

TRAJECTORY OPTIMIZATION OF A TACTICAL MISSILE BY USING
GENETIC ALGORITHM

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GENETIC ALGORITHM**

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ABSTRACT

TRAJECTORY OPTIMIZATION OF A TACTICAL MISSILE BY USING GENETIC ALGORITHM

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In this thesis, estimation of an optimal trajectory for a tactical missile is studied. Missile guidance algorithm is developed to achieve a desired mission goal according to some performance criteria and the imposed constraints. Guidance algorithms may include trajectory optimization to shape the whole trajectory in an optimal way, so that the desired performance needs such as maximum impact velocity, minimum time-of-flight or specific crossing angles can be satisfied. By performing missile path planning, an optimized trajectory helps to achieve missile mission with required performance criteria.

In order to optimize missile trajectory, various optimization algorithms can be utilized. In this thesis, Genetic Algorithm will be focused on mainly. Efficiency of gradient based optimization algorithms are also studied. Waypoints that missile must visit are taken as control parameters of the algorithm. A hypothetical missile is modeled for the analyzes.

Trajectory optimization is performed based on two cases. The first one is aimed to

reach an air target with maximum velocity and minimum flight time. In the second one, achieving a desired impact angle with maximum velocity against a stationary ground target is the purpose. Results are compared with reference models which uses conventional guidance algorithms.

Optimization algorithms are run offline initially and after that with the insights obtained, lookup tables are created for real time use in missile guidance. Scenario parameters are the inputs of lookup tables, and by interpolating the waypoints corresponding to these parameters, waypoints that the missile must visit can be obtained. This allows the missile trajectory to be improved without running the optimization algorithm in each scenario.

Keywords: Genetic Algorithm, Trajectory Optimization, Missile Guidance, Missile Flight Mechanics

ÖZ

TAKTİK BİR FÜZENİN GENETİK ALGORİTMA İLE YÖRÜNGE ENİYİLEMESİ

Özdil, Baran Dilan

Yüksek Lisans, Havacılık ve Uzay Mühendisliği Bölümü

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Bu tezde, taktik bir füze için optimal bir yörünge planlaması çalışılmıştır. Bazı performans kriterlerine ve uygulanan kısıtlamalara göre istenen görev hedefine ulaşmak için füze güdüm algoritmaları kullanılmaktadır. Güdüm algoritmaları ile istenen performansa ulaşabilmek için yörünge optimizasyonu yaygın bir yöntemdir. Böylece maksimum son hız, minimum uçuş süresi veya belirli çarpma açısı gibi istenen performans ihtiyaçları karşılanabilmektedir. Füze yörünge planlaması ile optimize edilmiş bir yörünge, gerekli performans kriterlerini sağlayarak füze görevini gerçekleştirmeye yardımcı olmaktadır.

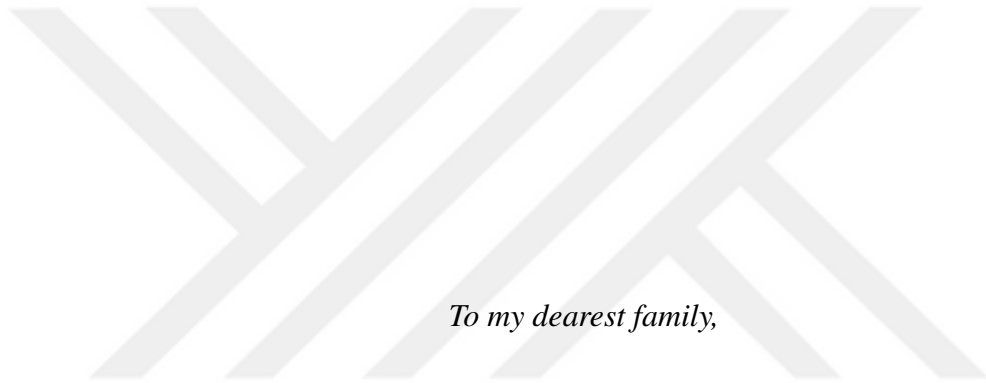
Füze yörüngesini optimize etmek için çeşitli optimizasyon algoritmaları kullanılabilir. Bu tezde Genetik Algoritma esas olarak ele alınacaktır. Gradyan tabanlı optimizasyon algoritmalarının da füze yörünge optimizasyonu üzerinde etkinliği incelenmiştir. Füzenin ziyaret etmesi gereken yol noktaları, algoritmanın kontrol parametreleri olarak alınmıştır. Analizler için bir varsayımsal füze modellenmiştir.

Yörünge optimizasyon problemi iki senaryo üzerinde ele alınmıştır. Bunlardan birin-

cisinde maksimum hız ve minimum uçuş süresi ile bir hava hedefine ulaşmak amaçlanmıştır. İkincisinde ise sabit bir yer hedefine karşı maksimum hız ile istenilen bir çarpma açısının elde edilmesi amaçlanmıştır. Sonuçlar geleneksel güdüm algoritmalarını kullanan referans modellerle karşılaştırılmıştır.

Optimizasyon algoritmaları çevrimdışı koşturulmuş olup, elde edilen öngörülerle füze güdümünde gerçek zamanlı kullanılmak üzere tablolar oluşturulmuştur. Senaryo parametreleri tablonun girdileridir ve bu parametrelere karşılık gelen yol noktaları interpolate ederek, füzenin ziyaret etmesi gereken yol noktaları elde edilebilmektedir. Bu sayede, her bir senaryoda optimizasyon algoritmasını çalıştırmadan füze yörüngesinin iyileştirilmesine olanak sağlanmıştır.

Anahtar Kelimeler: Genetik Algoritma, Yörünge Optimizasyonu, Füze Güdümü, Füze Uçuş Mekaniği



To my dearest family,

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LIST OF ABBREVIATIONS

dof	Degree of Freedom
BPPN	Biased Pure Proportional Navigation
CGM	Conjugate Gradient Method
GA	Genetic Algorithm
PNG	Proportional Navigation Guidance
PSO	Particle Swarm Optimization

LIST OF SYMBOLS

a_n	Acceleration Command
C_x	Static Force Coefficient in x Direction
C_y	Static Force Coefficient in y Direction
C_z	Static Force Coefficient in z Direction
h	Altitude
J	Cost Function
F_{aero}	Aerodynamic Force
F_{grav}	Gravitational Force
F_{prop}	Propulsive Force
F_{tot}	Total Force
M_{ter}	Terminal Mach number
m	Mass
m_p	Propellant Mass
m_t	Total Mass
N	Effective Navigation Ratio Constant
R	Range
S	Aerodynamic Reference Area
T	Thrust
t	Time
t_b	Burnout Time
t_f	Total Flight Time
q	Dynamic Pressure
V_c	Closing Velocity

V_m	Missile Velocity
α	Angle of Attack
θ	Pitch Angle of Missile
γ	Flight Path Angle





CHAPTER 1

INTRODUCTION

1.1 Objective of the Study

The operational success of a tactical missile depends on determinative decisions and trade-offs according to missile mission. Proper modelling of the missile and determination of parameters that will directly affect the optimal behavior of the missile should be considered in the beginning of the missile initial design phase.

For a tactical missile, determining a proper flight path has a significant impact on the accomplishment of the missile mission. Trajectory planning of a missile is an efficient way for missile guidance to evaluate the optimum missile trajectory. For the operational effectiveness of the system, trajectory optimization should be considered not only during the operations, but since the beginning of the conceptual design phase, so that the trade offs for the missile design can be achieved more efficiently.

Basically, missile guidance algorithm is developed to achieve a desired mission goal according to some performance criteria and the imposed constraints. Guidance algorithms may include trajectory optimization to shape the whole trajectory in an optimal way, so that the desired performance needs such as maximum impact velocity, minimum time-of-flight or specific crossing angles can be satisfied. Considering the missile path planning, an optimized trajectory helps to achieve missile mission with required performance capability.

Trajectory optimization of a tactical missile is a challenging issue. Many different optimization algorithms have been studied so far in this area. In this thesis, in order find an optimum path for the missile, various optimization methods are investigated,

however mainly focused on Genetic Algorithm. In recent years, Genetic Algorithm is a remarkably used method in the optimization problems due to its efficiency in wide search areas to find global optimum. The method is inspired by genetics and evolution of the species in nature.

The main purpose of this thesis is to propose an optimization tool by using Genetic Algorithm. Thus, important insights about missile guidance design can be obtained. Once the optimal trajectories are achieved, they are embedded into guidance algorithm which makes the missile to be capable of optimize trajectories in the operational use also.

1.2 Literature Review

In the trajectory optimization problem, the purpose is to maximize or minimize the objective function of the system while satisfying the flight constraints and also by considering the path boundaries. Many different numerical optimization methods are used in the optimization of aircraft trajectories in order to achieve a certain performance criteria or improve the current performance status.

Trajectory optimization can be treated as an optimal control problem. Optimal control solutions may contain direct or indirect methods. As it is stated in the study of Manickavasagam [2], in the indirect methods, solution is based on boundary value problem solving the optimal control state equations, boundary conditions, adjoint equations, control equations, transversality conditions and locates the roots of necessary conditions. Compared to direct methods, indirect methods has greater accuracy. On the other hand in direct methods, the state and control histories are parameterized. The minimum of the objective function is obtained without dealing with adjoint equations, control equations or boundary conditions[2]. Direct methods are superior in convergence properties when compared to indirect methods.

As it is stated in the study of Shippey in 2008[3], Nonlinear programming is a commonly used method for trajectory optimization problem. NLP uses gradient information of the problem and is often good at convergence and accuracy issues. However one problem with NLP is due to the need of gradient information, the method also

requires initial guess of the parameters. A poorly made initial guess may cause convergence to a wrong global optimum. Hence in order to overcome this, alternative approaches for trajectory optimization problem such as evolutionary programming is proposed.[3].

Heuristic algorithms are also used in the field of trajectory optimization. Among these, Genetic Algorithm is a widely used method. Genetic Algorithm was developed by Holland in 1970s. Holland's purpose is to study the phenomenon of adaptation as it occurs in nature and impose this phenomenon into computer algorithms [3]. Since then, genetic algorithm is developed and used in many optimization problems.

In the study of Cribbs carried out in 2004 [4], GA is used in two optimization problems. In the first one, the purpose is to achieve maximum closing velocity against a maneuvering target. To achieve this, line of sight rate bias based on range-to-go is used as a control parameter. For the second case, crossing angle in the intercept is optimized due to the needs for the intercept effectiveness. Control parameter is chosen as line of sight rate bias again.

In 2017, a study was carried out by Zandavi [5], about path planning by using Genetic and PSO Algorithms. This study investigates the path planning by using heuristic algorithms, genetic algorithm and particle swarm optimization in the design of midcourse phase of the missile. Both methods use design variables based on pitch programming and compares the results to obtain the maximum range and optimal height.

There are some other studies which uses hybrid algorithms for trajectory optimization. In 2013, Wang and Dong [6] carried out a study "Fast Intercept Trajectory Optimization for Multi-stage Air Defense Missile Using Hybrid Algorithm" which combines particle swarm optimization and sequential quadratic programming in order to find an optimal path with maximum terminal velocity. The proposed hybrid algorithm uses PSO as a global optimizer. Since PSO has poor accuracy and SQP needs to have a good initial guess to perform its better accuracy characteristic, initial guess comes from the global optimizer PSO algorithm and SQP enhances the accuracy in later phase.

Yokoyama and Suzuki [7] performed a study in 2005 namely "Flight Trajectory Optimization Using Genetic Algorithm Combined with Gradient Method". They claimed that gradient based optimization algorithms are sensitive to initial solutions due to the nonlinear nature of the problem, while convergence characteristics of genetic algorithm is not good as gradient based methods. Hence proposed algorithm combines genetic algorithm with gradient approach. While Genetic Algorithm is used to provide initial solutions, gradient based algorithm deals with finding local minimum.

In the thesis of Soyloğlu [8], "Missile Trajectory Optimization Using Genetic Algorithm", two problems are studied. These problems are based on maximum range and minimum flight time with a specific range. Angle of attack values are used as control parameters that defines the trajectories. The effects of the parameters of Genetic Algorithm such as population size, number of genes, crossover and mutation probabilities in the trajectory optimization problem are also studied.

In 2015, a study is conducted by Manickavasagam, Sarkar and Vaithyanathan [2] namely, "Trajectory Optimisation of Long Range and Air-to-Air Tactical Flight Vehicles". Trajectory optimization problem is handled in two scenarios. First, maximum range problem is studied by taking steer programming as control input. In the second case, minimum flight time for specified range and specified air-to-air engagement scenario is achieved by using pitch lateral acceleration as control input. In both problems, nonlinear programming is used for the optimization tool.

In the study carried by Kung and Chen [9] namely, "Missile Guidance Algorithm Design Using Particle Swarm Optimization", a pursuit-evasion optimization problem is studied by using PSO. As it is mentioned in this paper, while solving optimal feedback guidance law, artificial intelligence methods are very efficient in complex, nonlinear and dynamic pursuit evasion problems. Also, combination of Neural Networks and classical PN guidance laws are used to improve the intercept performance. In this paper it is also stated that, PSO algorithm has advantages in missile guidance algorithm due to the fast convergence characteristic of the algorithm and without the needs of being differentiable and continuous for objective function and constraints. The algorithm aims to find optimal guidance commands. Results are promising by means of miss distance, time of flight and pursuit capability performances.

Tekinalp and Arslan [10] performed a study about trajectory optimization by using direct collocation and nonlinear programming. In this study, an air to surface missile midcourse trajectory is optimized. The purpose is to maximize the range and the control parameter is taken as angle of attack.

Another study [11] namely "Missile Guidance Design Using Optimal Trajectory Shaping and Neural Network" is conducted by Jan, Lin, Chen, Lai and Hwang. In order to intercept and hypersonic threat, it is proposed to combine optimal trajectory shaping guidance to maximize final speed of midcourse at lock-on and neural networks with PNG in the terminal phase to minimize the tracking error.

Vavrina and Howell [12] carried out a study in 2018, namely "Global Low-Thrust Trajectory Optimization through Hybridization of a Genetic Algorithm and a Direct Method". The purpose is to obtain optimal low-thrust trajectories . A hybrid algorithm which combines the genetic algorithm for global search and direct gradient-based method for robust convergence is used.

1.3 Contribution

The contributions that this thesis will provide are as follows:

- Missile trajectory optimization problem will be studied by considering different optimization algorithms. Genetic Algorithm as an evolutionary method and Gradient based optimization algorithms are investigated, also combination of these methods for trajectory optimization will be discussed depending on a specific optimization mission.
- Waypoints on the trajectory are used as control variables of optimization problems. Effect of waypoint numbers are investigated also.
- Compared to other studies in the literature, missile trajectory optimization is achieved by controlling a few number of waypoints rather than dealing with several parameters that create a missile trajectory such as angle of attack, pitch angle etc.

- With the insights about choosing right algorithm and waypoint number, trajectory optimization problem will be discussed based on two main scenarios. Intercepting an air target with maximum terminal velocity and minimum flight time and hitting a ground target with a specific impact angle will be two different objectives in trajectory optimization.
- Look up tables based on several target points are generated to be embedded into the missile model so that the optimization algorithm will work online in operational use.

1.4 Scope

In the following sections, the methodology for the trajectory optimization of a tactical missile is described in Chapter 2 which contains the general information about Genetic Algorithm, Gradient Based Methods, missile modeling, and the application of Genetic Algorithm and a Gradient Based Algorithm in trajectory shaping. In Chapter 3, results to obtain optimum missile path based on two main scenarios are presented and discussed. Finally in Chapter 4 conclusion for the problem is stated and possible future work is mentioned.

CHAPTER 2

METHODOLOGY

As described in the previous chapter, there are different optimization algorithms available to solve trajectory optimization problem of a missile. In this chapter, different optimization algorithms are introduced as an optimization method for the trajectory optimization of a tactical missile. Effects of these methods in a specific optimization problem are studied. Also, modeling of the tactical missile is explained in the following sections.

2.1 Genetic Algorithm

Genetic Algorithm is a global optimization and search method which is a field of evolutionary computation derived from genetics and natural selection. Genetic Algorithm mimics the behavior of the species in the nature. As it can be observed from the nature itself, every species are tend to adapt themselves to the surrounding environment by evolving. Like this biological analogy, the idea behind the Genetic Algorithm is based on the "*survival of the fittest*" theory.

In order to introduce Genetic Algorithm, the most common terms are described as follows:

- **Fitness Function** is the function that evaluates how close a given solution to the desired solution of the problem. It actually determines how fit a solution is. According to Kinnear, the editor of *Advances in Genetic Programming*, [13] "fitness function is the only chance that you have to communicate your intentions to the powerful process that genetic programming represents. Make

sure that it communicates precisely what you desire.”

The fitness function should be created such that, each solution is awarded with a score to indicate how close it is to the desired solution. Also the function should be easy to implement to the problem.

- **Gene** is the term for the parameters that create the particular solution, which means chromosomes are made up of genes.
- **Chromosome** refers to a particular proposed solution in a population that the genetic algorithm solves. Chromosomes are usually represented in binary as strings. However, other encodings are also possible depending on the optimization problem.
- **Population** is the term used for all of the proposed solutions of the current generation. It is the subset of all candidate solutions of a given problem.
- **Generation** refers to the full set of the solutions for the one iteration of the algorithm.

Genetic Algorithm starts with creating a random population of chromosomes in the problem space. Although the individuals are selected random initially, the process continues with a guided search by using historical information of the previous populations. By obtaining a measure of goodness for the chromosomes from fitness function, a selection criteria is built to transmit the genetic information of the fittest individuals to the next generations. Algorithm evolves through the iterations by the composition of selection, crossover and mutation processes until the certain optimization criteria is achieved. In Figure 2.1 evolutionary steps of the Genetic Algorithm is represented.

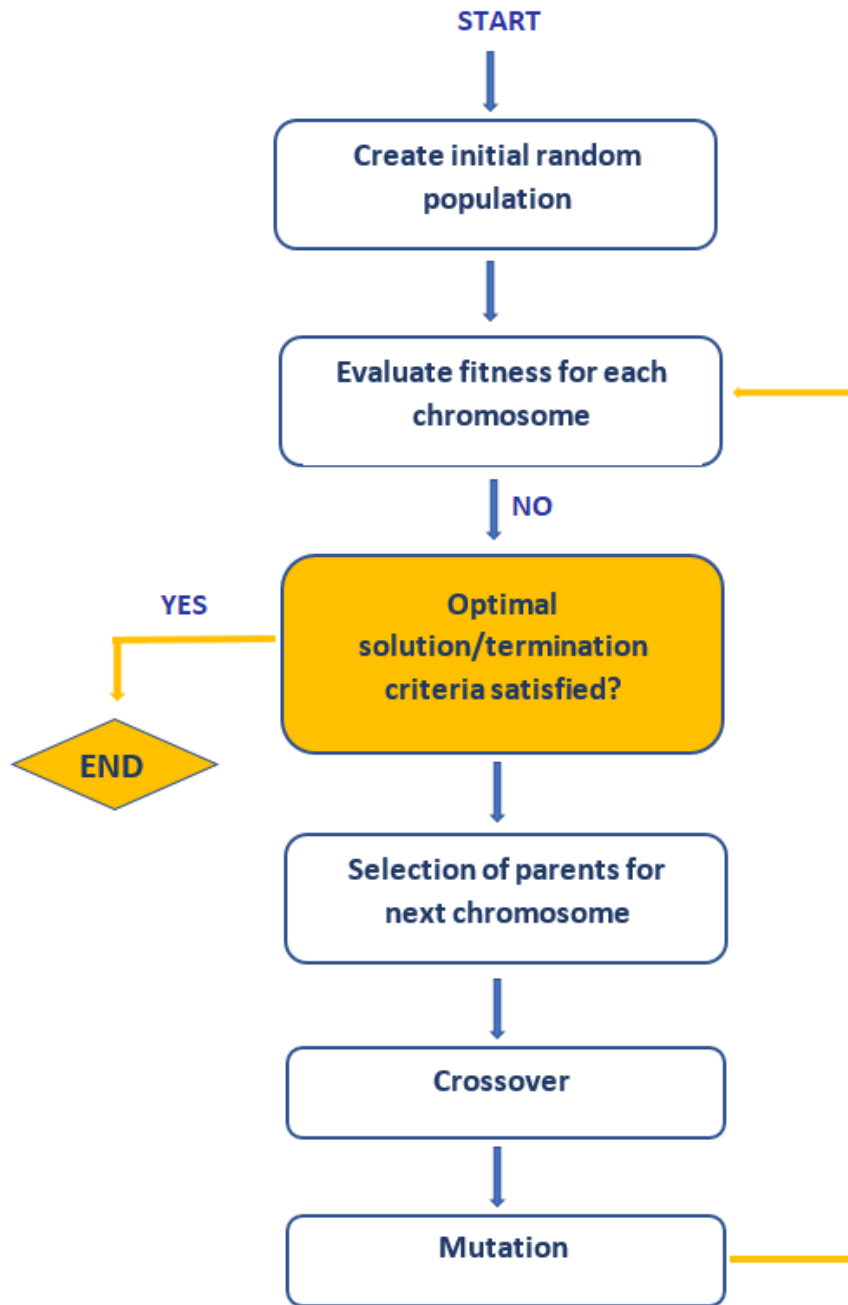


Figure 2.1: Genetic Algorithm Flowchart

There are several advantages of Genetic Algorithm to be applied for an optimization problem. It can be easily implemented in both constrained and unconstrained problems due to its derivative-free feature and not requiring to deal with much algebra. Genetic Algorithm is also applicable to any kind of problem like with a linear or

nonlinear, discrete or continuous character. And thanks to the evolutionary operators, Genetic Algorithm is highly effective in finding global optimum of the problem.

2.1.1 Selection

Selecting parents which will mate and recombine to create offsprings is usually regarded as the first operator of the Genetic Algorithm. In this process chromosomes are ranked according to their fitness values in order to decide whether they are fit enough to survive and to be sufficient enough for being a parent for the next generation. The individuals whose fitness values are higher relative to the population's average fitness value have the higher chance to proceed to the next generation. In order to perform selection process, a mating pool is built initially. Mating pool consists of the certain amount of individuals by taking into account that the individuals with higher fitness values will take more place to mate.

Selection process is highly important since selecting good parents drive the algorithm to a better convergence. There are different selection methods such as Roulette Wheel Selection, Stochastic Universal Sampling, Tournament Selection, Rank Selection, Random Selection etc.

2.1.2 Crossover

The crossover operator is actually analogous to the reproduction process in biology. This process is a significant part of the algorithm and distinguished factor from other optimization methods. Crossover process is used to recombine the genetic materials of two individuals. These individuals are selected by using the selection operator. For each pair of individuals, a random crossover point is chosen within the genes. After that the genetic material before this crossover point is copied from one parent and the rest copied from the second parent chromosome. The two new offspring created are added to the next generation of the population. In the following Figure 2.2 single point Crossover process is illustrated.

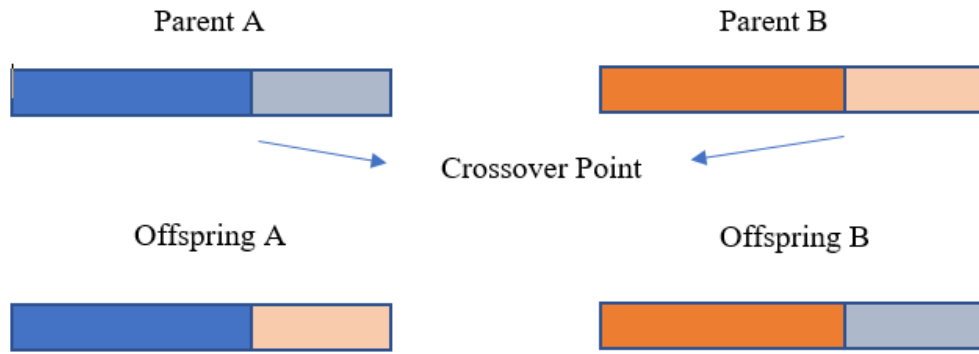


Figure 2.2: Crossover

Apart from the single point method, there are other crossover ways such as multipoint crossover or uniform crossover etc.

2.1.3 Mutation

After crossover process mutation takes place, mutation is the operator which makes random changes in chromosomes. Mutation can be achieved by altering the genes in a chromosome according to mutation probability. The mutation operator provides diversity in the population whenever it starts to repeats itself because of the repeatedly crossover and selections processes. It is important to choose a proper mutation probability in the algorithm. If the mutation probability is too low, there may be many genes which have been useful for the solution but cannot be discovered, also if the mutation probability is too high, the offsprings begin to lose the similarity with the parents which prevents to obtain the historical gains through the generations. Mutation also plays an important role to prevent being trapped in local minima and also it prevents the losses of the genetic material.

2.2 Gradient Based Optimization

Gradient based algorithms utilize the derivatives of the functions in optimization. Gradient based algorithms uses line search which provides a direction that helps to reach a better point in multidimensional space for the objective function. The basic

working principle of the Gradient Based Algorithms is depicted in Figure 2.3

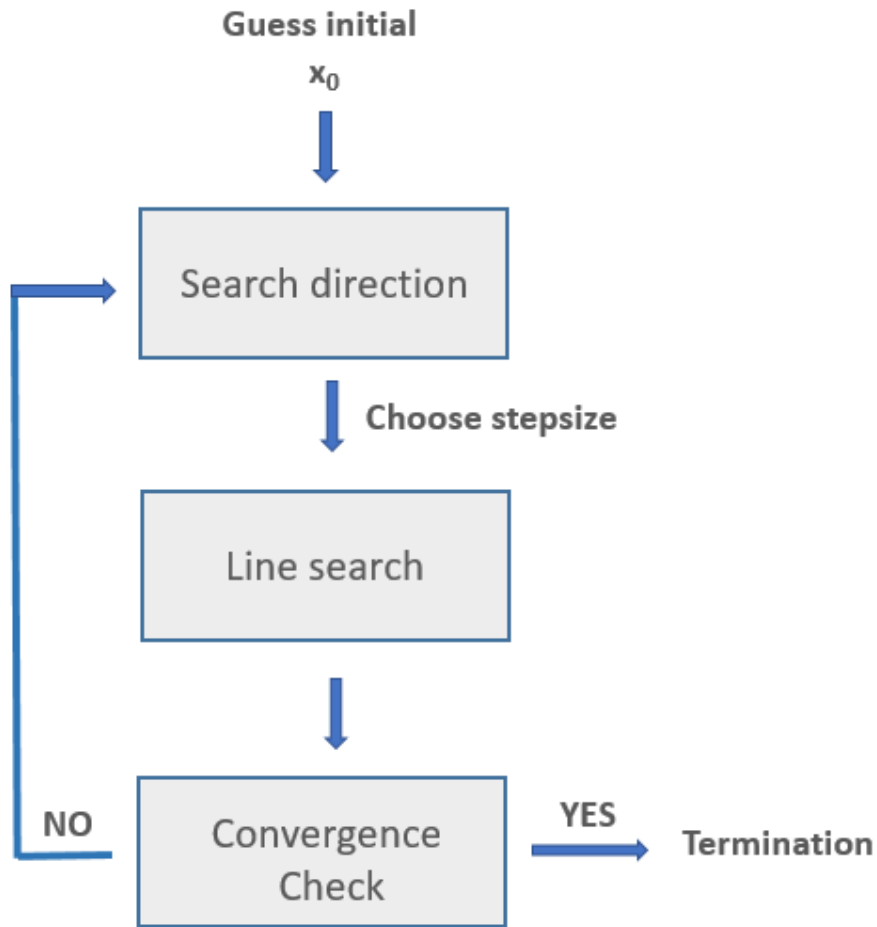


Figure 2.3: Gradient Based Algorithms Flowchart

The optimization process starts with assigning an initial value x_0 and then continues with computing a search direction. Depending on the method, search direction can be based on first or second derivatives. Search direction should be such that it provides a sufficient amount of decrease in the cost function. Iterations proceed until a certain optimization criteria is achieved. For an unconstrained gradient based optimization problem, most algorithms use the following formulation.

$$x_{i+1} = x_i + \alpha_i s_i \quad (2.1)$$

where x is the state variable, i is the iteration number, α_i is stepsize and s_i is the search direction. The updated state variables become the initial condition of next iteration.

2.2.1 Conjugate Gradient Method

Conjugate Gradient Method is one of the optimization algorithms which utilizes the gradient information in line search. An important modification that the method has the algorithm uses not only current gradient information, it combines gradient history of previous step with current information. This modification provides an improvement in convergence rate when compared to other gradient based algorithms, also without dealing with second derivatives.

$$x_1 = x_0 + \alpha s_0 \quad (2.2)$$

$$s_0 = -\nabla f(x_0) \quad (2.3)$$

$$s_1 = -\nabla f(x_1) + s_0 \frac{\nabla^T f(x_1) \nabla f(x_1)}{\nabla^T f(x_0) \nabla f(x_0)} \quad (2.4)$$

$$s_{i+1} = -\nabla f(x_{i+1}) + s_i \frac{\nabla^T f(x_{i+1}) \nabla f(x_{i+1})}{\nabla^T f(x_i) \nabla f(x_i)} \quad (2.5)$$

$$x_{i+1} = x_i + \alpha s_{i+1} \quad (2.6)$$

In this thesis, objective function is based on finite values comes from simulation data. Hence it appropriate to take the derivatives numerically in gradient calculations.

Let $y = f(x)$,

$$h = x_{i+1} - x_i = x_i - x_{i-1}$$

Therefore, numerical differentiation of $f(x)$ can be as follows:

$$\nabla f(x_i) = \frac{y_{i+1} - y_{i-1}}{2h} \quad (2.7)$$

2.3 Mathematical Modeling

In this section a mathematical model for a generic tactical missile is presented. Flight vehicles can be modeled in different fidelity levels. When the complexity of the missile itself and its subsystems are considered, fidelity level is a critical cost and time driver of the model to be used. Hence it is important to choose the right fidelity level according to problem characteristics and needs.

2.3.1 Mathematical Modeling of a Tactical Missile

Mathematical modeling of the tactical missile is defined with equations of motion by considering the aerodynamic, propulsion and mass properties of the vehicle, also the method used for the guidance is mentioned in this section.

Pseudo 5 dof simulation model is chosen to model the missile that is used in the trajectory optimization problem. Zipfel's study [14] about Pseudo 5 dof simulation states that, missile is modeled by 3 translational dynamics incorporating with pitch and yaw rates, so that missile is modeled with two pseudo motions.

In Pseudo 5 dof simulation model, yaw and pitch rates are estimated by using flight path frame kinematic equations while roll rate is discarded since the missile is usually stabilized around zero [15].

In order to establish the missile mathematical model, firstly the forces acting on the missile is represented. Aerodynamic, propulsive and gravitational forces are the main forces that the missile has in flight before burnout phase.

$$F_{tot}^b = F_{aero}^b + F_{prop}^b + F_{grav}^b \quad (2.8)$$

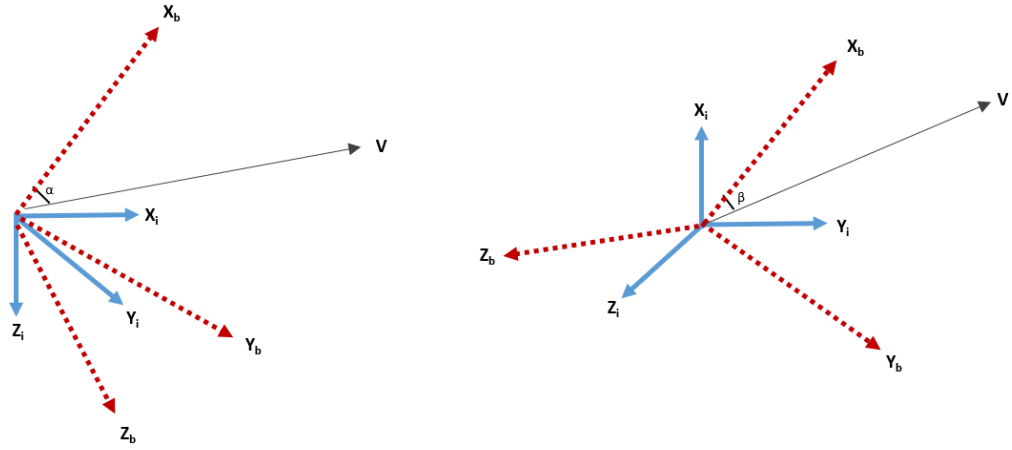


Figure 2.4: Inertial and Body Coordinate Frames

$$F_{aero}^b = qS \begin{Bmatrix} C_x \\ C_y \\ C_z \end{Bmatrix} \quad (2.9)$$

Propulsive forces before burnout phase of flight is given in Equation 2.10.

$$F_{prop}^b = \begin{Bmatrix} T \\ 0 \\ 0 \end{Bmatrix} \quad (2.10)$$

Generally, equations of motion is expressed in body coordinate frame. Gravitational force acts in inertial coordinate frame in +Z axis, so that it is converted into body coordinate frame by using equations 2.11 to 2.13.

$$F_{grav}^b = C^{b,i} F_{grav}^i \quad (2.11)$$

$$C^{b,i} = \begin{bmatrix} c\psi c\theta & s\psi c\theta & -s\theta \\ c\psi s\theta s\phi - s\psi c\phi & s\psi s\theta s\phi + c\psi c\phi & c\theta s\phi \\ c\psi s\theta c\phi + s\psi s\phi & s\psi s\theta c\phi - c\psi s\phi & c\theta c\phi \end{bmatrix} \quad (2.12)$$

$$F_{grav}^b = mg \begin{Bmatrix} -\sin \theta \\ \sin \phi \cos \theta \\ \cos \phi \cos \theta \end{Bmatrix} \quad (2.13)$$

Note that the missile mass is changing linearly with time in burnout phase as it is represented as Equation 2.14. Gravity constant g is calculated by using Matlab WGS84 model based on missile altitude and latitude properties.

$$m = m_t - \frac{t}{t_b} m \quad (2.14)$$

An important simplification that the pseudo 5 model has over 6 dof simulation is modeling of acceleration dynamics with a transfer function. The acceleration response for the guidance commands are represented as transfer function which makes the model simple and easy to implement.

Once the acceleration response is evaluated, the lateral forces can also be obtained. Lateral aerodynamic coefficients can be derived by using equations 2.15 and 2.16

$$ma_y = qSC'_y \quad (2.15)$$

$$ma_z = qSC'_z \quad (2.16)$$

Incidence angles can be evaluated by utilizing $C_y(M, \beta)$ and $C_z(M, \alpha)$ database [15].

As regards the estimation of C_x values, they are obtained from aerodynamics database in trim condition which means C_x only depends on Mach and incidence angles α and β [15]. Once the incidence angles are already obtained from lateral aerodynamic coefficient database, C_x can be estimated.

$$C_x(M, \alpha, \beta) \quad (2.17)$$

The missile is modeled in the Matlab Simulink environment. The model structure is summarized in Figure 2.5:

Table 2.1: Parameters of Hypothetical Missile Model

T	500 kN.s
t_b	10 s
m_t	400 kg
m_f	230 kg

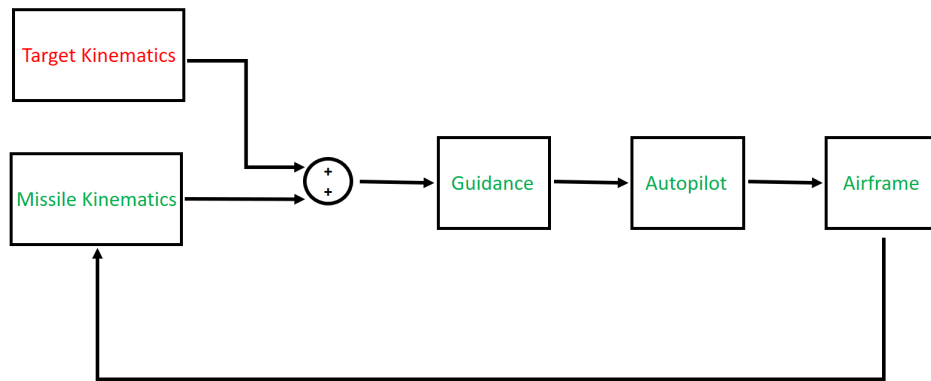


Figure 2.5: Model Structure of Tactical Missile

2.3.2 A Hypothetical Missile Model

For this study, a hypothetical surface to air missile configuration is modeled. The parameters that is used in this configuration is indicated in Table 2.1. In this hypothetical model, missile is assumed to use solid propellant rocket motor and aerodynamic coefficients are obtained by using DATCOM.

During the analyzes in this study, scenario parameters, optimization parameters and methods will change and the effects of these varying parameters are to be investigated. However, tactical missile parameters are kept constant and always be based on the parameters in Table 2.1.

2.3.3 Proportional Navigation Guidance

In the analyses, proportional navigation is used for guidance method. Proportional navigation is a widely used guidance method in homing missiles. The theory behind PNG is to generate acceleration command perpendicular to missile-target line of sight and also proportional to line-of-sight rate and closing velocity[1]. More specifically, if the missile and a non maneuvering target are closing at each other, interception occurs when the line of sight rate is nullified.

According to Zarchan [1], the engagement geometry to be linearized is illustrated in Figure 2.6. In this figure, n_t , n_c , β , λ , L , HE represent target acceleration, missile acceleration, heading angle of target, line-of-sight angle, missile leading angle and heading error respectively.

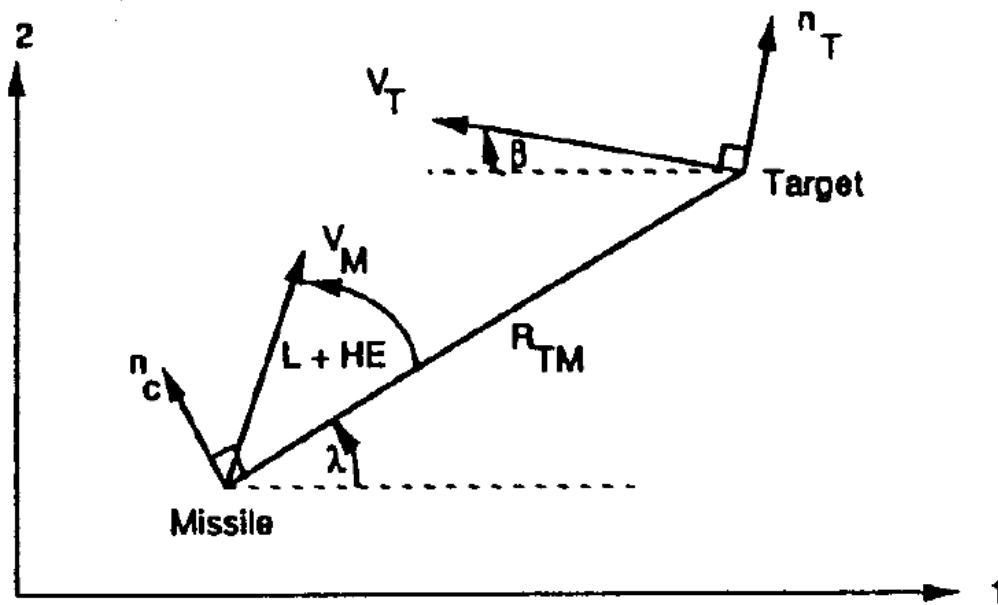


Figure 2.6: Engagement Model for Linearization (Zarchan, 1997)

According to missile-target engagement model, the relation for the commanded acceleration is described as follows:

$$a_n = NVc\dot{\lambda} \quad (2.18)$$

where

a_n is the acceleration command, V_c is the closing velocity, $\dot{\lambda}$ is the line of sight rate, and N is the effective navigation ratio constant.

2.4 Genetic Algorithm for Trajectory Optimization

In this section, implementation of Genetic Algorithm for trajectory optimization of a tactical missile is studied. The desired condition for the problem is to reach the optimum trajectory with maximum terminal velocity and minimum flight time. Missile trajectory optimization is achieved by using waypoint approach. Waypoints are a sequence of points describing the positions that the missile must visit. The missile is guided to waypoints gradually instead of directly being guided to final target point. Location of the waypoints is selected by using Genetic Algorithm.

In this section the Genetic Algorithm structure which is explained in the previous sections is studied in more detail by giving numerical values of the problem also. The algorithm is written in Matlab and also uses Simulink for the missile model. It also follows the structure as it is illustrated in Figure 2.5 In this study, the objective is to find optimum flight path with maximum velocity and minimum flight time for a specified target. Thus cost function of the problem is created as:

$$J = -M_{ter} + k_1 t_f \quad (2.19)$$

where

M_{ter} is the terminal mach number, t_f is the time of flight and k_1 is the penalty constant.

2.4.1 Initialize

Algorithm starts with creating a random initial population. Since the main purpose is to reach optimum missile trajectory with maximum terminal velocity and minimum

Table 2.2: Initial Population

P1x	P1z	Cost
23738.64	14443.17	0.1073
25833.21	8309.88	7292.64
7920.69	12403.64	-0.5276
26007.64	3844.52	12794.45
19544.26	7482.89	6178.60
7243.42	13904.56	-0.4962
11405.45	12298.69	-0.3725
17578.27	14473.40	-0.2215
27022.65	10524.62	4999.46
27192.43	2464.25	14400.61
8625.10	13038.68	-0.5211
27323.63	14141.91	0.2831

flight time by using waypoints, waypoints are represented as genes which create chromosomes. Waypoints are defined in the xz coordinate system since the missile and target motion is assumed to be planar. Therefore, for each waypoint two control variables are defined.

Initial population is represented as a full matrix size of $N_{pop} * N_{var}$. Populations are generated with N_{pop} chromosomes each having N_{var} variables. In our case, there are two waypoints to generate a chromosome and 12 different flight trajectories to create the populations. The initial random population created and corresponding cost values of the chromosomes is represented in Table 2.2:

2.4.2 Selection

In order to decide which chromosomes are selected to reproduce for the next generation, cost values are sorted in increasing order, initial population and corresponding cost values becomes as in the Table 2.3:

After ranking the chromosomes by evaluating their cost values, according to the se-

Table 2.3: Initial Population Sorted According to Cost Values

P1x	P1z	Cost
7920.696775	12403.6461	-0.5276
8625.100879	13038.68098	-0.5211
7243.429315	13904.56183	-0.4962
11405.45903	12298.69528	-0.3725
17578.27494	14473.40154	-0.2215
23738.64479	14443.17033	0.1073
27323.63398	14141.91222	0.2831
27022.65721	10524.62909	4999.46
19544.26266	7482.896674	6178.60
25833.21455	8309.883433	7292.64
26007.64469	3844.522402	12794.45
27192.43631	2464.251821	14400.61

lection rate the fittest chromosomes in the population are kept for the next generation and the rest is discarded. In our problem half of the fittest chromosomes are selected to be in the mating pool.

2.4.3 Crossover

From mating pool, parents are selected random to create new offsprings. There are several crossover techniques depending on how many parts to be divided and exchange the genetic material between parent chromosomes. Crossover points are randomly selected in the parent chromosomes. From previous step, 6 fit chromosomes are selected to be parents as in the Table 2.4:

Among these, p_1 and p_2 vectors represent the randomly chosen indices of the chromosomes from Table 2.4 to mate.

$$p_1=[3 \ 2 \ 4]$$

$$p_2=[5 \ 6 \ 2]$$

Table 2.4: Selected Chromosomes

Chromosome	P1x	P1z
1	7920.696775	12403.6461
2	8625.100879	13038.68098
3	7243.429315	13904.56183
4	11405.45903	12298.69528
5	17578.27494	14473.40154
6	23738.64479	14443.17033

By using index vectors p_1 and p_2 , the parent individuals that will mate are as follows:

Parent₁=Chromosome₃

Parent₂=Chromosome₅

Parent₁=[7243.429315 13904.56183]

Parent₂=[17578.27494 14473.40154]

Parent₃=Chromosome₂

Parent₄=Chromosome₆

Parent₃=[8625.100879 13038.68098]

Parent₄=[23738.64479 14443.17033]

Parent₅=Chromosome₄

Parent₆=Chromosome₂

Parent₅=[11405.45903 12298.69528]

Parent₆=[8625.100879 13038.68098]

After randomly selecting crossover points, variables in between these points are replaced by each other [16]. In our case, individuals are already consist of two genes therefore single point crossover is used. By using this combination approach, vari-

ables in a single offspring can be obtained as follows:

Offspring₁=[Parent₁₁ Parent₂₂]

Offspring₂=[Parent₂₁ Parent₁₂]

Offspring₁=[7243.429315 14473.40154]

Offspring₂=[17578.27494 13904.56183]

Offspring₃=[Parent₃₁ Parent₄₂]

Offspring₄=[Parent₄₁ Parent₃₂]

Offspring₃=[8625.100879 14443.17033]

Offspring₄=[23738.64479 13038.68098]

Offspring₅=[Parent₅₁ Parent₆₂]

Offspring₆=[Parent₆₁ Parent₅₂]

Offspring₅=[11405.45903 13038.68098]

Offspring₆=[8625.100879 12298.69528]

The whole population after crossover process is as in the Table 2.5:

2.4.4 Mutation

With a population size of 12, and 2 variables for each chromosome. There are 24 variables in a population and with a mutation rate of 0.2, the total mutated variables becomes 5. The variables that will experience mutation are selected random again.

The indices in Table 2.6 represent the rows and columns of selected mutated variables. After selecting the indices for mutation, they are replaced by random variables which are in the limits of the corresponding variables.

Population transforms into form as in Table 2.7: after mutation process. The values stated in bold are the ones being replaced by random values in mutation process.

Table 2.5: Population after crossover

P1x	P1z
7920.696775	12403.6461
8625.100879	13038.68098
7243.429315	13904.56183
11405.45903	12298.69528
17578.27494	14473.40154
23738.64479	14443.17033
7243.429315	14473.40154
17578.27494	13904.56183
8625.100879	14443.17033
23738.64479	13038.68098
11405.45903	13038.68098
8625.100879	12298.69528

Table 2.6: Selected Genes for the Mutation

Rows	Columns	Corresponding Variable
4	2	12298.69528
6	1	23738.64479
7	2	14473.40154
9	2	14443.17033
9	1	8625.100879

Table 2.7: Population after mutation

P1x	P1z
7920.696775	12403.6461
8625.100879	13038.68098
7243.429315	13904.56183
11405.45903	3546.96986
17578.27494	14473.40154
16462.3732	14443.17033
7243.429315	14476.67146
17578.2749	13904.5618
18461.15827	6425.01444
23738.64479	13038.68098
11405.45903	13038.68098
8625.100879	12298.69528

As mentioned before, by introducing these mutated genes to the chromosomes, in addition to providing diversity in the solution space, to be trapped in local minimas is also prevented.

This evolutionary processes of the Genetic Algorithm continues after a certain optimization criteria or the maximum iteration number is reached through the generations.

2.5 Conjugate Gradient Algorithm for Trajectory Optimization

In this section, an example iteration process is carried out for the application of Conjugate Gradient Algorithm in missile trajectory optimization.

In Figure 2.7 the working principle of the combination of Genetic and Conjugate Gradient Algorithms is illustrated.

$$x_0 = \begin{bmatrix} P_x \\ P_z \end{bmatrix} \quad (2.20)$$

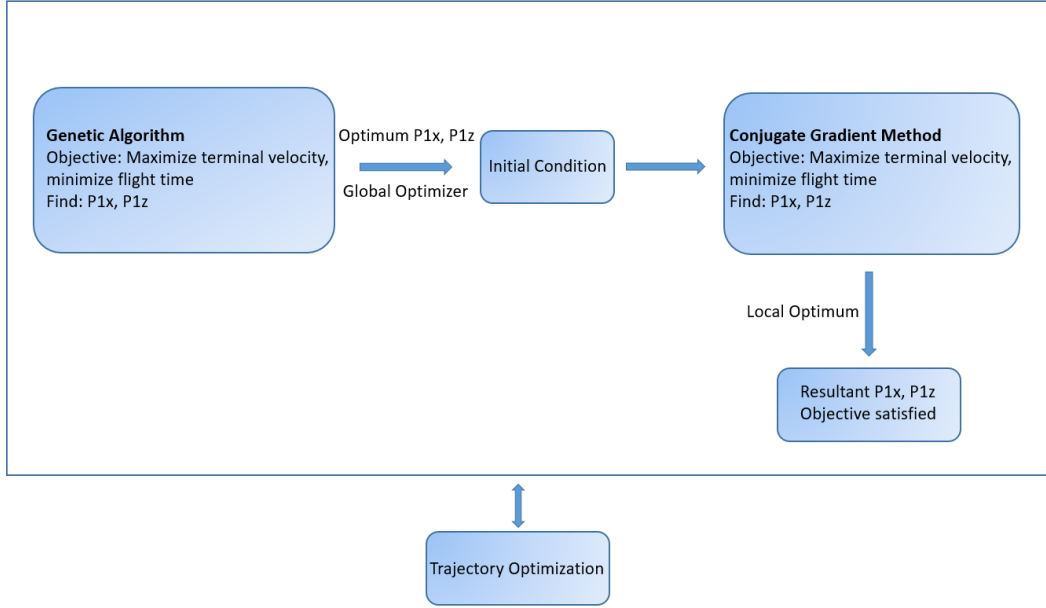


Figure 2.7: Combination of Genetic and Gradient Based Algorithms for Trajectory Optimization

In this problem, waypoint obtained from Genetic Algorithm serves as the initial condition for Conjugate Gradient Algorithm. Let the initial condition are evaluated as follows:

$$x_0 = \begin{bmatrix} 7039.31 \\ 12417.51 \end{bmatrix} \quad (2.21)$$

$$s_0 = - \begin{bmatrix} \nabla f(P_{x_0}) \\ \nabla f(P_{z_0}) \end{bmatrix} \quad (2.22)$$

For the numerical differentiation stated in equation 2.7 h value is selected as:

$$h=5m$$

$$\nabla f(P_{x_0}) = \frac{f(7039.31 + 5) - f(7039.31 - 5)}{10} \quad (2.23)$$

$$\nabla f(P_{z_0}) = \frac{f(12417.51 + 5) - f(12417.51 - 5)}{10} \quad (2.24)$$

$$s_0 = 10^{-6} \begin{bmatrix} -0.1917 \\ 0.9510 \end{bmatrix} \quad (2.25)$$

Let $\alpha=10e6$

$$P_{x_1} = P_{x_1} + \alpha s_0 \quad (2.26)$$

$$P_{z_1} = P_{z_1} + \alpha s_0 \quad (2.27)$$

$$P_{x_1}=7037.397$$

$$P_{z_1}=12408.008$$

$$\nabla f(P_{x_1}) = \frac{f(7037.397 + 5) - f(7037.397 - 5)}{10} \quad (2.28)$$

$$\nabla f(P_{z_1}) = \frac{f(12408.008 + 5) - f(12408.008 - 5)}{10} \quad (2.29)$$

$$\nabla f(x_1) = 10^{-6} \begin{bmatrix} 0.3620 \\ -0.3910 \end{bmatrix} \quad (2.30)$$

By inserting $\nabla f(x_0)$ and $\nabla f(x_1)$ into equation 2.5, s_1 can be obtained as follows:

$$s_1 = 10^{-6} \begin{bmatrix} -0.4198 \\ 0.6779 \end{bmatrix} \quad (2.31)$$

$$P_{x_2}=7033.199$$

$$P_{z_2}=12401.229$$

By repeating the iteration process, after reaching a certain performance criteria or maximum iteration number, Conjugate Gradient Algorithm terminates.

With the insights obtained from Genetic Algorithm as a global optimizer, Conjugate Gradient Method tries to achieve a local search and improve the convergence characteristics.



CHAPTER 3

RESULTS AND DISCUSSION

3.1 Problem Definition

In this thesis, trajectory optimization of a tactical missile is tried to be achieved. Missiles may reach a desired target point by using conventional guidance algorithms, however this algorithms may not always provide the best performance in terms of terminal velocity, time of flight, impact angle etc. There may be various optimization criteria for a missile trajectory depending on the performans needs. In this study, optimization problem is handled by considering two criteria. In the first problem missile is desired to reach target position with a maximum terminal velocity and minimum flight time. In the second one, missile is tried to achieve a specific impact angle with maximum velocity.

As it is stated in Chapter 2, waypoints are considered as control variables which build the trajectory. Selection of waypoints location is the problem to be solved by optimization algorithms. As the first phase of the study, a combination of two methods Genetic Algorithm and Conjugate Gradient Method will be used. Genetic Algorithm is applied to search the global optimum. After that, the obtained solution is used as an initial solution for the Conjugate Gradient Method. An in-depth search process is carried out by Conjugate Gradient Algorithm for the fine tuning of the results obtained by Genetic Algorithm. In the light of this first phase of the study, a comparison can be made among the algorithms to figure out which one is more effective in finding the optimum. The following analyses will take place based on these inferences.

Before implementing the optimization algorithms, in order to understand the effects of waypoints location to an optimized missile trajectory, firstly a brute force is applied

to the problem. After that contour and surface plots which represent the cost values according to waypoint location are created. As an example case, missile trajectory is built by using one waypoint between missile launch position and target position. After launch, missile must visit this waypoint and then steer to the target point. For this example, in order to reach 15000 meters altitude and 30000 meters range with a maximum velocity, the only one waypoint is used between 1000 - 29000 meters in downrange and 1000 - 18000 meters in altitude. By using the intervals of 250 meters, a wide set of waypoint location is tested in order to see the effect in cost function.

In Figure 3.1 and 3.2 the effect of waypoint location in cost function is illustrated. When applying a brute force in a wide search space, there may be some trajectories which can not even get close to the target point, therefore these solutions are not included into the contour plot in Figure 3.1 for an efficient illustration of the solutions. The white region below the contours represents these solutions.

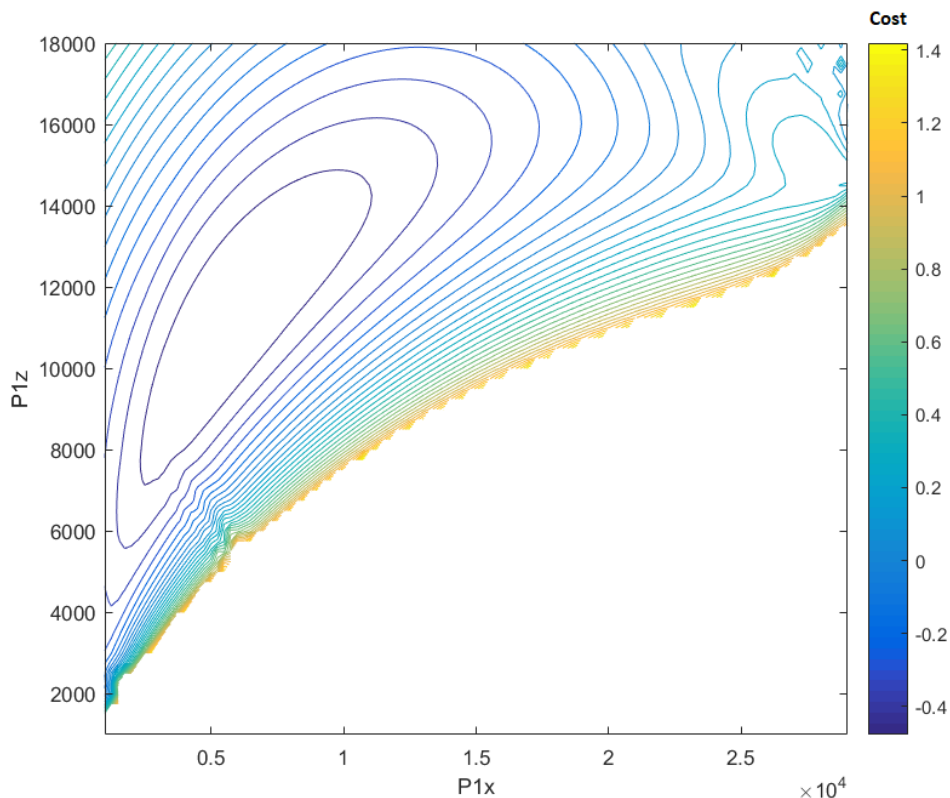


Figure 3.1: Effects of Waypoint Location in Cost Function by Using Brute Force

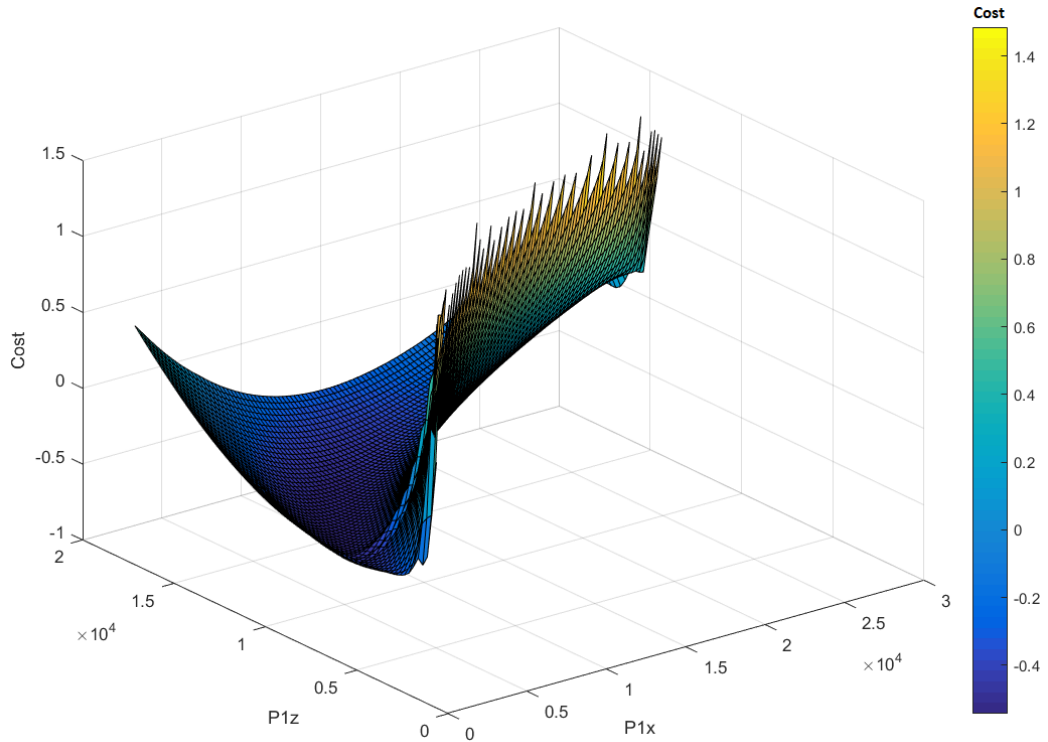


Figure 3.2: Effects of Waypoint Location inon Cost Function by Using Brute Force-2

The final objective is to obtain the waypoint that provides the optimum solution. Optimization algorithms will be used to find the minimum cost in this search space later.

3.2 Results

In this section, results obtained by the combination of Genetic Algorithm and Conjugate Gradient Method will be discussed. A reference initial model is also developed in order to compare the results with the trajectories obtained by conventional guidance algorithms (PNG) which guides the missile to the actual target point directly.

In order to reach the optimum solution, first Genetic Algorithm is used for global search. After that, the results obtained from Genetic Algorithm is used as an initial condition for Conjugate Gradient Algorithm for the fine tuning. The effect of working with these two methods together is examined.

As an example scenario, a tactical missile tries to intercept an air target which is at

Table 3.1: Parameters of Genetic Algorithm

Number of Generations	50
Population size	12
Selection rate	0.5
Mutation rate	0.2

15000 meters altitude and 30000 downrange. The objective is to intercept the target with maximum terminal velocity by minimizing the flight time also. The trajectory of the missile is tried to be optimized by using one waypoint. The effect of number of waypoints which the missile must visit is examined in the following sections.

3.2.1 Genetic Algorithm

In this trajectory optimization problem, Genetic Algorithm is used to provide an initial condition for the Conjugate Gradient Algorithm. The cost function for the trajectory is as in the stated in Section 2.4

$$J = -M_{ter} + k_1 t_f \quad (3.1)$$

Where k_1 is the penalty coefficient for the cost function. The Algorithm parameters are summarized in Table 3.1.

After 50 iterations, change in the cost function is demonstrated in Figure 3.3. The resultant waypoint locations and the final cost value are indicated in Table 3.2.

Since the solution obtained by GA will be improved by CGA, number of iterations is kept intentionally low. By this way, computation burden of the combination method can be reduced.

As it can be observed from Table 3.3 and Figure 3.4, waypoints obtained by Genetic Algorithm improves the missile trajectory greatly by means of terminal velocity and time of flight.

Table 3.2: Genetic Algorithm Results

Cost	-0.53628
P1x (m)	7039.31432
P1z (m)	12417.5189
Elapsed Time (s)	2698.591

Table 3.3: Comparison of GA and Initial Model

	Optimized Model (GA)	Initial Model (PNG)
Mach terminal	1.626	0.99
Time of flight (s)	54.48	55.65

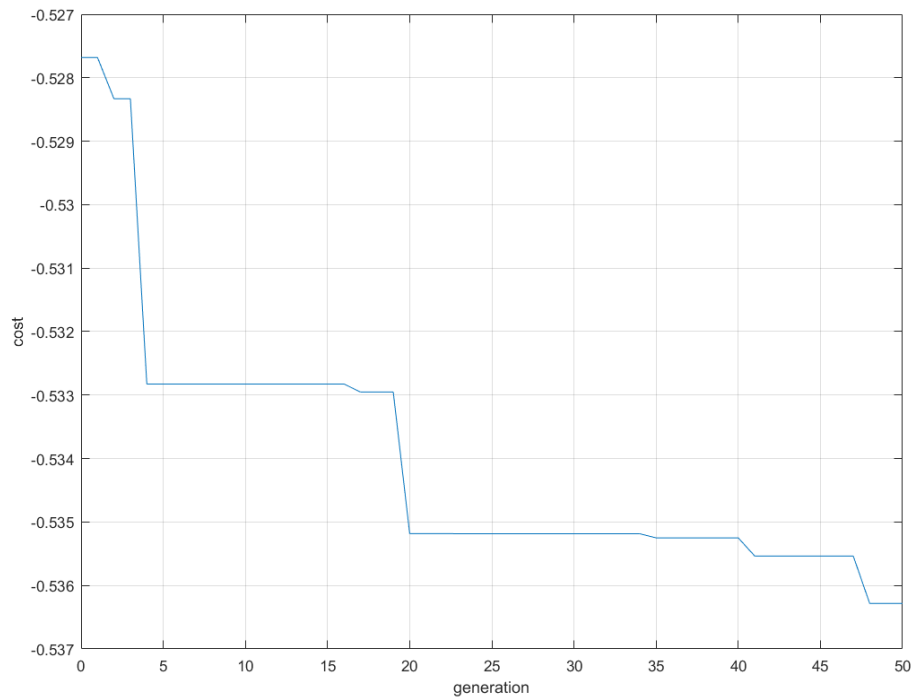


Figure 3.3: Cost function with respect to generations

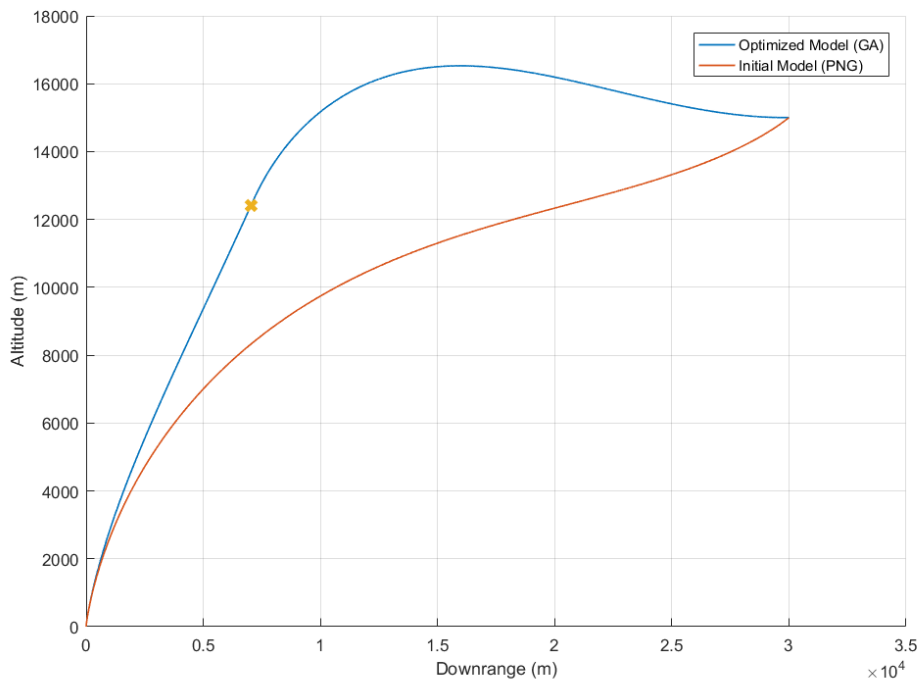


Figure 3.4: Trajectories Obtained by Genetic Algorithm and the Initial Model

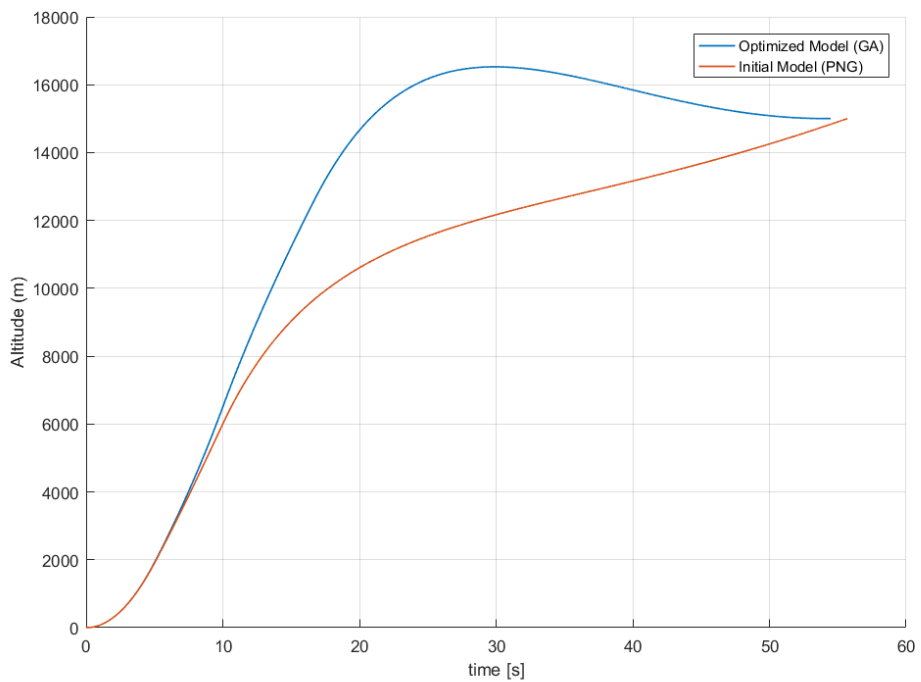


Figure 3.5: Altitude of Missile

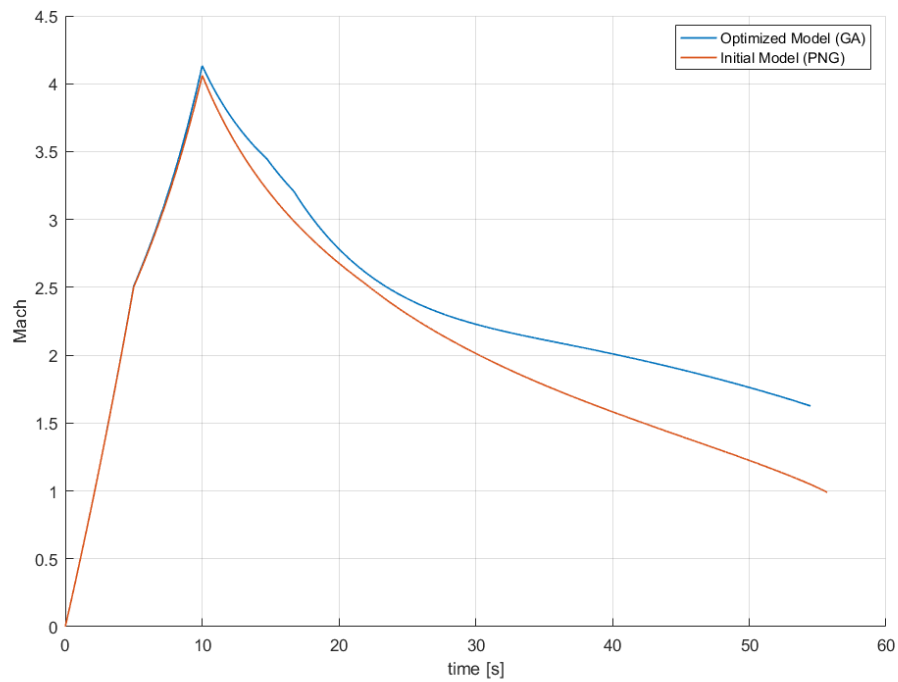


Figure 3.6: Mach Obtained by Genetic Algorithm and Initial Model

3.2.2 Conjugate Gradient Method

Since the Gradient Based Optimization Algorithms are superior in local searching, Conjugate Gradient Based Algorithm is used to improve the accuracy of the solution found by Genetic Algorithm.

With the initial conditions obtained from Genetic Algorithm, Conjugate Gradient Method is applied to the trajectory optimization problem. The results obtained by the combination of these two methods are stated in Table 3.4

Table 3.4: Results Obtained by Conjugate Gradient Method

Cost	-0.5429
P1x (m)	5925.595
P1z (m)	11384.658
Iteration number	207
Elapsed Time (s)	3995.225880

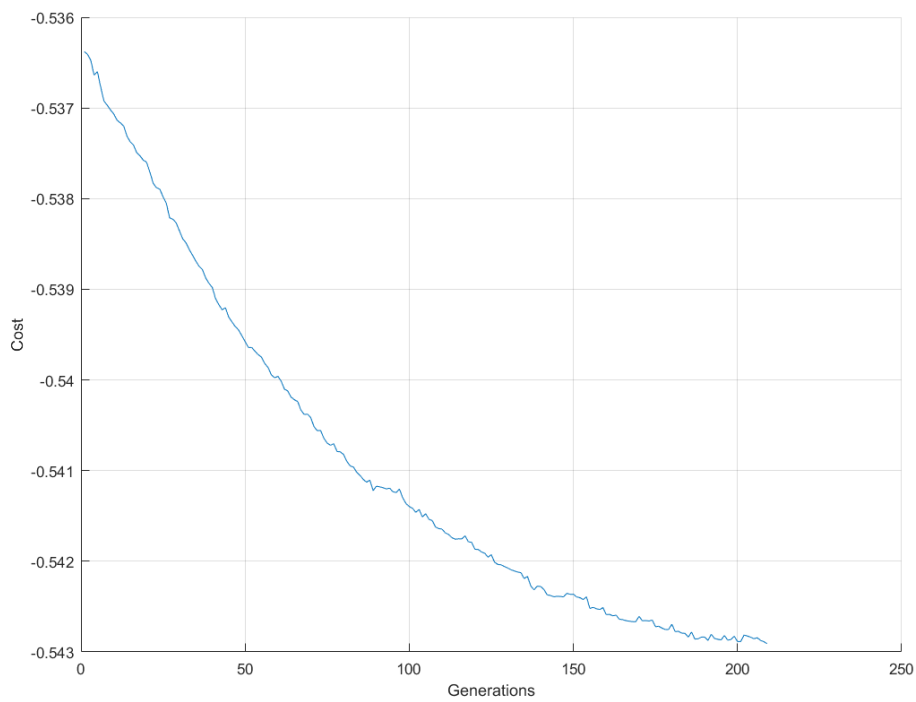


Figure 3.7: Cost function with respect to generations

The algorithm terminates after a certain amount of increase in cost function is captured. In this problem, after 207 iterations cost function is minimized as possible.

Table 3.5: Results Obtained by Initial and Optimized Models

	Initial Model (PNG)	Optimized Model (GA)	Optimized Model (CGA)
Cost	0.1230	-0.53628	-0.5429
$Mach_{ter}$	0.99	1.626	1.622
tof (s)	55.65	54.48	53.97

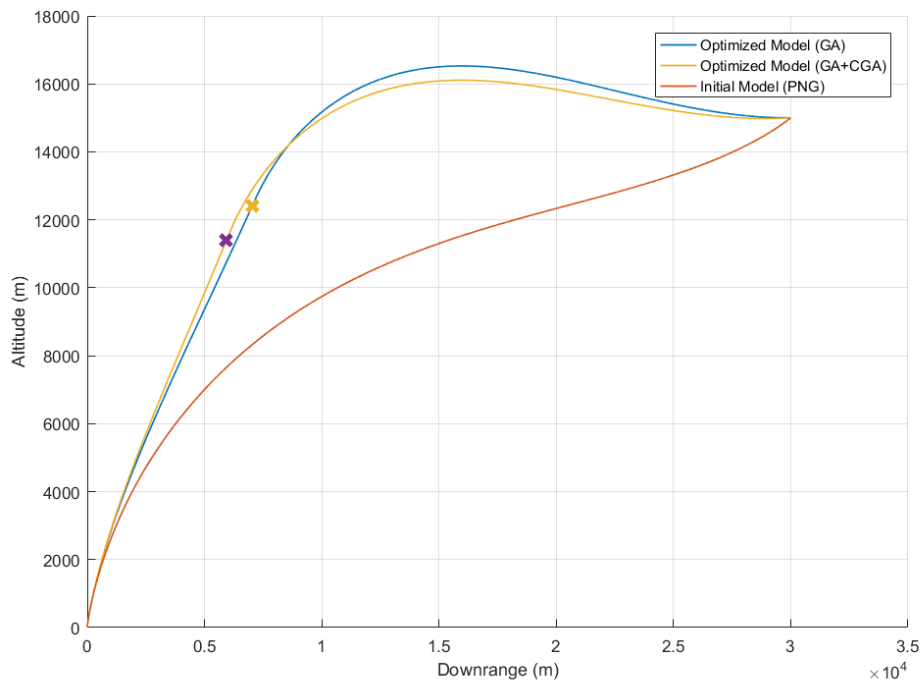


Figure 3.8: Trajectories Obtained by Genetic Algorithm, Gradient Based Algorithm and the Initial Model

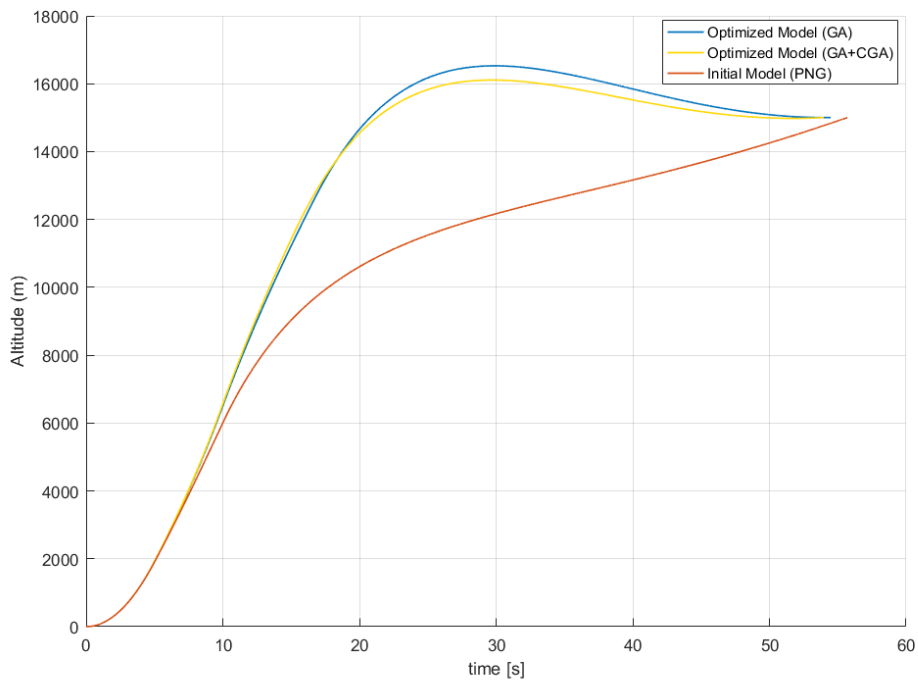


Figure 3.9: Altitude of Missile

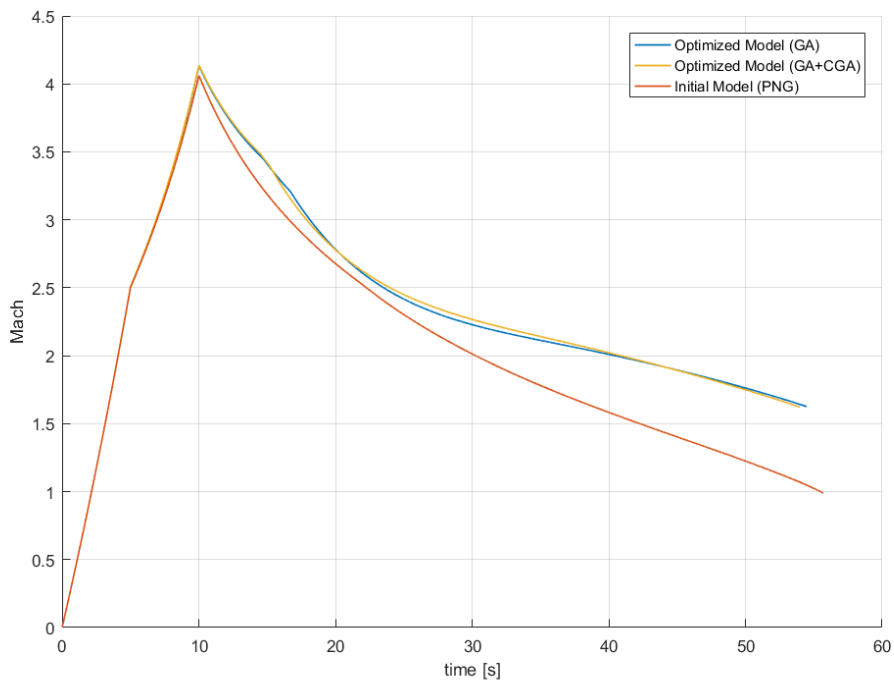


Figure 3.10: Mach Obtained by Genetic Algorithm, Gradient Based Algorithm and Initial Model

In order to examine the results and the efficiency of the algorithms, solutions are summarized in Table 3.5 . It can be clearly stated that, Genetic Algorithm has more contribution when compared to Conjugate Gradient Method. Although the purpose of using Conjugate Gradient Method is to increase the accuracy of the solution, it is seen that the method does not contribute much especially when the computation time is taken into consideration. Actually, the reason why the elapsed time is so high for Conjugate Gradient Method is to run the simulation in each step while taking numerical derivatives, and resulting in longer computation time. Hence it is decided to continue only with the Genetic Algorithm for the further analyses.

In Figure 3.11, the optimization process is illustrated in the search space of the problem. With the Genetic Algorithm, the cost is obtained as -0.53628 stated in the blue dot. After that, Conjugate Gradient Method follows the path which is represented by the red line, and the minimum cost is obtained as -0.5429 with the combination of these two methods.

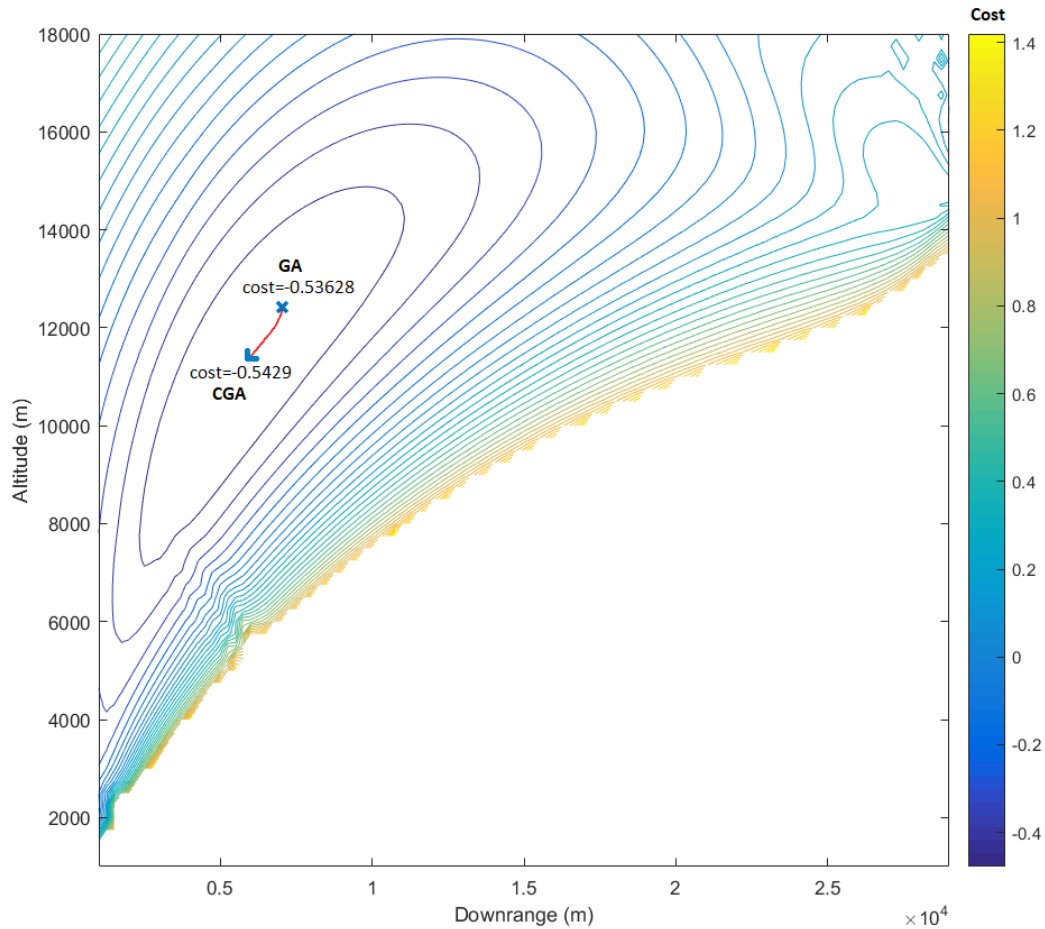


Figure 3.11: Change in the cost

3.2.3 The Effect of Number of Waypoints in Trajectory Optimization Problem

In this trajectory optimization problem, waypoints are taken as control parameters. While creating the trajectories, multiple waypoints can be used. If a small number of waypoints are used, optimal trajectory may be difficult to achieve, and also using large number of waypoints may bring additional computational load to the algorithm. Hence, it is important to choose the number of waypoints properly..

In order to understand the effect of waypoint number, several number of waypoints are utilized to build the missile trajectory and the effect on the cost function is examined. As an example scenario for intercepting a target at 15000 m altitude and 30000 m range, Genetic Algorithm is used with different numbers of waypoints as control parameters. The cost function is same as in the equation 3.1.

Table 3.6: The effect of number of waypoints in cost function

	Cost
1 waypoint	-0.53628
2 waypoints	-0.5843
3 waypoints	-0.59695
4 waypoints	-0.60511

From a single waypoint to four waypoints, cost function for the trajectory optimization problem is tested. From Table 3.6 and Figure 3.12 the effects of the waypoint numbers are indicated.

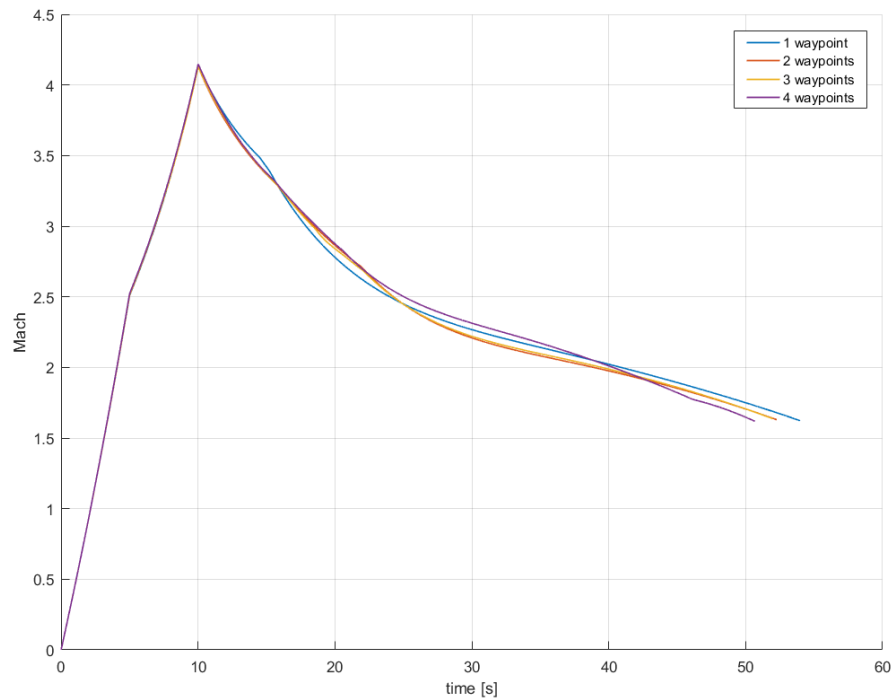


Figure 3.12: The effect of number of waypoints in missile velocity

As it can be observed in Figure 3.12, as the number of waypoints increase, there is not a significant change in the terminal velocity. However required time of flight to reach the same target decreases. From Table 3.6 final cost values after 50 iterations can be observed. A significant change in cost occurs as the number of waypoints increase

from one to two waypoints. Hence, it is decided to chose two waypoints to construct the trajectories for the further analyses.

Up to this point, analyzes are conducted in order to decide which algorithm is more beneficial, how many numbers of waypoints are to be used in order to handle the trajectory optimization problem.

After the analyzes, it is concluded that the use of the Genetic Algorithm alone will be sufficient for the missile trajectory optimization problem. In addition, using two waypoints as control parameters is convenient in order to build the trajectory.

3.2.4 Maximum Terminal Velocity Problem

In this section, a trajectory optimization problem which aims to intercept an air target as in Figure 3.13, with a maximum velocity and minimum flight time is examined. Especially for the air defense missiles, it is important to reach the intercept point with high velocities when the target maneuvers and the lethality issues are considered. Also flight time to meet an incoming air threat is a significant parameter. Hence, the cost function is taken as in the equation 3.1 which includes both the terminal velocity and the time of flight.



Figure 3.13: Tactical missile against an air target

From Figure 3.14 to Figure 3.20 results obtained by Genetic Algorithm for Maximum

Table 3.7: Waypoints Obtained by GA for Maximum Terminal Velocity Problem

P1x (m)	3033.03635
P1z (m)	5964.39287
P2x (m)	11866.8311
P2z (m)	14198.6486

Table 3.8: Results of Maximum Terminal Velocity Problem

	Optimized Model (GA)	Initial Model (PNG)
Mach terminal	1.63	0.98
Time of flight (s)	52.26	55.7

Terminal Velocity Problem are illustrated. Detailed discussions about these results take place in Section 3.3.

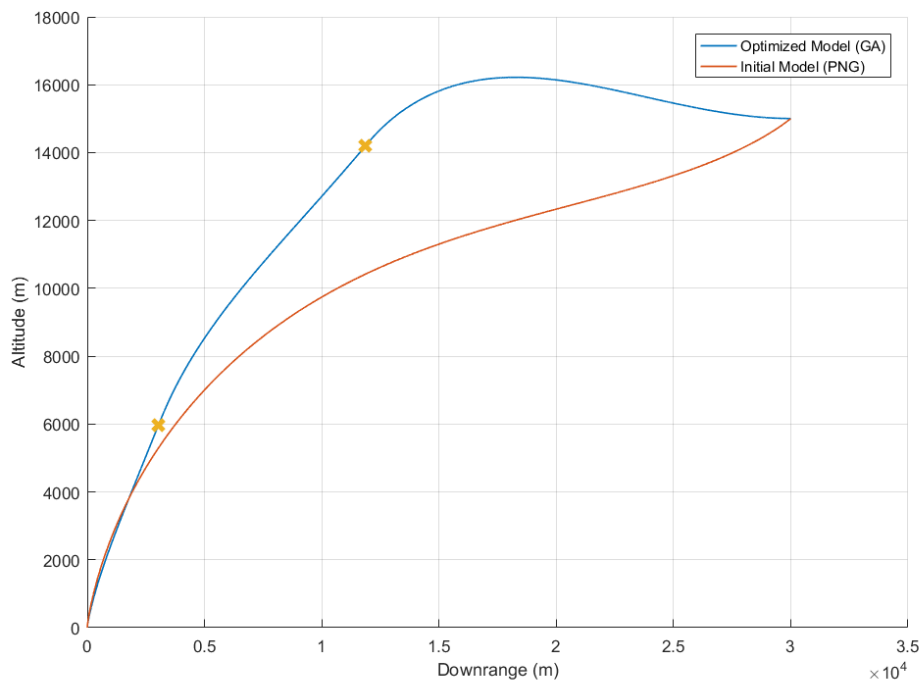


Figure 3.14: Trajectory of the missile

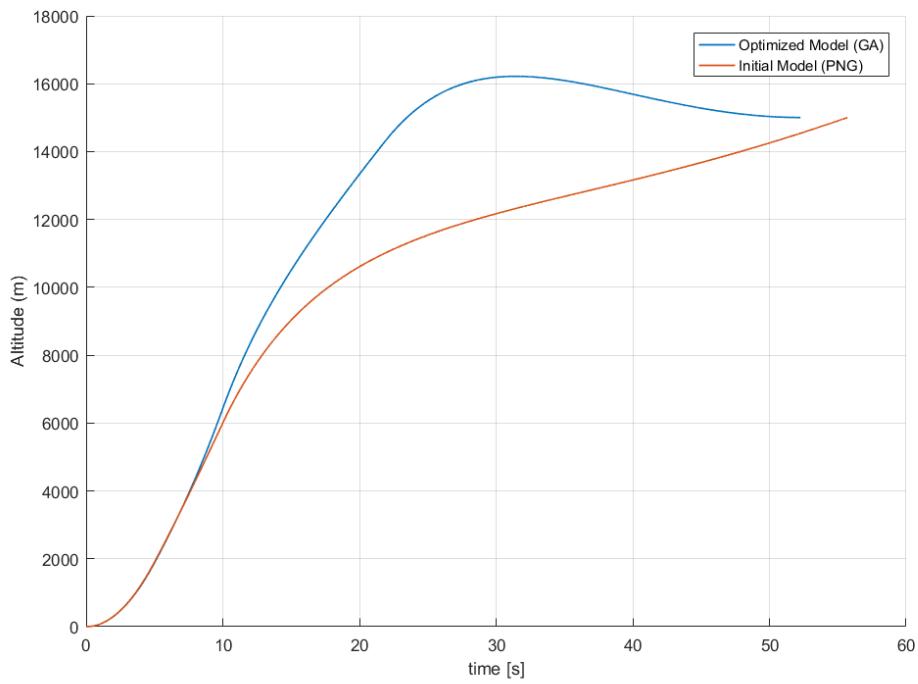


Figure 3.15: Altitude of the missile

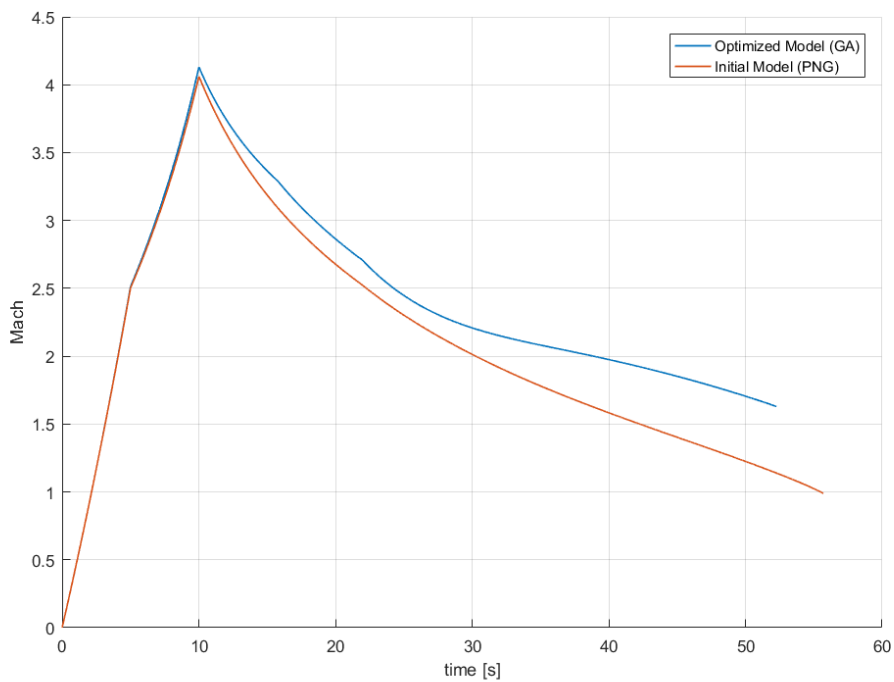


Figure 3.16: Mach profile of the missile

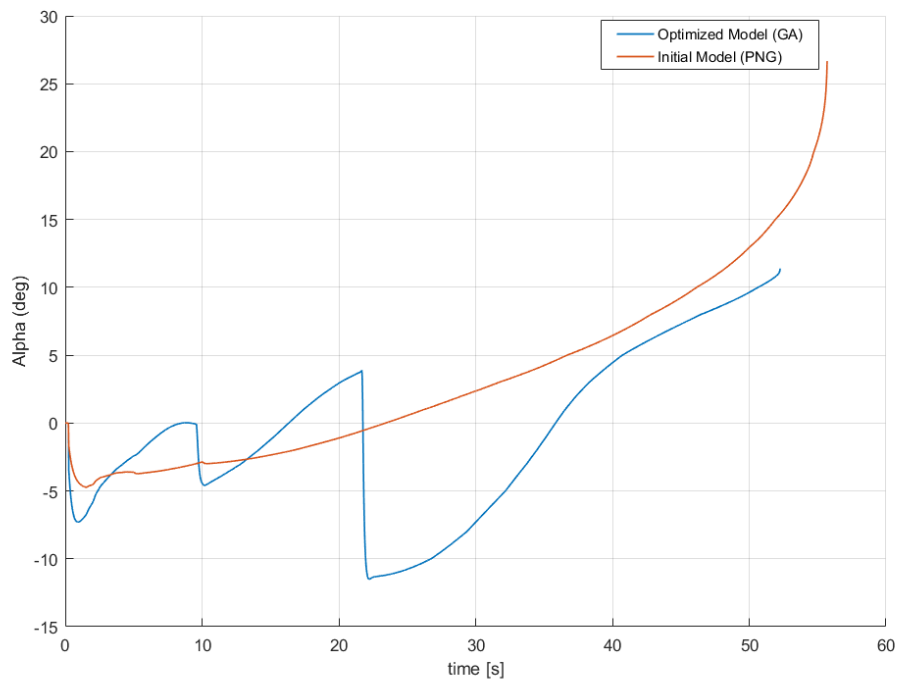


Figure 3.17: Alpha profile of the missile

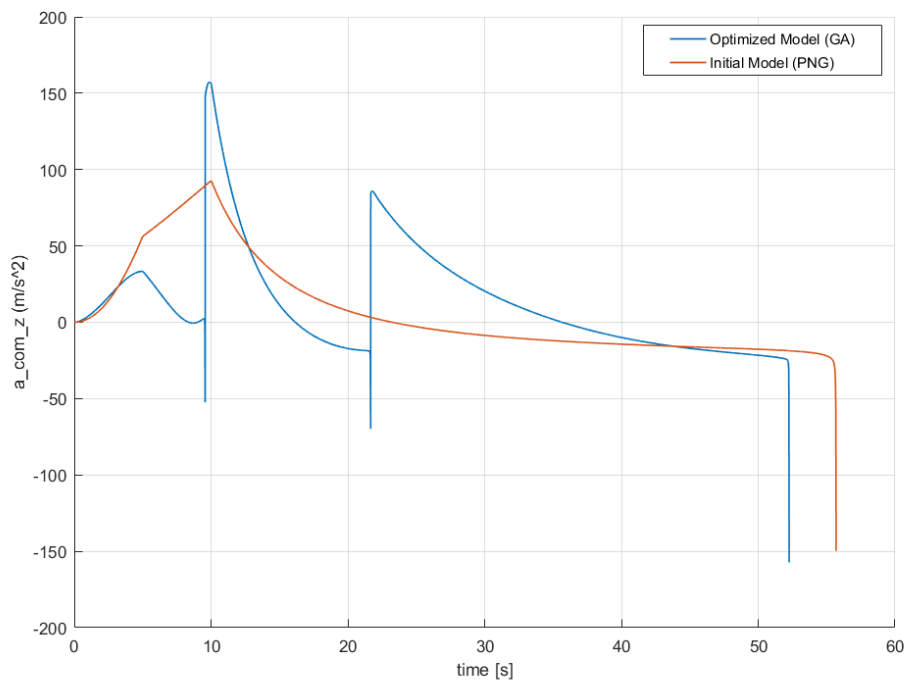


Figure 3.18: Acceleration command of the missile

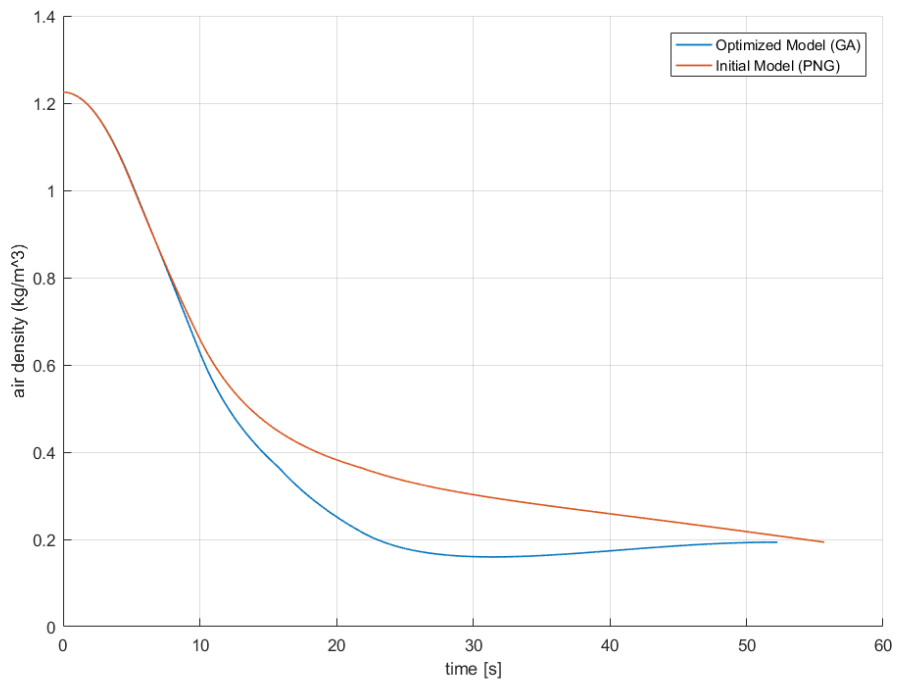


Figure 3.19: Air density

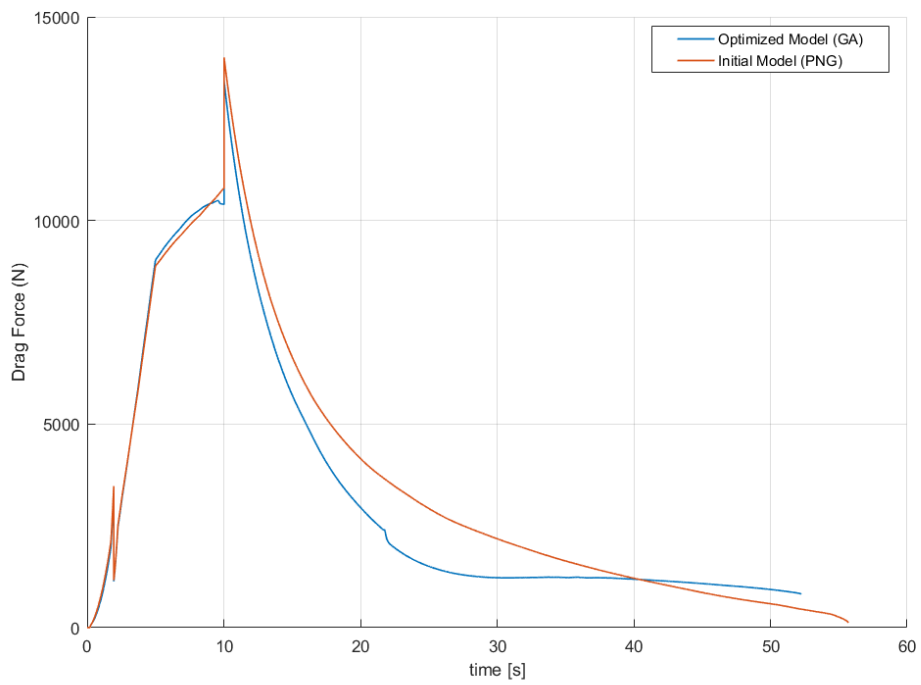


Figure 3.20: Drag force of the missile

3.2.5 Terminal Impact Angle Problem

In this section, trajectory of a missile is optimized to reach a specific impact angle against a ground target as in Figure 3.21. It is important to tune the terminal impact angle in order to increase warhead efficiency and lethality. By considering the impact velocity also, cost function of the problem is generated as in the equation 3.2

$$J = -M_{ter} + |\gamma - \gamma_{ref}| \quad (3.2)$$

Where γ is the flight path angle, and γ_{ref} is the desired terminal impact angle that the missile tries to achieve.



Figure 3.21: Tactical missile against a ground target

Impact angle constraints may be achieved by using biased pure proportional navigation guidance. In Figure 3.22 engagement geometry against a stationary target is indicated. When the purpose is to hit a stationary target with a desired flight path angle γ_F while having an initial flight path angle γ_{IC} , an optimal guidance law can be expressed as including a bias term.

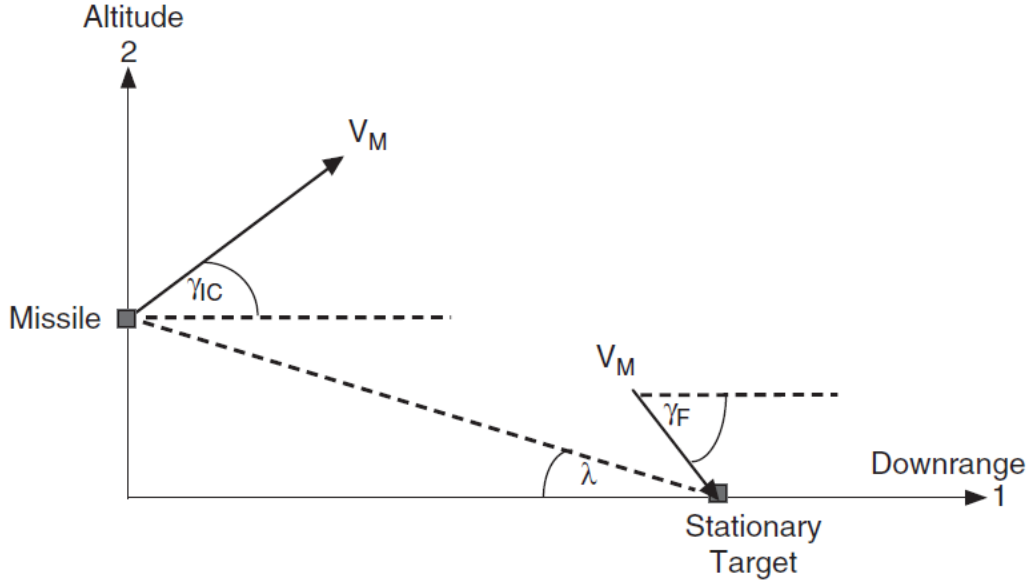


Figure 3.22: Engagement geometry for stationary target [1]

$$a_c = V_m \dot{\gamma} \quad (3.3)$$

$$\dot{\gamma} = N \dot{\lambda} + b \quad (3.4)$$

where b is the bias term. This bias term can be included as a constant as well as being calculated by the following equation 3.5 as stated in [1].

$$b = \frac{-\gamma_F(N-1) + N\lambda_{IC} - \gamma_{IC}}{\Delta t} \quad (3.5)$$

For a missile initially having a flight path angle γ_{IC} , in order to hit a target with a desired final impact angle γ_F , a bias value is calculated for a specific amount of time Δt for the commanded acceleration [1].

In this problem the purpose is to achieve -90 degrees impact angle while maximizing the terminal velocity. The results obtained by Genetic Algorithm and BPPN are compared.

From Figure 3.23 to Figure 3.27 results obtained by Genetic Algorithm for Terminal Impact Angle Problem are illustrated. Detailed discussions about these results take place in Section 3.3.

Table 3.9: Results of Terminal Impact Angle Problem

	Optimized Model (GA)	Initial Model (BPPN)
Mach terminal	1.69	0.56
Terminal flight path angle (deg)	-89.89	-81.77

Table 3.10: Waypoints Obtained by GA for Terminal Impact Angle Problem

P1x (m)	1416.34874
P1z (m)	1612.4366
P2x (m)	12502.6603
P2z (m)	4407.37958

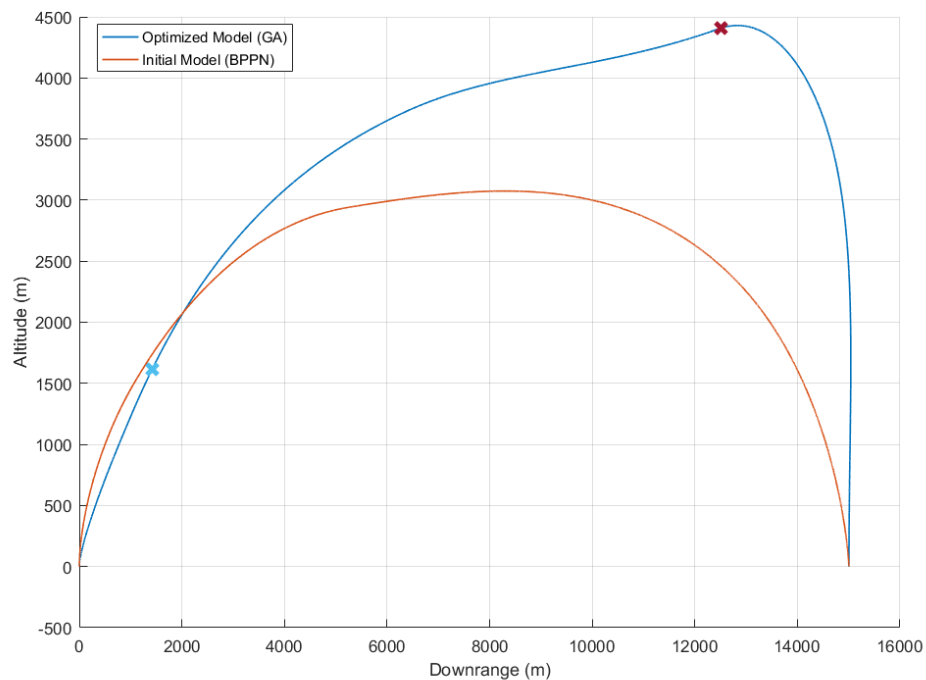


Figure 3.23: Trajectory of the missile

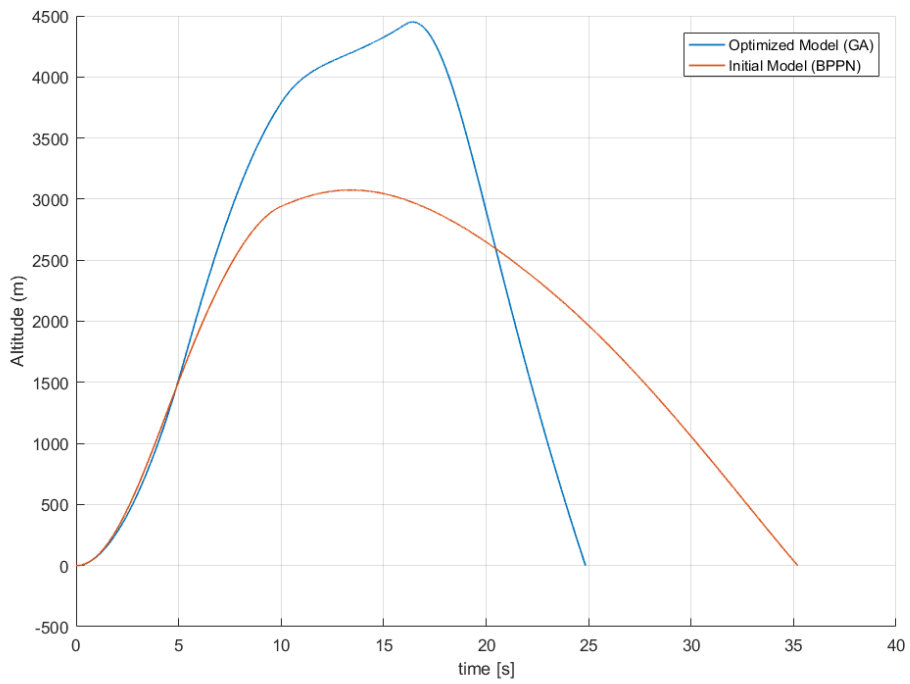


Figure 3.24: Altitude of the missile

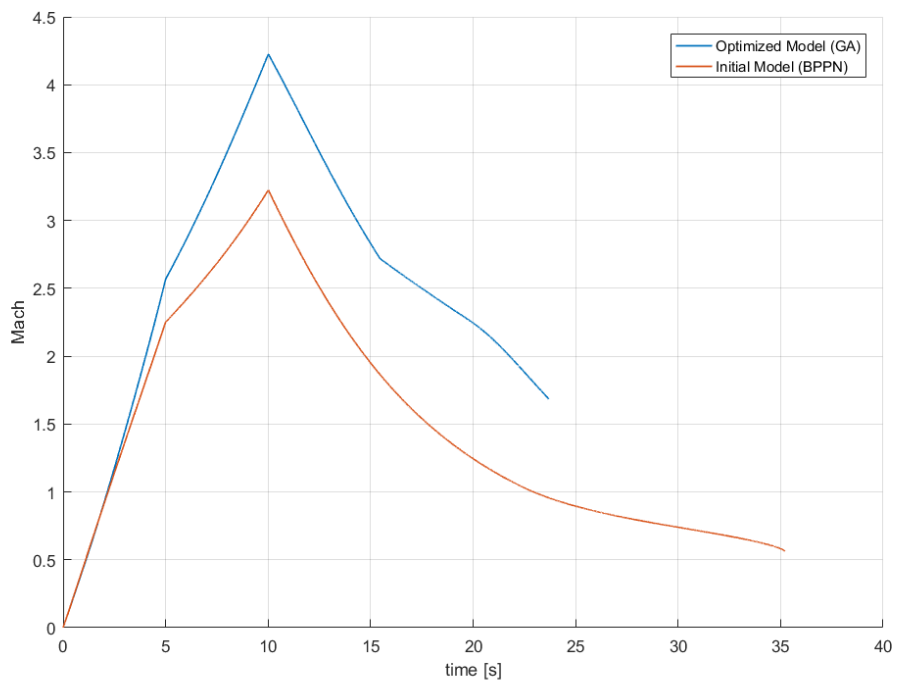


Figure 3.25: Mach profile of the missile

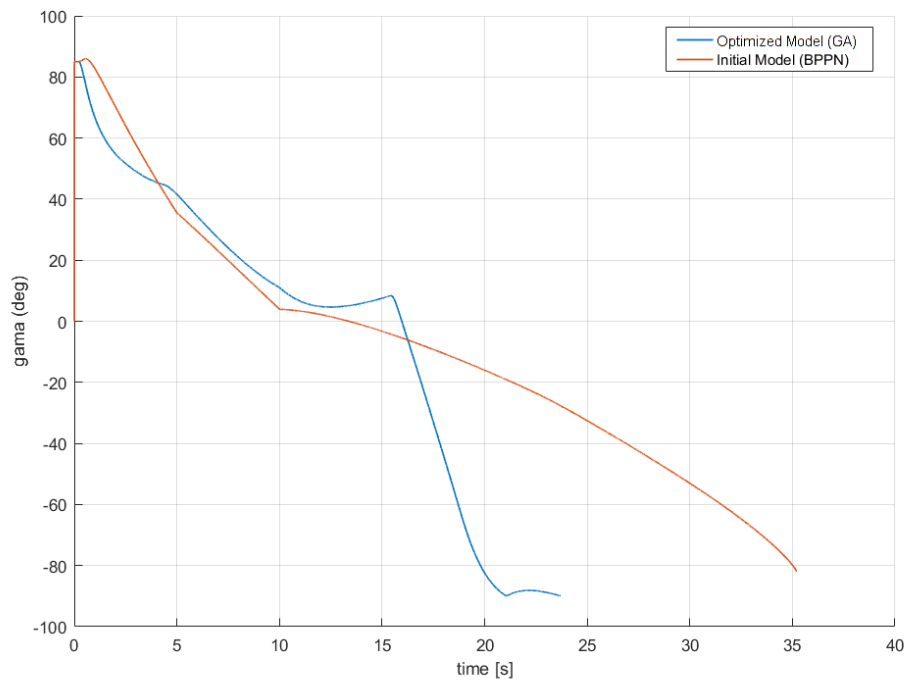


Figure 3.26: Flight Path Angle of the missile

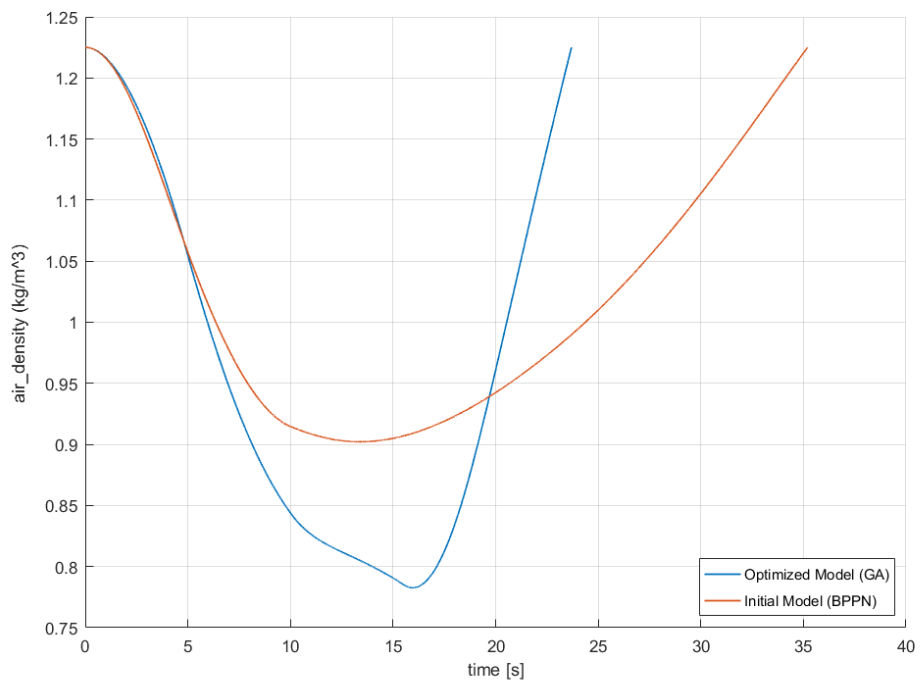


Figure 3.27: Air density of the missile

3.2.6 Generating Tables for the Missile Guidance

Up to this point, optimization algorithms to improve missile trajectory were run depending on the particular scenarios. The results obtained provide important insights about the missile guidance. However these results can not be used for every scenario.

In order to improve trajectories for different cases, lookup tables are generated so that a wide set of scenarios can be handled in missile guidance without running the optimization algorithm every single time.

For the maximum terminal velocity problem, from 20000 to 40000 meters in down-range with 5000 meters intervals and from 10000 to 18000 meters in altitude with 2000 meters intervals lookup tables are generated by using Genetic Algorithm.

As it can be observed in Figure 3.28, interception range and the altitude are the input parameters of the lookup table, and the two waypoint locations that the missile must visit are estimated by lookup table.



Figure 3.28: Input and outputs of the lookup table

As an example case, let the missile tries to intercept an incoming threat at 24000 meters downrange and at 11000 meters altitude. From the lookup tables embedded in missile guidance, waypoints that the missile must visit is obtained as in Table 3.11.

Results obtained by using waypoints that are calculated from look up tables are compared with initial model which guides the missile directly to the actual target position

Table 3.11: Waypoints Obtained from Lookup Tables

P1x	P1z	P2x	P2z
3515.3	5789.6	10906	11831

with proportional navigation guidance.

As it can be observed in Figures 3.29 and 3.30, Although Genetic Algorithm is not run specifically for this scenario, with the waypoints obtained from lookup tables, trajectories are quite optimized in terms of terminal velocity and flight time.

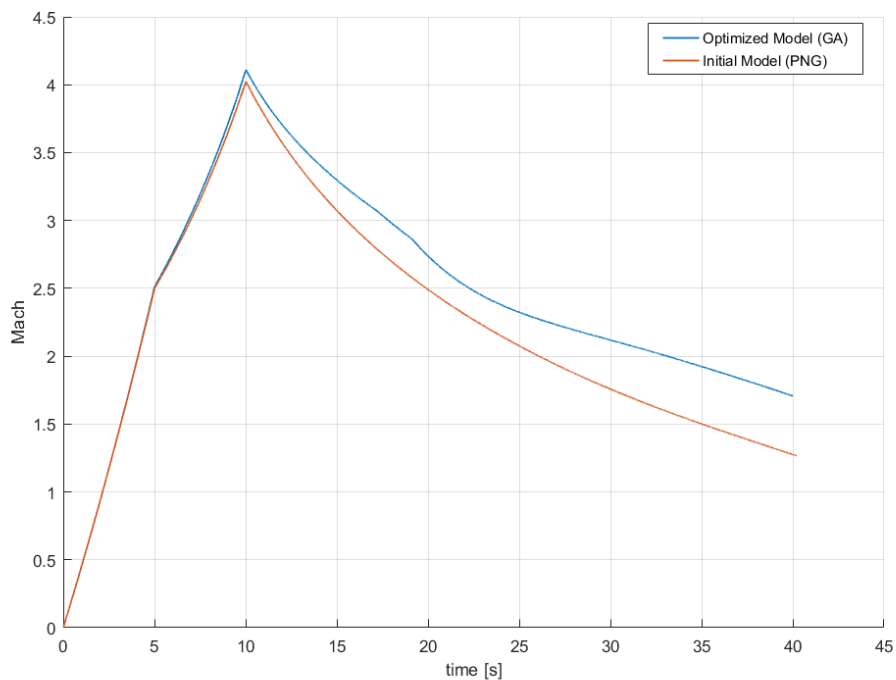


Figure 3.29: Mach profile of the missile

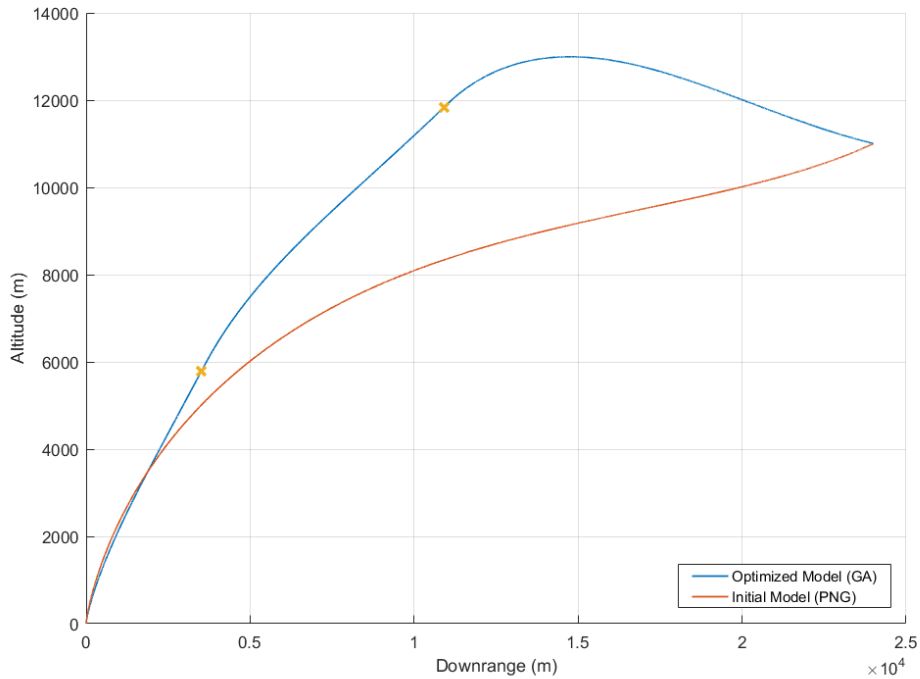


Figure 3.30: Trajectory of the missile

3.3 Discussion

The purpose of this study is to optimize the missile trajectory by considering different performance indices depending on the particular scenarios. As it is mentioned in section 1.2, there are several algorithms used in trajectory optimization problems. In order to choose the right algorithm and parameters that will be used in the algorithm, some analyses are conducted initially.

Since the Genetic Algorithm is known to be good at global search while the gradient based methods are superior in convergence accuracy, combination of these two algorithms is tried. The purpose is to utilize the solutions obtained by Genetic Algorithm as an initial guess for the Conjugate Gradient Method. Therefore fine tuning for the solutions come from Genetic Algorithm would be achieved. When the results of these hybrid method are investigated in Table 3.5, it is concluded that contribution of the gradient based method for the fine tuning is not sufficient enough to consider. Especially when the computation time of the algorithms are compared, Genetic Al-

gorithm achieves optimal results in a much shorter time. Even if a meaningful initial guess is provided to the gradient based method, the result does not improve much compared to GA. Hence it is decided to continue with Genetic Algorithm for the further analysis when the convergence performance and computation time are taken into consideration.

Once the algorithm to be used is selected, the effect of the waypoint number in missile trajectory is examined. As mentioned before, waypoints are accepted as the control parameters of the optimization algorithm. Cases in which the trajectories consist of several numbers of waypoints are investigated. As it can be observed in Table 3.6, a significant change occurs in cost function from one waypoint to two waypoints while building the missile trajectory. Hence, it is decided that two waypoints would be sufficient for being control parameters of the Genetic Algorithm.

After deciding the optimization method and the number of control variables, optimization problem is studied based on two problems namely, Maximum Terminal Velocity and Terminal Impact Angle Problem.

In the maximum terminal velocity problem, using Genetic Algorithm significantly improved the missile performance by means of terminal velocity and flight time. As it can be observed in Table 3.8 when compared to initial model by visiting waypoints selected by Genetic Algorithm, terminal velocity increases from 0.98 Mach to 1.63 Mach while the flight time decreases from 55.7 s to 52.26 s. It is apparent that with the optimized trajectory, there is an important improvement in the missile performance. In the initial model, the dynamic conditions to which the missile is exposed are not considered by PNG. It only attempts to reduce the LOS rate according to missile and target engagement geometry. From Figure 3.14 to Figure 3.20 it can be seen that, by visiting the selected waypoints, the missile flies at higher altitudes which result in being subject to less air density 3.19 so that less aerodynamic drag force 3.20. For this reason, the terminal velocity of optimized trajectory is much higher when compared to the initial model.

For the second problem, desired terminal angle is achieved by generating trajectory with waypoints by maximizing terminal velocity also. Results are compared with the ones obtained by biased pure proportional guidance. Table 3.9 presents the re-

sults of two methods. Genetic Algorithm provides impact angle very close to the desired value, while terminal velocity is also much greater than in the BPPN method. From Figure 3.23 to Figure 3.27 it can be understood that the optimized trajectory is achieved by flying at higher altitudes as in the first problem.

Guided by the insights we obtain from these analyzes, it is intended to optimize missile trajectories for several scenarios without running Genetic Algorithm for every single one. With the lookup tables mentioned in Section 3.2.6, trajectory optimization can be performed online in missile guidance.



CHAPTER 4

CONCLUSION

In this thesis, trajectory optimization of a tactical missile is investigated by using different optimization algorithms. To perform the analyzes, a hypothetical missile is modeled by using Pseudo 5 dof simulation model in Matlab, Simulink. Aerodynamic, propulsive and mass data is modeled for this hypothetical missile, then the optimization algorithms are performed based on this missile model.

Control parameters for the optimization algorithms are used as waypoints. These are the points that the missile must visit before reaching the final target position. Missile is guided to waypoints instead of actual target position. The optimum waypoint locations which satisfy the trajectory optimization criteria are obtained by using optimization algorithms.

From the insights gained from literature survey, there are various optimization algorithms to be used in trajectory optimization. In the beginning of the study, it is decided to investigate the effects of a gradient based algorithm together with an evolutionary optimization algorithm. With this hybrid algorithm, Genetic Algorithm is used for a global search and the solution obtained from Genetic Algorithm serves as an initial condition of Conjugate Gradient Algorithm. To improve the accuracy, Conjugate Gradient Method is used for fine tuning. The results indicate that, Genetic Algorithm is actually sufficient enough to meet the optimization criteria when the computational time and the complexity of an optimization for highly nonlinear system is considered. Hence, it is decided to continue with the analyzes with Genetic Algorithm due to its efficiency and the ease of implementation.

In this thesis, another study is carried out to decide how many numbers of waypoints

are sufficient for the optimization problem. This study shows that two waypoints to generate the missile trajectory is acceptable. The proceeding analyzes are performed by using two waypoints as control parameters of the Genetic Algorithm.

After deciding the optimization algorithm to be used and number of waypoints as control parameters, missile trajectory optimization problem is performed based on two scenarios. First, maximum terminal velocity and minimum flight time to intercept an air target is achieved. The results obtained by Genetic Algorithm is compared by a reference mode. The reference model uses conventional guidance algorithms to reach the target location. In the second problem trajectory is optimized to achieve a specific impact angle with maximum velocity against a stationary ground target. For this scenario, reference model is built by using BPPN guidance for comparison.

The results for the two problems indicate that using Genetic Algorithm with waypoints as control parameters provides promising solutions to optimize the missile trajectory. Genetic Algorithm allows the missile to fly at a more advantageous region in dynamic sense, so that optimized trajectories can be achieved.

With the insights from trajectory optimization study, it is decided to generate tables which will be used in missile guidance online. For this purpose, Genetic Algorithm is used in several scenarios. Waypoints are obtained based on different scenario parameters. By this way, for random scenario parameters, trajectory can be optimized without running Genetic Algorithm every single time.

4.1 Future Work

- In the future studies, different scenarios can be carried out for the operation of trajectory optimization in missile guidance.
- Depending on the missile mission and the requirements, detailed tables covering more scenarios can be created for the operational use.
- In the beginning of a missile design phase, the requirements can be studied by considering an optimized trajectory. Hence, trajectory optimization with the methods involved in this thesis can be used cooperatively in the initial design

phase of the missile.

- Apart from the methods studied in this thesis, other evolutionary algorithms can be used in trajectory optimization. Especially for more complex flight conditions or higher-level operational tasks, an intelligence autonomy of the missile system can be developed to generate trajectories.





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APPENDIX A

MISSILE PARAMETERS

A generic tactical missile which is created for this thesis is illustrated in Figure A.1. Missile is assumed to use solid propellant rocket motor and has guidance and warhead sections also. Tail parameters that provide control efficiency are also shown in the Figure A.2.

In order to obtain the aerodynamic coefficients, Missile DATCOM is used in the aerodynamic design. For preliminary design phase, it provides a convenient level of accuracy. An input file is generated which defines the flight conditions and geometry of the missile in order to be run. The parameters that is used to generate the input file is listed below A.1.

In Figure A.3 axial force coefficient based on mach number is plotted in zero alpha condition.

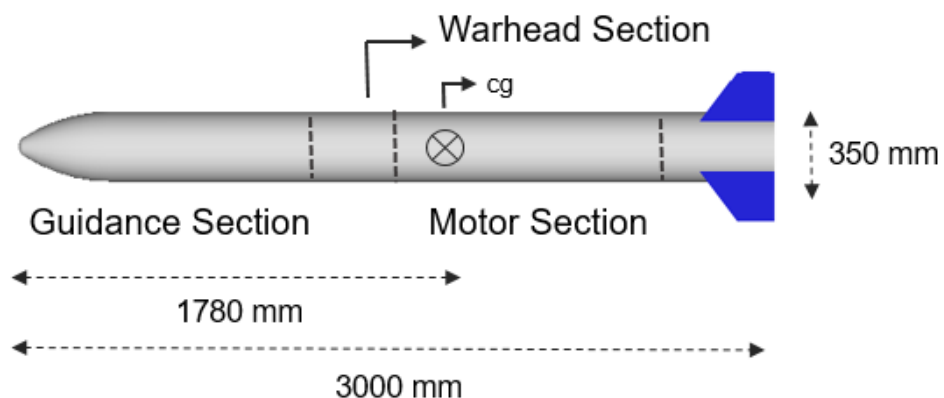


Figure A.1: Hypothetical Missile Geometry

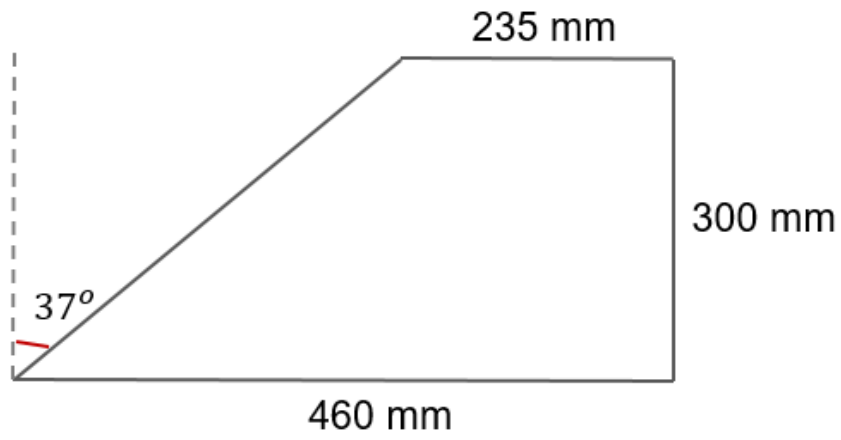


Figure A.2: Tail of the Missile

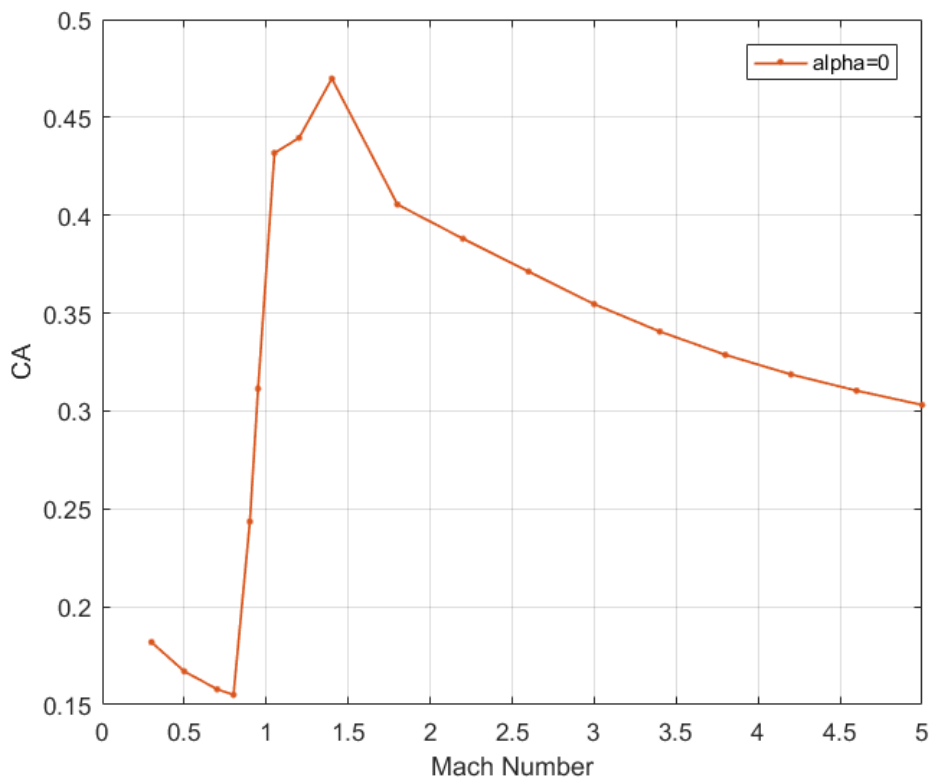


Figure A.3: Trimmed Axial force coefficient

A.1 DATCOM Parameters

```
TEST CASE   : Thesis Dilan Ozdil
DIM M
SOSE
PLOT3
PART
FORMAT (F9.4)
REFQ
XCG=1.78. ,
FLTCON
NALPHA=19.0 ,
ALPHA= 20.0 , 15.0 , 13.0 , 10.0 , 8.0 , 5.0 , 3.0 , 2.0 , 1.0 ,0.0 ,
ALPHA(11)=1.0 ,2.0 ,3.0 ,5.0 ,8.0 ,10.0 ,13.0 ,15.0 ,20.0
NMACH=18.0 ,
MACH=0.30 ,0.50 ,0.70 ,0.80 ,0.90 ,0.95 ,1.05 ,1.20 ,1.40 ,1.80 ,
MACH(11)=2.20 ,2.60 ,3.00 ,3.40 ,3.80 ,4.20 ,4.60 ,5.00 ,
ALT=10000.0 ,
AXIBOD
X0=0. ,
TNOSE=OGIVE ,
LNOSE=0.6 ,
DNOSE=0.35 ,
LCENTR=1.94 ,DCENTR=0.35 ,
FINSET1
SSPAN=0.175 ,0.475 ,
CHORD=0.46 ,0.235 ,
XLE=2.54 ,2.765 ,
NPANEL=4.0000 ,
PHIF=0. ,90. ,180. ,270. ,
DEFLCT
DELTA1=0.0 ,0.0 ,0.0 ,0.0 ,
DELTA2=0.0 ,0.0 ,0.0 ,0.0 ,
SAVE
NEXT CASE
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