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THREE ESSAYS ON GAME THEORY

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To my grandmother, my mother and my sister

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It was the longest journey of my life. It is hard to believe that I completed this thesis. Now, it is the time to remember the people who were with me along the way, and to share my gratefulness with them.

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Abstract

This thesis is composed of three papers on Game Theory. In these papers, the three fundamentals I am treating are:

1. The problem of implementation of cooperative solutions,

2. The dependence of the solution of a problem on its modelling,

3. The right concept of dual TU games.

The first chapter is a joint work with my supervisor Prof. Dr. Walter Trockel and was published in Journal of Mechanism and Institution Design, 2016. In this paper, we provide exact non-cooperative foundations first via weakly subgame perfect equilibria of a game which is a modification of Rubinstein's game, and then via subgame perfect equilibria of a game which is a further modification of our first game. Moreover, these two games provide a general rule how to transform approximate support results into exact ones. We discuss the relation of the support results in the related literature, including our present ones, with mechanism theoretic implementation in (weakly) subgame perfect equilibrium of the Nash solution. There, we come to the conclusion that a sound interpretation as an implementation can hardly be found except in very rare cases of extremely restricted domains of players' preferences.

In the second chapter, the purpose is to test experimentally a version of the classical Chain Store Game (CSG), proposed by Trockel (1986), and determine whether one of the two theories of *Induction* and *Deterrence*, which were originally tested competitively by Selten (1978), may better account for the results. With complete and perfect information, the CSG of Selten (1978) was designed to analyze the role of reputation in repeated market interactions. Its results were discussed in two different ways: one is based on backward induction, and the other is intuitively derived from a deterrence argument. As the two explanations are incompatible, alternative models have been proposed to understand them bet-

ter. The alternative game proposed by Trockel is an imperfect information version of the CSG in which the order of the two players is changed in each round and the ”Aggressive-Stay Out” equilibrium is used to build reputation. The existence of more than one equilibrium is the basis for the building of reputation. To the best of our knowledge, this study is the first attempt to experimentally test this alternative game with the same purpose.

The third chapter is a joint work with Fatma Aslan. In the paper, we present a new definition of the duality notion for TU-games, namely *c-duality* that works for superadditive games in the same way, while preserving its essence when it is extended to the class of not necessarily superadditive games. Moreover, we define the *anti-c-dual* of a coalitional TU-game and investigate whether solution concepts such as the Core and the c-Core of anti-c-dual game can be derived from the original game.

Keywords: non-cooperative foundation, Nash program, Chain Store Game, entry deterrence, dual of TU-games.

Özet

Bu tez Oyun Teorisi üzerine üç makaleden oluşmaktadır. Bu makalelerde aşağıdaki üç konu işlenmiştir:

1. İşbirlikçi çözümlerin implementasyon problemi,

2. Bir problemin çözümünün, problemin modellenme şekline bağımlılığı,

3. Transfer edilebilir faydalı oyunların dualinin doğru tanımı.

Birinci bölüm danışmanım Prof. Dr. Walter Trockel ile ortak çalışmamızdır. Journal of Mechanism and Institution Design isimli derginin 2016 sayısında yayımlanmıştır. Bu çalışmada, Rubinstein'ın oyununun versiyonu olan yeni bir oyunun zayıf alt-oyun mükemmel dengeleri aracılığıyla "tam işbirliksiz yapılar" sunulmuştur. Daha sonra, bu oyunun yeni bir versiyonunun altoyun mükemmel dengeleri aracılığıyla "tam işbirliksiz yapılar" önerilmiştir. Hatta bu iki oyun "yaklaşık destek" sonuçlarının "tam desteğe" nasıl dönüştürüleceği hususunda genel bir kural önerirler. Bu çalışmada elde edilen iki sonucu da içine koyarak ilgili literatürdeki destek sonuçlarının, Nash Çözümü'nün (zayıf) altoyun mükemmmel denge koseptindeki mekanizm teorik implementasyonu ile ilişkisi tartışılmıştır. Bu çalışmalar sonucunda şu sonuca varılmıştır: Sağlam bir implementasyon, oyuncuların tercihlerinin oldukça kısıtlandığı nadir durumlar dışında çok zor bulunur.

İkinci bölüm deneysel bir çalışmayı içerir. Tam ve kusursuz bilgili bir oyun olan Zincir Mağaza Oyunu(ZMO), Selten (1978) çalışmasında itibarın pazarlardaki rolünü anlamak amacıyla tasarlanmıştır. Bu oyunun nasıl oynanabileceği iki farklı şekilde tartışılmıştır: biri geriye doğru tümevarıma dayanır, diğeri ise sezgisel tabanlı caydırma argümanına. Bu iki yöntemin arasında farklılıklar sebebiyle, durumu daha iyi anlayabilmek ya da açıklayabilmek için önerilen yeni modeller literatürde mevcuttur. Trockel (1986) çalışmasında önerilen ZMO'daki hamle sırasının değiştirilmesiyle elde edilen kusurlu bilgili alternatif oyun bu

çalışmada kullanılmıştır. Bu oyunda ”Sert-Pazara Girme” strateji kümesi bir dengedir ve bu oyunun itibar yaratmadaki avantajıdır. Bu çalışma, ZMO’nun Trockel (1986) yer alan versiyonunu deneysel yöntemlerle test eden ilk çalışmadır ve Selten (1978)’de bu ünlü oyun için öngörülen Tümevarım ve Caydırma Hipotezleri’nden hangisinin elde edilen deneysel sonuçları daha iyi açıkladığı üzerine yapılan analizleri kapsamaktadır.

Sonuncu bölüm olan üçüncü bölüm Fatma Aslan ile ortak çalışmamızdır. Bu çalışmada, transfer edilebilir faydalı oyunların dual kavramına yeni bir tanım verilmiştir ve yeni tanım ’c-dual’ olarak adlandırılmıştır. C-dual ”süpertoplanır” oyunlar üzerinde dual ile aynı şekilde çalışır ve ”süpertoplanır” olmayan oyunlar üzerine genişletildiğinde de dualin ”süpertoplanır” oyunlar üzerindeki esasını/ ruhunu taşımaya devam eder. Ek olarak, transfer edilebilir faydalı oyunların anti-dual kavramı da anti-c-dual olarak yeniden tanımlanmıştır ve Çekirdek, c-Çekirdek gibi çözüm konseptlerinin anti-c-duallerinin özgün oyundan çıkarılıp çıkarılamayacağı araştırılmıştır.

Anahtar Kelimeler: tam işbirliksiz yapı, Nash programı, Zincir Mağaza Oyunu, girişimden caydırma, TU-oyunlarn duali.

1 On Non-Cooperative Foundation and Implementation of the Nash Solution in Subgame Perfect Equilibrium via Rubinstein's Game

The article starts with a discussion of the history and importance of the problem of bilateral exchange or negotiation. Next, we introduce the alternating offers game due to Rubinstein (1982) and its use by Binmore (1980) and by Binmore et al. (1986) to provide via its unique subgame perfect equilibrium an **approximate** non-cooperative support for the Nash bargaining solution of associated cooperative two-person bargaining games. These results had strengthened the prominent role of the Nash bargaining solution in cooperative axiomatic bargaining theory and its application, for instance in labor markets, and have often even been interpreted as a mechanism theoretical implementation of the Nash solution.

Our results in the present paper provide **exact** non-cooperative foundations first, in our Proposition, via **weakly** subgame perfect equilibria of a game which is a modification of Rubinstein's game, then in our Theorem, via subgame perfect equilibria of a game which is a further modification of our first game. Moreover, they provide a general rule how to transform approximate support results into exact ones.

Finally, we discuss the relation of the above mentioned support results, including our present ones, with mechanism theoretic implementation in (weakly) subgame perfect equilibrium of the Nash solution. There, we come to the conclusion that a sound interpretation as an implementation can hardly be found except in very rare cases of extremely restricted domains of players' preferences.

1.1 Introduction

The complex title of this article describes precisely its contents and goals, but it hides the enormous importance of the underlying problem of allocating justly the available resources among some population of individual agents.

In its most simple form, it is the problem of fair division of some divisible object among two persons. Here ”fair” is a non-technical term whose formal specification depends on the situation and on potential property rights of the negotiating persons. Accordingly, either distribution or exchange may best describe the activity to be analyzed.

Although this fundamental problem of bilateral negotiation is still at the heart of economics and game theory, it has a very long history. This is competently and transparently described by Dos Santos Ferreira (2002) which, referring to Stuart (1892) and Burnet (1900), traces back modern treatments of bilateral exchange and bargaining to Aristotle's (ca.335 B.C.) *Nichomachean Ethics*. He convincingly argues that underlying ideas about proportionality, arithmetic and geometric means of modern axiomatic bargaining solutions are appearing already in Aristotle's analysis.

Dos Santos Ferreira (2002, p.568) considers ”the Nichomachean Ethics in which Aristotle presents his analysis of bilateral exchange” as ”undoubtedly one of the most influential writings in the whole history of economic thought” that ”through the commentaries of Albertus Magnus and ... of his pupil Thomas Aquinas was one of the main sources of the Scholastic doctrine of just prices”. He then follows this influence via Turgot (1766, 1769), Marx (1867), Menger (1871) and Edgeworth (1881) to the modern treatments, in particular the seminal contributions by John F. Nash (1950, 1953) and Rubinstein (1982) underlying our present analysis. Shubik (1985) mentions Böhm-Bawerk (1891) horse market model, which had become the forerunner of assignment games, as another 19th century work concerned with bilateral exchange. The contract curve offered by Edgeworth and the price interval of Böhm - Bawerk reflect an indeterminacy of those early approaches that was only solved by Zeuthen (1930) and Hicks (1932).

Harsanyi (1956) compared the modellings of bilateral bargaining before and after the appearance of the theory of games [cf. Bishop (1963)] and found

Zeuthen's approach, that he presented in the language of game theory, superior to that of Hicks and proved that Zeuthen's solution coincides with the later by Nash (1950,1953) defined and axiomatically characterized Nash bargaining solution.

The first contributions to an analysis of bilateral bargaining via strategic non-cooperative games were independently presented by Ståhl (1972) and Krelle (1976), who provided a model of finite horizon alternate offers consecutive bargaining. That model was extended to infinite horizon sequential bargaining with discounting payoffs in the seminal article by Rubinstein (1982).

While cooperative axiomatic and non-cooperative strategic game theoretic models are based on quite different implicit assumptions about legal and institutional environments, the interesting question arose as to the relation between the solutions of the bargaining problem offered by either approach. That question belongs to what, based on some short passages in Nash's work, has been termed *Nash Program* in the literature [cf. Binmore and Dasgupta (1987)]. According to Reinhard Selten [private communication], it had been Robert Aumann who first had used that expression in some lecture.

A first contribution to the Nash program had been provided by Nash (1953) himself when he compared the Nash solution with the payoffs of his so called *simple demand game*. The continuum of Nash equilibria of this game, however, cannot be used as a support for the Nash solution, that corresponds to just one of them. Therefore Nash used a sequence of increasingly less distorted smooth games that converges to the simple demand game and whose sets of infinitely many Nash equilibria converges to the unique (in the words of van Damme (1991)) *essential Nash equilibrium* of the simple demand game with the payoffs of the Nash solution. This provided the first *approximate* non-cooperative support of the Nash solution. While Nash's underlying analysis for this result is vague and incomplete later contributions, among them van Damme (1991), rendered more precise arguments.

As to an *exact* as opposed to an only *approximate* support Binmore and Dasgupta (1987) close their article with the following passage:

*"Finally, it is necessary to comment on the fact that none of of the non-cooperative bargaining models which have been studied implement the Nash bargaining solution exactly. In each case, the implementation is **approximate** (or exact only in the limit).*

Notice that the "implementation" here is meant as a non-technical alternative for *support* or *foundation* and needs to be distinguished from the more challenging mechanism theoretic technical term *implementation*. **Both**, *exact support* and *exact implementation* of the Nash solution will be analyzed in this paper based on a modification of Rubinstein's game.

In contradiction to the above quotation of Binmore and Dasgupta (1987), the first *exact* non-cooperative support for the Nash solution to our knowledge has been provided already by van Damme (1986) via a unique Nash equilibrium payoff vector of a Meta-Bargaining game with specific subsets of the set of bargaining solutions being the players' strategy sets. With slight modifications of the sets of solutions admissible as strategies Naeve-Steinweg (2002) proved an analogous result for the Kalai-Smorodinsky solution. Admitting all bargaining solutions as strategies Trockel (2002) confirmed the support of the Nash solution. In quite a different approach, using a Walrasian modification of Nash's simple demand game, Trockel (2000) proved a unique Nash equilibrium support for the Nash solution.

An exact support of the Nash solution by unique subgame - perfect equilibrium payoff vectors is due to Howard (1992).

The relation between the Nash solution and the Rubinstein game was finally clarified in a seminal article by Binmore, Rubinstein and Wolinsky (1986), based on an earlier article by Binmore (1980) published in Binmore and Dasgupta (1987). For two different two-person cooperative bargaining games generated via different types of utility functions (*time discounting versus risk averse von-Neumann-Morgenstern*) imposed on the basic dynamic model of Rubinstein (1982) they provide approximations of the two respective Nash solution payoff vectors by the two unique subgame-perfect equilibrium outcomes.

In order to proceed from the Nash program aspect of the Nash solution support by non-cooperative equilibria to the mechanism theoretic implementation of the Nash solution in some equilibrium concept, one needs to clarify the relation between the Nash program and implementation of solutions of cooperative games (rather than just of social choice rules)! Strictly speaking, a solution can possibly *implemented* in some equilibrium (concept) only if it can be identified (which already implies a *restricted domain*) with some social choice rule, and in the special context of bargaining games with their traditional point- rather than set-valued solutions with some *social choice function*.

As already remarked above, the Nash program has its roots in some passages of Nash's work. It is discussed in great detail by Serrano (2004) who writes in the in the introduction:

"Similar to the microfoundations of the macroeconomics, which aim to bring closer the two branches of economic theory, the Nash program is an attempt to bridge the gap between the two counterparts of game theory (cooperative and non-cooperative). This is accomplished by investigating non-cooperative procedures that yield cooperative solutions as their equilibrium outcomes."

He then quotes the following passage from Harsanyi (1974):

"Nash (1953) has suggested that we can obtain a clear understanding of the alternative solution concepts proposed for cooperative games and can better identify and evaluate the assumptions to make about the players' bargaining behavior if we reconstruct them as equilibrium points in suitably defined bargaining games, treating the latter formally as non-cooperative games."

The relation between implementation theory and the Nash program has been extensively analyzed and discussed in Serrano (1997, 2004), Bergin and Duggan (1999), Trockel (2000, 2002a, 2002b, 2003).

As our modification of the Rubinstein game works for the diverse variants of the Rubinstein model with varying generality and complexity, we will work with a particularly simple and transparent special version that allows it to interpret the discount factor in both ways discussed in Binmore et al. (1986), namely as an indicator of either players' impatience or their risk aversion. Furthermore, we shall discuss the impact on implementability of the Nash solution of our exact and, in fact, also the approximate support results in Binmore (1980) and Binmore et al. (1986). This mechanism theoretical aspect is enormously relevant for applications of axiomatic bargaining solutions (for example, in wage bargaining) as discussed, for instance in Binmore et al. (1986) and in Gerber and Upmann (2006).

In section 2, some basic notions of bargaining theory are introduced. Section 3 presents our version of the Rubinstein game. Section 4 with our Proposition on weakly subgame perfect support of the Nash solution is followed by the very short section 5 on the concept of *weakly subgame-perfect equilibrium*. In section 6, we establish by our Theorem a subgame-perfect equilibrium support of the Nash solution. The final section 7 explains the impact of our results on the problem of implementing (a social choice function representing) the Nash solution in (weakly) subgame-perfect equilibrium and concludes.

1.2 Basic Concepts and Notation

We shall use the following two different types of games, namely two-person cooperative bargaining games and two-person non-cooperative games in extensive form, briefly extensive games. The definition of the latter ones is quite intricate though their illustrations via game trees are very intuitive. We shall use this notion as treated in Myerson (1991) (chapter 2) or in Mas-Colell et al. (1995)(chapter 7).

As to cooperative bargaining games, we shall use the following:

Definition 1 *A two-person bargaining game is a pair (U, d) where $d \in U \subset \mathbb{R}^2_+$ and U is non-empty, convex, compact and there exists an $x \in U$ such that $x >> d$. The set of two-person bargaining games is denoted by $\mathbb{B}$.*

Definition 2 *A bargaining solution is a mapping*

$$L : \mathbb{B} \longrightarrow \mathbb{R}^2$$
$$(U, d) \longmapsto L(U, d) \in U$$

If we can associate any $(U, d) \in \mathbb{B}$ with some extensive game $G^{U,d}$ whose subgame perfect equilibrium payoff vectors coincide with $L(U, d)$, then the game $G^{U,d}$ *supports* the solution $L(U, d)$ of (U, d) by subgame perfect equilibrium. Such a support provides *an exact non-cooperative foundation* for the solution L in the sense of the Nash program (cf. Binmore et al. (1986), Serrano (2004)). Exact non-cooperative foundations for the Nash solution have been provided in van Damme (1986) [see also Naeve-Steinweg (1999) for a generalization], Howard (1992), Naeve (1999), Trockel (2000, 2002b).

In the present paper, we want to present an exact non-cooperative foundation for *the Nash Solution* based on the Rubinstein game.

The relevant notion of a subgame perfect Nash equilibrium due to Selten (1965) is defined as a Nash equilibrium on an extensive game which induces a Nash equilibrium on any subgame.

1.3 The Rubinstein Game

The Rubinstein infinite horizon strategic bargaining model with the two players' alternating offers is concerned with how to divide a unit of some perfectly divisible good with a resulting allocation for the two players. This game introduced by Rubinstein (1982) was meant to analyze "what 'will be' the agreed contract, assuming that both parties behave rationally". No link to axiomatic cooperative bargaining or even the Nash solution is indicated or, at least it appears so, intended. Discount factors δ_1, δ_2 are assumed to be fix for both players. Possible consequences for the subgame perfect equilibrium regarding a relation to the Nash solution if δ_1, δ_2 are close to 1 are not an issue.

It was Binmore (1980) who related a dynamic version of Nash's simple demand game that he called "Modified Nash Demand Game II" to the (asymmetric)

Nash solution, by approximating it by unique subgame perfect equilibrium payoff vectors of his strategic games where the discount factors δ_1, δ_2 come close to 1. There are various versions of the original model of Rubinstein (1982) which has finite horizon predecessors in Ståhl (1972) and Krelle (1976). The most general version is used in Osborne and Rubinstein (1994) (chapter 7) where the set of feasible agreements X is a non-empty compact, connected set of some Euclidean space. In Binmore et al. (1986), in the introduction, the set X represents "physical outcomes" building the two players' "possible aggrements". The formal model in their section 2 "strategic bargaining models" specifies this set as $\bar{X} = \{x \in \mathbb{R}^2 | x_1 + x_2 \leq 1\}$. In both cases, inefficient feasible agreements are principally possible. Rubinstein (1982) uses $X = [0, 1]$ where the "split the pie" assumption excludes $x_1 + x_2 < 1$ and implies efficiency of the agreements.

Binmore (1980) implicitly expresses the feasible set of alternatives as the set of payoff vectors in the utility image of some unspecified outcome set and explicitly assumes the set U of feasible payoff vectors to be a non-empty compact, convex set in $\mathbb{R}^2$. In this framework, the feasible proposal pairs (x_1, x_2) of the two players are payoff vectors and thus are directly comparable to the Nash solution point of (U, d). Here $d \in U$ is the disagreement payoff vector of the cooperative bargaining problem.

In the framework of the other approaches based on X, histories without any agreement at any time are mapped by the players' utility functions π_1 and π_2 onto such a $d \in U$, where U is $\pi(X) = \{(\pi_1(x), \pi_2(x)) | x \in X\}$.

As shown and in fact exploited in Binmore et al. (1986), the same underlying X may lead via different sets of players' utility functions to different sets U and accordingly different Nash solution points $N(U, d)$. Binmore et al. (1986) presents two detailed versions of strategic bargaining a la Rubinstein: one with time preferences and impatient players, the other one with exogenous risk of breakdown of negotiations with risk averse players who have von Neumann-Morgenstern utility functions. In both models, the subgame perfect equilibrium payoff vectors converge to the respective Nash solution points of the induced utility possibility set U, where $\delta \in (0,1)$ converges to 1.

We shall use a particularly simple and transparent version of the strategic bargaining that simultaneously allows both interpretations, namely impatience or risk aversion of players as represented by δ as a discount factor as a probability of continuation of the negotiation process. This model is essentially that of Binmore (1980) and of Example 125.1 in Osborne and Rubinstein (1994). That will not affect the validity of our analysis for the more complex versions mentioned above. The interpretation of the discoun factors we choose will have, however, a crucial impact on the application of our results to implementability in the mechanism theoretic sense. A reference for this model is also the collection of sections 6.7.1, 6.7.2 and 10.1.2 in Peters (2015).

Let $X = [0,1]$ represent the pie to be split among the two players. The payoff vector resulting from a division $x = (x_1, x_2)$ with $x_1 + x_2 = 1$ is determined by the utility functions $u_i : X \to \mathbb{R}$ with $u_i(x_i) = x_i$ for all $i = 1, 2$. Discounting utilities with $\delta \in (0,1)$ results in a payoff vector $\delta^t x \in U$ for an agreement $x \in X$ at time $t \in \mathbb{N}_0 := \mathbb{N} \cup \{0\}$. We assume $U = X$ and $d = u(0,0) = 0 \in U \subset \mathbb{R}^2$.

For notational convenience, we shall identify players' proposals $x_1, y_2 \in U$ with the payoff vectors $(x_1, x_2) := (x_1, 1 - x_1)$ and $(1 - y_2, y_2) \in \triangle := \{(z_1, z_2 | z_1 + z_2 = 1)\}$. This identifications of $\triangle$ with U via the two projections on the first and second coordinate, respectively, allows us to speak of proposals in U or in $\triangle$ without creating confusion.

We treat only the symmetric case with the discount factor δ being the same for both players. The extension to the asymmetric case is possible like in the quoted literature and is straightforward.

In contrast to the finite horizon version of Ståhl (1972), in the Rubinstein game backward induction cannot be used for determining subgame perfect equilibria.

In our specific "split the pie" framework, there exists a "unique (not just essentially unique)" subgame perfect equilibrium (cf. Osborne and Rubinstein

(1994)(p.125)). This unique subgame perfect Nash equilibrium σ^*_δ is characterized as follows:

$\sigma^*_{\delta,1}$: At $t \in 2\mathbb{N}_0$, propose $x^*_\delta := (\frac{1}{1+\delta}, \frac{\delta}{1+\delta})$; at $t \in 2\mathbb{N}_0 + 1$, accept any proposal $z \in U$ of player 2

if and only if $z_1 \geq \delta \widetilde{z}_{\delta,1}$.

$\sigma^*_{\delta,2}$: At $t \in 2\mathbb{N}_0 + 1$, propose $y^*_\delta := (\frac{\delta}{1+\delta}, \frac{1}{1+\delta})$; at $t \in 2\mathbb{N}_0$, accept any proposal $z \in U$ of player 1

if and only if $z_2 \geq \delta y^*_{\delta,2}$.

x^*_δ and y^*_δ build the unique solution of the two equations $x_2 = \delta y_2$, $y_1 = \delta x_1$, for $x, y \in U$.

The stationarity of these equilibrium strategies is a result rather than an assumption (cf. Osborne and Rubinstein (1994)(p.126)).

It can be easily verified that the Nash products $x^*_{\delta,1}x^*_{\delta,2}$ and $y^*_{\delta,1}y^*_{\delta,2}$ of x^*_δ and y^*_δ are the same. As both points are on the efficient boundary of U- this is also true in the more general case where the u_i's are not identity functions; see for instance Figure 311.1 in Osborne and Rubinstein (1994)- with δ converging to 1, both of x^*_δ and y^*_δ converge to $z^* := N(U, 0)$, the Nash solution point of $(U, 0)$.

The choice of $X = [0, 1]$ like in Rubinstein (1982) used also in Example 120.1 of Osborne and Rubinstein (1994), is less natural than the $\bar{X} := \{(x_1, x_2) \in \mathbb{R}^2_+ | x_1 + x_2 \leq 1\}$ chosen in Binmore et al. (1986), if one wants to compare the subgame perfect equilibrium payoffs with the Nash solution of cooperative games for classes considered usually in the literature. There U is generally a compact, convex (often strictly convex) set with $d \in U$. Our special case deals only with the efficient boundaries of such sets and $d = (0, 0)$ does not satisfy $d_1 + d_2 = 1$. Making a proposal in our model corresponds to making a Pareto efficient proposal in the general case. In fact nothing relevant would change, if we replaced $(U, 0) = (X, 0)$ by $(\bar{X}, 0)$.

1.4 An Exact Non-Cooperative Foundation

Denote the extensive form game of Rubinstein with discount factor $\delta \in (0, 1)$ described in the previous section by G^{δ} and its subgame perfect equilibrium payoff vector by $\hat{z}^{\delta}$. Notice that the limit cases of $\delta = 0$ and $\delta = 1$ correspond to the ultimatum game and the Nash simple demand game, respectively.

In G°, the whole cake goes to the proposer in the unique subgame perfect equilibrium. In G^1, every Nash equilibrium payoff vector of the Nash simple demand game can be realized via some subgame perfect equilibrium. This discontinuity of the subgame perfect equilibrium correspondence at G^1 excludes its use for an exact support of the Nash solution.

It is our goal in this article to define a game that is based on the Rubinstein game and can play the role of a missing limiting game that turns the approximate support of the Nash solution due to Binmore et al. (1986) into an exact one.

In order to gain some intuition for the game G to be defined, we consider a game G_{δ} -for an arbitrary $\delta \in (0, 1)$- defined as follows: At stage 0, player 1 proposes some $x \in \triangle$, then player 2 either accepts, in which case the play ends with paying out the proposed payoffs, or she rejects with his only alternative move by which she decides that the Rubinstein game G^{δ} has to be played with player 1 starting as the proposer. Obviously, G_{δ} has the same unique subgame perfect equilibrium payoff vector as G^{δ}.

Next, we get the same result via replacing G_{δ} by $\hat{G}_{\delta}$ which we define as follows: At stage 0, player 1 proposes $x \in \triangle$. Player 2 then reacts by either accepting, which yields the end of the play and payoff vector x, or she reacts by choosing some $\rho \in (0, \delta]$ which means that G^{ρ} has to be played. Again it is clear that the unique subgame perfect equilibrium is $\hat{z}^{\delta}$.

In all those games, the respective $\hat{z}^{\delta}$ can be reached in two different ways: Either by the proposal $x := \hat{z}^{\delta}$ of player 1 being accepted by player 2, or by rejection of player 2 via the unique subgame-perfect equilibrium of G^{δ}.

What happens if we replace the choice set $(0, \delta]$ used in $\hat{G}_{\delta}$ by $(0, 1)$? Let us denote the game resulting from doing so by $\hat{G}$. The Nash equilibrium payoff vector $z^* = \lim_{\delta \to 1} \hat{z}^\delta$ can be realized in $\hat{G}$ only as an accepted proposal. No choice of $\rho \in (0,1)$ prescribing the play of $\hat{G}_\rho$ and its unique subgame perfect equilibrium payoff vector $\hat{z}^\rho$ could possibly justify a rejection of the proposal z^*. But unfortunately, this equilibrium fails to be subgame-perfect! Off the equilibrium path, any proposal $x \neq z^*$ would be to the disadvantage of player 1 or could be rejected by player 2 via a suitable choice of $\rho \in (0,1)$. But as there is no optimal way to choose such ρ, the game G does not have any subgame-perfect equilibrium.

We can establish, however, that z^* is the unique *weakly subgame perfect* equilibrium payoff vector of $\hat{G}$. And we will argue that the use of weakly subgame perfect equilibria secures the credibility of threats sufficiently well in order to justify this concept.

Moreover, we shall for convenience constrain ourselves to the countable set of $\delta_k \in (0,1)$ with $\delta_k := k/(k+1)$, $k \in \mathbb{N}$. Then $\lim_{k \to \infty} \delta_k = 1$. Accordingly, we denote G^{δ_k} and $\hat{z}^{\delta_k}$ by G^k and $\hat{z}^k$, respectively.

Although our main result in our Theorem will provide a subgame perfect equilibrium support for the Nash solution, we consider our Proposition, that we are going to state and prove next, an interesting support result by itself. It will, however, also build the basis for proving our Theorem.

We shall now introduce first our game G and then the concept of a weakly subgame perfect equilibrium that coincides with that of a subgame perfect equilibrium on finite games.

At round 0, one of the two players of the bargaining game (U, d) is selected randomly with probability $1/2$ to make a proposal $z \in \triangle$. After the first player makes a proposal, the other reacts by choosing an element $k \in \mathbb{N}_0 := \mathbb{N} \cup \{0\}$.

If she chooses 0, the proposal is accepted and the payoffs will be realized. If she chooses $k \in \mathbb{N}$, the proposal is rejected and the game G^k will be played, whose unique subgame perfect equilibrium payoff vector $\widehat{z}^k$ at $t = 1$ results in the discounted payoff vector $\delta_0 \widehat{z}^k$ at $t = 0$. In order to simplify the notation, we assume, w.l.o.g., $\delta_0 := 1$.

Like the Rubinstein games G^k,$k \in \mathbb{N}$, the game G has an infinity of Nash equilibria, among them the one where every player always chooses proposal $N(U, d)$ and always accepts this proposal and rejects any other one. Any other bargaining solution can be supported by Nash equilibria of G in an anologous way. This is essentially the situation we have in *Nash's simple demand game*.

Our final solution in the theorem will be based on subgame perfect equilibrium. But, as in contrast to the G^k,$k \in \mathbb{N}$, the game G does not have any subgame perfect equilibrium, we shall first prove a proposition where we use the weaker concept of weak subgame perfectness due to Trockel (2011).

Definition 3 *A Nash equilibrium of an extensive game is called* ***weakly subgame perfect*** *when it induces some Nash equilibrium in every subgame in which a Nash equilibrium exists.*

We shall very briefly discuss this concept in section 5. Here we will use it in order to state and prove our first non-cooperative support result.

Proposition *For the bargaining game $(U, d) = (U, 0)$, the extensive game $G(= G^{U,d})$ as defined above has an infinity of weakly subgame perfect equilibria with identical equilibrium path and equilibrium payoff vector $z^* = N(U, d)$.*

Proof *The proof is decomposed into several steps:*

- *One type of Nash equilibria is defined by the following rule for both players:*

  As the proposer choose z^*, as the follower accept exactly those proposals that are at least as good as z^*. In any Rubinstein subgame G^k, play according to the unique subgame perfect equilibrium.

  It is obivous that no other proposal nor any other reaction to a proposal can constitute an advantageous unilateral deviation for any player. In these Nash equilibria, z^* is realized in the first round.

- *There does not exist any subgame perfect equilibrium in this game.*

  This follows from the fact that G has subgames without Nash equilibria, namely those that directly follow any proposal $z \in \triangle$ that offers to the other player a payoff smaller than her coordinate of z^*. Although this player should reject, there is no optimal $k \in \mathbb{N}$ to do so.

- *The Nash equilibria described in the first step are weakly subgame perfect.*

  The trivial subgame G has Nash equilibria according to the first step. Each of those induces on every Rubinstein subgame G^k, $k \in \mathbb{N}$, a (subgame perfect) Nash equilibrium. All subgames starting at proposals that offer the other player a payoff at least as high as her coordinate of z^* have "acceptance" as the optimal, hence Nash equilibrium choice. Off the equilibrium path, these equilibria induce subgame perfect equilibria of Rubinstein games G^k, $k \in \mathbb{N}$.

  The only remaining subgames are those without equilibria described in the second step. So the Nash equilibria described in the first step are weakly subgame perfect.

- *Any other Nash equilibrium fails to be weakly subgame perfect.*

  In order to establish this claim, consider a Nash equilibrium payoff vector $\widetilde{z} \neq z^*$.

  There are two possible ways how $\widetilde{z}$ may have been realized:

  a) as an accepted proposal in the first round.
  b) as the result of a subgame G^k that started right after a first proposal has been rejected.

  W.l.o.g., let player 1 be the first proposer in the first round, and hence in every G^k if it is played after rejection of player 2.

  Case (a): If player 1 proposes $\widetilde{z}$ with $\widetilde{z}_2 < z_2^*$ and player 2 accepts, this cannot possibly be a part of a Nash equilibrium, since player 2 could just *reject with a sufficiently large k and ensure herself $\widehat{z}_2^k$ arbitrary close to $N_2(U, d)$, and $\widehat{z}_2^k > \widetilde{z}_2$.*

If player 1 proposes $\widetilde{z}$ with $\widetilde{z}_2 > z_2^*$ and player 2 accepts, player 1 could have improved by proposing z^*-unless player 2 rejects z^*. So only if player 2's strategy contains rejection of z^*, then $\widetilde{z}$ could possibly be a Nash equilibrium payoff vector. But rejecting z^* is only possible via choosing some G^k, $k \in \mathbb{N}$. As in each G^k, the unique subgame perfect equilibrium payoff for player 2 would be $\widehat{z}_2^k < z_2^* < \widetilde{z}_2$, none of them would justify rejection of z^*. Therefore, the payoff vector $\widetilde{z}$ can possibly result only from a Nash equilibrium that is not weakly subgame perfect and satisfies $\delta_0 \widetilde{z}_2 = \widetilde{z}_2 > \widehat{z}_2^k = \delta_0 \widehat{z}_2^k$.

Case (b): Suppose $\widetilde{z}$ is a weakly subgame perfect equilibrium payoff vector of G. Then $\widetilde{z} = \delta_0 \widehat{z}^k = \widehat{z}^k$ for some $k \in \mathbb{N}$. But then player 2 could improve by choosing $k+1$ with discounted payoff vector $\delta_0 \widehat{z}^{k+1} = \widehat{z}^{k+1}$. This implies that $\widetilde{z}$ is not a Nash equilibrium payoff vector of G, a contradiction. □

1.5 Remarks on Weakly Subgame Perfect Equilibria

In contrast to the games G^k, $k \in \mathbb{N}$, the game G has an infinity of weakly subgame perfect equilibria. How bad is this? There is no coordination problem involved as long as both players stay on the equilibrium path and the equilibirium payoff is uniquely determined. So the multiplicity of those equilibria appears to be harmless, in particular as subgame perfect equilibria also may have multiple ways of behavior off the equilibrium path.

So the critism could only be based on the lack of credible threats to reject, because there is no optimal way of rejecting! But from a decision theoritical point of view, this critisim is dubious. If there is a choice between money amounts $\{-50, 1, 2, ..., 10\}$, we take it for granted that -50 is rejected(via accepting 10). If the choice is among $\{-50\} \cup \mathbb{N}$, do we think that -50 will be accepted just because there is no best alternative? In real life, we avoid very bad or worst cases even if we are unable to do that in an optimal way.

But as this is a controversial point, we will provide a modified non-cooperative support result via subgame perfect equilibria in the next section.

1.6 Subgame Perfect Exact Support

When attempting to base an exact subgame perfect equilibrium foundation for the Nash solution based on Rubinstein's games G^k, $k \in \mathbb{N}$, the dilemma is the appearance of those subgames starting right after an initial proposal that do not have any Nash equilibrium. There are two potential ways out, one may consider:

1. Add a best alternative to the set $\mathbb{N}_0$. We did not see any natural way to do so. We might end up with no or a multiplicity of subgame perfect equilibria. Anyway, we did not follow this approach.

2. Stop those subgames starting right after the first proposals from being subgames. In order to do so, we modify our original game G in the following way: in the beginning, the proposer is chosen randomly with probability $1/2$, but both players do not observe that random choice. So each player has probability $1/2$ that she is the chosen proposer and $1/2$ that she has to react to her opponent's proposal. Accordingly, both players' strategies have to contain full descriptions of what they would propose and how they would react to any possible proposal $z \in \triangle$.

   We follow that approach!

In the beginning, the referee throws a fair coin in order to decide who of the two players starts with a proposal $z \in \triangle$. But the result of this random choice is not observed by the players. Then, not knowing whether they will start with a proposal or a rejection to the proposal, but knowing that any Rubinstein game played after a rejection starts with another proposal of the proposer, both players simultaneously and independently choose a pair consisting of a proposal and a reaction function on the set of possible proposals.

On this basis, we define now an extensive game $\widetilde{G}$ as follows:

At stage 0, the following things happen:

1. The referee throws a fair coin in order to randomly but privately determine which player will act as the proposer.

2. Both players, not knowing which of them will act as the proposer, simultaneously submit pairs (x_1, f_1), $(x_2, f_2) \in U \times \mathbb{N}_0^U$ to the referee, where x_i are their proposals, f_i are reaction function to their opponents proposal, for $i = 1, 2$.

3. The referee informs both players on their randomly determined roles. The proposer whose role is now common knowledge is w.l.o.g. called player 1.

At stage 1, the game either ends if $f_2(x_1) = 0$ or continues to stage 2 if $f_2(x_1) = k \in N$. In this case, player 1 starts at stage 2 with a proposal $x_1 \in U$ in the Rubinstein game G^k. The rest of the game is just playing this Rubinstein game G^k.

The specific structure of $\widetilde{G}$ at stage 0 has two consequences that are crucial for our Theorem:

First, it guarantees that the players' choices of proposals together with their reaction functions build actions at $t = 0$ in non-singleton information sets. Therefore, no subgame starts with such actions. Secondly, the full information of both players about their respective roles as proposer and reactor before the start of actions at stage 2 prevents the annoying situation that all G^k, $k \in N$ would start only at non-singleton information sets. In that case, the only subgame of $\widetilde{G}$ would be $\widetilde{G}$ itself.

Clearly, $N(U, d)$ would still be a subgame perfect equilibrium payoff vector. But the whole plethora of Nash equilibria would become subgame perfect ones, too!

Under the aspect of trying to support the Nash solution, we would essentially be back at Nash's simple demand game. Notice that in $\widetilde{G}$, the Rubinstein subgames G^k of G, $k \in \mathbb{N}$ will reappear twice : once via a rejection of player 2 and once as a rejection of player 1 in that part of $\widetilde{G}$ that follows the non-realized choice of player 2 as the proposer. Only those G^k, $k \in \mathbb{N}$ following a rejection by player 2 are potentially effective for the outcome of the game. But also in the other (now irrelevant) Rubinstein games, following rejections by player 1 of proposals by player 2, subgame perfectness of $\widetilde{G}$ requires the players to play subgame perfect equilibria.

It is impossible to just cancel that at stage 1 irrelevant part of the game tree as it is relevant for the players' choices at stage 0 before they are informed about their respective later roles.

This modification of our original game G is illustrated in the following two figures:

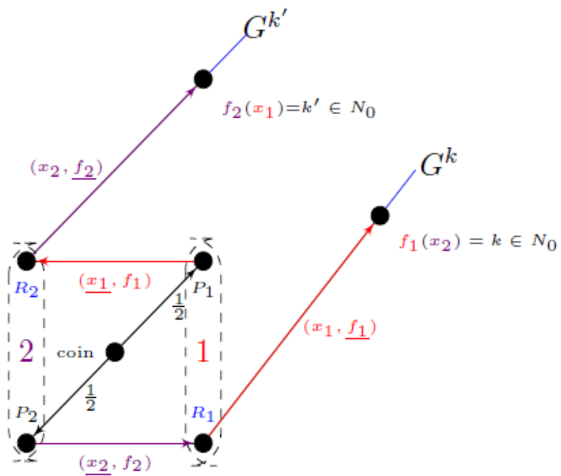


Figure 1

Figure 1 and 2 are equivalent stylized illustrations of stages 0 and 1 of $\widetilde{G}$.

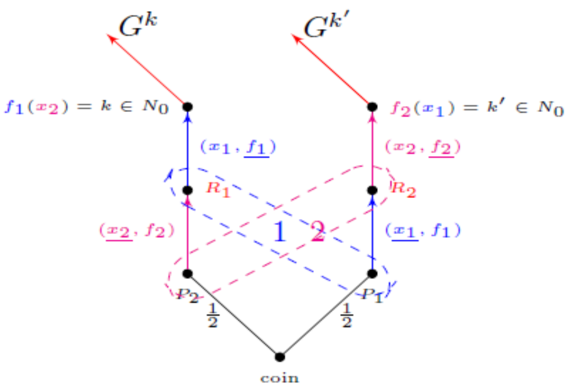


Figure 2

Notice that these figures are only schematic illustrations of the game $\widetilde{G}$ in its first stages $t = 0, 1$ rather than complete game trees. The infinite action sets for both players are represented in these figures only by one typical action for each player, namely (x_1, f_1), (x_2, f_2). In the Figures G^0 is the degenerate game consisting of the singleton set $\{0\}$ representing acceptance of a proposal.

The construction in Figure 2 is similar to the way in which Sudhölter et al. (2000) define the canonical extensive form for the Battle of Sexes game. It has precisely the intended effect in our present context. The one-player subgames without optimal actions in the reduced game have vanished now.

The only remaining subgames of the modified game $\widetilde{G}$ are $\widetilde{G}$ itself and the Rubinstein games G^k, $k \in \mathbb{N}$.

In $\widetilde{G}$, any subgame perfect equilibrium is a pair $(x_1, f_{1,k^1}; x_2, f_{2,k^2})$ with $(x_1, x_2) = z^*$ and $f_{i,k^i} : U \to \mathbb{N}_0$ such that

$$f_{i,k^i}(x) = \begin{cases} 0 & : x_i \geq z_i^* \\ k^i & : x_i < z_i^* \end{cases}$$

and $k^i \in \{k \in \mathbb{N} | \widehat{z}_i^k > x_i\}$, $i = 1, 2$.

After the first moves of both players, the game ends either in its equilibrium with payoff vector z^* or, else, in some game G^k, $k \in \mathbb{N}$, where the unique subgame perfect equilibrium is induced. The multiplicity of subgame perfect equilibria arises from the various k^i that may be chosen for f_{i,k^i}, $i = 1, 2$. But the equilibrium path is unique.

We can formulate the modified version of our support result now as follows:

Theorem *For the bargaining game* $(U, d) = (U, 0)$*, the extensive game* $\widetilde{G}(= \widetilde{G}^{U,d})$ *as defined above has an infinity of subgame perfect equilibria with identical equilibrium path and equilibrium payoff vector* z^**.*

Proof *This theorem is in fact a corollary to the Proposition.*

All Rubinstein games G_k*,* $k \in \mathbb{N}$ *occur as subgames in* $\widetilde{G}$*. There is no subgame anymore however, that starts after a proposal with actions* $k \in \mathbb{N}_0$*. The* k *chosen after a proposal* x_1 *of player 1 is now determined by the simultaneous choice* (x_2, f_2) *of player 2 at both points of her information set at* $t = 0$ *via* $k := f_2(x_1)$*.* $k' := f_1(x_2)$ *has been eliminated from further consideration by the referee's random choice of player 1 as the proposer. Accordingly, a lack of an optimal way of rejecting a proposal cannot destroy the subgame perfectness of a weakly subgame perfect Nash equilibrium. As at* $t = 0$*, there do not exist singleton information sets for the players, the games* $\widetilde{G}$ *and* G_k*,* $k \in \mathbb{N}$ *build all subgames of* $\widetilde{G}$*.*

Now, we define a Nash equilibrium of $\widetilde{G}$ *literally as in the first step of the proof of our Proposition. Any weakly subgame perfect equilibrium of* G *"becomes" (corresponds to) subgame perfect equilibria of* $\widetilde{G}$*. Any other Nash equilibrium payoff vector could in* G *only result from non weakly subgame perfect behavior in some* G^k*, thus in* $\widetilde{G}$ *only from violating subgame perfectness in that* G^k*. This proves our Theorem.* $\square$

Remark *The subgame perfect Nash equilibria of* $\widetilde{G}$ *are in fact even sequential Nash equilibria! The beliefs of both players can be expressed by probabilities on their non-singleton information sets. Whatever probabilities there may be, however, they do not have any influence on their decisions at these informations sets, as every choice* (x_i, f_i)*,* $i = 1, 2$ *is intended to be optimal at each point of the information set, respectively. Only the relevant parts of those pairs* (x_i, f_i)*,* $i = 1, 2$ *are different at the different points of the information sets.*

1.7 From Support to Implementation

The step from a non-cooperative support of a cooperative solution to a mechanism theoretic implementation is not trivial and requires some care and in fact certain assumptions.

In Serrano (1997), we find the following passage:

"The Nash Program and the abstract theory of implementation are often regarded as unrelated research agendas. Indeed, their goals are quite different: while the former attempts to gain additional support for cooperative solutions based on the specification of certain non-cooperative games, the latter tries to help an incompletely informed designer implement certain desirable outcomes. However, it is misleading to think that their methodologies cannot be reconciled. A common critism that is raised against the mechanisms in the Nash program is that they are not performing real "implementations" since their rules depend on the data of the underlying problem (say the characteristic function) that the designer is not supposed to know."

Bergin and Duggan (1999) also emphasizes the importance of independence of the game rules expressed by a mechanism of the players' preference profiles. And it is crucial now that payoffs are in utils representing preferences rather than in money.

The very fact that the presence of an outcome space is an additional ingredient in mechanism theory as compared to the Nash program indicates that these two can hardly be considered "equivalent", as claimed, for instance in Dagan and Serrano (1998)(abstract). Detailed treatments of the relation between the Nash program and implementation theory can be found in Bergin and Duggan (1999), Trockel (2000, 2002a) and Serrano (2004).

The conditions that are necessary in order to have a non-cooperative support that automatically provides an implementation in some equilibrium concepts are often satisfied in models in the literature [cf. Moulin (1984), Howard (1992)]. However, strictly speaking, there *solution based social choice rules* are implemented.[cf. Trockel (2003)].

This holds also true in principle for the model of Rubinstein (1982), as used in Binmore et al. (1986) and Osborne and Rubinstein (1994). However, the situation there is different. In a strict sense, these results do not provide a non-cooperative implementation for the Nash solution on a given prespecified class of two person cooperative bargaining games. They rather just define such an implementation for the classes of those bargaining games generated by their game forms together with their different types of utility functions. From a puristic point of view, there is missing an axiomation of the Nash solution on those classes. Clearly, this solution is still well defined as the maximizer of the Nash product. In fact, the section 3 in Binmore et al. (1986) has the heading *Nash solution as an approximation to the equilibria*. This terminology differs from the one prevailing in the literature on non-cooperative foundation where one thinks of supporting the Nash solution by equilibria of non-cooperative games, exactly or by approximation. In fact, the equilibria do approximate the Nash solution.

The Nash program is based on two passages in Nash (1951) and Nash (1953). While the first one may be interpreted as giving a higher priority to the strategic than to the axiomatic approach to bargaining, the second one from the introduction in Nash (1953) emphasizes the equal importance of both approaches:

"*We give two independent derivations of our solution of the two-person cooperative game. In the first, the cooperative game is reduced to a non-cooperative game. To do this, one makes the players' steps of negotiation in the cooperative game become moves in the non-cooperative model. Of course, one cannot represent all possible bargaining devices as moves in the non-cooperative game. The negotiation process must be formalized and restricted, but in such a way that each participant is still able to utilize all the essential strengths of his position. The second approach is by the axiomatic method. One states as axioms several properties that it would seem natural for the solution to have and then one discovers that the axioms actually determine the solution uniquely. The two approaches to the problem, via the negotiation model or via the axioms, are com-*

plementary; each helps to justify and clarify the other."

A *justification* in Nash's sense of the Nash bargaining solution concept on a specified *whole class of bargaining games* via the non-cooperative approach as opposed to just as the Nash solution point of *one specific bargaining game* requires intuitively that each game in that class can be generated or represented by a game in a family of "similar" strategic games. Such kind of *uniform* support for a whole class leads naturally to a common game form underlying that family of strategic games which is already an important step towards implementation.

Binmore et al. (1986) by using the terms *time-preference Nash solution* and *von Neumann-Morgenstern Nash solution* stress the fact that their two models provide approximate supports for the Nash solution on *different sets* of bargaining games.

The other direction of the Nash program, namely *justification* and *clarification* of the Rubinstein approach, is not so obvious. This model is defined with *discount factors* as important ingredients, technically and conceptually. But as soon as the discounting is taken serious rather than almost neglected the Nash solution of the induced cooperative bargaining games may differ significantly from the subgame perfect equilibrium payoff vectors of the strategic games.

As far as *implementation* is concerned, this fact is not disturbing, however. What in implementation theory has to be implemented is a social choice rule. And the implementation is not conceived as a *justification* or *clarification* like in the Nash program. Rather the social choice rule is the inherently justified solution method for a social choice problem with a specified outcome set. Its implementation in a strategic equilibrium concept represents the idea of realizing something already accepted as socially desirable via strategic interaction in the society according to certain rules, namely a *mechanism* or *game form*.

In situations where solutions of certain games can be identified with social choice rules, or as Hurwicz (1994) termed them, *desirability correspondences*, non-cooperative foundation may extend to implementation.

In our context, the Nash solution has to play the role of the social choice rule. One can easily see that the search for a natural and adequate outcome set needed for implementation leads to different results for the various versions of sequential bargaining in Binmore (1980), Binmore et al. (1986) or Osborne and Rubinstein (1994). Consequently, the generated utility spaces or cooperative bargaining games may be quite different.

In this context, it is important to notice that our use of the payoff set U (identified with the underlying $X = [0, 1]$) in this paper on which the negotiation is modeled rather than on the set $\bar{X} = \{x \in \mathbb{R}^2_+ | x_1 + x_2 \leq 1\}$ is just for convenience and simplicity of presentation, like the treatment in Binmore (1980). It could as well have been formulated via the models in Binmore et al. (1986) like for instance in Osborne and Rubinstein (1994)(Proposition 310.3).

From the conceptual point of view, there is a fundamental difference between the two models in Binmore et al. (1986) and therefore also in the two interpretations of the factor δ in the results of our present paper as far as the implementability problem is concerned.

If the Nash solution should be implemented on a class of bargaining games resulting from players who are really impatient then the time preference Nash solution cannot even approximately be implemented because the discounting factor cannot be made close to 1 by the designer. So the time preference Rubinstein approach could lead to implementation of the Nash solution only on a class of bargaining games generated by the strategic games with players whose idiosyncratic discount factors would have to be close enough to 1. Moreover, because of symmetry of the Nash solution, these discount factors would have to be the same for any pair of players negotiating. That means that practically we can forget about approximate implementation in that context.

A similar argumentation holds for those bargaining games in the second model of Binmore et al. (1986) that are generated when players are strongly risk averse. Only uniform weak enough risk aversion would allow implementation of the Nash solution for that induced class of bargaining games.

A third possibility is to think of a pool of risk neutral perfectly patient players.

In that case, the second model of Binmore et al. (1986) allows to give the breakdown probabilities as instruments into the hands of the designer. This fits well the structure of our games G and $\tilde{G}$. And only then we would get arbitrarily close to implementation of the Nash solution on the induced class of cooperative bargaining games. Only in that case our two present results could be transformed into conceptually meaningful implementation results. Yet, this domain of the Nash social rule would be very specific and very small.

Our exact non-cooperative foundation results for the Nash bargaining solution that we have presented in this article are, as we mentioned earlier, not the only ones in the literature.

For an approximate support via a modification of Rubinsteins game that ,differently from Rubinstein, works also for $n > 2$, see Moulin (1984).

As to the tasks of *clarification* and *justification*, it is interesting to point out to the similarity of insights into the meaning of the Nash solution provided by the Rubinstein alternating offer approach and by the non-cooperative strategic unique Nash equilibrium support and implementation in Trockel (2000) based on *Walrasian* payoff functions.

In the latter one, this Walrasian property of the Nash solution [cf. Trockel (1996)] represents perfect competition that simulates an abundance of *outside options* for both players in a game protecting them from exploitation by their respective opponents. In the Rubinstein game with almost negligible discounting and in our modified games, it is the *infinity of future options*, (almost) equally valuable, that creates the same effect. That suggests the interpretation of implementing the Nash solution as a sort of surrogate for sufficient competitive pressure.

When the implementation is not an issue but only a non-cooperative foundation that helps to *justify* the Nash bargaining solution and to *clarify* its meaning then our results will work equally well (but as we think not better) as the approximate results based on the Rubinstein game. The advantage of our modified Rubinstein game forms lies in the fact that they somehow make the subgame perfect equilibrium payoff function continous at $\delta = 1$.

While the Rubinstein games with δ converging to 1 induce a limit for the associated sequence of subgame perfect equilibrium payoffs, there is no associate *limit model* whose subgame perfect payoff equilibrium vectors would confirm this result. As we have shown in section 4, our game G is the limiting game for a sequence of modified versions of Rubinstein games having the same subgame perfect equilibrium outcomes as these.

Concluding Remark: *Totally analogous results to our Proposition and our Theorem can be proved via using Ståhl's rather than Rubinstein's model. In the games G^k, k would have been to be replaced by a double index (k, l), where k represents the discount factor δ, and l represents the number of stages of a (finite horizon!) Ståhl game.*

2 Does Informational Equivalence Preserve the Strategic Behavior? An Experimental Study on Trockel's Game

The purpose of the present study is to test experimentally a version of the classical Chain Store Game (CSG), proposed by Trockel (1986), and determine whether one of the two theories of *Induction* and *Deterrence*, which were originally tested competitively by Selten (1978), may better account for the results. With complete and perfect information, the CSG of Selten (1978) was designed to analyze the role of reputation in repeated market interactions. Its results were discussed in two different ways: one is based on backward induction, and the other is intuitively derived from a deterrence argument. As the two explanations are incompatible, alternative models have been proposed to understand them better. The alternative game proposed by Trockel is an imperfect information version of the CSG in which the order of the two players is changed in each round and the "Aggressive-Stay Out" equilibrium is used to build reputation. The existence of more than one equilibrium is the basis for the building of reputation. To the best of my knowledge, this study is the first attempt to experimentally test this alternative game with the same purpose.

2.1 Introduction

Purposeful "irrational" behavior is a means by which a person can easily influence people's opinions in such a way that the conclusions they draw about that person's preferences and strategies are incorrect. The building of reputation with this purpose may benefit the person from the conclusions others make about her and the situations where the short-term cost is overcompensated by the long-term benefit. Observations in daily life suggest including "irrational" behavior in the modeling of markets. The CSG with complete and perfect information was designed by Selten (1978) to analyse the role of reputation in repeated market interactions. This is a game that models the following scenario: A chain store located in 20 different towns acts as a local monopoly. In each town, there is a potential entrant who collects capital to establish a second store of the same kind as the monopoly in that town. Initially, only one potential entrant has enough capital, but gradually other entrants will have saved the necessary capital. The entrant, who has enough capital, decides whether to enter the market by establishing a second store or to stay out of the market completely. If this entrant decides to enter the market, then the monopoly employs one of two different monetary policies: Cooperative and Aggressive. A cooperative response yields considerably higher (duopoly) profits in that town for both the monopoly and the entrant when compared to an aggressive response. But in the latter case, however, monopoly profits in that town are much higher, if the entrant does not establish a second store. The entrant prefers to stay out of the market and use the capital in a different field instead, rather than establish a store only to be met by an aggressive response.

Selten (1978) discussed the CSG in two different ways: one is based on induction and the other one is derived intuitively from a deterrence argument. The Induction Hypothesis, represented by the subgame perfect equilibrium concept in a perfect information game, is not compatible with the Deterrence Hypothesis. To prove this incongruity, Rapoport and Sundali (1997) experimentally tested the CSG. The data obtained by this experiment is congruent with the predictions of the Induction Hypothesis, but not with the predictions of the Deterrence Hypothesis. Therefore, discussions on predatory pricing policy, reputation and the value of backward induction arise naturally; thus, it is important to understand the following experimental framework that is designed to find out if it is possible to deter entry into the market by building reputation in an alternative game where there is room for enticing players' beliefs. The alternative game that is used in this project is an imperfect information version of the CSG by Trockel (1986).

The CSG is a complete and perfect information game resulting from the limited number of repetitions of an extensive two-player game. The unique subgame perfect equilibrium computed via backward induction stipulates that each entrant establishes the second store in his town and that the monopoly always reacts with a cooperative monetary policy. On the other hand, according to the Deterrence Hypothesis, the monopoly should not behave in accordance with this equilibrium. By playing aggressively in the earlier stages, the monopoly may influence what later potential entrants think about his type. If these beliefs turn into ”aggressive monopoly beliefs” early enough in the sequentially repeated game, the aggressive policy increases the chain store’s profits in the long run. If that is the case, it is optimal for all potential entrants to stay out from the beginning. By this way, the chain store earns higher long run profits than those achievable by a cooperative policy.

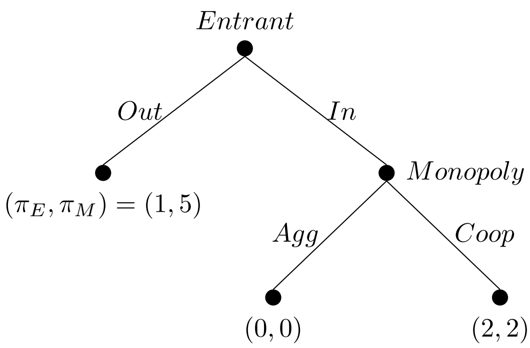
The results obtained by Rapoport and Sundali (1997) do not provide a statistically significant evidence in support of the Deterrence Theory. Hence the idea of the possibly observing the Deterrence Hypothesis with some modifications of CSG arises. In Trockel’s game, the order of play is changed in each stage game in an informationally equivalent way, and the ”stay out-aggressive” equilibrium can be (re)enforced via building reputation. The existence of more than a single equilibrium allows for the chosen equilibrium strategy to be interpreted as a signal that reveals the kind of behavior that will consistently be displayed when the game is repeated. If that behavior can effectively be repeated in the next stages of the game, then reputation is built up via the entrants’ Bayesian updating of their beliefs. The other advantage of the game is the imperfect information. The type of reputation that might be built in Trockel’s game organizes the coordination of the choice in the respective equilibrium. On the other hand, Trockel (1986) claims that this alternative game is as compatible with the scenario in Selten (1978) as the Chain Store Game itself. It is inevitable to agree with this claim which is based on the Sure Thing Principle that underlies the use of von Neumann-Morgenstern utilities assumed in the standard game theory [cf. Malinvaud (1952)].

It is the purpose of this paper to determine whether it is possible to deter the entry of potential entrants by building reputation with the help of the observations obtained through experimental methods. Understanding what affects the behavior of the players, how the behavior of the players changes, and whether the strategic behavior is preserved as a result of the equivalent information with the CSG are the main motivations of the study. To the best of my knowledge, this study is the second experiment which clings to the scenario, and the first attempt in the literature that experimentally tests this alternative game with the same purpose of understanding whether there is any possibility for entry deterrence. The original scenario is tested in the laboratory for the first time, as Selten (1978) suggests, and 20 entrants are used for the first time. The introduction of investment in the design is the major difference from the main design in Rapoport and Sundali (1997).

Political Science uses Game Theory to study the strategic interaction. According to Gates and Humes (1997), politics is inherently strategic, and bluffing is the basic ingredient of this strategy. The CSG and its multiple versions are introduced by Gates and Humes (1997) to explain the role of reputation on the basis of bluffing and commitment in international politics. Both bluffing and commitment are defined as the tools to manipulate the expectation(s) of the opponent and, consequently, lead to a change in the opponent's behavior. They explain the modelling of Kreps and Wilson (1982), Milgrom and Roberts (1982) with bluffing, and the modelling of Trockel (1986) with commitment(threat). They support the plausibility of benefit via reputation by real-life examples on developing and maintaining leadership. Thus, my paper contributes to the Political Science literature as well.

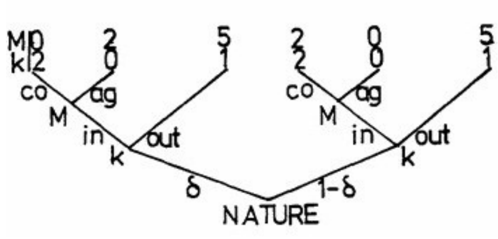
2.2 Literature Review

The CSG is a two-player game, which consists of a finite number of repetitions in a certain extensive game and used by Selten (1978) for game theoretically modelling in his Chain Store scenario. In this scenario, a chain store located in 20 different towns acts as a local monopoly. In each town, there is a potential entrant who considers establishing a second store of the same kind as the monopoly in that town. The potential entrants,in succession, have enough capital to establish the second store in their respective towns. The potential entrant, who has enough capital, decides whether to enter the market by establishing a second store or to stay out of the market. If she decides to enter the market, then the chain store could respond to this decision with one of two different monetary policies: Co-operative and Aggressive. A cooperative response yields considerably higher profits in that town for both the chain store and the entrant, than an aggressive one. But in the latter case, however, the profit of the chain store in that town is much higher if the entrant does not establish a second store. It is assumed that all the players wish to maximize their profits: for the chain store, the total of the profits in each town and for an entrant, the profit when she decides against to compete the chain store. Selten (1978) discusses the CSG in two different ways: one is based on backward induction and the other on an intuitive deterrence argument. Represented by the use of the subgame perfect equilibrium concept in this perfect information game, the Induction Hypothesis is not compatible with the Deterrence Hypothesis. According to the Induction Hypothesis, each entrant establishes the second store of the town and then the chain store decides not to fight the entrant. According to the Deterrence Hypothesis, the chain store should not behave in accordance with the Induction Hypothesis, but rather decide in how many of the last rounds she played 'Cooperative' and then play 'Aggressive' against the entrants in the first rounds to build reputation for deterring the next potential entrants. This way, the chain store would earn higher profits in the long run than she would by following the Induction Hypothesis. The conflict between these two hypotheses is called 'Chain Store Paradox' in Selten (1978) and in the following related literature.

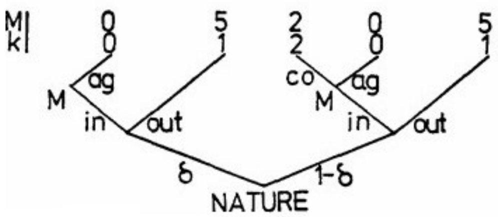


Graph 1. The Chain Store Game

In order to solve the Chain Store Paradox, Kreps and Wilson (1982) and Milgrom and Roberts (1982) relax the assumption of complete information and present versions of the CSG with incomplete/asymmetric information. In both papers, the aim is to disable the use of backward induction so that an environment is created in which learning is based on experience. This implies that reputation building is possible. In Milgrom and Roberts (1982), it is assumed that entrants have doubts about the strategies of the monopoly. Therefore, entrants believe that there are two types of monopoly: a ”weak” monopoly who shares the market, and a ”strong” monopoly who only reacts with ’Aggressive’ monetary policy. In Kreps and Wilson (1982), it is assumed that entrants have doubts about the payoffs of the monopoly. Again, entrants believe that there are two types of monopoly: a ”weak” monopoly who has the payoff options as in Selten (1978) and a ”strong” monopoly who is better off when ’Aggressive’ rather than ’Cooperative’ is played. Because of the incomplete information in the two versions, the entrants need to understand the type of the monopoly that they are facing. Any uncertainty regarding the monopoly player may trigger a fear in ’Aggressive’ monetary policy by the monopoly, and may therefore deter the entrants from entering the market. As long as the uncertainty exists, it is sufficient to deter almost all entrants from entering the market by ’Aggressive’ plays and building ”strong” monopoly impression/reputation even if she is a ”weak” monopoly.



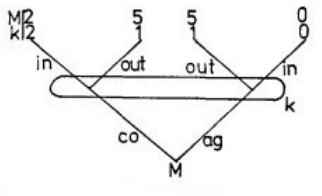
Graph 2. Kreps and Wilson (1982)



Graph 3. Milgrom and Roberts (1982) [1]

The CSG is a game with complete and perfect information. While the models of Kreps and Wilson (1982) and of Milgrom and Roberts (1982) relax the complete information assumption, Trockel (1986) keeps the complete information assumption but introduces an imperfect information modification of Selten's game. Selten's scenario is preserved in this version; the only difference is in the order of moves made by the players. In each entrant-monopoly encounter, first the monopoly decides how to play. Without knowing the decision of the monopoly, the entrant then decides whether to enter the market or to stay out of it. The two-player stage games of Selten (1978) and Trockel (1986) are equivalent in terms of decision-making, they differ only in the game structure. Beliefs explicitly appear in the model and in the analysis when non-singleton information sets appear. Then, in this game the strategy set "Aggressive-Stay out" is also a sequential equilibrium based on effective deterrence via reputation building. The existence of more than one equilibrium is crucial for the possibility of reputation building.

[1]These graphs are taken from Trockel (1986).



Graph 4. Trockel (1986) [2]

The literature includes several experimental studies on the CSG and its variants. Jung et al. (1994) test the asymmetric information version in Kreps and Wilson (1982) without a ”strong” monopoly with 7 participants (3 monopolies-4 entrants). One monopoly player faces 4 entrants, then the other 3 monopoly players face with the entrants, and 4 entrants face the first monopoly player again by shuffling the order of the entrants (8 rounds). In 4-periods phases, usually ’Aggressive’ play was preferred. Thus, the possibility of deterring the entrants from entering the market by building reputation is the outcome of the paper. Rapoport and Sundali (1997) is the first to attempt, in the literature, the study of the original CSG by employing experimental methods. Their study contains two treatments with 10 and 15 entrants. Although the statistically significant support for the Deterrence Hypothesis amplifies as the number of the entrants increase, the support for entry deterrence with ’Aggressive’ play is not observed. Sundali,Israeli and Janicki (2000) present a new version of the game with an additional move for the entrants in which they have the chance to either stay in the market or exit the market after the monopoly player’s ’Aggressive’ play. Unlike the other experimental studies, the participants may communicate with each other before submitting their decision. They were informed about the following concepts: backward induction, the Nash equilibrium, and the Prisoner’s Dilemma. Sundali,Israeli and Janicki (2000) report that none of the monopoly type participants responded aggressively on a consistent basis.

The data obtained by the experimental methods has led to discussions about the value of the predatory pricing, backward induction, and reputation. Therefore, it is warranted to understand the following experimental framework that

[2]This graph is taken from Trockel (1986).

has been designed to find out if it is possible to deter entry into the market by building reputation in an alternative game where there is room for building and influencing the beliefs of the players. When Trockel's game – equivalent to the original game in terms of decision-making – is tested experimentally, whether the results are the same as the results of the original game or whether the new game triggers different strategic behavior are important questions that merit further investigation.

2.3 Experimental Design

A laboratory experiment was designed to simulate the game introduced by Trockel (1986). The design is almost the same as the last version of the design in Rapoport and Sundali (1997). The z-tree program was used (Fischbacher (2007)). The experiment was conducted in two laboratories: BELİS (Bilgi Economics Lab İstanbul) and BAEL (Behavioral and Experimental Lab, METU). Five hundred and seventy-six undergraduate students from several faculties of İstanbul Bilgi University (İBÜ) and Middle East Technical University (METU) volunteered to participate in Spring 2017. The students were recruited by ORSEE (Greiner (2004)) used by both of the laboratories. In either university 12 sessions were conducted, for 24 sessions in total. In these 12 sessions, there were two treatments: the first treatment T1 uses the game of Trockel (1986), and the second treatment T2 differs from T1 in the payoff of the monopoly. The payoff was increased from 5 to 10 when the entrant choosed 'Out' to enhance the appeal of 'Aggressive' play.

Twenty-four participants were invited to each session. Written instructions were placed in every computer cubicle, so that the participants could follow them when they were aloud. After listening to the instructions, the twenty-four participants were assigned randomly to two roles: four of them were assigned to type A (representing the monopoly player in the scenario) and 20 of them were assigned to type B (representing the entrants in the scenario). In order to distinguish between the players, a positive number was assigned to each player (numbers between 1-4 for type A players; and numbers between 1-20 to the type B players.) Hence, the players in each session were numbered A1, A2, A3, A4, B1, B2,. . ., B20.

Each session included 20 rounds of play, and each round contained 4 independent two-player Trockel's games. Consequently, four data points were collected in each session. It was necessary to control the matchings of the two types of players in each round to conduct four games simultaneously. Four type A participants were activated in all 20 rounds, whereas the 20 type B participants were activated in only four rounds. That is why the matchings were designed. The twenty rounds were divided into four sections each containing 5 rounds. Each type B participant played once in each section and in each section, she was faced with a different type A participant. As soon as the experiment started, the program announced to all the type B participants the round where they would be active, and which type A participant would be matched with them on these rounds. In each round, all 20 type A participants and four different type B participants were activated. The other 16 type B participants waited for the next round in which they would be activated.

Each type A participant was matched in each round with a different type B participant and, consequently, she played with all the 20 type B participants once. Additionally, each type B participant played against each type A participant; i.e., whenever they were active, the "opponent" type A participants were different. For type A participants, the experiment contained 20 rounds against the 20 type B participants that were mutually independent. For this reason, they were not informed about the choices of the other type A participants.

Each of the activated participants in every round was given 1 point representing the investment in the scenario. In each round, type A participant moved first. Type A participants chose one of the actions, namely 'Z' and 'T'. Action 'Z' represents 'Aggressive' play, whereas Action 'T' represents 'Cooperative' play. Importantly, the decisions of type A participants were not announced to the type B participants. After the type A participant had decided, it was the decision time for the type B participant activated in this round. Without knowing the decision of their ”opponent”, type B participants chose one of the actions, namely ’X’ (’In’, enter the market) and ’Y’ (’Out’, stay out of the market). After type B participants submitted their decisions, they were informed about their payoffs in the round.

History tables were exhibited on individual screens. Type A tables portrayed information about type A decisions and payoffs for all previous rounds as well as the payoffs of her type B opponents in all the previous rounds. The history tables for active type B participants provided information about the decisions and payoffs of the active type B participants, who already had played against this type A participant in previous rounds as well as the payoffs of this type A participant. A waiting type B participant’s screen showed the same as that of the active type B participant who was scheduled to participate next with a type A participant. By the help of these history screens, the participants could follow the decisions/payoffs of the previous rounds. At the end of any round, all decisions made during the round were saved to the history tables of the type A and type B participants. In summary, whichever round was played, it was possible for all the participants to be informed about all previous rounds at the rate allowed by their types. After the payoffs were announced via feedback screens, the experiment continued with the next round in which type A participants would be matched with four new type B participants. After the completion of the 20th round, the experiment ended.

After the completion of all 20 rounds, a questionnaire was distributed to the participants to help understanding the reasons for their behavior during the experiment. This questionnaire was the version of the one in Sundali (1995). The questions were adapted to the modification of Trockel’s game, but their aims were preserved (see Appendix). Following the questionnaire, the participants were paid their earnings and the session was completed. The amount paid included the total earnings in the experiment. The total amount of type A participant was paid in TL and the total amount of type B participant was first multiplied by 5 and then paid in TL.

Each session was completed in approximately 75 minutes. The mean payment of type A participants was 46.86TL (İBÜ 43.71, METU 49.96) in T1 and 71.5TL (İBÜ 69.59, METU 73.42) in T2. The mean payment of type B participants was 28.17TL (İBÜ 28.71, METU 27.62) in T1 and 27.06TL (İBÜ 26.63, METU 27.5) in T2.

2.4 Results

Two hundred and eighty-eight undergraduate students from each of the two universities took part in the sessions. Hence, 240 type B data points and 48 type A data points were collected.

The data collected in the experiment are used to test the following hypotheses:

- **Ind1:** Monopoly players always choose 'Cooperative'.
- **Ind2:** Entrants always choose 'In'.
- **Det1:** Monopoly players choose 'Aggressive' in early rounds, choose 'Aggressive' less frequently in medium rounds, and choose 'Cooperative' in relatively last rounds.
- **Det2:** The behavior of the players is not invariant across all rounds.
- **Det3:** The probability of entering the market decreases in the relatively early rounds, and then increases.

Table 1. Pooled Data [3]

Entrant Order	Agg. Frequency	P(Agg)	In Frequency	P(In)
1	17	0,18	65	0,68
2	35	0,36	62	0,65
3	22	0,23	64	0,67
4	33	0,34	66	0,69
5	32	0,33	68	0,71
6	32	0,33	72	0,75
7	34	0,35	69	0,72
8	25	0,26	78	0,81
9	27	0,28	74	0,77
10	21	0,22	84	0,88
11	32	0,33	77	0,80
12	31	0,32	70	0,73
13	32	0,33	81	0,84
14	34	0,35	74	0,77
15	25	0,26	82	0,85
16	28	0,29	70	0,73
17	26	0,27	79	0,82
18	22	0,23	73	0,76
19	20	0,21	77	0,80
20	16	0,17	82	0,85
Total	544		1467	
Mean	27,20	0,29	73,35	0,73
S.D	5,98	0,07	6,55	0,07

According to Hypothesis Ind1, each monopoly player chooses 'Aggressive' 20 times. In order to determine whether the observed data support this hypothesis, a statistical test was conducted to test the null hypothesis that the mean number of 'Aggressive' decisions of the monopoly players is 20. The null hypothesis is rejected for both data sets (p-value 5.22E-11). Hence, the following result is obtained:

Result 1: Monopoly players do not always choose to play 'Cooperative'.

According to Hypothesis Ind2, each entrant chooses 'In' 4 times. A statistical test was used to determine whether the data support this null hypothesis that the mean number of 'In' decisions of the entrants is 4. The null hypothesis was rejected for both sets of data (p-value 1.35705E-67). Hence, the following result is obtained:

[3] The total number of decisions per round is 96.

Result 2: Entrants do not always choose 'In'.

It is clear from these two latest results that the experimental data do not support the Induction Hypothesis.

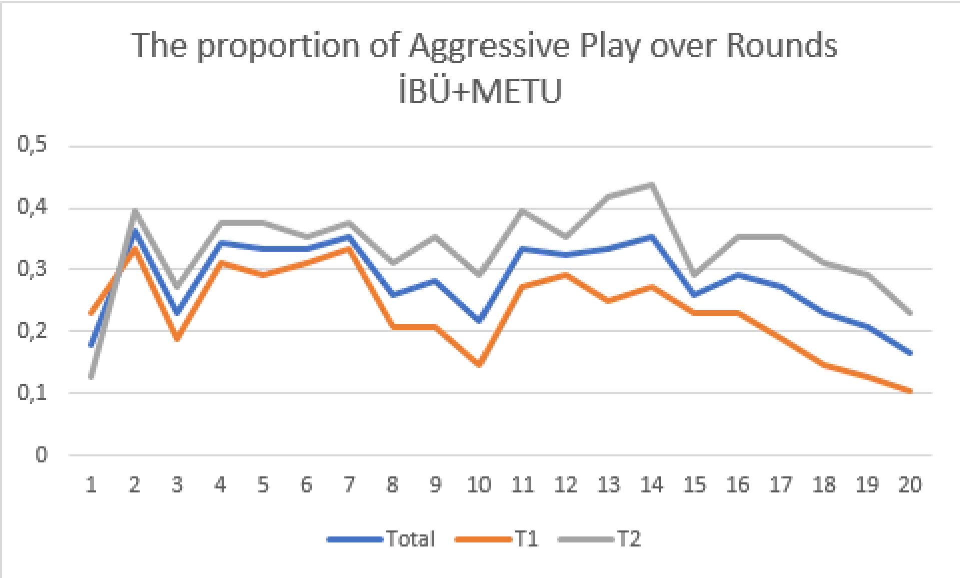
In order to further investigate whether support the Deterrence Hypothesis, we focus next on Hypotheses Det1, Det2, and Det3.

The Deterrence Hypothesis states that the behavior of the entrants is invariant to their order of play (Det2). To test this hypothesis, all the decisions of each round were listed in such a way that the 'Aggressive' ('Cooperative') decision is coded as 1 (0), so that 20 different lists were such constructed. Kruskal-Wallis H (one way) test was conducted on these lists to determine whether there are any statistically significant differences between their means. Rejecting the K-W test, the results show that the behavior of the monopoly players is not invariant across rounds (p-value 0.0147), nor is the behavior of the entrants (p-value 0.0004).

Result 3: The behavior of the monopoly players is not the across in all rounds, nor is the behavior of the entrants the same in all four rounds of play.

It is important to understand whether the variability in the decisions of both monopoly players and entrants is due to the round number, or the change in the information owing to the round number might be the basis of the variability. Later on, I employ logistic regression analysis to answer this question.

The proportion of 'Aggressive' play from round 1 to 16 by the monopoly players significantly exceeds the same proportion in rounds 17-20 (two-sided t-test, p-value 0.00037) (see Graph 5). This result suggests that Hypothesis Det1 might be supported. Note that this difference may also be attributed to end effect [cf. Selten and Stoecker (1986)].

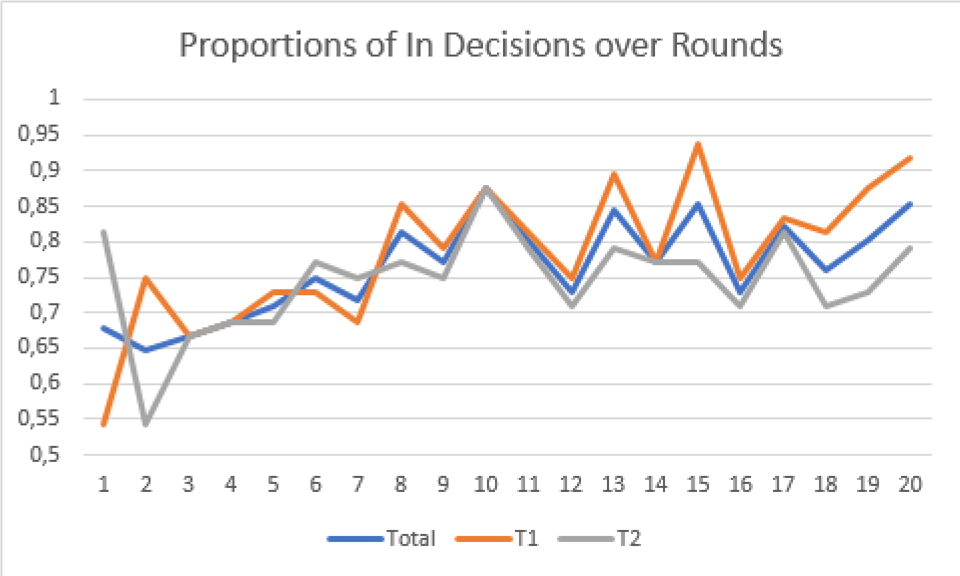


Graph 5. The proportion of Aggressive Play over Rounds

As shown in Graph 5, the proportion of 'Aggressive' play by the monopoly players from round 2 to round 4 [4] is greater than the corresponding proportion in last 4 rounds. This difference is significant (two-sided t-test, p-value 0.0072). Additionally, there is a significant difference between the proportion of 'Aggressive' play from round 5 to round 16 and the corresponding proportion in last 4 rounds (two-sided t-test, p-value 8.59E-05), the proportion between rounds 5 and 16 is greater. Finally, there is not a significant difference between the proportion of 'Aggressive' play by the monopoly players from round 2 to round 4 and the corresponding proportion between the rounds 5 and 16. Hence, we obtain only partial support for Hypothesis Det1.

Result 4: The mean frequency of 'Aggressive' play in the first 16 rounds statistically exceeds mean frequency of 'Aggressive' play in the last 4 rounds.

[4] The first round is interpreted as learning. Accordingly, it is excluded from this stage of the analysis.



Graph 6. The proportion of In Play over Rounds

It is also important to determine whether or not the behavior of the entrants varies over sections. For this purpose, for each section we list the numbers of 'In' responses to each monopoly player during the section. Kruskal-Wallis H test (one way) is conducted on these four lists in order to determine whether there are statistically significant differences between their means. The null hypothesis of equal means across all four sessions is soundly rejected (p-value 4.08E-05).

Result 5: The choices of the entrants are not the same across the four sections.

The present experiment includes two treatments, which differ from each other in their payoff (either 5 or 10) to the monopoly player when the outcome is 'Aggressive-Out'. Increase in the proportion of 'Aggressive' choices is expected to increase with the increase in payoff from 5 to 10, and the proportion of 'In' play is expected to decrease. A Kruskal-Wallis H (one way) test was conducted on the number of 'Aggressive' play and separately on the number of 'In' play to test these two hypotheses about the effect of change in payoff. The results of the test show that monopoly players do not play in the same way in the two treatments (p-value 0.0001). As expected, they support an increase in the proportion of 'Aggressive' play for the monopoly player. However, the expected statistically significant decrease in the 'In' play of the entrants is not supported (p-value 0.28).

Table 2. Pooled Data of T1

Entrant Order	Agg. Frequency	P(Agg)	In Frequency	P(In)
1	11	0,23	26	0,54
2	16	0,33	36	0,75
3	9	0,19	32	0,67
4	15	0,31	33	0,69
5	14	0,29	35	0,73
6	15	0,31	35	0,73
7	16	0,33	33	0,69
8	10	0,21	41	0,85
9	10	0,21	38	0,79
10	7	0,15	42	0,88
11	13	0,27	39	0,81
12	14	0,29	36	0,75
13	12	0,25	43	0,90
14	13	0,27	37	0,77
15	11	0,23	45	0,94
16	11	0,23	36	0,75
17	9	0,19	40	0,83
18	7	0,15	39	0,81
19	6	0,13	42	0,88
20	5	0,10	44	0,92
Total	224		752	
Mean	11,20	0,26	37,60	0,78
S.D	3,33	0,07	4,67	0,10

Table 3. Pooled Data of T2

Entrant Order	Agg. Frequency	P(Agg)	In Frequency	P(In)
1	6	0,13	39	0,81
2	19	0,40	26	0,54
3	13	0,27	32	0,67
4	18	0,38	33	0,69
5	18	0,38	33	0,69
6	17	0,35	37	0,77
7	18	0,38	36	0,75
8	15	0,31	37	0,77
9	17	0,35	36	0,75
10	14	0,29	42	0,88
11	19	0,40	38	0,79
12	17	0,35	34	0,71
13	20	0,42	38	0,79
14	21	0,44	37	0,77
15	14	0,29	37	0,77
16	17	0,35	34	0,71
17	17	0,35	39	0,81
18	15	0,31	34	0,71
19	14	0,29	35	0,73
20	11	0,23	38	0,79
Total	320		715	
Mean	15,50	0,32	35,10	0,73
S.D	3,87	0,08	4,38	0,09

Result 6: In 'Aggressive-Out' games, the choice of the entrants is not affected by the payoff of the monopoly player while the choice of the monopolies is affected.

2.4.1 Logistic Regression Analysis

A logistic regression analysis was conducted to estimate the probability of an entrant choosing 'In' with nine independent variables and the panel data obtained by combining data from İBÜ and METU. The binary dependent variable is 1, if the decision is 'In', and 0, if it is 'Out'. The independent variables are as followings:

#Agg is the number of 'Aggressive' decisions of the current monopoly player in the previous rounds,

#Unc is the number of uncertain decisions of the current monopoly player in the previous rounds[5],

[5]If the payoff of a monopoly player is 1, this means that the entrant of that round plays 'Out'. Hence, this payoff does not contain any information related to the decision of the monopoly

Round represents the number of playing round,

PRP is the payoff gained from the last play,

LRA is a binary variable indicating the last decision if the current monopoly player is 'Aggressive' or not,

LRU is a binary variable indicating if the last decision of the current monopoly player is 'Uncertain' or not,

Order represents the total number of consecutive decisions of the entrants (i.e., 1st or 2nd decision),

Id is the identity number of the student,

Uni indicates in which university the entrant studies (from METU or not).

Table 4. Entrants-Logistic Regression [6]

Model 1: Logit, using 1920 observations
Dependent variable: Action
QML standard errors

	Coefficient	*Std. Error*	*z*	*p-value*	
const	0.910774	0.143919	6.3284	<0.0001	***
Uni	−0.197755	0.113209	−1.7468	0.0807	
LRU	−0.406519	0.146071	−2.7830	0.0054	***
LRA	−0.395584	0.141935	−2.7871	0.0053	***
PRP	0.364364	0.0672427	5.4186	<0.0001	***
Round	0.0869297	0.0196424	4.4256	<0.0001	***
unidentified	−0.106826	0.0438641	−2.4354	0.0149	**
Agg	−0.192578	0.0358211	−5.3761	<0.0001	***

Mean dependent var	0.764062	S.D. dependent var	0.424694
McFadden R-squared	0.059584	Adjusted R-squared	0.051958
Log-likelihood	−986.4918	Akaike criterion	1988.984
Schwarz criterion	2033.464	Hannan-Quinn	2005.350

Number of cases 'correctly predicted' = 1473 (76.7%)
f(beta'x) at mean of independent vars = 0.425
Likelihood ratio test: Chi-square(7) = 125.007 [0.0000]

Table 4 presents the result of logistic regression analysis on the panel data gathered from the entrants in İBÜ and METU. The model is meaningful (Chi-sq.

player.

[6]The number of decisions made by the entrants in a session is 4*20=80. Since 24 sessions were conducted, there are 1920(=80*24) observations.

125.007, p-value $<$ 0.01) and all variables, except Uni, contribute significantly to the probability of 'In' decision. While Round and PRP have positive effects on the probability, the other variables have negative effect.

If LRA (or LRU) takes 0 value, the probability of 'In' decision is higher in comparison to the situation where LRA (or LRU) is 1. That is, the probability of 'In' decision decreases when the entrants observe that the last round action is not 'Cooperative'.

Increase in #Agg observations results in a decrease in the probability of 'In' decision. If #Agg increases by 1 unit, then the odd ratio of 'In' decision decreases by a factor 17%. Hence, it is evident that the entrants are affected by the decisions of the monopoly players and, accordingly, they decide to enter/stay out of the market. Also, the entrants are affected by the undetermined decisions of the monopoly players in the same manner as they are affected by the 'Aggressive' decisions. In other words, increase in #Unc observations decreases the probability of an 'In' decision.

If #Unc increases by 1 unit, then the odd ratio of an 'In' decision decreases approximately by a factor 10%. The entrants are reinforced to enter the market when they observe more 'Cooperative' decisions; alternatively, they decide not to enter the market (i.e., are deterred entering the market) when they observe more 'Aggressive' play. This supports the Deterrence Hypothesis/Argument.

The probability of an 'In' decision increases if Round increases. Whenever Round increases by 1 unit, the odd ratio of 'In' decision increases approximately by a factor 20%.

The other positive-effect-predictor is PRP: The probability of an 'In' decision is higher if PRP is high. This can be interpreted as risk attitude. If there is something in the "pocket", one may assume risk more often.

A second logistic regression analysis was conducted to estimate the probability of a monopoly player choosing 'Aggressive' with seven independent variables on the panel data gathered from İBÜ and METU monopolies. The binary dependent variable is 1, if the decision is 'Aggressive', and 0, if it is 'Cooperative'. The independent variables are the followings:

Agg-In is the number of Agg-In outcomes observed in the previous rounds,

Coop-In is the number of Coop-In outcomes observed in the previous rounds,

Agg-Out is the number of Agg-Out outcomes observed in the previous rounds,

Round represents the number of playing round,

LRI is binary variable indicating whether the last decision of the previous entrant is 'In' or not,

Uni indicates in which university the entrants studies (from METU or not),

Id is the identity number of the student.

Table 5. Monopoly-Logistic Regression [7]

Model 2: Logit, using 1824 observations
Dependent variable: Action
QML standard errors

	Coefficient	*Std. Error*	*z*	*p-value*	
const	−0.87034	0.148123	−5.8758	<0.0001	***
uni	−0.0476695	0.114069	−0.4179	0.6760	
LRI	0.255319	0.132378	1.9287	0.0538	
Coop-In	−0.201414	0.0175322	−11.4882	<0.0001	***
Agg-Out	0.454096	0.0541464	8.3864	<0.0001	***
Agg-In	0.141526	0.0303219	4.6675	<0.0001	***

Mean dependent var	0.288925	S.D. dependent var	0.453388
McFadden R-squared	0.144109	Adjusted R-squared	0.138637
Log-likelihood	−938.5398	Akaike criterion	1889.080
Schwarz criterion	1922.132	Hannan-Quinn	1901.273

Number of cases 'correctly predicted' = 1355 (74.3%)
f(beta'x) at mean of independent vars = 0.453
Likelihood ratio test: Chi-square(5) = 316.05 [0.0000]

Table 5 summarizes the results of the logistic regression analysis of the monopoly player. The model is meaningful (Chi-sq. 316.05, p-value < 0.01) and Agg-In, Agg-Out, and Coop-In are statistically significant predictors of the

[7]The first decisions of the monopoly players cannot be used in the logistic regression anaylsis by the lack of information. There were 96 monopoly players who decided 19 times (except the first decisions), so there are 1824(=96*19) observations.

probability of 'Aggressive' decision. Except Coop-In, the other two variables have positive effect on the probability.

Increase in the number of Agg-In observations causes higher probability of 'Aggressive' decision. If Agg-In increases by 1 unit, the odd ratio of 'Aggressive' decision increases approximately approximately by a factor 40%. The probability of 'Aggressive' decision increases if the number of Agg-Out observations increases. If the number of Agg-Out increases by 1 unit, the odd ratio of 'Aggressive' decision approximately doubles in size. In light of these two results, it is convenient to deduce that the monopoly players support the Deterrence Hypothesis. Precisely, as the number of Agg-In increases, increasing 'Aggressive' play is the way of insisting to convince entrants to decide to stay out of the market and consequently to earn more payoff; i.e., the strategy of reputation building.

An increase in the number of Coop-In observations leads to decrease in probability of 'Aggressive' decision. If the number of Coop-In increases by one unit, the odd ratio of 'Aggressive' decision decreases approximately by a factor 17%. This result, too, supports the Deterrence Hypothesis.

2.5 Discussions

The version of the Chain Store Game proposed by Trockel (1986) was tested experimentally and the data collected in my experiment were analyzed to determine which of the two theories of Selten and Trockel better account for the outcome: The Induction Hypothesis or the Deterrence Hypothesis. My main purpose is to experimentally study the dynamics of play, which aspects of the CSG affect behavior, and whether the same actions are chosen in both Trockel's and Selten's environments that are equivalent in terms of information. Put differently, my goal is to study whether equivalence in information preserves strategic behavior.

My starting point was the question whether equivalence of information in two different games preserves strategic behavior, and I conjecture that the answer is negative by identifying it with the framing effect [cf. Kahneman and Tversky (1985)]. More specifically, Trockel (1986) changes the order of play in each stage game of the original game in an informationally equivalent way, and this change can be interpreted as framing effect. Albeit the Game Theory neglects the difference in the information flow in these two games, there are studies which answer similar questions negatively. Rapoport (1997) conducts three experiments with the strategically equivalent games in extensive form, and the results show that there is a significant difference in how the games are played. Güth et al. (1998) presents support for the importance of order of play. According to the their results, the advantage of a player in a game can be destroyed by changing the order of play.

There are other experimental papers in the literature on the CSG and predatory pricing see, e.g., Camerer and Weigelt (1988), Jung et al. (1994), Rapoport and Sundali (1997), Sundali,Israeli and Janicki (2000). These studies are conducted as paper-and-pencil experiments. Unlike these, I conduct a laboratory experiment. Camerer and Weigelt (1988) and Jung et al. (1994) implemented the setup of Kreps and Wilson (1982). Both papers present clear evidence in support of the effectiveness of price cutting in entry deterrence and the possibility of reputation building in an environment with incomplete information. Rapoport and Sundali (1997) is the only paper in which the original scenario is used, but they do not find clear evidence as to how effective price-cutting is in deterring entry. My design is closer to the design of Experiment 3 in Rapoport and Sundali (1997). The main difference is in the use of the investment. Even if I do not have the data to examine the difference between the design with and the design without the investment, the results are expected to be different in each, based on the endowment effect in the literature [cf. Kahneman and Tversky (1979)]. In Sundali,Israeli and Janicki (2000), the possibility of reputation building for predatory behavior is provided in the environment with complete information.

Rapoport and Sundali (1997) and Sundali (1995) offer suggestions for future studies of the CSG. Some of them call for a more than 20 increase in the number of entrants, another design of a laboratory experiment, and the use of the Trockel's game. All of these suggestions have been implemented in the present study.

My results do not support the Induction Hypothesis. This conclusion is further supported by the post-experiment questionnaire, which shows that the monopoly players tried another way to earn more money, rather than utilize the strategies compatible with the Induction Hypothesis. Thus, the lowest payoff earned by the METU monopoly players is 41 TL, exceeding the 40 TL that might have been earned by adhering to the Induction Hypothesis. Except for five of the İBÜ monopoly players, all gained at least 41 TL. Two of the five İBÜ monopoly players chose 'Cooperative' in the initial rounds and 'Aggressive' in the last few rounds. Moreover, the entrants did not follow the path of the Induction Hypothesis to earn more money or not to lose their investment. The mean entrant payoff is approximately 12 TL less than 40 TL – the amount earned by following the Induction Hypothesis. Based on these results, I conclude that accepting short-term costs to gain a long-term benefit is rational behavior. Also, I infer that price cutting is a viable threat to firms that want to maintain the monopoly position and keep prices as high as possible. As expected by Sundali (1995), Trockel's game presents an environment which allows for reputation building.

At this juncture, the notion of reputation warrants additional clarification. Reputation denotes the beliefs/assumptions of other group members on the possible future decisions of an agent, given her previous decisions. Gates and Humes (1997) write: "reputation is used to manipulate the beliefs of another". If an entrant does not enter the market by examining the past behavior of the monopoly player, I interpret it as evidence for successful reputation building.

The logistic regression that I conducted suggests that both the entrants and the monopoly players considered the history of the session when making their choice. The entrants considered the choices of the monopoly players and, directly or indirectly, their decisions were affected by the round number. The monopoly players considered the outcomes of the previous rounds because they had no information about the entrants. In addition, LRI was not a significant predictor of 'Aggressive' decisions. This suggests that the monopoly players are farsighted. When considered together, these constitute the evidence that successive rounds were not perceived as independent in contrast to the Induction Argument. As the rounds progressed, the information on hand expanded, and the participants played in relation to the information they obtained. Furthermore, the monopoly players, who chose 'Aggressive' in response to the reactions to their past 'Aggressive' decisions, declared that they played in a compatible way with the Deterrence Hypothesis. This is not a surprising result, and is supported by Davis (1985). The Induction Hypothesis ignores what a player thinks about the other players because of the assumption of common knowledge of rationality. Staying out of the market might be more "rational" in certain situations, depending on the reputation of the monopoly player. As Davis (1985) notes, "It cannot be maintained that the Induction Theory is any more game theoretically correct than the Deterrence Theory".

It seems to be easier for monopoly players than for the entrants to accept the idea of reputation building in both the CSG and Trockel's game. This is the case because monopoly players know that they will participate in a 20-stage game in contrast to the entrants who only focus on a single-stage game. Result 6 supports this conjecture. There is a statistically significant increase in the number of 'Aggressive' plays from T1 to T2, while the difference between the outcome payoffs in the two treatments does not affect the entrants. Yet, the results of the logistic regression analysis indicate that the number of 'Non-Cooperative' plays affected the entrants. I explain this by arguing that entrants were aware of the possibility of entry deterrence even if they only made four choices.

3 c-Duality and Anti-c-Duality

The duality concept for coalitional TU-games has a natural interpretation if the game is superadditive. Nevertheless, this concept losses its meaning when it is adopted to the class of non-superadditive games. With this motivation, we present a new definition of the duality notion for TU-games, namely *c-duality* that works for superadditive games in the same way, while preserving its essence when it is extended to the class of not necessarily superadditive games. Moreover, we define the *anti-c-dual* of a coalitional TU-game and investigate whether solution concepts such as the Core and the c-Core of anti-c-dual game can be derived from the original game.

3.1 Introduction

The notion of duality of games with transferable utility is widely used in the literature for analyzing some classical solutions to bankruptcy problems. Aumann and Maschler (1985) use the duality concept to show the existence of certain types of division problems for which it is more appropriate to apply dual solution rules. Herrero and Villar (2001) use the duality relationship to provide new properties and new characterizations of some well-known rules to solve bankruptcy problems from an axiomatic viewpoint. They claim that the new characterizations and properties help to reveal the class of real life problems for which each of these rules might be better. Thomson and Yeh (2008) determine the property of various rules such as constrained equal awards rule, constrained equal losses rule, and Talmud rule, which are preserved under the duality operators.

A coalitional game with transferable utility (hereafter a coalitional TU-game or simply a game) is a pair (N, v) where N is the set of players and where v is a characteristic function that associates every nonempty subset S of N with a real number $v(S)$, called the "worth" of the coalition S. The dual of a coalitional TU-game is usually defined by assigning, to any coalition, what is left from the worth of the grand coalition, after the worth of its complement coalition is deducted. Funaki (1998) uses this duality concept to derive an axiomatization of a dual solution from an axiomatization of some original solution. Oishi and Nakayama (2009) introduce the anti-dual of a coalitional TU-game, obtained by multiplying its dual by -1, and present a class of games that fails to be invariant under the duality operation, but is invariant under anti-duality operation. Oishi et al. (2016) use these notions on solution concepts and axioms, and they find new axiomatizations of some solutions.

In the dual of a game, the worth of a coalition is calculated by substracting the worth of the complement coalition from the worth of the grand coalition in the original game; i.e. $v(N) - v(N \setminus S)$. Oishi and Nakayama (2009) interpret the dual worth of a coalition S as the amount that the complement set cannot prevent S from obtaining in the original game. Hereafter, we call this amount the unavoidable amount of S. However, this natural interpretation of the duality definition is valid only if the game is superadditive. In fact, their interpretation requires the computation of the following: i) What is the maximum worth that can be generated in the original game when the members of S stay together? We will call this maximum worth as the grand value for S. ii) What is the largest harm for S that the set $N \setminus S$ could cause? Hence, the coalition S should consider the case in which the members of the set $N \setminus S$ may form into a coalition structure that creates the maximum aggregate payoff. Since superadditivity is implicitly assumed and it requires that any pair of coalitions is best off by merging into one, the grand value for S is worth the all-player coalition (the so called grand coalition), which is $v(N)$. Moreover, the members in the complement set $N \setminus S$ reach their maximum joint payoff by forming the coalition structure $\{N \setminus S\}$, since they cannot guarantee more for themselves by having their members form subcoalitions under the superadditivity assumption. Therefore, the maximum aggregate worth that they can reach is $v(N \setminus S)$. Hence, the unavoidable amount of S - the part of the grand value for S which cannot be absorbed by the complement set $N \setminus S$ - is the difference between $v(N)$ and $v(N \setminus S)$.

We show in this paper that the unavoidable amount of S may differ in the case in which the game is not superadditive. Indeed, there are plenitude of real-life applications where the emergence of the grand coalition is either not guaranteed, might be perceivably harmful, or is plainly impossible (Sandhlom and Lesser (1997)). Aumann and Dreze (1985) give a detailed explanation for cases where superadditivity of a characteristic function is deficient and provide an insightful discussion on how a different coalition structure than the all-player coalition may arise. They define the superadditive cover of a game as follows: any coalition is assigned to the maximum aggregate worth of subcoalitions over the set of all possible smaller disjoint coalitions. Given the arguments outlined above, when defining the unavoidable amount of S for the games which are not necessarily superadditive, we should consider two things. First, the grand value for S may not be produced by the all-player coalition. Second, there might be a better coalition structure of the set $N \setminus S$ rather than $\{N \setminus S\}$ that allows their members to create larger total payoff than $v(N \setminus S)$ by building a partition of suitable subcoalitions and harvest the worth for each of them.

Based on the arguments above, we present a new duality concept for TU-games, namely *c-duality* which coincides with the standard duality concept and holds the same interpretation as in Oishi and Nakayama (2009) on the class of superadditive games and preserves its essence when it is extended to the class of non-superadditive games. In order to compute the unavoidable amount of a coalition S, we first define the grand value for the coalition S which is the maximum aggregate worth over all partitions of N that do not include proper subsets of S. In other words, we compute for a coalition S what can be achieved maximally by building all potential partitions of N in the case where S made a binding contract to stay together (possibly together with additional players). Next, we calculate the worth of $N \setminus S$ under the superadditive cover of v which is the amount that the players in $N \setminus S$ could maximally get as aggregate payoff by organizing themselves in a partition. In other words, we consider the worst-case scenario for coalition S by calculating the maximum aggregate worth of all potential partitions of the set $N \setminus S$. Hence, when one subtracts the maximum aggregate worth of $N \setminus S$ from the grand value for S, what remains is the unavoidable amount of the coalition S which is what the c-dual of a game v assigns to coalition S.

We also define a new anti-dual operator, *anti-c-dual* for the games which are not necessarily superadditive, and it coincides with the anti-dual operator on the set of superadditive games. We show that *Core* is *self anti-c-dual* on the domain of balanced games, *c-Core* (Sun et al. (2008)) is *self anti-c-dual* on the domain of c-balanced games. It is also shown that c-duality/anti-c-duality meets some of the properties satisfied by duality/ anti-duality while it fails to fulfill some of them.

The paper is organized as follows: in Section 2, we define basic concepts and introduce the *c-duality* (and corresponding the *anti-c-duality*) concept. In Section 3, we provide some concrete examples. In Section 4, we show that *Core* and *c-Core* are *self-anti-c-dual* on the domain of balanced games and c-balanced games, respectively.

3.2 Definitions and Notations

For a finite set N, the set $\mathbb{R}^N$ denotes the $|N|$-dimensional Euclidean space where the coordinates are indexed by the elements in N. Let $N = \{1, 2, ..., n\}$ be the set of players. A coalitional game with transferable utility is a pair (N, v) where v is the characteristic function such that $v : 2^N \to \mathbb{R}$ with $v(\emptyset) = 0$. Here, 2^N represents all possible coalitions that the agents in N can form and $v(S)$ represents the amount of transferable utility that S gets by acting together, so that the members of S can distribute $v(S)$ among themselves. A payoff vector x of (N, v) is an element of $\mathbb{R}^N$.

Definition 1 *Given a game (N, v), the dual of (N, v) is the game (N, v^*) such that for any coalition $S \subseteq N$,*

$$v^*(S) = v(N) - v(N \setminus S).$$

.

Definition 2 *Given a game (N, v), the completion v^c of v as follows:*

- $v^c(S) = v(S)$ *for* $S \subset N$
- $v^c(N) = \bar{v}(N)$ [8]

where $\mathbb{P}(N)$ is the set of partitions on N.

Definition 3 *Given a game (N, v), the c-dual of (N, v) is the game (N, v°) such that for any coalition S*

$$v^\circ(S) = v^S(N) - \bar{v}(N \setminus S)$$

where $v^S(N) = \max_{P \in \Delta_S} \sum_{T \in P} v(T)$ which is the "grand" value of S and the set of partitions in which S stay together $\Delta_S = \{P \in \mathbb{P}(N) | T \cap S \in \{\emptyset, S\}$ for all $T \in P\}$.

The characteristic function v° of the c-dual game assigns $v^\circ(S)$ to a coalition S, which is the part of the maximum worth of the game which cannot be absorbed by the complement set $N \setminus S$ via $\bar{v}(N \setminus S)$

Definition 4 *Given a game (N, v), the anti-c-dual of (N, v) is the game $(N, -(v^c)^\circ)$.*

3.3 Examples

Example 1 *For any N, define v such that $v(S) = 1$ for all nonempty $S \subseteq N$.*

The dual of this non-superadditive game, (N, v^*) is given by $v^*(S) = 0$ for all $S \subset N$ and $v^*(N) = 1$. In fact, any coalition S can get at least 1 by cooperating, so that the other players $N \setminus S$ cannot prevent S from obtaining 1. Then $v^*(S)$ does not express the unavoidable amount of S.

Let's consider c-dual game (N, v°). For all nonempty S, $v^\circ(S) = 1$ since the grand value of S $v^S(N) = 1 + |N \setminus S|$ and $\bar{v}(N \setminus S) = |N \setminus S|$.

[8] $\bar{v}(N)$ is the worth of the grand coalition in the superadditive hull $(N, \bar{v})$ of (N, v) defined by $\bar{v}(S) = \max_{\pi \in \mathbb{P}(S)} \sum_{C \in \pi} v(C)$ for all $S \subseteq N$.

The following example is to show that the two duality notions coincide on superadditive games.

Example 2 *Let* $N = \{1, 2, 3\}$. *Consider the superadditive game* (N, v) *where* $v(S) = 0$ *if* $|S| \leq 1$, *else* $v(S) = 1$. *Its dual game is given by* $v^*(S) = 0$ *if* $|S| \leq 1$, *else* $v^*(S) = 1$.

Consider the c-dual game (N, v°). *Let's start with* $v^S(N)$ *where* S *is singleton. Wlog, choose* $S = \{1\}$. *The grand value of* $\{1\}$ *is 1 since* $\Delta_{\{1\}} = \{\{N\}, \{\{1,2\},\{3\}\}, \{\{1,3\},\{2\}\}, \{\{1\},\{2,3\}\}, \{\{1\},\{2\},\{3\}\}\}$ *and* $\bar{v}(\{2,3\}) = 1$. *Hence,* $v^\circ(S) = 1 - 1 = 0$ *for all* S *singleton.*

Now, consider S *with two elements. Wlog, choose* $S = \{1, 2\}$. *The grand value of* $\{1, 2\}$ *is 1 since* $\Delta_{\{1,2\}} = \{\{N\}, \{\{1,2\},\{3\}\}\}$ *and* $\bar{v}(\{3\}) = 0$. *Thus* $v^\circ(S) = 1 - 0 = 1$ *for all* S *with two elements.*

It is clear that $v^\circ(N) = 1$ *since* $\Delta_N = \{\{N\}\}$ *and* $\bar{v}(\emptyset) = 0$. *Hence the dual and the c-dual of the given game is the same.*

3.4 Some Results on c-Duality and Anti-c-Duality

The c-duality and duality coincide on the domain of superadditive games.

Proposition 1 *If* (N, v) *is superadditive,* $v^\circ(S) = v(N) - v(N \setminus S) = v^*(S)$ *for all* $S \subseteq N$.

Proof *For any* $S \subseteq N$,

$v^\circ(S) = v^S(N) - \bar{v}(N \backslash S) \leq v^S(N) - v(N \backslash S) \underset{\Delta_S \subseteq \mathbb{P}(N)}{\leq} \max_{P \in \mathbb{P}(N)} \sum_{T \in P} v(T) - v(N \setminus S) = \bar{v}(N) - v(N \setminus S)$.

The superadditivity of (N, v) *implies* $\bar{v}(N) = v(N)$, *so* $v^\circ(S) = v(N) - v(N \setminus S) = v^*(S)$. $\square$

Convex cooperative games capture the intuitive property: "the incentives for joining a coalition increase as the coalition grows". Formally, a game is convex if $v(S \cup T) + v(S \cap T) \geq v(S) + v(T)$, for all $S, T \subseteq N$. If the inequality is reversed, then the resulting game is called concave. Oishi and Nakayama (2009) show that a game is convex if and only if its dual game is concave, or equally, its anti-dual is convex. We show that the c-dual game is concave and its anti-c-dual game is convex if the game is convex.

Proposition 2 *If (N, v) is a convex game, then the c-dual game (N, v°) is concave.*

Proof *If a game is convex, then it is superaddtive. By Proposition 1, $v^\circ(S) = v(N) - v(N \setminus S)$ for all $S \subseteq N$.*

Let S, T be two coalitions.

$$\begin{aligned} v^\circ(S) + v^\circ(T) &= v(N) - v(N \setminus S) + v(N) - v(N \setminus T) \\ &= 2v(N) - (v(N \setminus S) + v(N \setminus T)) \\ &\underset{convexity}{\geq} 2v(N) - (v(N \setminus (S \cup T)) + v(N \setminus (S \cap T))) \\ &= v^\circ(S \cup T) + v^\circ(S \cap T). \square \end{aligned}$$

Proposition 3 *If (N, v) is a convex game, then so is its anti-c-dual game $(N, -(v^c)^\circ)$.*

Proof *It is clear that (N, v^c) is convex when (N, v) is convex. Hence, (N, v^c) is superadditive game. Then by proposition 1, $(v^c)^\circ(S) = v^c(N) - v^c(N \setminus S) = \bar{v}(N) - v(N \setminus S)$ for all non-empty S and $(v^c)^\circ(\emptyset) = 0$.*

Let S, T be two coalitions suct that $S \cap T \neq \emptyset$.

$$\begin{aligned} &(v^c)^\circ(S \cap T) + (v^c)^\circ(S \cup T) = \bar{v}(N) - v(N \setminus (S \cap T)) + v(N) - v(N \setminus (S \cup T)) \\ &= 2\bar{v}(N) - (v((N \setminus S) \cap (N \setminus T)) + v((N \setminus S) \cup (N \setminus T))) \\ &\underset{convexity}{\leq} 2\bar{v}(N) - v(N \setminus S) - v(N \setminus T) = (v^c)^\circ(S) + (v^c)^\circ(T). \end{aligned}$$

If $S \cap T = \emptyset$, $(v^c)^\circ(S \cap T) + (v^c)^\circ(S \cup T) = (v^c)^\circ(S \cup T)$

$$\begin{aligned} &= \bar{v}(N) - v(N \setminus (S \cup T)) \\ &\leq \bar{v}(N) - v(N \setminus (S \cup T)) + \bar{v}(N) - v(N) \\ &= 2\bar{v}(N) - (v(N \setminus (S \cup T)) + v(N \setminus (S \cap T))) \\ &\underset{convexity}{\leq} 2\bar{v}(N) - v(N \setminus S) - v(N \setminus T) = (v^c)^\circ(S) + (v^c)^\circ(T) \end{aligned}$$

Hence, $(N, -(v^c)^\circ)$ is convex game. $\square$

By the well-known core existence theorem by Shapley (1971), we have the following result:

$$Core(N, -(v^c)^\circ) \neq \emptyset \text{ if } (N, v) \text{ is convex game.}$$

Oishi and Nakayama (2009) implicitly show in Theorem 2 that a game is balanced if its anti-dual is balanced, and vice versa. For c-duality, we have only the sufficient part of this statement.

Proposition 4 *If (N, v) is balanced, then $(N, -(v^c)^\circ)$ is balanced.*

Proof *Let (N, v) be balanced. Then by the Bondareva-Shapley Theorem, $Core(N, v) \neq \emptyset$.*

Let $x \in Core(N, v)$. For any coalition S, denote the sum of payoffs of all members of S by $x(S)$; i.e., $x(S) = \sum\limits_{i \in S} x_i$.

Then $x(N) = v(N) = \bar{v}(N)$(balancedness implies completeness) and $x(S) \geq v(S)$ for all $S \subseteq N$.

If $Core(N, -(v^c)^\circ) \neq \emptyset$, we are done. Consider $-x$.

$-x(N) = -\bar{v}(N) = -v^c(N) = -(v^c)^N(N) = -(v^c)^\circ(N)$.

$-(v^c)^\circ(S) = -(v^c)^S(N) + \overline{v^c}(N \setminus S) \leq -v^S(N) + \bar{v}(N \setminus S)$

$\underset{N \in \Delta_S}{\leq} -v(N) + \max\limits_{P \in \mathbb{P}(N \setminus S)} \sum_{A \in P} v(A)$

$\underset{x(A) \geq v(A)}{\leq} -v(N) + \max\limits_{P \in \mathbb{P}(N \setminus S)} \sum_{A \in P} x(A) = -v(N) + x(N \setminus S)$

$= -x(N) + x(N \setminus S) = -x(S)$ *for any coalition S.*

Thus $-x \in Core(N, -(v^c)^\circ)$. $\square$

Remark *By Proposition 4, we have the following results:*

- *The set of balanced games is invariant under the anti-c-dual operator,*
- *$Core(N, v)$ and $Core(N, -(v^c)^\circ)$ are non-empty if (N, v) is a balanced game.*

Furthermore, the Core of the original game and the -Core of its anti-c-dual are the same if the game is balanced.

Proposition 5 *If* (N, v) *is a balanced game, then* $Core(N, v) = -Core(N, -(v^c)^\circ)$.

Proof *By Proposition 4, we have* $Core(N, v) \subseteq -Core(N, -(v^c)^\circ)$.

Let $x \in Core(N, -(v^c)^\circ)$.

Then $x(N) = -(v^c)^\circ(N) = -v(N)$ *and* $x(S) \geq -(v^c)^\circ(S)$.

Consider $-x$.

$-x(N \setminus S) \underset{hypothesis}{\leq} (v^c)^\circ(N \setminus S) = (v^c)^{N \setminus S}(N) - \overline{v^c}(N \setminus (N \setminus S)) \leq \bar{v}(N) - \bar{v}(S) \leq \bar{v}(N) - v(S) = v(N) - v(S) = -x(N) - v(S)$.

Then $-x(N \setminus S) \leq -x(N) - v(S)$ *which implies* $x(S) \leq -v(S)$.

Since $-x(S) \geq v(S)$ *and* $-x(N) = v(N)$, *we have* $-x \in Core(N, v)$. □

Definition 5 *(Oishi et al. (2016)) The anti-c-dual of a solution* ϕ *on TU games is denoted by* ϕ^{acd} *and defined as* $\phi^{acd}(N, v) = -\phi(N, -(v^c)^\circ)$. *A solution* ϕ *is called self-anti-c-dual on a set* V *if* $\phi(N, v) = \phi^{acd}(N, v)$ *for all* $v \in V$ *where* V *is a set of TU games which is invariant under the anti-c-dual operator.*

By Proposition 5, we can conclude that *Core* is self-anti-c-dual on the domain of balanced games.

Definition 6 *(Sun et al. (2008)) The solution concept* $c - Core$ *is defined as the set of all payoff vectors* x *of* (N, v^c) *which are coalitionally rational for all* $S \subseteq N$ *and satisfy* $x(N) = v^c(N) = \bar{v}(N)$.

Hence *c-Core* of a TU game is $Core$ of its completion:

$$c - Core(N, v) = \{x \in \mathbb{R}^N | \; x(S) \geq v(S) \text{ for all } S \subseteq N \text{ and } x(N) = v^c(N) = \bar{v}(N)\}$$

The solution concept $c - Core$ leads to a new class of games : A TU game is called *c-balanced* if its completion is balanced. The Bondavera-Shapley Theorem implies that a TU game (N, v) has a non-empty $c - Core$ if and only if it is c-balanced.

In the following results, we show that the set of c-balanced games is invariant under the anti-c-dual operator, but the *Core* is not self-anti-c-dual on this set.

Proposition 6 *If* (N, v) *is a c-balanced game, so is* $(N, -(v^c)^\circ)$.

Proof *We know* (N, v^c) *and its anti-c-dual are balanced games. The characteristic function of the anti-c-dual game is* $-((v^c)^c)^\circ = -(v^c)^\circ$ *on any coalition, by the completeness of* (N, v^c). *Hence,* $(N, -(v^c)^\circ)$ *is a balanced game.*

Since a balanced game is also c-balanced, $(N, -(v^c)^\circ)$ *is c-balanced game.*

Proposition 7 *The Core is not self-anti-c-dual on the set of c-balanced games.*

Proof *Let* (N, v) *be a c-balanced TU game, but not balanced.*

$Core^{acd}(N, v) = -Core(N, -(v^c)^\circ) = -Core(N, -((v^c)^c)^\circ) = Core(N, v^c) \neq \emptyset$, *but* $Core(N, v) = \emptyset$.

Hence Core is not self-anti-c-dual on c-balanced games.

Combining the results above, $c - core(N, v) = -Core(N, -(v^c)^\circ) = -c - Core(N, -(v^c)^\circ) = c - Core^{acd}(N, v)$ if (N, v) is a c-balanced game. Hence unlike *Core*, *c-Core* is self-anti-c-dual on the set of c-balanced games.

3.5 Conclusion

The duality concept for games have been recently used in Oishi and Nakayama (2009) and Oishi et al. (2016) to develop the duality for solution concepts. Oishi and Nakayama (2009) propose the anti-duality notion under which some important classes of games are invariant while they are not invariant under the duality operator. Oishi et al. (2016) define self-anti-dual solutions, and show that some solution concepts satisfy self-anti-duality.

The interpretation in Oishi and Nakayama (2009) related to the worth of a coalition S under the dual game cannot make sense unless the game is superadditive. In order to extend the duality concept to the set of all TU games and so to strengthen the concept, in this paper, we introduce c-duality which keeps the same intuition as the duality concept on the set of superadditive games. Moreover, it maintains this natural intuition on the set of non-superadditive games.

The need for this notion arises from the inadequacy of the standard duality concept on non-superadditive games.

Furthermore, we present the anti-c-dual of a solution in the spirit of Oishi et al. (2016), then show that some of the properties which are satisfied by duality (anti-duality) are also satisfied by c-duality (anti-c-duality), while some of them are not. By Oishi and Nakayama (2009), it is known that a game is convex if and only if its anti-dual is convex (equally its dual is concave), and a game is balanced if and only if its anti-dual is balanced. For c-duality and anti-c-duality, we have the ”only if” part of the above statements, respectively. The balancedness immediately leads to the following results for c-duality: Core of the anti-dual game is not empty if the game is balanced. According to Oishi et al. (2016), the set of balanced games is invariant under the anti-dual operator. We have the same result for the c-dual operator. By using the balancedness property of duality, Oishi et al. (2016) show that Core is self-anti-dual on the set of balanced games. Self-anti-c-duality leads to the same property: Core is self-anti-c-dual on the set of balanced games.

The analysis of Oishi et al. (2016) covers the following assumption: the grand coalition eventually is formed. By dropping this assumption, we investigate whether the duality approach is applicable to the solution c-Core (Guesnerie and Oddou (1979), Sun et al. (2008)). We point out that the set of c-balanced games is invariant under the anti-c-dual operator and c-Core is self-anti-c-dual on the set of c-balanced games and on the set of balanced games.

4 References

Section I

Bergin, J., & Duggan, J. (1999). An implementation-theoretic approach to non-cooperative foundations. Journal of Economic Theory, 86(1), 50-76.

Binmore, K. (1980). The Nash bargaining theory II. : Chapter 4 in Binmore and Dasgupta (eds): The Economics of Bargaining, Oxford, Basil Blackwell : 61-76.

Binmore, K., & Dasgupta, P. (1987). Economics of bargaining. B. Blackwell.

Binmore, K., & Dasgupta, P. (1987). Nash Bargaining Theory: An Introduction. in: Binmore, Ken and Dasgupta, Partha (Eds.). The Economics of Bargaining. 1987a: Oxford: Basil Blackwell, 1-24.

Binmore, K., Rubinstein, A., & Wolinsky, A. (1986). The Nash bargaining solution in economic modelling. The RAND Journal of Economics, 176-188.

Binmore, K., Osborne, M. J., & Rubinstein, A. (1992). Noncooperative models of bargaining. Handbook of game theory with economic applications, 1, 179-225.

Bishop, R. L. (1963). Game-theoretic analyses of bargaining. The Quarterly Journal of Economics, 77(4), 559-602.

von Böhm-Bawerk, E. (1891). The positive theory of capital. GE Stechert.

Burnet, J. (Ed.). (1900). The ethics of Aristotle. Methuen.

Crawford,V.P. (2013). Behavioural Economics: Theory and Evidence on Bargaining. University of Oxford, Michaelmas Term 2013 and Asia Summer Institute in Behavioural economics, National University of Singapore,22 July 2 August 2013.

Cross, J.G. (1965). A theory of the bargaining process. American Economic Review 55 (1): 67 94.

Dagan, N., & Serrano, R. (1998). Invariance and randomness in the Nash program for coalitional games. Economics Letters, 58(1), 43-49.

Dos Santos Ferreira, R. (2002). Aristotles analysis of bilateral exchange: an early formal approach to the bargaining problem. European Journal of History of Economic Thought 9.4: 568 - 590.

Edgeworth,F. Y. (1881). Mathematical Psychics: An Essay on the Application of Mathematics to the Moral Sciences. reprinted 1961 New York: A.M. Kelley.

Gerber, A., & Upmann, T. (2006). Bargaining solutions at work: Qualitative differences in policy implications. Mathematical Social Sciences, 52(2), 162-175.

Gth, W., & Kocher, M. G. (2014). More than thirty years of ultimatum bargaining experiments: Motives, variations, and a survey of the recent literature. Journal of Economic Behavior & Organization, 108, 396-409.

Harsanyi, J. C. (1956). Approaches to the bargaining problem before and after the theory of games: A critical discussion of Zeuthen's, Hicks', and Nash's theories. Econometrica 24.2: 144 - 157.

Harsanyi, J. C. (1974). An equilibrium-point interpretation of stable sets and a proposed alternative definition. Management Science 20.11 : 1472-1495.

Hicks, J. R. (1932). The Theory of Wages. New York: Macmillan.

Howard, J. V. (1992). A social choice rule and its implementation in perfect equilibrium. Journal of Economic Theory 56.1: 142-159.

Hurwicz, L. (1994). Economic design, adjustment processes, mechanisms, and institutions. Review of Economic Design 1.1: 1-14.

Krelle, W. (1975). A New Theory of Bargaining. Working Paper 70, Institut fr Gesellschaftschafts - und Wirtschaftswissenschaften, Universitt Bonn.

Krelle, W. (1976). Preistheorie (Part 2). Tübingen: JCB Mohr (Paul Siebeck).

Marx, K. (1867). Das Kapital: Kritik der politischen konomie. Berlin: Dietz Verlag 1962.

Mas-Collel, W., & Green, M. JR (1995). Microeconomic Theory.

Menger, C. (1871). Grundsätze der Volkswirtschaftslhre. Wien 2.Auflage 1923.

Moulin, H. (1984). Implementing the Kalai-Smorodinsky bargaining solution. Journal of Economic Theory 33.1: 32-45.

Muthoo, A. (1999). Bargaining Theory with Applications. Cambridge University Press.

Myerson, R. B. (1991). Game theory: Analysis of Conflict. Harvard University.

Nash, J. (1950). The bargaining problem. Econometrica 18.2: 155 - 162.

Nash, J. (1951). Non-cooperative games. Annals of Mathematics: 286-295.

Nash, J. (1953). Two-person cooperative games. Econometrica: 128-140.

Naeve, J. (1999). Nash implementation of the Nash bargaining solution using intuitive message spaces. Economics Letters 62.1: 23-28.

Naeve-Steinweg, E. (1999). A note on van Damme's mechanism. Review of Economic Design 4.2: 179-187.

Naeve-Steinweg, E. (2002). Mechanisms supporting the KalaiSmorodinsky solution. Mathematical Social Sciences 44.1: 25-36.

Napel, S. (2002). Bilateral Bargaining. Lecture Notes in Economics and Mathematical Systems, No. 518, Berlin: Springer Verlag.

Nydegger, R. V., & Owen, G. (1974). Two-person bargaining: An experimental test of the Nash axioms. International Journal of game theory, 3(4), 239-249.

Ochs, J., & Roth, A. E. (1989). An experimental study of sequential bargaining. The American Economic Review, 355-384.

Osborne, M. J., & Rubinstein, A. (1990). Bargaining and markets. Academic press.

Osborne, M. J., & Rubinstein, A. (1994). A course in game theory. MIT press.

Peters, H. (1992). Axiomatic Bargaining Theory. Kluwer Academic Press.

Peters, H. (2015). Game Theory. A Multi-Leveled Approach, Second Edition. Springer-Verlag, Berlin Heidelberg.

Roth, A. (1979). Axiomatic Models of Bargaining. Lecture Notes in Economics and Mathematical Systems, No. 170, Berlin: Springer Verlag.

Roth, A. (1995). Bargaining Experiments. In: Kagel, J., Roth, A. (eds.): Handbook of Experimental Economics. Princeton University Press.

Rubinstein, A. (1982). Perfect equilibrium in a bargaining model. Econometrica: 97-109.

Selten, R. (1965). Pieltheoretische Behandlung eines Oligopolmodells mit Nachfrageträgheit: Teil 1: Bestimmung des dynamischen Preisgleichgewichts. Zeitschrift für die gesamte Staatswissenschaft/Journal of Institutional and Theoretical Economics: 301-324.

Serrano, R. (1997). A comment on the Nash program and the theory of implementation. Economics Letters 55.2: 203-208.

Serrano, R. (2004). Fifty years of the Nash program, 1953-2003. Investigaciones Economicas 29: 219-258.

Shubik, M. (1985). Game Theory in the Social Sciences. Concepts and Solutions. Cambridge, Mass.: The MIT Press.

Siegal, S., & Fouraker, L. E. (1960). Bargaining and group decision making: Experiments in bilateral monopoly.

Simon,H. A. (1955). A behavioral model of rational choice. Quarterly Journal of Economics 69.1: 99 - 118.

Ståhl, I. (1972). Bargaining theory. Economics Research Institute at the Stockholm School of Economics.

Stuart,J. A. (1892). Notes on the Nichomachean Ethics of Aristotle. Oxford,Clarendon Press.

Sudhlter, P., Rosenmller, J., & Peleg, B. (2000). The canonical extensive form of a game form: Part II. Representation. Journal of Mathematical Economics, 33(3), 299-338.

Thomson, W. (1994). Cooperative Models of Bargaining. Chapter 35 in: Aumann and Hart (eds.): Handbook of Game Theory, Volume II Amsterdam, Elsevier.

Thomson, W., & Lensberg, T. (2006). Axiomatic theory of bargaining with a variable number of agents. Cambridge University Press.

Trockel, W. (1996). A Walrasian approach to bargaining games. Economics Letters 51.3: 295-301.

Trockel, W. (2000). Implementations of the Nash solution based on its Walrasian characterization. Economic Theory 16.2: 277-294.

Trockel, W. (2002). Integrating the Nash program into mechanism theory. Review of Economic Design 7.1: 27-43.

Trockel, W. (2002). A universal meta bargaining implementation of the Nash solution. Social Choice and Welfare 19.3: 581-586.

Trockel, W. (2003). Can and should the Nash program be looked at as a part of mechanism theory?. Murat R. Sertel and Semih Koray (eds): Advances in Economic Design, Springer-Verlag, Berlin Heidelberg: 153-174.

Trockel, W. (2011). An exact non-cooperative support for the sequential Raiffa solution. Journal of Mathematical Economics 47(1): 77-83.

Turgot, A. R. J. (1766). Réflexions sur la Formation et la Distribution des Richesses. in: Formation et la Distribution des Richesses. Textes choisis et présentés par Joel-Thomas Ravix et Paul-Marie Romani.Paris: Flammarion 1997.

Turgot, A. R. J. (1769). Valeurs et Monnaies. in: Formation et la Distribution des Richesses. Textes choisis et présentés par Joel-Thomas Ravix et Paul-Marie Romani.Paris: Flammarion 1997.

van Damme, E. (1986). The Nash bargaining solution is optimal. Journal of Economic Theory 38.1: 78-100.

van Damme, E. (1991). Stability and Perfection of Nash Equilibria. Berlin: Springer-Verlag.

Young, H.P. (1993). An Evolutionary Model of Bargaining. Econometrica 61: 57 84.

Yaari, M. E., & Bar-Hillel, M. (1984). On dividing justly. Social choice and welfare, 1(1), 1-24.

Zeuthen, F. (1930). Problems of Monopoly and Economic Warfare. London:Routledge and Sons.

Section II

Camerer, C., & Weigelt, Keith. (1988) Experimental tests of a sequential equilibrium reputation model. Econometrica: Journal of the Econometric Society: 1-36.

Davis, L. H. (1985) No chain store paradox. Theory and Decision 18.2: 139-144.

Fischbacher, U. (2007) z-Tree: Zurich toolbox for ready-made economic experiments. Experimental economics 10.2: 171-178.

Gates, S., & Humes, B. D. (1997) Games, information, and politics: Applying game theoretic models to political science. University of Michigan Press.

Greiner, B. (2004) The online recruitment system ORSEE 2.0. A Guide for the Organization of Experiments in Economics.

Güth, W., Huck, S., & Rapoport, A. (1998). The limitations of the positional order effect: Can it support silent threats and non-equilibrium behavior?. Journal of Economic Behavior & Organization, 34(2), 313-325.

Jung, Y. J., Kagel, J.H., & Levin, D. (1994) On the existence of predatory pricing: An experimental study of reputation and entry deterrence in the chain-store game. The RAND Journal of Economics: 72-93.

Kahneman, D., & Tversky, A. (1979) Prospect theory: An analysis of decision under risk. Econometrica: Journal of the econometric society: 263-291.

Tversky, A., & Kahneman, D. (1985). The framing of decisions and the psychology of choice. In Environmental Impact assessment, technology assessment, and risk analysis (pp. 107-129). Springer, Berlin, Heidelberg.

Kreps, D. M., & Wilson, R. (1982) Reputation and imperfect information. Journal of economic theory 27.2: 253-279.

Malinvaud, E. (1952) Note on von Neumann-Morgenstern's strong independence axiom. Econometrica 20.4 : 679.

Milgrom, P., & Roberts, W. (1982) Predation, reputation, and entry deterrence. Journal of economic theory 27.2: 280-312.

Rapoport, A. (1997). Order of play in strategically equivalent games in extensive form. International Journal of Game Theory, 26(1), 113-136.

Rapoport, A., & Sundali, J. A. (1997) Induction vs. Deterrence in the Chain Store Game: How Many Potential Entrants Are Needed to Deter Entry?. Understanding Strategic Behavior: Essays in Honor of Reinhard Selten.

Selten, R. (1978) The chain store paradox. Theory and decision 9.2: 127-159.

Selten, R., & Stoecker, R. (1986) End behavior in sequences of finite Prisoner's Dilemma supergames: A learning theory approach. Journal of Economic Behavior & Organization 7.1: 47-70.

Sundali, J. A. (1995) An experimental investigation of market entry problems.

Sundali, J., Israeli, A., & Janicki, T. (2000). Reputation and deterrence: Experimental evidence from the Chain Store Game. The Journal of Business and Economic Studies, 6(1), 1.

Trockel, W. (1986) The chain-store paradox revisited. Theory and decision 21.2: 163-179.

Section III

Aumann, R.J. & Dreze, J.H. (1985). Cooperative games with coalition structures. International Journal of game theory 3.4: 217-237.

Aumann, R.J. & Maschler, M. (1985). Game theoretic analysis of a bankruptcy problem from the Talmud. Journal of economic theory 36.2: 195-213.

Bejan, C. & Gomez, J.C. (2012). Axiomatizing core extensions. International Journal of Game Theory 41.4: 885-898.

Funaki, Yukihiko. (1998). Dual axiomatizations of solutions of cooperative games. mimeo.

Guesnerie, R. & Oddou, C. (1979). On economic games which are not necessarily superadditive: solution concepts and application to a local public good problem with few a agents. Economics Letters 3.4: 301-306.

Herrero, C. & Villar, A. (2001). The three musketeers: four classical solutions to bankruptcy problems. Mathematical Social Sciences 42(3): 307-328.

Littlechild, Stephen C., & Owen, G. (1973). A simple expression for the Shapley value in a special case. Management Science 20.3: 370-372.

Myerson, R. B. (1991). Game theory: analysis of conict. Harvard University.

Oishi, T. & Nakayama, M. (2009). Antidual of economic coalitional TU Games. Japanese Economic Review 60.4: 560-566.

Oishi, T., Nakayama, M., Hokari, T. & Funaki, Y. (2016). Duality and anti-duality in TU games applied to solutions, axioms, and axiomatizations. Journal of Mathematical Economics 63: 44-53.

Sandhlom, T. W., & Lesser, V. R. (1997). Coalitions among computationally bounded agents. Artificial intelligence, 94(1-2), 99-137.

Shapley, L. S. (1971). Cores of convex games. International journal of game theory 1.1: 11-26.

Sun, N., Trockel, W. & Yang, Z. (2008). Competitive outcomes and endogenous coalition formation in an n-person game. Journal of Mathematical Economics 44.7: 853-860.

Thomson, W. & Yeh, C. (2008). Operators for the adjudication of conflicting claims. Journal of Economic Theory 143.1: 177-198.

Appendix

The instructions and the questionnaires of Section II are translated from Turkish.

Participant Instructions

Thank you very much for your participation.

This is a decision-making experiment and your decisions will be used in a scientific project. During the experiment, in each step you will make a decision, there is no ”correct” or ”incorrect” answer, the only answer is ”your” answer. The decisions you made, your answers and your earnings will not be matched with your name.

Your experiment earnings depend on the decisions you made, and it is explained how your earnings will be determined in this experiment booklet in details. That is why it is important to follow/read the instructions.

Until the experiment is finished, it is forbidden to communicate with the other participants in any possible way. If you have a question or a problem, please raise your hand and wait for the experimenter. Please do not ask your questions aloud and avoid distracting the others. Please turn your mobiles' volume and vibration off.

Types of Participants

There are two types of participants: Type A and Type B. These types will be assigned in the beginning of the experiment and will not change during the experiment.

Four of 24 participants in this room will be type A, the remaining 20 participants will be type B.

To each participant, also an positive number will be assigned randomly with respect to her type and this order number will not be changed during the experiment. The purpose of this number is to save/follow your decisions. So the participants in the room are A-1, A-2, A-3, A-4 and B-1, B-2, B-3, . . . , B-20.

In-experiment Matchings

The experiment contains 20 rounds. 4 type A participants will be active in all 20 rounds. 20 type B participants will be active only in 4 rounds. When the experiment starts, it will be determined randomly in which rounds which type B participants will be active, which type B participants will wait and whenever they are active, they will be matched with which of type A participants. Then, all type B participants will be informed on this via their screen. The periods and how the experiment will proceed are explained in details in the next sections.

In each round, all type A participants and 4 type B participants chosen by the computer will be active. So 4 encounters will take place in every round. The remaining 16 type B participants will wait until the next round in which they will be active. Each of the 16 type B participants can see in which rounds she will be active and which of type A participants she will face when she is active.

Each type A faces with a different type B participant in each round, so they face with all 20 type B participants one by one. Also, each type B participant faces with all type A participants; i.e., whenever she is active, the ”opponent” type A participant is a different participant.

For type A participants, the experiment only contains 20 rounds in which they face 20 type B participants and is independent from the matches of other type A participants. For this reason, they are not informed about the other type A participants' matches.

Periods

In each round, each of the activated participants will be given 1 point as a beginning point. So every type A participant will be given 20 points in total and every type B participant will be given 4 points in total. You can change your points by the decisions you will make during the experiment.

The experiment starts with the 1st round. Each of the activated participants' points are activated. Type A participants start first. They are asked to decide between 'Z' and 'T' and their decisions will not be announced to any of the participants (including actives and not actives), will be kept secret. After all type

A participants decide, the active type B participants decide. They are asked to decide between 'X' and 'Y' without knowing the decision of the related type A participant.

	A's Point(s)	B's Point(s)
A's Decision is 'Z' & B's Decision is 'X'	0	0
A's Decision is 'Z' & B's Decision is 'Y'	5	1
A's Decision is 'T' & B's Decision is 'X'	2	2
A's Decision is 'T' & B's Decision is 'Y'	5	1

- If the decision of type A participant is Z and the decision of type B participant is X, the points of type A participant decreases by 1 point. Her earnings in this round are saved as 0 point.
- If the decision of type A participant is Z and the decision of type B participant is Y, the points of type A participant increases by 4 points. Her earnings in this round are saved as 5 points.
- If the decision of type A participant is T and the decision of type B participant is X, the points of type A participant increases by 1 point. Her earnings in this round are saved as 2 points.
- If the decision of type A participant is T and the decision of type B participant is Y, the points of type A participant increases by 4 points. Her earnings in this round are saved as 5 points.
- If the decision of type A participant is Z and the decision of type B participant is X, the points of type B participant decreases by 1 point. Her earnings in this round are saved as 0 point.
- If the decision of type A participant is Z and the decision of type B participant is Y, the points of type B participant remains constant. Her earnings in this round are saved as 1 point.
- If the decision of type A participant is T and the decision of type B participant is X, the points of type B participant increases by 1 point. Her earnings in this round are saved as 2 points.

- If the decision of type A participant is T and the decision of type B participant is Y, the points of type B participant remains constant. Her earnings in this round are saved as 1 point.

After the decisions made and earnings saved, the earnings of each encounter are announced to the related active participants. Below, you can find an example of the announcement screens. (The values are changed.)

My Points	0
B's Points	0

A's Points	0
My Points	0

After the announcement of the earnings, the 1st round ends. The 2nd round starts with the new activated 4 type B participants. In the 2nd round, type A participants can view their own 1st round results and active type B participants can view 1st round results of the related (faced) type A participant. Below, you can find the history table of the results. (The names of the decisions and the values are changed.)

Tur	Benim Kararım	Turdaki B'nin Kararı	Bu Turdaki Puanım
1	M	P	9
2	?	?	?

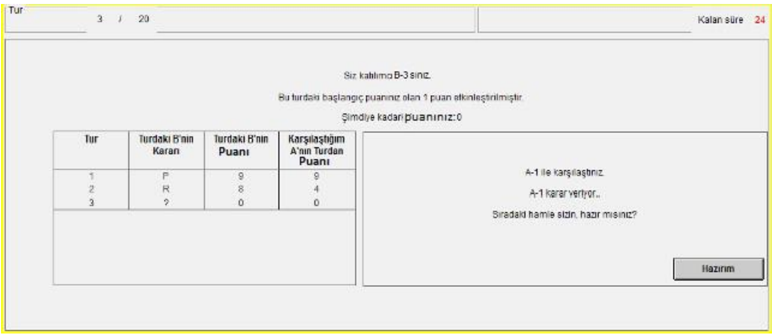
Tur	Turdaki B'nin Kararı	Turdaki B'nin Puanı	Karşılaştığım A'nın Turdan Puanım
1	P	9	9
2	?	?	?

The first table is an example to the history table which appears on the screen of each type A participant. By the help of this table, they can follow her own 1st round decision (1st column) as well as the earnings (2nd column), and also the decision of the 1st round's active type B participant (3rd column). The second table is an example to the history table which appears on the screen of each active type B participant. Since the decisions of all type A participants are kept secret, each type B participant can only see the related type A participant's 1st round earnings (3rd column), but she can follow the decision (1st column) and the earnings (2nd column) of the 1st round's active type B participant.

After the 2nd round which is identical to the 1st round in terms of stream, the 3rd round starts. The screens of the 3rd round's active participants are as follows: (The names of the decisions and the values are changed.)

From the writings above the tables, we can understand that it is A-1 who can view this screen. She earned 13 points in total from the first 2 rounds. From the box on the right side, we can see which of type B participant A-1 faced: in the 3rd round, A-1 and B-3 are matched and now, A-1 have to decide. If we examine the history table on the left side, in the 1st round, A-1 chose 'M' and the active type B participant of that round chose 'P'. As the result of these decisions, A-1 added 9 points to her earnings. In the 2nd round, A-1 chose 'N' and the active type B participant of that round chose 'R'. As the result of these decisions, A-1 added 4 points to her earnings. The total earnings 13 is the sum of the earnings gained in the previous rounds. As it is clear from the screen, type A participant does not have any information related to this round's active type B participant, except her order number.

At the same time, B-3 views the screen below:



The total earnings of B-3 up to now are 0. Either she has it as a result of her decision in one of the previous rounds or she has not decided yet. As we can understand from the box on the left side, type B participant who faced with A-1 in the 1st round chose ’P’ and as a result, she added 9 points to her earnings and A-1 added 9 points to her earnings. Type B participant who faced with A-1 in the 2nd round chose ’R’ and as a result, she added 8 points to her earnings and A-1 added 4 points to her earnings. If we focus on the box on the right side, B-3 is waiting for the decision of A-1 and must push the button ’Ready’.

After the decisions of all type A participants, the active type B participants decide. While each of type B participants deciding, the box on the right side of the screen of each type A participant changes to the new box ”B-3 decides.. After the decision, you will be informed related to your earnings.” written on it.

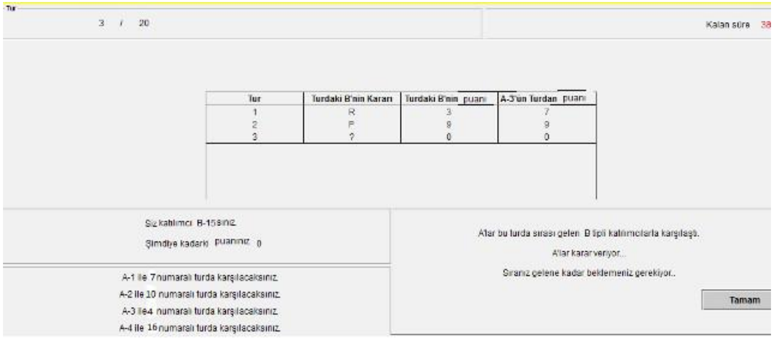
When each type B participant decides, the box on the right side of her screen changes to the new box ”A-1 decided. What is your decision?” written on it and with the decision buttons ’R’ and ’P’.

After the decisions of all active participants and the earnings are saved as described above, the earnings are announced to the related active participants.

After the 3rd round, the 4th round starts. In any new-beginning round, the history table will be updated. In summary, whichever round you are in, you can get information related to the all previous rounds. If your type is A, you can reach all information. If your type is B, you can reach all information except the decisions of the related type A participant.

After the realization of the 20th round as described above, the experiment will be finalised and a short questionnaire will be started.

Each type B participant will be active only in 4 of 20 rounds. If it is the round in which a type B participant is not active, she can view the screen below: (the names of the decisions and the values are changed.)



The screen above belongs to B-15 who is not active in the 3rd round. From the box on the left side, we can see in which of the rounds B-15 is active. B-15 will be active in the 4th round for the first time, then 7th, 10th and 16th round. When she is active in the 4th round, she will face with A-3. The box on the top is the history table of the previous rounds of A-3. This table will be seen and will be updated in each round until B-15 faces with A-3. Starting from the 5th round, the history table of the all previous rounds of A-1 will appear on the screen of B-15, since the next encounter will be with A-1. From the 8th to the 10th round, the history table of the all previous rounds of A-2 and from the 11th to the 20th round, the history table of the all previous rounds of A-4 will appear on her screen.

In summary, each of the waiting 16 type B participants can view the history table of type A participant with whom she will face next. In these tables, while the order number of type A participant can be found, the order number of type B participant who faced with the same type A participant before cannot be seen.

After the active participants decide, their earnings and the decisions of type B participants will be announced to all related type B participants who will face next with this related type A participant.

Since type A participants will not be informed about the other type A participants' encounters, the decisions of active type B participants is available only to type A participant with whom she faces, and these decisions cannot be reached by the other type A participants.

In-experiment Earnings

In each encounter, first type A participant decides between 'Z' and 'T' (Do not forget, this decision will not be announced to anybody). Then type B participant decides. Without knowing the decision of type A participant, she will decide between 'X' and 'Y'. The earnings in each round will be determined as we mentioned above.

The final earnings of a type A participant are the sum of the earnings she gained in each round. For example, if a type A participant gains $a_1, a_2, ..., a_{20}$ from the rounds respectively, her final earnings are $a_1 + a_2 + ... + a_{20}$ TL and this amount will be paid in cash in return for the receipts on your table.

The final earnings of a type B participant are the sum of the earnings she gained in each round. For example, if a type B participant gains b_1, b_2, b_3, b_4 from the rounds respectively, her final earnings are $5 * (b_1 + b_2 + b_3 + b_4)$ TL and this amount will be paid in cash in return for the receipts on your table.

Questionnaires

Independently from the types of the participants, the participants are asked to give information related to their age, gender, faculty and whether they have information on Game Theory.

Type A Questionnaire 1

Please read carefully and answer the questions below considering how you decided during the experiment.

(A) I tried to earn as much as I can. I played by trying to maximize my earnings.

(B) I tried to be fair towards type B participants. For me, the total of the earnings of all participants was also important, I did not consider only myself.

(C) As a type A participant, the most important thing was my reputation. I tried to be a ’tough’ player not to make type B participants benefit.

(D) My decisions depend on other reasons.

Could you please compare the given two options below considering which of the options explains your behavior during the experiment? Please mark your choice.

If you cannot decide between two options, please mark INDIFFERENT.

o A ( ) B ( ) Indifferent ( )

o B ( ) C ( ) Indifferent ( )

o C ( ) A ( ) Indifferent ( )

o A ( ) D ( ) Indifferent ( )

o D ( ) C ( ) Indifferent ( )

o B ( ) D ( ) Indifferent ( )

Any comments?

Type B Questionnaire

Please read carefully and answer the questions below considering how you decided during the experiment.

(A) Whomever I faced, my strategy was to play always ’X’ or always ’Y’.

(B) I did my choices between ’X’ and ’Y’ considering the number of round in which I decided.

(C) My decisions depend on other reasons.

(D) I did my choices between ’X’ and ’Y’ considering the ideas/the beliefs on type A participant with whom I face. I examined how she played against to the other type B participants before me.

Could you please compare the given two options below considering which of the options explains your behavior during the experiment? Please mark your choice.

If you cannot decide between two options, please mark INDIFFERENT.

o A ( ) B ( ) Indifferent ( )

o B ( ) C ( ) Indifferent ( )

o C ( ) A ( ) Indifferent ( )

o A ( ) D ( ) Indifferent ( )

o D ( ) C ( ) Indifferent ( )

o B ( ) D ( ) Indifferent ( )

Any comments?

Type A Questionnaire 2

Please read carefully and answer the questions below considering how you decided during the experiment.

(A) I decided in each round independently from the other rounds. I did not think that my decision to a type B participant could effect the other type B participants' decisions.

(B) I thought that my decision to a type B participant could effect the following type B participant's decision and only this participant.

(C) I thought that my decision to a type B participant could effect all of the following type B participants' decisions.

(D) I thought that my decision to a type B participant could effect all of the following type B participants' decisions, except a few of them from the last.

Could you please compare the given two options below considering which of the options explains your behavior during the experiment? Please mark your choice.

If you cannot decide between two options, please mark INDIFFERENT.

o A ( ) B ( ) Indifferent ( )

o B ( ) C ( ) Indifferent ( )

o C ( ) A ( ) Indifferent ( )

o A ( ) D ( ) Indifferent ( )

o D ( ) C ( ) Indifferent ( )

o B ( ) D ( ) Indifferent ( )

Any comments?