ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL

A REVIEW OF FFT ALGORITHMS AND A REAL-TIME ALGORITHM DEVELOPMENT FOR AIRBORNE VIBRATION TESTING APPLICATIONS

M.Sc. THESIS

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Department of Communication Systems

Satellite Communication and Remote Sensing Programme

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FFT ALGORİTMALARININ İNCELENMESİ VE UÇUŞTA TİTREŞİM TESTİ UYGULAMALARI İÇİN GERÇEK ZAMANLI ALGORİTMA GELİŞTİRİLMESİ

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To Mustafa Kemal ATATÜRK,

FOREWORD

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November 2023

Osman Birkan ÖZSEVEN
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ABBREVIATIONS

CFT	: Continuous Fourier Transform
CZT	: Chirp Z Transform
DAQ	: Data Acquisition System
DFT	: Discrete Fourier Transform
FFT	: Fast Fourier Transform
FT	: Fourier Transform
FTI	: Flight Test Instrumentation
HHT	: Hilbert Huang Transform
ICFT	: Inverse Continuous Fourier Transform
SNR	: Signal to Noise Ratio
STFT	: Short Time Fourier Transform
TDMS	: Technical Data Management Streaming
WT	: Wavelet Transform
ZT	: Z Transform

SYMBOLS

A	: Arbitrary Complex Number
$\mathbf{f}$	: Frequency
$\mathbf{f_{fundamental}}$	: Fundamental Frequency
$\mathbf{f_{harmonic}}$	: Harmonic Frequency
$\mathbf{f_{max}}$	: Highest Frequency Component
$\mathbf{f_{Nyquist}}$	: Nyquist Frequency
$\mathbf{f_{s}}$	: Starting Frequency
$\mathbf{f_{start}}$	: Stopping Frequency
$\mathbf{f_{end}}$	: Harmonic Number
j	: Imaginary Unit
k	: Discrete Frequency Index
N	: Number of Data Points
t	: Time
W	: Arbitrary Complex Number

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A REVIEW OF FFT ALGORITHMS AND A REAL-TIME ALGORITHM DEVELOPMENT FOR AIRBORNE VIBRATION TESTING APPLICATIONS

SUMMARY

Data collection is a critical aspect of flight test instrumentation as it enables the evaluation of aircraft performance and safety through the monitoring of critical parameters such as vibrations, temperature, pressure, and force, as well as digital bus operations. However, effective communication strategies are necessary to transmit this data, particularly high-frequency vibration data, during prototype testing of full-scale air vehicles. The Fourier Transform is a fundamental technique that decomposes signals into their constituent frequencies, thereby facilitating the analysis of specific components within continuous data.

The primary objective of this study is to conduct a comprehensive analysis and assessment of multiple Fast Fourier Transform (FFT) algorithms to determine the optimal approach for addressing practical aviation challenges. The selection of the most appropriate FFT algorithm can significantly influence the efficiency and precision of data analysis in the context of flight test instrumentation. The case study offers comprehensive insights into the utilization of the chosen FFT technique to handle a specific difficulty related to aviation. This indicates the practical importance of this research in the analysis of real-time vibration measurements.

This study introduces an algorithm that has been extensively developed for the purpose of analysing real-time vibration data. This algorithm provides a comprehensive understanding of the many features of signals within the frequency domain. The technique highlights the Chirp-Z Transform (CZT), which facilitates the accurate finding of frequency elements. The selection of the Hann window in the CZT analysis is crucial for mitigating spectral leakage, hence enhancing the precision of frequency analysis. During the spectrum resolution and pre-processing phase, the vibration data is carefully divided into overlapping windows, with parameters computed to improve frequency resolution. By applying oversampling with a 512-point Hann window at a sampling rate of 2048 Hz, the resolution is enhanced, enabling a more comprehensive analysis of the frequency spectrum. The increased level of accuracy demonstrated by this measurement technique serves to validate its importance in the detection and analysis of high-frequency harmonics. Thus, it facilitates a greater understanding of engine functionality and provides possibilities for potential progress in this domain.

The technique of collecting vibration data employs the usage of the PCB 33931 vibration sensor, which effectively translates mechanical vibrations into corresponding electrical impulses. To enhance precision, the raw data undergoes signal conditioning utilizing Curtiss-Wright hardware. This process involves noise reduction, amplification, and filtering. The efficiency of the algorithm is achieved by optimising computations across numerous CPU cores to minimize processing time. The real-time simulation of CZT is executed on a personal computer equipped with a 12th-generation Intel Core processor, particularly the i7-12700H model, operating at a

frequency of 2.30 GHz. In an extensive assessment including the central processing unit (CPU), hardware components, and processor, the mean duration per iteration, which contains the process of data collecting, is 0.0315 seconds. The comprehensive analysis offers an overall view of the computing process, considering multiple elements that influence performance. However, after conducting a more detailed examination, with a specific emphasis on the CZT analysis, it becomes obvious that there is a significant increase in efficiency. When isolating this one aspect from the whole algorithm, the time required for each iteration exhibits a considerable decrease to only 0.0017 seconds, indicating a significant boost in efficiency.

FFT ALGORİTMALARININ İNCELENMESİ VE UÇUŞTA TİTREŞİM TESTİ UYGULAMALARI İÇİN GERÇEK ZAMANLI ALGORİTMA GELİŞTİRİLMESİ

ÖZET

Uçuş test enstrümantasyonu alanında, çeşitli veri türlerinin toplanması, uçakların ve bileşenlerinin güvenliği ve performansı için büyük öneme sahiptir. Bu veriler sıcaklık, basınç, kuvvet, dijital veri izleme ve daha birçok şeyi içerebilir. Ancak, özellikle havacılıkta kompozit yapıların artan kullanımıyla son yıllarda belirli bir veri türü daha fazla önem kazanmıştır: titreşim. Titreşim ölçümleri, motor ve itki sistemi değerlendirmeleri, flutter analizi ve yapısal rezonans çalışmaları gibi bir dizi test için vazgeçilmezdir. Bu testler, uçağın farklı bileşenlerinin titreşimlere ve salınım hareketlerine nasıl tepki verdiğini anlamayı amaçlar ve tasarımı iyileştirmeye ve güvenliği artırmaya yardımcı olabilir.

Hava araçlarının prototip test uçuş faaliyetleri kapsamında, yer istasyonlarına telemetri yoluyla iletilmesi gereken büyük miktarda veri vardır. Ancak bu verileri toplamak ve analiz etmek büyük bir zorluk oluşturur. Çünkü iletim sırasında yüksek miktarda verinin hızlı ve güvenilir bir şekilde aktarılması gerekir. Ayrıca, yüksek frekansta titreşim verileri ile uğraşırken iletim kanalının bant genişliği önemlidir. Bu tür veriler genellikle karmaşık, yüksek frekansta davranışlar sergiler ve bu nedenle veri iletişimin verimliliğini optimize etmek ve iletişim akışındaki yüzü azaltmak için çeşitli teknikler kullanılabilir. Bu temel tekniklerden biri Fourier Dönüşümü ‘dür. Fourier Dönüşümü, sürekli zamanlı bir sinyali temel sinüzoidal frekansta bileşenlerine ayırarak zaman ve frekans alanları arasında bir köprü oluşturan matematiksel bir araçtır. Bir sinyalin spektrumunu açığa çıkararak her frekans bileşeninin gücünü ve fazını gösterir. Bu dönüşüm, sürekli sinyaller içinde belirli frekans bileşenlerini anlamak ve izole etmek için temel bir öneme sahiptir.

Ayrık zamanlı sinyaller bağlamında, genellikle Ayrık Fourier Dönüşümü ‘nü (DFT) kullanmak yaygın bir uygulamadır. DFT, sinyal örneklerinin bir ayrık frekans spektrumuna eşlenmesini sağlar ve dijital sinyaller içinde farklı frekansta sinüzoitlerin

genliklerini ve fazlarını belirler. Bu süreç dijital sinyal işleme için merkezi bir öneme sahiptir ve mühendislerin ve bilim insanlarının toplanan verilerden önemli bilgiler çıkarmalarına olanak tanır. Özellikle DFT'nin verimliliği, DFT'nin hesaplamasını büyük ölçüde hızlandıran Hızlı Fourier Dönüşümü (FFT) algoritması tarafından kullanılır ve daha pratik hale getirilir. Geleneksel FFT algoritmaları genellikle gerçek zamanlı uygulamalarda tercih edilirken, bu çalışma, frekans alanındaki sinyal özelliklerine ayrıntılı bir bakış sunarak gerçek zamanlı titreşim verilerinin işlenmesi için dikkatle tasarlanmış bir algoritma sunar. Bu algoritmanın merkezinde Chirp-Z Dönüşümü (CZT) bulunur ve belirli frekans bileşenlerinin hassas bir şekilde çıkarılmasına olanak tanır.

CZT analizinde Hann penceresi seçimi, frekans analizi sırasında spektral sızıntıyı azaltma rolü oynayarak frekans analizi sırasında yüksek bir doğruluk seviyesini sağlar. Spektral çözünürlük ve ön işleme aşamasında titreşim verileri, frekans çözünürlüğünü artırmak için hesaplanan parametrelerle örtüşen pencerelere bölünür. 2048 Hz örnekleme hızıyla 512 noktalı bir Hann penceresi ile aşırı örnekleme kullanılması, çözünürlüğü daha da iyileştirir ve frekans spektrumunun daha derinlemesine incelenmesini kolaylaştırır. Bu yüksek hassasiyet, yüksek frekanslı harmonikleri tanımada etkili olup motor performansının daha derin bir anlayışını sağlar ve sahadaki potansiyel gelişim fırsatlarına katkıda bulunur.

Üçüncü bölümde, CZT algoritması, prototip tahrik sistemini kapsamlı bir şekilde analiz etmek için uygulanmıştır. Bu süreç, harmonik hesaplama yoluyla istenen spektral yakınlaştırma aralığını belirlemeyi içerir ve bunun sonucunda f_{start} ve f_{end} gibi parametrelerin tanımlanmasına yol açar. İkinci bölümde belirtildiği gibi, f_s, aşırı örnekleme yöntemi kullanılarak hesaplanır. f_s belirlendikten sonra istenen çözünürlük de hesaplanır. Bu parametreler, CZT özel katsayılarını belirlemede önemli bir rol oynar, bunlar arasında M, α ve ω yer alır, bu da motor harmonik analizinin hassasiyetini ve netliğini önemli ölçüde artırır.

Bu makale, uçuş test enstrümantasyonundaki gerçek hayat havacılık sorunlarını çözmek için çeşitli FFT (Hızlı Fourier Dönüşümü) algoritmalarını gözden geçirerek en etkili yöntemi belirlemeye odaklanmaktadır. Uçuş testi sırasında elde edilen büyük veri setlerinin analizi hem hızlı hem de doğru sonuçlar elde etmek için uygun bir FFT algoritması seçiminin kritik öneme sahip olduğunu vurgular. Farklı FFT algoritmalarını karşılaştırarak en uygun olanını belirlemek ve bu seçimin uçuş test enstrümantasyonundaki veri analizi performansı üzerindeki etkilerini incelemektir. Uygulanan algoritmalar arasında Cooley-Tukey FFT, Radix-2 FFT ve Chirp Z Dönüşümü (CZT) gibi çeşitli seçenekler bulunmaktadır. Makalenin vaka çalışması bölümü, CZT algoritmasının seçilen bir havacılık sorununu çözmek için nasıl kullanıldığını detaylı bir şekilde açıklar. Gerçek zamanlı veri analizi, hızlı tepki ve doğruluk gerektiren uçuş testleri için CZT'nin neden tercih edildiğini gösterir. Bu bölüm, teorik bilgilerin pratik uygulamada nasıl kullanıldığını ve havacılık endüstrisindeki zorlukları nasıl aştığını açıklayarak araştırmanın somut önemini vurgular.

Titreşim verileri, titreşimleri elektrik sinyallerine dönüştüren PCB 33931 titreşim sensörü kullanılarak toplanır. Doğruluk sağlamak için ham veriler önce Curtiss-Wright donanımı ile sinyal işlemeden geçer; bu işlem, kuvvetlendirme, filtreleme ve gürültü azaltmayı içerir. Algoritmanın verimliliği, çoklu CPU çekirdeklerinde hesaplamaları optimize etmeye dayanır ve işleme süresini azaltır. Gerçek zamanlı CZT simülasyonu, 12. Nesil Intel Core, i7-12700H, 2.30 GHz bir bilgisayarda çalıştırılır. CPU, donanım ve işlemciyi içeren kapsamlı bir değerlendirme sonucunda, veri toplama dahil her işlem için ortalama iterasyon süresi 0.0315 saniye olarak hesaplanır. Bu kapsamlı analiz, performansı etkileyen çeşitli faktörleri hesaba katarak hesaplama sürecine geniş bir perspektif sunar. Ancak özellikle CZT analizine odaklanarak, her işlem için geçen süre sadece 0.0017 saniyeye önemli ölçüde azalır, bu da dikkate değer bir verimlilik gelişimini gösterir.

CZT'nin sunduğu odaklı ve verimli veri analizi ve temsili, özellikle belirli frekans bantlarına odaklandığında iletilmesi gereken veri miktarında önemli düşüşlere neden olabilir. Bu durum, Ölçüm ve Veri Toplama (DAQ) sistemlerinde iletişim kanallarındaki yükü azaltarak etkili bir veri iletimi sağlar. Ancak, gerçek zamanlı CZT analizi bir dizi avantajın yanı sıra bazı zorluklara da yol açabilir. Büyük veri setleri için hesaplama karmaşıklığı, gerçek zamanlı veri işlemek için optimize edilmiş algoritmaların gerekliliği gibi teknik zorluklar ortaya çıkabilir.

Sensör kalitesi, veri iletim hızı ve gürültü müdahalesi gibi faktörler sonuçların doğruluğunu etkileyebilir. Bu zorlukları etkili bir şekilde aşmak için, özel bir CZT fonksiyonu geliştirilebilir. Bu özel fonksiyon, belirli araştırma gereksinimlerini karşılamak üzere tasarlanacak ve analizde doğruluğu artırarak hesaplama verimliliğini optimize edebilecektir.

Önümüzdeki çalışmalarda, gerçek zamanlı CZT analizinin sağladığı birçok avantaja rağmen, özellikle büyük veri setleri için hesaplama karmaşıklığı ve gerçek zamanlı veri işleme algoritmalarının geliştirilmesi gibi bazı zorluklar ele alınması planlanmaktadır. Ayrıca, sensör kalitesi, veri iletim hızı ve gürültü müdahalesi gibi faktörlerin sonuçların doğruluğunu etkileyebileceği göz önünde bulundurularak özel bir CZT fonksiyonu geliştirilmesi düşünülmektedir. Bu özel fonksiyon, belirli araştırma gereksinimlerini karşılamak üzere tasarlanacak ve analizde daha yüksek doğruluk ve hesaplama verimliliği sağlayarak sonuçları iyileştirebilecektir.

1. INTRODUCTION

In the aerospace industry, data collection from air vehicles is crucial during the development or modernization stages while performing testing activities. The data which is obtained via air vehicle parts provides information about the efficiency of the systems. Although the integrated systems have some standards, it is critical to see the systems' integrability with others [1]. To validate, verify, and certify the air vehicle in accordance with relevant standards, it is critical to implement a flight test instrumentation system design. This design should include a data acquisition unit, network switch, and recorder, all smoothly integrated with the existing aircraft systems throughout the various stages of air vehicle development [2]. Since, there can be a requirement for the installation of additional equipment such as a sensor, transducer, and data bus systems to be able to collect temperature, vibration, strain, current, voltage, pressure, etc. data from the avionics or mechanical parts [3] [4]. In this paper, vibration data is analysed in detail.

Vibration data collected from sensors contains information about the mechanical behaviour of air vehicle components such as avionics, propulsion systems, control surfaces, gearboxes, and airframe structures. In terms of safety and reliability, excessive vibration may be a reason for mechanical problems or component wear. By monitoring vibrations, aircraft operators can detect issues early, reducing the risk of in-flight failures, accidents, or the health of various components due to mechanical failures [5] [6]. Irregular vibrations can indicate problems such as imbalance, misalignment, fatigue, or damage to these components [7]. Additionally, in the matter of compliance, regulatory authorities generally require aircraft operators to monitor and report on various parameters, including vibration levels. Conformity with these regulations is crucial to ensure the airworthiness and safety of aircraft [8]. In connection with research and development, aircraft manufacturers and researchers may use the obtained vibration data to improve the performance of the aircraft design and material. This data aids in the development of more efficient and reliable aircraft [9].

Fourier Transform (FT) is a fundamental technique that breaks down continuous-time signals into their component sinusoidal frequencies. It connects the time domain (where signals vary over time) to the frequency domain (where signals are represented by their frequencies). In the time domain, signals are complex waveforms formed by overlapping waves, each with its frequency, phase, and amplitude [10]. FT transforms this into a frequency spectrum, showing the strengths and phases of each frequency in the original signal. This is useful for identifying dominant frequencies. For discrete signals in digital applications, DFT is used, which maps a sequence of numbers (discrete signal) to another sequence representing its discrete frequency components. This spectrum reveals amplitudes and phases of discrete frequencies that approximate the original signal when combined [11].

DFT algorithms enable the analysis of signals in the frequency domain. These algorithms are crucial tools for decomposing discrete-time signals into their underlying frequency components. The Cooley-Tukey Radix-2 FFT, a widely used DFT algorithm, excels at efficiently computing the DFT for data lengths that are powers of 2 [12]. Mixed-radix FFT extends this approach to handle data lengths that are not powers of 2 by employing mixed-radix stages. The Prime Factor FFT specializes in efficiently computing the DFT of data with prime factorization lengths. Furthermore, Bluestein's FFT, also known as the Chirp Z-Transform (CZT), is vital when working with signals of arbitrary lengths, as it transforms them into a format suitable for FFT computation [13]. In the theory section, these algorithms are compared, and the most appropriate method is chosen for addressing the problem under consideration.

In this paper, a practical aviation challenge related to the propulsion system test is addressed. Specifically, during the propulsion system test, the transmission of vibration data to the ground station leads to significant constraints due to communication channel limitations. In response to this challenge, transmitting the vibration signal in its frequency domain format has been preferred, rather than the time domain signal, to facilitate real-time testing [14]. It's worth noting that a subsequent time domain analysis is also conducted during the post-data processing phase for comprehensive evaluation and insights. This approach allows to efficiently address the complexity of real-time data transmission and subsequent analysis in the context of aviation testing.

In contrast to the widespread preference for conventional FFT algorithms in [15] and [16], this paper stands out by opting for the CZT Method, a well-established alternative, to attain a more precise analysis. Since, in spectral analysis, the frequency resolution of the FFT is intrinsically tied to the length of the signal window. This means that the resolution is determined by dividing the sampling rate by the number of points in the FFT. While zero-padding is a technique often employed to interpolate between FFT bins, it doesn't increase the genuine frequency resolution. On the other hand, the CZT introduces a significant flexibility. It allows for computation of the DFT samples within a specific region of the z-plane, enabling users to define a starting frequency, an ending frequency, and the desired number of points or frequency resolution between them. This capability of the CZT is particularly beneficial when the focus is on a specific frequency range rather than the entire spectrum. Band-Selectable Fast Fourier Transform (Zoom FFT) also provides targeted frequency analysis [17]. However, while Zoom FFT focuses on a specific frequency range through down-sampling after mixing to baseband, the CZT calculates spectral values across any chosen frequency interval directly. CZT also succeeds in handling non-uniformly sampled data, making it suitable for scenarios where data points are irregularly spaced in time [18].

This paper is organized as follows, a brief introduction to the topic covered by the paper is given in Chapter 1. A theoretical comparison is made between different FFT algorithms and the terms spectral leakage, windowing techniques, zero-padding and oversampling are given in Chapter 2. Case study including the subjection of the propulsion system vibration analysis, the methods and techniques used in the CZT algorithm are given in Chapter 3. Result analysis is performed in Chapter 4. Concluding remarks are stated in Chapter 5 as performing CZT analysis in real-time.

2. THEORY

In vibration analysis, the vibration data obtained from sensors placed on system components, continuously or at regular intervals are analysed to predict whether there is a fault or not. Vibration signals taken from different points contain complex waveforms due to various forces and factors. Therefore, determining faults by examining time-waveform graphs is very challenging. The signals obtained in the time domain from vibration sensors are transformed into the frequency domain methods using the Fourier transform [19]. Concurrently, the amplitudes associated with these frequencies serve as a clear representation of the damage's severity, allowing for an accurate assessment of its extent. Faults exhibit various symptoms such as increased vibration, temperature, noise, and excessive current until the system malfunctions [7] [20]. Condition monitoring-based maintenance is performed by measuring and evaluating these symptoms. In this way, the risk of economic loss, operation problems and re-work caused by component failure may be reduced.

The Fourier Transform is a mathematical technique used to analyse signals or functions in terms of their frequency components. It decomposes a signal from the time domain into the frequency domain, revealing the individual sinusoidal components that make up the signal [21].

The Continuous Fourier Transform (CFT) is defined by using Eq. 2.1 [22].

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt \tag{2.1}$$

Where:

$X(f)$ is the complex-valued representation of the signal $x(t)$ in the frequency domain.

$x(t)$ is the signal in the time domain.

f is the frequency (in Hertz).

j is the imaginary unit.

The Inverse Continuous Fourier Transform (ICFT) is defined by using Eq. 2.2 [21].

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df \tag{2.2}$$

The DFT holds a significant position in the examination, creation, and realization of algorithms and systems in discrete-time signal processing applications, including but not limited to linear filtering, correlation analysis, and spectrum analysis DFT is equivalent to the samples of the FT at equally spaced frequencies. As a result, computing an N-point DFT involves calculating N samples of the FT of N equally spaced frequencies (2πk/N for k = 0 to N-1) on the unit circle in the z-plane. The main purpose is to use efficient algorithms for computing the N-point DFT, and these algorithms are commonly known as FFT algorithms [22].

When performing the computation of the DFT as outlined in Equation (3), there is an essential involvement of *N* complex multiplications and *N-1* complex additions for each X(k) value. Consequently, in the process of calculating *N* DFT values, the necessity arises for N^2 complex multiplications and *N(N-1)* complex additions. It's also worth noting that within each complex multiplication, there is an implicit utilization of four real multiplications and two real additions, whereas every complex addition is achieved with two real additions [23]. As a result, for large values of N (the length of the array), computing DFT directly requires a considerable amount of processing. As *N* increases, the number of operations grows rapidly.

The DFT is defined by using Eq. 2.3 [21].

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{j2\pi kn/N} \tag{2.3}$$

Where:

$X[k]$ is the complex−valued representation of the signal $x[n]$ in the frequency domain.

$x[n]$ is the discrete signal in the time domain.

k is the discrete frequency index (0 to $N-1$).

N is the number of data points in the time domain signal.

The Inverse Discrete Fourier Transform (IDFT) is defined by using Eq. 2.4 [22].

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j2\pi kn/N} \tag{2.4}$$

The number of applications of the DFT in digital signal processing and other domains increased significantly because of the method created by Cooley and Tukey in 1965 to reduce the computation effort related to the DFT. The previously mentioned method additionally established the foundation for the development of diverse algorithms, commonly referred to as the FFT [24]. They significantly decrease the quantity of operations required for the computation of the DFT, hence enhancing the feasibility of the procedure. The FFT is a very fast and cost-effective algorithm utilized for the computation of the DFT. It offers useful simplification and computational convenience [25].

Spectral leakage is observed when the FFT is applied to a signal that does not contain an exact integer number of cycles within the window utilized for the transformation. This causes the signal's energy, which is concentrated at a single frequency, to be distributed across multiple frequency divisions in the FFT output [26]. The issue of FFT spectral leakage arises when the discrete frequency bins of the FFT do not fully line up with the frequency components of a signal. In order words, spectral leakage is especially pronounced when the signal frequency is not an integer multiple of the frequency bin spacing (bin width), the signal duration is not an integer multiple of the sampling period and the signal is not properly windowed before applying the FFT. To address spectral leakage, it is common practice to employ windowing techniques. These involve multiplying the original signal by a window function that reduces the signal's amplitude towards the edges [27]. This effectively reduces the influence of signal values at the edges of the window, which contributes to spectral leakage. Common window functions include Hamming, Hanning (or Hann), Blackman, and more. These windows taper the signal towards the edges, reducing abrupt changes that contribute to spectral leakage [19]. No leakage, leakage, and windowed sine wave comparison is shown in Fig. 2.1.

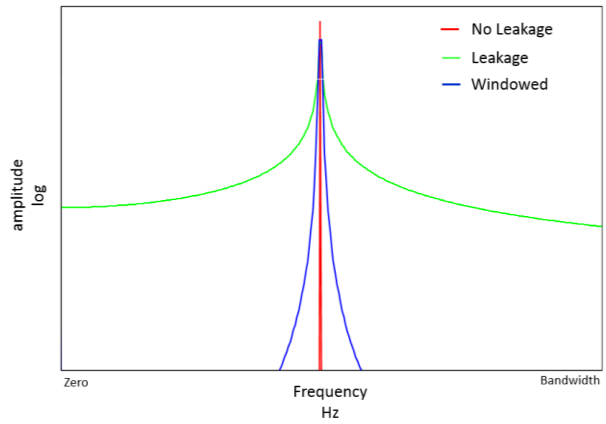

Figure 2.1 : Sine wave: no leakage, leakage and windowed [28].

The Hamming window, whose formula is given in Eq. 2.5 has a main lobe that's wider than the main lobes of the Hanning and Blackman windows. It offers a good balance between main lobe width and side lobe suppression compared to rectangular (no window) but not as good as Hanning and Blackman [29].

$$w(n) = 0.54 - 0.46 * \cos\left(\frac{(2 * \pi * n)}{N + 1}\right) \tag{2.5}$$

Where:

$w(n)$ is the value of the Hamming window at the sample.

N is the total number of samples in the window.

n takes on integer values starting from 0 and goes up to N-*1*.

The Hanning window, which is mathematically defined in Equation 2.6, is frequently known as the "raised cosine" window. The term "raised" or "shifted upwards" to initialize at 0 and conclude at 1 is derived from the geometric configuration of the function. The amplitudes of the side lobes have a reduced magnitude in comparison to the Hamming function [27].

$$w(n) = 0.5 - 0.5 * \cos\left(\frac{(2 * \pi * n)}{N + 1}\right) \tag{2.6}$$

Where:

$w(n)$ is the value of the Hanning window at the sample.

N is the total number of samples in the window.

n takes on integer values starting from 0 and goes up to N-*1*.

The Blackman window, as defined by Equation 2.7, has the most narrow primary lobe when compared to the other three windows. This feature offers optimal side lobe suppression, which means facilitating the reduction of spectral leakage [29]. Due to its strong side lobe suppression, the Blackman window can provide good results for precise frequency analysis.

$$w(n) = 0.42 - 0.5 * \cos\left(\frac{(2 * \pi * n)}{N - 1}\right) + 0.08 * \cos\left(\frac{(4 * \pi * n)}{N - 1}\right) \quad (2.7)$$

Where:

$w(n)$ is the value of the Blackman window at the sample.

N is the total number of samples in the window.

n takes on integer values starting from 0 and goes up to N-*1*.

All three windowing techniques (Hamming, Hanning, and Blackman) are designed to reduce spectral leakage when performing FFT analysis. The selection of the window to use is contingent on the particular demands and prerequisites. If frequency resolution and minimizing side lobes are concerned, Hanning or Blackman might be preferred. If a balance between frequency resolution and side lobe suppression is being looked for, Hamming could be a good choice [30]. Experimentation and understanding the signal characteristics help determine the best window for the application. There are some windowing techniques given in Table 1.1.

Table 1.1 : Comparison of windowing techniques [22-24].

Signal Content	**Window**
Sine wave or combination of a mix of sinusoidal waves	Hann
Sine wave (with a focus on amplitude precision)	Flat Top
Narrowband stochastic signal (pertaining to vibration data)	Hann
Wideband stochastic signal (resembling white noise)	Uniform
Closely situated sinusoidal waves	Uniform, Hamming
Stimulus signals (akin to a hammer strike)	Force
Outcome signals	Exponential
Signals with undisclosed content	Hann
Two sinusoidal signals with proximate frequencies but distinct amplitudes	Kaiser - Bessel
Two sinusoidal signals with proximate frequencies and nearly identical amplitudes	Uniform
Precise measurement of amplitude in single-tone signals	Flat Top

When working with vibration signals that contain high-frequency components, it is important to consider the specific characteristics of the signal and the requirements of the application. There are some other methods rather than the windowing method efficiently used while selecting a suitable FFT method such as zero padding, oversampling, etc.

In real-time FFT analysis, zero padding is commonly used to improve the accuracy and clarity of frequency analysis. It refers to the process of adding extra zeros to the end of a time-domain signal before performing the FFT [31]. The frequency resolution of the FFT depends on two critical factors: the length of the input signal and the sampling rate (or the signal's time duration). When the signal is longer, it offers enhanced frequency resolution, enabling a more precise distinction between closely spaced frequencies. By adding zeros and increasing the signal length, the frequency resolution of the FFT is enhanced. Zero padding can also be seen as a form of interpolation between the existing signal samples. This can result in smoother spectral peaks in the frequency-domain representation, making it easier to visualize and analyse the frequency content of the signal [32]. Zero padding is often used in scenarios where frequency components with higher precision are aimed to be analysed, such as identifying closely spaced harmonics in a signal, or visualization of the frequency spectrum with more detail.

Oversampling encompasses the act of obtaining and handling an increased number of samples from the vibration signal by utilizing a sampling rate that surpasses the Nyquist rate, equivalent to twice the maximum frequency component found within the signal. In mathematical terms, if the highest frequency component in the signal is f_{max}, then the Nyquist rate $f_{Nyquist}$ is given in Eq. 2.8 [33]. With more samples, better frequency resolution can be achieved. This means that closely spaced frequencies are distinguished more accurately, which can be beneficial in applications like frequency analysis or spectral leakage reduction. Also, oversampling can help in reducing the impact of noise and quantization errors [34]. By capturing more samples, the noise gets spread over a wider range of samples, leading to a better signal-to-noise ratio (SNR) in the resulting FFT. The other advantage of oversampling is to reduce the effects of aliasing, which occurs when high-frequency components in the signal fold back into the frequency range of interest due to inadequate sampling [35].

$$f_{Nyquist} = 2 \times f_{max} \tag{2.8}$$

There are several other methods and algorithms rather than FFT and DFT used for analysing and processing signals, especially in the field of signal processing and spectral analysis. Although the FFT and DFT are extensively utilized for the transformation of signals from the time domain to the frequency domain, it's important to note the existence of alternative methods like the Wavelet Transform, Short-Time Fourier Transform (STFT), and Hilbert-Huang Transform (HHT). These alternative techniques are designed to address signal processing challenges and provide specific analytical requirements [36]. The WT is a mathematical method and signal processing tool employed to analyse signals and data in both the time and frequency domains. The utilization of this technique proves to be highly advantageous in the capture and analysis of high-frequency, non-stationary, and transient components within signals. The set of fundamental functions, commonly referred to as wavelets, are typically characterized by short durations and are defined at various scales [37]. Additionally, the STFT is a widely used technique in the field of signal processing that is utilized to analyse the changes in the frequency properties of a signal over time. The analysis of non-stationary signals, which exhibit time-varying frequency components, is significant [38]. Moreover, the HHT is a widely employed signal processing methodology utilized for the examination of non-linear and non-stationary data. The purpose of its development is to address the constraints inherent in conventional techniques such as the FFT and the STFT when dealing with intricate signals that change over time. The aim of this technique is to extract the inherent modes, uncover patterns, and analyse the instantaneous frequencies within signals [39]. FFT and DFT offer several advantages compared to other approaches and algorithms stated earlier. These advantages include improved efficiency and speed, enhanced frequency accuracy, wide applicability, compliance with linearity principles, and simplified implementation [40].

In the circumstance of high-frequency applications, it may be reasonable to explore higher-level algorithms that provide better performance or specific functionalities. An approach that can be utilized for high-resolution frequency analysis of signals with non-uniformly spaced frequency components is the Chirp Z-Transform (CZT) which is an extension of the DFT [41].

The Z-transform (ZT) of a finite-duration sequence x[n] is determined at a specified set of locations $\{z_k\}$. The CZT is defined by using Eq. 2.9 [42].

$$X(z_k) = \sum_{n=0}^{N-1} x[n]{z_k}^{-n} \tag{2.9}$$

Where:

$X(z_k)$ is the CZT value at frequency point k.

$x[n]$ stands for the discrete sequence.

k denotes the frequency index within the range of 0 to $N-1$.

N represents the overall count of samples in the sequence.

Consider that the points z_k along a contour adhere to the pattern as specified in Eq. 2.10.

$$z_k = AW^{-k} \tag{2.10}$$

Where:

k serves as the frequency index, spanning from 0 to $M-1$.

M is an arbitrary integer (not always equal to N, the point

number of the desired complex spectrum).

A and W represent arbitrary complex numbers as defined in Eq. 2.11 [43].

$$A = A_0 e^{j\theta_0}, W = W_0 e^{j\varphi_0} \tag{2.11}$$

Where:

A is the contour starting point.

θ_0 is the starting angle.

W determines the rate at which the contour spirals inward or outward from a circle with a radius denoted as A_0.

φ_0 signifies the angular separation among the various frequencies.

When $A_0 = W_0 = 1$, $\theta_0 = 0$, $\varphi_0 = \frac{2\pi}{N}$ and $M = N$, the contour encompasses the full unit circle, and the CZT) is essentially identical to the DFT.

Firstly, it is excellent at detecting transient events in the data due to its ability to provide high-resolution time-frequency information. FFT and DFT may be less effective in isolating and characterizing these events. Secondly, the CZT provides significant utility in the analysis of non-stationary signals or signals exhibiting time-varying frequency components and is better suited to capture these variations accurately [44]. In contrast, the FFT and DFT assume constant frequency content over the entire signal duration. Thirdly, if the signal contains chirping or rapidly changing frequencies, the CZT can track these changes in a way that FFT and DFT cannot. The CZT's chirp modulation aligns with such frequency variations. Fourthly, the CZT can handle signals with arbitrary data lengths without requiring zero padding, making it versatile for various datasets [45]. FFT and DFT are most efficient when applied to data lengths that are powers of 2 [46].

The algorithm is used to compute the Z-transform of a sequence on a finely selected region of the Z-plane. When applied to vibration data analysis, especially in data acquisition (DAQ) systems the CZT can be beneficial in several ways such as focused frequency analysis, data compression, higher resolution, reduced noise, optimized data transmission and better feature extraction that can reduce the payload on a communication channel. In terms of focused frequency analysis, unlike the standard Fast Fourier Transform (FFT) that computes the spectrum uniformly across a wide frequency range, the CZT allows the user to focus on a specific frequency band of interest. This is especially useful when analysing vibration data where specific frequency bands are of interest [47]. Focusing on specific bands means fewer data points are needed, leading to less data to transmit. Regarding data compression, since only considering a particular frequency range, fewer data points are obtained than sending the entire frequency spectrum. By transmitting only, the relevant frequency components, the amount of data can be reduced sent over a communication channel. Concerning higher resolution, using the CZT can yield higher resolution results within the specific frequency band of interest compared to a standard FFT applied to the same data. The CZT has the potential to identify anomalies or features that could go unnoticed when using alternative methods. In connection with reduced noise, in the case of focusing on a specific frequency band, irrelevant frequency components (which might be just noise or other undesired vibrations) can be excluded. This can result in a cleaner signal representation, requiring less data for effective transmission or storage.

In the matter of optimized data transmission, in wireless DAQ systems, where bandwidth might be a constraint, sending only crucial or relevant information becomes a necessity. By utilizing CZT, it is essentially sending only the most pertinent information, thus optimizing the use of available bandwidth [41]. About better feature extraction, the CZT can help in extracting more meaningful features from vibration data by focusing on the relevant frequency components, which could be of a much smaller size than the original data, thus reducing the payload. A comparison of FFT, DFT and CZT is given in Table 2.1.

Table 2.1 :III Comparison of FFT, DFT and CZT. [22] [23] [41].

Feature	FFT	DFT	CZT
Computational Efficiency	Highly efficient (O (N log N))	Standard (O(N^2))	Depends on the algorithm used
Time-Domain Input	Finite-length sequence	Finite-length sequence	Finite-length sequence
Frequency-Domain Output	Discrete frequencies	Discrete frequencies	Discrete frequencies
Frequency Resolution	Fixed based on signal length	Fixed based on signal length	Adjustable
Time-Frequency Analysis	Limited (non-time-localized)	Limited (non-time-localized)	Effective (time-localized)
Application Range	General spectral analysis	General spectral analysis	Time-varying signals
Real vs. Complex Output	Complex	Complex	Complex

3. CASE STUDY

As detailed in Chapter 1, vibration testing has remarkable importance in the aerospace industry, surrounding the assessment of aircraft components, systems, and even entire airframes to ensure their dependable and safe operation across diverse flight conditions. In this case study, the primary objective is to subject the prototype propulsion system to strict vibration analysis. After a detailed evaluation of available analysis methods, the decision has been made to employ the CZT technique to handle the challenges that appeared in the case study.

The exact frequency range of interest for vibration tests using the CZT depends on the specific goals of the test and the components being analysed. The primary frequencies of interest here are associated with the engine's rotational speed and harmonics. The CZT algorithm's ability to outline the specified frequency range with remarkable resolution enables effective "zooming in" on this specific range to identify the engine harmonics relevant to the investigation [48].

Planned engine tests are set to run at 2000 RPM, meaning the engine completes 2000 rotations per minute, corresponding to a fundamental harmonic with a frequency determined by using Eq. 12. of 2000/60 = 33.33 Hz. This knowledge serves as a vital reference point for identifying the specific frequencies of interest during the frequency analysis. The choice of a frequency range spanning from 1 Hz to 512 Hz is planned, aimed at providing clear insight into the motor's harmonic behaviour. Even when considering the 10th harmonic, the corresponding frequency value is calculated as 330 Hz using Eq. 3.1.

$$f_{harmonic} = n \times f_{fundamental} \tag{3.1}$$

Where:

$f_{harmonic}$ is the frequency of the harmonic.

n is the harmonic number.

$f_{fundamental}$ is the fundamental frequency.

If the given parameters are substituted into Eq. 3.1,

$$f_{harmonic} = 10 \times 33.33$$

$$f_{harmonic} = 330.33\ Hz$$

$f_{harmonic}$ is rounded to 330 Hz.

The provided algorithm (see Appendix A) contains the methodology and computational steps for simulating real-time vibration data processing. Using the CZT, the provided algorithm emphasizes the specific frequency components, offering a detailed perspective on the signals in the frequency domain [49].

The given algorithm in the paper consists of vibration data extraction, defining key parameters, desired spectral resolution, pre-processing, CZT configuration, real-time processing simulation, visualization, playback, and performance assessment stages. In the data extraction stage, the algorithm loads vibration data from a TDMS (Technical Data Management Streaming) file. The acceleration values and corresponding time stamps are extracted from the data [50]. In the defining key parameters stage, key parameters are set, including the sampling frequency $f_s = 2048\ Hz$ denotes how frequently data is sampled, the $windowSize = 512\ samples$ defines the data segment length for spectral analysis, the $overlap\ = 256\ samples$ ensures smooth transitions between consecutive data segments and the $segmentSize = 512\ samples$, which denotes the data chunk processed in one iteration.

In the iteration establishment stage, to process the vibration data in segments, the number of iterations (chunks) is determined by using Eq. 3.2.

$$Iterations = \frac{length\ of\ full\ data}{segments\ size} - 1 \tag{3.2}$$

Eq. 3.2 ensures that each data chunk consists of non-overlapping segments, optimizing the computational efficiency.

In the spectral resolution and pre-processing stage, the vibration data is segmented into overlapping windows using the calculated parameters. Windowing helps reduce spectral leakage and improve frequency resolution. The process of oversampling with a 512-point Hann window and a sampling rate of 2048 Hz strengthens the resolution of frequency analysis [51].

The Hann window is utilized for the CZT analysis in this paper thanks to its numerous benefits. The selection of the Hann window is inspired by its ability to prevent spectral leakage, hence increasing the accuracy of frequency analysis [52]. It provides a consistent frequency response, facilitating the accurate identification of spectral peaks. Furthermore, this technique achieves a harmonious equilibrium between the width of the primary lobe and the suppression of side lobes, so increasing the resolution of frequencies while limiting the occurrence of unwanted sidelobe artefacts. Additionally, the Hann window is favoured for its simplicity and ease of implementation, requiring minimal computational resources [53].

In a CZT, the desired resolution is computed by using Eq. 3.3 [54].

$$The\ desired\ resolution\ =\ f_s\ /\ windowSize \tag{3.3}$$

If the given parameters are substituted into the Eq. 3.3,

$$The\ desired\ resolution = 2048\ /\ 512$$

$$The\ desired\ resolution = 4\ Hz$$

This means that each bin in the frequency spectrum represents a 4 Hz bandwidth.

In the CZT configuration, each data segment's processing time is clocked. This provides invaluable insights into computational requirements. Data segments, produced with 512 samples and 256-sample overlap, ensure a consistent data flow. Unlike traditional Fourier Transforms, CZT allows focused spectral analysis in the range of 1 Hz to 512 Hz (as defined by f_{start} and f_{end}).

If M is defined as the number of points between f_{start} and f_{end} based on a desired frequency resolution, then M is computed by using Eq. 3.4. As mentioned in Eq. 2.11, M is not equal to N.

$$M = \frac{f_{end} - f_{start}}{desired\ resolution} + 1 \tag{3.4}$$

If the given parameters are substituted into the Eq. 3.4,

$$M = \frac{512 - 1}{4} + 1$$

$$M = \ 128.75$$

M is rounded to 128 using the floor command.

The parameters α and ω for the CZT calculation are derived from Eq. 2.11 to apply Eq. 3.5 and Eq. 3.6. These parameters are essential for tuning the CZT to focus on the specified frequency range.

$$\alpha = e^{-1j \times 2\pi \times f_{start}/ f_s} \tag{3.5}$$

$$\omega = e^{-1j \times 2\pi \times (f_{end} - f_{start})/(M \times f_s)} \tag{3.6}$$

Instead of producing a spectrum with 512 points (which would have been the case with a regular FFT of a 512-point signal), the CZT method produces a spectrum with only 128 points. This reduction is possible because CZT focuses only on the frequencies between f_{start} and f_{end}, with the defined resolution. In essence, for each segment of the data, this reduction enables to focus on the frequency components that are most relevant to the analysis, reduces the computational overhead by processing fewer data points and provides a more streamlined visualization that emphasizes dominant frequencies within the desired range while discarding information outside the desired frequency range.

In the real-time processing simulation, the main loop simulates real-time data acquisition and processing. CZT is applied to the windowed segment, producing a frequency spectrum for the desired range. In Acra KAM-500 modules, signals are sampled many times faster than specified by the user [55]. The KAD/ADC/111 is a dedicated hard-wired state machine which performs oversampling on all available channels at a rate within the range of 16,000 sps (samples per second) to 32,000 sps. Additionally, the state machine incorporates digital filtering mechanisms to remove any noise. By using the information from the [56], the average sample per second is calculated as 24 ksps. A simulated acquisition delay of 0.021 seconds is added to replicate the real-world acquisition timings by using Eq. 3.7.

$$Acquisition\ Delay = \frac{Segment\ Size}{Average\ ksps} \tag{3.7}$$

If the given parameters are substituted into Eq. 3.7,

$$Acquisition\ Delay = \frac{512}{24}$$

$$Acquisition\ Delay = 0.021\ seconds$$

In the visualization stage, the algorithm generates two plots: a time domain plot and a frequency domain plot. In the time domain plot, the vibration data is plotted against time, providing insights into the acceleration values over time. In the frequency domain plot, the CZT-derived frequency spectrum highlights the top 5 magnitudes to emphasize dominant frequencies [29]. A simulation ‘pause’ introduces a realistic feel during playback.

In the performance assessment, the processing time for each segment is recorded, and the average segment processing time is computed by using Eq. 3.8.

$$average\ time = \frac{1}{iterations}\sum T \tag{3.8}$$

Where:

T: Elapsed time for each iteration.

Using the Chirp Z algorithm for propulsion system vibration analysis, despite its effectiveness and providing great solutions, presents challenges. These challenges include harmonic vibrations generated by rotating machinery, potential interactions with other propellers leading to instant frequencies, the presence of broadband noise unrelated to specific harmonics, and variations in vibration profiles across different operational states of the rotorcraft. To manage these, it's crucial to ensure sufficient resolution in frequency analysis, especially for distinguishing close frequencies when propeller interactions are a concern. Additionally, having data from other propellers, even for reference, can be advantageous. Higher resolution analysis in spectral analysis requires more distinct frequency bins over a given range, enabling better discrimination between closely spaced frequency components [47].

4. RESULT ANALYSIS

The real-time simulation has been performed using MATLAB® R2023a with the license number 41077201. The algorithm incorporates the following optimization techniques to enhance performance: memory pre-allocation, vectorization, and usage of built-in functions [57]. PARFOR library has been used to increase the real-time performance of the calculations.

Vibration data is collected using the PCB 33931 model vibration sensor [58]. This sensor is designed to capture vibrations in its surroundings and convert them into electrical signals. To ensure accurate data acquisition, the raw vibration signals are first subjected to signal conditioning. This process is facilitated by Curtiss-Wright hardware, which is specifically configured for various operations such as amplification, filtering, and noise reduction data acquisition to enhance the quality and relevance of the vibration data. The effectiveness of the algorithm hinges on its capacity to optimize computations across multiple CPU cores, resulting in a substantial reduction in processing time for handling large datasets [59]. The real-time CZT simulation has been run on a PC which has 12^{th} Gen Intel Core, i7-12700H and 2.30 GHz.

In a complete evaluation containing all relevant factors, such as the CPU, hardware, and processor, the average time per iteration, which includes acquisition time, is calculated as 0.0315 seconds. This holistic analysis offers a broad perspective on the computational process, accounting for the diverse variables that affect performance. However, a more detailed examination, specifically concentrating on the CZT analysis, shows a remarkable change in efficiency. When isolating this aspect from the overall computation, the time per iteration dramatically reduces to 0.0017 seconds.

Time domain vibration signals and frequency domain CZT magnitude plots have been presented in Fig. 4.1, Fig. 4.2, and Fig. 4.3, showing the results of a real-time simulation executed through the methods detailed in Chapter 3. These graphics are captured by taking screenshots at three randomly chosen points during the simulation.

This approach provides a dynamic look into the evolving behaviour of the system, offering valuable data that aids in the analysis and understanding of the simulated processes. The combination of time domain and CZT magnitude plots allows for a comprehensive examination of the system's response, making it an integral component of the case study in Chapter 3.

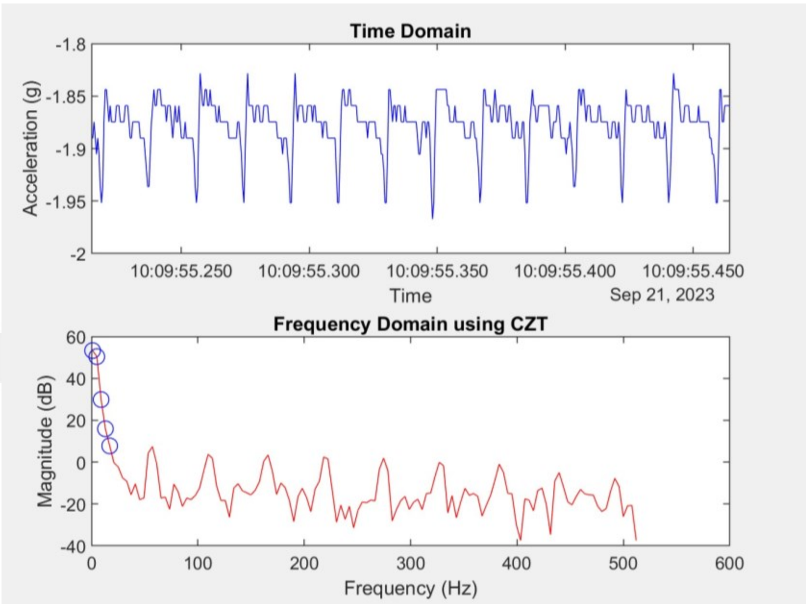

Figure 4.1 : Time domain vibration signals and frequency domain Chirp Z Transform (CZT) magnitude plots.

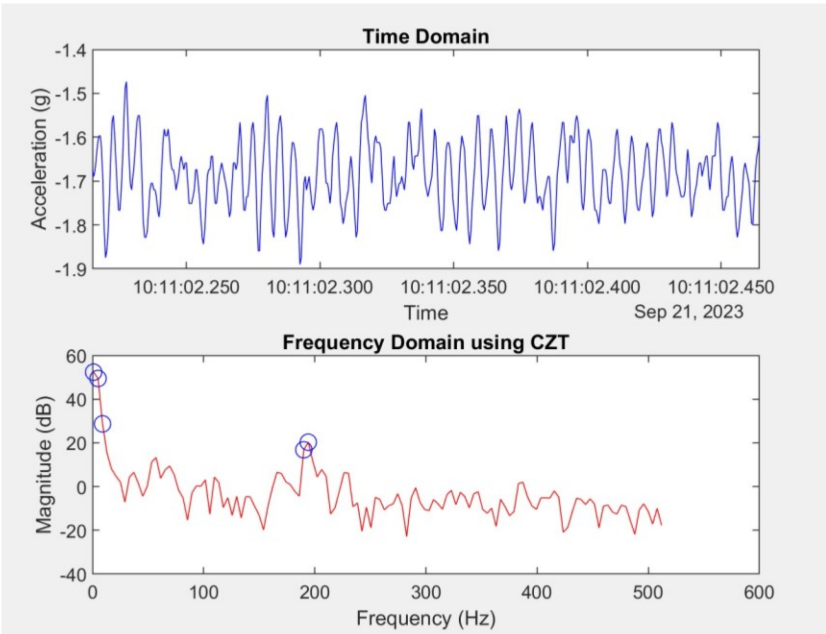

Figure 4.2 : Time domain vibration signals and frequency domain Chirp Z Transform (CZT) magnitude plots.

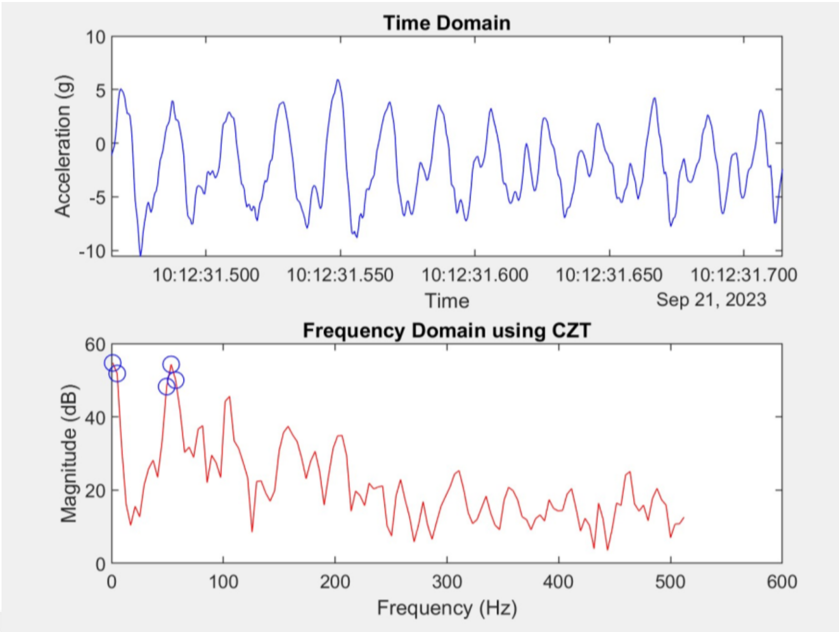

Figure 4.3 : Time domain vibration signals and frequency domain Chirp Z Transform (CZT) magnitude plots.

5. CONCLUSION

Vibration data collected from sensors contains essential information about the mechanical behaviour of structures and avionics, characterized by factors like frequency, amplitude, and phase. Time domain analysis reveals temporal behaviour, while frequency domain analysis through CZT uncovers underlying frequency components. Real-time CZT analysis continuously processes incoming vibration data, finding applications in anomaly detection, pattern identification, and spectral analysis. This research seeks to enhance real-time data analysis, especially where high-frequency data is vital for decision-making. In the context of propulsion systems, vibration analysis is crucial for early issue detection, ensuring safety and efficiency [60].

This paper primarily focuses on optimizing high-frequency data payload using the CZT algorithm, regardless of sensor quality. Given methodology reduces the number of data points by a factor of 4 for each segment. To enhance the CZT algorithm, the Hann windowing technique is incorporated which minimizes spectral leakage, ensures a smooth frequency response, and is easy to implement [61]. In Chapter 3, the CZT algorithm is applied to comprehensively analyse the prototype propulsion system. The process involves determining the desired spectral zoom range through harmonic calculation, leading to the definition of parameters like f_{start} and f_{end}. As outlined in Chapter 2, f_s is calculated using the oversampling method. After determining the f_s, the desired resolution is also calculated. These parameters play a crucial role in assigning CZT-specific coefficients, including M, α, and ω, significantly improving the precision and clarity of the engine harmonics analysis.

When existing similar studies are examined, it has been observed that FFT Algorithms are generally used in many real-time applications such as [12] and [16], and CZT is preferred for post-processing applications [62]. When the review results in Chapter II are evaluated, it is concluded that the best method for the case study given in Chapter III was a real-time CZT Algorithm. Additionally, this algorithm stands out among other CZT-based approaches by strategically utilising pre-calculated exponential factors and a careful buffering scheme, which collectively ensure a computationally efficient execution without compromising the transformative precision. These attributes are particularly advantageous when dealing with high-volume or streaming vibration data in dynamic systems.

In summary, the CZT offers a focused and efficient way to analyse and represent vibration data, which can lead to substantial reductions in the amount of data that needs to be transmitted, especially when specific frequency bands are of interest. This results in a reduced payload on communication channels in DAQ systems.

In a forthcoming study, while real-time CZT analysis offers numerous benefits, it also leads to some challenges, such as its computational complexity for large datasets and optimized algorithms to handle data in real-time. Additionally, sensor quality, data transmission speed, and noise interference can affect the accuracy of results. To manage these challenges effectively, a custom CZT function may be developed. This custom function will handle specific research requirements, potentially enhancing accuracy and computational efficiency in the analysis.

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APPENDICES

APPENDIX A: Pseudocode of the Algorithm

APPENDIX A

start

read the vibration data from the TDMS file

set vibrationData to data{1, 1}.vibrationData
set time to data{1, 1}.Time2048Hz

set fs (sampling frequency) to 2048 Hz
set windowSize to 512
set overlap to 256
set segmentSize to 512
set f_start (start frequency) to 1
set f_end (end frequncy) to 512

create window = hann(windowSize)

calculate desired_resolution = fs / windowSize
calculate M = (f_end - f_start) / desired_resolution + 1
calculate a = exp(-1j * 2 * pi * f_start / fs)
calculate w = exp(-1j * 2 * pi * (f_end - f_start) / (M * fs))

compute iterations = floor(length(fullVibrationData) / segmentSize) - 1

for iteration from 1 to iterations
start a timer (tic)

calculate startIndex = (iteration - 1) * segmentSize + 1
calculate endIndex = iteration * segmentSize

calculatevibrationData=fullVibrationData(startIndex:endIndex)
calculate timeSegment = fullTime(startIndex:endIndex)

reshape the vibration data into overlapping segments using the 'buffer' function
compute the CZT of the reshaped data with the Hann window, M, w, and a
store the results, time segment, and original vibration data in a data structure
pause for 0.021 seconds (simulated processing time)
record and store the elapsed time for this iteration

end of loop

create a frequency vector 'frequencies' from f_start to f_end with M points
initialize an array 'top5Handles' to store handles for the top 5 plotted points
create a figure with two subplots for time and frequency domains

for iteration from 1 to iterations

update the time domain plot with current time and vibration data
calculate the magnitude of the CZT in decibels
if 'top5Handles' is not empty, clear the previous top 5 points
find the top 5 indices with the highest magnitude
extract the corresponding frequencies and magnitudes
plot the top 5 points on the frequency domain subplot
pause for 0.25 seconds for visualization

end of loop

calculate the average processing time per segment
display the average processing time

end

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