ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE ENGINEERING AND TECHNOLOGY

AN INTERVAL VALUED PYTHAGOREAN FUZZY QUALITY FUNCTION DEPLOYMENT METHOD AND ITS APPLICATION TO SOLAR PHOTOVOLTAIC TECHNOLOGY DEVELOPMENT

M.Sc. THESIS

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Elif HAKTANIR, a M.Sc. student of İTU Graduate School of Science Engineering and Technology student ID 507161138, successfully defended the thesis entitled "AN INTERVAL VALUED PYTHAGOREAN FUZZY QUALITY FUNCTION DEPLOYMENT METHOD AND ITS APPLICATION TO SOLAR PHOTOVOLTAIC TECHNOLOGY DEVELOPMENT", which she prepared after fulfilling the requirements specified in the associated legislations, before the jury whose signatures are below.

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Jury Members :	**Assoc. Prof. Başar ÖZTAYŞİ** Istanbul Technical University	....
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Date of Submission : September 20, 2018
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To Neriman HAKTANIR, Mert HAKTANIR,
Batuhan AKTAŞ and KADIKÖY,

FOREWORD

First of all, I want to express my sincere gratitude to my thesis supervisor Prof. Dr. Cengiz KAHRAMAN for accepting me to study with. His guidance, support, wisdom and experience guided me to complete this thesis study. It was an honour to be his student.

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September 2018

Elif HAKTANIR
Res. Asst.

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ABBREVIATIONS

AI	: Absolute Importance
CI	: Correlation Impact Factor
CPR	: Combined Performance Rating Score
CR	: Customer Requirement
DM	: Decision Making
DR	: Design Requirement
FQFD	: Fuzzy Quality Function Deployment
HOQ	: House of Quality
IE	: Importance Evaluations
IVPFN	: Interval Valued Pythagorean Fuzzy Number
IVPFS	: Interval Valued Pythagorean Fuzzy Sets
MCDM	: Multiple Criteria Decision Making
NC	: Negative Correlation
OD	: Organizational Difficulty
PC	: Positive Correlation
PFS	: Pythagorean Fuzzy Sets
PV	: Photovoltaic
QFD	: Quality Function Deployment
R	: Terms in the Relationship Matrix
RAI	: Relative Absolute Importance
ROD	: Relative Organizational Difficulty

SYMBOLS

μ_L, μ_U : Lower and upper points of Membership Degree

v_L, v_U : Lower and upper points of Non-Membership Degree

π_L, π_U : Lower and upper points of Hesitancy Degree

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SUMMARY

Quality function deployment (QFD) is a focused methodology for carefully listening to the voice of the customer and then effectively responding to those needs customer requirements (CRs) and expectations through design requirements (DRs). In classical QFD, exact numbers are used to determine the priorities of the CRs and the position of the company among the competitors. However, vagueness and impreciseness are inevitable uncertainties in these kinds of human evaluations, which are generally realized by linguistic terms. In this thesis, the uncertainty in design processes are captured by a relatively new extension of ordinary fuzzy sets, Pythagorean fuzzy sets (PFS), aiming at presenting a larger domain to experts for assigning a membership degree and a non-membership degree together with their hesitancy. All the evaluation processes in the house of quality (HOQ) are performed based on interval-valued PFS (IVPFS) and present some novel definitions for measuring and prioritizing CRs and DRs, and determining the position of the company among the competitors. An application of the proposed PF-HOQ model for solar photovoltaic (PV) technology development is given. The hypothetical case study is based on three cutomers' and three experts' evaluations. The HOQ is built by considering 6 CRs and 8 DRs. Some optional parts of a HOQ such as correlation matrix, organizational difficulty, directions of improvement and competitor analysis are also taken into consideration.

The thesis is organized as follows. Section 1 is the introduction part. Section 2 introduces the concepts of QFD. Section 3 presents a literature review on Fuzzy QFD (FQFD). Section 4 gives the preliminaries for PFSs. Section 5 develops an IVPF QFD Model. Section 6 illustrates the application of the proposed model on solar photovoltaic technology development. Section 7 concludes the paper with future directions.

BULANIK KALİTE FONKSİYON GÖÇERİMİ VE FOTOVOLTAİK GÜNEŞ PANELİ TASARIMI UYGULAMASI

ÖZET

Günümüz rekabetçi piyasalarında ister üretici ister servis sağlayıcısı tüm şirketlerin varlık ve büyümelerini sürdürebilmek adına müşteri ihtiyaçlarına en kapsamlı ve hızlı şekilde cevap vermesi zorunluluğu kaçınılmaz bir hal almıştır. Rakipleri arasında hayatta kalmak isteyen şirketler için sürekli değişen ve yüksek çeşitliliğe sahip müşteri taleplerine devamlı olarak yanıt verebilmek ve sektörel yeniliklere ayak uydurabilmek büyük önem arz etmektedir. Teknolojik gelişmeler ve müşteri beklentilerine göre kişiselleştirilmiş ürün yelpazeleri talebi, şirketlerin global marketlerdeki rekabet avantajlarını korumaları amacıyla satış ve karlılıklarını arttırabilmeleri için daha düşük üretim maliyeti ve hazırlık süresiyle daha yüksek kaliteye ulaşmalarıyla doğrudan ilişkilidir.

Modern dünyanın tüketicileri artan teknoloji kullanımı ve küreselleşmenin getirisi olarak geçmişte hiç olmadığı kadar yüksek beklenti ve standartlara sahip hale gelmiştir. Üreticiler için estetik yaklaşımlar, kullanılabilirlik, ulaşılabilirlik gibi müşteri gereksinimleri (MGleri) üretilebilirlik, teknolojik olurluluk, işlevsellik gibi teknik özellikler (TÖler) arasındaki dengeyi sağlamak hayati önem taşır. Tüm MGlerini aynı anda karşılamak reel dünyada pek de mümkün olamadığından tüm şirketler sundukları tüm ürün veya hizmetler için daha tasarım aşamasındayken bir öncelik sırası belirlemelidir.

Bu bilgiler ışında MGlerini TÖlere yansıtma amacıyla faydalı bir ürün tasarım ve geliştirme tekniği olan Kalite Fonksiyon Göçerimi (KFG) 1996 yılında geliştirilmiş ve birçok kuruluşta uygulamacılar tarafından kullanılırken pek çok araştırmaya konu olmuştur.

Yoji Akao tarafından geliştirilip ilk olarak 1972'de Mitsubishi Ağır Sanayi Kobe Tershaneleri'nde uygulamaya konmuş olan KFG genel hatlarıyla MGlerinin dinlenerek TÖler belirlenirken hesaba katılmasına dayalı bir planlama aracı olarak tanımlanabilir. Bugüne kadar çok geniş bir yelpazede kendine uygulama ve araştırma alanı bulmuş olan KFGnden en sık ürün tasarımı, kalite yönetimi, karar verme gibi alanlarda faydalanılmıştır. KFGnin hem üretim hem de hizmet sektörlerinde kullanılması karşın yoğunlukla yazılım sistemleri, imalat, taşımacılık, tedarik zinciri, iletişim ve hizmet sektörlerinde ele alındığı görülmüştür. Ayrıca KFGnin en sık kullanıldığı süreçleri pazarlama, planlama, tasarım, üretim ve satış olarak sıralamak mümkündür.

KFGnin şirketlere sağladığı birçok avantaj bulunmaktadır. Yüksek müşteri memnuniyeti, şirket içi ekipler arası gelişmiş iletişim olanağı, artan ürün kalitesi, etkin süreç yönetimi, şirket gelirlerinde yükselme, üretim maliyetinde düşüş ve daha kısa ürün tasarlama süresi bunlardan başlıcalarıdır.
KFG süreci dört aşamadan oluşur. Her bir matrisin çıktısı kendisinden sonra gelen matrisin girdisini oluşturur. Dört aşama da birer matrisle ifade edilir. Aşamaları ürün planlama, parça planlama, süreç planlama ve üretim planlama olarak sıralamak mümkündür. Abdolshah ve Moradi (2013)'ye göre çoğu şirket ve araştırmacı KFGnin dört aşamasından yalnızca ilki olan ürün planlamayı uygulamaktadır. Bunu zaman ve maddi imkân kısıtlarına bağlamak mümkündür.

Her bir adımın şirketteki yetkili departmanlar tarafından yönetilmesi elde edilecek sonuçların başarısı açısından önem arz etmektedir. Üretim planlamanın pazarlama, parça planlamanın mühendislik ve tasarım, süreç planlamanın üretim ve üretim planlamanın kalite departmanları tarafından yürütülmesi gerekmektedir.

Girdileri MGleri ve çıktıları TÖler olan ilk matris çatısıyla birlikte bir evi andırdığından kalite evi olarak adlandırılır. Bu adım diğer tüm aşamaları doğrudan etkileyeceğinden sonraki basamakların doğru ve tutarlı olabilmesi için bu adımın hatasız uygulanması büyük önem taşımaktadır.

Kalite evinin bileşenleri zorunlu ve opsiyonel olmak üzere iki kısımda incelenebilir. Zorunlu kısımlar MGleri, TÖler, MGleri önem dereceleri, TÖler arası korelasyon matrisi ve kalite evinin gövdesini oluşturan MGleri ile TÖler arasındaki ilişki matrisi iken opsiyonel bileşenler Iqbal ve diğerleri (2016) tarafından rakip analizi, satış noktası verileri, MGleri arası korelasyon matrisi, uygulanabilirlik değerlendirmeleri, geliştirilme zamanı gibi bölümler olarak belirlenmiştir. Yine aynı çalışmalarında bu tercihe bağlı kısımların çoğu araştırmacı ve uygulamacı tarafından neden oldukları ekstra maliyet ve zaman gereksiniminden dolayı ihmal edildiklerine dikkat çekmişlerdir. Ayrıca sahip oldukları ölçüm zorlukları ve kalite evinin diğer zorunlu parçalarıyla entegre edilme zorlukları da yine uygulama alanlarının kısıtlı olmasına yol açmıştır. Ne var ki bu bölümlerin göz ardı edilmesi birçok açıdan sakıncalıdır. Ekstra bilgi sağlayan bu alanlar final sonuçlarını doğrudan etkileyeceği gibi hesaba katılmadıkları taktirde kalite evinden yanlış sonuçlar elde edilmesine de neden olabilir. Örneğin korelasyon matrisinde pozitif ilişkili iki TÖten birini iyileştirmek diğerini doğrudan etkileyeceğinden, bunun bilinmesi her iki TÖ yerine biri üzerine yoğunlaşarak gereksiz yere harcanacak ekstra zaman ve maliyetten kaçınmak anlamına gelebilir.

Bu çalışmanın literatüre kazandıracağı yeniliklerden biri de kalite evinde bulunması zorunlu olmamasına rağmen eksikliğinin veri kaybına yol açacağı korelasyon matrisi, geliştirilme yönü, organizasyonel zorluk, TÖlerin hedef değerleri, müşteri değerlendirmelerine göre rakip analizi ve TÖlere göre rakip analizi gibi opsiyonel kısımları da değerlendirmeye alarak özgün yöntem ve formüller sunmasıdır.

Kalite evinin kurulabilmesi için gereken MGlerinin toplanması genellikle anket, yüz yüze görüşme, odak grubu çalışmaları aracılığıyla olmaktadır. MGlerinin elde edileceği gruplar müşteriler veya son kullanıcılar olabileceği gibi ürün geliştirilmesinde taleplerini dile getiren uzmanlar ya da araştırmacılar da olabilir. Bu kanallarla toplanan taleplerde müşteriler genellikle kesin ve keskin sayısal veriler yerine sözel ifadeleri tercih ettiğinden cevaplar belirsiz, öznel ve bulanık olma eğilimi göstermektedirler. Bu durum da taraflı ve hatalı sonuçlara sebebiyet verebilmektedir. Klasik KFG bu noktada yetersiz kaldığından 1965 yılında L.A. Zadeh'in bulanık kümeler teorisini ortaya koymasıyla birlikte uygulamacılar tarafından Bulanık KFG (BKFG) yaklaşımı geliştirilmiştir. Klasik KFGnde sıklıkla kullanılan 1-5 arası kesin sayılı önem derecesi skalası bir kısım bilginin kaybına neden olabilmektedir. Oysa BKFG kesin sayılar yerine kesin veya tam olmayan sözel verileri kullandığından, bulanık kümelerin yardımıyla sayısal kesinliğe yaklaştırılabilir. Klasik KFGne oranla BKFGnin daha etkili, güvenilir, tutarlı ve anlamlı olduğuna dair literatürde görüş birliği bulunmaktadır.

Literatürde bu belirsizlikle baş etmede kullanılan yöntemler genellikle bulanık kümelerle ilişkilendirilmiştir. Geçtiğimiz son yirmi küsur yılda normal bulanık kümeler Tip-2 bulanık kümeler, sezgisel bulanık kümeler, Pisagor bulanık kümeler ve nötrosofik kümelere genişletilmiştir. Bu çalışmada tüm sözel terimler Pisagor bulanık kümelerle gösterilmiştir. Pisagor bulanık kümelerin henüz başka hiçbir çalışmada KFG ile beraber veya ürün geliştirme amacıyla kullanılmamış olması da bu çalışmanın yenilikçi ve özgün yönlerinden biridir.

Çalışmada öncelikle içerikle ilgili terimler hakkında bilgi verilmiş ve sonrasında BKFG hakkında kapsamlı bir literatür taramasıyla devam edilmiştir. İncelenecek yayınlarda en çok alıntı yapılma, en güncel olma gibi bazı kriterler gözetilmiştir. Çalışma Pisagor Bulanık Kümeler hakkında bilgiler sunularak devam etmiştir. Burada çalışma boyunca kullanılacak formüllere ve bulanık kümelerdeki bazı işlemlere yer verilmiştir. Ardından ortaya konan özgün BKFG modeli açıklanmış ve basamaklar halinde tüm aşamalar sunulmuş, özgün formüller ortaya konmuştur. Bu kısım iki bölümden oluşmuş, ilkinde KFGnin yukarıda açıklanan korelasyon matisi, organizasyonel zorluk, geliştirilme yönü gibi opsiyonel kısımları da hesaba katılarak MGleri ve TÖler arasındaki ilişkiler analiz edilmiş, TÖlerin önem sıralamalarına ulaşmak için bir metot önerilmiştir. İkinci kısımda ise MGleri ve TÖler bazında rakip analizi yapılarak iki kriteri ortak olarak ele alacak şekilde söz konusu şirketler için sıralamaya gidilmiştir. Çalışmanın ilerleyen kısmında ortaya konan bu BKFG yöntemi, Fotovoltaik Güneş Paneli uygulamasıyla örneklendirilmiştir. 3 müşteri ve 3 uzmanın görüşüne dayanan, 6 MG ve 8 TÖ ile bir kalite evi oluşturulmuş, önceki bölümde verilen aşamalar adım adım uygulanarak tüm TÖlerin öncelik dereceleri sıralanmıştır. Ardından 3 şirket için müsteri değerlendirmelerine göre rakip analizi yapılmıştır. Çalışma elde edilen sonuçların değerlendirmesiyle sonlandırılmıştır.

1. INTRODUCTION

Both manufacturers and service providers must satisfy customer needs in the most extensive and perpetual way to survive in today's extremely competitive business environment which obligates the companies to correspond to the dynamic and diversified CRs permanently and adapt themselves to the sectoral innovations to survive towards rivalries. The rapid changes in technology and increased customizations in product ranges impose lower production costs and shorter lead time, and higher product quality to be achieved in order to gain a sales and profit growth for sustaining competitive advantage in global markets.

Modern world's consumers have more expectancies and higher standards for a product than ever before. It is vital for the producers to sustain the balance between customer demands such as aesthetic concern, practical sufficiency, affordability and the engineering characteristic constraints like manufacturability, technological possibilities, functionalities, and cost. Since it is not possible to satisfy all customer demands in once, every company should decide on the priorities for each product or service they offer and regard those customer demands yet in the design phase.

In the light of these facts, a useful product design and development technique to translate the customer needs to the product design specifications, named QFD, was developed in 1966, practiced by many researchers in various cases and has become one of the most frequently applied methods for product development.

The rest of the thesis is organized as follows. Section 2 briefly introduces the concepts of QFD. Section 3 presents a literature review on FQFD. Section 4 gives the preliminaries for PFSs. Section 5 develops an IVPF QFD Model. Section 6 illustrates the application of the proposed model on solar PV technology development. Section 7 concludes the paper with future directions.

2. QUALITY FUNCTION DEPLOYMENT

QFD can be described as a useful tool to translate CRs, needs or demands “or known as voice of customer” to the DRs, that is introduced in 1966 in Japan by Yoji Akao. It was utilized at Kobe Shipyards of Mitsubishi Heavy Industries in 1972 (Brief, 2012) and has been applied in many areas so far such as product design, quality management, decision making (DM) etc. (Sivasamy et al., 2016).

QFD is a useful tool for both manufacturing and service sectors but it has mostly been applied in some specific sectors: software systems, manufacturing, transportation, supply chain, communication and service sectors (Onar et al., 2016; Abdolshah and Moradi, 2013). Within these sectors, it has mostly been imposed on the marketing, planning, product design, production and sales processes of the companies (Zaim and Şevkli, 2002).

QFD usage provides various advantages to the companies such as highest level of customer satisfaction and better communication opportunities among multifunctional teams. Its usage causes an increase on product quality, process efficiency and company revenue vice versa a decrease on production cost and product designing time.

QFD consists of four matrices that output of each matrix becomes the input of the next matrix. Four of the matrices are to represent the four phases of the processes: product planning, part deployment, process planning and product planning (Liu and Wang, 2010) as shown in Figure 2.1. Abdolshah and Moradi (2013) revealed that majority of the companies and the research papers apply only the first phase of the QFD according to the time and cost concerns.

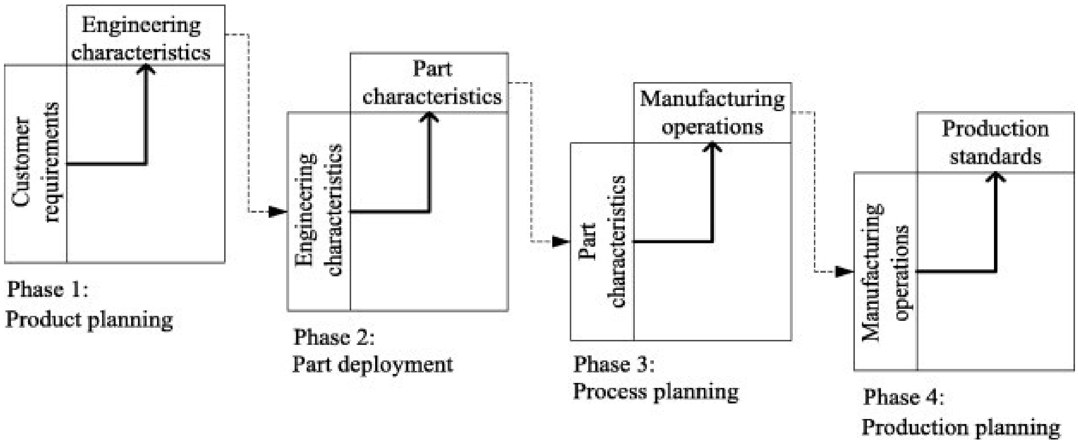


Figure 2.1: Phases of QFD (Liu and Wang, 2010).

Each step is better to be performed by the responsible departments in the firms. Brief (2012) proposed the product planning phase to be led by the marketing department, product design phase to be conducted by the engineering department, the process planning phase by manufacturing engineers and the last step, process control by the quality assurance department. Likewise, Zaim and Şevkli (2002) stated that QFD is a multidisciplinary process which needs to be done by diverse teams as marketing, design engineering, manufacturing engineering etc.

The first matrix of the process whose inputs (CRs) are being translated into the outputs as the DRs is also called as the HOQ matrix which is named by its shape that looks like a house with a roof. As it directly affects the next steps, the accuracy of this first matrix is vital for an accurate result at the end.

Iqbal et al. (2016) divide the parts of a HOQ into two as compulsory and optional. Compulsory parts are CRs, DRs, importance evaluations of CRs, and the relationship matrix between CRs and DRs which forms the body of the HOQ while the optional ones are the competitor analysis, sales point data, correlations between DRs, practical considerations, time to develop, etc. It is stated in their study that most of the researchers tend to ignore the optional parts of the HOQ presumably due to cost, time and evaluation difficulties concerns. It is also a major problem of those parts to integrate with the compulsory parts at the final computation of the absolute importance rankings of DRs. Nonetheless it is still important to take these optional parts into consideration since they all contain information that may affect the importance rankings of the DRs and misusage of them may cause inconvenient results. For instance, it is critical to know the correlation between DRs to reduce the duplication of effort if a positive relation exists between two DRs, which causes simultaneous satisfaction in both DRs when you enhance one; on the contrary if the correlation is negative, achievement of one may oppositely influence the other and lastly the third option is the low or zero relation which indicates mutually independence thus the necessity to satisfy the DRs on an individual basis to be able to meet entire CRs (Iqbal et al., 2016).

One of the novelties that this thesis provides to the literature is that it considers the optional parts like correlation matrix between DRs, direction of improvement for DRs, organizational difficulty, target values to be reached for the DRs, and the competitor analysis for CRs and DRs.

The most common way to collect the CRs to build the HOQ and gather its inputs is to conduct surveys, interviews or focus group interviews with the customers. Since the customers generally tend to define their expectations by using linguistic terms rather than exact or sharp numerical values such as *very long life, high quality but quite cheap or very low noise*; this causes a vagueness and impreciseness in these definitions. Hence the conventional QFD is inadequate to overcome this problem, FQFD is developed by the researchers after L.A. Zadeh presented the fuzzy set theory in 1965. Classical QFD's most common rate of importance scale is 1 to 5 crisp numbers, which usually causes a degree of information loss while FQFD uses linguistic variables that also might be imprecise or incomplete rather than crisp numbers that can be approximated to numerical precision and exactness under favor of fuzzy sets usage. There is a consensus in the literature for the effectiveness, reliability, accuracy and more meaningful results than classical QFD approaches that the FQFD brings.

The methods in the literature for capturing this uncertainty are generally extended by fuzzy sets. Fuzzy sets are one of the most used tools for handling the uncertainty. Ordinary fuzzy sets have been extended to type-2 fuzzy sets, hesitant fuzzy sets, intuitionistic fuzzy sets, PFSs, and neutrosophic sets in the last two decades and Liu (2011) classifies the methods integrated with FQFD under more than nine titles as follows: conventional QFD computation using fuzzy variables, fuzzy outranking, entropy, fuzzy tendency analysis, fuzzy multiple criteria decision making (MCDM), fuzzy integral, fuzzy analytic network process, fuzzy expected value, fuzzy goal programming, fuzzy expert systems etc. Another classification of the models to develop FQFD is asserted by Abdolshah and Moradi (2013). They categorize the models into eight titles: fuzzy linear and nonlinear programming models, MCDM models, fuzzy group DM models, metaheuristic methods, fuzzy regression models (linear and nonlinear), models proposed to prioritize CRs, hybrid models, and other methods.

It is a necessity to make an accurate decision on which FQFD method to use for the success of the implementation. In most of the studies, triangular fuzzy numbers are used due to their easiness on computations rather than other types such as trapezoidal fuzzy numbers and LR fuzzy numbers (Abdolshah and Moradi, 2013).

In this thesis, all these linguistic terms are represented by PFSs to give a larger domain for experts to assign membership and non-membership degrees. This is the superiority of PFS with respect to intuitionistic fuzzy sets.

PFSs have not yet been used in the design of products. The originality of this thesis is the first time usage of PFSs in a QFD study and a comprehensive fuzzy approach to model the relations in HOQ.

3. LITERATURE REVIEW ON FQFD

To gain a better understanding about the past studies on FQFD, a literature survey is conducted in this paper. Scopus is chosen as the main database to search for the related articles on FQFD. 128 document results are reached when the searching results are limited to the keyword "Fuzzy Quality Function Deployment" in article titles, abstracts, and keywords. The analyses of searching results are given by graphical illustrations in the following.As shown in Figure 3.1, the first article is published in 1994 on FQFD and the highest publication rate is reached in 2016 with 17 articles.

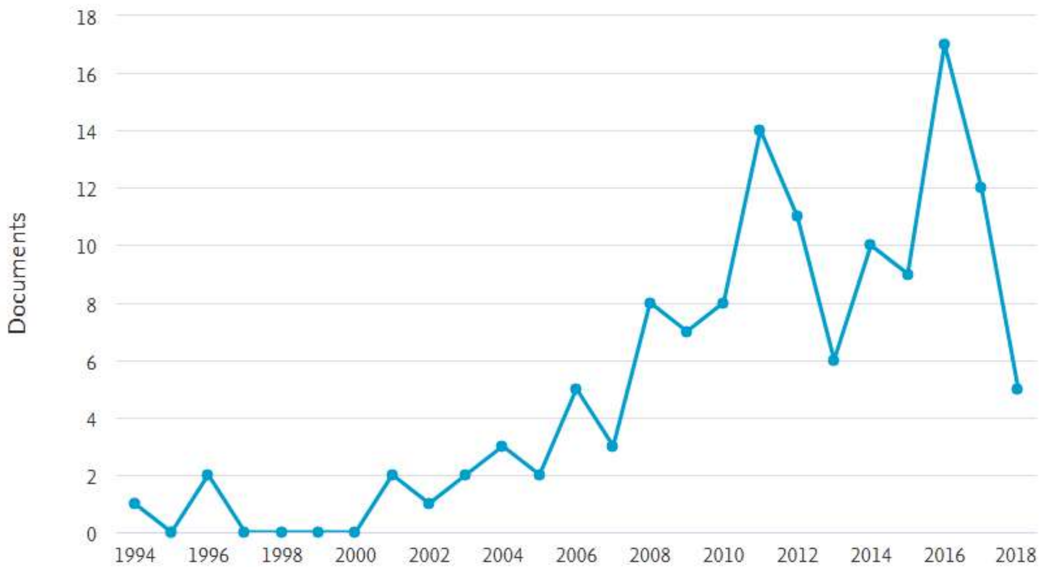


Figure 3.1: Frequencies of QFD publications over the years.

In Figure 3.2, the subject areas that FQFD is applied are illustrated. They are *engineering* with 26%, *computer science* with 18%, *business, management and accounting* with 17%, *mathematics* with 9%, *decision sciences* with 9%, *social sciences* with 8%, and *others* with 13%.

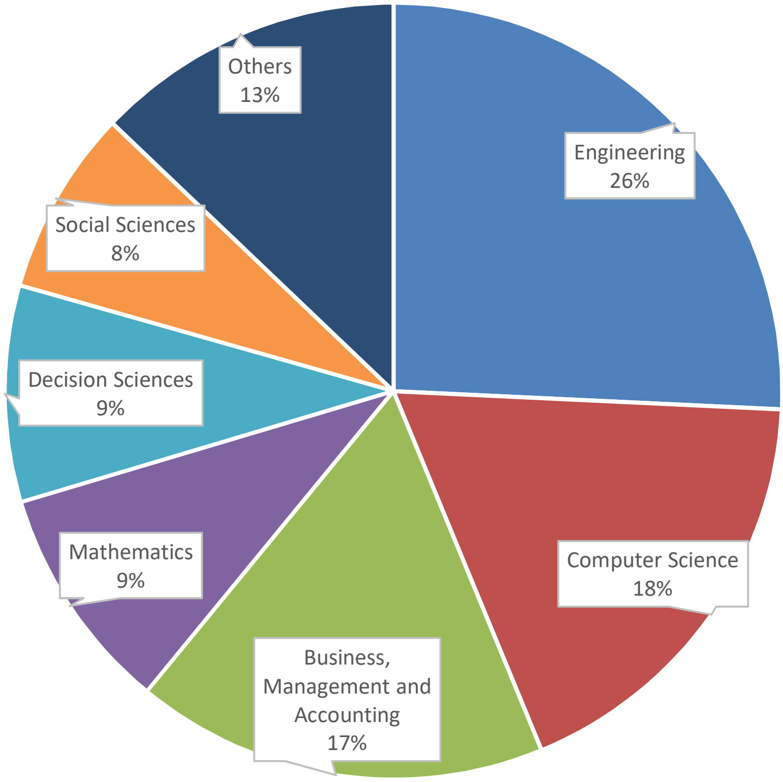


Figure 3.2: FQFD subject areas.

First publication on Scopus.com about FQFD was written by Bahrami (1994). He classified design into four: creative design, innovative design, redesign, and routine design. In the paper he focused on routine design and proposed a method by using information content and FQFD. He utilized from the probability values and information content to deal with design and validation issues of a manufacturing system that produces parts.

In Table 3.1 some of the most cited FQFD articles on Scopus.com are shown and below their reviews are given in publication year based order.

Table 3.1: Most cited FQFD articles on Scopus.com.

	Document title	Authors	Year	Source	Citation Frequency
1	Framework of a fuzzy quality function deployment system	Khoo, L.P., Hot, N.C.	1996	International Journal of Production Research 34(2), pp. 299-311	185
2	A Fuzzy Quality Function Deployment (FQFD) model for deriving optimum targets	Vanegas, L.V., Labib, A.W.	2001	International Journal of Production Research 39(1), pp. 99-120	142
3	Product design resources optimization using a non-linear fuzzy quality function deployment model	Fung, R.Y.K., Tu, Y., Tang, J., Wang, D.	2002	International Journal of Production Research 40(3), pp. 585-599	113
4	A fuzzy quality function deployment system for buildable design decision-makings	Yang, Y.Q., Wang, S.Q., Dulaimi, M., Low, S.P.	2003	Automation in Construction 12(4), pp. 381-393	98
5	Fuzzy QFD for supply chain management with reliability consideration	Sohn, S.Y., Choi, I.S.	2001	Reliability Engineering and System Safety 72(3), pp. 327-334	90
6	Fuzzy group decision-making to multiple preference formats in quality function deployment	Büyüközkan, G., Feyzioğlu, O., Ruan, D.	2007	Computers in Industry 58(5), pp. 392-402	84
7	An integrated fuzzy QFD model proposal on routing of shipping investment decisions in crude oil tanker market	Celik, M., Cebi, S., Kahraman, C., Er, I.D.	2009	Expert Systems with Applications 36(3 PART 2), pp. 6227-6235	60
8	Product design and selection using fuzzy QFD and fuzzy MCDM approaches	Liu, H.-T.	2011	Applied Mathematical Modelling 35(1), pp. 482-496	59
9	The extension of fuzzy QFD: From product planning to part deployment	Liu, H.-T.	2009	Expert Systems with Applications 36(8), pp. 11131-11144	53
10	Rating design requirements in fuzzy quality function deployment via a mathematical programming approach	Liu, S.-T.	2005	International Journal of Production Research 43(3), pp. 497-513	45

Khoo and Hot (1996) defined the QFD as a planning methodology to convert the CRs to the convenient DRs. They used relation matrices to show the interrelations between CRs, DRs, and within CRs-DRs. They pointed out the likelihood of expressing these relations by using either linguistic or crisp values. They associated the linguistic values with fuzziness and vagueness. Their study concentrated on facilitating the documentation process and computerization of QFD. They emphasized the deficiency of automatically interpreting the linguistic variables and offered a method by using the possibility theory and fuzzy arithmetic to overcome this fuzziness. An illustration was given to demonstrate the method.

Vanegas and Labib (2001) defined QFD and its purpose. They discoursed that conventional QFD is only capable of finding the design targets empirically and inadequate to reach the optimum values. In their paper they proposed a model to obtain the optimum values by integrating fuzzy numbers to deal with the subjective nature of the assessments. Constraints like cost, technical difficulty and market position was also taken into account and a car door example was given.

Sohn and Choi (2001) focused on developing a FQFD model on supply chains' reliability issues in product planning stage and supply chain management. They proposed a fuzzy MCDM procedure to run some reliability tests.

Fung et al. (2002) claimed that QFD is a useful DM technique for both highly customized, batch or mass production. They addressed that most of the existing studies ignore the constraints in product design and the correlation matrix among DRs. In other saying they assume that each DR has no effect on the others. In this study they proposed a nonlinear fuzzy model which incorporates the resource factors in QFD planning. They also took the correlations among DRs into consideration.

Yang et al. (2003) investigated the applicability of the QFD to the conceptual stage of a building design and adaptability of HOQ to meet the CRs for buildable designs. They benefitted from the fuzzy sets usage to overcome the fuzziness. They presented an example to illustrate their DM method for early design stages of buildable designs.

Liu (2005) claimed that the key factor of satisfying the CRs for product development is identifying the critical DRs. He said that if the weights of CRs and the relation measures between CRs and DRs are fuzzy numbers, assigning the DRs are even harder. He proposed a method to prioritize the DRs without knowing their membership functions. He determined the ratings of the DRs by two different nonlinear programs and he also considered the correlation between the DRs and budget factors. He gave an example of a flexible manufacturing system.

Büyüközkan et al. (2007) proposed a fuzzy group DM approach to fuse multiple preference styles to respond CRs in product development with QFD in a better way and illustrated with a numerical hatch door example.

Liu (2009) discoursed on the fact that majority of the past researches were only focused on the first phase of QFD which is the product planning phase and tended to ignore the subsequent phases (part deployment, process planning, and production planning). Distinctly he aimed to develop an extended FQFD approach which focuses on all the phases from product planning to part deployment. He developed a more improved method to gather CRs in product planning and he further considered the competitive analysis.

Celik et al. (2009) extended the QFD principles towards shipping investment process via the originally proposed Ship of Quality (SoQ) framework. They integrated the Fuzzy Analytic Hierarchy Process and Fuzzy Axiomatic Design algorithms into the SoQ frame to involve quantitative outcomes into the shipping investment decisions.

Liu (2011) stated that the past studies are mostly failed at three issues about QFD. They mostly ignored determining the subsequent prototype product selection, used fuzzy number algebraic operations to calculate the fuzzy sets in QFD which may cause a great deviation in the result from the correct value and did not apply competitive analysis in QFD. To overcome these issues, he integrated fuzzy QFD and the prototype product selection model and proposed MCDM approach to select the best prototype product.

FQFD has found a variety of research and application areas from past to the present. It still continues to get the attention in the current studies. Wang et al. (2017) developed the cloud-based production service concept for modeling a scalable and interoperable ordering system for self-service restaurants. They realized it through a structural and empirical service design using an integration of TRIZ, service QFD, and service blueprint approaches. He et al. (2017) proposed an improved Kano model named as importance-frequency Kano model and integrates it into QFD. The model adopts the logical Kano classification criteria to categorize CRs. Akkawuttiwanich and Yenradee (2018) proposed a new FQFD approach to manage the Supply Chain Operations Reference key performance indicators, which are widely used to measure supply chain performances by industrial practitioners. Jafarzadeh et al. (2018) proposed an integrated method which combines QFD, fuzzy logic, and data envelopment analysis. Babbar and Amin (2018) developed a novel QFD model to determine a set of suppliers and the order quantity. Their model is composed of a two-stage QFD and a stochastic multi-objective mathematical model.

4. PYTHAGOREAN FUZZY SETS

Atanassov (1999), introduced PFS by following the definition of the concept of an intuitionistic fuzzy set but named it as “Second Type Intuitionistic Fuzzy Sets” rather than PFSs. He defined the intuitionistic fuzzy sets of second type, the degree of membership, non-membership and non-determinacy, some operations and relations, and geometric interpretations. Yager (2013) introduced PFSs and its basic set operations. Furthermore, Yager and Abbasov (2013) investigated the link between Pythagorean membership degrees and complex numbers. They develop a MCDM method involving Pythagorean fuzzy geometric mean and the order weighted geometric operator. Yager (2014) introduced aggregation operations for PFS. Zhang and Xu (2014) developed Pythagorean fuzzy TOPSIS. Peng and Yang (2015) defined division and subtraction operations and their properties for PFSs. Moreover, they introduced a Pythagorean fuzzy superiority and inferiority ranking method in order to tackle multiple attribute group DM problems under uncertainty. Zhang (2016) proposed a closeness index-based ranking method for Pythagorean fuzzy numbers. Zhang et al. (2016) discussed IVPFS and their basic operations. Then, they introduced Pythagorean fuzzy QUALIFLEX method to deal with MCDM problems. Dick et al. (2016) studied complex valued membership grades by using PFS. Zhang et al. (2016) introduced Pythagorean fuzzy multi-granulation rough sets. Garg (2016) indicated the weakness of correlation coefficients between intuitionistic fuzzy sets and developed a new correlation coefficient and weighted correlation coefficient formulation between PFS. Garg (2017) proposed an improved accuracy function for the ranking order of IVPFS to eradicate the weakness of the current score and accuracy functions. Ilbahar et al. (2018) developed an integrated methodology including Pythagorean fuzzy AHP and a fuzzy inference system and used it for risk assessment in the field of occupational health and safety.

Yager (2013, 2014) introduced the PFS based on the idea that the sum of membership degree and non-membership degree may be larger than 1, but their square sum must be less than or equal 1.

Definition 1. Let X be a fix set. A single-valued PFS $\tilde{P}$ is an object having the form (Peng and Yang, 2016):

$$\tilde{P} = \{\langle x, (\mu_p(x), v_p(x)\rangle |\ x \in X\} \tag{4.1}$$

where the function $\mu_p\colon X \to [0,1]$ defines the degree of membership and $v_p\colon X \to [0,1]$ defines the degree of non-membership of the element $x \in X$ to P, respectively, and, for every $x \in X$, it holds that:

$$0 \leq \left(\mu_p(x)\right)^2 + \left(v_p(x)\right)^2 \leq 1 \tag{4.2}$$

The degree of hesitancy is given by equation 4.3:

$$\pi_P(x) = \sqrt{1 - \left(\mu_p(x)\right)^2 - \left(v_p(x)\right)^2} \tag{4.3}$$

or

$$\pi_p^2(x) = 1 - \left(\mu_p(x)\right)^2 - \left(v_p(x)\right)^2 \tag{4.4}$$

Definition 2. Let $\tilde{A} = \langle[\mu_A^L, \mu_A^U], [\ v_A^L,\ v_A^U]\rangle$, $\tilde{B} = \langle[\mu_B^L, \mu_B^U], [v_B^L, v_B^U]\rangle$ be two IVPFNs, and $\lambda > 0$, then the operations of these two IVPFNs are defined as follows (Peng and Yang, 2015):

$$\tilde{A} \oplus \tilde{B} = \left(\begin{bmatrix}\sqrt{(\mu_A^L)^2 + (\mu_B^L)^2 - (\mu_A^L)^2(\mu_B^L)^2}, \\ \sqrt{(\mu_A^U)^2 + (\mu_B^U)^2 - (\mu_A^U)^2(\mu_B^U)^2}\end{bmatrix}, [\ v_A^L v_B^L,\ v_A^U v_B^U]\right) \tag{4.5}$$

$$\tilde{A} \otimes \tilde{B} = \left(\begin{matrix}[\ \mu_A^L \mu_B^L,\ \mu_A^U \mu_B^U], \\ \left[\sqrt{(v_A^L)^2 + (v_B^L)^2 - (v_A^L)^2(v_B^L)^2}, \sqrt{(v_A^U)^2 + (v_B^U)^2 - (v_A^U)^2(v_B^U)^2}\right]\end{matrix}\right) \tag{4.6}$$

$$\lambda\tilde{A} = \left(\left[\sqrt{1 - (1 - (\mu_A^L)^2)^\lambda}, \sqrt{1 - (1 - (\mu_A^U)^2)^\lambda}\ \right], [\ (v_A^L)^\lambda, (v_A^U)^\lambda]\right) \tag{4.7}$$

$$(\tilde{A})^\lambda = \left([(\mu_A^L)^\lambda, (\mu_A^U)^\lambda], \left[\sqrt{1 - (1 - (v_A^L)^2)^\lambda}, \sqrt{1 - (1 - (v_A^U)^2)^\lambda}\right]\right) \tag{4.8}$$

Definition 3. Let $\tilde{A}_i$, j=1,2,…,k be a collection of IVPFNs. Then, their aggregated value using IVPF Weighted Geometric operator is also an IVPFN satisfying (Rahman et al., 2017)

$IVPFWG_{\lambda}(\tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \dots, \tilde{A}_k) =$

$$\left(\left[\prod_{i=1}^{k}(\mu_{A_i}{}^{L})^{\lambda_i}, \prod_{i=1}^{k}(\mu_{A_i}{}^{U})^{\lambda_i}\right], \left[\sqrt{1-\prod_{i=1}^{k}\left(1-\left(v_{A_i}^{L}\right)^2\right)^{\lambda_i}}, \sqrt{1-\prod_{i=1}^{k}\left(1-\left(v_{A_i}^{U}\right)^2\right)^{\lambda_i}}\right]\right) \quad (4.9)$$

where $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_k)^T$ is the weighted vector of $\tilde{A}_i$, i=1,2,…,k with $\lambda_i \epsilon [0,1]$ and $\sum_{i=1}^{k} \lambda_i = 1$.

Definition 4. Let $\tilde{A} = \langle[\mu_L, \mu_U], [v_L, v_U]\rangle$ be an IVPFN, π_L and π_U are the hesitancy degree of the lower and upper points of A, respectively, can be calculated as in equations 10 and 11:

$$\pi_U^2 = 1 - (\mu_L^2 + v_L^2) \quad (4.10)$$

$$\pi_L^2 = 1 - (\mu_U^2 + v_U^2) \quad (4.11)$$

It is known that the score functions or defuzzification functions are efficient when they are compared with PFNs in multiple attribute DM problems. However, the score functions in the literature (Ren et al., 2016; Zeng et al., 2017), are insufficient to indicate which PFN is higher than the other since they don't associate the hesitancy properly. Hence, a new defuzzification function is defined as in Definition 5.

Definition 5. Let $\tilde{A} = \langle[\mu_L, \mu_U], [v_L, v_U]\rangle$ be an IVPFN. The defuzzification of this number is calculated by equation 4.12:

$$P_D(\tilde{A}) = \frac{\left(\mu_L^2+\mu_U^2+(1-v_L^2-\pi_U^2)+(1-v_U^2-\pi_L^2)+\sqrt{\mu_L^2 * \mu_U^2}+\sqrt[4]{(1-v_L^2-\pi_U^2)*(1-v_U^2-\pi_L^2)}\right)}{6} \quad (4.12)$$

A larger value of $P_D(\tilde{A})$ indicates a larger $\tilde{A}$. Since $0 \leq \mu_U^2 + v_U^2 \leq 1$, $P_D(\tilde{A}) \epsilon$ [0,1].

Definition 6. Let $A = \langle[\mu_L^A, \mu_U^A], [v_L^A, v_U^A]\rangle$ and $B = \langle[\mu_L^B, \mu_U^B], [v_L^B, v_U^B]\rangle$ be IVPFNs and π_L^A, π_U^A, π_L^B and π_U^B are the hesitancy degrees of lower and upper points of the A and B, respectively. The distance between A and B can be calculated as in equation 4.13 (Rahman et al., 2017):

$$d(A,B) = \frac{\sqrt{2}}{4}\left(\sqrt{(\mu_A^L - \mu_B^L)^2 + (v_A^L - v_B^L)^2} + \sqrt{(\mu_A^U - \mu_B^U)^2 + (v_A^U - v_B^U)^2}\right) \quad (4.13)$$

5. AN IVPF QFD MODEL

In the following, an IVPF QFD model based on the evaluations of three experts and three customers is presented. If any of evaluators has no opinion about the considered CRs or DRs, the opinions of the others are processed only. The proposed model is given by two phases and 11 steps as follows.

5.1 CR&DR Relation Analysis

Step 1: Define linguistic CRs and assign customer importance ratings by using Pythagorean fuzzy scale given in Table 5.1.1. CRs are rated by three customers as in Figure 5.1.1 by using the scale in Table 5.1.1.

Table 5.1.1: Linguistic and corresponding numerical scale for the weights of criteria.

Linguistic Term	IVPF Number
Certainly Low Importance (CLI) / Certainly Low Satisfactory (CLS) / Certainly Low Relation (CLR) / Certainly Low Difficulty (CLD)	([0.10,0.30],[0.70,0.90])
Very Low Importance (VLI) / Very Low Satisfactory (VLS) / Very Low Relation (VLR) / Very Low Difficulty (VLD)	([0.20,0.40],[0.60,0.80])
Low Importance (LI) / Low Satisfactory (LS) / Low Relation (LR) / Low Difficulty (LD)	([0.30,0.50],[0.50,0.70])
Medium Level Importance (MLI) / Medium Level Satisfactory (MLS) / Medium Level Relation (MLR) / Medium Level Difficulty (MLD)	([0.40,0.60],[0.40,0.60])

Table 5.1.1 (continued): Linguistic and corresponding numerical scale for the weights of criteria.

Linguistic Term	IVPF Number
High Importance (HI) / High Satisfactory (HS) / High Relation (HR) / High Difficulty (HD)	([0.50,0.70],[0.30,0.50])
Very High Importance (VHI) / Very High Satisfactory (VHS) / Very High Relation (VHR) / Very High Difficulty (VHD)	([0.60,0.80],[0.20,0.40])
Certainly High Importance (CHI) / Certainly High Satisfactory (CHS) / Certainly High Relation (CHR) / Certainly High Difficulty (CHD)	([0.70,0.90],[0.10,0.30])

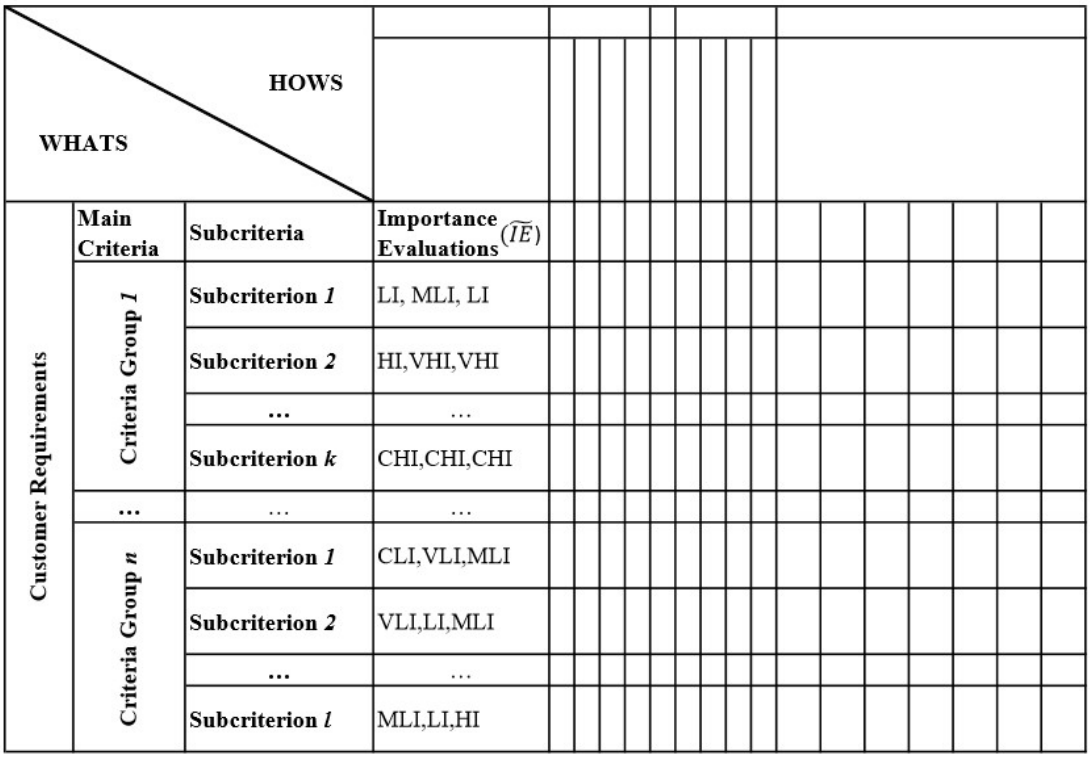


Figure 5.1.1: CRs and Customer Importance Ratings.

Step 2: Define the DRs; determine the direction of improvement of DRs and fill in the relationship matrix as in Figure 5.1.2.

Direction of Improvement				↑	↑		↓		↑	↓		↑
WHATS \ HOWS				Criteria Group *1*				…	Criteria Group *m*			
			Design Requirements	Subcriterion *1*	Subcriterion *2*	⋮	Subcriterion *r*	⋮	Subcriterion *1*	Subcriterion *2*	⋮	Subcriterion *s*
Customer Requirements	Main Criteria	Subcriteria	Importance Evaluations $(\widetilde{IE})$	$\tilde{R}_{11}$	$\tilde{R}_{12}$		$\tilde{R}_{1r}$		$\tilde{R}_{m1}$	$\tilde{R}_{m2}$		$\tilde{R}_{ms}$
	Criteria Group *1*	Subcriterion *1*	LI, MLI, LI	LR, MLR, LR			MLR, HR, LR		CHR, CHR, CHR			LR, LR, MLR
		Subcriterion *2*	HI,VHI,VHI		CLR, VLR		CLR, CLR, CLR			CHR, CHR, CHR		LR, HR, CHR
		…	…									
		Subcriterion *k*	CHI,CHI,CHI		MLR, MLR, VLR				LR, HR, HR			LR, CLR, VLR
	…	…	…									
	Criteria Group *n*	Subcriterion *1*	CLI,VLI,MLI		CLR, CLR, CLR		LR, MLR, LR			HR, HR, MLR		
		Subcriterion *2*	VLI,LI,MLI	VLR, CLR, MLR					HR, LR, LR	VHR, HR, VHR		
		…	…									
		Subcriterion *l*	MLI,LI,HI		HR, HR, CHR		VHR, VLR, MLR			VLR, LR, VLR		LR, LR, HR

Figure 5.1.2: DRs, their Direction of Improvement and the Relationship Matrix.

Step 3: Determine the level of organizational difficulty of the DRs by using the Pythagorean fuzzy scale given in Table 5.1.1 as in Figure 5.1.3.

WHATS \ HOWS				Criteria Group *1*				…	Criteria Group *m*			
			Design Requirements	Subcriterion *1*	Subcriterion *2*	⋮	Subcriterion *r*	⋮	Subcriterion *1*	Subcriterion *2*	⋮	Subcriterion *s*
Organizational Difficulty				MLD, HD, MLD	LD, VLD, MLD	…	CHD, CHD, VHD	…	MLD, LD, HD	CLD, CLD, CLD	…	LD, LD, VLD

Figure 5.1.3: Organizational Difficulty of the DRs.

Step 4: Determine the target values of DRs (How Muches) by using crisp numbers (α,β,…,η) in order to standardize the products as in Figure 5.1.4.

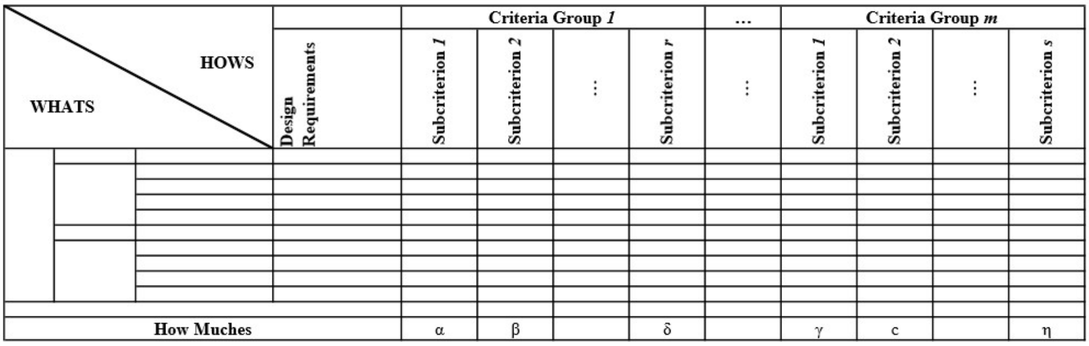


Figure 5.1.4: Target values of DRs.

Step 5: Construct the correlation matrix among DRs as given in Figure 5.1.5 by using the Pythagorean fuzzy scale given in Table 5.1.2. Positive correlation (blue color) is denoted with PC and negative correlation (red color) with NC in the correlation matrix where PC means that the values of two DRs increase or decrease simultaneously by the change of the other. NC means that while the value of one DR increases, it causes the value of the other to decrease or vice versa. If there is no correlation, the cell is left empty which means alteration in one DR does not affect the other DR that the relation is investigating with. In Figure 5.1.5, each cell shows three assessments from three evaluators. If there are two fuzzy linguistic terms in one cell, it represents that only two experts state a correlation between those two DRs but not the third one.

Table 5.1.2: IVPF Correlation Scale.

Linguistic Term for Positive Correlation	**IVPF Number**	**Linguistic Term for Negative Correlation**
Certainly Low + Correlation (CLPC)	([0.10,0.30],[0.70,0.90])	Certainly Low - Correlation (CLNC)
Very Low + Correlation (VLPC)	([0.20,0.40],[0.60,0.80])	Very Low - Correlation (VLNC)
Low + Correlation (LPC)	([0.30,0.50],[0.50,0.70])	Low - Correlation (LNC)

Table 5.1.2 (continued): IVPF Correlation Scale.

Linguistic Term for Positive Correlation	IVPF Number	Linguistic Term for Negative Correlation
Medium Level + Correlation (MLPC)	([0.40,0.60],[0.40,0.60])	Medium Level - Correlation (MLNC)
High + Correlation (HPC)	([0.50,0.70],[0.30,0.50])	High - Correlation (HNC)
Very High + Correlation (VHPC)	([0.60,0.80],[0.20,0.40])	Very High - Correlation (VHNC)
Certainly High + Correlation (CHPC)	([0.70,0.90],[0.10,0.30])	Certainly High - Correlation (CHNC)

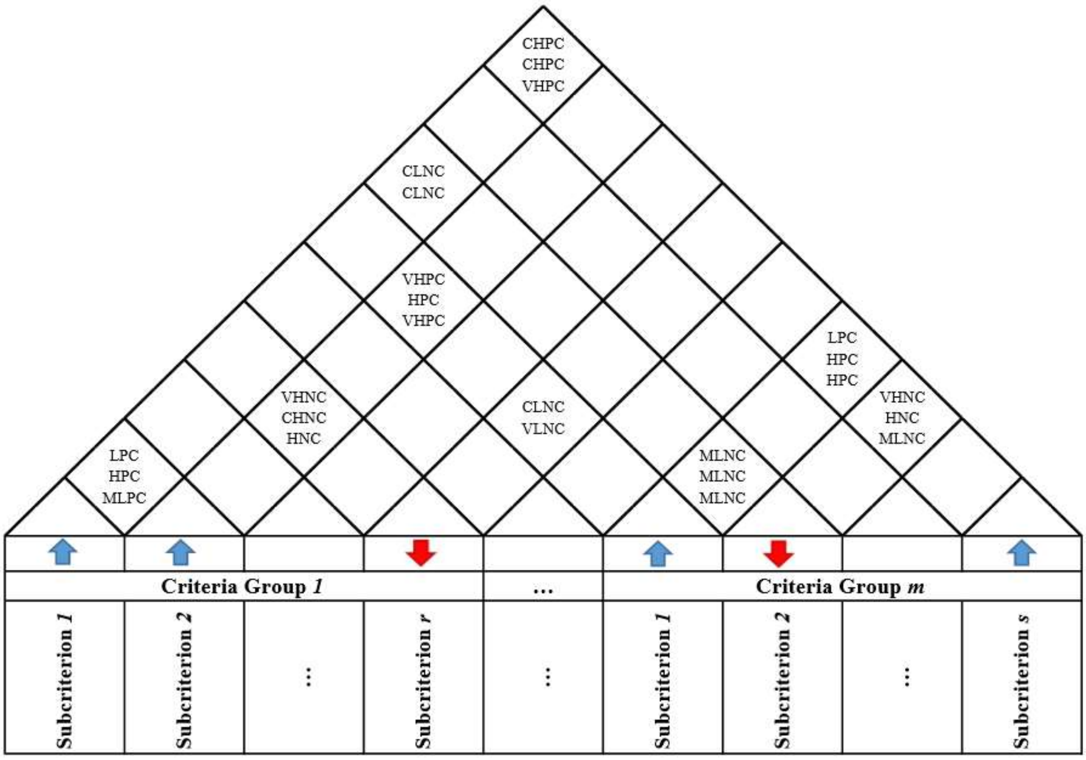


Figure 5.1.5: Correlation Matrix.

Step 6: To obtain the absolute importance $(\widetilde{AI})$ for each DR, first multiply the aggregated linguistic importance evaluations $(\widetilde{IE})$ of CRs and the aggregated linguistic terms $(\tilde{R})$ in the relationship matrix. Then multiple the result with the aggregated correlation impact factor $(\widetilde{CI})$ and then divide it by relative organizational difficulty

($\widetilde{ROD}$) as in equation 5.1.1. Organizational difficulty ($\widetilde{OD}$) means how difficult it is for an organization to achieve a certain DR. The linguistic assessments for $\widetilde{OD}$ of each DR are aggregated before calculating ($\widetilde{ROD}$). Hence, it is aimed to decrease of the impact of DRs whose organizational difficulty is larger. Larger $\widetilde{ROD_j}$ values cause smaller $\widetilde{AI_j}$. Relative absolute importance $\widetilde{(RAI)}$ is obtained by equation 5.1.4. Arithmetic operations of PFSs must be considered while applying multiplication and addition. Relative importance and absolute importance values are shown in HOQ in Figure 5.1.6.

$$\widetilde{AI}_{ij} = \{(\oplus_{i=1}^{T} \widetilde{IE}_i \otimes \tilde{R}_{hj}) \otimes (1 + \widetilde{CI}_j)\} \oslash (1 + \widetilde{ROD}_j) \qquad (5.1.1)$$

where $h = 1,2, \dots, m,\ j = 1, 2, \dots, (r + \dots + s)$ and $T = k + \dots + l$

$$\widetilde{CI}_j = \left(n_{c_j}/(j-1)\right) * (\widetilde{\widetilde{pc}}_j \ominus \widetilde{\widetilde{nc}}_j) \qquad (5.1.2)$$

$$\widetilde{ROD}_j = \left(\frac{\widetilde{OD}_j}{\oplus_1^{r+\dots+s} \widetilde{OD}_j}\right) \qquad (5.1.3)$$

where $\widetilde{-1} \leq \widetilde{CI}_j \leq \widetilde{+1}$ and

n_{c_j}: the number of correlations of DR_j with the other DRs

$\widetilde{\widetilde{pc}}_j$: average of the positive correlations of DR_j

$\widetilde{\widetilde{nc}}_j$: average of the negative correlations of DR_j

Fuzzy Relative absolute ($\widetilde{RA}$) importance values are obtained by equation 5.1.4.

$$\widetilde{RA}_{ij} = \widetilde{AI}_{ij} \oslash (\oplus_{j=1}^{r+\dots+s} \widetilde{AI}_{ij}), i = 1,2, \dots, T \qquad (5.1.4)$$

WHATS / HOWS		Design Requirements	Criteria Group *1*				…	Criteria Group *m*			
			Subcriterion *1*	Subcriterion *2*	⋮	Subcriterion *r*	⋮	Subcriterion *1*	Subcriterion *2*	⋮	Subcriterion *s*
Customer Requirements — Main Criteria	Subcriteria	Importance Evaluations ($\widetilde{IE}$)	$\tilde{R}_{11}$	$\tilde{R}_{12}$		$\tilde{R}_{1r}$		$\tilde{R}_{m1}$	$\tilde{R}_{m2}$		$\tilde{R}_{ms}$
Criteria Group *1*	Subcriterion *1*	LI, MLI, LI	LR, MLR, LR			MLR, HR, LR		CHR, CHR, CHR			LR, LR, MLR
	Subcriterion *2*	HI,VHI,VHI		CLR, VLR		CLR, CLR, CLR			CHR, CHR, CHR		LR, HR, CHR
	…	…									
	Subcriterion *k*	CHI,CHI,CHI		MLR, MLR, VLR				LR, HR, HR			LR, CLR, VLR
…	…	…									
Criteria Group *n*	Subcriterion *1*	CLI,VLI,MLI		CLR, CLR, CLR		LR, MLR, LR			HR, HR, MLR		
	Subcriterion *2*	VLI,LI,MLI	VLR, CLR, MLR					HR, LR, LR	VHR, HR, VHR		
	…	…									
	Subcriterion *l*	MLI,LI,HI		HR, HR, CHR		VHR, VLR, MLR			VLR, LR, VLR		LR, LR, HR
Absolute Importance ($\widetilde{AI}$)			$\widetilde{AI}_{11}$	$\widetilde{AI}_{12}$		$\widetilde{AI}_{1r}$		$\widetilde{AI}_{m1}$	$\widetilde{AI}_{m2}$		$\widetilde{AI}_{ms}$
Relative Absolute Importance (%)($\widetilde{RA}$)			$\widetilde{RA}_{11}$	$\widetilde{RA}_{12}$		$\widetilde{RA}_{1r}$		$\widetilde{RA}_{m1}$	$\widetilde{RA}_{m2}$		$\widetilde{RA}_{ms}$

Figure 5.1.6: Relative Importance and Absolute Importance Values.

Step 7: Rank the DRs with respect to $\widetilde{RA}_j$ values. Let $\widetilde{RA}_j = \langle [\mu^L_{\widetilde{RA}_j}, \mu^U_{\widetilde{RA}_j}], [v^L_{\widetilde{RA}_j}, v^U_{\widetilde{RA}_j}] \rangle$ be an IVPFN and $\pi^L_{\widetilde{RA}_j}, \pi^U_{\widetilde{RA}_j}$ are the hesitancy degree of lower and upper points of $\widetilde{RA}_j$, then the deffuzzifying procedure of this number calculated by equation 4.12. The highest RA_j indicates the most important DR that the design engineers must take into account in the design phase of a new product.

5.2 Competitive Analysis

Step 8: Determine the linguistic customer ratings of the competition by using the Pythagorean fuzzy scale given in Table 5.1.1 as in Figure 5.2.1. The linguistic customer ratings of the competition are assigned by multiple experts by using the scale in Table 5.1.1.

WHATS \ HOWS			Design Requirements	Customer Rating (O: Our Company; C1: Company 1; C2: Company 2)						
	Main Criteria	Subcriteria		CLS	VLS	LS	MLS	HS	VHS	CHS
Customer Requirements	Criteria Group *1*	Subcriterion *1*			O,C1	C1, C1,O,O	C2	C2,C2		
		Subcriterion *2*				C2	C2	C1,C2	O,O,C1,C1	O
		…								
		Subcriterion *k*			C1,C2	C1,C2,C2	O,O,C1	O		
	…	…								
	Criteria Group *n*	Subcriterion *1*		C1	C1,C2	O,C2	O,C1,C2	O		
		Subcriterion *2*					O,O,C1,C2	O,C1,C1,C2	C2	
		…								
		Subcriterion *l*						C1	O,C1,C2,C2	O,O,C1,C2

Figure 5.2.1: Linguistic Customer Ratings of the Competition.

To determine our position among the competitors, first linguistic customer ratings are aggregated with respect to the corresponding CR by using equation 4.9 and then the distances between our company and other companies ($\widetilde{D}_{O-C\ell}^{CR}$) are measured by using equation 5.1.5.

$$\widetilde{D}_{O-C\ell}^{CR} = \oplus_{t=1}^{T=k+\cdots+l}\left(\kappa_{O-C\ell}^{CR} \times d_t^{CR}(O, C\ell) \times \widetilde{w}_t^{CR}\right) \;\; , \ell = 1, \dots, \mathfrak{y} \quad (5.1.5)$$

where O and Cℓ represents our company and competitor ℓ, respectively. T represents the total number of sub-criteria of CRs. $\widetilde{w}_t^{CR}$ is the aggregated linguistic customer ratings with respect to the corresponding CR. $\kappa_{O-C\ell}^{CR}$ is defined as in equation 5.1.6.

$$\kappa_{O-C\ell}^{CR} = \begin{cases} +1, if\ O\ is\ better\ than\ C\ell \\ -1,\ if\ C\ell\ is\ better\ than\ O \\ 0, if\ O\ \ is\ equal\ to\ C\ell \end{cases} \quad (5.1.6)$$

$d_t^{CR}(O, C\ell)$ is calculated by equation 5.1.7:

$$d_t^{CR}(O, C\ell) = \frac{\sqrt{2}}{4}\left(\sqrt{(\mu_O^L - \mu_{Cl}^L)^2 + (v_O^L - v_{Cl}^L)^2} + \sqrt{(\mu_O^U - \mu_{Cl}^U)^2 + (v_O^U - v_{Cl}^U)^2}\right) \quad (5.1.7)$$

Step 9: Compare the DRs of our company with the other competitors by using the Pythagorean fuzzy scale given in Table 5.1.1 as in Figure 5.2.2.

WHATS \ HOWS		Design Requirements	Criteria Group *1*				…	Criteria Group *m*			
			Subcriterion *1*	Subcriterion *2*	…	Subcriterion *r*	…	Subcriterion *1*	Subcriterion *2*	…	Subcriterion *s*
Engineering Assessment	O: Our Company C1: Company 1 C2: Company 2	CLS							C1		
		VLS	C2					C1	O,C2		
		LS	C2			C2,C2		C2	C1,C1,C2		
		MLS	O,C1,C2		…		…	O,C1	O,C2	…	C1,C2,C2
		HS	O,O,C1	O,C1,C1,C2		O,O,C2		O,C1,C2	O		O,O,C1,C2
		VHS	C1	O,C1,C2		C1,C1		C2			
		CHS		O,C2		O,C1		O			O,C1

Figure 5.2.2: Linguistic DR Ratings of the Competition.

To determine our position among the competitors, first linguistic engineering assessments are aggregated with respect to the corresponding DR and then the distances between our company and other companies ($\widetilde{D}_{O-C\ell}^{DR}$) are measured by using equation 5.1.8.

$$\widetilde{D}_{O-C\ell}^{DR} = \oplus_{t=1}^{T=r+\cdots+s}\left(\kappa_{O-C\ell}^{DR} \times d_t^{DR}(O,C\ell) \times \widetilde{AI}_{ij}\right) \ , \ \ell = 1,\ldots,\mathfrak{y} \qquad (5.1.8)$$

where O and Cℓ represents our company and competitor ℓ, respectively. T represents the total number of sub-criteria of DRs. $\kappa_{O-C\ell}^{DR}$ is defined as in equation 5.1.9.

$$\kappa_{O-C\ell}^{DR} = \begin{cases} +1, if\ O\ is\ better\ than\ C\ell \\ -1,\ if\ C\ell\ is\ better\ than\ O \\ 0, if\ O\ \ is\ equal\ to\ C\ell \end{cases} \qquad (5.1.9)$$

$d_t^{DR}(O,C\ell)$ is calculated by equation 5.1.10:

$$d_t^{DR}(O,C\ell) = \frac{\sqrt{2}}{4}\left(\sqrt{(\mu_O^L - \mu_{Cl}^L)^2 + (v_O^L - v_{Cl}^L)^2} + \sqrt{(\mu_O^U - \mu_{Cl}^U)^2 + (v_O^U - v_{Cl}^U)^2}\right) \qquad (5.1.10)$$

Step 10: Obtain the combined performance rating score ($\widetilde{CPR}$) of our company in order to determine our position among the competitors by considering both customer ratings and engineering assessments as in equation 5.1.11.

$$\widetilde{CPR} = \chi\widetilde{D}_{O-C\ell}^{CR} \oplus \theta\widetilde{D}_{O-C\ell}^{DR} \qquad (5.1.11)$$

where χ and θ are the importance coefficients of CR and DR, respectively, and $\chi + \theta = 1$.

Step 11: Determine the relative position of our company on a scale as in Figure 5.2.3.

Figure 5.2.3: Scale indicating the location of our company.

6. APPLICATION: SOLAR PV TECHNOLOGY DEVELOPMENT

6.1 Problem definition

To demonstrate the proposed Pythagorean FQFD methodology, a case study given by Lee et al. (2017) is selected that focuses on new product development for a PV solar cell manufacturer in Taiwan. The study claims that increasing environmental awareness and economical concerns direct more and more people to prefer solar energy. Nevertheless, the high cost and low conversion efficiency require producers to accomplish an active R&D study to overcome these issues.

In their study, after the literature review, the authors carry out interviews with experts and the Taiwanese PV solar cell firm managers to list the possible CRs and DRs. Since it is not possible to list all the potential requirements, Fuzzy Delphi Method is applied to limit the list to the most important ones. Then, the prepared questionnaire is directed to the customers to weigh both CRs and DRs by their importance degrees correspondingly. After a series of calculations, by the answers of 6 evaluators, 12 CR candidates are narrowed down to the most important 6 ones while 16 DR candidates are reduced to 8.
Their results indicate that the most important 6 CRs for a solar cell are as follows: *Conversion efficiency, Manufacturing process, Modular design, Material quality, Product stability,* and *Government policy*. The experts determine the following 8 DRs to meet the stated CRs: *Battery array density, Antireflection film, Velvet surface, Quality control measures of cells, Series elements in the module, Pass of IEC 61215 and UL1703 standards, Antireflection coated glass,* and *Thermal expansion treatment.* The explanations of CRs and DRs are also given by Lee et al. (2017) in their article as shown in Table 6.1.1 and Table 6.1.2.

Table 6.1.1: CRs' explanations (Lee et al., 2017).

CRs	Explanation
High conversion efficiency	Solar energy conversion efficiency of the new module is higher than that of the current module
Manufacturing process	Maturity of solar module sealing and assembly technology
Modular design	Design of the new module (appearance and size) allows for part sharing for future assembly of solar products
Material quality	Quality of materials to meet the needs of the module, such as the quality of inner and outer material and reinforced glass material to resist scratches and bending
Product stability	Product stability when operating the solar module. Stability will affect the life of the solar cell system under normal use
Government policy	Relevant laws, subsidies, tax breaks and other measures that could affect the production of modules demanded by solar assembly manufacturer

Table 6.1.2: DRs' explanations (Lee et al., 2017).

DRs	Explanation
Battery array density	Effective layout of cells in order to improve the conversion efficiency of the modul
Anti-reflection film	The use of anti-reflection film in the module in order to reduce the reflection of sunlight and thus to improve the efficiency of the module
Velvet surface	The use of velvet surface to increase the sunlight radiating into the module and to improve the efficiency of the module
Quality control measures of cells	A module contains a series of solar cells, and the malfunction of a solar cell will affect the efficiency of the entire module
Series elements in the module	When a short circuit occurs, the output of good solar cells will be exhausted on the bad solar cells, and the overall output of the module will be affected

Table 6.1.2 (continued): DRs' explanations (Lee et al., 2017).

DRs	Explanation
Pass of iec 61215 and ul1703 standards	The certifications of solar panel module qualification iec 61215 and flat-plate PV modules and panels ul1703 are required for module manufacturers
Anti-reflection-coated glass	Anti-reflective coating can provide a better light transmittance
Thermal expansion treatment	The treatment is required to prevent battery expansion due to temperature increase

These DRs and CRs are applied to the case study in order to illustrate the proposed model. However, the DRs in Lee et al.'s (2017) case study are slightly modified to adopt them to the model in this study. The DRs *antireflection film* and *antireflection coated glass* are modified to *reflective film* and *reflective coated glass*, respectively. The reason why the directions of improvement are converted to negative for these two DRs is to obtain negative correlations in the correlation matrix. The rest of the case continues with empirical analyses.

In the following, the IVPF QFD model is illustrated by applying the 11 steps given in Part 5 based on the evaluations of three customers and three experts.

6.2 CR&DR Relation Analysis

Step 1: Define linguistic CRs and assign customer importance ratings by using Pythagorean fuzzy scale given in Table 5.1.1. CRs are rated by three customers as in Figure 6.2.1 by using the scale in Table 5.1.1.

WHATS \ HOWS										
Customer Requirements	Subcriteria	Importance Evaluations								
	High conversion efficiency	VHI, CHI, CHI								
	Manufacturing process	VHI, HI,MLI								
	Modular design	LI, LI,LI								
	Material quality	MLI, LI, LI								
	Product stability	LI, VLI, LI								
	Government policy	CLI, CLI, VLI								

Figure 6.2.1: CRs and Customer Importance Ratings.

Step 2: Define the DRs; determine the direction of improvement of DRs and fill in the relationship matrix as in Figure 6.2.2.

Direction of Improvement			↑	↓	↑	↑	↑	↑	↓	↑
WHATS \ HOWS		Design Requirements	Battery array density	Reflective film	Velvet surface	Quality control measures of cells	Series elements in the module	Pass of IEC 61215 and UL1703 standards	Reflective coated glass	Thermal expansion treatment
Customer Requirements	Subcriteria	Importance Evaluations	$\tilde{R}_1$	$\tilde{R}_2$	$\tilde{R}_3$	$\tilde{R}_4$	$\tilde{R}_5$	$\tilde{R}_6$	$\tilde{R}_7$	$\tilde{R}_8$
	High conversion efficiency	VHI, CHI, CHI	MLR LR LR	VLR LR MLR	CLR VLR CLR	LR VLR VLR	CLR CLR	CLR CLR CLR	MLR LR VLR	LR LR VLR
	Manufacturing process	VHI, HI,MLI				LR MLR	VLR VLR LR	VHR VHR VHR		
	Modular design	LI, LI,LI	VLR VLR MLR				VLR LR VLR			VHR VHR CHR
	Material quality	MLI, LI, LI		MLR HR MLR	LR LR LR		CLR CLR VLR	VLR VLR VLR	VLR VLR CLR	MLR HR LR
	Product stability	LI, VLI, LI				VHR HR HR	VLR VLR VLR	CLR VLR VLR		HR HR
	Government policy	CLI, CLI, VLI						CHR CHR CHR		

Figure 6.2.2: DRs, their Direction of Improvement and the Relationship Matrix.

Step 3: Determine the level of organizational difficulty of the DRs by using the Pythagorean fuzzy scale given in Table 5.1.1 as in Figure 6.2.3.

WHATS / HOWS		Design Requirements	Battery array density	Reflective film	Velvet surface	Quality control measures of cells	Series elements in the module	Pass of IEC 61215 and UL1703 standards	Reflective coated glass	Thermal expansion treatment
Organizational Difficulty			VHD, VHD, HD	CLD,VLD, LD	LD, VLD, LD	MLD, MLD, LD	HD, MLD, VHD	MLD, VHD, HD	VLD, LD, CLD	CHD, CHD, VHD

Figure 6.2.3: Organizational Difficulty of the DRs.

Step 4: Determine the target values of DRs (How Muches) in order to standardize the products as in Figure 6.2.4.

WHATS / HOWS		Design Requirements	Battery array density	Reflective film	Velvet surface	Quality control measures of cells	Series elements in the module	Pass of IEC 61215 and UL1703 standards	Reflective coated glass	Thermal expansion treatment
How Muches			90 (Wh/Kg)	Relative transmission increase of 3.1% over the spectral range of 400–1100 nm	<5% reflection	100% In-line Inspection	72PV cells in series to charge 24V panel	Benchmark of 30% degradation	Increased transmission by up to 3.8%	20.3 (CTE) (10-6/K-1)

Figure 6.2.4: Target values of DRs.

Step 5: Construct the correlation matrix among DRs as given in Figure 6.2.5 by using the Pythagorean fuzzy scale given in Table 5.1.2. Positive correlation (blue color) is denoted with PC and negative correlation (red color) with NC in the correlation matrix where PC means that the values of two DRs increase or decrease simultaneously by the change of the other. NC means that while the value of one DR increases, it causes the value of the other to decrease or vice versa. If there is no correlation, the cell is left empty which means alteration in one DR does not affect the other DR that the relation is investigated with. In Figure 6.2.5, each cell shows three assessments from three evaluators. If there are two fuzzy linguistic terms in one cell, it represents that only two experts state a correlation between those two DRs but not the third one.

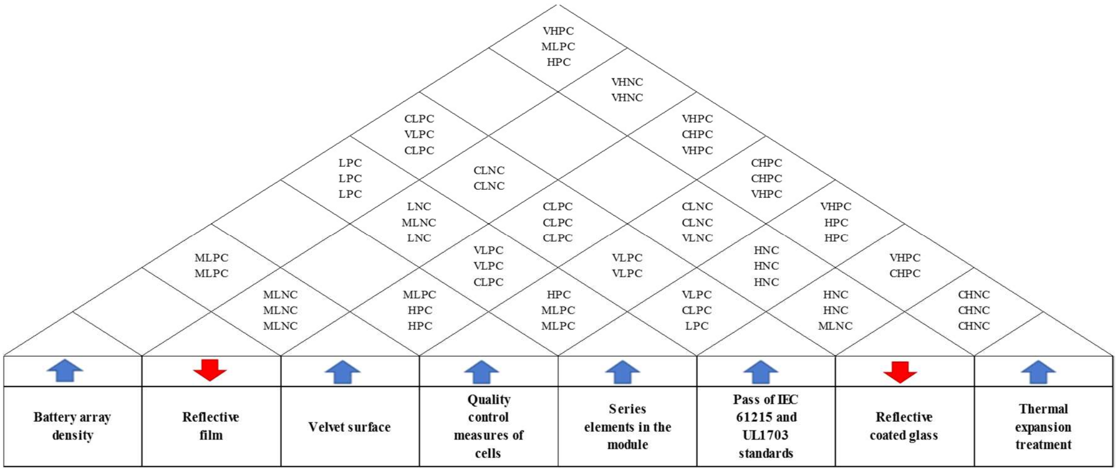


Figure 6.2.5: Correlation Matrix.

Step 6: To be able to apply this step and the formulations, first convert all the linguistic terms to the IVPF numbers by using Table 5.1.1 and aggregate all the views of the evaluators correspondingly as shown in the figures below. Each formula in Part 4 and Part 5 is illustrated at least once to provide a better understanding of the application.

Figure 6.2.6. represents each CR's IVPF values by each evaluator and their aggregated values.

			IVPF Importance Evaluations												Aggregated IVPF Importance Evaluations			
			Evaluator 1				Evaluator 2				Evaluator 3							
	Subcriteria	Linguistic Importance Evaluations	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U
Customer Requirements	High conversion efficiency	VHI, CHI, CHI	0.6000	0.8000	0.2000	0.4000	0.7000	0.9000	0.1000	0.3000	0.7000	0.9000	0.1000	0.3000	0.6649	0.8653	0.1418	0.3376
	Manufacturing process	VHI, HI,MLI	0.6000	0.8000	0.2000	0.4000	0.5000	0.7000	0.3000	0.5000	0.4000	0.6000	0.4000	0.6000	0.4932	0.6952	0.3131	0.5111
	Modular design	LI, LI,LI	0.3000	0.5000	0.5000	0.7000	0.3000	0.5000	0.5000	0.7000	0.3000	0.5000	0.5000	0.7000	0.3000	0.5000	0.5000	0.7000
	Material quality	MLI, LI, LI	0.4000	0.6000	0.4000	0.6000	0.3000	0.5000	0.5000	0.7000	0.3000	0.5000	0.5000	0.7000	0.3302	0.5313	0.4702	0.6707
	Product stability	LI, VLI, LI	0.3000	0.5000	0.5000	0.7000	0.2000	0.4000	0.6000	0.8000	0.3000	0.5000	0.5000	0.7000	0.2621	0.4642	0.5372	0.7389
	Government policy	CLI, CLI, VLI	0.1000	0.3000	0.7000	0.9000	0.1000	0.3000	0.7000	0.9000	0.2000	0.4000	0.6000	0.8000	0.1260	0.3302	0.6707	0.8746

Figure 6.2.6: Aggregated IVPF values of CRs.

To find the aggregated IVPF importance values for each CR, equation 4.9 is applied and the values encircled with red lines in Figure 6.2.6 are calculated as follows:

$$\mathbf{0.6649} = \ 0.6)^{1/3} \times (0.7)^{1/3} \times (0.7)^{1/3}$$

$$\mathbf{0.8653} = (0.8)^{1/3} \times (0.9)^{1/3} \times (0.9)^{1/3}$$

$$\mathbf{0.1418} = \sqrt{1-((1-((0.2)^2)^{1/3})\times(1-((0.1)^2)^{1/3})\times(1-((0.1)^2)^{1/3}))}$$

$$\mathbf{0.3376} = \sqrt{1-((1-((0.4)^2)^{1/3})\times(1-((0.3)^2)^{1/3})\times(1-((0.3)^2)^{1/3}))}$$

Figure 6.2.7 shows the aggregated IVPF values of the relationship matrix by using equation 4.9.

WHATS \ HOWS		Design Requirements															
		Battery array density				Reflective film				Velvet surface				Quality control measures of			
Customer Requirements	Subcriteria	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U
	High conversion efficiency	0.6649	0.8653	0.1418	0.3376	0.2884	0.4932	0.5111	0.7143	0.1260	0.3302	0.6707	0.8746	0.2289	0.4309	0.5703	0.7718
	Manufacturing process													0.3464	0.5477	0.4542	0.6547
	Modular design	0.2000	0.4000	0.6000	0.8000												
	Material quality					0.4309	0.6316	0.3705	0.5703	0.3000	0.5000	0.5000	0.7000				
	Product stability													0.5313	0.7319	0.2713	0.4702
	Government policy																

WHATS \ HOWS		Design Requirements															
		Series elements in the module				Pass of IEC 61215 and UL1703				Reflective coated glass				Thermal expansion treatment			
Customer Requirements	Subcriteria	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U
	High conversion efficiency	0.1000	0.3000	0.7000	0.9000	0.1000	0.3000	0.7000	0.9000	0.2884	0.4932	0.5111	0.7143	0.2621	0.4642	0.5372	0.7389
	Manufacturing process	0.2289	0.4309	0.5703	0.7718	0.6000	0.8000	0.2000	0.4000								
	Modular design	0.2289	0.4309	0.5703	0.7718									0.6316	0.8320	0.1735	0.3705
	Material quality	0.1260	0.3302	0.6707	0.8746	0.2000	0.4000	0.6000	0.8000	0.1587	0.3634	0.6377	0.8421	0.3915	0.5944	0.4114	0.6119
	Product stability	0.2000	0.4000	0.6000	0.8000	0.1587	0.3634	0.6377	0.8421					0.5000	0.7000	0.3000	0.5000
	Government policy					0.7000	0.9000	0.1000	0.3000								

Figure 6.2.7: Aggregated IVPF values of the Relationship Matrix.

Figure 6.2.8 shows the IVPF values of aggregated correlation matrix that calculated by using equation 4.9.

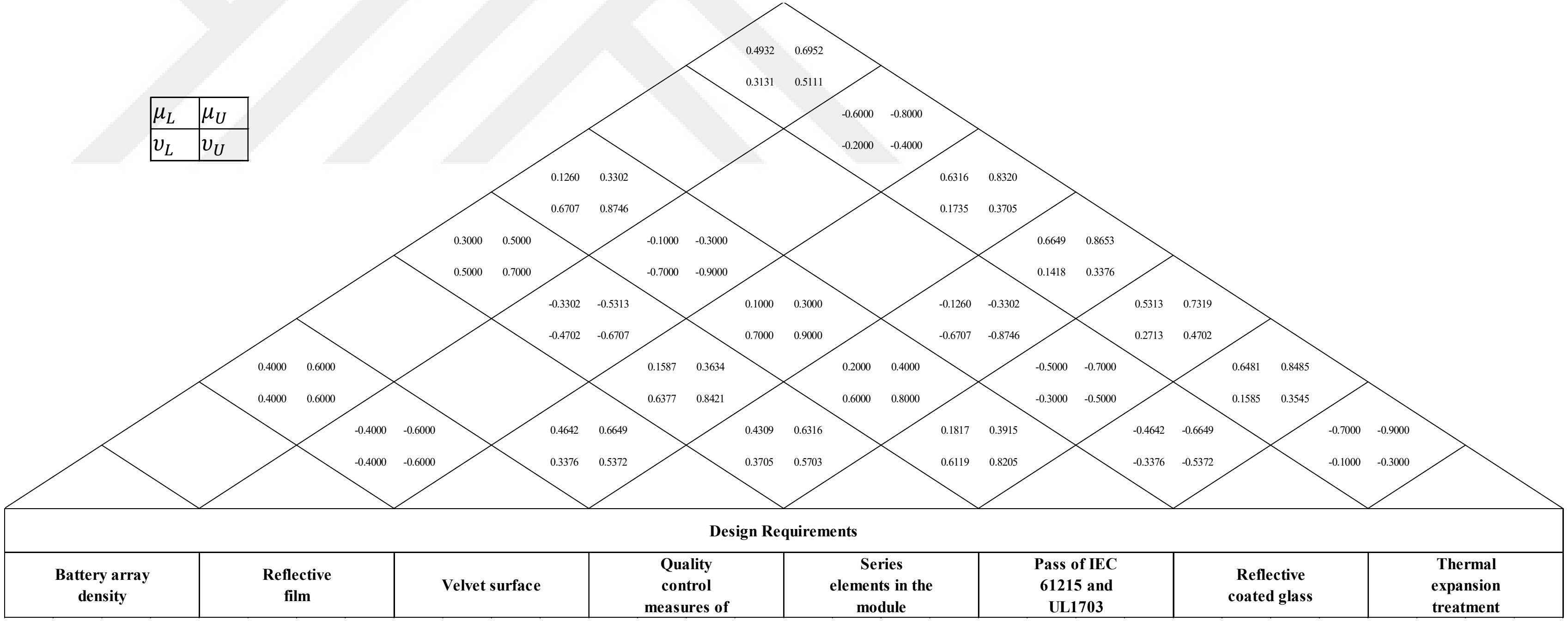


Figure 6.2.8: IVPF values of Aggregated Correlation Matrix.

Figure 6.2.9 shows the IVPF values of aggregated organizational difficulty values found by using equation 4.9.

	Design Requirements																															
	Battery array density				**Reflective film**				**Velvet surface**				**Quality control measures of**				**Series elements in the module**				**Pass of IEC 61215 and UL1703**				**Reflective coated glass**				**Thermal expansion treatment**			
	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U
Organizational Difficulty	0.5646	0.7652	0.2387	0.4372	0.1817	0.3915	0.6119	0.8205	0.2621	0.4642	0.5372	0.7389	0.3634	0.5646	0.4372	0.6377	0.4932	0.6952	0.3131	0.5111	0.4932	0.6952	0.3131	0.5111	0.1817	0.3915	0.6119	0.8205	0.6649	0.8653	0.1418	0.3376

Figure 6.2.9: IVPF values of Aggregated Organizational Difficulty.

To obtain the $(\widetilde{AI})$ for each DR, equation 5.1.1 must be applied but to do so first equations 5.1.2 and 5.1.3 should be applied to find $(\widetilde{CI})$ and $(\widetilde{ROD})$ that defined in Section 5 - Step 6 as shown in figures below.

Figure 6.2.10 shows the calculation of equation 5.1.2 step by step. The "yellow" variables belong to the positive correlation values while the "blue" ones belong to the negative ones.

DRs	Correlations of ech DR with other DRs				Sum of positive and negative correlations seperately						Deffuzified sum of positive and negative correlations seperately	Average of deffuzified positive and negative correlations seperately	$(\overline{\widetilde{pc}}_j \ominus \overline{\widetilde{nc}}_j)$	$\left(n_{c_j}/(j-1)\right) * (\overline{\widetilde{pc}}_j \ominus \overline{\widetilde{nc}}_j)$
	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	π_L	π_U				
Battery array density	0.4000	0.6000	0.4000	0.6000										
	0.3000	0.5000	0.5000	0.7000	0.4854	0.7211	0.2000	0.4200						
	0.1260	0.3302	0.6707	0.8746	0.4977	0.7565	0.1341	0.3673						
	0.4932	0.6952	0.3131	0.5111	0.6563	0.8826	0.0420	0.1877	0.1857	0.5675	0.6267	0.1567	0.1567	0.0895
Reflective film	0.4000	0.6000	0.4000	0.6000										
	0.3302	0.5313	0.4702	0.6707	0.5016	0.7353	0.1881	0.4024						
	0.1000	0.3000	0.7000	0.9000	0.5090	0.7629	0.1317	0.3622						
	0.6000	0.8000	0.2000	0.4000	0.7251	0.9217	0.0263	0.1449	0.1295	0.4735	0.7061	0.1765	-0.1765	-0.1009
Velvet surface	0.4000	0.6000	0.4000	0.6000										
	0.4000	0.6000	0.4000	0.6000	0.4000	0.6000	0.4000	0.6000	0.2800	0.6800	0.2950	0.2950		
	0.4642	0.6649	0.3376	0.5372	0.5839	0.8019	0.1350	0.3223						
	0.1587	0.3634	0.6377	0.8421	0.5980	0.8307	0.0861	0.2714						
	0.1000	0.3000	0.7000	0.9000	0.6033	0.8474	0.0603	0.2443						
	0.6316	0.8320	0.1735	0.3705	0.7860	0.9556	0.0105	0.0905	0.0786	0.3821	0.7799	0.1560	-0.1390	-0.1191
Quality control measures of cells	0.4642	0.6649	0.3376	0.5372										
	0.4309	0.6316	0.3705	0.5703	0.6009	0.8153	0.1251	0.3064						
	0.2000	0.4000	0.6000	0.8000	0.6218	0.8476	0.0750	0.2451						
	0.1260	0.3302	0.6707	0.8746	0.1260	0.3302	0.6707	0.8746	0.1261	0.5342	0.0826	0.0826		
	0.6649	0.8653	0.1418	0.3376	0.8111	0.9640	0.0106	0.0827	0.0639	0.3420	0.8067	0.2017	0.1191	0.0851
Series elements in the module	0.3000	0.5000	0.5000	0.7000										
	0.3302	0.5313	0.4702	0.6707										
	0.1587	0.3634	0.6377	0.8421	0.3361	0.5908	0.3188	0.5894						
	0.4309	0.6316	0.3705	0.5703	0.5269	0.7802	0.1181	0.3362						
	0.1817	0.3915	0.6119	0.8205	0.5491	0.8178	0.0723	0.2758						
	0.5000	0.7000	0.3000	0.5000	0.5760	0.7962	0.1411	0.3354	0.2535	0.6483	0.5112	0.2556		
	0.5313	0.7319	0.2713	0.4702	0.5532	0.7790	0.1660	0.3858	0.2443	0.6664	0.4855	0.0971	-0.1585	-0.1585
Pass of IEC 61215 and UL1703 standards	0.1260	0.3302	0.6707	0.8746										
	0.1000	0.3000	0.7000	0.9000										
	0.1000	0.3000	0.7000	0.9000	0.1604	0.4350	0.4695	0.7871						
	0.2000	0.4000	0.6000	0.8000	0.2543	0.5647	0.2817	0.6297						
	0.1817	0.3915	0.6119	0.8205	0.3091	0.6506	0.1724	0.5167						
	0.4642	0.6649	0.3376	0.5372	0.4725	0.7017	0.2363	0.4835	0.2739	0.7209	0.3898	0.1949		
	0.6481	0.8485	0.1585	0.3545	0.6895	0.9157	0.0273	0.1832	0.1279	0.5238	0.6757	0.1351	-0.0598	-0.0598
Reflective coated glass	0.1260	0.3302	0.6707	0.8746										
	0.5000	0.7000	0.3000	0.5000	0.5118	0.7387	0.2012	0.4373						
	0.4642	0.6649	0.3376	0.5372	0.6488	0.8640	0.0679	0.2349						
	0.7000	0.9000	0.1000	0.3000	0.8394	0.9756	0.0068	0.0705	0.0432	0.2953	0.8395	0.2099	0.2099	0.1199
Thermal expansion treatment	0.4932	0.6952	0.3131	0.5111										
	0.6000	0.8000	0.2000	0.4000										
	0.6316	0.8320	0.1735	0.3705	0.7384	0.9171	0.0543	0.1894						
	0.6649	0.8653	0.1418	0.3376	0.8639	0.9798	0.0077	0.0639						
	0.5313	0.7319	0.2713	0.4702	0.9044	0.9907	0.0021	0.0301						
	0.6481	0.8485	0.1585	0.3545	0.9457	0.9974	0.0003	0.0107	0.0051	0.1056	0.9488	0.1898		
	0.7000	0.9000	0.1000	0.3000	0.8207	0.9652	0.0200	0.1200	0.0540	0.3260	0.8154	0.4077	-0.2180	-0.2180

Figure 6.2.10: Correlation Impact Factor Calculations.

To have a better understanding, these calculations will be explained on an example in Figure 6.2.11. Thermal expansion treatment is selected because it has both positive and negative correlation values.

DRs	Correlations of ech DR with other DRs				Sum of positive and negative correlations seperately						Deffuzified sum of positive and negative correlations seperately	Average of deffuzified positive and negative correlations seperately	$(\overline{\widetilde{pc}}_j \ominus \overline{\widetilde{nc}}_j)$	$(n_{c_j}/(j-1)) * (\overline{\widetilde{pc}}_j \ominus \overline{\widetilde{nc}}_j)$
	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	π_L	π_U				
Thermal expansion treatment	0.4932	0.6952	0.3131	0.5111										
	0.6000	0.8000	0.2000	0.4000										
	0.6316	0.8320	0.1735	0.3705	0.7384	0.9171	0.0543	0.1894						
	0.6649	0.8653	0.1418	0.3376	0.8639	0.9798	0.0077	0.0639						
	0.5313	0.7319	0.2713	0.4702	0.9044	0.9907	0.0021	0.0301						
	0.6481	0.8485	0.1585	0.3545	0.9457	0.9974	0.0003	0.0107	0.0051	0.1056	0.9488	0.1898		
	0.7000	0.9000	0.1000	0.3000	0.8207	0.9652	0.0200	0.1200	0.0540	0.3260	0.8154	0.4077	-0.2180	-0.2180

Figure 6.2.11 Correlation Impact Factor Calculations on Thermal Expansion Treatment DR.

The values encircled with red lines in Figure 6.2.11 represents the sum of all positive correlations of Thermal Expansion Treatment with other DRs. The sums are found by the equation 4.5. And the results are achieved by first adding the two lines, then adding the third line to the summation of the first two and so on. For example, to achieve the values encircled with red lines in Figure 6.2.11, the summation of the first four positive correlations added to the fifth one by using equation 4.5.

$$[0.9457, 0.9974], [0.0003, 0.0107] = \left(\begin{bmatrix} \sqrt{(0,9044)^2 + (0.6481)^2 - (0.9044)^2(0.6481)^2}, \\ \sqrt{(0.9907)^2 + (0.8485)^2 - (0.9907)^2(0.8485)^2} \end{bmatrix}, \atop [\,0.0021 \times 0.1585, 0.0301 \times 0.3545] \right)$$

The values encircled with green lines in Figure 6.2.11 are found by applying equations 4.10 and 4.11 respectively as shown below.

$$0.0051 = 1 - (0.9457^2 + 0.0003^2)$$

$$0.1056 = 1 - (0.9974^2 + 0.0107^2)$$

The values encircled with purple lines in Figure 6.2.11 represent defuzzified sum of positive and negative correlations separately and average of defuzzified positive and negative correlations separately. Their calculations are shown below respectively.

$$0.9488 = \frac{\left(\begin{array}{c}0.9457^2+0.9974^2+(1-0.0003^2-0.1056^2)+(1-0.0107^2-0.0051^2)+\sqrt{0.0051^2\times0.1056^2}+ \\ \sqrt[4]{(1-0.0003^2-0.1056^2)\times(1-0.0107^2-0.0051^2)}\end{array}\right)}{6}$$

0.1898 = 0.9488/5 (There are 5 positive correlation values. To take the average, the sum has divided to 5.)

The values encircled with black lines in Figure 6.2.11 are calculated by using equation 5.1.2 as shown below.

$$-0.2180 = \left(\widetilde{\widetilde{pc}}_j \ominus \widetilde{\widetilde{nc}}_j\right) = 0.1898 - 0.4077$$

$$-0.2180 = \left(n_{c_j}/(j-1)\right) * \left(\widetilde{\widetilde{pc}}_j \ominus \widetilde{\widetilde{nc}}_j\right) = (7/(8-1)) * (-0.2180)$$

To obtain the absolute importance ($\widetilde{AI}$) for each DR, first multiply the aggregated linguistic importance evaluations ($\widetilde{IE}$) of CRs and the aggregated linguistic terms ($\tilde{R}$) in the relationship matrix. Then multiple the result with the aggregated correlation impact factor ($\widetilde{CI}$) and then divide it by relative organizational difficulty ($\widetilde{ROD}$) as in equation 5.1.1. Organizational difficulty ($\widetilde{OD}$) means how difficult it is for an organization to achieve a certain DR. The linguistic assessments for $\widetilde{OD}$ of each DR are aggregated before calculating ($\widetilde{ROD}$). Figure 6.2.12 shows how to calculate the relative organizational difficulty values for each DR.

DRs	Organizational Difficulty						Defuzzified Organizational Difficulty	Sum of Organizational Difficulty						Defuzzified Sum of Organizational Difficulty	Relative Organizational Difficulty
	μ_L	μ_U	ν_L	ν_U	π_L	π_U		μ_L	μ_U	ν_L	ν_U	π_L	π_U		
Battery array density	0.5646	0.7652	0.2387	0.4372	0.2234	0.6242	0.4830								0.5257
Reflective film	0.1817	0.3915	0.6119	0.8205	0.1735	0.5925	0.1184	0.5842	0.8056	0.1460	0.3587				0.1289
Velvet surface	0.2621	0.4642	0.5372	0.7389	0.2387	0.6427	0.1731	0.6217	0.8513	0.0785	0.2650				0.1884
Quality control measures of cells	0.3634	0.5646	0.4372	0.6377	0.2746	0.6768	0.2600	0.6838	0.9013	0.0343	0.1690				0.2830
Series elements in the module	0.4932	0.6952	0.3131	0.5111	0.2555	0.6587	0.3969	0.7727	0.9503	0.0107	0.0864				0.4321
Pass of IEC 61215 and UL1703	0.4932	0.6952	0.3131	0.5111	0.2555	0.6587	0.3969	0.8337	0.9746	0.0034	0.0442				0.4321
Reflective coated glass	0.1817	0.3915	0.6119	0.8205	0.1735	0.5925	0.1184	0.8398	0.9786	0.0021	0.0362				0.1289
Thermal expansion treatment	0.6649	0.8653	0.1418	0.3376	0.1372	0.5378	0.6193	0.9141	0.9947	0.0003	0.0122	0.0105	0.1645	0.9187	0.6741

Figure 6.2.12: Computing the Relative Organizational Difficulty.

Figure 6.2.13 represents the multiplication of the aggregated linguistic importance evaluations ($\widetilde{IE}$) of CRs and the aggregated linguistic terms ($\tilde{R}$) in the relationship matrix.

DR	$\widetilde{IE}_i$				$\tilde{R}_{hj}$				$\widetilde{IE}_i x \tilde{R}_{hj}$				$\oplus_{i=1}^{T} \widetilde{IE}_i \otimes \tilde{R}_{hj}$						Defuzzified
	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	π_L	π_U	
Battery array density	0.6649	0.8653	0.1418	0.3376	0.6649	0.8653	0.1418	0.3376	0.4421	0.7488	0.1995	0.4636							
	0.3000	0.5000	0.5000	0.7000	0.2000	0.4000	0.6000	0.8000	0.0600	0.2000	0.7211	0.9035	0.4454	0.7605	0.1439	0.4189	0.2462	0.7809	0.4124
Reflective film	0.6649	0.8653	0.1418	0.3376	0.2884	0.4932	0.5111	0.7143	0.1918	0.4268	0.5254	0.7523							
	0.3302	0.5313	0.4702	0.6707	0.4309	0.6316	0.3705	0.5703	0.1423	0.3356	0.5728	0.7930	0.2372	0.5237	0.3010	0.5966	0.3698	0.8531	0.1897
Velvet surface	0.6649	0.8653	0.1418	0.3376	0.1260	0.3302	0.6707	0.8746	0.0838	0.2857	0.6789	0.8898							
	0.3302	0.5313	0.4702	0.6707	0.3000	0.5000	0.5000	0.7000	0.0991	0.2657	0.6449	0.8482	0.1295	0.3827	0.4378	0.7547	0.2840	0.7915	0.0998
Quality control measures of cells	0.6649	0.8653	0.1418	0.3376	0.2289	0.4309	0.5703	0.7718	0.1522	0.3729	0.5821	0.8011							
	0.4932	0.6952	0.3131	0.5111	0.3464	0.5477	0.4542	0.6547	0.1709	0.3808	0.5330	0.7602	0.2274	0.5137	0.3102	0.6090			
	0.2621	0.4642	0.5372	0.7389	0.5313	0.7319	0.2713	0.4702	0.1392	0.3397	0.5840	0.8039	0.2647	0.5906	0.1812	0.4896	0.4115	0.8971	0.2316
Series elements in the module	0.6649	0.8653	0.1418	0.3376	0.1000	0.3000	0.7000	0.9000	0.0665	0.2596	0.7073	0.9119							
	0.4932	0.6952	0.3131	0.5111	0.2289	0.4309	0.5703	0.7718	0.1129	0.2996	0.6256	0.8374	0.1308	0.3887	0.4425	0.7637			
	0.3000	0.5000	0.5000	0.7000	0.2289	0.4309	0.5703	0.7718	0.0687	0.2154	0.7028	0.8910	0.1475	0.4364	0.3110	0.6804			
	0.3302	0.5313	0.4702	0.6707	0.1260	0.3302	0.6707	0.8746	0.0416	0.1754	0.7560	0.9331	0.1531	0.4641	0.2351	0.6349			
	0.2621	0.4642	0.5372	0.7389	0.2000	0.4000	0.6000	0.8000	0.0524	0.1857	0.7380	0.9146	0.1616	0.4924	0.1735	0.5807	0.4204	0.9438	0.1498
Pass of IEC 61215 and UL1703 standards	0.6649	0.8653	0.1418	0.3376	0.1000	0.3000	0.7000	0.9000	0.0665	0.2596	0.7073	0.9119							
	0.4932	0.6952	0.3131	0.5111	0.6000	0.8000	0.2000	0.4000	0.2959	0.5562	0.3662	0.6160	0.3027	0.5965	0.2590	0.5617			
	0.3302	0.5313	0.4702	0.6707	0.2000	0.4000	0.6000	0.8000	0.0660	0.2125	0.7082	0.8955	0.3092	0.6205	0.1834	0.5031			
	0.2621	0.4642	0.5372	0.7389	0.1587	0.3634	0.6377	0.8421	0.0416	0.1687	0.7602	0.9316	0.3117	0.6344	0.1394	0.4687			
	0.1260	0.3302	0.6707	0.8746	0.7000	0.9000	0.1000	0.3000	0.0882	0.2972	0.6748	0.8866	0.3227	0.6747	0.0941	0.4155	0.3721	0.8870	0.3005
Reflective coated glass	0.6649	0.8653	0.1418	0.3376	0.2884	0.4932	0.5111	0.7143	0.1918	0.4268	0.5254	0.7523							
	0.3302	0.5313	0.4702	0.6707	0.1587	0.3634	0.6377	0.8421	0.0524	0.1931	0.7334	0.9165	0.1986	0.4612	0.3853	0.6895	0.3119	0.8121	0.1497
Thermal expansion treatment	0.6649	0.8653	0.1418	0.3376	0.2621	0.4642	0.5372	0.7389	0.1743	0.4017	0.5504	0.7731							
	0.3000	0.5000	0.5000	0.7000	0.6316	0.8320	0.1735	0.3705	0.1895	0.4160	0.5221	0.7483	0.2553	0.5536	0.2873	0.5785			
	0.3302	0.5313	0.4702	0.6707	0.3915	0.5944	0.4114	0.6119	0.1293	0.3158	0.5941	0.8099	0.2843	0.6129	0.1707	0.4685			
	0.2621	0.4642	0.5372	0.7389	0.5000	0.7000	0.3000	0.5000	0.1310	0.3249	0.5938	0.8121	0.3108	0.6645	0.1014	0.3805	0.4137	0.8931	0.2895

Figure 6.2.13: Multiplication of $\widetilde{IEs}$ and $\tilde{R}$s.

The terms encircled with red lines in Figure 6.2.13 explained below to illustrate the multiplication operation for IVPF numbers that given in equation 4.6:

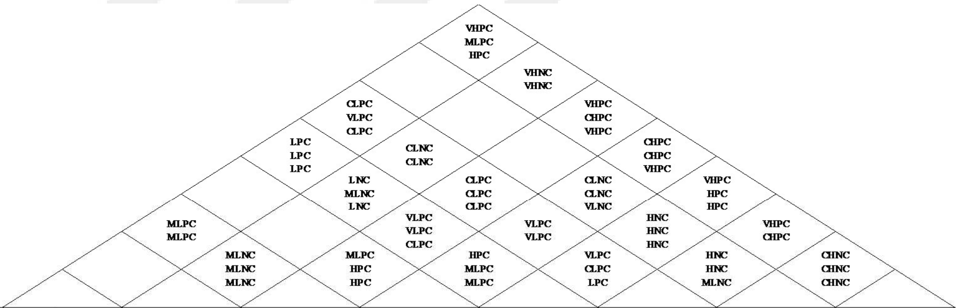
$$[0.4421, 0.7488], [0.1995, 0.4636] = \left(\begin{array}{c} [0.6649 \times 0.6649, 0.8653 \times 0.8653], \\ \left[\begin{array}{l} \sqrt{(0.1418)^2 + (0.1418)^2 - (0.1418)^2 (0.1418)^2}, \\ \sqrt{(0.3376)^2 + (0.3376)^2 - (0.3376)^2 (0.3376)^2} \end{array} \right] \end{array} \right)$$

Relative absolute importance ($\widetilde{RAI}$) values are obtained by equation 5.1.4. Final relative importance and absolute importance values are shown in Figure 6.2.14.

DRs	Defuzzified Multiplication of Importance Evaluations and terms in the Relationship Matrix	Defuzzified Correlation Impact Factor	Defuzzified Relative Organizational Difficulty	Absolute Importance	Relative Absolute Importance
Battery array density	0.4124	0.0895	0.5257	0.2945	**0.2293**
Reflective film	0.3005	-0.0598	0.4321	0.1973	0.1536
Velvet surface	0.2316	0.0851	0.2830	0.1959	0.1525
Quality control measures of cells	0.1897	-0.1009	0.1289	0.1511	0.1176
Series elements in the module	0.1497	0.1199	0.1289	0.1485	0.1156
Pass of IEC 61215 and UL1703 standards	0.2895	0.2180	0.6741	0.1353	0.1053
Reflective coated glass	0.1498	-0.1585	0.4321	0.0880	0.0685
Thermal expansion treatment	0.0998	-0.1191	0.1884	0.0739	0.0576

Figure 6.2.14: Absolute Importance and Relative Absolute Importance values.

Step 7: Rank the DRs with respect to $\widetilde{RA}_j$ values. Figure 6.2.15 shows the HOQ filled by three experts using the linguistic terms given in Table 5.1.1 and Table 5.1.2. Figure 6.2.16 presents the aggregated linguistic terms and their corresponding IVPF numerical values together with the final results of all computations given in the steps of the proposed model. Based on these results, the priorities of the DRs are ranked as follows: Battery array density, Thermal expansion treatment, Velvet surface, Reflective film, Series elements in the module, Pass of IEC 61215 and UL1703 standards, Reflective coated glass, Quality control measures of cells. The engineers should give the highest importance to Battery array density and Thermal expansion treatment to meet the CRs at the maximum level.

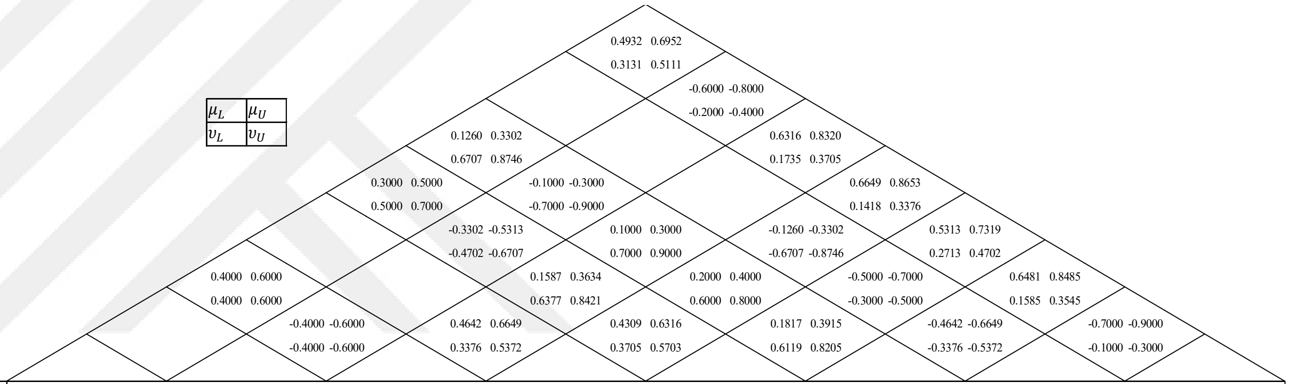


Direction of Improvement		↑	↓	↑	↑	↑	↑	↓	↑
HOWS / WHATS	Design Requirements	Battery array density	Reflective film	Velvet surface	Quality control measures of cells	Series elements in the module	Pass of IEC 61215 and UL1703 standards	Reflective coated glass	Thermal expansion treatment
Subcriteria (Customer Requirements)	Importance Evaluations	$\tilde{R}_1$	$\tilde{R}_2$	$\tilde{R}_3$	$\tilde{R}_4$	$\tilde{R}_5$	$\tilde{R}_6$	$\tilde{R}_7$	$\tilde{R}_8$
High conversion efficiency	VHI, CHI, CHI	MLR LR LR	VLR LR MLR	CLR VLR CLR	LR VLR VLR	CLR CLR	CLR CLR CLR	MLR LR VLR	LR LR VLR
Manufacturing process	VHI, HI,MLI				LR MLR	VLR VLR LR	VHR VHR VHR		
Modular design	LI, LI,LI	VLR VLR MLR				VLR LR VLR			VHR VHR CHR
Material quality	MLI, LI, LI		MLR HR MLR	LR LR LR		CLR CLR VLR	VLR VLR VLR	VLR VLR CLR	MLR HR LR
Product stability	LI, VLI, LI				VHR HR HR	VLR VLR VLR	CLR VLR VLR		HR HR
Government policy	CLI, CLI, VLI						CHR CHR CHR		
Organizational Difficulty		VHD, VHD, HD	CLD,VLD, LD	LD, VLD, LD	MLD, MLD, LD	HD, MLD, VHD	MLD, VHD, HD	VLD, LD, CLD	CHD, CHD, VHD
Target Values (How Muches)		90 (Wh/Kg)	Relative transmission increase of 3.1% over the spectral range of 400–1100 nm	<5% reflection	100% In-line Inspection	72PV cells in series to charge 24V panel	Benchmark of 30% degradation	Increased transmission by up to 3.8%	20.3 (CTE) (10-6/K-1)
Engineering Assessment (O: Our Company, C1: Company 1, C2: Company 2)	CLS					C1, C1			
	VLS					O, C1, C2			O, C1, C1,C2
	LS	O, O, C1, C2, C2			C1	O, Q, 2			O, C2
	MLS	O, C1, C1, C2			O, C1, C1, C2	C2			O, C2
	HS		C1, C2	O, O, O, C2, C2	O, O, C2, C2				
	VHS		O, O, C1,C1, C2	C1,C1,C1,C2			C1	O, C1, C2, C2	
	CHS		O, C2				O, O, O, C1, C2	O, O, C1, C1,C2	
Absolute Importance									
Relative Absolute Importance (%)									

Customer Rating
O: Our Company
C1: Company 1
C2: Company 2

Subcriteria	CLS	VLS	LS	MLS	HS	VHS	CHS
High conversion efficiency		C1, C1	O, C1, C2	O, O, C2			
Manufacturing process			O, C1	O, O, C1, C2	C1, C2, C2		
Modular design					O, C2	O, O, C1, C2	C1, C2
Material quality		C1	O, C1, C1, C2	O, C2, C2	O		
Product stability			O, C1, C1	O, C2, C2	O, C2		
Government policy						C1	O, O, O, C1, C2

Figure 6.2.15: HOQ Evaluated by three customers and three experts.



HOWS / WHATS		Importance Evaluations				Design Requirements																															
						Battery array density				Reflective film				Velvet surface				Quality control measures of cells				Series elements in the module				Pass of IEC 61215 and UL1703 standards				Reflective coated glass				Thermal expansion treatment			
Customer Requirements	Subcriteria	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U
	High conversion efficiency	0.6649	0.8653	0.1418	0.3376	0.6649	0.8653	0.1418	0.3376	0.2884	0.4932	0.5111	0.7143	0.1260	0.3302	0.6707	0.8746	0.2289	0.4309	0.5703	0.7718	0.1000	0.3000	0.7000	0.9000	0.1000	0.3000	0.7000	0.9000	0.2884	0.4932	0.5111	0.7143	0.2621	0.4642	0.5372	0.7389
	Manufacturing process	0.4932	0.6952	0.3131	0.5111													0.3464	0.5477	0.4542	0.6547	0.2289	0.4309	0.5703	0.7718	0.6000	0.8000	0.2000	0.4000								
	Modular design	0.3000	0.5000	0.5000	0.7000	0.2000	0.4000	0.6000	0.8000													0.2289	0.4309	0.5703	0.7718									0.6316	0.8320	0.1735	0.3705
	Material quality	0.3302	0.5313	0.4702	0.6707					0.4309	0.6316	0.3705	0.5703	0.3000	0.5000	0.5000	0.7000					0.1260	0.3302	0.6707	0.8746	0.2000	0.4000	0.6000	0.8000	0.1587	0.3634	0.6377	0.8421	0.3915	0.5944	0.4114	0.6119
	Product stability	0.2621	0.4642	0.5372	0.7389													0.5313	0.7319	0.2713	0.4702	0.2000	0.4000	0.6000	0.8000	0.1587	0.3634	0.6377	0.8421					0.5000	0.7000	0.3000	0.5000
	Government policy	0.1260	0.3302	0.6707	0.8746																					0.7000	0.9000	0.1000	0.3000								
Organizational Difficulty						0.5646	0.7652	0.2387	0.4372	0.1817	0.3915	0.6119	0.8205	0.2621	0.4642	0.5372	0.7389	0.3634	0.5646	0.4372	0.6377	0.4932	0.6952	0.3131	0.5111	0.4932	0.6952	0.3131	0.5111	0.1817	0.3915	0.6119	0.8205	0.6649	0.8653	0.1418	0.3376
Defuzzified Organizational Difficulty						0.4830				0.1184				0.1731				0.2600				0.3969				0.3969				0.1184				0.6193			
Defuzzified Relative Organizational Difficulty						0.5257				0.1289				0.1884				0.2830				0.4321				0.4321				0.1289				0.6741			
Absolute Importance						0.2945				0.1511				0.0739				0.1959				0.0880				0.1973				0.1485				0.1353			
Relative Absolute Importance (%)						0.2292				0.1176				0.0576				0.1525				0.0685				0.1536				0.1156				0.1053			

Figure 6.2.16: Aggregated linguistic terms and corresponding IVPFS.

6.3 Competitive Analysis

Step 8: Determine the linguistic customer ratings of the competition by using the Pythagorean fuzzy scale given in Table 5.1.1 as in Figure 6.3.1. The linguistic customer ratings of the competition are assigned by three experts by using the scale in Table 5.1.1.

			Customer Rating						
			O: Our Company C1: Company 1 C2: Company 2						
Customer Requirements	Subcriteria	Importance Evaluations	CLS	VLS	LS	MLS	HS	VHS	CHS
	High conversion efficiency	VHI, CHI, CHI		C1, C1	O, C1, C2	O, O, C2			
	Manufacturing process	VHI, HI, MLI			O, C1	O, O, C1, C2	C1, C2, C2		
	Modular design	LI, LI, LI					O, C2	O, O, C1, C2	C1, C2
	Material quality	MLI, LI, LI		C1	O, C1, C1, C2	O, C2, C2	O		
	Product stability	LI, VLI, LI			O, C1, C1	O, C2, C2	O, C2		
	Government policy	CLI, CLI, VLI						C1	O, O, O, C1, C2

Figure 6.3.1: Linguistic Customer Ratings of the Competition.

To determine our position among the competitors, first linguistic customer ratings are aggregated with respect to the corresponding CR by using equation 4.9 as in Figure 6.3.2 and then the distances between our company and other companies ($\widetilde{D}^{CR}_{O-C\ell}$) are measured by using equation 5.1.5 where O represents Our Company, C1 represents Company 1, and C2 represents Company 2. Figure 6.3.3 shows the distance between Our Company and Company 1. Figure 6.3.4 shows the distance between Our Company and Company 2.

							Aggregated IVPF Customer Ratings of the Competition						Defuzzified IVPF Customer Ratings of the Competition
	Subcriteria	**Importance Evaluations**	μ_L	μ_U	ν_L	ν_U	μ_L	μ_U	ν_L	ν_U	π_L	π_U	
Our Company	**High conversion efficiency**	LS	0.3	0.5	0.5	0.7	0.3634	0.5646	0.4372	0.6377	0.2746	0.6768	0.2600
		MLS	0.4	0.6	0.4	0.6							
		MLS	0.4	0.6	0.4	0.6							
	Manufacturing process	LS	0.3	0.5	0.5	0.7	0.3634	0.5646	0.4372	0.6377	0.2746	0.6768	0.2600
		MLS	0.4	0.6	0.4	0.6							
		MLS	0.4	0.6	0.4	0.6							
	Modular design	HS	0.5	0.7	0.3	0.5	0.5646	0.7652	0.2387	0.4372	0.2234	0.6242	0.4830
		VHS	0.6	0.8	0.2	0.4							
		VHS	0.6	0.8	0.2	0.4							
	Material quality	LS	0.3	0.5	0.5	0.7	0.3915	0.5944	0.4114	0.6119	0.2723	0.6775	0.2880
		MLS	0.4	0.6	0.4	0.6							
		HS	0.5	0.7	0.3	0.5							
	Product stability	LS	0.3	0.5	0.5	0.7	0.3915	0.5944	0.4114	0.6119	0.2723	0.6775	0.2880
		MLS	0.4	0.6	0.4	0.6							
		HS	0.5	0.7	0.3	0.5							
	Government policy	CHS	0.7	0.9	0.1	0.3	0.7000	0.9000	0.1000	0.3000	0.1000	0.5000	0.6706
		CHS	0.7	0.9	0.1	0.3							
		CHS	0.7	0.9	0.1	0.3							
Company 1	**High conversion efficiency**	VLS	0.2	0.4	0.6	0.8	0.2289	0.4309	0.5703	0.7718	0.2187	0.6223	0.1481
		VLS	0.2	0.4	0.6	0.8							
		LS	0.3	0.5	0.5	0.7							
	Manufacturing process	LS	0.3	0.5	0.5	0.7	0.3915	0.5944	0.4114	0.6119	0.2723	0.6775	0.2880
		MLS	0.4	0.6	0.4	0.6							
		HS	0.5	0.7	0.3	0.5							
	Modular design	VHS	0.6	0.8	0.2	0.4	0.6481	0.8485	0.1585	0.3545	0.1543	0.5549	0.5952
		CHS	0.7	0.9	0.1	0.3							
	Material quality	VLS	0.2	0.4	0.6	0.8	0.2621	0.4642	0.5372	0.7389	0.2387	0.6427	0.1731
		LS	0.3	0.5	0.5	0.7							
		LS	0.3	0.5	0.5	0.7							
	Product stability	LS	0.3	0.5	0.5	0.7	0.3000	0.5000	0.5000	0.7000	0.2600	0.6600	0.2029
		LS	0.3	0.5	0.5	0.7							
	Government policy	VHS	0.6	0.8	0.2	0.4	0.6481	0.8485	0.1585	0.3545	0.1543	0.5549	0.5952
		CHS	0.7	0.9	0.1	0.3							
Company 2	**High conversion efficiency**	LS	0.3	0.5	0.5	0.7	0.3464	0.5477	0.4542	0.6547	0.2713	0.6737	0.2442
		MLS	0.4	0.6	0.4	0.6							
	Manufacturing process	MLS	0.4	0.6	0.4	0.6	0.4642	0.6649	0.3376	0.5372	0.2692	0.6706	0.3632
		HS	0.5	0.7	0.3	0.5							
		HS	0.5	0.7	0.3	0.5							
	Modular design	HS	0.5	0.7	0.3	0.5	0.5944	0.7958	0.2174	0.4114	0.1974	0.5995	0.5223
		VHS	0.6	0.8	0.2	0.4							
		CHS	0.7	0.9	0.1	0.3							
	Material quality	LS	0.3	0.5	0.5	0.7	0.3634	0.5646	0.4372	0.6377	0.2746	0.6768	0.2600
		MLS	0.4	0.6	0.4	0.6							
		MLS	0.4	0.6	0.4	0.6							
	Product stability	MLS	0.4	0.6	0.4	0.6	0.4309	0.6316	0.3705	0.5703	0.2758	0.6771	0.3272
		MLS	0.4	0.6	0.4	0.6							
		HS	0.5	0.7	0.3	0.5							
	Government policy	CHS	0.7	0.9	0.1	0.3	0.7000	0.9000	0.1000	0.3000	0.1000	0.5000	0.6706

Figure 6.3.2: Aggregated IVPF Customer Ratings of Competition and Defuzzification.

CRs	Our Company							Company 1						
Subcriteria	μ_L	μ_U	ν_L	ν_U	π_L	π_U	Defuzzified Customer Ratings of the Competition	μ_L	μ_U	ν_L	ν_U	π_L	π_U	Defuzzified Customer Ratings of the Competition
High conversion efficiency	0.3634	0.5646	0.4372	0.6377	0.2746	0.6768	0.2600	0.2289	0.4309	0.5703	0.7718	0.2187	0.6223	0.1481
Manufacturing process	0.3634	0.5646	0.4372	0.6377	0.2746	0.6768	0.2600	0.3915	0.5944	0.4114	0.6119	0.2723	0.6775	0.2880
Modular design	0.5646	0.7652	0.2387	0.4372	0.2234	0.6242	0.4830	0.6481	0.8485	0.1585	0.3545	0.1543	0.5549	0.5952
Material quality	0.3915	0.5944	0.4114	0.6119	0.2723	0.6775	0.2880	0.2621	0.4642	0.5372	0.7389	0.2387	0.6427	0.1731
Product stability	0.3915	0.5944	0.4114	0.6119	0.2723	0.6775	0.2880	0.3000	0.5000	0.5000	0.7000	0.2600	0.6600	0.2029
Government policy	0.7000	0.9000	0.1000	0.3000	0.1000	0.5000	0.6706	0.6481	0.8485	0.1585	0.3545	0.1543	0.5549	0.5952

CRs								$d_t^{CR}(O,C1) \times \tilde{w}_t^{CR}$							Defuzzified
Subcriteria	$d_t^{CR}(O,C1)$	κ_{O-C1}^{CR}	$\kappa_{O-C\ell}^{CR} \times d_t^{CR}(O,C1)$	Aggregated Importance Evaluations of CRs				μ_L	μ_U	ν_L	ν_U	π_L	π_U	Defuzzified	$\widetilde{D}_{O-C1}^{CR}$
High conversion efficiency	0.1075	1	0.1075	0.6649	0.8653	0.1418	0.3376	0.2466	0.3714	0.8107	0.8899	0.0702	0.2820	0.1320	0.1320
Manufacturing process	0.0209	-1	-0.0209	0.4932	0.6952	0.3131	0.5111	0.0763	0.1171	0.9760	1.0141	-0.0422	0.0416	0.0237	-0.0237
Modular design	0.0637	-1	-0.0637	0.3000	0.5000	0.5000	0.7000	0.0774	0.1347	0.9568	1.0230	-0.0646	0.0785	0.0268	-0.0268
Material quality	0.1031	1	0.1031	0.3302	0.5313	0.4702	0.6707	0.1088	0.1834	0.9252	0.9597	0.0454	0.1322	0.0420	0.0420
Product stability	0.0712	1	0.0712	0.2621	0.4642	0.5372	0.7389	0.0711	0.1309	0.9567	0.9787	0.0251	0.0797	0.0250	0.0250
Government policy	0.0388	1	0.0388	0.1260	0.3302	0.6707	0.8746	0.0249	0.0669	0.9846	0.9948	0.0059	0.0299	0.0088	0.0088

Figure 6.3.3: Distance between Our Company and Company 1.

The terms encircled with red lines in Figure 6.3.3 explained below to illustrate the operations in equations 5.1.6 and 5.1.7 respectively:

$\kappa_{O-C\ell}^{CR} = 1$ for High Conversion Efficiency because defuzzified customer ratings of the competition of Our Company is greater than Company 1's.

$$d_t^{CR}(O,C\ell) = 0.1075 = \frac{\sqrt{2}}{4}\left(\sqrt{(0.3634-0.2289)^2+(0.4372-0.5703)^2}+\sqrt{(0.5646-0.4309)^2+(0.6377-0.7718)^2}\right)$$

The terms encircled with blue lines in Figure 6.3.3 explained below to illustrate the operation in equation 4.7:

$$\lambda\tilde{A} = 0.1075 \times ([0.6649, 0.8653], [0.1418, 0.3376])$$

$$= \left(\left[\sqrt{1-(1-(0.6649)^2)^{0.1075}}, \sqrt{1-(1-(0.8653)^2)^{0.1075}}\right], [(0.1418)^{0.1075}, (0.3376)^{0.1075}]\right)$$

$$= [0.2466, 0.3714], [0.8107, 0.8899]$$

CRs	Our Company							Company 2							$d_t^{CR}(O,C2)$	κ_{O-C2}^{CR}	$\kappa_{O-C\ell}^{CR} \times d_t^{CR}(O,C2)$	Aggregated Importance Evaluations of CRs				$d_t^{CR}(O,C2) \times \widetilde{w}_t^{CR}$							Defuzzified $\widetilde{D}_{O-C2}^{CR}$
Subcriteria	μ_L	μ_U	ν_L	ν_U	π_L	π_U	Defuzzified Customer Ratings of the Competition	μ_L	μ_U	ν_L	ν_U	π_L	π_U	Defuzzified Customer Ratings of the Competition								μ_L	μ_U	ν_L	ν_U	π_L	π_U	Defuzzified	
High conversion efficiency	0.3634	0.5646	0.4372	0.6377	0.2746	0.6768	0.2600	0.3464	0.5477	0.4542	0.6547	0.2713	0.6737	0.2442	0.0122	1	0.0122	0.6649	0.8653	0.1418	0.3376	0.0842	0.1293	0.9765	0.9868	0.0094	0.0394	0.0271	0.0271
Manufacturing process	0.3634	0.5646	0.4372	0.6377	0.2746	0.6768	0.2600	0.4642	0.6649	0.3376	0.5372	0.2692	0.6706	0.3632	0.0782	-1	-0.0782	0.4932	0.6952	0.3131	0.5111	0.1468	0.2243	0.9132	0.9489	0.0493	0.1445	0.0597	-0.0597
Modular design	0.5646	0.7652	0.2387	0.4372	0.2234	0.6242	0.4830	0.5944	0.7958	0.2174	0.4114	0.1974	0.5995	0.5223	0.0218	-1	-0.0218	0.3000	0.5000	0.5000	0.7000	0.0453	0.0790	0.9850	0.9923	0.0092	0.0277	0.0133	-0.0133
Material quality	0.3915	0.5944	0.4114	0.6119	0.2723	0.6775	0.2880	0.3634	0.5646	0.4372	0.6377	0.2746	0.6768	0.2600	0.0209	1	0.0209	0.3302	0.5313	0.4702	0.6707	0.0491	0.0832	0.9843	0.9917	0.0097	0.0287	0.0144	0.0144
Product stability	0.3915	0.5944	0.4114	0.6119	0.2723	0.6775	0.2880	0.4309	0.6316	0.3705	0.5703	0.2758	0.6771	0.3272	0.0283	-1	-0.0283	0.2621	0.4642	0.5372	0.7389	0.0449	0.0827	0.9826	0.9915	0.0101	0.0325	0.0137	-0.0137
Government policy	0.7000	0.9000	0.1000	0.3000	0.1000	0.5000	0.6706	0.7000	0.9000	0.1000	0.3000	0.1000	0.5000	0.6706	0.0000	0	0.0000	0.1260	0.3302	0.6707	0.8746	0.0000	0.0000	1.0000	1.0000	0.0000	0.0000	0.0000	0.0000

Figure 6.3.4: Distance between Our Company and Company 2.

Step 9: Compare the DRs of Our Company with the other competitors by using the Pythagorean fuzzy scale given in Table 5.1.1 as in Figure 6.3.5.

		Design Requirements	Battery array density	Reflective film	Velvet surface	Quality control measures of cells	Series elements in the module	Pass of IEC 61215 and UL1703 standards	Reflective coated glass	Thermal expansion treatment
Engineering Assessment	O: Our Company C1: Company 1 C2: Company 2	**CLS**					C1, C1			
		VLS					O, C1, C2			O, C1, C1,C2
		LS	O, O, C1, C2, C2			C1	O, O, 2			O, C2
		MLS	O, C1, C1, C2			O, C1, C1, C2	C2			O, C2
		HS		C1, C2	O, O, O, C2, C2	O, O, C2, C2				
		VHS		O, O, C1,C1, C2	C1,C1,C1,C2			C1	O, C1, C2, C2	
		CHS		O, C2				O, O, O, C1, C2	O, O, C1, C1,C2	

Figure 6.3.5: Linguistic DR Ratings of the Competition.

To determine our position among the competitors, first linguistic engineering assessments are aggregated with respect to the corresponding DR by using equation 4.9 as in Figure 6.3.6 and then the distances between our company and other companies ($\widetilde{D}^{DR}_{O-C\ell}$) are measured by using equation 5.1.8 where O represents Our Company, C1 represents Company 1, and C2 represents Company 2. Figure 6.3.7 shows the distance between Our Company and Company 1. Figure 6.3.8 shows the distance between Our Company and Company 2.

	Subcriteria	Importance Evaluations	μ_L	μ_U	ν_L	ν_U	Aggregated IVPF Engineering Assesments of the Competition μ_L	μ_U	ν_L	ν_U	π_L	π_U	Defuzzified Engineering Assesments of the Competition
Our Company	Battery array density	LS	0.3	0.5	0.5	0.7	0.3302	0.5313	0.4702	0.6707	0.2678	0.6698	0.2295
		LS	0.3	0.5	0.5	0.7							
		MLS	0.4	0.6	0.4	0.6							
	Reflective film	VHS	0.6	0.8	0.2	0.4	0.6316	0.8320	0.1735	0.3705	0.1704	0.5709	0.5722
		VHS	0.6	0.8	0.2	0.4							
		CHS	0.7	0.9	0.1	0.3							
	Velvet surface	HS	0.5	0.7	0.3	0.5	0.5000	0.7000	0.3000	0.5000	0.2600	0.6600	0.4036
		HS	0.5	0.7	0.3	0.5							
		HS	0.5	0.7	0.3	0.5							
	Quality control measures of cells	MLS	0.4	0.6	0.4	0.6	0.4642	0.6649	0.3376	0.5372	0.2692	0.6706	0.3632
		HS	0.5	0.7	0.3	0.5							
		HS	0.5	0.7	0.3	0.5							
	Series elements in the module	VLS	0.2	0.4	0.6	0.8	0.2621	0.4642	0.5372	0.7389	0.2387	0.6427	0.1731
		LS	0.3	0.5	0.5	0.7							
		LS	0.3	0.5	0.5	0.7							
	Pass of IEC 61215 and UL1703 standards	CHS	0.7	0.9	0.1	0.3	0.7000	0.9000	0.1000	0.3000	0.1000	0.5000	0.6706
		CHS	0.7	0.9	0.1	0.3							
		CHS	0.7	0.9	0.1	0.3							
	Reflective coated glass	VHS	0.6	0.8	0.2	0.4	0.6649	0.8653	0.1418	0.3376	0.1372	0.5378	0.6193
		CHS	0.7	0.9	0.1	0.3							
		CHS	0.7	0.9	0.1	0.3							
	Thermal expansion treatment	VLS	0.2	0.4	0.6	0.8	0.2884	0.4932	0.5111	0.7143	0.2465	0.6556	0.1954
		LS	0.3	0.5	0.5	0.7							
		MLS	0.4	0.6	0.4	0.6							
Company 1	Battery array density	LS	0.3	0.5	0.5	0.7	0.3634	0.5646	0.4372	0.6377	0.2746	0.6768	0.2600
		MLS	0.4	0.6	0.4	0.6							
		MLS	0.4	0.6	0.4	0.6							
	Reflective film	HS	0.5	0.7	0.3	0.5	0.5646	0.7652	0.2387	0.4372	0.2234	0.6242	0.4830
		VHS	0.6	0.8	0.2	0.4							
		VHS	0.6	0.8	0.2	0.4							
	Velvet surface	VHS	0.6	0.8	0.2	0.4	0.6000	0.8000	0.2000	0.4000	0.2000	0.6000	0.5288
		VHS	0.6	0.8	0.2	0.4							
		VHS	0.6	0.8	0.2	0.4							
	Quality control measures of cells	LS	0.3	0.5	0.5	0.7	0.3634	0.5646	0.4372	0.6377	0.2746	0.6768	0.2600
		MLS	0.4	0.6	0.4	0.6							
		MLS	0.4	0.6	0.4	0.6							
	Series elements in the module	CLS	0.1	0.3	0.7	0.9	0.1260	0.3302	0.6707	0.8746	0.1261	0.5342	0.0826
		CLS	0.1	0.3	0.7	0.9							
		VLS	0.2	0.4	0.6	0.8							
	Pass of IEC 61215 and UL1703 standards	VHS	0.6	0.8	0.2	0.4	0.6481	0.8485	0.1585	0.3545	0.1543	0.5549	0.5952
		CHS	0.7	0.9	0.1	0.3							
	Reflective coated glass	VHS	0.6	0.8	0.2	0.4	0.6649	0.8653	0.1418	0.3376	0.1372	0.5378	0.6193
		CHS	0.7	0.9	0.1	0.3							
		CHS	0.7	0.9	0.1	0.3							
	Thermal expansion treatment	VLS	0.2	0.4	0.6	0.8	0.2000	0.4000	0.6000	0.8000	0.2000	0.6000	0.1271
		VLS	0.2	0.4	0.6	0.8							
Company 2	Battery array density	LS	0.3	0.5	0.5	0.7	0.3634	0.5646	0.4372	0.6377	0.2746	0.6768	0.2600
		MLS	0.4	0.6	0.4	0.6							
		MLS	0.4	0.6	0.4	0.6							
	Reflective film	HS	0.5	0.7	0.3	0.5	0.5944	0.7958	0.2174	0.4114	0.1974	0.5995	0.5223
		VHS	0.6	0.8	0.2	0.4							
		CHS	0.7	0.9	0.1	0.3							
	Velvet surface	HS	0.5	0.7	0.3	0.5	0.5313	0.7319	0.2713	0.4702	0.2433	0.6441	0.4414
		HS	0.5	0.7	0.3	0.5							
		VHS	0.6	0.8	0.2	0.4							
	Quality control measures of cells	MLS	0.4	0.6	0.4	0.6	0.4642	0.6649	0.3376	0.5372	0.2692	0.6706	0.3632
		HS	0.5	0.7	0.3	0.5							
		HS	0.5	0.7	0.3	0.5							
	Series elements in the module	VLS	0.2	0.4	0.6	0.8	0.2884	0.4932	0.5111	0.7143	0.2465	0.6556	0.1954
		LS	0.3	0.5	0.5	0.7							
		MLS	0.4	0.6	0.4	0.6							
	Pass of IEC 61215 and UL1703 standards	CHS	0.7	0.9	0.1	0.3	0.7000	0.9000	0.1000	0.3000	0.1000	0.5000	0.6706
	Reflective coated glass	VHS	0.6	0.8	0.2	0.4	0.6316	0.8320	0.1735	0.3705	0.1704	0.5709	0.5722
		VHS	0.6	0.8	0.2	0.4							
		CHS	0.7	0.9	0.1	0.3							
	Thermal expansion treatment	VLS	0.2	0.4	0.6	0.8	0.2884	0.4932	0.5111	0.7143	0.2465	0.6556	0.1954
		LS	0.3	0.5	0.5	0.7							
		MLS	0.4	0.6	0.4	0.6							

Figure 6.3.6: Aggregated IVPF Engineering Assesments of Competition and Defuzzification.

	Our Company							Company 1											
Subcriteria	μ_L	μ_U	ν_L	ν_U	π_L	π_U	Defuzzified Aggregated Linguistic Customer Ratings of the Competition	μ_L	μ_U	ν_L	ν_U	π_L	π_U	Defuzzified Aggregated Linguistic Customer Ratings of the Competition	$d_t^{DR}(O,C1)$	κ_{O-C1}^{DR}	$\kappa_{O-C1}^{DR} \times d_t^{DR}(O,C1)$	$\widetilde{AI}_{ij}$	$\tilde{D}_{O-C1}^{DR}$
Battery array density	0.3302	0.5313	0.4642	0.6649	0.2678	0.6698	0.2295	0.3634	0.5646	0.4309	0.2057	0.2746	0.6768	0.2600	0.0985	-1	-0.0985	0.294468	-0.0290
Reflective film	0.6316	0.8320	0.1587	0.3634	0.1704	0.5709	0.5722	0.5646	0.7652	0.2289	0.2064	0.2234	0.6242	0.4830	0.0578	1	0.0578	0.151054	0.0087
Velvet surface	0.5000	0.7000	0.3000	0.5000	0.2600	0.6600	0.4036	0.6000	0.8000	0.2000	0.2000	0.2000	0.6000	0.5288	0.1061	-1	-0.1061	0.073945	-0.0078
Quality control measures of cells	0.4642	0.6649	0.3302	0.5313	0.2692	0.6706	0.3632	0.3634	0.5646	0.4309	0.2057	0.2746	0.6768	0.2600	0.1122	1	0.1122	0.195868	0.0220
Series elements in the module	0.2621	0.4642	0.5313	0.7319	0.2387	0.6427	0.1731	0.1260	0.3302	0.6649	0.2091	0.1261	0.5342	0.0826	0.1984	1	0.1984	0.08803	0.0175
Pass of IEC 61215 and UL1703 standards	0.7000	0.9000	0.1000	0.3000	0.1000	0.5000	0.6706	0.6481	0.8485	0.1414	0.2105	0.1543	0.5549	0.5952	0.0400	1	0.0400	0.197322	0.0079
Reflective coated glass	0.6649	0.8653	0.1260	0.3302	0.1372	0.5378	0.6193	0.6649	0.8653	0.1260	0.2091	0.1372	0.5378	0.6193	0.0052	0	0.0000	0.14855	0.0000
Thermal expansion treatment	0.2884	0.4932	0.4932	0.6952	0.2465	0.6556	0.1954	0.2000	0.4000	0.6000	0.2000	0.2000	0.6000	0.1271	0.1550	1	0.1550	0.135259	0.0210

Figure 6.3.7: Distance between Our Company and Company 1.

	Our Company							Company 2											
Subcriteria	μ_L	μ_U	ν_L	ν_U	π_L	π_U	Defuzzified Aggregated Linguistic Customer Ratings of the Competition	μ_L	μ_U	ν_L	ν_U	π_L	π_U	Defuzzified Aggregated Linguistic Customer Ratings of the Competition	$d_t^{DR}(O,C2)$	κ_{O-C2}^{DR}	$\kappa_{O-C2}^{DR} \times d_t^{DR}(O,C2)$	$\widetilde{AI}_{ij}$	$\tilde{D}_{O-C2}^{DR}$
Battery array density	0.3302	0.5313	0.4642	0.6649	0.2678	0.6698	0.2295	0.3634	0.5646	0.4309	0.6316	0.2746	0.6768	0.2600	0.0243	-1	-0.0243	0.294468	-0.0072
Reflective film	0.6316	0.8320	0.1587	0.3634	0.1704	0.5709	0.5722	0.5944	0.7958	0.1817	0.3915	0.1974	0.5995	0.5223	0.0264	1	0.0264	0.151054	0.0040
Velvet surface	0.5000	0.7000	0.3000	0.5000	0.2600	0.6600	0.4036	0.5313	0.7319	0.2621	0.4642	0.2433	0.6441	0.4414	0.0233	-1	-0.0233	0.073945	-0.0017
Quality control measures of cells	0.4642	0.6649	0.3302	0.5313	0.2692	0.6706	0.3632	0.4642	0.6649	0.3302	0.5313	0.2692	0.6706	0.3632	0.0000	0	0.0000	0.195868	0.0000
Series elements in the module	0.2621	0.4642	0.5313	0.7319	0.2387	0.6427	0.1731	0.2884	0.4932	0.4932	0.6952	0.2465	0.6556	0.1954	0.0206	-1	-0.0206	0.08803	-0.0018
Pass of IEC 61215 and UL1703 standards	0.7000	0.9000	0.1000	0.3000	0.1000	0.5000	0.6706	0.7000	0.9000	0.1000	0.3000	0.1000	0.5000	0.6706	0.0000	0	0.0000	0.197322	0.0000
Reflective coated glass	0.6649	0.8653	0.1260	0.3302	0.1372	0.5378	0.6193	0.6316	0.8320	0.1587	0.3634	0.1704	0.5709	0.5722	0.0243	1	0.0243	0.14855	0.0036
Thermal expansion treatment	0.2884	0.4932	0.4932	0.6952	0.2465	0.6556	0.1954	0.2884	0.4932	0.4932	0.6952	0.2465	0.6556	0.1954	0.0000	0	0.0000	0.135259	0.0000

Figure 6.3.8: Distance between Our Company and Company 2.

Table 6.3.1 and Table 6.3.2 represent the total distances between our company and competitors based on CRs and DRs respectively.

Table 6.3.1: Total distance between Our Company and competitors based on CRs.

CRs	Defuzzified Distances	
	O-C1	O-C2
High conversion efficiency	0.1320	0.0271
Manufacturing process	-0.0237	-0.0597
Modular design	-0.0268	-0.0133
Material quality	0.0420	0.0144
Product stability	0.0250	-0.0137
Government policy	0.0088	0.0000
TOTAL	**0.1572**	**-0.0452**

Table 6.3.2: Distances between Our Company and competitors based on DRs.

DRs	Defuzzified Distances	
	O-C1	O-C2
Battery array density	-0.0290	-0.0072
Reflective film	0.0087	0.0040
Velvet surface	-0.0078	-0.0017
Quality control measures of cells	0.0220	0.0000
Series elements in the module	0.0175	-0.0018
Pass of IEC 61215 and UL1703 standards	0.0079	0.0000
Reflective coated glass	0.0000	0.0036
Thermal expansion treatment	0.0210	0.0000
TOTAL	**0.0402**	**-0.0031**

Step 10: Equation 5.1.5 results in 0.1573 for O-C1 and -0.0452 for O-C2. equation 5.1.8 results in 0.0402 for O-C1 and -0.0031 for O-C2. Based on equation 5.1.11, the integrated CPR results (0.0987 for O-C1 and -0.0241 for O-C2) indicate that our company is better than C1 and worse than C2 by assuming that χ=0.5 and θ=0.5. The

same conclusion can be observed in Figure 6.3.9. The figure shows that our company is at the front of C1 and at the behind of C2.

Step 11: Determine the relative position of our company on a scale as in Figure 6.3.9.

Figure 6.3.9: Relative position of Our Company.

7. CONCLUSION

In today's business world there is an undeniable fast shift in CRs in both manufacturing and service sectors that forces all the companies to listen and respond those requirements starting by the designing phase. The CRs are evolving to be highly customized and diversified day by day. To resist the external challenges and competitors, companies should aim to lower production costs and designing time while increasing their service quality. To do so, they have to put their priorities in order since it not possible to answer all the customer needs in once. To define CRs and translating them into specific tasks for producing products meeting those needs, a prevalent method QFD is developed.

QFD consists of four associative matrices. The first matrix is denominated as HOQ where the CRs are summarized to be translated into DRs. HOQ's parts are divided into two as compulsory and optional. In this thesis study the optional parts (correlation matrix, organizational difficulty, directions of improvement and competitor analysis) are also taken into consideration since their absence may cause failures in the results nevertheless they are mostly ignored by researchers and practitioners because of the timewise and economic burden it brings.

When assessments of the voice of customers are made by linguistic terms and by multiple experts, a fuzzy approach is needed together with an aggregation operator due to the vague and imprecise nature of the linguistic evaluations. After the introduction of intuitionistic fuzzy sets to the literature, PFSs have been proposed since they have a larger domain for the determination of membership and non-membership degrees. This QFD study has been based on multi-expert linguistic assessments and these data have been processed by IVPFS. Through the aggregation operators and Pythagorean fuzzy operations, the relations among the elements of HOQ have been defined based on three customers' and three experts' evaluations for PV solar technology development. Performance analysis of our company among competitors has been formulated by PFS. The proposed equations have successfully measured the absolute importance of CRs and DRs and the distances between our company and the competitors. The study has been aimed at setting the relations in a HOQ in a comprehensive way under Pythagorean fuzziness.

For further research, a comparative analysis through neutrosophic FQFD analysis is suggested. Neutrosophic fuzzy sets are similarly based on a three-dimensional definition for membership functions: truthiness, indeterminacy, and falsity corresponding to membership, hesitancy, and non-membership, respectively.

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