COORDINATION UNDER RANDOM YIELD AND RANDOM DEMAND

by

Mehmet Güray Güler

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APPROVED BY:

Assoc. Prof. Taner Bilgiç ....
(Thesis Supervisor)

Assist. Prof. Aslı Erdem ....

Prof. Refik Güllü ....

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ABSTRACT

COORDINATION UNDER RANDOM YIELD AND RANDOM DEMAND

The aim of this study is to fill a gap in coordination literature by providing answers to two distinct research questions on two different systems: A newsvendor system and an assembly system.

First part of the study analyzes a newsvendor problem with random yield and random demand. Recalling the centralized solution, the system is decentralized and five different contracts are studied. It is shown that some of the contracts coordinate the chain while the others can coordinate with additional assumptions. In addition to the expected chain profit, variance of expected chain profit is written in closed form and the relation between the parameters and the expected profit and variance of the expected profit and the optimal order quantity is illustrated with numerical examples.

Second part of the study deals with an assembly system. As well as the demand, the yield of the suppliers are random. Concavity of expected chain profit of both two-supplier and N-supplier assembly system is shown. Instead of solving the optimal order quantities explicitly, the expected profit of the manufacturer and the chain is written is such a way that the manufacturer's function becomes a portion of that of the chain's. Four different contracts are proposed which are shown to coordinate the chain under forced compliance. The contracts are mixed type of contracts which includes payments from different contract schemes. Several different scenarios are created and numerical examples for centralized solution and contract schemes are provided.

ÖZET

RASSAL ARZ VE RASSAL TALEP ALTINDA KOORDiNASYON

Bu tezin amacı iki ayrı sistem üzerinde yapılan çalışmalara cevaplar bularak koordinasyon literatüründeki bir boşluğu doldurmaktır.

Çalışmanın ilk bölümü rassal talep ve rassal arz altındaki bir gazeteci çocuk problemini analiz etmektedir. Sistemin merkezi çözümü hatırlatıldıktan sonra dağıtık çözüm ve beş ayrı kontrat incelenmiştir. Bazı kontratların sistemi koordine ettiği, bazılarının ise ancak ek varsayımlar ile koordine edebildiği gösterilmiştir. Beklenen zincir kârına ek olarak, bu kârın varyansı kapalı formda yazılmış ve parametreler ile beklenen kâr, beklenen kârın varyansı ve optimal sipariş miktarı arasındaki ilişkiler sayısal örneklerle gsterilmiştir.

Tezin ikinci kısmı ise montaj sistemlerini incelemektedir. Talebin yanı sıra, tedarikçilerin ürünleri de rassaldır. İki tedarikçi ve N-tedarikçi sistemlerin beklenen kâr fonksiyonlarının dış bükey olduğu gösterilmiştir. Optimal sipariş miktarlarını teker teker çözmek yerine beklenen kâr fonksiyonları, tedarik zincirinin kâr fonksiyonu, üreticinin kâr fonksiyonunun bir katı haline gelecek şekilde yazılmıştır. Zorunlu uyum rejimi altında koordinasyon sağlayan dört kontrat sunulmuştur. Bu kontratlar farklı kontratların ödemelerini içeren karma kontratlardır. Farklı seneryolar ile merkezi çözüm ile konratlar üzerine sayısal örnekler sunulmuştur.

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α	Portion of received order(yield), $\alpha \in$(0,1]
α_i	Portion of received order(yield) from supplier i, $\alpha_i \in$ (0,1]
αQ	Stochastically proportional yield
$\alpha_i Q_i$	Stochastically proportional yield of supplier i
μ_α	Expected value of α
μ_D	Expected value of demand
ϕ	Revenue share of the supplier(s)
$\overline{\phi}$	1 - ϕ
ϕ_i	Revenue share of the supplier i
λ	Adjusting parameter for the contracts
$\pi()$	Profit function
A	Sum of selling price, r and cost of lost sales, g_r less salvage, h
b	Buy-back price
b_i	Buy-back price for supplier i
c	Cost of production of the supplier per unit
c_i	Cost of production of the supplier i per unit
D	Single period random demand
$f()$	Density function of α
$f_i()$	Density function of α_i
$F()$	CDF of α
$F_i()$	CDF of α_i
$g()$	Density function of demand
$G()$	CDF of demand
g_r	Opportunity loss of the retailer per unit
h	Holding cost / salvage value of the retailer per unit
h_i	Holding cost of component i of the retailer per unit
k	Inventory crediting parameter for quantity flexibility contract
m_i	Buy-back price in modified buy-back contracts

Q	Order size(Decision Variable)
Q_i	Order size from supplier i(Decision Variable)
r	Selling price
$T()$	Transfer payment to the supplier
$T_i()$	Transfer payment to the supplier i
V	Recovery payment
w	Wholesale price in wholesale contract
w_b	Wholesale price in buy-back contract
$w_{b,i}$	Wholesale price for supplier i in buy-back contracts
w_d	Wholesale price in quantity discount contract
w_f	Wholesale price in quantity flexibility contract
w_i	Wholesale price for supplier i in wholesale contract
$w_{m,i}$	Wholesale price in modified buy-back contract
w_r	Wholesale price in revenue share contract
Y_i	Random yield

BBRS	Buy-back contract with revenue share
BBSRS	Buy-back contract with sales revenue share
MBBRS	Modified buy-back contract with revenue share
MBBSRS	Modified Buy-back contract with sales revenue share

1. INTRODUCTION

With improvement of production techniques and technology, variety of all products increased to a significant level. Today many products, from cars to cellular phones, offer so many alternatives that nobody can resist. However this variety brought difficulty to the production processes as well. Thus the companies started to diversify and focused on the areas at which they do best. Some started to manufacture products, some started to transport products, some started to market products.

The companies gradually became an expert at one topic rather than doing all the processes for selling a product. This change caused the formation of chains in which every company provides a certain aspect of the whole supply process: Supply Chains.

However the companies saw that the overall profits of the supply chains may be lower than the expectation. This difference is mainly due to the fact that every company tries to maximize their own profits. In the past, since supply process usually was completely owned by a single company, there was a single decision maker. However, today there are various *players* who want more income.

This problem increased the importance of contracts which are simply the payments between the firms in a supply chain. With carefully designed contracts, everybody acts in *coordination* such that the overall profit of a chain increases to the case where there is a single decision maker.

This thesis deals with two problems. First problem is to implement well-known contracts in a random yield newsvendor setting and study whether the contracts provide coordination in the chain or not. The second problem is to design and implement contracts for an assembly system in which each supplier produces distinct components which are then assembled by a manufacturer in order to meet the random demand. As well the randomness in the demand, the suppliers' yields are random. Stochastically proportional yield structure is used in both of the problems.

The rest of the thesis is organized as follows. Chapter 2 provides an overlook for the studies about both of the problems. Contracts and their performances in the newsvendor problem with random yield are studied in Chapter 3. Chapter 4 deals with coordination in two and N supplier assembly systems. After theoretical work, we present numerical illustration in Chapter 5. The last chapter is conclusion for the thesis and provides some further research topics.

2. LITERATURE REVIEW

This work mainly focuses on two main problems. First problem is selling to the newsvendor with random yield. Newsvendor problem is studied by many researchers in recent years and coordination is one of the topics which has attracted attention. However coordination in random yield and random supply systems has not been studied yet. The second problem is an assembly system with random demand and random yield. The suppliers produce distinct components which are later assembled by the manufacturer to meet the random demand. The suppliers are unreliable and the yield comes out to be lower than the order quantity. The research on assembly systems focuses on choosing the optimal order quantity. There are some papers which study coordination but there is no work that studies coordination in assembly systems with random yield and random demand.

2.1. Newsvendor Problem

The first problem is a system with single retailer and single supplier having random yield and random demand. The literature about this system can be categorized into three main groups. In the first group, the centralized solution for the system is reviewed. In the second group, coordination mechanisms for systems having only random demand (classical newsvendor problem) are investigated. The third group includes reviews and other helpful papers about random yield.

2.1.1. Centralized Solution

Shih[1] studies both EOQ and newsvendor problem under random yield and random demand. Shih includes holding and shortage costs and shows that the total cost function of the newsvendor problem is convex in quantity ordered. Shih shows that deciding the optimal order quantity by assuming perfect yield instead of random yield results in a higher cost. Noori and Keller [2] also investigate a system with stochastic demand and random yield. Introducing the bias factor(amount received/amount ordered), they analyze the system when the demand is uniformly distributed and normally distributed. It is found that the quantity ordered depends on mean and most of the time the variance of demand, and is inversely proportional to the bias factor. Ehrhardt and Taube [3] work on a model in which the replenishment quantity is a random fraction of the quantity ordered. Linear cost structure is employed. They show that a very simple heuristic accounting only for the expected value of the replenishment quantity, not the variability, performs quite well for normally and negative binomially distributed demand.

Gerchak, Vickson and Parlar [4] study a periodic review production model with variable yield and uncertain demand. The optimal order quantity solves a ratio which resembles the one in this study(the random yield, u, is between (0,1] and there is no inventory I because this work deals with only single period). They found a critical ratio including the distribution of the yield and demand which depends on price, salvage value, cost of production and mean of the yield. They show that the order point does not change even the yield is random. However, the quantity ordered is not simply the difference of the order point and the available stock. As well as the final period(single period), two period and n-period problems are also investigated. It is found that the optimal policy for the general finite-horizon problem is not myopic, making the multi-period case hard to solve explicitly. It is shown that the order-up-to policies are not optimal.

Henig and Gerchak [5] work on a system with random yield and random demand. They prove that the ordering point does not change with randomness of the yield by using a very general cost structure. They use the stochastically proportional yield model which is also used in the previous papers that are cited above. For multi-period problem they show the existence of a critical reorder point and nonorder-up to optimal policy. Infinite horizon problem is shown to have a solution which approximates a long horizon problem. It is also shown that the critical reorder points are equal or greater than the the reorder points in the perfect yield models.

This group of papers analyze random yield newsvendor models but they do not include any study regarding coordination. In this research, coordination in newsvendor problem with random yield and random demand is analyzed.

2.1.2. Decentralized Models and Coordination

To cite papers about coordination, Lariviere and Porteus [6] deal with coordination through wholesale price contract, Weng [7] studies quantity discount contracts and Giannoccaro and Pontrandolfo [8] and Cachon and Lariviere [9] work on revenue sharing contracts. Cachon [10] reviews the contracts in details. In this research several contracts in classical newsvendor setting are investigated. Wholesale price, buy-back, revenue sharing, quantity flexibility, sales rebate and quantity discount contracts are studied in this work. As well as dealing with classical newsvendor problem, Cachon works on price dependent demand and effort dependent demand. He also studies coordinating multiple newsvendors and coordinating with demand updating. The framework in the first part of this thesis mainly stems from this review of Cachon's.

This group of papers deals with coordination but they do not study random yield models. This thesis studies random yield as well as the coordination.

2.1.3. Other Papers

Yano and Lee [11] provide a review of literature about the random yield models. They classify the papers as general, single stage continious-time models, discrete time models and complex manufacturing systems. They study the random yield newsvendor problem in the discrete time models, single stage - single period part. It is shown that the cost function of this system is convex. Khouja [12] gives a literature review and a classification of the papers dealing with the newsvendor problem, including the random yield literature.

Parlar and Wang [13] study diversification between the suppliers for single period when the yield is random. The concavity of the expected profit function is proved and they propose an approximation to find the global optimum. Anupindi and Akella [14] also study diversification of the suppliers when the yield is random. They analyze three different models with single and multi period horizons. It is found that there are two critical numbers which indicates from whom to order(both, one or none). Agnihothri, Lee and Kim [15] study a single period random yield model with a known and fixed demand, and a penalty cost when the demand is not met. They find distribution free results and show that there are two critical numbers for optimal ordering.

Ciarallo, Akella and Thomas [16] work on a single product problem having random demand and random capacity. They study single, multi and infinite horizon problems. Wang and Gerchak [17] extend this problem to a random yield environment. They show that for finite horizon problem there is a single critical point and solution of the finite horizon problem converges to infinite horizon problem's solution.

To summarize, the papers about random yield and random demand do not consider coordination. Additionally the papers about coordination do not study systems having randomness both in yield and demand. Our model deals with coordination in systems having both random demand and random yield.

2.2. Assembly Problem

The second problem is coordination assembly systems having random yield. The assembly system studied in this thesis has two distinct components produced by the suppliers. These components are then assembled by the retailer to meet the demand. As well as the demand, the suppliers' yields are random. The papers can be categorized into two main groups. First groups deals with the centralized solution where as the second group studies coordination and contracts.

2.2.1. Centralized Solution

Yao [18] works on assembly systems to figure out the optimum run quantities. Solution procedures are developed under yield distributions having increasing failure rates and convex setup cost functions. Gerchak, Wang and Yano [19] model an assembly system for a single period. They work on two models: components with identical costs and yield distributions and components with non-identical cost and yield distributions. They formulate the cost function and show the optimality conditions. Gurnani, Akella and Lehoczky [20] add a choice of joint supplier from whom the assembler can supply a set(both) of the components. They work on single and multi periods and show that it might be optimal to order more due to the randomness in the supply and sourcing from the joint supplier is optimal if the inventory level is below a critical ratio.

Gurnani, Akella and Lehoczky [21] study on an assembly system facing a random demand and random yield due to production yield losses. They formulate the exact cost functions with target level of finished products to assemble and the order quantity of the components from the suppliers as the decision variables. Then they propose a modified cost function to find the optimal ordering quantity and target level to assemble. In multi-period case it is found that it might be optimal to order extra components for future use. Also the optimal ordering policy and assembly target level policy are shown to be an order-up-to type of policy.

This group of papers focus on establishing the profit function of the chain and finding the optmimal order quantity. However they do not consider decentralized setting and coordination in the chain.

2.2.2. Decentralized System and Contracts

Gurnani and Gerchak [22] study assembly systems where demand is deterministic but supply is random due to yield losses. They propose two contracts, one with only punishment for undelivered items and other one with extra punishment to the worst one. They show that with extra penalty the suppliers interact and there is a nash equilibrium when coordination is achieved. Gerchak and Wang [23] were the first that study coordination in decentralized assembly systems having random demand. They worked on two systems; vendor managed inventory systems with revenue-sharing contracts and wholesale price based systems. They proposed two new contracts: Revenue sharing plus surplus subsidy contract and wholesale price plus buy back contract. They showed that these contracts can coordinate the chain and there is a continuum of such contracts which allows continuum of equilibrium allocation of channel profits. This papers deal with coordination in assembly systems but they do not study both random yield and random supply. When the papers are considered as a whole, there is no study that consider coordination in a random yield and random demand assembly system. This thesis deals with coordination in assembly systems with randomness both in demand and yield.

3. SELLING TO THE NEWSVENDOR WITH RANDOM YIELD

3.1. Problem Definiton

In this single period inventory control problem, there are two entities. The retailer faces a random demand. In order to meet this demand, he orders to the supplier. However, the retailer cannot receive the full order. Because of the randomness in the supply process, the supplier receives a portion of the order he placed due to quality problems. The distribution of the demand and the distribution of the fraction of the received order is independent and known by all the players. The cost parameters and the price is also known by the supplier and the retailer. The notation is as follows:

Q: Order size (Decision Variable)

r: Selling price per unit(Exogenous)

g_r: Opportunity loss of the retailer per unit

h: Holding cost of the retailer per unit

c: Cost of production per unit for the supplier

D: Single period random demand

α: portion of received order(yield), (0,1]

f(),F(): Density and CDF of α, respectively

g(), G(): Density and CDF of D, respectively

αQ : Stochastically Proportional Yield

μ_α : expected value of α

μ_D: expected value of the demand

We need following assumption because only μ_α portion of order is received on the average. That is the cost of a delivered order comes out to be c/μ_α. So the retailer has to sell the units more than the expected cost of that unit:

$$r \geq \frac{c}{\mu_\alpha} \tag{3.1}$$

The profit of the chain can be written as

$$\pi_c(Q) = r[\mathit{Sales}] - g_r[\mathit{Lost\ Sales}] + h[\mathit{End\text{-}product\ Inventory}] - cQ \tag{3.2}$$

Sales of the system is $\min(\alpha Q, D)$. The expected sales is:

$$\begin{aligned} S(\alpha, Q) &= E[\min(\alpha Q, D)] \\ &= \int_0^1 \int_{\alpha Q}^{\infty} \alpha Q f(\alpha) g(D) dD d\alpha + \int_0^1 \int_0^{\alpha Q} D f(\alpha) g(D) dD d\alpha \\ &= \mu_\alpha Q + \int_0^1 \int_0^{\alpha Q} [D - \alpha Q] f(\alpha) g(D) dD d\alpha \end{aligned} \tag{3.3}$$

3.2. Centralized Setting

In centralized setting, the decisions is given by a single decision maker. The system acts as a whole, such that they belong to a single owner. Under this setting, decisions which maximize the profit of the chain are made. The centralized profit function of the system is given as:

$$\pi(Q) = r \min(\alpha Q, D) - g_r [D - \alpha Q]^+ - h [\alpha Q - D]^+ - cQ \tag{3.4}$$

Taking the expectation, we have:

$$\begin{aligned} E[\pi(Q)] &= r \left\{ \int_0^1 \int_{\alpha Q}^{\infty} \alpha Q f(\alpha) g(D) dD d\alpha + \int_0^1 \int_0^{\alpha Q} D f(\alpha) g(D) dD d\alpha \right\} \\ &- g_r \int_0^1 \int_{\alpha Q}^{\infty} (D - \alpha Q) f(\alpha) g(D) dD d\alpha \\ &- h \int_0^1 \int_{\alpha Q}^{\infty} (\alpha Q - D) f(\alpha) g(D) dD d\alpha - cQ \end{aligned}$$

$$
\begin{aligned}
&= \int_0^1 \int_{\alpha Q}^{\infty} \left[\alpha Q\left(r+g_r\right) - g_r D\right] f(\alpha) g(D) dD d\alpha \\
&+ \int_0^1 \int_0^{\alpha Q} \left[D\left(r+h\right) - h\alpha Q\right] f(\alpha) g(D) dD d\alpha - cQ \\
&= \mu_\alpha Q\left(r+g_r\right) - g_r \mu_D - cQ \\
&+ \left(r+g_r+h\right) \int_0^1 \int_0^{\alpha Q} \left[D - \alpha Q\right] f(\alpha) g(D) dD d\alpha \\
&= (r+g_r+h) S(\alpha, Q) - h\mu_\alpha Q - g_r \mu_D - cQ \qquad (3.5)
\end{aligned}
$$

We now establish the shape of expected profit function:

Proposition 3.1 *Expected profit function given in (3.5) is concave in Q if* $r+g_r+h \geq 0$.

Proof:

$$\frac{\partial\left(E\left[\pi\left(Q\right)\right]\right)}{\partial Q} = \mu_\alpha\left(r+g_r\right) - c - \left(r+g_r+h\right) \int_0^1 \alpha f(\alpha) G(\alpha Q) d\alpha$$

$$\frac{\partial^2\left(E\left[\pi\left(Q\right)\right]\right)}{\partial Q^2} = -\left(r+g_r+h\right) \int_0^1 \alpha^2 f(\alpha) g(\alpha Q) d\alpha \qquad (3.6)$$

Every term in the integral is positive and the cost parameters are positive by assumption. Hence the expected profit function is always concave. □

As the function is concave, first order conditions are necessary and sufficient for the global optimum. If Q* is the optimum quantity for the centralized setting:

$$
\begin{aligned}
\frac{d\left(E\left[\pi\left(Q\right)\right]\right)}{dQ} &= 0 \\
\mu_\alpha\left(r+g_r\right) - c - \left(r+g_r+h\right) \int_0^1 \alpha f(\alpha) G(\alpha Q^*) d\alpha &= 0
\end{aligned}
$$

$$\int_0^1 \alpha f(\alpha)G(\alpha Q^*)d\alpha = \frac{\mu_\alpha(r+g_r)-c}{r+g_r+h} \tag{3.7}$$

In fact, (3.7) resembles the critical ratio in newsvendor problem. We make make the following definitions:

$$\begin{aligned} K(a) &= \int_0^1 \alpha f(\alpha)G(\alpha a)d\alpha \\ K(Q^*) &= \int_0^1 \alpha f(\alpha)G(\alpha Q^*)d\alpha \\ K(Q^*) &= \frac{\mu_\alpha(r+g_r)-c}{r+g_r+h} \end{aligned} \tag{3.8}$$

Proposition 3.2 *Optimal Q^* found from (3.7) is unique.*

Proof:

$$\frac{\partial K(Q)}{\partial Q} = \int_0^1 \alpha^2 f(\alpha)g(\alpha Q) > 0 \tag{3.9}$$

The function is strictly increasing in Q and $K(0) = 0$. For $Q = \infty$:

$$\lim_{Q\to\infty} K(Q) = \mu_\alpha < \infty$$

then there is a solution if:

$$\mu_\alpha > \frac{\mu_\alpha(r+g_r)-c}{r+g_r+h}$$
$$\mu_\alpha h > -c$$

which always holds since μ_α and h are nonnegative. Thus Q* is unique and well defined.□.

When we analyze the classical newsvendor problem, that is when there is no random yield, the optimal order quantity turns out to be:

$$G(Q^{*}) = \frac{r + g_r - c}{r + g_r + h} \tag{3.10}$$

So the randomness in the yield effects the left and the right hand side of the equation. α and $f(\alpha)$ appear at the left hand side of the equation and the μ_α appears at the right hand side. So when $\mu_\alpha = 1$, then the right hand side of (3.7) is just equal to that of newsvendor problem.

3.3. Decentralized Setting

In the decentralized setting, the retailer and supplier act as independent decision makers. They try to maximize their own profits. There is a transfer payment, *T()*, between two parties paid by the retailer to the supplier. The profit function of the retailer and its expected value are:

$$\begin{aligned}
\pi_r(Q) &= r \min(\alpha Q, D) - g_r [D - \alpha Q]^{+} - h [\alpha Q - D]^{+} - T(\cdot) \\
E[\pi_r(Q)] &= \mu_\alpha Q (r + g_r) - g_r \mu_D \\
&+ (r + g_r + h) \int_0^1 \int_0^{\alpha Q} [D - \alpha Q] f(\alpha) g(D) dD d\alpha - E[T(\cdot)] \qquad (3.11)
\end{aligned}$$

The profit function of the supplier and the expected value of it are:

$$\begin{aligned}
\pi_s(Q) &= -cQ + T(\cdot) \\
E[\pi_s(Q)] &= -cQ + E[T(\cdot)] \qquad (3.12)
\end{aligned}$$

3.4. Contracts

Contracts are different transfer payments. Several different transfer payments described in the previous section comes out to be the contracts between the supply chain entites. The profit of the chain under decentralized setting is always less than or equal to the profit of the chain in centralized setting. When the system is decentralized, every player tries to maximize its own profit. As a result of that, for example the retailer orders a quantity that is different from the one which maximizes the chain's profit. The aim of the contracts is to establish transfer payments between the players so that the retailer chooses the order quantity that maximizes the chain's profit. In this section we consider wholesale price contract, buy-back contract, revenue sharing contract, quantity flexibility contract and quantity discount contract. Main issue is to discuss whether they can coordinate the chain or not.

3.4.1. Wholesale Price Contract

In wholesale contract, the retailer pays to the supplier a wholesale price of w units.

$$E[T_w(Q,w)] \;=\; w\mu_\alpha Q$$

The profit function of the retailer and the expected value of this function are:

$$\begin{aligned} \pi_r\left(Q,w\right) &= r\min\left(\alpha Q, D\right) - g_r\left[D-\alpha Q\right]^+ - h\left[\alpha Q - D\right]^+ - w\mu_\alpha Q \\ E\left[\pi_r\left(Q,w\right)\right] &= \mu_\alpha Q\left(r+g_r\right) - g_r\mu_D \\ &+ \left(r+g_r+h\right)\int_0^1\int_0^{\alpha Q}\left[D-\alpha Q\right]f(\alpha)g(D)dDd\alpha - w\mu_\alpha Q \quad (3.13) \end{aligned}$$

The expected profit function of the supplier is:

$$
\begin{aligned}
\pi_s\left(Q, w\right) &= w\mu_\alpha Q - cQ \\
E\left[\pi_s\left(Q, w\right)\right] &= w\mu_\alpha Q - cQ
\end{aligned}
\tag{3.14}
$$

Concavity check of $E\left[\pi_r\left(Q, w\right)\right]$ results in:

Proposition 3.3 *Expected profit function given in (3.13) is concave in Q if $r+g_r+h \geq 0$.*

Proof:

$$
\begin{aligned}
\frac{\partial\left(E\left[\pi_r\left(Q, w\right)\right]\right)}{\partial Q} &= \mu_\alpha\left(r+g_r\right) - \left(r+g_r+h\right)\int_0^1 \alpha f(\alpha)G(\alpha Q)d\alpha - w\mu_\alpha \\
\frac{\partial^2\left(E\left[\pi_r\left(Q, w\right)\right]\right)}{\partial Q^2} &= -\left(r+g_r+h\right)\int_0^1 \alpha^2 f(\alpha)g(\alpha Q)d\alpha \leq 0
\end{aligned}
$$

□

Let $\overline{Q}$ be the optimal quantity for the retailer. Then:

$$
\frac{\partial\left(E\left[\pi_r\left(Q, w\right)\right]\right)}{\partial Q} = 0
$$

$$
\mu_\alpha\left(r+g_r\right) - \left(r+g_r+h\right)\int_0^1 \alpha f(\alpha)G(\alpha\overline{Q})d\alpha - w\mu_\alpha = 0
$$

Then for $\overline{Q}$ to be equal to Q^*:

$$
K(\overline{Q}) = \frac{\mu_\alpha\left(r+g_r\right) - w\mu_\alpha}{\left(r+g_r+h\right)} = \frac{\mu_\alpha\left(r+g_r\right) - c}{\left(r+g_r+h\right)} = K(Q^*)
$$

$$
w = \frac{c}{\mu_\alpha} \tag{3.15}
$$

So simply the retailer gives back the supplier the cost that the supplier spends for the order. Then clearly the profit of the supplier is zero. This phenomenon is known as double marginalization [24].

3.4.2. Buy-back Contract

In the buy-back contract the retailer pays w_b for every unit that comes to her, at the end of the season the supplier pays a premium of b to the retailer for every unit that is not sold.

$$T_b(Q, w_b, b) = w_b \alpha Q - b[\alpha Q - \min(\alpha Q, D)]$$

$$\begin{aligned} E[T(Q, w_b, b)] &= w_b \mu_\alpha Q - bE[\alpha Q - \min(\alpha Q, D)] \\ &= w_b \mu_\alpha Q + b \int_0^1 \int_0^{\alpha Q} [D - \alpha Q] \, f(\alpha) g(D) dD d\alpha \end{aligned}$$

The profit function of the retailer and its expected value are given as:

$$\begin{aligned} \pi_r(Q, w_b, b) &= r \min(\alpha Q, D) - g_r [D - \alpha Q]^+ - h [\alpha Q - D]^+ \\ &- w_b \alpha Q + b[\alpha Q - \min(\alpha Q, D)] \\ E[\pi_r(Q, w_b, b)] &= \mu_\alpha Q (r + g_r - w_b) - g_r \mu_D \\ &+ (r + g_r + h - b) \int_0^1 \int_0^{\alpha Q} [D - \alpha Q] \, f(\alpha) g(D) dD d\alpha \quad (3.16) \end{aligned}$$

The profit function of the supplier and the expected value of this function are:

$$\begin{aligned} \pi_s(Q, w_b, b) &= w_b \alpha Q - b[\alpha Q - \min(\alpha Q, D)] - cQ \\ E[\pi_s(Q, w_b, b)] &= w_b \mu_\alpha Q + b \int_0^1 \int_0^{\alpha Q} [D - \alpha Q] \, f(\alpha) g(D) dD d\alpha - cQ \quad (3.17) \end{aligned}$$

Proposition 3.4 *Expected profit function given in (3.16) is concave in Q if $r + g_r + h - b \geq 0$.*

Proof:

$$\begin{aligned}
\frac{\partial \left(E\left[\pi_r\left(Q, w_b, b\right)\right]\right)}{\partial Q} &= \mu_\alpha \left(r + g_r - w_b\right) \\
&\quad - \left(r + g_r + h - b\right) \int_0^1 \alpha f(\alpha) G(\alpha Q) d\alpha \\
\frac{\partial^2 \left(E\left[\pi_r\left(Q, w_b, b\right)\right]\right)}{\partial Q^2} &= - \left(r + g_r + h - b\right) \int_0^1 \alpha^2 f(\alpha) g(\alpha Q) d\alpha
\end{aligned}$$

b is the buy-back price of the left over inventory. So it is meaningless for b to be greater then the selling price r. This verifies the concavity of the function given in (3.16). □

Now let $\overline{Q}$ be the optimal quantity for the retailer.

$$\begin{aligned}
\frac{\partial \left(E\left[\pi_r\left(Q, w_b, b\right)\right]\right)}{\partial Q} &= 0 \\
\mu_\alpha \left(r + g_r - w_b\right) - \left(r + g_r + h - b\right) \int_0^1 \alpha f(\alpha) G(\alpha Q) d\alpha &= 0
\end{aligned}$$

Then for $\overline{Q}$ to be equal to Q^*:

$$K(\overline{Q}) = \frac{\mu_\alpha \left(r + g_r - w_b\right)}{r + g_r + h - b} = \frac{\mu_\alpha \left(r + g_r\right) - c}{\left(r + g_r + h\right)} = K(Q^*)$$

$$\begin{aligned}
w_b &= b \frac{K(Q^*)}{\mu_\alpha} + \frac{c}{\mu_\alpha} \\
w_b &= b \frac{\mu_\alpha (r + g_r) - c}{\mu_\alpha (r + g_r + h)} + \frac{c}{\mu_\alpha} \qquad (3.18)
\end{aligned}$$

When (3.18) is analyzed, it can be seen that w_b decreases with an increasing μ_α:

$$\frac{\partial w_b}{\partial \mu_\alpha} = \frac{c[b-(r+g_r+h)]}{\mu_\alpha^2(r+g_r+h)} < 0$$

However, we cannot figure out what the optimum quantity will be with an increasing μ_α. One point we can emphasize, increasing yield decreases the wholesale price. The retailer pays to the supplier w_b per unit delivered, $\mu_\alpha Q$. That is, when μ_α increases, then the quantity delivered by the retailer increases, and w_b decreases. So increasing w_b means that overall the retailer tries to stabilize the payment for the whole delivery.

The contract parameters that satisfies (3.18) coordinate the system. However not all of them are acceptable. For the parameters to be fine for the players, the contract must allow them to make profits. To find the acceptable range, we define the following:

$$\begin{aligned}
\mu_\alpha(r+g_r-w_b) &= \lambda[\mu_\alpha(r+g_r)-c] \\
r+g_r+h-b &= \lambda(r+g_r+h) \\
b &= (1-\lambda)(r+g_r+h) \\
w_b &= \frac{(1-\lambda)\mu_\alpha(r+g_r)+\lambda c}{\mu_\alpha}
\end{aligned}$$

which means that when $\lambda = 1$ there is no buy-back process and w_b just becomes the *wholesale price*. So when $\lambda = 1$ this contract becomes a wholesale price contract. $E\left[\pi_r\left(Q, w_b, b\right)\right]$ becomes a fraction of the whole chain's profit function:

$$E\left[\pi_r\left(Q, w_b, b\right)\right] = \lambda\left\{E[\pi_c(Q)]\right\} + (\lambda-1)g_r\mu_D \tag{3.19}$$

Also the supplier's profit function and its expected value are:

$$E\left[\pi_s\left(Q, w_b, b\right)\right] = (1-\lambda)\left\{E[\pi_c(Q)]\right\} + (1-\lambda)g_r\mu_D \tag{3.20}$$

Retailer gets all the profit when $\lambda = 1$ while supplier gets it with:

$$\lambda = \frac{g_r \mu_D}{g_r \mu_D + E[\pi(Q)]} \leq 1$$

Therefore, a buy-back contract coordinates the chain under random yield and can arbitrarily allocate profits between the supplier and the retailer.

When the classical newsvendor problem is considered, the parameters are:

$$w_b = b\frac{r + g_r - c}{r + g_r + h} + c \tag{3.21}$$

which shows that the buy-back parameters found from (3.18) simplifies to that of classical newsvendor when $\mu_\alpha = 1$. Also in classical newsvendor the parameters which are set as:

$$\begin{aligned} b &= (1-\lambda)(r + g_r + h) \\ w_b &= (1-\lambda)(r + g_r) + \lambda c \end{aligned} \tag{3.22}$$

result in same profit share as in (3.19) and (3.20). This result reduces to that of Cachon's[10].

This shows that the contract parameters are only affected by distribution of α, in fact only by μ_α. Although one can think that the variance of yield does not have a significant effect, it can be seen that this is not true when the critical ratio in (3.7) is analyzed. The optimal order quantity is dependent on the distribution of α. So the variance effects the selection of optimal order quantity. The retailer orders more (or less) according to the variance and distribution of α.

If $h < 0$, that is h stands for salvage value. Transfer payments are not affected since buy-back process is not related with the salvage value or holding cost.

w_b gets its lowest value when $b = 0$, that is when the contract becomes a wholesale price contract. Thus following inequality always holds:

$$w_b > w \tag{3.23}$$

3.4.3. Revenue Sharing Contract

With a revenue sharing contract the retailer pays w_r for every unit that she purchases. She also pays a part of her revenue, which is $(1-\phi)\, r.[\min(\alpha Q, D)]$, to the supplier, keeping ϕ portion of the revenue. The transfer function is:

$$T_r(Q, w_r, \phi) = w_r \alpha Q + (1-\phi) r[\min(\alpha Q, D)]$$

$$\begin{aligned} E[T_r(Q, w_r, \phi)] &= w_r \mu_\alpha Q + (1-\phi) r S(\alpha, Q) \\ &= w_r \mu_\alpha Q + (1-\phi) r \int_0^1 \int_0^{\alpha Q} [D - \alpha Q]\, f(\alpha) g(D) dD d\alpha \end{aligned}$$

The profit function of the retailer and the expected value of this function are:

$$\begin{aligned} \pi_r\,(Q, w_r, \phi) &= r \min(\alpha Q, D) - g_r\,[D-\alpha Q]^+ - h\,[\alpha Q - D]^+ \\ &- w_r \alpha Q - (1-\phi) r[\min(\alpha Q, D)] \\ E[\pi_r\,(Q, w_r, \phi)] &= \mu_\alpha Q\,(\phi r + g_r - w_r) - g_r \mu_D \\ &+ (\phi r + g_r + h) \int_0^1 \int_0^{\alpha Q} [D - \alpha Q]\, f(\alpha) g(D) dD d\alpha \end{aligned} \tag{3.24}$$

The profit function of the supplier and the expected value of this function are:

$$\pi_s\,(Q, w_r, \phi) = w_r \alpha Q + (1-\phi) r[\min(\alpha Q, D)] - cQ$$

$$\begin{aligned} E\left[\pi_s\left(Q, w_r, \phi\right)\right] &= w_r \mu_\alpha Q \\ &+ (1-\phi) r \int_0^1 \int_0^{\alpha Q} \left[D - \alpha Q\right] f(\alpha) g(D) dD d\alpha - cQ \end{aligned} \quad (3.25)$$

Proposition 3.5 *Expected profit function given in (3.25) is concave in Q if $\phi r + g_r + h \geq 0$.*

Proof:

$$\begin{aligned} \frac{\partial\left(E\left[\pi_r\left(Q, w_r, \phi\right)\right]\right)}{\partial Q} &= \mu_\alpha\left(\phi r + g_r - w_r\right) \\ &- \left(\phi r + g_r + h\right) \int_0^1 \alpha f(\alpha) G(\alpha Q) d\alpha \\ \frac{\partial^2\left(E\left[\pi_r\left(Q, w_r, \phi\right)\right]\right)}{\partial Q^2} &= -\left(\phi r + g_r + h\right) \int_0^1 \alpha^2 f(\alpha) g(\alpha Q) d\alpha \end{aligned}$$

As all the cost parameters and the functions in the integral is positive, expected profit function given in (3.25) is concave in Q.□

Now if we define $\overline{Q}$ be the optimal quantity for the retailer:

$$\begin{aligned} \frac{\partial\left(E\left[\pi_r\left(Q, w_r, \phi\right)\right]\right)}{\partial Q} &= 0 \\ \mu_\alpha\left(\phi r + g_r - w_r\right) - \left(\phi r + g_r + h\right) \int_0^1 \alpha f(\alpha) G(\alpha Q) d\alpha &= 0 \end{aligned}$$

then for $\overline{Q}$ to be equal to Q^*:

$$K[\overline{Q}] = \frac{\mu_\alpha\left(\phi r + g_r - w_r\right)}{\phi r + g_r + h} = \frac{\mu_\alpha\left(r + g_r\right) - c}{\left(r + g_r + h\right)} = K(Q^*)$$

$$\begin{aligned} w_r &= (1-\phi) r \left[\frac{K(Q^*)}{\mu_\alpha} - 1\right] + \frac{c}{\mu_\alpha} \\ w_r &= \phi \frac{r(\mu_\alpha + c)}{\mu_\alpha(r + g_r + h)} + \frac{h(c - \mu_\alpha r) + g_r c}{\mu_\alpha(r + g_r + h)} \end{aligned} \quad (3.26)$$

For the parameters satisfying (3.26), coordination is satisfied. As it can be seen from the equation, w_r increases with ϕ. This means the retailer gets higher revenue share when w_r gets higher. One another critical point is that the sign of $h(c-\mu_\alpha r)+g_r c$ is not clear. Following condition must hold for w_r to be nonnegative:

$$\begin{aligned} h(c-\mu_\alpha r)+g_r c &> 0 \\ \frac{c}{\mu_\alpha} &> r\frac{h}{h+g_r} \end{aligned} \tag{3.27}$$

If we analyze the limits of the parameters:

$$h >> g_r \quad \Rightarrow \quad r\frac{h}{h+g_r} = r \tag{3.28}$$

which means that from (3.1) w_r can be negative when ϕ is small enough. However if:

$$h << g_r \quad \Rightarrow \quad r\frac{h}{h+g_r} = 0 \tag{3.29}$$

then w_r is positive. However the supplier gets more revenue share with a small ϕ. Thus it is not an unacceptable offer for the supplier.

When (3.26) is analyzed, it can be seen that w_r decreases with an μ_α:

$$\frac{\partial w_r}{\partial \mu_\alpha} = \frac{-c(\phi r+h+c)}{\mu_\alpha^2(r+g_r+h)} < 0$$

Again, we do not know what the optimal order quantity be with an increasing μ_α. So the only that can be derived from here is that staying everything the same, the wholesale price decreases when the uncertainity in the yield decreases. This is the same result we get from the buy-back contract.

The parameters satisfying (3.26) coordinates the chain. However, like the buy-back contract, one another point is to see which range is applicable:

$$\begin{aligned} \mu_\alpha\left(\phi r + g_r - w_r\right) &= \lambda[\mu_\alpha(r+g_r) - c] \\ \phi r + g_r + h &= \lambda(r + g_r + h) \\ \phi &= \frac{\lambda r - (1-\lambda)(h+g_r)}{r} \\ w_r &= \frac{\lambda c - h\mu_\alpha(1-\lambda)}{\mu_\alpha} \end{aligned} \tag{3.30}$$

which means that when $\lambda = 1$ all the sales revenue goes to the retailer and contract becomes a wholesale price contract just like buy-back contract. It is found from (3.26) that w_r increases with ϕ. So it can be seen that both results agree. When λ gets larger, w_r gets larger. As w_r gets larger, the revenue share of the retailer ϕ gets larger because the closer the contract is to the wholesale price contract, the more the retailer gets. $E\left[\pi_r\left(Q, w_r, \phi\right)\right]$ becomes a fraction of the whole chain's profit function:

$$E\left[\pi_r\left(Q, w_r, \phi\right)\right] = \lambda\left\{E[\pi_c(Q)]\right\} + (\lambda - 1)g_r\mu_D$$

and the supplier's profit function is:

$$E\left[\pi_s\left(Q, w_r, \phi\right)\right] = (1-\lambda)\left\{E[\pi_c(Q)]\right\} + (1-\lambda)g_r\mu_D$$

Retailer gets all the profit when $\lambda = 1$ while supplier gets it with:

$$\lambda = \frac{g_r\mu_D}{g_r\mu_D + E[\pi(Q)]} \leq 1$$

Another important point is that Buy-Back and Revenue Sharing contracts are equivalent when the parameters have following relations:

$$w_r = w_b - b \tag{3.31}$$

$$b = (1-\phi)r \tag{3.32}$$

which means that w_b is greater than w_r. Also it can be seen from (3.30) that w_r gets it highest value,c/μ_α when $\lambda = 1$. So following condition can be written for the wholesale prices of the contracts:

$$w_b > w > w_r$$

The relation in (3.32) is also valid in classical newsvendor problem without random yield[10]. It is preserved under random yield as well. When the classical newsvendor problem is considered, the parameters are as follows:

$$w_r = \phi\frac{r(1+c)}{r+g_r+h} + \frac{h(c-r)+g_r c}{r+g_r+h} \tag{3.33}$$

which shows that the parameters found from (3.26) simplifies to that of classical newsvendor when $\mu_\alpha = 1$. Also in classical newsvendor, when the parameters are set as:

$$\begin{aligned} \phi &= \frac{\lambda r-(1-\lambda)(h+g_r)}{r} \\ w_r &= \lambda c - h(1-\lambda) \end{aligned} \tag{3.34}$$

then the supplier and the retailer has same profit shares as in (3.19) and (3.20).

The affect of α and its distribution is explained in buy-back contract. When μ_α increases, w_r decreases which means that the retailer tries to stabilize the payment for the whole delivery. Additionally there is one point that must be analyzed carefully.

The wholesale price w_r can be negative when λ becomes less than $h/h+c$. For this to be happen, the following inequality must hold:

$$cg_r\mu_D < hE[\pi_c] \tag{3.35}$$

This case may happen when selling price, r, has a very big value or when production cost, c, has a very small value. So this means that when r gets higher or c gets lower, the retailer wants some compensation from the supplier. In fact, this is a case that a supplier does not accept. However, for w_r to be negative λ should have a very small value and from (3.20) we know that the profit is allocated to supplier more, when λ gets smaller. So this will not be a problem for the supplier since negativity in the wholesale price in fact means that at the end he gets more profit. Additionally, when h stands for a salvage value, then there will not be any negativity for w_r.

The transfer payments and the structure of the contract changes if $h < 0$, that is h stands for salvage value and the retailer gives a share from the salvage addition to the revenue. Transfer payment includes an additional $-(1-\phi)h[\alpha Q - D]^+$ units. Retailer's expected profit function is:

$$\begin{aligned} E[\pi_r\left(Q, w_r, \phi\right)] &= \mu_\alpha Q\left(\phi r + g_r - w_r\right) - g_r\mu_D \\ &+ \left(\phi r + g_r + \phi h\right)\int_0^1\int_0^{\alpha Q}\left[D - \alpha Q\right]f(\alpha)g(D)dDd\alpha \end{aligned}$$

which is concave since $\phi r + g_r + \phi h > 0$ because $r > |h|$ since salvage cannot be more than selling price. The contract parameters are:

$$w_r = \frac{\phi(r-h)(c-\mu_\alpha h)}{r+g_r+h)\mu_\alpha} + \frac{g_r c + h\mu_\alpha(r-h)}{r+g_r+h)\mu_\alpha}$$

which shows that w_r increases with ϕ like the setting with holding cost.

The behaviour of w_r with respect to μ_α is also same when compared with the setting with holding cost:

$$\frac{\partial w_r}{\partial \mu_\alpha} = -\frac{c[\phi(r-h)+g_r]}{(r+g_r+h)\mu_\alpha^2}$$

For the applicable range, define following:

$$\begin{aligned} \phi &= \frac{\lambda(r+g_r-h)}{r-h} \\ w_r &= (1-\lambda)h + \frac{\lambda c}{\mu_\alpha} \end{aligned} \tag{3.36}$$

With these transformations, the expected profit function of the retailer and supplier becomes the ones presented in (3.19) and (3.20) respectively. Allocation of the profits with λ is same with the values of the setting with holding cost. Additionally, (3.36) reduces to the result in Cachon[10].

3.4.4. Quantity Flexibility Contract

In this contract, the retailer pays w_f for every unit received. Then supplier gives a credit for either the left over inventory, or a predetermined portion of the order that the retailer received. The choice is made by looking at which one is the smaller part. k is the parameter that determines the quantity to be credited.

$$T_q(Q,k,w_f) \quad = \quad w_f\alpha Q - (w_f+h)\ \min[(\alpha Q - D)^+, k\alpha Q]$$

$$\begin{aligned} E[T_f(Q,k,w_f)] = w_f\mu_\alpha Q \quad & -(w_f+h)\int_0^1\int_{\alpha Q(1-k)}^{\alpha Q}[\alpha Q - D]\, f(\alpha)g(D)dDd\alpha \\ & -(w_f+h)\int_0^1\int_0^{\alpha Q(1-k)} k\alpha Q f(\alpha)g(D)dDd\alpha \end{aligned}$$

The profit function of the retailer and the expected value of this function are:

$$
\begin{aligned}
\pi_r(Q, w_f, k) &= r \min(\alpha Q, D) - g_r [D - \alpha Q]^+ - h[\alpha Q - D]^+ \\
&- w_f \alpha Q + (w_f + h) \min[(\alpha Q - D)^+, k\alpha Q]
\end{aligned}
$$

$$
\begin{aligned}
E[\pi_r(Q, w_f, k)] &= \mu_\alpha Q (r + g_r - w_f) - g_r \mu_D \\
&+ (r + g_r - w_f) \int_0^1 \int_0^{\alpha Q} [D - \alpha Q] f(\alpha) g(D) dD d\alpha \\
&-(w_f + h) \int_0^1 \int_0^{\alpha Q(1-k)} [\alpha Q(1-k) - D] f(\alpha) g(D) dD d\alpha \quad (3.37)
\end{aligned}
$$

The profit function of the supplier and the expected value of this function are:

$$
\pi_s(Q, w_f, k) = w_f \alpha Q - (w_f + h) \min[(\alpha Q - D)^+, k\alpha Q] - cQ
$$

$$
\begin{aligned}
E[\pi_s(Q, w_f, k)] &= w_f \mu_\alpha Q - cQ \\
&- (w_f + h) \int_0^1 \int_{\alpha Q(1-k)}^{\alpha Q} [\alpha Q - D] f(\alpha) g(D) dD d\alpha \\
&- (w_f + h) \int_0^1 \int_0^{\alpha Q(1-k)} k\alpha Q f(\alpha) g(D) dD d\alpha \quad (3.38)
\end{aligned}
$$

Proposition 3.6 *Expected profit function given in (3.37) is concave in Q if $h + w_f \geq 0$ and $r + g_r - w_f \geq 0$*

Proof:

$$
\begin{aligned}
\frac{\partial (E[\pi_r(Q, w_f, k)])}{\partial Q} &= \mu_\alpha (r + g_r - w_f) \\
&- (r + g_r - w_f) \int_0^1 \alpha f(\alpha) G(\alpha Q) d\alpha \\
&- (w_f + h) \int_0^1 \alpha(1-k) f(\alpha) G[Q\alpha(1-k)] d\alpha
\end{aligned}
$$

$$\begin{aligned}\frac{\partial^2 \left(E\left[\pi_r\left(Q, w_f, k\right)\right]\right)}{\partial Q^2} &= -\left(r+g_r-w_f\right)\int_0^1 \alpha^2 f(\alpha) g(\alpha Q) d\alpha \\ &\quad -(w_f+h)\int_0^1 \alpha^2(1-k)^2 f(\alpha) g[Q\alpha(1-k)]d\alpha\end{aligned} \tag{3.39}$$

h and w_f are positive parameters. w_f is expected not to be greater than r, because the retailer cannot pay to the supplier more than he earns for a product. So the function is concave meaning that first order conditions are necessary and sufficient to find the optimal order quantity.□

Now let $\overline{Q}$ be the optimal quantity for the retailer.

$$\begin{aligned}\frac{\partial \left(E\left[\pi_r\left(Q, w_f, k\right)\right]\right)}{\partial Q} &= 0 \\ \mu_\alpha\left(r+g_r-w_f\right) & \\ -\left(r+g_r+h-w_f\right)\int_0^1 \alpha f(\alpha) G(\alpha\overline{Q}) d\alpha & \\ -(w_f+h)\int_0^1 \alpha(1-k) f(\alpha) G[\overline{Q}\alpha(1-k)]d\alpha &= 0\end{aligned} \tag{3.40}$$

Then for $\overline{Q}$ to be equal to Q^*:

$$K(\overline{Q}) = K(Q^*) = \frac{\mu_\alpha\left(r+g_r\right)-c}{\left(r+g_r+h\right)}$$

We want retailer to choose $\overline{Q}$ as the centralized optimal Q^*. Substitute the value in (3.8) in (3.40) to find the value of w_f. If we define the function X(Q) as:

$$X(Q) = \int_0^1 \alpha(1-k) f(\alpha) G[\alpha Q^*(1-k)]d\alpha \tag{3.41}$$

Then w_f is found as follows:

$$w_f = \frac{h[K(Q^*) - X(Q^*)] + c}{\mu_\alpha + X(Q^*) - K(Q^*)}$$
$$w_f = \frac{(\mu_\alpha h + c)(r + g_r + h)}{\mu_\alpha h + c + (r + g_r + h)X(Q^*)} - h \tag{3.42}$$

w_f does not only depend on μ_α. There is a closed form of function of k and α which does not let us to take the derivative of w_f with respect to μ_α. However we can see the behaviour of w_f with respect to k:

$$\frac{\partial w_f}{\partial k} = \frac{(\mu_\alpha h + c)(r + g_r + h)^2 \int_0^1 \alpha f(\alpha) \left[G(\alpha Q(1-k)) + \alpha Q(1-k) g(\alpha Q(1-k))\right]}{\left[\mu_\alpha h + c + (r + g_r + h)X(Q^*)\right]^2}$$

which is a positive value. This means that w_f is strictly increasing in k which is meaningful. If the supplier gives credit for more, obviously he requests more wholesale price to compensate that. The extreme values of k is important to see the range where the parameters are applicable and the allocation of profit. When k=*0*:

$$X(Q^*) = \int_o^1 \alpha f(\alpha) G(\alpha Q) d\alpha$$
$$= K(Q^*)$$
$$= \frac{\mu_\alpha (r + g_r) - c}{r + g_r + h} \tag{3.43}$$

$$w_f = \frac{(\mu_\alpha h + c)(r + g_r + h)}{\mu_\alpha h + c + (r + g_r + h)\frac{\mu_\alpha (r + g_r) - c}{r + g_r + h}} - h$$
$$= \frac{c}{\mu_\alpha} \tag{3.44}$$

which is simply the wholesale price.

The profit of the supplier is:

$$\begin{aligned} E\left[\pi_s\left(Q, w_f, k\right)\right] &= w_f \mu_\alpha Q - cQ \\ &- (w_f + h) \int_0^1 \int_{\alpha Q(1-0)}^{\alpha Q} \left[\alpha Q - D\right] f(\alpha) g(D) dD d\alpha \\ &- (w_f + h) \int_0^1 \int_0^{\alpha Q(1-0)} 0 \alpha Q f(\alpha) g(D) dD d\alpha \\ &= (\frac{c}{\mu_\alpha}) \mu_\alpha Q - cQ \\ &= 0 \end{aligned}$$

The supplier gets no profit while the retailer takes the whole chain profit, just like the wholesale price contract.

When *k=1*:

$$\begin{aligned} X(Q^*) &= \int_0^1 \alpha f(\alpha)(1-1) G[\alpha Q(1-1)] \\ &= 0 \end{aligned}$$

$$\begin{aligned} w_f &= \frac{(\mu_\alpha h + c)(r + g_r + h)}{\mu_\alpha h + c + (r + g_r + h)0} - h \\ &= r + g_r \end{aligned}$$

This time the wholesale price is higher than the sales price. So the wholesale price is manufacturing cost and a premium for random yield. The expected profit of the supplier is:

$$\begin{aligned} E\left[\pi_s\left(Q, w_f, k\right)\right] &= w_f \mu_\alpha Q - cQ - \\ &\quad (w_f + h) \int_0^1 \int_{\alpha Q(1-1)}^{\alpha Q} \left[\alpha Q - D\right] f(\alpha) g(D) dD d\alpha - \\ &\quad (w_f + h) \int_0^1 \int_0^{\alpha Q(1-1)} 1 \alpha Q f(\alpha) g(D) dD d\alpha \end{aligned}$$

$$
\begin{aligned}
&= (w_f + h)(\mu_\alpha Q - \int_0^1 \int_0^\alpha [\alpha Q - D] f(\alpha) g(D) dD d\alpha) \\
&\quad -h\mu_\alpha Q - cQ \\
&= E[\pi_c(Q)] + g_r \mu_D \qquad (3.45)
\end{aligned}
$$

So supplier gets more than the supply chain's profit, meaning that the retailer is in loss. The supplier's profit function is continuous in k; so every type of profit share is possible for k in [0,1). Another main point is that both of the extremes do not violate the concavity conditions given in Proposition 3.6.

When we consider the system without random yield, contract parameters are:

$$
\begin{aligned}
w_f &= \frac{(1 - G(Q^*))(r + g_r + h)}{1 + (1 - k)G(Q^*(1 - k)) - G(Q^*)} - h \\
&= \frac{(r + g_r + h)(h + c)}{(r + g_r + h)(1 - k)G(Q^*(1 - k)) + h + c} \qquad (3.46)
\end{aligned}
$$

which is the simplified form of the (3.42) when:

$$
\begin{aligned}
\mu_\alpha &= 1 \\
X(Q^*) &= (1 - k)G(Q^*(1 - k))
\end{aligned}
$$

The parameters result in same values for extreme cases of k. When k = 0:

$$
\begin{aligned}
w_f &= c \\
E[\pi_s(Q, w_f, k)] &= 0
\end{aligned}
$$

which is the same result with random yield when $\mu_\alpha = 1$. When k =1:

$$
\begin{aligned}
w_f &= r + g_r \\
E[\pi_s(Q, w_f, k)] &= E[\pi_c(Q)] + g_r \mu_D
\end{aligned}
$$

which is completely same with that of random yield model since there is no relation with μ_α in this case.

When h stands for salvage value instead of inventory holding cost, everything result stays same if $\frac{c}{\mu_\alpha} > |h|$. This condition holds because it is a *must* for this contract. The average production cost is $\frac{c}{\mu_\alpha}$ and the salvage value cannot be bigger than the production cost because the system orders infinitely many quantities if the products can be salvaged more than the production cost.

3.4.5. Quantity Discount Contract

In this contract, the unit price of a product is decreasing with respect to order quantity. So the more the retailer orders, the less she pays for a unit:

$$\begin{aligned} T_d(Q, w_d(Q)) &= w_d(Q)\alpha Q \\ E[T_d(Q, w_d(Q))] &= w_d(Q)\mu_\alpha Q \end{aligned}$$

Retailer's profit function and its expected value are:

$$\begin{aligned} \pi_r\left(Q, w_d(Q)\right) &= r\min\left(\alpha Q, D\right) - g_r\left[D - \alpha Q\right]^+ - h\left[\alpha Q - D\right]^+ \\ &- w_d(Q)\alpha Q \end{aligned}$$

$$\begin{aligned} E[\pi_r\left(Q, w_d(Q)\right)] &= \mu_\alpha Q\left(r + g_r\right) - g_r\mu_D - w_d(Q)\mu_\alpha Q + \\ &(r + g_r + h)\int_0^1\int_0^{\alpha Q}\left[D - \alpha Q\right]f(\alpha)g(D)dDd\alpha \end{aligned} \tag{3.47}$$

Supplier's profit function and its expected value are:

$$\begin{aligned} \pi_s\left(Q, w_d(Q)\right) &= w_d(Q)\alpha Q - cQ \\ E\left[\pi_s\left(Q, w_d(Q)\right)\right] &= w_d(Q)\mu_\alpha Q - cQ \end{aligned} \tag{3.48}$$

One way is to write the price $w_d(Q)$ as a portion of the chain's profit function. We define:

$$\begin{aligned} w_d(Q) &= \frac{1}{\mu_\alpha Q}(1-\lambda)(r+g_r+h)\int_0^1\int_0^{\alpha Q}[D-\alpha Q]f(\alpha)g(D)dDd\alpha \\ &+ \frac{\lambda}{\mu_\alpha}c+(r+g_r)(1-\lambda) \\ &= (1-\lambda)\frac{(r+g_r+h)}{\mu_\alpha}\frac{S(\alpha,Q)}{Q}-(1-\lambda)h+\frac{\lambda c}{\mu_\alpha} \end{aligned} \tag{3.49}$$

where λ is a parameter between 0 and 1. $\frac{S(\alpha,Q)}{Q}$ is decreasing in Q. If the coefficient of $\frac{S(\alpha,Q)}{Q}$ is positive, then $w_d(Q)$ is decreasing in Q. Then the coefficient should satisfy the following conditions:

$$\begin{aligned} (1-\lambda)\frac{(r+g_r+h)}{\mu_\alpha} &> 0 \\ 1 &> \lambda \\ r+g_r+h &> 0 \end{aligned}$$

all of which hold.

As it is expected, when $\lambda = 1$, the contract becomes a wholesale price contract and all the profit goes to the retailer.

When we define $w_d(Q)$ as in (3.49), the expected profit function of retailer and the supplier are the same with (3.19) and (3.20). Retailer gets all the profit when $\lambda = 1$ while supplier gets all when $\lambda = \frac{g_r\mu_D}{\pi_c+g_r\mu_D}$.

Another way to find $w_q(Q)$ for retailer to choose Q^*, is to set $\frac{\partial E[\pi_r(Q,w_d(Q))]}{\partial Q}$ to zero. It is a necessary but not a sufficient condition:

$$\begin{aligned} \mu_\alpha(r+g_r)-w_d(Q)\mu_\alpha-w_d^{'}(Q)Q\mu_\alpha-(r+g_r+h)K(Q)=0 \\ K(Q)=\frac{\mu_\alpha(r+g_r)-w_d(Q)\mu_\alpha-w_d^{'}(Q)Q\mu_\alpha}{r+g_r+h} \end{aligned}$$

For the retailer to chose Q^*, we substitute (3.8) into the equation above:

$$\frac{\mu_\alpha (r+g_r) - w_d(Q)\mu_\alpha - w_d^{'}(Q)Q\mu_\alpha}{r+g_r+h} = \frac{\mu_\alpha (r+g_r) - c}{r+g_r+h}$$

Then a simple differential equation comes out:

$$\begin{aligned} w_d^{'}(Q) + \frac{w_d(Q)}{Q} &= \frac{c}{\mu_\alpha Q} \\ w_d(Q) &= \frac{c}{\mu_\alpha} + \frac{\kappa}{Q} \end{aligned} \tag{3.50}$$

where κ is a constant. When we analyze w_d it can be seen that it decreases with Q which is the main condition of this contract. Additionally, Q has a decreasing function in μ_α. In fact, the result that occurs in buy-back and revenue sharing contracts holds here again. The retailer tries to stabilize the money that he pays for the whole delivery. Actually he does. The retailer pays w_d for the delivered units. The supplier's profit is:

$$\begin{aligned} E[\pi_s[Q,w_d]] &= Q\mu_\alpha(\frac{c}{\mu_\alpha} + \frac{\kappa}{Q}) - cQ \\ &= cQ + \kappa\mu_\alpha - cQ \\ &= \kappa\mu_\alpha \end{aligned} \tag{3.51}$$

which means that whatever the yield is, the retailer gives the production cost to the supplier and then gives a fixed price of $\kappa\mu_\alpha$. The fixed price, however, decreases with decreasing yield. So retailer makes a higher payment when the yield of the supplier is high.

If two different $w_d(Q)$ solutions are compared, (3.49) and (3.50), the similarity can be seen easily. For sake of completeness they are as follows:

$$\begin{aligned} w_d(Q) &= \frac{\lambda c}{\mu_\alpha} + (1-\lambda)\frac{(r+g_r+h)}{\mu_\alpha}\frac{S(\alpha,Q)}{Q} - (1-\lambda)h \\ w_d(Q) &= \frac{c}{\mu_\alpha} + \frac{\kappa}{Q} \end{aligned}$$

In the first equation, everything except the order quantity is fixed. When λ is set to a value then the equation becomes $\frac{\lambda c}{\mu_\alpha} + \frac{Constant}{Q}$. So both of the contracts has one part which compensates for the production cost of the supplier and one part which decreases with Q. One difference is that while the first one compensates partially the second one completely pays the production cost.

Now we have to establish whether $E[\pi_r(Q, w_d(Q))]$ is concave. The second derivative is:

$$\begin{aligned}
\frac{\partial^2 E[\pi_r(Q, w_d(Q))]}{\partial Q^2} &= -(r + g_r + h)\int_0^1 \alpha^2 f(\alpha)g(D)d\alpha \\
&\quad -2w_d^{'}(Q)\mu_\alpha - w_d^{''}(Q)Q\mu_\alpha
\end{aligned}$$

$$\begin{aligned}
&= -(r + g_r + h)\int_0^1 \alpha^2 f(\alpha)g(D)d\alpha \\
&\quad -2(-\frac{\kappa}{Q^2}\mu_\alpha) - \frac{2\kappa}{Q^3}Q\mu_\alpha \\
&= -(r + g_r + h)\int_0^1 \alpha^2 f(\alpha)g(D)d\alpha \leq 0
\end{aligned}$$

Then for every value of κ the function is concave and for different values of κ, the contract is a coordinating contract. If we compare it with the classical newsvendor:

$$\begin{aligned}
w_d(Q) &= (1-\lambda)(r + g_r + h)\frac{S(Q)}{Q} - (1-\lambda)h + \lambda c \\
w_d(Q) &= \frac{\kappa}{Q} + c
\end{aligned}$$

we see that the parameter μ_α drops from both of the equations. Additionally, $S(\alpha, Q)$ simplifies to $S(Q)$ since there is no random yield.

3.5. Supplier's Action under Voluntary Compliance

For a contract to coordinate the chain, the retailer should order the optimal order quantity that maximizes the chain's profit. However the quantity that the retailer orders may not be the optimal quantity for the supplier. In other words the supplier's profit function may reach to optimum with another quantity. With *forced compliance*, the supplier has to produce the quantity that retailer orders, whether or not this quantity is optimum for its profit function. If the order quantity of the retailer(or optimal order quantity of the chain) is also optimal for the supplier, then the supplier produces the ordered quantity voluntarily. This is called *voluntary compliance*. Until this point we presented which contract coordinates the chain under a forced compliance regime. We analyze the supplier's action to see whether the contracts can also coordinate under voluntary compliance.

3.5.1. Buy-back Contract

The supplier's expected profit function is given in (3.17). To establish its concavity with respect to Q:

Proposition 3.7 *Expected profit function given in (3.17) is concave in Q if* $b \geq 0$.

Proof:

$$\begin{aligned} \frac{\partial E\left[\pi_s\left(Q, w_b, b\right)\right]}{\partial Q} &= w_b\mu_\alpha - c \\ &- b\int_0^1 \alpha f(\alpha)G(\alpha Q)d\alpha \\ \frac{\partial^2 E\left[\pi_s\left(Q, w_b, b\right)\right]}{\partial Q^2} &= -b\int_0^1 \alpha^2 f(\alpha)g(\alpha Q)d\alpha \end{aligned} \tag{3.52}$$

is negative since b is nonnegative which provides the concavity of supplier's function in Q.□

Now for Q^* to be optimal:

$$w_b\mu_\alpha - c - b\int_0^1 \alpha f(\alpha)G(\alpha Q)d\alpha = 0$$
$$w_b = b\frac{K(Q^*)}{\mu_\alpha} + \frac{c}{\mu_\alpha}$$

which is the same with retailer's first order condition in (3.18). So supplier chooses Q^* as well meaning that even under voluntary compliance buy-back contract coordinates the chain.

3.5.2. Revenue Sharing Contract

Revenue sharing contract coordinates the chain under forced compliance like buy-back. For voluntary compliance, optimum order quantity for the supplier's expected profit function in (3.25) must be same with Q^*

Proposition 3.8 *Expected profit function given in (3.53) is concave in Q if $r \geq 0$.*

Proof:

$$\begin{aligned}
\frac{\partial E\left[\pi_s\left(Q, w_r, \phi\right)\right]}{\partial Q} &= \mu_\alpha(w_r + (1-\phi r)) - c \\
&- (1-\phi)r\int_0^1 \alpha f(\alpha)G(\alpha Q)d\alpha \\
\frac{\partial^2 E\left[\pi_s\left(Q, w_r, \phi\right)\right]}{\partial Q^2} = &- (1-\phi)r\int_0^1 \alpha^2 f(\alpha)g(\alpha Q)d\alpha \qquad (3.53)
\end{aligned}$$

Since r and ϕ are nonnegative, function is concave. □

Now for supplier to chose Q^* we see that same condition is needed with the retailer as it is proposed in (3.26):

$$w_r = \frac{(1-\phi)r[K(Q^*) - \mu_\alpha]}{\mu_\alpha} + \frac{c}{\mu_\alpha}$$

Hence supplier chooses the same quantity with the retailer which means that revenue share contract coordinates the chain under voluntary compliance regime.

3.5.3. Quantity Flexibility Contract

Quantity flexibility contract is found to coordinate the chain under forced compliance. The supplier should produce Q^* for voluntary compliance. To find the optimal order quantity for supplier we need to check the concavity of the expected profit function of the supplier given in (3.37)

$$\frac{\partial(E[\pi_s(Q,w_f,k)])}{\partial Q} = -c + w_f\mu_\alpha - (w_f+h)\int_0^1 \alpha f(\alpha)G(\alpha Q)d\alpha$$
$$(w_f+h)\int_0^1 \alpha(1-k)f(\alpha)G[Q\alpha(1-k)]d\alpha$$
$$\frac{\partial^2(E[\pi_s(Q,w_f,k)])}{\partial Q^2} = (w_f+h)\int_0^1 \alpha^2 f(\alpha)[(1-k)^2 g[\alpha Q(1-k)] - g(\alpha Q)]d\alpha$$

The concavity is not guaranteed. Therefore we cannot say that this contract is always a coordinating contract without forced compliance. Nevertheless it is helpful to see the first order conditions of the supplier's expected profit functions. For supplier to choose Q^*

$$\frac{\partial(E[\pi_s(Q,w_f,k)])}{\partial Q} = 0$$

$$\begin{aligned} -c + w_f\mu_\alpha - (w_f+h)\int_0^1 \alpha f(\alpha)G(\alpha Q)d\alpha & \\ (w_f+h)\int_0^1 \alpha(1-k)f(\alpha)G[Q\alpha(1-k)]d\alpha &= 0 \end{aligned}$$

$$w_f = \frac{c + h[K(Q^*) - X(Q^*)]}{\mu_\alpha + K(Q^*) - X(Q^*)}$$

where $X(Q)$ is defined in (3.41). So again, both retailer and supplier have the same first order conditions. Hence if this point can be found to be global optimum, then quantity flexibility contract can coordinate the chain under voluntary compliance. Also, if the function of expected profit of the supplier is found to be concave when distribution of yield and demand are realized, then the contract again coordinates the chain under voluntary compliance.

3.5.4. Quantity Discount Contract

Quantity discount contract coordinates the chain under voluntary compliance. For voluntary compliance, Q^* must maximize supplier's expected profit. For the contract parameters in (3.49), the profit of the supplier is a portion of the chain. So Q^* optimizes the expected profit function of the supplier. If the contract parameter in (3.50) is employed, then:

$$\begin{aligned} E[\pi_s(Q, w_d)] &= -cQ + \left\{ \frac{\kappa}{Q} + \frac{c}{\mu_\alpha} \right\} Q\mu_\alpha \\ &= \kappa\mu_\alpha \end{aligned}$$

which means that the profit for the supplier is independent of the ordered quantity, Q, but it is proportional to the constant κ. So the supplier accepts every quantity the retailer orders.

4. ASSEMBLY SYSTEMS UNDER RANDOM YIELD

4.1. Two Suppliers

4.1.1. Problem Definition

In this part, an assembly system with two suppliers and an assembler (manufacturer) is studied. A single product is produced by assembling two subcomponents obtained from two suppliers. As it is discussed in the literature review part, there is no such work that studied coordination in assembly systems with random demand and random yield. In this study, the centralized solution of the system is derived. Then the system is decentralized and contracts are discussed in order to see whether they are able to coordinate or not. Coordination with forced compliance regime is considered, that is the coordinating contracts assure that in decentralized solution, the manufacturer orders the optimal order quantity of the centralized system. The suppliers either accept to produce the ordered quantity under the contract conditions or reject the contracts all together. It turns out that some of the contracts that coordinate this chain are mixtures of well known contracts.

The centralized profit function is shown to be concave. Then, instead of solving the order quantity explicitly, the profit function of the manufacturer is written is such a way that it becomes a fraction of the chain's profit which guarantees that the quantity which maximizes the profit of the chain also maximizes the profit of the manufacturer.

The system is shown in Figure 4.1. The demand, D, is random and the density function and CDF of the demand is known by all the players. The manufacturer orders two distinct components from two suppliers, Q_1 and Q_2 in order to satisfy the demand. Each supplier produces different components and these components are used by the manufacturer to produce the final product. As well as the demand being random, due to the unreliability of the suppliers, the yield is also random. That is, when the manufacturer orders components from the suppliers, only a fraction of the ordered

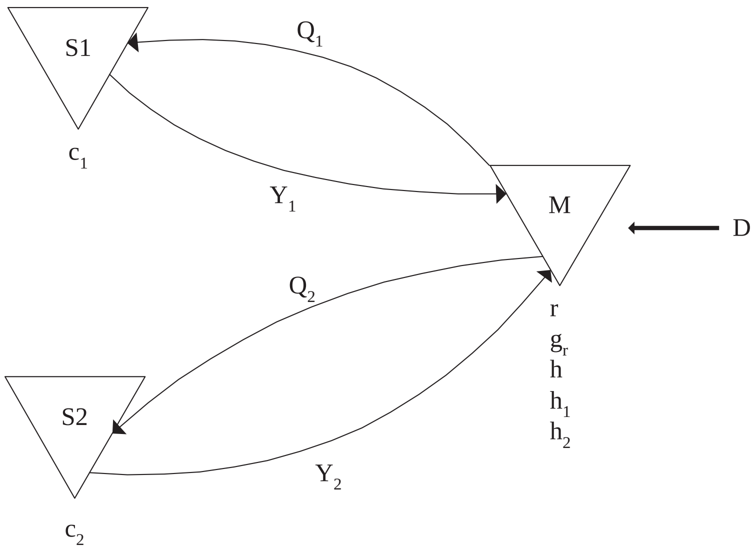

Figure 4.1. Assembly System

quantity is received by the manufacturer, $Y_1 = \alpha_1 Q_1$ and $Y_2 = \alpha_2 Q_2$ where α_1 and α_2 are random variables taking values in (0,1]. There is a cost for lost sales, g_r and holding the inventory of the components, h_1 and h_2. The unsold finished products are salvaged with a value of h. The notation is:

Q_i: *Order size for supplier i (Decision Variable)*

r: Selling price of the end product

g_r: *Opportunity loss of the manufacturer*

h: The salvage value of the end product, if not sold

h_i: *Holding cost of the component i; i=1,2*

c_i: *Cost of production per unit of supplier i*

D: Demand

α_i: *Portion of received order from supplier i, between 0 and 1*

$f_i(), F_i()$: *Distribution function and CDF of* α

g(), G(): Distribution and CDF of D, respectively

Y_i: αQ_i, *Stochastically proportional yield of supplier* i

μ_i : *Expected value of* α *for supplier i*

μ_D: *Expected value of the demand*

The profit of the chain is:

$$\begin{aligned}\pi_c(Q_1,Q_2) &= r[\textit{Sales}] - g_r[\textit{Lost Sales}] + h[\textit{End-product Inventory}] - c_1Q_1 - c_2Q_2\\ &- h_1[\textit{Inventory of component 1}] - h_2[\textit{Inventory of component 2}]\end{aligned}$$

where sales is minimum of delivered units or demand. Let's define the expected sales of the assembly system under random yield as following:

$$S_n(\alpha_i, Q_i) = E[\min(\alpha_1 Q_1, \alpha_2 Q_2, \ldots, \alpha_n Q_n, D)] \tag{4.1}$$

4.1.2. Centralized Setting

As it is cited in the newsvendor section, the system is said to have centralized setting if there is a single decision maker. When the parties in the system decide on their own to maximize their own profit, then the system is decentralized. The optimal solution of the centralized system is the maximum profit that the chain can make. So before decentralizing the chain, centralized solution of the system must be evaluated in order to analyze performance of the decentralized solution:

$$\begin{aligned}\pi_c(Q_1,Q_2) &= r\,[\min(\alpha_1Q_1,\alpha_2Q_2,D)] - g_r[D-\min(\alpha_1Q_1,\alpha_2Q_2)]^+ - c_1Q_1 - c_2Q_2\\ &+ h[\min(\alpha_1Q_1,\alpha_2Q_2)-D]^+\\ &- h_1[\alpha_1Q_1-\alpha_2Q_2]^+ - h_2[\alpha_2Q_2-\alpha_1Q_1]^+ \end{aligned}\tag{4.2}$$

When (4.2) is transformed to the following,the expected profit function of the chain becomes easier to analyse. For details please see Appendix B.

$$\begin{aligned}\pi_c(Q_1,Q_2) &= (r+g_r-h)[\min(\alpha_1Q_1,\alpha_2Q_2,D)] + h[\min(\alpha_1Q_1,\alpha_2Q_2)]\\ &+ h_1[\alpha_2Q_2-\alpha_1Q_1]^- + h_2[\alpha_1Q_1-\alpha_2Q_2]^-\\ &- g_rD - c_1Q_1 - c_2Q_2\end{aligned}$$

$$E[\pi_c(Q_1,Q_2)] = (r+g_r-h)S_2(\alpha_i,Q_i) + hE[\min(\alpha_1 Q_1,\alpha_2 Q_2)] + h_1 E\left[[\alpha_2 Q_2-\alpha_1 Q_1]^-\right] + h_2 E\left[[\alpha_1 Q_1-\alpha_2 Q_2]^-\right] - g_r\mu_D - c_1 Q_1 - c_2 Q_2 \quad (4.3)$$

where $[x]^-$ denotes $\min(0,x)$.

We first prove an intermediary result:

Lemma 4.1 *The hessian matrix of linear functions are both negative semi-definite and positive semi-definite.*

Proof: We take a linear function:

$$f(x_1,x_2...x_n) = a_1x_1 + a_1x_1... + a_nx_n$$

where a_i are constants. When we take the partial derivative with respect to any variable, x_k:

$$\frac{\partial f(x_1,x_2...x_n)}{\partial x_k} = a_k$$

we see that the result is a constant. Thus, the derivatives having degree of more than two, result in *zero* which means that the hessian matrix of *f()* are composed of *zeros*. So f() is jointly concave in $x_1, x_2...x_n$.$\Box$

Proposition 4.1 *Expected profit function given in (4.3) is jointly concave in* Q_1 *and* Q_2 *if* $r+g_r-h \geq 0$.

Proof: We analyze the function one by one:

$\underline{S_2(\alpha_i Q_i)}$

This function is $\min(\alpha_1Q_1,\alpha_2Q_2,D)$. The functions under minimum operation are linear functions of Q_1 and Q_2 and D is a constant and independent of Q. Linear functions are jointly concave by Lemma 4.1 Concavity is preserved under minimum operation[25]. So $S_2(\alpha_i Q_i)$ is concave when $r + g_r - h > 0$

$\underline{\min(\alpha_1 Q_1, \alpha_2 Q_2)}$

This function is again minimum of two linear functions. They are jointly concave by Lemma 4.1. As concavity is preserved under minimum operation, $\min(\alpha_1 Q_1, \alpha_2 Q_2)$ is concave.

$\underline{[\alpha_j Q_j - \alpha_i Q_i]^-}$

This function is in fact $\min[(\alpha_j Q_j - \alpha_i Q_i), 0]$. The part $(\alpha_j Q_j - \alpha_i Q_i)$ is a linear function and jointly concave in $Q_1 and Q_2$ by Lemma 4.1. Thus it is a linear function of Q_1 and Q_2 and concave *0* is a constant, so a linear function. Then the whole part, $[\alpha_j Q_j - \alpha_i Q_i]^-$ is concave since concavity is preserved under minimum operation.

$\underline{-g_r D - c_1 Q_1 - c_2 Q_2}$

This function is a linear function of Q_1 and Q_2. So it is concave by Lemma 4.1

So all parts are concave. Since sum of concave functions are concave[25], the profit function of the chain given is concave. Also since concavity is preserved under expectation, the chain's expected profit function in (4.1) is concave.

Although it may seem that this proof can be made also for convexity, it cannot be made since *minimum* operation violates convexity. □

4.1.3. Decentralized Setting

like the newsvendor problem, the players make their own decisions in decentralized setting. In the decentralized setting the profit function and the expected profit function of the players are shown in the equation below:

$$\begin{aligned}\pi_r(Q_1, Q_2) &= (r + g_r - h)[\min(\alpha_1 Q_1, \alpha_2 Q_2, D)] + h[\min(\alpha_1 Q_1, \alpha_2 Q_2)] \\ &+ h_1[\alpha_2 Q_2 - \alpha_1 Q_1]^- + h_2[\alpha_1 Q_1 - \alpha_2 Q_2]^- - g_r D \\ &- T_1() - T_2()\end{aligned}$$

$$\begin{aligned}E[\pi_r(Q_1, Q_2)] &= (r + g_r - h)S_2(\alpha_i, Q_i) + hE\left[\min(\alpha_1 Q_1, \alpha_2 Q_2)\right] \\ &+ h_1 E\left[[\alpha_2 Q_2 - \alpha_1 Q_1]^-\right] + h_2 E\left[[\alpha_1 Q_1 - \alpha_2 Q_2]^-\right] - g_r \mu_D \\ &- E[T_1()] - E[T_2()]\end{aligned}$$

$$\begin{aligned}\pi_{s_i}(Q_1, Q_2) &= T_i(\cdot) - c_i Q_i \\ E[\pi_{s_i}(Q_1, Q_2)] &= E[T_i(\cdot)] - c_i Q_i\end{aligned}$$

4.1.4. Contracts

The profit in decentralized setting is always less than or equal to the centralized profit. Since the aim of each player is to maximize its own profit, they deviate from the solution of the centralized system. Aim of the contracts is to modify the players' functions such that they choose the optimal order quantity which maximizes the whole chain's profit. In these contracts forced compliance is employed.

<u>4.1.4.1. Wholesale Price Contract.</u> The manufacturer only pays a wholesale price for the products. The profit function of the manufacturer is:

$$\begin{aligned} \pi_r &= (r+g_r-h)[\min(\alpha_1 Q_1, \alpha_2 Q_2, D)] + h[\min(\alpha_1 Q_1, \alpha_2 Q_2)] \\ &+ h_1[\alpha_2 Q_2 - \alpha_1 Q_1]^- + h_2[\alpha_1 Q_1 - \alpha_2 Q_2]^- - g_r D \\ &- w_1 Q_1 - w_2 Q_2 \end{aligned} \tag{4.4}$$

$$\begin{aligned} E[\pi_r] &= (r+g_r-h)S_2(\alpha_i, Q_i) + hE[\min(\alpha_1 Q_1, \alpha_2 Q_2)] \\ &+ h_1 E\left[[\alpha_2 Q_2 - \alpha_1 Q_1]^-\right] + h_2 E\left[[\alpha_1 Q_1 - \alpha_2 Q_2]^-\right] - g_r \mu_D \\ &- w_1 Q_1 - w_2 Q_2 \end{aligned} \tag{4.5}$$

Proposition 4.2 *Expected profit function given in (4.5) is jointly concave in* Q_1 *and* Q_2 *if* $r + g_r - h \geq 0$.

Proof:

Please see the proof of Proposition 4.1.

When the equation above is compared with (4.3), it can be seen that the only way to coordinate is, the wholesale prices should be equal to the cost of production, that is $w_i = c_i/\mu_{\alpha i}$. This means that the suppliers make *zero* profit. Coordination can be achieved via franchising payments.

<u>4.1.4.2. Buy-Back Contract.</u> One of the contracts which coordinate the chain in the previous chapter is the buy-back contract. So the buy-back contract is worth studying for the assembly system. The aim is to write the contract in such a way that the manufacturer's profit becomes a portion of the chain.

In this contract, the manufacturer pays to the suppliers a wholesale price, $w_{b,i}$ for each delivered units. The suppliers pay b_i to each manufacturer for the components which are not assembled(or in other words, not sold). The transfer payment is:

$$T_i(Q_1, Q_2, w_{b,i}, b_i) = w_{b,i}Q_i - b_i[\alpha_i Q_i - \min(\alpha_1 Q_1, \alpha_2 Q_2, D)]^+$$

Then the manufacturer's profit function is:

$$\begin{aligned}
\pi_r &= (r + g_r - h - b_1 - b_2)[\min(\alpha_1 Q_1, \alpha_2 Q_2, D)] \\
&+ (h + b_1 + b_2)[\min(\alpha_1 Q_1, \alpha_2 Q_2)] \\
&+ (h_1 - b_1)[\alpha_2 Q_2 - \alpha_1 Q_1]^- + (h_2 - b_2)[\alpha_1 Q_1 - \alpha_2 Q_2]^- \\
&- g_r D - w_{b,1}Q_1 - w_{b,2}Q_2
\end{aligned}$$

$$\begin{aligned}
E[\pi_r] &= (r + g_r - h - b_1 - b_2)S_2(\alpha_i, Q_i) \\
&+ (h + b_1 + b_2)E\left[\min(\alpha_1 Q_1, \alpha_2 Q_2)\right] \\
&+ (h_1 - b_1)E\left[[\alpha_2 Q_2 - \alpha_1 Q_1]^-\right] + (h_2 - b_2)E\left[[\alpha_1 Q_1 - \alpha_2 Q_2]^-\right] \\
&- g_r \mu_D - w_{b,1}Q_1 - w_{b,2}Q_2 \qquad (4.6)
\end{aligned}$$

Proposition 4.3 *Expected profit function given in (4.6) is jointly concave in* Q_1 *and* Q_2 *if* $r + g_r - h - b_1 - b_2 \geq 0$, $h + b_1 + b_2 \geq 0$, $h_1 - b_1 \geq 0$ *and* $h_2 - b_2 \geq 0$.

Proof:

Please see the proof of Proposition 4.1. □

Although the manufacturer's expected profit function is concave, it cannot be written as a fraction of the chain's profit which is the solution procedure we are trying to employ. However we cannot say it can coordinate or not with this information. First of all, the derivative of the expected profit function in (4.6) with respect to Q_1 and Q_2 must be found. Then both of the derivatives must be set to zero by substituting the optimal order quantity which is derived from (4.3). After substituting the optimal order quantity, the contract parameters which provides that derivative to be equal to zero have to be found. If such parameters exist, then buy-back contract is said to coordinate the chain. However, instead of solving all the derivatives, we try a buy-back contract with additional features.

4.1.4.3. Buy-Back with Sales Revenue Share and Recovery Payment. This contract is a mixed type of revenue share and buy - back. The manufacturer only shares the revenue for the sales, not for the salvage values. There is also a recovery payment for the worst player. That is the worst player gets a payment of V for every component it actually delivers.

Define $\overline{\phi} = 1 - \phi_1 - \phi_2$ where ϕ_i is the share of the revenue for the supplier i. The transfer payment and manufacturer's profit function respectively are:

$$
\begin{aligned}
T_i(Q_1, Q_2, w_{b,i}, b_i, \phi_i, V) &= w_{b,i}Q_i - b_i[\alpha_i Q_i - \min(\alpha_1 Q_1, \alpha_2 Q_2, D)]^+ \\
&+ r\phi_i[\min(\alpha_1 Q_1, \alpha_2 Q_2, D)] \\
&+ \begin{cases} V\alpha_i Q_i & \alpha_i Q_i < \alpha_j Q_j \\ 0 & o/w \end{cases}
\end{aligned} \tag{4.7}
$$

$$
\begin{aligned}
\pi_r &= (r(1-\phi_1-\phi_2) + g_r - h - b_1 - b_2)[\min(\alpha_1 Q_1, \alpha_2 Q_2, D)] \\
&+ (h + b_1 + b_2 - V)[\min(\alpha_1 Q_1, \alpha_2 Q_2)] \\
&+ (h_1 - b_1)[\alpha_2 Q_2 - \alpha_1 Q_1]^- + (h_2 - b_2)[\alpha_1 Q_1 - \alpha_2 Q_2]^- - g_r D \\
&- w_{b,1}Q_1 - w_{b,2}Q_2
\end{aligned}
$$

$$
\begin{aligned}
E[\pi_r] &= (r(1-\phi_1-\phi_2) + g_r - h - b_1 - b_2)S_2(\alpha_i, Q_i) \\
&+ (h + b_1 + b_2 - V)E[\min(\alpha_1 Q_1, \alpha_2 Q_2)] \\
&+ (h_1 - b_1)E\left[[\alpha_2 Q_2 - \alpha_1 Q_1]^-\right] + (h_2 - b_2)E\left[[\alpha_1 Q_1 - \alpha_2 Q_2]^-\right] \\
&- w_{b,1}Q_1 - w_{b,2}Q_2 - g_r\mu_D
\end{aligned} \tag{4.8}
$$

Proposition 4.4 *Expected profit function given in (4.8) is jointly concave in* Q_1 *and* Q_2 *if* $r(1-\phi_1-\phi_2)+g_r-h-b_1-b_2 \geq 0$ *,* $h+b_1+b_2-V \geq 0$*,* $h_1-b_1 \geq 0$ *and* $h_2-b_2 \geq 0$.

Proof:

Please see the proof of Proposition 4.1.

Now if $\phi = \phi_1+\phi_2$ and $A = r+g_r-h$ we make the following definitions:

$$\begin{aligned}
(A-r\phi)-b_1-b_2 &= \lambda(r+g_r-h) &\Rightarrow\quad r\phi &= (1-\lambda)(A-h_1-h_2)\\
h+b_1+b_2-V &= \lambda h &\Rightarrow\quad V &= (1-\lambda)(h+h_1+h_2)\\
h_1-b_1 &= \lambda h_1 &\Rightarrow\quad b_1 &= (1-\lambda)h_1\\
h_2-b_2 &= \lambda h_2 &\Rightarrow\quad b_2 &= (1-\lambda)h_2\\
& & w_{b,1} &= \lambda c_1\\
& & w_{b,2} &= \lambda c_2
\end{aligned} \tag{4.9}$$

In the relations above, λ is a parameter between 0 and 1. In the Proposition 4.4, it can be seen that several assumptions must hold for concavity. By changing the value of λ the payment scheme changes. As long as λ stays between the specified values(0 and 1), the expected profit function is concave and by definition λ is between 0 and 1 meaning that the expected profit function of the manufacturer is concave.

If $r+g_r > h+h_1+h_2$, then this contract is a buy-back and revenue share mix. However, if the reverse is true, then this contract is a buy-back and sales-rebate type of contract. Both coordinate the chain but the first one is more elegant.

Then the manufacturer's expected profit is:

$$E[\pi_r] = \lambda E[\pi_c] - (1-\lambda)g_r\mu_D \tag{4.10}$$

As for the profit share we can see that manufacturer gets all the profit with $\lambda = 1$ while the suppliers get it when $\lambda = \frac{g_r \mu_D}{E[\pi_c] + g_r \mu_D}$.

<u>4.1.4.4. Buy-Back with Revenue Share and Recovery Payment.</u> This contract is just like the previous one, but this time there is revenue share for both sales r and salvage h.

$$\begin{aligned} T_i(Q_1, Q_2, w_{b,i}, b_i, \phi_i, V) &= w_{b,i}Q_i - b_i[\alpha_i Q_i - min(\alpha_1 Q_1, \alpha_2 Q_2, D)]^+ \\ &+ r\phi_i[min(\alpha_1 Q_1, \alpha_2 Q_2, D)] \\ &+ h\phi_i[\min(\alpha_i Q_i, \alpha_j Q_j) - D]^+ \\ &+ \begin{cases} V\alpha_i Q_i & \alpha_i Q_i < \alpha_j Q_j \\ 0 & o/w \end{cases} \end{aligned} \tag{4.11}$$

If we define $\overline{\phi} = 1 - \phi_1 - \phi_2$, the profit function and its expected value are:

$$\begin{aligned} \pi_r &= (r\overline{\phi} + g_r - h\overline{\phi} - b_1 - b_2)[\min(\alpha_1 Q_1, \alpha_2 Q_2, D)] \\ &+ (h\overline{\phi} + b_1 + b_2 - V)[\min(\alpha_1 Q_1, \alpha_2 Q_2)] \\ &+ (h_1 - b_1)[\alpha_2 Q_2 - \alpha_1 Q_1]^- + (h_2 - b_2)[\alpha_1 Q_1 - \alpha_2 Q_2]^- \\ &- w_{b,1}Q_1 - w_{b,2}Q_2 - g_r D \end{aligned}$$

$$\begin{aligned} E[\pi_r] &= (r\overline{\phi} + g_r - h\overline{\phi} - b_1 - b_2)S_2(\alpha_i, Q_i) \\ &+ (h(1-\phi) + b_1 + b_2 - V)E[[\min(\alpha_1 Q_1, \alpha_2 Q_2)]] \\ &+ (h_1 - b_1)E\left[[\alpha_2 Q_2 - \alpha_1 Q_1]^-\right] + (h_2 - b_2)E\left[[\alpha_1 Q_1 - \alpha_2 Q_2]^-\right] \\ &- w_{b,1}Q_1 - w_{b,2}Q_2 - g_r \mu_D \end{aligned} \tag{4.12}$$

Proposition 4.5 *Expected profit function given in (4.12) is jointly concave in* Q_1 *and* Q_2 *if* $r(1-\phi_1-\phi_2)+g_r-h(1-\phi_1-\phi_2)-b_1-b_2 \geq 0$, $h(1-\phi)+b_1+b_2-V \geq 0$, $h_1-b_1 \geq 0$ *and* $h_2-b_2 \geq 0$.

Proof:

Please see the proof of Proposition 4.1.

Now if $\phi = \phi_1+\phi_2$ and $\bar{\phi} = 1-(\phi_1+\phi_2)$ we make the following definitions:

$$
\begin{aligned}
r\bar{\phi}+g_r-h\bar{\phi}-b_1-b_2 = &\ \lambda(r+g_r-h) & \Rightarrow \quad \phi = &\ (1-\lambda)\tfrac{r+g_r-h-h_1-h_2}{r-h} \\
h\bar{\phi}+b_1+b_2-V = &\ \lambda h & \Rightarrow \quad V = &\ (1-\lambda)\tfrac{r(h_1+h_2)-g_r h}{r-h} \\
h_1-b_1 = &\ \lambda h_1 & \Rightarrow \quad b_1 = &\ (1-\lambda)h_1 \\
h_2-b_2 = &\ \lambda h_2 & \Rightarrow \quad b_2 = &\ (1-\lambda)h_2 \\
& & w_{b,1} = &\ \lambda c_1 \\
& & w_{b,2} = &\ \lambda c_2
\end{aligned}
\tag{4.13}
$$

λ being a parameter between 0 and 1, assures the concavity of the system, like the previous contract.

According to the parameters, paying scheme can be different. Again, we need to cite that if $r+g_r-h-h_1-h_2>0$, then we have a revenue share. However, we need to take care of one more point. If $r(h_1+h_2)-g_r h>0$ then this contract includes a recovery payment. If the reverse is true, the worst one is punished per delivered unit, meaning its wholesale price is cut down. The manufacturer's expected profit is:

$$E[\pi_r] = \lambda E[\pi_c] - (1-\lambda)g_r\mu_D \tag{4.14}$$

When we look at the profit share we can see that manufacturer gets all the profit with $\lambda = 1$ while the suppliers get it when $\lambda = \frac{g_r\mu_D}{E[\pi_c]+g_r\mu_D}$.

4.1.4.5. Modified Buy-Back Contract. In a single supplier production system, the sales is just the minimum of the demand and the delivered units, so there is just one way for a buy-back contract. However the system we studied here is an assembly system meaning that there may be different types of buy-back contracts. In the previous contracts, the suppliers buy back the units which are more than sales. For example, if the realized sales is $\alpha_1 Q_1$, remember sales is $\min(\alpha_1 Q_1, \alpha_2 Q_2, D)$, then supplier 2 buys back $(\alpha_2 Q_2 - \alpha_1 Q_1)$ units. If sales is D, then supplier 2 buys back $(\alpha_2 Q_2 - D)$ units. However if the sales is D, then it means that there is some inventory left which is salvaged. Let's assume the realized values are $D < \alpha_1 Q_1 < \alpha_2 Q_2$. Then $(\alpha_1 Q_1 - D)$ units are salvaged and $(\alpha_2 Q_2 - \alpha_1 Q_1)$ units are left for inventory. So supplier buys back $(\alpha_2 Q_2 - \alpha_1 Q_1)$ units. As well as buy-back price, the manufacturer pays a wholesale price w_m to the suppliers. The transfer payments are:

$$T_i(Q_1, Q_2, w_{m_i}, m_i) = w_{m,i} Q_i - m_i [\alpha_i Q_i - \alpha_j Q_j]^+$$

The manufacturer's profit function is:

$$\begin{aligned}
\pi_r &= (r + g_r - h)[\min(\alpha_1 Q_1, \alpha_2 Q_2, D)] + h[\min(\alpha_1 Q_1, \alpha_2 Q_2)] \\
&+ (h_1 - m_1)[\alpha_2 Q_2 - \alpha_1 Q_1]^- + (h_2 - m_2)[\alpha_1 Q_1 - \alpha_2 Q_2]^- \\
&- w_{m,1} Q_1 - w_{m,2} Q_2 - g_r D
\end{aligned}$$

$$\begin{aligned}
E[\pi_r] &= (r + g_r - h)E[\min(\alpha_1 Q_1, \alpha_2 Q_2, D)] \\
&+ hE[\min(\alpha_1 Q_1, \alpha_2 Q_2)] \\
&+ (h_1 - m_1)E\left[[\alpha_2 Q_2 - \alpha_1 Q_1]^-\right] + (h_2 - m_2)E\left[[\alpha_1 Q_1 - \alpha_2 Q_2]^-\right] \\
&- w_{m,1} Q_1 - w_{m,2} Q_2 - g_r \mu_D \qquad (4.15)
\end{aligned}$$

Proposition 4.6 *Expected profit function given in 4.15 is jointly concave in Q_1 and Q_2 if $r + g_r - h \geq 0$, $h \geq 0$, $h_1 - m_1 \geq 0$ and $h_2 - m_2 \geq 0$.*

Proof:

Please see the proof of Proposition 4.1.

As the manufacturer's function is concave, her contract parameters can be found for optimum Q. However we cannot write this function as a fraction of the chain's expected profit function. The procedure that is cited in Section 4.1.4.2 applies here too. Hence, again, we add more mechanism.

4.1.4.6. Modified Buy-Back with Sales Revenue Share and Recovery Payment. This contract includes, addition to the contract in Section 4.1.4.5, a recovery payment to the worst supplier for every unit it ships. The manufacturer pays a revenue share to the suppliers for the sold units, but not for the salvaged units. Transfer payment is:

$$\begin{aligned} T_i(Q_1, Q_2, w_{m,i}, m_i, \phi_i, V) &= w_{m,i}Q_i - m_i[\alpha_i Q_i - \alpha_j Q_j]^+ \\ &+ r\phi_i \min(\alpha_1 Q_1, \alpha_2 Q_2, D) \\ &+ \begin{cases} V\alpha_i Q_i & \alpha_i Q_i < \alpha_j Q_j \\ 0 & o/w \end{cases} \end{aligned} \tag{4.16}$$

The manufacturer's profit function is be as follows:

$$\begin{aligned} \pi_r &= (r\overline{\phi} + g_r - h)[\min(\alpha_1 Q_1, \alpha_2 Q_2, D)] \\ &+ (h - V)[\min(\alpha_1 Q_1, \alpha_2 Q_2)] \\ &+ (h_1 - m_1)[\alpha_2 Q_2 - \alpha_1 Q_1]^- + (h_2 - m_2)[\alpha_1 Q_1 - \alpha_2 Q_2]^- \\ &- w_{m,1}Q_1 - w_{m,2}Q_2 - g_r D \end{aligned}$$

$$\begin{aligned} E\left[\pi_r\right] &= (r(\overline{\phi} + g_r - h)S_2(\alpha_i, Q_i) + (h - V)E[\min(\alpha_1 Q_1, \alpha_2 Q_2)] \\ &+ (h_1 - m_1)E[\alpha_2 Q_2 - \alpha_1 Q_1]^- + (h_2 - m_2)E[\alpha_1 Q_1 - \alpha_2 Q_2]^- \\ &- w_{m,1}Q_1 - w_{m,2}Q_2 - g_r \mu_D \end{aligned} \tag{4.17}$$

Proposition 4.7 *Expected profit function given in (4.17) is jointly concave in* Q_1 *and* Q_2 *if* $r(1-\phi_1-\phi_2)+g_r-h \geq 0$, $h-V \geq 0$, $h_1-m_1 \geq 0$ *and* $h_2-m_2 \geq 0$.

Proof:

Please see the proof of Proposition 4.1.

Now if $\phi = \phi_1 + \phi_2$ we make the following definitions:

$$\begin{aligned}
r(1-\phi)+g_r-h = &\ \lambda(r+g_r-h) & \Rightarrow \quad r\phi = &\ (1-\lambda)(r+g_r-h) \\
h-V = &\ \lambda h & \Rightarrow \quad V = &\ (1-\lambda)h \\
h_1-m_1 = &\ \lambda h_1 & \Rightarrow \quad m_1 = &\ (1-\lambda)h_1 \\
h_2-m_2 = &\ \lambda h_2 & \Rightarrow \quad m_2 = &\ (1-\lambda)h_2 \\
& & w_{m,1} = &\ \lambda c_1 \\
& & w_{m,2} = &\ \lambda c_2
\end{aligned} \tag{4.18}$$

With the transformations above, the manufacturer's expected profit function can be written just like (4.10) showing that the contract can coordinate the chain. Manufacturer gets all the profit with $\lambda = 1$ while the all the profit goes to the suppliers when $\lambda = \frac{g_r\mu_D}{E[\pi_c]+g_r\mu_D}$.

4.1.4.7. Modified Buy-Back with Revenue Share and Recovery Payment. This contract is just like the previous one, additionally there is a revenue share for salvage as well as the sales.

$$\begin{aligned}
T_i(Q_1,Q_2,w_{m,i},m_i,\phi_i,V) \ = &\ \ w_{r,i}Q_i - m_i[\alpha_iQ_i-\alpha_jQ_j] \\
& + \ r\phi_i\min(\alpha_1Q_1,\alpha_2Q_2,D) + h\phi_i[\min(\alpha_iQ_i,\alpha_jQ_j)-D]^+ \\
& + \begin{cases} V\alpha_iQ_i & \alpha_iQ_i < \alpha_jQ_j \\ 0 & o/w \end{cases}
\end{aligned} \tag{4.19}$$

If we define $\overline{\phi} = 1 - \phi_1 - \phi_2$, manufacturer's profit function is:

$$\begin{aligned}
\pi_r &= (r\overline{\phi} + g_r - h\overline{\phi})[\min(\alpha_1 Q_1, \alpha_2 Q_2, D)] \\
&+ (h\overline{\phi} - V)[\min(\alpha_1 Q_1, \alpha_2 Q_2)] \\
&+ (h_1 - m_1)[\alpha_2 Q_2 - \alpha_1 Q_1]^- + (h_2 - m_2)[\alpha_1 Q_1 - \alpha_2 Q_2]^- \\
&- w_{r,1} Q_1 - w_{r,2} Q_2 - g_r D
\end{aligned}$$

$$\begin{aligned}
E\left[\pi_r\right] &= (r\overline{\phi} + g_r - h\overline{\phi}) S_2(\alpha_i, Q_i) \\
&+ (h\overline{\phi} - V) E[\min(\alpha_1 Q_1, \alpha_2 Q_2)] \\
&+ (h_1 - m_1) E\left[[\alpha_2 Q_2 - \alpha_1 Q_1]^-\right] + (h_2 - m_2) E\left[[\alpha_1 Q_1 - \alpha_2 Q_2]^-\right] \\
&- w_{r,1} Q_1 - w_{r,2} Q_2 - g_r \mu_D
\end{aligned} \tag{4.20}$$

Proposition 4.8 *Expected profit function given in (4.20) is jointly concave in Q_1 and Q_2 if $r\overline{\phi} + g_r - h\overline{\phi} \geq 0$, $h\overline{\phi} - V \geq 0$, $h_1 - m_1 \geq 0$ and $h_2 - m_2 \geq 0$.*

Proof:

Please see the proof of Proposition 4.1.

Now if $\phi = \phi_1 + \phi_2$ let's make the following definitions:

$$\begin{array}{rlcrl}
r\overline{\phi} + g_r - h\overline{\phi} = & \lambda(r + g_r - h) & \Rightarrow & \phi = & \frac{(1-\lambda)(r+g_r-h)}{r-h} \\
h\overline{\phi} - V = & \lambda h & \Rightarrow & V = & -h\frac{(1-\lambda)g_r}{r-h} \\
h_1 - m_1 - s_2 = & \lambda h_1 & \Rightarrow & m_1 = & (1-\lambda)h_1 \\
h_2 - m_2 - s_1 = & \lambda h_2 & \Rightarrow & m_2 = & (1-\lambda)h_2 \\
 & & & w_{r,1} = & \lambda c_1 \\
 & & & w_{r,2} = & \lambda c_2
\end{array} \tag{4.21}$$

Here we found out that V should be negative, meaning that the worst one should be punished for every unit it send. That is the worst supplier's wholesale price is cut down V many units. The profit of the manufacturer is the same as in (4.10) so the value of λ is same for profit shares, that is manufacturer gets all the profit with $\lambda = 1$ while the all the profit goes to the suppliers when $\lambda = \frac{g_r \mu_D}{E[\pi_c] + g_r \mu_D}$.

4.1.4.8. Revenue Share with Modified Buy-Back without Salvage . The setting is same with the contract presented in Section 4.1.4.7. Now we assume that the end items are not salvaged. In fact, as there is no holding cost, it can be assumed that there is a salvage value so that the cost of assembly and maintenance is netted from the salvage. As h=0, this setting is also same with Section 4.1.4.6. There is no recovery payment to the worst, because the manufacturer now makes less profit so an extra payment distorts manufacturer's behavior. Staying consistent with the notation manufacturer's profit function:

$$T_i(Q_1, Q_2, w_{m_i}, m_i, \phi_i) = w_{r,i}Q_i - m_i[\alpha_i Q_i - \alpha_j Q_j] - s_i[\alpha_j Q_j - \alpha_i Q_i]^+ + r\phi_i[Sales]$$

$$\begin{aligned}
\pi_r &= (r\overline{\phi} + g_r)[\min(\alpha_1 Q_1, \alpha_2 Q_2, D)] \\
&+ (h_1 - m_1)[\alpha_2 Q_2 - \alpha_1 Q_1]^- + (h_2 - m_2)[\alpha_1 Q_1 - \alpha_2 Q_2]^- \\
&- w_{r,1}Q_1 - w_{r,2}Q_2 - g_r D
\end{aligned}$$

$$\begin{aligned}
E[\pi_r] &= (r\overline{\phi} + g_r)S_2(\alpha_i, Q_i) \\
&+ (h_1 - m_1)E\left[[\alpha_2 Q_2 - \alpha_1 Q_1]^-\right] + (h_2 - m_2)E\left[[\alpha_1 Q_1 - \alpha_2 Q_2]^-\right] \\
&- w_{r,1}Q_1 - w_{r,2}Q_2 - g_r \mu_D \qquad (4.22)
\end{aligned}$$

Proposition 4.9 *Expected profit function given in (4.22) is jointly concave in Q_1 and Q_2 if $r\overline{\phi} + g_r \geq 0$, $h_1 - m_1 \geq 0$ and $h_2 - m_2 \geq 0$.*

Proof:

Please see the proof of Proposition 4.1.

Like previous coordinating contracts, we make following statements:

$$
\begin{aligned}
r\overline{\phi} + g_r = \;& \lambda(r + g_r) & \Rightarrow \quad \phi = \;& \tfrac{(1-\lambda)(r+g_r)}{r} \\
h_1 - m_1 = \;& \lambda h_1 & \Rightarrow \quad m_1 = \;& (1-\lambda)h_1 \\
h_2 - m_2 = \;& \lambda h_2 & \Rightarrow \quad m_2 = \;& (1-\lambda)h_2 \\
& & w_{r,1} = \;& \lambda c_1 \\
& & w_{r,2} = \;& \lambda c_2
\end{aligned}
$$

The manufacturer's profit function is a portion of the chain's, as defined in (4.10). The lambda values for profit share is same.

4.2. N Suppliers

4.2.1. Problem Definition

We now extend the results of two-supplier models to N suppliers. First of all, the profit function and its expected value is derived and then its concavity is established. Then without solving the ordering quantities explicitly, the contracts proposed in two supplier system are implemented and coordination mechanisms are studied. The contracts proposed in the previous section seem to suit well for N-supplier system.

In short, the system is same with the previous assembly system, except, now there are N suppliers. Staying consistent with the notation, the profit of the chain is:

$$\begin{aligned}\pi_c &= r[\mathit{Sales}] + h[\mathit{FinishedGoodsInventory}] \\ &- g_r[\mathit{Lost\ Sales}] - \sum_N c_i Q_i - \sum_N h_i[\mathit{Inventory\ of\ component\ i}]\end{aligned}$$

4.2.2. Centralized Setting

The chain's profit function can be written as:

$$\begin{aligned}\pi_c &= r[\min_i(D, \alpha_i Q_i)] + h[\min_i(\alpha_i Q_i) - D)]^+ \\ &- g_r[D - \min_i(\alpha_i Q_i]^+ - \sum_N c_i Q_i \\ &- \sum_N h_i[\alpha_i Q_i - \min_i(\alpha_i Q_i)]\end{aligned}$$

which, by the transformatin in Appendix C, can be written as:

$$\begin{aligned}E[\pi_c] &= (r + g_r - h)S_N(\alpha_i, Q_i) - g_r\mu_D + hE[\min_i(\alpha_i Q_i)] \\ &- E\left[\sum_N (h_i \alpha_i Q_i)\right] + E\left[\min_i(\alpha_i Q_i)\right] \sum_N h_i - \sum_N c_i Q_i \qquad (4.23)\end{aligned}$$

Proposition 4.10 *Expected profit function given in (4.23) is jointly concave in* $Q_1,\ldots,$ Q_N *if* $r + g_r - h \geq 0$ *,*$h \geq 0$ *and* $\sum_i h_i \geq 0$ *.*

Proof:

For sake of simplicity, now let's take one by one the components of the function:

$\underline{S_N(\alpha_i, Q_i)}$

This is the sales which is in fact $\min(\alpha_1 Q_1, \ldots, \alpha_N Q_N, D)$. The functions under the minimum operation are linear functions which are jointly concave in $Q_1, ..., Q_N$ by Lemma 4.1. Concavity is preserved under minimum operation[25], so this part of the function is concave.

$\underline{\min_i(\alpha_i Q_i)}$

Linear functions are jointly concave by Lemma 4.1 and concavity is preserved under minimum operation.

$\underline{\sum_N (h_i \alpha_i Q_i)}$

This part is simply sum of linear functions. In fact this is a linear function of $Q_1, \ldots, Q_N$. Thus this part is concave.

$\underline{\min_i(\alpha_i Q_i) \sum_N h_i}$

It is known that $min_i(\alpha_i Q_i)$ is concave. This part is simply product of a concave function with a constant, $\sum_N h_i$. So this part is concave.

$\underline{-\sum_N c_i Q_i - g_r D}$

Simply a linear function of $Q_1, \ldots, Q_N$. Linear functions are jointly concave by Lemma 4.1.

Since sum of concave functions are concave[25], the profit function of the chain is concave. Concavity is preserved under expectation, so the expected profit function is concave.□

One might think that since linear functions have a hessian of *zero*s, this function is also a convex function too. However, minimum operation only preserves concavity while convexity is preserved under maximum operation.

4.2.3. Decentralized Setting

Manufacturer's and suppliers' profit functions are:

$$\begin{aligned}
\pi_r(Q_1,...,Q_N) &= r[\min_i(D,\alpha_i Q_i)] + h[\min_i(\alpha_i Q_i) - D]^+ \\
&- g_r[D - \min_i(\alpha_i Q_i]^+ - \sum_N h_i[\alpha_i - \min_i(\alpha_i Q_i)] - \sum_N T_i(\cdot) \\
\pi_{s_i}(Q_1,...,Q_N) &= T_i(\cdot) - c_i Q_i
\end{aligned}$$

4.2.4. Contracts

<u>4.2.4.1. WholeSale Price Contract.</u> The manufacturer pays the suppliers only a wholesale price. The profit function of the manufacturer is:

$$\begin{aligned}
\pi_r &= (r + g_r - h)[\min_i(D,\alpha_i Q_i)] - g_r D + h[\min_i(\alpha_i Q_i)] \\
&- \sum_N(h_i\alpha_i Q_i) + \min_i(\alpha_i Q_i)\sum_N h_i - \sum_N w_i Q_i
\end{aligned}$$

$$\begin{aligned}
E\left[\pi_r\right] &= (r + g_r - h)S_N(\alpha_i, Q_i) - g_r\mu_D + hE[\min_i(\alpha_i Q_i)] \\
&- E\left[\sum_N(h_i\alpha_i Q_i)\right] + E\left[\min_i(\alpha_i Q_i)\right]\sum_N h_i - \sum_N w_i Q_i \qquad (4.24)
\end{aligned}$$

Proposition 4.11 *Expected profit function given in (4.24) is jointly concave in* Q_1,..., Q_N *if* $r + g_r - h \geq 0$,$h \geq 0$ *and* $\sum_i h_i \geq 0$.

Proof:

Please see the proof of Proposition 4.23

When the profit function is compared with the decentralized solution in (4.23), it can be seen that, the only way to coordinate is to set the wholesale prices to the production cost of the suppliers, which does not let the suppliers to make profit. As the suppliers get zero profit, this contract cannot coordinate the chain as long as there is no end-of-term or franchising type of payments.

4.2.4.2. Buy-Back with Sales Revenue Share and Recovery Payment. This contract is N supplier type of the contract defined in Section 4.1.4.3. The transfer payment and the manufacturer's profit function are:

$$\begin{aligned} T_i(Q_i, w_{b,i}, b_i, \phi_i, V) &= w_iQ_i - b_i[\alpha_iQ_i - \min_i(D, \alpha_iQ_i)] + r\phi_i \min_i(D, \alpha_iQ_i) \\ &+ \begin{cases} V\alpha_iQ_i & \alpha_iQ_i < \alpha_jQ_j \quad \forall \quad j \neq i \\ 0 & o/w \end{cases} \end{aligned} \tag{4.25}$$

$$\begin{aligned} \pi_r &= (r(1 - \sum_N \phi_i) + g_r - h - \sum_N b_i)[\min_i(D, \alpha_iQ_i)] \\ &+ (h - V + \sum_N b_i)[\min_i(\alpha_iQ_i)] - \sum_N (h_i - b_i)[\alpha_iQ_i - \min_i(\alpha_iQ_i)] \\ &- g_rD - \sum_N w_{b,i}Q_i \end{aligned}$$

$$\begin{aligned} E[\pi_r] &= (r(1 - \sum_N \phi_i) + g_r - h - \sum_N b_i)S_N(\alpha_i, Q_i) \\ &+ (h - V + \sum_N b_i)E[\min_i(\alpha_iQ_i)] - \sum_N (h_i - b_i)E[\alpha_iQ_i - \min_i(\alpha_iQ_i)] \\ &- g_r\mu_D - \sum_N w_{b,i}Q_i \end{aligned} \tag{4.26}$$

Proposition 4.12 *Expected profit function given in (4.26) is jointly concave in* $Q_1, \ldots,$ Q_N *if* $r(1 - \sum_N \phi_i) + g_r - h - \sum_N b_i \geq 0$ *,* $h - V + \sum_N b_i \geq 0$ *and* $\sum_i (h_i - b_i) \geq 0$ *.*

Proof:

Please see the proof of Proposition 4.23

When we define $\sum_N \phi_i = \phi$ and $1 - \sum_N \phi_i = \bar{\phi}$, we can write the following:

$$
\begin{array}{rlcrl}
(r\bar{\phi} + g_r - h - \sum_N b_i) = & \lambda(r + g_r - h) & \Rightarrow & \phi = & \frac{(1-\lambda)(r+g_r-h-\sum_N h_i)}{r} \\
h - V + \sum_N b_i = & \lambda h & \Rightarrow & V = & (1-\lambda)(h + \sum_N h_i) \\
h_i - b_i = & \lambda h_i & \Rightarrow & b_i = & (1-\lambda)h_i \\
 & & & w_{b,i} = & \lambda c_i
\end{array}
$$

Like the contract in Section 4.2.4.5, there is a point that must be clarified in this contract. The contract payment scheme changes according to the parameter values. If $r + g_r > h + \sum_N h_i$ then related part of the payment scheme is a revenue share. But if not, then the related part resembles a sales rebate contract. Both coordinate the chain but the first one seems more relevant.

Then, manufacturer's profit function is the same with (4.10). The λ values are also the same. The manufacturer gets all the profit with $\lambda = 1$ and zero profit when $\frac{g_r \mu_D}{E[\pi_c] + g_r \mu_D}$.

<u>4.2.4.3. Buy-Back with Revenue Share and Recovery Payment.</u> Contract presented in this section is N supplier type of the contract defined in Section 4.1.4.4. The transfer payment and the manufacturer's profit function are:

$$
\begin{aligned}
T_i(Q_i, w_{b,i}, b_i, \phi_i, V) \quad = \quad & w_{b_i} Q_i - b_i[\alpha_i - min_i(D, \alpha_i Q_i)] \\
+ \quad & h\phi_i[\min_i(\alpha_i Q_i) - min_i(D, \alpha_i Q_i)]^+ + r\phi_i[min_i(D, \alpha_i Q_i)] \\
+ \quad & \begin{cases} V\alpha_i Q_i & \alpha_i Q_i < \alpha_j Q_j \quad \forall \quad j \neq i \\ 0 & o/w \end{cases}
\end{aligned} \tag{4.27}
$$

$$\begin{aligned}\pi_r &= (r(1-\sum_N \phi_i)+g_r-h(1-\sum_N \phi_i)-\sum_N b_i)[\min_i(D,\alpha_i Q_i)]\\ &+ (h-V+\sum_N b_i)[\min_i(\alpha_i Q_i)]-\sum_N (h_i-b_i)[\alpha_i Q_i-\min_i(\alpha_i Q_i)]\\ &- g_r D-\sum_N w_{b,i}Q_i\end{aligned}$$

$$\begin{aligned}E[\pi_r] &= (r(1-\sum_N \phi_i)+g_r-h(1-\sum_N \phi_i)-\sum_N b_i)S_N(\alpha_i,Q_i)\\ &+ (h-V+\sum_N b_i)E[\min_i(\alpha_i Q_i)]-\sum_N (h_i-b_i)E[\alpha_i Q_i-\min_i(\alpha_i Q_i)]\\ &- g_r\mu_D-\sum_N w_{b,i}Q_i \qquad (4.28)\end{aligned}$$

Proposition 4.13 *Expected profit function given in (4.28) is jointly concave in* $Q_1,\dots,$ Q_N *if* $r(1-\sum_N \phi_i)+g_r-h(1-\sum_N \phi_i)-\sum_N b_i \geq 0$ *,*$h-V+\sum_N b_i \geq 0$ *and* $\sum_i(h_i-b_i)\geq 0$ *.*

Proof:

Please see the proof of Proposition 4.23

When we define $\phi=\sum_N \phi_i$ and $\bar{\phi}=1-\phi$, we can write the following:

$$\begin{aligned}(r\bar{\phi}+g_r-h\bar{\phi}-\textstyle\sum_N b_i) &= \lambda(r+g_r-h) &\Rightarrow\quad \phi &= \tfrac{(1-\lambda)(r+g_r-h-\sum_N h_i)}{r-h}\\ h\bar{\phi}+\textstyle\sum_N b_i-V &= \lambda h &\Rightarrow\quad V &= (1-\lambda)\tfrac{r\sum_N h_i-g_r h}{r-h}\\ h_i-b_i &= \lambda h_i &\Rightarrow\quad b_i &= (1-\lambda)h_i\\ & & w_{b,i} &= \lambda c_i\end{aligned}$$

This contract is a bit different from the others. The paying scheme changes according to the parameters. If $r+g_r>h+\sum_N h_i$, then the contract is a revenue share, otherwise a sales rebate contract; like the previous one. Also, like the contract in Section 4.1.4.4, we need to check $r\sum_N h_i>g_r h$. If so, then there is recovery payment. Otherwise V is a punishment, rather than being a recovery payment to the worst. It is such as cutting down the wholesale price of that(the worst) supplier.

The manufacturer's profit function and the lambda values are the same with (4.10). Manufacturer gets all the profit with $\lambda = 1$ and zero profit when $\frac{g_r \mu_D}{E[\pi_c]+g_r\mu_D}$.

4.2.4.4. Modified Buy-Back with Sales Revenue Share and Recovery Payment. This contract is N-Supplier form of the contract defined in Section 4.1.4.6. Every supplier is paid $w_{m,i}$ for each unit ordered, plus a revenue share from the sales(not salvage). Each supplier then pays a penalty of m_i for each units that excess the sales.

Transfer payment and the manufacturer's profit function is shown below:

$$\begin{aligned} T_i(Q_i, w_{m,i}, m_i, \phi_i, V) &= w_i Q_i + r\phi_i[\min_i(D, \alpha_i Q_i)] \\ &+ \begin{cases} V\alpha_i Q_i & \alpha_i Q_i < \alpha_j Q_j \quad \forall \quad j \neq i \\ 0 & o/w \end{cases} \\ &- m_i[\alpha_i Q_i - \min_j(\alpha_j Q_j)]^+ \end{aligned}$$

$$\begin{aligned} \pi_r &= (r(1-\sum_N \phi_i) + g_r - h)[\min_i(D, \alpha_i Q_i)] - g_r D \\ &+ (h-V)[\min_i(\alpha_i Q_i)] \\ &- \sum_N (h_i - m_i)[\alpha_i Q_i - \min_i(\alpha_i Q_i)] - \sum_N w_i Q_i \end{aligned}$$

$$\begin{aligned} E[\pi_r] &= (r(1-\sum_N \phi_i) + g_r - h)S_N(\alpha_i, Q_i) - g_r \mu_D \\ &+ +(h-V)[\min_i(\alpha_i Q_i)] \\ &- \sum_N (h_i - m_i)[\alpha_i Q_i - \min_i(\alpha_i Q_i)] - \sum_N w_i Q_i \end{aligned} \tag{4.29}$$

Proposition 4.14 *Expected profit function given in (4.29) is jointly concave in* $Q_1, \ldots,$ Q_N *if* $r(1-\sum_N \phi_i) + g_r - h \geq 0$ *,*$h \geq 0$ *and* $\sum_i (h_i - m_i) \geq 0$ *.*

Proof:

Please see the proof of Proposition 4.23

Now, staying consistent with the previous notation, if we write:

$$\begin{array}{rlcrl} (r(1-\sum_N \phi_i)+g_r-h)= & \lambda(r+g_r-h) & \Rightarrow & \sum_N \phi_i = & \frac{(1-\lambda)(r+g_r-h)}{r} \\ h-V= & \lambda h & \Rightarrow & V= & (1-\lambda)h \\ h_i-m_i= & \lambda h_i & \Rightarrow & m_i= & (1-\lambda)h_i \\ & & & w_{m,i}= & \lambda c_i \end{array}$$

then the profit of the manufacturer is the same as in (4.10) that is manufacturer gets all the profit with $\lambda = 1$ while the all the profit goes to the suppliers when $\lambda = \frac{g_r\mu_D}{E[\pi_c]+g_r\mu_D}$.

4.2.4.5. Modified Buy-back with Revenue Share and Recovery Payment. Like the previous contracts, this one is N suppliers type of the contract defined in Section 4.1.4.7 . The transfer payment and manufacturer's expected profit function are as follows:

$$\begin{aligned} T_i(Q_i, w_{m,i}, m_i, \phi_i, V) &= w_iQ_i + r\phi_i[\min_i(D, \alpha_iQ_i)] + h\phi_i[\min_j(\alpha_iQ_i) - D]^+ \\ &+ \begin{cases} V\alpha_iQ_i & \alpha_iQ_i < \alpha_jQ_j \quad \forall \quad j \neq i \\ 0 & o/w \end{cases} \\ &- m_i[\alpha_iQ_i - \min_j(\alpha_jQ_j)]^+ \end{aligned}$$

$$\begin{aligned} \pi_r &= (r(1-\sum_N \phi_i) + g_r - h(1-\sum_N \phi_i))[\min_i(D, \alpha_iQ_i)] \\ &+ (h-V)[\min_i(\alpha_iQ_i)] - \sum_N (h_i - m_i)[\alpha_iQ_i - \min_i(\alpha_iQ_i)] \\ &- g_rD - \sum_N w_iQ_i \end{aligned}$$

$$\begin{aligned} E[\pi_r] &= (r(1-\sum_N \phi_i) + g_r - h(1-\sum_N \phi_i))S_N(\alpha_i, Q_i) \\ &+ (h-V)E[\min_i(\alpha_iQ_i)] - \sum_N (h_i - m_i)E[\alpha_iQ_i - \min_i(\alpha_iQ_i)] \\ &- g_r\mu_D - \sum_N w_iQ_i \end{aligned} \quad (4.30)$$

Proposition 4.15 *Expected profit function given in (4.30) is jointly concave in* $Q_1,\ldots,$ Q_N *if* $r(1-\sum_N \phi_i)+g_r-h(1-\sum_N \phi_i)\geq 0$ *,*$h-V\geq 0$ *and* $\sum_i (h_i-m_i)\geq 0$ *.*

Proof:

Please see the proof of Proposition 4.23

When we define the following:

$$\begin{aligned}
(r(1-\textstyle\sum_N \phi_i)+g_r-h(1-\sum_N \phi_i)) &= \lambda(r+g_r-h) & \Rightarrow \quad \textstyle\sum_N \phi_i &= \tfrac{(1-\lambda)(r+g_r-h)}{r-h}\\
h(1-\textstyle\sum_N \phi_i)-V &= \lambda h & \Rightarrow \quad V &= -\tfrac{g_r(1-\lambda)h}{r-h}\\
h_i-m_i &= \lambda h_i & \Rightarrow \quad m_i &= (1-\lambda)h_i\\
& & w_{m,i} &= \lambda c_i
\end{aligned}$$

Here we can see that the recovery payment in fact comes out to be a punishment to the worst one. It can be thought as cutting down the whole sale price of the worst one, as it comes out to be in the contract defined in Section 4.1.4.7. The manufacturer's profit function is the one defined in (4.10). The lambda values are same like the others, that is manufacturer gets all when $\lambda=1$ and suppliers get all when $\frac{g_r\mu_D}{E[\pi_c]+g_r\mu_D}$.

4.3. Observations

In the assembly systems with random yield, coordinating mechanisms are found by starting to implement buy-back contracts. It is seen that buy-back contract alone cannot achieve coordination using the way we employed. Then an addition is made to the contract, *a recovery payment(V)*. It is the payment that is made to the worst supplier per unit it delivers.

In a single supplier model, buy-back contracts are easy to implement because the sales either equals to demand or delivered units. So if the retailer has an inventory after the demand is realized, then the supplier buys them back. If there is no inventory, then there is no buy-back process. Buy-back is a guarantee to the retailer that the supplier compensates the loss by ordering more. So the supplier encourages the retailer to order more. If there is any inventory left at the end of the season, then it is more likely to be due to the supplier's action. In short, like all the other contracts, it is a form of risk sharing between the players.

However, in an assembly system, this is not only the case. If there is an inventory at the end of the period, this might be due to supplier's behavior. Let's say there are suppliers A and B and let the demand realized be 145. If A sends 145 and B sends 10, there is an inventory of 135 because of the supplier B.

The buy-back contract proposed is such that the supplier pays for the component inventory if delivered quantity is larger than the sales. That is, if supplier's quantity delivered is larger than the sales, then it is punished. In modified-buy-back contracts, the suppliers are punished for the real inventory, that is for the items that cannot be assembled. This means that even if there are end-products that are not sold, the suppliers are not punished for those, because they are salvaged. In short, the suppliers are punished for the components which costs the manufacturer for holding inventory.

Although the names of the contract implies that the contracts are composed of buy-back and revenue sharing contracts plus a recovery payment, after solving the parameters it comes out that in both of the buy-back contracts, the scheme can be a sales rebate depending on the salvage, inventory holding, lost sales and revenue parameters. Additionally, the recovery payment in buy back with revenue share contract can be a cut-down price. This scheme cannot be manipulated because these contract parameters are found from the exogenous cost and price parameters. Thus one cannot determine a contract to be a sales rebate or to be a revenue share.

Different from the others, modified buy-back contract with revenue share certainly has a cut-down price whatever the exogenous parameters are, instead of a recovery payment.

The contracts are written in such a way that the manufacturer's profit function becomes a portion of the chain's profit, that is:

$$E\left[\pi_r\right] = \lambda E\left[\pi_c\right] - (1-\lambda)g_r\mu_D$$

There are common points in the contracts. The contract parameters like revenue share parameter(ϕ) and recovery payment(V) change from contract to contract. However the buy-back and modified buy-back parameters are same in all of the contracts:

$$m_i = b_i = (1-\lambda)h_i \tag{4.31}$$

If the contracts are analyzed carefully, it can be seen that all the contracts turn out to be a wholesale contract in which the manufacturer gets all the profit when λ equals to one. For example let's check the modified buy-back contract parameters:

$$\begin{aligned}
(r\bar{\phi} + g_r - h\bar{\phi} - \textstyle\sum_N b_i) &= \lambda(r + g_r - h) &\Rightarrow\quad \phi &= \tfrac{(1-\lambda)(r+g_r-h-\sum_N h_i)}{r-h} \\
h\bar{\phi} + \textstyle\sum_N b_i - V &= \lambda h &\Rightarrow\quad V &= (1-\lambda)\tfrac{r\sum_N h_i - g_r h}{r-h} \\
h_i - b_i &= \lambda h_i &\Rightarrow\quad b_i &= (1-\lambda)h_i \\
& & w_{b,i} &= \lambda c_i
\end{aligned}$$

When λ equals to one, all the other payments become *zero*, thus the contract turns into a *wholesale price* contract, meaning that the profit goes to the manufacturer. However when λ is less than one, then the other payments come to the scene and at the point where λ hits $\frac{g_r\mu_D}{E[\pi_c]+g_r\mu_D}$, all the profit goes to the suppliers.

The profit can be placed entirely either to the manufacturer or to the suppliers. However, it is not completely figured out what is the share between the suppliers. The profit can be allocated between the suppliers up to a limit with the parameter ϕ. The sum of the ϕs is found to be equal to a value, but there is no restriction over the individual ϕs. So this allows the contract allocate the profit at least to some extent.

One other point is that, why does the manufacturer order different quantities from the suppliers instead ordering the same quantity like the one in Gurnani and Gerchak[22]? To understand this point, let's start from the beginning. The system has a random yield structure. That is the suppliers are unreliable and they do not deliver the exact quantity the manufacturer orders. So the delivered quantity has to be, somehow, corrected. What is important is that, this correction can either be made by the supplier, or by the manufacturer.

When the manufacturer orders the same amount from all the suppliers, then the suppliers should correct the amount because randomness of yield at each supplier is not necessarily the same. For example if one supplier ships 90 percent and the other one ships 50 percent of the order quantities on average, then it is ridiculous to order the same amount from both under a forced compliance regime. If the suppliers' randomness are different and the manufacturer orders the same amount from all of them, then the suppliers should be allowed to change the amount for sake of profitability of the chain. Accepting quantities that are more than the quantity ordered is up to the manufacturer.

If the correction is made by the manufacturer, then the manufacturer should order different quantities from the suppliers, accounting for their different randomness structure. As the manufacturer makes the correction, the suppliers should obey the order quantity. If suppliers also change the quantity, then there is double correction in the order quantity which corrupts the system, especially the system in which all the players know all the parameters and distributions. This double correction may work better in systems in which the players have limited information about each other. So in our model, the manufacturer makes the correction by ordering different quantities from the suppliers taking care of their different randomness structures. This coordination mechanism is completed by the forced compliance.

5. NUMERICAL ILLUSTRATIONS

So far we have analyzed the systems in terms of closed form functions. Further insight can be obtained by using numerical examples. For example the optimal order quantity's behavior according to mean of demand and yield or the expected profit function's pattern with respect to the variance of demand cannot be evaluated from closed form functions. Additionally they are also some kind of validation for the results that are derived in the previous chapters. We used MatLab 7.0.1 to prepare this numerical study.

5.1. Newsvendor Problem

Numerical examples are helpful to see the behavior of the parameters over the contract. The centralized system is investigated under a predetermined setting and then sensitivity analysis is performed on the parameters. In the sensitivity analysis, the optimal order quantity and the profit are studied with respect to changing cost and revenue parameters. In the following example the parameters are set as $r = 25$, $c = 5$, $h = 4$ and $g_r = 3$. Demand is assumed to be normally distributed with $\mu = 100$ and $\sigma = 10$. The supplier's yield has a uniform distribution between (0,1]. One parameter is changed gradually while keeping others constant and the expected chain profit and optimal order quantity are observed. All of the results are found as expected. One critical point is that, when changing the parameters, the conditions of concavity in Proposition 3.1 should not be violated.

Figures 5.1 and 5.2 show the relation between the selling price (r) versus optimal order quantity and versus chain profit.

As it is expected, both the quantity and the optimal profit increases when r increases. When the manufacturer is able to sell a good with a higher price, then it orders more and, of course, makes a higher profit.

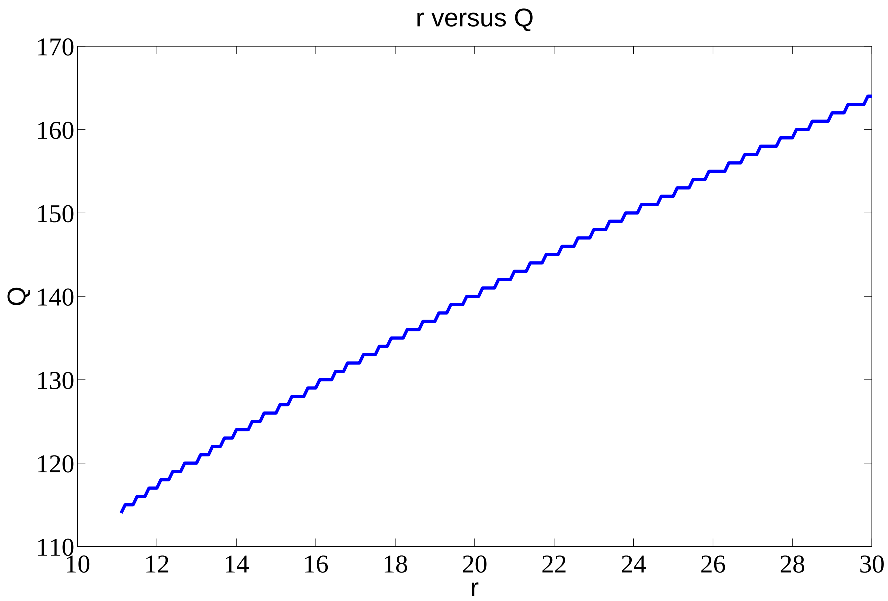

Figure 5.1. r versus Optimal Order Quantity

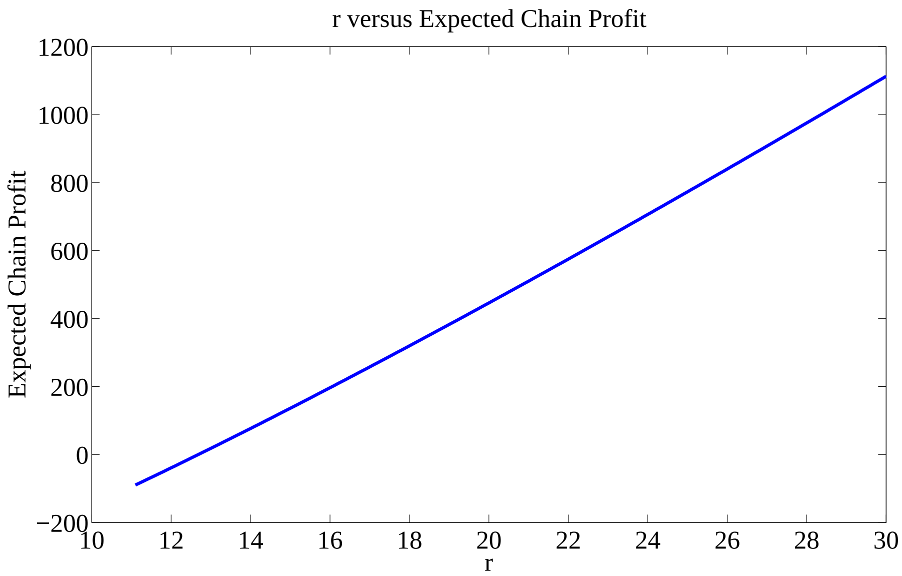

Figure 5.2. r versus Chain Profit

Other figures of sensitivity analysis are in Appendix D. Figures D.1 and D.2 show the relation between the production cost (c) versus optimal order quantity and versus chain profit.

When production cost increases, the manufacturer orders less because the system pays for every unit ordered but only sells the goods which are delivered. When every parameter stays constant but only production cost increases, this obviously decreases the chain's expected profit.

The same sensitivity analysis for holding cost(h) is shown in Figures D.3 and D.4. When holding cost increases, the manufacturer is not eager to hold inventory so the optimal order quantity decreases. Holding an inventory is a cost so when h increases the optimal profit decreases as well.

Figures D.5 and D.6 show how the optimal order quantity and expected chain profit change with g_r. When g_r increases, the manufacturer orders more in order to meet the demand and not to fall into lost sales. This increases the optimal order quantity. However g_r is a cost and a higher cost decreases the profit.

The stair like shapes of the figures in optimal order quantity graphs is due to the fact that order quantities are assumed to be integers. Thus when the range is small like in the graph of g_r, the plot is a step function.

Another main issue is to find the expected chain profit and optimal order quantity when the variance of the demand changes. Calculation of variance is given in Appendix A. Holding other parameters constant, Figure 5.3 shows how the optimal order quantity changes when variance of demand changes. The order quantity increases when the variance of the demand increases. At first this may seem unreasonable, however the distribution used for demand is a truncated normal. When the variance increases, (untruncated) normal distribution becomes flatter and its tail hits the y axis. Continuing to increase the variance causes negative demands. However negative demand is meaningless, so the probability that belongs to negative values are distributed over the

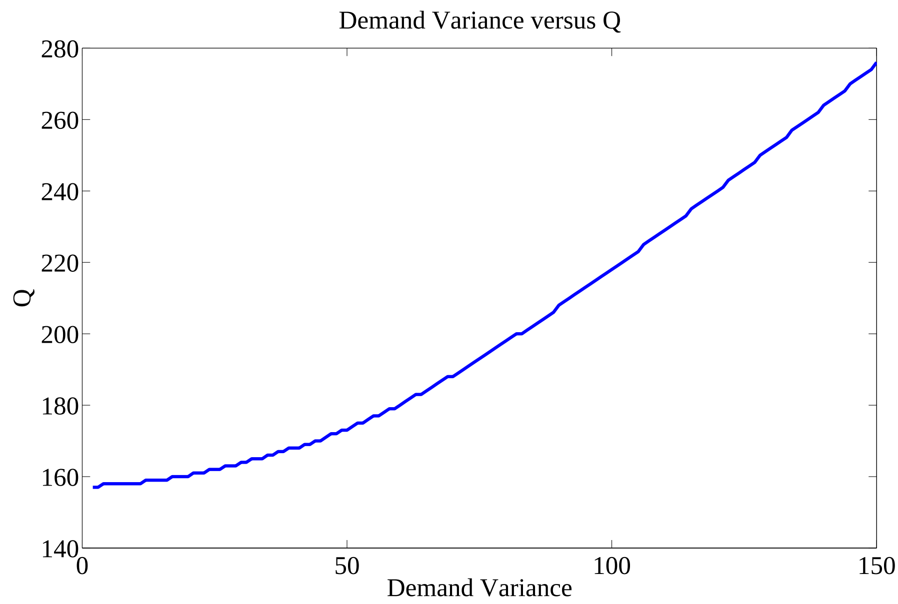

Figure 5.3. Demand Variance versus Optimal Order Quantity

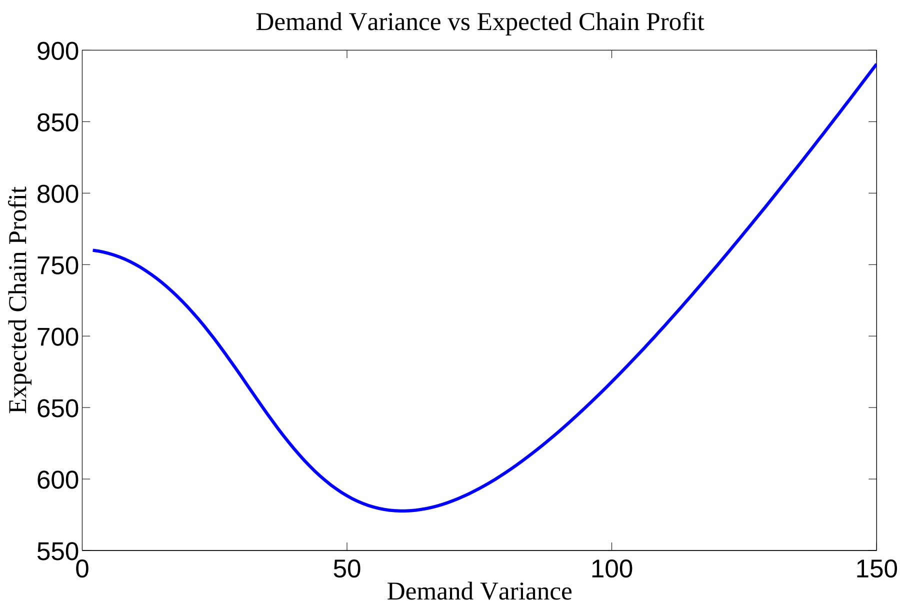

Figure 5.4. Demand Variance versus Expected Chain Profit

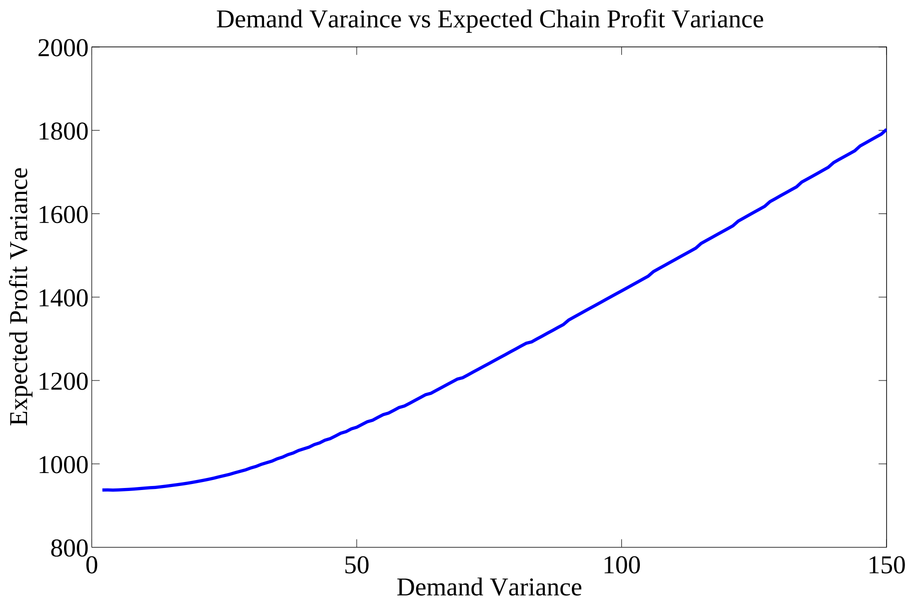

Figure 5.5. Demand Variance versus Expected Chain Profit Variance

positive values proportionally to the probability values of positive values not allowing any occurrence of negative demand. That is how the distribution is truncated. Hence, the actual variance and mean is different.

The behavior of profit when variance increases is shown in Figure 5.4. The profit first decreases but then increases. This can be explained again by the truncated distribution. The tail of the normal distribution hits the negative part after a variance of sixty, as it is seen on in Figure D.7. After computing the expected profit, the variance of the expected chain profit is also calculated and the relation between two variances is shown in Figure 5.5. As it is expected, the variance of the expected profit function increases with the increasing variance of the demand.

5.2. Assembly Systems

Numerical examples for the assembly system are quite important to observe the performance of the contracts. To achieve that, first the centralized system profit is plotted. Optimal order quantities and expected chain profit is studied under several scenarios which are shown in Table 5.1. The demand has a normal distribution with $\mu = 100$ and $\sigma = 25$. Both of the suppliers' yield has a uniform distribution between (0,1].

The scenarios are shown in Table 5.1. First run is the base case that we compare the other runs with. If we start from that it can be seen that the optimal order quantity for the first component is lower than the second one. This is because the first component is more expensive than the second one, regarding both for holding and production costs. If the selling price decreases(r), then obviously we expect the profit to decrease and the optimal order quantities to decrease, which is the case. Then salvage value is set to zero. Salvage is a revenue for the manufacturer, so setting it to zero forces the system to order less. Obviously this decreases the chain's expected profit. In the fourth run, the production cost of the first component is set to a lower value. This dramatically increased both of the order quantities, while the increase in the first component is higher as expected. This huge increase is due to the fact that this cost is paid for every unit ordered which increases the cost seriously. Then the same thing is done with the production cost of the second component. The optimal order quantity for both of the components increases while the second one has a much more increase. Both actions (decrease in the production costs) increases the profit as expected. Setting holding cost of the first component to a lower value results in an increase in the order quantity of the first component but a slight decrease in order quantity of the second one. Setting this to zero yields the same results. Component 1 increases however component 2 slightly decreases with respect to the basic first run. The same thing is observed when we play with the holding cost of the second component. When it is set to half of the original value, there is a slight decrease in component 1, but an increase in component 2 quantities. Setting it to zero results again in an increase in order quantity of the second component. Component 1 stays same with respect to the basic run. Decreasing the holding costs increases the profit as it is expected. The last parameter to change is the lost sales. When cost of lost sales is set to zero, it can be seen that the order quantities decrease and the profit increase because as there is no punishment for the lost sales, the system now tries to keep less inventory. The reverse also works fine. When cost of lost sales increases, both of the order quantities increase, component 1 having a less increase. This is an expected result because lost sales is punished severely so system orders more not to fall into lost sales. Expected profit decreases with respect to the basic setting when cost of lost sales has a higher value.

Table 5.1. Assembly System Solution with Two Suppliers having Random Yield

	r	h	c_1	c_2	h_1	h_2	g_r	Q_1	Q_2	$Profit$
1	40	5	4	3	2	1	2	198	224	743.78
2	30	5	4	3	2	1	2	156	175	197.97
3	40	0	4	3	2	1	2	180	205	695.16
4	40	5	2	3	2	1	2	281	266	1210.6
5	40	5	1	3	2	1	2	368	303	1530.2
6	40	5	4	1.5	2	1	2	226	317	1139
7	40	5	4	3	1	1	2	203	222	773.85
8	40	5	4	3	0	1	2	214	223	806.21
9	40	5	4	3	2	0.5	2	197	228	765.36
10	40	5	4	3	2	0	2	198	237	788.2
11	40	5	4	3	2	1	0	190	215	825.56
12	40	5	4	3	2	1	7	214	243	542.18

One important point is that, what is the effect of variance of the yield over the chain? In other words, what does a reliable supplier bring to the chain? In Table 5.2, supplier one has a constant yield of 0.5 and the other supplier's yield is same with the previous setting, having a uniform distribution between (0,1].

First of all, the significant change in the profit can be seen easily. Thus a reliable supplier increases the profit of the supply chain. Another effect can be seen when the fourth and the fifth settings are compared. There is an increase in the order quantities in this system, however this increase is smaller than the one in the previous system. This shows that the system gives more reaction to the production cost of a component which has random yield.

Table 5.2. Assembly System Solution with Constant Yield at Supplier One

	r	h	c_1	c_2	h_1	h_2	g_r	Q_1	Q_2	$Profit$	$Increase$
1	40	5	4	3	2	1	2	231	244	1285.3	72.8 %
2	30	5	4	3	2	1	2	210	205	547.1	176.4 %
3	40	0	4	3	2	1	2	219	233	1233.2	77.4 %
4	40	5	2	3	2	1	2	267	260	1778.8	46.9 %
5	40	5	1	3	2	1	2	318	278	2065.7	35 %
6	40	5	4	1.5	2	1	2	243	316	1698.6	49 %
7	40	5	4	3	1	1	2	233	241	1313.1	69.7 %
8	40	5	4	3	0	1	2	236	240	1341.7	66.4 %
9	40	5	4	3	2	0.5	2	230	256	1321.5	72.7 %
10	40	5	4	3	2	0	2	229	272	1363.6	73 %
11	40	5	4	3	2	1	0	227	236	1333.7	61.6 %
12	40	5	4	3	2	1	7	238	259	1170.8	115.9 %

Table 5.3 shows the optimal order quantities and expected profit of the chain when the first supplier has perfect yield. *Increase1* column shows the increase of the profit with respect to the random yield system and *Increase2* column shows the increase of the expected chain profit with respect to the model with supplier one having constrant yield in Table 5.2. The significant increase in the profit with respect to the random yield model can be seen easily. There are two important points. When selling price is low, then increase in the yield makes a greater increase in the profit. However when the production cost of the component which has a perfect yield is low like the one in

Table 5.3. Assembly System Solution with Perfect Yield at Supplier One

	r	h	c_1	c_2	h_1	h_2	g_r	Q_1	Q_2	$Profit$	$Increase1$	$Increase2$
1	40	5	4	3	2	1	2	133	251	1715	130.6 %	33.4 %
2	30	5	4	3	2	1	2	124	217	948.27	379 %	73.3 %
3	40	0	4	3	2	1	2	121	235	1634.3	135.1 %	32.5 %
4	40	5	2	3	2	1	2	158	266	1999.5	65.1 %	12.4 %
5	40	5	1	3	2	1	2	213	298	2180.6	42.5 %	5.5 %
6	40	5	4	1.5	2	1	2	141	325	2140.2	87.9 %	25.9 %
7	40	5	4	3	1	1	2	136	249	1751	126.2 %	33.3 %
8	40	5	4	3	0	1	2	140	246	1789.4	121.9 %	33.3 %
9	40	5	4	3	2	0.5	2	132	268	1778.5	132.3 %	34.5 %
10	40	5	4	3	2	0	2	131	289	1851.7	134.9 %	35.7 %
11	40	5	4	3	2	1	0	131	245	1758.9	113 %	31.8 %
12	40	5	4	3	2	1	7	136	266	1610	196.9 %	37.5 %

fifth setting, then the increase in the yield does not significantly affect the profit of the chain.

Table 5.4 shows the optimal order quantities and the expected chain profit when two suppliers have constant yield of *0.5*. Again the significant increase in the chain's expected profit can be seen from the table. The increase of the chain's profit with respect to the system having one supplier with constant yield (Table 5.2) is shown in the last column. As well as the significant increase in the profit, the reaction of the system to the change in production cost can be seen in the sixth run. In the system with supplier two having random yield, the change in optimal order quantity of component two is 82 . In this system, the change in the optimal order quantities are 15 in both. So randomness in the yield causes the system to give more reaction to the change in the production costs. Another important point in this system is that, in all of the settings, the optimal order quantities for two components are equal, so the system does not have any inventory holding costs which explains the same order quantities in the settings having different holding costs.

Table 5.4. Assembly System Solution with Constant Yields

	r	h	c_1	c_2	h_1	h_2	g_r	Q_1	Q_2	$Profit$	$Increase$
1	40	5	4	3	2	1	2	235	235	2310.5	79.8 %
2	30	5	4	3	2	1	2	222	222	1354.6	147.6 %
3	40	0	4	3	2	1	2	221	221	2218.3	79.9 %
4	40	5	2	3	2	1	2	255	255	2799.1	57.4 %
5	40	5	1	3	2	1	2	270	270	3061.2	48.2 %
6	40	5	4	1.5	2	1	2	250	250	2673.1	57.4 %
7	40	5	4	3	1	1	2	235	235	2310.5	76 %
8	40	5	4	3	0	1	2	235	235	2310.5	72.2 %
9	40	5	4	3	2	0.5	2	235	235	2310.5	74.8 %
10	40	5	4	3	2	0	2	235	235	2310.5	69.4 %
11	40	5	4	3	2	1	0	232	232	2317.9	73.8 %
12	40	5	4	3	2	1	7	240	240	2293.9	95.9 %

Table 5.5 shows the optimal order quantities and the expected chain profit of the assembly system with two suppliers having perfect yield. Significant increase in the chain's expected profit with respect to the random yield structure (Table 5.1) is shown in *Increase1* column and the increase with respect to the assembly system with suppliers having constant yield of *0.5* (Table 5.4) is shown in *Increase2* column. In addition to the increase in the profit of the chain, there is no inventory holding cost in this system like the model with two suppliers having constant yield in Table 5.4. In the fifth setting, since the salvage value is bigger than the production cost of an end product, the system produces infinitely many products. Same reason causes the multiple optima in fourth run. Thus the production of the first product is changed to *2.1* from 2 just for this fifth setting.

Figure 5.6 shows the graph of expected chain profit versus order quantities under the first setting. As it can be seen from the graph, the expected profit function of the chain is concave.

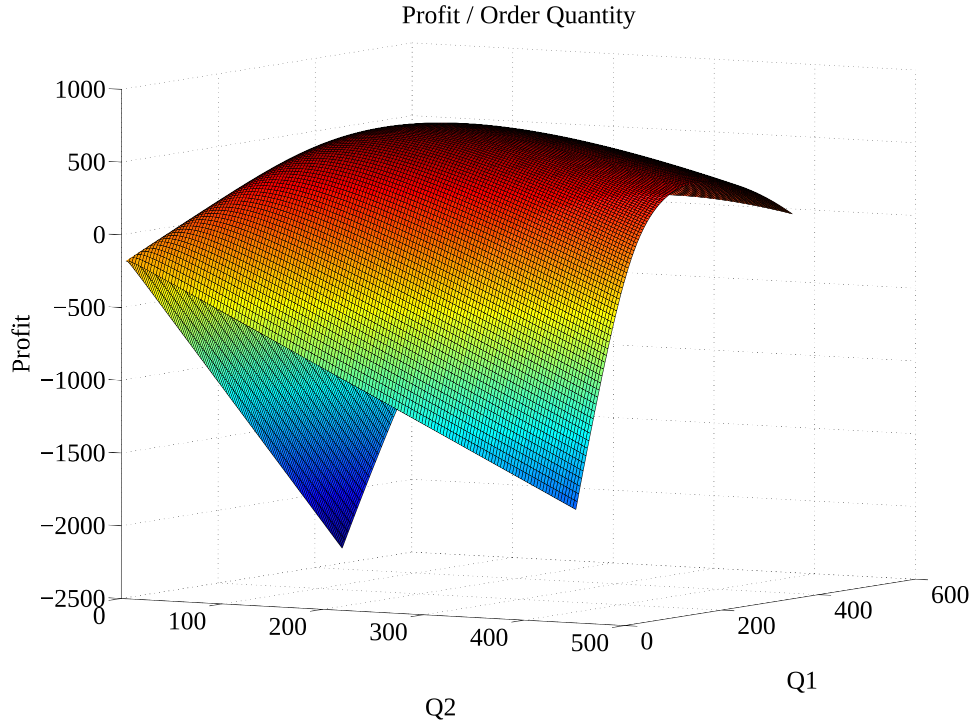

Figure 5.6. Expected Chain Profit versus Q_1 and Q_2

After plotting the expected chain profit, it is important to see how the contracts perform. We showed that all of the profit can be allocated either to the manufacturer or to the suppliers but we can not say anything about what the share is between the suppliers. For all of the contracts, when $\lambda = 1$ all the profit goes to the manufacturer and for $\lambda = \frac{g_r \mu_D}{E[\pi_c] g_r \mu D}$ all the profit goes to the suppliers. The λ value is the same for all of the contracts because it is independent of the contract parameters. All of the parameters of the contracts are shown in Table 5.6. The parameters are calculated from (4.9), (4.13), (4.18) and (4.21).

When λ is set to 0.2119 all the profit is allocated to the suppliers. All the contract parameters are calculated from the exogenous parameters and λ. This means that all of the parameters are fixed at a certain value for suppliers to get all of the profit. This condition also holds for the total revenue share parameter ϕ. However, individual revenue share parameters are free to choose as long as their sum stays constant, meaning there is one degree of freedom to choose the revenue share parameters.

Table 5.5. Assembly System Solution with Perfect Yields

	r	h	c_1	c_2	h_1	h_2	g_r	Q_1	Q_2	$Profit$	$Increase1$	$Increase2$
1	40	5	4	3	2	1	2	140	140	3198.6	330 %	38.4 %
2	30	5	4	3	2	1	2	136	136	2205.4	1014 %	62.8 %
3	40	0	4	3	2	1	2	124	124	3037.8	337.0 %	36.9 %
4	40	5	2	3	2	1	2	170	170	3482.4	187.7 %	24.4 %
5	40	5	1	3	2	1	2	-	-	-	-	-
6	40	5	4	1.5	2	1	2	155	155	3418	200.1 %	27.9 %
7	40	5	4	3	1	1	2	140	140	3198.6	313.3 %	38.4 %
8	40	5	4	3	0	1	2	140	140	3198.6	296.7 %	38.4 %
9	40	5	4	3	2	0.5	2	140	140	3198.6	317.9 %	38.4 %
10	40	5	4	3	2	0	2	140	140	3198.6	305.8 %	38.4 %
11	40	5	4	3	2	1	0	140	140	3199.8	287.6 %	38.0 %
12	40	5	4	3	2	1	7	142	142	3196	489.5 %	39.3 %

Tables 5.7 and 5.8 show the profit shares of the suppliers when revenue share parameters(ϕ_1 and ϕ_2) change from one extreme point, *0*, to another extreme, ϕ. It can be seen that when the revenue share parameter of the suppliers are at level *zero* then the suppliers make a loss, meaning that they pay to the other supplier. Reverse is also true. If revenue share of a supplier is at its maximum level, that is $\phi_i = \phi$, then this supplier makes a higher profit than the chain, meaning that it takes extra payment from the other supplier. In the tables, there are also the values of the ϕs at which the supplier has *zero* profit or takes all the profit.

Finally, the last figure show how the profit share occurs with respect to the profit share of first supplier in buy-back with sales revenue share contract. The figure for the other contracts are in Appendix E. The dotted line is the profit share of the first supplier. The graph is drawn with respect to the revenue share of first supplier, thus the dotted line increases with increasing profit share. The other line, dashed one is the profit share of the second supplier. As it is expected, the profit starts at its maximum point when $\phi_1 = 0$ and then comes to its minimum point when $\phi_1 = \phi$. The black box

Table 5.6. Contract Parameters

Contracts	ϕ	V	$b_1(m_1)$	$b_2(m_2)$	w_1	w_2	λ
BBSRS	0.6699	6.3047	1.5762	0.7881	0.8476	0.6357	0.2119
BBRS	0.7656	2.4768	1.5762	0.7881	0.8476	0.6357	0.2119
MBBSRS	0.729	3.9404	1.5762	0.7881	0.8476	0.6357	0.2119
MBBRS	0.8331	-0.2252	1.5762	0.7881	0.8476	0.6357	0.2119

Table 5.7. Profit Share of Supplier 1

Supplier 1				
Contracts	$\phi_1 = \phi$	$\phi_1 = 0$	ϕ_1 **for Zero Profit**	ϕ_1 **for Full Profit**
BBSRS	1131.7000	-431.8748	0.1850	0.5037
BBRS	1243.4000	-587.5180	0.2457	0.5567
MBBSRS	1191.7000	-509.9204	0.2185	0.5371
MBBRS	1313.2000	-679.2968	0.2840	0.5950

is the applicable range because out of that box the suppliers make negative profits. The straight line in the figures is just the sum of the profits of two suppliers. It stays constant at the value of 743.78 which is the expected profit of total chain. As it is mentioned before, the parameter λ is adjusted such that all the profit is allocated to the suppliers.

One might think that, this profit share can only be achieved when the suppliers have similar cost structures. So we try another run in which everything stays same but cost of production for supplier 1 increased to 20 and its holding cost increased to 4. For the system not to make loss, the revenue is also increased to 80. The centralized profit comes to be 476,53 with 117 units of component1 and 224 units of component2. The results are shown in Tables 5.9, 5.10 and 5.11.

Table 5.8. Profit Share of Supplier 2

Supplier 2				
Contracts	$\phi_2 = \phi$	$\phi_2 = 0$	ϕ_2 **for Zero Profit**	ϕ_2 **for Full Profit**
BBSRS	1175.7000	-387.9655	0.1662	0.4849
BBRS	1331.3000	-499.6226	0.2089	0.5199
MBBSRS	1253.7000	-447.8867	0.1919	0.5105
MBBRS	1423.1000	-569.3959	0.2381	0.5491

Table 5.9. Contract Parameters

Contracts	ϕ	V	$b_1(m_1)$	$b_2(m_2)$	w_1	w_2	λ
BBSRS	0.6339	7.0437	2.8175	0.70437	5.9125	0.8869	0.2956
BBRS	0.6762	3.6627	2.8175	0.70437	5.9125	0.8869	0.2956
MBBSRS	0.678	3.5219	2.8175	0.70437	5.9125	0.8869	0.2956
MBBRS	0.7232	-0.0939	2.8175	0.70437	5.9125	0.8869	0.2956

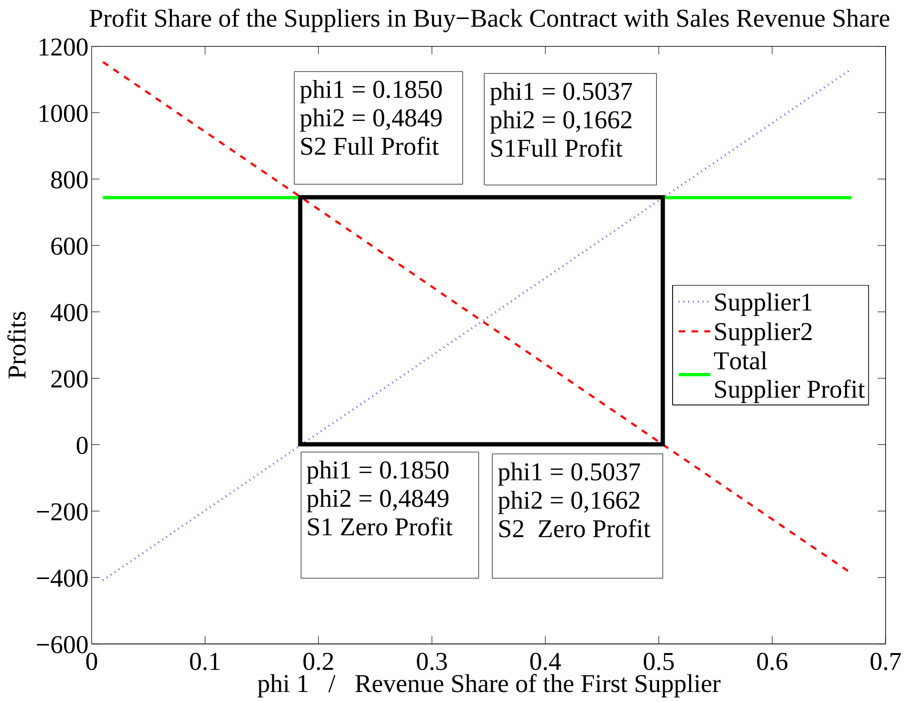

Figure 5.7. ϕ_1 versus Profit Shares in BBSRS

Table 5.10. Profit Share of Supplier 1

Supplier 1					
Contracts	$\phi_1 = \phi$	$\phi_1 = 0$	ϕ_1 **for Zero Profit**	ϕ_1 **for Full Profit**	ϕ
BBSRS	924.5303	-1414.5000	0.3833	0.5125	0.6339
BBRS	958.9667	-1543.5000	0.4171	0.5458	0.6762
MBBSRS	958.8569	-1542.6000	0.4181	0.5473	0.6780
MBBRS	995.6847	-1680.5000	0.4541	0.5829	0.7232

Table 5.11. Profit Share of Supplier2

Supplier 2					
Contracts	$\phi_2 = \phi$	$\phi_2 = 0$	ϕ_2 **for Zero Profit**	ϕ_2 **for Full Profit**	ϕ
BBSRS	1891.1000	-447.9997	0.1214	0.2506	0.6339
BBRS	2020.0000	-482.4360	0.1304	0.2591	0.6762
MBBSRS	2019.2000	-482.3262	0.1307	0.2599	0.6780
MBBRS	2157.0000	-519.1540	0.1403	0.2691	0.7232

6. CONCLUSIONS

6.1. Selling to the Newsvendor with Random Yield

With this study, coordination of a system consisting of a supplier and a retailer is investigated. There is a random yield in addition to the random demand. Different known contracts are studied to establish coordination. As far as it is known, this is the first study that analyzes the coordination of newsvendor under random demand and random yield. To summarize the coordination success of contracts:

- Buy-back, revenue-sharing and quantity discount contracts coordinate the chain under both forced and voluntary compliance regimes.
- Quantity flexibility contract can coordinate the chain under forced compliance regime. Under voluntary compliance, concavity of the supplier's profit function cannot be established. Thus we cannot make any conclusions about voluntary compliance.
- Wholesale price can coordinate only in forced compliance regime because the supplier makes zero profit under this contract. Thus, coordination in voluntary compliance can only be achieved via franchising payments or end-of-term payments.

There are also some interesting notes when we look over the whole work:

- The optimal order quantity is found from a critical ratio like the classical newsvendor problem.
- Buy-back and revenue sharing contracts are equivalent when the needed transformations are made.
- In quantity flexibility contract, the supplier's action cannot be characterized. However, it is interesting to note that the supplier's and the retailer's first order conditions are same. Thus the quantity that optimizes the chain's profit is a critical point for the supplier's expected profit function.

- Wholesale price contract is found to coordinate with franchising payments or end-of-term payments. Quantity discount contract just show the validity of this comment. The parameters of the quantity discount are such that the suppliers get a payment for their production cost and then a constant payment which can be thought as a franchising payment with a wholesale price contract.
- The parameters which coordinate the chain are found to be dependent only on μ_α. However the optimal order quantity depends on all the parameters of yield and demand. Also the random yield contract parameters turn out to be equal to classical newsboy parameters when $\mu_\alpha = 1$.
- When the relation of contract parameters with respect to μ_α is analyzed, it can be seen that there is a tendency of the retailer to stabilize the wholesale price payment for the whole delivery.
- When the contract parameters is written with λ, then we see that when λ become one, the retailer gets all the profit. In fact, all those contracts whose parameters can be written in λ become a wholesale price contract when $\lambda = 1$.
- In the numerical examples part, the relations with the exogenous cost parameters and the decision variable Q can be seen clearly. One interesting thing is that the profit decreases when the variance of the demand increases, but then starts to increase, which stems from the fact the we used a truncated normal distribution. When the graph of truncated normal distribution is analyzed, it can be seen that after some threshold, the left tail of the distribution hits *zero*, so the probability of negative demands are distributed proportionally to the positive values which means that after some value of variance, mean of the distribution in fact moves to a higher value.

6.2. Assembly Systems under Random Yield

In the second part of this thesis, coordination of an assembly system is analyzed. Unlike the newsvendor problem, we only analyzed forced compliance regime for coordination. Again, as far as we know, there is no previous work that studied coordination of an assembly system with random demand and random yield. There are two ways followed in this study.

- The first one is to establish the concavity of the assembly system's profit function and expected value of this function. Instead of writing huge integrals and drowning in pages of equation, a simple way is employed.
- The second way is that after establishing the concavity, instead of finding the optimal order quantity explicitly, the manufacturer's function is written is such a way that it becomes a fraction of the chain's profit function. Thus, the parameters that can satisfy this condition are said to coordinate the chain, as in Cachon[10]

By following this way, four coordinating contract mechanisms has been proposed. The contracts are derived from buy-back and revenue sharing contracts by adding a transfer payment, *recovery payment(V)*. Addition to its use in single period model, in a multi period model the worst suppliers can benefit from this recovery payment and can *recover* from that worst position among the suppliers to a better one. So at each period the worst supplier is given a recovery payment which is used to enhance the improvement of the quality of production in that supplier. In addition to that, in a game-theoretic analysis, the suppliers try to produce less than the other to get this payment but since recovery payment increases with the quantity delivered, the suppliers produce more to get more recovery which will enhance the performance of voluntary compliance. The important points in the contracts are:

- All of the contract allow the profit to be allocated either to the suppliers or to the manufacturer.
- Although tha profit share between the suppliers cannot be completely established, numerical results show that all of the contracts perform well.
- The profit is allocated according to the parameter λ. When $\lambda = 1$, all the profit goes to the manufacturer. In fact, all the contracts become a wholesale contract and it is shown that in wholesale contract the suppliers cannot make profit.
- The contracts have equal payments as well as different payments. In all of the contracts, buy-back parameters and wholesale prices are same. However, the revenue share and recovery payment parameters change in each contract.

- Although the parameters in the contracts are called revenue share or recovery payments, sometimes these parameters turns out to be sales rebates or cut-down prices when the parameters are solved.
- From numerical examples, the effect of random yield can be seen easily. There is a significant increase in the profit when the supplier one has a constant yield. Additionally, the system gives more reaction to the change in the production costs when the supplier has random yield. When the suppliers have constant yields, the system does not hold any inventory of the components as expected.
- Since random yield decreases the profit significantly, the recovery payment becomes more important especially in multi period analysis. Thus the contracts having recovery payments are the best for use in multi-periods. If the exogenous parameters makes the recovery payment a cut-down price, then that contract can be used for single period systems, although all contracts are suitable because they provide coordination in the chain.

In both newsvendor and assembly systems, when the profit is written as a fraction of the expected chain profit, all of the profit goes to the supplier(s) when $\lambda = \frac{g_r D}{g_r D + E[\pi_c()]}$ Although this ratio seems to be a random one, in fact it is not. $g_r \mu_D$ is the expected cost of the system when there is no order. $E[\pi_c]$ is the expected chain profit when optimal quantities are ordered. So this ratio is: *no order / (no order + optimal order).*

For further studies, two extensions can be added:

- Multi-period analysis: This study deals with a single period model. Solution and contract designs for multi-period can be analyzed. In fact, the contract parameter *Recovery Payment, V,* in the assembly system is an important point to analyze in multi-period systems especially when the yield is modeled to be related with *V.* That is, if the yield becomes larger with a recovery payment, the performance of all the system can improve period bu period.
- Game-Theoretical Analysis: In the assembly systems the contracts are analyzed under a forced compliance regime. The scheme can be extended to a game between suppliers by letting the contract be implemented under a voluntary com-

pliance regime. Applying both multi-period and game-theoretic analysis will be a complete and a hard study.

APPENDIX A: VARIANCE OF EXPECTED CHAIN PROFIT

The variance of the expected profit function of the chain is found as follows:

$$\begin{aligned}
g(\alpha, Q) &= (r + g_r + h)min(\alpha Q, D) - h\alpha Q - g_r D \\
Var(g(\alpha, Q)) &= E_\alpha[Var(g(\alpha, Q)|\alpha)] + Var_\alpha(E[g(\alpha, Q)]|\alpha)
\end{aligned}$$

$\underline{E_\alpha[Var(g(\alpha, Q)|\alpha)]}$:

$$\begin{aligned}
Var(g(\alpha, Q)|\alpha) &= E[g(\alpha, Q)^2|\alpha] - E[g(\alpha, Q)|\alpha]^2 \\
E\left[Var(g(\alpha, Q)|\alpha)\right] &= E\left[E[g(\alpha, Q)^2|\alpha]\right] - E\left[E[g(\alpha, Q)|\alpha]^2\right]
\end{aligned}$$

$$\begin{aligned}
E[g(\alpha, Q)|\alpha] &= (r + g_r + h)\left[\int_0^{\alpha Q} Dg(D)dD + \int_{\alpha Q}^{\infty} \alpha Q g(D)dD\right] - h\alpha Q - g_r \mu_D \\
E[g(\alpha, Q)|\alpha] &= \int_0^{\alpha Q} [(r + h)D - h\alpha Q]g(D)dD + \int_{\alpha Q}^{\infty} [(r + g_r)\alpha Q - g_r D]g(D)dD \\
E[g(\alpha, Q)^2|\alpha] &= \int_0^{\alpha Q} [(r + h)D - h\alpha Q]^2 g(D)dD + \int_{\alpha Q}^{\infty} [(r + g_r)\alpha Q - g_r D]^2 g(D)dD
\end{aligned}$$

$$\begin{aligned}
E_\alpha[Var(g(\alpha, Q)|\alpha)] &= \int_0^1 \left\{\int_0^{\alpha Q} [(r + h)D - h\alpha Q]^2 g(D)dD\right. \\
&+ \left.\int_{\alpha Q}^{\infty} [(r + g_r)\alpha Q - g_r D]^2 g(D)dD\right\} f(\alpha)d\alpha \\
&- \int_0^1 \left\{(r + g_r + h)\left[\int_0^{\alpha Q} Dg(D)dD\right.\right. \\
&+ \left.\left.\int_{\alpha Q}^{\infty} \alpha Q g(D)dD\right] - h\alpha Q - g_r \mu_D\right\}^2 f(\alpha)d\alpha
\end{aligned}$$

$$\begin{aligned} E_\alpha[Var(g(\alpha,Q)|\alpha)] &= \int_0^1 \left\{ \int_0^{\alpha Q} [(r+h)D - h\alpha Q]^2 g(D)dD \right. \\ &+ \left. \int_{\alpha Q}^{\infty} [(r+g_r)\alpha Q - g_r D]^2 g(D)dD \right\} f(\alpha)d\alpha \\ &- \int_0^1 \left\{ \int_0^{\alpha Q} [(r+h)D - h\alpha Q] g(D)dD \right. \\ &+ \left. \int_{\alpha Q}^{\infty} [(r+g_r)\alpha Q - g_r D] g(D)dD \right\}^2 f(\alpha)d\alpha \end{aligned}$$

$\underline{Var_\alpha(E[g(\alpha,Q)]|\alpha)}$

$$Var_\alpha(E[g(\alpha,Q)]|\alpha) = E[E[g(\alpha,Q)|\alpha]^2] - E[E[g(\alpha,Q)|\alpha]]^2$$

$$\begin{aligned} E[g(\alpha,Q)|\alpha] &= (r+g_r+h)\left[\int_0^{\alpha Q} Dg(D)dD + \int_{\alpha Q}^{\infty} \alpha Q g(D)dD\right] - h\alpha Q - g_r\mu_D \\ &= \int_0^{\alpha Q} [(r+h)D - h\alpha Q] g(D)dD + \int_{\alpha Q}^{\infty} [(r+g_r)\alpha Q - g_r D] g(D)dD \end{aligned}$$

$$\begin{aligned} Var_\alpha(E[g(\alpha,Q)]|\alpha &= \int_0^1 \left\{ (r+g_r+h)\left[\int_0^{\alpha Q} Dg(D)dD \right.\right. \\ &+ \left.\left. \int_{\alpha Q}^{\infty} \alpha Q g(D)dD\right] - h\alpha Q - g_r\mu_D \right\}^2 f(\alpha)d\alpha \\ &- \left\{ \int_0^1 \left((r+g_r+h)\left[\int_0^{\alpha Q} Dg(D)dD \right.\right.\right. \\ &+ \left.\left.\left. \int_{\alpha Q}^{\infty} \alpha Q g(D)dD\right] - h\alpha Q - g_r\mu_D \right) f(\alpha)d\alpha \right\}^2 \end{aligned}$$

$$
\begin{aligned}
&= \int_0^1 \left(\int_0^{\alpha Q} [(r+h)D - h\alpha Q] g(D) dD \right. \\
&+ \left. \int_{\alpha Q}^{\infty} [(r+g_r)\alpha Q - g_r D] g(D) dD \right)^2 f(\alpha) d\alpha \\
&- \left\{ \int_0^1 \left(\int_0^{\alpha Q} [(r+h)D - h\alpha Q] g(D) dD \right. \right. \\
&+ \left. \left. \int_{\alpha Q}^{\infty} [(r+g_r)\alpha Q - g_r D] g(D) dD \right) f(\alpha) d\alpha \right\}^2
\end{aligned}
$$

Variance

$$
\begin{aligned}
Var(g(\alpha, Q)) &= E_\alpha[Var(g(\alpha, Q)|\alpha)] + Var_\alpha(E[g(\alpha, Q)]|\alpha) \\
&= E\left[E[g(\alpha, Q)^2|\alpha]\right] \\
&- E\left[E[g(\alpha, Q)|\alpha]^2\right] + E[E[g(\alpha, Q)|\alpha]^2] - E[E[g(\alpha, Q)|\alpha]]^2
\end{aligned}
$$

$$
\begin{aligned}
&= \int_0^1 \left\{ \int_0^{\alpha Q} [(r+h)D - h\alpha Q]^2 g(D) dD \right. \\
&+ \left. \int_{\alpha Q}^{\infty} [(r+g_r)\alpha Q - g_r D]^2 g(D) dD \right\} f(\alpha) d\alpha \\
&- \int_0^1 \left\{ (r+g_r+h) \left[\int_0^{\alpha Q} Dg(D) dD \right. \right. \\
&+ \left. \left. \int_{\alpha Q}^{\infty} \alpha Q g(D) dD \right] - h\alpha Q - g_r \mu_D \right\}^2 f(\alpha) d\alpha \\
&+ \int_0^1 \left\{ (r+g_r+h) \left[\int_0^{\alpha Q} Dg(D) dD \right. \right. \\
&+ \left. \left. \int_{\alpha Q}^{\infty} \alpha Q g(D) dD \right] - h\alpha Q - g_r \mu_D \right\}^2 f(\alpha) d\alpha \\
&- \left\{ \int_0^1 \left((r+g_r+h) \left[\int_0^{\alpha Q} Dg(D) dD \right. \right. \right. \\
&+ \left. \left. \left. \int_{\alpha Q}^{\infty} \alpha Q g(D) dD \right] - h\alpha Q - g_r \mu_D \right) f(\alpha) d\alpha \right\}^2
\end{aligned}
$$

$$
\begin{aligned}
= & \int_0^1 \left\{ \int_0^{\alpha Q} [(r+h)D - h\alpha Q]^2 g(D) dD \right. \\
+ & \left. \int_{\alpha Q}^{\infty} [(r+g_r)\alpha Q - g_r D]^2 g(D) dD \right\} f(\alpha) d\alpha \\
- & \left\{ \int_0^1 \left((r+g_r+h) \left[\int_0^{\alpha Q} D g(D) dD \right. \right. \right. \\
+ & \left. \left. \left. \int_{\alpha Q}^{\infty} \alpha Q g(D) dD \right] - h\alpha Q - g_r \mu_D \right) f(\alpha) d\alpha \right\}^2
\end{aligned}
$$

$$
\begin{aligned}
= & \int_0^1 \left\{ \int_0^{\alpha Q} [(r+h)D - h\alpha Q]^2 g(D) dD \right. \\
+ & \left. \int_{\alpha Q}^{\infty} [(r+g_r)\alpha Q - g_r D]^2 g(D) dD \right\} f(\alpha) d\alpha \\
- & \left\{ \int_0^1 \left(\int_0^{\alpha Q} [(r+h)D - h\alpha Q] g(D) dD \right. \right. \\
+ & \left. \left. \int_{\alpha Q}^{\infty} [(r+g_r)\alpha Q - g_r D] g(D) dD \right) f(\alpha) d\alpha \right\}^2
\end{aligned}
$$

APPENDIX B: ASSEMBLY SYSTEM WITH TWO SUPPLIERS

When taking expectation, first of all the cases are written. In two supplier case, there are six cases. Let's write the profit function of the chain in (4.2) again:

$$\begin{aligned}\pi_c(Q_1, Q_2) &= r\left[\min(\alpha_1 Q_1, \alpha_2 Q_2, D)\right] - g_r[D - \min(\alpha_1 Q_1, \alpha_2 Q_2)]^+ - c_1 Q_1 - c_2 Q_2 \\ &+ h[\min(\alpha_1 Q_1, \alpha_2 Q_2) - D]^+ - h_1[\alpha_1 Q_1 - \alpha_2 Q_2]^+ - h_2[\alpha_2 Q_2 - \alpha_1 Q_1]^+\end{aligned}$$

In the following formulations we neglect the production cost part, $-c_1 Q_1 - c_2 Q_2$, because they are constant at all cases:

Case 1: $\alpha_1 Q_1 < \alpha_2 Q_2 < D$

$$r\alpha_1 Q_1 - g_r(D - \alpha_1 Q_1) - h_2(\alpha_2 Q_2 - \alpha_1 Q_1)$$

Case 2: $\alpha_1 Q_1 < D < \alpha_2 Q_2$

$$r\alpha_1 Q_1 - g_r(D - \alpha_1 Q_1) - h_2(\alpha_2 Q_2 - \alpha_1 Q_1)$$

Case 3: $\alpha_2 Q_2 < \alpha_1 Q_1 < D$

$$r\alpha_2 Q_2 - g_r(D - \alpha_2 Q_2) - h_1(\alpha_1 Q_1 - \alpha_2 Q_2)$$

Case 4: $\alpha_2 Q_2 < D < \alpha_1 Q_1$

$$r\alpha_2 Q_2 - g_r(D - \alpha_2 Q_2) - h_1(\alpha_1 Q_1 - \alpha_2 Q_2)$$

Case 5: $D < \alpha_1 Q_1 < \alpha_2 Q_2$

$$rD + h(\alpha_1 Q_1 - D) - h_2(\alpha_2 Q_2 - \alpha_1 Q_1)$$

Case 6: $D < \alpha_2 Q_2 < \alpha_1 Q_1$

$$rD + h(\alpha_2 Q_2 - D) - h_1(\alpha_1 Q_1 - \alpha_2 Q_2)$$

It can be seen that *Case 1* and *Case 2* are same, like *Case 3* and *Case 4*. Now let's rename the cases and call *Case 1* and *Case 2* as *Case A*, *Case 3* and *Case 4* as *Case B*, *Case 5* as *Case C* and *Case 6* as *Case D*:

Case A: $\alpha_1 Q_1$ is minimum

$$r\alpha_1 Q_1 - g_r(D - \alpha_1 Q_1) - h_2(\alpha_2 Q_2 - \alpha_1 Q_1)$$

Case B: $\alpha_2 Q_2$ is minimum

$$r\alpha_2 Q_2 - g_r(D - \alpha_2 Q_2) - h_1(\alpha_1 Q_1 - \alpha_2 Q_2)$$

Case C: $D < \alpha_1 Q_1 < \alpha_2 Q_2$

$$rD + h(\alpha_1 Q_1 - D) - h_2(\alpha_2 Q_2 - \alpha_1 Q_1)$$

Case D: $D < \alpha_2 Q_2 < \alpha_1 Q_1$

$$rD + h(\alpha_2 Q_2 - D) - h_1(\alpha_1 Q_1 - \alpha_2 Q_2)$$

With this notation, the function cannot be re-organized or no comments can be made for concavity. So let's add and subtract some values which sum up to zero in order to get some insight. Every line is one case and every change in each step is underlined to be followed easily:

$$\begin{aligned}
\Rightarrow\ & r\alpha_1 Q_1 && -g_r(D-\alpha_1 Q_1) && && -h_2(\alpha_2 Q_2 - \alpha_1 Q_1)\\
& r\alpha_2 Q_2 && -g_r(D-\alpha_2 Q_2) && && -h_1(\alpha_1 Q_1 - \alpha_2 Q_2)\\
& rD && && +h(\alpha_1 Q_1 - D) && -h_2(\alpha_2 Q_2 - \alpha_1 Q_1)\\
& rD && && +h(\alpha_2 Q_2 - D) && -h_1(\alpha_1 Q_1 - \alpha_2 Q_2)
\end{aligned}$$

$$\begin{aligned}
\Rightarrow\ & r\alpha_1 Q_1 && -g_r(D-\alpha_1 Q_1) && +\underline{\alpha_1 Q_1(h-h)} && -h_2(\alpha_2 Q_2 - \alpha_1 Q_1)\\
& r\alpha_2 Q_2 && -g_r(D-\alpha_2 Q_2) && +\underline{\alpha_2 Q_2(h-h)} && -h_1(\alpha_1 Q_1 - \alpha_2 Q_2)\\
& rD && \underline{-g_r D + g_r D} && +h(\alpha_1 Q_1 - D) && -h_2(\alpha_2 Q_2 - \alpha_1 Q_1)\\
& rD && \underline{-g_r D + g_r D} && +h(\alpha_2 Q_2 - D) && -h_1(\alpha_1 Q_1 - \alpha_2 Q_2)
\end{aligned}$$

$$\begin{aligned}
\Rightarrow\ & (r+g_r-h)\alpha_1 Q_1 && -g_r D && +h\alpha_1 Q_1 && -h_2(\alpha_2 Q_2 - \alpha_1 Q_1)\\
& (r+g_r-h)\alpha_2 Q_2 && -g_r D && +h\alpha_2 Q_2 && -h_1(\alpha_1 Q_1 - \alpha_2 Q_2)\\
& (r+g_r-h)D && -g_r D && +h\alpha_1 Q_1 && -h_2(\alpha_2 Q_2 - \alpha_1 Q_1)\\
& (r+g_r-h)D && -g_r D && +h\alpha_2 Q_2 && -h_1(\alpha_1 Q_1 - \alpha_2 Q_2)
\end{aligned}$$

which in fact turns out to be:

$$\begin{aligned}
\Rightarrow \quad & (r + g_r - h)\min(\alpha_1 Q_1, \alpha_2 Q_2, D) - g_r D + h\min(\alpha_1 Q_1, \alpha_2 Q_2) \\
& -h_1 \max[(\alpha_1 Q_1 - \alpha_2 Q_2), 0] - h_2 \max[(\alpha_2 Q_2 - \alpha_1 Q_1), 0] \\
\Rightarrow \quad & (r + g_r - h)\min(\alpha_1 Q_1, \alpha_2 Q_2, D) - g_r D + h\min(\alpha_1 Q_1, \alpha_2 Q_2) \\
& +h_1 \min[(\alpha_2 Q_2 - \alpha_1 Q_1), 0] + h_2 \min[(\alpha_1 Q_1 - \alpha_2 Q_2), 0]
\end{aligned} \tag{B.1}$$

APPENDIX C: ASSEMBLY SYSTEM WITH N SUPPLIERS

Arrangement of the function resembles that of two supplier system. To find the expectation of the profit function of N supplier system, *(N+1)!* cases have to be evaluated. We neglect the production costs $\sum_N c_i Q_i$ because they are constant at all cases. Let's establish three cases in which delivery of supplier 1 is the smallest:

Case 1: $\alpha_1 Q_1 < \alpha_2 Q_2 < \ldots < \alpha_k Q_k < \alpha_{k+1} Q_{k+1} < \ldots < \alpha_N Q_N < D$

$$\Rightarrow \quad r\alpha_1 Q_1 - g_r(D - \alpha_1 Q_1) - h_2(\alpha_2 Q_2 - \alpha_1 Q_1)\ldots$$
$$-h_k(\alpha_k Q_k - \alpha_k Q_k) - h_{k+1}(\alpha_{k+1} Q_{k+1} - \alpha_1 Q_1)\ldots - h_N(\alpha_N Q_N - \alpha_1 Q_1)$$

Case 2: $\alpha_1 Q_1 < \alpha_2 Q_2 < \ldots \alpha_k Q_k < D < \alpha_{k+1} Q_{k+1} < \ldots < \alpha_N Q_N$

$$\Rightarrow \quad r\alpha_1 Q_1 - g_r(D - \alpha_1 Q_1) - h_2(\alpha_2 Q_2 - \alpha_1 Q_1)\ldots$$
$$-h_k(\alpha_k Q_k - \alpha_k Q_k) - h_{k+1}(\alpha_{k+1} Q_{k+1} - \alpha_1 Q_1)\ldots - h_N(\alpha_N Q_N - \alpha_1 Q_1)$$

Case 3: $\alpha_1 Q_1 < D < \alpha_2 Q_2 < \ldots < \alpha_k Q_k < \alpha_{k+1} Q_{k+1} < \ldots < \alpha_N Q_N$

$$\Rightarrow \quad r\alpha_1 Q_1 - g_r(D - \alpha_1 Q_1) - h_2(\alpha_2 Q_2 - \alpha_1 Q_1)\ldots$$
$$-h_k(\alpha_k Q_k - \alpha_k Q_k) - h_{k+1}(\alpha_{k+1} Q_{k+1} - \alpha_1 Q_1)\ldots - h_N(\alpha_N Q_N - \alpha_1 Q_1)$$

It can be seen that all of the cases results in same values. Thus let's merge the cases where a supplier has the minimum delivery and call them *Case* min_i.

Now let's write Case 1, that is the case where delivery of supplier 1 is minimum. Then add and subtract some values which sum up to zero and does not affect the result but helpful to manipulate the function easily:

Case min_1: $\alpha_1 Q_1$ is minimum

$$\Rightarrow \quad r\alpha_1 Q_1 - g_r(D - \alpha_1 Q_1) - h_2(\alpha_2 Q_2 - \alpha_1 Q_1)\ldots$$
$$-h_k(\alpha_k Q_k - \alpha_k Q_k) - h_{k+1}(\alpha_{k+1} Q_{k+1} - \alpha_1 Q_1)\ldots - h_N(\alpha_N Q_N - \alpha_1 Q_1)$$
$$\Rightarrow \quad r\alpha_1 Q_1 - g_r(D - \alpha_1 Q_1)\underline{-h_1(\alpha_1 Q_1 - \alpha_1 Q_1)} - h_2(\alpha_2 Q_2 - \alpha_1 Q_1)\ldots$$
$$-h_k(\alpha_k Q_k - \alpha_k Q_k) - h_{k+1}(\alpha_{k+1} Q_{k+1} - \alpha_1 Q_1)\ldots - h_N(\alpha_N Q_N - \alpha_1 Q_1)$$
$$\Rightarrow \quad r\alpha_1 Q_1 - g_r(D - \alpha_1 Q_1) - \sum_N (h_i \alpha_i Q_i) + \alpha_1 Q_1) \sum_N h_i$$

which can be generalized as:

Case min_i: $\alpha_i Q_i$ is minimum

$$\Rightarrow \quad r\alpha_i Q_i - g_r(D - \alpha_i Q_i) - \sum_N (h_i \alpha_i Q_i) + \min_i(\alpha_i Q_i) \sum_N h_i \tag{C.1}$$

Now again let's write three cases where yield of supplier 1 is greater than demand but less than others:

Case 1: $D < \alpha_1 Q_1 < \alpha_2 Q_2 < \ldots < \alpha_k Q_k < \alpha_{k+1} Q_{k+1} < \ldots < \alpha_N Q_N$

$$\Rightarrow \quad rD + h(\alpha_1 Q_1 - D) - h_2(\alpha_2 Q_2 - \alpha_1 Q_1)\ldots$$
$$-h_k(\alpha_k Q_k - \alpha_k Q_k) - h_{k+1}(\alpha_{k+1} Q_{k+1} - \alpha_1 Q_1)\ldots - h_N(\alpha_N Q_N - \alpha_1 Q_1)$$

Case 2: $D < \alpha_1 Q_1 < \ldots \alpha_k Q_k < \alpha_{k+1} Q_{k+1} < \ldots < \alpha_N Q_N < \alpha_2 Q_2$

$$\Rightarrow \quad rD + h(\alpha_1 Q_1 - D) - h_2(\alpha_2 Q_2 - \alpha_1 Q_1) \ldots$$
$$-h_k(\alpha_k Q_k - \alpha_k Q_k) - h_{k+1}(\alpha_{k+1} Q_{k+1} - \alpha_1 Q_1) \ldots - h_N(\alpha_N Q_N - \alpha_1 Q_1)$$

Case 3: $D < \alpha_1 Q_1 < \alpha_2 Q_2 < \ldots < \alpha_{k+1} Q_{k+1} < \ldots < \alpha_N Q_N < \alpha_k Q_k$

$$\Rightarrow \quad rD + h(\alpha_1 Q_1 - D) - h_2(\alpha_2 Q_2 - \alpha_1 Q_1) \ldots$$
$$-h_k(\alpha_k Q_k - \alpha_k Q_k) - h_{k+1}(\alpha_{k+1} Q_{k+1} - \alpha_1 Q_1) \ldots - h_N(\alpha_N Q_N - \alpha_1 Q_1)$$

Like the previous cases, the results turn out to be same. So let these results merge into one called *Case* $min_{D,1}$ and write it as a single case:

Case $min_{D,1}$: $D < \alpha_1 Q_1 < \ldots$

$$\Rightarrow \quad rD + h(\alpha_1 Q_1 - D) - \textstyle\sum_{t=1}^{N}(h_t \alpha_t Q_t) + \alpha_1 Q_1 \sum_{t=1}^{N} h_t$$

which can be generalized as:

Case $min_{D,i}$: $D < \alpha_i Q_i < \ldots$

$$\Rightarrow \quad rD + h(\alpha_i Q_i - D) - \textstyle\sum_{t=1}^{N}(h_t \alpha_t Q_t) + \alpha_i Q_i \sum_{t=1}^{N} h_t \tag{C.2}$$

We have generalized the cases of the profit function. To be able to comment on the profit function, it must be modified into an elegant form. In the equations below, every line is a case and we add and subtract some value which sum up to zero:

$$\begin{aligned} \Rightarrow \quad & rD & & +h(\alpha_i Q_i - D) & -\textstyle\sum_{t=1}^{N}(h_t \alpha_t Q_t) + \alpha_i Q_i \sum_{t=1}^{N} h_t \\ & r\alpha_i Q_i & -g_r(D - \alpha_i Q_i) & & -\textstyle\sum_{t=1}^{N}(h_t \alpha_t Q_t) + \alpha_i Q_i \sum_{t=1}^{N} h_t \end{aligned}$$

We can see that the summations for component holding costs are common. So let take them out for simplifying

$$
\begin{array}{llll}
\Rightarrow & rD & \underline{-g_r(D-D)} & +h(\alpha_i Q_i - D) \\
 & r\alpha_i Q_i & -g_r(D-\alpha_i Q_i) & \underline{+h(\alpha_i Q_i - \alpha_i Q_i)} \\
\Rightarrow & (r+g_r-h)D & -g_r D & +h\alpha_i Q_i \\
 & (r+g_r-h)\alpha_i Q_i & -g_r D & +h\alpha_i Q_i
\end{array}
$$

which simplifies into the following equation:

$$
\begin{aligned}
\pi_c() = (r+g_r-h)\min_i(D, \alpha_i Q_i) - g_r D + h\min_i(\alpha_i Q_i) \\
- \sum_{t=1}^{N}(h_t \alpha_t Q_t) + \alpha_i Q_i \sum_{t=1}^{N} h_t
\end{aligned}
\tag{C.3}
$$

APPENDIX D: NUMERICAL ILLUSTRATIONS - NEWSVENDOR PROBLEM

In this part, there are results of numerical examples for the newsvendor problem. The figures below shows the relation between the exogenous parameters and the expected chain profit and optimal order quantity. Figure D.7 shows the probability density function of a normal distribution with $\mu = 100$ and $\sigma = 60$:

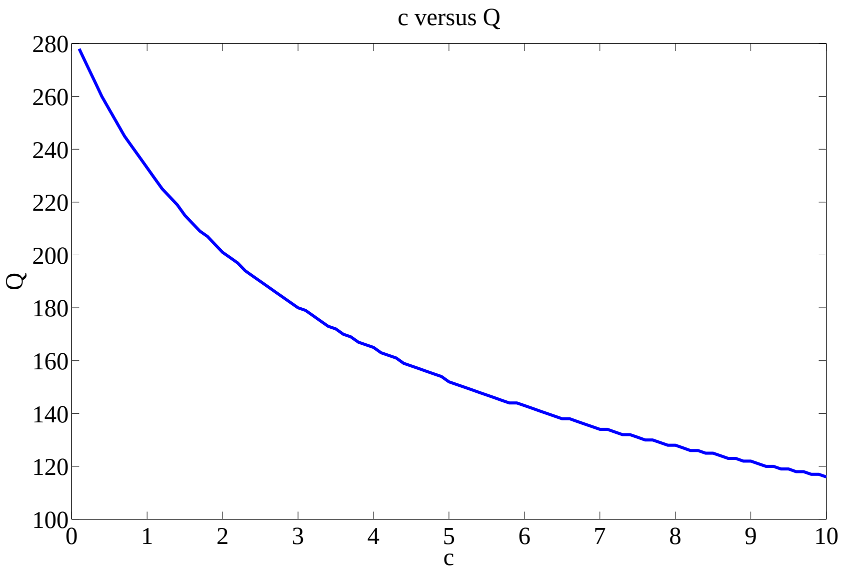

Figure D.1. c versus Optimal Order Quantity

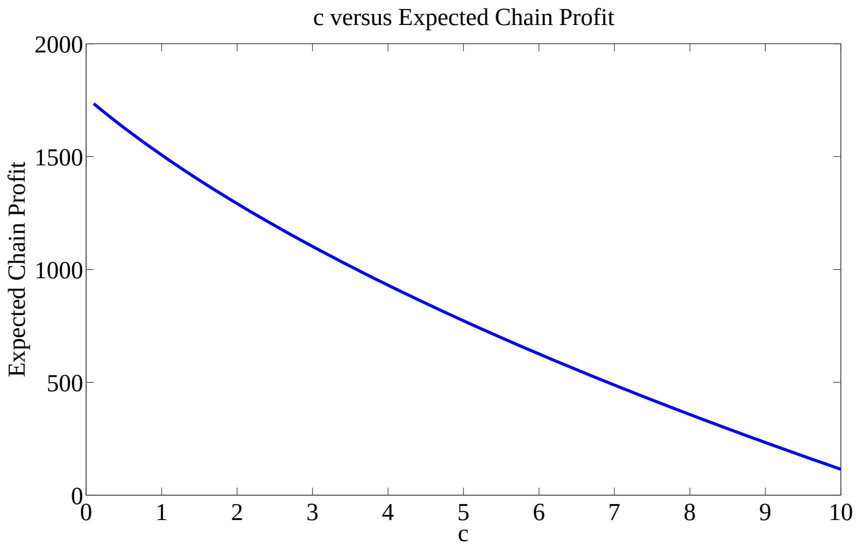

Figure D.2. c versus Chain Profit

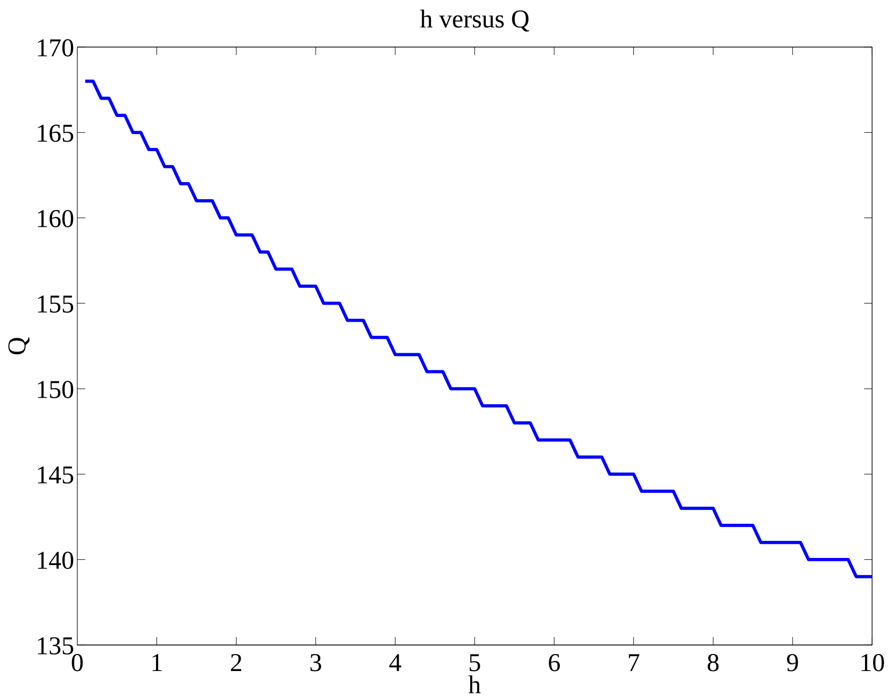

Figure D.3. h versus Optimal Order Quantity

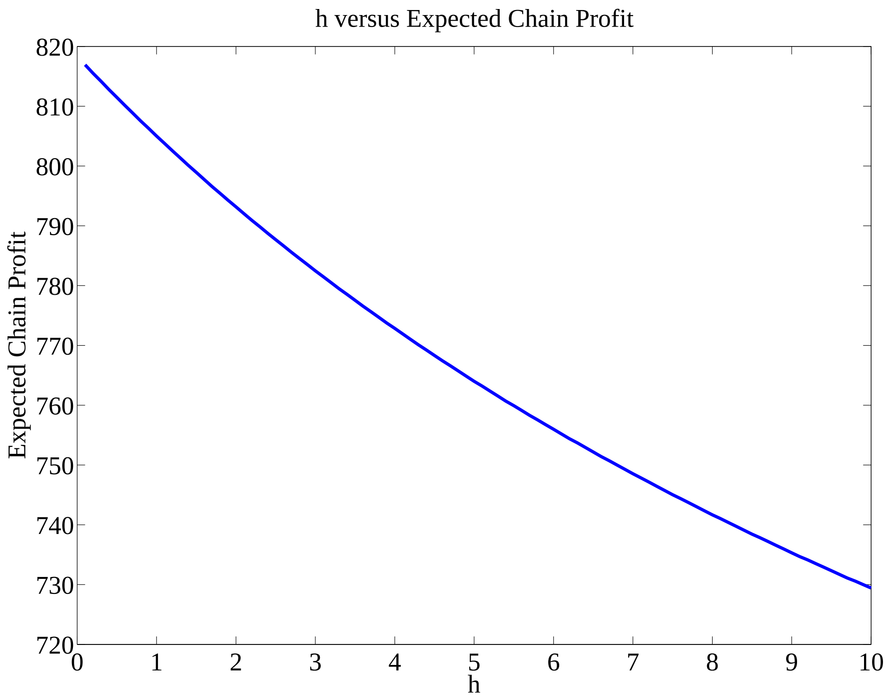

Figure D.4. h versus Chain Profit

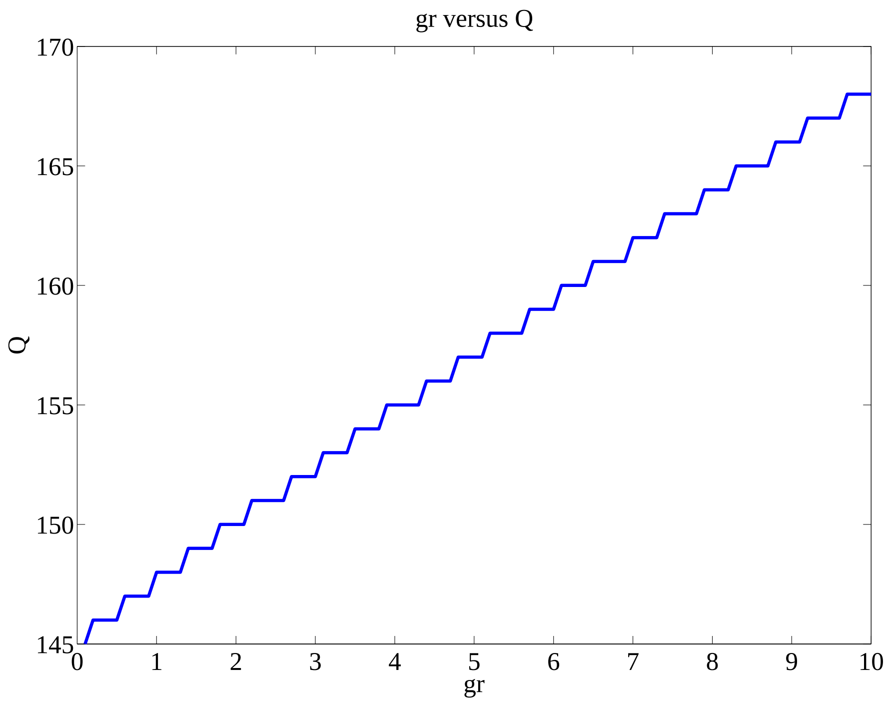

Figure D.5. g_r versus Optimal Order Quantity

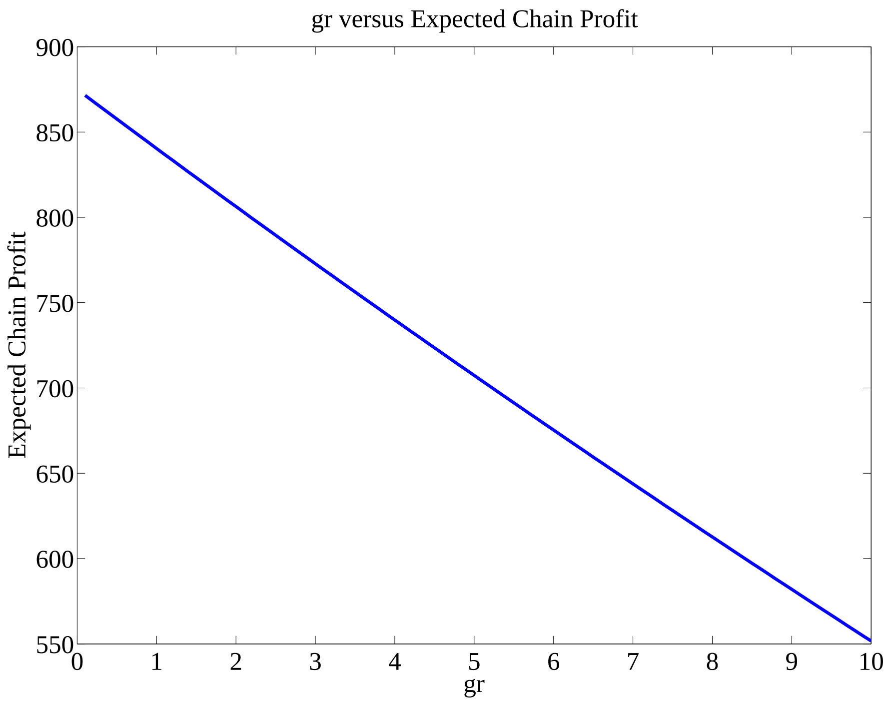

Figure D.6. g_r versus Chain Profit

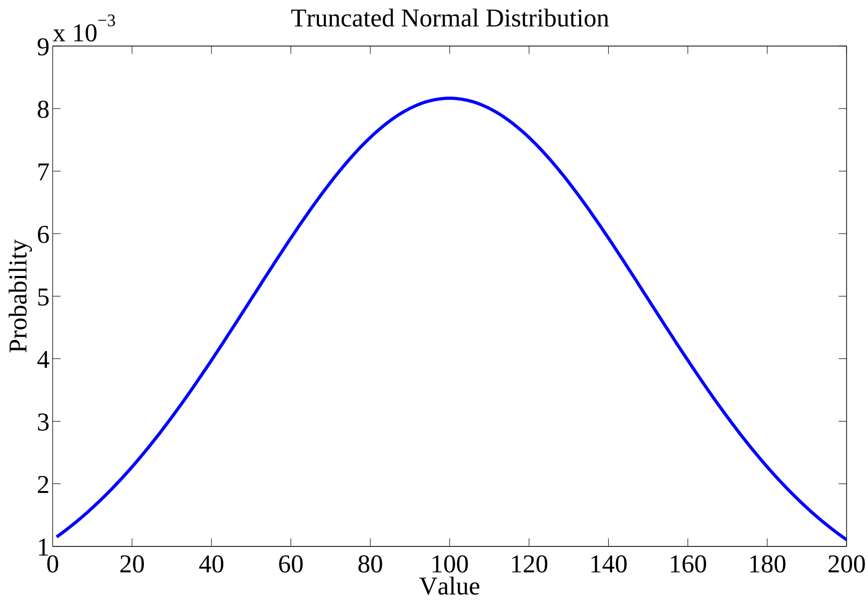

Figure D.7. Truncated Normal Distribution with $\mu = 100$ and $\sigma = 60$

APPENDIX E: NUMERICAL ILLUSTRATIONS - ASSEMBLY SYSTEM

Following figures show the profit allocation of suppliers in the two-supplier assembly system under different contracts. The figures show the profit share of the suppliers versus the revenue share parameter of the first supplier, ϕ_1

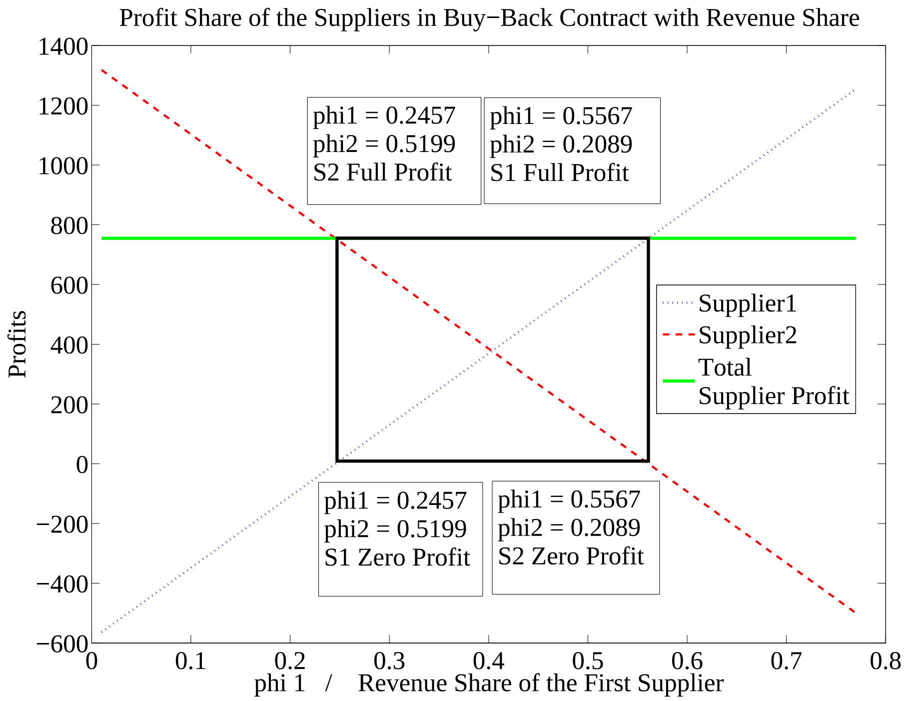

Figure E.1. ϕ_1 versus Profit Shares in BBRS

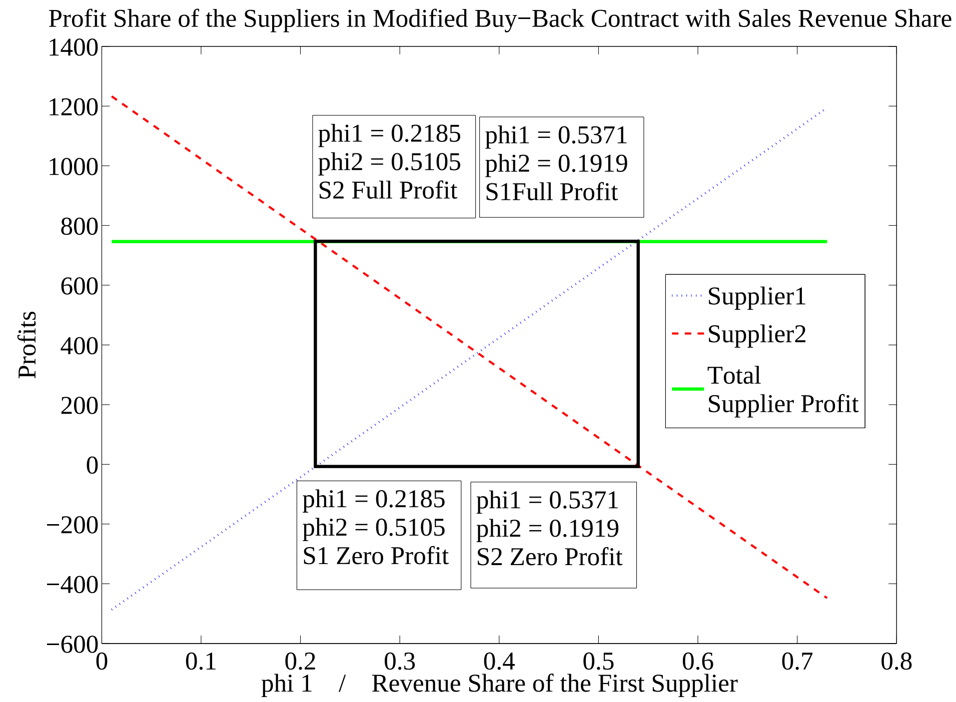

Figure E.2. ϕ_1 versus Profit Shares in MBBSRS

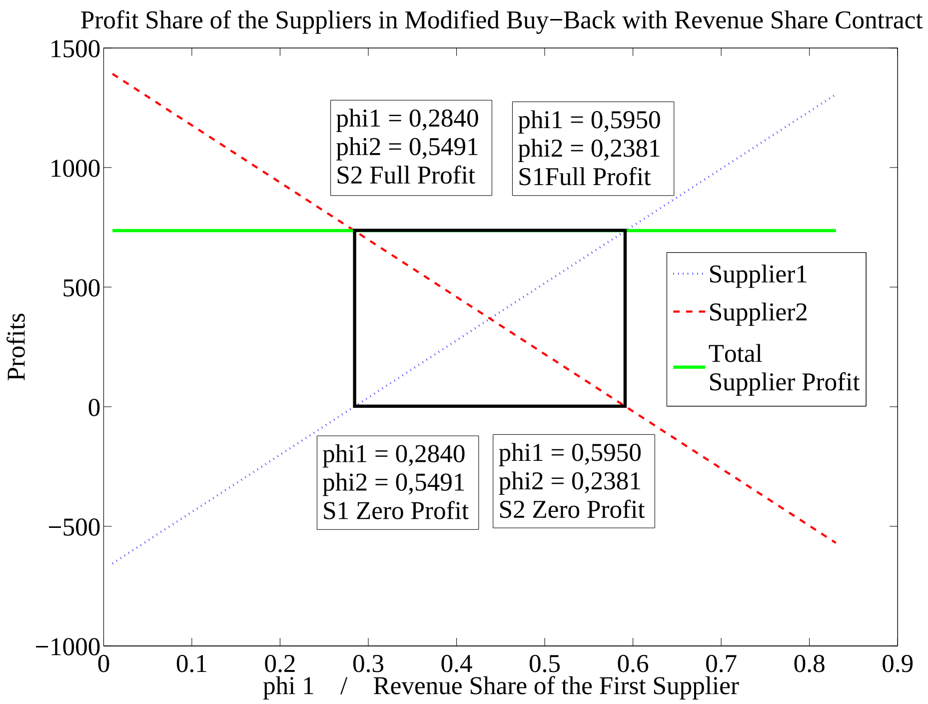

Figure E.3. ϕ_1 versus Profit Shares in MBBRS

REFERENCES

1. Shih, W., “Optimal inventory policies when stockouts results from defective products”, *International Journal of Production Research*, Vol. 18, No. 6, pp. 677–685, 1980.

2. Noori, A. H. and G. Keller, “One period order quantity strategy with uncertain match between the amount received and quantity requisitioned”, *INFOR*, Vol. 24, No. 1, pp. 1–11, 1986.

3. Ehrhardt, R. and L. Taube, “An inventory model with random replenishment quantities”, *International Journal of Production Research*, Vol. 25, No. 12, pp. 1795–1804, 1987.

4. Gerchak, Y., R. G. Vickson, and M. Parlar, “Periodic review production models with variable yield and uncertain demand”, *IIE Transactions*, Vol. 20, No. 2, pp. 144–150, 1988.

5. Henig, M. and Y. Gerchak, “The structure of periodic review policies in the presence of random yield”, *Operations Research*, Vol. 38, No. 4, pp. 634–643, 1990.

6. Lariviere, A. M. and E. L. Porteus, “Selling to the newsvendor: An analysis of price-only contracts”, *Manufacturing and Service Operations Management*, Vol. 3, No. 4, pp. 293–305, 2001.

7. Weng, Z. K., “Coordinating order quantities between the manufacturer and the buyer: A generalized newsvendor model”, *European Journal of Operations Research*, Vol. 156, pp. 148–161, 2004.

8. Giannoccaro, I. and P. Pontrandolfo, “Supply chain coordination by revenue sharing contracts”, *International Journal of Production Economics*, Vol. 89, pp. 131–139, 2004.

9. Cachon, G. P. and M. A. Lariviere, "Supply Chain Coordination with Revenue Sharing Contracts: Strengths and Limitations", *Management Science*, Vol. 51, No. 1, pp. 30–44, 2005.

10. Cachon, G. P., *Handbooks in Operations Research and Management Science: Supply Chain Management*, chapter Supply Chain Coordination with Contracts, 11, pp. 229–339, North Holland, 2004.

11. Yano, C. A. and H. L. Lee, "Lot sizing with random yields: A review", *Operations Research*, Vol. 43, No. 2, pp. 311–334, 1995.

12. Khouja, M., "The single period (news-vendor) problem: Literature review and suggestions for future research", *Management Scince*, Vol. 27, pp. 537–553, 1999.

13. Parlar, M. and D. Wang, "Diversification under yield randomness in inventory models", *European Journal of Operational Research*, Vol. 66, pp. 52–64, 1993.

14. Anupindi, R. and R. Akella, "Diversification under supply uncertainity", *Management Science*, Vol. 39, No. 8, pp. 944–963, 1993.

15. Agnihothri, S., J. S. Lee, and J. Kim, "Lot sizing with random yields and tardiness costs", *Computers and Operations Research*, Vol. 27, pp. 437–459, 2000.

16. Ciarallo, F. W., R. Akella, and E. M. Thomas, "A periodic review production planning model with uncertain capacity and uncertain demand - optimality of extended myopic policies", *Management Science*, Vol. 40, No. 3, pp. 320–332, 1994.

17. Wang, Y. and Y. Gerchak, "Periodic review production models with variable capacity, random yield, and uncertain demand", *Management Science*, Vol. 42, No. 1, pp. 130–137, 1996.

18. Yao, D. D., "Optimal run quantities for an assembly system with random yields", *IIE Transactions*, Vol. 20, No. 4, pp. 399–403, 1988.

19. Gerchak, Y., Y. Wang, and C. A. Yano, "Lot sizing in assembly systems with random component yields", *IIE Transactions*, Vol. 26, No. 2, pp. 19–24, 1994.

20. Gurnani, H., R. Akella, and J. Lehoczky, "Optimal order policies in assembly systems with random demand and random supplier delivery", *IIE Transactions*, Vol. 28, pp. 865–878, 1996.

21. Gurnani, H., R. Akella, and J. Lehoczky, "Supply management in assembly systems with random yield and random demand", *IIE Transactions*, Vol. 32, pp. 701–714, 2000.

22. Gurnani, H. and Y. Gerchak, "Coordination in decentralized assembly systems with uncertain component yields", 1998, dept. of Management Sciences, University of Waterloo.

23. Gerchak, Y. and Y. Wang, "Revenue-sharing vs. wholesale-price contracts in assembly systems with random demand", *Production and Operations Management*, Vol. 13, No. 1, pp. 23–33, 2004.

24. Spengler, J., "Vertical Integration and Antitrust Policy", *Journal of Political Economy*, pp. 347–352, 1950.

25. Bertsekas, D. P., A. Nedic, and A. E. Ozdaglar, *Convex Analysis and Optimization*, Athena Scientific, 2003.