

ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE
ENGINEERING AND TECHNOLOGY

**A METHOD FOR CORRECTING PARAMETERS OF RHEOLOGICAL
MODELS FACILITATING COUETTE TYPE VISCOMETERS**



M.Sc. THESIS

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Department of Petroleum and Natural Gas Engineering

Petroleum and Natural Gas Engineering Program

DECEMBER 2019

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İSTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ

**COUETTE TİPİ VİSKOMETRELERDEN ELDE EDİLEN REOLOJİK MODEL
PARAMETRELERİNİ DÜZELTMEK İÇİN BİR YÖNTEM**

YÜKSEK LİSANS TEZİ

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ARALIK 2019

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To my family,



FOREWORD

The oil and gas drilling establishment is essentially based on basic applied sciences such as applied physics, fluids mechanics, chemistry, and so forth. With the increasing number of studies and research papers published every day regarding those streams of science, a necessity arises in promoting the new findings in related fields and integrating them with the day-to-day drilling operation routine.

Taking a deep dive in understanding the rheological behavior of drilling fluids and expanding the knowledge in the grey areas such as plug flow to enhance measurement and interpretation techniques using basic field viscometer.

With a straightforward procedure that can be applied via simple software such as Excel and MATLAB, better description and enhanced interpretation of drilling fluids' rheological behavior can be obtained by facilitating simple Couette type viscometer data. This study resulted in remarkable findings that can be utilized in field drilling applications in exchange for technical and cost related advancements.

This study would not have taken place without the vision and continuous encouragement by Assoc. Prof. Dr. Gürsat ALTUN, by whom I received an outstanding guidance.

I also want to thank Eng. Rami SABBAGH for his incredible support, and for having faith in me throughout my studies.

December 2019

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ABBREVIATIONS

API	: American Petroleum Institute
BP	: Bingham Plastic
GPM	: Gallon per minute
HB	: Herschel-Bulkley
MSE	: Mean square error
OBM	: Oil based mud
PL	: Power Law
R#	: Fann viscometer reading at #rpm
Rel. Err.	: Relative error
RMSE	: Root mean square error
RPM	: Revolution per minute
SBM	: Synthetic based mud
SPP	: Stand pipe pressure
WBM	: Water based mud



NOMENCLATURE

A	: Area (m^2)
A_n	: Total bit nozzle area ($in.^2$)
C_1	: Constant of proportionality (Pa.s)
Ca	: Annular Reynold's number correction coefficient (dimensionless)
Cc	: Pipe Reynold's number correction coefficient (dimensionless)
d	: Pipe diameter (in.)
d_1	: Pipe outside diameter (in.)
d_2	: Hole inside diameter (in.)
F	: Force (N)
G	: Shear Modulus (Pa)
G_{BP}	: BP flow resistance constant-A (N.m)
$G_{BP,Fann}$	: BP Fann flow resistance constant-A (degrees)
G_{HB}	: HB flow resistance constant-A (N.m)
$G_{HB,Fann}$	: HB Fann flow resistance constant-A (degrees)
$G_{HB,Fann}$	: HB Fann flow resistance constant-A (degrees)
h	: Bob effective height (m, cm)
H_{BP}	: BP flow resistance constant-B (N.m.s)
$H_{BP,Fann}$	: BP Fann flow resistance constant-B (degrees. s^n)
H_{HB}	: HB flow resistance constant-B (N.m. n)
$H_{HB,Fann}$	: HB Fann flow resistance constant-B (degrees. s^n)
$H_{PL,Fann}$	: PL Fann flow resistance constant-B (degrees. s^n)
J	: Flow behaviour constant (dimensionless)
K	: Consistency index (Pa. s^n , lbf $s^n/100 ft^2$)
N	: Rotational speed (rps, rpm)
$N_{c,BP}$	: Bingham Plastic fluid critical rotational speed (rps, rpm)
$N_{c,HB}$	: HB fluid critical rotational speed (rps, rpm)
n	: Flow behaviour index (dimensionless)
q	: Flow rate (m^3/s , lt/min, gpm)
R	: Pipe radius (m)
r	: Viscometer radius (m, cm)
R^2	: Coefficient of determination (dimensionless)
R_1	: Pipe outside radius (m)
R_2	: Hole inside radius (m)
r_1	: Bob radius (m, cm)
r_2	: Rotor radius (m, cm)
R_s	: Plug radius (m)
Re_{eq}	: Equivalent Reynold's number (dimensionless)
Re_{eqcr}	: Critical equivalent Reynold's number (dimensionless)
T	: Torque (N.m, dyne.cm)
T°	: Temperature ($^\circ C$)
ΔL	: Pipe interval length (m, ft.)

ΔP	: Frictional pressure loss change (MPa, psi)
γ	: Shear Strain (dimensionless)
$\dot{\gamma}$	: Shear stress (1/s)
μ	: Viscosity (Pa.s, cp)
μ_p	: Plastic viscosity (Pa.s, mPa.s, cp)
θ	: Fann 35 dial reading (degrees)
v	: Velocity (m/s)
$\bar{v}$	: Average velocity (ft/s)
v_a	: Annular velocity (m/s)
v_p	: Pipe velocity (m/s)
τ	: Shear stress (Pa, lb/100sqf)
τ_0	: Herschel-Bulkley yield stress (Pa, lb/100sqf)
τ_c	: Calculated shear stress (Pa)
τ_m	: Measured shear stress (Pa)
τ_y	: Bingham Plastic yield stress (Pa, lb/100sqf)
$\bar{\tau}$	: Average shear stress (Pa)
ω	: Rotational speed (rad/s)
ρ	: Mud density (kg/m ³ , ppg)

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A METHOD FOR CORRECTING PARAMETERS OF RHEOLOGICAL MODELS FACILITATING COUETTE TYPE VISCOMETERS

SUMMARY

The success of a drilling operation is measured by two things; a well-engineered planning and experienced monitoring of operational parameters. Yet, these operational parameters form a framework and are incredibly dependent on each other. Failing to control the drilling fluid properties for instance, can limit hole cleaning, increase torque and drag, result in stuck pipe, or ultimately, an uncontrolled kick. The drilling fluids are an integral part of this framework, and controlling their rheological properties is vital to ensure a safe and cost efficient drilling operation.

Although, technical supply companies have always been competing to reveal whom can put up better equipment to test the rheological properties of drilling fluids, and while that resulted in cutting edge rheometers that can measure to high-end accuracy and precision, the simple field rotational viscometers are, and will be the forerunner for *routine* field testing of drilling fluids. This is due to numerous reasons, when compared with rheometers, field viscometers are more durable, easy to use, portable, cheap, and above all, allow for real-time measurement of the drilling fluids' rheology, which is vital for monitoring and adjusting the drilling fluid's parameters. The Fann 35 viscometer is the most used Couette type viscometer in the drilling industry, hence it was used as the prototype model in this thesis.

Upon testing drilling fluids using Couette type viscometers, a plug flow occurs at low rates of shear. Depending on the properties of the fluid, this incident can have a crucial impact on the dial reading values obtained, since the rheological models used to interpret these data assume a fully developed laminar flow. A few authors have suggested methods to determine the rpm at which the plug flow effect ends using the Bingham Plastic rheological model. However, not many researches have been conducted in this regard, and the plug flow occurrence have stayed a gray topic, until very recently, as it got more attention within the civil engineering and food engineering societies. Nonetheless, the traditional methods of field viscometer data interpretation are still being followed until today.

On the other hand, the three-parameter Herschel-Bulkley rheological model is being increasingly employed in the testing of drilling fluids' rheology rather than the conventional two-parameter Bingham Plastic and Power-Law models that have been observed to drastically misestimate or fail to describe the rheological behavior of some drilling fluids, especially with the rising variety of mud additives and extreme drilling conditions.

The American Petroleum Institute (API) have suggested simple equations to determine the rheological model parameters using some of the Fann 35 viscometer readings. However, research have shown that utilizing regression analysis to estimate those parameters yielded better results. Yet, non-linear regression of the Herschel-Bulkley

model have been noticed to give meaningless values for the yield stress. A more recent research has advised the utilization of somewhat, a unique approach that rather takes into account the pseudoplastic behavior for determining those parameters more accurately.

In this thesis, a straightforward method is proposed to correct the rheological parameters estimated using Fann 35 viscometer data. This has been done by determining the critical rpm at which the plug flow effect ends for both the Bingham Plastic and Herschel-Bulkley models using the parameters determined by the corrected regression approach mention earlier. The viscometer reading that falls below the critical rpm were then discarded, and the corrected regression is carried out once more using the remaining data. In order to hypothesize, three mud sample of different types, namely; water-based mud, synthetic based mud, and oil-based mud were tested. The method gave positive results for all three mud samples, and it was concluded that applying this method becomes more compelling for high-yield, and thick drilling fluids.

Factors affecting the plug flow evolution have been then investigated. Employing the Bingham Plastic model, it was found that the critical rpm increases proportionally with the yield stress-to-plastic viscosity ratio. As for the Herschel-Bulkley model, the effect of the flow behavior parameter has been illustrated to have a more dominant impact than the yield stress-to-consistency index ratio. Which agrees with the previous finding; that the more high-yield and thicker the fluid is, the more pronounced the plug flow consequence becomes.

The temperature effect was then studied on a sepiolite based drilling fluid, and it was observed that the critical rpm increases with temperature for such types of muds. Furthermore, at chemical alteration temperatures, the critical rpm spiked to as high as 150 rpm, and declined again upon reaching settling temperatures. It was concluded that for such types of muds, excluding the R3 and R6 when using the Bingham Plastic model, and often the R3 when using the Herschel-Bulkley model, should reduce the uncertainty while determining the model parameters. Taking the investigation one step further, the mud composition effect was inspected. The sample's critical rpm increased upon weighting the sepiolite mud, and decreased while increasing the salt concentration.

As per practical validation and application, the proposed method was tested for the stand pipe pressure calculation of an actual mud circulation test carried out on a well in the Po Valley, Italy. Utilizing the Bingham Plastic model, the error in frictional pressure loss prediction was considerably decreased upon applying the proposed correction method. As for the Herschel-Bulkley model however, the critical speed determined was below the lowest viscometer reading, and hence correcting the parameters was deemed unnecessary. It was noticed that the Bingham Plastic model overestimated the stand pipe pressure, whereas the Herschel-Bulkley model underestimated it, with the corrected Bingham Plastic model giving the best fit for the frictional pressure loss to the field measured values. Both models showed decreased accuracy at high flow rates, with high flow rates being crucial in modern drilling applications such as deep drilling, use of PDC bits, directional and horizontal drilling. Yet, correcting the model parameter decreased the error substantially.

To investigate the impact of correcting the Herschel-Bulkley model parameters, the water based mud used in this thesis was chosen to be the circulating fluid instead of the original mud. Notable pressure differences were observed subsequent to correcting

the model parameters indicating that proposed method is also valid for Herschel-Bulkley fluids as well.

This is absolutely important since the frictional pressure losses control the equivalent circulating density, which in turn affects almost every single aspect in the drilling application such as hole cleaning, bit hydraulics, surge and swab pressures, and most importantly, deep drilling applications (offshore drilling) where the mud window is relatively narrow.

Ultimately, regardless of the type of the mud and the temperature applied, following the approach presented in this thesis, will undoubtedly decrease the error in parameter determination of rheological models.





COUETTE TİPİ VİSKOMETRELERDEN ELDE EDİLEN REOLOJİK MODEL PARAMETRELERİNİ DÜZELTMEK İÇİN BİR YÖNTEM

ÖZET

Bir kuyunun sondajında operasyonel başarı iki şeyle ölçülür; iyi tasarlanmış bir mühendislik tasarımı ve operasyonel parametrelerin etkin bir şekilde izlenmesi. Bu işlemsel parametreler bir çerçeve oluşturmalarına rağmen birbirlerine oldukça bağımlıdır. Örneğin sondaj akışkanı özelliklerini kontrol edememek, kuyu temizliğini sınırlayabilir, tork ve sürtünmeyi artırabilir, sondaj dizisinin sıkışmasına veya nihayetinde kontrolsüz bir akışa (blowout) neden olabilir. Sondaj sıvıları bu yapının ayrılmaz bir parçasıdır ve reolojik özelliklerini kontrol etmek, güvenli ve düşük maliyetli bir sondaj işlemi sağlamak için hayati öneme sahiptir.

Her ne kadar, teknik donanım tedarik şirketleri her zaman sondaj akışkanlarının reolojik özelliklerini test etmek için hangi şirketin daha iyi test ekipmanı piyasaya sürebileceğini ortaya koymak için her zaman rekabet halindedirler. Bununla birlikte, ileri teknoloji ürünleri olan reometreler yüksek hassasiyet ve doğrulukta ölçüm yapabilmesine rağmen, basit saha tipi döner viskometreler sondaj sıvılarının sahada rutin testlerinde öncü konumdadır. Bu sayısız sebeplerden kaynaklanmaktadır. Reometrelerle karşılaştırıldığında, saha viskometreleri daha dayanıklı, kullanımı kolay, taşınabilir, ucuz ve hepsinden öte, sondaj akışkanının parametrelerinin izlenmesi ve ayarlanması için hayati olan sondaj akışkanları reolojisinin gerçek zamanlı olarak ölçülmesine izin verir. Fann 35 viskometre sondaj endüstrisinde en çok kullanılan Couette tip viskometredir, bu nedenle bu tezde prototip model olarak kullanılmıştır.

Couette tipi viskometreler kullanılarak sondaj akışkanları test edildiğinde, düşük kayma hızlarında bir tapa akışı meydana gelir. Akışkanın özelliklerine bağlı olarak, elde edilen kadran okuma değerleri üzerinde bu durum önemli bir etkiye sahip olabilir, çünkü bu verileri yorumlamak için kullanılan reolojik modeller tam gelişmiş bir tabakalı (laminer) akış varsaymaktadır. Birkaç araştırmacı, Bingham Plastik reolojik modelini kullanarak tapa akışının sona erdiği viskometre hızını belirlemek için bir yöntem önermiştir. Ancak, bu konuda tapa akışını göz önüne alan pek fazla araştırma yapılmamıştır. Tapa akışının oluşumu ve sınırlarının belirlenmesi gri bir konu olarak kalmasına rağmen, bu konu inşaat mühendisliği ve gıda mühendisliği alanlarında yakın zamanlarda daha fazla dikkat çeken bir durumdur. Bununla birlikte, geleneksel saha viskometresi ölçümlerine bağlı veri yorumlama yöntemleri günümüze kadar uygulanan bir yaklaşımdır.

Öte yandan, özellikle artan çeşitlilikte katkı maddeleri içeren ve zorlu sondaj koşullarında kullanılan sondaj akışkanlarının reolojik özelliklerini son derece yanlış bir şekilde tanımladığı veya tanımlayamadığı gözlemlenen geleneksel iki parametrelili

Bingham Plastik ve Power Law modelleri yerine üç parametrelili Herschel-Bulkley reolojik modeli sondaj akışkanlarının testinde giderek daha fazla kullanılmaktadır.

Amerikan Petrol Enstitüsü (API), Fann 35 viskometre okumalarından bazılarını kullanarak reolojik model parametrelerini belirleyen basit denklemler önermiştir. Ancak, araştırmalar bu parametrelerin tahmininde regresyon analizinin kullanılmasının daha iyi sonuçlar verdiğini göstermiştir. Bununla birlikte, Herschel-Bulkley modelinin doğrusal olmayan regresyonunun, akma noktası için anlamsız değerler verdiği fark edilmiştir. Yakın dönemde yapılan özgün bir araştırma, südoplastik (pseudoplastic) davranışı hesaba katarak reolojik parametreleri daha doğru bir şekilde belirlenmesinde yeni bir yaklaşımın kullanılmasını önermiştir.

Bu tezde, Fann 35 viskometre verilerini kullanılarak tahmin edilen reolojik parametreleri düzeltmek için basit bir yöntem önerilmiştir. Bu yöntemde, daha önce bahsedilen düzeltilmiş regresyon yaklaşımı kullanılarak hem Bingham Plastik hem de Herschel-Bulkley modelleri için tapa akış etkisinin sona erdiği kritik hızı belirleyerek parametrelerin bulunması gerçekleştirilmiştir. Kritik hızın altına düşen viskometre okumaları çıkarılmıştır ve daha sonra geriye kalan veriler kullanılarak düzeltilmiş regresyon bir kez daha gerçekleştirilmiştir. Hipotezin kontrolü için üç farklı çamur örneği; su bazlı çamur, sentetik bazlı çamur ve petrol bazlı çamur test edilmiştir. Yöntem her üç örnek için de pozitif sonuçlar vermiştir ve bu yöntemin uygulanmasının yüksek verimli (high yield) ve yüksek yoğunluklu (thick) sondaj akışkanları için daha çekici olduğu sonucuna varılmıştır.

Daha sonra, tapa akışının gelişimini etkileyen faktörler araştırılmıştır. Bingham Plastik modelini kullanarak, akma noktası-plastik viskozite oranı ile orantılı olarak kritik hızın arttığı bulunmuştur. Herschel-Bulkley modelinde, akış davranışı parametresinin etkisinin akma noktası-kıvamlilik indeksi oranından daha baskın bir etkiye sahip olduğu gösterilmiştir. Önceki bulgu ile aynı uygunlukta olarak; akışkan ne kadar yüksek akma noktalı ve yoğunsa, tapa akışı sonucunun o kadar belirgin olacaktır.

Daha sonra, sepiyolit temelli bir sondaj sıvısı üzerinde sıcaklık etkisi çalışılmıştır ve bu çamur türleri için kritik hızın sıcaklıkla arttığı gözlenmiştir. Ayrıca, flokülasyon sıcaklıklarında kritik hızın 150 dev/dak.'ya kadar yükseldiği ve deflokülasyon sıcaklığına ulaşıldığında ise tekrar düştüğü belirlenmiştir. Bu tip çamurlar için, Bingham Plastik modelini kullanırken R3 ve R6 hız okumalarının ve Herschel-Bulkley modelini kullanırken yoğunlukla R3 hız okumasının model parametrelerini belirlerken belirsizliği azaltması için hesaplamalarda kullanılmaması ve veri setinden çıkarılması gerektiği sonucuna varılmıştır. İnceleme bir adım daha ileri götürülerek, çamur kompozisyonu etkisi de incelenmiştir. Numunenin kritik hızının artan yoğunlukla arttığı ve artan tuz konsantrasyonunu ile azaldığı belirlenmiştir.

Bu çalışmada önerilen yöntemin doğrulaması ve pratik uygulaması için İtalya'da Po Vadisinde açılmış olan bir kuyuda sondaj çamurunun sirkülasyonu sırasında kuyudan ölçülen sürtünme basınç kayıpları hesaplanmıştır. Bingham Plastik (BP) model kullanılarak yapılan hesaplamalarda bu çalışmada önerilen yöntemin kullanılmasıyla elde edilen sürtünme basınç kayıplarının kuyuda ölçülen basınç kayıplarına daha yakın değerler verdiği ve hata payının klasik yöntemden hesaplanan sonuçlara göre önemli oranda düşük olduğu gösterilmiştir. Diğer taraftan, bu kuyuda kullanılan çamurun Herschel Bulkley (HB) model kullanılarak kritik hızının viskometreden elde edilen en düşük hız değerinden daha düşük olması nedeniyle HB modelinin kullanılmasına gerek kalmamıştır. Analiz sonucunda, BP modelinden hesaplanan sürtünme basınç kayıplarının kuyuda gözlemlenene basınç kayıplarından daha yüksek sonuçlar verdiği,

diğer taraftan HB model sonuçlarının ise gözlemlenenen daha düşük değerler verdiği belirlenmiştir. Bununla birlikte, gözlemlenen veya ölçülen sürtünme basınç kayıplarına en yakın sonuçların bu çalışmada önerilen ve BP modeline uygulanan yöntemden elde edildiği gösterilmiştir. Aynı zamanda, her iki yöntemden elde edilen sürtünme basınç kayıplarının çamur sirkülasyon debisinin artmasıyla kuyudan ölçülen değerlere ıraksama gösterdiği, hata payının arttığı gözlenmektedir. Bu gözlem çok önemlidir çünkü günümüzde PDC matkaplarının yaygın bir şekilde kullanılması nedeniyle sirkülasyon debileri oldukça yüksek olmak zorundadır ve klasik yöntemlerden elde edilen hesaplama sonuçlarının doğru değerden daha fazla ıraksadığını ve daha fazla hata değeri oluşturduğu çıkarımı yapılmaktadır. Bu nedenle, önerilen yöntem (model parametrelerini düzeltmek) bu hata payını önemli ölçüde düşürmektedir.

Düzeltilmiş HB model parametrelerinin etkisini göstermek amacıyla çalışmada literatürden elde edilen su bazlı çamur viskometre ölçüm değerleri kullanılmış ve Po Vadisi kuyusuna uygulanmıştır. Diğer bir ifadeyle, bu kuyuda kullanılan çamur yerine başka bir çamur kullanılsaydı HB model sonuçları nasıl değişirdi incelenmiştir. Literatürde uygulanan klasik HB model parametreleri ve bu çalışmada önerilen düzeltilmiş HB model parametrelerinin kullanılmasıyla elde edilen sonuçlar arasında önemli farklılık elde edilmiştir. Bu çalışmada önerilen yöntemden elde edilen sonuçların doğrulaması bu analiz ile gösterilmiştir.

Bu sonuç son derece önemlidir, çünkü sürtünme basınç kayıpları eşdeğer sondaj sirkülasyon yoğunluğunu (equivalent circulation density, ECD) doğrudan etkilemektedir ve bu yoğunluk değişimi ise kuyu temizliği, matkap hidroliği, dizi hareketi nedeniyle kuyu içi basınç dalgalanması, daha da önemlisi çamur penceresi aralığının göreceli olarak dar (sınırlı) olduğu derin su sondajı üzerine doğrudan etki edecektir.

Bu çalışmadan elde edilen diğer önemli bir sonuç ise çamur türünden bağımsız olarak hem Bingham Plastik hem de Herschel Bulkley modelleri için model sabitlerine bağlı olarak kritik hız değerlerini veren genelleştirilmiş grafikler verilmektedir. Bu grafikler kullanılarak, klasik yöntemlerden hesaplanan model sabitleri hesaplanarak kritik hızlar belirlenebilir ve bu hız değeri altındaki viskometre okumaları kullanılmadan model sabitleri düzeltilebilir.

Sonuçta, bu tezde sunulan yaklaşımın kullanılmasıyla, çamurun cinsi ve sıcaklığa bakılmaksızın, reolojik modellerin parametrelerinin belirlenmesindeki hata şüphesiz bir şekilde azaltacaktır.



1. INTRODUCTION

1.1 Drilling Fluids

Drilling operations in the energy sector are extremely stressful tasks, and arguably the most crucial aspect in field development, that requires the employment of many engineering and earth sciences expertise. This is due to the high uncertainty in such processes which requires continuous sampling and testing constraints to reduce or avoid drilling related problems. That in turn, is the key to lower the operational cost, and most importantly perform safely (Proehl and Sabins 2006, Taugbol et al. 2005).

Drilling fluids, or drilling muds are probably among the most vital factor in the oil and gas well drilling. The use of a drilling fluid makes the drilling process feasible, as it has the ability to suspend solids and persist in stability against high pressure and high temperature environments (Maghrabi et al. 2011, Sushant et al. 2011). Khodja, et al. (2010) showed that around 15% to 18% of the cost of drilling a well are related to drilling fluids.

Starting from the mud pits, the drilling mud goes through the mud pumps, where it receives enough momentum to travel down the drill pipe and out of the drill bit nozzles, only with sufficient pressure to carry away the drill cuttings back to the surface through the annulus, and discharge via the return line as demonstrated in Figure 1.1. The mud is then poured onto the shale shakers where the mesh-like sand screen separates drill cuttings and coarse particles from the drilling fluid. The mud continues afterwards through other filtration and purification processes such as the desander, desilter and the degasser, depending on the type of contamination.

After passing through the necessary filtration processes, the drilling mud is poured again into the mud pits, where continuous sampling and testing of the drilling fluid's physical parameters and chemical composition take place.

Due to lubricity ability of the drilling fluid, operational problems such as differential sticking and high torque and drag can be reduced (Mohamed et al. 2005). In fact, this

is directly related to the rheological description of the drilling fluid, which will be discussed in detail in following sections.

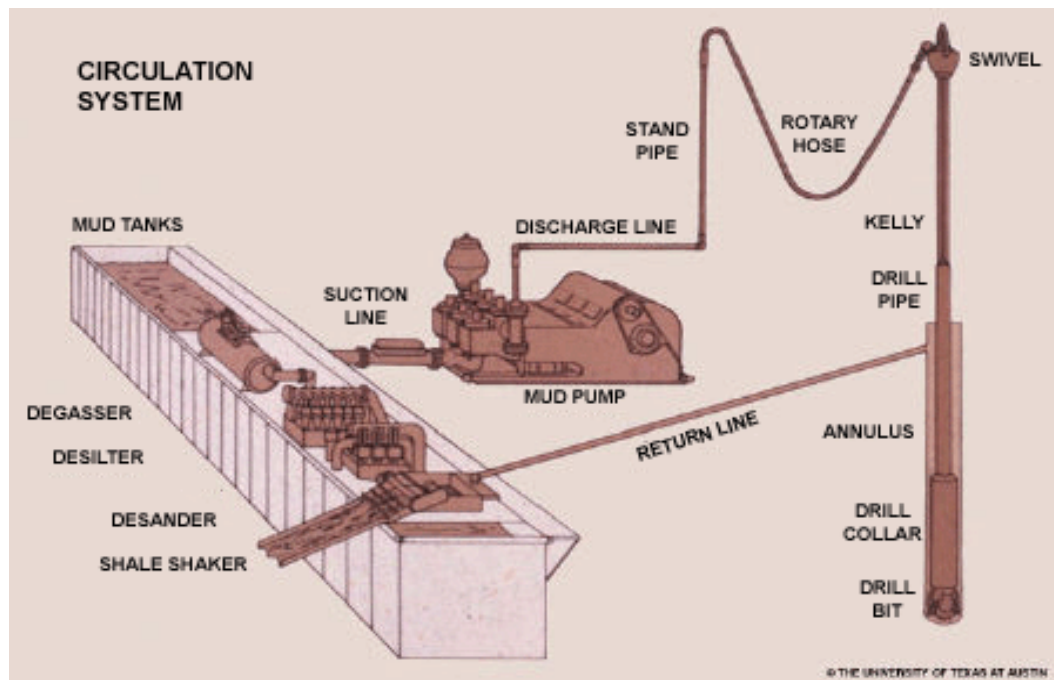


Figure 1. 1: Drilling fluid circulation system, (The University of Texas at Austin).

1.1.1 Types of drilling fluids

Depending on the depth of the prospect well, expected temperatures, type of formations to be encountered, and many other factors, the appropriate type of drilling fluids must be chosen to be compatible with the formerly mentioned conditions. These drilling fluid categories are:

- Gases: Natural gas or compressed air pumped through the drill string or the annulus in some special operations.
- Liquid/gas mixtures: Similar to the gas based drilling fluids. However, in this case, water is added to increase the viscosity and aids in cooling the drill string and drill bit. Polymers as foaming agents can be added in specific applications, as are claimed to increase hole cleaning ability.
- Water based muds (WBM): Consisting of either fresh or salt water with added clays and additives that help in resembling the homogenous structure of such muds. WBMs are possibly the most used type of drilling fluids due to many reasons such as efficiency in adjusting physical and chemical properties like the viscosity, density filtrate loss, pH, and salt concentration. The use of

smectite type clays such as bentonite and sepiolite in WBM enhances important features as solid suspension ability and gel strength. Adding potassium formate for instance, adds a set of lineaments to a WBM, those include viscosity control, lubrication of the drillstring, clay stability and enhancing rate of penetration. Lastly, WBMs stand first in terms of environmental considerations.

- Oil based muds (OBM): Oil based drilling fluids on the other hand, employ petroleum products like diesel as their base. The use of an OBM can be quite advantageous in some applications, as it increases the drilling fluid's lubricity, adds better shale stability and can withstand higher temperatures before deteriorating. However, in general, the disadvantages of OBMs outweigh their advantages, being extremely expensive and can easily lead to misinterpretation of logging data. On top of that, disposal of OBM cuttings can and handling is quite difficult, bringing the environmental considerations into the frame.
- Synthetic based muds (SBM): These types of drilling fluids are famous to feature the properties of an OBM with decreased toxicity, which is why the name "Low Toxicity Oil Based Mud" can be used interchangeably. Hence, an SBM is preferred for relatively confined spaces like offshore rigs, where most of the wells drilled are deep, high temperature holes. An SBM, however, fails to sort out the environmental and interpretations issues that the OBM presents.

Though, the use of water based muds is the most beneficial and cost efficient, and above all, can safely be disposed. It is almost impossible to maintain the quality of such muds under high pressure and high temperature conditions. Hence, the use of an oil based and synthetic based mud comes in handy.

In this thesis, the above mentioned three main types of drilling fluids, namely; WBM, SBM, and OBM are tested to justify that the suggested approach can be generalized in validity to any given type of drilling fluids. Figure 1.2 illustrates the types of drilling fluids in a hierarchy fashion.

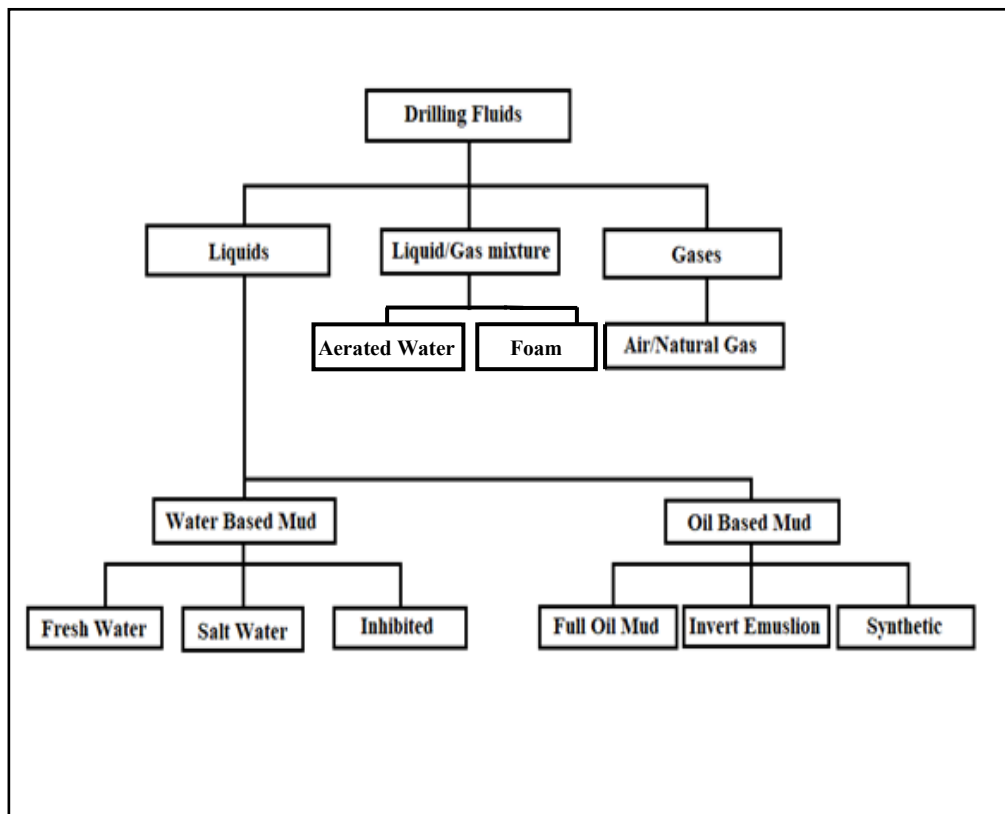


Figure 1. 2: Types of drilling fluids by composition.

1.1.2 Drilling fluid functions

As outlined in the previous section, the role of a drilling fluid is indispensable due to the numerous tasks and functions that it performs, and the contribution it adds to a various other drilling aspect. These functions include:

- Lubrication of the drill string and the drill bit, and decreasing friction in horizontal sections.
- Transporting the drill cuttings to the surface, hence ensuring hole cleaning.
- Increasing bit life and preserving mechanical features of the bit by continuous cooling.
- Maintaining adequate hydrostatic pressure to prevent the well from kicking.
- Ensuring a conductive medium for current and/or other signal transmission while conveying real time measurements and logging.
- Reducing rig hook load requirement through buoyancy-imposed drill string effective weight reduction.

- Ensuring shale stability and limiting the reactions with encountered formations. Sloughing shales are a good example of such.
- Transmitting energy to the drill bit.
- The use of an appropriate drilling fluid contributes to a huge cost reduction of other aspects in drilling applications.

1.1.3 Flow regimes

Fluid flow in the drilling industry, or in any other discipline is conducted via circular conduits. Çengel and Cimbala (2010), stated that fluids are most efficiently carried through circular conduits. This is because pipes can withstand higher differences in pressure between the inside and the outside, without substantial deformation.

By virtue of numerous studies and experiments conducted in regard to the fluid flow in conduits, we have developed a great understanding of the flow regime in the laminar region. However, until this day, there has not been any theoretical description to the behavior of fluid flow under turbulence conditions. Many have tried to simulate this type of flow, yet no analytical expression is proposed.

1.1.3.1 Laminar flow regime

Under laminar flow conditions, it is fairly easy to express the flow behavior mathematically and graphically, as the pressure change is proportional to viscosity and density. For circular conduits, the flow regime is laminar as far as Reynold's number is less than 2300, (Çengel and Cimbala, 2010). The velocity profile of circular conduits is simply predictable under laminar flow regime (parabola-like). As will be discussed in the coming chapters, this thesis focuses on the laminar flow regime as low rates of shear are investigated.

1.1.3.2 Turbulent flow regime

Turbulent flow is characterized by the chaotic mixing among the fluid particle, rather than moving in a predicted laminae-like fashion. This could be explained by the disturbing of the chemical bonds holding the fluid's laminae as the Reynold's number increases. Higher pressure drops are observed under this regime, due to the increased friction. Many experiment-based empirical relations exist in the literature to simulate the pressure drop behavior for turbulent flow.

As a rule of thumb, the fluid flow in a circular conduct is expected to be turbulent if the Reynold's number is above 4000.

1.1.3.3 Transitional flow regime

It has been noticed throughout experiments that the transition from laminar to turbulent regime does not happen abruptly, rather over a series of flow rate fluctuations, hence the term transitional region or transitional flow is used.

1.2 Rheology

All materials deform, and some start to flow when subject to a satisfactory magnitude of force and an amount of time. Rheology is the branch of physics that is concerned mainly with the flow and deformation of fluids (liquids and gasses) as a function of shear rate and/or shear time, under certain pressure and temperature.

In fact, rheology is also applicable to solids in terms of deformation of matter. Another way to regard this concept, is to remember that at a microscopic level, the primary discrimination between solids, liquids and gasses is the level of attraction between a substance's atoms, molecules, ions, etc.

Governed by Hooke's law, the elastic shear deformation of a solid is linearly related to the shear stress, and since it is an elastic deformation, once the stress is removed or decreased the strain will adjust respectively; i.e., reversible (Hibbeler and Sekar, 2013).

$$\tau = G\gamma \quad (1.1)$$

where,

τ : Shear Stress

γ : Shear Strain

G: Shear Modulus (Modulus of Rigidity)

By increasing the shear stress on the solid substance, it will begin to deform plastically, i.e., an non-reversible process. Most of the times a non-linear behavior is noticed in the plastic region. Meaning that the ability to resist deformation by shear stress is related to another property of the material called *ductility*. However, further shearing will lead to the substance failing to withstand the applied stress (Rupture Point).

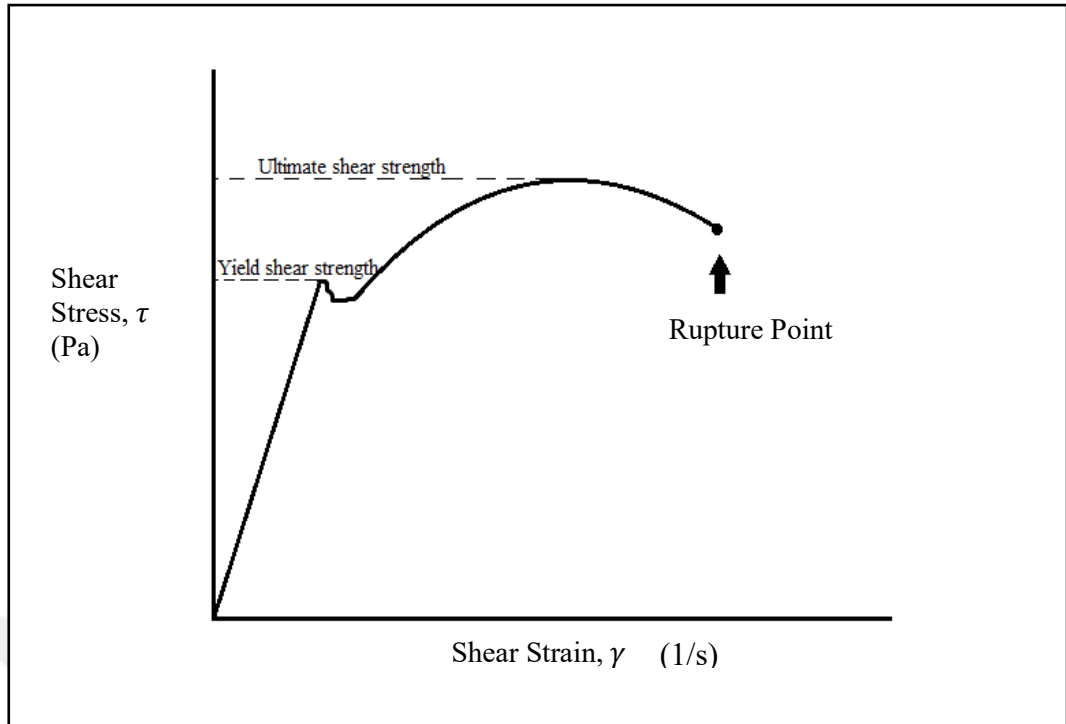


Figure 1. 3: Stress vs. strain behavior for most solids.

1.2.1 Newtonian fluids

Unlike solids, where the substance is resisting shear stress by deforming discretely, fluids on the other hand, will be deformed continuously “flow” when sufficient shear stress is applied. Meaning that, when the substance is sheared, the fluid’s particle are sliding past each other at a rate. Thus, the shear stress will be a function of the shear strain rate instead of the shear strain, (Bourgoyne et al., 1991).

To better understand this, let us consider a fluid contained between two slightly rough plates, (Figure 1.4). Where the bottom plate is stationary, and the upper plate is a distance L from the bottom plate and free to move along the horizontal. Now, we start moving the upper plate with a force F in the x -direction so that a constant velocity (v) is obtained, and upon waiting until a steady state is reached, the value of the velocity is found to increase proportionally with the magnitude of the force vector so that,

$$F = C_1 \frac{vA}{L} \quad (1.2)$$

where,

C_1 : Constant of proportionality or viscosity “ μ ”

A : Area of the plates

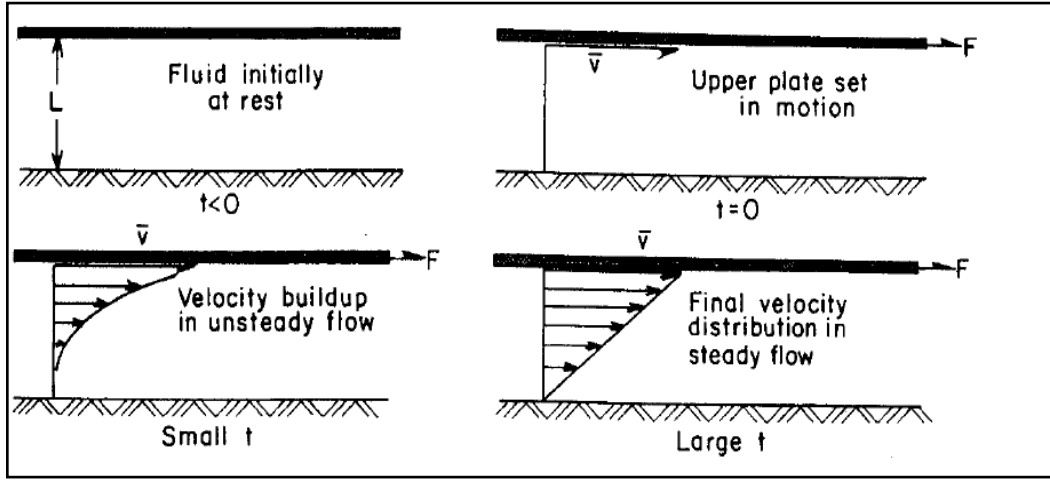


Figure 1. 4: Velocity distribution between the plates with time, (Bourgoyne et al., 1991).

Thus,

$$\frac{F}{A} = \mu \frac{v}{L} \quad (1.3)$$

We also know that the shear stress is defined as follows,

$$\tau = \frac{F}{A} \quad (1.4)$$

And that the velocity gradient or the shear rate as described before will be,

$$\dot{\gamma} = \frac{dv}{dL} \quad (1.5)$$

Hence, the above relation becomes,

$$\tau = \mu \dot{\gamma} \quad (1.6)$$

which is known as Newton's law of viscosity.

As noticed, the Newtonian's law of viscosity is somehow similar to Hooke's law, as the resistance to deform or flow is the constant of proportionality in both.

However, we should not confuse the concept of elasticity in Hooke's law with Newton's law, because even though some complex polymers might exhibit viscoelastic behavior, fluids in general can only behave elastically due to their compressibility, which is out of the scope of this thesis since constant thermodynamic conditions are assumed.

As the name suggests, this model is only applicable to Newtonian fluids, where the viscosity is always constant and independent of the stress applied. Newtonian fluids include water, gasses and some heavy oils to name a few. However, most of the fluids used in the oil industry are non-Newtonian fluids, as in most of drilling fluids, cement slurries and workover fluids for instance.

1.2.2 Non-Newtonian fluids

Contrary to Newtonian fluids, non-Newtonian fluids, predominantly exhibit flow resistance behavior that is non-linear to the applied stress, and in most cases non-Newtonian fluids will only start to flow upon exceeding a certain value of stress, called the yield stress or τ_y .

This non-linear behavior can be explained on a microscopic level as the varying shear stress can trigger the molecular or atomic structure and result in reorganization and altering the chemical framework of the substance, flocculation in polymers is an example of this. Same phenomena is observed in the plastic deformation of solids, as an increase of applied stress can amend the microstructure of a solid material.

Non-Newtonian fluids can be categorized as shear-thinning (pseudoplastic) and shear-thickening (dilatant) depending on the behavior these fluids demonstrate as a function of applied stress. Pseudoplastic fluids will show a decreasing flow resistance or apparent viscosity with increased shearing, whereas dilatant fluids will exhibit an increasing apparent viscosity with increasing shear stress. Most of drilling fluids and cement slurries are classified as pseudoplastic.

1.2.3 Time-dependent behavior

Apart from shear rate related deformation, some fluids exhibit a time-dependent behavior as they flow even when the shear rate is held constant. The viscosity of such fluids in this case is a function of the shear rate time as shown in Figure 1.5. This phenomenon is explained by the breaking of the cohesive forces between the substance's molecules with a force held constant for an amount of time, similar to fatigue aspect in solid materials.

Fluids that show decreasing viscosity with time are called thixotropic fluids, while fluids that manifest an increasing viscosity with time are called rheopectic fluids. Analogous to shear dependent fluids rheopexy can be related to dilatancy, where

thixotropy is related to pseudoplasticity. Mayonnaise and ketchup are examples of thixotropic fluids.

A number of drilling fluids in fact illustrate a thixotropic behavior. However, the effect of such anomaly has not gained considerable interest in the literature, since most rheological testing of drilling fluids and cement slurries is usually carried out upon shearing the substance for a sufficient amount of time.

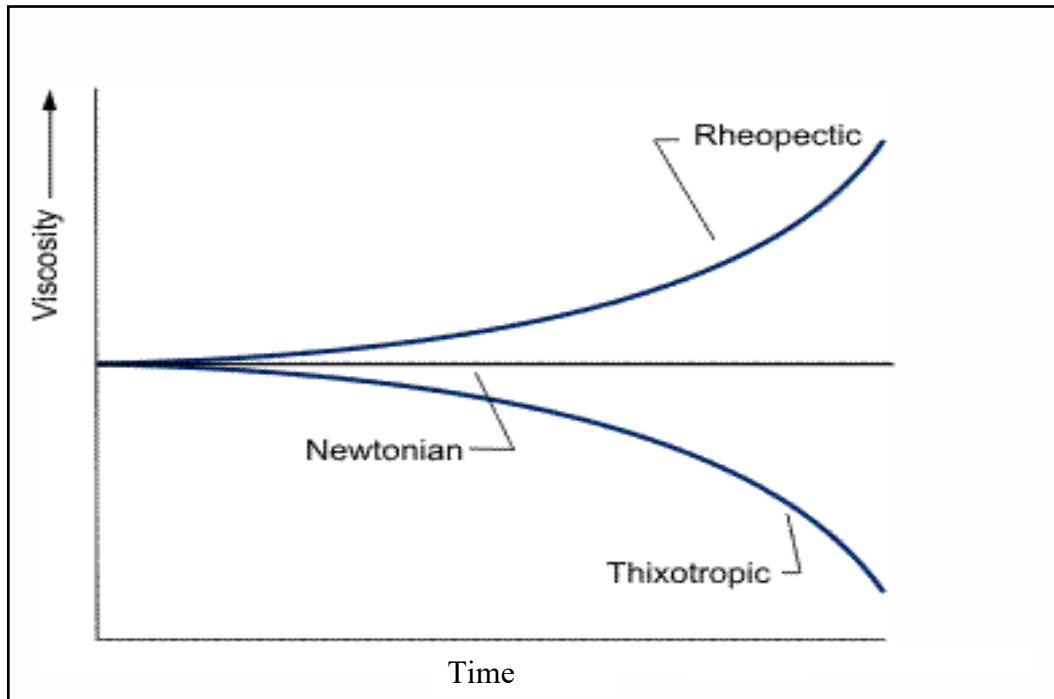


Figure 1. 5: Behavior of shear time dependent fluids, (Richardson, 2011).

1.3 Rheometry

In this section, the experimental techniques and apparatus used to determine the rheological properties of a fluid will be discussed briefly. Basically, there exist two main types of equipment involved in interpreting fluids' rheological parameters. These are the rheometers and the viscometers.

1.3.1 Rheometers

Rheometers are highly shear sensitive equipment that are mainly purposed to examine the response of shear rate for a wide range of applied shear stress, providing extensive information about the structure of the fluid. Generally, three types of rheometers are found in the market. These are the capillary rheometers, torque rheometers, and the dynamic oscillation rheometers, (Sherman, 2004).

Capillary rheometers can usually be operated to very high shear rates, and measure the viscosity as a function of pressure and temperature. Rotational rheometers on the other hand, come in handy for measuring the viscoelastic and polymer molecular structure at low rates of shear. Both, capillary rheometers and rotational rheometers can be used together to interpret the viscosity data for a wide range of shear rates, because they both measure the true viscosity of a fluid. Unlike the torque rheometers, where the viscosity is only recorded as a function of torque. Most of these rheometers can be connected to a data acquisition software. Figures 1.6 and 1.7 are examples of a capillary and an oscillatory rheometer, respectively.



Figure 1. 6: Capillary rheometer, (GmbH, 2019).



Figure 1. 7: Oscillatory rheometer, (Thermo Fisher, 2019).

1.3.2 Viscometers

Viscometers are generally designed to attain the viscosity of a fluid, rather than continuous shear stress vs. shear rate measurements. Elementary viscometers such as the orifice viscometers, capillary viscometers, falling piston viscometers etc. are types of viscometers that rely on the measurement of flow consistency relative to time, in order to determine the viscosity. Since these measurements depend on Poiseuille's law, it is not known to what extent it will be applicable to non-Newtonian fluids.

For this reason, the more advanced direct-indicating rotational viscometer was selected to be the viscometer of choice in the oil industry, and mostly in field measurements.

The direct-indicating viscometer yields direct measurements of the rotational speed and applied torque, which can be translated into shear rate and shear stress measurements respectively. In addition, having accuracy and simplicity combined in an easily portable viscometer, made the direct-indicating viscometer of most value among other viscometers

1.3.2.1 Fann 35 viscometer

Probably the viscometer used the most for field testing of drilling fluids is the Fann 35. As shown in Figure 1.7, this Couette rotational viscometer contains a rotating sleeve (rotor) and a coaxial stationary bob inside, where the testing fluid is placed in between. Upon switching on the motor to the specified RPM, the outer rotor will rotate at the respective angular velocity, shearing the fluid.

Assuming laminar flow, the fluid layers will begin to shear each other in a sliding-like motion towards the rotor. If we assume no slip is occurring at the rotor's inside wall and the bob's outside wall, the angular velocity of the fluid layer adjacent to the rotor must be identical to the rotor speed, and the fluid layer adjacent to the bob must be stationary, (Figure 1.9).

The bob in the Fann viscometer is connected to a torsion spring of a known constant that translates the shear stress into torque. Finally, the dial will revolve to display the magnitude of that torque, (Figure 1.8).

The shear rate in this case will be a function of the radius,

$$\dot{\gamma} = r \frac{d\omega}{dr} + \omega \quad (1.7)$$

Considering no slip is occurring between the fluid layers, Equation (1.7) becomes,

$$\dot{\gamma} = r \frac{d\omega}{dr} \quad (1.8)$$

As for the shear stress,

$$T = \tau(2\pi rh)r \quad (1.9a)$$

$$\tau = \frac{T}{2\pi hr^2} \quad (1.9b)$$

where,

h : is the height of the fluid, cm



Figure 1. 8: Fann 35 viscometer, (Fann, 2015).

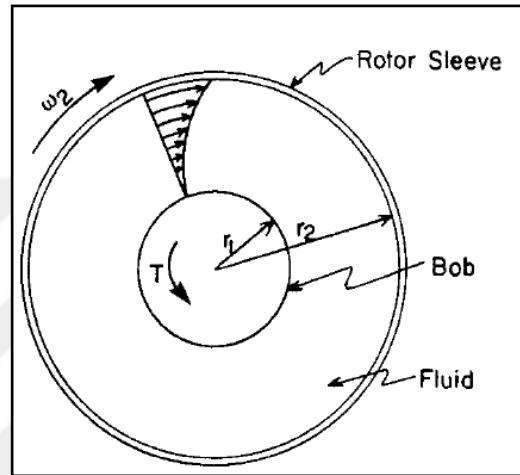


Figure 1. 9: Fann 35 viscometer velocity profile build up, (Bourgoyne et al., 1991).

It must be noted that the Fann 35 viscometers operate at either 6 or 12 speed variations which are 600, 300, 200 100, 6, 3 RPM or 600, 300, 200, 180, 100, 90, 60, 30, 6, 3, 1.8, 0.9 RPM. The dimension of the Fann viscometers are listed in Table 1.1, where Type 1 is the most commonly used type in the drilling industry.

Table 1. 1: Fann 35 viscometer dimensions.

Type	Bob Dimensions		Rotor Dimensions
	Radius, cm	Length, cm	Radius, cm
1	1.725	3.80	1.842
2	1.228	3.80	1.759
3	0.862	3.80	2.587
4	0.862	1.89	-

1.4 Rheological Models

A number of rheological models have been proposed in the literature to describe the flow behavior of non-Newtonian fluids. These models vary in complexity, while some are designed to be handy in practical or field use, other models are relatively more

complicated as they are proposed to simulate the rheological behavior more accurately. It is important to note that all the following rheological models, including the Newtonian model are only applicable in the case of a fully developed laminar flow. For turbulence flow regime, however, it is not possible to model the rheological behavior of a fluid analytically.

1.4.1 Bingham Plastic model

The Bingham Plastic model suggests that the fluid will only begin flow when sheared to certain value, called the yield stress. After exceeding the yield stress, the flow resistance is described to be proportional to the applied shear stress, and it is denoted as the plastic viscosity, μ_p . The model is given as:

$$\tau = \tau_y + \mu_p \dot{\gamma} \quad (1.10)$$

1.4.2 Power Law model

This model is quite useful in analyzing fluids of which, viscosity is shear rate dependent. The power law (PL) model is described as follows,

$$\tau = K \dot{\gamma}^n \quad (1.11)$$

The PL model, or so-called Ostwald-de Waele model, is a mathematical expression that involves two parameters to describe the non-Newtonian flow behavior. K is the consistency index, and n is the flow-behavior index. The value of n gives an idea about to what level the fluid behavior diverges from the Newtonian behavior. The Power Law model can reduce to the Newtonian model when the value of n is 1. Nevertheless, dilatant fluids are described with $n > 1$, where pseudoplastic fluids are expressed with a value of $n < 1$.

Both the Bingham Plastic and the Power Law models have been widely used in the oil industry in the past and still being in use today due to their simplicity, adequate rheological representation of drilling fluids and slurries, and to some level, being efficient and practical in field applications.

Nonetheless, both the Bingham Plastic and the Power Law models fail to describe the fluid flow behavior athwart the entire rheological spectrum.

1.4.3 Herschel-Bulkley model

Also known as the Yield Power Law model, this model was introduced by Winslow Herschel and Ronald Bulkley in 1926 as a generalized non-Newtonian fluid model, (Herschel and Bulkley, 1926).

$$\tau = \tau_0 + K\dot{\gamma}^n \quad (1.12)$$

The HB model implicates three parameters to describe the non-Newtonian behavior of fluids. These are τ_0 , K , and n . Similar to the Power Law, the K parameter is the HB consistency index, n is the flow index describing the pseudoplasticity of the fluid, and analogous to the BP model, a HB fluid will begin to flow once it is sheared to a certain yield stress, τ_0 .

As mentioned above, the HB model was constructed as a general expression to describe the rheological behavior in a broad range of fluids. The Newtonian model, Bingham Plastic model, and the Power Law model are all in fact special cases of the Herschel-Bulkley model.

Due to its relatively accurate rheological representation, the HB model had been adopted by many industries such as civil engineering and food engineering industries, pharmaceutical industry, medicine, so on and so forth. However, due to its analytical complexity, the BP and PL models, for years, thought to be satisfactory in the oil industry.

In recent years, and due to the advancement in drilling technologies, computational software, and the need for more predictive rheological modeling, the HB model had been receiving an increased attention, as in conducting a decent number of researches involving the integration of this model in drilling applications.

Figure 1.10 is a graphical representation of the rheological models mentioned above. Other models such as Casson model, Robertson-Stiff model, Sisko model are also used to describe the flow behavior of some fluids. However, these models are seldom used in the drilling fluids industry due to their relative inefficiency to assemble the pseudoplastic behavior of drilling fluids. Hence, these models will not be within the scope of this study.

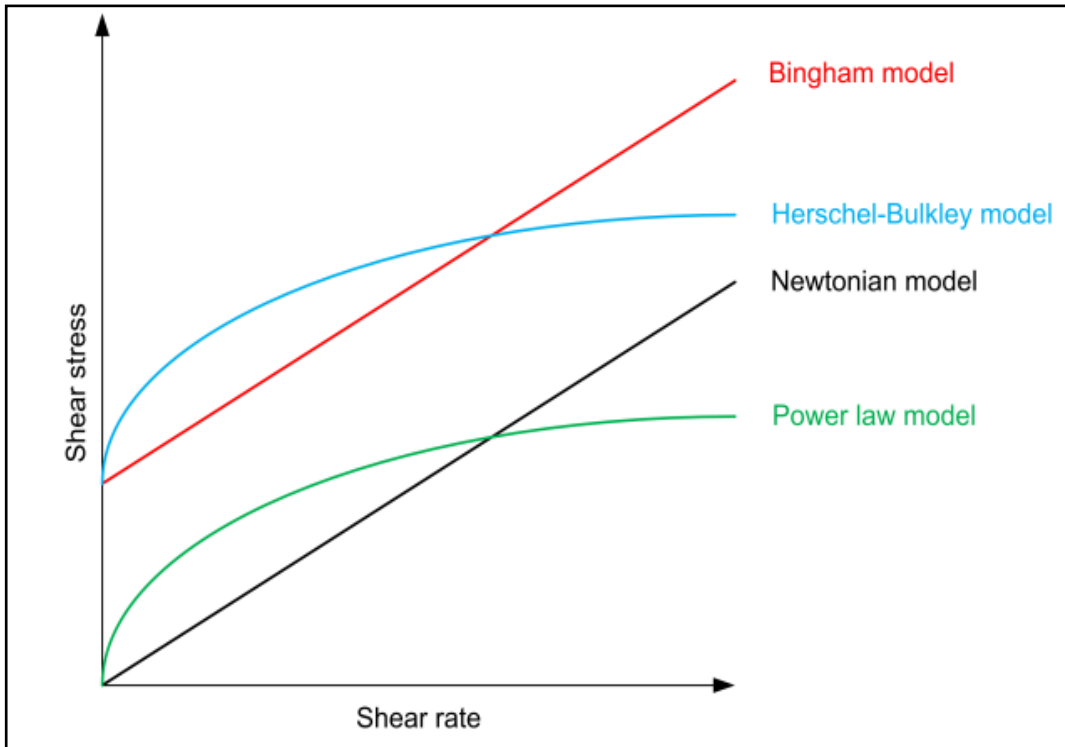


Figure 1. 10: Commonly used rheological models in the oil industry.

2. LITERATURE REVIEW

2.1 The Couette Inverse Problem

The Couette inverse problem arises when attempting to derive back the relationship between the shear stress and the shear rate by utilizing the viscometer T-N (Torque – Speed) data to fit into the rheological model of choice.

Once the solution for this inverse problem is obtained, the parameters of the rheological model can be estimated and the flow rheogram can be simulated. Hence, enabling the prediction of the rheological behavior of the fluid and forecast related operational parameters under the conditions of interest. The previously mentioned Fann 35 viscometer is frequently used to determine the fluid rheological properties.

The Couette inverse problem is basically a partial differential equation, the solution of which, is well known as the Reiner-Riwlin equation.

2.1.1 The Bingham Plastic solution

The shear rate for the Fann viscometer was derived earlier as,

$$\dot{\gamma} = r \frac{d\omega}{dr} \quad (2.1)$$

as well as the shear stress was related to the torque,

$$\tau = \frac{T}{2\pi hr^2} \quad (1.9b)$$

It is important to review the following assumptions:

- A fully developed laminar flow takes place between the rotor and the bob,
- No slip occurs at the walls or between the fluid laminae,
- Steady state flow and
- Time independent flow behavior.

Rewriting the Bingham Plastic model (Equation 1.10) using the above equations, namely; Equations (2.1 and 1.9b) for $\tau > \tau_y$ yields,

$$\frac{T}{2\pi hr^2} = \tau_y + \mu_p r \frac{d\omega}{dr} \quad (2.2)$$

Separating terms and integrating the above equation,

$$\int_{r_1}^{r_2} \left(\frac{T}{2\pi r^3 h} - \frac{\tau_y}{r} \right) dr = \mu_p \int_0^{\omega_2} d\omega \quad (2.3)$$

where,

r_1 : Bob radius, cm

r_2 : Rotor radius, cm

ω_2 : Rotor angular velocity, rad/s

T : Translated torque. dyne.cm

upon integrating,

$$\frac{T}{\pi h} = \frac{4(\omega_2 \mu_p + \tau_y \ln(\frac{r_2}{r_1}))}{(\frac{1}{r_1^2} - \frac{1}{r_2^2})} \quad (2.4)$$

The value of T can be obtained in units of dyne-cm using the Fann viscometer, by the following equation, (Bourgoyne et al., 1991).

$$T = 360.5\theta \quad (2.5)$$

Where, θ is the dial reading of the Fann viscometer,

$$\frac{360.5\theta}{\pi h} = \frac{4(\omega_2 \mu_p + \tau_y \ln(\frac{r_2}{r_1}))}{(\frac{1}{r_1^2} - \frac{1}{r_2^2})} \quad (2.6)$$

Heirman et al. (2008) suggested another form for the Reiner-Riwlin equation that directly utilize the T-N data, to determine the Bingham Plastic model parameters. This solution is presented in metric units as follows,

$$T = G_{BP} + H_{BP}N \quad (2.7)$$

$$G_{BP} = \frac{4\pi h \tau_y}{\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)} \ln\left(\frac{r_2}{r_1}\right) \quad (2.8)$$

$$H_{BP} = \frac{8\mu_p \pi^2 h}{\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)} \quad (2.9)$$

where,

T : Translated torque, N-m

N : Rotational speed, rps

r_1 : Bob radius, m

r_2 : Rotor radius, m

μ_p : Plastic viscosity, Pa.s

τ_y : Bingham Plastic yield stress, Pa

In order to employ this equation for field testing, field units must be used instead of the SI metric system units. Upon substituting the Fann viscometer's specified dimensions, the above equation can be expressed as follow,

$$\theta = G_{BP,Fann} + H_{BP,Fann} N \quad (2.10)$$

$$G_{BP,Fann} = 1.006 \tau_y \quad (2.11)$$

$$H_{BP,Fann} = 3.345 \times 10^{-3} \mu_p \quad (2.12)$$

where,

θ : Fann 35 Dial Reading

N : rpm

μ_p : cp

τ_y : lbf/100 sqft

Using linear regression least square method, the above equation can be rather convenient in evaluating the Bingham Plastic parameters using the $\theta - N$ data directly.

2.1.2 The Power Law solution

Again, substituting Equations (2.1 and 1.9b) in the Power Law model to construct the Couette inverse problem, we arrive to the following,

$$\frac{T}{2\pi hr^2} = Kr^n \left(\frac{d\omega}{dr} \right)^n \quad (2.13)$$

$$\int_0^{\omega_2} d\omega = \left(\frac{T}{2\pi hK} \right)^{1/n} \int_{r_1}^{r_2} \frac{dr}{r^{2n+1}} \quad (2.14)$$

With the assumptions made earlier in the case of Bingham Plastic flow are valid in the case, and integrating between the same boundary conditions for the inner and outer cylinders of the rotational viscometer:

$$\omega_2 = \left(\frac{T}{2\pi hK} \right)^{1/n} \left(\frac{n}{2} \right) \left(\frac{1}{r_1^{2/n}} - \frac{1}{r_2^{2/n}} \right) \quad (2.15)$$

Converting the above solution to field units, and substituting the Fann 35 viscometer dimensions as has been done in the previous section, the following can be obtained,

$$\theta = H_{PL,Fann} N^J \quad (2.16)$$

$$H_{PL,Fann} = \frac{504.6 * 2^{2n+1} \pi^{n+1} K}{(60n)^n \left(\frac{1}{0.01725^{2/n}} - \frac{1}{0.01842^{2/n}} \right)^n} \quad (2.17)$$

$$J = n \quad (2.18)$$

Where,

N : Fann 35 rotational speed, rpm

n : Flow Behavior Index, Dimensionless

K : Consistency Index, $\text{lbf s}^n/100 \text{ ft}^2$

2.1.3 The Herschel-Bulkley solution

Constructing the Couette inverse problem for the Herschel-Bulkley model by substituting the Equations (2.1 and 1.9b), for the shear rate and shear stress respectively, we arrive to the following differential equation.

$$\frac{T}{2\pi hr^2} = \tau_0 + K \left(r \frac{d\omega}{dr} \right)^n \quad (2.19)$$

Again, accepting the same boundary conditions and flow assumptions discussed in the Bingham Plastic model derivation are valid.

$$\int_{r_1}^{r_2} \left[\left(\frac{T}{2\pi r^3 h K} - \frac{\tau_0}{K} \right)^{1/n} \frac{1}{r} \right] dr = \int_0^{\omega_2} d\omega \quad (2.20)$$

Integrating the above differential equation for the same boundary condition noted earlier, yields a hypergeometric function, which can be represented by a Taylor series. However, in this case, the Herschel-Bulkley model parameters can only be determined numerically.

In their paper, Heirman et al. (2007) upon solving the above integral, they arrived to LerchPhi function. The LerchPhi function, even though being a continuous function and yielding a single value, it is however a non-polynomial function. Similar to the hypergeometric function, it can only be utilized to find the model parameters if a numerical software is used.

$$\left[\left(\frac{T}{2\pi r_1^2 h K} - \frac{\tau_0}{K} \right)^{\frac{1}{n}} \left[n - \hat{\varphi} \left(1 - \frac{T}{2\pi r_1^2 h \tau_0}, \quad 1, \quad \frac{1}{n} \right) \right] \right. \\ \left. - \left(\frac{T}{2\pi r_2^2 h K} - \frac{\tau_0}{K} \right)^{\frac{1}{n}} \left[n - \hat{\varphi} \left(1 - \frac{T}{2\pi r_2^2 h \tau_0}, \quad 1, \quad \frac{1}{n} \right) \right] \right] = 3\pi N \quad (2.21)$$

where,

$\hat{\varphi}(x, 1, a)$: Lerch-Phi function

N: Rotational speed, rps

The authors however, employed the knowledge of the HB model behavior (experimental data) in deriving a new mathematical solution by the use of decoupling of terms method in integrating the Couette inverse problem to yield the following constitutive equation (additive function),

$$T = G_{HB} + H_{HB}N^J \quad (2.22)$$

$$T' = G_{HB} \quad (2.23)$$

$$T'' = H_{HB}N^J \quad (2.24)$$

In the above equations, the term G_{HB} represents the flow resistance, while H_{HB} represents the viscosity factor, and similar to n , J is an indicator of the flow behavior.

According to the authors, and as proven by experimental data, the same flow resistance behavior of Bingham Plastic model can be applied for the integration of the first derivative function. Thus,

$$G_{HB} = \frac{4\pi h \tau_0}{\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)} \ln\left(\frac{r_2}{r_1}\right) \quad (2.25)$$

In the case where there is no flow resistance ($G_{HB} = 0$), the constitutive equation reduces to the Power Law model,

$$\frac{T''}{2\pi r^2 h} = K \left(r \frac{d\omega}{dr} \right)^n \quad (2.26)$$

Solving the above differential equation yields the following,

$$T'' = \frac{2^{2n+1} \pi^{n+1} h K}{n^n \left(\frac{1}{r_1^{2/n}} - \frac{1}{r_2^{2/n}} \right)^n} N^n \equiv H_{HB} N^J \quad (2.27)$$

So that,

$$K = \frac{H_{HB}n^n}{2^{2n+1}\pi^{n+1}h} \left(\frac{1}{r_1^{2/n}} - \frac{1}{r_2^{2/n}} \right)^n \quad (2.28)$$

$$n = J \quad (2.29)$$

By adding up the two solutions above to the additive function, the following analytical solution to the Couette inverse problem can be derived:

$$\begin{aligned} T &= T' + T'' = G_{HB} + H_{HB}N^J \\ &= \frac{4\pi h \tau_0}{\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)} \ln\left(\frac{r_2}{r_1}\right) + \frac{2^{2n+1}\pi^{n+1}hK}{n^n \left(\frac{1}{r_1^{2/n}} - \frac{1}{r_2^{2/n}}\right)^n} N^n \end{aligned} \quad (2.30)$$

Using the above equation is claimed to provide a reliable representation of the $\tau(\dot{\gamma})$ curve after exceeding the yield stress (τ_0).

Converting the above solution to field units, and substituting the Fann 35 viscometer dimensions, the following expression is obtained:

$$\theta = G_{HB,Fann} + H_{HB,Fann}N^J \quad (2.31)$$

$$H_{HB,Fann} = \frac{504.6 * 2^{2n+1}\pi^{n+1}K}{(60n)^n \left(\frac{1}{0.01725^{2/n}} - \frac{1}{0.01842^{2/n}} \right)^n} \quad (2.32)$$

$$G_{HB,Fann} = 1.006\tau_y \quad (2.33)$$

$$n = J \quad (2.34)$$

where,

N : Fann 35 rotational speed, rpm

τ_y : Yield stress, lbf/100ft²

n : Flow Behavior Index, Dimensionless

K : Consistency Index, lbf. s ^{n} /100 ft²

It is also noted by the authors that this solution is actually valid for both dilatant fluids, $n > 1$ and the shear-thinning pseudoplastic fluids $n < 1$ as well.

2.2 Model Parameter Determination

Real-time measurement of the drilling fluid's rheological properties plays a vital role in almost every single aspect of the drilling process. And while continuous monitoring of the drilling fluid's properties is needed, the urge of determining the rheological parameters of the fluid model as rapidly and accurate as possible arises. In this section, the methods of determining these parameters will be discussed.

A number of simplified methods and API recommended formulas which are intended to estimate the rheological parameters using only two or sometimes four points of measurements exist in the literature. Though these methods proposed for real-time field measurement, the accuracy of the methods is widely debated in literature. Yet these methods will be briefly discussed at the end of this section.

With the increased accessibility of computers and smart calculators in the field, it is more convenient to facilitate all the viscometer measurement points in the parameter calculation approach, believing it will yield more representative results than using only two or three points of relatively high shear rate values.

2.2.1 Least square method

Perhaps the most adequate statistical method in estimating the constants of various types of functions, whether linear or non-linear. Commencing from simple calculus, statistics and linear algebra, the least squares method assumes a function that goes through the data points of interest. Similar to the variance concepts in statistics, an error function, which describes the squared vertical distance between the estimated function value and the actual value for each data point is then minimized to obtain the

optimum fit. The method of least squares will be briefly explained in the following paragraph.

As seen in Figure 2.1, having a set of points; y_i, x_i where $1 \leq i \leq m$, m : the number of data points. It is possible to fit a function to represent these data points by minimizing the squared distance between each data point (y_i), and the function's value at that same point ($f(\theta_j, x_i)$), where θ_j denotes the vector of the function's constants, (Miller, 2007).

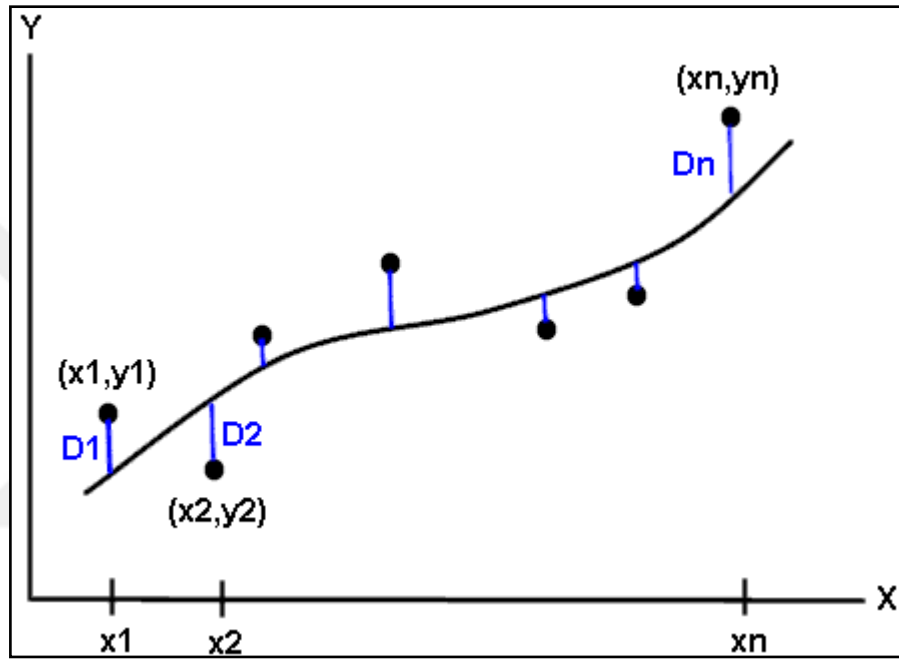


Figure 2. 1: Graphical representation of the least square method, (Miller, 2007).

This is achieved by equating the derivatives of the error function ($E = \sum_{i=1}^m [y_i - f(\theta_j, x_i)]^2$) with respect to the function constants to zero.

For a function of the form; $y = ax + b$,

$$E(a, b) = \sum_{i=1}^m [y_i - (ax_i + b)]^2 \quad (2.35)$$

Minimizing the error function,

$$\frac{\partial E}{\partial a} = \frac{\partial E}{\partial b} = 0 \quad (2.36)$$

$$\frac{\partial E}{\partial a} = \sum_{i=1}^m 2[y_i - (ax_i + b)](-x_i) = 0 \quad (2.37)$$

$$\frac{\partial E}{\partial b} = \sum_{i=1}^m 2[y_i - (ax_i + b)](-1) = 0 \quad (2.38)$$

Equations (2.37 and 2.38) can be rewritten in the following form, respectively,

$$\left(\sum_{i=1}^m x_i^2\right)a + \left(\sum_{i=1}^m x_i\right)b = \sum_{i=1}^m x_i y_i \quad (2.39)$$

$$\left(\sum_{i=1}^m x_i\right)a + \left(\sum_{i=1}^m 1\right)b = \sum_{i=1}^m y_i \quad (2.40)$$

Thus, we obtain a linear system of equations that can be solved for constant ‘a’ and ‘b’ knowing the values of the data point set, m.

$$\begin{pmatrix} \sum_{i=1}^m x_i^2 & \sum_{i=1}^m x_i \\ \sum_{i=1}^m x_i & \sum_{i=1}^m 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^m x_i y_i \\ \sum_{i=1}^m y_i \end{pmatrix} \quad (2.41)$$

2.2.1.1 BP model parameter determination

The Bingham Plastic model is a function of the form; $y = ax + b$, hence by solving the above-mentioned homogenous system of equations, and with the appropriate annotations,

$$\begin{pmatrix} \sum_{i=1}^m \dot{\gamma}_i^2 & \sum_{i=1}^m \dot{\gamma}_i \\ \sum_{i=1}^m \dot{\gamma}_i & \sum_{i=1}^m 1 \end{pmatrix} \begin{pmatrix} \mu_p \\ \tau_y \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^m \dot{\gamma}_i \tau_i \\ \sum_{i=1}^m \tau_i \end{pmatrix} \quad (2.42)$$

$$\tau_y = \frac{(\sum_{i=1}^m \dot{\gamma}_i^2)(\sum_{i=1}^m \tau_i) - (\sum_{i=1}^m \dot{\gamma}_i \tau_i)(\sum_{i=1}^m \dot{\gamma}_i)}{m(\sum_{i=1}^m \dot{\gamma}_i^2) - (\sum_{i=1}^m \dot{\gamma}_i)^2} \quad (2.43)$$

$$\mu_p = \frac{m(\sum_{i=1}^m \dot{\gamma}_i \tau_i) - (\sum_{i=1}^m \dot{\gamma}_i)(\sum_{i=1}^m \tau_i)}{m(\sum_{i=1}^m \dot{\gamma}_i^2) - (\sum_{i=1}^m \dot{\gamma}_i)^2} \quad (2.44)$$

For direct use of viscometer data, the above expressions can be transformed to the following form,

$$\begin{pmatrix} \sum_{i=1}^m N_i^2 & \sum_{i=1}^m N_i \\ \sum_{i=1}^m N_i & \sum_{i=1}^m 1 \end{pmatrix} \begin{pmatrix} H_{B,Fann} \\ G_{B,Fann} \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^m N_i \theta_i \\ \sum_{i=1}^m \theta_i \end{pmatrix} \quad (2.45)$$

$$G_{BP,Fann} = \frac{(\sum_{i=1}^m N_i^2)(\sum_{i=1}^m \theta_i) - (\sum_{i=1}^m N_i \theta_i)(\sum_{i=1}^m N_i)}{m(\sum_{i=1}^m N_i^2) - (\sum_{i=1}^m N_i)^2} \quad (2.46)$$

$$H_{BP,Fann} = \frac{m(\sum_{i=1}^m N_i \theta_i) - (\sum_{i=1}^m N_i)(\sum_{i=1}^m \theta_i)}{m(\sum_{i=1}^m N_i^2) - (\sum_{i=1}^m N_i)^2} \quad (2.47)$$

The values of τ_y and μ_p can be determined from Equations (2.11 and 2.12) respectively.

2.2.1.2 PL model parameter determination

As seen from Equation (1.11), the Power Law model is a non-linear model, hence the function has to be linearized as follows,

$$\ln(\tau) = \ln(K) + n \ln(\dot{\gamma}) \quad (2.48)$$

Similar to the $y = ax + b$ form, where,

$$y = \ln(\tau)$$

$$x = \ln(\dot{\gamma})$$

$$b = \ln(K)$$

$$a = n$$

Thus, the following system of equations must be solved linearly,

$$\begin{pmatrix} \sum_{i=1}^m 1 & \sum_{i=1}^m \ln(\dot{\gamma}_i) \\ \sum_{i=1}^m \ln(\dot{\gamma}_i) & \sum_{i=1}^m \ln^2(\dot{\gamma}_i) \end{pmatrix} \begin{pmatrix} \ln(K) \\ n \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^m \ln(\tau_i) \\ \sum_{i=1}^m \ln(\dot{\gamma}_i) \ln(\tau_i) \end{pmatrix} \quad (2.49)$$

The Power Law model parameters can hence be determined from the equations below,

$$K = \exp \left[\frac{(\sum_{i=1}^m \ln(\tau_i))(\sum_{i=1}^m \ln^2(\dot{\gamma}_i)) - (\sum_{i=1}^m \ln(\dot{\gamma}_i) \ln(\tau_i))(\sum_{i=1}^m \ln(\dot{\gamma}_i))}{m(\sum_{i=1}^m \ln^2(\dot{\gamma}_i)) - (\sum_{i=1}^m \ln(\dot{\gamma}_i))^2} \right] \quad (2.50)$$

$$n = \frac{m(\sum_{i=1}^m \ln(\dot{\gamma}_i) \ln(\tau_i)) - (\sum_{i=1}^m \ln(\dot{\gamma}_i))(\sum_{i=1}^m \ln(\tau_i))}{m(\sum_{i=1}^m \ln^2(\dot{\gamma}_i)) - (\sum_{i=1}^m \ln(\dot{\gamma}_i))^2} \quad (2.51)$$

Or by directly using the Fann 35 viscometer $\theta - N$ data:

$$H_{PL,Fann} = \exp \left[\frac{(\sum_{i=1}^m \ln(\theta_i))(\sum_{i=1}^m \ln^2(N_i)) - (\sum_{i=1}^m \ln(N_i) \ln(\theta_i))(\sum_{i=1}^m \ln(N_i))}{m(\sum_{i=1}^m \ln^2(N_i)) - (\sum_{i=1}^m \ln(N_i))^2} \right] \quad (2.52)$$

$$J = \frac{m(\sum_{i=1}^m \ln(N_i) \ln(\theta_i)) - (\sum_{i=1}^m \ln(N_i))(\sum_{i=1}^m \ln(\theta_i))}{m(\sum_{i=1}^m \ln^2(N_i)) - (\sum_{i=1}^m \ln(N_i))^2} \quad (2.53)$$

Again, the values of 'K' and 'n' parameters can be determined from Equations (2.17 and 2.18) respectively.

2.2.1.3 HB model parameter determination

As seen from Equation 1.12, the Herschel-Bulkley model is a three-parameter, non-linear function of shear rate, hence applying the least square method to obtain the model parameters will not be a straightforward problem. To lay it on the line, there are basically three ways of approaching such problems. One way is to determine the parameters by plain computational mathematics using iterative methods such as Newton-Gauss where an initial estimate of one of the model's parameters, in most

cases the flow behavior index ‘n’ will be required, which as it sounds, should not be expected to produce the most accurate estimates.

Setting the error function for the Herschel-Bulkley model to fit through measured data points,

$$E(K, n, \tau_y) = \sum_{i=1}^m [\tau_i - (K\dot{\gamma}_i^n + \tau_0)]^2 \quad (2.54)$$

$$\frac{\partial E}{\partial K} = 2 \left[\sum_{i=1}^m K\dot{\gamma}_i^{2n} + \sum_{i=1}^m \tau_0\dot{\gamma}_i^n - \sum_{i=1}^m \tau_i\dot{\gamma}_i^n \right] = 0 \quad (2.55)$$

$$\frac{\partial E}{\partial n} = 2K \left[\sum_{i=1}^m K\dot{\gamma}_i^{2n} \ln(\dot{\gamma}_i) + \sum_{i=1}^m \tau_0\dot{\gamma}_i^n \ln(\dot{\gamma}_i) - \sum_{i=1}^m \tau_i\dot{\gamma}_i^n \ln(\dot{\gamma}_i) \right] = 0 \quad (2.56)$$

$$\frac{\partial E}{\partial \tau_0} = 2 \left[\sum_{i=1}^m K\dot{\gamma}_i^n - \sum_{i=1}^m \tau_i + \sum_{i=1}^m \tau_0 \right] = 0 \quad (2.57)$$

Simplifying the above equation, and solving for the yield stress,

$$\tau_0 = \frac{\sum_{i=1}^m \tau_i \dot{\gamma}_i^n - K \sum_{i=1}^m \dot{\gamma}_i^{2n}}{\sum_{i=1}^m \dot{\gamma}_i^n} \quad (2.58)$$

$$\tau_0 = \frac{\sum_{i=1}^m \tau_i \dot{\gamma}_i^n \ln(\dot{\gamma}_i) - K \sum_{i=1}^m \dot{\gamma}_i^{2n} \ln(\dot{\gamma}_i)}{\sum_{i=1}^m \dot{\gamma}_i^n \ln(\dot{\gamma}_i)} \quad (2.59)$$

$$\tau_0 = \frac{\sum_{i=1}^m \tau_i - K \sum_{i=1}^m \dot{\gamma}_i^n}{\sum_{i=1}^m 1} \quad (2.60)$$

Equating the above equations and solving for K,

$$K = \frac{m \sum_{i=1}^m \tau_i \dot{\gamma}_i^n - \sum_{i=1}^m \dot{\gamma}_i^n \sum_{i=1}^m \tau_i}{m \sum_{i=1}^m \dot{\gamma}_i^{2n} - (\sum_{i=1}^m \dot{\gamma}_i^n)^n} \quad (2.61)$$

$$K = \frac{m \sum_{i=1}^m \tau_i \dot{\gamma}_i^n \ln(\dot{\gamma}_i) - \sum_{i=1}^m \dot{\gamma}_i^n \ln(\dot{\gamma}_i) \sum_{i=1}^m \tau_i}{m \sum_{i=1}^m \dot{\gamma}_i^{2n} \ln(\dot{\gamma}_i) - \sum_{i=1}^m \dot{\gamma}_i^n \sum_{i=1}^m \dot{\gamma}_i^n \ln(\dot{\gamma}_i)} \quad (2.62)$$

Using iteration methods to quantify the value of the flow behavior index ‘n’ by equating the two equations above, namely; Equations (2.61 and 2.62) can actually transform the Herschel-Bulkley into a polynomial of two parameters that can be solve linearly in terms of τ vs. $\dot{\gamma}^n$.

However, as shown by Nguimatsia, J. D. (2019), applying this method can sometimes produce erroneous yield stress estimates (negative yield stress). That can be explained by the lack of a fully laminar flow developments at such low rates of shear.

Another way of approaching this problem, is to remember that the system of equations can be solved for the parameters analytically if the model is linearized. This is basically done by guessing the value of the yield stress from the low shear rate data trend. The use of estimated yield stress is demonstrated as follows,

$$\ln(\tau - \tau_y) = \ln(K) + n \ln(\dot{\gamma}) \quad (2.63)$$

Knowing the value of the yield stress can reduce the model to a linear form of two unknown parameters,

$$\begin{pmatrix} \sum_{i=1}^m 1 & \sum_{i=1}^m \ln(\dot{\gamma}_i) \\ \sum_{i=1}^m \ln(\dot{\gamma}_i) & \sum_{i=1}^m \ln^2(\dot{\gamma}_i) \end{pmatrix} \begin{pmatrix} \ln(K) \\ n \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^m \ln(\tau_i - \tau_y) \\ \sum_{i=1}^m \ln(\dot{\gamma}_i) \ln(\tau_i - \tau_y) \end{pmatrix} \quad (2.64)$$

Where;

$$K = e^{\left[\frac{(\sum_{i=1}^m \ln(\tau_i - \tau_y))(\sum_{i=1}^m \ln^2(\dot{\gamma}_i)) - (\sum_{i=1}^m \ln(\dot{\gamma}_i) \ln(\tau_i - \tau_y))(\sum_{i=1}^m \ln(\dot{\gamma}_i))}{m(\sum_{i=1}^m \ln^2(\dot{\gamma}_i)) - (\sum_{i=1}^m \ln(\dot{\gamma}_i))^2} \right]} \quad (2.65)$$

$$n = \frac{m(\sum_{i=1}^m \ln(\dot{\gamma}_i) \ln(\tau_i - \tau_y)) - (\sum_{i=1}^m \ln(\dot{\gamma}_i))(\sum_{i=1}^m \ln(\tau_i - \tau_y))}{m(\sum_{i=1}^m \ln^2(\dot{\gamma}_i)) - (\sum_{i=1}^m \ln(\dot{\gamma}_i))^2} \quad (2.66)$$

The parameters can then be optimized iteratively by comparing the calculated shear stress values to the measured ones, as the assumed yield stress value will be adjusted at each iteration. This is done by minimizing the relative error function:

$$\Delta E = \sum_{i=1}^m (\tau_{i,measured} - \tau_{i,calculated})^2 \quad (2.67)$$

However, Mullineux (2007) showed in his study on applied mathematical modeling that following the above method can lead to inaccurate estimations of the Herschel-Bulkley model parameters. He also demonstrated the sensitivity of the ‘n’ and ‘k’ parameters to the initial yield stress estimate as shown in Figure 2.2.

He instead suggested an alternative method in determining the parameters of this model. Facilitating the knowledge of the model’s behavior, while the Herschel-Bulkley model is mostly used for pseudoplastic fluids, i.e. $0 < n < 1$. The author claims that the set of normal equations concerning the K and τ_y can be invoked to generate a function of ‘n’ alone that has a root at the convenient value of n. The function F(n) is in fact a determinant function,

$$F(n) = \begin{vmatrix} m & \sum_{i=1}^m \dot{\gamma}_i^n & \sum_{i=1}^m \dot{\gamma}_i^n \ln(\dot{\gamma}_i) \\ \sum_{i=1}^m \dot{\gamma}_i^n & \sum_{i=1}^m \dot{\gamma}_i^{2n} & \sum_{i=1}^m \dot{\gamma}_i^{2n} \ln(\dot{\gamma}_i) \\ \sum_{i=1}^m \tau_i & \sum_{i=1}^m \tau_i \dot{\gamma}_i^n & \sum_{i=1}^m \tau_i \dot{\gamma}_i^n \ln(\dot{\gamma}_i) \end{vmatrix} \quad (2.68)$$

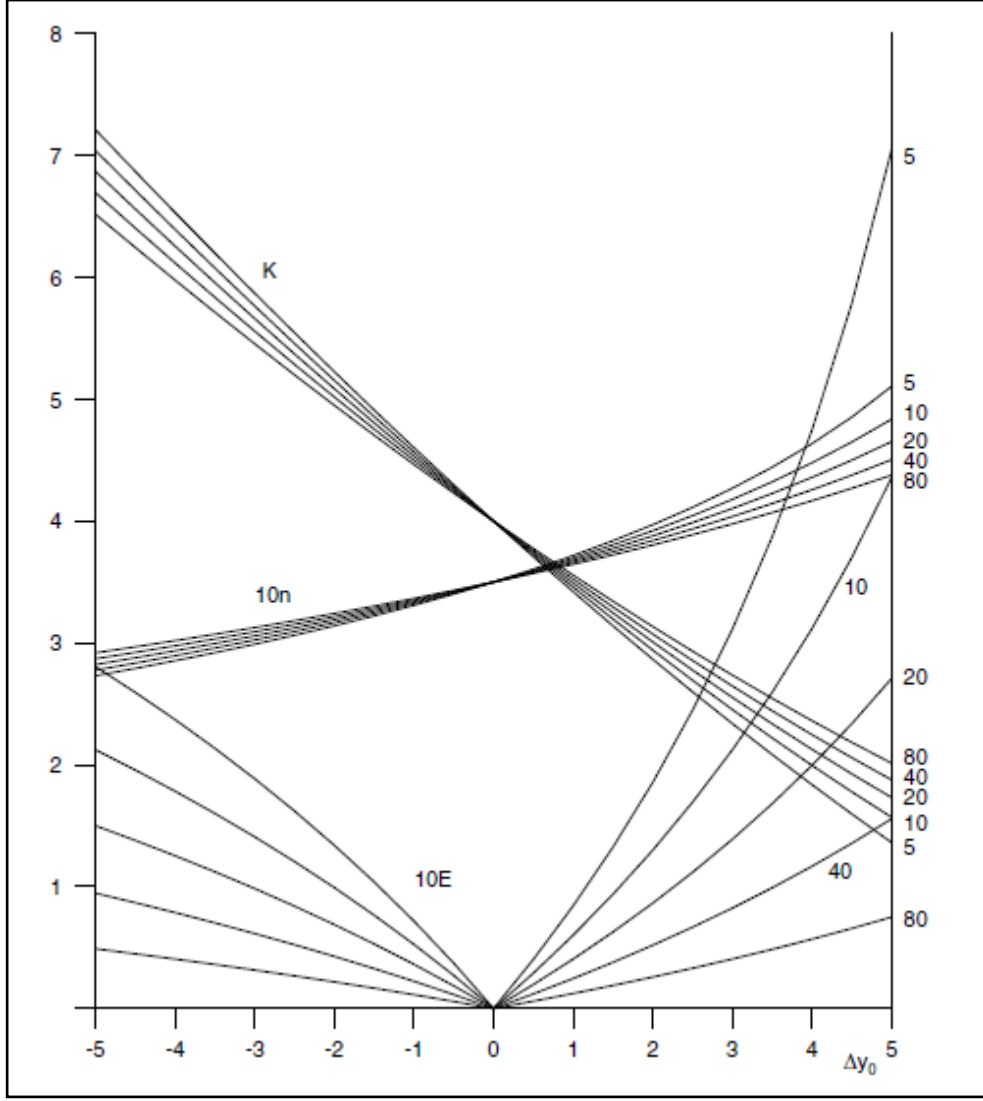


Figure 2. 2: Herschel-Bulkley parameter sensitivity to change in yield stress estimates, (Mullineux, G., 2007).

Expanding the above determinant function,

$$\begin{aligned}
 F(n) = m & \left[\sum_{i=1}^m \dot{\gamma}_i^{2n} \sum_{i=1}^m \tau_i \dot{\gamma}_i^n \ln(\dot{\gamma}_i) - \sum_{i=1}^m \tau_i \dot{\gamma}_i^n \sum_{i=1}^m \dot{\gamma}_i^{2n} \ln(\dot{\gamma}_i) \right] \\
 & - \sum_{i=1}^m \dot{\gamma}_i^n \left[\sum_{i=1}^m \dot{\gamma}_i^n \sum_{i=1}^m \tau_i \dot{\gamma}_i^n \ln(\dot{\gamma}_i) \right. \\
 & \left. - \sum_{i=1}^m \tau_i \sum_{i=1}^m \dot{\gamma}_i^{2n} \ln(\dot{\gamma}_i) \right] \\
 & + \sum_{i=1}^m \dot{\gamma}_i^n \ln(\dot{\gamma}_i) \left[\sum_{i=1}^m \dot{\gamma}_i^n \sum_{i=1}^m \tau_i \dot{\gamma}_i^n - \sum_{i=1}^m \tau_i \sum_{i=1}^m \dot{\gamma}_i^{2n} \right]
 \end{aligned} \tag{2.69}$$

The above function is believed to have two roots, one of which is the true parameter value of interest, and a second root at infinity, (Figure 2.3).

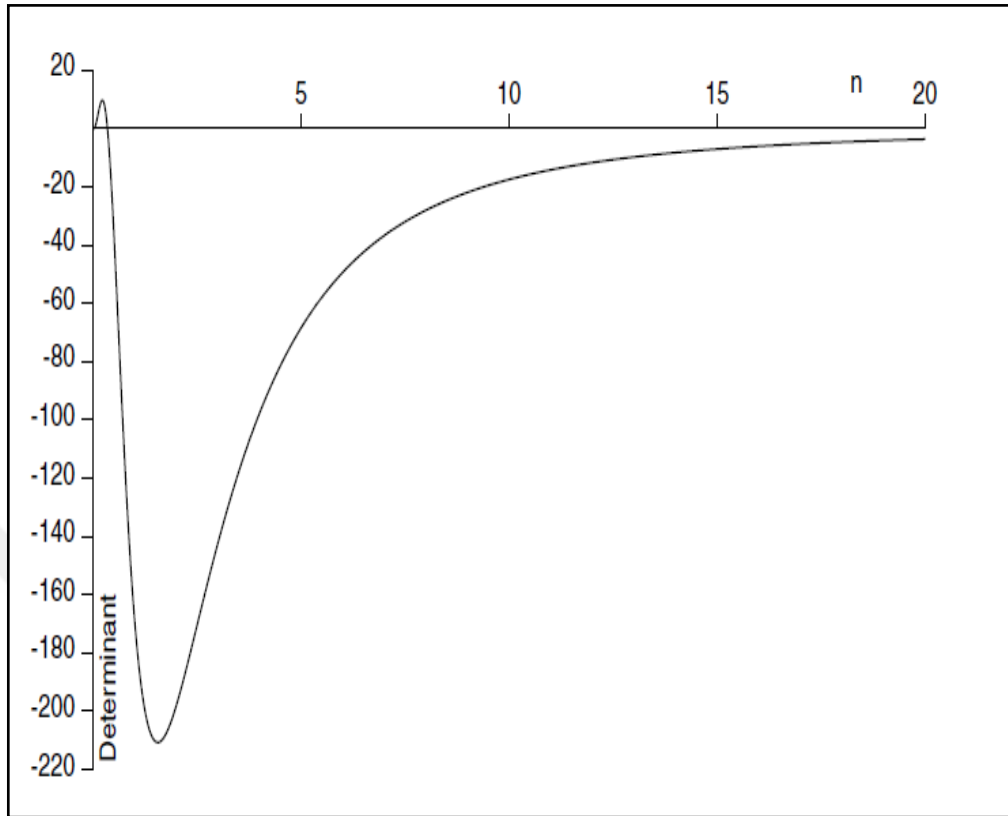


Figure 2. 3: F(n) function typical behavior, (Mullineux, G., 2007).

Again, once the flow behavior index is estimated, determination of the other two parameters is a straightforward procedure utilizing the least square method for a linear function as has been demonstrated earlier.

2.2.2 API recommended practice

2.2.2.1 BP model API recommended equations

The following are the field recommended API formulas, derived from the solution of the Couette inverse problem in field units that facilitate only two data points, (API ,2009).

$$\mu_p = \theta_{600} - \theta_{300} \quad (2.70)$$

$$\tau_y = \theta_{300} - \mu_p \quad (2.71)$$

The use of the 600 rpm and 300 rpm pair of data points is recommended. Since it is believed that at such rates of shear, a fully developed laminar flow must exist, where the viscoplastic behavior is observed.

2.2.2.2 PL model API recommended equations

As in the case of the Bingham Plastic solution, the API recommends using the 300 rpm and 600 rpm dial readings to determine the two parameters n and K (in field units) as follows, (API, 2009).

$$n = 3.322 \log \left(\frac{\theta_{600}}{\theta_{300}} \right) \quad (2.72)$$

$$K = \frac{\theta_{300}}{511^n} \quad (2.73)$$

2.2.2.3 HB model API recommended equations

On the other hand, the API recommended the use of the equations below to determine the HB model parameters for the field use of the Fann 35 rotational viscometer in field units, (API, 2009).

$$\tau_0 = 2\theta_3 - \theta_6 \quad (2.74)$$

$$n = 3.32 \log \left(\frac{\theta_{600} - \tau_y}{\theta_{300} - \tau_y} \right) \quad (2.75)$$

$$K = \left(\frac{\theta_{300} - \tau_y}{511^n} \right) \quad (2.76)$$

2.3 Plug Flow

As discussed earlier in this study, the solutions to the Couette inverse problem for the previously mentioned rheological models were derived assuming a stable laminar flow of homogenous fluid independent of the z-direction.

Since these assumptions do not really hold in real-life applications, especially for pseudoplastic (shear-thinning) drilling fluids, since most of these fluids whether water-base or emulsions, exhibit a yield behavior, meaning that the fluid will only begin to flow under laminar regime upon exceeding a threshold of shear stress which beyond doubt, complicates adequate rheological modeling via rotational viscometers. This is because the fluid is behaving as an elastic solid below such stresses.

This effect becomes more noticeable when evaluating low shear rate data. While it is difficult even for low-shear-range laboratory plate rheometers to correctly interpret low rates of shear, it becomes arguably tricky for field rotational viscometers that are not sensitive to low shear rates such as the Fann viscometer to do so.

This is explained by the plug flow phenomena as seen in Figure 2.4, where the fluid between the two cylinders is partially or fully in a solid state as the shear stress applied is not sufficient to break the cohesive forces along the radius. It is the appropriate time to remember that the shear stress applied on the fluid in a rotational viscometer decreases with increasing radius as expressed in Equation (1.9b).

Simply put; upon shearing the fluid, the region adjacent to the bob will experience the yield stress first and break loose, while the region adjacent to the rotor will experience the yield stress at last, which only by then it is adequate to say that the fluid is under laminar flow regime. This effect become more pronounced as the gap between the two cylinders gets wider.

This effect can be better seen in Figure 2.5, where the plug flow regime is highlighted in terms of shear stress vs. shear rate relationship (rheogram). A transitional flow regime occurs upon ending the plug flow effect, similar to the transitional flow that occurs between laminar and turbulent flow regimes consequent to the resettling of fluid particles.

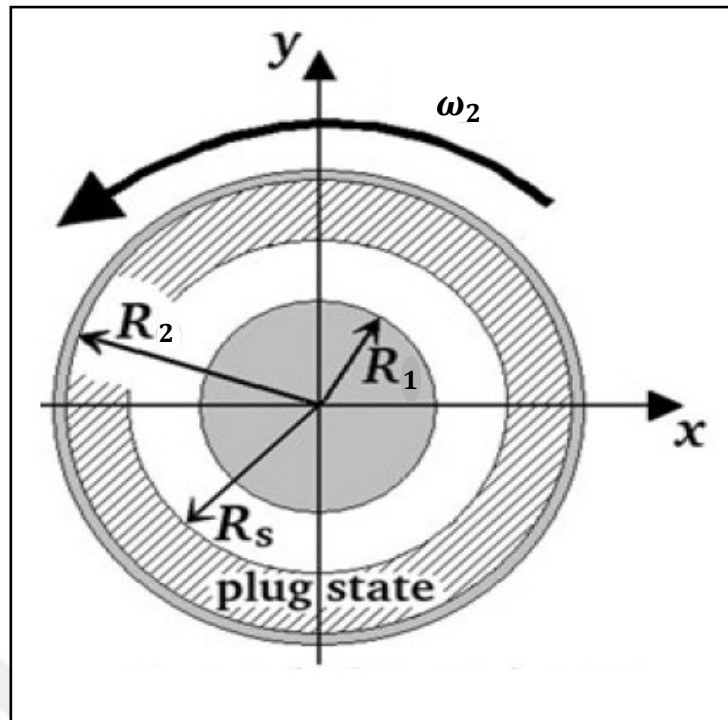


Figure 2. 4: Visualization of the plug flow in the viscometer gap, (Heirman et al. 2008).

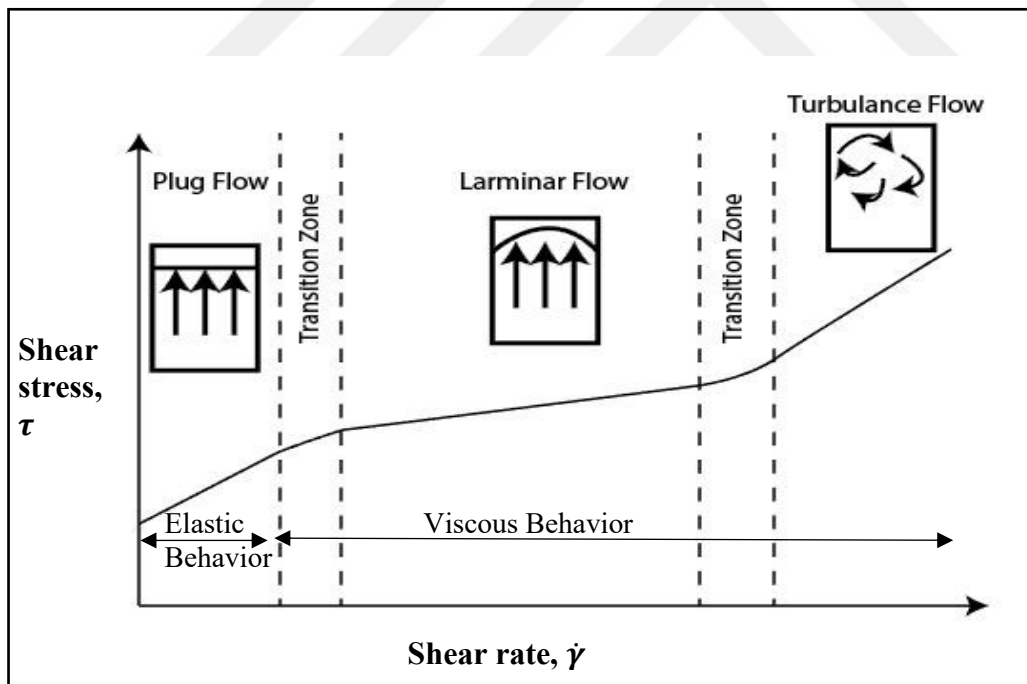


Figure 2. 5: Rheogram representation of rotational viscometer flow regimes.

3. OBJECTIVE OF THE STUDY

Various methods involving the parameter determination of rheological models are out there to choose from. And with the urge of continuous field testing of the rheological properties of the drilling fluids, simple, practical, and rapid calculation methods of these properties, unnoticeably deemed more important and received rather great deal of attention when compared to the efficiency of those methods. While ‘practical’ alone could be misleading. This study suggests a practical, yet pragmatic approach to better describe the rheology of drilling fluids.

With technology advancement, and computer accessibility in the field, the utilization of regression analysis via simple software or platforms such as Excel and MATLAB should not be considered a time-consuming task, rather a more efficient way of describing curve-like behavior of the drilling fluid rheograms. However, evaluating the rotational viscometer data using such techniques must be carried out with care, as some of the data points, in fact, do not represent the flow curve and must be omitted. As it will be shown, including all measurement points can yield inaccurate estimation of the model parameters and misinterpretation of the drilling fluid’s behavior.

In the following chapter, the concept of critical rotational speed and corrected shear rate will be applied to enhance the rheological description through the models mentioned earlier. This will be justified by comparing the measured data to the calculated data. Further verification will be demonstrated by the employment of the enhanced parameters in pressure loss calculation.



4. METHOD PROPOSAL AND INVESTIGATION

4.1 Critical Rotational Speed

In this section, the minimum rotational speed sufficient to overcome the plug flow, which relates to the critical shear rate, will be investigated for the Bingham Plastic and Herschel-Bulkley fluids.

4.1.1 BP model critical speed

When the yield stress is felt at the outer cylinder (rotor), Equation (1.9b) becomes,

$$\tau_y = \frac{T}{2\pi hr_2^2} \quad (4.1)$$

Substituting the above equation in the Bingham Plastic Reiner-Riwlin Equation (2.4) and solving for ω_2 :

$$\omega_2 = \frac{\tau_y}{2\mu_p} \left[\left(\frac{r_2}{r_1} \right)^2 - 1 - 2 \ln \left(\frac{r_2}{r_1} \right) \right] \quad (4.2)$$

Substituting the values of r_1 and r_2 of the Fann 35 viscometer and $2\pi N$ for ω_2 , and expressing in SI units,

$$N_{c,BP} = 7.165 \times 10^{-4} \frac{\tau_y}{\mu_p} \quad (4.3)$$

where,

$N_{c,BP}$: Bingham Plastic fluid critical rotational speed, rps

τ_y : Bingham Plastic yield stress, Pa

μ_p : Plastic viscosity, Pa.s

4.1.2 HB model critical speed

The same approach used in the Bingham Plastic fluid will be applied here. Substituting Equation (4.1) into the HB Reiner-Riwlin Equation (2.30) and solving for N,

$$N_{c,HB} = \left(\frac{n}{4\pi}\right) \left(\frac{1}{r_1^{2/n}} - \frac{1}{r_2^{2/n}}\right) \sqrt[n]{\left(r_2^2 - \frac{2}{\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)} \ln\left(\frac{r_2}{r_1}\right)\right) \frac{\tau_0}{K}} \quad (4.4)$$

$$\begin{aligned} N_{c,HB} &= \left(\frac{n}{4\pi}\right) \left(\frac{1}{0.01725^{2/n}} - \frac{1}{0.01842^{2/n}}\right) \sqrt[n]{\left(3.4 \times 10^{-4} - \frac{0.113}{\left(\frac{1}{2.98 \times 10^{-4}} - \frac{1}{3.4 \times 10^{-4}}\right)}\right) \frac{\tau_0}{K}} \end{aligned} \quad (4.5)$$

where,

τ_0 : HB yield stress, Pa

K: Consistency index, Pa sⁿ

$N_{c,HB}$: HB fluid critical rotational speed, rps

Thus, if the applied shear stress is below the yield stress, plug flow dominates, and if it is between the yield stress and critical stress, part of the fluid is in plug flow and part is viscous flow. Whereas above the critical speed, the whole fluid is regarded to be under laminar flow.

It is believed that interpreting the data points of rotational speeds above the critical speed would enhance the rheological model fitting and hence, the model parameter estimates. Furthermore, it can be seen that determining the critical rotational speed requires a previous estimate of the model's parameters. The utilization of the critical speed for interpretation accuracy will be applied as follows:

1. Initial estimate of model parameters by regressing all data points,
2. Using the initial parameters to determine the critical speed,
3. Omitting data points that fall below the critical speed,
4. Determining the corrected model parameters by regressing the reminder of data points.

4.2 Mud Samples

As the type of the drilling fluids is believed to be the biggest factor affecting its rheology, to avoid generalization fallacies, this study examines three mud samples of different origins, namely; synthetic based, oil based, and water based muds. These mud samples, as shown in the table below, are chosen to be high-yield and thick drilling fluids, in order to magnify the plug flow effect.

Table 4. 1: Mud sample dial reading of Fann 35 viscometer, (Power and Zamora, 2003).

MUD TYPE	R600	R300	R200	R100	R6	R3
SBM	58	41	34	26	16	15
OBM	76	42	31	19	8	7
WBM	77	56	47	39	31	30

4.3 SBM Evaluation

4.3.1 Utilizing BP model

In his paper, Speers, (1983) suggested the use of the following equation for the shear rate calculation using the Fann 35 viscometer data. However, Equation (4.6) assumes Newtonian shear rates in the viscometer gap, hence it is not the best equation to bet on for non-Newtonian models, (Speers, 1983).

$$\dot{\gamma} = 1.703 N \text{ (rpm)} \quad (4.6)$$

The shear stress on the other hand, is dependent only on the geometry of the viscometer as shown in Equation (1.9b). The following equation expresses the shear stress at the bob in SI units (Pa),

$$\tau = 0.5077 \theta \quad (4.7)$$

The θ -N data are transformed into $\tau - \dot{\gamma}$ data via the above equations, and the least square method Equations (2.43 and 2.44) are used to conduct a regression analysis on the direct $\tau - \dot{\gamma}$ data to find the plastic viscosity and yield stress.

The following table shows the calculated shear stresses by 6-reading Regression, in addition to the extrapolated shear stresses using the API recommended equation. Also, the relative error for each method is calculated and displayed respectively.

$$Rel. Err \% = \frac{(\tau_m - \tau_c)}{\tau_m} \times 100\% \quad (4.8)$$

Table 4. 2: 6-reading Bingham Plastic regression results for SBM.

$\dot{\gamma} \text{ (s}^{-1}\text{)}$	$\tau_m \text{ (Pa)}$	BP $\tau_c \text{ (Pa)}$	API $\tau_c \text{ (Pa)}$	BP Rel. Err. (%)	API Rel. Err. (%)
1021.8	29.447	30.560	28.860	3.780	1.992
510.9	20.816	19.657	20.175	5.567	3.080
340.6	17.262	16.023	17.280	7.179	0.103
170.3	13.200	12.388	14.384	6.150	8.971
10.2	8.123	8.972	11.663	10.452	43.576
5.1	7.616	8.863	11.576	16.384	52.008
Avg. Relative Error (%)				8.252	18.288

The critical speed was then calculated using the parameters obtained from the 6-reading regression technique via Equation (4.3) to be **17.6 rpm**. Meaning that the R3 and R6 data points have to be discarded, and the remaining data will be regressed again to determine the corrected model parameters.

4.3.1.1 BP Invalid data elimination

Using the $\tau - \dot{\gamma}$ data again to determine the model parameters, but this time excluding the R6 and R3 readings, a noticeable decrease in relative error was observed. Table (4.3) include the calculated shear stresses of the 4-reading using the least square method utilizing the Bingham Plastic model.

Table 4. 3: 4-reading Bingham Plastic regression results for SBM.

$\dot{\gamma} \text{ (s}^{-1}\text{)}$	$\tau_m \text{ (Pa)}$	BP $\tau_c \text{ (Pa)}$	BP Rel. Err. (%)
1021.800	29.447	29.755	1.047
510.900	20.816	20.181	3.049
340.600	17.262	16.990	1.576
170.300	13.200	13.799	4.533
Avg. Relative Error (%)			2.551

As suggested, the average error decreased by eliminating the non-representative data points from 8.252% to 2.551%.

4.3.2 Utilizing PL model

For comparison reasons, the data were regressed again for the Power Law model as demonstrated in earlier sections. The calculated shear stresses, along with the

extrapolated shear stresses using the API formula for the R600 and R300 readings are listed in Table 4.4.

Table 4. 4: 6-reading Power Law regression results for SBM.

$\dot{\gamma} \text{ (s}^{-1}\text{)}$	$\tau_m \text{ (Pa)}$	PL $\tau_c \text{ (Pa)}$	API $\tau_c \text{ (Pa)}$	PL Rel. Err. (%)	API Rel. Err. (%)
1021.8	29.447	24.374	27.761	17.225	5.726
510.9	20.816	20.689	19.624	0.607	5.726
340.6	17.262	18.798	16.020	8.898	7.194
170.3	13.200	15.956	11.324	20.877	14.210
10.2	8.123	8.203	2.771	0.988	65.894
5.1	7.616	6.963	1.958	8.565	74.283
Avg. Relative Error (%)				9.527	28.839

Using the least square method to regress the shear stress vs. shear rate data for the PL model yielded almost 10% error. Yet the use of the API recommended equations increased that error by almost 3 folds.

4.3.3 Utilizing HB model

For the sake of applying the least square method to regress the SBM data, equation (2.68) was first used to determine the flow behavior exponent ‘n’. Figure 4.1 illustrates the behavior of the F(n) function with changing n. as noticed, values of n generated with 0.001 intervals in the range of 0-1 only, this is justified by the pseudoplastic behavior that the model was developed for. Also, to clearly visualize the interception point at which the true value of n is retrieved.

Once the n-exponent is known, Equations (2.60 and 2.61) can be used to find τ_0 and K, respectively. The results of the regression are listed in the Table 4.5, accompanied with the respective shear stress values calculated using the API equations

Table 4. 5: 6-reading Herschel-Bulkley regression results for SBM.

$\dot{\gamma} \text{ (s}^{-1}\text{)}$	$\tau_m \text{ (Pa)}$	HB $\tau_c \text{ (Pa)}$	API $\tau_c \text{ (Pa)}$	HB Rel. Err. (%)	API Rel. Err. (%)
1021.8	29.447	29.483	27.766	0.125	5.708
510.9	20.816	20.703	19.631	0.543	5.691
340.6	17.262	17.274	16.420	0.068	4.878
170.3	13.200	13.306	12.667	0.801	4.041
10.2	8.123	8.006	7.525	1.444	7.370
5.1	7.616	7.692	7.207	1.000	5.367
Avg. Relative Error (%)				0.663	5.509

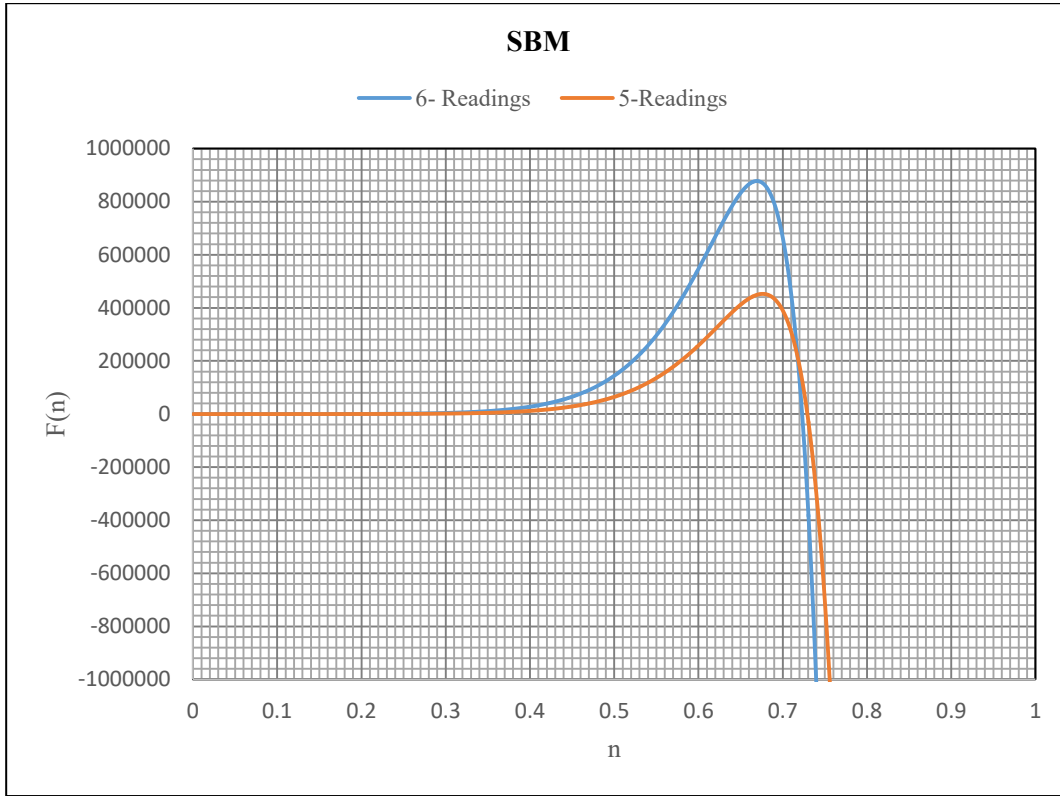


Figure 4. 1: $F(n)$ behavior for the 6-reading and 5-reading data.

4.3.3.1 HB Invalid data elimination

Plug flow data can be eliminated by determining the critical RPM using Equation (4.4) with the appropriate units, which yields $N_{c,HB} = 3.3$ RPM.

Thus, the $\tau - \dot{\gamma}$ data will be evaluated once more. This time, excluding the R3 reading. The results are listed in Table 4.6.

Table 4. 6: 5-reading Herschel-Bulkley regression results for SBM.

$\dot{\gamma} \text{ (s}^{-1}\text{)}$	$\tau_m \text{ (Pa)}$	HB $\tau_c \text{ (Pa)}$	HB Rel. Err. (%)
1021.8	29.447	29.490	0.148
510.9	20.816	20.690	0.602
340.6	17.262	17.266	0.023
170.3	13.200	13.317	0.881
10.218	8.123	8.085	0.476
Avg. Relative Error (%)			0.426

Excluding the R3 data point resulted in a decreased relative error, hence increased accuracy in predicting shear stresses, especially at *low values*.

In order to quantitatively compare the results of these models, Table 2.7 was constructed to include the parameters guessed by each method. Moreover, the Root Mean Square Error (RMSE) and the Coefficient of Determination (R^2) of each approach.

$$RMSE = \sqrt{MSE} = \sqrt{\frac{\sum_{i=1}^m (\tau_{m_i} - \tau_{c_i})^2}{n}} \quad (4.10)$$

$$R^2 = \frac{n * MSE}{\sum_{i=1}^m (\tau_{m_i} - \bar{\tau})^2} \quad (4.11)$$

$\bar{\tau}$: Average shear stress, Pa

τ_{m_i} : Measured shear stress, Pa

τ_{c_i} : Calculated shear stress, Pa

Table 4. 7: Summarized parameter determination results for SBM.

PARAMETER	PL		BP			HB		
	Regression	API	Regression	Corrected	API	Regression	Corrected	API
K, Pa. s ⁿ	4.73492	0.86585	-	-	-	0.14851	0.14164	0.16015
n	0.23647	0.50044	-	-	-	0.72310	0.72930	0.70414
τ_y , Pa	-	-	8.75417	10.60730	11.48930	-	-	-
μ_p , Pa. s	-	-	0.02134	0.01874	0.01700	-	-	-
τ_0 , Pa	-	-	-	-	-	7.20865	7.31310	6.70177
RMSE	2.45391	3.41509	1.08426	0.48212	2.25000	0.08661	0.08085	0.97877
R ²	0.89550	0.79760	0.97960	0.99352	0.91215	0.99987	0.99987	0.98337

4.3.4 Results evaluation

Figure 4.2 is a bar chart illustrating the accuracy of the above-mentioned methods, in addition to the API recommended practice for parameter determination.

$$\text{Accuracy \%} = 100\% - \text{Avg. Relative Error (\%)} \quad (4.9)$$

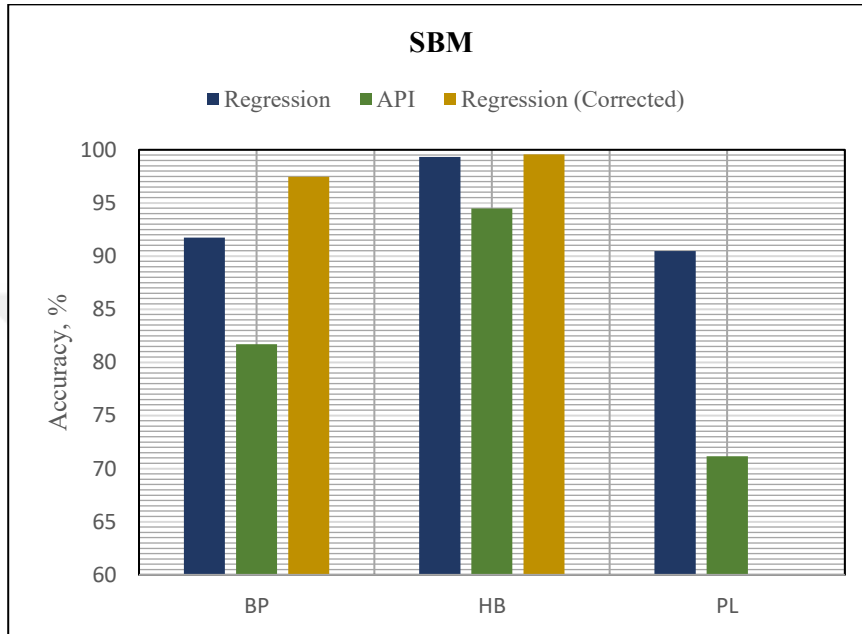


Figure 4. 2: Accuracy of model parameter determination methods for SBM.

As observed from Figure 4.2, using the API recommended practice can provide highly erroneous outcomes, especially in the case of Power Law model. Furthermore, in the case of regressing the shear rate vs. shear stress data, more reasonable results can be obtained. And lastly, correcting the regression data by eliminating the faulty readings that occur due to plug flow is shown to provide the most accurate fit for both Bingham Plastic and Herschel-Bulkley fluids.

4.4 OBM Evaluation

4.4.1 Utilizing BP model

Similar fashion as in the case of SBM is applied for the oil based mud. The θ -N data were transformed into shear stress vs. shear rate. Then these data were regressed using the method of least squares. The plastic viscosity and BP yield stress were determined, using Equations (2.43 and 2.44).

Table 4.8 lists the calculated shear stress values for the 6-reading Fann 35 viscometer data, along with shear stress values calculated using the API equations, with their relative error respectively.

Table 4. 8: 6-reading Bingham Plastic regression results for OBM.

$\dot{\gamma} \text{ (s}^{-1}\text{)}$	$\tau_m \text{ (Pa)}$	BP $\tau_c \text{ (Pa)}$	API $\tau_c \text{ (Pa)}$	BP Rel. Err. (%)	API Rel. Err. (%)
1021.8	38.585	38.755	38.571	0.441	0.037
510.9	21.323	21.237	21.200	0.406	0.577
340.6	15.739	15.397	15.410	2.169	2.087
170.3	9.646	9.558	9.620	0.918	0.273
10.218	4.062	4.069	4.177	0.172	2.846
5.109	3.554	3.893	4.003	9.553	12.650
Avg. Relative Error (%)				2.277	3.078

Using the parameters determined conventionally using the 6-reading data (μ_p and τ_y), the Bingham Plastic critical speed was calculated to be **4.7 rpm**, which means that the R3 data was in fact affected by the plug flow, hence has to be discarded.

The remaining 5-reading data will be regressed again to determine the corrected model parameters.

4.4.1.1 BP Invalid data elimination

Having regressed the 5-reading data using the least square method as described in the previous section, relative error of the calculated-to-measured shear stress was determined for each reading, in addition to the average relative error for comparison reasons as seen in Table 4.9.

Table 4. 9: 5-reading Bingham Plastic regression results for OBM.

$\dot{\gamma} \text{ (s}^{-1}\text{)}$	$\tau_m \text{ (Pa)}$	BP $\tau_c \text{ (Pa)}$	BP Rel. Err. (%)
1021.800	38.585	38.684	0.257
510.900	21.323	21.282	0.194
340.600	15.739	15.481	1.637
170.300	9.646	9.680	0.352
10.218	4.062	4.227	4.083
Avg. Relative Error (%)			1.305

The average relative error was observed to drop by almost half of its original value.

4.4.2 Utilizing PL model

Similar approach was carried out to investigate the parameter determination accountability using the Power Law model. The 6-reading OBM data were regressed utilizing the least square methods. In similar manner, the same parameters calculated

using the API recommended practice, namely (K, n) were used to generate shear stress data as seen in Table 4.10. Finally, the relative error of each method was determined.

Table 4. 10: 6-reading Power Law regression results for OBM.

$\dot{\gamma} \text{ (s}^{-1}\text{)}$	$\tau_m \text{ (Pa)}$	PL $\tau_c \text{ (Pa)}$	API $\tau_c \text{ (Pa)}$	PL Rel. Err. (%)	API Rel. Err. (%)
1021.8	38.585	28.293	36.375	26.674	5.727
510.9	21.323	21.162	20.102	0.758	5.728
340.6	15.739	17.856	14.209	13.451	9.718
170.3	9.646	13.355	7.852	38.450	18.597
10.218	4.062	4.109	0.707	1.167	82.588
5.109	3.554	3.073	0.391	13.522	89.003
Avg. Relative Error (%)				15.670	35.227

4.4.3 Utilizing HB model

The regression methodology proposed by Mullineux, G. (2007) will be applied to determine the model parameters. Hence, the F(n) function must be graphed to obtain the true value of n at which F(n) = 0. Figure 4.3 below illustrates the behavior of the F(n) function for the OBM rheological data for both the 6-reading and the 5-reading data.

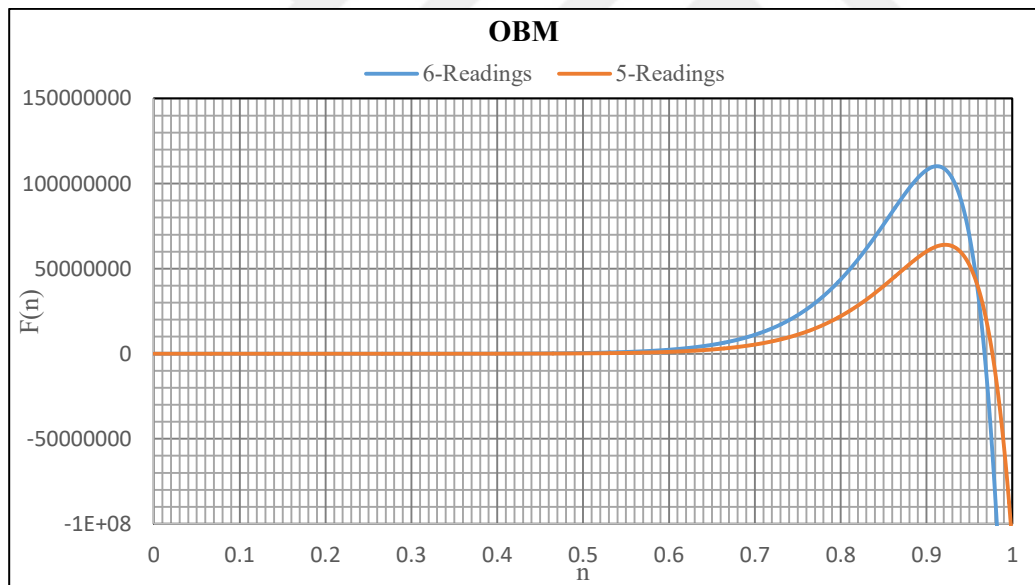


Figure 4. 3: F(n) behavior for the 6-reading and 5-reading data for OBM.

The K and τ_0 parameter were then determined using Equations (2.60 and 2.61). Consequently, the shear stress values were generated for the obtained HB model parameters. Table 4.11 below lists the results of the 6-reading regression utilizing the Herschel-Bulkley model, along with accordant relative errors.

Table 4. 11: 6-reading Herschel-Bulkley regression results for OBM.

$\dot{\gamma}$ (s ⁻¹)	τ_m (Pa)	HB τ_c (Pa)	API τ_c (Pa)	HB Rel. Err. (%)	API Rel. Err. (%)
1021.8	38.585	38.568	36.378	0.046	5.721
510.9	21.323	21.430	20.110	0.502	5.689
340.6	15.739	15.611	14.558	0.813	7.503
170.3	9.646	9.691	8.884	0.467	7.900
10.218	4.062	3.904	3.277	3.881	19.311
5.109	3.554	3.705	3.081	4.252	13.318
Avg. Relative Error (%)				1.660	9.907

4.4.3.1 HB Invalid data elimination

The Herschel-Bulkley critical speed was calculated using the regression parameters through Equation (4.4) which was $N_{c,HB} = 3.7$ rpm. On that account, the R3 reading is regarded in the plug flow region and hence must be eliminated. The remaining 5-reading data were regressed again in similar manner as discussed in the previous section, and the results are listed in the following table, (Table 4.12).

Table 4. 12: 5-reading Herschel-Bulkley regression results for OBM.

$\dot{\gamma}$ (s ⁻¹)	τ_m (Pa)	HB τ_c (Pa)	HB Rel. Err. (%)
38.585	38.579	0.016	38.585
21.323	21.402	0.370	21.323
15.739	15.600	0.883	15.739
9.646	9.726	0.830	9.646
4.062	4.048	0.335	4.062
Avg. Relative Error (%)			0.487

Thereupon, the relative error significantly decreased and hence, the accuracy of the model parameters increased.

4.4.4 Results evaluation

In this section, the overall results of the proposed approaches will be examined closely and compared for accuracy. The accuracy of each method is calculated and represented in a bar chart (Figure 4.4) to better visualize their accountability in the case of OBM analysis.

The Bingham Plastic model parameters determined facilitating all data readings resulted in fair estimation of shear stress values. Whereas, as in the case of SBM, the API recommended practice produced less accurate estimations to a degree, yet still acceptable. The elimination of data readings affected by the plug flow however, produced the most accurate parameter estimates for the BP model.

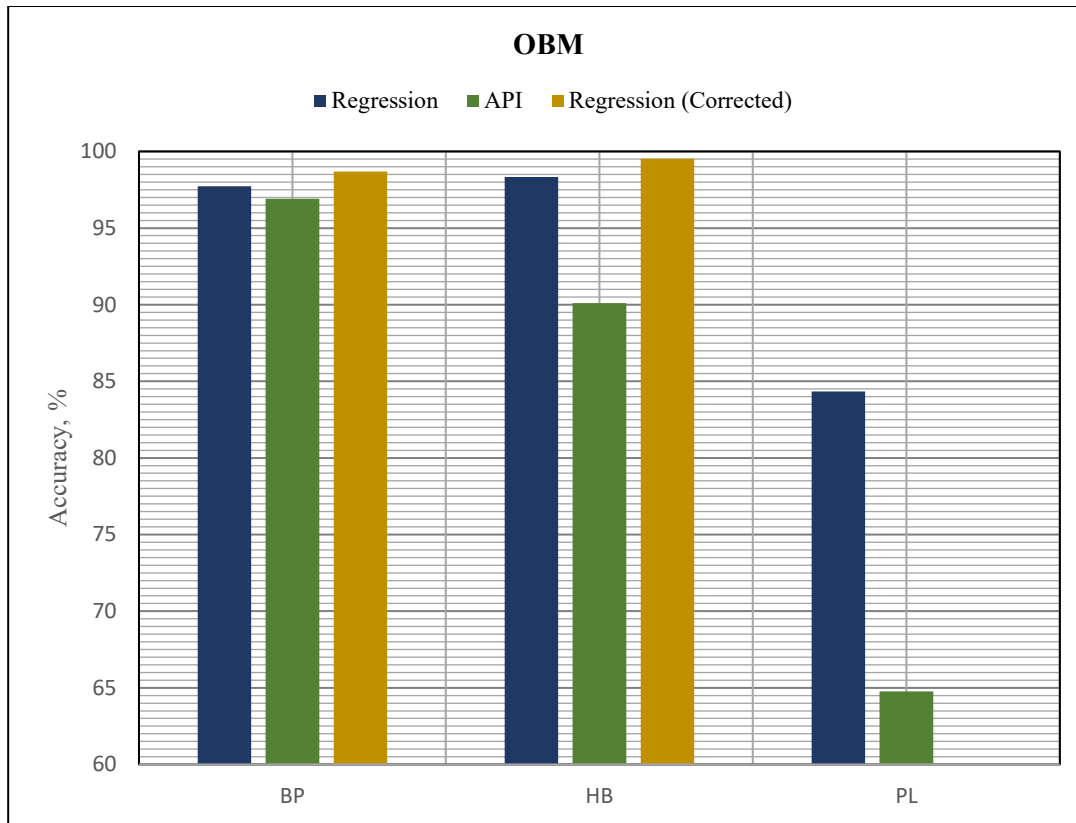


Figure 4. 4: Accuracy of model parameter determination methods for OBM.

Regressing the data to fit the Herschel-Bulkley model accounted for more accurate results. Be that as it may, it was still believed that the R3 data point is arousing negative effects on the parameter estimation, since it was determined to be a faulty reading, hence correcting the data by eliminating the R3 resulted in the most accurate model parameter estimation for the Herschel-Bulkley model, as well as for the other models. Again, using the API recommended equations produced arguably inaccurate estimations of the parameters in comparison to the least square method approach.

The Power Law model was apparently not a good fit for the oil based mud as it resulted in less than 90% accuracy. Whereas, the API equation decreased the accuracy to less than 65%.

Table 4.13 is constructed to summarize the results offered by the methods mentioned formerly. The model parameters of each method, in addition to the Root Mean Square Error (RMSE), and the Coefficient of Determination of the model fit are also listed.

Table 4. 13: Summarized parameter determination results for OBM.

	PL		BP			HB		
PARAMETER	Regression	API	Regression	Corrected	API	Regression	Corrected	API
K, Pa. sⁿ	1.5518	0.0968	-	-	-	0.0430	0.0401	0.0436
n	0.4190	0.8556	-	-	-	0.9676	0.9768	0.9588
τ_y, Pa	-	-	3.7182	3.8794	3.8298	-	-	-
μ_p, Pa. s	-	-	0.0343	0.0341	0.0340	-	-	-
τ_0, Pa	-	-	-	-	-	3.4968	3.6594	2.8722
RMSE	4.5539	2.3519	0.2145	0.1460	0.2379	0.1139	0.0802	1.2355
R²	0.8577	0.9621	0.9997	0.9998	0.9996	0.9999	1.0000	0.9895

4.5 WBM Evaluation

4.5.1 Utilizing BP model

Analogous to the procedure executed for the SBM and OBM, the WBM data were regressed using all 6 readings, facilitating the Bingham Plastic model and the results are listed in Table 4.14.

Table 4. 14: 6-reading Bingham Plastic regression results for WBM.

$\dot{\gamma} (s^{-1})$	$\tau_m (Pa)$	BP $\tau_c (Pa)$	API $\tau_c (Pa)$	BP Rel.Err. (%)	API Rel.Err. (%)
1021.8	39.093	39.595	38.213	1.285	2.251
510.9	28.431	27.623	27.484	2.841	3.332
340.6	23.862	23.633	23.908	0.960	0.192
170.3	19.800	19.642	20.331	0.799	2.682
10.2	15.739	15.891	16.970	0.968	7.821
5.1	15.231	15.771	16.862	3.547	10.710
Avg. Relative Error (%)				1.733	4.498

The Bingham Plastic critical speed was calculated using the regression parameters to be **28.8 rpm**. Thus, the regression analysis will be carried out again for only 4 readings, namely; R600, R300, R200, and R100.

4.5.1.1 BP Invalid data elimination

Upon correcting the determined model parameters by excluding the readings affected by the plug flow, the relative error has been noticed to drop as shown in Table 4.15.

Table 4. 15: 4-reading Bingham Plastic regression results for WBM.

$\dot{\gamma} (s^{-1})$	$\tau_m (Pa)$	BP $\tau_c (Pa)$	BP Rel. Err. (%)
1021.8	39.093	39.329	0.603
510.9	28.431	27.797	2.232
340.6	23.862	23.953	0.380
170.3	19.800	20.109	1.557
Avg. Relative Error (%)			1.193

4.5.2 Utilizing PL model

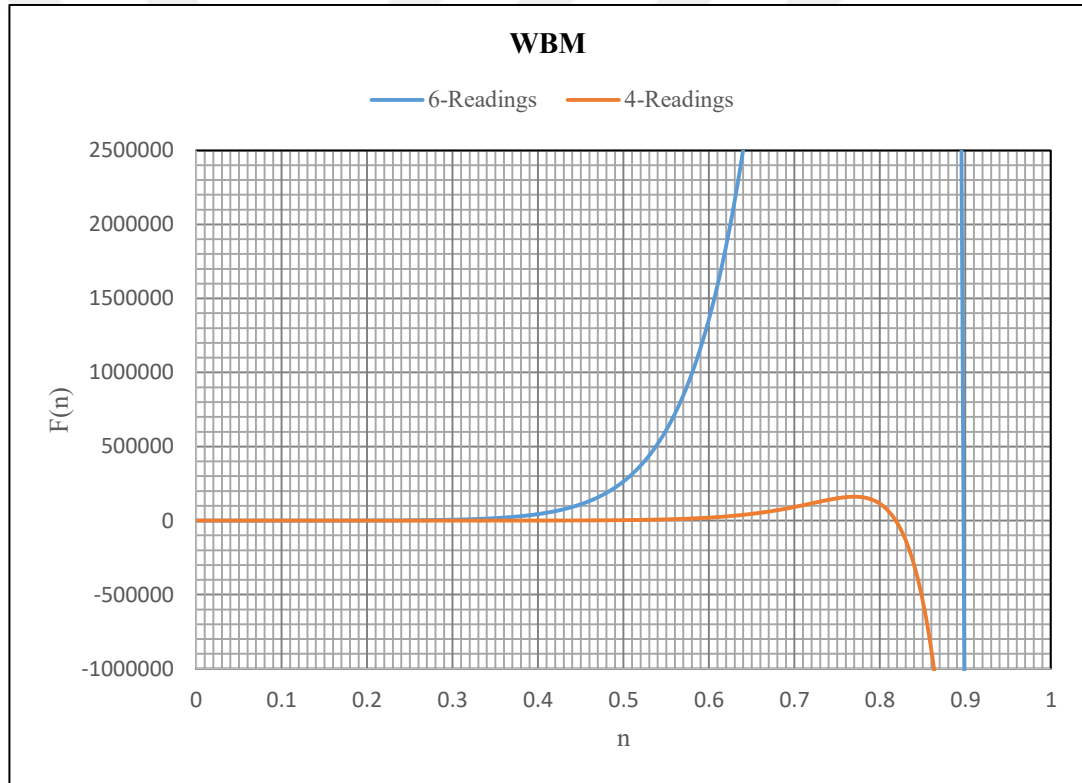
All the 6-reading data were regressed for the Power Law model parameter determination. The shear stresses were then generated using those parameter values and compared to the measured shear stress values along with shear stresses calculated using the API recommended formulas. The results are shown in Table 4.16.

Table 4. 16: 6-reading Power Law regression results for WBM.

$\dot{\gamma} \text{ (s}^{-1}\text{)}$	$\tau_m \text{ (Pa)}$	PL $\tau_c \text{ (Pa)}$	API $\tau_c \text{ (Pa)}$	PL Rel. Err. (%)	API Rel. Err. (%)
1021.8	39.093	31.566	36.857	19.253	5.720
510.9	28.431	28.393	26.805	0.135	5.721
340.6	23.862	26.686	22.249	11.837	6.760
170.3	19.800	24.003	16.181	21.227	18.280
10.218	15.739	15.613	4.443	0.797	71.773
5.109	15.231	14.044	3.231	7.796	78.787
Avg. Relative Error (%)				10.174	31.174

4.5.3 Utilizing HB model

Similarly, the $F(n)$ function is graphed for the water based mud in order to determine the true n value. Figure 4.5 exhibits the behavior and intersection point of both the 6-reading data and the 4-reading (corrected) data. As noticed, a considerable difference in the predicted n value is encountered for the water based mud.

**Figure 4. 5:** $F(n)$ behavior for the 6-reading and 4-reading data for WBM.

Using Equations (1.60 and 1.61), the K and τ_0 parameters were determined, thereafter, the shear stress values for their respective viscometer speeds were generated.

Similar to the procedure conducted on SBM and OBM, the API-generated shear stresses are also compared, (Table 4.17).

Table 4. 17: 6-reading Herschel-Bulkley regression results for WBM.

$\dot{\gamma} \text{ (s}^{-1}\text{)}$	$\tau_m \text{ (Pa)}$	HB $\tau_c \text{ (Pa)}$	API $\tau_c \text{ (Pa)}$	HB Rel. Err. (%)	API Rel. Err. (%)
1021.8	39.093	39.182	36.859	0.227	5.713
510.9	28.431	28.043	26.811	1.364	5.698
340.6	23.862	24.106	23.118	1.024	3.117
170.3	19.800	19.952	19.079	0.768	3.642
10.218	15.739	15.525	14.386	1.357	8.596
5.109	15.231	15.347	14.166	0.760	6.995
Avg. Relative Error (%)				0.917	5.627

Almost identical to the OBM and SBM cases, in that the API recommended practice resulted in unjustifiable errors.

4.5.3.1 HB Invalid data elimination

In order to check for the accuracy of the shear stress data listed in Table 4.17, and hence the model parameters, the critical speed must be calculated. A critical speed of **19.4 rpm** was given by Equation (4.4). Henceforth, both the R3 and R6 data must be discarded, and the remaining data should be regressed again. The F(n) function as shown in Figure (4.5) was used to determine the new n value, with that a new K and τ_0 are determined. Table 4.18 lists the shear stresses calculated using the corrected HB model parameters.

Table 4. 18: 4-reading Herschel-Bulkley regression results for WBM.

$\dot{\gamma} \text{ (s}^{-1}\text{)}$	$\tau_m \text{ (Pa)}$	HB $\tau_c \text{ (Pa)}$	HB Rel. Err. (%)
1021.8	39.093	39.128	0.090
510.9	28.431	28.197	0.823
340.6	23.862	24.154	1.225
170.3	19.800	19.707	0.473
Avg. Relative Error (%)			0.653

4.5.4 Results evaluation

It has been clear from the data presented up to this section that the water based muds are not different than synthetic based or even oil based muds when it comes to the concept of plug flow. As seen from Figure 4.6, and similar to the OBM and SBM cases, the followings can be stated:

- Regressing the Fann 35 viscometer data to obtain the model parameters reduces the error in those parameters, regardless of the model being used.
- The Power Law model might not be the best candidate for evaluating high-yield drilling fluids, especially when the low rates of shear are in concern.

- The Bingham Plastic model provides reasonable estimations of the model parameters through the API recommended practice if conducting a regression analysis is not feasible.
- The Herschel-Bulkley model is so far the best generalized rheological model for predicting the drilling fluid parameters, and it is recommended against using the API equation since it might result in considerable errors.
- Accounting for the plug flow in the Fann 35 viscometer at low rates of shear reduces the errors in estimating the model parameters; hence, adding more accuracy to the regression analysis regardless of the model used.

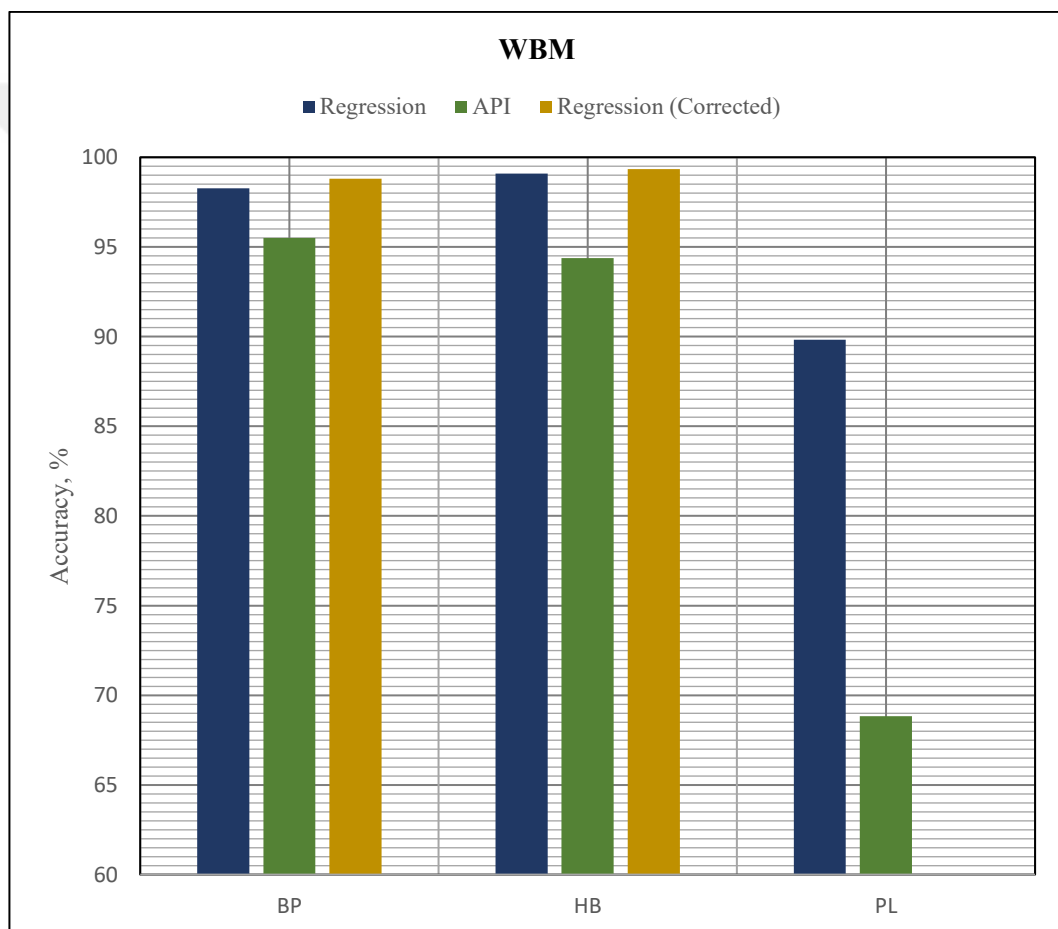


Figure 4. 6: Accuracy of model parameter determination methods for WBM.

Table 4.19 is a summary of the model parameters determined via the previously described methods, along with RMSE and R^2 of each method. An important change in HB model parameters is observed upon applying the proposed method.

Table 4. 19: Summarized parameter determination results for WBM.

	PL		BP			HB		
PARAMETER	Regression	API	Regression	Corrected	API	Regression	Corrected	API
K, Pa. sⁿ	10.9446	1.5272	-	-	-	0.0478	0.086903	0.0732
n	0.1529	0.4594	-	-	-	0.8978	0.8185	0.8296
τ_y, Pa			15.6516	16.2645	16.7552	-	-	-
μ_p, Pa. s	-	-	0.0234	0.0226	0.0210	-	-	-
τ_0, Pa	-	-	-	-	-	15.1402	13.88172	13.8822
RMSE	3.7354	7.0113	0.4650	0.3747	1.0110	0.2237	0.1938	1.3935
R²	0.7956	0.2800	0.9968	0.9973	0.9850	0.9993	0.9993	0.9716

4.6 Factors Affecting Critical Speed

It has been clear up to this point that the occurrence of plug flow in the rotational viscometer gap can lead to considerably erroneous readings, hence misinterpretation of the rheological parameters to fit the fluid model in question. This section is devoted to the investigation of the factors that affect the critical speed.

4.6.1 Mud type

Equation (4.3) suggests a proportional relationship between the critical speed and τ_y/μ_p ratio. Thus, an increase in this ratio will proportionally elevate the critical speed, hence the plug flow shear rates.

This effect is quite emphasized in the case of high-yield and thick fluids. This means that higher shear rates are required for the rotor to experience such high yield stresses. Figure (4.7) below illustrates the behavior of the Bingham Plastic critical speed with changing τ_y/μ_p ratio, and hence can be used as a quick correlation of BP model critical speed for any mud type.

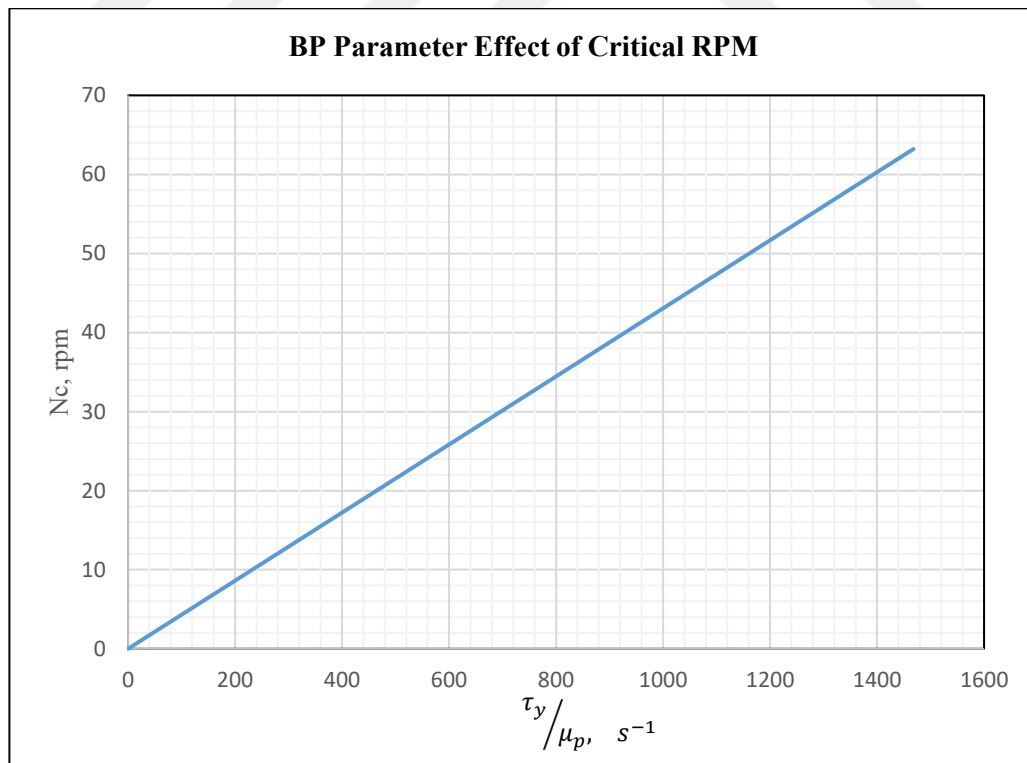


Figure 4. 7: Effect of the yield shear stress to the plastic viscosity ratio on BP critical speed.

Equation (4.4) shows rather a non-linear relationship between the HB model parameters and the critical rotational speed. Figure 4.8 exhibits the behavior of the critical speed with respect to the τ_0/K ratio. The trend has been plotted for various n values. Hence, it can be used as a quick correlation of HB critical speed for any mud type.

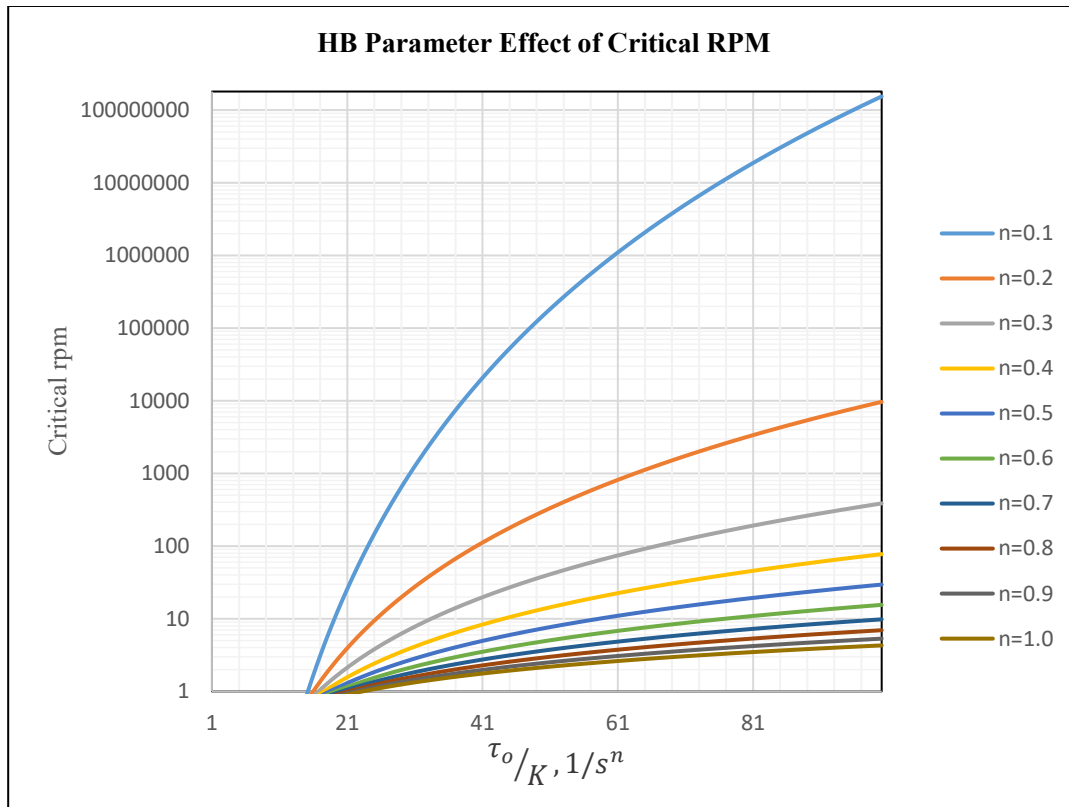


Figure 4. 8: Effect of Herschel-Bulkley parameters on critical speed.

As seen from the figure above, the parameter n has the most dominant effect on the critical speed, and it is magnified as n decreases. This means that the more pseudoplastic is the fluid, the higher the critical speed should be.

Similar to the Bingham Plastic case, the τ_0/K ratio positively affects the critical speed. However, this effect is exponentially controlled by the n parameter.

Hence, high-yield and thicker fluids are expected to elevate the Fann 35 viscometer critical speed.

4.6.2 Temperature effect

As the drilling fluid travels along the wellbore, it experiences a range of temperature values. Depending on the well depth, geothermal gradient, type of fluid, and other

factors, these temperatures are believed to alter the fluids rheological behavior, as it has been shown throughout various studies to be a function of applied temperature.

In his PhD dissertation, Ettehadi (2015), has investigated the rheological description of a number of drilling fluid samples emphasizing the temperature effect on the model parameter estimation. Those samples are:

- Sepiolite + fresh Water.
- Unweighted fresh water sepiolite mud + additives.
- Sepiolite mud with 171 kg/m³ NaCl intrusion + additives.
- Sepiolite mud with 342 kg/m³ NaCl intrusion + additives.
- Weighted fresh water sepiolite mud + additives.

In his study, he used both the Fann 35 viscometer and a plate-type rheometer in order to justify his findings.

Since rheometers usually reflect more accurate readings, and due to the unreliability of the Fann 35 viscometer at varying temperatures, the model parameters for both the Bingham Plastic and Herschel-Bulkley models determined using the rheometer will be used to emphasize the effect of temperature on the critical rotational speed for the Fann 35 viscometer. Equation (4.3) was used to calculate the BP critical rpm speed for the case of simple sepiolite clay plus fresh water for temperatures ranging from 38° C to 260° C. The results are listed in Table 4.20 and illustrated in Figure 4.9.

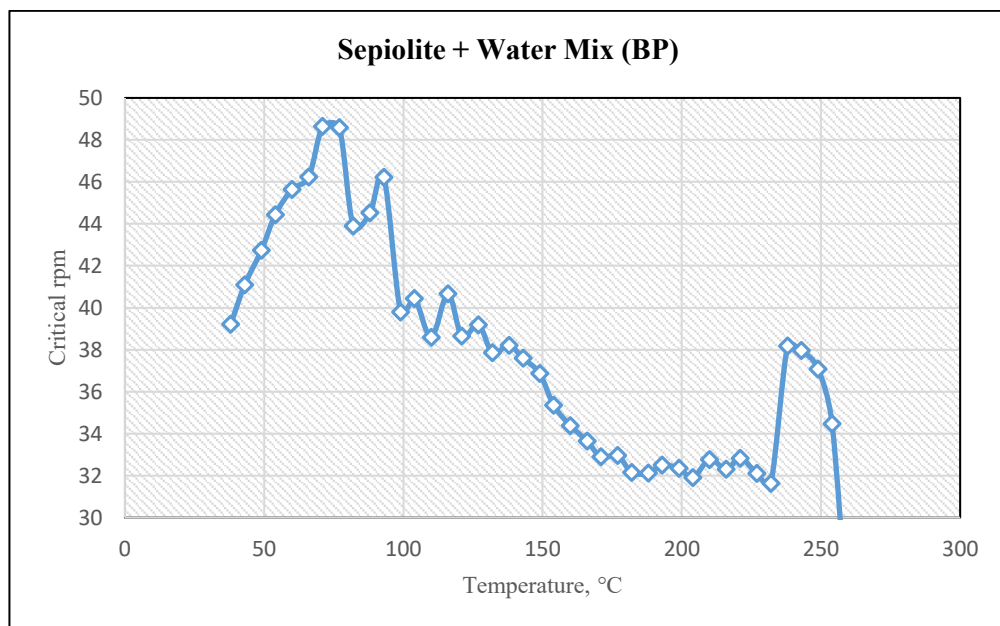


Figure 4. 9: BP Critical rpm and temperature data for sepiolite and water mud.

Table 4. 20: BP Critical rpm and temperature data for sepiolite and water mud.

$T^{\circ}, ^{\circ}\text{C}$	τ_y, Pa	$\mu_p, \text{mPa.s}$	N_c, rpm
38	8.11	8.90	39.22
43	6.78	7.11	41.09
49	5.72	5.77	42.74
54	5.25	5.09	44.44
60	4.75	4.49	45.64
66	4.30	4.01	46.23
71	3.99	3.54	48.64
77	3.63	3.22	48.58
82	3.23	3.17	43.89
88	3.03	2.93	44.52
93	2.90	2.70	46.22
99	2.55	2.75	39.80
104	2.46	2.62	40.44
110	2.33	2.60	38.58
116	2.32	2.45	40.67
121	2.21	2.46	38.64
127	2.15	2.37	39.19
132	2.08	2.37	37.86
138	2.07	2.34	38.20
143	2.05	2.35	37.60
149	2.02	2.36	36.86
154	1.99	2.43	35.35
160	1.98	2.47	34.38
166	2.00	2.56	33.65
171	1.99	2.61	32.89
177	2.07	2.71	32.96
182	2.08	2.78	32.15
188	2.11	2.83	32.12
193	2.17	2.87	32.50
199	2.17	2.89	32.35
204	2.15	2.90	31.91
210	2.18	2.87	32.77
216	2.17	2.90	32.30
221	2.14	2.81	32.83
227	2.07	2.78	32.10
232	2.00	2.72	31.63
238	2.14	2.42	38.17
243	2.10	2.38	37.96
249	2.03	2.36	37.08
254	1.95	2.43	34.47
260	1.46	2.58	24.32

The behavior suggested by Figure 4.9 shows that the critical rpm to terminate the plug flow in the viscometer gap increases with increasing temperature up to a certain value, then it starts to steeply decrease again to finally meet a plateau, where the variations in critical speed are further reduced. This behavior is better illustrated in Figures 4.10 and 4.11.

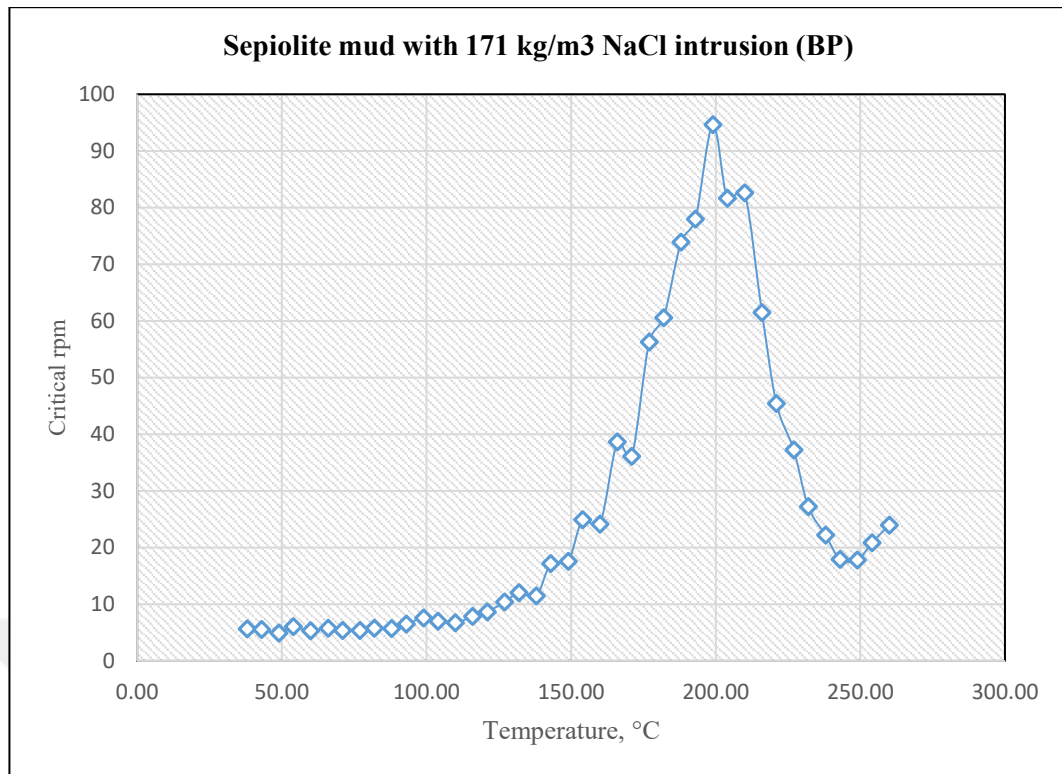


Figure 4. 10: BP Critical rpm change with temperature for sepiolite mud with 171 kg/m³ NaCl intrusion.

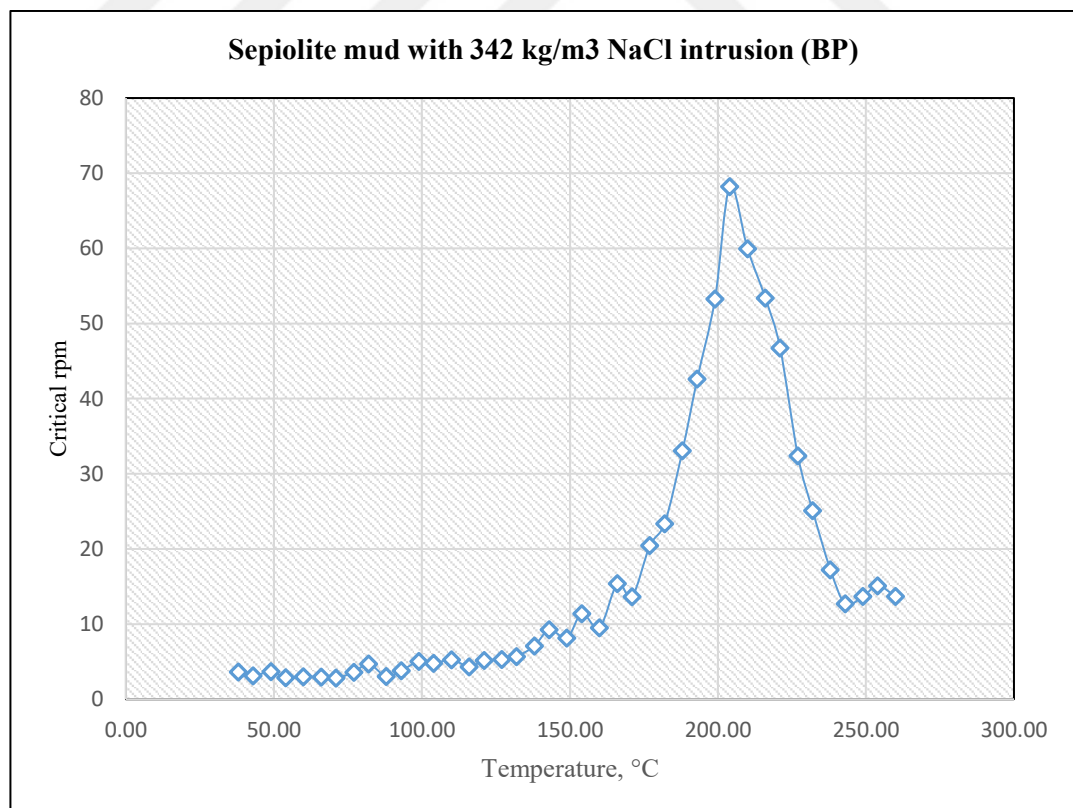


Figure 4. 11: BP Critical rpm change with temperature for sepiolite mud with 342 kg/m³ NaCl intrusion.

The hump-like behavior is better observed when adding salt to the drilling fluid. The critical speed starts to vary gradually and exponentially increase in both cases until it hits the maximum critical speed value. Thereafter, it steeply decreases again to reach a plateau of critical speed values slightly higher than initial values at lower temperatures.

This behavior could be justified by the chemical alteration of sepiolite clay particles, and other additives at certain temperatures. This behavior, however, is affected by the composition and additives present in the drilling fluid, as in some instances the critical speed could be delayed or reduced in magnitude. The composition effect on critical speed will be discussed in a separate section.

Using the Herschel-Bulkley model has already been proved to produce better description of the rheological behavior of pseudoplastic drilling fluids. Similar to the case of Bingham Plastic critical speed calculations, the rheometer data will be used to determine the Fann 35 viscometer critical rpm using Equation (4.4) for the same range of temperature using the HB model parameters suggested by the author. Table 4.21 and Figure 4.12 demonstrate the change of the critical speed with temperature for the case of plain sepiolite and fresh water.

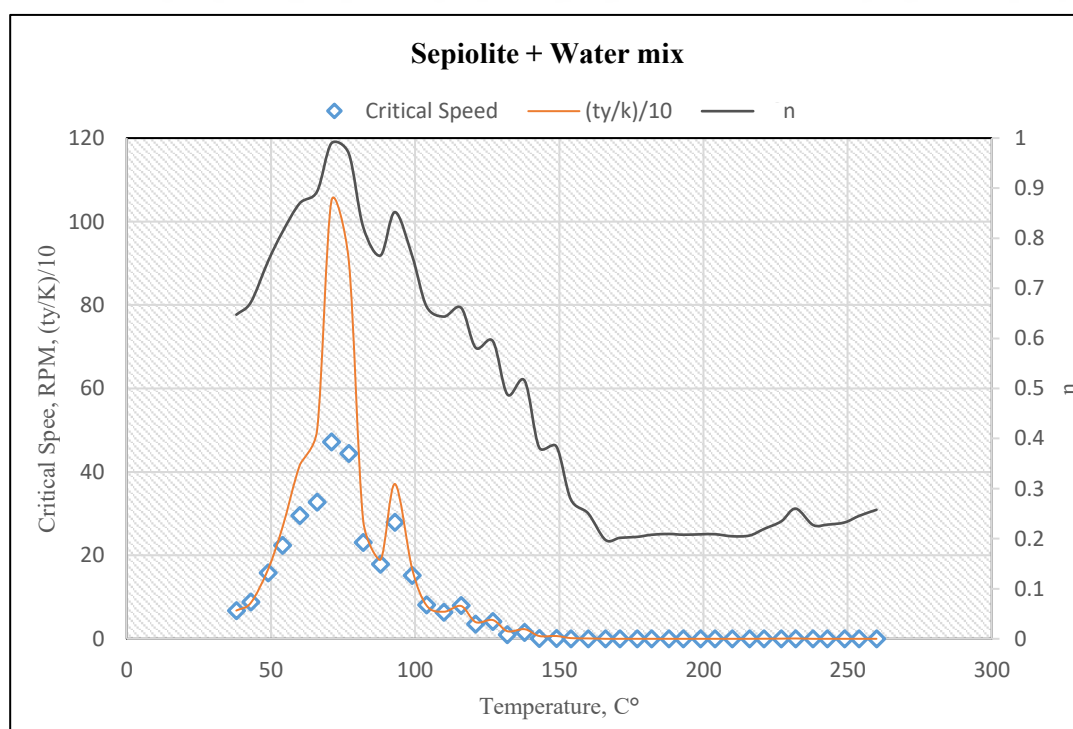


Figure 4. 12: HB Critical rpm change with temperature for sepiolite and water mixture.

Table 4. 21: HB Critical rpm and temperature data for sepiolite and water mud.

$T^{\circ} \text{C}$	τ_0, Pa	n	$K, Pa \cdot s^n$	N_c, rpm
38	7.1994	0.6473	0.106194	6.737
43	6.1312	0.6716	0.071275	8.813
49	5.3696	0.7538	0.032309	15.822
54	5.0295	0.8125	0.018849	22.439
60	4.6262	0.8698	0.011122	29.528
66	4.2123	0.8932	0.008435	32.756
71	3.9872	0.9897	0.003799	47.169
77	3.6131	0.9693	0.003987	44.402
82	3.0987	0.8224	0.010936	23.122
88	2.8606	0.7656	0.015089	17.879
93	2.8103	0.8522	0.007573	27.969
99	2.3869	0.7645	0.014289	15.225
104	2.2159	0.6634	0.027816	8.110
110	2.0616	0.6435	0.031753	6.388
116	2.0817	0.6609	0.026484	8.025
121	1.8825	0.5813	0.04698	3.571
127	1.8533	0.5942	0.041218	4.174
132	1.6286	0.488	0.089477	0.989
138	1.6669	0.5152	0.072233	1.530
143	1.3484	0.3824	0.197923	0.086
149	1.3148	0.3841	0.196198	0.083
154	0.8052	0.2785	0.478904	0.000
160	0.5829	0.2506	0.621263	0.000
166	0	0.1976	1.064338	0.000
171	0	0.2017	1.043262	0.000
177	0	0.2036	1.075834	0.000
182	0	0.2078	1.062901	0.000
188	0	0.2094	1.069607	0.000
193	0	0.2079	1.103137	0.000
199	0	0.2087	1.102179	0.000
204	0	0.2089	1.094994	0.000
210	0	0.2048	1.126129	0.000
216	0	0.2063	1.116549	0.000
221	0.19	0.2194	0.955605	0.000
227	0.3171	0.2347	0.817653	0.000
232	0.5082	0.2600	0.638507	0.000
238	0	0.2273	0.889503	0.000
243	0	0.2281	0.867948	0.000
249	0	0.2325	0.826275	0.000
254	0	0.2456	0.752509	0.000
260	0	0.2577	0.624616	0.000

Concordant with the results obtained facilitating the Bingham Plastic model, the hump-like behavior is undoubtedly noticeable for the HB critical rpm. However, as predicted,

the use of the Herschel-Bulkley model produced less noisy data, and a better visualized plateau effect. It must be noted that negative yield stress values cause negative critical speeds. However, this is physically meaningless.

It is also clear from Figure 4.12 that this hump-like behavior occurs when n value starts to approach 1, which in fact has the most dominant impact on the critical speed among other parameters.

The effect of salt addition is even more evident in the case of applying the Herschel-Bulkley model. Figures 4.13 and 4.14 indicate the HB critical rpm behavior with ranging temperature, upon adding NaCl. It is shown that the amount of salt added affects the magnitude of the critical speed, as well as chemical alteration temperatures.

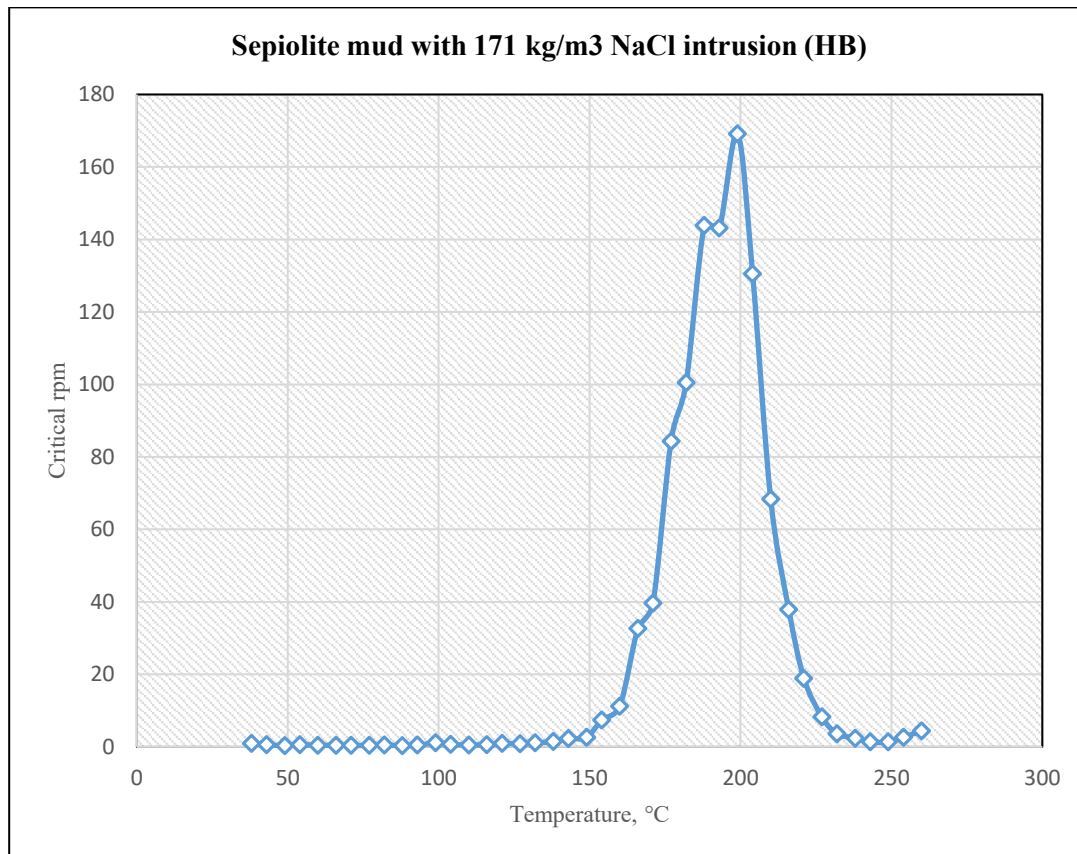


Figure 4. 13: HB Critical rpm change with temperature for sepiolite mud with 171 kg/m³ NaCl intrusion.

Other graphs and calculation results regarding the other mud samples can be found in Appendix A.

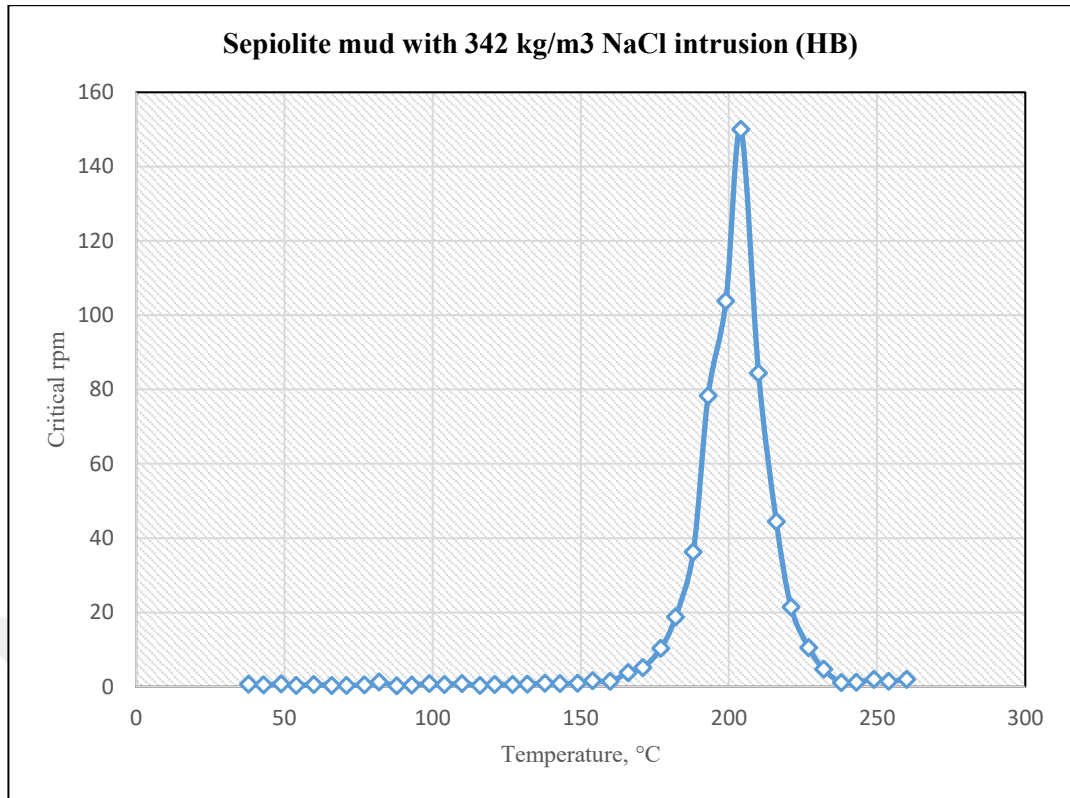


Figure 4. 14: HB Critical rpm change with temperature for sepiolite mud with 342 kg/m³ NaCl intrusion.

4.6.3 Composition effect

In the previous section, the effect of salt addition to drilling fluids on critical viscometer rpm using BP and HB models was briefly discussed. Accounting for the temperature effect, the 5 mud samples mentioned earlier will be studied closely for the critical viscometer speed for both BP and HB models.

For this reason, the critical rpm values were first calculated using Equation (4.3) at three temperature; 38° C, 77° C, and 199° C to eliminate the falsehood of chemical alteration consequence. Figure 4.15 and Table 4.22 are the results of the critical rpm change with mud composition at 38 °C.

Table 4. 22: BP Critical rpm and mud composition data @38° C.

Mud Composition	Nc, Rrpm
Sepiolite + Fresh Water	39.215
Unweighted fresh water sepiolite mud	3.839
Sepiolite mud with 171 kg/m ³ NaCl intrusion	5.623
Sepiolite mud with 342 kg/m ³ NaCl intrusion	3.635
Weighted fresh water sepiolite mud	5.351

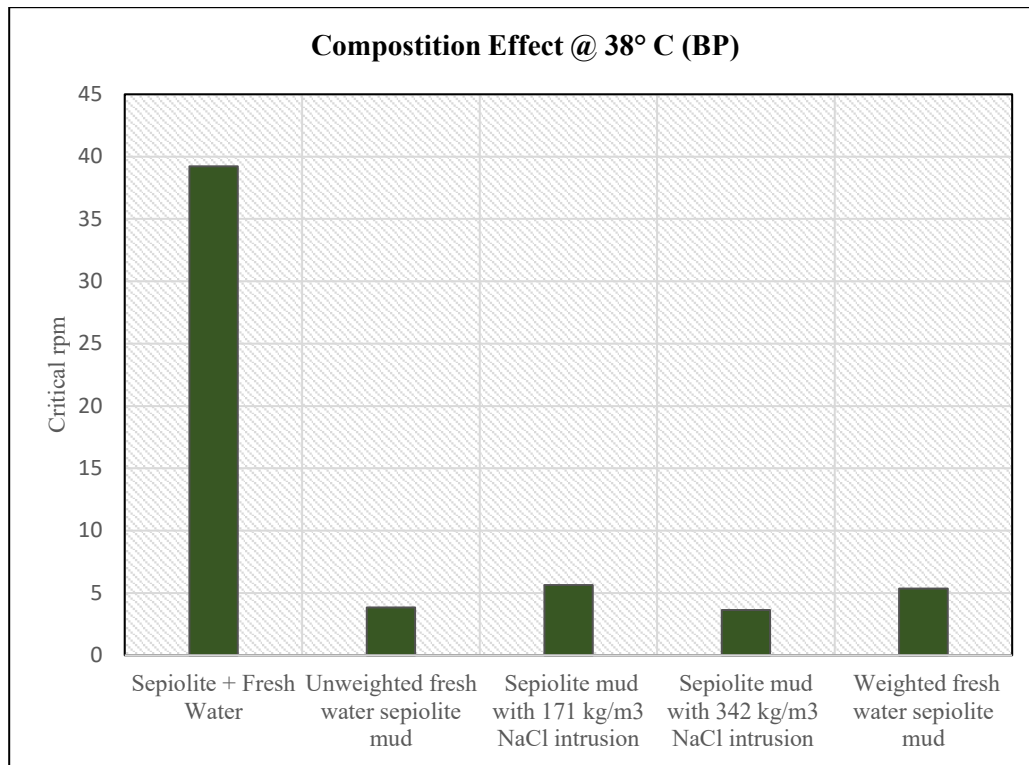


Figure 4. 15: BP critical rpm change with mud composition @ 38° C.

At 38° C, adding additives to the drilling fluid is shown to drastically decrease the critical rpm. Where adding salt will reflect an increase in the critical rpm, in addition to alteration effect delay. However, the critical rpm is noticed to decrease with further intermittent addition of salt. This is observed clearly from Figure 4.16. Lastly adding weighing agents to the drilling mud, is observed to slightly increase the critical rpm.

The temperature effect is clearly visible in Figure 4.16 and explained by the composition-based delay of alteration temperatures.

Thus, regardless of the temperature effect, it is safe to state that as a general rule, addition of additives play a huge factor in decreasing the necessary rpm at which plug flow effect ends. However Initial intrusion of salt to the mud is shown to slightly increase this value. Furthermore, any further addition of salt will gradually decrease critical rpm value. Finally, weighing agents like barite and sands increase the critical rpm.

In fact, similar behavior was observed at 77° C and 199° C as seen from Figures (4.17 and 4.18).

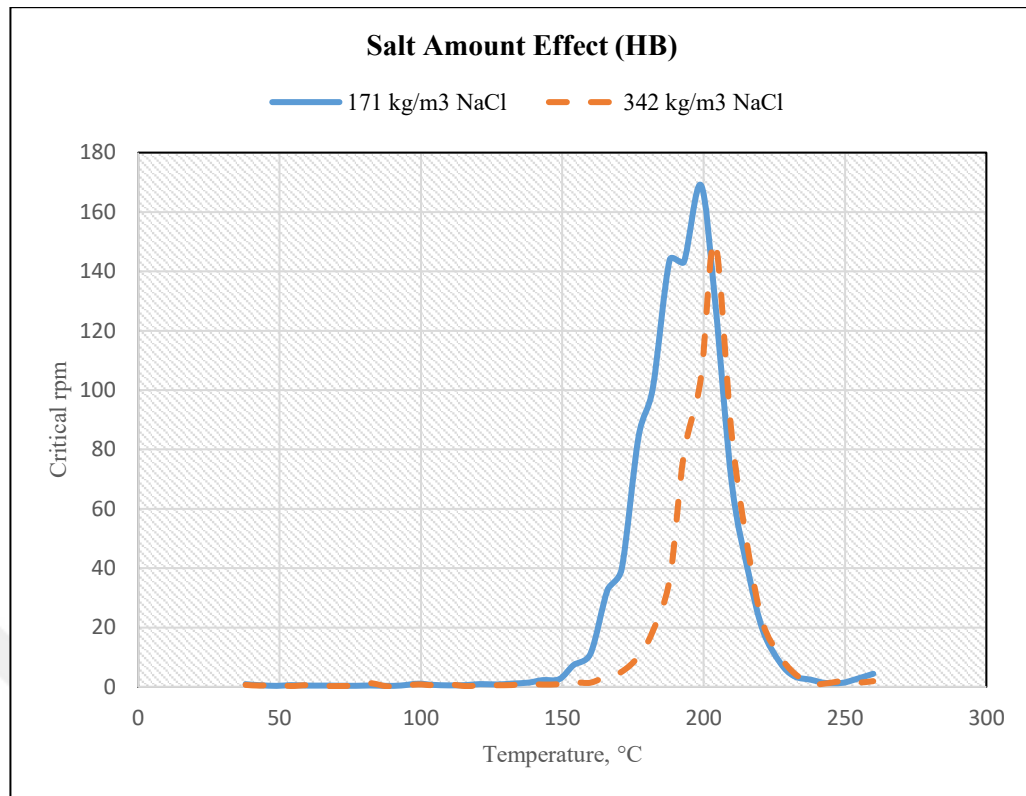


Figure 4. 16: Salt intrusion effect on critical speed for HB model.

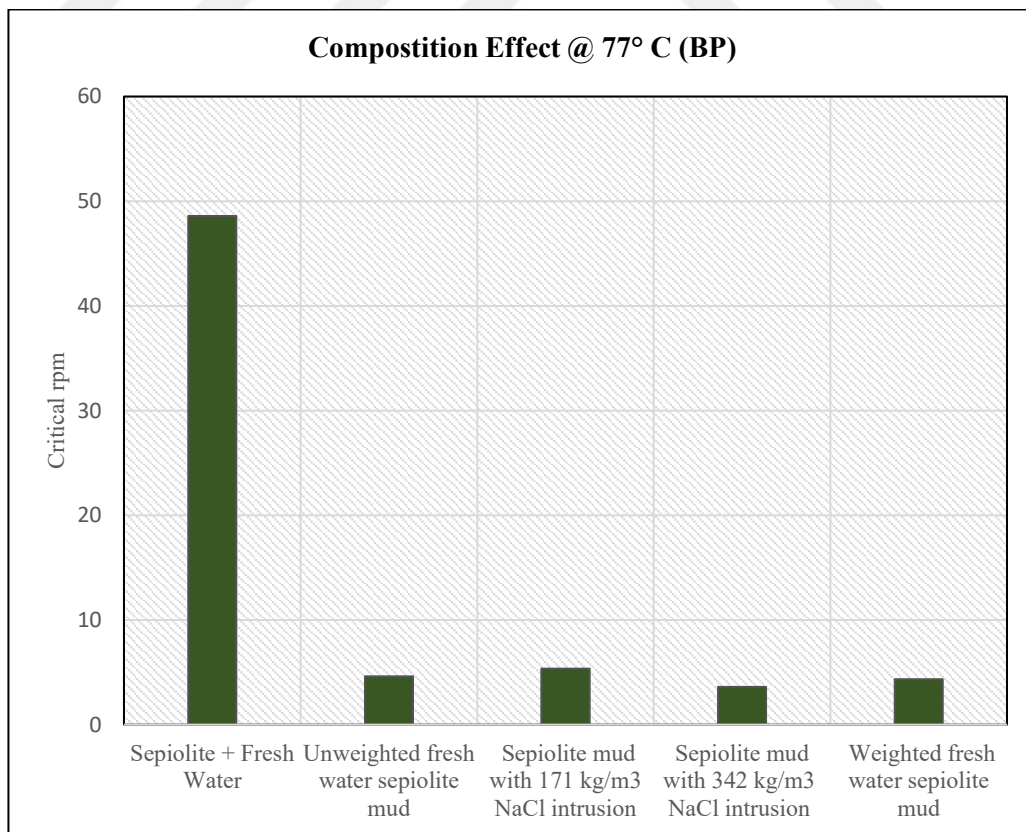


Figure 4. 17: BP critical rpm change with mud composition @ 77° C.

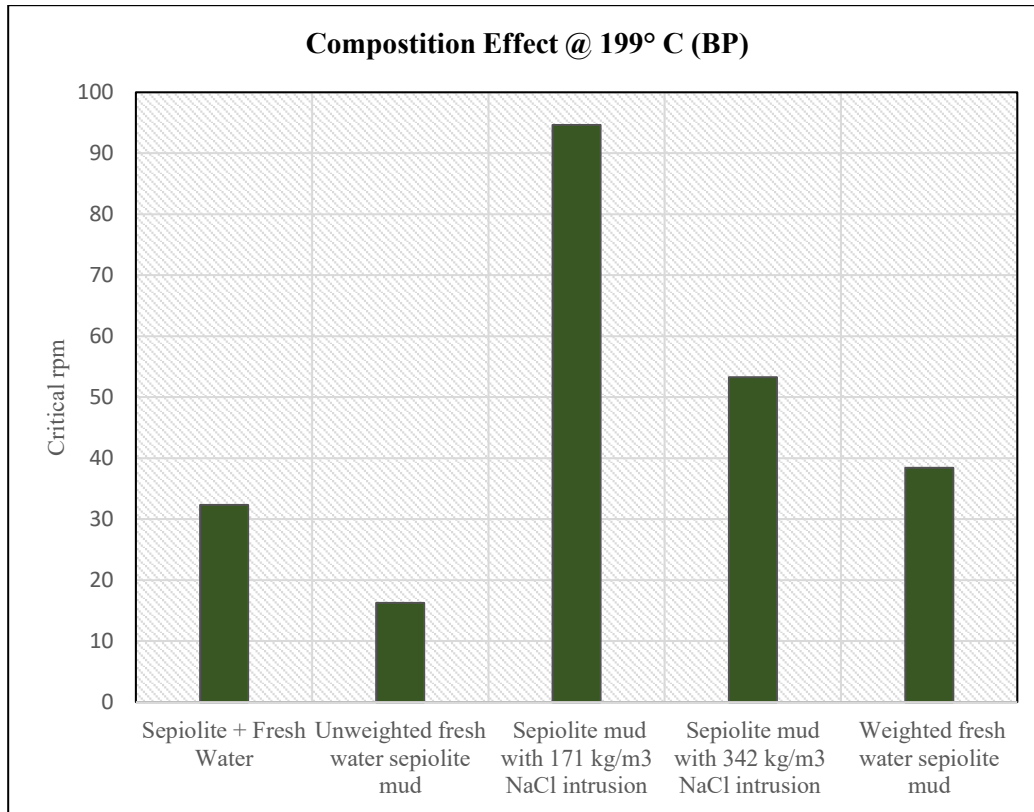


Figure 4. 18: BP critical rpm change with mud composition @ 199° C.

The use of the Herschel-Bulkley model to account for the composition effect is laid out to present similar conclusions, yet pushing the accuracy of the critical rpm determination one step further. Figure 4.19 demonstrates the effect of mud composition on critical rpm using Herschel-Bulkley model.

Mud composition effect calculation results involving other temperatures can be reviewed from appendix A.

To better visualize the effect of the weighing agent on the critical speed of the drilling fluid, the critical speed determined using the Herschel-Bulkley model was plotted against the temperature for both the weighted and the unweighted mud samples as seen in Figure 4.20. The addition of weighing agent is noticed to steepen the increase to the maximum critical speed upon alteration, which is considerably higher than the former case.

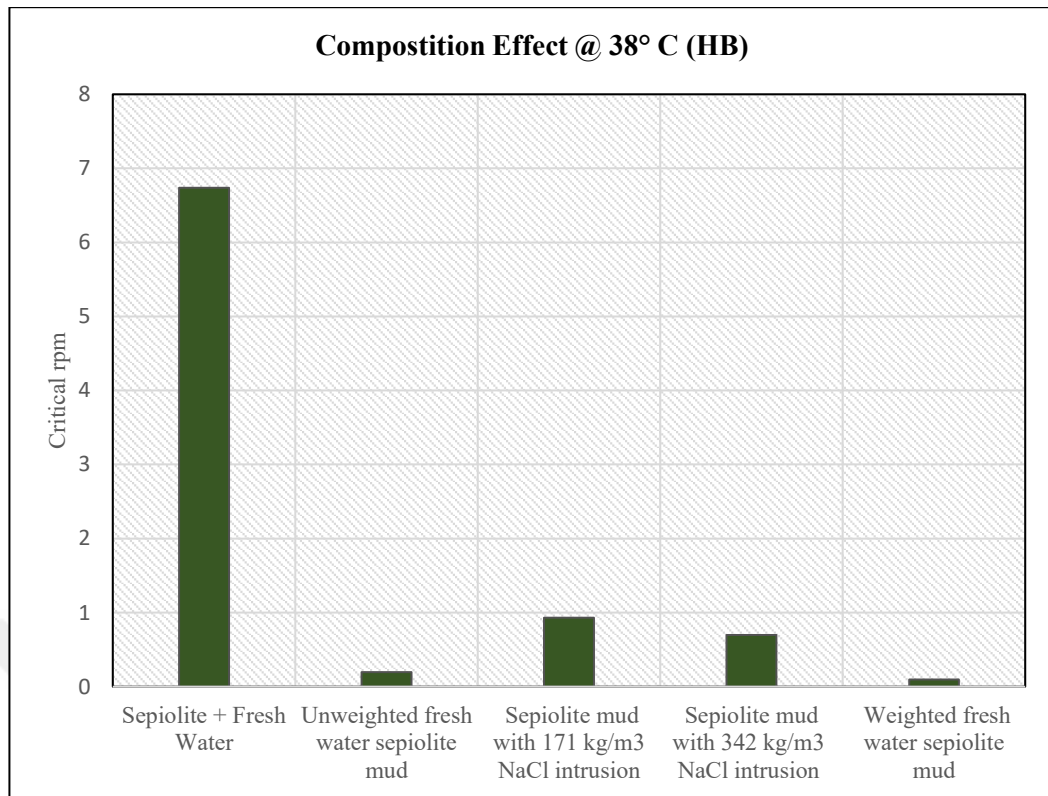


Figure 4. 19: HP critical rpm change with mud composition @ 38 °C.

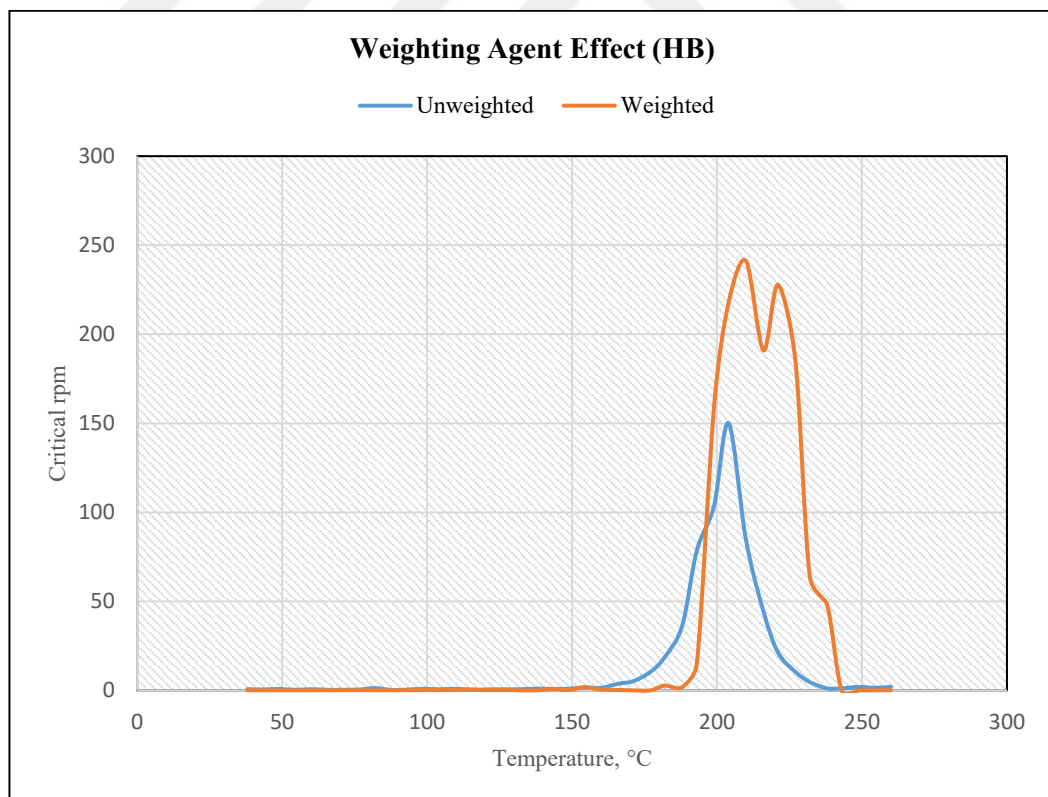


Figure 4. 20: Weighing agent effect on critical speed for HB model.



5. APPLICATION

Having studied the effect of the plug flow region on field viscometer readings, it was established that excluding the data distorted by such flow did in fact decrease the *average relative error*, and hence ensured better description of drilling fluids' rheology regardless of their type. Yet, it is extensively as important to analyze the impact of the proposed method on actual field applications. This section is dedicated to studying the influence of correcting the rheological model parameters on frictional pressure drop through the mud circulating system, (Figure 1.1).

In their paper, Maglione and Robotti (1996) published the circulation test data conducted on a vertical well located in the Po Valley, Northern Italy. At a depth of 3060 m, the geometry of the well is described in Table 5.1.

Table 5. 1: Circulation test well geometry.

Casing Size, in.		Depth, m		Open Hole, in
OD	ID	from	to	
30	29	0	52	-
20	19	0	598	26
-	-	598	3060	17 1/2

The circulation test was run at a depth of **798 m**, with the following operational conditions:

- Whilst running the drill string in hole (bit off bottom) with the drill string not rotating.
- The stand pipe pressure was monitored by the mud logging unit.
- The mud samples were taken from the shale shaker outlet while the test is running.

A water based mud with a density of 1190 kg/m^3 was circulating the hole. Table 5.2 lists the Fann 35 viscometer readings of the mud samples.

Table 5. 2: Fann 35 viscometer reading of the mud samples.

Rotor speed, rpm	Fann dial reading, °
600	38
300	26
200	22
100	15
6	5
3	4.5

The drill string was composed of drill pipes, heavy weight drill pipes, drill collars, and the bit. Other drill string or bottom-hole assembly subs such as stabilizers, bumpers, jars etc. will not be considered in the pressure loss calculations due to their minimal effect on frictional pressure loss. The drill string is outlined in Table 5.3, as well as other conduits in regard to the circulation test.

Table 5. 3: Drill string components and surface conduits concerned in the circulation test.

Section	OD, in.	ID, in.	Borehole ID, in.	Length, m
DP	5	4.28	19	513.6
HWDP	5	3	19	84.4
HWDP	5	3	17.5	52.6
DC	9	3	17.5	70
DC	11.25	3	17.5	77
Stand Pipe	-	4	17.5	20
Rotary Hose	-	3.5	17.5	20
Swivel	-	3.5	17.5	3.5
Kelly	-	3.5	17.5	12
Bit	-	3x15/32 + 1x14/32	17.5	0.4

The same heavy weight drill pipe diameter was mentioned twice in Table 5.3. This is because it was divided into two sections, where the first section and the drill pipes above it, are facing the inner wall of the surface casing, whereas the second section and the rest of the drill string are facing the inner wall of the 17 ½ uncased hole.

In fact, the circulation test was carried out at three different flow rates, and their corresponding stand pipe pressures were recorded and listed in the following table, (Table 5.4).

Table 5. 4: Actual mud circulation test results.

Pump rate, lt/min	Pump rate, gpm	SPP, Bar	SPP, MPa
1640	433.24	46.6	4.66
2460	649.87	103.3	10.33
3270	863.85	176.3	17.63

5.1 Stand Pipe Pressure Calculation Using BP Model

The viscometer data were first transformed into $\tau - \dot{\gamma}$ data via Equations (4.6 and 4.7), then, as has been illustrated in Chapter 4, the method of the least squares is applied through Equations (2.43 and 2.44) to obtain the BP yield stress and plastic viscosity. Having determined these parameters, Equation (4.3) was then used to check for the critical Fann rotational speed as this thesis provided. In fact, the critical rotational speed was calculated to be **9.5 rpm**, which means that to obtain the corrected parameters through the Bingham Plastic model the R3 and R6 readings must be omitted, and the remaining data should be regressed again to determine those parameters. Table 5.5 lists the regression analysis results both for the 6 readings (non-corrected), as well as the 4 readings (corrected) scenarios in SI units and field units.

Table 5. 5: Bingham Plastic regression analysis results.

	Non-corrected	Corrected
μ_p , Pa.s	0.0166	0.0132
τ_y , Pa	3.65	6.07
μ_p , cp	16.60	13.20
τ_y , lb/100ft ²	7.63	12.69

5.1.1 Pipe frictional pressure loss

Bourgoyne et al. (1991) related the wall shear stress to the frictional pressure drop analytically, and suggested the use of the following equations, in field units, to calculate the frictional pressure drop for Bingham Plastic fluids. First of all, the fluid flow regime must be determined facilitating the Reynold's number as expressed in the following equation,

$$N_{Re} = \frac{928 \rho \bar{v} d}{\mu_p} \quad (5.1)$$

where,

ρ : Mud density, ppg

$\bar{v}$: Average velocity, ft/s

d : Pipe diameter, in.

The average velocity for a pipe flow is determined from the flow rate by Equation (5.2), where q is in units of gallon per minute (gpm).

$$\bar{v} = \frac{q}{2.448d^2} \quad (5.2)$$

The authors advised the use of the Hedstrom number to assess the critical Reynold's number above which the flow regime is believed to be turbulent. The Hedstrom number is calculated by Equation (5.3), and the critical Reynold's number is determined utilizing the correlation suggested by Hanks (1967), (Figure 5.1).

$$N_{He} = \frac{37100 \rho \tau_y d^2}{\mu_p^2} \quad (5.3)$$

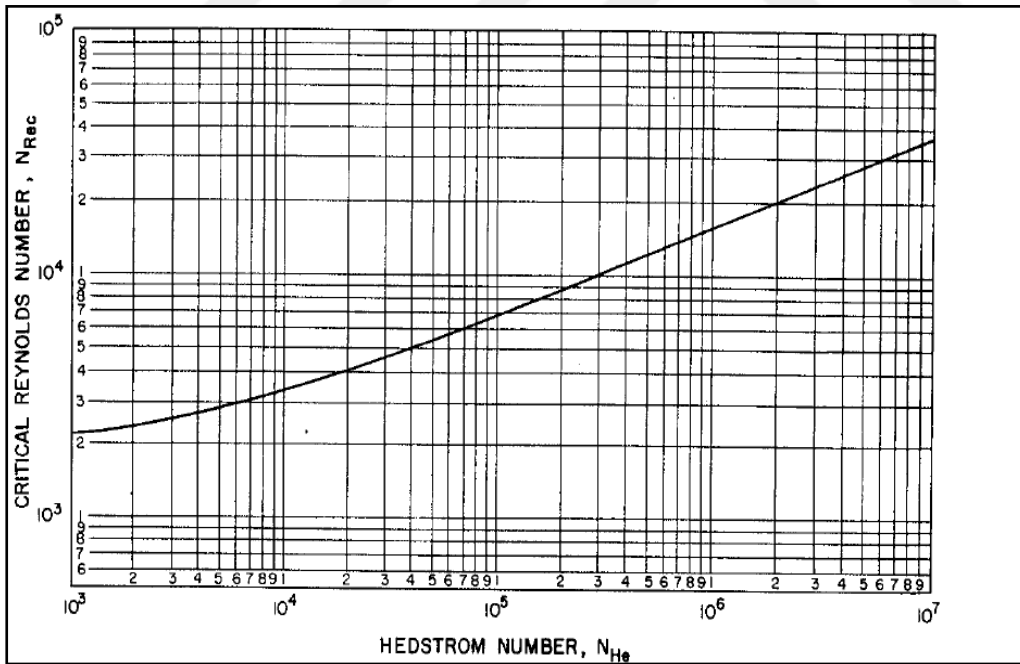


Figure 5. 1: Critical Reynold's number correlation, (Hanks, 1967).

The Reynold's number calculated from Equation (5.1) must be checked against the critical Reynold's number (N_{Rec}). In case $N_{Re} > N_{Rec}$, the flow is Turbulent, whereas

in case $N_{Re} < N_{Rec}$, the flow could be either laminar or turbulent. However, the flow regime that yields higher frictional pressure value will be chosen.

Once the flow regime is ascertained, the pipe frictional pressure drop can be calculated from Equations (5.4 and 5.5) for laminar flow regime and turbulent flow regime respectively.

$$\frac{\Delta P}{\Delta L} = \frac{\mu_p \bar{v}}{1500 d^2} + \frac{\tau_y}{225 d} \quad (5.4)$$

$$\frac{\Delta P}{\Delta L} = \frac{\rho^{0.75} \bar{v}^{1.75} \mu_p^{0.25}}{1800 d^{1.25}} \quad (5.5)$$

where,

ΔP : Frictional pressure loss change, psi

ΔL : Pipe interval length, ft.

5.1.2 Annular frictional pressure loss

For the calculation of frictional pressure loss in the annular section, Bourgoyne et al. (1991) recommended equating the annular cross-sectional area to area of a pipe. Hence, Equations (5.1, 5.2 and 5.3) become representative in the annular section case through Equations (5.6, 5.7 and 5.8) respectively.

$$N_{Re} = \frac{757 \rho \bar{v} (d_2 - d_1)}{\mu_p} \quad (5.6)$$

$$\bar{v} = \frac{q}{2.448 (d_2^2 - d_1^2)} \quad (5.7)$$

$$N_{He} = \frac{24700 \rho \tau_y (d_2 - d_1)^2}{\mu_p^2} \quad (5.8)$$

where,

d_1 : Pipe outside diameter, in.

d_2 : Hole inside diameter, in.

The critical Reynold's number is determined through the correlation represented in Figure 5.1, and similar to the pipe flow case, the critical Reynold's number is used to check for the flow regime.

After the flow regime has been determined, Equations (5.9 and 5.10) can be used to calculate the annular frictional pressure loss for the laminar flow regime and turbulent flow regime, respectively.

$$\frac{\Delta P}{\Delta L} = \frac{\mu_p \bar{v}}{1000 (d_2 - d_1)^2} + \frac{\tau_y}{200 (d_2 - d_1)} \quad (5.9)$$

$$\frac{\Delta P}{\Delta L} = \frac{\rho^{0.75} \bar{v}^{1.75} \mu_p^{0.25}}{1396 (d_2 - d_1)^{1.25}} \quad (5.10)$$

5.1.3 Pressure drop across the drill bit

As the drilling fluid discharges through the drill bit nozzles, the fluid velocity increases rapidly, which is essential for adequate hole cleaning, yet these high velocities result in great pressure drop across the drill bit. This pressure drop can be estimated from Equation (5.11).

$$\Delta P_b = \frac{q^2 \rho}{12031 A_n} \quad (5.11)$$

where,

A_n : Total bit nozzle area, in^2 .

5.1.4 SPP calculation

The stand pipe pressure measured during the circulation test is basically comprised of the annular frictional loss pressures, pipe frictional loss pressures, and the pressure drop across the bit. Considering the first part of the circulation test ($q = 1640 \text{ lt/min}$), those pressure losses are first calculated for the non-corrected Bingham Plastic model parameters, and the results are listed in Tables 5.6 and 5.7.

Similarly, the pressure drop calculation results obtained upon using the corrected Bingham Plastic model parameters can be seen in Tables 5.8 and 5.9.

Finally, a comparison of the stand pipe pressure calculated using both sets of model parameters to the actual value of the stand pipe pressure measured at 1640 lt/min along with respective relative errors are demonstrated in Table 5.10.

Table 5. 6: Pipe frictional pressure loss results utilizing non-corrected BP model parameters @1640 lt/min.

Section	$\bar{v}$ pipe, ft/s	N_{He} (Pipe)	N_{Rec} (Pipe)	N_{Re} (Pipe)	Pipe Flow Regime	Pressure loss (Pipe), psi
DP	9.66	186989.31	8500	22958.22	Turbulent	90.91
HWDP	19.66	91869.59	6800	32753.73	Turbulent	80.79
HWDP	19.66	91869.59	6800	32753.73	Turbulent	50.35
DC	19.66	91869.59	6800	32753.73	Turbulent	67.01
DC	19.66	91869.59	6800	32753.73	Turbulent	73.71
Stand Pipe	11.06	163323.71	8200	24565.30	Turbulent	4.88
Rotary Hose	14.45	125044.71	7300	28074.63	Turbulent	9.21
Swivel	14.45	125044.71	7300	28074.63	Turbulent	1.61
Kelly	14.45	125044.71	7300	28074.63	Turbulent	5.52

Table 5. 7: Annulus frictional pressure loss results utilizing non-corrected BP model parameters @1640 lt/min.

Section	$\bar{v}$ ann, ft/s	N_{He} (Annulus)	N_{Rec} (Annulus)	N_{Re} (Annulus)	Annular Flow Regime	Pressure Loss (Annulus), psi
DP	0.53	1332012.69	18200	3339.79	Laminar	4.67
HWDP	0.53	1332012.69	18200	3339.79	Laminar	0.77
HWDP	0.63	1061872.36	17100	3562.44	Laminar	0.54
DC	0.79	491009.78	12000	3024.71	Laminar	1.07
DC	0.98	265468.09	9000	2788.00	Laminar	1.65

Table 5. 8: Pipe frictional pressure loss results utilizing corrected BP model parameters @1640 lt/min.

Section	$\bar{v}$ pipe, ft/s	N_{He} (Pipe)	N_{Rec} (Pipe)	N_{Re} (Pipe)	Pipe Flow Regime	Pressure loss (Pipe), psi
DP	9.66	491332.88	13200	28864.43	Turbulent	85.85
HWDP	19.66	241396.40	9300	41179.92	Turbulent	76.30
HWDP	19.66	241396.40	9300	41179.92	Turbulent	47.55
DC	19.66	241396.40	9300	41179.92	Turbulent	63.28
DC	19.66	241396.40	9300	41179.92	Turbulent	69.61
Stand Pipe	11.06	429149.16	12900	30884.94	Turbulent	4.61
Rotary Hose	14.45	328567.33	12500	35297.08	Turbulent	8.69
Swivel	14.45	328567.33	12500	35297.08	Turbulent	1.52
Kelly	14.45	328567.33	12500	35297.08	Turbulent	5.22

Table 5. 9: Annulus frictional pressure loss results utilizing corrected BP model parameters @1640 lt/min.

Section	$\bar{v}_{ann}$, ft/s	N_{He} (Annulus)	N_{Rec} (Annulus)	N_{Re} (Annulus)	Annular Flow Regime	Pressure Loss (Annulus), psi
DP	0.53	3499994.83	23200	4198.98	Laminar	7.70
HWDP	0.53	3499994.83	23200	4198.98	Laminar	1.26
HWDP	0.63	2790174.45	20800	4478.91	Laminar	0.89
DC	0.79	1290176.67	18000	3802.85	Laminar	1.75
DC	0.98	697543.61	15500	3505.23	Laminar	2.65

Table 5. 10: Actual and measured stand pipe pressure results @1640 lt/min.

	Non-corrected Parameters	Corrected Parameters
Bit Pressure drop, psi	347.13	347.13
Pipe pressure loss, psi	383.98	362.62
Annular pressure loss, psi	8.70	14.24
Measured SPP, psi	739.81	724.00
Measured SPP, Mpa	5.10	4.99
Actual SPP, Mpa	4.66	4.66
Relative Error, %	9.46	7.12

Having corrected the Bingham Plastic model parameters did in fact reduce the relative error in the frictional pressure loss prediction as evident from the results presented in Table 5.10.

Following similar fashion, the pressure losses were also calculated for the other flow rates involved in the circulation test. The calculation results can be reviewed from Appendix B. However, Table 5.11 was constructed to demonstrate the calculated stand pipe pressures at those flow rates in comparison to the actual measured stand pipe pressures.

Table 5. 11: Circulation test conclusive results utilizing BP model.

	Non-corrected Parameters			Corrected Parameters		
Flow rate, lt/min	1640	2460	3270	1640	2460	3270
Bit Pressure drop, psi	347.13	781.05	1380.08	347.13	781.05	1380.08
Pipe pressure loss, psi	383.98	780.68	1284.67	362.62	737.25	1213.21
Annular pressure loss, psi	8.70	8.82	8.94	14.24	14.34	14.44
Measured SPP, psi	739.81	1570.54	2673.69	724.00	1532.64	2607.73
Measured SPP, Mpa	5.10	10.83	18.43	4.99	10.57	17.98
Actual SPP, Mpa	4.66	10.33	17.63	4.66	10.33	17.63
Relative error, %	9.46	4.83	4.56	7.12	2.30	1.98

Although, the relative error is observed to decrease consequent to the increase in flow rate, correcting the Bingham Plastic model parameter decreased that error to a further extent, which validates the method proposed by this thesis work. Figure 5.2 illustrates graphically the improvement of the frictional pressure estimation simultaneous to correcting the model parameters.

Furthermore, it is clear from the relative error that at higher flow rates, the influence of correcting the parameters becomes more pronounced.

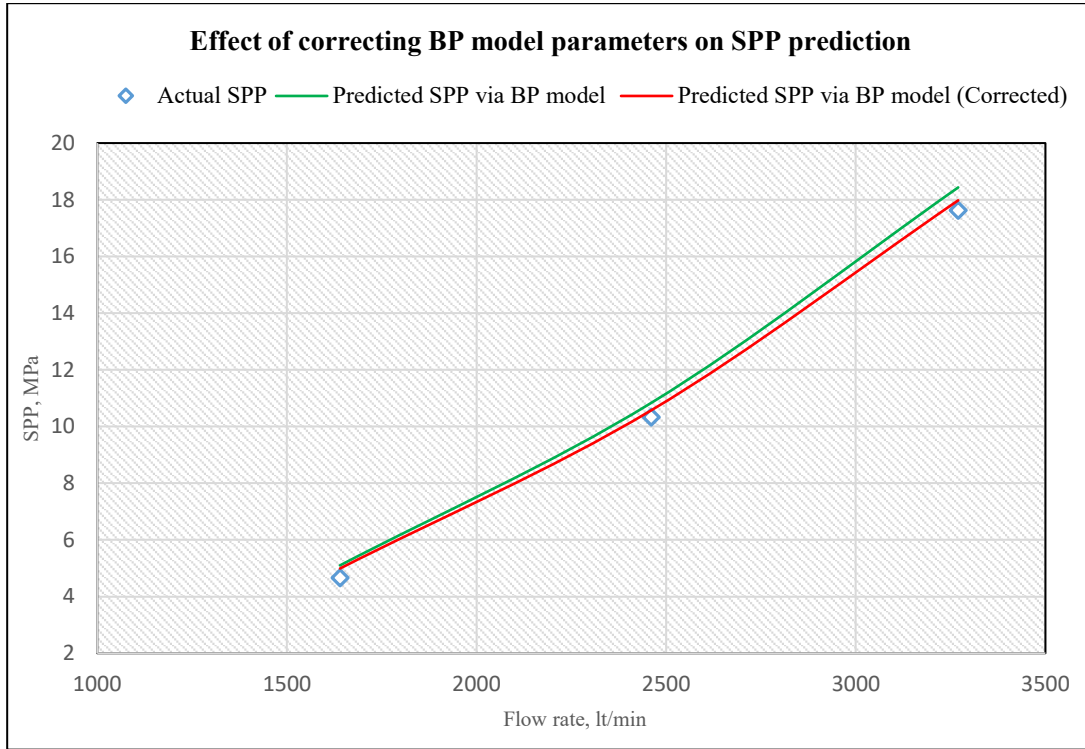


Figure 5. 2: Effect of correcting Bingham Plastic model parameters on SPP prediction.

5.2 Stand Pipe Pressure Calculation Using HB Model

Consistent with the followed approach in the previous section, the HB critical speed must be assessed first. Using Equation (2.68), the flow behavior exponent was determined to be **0.576**. Afterwards, the other parameters, namely; τ_0 and K are calculated through Equations (2.60 and 2.61) to be **1.354 Pa** and **0.332 Pa.sⁿ** Respectively.

Hence, the HB critical rotational speed was calculated from Equation (4.4), and a value of **0.069 rpm** was obtained. Since this value is below the R3 reading of the Fann 35 viscometer. The HB model suggests that the viscometer readings are not affected by the plug flow, thus the model parameters shall not be corrected.

It is quite normal (as observed from Chapter 4 of this thesis) for the critical speed to vary according to the model being used. This can be related to the nature of the model in the first level, as the Bingham Plastic model is argued to overestimate the yield stress value, whereas the Herschel-Bulkley model underestimates that value, (Feys, D. et al., 2012).

The yield stress is a physical quantity of the fluid in question, and it should be measured rather than estimated through models. However, measuring the yield stress directly is still unfortunately not feasible. Nevertheless, the drilling mud used in the circulation test is not considered a high-yield or even a thick mud. For this reason, and to illustrate the effect of correcting the HB parameters on pressure loss calculations, the previously studied water based mud from Table 4.1 will be considered in place of the actual mud in a hypothetical case study for the same well.

The stand pipe pressures predicted using the Herschel-Bulkley model were actually published by the authors and listed in Table 5.12. It is interesting to note that utilizing the HB model produced higher errors than utilizing the *corrected* BP parameters.

Table 5. 12: Circulation test results utilizing Herschel-Bulkley model.

Used model	HB		
Flow rate, lt/min	1640	2460	3270
Measured SPP, Mpa	4.74	9.91	16.70
Actual SPP, Mpa	4.66	10.33	17.63
Relative Error, %	1.72	4.07	5.28

5.2.1 Hypothetical case study

In their study on drilling hydraulics, Merlo, A. et al. (1995), proposed a numerical method to calculate the frictional pressure losses utilizing the HB model. Interestingly, they proved the reliability of their approach with testing on another well from the Po Valley, very similar to the well tested by Maglione and Robotti (1996).

Their approach will be applied to analyze the effect of correcting the model parameters on stand pipe pressure testing using the WBM from Table 4.1 as stated earlier. All the equation expressed in the following sections are in SI units.

5.2.1.1 Pipe frictional pressure loss

As a first step before calculating the frictional pressure loss, the pipe flow regime must be determined through the following equations.

$$Re_{eq} = \left(2 \frac{3n+1}{n}\right) \frac{\rho v_p^{(2-n)} R^n}{\tau_0 \left(\frac{R}{v_p}\right)^n + K \left(\frac{3n+1}{n Cc}\right)^n} \quad (5.12)$$

$$Cc = 1 - \frac{1}{2n+1} \left\{ \frac{\tau_0}{\tau_0 + K \left[\frac{(3n+1)q}{n\pi R^3} \right]^n} \right\} \quad (5.13)$$

where,

Re_{eq} : Equivalent Reynold's number

v_p : Pipe velocity, m/s

R : Pipe radius, m

Cc : Reynold's number correction coefficient, dimensionless

The equivalent Reynold's number must be checked against the critical equivalent Reynold's number (Re_{eqcr}). In case $Re_{eq} > Re_{eqcr}$ the flow is Turbulent, whereas in case $Re_{eq} < Re_{eqcr}$, the flow is said to be laminar.

$$Re_{eqcr} = \left[\frac{4(3n+1)}{ny} \right]^{\left(\frac{1}{1-z} \right)} \quad (5.14)$$

where,

$$y = \frac{\log(n) + 3.93}{50} \quad (5.15)$$

and,

$$z = \frac{1.75 - \log(n)}{7} \quad (5.16)$$

The frictional pressure loss within the pipe section can then be calculated through Equations (5.17 and 5.18), respectively.

$$\Delta P = \frac{2 \Delta L K}{R} \left[\frac{\tau_0}{K} + \left[\left(\frac{3n+1}{n Cc} \right) \left(\frac{q}{\pi R^3} \right) \right]^n \right] \quad (5.17)$$

$$\Delta P = f \rho \frac{q^2}{\pi^2 R^5} \Delta L \quad (5.18)$$

where f is the Fanning friction factor, and calculated from,

$$f = y(Re_{eq})^{-z} \quad (5.19)$$

5.2.1.2 Annular frictional pressure loss

In the case of an annular section, Equations (5.12 and 5.13) become,

$$Re_{eq} = \left(4 \frac{2n+1}{n} \right) \frac{\rho v_a^{(2-n)} (R_2 - R_1)^n}{\tau_0 \left(\frac{R_2 - R_1}{v_a} \right)^n + K \left(2 \frac{2n+1}{n Cc} \right)^n} \quad (5.20)$$

where,

v_a : Annular velocity, m/s

R_1 : Pipe outside radius, m

R_2 : Hole inside radius, m

$$Ca = 1 - \frac{1}{n+1} \left\{ \frac{\tau_0}{\tau_0 + K \left[\left(\frac{2(2n+1)}{n(R_2 - R_1)} \right) \left(\frac{q}{\pi(R_2^2 - R_1^2)} \right) \right]^n} \right\} \quad (5.21)$$

The critical equivalent Reynold's number (Re_{eqcr}), becomes,

$$Re_{eqcr} = \left[\frac{8(2n+1)}{ny} \right]^{\left(\frac{1}{1-z} \right)} \quad (5.22)$$

The z and y coefficients are the same as expressed in Equations (5.15 and 5.16). in case the equivalent Reynold's number was below the critical equivalent Reynold's number, and the flow is laminar, Equation (5.23) can be used to determine the frictional pressure loss in the annular section of length L.

$$\Delta P = \frac{2 \Delta L K}{(R_2 - R_1)} \left[\frac{\tau_0}{K} + \left[\frac{2(2n+1)}{n Ca (R_2 - R_1)} \frac{q}{\pi (R_2^2 - R_1^2)} \right]^n \right] \quad (5.23)$$

$$\Delta P = f \rho \frac{q^2}{\pi^2 (R_2 - R_1) (R_2^2 - R_1^2)^2} \Delta L \quad (5.24)$$

The Fanning friction factor can be determined from Equation (5.19).

5.2.1.3 Pressure loss across the bit

The impact of the drilling mud's viscosity on the pressure drop across the bit nozzles is believed to be very minimal, and hence Equation (5.11) will be used to determine the pressure bit pressure drop as demonstrated in the previous section.

5.2.1.4 SPP calculation

Both, the corrected, and uncorrected Herschel-Bulkley model parameters for the water based mud determined through regression analysis (Table 4.19), will be used for the stand pipe pressure calculation.

As stressed in earlier statement, the same well geometry (Tables 5.1 and 5.3), and operational conditions are considered valid in this case. Figure 5.3 illustrates the difference in stand pipe pressure calculation via HB model due to parameter correction.

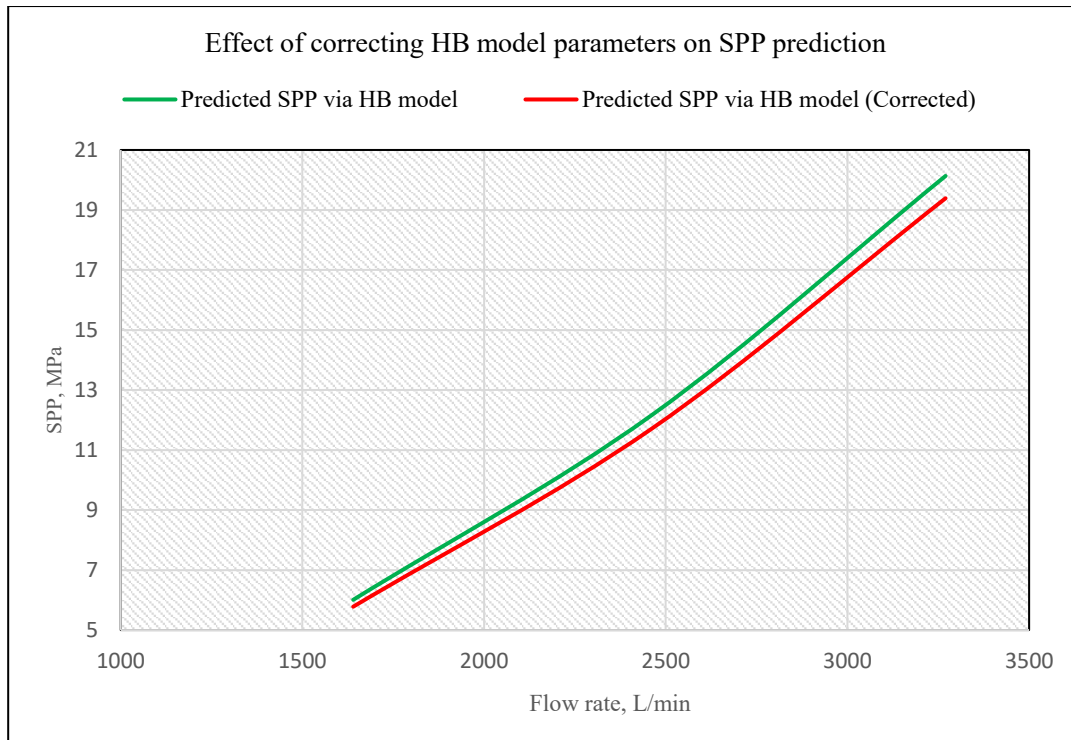


Figure 5. 3: Effect of correcting HB model parameters on SPP prediction.

The divergence of the SPP curve determined using the corrected parameters from the non-corrected parameter curve is clearly visible in Figure 5.3, and similar to the case where the SPP was determined utilizing the Bingham Plastic model, the difference of the two curves was increasing as the flow rate increased.

While the SPP curve generated using the corrected Herschel-Bulkley parameters is conceivable to be a more adequate representation of the actual SPP of such hypothetical case, difference in predicted pressures reached more than what is equivalent to a **100 psi**. Such differences in predicted pressure losses directly affect the equivalent mud circulating density (ECD). Where, in turn, can be reflected on numerous design and operational parameters, such as surge and swab pressures, bit hydraulics, and can be vital in deep drilling applications as the mud window get narrower. The calculation results concerning the SPP prediction utilizing the HB model can be reviewed from Appendix B.

6. CONCLUSIONS AND RECOMMENDATIONS

Predicting and controlling the rheological properties of a drilling fluid is a routine task in any drilling operation, and although a number of high precision and accuracy rheometers are available in the market, their use is still somewhat limited to laboratory testing and measurements. Rotational field viscometers, such as the Fann 35 rotational viscometer, are still the forerunner for daily testing and monitoring for the rheological properties of drilling fluids, due to their durability, easy usage, portability, low cost, and above all, tolerability for real-time measurement of the drilling fluids' rheology.

These viscometers, however, can produce erroneous readings, hence misinterpretation of the drilling fluid's rheological properties. Apart from measurement errors, equipment calibration etc., the occurrence of a plug flow at low shear rates when testing drilling fluids via rotational viscometer can incredibly affect the measurement results.

In this study, the effect of the plug flow is investigated, and a method for eliminating its effect by correcting the rheological parameters was presented, and the following can be concluded,

- Using regression analysis when determining parameters of rheological models, resulted in a more accurate estimation of those parameters regardless of the rheological model utilized.
- The use of Power Law model for high-yield drilling fluids can lead to extreme misinterpretation of their rheological properties.
- Regardless of the mud type and rheological model utilized, the API recommended equations for predicting the model parameters resulted in the highest error when compared to other predicting methods, and it is recommended against using them.
- Conducting a traditional regression analysis for the three-parameter Herschel-Bulkley model can misestimate the yield stress. The effect of such misestimation is shown by Mullineux, G. (2007), and it is recommended to employ his method when regressing the data for the HB model.

- The critical rotational rpm at which the plug flow effect ends is determined for both Bingham Plastic and Herschel-Bulkley models, and it has been shown that excluding the viscometer rpm values less than the critical speed, and regressing the remaining data, increased the accuracy of model by correcting its parameters, with the HB model outperforming other models when applying this method, regardless of the mud type.
- The effect of mud type was investigated, and it has been demonstrated that the critical rpm increases proportionally with the “yield stress to plastic viscosity ratio” for Bingham Plastic fluids. Whereas for HB fluids, the flow behavior exponent has been shown to have the most dominant effect on the critical speed, in comparison to the “yield stress-to-consistency index ratio”.
- The critical rpm increases with temperature for sepiolite based drilling fluids, and at sufficient high temperature (upon clay chemical alteration), the critical rpm increases greatly, as the flow behavior exponent approaches 1 (Newtonian behavior). Where the apparent viscosity of the fluid is hypothesized to be controlled by the consistency index.
- It was concluded that, for such types of muds, excluding the R3 and R6 when using the Bingham Plastic model, and often the R3 when using the Herschel-Bulkley model, should reduce the uncertainty while determining the model parameters. Taking the investigation one step further, the mud composition effect was inspected. The sample’s critical rpm increased upon weighting the sepiolite mud, and decreased while increasing the salt concentration.
- As demonstrated in Chapter 5, the proposed method is validated by predicting the stand pipe pressure of an actual mud circulating test and comparing the results with the conventional non-corrected method, where considerable differences in pressure were noticed. This is a crucial matter in narrow mud window applications.
- Regardless of the model being used, the influence of correcting the model parameters becomes more pronounced as the flow rate increases, which is substantial in directional, deep drilling and use of PDC bits.

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APPENDICES

APPENDIX A: Temperature and Composition Effect on Critical Speed Data and Plots.

APPENDIX B: Stand pipe calculation results using Bingham Plastic and Herschel-Bulkley models





APPENDIX A

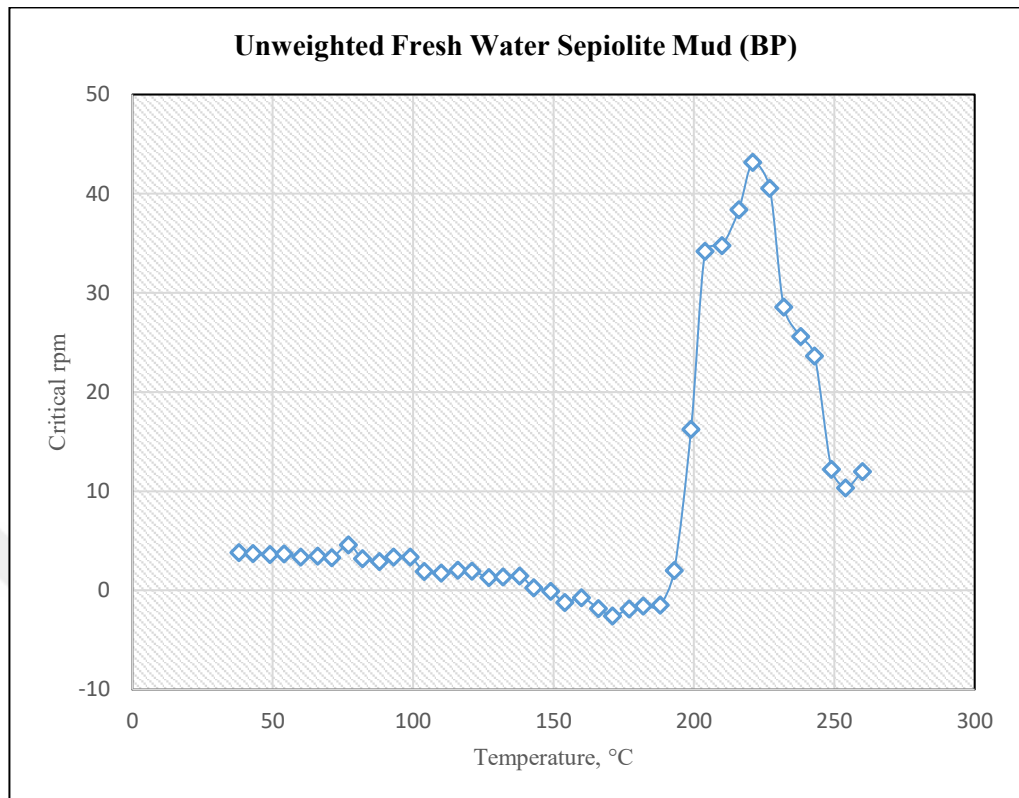


Figure A. 1: BP Critical rpm change with temperature for unweighted fresh water sepiolite mud.

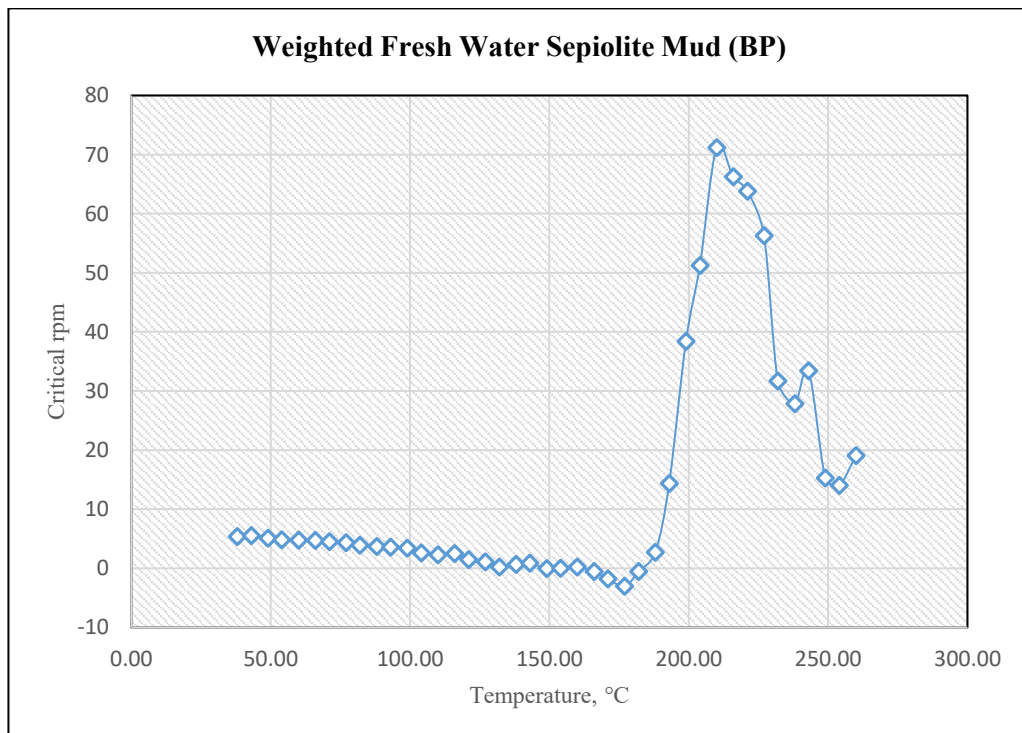


Figure A. 2: BP Critical rpm change with temperature for weighted fresh water sepiolite mud.

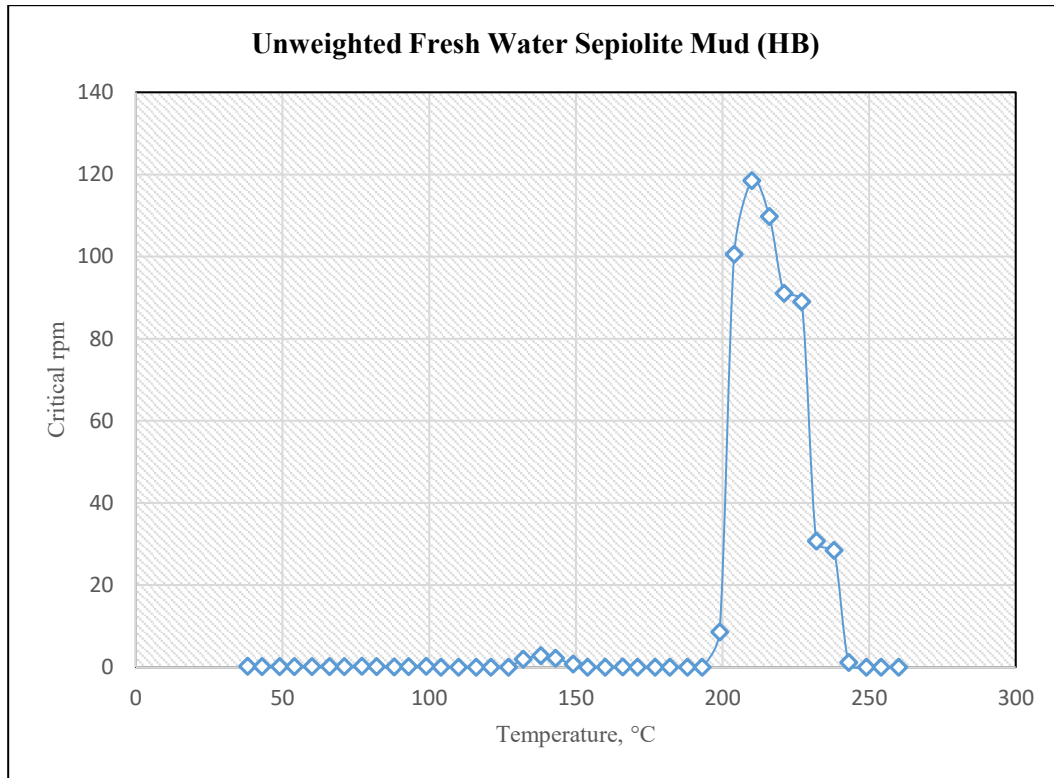


Figure A. 3: HB Critical rpm change with temperature for unweighted fresh water sepiolite mud.

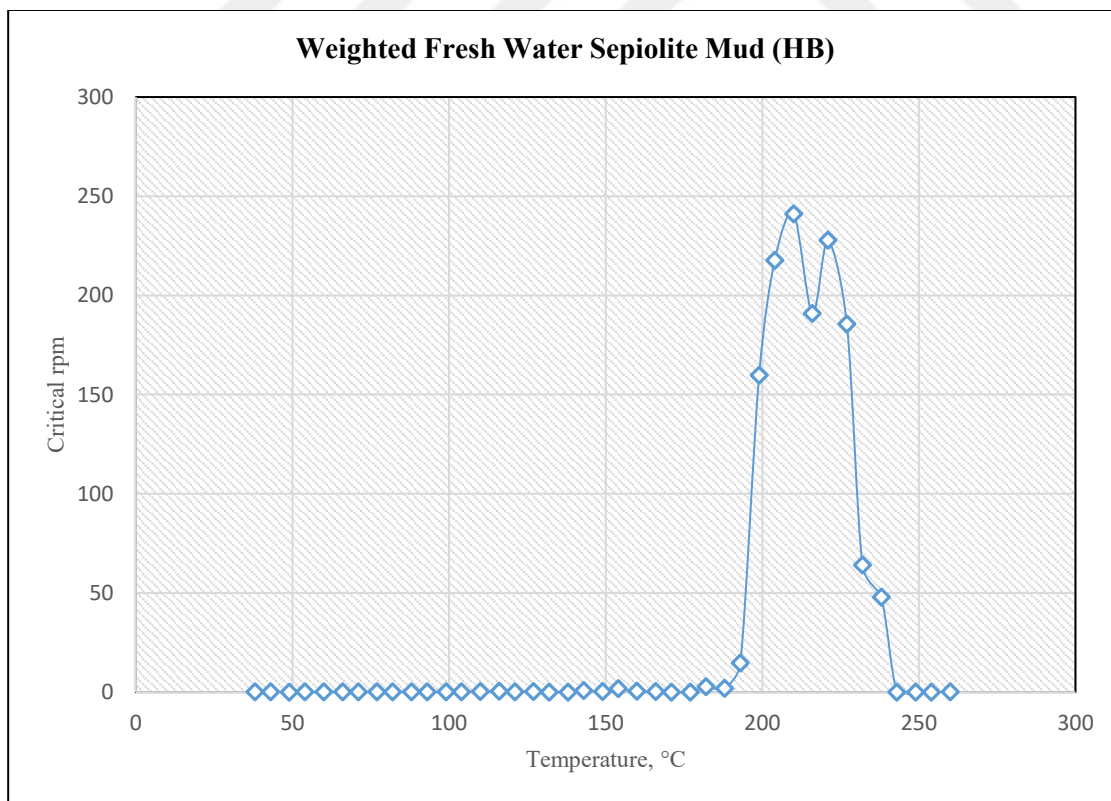


Figure A. 4: HB Critical rpm change with temperature for weighted fresh water sepiolite mud.

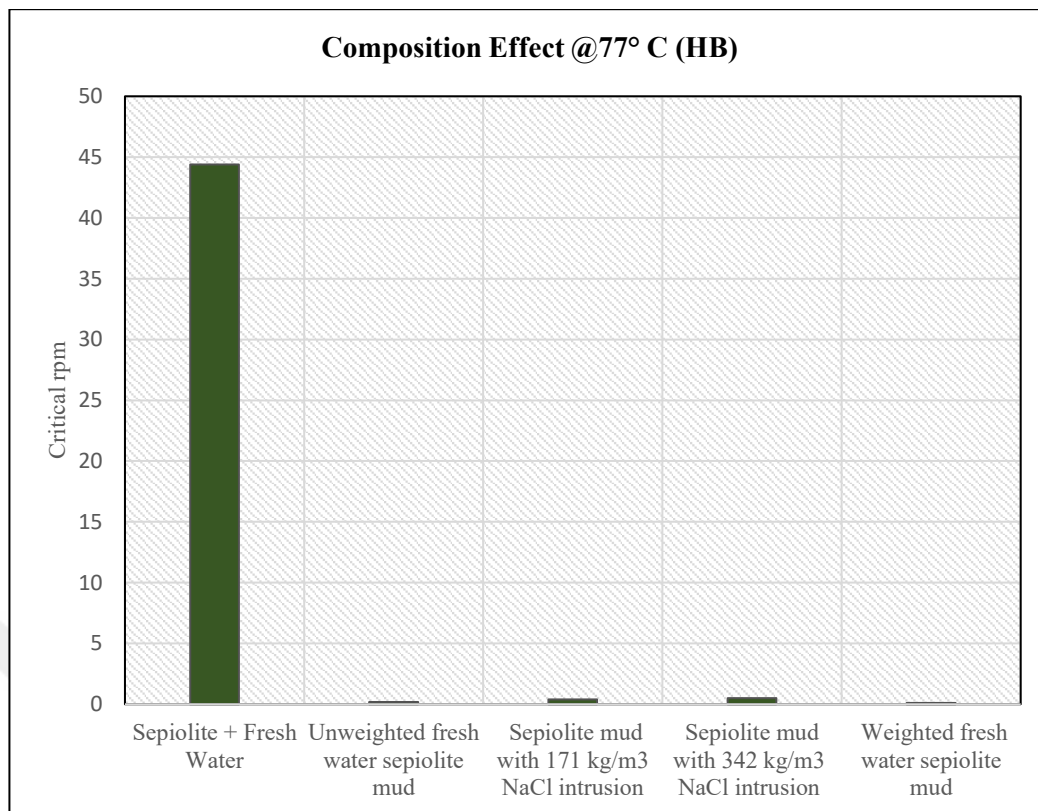


Figure A. 5: HB critical rpm change with mud composition @ 77° C.

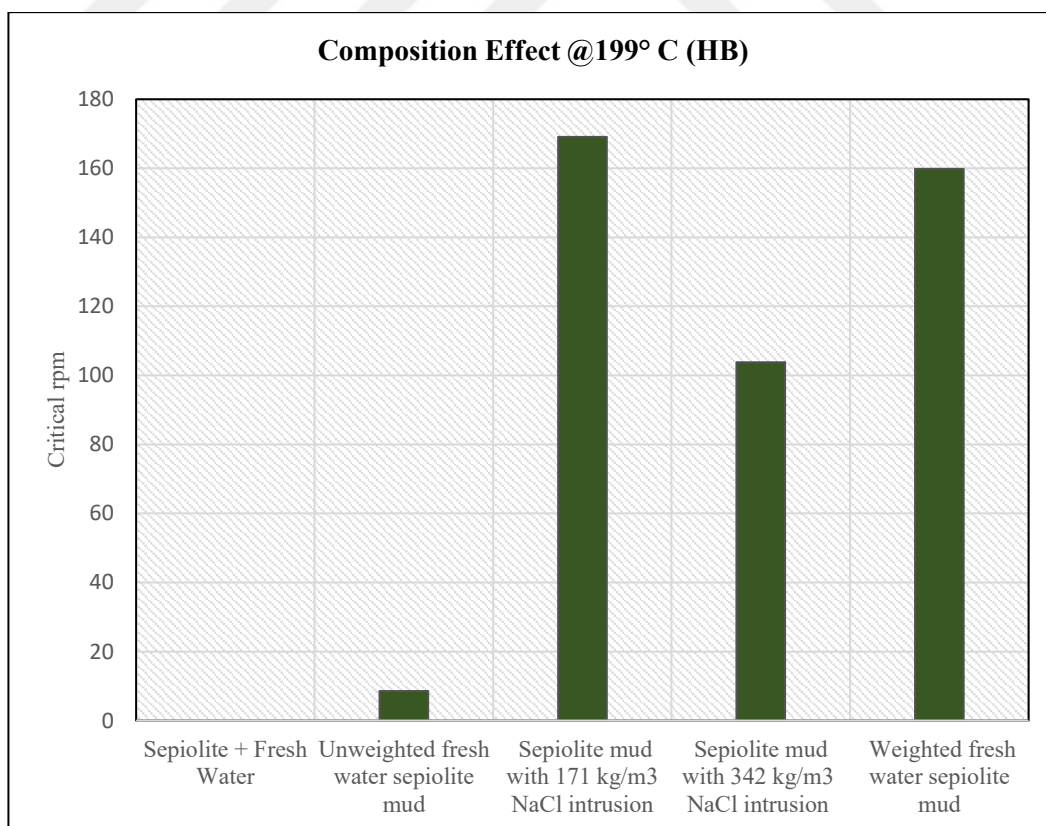


Figure A. 6: HB critical rpm change with mud composition @ 199° C.

Table A. 1: BP and HB Critical rpm and temperature data for sepiolite and water mud.

BP				HB			
$T^\circ, ^\circ\text{C}$	τ_y, Pa	$\mu_p, \text{mPa}\cdot\text{s}$	N_c, rpm	τ_0, Pa	n	$K, \text{Pa}\cdot\text{s}^n$	N_c, rpm
38	8.11	8.90	39.22	7.20	0.65	0.11	6.74
43	6.78	7.11	41.09	6.13	0.67	0.07	8.81
49	5.72	5.77	42.74	5.37	0.75	0.03	15.82
54	5.25	5.09	44.44	5.03	0.81	0.02	22.44
60	4.75	4.49	45.64	4.63	0.87	0.01	29.53
66	4.30	4.01	46.23	4.21	0.89	0.01	32.76
71	3.99	3.54	48.64	3.99	0.99	0.00	47.17
77	3.63	3.22	48.58	3.61	0.97	0.00	44.40
82	3.23	3.17	43.89	3.10	0.82	0.01	23.12
88	3.03	2.93	44.52	2.86	0.77	0.02	17.88
93	2.90	2.70	46.22	2.81	0.85	0.01	27.97
99	2.55	2.75	39.80	2.39	0.76	0.01	15.22
104	2.46	2.62	40.44	2.22	0.66	0.03	8.11
110	2.33	2.60	38.58	2.06	0.64	0.03	6.39
116	2.32	2.45	40.67	2.08	0.66	0.03	8.02
121	2.21	2.46	38.64	1.88	0.58	0.05	3.57
127	2.15	2.37	39.19	1.85	0.59	0.04	4.17
132	2.08	2.37	37.86	1.63	0.49	0.09	0.99
138	2.07	2.34	38.20	1.67	0.52	0.07	1.53
143	2.05	2.35	37.60	1.35	0.38	0.20	0.09
149	2.02	2.36	36.86	1.31	0.38	0.20	0.08
154	1.99	2.43	35.35	0.81	0.28	0.48	0.00
160	1.98	2.47	34.38	0.58	0.25	0.62	0.00
166	2.00	2.56	33.65	0.00	0.20	1.06	0.00
171	1.99	2.61	32.89	0.00	0.20	1.04	0.00
177	2.07	2.71	32.96	0.00	0.20	1.08	0.00
182	2.08	2.78	32.15	0.00	0.21	1.06	0.00
188	2.11	2.83	32.12	0.00	0.21	1.07	0.00
193	2.17	2.87	32.50	0.00	0.21	1.10	0.00
199	2.17	2.89	32.35	0.00	0.21	1.10	0.00
204	2.15	2.90	31.91	0.00	0.21	1.09	0.00
210	2.18	2.87	32.77	0.00	0.20	1.13	0.00
216	2.17	2.90	32.30	0.00	0.21	1.12	0.00
221	2.14	2.81	32.83	0.19	0.22	0.96	0.00
227	2.07	2.78	32.10	0.32	0.23	0.82	0.00
232	2.00	2.72	31.63	0.51	0.26	0.64	0.00
238	2.14	2.42	38.17	0.00	0.23	0.89	0.00
243	2.10	2.38	37.96	0.00	0.23	0.87	0.00
249	2.03	2.36	37.08	0.00	0.23	0.83	0.00
254	1.95	2.43	34.47	0.00	0.25	0.75	0.00
260	1.46	2.58	24.32	0.00	0.26	0.62	0.00

Table A. 2: BP and HB Critical rpm and temperature data for unweighted fresh water sepiolite mud.

$T^\circ, ^\circ\text{C}$	BP			HB			
	τ_y, Pa	$\mu_p, mPa.s$	N_c, rpm	τ_0, Pa	n	$K, Pa.s^n$	N_c, rpm
38	2.75	30.87	3.84	0.96	0.76	0.16	0.20
43	2.55	29.46	3.73	0.72	0.75	0.17	0.12
49	2.33	27.54	3.64	0.71	0.76	0.15	0.15
54	2.16	25.12	3.70	0.60	0.75	0.14	0.12
60	1.83	23.43	3.37	0.54	0.77	0.12	0.14
66	1.75	21.72	3.47	0.52	0.77	0.11	0.14
71	1.56	20.39	3.30	0.43	0.77	0.10	0.13
77	1.95	18.20	4.61	0.51	0.78	0.09	0.20
82	1.33	17.87	3.21	0.34	0.77	0.09	0.12
88	1.14	16.71	2.93	0.23	0.78	0.08	0.08
93	1.21	15.38	3.39	0.33	0.77	0.08	0.13
99	1.12	14.33	3.36	0.38	0.78	0.06	0.19
104	0.60	13.27	1.94	0.00	0.81	0.05	0.00
110	0.50	12.15	1.77	0.00	0.82	0.04	0.00
116	0.52	10.88	2.05	0.00	0.85	0.03	0.00
121	0.44	9.85	1.94	0.00	0.86	0.03	0.00
127	0.27	8.89	1.31	0.00	0.89	0.02	0.00
132	0.25	7.73	1.38	0.31	1.02	0.01	1.92
138	0.24	7.04	1.48	0.36	1.04	0.01	2.80
143	0.04	6.39	0.28	0.22	1.06	0.00	2.17
149	-0.01	5.90	-0.09	0.08	1.04	0.00	0.74
154	-0.15	5.35	-1.20	0.00	1.06	0.00	0.00
160	-0.08	5.06	-0.72	0.00	1.03	0.00	0.00
166	-0.19	4.60	-1.82	0.00	1.10	0.00	0.04
171	-0.26	4.37	-2.58	0.00	1.13	0.00	0.00
177	-0.17	3.95	-1.84	0.00	1.09	0.00	0.00
182	-0.13	3.68	-1.56	0.00	1.08	0.00	0.00
188	-0.12	3.54	-1.47	0.00	1.07	0.00	0.00
193	0.17	3.62	2.03	0.00	0.85	0.01	0.00
199	1.35	3.58	16.25	1.22	0.88	0.01	8.60
204	3.20	4.03	34.20	3.57	1.51	0.00	100.56
210	4.26	5.27	34.78	5.09	1.54	0.00	118.52
216	5.11	5.73	38.38	5.91	1.44	0.00	109.75
221	5.56	5.55	43.17	6.13	1.29	0.00	91.07
227	5.12	5.44	40.56	5.69	1.30	0.00	89.00
232	3.97	5.98	28.59	4.02	1.02	0.01	30.70
238	3.95	6.64	25.62	4.00	1.03	0.01	28.42
243	4.85	8.83	23.66	3.65	0.58	0.17	1.18
249	3.59	12.63	12.23	1.08	0.48	0.50	0.01
254	2.19	9.12	10.35	0.46	0.49	0.33	0.01
260	1.85	6.65	11.98	0.00	0.40	0.49	0.00

Table A. 3: BP and HB Critical rpm and temperature data for sepiolite mud with 171 kg/m³ NaCl intrusion.

$T^\circ, ^\circ\text{C}$	BP			HB			
	τ_y, Pa	$\mu_p, mPa.s$	N_c, rpm	τ_0, Pa	n	$K, Pa.s^n$	N_c, rpm
38	2.05	15.72	5.62	1.30	0.80	0.06	0.93
43	2.07	16.04	5.55	1.15	0.77	0.08	0.59
49	1.74	15.17	4.93	0.82	0.76	0.08	0.37
54	1.75	12.50	6.02	0.97	0.77	0.07	0.61
60	1.64	13.30	5.30	0.83	0.76	0.07	0.45
66	1.67	12.48	5.74	0.86	0.75	0.07	0.47
71	1.49	11.96	5.35	0.74	0.75	0.07	0.42
77	1.42	11.40	5.35	0.69	0.75	0.07	0.39
82	1.45	10.77	5.79	0.76	0.75	0.06	0.49
88	1.38	10.35	5.73	0.67	0.73	0.07	0.37
93	1.48	9.79	6.51	0.80	0.73	0.07	0.49
99	1.63	9.24	7.59	1.09	0.76	0.05	1.10
104	1.44	8.91	6.98	0.87	0.74	0.05	0.72
110	1.37	8.80	6.72	0.76	0.73	0.06	0.53
116	1.53	8.38	7.87	0.91	0.72	0.06	0.64
121	1.64	8.17	8.67	1.08	0.73	0.05	0.96
127	1.88	7.80	10.38	1.06	0.72	0.06	0.85
132	2.12	7.57	12.04	1.31	0.71	0.06	1.13
138	2.08	7.82	11.45	1.35	0.74	0.05	1.45
143	2.79	6.99	17.20	2.03	0.71	0.06	2.30
149	2.91	7.12	17.60	2.17	0.72	0.05	2.65
154	3.71	6.42	24.90	3.20	0.77	0.03	7.37
160	3.66	6.53	24.14	3.33	0.84	0.02	11.21
166	4.90	5.46	38.64	4.83	0.95	0.01	32.65
171	4.65	5.55	36.10	4.70	1.03	0.00	39.68
177	6.04	4.62	56.30	6.23	1.18	0.00	84.37
182	6.27	4.46	60.55	6.50	1.24	0.00	100.52
188	6.99	4.07	73.89	7.31	1.40	0.00	143.89
193	7.40	4.08	77.99	7.69	1.36	0.00	143.12
199	8.49	3.86	94.66	8.79	1.39	0.00	169.10
204	8.47	4.47	81.64	8.73	1.26	0.00	130.54
210	9.33	4.86	82.60	9.24	0.92	0.01	68.34
216	8.64	6.05	61.46	8.42	0.84	0.02	37.90
221	7.60	7.20	45.44	7.19	0.77	0.04	18.93
227	6.30	7.29	37.23	5.48	0.70	0.06	8.26
232	4.61	7.30	27.22	3.89	0.65	0.08	3.64
238	3.54	6.88	22.19	2.84	0.65	0.08	2.41
243	2.76	6.65	17.91	2.05	0.64	0.09	1.36
249	2.58	6.26	17.79	1.92	0.64	0.08	1.38
254	2.80	5.79	20.86	2.27	0.67	0.06	2.66
260	3.11	5.59	23.96	2.66	0.70	0.05	4.43

Table A. 4: BP and HB Critical rpm and temperature data for sepiolite mud with 342 kg/m³ NaCl intrusion.

$T^{\circ}, ^{\circ}\text{C}$	BP			HB			
	τ_y, Pa	$\mu_p, mPa.s$	N_c, rpm	τ_0, Pa	n	$K, Pa.s^n$	N_c, rpm
38	1.26	14.94	3.64	0.72	0.84	0.05	0.70
43	1.09	15.06	3.13	0.54	0.84	0.05	0.47
49	1.20	14.02	3.69	0.72	0.85	0.04	0.78
54	0.89	13.28	2.88	0.37	0.83	0.04	0.33
60	0.88	12.50	3.02	0.48	0.86	0.03	0.61
66	0.81	11.89	2.95	0.34	0.83	0.04	0.33
71	0.74	11.32	2.82	0.31	0.83	0.04	0.32
77	0.89	10.64	3.60	0.45	0.82	0.04	0.50
82	1.09	10.01	4.68	0.76	0.85	0.03	1.28
88	0.71	10.04	3.02	0.25	0.81	0.04	0.22
93	0.85	9.60	3.80	0.41	0.80	0.04	0.43
99	1.07	9.15	5.05	0.66	0.81	0.04	0.86
104	0.99	8.85	4.79	0.54	0.79	0.04	0.60
110	1.05	8.64	5.25	0.65	0.80	0.03	0.86
116	0.86	8.58	4.34	0.36	0.76	0.05	0.28
121	1.00	8.31	5.17	0.54	0.77	0.04	0.56
127	1.00	8.16	5.28	0.55	0.77	0.04	0.58
132	1.05	7.98	5.68	0.60	0.77	0.04	0.63
138	1.30	7.86	7.10	0.73	0.79	0.04	0.96
143	1.57	7.30	9.25	0.86	0.73	0.05	0.82
149	1.45	7.66	8.16	0.82	0.76	0.04	0.94
154	1.83	6.91	11.40	1.21	0.75	0.04	1.60
160	1.64	7.42	9.52	1.05	0.77	0.04	1.45
166	2.27	6.36	15.38	1.79	0.78	0.03	3.77
171	2.12	6.70	13.66	1.78	0.84	0.02	5.11
177	2.75	5.78	20.47	2.50	0.86	0.02	10.31
182	3.00	5.53	23.38	2.92	0.95	0.01	18.75
188	3.73	4.86	33.06	3.77	1.03	0.00	36.28
193	4.38	4.42	42.62	4.62	1.25	0.00	78.32
199	5.05	4.08	53.26	5.32	1.32	0.00	103.81
204	5.84	3.69	68.21	6.18	1.48	0.00	149.98
210	5.75	4.13	59.94	5.90	1.15	0.00	84.47
216	5.46	4.40	53.39	5.38	0.94	0.01	44.48
221	5.13	4.72	46.75	4.81	0.80	0.02	21.39
227	4.19	5.57	32.41	3.85	0.75	0.03	10.47
232	3.37	5.78	25.09	2.89	0.70	0.05	4.67
238	2.47	6.18	17.23	1.78	0.63	0.09	1.09
243	1.83	6.19	12.71	1.28	0.68	0.06	1.15
249	1.81	5.70	13.69	1.38	0.71	0.04	1.94
254	1.92	5.48	15.11	1.42	0.67	0.06	1.43
260	1.81	5.70	13.68	1.38	0.71	0.04	1.94

Table A. 5: BP and HB Critical rpm and temperature data for weighted fresh water sepiolite mud.

$T^\circ, ^\circ\text{C}$	BP			HB			
	τ_y, Pa	$\mu_p, mPa.s$	N_c, rpm	τ_0, Pa	n	$K, Pa.s^n$	N_c, rpm
38	5.54	44.56	5.35	1.67	0.68	0.41	0.10
43	5.06	39.58	5.51	1.44	0.67	0.39	0.08
49	4.24	36.01	5.07	0.90	0.67	0.37	0.04
54	3.71	33.26	4.80	0.81	0.68	0.31	0.05
60	3.38	30.62	4.76	0.71	0.68	0.28	0.05
66	3.10	28.36	4.71	0.86	0.70	0.23	0.09
71	2.75	26.21	4.52	0.74	0.71	0.20	0.09
77	2.44	24.19	4.35	0.68	0.72	0.17	0.10
82	2.05	22.65	3.90	0.42	0.72	0.16	0.06
88	1.79	20.90	3.68	0.42	0.74	0.13	0.08
93	1.61	19.21	3.60	0.42	0.75	0.11	0.11
99	1.40	17.47	3.44	0.46	0.78	0.08	0.18
104	0.97	16.01	2.61	0.28	0.81	0.06	0.16
110	0.77	14.52	2.29	0.32	0.86	0.04	0.32
116	0.75	13.10	2.47	0.39	0.87	0.03	0.51
121	0.41	12.11	1.45	0.11	0.89	0.03	0.15
127	0.28	10.78	1.11	0.13	0.93	0.02	0.32
132	0.05	9.58	0.22	0.00	0.98	0.01	0.00
138	0.13	8.95	0.63	0.00	0.94	0.01	0.00
143	0.15	7.57	0.86	0.14	0.99	0.01	0.76
149	0.00	7.02	-0.02	0.04	1.02	0.01	0.26
154	0.00	6.19	0.03	0.14	1.09	0.00	1.74
160	0.03	6.00	0.22	0.06	1.02	0.01	0.49
166	-0.07	5.28	-0.55	0.02	1.06	0.00	0.24
171	-0.22	5.26	-1.80	0.00	1.10	0.00	0.00
177	-0.34	4.82	-3.05	0.00	1.17	0.00	0.00
182	-0.06	4.50	-0.56	0.15	1.11	0.00	2.67
188	0.27	4.35	2.71	0.23	0.96	0.01	1.83
193	1.46	4.36	14.37	1.46	1.00	0.00	14.61
199	3.97	4.45	38.40	4.82	1.77	0.00	159.81
204	6.10	5.13	51.22	7.22	2.00	0.00	217.73
210	8.17	4.94	71.14	9.22	1.93	0.00	241.09
216	8.02	5.22	66.23	8.95	1.66	0.00	190.94
221	7.83	5.28	63.83	8.95	1.93	0.00	227.78
227	7.16	5.48	56.28	8.18	1.72	0.00	185.59
232	5.10	6.93	31.69	5.69	1.23	0.00	64.02
238	4.86	7.52	27.83	5.17	1.18	0.00	47.95
243	7.61	9.81	33.41	1.66	0.24	2.74	0.00
249	6.19	17.47	15.26	0.71	0.37	1.59	0.00
254	4.55	13.97	14.02	1.69	0.48	0.58	0.02
260	4.61	10.42	19.06	2.04	0.51	0.35	0.10

APPENDIX B

Table B. 1: Pipe frictional pressure loss results utilizing non-corrected BP model parameters @2460 lt/min.

Section	$\bar{v}$ pipe, ft/s	N_{He} (Pipe)	N_{Rec} (Pipe)	N_{Re} (Pipe)	Pipe Flow Regime	Pressure loss (Pipe), psi
DP	14.49	186989.31	8500	34437.34	Turbulent	184.83
HWDP	29.50	91869.59	6800	49130.60	Turbulent	164.25
HWDP	29.50	91869.59	6800	49130.60	Turbulent	102.37
DC	29.50	91869.59	6800	49130.60	Turbulent	136.23
DC	29.50	91869.59	6800	49130.60	Turbulent	149.85
Stand Pipe	16.59	163323.71	8200	36847.95	Turbulent	9.93
Rotary Hose	21.67	125044.71	7300	42111.94	Turbulent	18.72
Swivel	21.67	125044.71	7300	42111.94	Turbulent	3.28
Kelly	21.67	125044.71	7300	42111.94	Turbulent	11.23

Table B. 2: Annulus frictional pressure loss results utilizing non-corrected BP model parameters @2460 lt/min.

Section	$\bar{v}$ ann, ft/s	N_{He} (Annulus)	N_{Rec} (Annulus)	N_{Re} (Annulus)	Annular Flow Regime	Pressure Loss (Annulus), psi
DP	0.79	1332012.69	18200	5009.68	Laminar	4.71
HWDP	0.79	1332012.69	18200	5009.68	Laminar	0.77
HWDP	0.94	1061872.36	17100	5343.66	Laminar	0.54
DC	1.18	491009.78	12000	4537.07	Laminar	1.09
DC	1.48	265468.09	9000	4181.99	Laminar	1.70

Table B. 3: Pipe frictional pressure loss results utilizing corrected BP model parameters @2460 lt/min.

Section	$\bar{v}$ pipe, ft/s	N_{He} (Pipe)	N_{Rec} (Pipe)	N_{Re} (Pipe)	Pipe Flow Regime	Pressure loss (Pipe), psi
DP	14.49	491332.88	13200	43296.65	Turbulent	174.55
HWDP	29.50	241396.40	9300	61769.89	Turbulent	155.12
HWDP	29.50	241396.40	9300	61769.89	Turbulent	96.67
DC	29.50	241396.40	9300	61769.89	Turbulent	128.65
DC	29.50	241396.40	9300	61769.89	Turbulent	141.52
Stand Pipe	16.59	429149.16	12900	46327.42	Turbulent	9.37
Rotary Hose	21.67	328567.33	12500	52945.62	Turbulent	17.67
Swivel	21.67	328567.33	12500	52945.62	Turbulent	3.09
Kelly	21.67	328567.33	12500	52945.62	Turbulent	10.60

Table B. 4: Annulus frictional pressure loss results utilizing corrected BP model parameters @2460 lt/min.

Section	$\bar{v}$ ann, ft/s	N_{He} (Annulus)	N_{Rec} (Annulus)	N_{Re} (Annulus)	Annular Flow Regime	Pressure Loss (Annulus), psi
DP	0.79	3499994.83	23200	6298.47	Laminar	7.73
HWDP	0.79	3499994.83	23200	6298.47	Laminar	1.27
HWDP	0.94	2790174.45	20800	6718.36	Laminar	0.89
DC	1.18	1290176.67	18000	5704.27	Laminar	1.76
DC	1.48	697543.61	15500	5257.85	Laminar	2.69

Table B. 5: Pipe frictional pressure loss results utilizing non-corrected BP model parameters @3270 lt/min.

Section	$\bar{v}$ pipe, ft/s	N_{He} (Pipe)	N_{Rec} (Pipe)	N_{Re} (Pipe)	Pipe Flow Regime	Pressure loss (Pipe), psi
DP	19.26	186989.31	8500	45776.46	Turbulent	304.15
HWDP	39.21	91869.59	6800	65307.75	Turbulent	270.29
HWDP	39.21	91869.59	6800	65307.75	Turbulent	168.45
DC	39.21	91869.59	6800	65307.75	Turbulent	224.18
DC	39.21	91869.59	6800	65307.75	Turbulent	246.60
Stand Pipe	22.05	163323.71	8200	48980.81	Turbulent	16.33
Rotary Hose	28.81	125044.71	7300	55978.07	Turbulent	30.80
Swivel	28.81	125044.71	7300	55978.07	Turbulent	5.39
Kelly	28.81	125044.71	7300	55978.07	Turbulent	18.48

Table B. 6: Annulus frictional pressure loss results utilizing non-corrected BP model parameters @3270 lt/min.

Section	$\bar{v}$ ann, ft/s	N_{He} (Annulus)	N_{Rec} (Annulus)	N_{Re} (Annulus)	Annular Flow Regime	Pressure Loss (Annulus), psi
DP	1.05	1332012.69	18200	6659.21	Laminar	4.74
HWDP	1.05	1332012.69	18200	6659.21	Laminar	0.78
HWDP	1.25	1061872.36	17100	7103.16	Laminar	0.55
DC	1.57	491009.78	12000	6030.98	Laminar	1.11
DC	1.96	265468.09	9000	5558.99	Laminar	1.75

Table B. 7: Pipe frictional pressure loss results utilizing corrected BP model parameters @3270 lt/min.

Section	$\bar{v}$ pipe, ft/s	N_{He} (Pipe)	N_{Rec} (Pipe)	N_{Re} (Pipe)	Pipe Flow Regime	Pressure loss (Pipe), psi
DP	19.26	491332.88	13200	57552.86	Turbulent	287.23
HWDP	39.21	241396.40	9300	82108.75	Turbulent	255.26
HWDP	39.21	241396.40	9300	82108.75	Turbulent	159.08
DC	39.21	241396.40	9300	82108.75	Turbulent	211.71
DC	39.21	241396.40	9300	82108.75	Turbulent	232.88
Stand Pipe	22.05	429149.16	12900	61581.56	Turbulent	15.42
Rotary Hose	28.81	328567.33	12500	70378.93	Turbulent	29.09
Swivel	28.81	328567.33	12500	70378.93	Turbulent	5.09
Kelly	28.81	328567.33	12500	70378.93	Turbulent	17.45

Table B. 8: Annulus frictional pressure loss results utilizing corrected BP model parameters @3270 lt/min.

Section	$\bar{v}$ ann, ft/s	N_{He} (Annulus)	N_{Rec} (Annulus)	N_{Re} (Annulus)	Annular Flow Regime	Pressure Loss (Annulus), psi
DP	1.05	3499994.83	23200	8372.35	Laminar	7.76
HWDP	1.05	3499994.83	23200	8372.35	Laminar	1.27
HWDP	1.25	2790174.45	20800	8930.51	Laminar	0.89
DC	1.57	1290176.67	18000	7582.51	Laminar	1.78
DC	1.96	697543.61	15500	6989.09	Laminar	2.73

Table B. 9: Pipe frictional pressure loss results utilizing non-corrected HB model parameters @1640 lt/min.

Section	v_p , m/s	R_{eq} (Pipe)	R_{eqcr} (Pipe)	Pipe Flow Regime	Turbulent Pressure Loss, Pa
DP	2.94	3672.00	1346.98	Turbulent	920808.40
HWDP	5.99	10315.84	1346.98	Turbulent	686038.01
HWDP	5.99	10315.84	1346.98	Turbulent	427554.50
DC	5.99	10315.84	1346.98	Turbulent	568988.87
DC	5.99	10315.84	1346.98	Turbulent	625887.76
Stand Pipe	3.37	4531.38	1346.98	Turbulent	47648.61
Rotary Hose	4.40	6737.80	1346.98	Turbulent	83904.60
Swivel	4.40	6737.80	1346.98	Turbulent	14683.31
Kelly	4.40	6737.80	1346.98	Turbulent	50342.76

Table B. 10: Annulus frictional pressure loss results utilizing non-corrected HB model parameters @1640 lt/min.

Section	$\bar{v}_{ann}$, ft/s	R_{eq} (Annulus)	R_{eqcr} (Annulus)	Annular Flow Regime	Annular Flow Regime
DP	0.16	24.53	2353.02	Laminar	89981.17
HWDP	0.16	24.53	2353.02	Laminar	14786.63
HWDP	0.19	34.72	2353.02	Laminar	10405.72
DC	0.24	52.80	2353.02	Laminar	20877.61
DC	0.30	80.18	2353.02	Laminar	32321.29

Table B. 11: Pipe frictional pressure loss results utilizing corrected HB model parameters @1640 lt/min.

Section	v_p , m/s	R_{eq} (Pipe)	R_{eqcr} (Pipe)	Pipe Flow Regime	Turbulent Pressure Loss, Pa
DP	2.94	3749.65	1496.48	Turbulent	864602.75
HWDP	5.99	10469.42	1496.48	Turbulent	641390.23
HWDP	5.99	10469.42	1496.48	Turbulent	399728.98
DC	5.99	10469.42	1496.48	Turbulent	531958.72
DC	5.99	10469.42	1496.48	Turbulent	585154.60
Stand Pipe	3.37	4613.61	1496.48	Turbulent	44720.73
Rotary Hose	4.40	6835.76	1496.48	Turbulent	78643.06
Swivel	4.40	6835.76	1496.48	Turbulent	13762.54
Kelly	4.40	6835.76	1496.48	Turbulent	47185.84

Table B. 12: Annulus frictional pressure loss results utilizing corrected HB model parameters @1640 lt/min.

Section	$\bar{v}_{ann}$, ft/s	R_{eq} (Annulus)	R_{eqcr} (Annulus)	Annular Flow Regime	Annular Flow Regime
DP	0.16	27.13	2654.81	Laminar	84174.78
HWDP	0.16	27.13	2654.81	Laminar	13832.46
HWDP	0.19	38.25	2654.81	Laminar	9774.21
DC	0.24	57.55	2654.81	Laminar	19819.43
DC	0.30	86.40	2654.81	Laminar	31036.03

Table B. 13: Pipe frictional pressure loss results utilizing non-corrected HB model parameters @2460 lt/min.

Section	v_p , m/s	R_{eq} (Pipe)	R_{eqcr} (Pipe)	Pipe Flow Regime	Turbulent Pressure Loss, Pa
DP	2.94	7278.88	1346.98	Turbulent	1738095.69
HWDP	5.99	19067.90	1346.98	Turbulent	1318398.27
HWDP	5.99	19067.90	1346.98	Turbulent	821655.79
DC	5.99	19067.90	1346.98	Turbulent	1093458.28
DC	5.99	19067.90	1346.98	Turbulent	1202804.11
Stand Pipe	3.37	8870.33	1346.98	Turbulent	90230.71
Rotary Hose	4.40	12850.94	1346.98	Turbulent	159951.56
Swivel	4.40	12850.94	1346.98	Turbulent	27991.52
Kelly	4.40	12850.94	1346.98	Turbulent	95970.93

Table B. 14: Annulus frictional pressure loss results utilizing non-corrected HB model parameters @2460 lt/min.

Section	$\bar{v}_{ann}$, ft/s	R_{eq} (Annulus)	R_{eqcr} (Annulus)	Annular Flow Regime	Annular Flow Regime
DP	0.16	54.53	2353.02	Laminar	91061.96
HWDP	0.16	54.53	2353.02	Laminar	14964.23
HWDP	0.19	76.95	2353.02	Laminar	10565.14
DC	0.24	115.91	2353.02	Laminar	21400.11
DC	0.30	173.89	2353.02	Laminar	33533.47

Table B. 15: Pipe frictional pressure loss results utilizing corrected HB model parameters @2460 lt/min.

Section	v_p , m/s	R_{eq} (Pipe)	R_{eqcr} (Pipe)	Pipe Flow Regime	Turbulent Pressure Loss, Pa
DP	2.94	7394.71	1496.48	Turbulent	1627802.25
HWDP	5.99	19447.38	1496.48	Turbulent	1226672.16
HWDP	5.99	19447.38	1496.48	Turbulent	764489.99
DC	5.99	19447.38	1496.48	Turbulent	1017382.12
DC	5.99	19447.38	1496.48	Turbulent	1119120.33
Stand Pipe	3.37	8999.09	1496.48	Turbulent	84439.65
Rotary Hose	4.40	13037.96	1496.48	Turbulent	149366.34
Swivel	4.40	13037.96	1496.48	Turbulent	26139.11
Kelly	4.40	13037.96	1496.48	Turbulent	89619.80

Table B. 16: Annulus frictional pressure loss results utilizing corrected HB model parameters @2460 lt/min.

Section	$\bar{v}_{ann}$, ft/s	R_{eq} (Annulus)	R_{eqcr} (Annulus)	Annular Flow Regime	Annular Flow Regime
DP	0.16	59.96	2654.81	Laminar	85687.75
HWDP	0.16	59.96	2654.81	Laminar	14081.09
HWDP	0.19	84.19	2654.81	Laminar	9991.26
DC	0.24	125.25	2654.81	Laminar	20489.67
DC	0.30	185.58	2654.81	Laminar	32510.51

Table B. 17: Pipe frictional pressure loss results utilizing non-corrected HB model parameters @3270 lt/min.

Section	v_p , m/s	R_{eq} (Pipe)	R_{eqcr} (Pipe)	Pipe Flow Regime	Turbulent Pressure Loss, Pa
DP	2.94	11590.15	1346.98	Turbulent	2725471.02
HWDP	5.99	28866.84	1346.98	Turbulent	2094321.00
HWDP	5.99	28866.84	1346.98	Turbulent	1305228.49
DC	5.99	28866.84	1346.98	Turbulent	1736996.09
DC	5.99	28866.84	1346.98	Turbulent	1910695.70
Stand Pipe	3.37	13993.27	1346.98	Turbulent	141827.52
Rotary Hose	4.40	19891.83	1346.98	Turbulent	252644.40
Swivel	4.40	19891.83	1346.98	Turbulent	44212.77
Kelly	4.40	19891.83	1346.98	Turbulent	151586.64

Table B. 18: Annulus frictional pressure loss results utilizing non-corrected HB model parameters @3270 lt/min.

Section	$\bar{v}_{ann}$, ft/s	R_{eq} (Annulus)	R_{eqcr} (Annulus)	Annular Flow Regime	Annular Flow Regime
DP	0.16	95.29	2353.02	Laminar	92081.27
HWDP	0.16	95.29	2353.02	Laminar	15131.74
HWDP	0.19	134.06	2353.02	Laminar	10715.03
DC	0.24	200.25	2353.02	Laminar	21887.08
DC	0.30	297.34	2353.02	Laminar	34651.53

Table B. 19: Pipe frictional pressure loss results utilizing corrected HB model parameters @3270 lt/min.

Section	v_p , m/s	R_{eq} (Pipe)	R_{eqcr} (Pipe)	Pipe Flow Regime	Turbulent Pressure Loss, Pa
DP	2.94	11754.85	1496.48	Turbulent	2546839.55
HWDP	5.99	29628.30	1496.48	Turbulent	1940753.38
HWDP	5.99	29628.30	1496.48	Turbulent	1209521.66
DC	5.99	29628.30	1496.48	Turbulent	1609629.58
DC	5.99	29628.30	1496.48	Turbulent	1770592.54
Stand Pipe	3.37	14191.06	1496.48	Turbulent	132391.35
Rotary Hose	4.40	20232.32	1496.48	Turbulent	235178.53
Swivel	4.40	20232.32	1496.48	Turbulent	41156.24
Kelly	4.40	20232.32	1496.48	Turbulent	141107.12

Table B. 20: Annulus frictional pressure loss results utilizing corrected HB model parameters @3270 lt/min.

Section	$\bar{v}_{ann}$, ft/s	R_{eq} (Annulus)	R_{eqcr} (Annulus)	Annular Flow Regime	Annular Flow Regime
DP	0.16	104.27	2654.81	Laminar	87069.19
HWDP	0.16	104.27	2654.81	Laminar	14308.10
HWDP	0.19	145.87	2654.81	Laminar	10188.71
DC	0.24	214.98	2654.81	Laminar	21093.51
DC	0.30	315.16	2654.81	Laminar	33826.11

CURRICULUM VITAE



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EDUCATION:

- **B.Sc.** : 2017, Middle East Technical University NCC, Faculty of Engineering, Department of Petroleum and Natural Gas Engineering
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PROFESSIONAL EXPERIENCE AND REWARDS:

- 2018-2019 Business Trainer in Oil Drilling and Artificial Lift at WideScope Academy, Istanbul/Turkey.
- 2016 Internship at Turkish Petroleum Co., Batman/Turkey.
- 2015 Internship at Turkish Petroleum Co., Adiyaman/Turkey.
- High Honor awards for four consecutive semesters at Middle East Technical University NCC.