MODELS FOR PRODUCTION PLANNING AND POWER PROCUREMENT PORTFOLIOS

by

Çağrı Latifoğlu

Presented to the Graduate and Research Committee

of Lehigh University

in Candidacy for the Degree of

Doctor of Philosophy

in

INDUSTRIAL AND SYSTEMS ENGINEERING

Lehigh University

May 2012

Approved and recommended for acceptance as a dissertation in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

Date

Accepted Date

Committee Members:

Lawrence V. Snyder (Dissertation Advisor)

Pietro Belotti (Dissertation Advisor)

Theodore K. Ralphs

Shalinee Kishore

Abstract

In this dissertation our objective is to characterize and measure the impact of various sources of uncertainty in the electricity market from the end-user perspective and develop optimization methodologies to mitigate the final impact.

In Chapter 2, we study interruptible load contracts from the perspective of a participating manufacturing company. We develop a production planning framework that mitigates the uncertainty created by the contractual clauses. We present a mathematical modeling approach and computational results.

In Chapter 3, we conduct an experiment using real time electricity prices from the two regional U.S. markets to test the for inherent patterns in real-time locational marginal prices (LMPs) that could be used for constructing the uncertainty sets for the optimization problems. We present the statistical results and findings to characterize these patterns. Next, another experiment is conducted to compare the information content of various data selection rules and the accuracy of various forecasting techniques.

In Chapter 4, we conduct an experiment to quantify the value of information using two problem classes: the production planning problem and the job shop scheduling problem. We present various mathematical models to represent a limited set of protoypical optimization problems for each problem class, a comparison

of various methods that can be used to construct these optimization problems, simulations of these models with real prices, and finally a numerical analysis of the impact of price uncertainty on optimal solutions. In this chapter, the value of information is quantified as the reflection of the price uncertainty on the optimal objective function value's deviation from a solution obtained by solving an optimization problem with imperfect information.

Our findings indicate that depending on the production and the manufacturing environment, the impact of the price uncertainty on the optimal solutions varies significantly. Without conducting a similar analysis to ours, negotiating terms and prices purely based on price uncertainty may be speculative, illusory and misleading for the contract taking parties.

Acknowledgements

I would like to thank my committee for their support, guidance and support throughout the preparation of this dissertation. I owe the worlds to Google, Wikipedia and developers of R for helping me like no one else ever did by providing the guidance, knowledge and tools, without which this dissertation would not be possible. I also would like to thank R-Bloggers and StackOverflow communities for their invaluable support. Above all, I would like to thank my family for always standing with me and keeping me afloat when everything else is sinking.

Contents

List of Tables

List of Figures

Chapter 1

Introduction

In this dissertation our objective is to characterize and measure the impact of various sources of uncertainty in the electricity market from the end-user perspective and develop optimization methodologies to mitigate the final impact. The end-user, a manufacturing company in our case, is usually the market participant with the least amount of market power. This is due to the fact that most of the electricity markets are still heavily regulated and vertically integrated. When monopolistic entities control the generation, the transmission, the distribution and the trade of the electricity, the power procurement menu for the end-user is usually very limited. However the situation is changing with increasing deregulation, competition, market restructuring and penetration of the renewable energy resources. Under these circumstances, the least the end-user can only do better by measuring and characterizing the reflection of the uncertainty in electricity prices and availability onto the production/manufacturing output and optimize her operations accordingly.

In Chapter 2, we provide a production planning framework for rate-paying industrial production companies whose production operations strongly depend on electricity. The problem we study is an operational-level, aggregate production and inventory planning problem with electricity supply uncertainty and deterministic demand. In particular, our methodology provides a feasible production plan, if one exists under the given production and storage capabilities, that satisfies customer demands under all possible interruption scenarios using no information on uncertainty other than the contractual clauses. Our modeling approach has several benefits. First, the interruption uncertainty framework we describe allows different contract rules and operational rules to be embedded in the production planning problem simultaneously. We first develop a robust optimization model to solve this problem. We show that this problem can be modeled as a single linear program. Next we develop a novel heuristic that has roots on list processing and works at least an order of magnitude faster than the linear programming approach under certain assumptions.

In Chapter 3, we conduct an empirical study to characterize the LMPs in two regional U.S. markets: ISONE and PJM. The intent of this study is to analyze the behaviour, time-dependence and patterns of prices. The data shows that for accurate forecasts, one needs to choose the data horizon in close proximity of the forecast target date. We analyze the data, the daily price profiles and intraday price correlations using different time scales such as days, weeks, months and years. We provide aggregate statistics and test the data for independence, stationarity and normality. Finally, an experiment is conducted to test the accuracy of dynamically fit ARIMA models for price forecasts. The ARIMA models are fit using rolling time windows with different horizon lengths since one of the objectives of this study to test the information content of different horizons from the perspective of the optimization problem. Two different horizon selection rules are compared using five horizon lengths and various error measures. As a result we conclude that dynamically fit ARIMA models with varying horizons in the proximity of the forecast date (i.e. short data horizons) is the better choice for forecasting daily prices.

In Chapter 4, we conduct an experiment to measure the value of the information from the production planner's perspective. Various demand scenarios and rolling time horizons of varying length for the electricity price data are used to create and compare optimization instances of various mathematical models under the assumption that a production planning problem is solved at the beginning of the week, i.e. before the actual prices are revealed. The production planner may use various approaches in anticipation of the uncertainty and the value of information regarding the price uncertainty is quantized using optimization models assuming that, first the production planner chooses a methodology such as using the data for estimation and solving an LP using the estimated cost; or using the data to calculate various statistics and use these statistical calculations to create robust optimization instances; or use the prior week's prices as an estimator for the current week. The details of the models that were investigated in this study are summarized next.

The baseline model is a simple linear production planning model. First for all weeks and demand scenarios, the baseline LP is solved to find the true objective function value and the optimal production plan. Next for any given week, instances of this LP is created using various estimation rules (such as the prior week's prices, estimated prices obtained with dynamically fit ARIMA models) and solved. Similarly a simple robust optimization model with a budget of uncertainty is instantiated using various statistics obtained from the rolling time horizon and solved. There are five such variations for the robust optimization model. Finally a stochastic optimization model is instantiated and solved. For linear programs and robust models, the obtained production plan is combined with actual prices to calculate the would-be optimal objective function value where the deviation of this value from the true objective function value is calculated and used to compare these approaches. For the stochastic programming model, the objective function is directly used in the deviation calculations. Furthermore a job-shop scheduling model is also developed and tested using the electricity prices. Unlike the previous models, this model is only used analyze the impact of the electricity price uncertainty on processing schedule changes and calculate the optimal objective function value statistics. Our findings indicate that estimates for current week's prices obtained by using ARIMA models with short time horizons and using past week's price directly as an estimate and coupling this estimate with simpler mathematical models is slightly favorable to using complicated optimization models where the data is used to construct an uncertainty set and this uncertainty set is embedded in the model.

Chapter 2

Planning under Interruption Uncertainty

2.1 Introduction

Under deregulation, the electricity industry is continuously evolving and changing as different markets, such as the derivative, forward and spot markets, become more common. Within these markets, the availability of diversified services and pricing menus is increasing. Participants in these markets have developed and adopted many financial instruments for electricity transactions so that electricity is produced, priced and traded more efficiently. Interruptible load contracts (ILCs) are one type of these instruments which are employed to increase demand-side involvement so that the adverse effects of supply shortages at the utility are mitigated.

As defined in Baldick et al. [2006], an interruptible load contract between a utility and an industrial/commercial customer allows the utility to interrupt part or all of the supply of electricity to the customer over some period of time in exchange for some form of monetary reward. Typically, the maximum number of interruptions that can occur during the specified time interval is defined in the contract. Usually the utility does not physically interrupt the customer, but rather gives the customer advance notice to reduce loads or face a significantly increased cost rate during the interruption period.

Companies that participate in such ILCs benefit from the discounted rate structure, while the electricity utility enjoys the benefits of priority service. As defined in Chao and Wilson [1987], priority service refers to a menu of contingent contracts in which service provision is prioritized according to the customer's valuation of the service under supply scarcity. This helps the utility to reduce its exposure to spot market prices in the event of supply shortages. However, customers now bear additional risks that arise from the interruption of power, such as backorder costs, reduced production and storage capacities. Participation in ILCs requires flexibility from customers in order to be able to honor this requirement along with their requirement to satisfy the demand. To do this, customers must account for potential interruptions in their production plans. As demonstrated in a report by California Public Utilities Commission [2001], some companies that participated in ILCs were unable to honor the load reduction requirement due to a lack of capabilities and/or a lack of necessary planning. This was one of the triggering events that caused the California electricity crisis in 2001 [California Public Utilities Commission, 2001].

The purpose of this chapter is to provide a production planning framework for rate-paying industrial production companies whose production operations strongly depend on electricity. The problem we study is an operational-level, aggregate production and inventory planning problem with electricity supply uncertainty and deterministic demand. In particular, our methodology provides a feasible production plan, if one exists under the given production and storage capabilities, that satisfies customer demands under all possible interruption scenarios. Furthermore, we avoid using any probability distribution to characterize the interruptions, since our objective is to guarantee feasibility in any possible interruption realization, and since the distribution of interruptions is typically unknown. Therefore we use the only pieces of information that are available to the industrial company through the ILC: the length of the planning horizon (which is equal to the duration of the ILC), the unit length of interruptions, the limit on the number of interruptions and the reward scheme.

Our modeling approach has several benefits. First, the interruption uncertainty framework we describe allows different contract rules and operational rules to be embedded in the production planning problem simultaneously. Companies might have different operational procedures in the event of interruptions, such as limiting the production in post-interruption recovery or prohibiting production level increases in some periods. Second, the methodology we describe can be used for different types of ILCs, possibly with different reward schemes, such as the pay-in-advance and pay-as-you-go schemes described by Baldick et al. [2006]. Third, information regarding the utility's interruption dispatch behavior can easily be embedded into our production planning framework.

Although a number of papers on ILCs have appeared in the literature (see §2.2.1), these papers all consider the problem faced by the interrupting party, that is, the power provider, which can be a power generator or the utility. We chose to consider the impact of ILCs on the production planning process of the interrupted party, that is, the industrial company.

Our study is motivated by an air-separation process, and many of the elements in our model come from that setting; however, the robust methodology we provide is not restricted to this setting. In an air-separation process, a mass production system is used to produce liquid nitrogen and liquid oxygen in large volumes using special-purpose equipment. Both products are produced simultaneously through a single process. Virtually the only raw material needed is air, which is available in very large quantities at a very low cost, leaving electricity as the most critical resource that is required for production. The products generated by the air separation process are critical for medical treatment in hospitals, as well as for other high-impact applications. The availability of the products is therefore critical and this motivates the assumption in our model that stockouts are not allowed under any interruption scenario.

The remainder of this chapter is organized as follows. We review the literature on ILCs and production planning under supply uncertainty in §2. In §3, we formulate the robust production planning problem under supply uncertainty. §4 gives a real world application of the robust model in which we demonstrate how to construct the *uncertainty set* in such a way that it includes interruption-related operational rules. Computational results for both models are discussed in §5. §6 concludes the chapter and discusses future research.

2.2 Literature Review

2.2.1 Interruptible Load Contracts

The literature on ILCs is focused on pricing and has its roots in the body of research on service priority [Tschirhart and Jen, 1979, Chao and Wilson, 1987]. The former paper studies the impact of the division of customers into priority classes in which interruptible service is priced according to service reliability. The latter extends this study by adding an optimal price menu for better segmentation. Strauss and Oren [1993] further extends these studies by introducing an early notification option. Caves and Herriges [1992] addresses the optimal interruption dispatch behavior of the utility within a stochastic dynamic programming (SDP) framework when there is a limit on the interruptions that can be called.

Kamat and Oren [2002] studies the design and pricing of financial contracts for interruptible electricity service, emulating three types of ILCs using financial instruments such as forwards and options. Baldick et al. [2006] further extends the works by Caves and Herriges [1992], and Kamat and Oren [2002] and creates a structural model that is calibrated using temperature data. This model is then embedded into an SDP framework, which is used to valuate the ILC and to find the optimal interruption dispatch policy under different reward schemes. The authors also explore the performance of ILCs under retailer competition.

Fahrioğlu and Alvarado [2000, 2001] study incentive-compatible ILCs and describe a methodology for the electricity utility to estimate the customer demand and the value of interruptibility for the customer through utility data. The authors' objective is to design the contract incentives in a way that enables the revelation of the customers' actual valuation of the interruption.

2.2.2 Production Planning Literature

According to Rosenhead et al. [1972], decision-making can be classified into three different categories: certainty, risk, and uncertainty. In the certainty case, all decision-making elements are deterministic and known. Risk and uncertainty arise when the complete information that describes a situation to its full extent is lost. In risk situations, the information that the decision-maker has still contains some descriptive elements, such as probability distributions, that explain the situation to some extent. In the absence of these descriptive elements, uncertainty arises. Consistent with these definitions, we use the term “uncertainty” throughout this chapter to refer to problems in which no probabilistic information is available about the random parameters. Our problem is an operational-level, aggregate production and inventory planning problem with supply uncertainty.

Production planning under supply uncertainty has received a lot of attention from the supply chain, production and inventory theory communities. For example, Chao [1987] develops an optimal dynamic inventory policy in the presence of market disruptions. The model is based on the framework of a continuous-time Markov decision process with a finite state space in which the rate of inventory accumulation or reduction can be continuously adjusted. An economic analysis is provided for both elastic and inelastic demand. Weiss and Rosenthal [1992] develops an optimal inventory policy for EOQ systems where the start time of the disruption is known beforehand but the duration is unknown.

Parlar et al. [1995] studies a continuous-review stochastic inventory problem with random demand and random lead-time where supply availability is an alternating renewal process; see also Parlar and Perry [1996], Parlar [1997], Gürler and Parlar [1997]. Arreola-Risa and DeCroix [1998] consider a continuous-review inventory system in which partial backorders are allowed; see also Moinzadeh and Aggarwal [1997]. Güllü et al. [1997] study a periodic-review inventory model, shows the optimality of an order-up-to policy, and obtains a newsboy-like formula that determines the optimal order-up-to levels under deterministic dynamic demand and stochastic supply unavailability.

Most of the related work on supply uncertainty in the supply chain literature focuses on either random yields in the supply processes or complete supply disruptions, with known probabilistic information about disruptions. We refer the reader to Snyder et al. [2010], Peidro et al. [2009], Yano and Lee [1995] for through reviews of the literature on supply uncertainty under various supply chain settings. For a general review of production planning under uncertainty, see Mula et al. [2006].

2.2.3 Robust Optimization

One of the seminal works in robust optimization is by Soyster [1973], who proposed a linear model to construct a solution that is feasible for all the data that belong to a convex set. Optimizing over the worst-case scenario might produce solutions that are too conservative, and Soyster's approach has been criticized for being ultraconservative by Bertsimas and Sim [2004].

To cure over-conservativeness, El Ghaoui and Lebret [1997], El Ghaoui et al. [1998], Ben-Tal and Nemirovski [2000], Ben-Tal and Nemirovski [2002] consider uncertain convex optimization problems under ellipsoidal uncertainty sets. One drawback of this methodology is that the complexity of the original problem, with no uncertainty included, is increased when it is transformed into its robust counterpart (RC). To a limited extent, Kouvelis and Yu [1996] extend robust optimization to include discrete variables. When the objective is to optimize the worst-case performance over a set of scenarios, they show that the RCs of many polynomially solvable discrete optimization problems are NP-hard. Methodologies other than Soyster's require us to make additional assumptions on the structure of the uncertainty set and in this study we focus on uncertainty sets that can be directly constructed from contractual rules. Therefore, we construct an uncertainty polyhedron using only the contract rules, and for this type of construction Soyster's methodology is appropriate. Another benefit of using this approach is the ease of incorporating the impact of interruptions on production modes.

Bertsimas and Sim [2003, 2004, 2006] introduce and study the "budget of uncertainty" concept. This approach provides a mechanism to control the level of conservativeness by allowing only a subset (the size of which is controlled by the "budget of uncertainty") of the uncertain parameters to deviate from their nominal values simultaneously. Furthermore, the RC preserves the complexity of its nominal problem and thus can easily be extended to discrete optimization problems.

The studies we have mentioned until now are all static decision-making problems, in the sense that all the decision variables are determined before the realization of uncertainty, i.e., "here and now" decisions. Ben-Tal et al. [2004] introduce and study the adjustable RC (ARC) of multistage uncertain linear programming (LP) problems in which some of the decisions can be delayed until after some of the uncertain parameters have been observed, i.e., "wait-and-see decisions." The authors use the intersection of ellipsoids to define the uncertainty set and find that, often, the ARC is significantly less conservative than the usual RC. However, in most cases the ARC is NP-hard. To address this issue, the authors introduce affinely adjustable RCs (AARC) in which the wait-and-see decisions are formulated as affine functions of the uncertain parameters. Ben-Tal et al. [2006] further extends AARCs to include controlled deterioration in performance for large deviations in the uncertain data. For detailed theoretical background we refer to Ben-Tal and Nemirovski [2008], Ben-Tal et al. [2009].

2.3 Robust Production-Planning Models

2.3.1 Electricity Supply, Prices and Interruptions

Following the setting in Baldick et al. [2006], we allow the utility to place multiple interruptions over the planning horizon. We assume that every interruption lasts for exactly one period; however, multiple consecutive interruptions are allowed, as long as the total number of interrupted periods does not exceed the maximum specified by the contract. The uncertainty in the model stems from a lack of information regarding the exact timing of interruptions. Furthermore, we assume there is no prior historical information available to the company regarding interruptions. Moreover, we assume that the production company has production equipment that can be shut down instantly; therefore, there are no depreciation- and maintenance-related costs caused by interruptions. Other than the machinery and labor, the majority of the variable production cost is comprised by electricity consumption. Typically, air separation plants, aluminum foundries and paper mills have such cost characteristics [Sioshansi, 2008]. We also assume that the company can only purchase electricity from the utility and that electricity is only used for production.

2.3.2 Production Setting

We consider a company that has multiple plants and, due to technological and physical reasons, production and storage capacities of the plants are limited. All processing tasks are performed in batch mode and the processing time is negligible; however, there is a finite daily production capacity. The plants are geographically distinct, but each plant has the capability to serve the other plants' customers. This means the demand for products is aggregated over all plants. This is depicted in Figure 2.1 for a two-plant example.

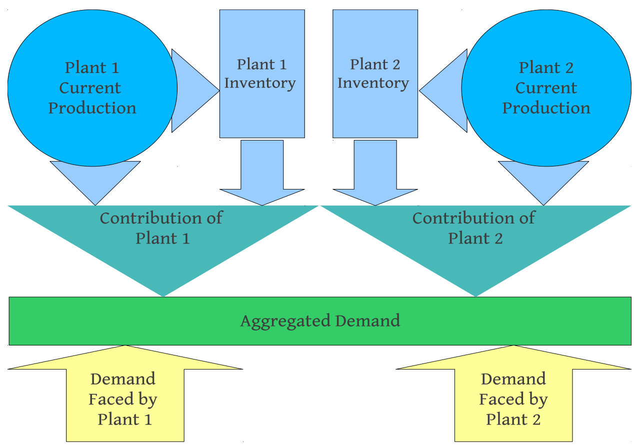

Figure 2.1: Demand Aggregation and Contribution of Plants

In general, the utility charges a fixed rate for electricity, as is typical of a fixed-price retail contract. As an incentive for participating in the ILC, the electricity retailer offers two types of rewards: pay-as-you-go and pay-in-advance, as defined in Baldick et al. [2006]. In a pay-as-you-go scheme, the utility pays a penalty for each interruption it dispatches. In a pay-in-advance scheme, there is no per-interruption penalty for the utility, however, the utility provides an overall discount in the electricity price for the contract horizon. For both payment schemes, the maximum number of interruptions that can be dispatched is contractually defined, though the exact times of the interruptions are not known in advance (by either party). In this study, we assume that the industrial company has accepted the pay-in-advance scheme, and therefore it faces a discounted fixed rate, r.

In our setting, the industrial company has negotiated a joint ILC contract for all plants, and this contract stipulates that at most one plant will be interrupted in a given period, where interruptions last exactly one period. The production planning horizon is equal to the contract horizon, which is terminated by the expiration of the contract. The objective of the industrial company is to minimize production costs subject to the following constraints:

- Daily production in each plant is less than or equal to the daily capacity;
- Inventory capacity is limited;
- Daily aggregated demand must be satisfied through production and accumulated inventory, i.e., no stock-outs are allowed.

The industrial company plans its production anticipating the interruptions, and the production plan is set at the beginning of the planning horizon. Once the production plan is set, it cannot be changed until the end of the horizon. In particular, the production plan cannot be changed in reaction to interruptions, other than to zero out the production at an interrupted plant. A robust solution for this problem has one important characteristic: It stays feasible under all possible interruption scenarios. We use these characteristics as our foundation for the robust modeling approach. A production planner can choose to "robustify" the aggregated inventory over plants, i.e., make sure that the sum of the inventory levels at different plants stays non-negative throughout the horizon, or s/he can choose to "robustify" individual inventories, i.e., make sure that individual inventories at plants are never exhausted throughout the horizon. In our model, we robustify the aggregated inventory over plants, but we describe how the same logic can be used to robustify individual inventories in §2.3.4. Robustifying individual inventories produces more conservative and potentially more costly solutions given that inventory at the other plants can no longer be used as a buffer for the adverse effects of an interruption at a particular plant. We enforce the at-most-one-interruption-per-period rule and denote the maximum number of interruptions as K. We use the notation in Table 2.1 throughout the remainder of the chapter.

Indices & Sets	
p	plant index, $p \in \mathcal{P} := \{1, \ldots, P\}$
t	time index, $t \in \mathcal{T} := \{1, \ldots, T\}$
g	product index, $g \in \mathcal{G} := \{1, \ldots, G\}$
$\mathcal{U}$	uncertainty set
Parameters	
$c^{pro}_{p,g}$	production capacity at plant p for product g
$c^{inv}_{p,g}$	inventory capacity at plant p for product g
$inv_{0,p,g}$	initial inventory of product g at plant p
$d_{t,g}$	aggregated demand for product g in period t
K	maximum number of interruptions
v	power-to-unit conversion factor (units/kWh)
r	electricity price ($/kWh)
Variables	
$z(\cdot)$	objective function value
$x_{t,p,g}$	amount of product g produced at plant p in period t
$w_{t,p,g}$	amount of demand for product g satisfied by plant p in period t
$inv_{t,p,g}$	inventory of product g at plant p at the end of period t

Table 2.1: Notation

2.3.3 Deterministic Production Planning Model

In this section, we present the deterministic production planning model, which contains no interruptions. We will use this model as a baseline and introduce the impact of interruptions in §2.3.4.

Note that Table 2.1 contains two sets of production-related decision variables, $x_{t,p,g}$ and $w_{t,p,g}$. The former represents the production of product g at plant p in period t, while the latter represents the demand for product g in period t that is satisfied by plant p. The two quantities may differ because production in period t may be used to satisfy demand *or* to be stored in inventory for future periods. The w variables are represented in Figure 2.1 by the arrows labeled "Contribution of Plant p."

Using this notation, we characterize the inventory levels that accumulate in each time period in (2.1a)–(2.1b). In (2.1a), the ending inventory in period t is calculated from the initial horizon inventory by adding the total production and subtracting the total items used to satisfy demand through period t. (Recall that the plants maintain separate inventories.) Equation (2.1b) reflects the relationship between the ending inventories in periods $t-1$ and t.

$$inv_{t,p,g} = \sum_{i=1}^{t} x_{i,p,g} - \sum_{i=1}^{t} w_{i,p,g} + inv_{0,p,g} \tag{2.1a}$$

$$= inv_{t-1,p,g} + x_{t,p,g} - w_{t,p,g} \quad \forall t, p, g \tag{2.1b}$$

We first present the deterministic production planning model, which we refer to as the Outer Problem (OP). (An "inner problem", solved by the interrupting party, will be discussed in §2.3.4.)

$$OP\colon \ \min \sum_{p=1}^{P}\sum_{t=1}^{T}\sum_{g=1}^{G} vr x_{t,p,g} \tag{2.2a}$$

$$\text{s.t.} \ inv_{t,p,g} = inv_{t-1,p,g} + x_{t,p,g} - w_{t,p,g} \ \ \forall t,p,g \tag{2.2b}$$

$$\sum_{p=1}^{P} w_{t,p,g} \geq d_{t,g} \ \ \forall t,g \tag{2.2c}$$

$$x_{t,p,g} \leq c_{p,g}^{pro} \ \ \forall t,p,g \tag{2.2d}$$

$$\sum_{p=1}^{P} inv_{t,p,g} \geq 0 \ \ \forall t,g \tag{2.2e}$$

$$inv_{t,p,g} \leq c_{p,g}^{inv} \ \ \forall t,p,g \tag{2.2f}$$

$$x_{t,p,g}, w_{t,p,g} \geq 0 \ \ \forall t,p,g \tag{2.2g}$$

The objective function (2.2a) is simply the cost of electricity used for production—the multiplier v converts production units to electricity consumption (in kWh). Constraints (2.2d) and (2.2f) enforce the production and inventory capacities. Constraints (2.2b) and (2.2c) enforce the relationship between actual demand, $d_{t,p,g}$, and the demand-satisfaction variable, $w_{t,p,g}$. In particular, for each time period and product, the sum of the units coming from all plants must equal the aggregated demand. Constraints (2.2e) enforce the no-stock-out condition on pooled inventory. Finally, constraints (2.2g) are non-negativity constraints.

2.3.4 Robust Simple Model

In this section, we robustify the OP against interruptions, giving rise to a problem we refer to as the Robust Outer Problem (ROP). Since we have no information regarding the utility's interruption dispatch policy, the robust problem must ensure feasibility in all possible interruption scenarios. An interruption scenario is defined by the times and locations of all K interruptions and may be constructed as follows. One would first choose K interruption times from the set $t = 1, ..., T$. There are $\binom{T}{K}$ such choices of interruption times. For a given interrupted time period, exactly one of the plants must be interrupted (due to the contractual agreement). There are P^K such possibilities. Therefore, there are exactly $\binom{T}{K}P^K$ interruption scenarios, which is combinatorial in size. The stochastic programming approach, which depends on individual scenarios rather than on a description of the uncertainty set, is computationally expensive. Hence, we use the robust optimization approach for handling the uncertainty. Note that the analysis above assumes that a scenario has exactly K interruptions. It is possible that fewer than K interruptions will occur over the horizon, but since we are interested in optimizing over the worst case, it is sufficient to assume that exactly K interruptions occur.

The uncertainty set, $\mathcal{U}$, contains all of the uncertainty scenarios. Our objective is to ensure that the aggregate inventory level is non-negative in every scenario. Another way to think about $\mathcal{U}$ is as the feasible set of an optimization problem aimed at determining the minimum inventory levels as a function of the (a priori unknown) interruptions and the production levels given by x. Consequently, we introduce a separate class of variables, $\xi_{t,p}$, to model the utility's choice of interruptions, and we describe the aggregate inventory as a function of those variables subject to constraints derived from the contract clauses. The $\xi_{t,p}$ are defined as

$$\xi_{t,p} = \begin{cases} 1 & \text{if an interruption occurs in period } t \text{ at plant } p \\ 0 & \text{otherwise.} \end{cases}$$

Then,

$$\mathcal{U} = \left\{ \xi \in \{0,1\}^{T \times P} \middle| \sum_{p=1}^{P} \xi_{t,p} \leq 1 \ \ \forall t, \ \ \sum_{p=1}^{P} \sum_{t=1}^{T} \xi_{t,p} \leq K \right\}.$$

Each $\xi \in \mathcal{U}$ corresponds to an interruption scenario since the vector ξ characterizes the time and location of every interruption. The first constraint ensures that no simultaneous interruptions occur, while the second constraint ensures that the number of interruptions doesn't exceed the contractual limit. Therefore, we replace the inventory constraint (2.2e) with

$$\min_{\xi \in \mathcal{U}} \left\{ \sum_{p=1}^{P} inv_{t,p,g}(\xi) \right\} \geq 0 \ \ \forall t, g \tag{2.4}$$

This approach allow us to embed the uncertainty into the OP to obtain the ROP, which is formulated below in (2.5a)–(2.5g).

This relationship between the company and the utility can be thought of as a leader-follower game in which the utility is the follower. This is reflected in the relationship between (2.4) and the ROP, formulated in (2.5a)–(2.5g) below. In particular, the company solves the ROP at the beginning of the planning horizon. This problem anticipates the interruptions and determines optimal production levels while maintaining feasibility in all interruption scenarios. The scenarios are determined by a virtual "opponent" whose objective is to sabotage the company's production so that the demand is not satisfied. The opponent does this by solving an inner optimization problem (IOP), which aims to cause infeasibilities by placing interruptions carefully throughout the time horizon. The IOP for a given t and g is given by (2.4). This problem is embedded into the constraints of the ROP; thus, the ROP contains constraints that can only be instantiated, evaluated, and enforced by solving the optimization problems described in (2.4). Note that the feasible region that contains the opponent's possible interruption decisions is not affected by the production decisions made by the company in the ROP.

Once a plant is interrupted, all production ceases. The ROP anticipates the opponent's behavior, so that all "optimal" actions of the opponent (worst-case scenarios for the planner) are considered and a production plan that ensures non-negative inventory, which implies feasibility in all possible scenarios, is found if one exists. This is characterized in constraint (2.5e) of the ROP. Furthermore, all outer problem variables are regarded as parameters in all inner problems.

The opponent solves as many optimization problems as there are inventory pools; that is, one for each $(t, g) \in \mathcal{T} \times \mathcal{G}$. The feasible region is the same for all IOPs; it is constructed using only the contractual obligations on interruptions. However the objective functions of a given IOP is the inventory level of the corresponding product at the corresponding time period, which is jointly characterized by production and interruption decisions. A negative inventory level for some (t, g) would mean success for the opponent since it would imply a stock-out for the company, which in turn implies the infeasibility of the production plan. The set of feasible actions of the opponent is characterized by the uncertainty set, $\mathcal{U}$, and each scenario in this uncertainty set corresponds to a possible action in the arsenal of the opponent.

Since production and inventory decisions belong to the planner while interruption decisions belong to the electricity utility, the bi-level modeling approach is appropriate for our problem. We formulate the bi-level form robust outer problem (ROP) as:

$$ROP\colon \ \min \ \sum_{p=1}^{P}\sum_{t=1}^{T}\sum_{g=1}^{G} vrx_{t,p,g} \tag{2.5a}$$

$$\text{s.t.} \ \ w_{t,p,g} \leq inv_{t-1,p,g} + x_{t,p,g} \ \ \forall t,p,g \tag{2.5b}$$

$$\sum_{p=1}^{P} w_{t,p,g} \geq \sum_{p=1}^{P} d_{t,p,g} \ \ \forall t,g \tag{2.5c}$$

$$x_{t,p,g} \leq c_{t,p}^{pro} \ \ \forall t,p,g \tag{2.5d}$$

$$\min_{\xi\in\mathcal{U}} \left\{ \sum_{p=1}^{P} inv_{t,p,g}(\xi) \right\} \geq 0 \ \ \forall t,g \tag{2.5e}$$

$$inv_{t,p,g} \leq c_{p,g}^{inv} \ \ \forall t \tag{2.5f}$$

$$x_{t,p,g}, w_{t,p,g} \geq 0 \ \ \forall t,p,g \tag{2.5g}$$

Constraints (2.5e) state that, for each t and g, the pooled inventory at plants, $\sum_{p=1}^{P} inv_{t,p,g}$, must be non-negative in every possible interruption scenario ξ. To robustify the individual inventories instead of the pooled inventory, one can replace constraints (2.5e) with

$$\min_{\xi\in\mathcal{U}}\{inv_{t,p,g}(\xi)\} \geq 0 \ \ \forall t,p,g.$$

Other than (2.5e), the ROP is identical to the OP. Moreover, note that setting $K = 0$ reduces the ROP to the OP.

In general bi-level programs (BLPs) are non-convex [Dempe, 2002] due to the fact that feasible regions of lower level problems are not necessarily convex and connected. In our case the ROP is a bi-level mixed-integer problem which is obviously non-convex. However we can represent inventory levels as bi-linear functions of x and ξ (as in (2.6, 2.7)), and we can convert the BLP into an LP using the optimality conditions of the inner problems; this approach is described in detail later in this section.

The opponent evaluates the quality of his interruption decisions by their effect on the actual production and inventory; however, the actual production is decided by the planner, not the opponent. Therefore, the impact of the opponent's interruption decision on the production plan needs to be represented in the inner problem. To this end, we define an auxiliary variable $\bar{x}_{t,p,g}$ for the inner problem as:

$$\bar{x}_{t,p,g} \quad = (1 - \xi_{t,p}) x_{t,p,g} \;\; \forall t, p, g \tag{2.6}$$

Intuitively, $\bar{x}_{t,p,g}$ represents the actual production of product g at plant p in period t—as planned, if there is no interruption ($\xi_{t,p} = 0$) or zero, if there is ($\xi_{t,p} = 1$). Note that from the IOP's perspective only $\xi_{t,p}$ is a variable and $x_{t,p,g}$ is a parameter. For the sake of compactness we write the bi-linear term $inv_{t,p,g}(\xi)$ as $\overline{inv}_{t,p,g}$. Inserting (2.6) into (2.1a) gives the actual objective for each IOP:

$$\overline{inv}_{t,p,g} \quad = \sum_{i=1}^{t} \bar{x}_{i,p,g} - \sum_{i=1}^{t} w_{i,p,g} + inv_{0,p,g}. \tag{2.7}$$

Then the $IOP(t, g) \; \forall t, g$ becomes:

$$\min_{\mathcal{U}} \left\{ \sum_{p=1}^{P} \overline{inv}_{t,p,g} \right\}. \tag{2.8}$$

These inner optimization problems, which capture the uncertainty, need to be solved simultaneously within the ROP. Note that the feasible region, $\mathcal{U}$, is non-empty and bounded; therefore the IOPs are always feasible with finite optimum. However, the binary variables ξ prevent using linear duality directly as suggested by Soyster [1973]. Instead, we relax the integrality of the uncertainty set to obtain the relaxed uncertainty set, $\mathcal{U}^R$:

$$\mathcal{U}^R = \left\{ \xi \in [0,1]^{T\times P} \middle| \sum_{p=1}^{P} \xi_{t,p} \leq 1 \ \ \forall t, \ \ \sum_{p=1}^{P} \sum_{t=1}^{T} \xi_{t,p} \leq K \right\}. \tag{2.9}$$

The following holds since $\mathcal{U} \subset \mathcal{U}^R$:

$$\min_{\mathcal{U}^R} \left\{ \sum_{p=1}^{P} \overline{inv}_{t,p,g} \right\} \leq \min_{\mathcal{U}} \left\{ \sum_{p=1}^{P} \overline{inv}_{t,p,g} \right\} \quad \forall t, g. \tag{2.10}$$

Replacing $\mathcal{U}$ with $\mathcal{U}^R$ in (2.8) gives us the relaxed inner optimization problem (RIOP). This relaxation allows us to convert the ROP from a BLP to an LP (as we will show below) and effectively reduce the complexity of the problem. In general, due to this relaxation, the real impact of interruptions on inventory levels will be amplified. This is clearly demonstrated in inequality (2.10). An intuitive way of describing this is as follows: with this linear relaxation, the planner no longer perceives the opponent's interruption strategies, ξ, as $0-1$ decisions but continuous decisions in $[0,1]$. The inventory levels obtained by solving the $RIOP$s will be lower bounds for the inventory levels obtained by solving the IOPs. Tightening the uncertainty set using cuts or providing the tightest linear programming relaxation of IOP mitigates the impact of the relaxation. However, it turns out that for this problem, these modifications are not required; the relaxation is actually equivalent to the original problem, because of the following:

Proposition 2.3.1. *The constraint matrix of IOP is totally unimodular (TU).*

The proof of Proposition (2.3.1) is given in Appendix A.1. This property completely cures the side-effects of the relaxation, i.e., all basic feasible solutions of IOPs are integral, which implies that the optimal solutions of $RIOP$ are integral. Moreover, there is no duality gap:

$$\min_{\mathcal{U}^R} \left\{ \sum_{p=1}^{P} \overline{inv}_{t,p,g} \right\} = \min_{\mathcal{U}} \left\{ \sum_{p=1}^{P} \overline{inv}_{t,p,g} \right\} \quad \forall (t,g) \tag{2.11}$$

Therefore,

$$z^*(DRIOP(t,g)) = z^*(RIOP(t,g)) = z^*(IOP(t,g)) \quad \forall (t,g). \tag{2.12}$$

Following Soyster's approach and noting that the problems ROP, IOP and $RIOP$ are all bounded, we replace $\mathcal{U}$ with $\mathcal{U}^R$ in constraints (2.5e) to obtain $RIOP(t,g)$. The explicit form and the dual variables corresponding to the constraints are given below:

$$
\begin{aligned}
RIOP(t,g)\text{: } \min\ & \sum_{p=1}^{P}\left((1-\xi_{t,p})x_{t,p,g}+\sum_{\hat{t}=1}^{t-1}\left((1-\xi_{\hat{t},p})x_{\hat{t},p,g}-w_{\hat{t},p,g}\right)+inv_{0,p,g}\right) & \\
s.t.\ & \sum_{p=1}^{P}\xi_{\hat{t},p}\leq 1\ \ \forall\hat{t} & \text{Dual: }\ \beta_{\hat{t}}^{t,g}\leq 0\\
& \sum_{p=1}^{P}\sum_{\hat{t}=1}^{T}\xi_{t,p}\leq K & \gamma^{t,g}\leq 0\\
& 0\leq\xi_{\hat{t},p}\leq 1\ \ \forall\hat{t},p & \theta_{\hat{t},\hat{p}}^{t,g}\leq 0
\end{aligned}
$$

The objective function of $RIOP(t,g)$ can be written as:

$$
\sum_{p=1}^{P}\left(\sum_{\hat{t}=1}^{t}(-\xi_{\hat{t},p}x_{\hat{t},p,g})+\sum_{\hat{t}=1}^{t}x_{\hat{t},p,g}-\sum_{\hat{t}=1}^{t-1}w_{\hat{t},p,g}+inv_{0,p,g}\right), \tag{2.14}
$$

where

$$
\sum_{p=1}^{P}\left(\sum_{\hat{t}=1}^{t}x_{i,p,g}-\sum_{\hat{t}=1}^{t-1}w_{\hat{t},p,g}+inv_{0,p,g}\right) \tag{2.15}
$$

is a constant from the perspective of $RIOP(t,g)$.

The dual variables (β,γ,θ) are superscripted with t,g since they belong to the problem for given t and g, i.e., one IOP must be solved for each t,g. The explicit form of the dual problem, called $DRIOP(t,g)$, is given below:

$$
\begin{aligned}
DRIOP(t,g)\text{:}\ \max\ & K\gamma^{t,g} + \sum_{\hat{t}=1}^{T}\left(\beta_{\hat{t}}^{t,g} + \sum_{\hat{p}=1}^{P}\theta_{\hat{t},\hat{p}}^{t,g}\right) + \\
& \sum_{p=1}^{P}\left(\sum_{\hat{t}=1}^{t} x_{\hat{t},p,g} - \sum_{\hat{t}=1}^{t-1} w_{\hat{t},p,g} + inv_{0,p,g}\right) \\
s.t.\ \ & \gamma^{t,g} + \beta_{\hat{t}}^{t,g} + \theta_{\hat{t},\hat{p}}^{t,g} \le -x_{\hat{t},\hat{p},g} && \forall \hat{t} \le t,\ \ \forall \hat{p} \\
& \gamma^{t,g} + \beta_{\hat{t}}^{t,g} + \theta_{\hat{t},\hat{p}}^{t,g} \le 0 && \forall \hat{t} > t,\ \ \forall \hat{p} \\
& \gamma^{t,g} \le 0\ \ \beta_{\hat{t}}^{t,g} \le 0\ \ \theta_{\hat{t},\hat{p}}^{t,g} \le 0 && \forall \hat{t}, \hat{p}
\end{aligned}
$$

By weak duality, $z(DRIOP(t,g)) \le z(RIOP(t,g))$ for all t,g since $DRIOP(t,g)$ is a maximization problem and $RIOP(t,g)$ is a minimization problem. By enforcing $z^*(DRIOP(t,g)) \ge 0$ for all t,g, we force the lower bounds on the real worst-case inventory levels to be greater than or equal to zero. By embedding $DRIOP(t,g)$ into the ROP, we obtain the following:

$$
\begin{aligned}
ROP\text{:}\ \min\ & \sum_{p=1}^{P}\sum_{t=1}^{T}\sum_{g=1}^{G} vrx_{t,p,g} \\
\text{s.t.}\ \ & w_{t,p,g} \le \sum_{i=1}^{t} x_{i,p,g} - \sum_{i=1}^{t-1} w_{i,p,g} + inv_{0,p,g} && \forall t,p,g \\
& \sum_{p=1}^{P} w_{t,p,g} \ge \sum_{p=1}^{P} d_{t,p,g} && \forall t,g \\
& x_{t,p,g} \le c_{p,g}^{pro} && \forall t,p,g \\
& K\gamma^{t,g} + \sum_{\hat{t}=1}^{T}(\beta_{\hat{t}}^{t,g} + \sum_{\hat{p}=1}^{P}\theta_{\hat{t},\hat{p}}^{t,g}) +
\end{aligned}
$$

$$\sum_{p=1}^{P}\left(\sum_{\hat{t}=1}^{t} x_{\hat{t},p,g} - \sum_{\hat{t}=1}^{t-1} w_{\hat{t},p,g} + inv_{0,p,g}\right) \geq 0 \qquad \forall t, g$$

$$\gamma^{t,g} + \beta_{\hat{t}}^{t,g} + \theta_{\hat{t},\hat{p}}^{t,g} \;\leq\; -x_{\hat{t},\hat{p},g} \qquad \forall t, g,\;\; \forall \hat{t} \leq t,\;\; \forall \hat{p}$$

$$\gamma^{t,g} + \beta_{\hat{t}}^{t,g} + \theta_{\hat{t},\hat{p}}^{t,g} \;\leq\; 0 \qquad \forall t, g,\;\; \forall \hat{t} > t,\;\; \forall \hat{p}$$

$$\sum_{i=1}^{t} x_{i,p,g} - \sum_{i=1}^{t-1} w_{i,p,g} + inv_{0,p,g} \leq c_{p,g}^{inv} \qquad \forall t, p, g$$

$$x_{t,p,g}, w_{t,p,g} \geq 0 \qquad \forall t, p, g$$

$$\gamma^{t,g} \leq 0 \quad \beta_{\hat{t}}^{t,g} \leq 0 \quad \theta_{\hat{t},\hat{p}}^{t,g} \leq 0 \qquad \forall t, g, \hat{t}, \hat{p}$$

Proposition 2.3.2. *If the ROP is solved to optimality, then the following hold:*

- $DRIOP(t, g)$ *and* $RIOP(t, g)$ *are feasible for all* t, g.
- $z(DRIOP(t, g)) = z(RIOP(t, g)) \geq 0$ *for all* t, g.

Proof. When the ROP is solved to optimality, this implies the feasibility of the ROP. Therefore $DRIOP(t, g)$ is feasible for all t, g [1].

Furthermore, $z(DRIOP(t, g)) \geq 0$ for all t, g, since the opposite would imply that the ROP is infeasible. But $IOP(t, g)$ is feasible for all t, g since $\mathcal{U} \neq \emptyset$, hence all subproblems are primal and dual feasible. By strong duality, all subproblems are solved to optimality. □

[1] Since the constraints of $DRIOP(t, g)$ are contained in those of the ROP.

2.3.5 Robust Methodology in Canonical Form

The approach we have described in the previous sections is in fact suitable for a general class of bi-level programs with following characteristics: (i) the outer problem is an LP, (ii) the inner problems are also LPs with common feasible regions. To clarify this connection, in this section we describe our approach using the canonical LP form. We start with the canonical formulation considering no uncertainty. We consider two subsets of constraints to distinguish the constraints that are directly affected by uncertainty (2.18c) from those that are not (2.18b).

$$\min \quad c^T x \tag{2.18a}$$

$$s.t. \quad Ax \geq b \tag{2.18b}$$

$$h^i(x) \geq 0 \ \ \forall i \in \mathcal{I} \tag{2.18c}$$

$$x \in \mathbb{R}^n_+ \tag{2.18d}$$

Then we introduce the uncertainty by reformulating constraints (2.18c) as inner problems (2.19c) which transforms the initial formulation to a bi-level problem:

$$\min \quad c^T x \tag{2.19a}$$

$$s.t. \quad Ax \geq b \tag{2.19b}$$

$$\min_{\xi \in \mathcal{U}} \{f^i(x, \xi)\} \geq 0 \ \ \forall i \in \mathcal{I} \tag{2.19c}$$

$$x \in \mathbb{R}^n_+ \tag{2.19d}$$

where

$$\mathcal{U} = \{\xi \in \mathbb{R}^p_+ | H\xi \geq r\}, \tag{2.20a}$$

$$f^i(x, \xi) = x^T G^i \xi + x^T d^i + \xi^T (q^i) \;\; \forall i \in \mathcal{I}. \tag{2.20b}$$

In (2.20a), H is the constraint coefficient matrix for the inner problems; in (2.20b), G^i are the matrices; and in (2.20a), d^i and (q^i) are constant vectors of dimension n and p respectively. These help us represent the bi-linear form $f^i(x, \xi)$ in a compact manner. Note that we assumed that, when reformulated using the inner problem variables, constraints (2.18c) will be written as the optimization problem (2.19c). The objective function in bi-linear form is given in (2.20b) and is linear from the perspective of the inner problem since the x variables are fixed. Therefore the inner problems are convex optimization problems with linear objectives, which implies the optimality conditions of the inner problems are all linear. The inner problem i is:

$$\min \quad \left(x^T (G^i) + (q^i)^T\right) \xi \tag{2.21a}$$

$$s.t. \quad H\xi \geq r \quad Dual\ Var.: \quad \lambda^i \geq 0 \tag{2.21b}$$

$$\xi \in \mathbb{R}^p_+ \tag{2.21c}$$

We first take the dual of each inner problem i,

$$\max \quad r^T \lambda^i \tag{2.22a}$$

$$s.t. \quad H^T \lambda^i \leq x^T (G^i) + (q^i)^T \tag{2.22b}$$

$$\lambda^i \in R_+^m \tag{2.22c}$$

and enforce the dual feasibility conditions in the outer problem. By weak duality, $\left(G^T x\right)^T \xi \geq r^T \lambda$, and the outer problem becomes

$$\min \quad c^T x \tag{2.23a}$$

$$s.t. \quad Ax \geq b \tag{2.23b}$$

$$x^T d^i + r^T \lambda^i \geq 0 \quad \forall i \in \mathcal{I} \tag{2.23c}$$

$$H^T \lambda^i \leq x^T (G^i) + (q^i)^T \quad \forall i \in \mathcal{I} \tag{2.23d}$$

$$\lambda^i \in R_+^m \quad \forall i \in \mathcal{I}, \quad x \in R_+^n \tag{2.23e}$$

2.3.6 Numerical Example

Consider an instance of the ROP with 2 plants, 7 time periods and 2 products. The production capacity is $c_{p,g}^{pro} = 5 \cdot 10^5$ and the inventory capacity is $c_{p,g}^{inv} = 10^6$. $K = 3$ interruptions are expected. Since we assumed a fixed rate for electricity, we solve the numerical instances of ROP with cost vectors where all the elements of the vector are identical to the fixed rate. The production cost times the conversion factor is 1, i.e., $vr = 1$. However, the ROP is general enough to handle the case where the cost rate of electricity changes for each time period. The demand data and the optimal solution of the ROP are given in Table 2.3. Since we assumed the plants are identical, in Table 2.3 we observe identical production levels for both plants. Furthermore, we also observe identical production levels for all time periods, but this behavior is not observed for all data sets. We tested these solutions under all possible interruption scenarios and confirmed that, indeed, the production plan is feasible, i.e., no stock-outs occur. The behavior of the solution under two sample interruption scenarios is given in Tables 2.4 and 2.5. In the first scenario (Table 2.4), plant 1 is interrupted three times consecutively at the beginning of the horizon. In the second scenario (Table 2.5), plant 1 is interrupted in periods 5 and 7, and plant 2 is interrupted in period 6. In the interrupted time periods, the planned production levels for both products are replaced with 0 and the inventory pool levels for each time period/product are calculated using the updated production levels. The ROP provides a production plan such that under any interruption scenario, the inventory pool levels will be always non-negative. If the number of interruptions in a scenario is less than the anticipated level of interruptions, K, the ending inventory pool level will be positive unless there is at least one period where the demand is 0. Since we assumed identical plants, we have reported the robust solution that provides identical production levels for both plants. In general there can be multiple robust solutions and cases where the robust optimal model assigns different production levels to different plants depending on the constraints.

Data	Total Demand	
Period	Prod. 1	Prod. 2
1	78337	22086
2	113422	21967
3	172944	42249
4	122049	55444
5	147796	34464
6	140045	38057
7	129098	35711

Table 2.2: Demands for Products

Solution	Product 1		Product 2	
Period	Plant 1	Plant 2	Plant 1	Plant 2
1	63972	63972	4544	4544
2	63972	63972	4544	4544
3	63972	63972	4544	4544
4	63972	63972	4544	4544
5	63972	63972	4544	4544
6	63972	63972	4544	4544
7	63972	63972	4544	4544

Table 2.3: Optimal Robust Solution

2.3.7 Robust Production Planning Heuristic

Depending on the magnitude of the input sets, the ROP may be a very large problem. For example, if there are 10 products, 10 plants, and 100 periods, the ROP has approximately one million variables and one million constraints. This problem may be difficult to solve exactly. Therefore, in this section we propose a heuristic that mimics the solution of the ROP. This heuristic is a variant of list scheduling augmented with an auxiliary linear program and applies only to the special case in which the plants are all identical. It also assumes that the production cost is fixed in all periods and the same at all plants. The heuristic evaluates the damage that would be caused by an interruption on each day, using each day's demand

Product 1					
Period	Plant 1	Plant 2	Total Production	Inventory Pool	Demand
0	-	-	-	200000	-
1	0	63972	63972	185635	78337
2	0	63972	63972	136185	113422
3	0	63972	63972	27213	172944
4	63972	63972	127944	33108	122049
5	63972	63972	127944	13255	147796
6	63972	63972	127944	1154	140045
7	63972	63972	127944	0	129098
Product 2					
Period	Plant 1	Plant 2	Total Production	Inventory Pool	Demand
0	-	-	-	200000	-
1	0	4544	4544	182458	22086
2	0	4544	4544	165034	21967
3	0	4544	4544	127328	42249
4	4544	4544	9087	80972	55444
5	4544	4544	9087	55594	34464
6	4544	4544	9087	26624	38057
7	4544	4544	9087	0	35711

Table 2.4: Scenario 1: 3 consecutive interruptions at the beginning of the horizon at plant 1

and its position in the horizon. Note that interruptions earlier in the horizon are more dangerous since early interruptions give the plants less time to build up their inventory. Therefore, the heuristic tries to “front-load” the production schedule to anticipate the most problematic interruptions.

We have assumed each product is subject to individual production and inventory capacity constraints at each plant; however, this might not need to be true in general. There might be conditions that affect the entire set of products such as

Product 1					
Period	Plant 1	Plant 2	Total Production	Inventory Pool	Demand
0	-	-	-	200000	-
1	63972	63972	127944	249607	78337
2	63972	63972	127944	264129	113422
3	63972	63972	127944	219128	172944
4	63972	63972	127944	225023	122049
5	0	63972	63972	141199	147796
6	63972	0	63972	65126	140045
7	0	63972	63972	0	129098

Product 2					
Period	Plant 1	Plant 2	Total Production	Inventory Pool	Demand
0	-	-	-	200000	-
1	4544	4544	9087	187001	22086
2	4544	4544	9087	174121	21967
3	4544	4544	9087	140959	42249
4	4544	4544	9087	94602	55444
5	0	4544	4544	64682	34464
6	4544	0	4544	31168	38057
7	0	4544	4544	0	35711

Table 2.5: Scenario 2: Plant 1 is interrupted in periods 5 and 7, and plant 2 is interrupted in period 6

joint inventory and capacity constraints:

$$\sum_{g \in \mathcal{G}} x_{t,p,g} \leq c^{pro} \quad \forall t, p \quad \text{(joint capacity)} \tag{2.24}$$

$$\sum_{g \in \mathcal{G}} inv_{t,p,g} \leq c^{inv} \quad \forall t, p \quad \text{(joint inventory)} \tag{2.25}$$

The formulation given for ROP does not contain constraints of these types, therefore one can effectively replace the given formulation with a set of formulations, each for a single product. In Algorithm 1, we describe a heuristic that can be applied when the outer problem is separable in terms of products.

Our heuristic consists of two steps. In the first step, the heuristic calculates, for each period t, the aggregate demand faced so far by all plants, represented by the non-decreasing sequence $\{a_t\}$. Next, for each period t, the heuristic calculates the number of available production slots in periods $1, \ldots, t$, assuming that the maximum possible number of interruptions occur by time t (at most one per period). This quantity is represented by the increasing sequence $\{s_t\}$. Up to time period $t = K$ there can be one interruption per time period, hence $s_1 = 1(p-1), s_2 = 2(p-1), \ldots, s_K = K(p-1)$. For time periods $t > K$, $s_t = tp - K$, so the number of available production slots for each time period t can be characterized as $s_t = tp - \min\{t, K\}$. Next, for each time period t, the heuristic calculates the "average production" required on each available plant to be able to satisfy the running total demand a_t and characterizes it as the sequence $\{l_t\}$ where $l_t = \dfrac{a_t}{s_t}$. Then, the heuristic determines the period t^* with the largest l_t value, in an attempt to detect the period in which infeasibility is most likely, accounting for the timing of the interruptions and the quantity of the demands.

In the second step, the heuristic compares t^* to K. If $t^* \leq K$ then it assigns the production levels as l_{t^*} at each plant for all time periods up to t^*. Then it updates initial inventory as $inv_0 + t^*(p-1)l_{t^*} - a_{t^*}$, which is calculated assuming t^* interruptions happened so far, therefore it updates the number of interruptions as $K - t^*$. If $t^* > K$ then again it assigns the production levels to l_{t^*} at each plant for all time periods up to t^*. But this time, it updates the initial inventory as $inv_0 + (t^*p - K)l_{t^*} - a_{t^*}$. Finally, it updates the set of time periods as $\mathcal{T} : \{ord(t^*+1), \ldots, (T)\}$, where $ord(t^*+1) = 1, \ldots, ord(T) = T - t^*$. After this step the heuristic goes back to step one and continues recursively until $K = 0$. When there are no interruptions left, it solves the standard production planning problem (2.2a)-(2.2g) and merges its solution into the production plan. This heuristic takes at most K iterations, where

Algorithm 1 Robust Production Planning Heuristic: Single Product

Require: Indices, Sets and Parameters as defined in Table 2.1

STEP 1

$a_t := \sum_{i=1}^{t} d_i \ \ \forall t \in \mathcal{T}$

$s_t := pt - \min\{t, K\} \ \ \forall t \in \mathcal{T}$

$l_t := \frac{a_t}{s_t} \ \ \forall t \in \mathcal{T}$

$t^* = arg \max_{t \in \mathcal{T}} l_t$

STEP 2

if $t^* \leq K$ **then**

$\forall t \leq t^*, \forall p \ x_{t,p} \leftarrow l_{t^*}$

$inv_0 \leftarrow inv_0 + t^*(p-1)l_{t^*} - a_{t^*}$

if $\max_{t,p} x_{t,p} > c_p^{pro}$ OR $\exists \hat{t} \in \{1, \ldots, t^*\}, \ \hat{p} \in \mathcal{P} \ \ s.t. \ \ inv_{\hat{t},\hat{p}} > c_{\hat{p}}^{inv}$ **then**

Declare Failure and HALT

end if

$K \leftarrow K - t^*$

$\mathcal{T} \leftarrow \{ord(t^* + 1), \ldots, ord(T)\}$

Go to STEP 1

else

$\forall t \leq t^*, \forall p \ x_{t,p} \leftarrow l_{t^*}$

$inv_0 \leftarrow inv_0 + (t^* p - K)l_{t^*} - a_{t^*}$

$K \leftarrow 0$

$\mathcal{T} \leftarrow \{ord(t^* + 1), \ldots, ord(T)\}$

Update parameters in Table 2.1 according to $\mathcal{T}$

Solve Standard Production Planning Problem (2.2) to obtain:

$y_{t,p} \ \forall t \in \mathcal{T}, p$ (the optimal solution)

if $\max_{t,p} x_{t,p} > c_p^{pro}$ OR $\exists \hat{t} \in \{1, \ldots, t^*\}, \ \hat{p} \in \mathcal{P} \ \ s.t. \ \ inv_{\hat{t},\hat{p}} > c_{\hat{p}}^{inv}$ OR Problem (2.2) is Infeasible **then**

Declare Failure and HALT

end if

$\forall t > t^*, \forall p \ x_{t,p} \leftarrow y_{t,p}$

end if

each iteration consists of the aforementioned two steps. In step 2 of every iteration, the algorithm compares $x_{t,p}$ values to c_p^{pro} and inventory levels to c_p^{inv}. If $x_{t,p} > c_p^{pro}$ or $inv_{t,p} > c_p^{inv}$ then the algorithm declares failure and terminates. Because of this termination rule, by construction every solution built by this heuristic is a feasible solution to ROP. The heuristic is summarized in Algorithm 1.

For the data given in Table 2.3, our heuristic calculates the exact optimal solution for the robust problem, as given in Table 2.3. Further numerical results are reported in §2.3.8, and again, the heuristic found the optimal solution in all instances tested. While our numerical results are hopeful on the optimality of this heuristic, we were unable to prove or disprove the optimality of the heuristic. Hence it stays as our conjecture that Algorithm 1 is actually an exact algorithm for finding the optimal solution of the ROP under the following conditions: (i) Products are not jointly constrained, i.e., ROP is separable in products, (ii) Plants are identical, (iii) Production costs and capacities are fixed throughout the horizon, (iv) Inventory capacities are fixed throughout the horizon. In all of the instances that we have tested, the heuristic either found the optimal solution or declared failure in the infeasible instances.

2.3.8 Computational Results

In this section, we report the results of a computational study designed to test the effectiveness of the robust model (ROP) as well as the heuristic presented in §2.3.7. Tables A.1 and A.2 summarize the computational results for a setting with 2 Plants, 5 Demand Patterns, $inv_0 = 2 \cdot 10^5$ and various time periods such as:

$$T \in \{5 \cdot 2^0, \ldots, 5 \cdot 2^5\}$$

Experiments are conducted to observe the behaviour under 20% and 40% interruption rates. For each instance type, i.e., a horizon length and an interruption rate, we created 5 random instances in which the demands are generated as $d_t \sim unif(0, 3c^{pro})$. Costs and capacities are as defined in the numerical example given in §2.3.6. We modeled all of the problems using AMPL and solved them to optimality using the solver Gurobi 4.0. The heuristic was implemented in Matlab r2010a.

The first column reports the instance name in the form T.di, where $i = 1, \ldots, 5$ is the random demand pattern. The next column reports the total demand for that instance. The table then lists, for the 20% interruption rate ($K = 0.2T$), the optimal objective value (found by Gurobi) and the associated CPU time and the objective value of the solution found by the heuristic (Algorithm 1) and the associated CPU time. The column labeled "Δ" reports the optimality gap, where $\Delta = |z(ALG1) - z(ROP)|$. The column labeled "Ψ" reports the ratio of total production to total demand (expressed as a percentage); that is, $\Psi = \frac{\text{Total Production}}{\text{Total Demand}}$. The last set of columns repeats this information for the 40% interruption rate.

Our heuristic found the optimal solution for the ROP for every instance (within the tolerance of $\Delta \leq 1$). Moreover, on average it executes an order of magnitude ($10\times$) faster than solving the ROP directly with Gurobi. Note also from Tables A.1 and A.2 that Ψ is greater for $K = 0.4T$ than for $K = 0.2T$; that is, as the number of interruptions increases, so does the total production. This result is also displayed in Figure 2.2, which plots Ψ for $K = 0.2T$ (lower point) and for $K = 0.4T$ (upper point) for each instance.

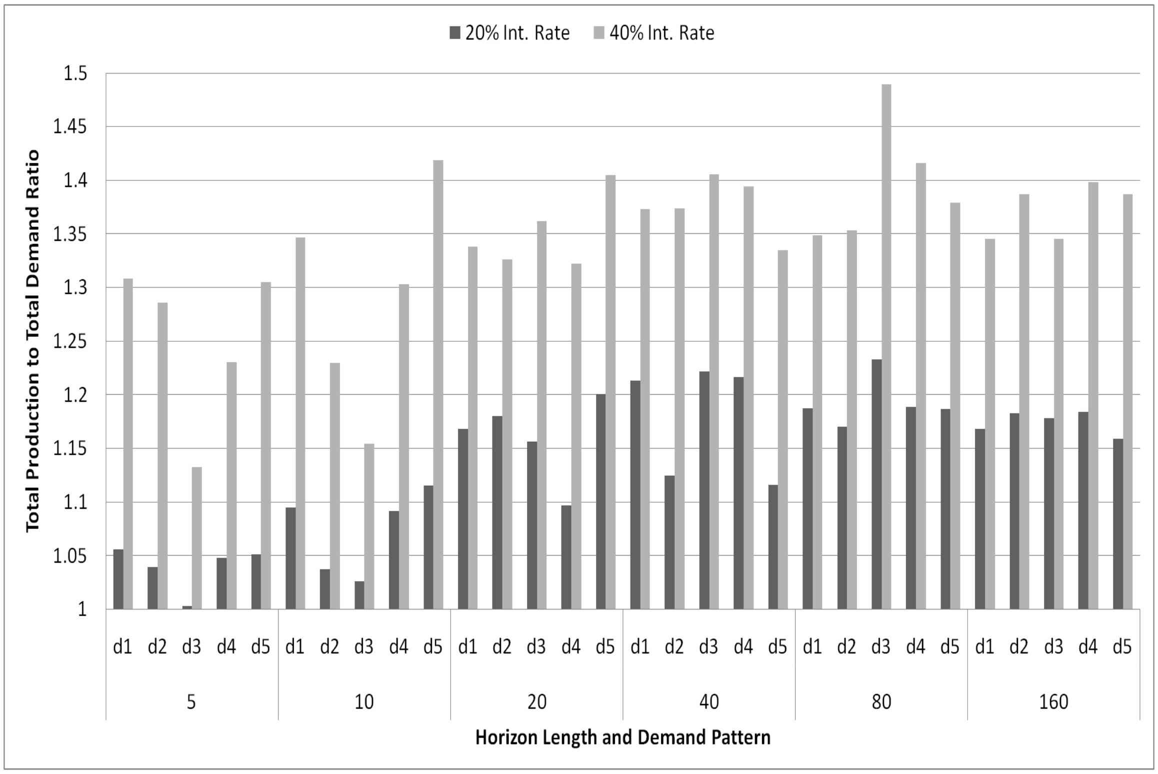

Figure 2.2: Total Production to Total Demand Ratio (Ψ) from Tables A.1 and A.2

2.4 Production Modes

So far, we have assumed a very simple form for the interruptions and the firm's reaction to them. However, interruptions might have more complicated effects on production, at more than just the interrupted plant, and/or in more than just the interrupted period. In this section, we show how to embed operational rules that govern how the system operates during or after interruptions into the ROP. Each plant may operate in various *production modes* that are governed by the operational rules. In §2.3, we considered only the simplest possible operational rule (no production is allowed at interrupted plants) and only two production modes (interrupted and unaffected).

For example, consider the following operational rule: Once a plant recovers from an interruption, for one period the plant is in “recovery mode” in which its production rate is temporarily reduced. This rule may be imposed, for example, to give the interrupted plant time to ramp its production back to normal. (Such an operational rule is imposed for the air-separation plants that motivated this study.) This rule induces 3 production modes: interrupted, recovery and unaffected. The application of this logical rule for a given interruption scenario is depicted in Figure 2.3.

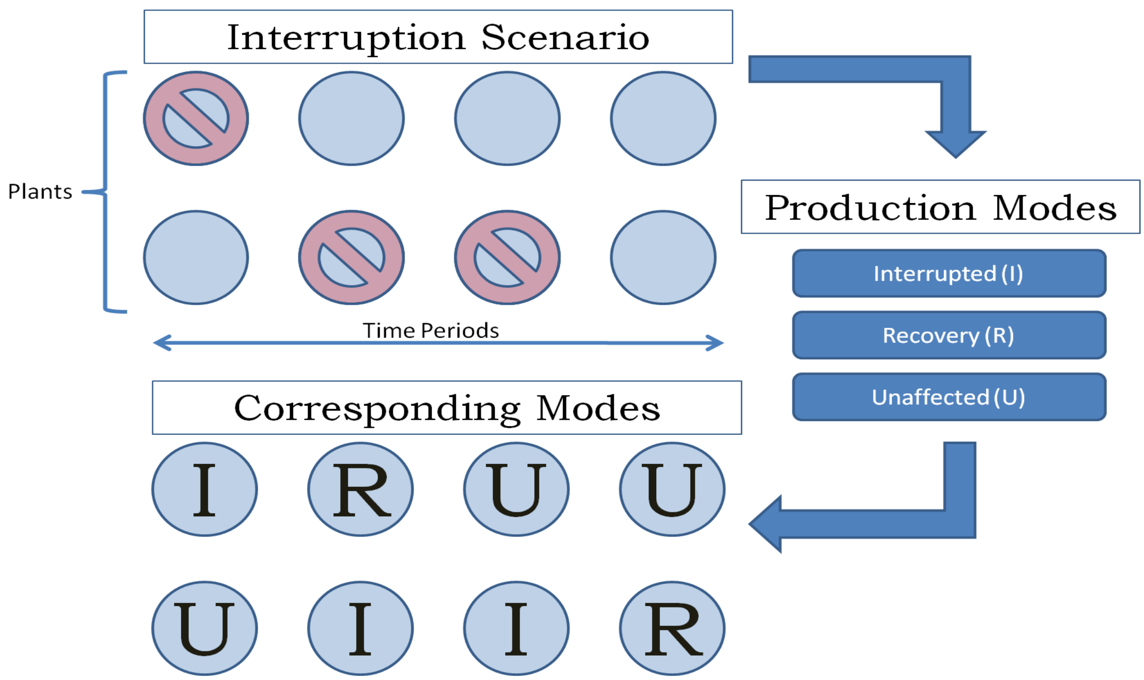

Figure 2.3: Interruptions and Associated Production Modes

We attempt to provide as general a framework as possible for modeling operational rules. Let $\mathcal{M}$ denote the set of production modes, and let

$$\xi_{m,t,p} = \begin{cases} 1, & \text{if plant } p \text{ is in production mode } m \text{ in period } t \\ 0, & \text{otherwise} \end{cases}$$

for $m \in \mathcal{M}$, $t \in \mathcal{T}$, $p \in \mathcal{P}$. The ξ variables indicate which production mode each plant is in, and they are a generalization of the ξ variables in 2.3.4. A production mode is defined by its effect on the production rate; in particular, let θ_m be the fraction of the normal production rate that a plant experiences when in production mode m. If $\theta_m = 0$, the production is completely interrupted; if $\theta_m = 1$, the plant is functioning normally; and if $0 < \theta_m < 1$, the plant is operating at a reduced rate. The θ parameters may be indexed by p and t if they are plant- and/or period-dependent, but for ease of exposition we assume they depend only on m. Next, we relate the actual production to the planned production levels and the production modes, in a generalization of (2.6):

$$\bar{x}_{t,p,g} = \sum_{m \in \mathcal{M}} \theta_m \xi_{m,t,p} x_{t,p,g} \quad \forall t, p, g$$

Finally, the operational rules that govern the production modes must be represented as linear constraints on the ξ variables, and these constraints must be added to the uncertainty set $\mathcal{U}$.

To take an example, consider the "recovery mode" outlined above, and suppose that a plant in recovery mode experiences half its normal production rate. Then we have $\mathcal{M} = \{I, R, U\}$, $\theta_I = 0$, $\theta_R = 0.5$, and $\theta_U = 1$. The operational rules can be enforced by including the following constraints in $\mathcal{U}$:

$$\xi_{I,t,p} + \xi_{R,t,p} + \xi_{U,t,p} = 1 \quad \forall t, p \tag{2.26a}$$

$$\xi_{R,t,p} \geq \xi_{I,t-1,p} - \xi_{I,t,p} \quad \forall t, p \tag{2.26b}$$

$$\xi_{R,t,p} \leq \xi_{I,t-1,p} \quad \forall t, p \tag{2.26c}$$

Constraints (2.26a) require each plant to be in exactly one recovery mode in each period. Constraints (2.26b-2.26c) require plant p to be in recovery mode in period t if and only if it was interrupted in period $t-1$ but it is not interrupted in period t. This approach for embedding operational rules into the ROP maintains the tractability of the problem.

As long as the opponent's decision space ($\mathcal{U}$) does not depend on the outer problem variables, i.e., the production variables x, and the opponent's objective function can be expressed as a bi-linear function of outer problem variables and inner problem variables, our methodology can continue to be used to combine the outer and inner problems into one linear program. In contrast, if the opponent's feasible region depends on the outer variables, then the resulting model will be nonlinear in general, although this may not be insurmountable. The inner problems can be reformulated as a set of optimality conditions, still one can cast the BLP as a single non-linear program and choose the appropriate solution method depending on the structure of this new non-linear program. Moreover, if the inner problems are non-convex optimization problems, then this approach is not appropriate since the optimality conditions of the inner problems are necessary but not sufficient for inner problem optimality.

2.5 Conclusion

In this chapter, we present a production planning framework for a rate-paying industrial production company whose production operations strongly depend on electricity. The problem we study is an operational-level, aggregate production and inventory planning problem with electricity supply uncertainty and deterministic demand. We assume that participation in an ILC provides a discounted and fixed rate to the production company, which effectively mitigates the negative impact of electricity price volatility but introduces supply uncertainty into the production system in the form of interruptions. Our robust production planning model accounts for this electricity supply uncertainty. In this model, we separate production decisions and interruption decisions, since production decisions belong to the industrial company while interruption decisions belong to the electricity retailer. The model can be solved using standard optimization techniques and software, but we also developed a heuristic that attempts to mimic the solution of the robust optimization model for a special case. In our computational experiment, our heuristic found the optimal solution for every instance approximately ten times faster than the optimization approach.

The interruption uncertainty framework we describe allows different contract rules and operational rules to be embedded into the production planning problem simultaneously. As we discussed in §2.4, it is straightforward to embed operational procedures that companies may implement in the case of interruptions, such as limiting the production in post-interruption recovery or prohibiting production level increases in some periods. Similarly, our framework could be used under different types of ILCs, such as the pay-in-advance and pay-as-you-go reward schemes described by Baldick et al. [2006]. Moreover, information regarding the utility's optimal interruption dispatch behavior can be embedded into our Stackelberg-like production planning framework. However, the extent to which the theoretical results and computational performance presented above will be preserved under different ILC types or interruption dispatch behaviors is a topic for future study. An important future avenue for study is to include demand uncertainty into the model.

Chapter 3

Empirical Analysis

3.1 Introduction

Our contribution in this chapter is an experimentation effort using real prices from the real-time markets in ISO New England (ISONE) and The PJM Interconnection, L.L.C. (PJM) regional transmission operators (RTOs). Throughout this dissertation, we use the terms “RTO” and “real-time market” interchangeably. The objective of this experiment is to test the for inherent patterns in real-time locational marginal prices (LMPs) that could be used for constructing the uncertainty sets for the optimization problems which are studied in §4. First, the data is visually described and LMP data from PJM and ISONE is analyzed using daily and weekly price curves. The aggregate statistics, the correlation between hourly prices, and the dependence and stationarity of hourly price series are analyzed using different time scales such as days, weeks, months and years. Next the data is analyzed for price spikes using spike detection thresholds. Finally, an experiment is conducted to test the accuracy of dynamically fit ARIMA models for price forecasts. The ARIMA models are fit using rolling time windows with different horizon lengths since one of the objectives of this study to test the information content of different horizons from the perspective of the optimization problem. Two different horizon selection rules are compared using five horizon lengths and various error measures.

The results show that daily price profiles in both markets are significantly time dependent. Short term data is found to be a good candidate for describing daily and weekly patterns. The one-step difference hourly price time series[1] are found to be stationary in both markets. Intraday prices and same-hour inter-day prices are found to be highly correlated; however, the two markets show different patterns in terms of intraday price correlation. Dynamically fit ARIMA models with short time horizons as input are found to be appropriate for forecasting daily prices, and the experimental results show that for a fixed list of horizons, the ARIMA models that would be fit will be of different parameters with different horizon lengths. It is also observed that for short term horizons, ARIMA models with constant mean terms are fit while for longer horizons ARIMA models with drift terms were fit.

3.1.1 Electricity Markets

We begin by providing an overview of RTOs and the markets.

PJM

Founded in 1927, PJM, is an independent, federally regulated RTO headquartered at Valley Forge, Pa. Prior to beginning of operation of the competitive wholesale market in 1997, PJM LLC [a] was a vertically integrated entity. It covers an area of 214,000 square miles that contains all or parts of Delaware, Indiana, Illinois,

[1] We slice the price data into multiple time series using different horizons.

Kentucky, Maryland, Michigan, New Jersey, North Carolina, Ohio, Pennsylvania, Tennessee, Virginia, West Virginia and the District of Columbia, with 6,145 substations and 61,591 miles of transmission lines. It has an installed generation capacity of 185,600 MW. PJM operates a centrally dispatched, competitive wholesale electric power market which has more than 700 participants (electricity buyers, sellers and traders) and serves about 60 million customers. According to State of the Market Reports [PJM LLC, e] in the first three months of 2011, PJM had total billings of $9.58 billion.

For prior history of FERC Regulations and PJM Markets refer to PJM LLC [b]. PJM received full RTO status from the Federal Energy Regulatory Commission (FERC) in December 2002. As an RTO, PJM meets independence, scope and regional configuration, operational authority and short term reliability characteristics required by FERC. According to PJM LLC [b], as an RTO, PJM provides the following functions:

- Tariff administration and design
- Congestion management
- Parallel path flow
- Ancillary services
- OASIS, Total Transmission Capability (TTC) and Available Transmission Capability (ATC)
- Market monitoring
- Planning and expansion

- Inter-regional coordination

PJM operates the energy market, the capacity market, the ancillary services market, and the Financial Transmission Rights (FTRs) market to fulfill its functions as an RTO [PJM LLC, c],[PJM LLC, d].

ISONE

ISONE was created by FERC in 1997 to oversee the restructuring of New England's wholesale electric power industry. At the time the New England Power Pool (NEPOOL) was facilitating the cooperation and the coordination of vertically integrated market participants, mainly private and municipal utilities that worked together to ensure electricity supply dependability, within six states: Maine, New Hampshire, Vermont, Massachusetts, Connecticut and Rhode Island. With the support of the New England Conference of Public Utilities Commissioners (NECPUC) and the New England Power Pool (NEPOOL), ISONE was found as an autonomous and financially independent system operator to administer the whole power market [ISONE, a],[ISONE, b].

ISONE implemented wholesale markets and it serves an area of 128,000 square miles, with more than 350 generators, 32,000 MW of generating capacity and 8,000 miles of transmission lines. ISONE wholesale electric power market has more than 500 participants (electricity buyers, sellers and traders) and serves about 6.5 million households and businesses (a population of 14 million people).

In 2003, ISONE added new markets and services per adoption of the Standard Market Design. During the August 2003 system failures, New England's grid stayed operational while much of the Northeast, Midwest and Canada was affected severely. In 2005, FERC designated ISO New England as the regional transmission organization for the six-state region. In this role, ISO New England continues to fulfill its responsibilities, but with broader authority over the day-to-day operation of the transmission system and greater independence to manage the power grid and wholesale markets.

In 2010, a volume of $9.1 billion was traded in wholesale electricity markets which includes the energy market, the capacity market, and the ancillary services market.

Energy

The role of PJM in the energy market is to coordinate and monitor market participant's electricity trade and delivery activities to provide open, fair and equitable access. LMPs are used to valuate the energy in a way that the time and location of the delivery is accounted for. LMP methodology combines the system-level energy cost, the congestion and marginal losses. Given two different locations with different levels of congestion, the LMP is higher in the congested area. There are two components of the Energy Market: Day-Ahead markets (forward market) and Real-Time markets (spot market). Day ahead market participants submit their hourly generation offers, demand bids and scheduled transactions to the auction for the next day to the market operator, and the market operator balances the prices and the transmissions. The next day, according to the operating conditions of the transmission systems and the actualized prices, current LMPs are calculated and published in five-minute intervals. The transactions are settled hourly. The market participants also have the option to participate in two-settlement markets where participants submits their generation/demand bids and scheduled transactions in the day-ahead market. Then they settle in the spot market based on deviations of the real time prices from the day-ahead positions. PJM also operates a Day-Ahead Scheduling Reserve Market where supplemental, thirty-minutes reserves that may be needed to deal with unanticipated system conditions during the actual operating day. The reserves are traded in a forward fashion.

The role of ISONE in the energy market is to coordinate the commitment and dispatch of generation and demand resources and facilitate electric energy trading to provide open, fair and equitable access. Similar to PJM, it consists of the Day-Ahead Market and the Real-Time Market. Locational marginal pricing is used to set the prices on the power grid in a way that congestion, transmission loss and electricity production costs are accounted for. The day-ahead clearing price is determined through an auction process where market participants submit their hourly generation offers, demand bids and scheduled transactions to the auction. Starting from the lowest price, generators are selected until the necessary supply to meet the demand and contingencies is committed. The price offered by the last chosen generator is set as the "wholesale clearing price" and all previously selected generators are awarded the clearing price. Day ahead markets and two-settlement markets let the market participants hedge against real-time price fluctuations.

For Day Ahead Markets, ISONE publishes a weekly report containing descriptive statistics for Locational Marginal Prices (LMPs) at the Hub, Load Zones, and External Nodes; a graph comparing day-ahead cleared demand, day-ahead cleared MWs (defined as the sum of cleared fixed demand, price sensitive demand and virtual bids, minus cleared virtual offers), forecast load, and actual load; and a graph that compares the variable production costs (based on fuel costs) for hypothetical gas and oil plants with the energy component of the LMP. For Real Time Markets, it publishes information about bid-in and cleared demand, virtual demand and virtual supply, as well as exhibits showing hourly day-ahead LMPs for the Hub, the eight Load Zones, and the five External Nodes [ISONE, c].

Capacity Market

The capacity markets exists to ensure the availability of resources to maintain regional power grid reliability. In this market, Reliability Pricing Model (RPM) which is developed by PJM, is implemented. In this model utilities and electricity suppliers are required to have necessary and adequate generation resources (capacity) to meet customer demand and safety reserves three years before it is needed. It also includes incentives to motivate development and deployment of Demand Response (DR) and energy efficiency programs. Load serving entities (LSEs) and generators can fulfill these requirements through capacity obtained through contracts, auctions or generating capacity owned/installed. All these elements are included in the RPM in such a way that locational transmission and capacity constraints are accounted for.

Similarly, the ISONE Capacity Market provides the necessary environment to ensure that enough supply exists to satisfy regional reliability and contingency requirements[2]. ISONE holds an annual auction for the projected capacity requirements for the next three years. This provides a means to include long-term signals in investment decisions for new generation and demand resources. An uncommon feature of this market is the fact that it allows bids from demand-side resources along with power plants for the necessary supply requirements.

[2]Reliability Requirements and Contingency Requirements differ in the sense that former is designed to ensure additional reserves that are required to ensure day-to-day smooth operation while the latter is designed to mitigate contingencies such as blackouts, line failures, . . ., etc.

Ancillary Services

In PJM, ancillary services encompasses two markets which operate to ensure and support the reliability of the transmission system: regulation services and synchronized reserve services. Regulation service provides the necessary mechanism to correct the short-term adjustments required for the stability of the power system. Matching generation and load along with output frequency adjustments are two mechanisms that are provided by this service. Synchronized reserve service provides the necessary mechanisms to provide synchronized emergency power on short notice. LSEs can meet their obligation to provide both services by in-house generation, by bi-lateral contracts and by market purchases.

In ISONE, ancillary services operate to ensure and support the reliability of the transmission grid where unforeseen transmission line and power plant failures may cause catastrophic failure. Furthermore, the minute-to-minute balance of electricity flow partially depends on these systems. Forward and Real-Time Operating Reserves provide an additional layer of protection of reliability through access to resources that can be called on quick notice. It also include reserves that can be acquired through Demand Side Management (DSM) and Demand Response (DR) programs that allow inclusion of the locational component of the price to address heavy demand and congestion. System frequency control is achieved through regulation of power plant output, and voltage support services provide a necessary means to maintain transmission voltages within safe limits. Some specific power plants provide black start capability for restarting the system after system-wide blackouts.

Financial Transmission Rights

Financial Transmission Rights (FTRs) help market participants hedge their price risk at the time of delivery. The holder of the FTRs has a right to collect a fee according to hourly energy-price differences across a transmission path in the Day-Ahead Market. In the PJM market, there are three types of FTR auctions: long-term (1-3 years), annual and monthly. FTRs can also be bought from a secondary market.

3.2 Literature Review

For select articles on the market characteristics for various energy markets, testing market structures, auctioning and bidding strategies, operational strategies, forecasting prices and modeling the risk of power portfolios, refer to Sorokin et al. [2012a], Sorokin et al. [2012b]. For similar studies to ours, see Longstaff and Wang [2004], Shawky et al. [2003], Popova [2004].

Future prices are required to conduct a complete analysis from the financial perspective. However the objective of this chapter to conduct an analysis which will be used to construct uncertainty sets for operational level optimization problems. More importantly the impact of the price uncertainty on the final operational cost will be measured through deviations in the optimal objective function value.

3.3 Data & Empirical Analysis

In this section, hourly Real-Time LMPs in PJM, [PJM LLC, f], and ISONE, [ISONE, d], are described, analyzed and compared. To be consistent with the production planning model, daily and weekly scopes are used. ISONE real time LMPs (prices) covers the interval from 2003-05-03 to 2011-10-06 (452 weeks) while PJM prices covers the interval from 2004-05-03 to 2011-10-30 (392 weeks). One of our objectives is to investigate the impact of using this information to structure the price uncertainty sets for the optimization model and compare it to the cases where more granular information can be acquired and used in a streaming manner.

All computations are done in CRAN-R from [R Development Core Team, 2011] and the following packages:
data.table [Dowle et al., 2012], *plyr* [Wickham, 2011],
forecast [Rob J. Hyndman and Schmidt, 2012],
fields [Furrer et al., 2012], *matrixStats* [Bengtsson et al., 2011],
lubridate [Grolemund and Wickham, 2011], *timeDate* [Wuertz et al., 2011],
reshape [Wickham and Hadley, 2007], *diagnostic* [Zeileis and Hothorn, 2002],
zoo [Zeileis and Grothendieck, 2005], *xts* [Ryan and Ulrich, 2011],
tseries [Trapletti and Hornik, 2011], and *timeSeries* [Wuertz and Chalabi, 2011].

All tables and plots are made using:
xtable [Dahl, 2012], *corrplot* [Wei, 2011],
ggplot2 [Wickham, 2009] and *gridExtra* [Auguie, 2012].

The data can be framed using different time perspectives, such as yearly, seasonal, monthly, weekly and daily data views. In this study, weekly and daily views are found more appropriate since the optimization problems that are used are usually instantiated using weekly and daily horizons. Any systematic component which comes into effect in time intervals larger than a week is irrelevant from the perspective of the optimization model instance. Hence the data is tested for stationarity, independence and systematic components only for daily and weekly perspectives.

3.3.1 Aggregate Statistics

Hourly statistics aggregated over all years are given in Table 3.1 and depicted in Figure 3.1. Tables B.1 and B.2 provide the hourly statistics for ISONE and PJM markets from the weekly perspective and can be found in Appendix B. Both markets have similar hourly LMP profiles when means and quantiles are considered. The negative values in minimum prices for the early hours in the day might be evidence for some generation inflexibility. When power plants have high start-up and shut-down costs, or due to system requirements, the operators might settle for negative LMPs, which means the power generators are paying the market to keep the generators remained on. This does not occur in the ISONE market. Prices in the ISONE market are higher in general, and price differences in the two markets are most pronounced in the early hours of the day. However, during the peak hours the gap becomes less than 4 %. The volatility in prices is slightly higher in the ISONE market but for hours 12, 13, 14, 18 and 19 the volatility difference in the two markets is less than 5 %. The daily pattern is clear in the aggregated data.

Stationarity

Let $\{P_{d,h}\}$ be the price process indexed on days and hours. For testing stationarity, we used the original price process and a set of subprocesses that we constructed using the same hours on different days. Formally,

$$P_{d,h} = \{P_{1,1}, P_{1,2}, \ldots, P_{1,24}, P_{2,1}, \ldots, P_{2,24}, \ldots\} \tag{3.1}$$

$$P_d^h = \{P_1^1, P_2^1, \ldots\} \ \ \forall h \in \{1, \ldots, 24\} \tag{3.2}$$

Hour	mean	Std.Dev.	min	Q1	median	Q3	max	mean	Std.Dev.	min	Q1	median	Q3	max
1	51.59	23.49	0	39.81	47.97	56.80	398.60	33.88	16.25	-12.61	25.50	30.04	37.40	227.47
2	44.54	12.01	0	37.99	44.75	51.34	153.68	32.92	17.03	-19.13	24.52	29.05	36.41	219.03
3	54.56	35.10	0	41.50	50.27	60.48	998.41	30.24	16.41	-26.22	23.13	27.60	34.05	217.74
4	50.49	13.88	0	42.85	49.79	57.84	166.16	28.55	16.13	-38.44	22.09	26.73	32.67	192.98
5	50.26	20.56	0	40.76	48.36	58.27	645.99	29.74	15.44	-45.63	23.15	27.40	33.09	177.55
6	60.78	18.13	0	49.99	58.12	69.10	175.98	34.91	20.03	-26.76	25.95	30.50	37.88	235.55
7	73.05	28.78	0	56.64	68.69	86.78	341.39	46.94	31.10	-39.62	28.95	37.76	56.33	272.99
8	94.21	37.77	10.96	72.43	93.06	115.13	856.06	49.40	31.01	-32.58	30.68	39.97	58.22	430.69
9	61.50	18.14	0	50.76	61.27	71.05	195.12	50.38	24.55	-0.16	33.63	42.53	60.28	223.95
10	55.97	46.36	0	41.45	51.41	63.14	1015.86	53.99	25.34	16.15	36.37	46.13	64.35	223.30
11	63.15	23.10	0	48.53	61.83	75.73	297.92	58.49	27.66	18.43	38.38	50.24	71.60	248.35
12	65.32	19.37	0	56.48	64.34	74.47	238.57	58.74	28.75	14.94	38.81	50.32	70.94	354.84
13	61.67	21.35	0	49.82	58.03	72.17	295.79	58.83	30.71	12.93	37.80	49.08	72.02	397.82
14	79.37	31.83	0	58.87	74.81	95.40	288.08	60.37	35.04	5.81	37.07	48.85	72.20	395.71
15	100.74	40.14	0	79.49	93.91	115.55	403.23	59.70	39.10	2.83	35.51	46.25	71.65	502.87
16	65.10	17.39	0	54.29	62.54	72.52	210.88	60.48	44.63	7.01	34.71	45.42	71.96	716.44
17	50.91	20.26	0	37.06	46.05	60.55	234.58	64.70	46.50	16.55	36.62	49.82	77.32	770.65
18	35.60	13.84	0	28.97	33.92	39.95	260.33	69.33	43.57	19.14	39.62	58.11	86	763.78
19	44.59	24.93	0	30.07	37.41	51.58	278.74	64.44	35.06	15.22	39.36	54.51	78.69	423.11
20	45.83	19.03	0	34.45	41.84	51.55	221.66	61.75	30.86	15.81	39.51	52.80	74.61	247.08
21	53.20	29.36	0	38.32	44.42	58.01	499.71	64.39	32.36	18.77	40.95	55.36	78.08	345.89
22	53.53	27.27	0	36.15	44.80	61.13	253.27	57.07	27.76	17.99	37.16	48.62	69.34	222.05
23	45.71	17.38	0	36.54	41.23	49.37	315.65	42.23	18.05	12	30.67	36.60	47.26	159.87
24	45.86	29.15	0	34.25	39.83	49.60	558.55	37.41	16.45	-1.62	27.80	32.60	41.55	191.62
	ISONE Hourly Statistics							PJM Hourly Statistics						

Table 3.1: Hourly Statistics

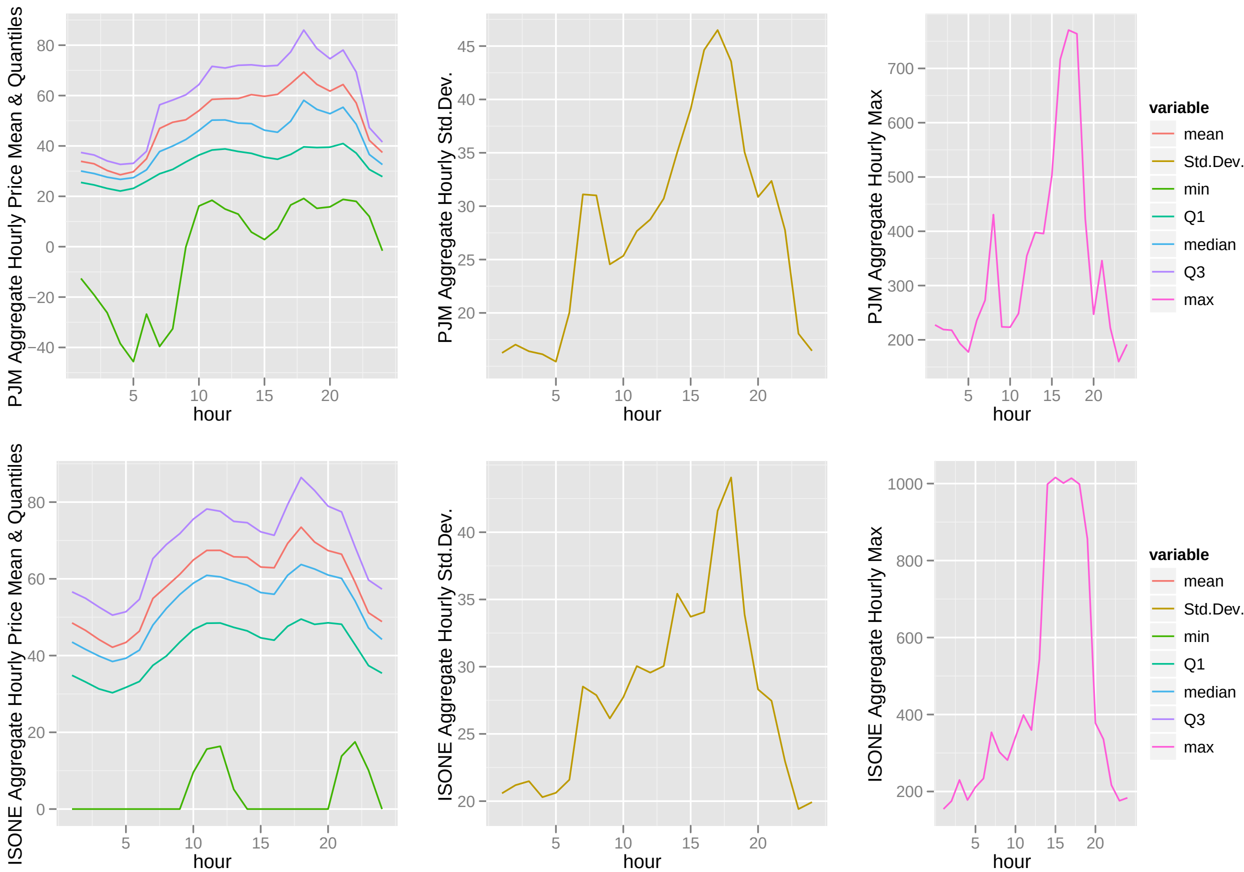

Figure 3.1: PJM vs ISONE Hourly Statistics

The following tests are conducted on the ISONE market hourly price time series using the R Development Core Team [2011] functions:

- Stationarity: null hypothesis that an observable series is trend stationary, *kpss.test* [Kwiatkowski et al., 1992]

- Stationarity: null hypothesis that an observable series is level stationary, *kpss.test* [Kwiatkowski et al., 1992]

- Unit Roots: the hypothesis that the time series has a unit root, *adf.test* [Banerjee et al., 1993],[Said and Dickey, 1984], *pp.test* [Banerjee et al., 1993],[Perron, 1988].

For both of the time series configurations, stationarity (both level and trend) is rejected; however, the null hypothesis that the series has a unit root is also rejected. Below are the results for KPSS level and trend stationarity tests along with a unit root test.

```
KPSS Test for Level Stationarity
KPSS Level = 8.5123, Truncation lag parameter = 63,
p-value = 0.01
KPSS Test for Trend Stationarity
KPSS Trend = 6.3332, Truncation lag parameter = 63,
p-value = 0.01
Augmented Dickey-Fuller Test
Dickey-Fuller = -19.0718, Lag order = 42,
p-value = 0.01
Phillips-Perron Unit Root Test
Dickey-Fuller Z(alpha) = -12851.81, Truncation lag parameter = 21,
```

```
p-value = 0.01
```

However, by taking one-step differences, kpss-test fails to accept the null hypothesis that the time series has a unit root (i.e. non-stationary) at $\alpha = 0.05$ level. So kpss-test finds the one-step difference stationary and, Dickey-Fuller test rejects the null hypothesis that there is a unit root, which are given below:

```
KPSS Test for Level Stationarity
KPSS Level = 7e-04, Truncation lag parameter = 63,
p-value = 0.1
KPSS Test for Trend Stationarity
KPSS Trend = 7e-04, Truncation lag parameter = 63,
p-value = 0.1
Augmented Dickey-Fuller Test
Dickey-Fuller = -53.9025, Lag order = 42,
p-value = 0.01
Phillips-Perron Unit Root Test
Dickey-Fuller Z(alpha) = -51161.41, Truncation lag parameter = 21,
p-value = 0.01
```

The results are similar for the PJM Market.

Independence

The hourly price data is clearly not a time series with independent elements. The hourly autocorrelations are revealed in Figure 3.2, where one can clearly observe the dependence of the hourly prices on the next day's prices for the same hour. In Figures 3.3 and 3.4 intraday hourly Pearson [1901], Kendall [1938], and Spearman

[1907] correlations are compared for P_d^h configurations (first row) and one-step difference P_d^h configurations (second row). The Pearson correlation coefficient is more sensitive to linear relationships whereas Kendall describes how similar the ranks are when the data is ranked by each of the quantiles. Spearman correlation coefficients are more sensitive monotone yet linear associations and relationships.

In the ISONE market, for P_d^h configurations Pearson correlation coefficient for hourly prices are highly correlated, whereas Kendall's rank-τ coefficient detects less correlation among all hours compared to the other two coefficients and Spearman coefficient detects higher correlation among all hours. By visual inspection, one can see two blocks where correlations are clustered together. For the 1-step difference P_d^h configuration, one can observe that the correlation among all hours are reduced significantly and the matrices are almost diagonal, yet clusters of correlated hours are still observable. In the PJM market, hourly correlations are weaker compared to those in the ISONE market. The block structures that can be observed in the ISONE market are replaced by less emphasized local patches in the correlation matrix.

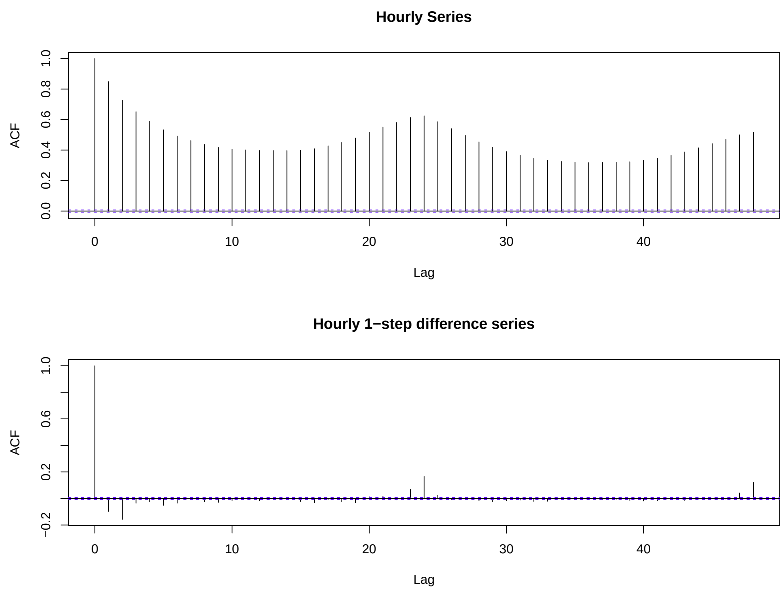

Figure 3.2: Autocorrelation of Hourly Prices and Hourly Price Differences

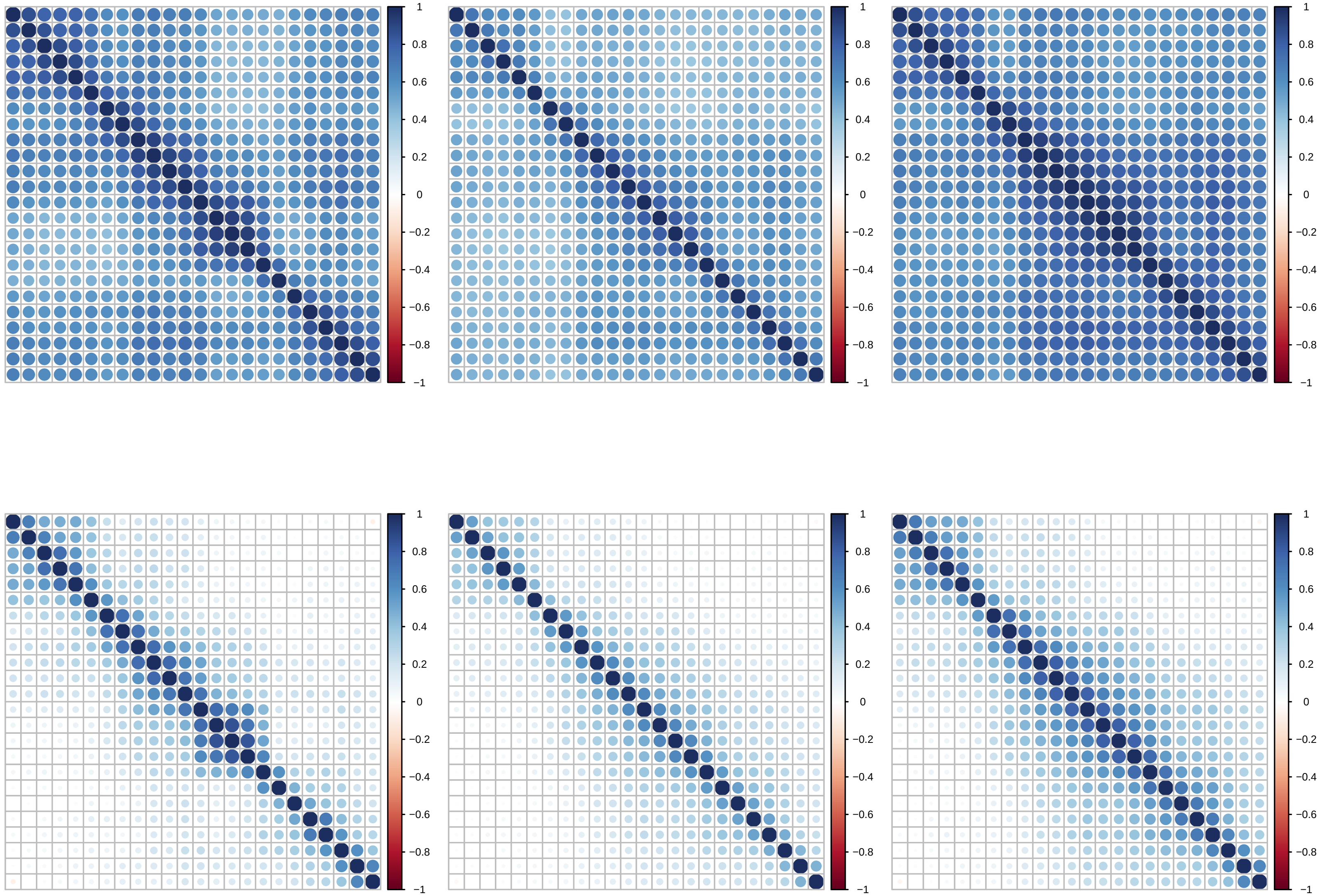

Figure 3.3: ISONE Intraday Corr. within Hours (Pearson,Kendall and Spearman)

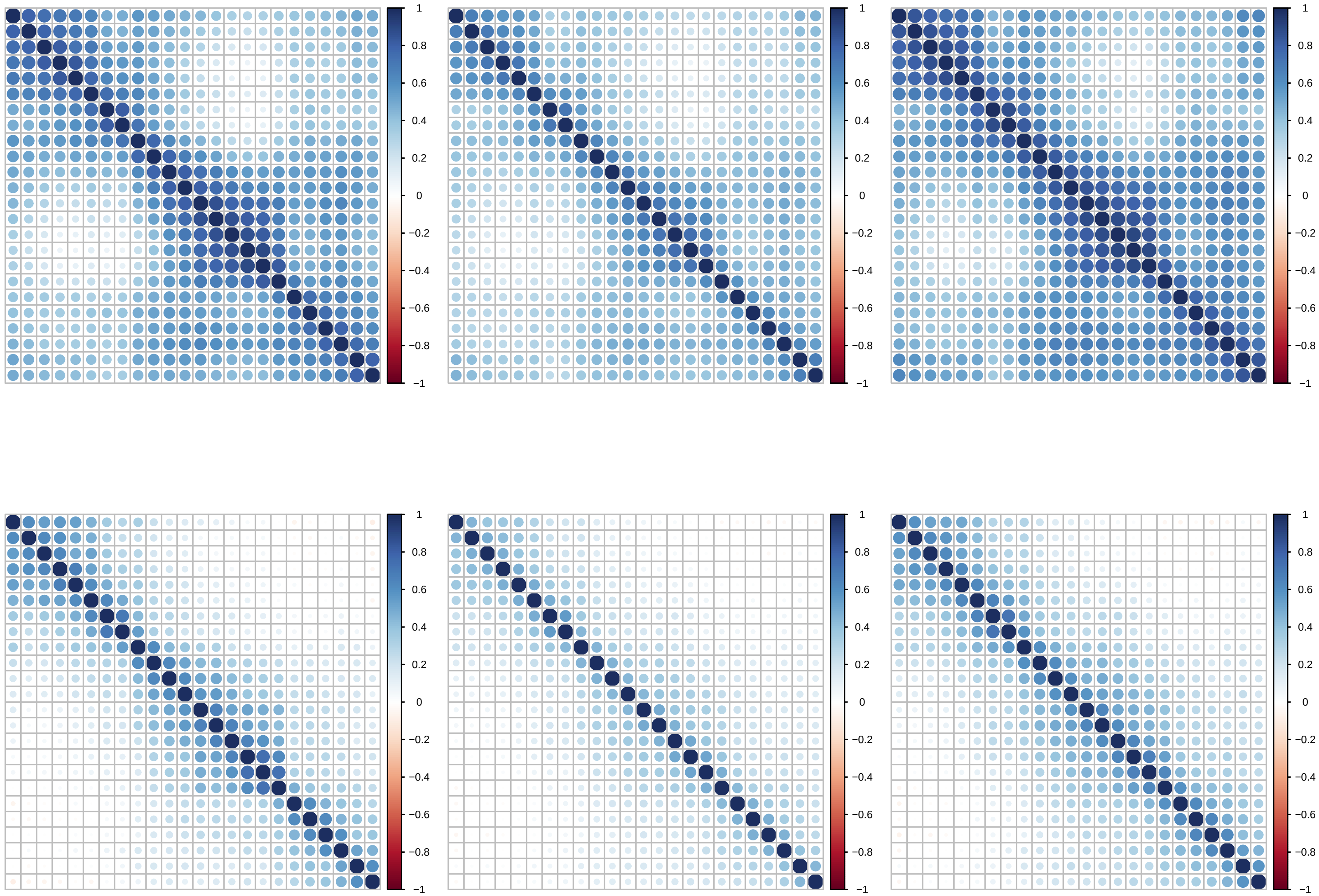

Figure 3.4: PJM Intraday Corr. within Hours (Pearson,Kendall and Spearman)

3.3.2 Larger Time Scales

In this section, we present the data using weekly and monthly perspectives, where it is analyzed separately for different days of the week. In Figures B.1 and B.2 every hour within a week is labeled; where 0 stands for Sunday and 6 stands for Saturday. The prices for the whole week are represented in a single curve, which fits the scope of the production planning problem's cost vector. In boxplots, the lower end of the middle rectangle corresponds to the first quantile and the top end corresponds to the third quantile. The ends of the whiskers corresponds to $Q1 - 1.5\ IQR$ and $Q3 + 1.5\ IQR$ where IQR stands for the inter-quantile range. ISONE has slightly higher but less volatile hourly prices compared to the PJM market. In the PJM market peak hours are more pronounced and prices have a skewed distribution due to medians being closer to the lower hinges.

Daily and weekly periodicity are clearly present in both plots. Since our objective is not to determine a prediction model, but to measure the sensitivity of the optimal production solution to given prices and develop a method to appropriately select and structure the data into an uncertainty set, prices are used in their raw form.

Next a weekly breakdown of the data from monthly and seasonal perspectives are presented in Figures B.3, B.4 and B.5. During shoulder seasons, fall and spring, prices tend to be statistically lower with less volatility. The data shows that 2009 Fall and Spring were the best amongst all others for both markets. In terms of price quantiles, Summer 2008 and Fall 2005 were the worst for both markets. Both markets show similar levels for monthly comparison of prices in two markets, for instance, in 2008, December and January are found to be the months with the most intraday variability in the PJM market. Furthermore, the monthly change in the hourly price profile is more evident in years 2008, 2010 and 2011 in both markets. The prices increase significantly in the afternoon for summer months compared to the rest.

3.3.3 Spikes

Due to inherent daily patterns, choosing a method to identify spikes becomes tricky. If one were to accept that there is a daily price profile, then peak price points within the day are not spikes but rather a realization of the price characteristics for that specific hour. Since the data is analyzed from the hourly perspective, two ideas to detect spikes are combined: First, calculate a dynamic threshold price point and identify a spike whenever prices pass that threshold. Second, filter the data to identify high spike percentages or anomalous maximum prices.

For identifying a threshold, the ratios of the day-ahead LMPs and real-time LMPs are analyzed. Instead of using the $Q3 + 1.5IQR$ formula for the thresholds, which is depicted by the top end of the boxplot whiskers, $2\ Q3$ is used as a conservative approximation as the spike threshold. This quantity seems to coincide with the maximum prices for non-peaking hours but to fall short of maximum prices observed. Maximum prices above $\$300$/MWh are also identified as spikes, since that price point looks like a natural cut-off point for the data (see Figure 3.5).

A summary of results with notable maximum prices or high spike percentages are given in Tables B.3 and B.4. For every (year, hour) pair, "size" indicates the number of data points that belong to that specific pair, "threshold" indicates the twice the median of all prices, "N" indicates number of spikes identified. The rest of the columns provide statistical information about the subset of prices that are flagged as spikes. For a comparison of dynamically calculated spike counts refer to

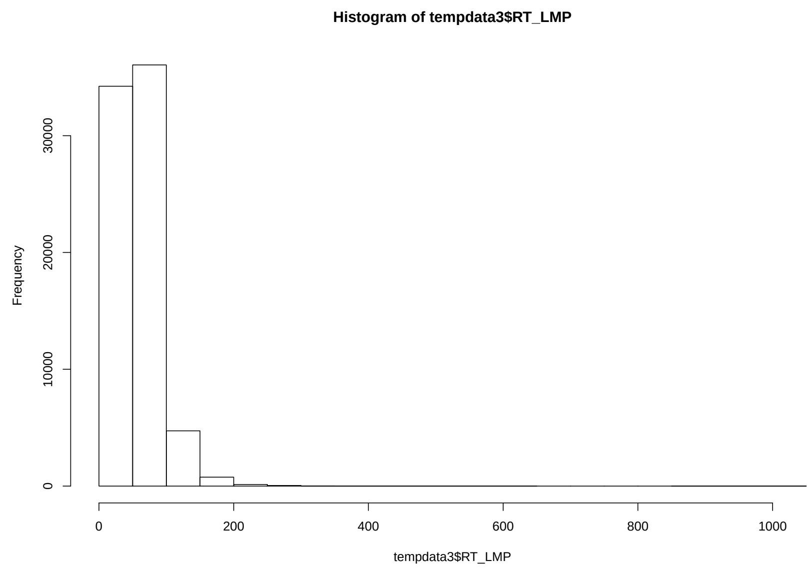

Figure 3.5: Price Histogram

Figure B.6.

Two different threshold calculation methods are tested, and yearly aggregated results are presented for both methods in Table B.5. The first method calculates the threshold as twice the median of the prices minus the standard deviation for all prices within a year. The second method is similar to the first one, with the only difference being the multiplier of the median is now three. One important observation from these tables is the significant spike volatility in 2006, which is detected with both the high threshold and the low threshold in ISONE and PJM markets. In 2010 and 2011, the number of spikes detected by both thresholds increased; however, the volatility is not as severe compared to 2006 spikes. The PJM market is more spiky in all years which is evident from the spike counts given in Table B.5 and depicted in Figure B.6. Even though there is no significant pattern in the yearly price thresholds, other than the fact that the thresholds are high for years with high volatility such as 2006 and 2008, both of the threshold methods flag more spikes for years 2010 and 2011 in both markets. While both markets have similar high/low threshold levels, the PJM market shows a higher number of spikes, which is evident in Figure B.6. In this figure, a comparison of daily spike counts for both markets is presented from a year-hour perspective. The daily spike profile is flatter for ISONE; however, for PJM it changes from year to year.

3.4 Forecasting and Information Criteria

3.4.1 ARIMA Models and Error Measures

The final step in this analysis is an experiment to find an appropriate model for daily forecasts and an appropriate data horizon. We test dynamically calculated ARIMA models [Mills, 1991] with different horizon lengths of $\{2^0, \ldots, 2^5\}$ days. For each day and horizon length, the R package *forecast* [Rob J. Hyndman and Schmidt, 2012] is used to fit an ARIMA model. Next, the fitted ARIMA model is used to forecast the prices for the day in question in a rolling manner. *forecast* automatically estimates ARIMA parameters using the following algorithms:

- p: Order of autoregressive component, q: Moving average;
  (selected using Hyndman-Khandakar algorithm [Hyndman et al., 2007])

- d: Order of integrated component, (MLE Estimation) (constant trend, d=0; linear trend, d=1; quadratic trend, d=2)

For comparing the candidate ARIMA models, *forecast* use Akaike's information criterion, Akaike [1973]. This criterion is based on the entropy and calculated using the formula $2k - 2ln(L)$, where k is the number of parameters used in the models and L is the maximum likelihood estimate. The objective of this experiment is to evaluate the forecasting performance of the dynamically configured ARIMA models in both markets by using different in horizon lengths.

The algorithm is as follows: For every daily price instance p_i, auto-fit an ARIMA model using horizon lengths $h_j \in \{2^0, 2^1, 2^2, \ldots, 2^5\}$. The accuracy of the forecast which is characterized using error measures such as ME, RMSE, MAE, MPE, MAPE and MASE which are formulated below [Hyndman and Koehler, 2006]. Let A_t be the actual price and B_t the forecast price at time t. Then

$$e_t = A_t - B_t \tag{3.3}$$

$$k_t = 100\ e_t / A_t \tag{3.4}$$

$$l_t = \frac{e_t}{\frac{1}{n-1}\ \sum_{i=2}^{n} |A_i - A_{i-1}|} \tag{3.5}$$

$$\text{Mean Error} \quad mean(e_t) \tag{3.6}$$

$$\text{Root Mean Square Error} \quad mean(e_t^2) \tag{3.7}$$

$$\text{Mean Absolute Error} \quad \sqrt{MSE} \tag{3.8}$$

$$\text{Mean Percentage Error} \quad mean(|e_t|) \tag{3.9}$$

$$\text{Mean Absolute Percentage Error} \quad mean(|k_t|) \tag{3.10}$$

$$\text{Mean Absolute Scaled Error} \quad mean(|l_t|) \tag{3.11}$$

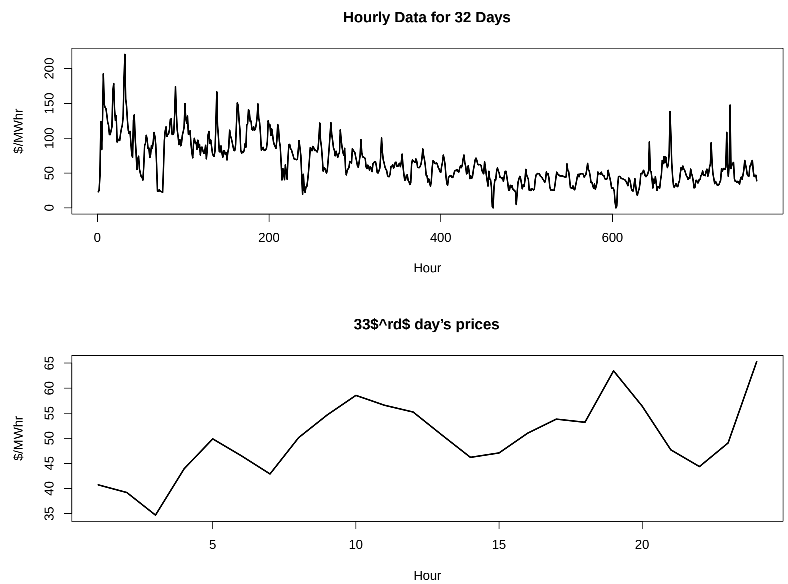

Figure 3.6: Historical Prices for 32 days and The Real Prices for the 33^{rd} Day

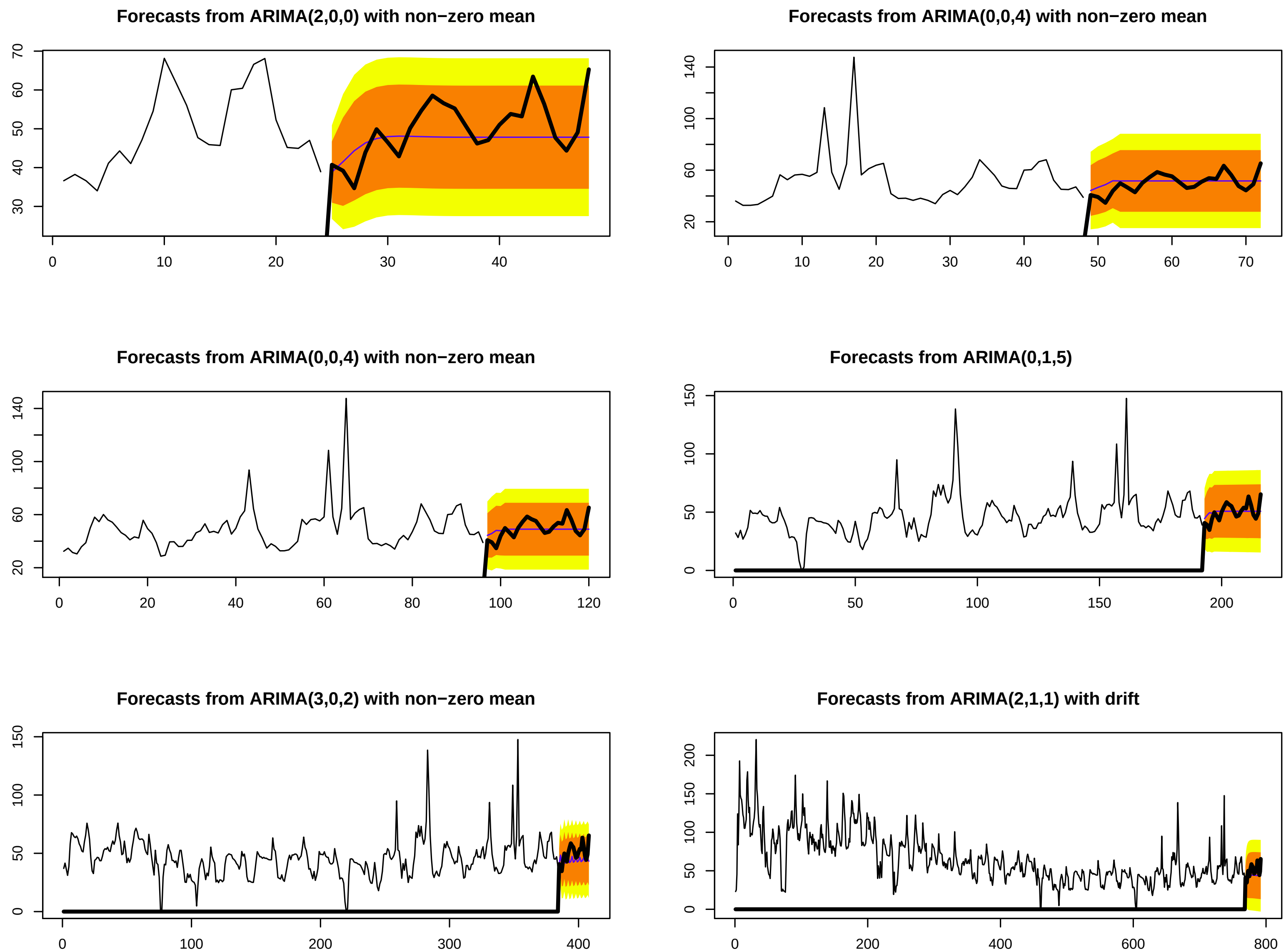

Figure 3.7: Forecasts for Different Horizons

3.4.2 Horizon Selection and Evaluation

Horizons are evaluated according to two rules. The first rule chooses the ones[3] that score lowest on most of the measures, and the second rule calculates a final measure, which is the product of the absolute values of all above measures and chooses the minimum. Figure 3.6 presents the the hourly data from the ISONE Market starting from 2003-03-03, Hour 1 until 2003-04-03, Hour 24.

The forecasting accuracy is measured using the real prices realized on day 2003-04-04. The forecasts for different horizons are given in Figure 3.7. The calculated errors and horizon decisions by both rules are given in Table 3.2. Both horizons, namely 2-day and 8-day horizons, capture the the actual prices with very high confidence, which can be observed in Figure 3.7. Note that for shorter time horizons, the fitted model has a constant mean, while for the 32-day horizon a model with a drift term is fit.

Horizon	ME	RMSE	MAE	MPE	MAPE	MASE
2 days	-0.98	6.50	5.33	-3.83	11.09	0.42
8 days	-0.18	6.59	5.23	-2.29	10.78	0.44

Table 3.2: Horizons and Errors Choosen by Rules

The results for both markets are summarized above in the confusion matrices. The matrices show the number of cases where the column value is chosen by Rule 2 and the row value is chosen by Rule 1. The second rule tends to choose shorter horizons while offering similar error measures as the first rule.

[3] Plural, since there can be ties.

```
ISONE DATA
         Rule 2
Rule 1       1   2   4   8  16
       1   300  57  51  64  59
       2    65 233  47  44  58
       4    44  54 250  48  54
       8    41  53  52 276  66
       16   48  43  56  65 310
       32   87  89 119 157 242
mismatch
[1] 0.2215837
```

```
PJM DATA
         Rule 2
Rule 1       1   2   4   8  16
       1   265  39  34  42  47
       2    46 183  44  42  47
       4    44  54 194  69  51
       8    52  52  47 219  57
       16   37  35  55  64 290
       32   84  88 100 145 186
mismatch
[1] 0.2223451
```

For instance, there were 300 cases in the ISONE market where both rules chose the 1-day horizon, 65 cases where Rule 2 chose the 1-day horizon and Rule 1 chose the 2-day horizon and so forth. Rule 2 never chose the 32-day horizon in both markets, and it tends to balance all six measures, while Rule 1 tends to choose horizons in which one of the error measures might be extremely off while others are favorable. This is due to the fact that Rule 1 is based on the number of favorable error scores, regardless of how unfavorable the remaining error metric scores are. Rule 2 outperforms Rule 1 in both markets and provides the better data horizon selection with respect to the MASE error measure. The performances are summarized in Table 3.3. Values less than 1 mean that the forecasting method used provides a better performance compared to using the $B_t = A_{t-1}$ method. Both rules perform better in the ISONE market in terms of minimum, median and worst-case maximum value of MASE.

market	rule	mean	Std.Dev.	min	Q1	median	Q3	max
ISONE	1	0.947	0.557	0.0521	0.608	0.828	1.123	8.912
ISONE	2	0.980	0.528	0.0521	0.652	0.875	1.173	8.912
PJM	1	0.891	0.504	0.0994	0.596	0.784	1.068	10.274
PJM	2	0.926	0.462	0.0994	0.624	0.837	1.117	4.215

Table 3.3: MASE for Rule 1 and 2 in both markets

3.5 Identified Cues for the Uncertainty Set

The daily perspective clearly demonstrates that there are changing daily price profiles in both markets. This can be observed in the yearly-hourly overlay of LMPs colored according to the month; which is depicted in Figure B.7. In this figure, spikes are filtered out to make sure hours that are more variable are emphasized. There is a clear separation of highly volatile hours from hours with less volatility. Following years with large fluctuations, the daily range of prices decreases, yet daily patterns in different months might be similar or substantially different according to years. Furthermore, existence of negative prices in PJM creates an interesting situation from the customer's perspective. Mainly by producing at those hours, a manufacturer would be making money by using electricity. For instance in 2009-06-07 in an interval from 4:00 AM to 6:00 AM the prices were in the range between $-\$45$ /MWh, $-\$4.5$ /MWh which provides a two hour window where manufacturer can actually earn money just from producing. While such occasions are not frequent and only 0.5 % of the PJM data set displays negative prices, these incidents can be observed in all years and almost all months.

By using intraday quantiles and standard deviations, one can easily identify the peaking hours; however the data shows that throughout years and seasons, price ranges and peaking hours can change significantly (compare ISONE 2003 to ISONE

2011 data) in the daily profiles. Intraday prices and same-hour inter-day prices are found to be highly correlated; however, the two markets show different patterns in terms of intraday price correlation. Furthermore, after significantly volatile years, there is evidence for actions taken (by regulation or by market participants) which would mitigate the situation for the next year. There are at least two such cycles that can be observed in both ISONE and PJM data.

The data suggests that uncertainty sets constructed by aggregating the data over years or months will obscure the characteristics of the data that is significantly dependent on the chosen time frame. Therefore, in both markets, short term data is found to be a good candidate for describing daily and weekly patterns. Interpreting the data using long time scales is hard due to the changes in regulation, the markets and market features.

The one-step difference hourly price time series are found to be stationary in both markets. Intraday prices and same-hour inter-day prices are highly correlated. Dynamically fit ARIMA models with short time horizons as input are found appropriate for forecasting the daily prices, and the experimental results show that for a fixed list of horizons, the ARIMA models that would be fit will be of different parameters with different horizon lengths. It is also observed that for short term horizons, ARIMA models with constant mean terms are fit while for longer horizons ARIMA models with drift terms were fit.

Chapter 4

The Value of Information: A Production Model and A Scheduling Model

4.1 Introduction

In this chapter, an experiment is conducted to quantify the value of information from two perspectives: a production planner's perspective and a job shop scheduler's perspective. In this research, the value of information is quantified as the reflection of the price uncertainty on the optimal objective function value's deviation from a solution obtained by solving an optimization problem with imperfect information.

For the production planner's perspective, various demand scenarios and rolling time horizons of varying length for electricity price data are used to create and compare optimization instances of various mathematical models under the assumption that a production planning problem is solved at the beginning of the week, i.e. before the actual prices are revealed. There are alternative approaches to a production planner can take. Some of them are: (1) The production planner uses the data for estimation and solves an LP using the estimated cost, (2) uses the data to calculate various statistics and use these statistical calculations to create robust optimization instances; (3) uses the prior week's prices as an estimator for the current week. The details of the models that were investigated in this study are summarized next.

Let us define the data selection approaches first. Given a scope (week/day) i, p_i^0 is the vector that contains the actual prices that are observed in that scope. We use a weekly scope for production planning problems. p_i^1 is the vector that contains the actual prices that were observed in scope $i-1$, hence it characterizes the past scope's prices for any given scope. p_i^2 is the vector that contains the price estimate obtained by dynamically fitting an ARIMA model for scope i using variable length horizons, say l_i, which means using the data in $\mathcal{Q}_i = \{p_j^0 \; s.t. \; i - l_i \leq j \leq i-1\}$ for fitting and forecasting for scope i. All of the $\mathbf{p}_i$ vectors can be modularly used with the baseline LP model.

Due to the non-stationary nature of the LMPs, ARIMA models using different horizon lengths (2 weeks, 4 weeks and 8 weeks) are used to estimate the current week's prices. Except for the fact that now time horizons are in terms of weeks, this approach is exactly the same approach used in Chapter 3. For instance, choosing an eight week horizon for the week of 2000-08-31 means selecting the electricity prices from past eight weeks prior to day 2000-08-31. These prices are depicted in Figure 4.2.

Next we define the optimization models. The baseline model is a deterministic linear production planning model, which we call the standard production planning model, [SPP]. The details of [SPP] is given in §4.3.3. Note that this model takes the prices as the objective function vector and the demand scenario as the right hand side vector. Next a robust optimization model with a budget of uncertainty, [RPP], is developed. This model is described in in §4.3.3 and it takes a nominal cost vector, a cost deviation vector and a demand scenario right hand side vector as parameters. Lastly, a stochastic programming model [SOPP], also in §4.3.3, is developed. The parameters for this model are multiple cost vectors depending on the horizon and a single demand scenario vector.

First for all weeks i and demand scenarios $d_{1,\ldots,10}$, the [SPP]$_i$ is solved using p_i^0 as the objective function coefficients and $\mathbf{d}$ as the right hand side to find the true objective function value, $z^*_{0,i}$, and the optimal production plan, $S^*_{0,i}$. Note the there is no index for the demand scenario, and the reason is we have dropped it for the ease of exposition. Next [SPP]$_i$ is solved using p_i^1 and p_i^2. Say the corresponding optimal solution vectors are $S^*_{1,i}$ and $S^*_{2,i}$. Now the deviations in [SPP]$_i$+ p_i^1 and [SPP]$_i$+ p_i^2 are:

$$\rho_{i,1}^{SPP} = \frac{|z^*_{0,i} - (p_i^0)^T S^*_{1,i}|}{z^*_{0,i}} \quad \forall i \tag{4.1}$$

$$\rho_{i,2}^{SPP} = \frac{|z^*_{0,i} - (p_i^0)^T S^*_{2,i}|}{z^*_{0,i}} \quad \forall i \tag{4.2}$$

Similarly [RPP]$_i$ is instantiated using the data in $\mathcal{Q}_i$. Five different approaches which are discussed in §4.3.2 are used to calculate various statistics on $\mathcal{Q}_i$ where these statistics are used to calculate nominal and deviation vectors. Using these vectors [RPP]$_i$, $i \in (1, \ldots, 5)$ are instantiated and solved to obtain

$$\rho_{i,j}^{RPP} \quad 1 \leq j \leq 5 \ \ \forall i \tag{4.3}$$

Finally [SOPP] is instantiated using $\mathcal{Q}_i$. This time all the price vectors in $\mathcal{Q}_i$ are used as scenarios for the [SOPP]. For [SPP] and [RPP] instances, the obtained production plan, S^*, is combined with actual prices, p^0, to calculate the would-be optimal objective function value; however, this is not the case for [SOPP] since this model only gives information about the expected objective function value. Therefore the corresponding ρ^{SPP} is calculated as

$$\rho_i^{SOPP} = \frac{|z_{0,i}^* - z(SOPP(\mathcal{Q}_i))^*|}{z_{0,i}^*} \quad \forall i \tag{4.4}$$

For the scheduler's perspective, a job-shop scheduling model is developed and tested using the electricity prices in a similar fashion to the production planning models using a daily scope. Due to prohibitive computation times and model complexity, a single 0-1 Integer Programming Model (BIP) is first solved with real prices to obtain the baseline, true solutions. Next, the BIP is solved with forecast prices in a rolling fashion. As for forecasting techniques, the past week's prices and dynamically fit varying horizon ARIMA models are used.

4.2 Literature Review

Sorokin et al. [2012a,b] contains an exhaustive list of optimization models and approaches on different aspects of operational planning in power markets from various perspectives such as power generators, power marketers and customers. Zhu et al. [2010], Karwan and Keblis [2007], and Ierapetritou et al. [2002] provide models that are similar to ours in terms of problem setting, practicality and uncertainty[1].

[1] Also see Engell et al. [2010]

A robust optimization model developed by Conejo and Carrion [2006] is relevant to our research since it addresses robust power procurement. Another similar model worth mentioning is by Zheng et al. [2010] where the authors discuss a portfolio optimization problem occurring in the energy market. Energy distributing public services have to decide how much of the requested energy demand has to be produced in their own power plant, and which complementary amount has to be bought from the spot market and from load following contracts. This problem is formulated as a mixed-integer linear programming problem. These two models were initially found appropriate for our assessment since they both simultaneously cover day-to-day power asset execution and operational planning. Refer to relevant papers for the results.

All data manipulation and statistical computational implementations are done in [R Development Core Team, 2011] using the following packages in addition to the packages we have used in §3: *gputools* [Buckner et al., 2011], *rje* [Evans, 2012], *slam* [Hornik et al., 2011] and *Matrix* [Bates and Maechler, 2012].

The optimization model components such as the objective functions, constraint matrices, right hand sides and such are built in [R Development Core Team, 2011], the optimization instances are solved in Gurobi 4.6.1 [Gurobi Optimization, 2012] and the results are then transferred back to R. We have written the interfaces to Gurobi libraries in C++ and registered as R plugins using *inline* [Sklyar et al., 2010] and *Rcpp* [Eddelbuettel and François, 2011]. Just recently, after we have finished writing the interfaces and finished running the simulations, Gurobi started supporting R in version 5.0.

4.3 Data & Data Selection

4.3.1 Demand Data

10 demand scenarios which are given in Table C.1 and depicted in Figure 4.1 are used. The scenarios start from a perfectly predictable pattern (a sine curve) in scenario 1 and more noise is introduced progressively for each scenario. The formulas that are used to create each scenario are given below:

$$\begin{aligned}
sce1 &\quad 100\ \sin\ \pi/42 \\
sce2 &\quad 100\ \sin\ \pi/84 \\
sce3 &\quad 100\ \sin\ \pi/84 + 10\ U[-1,1] \\
sce4 &\quad 100\ \sin\ \pi/84 + 10\ N(0,1) \\
sce5 &\quad 100\ \sin\ \pi/42 + 100\ \sin\ \pi/84 + 30\ U[-1,1] + 10\ N(0,1) \\
sce6 &\quad 100\ \sin\ \pi/42 + 100\ \sin\ \pi/84 + 10\ U[-1,1] + 30\ N(0,1) \\
sce7 &\quad 100\ U[-1,1] \\
sce8 &\quad 100\ N(0,1) \\
sce9 &\quad 100\ U[-1,1] + 100\ N(0,1) \\
sce10 &\quad 100\ \sin\ \pi/42 + 100\ \sin\ \pi/84 + 100\ U[-1,1] + 100\ N(0,1)
\end{aligned}$$

Four demand signals, $\{\sin\ \pi/42, \sin\ \pi/84, U[-1,1], N(0,1)\}$, are mixed in ten different ways to obtain the above demand scenarios. The $\sin$ curves are used to shaped the demand while uniform and normal random variables are used to introduce noise to the curves. To remove the negative demands, the final mixture matrix is translated by the absolute value of the smallest negative demand. Note that the

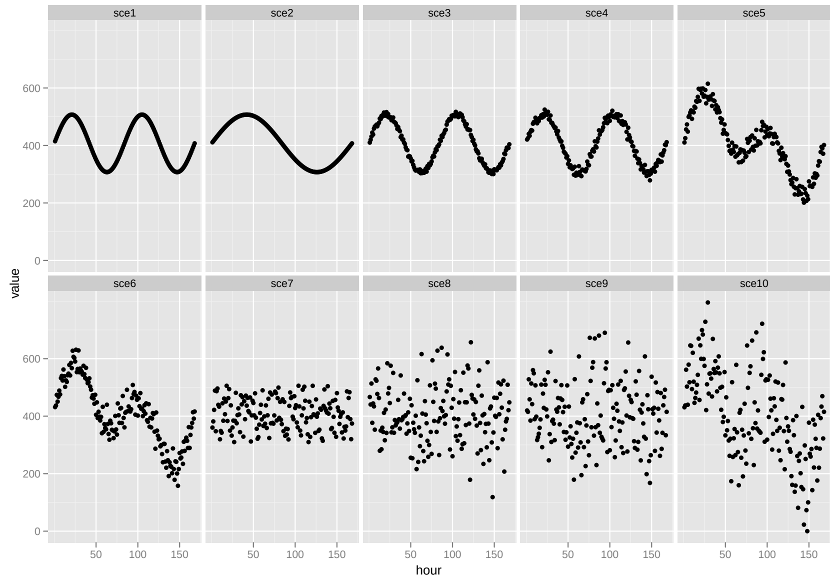

Figure 4.1: Demand Data

same demand for a given week is used along with price curves from both PJM and ISONE markets in optimization problems. It is also assumed that the demand process is independent of the electricity prices and the objective of the comparison is to measure the potency of different joint data selection and optimization rules. The demand scenario statistics are presented in Table 4.1.

4.3.2 Price Data: Selection & Usage

Robust Uncertainty Sets

The robust approach described by Bertsimas et al. [2010] is used for building [RPP] prototypes. Every price point has a lower bound and an upper bound. Following the budget of uncertainty concept of [Bertsimas et al., 2010], for a predefined fraction of the time, the prices will be realized at their upper bounds. If the

Demand Stats for Scenarios					
	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
mean	407.29	407.29	407.36	407.21	407.43
Std.Dev.	70.92	70.92	71.35	73.43	103.65
min	307.29	307.29	300.51	278.68	201.05
Q1	337.95	336.58	335.72	334.99	344.89
median	407.29	407.29	405.92	412.86	406.80
Q3	476.63	478.00	475.36	478.63	474.70
max	507.29	507.29	516.86	524.68	614.83
	Scenario 6	Scenario 7	Scenario 8	Scenario 9	Scenario 10
mean	407.12	408.02	406.48	407.21	407.21
Std.Dev.	107.48	53.64	98.47	107.23	154.97
min	157.82	309.91	118.40	167.74	0.00
Q1	335.66	362.47	344.94	324.90	296.10
median	409.86	405.08	399.75	400.62	402.58
Q3	477.83	452.31	464.93	481.40	521.48
max	631.19	505.83	657.06	690.07	795.49

Table 4.1: Demand Statistics for Each Scenario

fraction is defined as 7 %, there will be 12 points within 168 price points that will be realized as their maximum value. A 10 % spike fraction is chosen for our experiment hence for this approach, lower and upper bounds for each price point have to be defined. In our notation, nominal LMPs are $\bar{c}$, and the deviations are $\hat{c}$. We test five different methods to determine the lower bounds (lb) and upper bounds (ub) of price points:

1. lb=min, ub=max
2. lb=Quantile 1, ub=Quantile 3
3. $\mu \pm \sigma$
4. $\mu \pm 1.96\ \sigma$
5. $\mu \pm 6\ \sigma$

Once lb and ub are calculated, the nominal curve is set as $\bar{c}$=(ub+lb)$/2$ and the deviation is set as $\hat{c}$=(ub-lb)$/2$. Depending on the uncertainty set selected, the robust model is named as RPP.1, RPP.2, . . . , RPP.5.

4.3.3 Production Planning Models

In this section, we formulate the standard production planning model [SPP], the robust production planning model [RPP], and the stochastic production planning model [SOPP].

Standard Production Planning Model

$$
\begin{aligned}
[SPP] \quad \min \quad & c^T x + [10\ e^T] inv && \text{(4.5a)} \\
s.t. \quad & inv_0 = 0, \quad inv_{168} = 0 && \text{(4.5b)} \\
& inv_i = inv_{i-1} + x_i - d_i \quad \forall i \in \{1...168\} && \text{(4.5c)} \\
& x \geq 0, \quad inv \geq \quad 0, && \text{(4.5d)}
\end{aligned}
$$

where $e = [1, \ldots, 1]^T_{168\times 1}$. The vector $\mathbf{c}$ denotes the weekly LMPs. The vectors $\mathbf{x}, \mathbf{inv}$ are the production and inventory variables respectively. The parameter d gets its value from the demand scenario set. For ISONE, there are 452 instances of vector c. This model is used due to its simplicity and to be able to isolate the effects of price changes on the production values. Further constraining this model would dampen the effect of the price uncertainty. [2]

[2]Consider a hypothetical case where the inv variables all have an upper bound of 0. This would be equivalent to just in time production, where the demand for a certain hour, would be met by the production in that particular hour, no matter what the price is. Therefore, it wouldn't be possible to measure the impact of the LMPs on the production schedules, since they would be irrelevant.

Robust Production Planning Model

The robust production planning model is formulated as follows:

$$[RPP] \quad \min \quad \bar{c}^T x + [10\ e^T] inv + \left\{ \max_{\eta \in \mathcal{U}} \quad \hat{c}^T (\eta \cdot x) \right\} \tag{4.6a}$$

$$s.t. \quad inv_0 = 0, \quad inv_{168} = 0 \tag{4.6b}$$

$$inv_i = inv_{i-1} + x_i - d_i \ \ \forall i \in \{1...168\} \tag{4.6c}$$

$$x \geq 0, \quad inv \geq 0, \tag{4.6d}$$

η is the robust multiplier vector for the production variables, x and $\eta \cdot x$ represents an element-wise multiplication. Let $frac$ be the number of hours (analogous to budget of uncertainty in the objective) in which LMPs hit their maximum value. Then the inner problem is:

$$\max \quad \hat{c}^T \eta \cdot x \qquad (DUAL) \tag{4.7a}$$

$$s.t. \quad \eta_t \leq 1 \qquad \alpha_t \tag{4.7b}$$

$$-\eta_t \leq 1 \qquad \beta_t \tag{4.7c}$$

$$\eta_t - v_t \leq 0 \qquad \gamma_t \tag{4.7d}$$

$$-\eta_t - v_t \leq 0 \qquad \theta_t \tag{4.7e}$$

$$0 \leq v_t \leq 1 \qquad \rho_t \tag{4.7f}$$

$$\sum_t v_t \leq frac \qquad \psi \tag{4.7g}$$

The dual is as follows:

$$\min \quad \sum_t (\alpha_t + \beta_t + \rho_t) + \psi\, frac \tag{4.8a}$$

$$s.t. \quad \alpha_t - \beta_t + \gamma_t - \theta_t = \hat{c}_t * x_t \tag{4.8b}$$

$$-\gamma_t - \theta_t + \rho_t + \psi \geq 0 \tag{4.8c}$$

Inserting the dual back into the main problem using strong duality, we get the following model:

$$\min \quad \bar{c}^T x + [10\ e^T]inv + \sum_i (\alpha_i + \beta_i + \rho_i) + \psi\, frac \tag{4.9a}$$

$$s.i. \quad inv_0 = 0, \quad inv_{168} = 0 \tag{4.9b}$$

$$inv_i = inv_{i-1} + x_i - d_i \quad \forall i \in \{1...168\} \tag{4.9c}$$

$$\alpha_i - \beta_i + \gamma_i - \theta_i = \hat{c}_i\ x_i \quad \forall i \in \{1...168\} \tag{4.9d}$$

$$x \geq 0, \quad inv \geq \quad 0, \quad \alpha, \beta, \gamma, \theta, \rho, \psi \geq 0. \tag{4.9e}$$

Stochastic Production Planning Model

In the [SOPP] prototype, we allow the cardinality of the price scenario set, N, to be either 2,4 or 8 weeks. For each demand scenario, the prices from the appropriate horizon length are collected in the uncertainty set. In the stochastic model, the objective is to minimize the expected cost. The difference of this approach is that it is only informative, in the sense that one uses only the resulting objective function to measure the deviation of from the real cost. The resulting optimal solution vector of [SOPP] is not an actual production plan, but multiple production plans, the number of which depends on the cardinality of the price scenario set, stitched together. In the previous models and techniques the resulting production schedule were used to assess the proximity to real optimal cost.

For [SOPP] we assume equal probabilities for each scenario. For instance, when LMP information of the past 8 weeks is included, both scenarios have a probability of $\frac{1}{8}$.

$$[SOPP] \quad \min \quad \frac{1}{N}\{\sum_{S=1}^{N} c^T x_S + [10\ e^T] inv_S\} \tag{4.10a}$$

$$s.t. \quad inv_{0,S} = 0, \quad inv_{168,S} = 0 \tag{4.10b}$$

$$inv_{i,S} = inv_{i-1,S} + x_{i,S} - d_{i,S} \quad \forall i \in \{1...168\} \quad S \in \{1,\dots,N\} \tag{4.10c}$$

$$x \geq 0, \quad inv \geq \quad 0, \tag{4.10d}$$

The resulting formulation clearly has a block angular structure and is separable.

4.4 Numerical Study

4.4.1 Baseline Solutions

First the SPP Model with actual prices for all of the demand scenarios and for all of the price instances to obtain is solved to obtain a comparison baseline. A solution summary for one price and one demand instance is demonstrated as an example below in Figures C.6 and C.7. In this example, the prices for the week of 2009-08-31 for ISONE and PJM markets are used. Demand scenario ten is used Figure

4.1. In the ISONE solution, the production is in just-in-time mode until the latest and the lowest price point. For that instant it is maxed out to cover the demand until the 75^{th} hour. When the LMPs are spiking towards the end, one can observe the model chooses to produce right before the spikes. In the PJM market a similar behaviour can be observed. These models represent the "real" optimal solutions, where LMPs for a particular week are perfectly known at the beginning of the week. The objective function values for each week will be used as the baseline to measure the ρ value of the proposed methodology. The progression of the optimal objective function values for every week for each demand scenario are given in Figures C.8, C.9 and summarized in Table 4.2. For all of the demand scenarios, the progression of the objective function values has a similar statistics and patterns. One interesting observation is, when the optimal weekly solution vectors are overlayed on top of each other, one can observe the original demand signal and integer multipliers of this signal in the resulting graph. This may have connections to Wagner-Whitin algorithm, Wagner and Whitin [1958], however we do not investigate. This phenomenon is most clear for the demand scenarios 1:3 and it can be observed in Figures C.1 and C.2. This is due to the fact that, there is no upper bound for the inventory so the production that occurs to meet a particular demand at some time is exactly equal to the demand quantity, i.e. demand quantities are not divided amongst time points.

Let us give an example for the estimation procedure using automated ARIMA. Given the 8-week history before 2009-08-31, automated ARIMA fits the following models:

```
MODEL                                   AIC VALUE
ARIMA(0,1,0)                          : 9988.885
```

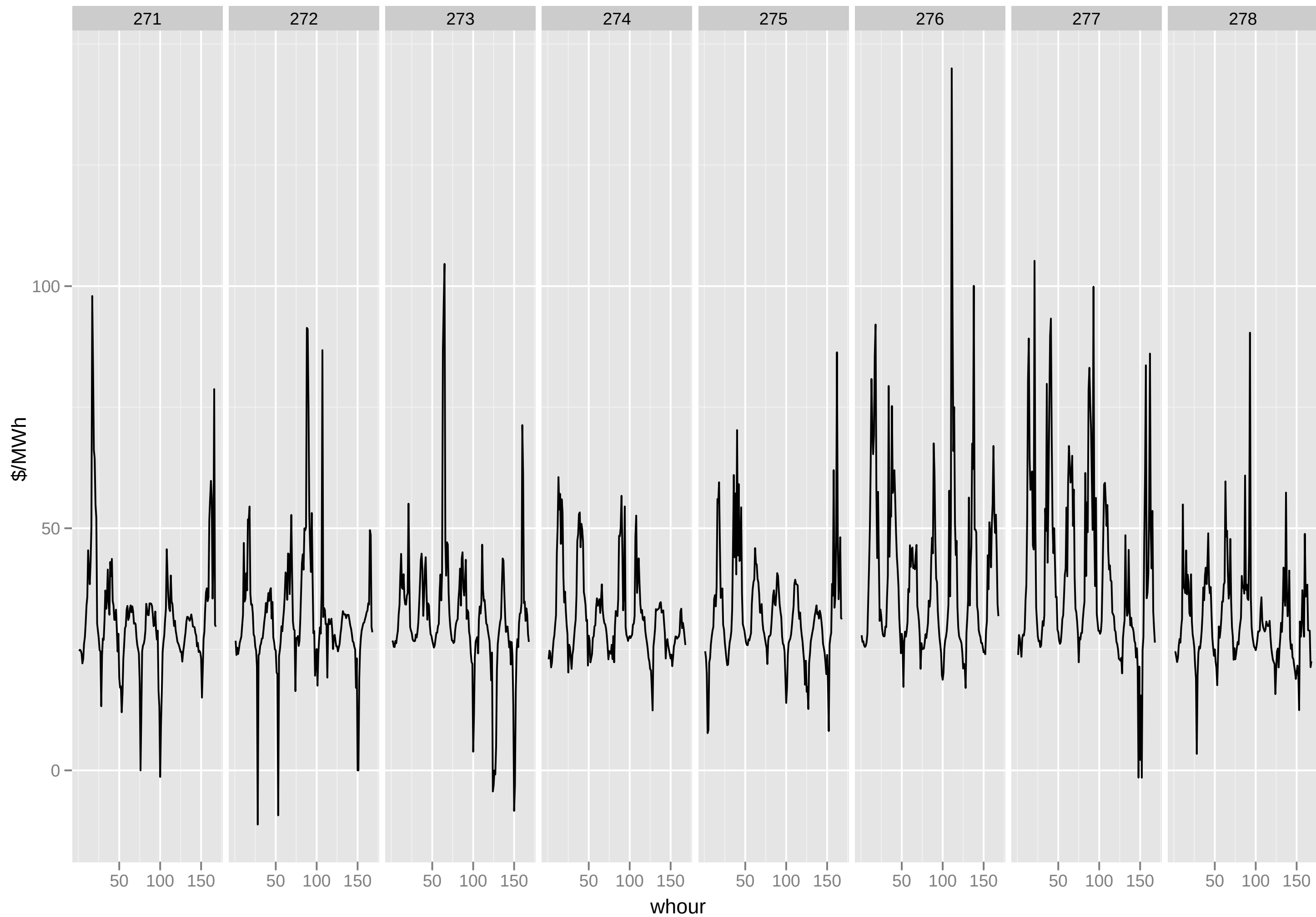

Figure 4.2: 8 week history

ISONE Objective Stats for Scenarios					
	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
mean	3.827544e+06	3.827306e+06	3.828388e+06	3.826102e+06	3.839213e+06
Std.Dev.	1.137495e+06	1.138537e+06	1.137875e+06	1.137131e+06	1.144011e+06
min	1.628839e+06	1.617861e+06	1.629808e+06	1.627454e+06	1.619847e+06
Q1	3.020936e+06	3.009510e+06	3.021104e+06	3.019421e+06	3.001192e+06
median	3.662170e+06	3.653735e+06	3.663521e+06	3.659914e+06	3.678063e+06
Q3	4.321423e+06	4.333839e+06	4.322160e+06	4.320348e+06	4.331212e+06
max	8.357823e+06	8.476668e+06	8.361970e+06	8.354465e+06	8.517080e+06
	Scenario 6	Scenario 7	Scenario 8	Scenario 9	Scenario 10
mean	3.834641e+06	3.825164e+06	3.802305e+06	3.810742e+06	3.832140e+06
Std.Dev.	1.142476e+06	1.137176e+06	1.129541e+06	1.133274e+06	1.143546e+06
min	1.615140e+06	1.638061e+06	1.614524e+06	1.624211e+06	1.614163e+06
Q1	3.001991e+06	3.010140e+06	2.994547e+06	3.000348e+06	3.016979e+06
median	3.672771e+06	3.650434e+06	3.624010e+06	3.632398e+06	3.661117e+06
Q3	4.327014e+06	4.307442e+06	4.278379e+06	4.292487e+06	4.332229e+06
max	8.502068e+06	8.367972e+06	8.292916e+06	8.334392e+06	8.515891e+06
PJM Objective Stats for Scenarios					
	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
mean	3.137882e+06	3.145205e+06	3.138770e+06	3.136757e+06	3.164376e+06
Std.Dev.	8.231876e+05	8.256538e+05	8.235892e+05	8.226660e+05	8.313057e+05
min	1.745212e+06	1.735550e+06	1.746321e+06	1.745993e+06	1.742129e+06
Q1	2.514336e+06	2.508433e+06	2.514639e+06	2.513200e+06	2.526135e+06
median	2.988951e+06	3.006458e+06	2.990120e+06	2.988832e+06	3.019086e+06
Q3	3.607752e+06	3.612891e+06	3.608127e+06	3.606116e+06	3.631880e+06
max	6.192862e+06	6.199714e+06	6.193827e+06	6.186768e+06	6.295933e+06
	Scenario 6	Scenario 7	Scenario 8	Scenario 9	Scenario 10
mean	3.160350e+06	3.129131e+06	3.109000e+06	3.117880e+06	3.160467e+06
Std.Dev.	8.293845e+05	8.227337e+05	8.132936e+05	8.172428e+05	8.295300e+05
min	1.739916e+06	1.723565e+06	1.720281e+06	1.731370e+06	1.737143e+06
Q1	2.523994e+06	2.501918e+06	2.487937e+06	2.491905e+06	2.521550e+06
median	3.010493e+06	2.979770e+06	2.960016e+06	2.970150e+06	2.995675e+06
Q3	3.623121e+06	3.586126e+06	3.567741e+06	3.568434e+06	3.613529e+06
max	6.283106e+06	6.248204e+06	6.177622e+06	6.187270e+06	6.308363e+06

Table 4.2: Baseline Objective Function Value Statistics for Each Scenario

```
ARIMA(0,1,2)                    : 9897.353

ARIMA(0,1,4)                    : 9896.282

ARIMA(1,1,0)                    : 9923.892

ARIMA(0,1,1)                    : 9902.669

ARIMA(1,1,2)                    : 9899.684

ARIMA(0,1,0) with drift         : 9990.913

ARIMA(1,1,4)                    : 9899.361

ARIMA(0,1,2) with drift         : 9899.416

ARIMA(2,1,0)                    : 9898.868
```

```
ARIMA(0,1,3)                    : 9897.738
ARIMA(2,1,2)                    : 9894.865
ARIMA(0,1,4) with drift         : 9898.343
ARIMA(0,1,5)                    : 9898.283
ARIMA(3,1,0)                    : 9899.42
ARIMA(1,1,0) with drift         : 9925.936
ARIMA(0,1,1) with drift         : 9904.724
ARIMA(1,1,1)                    : 9900.117
ARIMA(1,1,3)                    : 9899.531
ARIMA(1,1,2) with drift         : 9901.75
ARIMA(2,1,1)                    : 9899.136
ARIMA(3,1,2)                    : 9897.022
ARIMA(0,1,3) with drift         : 9899.801
ARIMA(1,1,4) with drift         : 9901.426
ARIMA(4,1,0)                    : 9901.683
ARIMA(2,1,0) with drift         : 9900.925
ARIMA(5,1,0)                    : 9904.55
ARIMA(2,1,3)                    : 9895.287
ARIMA(0,1,5) with drift         : 9900.346
ARIMA(3,1,1)                    : 9899.832
ARIMA(1,1,1) with drift         : 9902.18
ARIMA(4,1,1)                    : 9903.46
ARIMA(1,1,3) with drift         : 9901.593
ARIMA(2,1,1) with drift         : 9901.2
ARIMA(2,1,2) with drift         : 9896.914
ARIMA(3,1,0) with drift         : 9901.484
ARIMA(3,1,2) with drift         : 9899.071
```

```
ARIMA(4,1,0) with drift         : 9903.746
ARIMA(5,1,0) with drift         : 9906.614
ARIMA(2,1,3) with drift         : 9897.332
ARIMA(3,1,1) with drift         : 9901.899
ARIMA(4,1,1) with drift         : 9905.519
```

The data used for estimation, the forecast and the real prices are illustrated in Figure C.3. All models assumes non-stationarity and allows for the drift terms. Corresponding AIC values are listed in the right. These models are compared according to their AIC values and the model with the maximum AIC value is chosen.

```
ARIMA(2,1,2)

Coefficients:
         ar1      ar2      ma1     ma2
      0.9625  -0.4337  -1.2176  0.5979
s.e.  0.1663   0.0818   0.1577  0.1008

sigma^2 estimated as 92.23:  log likelihood=-4943.75
AIC=9897.5   AICc=9897.54   BIC=9923.51
```

By using this method the following production plan depicted in Figure 4.3 is obtained. Evaluating the production solution obtained from estimation using real prices, it is observed that, if one were to follow that schedule, the actualized objective function value would be 2.045318 % greater than the real optimal cost.

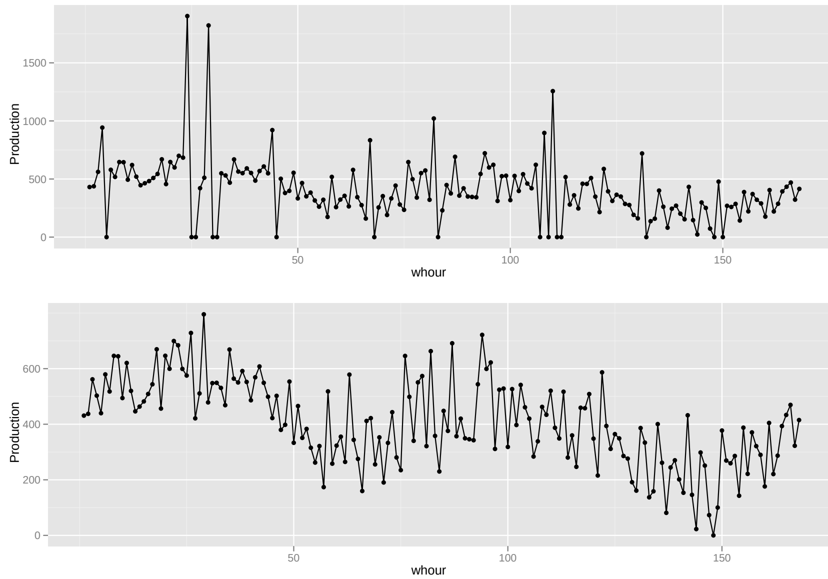

Figure 4.3: Optimal Solution Comparison: $S^*([SPP] + p^0)$ vs $S^*([SPP] + p^1)$

4.4.2 Value of the Information

For comparing the previously introduced data selection and modeling approaches, we have conducted a simulation and calculated the ρ values for all price instances in ISONE and PJM markets. Basically, everything is calculated in a large for-loop. All optimization models are initialized before the simulation so that we don't recalculate the components that will stay the same throughout the loop. For the LP approaches (last week's price and estimated price), the constraint matrix stays the same, only the objective function coefficients change. For the robust LP approaches (RPP.1-5), uncertainty sets change for all rolling windows so the dualized constraint right hand sides change along with the nominal objective coefficients. For the stochastic LP approach only objective coefficients change. For each iteration the time windows are set according to each horizon and the price statistics for each hour are calculated. Then the appropriate objective function and right hand side vectors for all of the models are constructed, the models are solved and the results are recorded in a rolling fashion. Overall results are presented in Figures C.4, C.5 and Table 4.3. In both markets, in terms of ρ values, the following performance in descending order is observed: [SPP] + p^2, [RPP.3], [RPP.4], [RPP.1] or [RPP.2], [SPP] + p^1, [SOPP].

In ISONE market, [SPP] + p^2 is found to be the best approach. On average, the ρ values are less than 5 % on average. However in PJM market ρ values are larger due to reduced forecasting accuracy. [SPP] + p^2 approach still performs better than the other approaches. However, due to highly volatile market, the ρ values are around 10 % of the optimality on average. Regardless of the approach, PJM market provides lower optimal objective function values due to the lower prices which is evident from Table 4.2. Even in the case where PJM prices are increased by 10 %, optimal production costs in the PJM market are around 90 % of what they would be in the ISONE market.

	[SPP] + p^1	[SPP] + p^2	RPP.1	RPP.2	RPP.3	RPP.4	RPP.5	SOPP
mean	6.868946e-02	4.788795e-02	6.215265e-02	6.215238e-02	5.455468e-02	5.861122e-02	7.336887e-02	1.273968e-01
Std.Dev.	6.213379e-02	5.864475e-02	5.689801e-02	5.689813e-02	5.532790e-02	5.588585e-02	5.775638e-02	1.110870e-01
min	5.153874e-03	2.001152e-04	1.744689e-03	1.744689e-03	1.963270e-03	2.904568e-03	8.926670e-03	1.652865e-06
Q1	3.282688e-02	1.613529e-02	3.008254e-02	3.008254e-02	2.514818e-02	2.838942e-02	3.983608e-02	4.678760e-02
median	5.372225e-02	3.176688e-02	4.672603e-02	4.672603e-02	4.075348e-02	4.445811e-02	5.811349e-02	9.709147e-02
Q3	8.242418e-02	5.747404e-02	7.617543e-02	7.617543e-02	6.450988e-02	6.911318e-02	8.860490e-02	1.753278e-01
max	6.621553e-01	6.507335e-01	6.526531e-01	6.526531e-01	6.580398e-01	6.735431e-01	6.896353e-01	7.369913e-01
ISONE Aggregate Summary of ρ Values								
	[SPP] + p^1	[SPP] + p^2	RPP.1	RPP.2	RPP.3	RPP.4	RPP.5	SOPP
mean	1.256682e-01	1.002800e-01	1.233003e-01	1.232992e-01	1.108869e-01	1.166368e-01	1.393535e-01	1.408342e-01
Std.Dev.	7.768015e-02	8.878317e-02	7.746448e-02	7.746437e-02	7.826571e-02	7.814301e-02	7.969383e-02	1.160820e-01
min	1.999689e-02	2.716156e-03	1.671855e-02	1.671855e-02	1.221175e-02	1.428012e-02	2.770020e-02	5.634148e-06
Q1	7.022129e-02	4.322721e-02	6.951197e-02	6.951197e-02	5.907297e-02	6.525821e-02	8.592444e-02	5.101813e-02
median	1.105309e-01	8.178784e-02	1.106421e-01	1.106421e-01	9.486954e-02	1.009599e-01	1.229888e-01	1.108401e-01
Q3	1.657080e-01	1.272263e-01	1.551851e-01	1.551851e-01	1.415035e-01	1.475089e-01	1.721503e-01	2.048446e-01
max	7.470098e-01	9.886768e-01	8.168622e-01	8.168622e-01	8.435617e-01	8.332765e-01	8.459896e-01	8.153722e-01
PJM Aggregate Summary of ρ Values								

Table 4.3: Aggregate Summary of Proximities

4.5 Job Shop Scheduling Model

In this section, a job-shop scheduling model ([JSP]) prototype is used to conduct an extensive numerical study using the real spot prices. The nature of this model is significantly different from the previous ones in the sense that there are no demands, the resources are more limited yet the decisions to be made are similar: instead of "when to produce" it is "when and where to process". There are no alternative sources of electricity procurement other than the spot market. This model is developed purely for assessing the impact of spot prices on the schedules. For assessing the operational impact on a job-shop environment, we develop a novel model for scheduling multiple machines, jobs and processes. This model captures the inherent complexity of scheduling machine-mode changes, scheduling the production process and product movements within machinery. While it is not appropriate for the large-scale problems due to the inherent complexity of the problem (NP-Complete, Garey et al. [1976]), this model is used to assess the impact of the price uncertainty on the operational schedules. Our objective is to find a feasible schedule within the given time horizon and processing capabilities that require minimum cost and finding the optimal start times. implies and facility requires finding the correct configuration of the machines during the production horizon.

4.5.1 Notation

- Processes: Tasks to be performed, $p \in \mathcal{P}$
- Products: A sequetial instantiation of processes, $e \in \mathcal{E}$
- Machines: Facilities capable of hosting multiple jobs, $f \in \mathcal{F}$

- Capabilities: Machines with different sets of processing capabilities, $c_f \subset \mathcal{P}$
- Horizon: T time units, $t \in \{1, ..., T\}$

The notable differences from classical job shop models are listed as: (1) The model integrates configuration of the production environment since machines can change modes. (2) The objective is to minimize energy cost of mode changes and processing.

Classical scheduling problems, depending on the shop environment, may have single/multiple machines that are similar/dissimilar where the objective is to find a sequence of visits/start times of products to minimize tardiness/minimize maximum job length/minimize total time spent/minimize maximum tardiness . . . [Pinedo, 2012]. Job-shop scheduling problems are the most general problems which are amongst the notoriously hard combinatorial optimization problems.

This problem is a variant of job-shop scheduling. In our model, a task is a process execution for a unit length of time, jobs are sets of tasks with precedence constraints and machines are processing centers where execution occurs. Furthermore machines may or may not have the equivalent processing capabilities. Machines with different processes and process transition costs are allowed in this model. A setup time of one unit for a machine to change current processing capability is incurred, along with the power requirement. Machines can process multiple products simultaneously and product movements require one unit of time. In this setting, each product is represented as a sequence of processes that needs to finished in order for completion. Each task takes exactly one unit of time. Products can be moved from machine to machine at no cost within negligible time. The facility network is fully connected, so products can be moved from every facility to every other facility. The objective is to find the optimal starting time and facility of each task in each product sequence which incurs the minimum power cost. Facilities consume power to change modes, i.e. $fac_f^{p,p'}$. Every facility has a different capability set and each facility can be in only one mode at a given time. For a product task to be able to be processed by the facility, the facility's mode must match the current product process, i.e. if the 3^{rd} task of product 1 is process 2, then this task can only start at a facility which is in mode 2. Furthermore, if multiple different jobs are present at the facility simultaneously with their respective task processes matching the facility mode, these tasks can be processed simultaneously. Therefore, routing of products, scheduling of job tasks and machine modes are integrated in one model.

The variables are given below and are explained in the next paragraph:

- Start Time: $\beta_{t,f}^{e,k}$, $x_t^{e,k} \in \{0,1\}$
- Product Location and Movement: $\hat{\alpha}_{e,t}$, $\alpha_{e,t}^{f,f'}$,$y_{t,f}^e \in \{0,1\}$
- Machine Mode: $\hat{\gamma}_{t,f}$, $\gamma_{t,f}^{p,p'}$,$z_{t,f}^p \in \{0,1\}$

The $\beta_{t,f}^{e,k}$ variables are non-zero if the k^{th} process of sequence e starts at time t at facility f. These variables are aggregated over f to create the $x_t^{e,k}$ variables since a task can only start in a single facility for any given time. The location of the product is captured in variables $y_{t,f}^e$ which are non-zero if sequence e is at facility f at time t. The $\alpha_{e,t}^{f,f'}$ track movement of the products and are non-zero if $y_{t,f}^e = y_{t+1,f'}^e = 1$, for $f' \neq f$. These variables are aggregated over f to create the $\hat{\alpha}^{e,t}$ variables which are non-zero if the product e is moving at time t. The $z_{t,f}^p$ variables capture machine modes at facilities and are non-zero if facility f is in mode p at time t. The $\gamma_{t,f}^{p,p'}$ variables track mode changes at facilities and are non-zero if $z_{t,f}^p = z_{t,f}^{p'} = 1$, for $p' \neq p$. They are aggregated over modes to obtain the $\hat{\gamma}_{t,f}$ variables, which simply track if a machine is changing mode at time t or not. The [JSP] is given below and

$price$ stands for the daily price vector:

$$[JSP] \quad \min \sum_{t,f} price_t \Big(\sum_{\{e,k\}} \beta_{t,f}^{e,k} + \sum_{\{p,p'\}} \gamma_{t,f}^{p,p'} \; fac_f^{p,p'} \Big) \tag{4.11a}$$

$$\sum_t x_t^{e,k} = 1 \;\; \forall e,k \tag{4.11b}$$

$$\sum_k x_t^{e,k} \leq 1 \;\; \forall e,t \tag{4.11c}$$

$$\sum_f y_{t,f}^{e} = 1 \;\; \forall e,t \tag{4.11d}$$

$$\sum_p z_{t,f}^{p} = 1 \;\; \forall t,f \tag{4.11e}$$

$$\sum_f \beta_{t,f}^{e,k} - x_t^{e,k} = 0 \;\; \forall e,k,t,f \tag{4.11f}$$

$$\sum_f x_{t'}^{e,k'} + x_t^{e,k} \leq 1 \;\; \forall e,k,t \;\; \forall k' > k \;\; \forall t' \leq t-1 \tag{4.11g}$$

$$\beta_{t,f}^{e,k} - y_{t,f}^{e} \leq 0 \;\; \forall e,t,f \tag{4.11h}$$

$$\beta_{t,f}^{e,k} - y_{t+1,f}^{e} \leq 0 \;\; \forall e,t,f \tag{4.11i}$$

$$\beta_{t,f}^{e,k} - z_{t,f}^{e,k} \leq 0 \;\; \forall e,k,t,f \tag{4.11j}$$

$$\beta_{t,f}^{e,k} + \hat{\alpha}_t^{e} \leq 1 \;\; \forall e,t \tag{4.11k}$$

$$\beta_{t,f}^{e,k} + \hat{\gamma}_t^{e} \leq 1 \;\; \forall t,f \tag{4.11l}$$

$$\hat{\gamma}_{t,f} - \gamma_{t,f}^{p,p'} \geq 0 \;\; \forall t,f,p \neq p' \tag{4.11m}$$

$$z_{t,f}^{p} - \gamma_{t,f}^{p,p'} \geq 0 \;\; \forall t,f,p \neq p' \tag{4.11n}$$

$$z_{t+1,f}^{p} - \gamma_{t,f}^{p,p'} \geq 0 \;\; \forall t,f,p \neq p' \tag{4.11o}$$

$$z_{t,f}^{p} + z_{t+1,f}^{p'} + \gamma_{t,f}^{p,p'} \geq -1 \;\; \forall t,f,p \neq p' \tag{4.11p}$$

$$\hat{\alpha}_{e,t} - \alpha_{e,t}^{f,f'} \geq 0 \;\; \forall e,t,f \neq f' \tag{4.11q}$$

$$y_{t,f}^{e} - \alpha_{e,t}^{f,f'} \geq 0 \;\; \forall e,t,f \neq f' \tag{4.11r}$$

$$y_{t+1,f}^{e} - \alpha_{e,t}^{f,f'} \geq 0 \;\; \forall e,t,f \neq f' \tag{4.11s}$$

$$- y^{e}_{t+1,f'} - y^{e}_{t,f} + \alpha^{f,f'}_{e,t} \geq -1 \;\; \forall e, t, f \neq f' \tag{4.11t}$$

The constraints [4.11b] states that every task should have a start time. The constraints [4.11c] enforces the fact that the tasks of a process can not start simultaneously. The constraints [4.11d] states that every product must be at exactly one facility in each time period. The constraints [4.11e] states that every facility must be in exactly one mode in each time period. The constraints [4.11f] states that β variables are aggregated to obtain x variables. The constraints [4.11g] enforces that the steps that follow a certain task can not start earlier than that task.

The constraints [4.11h-l] enforce that a product needs to be at the facility at times t and $t+1$, the mode of the facility should match the mode of the current product task, the facility should not be changing modes and the product should not be moving, if the k^{th} task of product e is to be started at time t at facility f. The constraints [4.11m-p] govern the facility modes whereas The constraints [4.11q-t] govern product movement.

4.5.2 Model Stats, Possible Reductions, Infeasibility Certificates

Depending on the length of the time horizon and the product sequence length, tasks can be restricted to certain time windows in preprocessing. These windows naturally arise and can be used to reduce the problem size. Combined with the fact that mode changes or movements both require a unit of time, this piece of information is used to do initial capacity assessments. By exploiting the fact that a movement and mode change both requires a unit of time, one can develop supplementary models that would provide infeasibility certificates. One possible method for assessing infeasibility is as follows. Let:

$$e_i := \{P_{(1)}, P_{(2)}, P_{(3)}, ..., P_{(k_i)}\} \tag{4.12a}$$

where e_i stands for the i^{th} job and k_i stands for the number of tasks for job i. Noting that some of the job tasks might be the same, first calculate the total number of unique processes, a_i, that are required for the job as well as $a_{i,l}$, which indicates the number of times process l has to be performed for product i. Next calculate the number of process changes, b_i. Note that if all the unique process are repeated in clustered blocks, the minimum number of process changes would be required; however, if processes alternate at every step, the maximum number of process changes will occur. The importance of this observation is that, depending on the particular ordering of the sequences in the jobs the required time can change significantly. For instance, for a sequence where the same processes are clustered together to follow each other, the minimum time requirement would be $k_i + a_i - 1$. However if the processes are alternating, even with only 2 processes, it would be $2\ k_i - 1$. So the minimum time requirements can be generalized as follows: $d_i = k_i + b_i - 1$.

This piece of information can be used in two ways: first to construct the time windows and to construct a supplementary model to assess the capacity of the processing environment and time horizon.

First solving the following auxiliary problem one can assess the feasibility of the processing environment in the preprocessing stage:

$$[JSP-A] \quad \min\ 1 \tag{4.13a}$$

$$\sum_{\{t,f\}} x_{t,f,p_l} \geq \sum_i a_{i,l} \ \ \forall l \in \{1,...,p\} \tag{4.13b}$$

$$\sum_{p_l \in C_f} x_{t,f,p_l} = 1 \ \ \forall t,f \tag{4.13c}$$

$$\hat{\gamma}_{t,f} - \gamma_{t,f}^{p,p'} \geq 0 \ \ \forall t,f,p \neq p' \tag{4.13d}$$

$$z_{t,f}^{p} - \gamma_{t,f}^{p,p'} \geq 0 \ \ \forall t,f,p \neq p' \tag{4.13e}$$

$$z_{t+1,f}^{p} - \gamma_{t,f}^{p,p'} \geq 0 \ \ \forall t,f,p \neq p' \tag{4.13f}$$

$$z_{t,f}^{p} + z_{t+1,f}^{p'} + \gamma_{t,f}^{p,p'} \geq -1 \ \ \forall t,f,p \neq p' \tag{4.13g}$$

This simple capacity check does not create a proper infeasibility certificate, i.e. a feasible solution to this model does not imply the original model has a feasible solution since it ignores product movements. However, if the original problem is infeasible in the sense that the current processing environment does not provide enough capacity or the time horizon is too short to be able to have a solution, it would be detected by the infeasibility of this problem.

Next, given job i, one can calculate the latest starting times using the minimum time requirements, d_i. If $d_i \geq T+1$, the problem is infeasible. Otherwise the latest start time for task 1 of job i is $T-d_i+1$; for task 2, it is $T-d_i+2$, ... , for task k_i, it is $T-d_i+k_i$. This implies all the β, x variables that correspond to time points after the latest start time can be safely set to 0. The solution of the supplementary capacity checking problem can be used as a starting point; however, it doesn't guarantee a feasible start since it ignores product movements and order structure of the job sequences.

4.6 Numerical Study

The [JSP] may have significantly different instances depending on the production environment and problem parameters. To demonstrate the significance of this, consider a very simple problem where there are only two processes and two products with the following process sequences:

$$\{e_1\} := \{P_1, P_1, P_1, P_2, P_2, P_2\} \tag{4.14a}$$

$$\{e_2\} := \{P_2, P_2, P_2, P_1, P_1, P_1\} \tag{4.14b}$$

If there is only one machine with processing capability $\{P_1, P_2\}$, the optimal solution can be characterized as the following sequence:

$$\underbrace{P_1, P_1, P_1 \Big|\Big| P_2, P_2, P_2}_{\{e_1\}}, \underbrace{P_2, P_2, P_2 \Big|\Big| P_1, P_1, P_1}_{\{e_2\}} \tag{4.15a}$$

where the separating double lines indicate process mode changes. The minimal completion time is 14 time units, i.e. any time horizon with $T \leq 13$ is infeasible. Let us introduce another machine with the same capabilities.

$$M1 \quad \underbrace{P_1, P_1, P_1}_{\{e_1\}} >> \underbrace{P_1, P_1, P_1}_{\{e_2\}} \tag{4.16a}$$

$$M2 \quad \underbrace{P_2, P_2, P_2}_{\{e_2\}} >> \underbrace{P_2, P_2, P_2,}_{\{e_1\}} \tag{4.16b}$$

where $>>$ stands for the product movement from one machine to another. By keeping machine 1 in mode $P1$ and machine 2 in mode $P2$ at all times, now a minimal completion time of 7 time units is achieved considering the 1 unit movement time of product 1 to machine 2 and product 2 to machine 1 at time interval 3. If the process sequences were changed to

$$\{e_1\} := \{P_1, P_2, P_1, P_2, P_1, P_2\} \tag{4.17a}$$

$$\{e_2\} := \{P_2, P_1, P_2, P_1, P_2, P_1\}, \tag{4.17b}$$

the minimal completion time in both one- and two-machine environments would be 12 time units. Even for an example as simple as this, small changes in machine numbers, processing capabilities, product tasks, order of product tasks and other parameters of the problem usually affect the problem structure, its feasibility and the corresponding optimal solution significantly. Hence, to be able to conduct a similar experiment as earlier, the scope of the problem is constrained to one single instance with 4 processes, 2 products of 7 tasks each and 2 facilities. The notation for the production environment and the product tasks is given below:

$$\mathcal{P} = \{P_1, P_2, P_3, P_4\} \tag{4.18a}$$

$$\mathcal{M} = \{M_1 = \{P_1, P_2, P_3\}, M_2 = \{P_2, P_3, P_4\}\} \tag{4.18b}$$

$$\{e_1\} = \{P_1, P_2, P_3, P_4, P_3, P_2, P_4\} \tag{4.18c}$$

$$\{e_2\} = \{P_1, P_4, P_2, P_3, P_3, P_4, P_1\} \tag{4.18d}$$

Each process draws the same level of power rate in each machine. We instantiated this problem using two random seeds where power requirements are randomly chosen. For the first instance the requirements are $\{400, 200, 400, 100\}$ MWs respectively; however process mode changes in different machines draw different amounts of power. The facility mode switch requirements for the first instance are (MWs):

```
facility 1       facility 2
    1   2   3          2   3   4
1   0 300 800       2   0 100 100
2 600   0 200       3 800   0 100
3 300 200   0       4 100 300   0
```

4.6.1 Example Solution

An example solution for the price curve in Figure 4.4 is given in Table 4.4 for instance 1. The first product starts being processed at facility 1 at hours $3, 5, 7$, travels to facility 2 at hour 8, gets processed at hour 9, travels back to facility 1 at hour 10, gets processed at hours $11, 19$, travels to facility 2 to be processed at hour 23. The schedule for product 2 can be read in a similar way. Products 1 and 2 are processed simultaneously at hours $5, 7$ in facility 1.

Product	Hour	Facility	Process
1	3	1	1
1	5	1	2
1	7	1	3
1	9	2	4
1	11	1	3
1	19	1	2
1	23	2	4
2	1	1	1
2	3	2	4
2	5	1	2
2	7	1	3
2	8	1	3
2	10	2	4
2	23	1	1

Table 4.4: Example Schedule for the price curve in Figure 4.4

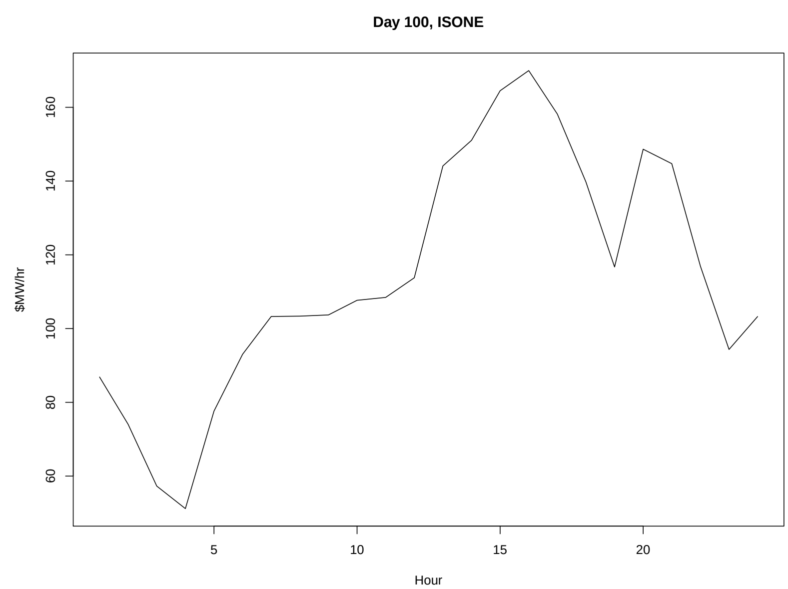

Figure 4.4: Day 100, ISONE

4.6.2 Results

In this section, the results of the numerical study using the model $[JSP]$ and the data selection methods p^1 and p^2. The forecasting power of using p^1 and p^2 methodologies in PJM and ISONE markets are tested and are characterized as a summary of the previously defined accuracy measures, in Tables C.2, C.4, C.6, C.8, C.3, C.5, C.7, C.9. These tables can be found in §C.2. The [JSP] is instantiated twice using different random seeds, hence the instances are similar yet different in terms of power requirements. The first instance is solved using the first 1583 curves from the PJM market and the first 1808 curves from the ISONE market which implies around 58 % of the data is used for instance 1 and the rest is is used for instance 2. Next the ρ values are calculated. The statistical summary of ρ values are given in Tables 4.5 and 4.6.

instance	ρ	mean	Std.Dev.	min	Q1	median	Q3	max
1	$\rho_{p^1}^{[JSP]}$	1.18	1.70	-5.19	0.81	1.02	1.32	58.85
1	$\rho_{p^2}^{[JSP]}$	1.36	2.35	-6.01	0.91	1.15	1.51	84.06
2	$\rho_{p^1}^{[JSP]}$	1.29	2.56	-33.69	1.02	1.14	1.37	70.27
2	$\rho_{p^2}^{[JSP]}$	1.35	2.26	-45.13	1.10	1.23	1.49	42.27

Table 4.5: PJM: $\rho_{p^1}^{[JSP]}$ vs $\rho_{p^2}^{[JSP]}$ statistics, anomalies allowed, instances 1,2

In ISONE market instance 2 ρ values are significantly better than the instance 1 ρ values. By using these tables it is hard to separate p^1 and p^2 data selection rules since the results show conflicting results for different markets. For instance, in PJM market, p^1 is the best choice for both instances; however, both data selection rules perform equally worse in ISONE market for instance 1 whereas for instance 2, again p^1 is the better choice.

instance	ρ	mean	Std.Dev.	min	Q1	median	Q3	max
1	$\rho_{p^1}^{[JSP]}$	1.85	1.88	0.56	1.43	1.65	1.96	76.83
1	$\rho_{p^2}^{[JSP]}$	1.85	1.25	0.92	1.47	1.68	1.97	47.07
2	$\rho_{p^1}^{[JSP]}$	0.48	0.38	0.05	0.25	0.37	0.54	5.21
2	$\rho_{p^2}^{[JSP]}$	0.61	0.46	0.02	0.32	0.46	0.70	4.48

Table 4.6: ISONE: $\rho_{p^1}^{[JSP]}$ vs $\rho_{p^2}^{[JSP]}$ statistics, anomalies allowed, instances 1,2

In PJM market for instances 1 and 2, the mean ρ value is around 1.36 if one were to use [JSP]+p^2, which implies on average by using this policy, one will end up paying 36 % more than the best achievable cost. The anomalous cases are present in both markets as one can observe in the minimum and maximum values of ρ in Tables 4.5 and 4.6. In PJM market, the cases with $\rho \geq 3$ are tagged as anomalous and in ISONE market, the cases with $\rho \geq 5$ are tagged as anomalous. These cases usually correspond to extreme spikes, negative prices and days where both forecasting methodologies failed significantly. The percentage of the anomalous cases are summarized next.

```
# Anomalies in PJM:
# Percentage of data with negative prices (profitable production)
[1] 0.004424779
# Percentage of data where scheduling according to  ARIMA estimates
# incurs a cost that is at least 3 x the real optimal cost
[1] 0.0140118
# Percentage of data where scheduling according to last week's prices
# incurs a cost that is at least 3 x the real optimal cost
[1] 0.01696165
```

```
# Anomalies in ISONE:
# Percentage of data with negative prices (profitable production)
[1] 0
# Percentage of data where scheduling according to  ARIMA estimates
# incurs a cost that is at least 5 x the real optimal cost
[1] 0.002873563
# Percentage of data where scheduling according to last week's prices
# incurs a cost that is at least 5 x the real optimal cost
[1] 0.002554278
```

Next the data points that are tagged as anomalous are removed from the dataset and forecasting accuracy statistics and ρ values are recalculated. For forecasting accuracy, in Tables C.2, C.4, C.3, C.5, the anomalous observations are included in accuracy calculations and in Tables C.6, C.8, C.7, C.9, those observations are excluded. In Tables 4.7 and 4.8, the results of ρ recalculations are presented.

instance	ρ	mean	Std.Dev.	min	Q1	median	Q3	max
1	$\rho_{p^1}^{[JSP]}$	1.09	0.42	0.02	0.81	1.01	1.30	2.98
1	$\rho_{p^2}^{[JSP]}$	1.24	0.48	0.09	0.91	1.14	1.49	2.98
2	$\rho_{p^1}^{[JSP]}$	1.24	0.36	0.03	1.02	1.14	1.35	2.93
2	$\rho_{p^2}^{[JSP]}$	1.34	0.38	0.38	1.10	1.23	1.47	2.99

Table 4.7: PJM: $\rho_{p^1}^{[JSP]}$ vs $\rho_{p^2}^{[JSP]}$ statistics, anomalies removed, instances 1,2

With anomalies removed, the ρ values in both markets has improved. However the results of the previous analysis with anomalies included are still valid.

instance	ρ	mean	Std.Dev.	min	Q1	median	Q3	max
1	$\rho_{p^1}^{[JSP]}$	1.79	0.56	0.56	1.43	1.65	1.96	4.82
1	$\rho_{p^2}^{[JSP]}$	1.80	0.51	0.92	1.47	1.68	1.95	4.73
2	$\rho_{p^1}^{[JSP]}$	0.48	0.36	0.05	0.25	0.37	0.54	3.61
2	$\rho_{p^2}^{[JSP]}$	0.61	0.46	0.02	0.32	0.46	0.69	4.48

Table 4.8: ISONE: $\rho_{p^1}^{[JSP]}$ vs $\rho_{p^2}^{[JSP]}$ statistics, anomalies removed, instances 1,2

4.7 Conclusion

In this chapter, an experiment is conducted to compare various optimization models and data selection approaches to measure the impact of the price uncertainty on the optimal solutions of two classes of operational problems: production planning and job-shop scheduling. For each class of problem, some specific instances are chosen and the models are simulated using real prices from ISONE and PJM markets. For instantiating the problem classes, representative optimization model templates such as Linear Programming, Robust Optimization and Stochastic Programming are used. Due to this fact, we don't claim that, for different modeling approaches, the numerical results will be similar. However the templates we have chosen have two important characteristics: First, the models capture the gist of the problem class and chosen optimization methodology; second, the models are kept simple so that the dependence on the particular characteristics of the chosen production/scheduling setting is weak.

For the simulation part of the experiment, different horizons for estimation accuracy are compared. The reason for this comparison is to test the hypothesis that there might be common time intervals within years where the forecasts consistently require more data which can be classified as a pattern. However the same patterns, if there are any, might not be valid for the optimal production plans and schedules. We didn't test the formal hypothesis. So it stays as our conjecture; however, the automatically fit ARIMA models show significant changes in terms of selected model parameters depending on the time horizon chosen for a particular day and the time horizon length, which in our opinion is supporting evidence.

For production planning problems, our findings indicate that using [SPP]+p^1 or [SPP]+p^2 is favorable to using [RPP.1-5] or SOPP. The [SPP]+p_i $i = \{1, 2\}$ is advantageous for the following reasons:

- The uncertainty modeling step and optimization step are modular in the sense that one can change either of them without affecting the other step. In the latter approach, a change in the uncertainty model may also require a significant change in the optimization model.

- Both steps can be unified in a simulation optimization approach which provides a flexible decision making-foundation. The latter approach is by construction integrated which implies it might not be easily separated as the former approach.

- The latter approach might be favorable when the uncertainty is well-structured or well-known, integration with the optimization model does not cause emergent complexity and data size is relatively small. However as shown in §3, the uncertainty in electricity prices in both markets is quite hard to model and depends significantly on the time interval selected.

- In the former approach, optimization models are considerably smaller, which makes a significant impact since some of these models are mixed integer linear (or quadratic) models. In the latter approach, due to the model's integrated nature, there are usually more constraints and variables required to weave the uncertain parameters into the optimization model. The nature of the uncertainty might be very sophisticated and complex. For example statistical methods may fail to identify the underlying factors, the majority of the forecasting models may fail to provide accurate estimates or the uncertainty may have a well-characterizable structure; however, the statistical characterization may be too complex for integration into mathematical models. In these cases the resulting complexity will emerge from the complexity of the uncertainty and complexity of the optimization, which will render the integrated estimation-optimization approach inapplicable.

- Since optimization problems are instantiated for specific price instances, the former approach is naturally parallelizable and more appropriate for massive data sets.

For the job scheduling model, we found that, using last week's prices as estimates is favorable to using ARIMA estimates. Comparing ρ values of production planning and scheduling problems, we observe significant differences. For the production planning problems, [SPP]$+p^2$ is found to be best approach in both markets. This method performs better in ISONE market compared to PJM market, however only marginally. The ρ values are significantly higher for the [JSP] which indicates this model is more sensitive to price uncertainty compared to production planning models.

Chapter 5

Conclusion & Future Study

The interruption uncertainty framework we describe in Chapter 2 allows different contract rules and operational rules to be embedded into the production planning problem simultaneously. As we discussed in §2.4, it is straightforward to embed operational procedures that companies may implement in the case of interruptions, such as limiting the production in post-interruption recovery or prohibiting production level increases in some periods. Our study shows that the right modeling approach coupled with simple heuristic rules is very effective solving in this problem. Our framework could be used under different types of ILCs, such as the pay-in-advance and pay-as-you-go reward schemes described by Baldick et al. [2006]. Moreover, information regarding the utility's optimal interruption dispatch behavior can be embedded into our Stackelberg-like production planning framework. However, the extent to which the theoretical results and computational performance presented above will be preserved under different ILC types or interruption dispatch behaviors is a topic for future study. An important future avenue for study is to include demand uncertainty into the model.

In Chapter 3, we conducted an empirical analysis of electricity market prices in the PJM and ISONE markets using different time scales and horizons. The daily perspective clearly shows that there are changing daily price profiles in both markets. By using intraday quantiles and standard deviations, one can easily identify the peaking hours; however, the data shows that through out years and seasons, peaking hours can change significantly. Intraday prices and same-hour inter-day prices are found to be highly correlated however two markets show different patterns in terms of intraday price correlation. Furthermore, after significantly volatile years, there is evidence for actions taken (by regulation or by market participants) which would mitigate the situation for the next year. The data suggests that uncertainty sets constructed by aggregating the data over years or months will obscure the characteristics of the data that is significantly dependent on the chosen time frame. Therefore in both markets, short term data is found to be a good candidate for describing daily and weekly patterns. Dynamically fit ARIMA models with varying short horizons are found to be accurate in terms of forecasting daily LMPs. Automatically fit ARIMA models show significant changes in terms of selected model parameters depending on the time horizon chosen for a particular day and the time horizon length. For future study, we aim to focus on using the machine learning techniques to dynamically structure the uncertainty sets and optimization problems. Our initial experiments with this approach were not successful in terms of identifying spikes however clustering price curves and extracting the information from the attributes (i.e. labels) of these curves still looks like a promising way to structure the uncertainty sets.

In Chapter 4, an experiment is conducted to compare various optimization models and data selection approaches and to measure the impact of the price uncertainty on the optimal solutions of two classes of operational problems: production planning and job-shop scheduling. For each class of problem, specific instances are chosen and the models are simulated using real prices from the ISONE and PJM markets. For instantiating the problem classes, representative optimization model templates such as Linear Programming, Robust Optimization and Stochastic Programming are used. For the simulation part of the experiment, different horizons for estimation accuracy are compared. Our findings indicate that using ARIMA models with short time horizons to estimate the objective function coupled with simpler mathematical models is slightly favorable to using complicated optimization models where the data is used to construct an uncertainty set and this uncertainty set is embedded in the model. As for future study, using pattern recognition to identify representative price curves and measuring the proximity of the representative optimal solutions to real the optimal solutions is another direction we are planning to explore.

Appendix A

Proofs and Tables

A.1 Proof of Proposition 2.3.1

Note that for given t, g, $IOP(t, g)$ is:

$$\min_{\mathcal{U}} \left\{ \sum_{p=1}^{P} \overline{inv}_{t,p,g} \right\} \geq 0 \ \ \forall t, g \tag{A.1}$$

where

$$\mathcal{U} = \left\{ \xi \in \{0,1\}^{T \times P} \middle| \sum_{p=1}^{P} \sum_{t=1}^{T} \xi_{t,p} \leq K \ \ , \ \ \sum_{p=1}^{P} \xi_{t,p} \leq 1 \ \ \forall t \ \ \right\} \tag{A.2}$$

Before proving Proposition 2.3.1 (total unimodularity of the constraint matrix of IOP), we first give three well known properties of TU matrices (see, e.g., Hoffman and Kruskal [1956]).

Lemma A.1.1. *A matrix* $\mathbf{A}$ *is TU* $\iff$ $\mathbf{A^T}$ *is TU.*

Lemma A.1.2. *A matrix* $\mathbf{A}$ *is TU* $\iff$ $[\mathbf{A} \quad \mathbf{I}]$ *is TU.*

Lemma A.1.3. *Let $\mathbf{A}$ be an $m \times n$ matrix whose rows can be partitioned into two disjoint sets $\mathcal{B}$ and $\mathcal{C}$ with the following properties:*

1. *Every column of $\mathbf{A}$ contains at most two non-zero entries;*
2. *Each entry is 0, 1, or -1;*
3. *If two non-zero entries in a column of $\mathbf{A}$ have the same sign, then the row of one is in $\mathcal{B}$, and the other in $\mathcal{C}$;*
4. *If two non-zero entries in a column of $\mathbf{A}$ have opposite signs, then the rows of both are in $\mathcal{B}$, or both in $\mathcal{C}$.*

Then $\mathbf{A}$ is TU.

Consider the constraint matrix $\mathbf{A}$ defined by $\mathcal{U}$. It has the following form:

$$\mathbf{A} = \begin{bmatrix} \mathbf{e}^T \\ \mathbf{A}_T \quad \ldots \quad \mathbf{I}_T \\ \mathbf{I}_{T\times P} \end{bmatrix} \tag{A.3}$$

where

$$\mathbf{e}^T = [1 \ldots 1]_{T\times P}$$

. The first row of $\mathbf{A}$, $\mathbf{e}^T$, captures the coefficients from the second inequality in $\mathcal{U}$. The second part, which is constructed by repeating $\mathbf{I}_T$ P times, captures the coefficients from the first set of inequalities in $\mathcal{U}$. Finally, $\mathbf{I}_{T\times P}$ is an identity matrix and captures the coefficients from the upper-bound inequalities for $\xi_{t,p}$. Now, define $\hat{\mathbf{A}}$

such that

$$\hat{\mathbf{A}} = \begin{bmatrix} & \mathbf{e}^T & \\ \mathbf{I}_T & \ldots & \mathbf{I}_T \end{bmatrix} \tag{A.4}$$

Observe that $\hat{\mathbf{A}}$ is TU since it satisfies the conditions of Lemma A.1.3. The first two conditions of Lemma A.1.3 are satisfied trivially. For the third condition, all the nonzero elements of $\hat{\mathbf{A}}$ are positive, therefore one can construct an appropriate partition of the rows by putting the first row of $\hat{\mathbf{A}}$ in the set $\mathcal{B}$ and the rest of the rows in the set $\mathcal{C}$. The fourth condition doesn't apply since there are no negative elements in $\hat{\mathbf{A}}$. Now, by Lemma A.1.1, $\hat{\mathbf{A}}^T$ is also TU. By Lemma A.1.2, we can augment this matrix with $\mathbf{I}_{T\times P}$ to obtain $\mathbf{A}^T = [\hat{\mathbf{A}}^T \quad \mathbf{I}_{T\times P}]$ and still retain the TU property. Finally, by Lemma A.1.1, we see that $\mathbf{A}$ is indeed TU. □

A.2 Tables

Problem	Total Demand (units)	Z(ALG1) (units)	Time (millisecs)	Z(ROP) (units)	Time (millisecs)	Δ	Ψ
5.d1	1,796,364	1,896,364.0	153.6	1,896,364	0	0	105.57%
5.d2	2,026,226	2,105,192.0	1.6	2,105,192	0	0	103.90%
5.d3	2,049,154	2,054,615.6	3.1	2,054,615	0	0.6	100.27%
5.d4	2,082,395	2,182,395.0	0.4	2,182,395	0	0	104.80%
5.d5	1,965,692	2,065,692.0	0.4	2,065,692	0	0	105.09%
10.d1	3,106,557	3,401,842.2	4.1	3,401,842	0	0.2	109.51%
10.d2	2,555,252	2,650,541.8	0.4	2,650,541	0	0.8	103.73%
10.d3	2,611,483	2,679,425.6	0.4	2,679,425	0	0.6	102.60%
10.d4	3,264,872	3,563,316.0	0.6	3,563,316	0	0	109.14%
10.d5	3,187,867	3,555,612.0	0.6	3,555,612	10	0	111.54%
20.d1	5,411,358	6,321,459.0	0.8	6,321,459	10	0	116.82%
20.d2	4,492,461	5,301,452.0	0.6	5,301,452	10	0	118.01%
20.d3	5,135,481	5,937,552.0	3.5	5,937,552	10	0	115.62%
20.d4	5,360,054	5,877,101.5	0.6	5,877,101	10	0.5	109.65%
20.d5	5,821,935	6,987,935.0	0.6	6,987,935	10	0	120.03%
40.d1	10,151,455	12,311,710.5	1.7	12,311,710	40	0.5	121.28%
40.d2	10,533,148	11,843,507.1	1.5	11,843,507	40	0.1	112.44%
40.d3	10,224,667	12,492,212.0	1.1	12,492,212	50	0	122.18%
40.d4	10,389,448	12,639,871.4	1.1	12,639,871	40	0.4	121.66%
40.d5	10,252,456	11,444,351.6	1.1	11,444,351	50	0.6	111.63%
80.d1	20,538,055	24,389,433.0	3.2	24,389,433	190	0	118.75%
80.d2	20,586,039	24,093,313.8	1.9	24,093,313	180	0.8	117.04%
80.d3	21,014,425	25,910,897.5	3	25,910,897	190	0.5	123.30%
80.d4	18,607,001	22,115,780.0	2.5	22,115,779	180	1	118.86%
80.d5	20,092,741	23,837,931.0	1.8	23,837,931	190	0	118.64%
160.d1	38,474,902	44,946,580.7	6.4	44,946,580	860	0.7	116.82%
160.d2	42,893,082	50,718,101.4	5.7	50,718,101	880	0.4	118.24%
160.d3	43,904,907	51,734,708.0	3.5	51,734,708	890	0	117.83%
160.d4	41,125,495	48,696,578.7	4.3	48,696,578	850	0.7	118.41%
160.d5	38,225,164	44,297,082.3	3.3	44,297,082	890	0.3	115.88%

Table A.1: 20% Interruption Rate

Problem	Total Demand (units)	Z(ALG1) (units)	Time (millisecs)	Z(ROP) (units)	Time (millisecs)	Δ	Ψ
5.d1	1,796,364	2349731.0	7.4	2,349,731	0	0	130.80%
5.d2	2,026,226	2605192.0	1	2,605,192	0	0	128.57%
5.d3	2,049,154	2321216.0	0.4	2,321,215	0	1	113.28%
5.d4	2,082,395	2562477.0	0.4	2,562,477	0	0	123.05%
5.d5	1,965,692	2565692.0	0.5	2,565,692	0	0	130.52%
10.d1	3,106,557	4182983.0	0.5	4,182,983	0	0	134.65%
10.d2	2,555,252	3142691.3	0.5	3,142,691	0	0.3	122.99%
10.d3	2,611,483	3014353.8	0.5	3,014,353	0	0.8	115.43%
10.d4	3,264,872	4254910.0	3.7	4,254,910	0	0	130.32%
10.d5	3,187,867	4522948.0	0.5	4,522,948	0	0	141.88%
20.d1	5,411,358	7239097.0	1.3	7,239,097	10	0	133.78%
20.d2	4,492,461	5959097.5	0.8	5,959,097	10	0.5	132.65%
20.d3	5,135,481	6995207.0	0.8	6,995,207	10	0	136.21%
20.d4	5,360,054	7087144.3	1.2	7,087,144	10	0.3	132.22%
20.d5	5,821,935	8178779.2	1.1	8,178,779	10	0.2	140.48%
40.d1	10,151,455	13936488.9	2.1	13,936,488	50	0.9	137.29%
40.d2	10,533,148	14473102.0	1.1	14,473,101	40	1	137.41%
40.d3	10,224,667	14368459.0	1.8	14,368,459	50	0	140.53%
40.d4	10,389,448	14484549.9	1.5	14,484,549	30	0.9	139.42%
40.d5	10,252,456	13686400.1	1.1	13,686,400	50	0.1	133.49%
80.d1	20,538,055	27693071.0	4.8	27,693,071	190	0	134.84%
80.d2	20,586,039	27865140.3	2.9	27,865,140	190	0.3	135.36%
80.d3	21,014,425	31301283.0	1.9	31,301,283	180	0	148.95%
80.d4	18,607,001	26346772.0	4.2	26,346,771	180	1	141.60%
80.d5	20,092,741	27710980.0	1.7	27,710,979	180	1	137.92%
160.d1	38,474,902	51751744.7	6.7	51,751,744	870	0.7	134.51%
160.d2	42,893,082	59498000.3	11	59,498,000	950	0.3	138.71%
160.d3	43,904,907	59081412.7	4	59,081,412	870	0.7	134.57%
160.d4	41,125,495	57495417.0	2.6	57,495,417	870	0	139.80%
160.d5	38,225,164	53016172.0	2.7	53,016,171	870	1	138.69%

Table A.2: 40% Interruption Rate

Appendix B

Empirical Study: Tables and Plots

	whour	mean	Std.Dev.	min	Q1	median	Q3	max
1	1.00	45.08	20.75	0.00	31.52	40.03	54.26	154.30
2	2.00	43.25	21.22	0.00	30.07	38.66	51.52	166.42
3	3.00	41.79	21.12	0.00	30.08	38.62	48.67	161.32
4	4.00	40.23	21.50	0.00	29.02	37.91	47.16	177.85
5	5.00	41.10	21.66	0.00	30.55	38.07	48.47	170.69
6	6.00	46.07	24.56	0.00	32.39	40.47	52.62	233.80
7	7.00	59.45	32.04	0.00	40.33	50.03	69.59	253.27
8	8.00	63.63	30.49	0.00	43.57	55.52	75.06	217.34
9	9.00	64.63	29.27	0.00	45.28	58.10	75.41	243.81
10	10.00	67.56	28.82	18.48	48.02	60.15	78.80	208.16
11	11.00	70.37	30.09	22.07	50.02	62.95	81.18	243.19
12	12.00	70.31	30.90	19.91	49.73	62.67	81.58	263.41
13	13.00	67.64	29.72	20.37	48.65	60.40	78.84	250.21
14	14.00	69.09	33.20	19.52	48.06	60.73	78.34	307.40
15	15.00	66.11	30.21	20.11	46.36	58.45	75.42	248.38

16	16.00	65.97	34.19	20.77	46.39	57.27	73.75	268.50
17	17.00	73.78	46.38	21.48	50.17	63.78	84.37	645.99
18	18.00	77.92	38.11	22.68	51.75	67.04	93.58	288.71
19	19.00	73.69	34.92	19.83	50.23	64.45	87.12	308.22
20	20.00	71.18	29.78	24.15	51.57	63.19	82.90	210.62
21	21.00	67.76	27.39	25.06	48.91	61.03	78.98	209.43
22	22.00	58.76	24.54	21.63	42.31	52.27	67.37	216.62
23	23.00	48.74	18.99	19.69	34.99	45.09	58.23	151.10
24	24.00	46.05	19.18	1.19	33.10	41.64	53.77	142.65
25	25.00	47.07	20.75	0.00	32.97	41.88	56.21	150.47
26	26.00	45.09	20.80	0.00	31.46	40.42	52.74	151.98
27	27.00	41.12	19.52	0.00	29.49	37.69	48.46	110.08
28	28.00	40.03	19.91	0.00	29.41	37.16	47.76	139.42
29	29.00	42.57	21.23	0.00	31.14	39.06	51.38	149.18
30	30.00	47.67	23.29	0.00	33.95	42.20	55.36	198.77
31	31.00	60.14	28.89	16.96	42.42	53.42	67.11	243.24
32	32.00	63.61	28.81	11.75	46.72	57.38	72.79	288.08
33	33.00	62.48	24.37	21.07	45.66	57.66	72.00	181.25
34	34.00	64.44	26.38	21.94	47.40	58.32	74.19	241.94
35	35.00	68.20	31.69	25.02	49.12	61.55	78.53	337.12
36	36.00	68.29	30.33	23.74	50.07	61.76	78.39	359.61
37	37.00	65.89	26.64	25.42	48.74	59.56	75.33	205.51
38	38.00	67.52	30.49	23.01	48.38	59.94	75.86	252.68
39	39.00	66.62	32.79	23.56	46.46	58.67	74.75	269.66
40	40.00	65.33	30.95	25.84	46.47	58.13	73.54	264.51
41	41.00	72.59	52.29	24.82	49.15	63.19	81.39	901.61
42	42.00	76.70	53.92	12.27	50.28	65.65	86.82	937.63

43	43.00	73.26	48.94	23.95	49.85	63.78	84.68	856.06
44	44.00	68.19	26.38	28.64	49.23	62.69	78.64	186.33
45	45.00	66.02	25.96	23.32	50.11	59.62	75.64	251.06
46	46.00	58.67	21.86	20.83	43.62	54.98	67.79	205.76
47	47.00	49.62	18.19	10.11	36.96	46.59	57.20	120.89
48	48.00	47.35	19.22	9.42	34.82	42.31	55.59	129.69
49	49.00	47.89	20.47	17.86	34.56	43.24	54.63	138.52
50	50.00	45.51	18.88	2.85	33.15	41.18	52.79	133.70
51	51.00	43.45	20.64	0.00	30.48	38.66	51.62	196.21
52	52.00	41.24	18.76	0.00	29.90	37.66	50.34	136.90
53	53.00	43.26	19.92	0.00	31.64	39.59	50.73	152.74
54	54.00	46.91	18.46	0.00	35.02	42.24	54.95	147.31
55	55.00	59.16	24.61	18.28	41.62	54.23	71.20	198.65
56	56.00	63.56	26.55	21.34	45.20	58.09	74.15	258.07
57	57.00	63.47	25.31	22.09	46.68	59.51	74.26	281.46
58	58.00	66.01	28.27	23.46	48.47	59.43	76.40	341.29
59	59.00	67.21	26.64	27.16	49.84	61.63	77.89	235.59
60	60.00	68.13	28.48	29.39	50.58	61.90	79.04	329.37
61	61.00	68.29	36.05	28.68	48.57	60.59	76.01	544.50
62	62.00	70.30	53.48	28.07	48.45	60.23	77.30	998.49
63	63.00	67.47	53.03	28.67	46.98	57.89	74.09	1015.86
64	64.00	67.97	53.60	28.10	47.01	57.83	73.58	1001.31
65	65.00	73.70	59.20	27.90	50.36	62.18	81.41	1014.02
66	66.00	77.07	54.61	26.55	50.50	65.50	90.89	920.29
67	67.00	70.14	29.91	22.01	48.25	63.50	83.01	279.22
68	68.00	68.08	27.09	23.88	49.11	62.63	78.86	212.98
69	69.00	67.86	27.58	26.62	50.12	62.09	78.50	248.29

70	70.00	59.10	22.44	22.53	42.82	54.31	68.20	161.76
71	71.00	50.45	18.61	16.05	36.87	46.19	58.30	133.11
72	72.00	47.94	19.26	3.23	35.62	43.52	55.47	153.10
73	73.00	48.35	19.69	5.70	35.95	42.94	56.13	154.18
74	74.00	46.62	19.75	2.98	34.14	41.53	54.75	149.98
75	75.00	43.52	21.92	0.00	31.48	38.57	50.88	229.99
76	76.00	41.87	19.61	0.00	29.91	37.46	49.88	141.16
77	77.00	44.21	19.44	0.00	32.47	39.45	52.15	185.75
78	78.00	47.54	21.02	0.00	34.11	42.28	55.70	203.05
79	79.00	59.85	31.58	0.00	41.01	52.86	68.42	353.78
80	80.00	62.27	28.11	0.00	43.88	56.60	72.20	302.36
81	81.00	63.02	26.39	0.00	45.14	58.05	72.07	266.94
82	82.00	65.84	26.36	24.83	47.71	60.28	76.61	195.43
83	83.00	67.95	27.95	15.63	49.26	62.35	79.03	222.42
84	84.00	67.95	28.92	27.45	48.56	61.02	77.63	215.78
85	85.00	66.34	28.38	26.24	47.79	59.88	75.64	218.78
86	86.00	67.23	35.87	26.82	47.47	58.55	75.61	499.71
87	87.00	64.17	29.13	5.98	45.39	57.44	73.34	270.74
88	88.00	63.49	27.89	0.00	44.54	57.69	73.37	216.48
89	89.00	68.05	29.85	18.89	48.05	61.51	78.48	223.66
90	90.00	70.66	32.19	23.85	48.76	62.73	83.11	265.49
91	91.00	68.17	28.75	26.31	48.92	61.80	81.69	256.81
92	92.00	68.31	30.56	26.50	48.79	61.70	80.83	377.80
93	93.00	67.62	27.12	27.72	48.61	62.02	77.69	208.07
94	94.00	60.63	23.38	24.23	44.09	56.34	68.67	196.08
95	95.00	51.57	18.66	21.30	38.58	48.40	60.00	135.74
96	96.00	48.97	19.86	14.74	35.05	44.19	57.57	149.57

97	97.00	48.55	19.33	0.00	35.43	44.64	56.77	125.67
98	98.00	46.42	19.71	0.00	33.11	42.37	55.61	146.52
99	99.00	43.65	19.59	0.00	31.54	40.60	51.67	149.79
100	100.00	41.70	19.11	0.00	30.27	38.71	49.39	125.47
101	101.00	43.86	18.75	0.00	32.83	39.80	51.23	117.08
102	102.00	47.60	20.85	0.00	34.12	42.42	55.69	183.57
103	103.00	59.46	29.38	7.41	40.81	52.20	68.73	261.37
104	104.00	63.77	28.50	0.00	44.08	56.31	75.11	200.16
105	105.00	64.69	27.06	20.25	46.27	58.35	76.63	251.46
106	106.00	67.45	27.28	23.72	48.91	61.49	78.08	240.24
107	107.00	70.28	30.64	27.53	49.83	63.66	80.88	356.70
108	108.00	69.40	29.34	27.47	50.23	61.51	80.38	265.31
109	109.00	67.83	33.55	26.88	48.72	61.23	77.31	422.29
110	110.00	68.32	36.38	28.07	48.44	60.84	76.88	558.55
111	111.00	65.33	33.61	22.59	45.62	59.15	74.11	474.29
112	112.00	63.66	31.57	24.99	44.17	57.23	72.17	455.18
113	113.00	67.41	29.10	26.11	47.73	60.73	77.23	201.00
114	114.00	69.90	51.95	28.50	48.15	60.30	81.60	998.41
115	115.00	63.77	25.69	13.11	45.67	59.44	75.96	267.88
116	116.00	61.46	23.58	24.05	45.26	56.22	71.17	246.02
117	117.00	61.22	22.22	23.56	45.88	56.13	72.75	178.31
118	118.00	57.90	20.43	20.26	43.42	54.13	65.95	145.30
119	119.00	53.04	19.26	11.50	39.48	50.24	61.03	146.26
120	120.00	51.96	20.06	17.41	38.10	47.09	60.30	143.70
121	121.00	52.72	21.31	19.13	38.46	47.09	61.56	137.36
122	122.00	51.75	23.16	2.91	36.03	45.80	60.46	175.14
123	123.00	49.07	23.40	0.00	33.64	43.34	56.83	211.88

124	124.00	46.49	21.18	0.00	33.03	41.27	55.46	129.62
125	125.00	46.31	22.47	0.00	32.86	41.33	55.41	211.37
126	126.00	46.14	20.96	0.00	32.88	41.05	56.33	131.33
127	127.00	45.93	22.63	0.00	32.95	41.42	54.89	204.46
128	128.00	48.48	21.66	0.00	34.83	44.28	58.18	148.74
129	129.00	60.43	25.53	17.99	42.36	55.51	73.22	221.65
130	130.00	66.89	29.95	23.41	47.38	61.56	79.03	297.92
131	131.00	68.99	34.65	26.97	48.76	63.49	78.36	398.60
132	132.00	67.44	31.45	23.36	48.23	60.82	76.51	359.18
133	133.00	63.91	28.47	11.09	46.06	57.66	74.75	267.10
134	134.00	59.14	23.25	6.82	44.06	53.23	69.47	186.26
135	135.00	56.75	22.88	17.62	41.54	51.51	66.57	192.40
136	136.00	57.69	25.92	21.81	41.89	51.38	66.78	266.60
137	137.00	64.33	31.46	22.53	44.41	56.31	74.81	303.36
138	138.00	69.26	32.67	22.88	47.19	60.88	83.61	251.97
139	139.00	66.33	27.86	22.84	46.89	59.61	80.06	221.29
140	140.00	63.54	24.51	17.35	46.78	58.05	74.33	169.44
141	141.00	63.96	25.20	24.05	46.13	58.40	75.61	189.92
142	142.00	59.26	23.11	22.10	42.05	54.24	70.69	177.53
143	143.00	54.39	21.19	20.91	38.95	50.13	63.53	175.95
144	144.00	52.96	21.93	19.87	37.81	47.77	61.09	183.52
145	145.00	49.94	21.00	9.29	35.87	44.45	57.84	154.41
146	146.00	47.32	23.50	0.00	33.62	43.37	55.72	157.70
147	147.00	46.67	22.92	0.00	33.30	42.05	54.45	197.26
148	148.00	43.60	21.20	0.00	31.96	39.48	51.05	169.59
149	149.00	42.64	20.38	0.00	31.29	38.71	50.80	149.41
150	150.00	42.65	21.18	0.00	30.68	38.68	51.59	141.92

151	151.00	40.16	21.36	0.00	28.62	36.83	48.73	156.13
152	152.00	40.64	19.43	0.00	29.38	36.32	50.46	148.09
153	153.00	49.61	21.36	0.00	35.73	44.73	58.63	189.48
154	154.00	56.18	25.15	9.56	39.61	49.94	65.05	196.74
155	155.00	58.82	26.39	18.19	42.38	52.95	66.45	278.74
156	156.00	60.45	26.33	16.36	43.61	54.37	69.25	232.42
157	157.00	60.30	25.51	5.13	43.55	55.85	68.54	214.40
158	158.00	57.83	24.66	0.00	41.82	53.13	67.68	214.68
159	159.00	55.14	22.77	0.00	39.76	50.73	65.17	173.51
160	160.00	55.98	23.95	0.00	39.99	51.35	66.17	181.92
161	161.00	64.84	30.25	0.00	43.94	58.66	75.83	218.51
162	162.00	72.75	37.34	0.00	47.09	64.83	88.58	260.77
163	163.00	71.77	34.64	0.00	47.63	64.45	86.80	258.29
164	164.00	70.77	33.68	0.00	47.88	63.69	82.77	275.28
165	165.00	70.48	34.42	13.79	48.97	62.77	82.47	335.89
166	166.00	59.56	24.76	17.51	42.09	53.38	70.78	192.50
167	167.00	50.35	20.32	15.93	35.89	46.16	58.75	152.31
168	168.00	46.64	18.85	0.00	33.14	42.87	54.99	151.50

Table B.1: ISONE Aggregate Week-Hourly Statistics

	whour	mean	Std.Dev.	min	Q1	median	Q3	max
1	1.00	31.19	15.20	-12.61	23.83	28.43	34.80	156.57
2	2.00	29.20	14.51	0.00	23.24	27.25	32.77	128.16
3	3.00	27.04	14.01	-19.47	21.68	25.75	31.93	133.41
4	4.00	26.78	15.69	-12.48	20.30	25.44	30.78	143.01
5	5.00	28.38	15.67	-5.36	22.22	26.50	31.30	177.55
6	6.00	34.91	20.84	-20.45	25.35	29.93	37.89	200.08
7	7.00	51.38	32.72	-3.60	30.88	41.29	62.08	228.86
8	8.00	53.11	37.79	-17.85	33.33	42.47	61.44	430.69
9	9.00	52.94	26.93	12.77	35.18	44.60	61.69	223.95
10	10.00	57.24	26.55	16.69	38.69	48.01	69.10	182.36
11	11.00	63.65	27.70	19.64	42.74	56.92	78.63	193.66
12	12.00	64.02	29.14	23.12	42.63	56.45	78.38	205.55
13	13.00	63.17	29.33	24.77	40.90	55.44	77.01	199.95
14	14.00	65.26	34.81	23.05	40.53	56.62	79.33	245.57
15	15.00	63.47	41.63	22.62	39.11	50.64	75.59	484.35
16	16.00	64.75	42.51	22.23	37.58	50.38	79.00	363.47
17	17.00	68.69	43.27	22.96	40.04	53.39	85.32	378.19
18	18.00	74.87	38.38	23.77	45.18	61.53	99.21	239.82
19	19.00	69.87	36.18	21.88	41.48	59.09	90.94	239.37
20	20.00	67.48	32.58	21.87	42.62	59.04	83.75	187.17
21	21.00	68.87	32.83	22.99	44.88	61.80	83.75	221.43
22	22.00	57.60	26.05	21.92	38.49	50.34	69.47	182.93
23	23.00	41.09	15.87	18.93	30.30	36.44	45.99	111.86
24	24.00	35.76	13.71	-1.62	27.62	31.82	41.00	120.02
25	25.00	32.65	13.29	-2.65	25.70	29.76	36.61	97.60

26	26.00	32.30	15.09	0.00	24.11	29.07	36.25	137.53
27	27.00	29.49	15.40	-26.22	22.63	27.32	33.32	118.12
28	28.00	27.40	15.02	-38.44	21.39	26.46	31.90	126.83
29	29.00	30.22	14.93	-32.49	23.67	27.75	33.57	108.30
30	30.00	36.79	19.19	0.00	27.45	31.66	39.08	198.30
31	31.00	53.84	30.25	14.22	33.85	44.63	64.31	251.59
32	32.00	54.80	29.36	19.86	35.06	47.03	63.33	218.96
33	33.00	50.48	21.29	23.03	35.24	44.23	59.82	157.29
34	34.00	53.76	23.34	25.89	37.39	47.42	59.83	175.57
35	35.00	59.10	26.24	25.17	40.15	52.55	70.49	209.70
36	36.00	60.04	26.64	25.29	40.16	52.94	71.38	195.92
37	37.00	62.23	35.18	16.85	39.68	52.48	74.08	397.82
38	38.00	65.21	39.89	5.81	40.23	53.85	75.46	395.71
39	39.00	65.73	45.34	2.83	37.78	51.05	77.69	502.87
40	40.00	67.22	59.56	7.01	36.74	50.45	79.67	716.44
41	41.00	72.49	66.67	23.40	39.40	56.29	83.97	770.65
42	42.00	76.58	62.58	19.96	44.42	63.71	89.16	763.78
43	43.00	69.13	35.69	20.73	43.62	60.37	81.43	255.71
44	44.00	65.66	29.70	24.11	42.44	57.77	81.05	166.64
45	45.00	66.53	29.08	26.85	44.90	58.33	79.03	210.64
46	46.00	58.91	28.25	26.31	38.32	50.65	71.52	222.05
47	47.00	42.74	17.68	18.76	31.52	37.47	47.64	128.93
48	48.00	38.38	15.75	14.41	28.42	33.73	42.33	111.86
49	49.00	34.65	15.42	-8.39	26.30	31.10	38.34	136.20
50	50.00	33.73	18.13	0.00	24.82	29.32	36.82	180.07
51	51.00	30.49	14.81	-5.16	23.47	28.12	35.05	138.48
52	52.00	29.07	14.02	-11.74	22.72	27.34	33.05	145.23

53	53.00	31.07	13.64	-9.31	24.57	28.80	33.89	154.93
54	54.00	38.71	18.68	9.15	28.09	33.02	41.11	166.43
55	55.00	55.82	30.68	-3.33	35.40	46.30	65.73	202.29
56	56.00	56.43	29.39	3.82	36.06	47.46	66.09	194.30
57	57.00	52.43	21.54	23.73	36.77	45.84	63.18	165.11
58	58.00	56.04	23.59	26.81	39.20	49.54	68.71	169.54
59	59.00	60.64	26.70	28.03	40.55	52.77	74.00	206.31
60	60.00	61.11	27.93	26.63	42.03	52.16	72.48	245.21
61	61.00	61.62	30.57	25.32	40.30	52.90	72.56	262.82
62	62.00	65.55	38.37	25.28	40.40	55.74	75.02	342.20
63	63.00	65.39	41.69	3.12	38.67	51.80	78.03	325.92
64	64.00	66.87	50.33	24.07	36.62	49.83	78.09	472.53
65	65.00	71.78	54.40	22.90	39.36	57.09	83.63	675.06
66	66.00	75.51	49.59	26.85	43.56	62.45	90.99	590.03
67	67.00	71.19	40.28	26.31	42.81	61.53	87.95	423.11
68	68.00	67.57	33.88	28.54	44.27	58.40	81.33	247.08
69	69.00	69.88	32.92	28.75	45.44	60.73	84.85	214.00
70	70.00	61.39	28.98	27.66	39.61	53.05	74.69	201.87
71	71.00	43.79	18.80	14.82	31.76	37.99	50.48	133.32
72	72.00	38.56	16.75	13.84	28.49	33.77	43.77	159.10
73	73.00	35.01	20.00	0.57	25.56	30.30	37.42	227.47
74	74.00	33.91	18.99	-18.58	25.24	29.88	36.73	219.03
75	75.00	30.75	15.73	-5.29	24.03	27.91	33.53	173.44
76	76.00	29.97	16.83	-11.18	23.55	26.82	32.56	192.98
77	77.00	31.27	15.23	1.32	24.34	27.77	33.62	171.73
78	78.00	37.94	18.82	9.50	28.04	32.52	39.30	153.54
79	79.00	56.34	34.01	14.58	34.78	45.49	62.79	228.45

80	80.00	57.61	33.14	22.18	36.48	47.32	68.59	234.68
81	81.00	53.68	23.68	22.32	36.21	47.26	65.03	173.88
82	82.00	56.52	24.79	26.27	38.72	48.57	67.51	169.05
83	83.00	61.73	26.89	22.91	41.05	53.47	75.91	179.22
84	84.00	61.05	28.19	27.65	41.40	53.28	72.13	204.51
85	85.00	61.84	28.73	27.34	40.08	52.80	77.22	218.58
86	86.00	65.18	35.47	9.74	40.16	52.66	83.31	275.48
87	87.00	64.31	39.47	9.30	38.36	49.99	79.23	342.39
88	88.00	63.74	45.62	15.68	36.57	48.63	77.47	483.34
89	89.00	67.16	41.91	16.55	38.70	51.74	86.03	368.68
90	90.00	69.99	36.18	19.14	42.35	61.19	87.93	240.47
91	91.00	67.06	34.97	15.22	41.77	59.90	81.69	261.34
92	92.00	65.75	33.18	19.78	43.93	57.69	78.64	216.94
93	93.00	69.52	36.74	21.15	42.73	62.15	85.55	345.89
94	94.00	60.51	30.27	20.98	39.23	50.32	72.65	215.21
95	95.00	43.02	18.53	12.00	31.12	36.91	47.20	137.28
96	96.00	38.26	17.06	3.36	27.90	32.90	41.99	119.62
97	97.00	34.09	14.81	1.01	25.80	30.41	37.89	118.86
98	98.00	32.73	14.63	-1.52	24.75	29.05	36.04	124.79
99	99.00	30.41	15.10	-11.90	23.46	27.77	33.75	131.19
100	100.00	28.59	15.50	-34.43	22.34	26.75	32.67	127.14
101	101.00	30.82	13.62	0.00	23.95	28.20	34.06	113.78
102	102.00	36.34	18.05	2.59	27.21	31.52	39.77	170.09
103	103.00	51.66	31.75	-5.10	32.80	41.62	58.55	272.99
104	104.00	54.32	30.68	-11.57	34.41	42.84	62.50	213.80
105	105.00	55.00	24.97	22.91	36.11	47.80	66.19	178.74
106	106.00	56.62	25.10	22.81	37.70	49.97	67.38	202.48

107	107.00	62.46	29.22	23.03	41.05	55.22	73.52	194.74
108	108.00	60.37	29.74	22.43	40.66	52.41	72.48	354.84
109	109.00	59.66	29.81	21.89	38.90	50.61	70.89	229.37
110	110.00	62.29	34.02	18.85	39.10	51.23	76.68	293.85
111	111.00	61.85	36.78	20.23	37.21	47.56	78.03	328.32
112	112.00	60.00	37.99	19.72	35.43	45.23	72.88	256.18
113	113.00	61.72	36.92	19.16	37.06	47.58	73.31	229.54
114	114.00	65.67	35.94	25.16	39.01	55.14	83.85	262.17
115	115.00	57.86	31.54	22.28	37.57	48.16	66.95	316.57
116	116.00	52.95	24.62	20.82	36.43	43.40	62.66	187.91
117	117.00	56.99	30.45	19.45	37.28	46.58	65.94	266.16
118	118.00	55.02	27.16	22.88	36.33	46.01	65.02	200.12
119	119.00	42.43	17.49	18.98	30.93	36.54	47.44	117.29
120	120.00	39.60	18.36	18.67	28.67	33.54	43.49	143.15
121	121.00	36.75	17.26	7.67	27.18	31.07	40.24	155.90
122	122.00	37.01	17.71	8.10	26.59	31.41	42.51	134.97
123	123.00	34.58	18.78	0.00	25.04	29.63	38.16	162.88
124	124.00	31.61	18.75	-20.65	23.33	28.05	35.38	162.34
125	125.00	31.21	17.51	-6.47	23.19	27.59	34.16	163.15
126	126.00	33.89	21.21	-18.89	24.18	28.95	36.56	197.56
127	127.00	33.28	19.66	-30.55	24.37	29.86	36.56	128.25
128	128.00	39.12	21.77	-32.58	27.52	33.12	43.12	189.42
129	129.00	49.32	28.49	-0.16	31.60	40.27	58.83	219.32
130	130.00	56.09	30.08	17.88	34.28	46.56	68.78	223.30
131	131.00	58.28	30.27	22.68	35.94	47.55	74.01	248.35
132	132.00	57.55	31.53	19.57	36.22	46.49	69.19	251.29
133	133.00	54.72	31.52	20.26	34.86	44.27	63.90	222.08

134	134.00	52.05	29.93	17.71	33.23	41.77	61.39	182.82
135	135.00	50.73	32.94	18.52	31.36	38.94	56.97	243.99
136	136.00	52.49	34.45	18.78	31.05	39.55	60.24	254.50
137	137.00	56.54	33.35	21.47	32.41	44.74	67.89	213.44
138	138.00	61.16	34.12	23.96	35.45	50.68	75.42	207.28
139	139.00	57.21	30.07	23.20	35.57	49.29	67.94	234.46
140	140.00	54.71	27.05	21.63	35.87	46.44	65.64	234.83
141	141.00	55.80	27.04	23.68	36.41	47.73	66.16	173.67
142	142.00	52.22	24.91	23.17	34.16	44.63	62.32	183.23
143	143.00	42.24	19.42	12.08	30.46	36.17	47.94	142.18
144	144.00	36.57	16.32	3.63	27.31	32.28	40.39	133.80
145	145.00	32.84	16.43	-6.02	25.46	29.61	35.59	153.28
146	146.00	31.57	18.56	-19.13	23.95	28.04	34.95	146.38
147	147.00	28.92	19.41	-13.93	21.73	26.61	32.55	217.74
148	148.00	26.39	16.16	-28.22	19.83	26.02	31.25	125.42
149	149.00	25.21	16.20	-45.63	18.82	25.16	30.41	172.14
150	150.00	25.75	20.51	-26.76	18.30	25.61	31.06	235.55
151	151.00	26.25	20.84	-39.62	18.09	25.66	31.59	140.08
152	152.00	30.38	20.62	-21.40	21.98	27.50	33.71	160.58
153	153.00	38.82	20.28	6.41	27.85	32.46	41.92	183.84
154	154.00	41.67	19.10	16.15	30.09	35.26	45.94	159.71
155	155.00	43.58	20.60	18.43	30.59	36.27	48.48	141.81
156	156.00	47.03	24.61	14.94	31.81	38.23	51.45	178.34
157	157.00	48.54	26.56	12.93	31.93	39.27	54.28	167.91
158	158.00	47.04	26.02	14.86	30.90	36.84	54.48	163.78
159	159.00	46.40	28.86	17.79	30.14	35.85	51.46	198.00
160	160.00	48.28	32.40	18.88	29.63	35.99	53.99	253.94

161	161.00	54.48	36.71	21.04	31.38	39.95	61.90	249.31
162	162.00	61.52	37.94	24.70	34.55	49.98	75.39	281.86
163	163.00	58.74	32.59	19.78	35.37	47.60	70.43	184.15
164	164.00	58.14	30.22	15.81	35.76	49.21	70.76	193.89
165	165.00	63.12	33.48	18.77	36.62	53.77	77.96	196.52
166	166.00	53.82	27.27	17.99	34.49	44.54	64.84	169.15
167	167.00	40.33	18.25	17.56	28.93	34.84	45.10	159.87
168	168.00	34.76	16.44	0.80	25.94	30.47	39.12	191.62

Table B.2: PJM Aggregate Week-Hourly Statistics

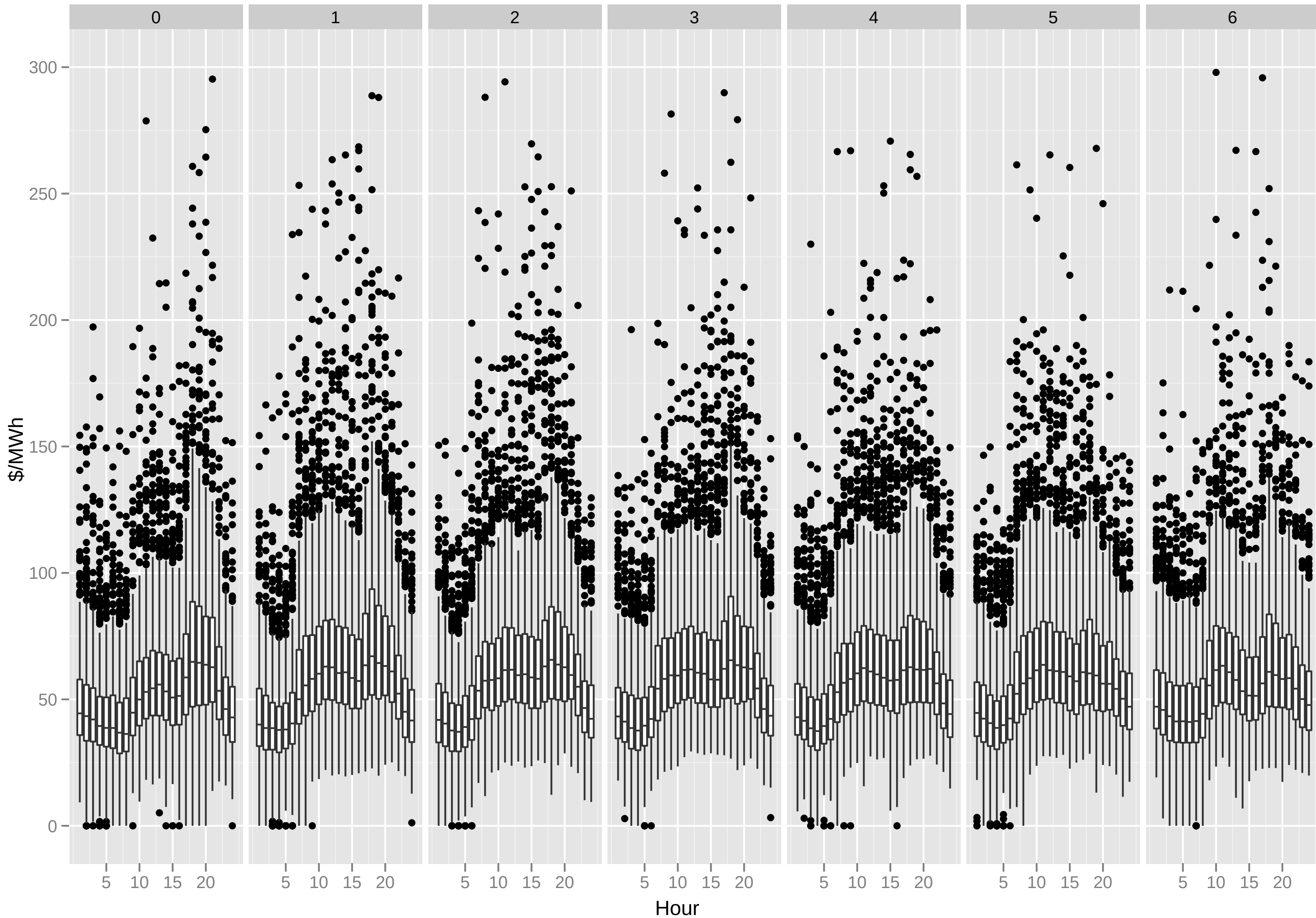

Figure B.1: ISONE Daily

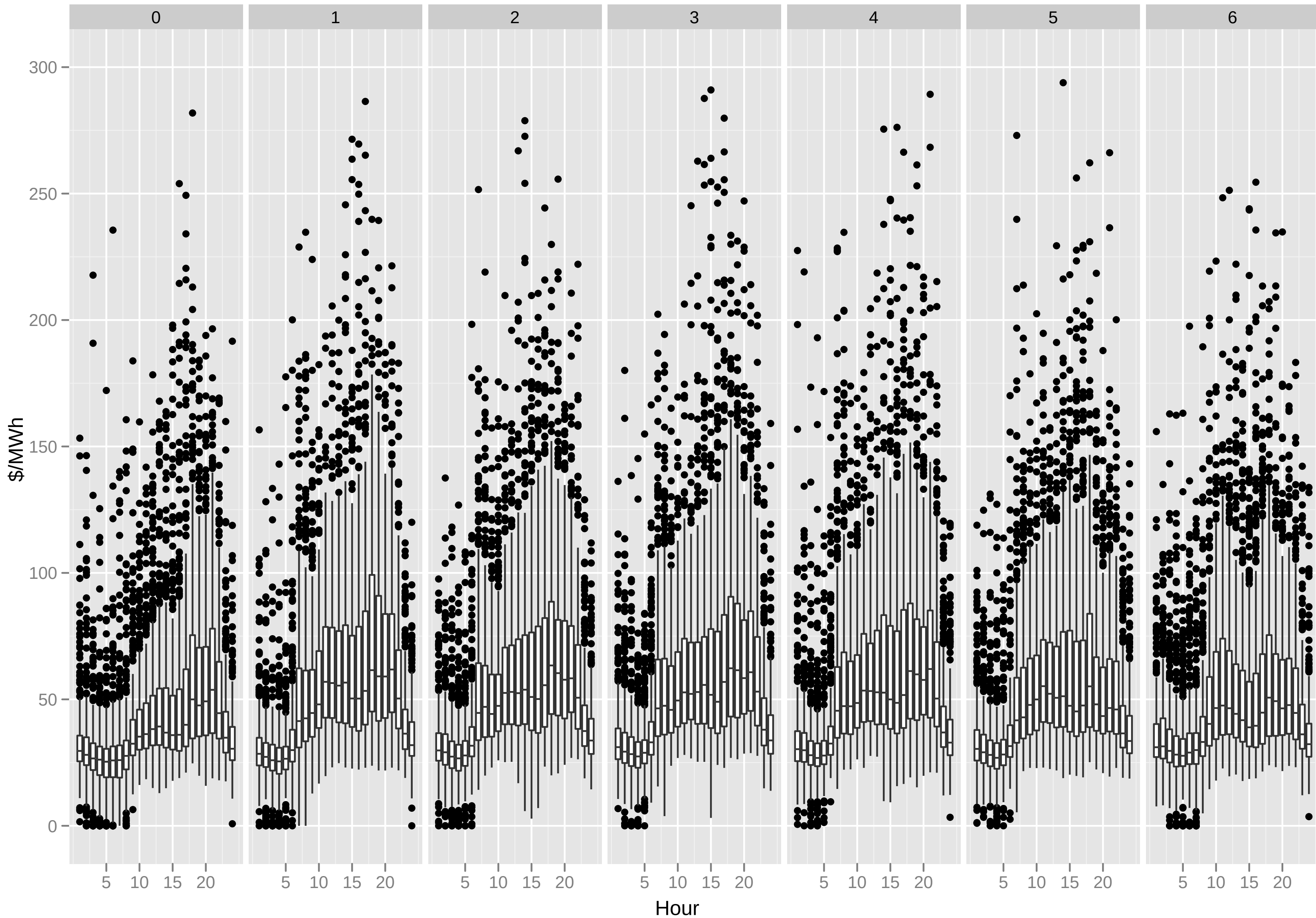

Figure B.2: PJM Daily

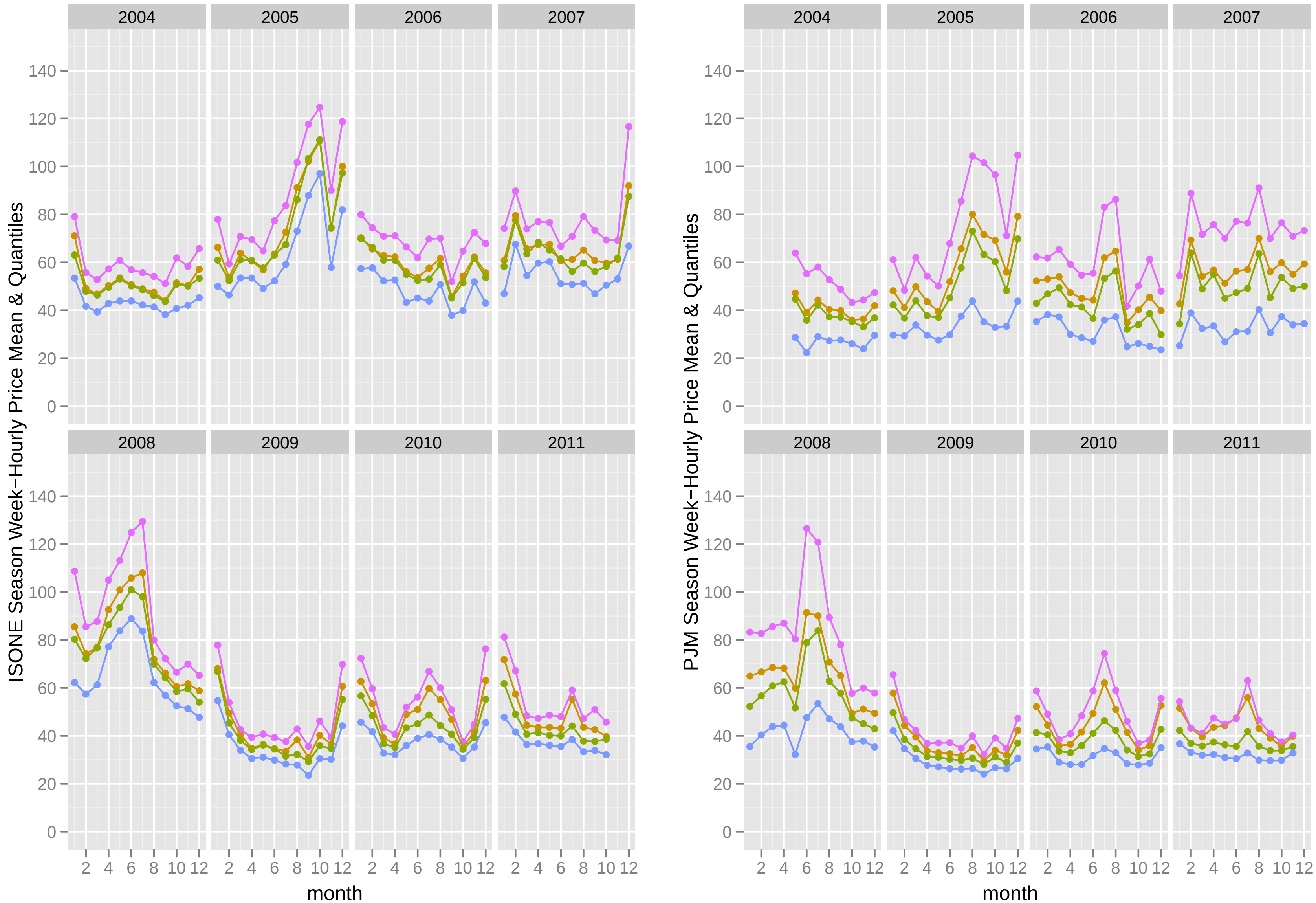

Figure B.3: Monthly Stats Comparison: Q1, median, mean, Q3

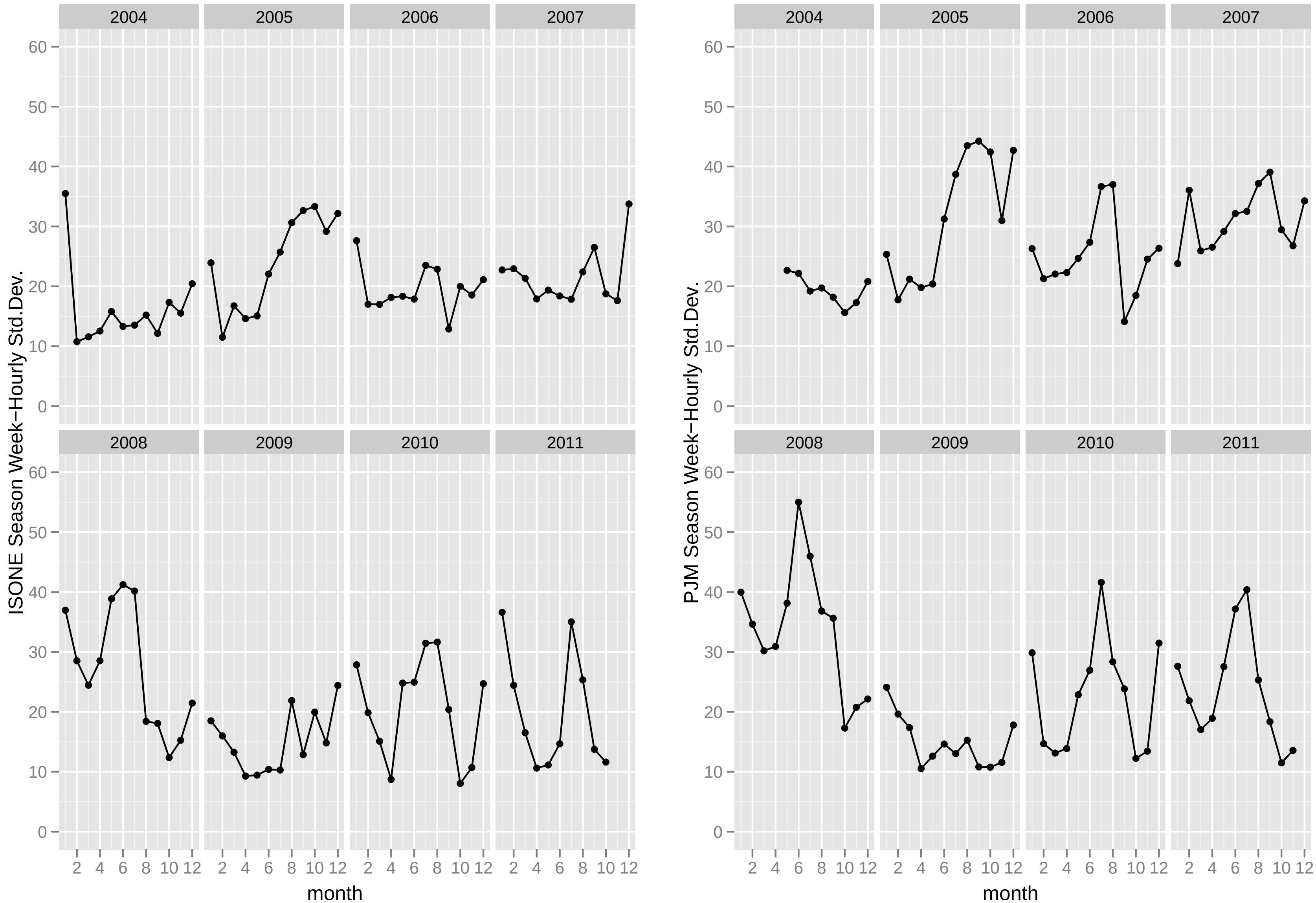

Figure B.4: Monthly Stats Comparison: Std.Dev.

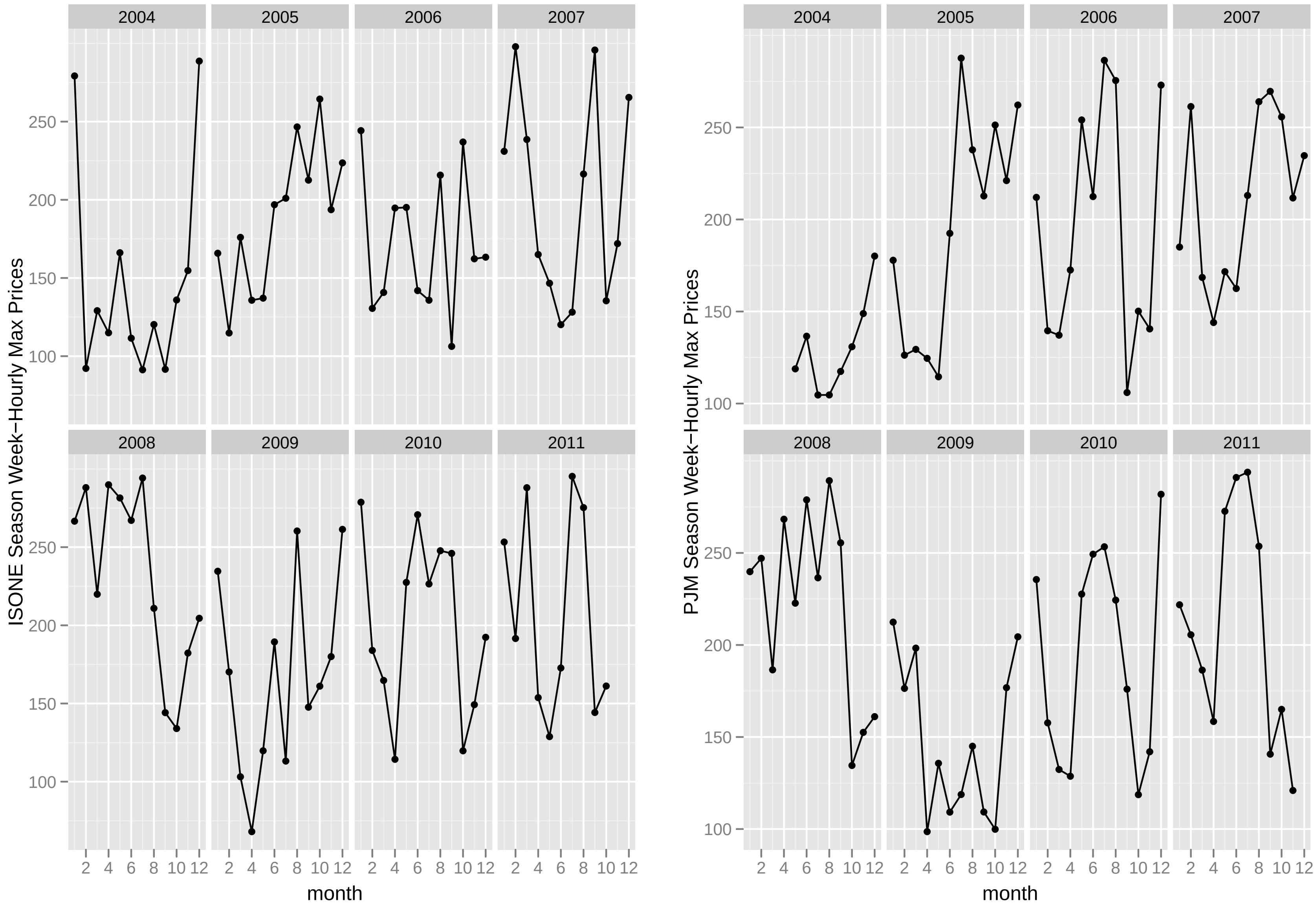

Figure B.5: Monthly Stats Comparison: Max Prices

year	hour	size	threshold	percentage	N	mean	Std.Dev.	min	Q1	median	Q3	max
2004	7	366	114.72	0.01	4	210.04	97.81	138.32	155.92	174.03	228.16	353.78
2004	8	366	119.05	0.01	5	187.79	71.03	126.78	130.15	189.37	190.29	302.36
2008	10	366	205.08	0.01	3	270.53	61.66	228.35	235.14	241.94	291.62	341.29
2003	11	304	118.35	0.02	5	192.39	117.43	124.65	125.78	133.76	179.18	398.60
2008	11	366	221.63	0.02	6	285.11	63.41	222.42	236.14	268.69	326.39	379.94
2011	11	303	119.96	0.04	11	171.57	65.82	120.94	132.94	153.68	181.05	356.70
2008	12	366	220.01	0.01	4	327.89	45.26	263.41	312.88	344.27	359.29	359.61
2006	13	365	148.36	0.01	2	363.67	255.73	182.84	273.25	363.67	454.08	544.50
2011	13	303	121.59	0.04	11	201.21	92.80	129.60	142.13	146.91	226.16	422.29
2006	14	365	146.98	0.01	2	574.96	598.96	151.43	363.19	574.96	786.73	998.49
2008	14	366	209.12	0.02	7	250.67	40.36	214.68	223.03	226.99	279.77	307.40
2010	14	365	121.68	0.05	18	177.34	87.36	123.95	130.79	154.85	176.94	499.71
2011	14	303	122.28	0.03	8	229.73	143.47	132.49	134.05	180.44	256.13	558.55
2006	15	365	142.12	0.01	2	590.52	601.51	165.19	377.86	590.52	803.19	1015.86
2011	15	303	117.54	0.03	8	179.65	120.02	118.75	127.58	139.56	150.80	474.29
2006	16	365	140.08	0.01	2	602.10	564.57	202.89	402.50	602.10	801.70	1001.31
2011	16	303	115.04	0.03	8	193.20	113.51	118.66	128.07	147.28	195.26	455.18
2004	17	366	139	0.01	3	471.14	220.33	223.66	383.71	543.76	594.88	645.99
2005	17	365	224.94	0.01	2	322.38	26.89	303.36	312.87	322.38	331.88	341.39
2006	17	365	157.80	0.01	2	957.82	79.49	901.61	929.71	957.82	985.92	1014.02
2008	17	366	211.41	0.02	9	266.66	69.10	214.56	221.29	229.39	289.89	403.23
2010	17	365	139.46	0.05	17	167.99	42.27	139.65	141.63	146.26	182.16	306.25
2003	18	304	127.12	0.02	5	322.13	378.67	129.22	135.02	167.70	180.28	998.41
2004	18	366	144.22	0.02	8	285.94	261.84	145.37	152.99	185.62	266.74	920.29
2006	18	365	173.36	0.01	4	441.95	341.13	193.12	231.46	318.52	529.01	937.63
2004	19	366	138.81	0.01	3	281.42	25.78	256.81	268.01	279.22	293.72	308.22
2005	19	365	215.66	0	1	856.06		856.06	856.06	856.06	856.06	856.06
2004	20	366	131.22	0.01	3	242.24	123.56	135.94	174.46	212.98	295.39	377.80
2011	21	303	113.99	0.04	11	178.47	73.34	122.88	131.97	134.05	195.42	335.89
ISONE Yearly-Hourly Spike Statistics												

Table B.3: ISONE Yearly-Hourly Spike Statistics

year	hour	size	threshold	percentage	N	mean	Std.Dev.	min	Q1	median	Q3	max
2007	8	365	140.56	0.03	10	198.87	85.94	146.01	150.35	173.26	191.98	430.69
2011	12	310	104.51	0.03	8	188.03	75.49	123.64	132.15	176.56	204.77	354.84
2006	13	365	136.72	0.02	8	191.36	87.14	136.98	145.90	157.74	183.94	397.82
2006	14	365	135.72	0.03	10	228.17	89.08	138.96	153.28	214.72	270.12	395.71
2006	15	365	139.18	0.04	13	220.83	109.70	142.21	152.22	169.28	271.48	502.87
2007	15	365	171.42	0.02	8	266.22	92.38	178.23	226.53	245.35	263.69	484.35
2008	15	366	207.32	0.02	8	236.53	34.14	207.86	215.63	223.62	246.88	310.29
2011	15	310	112.15	0.07	21	178.23	69.76	112.48	129.76	151.44	197.43	348.25
2006	16	365	130.86	0.04	13	264.04	147.06	132.58	151.24	210.54	372.55	627.55
2007	16	365	175.56	0.01	5	275.24	115.15	191.24	196.61	246.23	269.59	472.53
2008	16	366	203.12	0.02	8	290.83	92.93	204.08	232.54	254.22	322.13	483.34
2010	16	365	115.94	0.06	23	167.43	49.46	118.37	136.41	157.40	183.35	346.75
2011	16	310	115.27	0.08	25	195.65	124.35	117.16	126.72	156.79	240.33	716.44
2006	17	365	144.46	0.03	12	298.97	170.60	150.41	183.94	243.02	359.22	752.37
2007	17	365	192.34	0.02	6	328.83	180.54	199.28	230.88	246.87	346.26	675.06
2008	17	366	218.74	0.02	9	256.78	34.13	220.45	234.07	255.49	266.36	335.04
2011	17	310	128.50	0.08	25	191.52	125.79	133.08	139.35	162.36	187.70	770.65
2006	18	365	163.54	0.02	6	313.02	231.27	164.45	183.51	207.16	321.75	763.78
2007	18	365	196.62	0.01	4	302.97	191.51	197.13	208.05	212.37	307.29	590.03
2008	18	366	230.71	0.01	5	255.13	45.46	230.96	233.51	235.10	239.82	336.25
2010	18	365	142.60	0.04	14	189.03	49.81	150.36	155.94	168.92	200	311.77
2011	18	310	131.61	0.07	23	186.45	122.59	131.85	140.48	150.80	174.51	730
2007	19	365	183.84	0.03	10	233.72	71.94	190.38	191.33	200.84	249.59	423.11
2008	19	366	211.69	0.01	5	248.30	41.33	213.48	219.02	239.37	253.06	316.57
2007	21	365	179.16	0.01	3	241.92	90.28	183.35	189.94	196.52	271.20	345.89
PJM Yearly-Hourly Spike Statistics												

Table B.4: PJM Yearly-Hourly Spike Statistics

Low Spike Threshold											
year	size	threshold($/MWh)	percentage	N	mean	Std.Dev.	min	Q1	median	Q3	max
2003	7296	85.57	0.04	262	117	64.20	85.59	92.77	104.86	124.15	998.41
2004	8784	94.54	0.02	187	140.49	90.63	94.61	100.71	110.19	137.97	920.29
2005	8760	155.53	0.02	195	182.24	54.27	155.54	164.34	173.25	184.78	856.06
2006	8760	107.04	0.02	192	161.67	153.49	107.47	114.65	125.08	143.36	1015.86
2007	8760	131.74	0.02	140	156.49	32.71	131.78	137.32	143.61	160.18	297.92
2008	8784	155.05	0.03	305	193.12	41.16	155.14	166.48	180.92	202.25	403.23
2009	8760	75.24	0.06	544	95.47	23.96	75.26	80.35	87.88	101.97	261.37
2010	8760	84.20	0.07	594	119.59	38.61	84.22	92.64	107.37	133.47	499.71
2011	7272	79.67	0.06	471	122.60	55.92	79.70	87.12	103.17	139.40	558.55
High Spike Threshold											
year	size	threshold($/MWh)	percentage	N	mean	Std.Dev.	min	Q1	median	Q3	max
2003	7296	139.36	0.01	37	197.79	144.15	140.58	147.59	158.36	180.28	998.41
2004	8784	153.41	0	40	254.43	146.96	154.75	182.65	204	261.29	920.29
2005	8760	249.63	0	6	379.79	235.70	252.73	261.68	283.89	331.88	856.06
2006	8760	176.80	0	21	449.83	353.14	177.85	195.12	215.78	901.61	1015.86
2007	8760	209.58	0	14	241.05	29.76	211.88	218.60	228.19	262.11	297.92
2008	8784	249.91	0	28	295.27	45.09	250.21	261.72	274.98	331.31	403.23
2009	8760	122.55	0.01	46	157.35	32.88	122.78	135.45	151.87	161.54	261.37
2010	8760	138.69	0.02	132	175.91	43.45	138.97	148.62	162.89	189.59	499.71
2011	7272	132.87	0.02	135	186.44	68.36	133.12	145.42	161.32	200.21	558.55
ISONE Yearly Spike Statistics: High \ Low Threshold											
Low Spike Threshold											
year	size	threshold($/MWh)	percentage	N	mean	Std.Dev.	min	Q1	median	Q3	max
2004	5832	83.30	0.03	186	97.08	15.79	83.36	87.32	91.28	100.67	180.10
2005	8760	114.39	0.08	712	145.01	27.60	114.40	123.51	137.40	160.74	287.63
2006	8760	87.31	0.09	786	119.41	58.59	87.41	94.38	103.81	121.45	763.78
2007	8760	116.15	0.05	430	151.45	55.73	116.15	123.57	135.60	159.56	675.06
2008	8784	127.23	0.08	677	162.34	35.57	127.30	137.70	153.15	173.77	483.34
2009	8760	66.28	0.05	478	88.70	23.43	66.28	72.45	81.20	94.81	212.40
2010	8760	72.78	0.10	867	107.46	36.11	72.84	81.88	95.03	121.65	346.75
2011	7440	60.59	0.13	981	100.48	55.30	60.61	69.04	85.09	113.43	770.65
High Spike Threshold											
year	size	threshold($/MWh)	percentage	N	mean	Std.Dev.	min	Q1	median	Q3	max
2004	5832	134.88	0	8	149.71	14.14	136.56	139.66	147.96	153.12	180.10
2005	8760	189.58	0.01	51	213.63	21.74	191.12	198.28	206.56	219.80	287.63
2006	8760	147.35	0.01	79	240.47	126.41	147.97	161.26	190.04	272.24	763.78
2007	8760	191.54	0.01	45	266.89	109.37	192.18	201.08	227.08	261.34	675.06
2008	8784	210.17	0.01	55	250.04	47.17	211.56	222.38	234.07	255.51	483.34
2009	8760	107.99	0.01	74	133.24	24.22	108.29	115.57	124.78	144.53	212.40
2010	8760	122.30	0.02	211	158.45	36.51	122.32	132.19	146.27	177.02	346.75
2011	7440	106.24	0.04	285	156.76	75.32	106.69	119.03	136.69	166.33	770.65
PJM Yearly Spike Statistics: High \ Low Threshold											

Table B.5: Yearly Spike Stats

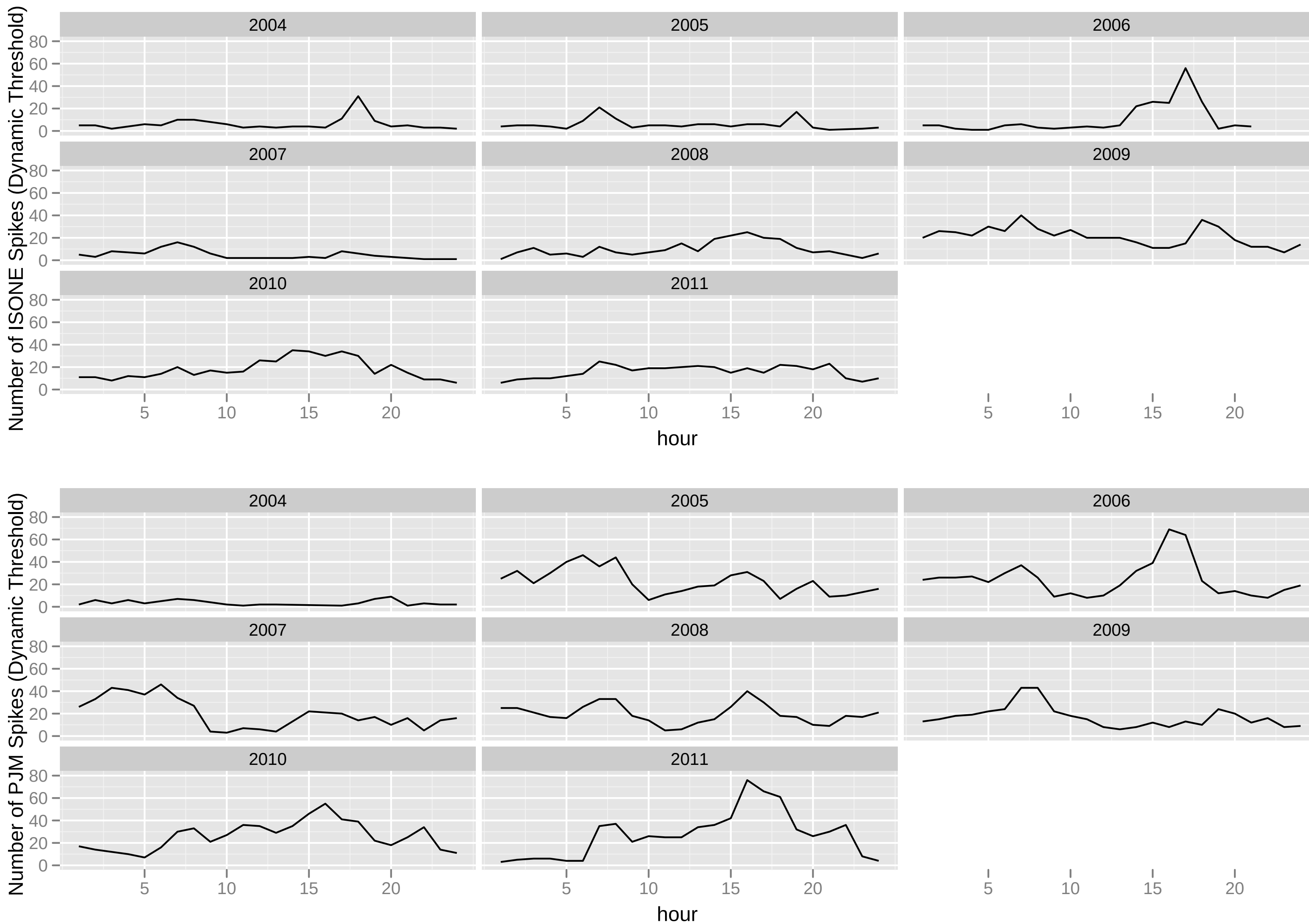

Figure B.6: PJM vs ISONE Yearly-Hourly Spike Counts

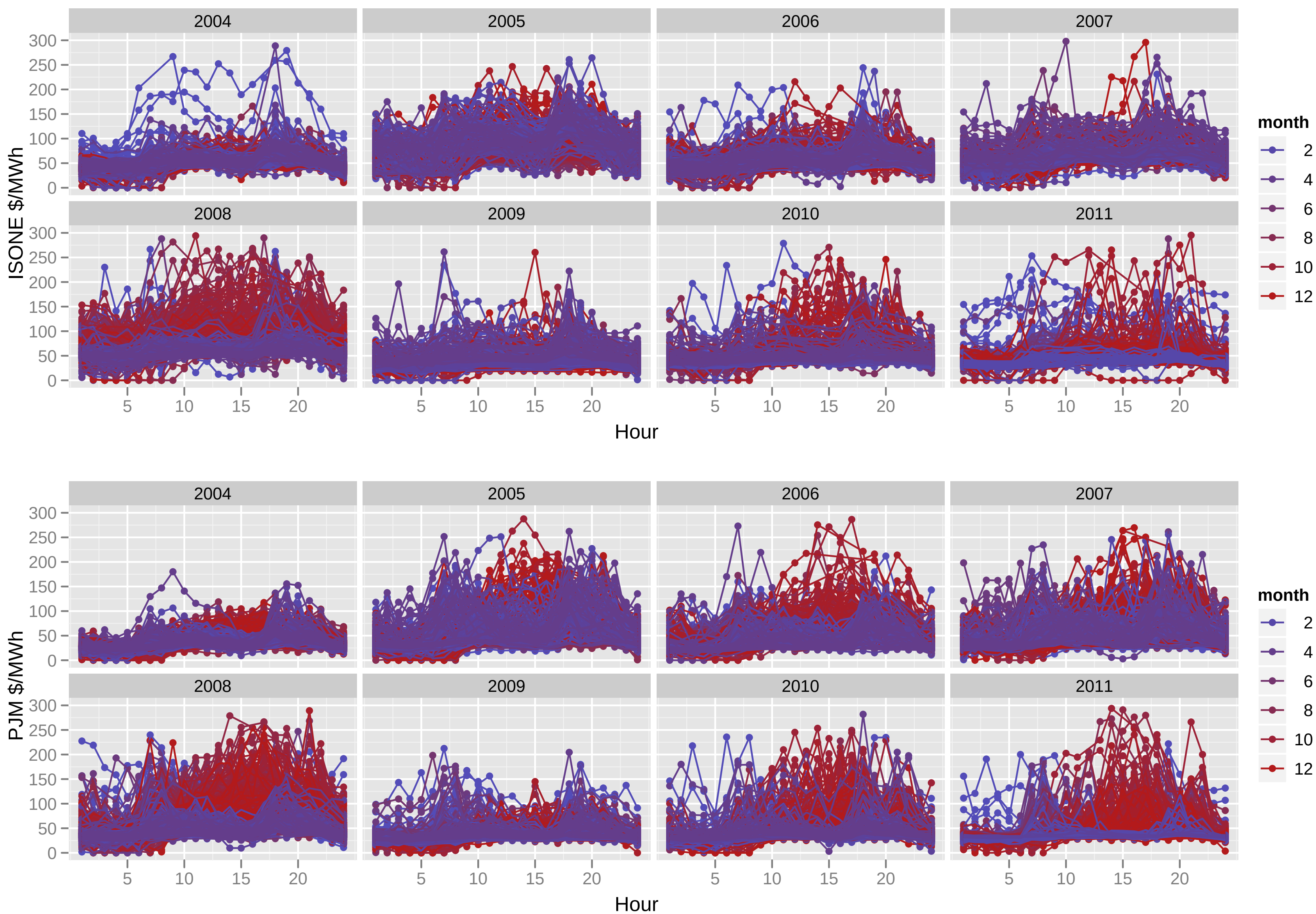

Figure B.7: Hourly Overlays.

Appendix C

Value of Information

C.1 Demand Scenarios

	hour	sce1	sce2	sce3	sce4	sce5	sce6	sce7	sce8	sce9	sce10
1	1.00	414.76	411.03	410.07	420.70	410.37	431.63	360.39	466.68	419.79	431.00
2	2.00	422.19	414.76	419.64	425.52	425.32	437.10	381.71	440.59	415.01	437.39
3	3.00	429.54	418.49	431.00	440.17	455.74	474.09	421.86	513.60	528.17	561.62
4	4.00	436.77	422.19	444.93	433.72	473.12	450.71	488.93	376.87	458.51	502.89
5	5.00	443.82	425.88	437.86	447.52	448.22	467.55	347.63	444.29	384.63	439.75
6	6.00	450.68	429.54	458.65	453.35	499.50	488.91	486.97	434.00	513.68	579.32
7	7.00	457.29	433.17	466.18	451.86	504.43	475.79	496.23	353.04	441.97	517.86
8	8.00	463.62	436.77	466.84	475.70	514.82	532.55	439.45	528.08	560.24	646.04
9	9.00	469.64	440.32	472.22	481.24	522.02	540.06	433.11	523.33	549.15	644.53
10	10.00	475.31	443.82	466.54	482.31	492.55	524.08	319.65	477.31	389.67	494.22
11	11.00	480.60	447.28	474.71	496.46	518.81	562.31	348.48	565.97	507.17	620.46
12	12.00	485.47	450.68	479.00	491.06	515.04	539.15	342.60	463.14	398.45	520.02
13	13.00	489.91	454.02	493.65	477.15	535.10	502.08	444.69	279.63	317.04	446.39
14	14.00	493.89	457.29	491.57	488.16	531.21	524.38	384.11	349.96	326.78	463.39
15	15.00	497.39	460.49	502.78	485.14	554.53	519.25	461.26	284.83	338.80	482.10
16	16.00	500.38	463.62	500.33	495.64	551.84	542.46	406.83	359.95	359.49	508.91
17	17.00	502.85	466.67	507.20	496.64	569.08	547.97	450.81	345.25	388.78	543.72
18	18.00	504.78	469.64	514.62	505.20	597.07	578.23	505.67	411.50	509.88	669.72
19	19.00	506.17	472.52	503.77	497.06	555.09	541.67	383.30	316.20	292.20	456.32

20	20.00	507.01	475.31	512.56	508.59	593.25	585.32	462.78	423.09	478.58	646.32
21	21.00	507.29	478.00	515.98	500.74	597.54	567.06	494.23	341.83	428.77	599.48
22	22.00	507.01	480.60	501.25	524.68	580.72	627.58	349.72	584.02	526.45	699.47
23	23.00	506.17	483.09	509.21	513.34	598.24	606.51	437.62	478.96	509.30	683.98
24	24.00	504.78	485.47	497.29	513.88	569.60	602.78	332.40	498.31	423.42	599.09
25	25.00	502.85	487.75	498.19	506.69	573.18	590.18	360.73	445.71	399.15	575.17
26	26.00	500.38	489.91	498.10	517.20	592.99	631.19	384.51	575.51	552.73	728.44
27	27.00	497.39	491.96	487.65	491.03	546.51	553.26	309.97	343.72	246.39	421.16
28	28.00	493.89	493.89	491.54	489.28	568.82	564.29	383.77	361.13	337.60	510.81
29	29.00	489.91	495.70	497.31	504.24	614.83	628.69	481.23	550.52	624.46	795.49
30	30.00	485.47	497.39	482.28	478.97	559.48	552.86	375.36	342.22	310.29	478.57
31	31.00	480.60	498.95	480.24	478.52	569.10	565.67	403.71	386.55	382.97	547.93
32	32.00	475.31	500.38	477.30	471.38	570.44	558.60	427.20	368.01	387.92	549.03
33	33.00	469.64	501.68	469.51	466.44	560.44	554.30	406.00	375.29	374.00	530.74
34	34.00	463.62	502.85	457.35	460.83	537.56	544.53	344.53	379.38	316.62	468.51
35	35.00	457.29	503.88	463.84	462.23	578.47	575.26	472.76	456.71	522.18	668.78
36	36.00	450.68	504.78	454.05	448.91	556.51	546.22	440.98	389.56	423.25	564.13
37	37.00	443.82	505.55	449.71	438.76	554.68	532.79	466.14	356.69	415.54	550.33
38	38.00	436.77	506.17	428.92	450.20	525.56	568.10	328.88	541.59	463.18	591.54
39	39.00	429.54	506.66	434.02	427.40	540.19	526.95	452.03	385.83	430.57	552.20
40	40.00	422.19	507.01	420.42	420.40	514.80	514.75	389.54	389.33	371.59	486.21
41	41.00	414.76	507.22	421.18	413.76	532.95	518.11	471.48	397.27	461.46	568.86
42	42.00	407.29	507.29	410.23	414.42	523.24	531.61	436.70	478.56	507.97	607.97
43	43.00	399.82	507.22	405.48	399.08	515.99	503.20	463.88	399.93	456.52	548.98
44	44.00	392.39	507.01	393.45	392.01	494.91	492.04	417.90	403.53	414.13	498.95
45	45.00	385.04	506.66	385.63	378.22	479.38	464.55	413.23	339.12	345.07	422.19
46	46.00	377.81	506.17	383.60	374.57	490.82	472.76	465.16	374.86	432.73	502.14
47	47.00	370.76	505.55	361.22	371.36	441.01	461.28	311.96	413.31	317.97	379.69
48	48.00	363.90	504.78	363.45	358.01	454.14	443.27	402.74	348.40	343.85	397.95
49	49.00	357.29	503.88	361.94	362.61	473.14	474.47	453.75	460.44	506.90	553.50
50	50.00	350.96	502.85	354.81	335.77	442.90	404.82	445.84	255.45	294.00	333.22
51	51.00	344.94	501.68	344.49	348.01	441.05	448.08	402.81	437.95	433.47	465.51
52	52.00	339.27	500.38	346.50	323.91	438.67	393.49	479.53	253.65	325.89	350.96
53	53.00	333.98	498.95	332.75	330.98	418.92	415.37	394.91	377.19	364.81	383.16
54	54.00	329.11	497.39	324.00	323.82	398.61	398.25	356.25	354.46	303.42	315.34
55	55.00	324.67	495.70	316.08	318.15	380.80	384.93	321.43	342.08	256.22	262.00
56	56.00	320.69	493.89	312.68	320.12	382.69	397.57	327.18	401.60	321.49	321.49
57	57.00	317.19	491.96	313.52	298.05	371.70	340.76	370.54	215.85	179.11	173.68
58	58.00	314.20	489.91	314.58	325.97	409.71	432.50	411.02	524.95	528.68	518.21

59	59.00	311.73	487.75	314.97	295.08	385.26	345.48	439.69	240.79	273.19	258.10
60	60.00	309.80	485.47	307.93	305.16	377.75	372.21	388.66	360.94	342.30	322.99
61	61.00	308.41	483.09	316.66	297.25	397.82	358.98	489.87	295.70	378.27	355.19
62	62.00	307.57	480.60	303.44	300.06	360.98	354.22	366.01	332.21	290.93	264.51
63	63.00	307.29	478.00	306.47	328.16	396.42	439.80	399.10	616.01	607.82	578.53
64	64.00	307.57	475.31	304.22	307.74	365.70	372.76	373.77	409.03	375.51	343.81
65	65.00	308.41	472.52	311.42	295.54	369.82	338.06	437.46	278.66	308.83	275.18
66	66.00	309.80	469.64	304.96	293.39	341.22	318.09	358.89	243.23	194.83	159.69
67	67.00	311.73	466.67	311.30	316.23	374.33	384.19	403.00	452.31	448.02	411.84
68	68.00	314.20	463.62	319.53	314.02	386.33	375.30	460.55	405.43	458.70	421.94
69	69.00	317.19	460.49	308.88	314.01	342.27	352.54	324.14	375.48	292.33	255.44
70	70.00	320.69	457.29	328.19	311.39	383.91	350.31	482.35	314.35	389.42	352.82
71	71.00	324.67	454.02	321.45	309.79	346.86	323.55	375.10	258.54	226.36	190.46
72	72.00	329.11	450.68	335.90	318.35	382.11	347.03	475.18	299.77	367.66	332.86
73	73.00	333.98	447.28	330.92	343.99	374.78	400.91	376.63	507.29	476.63	443.31
74	74.00	339.27	443.82	335.95	333.06	359.62	353.84	374.05	345.16	311.92	280.44
75	75.00	344.94	440.32	344.47	331.10	362.71	335.96	402.56	268.85	264.12	234.80
76	76.00	350.96	436.77	358.80	369.65	422.66	444.36	485.73	594.22	672.66	645.80
77	77.00	357.29	433.17	364.58	361.54	409.28	403.21	480.16	449.80	522.67	498.55
78	78.00	363.90	429.54	361.70	361.52	377.17	376.79	385.29	383.43	361.42	340.29
79	79.00	370.76	425.88	376.30	381.34	416.57	426.65	462.75	513.14	568.60	550.66
80	80.00	377.81	422.19	387.03	386.68	429.22	428.52	499.41	495.93	588.06	573.48
81	81.00	385.04	418.49	383.73	378.85	386.12	376.35	394.22	345.37	332.30	321.24
82	82.00	392.39	414.76	396.64	414.45	434.67	470.29	449.79	627.90	670.40	662.97
83	83.00	399.82	411.03	397.82	397.27	395.01	393.91	387.29	381.79	361.79	358.05
84	84.00	407.29	407.29	403.80	393.05	382.57	361.06	372.36	264.84	229.91	229.91
85	85.00	414.76	403.55	419.90	413.32	425.01	411.83	458.71	392.85	444.27	448.00
86	86.00	422.19	399.82	416.25	424.27	398.96	415.00	347.83	428.04	368.58	376.01
87	87.00	429.54	396.09	433.76	452.62	454.09	491.81	449.51	638.09	680.31	691.37
88	88.00	436.77	392.39	429.20	437.82	400.22	417.47	331.63	417.87	342.21	356.78
89	89.00	443.82	388.70	438.73	448.39	414.53	433.85	356.39	452.99	402.09	420.03
90	90.00	450.68	385.04	443.54	449.91	406.25	418.98	335.95	399.57	328.24	349.37
91	91.00	457.29	381.41	452.08	453.95	412.45	416.18	355.22	373.89	321.82	345.93
92	92.00	463.62	377.81	454.80	463.27	407.34	424.28	319.08	403.82	315.60	342.46
93	93.00	469.64	374.26	472.48	477.52	453.02	463.09	435.75	486.05	514.51	543.83
94	94.00	475.31	370.76	482.83	496.06	482.10	508.56	482.54	614.81	690.07	721.55
95	95.00	480.60	367.30	486.17	490.87	467.61	477.01	463.07	510.03	565.81	599.13
96	96.00	485.47	363.90	491.42	497.55	472.00	484.27	466.75	528.08	587.54	622.34
97	97.00	489.91	360.56	489.02	477.60	428.19	405.35	398.34	284.16	275.21	311.11

98	98.00	493.89	357.29	492.09	503.73	448.34	471.61	389.31	505.68	487.70	524.30
99	99.00	497.39	354.09	503.60	499.59	465.04	457.00	469.46	429.28	491.46	528.35
100	100.00	500.38	350.96	502.48	485.70	435.67	402.13	428.28	260.57	281.55	318.31
101	101.00	502.85	347.91	505.94	508.06	457.96	462.19	438.23	459.39	490.34	526.51
102	102.00	504.78	344.94	501.85	503.20	432.04	434.74	377.93	391.41	362.05	397.20
103	103.00	506.17	342.06	501.58	520.82	441.81	480.29	361.34	553.75	507.80	541.46
104	104.00	507.01	339.27	516.86	499.35	460.89	425.86	505.83	330.68	429.22	460.92
105	105.00	507.29	336.58	509.96	502.99	440.29	426.34	433.99	364.27	390.97	420.26
106	106.00	507.01	333.98	501.27	497.75	407.24	400.19	349.93	314.68	257.32	283.74
107	107.00	506.17	331.49	498.76	504.40	406.37	417.65	333.16	389.58	315.45	338.54
108	108.00	504.78	329.11	504.35	508.80	429.31	438.22	402.91	447.49	443.11	462.42
109	109.00	502.85	326.83	511.33	495.53	440.51	408.92	492.10	334.12	418.93	434.03
110	110.00	500.38	324.67	502.35	508.68	431.98	444.64	427.04	490.33	510.08	520.54
111	111.00	497.39	322.62	506.91	485.31	429.20	386.00	502.52	286.48	381.72	387.14
112	112.00	493.89	320.69	498.53	483.41	410.72	380.49	453.65	302.49	348.85	348.85
113	113.00	489.91	318.88	487.05	504.33	407.32	441.87	378.64	551.41	522.75	516.96
114	114.00	485.47	317.19	484.10	475.31	381.11	363.53	393.58	305.71	292.00	280.09
115	115.00	480.60	315.63	473.56	484.72	371.95	394.26	336.93	448.49	378.13	359.78
116	116.00	475.31	314.20	465.57	471.50	349.19	361.05	309.91	369.18	271.80	246.73
117	117.00	469.64	312.90	473.95	473.73	392.28	391.84	450.40	448.23	491.34	459.30
118	118.00	463.62	311.73	455.69	480.51	361.14	410.79	327.93	576.18	496.81	457.59
119	119.00	457.29	310.70	456.22	473.16	373.34	407.22	396.55	565.95	555.21	508.61
120	120.00	450.68	309.80	453.48	447.37	358.28	346.06	435.31	374.20	402.22	348.12
121	121.00	443.82	309.03	453.66	420.97	352.23	286.85	505.66	178.77	277.13	215.41
122	122.00	436.77	308.41	436.68	461.74	362.59	412.72	406.41	657.06	656.17	586.77
123	123.00	429.54	307.92	429.23	436.21	335.90	349.87	404.16	474.00	470.87	393.75
124	124.00	422.19	307.57	415.66	427.61	308.29	332.18	341.98	461.42	396.11	311.30
125	125.00	414.76	307.36	419.86	414.63	329.99	319.53	458.25	405.95	456.91	364.46
126	126.00	407.29	307.29	406.37	412.39	309.62	321.67	398.07	458.30	449.08	349.08
127	127.00	399.82	307.36	400.04	398.17	298.91	295.18	409.52	390.85	393.09	285.68
128	128.00	392.39	307.57	386.54	396.59	279.33	299.44	348.80	449.36	390.87	276.24
129	129.00	385.04	307.92	379.61	381.04	265.38	268.23	353.02	367.27	313.00	191.37
130	130.00	377.81	308.41	379.73	364.11	270.97	239.74	426.43	270.27	289.41	161.05
131	131.00	370.76	309.03	372.25	380.63	286.87	303.63	422.26	506.07	521.05	386.26
132	132.00	363.90	309.80	355.44	379.10	256.23	303.54	322.70	559.26	474.68	333.80
133	133.00	357.29	310.70	348.00	354.20	229.74	242.15	314.40	376.42	283.52	136.93
134	134.00	350.96	311.73	353.81	338.43	251.44	220.66	435.85	281.96	310.52	158.63
135	135.00	344.94	312.90	353.51	351.36	282.69	278.39	493.01	471.51	557.24	400.50
136	136.00	339.27	314.20	341.23	338.83	251.62	246.81	426.91	402.82	422.44	261.33

137	137.00	333.98	315.63	335.20	316.65	228.65	191.55	419.47	233.97	246.15	81.19
138	138.00	329.11	317.19	329.63	329.13	240.59	239.59	412.50	407.50	412.71	244.43
139	139.00	324.67	318.88	334.37	318.36	259.06	227.05	504.31	344.26	441.28	270.24
140	140.00	320.69	320.69	320.84	317.28	231.13	224.01	408.82	373.19	374.72	201.52
141	141.00	317.19	322.62	320.85	305.63	231.92	201.48	443.85	291.63	328.19	153.42
142	142.00	314.20	324.67	316.23	332.23	255.70	287.70	427.60	587.60	607.91	432.20
143	143.00	311.73	326.83	306.51	308.42	212.29	216.12	355.06	374.18	321.95	145.93
144	144.00	309.80	329.11	304.96	293.74	201.05	178.61	358.92	246.74	198.37	22.70
145	145.00	308.41	331.49	312.99	310.38	248.34	243.11	453.15	427.01	472.87	298.19
146	146.00	307.57	333.98	306.62	310.20	234.05	241.21	397.80	433.61	424.12	251.10
147	147.00	307.29	336.58	300.79	297.43	207.23	200.51	342.32	308.71	243.73	73.02
148	148.00	307.57	339.27	312.50	278.68	225.47	157.82	456.63	118.40	167.74	0.00
149	149.00	308.41	342.06	300.51	302.00	213.07	216.06	328.29	343.24	264.24	100.13
150	150.00	309.80	344.94	317.09	315.50	275.03	271.85	480.20	464.34	537.25	377.41
151	151.00	311.73	347.91	314.03	311.14	258.63	252.85	430.22	401.32	424.25	269.31
152	152.00	314.20	350.96	315.35	313.22	260.32	256.07	418.72	397.47	408.90	259.48
153	153.00	317.19	354.09	313.77	322.80	259.32	277.39	373.05	463.37	429.13	285.83
154	154.00	320.69	357.29	319.75	308.82	256.01	234.16	397.92	288.64	279.27	142.67
155	155.00	324.67	360.56	324.68	335.63	288.93	310.85	407.38	516.97	517.06	387.71
156	156.00	329.11	363.90	322.72	329.05	266.52	279.18	343.46	406.76	342.93	221.36
157	157.00	333.98	367.30	334.58	341.06	302.85	315.81	413.22	478.02	483.95	370.65
158	158.00	339.27	370.76	330.78	349.61	287.60	325.27	322.35	510.70	425.76	321.20
159	159.00	344.94	374.26	340.50	347.18	300.81	314.17	362.84	429.64	385.19	289.81
160	160.00	350.96	377.81	345.21	342.17	295.46	289.38	349.83	319.42	261.96	176.15
161	161.00	357.29	381.41	352.99	368.92	330.13	361.99	364.25	523.59	480.54	404.66
162	162.00	363.90	385.04	371.80	343.90	345.35	289.55	486.31	207.27	286.29	220.65
163	163.00	370.76	388.70	369.68	365.31	343.49	334.75	396.54	352.81	342.06	286.93
164	164.00	377.81	392.39	383.41	375.26	377.15	360.84	463.29	381.72	437.72	393.34
165	165.00	385.04	396.09	392.65	383.38	395.02	376.47	483.41	390.68	466.80	433.35
166	166.00	392.39	399.82	390.65	402.59	389.90	413.79	389.91	509.34	491.96	469.58
167	167.00	399.82	403.55	391.09	401.18	371.27	391.44	320.05	420.91	333.67	322.46
168	168.00	407.29	407.29	404.00	411.36	401.49	416.21	374.39	448.01	415.10	415.10

Table C.1: Hourly Demands for 10 Scenarios

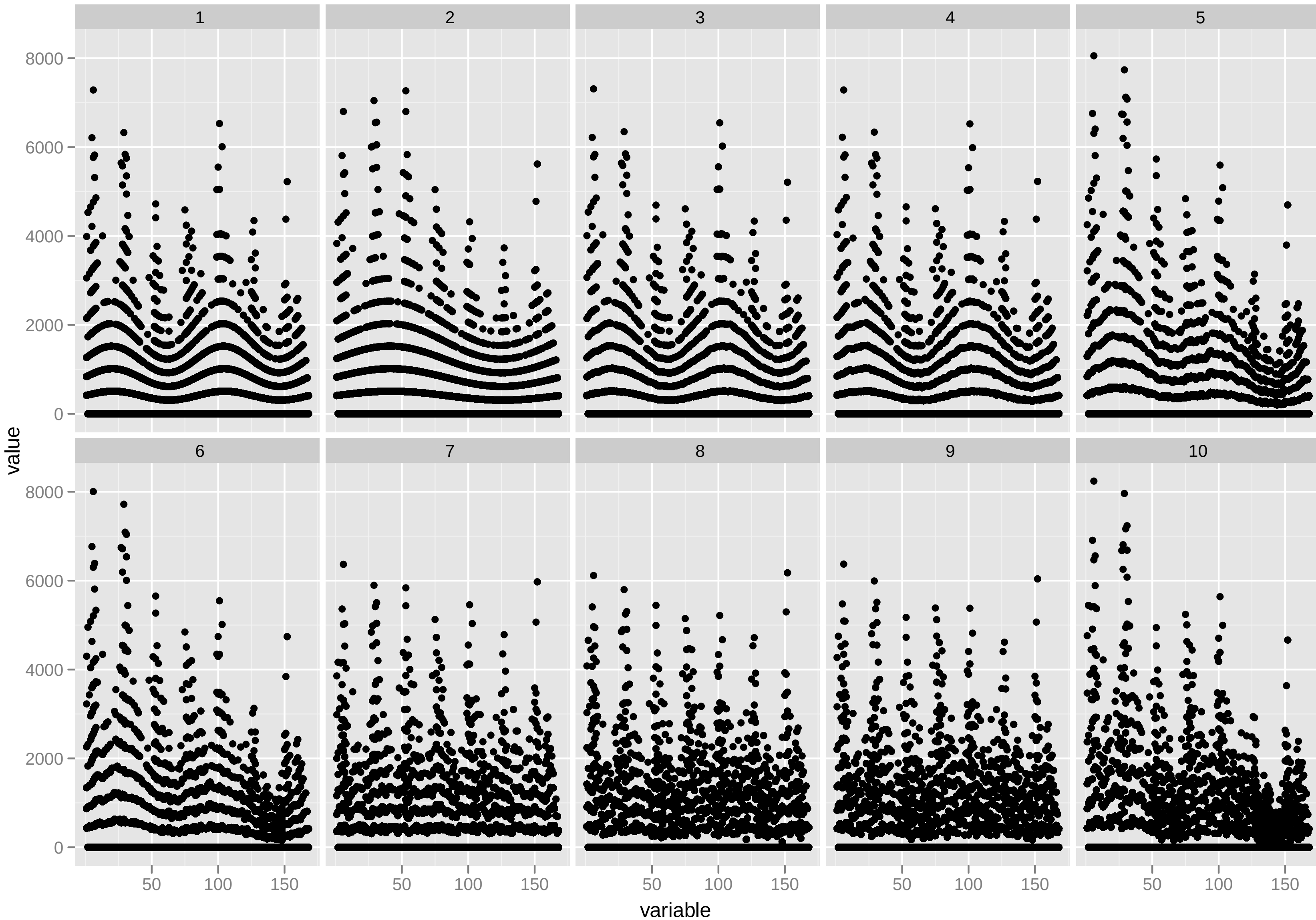

Figure C.1: Overlayed Production Vectors

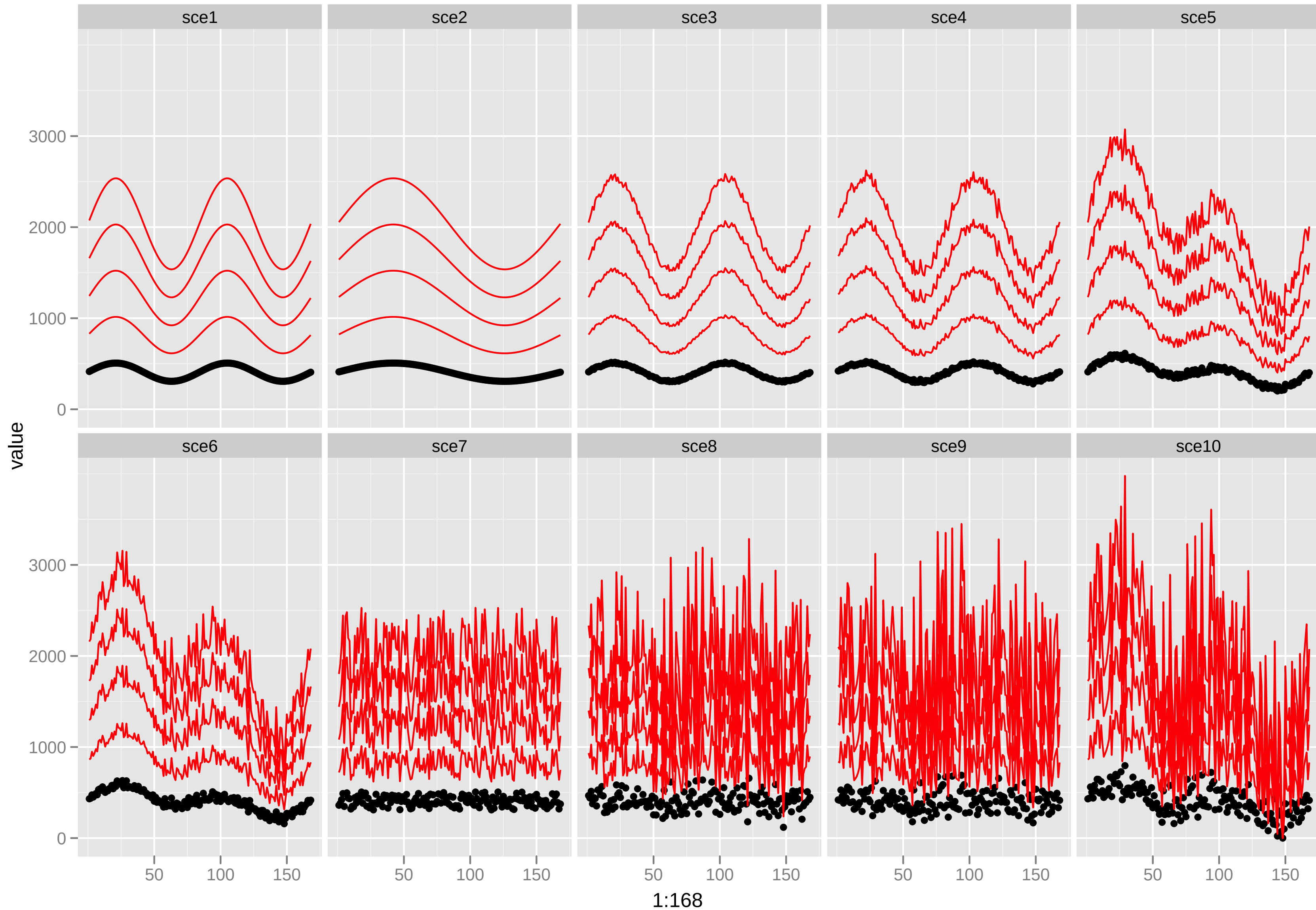

Figure C.2: Multiplied Demand Signals

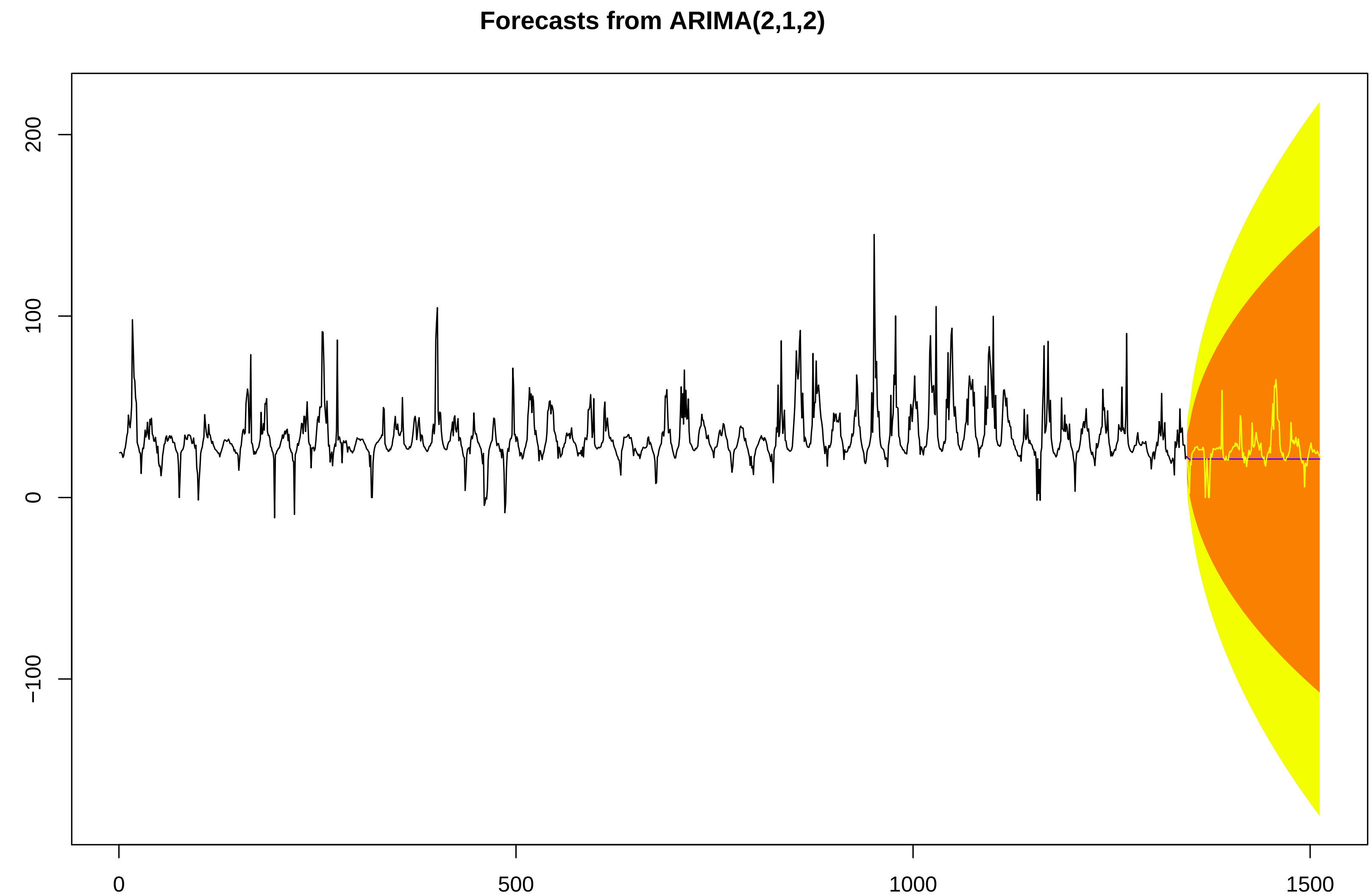

Figure C.3: 8 week history, the forecast, the confidence region and real prices

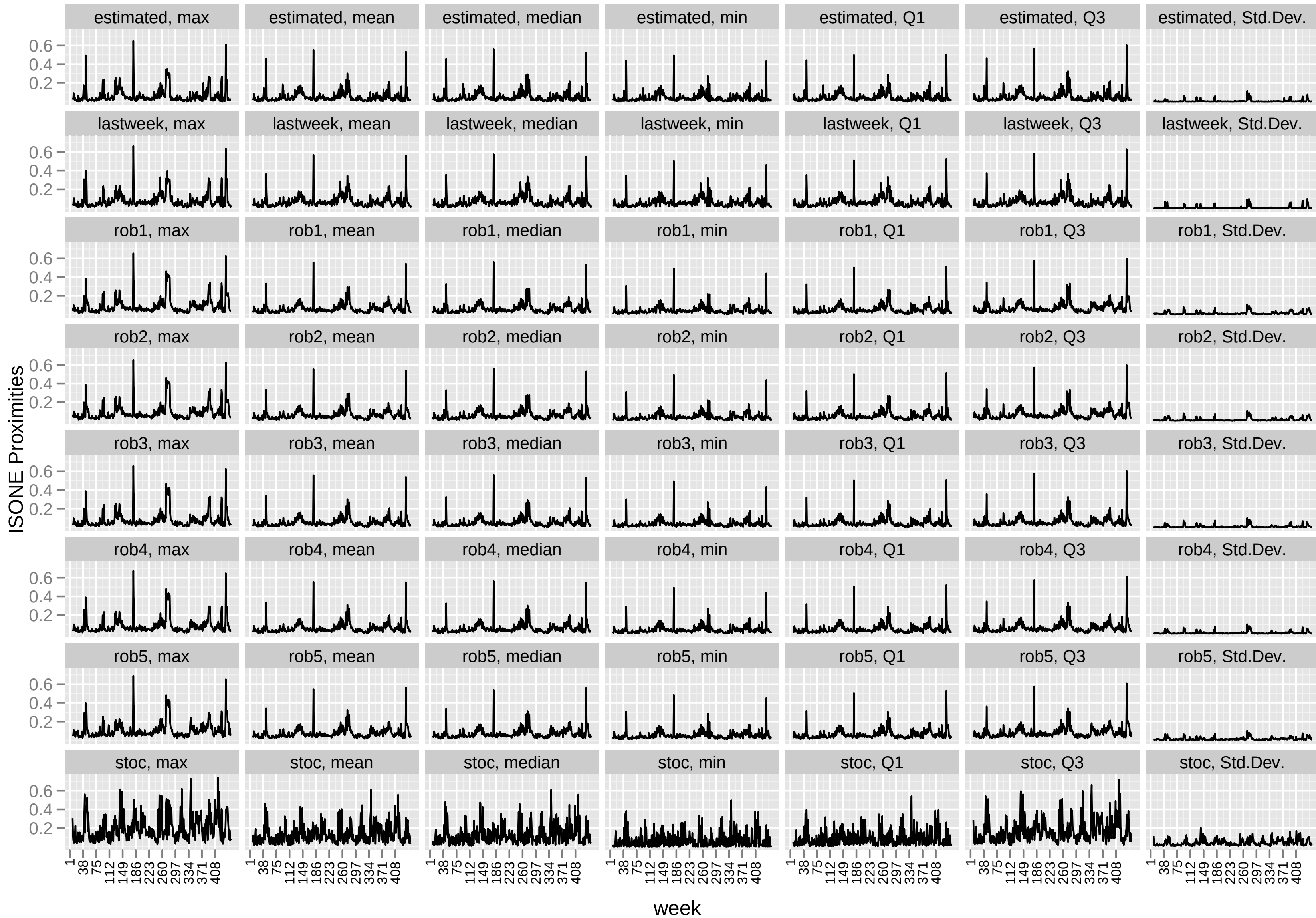

Figure C.4: ISONE Production Planning ρ values

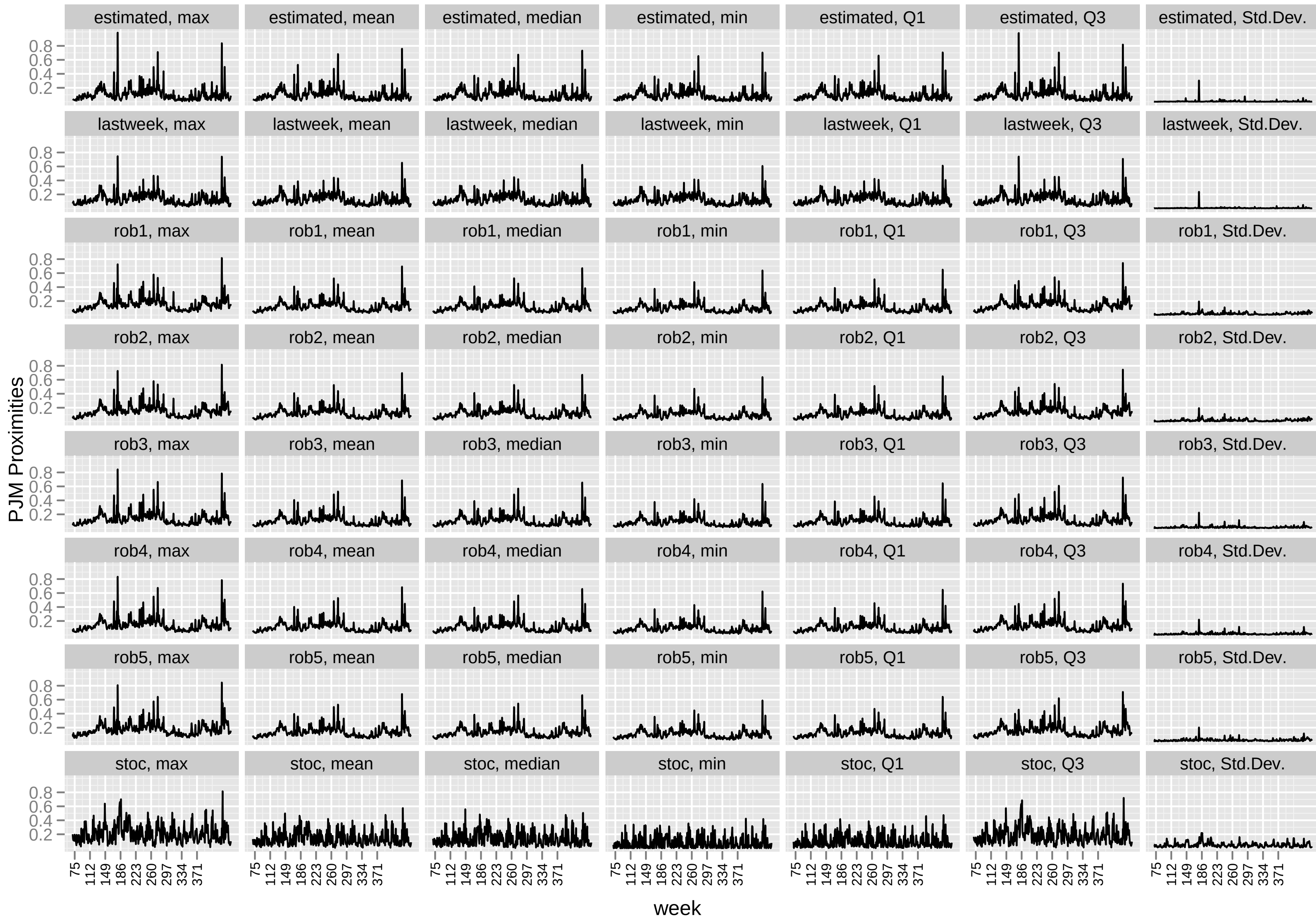

Figure C.5: PJM Production Planning ρ values

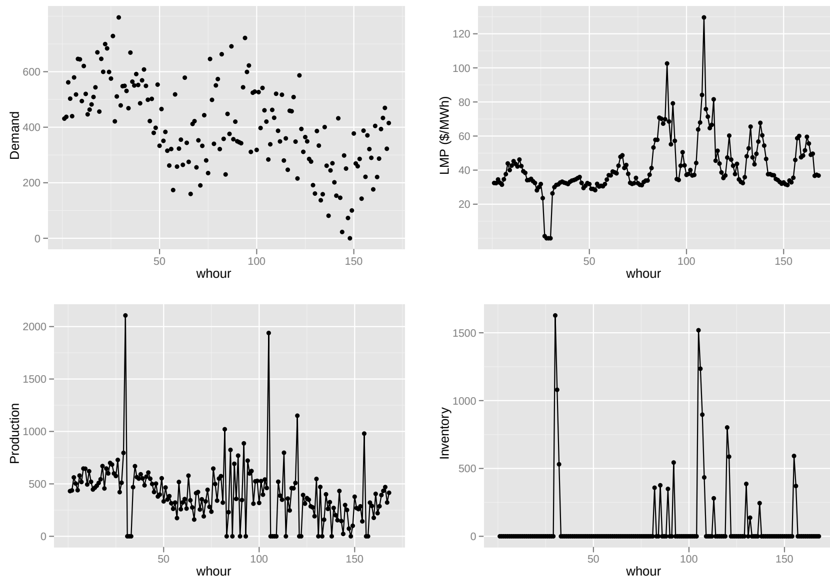

Figure C.6: ISONE Solution

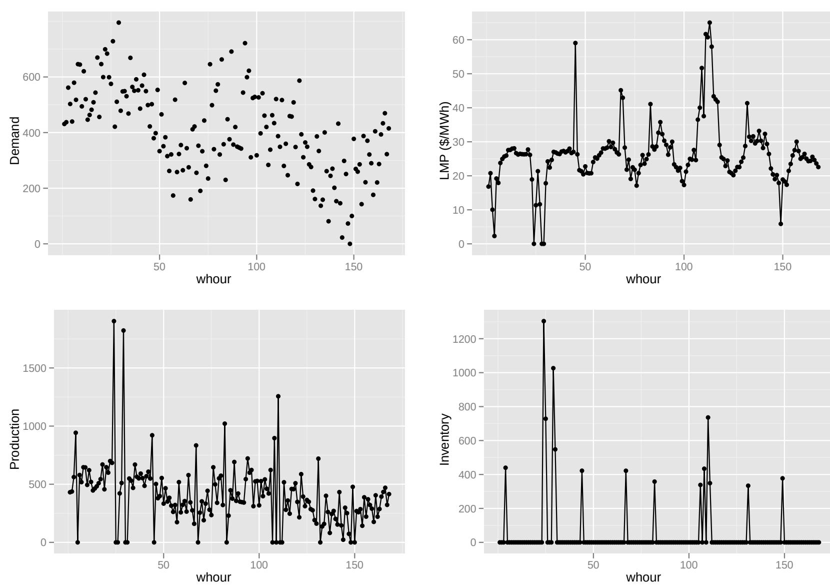

Figure C.7: PJM Solution

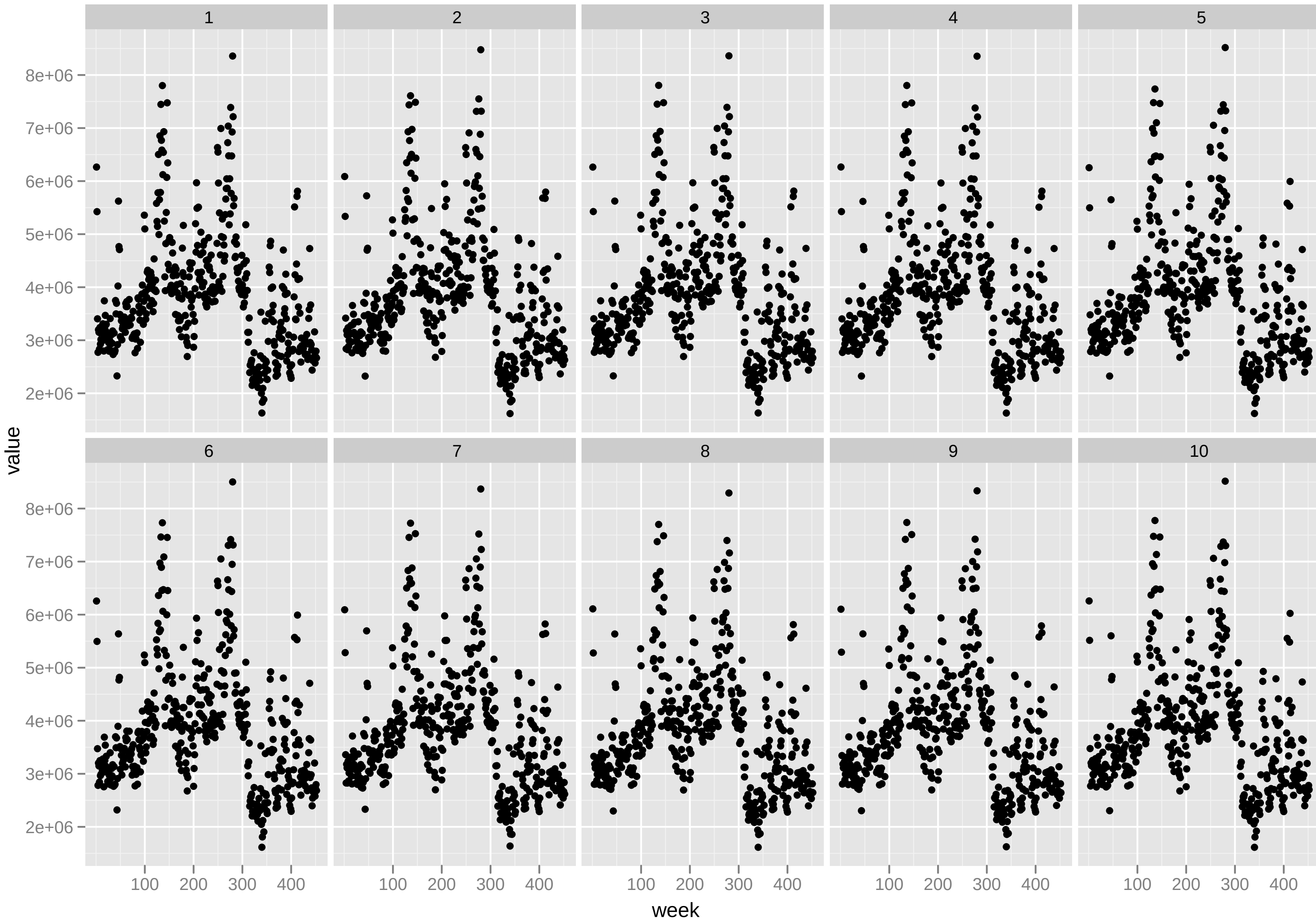

Figure C.8: ISONE Optimal Obj. Fn. Values

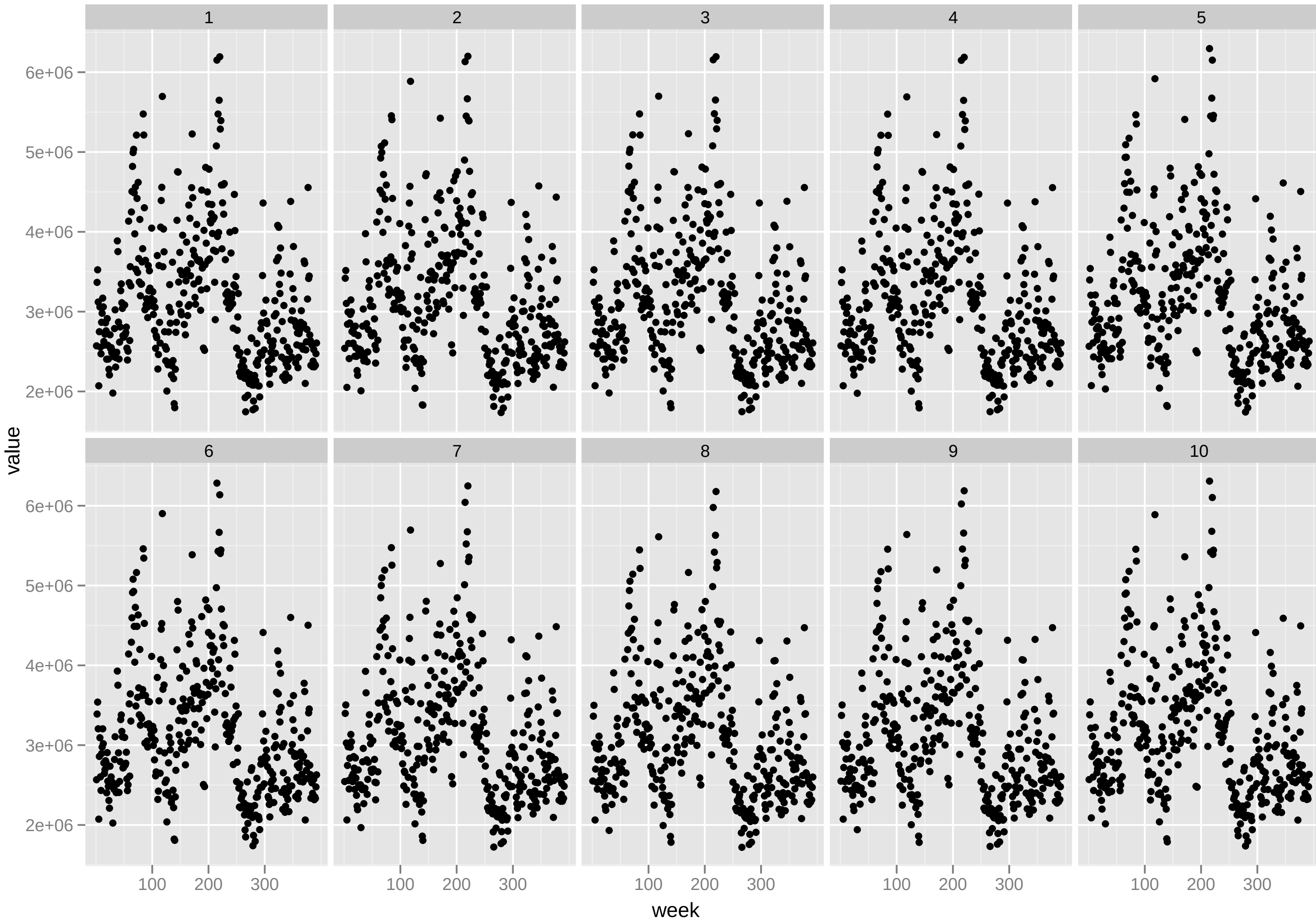

Figure C.9: PJM Optimal Obj. Fn. Values

C.2 Forecasting Accuracy

	ME	RMSE	MAE	MPE	MAPE
mean	-0.00	25.95	19.04		Inf
Std.Dev.	18.62	18.21	13.26		
min	-155.04	2.04	1.70	-Inf	5.06
Q1	-8.74	14.46	10.42	-30.34	23.55
median	-0.21	21.84	15.75	-6.25	33.38
Q3	9.45	32.11	24.06	13.07	47.68
max	154.92	262.86	155.83	Inf	Inf

Table C.2: PJM: p^1 accuracy statistics, anomalies allowed

	ME	RMSE	MAE	MPE	MAPE	MASE
mean	3.33	20.81	15.85	-Inf	Inf	0.93
Std.Dev.	11.05	15.51	11.11			0.46
min	-46.60	1.16	0.79	-Inf	2.99	0.10
Q1	-1.22	11.04	8.37	-18.99	21.65	0.62
median	1.41	17.71	13.57	-5.10	30.34	0.84
Q3	7.09	26.23	20.17	0.65	42.05	1.12
max	146.83	264.91	155.41	783.77	Inf	4.22

Table C.3: PJM: p^2 accuracy statistics, anomalies allowed

	ME	RMSE	MAE	MPE	MAPE
mean	-0.01	21.46	15.98	-Inf	Inf
Std.Dev.	17.52	20.42	13.30		
min	-220.08	3.01	2.16	-Inf	5.22
Q1	-7.26	11.16	8.39	-20.86	17.13
median	0.18	16.49	12.41	-3.04	23.42
Q3	7.23	25.77	19.46	9.36	33.93
max	217.19	400.92	220.08	72.23	Inf

Table C.4: ISONE: p^1 accuracy statistics, anomalies allowed

	ME	RMSE	MAE	MPE	MAPE	MASE
mean	2.04	16.15	12.24	-Inf	Inf	0.98
Std.Dev.	9.45	14.86	9.66			0.53
min	-50.76	2.00	1.71	-Inf	4.96	0.05
Q1	-1.28	8.66	6.84	-10.52	14.38	0.65
median	0.75	12.83	9.96	-2.64	19.29	0.87
Q3	4.50	19.37	14.92	1.36	26.41	1.17
max	145.63	358.29	217.80	84.02	Inf	8.91

Table C.5: ISONE: p^2 accuracy statistics, anomalies allowed

	ME	RMSE	MAE	MPE	MAPE
mean	0.00	25.63	18.83		Inf
Std.Dev.	18.26	17.62	12.94		
min	-155.04	2.04	1.70	-Inf	5.06
Q1	-8.61	14.35	10.33	-30.14	23.30
median	-0.09	21.70	15.57	-6.34	33.07
Q3	9.29	31.83	23.80	12.57	46.73
max	154.92	262.86	155.83	Inf	Inf

Table C.6: PJM: p^1 accuracy statistics, anomalies removed

	ME	RMSE	MAE	MPE	MAPE	MASE
mean	3.32	20.54	15.71	-Inf	Inf	0.92
Std.Dev.	10.78	14.76	10.77			0.46
min	-46.60	1.16	0.79	-Inf	2.99	0.10
Q1	-1.16	10.94	8.30	-18.84	21.52	0.62
median	1.44	17.60	13.47	-5.21	30.02	0.83
Q3	7.09	26.02	20.05	0.58	41.37	1.11
max	146.83	264.91	155.41	783.77	Inf	4.22

Table C.7: PJM: p^2 accuracy statistics, anomalies removed

	ME	RMSE	MAE	MPE	MAPE
mean	-0.12	21.22	15.86	-Inf	Inf
Std.Dev.	16.96	18.85	12.69		
min	-220.08	3.01	2.16	-Inf	5.22
Q1	-7.26	11.15	8.38	-20.68	17.11
median	0.17	16.42	12.38	-3.04	23.41
Q3	7.20	25.57	19.35	9.34	33.81
max	157.52	400.92	220.08	72.23	Inf

Table C.8: ISONE: p^1 accuracy statistics, anomalies removed

	ME	RMSE	MAE	MPE	MAPE	MASE
mean	1.98	15.92	12.13	-Inf	Inf	0.98
Std.Dev.	9.04	12.81	8.76			0.52
min	-50.76	2.00	1.71	-Inf	4.96	0.05
Q1	-1.26	8.63	6.83	-10.45	14.36	0.65
median	0.75	12.80	9.94	-2.62	19.25	0.87
Q3	4.50	19.32	14.88	1.39	26.34	1.17
max	145.63	228.81	145.63	84.02	Inf	8.91

Table C.9: ISONE: p^2 accuracy statistics, anomalies removed

Bibliography

H. Akaike. Information theory and an extension of the maximum likelihood principle. In *Second international symposium on information theory*, volume 1, pages 267–281. Springer Verlag, 1973.

A. Arreola-Risa and G. A. DeCroix. Inventory management under random supply disruptions and partial backorders. *Naval Research Logistics*, 45(7):687–703, 1998. ISSN 1520-6750.

B. Auguie. gridextra: functions in grid graphics, 2012. URL `http://CRAN.R-project.org/package=gridExtra`. R package version 0.9.

R. Baldick, S. Kolos, and S. Tompaidis. Interruptible electricity contracts from an electricity retailer's point of view: Valuation and optimal interruption. *Operations Research*, 54(4):627–642, 2006. ISSN 0030364X.

A. Banerjee, J. Dolado, J. Galbraith, and D. Hendry. Co-integration, error correction, and the econometric analysis of non-stationary data. *OUP Catalogue*, 1993.

D. Bates and M. Maechler. *Matrix: Sparse and Dense Matrix Classes and Methods*, 2012. URL `http://Matrix.R-forge.R-project.org/`. R package version 1.0-6.

A. Ben-Tal and A. Nemirovski. Robust solutions of linear programming problems contaminated with uncertain data. *Mathematical Programming*, 88(3):411–424, 2000.

A. Ben-Tal and A. Nemirovski. Robust optimization - methodology and spplications. *Mathematical Programming*, 92(3):453–480, May 2002.

A. Ben-Tal and A. Nemirovski. Selected topics in robust convex optimization. *Mathematical Programming*, 112(1):125–158, 2008.

A. Ben-Tal, A. . Goryashko, E. . Guslitzer, and A. Nemirovski. Adjustable robust solutions of uncertain linear programs. *Mathematical Programming*, 99(2):351–376, Mar 2004.

A. Ben-Tal, S. P. Boyd, and A. Nemirovski. Extending scope of robust optimization: Comprehensive robust counterparts of uncertain problems. *Mathematical Programming*, 107(1):63–89, 2006.

A. Ben-Tal, L. El Ghaoui, and A. Nemirovski. *Robust Optimization (Princeton Series in Applied Mathematics)*. Princeton University Press, 41 William Street, Princeton, New Jersey., 2009. ISBN 0691143684.

H. Bengtsson, H. C. Bravo, R. Gentleman, and H. Jaffee. matrixstats: Methods that apply to rows and columns of a matrix, 2011. URL `http://CRAN.R-project.org/package=matrixStats`. R package version 0.4.3.

D. Bertsimas and M. Sim. Robust discrete optimization and network flows. *Mathematical Programming*, 98(1):49–71, 2003.

D. Bertsimas and M. Sim. The price of robustness. *Operations Research*, 52(1):35–53, 2004.

D. Bertsimas and M. Sim. Tractable approximations to robust conic optimization problems. *Mathematical Programming*, 107(1):5–36, 2006.

D. Bertsimas, D. B. Brown, and C. Caramanis. Theory and applications of robust optimization, oct 2010.

J. Buckner, M. Seligman, and J. Wilson. *gputools: A few GPU enabled functions*, 2011. URL `http://CRAN.R-project.org/package=gputools`. R package version 0.26.

California Public Utilities Commission. Report on interruptible programs and rotating outages. Technical report, Energy Division, 2001.

D. W. Caves and J. A. Herriges. Optimal dispatch of interruptible and curtailable service options. *Operations Research*, 40(1):104–112, 1992. ISSN 0030364X.

H.-P. Chao. Inventory policy in the presence of market disruptions. *Operations Research*, 35(2):274–281, 1987. ISSN 0030364X.

H.-P. Chao and R. Wilson. Priority service: Pricing, investment, and market organization. *The American Economic Review*, 77(5):899–916, 1987. ISSN 00028282.

A. Conejo and M. Carrion. Risk-constrained electricity procurement for a large consumer. *Generation, Transmission And Distribution, Iee Proceedings-*, 153(4): 407 – 413, July 2006. ISSN 1350-2360.

D. B. Dahl. *xtable: Export tables to LaTeX or HTML*, 2012. URL `http://CRAN.R-project.org/package=xtable`. R package version 1.7-0.

S. Dempe. *Foundations of Bilevel Programming (Nonconvex Optimization and Its Applications)*. Springer, 2002. ISBN 1402006314.

M. Dowle, T. Short, and S. Lianoglou. data.table: Extension of data.frame for fast indexing, fast ordered joins, fast assignment, fast grouping and list columns., 2012. URL `http://CRAN.R-project.org/package=data.table`. R package version 1.7.10.

D. Eddelbuettel and R. François. Rcpp: Seamless R and C++ integration. *Journal of Statistical Software*, 40(8):1–18, 2011. URL `http://www.jstatsoft.org/v40/i08/`.

L. El Ghaoui and H. Lebret. Robust solutions to least-squares problems with uncertain data. *SIAM Journal on Matrix Analysis and Applications*, 18:1035–1064, 1997.

L. El Ghaoui, F. Oustry, and H. Lebret. Robust solutions to uncertain semidefinite programs. *SIAM Journal of Optimization*, 9:33–52, 1998.

S. Engell, E. Handschin, C. Rehtanz, and R. Schultz. Capacity planning and scheduling in electrical power systems and in chemical and metallurgical production plants. In M. Grtschel, K. Lucas, and V. Mehrmann, editors, *Production Factor Mathematics*, pages 279–305. Springer Berlin Heidelberg, 2010. ISBN 978-3-642-11248-5.

R. Evans. *rje: Miscellaneous useful functions*, 2012. URL `http://CRAN.R-project.org/package=rje`. R package version 1.2.

M. Fahrioğlu and F. L. Alvarado. Designing incentive compatible contracts for effective demand management. *IEEE Transactions on Power Systems*, 15(4):1255 –1260, nov 2000. ISSN 0885-8950.

M. Fahrioğlu and F. L. Alvarado. Using utility information to calibrate customer demand management behavior models. *IEEE Transactions on Power Systems*, 16 (2):317 –322, may 2001. ISSN 0885-8950.

R. Furrer, D. Nychka, and S. Sain. fields: Tools for spatial data, 2012. URL `http://CRAN.R-project.org/package=fields`. R package version 6.6.3.

M. R. Garey, D. S. Johnson, and R. Sethi. The complexity of flowshop and jobshop scheduling. *Mathematics of Operations Research*, 1(2):pp. 117–129, 1976. ISSN 0364765X. URL `http://www.jstor.org/stable/3689278`.

G. Grolemund and H. Wickham. Dates and times made easy with lubridate. *Journal of Statistical Software*, 40(3):1–25, 2011. URL `http://www.jstatsoft.org/v40/i03/`.

R. Güllü, E. Önol, and N. Erkip. Analysis of a deterministic demand production/inventory system under nonstationary supply uncertainty. *IIE Transactions*, 29(8):703–709, Aug 1997.

Ü. Gürler and M. Parlar. An inventory problem with two randomly available suppliers. *Operations Research*, 45(6):904–918, 1997. ISSN 0030364X.

I. Gurobi Optimization. Gurobi optimizer reference manual, 2012. URL `http://www.gurobi.com`.

A. J. Hoffman and J. B. Kruskal. *Integral boundary points of convex polyhedra*, volume 38. 1956.

K. Hornik, D. Meyer, and C. Buchta. *slam: Sparse Lightweight Arrays and Matrices*, 2011. URL `http://CRAN.R-project.org/package=slam`. R package version 0.1-23.

R. Hyndman and A. Koehler. Another look at measures of forecast accuracy. *International Journal of Forecasting*, 22(4):679–688, 2006.

R. Hyndman, Y. Khandakar, M. U. D. of Econometrics, and B. Statistics. *Automatic Time Series for Forecasting: The Forecast Package for R*. Monash University, Department of Econometrics and Business Statistics, 2007.

M. Ierapetritou, D. Wu, J. Vin, P. Sweeney, and M. Chigirinskiy. Cost minimization in an energy-intensive plant using mathematical programming approaches. *Industrial & Engineering Chemistry Research*, 41(21):5262–5277, 2002.

ISONE. `http://www.iso-ne.com/aboutiso/co_profile/history/index.html`, a.

ISONE. `http://www.iso-ne.com/img/wem.pdf`, b.

ISONE. `http://www.iso-ne.com/markets/mkt_anlys_rpts/wkly_mktops_rpts/index.html`, c.

ISONE. `http://www.iso-ne.com/markets/hstdata/index.html`, d.

R. Kamat and S. S. Oren. Exotic options for interruptible electricity supply contracts. *Operations Research*, 50(5):835–850, 2002. ISSN 0030364X.

M. Karwan and M. Keblis. Operations planning with real time pricing of a primary input. *Computers & Operations Research*, 34(3):848 – 867, 2007. ISSN 0305-0548.

M. Kendall. A new measure of rank correlation. *Biometrika*, 30(1/2):81–93, 1938.

P. Kouvelis and G. Yu. *Robust Discrete Optimization and Its Applications (Nonconvex Optimization and Its Applications)*. Springer-Verlag New York, LLC., 1996. ISBN 0792342917.

D. Kwiatkowski, P. Phillips, P. Schmidt, and Y. Shin. Testing the null hypothesis of stationarity against the alternative of a unit root* 1:: How sure are we that economic time series have a unit root? *Journal of econometrics*, 54(1-3):159–178, 1992.

F. Longstaff and A. Wang. Electricity forward prices: a high-frequency empirical analysis. *The Journal of Finance*, 59(4):1877–1900, 2004.

T. C. Mills. *Time Series Techniques for Economists*. Cambridge University Press, 1991. ISBN 0521405742. URL `http://www.amazon.com/Series-Techniques-Economists-Terence-Mills/dp/0521405742%3FSubscriptionId%3D0JYN1NVW651KCA56C102%26tag%3Dtechkie-20%26linkCode%3Dxm2%26camp%3D2025%26creative%3D165953%26creativeASIN%3D0521405742`.

K. Moinzadeh and P. Aggarwal. Analysis of a production/inventory system subject to random disruptions. *Management Science*, 43(11):1577–1588, 1997. ISSN 00251909.

J. Mula, R. Poler, J. P. García-Sabater, and F. C. Lario. Models for production planning under uncertainty: A review. *International Journal of Production Economics*, 103(1):271 – 285, 2006. ISSN 0925-5273.

M. Parlar. Continuous-review inventory problem with random supply interruptions. *European Journal of Operational Research*, 99(2):366 – 385, 1997. ISSN 0377-2217.

M. Parlar and D. Perry. Inventory models of future supply uncertainty with single and multiple suppliers. *Naval Research Logistics*, 43(2):191–210, 1996. ISSN 1520-6750.

M. Parlar, Y. Wang, and Y. Gerchak. A periodic review inventory model with markovian supply availability. *International Journal of Production Economics*, 42(2):131 – 136, 1995. ISSN 0925-5273.

K. Pearson. Liii. on lines and planes of closest fit to systems of points in space. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, 2 (11):559–572, 1901.

D. Peidro, J. Mula, R. Poler, and F.-C. Lario. Quantitative models for supply chain planning under uncertainty: a review. *The International Journal of Advanced Manufacturing Technology*, 43(3):400–420, 2009.

P. Perron. Trends and random walks in macroeconomic time series:: Further evidence from a new approach. *Journal of economic dynamics and control*, 12(2-3): 297–332, 1988.

M. L. Pinedo. *Scheduling: Theory, Algorithms, and Systems*. Springer, 2012. ISBN 1461419867. URL `http://www.amazon.com/Scheduling-Algorithms-Michael-L-Pinedo/dp/1461419867%3FSubscriptionId%3D0JYN1NVW651KCA56C102%26tag%3Dtechkie-20%26linkCode%3Dxm2%26camp%3D2025%26creative%3D165953%26creativeASIN%3D1461419867`.

PJM LLC. `http://www.pjm.com/~/media/about-pjm/newsroom/fact-sheets/pjm-at-a-glance.ashx`, a.

PJM LLC. http://www.pjm.com/documents/reports/~/media/documents/reports/state-of-market/2011/2011q1-som-pjm.ashx, b.

PJM LLC. http://www.pjm.com/~/media/about-pjm/newsroom/fact-sheets/pjms-role-as-an-rto-fact-sheet.ashx, c.

PJM LLC. http://pjm.com/training/~/media/training/core-curriculum/ip-trans-101/markets-overview.ashx, d.

PJM LLC. http://www.pjm.com/documents/reports/state-of-market-reports.aspx, e.

PJM LLC. http://www.pjm.com/markets-and-operations/energy/real-time/historical-bid-data.aspx, f.

J. Popova. Spatial pattern in modeling electricity prices: Evidence from the pjm market. In *Proc. 24th USAEE and IAEE North American Conf*, pages 8–10, 2004.

R Development Core Team. R: A language and environment for statistical computing, 2011. URL http://www.R-project.org/. ISBN 3-900051-07-0.

S. R. Rob J. Hyndman and D. Schmidt. forecast: Forecasting functions for time series, 2012. URL http://CRAN.R-project.org/package=forecast. R package version 3.18.

J. Rosenhead, M. Elton, and S. K. Gupta. Robustness and optimality as criteria for strategic decisions. *Operational Research Quarterly (1970-1977)*, 23(4):413–431, 1972. ISSN 00303623.

J. A. Ryan and J. M. Ulrich. xts: extensible time series, 2011. URL http://CRAN.R-project.org/package=xts. R package version 0.8-2.

S. Said and D. Dickey. Testing for unit roots in autoregressive-moving average models of unknown order. *Biometrika*, 71(3):599–607, 1984.

H. Shawky, A. Marathe, and C. Barrett. A first look at the empirical relation between spot and futures electricity prices in the united states. *Journal of Futures Markets*, 23(10):931–955, 2003.

F. P. Sioshansi. Competitive electricity markets - design, implementation, performance, 2008.

O. Sklyar, D. Murdoch, M. Smith, D. Eddelbuettel, and R. Francois. *inline: Inline C, C++, Fortran function calls from R*, 2010. URL `http://CRAN.R-project.org/package=inline`. R package version 0.3.8.

L. V. Snyder, Z. Atan, P. Peng, Y. Rong, A. J. Schmitt, and B. Sinsoysal. Or/ms models for supply chain disruptions: A review. *SSRN eLibrary*, 2010.

A. Sorokin, S. Rebennack, P. M. Pardalos, N. A. Iliadis, M. V. F. Pereira, and P. M. Pardalos, editors. *Handbook of Networks in Power Systems I*. Energy Systems. Springer Berlin Heidelberg, 2012a. ISBN 978-3-642-23193-3.

A. Sorokin, S. Rebennack, P. M. Pardalos, N. A. Iliadis, M. V. F. Pereira, and P. M. Pardalos, editors. *Handbook of Networks in Power Systems II*. Energy Systems. Springer Berlin Heidelberg, 2012b. ISBN 978-3-642-23406-4.

A. L. Soyster. Convex programming with set-inclusive constraints and applications to inexact linear programming. *Operations Research*, 21(5):1154–1157, 1973. ISSN 0030364X.

C. Spearman. Demonstration of formulae for true measurement of correlation. *The American Journal of Psychology*, pages 161–169, 1907.

T. Strauss and S. S. Oren. Priority pricing of interruptible electric service with an early notification option. *The Energy Journal*, 14(2):175–196, 1993.

A. Trapletti and K. Hornik. tseries: Time series analysis and computational finance, 2011. URL `http://CRAN.R-project.org/package=tseries`. R package version 0.10-27.

J. Tschirhart and F. Jen. Behavior of a monopoly offering interruptible service. *The Bell Journal of Economics*, 10(1):244–258, 1979. ISSN 0361915X.

H. Wagner and T. Whitin. Dynamic version of the economic lot size model. *Management science*, pages 89–96, 1958.

T. Wei. *corrplot: Visualization of a correlation matrix*, 2011. URL `http://CRAN.R-project.org/package=corrplot`. R package version 0.60.

H. J. Weiss and E. C. Rosenthal. Optimal ordering policies when anticipating a disruption in supply or demand. *European Journal of Operational Research*, 59 (3):370 – 382, 1992. ISSN 0377-2217.

Wickham and Hadley. Reshaping data with the reshape package. *Journal of Statistical Software*, 21(12), 2007. URL `http://www.jstatsoft.org/v21/i12/paper`.

H. Wickham. *ggplot2: elegant graphics for data analysis*. Springer New York, 2009. ISBN 978-0-387-98140-6. URL `http://had.co.nz/ggplot2/book`.

H. Wickham. The split-apply-combine strategy for data analysis. *Journal of Statistical Software*, 40(1):1–29, 2011. URL `http://www.jstatsoft.org/v40/i01/`.

D. Wuertz and Y. Chalabi. timeseries: Rmetrics - financial time series objects,

2011. URL http://CRAN.R-project.org/package=timeSeries. R package version 2130.92.

D. Wuertz, Y. Chalabi, and M. Maechler. timedate: Rmetrics - chronological and calendarical objects, 2011. URL http://CRAN.R-project.org/package=timeDate. R package version 2131.00.

C. A. Yano and H. A. Lee. Lot sizing with random yield: A review. *Operations Research*, 43(2):311–334, 1995.

A. Zeileis and G. Grothendieck. zoo: S3 infrastructure for regular and irregular time series. *Journal of Statistical Software*, 14(6):1–27, 2005. URL http://www.jstatsoft.org/v14/i06/.

A. Zeileis and T. Hothorn. Diagnostic checking in regression relationships. *R News*, 2(3):7–10, 2002. URL http://CRAN.R-project.org/doc/Rnews/.

Q. P. Zheng, S. Rebennack, N. A. Iliadis, and P. M. Pardalos. Optimization models in the natural gas industry. In P. M. Pardalos, S. Rebennack, M. V. F. Pereira, and N. A. Iliadis, editors, *Handbook of Power Systems I*, Energy Systems, pages 121–148. Springer Berlin Heidelberg, 2010. ISBN 978-3-642-02493-1.

Y. Zhu, S. Legg, and C. Laird. A multiperiod nonlinear programming approach for operation of air separation plants with variable power pricing. *Aiche Journal*, pages N/A–N/A, 2010. ISSN 1547-5905.