AN APPLICATION OF DEMAND FORECASTING AND FARE MANAGEMENT FOR AN AIRLINE COMPANY

by

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ABSTRACT

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In this thesis, we aim to propose a model for estimating demand and obtaining ticket fare that maximizes revenue, based on real market data. In this context, eight different single leg local market routes are examined, and various time series analysis methods and regression analysis are applied. The purpose of implementation of time series analysis is to reach the best forecasted demand values that are tested against historical realizations of 26 weeks of sales data. In order to generate models and compare results, R (programming language) is used as a tool. After obtaining the best fitted demand models for each market route, fare values are found by using a proposed fare production process. This process is developed to produce demand-based fare values. Forecasted revenue with forecasted demand and produced fare values are achieved for each market route, and all results are examined separately to analyze performance of demand models and fares.

ÖZET

TALEP TAHMİNİ VE ÜCRET YÖNETİMİNİN BİR HAVAYOLU ŞİRKETİNDE UYGULAMASI

Bu tezde, gerçek pazar verilerine dayalı, talebin tahmini ve geliri maksimuma çıkaran ücretin elde edilmesi için bir model sunmayı amaçlıyoruz. Bu kapsamda, sekiz farklı tek bacaklı yerel rotalar incelenmekte olup farklı zaman seri analizi yöntemleri ile doğrusal regresyon uygulanmaktadır. Zaman serilerinin uygulanmasındaki amaç, gerçekleşen 26 haftalık satış verisi ile test edildiğinde en iyi tahmin edilmiş talep değerlerine ulaşmaktır. Modellerin oluşturulması ve kıyaslanması için araç olarak R programlama dili kullanılmaktadır. Her bir rota için en iyi uyum sağlayan talep modelinin elde edilmesinden sonra, tasarlanan ücret üretme süreci kullanılarak ücret değerleri elde edilmektedir. Bu yöntem talep bazlı ücret üretmek için geliştirilmektedir. Her bir rota için tahmini talep ve üretilen ücret değerleri ile birlikte tahmini gelir elde edilmekte, talep modellerinin ve ücretlerin performanslarının analiz edilmesi için tüm sonuçlar ayrı ayrı incelenmektedir.

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LIST OF SYMBOLS

A	Additive parameter
a	Amount/Frequency parameter
A_d	Additive damped parameter
B	Backshift operator
$b_j^*(t)$	Nested booking limits
c	Capacity parameter
C_t	The cyclical component
D	Degree of differencing for the seasonal part of the ARIMA model
d	Degree of differencing
E	Expected value
F	Average fare
f	Fare class
f_r	Rival fare
i	Market route index
I_t	Irregular, random or residual component
j	Class index
K	Time series period
L	Length of the season
L_t	Last value parameter
M	Multiplicative parameter
m	Time intervals
M_d	Multiplicative damped parameter
N	None
n	Total fare class value
P	Order of the Autoregressive model for the seasonal part
mp	Market potential parameter
p	Order of the Autoregressive model
p_j	Price of class
$R(t)$	Random variable

Q	Order of the Moving Average for the seasonal part
q	Order of the Moving Average
s	Seasonality parameter
S_t	The seasonal component
SS	Seats Sold parameter
SS_{t-1}	Last Seats Sold parameter
T	Total periods
t	Time index
T_t	The trend component
u	Control parameter
$V_t(x)$	Value function
Y_t	Row series
$y_j^*(t)$	Optimal booking protection level
Z_t	The original non-seasonally adjusted series
$\hat{Z}_{t+1}$	The estimate value of series

α	Smoothing constant
β	Smoothing constant
β_i	Demand/Price elasticity
γ	Smoothing constant
θ	Smoothing constant
$\lambda_j(t)$	The probability of an arrival of class j in period t
$\pi_t(x)$	Marginal value function

LIST OF ACRONYMS/ABBREVIATION

ACF	Autocorrelation Function
AR	Autoregressive
ARIMA	Autoregressive Integrated Moving Average
Avfare	Average Fare
Cap	Capacity
DDS	Direct Data Solutions
ESB	Ankara Esenboğa Airport
EMSR	Expected Marginal Seat Revenue
ETS	Error, Trend, Seasonality
GDP	Gross Domestic Product
HW	Holt Winters'
IATA	International Air Transportation Association
IST	Istanbul Atatürk Airport
LCC	Low Cost Carrier
LR	Linear Regression
LYS	Lyon Airport
MA	Moving Average
MAPE	Mean Absolute Percentage Error
MIDT	Marketing Information Data Types
NRT	Tokyo Narita Airport
O&D	Origin to Destination
PACF	Partial Autocorrelation Function
SARIMA	Seasonal Autoregressive Integrated Moving Average
SES	Simple Exponential Smoothing

SS	Seats sold
VKO	Moscow Vnukovo Airport

1. INTRODUCTION

Revenue management provides the maximization of revenue by taking advantage of diversification of sales price. Some trade sectors are in need of revenue management systems because they have perishable goods (rooms in a hotel, seats in a flight, cars to rent in a car-dealership, seats in a theater, etc.), and they have to sell these products over a finite horizon.

In the air transportation sector, operation and marketing costs are very high. Hence, airlines cannot take the chance of wasting a seat. In order to cope with this problem, airlines tend to sell seats in a flight with differentiated and time dependent sales price. So, revenue management emerged and has been developing with various studies by academics and revenue management analysts in airlines.

Revenue management includes three main areas: pricing, demand forecasting and optimization. In literature, there are many studies about revenue management tools and these studies are always interested in airlines' own specific data or macro data values from different sources regardless of airlines' data. Airlines' data do not contain market values and competitors' sales structure. Because of this situation, demand forecasting tool in revenue management needs to be improved. The effect of competitors' sales price can be the main input to forecast demand in different markets where high competition and limited market size exist.

In revenue management systems, the most important area of research is demand forecasting. These systems can only use airlines' own data to forecast demand because the other airlines do not give permission to use its own data. However, other airlines' sales price is public, as they sell tickets through public sales channels such as websites, call centers, online travel agencies, etc. Therefore, these sales price data can be used to generate demand models. Beside the competitors' sales price, market size is very significant input for demand forecasting. From different market analysis programs such as MIDT (marketing information data types) and DDS (Direct Data Solutions), market potential can be obtained.

In the content of this thesis, unlike other studies in the literature, all different effects on demand such as competitors' sales position in the market and market passenger size with airlines own sales data are taken into account for the calculation of demand forecasting. Through all these parameters, the main aim of this thesis is to indicate the importance of market values in demand forecasting for revenue maximization in airline revenue management.

In this study, eight different market routes, which have different characteristics, are chosen to obtain general structure of demand models. There are one market route from the European continent, one route from the African continent, two routes from the Asian continent, two routes from the American continent and two routes from Turkey. All these market routes' data are available and some of them have very high competition. So as to build demand models, different time series models are implemented. The best fitted forecasting models are chosen after comparison of the realized data, then, weekly forecasting demand models are generated. After obtaining demand models, its effects on revenue with fare production are examined.

There are five chapters in the content of this thesis. In the next chapter, literature survey and overview about revenue management in air transportation sector are given. In chapter 3, the choice of market routes with market conditions and description of the data are located. Demand modeling with the application of time series models are presented and all models are compared with the real life data. In chapter 4, revenue maximization with fare production is present. Fare production and the implementation of its effect on revenue are elaborated separately. Finally, the last chapter includes the evaluation of this study's conclusions and future studies are mentioned.

2. LITERATURE SURVEY AND OVERVIEW

Revenue management system has a very crucial role in air transportation sector. It has included many significant tools such as pricing, demand forecasting, demand management, seat allocation, fare management, overbooking, decrement, upgrading etc. Especially, revenue management systems use time series analysis to forecast passenger demand in air travel and in order to reach optimal solution it has to take into consideration demand-price relationship which affects the choice of customers.

2.1. The Development of Revenue Management in Airlines

Many industries experience many problems of selling perishable goods. Especially, airlines, hotels, theaters and retailers face selling a fixed capacity of a product over a finite horizon [1]. In all these sectors, firms have to sell their goods before the determined time. Otherwise, it generates bad investments in seats, rooms, etc. In order to reach their targets, these industries have to maximize their revenues. In particular, airlines sector had to develop their own strategy to increase revenue because of their high costs. In 1970's, revenue management perspective started with reservations and space control system in airlines, it was about simple pricing with low price in the special market places without yield management. In 1980's, basic yield system was developed by American Airlines with 26 selling classes. These classes represented different fare levels. In 1985, Revenue management system was introduced with yield focus through fare class hierarchy, then, deregulation and aggressive competition occurred. In 1990's, inventory of many routes and pricing in the different places started to merge and very complicated revenue management systems emerged like that Sabre Airmax reservation systems. After 2000, major airlines established their own revenue management departments and started to use revenue management systems with focusing on pricing, fare class levels, inventory structure such as business cabin seats and economy cabin seats.

2.2. The Aviation Revenue Management Tools in Airlines

In the commercial airline sector, demand is stochastic and price sensitive, therefore, pricing is very important tool to increase selling of seats and income. In 2000's, many researches were about the optimal timing of price changes. Feng and Gallego provided a price path associated with a general Poisson process with Markovian time dependent, predictable intensities [1]. And they reached an efficient algorithm to compute the optimal value functions with optimal pricing policy.

In addition to pricing control, other important tool is inventory control to maximize revenue in the air transportation commercial. In 1987-1989, Peter Belobaba produced the marginal seat revenue principle to multiple fare classes in a nested inventory structure [2], it is called expected marginal seat revenue (EMSR), and this principle is also used to forecast the overbooking parameters. Overbooking is a significant tool so as to sell seats over capacity which stem from cancelled reservations and no-show passengers. If airline sector does not use overbooking, they must sell seats at a higher price because the profit rate is very low in commercial airlines. Sometimes in a flight, only a seat could be the profit for that special operation, and if two passengers cancel their reservations without ticketing, that generates loss and abuse in reservation systems.

In the commercial airline we can add many different tools to increase revenue, such as, upgrading passengers who have tickets in economy cabin with very low fares, overbooking which includes no-show passengers and decrement referring to passengers who cancel or reissue their tickets, upselling passengers forced to buy higher class price, promotion for empty seats, changing aircraft type for special operations, etc. Beside these all tools, the major tools can be mentioned as pricing, demand forecasting and optimization including seat allocation and fare strategy management in airline revenue management.

2.2.1. Pricing

The content of pricing came out before revenue management, and it did not contain yield management or seat allocation. This tool only focused on adjusting fare levels for different markets and different routes. After simple pricing with low price in varied markets, different selling classes emerged by American Airlines as 26 fare classes. This feature provides very convenient transition in classes. In airline sector, mono fare class always brought about high price because this fare calculation included only cost factors and profit rate per ticket. Due to high flight operation and marketing costs, they had to determine selling fare as above the standards in a country. Therefore, flights generally had 40-50% load factors with some cancellations, and firms generally had bad financial scorecards because of that selling method. With varied classes and revenue management system, planners tended to provide class variation. In order to reach perfect solution, planners had to follow market sales and inventory for all flights, in all selling days. This was very inefficient operation and airlines had to face difficult selling operation. Under competition factor, some new pricing methods were started to use. Firms have launched to use its price as a tool to induce demand with objective to maximize the total expected revenue when the sale ends [3]. Li and Ji-Hua examined major airlines in China. They presented the significance and necessity of application of revenue management in China's commercial airline sector. Li and Ji-Hua developed a continuous-time dynamic pricing for two competitive flights. They use only two price levels for each flight. And they assumed that there were only two parallel flights from the same origin to the same destination with the same time schedule. In order to reach a concrete solution without optimization, they ignored cancellation, no-show and overbooking parameters, they only focused on seat inventory and pricing. In 2000's, it was very difficult to have the information from the competitors in a real time due to business secrets. Therefore, they only determined two classes' fares for two airlines and reached a reasonable solution for practical use under some assumptions.

Nowadays, airlines can reach the rival's base fare and total price with general distribution systems, reservation systems and websites. With some sales channels, they can see all included fares. Airlines have to use these systems because of high competition and different market conditions. In order to reach customers, they have to generate different channels. For example, airlines have to use reservation systems to sell their seats with ticketing in all over the world. But, there are a few reservation systems, like Amadeus, Troya and Galileo. In the market, travel agencies usually use a single system. Hence, these reservation systems need to have communication each other because of that restriction. As seen in Figure 2.1 as a variation of airline general distribution system, customers can access airline reservation systems with different channels.

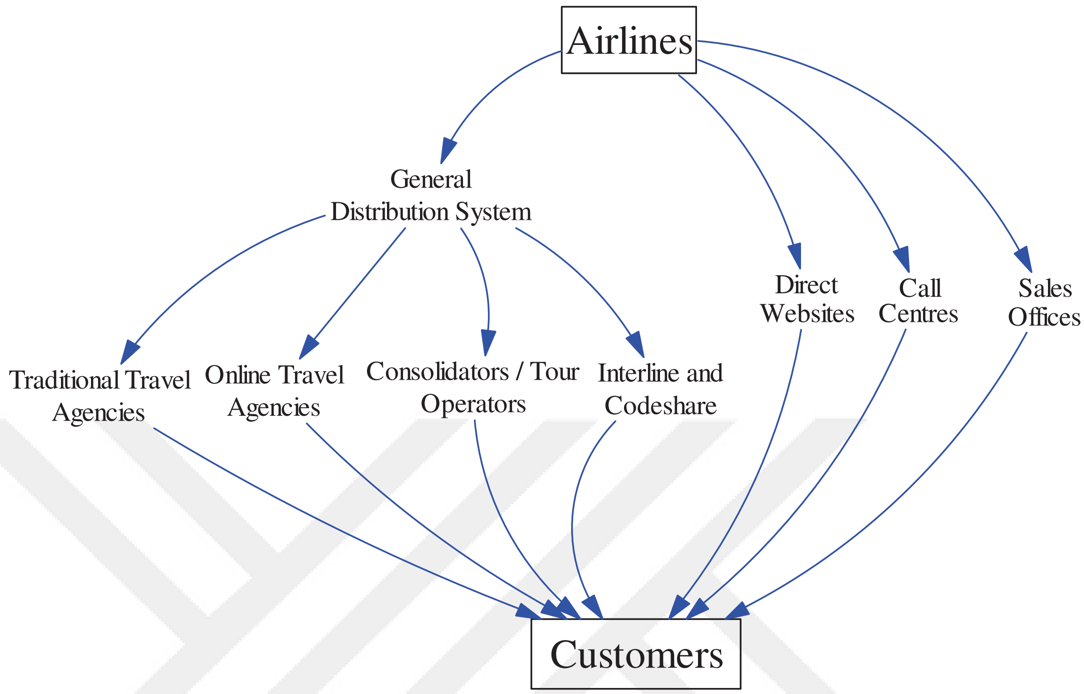

Figure 2.1: Airlines Sales Channels

At the present time, in the airline commercial sector, there is an open competition market with sales channels. After that, airlines head for similar fares. If there is a monopolistic carrier in a market, airline can determine their fares with consideration of market passenger profiles, gross domestic product per capita, population, socio-economic factors, frequency of flights, aircraft seat capacity, speed of aircrafts, distances between the airport, connection time, single or multi leg operation, season, week days, and location features (holiday places or not). But, if there is an alternative airline in the same market with the similar schedule, they have to use similar fares and they can reach all fares in their systems. This situation triggers airlines to tend to find different ways to acquire more customers. For example, efficient flight operations, good connection times, good schedules, high quality services such as foods, drinks, special entertainment tools, comfortable seats in the aircrafts, advertising, sponsorships, providing better luxury tools such as lounges.

In the revenue management perspective, this situation leads demand forecasting to gain more importance. Revenue management optimization with demand forecasting, yield management and dynamic pricing has become the main tool to reach high revenue target. Dynamic pricing refers to different fares for a good in the determined time. It has own boundaries; the lowest price and the highest price. But in the airline commercial sector, it is used as a fare management tool provides transition of fares with seat management. Besides pricing, dynamic pricing does not change fares; it only manages fare levels owing to fare classes.

2.2.2. Demand Forecasting

2.2.2.1. Demand Dependency in Revenue Management: In the air transportation sector, demand forecasting and the shape of demand distribution are very important because revenue management systems are supposed to provide good calculated number of seats for late booking high fare classes' demand [4]. Demand distribution for a flight leg is significant for figuring out net loads for different seating capacities [5]. Load is known as spill in airline sector. This situation can cause loss of customers exceed on flight capacity. In terms of revenue management, demand forecasting is crucial to allocate the right number of seats for booking classes. The older revenue management systems are based on unadjusted Expected Marginal Seat Revenue and independent demands have become less relevant owing to the global proliferation of simplified lightly restricted airfares since the year 2008 [6]. Revenue management systems based on dependent demand could out-perform prior revenue management systems which are not working as needed in the new lightly restricted fare environment. These tools can provide nearly 3+ percent revenue improvements. But, dependent demand tools are much more complex to operate due to numerous significant inputs.

We can classify three major types of techniques for the estimation of passenger demand:

- Single flight, single-class methods: Use single flight and class bookings and availability history.
- Single flight, multi-class methods: Use history for a single flight across multiple fare classes.

- Multi flight, multi-class methods: Consider activity across all flights and classes in a market simultaneously [6].

Airline revenue management systems are based on the second and the third type of these techniques and depend on the historical booking data of similar flights to forecast future demand. Historical data are always collected and examined in the form of time series with the aim of defining its alteration and estimating its trend in the future. In the addition to historical data, demand forecasting modules usually use reservations on the future flights to calculate future demands. As seen Figure 2.2 which is a variation of structure for a market-reactive airline RM systems (Bilegan, 2005), demand forecasting revises itself periodically.

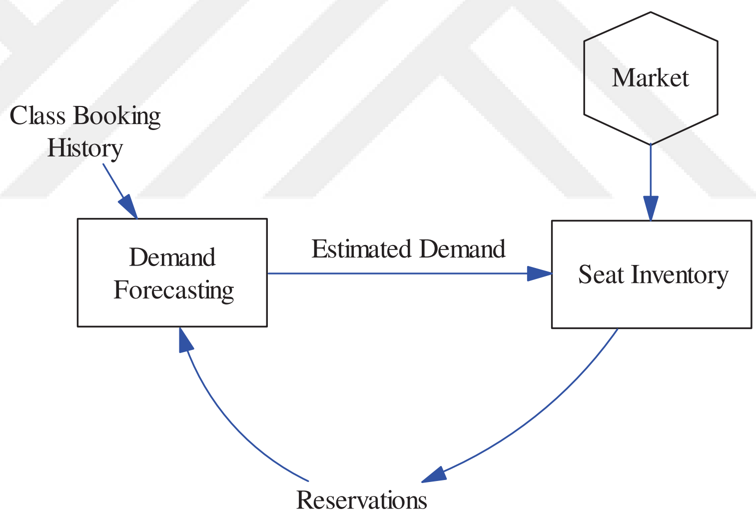

Figure 2.2: Demand Forecasting Structure

<u>2.2.2.2. Time Series Analysis:</u> The main forecasting tool in air transportation sector is the time series analysis. The method of time series decomposition, methods of moving averages and different smoothing methods belong to the methods of time series analysis [7].

We can categorize the time series analysis into four components as seen Figure 2.3 which is adopted from the statistical analysis of movement:

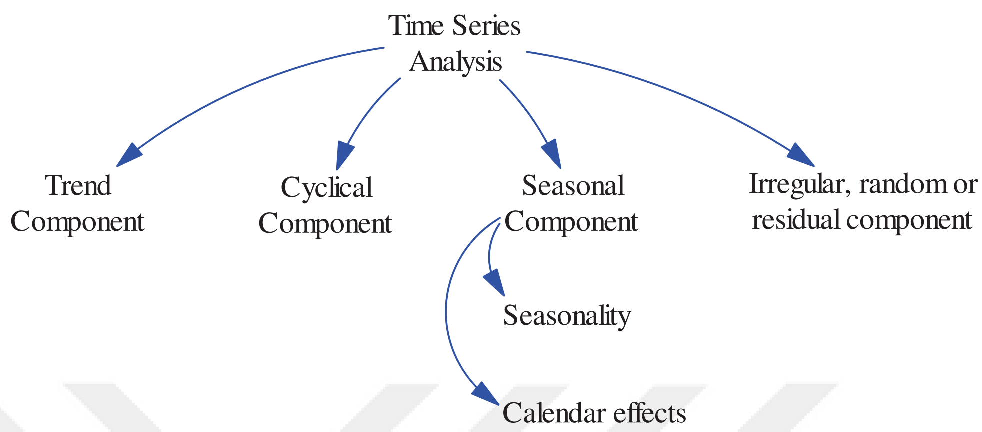

Figure 2.3: The Time Series Decomposition

- Trend component means the combined long-term tendency of phenomenon movement in time.
- Cyclical component means periodical repeating of particular values.
- Seasonal component means the systematic variations of the time series. The seasonality component can be divided into seasonality and calendar effects [8].
  1. Seasonality means the trend repeating in the period; in the same month or quarter every year.
  2. Calendar effects mean the alteration because of the structure of calendar. (For example, moving holiday effects, trading day effects).
- Irregular, random or residual component means the random fluctuations of short-term movements.

The time series decomposition relies on the estimation on the division of the basic components from the time series. For each component the estimation of the future values is practiced by forward extrapolation and the average estimate is achieved by the merging the different estimates [7].

- In additive model, there are some assumptions that the seasonal and irregular component are independent of the trend and the width of seasonal alterations does not change over time and the annual overall of seasonal fluctuations equals to zero. The original time-series Z_t can be shown as:

$$Z_t = T_t + C_t + S_t + I_t \tag{2.1}$$

Z_t: The original non-seasonally adjusted series
T_t: The trend component
C_t: The cyclical component
S_t: The seasonal component
I_t: Irregular, random or residual component

- In multiplicative model, the model is based on some assumptions that the seasonal component width is proportional to the trend level and the irregular component variance is proportional to the value of systematic components.

The original time-series Z_t can be shown as:

$$Z_t = T_t * C_t * S_t * I_t \tag{2.2}$$

<u>2.2.2.3. Forecasting Methods:</u> In order to calculate air transportation demand forecasts, Exponential Smoothing (Holt Winters, ETS (Error, Trend, and Seasonality)), ARIMA (Autoregressive Integrated Moving Average) and Regression are usually used as a time series tool by revenue management systems and demand analysts [9].

- Exponential Smoothing (Holt Winters and ETS): In the capacity management systems, the exponential smoothing methods are more prevalent owing to their simplicity, robustness and precision. Exponential smoothing methods were extended nearly fifteen methods until 2003 [10]. Some of these exponential smoothing algorithms are known as the simple exponential smoothing (N, N) cell in table, Holt's linear method (A, N) cell in the table, the damped trend method (A_d, N) cell in table, the additive Holt-Winters' method (A, A) cell and the multiplicative Holt-Winters' method (A, M) cell in the table, as seen Table 2.1 extended by Taylor (2003):

Table 2.1: The Exponential Smoothing Methods

	Seasonal Component		
Trend Component	N (None)	A (Additive)	M (Multiplicative)
N (None)	N, N	N, A	N, M
A (Additive)	A, N	A, A	A, M
A_d (Additive damped)	A_d, N	A_d, A	A_d, M
M (Multiplicative)	M, N	M, A	$M\ , M$
M_d (Multiplicative damped)	M_d, N	M_d, A	M_d, M

Simple exponential smoothing (SES) is the basic exponential smoothing method with the smoothing constant $\alpha \in [0,1]$. This constant is appointed to z_t as the weight.

$\hat{Z}_{t+1}$, the estimate value for period $t+1$, can be computed as the weighted overall of the present and estimate time series values and $1-\alpha$ is appointed to $\hat{Z}_t$ as the weight.

$$\hat{Z}_{t+1} = (1-\alpha)\hat{Z}_t + \alpha z_t \tag{2.3}$$

For the fixed period of forecast:

$$\hat{Z}_{t+k} = \hat{Z}_{t+1}\ , k = 1, \dots, K \tag{2.4}$$

And, the formula can be written as:

$$\hat{Z}_{t+1} = \ \alpha \textstyle\sum_{j=0}^{\infty} (1-\alpha)^{j}\, z_{t-j} \tag{2.5}$$

Holt's method which is linear exponential smoothing is utilized to smooth data that include the linear trend. Under the assumptions of $0 < \alpha < 1$ and $0 < \beta < 1$, the estimate for interval $t+1$ for L and T smoothing parameters is expressed as:

$$\hat{Z}_{t+1} = L_t + T_t \tag{2.6}$$

$$L_t = \alpha z_t + (1-\alpha)(L_{t-1} + T_{t-1}) \tag{2.7}$$

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1} \tag{2.8}$$

For the last value of L:

$$\hat{Z}_{t+k} = L_t + kT_t\,, \quad k = 1, \dots, K. \tag{2.9}$$

Holt-Winters' method which is exponential smoothing method with trend and seasonality is used for a series of data except for trend include the seasonal component. For L, T and S, $0 < \alpha < 1$, $0 < \beta < 1$ and $0 < \gamma < 1$ are smoothing parameters. L is the length of the season. This method breaks into two versions, multiplicative and additive.

For the Holt-Winters' multiplicative method, we can define the interval as $t + k$ and the formula can be explained:

$$\hat{Z}_{t+k} = (L_t + kT_t)S_{t+k-L},\ k = 1, \dots, K. \tag{2.10}$$

And three components of estimate values are:

$$L_t = \alpha(z_t \,/\, S_{t-L}) + (1-\alpha)(L_{t-1} + T_{t-1}) \tag{2.11}$$
$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1} \tag{2.12}$$
$$S_t = \gamma(z_t \,/\, L_t) + (1-\gamma)S_{t-L} \tag{2.13}$$

In the Holt-Winters' additive version,

$$\hat{Z}_{t+k} = A_t + kT_t + S_{t+k-L}, k = 1, \dots, K. \tag{2.14}$$

And three components of estimate values are:

$$A_t = \alpha(z_t - S_{t-L}) + (1-\alpha)(A_{t-1} + T_{t-1}) \tag{2.15}$$
$$T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1} \tag{2.16}$$
$$S_t = \gamma(z_t - A_t) + (1-\gamma)S_{t-L} \tag{2.17}$$

With the error parameter, exponential smoothing can be ensured in remembering the order in which the components are specified. ETS provides that convenience in exponential smoothing methods, and it consists of three components: error, trend and seasonality. The notion of $ETS(A, A, N)$ means a model with additive errors, additive trend and no seasonality and it is called Holt's linear method with additive errors.

Exponential smoothing method is very common in aviation demand forecasting analysis, in particular, Holt-winters' model is very useful and ETS is used when automatic forecasting is preferred. But, the optimal forecasts for all calculation with exponential smoothing cannot be obtained because a full three years of seasonal data are required to implement the seasonal forecasts using Holt Winters' method [10]. With deseasonalization technique, ETS forecasting without a full three years of seasonal data can be provided. Firstly, decomposition of the data is carried out and then deseasonalization is applied. After this step, seasonality multiplier are reached, and ETS method can be used to estimate without seasonality effect in data. By seasonality multiplier, demand forecasts with seasonality effects can be acquired. However, this technique sometimes cannot produce a good estimation like ARIMA or Regression methods.

- ARIMA: A time series demand model is stochastic process where an ordered set of random variables in the time index *t* located a finite or countable infinite sequence of value. Mean and variance of this process are utilized to determine it together with two functions: the Autocorrelation Function (ACF) and the Partial Autocorrelation Function (PACF). The ACF is a dimension of the correlation between two variables generating the stochastic process and The PACF dimension is the net correlation between two variables.

An ARIMA (Auto Regressive Integrated Moving Average) model is determined on the basis of the estimated ACF and PACF, starting from the original data. Especially, the characteristics of ACF and PACF enable the identification of the model order [11]. When the row series is given by Y_t, the differenced series is:

$$z_t = Y_t - Y_{t-1} \tag{2.18}$$

If there is an exponential decrease in the ACF, an AR (Autoregressive) model is suitable. If there are spikes in the first one or more lags of the ACF and an exponentially decreasing PACF, a MA (Moving Average) is appropriate. A mixed model is supposed to demonstrate exponential decreases in both functions. An $AR(1)$ process and with backshift operator B are:

$$z_t = \theta_1 z_{t-1} + a_t \tag{2.19}$$

$$(1 - \theta_1 B) z_t = a_t \tag{2.20}$$

An $AR(2)$ process and with backshift operator are:

$$z_t = \theta_1 z_{t-1} + \theta_2 z_{t-2} + a_t \tag{2.21}$$

$$(1 - \theta_1 B - \theta_2 B^2) z_t = a_t \tag{2.22}$$

A $MA(1)$ process and with backshift operator are:

$$z_t = a_t - \theta_1 a_{t-1} \tag{2.23}$$

$$z_t = (1 - \theta_1 B) a_t \tag{2.24}$$

A $MA(2)$ process and with backshift operator are:

$$z_t = a_t - \theta_1 a_{t-1} + \theta_2 a_{t-2} \tag{2.25}$$

$$z_t = (1 - \theta_1 B - \theta_2 B^2) a_t \tag{2.26}$$

For an $ARIMA(1,1)$ model can be written as:

$$z_t = \theta_1 z_{t-1} + a_t - \lambda\, a_{t-1} \tag{2.27}$$

$$(1 - \theta_1 B) z_t = (1 - \lambda\, B)\, a_t \tag{2.28}$$

If there is both stationary, serial dependence and unexplained components such as interdependence with other series after the raw data is analyzed, the derived ARIMA model could be the best to use [12].

When seasonal components are incorporated in the ARIMA model, the model is named as the SARIMA model. If there is no seasonal effect, a SARIMA model turns out to be pure $ARIMA(p,d,q)$ and if the dataset is stationary, ARIMA mitigates to $ARMA(p,q)$. In the airline passenger demand forecasting, non-seasonal $ARIMA(p,d,q)$ and seasonal multiplicative $ARIMA(p,d,q)(P,D,Q)^s$ are the most used [13].

- Regression Analysis: Demand forecasting of air passengers consists of two analysis perspectives. The first one is Macro-analysis. It is interested in air transportation activities and aims to estimate total levels of air transportation without any specific route analysis. And the second one is Micro-analysis [14]. It is concerned with more measures of air traffic and focused on more specific market routes such as origin-destination flows. To have feasible solutions with regression analysis, choice of model approach is very significant.

In Bangladesh, Jobair and Karim studied the condition of air transportation and they analyzed the variation of demand forecasting with the socio-economic, technological and transportation system parameters for the domestic air travel market [15]. In order to reach reasonable city-based solution, they focused on microanalysis. Socio-economic parameters are very important for the countries where air passenger traffic level is not satisfied and where an airline is a monopolistic carrier. Hence, they use population, employment, gross domestic product (GDP) beside speed of aircraft, speed of alternatives and distances parameters. In their study, they use multiple regression analysis technique for the application of a great range of estimation problems to comprehend the relationship between the dependent variable and a group of explanatory variables: socio-economic, demographic, market factors, travel impedance and intermodal competition. In the light of this study, air trip regression model can be generalized as similar to:

$$Demand_{ift} = A * (Population_{it})^{\beta_i} * (GDP_{it})^{\beta_2} * (ticket\ fare_{ift})^{\beta_3} * (capacity_{it})^{\beta_4} * (\ year_t)^{\beta_5} * (frequency_{it})^{\beta_6} * \exp(\gamma_1 Tourism_t + \gamma_2 HUB_i + \gamma_3 Competition_{it} + \gamma_4 Weekend_t + \gamma_5 Season_t + \varepsilon\ ) \tag{2.29}$$

i: Market route

t: The day of observation

f: Fare class

β_i: Demand-price elasticity

This is a non-linear equation and we can transform into linear form with natural logarithm. This situation facilitates to calculate the model. However, sometimes non-linear models (quadratic and log-linear models) cannot reach to the optimal forecast according to the data, and linear models (simple or multiple linear models) could out-perform. This status shows that the determination of model is very crucial for accurate forecasting.

2.2.3. Optimization

The seat inventory control in revenue management is very important problem which has to be resolved efficiently. Optimization tool emerged with a solution method for the seat inventory control problem for a single leg flight with two fare classes in 1972. Littlewood proposed shutting down the cheaper fare class while the certain income from selling the cheaper price was surpassed by the expected revenue of selling the same seat at the higher price. Each flight leg can be optimized separately with single leg seat inventory control method. In order to allocate seats properly, revenue management system needs a protection strategy. There are two seat protection perspectives: partitioned protection and nested case. A partitioned protection means a booking limit of 15 on class C sales is equal to protecting 15 seats only for class C. In the nested case, a booking limit of 15 seats on class C sales is equivalent to a protection level of 15 for classes A, B and C combined. In the addition to single leg optimization, network seat inventory control was formulated by Glover, Lorenzo and McMillan in 1982. They targeted to reach the flow on each flight leg in the network with maximizing revenue [16]. However, an obstacle of this method is the indiscrimination of the routes from an origin to a destination. Therefore, this method works only when passengers are indifferent on flight path. To handle this obstruction, network seat inventory control is differentiated over market routes passenger demand. In revenue management system, if more passengers use direct flights and the network contribution is very low, the optimal seat allocation strategy can be obtained by single leg optimization technique.

Single leg inventory control has two solution methods: static and dynamic solution methods. Static method produces an optimal seat assignment at a precise point in time. But, the authentic booking claims do not reach at one point in time, they happen gradually over the booking period. On account of this, in order to accomplish a better solution, the actual demand can be monitored and the booking control policy must be revised with the reservation instant feedback. This method is called dynamic solution method [17].

In the dynamic model, there are n fare classes, $p_1 \geqslant p_2 \geqslant p_2 \geqslant \cdots \geqslant p_n$. There are T total periods and time index runs from $t = 1$ to $t = T$, and t represents periods and j displays classes. The probability of an arrival of class j in period t is denoted $\lambda_j(t)$. The assumption of at most one arrival per period implies that we have [18]:

$$\sum_{j=1}^{n} \lambda_j(t) \leqslant 1 \tag{2.30}$$

$$P\left(R(t) = p_j \right) = \lambda_j(t) \tag{2.31}$$

$R(t)$ (Random variable) is equal to p_j if a demand for class j arrives in period t, otherwise $R(t) = 0$.

In order to generate value function ($V_t(x)$) in period t, u can be defined control parameter. If the arrival is accepted, u equals to 1, otherwise $u = 0$. The total maximization of current revenue,

$$R(t)u + V_{t+1}(x - u) \tag{2.32}$$

The Bellman equation is:

$$V_t(x) = E\left[\max_{u \in \{0,1\}} \{R(t)u + V_{t+1}(x - u)\}\right] \tag{2.33}$$

$$V_t(x) = V_{t+1}(x) + E\left[\max_{u \in \{0,1\}} \{\left(R(t) - \Delta V_{t+1}(x)\right)u\}\right] \tag{2.34}$$

$$\Delta V_{t+1}(x) = V_{t+1}(x) - V_{t+1}(x - 1) \tag{2.35}$$

This is the expected marginal value of capacity in period $t + 1$. The restrictions are:

$$V_{T+1}(x) = 0, \qquad x = 0,1, \dots, C \tag{2.36}$$

$$V_t(0) = 0, \qquad t = 0,1,\dots,T \tag{2.37}$$

In order to achieve optimal policy, the booking request is accepted if and only if $\Delta V_{t+1}(x) \leqslant p_j$. Optimal solution can be achieved via bid-price control. Bid-price shows the seat marginal value. If the offered price value surpasses this threshold, booking can be happened.

$$\Delta V_t(x) = \pi_t(x) \tag{2.38}$$

Thanks to bid-price control, time-dependent optimal protection levels can be defined as:

$$y_j^*(t) = \max \left\{x : p_{j+1} < \Delta V_{t+1}(x) \right\}, \quad j = 1,2,\dots,n-1 \tag{2.39}$$

The protection levels are nested and the optimal acceptation of j class exists if and only if the remaining capacity surpasses $y_{j-1}^*(t)$. $y_j^*(t)$ is the capacity and is protected for classes $j-1,\dots,1$.

$$y_1^*(t) \leqslant y_2^*(t) \leqslant \cdots \leqslant y_{j-1}^*(t) \tag{2.40}$$

And the nested booking limits can be described as:

$$b_j^*(t) = C - y_{j-1}^*(t), \quad j = 2,\dots,n \tag{2.41}$$

2.3. Revenue Management and Demand-Price Relationship

Fare management is a very important tool enabling an increase of income in airline operations. Thus, the determination of fare levels and the availability of seats based on fare levels need a better and more detailed understanding. If the price is arranged low, potential income will be lost, however, if the fare is determined to be high for that available seat, potential demand will be missed. When the low-price levels are not available, customers may choose to postpone purchasing a ticket or select the other airline or decide to travel with a substitute transportation vehicle such as their own cars, trains, or buses [19].

Because of these reasons, in our study, we are focusing on demand modelling by incorporating the effects of the airline's and its competitor's price. This situation triggers a more realistic solution. In revenue management studies, usually macro-analysis method is selected to forecast demand regardless of the effect of competitor's price. After online shopping started to widely use via internet, the competition in air transportation sector has exacerbated. Although there are a number of studies including the notion of competition, these studies generally only take into account power of the airline on the market (measured by capacity, frequency, speed of aircrafts), but not the competitor's price. However, if the rival fare is lower than you, these all parameters could have almost no effect on the choice of consumers [20]. In order to overcome the price effect, many researchers have been focused on price-demand elasticity in airlines and they have used demand functions including socio-economic factors, GDP, frequency, fare levels, capacity, year and dummy variables (holiday location or not, HUB point or not, etc.) [21]. Contrary to these studies, we have determined our parameters which affect the demand function directly and focused on microanalysis based on the market routes with direct flights. In order to achieve right demand models, we have used ETS, HW, ARIMA, SARIMA, and Regression Analysis from time series models and compared them for the determination of proper fixed model with the real data. After we reached to best fitted model, we have used it to develop the optimal revenue strategy.

3. DEMAND MODELLING IN AIRLINE COMMERCIAL SECTOR

In the commercial airline sector, demand forecasting has become the most important revenue management tool because of open competition in markets. To achieve a satisfactory forecast, one needs to consider market conditions in which the airline operates. Selected market routes can be investigated in terms of many different parameters including seats sold which refers to sales, capacity, amount of flights referring to frequency, seasonality, average fare with rival fare and potential demand. Time series analysis and R-Project are very useful to generate forecasting models with the opted routes' data. After comparing all models in the market, a satisfactory model can be achieved.

3.1. Market Conditions

Market conditions must be examined region by region because all regions have their own market trends and they can be differentiated from country to country. In all air travel markets, a conspicuous decreasing in air fares caused by deregulation and risen competition with macro-economic and socio-demographic developments have a key role in demand growth. In recent years, price transparency triggers low fares and produce new demand groups. Especially, leisure travel has developed with the increase of alternative air transportation and fare types such as Low Cost Carriers, Charters and promotion fares which are launched several months before the departure. In addition to the dramatically low fares, air travel passengers are dependent on their characteristics according to income, home ownership abroad, aim of traveling, travel budgets and the number of trips taken in previous twelve months [22].

In the air transportation sector, we can categorize the main headlines that airlines present their passengers and customers choose an airline company which is satisfied:

- Acceptable air travel price
- Enough aircrafts which have enough seat capacity for interested market
- Easy flight

- Good service in the aircraft
- The entertainment tools in the aircraft
- Relatively delicious food during the travel
- Relatively comfortable seats
- Enough luggage capacity per passenger
- Well luggage service
- Airport, travel and transfer security
- Transportation from the departure city to the airport and from the airport to the arrival city

These services are waited by passengers for all airlines. However, with very low fares, passengers could waive their some expectations for that travel. At the present time, many passengers' claims are very high. They want to have very good and comfortable travel and have good services irrespective of low fares.

When we look at market conditions region by region, according to the study launched by IATA (International Air Transportation Association), geographic market conditions indicate important differences [23].

- North American market has relatively enough capacity and it has unit elasticity. Its passenger traffic is very high. For short haul and medium haul, its air travel fares head for low fares. These conditions trigger very low growth for airlines in short and medium haul traffics. And they tend to increase their profitability with the cost efficient operations. For this region, capacity discipline is very crucial to improve premium traffic, and they focused on increase Load Factors with demand growth [24]. South America has more elasticity than North America. In this region, LCCs are increasing, especially in Brazil. Nonetheless, its passenger traffic is very low when compared to North America.
- In Europe, airlines use very low fares in varied market routes. This region is more elastic than North America. European routes have shorter overall travel distances and robust rivalry exists. Europe contains many airlines with LCCs and Charters. This situation triggers low fares and high competition.

- Asia has relatively inelastic demand when compared to North America. Low cost carriers are newly increasing but overall distances between airports are longer and middle class demand is moderately low in this region.
- Africa region has more inelastic demand than Asia. African economies slog on the production of middle class demand. And for this region, air travel is concentrated among individuals having higher revenue and less-fare sensitive [23].

Beside all these conditions, all markets have their own aviation industry's conditions. For example, technological improvements are very important situation for air transportation. Thanks to technology, passengers can access their flights easily and they can do online check-in before the departure. Wherever they are, they can buy a ticket with internet or mobile phones, and they can easily learn flight security and details. On the other hand, travel agencies need to widespread of the internet network because they have to buy a ticket when a customer requests and they can offer different airlines with different locations and fares. With internet applications such as Booking.com, Trip advisor or Expedia and others, all airlines can compete with the others, and new demand groups with low fares and advertising can occur. These circumstances provide more competitions and more air passengers. In addition to the technological development, airport's quality adds a positive value to the air transportation sector. Ground services, waiting rooms, the cleanliness of the airport, luggage transferring and loading to aircrafts are significant for passengers and they do not want to experience any troubles regarding these issues.

In the addition to the aviation industry's conditions, all markets have their own macro-economic background (the effects of Gross Domestic Product per Capita, interest rates, wealth and well-being) and socio-economic and demographic (holiday culture, universities including students and professors from various countries, migration rate) factors. These factors affect air transportation demands. People from the countries which have high well-being citizens ignore low fares and they search an acceptable price that can be high. However, customers whose countries have very low GDP are extremely price-sensitive, if fares a little high from their expectations; they can cancel their travelling plans.

All these factors considered, airlines must be very sensitive to the markets and they have to investigate consumers' expectations and produce tangible solutions. Sometimes it can be about inflight service or sometimes it can be agencies' development in the market or sometimes it can be the cleanliness of the airport because all these factors influence airlines the most.

3.2. The Choice of Market Routes

In the aviation sector, airlines must select their market routes with a meticulous study. In order to achieve right routes, market analysis based region must be done very well. Market analysis contains two main perspectives in air transportation sector.

- First one is local market including direct flights.
- The second one is transit market including beyond markets.

In order to reach the optimal decision, the research should be done with the regional factors because transit markets have their own specific regional characteristics. With the market examination, the fundamental aim is the estimation of demand potential on different paths. The main study spaces in the determination of the market routes are:

- The alteration of demand potential
- Seasonality
- Competitor analysis
- Competitors' market shares
- Flight frequency
- Passenger's profiles [22].

At the same time, these principles are very significant for revenue management mentality. Unlike revenue management, airlines have to incorporate cost factors to decide market routes. In the addition to these calculations, airport slots availability, flight permits, state policies, visa status, aircrafts availability, the distance between two airports, airport taxes, the facilities and difficulties of money transferring, regional security, and extra rights are taken into consideration and a decision is made.

In the light of these principles, we selected eight market routes based local markets which have high passenger traffics and reasonable good seat capacities. So as to design a good demand model, we focused on direct flights based a single leg operation. In the leg-based study, number of passengers for a market route, fares, competitors' fares and market potential can be better clearly analyzed. Otherwise, for example, in the LYS (Lyon) - IST (Istanbul) flight, there can be many passengers going from LYS to NRT (Tokyo) or from LYS to VKO (Moscow) or from LYS to ESB (Ankara) with IST hub connection beside the local passengers going from LYS to IST. In this situation, the determination of average fares is more formidable. Because of this inconvenient computation, we analyzed the market routes which have very little transit effects and high local passenger profile.

We launched market routes as Route A, Route B, Route C, Route D, Route E, Route F, Route G and Route H.

- Route A in The European Continent
- Route B in The African Continent
- Route C in Turkey
- Route D in The Asian Continent
- Route E in The Asian Continent
- Route F in The American Continent
- Route G in The American Continent
- Route H in Turkey

From Europe, we have opted only one location because its market characteristics are available, it has more local passengers and its transit rate on all passengers is very low. Demand variation and seasonality can be seen in all years as seen Figure 3.1.

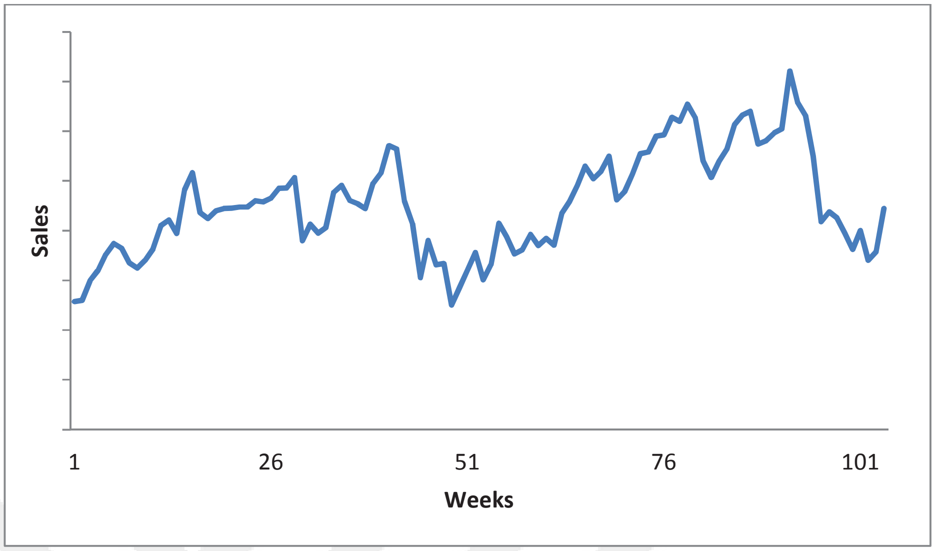

Figure 3.1: The Seasonality of Route A in Europe

Seat capacity and the amount of aircrafts are plausible. In this market, competitor has a significant role and it has nearly 30% of market rate and it has many flights on that way. Similarly, the market route B in Africa has the same characteristics as seen Figure 3.2.

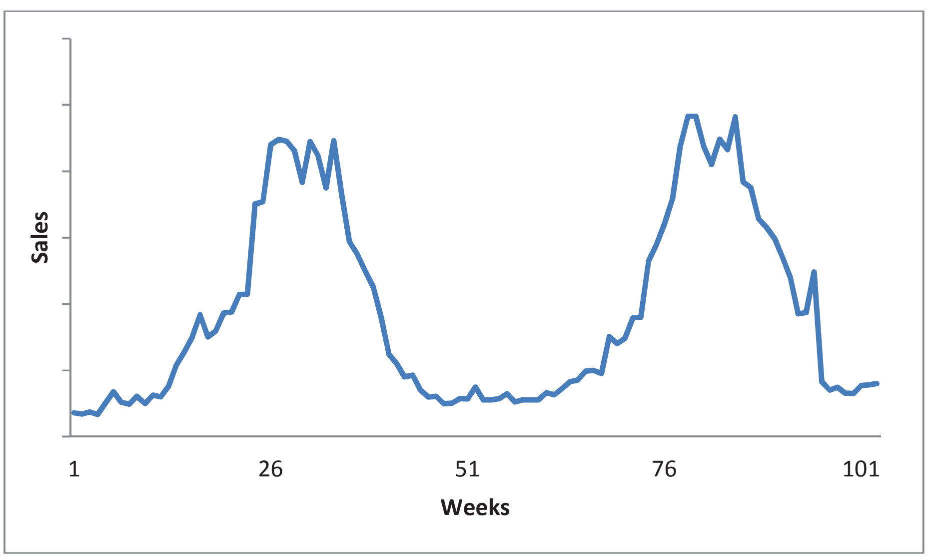

Figure 3.2: The Seasonality of Route B in Africa

From Asian and American Continents, we selected two market routes for each continent because there are capacity problems with inadequate aircrafts. Demand variation can be seen in all years but we cannot see seasonality exactly for these four market routes as seen Figure 3.3-3.6.

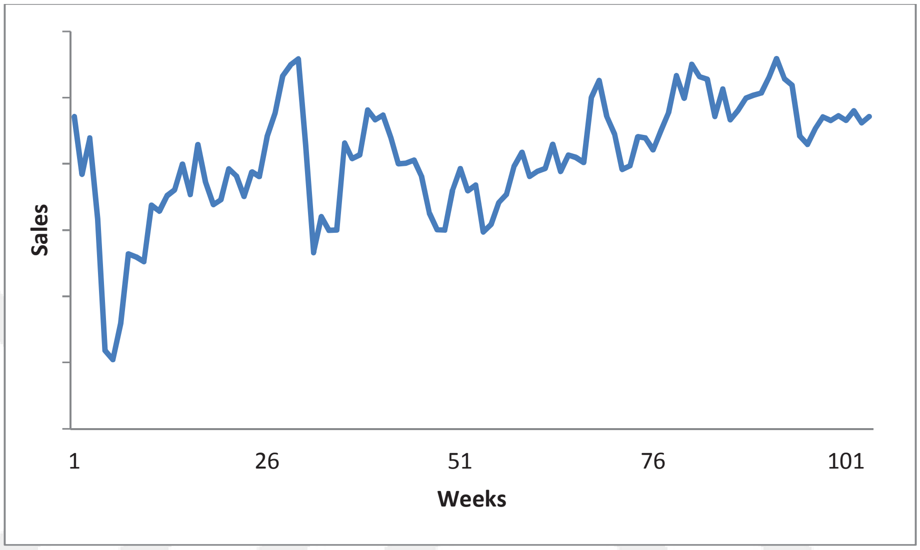

Figure 3.3: The Seasonality of Route D in Asia

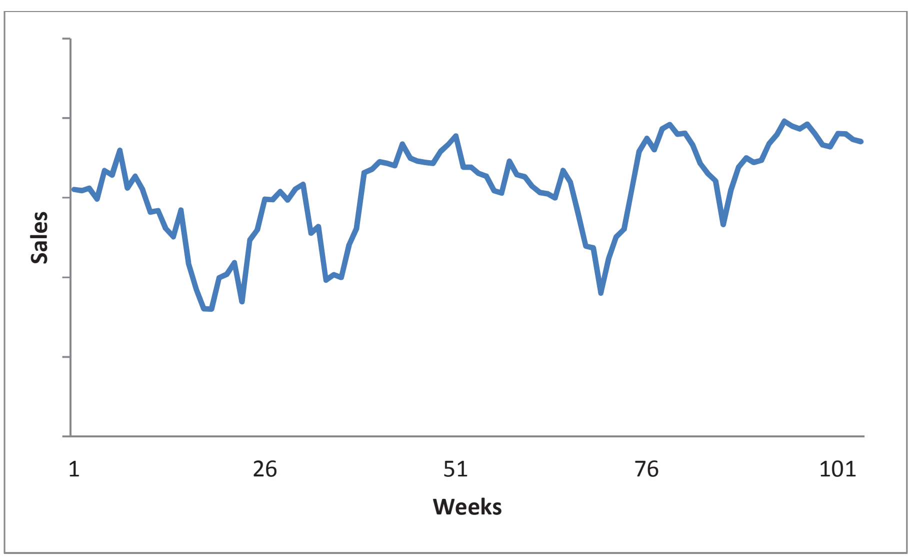

Figure 3.4: The Seasonality of Route E in Asia

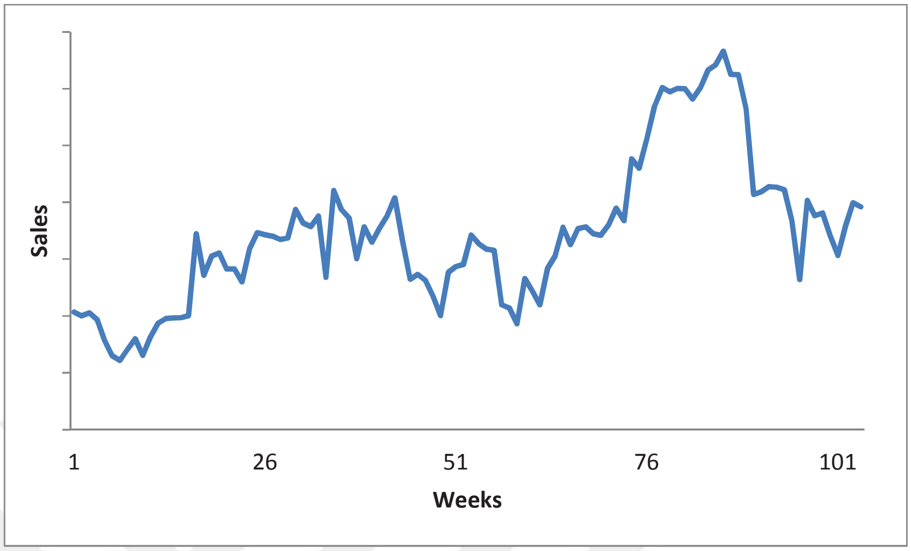

Figure 3.5: The Seasonality of Route F in America

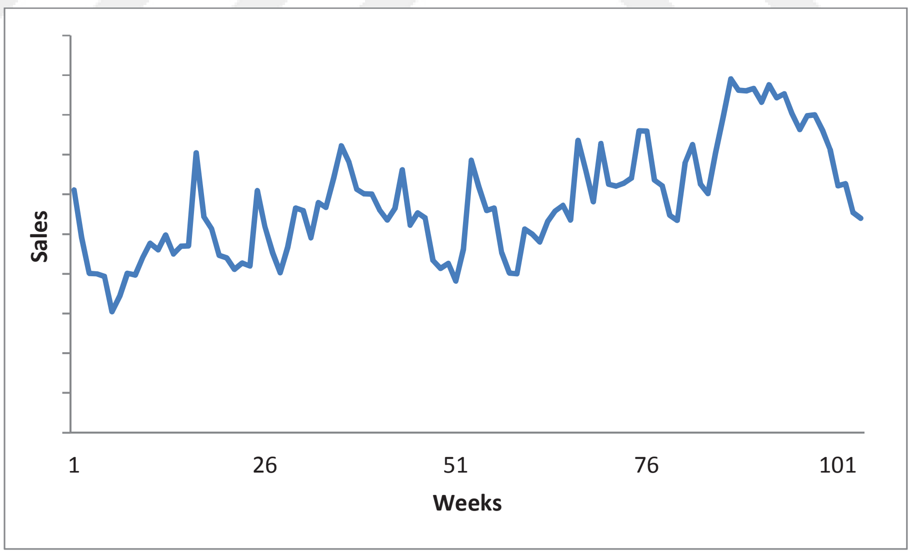

Figure 3.6: The Seasonality of Route G in America

In these markets, competitors have very significant roles and they have nearly 30-40% of market rate for each market route. To design a demand model with the right market conditions, these two similar markets for each region shed light on our study.

In addition to these market routes, we selected two destinations in Turkey to have specific characteristics in a specific country with two different markets. These routes have their own certain factors. In the market route C, the competitor has a very little role and it has nearly 20% of market share and it does not have enough capacity and aircrafts. For this market route, we can see seasonality and demand variation in all years as seen Figure 3.7. But, for market route H, we cannot see seasonality exactly as seen Figure 3.8. The competitor in this market is prevalent and it has over 50% of market share.

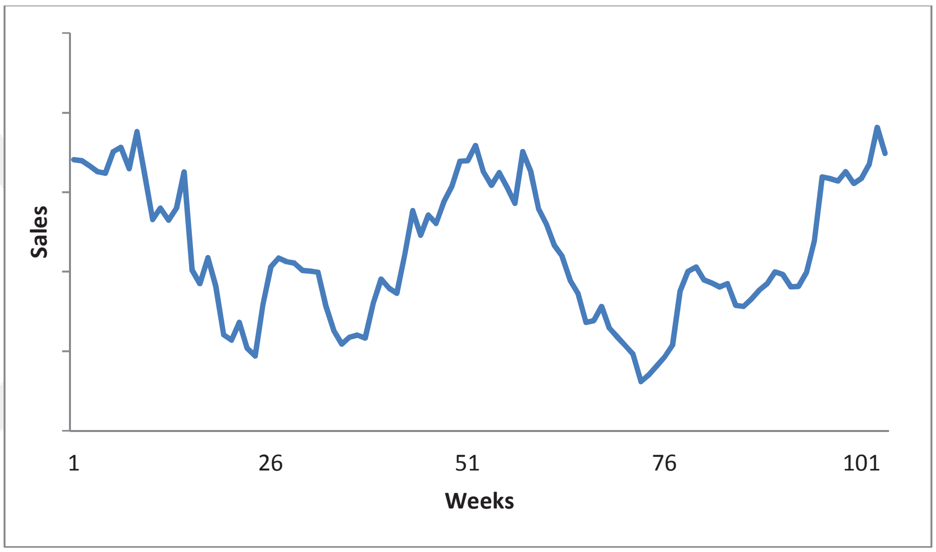

Figure 3.7: The Seasonality of Route C in Turkey

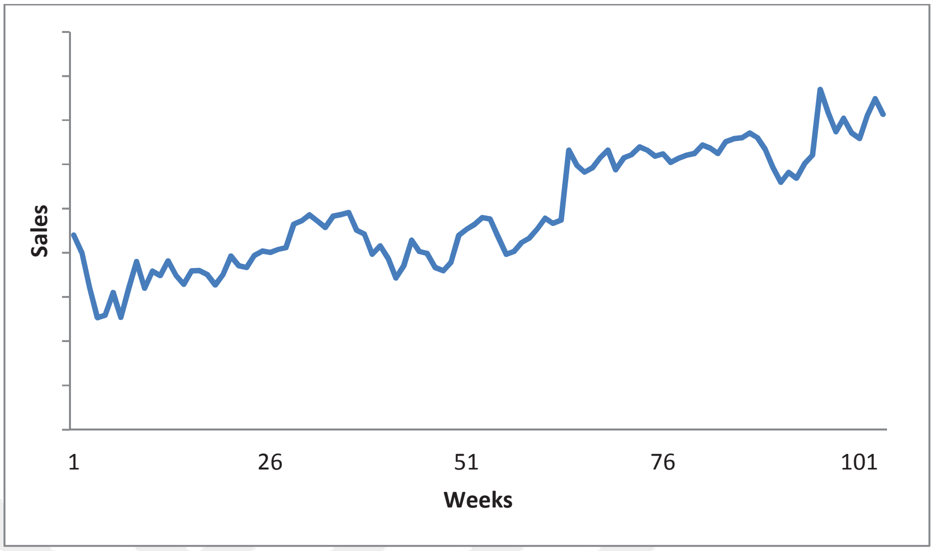

Figure 3.8: The Seasonality of Route H in Turkey

3.3. Description of the Data

3.3.1. Aggregation of the Data

The model of air transportation passenger demand can be classified with the several ways of data collection. Some studies are based on the airport passengers' data beside the data of airlines, transportation agencies, transportation associations, markets and the other official sources. If the airports passengers' data is taken into consideration, there can be some troubles in terms of the focusing on the specific markets. These data only includes airlines' arrivals and departures data unlike O&D (origin to destination) perspective. In order to examine market routes as a special route with the local market passenger traffic, we focus on the data based O&D and the interested markets.

The data for the study springs from several various sources. So as to compare and combine right parameters, all possibilities are considered and the passenger traffic data with the perspective of deduction is examined. In the air transportation sector, the research based O&D perspective is very hard to make because all flights include many passengers who have different O&Ds. In order to achieve right solutions with the passengers data, firstly O&D

structures are examined and market routes which have very little transit markets are chosen. Therefore, the study turns to be an O&D market analysis based local traffic with the passenger demands. The data contains weekly passengers SS (seats sold) with the last week data as a different parameter like lastSS for two years and this study contains 104 weeks.

According to the market conditions, amount of the flight namely flight frequency and total seat capacity are very significant for the passengers because inadequate seat allocation to the market affects passengers' demands directly. Hence, the amounts of aircrafts operated by the same airline company and total seats are included. These parameters indicate market growth and seats sufficiency. With Seats Sold and Capacity parameters, seasonality plays a key role to understand passengers demand variation. Some routes have seasonality exactly and it can be seen, however, some of them do not show their seasonality effects. This situation gives a direction to reach the optimal demand modelling.

Moreover, the price variables take place in the parameters as the most significant variables due to its effect on the customers. For the description of the competitors' effect on the market, the rival airline which has the most market share or the second one is considered and its effect on the market is shown via sales price. Its capacity and seats sold values are ignored because of the other parameter which is called demand potential containing all these effects. Demand potential parameter shows that the passengers who preferred the other airlines.

$$Demand\ Potential_{it} = Total\ Demand_{it} - SeatsSold_{it} \tag{3.1}$$

i: Market route

t: The week of observation

With the competitor's price and demand potential parameter, the price of the seat is indicated as an average fare of the available seats because the fare of the seats can be changed over time and if only one price can be presented for the model, it must be average fare in the interested week. Similarly, competitor's price is the overall submitted ticket fares in that market.

On the contrary of these parameters, the effects of aviation improvements, airport developments, macro-economic, socio-economic and demographic factors in the interested markets are ignored because the chosen market routes have reached a reasonable saturation about all these issues. These market routes' air passenger traffics are very high when interested regions are considered and the influence of the change in these factors is very little on the passenger demands in these markets.

3.3.2. Usage of the Data

In the air transportation sector, the passenger data is very confidential due to security. This situation triggers airlines to hide all data from the other institutions. With the price transparency and open competition in the air transportation, airlines turn to generate data pools with the other airlines. Especially, they want to utilize their aviation alliances in terms of data sharing.

Unlike decade years before, air traffic is very crowded as seen in Figure 3.9 [25].

Figure 3.9: The Air Traffic in the World

These air traffic shows that there are many different markets which air passenger demand can differentiate region by region. This circumstance spurs to open up new markets because airlines have to use their flights with maximum capacity and maximum utilization because of cost efficiency. In order to fly to a new destination, airlines have to analyze that market very cautiously and they need to the past air passenger traffic data. Nowadays, this sharing issue is currently being discussed among authorities.

As mentioned above, some restrictions still go on. Hence, the data cannot be expressed as original because of some security anxieties. But, all graphs without numbers indicate right lines in the light of original data. In this study, market routes' original names are not mentioned, and the data which is shown are all manipulated with some techniques.

Table 3.1: The Data History of Route A

Week	SS	LastSS	Cap	Amount	Season	Avfare	Rival Fare	Potential
1	3489	4216	5318	33	0	36.08	28.62	1102
2	3526	3489	5520	33	0	33.47	26.01	1114
3	4061	3526	5404	33	0	36.46	29.01	1282
4	4340	4061	5632	33	0	34.38	26.92	1370
5	4777	4340	5973	37	0	33.85	26.39	1508
6	5078	4777	6398	37	0	34.79	27.33	1603
7	4947	5078	6333	37	0	34.08	26.62	1562
8	4545	4947	6141	37	0	33.99	26.53	1435
9	4406	4545	6107	37	0	31.97	24.51	1391
10	4614	4406	6253	37	0	35.07	27.61	1457

As seen in Table 3.1;

- SS shows seats sold of the airline (realized demand),
- LastSS shows last week SS and it refers to SS_{t-1}
- Cap shows total capacity of the same airline in this market route and it refers to c,
- Amount shows total amount of flights of the same airline in this market route and it refers to a,
- Season shows 1 or 0 (summer or not) and it refers to s,

- Avfare shows presented average fare and it refers to F,
- Rival Fare shows presented competitor's average fare and it refers to f_r
- Potential shows the passengers who preferred the other airlines and it refers to mp.

3.4. The Application of Time Series Models

In order to analyze Time Series Models, R-project is used. R-project provides very fast and efficient solutions and developments of the models can be done. HW-ETS, ARIMA and Regression give different models with R-project. To reach minimum error with the maximum accuracy, the data which contains realized data in 26 weeks is used to compare the models.

3.4.1. ETS-Holt Winters

Exponential Smoothing method is widespread of usage for many decades. ETS can be explained as an extended of exponential smoothing methods and it provides state space likelihood calculation. ETS focuses on former gathering of ad hoc approaches. However, it needs to have a full three years of seasonal data to implement the seasonal forecast with the simple exponential smoothing (ETS (N, N)), Holt's Linear Method (ETS (A, N)), the additive damped trend method (ETS (A_d, N)), the additive Holt Winters' method (ETS (A, A)) and the multiplicative Holt-Winters' method (ETS (A, M)). First of all, we ignore this requirement of the method and apply all market routes data to reach correct forecasts of next 26 weeks.

- Route A: The simple exponential smoothing model occurs and forecasting of this Route with the realized SS values are seen in Figure 3.10 and Figure 3.11.

```
ETS(M,N,N)

Call:
 ets(y = mydata1$ss)

  Smoothing parameters:
    alpha = 0.9092

  Initial states:
    l = 5099.3913

  sigma:  0.1026

     AIC     AICc      BIC
1902.750 1902.868 1908.038
```

Figure 3.10: Route A-ETS R-Project result

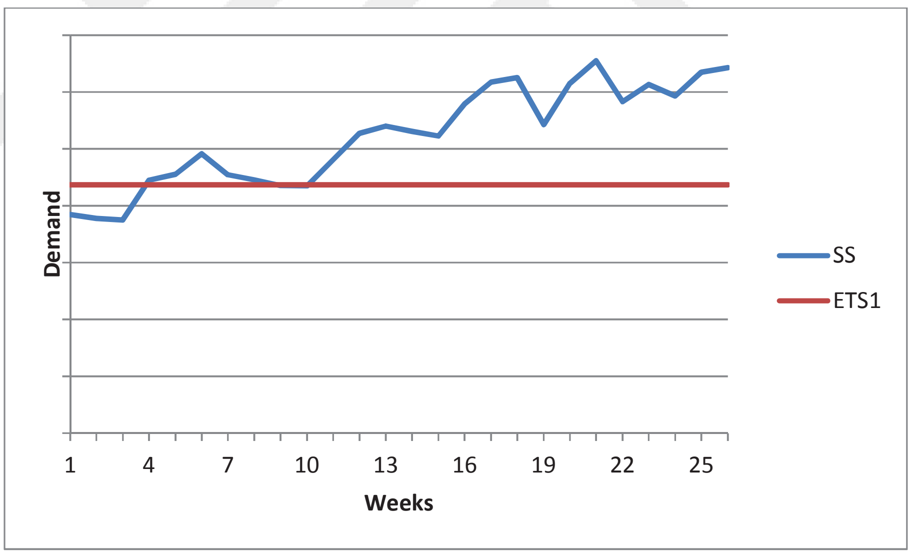

Figure 3.11: Route A-ETS forecasting result

This situation indicates that ETS forecasting model without any seasonality effect produces inefficient solution for Route A. Therefore, in order to calculate any reasonable forecasts, we apply deseasonalization technique to spur any seasonality.

i. Weekly data are turned out to be monthly data.

ii. Seasonal decomposition is subjected with the multiplicative perspective in R-project because multiplicative situations give more efficient solutions. Then, seasonality multipliers are created.

iii. Weekly-based Seats Solds are divided by these multipliers with the assumption that the effect of the seasonality in the same week is the same.

iv. These data series are used to forecast the next weeks' seats sold values.

v. The forecasting results are multiplied with the seasonality multipliers, and then they give us the forecast solutions, and the solution can be seen in Figure 3.12.

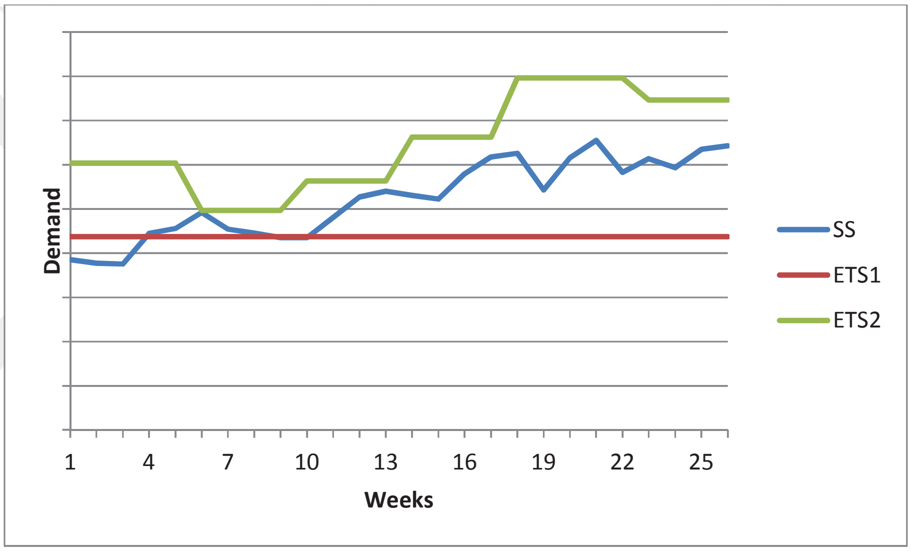

Figure 3.12: Route A-ETS2 forecasting result

- Route B: The multiplicative damped trend model occurs with ETS method and forecasting of this Route with the realized SS values is seen in Figure 3.13.

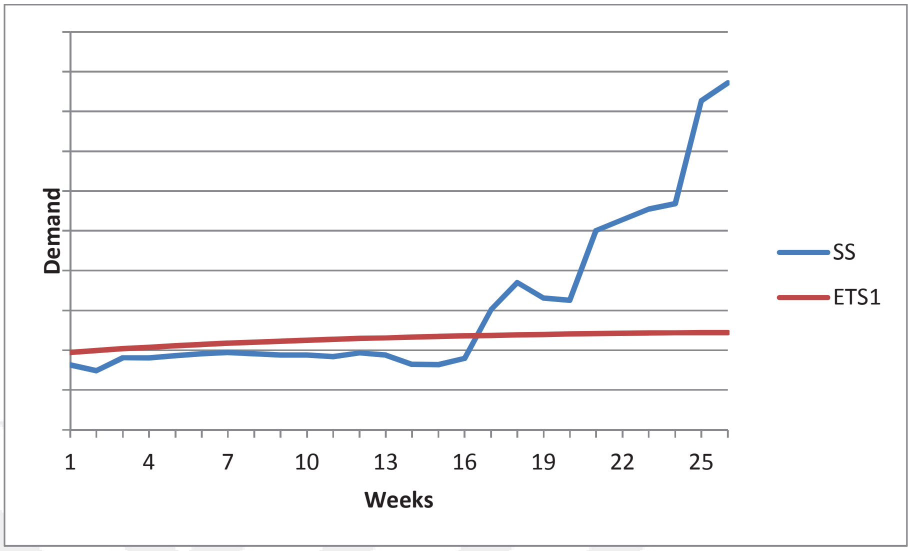

Figure 3.13: Route B-ETS forecasting result

With the deseasonalization technique, the forecasting result is seen in Figure 3.14.

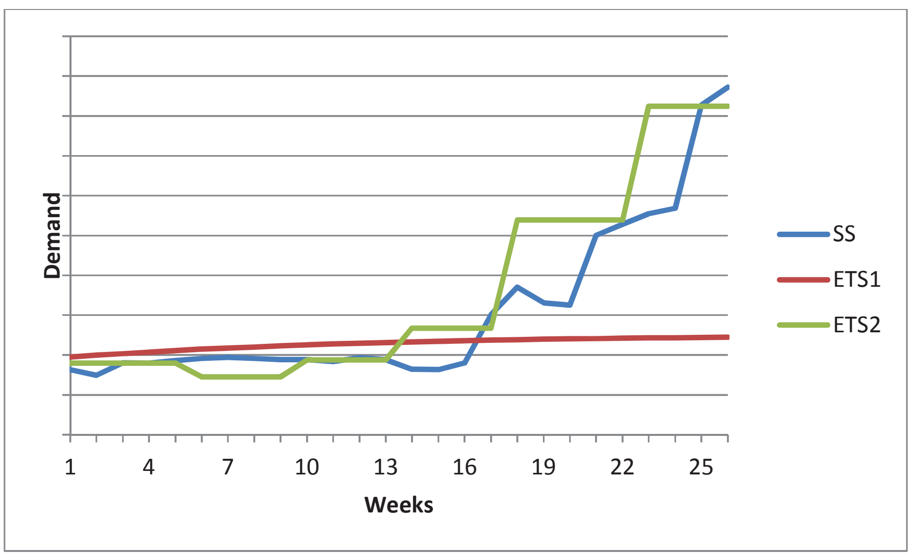

Figure 3.14: Route B-ETS2 forecasting result

- Route C: The simple exponential smoothing model appears with ETS method and forecasting of this Route with the realized SS values is seen in Figure 3.15.

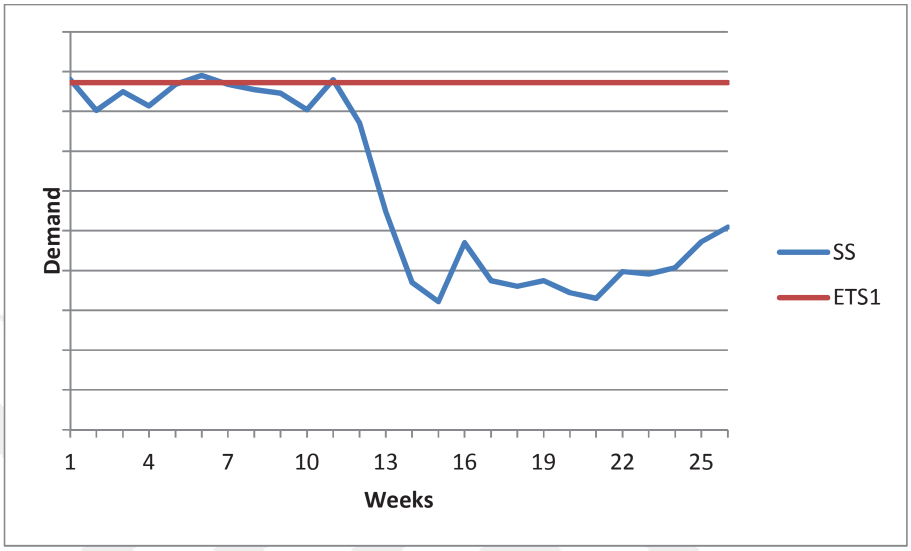

Figure 3.15: Route C-ETS forecasting result

With the deseasonalization technique, the forecasting result is seen in Figure 3.16.

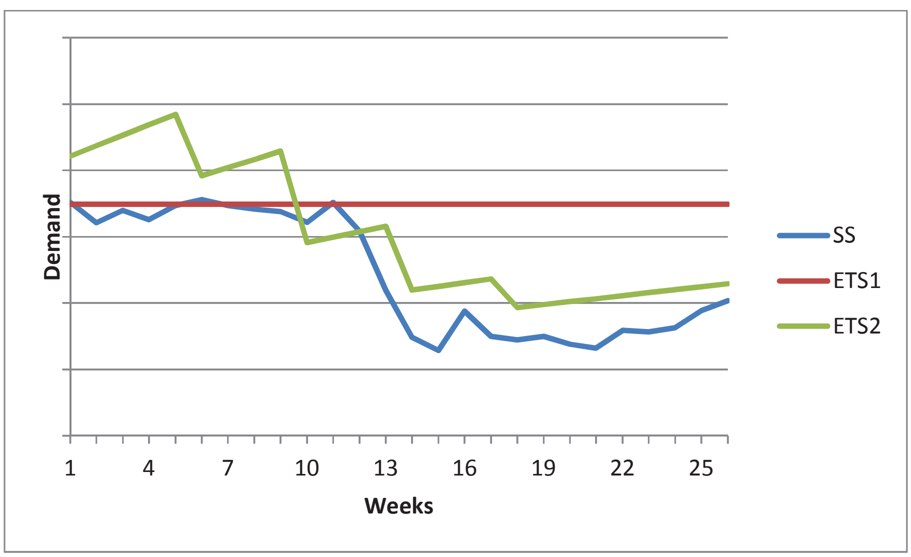

Figure 3.16: Route C-ETS2 forecasting result

- Route D: The simple exponential smoothing model seems with ETS method and forecasting of this Route with the realized SS values is seen in Figure 3.17.

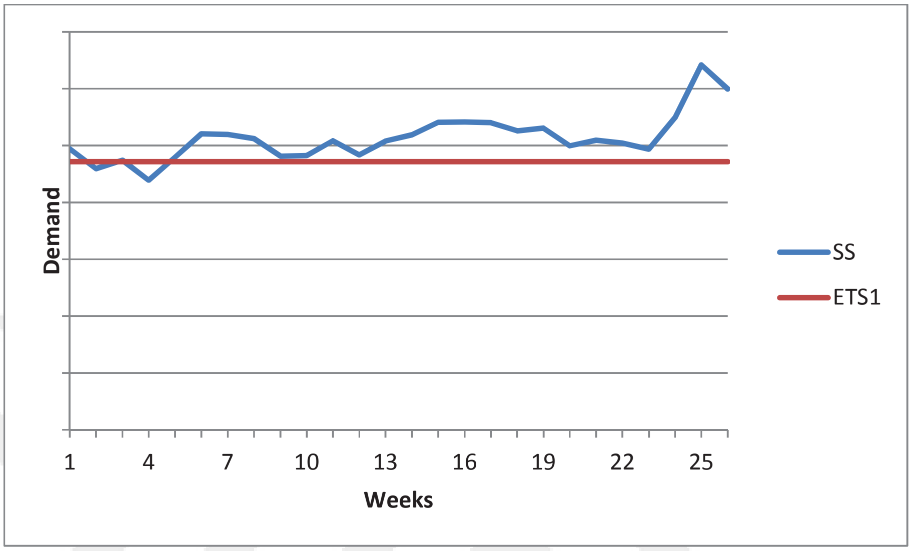

Figure 3.17: Route D-ETS forecasting result

With the deseasonalization technique, the forecasting result is seen in Figure 3.18.

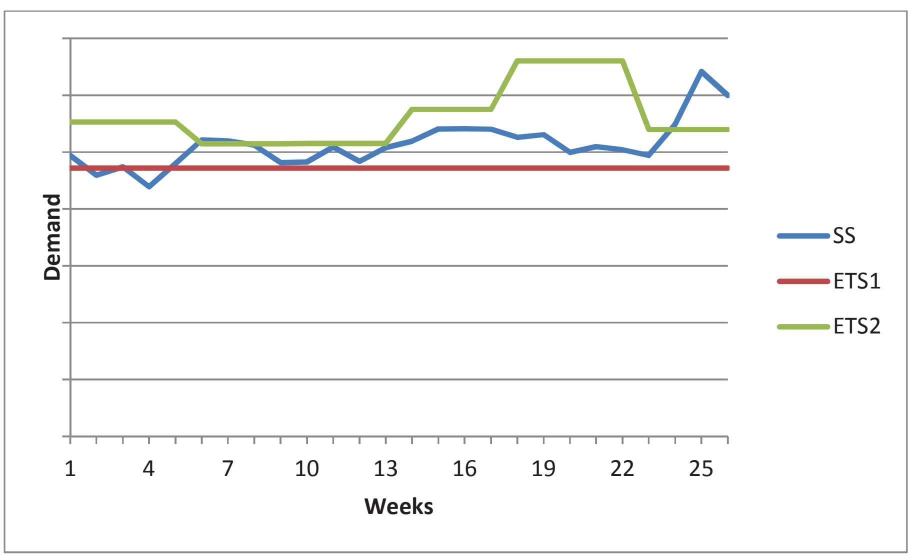

Figure 3.18: Route D-ETS2 forecasting result

- Route E: The simple exponential smoothing model occurs with ETS method and forecasting of this Route with the realized SS values is seen in Figure 3.19.

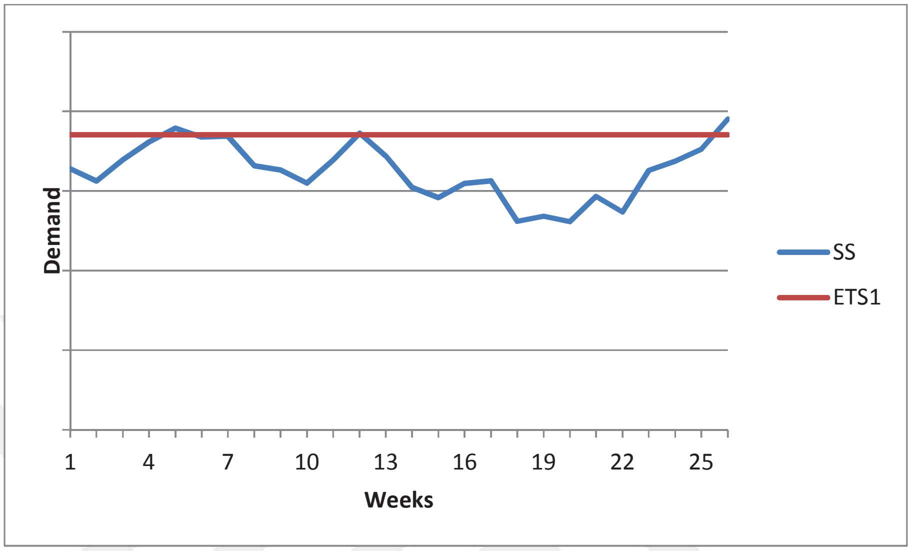

Figure 3.19: Route E-ETS forecasting result

With the deseasonalization technique, the forecasting result is seen in Figure 3.20.

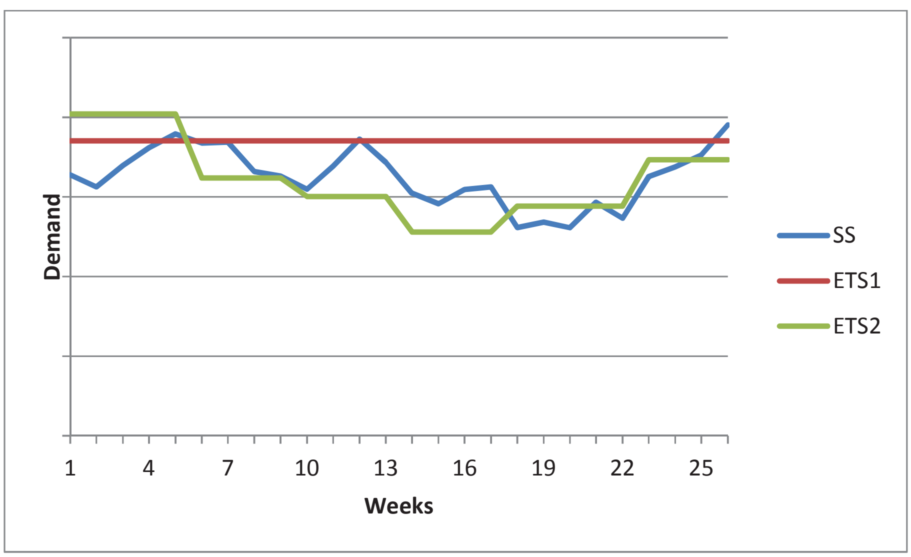

Figure 3.20: Route E-ETS2 forecasting result

- Route F: The simple exponential smoothing model appears with ETS method and forecasting of this Route with the realized SS values is seen in Figure 3.21.

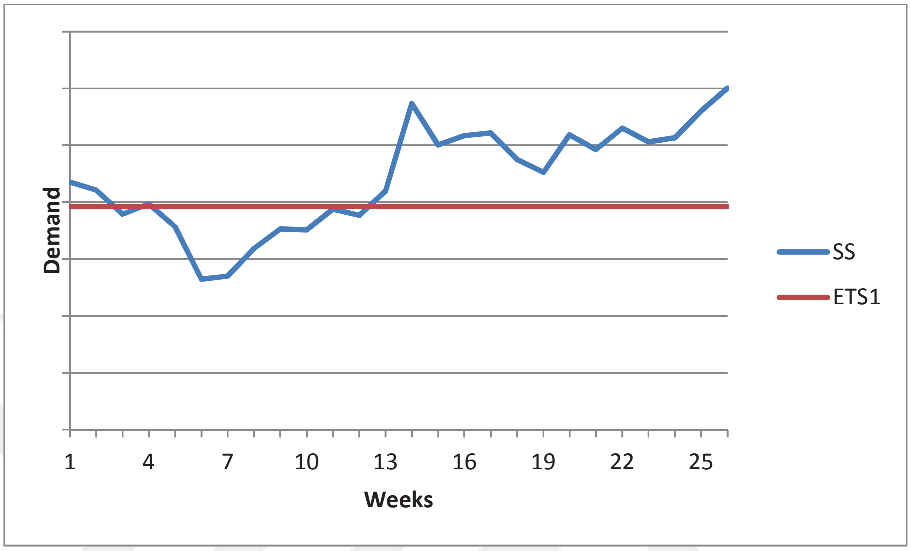

Figure 3.21: Route F-ETS forecasting result

With the deseasonalization technique, the forecasting result is seen in Figure 3.22.

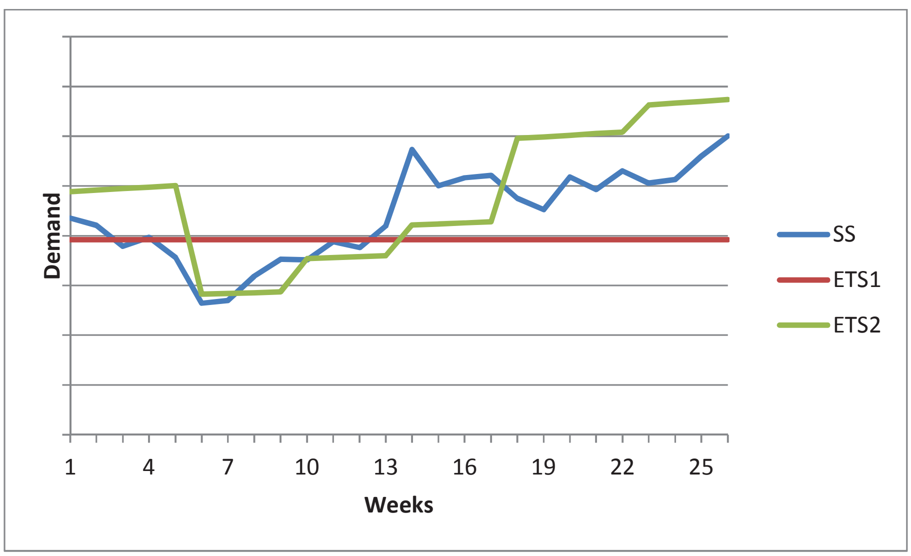

Figure 3.22: Route F-ETS2 forecasting result

- Route G: The simple exponential smoothing model seems with ETS method and forecasting of this Route with the realized SS values is seen in Figure 3.23.

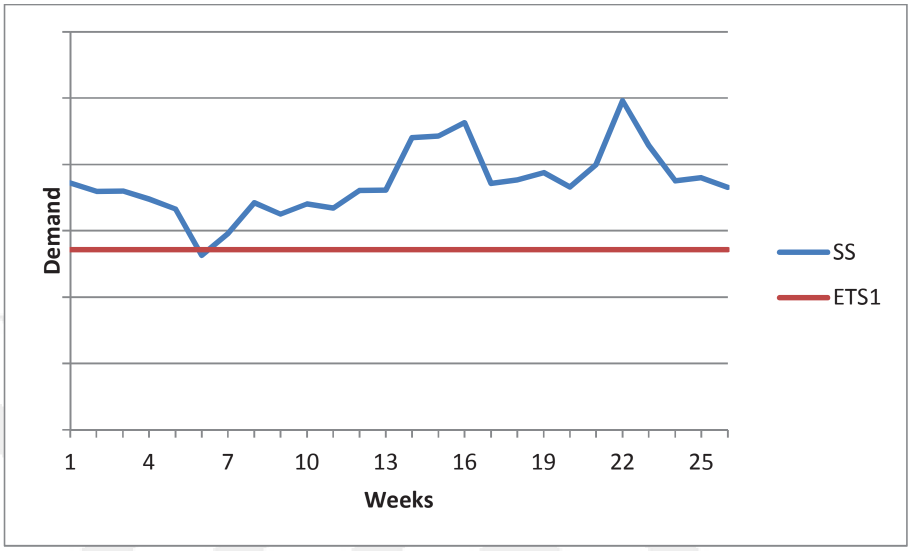

Figure 3.23: Route G-ETS forecasting result

With the deseasonalization technique, the forecasting result is seen in Figure 3.24.

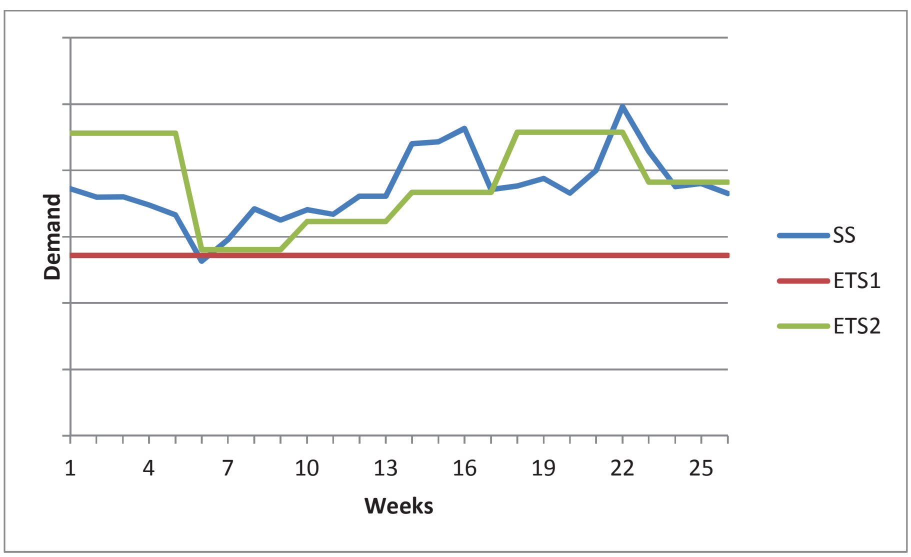

Figure 3.24: Route G-ETS2 forecasting result

- Route H: The simple exponential smoothing model occurs with ETS method and forecasting of this Route with the realized SS values is seen in Figure 3.25.

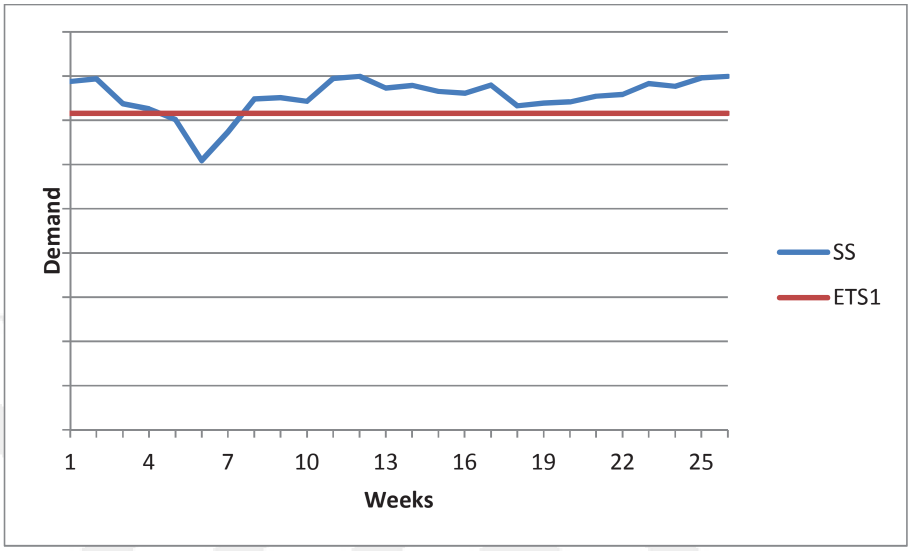

Figure 3.25: Route H-ETS forecasting result

With the deseasonalization technique, the forecasting result is seen in Figure 3.26.

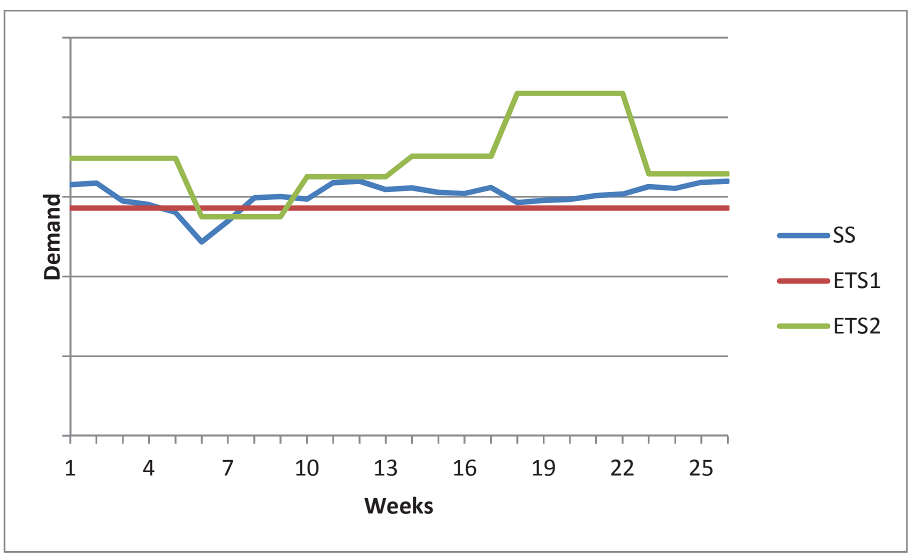

Figure 3.26: Route H-ETS2 forecasting result

3.4.2. ARIMA

Exponential smoothing method produce forecasting with the assumption that the forecast errors are uncorrelated and normally distributed with mean zero and constant variance. However, in some situations, if correlations are taken into consideration, a better forecasting model can be made. ARIMA provides this situation with non-zero correlation. In order to reach a better solution, we carry out ARIMA models for 8 market routes.

- Route A: ARIMA (0,1,0) is reached with this method and forecasting of this Route with the realized SS values are seen in Figure 3.27 and Figure 3.28.

```
Series: mydata1$ss
ARIMA(0,1,0)

sigma^2 estimated as 829790:  log likelihood=-848.04
AIC=1698.08   AICc=1698.12   BIC=1700.72
```

Figure 3.27: Route A-ARIMA R-Project result

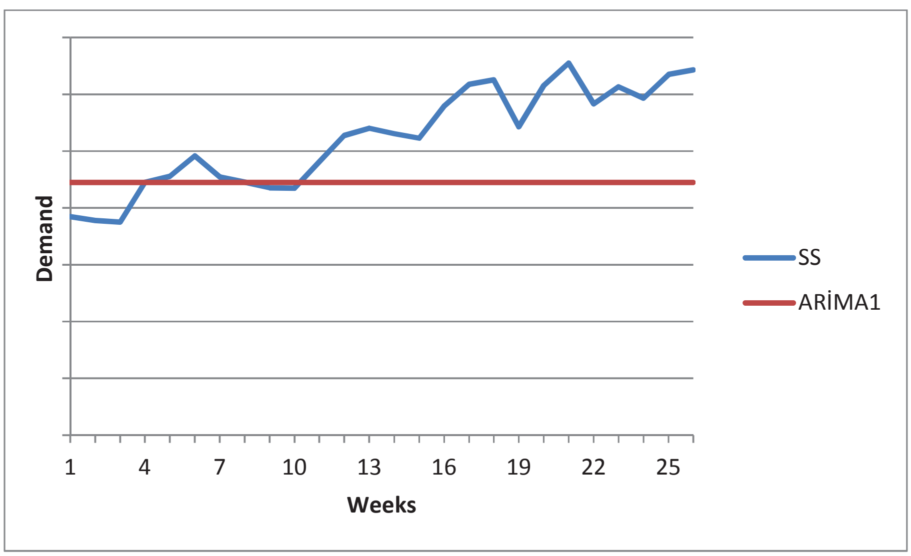

Figure 3.28: Route A-ARIMA forecasting result

It is seen that ARIMA forecasting result is very similar to ETS forecasts. Hence, SARIMA (seasonal ARIMA) is applied to all Routes and compared with ARIMA and realized SS values as seen in Figure 3.29 and Figure 3.30.

```
Series: arima1
ARIMA(0,1,0)(0,0,1)[13]

Coefficients:
        sma1
      0.2120
s.e.  0.1019

sigma^2 estimated as 792083:  log likelihood=-845.94
AIC=1695.89   AICc=1696.01   BIC=1701.16
```

Figure 3.29: Route A-SARIMA R-Project result

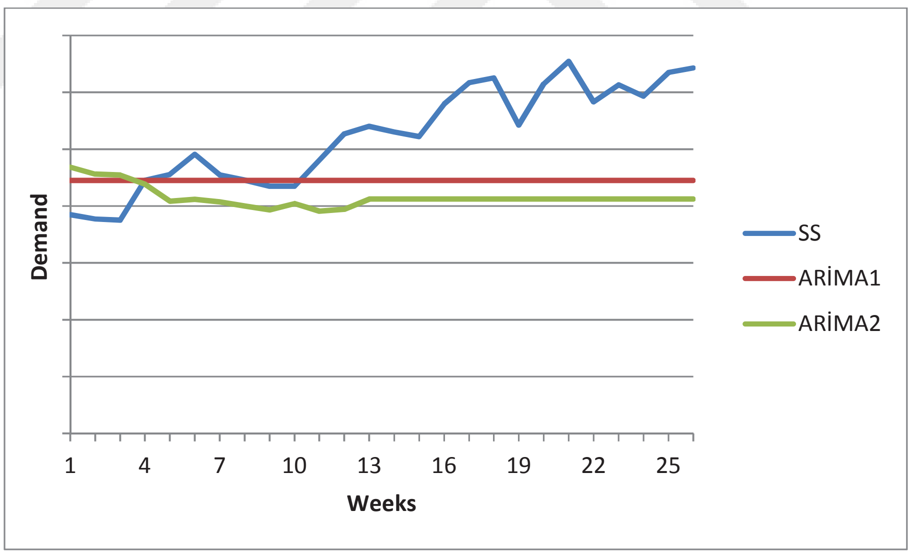

Figure 3.30: Route A-SARIMA (ARIMA2) forecasting result

- Route B: ARIMA (1,0,0) with zero mean is reached with ARIMA method and forecasting of this Route with the realized SS values is seen in Figure 3.31.

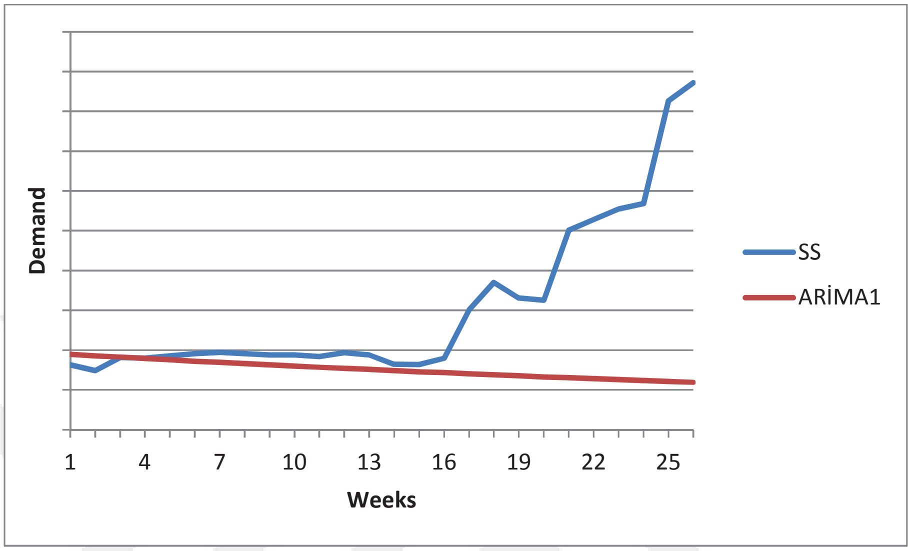

Figure 3.31: Route B-ARIMA forecasting result

With the SARIMA method, ARIMA (1,0,0) (0,0,1) [13] with non-zero mean is achieved and the forecasting result is seen in Figure 3.32.

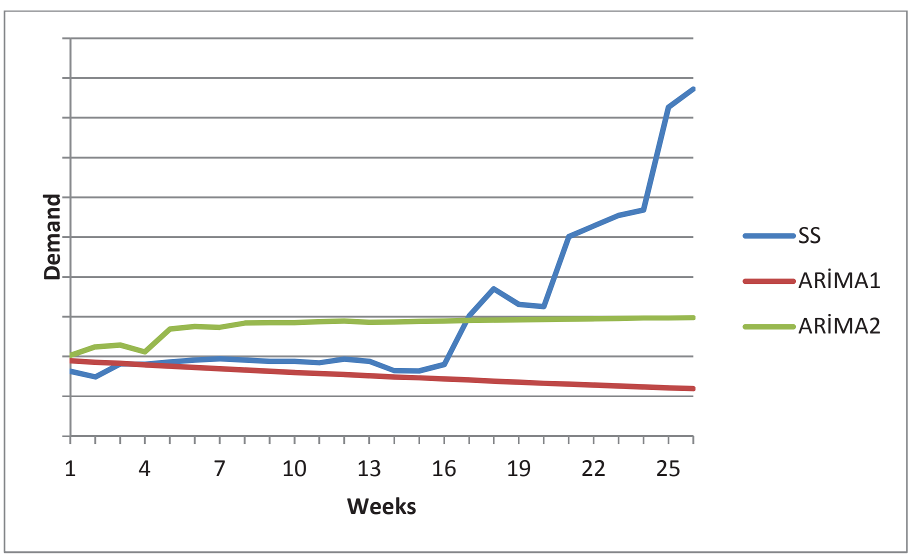

Figure 3.32: Route B-SARIMA (ARIMA2) forecasting result

- Route C: ARIMA (1,0,1) with non-zero mean is reached with ARIMA method and forecasting of this Route with the realized SS values is seen in Figure 3.33.

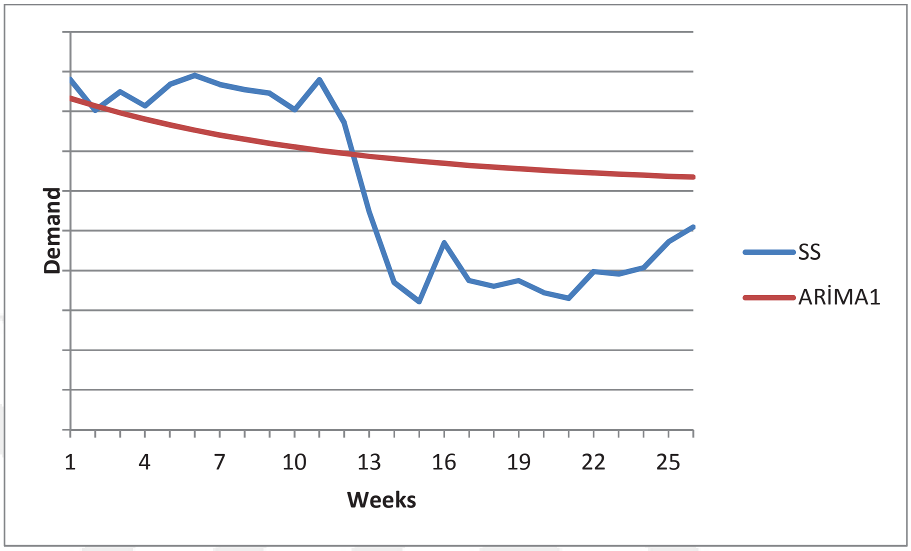

Figure 3.33: Route C-ARIMA forecasting result

With the SARIMA method, ARIMA (1,0,1) (2,0,1) [13] with non-zero mean is achieved and the forecasting result is seen in Figure 3.34.

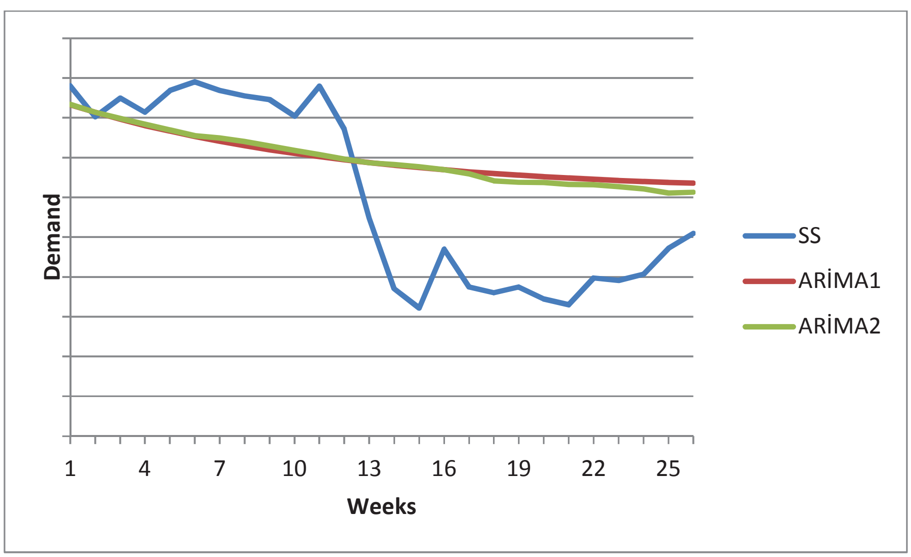

Figure 3.34: Route C-SARIMA (ARIMA2) forecasting result

- Route D: ARIMA (1,1,2) is reached with ARIMA method and forecasting of this Route with the realized SS values is seen in Figure 3.35.

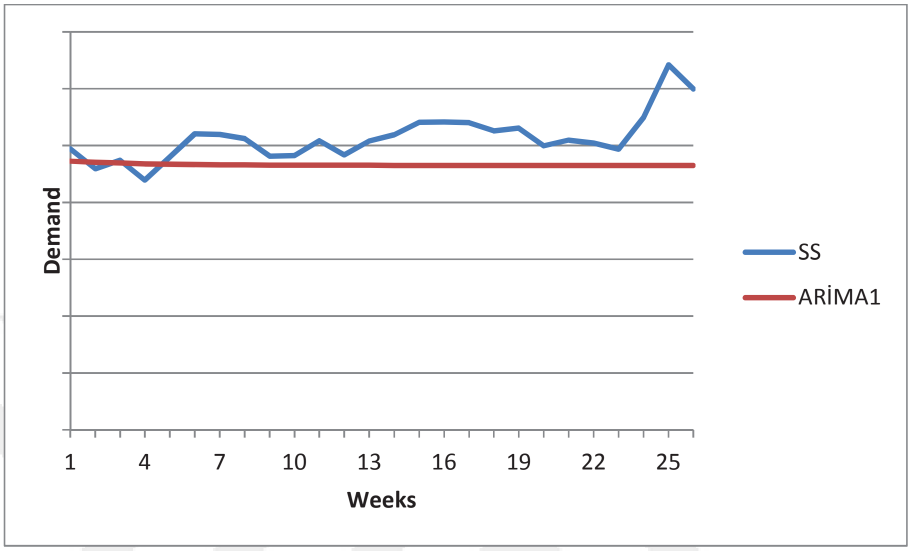

Figure 3.35: Route D-ARIMA forecasting result

With the SARIMA method, ARIMA (2,1,2) (1,0,0) [13] is achieved and the forecasting result is seen in Figure 3.36.

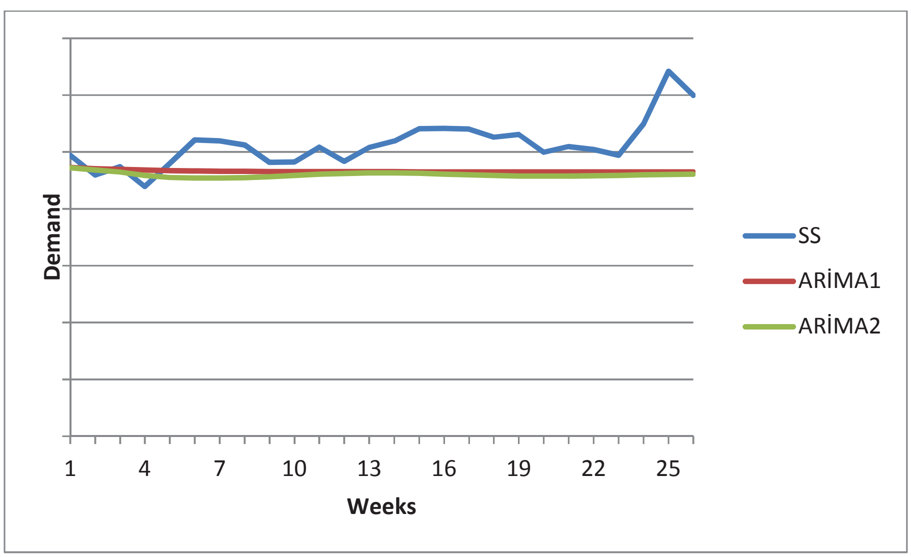

Figure 3.36: Route D-SARIMA (ARIMA2) forecasting result

- Route E: ARIMA (0,1,0) is reached with ARIMA method and forecasting of this Route with the realized SS values is seen in Figure 3.37.

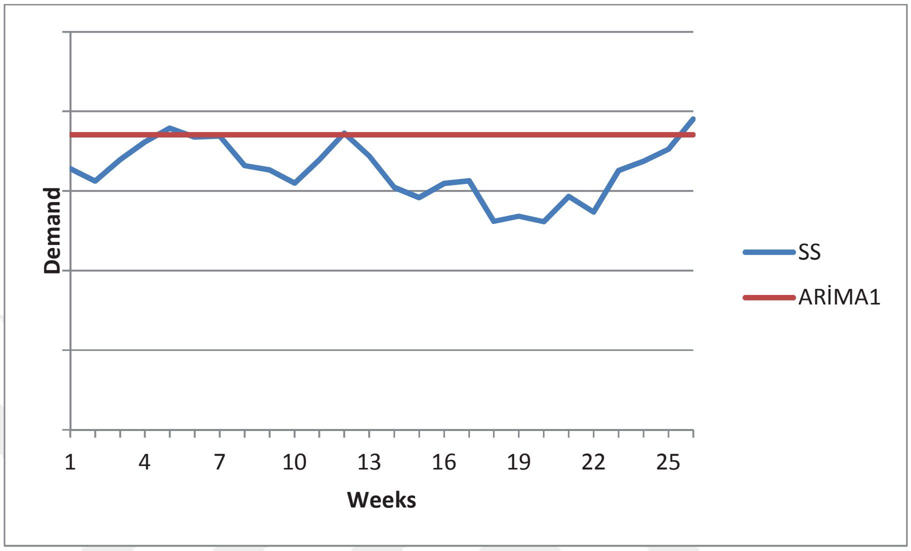

Figure 3.37: Route E- ARIMA forecasting result

With the SARIMA method, ARIMA (0,1,0) (1,0,0) [26] is achieved and the forecasting result is seen in Figure 3.38.

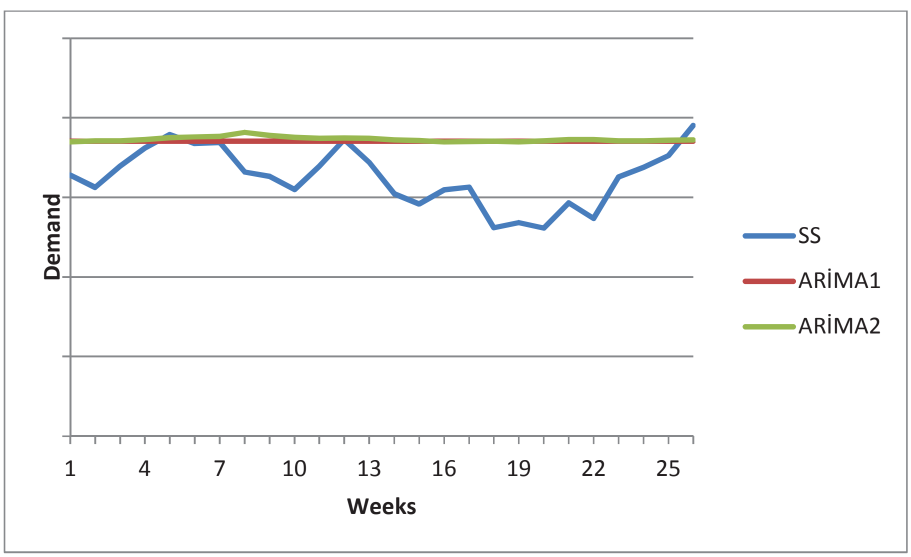

Figure 3.38: Route E- SARIMA (ARIMA2) forecasting result

- Route F: ARIMA (0,1,1) is reached with ARIMA method and forecasting of this Route with the realized SS values is seen in Figure 3.39.

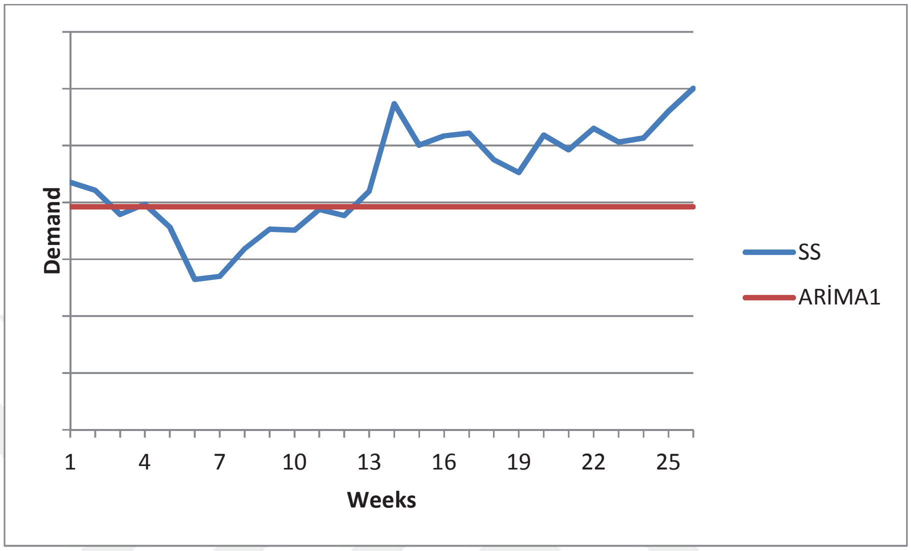

Figure 3.39: Route F- ARIMA forecasting result

With the SARIMA method, ARIMA (1,1,0) without any seasonality is achieved and the forecasting result is seen in Figure 3.40.

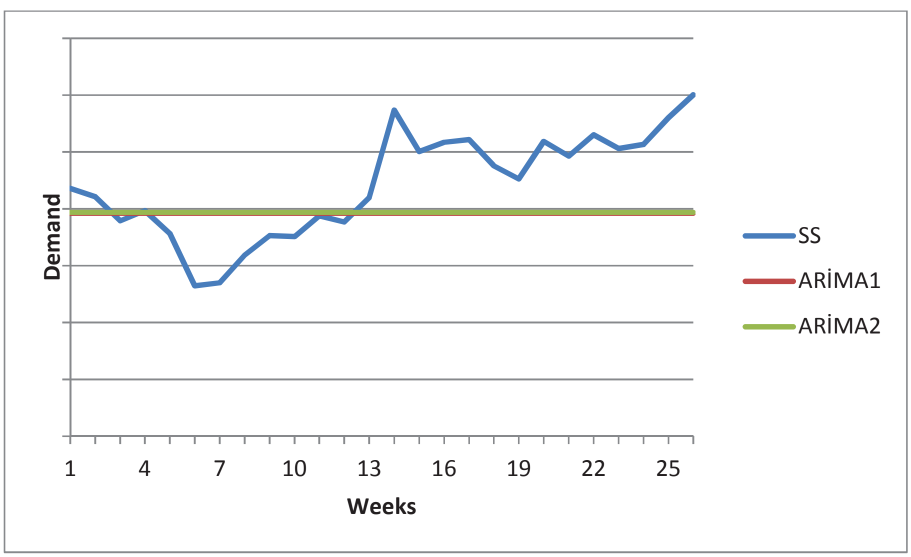

Figure 3.40: Route F- SARIMA (ARIMA2) forecasting result

- Route G: ARIMA (1,1,1) is reached with ARIMA method and forecasting of this Route with the realized SS values is seen in Figure 3.41.

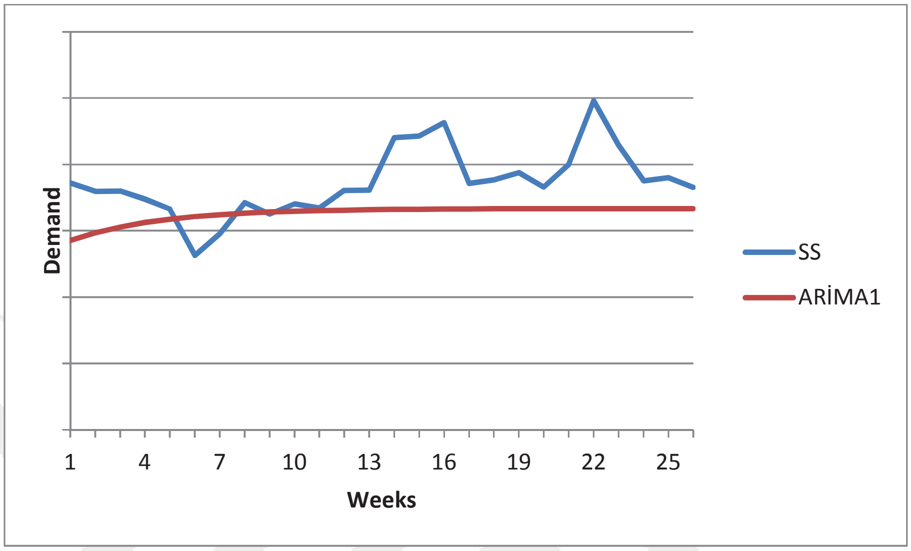

Figure 3.41: Route G-ARIMA forecasting result

With the SARIMA method, ARIMA (0,1,0) (1,0,0) [26] is achieved and the forecasting result is seen in Figure 3.42.

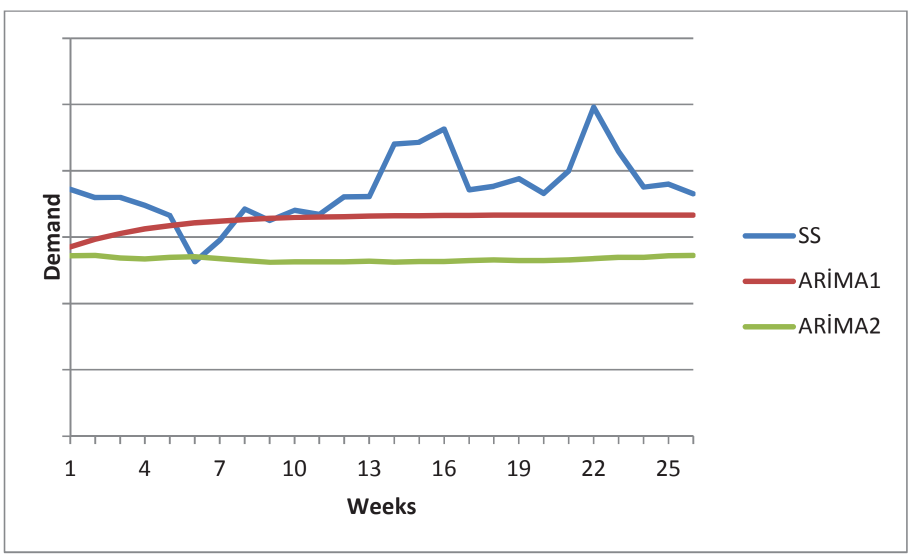

Figure 3.42: Route G-SARIMA (ARIMA2) forecasting result

- Route H: ARIMA (2,1,1) with drift is reached with ARIMA method and forecasting of this Route with the realized SS values is seen in Figure 3.43.

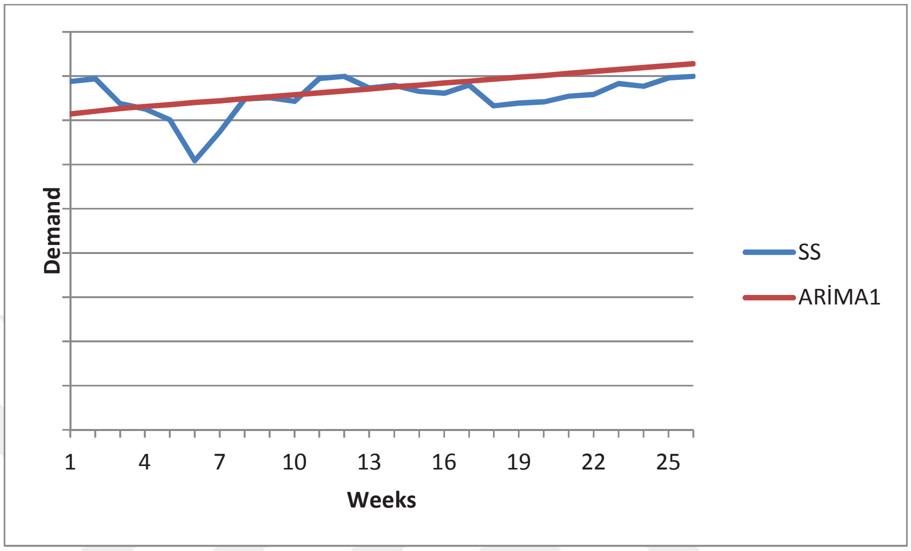

Figure 3.43: Route H-ARIMA forecasting result

With the SARIMA method, ARIMA (0,1,0) (1,0,0) [13] and ARIMA (0,1,2) (1,0,0) [26] with drift are achieved and the forecasting results are seen in Figure 3.44 and Figure 3.45.

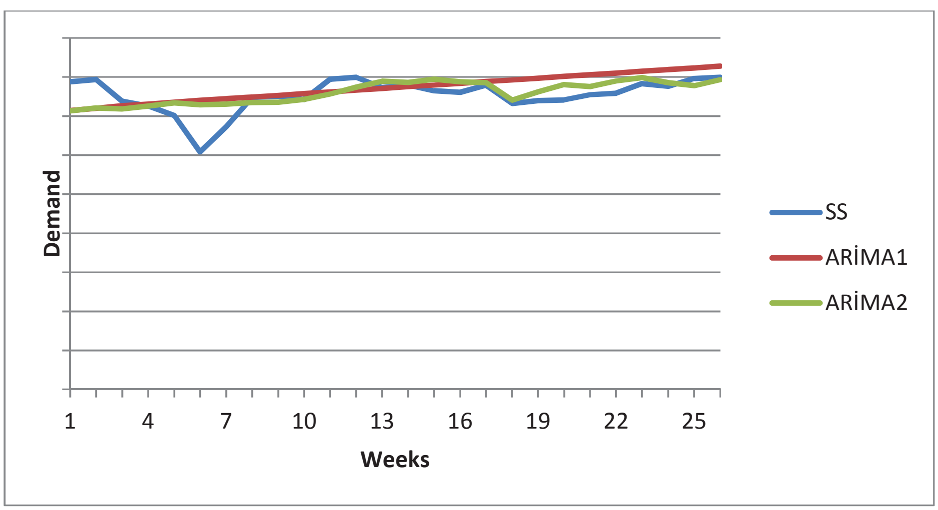

Figure 3.44: Route H-SARIMA (ARIMA2) forecasting result

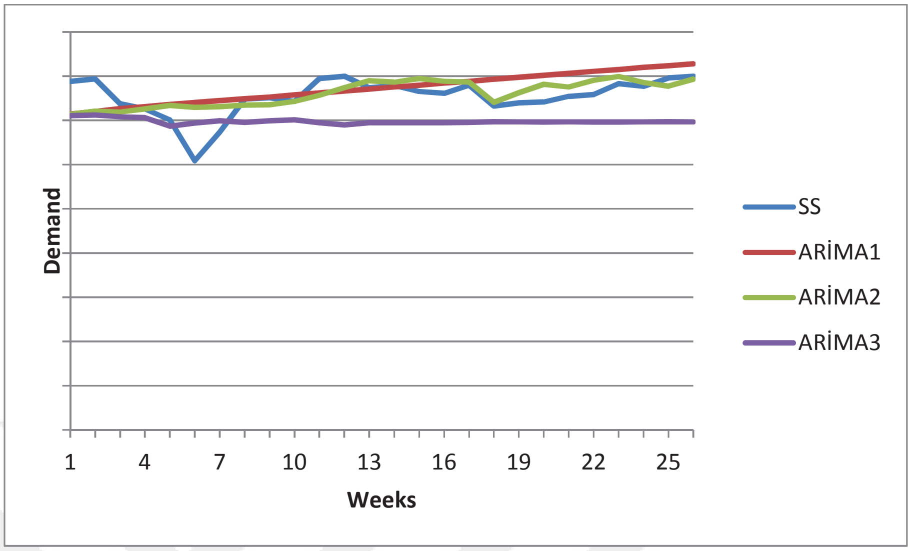

Figure 3.45: Route H-SARIMA (ARIMA3) forecasting result

3.4.3. Regression Analysis

Linear regression models have some assumptions about the predictor variables. Different extensions have been developed to ensure the relaxation of these assumptions and in some cases eliminated entirely. These assumptions are:

- Weak Exogeneity: The predictor variables can be treated as fixed values rather than random variables.
- Linearity: The response variable is a linear combination of the parameters and the predictor variables.
- Constant Variance (Homoscedasticity): The response variables have the same variance.
- Independence of the errors: The errors of the responses are uncorrelated with each other.
- Lack of multicollinearity: Linear independence in the predictor variables [26].

In the light of these assumptions, linear regression analysis is applied for 8 market routes and develop them separately according to the market condition perspective.

- For Route A, when linear regression is carried out and the formula $SS_{11}$is achieved as seen in Figure 3.46.

$$SS_{11} = -2256 + 0.195SS_{t-1} + 60F + 0.85c - 91a + 0.7mp \quad (3.2)$$

```
Call:
lm(formula = ss ~ lastss + avfare + cap + amount + pot, data = mydata1)

Residuals:
    Min      1Q  Median      3Q     Max
-1315.1  -300.0    23.3   395.4  1242.0

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -2.256e+03  7.972e+02  -2.830 0.005642 **
lastss       1.953e-01  6.531e-02   2.991 0.003518 **
avfare       6.003e+01  1.122e+01   5.348 5.82e-07 ***
cap          8.573e-01  9.691e-02   8.847 3.84e-14 ***
amount      -9.148e+01  2.279e+01  -4.014 0.000117 ***
pot          7.018e-01  1.251e-01   5.610 1.88e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 544.3 on 98 degrees of freedom
Multiple R-squared: 0.9343,     Adjusted R-squared: 0.9309
F-statistic: 278.7 on 5 and 98 DF,  p-value: < 2.2e-16
```

Figure 3.46: Route A-LR R-Project1 results

This model shows that if avfare increases, demand increases. This is illogical in practice. Therefore, we develop this model such as observed in Figure 3.47;

$$SS_{12} = -5306 + 0.17SS_{t-1} + 0.73c - 511s + 63f_r + 0.62mp \quad (3.3)$$

```
Call:
lm(formula = ss ~ lastss + cap + season + rivalfare + pot, data = mydata1)

Residuals:
    Min      1Q  Median      3Q     Max 
-1632.0  -384.6    23.7   449.4  1423.3 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -5306.5329   888.7177  -5.971 3.79e-08 ***
lastss          0.1756     0.0719   2.443   0.0164 *  
cap             0.7312     0.1031   7.090 2.10e-10 ***
season       -511.5479   255.1613  -2.005   0.0477 *  
rivalfare      63.3939    13.8200   4.587 1.33e-05 ***
pot             0.6243     0.1400   4.460 2.19e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 575.7 on 98 degrees of freedom
Multiple R-squared: 0.9265,     Adjusted R-squared: 0.9228 
F-statistic: 247.1 on 5 and 98 DF,  p-value: < 2.2e-16
```

Figure 3.47: Route A-LR R-Project2 results

This formula contains rivalfare which includes avfare effect in itself and season. Season is insignificant for SS_{11}. Hence, we remove this parameter from SS_{12}, and SS_{13} is achieved as seen in Figure 3.48. And all forecasting results can be seen in Figure 3.49.

$$SS_{13} = -3882 + 0.21SS_{t-1} + 0.58c + 47.8f_r + 0.71mp \quad (3.4)$$

```
Call:
lm(formula = ss ~ lastss + cap + rivalfare + pot, data = mydata1)

Residuals:
     Min       1Q   Median       3Q      Max
-1547.57  -414.06    82.72   517.43  1496.08

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -3.882e+03  5.417e+02  -7.166 1.40e-10 ***
lastss       2.173e-01  6.987e-02   3.111  0.00244 **
cap          5.860e-01  7.457e-02   7.859 4.81e-12 ***
rivalfare    4.781e+01  1.160e+01   4.122 7.82e-05 ***
pot          7.163e-01  1.343e-01   5.335 6.06e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 584.4 on 99 degrees of freedom
Multiple R-squared: 0.9235,     Adjusted R-squared: 0.9204
F-statistic: 298.7 on 4 and 99 DF,  p-value: < 2.2e-16
```

Figure 3.48: Route A-LR R-Project3 results

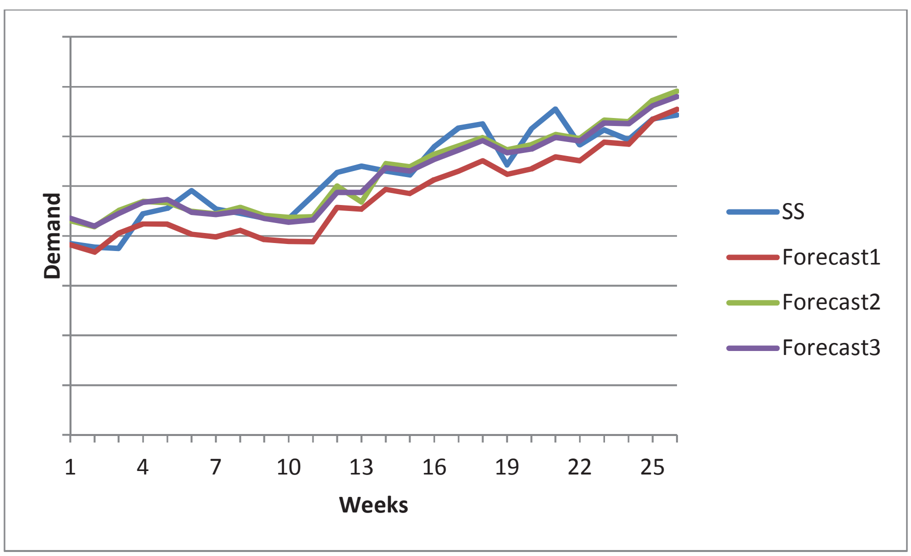

Figure 3.49: Route A-LR forecasting results

- For Route B, when linear regression is carried out and the formula SS_{21} is achieved.

$$SS_{21} = -1801 + 41F + 0.54c - 450s + 0.92mp \quad (3.5)$$

This model similarly shows that if avfare increases, demand increases. Therefore, we develop this model such as;

$$SS_{22} = -1309 + 0.54c - 450s + 41f_r + 0.92mp \tag{3.6}$$

In similar with Route A, we want to develop this formula without negative seasonality effect. Hence, we remove this parameter from SS_{22}, and SS_{23} is achieved. And all forecasting results can be seen in Figure 3.50.

$$SS_{23} = -1191 + 0.12SS_{t-1} + 0.42c + 35.8f_r + 0.91mp \tag{3.7}$$

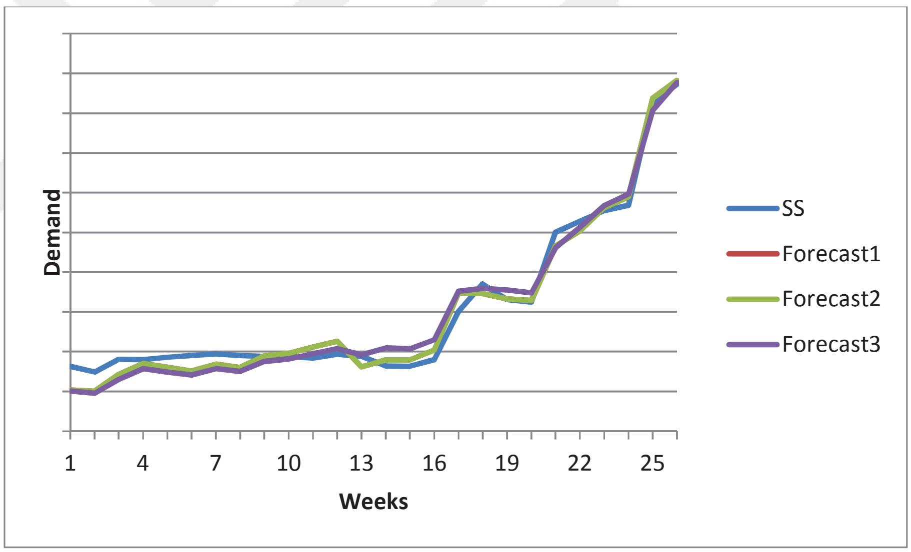

Figure 3.50: Route B-LR forecasting results

- For Route C, when linear regression is carried out and the formula SS_{31} is achieved.

$$SS_{31} = -47 + 0.45SS_{t-1} + 0.61F + 0.16c - 111s + 0.74mp \tag{3.8}$$

This model similarly shows that if avfare increases, demand increases. Therefore, we develop this model such as;

$$SS_{32} = -35 + 0.45SS_{t-1} + 0.15c - 114s + 0.59f_r + 0.74mp \quad (3.9)$$

In similar with Route B, we want to develop this formula without negative seasonality effect. Hence, we remove this parameter from $SS_{32}$and SS_{33} is achieved. And all forecasting results can be seen in Figure 3.51.

$$SS_{33} = -190 + 0.5SS_{t-1} + 0.2c + 0.42f_r + 0.77mp \quad (3.10)$$

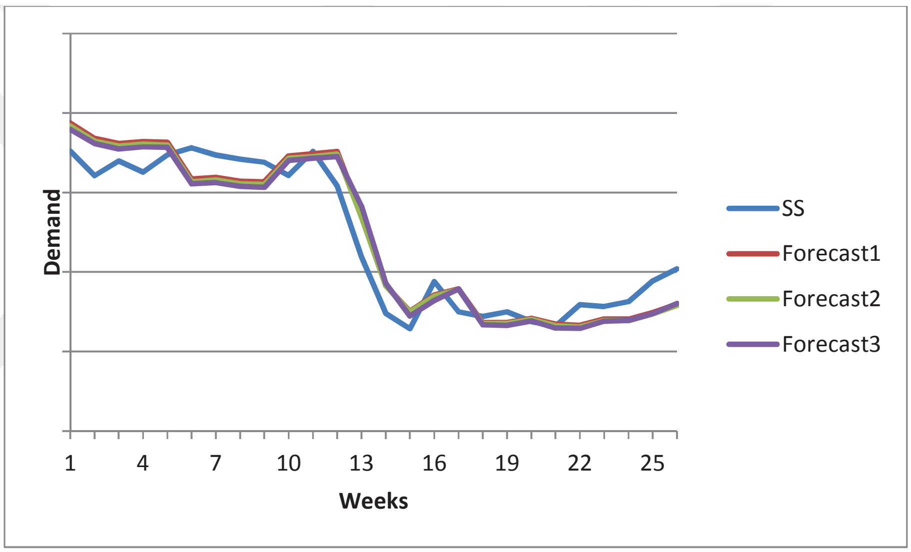

Figure 3.51: Route C-LR forecasting results

- For Route D, when linear regression is applied and the formula SS_{41} is achieved.

$$SS_{41} = -582 + 0.51SS_{t-1} + 0.5c + 162.6s + 0.11mp \quad (3.11)$$

This model shows that demand does not depend on the fare. In practice, demand is usually attached to the ticket price. Although fare parameters are insignificant, we develop this model such as;

$$SS_{42} = -687.5 + 0.51SS_{t-1} + 0.5c + 157.5s + 0.81f_r + 0.11mp \quad (3.12)$$

Unlike Route B and Route C, seasonality has positive effect on Route D and fares are seen insignificant, thus, we do not develop this formula in terms of seasonality effect. And forecasting results can be seen in Figure 3.52.

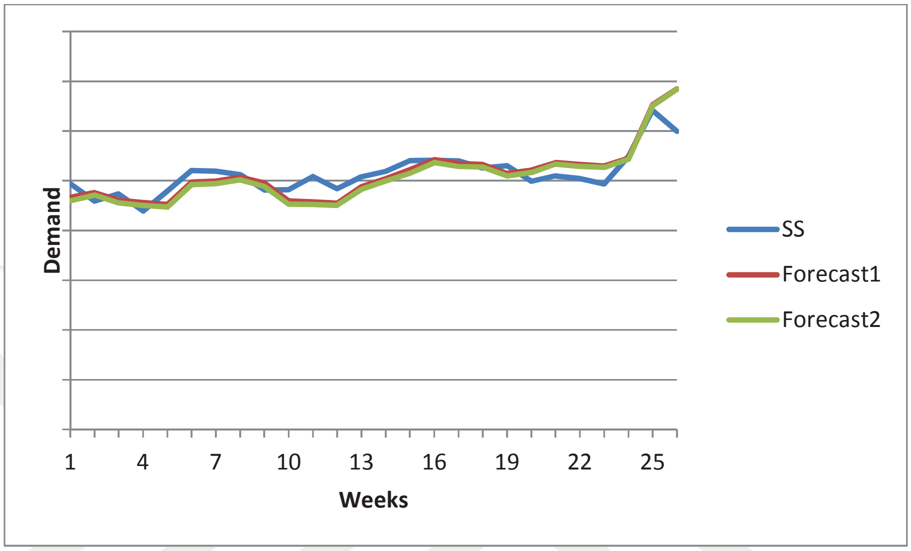

Figure 3.52: Route D-LR forecasting results

- For Route E, when linear regression is carried out and the formula SS_{51} is achieved.

$$SS_{51} = -78 + 0.77SS_{t-1} + 1.19F + 0.37mp \quad (3.13)$$

This model similarly shows that if avfare increases, demand increases. Therefore, we develop this model such as;

$$SS_{52} = -68 + 0.77SS_{t-1} + 1.19f_r + 0.37mp \quad (3.14)$$

For Route E, intercept, seasonality, amount and capacity factors are insignificant. When intercept was removed, the result worsened; therefore, it was kept in formula. In order to see capacity effect on the forecasting results, cap parameter is added and SS_{53} is achieved. And all forecasting results can be seen in Figure 3.53.

$$SS_{53} = -86 + 0.77SS_{t-1} + 1.19f_r + 0.37mp + 0.009c \quad (3.15)$$

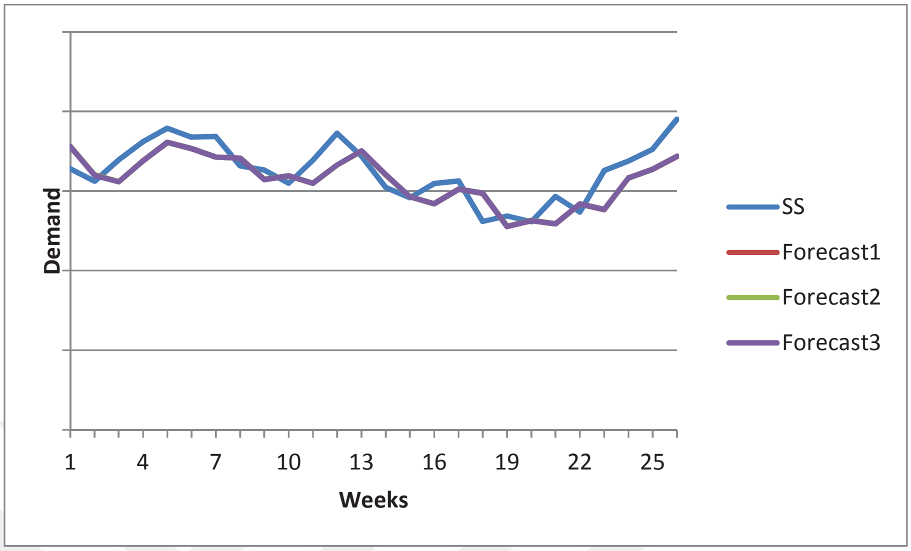

Figure 3.53: Route E-LR forecasting results

- For Route F, when linear regression is carried out and the formula SS_{61} is achieved.

$$SS_{61} = -1643 + 0.32SS_{t-1} - 13.3F + 199a - 223s + 15.6f_r + 0.24mp \qquad (3.16)$$

In this model, avfare and rivalfare have expected coefficients. But, it has negative seasonality; therefore, we develop this model such as;

$$SS_{62} = -1468 + 0.34SS_{t-1} - 12.6F + 188a + 14.7f_r + 0.16mp \qquad (3.17)$$

All forecasting results can be seen in Figure 3.54.

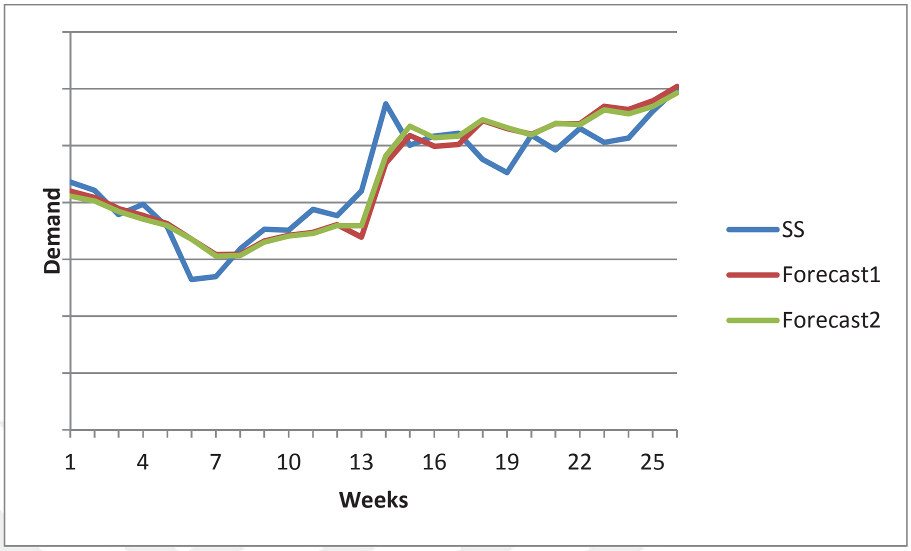

Figure 3.54: Route F-LR forecasting results

- For Route G, when linear regression is carried out and the formula SS_{71} is achieved.

$$SS_{71} = -1231 + 0.41SS_{t-1} + 8.04F + 0.54c \quad (3.18)$$

This model similarly shows that if avfare increases, demand increases. Therefore, we develop this model such as;

$$SS_{72} = -1186 + 0.4SS_{t-1} + 0.55c + 7.72f_r \quad (3.19)$$

For Route G, seasonality and potential factors are insignificant. In order to see potential effect on the forecasting results, market potential parameter is added and SS_{73} is reached. And all forecasting results can be seen in Figure 3.55.

$$SS_{73} = -1668 + 0.37SS_{t-1} + 0.59c + 6.32f_r + 0.056mp \quad (3.20)$$

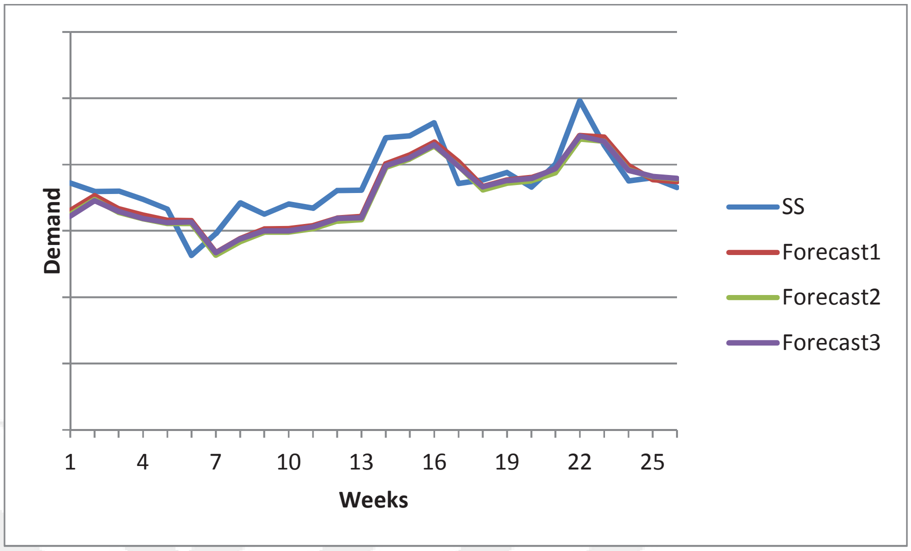

Figure 3.55: Route G-LR forecasting results

- For Route H, when linear regression is carried out and the formula SS_{81} is achieved.

$$SS_{81} = -261 + 0.43SS_{t-1} - 0.99c + 499a + 0.53f_r \quad (3.21)$$

In this model, capacity has a negative impact on SS because of positive amount effect. To see cap effect without amount parameter, model is developed as;

$$SS_{82} = -223 + 0.57SS_{t-1} + 0.37c + 0.44f_r \quad (3.22)$$

All forecasting results can be seen in Figure 3.56.

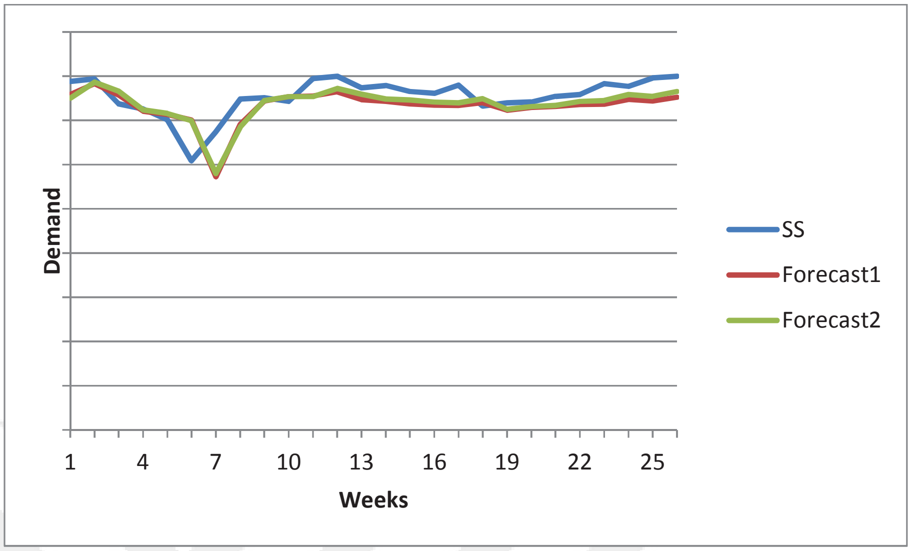

Figure 3.56: Route H-LR forecasting results

3.5. The Comparison of Forecasting Results

When the results of the estimation are compared, tables below show the error rates and they have positive and negative values which represent the difference between forecasted values and the realized sales during 26 weeks. In order to demonstrate the best estimate, we prefer to exhibit graphs which indicate forecasted values and realized sales during 26 weeks and we only display the best forecasting models with the realized sales.

- For Route A, LR forecasting results give better and close solutions to each other and forecast3 has the best fitting when compared to the realized SS values as seen in Table 3.2 and Figure 3.57.

Table 3.2: Route A-Error Rates for all methods

Week	forecast1	forecast2	forecast3	ETS1	ETS2	ARIMA1	ARIMA2
1	-0,61%	11,97%	13,20%	-13,55%	-57,05%	-15,64%	-21,75%
2	-2,79%	10,84%	11,29%	-15,73%	-60,07%	-17,86%	-20,80%
3	8,04%	20,20%	18,66%	-16,43%	-61,03%	-18,57%	-21,12%
4	-4,69%	5,46%	5,14%	1,80%	-35,81%	0,00%	1,36%
5	-7,10%	2,56%	3,83%	4,09%	-32,65%	2,33%	10,29%
6	-17,86%	-8,58%	-8,94%	11,09%	-1,00%	9,46%	16,19%
7	-12,41%	-2,20%	-2,51%	3,88%	-9,19%	2,11%	10,38%
8	-7,64%	2,66%	0,88%	1,93%	-11,41%	0,12%	10,14%
9	-9,79%	1,29%	-0,11%	-0,36%	-14,01%	-2,21%	9,69%
10	-10,63%	0,52%	-1,73%	-0,44%	-29,51%	-2,29%	7,03%
11	-19,15%	-8,76%	-10,14%	9,12%	-17,18%	7,45%	18,61%
12	-13,26%	-5,09%	-7,48%	17,14%	-6,84%	15,61%	25,16%
13	-16,04%	-13,30%	-9,70%	19,13%	-4,27%	17,64%	23,64%
14	-6,96%	2,80%	1,23%	17,66%	-24,89%	16,15%	22,25%
15	-7,21%	3,13%	1,52%	16,40%	-26,81%	14,86%	21,06%
16	-11,44%	-2,59%	-4,52%	24,62%	-14,33%	23,24%	28,82%
17	-14,19%	-5,80%	-7,20%	29,22%	-7,36%	27,92%	33,17%
18	-11,89%	-4,43%	-5,39%	30,15%	-27,40%	28,87%	34,04%
19	-3,52%	5,53%	4,58%	19,51%	-46,81%	18,03%	24,00%
20	-13,04%	-5,07%	-6,61%	28,99%	-29,53%	27,68%	32,94%
21	-14,60%	-7,78%	-8,62%	33,29%	-21,68%	32,06%	37,00%
22	-5,46%	2,22%	1,35%	25,07%	-36,67%	23,69%	29,24%
23	-4,05%	3,12%	2,29%	28,77%	-21,60%	27,46%	32,74%
24	-1,49%	6,18%	5,49%	26,36%	-25,72%	25,00%	30,46%
25	-0,06%	5,98%	4,24%	31,19%	-17,47%	29,93%	35,02%
26	1,82%	7,50%	5,76%	32,04%	-16,02%	30,79%	35,83%
MAPE	8,73%	5,79%	**5,67%**	19,10%	24,07%	18,15%	24,12%

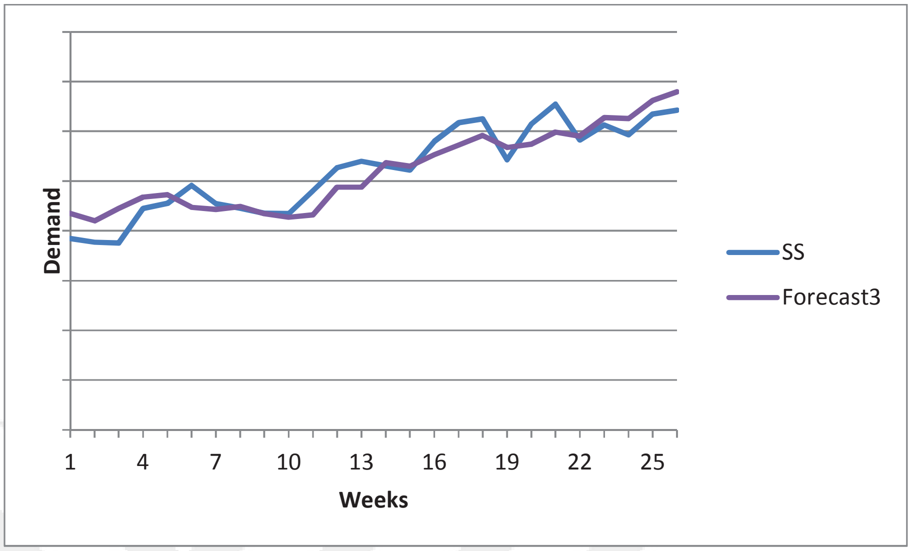

Figure 3.57: Route A - The Best forecasting result

- For Route B, LR forecasting results give better and very close solutions to each other. Forecast1 and forecast2 have the same error rates and the best fitting when compared to the realized SS values as seen in Table 3.3 and Figure 3.58.

Table 3.3: Route B-Error Rates for all methods

Week	forecast1	forecast2	forecast3	ETS1	ETS2	ARIMA1	ARIMA2
1	-36,57%	-36,57%	-37,92%	-19,33%	-10,09%	-15,90%	-24,26%
2	-32,27%	-32,27%	-35,95%	-33,89%	-20,74%	-24,78%	-50,78%
3	-20,80%	-20,80%	-28,11%	-12,52%	0,61%	-0,82%	-26,56%
4	-5,43%	-5,43%	-12,70%	-15,17%	0,17%	0,59%	-17,22%
5	-13,62%	-13,62%	-19,82%	-13,39%	3,38%	5,56%	-44,30%
6	-20,86%	-20,86%	-25,73%	-12,28%	23,82%	9,62%	-44,44%
7	-13,26%	-13,26%	-19,20%	-11,84%	25,19%	12,88%	-40,51%
8	-15,97%	-15,97%	-21,16%	-15,37%	23,82%	12,91%	-48,78%
9	1,05%	1,05%	-6,55%	-18,53%	22,64%	13,19%	-51,98%
10	4,19%	4,19%	-3,45%	-19,74%	0,46%	14,82%	-51,95%
11	14,94%	14,94%	5,96%	-23,67%	-1,82%	14,48%	-56,47%
12	16,71%	16,71%	7,41%	-18,52%	3,28%	20,25%	-49,50%
13	-14,04%	-14,04%	2,11%	-23,10%	0,35%	19,34%	-52,29%
14	8,91%	8,91%	27,58%	-41,73%	-62,78%	9,50%	-74,75%
15	9,49%	9,49%	26,67%	-43,46%	-63,68%	10,67%	-76,37%
16	13,85%	13,85%	27,83%	-31,40%	-49,01%	20,17%	-61,14%
17	15,16%	15,16%	16,68%	21,37%	11,32%	53,37%	3,78%
18	-6,47%	-6,47%	-2,91%	35,56%	-45,68%	62,67%	21,27%
19	0,48%	0,48%	7,39%	27,60%	-62,94%	59,01%	11,66%
20	1,03%	1,03%	6,97%	26,07%	-65,69%	59,09%	9,90%
21	-7,01%	-7,01%	-7,94%	51,78%	-7,66%	73,90%	41,28%
22	-4,70%	-4,70%	-2,94%	54,14%	-2,04%	75,72%	44,19%
23	1,34%	1,34%	2,30%	56,20%	-48,58%	77,30%	46,71%
24	3,77%	3,77%	4,87%	57,12%	-45,06%	78,25%	47,84%
25	1,26%	1,26%	-2,46%	70,45%	0,29%	85,32%	64,06%
26	1,16%	1,16%	0,56%	71,92%	5,47%	86,34%	65,84%
MAPE	**7,59%**	**7,59%**	9,54%	42,01%	22,25%	52,71%	44,90%

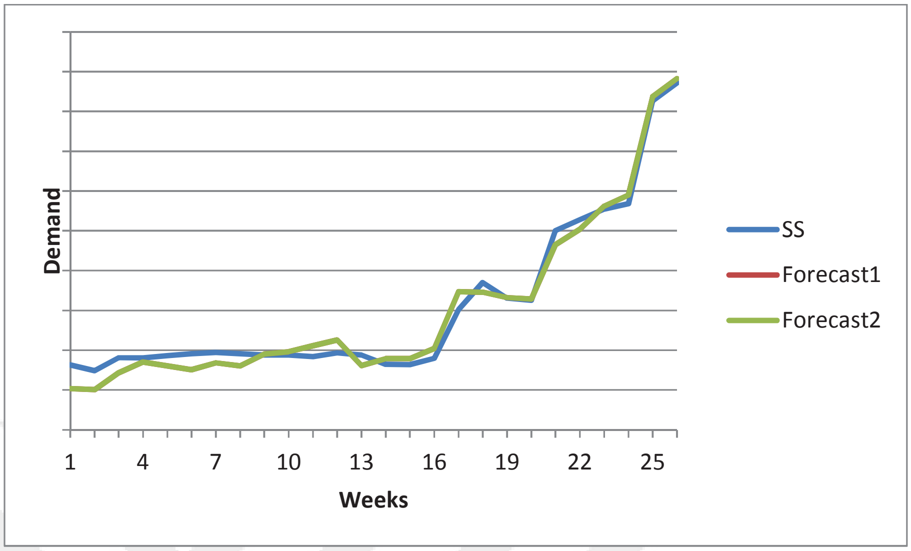

Figure 3.58: Route B - The Best forecasting result

- For Route C, LR forecasting results give better and close solutions to each other and forecast2 has the best fitting when compared to the realized SS values as seen in Table 3.4 and Figure 3.59.

Table 3.4: Route C-Error Rates for all methods

Week	forecast1	forecast2	forecast3	ETS1	ETS2	ARIMA1	ARIMA2
1	10,06%	9,04%	7,72%	0,96%	-19,68%	5,44%	5,32%
2	14,60%	13,60%	12,69%	-8,66%	-36,23%	-1,40%	-1,42%
3	6,44%	5,47%	4,37%	-2,65%	-33,34%	6,24%	6,09%
4	11,82%	10,82%	9,75%	-7,13%	-44,00%	4,09%	3,75%
5	4,54%	3,63%	2,72%	-0,46%	-39,59%	11,72%	11,37%
6	-11,04%	-11,96%	-12,72%	2,08%	-10,03%	15,44%	15,21%
7	-8,06%	-8,98%	-9,98%	-0,46%	-16,45%	14,64%	13,76%
8	-8,05%	-9,00%	-9,96%	-2,05%	-21,91%	14,59%	13,34%
9	-7,34%	-8,33%	-9,35%	-3,14%	-26,86%	14,87%	13,77%
10	7,64%	6,62%	5,80%	-8,46%	9,40%	11,63%	10,74%
11	-0,88%	-1,83%	-2,53%	0,85%	14,86%	20,18%	19,58%
12	14,04%	12,93%	11,92%	-13,10%	0,23%	9,95%	9,69%
13	23,77%	22,74%	29,21%	-59,13%	-44,09%	-25,40%	-25,32%
14	23,62%	23,01%	25,38%	-135,36%	-48,25%	-83,74%	-84,24%
15	17,17%	16,45%	12,55%	-171,23%	-75,15%	-109,90%	-110,50%
16	-9,03%	-9,80%	-12,54%	-85,53%	-22,75%	-42,43%	-42,41%
17	19,49%	18,63%	19,19%	-132,85%	-57,75%	-77,42%	-75,90%
18	-5,42%	-6,20%	-7,38%	-141,89%	-34,06%	-83,05%	-78,00%
19	-8,87%	-9,66%	-11,28%	-132,85%	-32,00%	-75,10%	-70,32%
20	3,05%	2,02%	0,48%	-153,12%	-46,71%	-89,25%	-84,80%
21	1,73%	0,30%	-1,83%	-164,24%	-56,50%	-96,52%	-91,65%
22	-16,40%	-17,53%	-18,91%	-119,37%	-32,71%	-62,35%	-58,84%
23	-10,08%	-11,22%	-11,76%	-122,74%	-37,77%	-64,09%	-60,14%
24	-13,58%	-14,72%	-14,41%	-114,25%	-35,25%	-57,19%	-52,55%
25	-20,97%	-22,02%	-21,80%	-84,75%	-18,97%	-35,03%	-29,30%
26	-21,56%	-22,48%	-21,17%	-71,15%	-12,39%	-24,65%	-20,23%
MAPE	10,75%	**10,71%**	10,93%	43,94%	28,18%	29,31%	27,92%

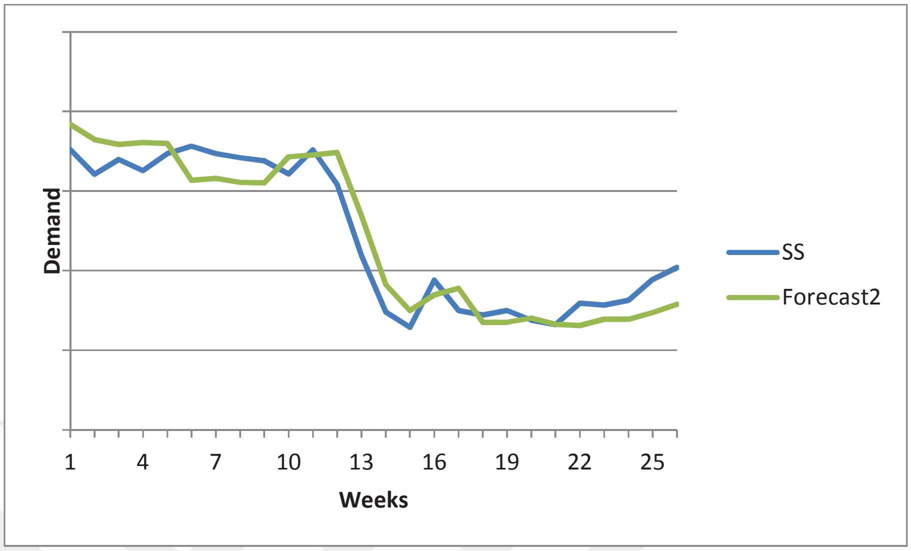

Figure 3.59: Route C - The Best forecasting result

- For Route D, LR forecasting results give better and close solutions to each other and forecast1 has the best fitting when compared to the realized SS values as seen in Table 3.5 and Figure 3.60.

Table 3.5: Route D-Error Rates for all methods

Week	forecast1	forecast2	ETS1	ETS2	ARIMA1	ARIMA2
1	-5,54%	-6,94%	4,57%	-11,90%	4,38%	4,56%
2	3,70%	2,62%	-2,70%	-20,43%	-2,45%	-1,97%
3	-2,75%	-3,93%	0,46%	-16,72%	1,02%	1,99%
4	3,81%	2,58%	-7,38%	-25,92%	-6,53%	-4,40%
5	-5,73%	-6,89%	1,71%	-15,26%	2,64%	5,15%
6	-4,49%	-5,44%	9,45%	1,18%	10,41%	12,85%
7	-3,82%	-4,93%	9,24%	0,95%	10,28%	12,65%
8	-0,98%	-2,01%	7,93%	-0,48%	9,04%	11,25%
9	2,92%	1,53%	2,08%	-6,87%	3,31%	5,21%
10	-4,72%	-6,09%	2,20%	-6,82%	3,46%	4,92%
11	-10,10%	-11,10%	7,28%	-1,28%	8,49%	9,38%
12	-5,98%	-6,81%	2,48%	-6,52%	3,78%	4,49%
13	-3,86%	-4,93%	7,13%	-1,44%	8,38%	8,84%
14	-2,77%	-3,74%	9,10%	-10,86%	10,33%	10,78%
15	-3,54%	-4,70%	12,80%	-6,35%	13,98%	14,55%
16	0,23%	-0,90%	12,86%	-6,27%	14,05%	14,83%
17	-1,01%	-2,16%	12,70%	-6,47%	13,89%	14,93%
18	1,40%	0,35%	10,27%	-25,63%	11,50%	12,78%
19	-3,01%	-3,86%	11,12%	-24,45%	12,34%	13,75%
20	4,42%	3,47%	5,53%	-32,28%	6,82%	8,37%
21	5,22%	4,63%	7,46%	-29,58%	8,73%	10,21%
22	5,72%	5,01%	6,47%	-30,96%	7,75%	9,14%
23	7,28%	6,67%	4,50%	-9,23%	5,81%	7,08%
24	-0,71%	-1,16%	14,13%	1,79%	15,31%	16,32%
25	1,64%	1,36%	26,52%	15,96%	27,53%	28,31%
26	14,22%	14,16%	21,30%	9,99%	22,38%	23,15%
MAPE	**4,22%**	4,55%	8,87%	12,38%	9,86%	10,92%

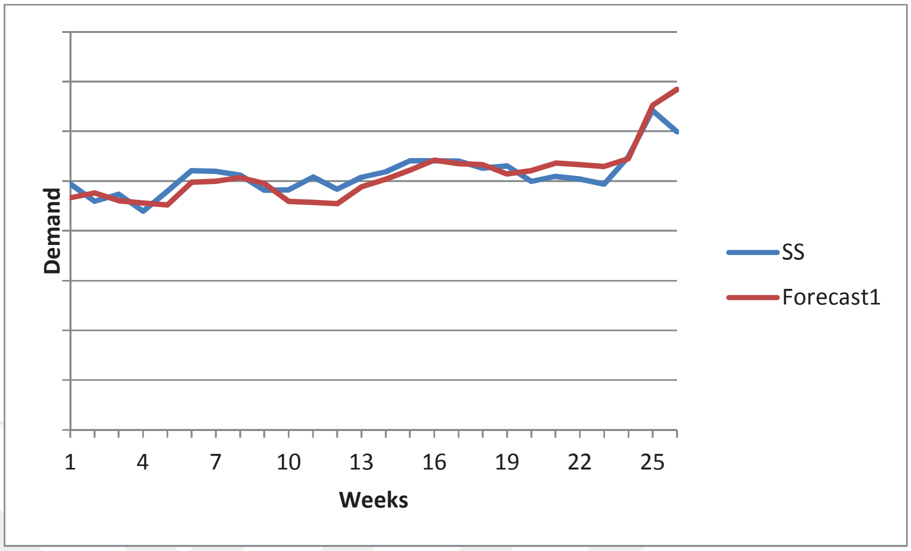

Figure 3.60: Route D - The Best forecasting result

- For Route E, LR forecasting results give better and very close solutions to each other and forecast3 has the best fitting when compared to the realized SS values as seen in Table 3.6 and Figure 3.61.

Table 3.6: Route E-Error Rates for all methods

Week	forecast1	forecast2	forecast3	ETS1	ETS2	ARIMA1	ARIMA2
1	8,73%	8,76%	8,73%	-13,07%	-23,33%	-13,06%	-12,90%
2	2,44%	2,47%	2,27%	-18,56%	-29,33%	-18,55%	-18,75%
3	-8,16%	-8,13%	-8,16%	-9,20%	-19,12%	-9,19%	-9,34%
4	-6,58%	-6,56%	-6,53%	-2,44%	-11,74%	-2,43%	-2,95%
5	-4,68%	-4,65%	-4,66%	2,21%	-6,67%	2,22%	1,14%
6	-3,99%	-3,97%	-3,97%	-0,77%	11,95%	-0,76%	-2,17%
7	-7,09%	-7,06%	-7,05%	-0,50%	12,19%	-0,49%	-2,12%
8	2,82%	2,85%	2,87%	-11,70%	2,39%	-11,69%	-15,03%
9	-3,64%	-3,61%	-3,65%	-13,55%	0,78%	-13,54%	-15,71%
10	2,96%	2,99%	2,99%	-19,64%	2,94%	-19,63%	-21,08%
11	-8,58%	-8,56%	-8,61%	-9,39%	11,24%	-9,39%	-10,37%
12	-10,68%	-10,65%	-10,64%	0,53%	19,29%	0,54%	-0,51%
13	1,88%	1,91%	1,93%	-7,80%	12,53%	-7,80%	-8,85%
14	5,11%	5,14%	5,14%	-21,68%	15,99%	-21,67%	-22,23%
15	0,41%	0,44%	0,46%	-27,10%	12,24%	-27,09%	-27,33%
16	-8,19%	-8,16%	-8,15%	-19,79%	17,29%	-19,78%	-19,49%
17	-3,39%	-3,36%	-3,40%	-18,49%	18,19%	-18,48%	-18,37%
18	13,48%	13,51%	13,51%	-41,57%	-10,15%	-41,56%	-41,57%
19	-4,81%	-4,77%	-4,80%	-38,09%	-7,44%	-38,08%	-37,87%
20	0,63%	0,67%	0,65%	-41,79%	-10,32%	-41,78%	-41,98%
21	-11,80%	-11,76%	-11,77%	-26,41%	1,65%	-26,40%	-27,03%
22	3,75%	3,78%	3,78%	-35,56%	-5,47%	-35,55%	-36,31%
23	-15,03%	-15,00%	-14,99%	-13,83%	-6,48%	-13,82%	-13,98%
24	-6,25%	-6,22%	-6,22%	-9,78%	-2,70%	-9,77%	-9,95%
25	-7,11%	-7,08%	-7,10%	-5,11%	1,67%	-5,10%	-5,46%
26	-12,00%	-11,97%	-11,98%	5,06%	11,19%	5,07%	4,69%
MAPE	6,40%	6,390%	**6,388%**	14,57%	10,99%	14,57%	15,10%

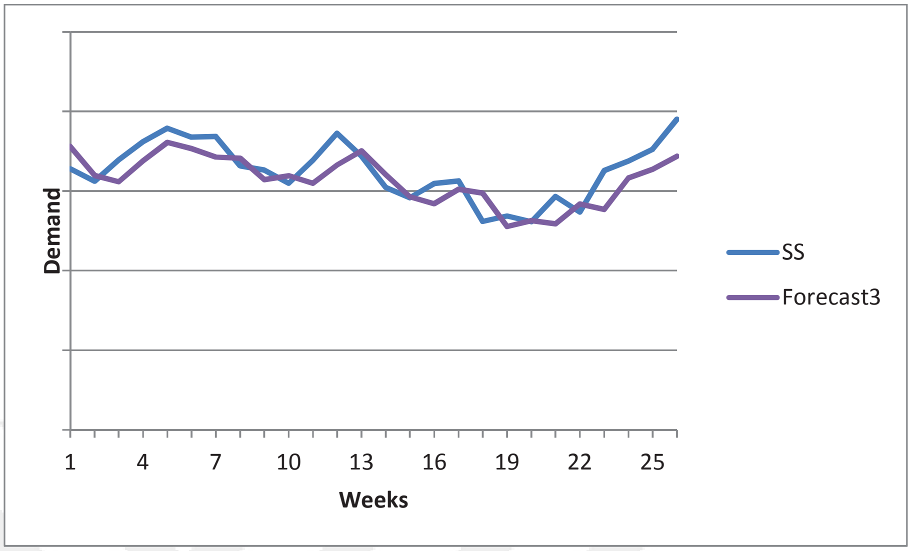

Figure 3.61: Route E - The Best forecasting result

- For Route F, LR forecasting results give better and close solutions to each other and forecast2 has the best fitting when compared to the realized SS values as seen in Table 3.7 and Figure 3.62.

Table 3.7: Route F-Error Rates for all methods

Week	forecast1	forecast2	ETS1	ETS2	ARIMA1	ARIMA2
1	-3,59%	-5,70%	10,05%	-12,26%	10,05%	9,65%
2	-2,83%	-4,41%	6,91%	-16,87%	6,90%	6,55%
3	2,75%	1,39%	-3,41%	-30,58%	-3,42%	-3,82%
4	-4,95%	-6,55%	1,23%	-25,45%	1,22%	0,84%
5	1,78%	0,72%	-9,88%	-40,37%	-9,88%	-10,31%
6	27,18%	26,98%	-48,31%	-6,87%	-48,31%	-48,89%
7	14,15%	13,21%	-45,17%	-5,21%	-45,18%	-45,75%
8	-3,01%	-3,81%	-22,98%	10,36%	-22,99%	-23,47%
9	-5,77%	-6,45%	-11,12%	18,55%	-11,13%	-11,56%
10	-2,59%	-3,05%	-11,57%	-0,83%	-11,57%	-12,01%
11	-10,42%	-11,02%	-1,01%	8,20%	-1,02%	-1,41%
12	-4,14%	-4,59%	-4,07%	4,89%	-4,07%	-4,48%
13	-19,22%	-14,46%	6,64%	14,21%	6,64%	6,27%
14	-18,28%	-16,00%	31,68%	26,52%	31,68%	31,41%
15	3,42%	6,74%	21,71%	15,35%	21,71%	21,40%
16	-3,49%	-0,65%	24,18%	17,57%	24,18%	23,88%
17	-3,81%	-0,90%	24,88%	17,88%	24,88%	24,58%
18	14,38%	14,71%	17,56%	-25,33%	17,56%	17,24%
19	17,22%	17,48%	13,37%	-32,41%	13,37%	13,03%
20	0,33%	0,11%	24,42%	-16,15%	24,41%	24,12%
21	9,26%	9,42%	20,47%	-22,85%	20,47%	20,16%
22	1,53%	1,31%	26,10%	-14,77%	26,10%	25,81%
23	12,62%	11,35%	22,50%	-31,17%	22,50%	22,20%
24	9,87%	8,43%	23,61%	-29,98%	23,60%	23,30%
25	3,26%	1,45%	30,08%	-19,58%	30,08%	29,80%
26	0,58%	-1,35%	34,76%	-12,16%	34,76%	34,50%
MAPE	7,38%	**7,00%**	19,52%	18,95%	19,52%	19,41%

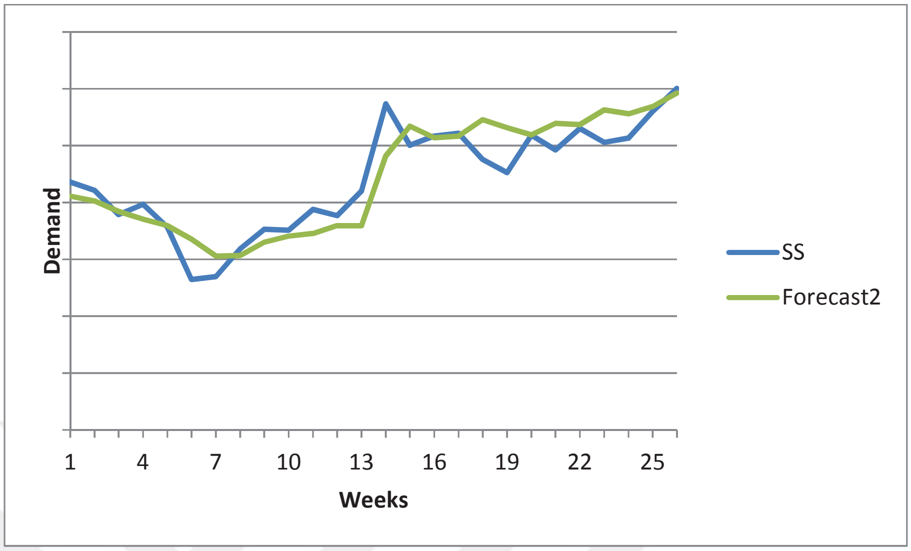

Figure 3.62: Route F - The Best forecasting result

- For Route G, LR forecasting results give better and very close solutions to each other and forecast1 has the best fitting when compared to the realized SS values as seen in Table 3.8 and Figure 3.63.

Table 3.8: Route G-Error Rates for all methods

Week	forecast1	forecast2	forecast3	ETS1	ETS2	ARIMA1	ARIMA2
1	-11,27%	-12,87%	-13,63%	27,02%	-22,50%	23,42%	26,95%
2	-1,74%	-3,41%	-3,94%	24,42%	-26,86%	17,47%	24,24%
3	-7,48%	-8,93%	-8,54%	24,51%	-26,72%	15,11%	25,50%
4	-6,91%	-8,47%	-8,41%	21,86%	-31,16%	10,21%	23,27%
5	-5,03%	-6,56%	-6,14%	18,36%	-37,04%	4,66%	18,96%
6	20,01%	18,28%	19,17%	-3,33%	-6,63%	-22,15%	-2,84%
7	-9,49%	-10,92%	-9,72%	8,03%	5,10%	-9,71%	9,48%
8	-15,72%	-17,02%	-15,87%	20,58%	18,05%	4,61%	22,61%
9	-6,93%	-8,43%	-7,72%	16,40%	13,74%	-0,93%	19,39%
10	-10,99%	-12,46%	-11,81%	20,19%	5,15%	3,26%	22,79%
11	-7,81%	-9,26%	-8,38%	18,66%	3,34%	1,11%	21,29%
12	-11,59%	-12,87%	-11,66%	24,70%	10,51%	8,25%	27,19%
13	-10,90%	-12,30%	-11,49%	24,76%	10,59%	8,17%	26,97%
14	-8,80%	-10,04%	-9,59%	38,25%	16,60%	24,53%	40,35%
15	-6,51%	-7,82%	-7,44%	38,68%	17,19%	24,99%	40,56%
16	-6,29%	-7,57%	-7,20%	41,32%	20,74%	28,16%	43,17%
17	8,87%	7,20%	6,97%	26,79%	1,12%	10,32%	28,72%
18	-2,56%	-3,90%	-2,77%	27,84%	-21,55%	11,57%	29,43%
19	-2,89%	-4,20%	-3,07%	29,94%	-18,01%	14,12%	31,74%
20	4,02%	2,63%	3,66%	25,69%	-25,18%	8,88%	27,62%
21	-1,97%	-3,12%	-1,50%	32,07%	-14,41%	16,70%	33,55%
22	-10,51%	-11,65%	-10,74%	45,23%	7,75%	32,82%	46,15%
23	2,95%	1,45%	1,33%	36,67%	10,92%	22,31%	37,10%
24	6,05%	4,42%	4,14%	27,64%	-1,77%	11,24%	28,18%
25	-0,75%	0,34%	0,60%	28,50%	-0,56%	12,28%	28,48%
26	2,38%	3,51%	3,71%	25,62%	-4,61%	8,75%	25,49%
MAPE	**7,12%**	7,87%	7,45%	27,26%	14,58%	14,45%	28,61%

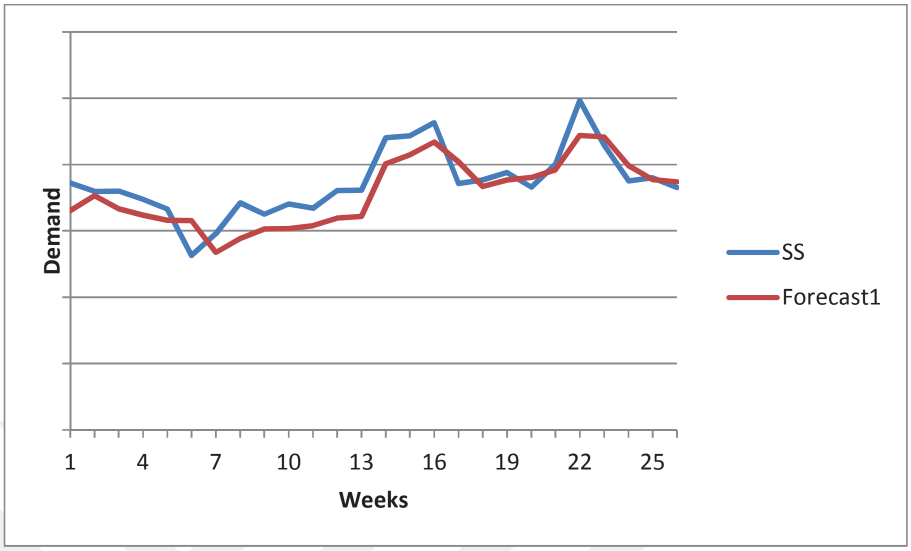

Figure 3.63: Route G - The Best forecasting result

- For Route H, LR forecasting results give better and very close solutions to each other, however, unlike the other routes, ARIMA2 has the minimum error rate when compared to the realized SS values as seen in Table 3.9. Nevertheless, when the graph is looked, forecast2 has the best fitting as seen in Figure 3.64.

Table 3.9: Route H-Error Rates for all methods

Week	forecast1	forecast2	ETS1	ETS2	ARIMA1	ARIMA2	ARIMA3
1	-3,74%	-4,82%	9,23%	-10,53%	9,40%	9,51%	9,82%
2	-1,43%	-1,02%	9,91%	-9,69%	9,27%	9,20%	10,29%
3	2,64%	3,85%	3,01%	-18,10%	1,53%	2,52%	3,97%
4	-0,74%	-0,32%	1,47%	-19,97%	-0,70%	0,08%	2,79%
5	1,72%	1,97%	-1,97%	-24,16%	-4,86%	-4,61%	2,07%
6	15,08%	14,89%	-17,55%	-13,14%	-21,62%	-19,85%	-13,96%
7	-15,03%	-13,98%	-6,21%	-2,22%	-10,53%	-8,57%	-3,80%
8	-7,67%	-8,41%	4,37%	7,96%	-0,11%	1,77%	7,06%
9	-1,01%	-0,82%	4,75%	8,32%	-0,29%	2,08%	6,88%
10	1,45%	1,50%	3,66%	-9,56%	-2,04%	-0,06%	5,59%
11	-4,98%	-5,20%	9,97%	-2,39%	4,09%	4,83%	12,61%
12	-4,39%	-3,42%	10,53%	-1,75%	4,14%	3,19%	13,74%
13	-3,37%	-1,85%	7,46%	-5,24%	0,28%	-2,11%	10,22%
14	-4,54%	-3,99%	8,18%	-12,66%	0,49%	-0,91%	10,86%
15	-3,65%	-2,46%	6,50%	-14,72%	-1,91%	-3,88%	9,26%
16	-3,47%	-2,61%	6,01%	-15,32%	-3,02%	-3,56%	8,71%
17	-5,87%	-5,12%	8,24%	-12,58%	-1,14%	-0,82%	10,84%
18	1,01%	2,20%	2,35%	-46,85%	-8,23%	-1,14%	4,79%
19	-2,14%	-1,93%	3,21%	-45,56%	-7,88%	-3,14%	5,74%
20	-1,65%	-1,40%	3,53%	-45,07%	-8,11%	-5,35%	6,16%
21	-3,03%	-2,76%	5,20%	-42,57%	-6,83%	-2,74%	7,71%
22	-3,01%	-2,07%	5,70%	-41,82%	-6,84%	-4,20%	8,26%
23	-5,91%	-4,92%	8,65%	-5,01%	-4,06%	-2,03%	11,16%
24	-3,78%	-2,34%	7,88%	-5,89%	-5,50%	-1,18%	10,31%
25	-6,52%	-5,22%	10,14%	-3,30%	-3,47%	2,32%	12,43%
26	-5,93%	-4,29%	10,53%	-2,85%	-3,57%	0,77%	12,88%
MAPE	4,28%	**3,87%**	6,78%	16,23%	4,84%	**3,73%**	8,61%

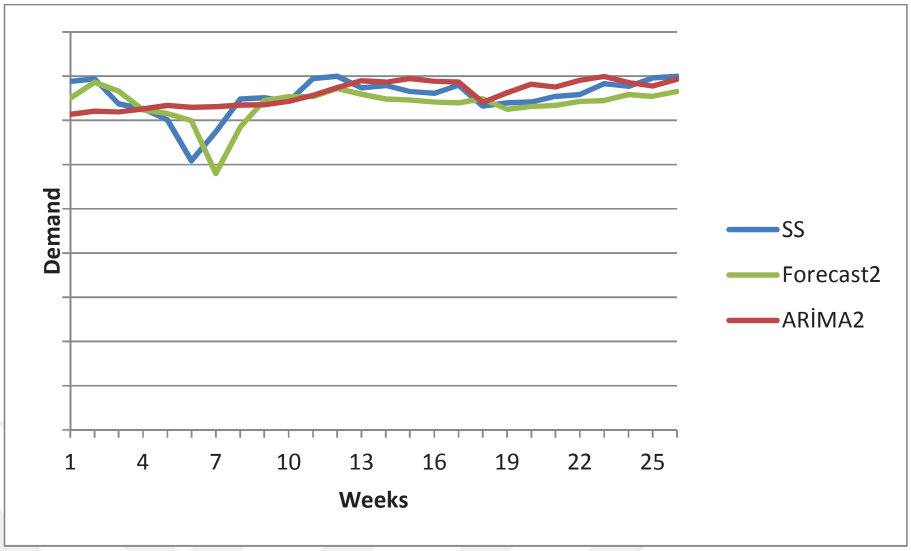

Figure 3.64: Route H - The Best forecasting result

4. REVENUE MANAGEMENT WITH FARE PRODUCTION

Sales of passenger and cargo service in air transportation sector generate airline's revenue. Revenue management has to maximize this revenue with correct forecasting of expected demand and optimum fare. Especially, for passenger service in the aviation sector, sale price has to struggle competitive conditions and demand models has to help demand analysts to manage class availability which refers to sale price levels. In order to present a good competitive price, we need to produce fare that gives the best price or plausible price in the market. In monopoly markets, price is not significant to compete with the others. Therefore, airlines should generate a fair price which determined by taking into account other factors such as socio-economic indicators, unit revenue per capita, and prevalence of air transportation in the country.

4.1. Fare Production

In the dynamic models of section 2.2.3, there are n fare classes, $p_1 \geqslant p_2 \geqslant p_2 \geqslant \cdots \geqslant p_n$, T total periods and time index runs from $t = 1$ to $t = T$, t represents periods and j displays classes. The probability of an arrival of class j in period t is denoted by $\lambda_j(t)$. The assumption of at most one arrival per period implies that;

$$\sum_{j=1}^{n} \lambda_j(t) \leqslant 1 \tag{4.1}$$

$$P\left(R(t) = p_j \right) = \lambda_j(t) \tag{4.2}$$

The sales data consist of 104 weeks and models focus on the average fare, therefore we assume that there is only one class ($n = 1$). We define $\lambda = demand$ as a function of different parameters (avfare, rivalfare, lastSS, capacity, amount/frequency, market potential and seasonality). Sales period of a flight starts nearly 350 days before departure and its total sales period can be described from now $t = 0$ to flight date $t = T$ with m intervals as seen in Figure 4.1 and m intervals represents weeks in the sales period.

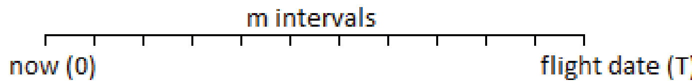

Figure 4.1: Time intervals

Demand models and fare production are dynamic and their forecasting can be revised all weeks through ongoing flights' data. For example, in order to figure out demand and fare for a flight of which departure date is 30 weeks later, rivalfare, lastSS and other parameters are needed. But, they can change weekly. To reach lastSS parameter, forecasted demand is used. Forecasted demand for week 29 turns into lastSS value for week 30, so when any week realized, its realized demand turns the next week's lastSS and it affects other weeks' variables respectively.

In demand modeling, the main aim is to have the best fitted forecasting model with minimum error rates. Therefore, it is not possible to include all the explanatory variables in the model and the best statistical model may not have all parameters. In different demand models, there can be many different parameters. Some of them include fare variables, avfare and/or rivalfare, but, some of them have no fare variable because of being niche market.

In the Section 3, the best fitted demand models of each market route are achieved with minimum error rate when compared to other models. Error rate can be numbered through the comparison of forecasted demand values with the realized 26 weeks data. After having forecasted demand values, avfare can be calculated from models to reach revenue. In order to find maximum revenue, optimum or plausible fare is necessary. Fare can sometimes reach an optimum value, but sometimes it is not optimal due to market conditions such as competitor's very low price (competitor's price is very significant to determine airline's sale price and that price is dynamic), the perception to the brand (people can meet the brand firstly), high supply in that market, low market share, launching new market, the perception of customers to airlines and air transportation. Because of these reasons, fare production is used instead of fare optimization as a term.

After having forecasted demand values in the air transportation sector, thanks to fare production, the main aim turns into maximum revenue. Good forecasted demand with accurate fare provides a very significant tool to airlines in very high competitive markets. In aviation sector, the level of competition is increasing day by day. To determine the correct fare, there are many parameters which affect customers' perception. These parameters' effects can be explained as below:

- Avfare (sale price) and rivalfare (competitor's price) have very high effect on customers. High ticket price is the main reason to give up or change traveling plan. Therefore, fare is very vital to customers in some markets. All demand models except market route D have fare variables as a parameter.
- Capacity has high effect on fare production. High capacity generates high pressure to reduce sale price because of the load factor meaning the percentage of passengers in that flight. All demand models except market route F have capacity variable as a parameter.
- Amount/frequency provides different passenger types to use that flight because of different connections to different destinations. Therefore, it has effect on the price and demand. Demand model of market route F has amount variable as a parameter.
- LastSS shows the last week sales. It presents the trend of the customers' choice and gives direction to fare and demand functions. All demand models except market route B have lastSS variables as a parameter.

These parameters are significant to find the maximum revenue because there is always a risk to lose customer, they can choose different airlines or give up travelling with air transportation or cancel their own plan to go another place. The best price, which maximizes revenue and does not cause a reason to lose a customer, is the goal.

4.2. The Implementation of Revenue Maximization with Fare Production

In order to reach forecasted fare values, the best fitted demand models are equalized with demand models which include fare parameter and have minimum error rates when compared to realized data.

Fare production process contains 5 steps:

i. At first, the best demand models which do not contain avfare are obtained because realized avfare is used as data source for avfare parameter when the best demand models are studied. To find expected avfare, it is assumed that avfare parameter is absent and the best model is available. All used models, whether avfare is present or not, have minimum error rates when compared to other models for a specific market. If avfare exists in the best model, new demand model is generated and its forecasted demand values are used. This situation only occurs in two different market routes' demand functions (market route F and market route G).
ii. After the demand models are obtained without avfare parameter, the alternative demand model is generated with avfare. This demand model has minimum error rate when compared to other demand models including avfare parameter.
iii. The best forecasted demand values (i) replace the alternative demand values (ii) and these models equalize each other.
iv. After this equalization, rivalfare, capacity, amount, lastSS and demand values are available and there is an only absent parameter which is avfare. Then, Avfare is reached with the equalization of the forecasted values with the alternative model.
v. Finally, forecasted revenue is formed with forecasted demand and forecasted avfare for the next 26 weeks and its maximization is checked owing to the comparison with realized revenue including 26 weeks realized data.

- For Route A, the best demand model is:

$$SS_{13} = -3882 + 0.21SS_{t-1} + 0.58c + 47.8f_r + 0.71mp \tag{4.3}$$

Because of the absence of the avfare parameter, it is not required to create a new demand function. In order to reach avfare value, the alternative demand function is formed. The alternative model is:

$$SS_{1a} = -2256 + 0.195SS_{t-1} + 60F + 0.85c - 91a + 0.7mp \tag{4.4}$$

After obtaining demand values from the best model, they replace with the alternative demand values as;

$$SS_{13} = SS_{1a} \tag{4.5}$$

Avfare is only absent parameter in this equation, then it can be obtained for other weeks. With demand values and fare values, Forecasted Revenue is generated as total revenue of 26 weeks. When compared to Realized Revenue of Market Route A, solutions are given as follows:

$$Forecasted\ Revenue_a = 12{,}175{,}426$$
$$Realized\ Revenue_a = 11{,}663{,}989$$

Forecasted Revenue gives better income than Realized Revenue with the difference of 4.38%.

- For route B, the best demand model is:

$$SS_{22} = -1309 + 0.54c - 450s + 41f_r + 0.92mp \tag{4.6}$$

And the alternative demand function is:

$$SS_{2a} = -1801 + 41F + 0.54c - 450s + 0.92mp \tag{4.7}$$

And the equalization of demand values from the best model with the alternative demand is:

$$SS_{22} = SS_{2a} \tag{4.8}$$

After this replacement, Avfare parameter can be obtained for other weeks. When compared to Forecasted Revenue with Realized Revenue of Market Route B, results can be seen as:

$$Forecasted\ Revenue_b = 4{,}041{,}050$$
$$Realized\ Revenue_b = 4{,}079{,}865$$

In this case, Forecasted Revenue is lower than realized one. In our calculation of fare values, the best demand models are used. Therefore, all produced fares are demand based. When this reason is analyzed, it is observed that, when forecasted demand values are lower than realized demand, it gives rise to low fare values and low revenue. However, in revenue management, if sale price is low, demand will be high and if sale price is high, demand might be low in general. Applied demand models are accepted the best after the comparison of different demand models with different forecasting methods. Because of this reason, they cannot be revised. Hence, fare values are supposed to be revised according to price elasticity, which gives passenger responses to the price changes, and demand difference rate between forecasted demand and realized demand.

Market Route B is in The African Continent and price multiplier of market route level in Africa is 0.6 as IATA [23]. The difference rate of demand is equal to -2%. Price elasticity in Market Route B can be figured out as:

$$elasticity_b = \text{price multiplier} * \text{demand difference rate} \tag{4.9}$$

And price elasticity is equal to 1.2. This proportion can be used to increase fare values weekly. Then, new Forecasted Revenue can be reached as:

$$Forecasted\ Revenue_{bnew} = 4{,}089{,}543$$

New Forecasted Revenue produces better income than Realized Revenue with the difference of 0.24%.

- For Route C, the best demand model is:

$$SS_{32} = -35 + 0.45SS_{t-1} + 0.15c - 114s + 0.59f_r + 0.74mp \tag{4.10}$$

And the alternative demand function is:

$$SS_{3a} = -47 + 0.45SS_{t-1} + 0.61F + 0.16c - 111s + 0.74mp \tag{4.11}$$

And the equalization of demand values from the best model with the alternative demand is:

$$SS_{32} = SS_{3a} \tag{4.12}$$

After this equalization, Avfare parameter of Market Route C can be obtained for other weeks. When compared to Forecasted Revenue with Realized Revenue of Market Route C, conclusions can be seen as:

$$Forecasted\ Revenue_c = 8{,}872{,}929$$
$$Realized\ Revenue_c = 9{,}428{,}106$$

As seen in the previous market route, Forecasted Revenue is lower than realized one with the same reason which is low demand values. Therefore, fare values can be revised according to price elasticity and demand difference rate.

Market Route C is in Turkey and price multiplier in Europe is 1.4. The difference rate of demand is equal to -5.34%. Price elasticity in Market Route C equals to 7.47%. This proportion can be used to rise up fare values. Then, new Forecasted Revenue can be achieved as:

$$Forecasted\ Revenue_{cnew} = 9{,}536{,}270$$

New Forecasted Revenue generates better income than Realized Revenue with the difference of 1.15%.

- For Route D, the best demand model is:

$$SS_{41} = -582 + 0.51SS_{t-1} + 0.5c + 162.6s + 0.11mp \tag{4.13}$$

And the alternative demand function is:

$$SS_{4a} = -695 + 0.51SS_{t-1} + 0.5c + 0.81F + 157.5s + 0.11mp \quad (4.14)$$

And the equalization of demand values from the best model with the alternative demand is:

$$SS_{41} = SS_{4a} \quad (4.15)$$

After this calculation, Avfare parameter of Market Route D can be reached for other weeks. When compared to Forecasted Revenue with Realized Revenue of Market Route D, consequences can be seen as:

$$Forecasted\ Revenue_d = 7{,}913{,}979$$
$$Realized\ Revenue_d = 6{,}296{,}276$$

Forecasted Revenue achieves much better income than Realized Revenue with the difference of 25.69%. For this market, produced fare is generated for the summer season and the winter season because this is a niche market and there is no powerful rival to take into consideration. There can be different causes regarding of being niche market, for example; war, slot permission problems, distances, security, aircraft deficiency, airport conditions, etc. If fares are plausible, airline can be full load factor with maximum revenue.

- For Route E, the best demand model is:

$$SS_{53} = -86 + 0.77SS_{t-1} + 1.19f_r + 0.37p + 0.009c \quad (4.16)$$

And the alternative demand function is:

$$SS_{5a} = -78 + 0.77SS_{t-1} + 1.19F + 0.37mp \quad (4.17)$$

And the equalization of demand values from the best model with the alternative demand is:

$$SS_{53} = SS_{5a} \quad (4.18)$$

After this replacement, Avfare parameter of Market Route E can be obtained for other weeks. When compared to Forecasted Revenue with Realized Revenue of Market Route E, sums can be seen as:

$$Forecasted\ Revenue_e = 8{,}274{,}636$$
$$Realized\ Revenue_e = 8{,}568{,}511$$

As seen in the previous market routes, Forecasted Revenue is lower than realized one with the similar reason which is low demand values. Therefore, fare values can be revised according to price elasticity and demand difference rate.

Market Route E is in The Asian Continent and price multiplier in Asia is 0.95. The difference rate of demand is equal to -4.9%. Price elasticity in Market Route E equals to 4.65%. This rate can be used to improve fare values. Then, new Forecasted Revenue can be achieved as:

$$Forecasted\ Revenue_{enew} = 8{,}659{,}407$$

New Forecasted Revenue gives better income than Realized Revenue with the difference of 1.06%.

- For Route F, the best demand model is:

$$SS_{62} = -1468 + 0.34SS_{t-1} - 12.6F + 188a + 14.7f_r + 0.16mp \quad (4.19)$$

In this model, avfare exists as a parameter, that’s why, new demand function is created.

$$SS_{6new} = -1528 + 0.3SS_{t-1} + 2.61f_r + 180a - 217.8s + 0.23mp \quad (4.20)$$

As to obtain avfare, we can use the best model as an alternative model.

$$SS_{6new} = SS_{62} \tag{4.21}$$

After this equalization, avfare parameter of Market Route F can be reached for other weeks. When compared to Forecasted Revenue with Realized Revenue of Market Route F, results can be seen as:

$$Forecasted\ Revenue_f = 39{,}310{,}042$$
$$Realized\ Revenue_f = 36{,}508{,}445$$

Forecasted Revenue creates better income than Realized Revenue with the difference of 7.67%.

- For Route G, the best demand model is:

$$SS_{71} = -1231 + 0.41SS_{t-1} + 8.04F + 0.54c \tag{4.22}$$

As similar to demand model of Market Route F, avfare exists as a parameter in the best demand model of market route G, hence, new demand function is created.

$$SS_{7new} = -1168 + 0.37SS_{t-1} + 6.32f_r + 0.59c + 0.056mp \tag{4.23}$$

In order to find avfare, we can use the best model as an alternative model.

$$SS_{7new} = SS_{71} \tag{4.24}$$

After this calculation, avfare parameter of Market Route G can be reached for other weeks. When compared to Forecasted Revenue with Realized Revenue of Market Route G, conclusions can be seen as:

$$Forecasted\ Revenue_g = 8{,}451{,}994$$
$$Realized\ Revenue_g = 9{,}085{,}800$$

As seen in the previous market routes, Forecasted Revenue is lower than realized one with the same reason which is low demand values. Hence, fare values can be revised according to price elasticity and demand difference rate.

Market Route G is in The American Continent and price multiplier in America is 1.25. The difference rate of demand is equal to -6.15%. Price elasticity in Market Route G is equal to 7.681%. This rate can be used to enhance fare values. Then, new Forecasted Revenue can be obtained as:

$$Forecasted\ Revenue_{gnew} = 9{,}101{,}213$$

New Forecasted Revenue generates better income than Realized Revenue with the difference of 0.17%.

- For Route H, the best demand model is:

$$SS_{82} = -223 + 0.57SS_{t-1} + 0.37c + 0.44f_r \qquad (4.25)$$

And the alternative demand function is:

$$SS_{8a} = -223 + 0.57SS_{t-1} + 0.37c + 0.44F \qquad (4.26)$$

And the equalization of demand values from the best model with the alternative demand is:

$$SS_{82} = SS_{8a} \qquad (4.27)$$

After this replacement, avfare parameter of Market Route H can be achieved for other weeks. When compared to Forecasted Revenue with Realized Revenue of Market Route H, results can be seen as:

$$Forecasted\ Revenue_h = 16{,}004{,}104$$
$$Realized\ Revenue_h = 16{,}384{,}121$$

As seen in the previous market routes, Forecasted Revenue is lower than realized one with the same reason which is low demand values. Therefore, fare values can be revised according to price elasticity and demand difference rate.

Market Route H is in Turkey and price multiplier in Europe is 1.4. The difference rate of demand is equal to -2.23%. Price elasticity in Market Route H is equal to 3.12%. This rate can be used to improve fare values. Then, new Forecasted Revenue can be obtained as:

$$Forecasted\ Revenue_{hnew} = 16{,}503{,}433$$

New Forecasted Revenue gives us better income than Realized Revenue with the difference of 0.73%.

Finally, all studied market routes indicate own characteristic features. Market Route A provides all rules and gives better forecasted revenue than realized revenue and its history data are very acceptable (Figure 3.1). Market Route B needs to improve fare values because of its situation. It has much more seasonality effect in history data (Figure 3.2). Market Route C needs to enhance fare values, too. As similar to Market Route B, it has high seasonality effect. Market Route D which is niche market ensures much better revenue when compared to Realized Revenue. Market Route E needs to develop fare values and it has fluctuations in the last 2 years' data. Market Route F which is in The American Continent gives us 7% better revenue than realized one, it refers to nearly 2.8 million $. Market Route G and H grow properly in history data. It is hard to show seasonality and turning demand points in the year. Hence, demand and fare forecasting are very hard. But, after obtained best demand models with improved fare values, they make better solutions for revenue.

As observed in Table 4.1 and Table 4.2, all developed demand models and produced fares provide better forecasted revenue in all markets, sometimes difference rate attains 25% and sometimes it reaches only 0.17%. This situation indicates that better revenue can be achieved owing to these best fitted demand models and produced fare values.

Table 4.1: Forecasted values of Market Routes for 26 weeks

Routes	Forecasted Average Fare	Forecasted Demand	Forecasted Revenue
A	44.58	225,099	12,175,426
B	47.08	65,590	4,089,543
C	306.57	26,232	9,536,270
D	118.60	55,297	7,913,979
E	211.53	33,920	8,659,407
F	337.63	96,336	39,310,042
G	96.14	77,093	9,101,213
H	429.32	31,814	16,503,433

Table 4.2: The Comparison of Forecasted Revenue and Realized Revenue

Routes	Forecasted Revenue	Realized Revenue	Revenue Difference	Difference Rate
A	12,175,426	11,663,989	511,437	4.38%
B	4,089,543	4,079,865	9,678	0.24%
C	9,536,270	9,428,106	108,164	1.15%
D	7,913,979	6,296,276	1,617,703	25.69%
E	8,659,407	8,568,511	90,895	1.06%
F	39,310,042	36,508,445	2,801,597	7.67%
G	9,101,213	9,085,800	15,413	0.17%
H	16,503,433	16,384,121	119,312	0.73%

5. CONCLUSION

In this thesis, we studied airline demand modeling for single leg local market routes and examined eight different markets from different continents. The main aim was to harmonize air transportation demand structure with market effects in modeling. In order to design demand forecasting models, time series analysis tools were implemented. Especially, regression analysis is very useful for airline revenue management. Regression analysis usually had minimum error rates in this study when compared to exponential smoothing, ETS, ARIMA and SARIMA via R-project. Regression indicated that some parameters can be substitute each other; average fare versus rival fare and amount versus capacity. Actually, this situation is very logical because of aviation market structure. Therefore, all regression models were differentiated in terms of market perspective. This process provided better forecasting for some markets.

After obtained the best fitted demand models through time series analysis, we studied revenue maximization. For the problem of fare optimization, we developed a fare production process to reach demand-based fare. This fare ensures taking a position regard to market demand potential and helps airlines to produce dynamic fares to maximize revenue in different markets. In some cases, produced fare did not give a satisfied value because of demand structure. In order to attain a good demand-based fare, we generated fare values through demand-price elasticity and reached better fare values. These fare values with the best fitted demand models gave us more revenue when compared to realized-revenue for the next 26 weeks.

In revenue management principles, the main aim is to maximize revenue with demand forecasting and fare optimization. With our best fitted demand models and fare production, airlines can get more revenue and these demand models depict demand and fare trend for demand and fare analysts. This situation provides airlines to draw a perspective to the markets long before and they can take a position for different markets for different competitors beforehand.

Considering the dynamics of the market, we proposed a different perspective to revenue management systems. In air transportation sector, profit margin is very little when compared to airlines' endorsements. Therefore, revenue management systems are requirement. But, these systems are in need of market perspective to incorporate demand forecasting. Through this thesis, we presented the importance of competitors' effect for revenue management.

Future research undertaking on demand modeling with fare production may be focused on improving demand model abilities for beyond O&Ds. Demand models in this thesis were based on weekly sales period. Hence, daily fare production and daily demand forecasting could be developed. In order to reach exact solution, we used average fare regardless of sales class. After this study, demand models based sales classes can be generated.

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