CONTROL OF INVERTED PENDULUM: COMPARISON OF VARIOUS STRATEGIES FOR SWING UP AND BALANCING

by

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ABSTRACT

CONTROL OF INVERTED PENDULUM: COMPARISON OF VARIOUS STRATEGIES FOR SWING UP AND BALANCING

In this thesis, two different control schemes are designed for swinging up and three different schemes for balancing of an inverted pendulum, a non-linear electro-mechanical system with open loop unstablity. The controllers which are implemented for experimental purpose are a heuristic control technique using fuzzy logic and an energy control technique for swinging up. In order to balance the pendulum in the upright equilibrium, state feedback control, LQR control which is an optimal control technique and well known traditional control technique PID control are implemented. The control objective is swinging up the pendulum from its initial downward equilibrium to the upward and then balance at this state while it is tracking of desired trajectory. Three different combinations of proposed control schemes are chosen and their performances are compared with each other. Having compared all combinations, it is observed that every controller has advantages and merits for different cases.

ÖZET

TERS SARKAÇ DENETİMİ: YUKARI KALDIRMA VE DENGEDE TUTMA İÇİN ÇEŞİTLİ STRATEJİLERİN KIYASLANMASI

Bu tezde, doğrusal olmayan bir elektro-mekanik sistem olan ve açık çevrimde kararsız özellik gösteren, ters sarkaçın yukarı kaldırılması için iki farklı denetim tasarımı dengede tutulması için üç farklı denetim tasarımı yapılmıştır. Deneysel amaçlı gerçekleştirilen denetleyiciler, yukarı kaldırma için bulanık mantık kullanan bir sezgisel denetleyici ve bir enerji denetleyicidir. Yukarıda dengede tutmak için durum değişkeni geri besleme denetleyicisi ve en uygun denetleme tekniği olan LQR denetleyici ve iyi bilinen geleneksel denetim tekniği olarak kullanılan PID denetleyici uygulanmıştır. Denetimin amacı sarkacı başlangıçtaki aşağı sarkık pozisyonundan yukarıya kaldırmak ve burada dengede tutarken dayanak yörüngede, konumunu izlemesidir. Önerilen tekniklerden üç farklı karışım seçilmiş ve başarımları incelenmiştir. Karşılaştırdığımız zaman görülmüştür ki, her denetleyicisi farklı durumlarda belli üstünlüklere ve faydalara sahiptir.

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e	Error Signal
F	Force
F_c	Friction Force
f_p	Friction Coefficient
K_D	Differential Gain
K_I	Integral Gain
K_P	Proportional Gain
l	Length of the Pendulum
m_c	Mass of the Cart
m_p	Mass of the Pendulum
t	Time
u	Controller Signal
V	Reaction Force of the Rail
x	The Position of the Cart
θ	The Angular Position

1. INTRODUCTION

In recent decades, with the development of technology, control engineering has been playing a very important role in the industrialization process. We meet technical systems everywhere nowadays, which need the existence of a controller. A large application area of automation engineering is to control electro-mechanical systems. In control engineering a mathematical model is used, which describes the electro-mechanical system and which makes possible to design of an automatic controller. Since the physical connections of a mechanical system can accept very complex shape, modeling is a considerably important task and often represents the most difficult part of the controller of a system. After a linear mathematical model is obtained design of an automatic controller is an easy task. However many system in real life cab only be modeled with a nonlinear mathematical model. As a typical control system, the control of an inverted pendulum is excellent in testing and evaluating different control methods since they are ideal examples for non-linear systems and also they are used to show some ideas in control engineering such as the stabilization of systems that show an unstable behavior. The fundamental principles within this control system can be found in many industrial applications, such as stability control of walking robots, vibration control of launching platform for shuttles etc.

Different pendulum systems have been investigated and many different control methods have been proposed for these systems in the literature.

In [1] it is stated that the inverted pendulum has been used as a benchmark for motivating the study of nonlinear control techniques. R.Lozano et.al. [1] propose a controller that uses a particular Lyapunov function and the stability of the system is carried out by LaSalles's invariance problem. The designed controller balances the inverted pendulum and raises it to its upper equilibrium position while the cart displacement is brought to zero. The control strategy is based on an energy approach of the cart-pendulum system. In [2] a dual inverted pendulum system, which is a cart with two independent inverted pendula, is analyzed and compared to the single inverted pendulum system using classical linear methods. K.Lundberg and J.Roberge propose a classical controller that uses only the angles of the pendula and the position of the cart. The designed controller stabilizes the pendula in the inverted position with the cart being positioned at the center of the track. Stonier et al. [3] propose a learning fuzzy controller and compare a two-layered rule set with a single fuzzy logic rule set. Muskinja and Tovornik [4] propose a fuzzy logic controller that aims to swing up the pendulum from lower position to upper position and balance it in the center. For this they use two sets of rules; one is for swinging, and the other one is for balancing. Vikramaditya and Rajamani [5] design a nonlinear control system for trajectory tracking of the trolley-crane system, by using Lyapunov functions and a modified version of sliding-surface control. Astrom and Furuta [6] propose the concept of energy control and show robust strategies for swinging up an inverted pendulum by using this idea. On the other hand, Furuta and Iwase [7] study some control laws in comparison of swing up time and the State Dependent Riccati Equation is found effective for designing the swing up control; the control law is formed as a linear combination of a sine function of the angle and the angular velocity, and a variable structure control with a sliding mode. In [8] a variant of double inverted pendulum is introduced, and a robust feedback controller that is designed by using partial feedback linearization in combination with a limit-cycle inducing control is proposed. J.Lam [9] proposes two methods for swing up, a nonlinear heuristic controller and an energy controller. After swinging up of the pendulum a linear quadratic regulator (LQR) state feedback optimal controller is implemented to maintain the balanced state. M. Bugeja [10] also proposes a similar methodology for the complete control system of an inverted pendulum (swing up and stabilizing of an inverted pendulum) by using an energy controller for swing up and a state feedback controller for balancing. Earl and D'Andrea [11] propose an optimal feedback controller for an initially inverted pendulum that starts from an initial cart position. The goal is to move the cart to a final position in minimum time with minimum oscillation of the pendulum. Sazonov et.al. [12] propose a hybrid system controller, in which a neural controller and an LQG controller are performing together. Genetic Algorithm methodologies are applied for the optimization of the neural controller. The quality of the regulation process is maintained by the neural controller and the wide range of the operation and the stability of transient processes are achieved by the LQG controller. The proposed controller has the benefits of the two constituent controllers. Schreiber et.al. [13] introduce the concept of the *"interactive null space motion"* which was explained as balancing the pendulum as primary task and modifying the configuration as a subtask. In other words a robust controller which is able to maintain the balanced position against perturbations is designed based on the well known LQR methodologies. Nair and Leonard [14] re-investigate the Furuta pendulum and propose that the Furuta pendulum can be expressed as a planar pendulum plus a gyroscopic force. An energy function is chosen as a Lyapunov function for stability, but the results are not so useful; therefore it is suggested to combine other nonlinear techniques to improve the results. Zhao and Spong [15] propose a hybrid controller which works with the idea of first designing a linear controller for the balancing which is a local stabilizing controller for a fixed region (neighborhood) of an initial point, then designing another controller which steers the cart-pendulum system from any given initial position to the neighborhood of the locally stable initial position. Yeh and Li [16] propose a multi-stage fuzzy logic controller. A multi-stage fuzzy logic controller is defined as a set of fuzzy rules that contains intermediate variables. Normally a set of fuzzy rules uses three kinds of linguistic variables. The input variables which appear only in the antecedent part (IF part) of the fuzzy rules, the output variables which appear only in the consequent part (THEN part) of the fuzzy rule and finally the intermediate variables that can appear in the both parts of the fuzzy logic rules. Through the use of the proposed multi-stage fuzzy controller, a satisfactory performance is reported in their work.

All studies mentioned above work on different types of the pendulum systems and deal with different processes with these pendulum systems. Some of them only work on swinging up the pendulum from an initially downward position to an upward position; some of them try to balance around a center point an initially inverted pendulum and some try to track a given trajectory of the pendulum while keeping the pendulum angle constant around the downward position (crane or trolley system) or the upward position. Also the mechanical configurations of the pendulum systems are different; some have double pendula, some have a single pendulum with horizontally moving cart whereas some systems are combination of two linked pendula. The most common definitions of an inverted pendulum are summarized below.

Consider a one stage inverted pendulum system with one degree of freedom. The cart with a slim stick fasted on, can move along a smooth track under the force generated from a servo motor. A controller which controls the servo motor needs to be designed such that the one stage inverted pendulum can be steadily balanced at its vertical position being robust to impulse disturbances.

According to University of Bremen Institute of Automation an inverted pendulum is an upright balanced stick mounted on a horizontally moving cart. The development and verification of new methodologies and techniques of control systems can be done clearly on those systems. Inverted pendulum systems with one angle deviation are described in the literature frequently. The pendulum with two angles deviation can be found very rarely. In this case, the motion of the pendulum in one direction effects the equation of motion for the other direction. For this, the decoupling of the two motions is necessary and can be done under some constrains. The result is two non-linear motion equations independent from the other angle deviation. The motion equation of an inverted pendulum is nonlinear. It can be linearized for an operating point. The linear control theory makes a large number of mathematical tools available to examine such systems and offers a large selection of design procedures, e.g. the method of state feedback. The dynamics of the system can be affected by the feedback condition. The eigenvalue location of the desired system can be specified for example by the procedure of the eigenvalue placement.

In [10] an inverted pendulum is defined pretty well actually: "Single rod Inverted Pendulum (SIP) consists of a freely pivoted rod, mounted on a motor driven cart. With the rod exactly centered above the motionless cart, there are no sidelong resultant forces on the rod and it remains balanced as shown in Figure 1.1.1. In principle it can stay this way indefinitely, but in practice it never does. Any disturbance that shifts the rod away from equilibrium, gives rise to forces that push the rod farther from this equilibrium point, implying that the upright equilibrium point is inherently unstable as shown in Figure 1.1.2. Under no external forces, the rod would always come to rest in the downward equilibrium point as a result of gravity, hanging down as shown in Figure 1.1.3. This is called the pendant position. This equilibrium point is stable as opposed to the upright equilibrium point." [10]

The target of the control process is to swing up the pendulum from its initial downward position and to stabilize it in the inverted position, after it comes to the upright equilibrium point. The cart must also be addressed to a reference position on the rail. All this can be done only by driving the cart back and forth within the limited cart travel range on the rail. The inverted pendulum system belongs to the class of under-actuated mechanical systems having fewer control inputs than degrees of freedom. This fact makes the design of the controller a challenging task and as a result the pendulum system becomes a classical benchmark for testing different linear and nonlinear control techniques.

J.Lam [9] which has a similar study also defines the control problem as the balancing of an inverted pendulum by moving a cart along a horizontal track. In this work two methods are described to swing a pendulum attached to a cart from an initial downwards position to an upright position and maintain that state: A nonlinear heuristic controller and an energy controller. After the pendulum is swung up, a linear quadratic regulator, on state feedback optimal controller is proposed to maintain the balanced state. The heuristic controller outputs a repetitive signal at the appropriate moment and is finely tuned for the specific experimental setup. The energy controller adds an appropriate amount of energy into the pendulum system in order to achieve a desired energy state. The optimal state feedback controller is a stabilizing controller based on a model linearized around the upright position and is effective when the cart-pendulum system is near the balanced state.

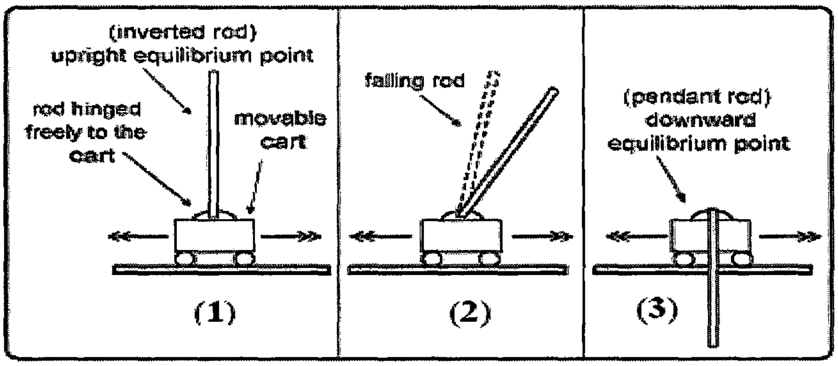

Figure 1.1. Equilibrium Points [10]

J.Lam [9] also states that, “The inverted pendulum system is a standard problem in the area of control systems since it is a naturally nonlinear system. Also it maintains a brief understanding of some complex ideas in nonlinear control. They are often useful to demonstrate terms in linear control such as the stabilization of unstable systems.”

In this system, an inverted pendulum is mounted to a cart equipped with motor that moves it along a horizontal rail. The user is able to dictate the position and velocity of the cart through the motor and the track restricts the cart to movement in the horizontal direction. Sensors are attached to the cart and the pivot in order to measure the cart position and pendulum joint angle, respectively. Measurements are taken with quadrature encoder connected to a general purpose data acquisition and control board. Matlab/Simulink is used to implement the controller and analyze data. The inverted pendulum system inherently has two equilibria, one of which is stable while the other is unstable. The stable equilibrium corresponds to a state in which the pendulum is pointing downwards. In the absence of any control force, the system will naturally return to this state. The stable equilibrium requires no control input to be achieved and, thus, is uninteresting from a control perspective. The unstable equilibrium corresponds to a state in which the pendulum points strictly upwards and, thus, requires a control force to maintain this position. The basic control objective of the inverted pendulum problem is to maintain the unstable equilibrium position when the pendulum initially starts in an upright position. The control objective for this project will focus on starting from the stable equilibrium position (pendulum pointing down), swinging it up to the unstable equilibrium position (pendulum upright), and maintaining this state.

The pendulum-cart set-up used in this study consists of a rod attached on a cart in such a way that the rod can pivot freely only in the vertical plane as shown in Figure 1.7. The figure shows double pendula, however they attached with a shaft through the cart so that they act like a single pendulum. A DC motor is equipped to the system to drive the cart. The cart is driven back and forth on a rail of limited range to swing and to balance the rod.

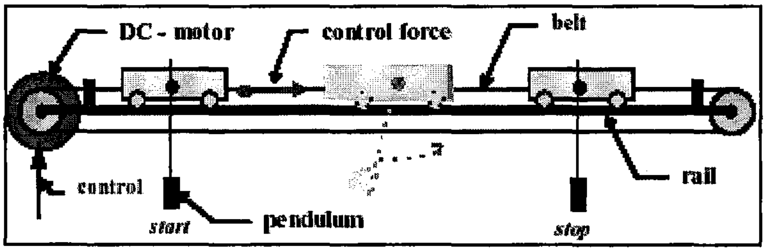

Figure 1.2. The pendulum / cart arrangement [17]

The perpendicular positions of the pendulum (upright and down) are equilibrium positions when no force is being applied. The presence of a small change from the upright equilibrium position results in an unstable motion. Generally the pendulum control problem is to bring the rod to one of the equilibrium positions and preferably to do so as fast as possible, with few oscillations, and without letting the angle and velocity become too large. After accomplishing the desired equilubria, keeping the system in this equilibrium point in the presence of some random disturbing effects becomes the main target. When we want to do simple tasks e.g. driving the cart from one position on the rail to another, manual control of the cart-pendulum system is applicable. On the other hand a state variable feedback control system must be designed as shown in Figure 1.3. for more challenging processes such as stabilizing the rod in an upright position. The intention in the inverted pendulum control algorithm is running a force sequence on the cart with a magnitude that is determined from some series of constraints, such that the cart does not pass over the limits of the rail and the rod does not exhibit swinging motion with an increasing amplitude.

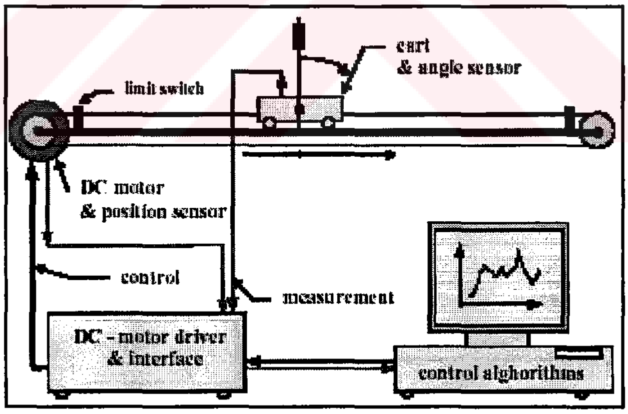

Figure 1.3. Pendulum Control System [17]

Firstly the main target is to swing up the rod to its upright position and then, once it has been succeeded, the control algorithm drives the cart back to the centre of the rail maintaining the rod in its perpendicular position at the same time. Therefore two independent control algorithms are applied for this purpose:

- a swinging algorithm, and
- a stabilizing algorithm

Only one of two control algorithms is acting in each control zone. These zones are shown in Figure 1.4.

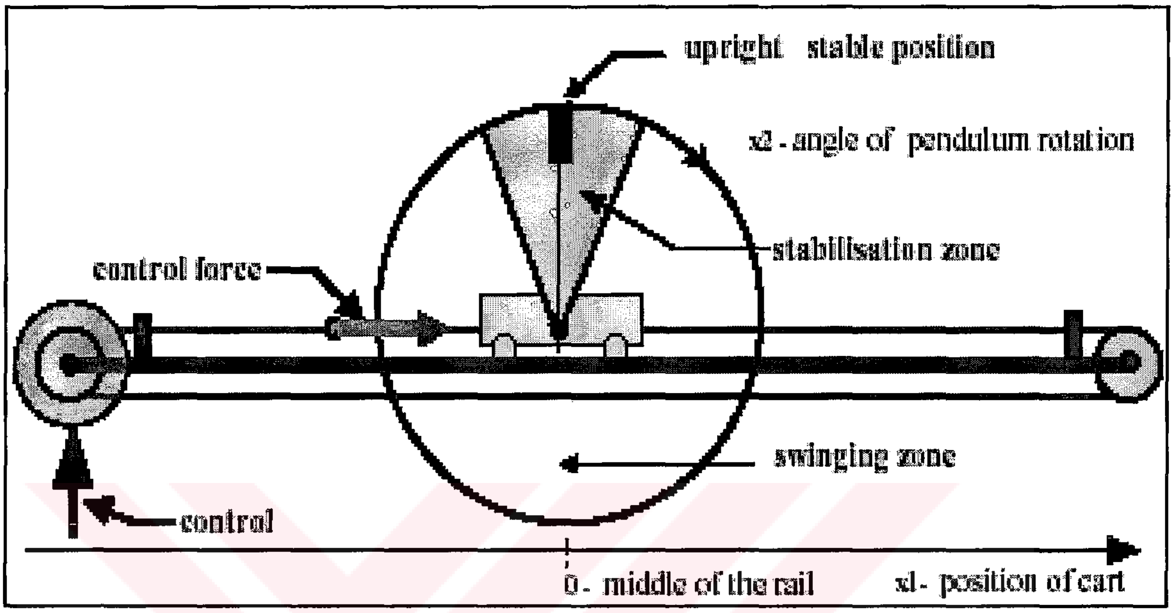

Figure 1.4. Activity zones of two control algorithms [17]

In order to have a smooth transition in landing around the neighborhood of the upright position (the stabilization zone in Figure 1.4), a functional block named “the soft landing arbiter” checks whether the energy difference of the rod is enough to maintain the centre of gravity of the rod to its upright position. If the condition is satisfied then the control is set to zero and the swing up control process is finished. After the rod has entered the stabilization zone, the system can be evaluated as linear and the control is switched to the stabilizing algorithm. Because of the limited driving range of the rail a functional block named “length control” is included, to reinforce the centering of the cart and avoid stepping-over the limits of the rail. The rule is very simple. When the positional value assigned by the "length control" functional block is attained, then a force that has the maximal amplitude is applied to the cart to drive it back far from the critical limit position.

The pendulum-cart set-up uses incremental encoders. Figure 1.5 shows the determination of the direction of rotation for an incremental optical encoder.

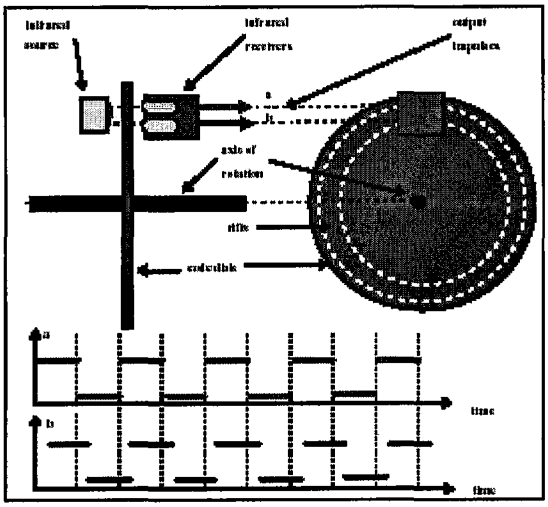

Figure 1.5. Position Sensor [17]

The light beams emitted by two light sources (A and B) go through two rings of slits on the disc. The slits have a phase difference, so that the electric outputs of the receivers (A and B) are rectangular waves with a phase difference. The sign of the phase difference allows the direction of rotation to be determined. The following point should be noted when using an incremental encoder. After each experiment or power switching the cart should be moved to the centre of the rail before starting the next experiment, to allow the zero value of the position to be set. The control signal flows from the computer through the D/A converter of the data acquisition board. The D/A output is wired to the power amplifier input which drives the DC motor. The power amplifier and encoder interface are located in the *Digital Pendulum Controller* box. This box is equipped with two switches: the main power switch and the switch for cutting off the DC motor power. At the rail ends there are two limit switches which cut off the DC motor power when the cart overruns the limit points (Figure 1.6).

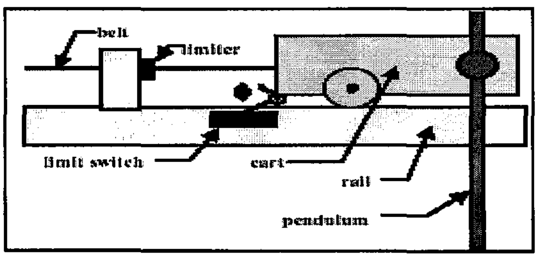

Figure 1.6. Sensors of the limit points [17]

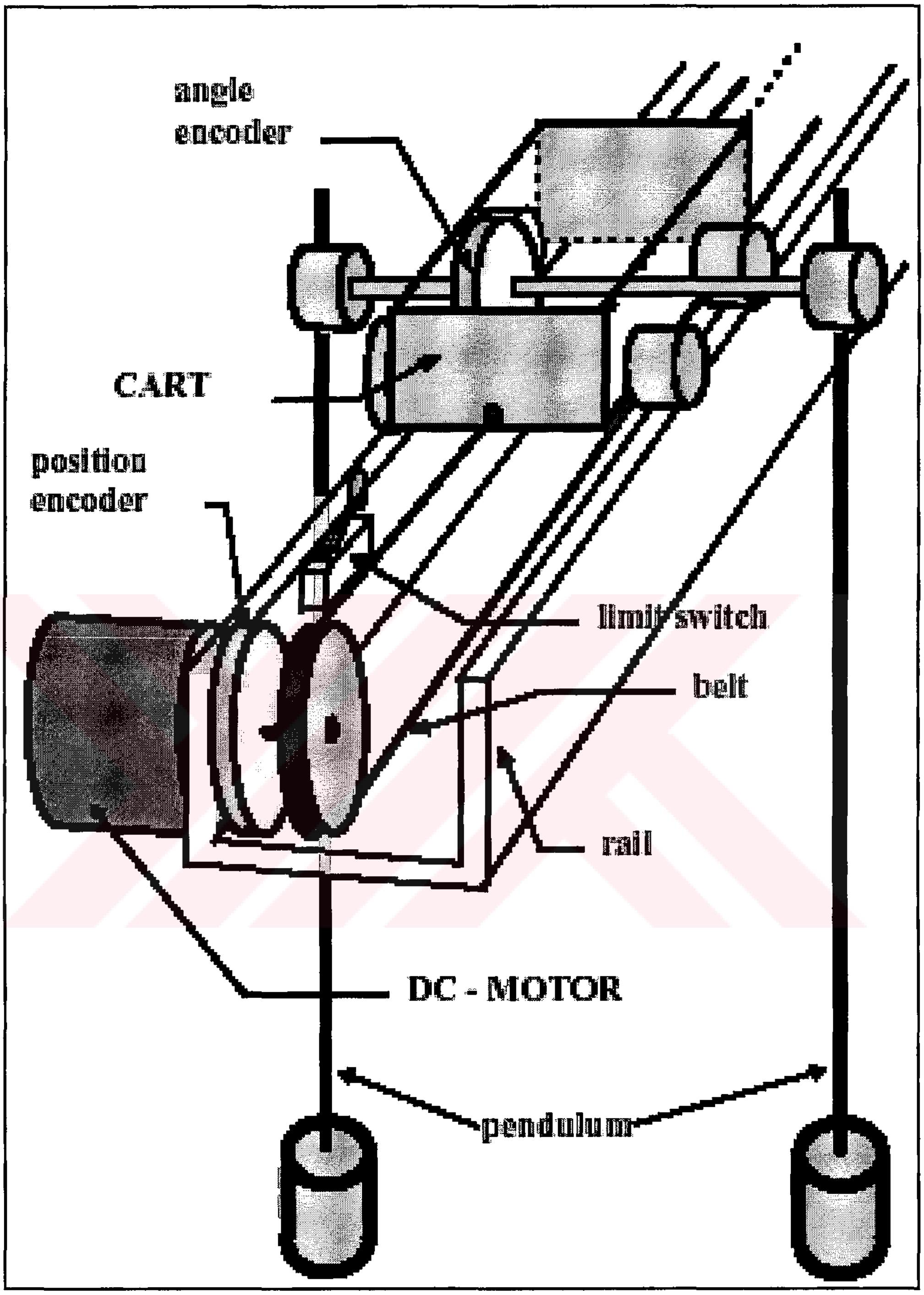

Figure 1.7. Mechanical part and sensors of pendulum-cart set-up [17]

2. PARAMETER IDENTIFICATION AND MODELLING

In the pendulum model, there are three groups of parameters:

- Constant (not to be identified)
- Well identified
- Poorly identified

For example we have the dual pendulum and the cart that are manufactured according to manufacturing tolerances and so, it is accepted that the masses of them are constant and known exactly. The moment of inertia of the rotating pendulum is also another example of this kind of parameters. It slightly differs in different plants. The reason is some small differences in the dimensions of the assembled set. Since the pendulum is set up by the user in the laboratory these kinds of dimension differences may occur. Unfortunately, in a real system there is usually a group of poorly identified parameters, characterized by random behavior; parameters related to the cart friction are in this group. The parameters listed in Table 2.1 are a combination of measured values and derived values obtained by identification experiments.

The pendulum-cart system is an ideal equipment to show some complex and important problems of control theory. The system has the following suggested characteristics:

- The dynamic model involves four state variables and one control variable and is described by one ordinary differential equation of the fourth order or by four ordinary differential equations of the first order.
- the system is non-linear
- the control is bounded
- the length of the rail is limited so the cart position is bounded

The Control problems that can be demonstrated with this system is the “Crane Problem” as steering from point to point with minimum oscillation of the pendulum and the “Pendulum Problem” as swinging up to the vicinity of the upright position and stabilization in the upright position.

Figures 2.1, and 2.2 illustrate a schematic of the pendulum-cart system, and acting forces on the cart respectively.

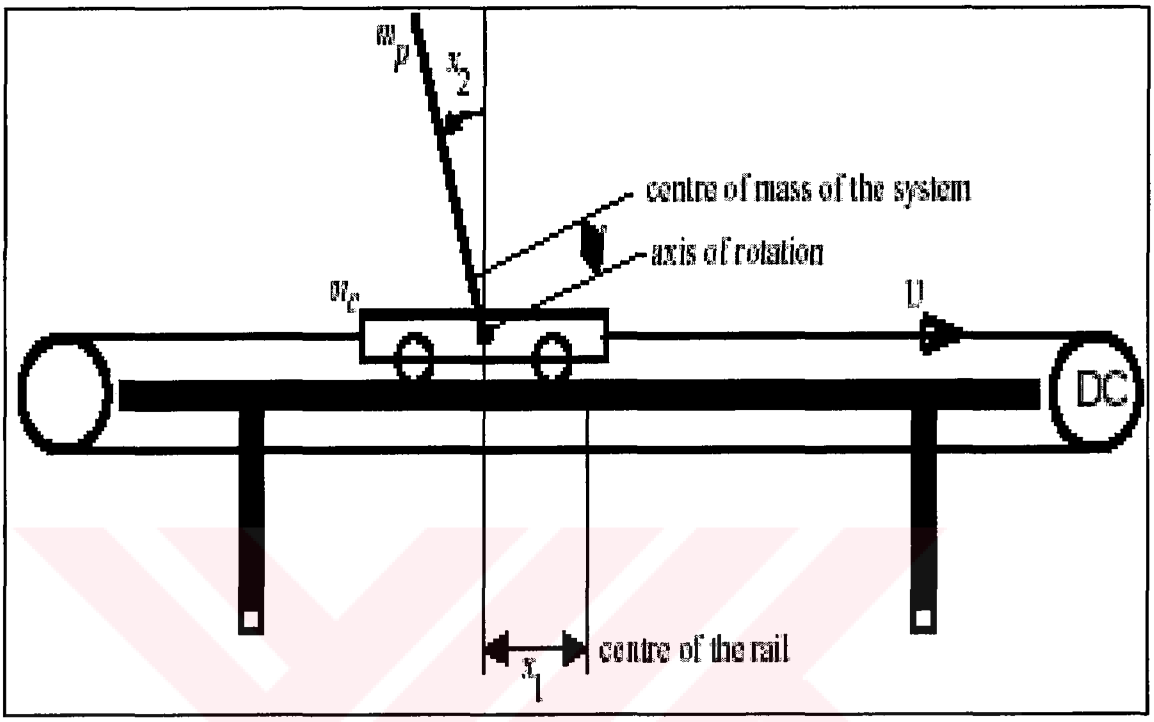

Figure 2.1. Laboratory model of Pendulum-Cart System [17]

The pendulum is pivoting vertically around an axis located on a cart. The cart is sliding along a horizontal track whose long axis is on the plane of rotation. U is the control force acting on the cart and parallel to the rail, m_c is the mass of the cart and the mass of the pendulum is m_p. The distance from the axis of rotation to the centre of mass of the pendulum-cart system is denoted by l. The moment of inertia of the pendulum-cart system with respect to the centre of mass is denoted by J. The state of the system is the vector $X = [x_1\ x_2\ x_3\ x_4]^T$

where:

x_1 is the cart position (distance from the centre of the rail)

x_2 is the angle between the upward vertical and the ray pointing at the centre of mass, measured counterclockwise from the cart (x_2 = 0 for the upright position of the pendulum)

x_3 is the cart velocity

x_4 is the pendulum angular velocity

T_c denotes the friction in the motion of the cart

D_p is the moment of friction in the angular motion of the pendulum, proportional to the angular velocity: $D_p = f_p \cdot x_4$

where f_p is the friction coefficient and l is the length of the rod

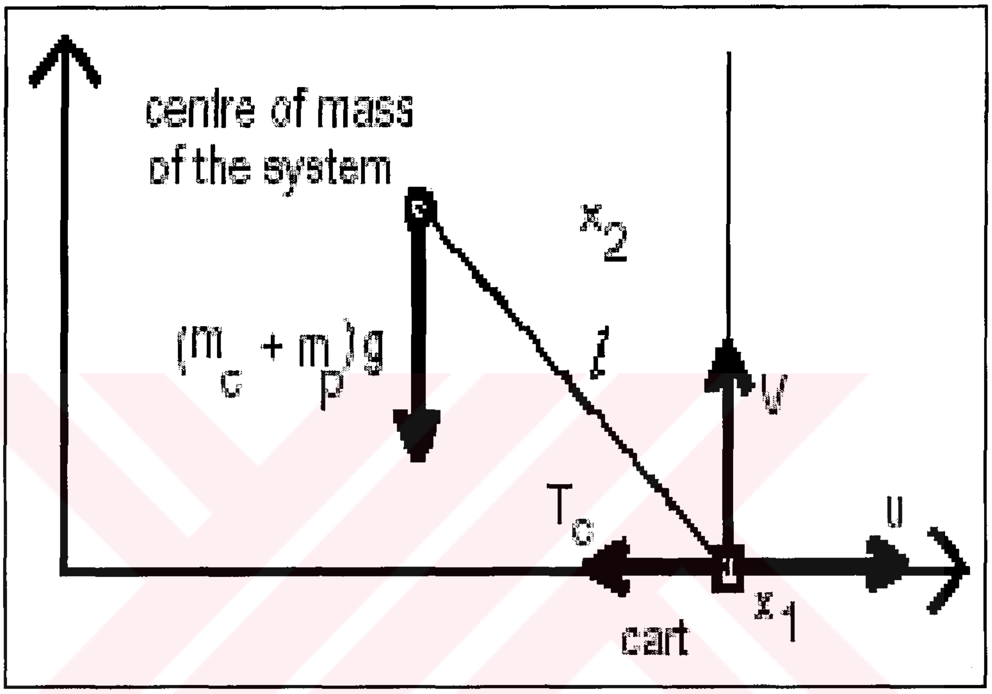

Figure 2.2. Showing the forces acting on the cart [17]

V is the reaction force of the rail that acts vertically on the cart. As the horizontal plane position of the centre of mass is equal to x_1 - $l\sin x_2$ and the vertical to $l\cos x_2$, the equations of dynamics are as follows:

$$(m_c + m_p)(x_1 - l\sin x_2)'' = u - T_c, \tag{2.1}$$

$$(m_c + m_p)(l\cos x_2)'' = V - (m_c + m_p)g, \tag{2.2}$$

$$J x_2'' = (u - T_c)\, l\cos x_2 + Vl\sin x_2 - D_p. \tag{2.3}$$

The second derivative terms with respect to time t are denoted by (.)"and with the same token (.)' denotes the first derivative with respect to time t. The movement of the centre of mass is defined with the first two equations, while the rotation of the whole system around the centre of mass is described by the third one. After some simple calculations the state equations (for $t \geq 0$) are obtained as follows:

In (2.1) taking the second derivative yields

$$x_1'' - l\,x_2''\cos x_2 + l(x_2')^2\sin x_2 = (u - T_c)\,/\,(m_c + m_p) \tag{2.4}$$

and expressing x_1'' as a function of other terms we have

$$x_1'' = [u - T_c + (m_c + m_p)\,l\,x_2''\cos x_2 - l(x_2')^2\sin x_2]\,/\,(m_c + m_p) \tag{2.5}$$

from (2.2) we express V as a function of other terms as follows:

$$V = (m_c + m_p)(g - l\,(x_2')^2\cos x_2 - l\,x_2''\sin x_2) \tag{2.6}$$

and replacing V in (2.3) x_2'' can be expressed as

$$x_2''=[(u-T_c)l\cos x_2+(m_c + m_p)(\,gl\sin x_2 - l^2x_4^2\sin x_2\cos x_2)-f_px_4]/[J+(m_c+m_p)l^2\sin^2 x_2) \tag{2.7}$$

and finally, by using (2.1)-(2.7) the states of the model are obtained as

$$x_1' = x_3 \tag{2.8}$$

$$x_2' = x_4 \tag{2.9}$$

$$x_3' = \frac{a(u - T_c - \mu x_4^2 \sin x_2) + l\cos x_2(\mu g \sin x_2 - f_p x_4)}{J + \mu \ell \sin^2 x_2} \tag{2.10}$$

$$x_4' = \frac{l\cos x_2(u - T_c - \mu x_4^2 \sin x_2) + \mu g \sin x_2 - f_p x_4}{J + \mu \ell \sin^2 x_2} \tag{2.11}$$

where

$$a = l^2 + \frac{J}{m_c + m_p}, \tag{2.12}$$

$$\mu = (m_c + m_p)l. \tag{2.13}$$

Table 2.1. System parameters [17]

PARAMETER DESCRIPTION	SYMBOL	VALUE
Cart mass [kg] :	mc	1.12
Cart mass without positioners [kg] :	mc_	1.045
Pendulum weight mass [kg] :	mpw	0.095
Pendulum stick mass [kg] :	mps	0.025
Rail length [m] :	Rl	1
Pendulum stick length [m] :	lp	0.402
Distance lpo [m] :	lpo	0.146
Length of load[m] :	lc	0.041
Distance lco [m] :	lco	0.347
Radius of the load [m] :	rc	0.02
Radius of the stick [m] :	rp	0.006
Theoretic pend. moment of inertia [kg*m2]:	Jpt	0.0123321
Static friction [N] :	FS	2.28133
Coulombic friction [N] :	FC	2.53165
Maximum control [N] :	M	17.463
Minimum control [N] :	DZu	1.37918
Minimum cart velocity [m/s] :	DZcv	-0.00793711
Minimum pend. velocity [rad/s] :	DZpv	3.37476
Pendulum friction [kg*m2/s] :	fp	0.000107443
Pendulum period [s] :	T	1.16
Pendulum moment of inertia [s] :	Jp	0.0139231
Moment of inertia [kg*m2] :	J	0.0135735
Distance: axis of rotation-mass centre [m] :	l	0.0167903

3. SWING UP CONTROL PROCESS

In order o have a clear understanding of the control process; a block diagram of the system is given in Figure 3.1.

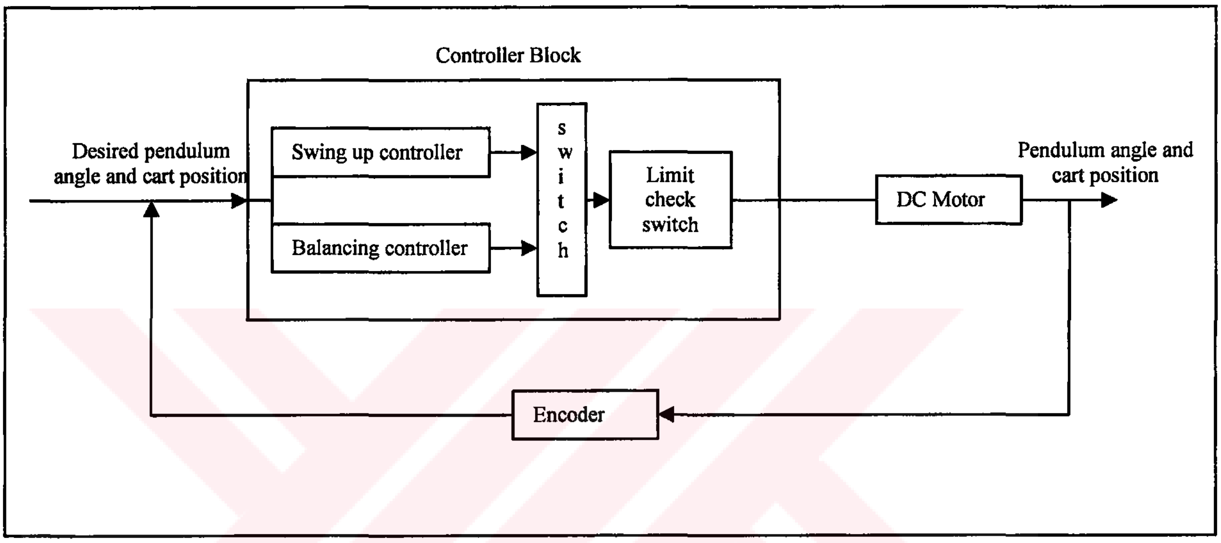

Figure 3.1. Block Diagram of the System

The control process starts with defining the desired trajectory for the cart and the desired angle for the pendulum (perpendicular to horizontal axis and pointing upward). The first action is to swing up the pendulum from the initially downward position to an upward position therefore first controller named "Swing up controller" is in action at this point. After the pendulum swings up another controller block named "Balancing controller" takes over the action. The transition between the controllers is maintained by a switch which checks the angular position of the pendulum (if the angle is far away from the upward position the swing up controller is active when it enters a specified region the balancing controller is active). While the control process is taking place another switch is checking the position of the cart to prevent the car from crossing over the limits of the rail. After all checks, a signal is sent to actuator (DC motor) and it drives the pendulum system to a new position and while the motor actuates on the system the new coordinates of the system are being fedback via encoders and the algorithm continues to run in pre-defined time period until the desired conditions are satisfied.

3.1. Swing Up Controller Design

As it is stated earlier the job of the "Swing up controller" is to bring the pendulum to an upward position. Two different controllers have been designed for this process. The first one is a heuristic controller that provides voltage in the appropriate polarity and, thus drives the cart back and forth along the limited rail repeatedly. It will repeat this action until the pendulum is close enough to the upright position such that the stabilizing controller can be triggered to maintain this balanced state. The controller is based on the Fuzzy Logic theory. The second scheme is an energy controller that regulates the amount of energy in the pendulum. This controller inputs energy into the cart-pendulum system until it attains the energy state that corresponds to the pendulum in the upright position. Similar to the heuristic control method, the energy control method will also switch to the stabilizing controller when the pendulum is close to the upright position. The switch that triggers the stabilizing controller in both cases is activated when the pendulum is within the upward zone.

3.2. Fuzzy Logic Controller

The heuristic controller is a fuzzy logic-based control design that determines the direction and the moment in time the cart should move depending on the state of the system. So, first of all it is necessary to understand fuzzy logic.

3.2.1. Fuzzy Sets and Basic principles of Fuzzy Logic

Zadeh makes a case that human beings reason not in terms of discrete symbols and numbers, but in terms of fuzzy sets. These fuzzy terms define general categories, but not rigid or fixed collections. The transition from one category-concept, idea, or problem state to the next is gradual with some states having greater or less membership in the one set and then another. From this idea of elastic sets, Zadeh proposed the concept of a fuzzy set. Fuzzy sets are functions that map a value that might be a member of the set to a number between zero and one indicating its actual degree of membership. A degree of zero means that the value is not in the set, and a degree of one means that the value is completely representative of the set. This produces a curve across the members of the set.

There are many resources that have been written on the subject of fuzzy sets since Zadeh introduced the fuzzy set concept in 1965.

A membership function is defined as follows. Let X be a set of objects, called the universe, whose elements are denoted by x. Membership in a subset A of X is the membership function, μ_A from X to the real interval [0,1]. The universe is all the possible elements of concern in the particular context. A is called a fuzzy set and is a subset of X that has no sharp boundary. μ_A is the grade of membership of x in A. The closer the value of μ_A is to 1, the more x belongs to A. In short, a membership function provides a measure of the degree of similarity of an element.

Example: A car can be viewed as "domestic" or "foreign" from different perspectives. One perspective is that a car is domestic if it has the name of a national auto manufacturer otherwise it is foreign. There is nothing fuzzy about this perspective; however, many domestic cars have some components produced outside of the country and some foreign cars that have domestic production components. So from this perspective, one could think of the membership functions for domestic and foreign cars looking like μ_D(x) and μ_F(x) depicted in Figure 3.2.

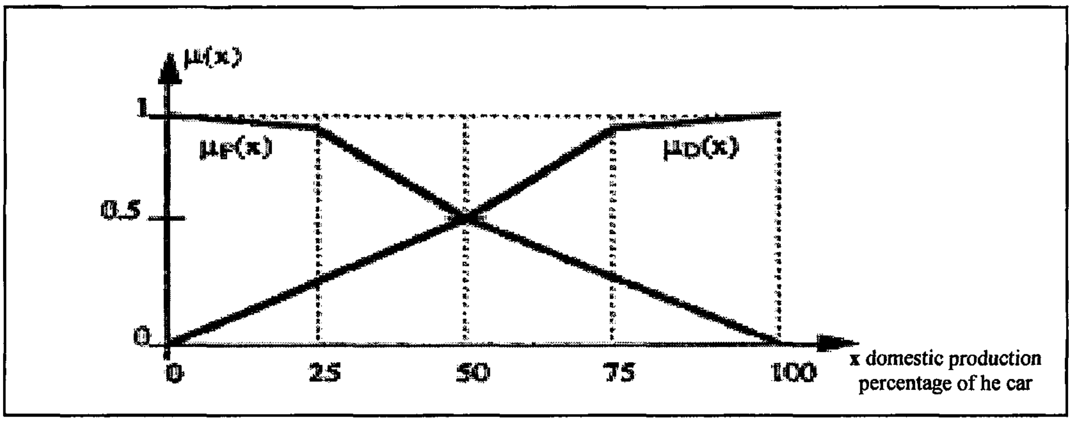

Figure 3.2. Membership Functions of the domestic and foreign cars. [22]

At this point, it is essential to explore the use of what might be called *"linguistic variables"*, that is, variables whose values are words instead of numbers. Let u denotes the name of a linguistic variable (e.g. pressure). Numerical values of a linguistic variable u are denoted by x. A linguistic variable is usually divided into regions, which cover its universe of discourse.

Example: Let pressure be a linguistic variable. It can be divided into the following set of regions: (weak, low, OK, strong, high) where each term is assigned by a fuzzy set in the universe of discourse U =[100 psi, 2300 psi]. Then this division might be interpreted as follows: weak is a pressure below 200 psi, low as a pressure close to 700 psi, OK as close to 1050 psi, strong is close to 1500 psi, and high as a pressure above 2200 psi. The membership functions of these terms are shown in Figure 3.3.

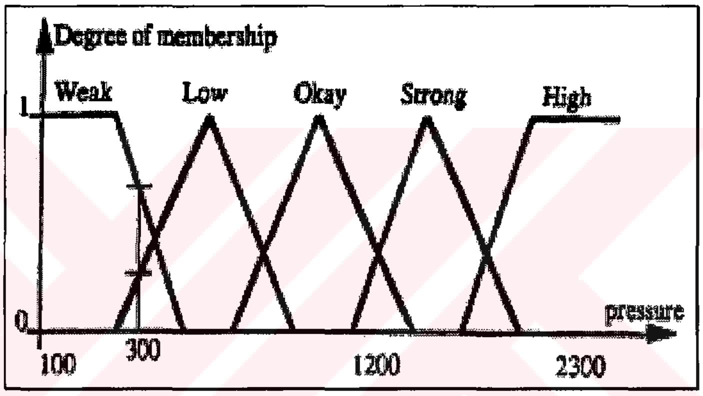

Figure 3.3. Membership Functions for the pressure [22]

The total allowable universe of values is called the *domain* of the fuzzy set. The domain is a set of real numbers, increasing monotonically from left to right where the values can be both positive and negative. The fuzzy set "A" is completely characterized by the set of pairs,

$$A = \{(x, \mu_A(x)), x \in X)\} \quad (3.1)$$

"Support" of a fuzzy set *A* in the universal set *X* is the crisp set that contains all the elements of *X* that have a nonzero membership grade in *A*. That is

$$\text{supp}\, A = \{ X \in x \mid \mu_A(x) > 0 \} \quad (3.2)$$

When *X* is an interval of real numbers, a fuzzy set *A* is expressed as

$$A = \int_x \frac{\mu_A(x)}{x} \tag{3.3}$$

An *"empty"* fuzzy set has an empty support which implies that the membership function assigns 0 to all elements of the universal set.

As in classical set theory, fuzzy set theory has set operations, namely union, intersection and complement. Fuzzy set operations are analogous to crisp set operations. The important thing in defining fuzzy set logical operators is that if we keep fuzzy values to the extremes 1 (True) or 0 (False), the standard logical operations should hold. In order to define fuzzy set logical operators, first consider crisp set operators. The most elementary crisp set operations are union, intersection, and complement, which essentially correspond to *OR, AND,* and *NOT* operators, respectively. Let *A* and *B* be two subsets of *U.* The union of *A* and *B*, denoted by A∪B, contains all elements in either *A* or *B*; that is, $\mu_{A\cup B}(x)=1$ if $x\in A$ or $x\in B$. The intersection of *A* and *B*, denoted by A∩B, contains all the elements that are simultaneously in *A* and *B*; that is, $\mu_{A\cap B}(x)=1$ if $x\in A$ and $x\in B$. The complement of *A* is denoted by Â, and it contains all elements that are not in *A*; that is $\mu_{\hat{A}}(x)=0$ if $x\in A$. The truth tables for these operators are shown in Table 3.1.

Table 3.1. Truth Tables of the logical operators

AND			OR			NOT	
A	B	A∪B	A	B	A∪B	A	$\overline{A}$
0	0	0	0	0	0	0	1
0	1	0	0	1	1	1	0
1	0	0	1	0	1		
1	1	1	1	1	1		

In FL, the truth of any statement is a matter of degree. In order to define FL operators, we have to find the corresponding operators that preserve the results of using *AND, OR,* and *NOT* operators. The answer is *min, max,* and *complement* operations. These operators are defined, respectively, as

$$\mu_{A\cup B}(x) = \max\left[\mu_A(x), \mu_B(x)\right] \tag{3.4}$$

$$\mu_{A\cap B}(x) = \min\left[\mu_A(x), \mu_B(x)\right] \tag{3.5}$$

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x) \tag{3.6}$$

A fuzzy model consists of a series of conditional and unconditional fuzzy propositions. A proposition or statement establishes a relationship between a value in the underlying domain and a fuzzy space. A *conditional fuzzy proposition* is one that is qualified as an *if* statement. The proposition following the *if* term is the antecedent or predicate and is an arbitrary fuzzy proposition. The proposition following the *then* term is the consequent and is also any arbitrary fuzzy proposition. "If w is Z then x is Y" interpreted as x is a member of Y to the degree that w is a member of Z.

This needs a nonlinear mapping of the input data vector into a scalar output, using fuzzy rules which are defined by the fuzzy inference system (FIS). The mapping process involves input/output membership functions, FL operators, fuzzy if–then rules, aggregation of output sets, and defuzzification. A FIS with multiple outputs can be considered as a collection of independent multi-input, single-output systems. A general model of a fuzzy inference system (FIS) is shown in Figure 3.4. The FLS maps crisp inputs into crisp outputs.

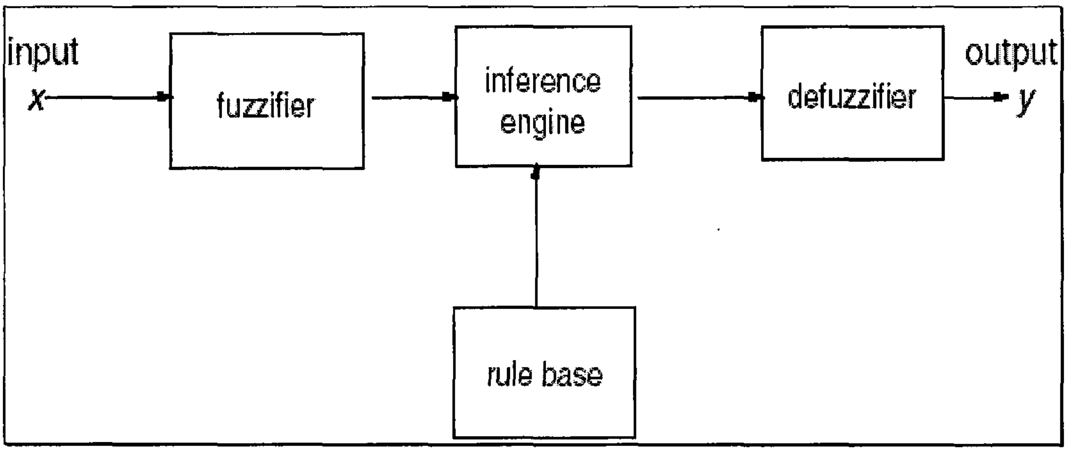

Figure 3.4. Block Diagram of Fuzzy Inference System

It can be seen from the figure that the FIS contains four components: the fuzzifier, inference engine, rule base, and defuzzifier. The rule base contains linguistic rules that are provided by experts. It is also possible to extract rules from numeric data.

Once the rules have been established, the FIS can be viewed as a system that maps an input vector to an output vector. The fuzzifier maps input numbers into corresponding fuzzy memberships. This is required in order to activate rules that are in terms of linguistic variables. The fuzzifier takes input values and determines the degree to which they belong to each of the fuzzy sets via membership functions. The inference engine defines mapping from input fuzzy sets into output fuzzy sets. It determines the degree to which the antecedent is satisfied for each rule. If the antecedent of a given rule has more than one clause, fuzzy operators are applied to obtain one number that represents the result of the antecedent for that rule. It is possible that one or more rules may fire at the same time. Outputs for all rules are then aggregated. During aggregation, fuzzy sets that represent the output of each rule are combined into a single fuzzy set. Fuzzy rules are fired in parallel, which is one of the important aspects of an FIS. In an FIS, the order in which rules are fired does not affect the output. The defuzzifier maps output fuzzy sets into a crisp number.

Given a fuzzy set that encompasses a range of output values, the defuzzifier returns one number, thereby moving from a fuzzy set to a crisp number. Several methods for defuzzification are used in practice, including the centroid, maximum, mean of maxima, height, and modified height defuzzifier. The most popular defuzzification method is the centroid, which calculates and returns the center of gravity of the aggregated fuzzy set. FISs employ rules. However, unlike rules in conventional expert systems, a fuzzy rule localizes a region of space along the function surface instead of isolating a point on the surface. For a given input, more than one rule may fire. Also, in an FIS, multiple regions are combined in the output space to produce a composite region. A general schematic of an FIS is shown in Figure 3.5.

3.2.2. Fuzzy Model of the Pendulum System

In the pendulum system, fuzzy theory is used to swing up the pendulum. The inputs of the system are the pendulum angle and the cart position and the output is the voltage applied to the cart. MATLAB's Fuzzy Logic Toolbox is used for the design process. In order to determine the membership functions and the limits of measured values for inputs and output. Reverse engineering methodology is applied. A built in control system for the control of whole system is observed. The changing in magnitude of the signals applied during the control process and the signal routings are recorded and a general pattern of methodology and the rule base are structured.

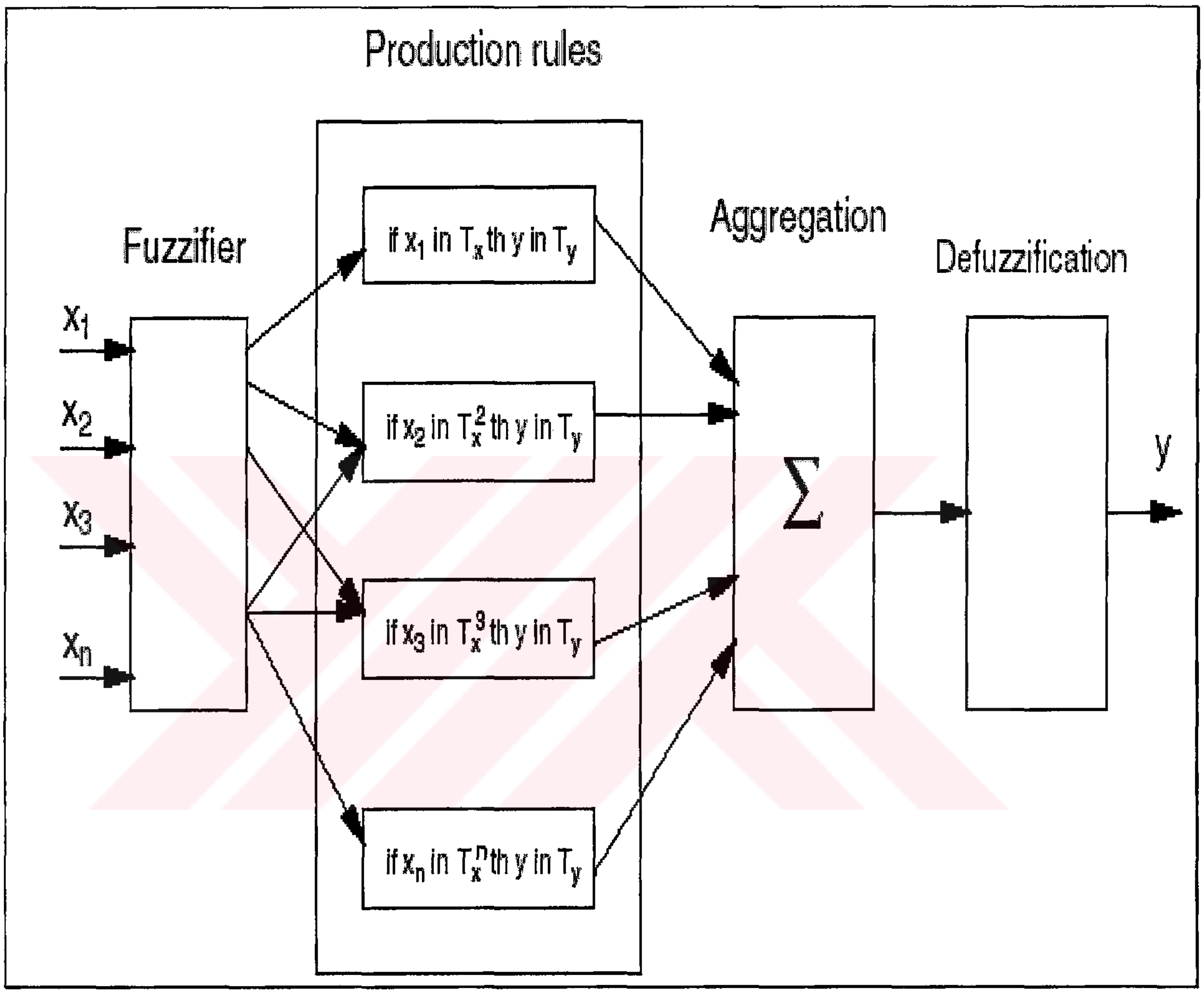

Figure 3.5. Schematic diagram of a fuzzy inference system.

The inputs are assigned with membership functions as follows. The position of the cart is assigned with the five triangular membership functions considering the rail limits in the range [-0.3, 0.3] meters, as nb (negative big), ns (negative small), z (zero), ps (positive small), and pb (positive big). The pendulum angle is assigned with five triangular membership functions in the range of [0, 2π] radians. In Figures 3.6 and 3.7 the membership functions of the inputs are shown.

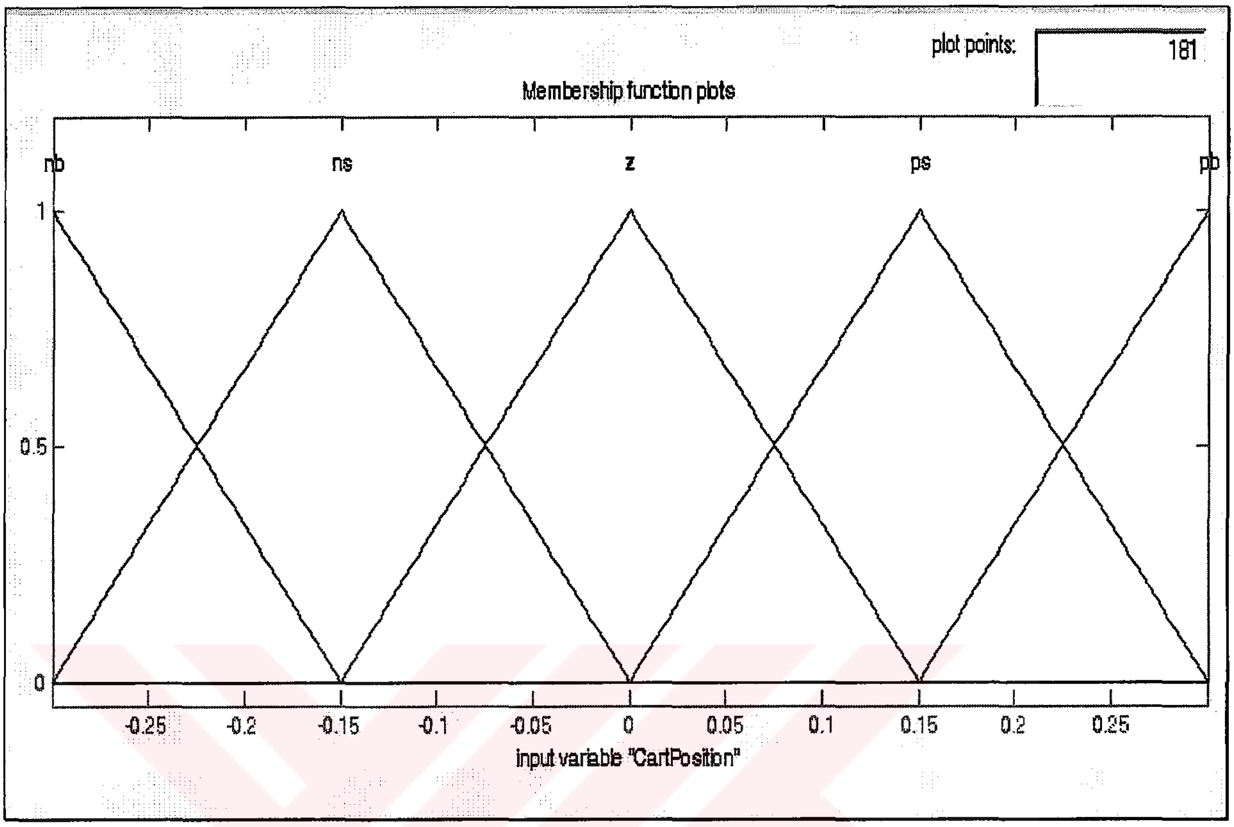

Figure 3.6. Membership Functions for the cart position

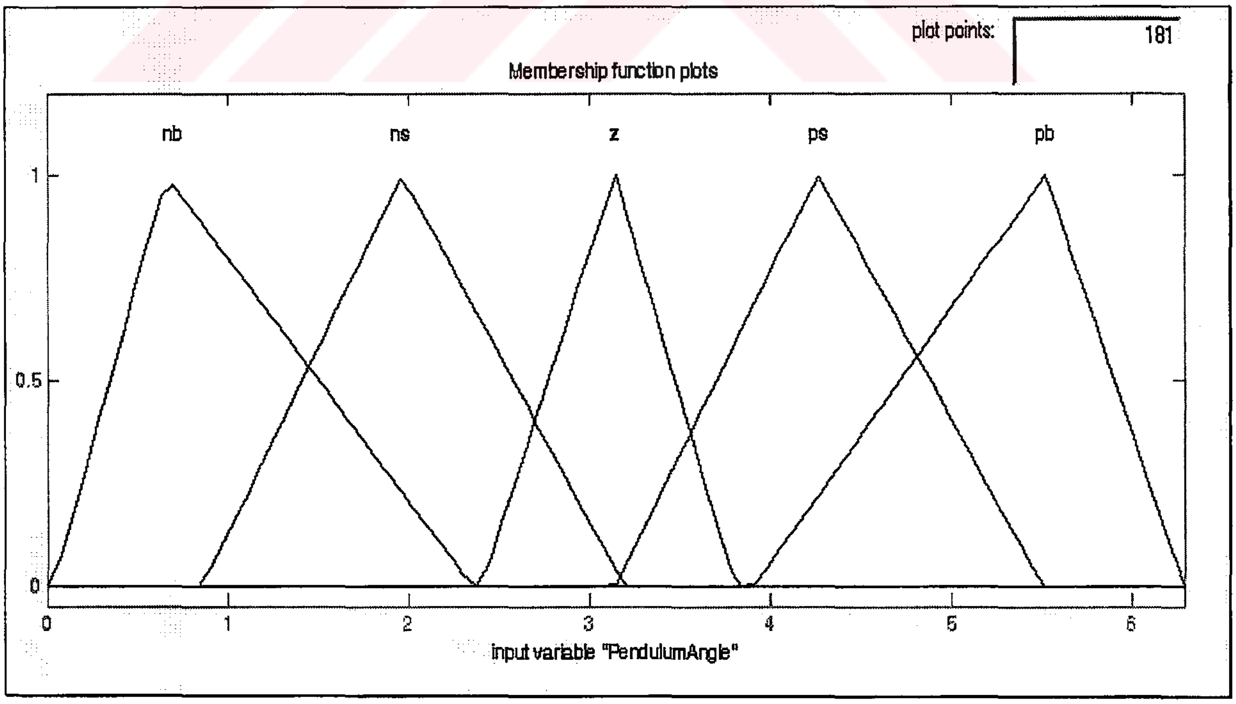

Figure 3.7. Membership Functions for the pendulum angle

The voltage has three membership functions as p (positive), z (zero), and n (negative) as shown in Figure 3.8. In order to apply a voltage in the appropriate polarity a rule base consists of 25 rules (since there are 5 memberships for each input the total number of the rules is equal to 5 x 5) is formed with the help of rule base editor shown in Figure 3.9.

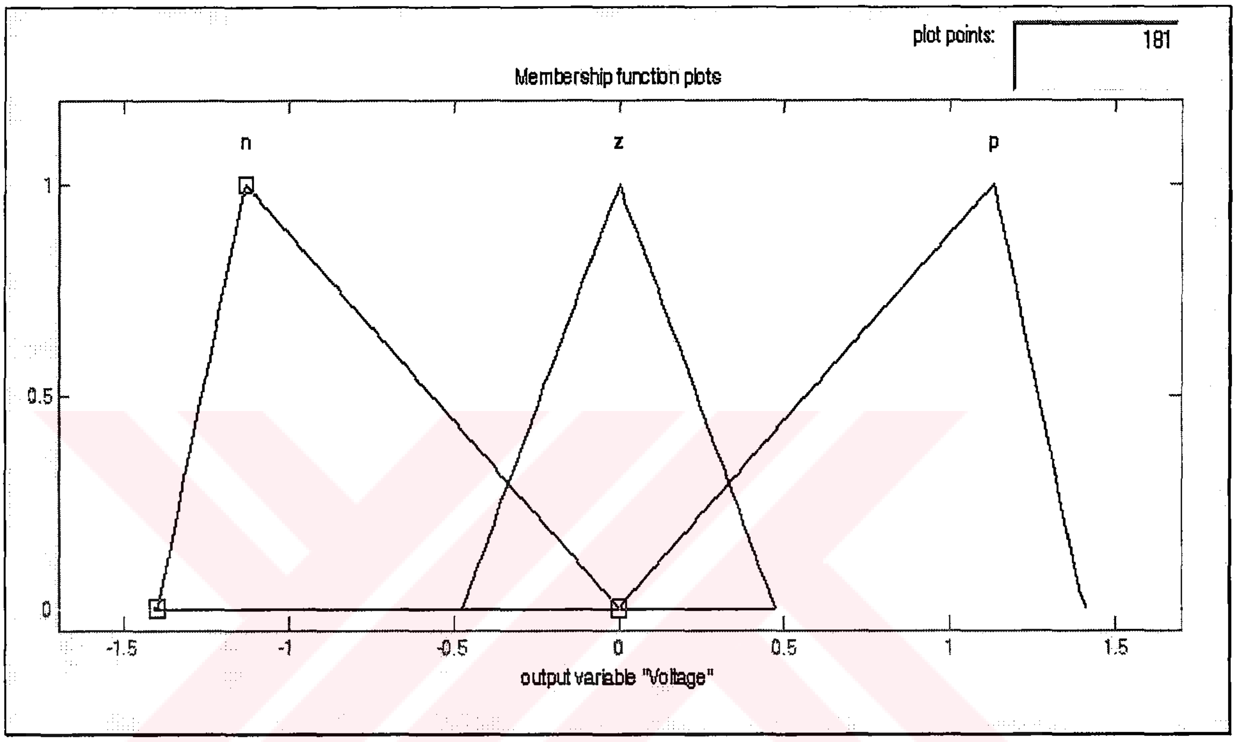

Figure 3.8. Membership Functions for the voltage

1. If (CartPosition is z) and (PendulumAngle is z) then (Voltage is p) (1)
2. If (CartPosition is z) and (PendulumAngle is ps) then (Voltage is n) (1)
3. If (CartPosition is z) and (PendulumAngle is pb) then (Voltage is n) (1)
4. If (CartPosition is z) and (PendulumAngle is ns) then (Voltage is p) (1)
5. If (CartPosition is z) and (PendulumAngle is nb) then (Voltage is p) (1)
6. If (CartPosition is ns) and (PendulumAngle is z) then (Voltage is p) (1)
7. If (CartPosition is ns) and (PendulumAngle is ps) then (Voltage is z) (1)
8. If (CartPosition is ns) and (PendulumAngle is pb) then (Voltage is z) (1)
9. If (CartPosition is ns) and (PendulumAngle is ns) then (Voltage is p) (1)
10. If (CartPosition is ns) and (PendulumAngle is nb) then (Voltage is p) (1)
11. If (CartPosition is nb) and (PendulumAngle is z) then (Voltage is z) (1)
12. If (CartPosition is nb) and (PendulumAngle is ps) then (Voltage is z) (1)
13. If (CartPosition is nb) and (PendulumAngle is pb) then (Voltage is z) (1)
14. If (CartPosition is nb) and (PendulumAngle is ns) then (Voltage is p) (1)
15. If (CartPosition is nb) and (PendulumAngle is nb) then (Voltage is p) (1)
16. If (CartPosition is ps) and (PendulumAngle is z) then (Voltage is z) (1)
17. If (CartPosition is ps) and (PendulumAngle is ps) then (Voltage is n) (1)
18. If (CartPosition is ps) and (PendulumAngle is pb) then (Voltage is n) (1)
19. If (CartPosition is ps) and (PendulumAngle is ns) then (Voltage is z) (1)
20. If (CartPosition is ps) and (PendulumAngle is nb) then (Voltage is z) (1)
21. If (CartPosition is pb) and (PendulumAngle is nb) then (Voltage is z) (1)
22. If (CartPosition is pb) and (PendulumAngle is ns) then (Voltage is z) (1)
23. If (CartPosition is pb) and (PendulumAngle is pb) then (Voltage is n) (1)
24. If (CartPosition is pb) and (PendulumAngle is ps) then (Voltage is n) (1)
25. If (CartPosition is pb) and (PendulumAngle is z) then (Voltage is z) (1)

Figure 3.9. The rule base

The fuzzy-logic-based control design is completely dependent on the pendulum angle, and cart position which are the available measured state variables. The principle of swinging up is based on having minimum number of swing (going back and forth) in minimum time. In order to achieve this, a maximum voltage is applied to the system to have maximum acceleration then according to the position of the pendulum-cart system the voltage is adjusted. The direction the cart moves is in the opposite direction of the pendulum deflection angle. When the direction of the cart movement is determined, a voltage is applied to the cart in the opposite direction of the pendulum angle so that it gets accelerated and while doing this the cart position must be watched to see if the cart approaches one of the limits. Apply the maximum voltage in the opposite direction of the limit to avoid the over riding of the limits. This control scheme will effectively move the cart back and forth along the track repeatedly until the pendulum swings close enough to the upright position.

The voltage gain of this control scheme is determined by repeated experimentation. There is a direct correlation between the time it takes to swing the pendulum to its upright position and the magnitude of the voltage gain. A gain that is too high, though, may make the pendulum approach the upright position with too high a velocity and, thus, the stabilizing controller will be unable to balance the pendulum. On the other hand, a gain too low may not provide enough energy to the pendulum so that it can reach the upright position. Also, the reliability of the controller in performing the task varies depending on the gain selected. Thus, repeated experimentation is required to finely tune the gain so that the pendulum approaches the upright position with just the right amount of velocity and in a reasonable amount of time with a high success rate.

3.3. Energy Based Controller

The swinging up of a pendulum from the downwards position can also be accomplished by controlling the total energy of the system. The energy in the pendulum system can be brought to a desired value through the use of feedback control. The pendulum can be swung up to its unstable equilibrium by adding in enough energy such that its value corresponds to the upright position. This technique is well known in literature, References [6], [7] contain a thorough explanation of this method. This technique aims at swinging up the pendulum, while keeping the cart within a limited horizontal travel on the rail. This is accomplished by satisfying a particular mathematical condition, derived from the energy equations of the pendulum.

The system is defined such that the energy, E, is zero in the upright position. The energy of the pendulum can be written as

$$E = \left(\frac{1}{2} J \left(\dot{\theta} \right)^2 + m_p g l \left(\cos\theta - 1 \right) \right) \tag{3.7}$$

where m_p is the mass of the pendulum, l is the half-length of the pendulum, g is the acceleration of gravity, and J is the rotational inertia with respect to the pivot point. Thus, the energy in the pendulum is a function of the pendulum angle and the pendulum angular velocity. Note also that the energy corresponding to the pendulum in the downwards position is $-2m_p gl$. The goal of the control scheme is to add energy into the system until the value corresponds to the pendulum in the upright position. The control law implemented to achieve the desired energy is as follows.

Since the pendulum is accelerated by the acceleration of the cart It is clear that the energy can be increased or decreased by changing the sign (*sgn*) of $\ddot{x}$ in accordance with that of $\dot{\theta}\cos\theta$.

Energy can be pumped into the pendulum by generating $\ddot{x}$ (acceleration on the cart). However, one cannot concentrate on swinging-up the pendulum only, without considering the finite cart travel (limited range for *x*). Therefore, $\ddot{x}$ has to be controlled while keeping the constraint on *x* in mind. Basically, the design method suggests; constructing a control law such that the resulting closed loop system is linear (through feedback linearization) and having a sinusoidal reference input to ensure the desired bounded nature of *x*. This reference input is derived from $(\theta, \dot{\theta})$, and generates $\ddot{x}$. This is done in order to control the total energy to the prescribed value corresponding to the energy of the pendulum at the upright equilibrium point.

The methodology described above is implemented to the system by using MATLAB/SIMULINK. The energy equations and the conditions are defined in a Simulink model file as shown in Figure 3.10.

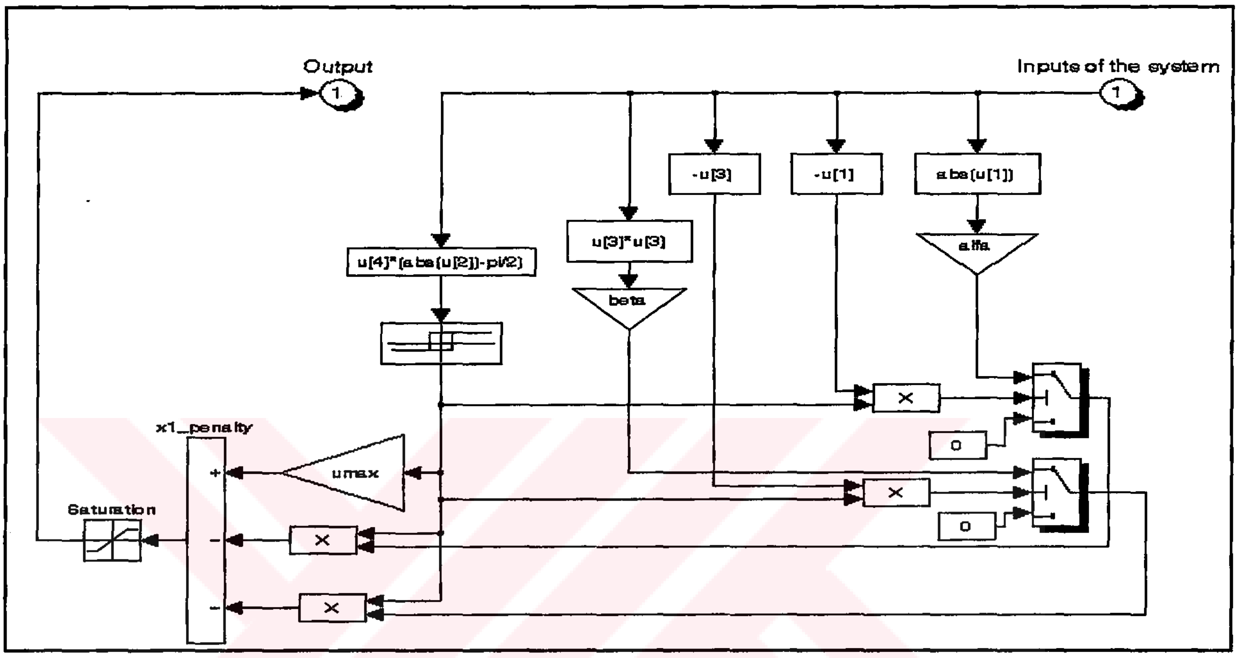

Figure 3.10. Swing Up Model

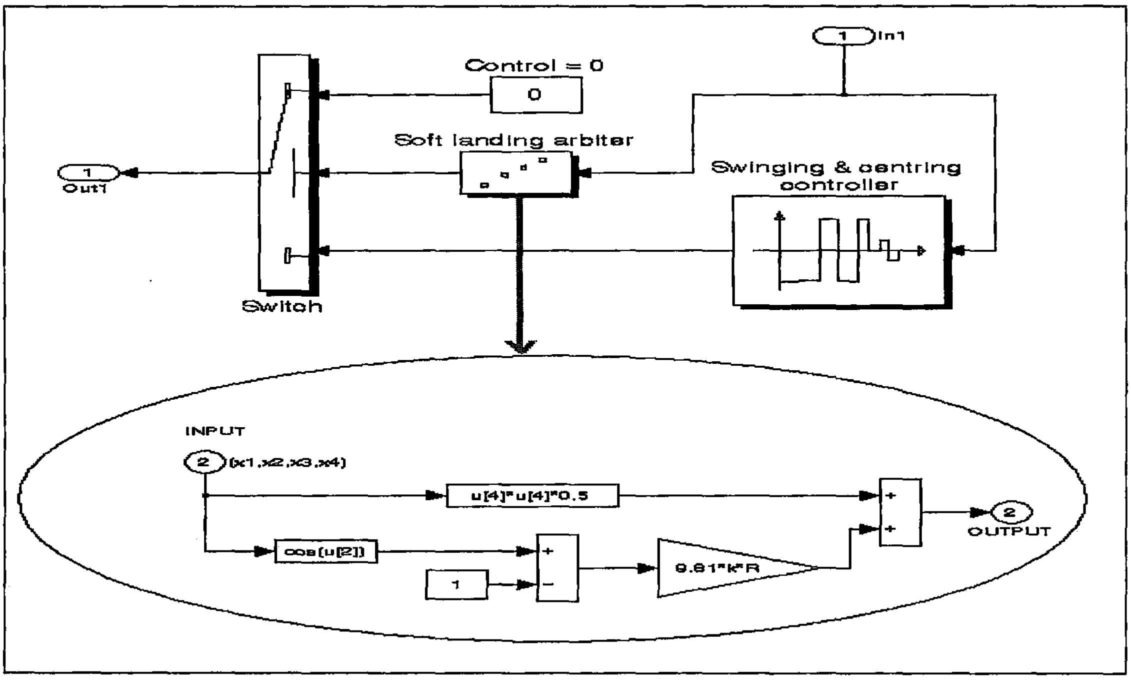

Figure 3.11. Soft Landing Arbiter Switch

The model increases the total energy of the system; while the energy of the system is increased a switch mechanism named as *"soft landing arbiter"* checks the amount of the energy. If the amount is equal to desired amount the output of the model become zero so that no more energy is pumped into the system. The switch mechanism is depicted in Figure 3.11.

4. BALANCE CONTROL

The balance control algorithm maintains the upright equilibrium of the pendulum after the pendulum is swung up by one of the swing up controllers that are proposed earlier. Three different control schemes are proposed for the balance control process;

- State variable feedback control
- Linear Quadratic Regulator (LQR) which is an extended version of the state variable feedback control to have an optimum performance
- PID control which is the most frequently applied control method for industrial process control problems

The overview and the theory of each scheme are given in the following parts.

4.1. State Variable Feedback Control

The theory and design techniques of control have come to be divided into two categories: Classical control methods use the Laplace or Fourier transforms and were the dominant methods for control design until about 1960, whereas modern control methods are based on ordinary differential equations (ODEs) in state form and were introduced into the field starting in the 1960s. Many connections have been discovered between the two categories, and many publications have been presented in the literature. [23], [24], [25], [26], [27]

The idea of using state variables is named as "state space". The state of a dynamic system is often directly describes the distribution of internal energy in the system. For example it is common to select the following as state variables: position (potential energy), velocity (kinetic energy), capacitor voltage (electric energy), and inductor current (magnetic energy). The internal energy can always be computed from the state variables. The states can be related to the system inputs and outputs and thus the internal variables are connected to the external inputs and to the sensed outputs. In contrast, the transfer function relates only the input to the output and does not show the internal behavior. The state space form keeps the latter information, which is sometimes very important.

Advantages of state-space design are especially apparent when the system to be controlled has more than one control input or more than one sensed output. The design approach used for the systems described in state form may be summarized as follows. First the control is designed as if all the state variables were measured and available for use in the control law. This provides the possibility of assigning arbitrary dynamics for the system. Having a satisfactory control law based a full-state feedback, finally external reference command inputs are introduced and the structure is complete.

It is necessary to develop some analytical results and tools before starting with a design process using state variable descriptions.

The state of a system is a set of variables such that the knowledge of these variables and the input functions with the equations describing the dynamics will provide the future state and the output of the system. For a dynamic system, the state of a system is described in terms of a set of state variables $[x_1(t), x_2(t) \ldots, x_n(t)]$. The state variables are those variables that determine the future behavior of a system when the present state of the system and the excitation signals are known. Consider the system shown in Figure 4.1, where $y_1(t)$ and $y_2(t)$ are the output signals and $u_1(t)$ and $u_2(t)$ are the input signals. A set of state variables $[x_1, x_2, \ldots\ldots. x_n]$ for the system shown in the figure is asset such that knowledge of the initial values of the state variables $[x_1(t_0), x_2(t_0) \ldots, x_n(t_0)]$ at the initial time t_0, and of the input signals $u_1(t)$ and $u_2(t)$ for $t \geq t_0$, suffices to determine the future values of the outputs and state variables.

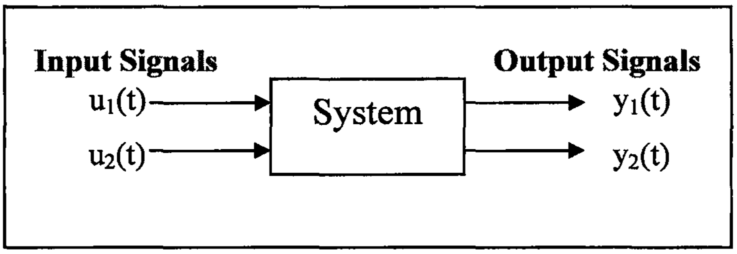

Figure 4.1. System Block Diagram

The concept of a set of state variables that represent the dynamic system can be illustrated in terms of the spring-mass-damper system shown in Figure 4.2. The number of state variables chosen to represent this system should be as small as possible in order to avoid redundant state variables. A set of state variables sufficient to describe this system includes the position and the velocity of the mass. Therefore the set of variables is defined as (x_1, x_2), where

$$x_1(t) = y(t) \quad \text{and} \quad x_2(t) = dy(t)/dt$$

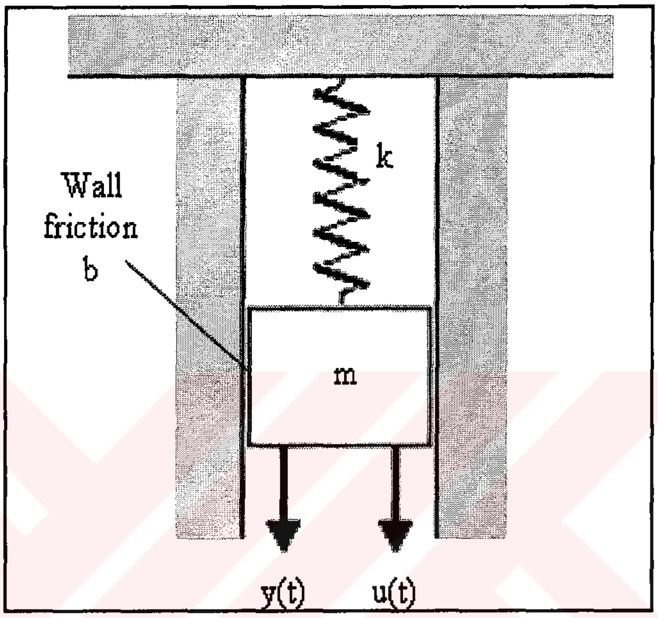

Figure 4.2. A spring-mass-damper system

The differential equation describes the behavior of the system and is usually written as

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = u(t) \tag{4.1}$$

To write the differential equation in terms of the state variables which are given above the following substitution is done.

$$m\frac{dx_2}{dt} + bx_2 + kx_1 = u(t) \tag{4.2}$$

Therefore the differential equation that describes the behavior of the spring-mass-damper system can be written as a set of two first order differential equations as follows:

$$\frac{dx_1}{dt} = x_2, \tag{4.3}$$

$$\frac{dx_2}{dt} = -\frac{b}{m}x_2 - \frac{k}{m}x_1 + \frac{1}{m}u \tag{4.4}$$

This set of differential equations describes the behavior of the state of the system in terms of the rate of change of each state variable.

Similarly for the pendulum system, the states of the system are obtained in Chapter 2 (Equation2.8-2.13) as follows:

$$x_1' = x_3 \tag{4.5}$$

$$x_2' = x_4 \tag{4.6}$$

$$x_3' = \frac{a(F - T_c - \mu x_4^2 \sin x_2) + l\cos x_2(\mu g \sin x_2 - f_p x_4)}{J + \mu \ell \sin^2 x_2} \tag{4.7}$$

$$x_4' = \frac{l\cos x_2(F - T_c - \mu x_4^2 \sin x_2) + \mu g \sin x_2 - f_p x_4}{J + \mu \ell \sin^2 x_2} \tag{4.8}$$

where

$$a = l^2 + \frac{J}{m_c + m_p}, \quad (4.9) \qquad \mu = (m_c + m_p)l. \quad (4.10)$$

where:

x_1 is the cart position (distance from the centre of the rail)

x_2 is the angle between the upward vertical and the ray pointing at the centre of mass, measured counterclockwise from the cart ($x_2 = 0$ for the upright position of the pendulum)

x_3 is the cart velocity

x_4 is the pendulum angular velocity

and other parameters are listed in Chapter 2.

It seems that there are two problems with the obtained states; the system is nonlinear and the states represent only the mechanical part of the pendulum system, however the system is equipped with a DC motor, which is not included in the model given above.

To overcome the nonlinearity a linearization process is necessary. In order to do that the concept of the "equilibrium point" should be introduced. An equilibrium point of a system is defined as a special state in which a nonlinear system can be linearized around that point. The inverted position of the pendulum corresponds to the unstable equilibrium point of the system as $(\theta,\dot{\theta}) = (0, 0)$. In the neighborhood of this equilibrium point both θ and $\dot{\theta}$ are very small (measured in rad and rad/sec respectively). In general, for small angles of θ and $\dot{\theta}$: $\sin(\theta) \approx \theta$, $\cos(\theta) \approx 1$ and $(\dot{\theta})^2 \approx 0$. Using these approximations a new linear model can be obtained for the pendulum system as follows.

$$x_1'' = l\, x_2'' + F / (m_c + m_p) \tag{4.11}$$

$$x_2'' = [F l + (m_c + m_p) g l x_2 - f_p x_4] / J \tag{4.12}$$

where $F = u - T_c$

And using (2.1)-(2.7), (4.11)-(4.12) the states of the linear model are obtained as

$$x_1' = x_3 \tag{4.13}$$

$$x_2' = x_4 \tag{4.14}$$

$$x_3' = aF + \frac{l\mu}{J} x_2 - l f_p x_4 \tag{4.15}$$

$$x_4' = \frac{l}{J} F + \frac{\mu}{J} x_2 - f_p x_4 \tag{4.16}$$

where

$$a = \frac{l^2}{J} + \frac{1}{m_c + m_p}, \quad (4.17) \quad \mu = (m_c + m_p) l\, g. \tag{4.18}$$

Although the linearization process is succeeded, the second problem is still waiting which is related with the DC motor. The model of the system includes only the mechanical part of the whole electro-mechanical system. Actually there is nothing difficult to add the electric part equations but the problem is unavailable parameter values such as

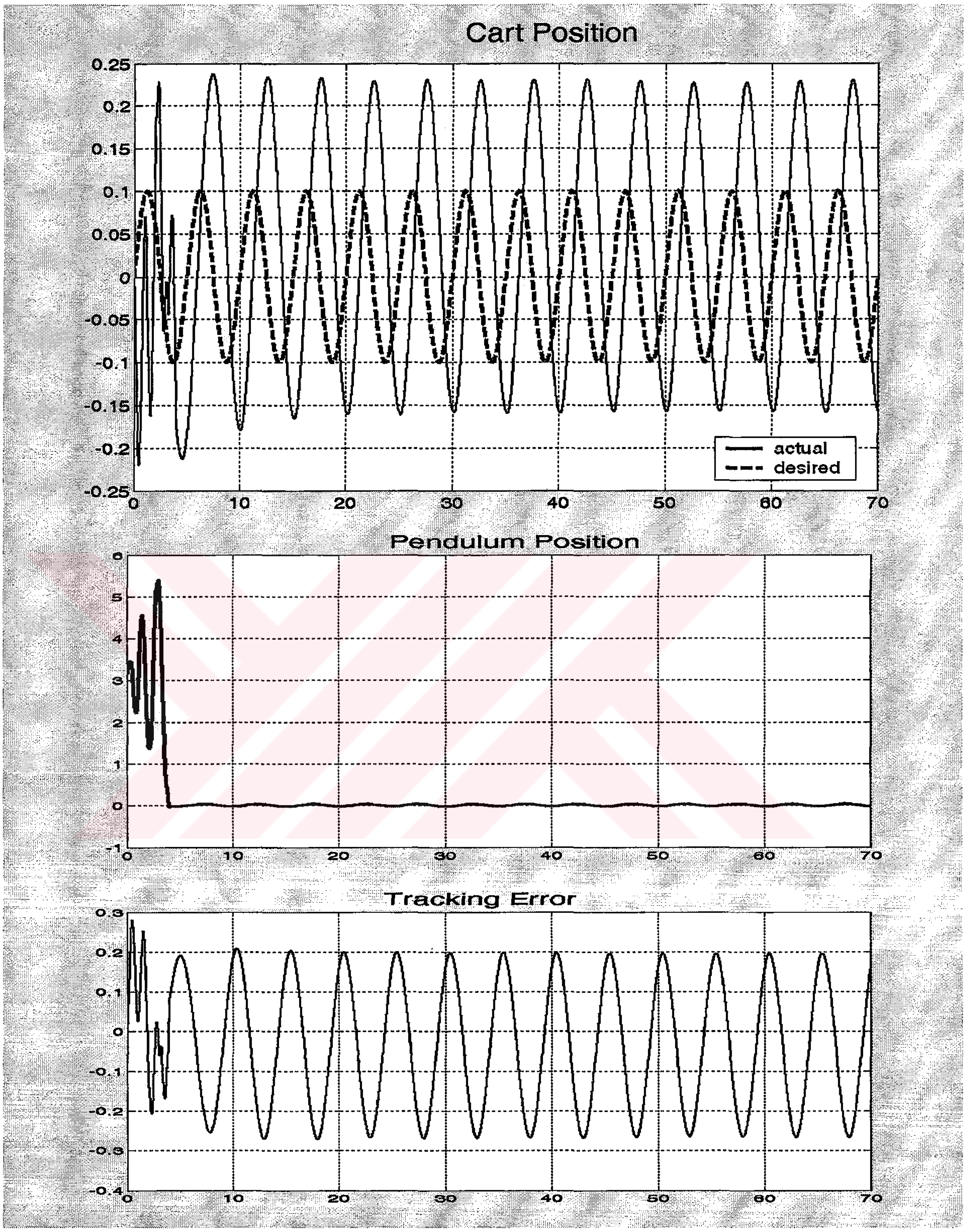

Figure 5.6. Response graphs of E-SVF control with sinusoidal input of 0.2 Hz

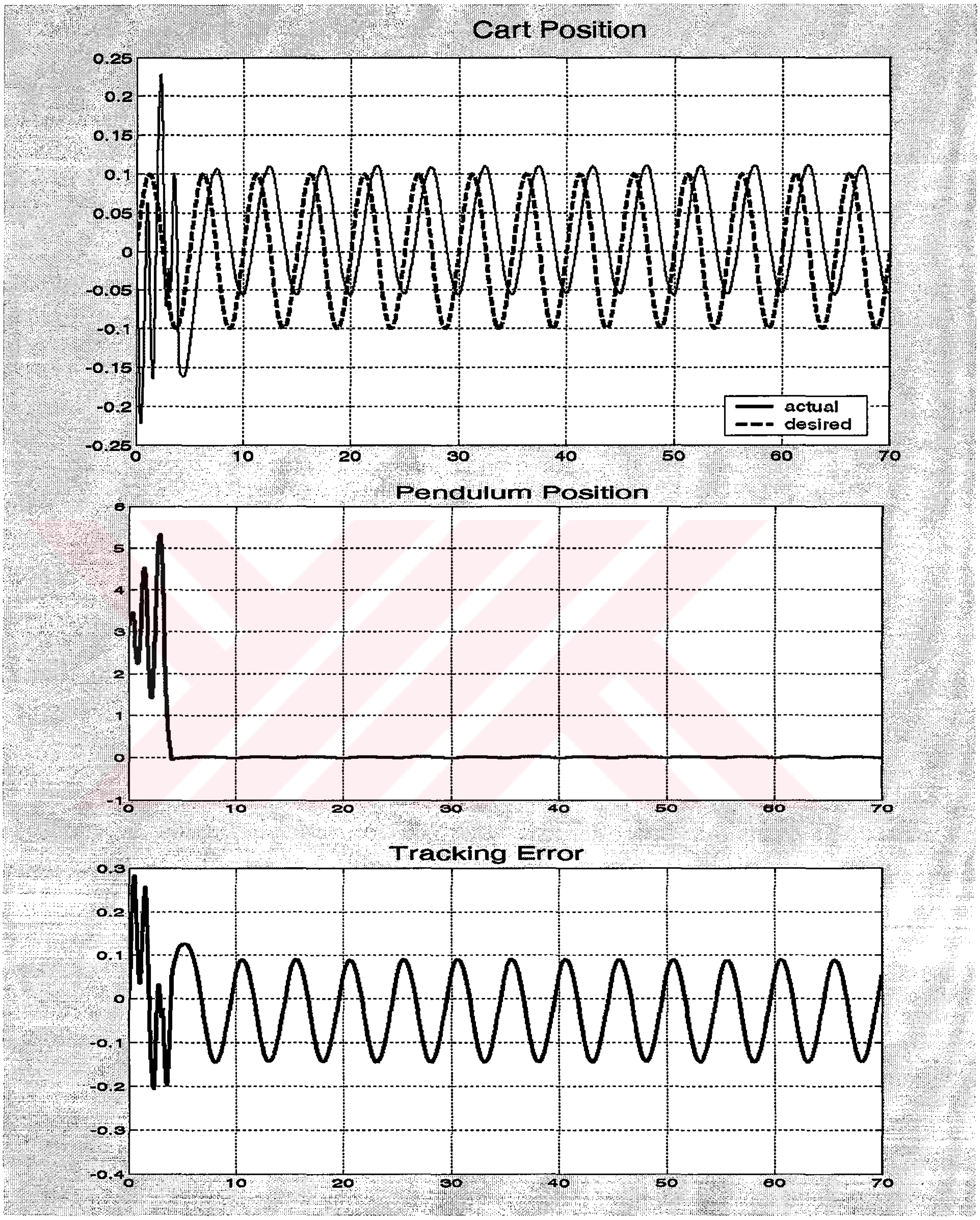

Figure 5.7. Response graphs of E-LQR control with sinusoidal input of 0.2 Hz

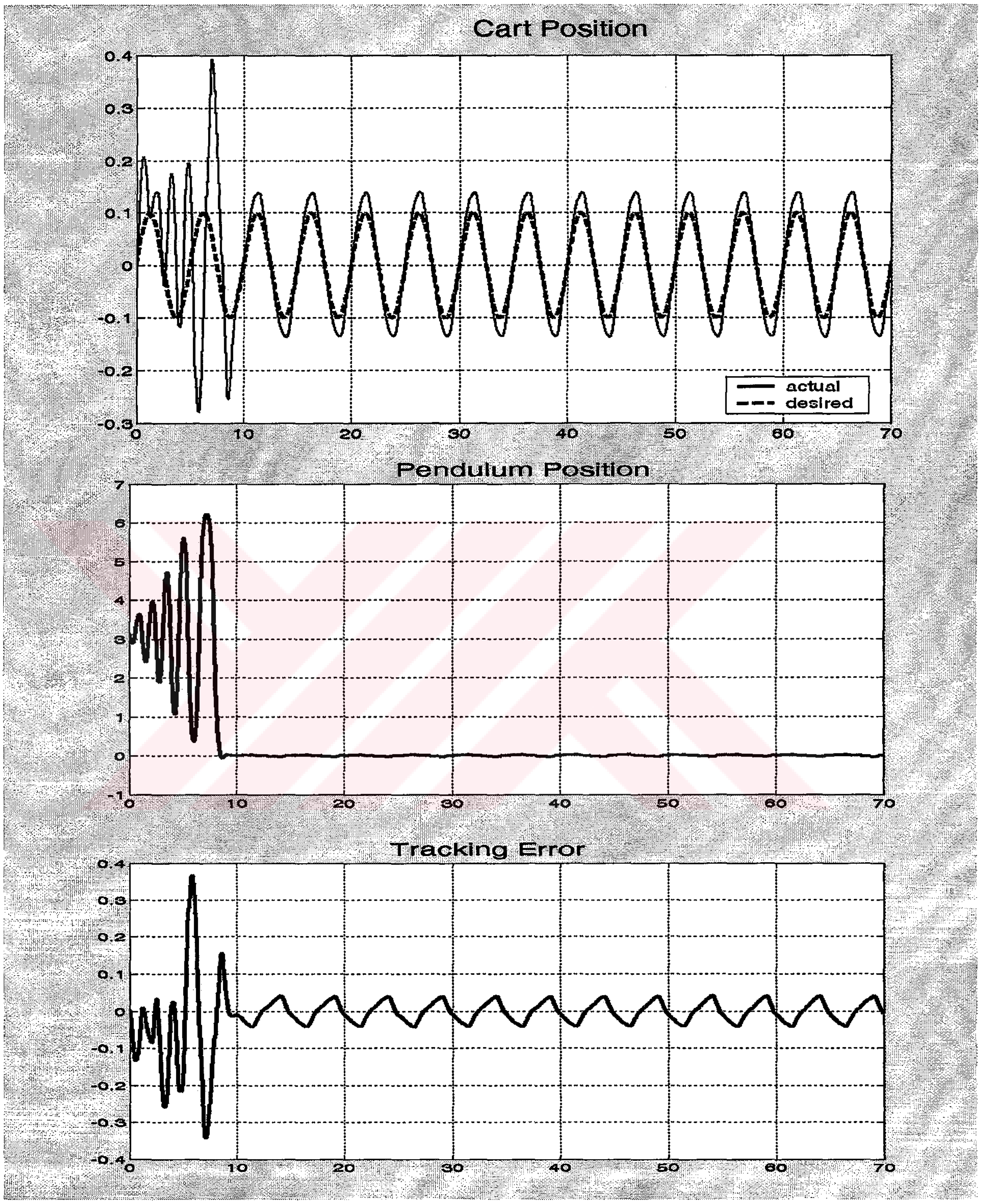

Figure 5.8. Response graphs of F-PID control with sinusoidal input of 0.2 Hz

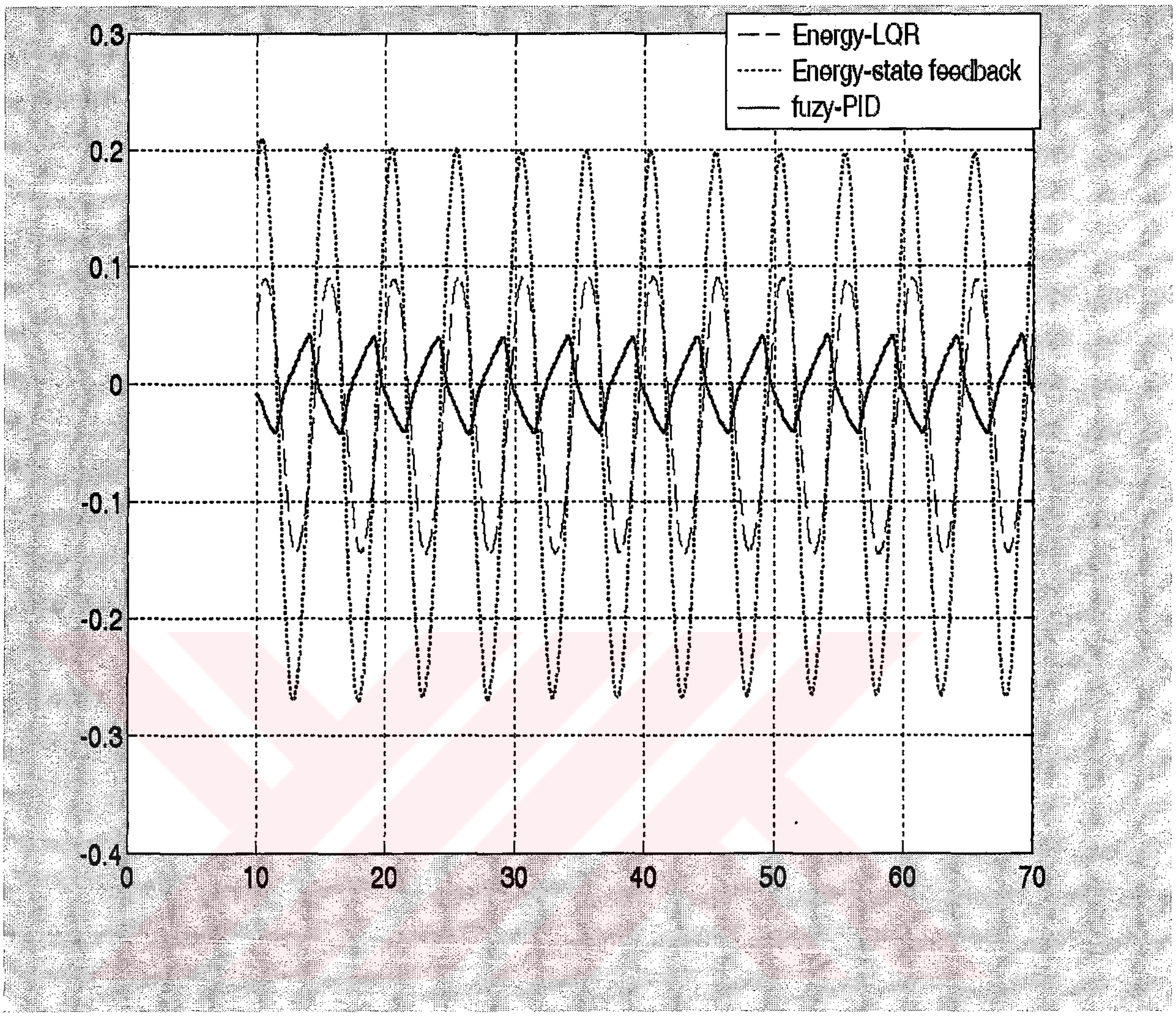

Figure 5.9. Tracking errors of all combinations

The responses graphs given in Figure 5.12-5.14 show that as input function is increased in frequency, state feedback controller shows a sensitive response, whereas other combinations maintain their stability better. The tracking error of the state variable feedback controller increases enormously in comparison with the others as shown in the tracking error graph given in figure 5.15. Fuzzy-PID combination still gives the minimum error.

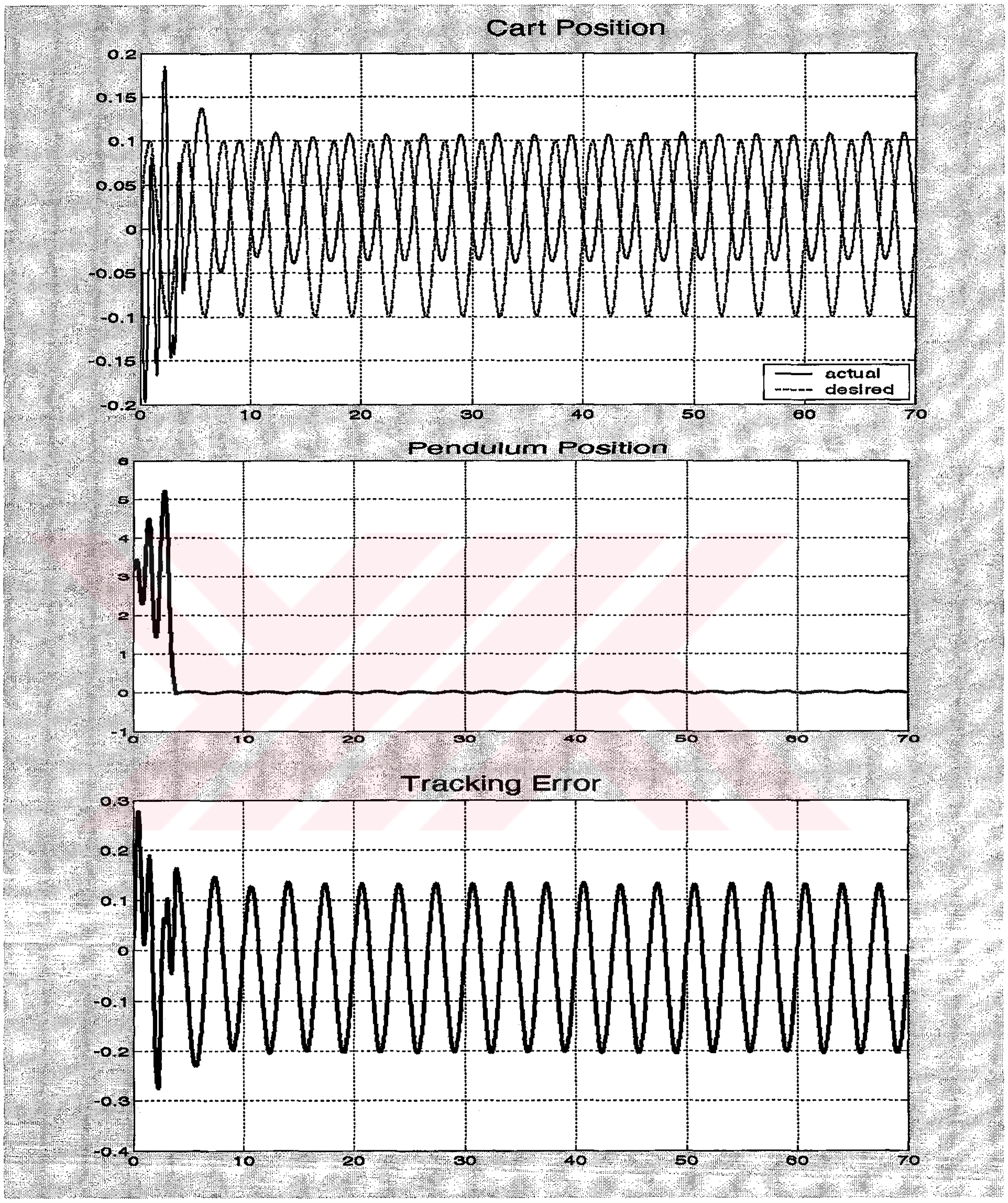

Figure 5.10. Response graphs of E-SVF control with sinusoidal input of 0.3 Hz

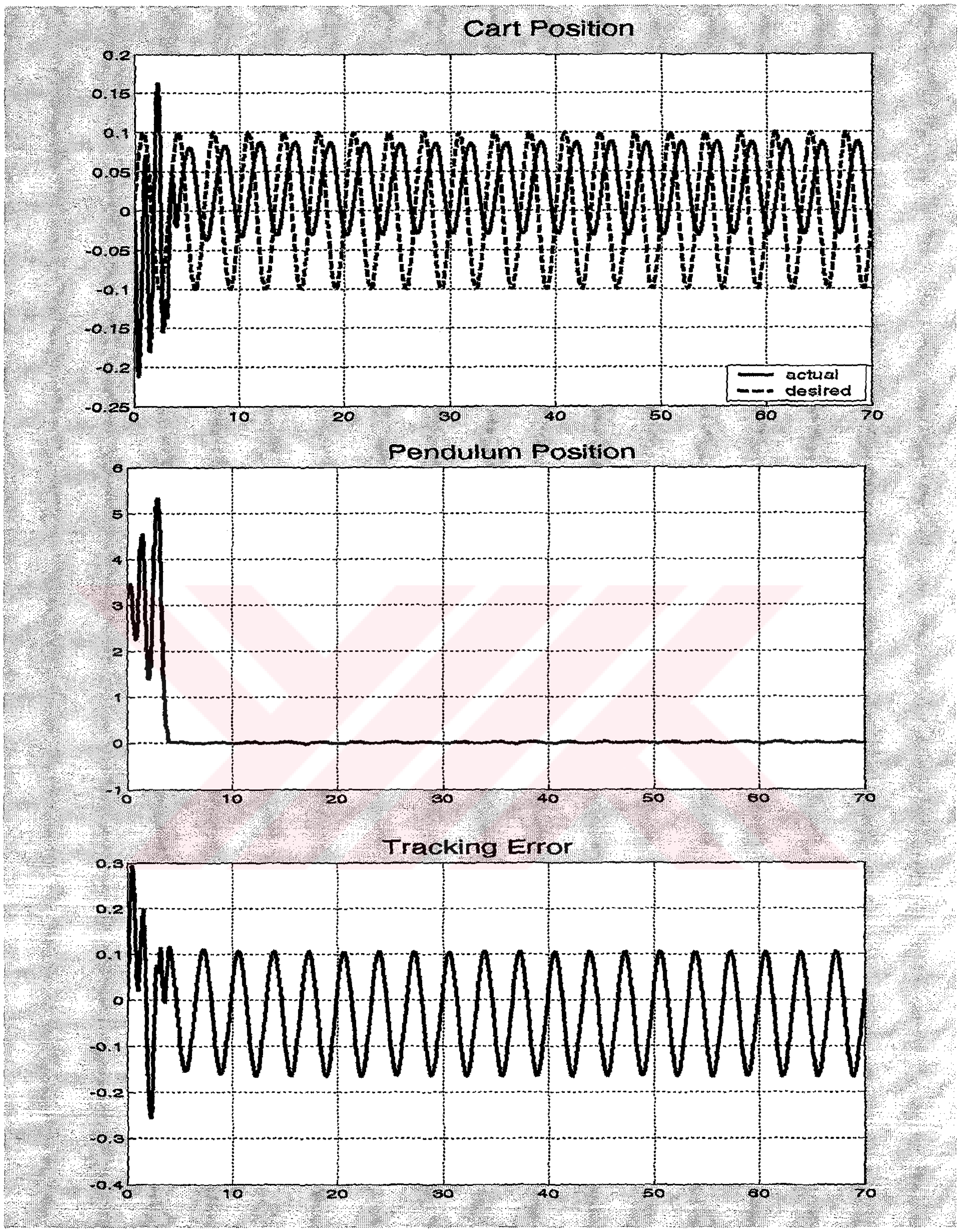

Figure 5.11. Response graphs of E-LQR control with sinusoidal input of 0.3 Hz

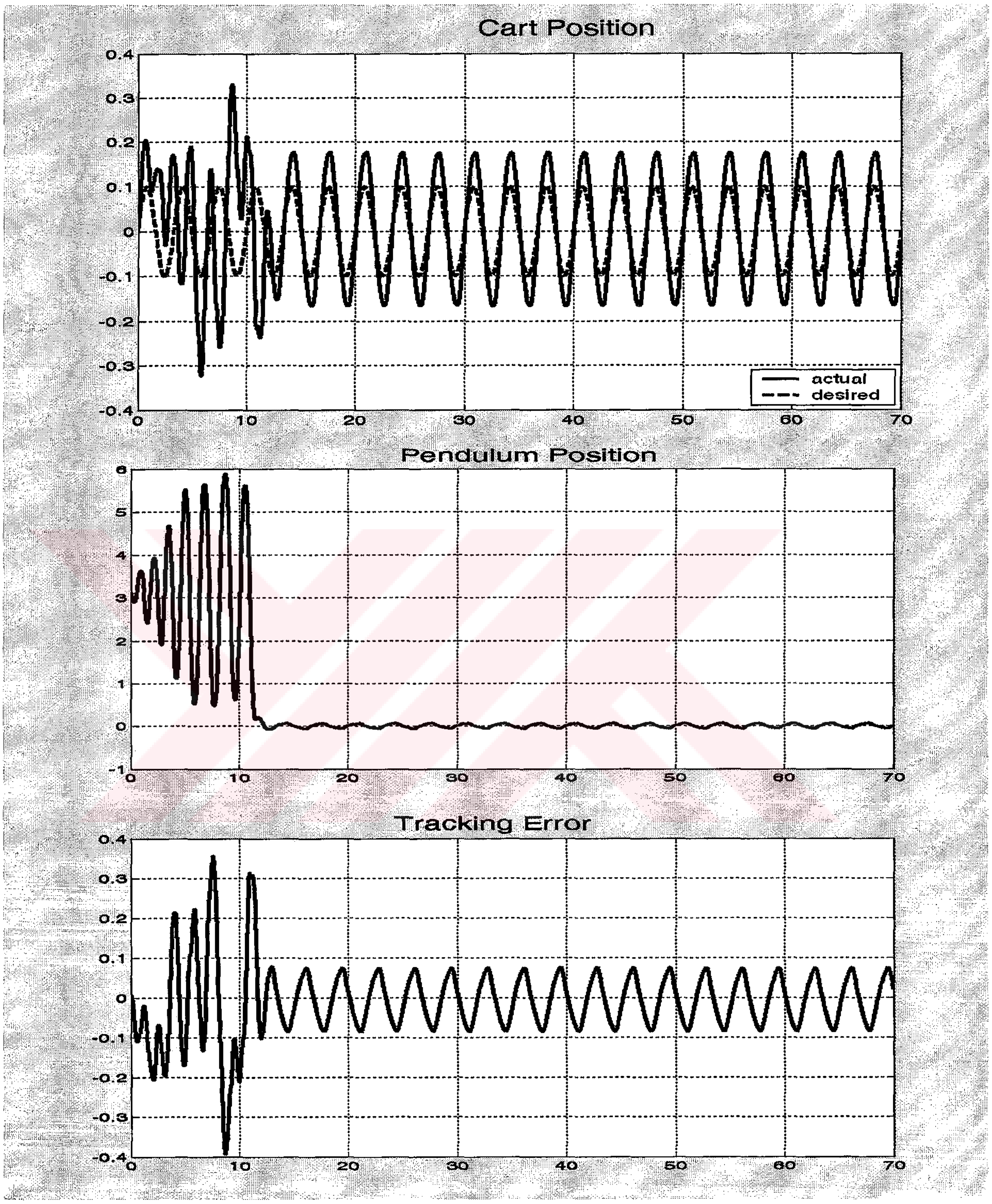

Figure 5.12. Response graphs of F-PID control with sinusoidal input of 0.3 Hz

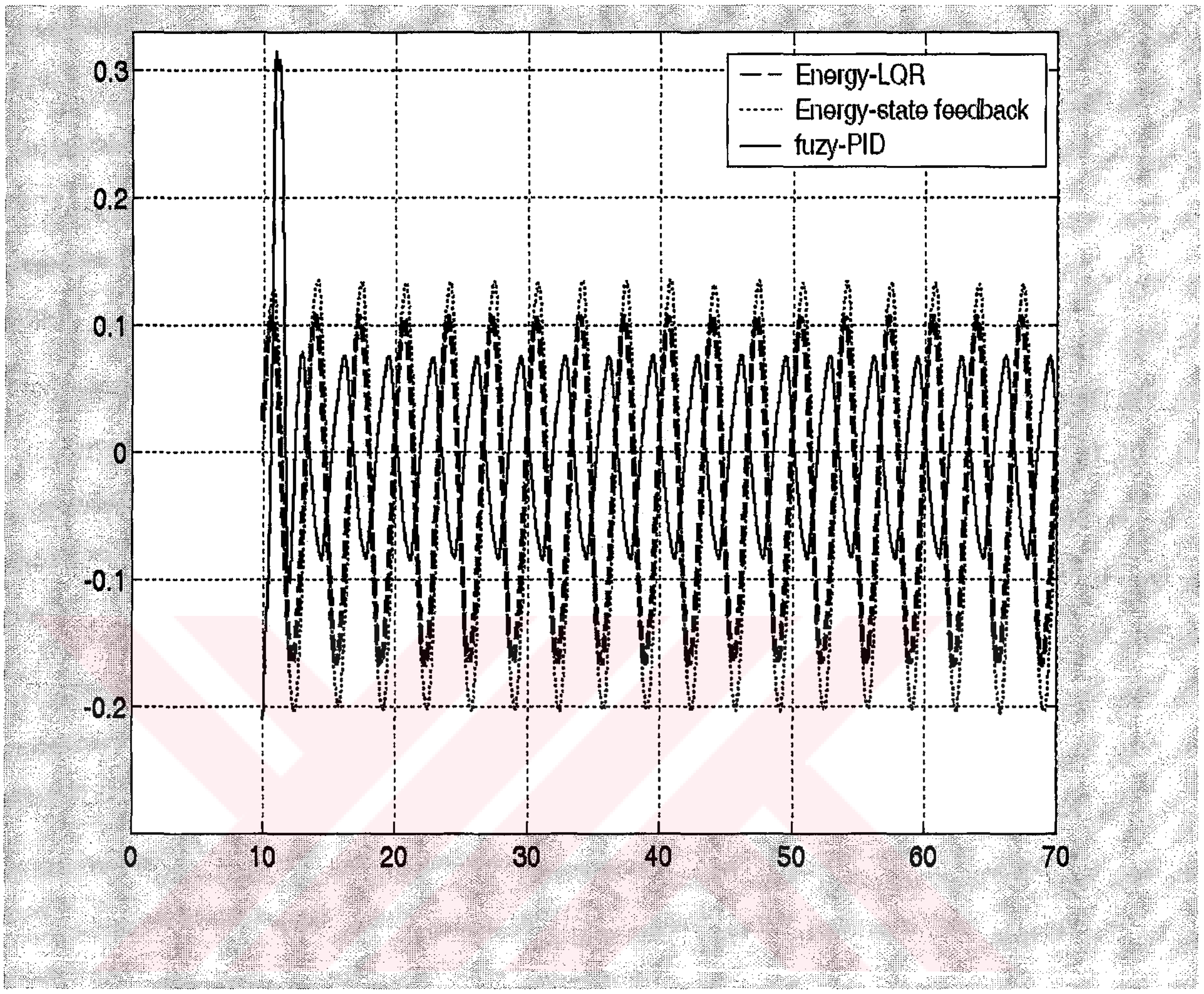

Figure 5.13. Tracking errors of all combinations

The response graphs are given through Figure 5.16-5-18 and the Figure 5.19 shows the tracking error of the three combinations. As a summary for frequency change experiment, the following observations are given. When the frequency of the sinusoidal input signal is increased to 0.3 Hertz the state feedback controller totally loses its effect on the cart, the cart is in the opposite direction of its desired position (it is at the right when it must be in the left and left when it must be in the right). However LQR control shows a more robust response to that frequency change. Fuzzy-PID shows the most robust response to the frequency change for trajectory tracking but some oscillations on the pendulum position are being observed at this point, but these oscillations are in an acceptable tolerance range.

5.2. Responses to Pulse Disturbance

In this section the sinusoidal input signal is applied to the system with constant frequency, and an extra pulse signal is applied to the system acting as a disturbance. The pulse is given to the system from the output voltage port of the system so that an extra voltage acts on the system and the pendulum-cart system shifts from its desired position with overloaded force, and the controller tries to overcome this unusual effect. The Simulink model of the application is shown in Figure 5.20. During the application the frequency level is kept constant and in order to have a good comparison the magnitude of the pulse is increased. The response of each combination in each step is plotted below.

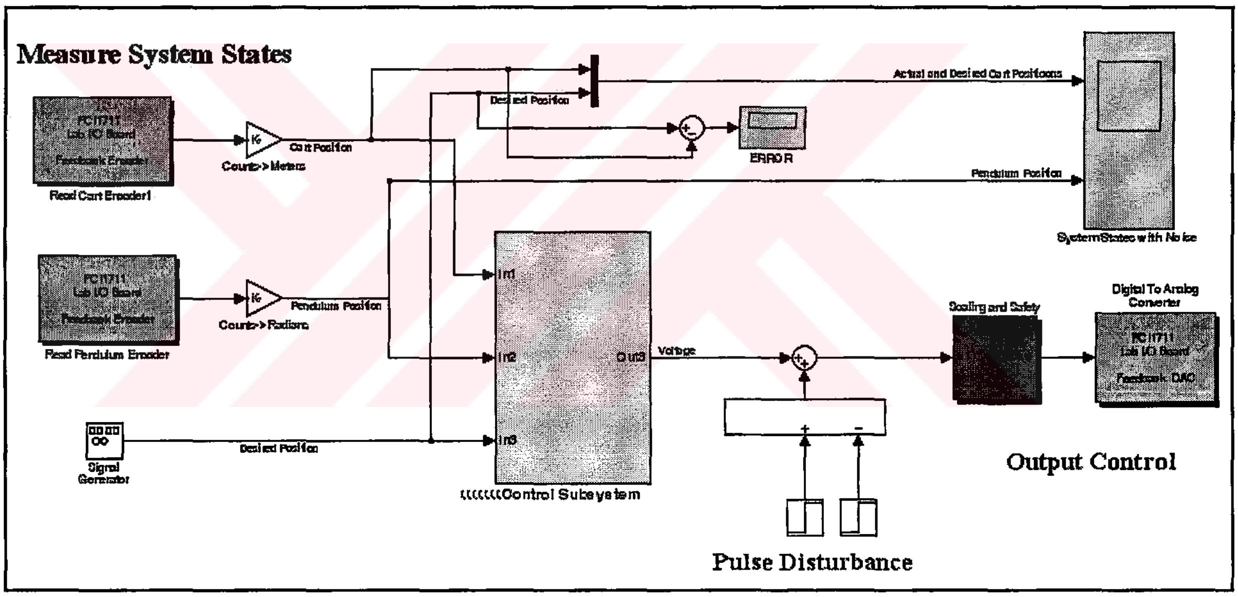

Figure 5.14. Simulink model of the system with pulse disturbance

The starting and ending time of the pulse is 33 and 34 seconds of the simulation respectively. This time range is chosen on purpose. Because, up to this time the pendulum is swung up and a satisfactory balancing movement in the upright position is maintained by the controller in an adequate period of time and when the pulse is applied the cart is in the one end of the limited rail, the cart flies off in the opposite direction with the effect of the disturbance and overriding of the limits is avoided.

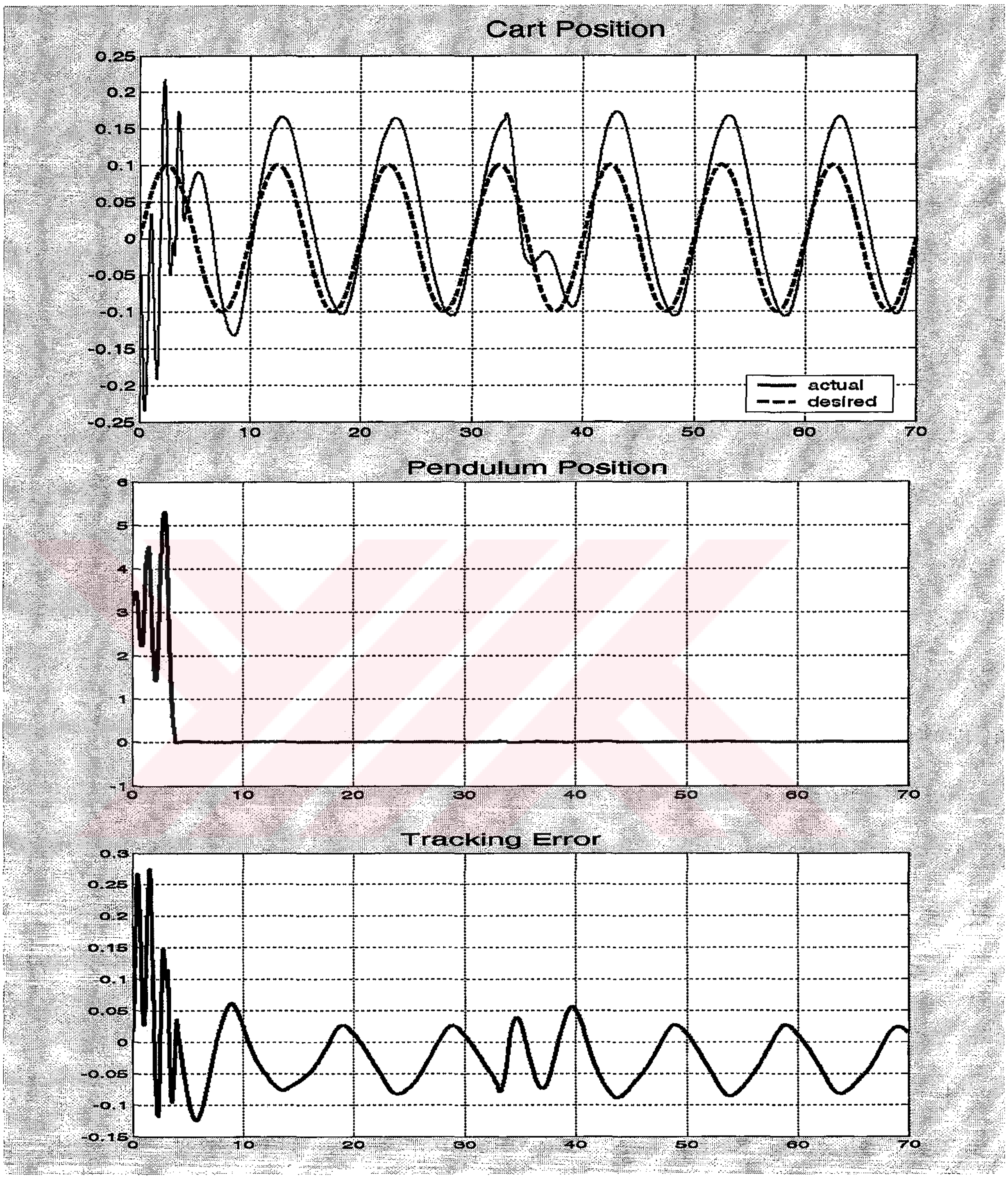

Figure 5.15. Response graphs of E-SVF control with pulse disturbance of magnitude 1

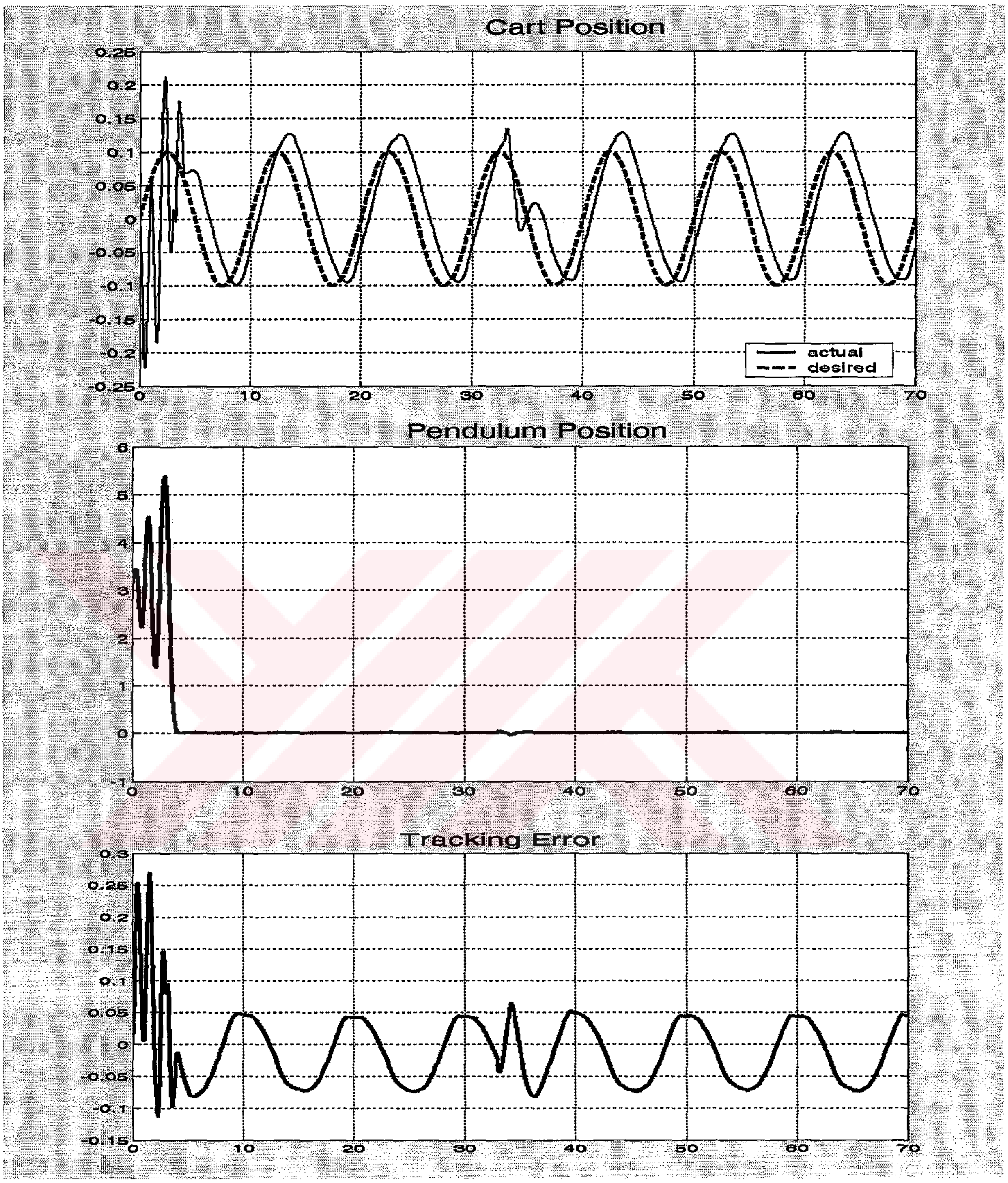

Figure 5.16. Response graphs of E-LQR control with pulse disturbance of magnitude 1

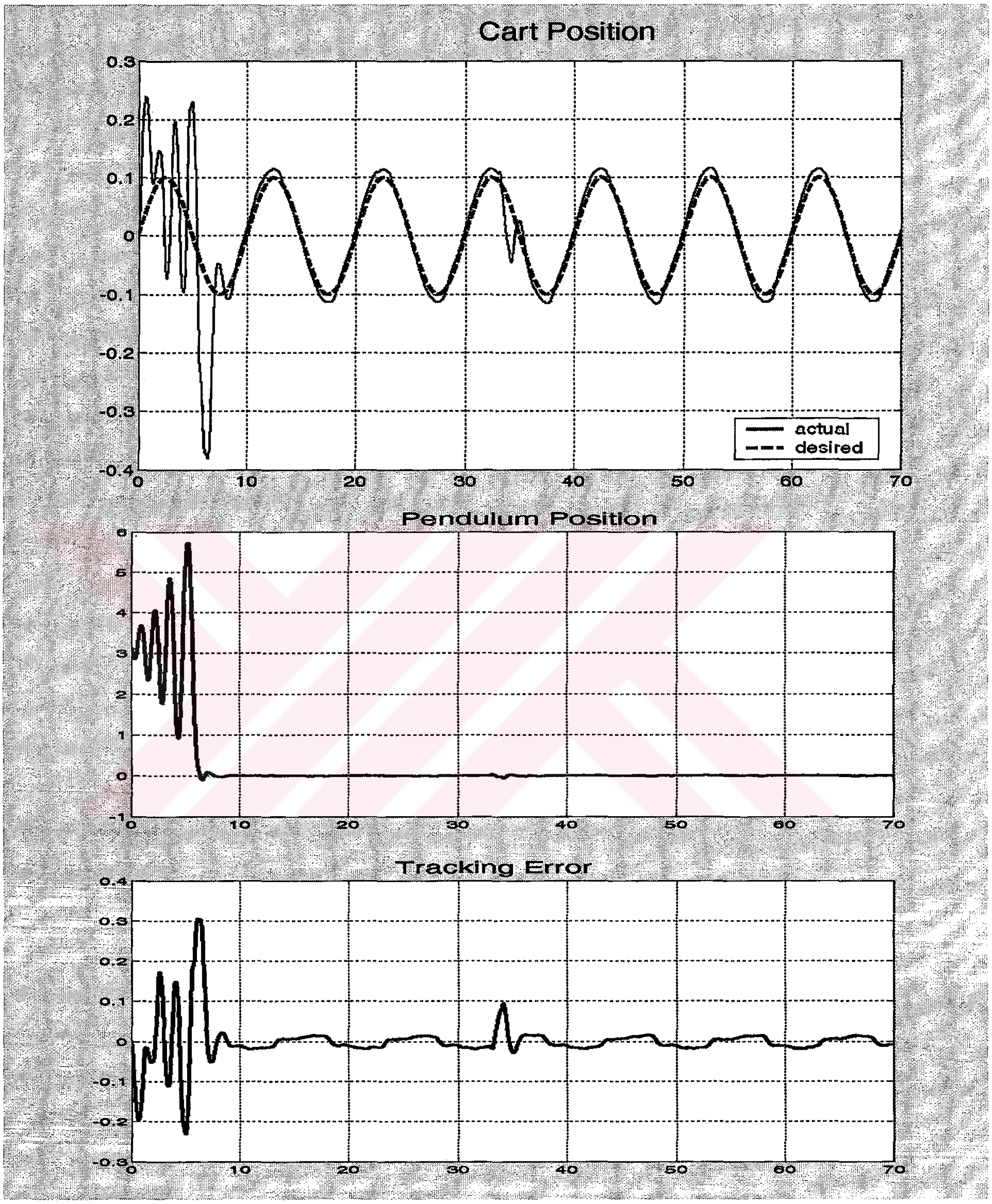

Figure 5.17. Response graphs of F-PID control with pulse disturbance of magnitude 1

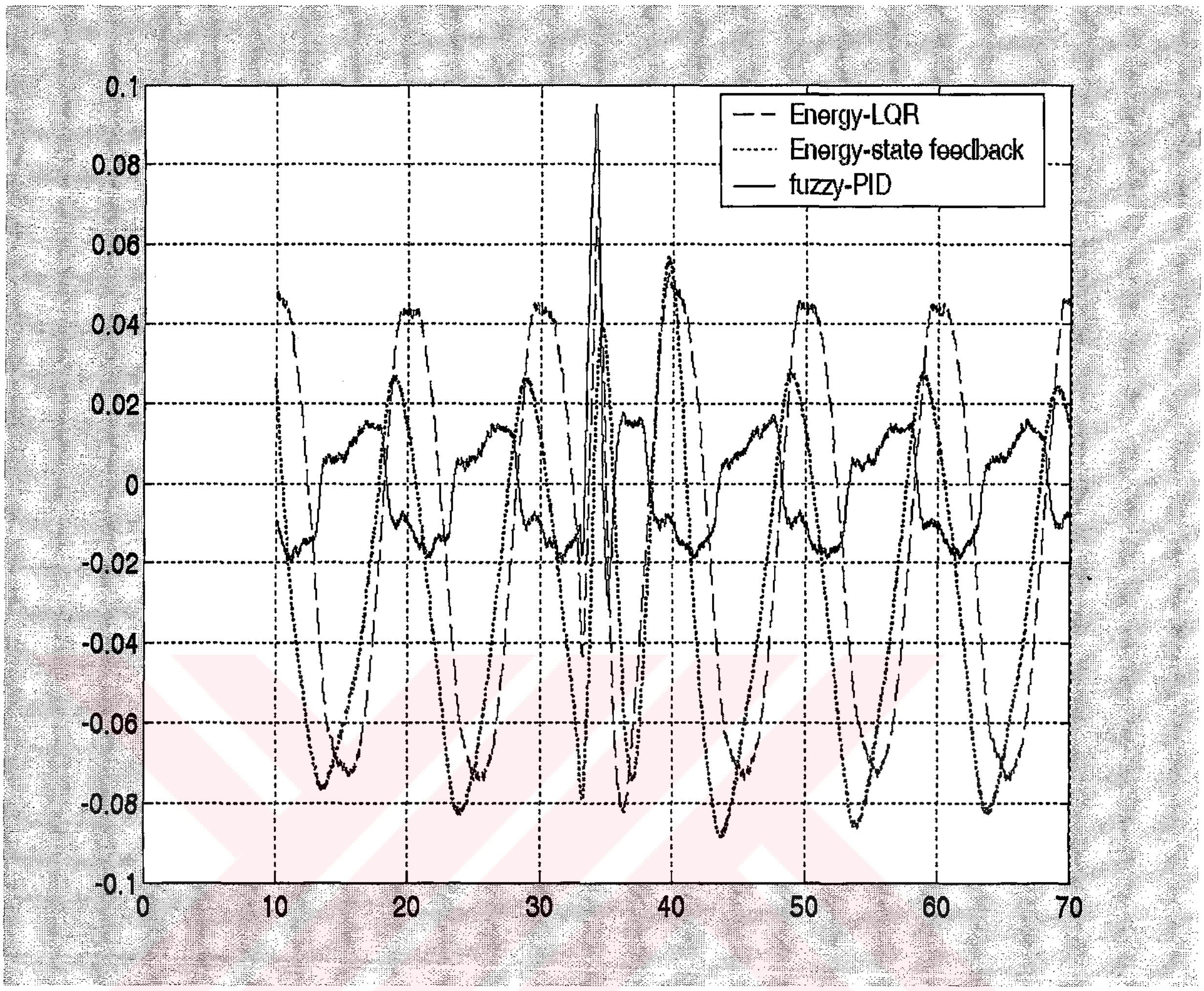

Figure 5.18. Tracking errors of all combinations

The response graphs of the each combination are plotted through Figures 5.15-5.17 and in Figure 5.18 the tracking errors of each combination are plotted in the same graph. The Energy-LQR and Energy-state feedback combinations show a robust behavior to the disturbance. On the other hand fuzzy-PID combination response has the minimum error on the average but it gives the maximum error when the disturbance is applied. This shows that the robustness of the fuzzy-PID controller is less than the others. In order to show that more clearly, the magnitude of the pulse disturbance is increased and the response graphs of each combination are given in the following section.

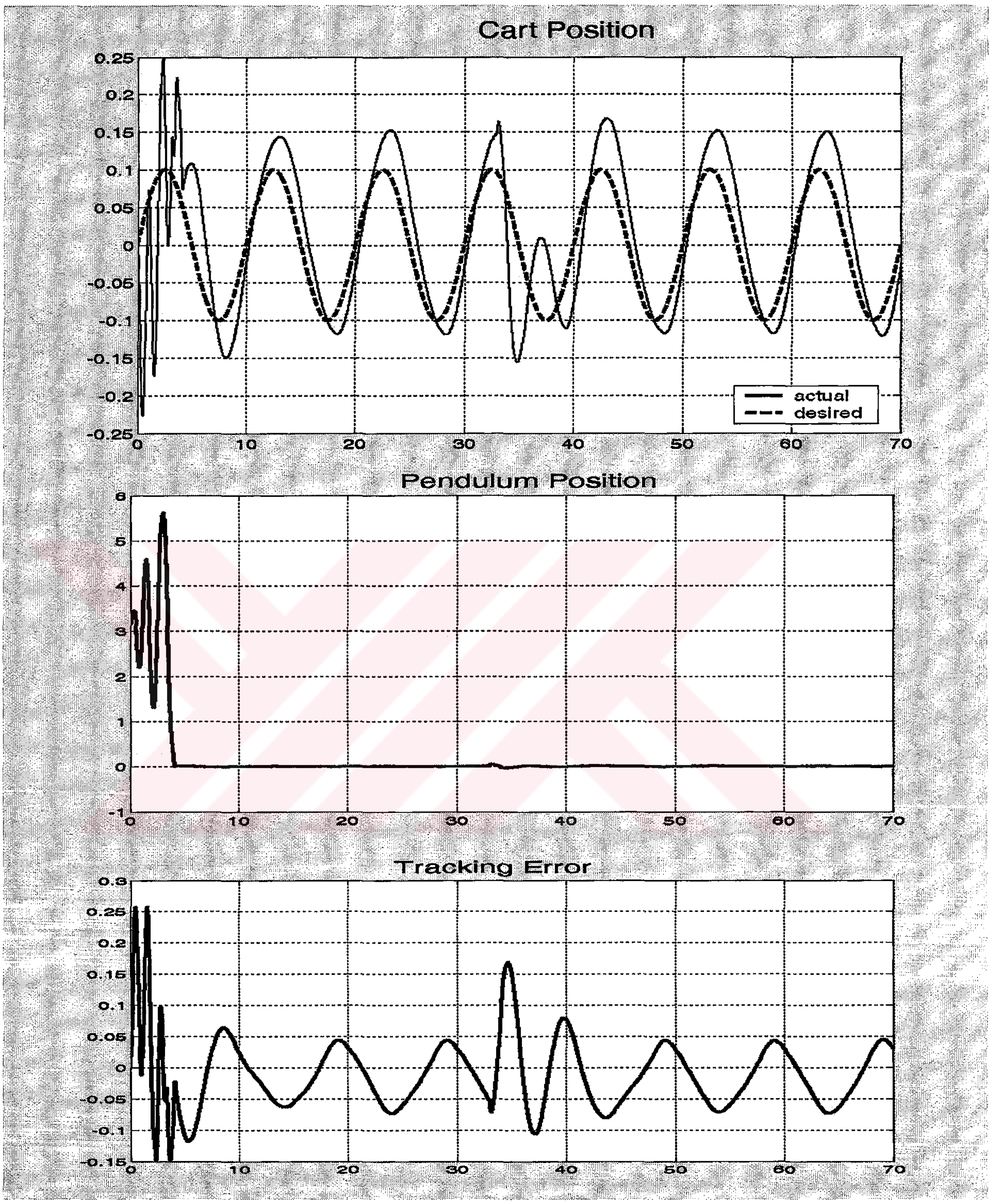

Figure 5.19. Response graphs of E-SVF control with pulse disturbance of magnitude 2

inductance, resistance, the gearbox ratio and the motor coefficients. Unfortunately, these parameter values are not given with the manuals of the system, furthermore the values of the parameters that are given in Table 2.1 have proved not to be reliable. The given values are changed slightly during the design process by a trial and error approach. For an accurate calculation of the linear model the linearization toolbox of the MATLAB is used. The nonlinear equations are defined in a Simulink model file and the approximated linear model is obtained as shown in Figure 4.3.

To overcome the second problem related with the DC motor the following methodology is applied. Considering the DC motor having a first order transfer function and considering that this transfer function can be approximated as a gain when it is evaluated with the mechanical part equations, a simulation model file is prepared with using MATLAB/Simulink as shown in Figure 4.4.

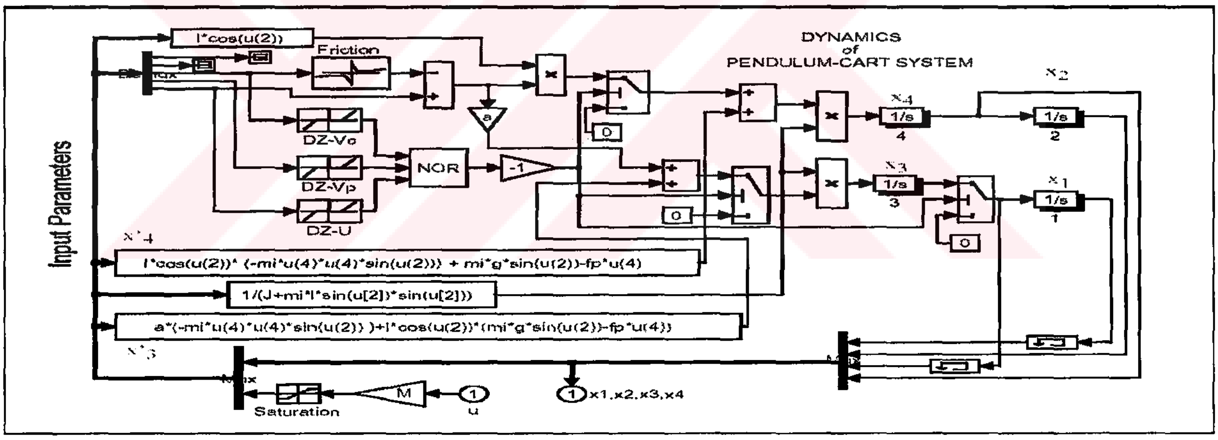

Figure 4.3. MATLAB/Simulink representation of the system

The prepared model includes both the virtual simulation and the real time simulation of the system. The virtual part includes only pre-modeled mechanical equations and the real time simulation includes the natural electro-mechanical behavior of the system inherently. An input step function is given to both systems and the responses of each system are obtained. Figure 4.5 shows the responses of two models in the same plot. It can be easily seen that a constant multiplier is enough to obtain the same response from the two models, which supports the idea that the motor equation can be considered as a gain. After a few trial steps the true coefficient value for the motor is obtained as K_m=0.75 as shown in Figure 4.6.

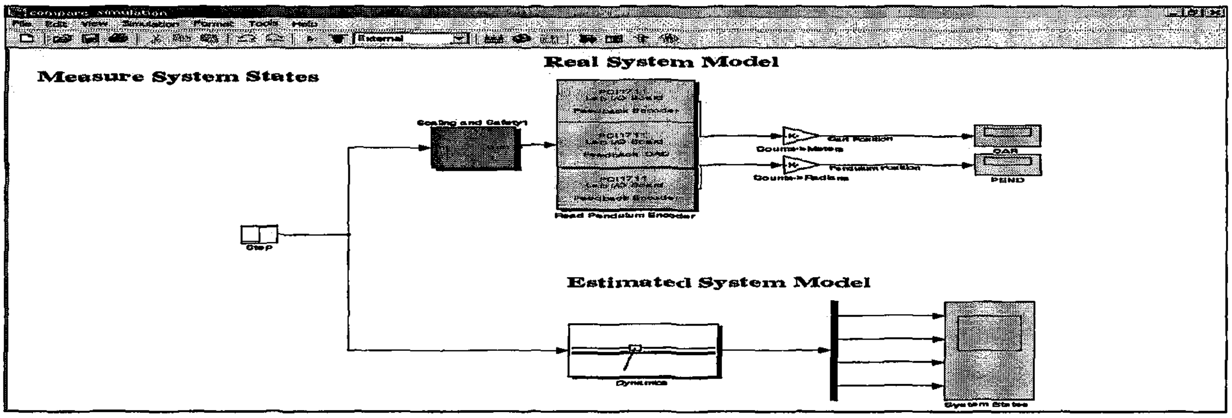

Figure 4.4. MATLAB/Simulink representation of the system

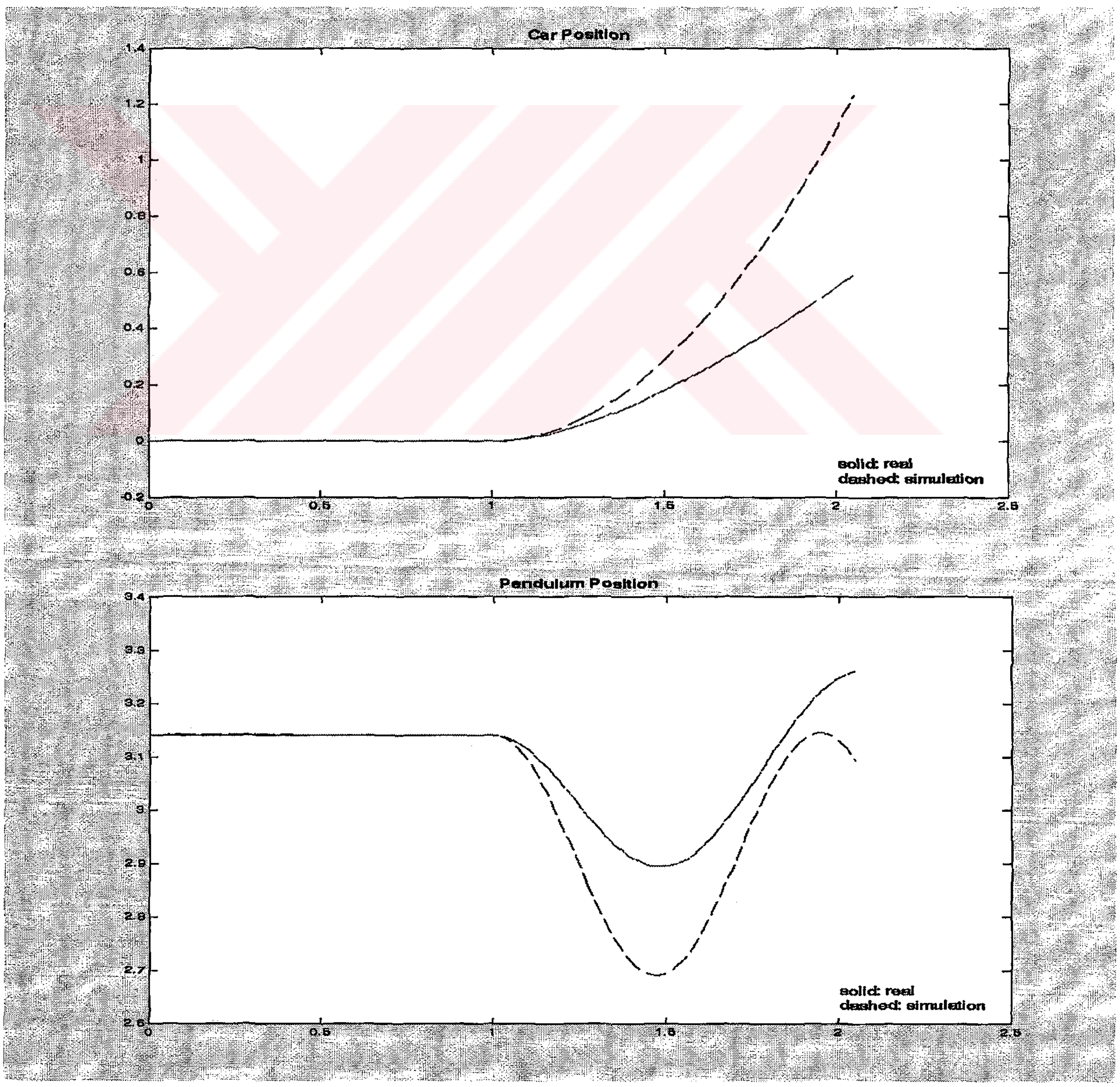

Figure 4.5. Responses of the models without including motor gain

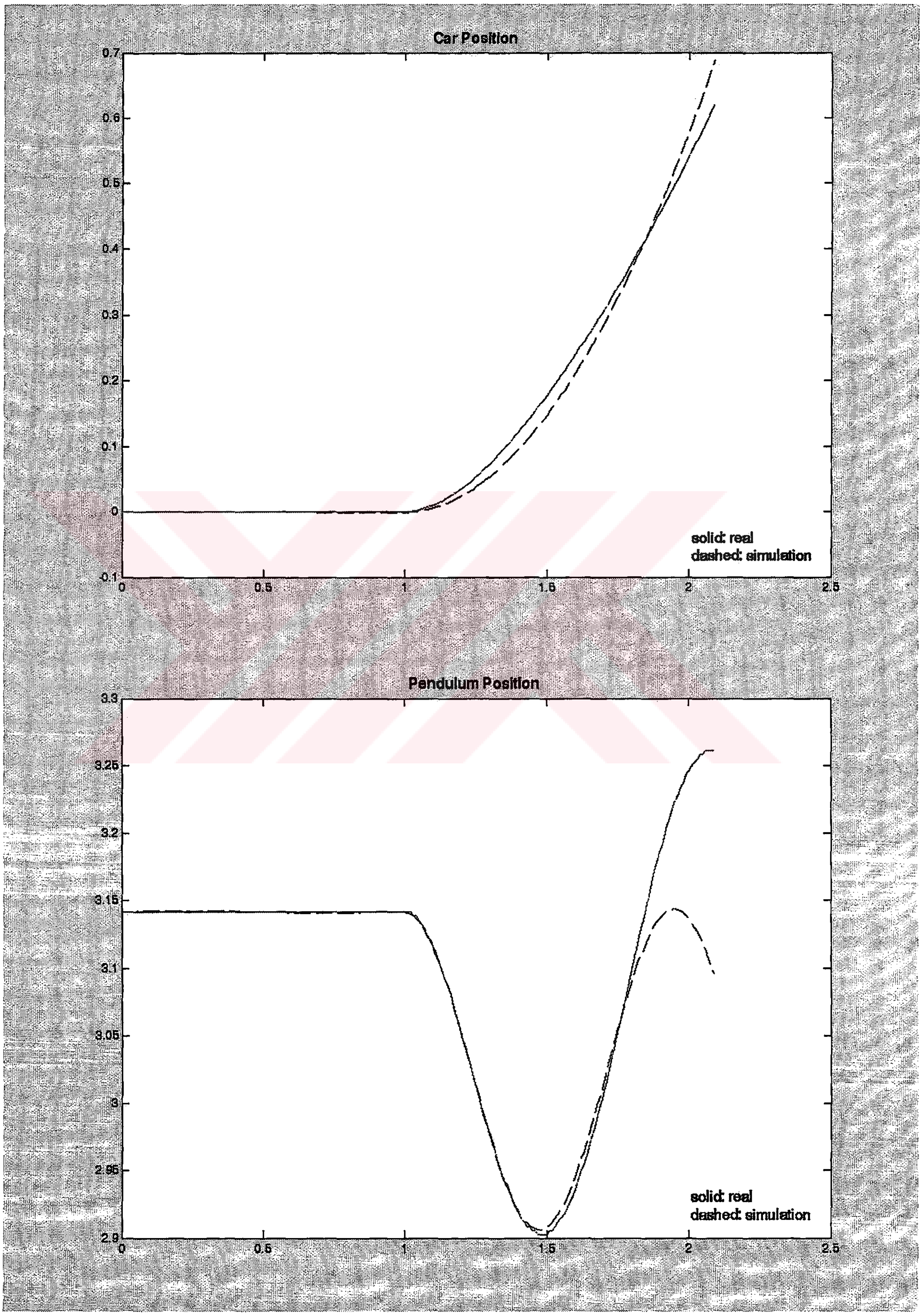

Figure 4.6. Responses of the models with motor gain

The resultant state space representation of the linear system is as follows

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 44.0707 & -0.0223 & -3.4576 \\ 0 & 0.7676 & -0.0004 & -0.8299 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 13.0667 \\ 3.1362 \end{pmatrix} F \qquad (4.19)$$

Using a control law where the input function is defined as

$$u(t)=Kx(t) \qquad (4.20)$$

Then the eigenvalues of the closed loop system can be determined by

$$\Delta(\lambda) = |\lambda I - A + BK| = 0 \qquad (4.21)$$

To maintain the stability of the system the closed loop poles must be in the left hand side of the plane. Weighting coefficients of the states which are the elements of the K matrix are defined in a range for stability purposes. To have a better system response a few trial and error steps were applied to see the effects of the weighting coefficients, and finally suitable K matrix values that maintain both stability and a satisfactory performance for both angle and trajectory tracking are obtained as

$$K = \begin{pmatrix} -7 \\ 50 \\ -4 \\ 3,5 \end{pmatrix}^T \qquad (4.22)$$

This state feedback coefficient matrix is built in the used set. Another practical way of determining the weighting coefficients is pole assignment. If the pole locations that maintain the system to have a stable and satisfactory performance are exactly known, the K matrix values are determined so as to force the system closed loop poles to that known pole locations. The pole placement method has not been implemented in this work.

The output states are fedback to the system with coefficients determined in Equation 4.10 as shown in Figure 4.7.

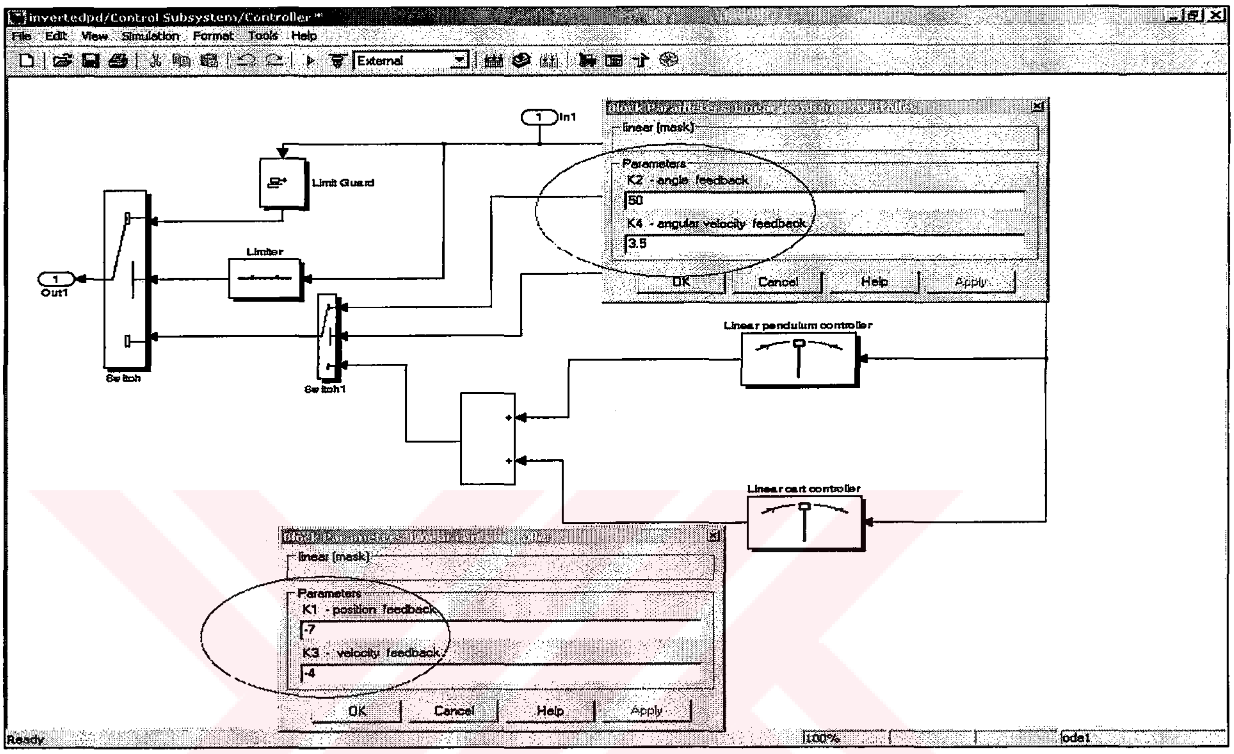

Figure 4.7. State feedback coefficients from Simulink model

4.2. Linear Quadratic Regulator (LQR)

The word optimal intuitively means to perform a job in the best possible way. Before starting a trial for such an optimal solution, the job must be well defined. A mathematical foundation must be established for quantifying what best means and the possible alternatives must be pointed out. Unless there is a clear understanding on these qualifiers, a suggestion that a system is optimal is really meaningless.

Linear-quadratic-Gaussian (LQG) control is a modern state-space technique for designing optimal dynamic regulators (in literature sometimes this terminology interpreted as LQR which nearly have the same meaning with LQG). It enables you to trade off regulation performance and control effort, and to take into account process disturbances and measurement noise. [28], [29], [30] Like pole placement, LQG design requires a state-space model of the plant which forms the mathematical foundation of the system that is pointed out in the preceding paragraph of this section. In section 4.1, a well identified mathematical model of the whole system including both electrical and mechanical components is defined for this purpose.

The main objective of optimal control is to determine control signals that will cause a system to satisfy some physical constraints and at the same time optimize (maximize or minimize) a chosen performance criterion. The formulation of optimal control problem requires

- a mathematical description (model) of the system to be controlled generally in state variable form (which is done in sections 2 and 4),
- a specification of the performance index (cost function), and
- a statement of the boundary conditions and the physical constraints on the states and controls (which is defined by plant, input ant state matrices).

In LQG control, the regulation performance is measured by a quadratic performance criterion (or simply the cost function of the system) of the form

$$J(u) = \frac{1}{2}\int_0^T \left(x^T Qx + u^T Ru\right) dt \qquad (4.23)$$

Subject to the constraint

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (4.24)$$

The weighting matrices Q, and R are user specified and define the trade-off between regulation performance (how fast x(t) goes to zero) and control effort. The design seeks a state-feedback law (u = Kx) that minimizes the cost function J(u). The minimizing gain matrix K is obtained by solving an algebraic Riccati equation.

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \qquad (4.25)$$

where K = $R^{-1}B^TP$, and P is from the solution of the algebraic Riccati equation:

The matrices A and B come from the state-space representation of the system. As already stated the Q and R matrices are specified by the control system designer where these matrices define the weighting measures for the errors of the states and input respectively. For the pendulum system these matrices defined as

$$Q = \begin{pmatrix} 20 & 0 & 0 & 0 \\ 0 & 200 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (4.26) \qquad \text{and} \qquad R = (0.4) \quad (4.27)$$

It should not be forgotten that one can define a different matrix configuration for a different desired performance and gets a different performance according to predefined and desired conditions. In the application of this method, various configurations of Q and R matrices have been tried but the desired performance is mostly on the stabilization of the pendulum in the upright position and tracking of the trajectory of the car. So, corresponding weighting coefficients of the states are higher than the others (velocities of the car and the pendulum).

For the solution of the algebraic cost function and Riccati equation, MATLAB is used for practical and fast results and accurate calculations. The obtained K matrix is defined as

$$K = \begin{pmatrix} -5 \\ 26.4676 \\ -7.2369 \\ 4.0814 \end{pmatrix}^T \quad (4.28)$$

As in section 4.1, the states of the system are fedback to system with these coefficients.

4.3. PID Control

The PID control rule is very common in control systems. It is the basic tool for solving most process control problems. The PID controllers are usually standard building blocks for industrial automation. The most basic PID controller has the form

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau + K_d \frac{de(t)}{dt} \tag{4.29}$$

where $u(t)$ is the control output and the error, $e(t)$, is defined as

$e(t)$ = desired value – measured value of quantity being controlled.

The control gains K_p, K_d, and K_i determine the weight of the contribution of the error, the integral of the error, and the derivative of the error to the control output and will dictate the response of the closed-loop system to the initial conditions and inputs.

There are a number of tuning methods for PID controllers. Some of them are based on transient response experiments e.g., the Ziegler-Nichols step response tuning rule, other are based on relay feedback when the parameters of a PID controller are determined from features of the limit cycle of the closed-loop system, some others are based on frequency analysis [30], [26], [27]. In the pendulum system under consideration, the candidate quantities for PID control are the cart position and pendulum angle.

It is considered that the pendulum-cart control system has to contain two PID controllers. The first operates based on the angle of the pendulum and the second operates based on the position of the cart. The outputs of the PID controllers are added to produce the final control value for the D/A converter, and as a result the output motor torque.

A schematic of the proposed double PID control system for the pendulum-cart system is shown in Figure 4.8, constructed as a Simulink block diagram.

The adjustment of the PID coefficients is done by trial and error method. The values of the state variable weighting coefficients are used as initial values to have an idea on the system response then these values are incrementally changed, so that the sensitivity of the system to each parameter is understood. The final values of the coefficients for a satisfactory system performance are given in Table 4.1.

The resulting configuration of the PID parameters shows that a PD controller is more suitable to achieve a satisfactory performance for the pendulum system.

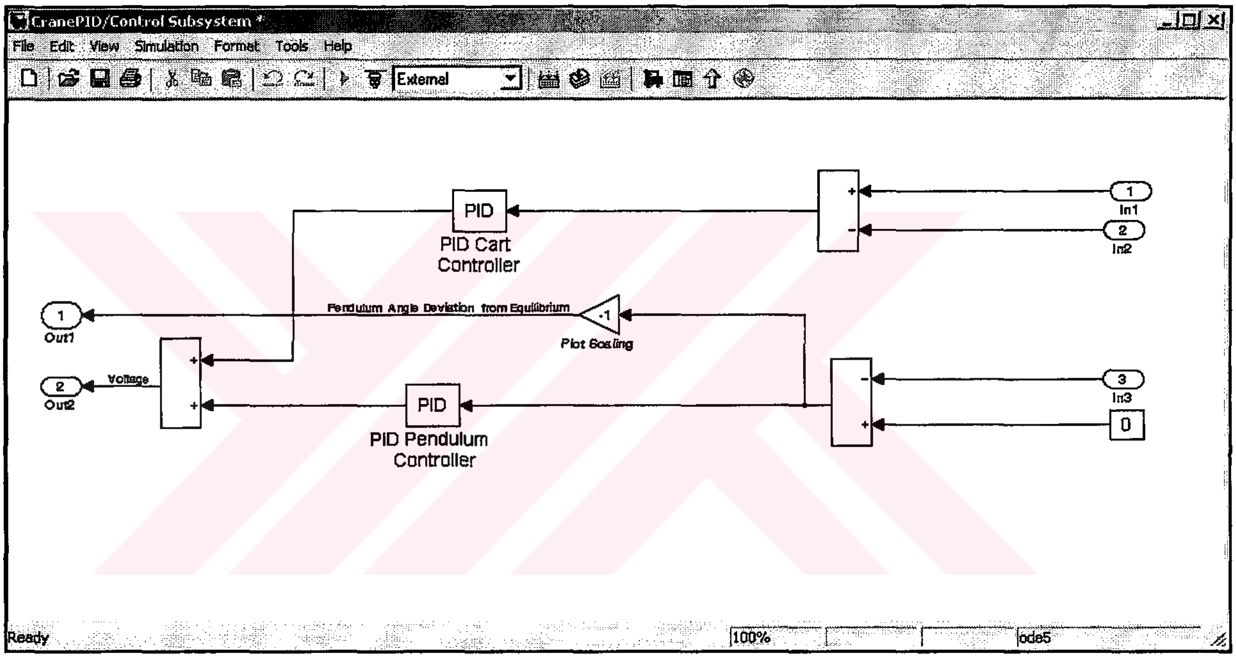

Figure 4.8. Block Diagram of the Double PID Control System from Simulink Model

Table 4.1. Parameters for the PID controllers

Cart controller			Pendulum Controller		
K_p	K_i	K_d	K_p	K_i	K_d
-15	0	-7	25	0	4

5. EXPERIMENTAL RESULTS

In this chapter, the implementation of the control methods, proposed in the previous chapters, has been studied. Three different combinations of the methodologies are implemented for the control of the whole system;

- Energy based controller for swing up – State variable feedback controller for balancing (E-SVF)
- Energy based controller for swing up – LQR controller for balancing (E-LQR)
- Fuzzy logic controller for swing up – PID controller for balancing (F-PID)

The simulation period of the each implementation is 70 second. The time is enough to get a satisfactory response. A number of various sinusoidal input signals that maintain back and forth movements along the rail are applied. The sinusoidal input signals differ in frequency to have a comparison between the control methodologies. Furthermore to see the robustness of each combination, an extra input signal is applied as a disturbance. The implementation is done by using the real time feature of MATLAB/Simulink; a Simulink model file is prepared for this purpose. The input signals are sent to the system via the model as shown in Figure 5.1. The responses and the errors of each input pattern are given in the following sections.

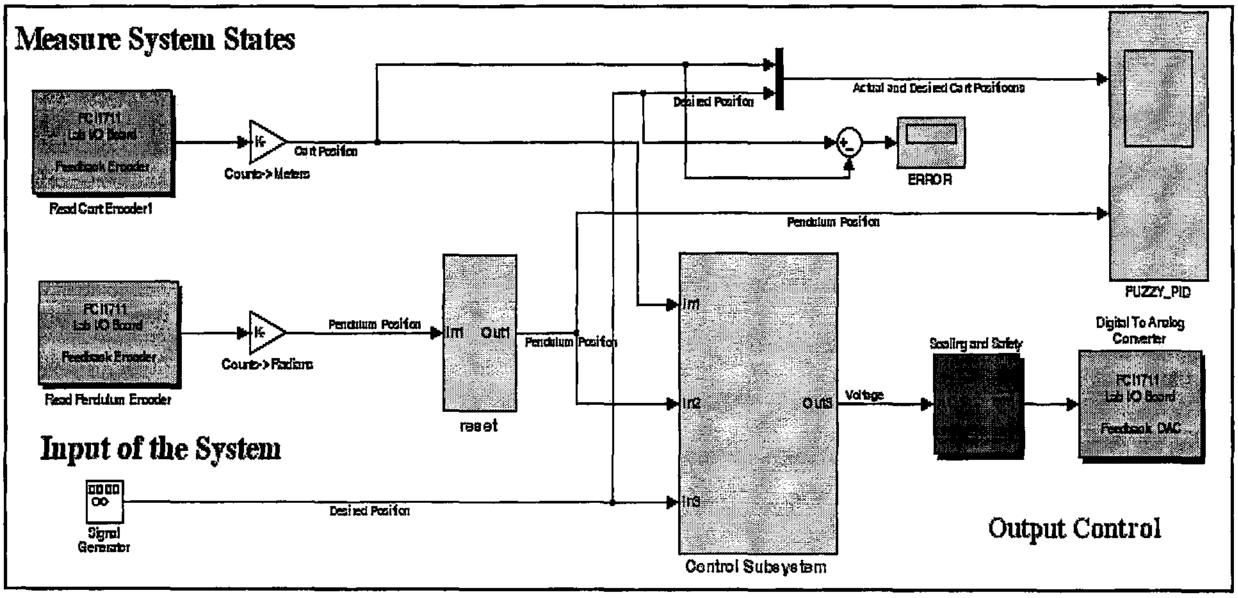

Figure 5.1. Simulink Model of the control system

5.1. Responses to Sinusoidal Signals with Different Frequencies

In this section the only input signal to the system is the desired trajectory of the cart. In order to maintain a well understanding, a back and forth movement along to rail is enough. For this purpose the input signal should be a sinus with a limited amplitude so that the cart can be driven on the rail without overriding the limits of the rail. The frequency of the sinus is slightly increased in each implementation. So that the capability of tracking the trajectory is seen for each combination. The responses and the errors are plotted for all cases. The frequency of the sinus is taken to be 0.1, 0.2 and 0.3 Hertz respectively.

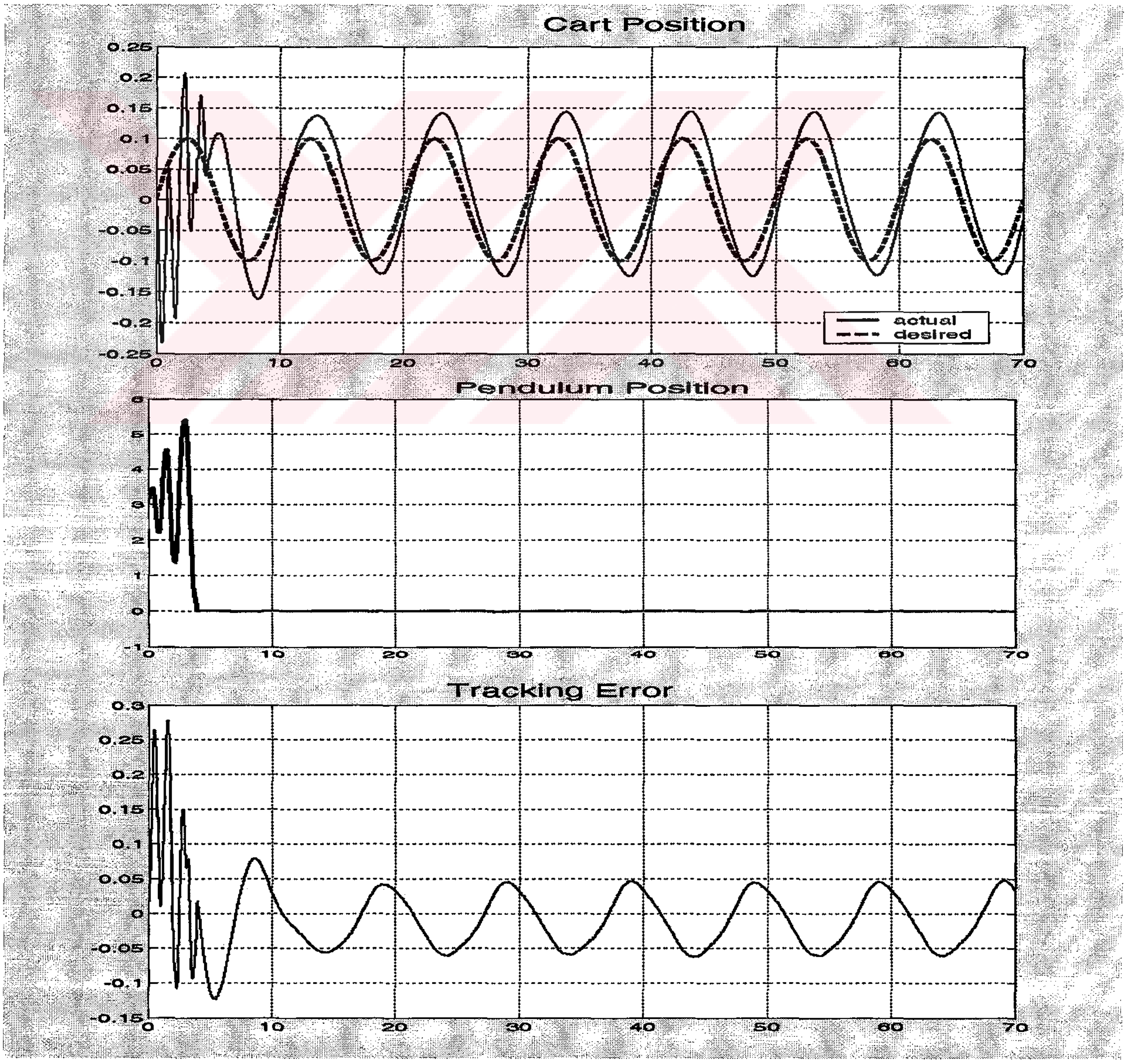

Figure 5.2. Response graphs of E-SVF control with sinusoidal input of 0.1 Hz

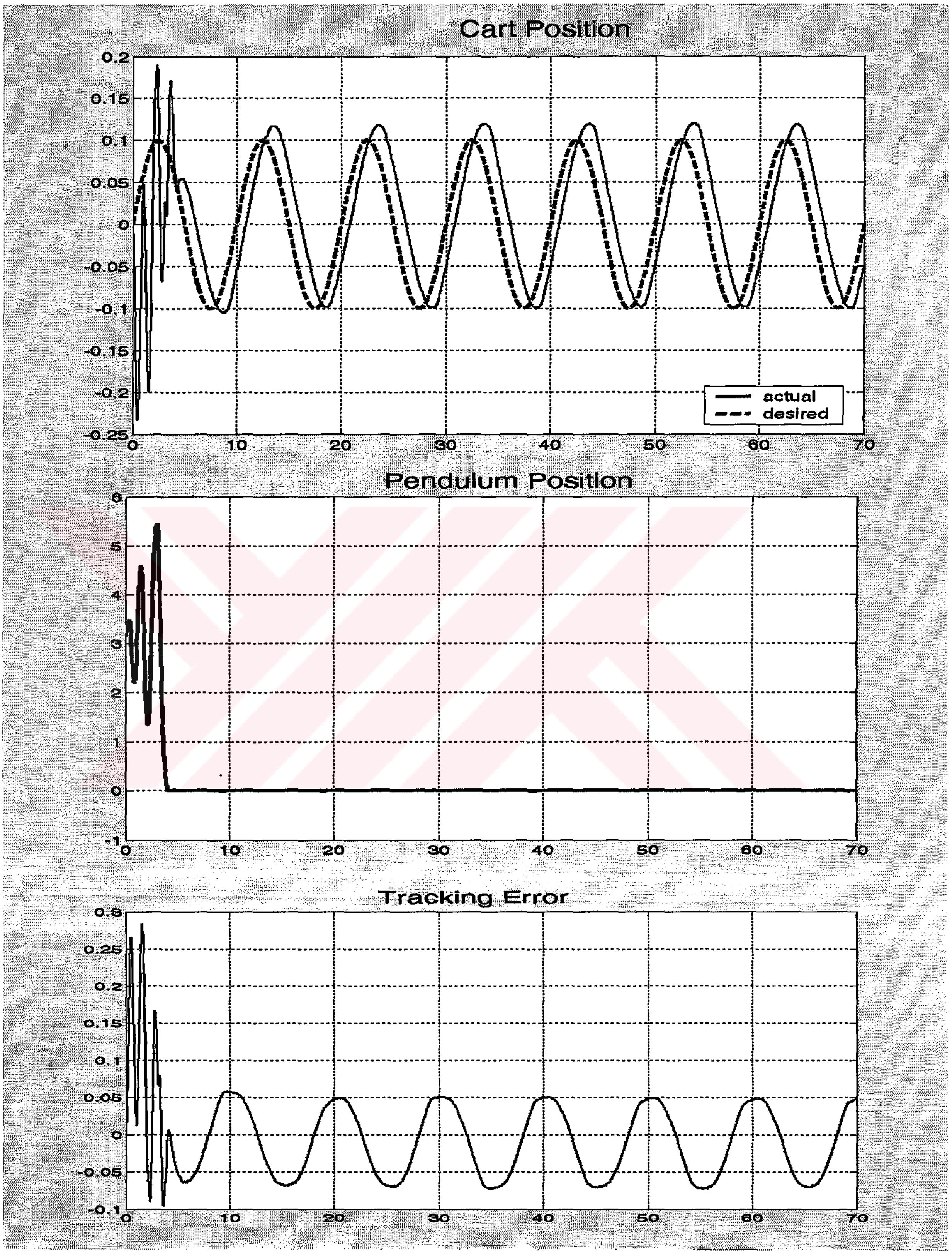

Figure 5.3. Response graphs of E-LQR control with sinusoidal input of 0.1 Hz

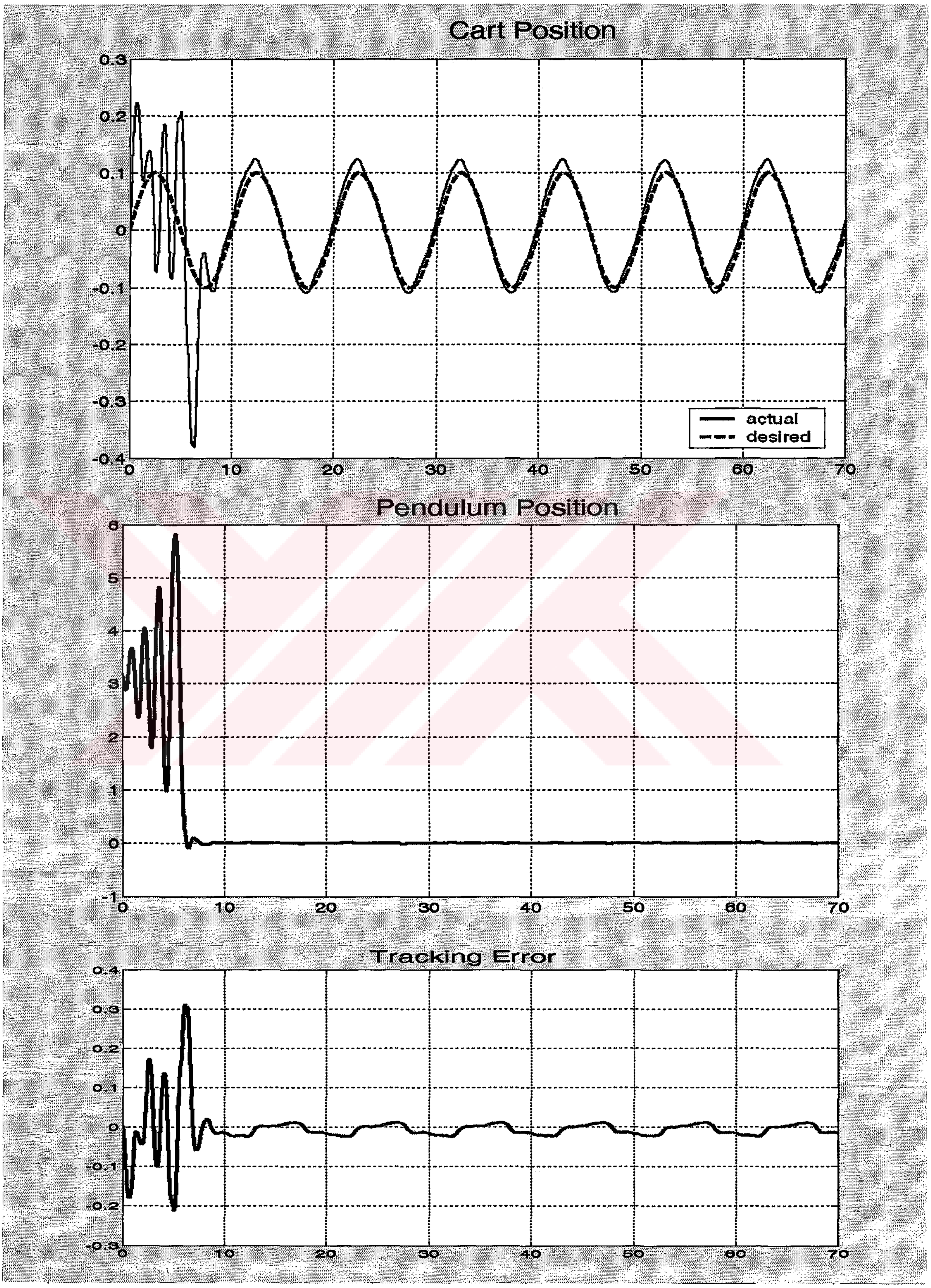

Figure 5.4. Response graphs of F-PID control with sinusoidal input of 0.1 Hz

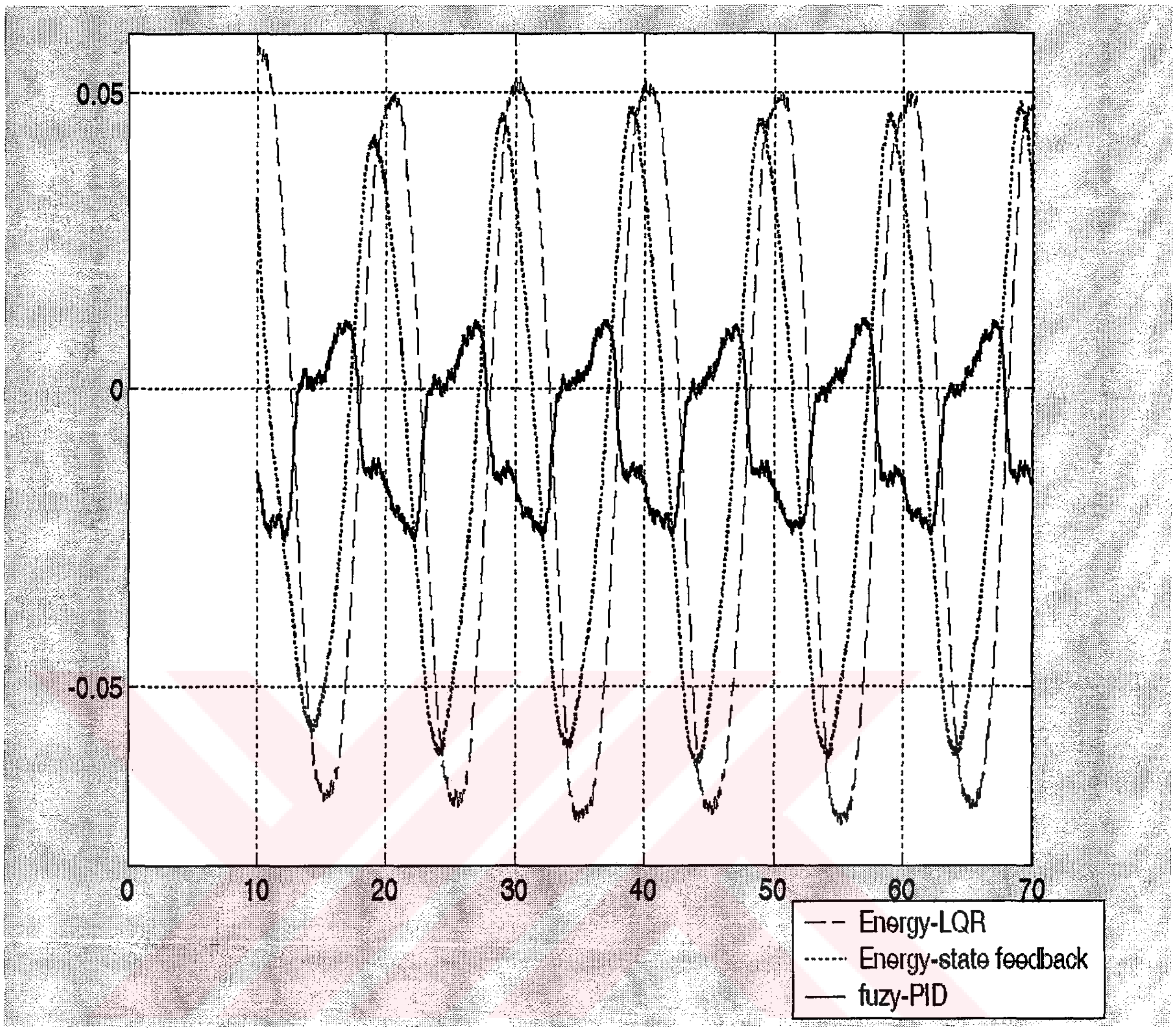

Figure 5.5. Tracking errors of all combinations

As shown in Figure 5.11, the minimum error is achieved by the fuzzy-PID controller, whereas the swing up performance of fuzzy-PID is not well as the others as seen from Figures 5.3, 5.6, and 5.9. The swing up process is done with 3 swings in energy based controller for about 5 seconds. Whereas in fuzzy controller swing up process takes about 7 seconds and needs 4 swings.

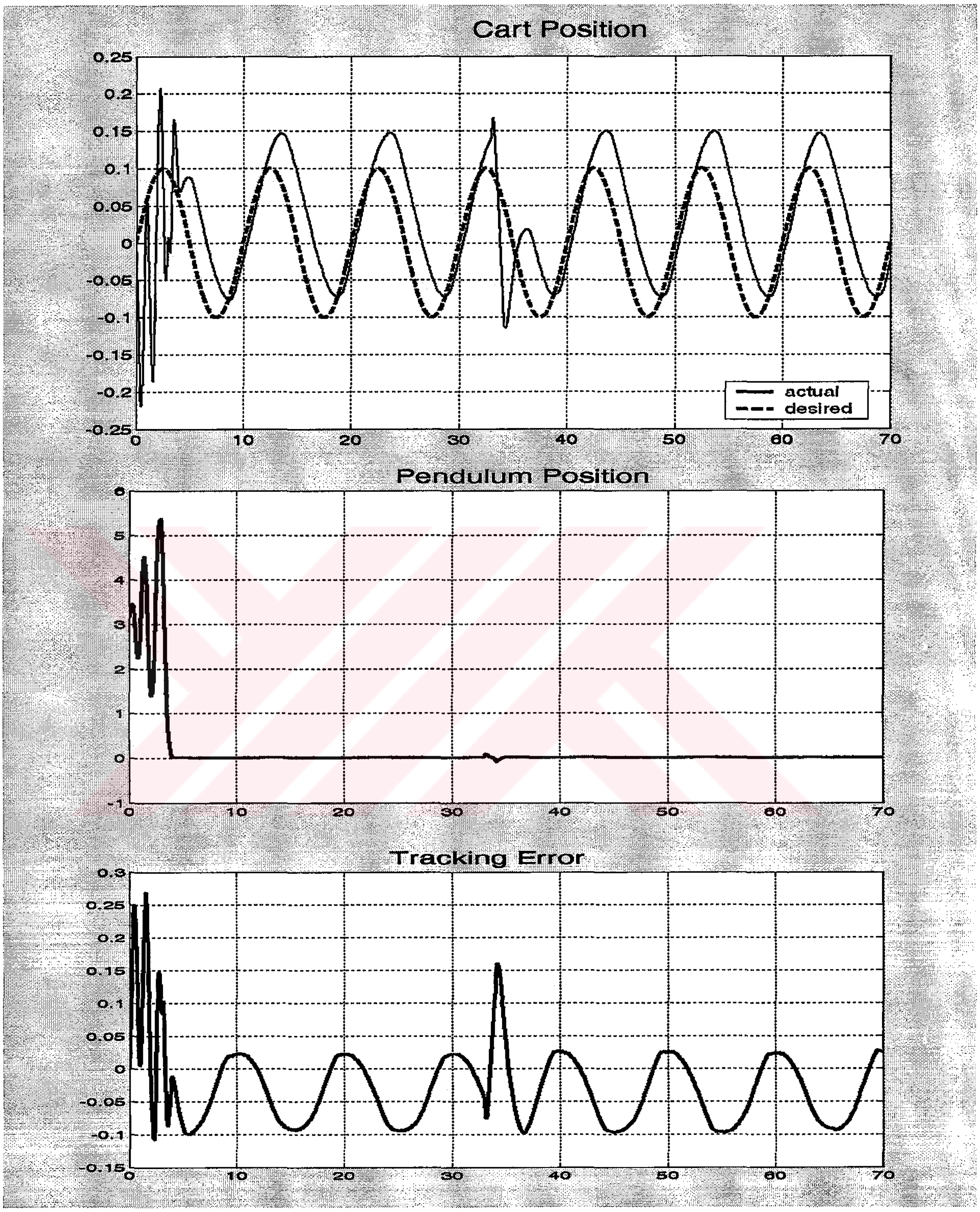

Figure 5.20. Response graphs of E-LQR control with pulse disturbance of magnitude 2

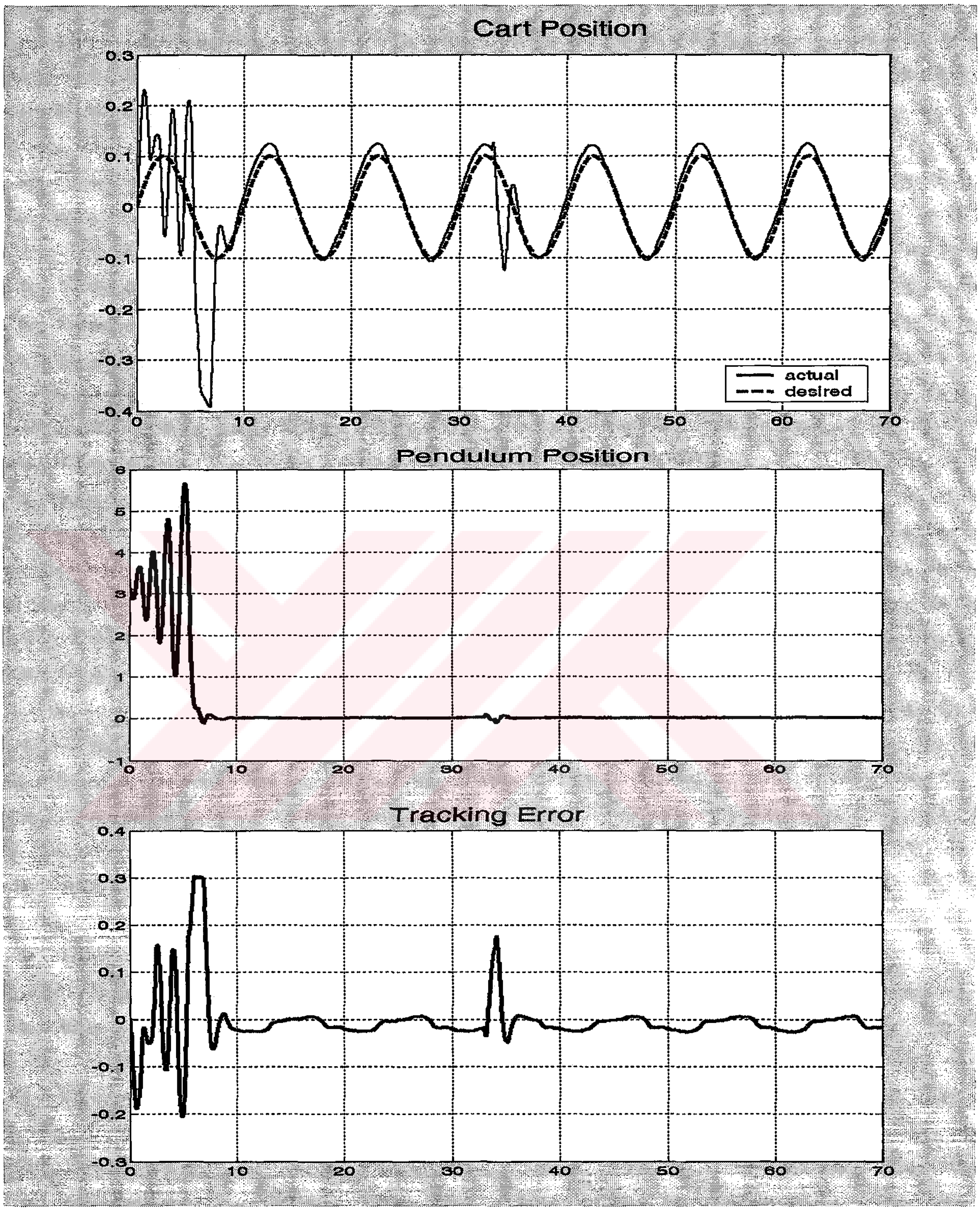

Figure 5.21. Response graphs of F-PID control with pulse disturbance of magnitude 2

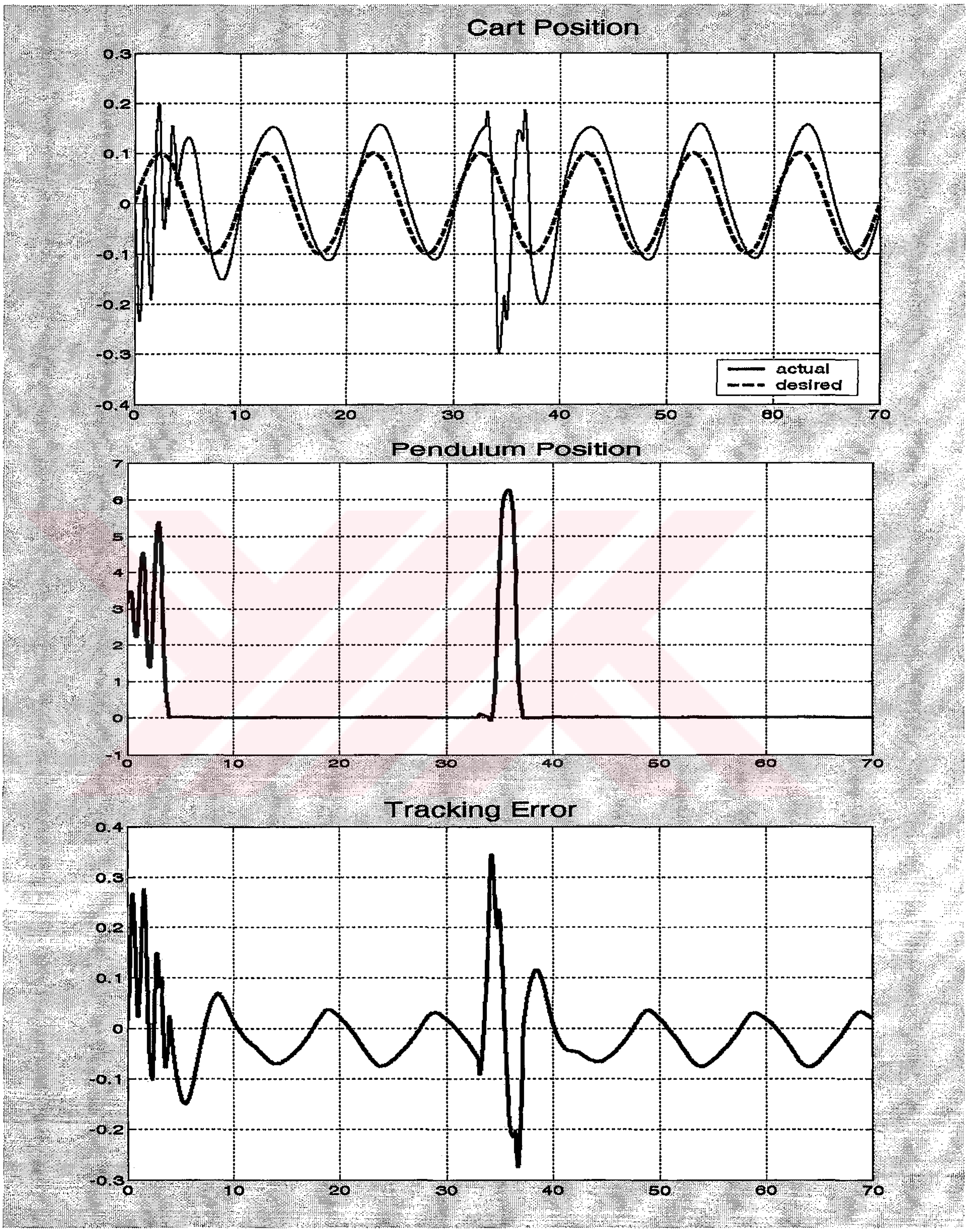

Figure 5.22. Response graphs of E-SVF control with pulse disturbance of magnitude 4.2

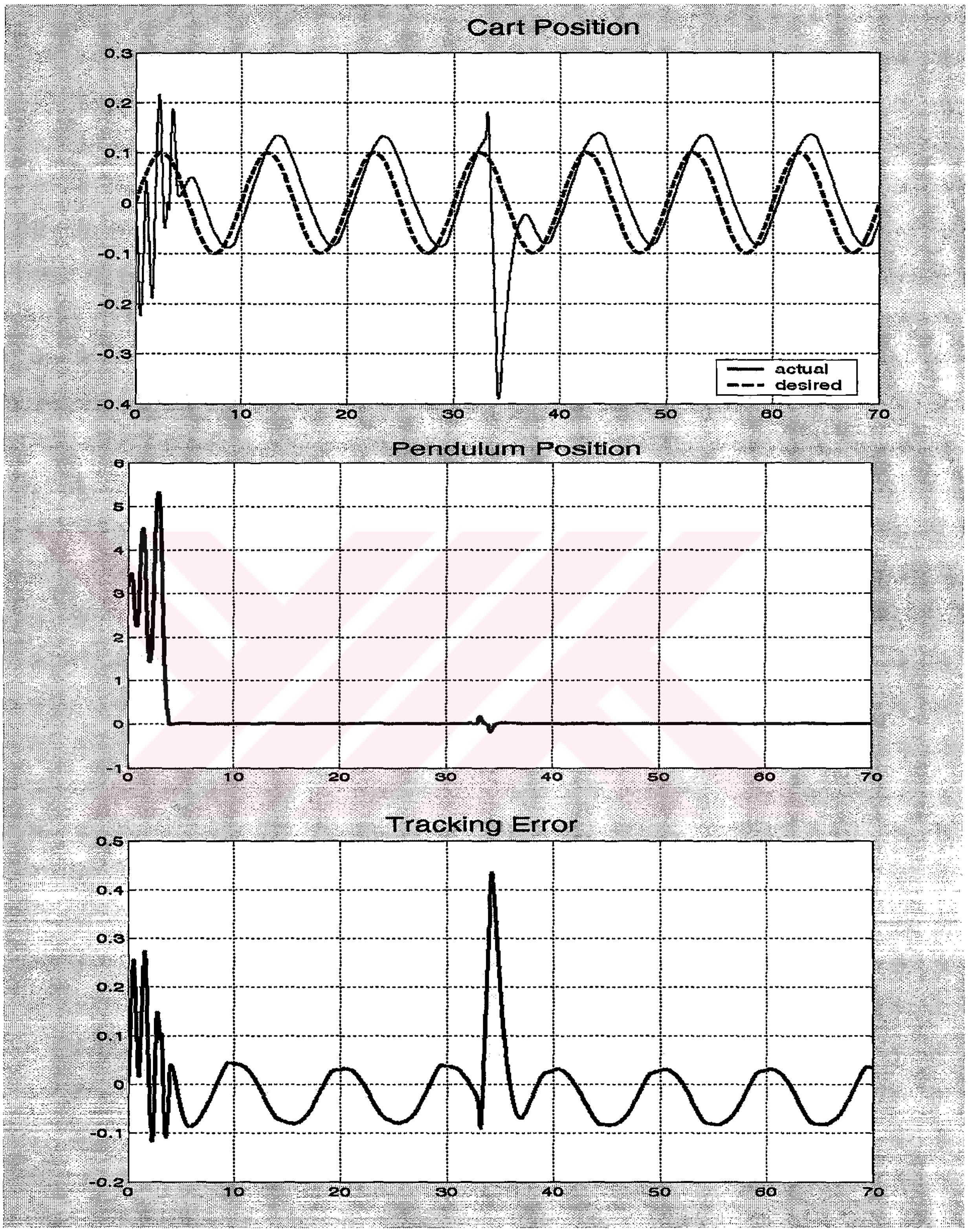

Figure 5.23. Response graphs of E-LQR control with pulse disturbance of magnitude 4.2

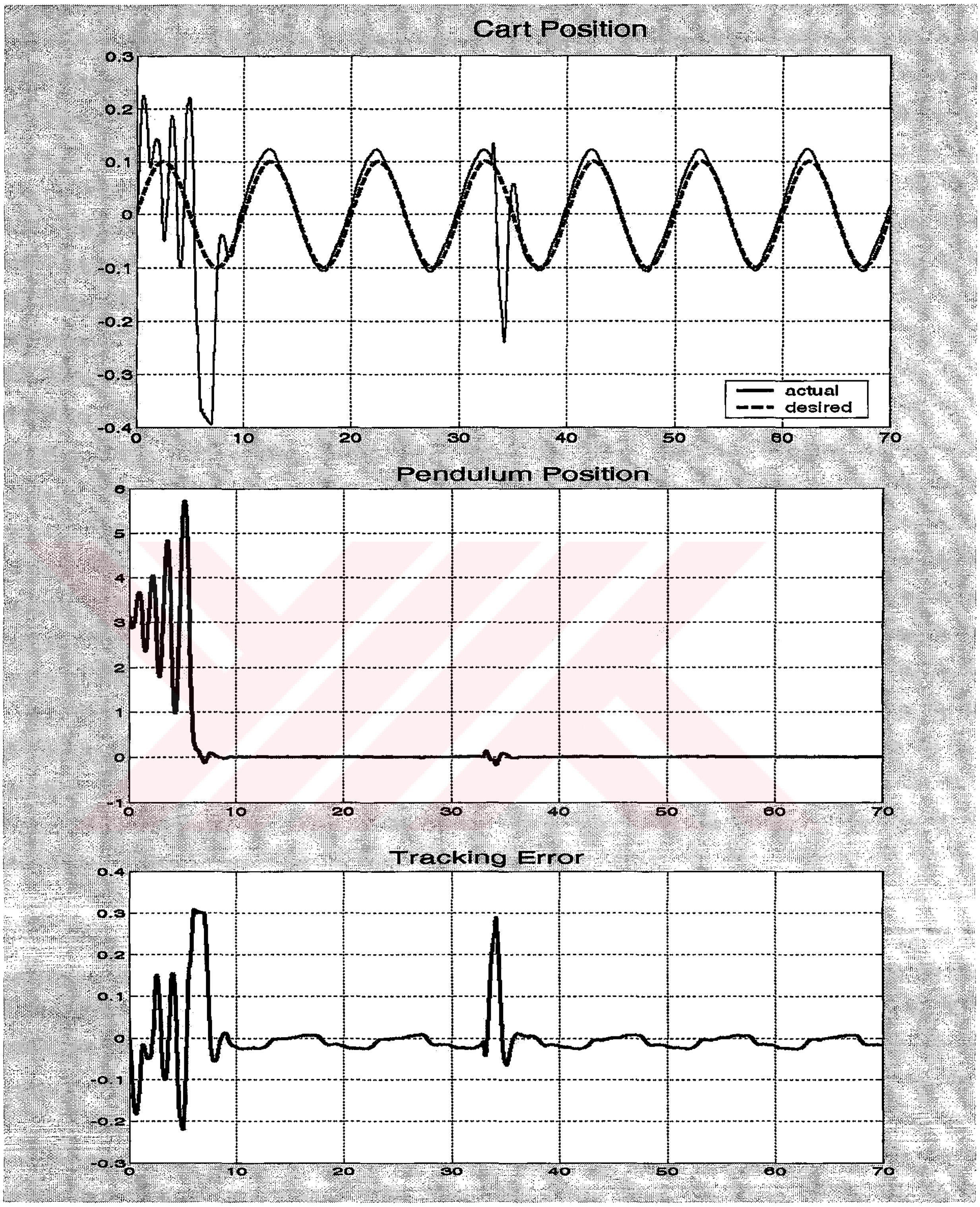

Figure 5.24. Response graphs of F-PID control with pulse disturbance of magnitude 3.2

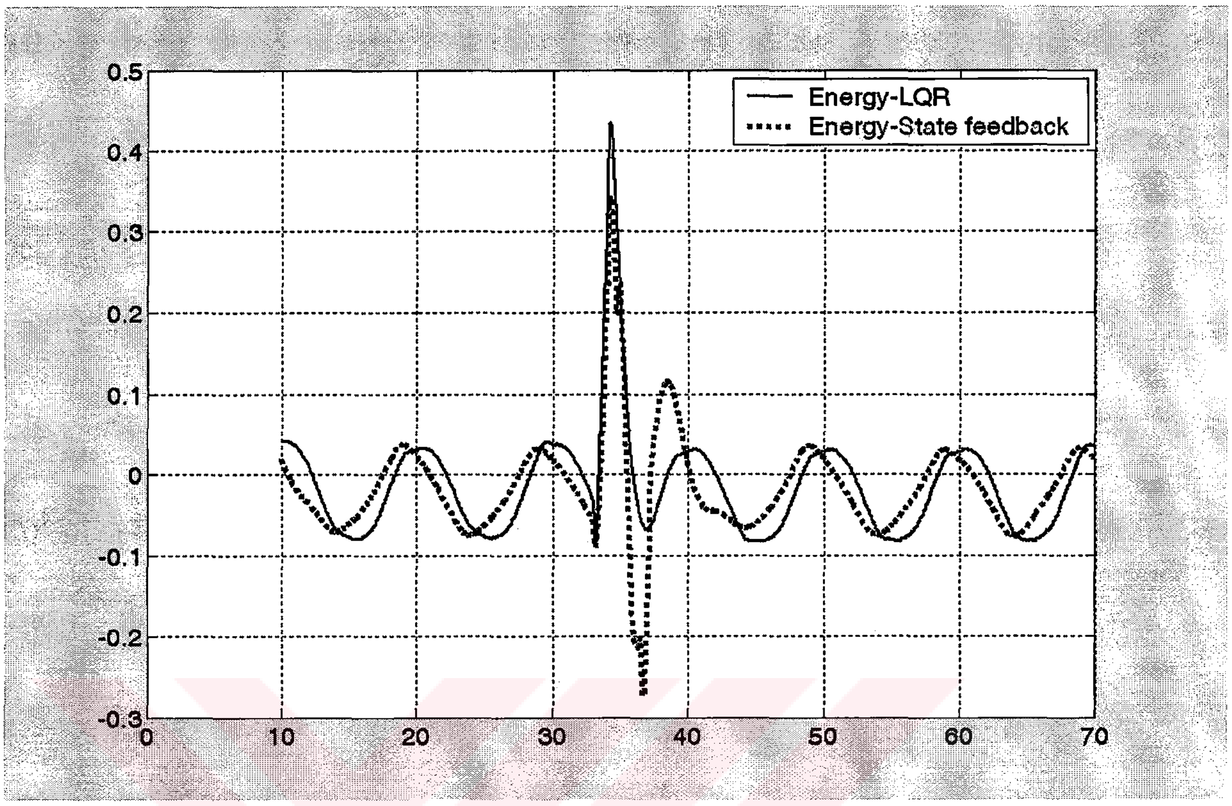

Figure 5.25. Tracking errors of the combinations with 4.2 magnitude pulse disturbance

From Figure 5.19 to Figure 5.25, the response graphs of the system to pulse disturbance changing in the level of magnitude are given. The maximum levels of the magnitude that the combinations can be capable of are 3.2 for F-PID and 4.2 for E-LQR and E-SVF. When the magnitude of the pulse is grater than these determined values the cart overrides the limits of the rail and the simulation stops. As understood from the limit magnitude values the F-PID combination is more sensitive to pulse disturbance. The other two combinations show more robust behavior. Among the two of them E-LQR combination is more robust. When the magnitude of the pulse is 4.2 E-SVF combination loses its equilibrium in the upright balanced position and the pendulum falls down but it achieves the upright equilibrium after one swing and maintains stability as seen from Figure 5.22. There is no such problem in E-LQR combination it keeps the upright equilibrium and overcome the effect of the disturbance as seen in Figure 5.23.

6. CONCLUSIONS

In this thesis, the focus is to compare various control techniques to swing up and balance the inverted pendulum in a given sinusoidal trajectory of the cart. The control process is non-linear and unstable in open loop. Furthermore, the process is divided into two sub-processes as swing up and balance; each job should be done by its own controller and there must be a switch mechanism between the jobs. Two control schemes are implemented for swing up process; a fuzzy logic based controller and an energy based controller. Similarly there are three control schemes for the balancing process; a state variable feedback controller, an LQR controller and a PID controller. Three selected combination of control schemes are implemented on the system by using the real time feature of the MATLAB/Simulink software. The experimental results are analyzed and compared.

The energy based controller is well known in the literature and actually built in with the set up used in this thesis work but several improvements are done on the system with inspiration from the related references. Fuzzy logic based controller is implemented by defining fuzzy inputs and outputs for the pendulum system and determining the membership functions for each. The fuzzy logic toolbox of the MATLAB is used for this purpose.

The state variable feedback controller is also built in with the system. An optimum version of the state feedback controller named as LQR is implemented by re-modeling the system for missing model of the electrical part and getting the linear model of the system. PID controller is also implemented for the balancing by following a trial and error methodology for the adjustment of the parameters of the controller.

Three combinations of the control schemes are chosen to control whole system as

- Energy-state variable feedback (E-SVF)
- Energy-LQR (E-LQR)
- Fuzzy-PID (F-PID)

In order to compare these three combinations two different tests is applied on the system for each combination. The first one is based on the frequency of the sinusoidal input signal. By changing the signal, the performance of each combination is observed. The second test based on the disturbance effect. The performance of each control combination is investigated using a pulse disturbance and by increasing the magnitude level of the pulse disturbance. The robustness characteristics of the combinations are observed.

For the robustness to the frequency change, F-PID has the most robust behavior, the tracking and balancing is properly maintained at each frequency. Among E-SVF and E-LQR, E-SVF shows a very sensitive response to frequency change the tracking is not achieved properly. On the other hand E-LQR is less sensitive. This result can be predicted because of the optimal characteristics of the LQR controller.

In the second test, the performances of the combinations are different than in the first test. When the system is given a pulse disturbance E-SVF and E-LQR have higher capability to overcome the disturbing effect of the pulse signal. Whereas, F-PID is sensitive to this effect. The magnitude of the pulse is increased to see how robust each system is. Among E-SVF and E-LQR, E-LQR is more robust than the E-SVF as in the frequency difference robustness, because of the optimal behavior of the LQR controller.

In conclusion, considering the inverted pendulum as one of the simplest problems in robotics is that of controlling the position of a single link using a steering force. Pole-balancing systems are also impressive demonstration models of missile stabilization problems. So the pendulum experiment is a good representation of some of the industrial process control problems. There are there different methodologies proposed in this study. Every methodology has its own advantages and merits according to various combination of system configuration. In other words there are three different solutions for one problem. According to the trade offs of the system user, a methodology can be preferable and be alternative to others. For a fast process which means that the frequency of the system is big, F-PID is a good choice to track the trajectory. On the other hand if the system is open to outside disturbing effects E-SVF or E-LQR is more suitable choices. Knowing the desired performance criterion enables the right choice.

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