FOURIER SERIES COMBINATORY EXPLANATION

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DECEMBER 2023

ÇANKAYA UNIVERSITY

GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS
M.Sc. Thesis in
MATHEMATICS

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ARALIK 2023

ABSTRACT

FOURIER SERIES COMBINATORY EXPLANATION

KARADENİZ, SELEN
M.Sc. in Mathematics

Supervisor: Assist. Prof. Dr. Erkan Murat Türkan
December 2023, 86 pages

The first topics we covered in this study were the causes of the earthquake and how to stay safe in the event of one. The study of surface and shallow water waves, p-waves, s-waves, Rayleigh waves, love waves, and other wave types was conducted at the epicenters of the earthquake. The findings of this research allowed for a comparison of these wave styles. The purpose of this thesis is to create mathematical models of earthquake waves using Fourier series and wavelet transform, which have been used to explain certain phenomena as a result of these studies.

Keywords: p-waves, s-waves, Rayleigh waves, wave equation, wavelet transform, Fourier transform, Fourier series

ÖZET

FOURIER SERİSİ BİRLEŞİK AÇIKLAMA

KARADENİZ, SELEN
Matematik Yüksek Lisans

Danışman: Dr. Öğr. Üy. Erkan Murat Türkan
Aralık 2023, 86 sayfa

Bu çalışmada ele aldığımız ilk konular depremin nedenleri ve deprem durumunda nasıl güvende kalınacağıydı. Depremin merkez üslerinde yüzey ve sığ su dalgaları, p dalgaları, s dalgaları, Rayleigh dalgaları, love dalgaları ve diğer dalga türlerinin incelenmesi gerçekleştirildi. Bu araştırmanın bulguları bu dalga stillerinin karşılaştırılmasına olanak sağladı. Bu tezin amacı, bu çalışmalar sonucunda bazı olguları açıklamada kullanılan Fourier serisi ve dalgacık dönüşümünü kullanarak deprem dalgalarının matematiksel modellerini oluşturmaktır.

Anahtar Kelimeler: p-dalgaları, s-dalgaları, Rayleigh dalgaları, dalga denklemi, dalgacık dönüşümü, Fourier dönüşümü, Fourier serisi

ACKNOWLEDGEMENT

I would like to thank Assist. Prof. Dr. Erkan Murat Türkan for his support, for his supervision, for his continued guidance, for his readiness to help at all time while reading the written work, for his encouragement through the development of this thesis.

I would like to thank all teachers in the Department of Mathematics.

I would like to thank my family and my grandmother for their love and support. I can never forget the things they have done standing beside me for better or worse.

I would like to thank my friend Taylan Demir, Kaan Öztürk, Ege Özmen, Ukte Yüksel,Yetkin Büyükyılmaz,Naz Dündar and Bilgehan Aktepe whom I always found around me in good and bad time.

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LIST OF SYMBOLS AND ABBREVIATIONS

SYMBOLS

∂:	Partial Derivative Operator
$\mathcal{F}$:	Fourier Transform
$\|\psi\|$:	Norm function
$\mathbb{R}$:	Set of Real Numbers
Δ:	Laplacian operator
λ:	Eigenvalues
X_n:	Eigenfunction
$*$:	Convolution
W:	Wavelet transform

CHAPTER I
INTRODUCTION

The wave equation is important to demonstrate the analysis of seismic wave propagation in real-world problems. In particular, it can be connected to the electromagnetic wave equation and simulated using Mathematica or Matlab. Many mathematicians have been working on the seismology and wave equation with simulations for a long time [1-15], and it has been linked to the analysis of seismological data and wave velocity. Additionally, Hajrulla, Sh, Demir, T, Hoxha, F, Hajrulla, D, and Bezati, L worked on the theory of shallow water waves. In their article in [10], they used the matrix method, and in their articles [10-15], they used numerous simulations. Because the intensity and wavelength of water waves depend on the intensity of earthquakes, we can use and connect seismic wave equations with shallow water waves.

Our current goal is to obtain a transformation based on the Fourier series, analyze the signals, their intensity, the earthquakes they cause, and the damages they cause. After doing this analysis, we will draw the necessary conclusions to reduce the earthquake's damage. In earthquake engineering, the use of wavelet transform for the interpretation of time-frequency analysis of seismic records has grown in popularity in recent years. One of the best techniques for examining how the frequency content of the signal under analysis changes over time is the wavelet transform. The computation of the earthquake's maximum and minimum wavelengths was desired in one of the studies. In the process, two distinct points were used as references; one point was identified as close, or high frequency, and the other as low frequency. As a result, wavelength data and graphs were produced for each point. A short-time Fourier transform was used to look at how the frequency changed over time. To do this, the signal under study was split up into different time windows, and the Fourier analysis was computed for each window. It has been feasible to track the frequency changes over time in this way. It has been observed that the wavelet transform makes it possible to analyze the data in both the time and frequency domains.

An approach for analyzing the time-frequency optimum resolution of non-stationary earthquake records is the wavelet transform.

Because they have a specific function, signals that can be measured by physical instruments or observation aid in our ability to draw conclusions. By increasing the diversity of the entered data, all possible outcomes regarding the signals can be achieved in this way.

Numerous industries, including sound engineering, display, and vibration, use Fourier series. Converting a signal from the time domain to the frequency domain is one of its applications, which makes signal understanding easier. In addition, it is employed in the processing of data from medical imaging systems like MRIs and CAT scans and even in earthquake protection systems to prevent building collapse.

Buildings vibrate at their inherent frequencies during an earthquake. If the earthquake's vibrations coincide with the building's natural frequency, the likelihood of the structure being damaged rises along with the vibrations.

In this thesis, we mentioned that earthquake and history of earthquake in chapter 2, types of waves in chapter 3, partial differential equation in chapter 4, Fourier series in chapter 5, Fourier transformation in chapter 6, wavelet transform in chapter 7, explanation of Fourier and wavelet transforms in chapter 8, conclusion in chapter 9 and reference was given after conclusion. Finally, appendix section was shared in final part in my thesis.

CHAPTER II
EARTHQUAKE

2.1 WHAT IS AN EARTHQUAKE?

Seismic waves propagate in all directions and create oscillations and vibrations in periods that may differ in the locations they reach [1]. In addition, they lose some of their energy depending on their specific periods at the time of propagation [1]. For this reason, it is seen that the one that is close to the seismic source is shaken more and the one that is far from the seismic source is less shaken for the same reason (see [1]). The vibrations caused by seismic waves can be at frequencies above the sound threshold level, as well as their effect can be sustained for a short or long time (see [1]). Severe earthquakes are the sudden release of the energy accumulated in the ground layer by the cracking of the ground layer, and as a result, seismic waves can occur (see [1]). There are two basic types of seismic waves [1]. These are body waves and surface waves [1]. If we compare body waves with surface waves, we can say that surface waves are much faster (see [1]). As body waves descend deep, surface waves propagate close to the earth. Body waves are of two types, [1]. These are P-waves and S-waves [1]. P-waves propagate in the sea, atmosphere and land, S-waves propagate in solid parts of the earth, [1]. One difference between P-waves and S-waves is that P-waves are faster. P waves are faster in solid media than S waves [1]. At the moment of the earthquake, we primarily perceive the P-wave [1]. Expansion, contraction or vice versa effects can be observed when the P wave propagates [1]. In S waves, shaking and rotation movements are observed [1].

Surface waves, on the other hand, are two different types of waves, Love and Rayleigh waves [1]. These waves create long-term and large amplitude movements on the earth [1]. It is slower than the S wave [1]. Rayleigh wave creates sea waves effect on earth.

Multidirectional shaking can be observed in the propagation [1]. The Love wave only oscillates horizontally to the propagation direction in the foundations of the structures [1]. Although the long-term ones are large, the damage rate is not so much, but causes the oscillation to be slow in the structures [1]. High-rise buildings are more affected by long-term seismic waves [1].

2.2 HOW DO SEISMIC WAVES FORM?

It is defined as a sudden release of energy in the earth's crust [1]. In other words, it is the release of energy accumulated during a fracture [1]. This released energy spreads in all directions and continues to increase from mild to severe during the spread, and this can be felt [1].

2.3 EARTHQUAKE

The effects of an earthquake on the ground surface are defined as the intensity of the earthquake, [1]. The measure of the severity is made by considering various factors such as the shaking of chandeliers during an earthquake, the movement of furniture, waking up from sleep in an earthquake during the night, and the total amount of damage, [1]. The magnitude, focal depth and distance of this earthquake can be different, even the resistance of the structures against earthquakes, [1]. The intensity of the earthquake is evaluated according to the “Intensity Charts” prepared as a result of the observed effects of the earthquakes and taking into account the experiences of many years, [1]. Each country has its own unique earthquake map, [1]. As a result of the data obtained by examining these maps, earthquake intensities are taken into consideration, [1].

In line with these, it is aimed to create an earthquake regulation and in line with this target and construction methods, iron and concrete to be used in the construction phase are selected according to this regulation [1]. Since earthquake intensities differ from country to country, the intensity maps that is formed as a result of earthquake data also differ [1]. For this reason, each country's earthquake intensity map guides how the country is continue the process in construction [1]. If we explain this with an example; When the earthquake maps of Turkey and Japan are examined, the frequency of earthquakes and the intensity of the earthquakes are seen to be higher in Japan [1]. This is a supportive situation for us that a building constructed in Turkey could be less durable than a building in Japan [1]. The intensity of the earthquake, which corresponds to the range on the Intensity Chart, is classified according to that severity level [1]. In these tables, intensities are shown with roman numerals. For example, the Mercalli Ruler (MM) covers XII severity [1]. Accordingly, it is known that earthquakes of magnitude V and small generally do not cause major damage to structures and are evaluated only according to the way people feel the earthquake [1]. Another severity between VI and XII was evaluated in the light of observable and measurable data such as earthquakes causing damage to structures and breaking and splitting in the field [1]. The most commonly used is Modified Mercalli Intensity Scale [1]. It has no basis that is not related to mathematics and is based on the reality of observed information [1].

2.3.1 Focal Point (Hypocentre)

It is the point where the energy that causes the earthquake to arise from in the earth [1]. In reality, it is a field, not a point where energy emerges, but in practical applications it is considered a point [1].

"Focal Point" or "Hypocentre" describes a specific point of the earthquake below the ground [2-4]. This point is considered to be where the earthquake started and is located at a depth within the Earth's crust [2-4]. This depth affects the distance that seismic waves travel through the Earth's crust and reach the surface [2-4].

The magnitude and effects of an earthquake can often vary depending on the depth of the hypocentre and its distance from the earth's surface [2-4]. Determining the hypocenters of earthquakes is especially important for earthquake scientists and seismologists [2-4]. This information is used to understand the causes of earthquakes, to assess earthquake risks and to identify measures that can be taken to minimize or eliminate the many losses caused by earthquakes [2-4].

2.3.2 External Center (Epicentre)

Epicentre is the area closest to the focal point the earthquake was the strongest and caused the most damage due to this reason [1].

Epicenter refers to the point on the earth's surface where an earthquake is most severe and where the most damage usually occurs [1-4]. It is the point on the earth's surface that is closest to the earthquake's focal point underground [1-4].

The epicenter can be in different places depending on the magnitude and depth of the earthquake [1-4]. Since an earthquake usually starts at a point closer to the earth's surface, a region on the earth's surface that is closer to the focal point will usually be the epicenter [1-4]. The magnitude of the earthquake and the geological characteristics of the location are important factors in determining the damage at the epicenter [1-4].

2.3.3 Focal Depth

The shortest distance from the earth's crust where the accumulated energy is released during an earthquake is called the focal depth [1]. Earthquakes are classified according to their focal depths[1]. This classification is valid for tectonic earthquakes and they are divided into three groups as shallow earthquake, medium earthquake and deep earthquake [1]. If it is 0-60 km deep, it is a shallow earthquake, if 70-300 km is a medium depth earthquake, and finally if it is more than 300 km, it is a deep earthquake [1]. The depth of focus determines how we will feel the effects of the earthquake on the ground. Earthquakes with a focal depth closer to the surface can often cause greater damage because more energy is transmitted to the surface as the earthquake waves propagate from a shallower focal point [2-4]. Earthquakes with a deeper focus can generally be less damaging because less energy reaches the surface because the energy is spread over a wider area [2-4].

Differences and Similarities of Focal Depth and External Center:

In both cases, the focal center of an earthquake is a point in the earth's crust, and the epicenter is a point on the surface caused by the energy radiating from this point [1-4].

2.3.4 Leading Earthquake (Foreshock)

It is a small tremor that occurs a few seconds or a few weeks before an earthquake of greater intensity and occurs near or close to this rupture area [1]. A foreshock is a smaller earthquake that occurs before a large earthquake. These earthquakes are considered to be harbingers of a major earthquake, but it is thought that such earthquakes release the tension in the earth's crust and cause the main earthquake to occur [16].

A foreshock usually occurs in or around the center of the main earthquake and is distinguished from other earthquakes in this region by the fact that it occurs shortly before a major earthquake occurs [16]. However, not all foreshocks always herald a major earthquake [16]. Some foreshocks may only cause small tremors [16].

Understanding the relationship between foreshocks and major earthquakes and predicting such events is an important topic for earthquake researchers [16].

Despite having fundamentally similar characteristics, foreshocks and mainshocks often differ in terms of magnitude, timing, energy and damage potential [1-4]. Foreshocks are usually smaller and less energetic, and can be seen as a precursor to a mainshock [1-4]. Main earthquakes, on the other hand, are usually larger, more intense and widely distributed [1-4].

2.3.5 Tsunami (Sea Waves)

They are giant sea waves that occur as a result of large-scale movement on the ocean floor [1]. Tsunamis are ocean surges that occur as a result of a sudden release of energy in a large body of water [17, 18]. They usually occur as a result of large-scale seafloor changes such as earthquakes, submarine volcanic eruptions, submarine landslides or meteor impacts [17, 18]. Tsunamis are generally high-energy and long-period waves and can spread for hundreds or thousands of kilometers in open seas [17, 18].

The main characteristics of a tsunami are given below:

- Tsunami waves usually travel at very high velocity (hundreds of kilometers per hour) and long wavelengths [17, 18].
- In the open sea, tsunami waves are usually low in height and go unnoticed [17, 18].
- Tsunamis increase in height as they approach the coast and can cause serious destruction in coastal areas [17, 18].
- The waves of tsunamis are not usually confused with other types of waves [17, 18]. The time between tsunami waves is usually longer than other waves [17, 18].

Tsunamis can cause serious disasters in many parts of the world, especially in the Pacific [17, 18].

Tsunamis are large surges caused by large-scale crustal changes, mostly on the seafloor [17, 18]. The causes of tsunamis are listed below [17, 18].

- Large earthquakes caused by the collision or separation of tectonic plates on the seafloor are one of the most common causes of tsunamis [17, 18].

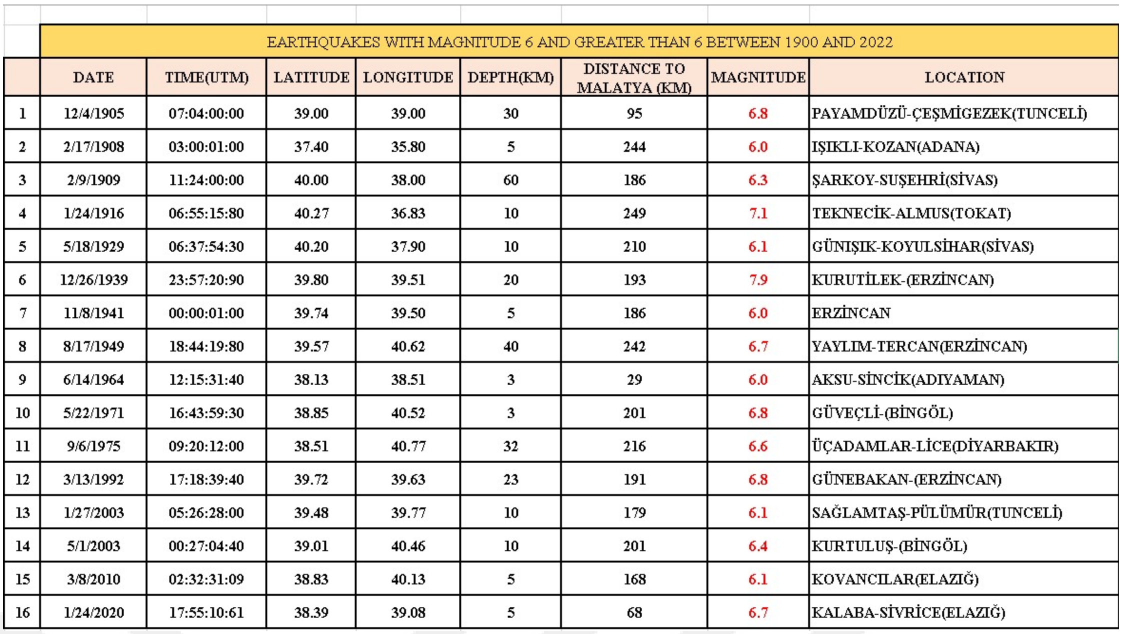
- The eruption of a volcano on the seabed can cause a sudden displacement of seawater and generate tsunamis [17, 18].
- A large landslide on the seabed can cause an unexpected displacement of seawater and generate tsunami waves [17, 18].
- When a large meteorite hits the sea, it can cause seawater to move strongly and cause tsunamis [17, 18].

For the Prevention of Tsunamis;

- Early warning systems that monitor seafloor earthquakes and other potential Tsunami triggers can alert coastal areas to potential hazards and save time accordingly [17, 18].
- Continuous monitoring of sea level is important for detecting sudden changes and warning tsunami-affected areas [17, 18].
- Societies with a high level of awareness of the tsunami hazard can take the right reactions and actions in the event of a disaster, thus minimizing the loss of life and property [17, 18].
- Structures (offices, buildings, residences, living spaces, etc.) in coastal areas should be designed to withstand tsunamis [17, 18].
- Individuals living in coastal areas at risk of tsunami should evacuate to safe areas quickly in case of a disaster [17, 18].

Earthquake in Turkey

Turkey is located in a region with an active tectonic structure and has therefore experienced many earthquakes throughout history. Here are some of the major earthquakes in Turkey and their history:



	EARTHQUAKES WITH MAGNITUDE 6 AND GREATER THAN 6 BETWEEN 1900 AND 2022							
	DATE	TIME(UTM)	LATITUDE	LONGITUDE	DEPTH(KM)	DISTANCE TO MALATYA (KM)	MAGNITUDE	LOCATION
1	12/4/1905	07:04:00:00	39.00	39.00	30	95	6.8	PAYAMDÜZÜ-ÇEŞMİGEZEK(TUNCELİ)
2	2/17/1908	03:00:01:00	37.40	35.80	5	244	6.0	IŞIKLI-KOZAN(ADANA)
3	2/9/1909	11:24:00:00	40.00	38.00	60	186	6.3	ŞARKOY-SUŞEHRİ(SİVAS)
4	1/24/1916	06:55:15:80	40.27	36.83	10	249	7.1	TEKNECİK-ALMUS(TOKAT)
5	5/18/1929	06:37:54:30	40.20	37.90	10	210	6.1	GÜNIŞIK-KOYULSİHAR(SİVAS)
6	12/26/1939	23:57:20:90	39.80	39.51	20	193	7.9	KURUTİLEK-(ERZİNCAN)
7	11/8/1941	00:00:01:00	39.74	39.50	5	186	6.0	ERZİNCAN
8	8/17/1949	18:44:19:80	39.57	40.62	40	242	6.7	YAYLIM-TERCAN(ERZİNCAN)
9	6/14/1964	12:15:31:40	38.13	38.51	3	29	6.0	AKSU-SİNCİK(ADIYAMAN)
10	5/22/1971	16:43:59:30	38.85	40.52	3	201	6.8	GÜVEÇLİ-(BİNGÖL)
11	9/6/1975	09:20:12:00	38.51	40.77	32	216	6.6	ÜÇADAMLAR-LİCE(DİYARBAKIR)
12	3/13/1992	17:18:39:40	39.72	39.63	23	191	6.8	GÜNEBAKAN-(ERZİNCAN)
13	1/27/2003	05:26:28:00	39.48	39.77	10	179	6.1	SAĞLAMTAŞ-PÜLÜMÜR(TUNCELİ)
14	5/1/2003	00:27:04:40	39.01	40.46	10	201	6.4	KURTULUŞ-(BİNGÖL)
15	3/8/2010	02:32:31:09	38.83	40.13	5	168	6.1	KOVANCILAR(ELAZIĞ)
16	1/24/2020	17:55:10:61	38.39	39.08	5	68	6.7	KALABA-SİVRİCE(ELAZIĞ)

Figure 1: Earthquake in Turkey

History of earthquake

NC State professor Ashly Cabas explains how the earthquake that hit Turkey resulted in so much death and destruction [19]. Turkey is known as one of the most active earthquake zones in the world, as it is located in a region where three tectonic plates interact [19]. It is seen that the loss of life and damage in both earthquakes are caused by various factors, both human and natural [19]. In the United States, due to both the fault lines passing over it and its geographical location, large and destructive earthquakes can occur that are severe and cause severe weathering as a result [19]. On Monday, February 6, at 04:17 local time, a 7.8 magnitude earthquake occurred in the south of Turkey, close to the Northern Syria border, and the energy equivalent to 8 million tons of TNT was released [19]. Nine hours later, a 7.5 magnitude earthquake occurred in the same area [19]. The reason why Turkey is one of the most active earthquake zones in the world lies in the Anatolian plate, which was compressed to the west by the collision of two tectonic plates (Arabian and Eurasian) in the northward direction [19]. These plates, which we mentioned above, are compressed from their edges due to friction and pressure (with each other) while they pass each other [19].

As a result, when the pressure is lifted, an earthquake occurs and energy waves are triggered that cause the ground to shake [19]. Ashly Cabas, an assistant professor of construction, environmental engineering and the Center for Geospatial Analytics, said the sheer strength of recent earthquakes provides "the perfect recipe for strong ground motions to happen [19]." Cabas added that earthquakes in Turkey are produced by tectonic environments that can also be found in the United States, particularly in California, where the San Andreas fault forms the boundary between the Pacific Plate and the North American Plate. Another example; there were several earthquakes have happened in the world [19]. Especially, earthquakes have damaged nature and peoples' life for a long time [19]. Together with there were many people were damaged from Gölcük earthquake and so many scientists researched about earthquakes happened on this area [20]. According to the seismologists, north anatolian fault is the most dangerous to produce high intensity earthquake. Many earthquakes lists were given below [21-23]:

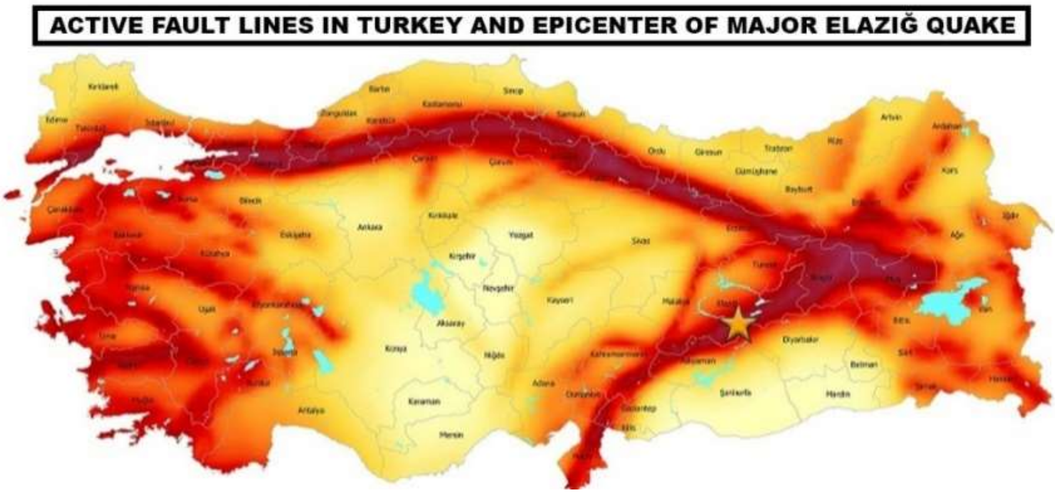


Figure 2: Earthquake Region

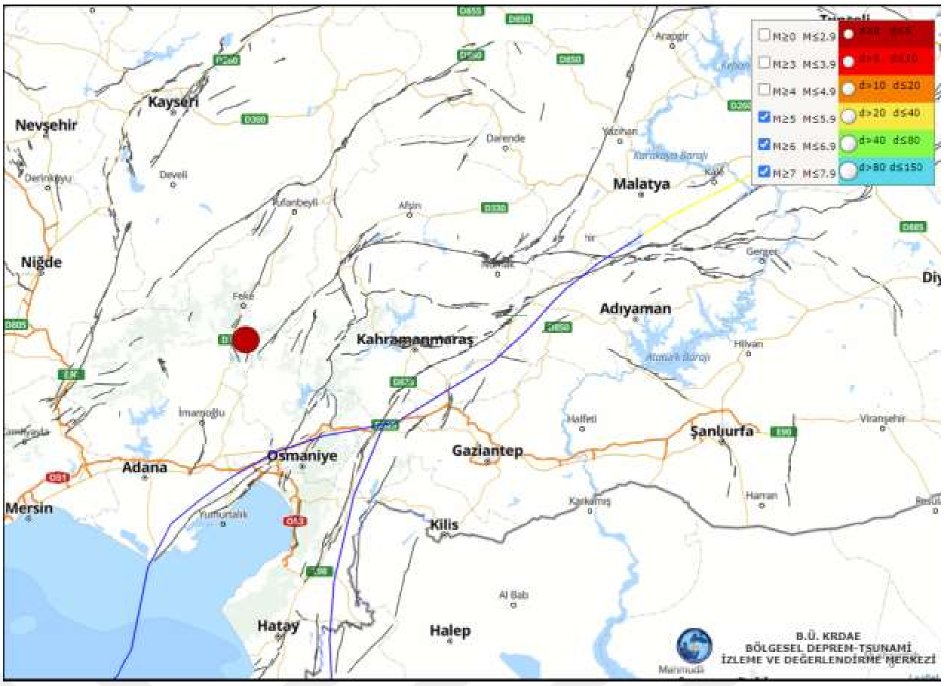


Figure 3: Adana Earthquake

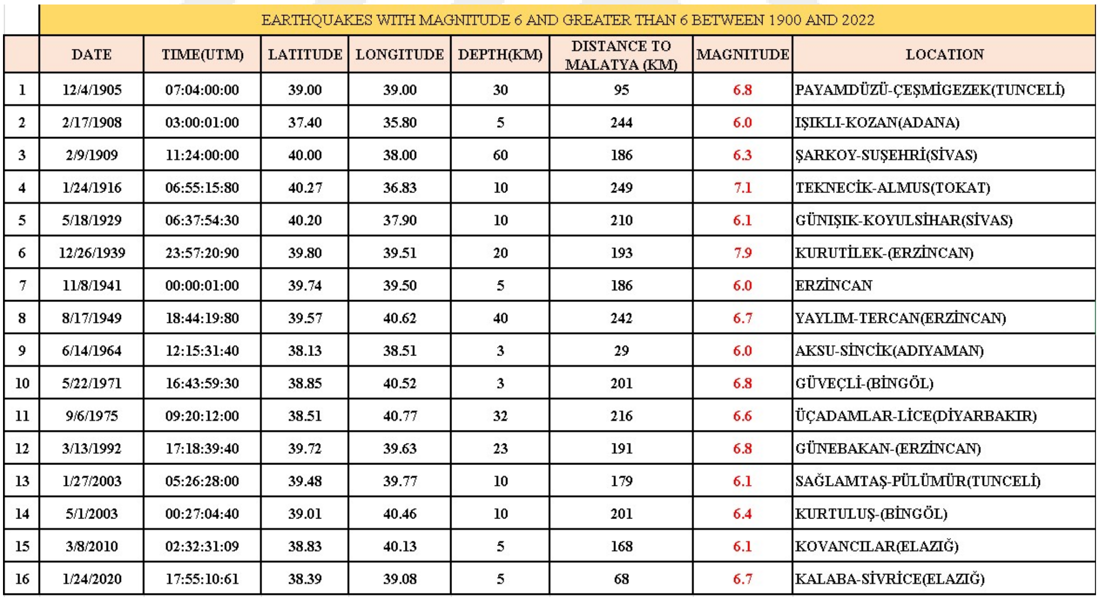


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Figure 4: Data of Earthquakes on Turkey

CHAPTER III

TYPES OF WAVES

3.1 EARTHQUAKE TYPES

Earthquakes can be of various types according to their causes [1, 2]. Most of the earthquakes in the world can occur as described in the previous sections, but there are rare types of earthquakes that occur due to other natural causes [1, 2]. Earthquakes as a result of the movement of the plates described above are generally described as tectonic earthquakes, and these earthquakes mostly occur at plate boundaries [1, 2]. 90% of earthquakes on earth fall into this group [1, 2]. Earthquakes in Turkey are mostly tectonic earthquakes. The second type of earthquakes are volcanic earthquakes [1, 2]. These earthquakes occur as a result of the eruption of volcanoes [1, 2]. It is known that such earthquakes happen as a result of the explosions made by the gases formed as a result of the physical and chemical events during the emergence of the substance from the solid state to the liquid state with the effect of heat in the depths of the earth [1, 2]. For that reason there are no active volcanoes in Turkey, such earthquakes are not seen [1, 2]. Another type of earthquake is collapse earthquakes [1, 2]. The cavities are formed by the collapse of the ceiling block as a result of the collapse of the underground caves, galleries in coal mines, in areas where water-soluble materials such as salt are abundant, as a result of the earthquake [1, 2]. Earthquake is a natural phenomenon. It is not possible to completely eliminate this natural phenomenon, but it may be possible to reduce the damage that may occur with some measures that can be taken [1, 2]. We can list some of these methods as follows [1, 2].

1) Residential areas should be chosen appropriately as the ground [1, 2]. In this way, it will be possible to create solid houses by avoiding slippery ground and loose soil [1, 2].

2) The buildings constructed may differ structurally and functionally according to the need [1, 2]. For example; As a result of the calculations made in the housing project design, the strengths of the materials to be used may be higher than the

strengths of the materials to be used in a single-storey office design [1, 2]. In daily life, some of us spend more time at home, some at work, and some in places such as parks and gardens [1, 2]. Regardless of time and place, the earthquake factor must be taken into account in the construction of every building [1, 2]. For that reason, the thing we should not forget is that the time and intensity of the earthquake are not known and that we can encounter an earthquake at any time, no matter where we are [1, 2]. For this reason, considering the construction regulations, in order to ensure the safety of the building, regardless of what it is, it should be made resistant to earthquake effects [1, 2].

3) It is necessary to stick to the areas reserved for residences in the zoning plan [1, 2].

4) Avoid slopes where there is a large amount of avalanche and snowfall [1, 2].

5) We should increase the security of the building in the housing purchase or construction processes [1, 2]. In addition, we should take care to fix the existing items in the area we are in and place them more robustly against earthquakes [1, 2].

6) We must fix toxic, flammable materials in a position where they will not fall and make sure that they do not break (see [1, 2]).

7) In case of emergency, there should be no dangerous elements in the corridors for exiting from the buildings [1, 2].

8) By preparing an earthquake plan, the drill of how we should act in an emergency should be applied repeatedly at certain periods [1, 2].

9) We should have conversations and exercises with family members about how to protect ourselves during an earthquake [1, 2].

10) Considering that the earthquake may occur at night, we should place our bed away from the window and places where things can spill [1, 2]. If we are inside the building at the time of the earthquake, we need to take the following steps. [1, 2].

1- Not to panic is our priority and most important item [1, 2]. We should stay away from unfixed items and squat or lie down next to items that can provide protection, such as under tables and sofas [1, 2]. We must protect our heads by holding them between our hands. If there is an extra material in the environment we are in, we can protect our heads by getting help from it [1, 2]. We should stay where we are and in the same position until the tremor ends, and we should not try to leave the environment we are in before the tremor ends [1, 2].

2- After the shaking has passed, we should cut off the electricity and turn off the gas and water values [1, 2].

3- Since it is not possible to know where and when the earthquake will occur, we must prepare an emergency kit beforehand. In the event of an earthquake, we should take only the necessary items and materials with the emergency bag and leave the environment in a controlled manner [1, 2].

4- In the event of an earthquake, predetermined emergency assembly areas should be visited as quickly and in a controlled manner as possible [1, 2].

5- We should stay away from stairs, corridors and columns [1, 2].

6- At school, we should stay in the classroom with the teacher and wait for the concussion to pass, by crouching under or next to the desk, to protect our heads [1, 2].

7- We should avoid the use of elevators [1, 2].

Steps to follow if we were outside the building at the time of the earthquake:

1- We should stay away from power lines, other buildings and walls [1, 2]. By crouching in the open field, we must be prepared for the dangers that will come from the environment [1, 2].

2- We should get away from the sea shore [1, 2].

3- We should be in open areas in order to be protected from any material that may fall from the buildings [1, 2].

If we are driving during an earthquake, we must follow these steps:

1-If our location is safe, we should stop and stay in the vehicle [1, 2]. If the vehicle is driving on the highway; We must turn right and stop before blocking the road. But after the shaking stops, we should go to the open areas [1, 2].

2- We should stay away from the traffic, trees, poles and power transmission lines as much as possible [1, 2].

If we are in public transport during the earthquake;

1-There should be no effort to get away from the environment we are in, and the steps shown in earthquake drills should be applied without panicking [1, 2].

2- We must hold on to a hanger or another place in the subway, bus that we are sure is solid [1, 2]. We must listen to the direction of the vehicle personnel we are in and follow the instructions exactly [1, 2].

Precautions that would be beneficial to apply after an earthquake

1- According to the directive of institutions and organizations that carry out search and rescue activities after the earthquake, we should help those in need of help, first aid intervention to injured creatures and debris removal works [1, 2].

2- We should prefer to be in the assembly area or open areas. For any reason, we should avoid entering the building and getting close to the building surroundings [1, 2].

3- We should avoid making phone lines busy unnecessarily [1, 2].

4- Considering that the transport of the sick or injured can be done easily, we should avoid behaviors that hinder traffic [1, 2].

5- Considering the individuals in the developmental period, it is necessary to keep them away from negative feelings and thoughts [1, 2]. For this purpose, it is important to talk with family members about why and why the situation arises, what to do or what not to do in the event of an earthquake [1, 2]. Communication ways should be tried to be created to facilitate the understanding of the individual who is in the developmental period [1, 2].

6- We need to help the officials so that panic and confusion do not occur during the rescue [1, 2].

7- If the building is damaged, we must wait at least an hour to enter [1, 2]. We must act in accordance with the information to be given by the relevant persons [1, 2]. We should avoid acting on our own will and direction [1, 2].

3.2 P-WAVES

P waves are also known as Primary waves or Compression waves. It is a type of seismic wave that is effective in the field of seismology and is accepted as the beginning of ground shaking. These waves are generated as a result of earth tremors and can show propagation movements in the three states of matter in nature: solid, liquid and gaseous phases.

The basic properties of P waves are given below [24-26]: Velocity of p-waves are shown by [24];

$$\upsilon_x = \sqrt{\frac{K + \frac{4}{3}u}{\rho}} = \sqrt{\frac{\lambda + 2\mu}{\rho}} \tag{3.1}$$

and K is mass module in here, μ is the shear modulus, ρ is the density of the material through that the wave propagates and λ is the first Láme parameter [24].

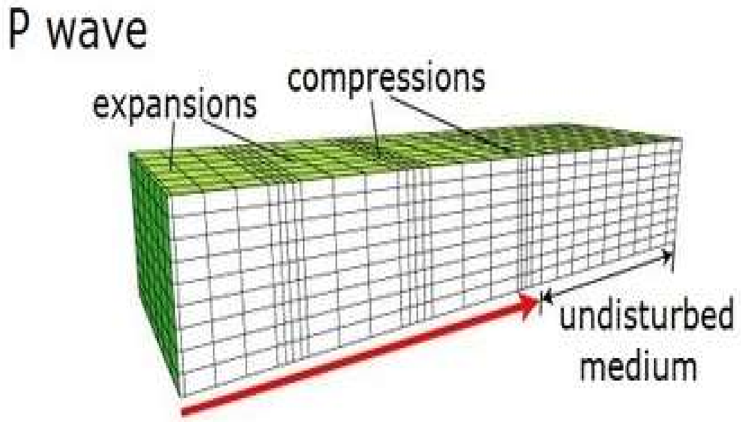

Figure 5: p-wave

Mode of Progress

P-waves make a progressive movement as a result of the propagation of the waves [24-26]. This means that the parts will move forward by showing compression and relaxation kinetics due to wave propagation [24-26].

Average Velocity

Generally, P waves are faster than S waves [24-26]. Their speed is variable [24-26]. Depending on the types of materials in the Earth's crust, their speed can vary, and P waves are usually faster than the speed of sound (343 m/s) [24-26].

Material Types

P waves can propagate in the three states of matter in nature: solid, liquid and gaseous phases [24-26]. However, when they propagate in gaseous media or materials, they usually retain the characteristic of being only compression waves and continue to make the compression motion [24-26].

Direction of Motion

P-waves can compress and expand particles in the direction of propagation [24-26]. Accordingly, as the waves pass through the material, the direction of motion of the particles is in the direction of the motion of the waves [24-26].

P waves are the type of waves that seismic waves warn about first before ground shaking [24-26]. This wave formation is considered an indicator that the ground shaking will not be mild [24-26]. These waves can reach layers deeper than the earth's surface and usually travel at a different speed than S (Secondary or Shear) waves [24-26]. In seismology, it is possible to obtain information about the structure of the Earth's crust by analyzing P waves [24-26].

3.3 S-WAVES

S-waves, also known as seismic waves, refer to the mechanical waves that occur during the propagation of earthquakes and travel in solid media [25, 26]. A mechanical wave is known as an energy-carrying wave type that is released as a result of the interaction of molecular particles in an environment [25, 26]. This wave provides the transmission of energy starting from a point as a result of the interaction of particles in the medium [25, 26]. Mechanical waves are generally known to propagate in an elastic medium [25, 26]. Accordingly, the particles in the environment interact with each other to form the wave [25, 26]. The basic principle in the propagation of these waves is the elastic forces between particles and the transmission of these forces by vibrations in the medium [25, 26]. These waves are often associated with deformation of the Earth's crust or its interior and earthquakes [25, 26].

Deformation is the change or distortion of the shape of a material [25, 26]. Earthquakes are large releases of energy in the Earth's crust and produce mechanical waves, often referred to as seismic waves [25, 26]. During earthquakes, large forces act on the rocks in the Earth's crust [25, 26]. These forces deform the rocks and release energy that propagates in waves into the surrounding environment [25, 26].

In the light of this information, mechanical waves and deformation are often associated with earthquakes because earthquakes cause deformation of the material in the ground layer, as well as the generation of mechanical waves [25, 26].

S-waves can penetrate deeper than surface waves (such as P and Love waves) [25, 26]. However, S-waves can only travel in solid media; therefore, they cannot propagate in liquids [25, 26].

S-waves (also called Shear waves or Secondary waves) are a type of seismic wave system in seismology that occurs as a result of ground shaking and propagates through solid media [25, 26]. Like P-waves, S-waves have important characteristics [25, 26]. There are some factors affecting S-waves [25, 26]. These are; propagation pattern, velocity, material types, direction of motion, damage effect and reaching the ground surface later [25, 26].

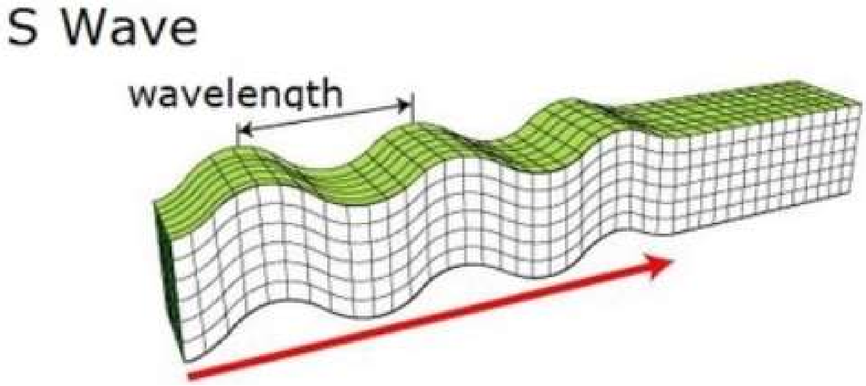

Figure 6: s-wave

Mode of Progress

S-waves make advancing motion as a result of the propagation of the waves horizontally [25, 26]. With this motion, the parts make progress by showing side tilting kinetics thanks to the propagation of the waves [25, 26].

Velocity

The propagation velocity of S waves is half as slow as the propagation velocity of P waves [25, 26].

Material Types

S waves propagate only in solid media and by their nature cannot propagate in liquid and gaseous media [25, 26].

Direction of Motion

Due to their dynamism, S waves move particles only in a horizontal direction [25, 26]. This means that the particles do not move up and down, but only in a horizontal plane [25, 26].

Damage Effect

The waves generated during shaking cause significant damage to the earth's crust [25, 26]. S waves show more displacement effect on the earth crust compared to P waves due to their motion structure [25, 26]. Accordingly, the damage they can cause during shaking is higher than P waves [25, 26].

Reaching the Earth Later

They reach the earth's surface later than P waves, and S waves are used to predict the intensity of an earthquake [25, 26].

S waves are known as seismic secondary waves and can transmit the effects of ground shaking on a deeper and larger scale [25, 26]. In seismology, it is possible to obtain information about the structure of the earth's crust by analyzing S waves [25, 26].

3.4 WHAT IS THE DIFFERENCE BETWEEN P AND S WAVES ?

Differences between p-wave and s-wave

In seismology, P (primary) waves and S (secondary or shear waves) are the two main types of seismic waves produced during the propagation of an earthquake [25, 26]. The main differences between the two types of waves are as follows [25, 26]:

Progression Shape

P-Waves: Longitudinal waves. They make compression and rarefaction movements in the direction of propagation [25, 26].

S-Waves: Transversal (horizontal) waves [25, 26]. They move in the direction of propagation by making horizontal wave movements [25, 26].

Velocity

P-Waves: They travel faster than the speed of sound and propagate faster than S waves [25, 26].

S-Waves: They travel at a slower speed than P waves and have half the speed of P waves [25, 26].

Material Types

P-Waves: They can propagate in solid, liquid and gaseous media [25, 26].

S-Waves: They can propagate only in solid media. They cannot travel in liquids and gases [25, 26].

Direction of Motion

P-Waves: Since they make compression and expansion movements in the direction of propagation, the movement of particles in the material is parallel to the direction of propagation of the waves [25, 26].

S-Waves: Since S waves propagate only in the horizontal direction, the motion of the particles in the material will be perpendicular to the direction of propagation of the wave [25, 26].

Damage Effect

P-Waves: They have a smaller displacement effect than s-waves [25, 26].

S-Waves: They have a larger displacement effect than p-waves and can cause more damage than p-waves during any shaking [25, 26].

Timing

P-Waves: They propagate before the formation of s-waves [25, 26].

S-Waves: They propagate after the p-waves have formed [25, 26]. The differences between p and s waves described above explain the basic characteristics of their behavior [25, 26]. In seismology, the study of both types of waves plays an important role in learning about the characteristics of earthquakes and the structure of the Earth's crust [25, 26].

3.5 RAYLEIGH WAVES

Waves that travel along the earth's surface and produce oscillations in the vertical and horizontal directions [27-29]. They move by rolling as a form of motion, and this is why it is also known as ground rolling in scientific articles [27-29]. They consist of particle motions along an elliptical orbit and are similar to P and S waves because of the motions of their orbits [27-29]. It can even be thought of as a combination of P and S waves [27-29]. The frequencies of the waves change with increasing frequency [27-29]. It starts with slow oscillations first and then makes rapid vibrations to form Rayleigh waves [27-29]. This type of movement starts with the Rayleigh waves being close to the surface and ends with the damping of the wave as it descends to the depths [27-29]. If we deduce from this, earthquake waves reduce their effect as we go down to the depths, and they fade out on their own as we go deeper [27-29].

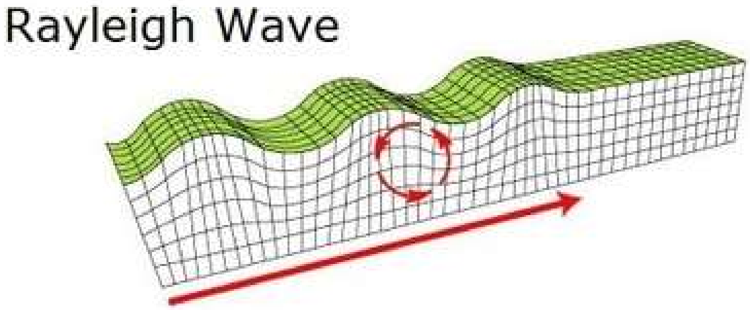

Figure 7: Rayleigh-Wave

Rayleigh waves are surface waves and play an important role in seismology [27-29]. These waves are produced during ground shaking at the surface of the Earth's crust and in deeper layers [27-29].

The main properties of Rayleigh waves are:

Surface Propagation

Rayleigh waves propagate on the surface of the Earth's crust and occur along with other surface waves [27-29].

Rotational Movement

They rotate on the surface and make circular or ellipsoidal movements [27-29]. This rotational movement causes a circular motion on the ground [27-29].

Rayleigh waves are known as seismic waves that exhibit rotational motion, which is the motion in which an object rotates around an axis [27-29]. The basic characteristics of rotational motion in Rayleigh waves are given in the following descriptions:

Horizontal and Vertical Motion

Rayleigh waves have the ability to move in both horizontal and vertical axis. Therefore, the displacement movements in the ground occur both horizontally and vertically and this movement has a rotational character [27-29].

Circular or Ellipsoidal Motion

Particles that make their rotational motion on the ground surface move in a circle or ellipse and this ellipsoidal motion continues as the wave progresses [27-29].

Expressed in Polar Coordinates

"Polar coordinates" refers to a coordinate system in which the position of a point is defined by the distance, r, from a starting point and the angle, θ, between the line segment formed by this starting point and the positive horizontal axis [27-29]. In this system, the position of the point is expressed as (r, θ) [27-29]. Since Rayleigh waves have a rotational motion, the circular or ellipsoidal nature of this wave motion can be expressed in polar coordinates [27-29]. This expression is also used to determine the position of a point rotating around a center of the wave motion at a given instant [27-29].

Relationship between Wave Direction and Direction of Motion

The motion of Rayleigh waves is related to the direction of wave propagation [27-29]. This motion is a circular motion rotating perpendicular to the wave direction [27-29]. Particles on the surface tend to rotate in directions perpendicular to the wave direction [27-29]. It explains that the motion of Rayleigh waves represents a circular motion in relation to the direction of wave propagation [27-29]. Some factors associated with this wave are wave direction, direction of motion, circular motion and rotational nature [27-29].

Wave Direction

Rayleigh waves are seismic waves propagating in elastic media at the ground surface. These waves play a particularly important role in ground vibration analysis [27-29]. Rayleigh waves usually occur in fourth-order elastic media and usually propagate from one surface to another [27-29]. The wave direction affects the wave speed and wave form, depending on the properties of the surface over which these waves travel and the elastic parameters of the medium [27-29]. The propagation speed of Rayleigh waves is generally slower from the surface to the ground [27-29]. This varies depending on the properties of the ground at the surface [27-29]. Wave direction is important in determining this speed difference and the overall behavior of the wave. It also affects how the energy of the wave is dissipated and how it responds to changes in the surface of the ground [27-29].

Direction of Motion

Particles on the surface tend to rotate, but in directions perpendicular to the direction of wave propagation [27-29]. Therefore, it performs a rotational motion diagonal to the direction of wave propagation [27-29].

Circular Motion

In [27-29], particles on the surface can have a circular motion character if they rotate in a direction perpendicular to the direction of propagation of the waves. In short, this motion of the particles can be on a circle or on the path that Rayleigh waves follow during the motion of the particles on the surface (along an ellipsoidal path) [27-29]. An ellipsoidal path refers to a curve in the shape of an ellipse. An ellipse is often thought of as a trajectory shape followed during a rotational motion [27-29]. Rayleigh waves realize the rotational motion of particles rotating in directions perpendicular to the direction of wave propagation by following this ellipsoidal path [27-29].

Rotational Nature

The fact that the motion of Rayleigh waves is perpendicular to the wave direction explains their rotational nature [27-29]. In other words, it shows that the rotational motion, which is one of the main features of Rayleigh waves, is related to the direction of wave propagation [27-29]. In the process of research in the field of seismology and obtaining information about the effects of ground shaking, there are factors of horizontal and vertical motion, high frequency, weak depth factor [27-29].

Horizontal and Vertical Motion

Rayleigh waves have both a horizontal and a vertical motion component [27-29]. As a result of having these two components of motion, they are capable of displacement both on the surface of the ground and in the layers below it [27-29].

High Frequency

What distinguishes them from other seismic waves is that they can propagate at a higher frequency [27-29].

Weak Dependence on Depth

The intensity of Rayleigh waves varies as the depth of the medium in which they propagate increases or decreases [27-29]. As the depth increases, the intensity of the waves weakens while the effects they create on the surface decrease [27-29]. The reason for this is related to the fact that the wave motion loses energy as well as the effect of the wave motion on the internal structure. This causes some effects (Energy dissipation, Resistance and Damping, Diffusion) [27-29]:

Energy Distribution

Rayleigh waves propagate at the surface. As they move from the surface to deeper layers, they can lose their available energy [27-29]. As a result, the wave energy is dissipated and a decrease in their impact on the surface can be observed [27-29].

Resistance and Damping

In [27-29], as the surface depth increases, the material in the ground is subjected to more resistance and damping. The resistance and dumping it is subjected to can dampen the wave motion and reduce its effects [27-29].

Diffusion

Diffusion means the homogeneous spreading or dispersion of a substance in another substance or in an environment [27-29]. The diffusion process takes place from where the density of matter is high to where it is low [27-29]. Wave energy diffusion, on the other hand, means the homogeneous distribution of energy in the environment where wave energy is spread and the spread of energy from one point to the surrounding environment [27-29]. In this process, energy is transferred from one place to another [27-29]. Diffusion of wave energy can be observed in various physical environments [27-29].

For example, if we think in terms of water waves:

Water Waves

When a stone is thrown into the water, its impact on the water surface creates wave energy [27-29]. This energy can spread on the water surface and transfer energy to its surroundings. Thanks to this possible transfer, the spread of energy from one point to other regions on the water surface is realized by diffusion [27-29].

3.6 LOVE WAVES

Love waves is a term used in seismology and earth sciences [27-29]. They are a type of seismic wave known as surface waves [27-29]. Love waves make a rotating motion at the surface of the earth, usually in a horizontal direction, and are often generated as part of seismic effects from large earthquakes [27-29].

The main characteristics of Love waves (horizontal motion, refraction at the ground surface and density-dependent velocity) are given below:

Horizontal Movement

Love waves move horizontally, forming the main component of wave motions [27-29]. The energy of these waves is generated as a result of the rotational motion at the ground surface and in the horizontal direction [27-29].

Propagation at the Earth's surface

Love waves generally propagate at the earth's surface and, together with other surface waves, are a factor in the seismic effects of earthquakes [27-29].

Density Dependent Velocity

The velocity of Love waves can vary depending on the density of the ground [27-29]. If the density of the material is high, they can have higher velocities [27-29]. Soil density, also commonly known as bulk density, is a measurement of the mass per unit volume of a soil. Density is usually calculated in kilograms [27-29].

Soil density is expressed by the following formula:

$$Soil\ Density = \frac{Total\ Mass}{Total\ Volume}$$

Soil Density=(Total Mass)/(Total Volume)

The variables that make up this formula are total mass and total volume.

Total Mass Refers to the total mass contained in the soil.

Total Volume

Refers to the total volume occupied by the ground. Soil density is important in fields such as civil engineering and geology. Soil density is related to many properties of a soil such as bearing capacity, water permeability, soil behavior and earthquake effects. Soil density is usually expressed in two ways:

Total Density

In [27-29], it includes the volume and mass of all the material in the soil. Total density is measured in kilograms per cubic meter (kg/m^3).

Effective Density

In [27-29], it only includes the volume and mass of solid particles in the soil. For this reason, it does not take into account the void volume. Effective density is also measured in kilograms per cubic meter (kg/m^3).

The Ability of Love Waves to Propagate on the Surface:

In [27-29], the waves traveling on the ground surface are Love waves and the wave energy has the ability to propagate on the surface of the ground. This propagation plays a role in the seismic effects that may occur in the ground during an earthquake.

Energy Transmission Ability

Love waves have the ability to carry energy and thus carry energy in circular motions on the ground surface. In addition, it refers to the spread of the energy released during an earthquake on the surface of the ground starting from a point [27-29].

Horizontal Motion

The main characteristic of Love waves is that they are characterized by a horizontal motion. These waves travel in a rotational motion in a horizontal direction [27-29]. Particles on the surface tend to rotate in directions perpendicular to each other. For this reason, it is often possible to feel the effects of large earthquakes at the surface [27-29].

Love waves are often considered together with Rayleigh waves and are used to understand and measure earthquake effects at the ground surface [27-29]. The properties of these waves are studied by seismologists to provide information about the intensity and effects of earthquakes [27-29].

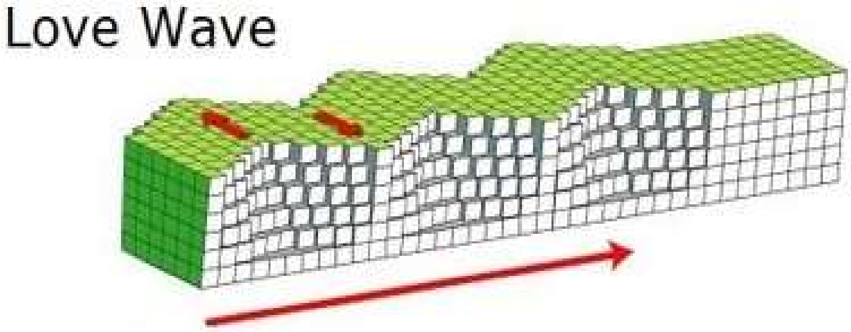

Figure 8: Love Wave

Similarities between Rayleigh Waves and Love Waves:

Rayleigh and love waves, both seismic waves, describe two different types of surface waves that are related to the propagation patterns of earthquakes [27-29]. Their common features are the following:

Surface Propagation

Both types of waves are surface waves and propagate on the surface of the earth layer. It is a result of seismic energy generated in the earth layer by the earthquake [27-29].

Rotational Motion

Both Rayleigh waves and Love waves follow a circular motion at the surface. The difference is that Rayleigh waves include both horizontal and vertical components, while Love waves focus on a horizontal rotational motion [27-29].

Ability to Transmission Energy at the Earth's Surface

Both types of waves (Rayleigh and Love waves) can carry energy on the surface of the ground. Accordingly, they enable the energy generated by the earthquake to spread and dissipate on the surface [27-29].

Velocity Depending on Ground Properties

Both types of waves (Rayleigh and love waves) propagate at different speeds due to the effect of the properties of the ground. The elastic properties of the ground are a factor affecting the wave speed [27-29].

3.7 SURFACE WAVES

Surface waves are the general name for seismic waves generated in the field of seismology (earthquake science) and propagating on the earth's surface [30, 31].

Surface waves are generated as a result of vibrational motions that occur during earthquakes and they continue to move on the earth's surface [30, 31].

Formation of surface:

The formation of surface waves occurs when the seismic energy generated at the source point of the earthquake propagates in the form of waves to the Earth's surface. The released energy propagates on the ground around the origin of the earthquake, which occurs in the inner layers of the Earth's surface [30, 31].

Surface waves are analyzed under two headings. These are Rayleigh waves and Love waves [30, 31].

3.8 SHALLOW WATER WAVES

This type of wave occurs when the water depth is limited or shallow. They are most common in shallow waters such as ocean coasts, bays, lakes or river deltas. Shallow water affects the movement of waves and as a result determines the characteristics of the wave type [17, 32]. Shallow water waves are a type of wave that occurs in an environment where the water depth determines the behavior of the waves [17, 32].

The shallow water wave is related with wave theory and velocity function f and the surface displacement function g must be found to show this theory (see [10]);

$$\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial z^2}\right) = 0 \tag{3.2}$$

and from (3.2) then the free surface boundary conditions introduced, in below part; (see [10]);

$$\frac{\partial f}{\partial z} = \frac{\partial g}{\partial t} + \frac{\partial f}{\partial x}\frac{\partial g}{\partial x}, \quad f = g \tag{3.3}$$

Shallow water wave equation is given by

$$\frac{\partial f}{\partial t} + \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2\right], \tag{3.4}$$

Wave Velocity and Length

In shallow water, waves generally move more slowly and can have shorter wave lengths. This is a feature where water depth affects the speed at which waves propagate and their characteristic length [10-32].

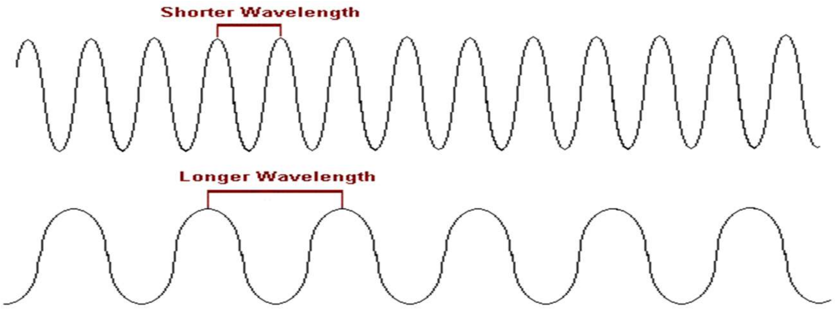

Figure 9: Wave velocity and wavelength

Wave Shape

Waves formed by the impact of shallow water have a symmetrical shape. This is due to the fact that the waves move perpendicular to the water depth and the height of the wave crests increases as the water depth decreases [10-32].

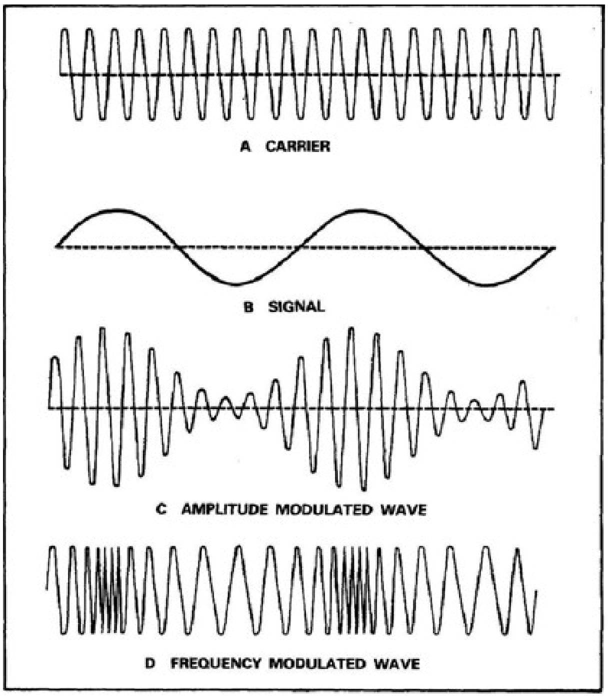

Figure 10: Frequency of wave

Movement of Water Particles at the Surface

Shallow water waves cause particles at the water surface to make a generally circular motion. This circular motion is caused by the reduction of wave energy as it rotates around the water depth in shallow water [10-32].

Shore Impacts:

Waves in shallow water can often cause significant impacts in coastal areas. When waves hit the shore, they can cause coastal erosion, water level fluctuations and other coastal impacts [10-32].

CHAPTER IV

PARTIAL DIFFERENTIAL EQUATION

An equation involving an unknown function of two or more variables and some of its partial derivatives is known as a partial differential equation (PDE).[33]

Definition: A form expression [33];

$$F(D^k u(x), D^{k-1}u(x), \dots, Du(x), u(x), x) = 0, \quad (x \in U) \qquad (4.1)$$

is referred to as a partial differential equation of order k, where,

$$\mathrm{F}: \mathbb{R}^{n^k} \times \mathbb{R}^{n^{k-1}} \times \dots \times \mathbb{R}^n \times \mathbb{R} \times U \to \mathbb{R}$$

is given and

$$u: U \subset \mathbb{R}^n \to \mathbb{R}$$

is the unidentified [33].

Definition**:** If the partial differential equation (4.1) has the following form, it is referred to as linear;

$$\sum_{|\alpha| \le k} a_\alpha(x) D^\alpha u = f(x)$$

For the specified function $a_\alpha \ (|\alpha| \le k), f$.

If $f \equiv 0$, then this lienar PDE is homogenous.

If the PDE (4.1) has the form

$$\sum_{|\alpha| = k} a_\alpha(x) D^\alpha u + a_0(D^{k-1}u, \dots, Du, u, x) = 0$$

then it is semilinear.

If the PDE (4.1) has this form

$$\sum_{|\alpha| = k} a_\alpha(D^{k-1}u, \dots, Du, u, x) D^\alpha u + a_0(D^{k-1}u, \dots, Du, u, x) = 0$$

it is quasilinear.

The PDE (4.1) is fully nonlinear if it is not dependent on the highest order derivatives.

Let k be a positive integer. The form's expressiveness

$$F(D^k u(x), D^{k-1}u(x), \dots, Du(x), u(x), x) = 0, \quad (x \in u)$$

is called a k^{th} order system of partial differential equations

$$\text{F}: \mathbb{R}^{mn^k} \times \mathbb{R}^{mn^{k-1}} \times \dots \times \mathbb{R}^{mn} \times \mathbb{R}^m \times u \to \mathbb{R}^m$$

is given and

$$u: U \to \mathbb{R}^m, \qquad u = (u^1, \dots, u^m)$$

is the unknown.

Notation: "PDE" is an abbreviation for both the singular "partial differential equation" and "partial differential equations".

Examples: There is no general theory that can be applied to all partial differential equations. Given the wide range of physical, geometric, and probabilistic phenomena that PDE can model, such a theory is extremely unlikely to exist. Instead, research focuses on a variety of specific partial differential equations that are important for applications both within and outside of mathematics, with the hope that insight into the origins of these PDEs will lead to solutions [33].

4.1.1 Single Partial Differential Equations

a) Linear Equations

1) Laplace's Equation

Laplace's equation, so named in honor of Pierre-Simon Laplace, the man who initially examined its characteristics, is a second-order partial differential equation in mathematics and physics.

This is frequently written as,

$$\Delta u = \nabla^2 u = \sum_{i=1}^{n} u_{x_i x_i} = 0$$

where $\Delta = \nabla . \nabla = \nabla^2$ is named by Laplace operator, or the divergence operator (also symbolized with “div”), ∇ is the gradient operator (also it can be shown as “ grad”), and $u(x, y, z)$ is a twice-differentiable real valued function.

-In rectangular coordinates,

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0.$$

-In cylindrical coordinates,

$$\nabla^2 u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 u}{\partial \phi^2}\right) + \frac{\partial^2 u}{\partial z^2} = 0.$$

-In spherical coordinates, using the (r, θ, φ) convention

$$\nabla^2 u = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial u}{\partial \theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2 u}{\partial \varphi^2} = 0.$$

2) Helmholtz’s (or eiegenvalue) Equation

$$-\Delta u = \lambda u$$

or

the above expression can be called by,

$$\nabla^2 u = -k^2 u$$

where ∇^2 is the Laplace operator, k^2 is the eigenvalue, and u is the eigenfunction. The Helmholtz equation is the Laplace operator's eigenvalue problem in mathematics. The linear partial differential equation is what it relates to. The equation as named by Hermann von Helmholtz.

3) Linear Transport Equation

$$u_t + \sum_{i=1}^{n} b^i u_{x_i} = 0$$

In [34], the linear transport equation are depending on time-variable and we can show introduce detail as,

$$u_t + bu_{x_1} + b^2u_{x_2+}b^3u_{x_3} + \cdots + b^nu_{x_n} = 0$$

4) Heat (or diffusion) Equation

$$u_t - \Delta u = 0.$$

The heat equation is a specific type of partial differential equation found in mathematics and science. Calorie functions are another name for solutions to the heat equation. Joseph Fourier created the theory of the heat equation in 1822 in order to simulate the diffusion of a quantity, such as heat, through a certain area. The heat equation, the archetypal parabolic partial differential equation, is one of the most extensively researched subjects in pure mathematics, and the study of it is seen to be essential to the study of partial differential equations in general.

5) Schröndinger's Equation

$$iu_t + \Delta u = 0.$$

A quantum-mechanical system's wave function is determined by the linear partial differential equation known as the Schrödinger equation. An important turning point in the evolution of quantum mechanics was reached with its discovery. It bears Erwin Schrödinger's name, who proposed the equation in 1925 and published it in 1926, providing the foundation for the research that led to his 1933 Nobel Prize in Physics.

6) Wave Equation

$$u_{tt} - \Delta u = 0$$

A second-order linear partial differential equation known as the "wave equation" is used to describe various types of waves or standing wave fields, including electromagnetic (such as light waves) and mechanical (such as sound, water, and seismic waves). It appears in domains such as fluid dynamics, electromagnetism, and acoustics.

7) Klein-Gordon Equation

$$u_{tt} - \Delta u + m^2u = 0$$

A relativistic wave equation connected to the Schrödinger equation is the Klein-Gordon equation, often known as the Klein–Fock–Gordon equation or occasionally the Klein–Gordon–Fock equation. It is clearly Lorentz-covariant and second-order in both space and time.

b) Nonlinear Equations

1) Nonlinear Poisson Equation

$$-\Delta u = f(u)$$

A widely used elliptic partial differential equation in theoretical physics is Poisson's equation. An electrostatic or gravitational (force) field can be calculated using the potential field that results from a given electric charge or mass density distribution, for instance, which is the solution to Poisson's equation. It is a physics-related version of Laplace's equation, which is also widely used. The French mathematician and scientist Siméon Denis Poisson is honored by the equation's name. In here, Δ is the Laplace operator, and u and f are real or complex-valued functions on manifold.

2) Monge-Ampere Equation

$$det(D^2u) = f$$

A nonlinear second-order partial differential equation of a particular form is known as a (real) Monge–Ampère equation in mathematics. If a second-order equation is linear in both the second-order partial derivatives and the determinant of the Hessian matrix of the unknown function u of two variables, x and y, it is of the Monge–Ampère type. Over a specific domain $\mathbb{D}$ of $\mathbb{R}^2$, the independent variables (x, y) fluctuate.

3) Hamilton-Jacobi Equation

$$u_t + H(Du, x) = 0$$

The Hamilton-Jacobi equation is a different way to express classical mechanics in physics. It is named after William Rowan Hamilton and Carl Gustav Jacob Jacobi and is comparable to other expressions like Newton's laws of motion, Lagrangian mechanics, and Hamiltonian mechanics.

4) Inviscid Burgers’ Equation

Burgers' equation, also known as the Bateman–Burgers equation, is a basic partial differential equation and convection–diffusion equation that appears in a number of applied mathematics domains, including traffic flow, nonlinear acoustics, fluid mechanics, and gas dynamics. Harry Bateman initially presented the equation in 1915 and subsequently investigated in 1948 by Johannes Martinus Burgers. For a given field $u(x,t)$ and diffusion coefficient (or kinematic viscosity, as in the original fluid mechanical context) v, The dissipative system is the generic form of Burgers' equation, commonly referred to as viscous Burgers' equation, in one space dimension:

$$u_t + uu_x = vu_{xx}$$

When the diffusion term is absent $(v = 0)$ Burgers' equation becomes the inviscid Burgers' equation:

$$u_t + uu_x = 0.$$

That is a prototype as conservation equations which, could improve discontinuities (shock waves).

Method of Seperation of Variables

A traditional strategy for resolving various kinds of partial differential equations is the method of separation of variables. We come up with a fix, say $u_n(x,t)$ to a partial differential equation as being a linear combination of simple component functions $u_n(x,t)$, $n = 0,1,2,\ldots$, which also satisfy some boundary conditions in the equation. To identify a constituent solution, $u_n(x,t)$, we assume it can be written with its variables separated; that is as, [33]

$$u_n(x,t) = x_n(x)T_n(t)$$

Substituting this form for a solution into the partial differential equation and using the boundary conditions leads, in many circumstances, two ordinary differential equations for the unknown functions $x_n(x)$ and $T_n(t)$ [33].

By doing this, we have simplified the issue of solving a partial differential equation to the easier issue of solving a single-variable differential equation. We will demonstrate this method for the wave equation and the heat equation in this section.[34].

The following initial-boundary value issue was solved in the preceding section as a mathematical model for the sourceless heat flow in a uniform wire with ends maintained at constant zero temperature:

1) $\frac{\partial u(x,t)}{\partial t} = \beta \frac{\partial^2 u(x,t)}{\partial x^2}, \quad 0 < x < 1, \quad t > 0$ (4.2)

2) $u(0,t) = u(L,t) = 0, \quad t > 0$ (4.3)

3) $u(x,0) = f(x), \quad 0 < x < L$ (4.4)

To solve this problem by the method of separation of variables, we begin by addressing equation (4.2). We propose that it has solutions of the form;

$$u(x,t) = X(x)T(t)$$

Assuming that T is a function of t alone and X is a function of x alone. We compute U's partial derivatives first to get the following values, which help us determine X and T:

$$\frac{\partial u}{\partial t} = X(x)T'(t) \text{ and } \frac{\partial^2 u}{\partial x^2} = X''(x)T(t)$$

These expressions can be substituted into (4.2) to obtain,

$$X(x)T'(t) = \beta X''(x)T(t)$$

And seperating variables yields,

$$\frac{T'(t)}{\beta T(t)} = \frac{X''(x)}{X(x)} \tag{4.5}$$

It is evident that the functions on the left side of equation (4.5) rely solely on t, whereas the functions on the right side depend solely on X. The ratio on the right must remain constant if we fix t and modify x [34].

$$\frac{X''(x)}{X(x)} = -\lambda, \qquad \frac{T'(t)}{\beta T(t)} = -\lambda$$

or $$X''(x) = -\lambda X(x), \quad T'(t) = -\lambda\, \beta T(t) \tag{4.6}$$

Therefore, we have simplified the issue of solving the partial differential equation (4.3) to solving the two ordinary differential equations in (4.6) for separable solutions. We then have a look at the boundary conditions in (4.3). Since $u(x,t) = X(x)T(t)$ these conditions are $X(0)T(t) = 0,\ X(L)T(t) = 0, \quad t > 0$.

Hence, either $T(t) = 0$ for all $t > 0$ which implies that $u(x,t) = 0$ or

$$X(0) = X(L) = 0. \tag{4.7}$$

(6) Ignoring the trivial solution $u(x,t) = 0$, we get the boundary value problem by combining the differential equation for X in (4.6) with the boundary conditions in (4.7).

$$X''(x) + \lambda X(x) = 0, \qquad X(0) = X(L) = 0 \tag{4.8}$$

where λ can be any constant.

Now, this expression could be solved by the method of constant coefficient in (4.7), we try $X(x) = e^{rx}$, derive the auxiliary equation $r^2 + \lambda = 0$, and consider three cases.

Case 1: $\lambda < 0$, since the auxiliary equation's roots in this instance are $\pm\sqrt{-\lambda}$, the differential equation in (4.7) has a general solution of

$$X(x) = C_1 e^{\sqrt{-\lambda}x} + C_2 e^{-\sqrt{-\lambda}x}$$

In order to ascertain C_1 and C_2, we utilize the boundaries.

$$X(0) = C_1 + C_2$$

$$X(L) = C_1 e^{\sqrt{-\lambda}L} + C_2 e^{-\sqrt{-\lambda}L} = 0.$$

Using the initial equation, we observe that $C_2 = -C_1$. Then, $C_1\left(e^{\sqrt{-\lambda}L} - e^{-\sqrt{-\lambda}L}\right) = 0$ or $C_1\left(2e^{\sqrt{-\lambda}L} - 1\right) = 0$. Since $-\lambda > 0$, it follows that $\left(2e^{\sqrt{-\lambda}L} - 1\right) < 0$.

Case 2: $\lambda = 0$, the differential equation can be solved generally as $X(x) = C_1 + C_2x,$ where r is a repeated root of the auxiliary equation. The boundary conditions in (4.8) obtain $C_1 = 0$ and $C_1 + C_2L = 0$, that satisfy that $C_1 = C_2 = 0$. Therefore, (4.8) has no nontrivial solution when $\lambda = 0$.

Case 3: $\lambda > 0$, The auxiliary equations' roots in this instance are $\pm i\sqrt{\lambda}$. Therefore, a general solution to $X''(x) + \lambda X(x) = 0$ is $X(x) = c_1 cos\sqrt{\lambda}x + c_2 sin\sqrt{\lambda}x$. The boundary conditions are $X(x) = X(L) = 0$ this time provide the system with $c_1 = 0$ and $c_1 cos\sqrt{\lambda}L + c_2 sin\sqrt{\lambda}L = 0$.

CHAPTER V
FOURIER SERIES

Definition: An expansion of a periodic function into the sum of trigonometric functions is known as a Fourier series. While not all trigonometric series are Fourier series, the Fourier series is an example of a trigonometric series [35]. Also, let f be a piecewise continuous function on the interval $[-L, L]$. The Fourier series of f is the trigonometric series:

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n cos \frac{n\pi x}{L} + b_n sin \frac{n\pi x}{L} \right\}$$

where the a_n's and b_n's are defined by;

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) cos \frac{n\pi x}{L} dx, \qquad n = 0,1,2,3, \dots$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) sin \frac{n\pi x}{L} dx, \quad n = 0,1,2,3, \dots$$

Theorem 1: When the function f is an even piecewise continuous function on $[-a, a]$, then

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$$

When the function f is an odd piecewise continuous function on $[-a, a]$, then

$$\int_{-a}^{a} f(x)dx = 0$$

Proof: When f is an even function, then $f(-x) = f(x)$. Hence,

$$\int_{-a}^{a} f(x)dx = \int_{-a}^{0} f(x)dx + \int_{0}^{a} f(x)dx = \int_{0}^{a} f(x)dx + \int_{0}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$$

and also if f is an odd function, then $f(-x) = -f(x)$. So,

$$\int_{-a}^{a} f(x)dx = 0$$

Simulation

Example 2: Compute the Fourier series for

$$f(x) = \begin{cases} 0, & -\pi < x < 0, \\ x, & 0 < x < \pi. \end{cases}$$

Solution 2: Here $L = \pi$, then, we have

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) cosnxdx = \frac{1}{\pi} \int_{0}^{\pi} xcosnxdx$$

$$= \frac{1}{\pi n^2} \int_{0}^{\pi n} ucosudu = \frac{1}{\pi n^2} [cosu + usinu]|_0^{\pi n}$$

$$= \frac{1}{\pi n^2} (cosn\pi - 1) = \frac{1}{\pi n^2} [(-1)^n - 1], \quad n = 1,2,3, \dots$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)dx = \frac{1}{\pi} \int_{0}^{\pi} xdx = \frac{x^2}{2\pi} |_0^{\pi} = \frac{\pi}{2},$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) sinnxdx = \frac{1}{\pi} \int_{0}^{\pi} xsinnxdx$$

$$= \frac{1}{\pi n^2} \int_{0}^{\pi n} usinudu = \frac{1}{\pi n^2} [sinu - ucosu] |_0^{\pi n}$$

$$= \frac{-cosn\pi}{n} = \frac{(-1)^{n+1}}{n}, \quad n = 1,2,3, \dots$$

Therefore,

$$f(x) \sim \frac{\pi}{4} + \sum_{n=1}^{\infty} \left\{ \frac{1}{\pi n^2} [(-1)^n - 1] cosnx + \frac{(-1)^{n+1}}{n} sinnx \right\}$$

$$= \frac{\pi}{4} - \frac{2}{\pi} \left\{ cosx + \frac{1}{9} cos3x + \frac{1}{25} cos5x + \cdots \right\} + \left\{ sinx - \frac{1}{2} sin2x + \frac{1}{3} sin3x + \cdots \right\}$$

Python code of Example 2:

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import quad

# Define the function f(x)
def f(x):
    if -np.pi < x < 0:
        return 0
    elif 0 < x < np.pi:
        return x
    else:
        return 0  # Outside the specified range

# Define the period and the number of terms in the series
T = 2 * np.pi
num_terms = 10

# Compute the Fourier series coefficients for the first part (0 for -pi < x < 0)
a0_first_part, _ = quad(lambda x: f(x), -np.pi, 0)
an_first_part = np.zeros(num_terms)
bn_first_part = np.zeros(num_terms)

# Compute the Fourier series coefficients for the second part (x for 0 < x < pi)
a0_second_part, _ = quad(lambda x: f(x), 0, np.pi)
an_second_part = np.zeros(num_terms)
bn_second_part = np.zeros(num_terms)

for n in range(1, num_terms + 1):
    an_first_part[n-1], _ = quad(lambda x: f(x) * np.cos(2 * np.pi * n * x / T), -np.pi, 0)
    bn_first_part[n-1], _ = quad(lambda x: f(x) * np.sin(2 * np.pi * n * x / T), -np.pi, 0)

    an_second_part[n-1], _ = quad(lambda x: f(x) * np.cos(2 * np.pi * n * x / T), 0, np.pi)
```

```
    bn_second_part[n-1], _ = quad(lambda x: f(x) * np.sin(2 * np.pi * n * x / T), 0,
np.pi)

# Function to compute the Fourier series approximation
def fourier_series_approx(x, a0_1, an_1, bn_1, a0_2, an_2, bn_2, T, num_terms):
    series_sum_1 = a0_1 / 2.0  # Initialize with a0/2 for the first part
    series_sum_2 = a0_2 / 2.0  # Initialize with a0/2 for the second part

    for n in range(1, num_terms + 1):
        series_sum_1 += an_1[n-1] * np.cos(2 * np.pi * n * x / T) + bn_1[n-1] * np.sin(2
* np.pi * n * x / T)
        series_sum_2 += an_2[n-1] * np.cos(2 * np.pi * n * x / T) + bn_2[n-1] * np.sin(2
* np.pi * n * x / T)

    return np.piecewise(x, [x < 0, x >= 0], [series_sum_1, series_sum_2])

# Generate x values
x_values = np.linspace(-np.pi, np.pi, 1000)

# Compute the Fourier series approximation for each x
y_values_approx = np.array([fourier_series_approx(x, a0_first_part, an_first_part,
bn_first_part,
                                                   a0_second_part, an_second_part, bn_second_part, T,
num_terms) for x in x_values])

# Plot the original function and its Fourier series approximation
plt.plot(x_values, [f(x) for x in x_values], label='Original function')
plt.plot(x_values, y_values_approx, label='Fourier series approximation')
plt.title('Fourier Series Approximation')
plt.xlabel('x')
plt.ylabel('y')
plt.legend()
plt.show()
```

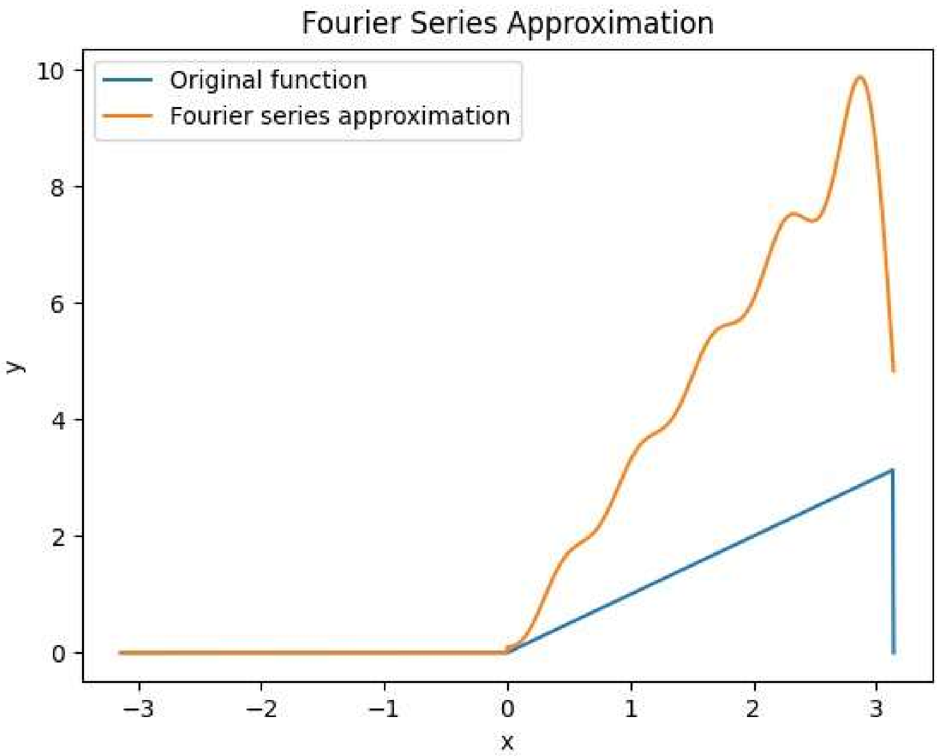

Figure 11: Fourier Series Approximation

Example 3: Compute the Fourier series for

$$f(x) = \begin{cases} -1, & -\pi < x < 0, \\ 1, & 0 < x < \pi. \end{cases}$$

Solution of Example 3: Our aim is $L = \pi$. f is an odd function, as you can see. As $f(x)\ cosnx$ is the product of an odd function and an even function, it is also an odd function. Therefore,

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) cosnxdx = 0, \quad n = 0,1,2,\ldots$$

Additionally, since $f(x) sin\, nx$ is the product of two odd functions, it is an even function [30]. Hence,

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) sinnxdx = \frac{2}{\pi}\int_{0}^{\pi} sinnxdx$$

$$= \frac{2}{\pi}\left[\frac{-cosnx}{n}\right]|_0^{\pi} = \frac{2}{\pi}\left[\frac{1}{n} - \frac{(-1)^n}{n}\right], \quad n = 1,2,3,\ldots$$

$$= \begin{cases} 0, & n\ even, \\ \dfrac{4}{\pi n}, & n\ odd. \end{cases}$$

Hence,

$$f(x) \sim \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{[1-(-1)^n]}{n} sinnx = \frac{4}{\pi}\left[sinx + \frac{1}{3} sin3x + \frac{1}{5} sin5x + \cdots\right]$$

Python code of Example 3:

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import quad

# Define the function f(x)
def f(x):
    if -np.pi < x < 0:
        return -1
    elif 0 < x < np.pi:
        return 1
    else:
        return 0  # Outside the specified range

# Define the period and the number of terms in the series
T = 2 * np.pi
num_terms = 10

# Compute the Fourier series coefficients
a0, _ = quad(lambda x: f(x), -np.pi, np.pi)
an = np.zeros(num_terms)
bn = np.zeros(num_terms)

for n in range(1, num_terms + 1):
    an[n-1], _ = quad(lambda x: f(x) * np.cos(2 * np.pi * n * x / T), -np.pi, np.pi)
    bn[n-1], _ = quad(lambda x: f(x) * np.sin(2 * np.pi * n * x / T), -np.pi, np.pi)
```

```
# Function to compute the Fourier series approximation
def fourier_series_approx(x, a0, an, bn, T, num_terms):
    series_sum = a0 / 2.0  # Initialize with a0/2

    for n in range(1, num_terms + 1):
        series_sum += an[n-1] * np.cos(2 * np.pi * n * x / T) + bn[n-1] * np.sin(2 * np.pi * n * x / T)

    return series_sum

# Generate x values
x_values = np.linspace(-np.pi, np.pi, 1000)

# Compute the Fourier series approximation for each x
y_values_approx = np.array([fourier_series_approx(x, a0, an, bn, T, num_terms) for x in x_values])

# Plot the original function and its Fourier series approximation
plt.plot(x_values, [f(x) for x in x_values], label='Original function')
plt.plot(x_values, y_values_approx, label='Fourier series approximation')
plt.title('Fourier Series Approximation')
plt.xlabel('x')
plt.ylabel('y')
plt.legend()
plt.show()
```

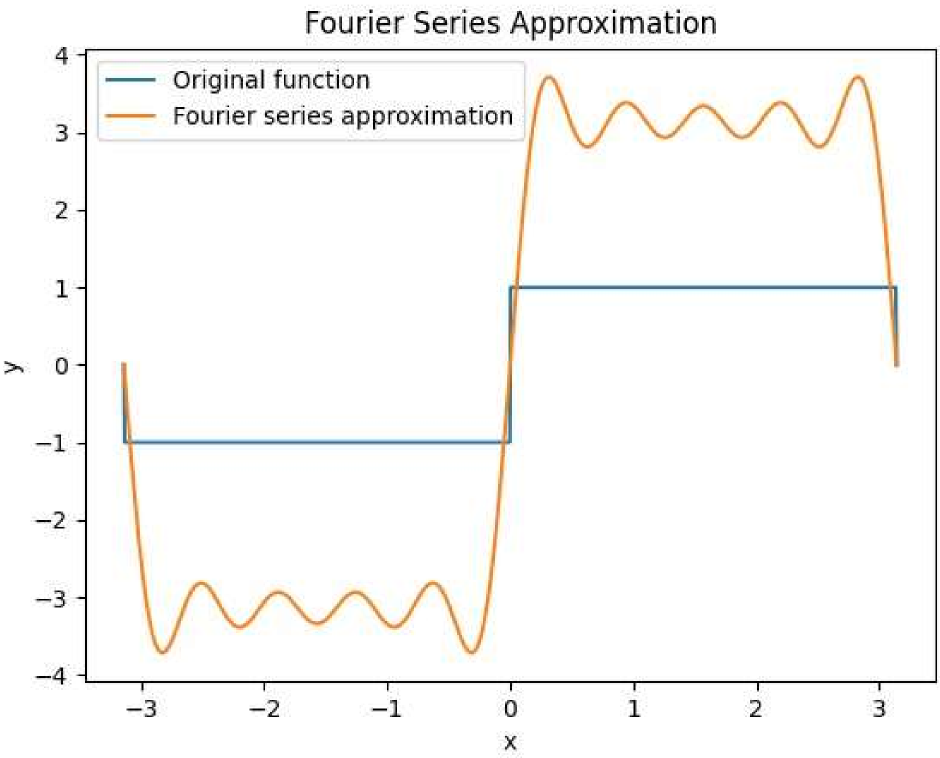

Figure 12: Fourier Series Approximation

Example 4: Determine the Fourier series for $f(x) = |x|, -1 < x < 1$.

Solution of Example 4: Our aim is $L = 1$. Because, f is an even function, $f(x)sin(n\pi x)$ is an odd function [35]. Hence,

$$b_n = \int_{-1}^{1} f(x)sin(n\pi x)dx = 0, \quad n = 1,2,3, \dots$$

Because, $f(x)cos(n\pi x)$ is an even function, then

$$a_0 = \int_{-1}^{1} f(x)dx = 2\int_{0}^{1} xdx = 1.$$

$$a_n = \int_{-1}^{1} f(x)cos(n\pi x)dx = 2\int_{0}^{1} xcos(n\pi x)dx$$

$$= \frac{2}{\pi^2 n^2}\int_{0}^{\pi n} ucosudu = \frac{2}{\pi^2 n^2}[cosu + usinu]|_0^{\pi n} = \frac{2}{\pi^2 n^2}(cosn\pi - 1)$$

$$= \frac{2}{\pi^2 n^2}[(-1)^n - 1], \quad n = 1,2,3, \dots$$

Hence,

$$f(x)\sim\frac{1}{2}+\sum_{n=1}^{\infty}\frac{2}{\pi^{2}n^{2}}[(-1)^{n}-1]\cos(n\pi x)$$

$$=\frac{1}{2}-\frac{4}{\pi^{2}}\left\{cos(\pi x)+\frac{1}{9}cos(3\pi x)+\frac{1}{25}cos(5\pi x)+\cdots\right\}.$$

Python code of Example 4:

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import quad
# Define the function |x|
def f(x):
    return np.abs(x)

# Define the period and the number of terms in the series
T = 2
num_terms = 10

# Compute the Fourier series coefficients
a0, _ = quad(lambda x: f(x), -T/2, T/2)
an = np.zeros(num_terms)
bn = np.zeros(num_terms)

for n in range(1, num_terms + 1):
    an[n-1], _ = quad(lambda x: f(x) * np.cos(2 * np.pi * n * x / T), -T/2, T/2)
    bn[n-1], _ = quad(lambda x: f(x) * np.sin(2 * np.pi * n * x / T), -T/2, T/2)

# Function to compute the Fourier series approximation
def fourier_series_approx(x, a0, an, bn, T, num_terms):
    series_sum = a0 / 2.0  # Initialize with a0/2

    for n in range(1, num_terms + 1):
```

```
        series_sum += an[n-1] * np.cos(2 * np.pi * n * x / T) + bn[n-1] * np.sin(2 * np.pi
* n * x / T)

    return series_sum

# Generate x values
x_values = np.linspace(-1, 1, 1000)

# Compute the Fourier series approximation for each x
y_values_approx = np.array([fourier_series_approx(x, a0, an, bn, T, num_terms) for x
in x_values])

# Plot the original function and its Fourier series approximation
plt.plot(x_values, f(x_values), label='Original function: $|x|$')
plt.plot(x_values, y_values_approx, label='Fourier series approximation')
plt.title('Fourier Series Approximation of $|x|$')
plt.xlabel('x')
plt.ylabel('y')
plt.legend()
plt.show()
```

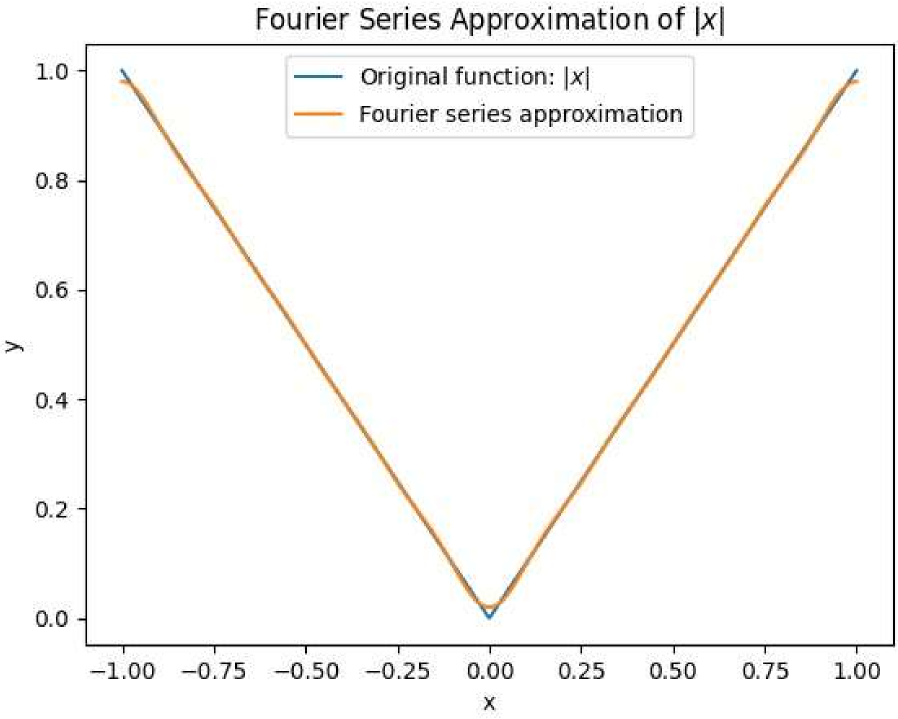

Figure 13: Fourier Series Approximation

5.1 WAVE EQUATION

The partial differential equation that gives the position $u(x,t)$ of a string, such as a fixed-end elastic guitar string, relative to its equilibrium position when it is broken is the wave equation [36].

$$u_{tt} = c^2 u_{xx,}$$

with $c^2 = T/\rho$

and boundary conditions at the ends of the rope at x=0 and L given by

$$u(0,t) = 0, \qquad u(L,t) = 0$$

To construct a boundary value problem using the Wave equation, which is a second-order partial differential equation with respect to time, initial conditions are required for both the displacement of the string due to the break and the initial velocity of the displacement. We assume,

$$u(x,0) = f(x), \quad u_t(x,0) = 0, \quad 0 \le x \le L$$

To solve this problem we use seperation of variables method and let

$$u(x,t) = X(x)T(t)$$

By using the above substitution in the wave equation we obtained the following equations:

$$\frac{X''}{X}=\frac{1}{c^2}\frac{T''}{T}=-\lambda$$

These equations gives the following ordinary differential equations:

$$X''+\lambda X=0,\ \ T''+\lambda c^2 T=0$$

From the boundary conditions of wave equation, for the first ordinary equation we obtain the following boundary conditions:

$$X(0)=0,\ \ X(L)=0$$

It is exactly an eigenvalue problem which has non-trivial solutions only if λ>0 and possible eigenvalues are:

$$\lambda_n=(n\pi/L)^2,\quad n=1,2,3,....$$

and the eigenfunctions are as follows

$$X_n=sin(n\pi x/L)$$

By putting the eigenvalues we found into the ode containing T, we obtain the ode's

$$T_n''+\frac{n^2\pi^2c^2}{L^2}T_n=0$$

With general solution given by

$$T_n(t)=Acos\frac{n\pi ct}{L}+Bsin\frac{n\pi ct}{L}$$

The second boundary condition in the wave equation implies

$$u_t(x,0)=X(x)T'(0)=0$$

Which implies $T'(0)\ =0$ and so $B=0$. Combining our solution for $x_n(x)$and $T_n(t)$, we get

$$u_n(x,t)=sin\frac{n\pi x}{L}cos\frac{n\pi ct}{L},\quad n=1,2,3,...$$

So the general solution for $u(x,t)$ is of the form

$$u(x,t)=\sum_{n=1}^{\infty}b_n sin\frac{n\pi x}{L}cos\frac{n\pi ct}{L}$$

By using the boundary conditions we obtain

$$f(x) = \sum_{n=1}^{\infty} b_n sin(n\pi x/L),$$

It is a Fourier Sine series. Hence, the coefficients are:

$$b_n = \frac{2}{L}\int_0^L f(x) sin\frac{n\pi x}{L} dx, \quad n = 1,2,3,\dots$$

By using the trigonometric identity

$$sinxcosy = \frac{1}{2}(\sin(x+y) + \sin(x-y))$$

The solution can be rewritten as follows:

$$u(x,t) = \frac{1}{2}\sum_{n=1}^{\infty} b_n \left(sin\frac{n\pi(x+ct)}{L} + sin\frac{n\pi(x-ct)}{L}\right)$$

In [36], it is possible to think of the first and second sine functions as travelling waves moving left or right at a speed of c. This can be observed by increasing the time, $t \to t + \delta$, and noting that the value of the first sine function remains constant as long as the position is shifted by $x \to x - c\delta$, as long as $x \to x + c\delta$, the second sine function remains unchanged. A standing wave is created by two waves of equal amplitude travelling in opposite directions:

Hammered String

A piano string is hammered, as opposed to a guitar string that is plucked. For a piano string, the proper boundary conditions would be

$$u(x,0) = 0, \;\; u_t(x,0) = g(x), \;\; 0 \le x \le L$$

With these boundary conditions we get $T(0) = 0$ and hence $A = 0$. So the general solution becomes:

$$u(x,t) = \sum_{n=1}^{\infty} b_n sin\frac{n\pi x}{L} sin\frac{n\pi ct}{L}$$

From the boundary conditions we get:

$$g(x) = \frac{\pi c}{L}\sum_{n=1}^{\infty} nb_n sin\frac{n\pi x}{L} dx$$

It is a Fourier Sine series. Hence, the coefficients are:

$$b_n = \frac{2}{n\pi c}\int_0^L g(x) sin\frac{n\pi x}{L} dx$$

General Initial Conditions

If we generalize the boundary conditions as follows:

$$u(x,0) = f(x), \quad u_t(x,0) = g(x), \quad 0 \leq x \leq L$$

By using superposition we can use the below substitution:

$$u(x,t) = v(x,t) + w(x,t)$$

Where $v(x,t)$ satisfies first type of boundary conditions and $w(x,t)$ satisfies the second type boundary conditions.

CHAPTER VI
FOURIER TRANSFORMATION

6.1 GENERAL DEFINITION AND PROPERTIES OF FOURIER TRANSFORMATION

In [21], Fourier transformation is used everywhere such as; signal processing, image processing, intensity of earthquake [37-46].

$$\mathcal{F}\{f(t);\omega\} = \int_{-\infty}^{\infty} e^{i\omega t} f(t)dt \qquad (6.1.1)$$

where f is a piecewise continuous function, [37].

Inverse Fourier transformation is defined as;

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_e(\omega)e^{-i\omega t}d\omega \qquad (6.1.2)$$

Fourier transformconvolution is defined as;

$$f(t) * y(t) = \int_{-\infty}^{\infty} f(t-\tau)y(\tau)d\tau$$

$f(t)$ and $y(t)$ are defined in $(-\infty,\infty)$. The produce of their Fourier transforms;

$$\mathcal{F}_e\{f(t) * y(t);\ \omega\} = f_e(\omega)Y_e(\omega) \qquad (6.1.3)$$

General properties of Fourier transformations;

By using (6.1.1) then we will give many proofs about the linearity property.

1. $\mathcal{F}_e\{xf(t) + yg(t);\omega\} = x\{f(t)\} + y\mathcal{F}_e\{g(t)\}$

Proof:

$$\mathcal{F}_e\{xf(t) + yg(t);\omega\} = \int_{-\infty}^{\infty} e^{i\omega t}[xf(t) + yg(t)]dt$$

$$= \int_{-\infty}^{\infty} \left(e^{iwt}xf(t) + e^{iwt}yg(t)\right)dt$$

$$= x \int_{-\infty}^{\infty} e^{i\omega t} f(t)dt + y \int_{-\infty}^{\infty} e^{i\omega t} g(t)dt$$

so we obtain this proof

2. $\mathcal{F}_e\{xf(t) - yg(t); \omega\} = x\mathcal{F}_e\{f(t)\} - y\mathcal{F}_e\{g(t)\}$

Proof:

$$\mathcal{F}_e\{xf(t) - yg(t); \omega\} = \int_{-\infty}^{\infty} e^{i\omega t}[xf(t) - yg(t)]dt$$

$$= \int_{-\infty}^{\infty} \left(e^{iwt} xf(t) - e^{iwt} yg(t)\right)dt$$

$$= x \int_{-\infty}^{\infty} e^{i\omega t} f(t)dt - y \int_{-\infty}^{\infty} e^{i\omega t} g(t)dt$$

CHAPTER VII
WAVELET TRANSFORM

7.1 DEFINITION OF WAVELET TRANSFORM

The main idea behind wavelets is to be able to analyze to scale. It may be possible to gain a completely new mindset or perspective by using wavelets in data processing. In addition, the scale we use to look at the data also plays an important role in wavelet analysis. Wavelet algorithms process data at different scales or resolutions. Here, your perspective and where you look are important. If we look at a signal through a large window, we notice large features, or if we look at a signal through a small window, we notice small features. This property makes wavelets both useful and interesting. Wavelets are mathematical functions that divide data into different frequency components, that can then describe each component with a resolution matched to the scale of the component. The wavelet transform is a function that separates functions, operators, or data into components of different frequencies and is independent on each component. that is, it is a tool that allows us to work separately. Wavelet analysis is successful in revealing distortion points, discontinuities and similarities in higher order derivatives that other analysis methods cannot capture. Wavelets have developed in the fields of mathematics and seismic geology. In recent years, as a result of the coming together of different disciplines and the exchange of information they have made among themselves, it has provided us with new wavelet applications such as radar and predicting the probability of an earthquake.

7.2 WAVELET TRANSFORM

In this part, continuous case of wavelet transform were used [47-58].

7.2.1 Continuous Wavelet Transform

In this part, $\psi \in L^2(\mathbb{R})$ be a window function. To illustrate, its Fourier transform ψ is defined as a window function [47], and also, the centres and widths of ψ and $\bar{\psi}$ are introduced by $t_c, 2\Delta_\psi$ and $\omega_c 2\Delta_{\bar{\psi}}$. It is shown as for $x, y \in \mathbb{R}, x \neq 0,$

$$\psi_{y,x}(t) = |y|^{-1/2}\psi\left(\frac{t-y}{x}\right) \tag{7.1}$$

Especially, continuous wavelet transform is used with signal processing, data of signal processing when we analyze seismic events on surface [47-49]. The continuous wavelet transform of a signal $h \in L^2(\mathbb{R})$ could be introduced as ([48, 49]);

$$\left(W_\psi h\right)(y,x) = \langle h, \psi_{y,x}\rangle = \int_{-\infty}^{\infty} h(t)\overline{\psi_{y,x}(t)}\,dt \tag{7.2}$$

Notice that the definitions of the parameter x gives $\psi_{y,x}$ to compress or expand depending on the choice of x [48]. We define $\psi_{y,x}$ as the total energy and it is independent of x and y,

$$\left\|\psi_{y,x}\right\|_2^2 = \|\psi\|_2^2.$$

Now, we consider proceed to view the effect of parameter x in the time frequency window of the wavelet transform. Because of, the centre and width of the window function ψ are t_c and $2\Delta_\psi$ respectively, these quantities for $\psi_{y,x}$ could be seen to as $y + xt_c$ and $2x\Delta_\psi$ respectively. Hence, the time window is written as;

$$\left[y + xt_c - x\Delta_\psi, y + xt_c + x\Delta_\psi\right].$$

We construct from the information then,

$$\left(W_\psi h\right)(y,x) = \langle h, \psi_{y,x}\rangle = \frac{1}{2\pi}\langle \hat{h}, \widehat{\psi_{y,x}}\rangle \tag{7.3}$$

and then

$$\psi_{y,x}(\omega) = |x|^{-1/2}\int_{-\infty}^{\infty} e^{-i\omega t}\psi\left(\frac{t-y}{x}\right)dt = x|x|^{-1/2}e^{-ib\omega}\hat{\psi}(x\omega) \tag{7.4}$$

$$\left(W_\psi h\right)(y,x) = \frac{x|x|^{-1/2}}{2\pi}\int_{-\infty}^{\infty} \hat{h}(\omega)e^{ib\omega}g(x\omega - \omega_c)d\omega \tag{7.5}$$

and $g(x\omega - \omega_c)$ will be introduced as $\frac{2}{x}\Delta_{\hat{\psi}}$. So the frequency window in this case is introduced as

$$\left[\frac{\omega_c}{x} - \frac{1}{x}\Delta_{\hat{\psi}}, \frac{\omega_c}{x} + \frac{1}{x}\Delta_{\hat{\psi}}\right].$$

Hence, the rectangular time frequency window is introduced by,

$$\left[y + xt_c - x\Delta_\psi, y + xt_c + x\Delta_\psi\right] \times \left[\frac{\omega_c}{x} - \frac{1}{x}\Delta_{\hat{\psi}}, \frac{\omega_c}{x} + \frac{1}{x}\Delta_{\hat{\psi}}\right], \qquad (7.6).$$

From the information of (7.2.6), the time frequency window area is again $4\Delta_{\psi}\Delta_{\hat{\psi}}$.

$$\hat{f}(x,y) = \left(W_{\psi}f\right)(y,x) = \langle f, \psi_{y,x}\rangle = \frac{1}{2\pi}\langle \hat{f}, \hat{\psi}_{y,x}\rangle$$

$$\hat{f}(x,y) = \left(W_{\psi}h\right)(y,x) = \frac{x|x|^{-1/2}}{2\pi}\int\limits_{-\infty}^{\infty} \hat{h}(\omega)e^{ib\omega}\overline{\psi(x\omega)}d\omega$$

$$= x|x|^{-1/2}\left\{\widehat{\hat{f}(.)\overline{\psi(x.)}}\right\}^{V}\text{(b),}$$

$$\int_{-\infty}^{\infty}\hat{f}\,(\mathrm{x,y})e^{-ib\omega}db = \; x|x|^{-1/2}\,\hat{f}(\omega)\psi\overline{(\alpha\omega).}$$

$$\int_{-\infty}^{\infty} q(x)\psi(\alpha\omega)\int_{-\infty}^{\infty} f\hat{f}(x,y)e^{-ib\omega}dydx = \left[\left|\int_{0}^{\infty} q(\alpha)x^{1/2}\;\lfloor\psi(\alpha\omega)\rfloor^{2}dx\right|\hat{f}\,(\omega)\right.$$

$$f(\omega) = \frac{1}{Y(\omega)}\;\widehat{\int_{0}^{\infty}} q(x)\psi(x\omega)\int_{-\infty}^{\infty}\hat{f}\,(x,y)e^{-iy\omega}dydx$$

$$= \frac{1}{Y(\omega)}\int_{0}^{\infty} q(x)\int_{-\infty}^{\infty} x^{-1/_{2}\psi_{y,x}}\;\hat{f}(x,y)dydx$$

Putting $\psi^{y,x}(\omega) = \frac{1}{Y(\omega)}\psi_{y,x}(\omega)$ we may write

$$\hat{f}(\omega) = q(x)x^{-1/_{2}}\int_{-\infty}^{\infty}\psi^{y,x}(\omega)\widehat{f(x,y)}dydx$$

One can easily see that $\psi^{y,x}\epsilon\mathcal{L}^{2}(\mathbb{R})$ with $\|\psi^{y,x}\|_{2}^{2} \quad \leq A^{-2}\|\psi\|_{2}^{2}$

Hence

$$\psi^{y,x}(u) = \frac{x^{1/2}}{2\pi}\int_{-\infty}^{\infty}\left(Y(\omega)\right)^{-1}\psi(x\omega)e^{i(u-b)\omega}d\omega = \psi^{x}(u-b)$$

Say with,

$$\psi^{y,x}(u) = \frac{x^{1/2}}{2\pi}\int_{-\infty}^{\infty}\left(Y\left(\frac{\omega}{\alpha}\right)\right)^{-1}\;\psi(\omega)e^{iu\omega/_{x}}d\omega$$

We now recover the original signal f from its Fourier transform $\hat{f}$ by applying the inverse Fourier transform on both sides of

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{0}^{\infty} q(x)x^{-1/2}\,\hat{f}(x,y)dydx\int_{-\infty}^{\infty}\psi^{y,x}\,(\omega)e^{it\omega}d\omega$$

$$= \int_{0}^{\infty} q(x)x^{-1/2}\int_{-\infty}^{\infty}\psi^{y,x}(t)\hat{f}(x,y)dydx$$

Therefore f can be obtained as superposition of the family of functions $\psi^{y,x}$ which are elements of $L^2(\mathbb{R})$.However ,this formula involves a whole new family of functions $\psi^{y,x}$ amd will increase the computational cost to a large extent.

$Y\left(\frac{\omega}{x}\right) = Y(\omega)$ for almost all $\omega \epsilon\ \mathbb{R}$ and $x > 0$. In this case $\psi^{x}(u) = x^{-1/2}\psi^{1}(\frac{u-y}{x})$ Y must be constant for $\omega > 0$ and $\omega < 0$. This reveals the role of the weight function $q(x)$.We choose q so that Y satisfies .For this we choose q to be such that $q(x)x^{-1/2} = \frac{1}{x}$. This gives

$$Y(\omega) = \int_{0}^{\infty}\frac{|\psi(x\omega)|^2}{x}dx = \int_{0}^{\infty}\frac{|\psi(\xi)|^2}{\xi}d\xi = C_+$$

for $\omega > 0$

And

$Y(\omega)$=$Y(\omega) = \int_0^\infty \frac{|\psi(x\omega)|^2}{x}dx = \int_0^\infty \frac{|\psi(-\xi)|^2}{\xi}d\xi = C_-$ for $\omega < 0$

Thus Y is bounded if and only if 0< $C_{\pm<\infty}$.

This means we require

$$\int_{0}^{\infty}\frac{|\psi(\pm\xi)|^2}{\xi}d\xi < \infty$$

this is known as admissibility condition.Thus the choice $q(x) = x^{-3/2}$ makes Y a piecewise function on one hand and simplifies the construction of ψ^1 on the other hand. If ψ is such that $C_+ = C_- = C/2$, then

$$\psi^1 = \frac{2}{C}\psi$$

Let $\psi \epsilon \mathcal{L}^2(\mathbb{R})$ be such that

$$C = \int_{-\infty}^{\infty}\frac{|\psi(\xi)|^2}{|\xi|}d\xi < \infty$$

Then $f \in \mathcal{L}^2(\mathbb{R})$ can be recovered from its wavelet transform $\hat{f}$ by

$$f(t) = \frac{1}{C}\int_{0}^{\infty}\int_{-\infty}^{\infty} x^{-2}\psi^{y,x}(t)\hat{f}(x,y)dydx$$

Infact, the same analysis can be perfomed for $a < 0$ also and we could integrate over the whole of $\mathbb{R}\backslash\{0\}$ with respect to a to get the General Reconstruction Formula as follows:

$$f(t) = \frac{1}{C}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^{-2}\,\psi^{y,x}\hat{f}(x,y)dydx$$

We are now in a position to define a basic wavelet or a mother wavelet.
Let $\psi \in \mathcal{L}^2$ ($\mathbb{R}$) be such that

$$C_\psi = \int_{-\infty}^{\infty} \frac{|\psi(\omega)|^2}{|\omega|} d\omega < \infty$$

Then ψ is called a basic wavelet (or mother wavelet).The function $\psi^1 = \frac{2}{c}\psi(t)$ is called dual mother wavelet of ψ. The family $\{\psi^{y,x}\}$ is called dual wavelet family to $\{\psi^{y,x}\}$. Note that the admissibility condition implies that $\psi(\omega) \rightarrow 0$ as $\omega \rightarrow 0.$ If ψ is continuous then $\psi(0) = 0$, which gives

$$\int_{-\infty}^{\infty} \psi(t)dt = 0$$

The above equation justifies the name wavelet to such window functions.

7.3 COMPARISON AMONG THE FOURIER TRANSFORM, SHORT-TIME FOURIER TRANSFORM(STFT) AND WAVELET TRANSFORM

We discuss the characteristics of the three transforms in this section [59].

7.3.1 Forward Transform

Fourier Transform

$$F(\omega) = \int_{-\infty}^{\infty} f(t)exp(-j\omega t)dt \tag{7.7}$$

Through full time axis integration, the Fourier transform, as we all know, converts signals from the time domain to the frequency domain. We are unable to determine when a particular frequency rises, though, if the signal is not stationary, that is, if the frequency composition varies over time [59].

STFT

$$Sf(u,\xi) = \int_{-\infty}^{\infty} f(t)w(t-u)\, exp(-j\xi t)\, dt \tag{7.8}$$

In order to address the Fourier transform problem, the STFT introduces a sliding window $w(t-u)$. The window's purpose is to extract a tiny percentage of the signal f(t) before performing a Fourier transform. There are two independent parameters in the transformed coefficient. The first one is the time parameter τ, which represents the instant in question. Similar to the Fourier transform, the other is the frequency parameter ξ. But then there's another issue. On the spectrum, the extremely low frequency component is undetectable. This is the rationale behind our usage of fixed-size windows. Let's say there is a single window. In the time domain, the extracted data in one second appears to be flat (DC) if there is a signal with a frequency of $0.1Hz$.

Wavelet Transform

$$Wf(s,u) = \int_{-\infty}^{\infty} f(t)\frac{1}{\sqrt{s}}\psi^{*}\left(\frac{t-u}{s}\right)dt \tag{7.9}$$

The prior issue is resolved by the wavelet transform. A balance between the frequency domain (finite bandwidth) and time domain (finite length) is achieved by the wavelet function's design. Very high frequency components can be precisely located at small s, while very low frequency components can be seen at large s as we translate and dilate the mother wavelet [59].

7.3.2 Inverse transform

Fourier Transform

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)exp(j\omega t)dt. \tag{7.10}$$

STFT

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Sf(u,\xi)w(t-u)exp(j\xi t)d\xi du. \tag{7.11}$$

Wavelet transform

$$f(t) = \frac{1}{C_{\psi}}\int_{0}^{\infty}\int_{-\infty}^{\infty} Wf(s,u)\frac{1}{\sqrt{s}}\psi\left(\frac{t-u}{s}\right)du\frac{ds}{s^{2}}, \tag{7.12}$$

$$C_{\psi} = \int_{0}^{\infty} \frac{|\Psi(\omega)|^2}{\omega} d\omega < \infty. \quad (7.13)$$

7.3.3 Basis

Fourier transform

Differential frequency complex exponential function:

$$exp(j\omega t). \quad (7.14)$$

STFT

Complex exponential function that has been truncated or windowed:

$$w(t-u)exp(j\xi t). \quad (7.15)$$

Wavelet transform

Mother Wavelets scaled and translated version:

$$\frac{1}{\sqrt{s}}\psi\left(\frac{t-u}{s}\right). \quad (7.16)$$

Now, we show the different basis for each transforms which are Fourier transform, STFT and wavelet transform [59];

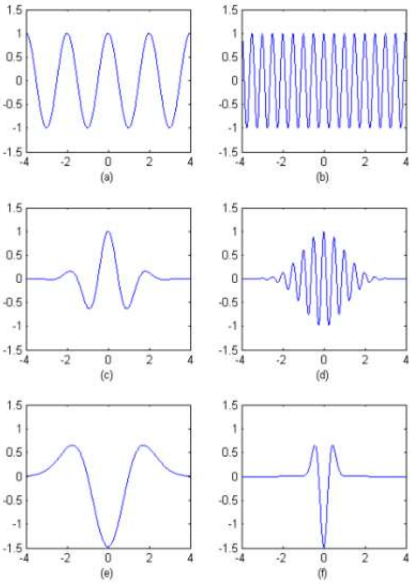

Figure 14: Different basis for the transforms:

(a) Real part of the basis for Fourier transform, $exp(j\pi t)$.

(b) Basis for different frequency, $exp(j4\pi t)$.

(c) Basis for STFT, using Gaussian window of $\sigma = 1.$. It is $exp(-t^2/2)\ exp(j\pi t)$.

(d) Basis for different frequency, $exp(-t^2/2)\ exp(j4\pi t)$.

(e) Mexican-hat mother wavelet function and

(f) $s = 4$.

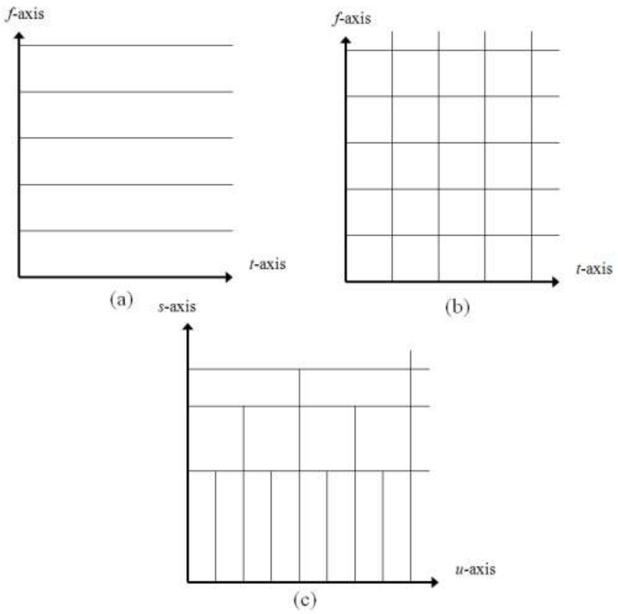

Figure 15: Different time-frequency tile allocation of the three transforms:

(a) Fourier Transform
(b) STFT
(c) Wavelet Transform

CHAPTER VIII

EXPLANATION OF FOURIER AND WAVELET TRANSFORMS

8.1 APPLICATION OF WAVELET ANALYSIS ON EARTHQUAKE RECORDS

In [60], An alternate method for dissecting a signal into its component pieces is wavelet analysis. A system that combines the temporal and spectral domains achieves this. The signal is divided into a set of local basis functions known as wavelets for wavelet analysis. A signal can be broken down even further into its wavelet components, or levels, which have numbers ranging from -1 to +1.

The original signal can be recovered by adding the individual wavelet levels. A wavelet is defined by its wavelet function, $W(x)$, which is obtained by taking differences from the Corresponding scaling function, $\phi(x)$. The wavelet function for dilation is:

$$W(x) = \sum_{k=0}^{N-1} (-1)^k c_k \phi(2x + k - N + 1)$$

where Q denotes the constants in numbers. The coefficients remain the same as in the definition of $\phi(x)$, but the terms are arranged differently and the signs of some of the terms are switched from positive to negative.

Their coefficients must be carefully selected, paying close attention to a number of requirements that these coefficients must meet, in order to generate suitable wavelets (Newland, 1993). The wavelet transform's primary goal is to split any given arbitrary signal $f(x)$ into an infinite summation of wavelets at various sizes in accordance with the expansion:

$$f(x) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} c_{j,k} W\left(2^j x - k\right)$$

Harmonic Wavelet transform: The harmonic wavelet transform, first presented by David Edward Newland in 1993, is a wavelet-based linear transformation that converts a given function into a time-frequency representation in signal processing mathematics. It combines the benefits of continuous wavelet transform with short-time Fourier transform. Its discrete counterpart may be calculated effectively with a rapid Fourier transform technique, and its expression can be formulated in terms of repeated Fourier transformations.

Harmonic Wavelets: The Fourier transform of harmonic wavelets is described by

$$\begin{cases} W_{m,n}(\omega) = 1/(n-m)(2\pi) \text{for } m(2\pi) \leq \omega < n(2\pi) \\ \qquad\quad = 0 \qquad\qquad\qquad\qquad \text{elsewhere} \end{cases}$$

where n and m are positive, real numbers. The function has a constant amplitude inside the range $m(2\,\pi)$ to $n(2\,\pi)$, which is normalized to guarantee that the enclosed area is unity, and 0 outside of this band. The definition of the matching wavelet function is as follows:

$$w(t) = \frac{e^{i4\pi t} - e^{i2\pi t}}{i2\pi t} \tag{8.1.1}$$

The Fourier transforms of these orthogonal functions are square window functions, which are 0 outside of a certain octave band and constant within. Specifically, they meet:

$$\int_{-\infty}^{\infty} w^*(2^j t - k).w(2^{j'} t - k')dt = \frac{1}{2^j}\delta_{j,j'}\delta_{k,k'}$$

$$\int_{-\infty}^{\infty} w(2^j t - k).w(2^{j'} t - k')dt = 0.$$

These wavelets become less concentrated in time (t) and more localized in Fourier space (frequency) and higher frequency bands as the order j grows. They thereby describe behaviors of the function on multiple timeframes (and at varying time offsets for different k) when they are used as a foundation for extending an arbitrary function. All of the negative orders $(j < 0)$ can, however, be combined into a single family of "scaling" functions, denoted as $\varphi(t-k)$, where

$$\varphi(t) = \frac{e^{i2\pi t} - 1}{i2\pi t}.$$

For varying k, the function φ is orthogonal to itself; for non-negative j, it is also orthogonal to the wavelet functions:

$$\int_{-\infty}^{\infty} \varphi^*(t-k).\varphi(t-k')dt = \delta_{k,k'}$$

$$\int_{-\infty}^{\infty} w^*(2^j t-k).\varphi(t-k')dt = 0 \ for\ j \geq 0$$

$$\int_{-\infty}^{\infty} \varphi(t-k).\varphi(t-k')dt = 0$$

$$\int_{-\infty}^{\infty} w(2^j t-k).\varphi(t-k')dt = 0 \ for\ j \geq 0.$$

Thus, an arbitrary real- or complex-valued function $f(t) \in L_2$ is enlarged on the basis of the harmonic wavelets (for all integers j) and their complex conjugates in the harmonic wavelet transform:

$$f(t) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \left[a_{j,k} w(2^j t-k) + \tilde{a}_{j,k} w^*(2^j t-k)\right],$$

Alternatively, the scaling functions φ can be added to the wavelets' basis for non-negative j.

$$f(t) = \sum_{k=-\infty}^{\infty} \left[a_k \varphi(t-k) + \tilde{a}_k \varphi^*(t-k)\right]$$
$$+ \sum_{j=0}^{\infty} \sum_{k=-\infty}^{\infty} \left[a_{j,k} w(2^j t-k) + \tilde{a}_{j,k} w^*(2^j t-k)\right].$$

Then, in theory, the expansion coefficients may be calculated using the orthogonality relationships:

$$a_{j,k} = 2^j \int_{-\infty}^{\infty} f(t).w^*(2^j t-k)dt$$

$$\tilde{a}_{j,k} = 2^j \int_{-\infty}^{\infty} f(t).w(2^j t-k)dt$$

$$a_k = \int_{-\infty}^{\infty} f(t).\varphi^*(t-k)dt$$

$$\tilde{a}_k = \int_{-\infty}^{\infty} f(t).\varphi(t-k)dt.$$

For a real-valued function $f(t)$, $\tilde{a}_{j,k} = a^*_{j,k}$ and $\tilde{a}_k = a^*_k$ so one can cut the number of independent expansion coefficients in half. This expansion has the property, analogous to Parseval's theorem, that:

$$\sum_{j=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} 2^{-j}\left(\left|a_{j,k}\right|^2 + \left|\tilde{a}_{j,k}\right|^2\right)$$
$$= \sum_{k=-\infty}^{\infty}(|a_k|^2 + |\tilde{a}_k|^2) + \sum_{j=0}^{\infty}\sum_{k=-\infty}^{\infty} 2^{-j}\left(\left|a_{j,k}\right|^2 + \left|\tilde{a}_{j,k}\right|^2\right)$$
$$= \int_{-\infty}^{\infty} |f(x)|^2 dx.$$

In [60], “A wavelet in the frequency range $m(2\pi)$ to $n(2\pi)$, where $n(2\pi)$, is indicated by levels $n > m$. Adjacent wavelet levels must have Fourier transforms with frequency bands contacting one another in order to encompass all values of o along the axis 0 to 00, forming a full set of wavelets”. (1995, Newland). For the scenario when $m = l$ and $n = 2$, Figures 16 and 17 depict the real and imaginary components of w(x), which are described by equation (8.1).

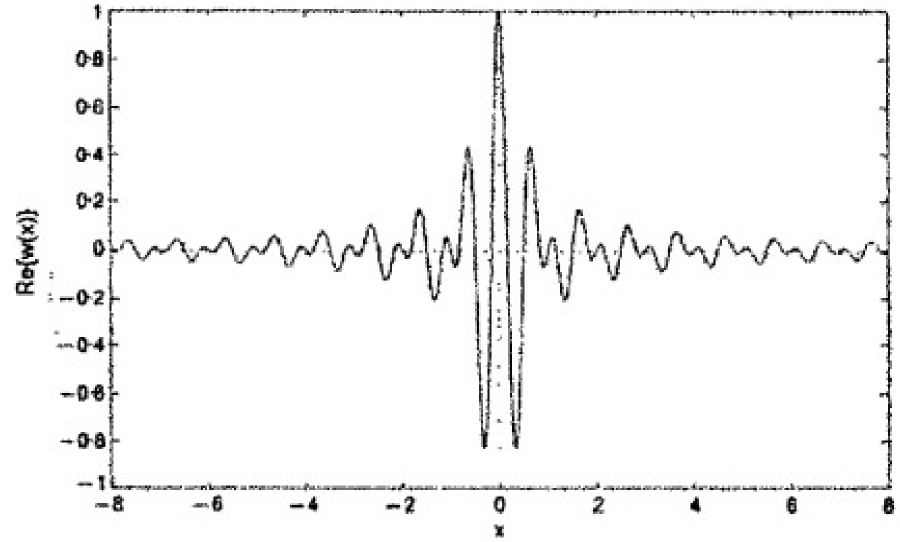

Figure 16: Real part of $w(x)$.

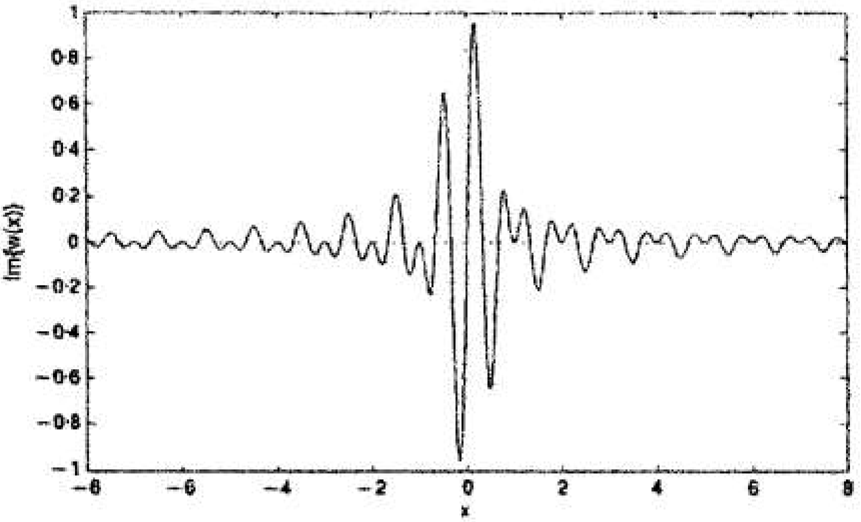

Figure 17: Imaginary part of $w(x)$.

Only for the whole duration of a signal f(t) can frequency information be recovered in Fourier analysis. The Fourier transform equation's integral runs from $-\infty$ to ∞, therefore the information it offers in the frequency domain comes from an average across the entire signal's duration. The Fourier transform $F(\omega)$ will benefit from any local oscillation in $f(t)$, but its position on the time axis will be lost. It is impossible to tell whether the value of $F(\omega)$ at a given o is driven by frequencies that are present only once or for a few chosen periods, or whether it is caused by frequencies present throughout the life of $f(t)$. Nevertheless, wavelet analysis overcomes this drawback. It is possible to see changes in frequency components over time by employing wavelets. We used the Fourier and wavelet transform to obtain the signal data of earthquakes to obtain the signal data of time-dependent earthquake waves. I used the Fourier and wavelet transform to obtain signal data of time-dependent earthquake waves in obtaining signal data of earthquakes. We have shown the Fourier series with its mathematical expression and as a result of this method, we obtained approximate solutions of the Fourier series and we have made its mathematical modeling using pyhton code. In the same way, if we have done frequency analysis in fourier and wavelet, we use the same modeling in seismic waves. We can take any seismic wave and take its mathematical expression and measure the intensity of the earthquake using the frequency analysis method of earthquake waves. The example I will give here is to solve the wave intensity of the Izmit earthquake using the wavelet analysis method.

Theory of wave motion

In [61], when a mechanical vibration source is present in an environment, energy is produced and radiates outward in all directions. There is a persistent presence in the surroundings during this spread. The energy wave equation is a differential equation that is derived even in the absence of distortion. The simple harmonic oscillation equation is the expression for this equation. Simple harmonic oscillation is a type of periodic motion in which the acceleration of the vibrating dot is proportionate to the displacement. Point if the distance from the center where the movement starts is denoted by x, simple harmonic motion is shown by the following differential equation.

$$\frac{\partial^2 x}{\partial t^2} = -\omega^2 x$$

$$x = x_1 cos(\omega t) + x_2 sin(\omega t)$$

In this case, the acceleration is pointing in the opposite direction as the displacement, as shown by the negative sign of the coefficient of proportionality, ω. Figure 18 shows the change in particle speed and displacement as a function of time.

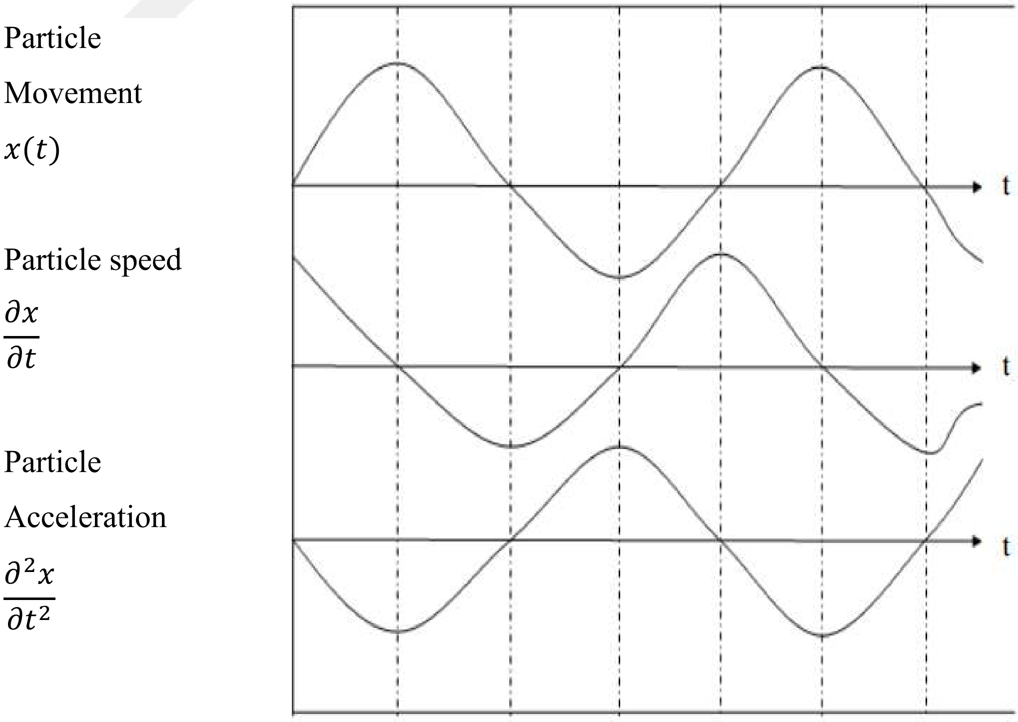

Figure 18: Time dependent displacement, velocity and acceleration of particle motion of simple harmonic oscillation chart [61].

CHAPTER IX
CONCLUSION

The aim of our thesis is to explain earthquake wave types and then to make mathematical modeling. First, what is an earthquake? We talked about how seismic waveforms take form, the occurrence of an earthquake and the waves that occur during an earthquake. Secondly, the history of earthquakes was explained. Thirdly, after talking about earthquake wave types, wave equations are introduced because this is important for us to establish mathematical modeling. The frequency analysis of the intensity of a possible earthquake wave using Fourier and Wavelet transform method is done and the modeling of the earthquake wave is shown on matlab by taking the example of Izmit earthquake.

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APPENDICES

APPENDIX A

1- **P-waves:** $v_x = \sqrt{\frac{K+\frac{4}{3}u}{\rho}} = \sqrt{\frac{\lambda+2\mu}{\rho}}$

2- **Density Dependent Velocity**: $Soil\ Density = \frac{Total\ Mass}{Total\ Volume}$

3- **Linear Equations**

Laplace's Equation: $\nabla^2 u = 0$

Helmholtz's (or eiegenvalue) Equation: $\nabla^2 u = -k^2 u$

Linear Transport Equation: $u_t + \sum_{i=1}^{n} b^i u_{x_i} = 0$

Heat (or diffusion) Equation: $u_t - \Delta u = 0$

Schröndinger's Equation: $i u_t + \Delta u = 0$

Wave Equation: $u_{tt} - \Delta u = 0$

Klein-Gordon Equation: $u_{tt} - \Delta u + m^2 u = 0$

APPENDIX B

4- **Nonlinear Equations**

Nonlinear Poisson Equation: $-\Delta u = f(u)$

Monge-Ampere Equation: $det(D^2 u) = f$

Hamilton-Jacobi Equation: $u_t + H(Du, x) = 0$

Inviscid Burgers' Equation: $u_t + u u_x = 0$

5- **Fourier Transform:** $F(\omega) = \int_{-\infty}^{\infty} f(t) exp(-j\omega t) dt$

6- **Wavelet Transform:** $Wf(s,u) = \int_{-\infty}^{\infty} f(t) \frac{1}{\sqrt{s}} \psi^* \left(\frac{t-u}{s}\right) dt$