, and the vector space spanned by the end-effector location, the end-effector space. For robot manipulators, the Jacobian matrix, or simply Jacobian, is defined as the matrix that transforms the joint rates in the actuator space to the velocity state in the end-effector space.

The Jacobian matrix is also useful in other applications. For some configurations of a manipulator, the Jacobian matrix may lose its full rank. Such conditions are called singular conditions. At a singular condition, a serial manipulator may lose one or more degrees of freedom while a parallel manipulator may gain one or more degrees of freedom.

2.1. Differential Kinematics of a Rigid Body

I will first study the differential kinematics of a rigid body. Then these kinematics properties are applied for a derivation of the differential kinematics of the links in a manipulator and for a development of the Jacobian matrix. Since we will be dealing with many frames of reference, the following rotations are made to identify the frame with respect to which a vector is defined. A vector **p** can be a function of time in one reference frame but constant in another reference frame. Thus, in general, we need two frames of references to describe the nature of a vector: one with respect to which the change of a vector is measured and another in which the vector is expressed. We use an inner leading superscript to denote the frame with respect to which a vector is being measured, and an outer leading superscript to indicate the frame in which the vector is expressed.

For example, ${}^{B}p$ denotes the position vector of a point P with respect to frame B, and ${}^{A}({}^{B}p)$ denotes ${}^{B}p$ expressed in frame A. Similarly, the velocity of P is defined by taking the derivative of B p with respect to time:

$$ {}^{B}v_{p} = \frac{d\,{}^{B}p}{dt} \tag{2.1} $$

However, once the differentiation is taken, the vector can be expressed in any other frame. Thus

$$ {}^{A}\left({}^{B}v_{p}\right) = {}^{A}\left(\frac{d\,{}^{B}p}{dt}\right) \tag{2.2} $$

Indicates that the differentiation is taken with respect to frame B and the resulting vector is expressed in frame A.

When the two leading superscripts are the same or when the frame with respect to which a vector quantity is being measured is clearly understood, the inner superscript will be omitted. For clarity, we often use the rotation matrix ${}^{A}R_{B}$ to transform a vector from one reference frame to another:

$$ {}^{A}\left({}^{B}p\right) \equiv {}^{A}R_{B}\,{}^{B}p \tag{2.3} $$

Furthermore, when no specific reference frame is mentioned, either the base frame is implied or any reference frame can be used. Note that all vectors in one equation must be expressed in the same reference frame.

2.1.1 Angular Velocity of a Rigid Body

While the linear velocity describes the rate of change of the position of a point in space, the angular velocity vector describes the rate of change of the orientation of a rigid body. Figure 2.1 shows that frame B is rotating with respect to frame A with a fixed point O. The orientation of frame B with respect to A can be described by a rotation matrix, ${}^{A}R_{B}$. Since the rotation matrix ${}^{A}R_{B}$ is orthogonal, the inverse transformation of ${}^{A}R_{B}$ is identical to the transpose. Hence

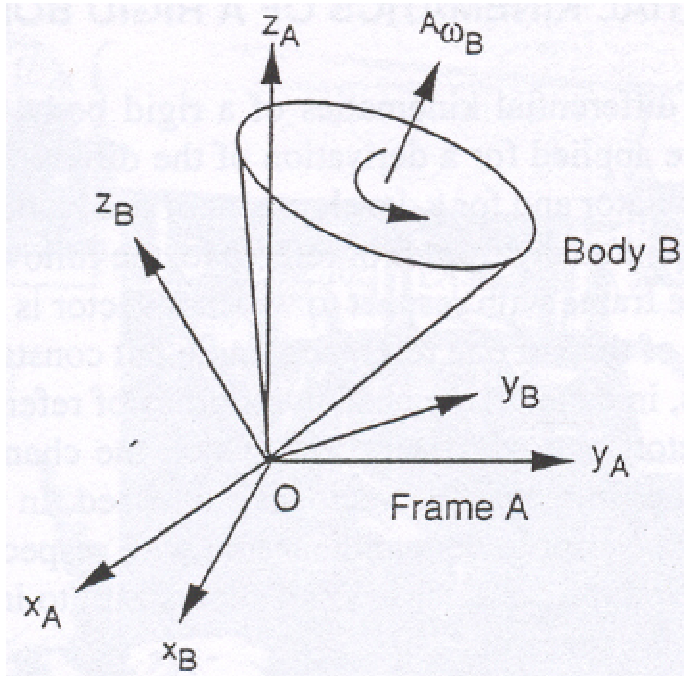

Figure 2.1 Instantaneous rotation of frame B with respect to A.

$$ {}^{A}R_{B} * {}^{A}R_{B}^{T} = I \tag{2.4}$$

where I is a 3 x 3 identity matrix.

Taking the derivative of Eq. (2.2) with respect to time, we obtain

$$ {}^{A}\dot{R}_{B} * {}^{A}R_{B}^{T} + {}^{A}R_{B} * {}^{A}\dot{R}_{B}^{T} = 0 \tag{2.5}$$

Substituting ${}^{A}R_{B}^{T} = {}^{A}R_{B}^{-1}$ and ${}^{A}R_{B} = \left({}^{A}R_{B}^{-1}\right)^{T}$ into Eq. (2.5), we obtain

$$ \left({}^{A}\dot{R}_{B} * {}^{A}R_{B}^{-1}\right) + \left({}^{A}R_{B} * {}^{A}\dot{R}_{B}^{-1}\right)^{T} = 0 \tag{2.6}$$

Hence ${}^{A}\dot{R}_{B}^{T}\,{}^{A}R_{B}^{-1}$ is a 3 x 3 skew-symmetric matrix. Without losing generality, we may define the skew-symmetric matrix as

$$\Omega \equiv {}^{A}\dot{R}_{B}^{T}\,{}^{A}R_{B}^{-1} = \begin{bmatrix} 0 & -\varpi_{z} & \varpi_{y} \\ \varpi_{z} & 0 & -\varpi_{x} \\ -\varpi_{y} & \varpi_{x} & 0 \end{bmatrix} \qquad (2.7)$$

Here ω_x , ω_y , ω_z and are to be identified as three independent parameters specifying the angular velocity of a rigid body. In what follows it will be shown that these three quantities form the components of a vector called the angular velocity vector of B in A.

The position vector of a point P that is embedded in frame B and measured with respect to frame A is given by

$$^{A}p = {}^{A}R_{B} * {}^{B}p \qquad (2.8)$$

Note that ^{B}p is a constant vector in frame B since P is embedded in B. The velocity of P with respect to frame A is obtained by taking the derivative of Eq. (2.8) with respect to time:

$$^{A}V_{p} = \frac{d}{dt}\left({}^{A}R_{B} * {}^{B}p\right) = {}^{A}\dot{R}_{B} * {}^{B}p \qquad (2.9)$$

Solving ^{B}P from Eq. (2.8) and substituting the resulting expression info Eq. (2.9) yields

$$^{A}V_{p} = {}^{A}R_{B} * {}^{A}R_{B}^{-1} * {}^{A}p \qquad (2.10)$$

Substituting Eq. (2.7) into (2.10) produces

$$^{A}V_{p} = \Omega * {}^{A}p \qquad (2.11)$$

We may ask ourselves the following question: Is there any point in B that has zero velocity at that instant. Assuming that P is such a point,

$$ {}^{A}V_{p} = \Omega * {}^{A}\overline{p} = 0 \tag{2.12} $$

Equation (2.12) consists of three homogeneous linear equations in three unknowns, $\overline{p}_X, \overline{p}_Y$, and $\overline{p}_Z$ The compatibility condition for the existence of nontrivial solutions is that the determinant of the coefficient matrix must vanish. Since Ω is a 3 x 3 skew-symmetric matrix, this condition is satisfied automatically:

$$ \begin{bmatrix} 0 & -\varpi_z & \varpi_y \\ \varpi_z & 0 & -\varpi_x \\ -\varpi_y & \varpi_x & 0 \end{bmatrix} = \varpi_x * \varpi_y * \varpi_z - \varpi_z \varpi_z \varpi_z = 0 $$

Hence only two of the three equations in Eq. (2.12) are independent. Solving Eq. (2.12) for the ratio $\overline{p}_x : \overline{p}_y : \overline{p}_z$ we obtain

$$ \overline{p}_x : \overline{p}_y : \overline{p}_z = \omega_x : \omega_y : \omega_z \tag{2.13} $$

We conclude that there exist infinitely many stationary points, and these points lie on a line that passes through the origin and is parallel to the vector ${}^{A}\omega_B = [\omega_x, \omega_y, \omega_z]^T$. We call the vector ${}^{A}\omega_{B}$ the angular velocity vector and the line the instantaneous screw axis. Using the vector notation, Eq. (2.11) can be written as

$$ {}^{A}V_P = {}^{A}\omega_B * {}^{A}p \tag{2.14} $$

2.1.2 Linear Velocity of a Point

Figure 2.3 shows a rigid body B that is making an instantaneous rotation as well as translation with respect to a reference frame A. The position vector of a point P, which is not necessarily fixed in frame B, relative to frame a can be written as

$$ {}^{A}p = {}^{A}q + {}^{A}R_{B} * {}^{B}p \tag{2.15} $$

where ${}^{A}q = \overline{0}\overline{Q}$ denotes the position vector of the origin Q of frame B with respect to frame A.

To derive the velocity of p, we first consider the rate of change of the second term in Eq. (2.15). This is essentially the ease when frame B is rotating with respect to frame A with the origin Q fixed in A. Differentiating the second term of Eq. (2.15) with respect to time yields

$$ \frac{d}{dt}\left({}^{A}R_{B} * {}^{B}p\right) = {}^{A}R_{B} * {}^{B}V_{p} + {}^{A}\dot{R}_{B} * {}^{B}p \tag{2.16} $$

where ${}^{B}v_{p} = \frac{d}{dt}{}^{B}p$ denotes the velocity of P with respect frame B.

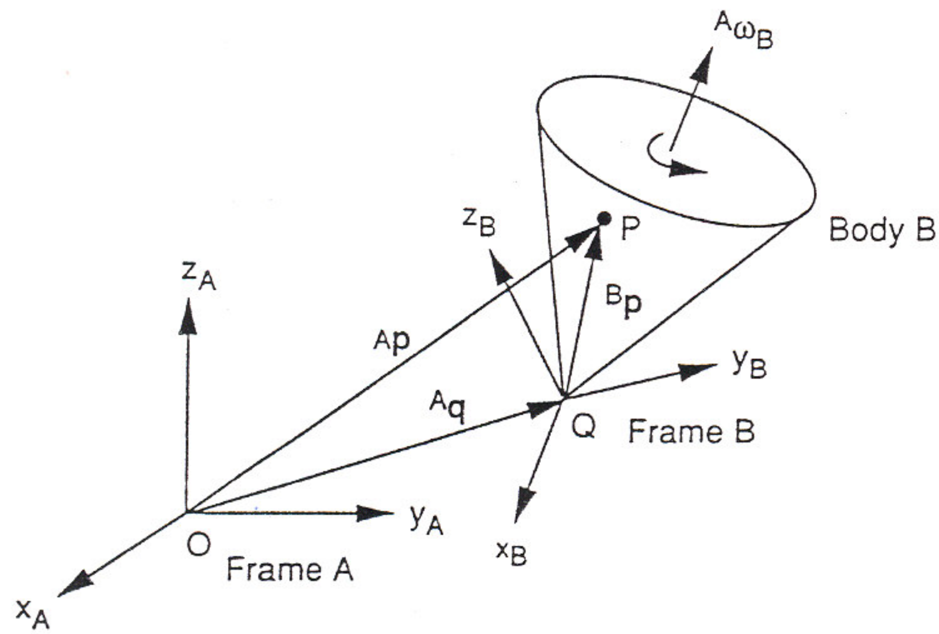

Figure 2.2 Instantaneous motion of a rigid body B with respect to frame A.

Post multiplying both sides of Eq. (2.7) by ^{A}R's, we obtain

$$^{A}\dot{R}_{B} = \Omega *{}^{A}R_{B} \tag{2.17}$$

Substituting Eq. (2.17) into (2.16) yields

$$\frac{d}{dt}\left({}^{A}R_{B} *{}^{B}p\right){}^{A}R_{B} *{}^{B}v_{p} + \Omega{}^{A}R_{B} *{}^{B}P \tag{2.18}$$

Equation (2.18) can be written in vector form as

$$\frac{d}{dt}\left({}^{A}R_{B} *{}^{B}p\right){}^{A}R_{B} *{}^{B}v_{p} + {}^{A}\omega_{B} * \left({}^{A}R_{B} *{}^{B}P\right) \tag{2.19}$$

When the origin Q of frame B is moving with respect to frame A, we simply add a component representing the linear velocity of Q in A to Eq. (2.19). Hence a general equation of motion can be written as

$$ {}^{A}v_{p} = {}^{A}v_{q} + {}^{A}R_{B} * {}^{B}v_{p} + {}^{A}\omega_{B} * \left({}^{A}R_{B} * {}^{B}P \right) \tag{2.20} $$

where ${}^{A}v_{q} = {}^{A}q$ denotes the velocity of Q relative to frame A. The first term in Eq. (2.20) is contributed by the linear velocity of Q with respect to frame A, the second term is contributed by the relative motion of P with respect to frame B, and the third term is contributed by the rotation of frame B with respect to A.

If point P is embedded in the moving frame B, ${}^{A}v_{p} = 0$ identically Equation (2.20) reduces to

$$ {}^{A}v_{p} = {}^{A}v_{q} + {}^{A}\omega_{B} * \left({}^{A}R_{B} * {}^{B}P \right) \tag{2.21} $$

Although Eq. (2.21) is derived for the case in which Q is the origin of a moving frame, it is equally applicable to any two points fixed on the moving frame. In general, if P and Q are two points embedded in a rigid body B, their velocities are related by the equation

$$ {}^{A}v_{p} = {}^{A}v_{q} + {}^{A}\omega_{B} * \left({}^{A}p - {}^{A}q \right) \tag{2.22} $$

2.1.3 Instantaneous Screw Axis

In this part I show that a general instantaneous motion of a rigid body can be described by a differential rotation about a unique axis and a differential translation along the same axis. This concept will be applied to the Jacobian analysis of serial manipulators.

For a general spatial motion of a rigid body B, are there any stationary points in B. If ${}^{B}\tilde{p}$ is a stationary point, ${}^{A}v_{\tilde{p}} = 0$ identically, and Eq. (2.21)

Reduces to

$$ {}^{A}\omega_{B} * \left({}^{A}R_{B} * {}^{B}\tilde{p} \right) = -{}^{A}v_{q} \tag{2.23} $$

Since the angular velocity ${}^{A}w_{B}$ is derived from a 3 x 3 skew-symmetric matrix Ω. The coefficients matrix of Eq. (2.23) is singular. It follows that in general, there are no solutions to Eq. (2.21). However, we may seek for those points whose linear velocity vectors point along the direction of the angular velocity. That is,

$$ {}^{A}v_{\tilde{p}} = \lambda * {}^{A}w_{B} \tag{2.24} $$

where A is called a pitch.

Substituting Eq. (2.22) into (2.21) yields

$$ {}^{A}v_{q} + {}^{A}\omega_{B} * \left({}^{A}R_{B} * {}^{B}\tilde{p} \right) = \lambda * {}^{A}w_{B} \tag{2.25} $$

Dot multiplying both sides of Eq. (2.25) by ${}^{A}w_{B}$ we obtain

$$ \lambda = \frac{{}^{A}w_{B} * {}^{A}v_{q}}{{}^{A}w_{B}^{2}} \tag{2.26} $$

Equation (2.25) can be written in the form

$$ {}^{A}w_{B} * \left({}^{A}R_{B} * {}^{B}\tilde{p} \right) = -{}^{A}v_{q}^{1} \tag{2.27} $$

where ${}^{A}v_{q}^{1} = {}^{A}v_{q} - \lambda {}^{A}w_{B}$ is orthogonal to ${}^{A}w_{B}$ that is.

$$ {}^{A}w_{B} * {}^{A}v_{q}^{1} = {}^{A}w_{B} * \left({}^{A}v_{q} - \lambda {}^{B}w_{B} \right) = 0 \tag{2.28} $$

We now make use of the following result derived from vector algebra. Let vectors a, b. and c in Fig. 2.2 satisfy the following two conditions:

a x c = b

a . b = 0

Then c has infinite number of solutions lying on a line:

$$c = \frac{a * b}{a^2} + \mu a$$

where μ is an arbitrary scalar constant.

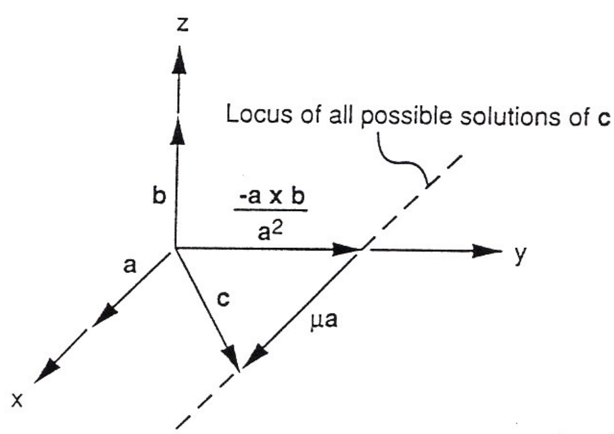

Figure 2.3 Vector relation

From the vector algebra above, we conclude that all solutions to Eqs. (2.27) and (2.28) are given by

$$^{A}R_{B}\,^{B}\tilde{p} = \frac{^{A}\varpi_{B} \times {}^{A}v'_{q}}{^{A}\varpi_{B}^{2}} + \mu\,^{A}\varpi_{B} \qquad (2.29)$$

Applying Eq. (2.15), Eq. (2.29) can be written as

$$ {}^{A}\tilde{p} = {}^{A}q + \frac{{}^{A}\varpi_{B} \times {}^{A}v'_{q}}{{}^{A}\varpi_{B}^{2}} + \mu\, {}^{A}\varpi_{B} \tag{2.30} $$

Equation (2.29) or (2.30) states that the locus of all points whose instantaneous linear velocities point along the direction of the angular velocity vector line. This line that is parallel to the angular velocity vector is called the instantaneous screw axis.

2.2 Differential Kinematics of Serial Manipulators

In this part, I explain the differential kinematics of a serial manipulator using the Denavit - Hartenberg transformation matrix. First, we study the differential motion of a link. Then we apply it to the differential motion of a serial manipulator.

2.2.1 Link Differential Transformation Matrix

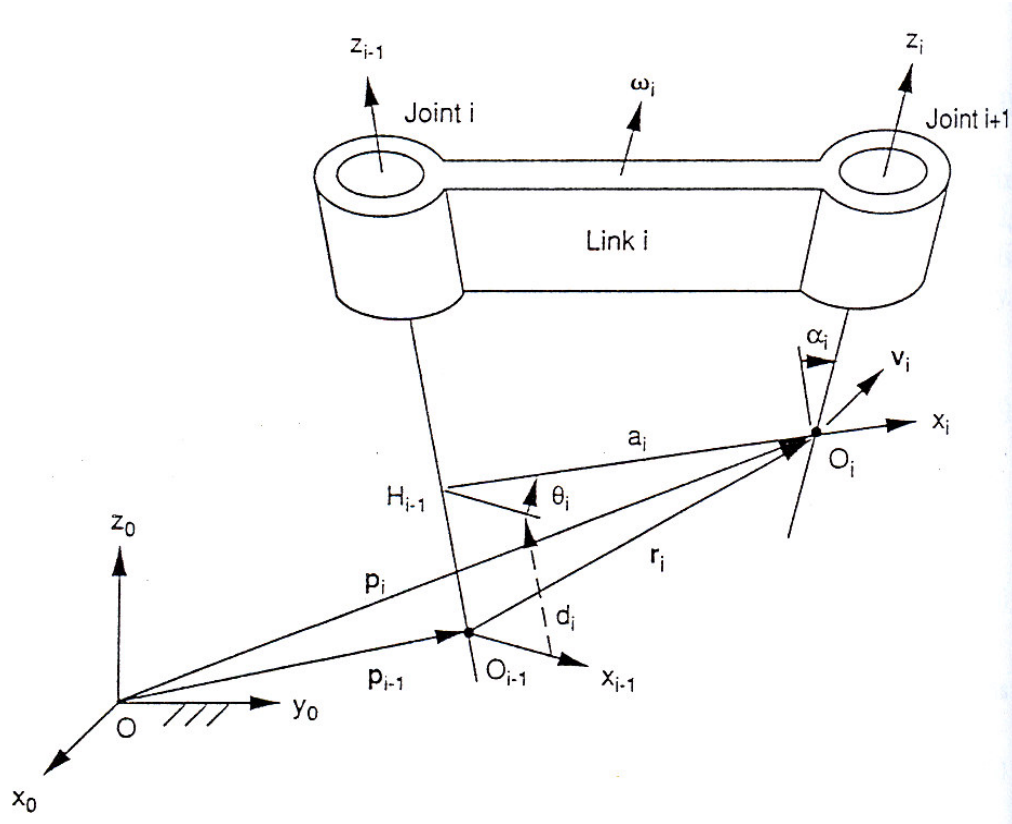

Figure 2.4 Geometry of link i and its motion state

Figure 2.4 shows a typical link, i. of a manipulator. According to the D-H convention; a cartesian coordinate system (x_i , y_i , z_i) is attached to the distal end of link i, and the fixed coordinate system is denoted by frame (x_0 , y_0 , z_0). The location of link i can be described by a position vector p_i of O_i and a rotation matrix ${}^0R_i$ of link i with respect to the fixed reference frame O. The velocity state of link i can be described by the linear velocity v_i of the origin O_i and the angular velocity w_i of link i relative to the fixed reference frame.

The D-H transformation matrix is given by Eq. (2.2) and the inverse transformation is given by Eq. (2.5). Taking the derivative of Eq. (2.2) with respect to time, we obtain

$$
{}^{i-1}\dot{A}_i = \begin{bmatrix} -\dot{\theta}_i s\theta_i & -\dot{\theta}_i c\alpha_i\theta_i & \dot{\theta}_i s\alpha_i\theta_i & -\dot{\theta}_i a_i\theta_i \\ \dot{\theta}_i c\theta_i & -\dot{\theta}_i c\alpha_i\theta_i & \dot{\theta}_i s\alpha_i\theta_i & \dot{\theta}_i a_i\theta_i \\ 0 & 0 & 0 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.31}
$$

in Eq. (2.31), both e_i and d_i are treated as variables. For a revolute joint, d_i =0, and for a prismatic joint, $\dot{\theta}_i = 0$ Post multiplying both sides of Eq. (2.31) by $\left({}^{i-1}A_i\right)^{-1}$.

$$
\left({}^{i-1}\dot{A}_i\right)\left({}^{i-1}A_i\right)^{-1} = \begin{bmatrix} \dot{\theta}_i\,{}^{i-1}Z_{i-1} & . & \dot{\theta}_i\,{}^{i-1}Z_{i-1} \\ \ldots\ldots\ldots\ldots & . & \ldots\ldots\ldots\ldots \\ 0 & . & 0 \end{bmatrix} \tag{2.32}
$$

where

$$
{}^{i-1}Z_{i-1} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{2.33}
$$

$$
{}^{i-1}Z_{i-1} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{2.34}
$$

Equation (2.33) represents a unit vector pointing along z_{i-1} axis. Similarly, Eq. (2.32) represents a 3 x 3 skew symmetric matrixes whose nonzero elements denote a unit angular velocity of link i with respect to link i-1 both ${}^{i-1}Z_{i-1}$ and ${}^{i-1}Z_{i-1}$ are expressed in the (i-1) the link frame. We conclude that the upper left 3 x 3 submatrix of $({}^{i-1}A_i)({}^{i-1}A_i)^{-1}$ represents the angular velocity of link i relative to link i-1 and the fourth column represents the linear velocity of a point, which is embedded in link i and instantaneously coincident with O_{i-1} relative to link i-1.

2.2.2 Differential Transformation Matrix

A standard serial manipulator as shown as Figure 2.5, where P_n denotes the position vector of the origin of the end-effector frame, and P_{i-1} denotes the position vector of the origin of the (i -l) the frame relative to the fixed frame. Further, ${}^{i-1}p^{*}_{n}$ denotes the vector pointing from O_{i-1} to O_n and expressed in the fixed frame. The loop closure equation for such an n-dof serial manipulator can be written as

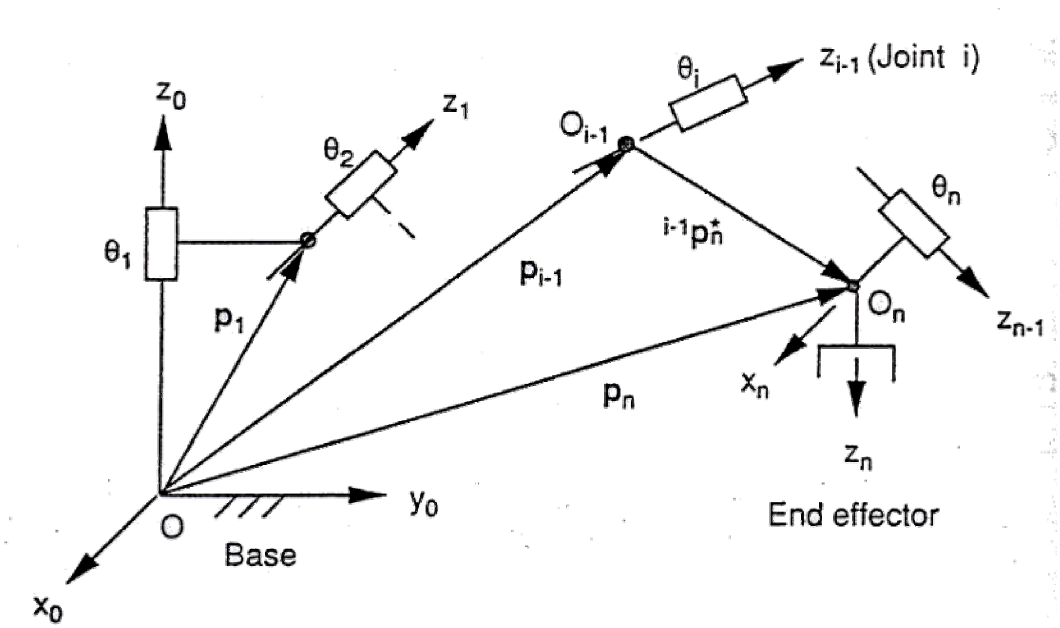

Figure 2.5 Link parameters of serial manipulator

$$ {}^{0}A_{n} = {}^{0}A_{1}\,{}^{1}A_{2}\,{}^{2}A_{3}\ldots\ldots{}^{n-1}A_{n} \tag{2.35}$$

Taking the derivative of Eq. (2.35) with respect to time, we obtain

$$ {}^{0}\dot{A}_{n} = \left({}^{0}\dot{A}_{1}\,{}^{1}A_{2}\ldots\ldots{}^{n-1}A_{n}\right) + \left({}^{0}A_{1}\,{}^{1}\dot{A}_{2}\ldots\ldots{}^{n-1}A_{n}\right) + \ldots\ldots + \left({}^{0}A_{1}\,{}^{1}A_{2}\ldots\ldots{}^{n-1}\dot{A}_{n}\right) \tag{2.36}$$

Equation (2.36) contains 12 nontrivial scalar equations that can be reduced to a system of six independent equations as follows. Post multiplying Eq. (2.36) by ${}^{0}A_{n}^{-1}$ we obtain

$$ {}^{0}\dot{A}_{n}\,{}^{0}A_{n}^{-1} = {}^{0}\dot{A}_{1}\,{}^{0}A_{1}^{-1} + {}^{0}A_{1}\left({}^{1}A_{2}\,{}^{1}A_{2}^{-1}\right){}^{0}A_{1}^{-1} + \left({}^{0}A_{1}\,{}^{1}A_{2}\right)\left({}^{2}A_{3}\,{}^{2}A_{3}^{-1}\right)\left({}^{0}A_{1}\,{}^{1}A_{2}\right)^{-1} + \ldots.. \tag{2.37} $$

The matrix ${}^{0}A_{n}$ can be decomposed into two submatrices:

$$ {}^{0}\dot{A}_{n} \equiv \begin{bmatrix} \dot{R}_{n} & v_{n} \\ 0 & 0 \end{bmatrix} \tag{2.38} $$

where R_n denotes the rate of change of the end-effector rotation matrix and $v_n=p_n$ denotes the linear velocity of the origin of the hand coordinate system.

Similar to Eq. (2.32), we can express the matrix products in Eq. (2.37) as

$$ {}^{0}\dot{A}_{n} \equiv \begin{bmatrix} \Omega_{n} & v_{0} \\ 0 & 0 \end{bmatrix} \tag{2.39} $$

$$ \left({}^{i-1}\dot{A}_{i}\right)\left({}^{i-1}A_{i}\right)^{-1} = \begin{bmatrix} \dot{\theta}_{i}\,{}^{i-1}Z_{i-1} & d_{i}\,{}^{i-1}Z_{i-1} \\ 0 & 0 \end{bmatrix} \tag{2.40} $$

Note that $\Omega_n=R_n\,R^{T}_{n}$ is a 3 x 3 skew-symmetric matrix whose elements represent the angular velocity of the end effector, and v_0 represents the linear velocity of a point in the end effector that is instantaneously coincident with the origin of the fixed reference frame. For convenience, we define

$$ {}^{0}A_{1}\,{}^{1}A_{2}\ldots..{}^{i-2}A_{i-1} \equiv \begin{bmatrix} R_{i-1} & p_{i-1} \\ 0 & 1 \end{bmatrix} \tag{2.41} $$

where R_{i-1} and P_{i-1} denote the rotation matrix and the position vector of the origin of the (i-1) the frame with respect to the fixed reference frame. Substituting Eq. (2.39) through (2.41) into (2.37) yields

$$\begin{bmatrix} \Omega_n & v_o \\ 0 & 0 \end{bmatrix}$$

$$= \sum_{i=1}^{n} \begin{bmatrix} \dot{\theta}_i \left(R_{i-1}{}^{i-1}Z_{i-1}R_{i-1}^T \right) & . & -\dot{\theta}_i \left(R_{i-1}{}^{i-1}Z_{i-1}R_{i-1}^T \right) p_{i-1} + \dot{d}_i R_{i-1}{}^{i-1}z_{i-1} \\ \cdots\cdots\cdots & . & \cdots\cdots\cdots \\ 0 & . & 0 \end{bmatrix}$$

$$= \sum_{i=1}^{n} \begin{bmatrix} \dot{\theta}_i Z_{i-1} & . & -\dot{\theta}_i Z_{i-1} p_{i-1} + \dot{d}_i z_{i-1} \\ \cdots\cdots & . & \cdots\cdots\cdots \\ 0 & . & 0 \end{bmatrix} \tag{2.42}$$

where

$$Z_{i-1} = R_{i-1}{}^{i-1}Z_{i-1}R_{i-1}^T \tag{2.43}$$

$$z_{i-1} = R_{i-1}{}^{i-1}Z_{i-1} \tag{2.44}$$

Equation (2.42) contains only six independent equations. The (3,2), (1,3), and (2,1) elements form the angular velocity vector w_n of the end effector, and the last column represents the linear velocity of a point in the end effector that is instantaneously coincident with the origin of the fixed frame. Writing Eq. (2.22) in vector form, we obtain

$$\omega = \sum_{i=1}^{n} \dot{\theta}_i z_{i-1} \tag{2.45}$$

$$v_o = \sum_{i=1}^{n} \left(-\dot{\theta}_i z_{i-1} \times p_{i-1} + \dot{d}_i z_{i-1} \right) \tag{2.46}$$

Equations (2.45) and (2.46) implies that the angular velocities of the links are additive. We may think of the end effector as rotating instantaneously about and translating along all the joint axes, and the effect of the instantaneous motion about each joint axis can be added linearly. We note that the velocity, v_n of a point located at the origin of the hand coordinate system is related to v_0 by the following transformation:

$$v_n = v_o + \omega_n \times p_n \tag{2.47}$$

2.3 Manipulator Jacobian Matrix

Let $x_i = f_i(q_1, q_2, q_3, ...q_n)$ for i = 1,2,3,..., m be a set of m equations, each a function of n independent variables. Then the time derivatives of x_i can be written as a function of q_i as follows:

$$\dot{x}_i = \frac{\partial f_i}{\partial q_1}\dot{q}_1 + \frac{\partial f_i}{\partial q_2}\dot{q}_2 + \frac{\partial f_i}{\partial q_3}\dot{q}_3 + ... + \frac{\partial f_i}{\partial q_n}\dot{q}_n$$

i=1,2,3,……….,m. (2.48)

We obtain

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ . \\ . \\ . \\ \dot{x}_m \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \frac{\partial f_1}{\partial q_2} & . & . & . & \frac{\partial f_1}{\partial q_n} \\ \frac{\partial f_2}{\partial q_1} & \frac{\partial f_2}{\partial q_2} & . & . & . & \frac{\partial f_2}{\partial q_n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \frac{\partial f_m}{\partial q_1} & \frac{\partial f_m}{\partial q_2} & . & . & . & \frac{\partial f_m}{\partial q_n} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ . \\ . \\ . \\ \dot{q}_n \end{bmatrix} \tag{2.49}$$

or simply

$$\dot{x} = J\dot{q} \tag{2.50}$$

where $x = [x_1, x_2, x_3, ..., x_m]^T$ denotes an m dimensional vector, $q = [q_1, q_2, q_3, ..., q_n]^T$ denotes an n dimensional vector, and J denotes the m*n matrix of the partial derivatives in Eq. (2.49).

We can J the Jacobian matrix, or simply Jacobian. The Jacobian matrix is a linear transformation matrix that maps an n-dimensional velocity vector q into an m-dimensional velocity vector x. We may think of the elements of J as the influence coefficients of the vector function x. The (i, J) element of J describes how a differential change in q_j affects the differential change in x_i. In general, the vector x is a nonlinear function of q. Hence the Jacobian matrix is also a function of q. Thus, the Jacobian matrix is configuration dependent.

For a robot manipulators, the Jacobian matrix is defined as the coefficient matrix of any set of equations that relates the velocity state of the end effector to the actuated joint rates. The joint rates are defined as

$$\dot{q}_i = \begin{cases} \dot{\theta}_i & \text{for a revolute joint} \\ \dot{d}_i & \text{for a prismatic joint} \end{cases}$$

(2.51)

The velocity state of the end effector, x, can be expressed in several different ways. Perhaps the most commonly used definitions are the conventional Jacobian and screw-based Jacobian.

1. *Conventional Jacobian*; In a conventional Jacobian, the end-effector velocity state is expressed in terms of the linear velocity of the origin of the end-effector coordinate frame, v_n and the angular velocity of the end effector, w_n

$$\dot{x} = \begin{bmatrix} v_n \\ \omega_n \end{bmatrix} \tag{2.52}$$

2. *Screw-based Jacobian*; The screw-based Jacobian is defined in terms of the angular velocity of the end effector, w_n, and the linear velocity of a reference point, v_0 in the end effector that is instantaneously coincident with the origin of a reference frame in which the screws are expressed:

$$\dot{x} = \begin{bmatrix} w_n \\ 0 \end{bmatrix} \tag{2.53}$$

The end-effector velocity, $\dot{x}$, for the screw-based Jacobian is defined with its angular and linear velocity vectors arranged in reverse order from the conventional Jacobian.

In general, the Jacobian matrix is an m x n matrix, where m denotes the degrees of freedom of the end-effector space and n denotes the number of actuated joint variables.

For a 6-dof spatial manipulator, m = n = 6, the Jacobian matrix is a 6 x 6 square matrix. For a manipulator with less than 6 degrees of freedom, the end-effector velocity state may contain just the linear velocity vector, or the angular velocity vector, or a combination of some linear and angular velocity components. For example, the working space of a planar manipulator is confined to a two-dimensional space. A three-component vector $\dot{x} = [w_x, w_y, w_z]^T$ is sufficient to describe the velocity state of the end effector. Hence the Jacobian reduces to a 3 x 3 matrix. Similarly for a point positioning device $\dot{x} = [v_x, v_y, v_z]^T$ and for a body-orienting mechanism, $\dot{x} = [w_x, w_y, w_z]^T$ on the other hand, for a manipulator with redundant degrees of freedom, we may have n > 6.

2.4 Conventional Jacobian

As mentioned earlier, any point in the end effector can be chosen as the reference point to describe the velocity start of the end effector. A logical choice is the origin, O_n , of the end-effector frame. Using this definition, the end-effector velocity start can be expressed in terms of the joint rates as follows:

$$v_n = \sum_{i=1}^{n} \left[\dot{\theta}_i \left(z_{i-1} \times {}^{i-1}p_n^* \right) + z_{i-1} d_i \right] \tag{2.54}$$

$$\omega_n = \sum_{i-1}^{n} \dot{\theta}_i z_{i-1} \tag{2.55}$$

where $\dot{\theta}_i$ and $\dot{d}_i$ are the rate of rotation about and translation along the i.th joint axis, z_{i-1} is a unit vector along the i.th joint axis, and ${}^{I-1}p_n^*$ is a vector defined from the origin of the (i-1) th link frame, O_{i-1} to the origin of the end effector frame, as shown in Fig. 2.6. Note that all vectors in Eqs. (2.54) and (2.55) are expressed in the fixed coordinate frame, (x_0 ,y_0 , z_0) writing Eqs. (2.54) and (2.55) in matrix form,

$$\dot{x} = \begin{bmatrix} v_n \\ w_n \end{bmatrix} = j\dot{q} \tag{2.56}$$

where

$$J = [J_1, J_2, \ldots\ldots\ldots, J_n]$$

$$J_i = \begin{bmatrix} z_{i-1} \times {}^{i-1}p_n^* \\ z_{i-1} \end{bmatrix} \qquad \text{for a revolute joint,}$$

$$J_i = \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix} \qquad \text{for a prismatic joint,}$$

The left-hand side of Eq. (2.56) is a 6x1 vector composed of the elements of v_n and w_n, while the right-hand side is a product of the Jacobian matrix and the vector of joint rates. The vector of joint rates consists of all the actuated joint rates $\dot{q}_i$, for i=1,2,3,…….n.

The i the column of the Jacobian matrix, J_i, represents the effect of the i.th joint rate on the velocity state of the end effector.

Equation (2.62) implies that to compute the Jacobian matrix, the direction and location of each joint axis should be determined first. This can be accomplished by the following matrix operations:

$$z_{i-1} = {}^{0}R_{i-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{2.57}$$

$${}^{i-1}p_n^* = {}^{0}R_{i-1}\, {}^{i-1}r_i + {}^{i}p_n^* \tag{2.58}$$

where

$${}^{i-1}r_i = \begin{bmatrix} a_i c\theta_i \\ a_i s\theta_i \\ d_i \end{bmatrix}$$

Denotes the vector $\overline{O_{i-1}O_i}$ expressed in the (i-1) the link frame, while ${}^{i-1}p_n^*$ denotes the vector $\overline{O_{i-1}O_i}$ expressed in the fixed frame. Once the Jacobian is known, the end-effector velocity can be computed directly from Eq. (2.56) for any given joint rates. On the other hand, given a desired end-effector velocity, the inverse transformation of Eq. (2.56) can be solved for the joint rates.

2.4.1 Jacobian of a Planar 2-DOF Manipulator

I show as the Jacobian of a planar 2-dof manipulator shown in Fig. 2.6. The manipulator is made up of two revolute joints, with both axes pointing out of the paper. A coordinate system is attached to each link according to the D-H convention for the purpose of analysis. The (x_0, y_0) coordinate system is attached to the base with its origin located at the fixed pivot O. The x_0 axis points to the right.

We first compute the vectors z_{i-1} and ${}^{i-1}p_2^*$ by applying Eqs. (2.57) and (2.58):

$$z_0 = z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$${}^1p_2^* = \begin{bmatrix} a_2 c\theta_{12} \\ a_2 s\theta_{12} \\ 0 \end{bmatrix}$$

$${}^0p_2^* = \begin{bmatrix} a_1 c\theta_1 + a_2 c\theta_{12} \\ a_1 c\theta_1 + a_2 s\theta_{12} \\ 0 \end{bmatrix}$$

where $\theta_{12} = \theta_1 + \theta_2$. We know that the expressions above can be obtained directly from the geometry of the links without using the Denavit-Hartenberg transformation matrices. Substituting the expressions above into Eq. (2.56), we obtain

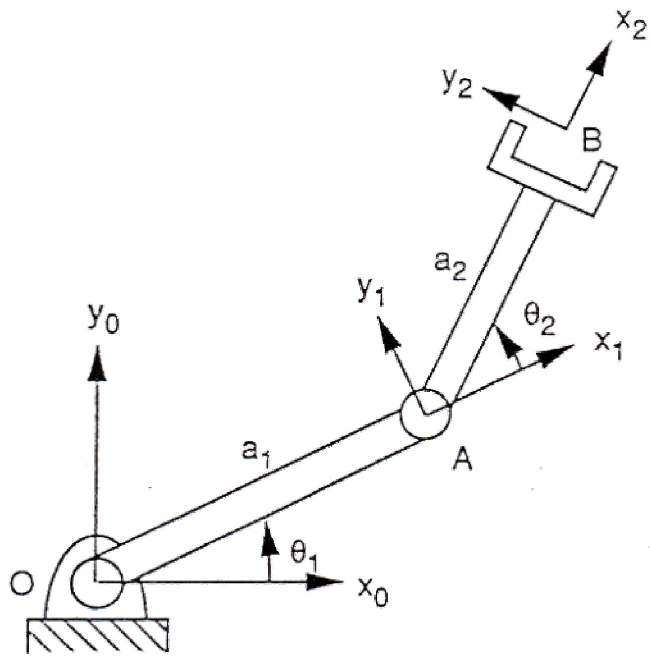

Figure 2.6 Serial manipulator of 2-dof

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} -a_1 s\theta_1 - a_2 s\theta_{12} & -a_2 s\theta_{12} \\ a_1 c\theta_1 + a_2 c\theta_{12} & a_2 c\theta_{12} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \tag{2.59}$$

Hence the Jacobian matrix is given by

$$J = \begin{bmatrix} -a_1 s\theta_1 - a_2 s\theta_{12} & -a_2 s\theta_{12} \\ a_1 c\theta_1 + a_2 c\theta_{12} & a_2 c\theta_{12} \end{bmatrix} \tag{2.60}$$

2.4.2 Jacobian of a Planar 3-DOF Manipulator

As a second example, we study the convention Jacobian of the planar 3-dof manipulator shown in Fig. 2.3. We first compute the vectors z_{i-1} and ${}^{i-1}p_3^*$ from Eqs. (2.57) and (2.56), for i=1.2 and 3 as fol1ows:

$$z_0 = z_1 = z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$^{2}p_3^* = \begin{bmatrix} a_3 c\theta_{123} \\ a_3 s\theta_{123} \\ 0 \end{bmatrix}$$

$$^{1}p_3^* = \begin{bmatrix} a_2 c\theta_{12} + a_3 c\theta_{123} \\ a_2 s\theta_{12} + a_3 s\theta_{123} \\ 0 \end{bmatrix}$$

where $\theta_{12}=\theta_1+\theta_2$ and $\theta_{12}=\theta_1+\theta_2+\theta_3$ substituting the expressions above into Eq.(2.56), we obtain

$$\begin{bmatrix} v_x \\ x_y \\ w_z \end{bmatrix} = J \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

where

$$J = \begin{bmatrix} -(a_1 s\theta_1 + a_2 s\theta_{12} + a_3 s\theta_{123}) & -(a_2 s\theta_{12} + a_3 s\theta_{123}) & a_3 s\theta_{123} \\ (a_1 c\theta_1 + a_2 c\theta_{12} + a_3 c\theta_{123}) & (a_2 c\theta_{12} + a_3 c\theta_{123}) & a_3 c\theta_{123} \\ 1 & 1 & 1 \end{bmatrix} \tag{2.61}$$

We note that if the reference point is chosen at origin of the (x_2, y_2) frame, the Jacobian matrix reduces to

$$J = \begin{bmatrix} -(a_1 s\theta_1 + a_2 s\theta_{12}) & -(a_2 s\theta_{12}) & 0 \\ (a_1 c\theta_1 + a_2 c\theta_{12}) & (a_2 c\theta_{12}) & 0 \\ 1 & 1 & 1 \end{bmatrix} \tag{2.62}$$

2.4.3 Jacobian of the Stanford Manipulator

Figure 2.7 show s a schematic diagram of the Stanford arm described. To simplify the analysis, the origin of the fixed coordinate frame is located at the point of intersection of the first two joint axes, and the origin of the (x_6 ,y_6 ,z_6) frame is located at the point of intersection of the last three joint axes.

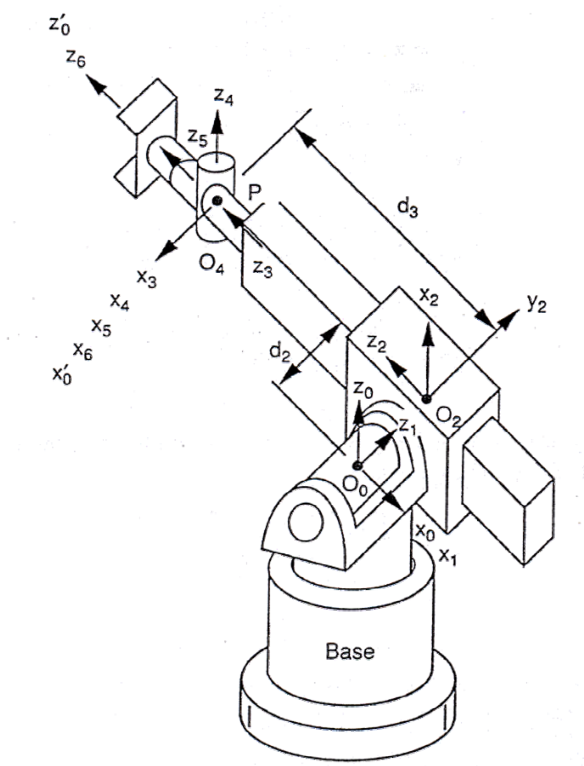

Figure 2.7 Stanford manipulator

The D-H link parameters are listed in Tab1e 2.1, from which the D-H transformation matrices are derived as follows:

Table 2.1 D-H link parameters of the Stanford arm

Joint i	**α_i**	**a_i**	**d_i**	**θ_i**
1	-90°	0	0	θ_1 (variable)
2	90°	0	d_2 (constant)	θ_2 (variable)
3	0°	0	d_3 (variable)	-90° (constant)
2	-90°	0	0	θ_4 (variable)
5	90°	0	0	θ_5 (variable)
6	0°	0	0	θ_6 (variable)

$$
{}^{0}A_1 = \begin{bmatrix} c\theta_1 & 0 & -s\theta_1 & 0 \\ s\theta_1 & 0 & c\theta_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}A_2 = \begin{bmatrix} c\theta_2 & 0 & s\theta_2 & 0 \\ s\theta_2 & 0 & -c\theta_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{3}A_4 = \begin{bmatrix} c\theta_4 & 0 & -s\theta_4 & 0 \\ s\theta_4 & 0 & c\theta_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$^{4}A_5 = \begin{bmatrix} c\theta_5 & 0 & s\theta_5 & 0 \\ s\theta_5 & 0 & -c\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{5}A_6 = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ s\theta_6 & c\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The directions of the joint axes, zi-ı, are derived by applying Eq. (2.63):

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{2.63}$$

$$z_1 = {}^{0}R_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -s\theta_1 \\ c\theta_1 \\ 0 \end{bmatrix} \tag{2.64}$$

$$z_2 = z_3 = {}^{0}R_2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} c\theta_1 s\theta_2 \\ s\theta_1 s\theta_2 \\ c\theta_2 \end{bmatrix} \tag{2.65}$$

$$z_4 = {}^{0}R_4 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -s\theta_1 s\theta_4 + c\theta_1 c\theta_2 c\theta_4 \\ c\theta_1 s\theta_4 + s\theta_1 c\theta_2 c\theta_4 \\ -s\theta_2 c\theta_4 \end{bmatrix} \tag{2.66}$$

$$z_5 = {}^{0}R_5 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -s\theta_1 c\theta_4 s\theta_5 + c\theta_1 c\theta_2 s\theta_4 s\theta_5 + c\theta_1 s\theta_2 c\theta_5 \\ -c\theta_1 c\theta_4 s\theta_5 + s\theta_1 c\theta_2 s\theta_4 s\theta_5 + s\theta_1 s\theta_2 c\theta_5 \\ -s\theta_2 s\theta_4 s\theta_5 + c\theta_2 c\theta_5 \end{bmatrix} \tag{2.67}$$

The position vectors, i-ıp6' are derived by applying Eq. (2.62)

$$ {}^{3}p_6^* = {}^{4}p_6^* = {}^{5}p_6^* = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{2.68} $$

$$ {}^{2}p_6^* = \begin{bmatrix} d_3 c\theta_1 s\theta_2 \\ d_3 s\theta_1 s\theta_2 \\ d_3 c\theta_2 \end{bmatrix} \tag{2.69} $$

$$ {}^{1}p_6^* = \begin{bmatrix} d_3 c\theta_1 s\theta_2 - d_2 s\theta_1 \\ d_3 s\theta_1 s\theta_2 + d_2 c\theta_1 \\ d_3 c\theta_2 \end{bmatrix} \tag{2.70} $$

$$ {}^{0}p_6^* = \begin{bmatrix} d_3 c\theta_1 s\theta_2 - d_2 s\theta_1 \\ d_3 s\theta_1 s\theta_2 + d_2 c\theta_1 \\ d_3 c\theta_2 \end{bmatrix} \tag{2.71} $$

The Jacobian matrix is derived by applying Eq. (2.62) column by column:

$$ \begin{bmatrix} v_{6x} \\ v_{6y} \\ v_{6z} \\ w_x \\ w_y \\ w_z \end{bmatrix} = J \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \\ \dot{\theta}_6 \end{bmatrix} $$

where the Jacobian matrix is given by

$$J = \begin{bmatrix} d_3 c\theta_1 s\theta_2 - d_2 c\theta_1 & d_3 c\theta_1 c\theta_2 & 0 & 0 & 0 \\ d_3 c\theta_1 s\theta_2 - d_2 s\theta_1 & d_3 s\theta_1 c\theta_2 & 0 & 0 & 0 \\ 0 & -d_3 s\theta_2 & 0 & 0 & 0 \\ 0 & -s\theta_1 & c\theta_1 s\theta_2 & j_{45} & j_{46} \\ 0 & c\theta_1 & s\theta_1 s\theta_2 & j_{55} & j_{56} \\ 1 & 0 & c\theta_2 & j_{65} & j_{66} \end{bmatrix} \tag{2.72}$$

where j_{45}, j_{55}, and j_{65} represent the x, y, and z components of the un it vector z_4, and j_{46}, j_{56}, and j_{66} are the x, y, and z components of z_5.

As a consequence of the concurrence of axes 2, 5 and 6, all the elements in the upper right 3 x 3 submatrix are equal to zero.

CHAPTER THREE

POSITION ANALYSIS OF SERIAL MANIPULATORS

3.1 Introduction

A serial manipulator consists of several links connected in series by various types of joints, typically revolute and prismatic joints. One end of the manipulator is attached to the ground and the other end is free to move in space. For this reason a serial manipulator is sometimes called an open-loop manipulator. We call the fixed link the base, and the free end where a gripper or a mechanical hand is attached, the end effector.

For a robot to perform a specific task, the location of the end effector relative to the base should be established first. This is called the position analysis problem. There are two types of position analysis problems: direct position or direct kinematics and inverse position or inverse kinematics problems. For direct kinematics, the joint variables are given and the problem is to find the location of the end effector. For inverse kinematics, the location of the end effector is given and the problem is to find the joint variables necessary to bring the end effector to the desired location. For a serial manipulator, direct kinematics is fairly straightforward, whereas inverse kinematics becomes very difficult. On the other hand, for a parallel manipulator, inverse kinematics is very straightforward, whereas direct kinematics becomes very difficult. We note that for a deficient manipulator the end effector cannot be positioned freely in space, and for a redundant manipulator there may be several infinitudes of inverse kinematic solutions corresponding to a given end-effector location, depending on the degrees of redundancy.

The number of possible inverse kinematics solutions depends on the type and location of a robot manipulator. In general, closed-form solutions can be found for manipulators with simple geometry, such as manipulators with three consecutive joint axes intersecting at a common point or three consecutive joint axes parallel to one another. For a manipulator of general geometry, the inverse kinematics problem becomes a very difficult task. Following is a brief review of the historical development of the inverse kinematics problem.

There are two commonly used methods, Denavit and Hartenberg's method and the method of successive screw displacements, are introduced. Both methods are systematic in nature and more suitable for the kinematic analysis of serial manipulators. The kinematics of several frequently used robotic structures will be analyzed to demonstrate the methodologies. The inverse kinematics of the 6R manipulator of general geometry is the most difficult problem in kinematics.

3.2 Link Parameters and Link Coordinate Systems

In general, an n-dof serial manipulator consists of a base link and n moving links connected in series by n joints without forming a closed loop. The relative motion associated with each joint can be controlled by an actuator such that the end effector can be positioned anywhere within its workspace.

The z_i -axis is aligned with the (i + l) the joint axis. The positive direction of rotation or translation can be chosen arbitrarily. Manipulator in which the first joint axis points up vertically, the second joint axis is perpendicular to the first with a small offset, the third and fourth joint axes are both parallel to the second, and the fifth joint axis intersects the fourth perpendicularly.

To describe the geometry of the links, starting from the base link, we number the links sequentially from 0 to n and the joints from 1 to n. Thus, except for the base link and the end-effector link, every link has two joints. Link 1 is connected to the base link by joint 1, link 2 is connected to link 1 by joint 2, and so on. Link i has joint i at its proximal end and joint i + 1 at the distal end, as shown in Fig. 3.1.

Following Denavit and Hartenberg's convention, a Cartesian coordinate system is attached to each link of a manipulator. Except for the base and end-effector link, coordinate system i is attached to link i according to the following rules:

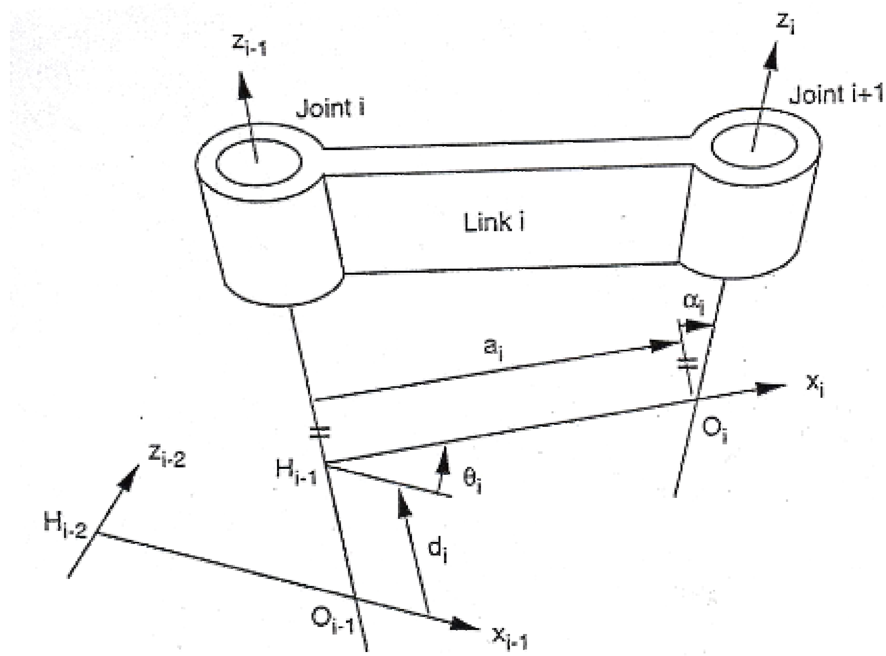

Figure 3.1 Link parameters

The xi-axis is defined along the common normal between the i^th and U+l the joint axes and points from the i^th to the U + 1 the joint axis. If the two joint axes are parallel, the xi-axis can be chosen anywhere perpendicular to the two joint axes. In case of two intersecting joint axes, the x_j axis can be defined either in the direction of the vector cross product Z_{i-1} x Z_i or in the opposite direction, and the origin is at the point of intersection. The y_j axis is determined by the right-hand role.

The zero^th coordinate system is attached to the base at any convenient location as long as the z_0 axis is aligned with the first joint axis. Furthermore, we attach a coordinate system to the end-effector link, called the end effector or hand coordinate system, to describe the location of the end effector. The hand coordinate system can be located anywhere in the end-effector as long as the x_n axis is normal to the last joint axis. For convenience, the z_n axis is often defined along the direction of approach of a gripper.

Let H_{i-1} be the point of intersection of the x_j and z_{j-1} axes, and let O_j , the origin of the ith coordinate system, be the point of intersection of the X_i and L_i axes as shown in Fig. 3.1. Then, regardless of how the links are constructed physically, the following parameters are uniquely determined by the geometry of the axes:

a_i: offset distance between two adjacent joint axes, where $a_i = [H_{j-1}\ O_j\ ]$

d_i : translational distance between two incident normals of a joint axis. di = $[0_{i-1}\ H_{i-1}]$ is positive if the vector $[O_{i-1}\ H_{i-1}]$ points in the positive z_{i-1} direction; otherwise, it is negative.

α_i: Twist angle between two adjacent joint axes. It is the angle required to rotate the z_i-1-axis into alignment with the z_i-axis about the positive x_j-axis according to the right-hand role.

e_i: Joint angle between two incident normals of a joint axis. It is the angle required to rotate the X_{i-1} axis into alignment with the x_i-axis about the positive z_{i-1}-axis according to the right-hand role.

For a revolute joint, a_i, α_i, and di are constant, and e_i is a variable that measures the relative location of link i with respect to link i-1. For a prismatic joint, a_i, α_i, and e_i are constant, and d_i is a variable that measures the relative location of link i with respect to link i-1. in what follows, we refer to e_i for a revolute joint and d_i for a prismatic joint as joint variables and to the constant parameters as link parameters. We note that a joint variable provides only the relative location between two adjacent links; it should not be confused with the term displacement. A displacement implies the amount of angle or distance needed to move a link from one location co another. Using the definitions above, the displacement required to move a link from one location to another is equal to the difference between two successive values of a joint variable.

For a prismatic joint, the direction of the joint axis defines the direction of relative translation between two links. Unlike a revolute joint, only the direction of the joint axis is important. Although the location of a prismatic joint has no effect on relative displacement, the physical location of the joint will be used to establish the foregoing link parameters. Note that several different coordinate systems can be defined for a manipulator, due to various possible choices of the positive z and x axes. A general procedure for establishing Denavit-Hartenberg coordinate systems can be summarized as follows: Starting from the base link, number the links and joints sequentially.

The base is numbered as link O and the last link is the end effector. Except for the base and end effector link, every link contains two joints. Joint i connects link i to link i-1. Draw the common normal between every two adjacent joint axes. Except for the first and last joint axes, every joint axis should have two incident common normals, one with the (i - l) th and the other with the (i + l) the joint axis.

Establish the base coordinate system such that the z_0-axis is aligned with the first joint axis, the x_0-axis is perpendicular to the z_0-axis, and the y_0-axis is determined by the right-hand role.

Establish the nth hand coordinate system such that the x_n axis is perpendicular to the last joint .axis. The z_n axis is usually chosen in the, direction of approach of the end effector.

Attach a Cartesian coordinate system to the distal end of all the other links as follows:

- The z_i axis is aligned with the (i+1) the joint axis.

- The x_i axis is defined along the common normal between the i^{th} and (i+1) the joint axes, pointing from the i^{th} to the (i-1) the joint axis. If the joint axes are parallel, the x_i axis can be chosen anywhere perpendicular to the two joint axes. in the case of two intersecting joint axes, the x_i axis can be defined either in the direction of the vector cross product z_{i-1} x z_i or in the opposite direction, and the origin is located at the point of intersection.

- The y_i axis is defined according to the right-hand role.

Determine the link parameters and joint variables, a_i, α_i, θ_i and d_i .

There are n + 1 coordinate systems for an n-dof manipulator However, if additional reference coordinate systems are defined, they can be related to one of the coordinate systems above by a transformation matrix. We note that John Craig used a different convention; Craig attached the i^{th} coordinate systems to the proximal end of link i, which results in a different homogeneous transformation matrix.

3.3 Denavith-Hartenberg Homogeneous Transformation Matrices

Having established a coordinate system to each link of a manipulator, a 4 x 4 transformation matrix relating two successive coordinate systems can be established. Observation of Fig. 3.2 reveals that the i^{th} coordinate system can be thought of as being displaced from the (i-1) the coordinate system by the following successive rotations and translations.

1. The (i+1) the coordinate system is translated along the z_{i-1} axis a distance d_i. This brings the origin O_{i-1} into coincidence with H_{i-1}. The corresponding transformation matrix is

$$T(z,d)=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & d_i\\ 0 & 0 & 0 & 1\end{bmatrix}$$

2. The displaced (i -1)the coordinate system is rotated about the z_{i-1} axis an angle θ_i which brings the x_{i-1} axis into alignment with the x_i axis. The corresponding transformation matrix is

$$T(z,\theta)=\begin{bmatrix}c\theta_i & -s\theta_i & 0 & 0\\ s\theta_i & c\theta_i & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

3. The displaced (i -1) the coordinate system is translated along the x_i axis a distance a_i. This brings the origin O_{i-1} into coincidence with O_i The corresponding transformation matrix is

$$T(x,a)=\begin{bmatrix}1 & 0 & 0 & a_i\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

4. The displaced (i - l) the coordinate system is rotated about the x_i axis an angle a_j , which brings the two coordinate systems into complete coincident. The corresponding transformation matrix is

$$T(x,\alpha)=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & c\alpha_i & -s\alpha_i & 0\\ 0 & s\alpha_i & c\alpha_i & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

We may think of the transformations above as four basic transformations about the moving coordinate axes. Therefore, the resulting transformation matrix, ${}^{i-1}A_i$ is given by

$${}^{i-1}A_i = T(z,d)T(z,\theta)T(x,a)T(x,\alpha) \tag{3.1}$$

Expanding Eq. (3.1), we obtain

$${}^{i-1}A_i = \begin{bmatrix} c\theta_i & -c\alpha_i s\theta_i & s\alpha_i s\theta_i & a_i c\theta_i \\ s\theta_i & c\alpha_i c\theta_i & -s\alpha_i c\theta_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.2}$$

Equation (3.2) is called the Denavit-Hartenberg (D-H) transformation matrix. The trailing subscript i and the leading superscript i-i denote that the transformation takes place from the i^{th} coordinate system to the (i - l) the coordinate system.

Let the homogeneous coordinates of the position vector of a point relative to the i^{th} coordinate system be denoted by ${}^{i}p$ = $[P_x, P_y, P_z, 1]^T$, Also let the homogeneous coordinates of a unit vector expressed in the i^{th} coordinate system be denoted by ${}^{i}U$ = $[U_x , U_y, U_z, O]^T$. Then the transformation of a position vector and a unit vector from the i^{th} to the (i - I) the coordinate system can be written as

$${}^{i-1}p = {}^{i-1}A_i \, {}^{i}p \tag{3.3}$$

$${}^{i-1}u = {}^{i-1}A_i \, {}^{i}u \tag{3.4}$$

Note that the leading superscript is used to indicate the coordinate system with respect to which a vector is expressed. Although the transformation matrix A is not orthogonal, the inverse transformation exists and is given by

$$ {}^{i}A_{i-1} = \left({}^{i-1}A_i\right)^{-1} = \begin{bmatrix} c\theta_i & s\theta_i & 0 & -a_i \\ -c\alpha_i s\theta_i & c\alpha_i c\theta_i & s\alpha_i & -d_i s\alpha_i \\ s\alpha_i s\theta_i & -s\alpha_i c\theta_i & c\alpha_i & -d_i c\alpha_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.5} $$

Example 3.1.

Planar 3-DOF Manipulator Figure 3.2 shows a 3-dof planar manipulator constructed with three revolute joints located at points 00, A, and P, respectively. A coordinate system is attached to each link. The (x_0, y_0, z_0) coordinate system is attached to the base with its origin located at the first joint pivot and the x-axis pointing to the right. Since the joint axes are all parallel to each other, all the twist angles IXi and translational distances d_i are zero.

For the coordinate systems chosen, the link parameters are given in Table 3.1. The D-H transformation matrices are obtained by substituting the D-H link parameters into Eq. (3.2):

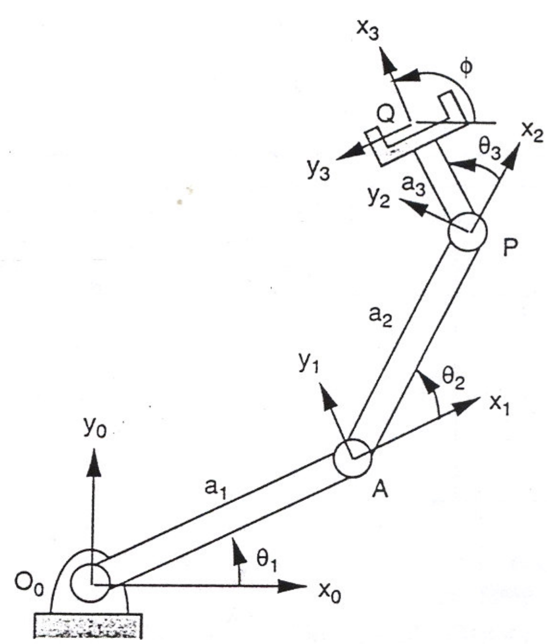

Figure 3.2 3-dof manipulator

$$
{}^{0}A_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & a_1 c\theta_1 \\ s\theta_1 & c\theta_1 & 0 & a_1 s\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.6}
$$

$$
{}^{1}A_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & a_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & a_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.7}
$$

$$
{}^{2}A_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_3 c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & a_3 s\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.8}
$$

Example 3.2

The SCARA arm is a type of 4dof manipulator. It has been produced by several companies,

Table 3.1 D-H parameters of a 3-DOF manipulator

Joint i	α_i	a_i	d_i	θ_i
1	0	a_1	0	θ_1
2	0	a_2	0	θ_2
3	0	a_3	0	θ_3

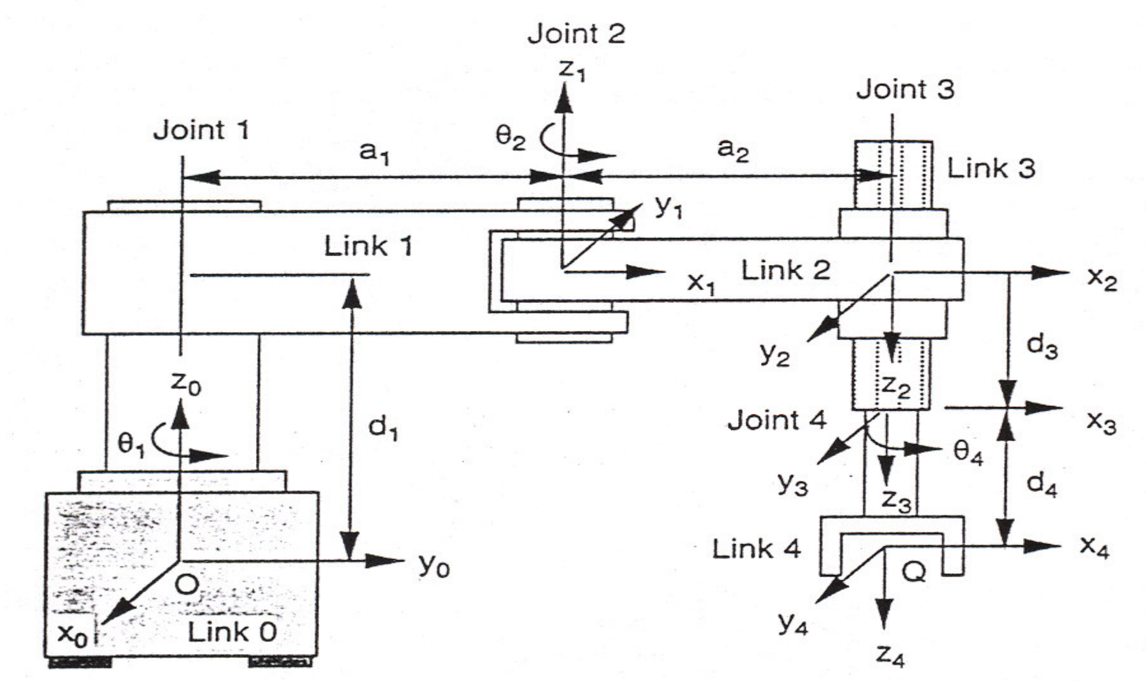

Figure 3.3 Link parameters of a SCARA arm.

A Scara arm is constructed with four joint axes parallel to each other. The first two and the fourth are revolute joints, and the third is a prismatic joint Figure 3.4 shows a schematic diagram of a SCARA arm. For the coordinate systems established in the figure, the corresponding link parameters are listed in Table 3.2.

Substituting the D-H link parameters into Eq. (3.2), we obtain the D-H transformation matrices:

$$^{0}A_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & a_1c\theta_1 \\ s\theta_1 & c\theta_1 & 0 & a_1s\theta_1 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.9}$$

Table 3.2 D-H parameters of the SCARA arm

Joint i	α_i	a_i	d_i	θ_i
1	0	a_1	d_1	θ_1
2	π	a_2	0	θ_2
3	0	0	d_3	0
4	0	0	d_4	θ_4

$$^{1}A_2 = \begin{bmatrix} c\theta_2 & s\theta_2 & 0 & a_2 c\theta_2 \\ s\theta_2 & -c\theta_2 & 0 & a_2 s\theta_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.10}$$

$$^{2}A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.11}$$

$$^{3}A_4 = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & 0 \\ s\theta_4 & c\theta_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.12}$$

In this robot, the joint variables are θ_1, θ_2, d_3, and θ_4 The first two joint variables control the x and y coordinates, the third joint variable controls the z coordinate, and the fourth joint variable controls the orientation of the end effector. Since the robot has only 4 degrees of freedom, the orientation of the end effector cannot be specified arbitrarily.

As a matter of fact, the z_4 axis must be always pointing in the negative z_0 direction. Although the SCARA robot has only 4 degrees of freedom, it is very useful for assembling components on a plane.

3.4 Loop-Closure Equations

In a study of the kinematics of robot manipulators, we are interested in deriving an a1gebraic equation relating the location of the end effector to the joint variables. The location of the end effector can be specified by the following 4 x 4 homogeneous transformation matrix:

$$ {}^{0}A_{n} = \begin{bmatrix} u & v & w & q \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.13}$$

where the upper right 3 x 1 submatrix describes the position of a reference point Q and the upper left 3 x 3 submatrix describe the orientation of the end effector. The orientation of the end effector can be specified in terms of three Euler angles, or the direction cosines of the three end effector coordinate axes, u, v, and w. If the ω-u-ω Euler angles are used, for example, the elements of the upper left 3 x 3 submatrix are given by

$$u_x = c\phi c\psi - s\phi c\theta s\psi$$
$$u_y = s\phi c\psi + c\phi c\theta s\psi$$
$$u_z = s\phi s\psi$$

$$v_x = -c\phi s\psi - s\phi c\theta c\psi$$
$$v_y = -s\phi s\psi + c\phi c\theta c\psi$$
$$v_z = s\phi c\psi$$

$$w_x = s\phi s\theta$$
$$w_y = -c\phi s\theta$$
$$w_z = c\theta \tag{3.14}$$

If the direction cosines are used, u, v, and w represent three unit vectors directed along the three coordinate axes of the hand coordinate system and. expressed in the base coordinate system.

From the geometry of the links, the transformation matrix ${}^{0}A_{n}$ above can be thought of as the resultant of a series of coordinate transformations beginning from the base coordinate system to the end-effector coordinate system. That is,

$$ {}^{0}A_{1}\,{}^{1}A_{2}\,{}^{2}A_{3}\,\ldots\ldots\ldots\,{}^{n-1}A_{n} = {}^{0}A_{n} \quad (3.15) $$

Equation (3.15) is called the loop-closure equation of a serial manipulator. It contains 16 scalar equations, four of which are trivial. Equating the upper right 3 x 1 submatrix results in three independent equations, representing the position of the end effector. Equating the elements of the upper left 3 x 3 submatrix result in nine equations, representing the orientation of the end effector. However, only three of the nine orientation equations are independent because of the orthogonal conditions.

The loop-closure equation, Eq. (3.15), can be used to solve both direct and inverse kinematics problems. For direct kinematics, the joint variables are given and the problem is to find where the end effector is with respect to the base coordinate system. This can be accomplished by multiplying the D-H matrices on the left-hand side of the equation. For the inverse kinematics, the end-effector location (i.e., ${}^{0}A_{n}$) is given and the problem is to find the joint variables needed to bring the end effector to the desired location. The problem becomes very nonlinear in what follows; we concentrate on the inverse kinematics problem.

Example 3.3.

Scorbot Robot Figure 3.5 shows a schematic diagram of the Scorbot robot. In this diagram, the second, third, and fourth joint axes are parallel to one another and point into the paper at points A, B, and P, respectively. The first joint axis points up vertically, and the fifth joint axis intersects the fourth perpendicularly. We wish to find the overall transformation matrix for the robot.

Using the coordinate systems established in Fig. 3.5, the corresponding link parameters are listed in Table 3.3. Substituting the D-H link parameters into Eq. (3.2), we obtain the D-H transformation matrices:

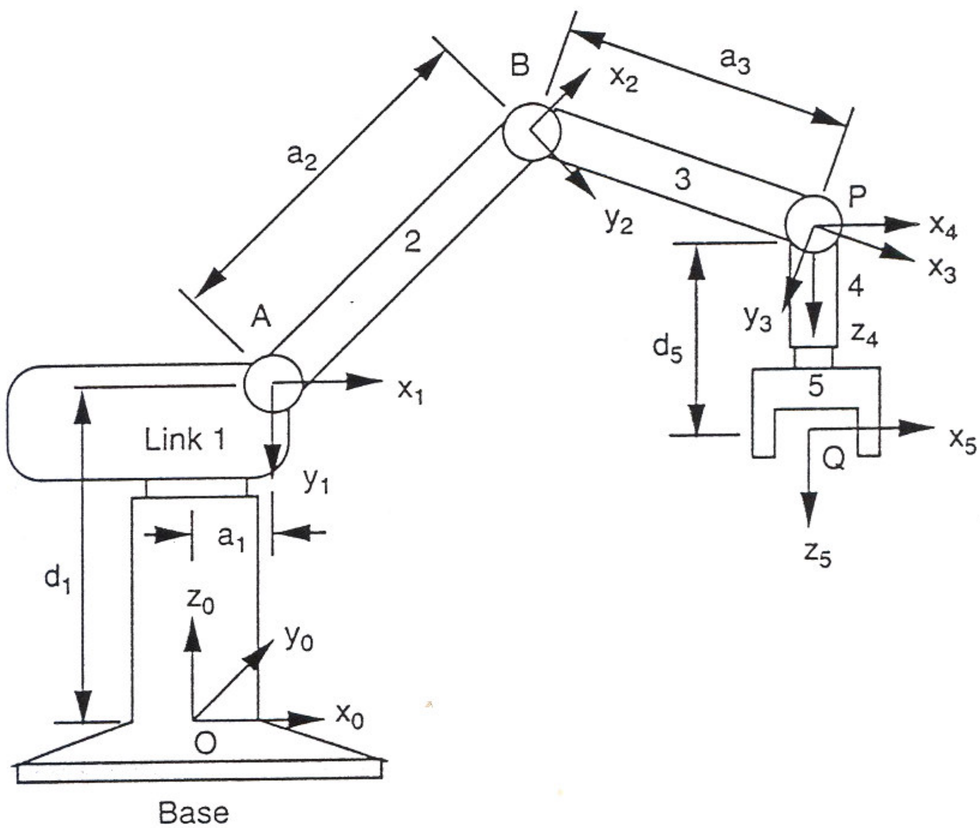

Figure 3.4 Scorbot robot link parameters diagram.

Table 3.3 D-H parameters of a 5 - DOF manipulator

Joint i	α_i	a_i	d_i	θ_i
1	$-\pi/2$	a_1	d_1	θ_1
2	0	a_2	0	θ_2
3	0	a_3	0	θ_3
4	$-\pi/2$	0	0	θ_4
5	0	0	d_5	θ_5

$$ {}^{0}A_1 = \begin{bmatrix} c\theta_1 & 0 & -s\theta_1 & a_1 c\theta_1 \\ s\theta_1 & 0 & c\theta_1 & a_1 s\theta_1 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.16} $$

$$ {}^{1}A_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & a_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & a_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.17} $$

$$ {}^{2}A_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_3 c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & a_3 s\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.18} $$

$$ {}^{3}A_4 = \begin{bmatrix} c\theta_4 & 0 & -s\theta_4 & 0 \\ s\theta_4 & 0 & c\theta_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.19} $$

$$^{4}A_5 = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ s\theta_5 & -s\theta_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.20}$$

Multiplying Eqs. (3.17), (3.18), and (3.19) yields

$$^{1}A_4 = \begin{bmatrix} c\theta_{234} & 0 & -s\theta_{234} & a_3c\theta_{23} + a_2c\theta_2 \\ s\theta_{234} & 0 & c\theta_{234} & a_3s\theta_{23} + a_2s\theta_2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.21}$$

where

$$c\theta_{ij} = \cos(\theta_i + \theta_j), s\theta_{ij} = \sin(\theta_i + \theta_j), c\theta_{ijk} = \cos(\theta_i + \theta_j + \theta_k), and . s\theta_{ijk} = \sin(\theta_i + \theta_j + \theta_k).$$

Note that Eq. (3.21) provides a transformation from the fourth coordinate system to the first coordinate system. We may treat θ_2 , θ_{23} , and θ_{234} as new variables. in this way, the orientation submatrix contains only one variable, θ_{234} , while the position submatrix contains two variables, θ_2 and θ_{23} This important fact can be used for deriving a closed form solution for any manipulator with three consecutive parallel joint axes.

Multiplying Eqs. (3.16), (3.21), and (3.20) yields the elements of the overall transformation matrix $^{0}A_5$:

$$u_x = c\theta_1 c\theta_{234} c\theta_5 + s\theta_1 s\theta_5$$

$$u_y = s\theta_1 c\theta_{234} c\theta_5 - c\theta_1 s\theta_5$$

$$u_z = -s\theta_{234} c\theta_5$$

$$v_x = -c\theta_1 c\theta_{234} s\theta_5 + s\theta_1 c\theta_5$$

$$v_y = -s\theta_1 c\theta_{234} s\theta_5 - c\theta_1 c\theta_5$$

$$v_z = s\theta_{234} s\theta_5$$

$$w_x = -c\theta_1 s\theta_{234}$$

$$w_y = -s\theta_1 s\theta_{234}$$

$$w_z = -c\theta_{234}$$

$$q_x = c\theta_1 (a_1 + a_2 c\theta_2 + a_3 c\theta_{23} - d_5 s\theta_{234})$$

$$q_y = s\theta_1 (a_1 + a_2 c\theta_2 + a_3 c\theta_{23} - d_5 s\theta_{234})$$

$$q_z = d_1 - a_2 s\theta_2 - a_3 s\theta_{23} - d_5 c\theta_{234} \tag{3.22}$$

Since this is a 5-dof manipulator, only five of the six parameters of the end effector can be specified. Very often, the desired position of a point and the direction of a line in the end effector (e.g., the position of point Q and the direction of x_5 axis) are specified. 5-dof manipulators are useful for spray painting, spot welding, and sealant applications for which only the position and direction of a line are essential.

3.5 Denavit-Hartenberg Method

Although the loop-closure equation, Eq. (3.15), can be applied to solve the inverse kinematics problem, in practice it is rarely solved in its present form. in general, if there are three intersecting joint axes, we may week with the position of the point of intersection first, thereby avoiding the joint variables associated with the three intersecting axes. If there are three parallel joint axes, we may combine the three joint variables as illustrated in the Scorbot robot example. We may also pre or postmultiply the loop-closure equation by the inverse of the matrix ${}^{i-1}A_i$ to obtain alterative loop closure equations, such as

$$\left({}^{0}A_{1}\right)^{-1}\,{}^{0}A_{n} = {}^{1}A_{2}\,{}^{2}A_{3}\ldots{}^{n-1}A_{n}\,, \tag{3.23}$$

$$\left({}^{1}A_{2}\right)^{-1}\left({}^{0}A_{1}\right)^{-1}\,{}^{0}A_{n} = {}^{2}A_{3}\,{}^{3}A_{4}\ldots{}^{n-1}A_{n}\,, \tag{3.24}$$

$$\left({}^{2}A_{3}\right)^{-1}\left({}^{1}A_{2}\right)^{-1}\left({}^{0}A_{1}\right)^{-1} = {}^{3}A_{4}\,{}^{4}A_{5}\ldots{}^{n-1}A_{n}\,. \tag{3.25}$$

One reason for rearranging the loop-closure equation is to redistribute the unknown variables on both sides of the equation as evenly as possible. Another reason is to take advantage of some special conditions, such as three consecutive intersecting joint axes or three consecutive parallel joint axes. In many cases, the equation becomes decoupled and a closed-form solution can be derived.

3.5.1 Analysis of a Planar 3-DOF Manipulator

For the planar 3-dof manipulator shown in Fig. 3.3, the overall transformation matrix is given by

$${}^{0}A_{3} = {}^{0}A_{1}\,{}^{1}A_{2}\,{}^{2}A_{3}\,. \tag{3.26}$$

Substituting Eqs. (3.6) through (3.8) into (3.26), we obtain

$${}^{0}A_{3} = \begin{bmatrix} c\theta_{123} & -s\theta_{123} & 0 & a_1 c\theta_1 + a_2 c\theta_{12} + a_3 c\theta_{123} \\ s\theta_{123} & c\theta_{123} & 0 & a_1 s\theta_1 + a_2 s\theta_{12} + a_3 s\theta_{123} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.27}$$

a. Direct Kinematics.

The position vector of the origin Q expressed in the end-effector coordinate system is given by $^{3}q = [\ 0,\ 0,\ 0,\ 1\ ]^{T}$ Let the position vector of Q with respect to the base coordinate system be $^{0}q = [\ q_x, q_y, q_z, 1\ ]^{T}$ Then we can relate ^{3}q to ^{0}q by the following transformation:

$$\begin{bmatrix} q_x \\ q_y \\ q_z \\ 1 \end{bmatrix} = {}^{0}A_3 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} a_1 c\theta_1 + a_2 c\theta_{12} + a_3 c\theta_{123} \\ a_1 s\theta_1 + a_2 s\theta_{12} + a_3 s\theta_{123} \\ 0 \\ 1 \end{bmatrix}. \tag{3.28}$$

Hence, given θ_1 , θ_2 , and θ_3 , the position of point Q can be computed by Eq. (3.28). Similarly, the position vector of any other point in the end effector,

$${}^{3}g = [g_u, g_v, 0, 1]^{T},$$

is given by

$$\begin{bmatrix} g_x \\ g_y \\ g_z \\ 1 \end{bmatrix} = {}^{0}A_3 \begin{bmatrix} g_u \\ g_v \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} g_u c\theta_{123} - g_v s\theta_{123} + a_1 c\theta_1 + a_2 c\theta_{12} + a_3 c\theta_{123} \\ g_u s\theta_{123} + g_v c\theta_{123} + a_1 s\theta_1 + a_2 s\theta_{12} + a_3 s\theta_{123} \\ 0 \\ 1 \end{bmatrix} \tag{3.29}$$

From Eq. (3.27), we conclude that the orientation angle of the end effector is equal to

$$\theta_1 + \theta_2 + \theta_3.$$

b. Inverse Kinematics,

For the inverse kinematics problem, the location of the end effector is given and the problem is to find the joint angles θ_1 , i = 1, 2, 3, necessary to bring the end effector to the desired location. For a planar 3-dof manipulator, the end effector can be specified in terms of the position of point Q and an orientation angle ϕ of the end effector. Hence the overall transformation matrix from the end-effector coordinate system to the base coordinate system, ${}^0A_3$, is given by

$$ {}^0A_3 = \begin{bmatrix} c\phi & -s\phi & 0 & q_x \\ s\phi & c\phi & 0 & q_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{3.30} $$

Inverse kinematics solutions can be obtained by equating the elements of Eq. (3.27) to that of (3.31). To find the orientation of the end effector, we equate the (1,1) and (2,1) elements of Eq. (3.27) to that of (3.30):

$$ c\theta_{123} = c\phi, \tag{3.31} $$

$$ s\theta_{123} = s\phi. \tag{3.32} $$

Hence

$$ \theta_{123} = \theta_1 + \theta_2 + \theta_3 = \phi. \tag{3.33} $$

Next, we equate the (1,4) and (3,4) elements of Eq. (3.29) to that of (3.30):

$$ p_x = a_1 c\theta_1 + a_2 c\theta_{12,} \tag{3.34} $$

$$ p_y = a_1 s\theta_1 + a_2 s\theta_{12,} \tag{3.35} $$

where $p_x = q_x - a_3 c\phi$ and $p_y = q_y - a_3 s\phi$ denote the position vector of the point P located at the third joint axis shown in Fig. 3.3. Note that by using this substitution, θ_3 disappears from Eqs. (3.34.) and (3.35.) . From Fig. 3.3 we observe that the distance from point O to P is independent of θ_1 Hence we can eliminate θ_1 by summing the squares of Eqs. (3.34.) and (3.35.); that is,

$$p_x^2 + p_y^2 = a_1^2 + a_2^2 + 2a_1 a_2 c\theta_2. \tag{3.36}$$

Solving Eq. (3.36.) for θ_2 we obtain

$$\theta_2 = \cos^{-1} \kappa, \tag{3.37}$$

where

$$\kappa = \frac{p_x^2 + p_y^2 + a_1^2 - a_2^2}{2a_1 a_2}$$

Equation (3.37) yields (1) two real roots if $|K| < 1$, (2) one double root if $|K| = 1$, and (3) no real roots if $|K| > 1$. In general, if $\theta_2 = \theta_2^*$; is a solution, $\theta_2 = -\theta_2^*$; is also a solution, where $\pi \geq \theta_2^* \geq 0$ We call $\theta_2 = \theta_2^*$; the elbow-down solution and $\theta_2 = -\theta_2^*$; the elbow-up solution. If $|K| = 1$, the arm is in a fully stretched or folded configuration. If $|K| > 1$, the position is not reachable.

Corresponding to each θ_2, we can solve θ_1 by expanding Eqs. (3.34) and (3.35) as follows:

$$(a_1 + a_2 c\theta_2) c\theta_1 - (a_2 s\theta_2) s\theta_1 = p_x, \tag{3.38}$$

$$(a_2 s\theta_2)c\theta_1 + (a_1 + a_2 c\theta_2)s\theta_1 = p_y. \tag{3.39}$$

Solving Eqs. (3.38) and (3.39) for $c\theta_1$ and $s\theta_1$, yields

$$c\theta_1 = \frac{p_x(a_1 + a_2 c\theta_2) + p_y a_2 s\theta_2}{\Delta},$$

$$s\theta_1 = \frac{-p_x a_2 s\theta_2 + p_y(a_1 + a_2 c\theta_2)}{\Delta}.$$

where $\Delta = a_1^2 + a_2^2 + 2a_1 a_2 c\theta_2$ Hence, corresponding to each θ_2, we obtain a unique solution for θ_1 :

$$\theta_1 = A\tan 2(s\theta_1, c\theta_1). \tag{3.40}$$

In a computer program we may use the function Atan2(x, y) to obtain a unique solution for θ_2. However, the solution may be real or complex. A complex solution corresponds to an end-effector location that is not reachable by the manipulator. Once θ_1 and θ_2 are known, Eq. (3.33) yields a unique solution for θ_3 Hence, corresponding to a given end-effector location, there are generally two real inverse kinematics solutions, one being the reflection of the other about a line connecting points O and P, as illustrated in Fig. 3.6.

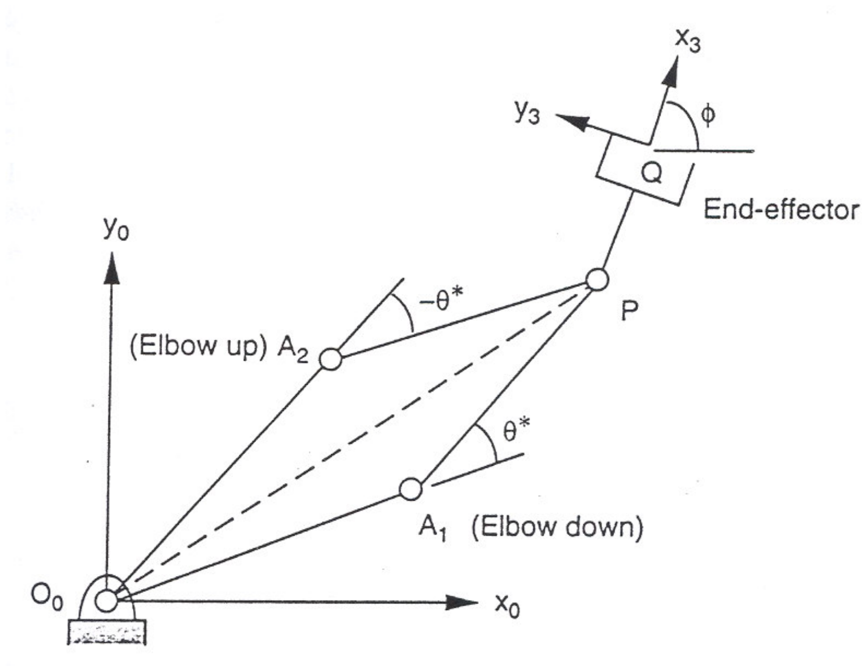

Figure 3.5 Two solution of inverse kinematics.

c. Vector-Loop Method.

Although the D-H method of analysis is a very powerful tool, the inverse kinematics problem can of ten be solved by other methods, such as the vector-loop method. For example, the vector-loop method becomes more efficient for analysis of the 3-dof planar manipulator shown in Fig. 2.3. For convenience, the manipulator has been respected as shown in Fig. 3.7.

Using vector algebra, the position vector of the wrist center P can be relate to the origin Q of the end effector by the equations

$$p_x = q_x - a_3 c\phi \tag{3.41}$$

$$p_y = q_y - a_3 s\phi \tag{3.42}$$

From Fig. 3.7, we observe that the orientation angle ϕ is re1ated to the joint angles by

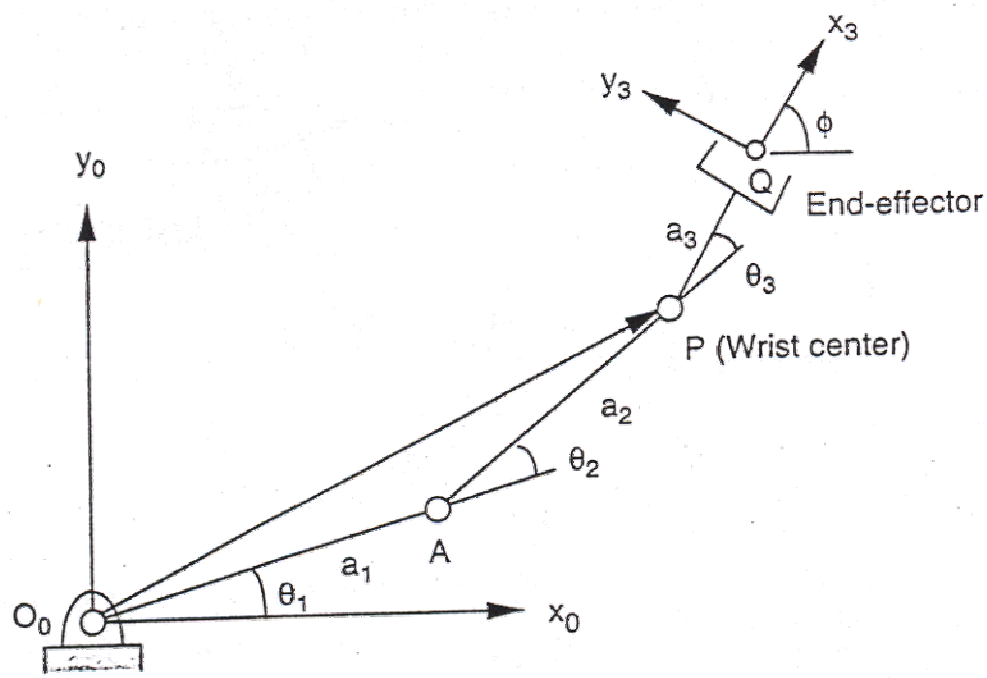

Figure 3.6 Vector loop 3-DOF manipulator.

$$\phi = \theta_1 + \theta_2 + \theta_3. \tag{3.43}$$

We now form a fictitious vector loop equation as follows:

$$\overline{OA} + \overline{AP} = \overline{OP}. \tag{3.44}$$

Taking the x and y components of Eq. (3.44) yields

$$p_x = a_1 c\theta_1 + a_2 c\theta_{12}, \tag{3.45}$$

$$p_y = a_1 s\theta_1 + a_2 s\theta_{12}. \tag{3.46}$$

Note that using the vector-loop method, we have derived Eqs. (3.34) and (3.35) with very little effort.

3.5.2 Position Analysis of the Scorbot Robot

For the Scorbot robot shown in Fig. 3.5, the overall transformation matrix is given by Eq. (3.22). We wish to solve the direct and inverse kinematics problems.

a. For Direct Kinematics Solutions;

For the direct kinematics problem, we simply substitute the given joint angles into Eq. (3.22) to obtain the end-effector position, (q_x ,q_y ,q_z), and the orientation in terms of the three unit vectors (u_x ,u_y ,u_z), (v_x ,v_y ,v_z) and (w_x ,w_y ,w_z).

b. For Inverse Kinematics Solutions;

For the inverse kinematics problem, only 5 of the 12 parameters associated with the end-effector position vector and rotation matrix can be specified at will. This is because the manipulator has only 5 degrees of freedom. It is obvious that the position vector q and the approach vector W cannot be specified simultaneously, due to the fact that q and w together depend only on 4 degrees of freedom of the manipulator. For this exercise we assume that q and u are specified and that the other two unit vectors, v and w, are to be determined after the joint angles are found.

Although Eq. (3.22) can be used to solve the inverse kinematics, in what follows we take a more straightforward approach by multiplying both sides of the loop-closure equation by (${}^0A_1$)$^{-1}$; that is,

$$\left({}^0A_1\right)^{-1}\,{}^0A_5 = {}^1A_2\,{}^2A_3\,{}^3A_4\,{}^4A_5 \tag{3.47}$$

Equating the first column of Eq. (3.47), we obtain

$$u_x c\theta_1 + u_y s\theta_1 = c\theta_{234}\theta_5 \tag{3.48}$$

$$-u_z = s\theta_{234} c\theta_5 \tag{3.49}$$

$$-u_x s\theta_1 + u_y c\theta_1 = -s\theta_5 . \tag{3.50}$$

Similarly, equating the fourth column of Eq. (3.47), we obtain

$$q_x c\theta_1 + q_y s\theta_1 - a_1 = a_2 c\theta_2 + a_3 c\theta_{23} - d_5 s\theta_{234}, \tag{3.51}$$

$$-q_2 + d_1 = a_2 s\theta_2 + a_3 s\theta_{23} + d_5 c\theta_{234}, \tag{3.52}$$

$$-q_x s\theta \;+ q_y c\theta_1 = 0. \tag{3.53}$$

The first joint angle, θ_1 , is obtained immediately from Eq. (3.53):

$$\theta_1 = \tan^{-1}\frac{q_y}{q_x} \tag{3.54}$$

There are two solutions; that is, if $\theta_1 = \theta_1^*$ is a solution, $\theta_1 = \Pi + \theta_1^*$ is also a solution. Once θ_1 is found, two solutions for θ_5 are obtained from Eq. (3.50):

$$\theta_5 = \sin^{-1}\left(u_x s\theta_1 - u_y c\theta_1\right). \tag{3.55}$$

That is, if $\theta_5 = \theta_5^*$ is a solution, $\theta_5 = \Pi - \theta_5^*$ is also a solution.

Corresponding to each solution set of (θ_1 , θ_5), Eqs. (3.48) and (3.49) produce a unique solution of θ_{234} :

$$\theta_{234} = A\tan 2\left[-u_z / c\theta_5, \left(u_x c\theta_1 + u_y s\theta_1\right)/ c\theta_5\right]. \tag{3.56}$$

Next, we solve Eqs. (3.51) and (3.52) for θ_2 and θ_3. Equations (3.51) and (3.52) can be written

$$a_2 c\theta_2 + a_3 c\theta_{23} = k_1. \tag{3.57}$$

$$a_2 s\theta_2 + a_3 s\theta_{23} = k_2. \tag{3.58}$$

where

$$k_1 = q_x c\theta_1 + q_y s\theta_1 - a_1 + d_5 s\theta_{234}$$

and

$$k_2 = -q_2 + d_1 - d_5\theta_{234}$$

Summing the squares of Eqs. (3.57) and (3.58) yields

$$a_2^2 + a_3^2 + 2a_2 a_3 c\theta_3 = k_1^2 + k_2^2. \tag{3.59}$$

Hence

$$\theta_3 = \cos^{-1}\frac{k_1^2 + k_2^2 - a_2^2 - a_3^2}{2a_2 a_3}. \tag{3.60}$$

and there are two solutions of θ_3; that is, if $\theta_3 = \theta_3^*$ is a solution, $\theta_3 = -\theta_3^*$; is also a solution.

Once θ_3 is known, we can solve θ_2 by expanding Eqs. (3.57) and (3.58) as follows:

$$(a_2 + a_3 c\theta_3)c\theta_2 - (a_3 s\theta_3)s\theta_2 = k_1. \tag{3.61}$$

$$(a_3 s\theta_3)c\theta_2 + (a_2 + a_3 c\theta_3)s\theta_2 = k_2. \tag{3.62}$$

Solving Eqs. (3.61) and (3.62) for $c\theta_2$ and $s\theta_2$ yields

$$c\theta_2 = \frac{k_1(a_2 + a_3 c\theta_3) + k_2 a_3 s\theta_3}{a_2^2 + a_3^2 + 2a_2 a_3 c\theta_3}$$

$$s\theta_2 = \frac{-k_1 a_3 s\theta_3 + k_2(a_2 + a_3 c\theta_3)}{a_2^2 + a_3^2 + 2a_2 a_3 c\theta_3}.$$

Hence, corresponding to each solution set of (θ_1 , θ_3 , θ_5 , θ_{234}) , we obtain a unique solution of θ_2 :

$$\theta_2 = A\tan 2(s\theta_2, c\theta_2). \tag{3.63}$$

Finally, θ_4 is obtained by

$$\theta_4 = \theta_{234} - \theta_2 - \theta_3. \tag{3.64}$$

We conc1ude that corresponding to each given end-effector location, there are at most eight inverse kinematics solutions.

3.5.3 Position Analysis of an Elbow Manipulator

Figure 3.15 shows the schematic diagram of an elbow manipulator. In this manipulator, the second joint axis intersects the first perpendicularly, the third and fourth joint axes are parallel to the second, the fifth joint axis is perpendicular to the fourth with a small offset distance a4, and the sixth joint axis intersects the fifth perpendicularly. We wish to solve the inverse kinematics problem of this manipulator using the method of successive screw displacements.

a. Reference Position;

First we identify a reference configuration with respect to which the displacement of the manipulator will be measured. Figure 3.16 shows such a reference configuration, where the first joint axis, Ψ_1 , points up vertically in the positive z-direction; the second, third, and fourth joint axes, Ψ_2 , Ψ_3 , and Ψ_4 , are all pointing out of the paper; the fifth joint axis, Ψ_5 , points in the positive z-direction; and the sixth joint axis, Ψ_6 , points in the positive x-direction. The hand coordinate system is located at point Q such that the w_0-axis points in the positive x-direction and the u_0-axis points in the positive z-direction.

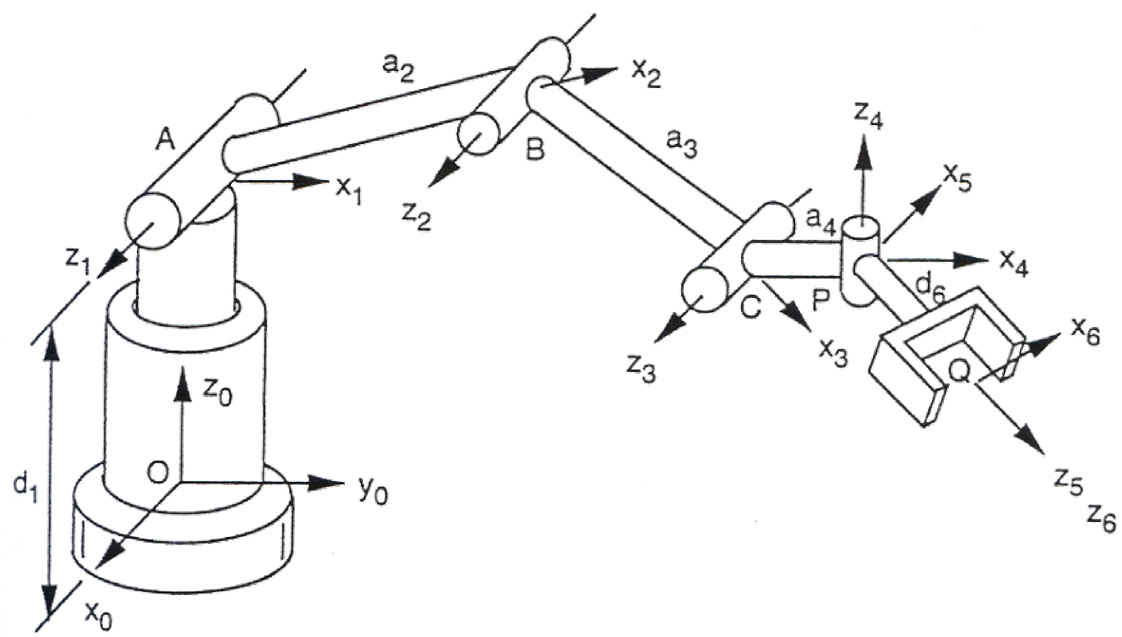

Figure 3.7 6-DOF manipulator

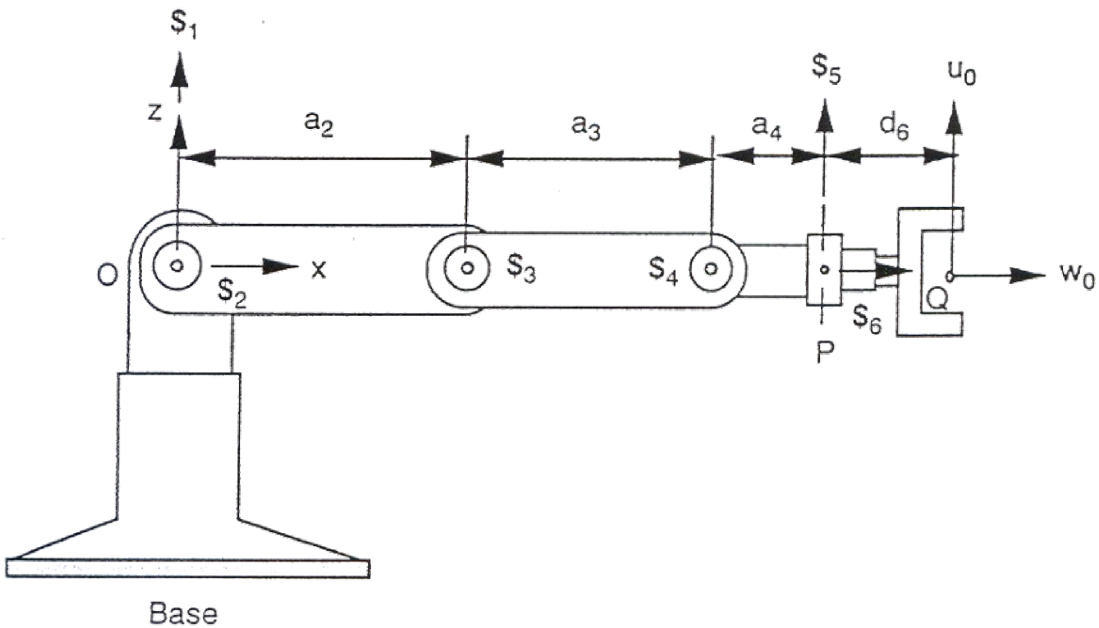

Figure 3.8 Links parameters of elbow manipulator

Table 3.5 Screw Axis Locations of the Elbow Manipulator

Joint i	s_i	s_{oi}
1	(0, 0, 1)	(0, 0, 0)
2	(0, -1, 0)	(0, 0, 0)
3	(0, -1, 0)	(a_2 , 0, 0)
4	(0, -1, 0)	($a_2 + a_3$, 0, 0)
5	(0, 0, 1)	($a_2 + a_3 + a_4$, 0, 0)
6	(1, 0, 0)	(0, 0, 0)

At this reference position, the locations of the screw axes with respect to the fixed reference frame are listed in Table 3.5. The reference position of the end effector is

$u_0 = [0, 0, 1]^T$,
$v_0 = [0, -1, 0]^T$,
$w_0 = [1 , 0, 0]^T$.

and

p_0=[$a_2+a_3+a_4$, 0, 0 $]^T$

b. Target Position

Let the target position of the end effector be

$u = [\ u_x\ ,\ u_y\ ,\ u_z\ ]^T$,
$v = [\ v_x\ ,\ v_y\ ,\ v_z\ ]^T$,
$w = [w_x\ ,\ w_y\ ,\ w_z\ ]^T$

and
$p = [\ p_x\ ,\ p_y\ ,\ p_z\ ]^T$

c. Transformation Matrices.

Substituting the coordinates of the joint axes into Eq. (3.132), we obtain the screw transformation matrices:

$$A_1 = \begin{bmatrix} c\theta_1 & -s\theta_i & 0 & 0 \\ s\theta_1 & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_1^{-1} = \begin{bmatrix} c\theta_1 & s\theta_i & 0 & 0 \\ -s\theta_1 & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c\theta_2 & 0 & -s\theta_2 & 0 \\ 0 & 1 & 0 & 0 \\ s\theta_2 & 0 & c\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} c\theta_3 & 0 & -s\theta_3 & a_2(1-c\theta_3) \\ 0 & 1 & 0 & 0 \\ s\theta_3 & 0 & c\theta_3 & -a_2 s\theta_3 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} c\theta_4 & 0 & -s\theta_4 & (a_2+a_3)(1-c\theta_4) \\ 0 & 1 & 0 & 0 \\ s\theta_4 & 0 & c\theta_4 & -(a_2+a_3)s\theta_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & (a_2+a_3+a_4)(1-c\theta_5) \\ s\theta_5 & c\theta_5 & 0 & -(a_2+a_3+a_4)s\theta_5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\theta_6 & -s\theta_6 & 0 \\ 0 & s\theta_6 & c\theta_6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The matrix products $A_2A_3A_4$ and $A_1A_2A_3A_4$ are computed as

$$A_2A_3A_4 = \begin{bmatrix} c\theta_{234} & 0 & -s\theta_{234} & a_2c\theta_2 + a_3c\theta_{23} - (a_2+a_3)c\theta_{234} \\ 0 & 1 & 0 & 0 \\ s\theta_{234} & 0 & c\theta_{234} & a_2s\theta_2 + a_3s\theta_{23} - (a_2+a_3)s\theta_{234} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.65}$$

$A_1A_2A_3A_4 =$

$$\begin{bmatrix} c\theta_1 c\theta_{234} & -s\theta_1 & -c\theta_1 s\theta_{234} & c\theta_1\left[a_2c\theta_2 + a_3c\theta_{23} - (a_2+a_3)c\theta_{234}\right] \\ s\theta_1 c\theta_{234} & c\theta_1 & -s\theta_1 s\theta_{234} & s\theta_1\left[a_2c\theta_2 + a_3c\theta_{23} - (a_2+a_3)c\theta_{234}\right] \\ s\theta_{234} & 0 & c\theta_{234} & \left[a_2c\theta_2 + a_3c\theta_{23} - (a_2+a_3)c\theta_{234}\right] \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(3.66)

d. Inverse Kinematics.

The transformation of the wrist center point P is given by

$$p = A_1A_2A_3A_4\,p_0. \qquad (3.67)$$

Multiplying both sides of the equation above by A_1^{-1} we obtain

$$A_1^{-1} = \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} = A_2A_3A_4 \begin{bmatrix} a_2+a_3+a_4 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad (3.68)$$

Substituting A_1^{-1} and Eq. (2.65) into (2.68) yields

$$p_x c\theta_1 + p_y s\theta_1 = a_2c\theta_2 + a_3c\theta_{23} + a_4c\theta_{234} \qquad (3.69)$$

$$-p_x s\theta_1 + p_y c\theta_1 = 0 \qquad (3.70)$$

$$p_z = a_2s\theta_2 + a_3s\theta_{23} + a_4s\theta_{234} \qquad (3.71)$$

From Eq. (3.70), two solutions of θ_1 are found immediately:

$$\theta_1 = \tan^{-1}\frac{p_y}{p_x} \tag{3.72}$$

For this manipulator, the position and orientation are not decoupled. Therefore, we need to work on both simultaneously. Applying the transformation matrix to the approach vector w gives

$$R_1^T w = R_2 R_3 R_4 R_5 w_0 \tag{3.73}$$

where R_1 denotes the upper left 3 x 3 submatrix of A_i. Expanding Eq. (3.73), we obtain

$$w_x c\theta_1 + w_y s\theta_1 = c\theta_{234} c\theta_5 \tag{3.74}$$

$$-w_x s\theta_1 + w_y c\theta_1 = s\theta_5 \tag{3.75}$$

$$w_z = s\theta_{234} c\theta_5 \tag{3.76}$$

Corresponding to each solution of θ_1 Eq. (3.75) yields two solutions of θ_5 :

$$\theta_5 = \sin^{-1}\left(-w_x s\theta_1 + w_y c\theta_1\right) \tag{3.77}$$

That is if $\theta_5 = \theta_5^*$ is a solution, $\theta_5 = \pi - \theta_5^*$ is also a solution. Once θ_1 and θ_5 are known, Eqs. (3.74) and (3.76) can be solved for $s\theta_{234}$ and $c\theta_{234}$. This leads to a unique solution for θ_{234} :

$$\theta_{234} = A\tan 2\left\lfloor w_z / c\theta_5, \left(w_x c\theta_1 + w_y s\theta_1\right)/ c\theta_5 \right\rfloor \tag{3.78}$$

We solve Eqs. (3.69) and (3.71) for θ_2 and θ_3 for convenience, we rewrite Eqs. (3.69) and (3.71) as follows:

$$a_2 c\theta_2 + a_3 c\theta_{23} = k_1 \tag{3.79}$$

$$a_2 s\theta_2 + a_3 s\theta_{23} = k_2 \tag{3.80}$$

where $k_1 = p_x c\theta_1 + p_y s\theta_1 - a_4 c\theta_{234}$ and $k_2 = p_z - a_4 s\theta_{234}$ summing the squares of Eqs. (3.79) and (3.80), we obtain

$$a_2^2 + a_3^2 + 2a_2 a_3 c\theta_3 = k_1^2 + k_2^2 \tag{3.81}$$

Hence

$$\theta_3 = \cos^{-1} \frac{k_1^2 + k_2^2 - a_2^2 - a_3^2}{2a_2 a_3} \tag{3.82}$$

Therefore, corresponding to each solution set of θ_1 , θ_5 and θ_{234} there are at most two real solutions of θ_3 Namely, if θ_3^* is a solution, $\theta_3 = -\theta_3^*$ is also a solution. Once θ_3 is known, θ_2 can be obtained by solving Eqs. (3.79) and (3.80) simultaneously for $s\theta_2$ and $c\theta_3$. This produces one solution of θ_2.

Finally, the solutions of θ_4 are obtained from the relation

$\theta_4 = \theta_{234} - \theta_2 - \theta_3$

To solve for θ_6 we apply the transformation to the unit vector u:

$$(R_1 R_2 R_3 R_4)^T u = R_5 R_6 u_0 \tag{3.83}$$

Expanding Eq. (3.83), we obtain

$$u_x c\theta_1 c\theta_{234} + u_y s\theta_1 c\theta_{234} + u_z s\theta_{234} = s\theta_5 s\theta_6 \tag{3.84}$$

$$-u_x s\theta_1 + u_y c\theta_1 = -c\theta_5 s\theta_6 \tag{3.85}$$

$$-u_x c\theta_1 s\theta_{234} - u_y s\theta_1 s\theta_{234} + u_z c\theta_{234} = s\theta_6 \tag{3.86}$$

We can so1ve Eqs. (3.84) and (3.85) for $s\theta_6$:

$$s\theta_6 = s\theta_5 \left(u_x c\theta_1 c\theta_{234} + u_y s\theta_1 c\theta_{234} + u_z s\theta_{234}\right) - c\theta_5 \left(-u_x s\theta_1 + u_y c\theta_1\right) \tag{3.87}$$

Equations (3.86) and (3.87) together determine a unique solution for θ_6 :

$$\theta_6 = A\tan 2\left(s\theta_6, c\theta_6\right) \tag{3.88}$$

We conclude that there are at most eight real inverse kinematic solutions.

CHAPTER FOUR

DC MOTOR SPEED CONTROL

4.1. Introduction

The second part of the model is electronic circuits. The control action which is planned to be occurred is speed control of the balancing mass according to derivation angle. Since it is obvious that analog outputs of the data acquisition cards are mostly within a range of –10 to 10V or smaller. Though it seems to be enough to control a 12 V DC motor, it is not possible to drive motors directly from the analog outputs because of the high operating and start up currents. Common way is to design a driver circuit between data acquisition card and motor.

4.2. DC Motors

Because of the simple reverse drive and good torque speed characteristic, a DC Type of motor is chosen for control application. The used DC motor consists of two parts: Stator and Rotor. Usually, the stator is the fixed outer part which rotor rotates inside.

DC motors are not used in ordinary applications because it is much easier to use the AC current which is readily available almost everywhere. However, in special heavy-duty applications like electric trains, steel mills, etc, the DC motors are preferred because their speed and torque may be easily varied without suffering a reduction in the efficiency.

A DC motor is driven by DC current supplied to the rotor windings. This is called armature current. The rotor is placed inside the magnetic field of the stator. The interaction between the current-carrying armature windings and the electro magnetic field of the stator produces a force (Lorentz Force) that rotates the rotor. The stator field is maintained by either permanent magnets or by a field current passing through the stator windings (field current).

In normal mode, a DC motor converts the electrical (armature current) to mechanical (rotor torque). However, as shown in the figure (4.1), the same machine can be used in generator mode when the mechanical power is transformed back to electrical. Regenerative braking mode is converting the potential energy of the bucket to electricity.

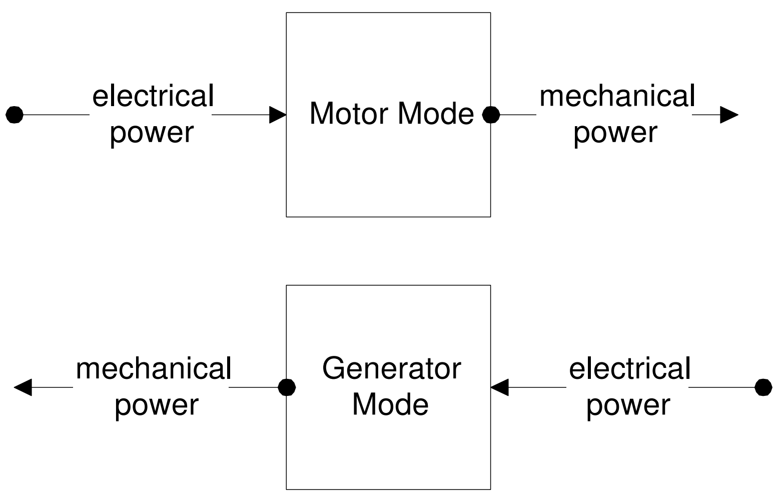

Figure 4.1 Operating modes of DC machines

4.2.1. DC Motor Types

The armature and field circuits can be connected in a number of ways, leading to different types of DC motors:

Separately Excited DC Motors: The field and armature circuits are totally separate. The field current is supplied from a secondary source (or by permanent magnets).

Self-excited DC Motors: The field and armature currents are provided by the same source. These are usually smaller motors and the field and armature may be connected in three different ways:

Shunt Motor: The field and armature windings are in parallel
Series Motor: They are in series
Compound Motor: Both shunt and series windings are used.

In a shunt motor, the field windings are "shunted" across the voltage supply to the armature. In other words, the field and armature windings are in two parallel paths as shown in the following figure (4.2): A shunt-connected DC motor is designed for a specific voltage and is rated to deliver a specific power at a specific speed. This speed is referred to as the base speed and it occurs when I_f is near its maximum value.

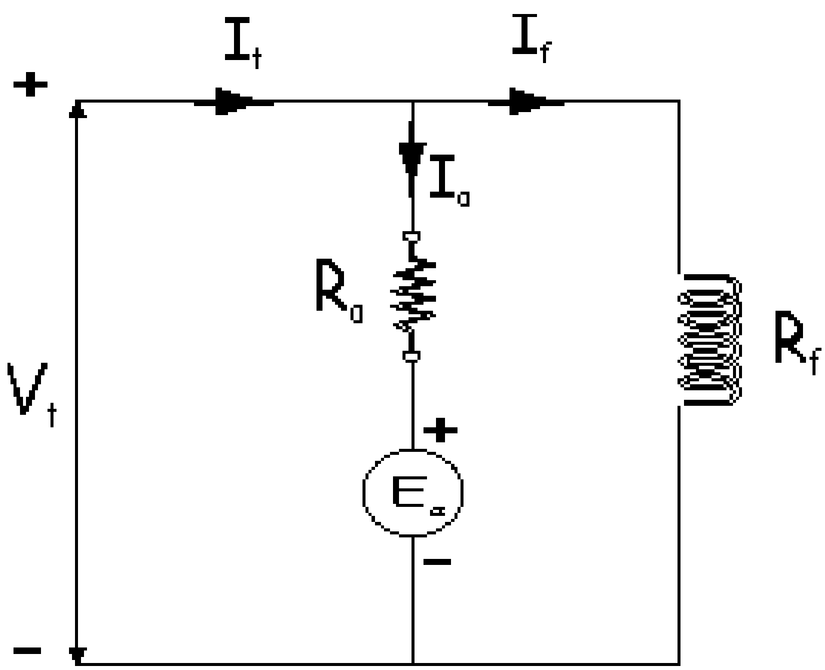

Figure 4.2 Electrical circuit of DC shunt motor

In the series-wound motor the armature and the field windings are in series. (figure 4.3) Compared to shunt motors, the field winding in series-wound motors are wound using wires of larger diameter. That is because the field winding sees the full load current in series motors. We will see that this arrangement gives excellent starting characteristics (high torque at the start) but tends to run away when the load vanishes.

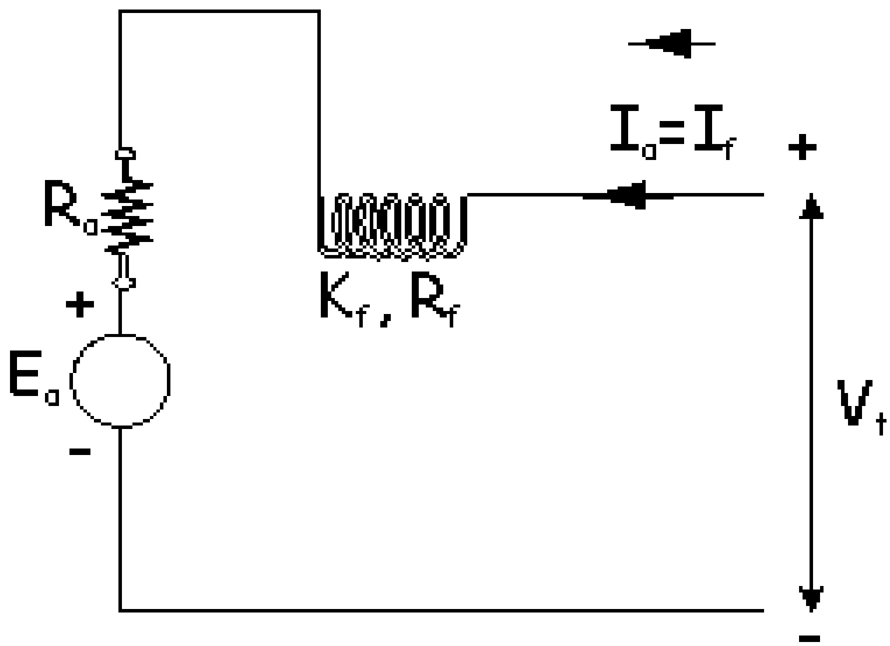

Figure 4.3 Electrical circuit of DC series motor.

The advantage of a series motor is the very high torque available at start-up. This makes them particularly suitable for applications like trains, automobile starter motors, hoists, industrial mixers, etc.

In a Permanent Magnet motor a coil of wire (called the armature) is arranged in the magnetic field of a permanent magnet in such a way that it rotates when a current is passed through it. When a coil of wire is moving in a magnetic field a voltage is induced in the coil - so the current (which is caused by applying a voltage to the coil) causes the armature to rotate and so generate a voltage.

The value of the back emf is determined by the speed of rotation and the strength of the magnet(s) such that if the magnet is strong the back emf increases and if the speed increases, so too does the back emf. It follows from this that if you use a weaker magnet to make a particular motor, you will get a higher speed motor.

If you apply now a load to the armature, it will slow down. The back emf will decrease so the difference between applied voltage and back emf will increase. It is this difference that causes the current in the armature to flow - so the current will increase as you increase the mechanical loading. It should be apparent therefore that an unloaded motor will take little current. It should also be clear that if you apply more voltage the motor will speed up, apply less and it will slow: this is what the speed controller does: it varies the voltage applied to the motor. Speed control methods will be explained in the next chapter.

4.1. Motor Driving Circuits

For designing driver circuits it is possible to deal with the problem by several technologies The DC motor speed can be controlled in a number of ways. The most common options are listed below:

Field Control
Armature resistance Control
Armature Voltage Control
Ward-Leonard Drive
Solid-State Control

4.2. Controlling the Speed by Controlling the Field

It has already demonstrated that the speed varies with the inverse of the field:

$$\omega_m = \frac{V_t}{K_a\Phi} - \frac{R_a}{(K_a\Phi)^2}T \tag{4.1}$$

Therefore, for a given torque, the speed should decrease if the field is weakened. This increase in speed will, however, will come at the expense of the torque because the torque is directly proportional to the field.

For a shunt motor, the field is weakened by putting a variable field resistance in series with the field windings (figure 5.1)

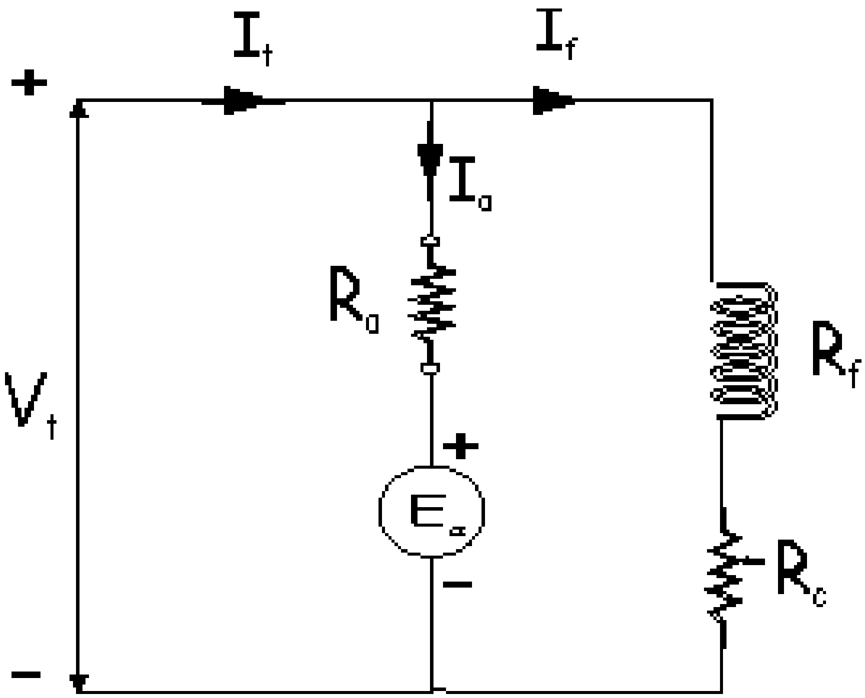

Figure 4.4 Field control in a shunt motor

This circuit can be implemented as a resistance switching mechanism similar to the armature resistance switching during start-up. It is easy to do and inexpensive because the control is implemented at the low-power part (in the field circuit) of the system. However, because of the large inductance in the field circuit, the response to resistance changes will be slow.

The same field weakening effect can be obtained by in series motors by diverting some of the field current from the field windings (figure 4.2) In this instance, the control device is referred to as a field diverter.

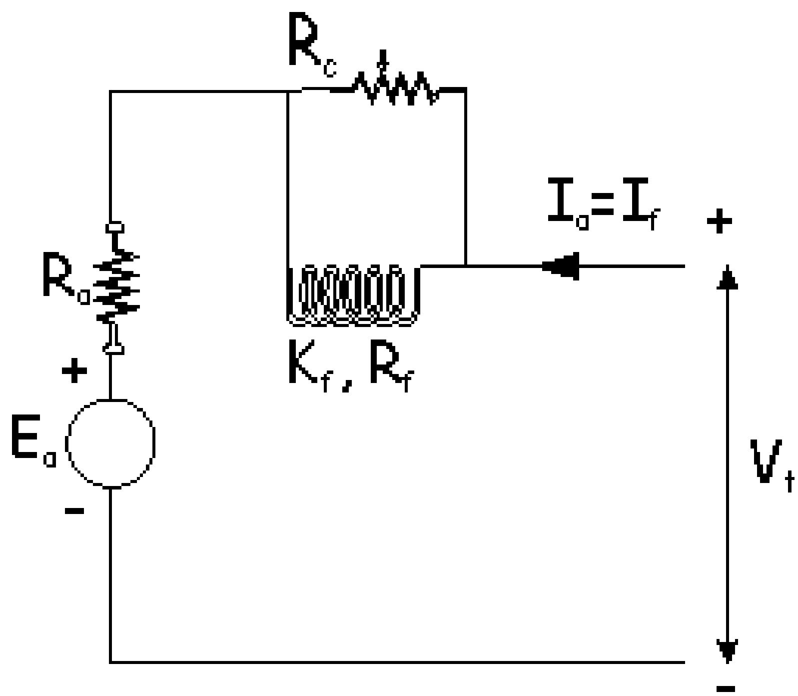

Figure 4.5 Field control in a series motor

4.3. Speed Control By Controlling The Armature Resistance

Changing the armature voltage is another way of controlling the speed. The easiest way to control the armature voltage is by connecting a variable external resistance in series with the armature.

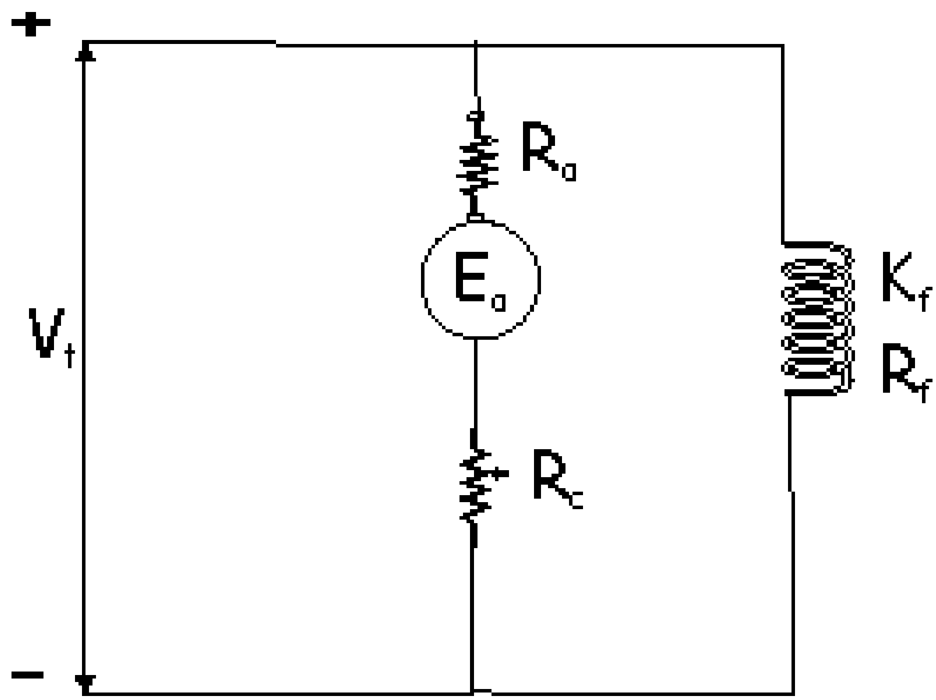

Figure 4.6 Speed control by controlling the armature resistance

This is easy to implement. It is not efficient because of the insertion of a higher resistance into the armature circuit and the subsequent I^2R losses on that resistance.

As the armature resistance is increased, and assuming the supply voltage and the field stay constant, the new speed is calculated by the following equation where Ra+ Rc is substituted for Ra.

$$\omega_m = \frac{V_t}{K_a\Phi} - \frac{R_a}{(K_a\Phi)^2}T \qquad (4.2)$$

4.4. Controlling the Speed by Modulating the Armature Voltage

The most common method of controlling the speed is by directly controlling the armature voltage:

$$\omega_m = \frac{V_t}{K_a\Phi} - \frac{R_a}{(K_a\Phi)^2}T \tag{4.3}$$

By changing the armature voltage from 0 to its rated value, the speed can be varied from 0 to the rated speed. The armature voltage control can be combined with field control to give the ability to control motor speed from 0 to several times its rated value. This is seen in the following chart:

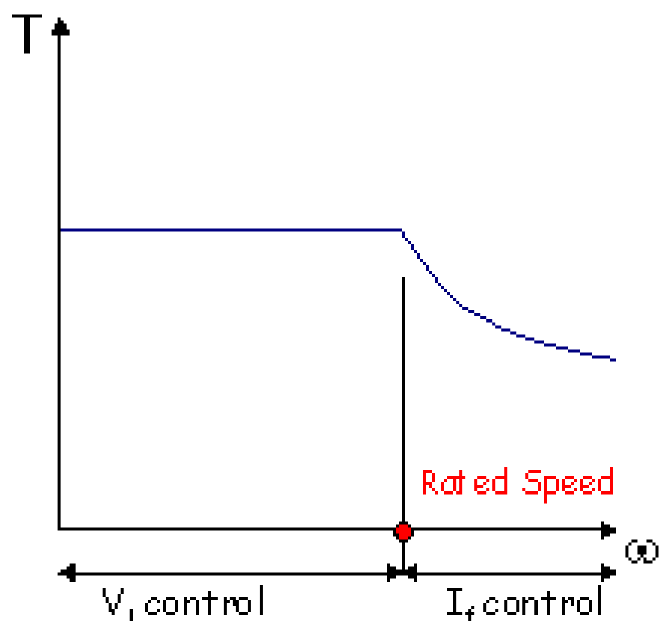

Figure 4.7 Armature voltage control combined with current control

A potential divided is commonly used in shunt motors to provide the armature voltage control. The use is limited to smaller motors because of the high current ratings for larger machines make such dividers impractical. For larger motors, the same effect is achieved by a slightly more complicated Ward-Leonard drive system.

4.5. Ward-Leonard Systems

Ward-Leonard systems were introduced in 1890s. Schematically, the operation of the system is as follows:

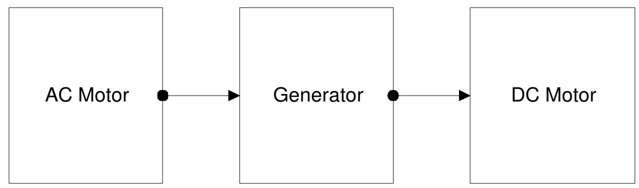

Figure 4.8 Ward Leonard system

The AC motor drives the generator. The generator generates the terminal voltage for the DC motor. This voltage can be modulated by modulating the field current on the Generator.

Ward-Leonard drive provides very smooth and reliable speed control with no loss of motor efficiency. They are highly complex systems, however, and this makes them suitable only for high-price high-quality applications. They are very common in mine sites driving shovels, mine cages and mills.

4.6. Solid-State Speed Control

Solid-state converters and rectifiers have become available in recent years even in high-power circuits. Such devices are gradually replacing the Ward-Leonard systems based on dedicated motor generator sets.

These controlled rectifiers are commonly referred to as Silicon Controlled Rectifiers or SCRs. By chopping the supply voltage, they produce a pulse train for the armature voltage rather than a continuous supply:

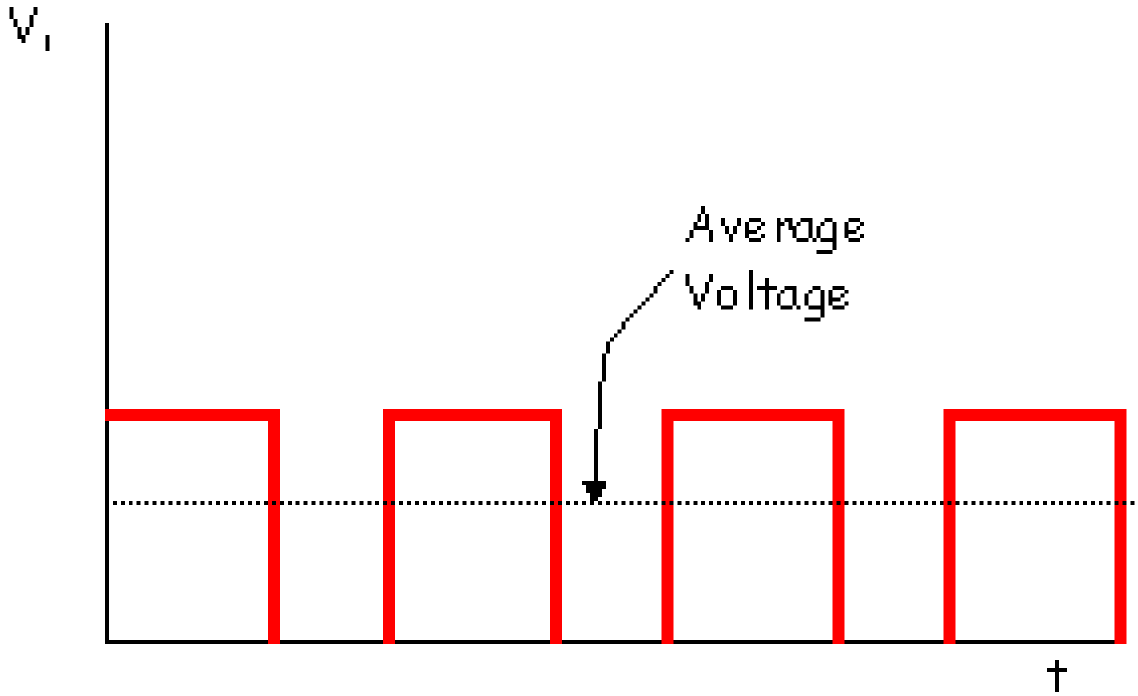

Figure 4.9 Pulse train

By modulating the pulse train frequency, the average armature voltage is modulated. The shape of the pulses is not always square. For example, a SCR chopping an AC signal to provide rectified excitation for a DC motor produces pulses of a half-sine shape.

The standard voltage-speed-torque relationships apply with the substitution of the average values:

$$\begin{aligned} V_t &= E_a + I_a R_a \\ E_a &= K_a \Phi \omega_m \\ T &= K_a \Phi I_a \end{aligned} \tag{4.4}$$

4.7. Basics of Pulse Width Modulation

The control of electric motors is something which interests nearly everyone involved with model building. On the face of it, a simple controller with a voltage supply and a switch or potentiometer is sufficient. Every model, however, its own motor requirements with regard to the space available, the power of the motor, its speed, whether it must stop and start frequently, and the need for reduction gearing.

On the face of it, simple methods of control are perfectly adequate, with a regulated voltage supply, a simple on off switch, and the means to reverse the motor. Speed can be controlled with a potentiometer. In reality, this provides very unrealistic results. The main problem is poor starting performance, the motor tending to jump almost instantly from a stationary position to what is often more than half speed. The main cause seems to be the starting characteristic of the motor itself which when under load seems reluctant to start. A motor has a relatively low resistance when it is stationary. As the speed control is advanced, the current through the motor increases, but the voltage across the motor remains quite low. The speed control therefore has to be well advanced before the voltage and power fed to the motor are high enough to overcome its reluctance to start. As the motor speed and the load on it changes, there are changes in the internal resistance. Speed regulation is not very good under these circumstances, particularly at low speed

PWM is a common technique for speed control. A good analogy is bicycle riding. You peddle (exert energy) and then coast (relax) using your momentum to carry you forward. As you slow down (due to wind resistance, friction, and road shape) you peddle to speed up and then coast again.

The duty cycle is the ratio of peddling time to the total time (peddle+coast time). A 100% duty cycle means you are peddling all the time and 50% only half the time.

PWM for motor speed control works in a very similar way. Instead of peddling, your motor is given a fixed voltage value (say +5 V) and starts spinning. The voltage is then removed and the motor "coasts". By continuing this voltage on-off duty cycle, motor speed is controlled.

In the above diagrams, V is the voltage across the motor and t is time. By switching quickly, we can create an average voltage across the motor. The speed of the motor can be adjusted by changing the pulse-width ratio:

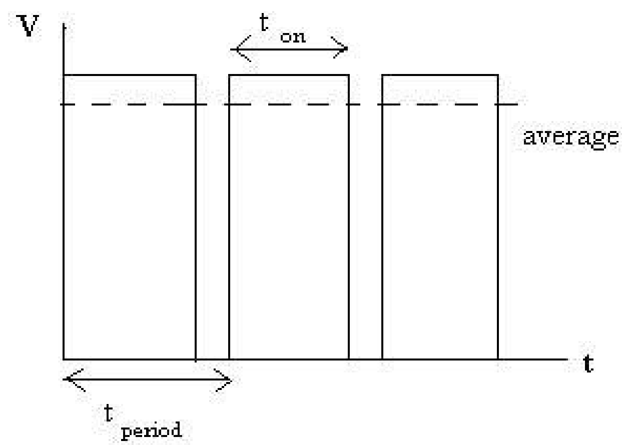

Figure 4.10 Pulse Width Modulation parameters

The concept of PWM inherently requires timing. The classic 555-timer chip, operational amplifiers and some potentiometers can be used to generate PWM. The pots are manually adjusted for the desired duty cycle. However, if a PC is used, it can automatically change the duty cycle and control your motor's speed.

4.8. General Description of Circuit

In this experiment, the PWM is used to control the duty cycle of PM motor armature voltage. A fifty- percent duty cycle is brake since effective voltage equals to zero and when it is less then 50 percent, the rotating direction is clockwise. Counterclockwise rotating direction is obtained with higher duty cycle.

Because the duty cycle determines the averaged DC voltage the armature winding of DC motor receives and the speed of the DC motor depends on the armature voltage, by changing the duty cycle of the PWM waveform, the speed of DC motor can be adjusted or controlled.

Electronic circuit for Pulse Width Modulation consists of three cascaded parts. The First is a precise timed trigger generator with LM 555 timer chip and the second is a sawtooth pulse oscillator with an RC and a transistor triggered by LM 555 output and the last one is an opamp circuit which compares our control voltage and sawtooth pulse then obtains PWM. At the output of this opamp there is a relay which controlled by one bit of the digital output channels to control the direction of the by applying modulated signal to the correct pin of H-Bridge.

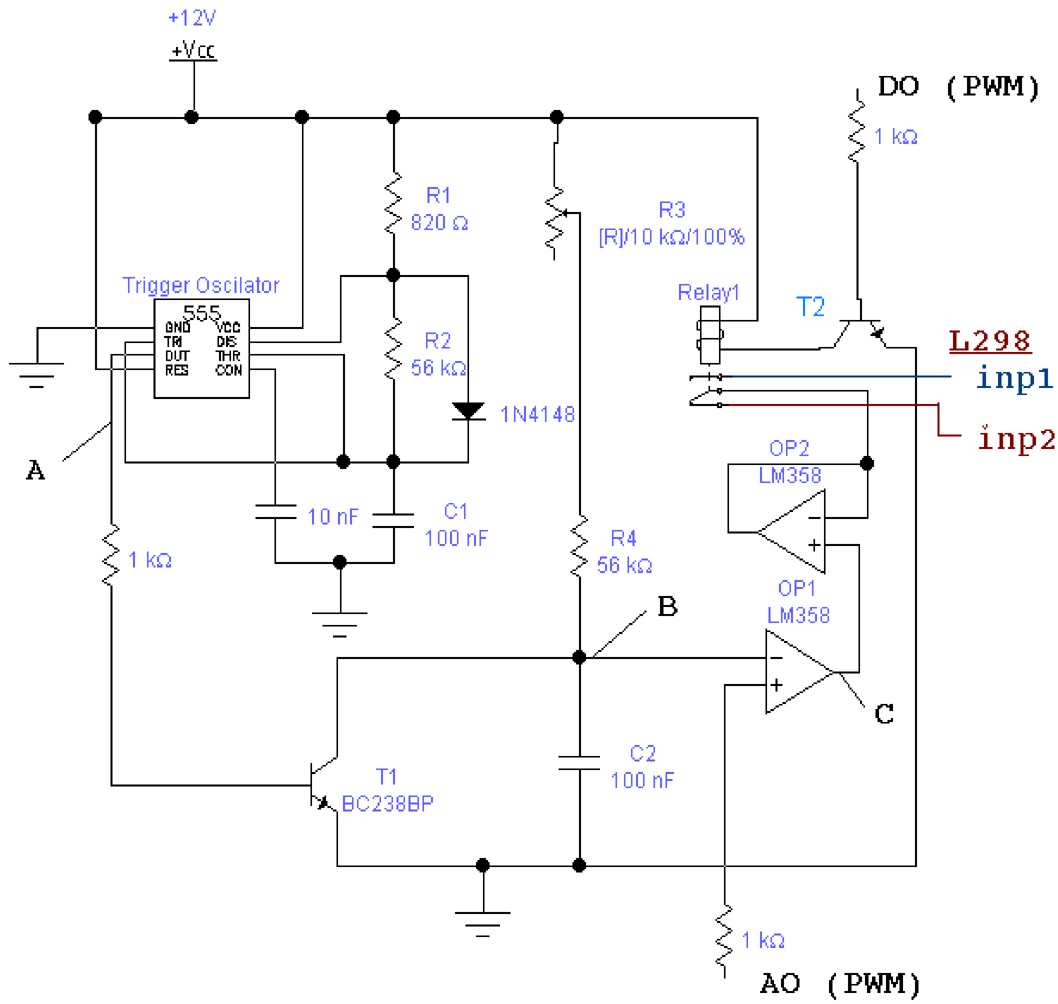

Figure 4.11 PWM Circuit for driving L298

4.9. Operation of Circuits

The first part of circuit is a trigger generating oscillator. A LM 555 Timer Chip is connected as in figure (pins 2 and 6 are connected each other) it will trigger itself and run as a multivibrator. External capacitor charges through $R_1 + R_2$ and discharges through R_1 .Thus the duty cycle can be precisely adjusted by the ratio of these two resistors according to given formulas. The charge time when the output is low is given by:

$$t_1 = 0.693(R_1 + R_2)C_1 \tag{4.5}$$

$$t_1 = 0.693.(820 + 56000).100.10^{-9}$$
$$t_1 = 3.938 \text{ ms}$$

And the discharge time when the output is high by:

$$t_2 = 0.693(R_1)C_1 \tag{4.6}$$
$$t_1 = 0.693.820.100.10^{-9}$$
$$t_1 = 0.0568 \text{ ms}$$

And the frequency of oscillation is:

$$f = \frac{1.44}{t_1 + t_2} \tag{4.7}$$
$$f = \frac{1.44}{3.938 + 0.0568}$$
$$f = 360 \text{ Hz}$$

This frequency determines the frequency of PWM signal. It must be chosen convenient to motor characteristics.

In this mode of operation, the capacitor charges and discharges between 1/5 V_{cc} and 2/5V_{cc} .The charge and discharge times are independent of the supply voltage.

Next part of the circuit a typical RC connection which is used for generating a almost linear sawtooth wave form. This wave form is used as the template of Pulse Width Modulation output.

When base of the transistor T_1, output of the LM 555, is in low state, the capacitor is begin to charge and the voltage of the point A begins to increase and when base of transistor T_1 goes high, the charge in capacitor flows directly to ground with in very short time because of the small resistance of the transistor. It is clear that obtaining a linear wave at maximum PWM voltage amplitude is quite important. Regarding this fact, linearity of the curve is tried to achieve by varying time constant choice adjusted from P_1 which also effects amplitude of the output curve. P_1 adjusts linear part of 12 V exponential by determining cut off time. After proper settings a quasi linear wave form and 5.124 Volt amplitude is obtained. Frequency of the wave is the same as the timer frequency which is a necessity for stable operation of the entire system.

The last part of the PWM circuit is a comparator with ½ of LM558 (Low power quadratic opamp), and a buffer with other ½ of LM 558 and a relay. Signal at, point B, is applied to inverting input of OP1 and analogue output of PCL 812 Data Acquisition Card which occurs as a result of controller calculations in PC is applied to non inverting input. Since LM 558 output is 0 V when inverting input is bigger than non-inverting input and V_{DC} when non-inverting input is bigger. As we compare the sawtooth wave with our analogue input, output is a signal that has a duty ratio (eg: %20 on %80 off) which is appreciate to AO voltage value.

This signal is connected to a buffer circuit, a unity gain amplifier (OP2), to protect the circuit voltage changes in L298 motor driver circuit. Relay is used for changing modulated signal pin from inp1 to inp2 which are logical control circuit inputs for motor driving circuits.

4.10. Principles of Motor Control

Figure5.9 illustrates driving a DC motor using a power transistor bridge. By driving the four transistor in the correct sequence the direction of current flow through the motor is reversed, consequently reversing the direction of the motor's rotation. The motor torque is a function of the current amplitude, the motor's internal parameters, and the external load. The resistive torque is dependent on the motor's internal friction. The current level can be controlled with current chopping. The controller checks the current level by monitoring the sense resistor voltage and then drives the appropriate power transistor. On the other hand this means that when current does not flow in the sense resistor it is not possible to measure the current level and thus limit it.

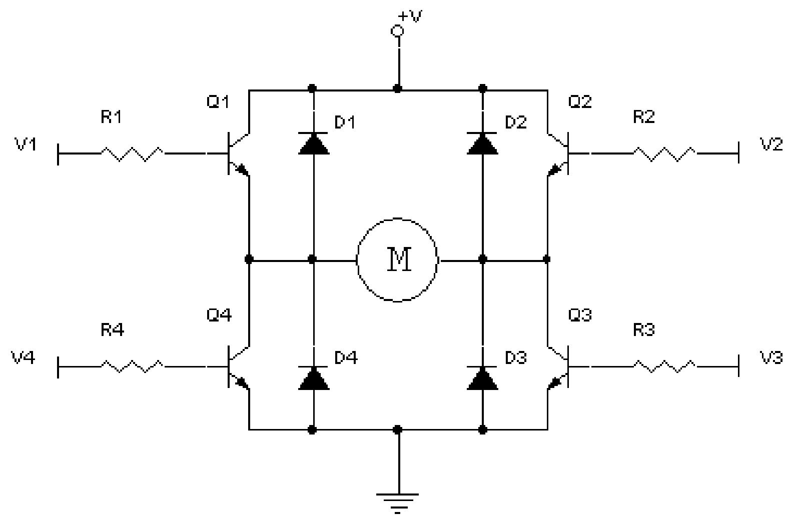

Figure 4.12 H Bridge circuit

A more general application circuit includes an external control loop. Data relating to the actual motor speed is transmitted to the controller by the system which stabilizes the current in the bridge as a function of the requested rotation velocity. In this case also the current is limited through chopping.

Electrically a DC motor can be viewed as a series RL network with a voltage generator V(w). The generator represents the back electromotive force (BEMF) generated by the motor's rotation and which opposes the electromotive force of the supply. The value of the BEMF is a function of the motor's angular velocity. If the motor has no external load and its velocity is not limited, it will accelerate up to the velocity w such that V(w) equals the supply voltage Vs. In this situation the two EMF's cancel each other and thus the motor torque responsible for acceleration will go away. In reality V(w) is always slightly less than Vs in which case a small motor torque is necessary to compensate resistive torque due to internal friction. Thus it can be seen that the motor's BEMF can reach elevated values which in some cases can create application problems due to a certain type of stress. Solution to this problem is the is to put what is known as a flyback diode in the reverse direction across the inductive load, so that the voltage spike will forward bias the diode, creating a return path for the current. Fig(c) shows how a flyback diode is connected.

4.11. Motor Driver Circuit

The L298 is an integrated monolithic circuit It is a high voltage, high current dual full-bridge driver designed to accept standard TTL logic levels and drive inductive loads such as relays, solenoids, DC and stepping motors. Two enable inputs are provided to enable or disable the device independently of the input signals. The emitters of the lower transistors of each bridge are connected together and the corresponding external terminal can be used for the connection of an external sensing resistor. An additional supply input is provided so that the logic works at a lower voltage.

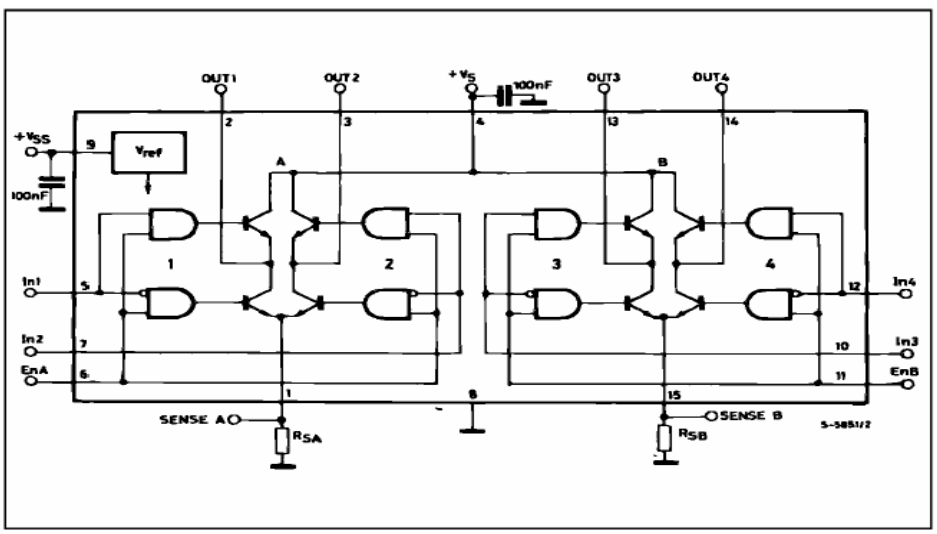

Figure 4.13 Schematic circuit of L298

The L298 integrates two power output stages (A; B).The power output stage is a bridge configuration and its outputs can drive an inductive load in common or differential mode, depending on the state of the inputs. The current that flows through the load comes out from the bridge at the sense output: an external resistor allows to detect the intensity of this current.

When the supply and the logic supply voltage applied and the motor is connected to output pins of any of two bridges. Circuit is ready to operate. Enable pins controls whether the bridge is on work. In order to explain operating of the circuits, it is assumed that input1 and enable A is high and input2 is low. This logic signals are connected to input pins of four And-logic gates directly or inverted. In given conditions, Transistor A1 and A4 is triggered and current flows through output1 to output2 and motor turns counter clockwise. If input2 is high and input1 is low, transistors A2 and A5 are conductive this causes current flows output2 to output1 and motor turns clockwise. In each case current flows on external resistor R_s, connected to current sense output, through ground. The sense output voltage can be used to control the current amplitude by chopping the inputs, or to provide over

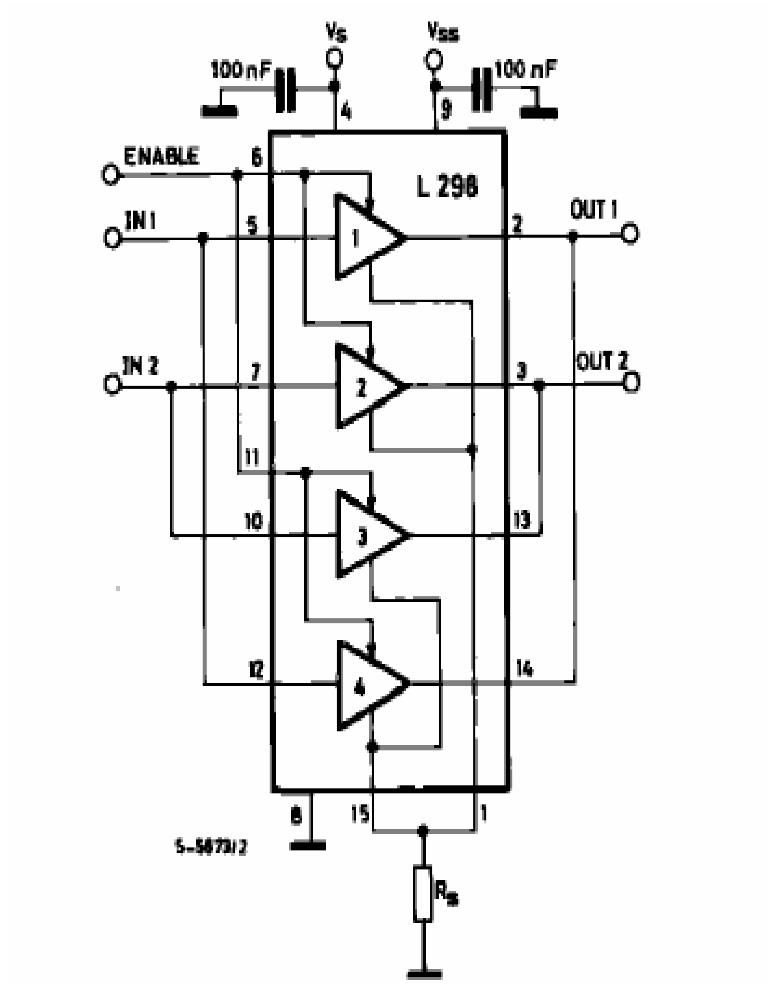

Figure 4.14 Power setup of L298

Current protection by switching low the enable input. As the repetitive peak current needed from the load is higher than 2 Amps, a paralleled configuration is chosen.(figure 4.11) This solution can drive motors up to 5 amperes in DC operation.

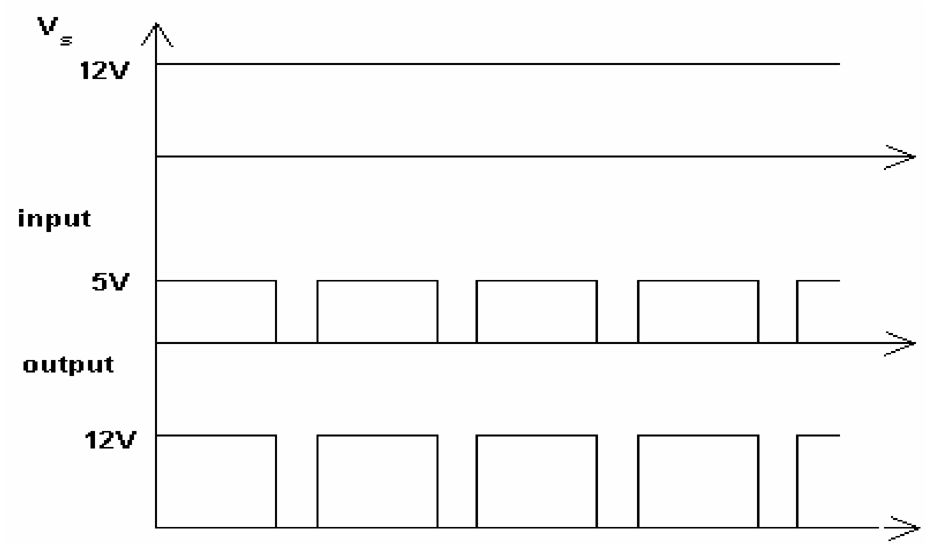

Figure 4.15 PWM signal at the output of L298

When L298 and PWM circuit combined modulated signal goes to one of the input pins and the other is connected to ground through relay. At this situation, supply voltage modulated by PWM signal and resulting signal flows on to motor. and determines the angular velocity of motor (Figure 4.12).

CHAPTER FIVE

MECHANICAL DESIGN AND LASER SINTERING SYSTEM

5. Introduction

A three-fingered end-effector has been designed for use in an unstructured environment. The design has three driven inputs per finger to move the tip to the desired position in order to make contact with an object. A theoretical analysis of the three-dimensional work space of the three-fingered gripper has been used to form a Jacobian matrix based on two inputs per finger, to provide a numerical inverse kinematics solution.

The end-effector discussed in this chapter has three controlled inputs for each of the three fingers, leading to a complex coordinated. When a solid object is picked up, the fingers will be moved to their open positions and then rotated to form the appropriate grip.

In practice, an object may not be symmetrically placed, it is anticipated that touch sensors will be used to detect that contact has been made with the object surface. Numerical solutions have been found using a Jacobian matrix for the inverse kinematics of a three-fingered gripper, with each finger having three degrees of freedom. Very good predictions of the input positions are made for given target positions of the fingertips. By resetting the iteration procedure and using the forward kinematics equations, the target errors are reduced. Quadratic functions of a linear dimension of an object determine the finger inputs that have the potential for an efficient and fast response.

5.1. Mechanical Design

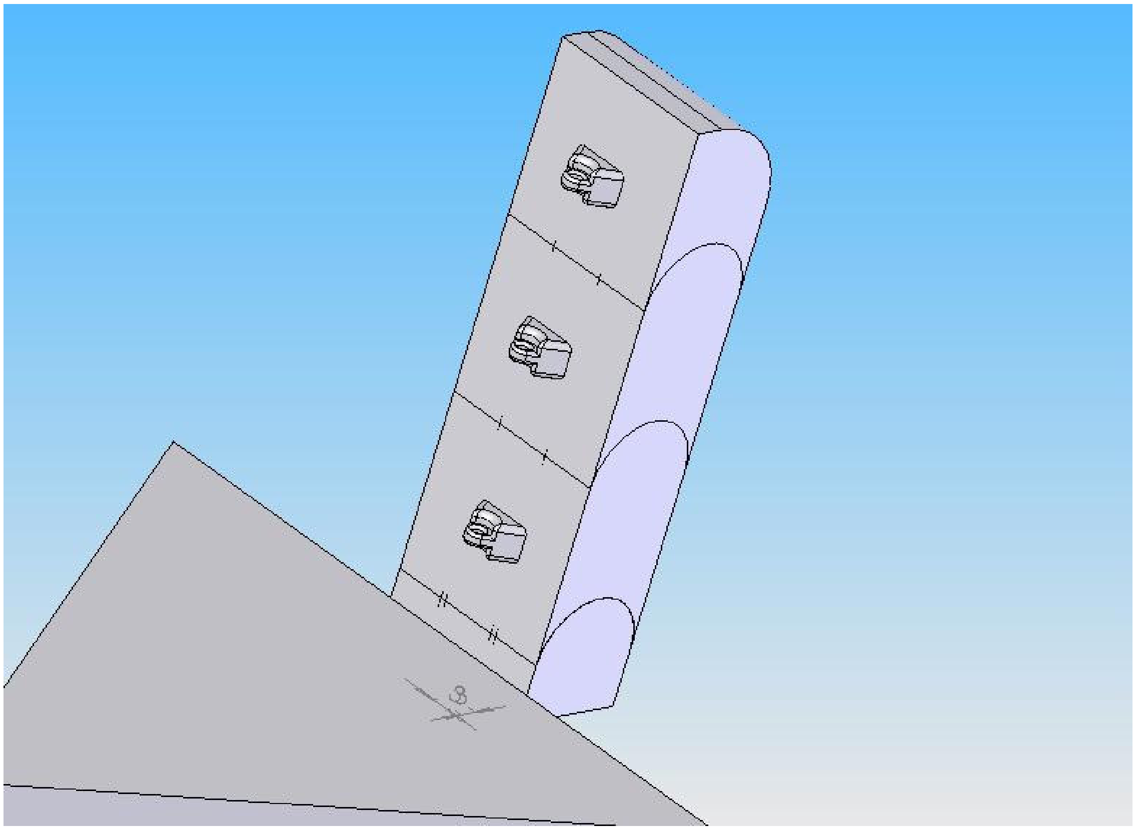

Figure 5.1 Joint of mechanical design

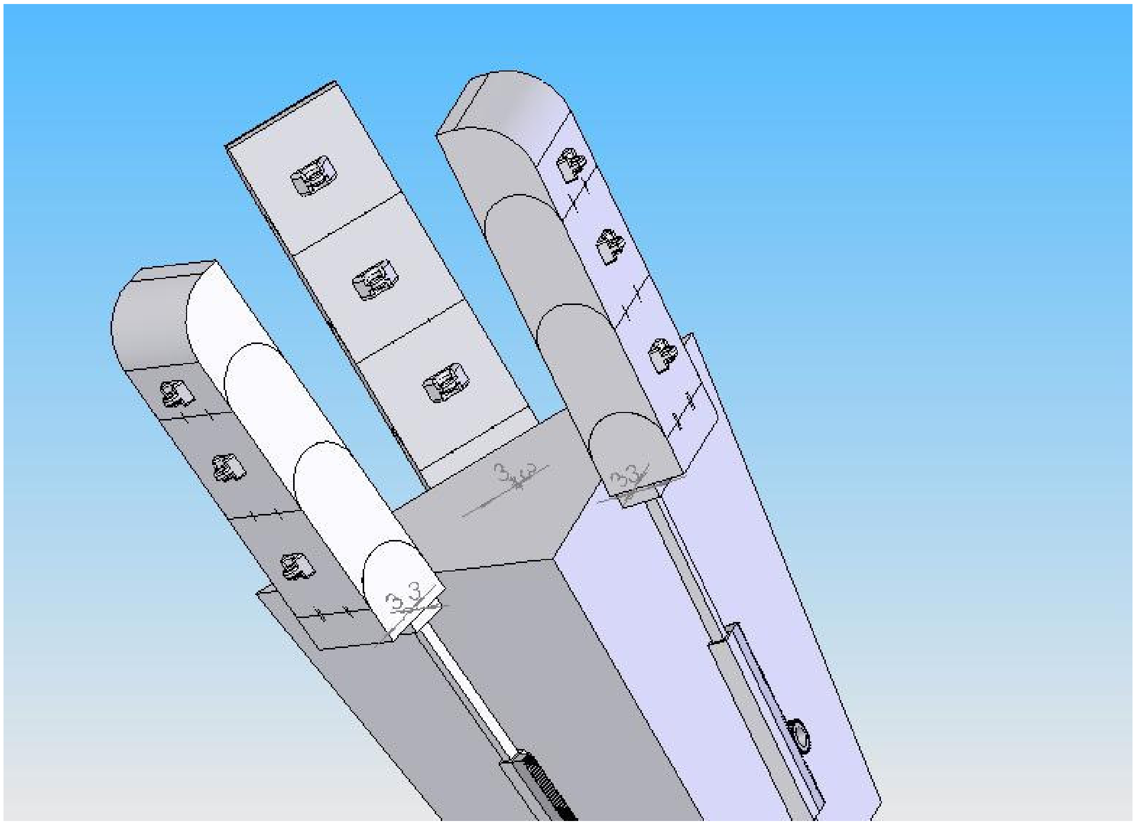

Figure 5.2 Link of mechanical design

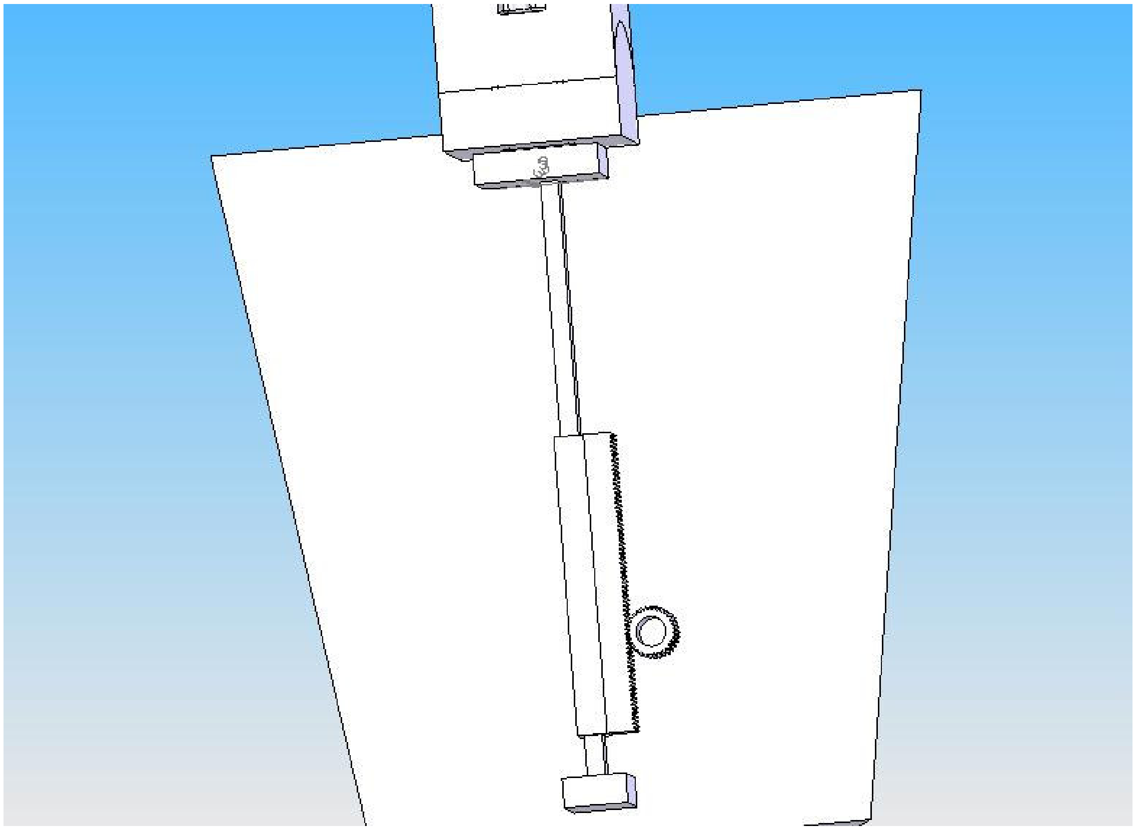

Figure 5.3 Motion mechanism of mechanical design

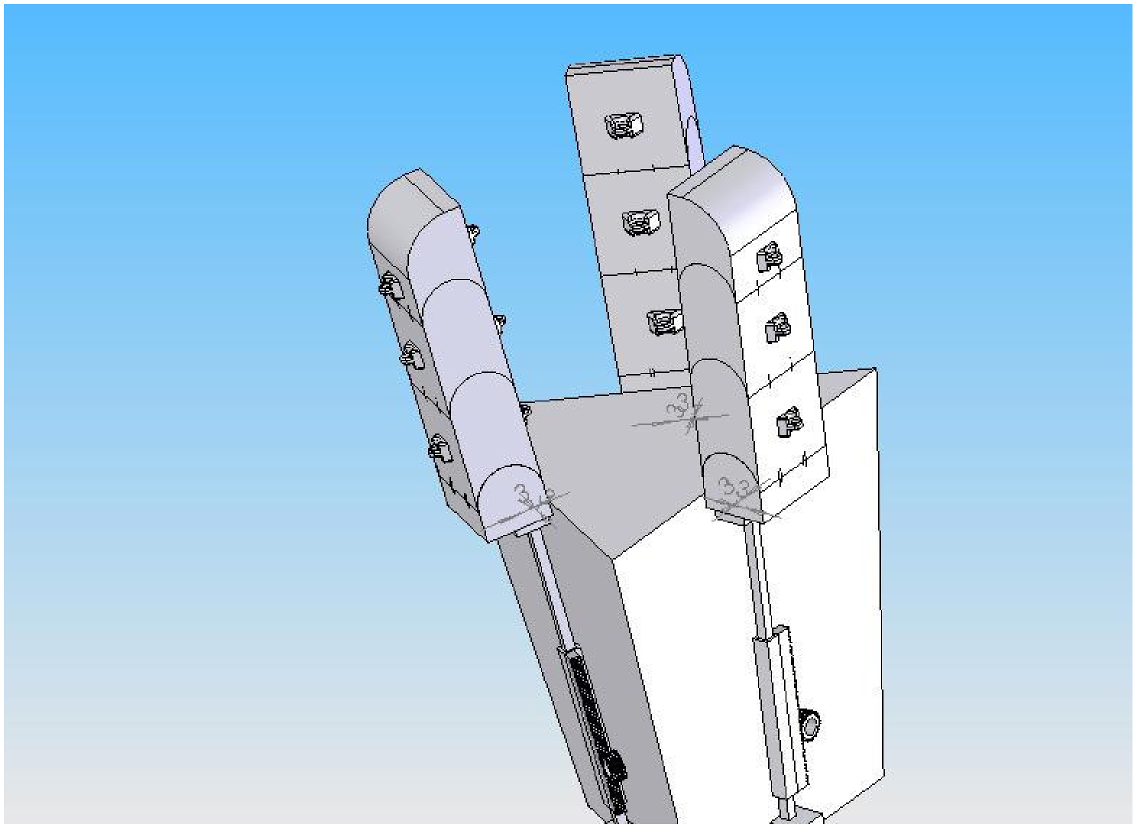

Figure 5.4 Mechanical design

5.2. Sintering Laser System. (SLS)

For a long time, completely sinterstation operation can make with metal and ceramics powder in industry. Before that, green state had mixed with transitory binder and compressed in a template therefore green state had been occupied one another. After mixing and compressing, Powder material had kept in a high temperature sinterstations kiln-dryer for sintering. Above pictures belong to this technology.

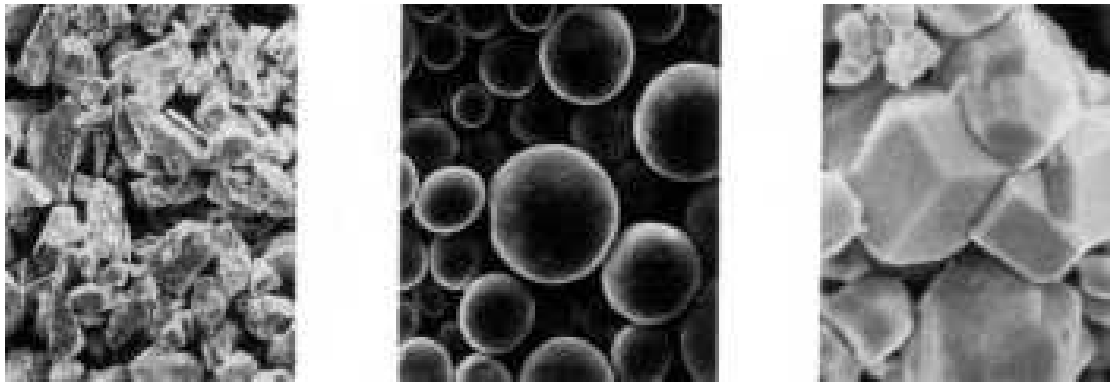

Figure 5.5 Metal and plastic particle

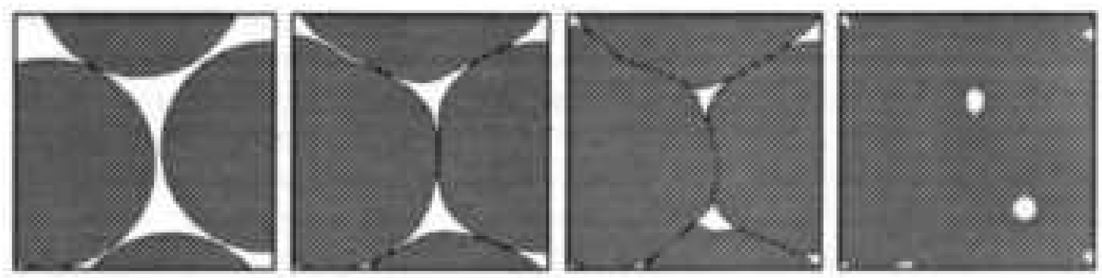

Figure 5.6 Four phases for sintering

In building industry's materials, such as brick, faience, ceramic etc., produced the same technology with sinterstation and it is called "ceramic bond".

Sintering time becomes shorter. Beam of heat source passes the material in a very short time. It is not possible to be ideal sintering operation in a short time. In generally sintering and melting-cooling materialize at the same time. Some auto construction, component can be subjected in a different kiln-dryer after construction. If beam energy is not sufficient for sintering or melting, metal powder with plastic covering can be employed. This stage is called "green state". After this, materials can sintered in a kiln-dryer.

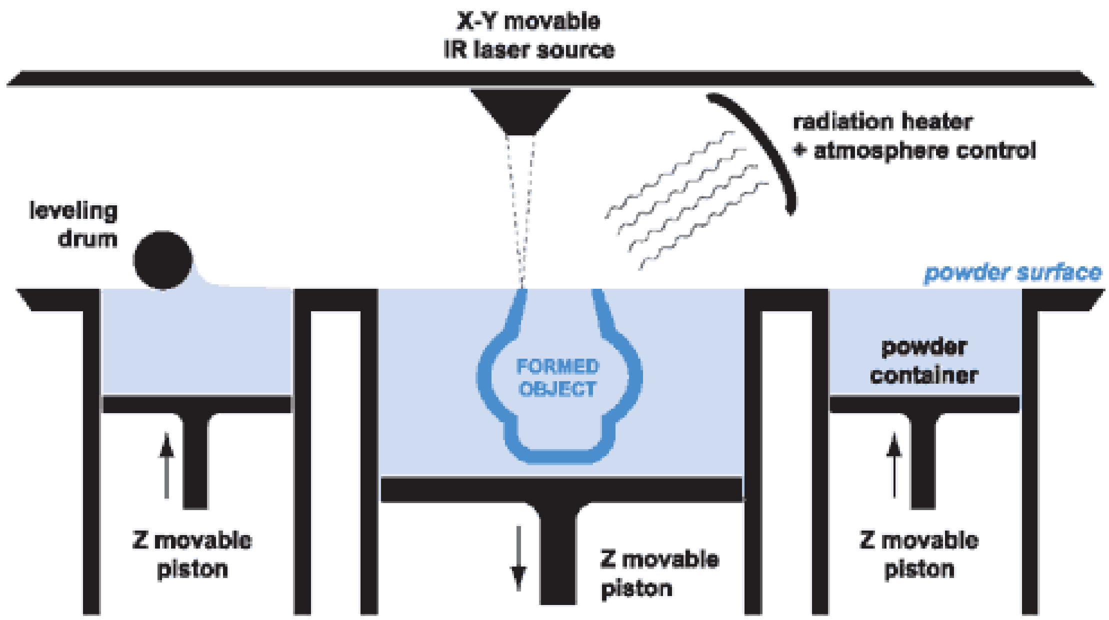

Figure 5.7 Sintering machine diagram

The data from a 3D CAD file is entered into the sinterstation. A precise, thin layer of heat fusible powder is deposited across the build platform. A computer controlled carbon dioxide laser scans a cross section of the 3D solid model (described in the CAD file). As the laser draws the cross section, the powder contacted by the laser is fused together, without actually melting the powder, forming a thin layer (typical 0.004”). A new layer of powder is deposited and the process of laser sintering and spreading powder is repeated until the 3D object is complete. Parts do not require post curing (except when ceramics are used). No support structures are required...so your parts are produced cleaner with less post-processing.

The parts are built in Polyamide (PA). The powder, being a solid material, has the attractive feature of being self-supporting for the generated product sections. This makes supports (typical for stereolithography) redundant. The polyamide material allows the production of fully functional prototypes with high mechanical and thermal resistance.

The use of PA powder with glass filling (PA-GF) has a much higher thermal resistance and is typically used in functional tests with high thermal loads.

The standard accuracy of SLS parts is +/- 0.2%; the general wall thickness should be at least 1mm, but in most cases a smaller local wall thickness is feasible.

Polyamide SLS parts have an excellent long-term stability and are resistant against most chemicals. They can be made watertight by impregnation. The PA material is certified as biocompatible and not harmful to health or the environment.

Table 5.1 Laser Sintering Properties

Material Properties	Density of sintered part	ASTM D792	0,95 g/cm³
	Moisture Absorption 73° F	ASTM D570	0,41 %
Mechanical Properties	Tensile Modulus	ASTM D638	1700 MPa
	Tensile Strength	ASTM D638	45 MPa
	Elongation at break	ASTM D638	15 %
	Flexural Modulus	ASTM D790	1300 MPa
	Izod – Impact Strength	ASTM 256	440 J/m
	Izod – Notched Impact	ASTM 256	220 J/m
Thermal Properties	Melting Point	DSC	363°F (184°C)
	HDT (Deflection Temp under Load) @ 66psi	ASTM D648	350°F (177°C)
	HDT (Deflection Temp under Load) @ 264psi	ASTM D648	187°F (86°C)
Sintering Parameters	Layer thickness		0,006" (0.150 mm.)
	Build Envelope (largest part size)		13,4"x13,4"x24,4" (340 x 340 x 620 mm.)
	Minimum feature size		0,030" (0,750 mm.)

REFERENCES

Tsai L.-W., (1999). *Robot Analysis* (2^{nd}. ed.). Maryland: John Wiley & Sons. Inc.

Groover, M. P., Weiss, M., Nagel, R. N., Odrev N. G., (1986) *Industrial Robotics* (3^{rd}.ed.) MA: McGraw-Hill Inc.

Dorpf, R., (1992). *Modern Control Systems.* (6^{th} ed.). Istanbul, Literatür Yayınları

Çetin L., (2001). *Modeling Of a Rotating Platform And Its Position Control Using Various Control Algorithms.* IZMIR: Dokuz Eylül University Graduate School Of Natural And Applied Sciences.

Kuo, B., (1987). *Automatic Control Systems* (5th ed.). New Jersey, Prentice Hall

Duffy, J., (1980). *Analysis of Mechanism and Robot Manipulator.* New York: Wiley.

Mckerrow, P. J., (1995). *Introduction to Robotics*. Australia: McKerrow.

Asada, H., & Kanade, T., (1981). *Design of Direct Drive Mechanical Arms*. CMU Robotics Institute Technical Reports.

Ayberk, A., (2001). *Vision and Tracking Control of a Robot Manipulator*. IZMIR: Dokuz Eylül University Graduate School Of Natural And Applied Sciences.

Featherstone, R., (1983). *Position and Velocity Transformations between Robot End-Effector Coordinate and Joint Angles.* The International Journal of Robotics Research.