UNIVERSITÉ PARIS 1 PANTHÉON-SORBONNE
U.F.R. DE SCIENCES ÉCONOMIQUES

Thèse pour le Doctorat de Sciences Économiques
présentée et soutenue publiquement par

Ahmet OZKARDAS

le 17 novembre 2014

ESSAYS ON WAGE BARGAINING

Directeur de thèse : Agnieszka RUSINOWSKA

COMPOSITION DU JURY :

Philippe BICH	Professeur, Université Paris 1 Panthéon-Sorbonne, PSE	Examinateur
Jacek MERCIK	Professeur, Wroclaw School of Banking, Wroclaw	Rapporteur
Stefano MORETTI	Chargé de Recherche au CNRS, Université Paris-Dauphine	Examinateur
Agnieszka RUSINOWSKA	Directeur de Recherche au CNRS, PSE Université Paris 1 Panthéon-Sorbonne	Directeur de thèse
Harrie de SWART	Professeur, Erasmus University, Rotterdam	Rapporteur

Acknowledgements

First and foremost I want to thank my supervisor Agnieszka Rusinowska. I am extremely grateful for her support and guidance starting from the project of my master thesis until my Ph.D. defense. Her thoughtful advices helped me to advance with publishing chapters of this thesis and preparing its final version for the defense. I appreciate all her contributions of time, ideas and enthusiasm throughout the last six years.

I am also very grateful to Professor Jacek Mercik, Professor Harrie de Swart, Professor Philippe Bich and Professor Stefano Moretti for having agreed to take part of my Ph.D. Committee and for having read this thesis. Additionally I would like to express my gratitude to Professor Jacek Mercik and Professor Harrie de Swart for having agreed to be referees of the thesis.

Furthermore, I would like to acknowledge the support from Turgut Ozal University. Especially I am thankful to Muhammet Kosecik and Ramazan Tas for helping and encouraging me during my Ph.D. thesis.

Special thanks also go to all my colleagues at Turgut Ozal University, in particular, to Ahmet Burak, Ahmet Sencer, Abdulmelik, Omer and Salih.

Last but not least, many thanks to my family, in particular to my mom, my dad, my sisters and my cats, for their continuous support which helped me cope with all 'ups' and 'downs' that came along the way during these four years. Siz olmasaydınız bu tez asla bitmezdi, teşekkür ederim.

Abstract

This Ph.D. dissertation develops important contributions to the literature on wage bargaining. We introduce discount rates varying in time to the wage bargaining models in order to model real life situations in a more accurate way.

In Chapter 1, we state the main objectives of this dissertation.

In Chapter 2, we deliver a brief literature overview of bargaining models, more precisely wage bargaining models. We recall axiomatic and strategic approaches to bargaining and then describe in details strategic approach to wage bargaining models.

In Chapter 3, we investigate the wage bargaining model with preferences varying in time. First, we analyze subgame perfect equilibria in the model and then determine the subgame perfect equilibria payoffs of the parties. Furthermore, we study the inefficient equilibria in the model.

In Chapter 4, we investigate some extensions of the generalized wage bargaining model. First, we analyze wage bargaining with the go-slow actions of the union and study the subgame perfect equilibrium payoffs. Next, we investigate a wage bargaining model where the firm has the lockout option.

In Chapter 5, we apply the generalized wage bargaining models to real life problems, such as price negotiations.

In Chapter 6, we present conclusions and give new insights to our future research.

Keywords*: union-firm wage bargaining, discount rates varying in time, subgame perfect equilibrium, strike, equilibrium payoffs, go-slow, lockout, price negotiation*

Résumé

Titre : Essais sur les négociations salariales

Cette dissertation de doctorat développe des contributions importantes à la littérature sur la négociation salariale. Nous introduisons des taux d'actualisation variant dans le temps pour les modèles de négociation salariale afin de modéliser des situations réelles d'une manière plus précise.

Dans le Chapitre 1, nous présentons les objectifs principaux de cette dissertation.

Dans le Chapitre 2, nous offrons un bref aperçu de la littérature sur les modèles de négociation, plus précisément des modèles de négociation salariale. Nous rappelons les approches axiomatiques et stratégiques des modèles de négociation et étudions en détail l'approche stratégique des modèles de négociation salariale.

Dans le Chapitre 3, nous étudions le modèle de négociation salariale avec des préférences qui varient dans le temps. Tout d'abord, nous analysons les équilibres en sous-jeu parfait dans le modèle, d'autre part, nous déterminons les gains d'équilibre en sous-jeux parfaits des parties. Par ailleurs, nous étudions les équilibres inefficaces dans le modèle.

Dans le Chapitre 4, nous étudions quelques extensions du modèle de négociation salariale généralisé. Premièrement, nous analysons les négociations salariales avec les actions de "go-slow" et étudions les gains d'équilibre en sous-jeux parfaits. Par ailleurs, nous étudions un modèle de négociation salariale où la firme a l'option de "lockouts".

Dans le Chapitre 5, nous appliquons les modèles de négociation de salaires généralisés aux problèmes de la vie réelle, comme les négociations de prix.

Dans le Chapitre 6, nous présentons les conclusions et donnons de nouvelles perspectives à nos recherches futures.

Mots clés : *Négociation salariale entre un syndicat et une firme, taux d'escompte variable, équilibre en sous-jeu parfait, paiements sur un équilibre, "go-slow", "lockout", negotiation de prix*

Résumé prolongé

La théorie de la négociation et de ses applications, par exemple, les négociations salariales entre les entreprises et les syndicats, sont largement analysées dans la littérature. L'une des approches pour expliquer l'interaction entre les négociateurs est basée sur la négociation statique de Nash (Nash [1950]), où l'analyse est axée sur les résultats et ses propriétés. Une autre approche initiée par Rubinstein [1982] analyse les stratégies de négociation et donne plus de perspicacité pour comprendre la procédure de négociation. Afin de modéliser des situations réelles, l'utilisation du modèle de négociation dynamique de Rubinstein permet de comprendre clairement les incitations des acteurs pour obtenir un accord dès que possible.

L'un des sujets sur la théorie de la négociation largement discuté dans la littérature économique concerne la négociation collective sur les salaires entre les entreprises et les travailleurs. Malgré de nombreux travaux sur ce sujet, à notre connaissance, la négociation salariale avec des taux d'actualisation variant dans le temps n'a pas été analysée avant. Cette thèse de doctorat est consacrée précisément à la *négociation salariale avec les préférences des parties déterminées par des taux d'actualisation variant dans le temps*, qui sera aussi appelée *la négociation salariale généralisée.* Plus précisément, les objectifs de cette thèse de doctorat sont les suivants :

1. Etudier les négociations salariales et fournir un aperçu des modèles de négociation salariale

2. Souligner l'importance de la négociation salariale généralisée pour modéliser des situations de la vie réelle

3. Etudier le modèle de négociation salariale avec les préférences (taux d'actualisation) variant dans le temps:

   a) Analyser des équilibres parfaits en sous-jeux dans le modèle

   b) Déterminer les gains des équilibres parfaits en sous-jeux des parties

   c) Etudier les équilibres inefficaces dans le modèle

4. Etudier des extensions du modèle de négociation salariale généralisée, comme les négociations salariales avec les actions "go-slow" et "lockouts"

5. Appliquer les modèles de négociation de salaires généralisées aux problèmes de la vie réelle, tels que la négociation de prix et les négociations des prix des produits pharmaceutiques.

Nous généralisons le modèle de négociation salariale introduit par Fernandez et Glazer [1991] et Haller et Holden [1990] et basé sur la négociation de Rubinstein (Rubinstein [1982]) en supposant que les préférences des parties dans leur cadre sont variables dans le temps. Dans ce modèle de négociation salariale non-coopérative, une entreprise monopolistique et un syndicat négocient le nouveau salaire pour les travailleurs. Il existe une négociation séquentielle en temps discret et un horizon potentiellement infini dans lequel les parties alternent en faisant des offres de contrats de salaire que l'autre partie peut soit accepter soit refuser. En cas de rejet du contrat de salaire proposé par l'une des parties, le syndicat doit décider de faire ou non la grève pendant cette période. Dans la version étendue de ce modèle, au lieu de la décision de grève du syndicat, nous considérons la décision de "lockout" de l'entreprise.

La réalisation de nos objectifs est liée aux chapitres 2, 3, 4 et 5. Dans le chapitre 2, nous fournissons un aperçu de la littérature de la théorie de la négociation. En particulier, nous récapitulons les deux approches statiques et stratégiques de la négociation, certaines généralisations et des extensions du modèle original de la négociation de Rubinstein et des modèles de négociation salariale. Nous soulignons également l'importance d'utiliser des taux d'actualisation variant dans le temps pour modéliser des situations de la vie réelle, et nous nous référons à d'autres travaux qui traitent de la question des préférences non-stationnaires.

Dans le chapitre 3, nous fournissons une analyse de l'équilibre détaillée du modèle de négociation salariale généralisé. Ce chapitre est basé sur Ozkardas et Rusinowska [2014a,à paraître,2014b]. Après une brève description de la négociation salariale entre le syndicat et l'entreprise présentée dans la section 3.2, dans la section 3.3 nous étudions différentes décisions de la grève du syndicat et comparons les cas exogènes. Notre analyse montre qu'en fait, il serait plus rentable pour le syndicat de prendre une décision de grève "mélangée": faire la grève si l'offre du syndicat est rejetée, et statu quo si le syndicat rejette une offre. Ce que le syndicat obtiendrait en équilibre dans un tel cas de décision de grève mixte est plus élevé que ce qu'il obtiendrait dans les équilibres de décisions de grève extrêmes (toujours en grève ou toujours en "hold-out"). Nos résultats pour les cas avec les décisions de grève exogènes généralisent des résultats précédents avec des taux d'actualisation constant (par exemple Fernandez et Glazer [1991], et

Haller et Holden [1990]).

En outre, nous relâchons l'hypothèse de la décision de grève exogène, et dans la section 3.4 nous fournissons l'analyse de l'équilibre dans le cas général. Nous trouvons les équilibres parfait en sous-jeux (que l'on désignera ici par SPE) dans lesquels les stratégies qui soutiennent les équilibres dans les cas exogènes (toujours en grève, et faire la grève seulement après le rejet de ses propres propositions) sont combinées avec les stratégies à salaire minimum, à condition que le syndicat soit suffisamment patient. Ce dernier SPE est limité aux situations où l'entreprise est au moins aussi patient que le syndicat. Si l'entreprise est plus impatient que le syndicat, il vaut mieux que l'entreprise joue la stratégie sans concession (rejeter toutes les offres et toujours faire une offre inacceptable).

Après avoir déterminé le SPE du modèle de négociation salariale avec des taux d'actualisation variant dans le temps, nous généralisons la méthode utilisée par Houba et Wen [2008] et l'appliquons à notre modèle afin de trouver les gains extrêmes des SPE dans le modèle de négociation salariale généralisé. Cette partie de la thèse est présentée dans la section 3.5. Nous déterminons les gains extrêmes dans les SPE pour des cas particuliers de séquences de taux d'actualisation variant dans le temps. A part dériver les limites exactes des gains d'équilibre, nous caractérisons également les profils de stratégies d'équilibre qui prennent en charge ces gains extrêmes. Nos résultats pour le modèle avec des taux d'actualisation variant généralisent les résultats de Houba et Wen [2008] obtenus pour le modèle avec des taux d'actualisation constants. Dans la section 3.6 nous présentons également d'autres résultats liés aux équilibres inefficaces dans la négociation salariale généralisée.

Chapitre 4 concerne certaines extensions de la négociation salariale généralisée: le modèle avec la décision de "go-slow" du syndicat présenté dans les sections 4.2 et le modèle avec les décisions de "lockout" de l'entreprise présenté dans la section 4.3. Ce chapitre est basé sur Ozkardas et Rusinowska [2014c,b]. Plus précisément, nous étendons le modèle de négociation salariale de Fernandez et Glazer [1991] et de notre négociation salariale généralisée par l'introduction de l'option "go-slow" du syndicat. Nous spécifions l'attitude du syndicat, qui peut être hostile ou altruiste, puis déterminons les équilibres parfaits en sous-jeux de la négociation salariale prolongée. Nous analysons également une extension du modèle en intégrant l'option de lockout de l'entreprise. Nous montrons que sous certaines hypothèses, il y a un SPE avec un accord immédiat qui donne au syndicat un contrat de salaire plus faible que le contrat de statu quo lorsque le syndicat n'est pas autorisé à menacer l'entreprise, mais l'entreprise a la possibilité de lockout.

Des applications du modèle de négociation salariale généralisé aux négociations du prix sont présentés dans le chapitre 5. La section 5.1 concerne la négociation du prix entre un vendeur et un acheteur avec les préférences décrites par des facteurs d'actualisation variant dans le temps. Cette section est basée sur Ozkardas et Rusinowska [2013]. Nous déterminons l'unique SPE pour les stratégies sans retard indépendantes de l'histoire passée du jeu et l'équilibre des gains extrêmes du vendeur et de l'acheteur dans le cas général. Il semble que les profils de la stratégie d'équilibre sans retard soutiennent ces gains extrêmes. Sous équilibre, ni le vendeur ni l'acheteur ne font une offre inacceptable. Enfin, nous proposons d'appliquer notre modèle à la négociation du prix des produits pharmaceutiques. Cette partie est présentée dans la section 5.2.

Quelques remarques finales, en particulier, une brève présentation de nouvelles recherches possibles sur les négociations salariales, sont présentées dans le chapitre 6.

Contents

Chapter 1

Introduction

Bargaining theory and its applications, e.g., wage bargaining between firms and unions, are vastly analyzed in the literature. One of the approaches to explain the interaction between bargainers is based on the static Nash bargaining (Nash [1950]), where the analysis is focused on the outcome and its properties. Another approach initiated by Rubinstein [1982] analyses bargaining strategies and gives more insight to understand the bargaining procedure. In order to model real life situations, using Rubinstein's dynamic bargaining model provides clear understanding of the incentives of the players to make an agreement as soon as possible.

One of the topics on bargaining theory broadly discussed in the economics literature concerns collective wage bargaining between firms and workers. Despite numerous works on this issue, to the best of our knowledge wage bargaining with discount rates varying in time has not been considered before. This Ph.D. thesis is devoted to such a *wage bargaining with preferences of the parties described by discount rates varying in time*, which will be also referred to as the *generalized wage bargaining*. More precisely, the objectives of this Ph.D. thesis are the following:

1. Studying wage bargaining and delivering an overview of wage bargaining models
2. Emphasizing the importance of the generalized wage bargaining to model real life situations
3. Investigating the wage bargaining model with preferences (discount rates) varying in time:
    a) Analyzing subgame perfect equilibria in the model

   b) Determining the subgame perfect equilibria payoffs of the parties
   c) Studying the inefficient equilibria in the model

4. Investigating some extensions of the generalized wage bargaining model, like wage bargaining with go-slow actions and lockouts
5. Applying the generalized wage bargaining models to real life problems, such as price negotiations.

We extend the wage bargaining model introduced in Fernandez and Glazer [1991] and Haller and Holden [1990] and based on Rubinstein's bargaining (Rubinstein [1982]) by assuming that the parties' preferences in their framework are varying in time. In this non-cooperative wage bargaining model, a monopolistic firm and a union bargain over the new wage for the workers. There exists a sequential bargaining over discrete time and a potentially infinite horizon in which the parties alternate in making offers of wage contracts that the other party can either accept or reject. In case of a rejection of the proposed wage contract by one of the parties, the union must decide whether or not to strike in that period. In the extended version of this model, instead of the union's strike decision, we consider the firm's lockout decision.

The realization of our objectives are related to Chapters 2, 3, 4 and 5. In Chapter 2, we provide the literature overview of the bargaining theory. Especially, we recapitulate both static and strategic approaches to bargaining, some generalizations and extensions of the original Rubinstein's bargaining model and wage bargaining models. We also emphasize the importance of using discount rates varying in time to model real life situations, and refer to other works that discuss the issue of non-stationary preferences.

In Chapter 3, we introduce and provide a detailed equilibrium analysis of the generalized wage bargaining model. This chapter is based on Ozkardas and Rusinowska [2014a, Forthcoming, 2014b]. After a brief description of the wage bargaining between the union and the firm presented in Section 3.2, in Section 3.3 we study different exogenous strike decisions of the union and compare the exogenous cases. Our analysis shows that, in fact, it would be more profitable for the union to use a "mixed" strike decision: striking if the union's offer is rejected, but holding out if the union rejects an offer. What the union would get under the equilibrium in such a case of the mixed strike decision is higher than what it would get under the equilibria of the extreme strike decisions (always striking or always holding out). Our results for the cases with the exogenous strike decisions generalize some previous results for constant discount rates (e.g. Fernandez and Glazer [1991] and Haller and Holden [1990]).

Furthermore, we relax the assumption of the exogenous strike decision, and in Section 3.4 we provide the equilibrium analysis for the general case. We find *subgame perfect equilibria* (that will be denoted here by *SPE*) in which the strategies supporting the equilibria in the exogenous cases (always strike, and strike only after rejection of own proposals) are combined with the minimum-wage strategies, provided that the union is sufficiently patient. The latter SPE is restricted to the situations when the firm is at least as patient as the union. If the firm is more impatient than the union, then the firm is better off by playing the no-concession strategy (reject all offers and always make an unacceptable offer).

After determining the SPE of the wage bargaining model with discount rates varying in time, we generalize the method used in Houba and Wen [2008] and apply it to our model in order to find the extreme payoffs under SPE in the generalized wage bargaining model. This part of the thesis is presented in Section 3.5. We determine the extreme payoffs under SPE for particular cases of sequences of discount rates varying in time. Apart from deriving the exact bounds of the equilibrium payoffs, we also characterize the equilibrium strategy profiles that support these extreme payoffs. Our findings for the model with varying discount rates generalize the results of Houba and Wen [2008] obtained for the model with constant discount rates. In Section 3.6 we also present further results related to inefficient equilibria in the generalized wage bargaining.

Chapter 4 concerns some extensions of the generalized wage bargaining: the model with go-slow decision of the union presented in Sections 4.2 and the model with lockout decisions of the firm presented in Section 4.3. This chapter is based on Ozkardas and Rusinowska [2014c,b]. More precisely, we extend the wage bargaining model of Fernandez and Glazer [1991] and our generalized wage bargaining by introducing the go-slow strategy of the union. We specify the attitude of the union, which can be either hostile or altruistic, and then determine the subgame perfect equilibria of the extended wage bargaining. We also analyze an extension of the model by incorporating the lockout option of the firm. We prove that under certain assumptions there is a SPE with an immediate agreement which yields the union a wage contract smaller than the status quo contract when the union is not allowed to threaten the firm but the firm has the lockout option.

Applications of the generalized wage bargaining model to price negotiations are presented in Chapter 5. Section 5.1 concerns the price negotiation between a seller and a buyer with preferences described by discount factors varying in time. This section is based on Ozkardas and Rusinowska [2013]. We determine the unique SPE for no-delay strategies independent of the former history of the game and the equilibrium extreme payoffs of the seller and the buyer for the general case. It appears that the no-delay equilibrium strategy profiles support these extreme payoffs. Under equilibrium, neither the seller nor the buyer makes an unacceptable offer. Finally, for a future research agenda we propose to apply our model to pharmaceutical product price negotiation. This part is presented in Section 5.2.

Some concluding remarks, in particular, a short presentation of more possible new research projects on wage bargaining, are presented in Chapter 6.

Chapter 2

Bargaining models - A brief literature overview

2.1 Introduction

In many economic, social and political issues one can frequently be confronted with bargaining situations. We refer to a bargaining situation as the interaction between two or more individuals/organizations in which they make cooperation for conflicting benefits. For example, one may analyze the bargaining between a seller and a buyer for price determination of a good or the bargaining between governments and international organizations on the reduction in the stockpiles of conventional armaments. It is straightforward to present numerous examples of many micro- or macro-scaled issues, where bargaining theories can be very suitable for modeling them. The reason for using a theoretical explanation of bargaining lies behind the necessity for understanding the basis of the human interactions and how the future interactions should be shaped.

In order to deal with the bargaining theories, one may ask the following questions on this subject: What are the variables and/or factors that determine the negotiation's outcome? How is it possible to maximize the bargaining outcomes? Which negotiation strategy gives more profitable bargaining outcome? How to apply these strategies? What is the source of the bargaining power? What affects the reduction or increase of this power? Why the bargaining power is different for each player? etc.

Due to the fact that the bargaining is a time consuming and costly process, theoretical works on bargaining must satisfy the properties of efficiency and distribution. The property of efficiency is defined as follows: the players (either individuals or institutions) need to reach an agreement with the highest utility levels by the fastest way. For instance, when the wage agreement is reached after a lengthy strike period, both the workers and the firm bear the cost of such a late agreement. The purpose of the bargaining theories is to analyze and determine the maximum utility level in the minimum time for both workers and firm. To illustrate this more precisely, one may investigate the peace agreements. Instead of signing a peace treaty after hundreds of deaths, avoiding wars between two states would be more profitable and reasonable. The property of distribution gives us the rules for the determination of the utilities.

In this survey, we recapitulate the bargaining theories analyzed in the literature. In Section 2.2, we first investigate the axiomatic approach derived from Nash [1950], where the solution satisfies a set of well-defined axioms. Then, we recall the dynamic approach to bargaining problems of Rubinstein [1982], where the players make alternating offers. Finally, we investigate some selected extensions of Rubinstein's model. Section 2.3 concerns the wage bargaining models based on Rubinstein's dynamic model. A brief conclusion is presented in Section 2.4.

2.2 Bargaining models

In this section, we analyze the bargaining models between two or more players over a division of a surplus. First, we concentrate on the axiomatic approach derived by Nash and explain the Nash bargaining solution. Next, we investigate the original Rubinstein's alternating offers bargaining model. Last part of this section is devoted to the generalizations and extensions of Rubinstein's model.

2.2.1 Axiomatic approach - Nash bargaining solution

Nash [1950] analyzes the bargaining problems by considering the set of outcomes or agreements that satisfy some properties instead of taking notice of the strategic aspects of bargaining. Nash [1950] states that *"One states as axioms several properties that would seem natural for the solution to have and then one discovers that axioms actually determine the solution uniquely."* Let us recapitulate the Nash bargaining solution after giving the postulated axioms.

Consider two players, labeled $i = 1, 2$, who bargain over a division of a cake (or surplus) of size 1. They try to come to an agreement over alternatives in some arbitrary set. Let X be the set of possible agreements, i.e.,

$$X = \{(x_1, x_2) : x_1 + x_2 = 1, x_i \geq 0\}$$

and let D denote the disagreement outcome, i.e., $D = (0, 0)$.

We assume that each player i's preferences are represented by a utility function u_i over $X \cup \{D\}$. Let U be the set of possible payoffs defined by

$$U = \{(v_1, v_2) : u_1(x) = v_1, u_2(x) = v_2 \text{ for some } x \in X\}$$

and $d = (u_1(D), u_2(D))$.

We assume that U is convex and compact set and there exists some $v \in U$ such that $v > d$. Under these assumptions, a *bargaining problem* is a pair (U, d) where $U \subset \mathbb{R}^2$ and $d \in U$.

A *bargaining solution* is a function $f : \mathcal{B} \to \mathbb{R}^2$ where $\mathcal{B}$ is the set of all possible bargaining problems. The bargaining solution f must satisfy the following axioms:

1. *Pareto Efficiency:* A bargaining solution $f(U, d)$ is Pareto efficient if there does not exist a $(v_1, v_2) \in U$ such that $v \geq f(U, d)$ and $v_i > f_i(U, d)$ for some i.

2. *Symmetry:* Let (U, d) be such that $(v_1, v_2) \in U$ if and only if $(v_2, v_1) \in U$ and $d_1 = d_2$. Then $f_1(U, d) = f_2(U, d)$.

3. *Invariance of Equivalent Payoff Representations:* Given a bargaining problem (U, d), consider a different bargaining problem (U', d') for some $\alpha > (0, 0)$ and β, where $U' = \{(\alpha_1 v_1 + \beta_1, \alpha_2 v_2 + \beta_2) : (v_1, v_2) \in U\}$ and $d' = (\alpha_1 d_1 + \beta_1, \alpha_2 d_2 + \beta_2)$. Then $f_i(U', d') = \alpha_i f_i(U, d) + \beta_i$ for $i = 1, 2$.

4. *Independence of Irrelevant Alternatives:* Let (U, d) and (U', d) be two bargaining problems such that $U' \subseteq U$. If $f(U, d) \in U'$, then $f(U', d) = f(U, d)$.

Theorem. *A pair of payoffs (v_1^*, v_2^*) is a Nash bargaining solution if it solves the following optimization problem:*

$$\max_{v_1, v_2} (v_1 - d_1)(v_2 - d_2) \text{ subject to } (v_1, v_2) \in U, (v_1, v_2) \geq (d_1, d_2)$$

The Nash bargaining solution denoted by $f^N(U, d)$ is the unique bargaining solution that satisfies the four axioms mentioned above.

2.2.2 Strategic approach - Rubinstein's model

Instead of using the axiomatic (static) model of Nash and the analysis of the properties of the solution, one can apply a dynamic approach to bargaining derived from Rubinstein [1982] and study a subgame perfect equilibrium (SPE).

Indeed, while the Nash approach to bargaining has some advantages such as tractability, one may need to analyze the strategic aspects of bargaining, such as rules and course of negotiating. In addition, in order to model real life situations, it might be difficult to establish the Nash bargaining solution without a good knowledge of the strategic aspects. Including the determination of the disagreement points and the bargaining power, one may observe some ambiguities in the Nash bargaining solution. It appears that Rubinstein's dynamic model gives clear understanding of bargaining situations. In particular, introducing the cost of bargaining represented by the player's discount factors, clarifies the incentives of the players to make an agreement as soon as possible. Consequently, one can define the Rubinstein's bargaining model as an explicit model of strategic bargaining, where the players makes offers and counter-offers. Moreover one can apply this model to real life bargaining more smoothly.

We can present the model as bargaining problem of a game in extensive form. Suppose that two players bargain over a division of a cake, which is infinitely divisible and normalized to 1. There is no deadline for the bargaining, hence the players can alternate the offers forever. An agreement is a pair $x = (x_1, x_2)$ where x_i is the share of the cake received by player i, for $i = 1, 2$. The set of all possible agreements is

$$X = \left\{(x_1, x_2) \in \mathbb{R}^2 : x_1 + x_2 = 1 \text{ and } x_i \geq 0, \text{ for } i = 1, 2\right\}$$

In period 0, player 1 makes an offer to player 2. If player 2 accepts, the offer is implemented and the game ends. The players divide the cake according to the agreement offer. If player 2 rejects the offer made by player 1, then she makes a counter-offer in the next period. If the counter-offer is accepted by player 1, then the game ends and they split the cake according to this offer. Otherwise, player 1 makes a counter-offer in period 2, etc.

The result of the game is a pair of (x, t), where $x = (x_1, x_2)$ is the agreement and $t \in \mathbb{N}$ is the number of proposals rejected in the bargaining. We denote the disagreement by D.

Rubinstein uses subgame perfection where the SPE of the bargaining game is a pair of strategies which constitute a Nash equilibrium in every subgame of the game.

Subgame perfection eliminates the equilibria based on incredible threats in which a player would not be willing to carry out. Rubinstein [1982] shows that there is a unique SPE of this game which satisfies the *No-Delay* and *Stationarity* properties, i.e., all equilibrium offers are accepted and a player makes the same offer in equilibrium whenever she has to make an offer.

Rubinstein analyzes a model of bargaining where the time preferences of each player i $(i = 1, 2)$ are expressed by a constant discount rate δ_i, where $0 < \delta_i < 1$. The utility function of each player i is defined as follows:

$$u_i\left(x,t\right) = x_i \delta_i^t \text{ for every } \left(x,t\right) \in X \times \mathbb{N} \text{ and } u_i\left(D\right) = 0$$

Consider the following pair of strategies (f^*, g^*):

Player 1 *proposes* x^* *and accepts* y *if and only if* $y_1 \geq y_1^*$ *and player* 2 *proposes* y^* *and accepts* x *if and only if* $x_2 \geq x_2^*$.

Rubinstein [1982] *shows that* (f^*, g^*) *is the unique SPE of the bargaining game of alternating offers where the agreement is obtained at the beginning of the game and*

$$x_1^* = \frac{1-\delta_2}{1-\delta_1\delta_2} \text{ and } x_2^* = \frac{\delta_2\left(1-\delta_1\right)}{1-\delta_1\delta_2}$$

$$y_1^* = \frac{1-\delta_1}{1-\delta_1\delta_2} \text{ and } y_2^* = \frac{\delta_1\left(1-\delta_2\right)}{1-\delta_1\delta_2}$$

The share of each player in the equilibrium depends on both players' discount factors δ_i $(i = 1, 2)$. In particular, the equilibrium share x_i^* obtained by player i is strictly increasing in her own discount factor and strictly decreasing in her opponent's discount factor. Namely, if a player is more patient, she can afford to wait.

If both players have the same discount factors, the model predicts an agreement with the payoffs $\left(\frac{1}{(1+\delta)}, \frac{\delta}{(1+\delta)}\right)$. It appears that the first-mover has an advantage as $\frac{1}{(1+\delta)} > \frac{\delta}{(1-\delta)}$. However, if $\delta \to 1$, the first-mover advantage disappears and the agreement payoffs are $\left(\frac{1}{2}, \frac{1}{2}\right)$. In case of the immediate counter-offers, i.e., $\delta_1 = \delta_2 = 0$, a continuum of SPE, including equilibria that are Pareto inefficient, exists.

2.2.3 Extensions of Rubinstein's model - nonstationary preferences

Numerous extensions of Rubinstein's original bargaining model are presented in the literature, see, e.g., Osborne and Rubinstein [1990]. Also Muthoo [1999] demonstrates the models with risk of breakdown, inside and outside options, etc. In this sub-section, we concentrate on the extensions of Rubinstein's bargaining model with non-stationary preferences of the parties. Binmore [1987b] analyzes preferences that do not necessarily satisfy the stationarity assumption and demonstrates a continuum of SPE for any time interval between consecutive offers. In Coles and Muthoo [2003] one can find a short survey of bargaining models in which players have time-varying payoffs.

A certain generalization of the original Rubinstein's model is presented by Rusinowska [2000, 2001] where she assumes that the parties' preferences are expressed not by a constant discount rate but by a sequence of discount rates varying in time $(\delta_{i,t})_{t\in\mathbb{N}}$, where $\delta_{i,t}$ denotes the discount rate of player i $(i = 1, 2)$ in period t, $\delta_{i,0} = 1$, $0 < \delta_{i,t} < 1$ for $t \geq 1$. In such a case, the payoff of player i in given period t is $x_i \prod_{k=0}^{t} \delta_{i,k}$. Consider the following pair of strategies:

(A) In each period $2t$ $(t \in \mathbb{N})$ player 1 submits an offer x^{2t} and in each period $2t+1$ accepts an offer s by player 2 if and only if $s_1 \geq y_1^{2t+1}$. In each period $2t+1$ player 2 submits an offer y^{2t+1} and in each period $2t$ accepts an offer r by player 1 if and only if $r_2 \geq x_2^{2t}$, where $x^{2t} = (x_1^{2t}, x_2^{2t})$ and $y^{2t+1} = \left(y_1^{2t+1}, y_2^{2t+1}\right)$.

Rusinowska [2001] proves that if players' preferences are expressed by sequences of discount rates $(\delta_{i,t})_{t\in\mathbb{N}}$, strategies do not depend on the former history and satisfy (A), and $\prod_{j=1}^{t+1} \delta_{1,2j}\delta_{2,2j-1} \rightarrow_{t\to+\infty} 0$, then there is only one SPE of the form defined in (A), where the offer of player 1 in period 0 is given by

$$x_1^0 = 1 - \delta_{2,1} + \sum_{n=1}^{+\infty} \left(\prod_{k=1}^{n} \delta_{1,2k}\delta_{2,2k-1} \right) (1 - \delta_{2,2n+1})$$

and the offers in every period are determined in a recursive way.

Houba and Wen [2006] generalize Rubinstein's model by assuming that the bargaining periods are not constant. They investigate a bargaining model where two players negotiate how to share an infinite sequence of pies, one per period, for infinitely many periods, where the discount factor $\delta_i = e^{-r_i \Delta}$ depends on the discount rates r_i of the parties and also on the time interval Δ between the bargaining periods.

In Rubinstein's bargaining model, the patience plays a key role for the equilibrium payoffs. On the contrary, in this modified model of Houba and Wen [2006], if the time interval between the periods shrinks to zero, the less patient player will always receive 50% of his utopia payoff, regardless of the difference between the players' discount rates. This result shows that if the non-stationary contracts are allowed, the patience is no longer an issue. Furthermore, allowing for non-stationary contracts makes both players better off but not evenly. In particular, if one of the players becomes more patient, it receives all the additional benefits from a larger difference between their time preferences.

The risk of breakdown is analyzed by Binmore et al. [1986] where the termination possibility is based on the following reasons: an agent may want to stop the bargaining immediately or an external invention may force the parties to finish the bargaining immediately. In both cases, the best and rational thing for the bargainers is to accept the last offer on the table.

Another generalization of Rubinstein's model with risk of breakdown is presented by Vidal-Puga [2008]. In his model, two agents bargain over a share of a pie by making alternating offers. There is a discount factor $\delta = e^{-s\Delta} < 1$ and the player's utility for a piece of size u at time t is $\delta^t u$. If the responder does not accept the last offer, both agents will get zero. There is a probability of $1 - \rho$ in which the last offer is the termination offer where $\rho = e^{-r\Delta}$. The author assumes the discount factor δ as an *internal* factor that shows the impatience and ρ as an *external* factor which determines the belief that the proposal on the table will become a *take-it-or-leave-it* offer. Δ is defined by ρ and δ as the delay between the offers and counteroffers. Hence, both external and internal factors of the model depend on *time*.

For arbitrary ρ and δ, Vidal-Puga [2008] determines three different regions for SPE of the bargaining model. If $(\rho, \delta) \in IA$ where $IA := \{(\rho, \delta) : \rho\,(1 - \delta^2) > \delta\,(1 - \rho)\}$ and stands for *Immediate Agreement*, then there exists a unique SPE payoff allocation $\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$ which coincides with Rubinstein's result. If $(\rho, \delta) \in PD$ where $PD := \{(\rho, \delta) : \delta\,(1 - \rho) > \rho\,(1 - \rho\delta^2)\}$ and stands for *Perpetual Disagreement,* then there does not exist any SPE with immediate agreement and there exists a unique SPE where the proposer always claims the whole pie, and the responder rejects when this proposal is final. And the last region is $DA := \{(\rho, \delta) : (\rho, \delta) \notin IA \cup PD\}$ where DA stands for *Delayed Agreement.* In DA, there is no stationary SPE, and there exists a continuum of SPE payoffs. In the *Delayed Agreement* region, the payoffs obtained when the agreement is immediate are potentially worse for the proposer than the payoffs obtained when the agreement is delayed. More precisely, if $(\rho, \delta) \in DA$, it is better for the first proposer to start the bargaining with unacceptable high offers.

Houba and Wen [2011] investigate a general bargaining model that involves the endogenous threats to the Rubinstein's bargaining model. Although Hicks paradox assumes that delay and strikes are Pareto inefficient, Houba and Wen [2011] indicate that when the players have different time preferences, reaching an agreement with delay is not necessarily inefficient. The authors get the following two crucial results for the negotiation models with different discount factors: the proposer may prefer to make an unacceptable offer in his worst SPE and the Pareto frontier of SPE is not necessarily the bargaining frontier.

Herings and Predtetchinski [2012] present a generalization of Rubinstein's bargaining model with n players where all players accept the agreement unanimously. In the model, the authors postulate an assumption that the players have no possibility for leaving the table with only partial agreements. For removing a potential source of multiplicity of equilibria, the shares of players are determined sequentially. Herings and Predtetchinski [2012] prove that there exists a unique SPE for the sequential share bargaining protocol with orderly voting mechanism. In equilibrium, there is no delay for the agreement and the results obtained in the n-player model are qualitatively equal to the results obtained in the 2-player model.

By using the Shaked and Sutton [1984] method modified by introducing unacceptable offers, Houba and Wen [2014, Forthcoming] indicate the existence of SPE, especially when a stationary SPE does not exist. They refer to the bargaining model of Vidal-Puga [2008] for demonstrating in details how to insert unacceptable offers in the backward induction technique and how to find the extreme equilibrium strategy profiles from the backward induction.

2.3 Wage bargaining models

In the economic theory one can analyze the reasons for preferring the wasteful mechanisms, such as strikes, instead of optimal distributions of the gains by the rational agents (see e.g. Hart [1989]). In other words, although there exist Pareto-optimal equilibria, why the rational parties get Pareto-inefficient outcomes? In the wage bargaining literature, several works have been devoted to this issue and explain it by the existence of asymmetric information. In general, strikes are assumed to be a signaling device of the firm's profitability. Since this profitability is unobserved, firms with lower profits can accept to bear the cost of strikes for making lower wage agreements. Some empirical evidences support such ideas and prove that the bargaining between two rational agents should be efficient if there is no asymmetric information.

2.3.1 Strategic approach to wage bargaining

A new perspective on the wage bargaining is presented by Fernandez and Glazer [1991] who prove that under complete and symmetric information one can observe the strikes. Moreover, irrationality or asymmetric information is not anymore a necessary condition for obtaining the inefficient equilibrium. The authors testify the multiplicity of SPE where some of them are Pareto efficient and some are not. Hence, assuming the unique SPE is not valid. In this sub-section, we recapitulate this wage bargaining model.

Fernandez and Glazer [1991] extend Rubinstein's bargaining model to wage bargaining. Two agents, referred to as union and firm, are assumed to bargain sequentially over discrete time and infinite horizon under complete information. They make alternating offers of wage contracts and each party is free to accept or reject the other's offer. Union consists of L identical workers and the number of the workers is normalized to 1, and the wage paid by the firm is entitled to per day work. Parties bargain over the division of F, where F is the revenue associated with the union's output. There exists a wage contract w_0 which has come up to renegotiation. Differently from the original Rubinstein's bargaining model, Fernandez and Glazer [1991] modify one party's i.e., the union's, strategy by introducing strike possibility. In case of rejection, the union decides whether to strike or to hold out in this period. If the union strikes, then both parties will get zero and the bargaining process advances to the next period. Otherwise, i.e., when the union holds out, it gets the existing wage w_0 for this period.

Bargaining mechanism is defined as follows: each party makes an alternating offer over discrete time $t \in \{1, 2, \ldots\}$. In each odd period, the union proposes x_t and the firm replies by accepting the offer or rejecting it. In case of an agreement, the new wage contract determines the utilities of parties till infinity. If the firm rejects, then the union makes a decision whether to make a strike or not in period t. If the union decides not to strike, then the union gets the existing wage w_0, where $0 \leq w_0 \leq F$, in this period and the firm gets $F - w_0$. Contrarily, if the union's decision is to make a strike, then both parties renounce their payoffs and both get zero. After the union's strike decision, time advanced one period and the firm makes an offer y_t in the even numbered period. The union replies by accepting or rejecting the offer. Accepting the offer means the establishment of the new wage contract. In case of rejection, a new strike decision of the union for this period is taken. Bargaining mechanism continues until an agreement is reached or to infinity if no agreement is reached.

Fernandez and Glazer [1991] use constant but different discount factors, where $0 <$

$\delta_f < 1$ is the discount factor of the firm and $0 < \delta_u < 1$ is the discount factor of the union. Haller and Holden [1990] present the same wage bargaining model, but they use equal discount rates for the firm and the union, where $0 < \delta = \delta_u = \delta_f < 1$.

The union maximizes wage earnings of the workers where the discounted sum of wage earnings is

$$\sum_{t=1}^{\infty} \delta_u^{t-1} w_t$$

and the firm's objective is the maximization of discounted sum of profits

$$\sum_{t=1}^{\infty} \delta_f^{t-1} (F - w_t)$$

where w_t is the new wage contract accepted in period t, $w_t = 0$ for strike periods, $w_t = w_0$ for holdout periods, and $w_t = w$ for every $t \geq T$ if the agreement w occurs in period T.

In their first result (Lemma 1), Fernandez and Glazer [1991] assume that the union is committed to strike in every period of disagreement. They determine the unique SPE of the bargaining game between the union and the firm with the agreement obtained in the first period of negotiation. Under this strategy, the new contract is $\bar{w}$ if bargaining starts in an odd-numbered period and the new contract is $\bar{z}$ if the bargaining commences in an even-numbered period, where

$$\bar{w} = \frac{(1 - \delta_f) F}{1 - \delta_u \delta_f} \quad \text{and} \quad \bar{z} = \frac{\delta_u (1 - \delta_f) F}{1 - \delta_u \delta_f}$$

This result gives the solution of Rubinstein's original bargaining game. For the proof, Fernandez and Glazer [1991] refer to Rubinstein [1982] and to Shaked and Sutton [1984]. Haller and Holden [1990] also obtain same results, with $\delta_u = \delta_f = \delta$ and $F = 1$, i.e., the new contract is $\frac{1}{1+\delta}$ if the union starts and $\frac{\delta}{1+\delta}$ if the firm starts the bargaining. They argue that $\bar{w} > \bar{z}$ which shows the first player's advantage.

Before characterizing the complete set of the Pareto-efficient SPE of the model, three particular equilibria are presented by Fernandez and Glazer [1991]. In their Lemma 2 on the minimum wage contract, they prove that there is a SPE in which an agreement of w_0 is reached in the first period. The pair of strategies which gives w_0 is as follows: the union's strategy is never to strike, to offer $x_t = w_0$ in every odd periods and to accept an offer of the firm in every even period if and only if $y_t \geq w_0$. The firm's strategy is to offer $y_t = w_0$ in every even period and to accept an offer of the union in every odd period if and only if $x_t \leq w_0$.

In their Lemma 3, Fernandez and Glazer [1991] show the existence of a SPE in which an agreement of $\bar{w}$ is reached in the first period if and only if $w_0 \leq \delta_u \bar{z}$. Strategy of the union is to offer $\bar{w}$ in every odd period, to accept any offer y_t in every even period if and only if $y_t \geq \bar{z}$ and to strike in every odd period if its offer is rejected and in every even period if the firm offers less than $\bar{z}$. If, at some point, the union deviates from this strategy, then both parties play thereafter according to the strategies described in Lemma 2, i.e., the union gets the minimum wage w_0.

Fernandez and Glazer [1991] demonstrate that Lemma 3 does not give the maximum wage contract to the union. They prove Lemma 4 which gives the SPE in which an agreement of w' is obtained in the first period. This equilibrium is the maximum wage contract of the union. Consequently, if the firm starts the bargaining, the wage contract is z' where

$$w' = \bar{w} + \frac{\delta_f w_0 \left(1 - \delta_u\right)}{1 - \delta_u \delta_f} \quad \text{and} \quad z' = \bar{z} + \frac{w_0 \left(1 - \delta_u\right)}{1 - \delta_u \delta_f}$$

To generate w' as the equilibrium outcome, the pair of subgame perfect equilibrium strategies, called alternating strike strategies, are as follows: the union offers w' and strikes if this offer is rejected in every odd periods and it accepts an offer y_t of the firm if and only if $y_t \geq z'$, and never strikes in an even period. If there exists a deviation of the union from this rule, then both players play according to the strategies described in Lemma 2 which gives the minimum wage contract. Fernandez and Glazer [1991] prove that w' is the maximum wage contract. By re-arranging $w' = w_0 + \frac{\left(1-\delta_f\right)(F-w_0)}{\left(1-\delta_u \delta_f\right)}$, one can obtain the result of the original Rubinstein's bargaining where the cake size is $(F - w_0)$ and the union ensures the minimum wage w_0.

In their Theorem 1, Fernandez and Glazer [1991] prove that any wage contract w such that $w_0 \leq w \leq w'$ can be generated as an equilibrium wage contract with agreement reached in the first period. According to their result, if one can obtain w' as the efficient SPE, then all wage contracts in the range between the minimum and the maximum wage contracts are also efficient subgame perfect equilibrium. For the formal proofs of Lemmas 1-4 and Theorem 1, we refer to Fernandez and Glazer [1991].

Bolt [1995] makes a comment on Fernandez and Glazer [1991] and argues that the alternating strike strategies constitute no Nash equilibrium if $\delta_f < \delta_u$. The maximum wage contract of Fernandez and Glazer [1991] holds if and only if $\delta_f \geq \delta_u$, otherwise, by playing an alternative strategy called no-concession strategy of the firm, the firm can increase its payoff. The no-concession strategy of the firm is described as follows: the firm rejects all offers of the union in odd periods and always proposes non-acceptable offers in an even numbered period. By ensuring the disagreement, the firm secures the discounted sum of its disagreement payoffs. More precisely, instead of giving the maximum wage to the union, the firm prefers to make unacceptable offers and alternate between strikes and giving w_0 to the union. One can show that if the firm is sufficiently impatient, depending on the size of the existing wage, the parties may not reach an agreement at all.

Bolt [1995] modifies the first theorem of Fernandez and Glazer [1991] as follows: for sufficiently large $\delta_u < 1$, if $\delta_f \geq \delta_u$ the alternating strike strategies of Fernandez and Glazer [1991] support a subgame perfect equilibrium in which an agreement of $(w', F - w')$ is obtained in the first period and this is the maximum wage contract for the union. On the other hand, if $\delta_f < \delta_u$, then the modified alternating strike strategies support a SPE in which the agreement is reached only in odd periods and holdouts occur in even periods as long as no agreement is reached. The pair of subgame perfect strategies are as follows: the union offers $\frac{F+\delta_f w_0}{1+\delta_f}$ in odd periods and accepts an offer z_t if and only if $z_t \geq (1 - \delta_u) w_0 + \delta_u \left(\frac{F+\delta_f w_0}{1+\delta_f}\right)$, it strikes in odd periods and holds out in even periods if there is no agreement; the firm offers 0 in even periods and accepts an offer w_t of the firm if and only if $w_t \leq \frac{F+\delta_f w_0}{1+\delta_f}$ in odd periods. In case of a deviation of the union, both parties play according to the minimum wage equilibrium strategies. For the formal proof, see Bolt [1995].

The modified alternating strike strategies give the maximum wage contract to the union and Bolt [1995] mentions that if the existing wage level, i.e., w_0 is sufficiently low, the union gets higher wage with a threat of strike in every period since the cost of strike is relatively low compared to the cost of holdout. The union can also reduce the no-concession payoff of the firm to zero by playing the always-strike strategy. Bolt [1995] remarks that if the discount factors of the players are constant and equal, then the maximum wage contract defined in Haller and Holden [1990] constitutes Nash equilibrium.

Holden [1994] criticizes the alternating strike strategies introduced in Fernandez and Glazer [1991] and comments that applying such a strategy to real life problems is not obvious. Hence, a new strategy is described by Holden [1994] as follows: in case of a disagreement, the union decides whether to strike for that period and commits to strike for the next period without looking who makes the offer or not. More precisely, if the union decides to make a strike at a given period, it will necessarily commit to strike in the next period. The author notes that the number of commitments to strike will not influence the result if it is even and more than two.

Holden [1994] investigates a bargaining game where the union's strike decision is endogenously given and shows that the unique SPE outcome is $W^* = \max\{w_0, W^S\}$ where $W^S = \frac{\delta}{(1+\delta)}$ if the firm makes the first offer. W^S is a SPE outcome under the following strategies: for $w_0 \leq W^S$, in odd periods the union accepts any $W \geq W^S$ and strikes or commits to strike in the following period if no agreement is reached and the firm proposes W^S; in even periods the union proposes $W^U = \frac{1}{(1+\delta)}$ and does not strike unless it is committed to strike, the firm accepts any offer $W \leq W^U$ if the union is committed to strike and it accepts any offer $W \leq W' = (1-\delta)\, w_0 + \delta W^S$ if the union is not committed to strike. For the detailed proof, see Holden [1994]. $\left(W^S, W^U\right)$ are the outcomes obtained in the original Rubinstein's bargaining game with equal discount rates i.e., if $\delta = \delta_u = \delta_f$. For $w_0 > W^S$, it is obvious that the union can obtain the unique SPE w_0 when it never strikes.

2.3.2 Backward induction technique to wage bargaining

Houba and Wen [2008] make new contributions to Haller and Holden [1990], Fernandez and Glazer [1991], Holden [1994] and Bolt [1995]. They analyze the wage bargaining between the union and the firm by using the backward induction technique introduced by Shaked and Sutton [1984]. Results obtained in Fernandez and Glazer [1991] when the union is less patient than the firm are confirmed. On the contrary, if the union is more patient then the firm, one may prove that the continuation payoffs are not always bounded by the bargaining frontier. Therefore, Houba and Wen [2008] characterize extreme equilibria profiles for all possible discount factors. More precisely, if the discount factor of the firm is below the union's, the firm follows the no-concession strategy against the union's alternating strike strategies for keeping the continuation payoff from delay above the bargaining frontier. Hence, when $\delta_f < \delta_u$, the alternating strike strategy is not effective when the firm adopts the no-concession strategy.

The bargaining model of Houba and Wen [2008] is based on Fernandez and Glazer [1991] with the firm's revenue F normalized to 1. Minimum wage, i.e., w_0, that the union can always obtain, is the stationary subgame perfect equilibrium outcome for all $(\delta_u, \delta_f) \in (0,1)^2$ and $(1 - w_0)$ is the firm's best equilibrium outcome. Before determining the union's best and the firm's worst equilibrium payoffs, Houba and Wen [2008] show that some feasible outcomes may lead to payoffs strictly above the bargaining frontier if the firm and the union have different discount factors.

Differently from the model of Fernandez and Glazer [1991], Houba and Wen [2008] use period 0 instead of period 1 as the starting period of the bargaining model. Firstly, they give the necessary conditions for M_u, where M_u denotes the supremum of the union's subgame perfect equilibrium payoffs in any even period where the union makes an offer, and the necessary conditions for m_f, where m_f denotes the infimum of the firm's subgame perfect equilibrium payoffs in any odd period where the firm makes an offer. Both M_u and m_f depend on $(\delta_u, \delta_f) \in (0,1)^2$ as well as on the minimum wage $w_0 \in [0,1]$. We have $w_0 \leq M_u \leq 1$ and $w_0 \leq 1 - m_f \leq 1$.

A first proposition of Houba and Wen [2008] gives the necessary conditions for m_f which cannot be less than the minimum of the firm's highest continuation payoff from making either the least irresistible or an unacceptable offer with the reference to either the strike decision or the holdout decision of the union. If the union decides not to strike after rejecting the firm's offer, the firm can get at least $1 - (1 - \delta_u) w_0 - \delta_u M_u$ from making the least irresistible offer and $1 - (1 - \delta_f) w_0 - \delta_f M_u$ from making an unacceptable offer. If the union makes a strike after rejecting the firm's offer, the firm can get at least $1 - \delta_u M_u$ from making the least irresistible offer and $\delta_f (1 - M_u)$ from making an unacceptable offer. Since $\delta_f (1 - M_u) \leq 1 - \delta_u M_u$, the firm will never make an unacceptable offer if the union strikes. Hence, Houba and Wen [2008] get the following Proposition 1:

For all $(\delta_{u,} \delta_f) \in (0,1)^2$ and $w_0 \in [0,1]$

$$m_f \geq \begin{cases} 1 - \delta_u M_u, & \text{if } (\delta_u - \delta_f) M_u \geq (1 - \delta_f) w_0, \\ 1 - (1 - \delta_f) w_0 - \delta_f M_u, & \text{if } (\delta_u - \delta_f) M_u < (1 - \delta_f) w_0, \quad \delta_f < \delta_u, \\ 1 - (1 - \delta_u) w_0 - \delta_u M_u, & \text{if } \delta_f \geq \delta_u. \end{cases}$$

Writing the necessary conditions for the supremum of the union's SPE payoffs in any even period is analogous to Proposition 1. If the union holds out after rejecting the firm's offer, it can obtain at most $1 - (1 - \delta_f)(1 - w_0) - \delta_f m_f$ from making the least acceptable offer or $(1 - \delta_u) w_0 + \delta_u (1 - m_f)$ from making an unacceptable offer. On the other hand, if the union strikes after rejecting the firm's offer, it gets at most $1 - \delta_f m_f$ from making the least acceptable offer or $\delta_u (1 - m_f)$ from making an unacceptable offer. Since $\delta_u (1 - m_f) \leq 1 - \delta_f m_f$, the union never makes unacceptable offer after striking. Hence, Proposition 2 gives the necessary conditions for M_u:

For all $(\delta_{u,} \delta_f) \in (0,1)^2$ and $w_0 \in [0,1]$

$$M_u \leq \begin{cases} 1-\delta_f m_f, & \text{if } \delta_u (1-m_f) \geq w_0 \\ 1-(1-\delta_f)(1-w_0)-\delta_f m_f, & \text{if } \delta_u (1-m_f) < w_0, \quad \delta_f \geq \delta_u \\ 1-(1-\delta_u)(1-w_0)-\delta_u m_f, & \text{if } \delta_u (1-m_f) < w_0, \quad \delta_f < \delta_u. \end{cases}$$

After determining the necessary conditions for the supremum of the union's SPE payoffs and the infimum of the firm's SPE payoffs, Houba and Wen [2008] derive extreme payoffs for both parties. They take into consideration the cases $\delta_u > \delta_f$ and $\delta_u \leq \delta_f$.

When $\delta_u \leq \delta_f$, Houba and Wen [2008] obtain the following result (Proposition 3 in Houba and Wen [2008]) which validates Lemma 4 in Fernandez and Glazer [1991]:

$$M_u = \begin{cases} w_0 + \frac{(1-\delta_f)(1-w_0)}{1-\delta_u\delta_f}, & \text{if } (\delta_u,\delta_f) \in A, \\ w_0, & \text{if } (\delta_u,\delta_f) \notin A, \end{cases} \text{ and } m_f = \begin{cases} \frac{(1-\delta_u)(1-w_0)}{1-\delta_u\delta_f}, & \text{if } (\delta_u,\delta_f) \in A, \\ 1-w_0, & \text{if } (\delta_u,\delta_f) \notin A, \end{cases}$$

where $A = \{(\delta_u,\delta_f) : \delta_u \leq \delta_f, \delta_u (\delta_u - w_0)\delta_f \leq (1-w_0)\delta_u^2 + w_0\delta_u - w_0\}$

On the other hand, when $\delta_u > \delta_f$ the union best subgame perfect equilibrium can be either below or above the bargaining frontier. Houba and Wen [2008] present the following result in their Proposition 4:

$$M_u = \begin{cases} \frac{1+w_0\delta_f}{1+\delta_f} & \text{if } (\delta_u,\delta_f) \in B \\ \frac{1-\delta_f}{1-\delta_u\delta_f} & \text{if } (\delta_u,\delta_f) \in C \\ w_0 & \text{if } (\delta_u,\delta_f) \notin B \cup C \end{cases} \text{ and } m_f = \begin{cases} \frac{1-w_0}{1+\delta_f} & \text{if } (\delta_u,\delta_f) \in B \\ \frac{1-\delta_u}{1-\delta_u\delta_f} & \text{if } (\delta_u,\delta_f) \in C \\ 1-w_0 & \text{if } (\delta_u,\delta_f) \notin B \cup C \end{cases}$$

where $B = \left\{(\delta_u,\delta_f) : \delta_u > \delta_f, \delta_f \geq \frac{\delta_u - w_0}{1-\delta_u w_0}, (\delta_u - w_0)\delta_f \geq w_0(1-\delta_u)\right\}$,

$C = \left\{(\delta_u,\delta_f) : \delta_f \leq \frac{\delta_u - w_0}{1-\delta_u w_0}, (\delta_u - w_0)\delta_f \leq \frac{\delta_u^2 - w_0}{\delta_u}\right\}$

For $(\delta_u,\delta_f) \in B$, following the union's alternating strike strategies and the firm's no-concession strategy of Bolt [1995] gives the union's best subgame perfect equilibrium. For $(\delta_u,\delta_f) \in C$, the union's best subgame perfect equilibrium is obtained by the always-strike strategy defined in Bolt [1995]. As a consequence, if $(\delta_u,\delta_f) \notin A \cup B \cup C$, then there is a unique subgame perfect equilibrium which gives the minimum wage to the union, i.e., w_0.

2.3.3 Extensions of the wage bargaining model

Fernandez and Glazer [1991] also consider in their model inefficient equilibria obtained after uninterrupted T periods of strike. One can obtain inefficient equilibria by having peaceful negotiations alternate with several periods of strikes. In their Theorem 2, Fernandez and Glazer [1991] determine the necessary conditions for a subgame perfect equilibrium in the play in which there is a strike of T periods followed by an agreement of $\hat{w}$ if and only if $\left(1-\delta_f^{1-T}\right)F+\delta_f^{1-T}\bar{z}\geq\hat{w}\geq\delta_u^{-T}w_0$. For obtaining the inefficient equilibria, Fernandez and Glazer [1991] consider the following strategies: In each period prior to $T+1$, both the union and the firm makes non serious offers, in period $T+1$, if it is an odd numbered period, the union offers $x_{T+1}=\hat{w}$ and the firm accepts such an offer, and if it is an even number period, the firm makes the offer $y_{T+1}=\hat{w}$ and the union accepts such an offer. In addition, the union strikes uninterruptedly T periods. In case of a deviation of the union, both parties play thereafter the minimum wage equilibrium. Instead of T periods of strikes, if an agreement of $\hat{w}$ is reached in periods prior to $T+1$, it will be Pareto improving, hence a SPE $\hat{w}$ after T periods of strike is obviously an inefficient equilibrium. Without making strikes the union can always get w_0, hence it prefers to make T periods of strikes followed by a wage $\hat{w}$ if and only if $\delta_u^T\hat{w}\geq w_0$. On the other hand, the firm prefers to bear the cost of T periods of strike instead of achieving the agreement of the lowest wage contract $\bar{z}$ for itself, i.e., the firm accepts T periods of strikes if and only if $F-\bar{z}\leq\delta_f^{T-1}\left(F-\hat{w}\right)$. By rearranging these two conditions, Fernandez and Glazer [1991] obtain the condition given in their Theorem 2. For the formal proof, see Fernandez and Glazer [1991].

Some extensions such as lockout possibilities or multiple contract renegotiations are also analyzed in Fernandez and Glazer [1991]. Firstly, the wage contract in case of the lockout possibility of the firm is investigated. The authors assume that if the union has no option for making a strike but the firm can lock out the union in every even period if there is no agreement, then the SPE is $\widetilde{w}$ if the union starts and $\widetilde{z}$ if the firm starts the bargaining procedure, where

$$\widetilde{w}=\frac{\left(1-\delta_f\right)w_0}{1-\delta_u\delta_f}\quad\text{and}\quad\widetilde{z}=\frac{\delta_u\left(1-\delta_f\right)w_0}{1-\delta_u\delta_f}$$

The strategies of the firm for obtaining the lockout equilibrium is analogous to the strategy of the union described in Lemma 4 in Fernandez and Glazer [1991]. By using this strategy, the bargaining game turns to the original Rubinstein's bargaining game where the cake size is w_0.

Another extension mentioned in Fernandez and Glazer [1991] concerns the possibility of multiple contract renegotiations. It is assumed that in every M periods, the union and the firm renegotiate the wage contract. One can prove that all equilibrium outcomes obtained by the previous theorems of Fernandez and Glazer [1991] are also the equilibrium outcomes of the multiple contract renegotiations model. The union can expect a high wage level in the future to compensate its loss for the periods that give less than w_0. Thereby, this modified wage bargaining model does not necessarily give the union a wage contract higher than w_0. Hence, one can accept a wage contract smaller than w_0 as a part of SPE.

2.4 Concluding remarks

Bargaining models discussed in this short survey give a brief insight into the way of how economic, social and political situations can be modeled. In particular, by applying the dynamic bargaining models, one may investigate strategic aspects of bargaining. In the literature, numerous extensions of bargaining models have been proposed. This survey presents only some of them, with a particular focus on the models with non-stationary preferences of the parties. Such a framework is more suitable to model reality than the original bargaining with constant discount rates. Patience of parties, represented by their discount rates, may obviously be changing over time, due to many circumstances, e.g., economic, financial, political, social, environmental, health or climatic issues.

Extending the dynamic model of Rubinstein [1982] to wage bargaining is one of the leading issues presented in the literature. Also considering varying discount rates in the wage bargaining setup is relevant and important. In the next chapters, we introduce and study a generalized wage bargaining between the union and the firm in which preferences of both parties are described by sequences of discount rates varying in time, as well as some extensions and applications of the model. When the number of periods without production during strikes increases, the firm may be losing its patience to wait for a late agreement. On the other hand, as the workers are not paid during the strike periods, the patience of the union may be also diminishing. During negotiations between a seller and a buyer, preferences of the buyer for a specific product may alternate instantly. Similarly, during the price determination of a pharmaceutical product, preferences of the parties may alternate quickly, depending, e.g., on the importance of illnesses. All such changes in real life situations are represented by discount rates varying in time.

Chapter 3

Wage bargaining with discount rates varying in time[1]

3.1 Introduction

Collective wage bargaining between firms and unions (workers' representatives) is one of the most central issues in labor economics. Both cooperative and non-cooperative approaches to collective wage bargaining are applied in the literature; for broader surveys of bargaining models see, e.g., Osborne and Rubinstein [1990], Muthoo [1999]. Some authors apply a dynamic (strategic) approach to wage bargaining and focus on the concept of *subgame perfect equilibrium*. Several modified versions of Rubinstein's game (Rubinstein [1982], Fishburn and Rubinstein [1982]) to union-firm negotiations are proposed. Haller and Holden [1990] extend Rubinstein's model to incorporate the choice of calling a strike in union-firm negotiations. It is assumed that in each period until an agreement is reached the union must decide whether or not it will strike in that period. Both parties have the same discount factor δ. Fernandez and Glazer [1991] consider essentially the same wage-contract sequential bargaining, but with the union and the firm using different discount factors δ_u, δ_f. We will refer to their model as the *F-G model*. Holden [1994] assumes a weaker type of commitment in the F-G model. Also Bolt [1995] studies the F-G model. Houba and Wen [2008] apply the method of Shaked and Sutton [1984] to derive the exact bounds of equilibrium payoffs in the F-G model and characterize the equilibrium strategy profiles that support these extreme

[1]This chapter is based on Ozkardas and Rusinowska [2014a], Ozkardas and Rusinowska [Forthcoming], and Ozkardas and Rusinowska [2014b].

equilibrium payoffs for all discount factors.

Although numerous versions of wage bargaining between unions and firms are presented in the literature, a common assumption is the stationarity of the parties' preferences that are described by constant discount factors. In real bargaining, however, due to time preferences, discount factors of the parties may vary in time. Cramton and Tracy [1994b] emphasize that stationary bargaining is very rare in real-life situations. In the framework of the original Rubinstein model, several other authors discuss non-stationarity of parties' preferences (see, for instance, Binmore [1987b] and Binmore et al. [1990], pages 187-188). Coles and Muthoo [2003] study an alternating offers bargaining model in which the set of utilities evolves through time in a non-stationary way, but additionally assume that this set evolves smoothly through time. They show that in the limit as the time interval between two consecutive offers becomes arbitrarily small, there exists a unique SPE. Rusinowska [2000, 2001, 2002b, 2004] generalizes the original model of Rubinstein to bargaining models with preferences described by sequences of discount rates or/and bargaining costs varying in time.

In this chapter, we investigate the *union-firm wage bargaining with discount rates varying in time* which generalizes the F-G wage bargaining with constant discount rates. While several generalizations of the original Rubinstein model with non-stationary preferences have been presented in the literature, to the best of our knowledge no such generalized F-G model has been analyzed before. First, we consider three games in this generalized setup, where the union's strike decision is taken as *exogenous*: the case when the union is committed to strike in each period in which there is a disagreement, the case when the union is committed to go on strike only when its own offer is rejected, and the case of "never strike" decision. We determine SPE for these games and compare the results among the three cases of the exogenous strike decisions. As mentioned in Section 3.3 and shown by Fact 3.1, while the F-G model coincides with Rubinstein's model under the "always-strike decision", the generalized wage bargaining model and the generalization of Rubinstein's model do not coincide.

The study of the exogenous strike decisions is aligning with some real-life observations. In some countries and in some sectors, workers do not have legal rights to make official strikes, and consequently, in some environments strikes never take place. On the contrary, if the strikes are formally allowed, sometimes unions call for the non-stop strikes. Our comparison of the exogenous cases shows that, in fact, it would be more profitable for unions to use a "mixed" strike decision: striking if the union's offer is rejected, but holding out if the union rejects an offer. We show that what the union would get under equilibrium in such a case of the mixed strike decision is higher than what it would get under equilibria of the extreme strike decisions (always striking or always holding out). Our results for the cases with the exogenous strike decisions (Theorems 3.1 and 3.2, and Fact 3.2) generalize some previous results for constant discount rates: Lemma 1 in Fernandez and Glazer [1991], formulas (3) and (4) in Haller and Holden [1990], and Lemma 2 in Fernandez and Glazer [1991].

After considering the exogenous strike decisions, we investigate a general model with no assumption on the commitment to strike. The analysis of the three exogenous cases helps us to investigate SPE for the general case. Our Fact 3.3 shows that Lemma 2 of Fernandez and Glazer [1991] on the minimum wage contract obtained in equilibrium remains valid for the general model. We find SPE in which the strategies supporting the equilibria in the exogenous cases (always strike, and strike only after rejection of own proposals) are combined with the minimum-wage strategies, provided that the union is sufficiently patient. The corresponding results (Propositions 3.3 and 3.4) generalize Lemmas 3 and 4 of Fernandez and Glazer [1991], and Proposition 1(i) of Bolt [1995]. The latter SPE is restricted to the situations when the firm is at least as patient as the union. If the firm is more impatient than the union, then the firm is better off by playing the no-concession strategy (reject all offers and always make an unacceptable offer). This result is presented in Proposition 3.5. We find a SPE for this case (Theorem 3.3) which generalizes Proposition 1(ii) by Bolt [1995].

The approach used in this chapter is based on generalizing the analytical method used in the works of the F-G model (Fernandez and Glazer [1991], Haller and Holden [1990], Holden [1994], Bolt [1995], Houba and Wen [2008]). Such an approach to wage bargaining is different from the approach to Rubinstein's bargaining game applied by Binmore [1987b]. He defines a model which is very similar to Rubinstein's model, except that in Binmore [1987b] it is not required that a player makes an offer in every period when there is his turn to do so. Then Binmore [1987b] proposes an alternative method which provides a geometric characterization of SPE for the introduced model. Such a "geometric technique" allows to refine the Rubinstein's results, in particular, by considering the case where the "cake" to be divided does not shrink steadily over time. We believe that in order to find SPE for the wage bargaining model with strike decisions and discount factors varying in time, it is more straightforward to use the "traditional" approach and to determine analytically SPE in the model.

After determining the SPE in the general wage bargaining model, we generalize the method used in Houba and Wen [2008] and apply it to our model in order to find the extreme payoffs under SPE in the wage bargaining with discount rates varying in time. First, we describe necessary conditions under arbitrary sequences of discount rates for the supremum of the union's SPE payoffs and the infimum of the firm's SPE payoffs in all periods when the given party makes an offer. Then, we determine the extreme payoffs under SPE for particular cases of sequences of discount rates varying in time. Apart from deriving the exact bounds of the equilibrium payoffs, we also characterize the equilibrium strategy profiles that support these extreme payoffs. Our findings for the model with varying discount rates generalize the results of Houba and Wen [2008] obtained for the model with constant discount rates.

Apart from the analysis of efficient equilibria in the wage bargaining with constant discount rates, Fernandez and Glazer [1991] also present a result on inefficient equilibria. To the best of our knowledge these issues have not been considered so far for the model with discount rates varying in time. We deliver further results related to inefficient equilibria in the generalized wage bargaining. More precisely, we show that there exist inefficient subgame perfect equilibria in the model where the union strikes for uninterrupted T periods prior to reaching a final agreement.

The remainder of this chapter is as follows. In Section 3.2, we present the generalized wage bargaining model with discount rates varying in time. Section 3.3 concerns different exogenous strike decisions when the union is supposed to go on strike in each period in which there is a disagreement, when the union goes on strike only after rejection of its own proposals, and when the union is supposed to go never on strike. Section 3.4 is devoted to SPE in the general model. In Section 3.5, we determine necessary conditions for the supremum of the union's SPE payoffs and the infimum of the firm's SPE payoffs, and then we calculate the extreme payoffs for particular cases of the sequences of discount rates varying in time. We also present equilibrium strategy profiles that support these payoffs. Section 3.6 analyzes inefficient equilibria in the generalized model with strikes. Some concluding remarks are presented in Section 3.7.

3.2 Wage bargaining model with discount factors varying in time

The bargaining procedure between the union and the firm is the following (Fernandez and Glazer [1991], Haller and Holden [1990]). There is an existing wage contract that specifies the wage that a worker is entitled to per day of work, which has come up for renegotiation. Two parties (union and firm) bargain sequentially over discrete time and a potentially infinite horizon. They alternate in making offers of wage contracts that the other party is free either to accept or to reject. Upon either party's rejection of a proposed wage contract, the union must decide whether or not to strike in that period. Under the previous contract $w_0 \in [0,1]$, the union receives w_0 and the firm receives $1-w_0$. By the new contract $W \in [0,1]$, the union and the firm will get W and $1-W$, respectively.

More precisely, the parties bargain as follows. In period 0 the union proposes W^0. If the firm accepts the new wage contract, then the agreement is reached and the payoffs are $(W^0, 1-W^0)$. If the firm rejects it, then the union can either go on strike, and then both parties get $(0,0)$ in the current period, or go on with the previous contract with payoffs $(w_0, 1-w_0)$. After the union goes on strike or holds out, it is the firm's turn to make a new offer Z^1 in period 1, which assigns Z^1 to the union and $(1-Z^1)$ to the firm. If the union accepts this offer, then the agreement is reached, otherwise the union either goes on strike or holds out, and then makes its offer W^2 in period 2. This procedure goes on until an agreement is reached and upon either party's rejection of a proposed contract the union decides whether or not to strike in that period. W^{2t} denotes the offer of the union made in an even-numbered period $2t$, and Z^{2t+1} denotes the offer of the firm made in an odd-numbered period $2t+1$.

The key difference between the F-G model and our wage bargaining lies in preferences of both parties and, as a consequence, in their utility functions. While Fernandez and Glazer [1991] assume stationary preferences described by constant discount rates δ_u and δ_f, we consider a model with preferences of the union and the firm described by *sequences of discount factors varying in time*, $(\delta_{u,t})_{t\in\mathbb{N}}$ and $(\delta_{f,t})_{t\in\mathbb{N}}$, respectively, where

$\delta_{u,t}$ = discount factor of the union in period $t \in \mathbb{N}$, $\delta_{u,0} = 1$, $0 < \delta_{u,t} < 1$ for $t \geq 1$

$\delta_{f,t}$ = discount factor of the firm in period $t \in \mathbb{N}$, $\delta_{f,0} = 1$, $0 < \delta_{f,t} < 1$ for $t \geq 1$

The *result* of the wage bargaining is either a pair (W,T), where W is the wage contract agreed upon and $T \in \mathbb{N}$ is the number of proposals rejected in the bargaining, or a *disagreement* $(0,\infty)$, i.e., the situation in which the parties never reach an agreement. The following notation for each $t \in \mathbb{N}$ is introduced:

$$\delta_u(t) := \prod_{k=0}^{t} \delta_{u,k}, \quad \delta_f(t) := \prod_{k=0}^{t} \delta_{f,k} \quad \text{and} \tag{3.2.1}$$

$$\text{for } 0 < t' \le t, \quad \delta_u(t',t) := \frac{\delta_u(t)}{\delta_u(t'-1)} = \prod_{k=t'}^{t} \delta_{u,k}, \quad \delta_f(t',t) := \frac{\delta_f(t)}{\delta_f(t'-1)} = \prod_{k=t'}^{t} \delta_{f,k} \tag{3.2.2}$$

The utility of the result (W, T) for the union is equal to the discounted sum of wage earnings

$$U(W,T) = \sum_{t=0}^{\infty} \delta_u(t) u_t \tag{3.2.3}$$

where $u_t = W$ for each $t \ge T$ and, if $T > 0$ then for each $0 \le t < T$:

$u_t = 0$ if there is a strike in period $t \in \mathbb{N}$

$u_t = w_0$ if there is no strike in period t.

The utility of the result (W, T) for the firm is equal to the discounted sum of profits

$$V(W,T) = \sum_{t=0}^{\infty} \delta_f(t) v_t \tag{3.2.4}$$

where $v_t = 1 - W$ for each $t \ge T$ and, if $T > 0$ then for each $0 \le t < T$:

$v_t = 0$ if there is a strike in period t

$v_t = 1 - w_0$ if there is no strike in period t.

We set $U(0, \infty) = V(0, \infty) = 0$. We assume that the series that define $U(W,T)$ and $V(W,T)$ in (3.2.3) and (3.2.4) are convergent. In particular, we analyze $(\delta_{u,t})_{t\in\mathbb{N}}$ and $(\delta_{f,t})_{t\in\mathbb{N}}$ that are bounded by a certain number smaller than 1, i.e., we assume that

there exist $a < 1$ *and* $b < 1$ *such that* $\delta_{u,t} \le a$ *and* $\delta_{f,t} \le b$ *for each* $t \in \mathbb{N}$. (3.2.5)

The conditions given in (3.2.5) are sufficient for the convergence of the series that define $U(W,T)$ and $V(W,T)$ in (3.2.3) and (3.2.4). The convergence follows immediately from the comparison test applied to the geometric series.

We also introduce a kind of generalized discount factors which take into account the sequences of discount rates varying in time and the fact that the utilities are defined by the discounted streams of payoffs. We have for every $t \in \mathbb{N}_+$

$$\Delta_u(t) := \frac{\sum_{k=t}^{\infty} \delta_u(t,k)}{1 + \sum_{k=t}^{\infty} \delta_u(t,k)}, \quad \Delta_f(t) := \frac{\sum_{k=t}^{\infty} \delta_f(t,k)}{1 + \sum_{k=t}^{\infty} \delta_f(t,k)} \tag{3.2.6}$$

and consequently, for every $t \in \mathbb{N}_+$

$$1 - \Delta_u(t) = \frac{1}{1 + \sum_{k=t}^{\infty} \delta_u(t,k)}, \quad 1 - \Delta_f(t) = \frac{1}{1 + \sum_{k=t}^{\infty} \delta_f(t,k)} \tag{3.2.7}$$

Note that for every $t \in \mathbb{N}_+$

$$\Delta_f(t) \geq \Delta_u(t) \quad \text{if and only if} \quad \sum_{k=t}^{\infty} \delta_f(t,k) \geq \sum_{k=t}^{\infty} \delta_u(t,k)$$

Obviously, for the special case of constant discount rates, i.e., if $\delta_{u,t} = \delta_u$ and $\delta_{f,t} = \delta_f$ for every $t \in \mathbb{N}_+$, we have $\Delta_u(t) = \delta_u$ and $\Delta_f(t) = \delta_f$.

In what follows, $\Delta_u(t)$ and $\Delta_f(t)$ will be called the *generalized discount factors* of the union and the firm in period t, respectively.

Furthermore, we introduce the additional definition and notation.

Definition 3.1. *Let (s_u, s_f) be the following family of strategies:*

- *Strategy of the union s_u: in period $2t$ ($t \in \mathbb{N}$) propose $\overline{W}^{2t}$; in period $2t+1$ accept an offer y if and only if $y \geq \overline{Z}^{2t+1}$;*
- *Strategy of the firm s_f: in period $2t+1$ propose $\overline{Z}^{2t+1}$; in period $2t$ accept an offer x if and only if $x \leq \overline{W}^{2t}$.*

A strategy of the union additionally specifies its strike decision.

3.3 Exogenous strike decisions of the union

In this section, we assume that the union commits to a specific strike decision and consider the family (s_u, s_f) of the parties' strategies given in Definition 3.1. This assumption will be then relaxed in Section 3.4, where SPE for the general model are presented.

3.3.1 Going always on strike under a disagreement

We analyze the case when the strike decision of the union is exogenous and the union is supposed to go on strike in each period in which there is a disagreement. Fernandez and Glazer [1991] show that in such a case, if preferences are defined by constant discount factors, then there is the unique SPE of the wage bargaining game. It coincides with the SPE in Rubinstein's model and leads to the agreement $\overline{W} = \frac{1-\delta_f}{1-\delta_u\delta_f}$ reached in period 0. We generalize the equilibrium result obtained in Fernandez and Glazer [1991] to the model with discount factors varying in time.

First of all, we deliver necessary and sufficient conditions for (s_u, s_f) to be a SPE. According to these conditions, in every even (odd, respectively) period the firm (the union, respectively) is indifferent between accepting the equilibrium offer of the union (of the firm, respectively) and rejecting that offer. This is formalized in the following proposition.

Proposition 3.1. *Consider the generalized wage bargaining model with preferences of the union and the firm described by the sequences of discount factors $(\delta_{i,t})_{t\in N}$, where $\delta_{i,0} = 1$, $0 < \delta_{i,t} < 1$ for $t \geq 1$, $i = u, f$. Assume that the strike decision is given exogenously and the union is committed to strike in every period in which there is a disagreement. Then (s_u, s_f) is a SPE of this game if and only if the offers satisfy the following infinite system of equations: for each $t \in \mathbb{N}$*

$$1 - \overline{W}^{2t} = \left(1 - \overline{Z}^{2t+1}\right) \Delta_f(2t+1) \quad \textit{and} \quad \overline{Z}^{2t+1} = \overline{W}^{2t+2} \Delta_u(2t+2) \tag{3.3.1}$$

Proof. ($\Leftarrow$) Let (s_u, s_f) be defined by (3.3.1) which can be equivalently written as

$$1 - \overline{W}^{2t} + \left(1 - \overline{W}^{2t}\right) \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k) = \left(1 - \overline{Z}^{2t+1}\right) \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k) \tag{3.3.2}$$

and

$$\overline{Z}^{2t+1} + \overline{Z}^{2t+1} \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k) = \overline{W}^{2t+2} \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k) \tag{3.3.3}$$

Consider an arbitrary subgame starting in period $2t$ with the union making an offer. Under (s_u, s_f) the union gets $\overline{W}^{2t} + \overline{W}^{2t} \sum_{k=2t+1}^{\infty} \delta_u(2t+1, k)$ and the firm gets $\left(1 - \overline{W}^{2t}\right) + \left(1 - \overline{W}^{2t}\right) \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k)$. Suppose that the union deviates from s_u. If it proposes a certain $x > \overline{W}^{2t}$, then it gets at most $\overline{Z}^{2t+1} \sum_{k=2t+1}^{\infty} \delta_u(2t+1, k)$. From (3.3.2), $0 \leq 1 - W^{2t} = \left(\overline{W}^{2t} - \overline{Z}^{2t+1}\right) \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k)$, and therefore $\overline{W}^{2t} \geq \overline{Z}^{2t+1}$. Consequently, $\overline{W}^{2t} + \overline{W}^{2t} \sum_{k=2t+1}^{\infty} \delta_u(2t+1, k) \geq \overline{Z}^{2t+1} \sum_{k=2t+1}^{\infty} \delta_u(2t+1, k)$, and hence the union would not be better off by this deviation. If the union proposes a certain $x < \overline{W}^{2t}$, then it gets $x + x \sum_{k=2t+1}^{\infty} \delta_u(2t+1, k)$, but then it is worse off, since $x + x \sum_{k=2t+1}^{\infty} \delta_u(2t+1, k) < \overline{W}^{2t} + \overline{W}^{2t} \sum_{k=2t+1}^{\infty} \delta_u(2t+1, k)$. If the firm rejects $\overline{W}^{2t}$, then it gets at most $\left(1 - \overline{Z}^{2t+1}\right) \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k)$, which by virtue of equation (3.3.2) is equal to $\left(1 - \overline{W}^{2t}\right) + \left(1 - \overline{W}^{2t}\right) \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k)$, so the firm would not be better off.

The analysis of a subgame starting in $2t + 1$ with the firm proposing is analogous to the study of a subgame starting in $2t$, except that we use (3.3.3) instead of (3.3.2).

Consider a subgame starting in period $2t$ with the firm replying to an offer x. Let $x \leq \overline{W}^{2t}$. Under (s_u, s_f) the firm accepts it and gets $(1 - x) + (1 - x) \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k)$.

Suppose that the firm rejects such x. We already know that it is optimal for the firm to propose Z^{2t+1} in $(2t+1)$, so the firm would get $(1-Z^{2t+1})\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k)$, but from (3.3.2), $(1-x)+(1-x)\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k) \geq \left(1-\overline{W}^{2t}\right)+\left(1-\overline{W}^{2t}\right)\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k) = \left(1-\overline{Z}^{2t+1}\right)\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k)$. Hence, the firm would not be better off by this deviation. Let $x > \overline{W}^{2t}$. Under (s_u, s_f) the firm rejects it and proposes $\overline{Z}^{2t+1}$ which is accepted. The union gets then $\overline{Z}^{2t+1}\sum_{k=2t+1}^{\infty}\delta_u(2t+1,k)$ and the firm $(1-Z^{2t+1})\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k)$. If the firm accepts such x, then it gets $(1-x)+(1-x)\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k)$. But from (3.3.2), $(1-x)+(1-x)\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k) < \left(1-\overline{W}^{2t}\right)+\left(1-\overline{W}^{2t}\right)\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k) = \left(1-\overline{Z}^{2t+1}\right)\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k)$, so the firm would be worse off by this deviation.

The analysis of subgames starting in period $2t+1$ by the union replying is analogous to the analysis of the corresponding subgames starting in period $2t$ by the firm replying.
($\Rightarrow$) Let (s_u, s_f) be a SPE. Consider a subgame starting in period $2t$ with the union making an offer. Using (s_u, s_f) gives $\left(1-\overline{W}^{2t}\right)+\left(1-\overline{W}^{2t}\right)\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k)$ to the firm. By rejecting $\overline{W}^{2t}$ the firm gets $\left(1-\overline{Z}^{2t+1}\right)\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k)$. Since (s_u, s_f) is a SPE, $\left(1-\overline{W}^{2t}\right)+\left(1-\overline{W}^{2t}\right)\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k) \geq \left(1-\overline{Z}^{2t+1}\right)\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k)$. Suppose $\left(1-\overline{W}^{2t}\right)+\left(1-\overline{W}^{2t}\right)\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k) > \left(1-\overline{Z}^{2t+1}\right)\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k)$. Then there exists $\tilde{x} > \overline{W}^{2t}$ such that $\left(1-\overline{W}^{2t}\right)+\left(1-\overline{W}^{2t}\right)\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k) > (1-\tilde{x})+(1-\tilde{x})\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k) > \left(1-\overline{Z}^{2t+1}\right)\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k)$. Since $\tilde{x} > \overline{W}^{2t}$, the firm rejects it and gets $\left(1-\overline{Z}^{2t+1}\right)\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k)$, but it would be better off by accepting this offer. Hence, we get a contradiction and prove (3.3.2). Proving (3.3.3) is analogous by considering a subgame starting in period $2t+1$ with the firm proposing. □

Rusinowska [2000, 2001] determines SPE for the generalized Rubinstein model in which preferences of player $i = 1, 2$ are expressed not by a constant discount rate $0 < \delta_i < 1$ (as in the original Rubinstein framework), but by a sequence of discount rates $(\delta_{i,t})_{t\in N}$ varying in time, where $\delta_{i,0} = 1$, $0 < \delta_{i,t} < 1$ for $t \geq 1$. In her model, the utility $\widetilde{U}_i$ to player $i = 1, 2$ of the result (W, T), where $W \in [0, 1]$ is the agreement and $T \in \mathbb{N}$ is the number of periods rejected in the bargaining, is equal to

$$\widetilde{U}_i(W,T) = W_i\prod_{k=0}^{T}\delta_{i,k}, \text{ where } W_1 = W \text{ and } W_2 = 1-W \tag{3.3.4}$$

and the utility of the disagreement $(0, \infty)$ is equal to $\widetilde{U}_i(0, \infty) = 0$. Note that this generalized bargaining model differs from the generalized wage bargaining proposed in the present paper, in particular, because in the latter the utility of the union is defined as the discounted *sum* of wage earnings (see formula (3.2.3)) and the utility of the firm is defined by the discounted *sum* of profits (see formula (3.2.4)). While the F-G model coincides with Rubinstein's model under the 'always-strike decision', the generalized wage bargaining model and the generalization of Rubinstein's model mentioned above do not coincide. Consequently, as shown in Fact 3.1, the result on SPE in the generalized Rubinstein model by Rusinowska [2000, 2001] cannot be applied to the generalized wage bargaining model introduced in the present work.

Fact 3.1. *The generalized wage bargaining model in which the strike decision is given exogenously and the union is committed to strike in every disagreement period does not coincide with the generalized Rubinstein model with discount rates varying in time, and in general the SPE of the two models are different.*

Proof. In order to find the SPE offers in the generalized Rubinstein model with players 1 and 2 being the union and the firm, respectively, we need to solve the following infinite system of equations for each $t \in \mathbb{N}$ (Rusinowska [2000, 2001])

$$1 - \overline{W}^{2t} = \left(1 - \overline{Z}^{2t+1}\right) \delta_{f,2t+1} \quad \text{and} \quad \overline{Z}^{2t+1} = \overline{W}^{2t+2} \delta_{u,2t+2} \tag{3.3.5}$$

In order to find the SPE offers in the generalized wage bargaining model with the exogenous "always strike" decision we need to solve (3.3.1) for each $t \in \mathbb{N}$. For the model with constant discount rates δ_u and δ_f these two infinite systems (3.3.1) and (3.3.5) are equivalent. For each $t \in \mathbb{N}$, $\Delta_f(2t+1) = \delta_f$ and $\Delta_u(2t+2) = \delta_u$, so inserting this into (3.3.1) gives equivalently (3.3.5), since $\delta_{f,2t+1} = \delta_f$, $\delta_{u,2t+2} = \delta_u$. However, these two infinite systems are not equivalent if we consider the generalized wage bargaining model, because

$$\Delta_f(2t+1) = \frac{\delta_{f,2t+1}(1 + \sum_{k=2t+2}^{\infty} \delta_f(2t+2, k))}{1 + \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k)}$$

$$\Delta_u(2t+2) = \frac{\delta_{u,2t+2}(1 + \sum_{k=2t+3}^{\infty} \delta_u(2t+3, k))}{1 + \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)}$$

and for any $t \neq t'$ usually

$$\sum_{k=t}^{\infty} \delta_f(t, k) \neq \sum_{k=t'}^{\infty} \delta_f(t', k), \quad \sum_{k=t}^{\infty} \delta_u(t, k) \neq \sum_{k=t'}^{\infty} \delta_u(t', k)$$

and therefore usually

$$\Delta_f(2t+1) \neq \delta_{f,2t+1}, \quad \Delta_u(2t+2) \neq \delta_{u,2t+2}$$

As an illustrative example, consider $\delta_{f,1} = \delta_{u,1} = \frac{1}{2}$, $\delta_{f,t} = \delta_{u,t} = \frac{1}{3}$ for each $t \geq 2$. Then

$$\sum_{k=1}^{\infty} \delta_f(1,k) = \frac{3}{4}, \quad \sum_{k=2t+1}^{\infty} \delta_f(2t+1,k) = \frac{1}{2} \text{ for each } t \geq 1$$

Solving the system (3.3.5) gives $\overline{W}^0 = \frac{5}{8}$, $\overline{W}^{2t} = \frac{3}{4}$ for each $t \geq 1$, $\overline{Z}^{2t+1} = \frac{1}{4}$ for each $t \in N$, but this solution does not satisfy the first equation of (3.3.1), i.e., $1 - \overline{W}^0 \neq \left(1 - \overline{Z}^1\right) \Delta_f(1)$. □

By solving the infinite system (3.3.1), we can determine the SPE offers made by the union and the firm, as presented in Theorem 3.1. Since we will compare the SPE offers under different exogenous strike decisions, in the statement of the corresponding results (but not in their proofs), we will use additional notations. For the 'always strike' decision case, the SPE offers will be denoted by $\overline{W}_{AS}^{2t}$ and $\overline{Z}_{AS}^{2t+1}$ for every $t \in \mathbb{N}$.

Theorem 3.1. *Consider the generalized wage bargaining model with preferences described by the sequences of discount factors $(\delta_{i,t})_{t\in N}$, where $\delta_{i,0} = 1$, $0 < \delta_{i,t} < 1$ for $t \geq 1$, $i = u, f$. Assume that the strike decision is given exogenously and the union is committed to strike in every disagreement period. Then there is the unique SPE of the form (s_u, s_f), in which the offers of the parties, for each $t \in \mathbb{N}$, are given by*

$$\overline{W}_{AS}^{2t} = 1 - \Delta_f(2t+1) + \sum_{m=t}^{\infty}(1 - \Delta_f(2m+3)) \prod_{j=t}^{m} \Delta_u(2j+2)\Delta_f(2j+1) \tag{3.3.6}$$

$$\overline{Z}_{AS}^{2t+1} = \overline{W}_{AS}^{2t+2} \Delta_u(2t+2) \tag{3.3.7}$$

Proof. We solve the system (3.3.1) which is equivalent, for each $t \in \mathbb{N}$, to

$$\overline{W}^{2t} - \overline{Z}^{2t+1}\Delta_f(2t+1) = 1 - \Delta_f(2t+1) \quad \text{and} \quad \overline{Z}^{2t+1} - \overline{W}^{2t+2}\Delta_u(2t+2) = 0 \tag{3.3.8}$$

and gives immediately (3.3.7). Note that (3.3.8) is a regular triangular system $AX = Y$, with $A = [a_{ij}]_{i,j\in\mathbb{N}^+}$, $X = [(x_i)_{i\in\mathbb{N}^+}]^T$, $Y = [(y_i)_{i\in\mathbb{N}^+}]^T$, where for each $t, j \geq 1$

$$a_{t,t} = 1, \; a_{t,j} = 0 \text{ for } j < t \text{ or } j > t+1 \tag{3.3.9}$$

and for each $t \in \mathbb{N}$

$$a_{2t+1,2t+2} = -\Delta_f(2t+1), \quad a_{2t+2,2t+3} = -\Delta_u(2t+2) \tag{3.3.10}$$

$$x_{2t+1} = \overline{W}^{2t}, \quad x_{2t+2} = \overline{Z}^{2t+1}, \quad y_{2t+1} = 1 - \Delta_f(2t+1), \quad y_{2t+2} = 0 \tag{3.3.11}$$

Any regular triangular matrix A possesses the (unique) inverse matrix B, i.e., there exists B such that $BA = I$, where I is the infinite identity matrix. The matrix $B = [b_{ij}]_{i,j \in \mathbb{N}^+}$ is also regular triangular, and its elements are the following:

$$b_{t,t} = 1, \ b_{t,j} = 0 \text{ for each } t, j \geq 1 \text{ such that } j < t \tag{3.3.12}$$

$$b_{2t+1,2t+2} = \Delta_f(2t+1), \ b_{2t+2,2t+3} = \Delta_u(2t+2) \text{ for each } t \in \mathbb{N} \tag{3.3.13}$$

and for each $t, m \in \mathbb{N}$ and $m > t$

$$b_{2t+2,2m+2} = \prod_{j=t}^{m-1} \Delta_u(2j+2)\Delta_f(2j+3), \quad b_{2t+2,2m+3} = \prod_{j=t}^{m-1} \Delta_u(2j+2)\Delta_f(2j+3)\Delta_u(2m+2) \tag{3.3.14}$$

$$b_{2t+1,2m+1} = \prod_{j=t}^{m-1} \Delta_u(2j+2)\Delta_f(2j+1), \quad b_{2t+1,2m+2} = \prod_{j=t}^{m-1} \Delta_u(2j+2)\Delta_f(2j+1)\Delta_f(2m+1) \tag{3.3.15}$$

Next, by applying $X = BY$ we get $\overline{W}^{2t}$ as given by (3.3.6). Obviously $\overline{W}^{2t} \geq 0$. Let us consider the sequence of partial sums for $k > t$

$$S_k = 1 - \Delta_f(2t+1) + \sum_{m=t}^{k-1} (1 - \Delta_f(2m+3)) \prod_{j=t}^{m} \Delta_u(2j+2)\Delta_f(2j+1)$$

The sequence is increasing and also $S_k \leq 1$ for each $k > t$, and therefore $\overline{W}^{2t} = \lim_{k \to +\infty} S_k \leq 1$. Since $0 \leq \overline{W}^{2t+2} \leq 1$, we have $0 \leq \overline{Z}^{2t+1} < 1$. □

Formula (3.3.6) presents the SPE offer made by the union in an even period. It is determined by the generalized discount factors of the union in all even periods following the given period and by the generalized discount factors of the firm in all odd periods following that period. Shaked and Sutton [1984] provide a nice interpretation of the solution in the wage bargaining à la Rubinstein for constant discount rates: the payoff of the firm (which is the first mover in their model) coincides with the sum of the shrinkages of the cake which occur during the time periods when the offers made in even periods are rejected. For the common discount rate δ, we have $\frac{1}{1+\delta} = (1-\delta)(1 + \delta^2 + \delta^4 + \cdots)$

which explains this interpretation, because the cake shrinks from δ^{2t} to δ^{2t+1}, i.e., by $(1-\delta)\delta^{2t}$ if it is rejected in period $2t$. As Shaked and Sutton [1984] mention, this also holds for the (constant) discount rates which are not equal. In our case, we notice a similar (but generalized) pattern, with the generalized discount factors.

According to (3.3.7), the SPE offer made by the firm in an odd period is equal to the SPE offer made by the union in the subsequent period, discounted by the generalized discount factor of the union. In other words, what the union can earn by accepting the SPE offer made by the firm in an odd period is equal to what the union could earn by rejecting that offer and submitting its SPE offer in the subsequent even period (that would be accepted by the firm).

Note that the more patient the union is in the subsequent periods, the more is proposed to the union in a given period under the SPE, both by the union and by the firm.

Example 3.1. When we apply our result to the wage bargaining studied by Fernandez and Glazer [1991], we get obviously their result (see Lemma 1 in Fernandez and Glazer [1991]). Let us calculate the share $\overline{W}^0$ that the union proposes for itself at the beginning of the game. We have $\delta_{f,2t+1} = \delta_f$ and $\delta_{u,2t+2} = \delta_u$ for each $t \in \mathbb{N}$. Hence, for each $t \in \mathbb{N}$

$$\overline{W}_{AS}^{2t} = (1-\delta_f) + (1-\delta_f)\left[\delta_f\delta_u + (\delta_f\delta_u)^2 + \cdots\right] = \frac{1-\delta_f}{1-\delta_f\delta_u}$$

Example 3.2. Let us analyze a model in which the union and the firm have the following sequences of discount factors varying in time: for each $t \in \mathbb{N}$

$$\delta_{f,2t+1} = \delta_{u,2t+1} = \frac{1}{2}, \quad \delta_{f,2t+2} = \delta_{u,2t+2} = \frac{1}{3}$$

Hence, for each $j \in \mathbb{N}$

$$\sum_{k=2j+1}^{\infty} \delta_f(2j+1,k) = \frac{1}{2} + \frac{1}{2}\cdot\frac{1}{3} + \frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2} + \cdots =$$

$$= \frac{1}{2}\left(1+\frac{1}{6}+\frac{1}{6^2}+\cdots\right) + \frac{1}{6}\left(1+\frac{1}{6}+\frac{1}{6^2}+\cdots\right) = \frac{4}{5},\ \Delta_f(2j+1) = \frac{4}{9}$$

$$\sum_{k=2j+2}^{\infty} \delta_u(2j+2,k) = \frac{1}{3} + \frac{1}{3}\cdot\frac{1}{2} + \frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{3} + \cdots =$$

$$= \frac{1}{3}\left(1+\frac{1}{6}+\frac{1}{6^2}+\cdots\right) + \frac{1}{6}\left(1+\frac{1}{6}+\frac{1}{6^2}+\cdots\right) = \frac{3}{5},\ \Delta_u(2j+2) = \frac{3}{8}$$

Hence, by virtue of (3.3.6) the offer of the union in period 0 in the SPE is equal to

$$\overline{W}^0_{AS} = \frac{5}{9} + \frac{4}{9} \cdot \frac{3}{8} \cdot \frac{5}{9} + \left(\frac{4}{9} \cdot \frac{3}{8}\right)^2 \cdot \frac{5}{9} + \cdots = \frac{5}{9}\left(1 + \frac{1}{6} + \frac{1}{6^2} + \cdots\right) = \frac{2}{3}$$

Note again that if we would apply the generalization of the original Rubinstein model to this example, then we would get $\overline{W}^0 \neq \frac{2}{3}$.

3.3.2 Going on strike only after rejection of own proposals

Haller and Holden [1990] consider also another game with the exogenous strike decision, in which the union goes on strike only after its own proposal is rejected and it holds out if a proposal of the firm is rejected. They analyze the model with the same discount factor δ and show that in such a game there is the unique SPE with the union's offer equal to $\overline{W} = \frac{1+\delta w_0}{1+\delta}$. We generalize this game to the model with discount rates varying in time.

Similarly as Proposition 3.1 for the case of always strike decision, Proposition 3.2 presents necessary and sufficient conditions for (s_u, s_f) to be a SPE for the case of "going on strike only after rejection of own proposals", if the firm is at least as patient as the union, i.e., more precisely, if the generalized discount factor of the firm in every even period is at least as high as the generalized discount factor of the union in this even period. According to these conditions, each party is indifferent between accepting and rejecting the equilibrium offer in every period in which it is the turn of that party to reply to the offer.

Proposition 3.2. *Consider the generalized wage bargaining model with preferences of the union and the firm described by the sequences of discount factors $(\delta_{i,t})_{t\in N}$, where $\delta_{i,0} = 1$, $0 < \delta_{i,t} < 1$ for $t \geq 1$, $i = u, f$, and*

$$\Delta_f(2t+2) \geq \Delta_u(2t+2) \text{ for each } t \in \mathbb{N} \tag{3.3.16}$$

Assume that the strike decision is given exogenously and the union is committed to strike only after rejection of its own proposals. Then (s_u, s_f) is a SPE of this game if and only if the offers satisfy the following infinite system of equations: for each $t \in \mathbb{N}$

$$1-\overline{W}^{2t} = \left(1 - \overline{Z}^{2t+1}\right)\Delta_f(2t+1) \quad \text{and} \quad \overline{Z}^{2t+1} = w_0\left(1 - \Delta_u(2t+2)\right)+\overline{W}^{2t+2}\Delta_u(2t+2) \tag{3.3.17}$$

Proof. (⇐) The analysis of subgames that start with replies to an offer as well as of a subgame starting in period $2t$ with the union making an offer is analogous to the analysis of the corresponding subgames of the going always on strike case.

Consider a subgame starting in period $2t+1$ with the firm making an offer. Under (s_u, s_f), the union gets $\overline{Z}^{2t+1} + \overline{Z}^{2t+1} \sum_{k=2t+2}^{\infty} \delta_u(2t+2,k)$ and the firm $\left(1-\overline{Z}^{2t+1}\right) + \left(1-\overline{Z}^{2t+1}\right) \sum_{k=2t+2}^{\infty} \delta_f(2t+2,k)$. Suppose that the firm deviates from s_f and proposes a certain $y < \overline{Z}^{2t+1}$. Then the firm gets $(1-w_0) + \left(1-\overline{W}^{2t+2}\right) \sum_{k=2t+2}^{\infty} \delta_f(2t+2,k)$. Note that $\overline{Z}^{2t+1} \geq w_0$, otherwise the union would prefer to reject $\overline{Z}^{2t+1}$ and to get w_0 in period $2t+1$. From (3.3.17), $0 \leq \overline{Z}^{2t+1} - w_0 = \left(\overline{W}^{2t+2} - \overline{Z}^{2t+1}\right) \sum_{k=2t+2}^{\infty} \delta_u(2t+2,k)$, and therefore $\overline{W}^{2t+2} \geq \overline{Z}^{2t+1}$. By virtue of (3.3.16), $\left(\overline{W}^{2t+2} - \overline{Z}^{2t+1}\right) \sum_{k=2t+2}^{\infty} \delta_f(2t+2,k) = \left(\overline{Z}^{2t+1} - w_0\right) \frac{\sum_{k=2t+2}^{\infty} \delta_f(2t+2,k)}{\sum_{k=2t+2}^{\infty} \delta_u(2t+2,k)} \geq \left(\overline{Z}^{2t+1} - w_0\right)$. Hence, we have $\left(1-\overline{Z}^{2t+1}\right) + \left(1-\overline{Z}^{2t+1}\right) \sum_{k=2t+2}^{\infty} \delta_f(2t+2,k) \geq (1-w_0) + \left(1-\overline{W}^{2t+2}\right) \sum_{k=2t+2}^{\infty} \delta_f(2t+2,k)$, so this deviation would not be profitable to the firm. The proofs that other deviations are not profitable to the deviating party are similar to the going always on strike case.

(⇒) The proof is analogous to the proof of Proposition 3.1. □

Remark 3.1. Note that if $\Delta_f(2t+2) < \Delta_u(2t+2)$ for some $t \in \mathbb{N}$, then in the corresponding subgame starting in period $2t+1$ with the firm making an offer, (s_u, s_f) as defined by (3.3.17) would not be a Nash equilibrium, and consequently would not be a SPE of the game.

By solving the infinite system (3.3.17), we determine the SPE offers made by the union and the firm, as presented in Theorem 3.2. For the "strike only after rejection" case, the SPE offers will be denoted by $\overline{W}_{SAR}^{2t}$ and $\overline{Z}_{SAR}^{2t+1}$ for every $t \in \mathbb{N}$.

Theorem 3.2. *Consider the generalized wage bargaining model with preferences described by the sequences of discount factors $(\delta_{i,t})_{t \in N}$, where $\delta_{i,0} = 1$, $0 < \delta_{i,t} < 1$ for $t \geq 1$, $i = u, f$ and condition (3.3.16) is satisfied, i.e.,*

$$\Delta_f(2t+2) \geq \Delta_u(2t+2) \text{ for each } t \in \mathbb{N}$$

Assume that the strike decision is given exogenously and the union is committed to strike only after rejection of its own proposals. Then there is the unique SPE of the form (s_u, s_f), in which the offers of the parties for each $t \in \mathbb{N}$ are given by

$$\overline{W}_{SAR}^{2t} = 1 - \Delta_f(2t+1) + w_0 \Delta_f(2t+1)(1 - \Delta_u(2t+2)) +$$

$$\sum_{m=t}^{\infty}(1-\Delta_f(2m+3)+w_0\Delta_f(2m+3)(1-\Delta_u(2m+4)))\prod_{j=t}^{m}\Delta_u(2j+2)\Delta_f(2j+1) \tag{3.3.18}$$

$$\overline{Z}_{SAR}^{2t+1}=w_0\left(1-\Delta_u(2t+2)\right)+\overline{W}_{SAR}^{2t+2}\Delta_u(2t+2) \tag{3.3.19}$$

Proof. We need to solve (3.3.17) for each $t\in\mathbb{N}$, which is equivalent for each $t\in\mathbb{N}$ to

$$\overline{W}^{2t}-\overline{Z}^{2t+1}\Delta_f(2t+1)=1-\Delta_f(2t+1)\quad\text{and} \tag{3.3.20}$$

$$\overline{Z}^{2t+1}-\overline{W}^{2t+2}\Delta_u(2t+2)=w_0\left(1-\Delta_u(2t+2)\right) \tag{3.3.21}$$

From (3.3.21) we get (3.3.19). (3.3.20) and (3.3.21) constitute a regular triangular system $AX=Y$ with $A=[a_{ij}]_{i,j\in\mathbb{N}^+}$, $X=[(x_i)_{i\in\mathbb{N}^+}]^T$, $Y=[(y_i)_{i\in\mathbb{N}^+}]^T$, where A is the same as for Theorem 3.1 and is described by (3.3.9) for $t,j\geq 1$ and (3.3.10) for $t\in\mathbb{N}$.

$$x_{2t+1}=\overline{W}^{2t},\quad x_{2t+2}=\overline{Z}^{2t+1},\quad y_{2t+1}=1-\Delta_f(2t+1),\quad y_{2t+2}=w_0\left(1-\Delta_u(2t+2)\right)$$

Since we have the same A as in the always-strike decision, its (unique) inverse matrix B is the same. By applying $X=BY$ we get $\overline{W}^{2t}$ as in (3.3.18). From (3.3.19) $0\leq\overline{Z}_{2t+1}\leq 1$. Also $\overline{W}^{2t}\geq 0$. The proof that $\overline{W}^{2t}\leq 1$ goes analogously as in Theorem 3.1. □

Remark 3.2. Note that $\overline{W}_{SAR}^{2t}$ given in (3.3.18) can be written equivalently as

$$\overline{W}_{SAR}^{2t}=\overline{W}_{AS}^{2t}+w_0\Big(\Delta_f(2t+1)(1-\Delta_u(2t+2))+$$
$$+\sum_{m=t}^{\infty}\Delta_f(2m+3)(1-\Delta_u(2m+4))\prod_{j=t}^{m}\Delta_u(2j+2)\Delta_f(2j+1)\Big) \tag{3.3.22}$$

and hence, $\overline{W}_{SAR}^{2t}>\overline{W}_{AS}^{2t}$. This has an intuitive interpretation. Going on strike only after rejection of own proposals (i.e., in even periods) gives a greater wage contract than going on strike in every disagreement period, because the first strategy creates an asymmetry in costs of rejecting. Under the first strategy, it is more costly for the firm to reject the union's offer (which leads to the strike) than it is for the union to reject the firm's offer (which leads to the holdout).

Since $\overline{W}_{SAR}^{2t+2}>\overline{W}_{AS}^{2t+2}$, we have also $\overline{Z}_{SAR}^{2t+1}=w_0\left(1-\Delta_u(2t+2)\right)+\overline{W}_{SAR}^{2t+2}\Delta_u(2t+2)>\overline{W}_{AS}^{2t+2}\Delta_u(2t+2)=\overline{Z}_{AS}^{2t+1}$, and therefore $\overline{Z}_{SAR}^{2t+1}>\overline{Z}_{AS}^{2t+1}$.

Example 3.3. Let us apply this result to the wage bargaining studied by Fernandez and Glazer [1991], i.e., we have $\delta_{f,t} = \delta_f$ and $\delta_{u,t} = \delta_u$ for each $t \in \mathbb{N}$. Hence, for each $t \in \mathbb{N}$

$$\overline{W}^{2t}_{SAR} = (1 - \delta_f + w_0\delta_f(1 - \delta_u))\left[1 + \delta_f\delta_u + (\delta_f\delta_u)^2 + \cdots\right] =$$
$$= \frac{1 - \delta_f + w_0\delta_f(1 - \delta_u)}{1 - \delta_f\delta_u} = w_0 + \frac{(1 - \delta_f)(1 - w_0)}{1 - \delta_f\delta_u}$$

If additionally we assume that $\delta_f = \delta_u = \delta$, then $\overline{W}^{2t}_{SAR} = \frac{1+\delta w_0}{1+\delta}$, which coincides with the result by Haller and Holden [1990].

Example 3.4. We analyze the model presented in Example 3.2. By virtue of (3.3.18) the offer of the union in period 0 in the SPE is equal to

$$\overline{W}^{0}_{SAR} = \left(\frac{5}{9} + \frac{4}{9}\cdot\frac{5}{8}\cdot w_0\right)\left[1 + \frac{4}{9}\cdot\frac{3}{8} + \left(\frac{4}{9}\cdot\frac{3}{8}\right)^2 + \cdots\right] = \frac{2 + w_0}{3} > \frac{2}{3} = \overline{W}^{0}_{AS}$$

3.3.3 Going never on strike

In case of the exogenous "never-strike" decision of the union, the unique SPE leads to the minimum wage contract w_0. The SPE offers for this case are denoted by $\overline{W}^{2t}_{NS}$ and $\overline{Z}^{2t+1}_{NS}$. We have the following:

Fact 3.2. *Consider the generalized wage bargaining model with preferences of the union and the firm described by the sequences of discount factors* $(\delta_{i,t})_{t\in N}$, *where* $\delta_{i,0} = 1$, $0 < \delta_{i,t} < 1$ *for* $t \geq 1$, $i = u, f$. *Assume that the no-strike decision is given exogenously and the union never goes on strike. Then there is the unique SPE of the form* (s_u, s_f), *where* $\overline{W}^{2t}_{NS} = \overline{Z}^{2t+1}_{NS} = w_0$ *for each* $t \in \mathbb{N}$.

Proof. Suppose that the union never goes on strike. Similar as in the proof of Proposition 3.1 one can show that if (s_u, s_f) is a SPE, then it must hold for each $t \in \mathbb{N}$

$$\left(1 - \overline{W}^{2t}\right) + \left(1 - \overline{W}^{2t}\right)\sum_{k=2t+1}^{\infty}\delta_f(2t+1, k) = (1 - w_0) + \left(1 - \overline{Z}^{2t+1}\right)\sum_{k=2t+1}^{\infty}\delta_f(2t+1, k) \tag{3.3.23}$$

and

$$\overline{Z}^{2t+1} + \overline{Z}^{2t+1}\sum_{k=2t+2}^{\infty}\delta_u(2t+2, k) = w_0 + \overline{W}^{2t+2}\sum_{k=2t+2}^{\infty}\delta_u(2t+2, k) \tag{3.3.24}$$

Obviously, $\overline{W}^{2t} = \overline{Z}^{2t+1} = w_0$ for each $t \in \mathbb{N}$ is a solution of this system of equations, and we know from the infinite matrices theory that this system has the only one solution. One can easily show that (s_u, s_f) with $\overline{W}^{2t} = \overline{Z}^{2t+1} = w_0$ for $t \in \mathbb{N}$ is a SPE. □

Remark 3.3. Note that $\overline{W}^{2t}_{SAR}$ given in (3.3.18) can also be written equivalently as

$$\overline{W}^{2t}_{SAR} = w_0 + (1-w_0)\left(1 - \Delta_f(2t+1) + \sum_{m=t}^{\infty}(1-\Delta_f(2m+3))\prod_{j=t}^{m}\Delta_u(2j+2)\Delta_f(2j+1)\right) \tag{3.3.25}$$

and therefore $\overline{W}^{2t}_{SAR} > w_0 = \overline{W}^{2t}_{NS}$ if $w_0 < 1$. This means that striking only after rejection of own proposals gives to the union the minimum wage contract plus the solution of the case "going always on strike" with the size of the "cake" equal to $1 - w_0$ instead of 1.

Moreover, $1 - \overline{W}^{2t}_{SAR} = (1 - w_0)(1 - \overline{W}^{2t}_{AS})$, which means that in this case the firm gets what it would have under the "going always on strike" equilibrium with the size of the cake equal to $1 - w_0$.

Since $\overline{W}^{2t+2}_{SAR} > w_0$, we have also $\overline{Z}^{2t+1}_{SAR} = w_0\left(1 - \Delta_u(2t+2)\right) + \overline{W}^{2t+2}_{SAR}\Delta_u(2t+2) = w_0 + \Delta_u(2t+2)(\overline{W}^{2t+2}_{SAR} - w_0) > w_0 = \overline{Z}^{2t+1}_{NS}$.

3.4 Subgame perfect equilibria in the general model

After finding the unique SPE for each of the three cases with the exogenous strike decisions, we show that the strategies forming these SPE also appear in the SPE for the general model, i.e., for the model with no assumption on the commitment to strike.

First of all, we consider the pair of strategies analyzed in Subsection 3.3.3. It appears that Lemma 2 of Fernandez and Glazer [1991] remains valid for the general wage bargaining model with discount factors varying in time. We have the following:

Fact 3.3. *Consider the generalized wage bargaining model with preferences of the union and the firm described by the sequences of discount factors $(\delta_{i,t})_{t\in N}$, where $\delta_{i,0} = 1$, $0 < \delta_{i,t} < 1$ for $t \geq 1$, $i = u, f$. There is a SPE in which an agreement of w_0 is reached immediately in period 0. This SPE is the following 'minimum-wage equilibrium':*

- *The union plays s_u with $\overline{W}^{2t} = w_0$ for each $t \in \mathbb{N}$ and never goes on strike;*
- *The firm plays s_f with $\overline{Z}^{2t+1} = w_0$ for each $t \in \mathbb{N}$.*

Proof. It is easy to show that the "minimum-wage" strategies form a SPE for the general wage bargaining game. If one party changes its strategy, with the strategy of the another party being fixed, then the deviating party cannot be better off: neither if at some point it makes an offer different from w_0, nor when it accepts (rejects) an offer which gives the party less (more) than the considered profile of strategies (w_0 for the union and $1 - w_0$ for the firm). The union will not be better off when it decides to change its "never strike" decision and goes on strike when there is a disagreement. □

Next, we consider the pair of strategies presented for the always strike case in Theorem 3.1 of Subsection 3.3.1. If we combine this pair of strategies with the "minimum-wage" strategies, then we find a SPE for the general wage bargaining, provided that the union is sufficiently patient (i.e., the generalized discount factors of the union in all odd periods are sufficiently high). The following proposition generalizes Lemma 3 of Fernandez and Glazer [1991].

Proposition 3.3. *Consider the generalized wage bargaining model with preferences of the union and the firm described by the sequences of discount factors* $(\delta_{i,t})_{t\in N}$*, where* $\delta_{i,0} = 1$*,* $0 < \delta_{i,t} < 1$ *for* $t \geq 1$*,* $i = u, f$*. If*

$$w_0 \leq \overline{Z}_{AS}^{2t+1} \Delta_u(2t+1) \text{ for every } t \in \mathbb{N} \tag{3.4.1}$$

then there exists a SPE in which the agreement of $\overline{W}_{AS}^{0}$ *is reached in period 0, where* $\overline{W}_{AS}^{0}$ *is given in Theorem 3.1. This SPE is formed by the following profile of strategies:*

- *The union plays* s_u *with* $\overline{W}^{2t} = \overline{W}_{AS}^{2t}$ *for each* $t \in \mathbb{N}$ *and always goes on strike if there is a disagreement, where* $\overline{W}_{AS}^{2t}$ *is given in (3.3.6);*
- *The firm plays* s_f *with* $\overline{Z}^{2t+1} = \overline{Z}_{AS}^{2t+1}$ *for each* $t \in \mathbb{N}$*, where* $\overline{Z}_{AS}^{2t+1}$ *is given in (3.3.7);*
- *If, however, at some point, the union deviates from the above rule, then both parties play thereafter according to the strategies given in the "minimum-wage equilibrium".*

Proof. Note that from assumption (3.4.1) it follows that $\overline{W}_{AS}^{2t} \geq w_0$ and $\overline{Z}_{AS}^{2t+1} \geq w_0$ for every $t \in \mathbb{N}$, because we have $\overline{Z}_{AS}^{2t+1} \geq \overline{Z}_{AS}^{2t+1}\Delta_u(2t+1) \geq w_0$, and from (3.3.1), $1-\overline{W}_{AS}^{2t} = \left(1 - \overline{Z}_{AS}^{2t+1}\right) \Delta_f(2t+1) \leq (1 - w_0)\, \Delta_f(2t+1)$. Hence, $\overline{W}_{AS}^{2t} \geq 1-(1 - w_0)\, \Delta_f(2t+1) = w_0 + (1 - \Delta_f(2t + 1))(1 - w_0) \geq w_0$

In order for the union not to deviate from its strike decision in any $2t$ period when no agreement is reached, it must hold $w_0 + w_0 \sum_{k=2t+1}^{\infty} \delta_u(2t+1,k) \leq \overline{Z}_{AS}^{2t+1} \sum_{k=2t+1}^{\infty} \delta_u(2t+1,k)$, which is equivalent to (3.4.1). Hence, the required condition holds.

In order for the union not to deviate from its strike decision in any $2t+1$ period when no agreement is reached, it must hold $w_0 \leq \overline{W}_{AS}^{2t+2} \Delta_u(2t+2)$, but this is satisfied, since from (3.4.1), $w_0 \leq \overline{Z}_{AS}^{2t+1} \Delta_u(2t+1) \leq \overline{Z}_{AS}^{2t+1} = \overline{W}_{AS}^{2t+2} \Delta_u(2t+2)$.

Consider a (proper) subgame such that the union has already deviated in an earlier period. Then, if the parties play the considered profile of strategies, then they use the minimum-wage equilibrium strategies. Hence, from Fact 3.3, this profile is a Nash equilibrium in every subgame starting after the subgame with the deviation.

Consider a subgame such that the union has not deviated before. If the union deviates now in period $2t$ and proposes $x \neq \overline{W}_{AS}^{2t} \geq w_0$, then the firm switches to the minimum-wage strategy and the union cannot be better off by this deviation. Also the firm cannot be better off by deviating in $2t+1$ and proposing $y \neq \overline{Z}_{AS}^{2t+1}$. Finally, it is easy to show that no party can be better off by a deviation when replying to an offer of the other party. □

Proposition 3.4. *Consider the generalized wage bargaining model with preferences of the union and the firm described by the sequences of discount factors $(\delta_{i,t})_{t\in N}$, where $\delta_{i,0} = 1$, $0 < \delta_{i,t} < 1$ for $t \geq 1$, $i = u, f$. If*

$$w_0 \leq \overline{Z}_{SAR}^{2t+1} \Delta_u(2t+1) \text{ for every } t \in \mathbb{N} \tag{3.4.2}$$

and condition (3.3.16) is satisfied, i.e.,

$$\Delta_f(2t+2) \geq \Delta_u(2t+2) \text{ for each } t \in \mathbb{N}$$

then there exists a SPE in which the agreement of $\overline{W}_{SAR}^{0}$ is reached in period 0, where $\overline{W}_{SAR}^{0}$ is given in Theorem 3.2. This SPE is supported by the following 'generalized alternating strike strategies':

- *The union plays s_u with $\overline{W}^{2t} = \overline{W}_{SAR}^{2t}$ for each $t \in \mathbb{N}$, goes on strike after rejection of its own proposals and holds out after rejecting firm's offers, where $\overline{W}_{SAR}^{2t}$ is given in (3.3.18);*

- *The firm plays s_f with $\overline{Z}^{2t+1} = \overline{Z}_{SAR}^{2t+1}$ for each $t \in \mathbb{N}$, where $\overline{Z}_{SAR}^{2t+1}$ is given in (3.3.19);*

- *If, however, at some point, the union deviates from the above rule, then both parties play thereafter according to the strategies given in the "minimum-wage equilibrium".*

Proof. From (3.3.25), if $w_0 < 1$ then we have $\overline{W}_{SAR}^{2t} > w_0$ and $\overline{Z}_{SAR}^{2t+1} > w_0$ for every $t \in \mathbb{N}$. If in period $2t$, when no agreement is reached, the union deviates from its strike decision, then it is not better of by virtue of condition (3.4.2). If in period $2t+1$, when no agreement is reached, the union deviates from its 'hold out' decision, then it is worse off, since $w_0 \sum_{k=2t+2}^{\infty} \delta_u(2t+2,k) < w_0 + \overline{W}_{SAR}^{2t+2} \sum_{k=2t+2}^{\infty} \delta_u(2t+2,k)$. The remaining parts of the proof goes similarly to the proof of Proposition 3.3. □

Next, we will find a SPE for a particular case of the wage bargaining when condition (3.3.16) is not satisfied, i.e., for the game with $\Delta_u(2t+2) > \Delta_f(2t+2)$ for each $t \in \mathbb{N}$. In such a case, given the generalized alternating strike strategy of the union, the firm is better off by playing the so called no-concession strategy instead of the generalized alternating strike strategy. The *no-concession strategy of the firm* is defined as follows:

- *Reject all offers of the union in every even period $2t$, and make an unacceptable offer (e.g., $Z_{NC}^{2t+1} = 0$) in every odd period $2t+1$.*

We can prove the following result.

Proposition 3.5. *If there exists $T \in \mathbb{N}$ such that $\Delta_u(2t+2) > \Delta_f(2t+2)$ for each $t \geq T$, then the pair of the generalized alternating strike strategies is not a SPE. In particular, for $T = 0$, this pair is not a Nash equilibrium.*

Proof. Assume that there exists $T \in \mathbb{N}$ such that $\Delta_u(2t+2) > \Delta_f(2t+2)$ for each $t \geq T$. Then we have the following:

$$\sum_{m=T}^{\infty} (1 - \Delta_f(2m+3)) \prod_{j=T}^{m} \Delta_f(2j+1)\Delta_u(2j+2) = \sum_{m=T}^{\infty} \frac{\prod_{j=T}^{m} \Delta_f(2j+1)\Delta_u(2j+2)}{1 + \sum_{k=2m+3}^{\infty} \delta_f(2m+3,k)} >$$

$$> \sum_{m=T}^{\infty} \frac{\prod_{j=T}^{m} \delta_{f,2j+1}\delta_{f,2j+2}}{1 + \sum_{k=2T+1}^{\infty} \delta_f(2T+1,k)} = \frac{\sum_{m=T}^{\infty} \delta_f(2T+1,2m+2)}{1 + \sum_{k=2T+1}^{\infty} \delta_f(2T+1,k)}$$

Hence, we have

$$\sum_{m=T}^{\infty} (1 - \Delta_f(2m+3)) \prod_{j=T}^{m} \Delta_f(2j+1)\Delta_u(2j+2) > \frac{\sum_{m=T}^{\infty} \delta_f(2T+1,2m+2)}{1 + \sum_{k=2T+1}^{\infty} \delta_f(2T+1,k)} \quad (3.4.3)$$

Consider a subgame starting in period $2T$ in which the union proposes $\overline{W}^{2T}_{SAR}$ and no deviation of the union has taken place before. Then, the generalized alternating strike strategies lead to the agreement $\overline{W}^{2T}_{SAR}$ reached in period $2T$. If the firm switches to the no-concession strategy, then it gets the (normalized) payoff $(1 - Y^{2T}_{NC})$ equal to

$$1 - Y^{2T}_{NC} = (1 - w_0)\frac{\sum_{m=T}^{\infty} \delta_f(2T+1, 2m+1)}{1 + \sum_{m=2T+1}^{\infty} \delta_f(2T+1, m)} =$$

$$= (1 - w_0)\left[\Delta_f(2T+1) - \frac{\sum_{m=T}^{\infty} \delta_f(2T+1, 2m+2)}{1 + \sum_{m=2T+1}^{\infty} \delta_f(2T+1, m)}\right]$$

Note that

$$1 - \overline{W}^{2T}_{SAR} = (1 - w_0)\left(1 - \overline{W}^{2T}_{AS}\right) =$$

$$(1 - w_0)\left(\Delta_f(2T+1) - \sum_{m=T}^{\infty}(1 - \Delta_f(2m+3))\prod_{j=T}^{m}\Delta_f(2j+1)\Delta_u(2j+2)\right)$$

Hence, $1 - Y^{2T}_{NC} > 1 - \overline{W}^{2T}_{SAR}$, as it is equivalent to (3.4.3), which shows that the firm is better off by switching to the no-concession strategy. □

The intuition behind this result is the following. Since the firm is more impatient than the union and its disagreement payoff in even periods is very low, the firm is willing to disagree forever, i.e., to make unacceptable offers and alternate between strikes and paying the old contract w_0, rather than paying the contract $\overline{W}^{0}_{SAR}$. For this case, the SPE is modified as presented in the following theorem.

Theorem 3.3. *Consider the generalized wage bargaining model with preferences of the union and the firm described by the sequences of discount factors $(\delta_{i,t})_{t\in N}$, where $\delta_{i,0} = 1$, $0 < \delta_{i,t} < 1$ for $t \geq 1$, $i = u, f$, where*

$$\Delta_u(2t+2) > \Delta_f(2t+2) \text{ for each } t \in \mathbb{N} \tag{3.4.4}$$

and for each $t \in \mathbb{N}$

$$w_0 \leq \Delta_u(2t+1)\left((1 - \Delta_u(2t+2))w_0 + \Delta_u(2t+2)\widetilde{W}^{2t+2}\right) \tag{3.4.5}$$

where

$$\widetilde{W}^{2t} = \frac{1 + \sum_{m=t}^{\infty} \delta_f(2t+1, 2m+2) + w_0 \sum_{m=t}^{\infty} \delta_f(2t+1, 2m+1)}{1 + \sum_{m=2t+1}^{\infty} \delta_f(2t+1, m)} \tag{3.4.6}$$

Then there exists a SPE in which an agreement is reached only in even periods. This SPE is supported by the following "modified generalized alternating strike strategies":

1. *Union:*
   - *In every period* $2t$ *propose* $\widetilde{W}^{2t}$ *given by (3.4.6);*
   - *In every period* $2t+1$ *accept an offer* y *if and only if* $y \geq (1-\Delta_u(2t+2))w_0 + \Delta_u(2t+2)\widetilde{W}^{2t+2}$*;*
   - *Strike in even periods and hold out in odd periods if no agreement is reached;*
   - *If the union deviates, then play the minimum-wage strategy.*
2. *Firm:*
   - *In every period* $2t+1$ *propose* $\widetilde{Z}^{2t+1} = 0$*;*
   - *In every period* $2t$ *accept an offer* x *if and only if* $x \leq \widetilde{W}^{2t}$*;*
   - *If the union deviates, then play the minimum-wage strategy.*

Proof. Note that for $\widetilde{W}^{2t}$ given by (3.4.6), if $w_0 < 1$, then we have $\widetilde{W}^{2t} > w_0$ for every $t \in \mathbb{N}$. If in period $2t$, when no agreement is reached, the union deviates from its strike decision, then it is not better off by virtue of condition (3.4.5). Moreover, as $\widetilde{W}^{2t} > w_0$, the union would be worse off by deviating from the hold out decision in period $2t+1$.

In any (proper) subgame, where the union has already deviated before, no party would be better off by deviating on its own from the required minimum-wage strategy.

Suppose that there was no deviation by the union before. In any even period $2t$, the union prefers to offer $\widetilde{W}^{2t}$: by proposing less than $\widetilde{W}^{2t}$ it would be worse off, and by proposing more than $\widetilde{W}^{2t}$, it would get at most $w_0 \sum_{k=2t+1}^{\infty} \delta_u(2t+1,k)$ which is less than $\widetilde{W}^{2t}\left(1+\sum_{k=2t+1}^{\infty} \delta_u(2t+1,k)\right)$. Consider any odd period $2t+1$. The firm's no-concession payoff from that period onward will be

$$\frac{(1-w_0)\left(1+\sum_{m=t}^{\infty} \delta_f(2t+2,2m+3)\right)}{1+\sum_{m=2t+2}^{\infty} \delta_f(2t+2,m)}$$

given the strategy of the union. Hence, the firm will not offer more to the union than

$$1-\frac{(1-w_0)\left(1+\sum_{m=t}^{\infty} \delta_f(2t+2,2m+3)\right)}{1+\sum_{m=2t+2}^{\infty} \delta_f(2t+2,m)} =$$

$$\frac{w_0+\sum_{m=t}^{\infty} \delta_f(2t+2,2m+2)+w_0\sum_{m=t}^{\infty} \delta_f(2t+2,2m+3)}{1+\sum_{m=2t+2}^{\infty} \delta_f(2t+2,m)}$$

$$Z^{2t+1} \leq \frac{w_0+\sum_{m=t}^{\infty} \delta_f(2t+2,2m+2)+w_0\sum_{m=t}^{\infty} \delta_f(2t+2,2m+3)}{1+\sum_{m=2t+2}^{\infty} \delta_f(2t+2,m)}$$

In period $2t+1$, the union will reject any offer and hold out, because

$$\frac{w_0 + \sum_{m=t}^{\infty} \delta_f(2t+2, 2m+2) + w_0 \sum_{m=t}^{\infty} \delta_f(2t+2, 2m+3)}{1 + \sum_{m=2t+2}^{\infty} \delta_f(2t+2, m)} =$$

$$= w_0(1 - \Delta_f(2t+2)) + \frac{\widetilde{W}^{2t+2}\left(\delta_{f,2t+2} + \delta_{f,2t+2} \sum_{m=2t+3}^{\infty} \delta_f(2t+3, m)\right)}{1 + \sum_{m=2t+2}^{\infty} \delta_f(2t+2, m)} <$$

$$< w_0(1 - \Delta_f(2t+2)) + \frac{\widetilde{W}^{2t+2} \sum_{m=2t+2}^{\infty} \delta_f(2t+2, m)}{1 + \sum_{m=2t+2}^{\infty} \delta_f(2t+2, m)} =$$

$$= w_0(1 - \Delta_f(2t+2)) + \widetilde{W}^{2t+2}\Delta_f(2t+2) = \Delta_f(2t+2)(\widetilde{W}^{2t+2} - w_0) + w_0 <$$

$$< \Delta_u(2t+2)(\widetilde{W}^{2t+2} - w_0) + w_0 = w_0(1 - \Delta_u(2t+2)) + \widetilde{W}^{2t+2}\Delta_u(2t+2)$$

The last inequality comes from (3.4.4) and from the fact that $\widetilde{W}^{2t+2} > w_0$.

□

Theorem 3.3 generalizes Proposition 1(ii) of Bolt [1995]. Under this SPE, the union offers $\widetilde{W}^{2t}$ in every period $2t$, and accepts an offer in period $2t+1$ only if it gives to the union at least as much as what the union would get by rejecting, holding out and getting its offer $\widetilde{W}^{2t+2}$ in $2t+2$. Note that the union's offer $\widetilde{W}^{2t}$ in period $2t$ is equal to its (normalized) payoff Y_{NC}^{2t} which it would get when the firm uses the no-concession strategy from period $2t$, i.e.,

$$\widetilde{W}^{2t} = Y_{NC}^{2t} = 1 - (1 - w_0)\frac{\sum_{m=t}^{\infty} \delta_f(2t+1, 2m+1)}{1 + \sum_{m=2t+1}^{\infty} \delta_f(2t+1, m)} =$$

$$= \frac{1 + \sum_{m=t}^{\infty} \delta_f(2t+1, 2m+2) + w_0 \sum_{m=t}^{\infty} \delta_f(2t+1, 2m+1)}{1 + \sum_{m=2t+1}^{\infty} \delta_f(2t+1, m)}$$

Moreover, under this SPE, the firm always makes unacceptable offers, but accepts an offer in period $2t$ if it gives to him at least its no-concession payoff $1 - Y_{NC}^{2t}$. Both parties switch to the minimum-wage strategies if the union deviates.

3.5 On equilibrium payoffs in wage bargaining with discount rates varying in time

3.5.1 Necessary conditions in the generalized wage bargaining

Houba and Wen [2008] apply the method of Shaked and Sutton [1984] to the F-G model to derive the supremum of the union's SPE payoffs in any even period and the infimum of the firm's SPE payoffs in any odd period. We generalize their method to the wage bargaining with sequences of discount rates varying in time. Let for $t \in \mathbb{N}$

M_u^{2t} = supremum of the union's SPE payoffs in any even period $2t$ where the union makes an offer

m_f^{2t+1} = infimum of the firm's SPE payoffs in any odd period $2t+1$ where the firm makes an offer

M_u^{2t} and m_f^{2t+1} depend on the sequences $(\delta_{u,t})_{t\in\mathbb{N}}$, $(\delta_{f,t})_{t\in\mathbb{N}}$, and $0 \leq w_0 \leq 1$. Since w_0 is the union's worst SPE payoff, we have for each $t \in \mathbb{N}$

$$w_0 \leq M_u^{2t} \leq 1 \quad \text{and} \quad w_0 \leq 1 - m_f^{2t+1} \leq 1$$

In this subsection, we determine necessary conditions for M_u^{2t} and m_f^{2t+1}, where $t \in \mathbb{N}$.

First, let us show that for all $(\delta_{u,t})_{t\in\mathbb{N}}$, $(\delta_{f,t})_{t\in\mathbb{N}}$, $0 \leq w_0 \leq 1$, and $t \in \mathbb{N}$

$$M_u^{2t} \leq \max \begin{cases} w_0(1-\Delta_f(2t+1)) + (1-m_f^{2t+1})\Delta_f(2t+1) & (3.5.1a) \\ w_0(1-\Delta_u(2t+1)) + (1-m_f^{2t+1})\Delta_u(2t+1) & (3.5.1b) \\ 1-m_f^{2t+1}\Delta_f(2t+1) \ \text{ subject to } \ (1-m_f^{2t+1})\Delta_u(2t+1) \geq w_0 & (3.5.1c) \end{cases} \tag{3.5.1}$$

To see that, consider an arbitrary even period $2t$, $t \in \mathbb{N}$. First of all, note that

$$1-(1-w_0)(1-\Delta_f(2t+1))-m_f^{2t+1}\Delta_f(2t+1) = w_0(1-\Delta_f(2t+1))+(1-m_f^{2t+1})\Delta_f(2t+1)$$

and

$$1-(1-w_0)(1-\Delta_u(2t+1))-m_f^{2t+1}\Delta_u(2t+1) = w_0(1-\Delta_u(2t+1))+(1-m_f^{2t+1})\Delta_u(2t+1)$$

(1) If the union holds out after its offer is rejected, the firm will get at least

$$(1-w_0)(1-\Delta_f(2t+1)) + m_f^{2t+1}\Delta_f(2t+1)$$

by rejecting the union's offer. Hence, the union's SPE payoffs must be smaller than or equal to

$$1-(1-w_0)(1-\Delta_f(2t+1)) - m_f^{2t+1}\Delta_f(2t+1) \tag{3.5.2}$$

from making the least acceptable offer, or

$$w_0(1-\Delta_u(2t+1)) + (1-m_f^{2t+1})\Delta_u(2t+1) \tag{3.5.3}$$

from making an unacceptable offer.
(2) The union may threaten to strike if the firm rejects its offer, which is credible if and only if $(1-m_f^{2t+1})\sum_{k=2t+1}^{\infty}\delta_u(2t+1,k) \geq w_0 + w_0\sum_{k=2t+1}^{\infty}\delta_u(2t+1,k)$, i.e., if and only if

$$(1-m_f^{2t+1})\Delta_u(2t+1) \geq w_0$$

In this case, the union's SPE payoffs must be smaller than or equal to

$$1-m_f^{2t+1}\Delta_f(2t+1) \tag{3.5.4}$$

from making the least acceptable offer, or

$$(1-m_f^{2t+1})\Delta_u(2t+1) \tag{3.5.5}$$

from making an unacceptable offer. Since $m_f^{2t+1} \leq 1$, note that we have always

$$1-\Delta_u(2t+1) \geq m_f^{2t+1}\left(\Delta_f(2t+1)-\Delta_u(2t+1)\right)$$

which is equivalent to

$$1-m_f^{2t+1}\Delta_f(2t+1) \geq (1-m_f^{2t+1})\Delta_u(2t+1)$$

This means that if the union threatens to strike in an even period, it will not make an unacceptable offer in that period. Hence, the union's SPE payoffs cannot be greater than the maximum of the three cases (3.5.2), (3.5.3) and (3.5.4), and therefore we obtain (3.5.1).

From (3.5.1) we get the following result:

Proposition 3.6. *We have for all $(\delta_{u,t})_{t\in\mathbb{N}}$, $(\delta_{f,t})_{t\in\mathbb{N}}$, $0 \leq w_0 \leq 1$, and $t \in \mathbb{N}$*

$$M_u^{2t} \leq \begin{cases} 1-m_f^{2t+1}\Delta_f(2t+1) & \textit{if } (3.5.7) \\ w_0(1-\Delta_f(2t+1)) + (1-m_f^{2t+1})\Delta_f(2t+1) & \textit{if } (3.5.8) \\ w_0(1-\Delta_u(2t+1)) + (1-m_f^{2t+1})\Delta_u(2t+1) & \textit{if } (3.5.9) \end{cases} \tag{3.5.6}$$

where

$$(1-m_f^{2t+1})\Delta_u(2t+1) \geq w_0 \tag{3.5.7}$$

$$(1-m_f^{2t+1})\Delta_u(2t+1) < w_0 \quad \textit{and} \quad \Delta_f(2t+1) \geq \Delta_u(2t+1) \tag{3.5.8}$$

$$(1-m_f^{2t+1})\Delta_u(2t+1) < w_0 \quad \textit{and} \quad \Delta_f(2t+1) < \Delta_u(2t+1) \tag{3.5.9}$$

Proof. Consider an arbitrary $t \in \mathbb{N}$.
(1) Suppose that strike is not credible, i.e., $(1 - m_f^{2t+1})\Delta_u(2t+1) < w_0$.
We have (3.5.1a) $\geq$ (3.5.1b) if and only if

$$w_0(1-\Delta_f(2t+1))+(1-m_f^{2t+1})\Delta_f(2t+1) \geq w_0(1-\Delta_u(2t+1))+(1-m_f^{2t+1})\Delta_u(2t+1) \quad \Leftrightarrow$$

$$(1 - m_f^{2t+1} - w_0)\Delta_f(2t+1) \geq (1 - m_f - w_0)\Delta_u(2t+1)$$

which establishes the second and the third cases of (3.5.6).
(2) Suppose that strike is credible, i.e., $(1 - m_f^{2t+1})\Delta_u(2t+1) \geq w_0$. Then, (3.5.1c) $\geq$ (3.5.1a). Moreover, (3.5.1c) $\geq$ (3.5.1b), because $1 - w_0 \geq m_f$ and (3.5.1c) $\geq$ (3.5.1b) if and only if

$$1 - m_f^{2t+1}\Delta_f(2t+1) \geq w_0(1 - \Delta_u(2t+1)) + (1 - m_f^{2t+1})\Delta_u(2t+1) \quad \Leftrightarrow$$

$$(1 - w_0)(1 - \Delta_u(2t+1)) \geq m_f^{2t+1}\left(\Delta_f(2t+1) - \Delta_u(2t+1)\right)$$

which is always true. Then, we obtain the first case of (3.5.6).

□

Similarly, we can show that for all $(\delta_{u,t})_{t\in\mathbb{N}}$, $(\delta_{f,t})_{t\in\mathbb{N}}$, $0 \leq w_0 \leq 1$, and $t \in \mathbb{N}$

$$m_f^{2t+1} \geq \min\begin{cases} \max\begin{cases} 1 - w_0(1 - \Delta_u(2t+2)) - M_u^{2t+2}\Delta_u(2t+2) & (3.5.10a) \\ 1 - w_0(1 - \Delta_f(2t+2)) - M_u^{2t+2}\Delta_f(2t+2) & (3.5.10b) \end{cases} \\ 1 - M_u^{2t+2}\Delta_u(2t+2) \text{ subject to } M_u^{2t+2}\Delta_u(2t+2) \geq w_0 & (3.5.10c) \end{cases} \tag{3.5.10}$$

To see that, consider an arbitrary odd period $2t+1$, $t \in \mathbb{N}$.
(1) If the union holds out after rejecting the firm's offer, the union will get at most

$$w_0(1 - \Delta_u(2t+2)) + M_u^{2t+2}\Delta_u(2t+2)$$

and hence the union will accept any higher offer. Hence, the firm could get at least

$$1 - w_0(1 - \Delta_u(2t+2)) - M_u^{2t+2}\Delta_u(2t+2) \tag{3.5.11}$$

from making the least irresistible offer. The firm could receive at least

$$(1-w_0)(1-\Delta_f(2t+2))+(1-M_u^{2t+2})\Delta_f(2t+2) = 1-w_0(1-\Delta_f(2t+2))-M_u^{2t+2}\Delta_f(2t+2) \tag{3.5.12}$$

from making any unacceptable offer. The firm will make either the least irresistible offer or an unacceptable offer, depending on whether (3.5.11) or (3.5.12) is greater.
(2) If the union strikes after rejecting the firm's offer, the union gets at most

$$M_u^{2t+2}\Delta_u(2t+2)$$

Hence, the firm will get at least

$$1-M_u^{2t+2}\Delta_u(2t+2)$$

from making the least irresistible offer, or

$$(1-M_u^{2t+2})\Delta_f(2t+2)$$

from making an unacceptable offer. Since $M_u^{2t+2}\leq 1$, note that

$$M_u^{2t+2}\left(\Delta_u(2t+2)-\Delta_f(2t+2)\right)\leq 1-\Delta_f(2t+2)$$

and therefore

$$1-M_u^{2t+2}\Delta_u(2t+2)\geq(1-M_u^{2t+2})\Delta_f(2t+2)$$

This implies that the firm will never make an unacceptable offer if the union threatens to strike after rejecting the firm's offer. Strike in period $2t+1$ is credible if and only if $M_u^{2t+2}\sum_{k=2t+2}^{\infty}\delta_u(2t+2,k)\geq w_0+w_0\sum_{k=2t+2}^{\infty}\delta_u(2t+2,k)$, i.e., if and only if

$$M_u^{2t+2}\Delta_u(2t+2)\geq w_0$$

Hence, we obtain (3.5.10).

From (3.5.10) we get the following:

Proposition 3.7. *We have for all $(\delta_{u,t})_{t\in\mathbb{N}}$, $(\delta_{f,t})_{t\in\mathbb{N}}$, $0\leq w_0\leq 1$, and $t\in\mathbb{N}$*

$$m_f^{2t+1}\geq\begin{cases}1-M_u^{2t+2}\Delta_u(2t+2) & \textit{if } (3.5.14)\\ 1-w_0(1-\Delta_f(2t+2))-M_u^{2t+2}\Delta_f(2t+2) & \textit{if } (3.5.15)\\ 1-w_0(1-\Delta_u(2t+2))-M_u^{2t+2}\Delta_u(2t+2) & \textit{if } (3.5.16)\end{cases}\tag{3.5.13}$$

where

$$M_u^{2t+2}\left(\Delta_u(2t+2)-\Delta_f(2t+2)\right)\geq w_0(1-\Delta_f(2t+2))\tag{3.5.14}$$

$$M_u^{2t+2}\left(\Delta_u(2t+2)-\Delta_f(2t+2)\right)<w_0(1-\Delta_f(2t+2))\quad \textit{and}\quad \Delta_u(2t+2)>\Delta_f(2t+2)\tag{3.5.15}$$

$$\Delta_u(2t+2)\leq\Delta_f(2t+2)\tag{3.5.16}$$

Proof. Consider an arbitrary $t \in \mathbb{N}$.
(1) Assume that $\Delta_u(2t+2) \leq \Delta_f(2t+2)$. We have

$$1 - w_0(1-\Delta_f(2t+2)) - M_u^{2t+2}\Delta_f(2t+2) = 1 - M_u^{2t+2} + (M_u^{2t+2} - w_0)(1-\Delta_f(2t+2)) \leq$$

$$1 - M_u^{2t+2} + (M_u^{2t+2} - w_0)(1-\Delta_u(2t+2)) = 1 - w_0(1-\Delta_u(2t+2)) - M_u^{2t+2}\Delta_u(2t+2)$$

Hence, (3.5.10a) $\geq$ (3.5.10b). Moreover, (3.5.10c) $>$ (3.5.10a), and we get the third case of (3.5.13).
(2) Assume that $\Delta_u(2t+2) > \Delta_f(2t+2)$. Then (3.5.10b) $>$ (3.5.10a). Moreover, (3.5.10c) $>$ (3.5.10b) if and only if

$$w_0(1-\Delta_f(2t+2)) + M_u^{2t+2}\Delta_f(2t+2) > M_u^{2t+2}\Delta_u(2t+2) \quad \Leftrightarrow$$

$$M_u^{2t+2}\left(\Delta_u(2t+2) - \Delta_f(2t+2)\right) < w_0(1-\Delta_f(2t+2))$$

which gives the second case of (3.5.13). On the other hand, if

$$M_u^{2t+2}\left(\Delta_u(2t+2) - \Delta_f(2t+2)\right) \geq w_0(1-\Delta_f(2t+2))$$

then (3.5.10c) $\leq$ (3.5.10b) and the strike is credible, because

$$M_u^{2t+2}\Delta_u(2t+2) - w_0 \geq M_u^{2t+2}\Delta_f(2t+2) + w_0(1-\Delta_f(2t+2)) - w_0 =$$

$$= \Delta_f(2t+2)(M_u^{2t+2} - w_0) \geq 0$$

We get then the first case of (3.5.13).

□

Remark 3.4. Note that our Propositions 3.6 and 3.7 generalize the corresponding results on necessary conditions for M_u and m_f for the model with constant discount rates presented in Houba and Wen [2008] (Propositions 2 and 1).

From Propositions 3.6 and 3.7, we can write the following fact that will be useful for determining M_u^{2t} and m_f^{2t+1} for some particular cases.

Fact 3.4. *Let $t \in \mathbb{N}$.*

1. *If $\Delta_u(t) \leq \Delta_f(t)$, then*

$$M_u^{2t} \leq \begin{cases} 1 - m_f^{2t+1}\Delta_f(2t+1) & \text{if } (1-m_f^{2t+1})\Delta_u(2t+1) \geq w_0 \\ w_0(1-\Delta_f(2t+1)) + (1-m_f^{2t+1})\Delta_f(2t+1) & \text{if } (1-m_f^{2t+1})\Delta_u(2t+1) < w_0 \end{cases}$$

and

$$m_f^{2t+1} \geq 1 - w_0(1-\Delta_u(2t+2)) - M_u^{2t+2}\Delta_u(2t+2)$$

2. *If* $\Delta_u(t) > \Delta_f(t)$*, then*

$$M_u^{2t} \leq \begin{cases} 1 - m_f^{2t+1}\Delta_f(2t+1) & \text{if } (1-m_f^{2t+1})\Delta_u(2t+1) \geq w_0 \\ w_0(1-\Delta_u(2t+1)) + (1-m_f^{2t+1})\Delta_u(2t+1) & \text{if } (1-m_f^{2t+1})\Delta_u(2t+1) < w_0 \end{cases}$$

and

$$m_f^{2t+1} \geq \begin{cases} 1 - M_u^{2t+2}\Delta_u(2t+2) & \text{if } (3.5.17) \\ 1 - M_u^{2t+2}\Delta_f(2t+2) - w_0(1-\Delta_f(2t+2)) & \text{if } (3.5.18) \end{cases}$$

where

$$M_u^{2t+2}\left(\Delta_u(2t+2) - \Delta_f(2t+2)\right) \geq w_0(1-\Delta_f(2t+2)) \tag{3.5.17}$$

$$M_u^{2t+2}\left(\Delta_u(2t+2) - \Delta_f(2t+2)\right) < w_0(1-\Delta_f(2t+2)) \tag{3.5.18}$$

3.5.2 Extreme equilibrium payoffs in the generalized model

From the necessary conditions presented in the previous subsection, we now determine M_u^{2t} and m_f^{2t+1} for $t \in \mathbb{N}$ for some particular cases of the discount rates varying in time. Let $\Delta_u(t)$ and $\Delta_f(t)$ for $t \in \mathbb{N}$ be the generalized discount rates of the union and the firm, respectively, as defined in (3.2.6).

In order to simplify the presentation of the results, first we introduce the notation for different sums of the generalized discount rates. We have for each $t \in \mathbb{N}$:

$$\widetilde{\Delta}(t) := 1 - \Delta_f(2t+1) + \sum_{m=t}^{\infty}(1-\Delta_f(2m+3))\prod_{j=t}^{m}\Delta_u(2j+2)\Delta_f(2j+1) \tag{3.5.19}$$

$$\overline{\Delta}(t) := 1 - \Delta_u(2t+2) + \sum_{m=t}^{\infty}(1-\Delta_u(2m+4))\prod_{j=t}^{m}\Delta_u(2j+2)\Delta_f(2j+3) \tag{3.5.20}$$

$$\widehat{\Delta}(t) := w_0 + (1-w_0)\left(1 - \Delta_f(2t+1) + \sum_{m=t}^{\infty}(1-\Delta_f(2m+3))\prod_{j=t}^{m}\Delta_f(2j+1)\Delta_f(2j+2)\right) \tag{3.5.21}$$

$$\breve{\Delta}(t) := (1-w_0)\left(1 - \Delta_f(2t+2) + \sum_{m=t}^{\infty}(1-\Delta_f(2m+4))\prod_{j=t}^{m}\Delta_f(2j+2)\Delta_f(2j+3)\right) \tag{3.5.22}$$

Remark 3.5. When we consider the model with constant discount rates, i.e., $\delta_{u,t} = \delta_u$ and $\delta_{f,t} = \delta_f$ for each $t \in \mathbb{N}$, we get for every $t \in \mathbb{N}$

$$\widetilde{\Delta}(t) = \frac{1-\delta_f}{1-\delta_u\delta_f}, \quad \overline{\Delta}(t) = \frac{1-\delta_u}{1-\delta_u\delta_f}, \quad \widehat{\Delta}(t) = \frac{1+w_0\delta_f}{1+\delta_f}, \quad \check{\Delta}(t) = \frac{1-w_0}{1+\delta_f}$$

Our first results concern the case when the generalized discount rate of the union is always not greater than the generalized discount rate of the firm in the same period. The following proposition presents the supremum of the union's SPE payoffs in any even period and the infimum of the firm's SPE payoffs in any odd period for the particular cases with $\Delta_u(t) \leq \Delta_f(t)$ for every $t \in \mathbb{N}$: when either the strike is always credible or the strike is never credible.

Proposition 3.8. *Let $\Delta_u(t) \leq \Delta_f(t)$ for every $t \in \mathbb{N}$.*

(i) If for every $t \in \mathbb{N}$

$$\left[w_0 + (1-w_0)\Delta_u(2t+2)\widetilde{\Delta}(t+1)\right]\Delta_u(2t+1) \geq w_0 \tag{3.5.23}$$

then

$$M_u^{2t} = w_0 + (1-w_0)\widetilde{\Delta}(t) \tag{3.5.24}$$

$$m_f^{2t+1} = (1-w_0)\left[1-\Delta_u(2t+2)\widetilde{\Delta}(t+1)\right] \tag{3.5.25}$$

The SPE strategy profile that supports these M_u^{2t} and m_f^{2t+1} defined in (3.5.24) and (3.5.25) is given by the following 'generalized alternating strike strategies':

- *In period $2t$ the union proposes $w_0 + (1-w_0)\widetilde{\Delta}(t)$, in period $2t+1$ it accepts an offer y if and only if $y \geq w_0 + (1-w_0)\Delta_u(2t+2)\widetilde{\Delta}(t+1)$, it goes on strike after rejection of its own proposals and holds out after rejecting firm's offers.*
- *In period $2t+1$ the firm proposes $w_0 + (1-w_0)\Delta_u(2t+2)\widetilde{\Delta}(t+1)$, in period $2t$ it accepts x if and only if $x \leq w_0 + (1-w_0)\widetilde{\Delta}(t)$.*
- *If, however, at some point, the union deviates from the above rule, then both parties play thereafter according to the following 'minimum-wage strategies':*
  - *The union always proposes w_0, accepts y if and only if $y \geq w_0$, and never goes on strike.*
  - *The firm always proposes w_0 and accepts x if and only if $x \leq w_0$.*

(ii) If for every $t \in \mathbb{N}$

$$\left[w_0 + (1 - w_0)\Delta_u(2t+2)\widetilde{\Delta}(t+1)\right] \Delta_u(2t+1) < w_0 \tag{3.5.26}$$

then

$$M_u^{2t} = w_0 \quad \textit{and} \quad m_f^{2t+1} = 1 - w_0 \tag{3.5.27}$$

The SPE strategy profile that supports these M_u^{2t} and m_f^{2t+1} defined in (3.5.27) is given by the minimum-wage strategies.

Proof. Let $\Delta_u(t) \leq \Delta_f(t)$ for every $t \in \mathbb{N}$. From Fact 3.4 we have for every $t \in \mathbb{N}$:

$$M_u^{2t} \leq \begin{cases} 1 - m_f^{2t+1}\Delta_f(2t+1) & \text{if } (1 - m_f^{2t+1})\Delta_u(2t+1) \geq w_0 \\ w_0(1 - \Delta_f(2t+1)) + (1 - m_f^{2t+1})\Delta_f(2t+1) & \text{if } (1 - m_f^{2t+1})\Delta_u(2t+1) < w_0 \end{cases}$$

and

$$m_f^{2t+1} \geq 1 - w_0(1 - \Delta_u(2t+2)) - M_u^{2t+2}\Delta_u(2t+2)$$

(i) Consider the case when the strike is always credible, i.e., $(1-m_f^{2t+1})\Delta_u(2t+1) \geq w_0$ for every $t \in \mathbb{N}$. We solve for every $t \in \mathbb{N}$

$$M_u^{2t} + m_f^{2t+1}\Delta_f(2t+1) = 1 \quad \text{and} \quad m_f^{2t+1} + M_u^{2t+2}\Delta_u(2t+2) = 1 - w_0(1 - \Delta_u(2t+2))$$

which is a regular triangular system $AX = Y$, with $A = [a_{ij}]_{i,j\in\mathbb{N}^+}$, $X = [(x_i)_{i\in\mathbb{N}^+}]^T$, $Y = [(y_i)_{i\in\mathbb{N}^+}]^T$, where for each $t, j \geq 1$

$$a_{t,t} = 1,\ a_{t,j} = 0 \text{ for } j < t \text{ or } j > t+1$$

and for each $t \in \mathbb{N}$

$$a_{2t+1,2t+2} = \Delta_f(2t+1), \quad a_{2t+2,2t+3} = \Delta_u(2t+2)$$

$$x_{2t+1} = M_u^{2t}, \quad x_{2t+2} = m_f^{2t+1}, \quad y_{2t+1} = 1, \quad y_{2t+2} = 1 - w_0(1 - \Delta_u(2t+2))$$

Any regular triangular matrix A possesses the (unique) inverse matrix B, i.e., there exists B such that $BA = I$, where I is the infinite identity matrix. The matrix $B = [b_{ij}]_{i,j\in\mathbb{N}^+}$ is also regular triangular, and its elements are the following:

$$b_{t,t} = 1,\ b_{t,j} = 0 \text{ for each } t, j \geq 1 \text{ such that } j < t$$

$$b_{2t+1,2t+2} = -\Delta_f(2t+1),\ b_{2t+2,2t+3} = -\Delta_u(2t+2) \text{ for each } t \in \mathbb{N}$$

and for each $t, m \in \mathbb{N}$ and $m > t$

$$b_{2t+1,2m+1} = \prod_{j=t}^{m-1} \Delta_f(2j+1)\Delta_u(2j+2), \quad b_{2t+1,2m+2} = -\prod_{j=t}^{m-1} \Delta_f(2j+1)\Delta_u(2j+2)\Delta_f(2m+1)$$

$$b_{2t+2,2m+2} = \prod_{j=t}^{m-1} \Delta_u(2j+2)\Delta_f(2j+3), \quad b_{2t+2,2m+3} = -\prod_{j=t}^{m-1} \Delta_u(2j+2)\Delta_f(2j+3)\Delta_u(2m+2)$$

Next, by applying $X = BY$ we get M_u^{2t} as given in (3.5.24) and m_f^{2t+1} as given in (3.5.25). The strike credibility condition $(1 - m_f^{2t+1})\Delta_u(2t+1) \geq w_0$ for every $t \in \mathbb{N}$ is then written as in (3.5.23). In Section 3.4 (Proposition 3.4) we show that under an equivalently expressed condition (3.5.23) and $\Delta_u(2t+2) \leq \Delta_f(2t+2)$ for every $t \in \mathbb{N}$, the proposed strategy profile (formed by the generalized alternating strike strategies) is a SPE.
(ii) Consider the case when the strike is never credible, i.e., $(1 - m_f^{2t+1})\Delta_u(2t+1) < w_0$ for every $t \in \mathbb{N}$. Then we have the infinite system for $t \in \mathbb{N}$

$$M_u^{2t} + m_f^{2t+1}\Delta_f(2t+1) = w_0(1 - \Delta_f(2t+1)) + \Delta_f(2t+1)$$

and

$$m_f^{2t+1} + M_u^{2t+2}\Delta_u(2t+2) = 1 - w_0(1 - \Delta_u(2t+2))$$

which as a regular triangular system possesses a unique solution. This solution is given by (3.5.27). It is supported by the minimum-wage strategies profile which is a SPE as shown in Section 3.4 (Fact 3.3).

□

Remark 3.6. Note that our Proposition 3.8 generalizes the corresponding results on M_u and m_f for the model with constant discount rates presented in Houba and Wen [2008] (Proposition 3). When we consider the model with constant discount rates, i.e., we put $\delta_{u,t} = \delta_u$ and $\delta_{f,t} = \delta_f$ for each $t \in \mathbb{N}$, and we assume that $\delta_u \leq \delta_f$, we get for every $t \in \mathbb{N}$

$$M_u^{2t} = w_0 + \frac{(1 - w_0)(1 - \delta_f)}{1 - \delta_u\delta_f}, \quad m_f^{2t+1} = \frac{(1 - w_0)(1 - \delta_u)}{1 - \delta_u\delta_f}$$

and the strike credibility condition (3.5.23) is equivalent to

$$(1 - w_0)\delta_u^2 + w_0\delta_u - w_0 \geq \delta_u\delta_f(\delta_u - w_0)$$

Our next results concern some particular cases when the generalized discount rate of the union is always greater than the generalized discount rate of the firm in the same period. Three particular cases are considered.

Proposition 3.9. *Let $\Delta_u(t) > \Delta_f(t)$ for every $t \in \mathbb{N}$.*

(i) If for every $t \in \mathbb{N}$

$$\left(1 - \overline{\Delta}(t)\right) \Delta_u(2t+1) \geq w_0 \tag{3.5.28}$$

and

$$\widetilde{\Delta}(t+1)\left(\Delta_u(2t+2) - \Delta_f(2t+2)\right) \geq w_0(1 - \Delta_f(2t+2)) \tag{3.5.29}$$

then

$$M_u^{2t} = \widetilde{\Delta}(t) \quad and \quad m_f^{2t+1} = \overline{\Delta}(t) \tag{3.5.30}$$

The SPE strategy profile that supports these M_u^{2t} and m_f^{2t+1} defined in (3.5.30) is given by the following 'always strike strategies':

- *In period $2t$ the union proposes $\widetilde{\Delta}(t)$, in period $2t+1$ it accepts an offer y if and only if $y \geq 1 - \overline{\Delta}(t)$, it always goes on strike if there is a disagreement.*
- *In period $2t+1$ the firm proposes $1 - \overline{\Delta}(t)$, in period $2t$ it accepts x if and only if $x \leq \widetilde{\Delta}(t)$.*
- *If, however, at some point, the union deviates from the above rule, then both parties play thereafter according to the 'minimum-wage strategies'.*

(ii) If for every $t \in \mathbb{N}$

$$\left(1 - \check{\Delta}(t)\right) \Delta_u(2t+1) \geq w_0 \tag{3.5.31}$$

and

$$\widehat{\Delta}(t+1)\left(\Delta_u(2t+2) - \Delta_f(2t+2)\right) < w_0(1 - \Delta_f(2t+2)) \tag{3.5.32}$$

then

$$M_u^{2t} = \widehat{\Delta}(t) \quad and \quad m_f^{2t+1} = \check{\Delta}(t) \tag{3.5.33}$$

The SPE strategy profile that supports these M_u^{2t} and m_f^{2t+1} defined in (3.5.33) is given by the following 'modified generalized alternating strike strategies':

- *In period $2t$ the union proposes $\widehat{\Delta}(t)$, in period $2t+1$ it accepts an offer y if and only if $y \geq (1 - \Delta_u(2t+2))w_0 + \Delta_u(2t+2)\widehat{\Delta}(t+1)$, it strikes in even periods and holds out in odd periods if no agreement is reached.*
- *In period $2t+1$ the firm proposes 0, in period $2t$ it accepts x if and only if $x \leq \widehat{\Delta}(t)$.*

- *If, however, at some point, the union deviates from the above rule, then both parties play thereafter according to the 'minimum-wage strategies'.*

(iii) If for every $t \in \mathbb{N}$

$$M_u^{2t+2}(\Delta_u(2t+2) - \Delta_f(2t+2)) < w_0(1 - \Delta_f(2t+2))$$

and

$$(1 - m_f^{2t+1})\Delta_u(2t+1) < w_0$$

then for each $t \in \mathbb{N}$

$$M_u^{2t} = w_0 \quad \textit{and} \quad m_f^{2t+1} = 1 - w_0 \tag{3.5.34}$$

The SPE strategy profile that supports these M_u^{2t} *and* m_f^{2t+1} *defined in (3.5.34) is given by the minimum-wage strategies.*

Proof. Let $\Delta_u(t) > \Delta_f(t)$ for every $t \in \mathbb{N}$. From Fact 3.4 we have for every $t \in \mathbb{N}$:

$$M_u^{2t} \leq \begin{cases} 1 - m_f^{2t+1}\Delta_f(2t+1) & \text{if } (1 - m_f^{2t+1})\Delta_u(2t+1) \geq w_0 \\ w_0(1 - \Delta_u(2t+1)) + (1 - m_f^{2t+1})\Delta_u(2t+1) & \text{if } (1 - m_f^{2t+1})\Delta_u(2t+1) < w_0 \end{cases}$$

and

$$m_f^{2t+1} \geq \begin{cases} 1 - M_u^{2t+2}\Delta_u(2t+2) & \text{if } (3.5.17) \\ 1 - M_u^{2t+2}\Delta_f(2t+2) - w_0(1 - \Delta_f(2t+2)) & \text{if } (3.5.18) \end{cases}$$

(i) Consider the case when for every $t \in \mathbb{N}$, $(1 - m_f^{2t+1})\Delta_u(2t+1) \geq w_0$ (i.e., strike is credible in period $2t$) and condition (3.5.17) holds. If (3.5.17) is satisfied, then strike is credible in period $2t + 1$. Then, we solve the infinite system for every $t \in \mathbb{N}$

$$M_u^{2t} + m_f^{2t+1}\Delta_f(2t+1) = 1 \quad \text{and} \quad m_f^{2t+1} + M_u^{2t+2}\Delta_u(2t+2) = 1$$

which is is a regular triangular system $AX = Y$, with $A = [a_{ij}]_{i,j \in \mathbb{N}^+}$ and $X = [(x_i)_{i \in \mathbb{N}^+}]^T$ the same as in the proof of Proposition 3.8, and with $Y = [(y_i)_{i \in \mathbb{N}^+}]^T$ such that $y_{2t+1} = y_{2t+2} = 1$. The (unique) inverse matrix B is the same as before, and by applying $X = BY$ we get M_u^{2t} and m_f^{2t+1} as given by (3.5.30). The conditions $(1 - m_f^{2t+1})\Delta_u(2t+1) \geq w_0$ and (3.5.17) are equivalent to (3.5.28) and (3.5.29). In Section 3.4 (Proposition 3.3) we show that the proposed strategy profile (formed by the "always strike strategies") is a SPE under an equivalently expressed condition (3.5.28).

(ii) Consider the case when for every $t \in \mathbb{N}$, $(1 - m_f^{2t+1})\Delta_u(2t+1) \geq w_0$ (i.e., strike is credible in period $2t$) and condition (3.5.18) holds. Then, we solve the infinite system for every $t \in \mathbb{N}$

$$M_u^{2t} + m_f^{2t+1}\Delta_f(2t+1) = 1 \quad \text{and} \quad m_f^{2t+1} + M_u^{2t+2}\Delta_f(2t+2) = 1 - w_0(1 - \Delta_f(2t+2))$$

which is is a regular triangular system $AX = Y$. By applying $X = BY$ we get M_u^{2t} and m_f^{2t+1} as given by (3.5.33). The conditions $(1 - m_f^{2t+1})\Delta_u(2t+1) \geq w_0$ and (3.5.18) are equivalent to (3.5.31) and (3.5.32). In Section 3.4 (Theorem 3.3) we show that if $\Delta_u(2t+2) > \Delta_f(2t+2)$ for each $t \in \mathbb{N}$, then the proposed strategy profile (formed by the "modified generalized alternating strike strategies") is a SPE under the following condition:

$$w_0 \leq \Delta_u(2t+1)\left((1 - \Delta_u(2t+2))w_0 + \Delta_u(2t+2)W^{2t+2}\right) \tag{3.5.35}$$

where

$$W^{2t} = \frac{1 + \sum_{m=t}^{\infty} \delta_f(2t+1, 2m+2) + w_0 \sum_{m=t}^{\infty} \delta_f(2t+1, 2m+1)}{1 + \sum_{m=2t+1}^{\infty} \delta_f(2t+1, m)}$$

One can show that $W^{2t} = M_u^{2t} = \widehat{\Delta}(t)$:

$$W^{2t} = w_0 + (1 - w_0)\left(\frac{1 + \sum_{m=t}^{\infty} \delta_f(2t+1, 2m+2)}{1 + \sum_{m=2t+1}^{\infty} \delta_f(2t+1, m)}\right) =$$

$$= w_0 + (1 - w_0)\left(1 - \Delta_f(2t+1) + \frac{\sum_{m=t}^{\infty} \delta_f(2t+1, 2m+2)}{1 + \sum_{m=2t+1}^{\infty} \delta_f(2t+1, m)}\right) =$$

$$= w_0 + (1 - w_0)\left(1 - \Delta_f(2t+1) + \sum_{m=t}^{\infty}(1 - \Delta_f(2m+3))\prod_{j=t}^{m}\Delta_f(2j+1)\Delta_f(2j+2)\right) = \widehat{\Delta}(t)$$

Moreover, note that (3.5.31) implies condition (3.5.35):

$$w_0 \leq \Delta_u(2t+1)\left(1 - \breve{\Delta}(t)\right) =$$

$$= \Delta_u(2t+1)\left[w_0 + (1 - w_0)\left(\Delta_f(2t+2) - \sum_{m=t}^{\infty}(1 - \Delta_f(2m+4))\prod_{j=t}^{m}\Delta_f(2j+2)\Delta_f(2j+3)\right)\right]$$

$$= \Delta_u(2t+1)\left[w_0 + (1 - w_0)\Delta_f(2t+2)\left(1 - \Delta_f(2t+3) + \sum_{m=t+1}^{\infty}(1 - \Delta_f(2m+3))\prod_{j=t+1}^{m}\Delta_f(2j+1)\Delta_f(2j+2)\right)\right]$$

$$= \Delta_u(2t+1)\left(w_0 + \Delta_f(2t+2)(\widehat{\Delta}(t+1) - w_0)\right) <$$

$$< \Delta_u(2t+1)\left(w_0 + \Delta_u(2t+2)(\widehat{\Delta}(t+1) - w_0)\right) =$$

$$= \Delta_u(2t+1)\left((1-\Delta_u(2t+2))w_0 + \Delta_u(2t+2)\widehat{\Delta}(t+1)\right)$$

(iii) Consider the case when for every $t \in \mathbb{N}$,

$(1 - m_f^{2t+1})\Delta_u(2t+1) < w_0$ and $M_u^{2t+2}(\Delta_u(2t+2) - \Delta_f(2t+2)) < w_0(1 - \Delta_f(2t+2))$

Then, we solve the infinite system for every $t \in \mathbb{N}$

$$M_u^{2t} + m_f^{2t+1}\Delta_u(2t+1) = w_0 + \Delta_u(2t+1)(1-w_0)$$

and

$$m_f^{2t+1} + M_u^{2t+2}\Delta_f(2t+2) = 1 - w_0(1 - \Delta_f(2t+2))$$

which is is a regular triangular system $AX = Y$ with the solution $M_u^{2t} = w_0$ and $m_f^{2t+1} = 1 - w_0$ for each $t \in \mathbb{N}$. The SPE supporting this solution is the minimum-wage strategies profile.

□

Remark 3.7. Note that our Proposition 3.9 generalizes the corresponding results on M_u and m_f for the model with constant discount rates presented in Houba and Wen [2008] (Proposition 4). Consider the model with constant discount rates, i.e., let $\delta_{u,t} = \delta_u$ and $\delta_{f,t} = \delta_f$ for each $t \in \mathbb{N}$, and assume that $\delta_u > \delta_f$. Then from Proposition 3.9(i), we get for every $t \in \mathbb{N}$

$$M_u^{2t} = \frac{1-\delta_f}{1-\delta_u\delta_f}, \quad m_f^{2t+1} = \frac{1-\delta_u}{1-\delta_u\delta_f}$$

and the strike credibility conditions (3.5.28) and (3.5.29) are equivalent to the set C in Houba and Wen [2008]:

$$(\delta_u - w_0)\delta_f \le \frac{\delta_u^2 - w_0}{\delta_u} \quad \text{and} \quad \delta_f \le \frac{\delta_u - w_0}{1 - w_0\delta_u}$$

respectively. From Proposition 3.9(ii), we get for every $t \in \mathbb{N}$

$$M_u^{2t} = \frac{1 + w_0\delta_f}{1+\delta_f}, \quad m_f^{2t+1} = \frac{1-w_0}{1+\delta_f}$$

and the conditions (3.5.31) and (3.5.32) are equivalent to the set B in Houba and Wen [2008]:

$$\delta_f(\delta_u - w_0) \ge w_0(1-\delta_u) \quad \text{and} \quad \delta_f > \frac{\delta_u - w_0}{1 - \delta_u w_0}$$

Remark 3.8. In Propositions 3.8 and 3.9, M_u^{2t} and m_f^{2t+1} for every $t \in \mathbb{N}$ are determined for several cases where particular conditions on the discount rates of both parties are satisfied. In order to calculate M_u^{2t} and m_f^{2t+1} for an arbitrary case, we can proceed as follows. Given the sequences of discount rates $(\delta_{u,t})_{t\in\mathbb{N}}$ and $(\delta_{f,t})_{t\in\mathbb{N}}$, the sequences of the generalized discount rates are then also given. Depending on which conditions hold, we apply Fact 3.4 to determine the infinite sequence of necessary conditions for M_u^{2t} and m_f^{2t+1} for every $t \in \mathbb{N}$. Note that we get always an infinite regular triangular system of equations which has a unique solution, being the sequence $(M_u^{2t}, m_f^{2t+1})_{t\in\mathbb{N}}$. However, the solution does not always satisfy the required conditions. To see that consider the case where for every $t \in \mathbb{N}$,

$$\Delta_u(t) > \Delta_f(t), \quad (1 - m_f^{2t+1})\Delta_u(2t+1) < w_0 \quad \text{and}$$
$$M_u^{2t+2}\left(\Delta_u(2t+2) - \Delta_f(2t+2)\right) \geq w_0(1 - \Delta_f(2t+2))$$

Then, solving for every $t \in \mathbb{N}$

$$M_u^{2t} = w_0(1 - \Delta_u(2t+1)) + (1 - m_f^{2t+1})\Delta_u(2t+1) \quad \text{and} \quad m_f^{2t+1} = 1 - M_u^{2t+2}\Delta_u(2t+2)$$

leads to

$$M_u^{2t} = w_0\left(1 - \Delta_u(2t+1) + \sum_{m=t}^{\infty}(1 - \Delta_u(2m+3))\prod_{j=t}^{m}\Delta_u(2j+1)\Delta_u(2j+2)\right)$$

but this means that $M_u^{2t} < w_0$, and therefore we get a contradiction.

3.6 Inefficient equilibria in the generalized model with strikes

In the previous sections, we considered only efficient equilibria in the generalized wage bargaining where the agreement is reached immediately in period 0. Now we will prove the result concerning inefficient subgame perfect equilibria in this model, where the union strikes for uninterrupted T periods prior to reaching a final agreement.

Theorem 3.4. *Consider the generalized wage bargaining model with preferences of the union and the firm described by the sequences of discount factors $(\delta_{i,t})_{t\in N}$, where $\delta_{i,0} = 1$, $0 < \delta_{i,t} < 1$ for $t \geq 1$, $i = u, f$. If $\hat{w} \in [0,1]$ and $T \geq 1$ are such that*

$$w_0 \leq \hat{w}\frac{\sum_{k=T}^{\infty}\delta_u(1,k)}{1 + \sum_{k=1}^{\infty}\delta_u(1,k)} \tag{3.6.1}$$

and for each $\tau \in \mathbb{N}$ such that $2\tau + 1 < T$

$$(1-\hat{w})\sum_{k=T}^{\infty}\delta_f(1,k) \geq \left(1-\overline{Z}^{2\tau+1}\right)\sum_{k=2\tau+1}^{\infty}\delta_f(1,k) \tag{3.6.2}$$

where $\overline{Z}^{2\tau+1}$ denotes the firm's offer in period $2\tau+1$ given in Theorem 3.1 (exogenous "always strike decision" case) then there is a subgame perfect equilibrium with a strike of T periods (from period 0 till $T-1$) followed by an agreement $\hat{w}$ reached in period T.

Proof. Let $\hat{w}$ and T be such that (3.6.1) and (3.6.2) are satisfied. Let $\overline{W}^{2t}$ and $\overline{Z}^{2t+1}$ denote the offers of the union and the firm, respectively, defined in Theorem 3.1 (formulas (3.3.6) and (3.3.7)). Consider the following pair of strategies:
Strategy of the union:

1. In every period $t < T$, where neither the union nor the firm has deviated before:
   - if t is even then make an unacceptable offer (that the firm rejects, e.g., 1 for the union)
   - if t is odd then accept y if and only if $y \geq \overline{Z}^{t}$
   - strike if there is a disagreement
2. In period T, where neither the union nor the firm has deviated before:
   - if T is even then propose $\hat{w}$
   - if T is odd then accept y if and only if $y \geq \hat{w}$
   - strike if there is a disagreement
3. In every period $t > T$, where neither the union nor the firm has deviated before:
   - if t is even then propose $\overline{W}^{t}$
   - if t is odd then accept y if and only if $y \geq \overline{Z}^{t}$
   - strike if there is a disagreement
4. If in period $t \leq T$ the union deviates, then play the minimum wage strategy thereafter
5. If in period $t \leq T$ the firm deviates, then play the always strike strategy thereafter

6. If in period $t > T$ any party deviates, then play the minimum wage strategy thereafter.

Strategy of the firm:

1. In every period $t < T$, where neither the union nor the firm has deviated before:
   - if t is odd then make an unacceptable offer (that the union rejects, e.g., w_0 for the union)
   - if t is even then accept x if and only if $x \leq w_0$
2. In period T, where neither the union nor the firm has deviated before:
   - if T is odd then propose $\hat{w}$
   - if T is even then accept x if and only if $x \leq \hat{w}$
3. In every period $t > T$, where neither the union nor the firm has deviated before:
   - if t is odd then propose $\overline{Z}^t$
   - if t is even then accept x if and only if $x \leq \overline{W}^t$
4. If in period $t \leq T$ the union deviates, then play the minimum wage strategy thereafter
5. If in period $t \leq T$ the firm deviates, then play the always strike strategy thereafter
6. If in period $t > T$ any party deviates, then play the minimum wage strategy thereafter.

One can show that this pair of strategies is the SPE. In every subgame such that a party has deviated before, this pair of strategies is the Nash equilibrium, since the minimum wage strategies, the always strike strategies, as well as the always strike strategies with the switch to the minimum wage strategies in case of a deviation, form the Nash equilibrium.

Also note that by virtue of (3.6.1), the union prefers to strike till period $T - 1$ instead of reaching an earlier agreement. Any deviation of the union prior to period T would not be better to the union, because if the union deviates, e.g., by trying to reach an earlier agreement that the firm would prefer than $\hat{w}$ in period T, then the parties play thereafter the minimum wage strategies that give w_0 to the union.

By virtue of (3.6.2), also the firm would not be better off by deviating and trying to reach an earlier agreement, because if the firm makes an offer before period T that the union would prefer, then the parties play the always strike strategies thereafter. □

Fernandez and Glazer [1991] prove (Theorem 2) that in the wage bargaining[2] with constant discount rates δ_u and δ_f, if $\hat{w}$ is such that

$$\left(1-\delta_f^{1-T}\right)F+\delta_f^{1-T}\overline{z}\geq\hat{w}\geq\delta_u^{-T}w_0 \tag{3.6.3}$$

where $\overline{w}=\frac{(1-\delta_f)F}{1-\delta_u\delta_f}$ and $\overline{z}=\frac{\delta_u(1-\delta_f)F}{1-\delta_u\delta_f}$ are the solutions to Rubinstein's original bargaining game [Rubinstein, 1982], then there is a subgame perfect equilibrium with a strike of T periods followed by an agreement of $\hat{w}$. Note that if we apply our Theorem 3.4 to the case of constant discount rates, $\delta_{u,t}=\delta_u$ and $\delta_{f,t}=\delta_f$ for every $t\in\mathbb{N}_+$, and assume that $F=1$, then we recover the result of Fernandez and Glazer [1991].

3.7 Concluding remarks

We calculated the equilibrium payoffs for the wage bargaining model between the union and the firm with preferences of the parties expressed by discount rates varying in time. First, we generalized the *F-G model* and determined SPE for three cases with *exogenous* strike decision: when the union is committed to go on strike in each period in which there is a disagreement, when the union is committed to go on strike only when its own offer is rejected and the case when the union is supposed to go never on strike. We presented the unique SPE for each of these three cases. Furthermore, we considered the general model where no commitment to strike is assumed and found SPE under particular assumptions on the discount rates.

We applied the method of Houba and Wen [2008] to our generalized wage bargaining model. Since we assume that the sequence of discount rates of a party can be arbitrary, with the only restriction that the infinite series that determines the utility for the given party must be convergent, first we described the conditions in a general case for the supremum of the union's SPE payoffs in any even period and for the infimum of the firm's SPE payoffs in any odd period. Then, we solved the conditions for particular cases of the sequences of discount rates. Furthermore, we analyzed the existence of

[2] In Fernandez and Glazer [1991] the wage offers are made over discrete time periods $t\in\{1,2,...\}$ with the union proposing in odd-numbered periods and the firm proposing in even-numbered periods. In our setup this is also the union that starts the bargaining but in period 0, i.e., it makes its offers in even-numbered periods.

inefficient SPE with a strike for some periods followed by agreement when the parties have varying discount factors.

In the following chapters, we investigate some extensions of the wage bargaining model, e.g., the case when the union can be on go-slow threats, and the case when the firm has the lockouts option. We also present some applications of the model to other bargaining issues such as price negotiations.

Chapter 4

Extensions of the generalized wage bargaining model[1]

4.1 Introduction

In collective wage bargaining between unions and firms, one can observe costly conflicts such as strikes or slowdown strikes. Kennan and Wilson [1989, 1993] emphasize that strikes are the signaling devices of the firm's willingness to pay to the workers. Therefore, if the firm is more profitable, workers have high wage expectations. Ingram et al. [1993] find empirical evidences both for and against this explanation of the occurrence of strikes.

By using noncooperative bargaining theories one may analyze wage expectations of unions and outcomes of union-firm negotiations in a better way (see e.g. Kennan and Wilson [1989, 1993], Osborne and Rubinstein [1990] and Binmore et al. [1990]). Especially, the private information of the firm's willingness to pay can stimulate the strikes. Other inefficiencies in the wage bargaining are shown, for instance, in Crawford [1982] who analyzes uncertain commitments and in Haller and Holden [1990] and Fernandez and Glazer [1991] who point multiple equilibria in bargaining game.

Although holdout threats of the union are frequently ignored in the literature on wage bargaining models (see e.g. Fudenberg et al. [1985], Hart [1989] and Kennan and Wilson [1989]), Cramton and Tracy [1992, 1994a] prove that, as well as the strikes, holdout threats after the expiration of the contract can also provide a significant wage increase. By investigating the labor negotiations in the US, they analyze the problem

[1] This chapter is based on Ozkardas and Rusinowska [2014c] and Ozkardas and Rusinowska [2014b].

of the firm's willingness to pay caused by the private information. They conclude that most of the conflicts during collective bargaining are ended off by holdout threats of the union such as work-to-rule or go-slow actions instead of strike. After the expiration of the actual contract, workers continue to work with the existing wage level until a new contract is signed. For instance, between 1970 and 1989 the holdout threats appeared four times more frequently then the strikes during the wage negotiations in the US labor market.

In order to analyze the effects of the union's threats on wage levels, Moene [1988] indicates four different threats: work-to-rule, go-slow, wild cat strikes and official strikes or lockouts. Work-to-rule is a non-official industrial action in which the workers severely slow down their working efforts to the minimum required level by the rules of their contract. Differently from work-to-rules, go-slow is an official threat of the union where the workers announce officially how much they reduce their work efforts. Moene [1988] argues that holdout threats of the union give a higher wage increase than strikes.

The analysis of the holdout threats of the union may help to study real world collective wage bargaining where the strikes are prohibited. For instance, Moene and Wallerstein [1997] examine the go-slow threats of the union in Scandinavian countries.

Fernandez and Glazer [1991] discuss an extension of their wage bargaining model in which the firm is allowed to lock out the union and neither strikes nor holdout threats of the union is feasible. To the best of our knowledge, the lockouts option has not been considered so far for the model with discount rates varying in time.

The aim of this chapter is, firstly, to examine the effects of the union's holdout threats, such as go-slow, on the wage determination when the parties' preferences vary in time. Secondly, we aim to investigate the generalized wage bargaining model with lockouts. In order to apply the go-slow strategies of the union, we modify the wage bargaining model of Fernandez and Glazer [1991]. First, we restrict our analysis to history independent strategies with no delay. We specify two different attitudes of the union, either *hostile* or *altruistic*, and determine the subgame perfect equilibria in the wage bargaining for each of the attitudes. More precisely, we say that the union is hostile if it is on go-slow in every period when there is no agreement. An altruistic union always holds out and continues to work with the same effort and wage during the disagreement periods. Then we generalize and apply the method used in Houba and Wen [2008] to the situation when the strikes are not allowed and the union can threaten the firm with being on go-slow. In the second part of this chapter, we consider a model in which the firm is allowed to engage in lockouts. More precisely, we examine a game in which only lockouts by the firm are feasible, i.e., the union is not allowed to strike. We prove that under certain assumptions there is a SPE with an immediate agreement which yields the union a wage contract smaller than the status quo contract. Under this equilibrium the firm always locks out the union after its own offer is rejected and holds out after rejecting an offer of the union.

The rest of the chapter is organized as follows. In Section 4.2, the generalized wage bargaining model where the union can threaten the firm with the go-slow action is described in details. We determine the subgame perfect equilibria of the wage bargaining depending on the union's attitude (hostile or altruistic). Furthermore, we derive the necessary conditions for the supremum of the union's SPE payoffs and the infimum of the firm's SPE payoffs, and calculate the extreme payoffs for some particular case of the discount rates. Section 4.3 concerns the generalized wage bargaining in which only lockouts are feasible, i.e., the union is not allowed neither to strike nor to go-slow. Our conclusions are presented in Section 4.4.

4.2 The generalized wage bargaining with the go-slow option

4.2.1 Description of the model

We consider a model of wage bargaining between a monopolistic firm and a union. As in the original model of Fernandez and Glazer [1991] and the generalized wage bargaining model investigated in Chapter 3, the union and the firm make alternating offers during the negotiations. There is an existing wage contract which has come up for renegotiation. We suppose that all workers are unionized and they have equal skills. We assume that the risk neutrality of both the firm and the union is relinquished, and hence the varying discount rates are introduced.

Inspired by the works of Rusinowska [2002a] and De Marco and Morgan [2008, 2011], we introduce in the model different attitudes of the union. Rusinowska [2002a] analyzes the bargaining model under an assumption of players' attitudes towards their opponents' payments. She determines the type of a player as jealous or friendly to examine the effects over his/her opponent's payoff while his/her own payoff is constant. De Marco and Morgan [2008, 2011] introduce and study the concepts of the (strong) friendliness equilibrium and the slightly altruistic (correlated) equilibrium.

In our wage bargaining model we assume that the union and the firm divide the added value normalized to 1. Under the existing wage contract, the firm makes a wage payment of w_0 on a daily basis where $w_0 \in [0, 1]$. By the new contract $W \in [0, 1]$, the union and the firm will get W and $1 - W$, respectively. We assume that the attitude of the union towards the firm can be either *hostile* or *altruistic*. The type of the union is a common knowledge. If the union is hostile, then it makes go-slow threats in every disagreement period. Under the go-slow decision, the payoff of the union is the existing wage w_0 and the payoff of the firm is the discounted added value according to the rate of go-slow minus wage spending, i.e., $\lambda - w_0$, where $\lambda \in [w_0, 1]$ is the given rate of go-slow. On the other hand, if the union is altruistic, then it does not make any threat to the firm in disagreement periods, i.e., the payoffs of the union and the firm are w_0 and $1 - w_0$, respectively[2]. Players bargain sequentially over discrete time and a potentially infinite horizon. They make new wage offers alternately in which the other party is free to accept or to reject. After a rejection of an offer, the union decides whether to go-slow or not according to its attitude.

More precisely, the bargaining procedure is as follows. In period 0, the union makes the first offer of W^0 where the firm is free to accept or to reject. If the firm accepts W^0, then the agreement is reached and the payoffs are $(W^0, 1 - W^0)$. Otherwise the hostile union makes the go-slow threat and the payoffs are $(w_0, \lambda - w_0)$, and the altruistic union continues with the existing contract and the payoffs are $(w_0, 1 - w_0)$. In case of a disagreement in this period, it is the firm's turn to make a new offer Z^1 to the union in period 1. This procedure continues until an agreement is reached. In every even numbered period $2t$ the union makes an offer W^{2t} and in every odd numbered period $2t + 1$ the firm makes an offer Z^{2t+1}.

Similarly to Chapter 3, we assume that the preferences of the union and the firm are described by sequences of discount factors varying in time. $(\delta_{u,t})_{t \in \mathbb{N}}$ is the discount factor of the union in period $t \in \mathbb{N}$ and $(\delta_{f,t})_{t \in \mathbb{N}}$ is the discount factor of the firm in period $t \in \mathbb{N}$ where $\delta_{i,0} = 1$, $0 < \delta_{i,t} < 1$ for $t \geq 1$ and $i = u, f$.

The result of the wage bargaining is either a pair (W, T) where W is the wage contract agreed upon and $T \in \mathbb{N}$ is the number of proposals rejected in the bargaining, or a disagreement denoted by $(0, \infty)$ where the parties never reach an agreement.

We use the same notations and definitions as in (3.2.1), (3.2.2), (3.2.6) and (3.5.19). Moreover, the family of strategies (s_u, s_f) is given by Definition 3.1, except that the

[2]Note that for $\lambda = 1$ we recover the case of the altruistic union.

union's attitude specifies additionally its go-slow decision.

The utility of the result (W, T) for the union is equal to

$$U(W,T) = \sum_{t=0}^{\infty} \delta_u(t) u_t \tag{4.2.1}$$

where $u_t = W$ for each $t \geq T$, and if $T > 0$ then for each $0 \leq t < T$

$u_t = w_0$ if there is no agreement in period $t \in \mathbb{N}$ regardless of the union's attitude.

The utility of the result (W, T) for the firm is equal to

$$V(W,T) = \sum_{t=0}^{\infty} \delta_f(t) v_t \tag{4.2.2}$$

where $v_t = 1 - W$ for each $t \geq T$, and if $T > 0$ then for each $0 \leq t < T$

$v_t = \lambda - w_0$ if the union is hostile,

$v_t = 1 - w_0$ if the union is altruistic.

The utility of the disagreement is equal to

$$U(0,\infty) = V(0,\infty) = 0 \tag{4.2.3}$$

We make the same assumption on the sequences of discount rates as in (3.2.5).

4.2.2 Subgame perfect equilibria under different attitudes of the union

Depending on labor laws, strike actions may not be protected legally in some countries. Although necessary federal legislations were accepted in 1930's workers' rights to strike, people who work for the federal government are not allowed to strike in the US. In particular, all public officers, including teachers, are forbidden to strike in New York state. In addition, railroad or airline workers in the US are not legally permitted to strike except under certain conditions. Also in some countries, such as Turkey, strikes are legally forbidden for the employees in sectors that have impact on the security of life and property, such as law enforcement officers or bank employees.

Since the wage bargaining models that include the strike option cannot explain properly the wage negotiation processes if the legal interdiction on making strikes exists, we investigate the holdout threats of the union. More precisely, we introduce a modification of the bargaining model of Fernandez and Glazer [1991]. We assume that the union cannot strike for threatening the firm, but it can decide to go-slow in a disagreement period. If an agreement is not reached, regardless of the union's attitude, the union gets w_0 (i.e., the existing wage), but the firms bear the go-slow decision of the union with a decrease of its payoff from $(1 - w_0)$ to $(\lambda - w_0)$ where $\lambda \in [w_0, 1]$. If the go-slow rate λ of the union is close to the minimum level w_0, then the union's go-slow threat has the maximum effect on the firm's payoff. Inversely, if $\lambda = 1$, then there is no threat of the union over the firm.

In this subsection, we analyze the SPE of the wage bargaining depending on the attitude of the union. First, consider the case of the hostile union. Let $\overline{W}_H^{2t}$ and $\overline{Z}_H^{2t+1}$ denote the SPE offers when the union is hostile.

Theorem 4.1. *Consider the generalized alternating offer model of wage bargaining with preferences of the union and the firm described by the sequences of discount factors* $(\delta_{i,t})_{t\in N}$, *where* $\delta_{i,0} = 1$, $0 < \delta_{i,t} < 1$ *for* $t \geq 1$, $i = u, f$. *Assume that the union is hostile. Then there is the unique SPE of the form* (s_u, s_f) *introduced in Definition 3.1, in which the offers of the parties are given by*

$$\overline{W}_H^{2t} = w_0 + (1 - \lambda)\, \tilde{\Delta}\,(t) \tag{4.2.4}$$

and for each $t \in \mathbb{N}$

$$\overline{Z}_H^{2t+1} = w_0 + (1 - \lambda)\, \Delta_u\,(2t + 2)\, \tilde{\Delta}\,(t + 1) \tag{4.2.5}$$

Proof. Similarly to the proof of Proposition 3.1 one can show that (s_u, s_f) is a SPE of this game if and only if the offers satisfy the following infinite system of equations, for each $t \in \mathbb{N}$

$$\left(1 - \overline{W}^{2t}\right) + \left(1 - \overline{W}^{2t}\right) \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k) = (\lambda - w_0) + \left(1 - \overline{Z}^{2t+1}\right) \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k) \tag{4.2.6}$$

and

$$\overline{Z}^{2t+1} + \overline{Z}^{2t+1} \sum_{k=2t+2}^{\infty} \delta_u(2t + 2, k) = w_0 + \overline{W}^{2t+2} \sum_{k=2t+2}^{\infty} \delta_u(2t + 2, k) \tag{4.2.7}$$

which can be equivalently written by

$$\overline{W}^{2t} - \overline{Z}^{2t+1} \Delta_f\,(2t + 1) = (1 - \lambda + w_0)\,(1 - \Delta_f\,(2t + 1)) \tag{4.2.8}$$

$$\overline{Z}^{2t+1} - \overline{W}^{2t+2} \Delta_u\,(2t + 2) = w_0\,(1 - \Delta_u\,(2t + 2)) \tag{4.2.9}$$

The infinite system of (4.2.8) and (4.2.9) is a regular triangular system $AX = Y$ with $A = [a_{ij}]_{i,j\in\mathbb{N}^+}$, $X = [(x_i)_{i\in\mathbb{N}^+}]^T$, $Y = [(y_i)_{i\in\mathbb{N}^+}]^T$, where for each $t, j \geq 1$, $a_{t,t} = 1$, $a_{t,j} = 0$, for $j < t$ or $j > t+1$ and for each $t \in \mathbb{N}$

$$a_{2t+1,2t+2} = -\Delta_f(2t+1), \; a_{2t+2,2t+3} = -\Delta_u(2t+2)$$

Moreover, we have

$$x_{2t+1} = \overline{W}^{2t}, \quad x_{2t+2} = \overline{Z}^{2t+1}$$

$$y_{2t+1} = (1-\lambda+w_0)(1-\Delta_f(2t+1)), \quad y_{2t+2} = w_0(1-\Delta_u(2t+2))$$

We know that any regular triangular matrix A possesses the (unique) inverse matrix B, i.e., there exists B such that $BA = I$, where I is the infinite identity matrix. The matrix $B = [b_{ij}]_{i,j\in\mathbb{N}^+}$ is also regular triangular, and its elements are the following:

$$b_{t,t} = 1, \; b_{t,j} = 0 \text{ for each } t, j \geq 1 \text{ such that } j < t \tag{4.2.10}$$

for each $t \in \mathbb{N}$

$$b_{2t+1,2t+2} = \Delta_f(2t+1), \quad b_{2t+2,2t+3} = \Delta_u(2t+2) \tag{4.2.11}$$

and for each $t, m \in \mathbb{N}$ and $m > t$

$$b_{2t+2,2m+2} = \prod_{j=t}^{m-1} \Delta_u(2j+2)\Delta_f(2j+3) \tag{4.2.12}$$

$$b_{2t+2,2m+3} = \prod_{j=t}^{m-1} \Delta_u(2j+2)\Delta_f(2j+3)\Delta_u(2m+2) \tag{4.2.13}$$

$$b_{2t+1,2m+1} = \prod_{j=t}^{m-1} \Delta_u(2j+2)\Delta_f(2j+1) \tag{4.2.14}$$

$$b_{2t+1,2m+2} = \prod_{j=t}^{m-1} \Delta_u(2j+2)\Delta_f(2j+1)\Delta_f(2m+1) \tag{4.2.15}$$

Hence, $AX = Y$ is equal to

$$\begin{bmatrix} 1 & -\Delta_f(1) & 0 & 0 & \cdots \\ 0 & 1 & -\Delta_u(2) & 0 & \cdots \\ 0 & 0 & 1 & -\Delta_f(3) & \cdots \\ \vdots & & & \ddots & \cdots \end{bmatrix} \begin{bmatrix} \overline{W}^0 \\ \overline{Z}^1 \\ \overline{W}^2 \\ \overline{Z}^3 \\ \vdots \end{bmatrix} = \begin{bmatrix} (1-\lambda+w_0)(1-\Delta_f(1)) \\ w_0(1-\Delta_u(2)) \\ (1-\lambda+w_0)(1-\Delta_f(3)) \\ w_0(1-\Delta_u(4)) \\ \vdots \end{bmatrix}$$

By applying $X = BY$, where

$$B = \begin{bmatrix} 1 & \Delta_f(1) & \Delta_f(1)\Delta_u(2) & \cdots & \cdots \\ 0 & 1 & \Delta_u(2) & \Delta_u(2)\Delta_f(3) & \cdots \\ 0 & 0 & 1 & \Delta_f(3) & \cdots \\ \vdots & & & \ddots & \cdots \end{bmatrix}$$

we have

$$\overline{W}_H^{2t} = (1-\lambda+w_0)(1-\Delta_f(2t+1)) + w_0\Delta_f(2t+1)(1-\Delta_u(2t+2)) +$$

$$+ (1-\lambda+w_0)\Delta_f(2t+1)\Delta_u(2t+2)(1-\Delta_f(2t+3)) + \cdots$$

and therefore $\overline{W}_H^{2t}$ and $\overline{Z}_H^{2t+1}$ are given by (4.2.4) and (4.2.5), respectively. □

Example 4.1. Let us apply this result to the wage bargaining with constant discount rates as in Example 3.1. We have $\delta_{f,t} = \delta_f$ and $\delta_{u,t} = \delta_u$ for each $t \in \mathbb{N}$, and therefore for each $j \in \mathbb{N}$, $\Delta_f(2t+1) = \delta_f$ and $\Delta_u(2t+2) = \delta_u$. By inserting this into (4.2.4), we get

$$\overline{W}_H^{2t} = w_0 + \frac{(1-\delta_f)(1-\lambda)}{1-\delta_f\delta_u}$$

If additionally we assume that $\delta_f = \delta_u = \delta$, then $\overline{W}_H^{2t} = w_0 + \frac{1-\lambda}{1+\delta}$.

Example 4.2. Consider Example 3.2, i.e., the model in which the union and the firm have the following sequences of discount factors varying in time: for each $t \in \mathbb{N}$

$$\delta_{f,2t+1} = \delta_{u,2t+1} = \frac{1}{2}, \quad \delta_{f,2t+2} = \delta_{u,2t+2} = \frac{1}{3}$$

By virtue of (4.2.4) the offer of the union in period $2t$ in the SPE is equal to

$$\overline{W}_H^{2t} = w_0 + \frac{2(1-\lambda)}{3}.$$

If the union is supposed to be altruistic, i.e., it is never on go slow in disagreement periods, then we obtain the unique SPE that leads to the minimum wage contract w_0. Let us denote the SPE offers when the union is altruistic as $\overline{W}_A^{2t}$ and $\overline{Z}_A^{2t+1}$. We have the following fact:

Fact 4.1. *Consider the generalized wage bargaining model with preferences of the union and the firm described by the sequences of discount factors* $(\delta_{i,t})_{t\in N}$, *where* $\delta_{i,0} = 1$,

$0 < \delta_{i,t} < 1$ for $t \geq 1$, $i = u, f$. Assume that the attitude of the union is altruistic. Then there is the unique SPE of the form (s_u, s_f), where

$$\overline{W}_A^{2t} = \overline{Z}_A^{2t+1} = w_0$$

for each $t \in \mathbb{N}$.

Proof. Suppose that the union is altruistic. One can show that if (s_u, s_f) is a SPE, then it must hold for each $t \in \mathbb{N}$

$$\left(1 - \overline{W}^{2t}\right) + \left(1 - \overline{W}^{2t}\right) \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k) = (1 - w_0) + \left(1 - \overline{Z}^{2t+1}\right) \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k) \tag{4.2.16}$$

and

$$\overline{Z}^{2t+1} + \overline{Z}^{2t+1} \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k) = w_0 + \overline{W}^{2t+2} \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k) \tag{4.2.17}$$

and hence we get

$$\overline{W}^{2t} - \overline{Z}^{2t+1} \Delta_f(2t+1) = w_0 (1 - \Delta_f(2t+1))$$

$$\overline{Z}^{2t+1} - \overline{W}^{2t+2} \Delta_u(2t+2) = w_0 (1 - \Delta_u(2t+2))$$

Obviously, $\overline{W}^{2t} = \overline{Z}^{2t+1} = w_0$ for each $t \in \mathbb{N}$ is a solution of this system of equations, and we know from the infinite matrices theory that this system has only one solution. One can also show that (s_u, s_f) with $\overline{W}_A^{2t} = \overline{Z}_A^{2t+1} = w_0$ for $t \in \mathbb{N}$ is a SPE. □

Remark 4.1. We have the following:

$$\overline{W}_H^{2t} = \overline{W}_A^{2t} + (1 - \lambda) \tilde{\Delta}(t)$$

where $(1 - \lambda) \tilde{\Delta}(t) \geq 0$, and therefore $\overline{W}_H^{2t} \geq \overline{W}_A^{2t}$.

4.2.3 On the subgame perfect equilibrium payoffs

By applying the Shaked and Sutton [1984] method to the wage bargaining model of Fernandez and Glazer [1991], Houba and Wen [2008] derive the extreme equilibrium payoffs. We generalize their method and apply it to the model with the sequences of discount rates varying in time, where the strikes are not allowed and the sole threat of the union is to be on go-slow during disagreement periods.

We use the same notation as in the previous chapter, i.e., let M_u^{2t} be the supremum of the union's SPE payoffs in any $2t$ period and m_f^{2t+1} be the infimum of the firm's SPE payoffs in any $2t+1$ periods, $t \in \mathbb{N}$. The following propositions present the necessary conditions on m_f^{2t+1} and M_u^{2t}, for $t \in \mathbb{N}$, respectively:

Proposition 4.1. *We have for all* $(\delta_{u,t})_{t\in\mathbb{N}}$, $(\delta_{f,t})_{t\in\mathbb{N}}$, $0 \le w_0 \le \lambda \le 1$ *and* $t \in \mathbb{N}$

$$m_f^{2t+1} \ge \begin{cases} 1 - w_0\left(1 - \Delta_u\left(2t+2\right)\right) - M_u^{2t+2}\Delta_u\left(2t+2\right) & \text{if (4.2.19)} \\ \left(\lambda - w_0\right)\left(1 - \Delta_f\left(2t+2\right)\right) + \left(1 - M_u^{2t+2}\right)\Delta_f\left(2t+2\right) & \text{if (4.2.20)} \end{cases} \tag{4.2.18}$$

$$\Delta_u\left(2t+2\right) \le \Delta_f\left(2t+2\right) \text{ or}$$

$$\Delta_u\left(2t+2\right) > \Delta_f\left(2t+2\right) \text{ and}$$

$$\left(1 - \Delta_f\left(2t+2\right)\right)\left(1-\lambda\right) > \left(M_u^{2t+2} - w_0\right)\left(\Delta_u\left(2t+2\right) - \Delta_f\left(2t+2\right)\right) \tag{4.2.19}$$

$$\Delta_u\left(2t+2\right) > \Delta_f\left(2t+2\right) \text{ and}$$

$$\left(1 - \Delta_f\left(2t+2\right)\right)\left(1-\lambda\right) \le \left(M_u^{2t+2} - w_0\right)\left(\Delta_u\left(2t+2\right) - \Delta_f\left(2t+2\right)\right) \tag{4.2.20}$$

Proof. We consider an arbitrary odd period $2t+1$, $t \in \mathbb{N}$. If the union holds out after rejecting the firm's offer, the union will get at most $w_0\left(1 - \Delta_u\left(2t+2\right)\right) + M_u^{2t+2}\Delta_u\left(2t+2\right)$. Hence the firm could get at least $1 - w_0\left(1 - \Delta_u\left(2t+2\right)\right) - M_u^{2t+2}\Delta_u\left(2t+2\right)$ from making an irresistible offer and at least $\left(1 - w_0\right)\left(1 - \Delta_f\left(2t+2\right)\right) + \left(1 - M_u^{2t+2}\right)\Delta_f\left(2t+2\right) = 1 - w_0\left(1 - \Delta_f\left(2t+2\right)\right) - M_u^{2t+2}\Delta_f\left(2t+2\right)$ from making an unacceptable offer. The firm will make either the least irresistible offer or an unacceptable offer, depending on these two payoffs.

If the union is on go slow after rejecting the firms's offer, the union will get at most $w_0\left(1 - \Delta_u\left(2t+2\right)\right) + M_u^{2t+2}\Delta_u\left(2t+2\right)$. Hence the firm will get at least $1 - w_0\left(1 - \Delta_u\left(2t+2\right)\right) - M_u^{2t+2}\Delta_u\left(2t+2\right)$ from making an irresistible offer or $\left(\lambda - w_0\right)\left(1 - \Delta_f\left(2t+2\right)\right) + \left(1 - M_u^{2t+2}\right)\Delta_f\left(2t+2\right)$ from making an unacceptable offer.

Consequently, we get the following: for all $(\delta_{u,t})_{t\in\mathbb{N}}$, $(\delta_{f,t})_{t\in\mathbb{N}}$, $0 \le w_0 \le \lambda \le 1$ and $t \in \mathbb{N}$

$$m_f^{2t+1} \ge \min \begin{cases} \max \begin{cases} 1 - w_0\left(1 - \Delta_f\left(2t+2\right)\right) - M_u^{2t+2}\Delta_f\left(2t+2\right) & (a) \\ 1 - w_0\left(1 - \Delta_u\left(2t+2\right)\right) - M_u^{2t+2}\Delta_u\left(2t+2\right) & (b) \end{cases} \\ \max \begin{cases} 1 - w_0\left(1 - \Delta_u\left(2t+2\right)\right) - M_u^{2t+2}\Delta_u\left(2t+2\right) & (b) \\ \left(\lambda - w_0\right)\left(1 - \Delta_f\left(2t+2\right) + \left(1 - M_u^{2t+2}\right)\Delta_f\left(2t+2\right)\right) & (c) \end{cases} \end{cases} \tag{4.2.21}$$

Consider now an arbitrary $t \in \mathbb{N}$. If $\lambda < 1$, then we have $1 - w_0 (1 - \Delta_f (2t+2)) > (\lambda - w_0)(1 - \Delta_f (2t+2)) + M_u^{2t+2} \Delta_f (2t+2)$. Hence we get (4.2.21$a$) > (4.2.21$c$).

Assume that $\Delta_u (2t+2) \leq \Delta_f (2t+2)$. Then we have $1 - w_0 (1 - \Delta_f (2t+2)) - M_u^{2t+2} \Delta_f (2t+2) \leq 1 - w_0 (1 - \Delta_u (2t+2)) - M_u^{2t+2} \Delta_u (2t+2)$, therefore we get (4.2.21a) ≤ (4.2.21b). Moreover, we have

$(\lambda - w_0)(1 - \Delta_f (2t+2)) + (1 - M_u^{2t+2}) \Delta_f (2t+2) \leq 1 - w_0 (1 - \Delta_u (2t+2)) - M_u^{2t+2} \Delta_u (2t+2)$, and hence (4.2.21$c$) ≤ (4.2.21$b$).

Assume that $\Delta_f (2t+2) < \Delta_u (2t+2)$. Then we have the following: $1 - w_0 (1 - \Delta_f (2t+2)) - M_u^{2t+2} \Delta_f (2t+2) > 1 - w_0 (1 - \Delta_u (2t+2)) - M_u^{2t+2} \Delta_u (2t+2)$, we get (4.2.21$a$) ≥ (4.2.21$b$) and $(\lambda - w_0)(1 - \Delta_f (2t+2)) + (1 - M_u^{2t+2}) \Delta_f (2t+2) < 1 - w_0 (1 - \Delta_u (2t+2)) - M_u^{2t+2} \Delta_u (2t+2)$ if and only if $(1 - \Delta_f (2t+2))(1 - \lambda) > (M_u^{2t+2} - w_0)(\Delta_u (2t+2) - \Delta_f (2t+2))$. Hence, we get (4.2.21$b$) > (4.2.21$c$), otherwise we have (4.2.21c) ≥ (4.2.21b). □

Proposition 4.2. *We have for all* $(\delta_{u,t})_{t \in \mathbb{N}}$, $(\delta_{f,t})_{t \in \mathbb{N}}$, $0 \leq w_0 \leq \lambda \leq 1$ *and* $t \in \mathbb{N}$

$$M_u^{2t} \leq \begin{cases} w_0 (1 - \Delta_u (2t+1)) + \left(1 - m_f^{2t+1}\right) \Delta_u (2t+1) & \text{if (4.2.23)} \\ 1 - (\lambda - w_0)(1 - \Delta_f (2t+1)) - m_f^{2t+1} \Delta_f (2t+1) & \text{if (4.2.24)} \end{cases} \tag{4.2.22}$$

$$\begin{gathered} \Delta_f (2t+1) < \Delta_u (2t+1) \ \ \textit{and} \\ \left(w_0 + m_f^{2t+1}\right)(\Delta_f (2t+1) - \Delta_u (2t+1)) > 1 - \lambda (1 - \Delta_f (2t+1)) - \Delta_u (2t+1) \end{gathered} \tag{4.2.23}$$

$$\begin{gathered} \Delta_f (2t+1) \geq \Delta_u (2t+1) \ \ \textit{or} \\ \Delta_f (2t+1) < \Delta_u (2t+1) \ \ \textit{and} \\ \left(w_0 + m_f^{2t+1}\right)(\Delta_f (2t+1) - \Delta_u (2t+1)) \leq 1 - \lambda (1 - \Delta_f (2t+1)) - \Delta_u (2t+1) \end{gathered} \tag{4.2.24}$$

Proof. We consider an arbitrary even period $2t$, $t \in \mathbb{N}$. If the union holds out after its offer is rejected, the firm will get at least $(1 - w_0)(1 - \Delta_f (2t+1)) + m_f^{2t+1} \Delta_f (2t+1)$. Hence the union's SPE payoffs must be smaller than or equal to $w_0 (1 - \Delta_f (2t+1)) + \left(1 - m_f^{2t+1}\right) \Delta_f (2t+1)$ from making the least acceptable offer or $w_0 (1 - \Delta_u (2t+1)) + \left(1 - m_f^{2t+1}\right) \Delta_u (2t+1)$ from making an unacceptable offer.

If the union is on go slow after its offer is rejected, the firm will get at least $(\lambda - w_0)(1 - \Delta_f (2t+1)) + m_f^{2t+1} \Delta_f (2t+1)$ by rejecting the union's offer. Hence the union's SPE payoffs must be smaller than or equal to $1 - (\lambda - w_0)(1 - \Delta_f (2t+1)) -$

$m_f^{2t+1}\Delta_f(2t+1)$ from making the least acceptable offer, or $w_0(1-\Delta_u(2t+1)) + \left(1-m_f^{2t+1}\right)\Delta_u(2t+1)$ from making an unacceptable offer.

Consequently, we have for all $(\delta_{u,t})_{t\in\mathbb{N}}$, $(\delta_{f,t})_{t\in\mathbb{N}}$, $0 \le w_0 \le \lambda \le 1$ and $t \in \mathbb{N}$

$$M_u^{2t} \le \max \begin{cases} \max \begin{cases} w_0(1-\Delta_f(2t+1)) + \left(1-m_f^{2t+1}\right)\Delta_f(2t+1) & (a) \\ w_0(1-\Delta_u(2t+1)) + \left(1-m_f^{2t+1}\right)\Delta_u(2t+1) & (b) \end{cases} \\ \max \begin{cases} w_0(1-\Delta_u(2t+1)) + \left(1-m_f^{2t+1}\right)\Delta_u(2t+1) & (b) \\ 1-(\lambda-w_0)(1-\Delta_f(2t+1)) - m_f^{2t+1}\Delta_f(2t+1) & (c) \end{cases} \end{cases} \tag{4.2.25}$$

For every $t \in \mathbb{N}$ and $\lambda < 1$, $1-(\lambda-w_0)(1-\Delta_f(2t+1)) - m_f^{2t+1}\Delta_f(2t+1) > w_0(1-\Delta_f(2t+1)) + \left(1-m_f^{2t+1}\right)\Delta_f(2t+1)$, and hence we get (4.2.25c) > (4.2.25a).

Assume that $\Delta_f(2t+1) \ge \Delta_u(2t+1)$. Then (4.2.25$a$) $\ge$ (4.2.25b), and since (4.2.25c) > (4.2.25a), we have $M_u^{2t} \le 1-(\lambda-w_0)(1-\Delta_f(2t+1)) - m_f^{2t+1}\Delta_f(2t+1)$.

If $\Delta_f(2t+1) < \Delta_u(2t+1)$, then (4.2.25$a$) < (4.2.25$b$) and $w_0(1-\Delta_u(2t+1)) + \left(1-m_f^{2t+1}\right)\Delta_u(2t+1) > 1-(\lambda-w_0)(1-\Delta_f(2t+1)) - m_f^{2t+1}\Delta_f(2t+1)$ if and only if $\left(w_0+m_f^{2t+1}\right)(\Delta_f(2t+1)-\Delta_u(2t+1)) > 1-\lambda(1-\Delta_f(2t+1)) - \Delta_u(2t+1)$. Hence, (4.2.25$b$) > (4.2.25$c$), otherwise we have (4.2.25c) > (4.2.25b). □

We can use Propositions 4.1 and 4.2 to determine the extreme equilibrium payoffs for particular cases of the discount rates varying in time. Fact 4.2 shows one of the cases, when in every period the generalized discount factor of the firm is not smaller than the generalized discount factor of the union.

Fact 4.2. *Let $0 \le w_0 \le \lambda \le 1$, and let $(\delta_{u,t})_{t\in\mathbb{N}}$ and $(\delta_{f,t})_{t\in\mathbb{N}}$ be the sequences of discount rates such that $\Delta_f(t) \ge \Delta_u(t)$ for every $t \in \mathbb{N}$. Then we have for every $t \in \mathbb{N}$*

$$M_u^{2t} = w_0 + (1-\lambda)\tilde{\Delta}(t) \tag{4.2.26}$$

$$m_f^{2t+1} = (1-w_0) - (1-\lambda)\Delta_u(2t+2)\tilde{\Delta}(t+1) \tag{4.2.27}$$

where $\tilde{\Delta}(t)$ is given in (3.5.19).

Proof. Let $\Delta_f(2t+2) \ge \Delta_u(2t+2)$ and $\Delta_f(2t+1) \ge \Delta_u(2t+1)$ for every $t \in \mathbb{N}$. From Propositions 4.1 and 4.2 we have for every $t \in \mathbb{N}$:
$m_f^{2t+1} + M_u^{2t+2}\Delta_u(2t+2) = 1 - w_0(1-\Delta_u(2t+2))$ and $M_u^{2t} + m_f^{2t+1}\Delta_f(2t+1) = 1-(\lambda-w_0)(1-\Delta_f(2t+1))$ which is a regular triangular system and possesses a unique solution. This solution is given by (4.2.26) and (4.2.27).

□

Remark 4.2. Note that M_u^{2t} and m_f^{2t+1} defined in (4.2.26) and (4.2.27) are equal to the SPE payoffs obtained by the union and the firm under the *"always going slow"* case. More precisely, this SPE strategy profile is given by the following strategies:

- In period $2t$ the union proposes $w_0 + (1-\lambda)\tilde{\Delta}(t)$, in period $2t+1$ it accepts an offer if and only if $y \geq w_0 + (1-\lambda)\Delta_u(2t+2)\tilde{\Delta}(t+1)$, it is always on go-slow if there is a disagreement.

- In period $2t+1$ the firm proposes $w_0 + (1-\lambda)\Delta_u(2t+2)\tilde{\Delta}(t+1)$, in period $2t$ it accepts x if and only if $x \leq w_0 + (1-\lambda)\tilde{\Delta}(t)$.

This $M_u^{2t} = \bar{W}_H^{2t} = w_0 + (1-\lambda)\tilde{\Delta}(t)$ can be interpreted as follows: the union gets the existing wage plus the gain from being on go-slow which depends on the go-slow rate λ and $\tilde{\Delta}(t)$ determined by the discount factors of both parties.

Remark 4.3. When the go-slow rate $\lambda = 1$, then $M_u^{2t} = w_0$ which gives the minimum wage contract. This SPE is acquired by the never-go-slow strategies of the union. On the other hand, when the go-slow rate $\lambda = w_0$, then we have $M_u^{2t} = w_0 + (1-w_0)\tilde{\Delta}(t)$ which is equal to the SPE payoff obtained by the generalized alternating strike strategies shown in Ozkardas and Rusinowska [2014a, Forthcoming].

Remark 4.4. Note that for some cases of the discount rates the solutions on M_u^{2t} and m_f^{2t+1} do not satisfy the necessary conditions. We give some examples below:

- Let $\Delta_f(2t+2) \geq \Delta_u(2t+2)$, $\Delta_f(2t+1) < \Delta_u(2t+1)$ and $\left(w_0 + m_f^{2t+1}\right)(\Delta_f(2t+1) - \Delta_u(2t+1)) > 1 - \lambda(1-\Delta_f(2t+1)) - \Delta_u(2t+1)$ for every $t \in \mathbb{N}$. We have the infinite system for $t \in \mathbb{N}$: $m_f^{2t+1} + M_u^{2t+2}\Delta_u(2t+2) = 1 - w_0(1-\Delta_u(2t+2))$ and $M_u^{2t} + m_f^{2t+1}\Delta_u(2t+1) = w_0(1-\Delta_u(2t+1)) + \Delta_u(2t+1)$ which is a regular triangular system and has a unique solution of $M_u^{2t} = w_0$. But this unique solution does not satisfy the necessary condition.

- Consider the case where $\Delta_f(2t+2) < \Delta_u(2t+2)$, $\Delta_f(2t+1) < \Delta_u(2t+1)$, $\left(w_0 + m_f^{2t+1}\right)(\Delta_f(2t+1) - \Delta_u(2t+1)) > 1 - \lambda(1-\Delta_f(2t+1)) - \Delta_u(2t+1)$ and $(1-\Delta_f(2t+2))(1-\lambda) > (M_u^{2t+2} - w_0)(\Delta_u(2t+2) - \Delta_f(2t+2))$ for every $t \in \mathbb{N}$. We have the infinite system for $t \in \mathbb{N}$: $m_f^{2t+1} + M_u^{2t+2}\Delta_u(2t+2) = 1 - w_0(1-\Delta_u(2t+2))$ and $M_u^{2t} + m_f^{2t+1}\Delta_u(2t+1) = w_0(1-\Delta_u(2t+1)) + \Delta_u(2t+1)$ which has a unique solution $M_u^{2t} = w_0$ and $m_f^{2t+1} = 1 - w_0$, but this solution does not satisfy one of the necessary conditions.

- Consider the case where $\Delta_f(2t+2) < \Delta_u(2t+2)$, $\Delta_f(2t+1) < \Delta_u(2t+1)$, $\left(w_0 + m_f^{2t+1}\right)(\Delta_f(2t+1) - \Delta_u(2t+1)) > 1 - \lambda(1 - \Delta_f(2t+1)) - \Delta_u(2t+1)$ and $(1 - \Delta_f(2t+2))(1-\lambda) \leq (M_u^{2t+2} - w_0)(\Delta_u(2t+2) - \Delta_f(2t+2))$ for every $t \in \mathbb{N}$. We obtain the following infinite system of equations, for $t \in \mathbb{N}$: $m_f^{2t+1} + M_u^{2t+2}\Delta_f(2t+2) = (\lambda - w_0)(1 - \Delta_f(2t+2)) + \Delta_f(2t+2)$ and $M_u^{2t} + m_f^{2t+1}\Delta_u(2t+1) = w_0(1 - \Delta_u(2t+1)) + \Delta_u(2t+1)$, and hence $M_u^{2t} = w_0 + (1-\lambda)\left(\Delta_u(2t+1) - \sum\limits_{m=t}^{\infty}(1 - \Delta_u(2m+3))\prod\limits_{j=t}^{m}\Delta_u(2j+1)\Delta_f(2j+2)\right)$, but it does not satisfy one of the necessary conditions.

- Let $\Delta_f(2t+2) < \Delta_u(2t+2)$, $\Delta_f(2t+1) \geq \Delta_u(2t+1)$ and $(1-\lambda)(1 - \Delta_f(2t+2)) \leq (M_u^{2t+2} - w_0)(\Delta_u(2t+2) - \Delta_f(2t+2))$ for every $t \in \mathbb{N}$. We have the infinite system for $t \in \mathbb{N}$: $m_f^{2t+1} + M_u^{2t+2}\Delta_f(2t+2) = (\lambda - w_0)(1 - \Delta_f(2t+2)) + \Delta_f(2t+2)$ and $M_u^{2t} + m_f^{2t+1}\Delta_f(2t+1) = 1 - (\lambda - w_0)(1 - \Delta_f(2t+1))$ and therefore $M_u^{2t} = 1 - \lambda + w_0$, but it does not satisfy the necessary condition.

- Let $\Delta_f(2t+2) < \Delta_u(2t+2)$, $\Delta_f(2t+1) < \Delta_u(2t+1)$, $\left(w_0 + m_f^{2t+1}\right)(\Delta_f(2t+1) - \Delta_u(2t+1)) \leq 1 - \lambda(1 - \Delta_f(2t+1)) - \Delta_u(2t+1)$ and $(1 - \Delta_f(2t+2))(1-\lambda) \leq (M_u^{2t+2} - w_0)(\Delta_u(2t+2) - \Delta_f(2t+2))$ for every $t \in \mathbb{N}$. We have the infinite system for $t \in \mathbb{N}$: $m_f^{2t+1} + M_u^{2t+2}\Delta_f(2t+2) = (\lambda - w_0)(1 - \Delta_f(2t+2)) + \Delta_f(2t+2)$ and $M_u^{2t} + m_f^{2t+1}\Delta_f(2t+1) = 1 - (\lambda - w_0)(1 - \Delta_f(2t+1))$ and hence $M_u^{2t} = 1 - \lambda + w_0$, but it does not satisfy one of the necessary conditions.

4.3 The generalized wage bargaining with lockouts

In the generalized wage bargaining considered in Ozkardas and Rusinowska [2014a, Forthcoming], only the union is allowed to engage in actions different from making offers and accepting/rejecting such as going on strike or holding out. Let us consider a model in which the firm is allowed to engage in lockouts and holdout. For simplicity and without affecting qualitatively our results, we assume that if the firm locks out the union, then the parties get $(0,0)$, and in case of holdout – as usual – they get $(w_0, 1 - w_0)$.

We examine a game in which only lockouts by the firm are feasible, i.e., the union is not allowed to strike. By $\overline{W}_{LAR}^{2t}$ and $\overline{Z}_{LAR}^{2t+1}$ we denote the SPE offers in this game.

We have the following result.

Theorem 4.2. *Consider the generalized wage bargaining model with lockouts and without strikes, in which preferences of the union and the firm are described by the sequences of discount factors* $(\delta_{i,t})_{t\in N}$*, where* $\delta_{i,0} = 1$*,* $0 < \delta_{i,t} < 1$ *for* $t \geq 1$*,* $i = u, f$*. If*

$$1 - w_0 \leq \left(1 - \overline{W}_{LAR}^{2t+2}\right) \Delta_f(2t+2) \text{ for every } t \in \mathbb{N} \tag{4.3.1}$$

and the following condition is satisfied

$$\Delta_f(2t+1) \leq \Delta_u(2t+1) \text{ for each } t \in \mathbb{N} \tag{4.3.2}$$

then there exists a SPE in which the agreement of $\overline{W}_{LAR}^{0}$ *is reached in period 0, where for each* $t \in \mathbb{N}$

$$\overline{W}_{LAR}^{2t} = w_0 \left(1 - \Delta_f(2t+1) + \sum_{m=t}^{\infty} (1 - \Delta_f(2m+3)) \prod_{j=t}^{m} \Delta_u(2j+2)\Delta_f(2j+1)\right) \tag{4.3.3}$$

$$\overline{Z}_{LAR}^{2t+1} = \overline{W}_{LAR}^{2t+2} \Delta_u(2t+2) \tag{4.3.4}$$

This SPE is supported by the following 'generalized alternating lockout strategies':

- *In period* $2t$ *the union proposes* $\overline{W}_{LAR}^{2t}$*, in period* $2t+1$ *it accepts an offer* y *if and only if* $y \geq \overline{Z}_{LAR}^{2t+1}$*.*
- *In period* $2t+1$ *the firm proposes* $\overline{Z}_{LAR}^{2t+1}$*, in period* $2t$ *it accepts an offer* x *if and only if* $x \leq \overline{W}_{LAR}^{2t}$*, it holds out after rejecting an offer of the union in period* $2t$ *and locks out after rejection of its own proposals in period* $2t+1$*.*
- *If, however, at some point, the firm deviates from the above rule, then both parties play thereafter according to the 'minimum-wage strategies':*
  - *The union offers* w_0 *for each* $t \in \mathbb{N}$ *and accepts* y *if and only if* $y \geq w_0$*.*
  - *The firm offers* w_0 *for each* $t \in \mathbb{N}$ *and accepts* x *if and only if* $x \leq w_0$*, and never locks out the union.*

Proof. In the proof we will write simply $\overline{W}^{2t}$ and $\overline{Z}^{2t+1}$ instead of $\overline{W}_{LAR}^{2t}$ and $\overline{Z}_{LAR}^{2t+1}$. We need to solve the following system, for each $t \in \mathbb{N}$:

$$1 - \overline{W}^{2t} = (1 - w_0)(1 - \Delta_f(2t+1)) + \left(1 - \overline{Z}^{2t+1}\right) \Delta_f(2t+1)$$

and

$$\overline{Z}^{2t+1} = \overline{W}^{2t+2} \Delta_u (2t+2)$$

which is equivalent, for each $t \in \mathbb{N}$, to

$$\overline{W}^{2t} - \overline{Z}^{2t+1} \Delta_f (2t+1) = w_0 (1 - \Delta_f (2t+1)) \text{ and } \overline{Z}^{2t+1} - \overline{W}^{2t+2} \Delta_u (2t+2) = 0 \tag{4.3.5}$$

and forms a regular triangular system $AX = Y$, with $A = [a_{ij}]_{i,j \in \mathbb{N}^+}$, $X = \left[(x_i)_{i \in \mathbb{N}^+}\right]^T$, $Y = \left[(y_i)_{i \in \mathbb{N}^+}\right]^T$, where for each $t, j \geq 1$

$$a_{t,t} = 1,\ a_{t,j} = 0 \text{ for } j < t \text{ or } j > t+1 \tag{4.3.6}$$

and for each $t \in \mathbb{N}$

$$a_{2t+1,2t+2} = -\Delta_f (2t+1),\ a_{2t+2,2t+3} = -\Delta_u (2t+2) \tag{4.3.7}$$

$$x_{2t+1} = \overline{W}^{2t},\ x_{2t+2} = \overline{Z}^{2t+1},\ y_{2t+1} = w_0 (1 - \Delta_f (2t+1)),\ y_{2t+2} = 0 \tag{4.3.8}$$

Since we have the same A as in the always strike decision, its (unique) inverse matrix B is the same. By applying $X = BY$ we get $\overline{W}^{2t}$ as in Theorem 4.2.

The 'generalized alternating lockout strategies' form a SPE. Using the similar method to the one applied in Chapter 3, one can easily show that no deviation would be profitable for the deviating party.

In particular, the firm gets $(1-w_0)\left(1 + \sum_{k=2t+2}^{\infty} \delta_f(2t+2,k)\right)$ when deviating from its lockouts decision in period $2t+1$, and $\left(1 - \overline{W}^{2t+2}\right) \sum_{k=2t+2}^{\infty} \delta_f(2t+2,k)$ when not deviating. Hence, by virtue of condition (4.3.1), the firm does not want to deviate. Also $1 - w_0 \leq \left(1 - \overline{W}^{2t+2}\right) \Delta_f(2t+2) \leq 1 - \overline{W}^{2t+2}$ and therefore we get $\overline{W}^{2t+2} \leq w_0$ and also $\overline{Z}^{2t+1} = \overline{W}^{2t+2} \Delta_u (2t+2) < w_0$. Furthermore, $\overline{W}^{2t} = \overline{Z}^{2t+1} \Delta_f (2t+1) + w_0 (1 - \Delta_f (2t+1)) > \overline{Z}^{2t+1}$.

If the union deviates and offers some $x > \overline{W}^{2t}$ in period $2t$, then it gets $w_0 + \overline{Z}^{2t+1} \sum_{k=2t+1}^{\infty} \delta_u(2t+1,k)$. But from (4.3.2) and (4.3.5) we have:
$\overline{W}^{2t} = \overline{Z}^{2t+1} \Delta_f (2t+1) + w_0 (1 - \Delta_f (2t+1)) = w_0 - \Delta_f (2t+1) \left(w_0 - \overline{Z}^{2t+1}\right) \geq w_0 - \Delta_u (2t+1) \left(w_0 - \overline{Z}^{2t+1}\right) = w_0(1 - \Delta_u (2t+1)) + \overline{Z}^{2t+1} \Delta_u (2t+1)$ and therefore $w_0 + \overline{Z}^{2t+1} \sum_{k=2t+1}^{\infty} \delta_u(2t+1,k) \leq \overline{W}^{2t}(1 + \sum_{k=2t+1}^{\infty} \delta_u(2t+1,k))$. Hence, the deviation would not be profitable for the union.

If the union deviates and offers some $x < \overline{W}^{2t}$ in period $2t$, then it gets $x(1 + \sum_{k=2t+1}^{\infty} \delta_u(2t+1,k)) < \overline{W}^{2t}(1 + \sum_{k=2t+1}^{\infty} \delta_u(2t+1,k))$, so the union would be worse off by this deviation.

If the union deviates in period $2t+1$ and accepts an offer that gives it less than $\overline{Z}^{2t+1}$ or rejects an offer that gives it at least $\overline{Z}^{2t+1}$, then from the second equation of (4.3.5), the union will not be better off.

If the firm deviates in period $2t+1$ when making an offer, then it gets at most $(1-w_0)\left(1+\sum_{k=2t+2}^{\infty}\delta_f(2t+2,k)\right) < \left(1-\overline{Z}^{2t+1}\right)\left(1+\sum_{k=2t+2}^{\infty}\delta_f(2t+2,k)\right)$ as $\overline{Z}^{2t+1} < w_0$, so the firm would not be better off by any deviation.

If the firm deviates in period $2t$ when replying to an offer, i.e., it accepts an offer that gives it less than $1-\overline{W}^{2t}$ or rejects an offer that gives it at least $1-\overline{W}^{2t}$, then from the first equation of (4.3.5), the firm will not be better off. □

Remark 4.5. Note that for every $t \in \mathbb{N}$, $\overline{W}^{2t}_{LAR} = w_0\overline{W}^{2t}_{AS} < w_0$ and also $\overline{Z}^{2t+1}_{LAR} = \overline{W}^{2t+2}_{LAR}\Delta_u(2t+2) < w_0$. Hence, under the SPE the union gets a wage contract smaller than the status quo contract w_0. For constant discount rates, we get $\overline{W}^{2t}_{LAR} = \frac{w_0(1-\delta_f)}{1-\delta_f\delta_u}$.

4.4 Concluding remarks

We investigated the SPE for the union-firm wage bargaining model with discount rates varying in time when the strikes are not allowed and the sole threat of the union is to decrease the output level by using the go-slow option. First, we modified the generalized bargaining model presented in Chapter 3 by introducing the go-slow action of the union and studied the SPE under different attitudes of the union. Then we used an extended version of the analysis presented in Houba and Wen [2008] to deliver the necessary conditions for the extreme payoffs and we calculated the extreme payoffs of the parties for a particular case of the discount rates when strikes are prohibited. We also investigated the generalized wage bargaining in which the firm can engage in lockouts and holdout.

In the wage bargaining literature, the union's threats different from strikes are usually not taken into consideration. An important feature of our model lies on introducing such threats in the union-firm bargaining. In order to model real life situations in a more accurate way, we also consider varying discount rates.

It is worthy of note that although strikes are not allowed, the union can achieve a wage increase during the wage bargaining. We show that threatening the firm with the go-slow decision in every disagreement periods gives a significant wage increase to the union. This result is also supported by the supremum of the union's subgame perfect equilibrium payoff for some particular cases of the sequences of discount rates. More precisely, the "*always going slow strategy*" leads in some cases to the maximum wage that the union can achieve. In other words, while the union always gets the existing wage, it prefers to threat and punish the firm by being on go slow in every period when there is no agreement. In this case, the firm's added value decreases with the go-slow rate. The firm's loss during the go-slow is equal to the actualized value of the union's wage increase. Furthermore, the subgame perfect equilibrium payoffs for some cases are the same as our results on the wage bargaining with strike decisions of the union (see e.g. Ozkardas and Rusinowska [2014a]). Depending on the go-slow rate λ, the supremum of the union's subgame perfect equilibrium payoffs can be supported by the generalized alternating strike strategy or the never strike strategy of the union defined in Ozkardas and Rusinowska [Forthcoming].

Chapter 5

Applications of the generalized wage bargaining model

In this chapter, we apply our generalized wage bargaining model with varying discount rates to price bargaining issues. Section 5.1 is dedicated to a general price negotiation. In Section 5.2 we propose a future research project on an application of our model to pharmaceutical product price negotiations.

5.1 Price negotiation with discount factors varying in time[1]

5.1.1 Introduction

This section concerns price bargaining – undoubtedly an important issue in most economic and market negotiations. In such a bargaining, a seller wants to sell his product at a highest price to maximize his profit whereas a buyer wants to buy it at a lowest price to maximize his surplus. If the seller and buyer do not agree on a price, then there will be no transaction.

Numerous works are devoted to price bargaining between sellers and buyers. Non-cooperative two-person sequential bargaining models are used to examine the bargaining behavior in different kinds of markets. Frequently the analysis takes notice of reference points – the concept introduced in prospect theory (Kahneman and Tversky [1979], Tversky and Kahneman [1991, 1992]). Some reference points are external such as pre-

[1] This section is based on Ozkardas and Rusinowska [2013].

vious paid prices or market values (Kahneman [1992], Kristensen and Gaerling [1997], Northcraft and Neale [1987]), and others are internal such as reservation price or aspiration price (Kristensen and Gaerling [1997]). In the price bargaining literature, it is still unclear what are the internal reference points. Kristensen and Gaerling [1997] use an experimental study for determining the reference points of price bargaining and show the importance of reservation prices of both sellers and buyers in a competitive market. A reservation price is the point at which the bargainers are indifferent to accept or to reject the offer of the other party. In other words, in a seller-buyer bargaining, it is the maximum (minimum) price at which the buyer (seller) is willing to buy (sell) the product. Kristensen and Gaerling [1997] find in their experiment that if the expected market price is lower and the first offer is higher than the reservation price, then using it as a reference point will not be significant. However, White et al. [1994] find that a buyer's reservation price is the most important reference point for the buyers. Kwon et al. [2009] create a reservation price reporting mechanism by using an experimental study. Van Poucke and Buelens [2002] introduce the notion of an offer zone, which is the difference between aspiration price and initial offer, and study its influence on the negotiated outcome, by running some simulated seller-buyer negotiations between managers.

Many works on non-cooperative two-person bargaining models are based on Rubinstein [1982] formulation of sequential bargaining process in discrete time with alternating offers and counteroffers and on the determination of subgame perfect equilibria (SPE). Time and information are important elements in these models. Some authors consider one-sided or two-sided asymmetric information and present models of sequential bargaining under incomplete information. Price bargaining between manufacturer and distributor under asymmetric and incomplete information of distributor's knowledge about buyers' reservation price is tested in an experimental study of sequential bargaining by Srivastava et al. [2000]. Feri and Gantner [2011] modify Rubinstein's sequential bargaining model by two-sided incomplete information and study experimentally price bargaining. Cramton [1991] adds transaction cost to Rubinstein's sequential bargaining model with asymmetric information. Gul and Sonnenschein [1988] identify the delay to agreement with a screening process of a price bargaining model between a buyer and a seller where there exists an uncertainty about the valuation of one party.

An important issue in non-cooperative bargaining models concerns preferences of bargainers, in particular, non-stationarity of preferences. Although several works emphasize that stationary bargaining models are rare in real-life situations (e.g., Cramton and Tracy [1994b]), models with discount factors varying in time do not receive enough attention so far. Non-stationarity of parties' preferences in the original Rubinstein model is discussed, e.g., in Binmore [1987b], Coles and Muthoo [2003], Rusinowska [2001, 2002b, 2004]. Trefler [1999] modifies Rubinstein and Wolinsky [1985] bargaining framework by adding the Markov process of pairwise matching to analyze the impact of market supply and demand on bilateral bargaining outcomes. Dickinson [2003] introduces the importance of risk preferences on the bargaining outcomes in price negotiation.

Price bargaining models are frequently tested by laboratory experiments (Roth and Kagel [1995]). For example, price bargaining on perishable goods market is studied experimentally by Moulet and Rouchier [2008] to determine the effects of time on sequential bargaining model. Cason et al. [2003] compare posted price versus bilateral bargaining price by using laboratory experiments and find that the bargaining price is higher and sticker than posted prices. Other studies use field experiments for reference points of price bargaining (Abdul-Muhmin [2001]).

Although price negotiation between a seller and a buyer can be seen as a microeconomic problem, several authors apply price negotiation models to macroeconomic issues. An application of price bargaining to international trade between two countries over two non-storable goods is analyzed by Fernández-Blanco [2012]. Oczkowski [1999] applies Nash bargaining framework to an econometric analysis of price and quantity bargaining model.

In this section we consider a monopolistic seller that sells a unique and indivisible good in a market with only one buyer. They bargain over the price of the product by making alternating offers. An initial offer is made by the seller and the buyer is free to either accept or reject it. If he rejects the offer, then it is his turn to make a new offer. We use therefore Rubinstein's bargaining procedure (Rubinstein [1982]), but similarly as in Rusinowska [2001] we generalize the model by assuming that preferences of each party are expressed by discount factors varying in time. There are several differences between the present model and the model analyzed in Rusinowska [2001]. In the latter, two players bargain over a division of one unit of infinitely divisible good and the utility of a player is given by the discounted agreement (i.e., the discounted part of the good received by the given player). In our model, the seller and the buyer bargain over the price of a good, the payoffs are different from the ones defined in Rusinowska [2001], and the utility of a bargainer is given by the discounted sums of the payoffs from period

0 to infinity. We assume that the sequence of discount rates of a party can be arbitrary, with the only restriction that the infinite series that determines the utility for the given party must be convergent. In Ozkardas and Rusinowska [2014a] we consider a wage bargaining in which a union and a firm bargain over a wage contract and the union may go on strike if an offer is rejected. Under some assumptions on the parameters in the model, the utilities of the seller and the buyer coincide with the utilities of the union and the firm in the wage bargaining in which the union commits to go on strike whenever there is a disagreement (Ozkardas and Rusinowska [2014a]). Consequently, the particular case of wage bargaining can be applied to the price negotiation model.

In this section, first we restrict our analysis to history independent strategies with no delay which means that an offer of a player is independent of the previous offers of the players and when a player has to make an offer, his equilibrium offer is accepted by the other party. Similarly as in Ozkardas and Rusinowska [2014a], we determine the unique subgame perfect equilibrium for no-delay strategies independent of the former history of the game. Then we relax the no-delay assumption and determine the highest equilibrium payoff of the seller and the lowest equilibrium payoff of the buyer for the general case (see e.g. Ozkardas and Rusinowska [Forthcoming]). We show that the no-delay equilibrium strategy profiles support these extreme payoffs. Our approach to the analysis of equilibrium payoffs in the price bargaining is similar to the one used in Houba and Wen [2008] who apply the method by Shaked and Sutton [1984] to derive the exact bounds of equilibrium payoffs in wage bargaining introduced in Fernandez and Glazer [1991]. However, while preferences of the union and of the firm in the model of Fernandez and Glazer [1991] are constant in time, in our model the seller and the buyer have preferences varying in time.

Section 5.1.2 describes the price bargaining model with discount rates varying in time. In Section 5.1.3 we determine the unique subgame perfect equilibrium of the model, when we restrict the analysis to history independent strategies with no delay. In Section 5.1.4 we analyze the equilibrium payoffs for the general model.

5.1.2 The model

We introduce a model of price negotiation between a seller and a buyer on a unique indivisible product. We suppose that the seller is in a monopolistic situation and the buyer is monopsone which means that the market is constituted by two players.

The buyer has a reservation price of R for the unique product and he buys it for personal satisfaction. His reservation price is an indicator of the buyer's willingness to buy. If the buyer cannot obtain the product, he pays a dissatisfaction cost of D. On the other hand, if he gets the product, he has a positive satisfaction gain of S, where $R \geq S \geq D \geq 0$. The seller desires to sell the product and to make a positive and maximum profit. If the seller cannot sell it, he pays a cost of $0 < C \leq S + D$ of producing the product. The bargaining procedure between the seller and the buyer is the following. The seller and the buyer bargain sequentially over discrete time and a potentially infinite horizon. They alternate in making offers of price that the other party is free either to accept or to reject.

Let P_s^{2t} denote the offer of the seller made in an even-numbered period $2t$, where $t \in \mathbb{N}$, and let P_b^{2t+1} denote the offer of the buyer made in an odd numbered period $2t+1$. The range of the proposed price is $[0, S + D]$, i.e., neither the seller nor the buyer can propose a price above the sum of the satisfaction value and the dissatisfaction cost. In period 0 the seller proposes P_s^0, and if the buyer accepts this price, than the agreement is reached and the payoffs in period 0 are $(P_s^0 - C, R - P_s^0 + S)$. If the buyer rejects it, then the payoffs in period 0 are $(-C, R - D)$, and it is the buyer's turn to make a counter-offer P_b^1 in period 1. If the seller accepts this offer, then the payoffs in period 1 are $(P_b^1 - C, R - P_b^1 + S)$. Otherwise, the payoffs in period 1 are $(-C, R - D)$, and the seller makes a new offer in the next period. This procedure goes on until an agreement is reached.

In the price negotiation, preferences of the seller and the buyer are described by sequences of discount factors varying in time, $(\delta_{s,t})_{t\in\mathbb{N}}$ and $(\delta_{b,t})_{t\in\mathbb{N}}$, respectively, where $\delta_{s,t}$ is the discount factor of the seller in period $t \in \mathbb{N}$, $\delta_{s,0} = 1$, $0 < \delta_{s,t} < 1$ for $t \geq 1$ and $\delta_{b,t}$ is the discount factor of the buyer in period $t \in \mathbb{N}$, $\delta_{b,0} = 1$, $0 < \delta_{b,t} < 1$ for $t \geq 1$.

The result of the price negotiation is either a pair (P, T), where $P \in [0, S + D]$ is the agreed price of the product and $T \in \mathbb{N}$ is the number of periods before reaching the agreement, or a disagreement denoted by (d, ∞) and meaning the situation in which the parties never reach an agreement.

For each $t \in \mathbb{N}$, let

$$\delta_s(t) := \prod_{k=0}^{t} \delta_{s,k},\ \delta_b(t) := \prod_{k=0}^{t} \delta_{b,k},\ \text{for } 0 < t' \leq t,\ \delta_s(t', t) = \prod_{k=t'}^{t} \delta_{s,k},\ \delta_b(t', t) = \prod_{k=t'}^{t} \delta_{b,k}$$

The utility of the result (P, T) for the seller, where $S + D \geq P \geq 0$ and $T \in \mathbb{N}$, is equal to

$$U_s(P,T) = \sum_{t=0}^{\infty} \delta_s(t)\, u_s(t) \tag{5.1.1}$$

where $u_s(t) = P - C$ for each $t \geq T$, and if $T > 0$ then $u_s(t) = -C$ for each $0 \leq t < T$.

The utility of the result (P,T) for the buyer is equal to

$$U_b(P,T) = \sum_{t=0}^{\infty} \delta_b(t)\, u_b(t) \tag{5.1.2}$$

where $u_b(t) = R - P + S$ for each $t \geq T$, and if $T > 0$ then $u_b(t) = R - D$ for each $0 \leq t < T$, where $R \geq S \geq D \geq 0$ and $S + D \geq P \geq 0$.

The utilities of the disagreement for the seller and the buyer are equal to

$$U_s(d,\infty) = -C \sum_{t=0}^{\infty} \delta_s(t)\,,\ U_b(d,\infty) = (R - D) \sum_{t=0}^{\infty} \delta_b(t)$$

At the seller's side, when the agreement (P,T) is reached, his payoff in every period $t \geq T$ will be equal to $u_s(t) = P - C$, i.e., to the difference between the price and the production cost. If $P \geq C$, the seller will make a profit from this agreement. On the other hand, if the agreement is not reached in period T, then the seller's payoff at period T will be $u_s(T) = -C$, i.e., the production cost which is equal to the loss of the seller. We therefore assume that the product can be used only within one period and must be produced each time when a new period starts.

For the buyer, the agreement (P,T) gives to the buyer in every period $t \geq T$ the payoff equal to $u_b(t) = R - P + S$, i.e., to the difference between his reservation price for that product and the agreement price, plus the satisfaction value for obtaining the product. Hence, the buyer's payoff in the agreement has two components: the surplus of the buyer which is the amount of money that stays in his pocket and the satisfaction value that comes from obtaining the product. In case of a disagreement, the payoff level of the buyer in period T is equal to $u_b(T) = R - D$, i.e., to the difference between the reservation price and the cost of the disagreement. This means that the buyer suffers from not obtaining the product, but he still has some money in his pocket.

Remark 5.1. Note that if $R = D = 1 - S$ and $C = 0$, then we recover the wage bargaining with discount rates varying in time, where the union commits to strike whenever there is a disagreement; see Ozkardas and Rusinowska [2014a].

The utilities for both parties depend on the infinite series, so we need to well define the sequences of discount rates.

Remark 5.2. The necessary conditions for the convergence of the infinite series which define $U_s(P,T)$ and $U_b(P,T)$ in (5.1.1) and (5.1.2) are

$$\delta_s(t) \to_{t\to+\infty} 0 \quad \text{and} \quad \delta_b(t) \to_{t\to+\infty} 0 \tag{5.1.3}$$

but these are not sufficient conditions. The necessary conditions come immediately from the necessary condition of the convergence of the infinite series. To see that these are not sufficient conditions, consider $\delta_{b,k} = \frac{k}{k+1}$ for each $k \geq 1$, $\delta_{b,0} = 1$. Then

$$\delta_b(t) = \frac{1}{2} \cdot \frac{2}{3} \cdots \frac{t}{t+1} = \frac{1}{t+1} \to_{t\to+\infty} 0$$

If the agreement P is reached immediately, then $U_b(P,0) = (R-P+S)\sum_{t=0}^{\infty} \frac{1}{t+1}$ which is a divergent series. Similarly, if P is reached in a certain period $T > 0$, then $U_b(P,T) = \sum_{t=0}^{T-1} \delta_b(t) u_t + (R-P+S)\sum_{t=T}^{\infty} \frac{1}{t+1}$.

If $(\delta_{s,t})_{t\in\mathbb{N}}$ and $(\delta_{b,t})_{t\in\mathbb{N}}$ are bounded by a certain number smaller than 1, i.e., if

$$\text{there exist } \Phi_s < 1 \text{ and } \Phi_b < 1 \text{ such that } \delta_{s,t} \leq \Phi_s \text{ and } \delta_{b,t} \leq \Phi_b \text{ for each } t \in \mathbb{N} \tag{5.1.4}$$

then the series which define $U_s(P,T)$ and $U_b(P,T)$ in (5.1.1) and (5.1.2) are convergent. We have for each $t \in \mathbb{N}$

$$0 \leq \delta_b(t)(R-P+S) \leq (\Phi_b)^t (R-P+S)$$

Let the agreement P be reached immediately. Since $\sum_{t=0}^{\infty} (\Phi_b)^t$ is the convergent geometric series, by virtue of the comparison test, $U_b(P,0)$ is also convergent. The proof is similar if P is reached in a certain period $T > 0$ and it is analogous for the seller. The sufficient conditions given in (5.1.4) are not necessary conditions. To see that, consider $\delta_{b,k} = \frac{k}{k+2}$ for each $k \geq 1$, $\delta_{b,0} = 1$. The sequence does not satisfy the condition (5.1.4). However, we have

$$\delta_b(t) = \frac{1}{3} \cdot \frac{2}{4} \cdots \frac{t}{t+2} = \frac{2}{(t+1)(t+2)} \to_{t\to+\infty} 0$$

If the agreement P is reached immediately, then $U_b(P,0) = (R-P+S)\sum_{t=1}^{\infty} \frac{2}{(t+1)(t+2)}$ which is convergent by virtue of the comparison test: $\frac{1}{t^2} \geq \frac{1}{(t+1)(t+2)}$ and we know that $\sum_{t=1}^{\infty} \frac{1}{t^2}$ is convergent. The proof is similar if P is reached in a certain period $T > 0$.

Not only every decreasing sequence $(\delta_{s,t})_{t\in\mathbb{N}}$ ($(\delta_{b,t})_{t\in\mathbb{N}}$, respectively) satisfies (5.1.4) and gives the convergent series defined in (5.1.1) ((5.1.2), respectively) but also some increasing sequences do that; see, e.g., $\delta_{b,k} = \frac{1}{3} - \frac{1}{3k+3}$ for each $k \geq 1$.

Remark 5.3. We restrict our analysis to the case in which the discount rates satisfy condition (5.1.4). Hence, in particular, for each $t \in \mathbb{N}$,

$$\sum_{k=2t+1}^{\infty} \delta_s(2t+1,k) \leq \frac{\Phi_s}{1-\Phi_s}, \quad \sum_{k=2t+2}^{\infty} \delta_b(2t+2,k) \leq \frac{\Phi_b}{1-\Phi_b} \tag{5.1.5}$$

5.1.3 Subgame perfect equilibrium

First, we find the unique SPE if we restrict our analysis to no-delay strategies independent of the former history of the game. The notation is similar to the one introduced and used in the previous chapters, i.e., for every $t \in \mathbb{N}_+$

$$\Delta_s(t) = \frac{\sum_{k=t}^{\infty} \delta_s(t,k)}{1+\sum_{k=t}^{\infty} \delta_s(t,k)}, \ \Delta_b(t) = \frac{\sum_{k=t}^{\infty} \delta_b(t,k)}{1+\sum_{k=t}^{\infty} \delta_b(t,k)} \tag{5.1.6}$$

and consequently, for every $t \in \mathbb{N}_+$

$$\Delta_s(t) \leq \Phi_s \text{ and } \Delta_b(t) \leq \Phi_b \tag{5.1.7}$$

Proposition 5.1. *Consider the price bargaining model in which preferences of the seller and the buyer are described by the sequences of discount factors $(\delta_{i,t})_{t\in\mathbb{N}}$, where $\delta_{i,0} = 1$, $0 < \delta_{i,t} < 1$ for $t \geq 1$, $i = s, b$. Consider the following family of strategies (s_s, s_b): in each period $2t+1$ the seller accepts an offer y of the buyer if and only if $y \geq P_b^{2t+1}$, and in each period $2t$ the buyer accepts an offer x of the seller if and only if $x \leq P_s^{2t}$, where P_s^{2t} is an offer of the seller in $2t$ and P_b^{2t+1} is an offer of the buyer in $2t+1$. Then (s_s, s_b) is a SPE of this game if and only if the offers satisfy the following infinite system of equations for each $t \in \mathbb{N}$:*

$$R - P_s^{2t} + S = (R-D)\left(1-\Delta_b(2t+1)\right) + \left(R - P_b^{2t+1} + S\right)\Delta_b\left(2t+1\right) \tag{5.1.8}$$

$$P_b^{2t+1} - C = -C(1-\Delta_s(2t+2)) + \left(P_s^{2t+2} - C\right)\Delta_s\left(2t+2\right) \tag{5.1.9}$$

Proof. The proof is analogous to the proof of Proposition 3.1, but for sake of completeness we present it as well.

($\Leftarrow$) Let (s_p, s_c) be defined by (5.1.8) and (5.1.9), which can be equivalently written as

$$\left(R - P_s^{2t} + S\right) + \left(R - P_s^{2t} + S\right)\sum_{k=2t+1}^{\infty} \delta_b(2t+1,k) =$$

$$(R-D) + \left(R - P_b^{2t+1} + S\right)\sum_{k=2t+1}^{\infty} \delta_b(2t+1,k) \tag{5.1.10}$$

$$\left(P_b^{2t+1}-C\right)+\left(P_b^{2t+1}-C\right)\sum_{k=2t+2}^{\infty}\delta_s(2t+2,k)=-C+\left(P_s^{2t+2}-C\right)\sum_{k=2t+2}^{\infty}\delta_s(2t+2,k) \tag{5.1.11}$$

We show that (s_s, s_b) is a SPE.

Consider an arbitrary subgame starting in period $2t$ with the seller making an offer. Under (s_s, s_b), the seller gets $(P_s^{2t}-C)+(P_s^{2t}-C)\sum_{k=2t+1}^{\infty}\delta_s(2t+1,k)$ and the buyer gets $(R-P_s^{2t}+S)+(R-P_s^{2t}+S)\sum_{k=2t+1}^{\infty}\delta_b(2t+1,k)$. If the seller deviates from s_s and proposes a certain $x > P_s^{2t}$, then the seller gets $-C+\left(P_b^{2t+1}-C\right)\sum_{k=2t+1}^{\infty}\delta_s(2t+1,k)$. From (5.1.10), $0 \leq (D+S-P_s^{2t}) = \left(P_s^{2t}-P_b^{2t+1}\right)\sum_{k=2t+1}^{\infty}\delta_b(2t+1,k)$, and hence $P_s^{2t} \geq P_b^{2t+1}$. The seller is then not better off by this deviation, because we have
$(P_s^{2t}-C)+(P_s^{2t}-C)\sum_{k=2t+1}^{\infty}\delta_s(2t+1,k) \geq -C+\left(P_b^{2t+1}-C\right)\sum_{k=2t+1}^{\infty}\delta_s(2t+1,k)$.

Suppose that the seller deviates from s_s and proposes a certain $x < P_s^{2t}$. Then the seller gets $(x-C)+(x-C)\sum_{k=2t+1}^{\infty}\delta_s(2t+1,k)$, but he is worse off since $(x-C)+(x-C)\sum_{k=2t+1}^{\infty}\delta_s(2t+1,k) < (P_s^{2t}-C)+(P_s^{2t}-C)\sum_{k=2t+1}^{\infty}\delta_s(2t+1,k)$.

Suppose that the buyer deviates from s_b and rejects P_s^{2t}. Then he gets at most $(R-D)+\left(R-P_b^{2t+1}+S\right)\sum_{k=2t+1}^{\infty}\delta_b(2t+1,k)$, which from (5.1.10) is equal to $R-P_s^{2t}+S+(R-P_s^{2t}+S)\sum_{k=2t+1}^{\infty}\delta_b(2t+1,k)$, so the buyer is not better off by this deviation.

The analysis of an arbitrary subgame starting in $2t+1$ with the buyer making an offer is analogous to the study of the subgame starting in $2t$, except that we use (5.1.11) instead of (5.1.10).

Consider an arbitrary subgame starting in period $2t$ with the buyer replying to an offer $x \leq P_s^{2t}$. Under (s_s, s_b) he gets $(R-x+S)+(R-x+S)\sum_{k=2t+1}^{\infty}\delta_b(2t+1,k)$. A deviation from s_s does not change the result for the seller. Suppose that the buyer deviates from s_b and rejects such x. We know that it is optimal for the buyer to propose P_b^{2t+1} in $2t+1$, so the buyer gets $(R-D)+\left(R-P_b^{2t+1}+S\right)\sum_{k=2t+1}^{\infty}\delta_b(2t+1,k)$. By virtue of (5.1.10), we have $(R-x+S)+(R-x+S)\sum_{k=2t+1}^{\infty}\delta_b(2t+1,k) \geq (R-P_s^{2t}+S)+(R-P_s^{2t}+S)\sum_{k=2t+1}^{\infty}\delta_b(2t+1,k) = (R-D)+\left(R-P_b^{2t+1}+S\right)\sum_{k=2t+1}^{\infty}\delta_b(2t+1,k)$, and hence the buyer is not better off by this deviation.

Consider an arbitrary subgame starting in period $2t$ with the buyer replying to an offer $x > P_s^{2t}$. Under (s_s, s_b) the buyer rejects it and proposes P_b^{2t+1} which is accepted. The seller gets then $-C + \left(P_b^{2t+1} - C\right) \sum_{k=2t+1}^{\infty} \delta_s (2t+1, k)$ and the buyer gets $(R - D) + \left(R - P_b^{2t+1} + S\right) \sum_{k=2t+1}^{\infty} \delta_b (2t+1)$. If the buyer deviates from s_b and accepts such x, then it gets $(R - x + S) + (R - x + S) \sum_{k=2t+1}^{\infty} \delta_b (2t+1, k)$. But from (5.1.10) we have $(R - x + S) + (R - x + S) \sum_{k=2t+1}^{\infty} \delta_b (2t+1, k) < (R - P_s^{2t} + S) +$ $+ (R - P_s^{2t} + S) \sum_{k=2t+1}^{\infty} \delta_b (2t+1, k) = (R - D) + \left(R - P_b^{2t+1} + S\right) \sum_{k=2t+1}^{\infty} \delta_b (2t+1, k)$, so the buyer is worse off.

The analysis of subgame starting in $2t+1$ by the seller replying to an offer $y \geq P_b^{2t+1}$ and to an offer $y < P_b^{2t+1}$ is analogous to the analysis of the corresponding subgames starting in period $2t$ by the buyer replying to x.

$(\Rightarrow)$ Let (s_s, s_b) be a SPE. We will show that it must be defined by (5.1.10) and (5.1.11) which are equivalent to (5.1.8) and (5.1.9). Consider an arbitrary subgame starting in period $2t$ with the seller making an offer. Under (s_s, s_b) the seller proposes P_s^{2t} which is accepted and gives $(R - P_s^{2t} + S) + (R - P_s^{2t} + S) \sum_{k=2t+1}^{\infty} \delta_b (2t+1, k)$ to the buyer. By rejecting P_s^{2t}, the buyer would get $(R - D) + \left(R - P_b^{2t+1} + S\right) \sum_{k=2t+1}^{\infty} \delta_b (2t+1, k)$. Since (s_s, s_b) is a SPE, it must be $(R - P_s^{2t} + S) + (R - P_s^{2t} + S) \sum_{k=2t+1}^{\infty} \delta_b (2t+1, k) \geq$ $(R - D) + \left(R - P_b^{2t+1} + S\right) \sum_{k=2t+1}^{\infty} \delta_b (2t+1, k)$.

Suppose that the following holds: $(R - P_s^{2t} + S) + (R - P_s^{2t} + S) \sum_{k=2t+1}^{\infty} \delta_b (2t+1, k) >$ $(R - D) + \left(R - P_b^{2t+1} + S\right) \sum_{k=2t+1}^{\infty} \delta_b (2t+1, k)$. Then there exists $\widetilde{x} > P_s^{2t}$ with $R - P_s^{2t} +$ $S + (R - P_s^{2t} + S) \sum_{k=2t+1}^{\infty} \delta_b (2t+1, k) > (R - \tilde{x} + S) + (R - \tilde{x} + S) \sum_{k=2t+1}^{\infty} \delta_b (2t+1, k) >$ $(R - D) + \left(R - P_b^{2t+1} + S\right) \sum_{k=2t+1}^{\infty} \delta_b (2t+1, k)$. Since $\tilde{x} > P_s^{2t}$, the buyer rejects it and gets $(R - D) + \left(R - P_b^{2t+1} + S\right) \sum_{k=2t+1}^{\infty} \delta_b (2t+1, k)$, but he would be better off if he accepted this offer. Hence we get a contradiction and prove (5.1.10). Proving (5.1.11) is analogous by considering an arbitrary subgame starting in period $2t+1$ with the buyer making an offer. $\square$

Proposition 5.1 presents necessary and sufficient conditions for the profile (s_s, s_b) to be a SPE. The first equation means that the buyer is indifferent between accepting the equilibrium offer of the seller and rejecting that offer. Similarly, the second equation expresses the indifference of the seller between accepting and rejecting the equilibrium offer of the buyer. By solving the infinite system (5.1.8) and (5.1.9), we determine the equilibrium offers proposed under the strategies (s_s, s_b).

Proposition 5.2. *Consider the price bargaining model with preferences of the seller and the buyer described by the sequences of discount factors $(\delta_{i,t})_{t\in\mathbb{N}}$, where $\delta_{i,0} = 1$, $0 < \delta_{i,t} < 1$ for $t \geq 1$, $i = s, b$. Then there is the unique SPE of the form (s_s, s_b) stated in Proposition 5.1, in which the offers of the parties, for every $t \in \mathbb{N}$, are given by*

$$P_s^{2t} = (S + D)\left(1 - \Delta_b(2t + 1) + \sum_{m=t}^{\infty}(1 - \Delta_b(2m + 3))\prod_{j=t}^{m}\Delta_s(2j + 2)\Delta_b(2j + 1)\right) \tag{5.1.12}$$

$$P_b^{2t+1} = P_s^{2t+2}\Delta_s\left(2t + 2\right) \tag{5.1.13}$$

Proof. By virtue of Proposition 5.1, we need to solve the infinite system of equations (5.1.8) and (5.1.9), which can be equivalently written for each $t \in \mathbb{N}$, as

$$P_s^{2t} - P_b^{2t+1}\Delta_b\left(2t + 1\right) = (S + D)(1 - \Delta_b(2t + 1)) \tag{5.1.14}$$

and

$$P_b^{2t+1} - P_s^{2t+2}\Delta_s\left(2t + 2\right) = 0 \tag{5.1.15}$$

From (5.1.15) we get immediately (5.1.13). In order to calculate P_s^{2t}, we use a similar matrix method as the one applied in the previous chapters for the union-firm wage bargaining. The infinite system of (5.1.14) and (5.1.15) is a regular triangular system $AX = Y$, where $A = [a_{ij}]_{i,j\in\mathbb{N}^+}$, $X = \left[(x_i)_{i\in\mathbb{N}^+}\right]^T$, $Y = \left[(y_i)_{i\in\mathbb{N}^+}\right]^T$, for each $t, j \geq 1$

$$a_{t,t} = 1, \quad a_{t,j} = 0 \text{ for } j < t \text{ or } j > t + 1 \tag{5.1.16}$$

for each $t \in \mathbb{N}$

$$a_{2t+1,2t+2} = -\Delta_b\left(2t + 1\right), \quad a_{2t+2,2t+3} = -\Delta_s\left(2t + 2\right) \tag{5.1.17}$$

$$x_{2t+1} = P_s^{2t},\ x_{2t+2} = P_b^{2t+1},\ y_{2t+1} = (S + D)(1 - \Delta_b(2t + 1)), \quad y_{2t+2} = 0 \tag{5.1.18}$$

Any regular triangular matrix A possesses the (unique) inverse matrix B, which is also regular triangular. In other words, there exists $B = [b_{ij}]_{i,j\in\mathbb{N}^+}$ such that $BA = I$, where I is the infinite identity matrix, and

$$b_{t,t} = 1,\ b_{t,j} = 0 \text{ for each } t, j \geq 1 \text{ such that } j < t \tag{5.1.19}$$

for each $t \in \mathbb{N}$

$$b_{2t+1,2t+2} = \Delta_b (2t+1), \quad b_{2t+2,2t+3} = \Delta_s (2t+2) \tag{5.1.20}$$

and for each $t, m \in \mathbb{N}$ and $m > t$

$$b_{2t+2,2m+2} = \prod_{j=t}^{m-1} \Delta_s (2j+2) \Delta_b (2j+3) \tag{5.1.21}$$

$$b_{2t+2,2m+3} = \prod_{j=t}^{m-1} \Delta_s (2j+2) \Delta_b (2j+3) \Delta_s (2m+2) \tag{5.1.22}$$

$$b_{2t+1,2m+1} = \prod_{j=t}^{m-1} \Delta_s (2j+2) \Delta_b (2j+1) \tag{5.1.23}$$

$$b_{2t+1,2m+2} = \prod_{j=t}^{m-1} \Delta_s (2j+2) \Delta_b (2j+1) \Delta_b (2m+1) \tag{5.1.24}$$

We have then

$$\begin{bmatrix} 1 & -\Delta_b(1) & 0 & 0 & \cdots \\ 0 & 1 & -\Delta_s(2) & 0 & \cdots \\ 0 & 0 & 1 & -\Delta_b(3) & \cdots \\ 0 & 0 & 0 & 1 & \ddots \\ \vdots & \vdots & \vdots & \vdots & \cdots \end{bmatrix} \begin{bmatrix} P_s^0 \\ P_b^1 \\ P_s^2 \\ P_b^3 \\ \vdots \end{bmatrix} = \begin{bmatrix} (S+D)(1-\Delta_b(1)) \\ 0 \\ (S+D)(1-\Delta_b(3)) \\ 0 \\ \vdots \end{bmatrix}$$

$$\begin{bmatrix} P_s^0 \\ P_b^1 \\ P_s^2 \\ P_b^3 \\ \vdots \end{bmatrix} = B \begin{bmatrix} (S+D)(1-\Delta_b(1)) \\ 0 \\ (S+D)(1-\Delta_b(3)) \\ 0 \\ \vdots \end{bmatrix}$$

where

$$B = \begin{bmatrix} 1 & \Delta_b(1) & \Delta_b(1)\Delta_s(2) & \Delta_b(1)\Delta_s(2)\Delta_b(3) & \cdots \\ 0 & 1 & \Delta_s(2) & \Delta_s(2)\Delta_b(3) & \cdots \\ 0 & 0 & 1 & \Delta_b(3) & \cdots \\ 0 & 0 & 0 & 1 & \ddots \\ \vdots & \vdots & \vdots & \vdots & \cdots \end{bmatrix}$$

and hence we get P_s^{2t} as given by (5.1.12).

Note that $P_s^{2t}, P_b^{2t+1} \in [0, S+D]$ for each $t \in \mathbb{N}$. Obviously $P_s^{2t} \geq 0$. Let us consider the sequence of partial sums for $k > t$:

$$S_k = (S+D)\left(1 - \Delta_b(2t+1) + \sum_{m=t}^{k-1}(1-\Delta_b(2m+3))\prod_{j=t}^{m}\Delta_s(2j+2)\Delta_b(2j+1)\right)$$

The sequence is obviously increasing, and also $S_k \leq S+D$ for each $k > t$. Hence, $P_s^{2t} = \lim_{k\to+\infty} S_k \leq S+D$. □

We could expect that in the price negotiation model the agreed prices (P_s^{2t}) and $\left(P_b^{2t+1}\right)$ would depend on the reservation price R, the dissatisfaction cost D, the satisfaction value S, the production cost C and the discount factors $(\delta_{s,t})$ and $(\delta_{b,t})$, since in the literature they are usually supposed to be the reference points of the price determination. However, the results obtained in our model show that there is no dependence of the agreement price level on some of these determinants. More precisely, the offered prices at the equilibrium depend only on the sum of the dissatisfaction cost and the satisfaction value of the buyer, and on the discount rates of both parties. This means that when proposing a price the seller does care about the (dis)satisfaction values of the buyer. The higher these values are, the higher the prices offered by the seller and the buyer are, i.e., if the buyer is highly attached to the product and the seller knows that, the seller will offer higher prices and the buyer will accept it. Moreover, the more patient the seller will be in the future, the higher the prices offered by both parties are.

In the market with only one seller and one buyer, both parties do not have any other alternatives and they want to reach an agreement quickly. If there were other buyers in the market that desired to buy the product, the monopolistic seller could make higher profits. On the other hand, if there were many sellers that wanted to sell their products, the buyer could find lower prices. The market with many sellers and buyers gives the perfect competition situation. In our model with one seller and one buyer it seems natural that the price does not depend on the production cost or the reservation price. However, the reservation price which indicates the buyer's willingness to buy and the production cost of the seller will determine the payoffs of the parties in every period as defined in (5.1.1) and (5.1.2). Indeed, note that in a single period the sum of the agreement payoffs is equal to $(R+S-C)$ and the sum of the disagreement payoffs is equal to $(R-D-C)$.

5.1.4 The equilibrium payoffs

Next we determine the highest SPE payoff of the seller and the lowest SPE payoff of the buyer for the general case when making an unacceptable offer is allowed.

Houba and Wen [2008] apply the method of Shaked and Sutton [1984] to the wage bargaining model of Fernandez and Glazer [1991] to derive the supremum of the union's SPE payoffs and the infimum of the firm's SPE payoffs. We generalize this method to the price negotiation model with sequences of discount rates varying in time.

Let M_s^{2t} denote the supremum of the seller's SPE payoff in any even period $2t$, where the seller makes an offer. Let m_b^{2t+1} denote the infimum of the buyer's SPE payoff in any odd period $(2t+1)$, where the buyer makes an offer.

First we will derive necessary conditions for M_s^{2t} and m_b^{2t+1}. We can notice that for every $t \in \mathbb{N}$

$$-C \leq M_s^{2t} \leq S+D-C, \quad R-D \leq m_b^{2t+1} \leq R+S$$

We have the following necessary conditions.

Proposition 5.3. *For all $(\delta_{s,t})_{t\in\mathbb{N}}$, $(\delta_{b,t})_{t\in\mathbb{N}}$, $R \geq S \geq D \geq 0$, $0 < C \leq S+D$, and $t \in \mathbb{N}$,*

$$M_s^{2t} \leq S+D-C+\left(R-D-m_b^{2t+1}\right)\Delta_b(2t+1) \tag{5.1.25}$$

and

$$m_b^{2t+1} \geq R+S-\left(C+M_s^{2t+2}\right)\Delta_s(2t+2) \tag{5.1.26}$$

Proof. Consider an arbitrary even period $2t$. The seller makes either an unacceptable offer or an irresistible offer. If the buyer rejects the seller's offer, then he will get at least $(R-D)(1-\Delta_b(2t+1))+m_b^{2t+1}\Delta_b(2t+1)$. Hence, the seller gets at most $R+S-C-(R-D)(1-\Delta_b(2t+1))-m_b^{2t+1}\Delta_b(2t+1)$ from making the least acceptable offer. Alternatively, the seller gets at most $-C(1-\Delta_s(2t+1))+(R+S-C-m_b^{2t+1})\Delta_s(2t+1)$ from making an unacceptable offer. Hence, we get

$$M_s^{2t} \leq \max\begin{cases} R+S-C-(R-D)(1-\Delta_b(2t+1)-m_b^{2t+1}\Delta_b(2t+1) \\ -C(1-\Delta_s(2t+1))+(R+S-C-m_b^{2t+1})\Delta_s(2t+1) \end{cases} \tag{5.1.27}$$

which can be equivalently written as

$$M_s^{2t} \leq \max\begin{cases} S+D-C+\left(R-D-m_b^{2t+1}\right)\Delta_b(2t+1) \\ -C+\left(R+S-m_b^{2t+1}\right)\Delta_s(2t+1) \end{cases} \tag{5.1.28}$$

which leads to

$$M_s^{2t} \leq \begin{cases} S + D - C + \left(R - D - m_b^{2t+1}\right) \Delta_b(2t+1) & \text{if (5.1.30)} \\ -C + \left(R + S - m_b^{2t+1}\right) \Delta_s(2t+1) & \text{otherwise} \end{cases} \tag{5.1.29}$$

where

$$S(1-\Delta_s(2t+1))+D(1-\Delta_b(2t+1)) \geq \left(R - m_b^{2t+1}\right)(\Delta_s(2t+1) - \Delta_b(2t+1)) \tag{5.1.30}$$

However, we can show that (5.1.30) always holds.

Let $\Delta_s(2t+1) \leq \Delta_b(2t+1)$. We know that $-S \leq R-m_b^{2t+1} \leq D$. If $0 \leq R-m_b^{2t+1} \leq D$, then the right hand side of (5.1.30) is not positive. Hence, since the left hand side of (5.1.30) is not negative, (5.1.30) holds. If $-S \leq R - m_b^{2t+1} < 0$, then we have $0 \leq \left(R - m_b^{2t+1}\right)(\Delta_s(2t+1) - \Delta_b(2t+1)) \leq -S(\Delta_s(2t+1) - \Delta_b(2t+1)) = S(\Delta_b(2t+1) - \Delta_s(2t+1)) \leq S(1 - \Delta_s(2t+1)) \leq S(1 - \Delta_s(2t+1)) + D(1 - \Delta_b(2t+1))$, and therefore (5.1.30) also holds.

Let $\Delta_s(2t+1) > \Delta_b(2t+1)$. If $-S \leq R - m_b^{2t+1} < 0$, then the right hand side of (5.1.30) is negative, and therefore (5.1.30) holds, since the left hand side of (5.1.30) is not negative. If $0 \leq R - m_b^{2t+1} \leq D$, then we have $0 \leq \left(R - m_b^{2t+1}\right)(\Delta_s(2t+1) - \Delta_b(2t+1)) \leq D(\Delta_s(2t+1) - \Delta_b(2t+1)) \leq D(1 - \Delta_b(2t+1)) \leq S(1 - \Delta_s(2t+1)) + D(1 - \Delta_b(2t+1))$, and therefore (5.1.30) also holds.

Consider an arbitrary odd period $2t+1$. The buyer makes either an unacceptable offer or an irresistible offer. If the seller rejects the buyer's offer, then he will get at most $-C(1-\Delta_s(2t+2))+M_s^{2t+2}\Delta_s(2t+2)$. Hence, the buyer gets at least $R+S-C+C(1-\Delta_s(2t+2)) - M_s^{2t+2}\Delta_s(2t+2)$ from making the least irresistible offer. Alternatively, the buyer gets at least $(R-D)(1-\Delta_b(2t+2)) + (R+S-C-M_s^{2t+2})\Delta_b(2t+2)$ from making an unacceptable offer. Hence, we get

$$m_b^{2t+1} \geq \max \begin{cases} R + S - (C + M_s^{2t+2})\Delta_s(2t+2) \\ R - D + (S + D - C - M_s^{2t+2})\Delta_b(2t+2) \end{cases} \tag{5.1.31}$$

which leads to

$$m_b^{2t+1} \geq \begin{cases} R + S - (C + M_s^{2t+2})\,\Delta_s(2t+2) & \text{if (5.1.33)} \\ R - D + (S + D - C - M_s^{2t+2})\,\Delta_b(2t+2) & \text{otherwise} \end{cases} \tag{5.1.32}$$

where

$$(S + D)\,(1 - \Delta_b(2t+2)) \geq \left(C + M_s^{2t+2}\right)(\Delta_s(2t+2) - \Delta_b(2t+2)) \tag{5.1.33}$$

However, note that (5.1.33) is always satisfied, since $S+D \geq C+M_s^{2t+2}$ and $1-\Delta_b(2t+2) \geq \Delta_s(2t+2) - \Delta_b(2t+2)$. This completes the proof. □

It appears that under SPE neither the seller nor the buyer makes an unacceptable offer, as making the least irresistible offer gives always a higher payoff than proposing an unacceptable offer.

Next, from Proposition 5.3 we will calculate M_s^{2t} and m_b^{2t+1} for $t \in \mathbb{N}$.

Proposition 5.4. *For all* $(\delta_{s,t})_{t\in\mathbb{N}}$, $(\delta_{b,t})_{t\in\mathbb{N}}$, $R \geq S \geq D \geq 0$, $0 < C \leq S+D$, *and* $t \in \mathbb{N}$,

$$M_s^{2t} = (S+D)\left(1-\Delta_b(2t+1)+\sum_{m=t}^{\infty}(1-\Delta_b(2m+3))\prod_{j=t}^{m}\Delta_s(2j+2)\Delta_b(2j+1)\right)-C \tag{5.1.34}$$

$$m_b^{2t+1} = R+S-(C+M_s^{2t+2})\Delta_s\,(2t+2) \tag{5.1.35}$$

Proof. When looking for the upper bound of M_s^{2t} and the lower bound of m_b^{2t+1}, we need to solve the following infinite system: for each $t \in \mathbb{N}$

$$M_s^{2t} = S+D-C+\left(R-D-m_b^{2t+1}\right)\Delta_b(2t+1)$$

and

$$m_b^{2t+1} = R+S-\left(C+M_s^{2t+2}\right)\Delta_s(2t+2)$$

Hence, we get immediately (5.1.35), and if $-C \leq M_s^{2t} \leq S+D-C$, then $R-D \leq m_b^{2t+1} \leq R+S$. Furthermore, we have

$$\begin{bmatrix} 1 & \Delta_b(1) & 0 & 0 & \cdots \\ 0 & 1 & \Delta_s(2) & 0 & \cdots \\ 0 & 0 & 1 & \Delta_b(3) & \cdots \\ 0 & 0 & 0 & 1 & \ddots \\ \vdots & \vdots & \vdots & \vdots & \cdots \end{bmatrix}\begin{bmatrix} M_s^0 \\ m_b^1 \\ M_s^2 \\ m_b^3 \\ \vdots \end{bmatrix} = \begin{bmatrix} S+D-C+(R-D)\Delta_b(1) \\ R+S-C\Delta_s(2) \\ S+D-C+(R-D)\Delta_b(3) \\ R+S-C\Delta_s(4) \\ \vdots \end{bmatrix}$$

$$\begin{bmatrix} M_s^0 \\ m_b^1 \\ M_s^2 \\ m_b^3 \\ \vdots \end{bmatrix} = B\begin{bmatrix} S+D-C+(R-D)\Delta_b(1) \\ R+S-C\Delta_s(2) \\ S+D-C+(R-D)\Delta_b(3) \\ R+S-C\Delta_s(4) \\ \vdots \end{bmatrix}$$

where

$$B = \begin{bmatrix} 1 & -\Delta_b(1) & \Delta_b(1)\Delta_s(2) & -\Delta_b(1)\Delta_s(2)\Delta_b(3) & \cdots \\ 0 & 1 & -\Delta_s(2) & \Delta_s(2)\Delta_b(3) & \cdots \\ 0 & 0 & 1 & -\Delta_b(3) & \cdots \\ 0 & 0 & 0 & 1 & \ddots \\ \vdots & \vdots & \vdots & \vdots & \cdots \end{bmatrix}$$

which gives us (5.1.34). Obviously, $M_s^{2t} \geq -C$, and similarly to the proof of Proposition 5.2, one can show that $M_s^{2t} \leq S + D - C$. □

Remark 5.4. Note that M_s^{2t} and m_b^{2t+1} calculated in Proposition 5.4 coincide with the results presented in Proposition 5.2 on the prices offered under the SPE with no-delay. Indeed, by combining Propositions 5.2 and 5.4 we get for each $t \in \mathbb{N}$,

$$M_s^{2t} = P_s^{2t} - C \text{ and } m_b^{2t+1} = R + S - P_b^{2t+1}$$

Consequently, the no-delay equilibrium strategies (s_s, s_b) presented in Proposition 5.2 support the extreme payoffs M_s^{2t} and m_b^{2t+1}.

5.2 Pharmaceutical product price negotiation with discount factors varying in time

5.2.1 Introduction

Competition between the firms in the pharmaceutical industry yields many important economic issues to discuss. Protecting the high-cost Research and Development activities by a patent seems reasonable for firms to make higher profits with the determination of high prices, generic substitutes also threaten branded pharmaceutical firms in the market. Previous studies concern the determinants of pharmaceutical product prices, but they do not make a generalization of the characteristics of drug prices.

Bhattacharya and Vogt [2003] create a simple model of pharmaceutical price dynamics by analyzing the drug's life cycle. They find out that, in fact of generic entry, prices of pharmaceutical products rise. They also underline the effects of patent protections on prescription drugs and conclude that prices of branded products continue to rise although their patents expire. This effect is based on the product differentiation in the market place.

A comparison of different regulations on pharmaceutical product prices of various countries is analyzed by Danzon and Towse [2003]. They test the correlation between price competition and regulations of manufacturer prices and retail pharmacy margins. They find out that the price competition between generic competitors is significant in unregulated or less regulated markets such as the United States, United Kingdom, Canada and Germany, but strict regulation systems, such as France, Italy and Japan, reduce generic competitions. These results also verify the findings of Giaccotto et al. [2005].

More specifically, Reekie and Allen [1985] analyze the UK pharmaceutical industry for comparing generic and brand products. They argue that generic substitution of less regulated pharmaceutical industries could increase competitive pressure.

Difficulties on Research and Development in pharmaceutical industry is analyzed by Giaccotto et al. [2005]. They analyze theoretically and empirically the existence of a positive and direct relation between R&D spending and real drug prices. According to their model and simulations, drug price control regime restricts the new drugs and reduce R&D spending on pharmaceutical industry.

Virts and Weston [1980] works on the returns to R&D in the US pharmaceutical industry. In their study, they focus on two main issues that affect the rate of return: possible resource mis-allocation and the drug innovation environment. They give evidence of a decrease in expected return with strict regulations on pharmaceutical industry.

Danzon and Towse [2003] review the economic effects of patents and differential pricing for pharmaceuticals. Ellison et al. [1997] analyze more specifically the characteristics of demand side of pharmaceutical products by examining four special cephalosporins. They create a model for demand as a multistage budgeting problem and find out that there exist high elasticities between generic substitutes and significant elasticities between some therapeutic substitutes.

Morton [1999] studies the entry decisions in generic pharmaceutical industry and uses drug entries in the period 1984-1994 to estimate the potential entrants. She argues that the market with more hospital sales, larger revenue markets and generics for chronic conditions attract more firms to enter the generic pharmaceutical industry.

Pavcnik [2002] analyzes the potential patient out-of-pocket expenses. For understanding the impact of patient reimbursement on price determination, the author uses a unique policy experiment from Germany. She gives some evidences of significant decreases on the pharmaceutical product prices, mostly for brand-name products, after the change in potential out-of-pocket expenses.

Patient's co-payment for buying pharmaceuticals and the price of a patented drug is analyzed by Jelovac [2010]. In her paper, Nash bargaining model is used to explain the determination of pharmaceutical product between a health authority and a monopoly producer. Also, an optimal co-payment degree is determined in this study.

External referencing for the price determination of pharmaceuticals is another important point for health economics. Garcia Mariñoso et al. [2011] create a pricing mechanism with adoption of external referencing. Kanavos and Costa-Font [2005] study the effects of pharmaceutical parallel trade in European Union. Expectations in parallel trade is based on the reduction of prices paid by health insurance and consumers. But the evidences obtained from the study of Kanavos and Costa-Font [2005] show that the gain from parallel trade helps mostly the distributors rather than the consumers. Also they prove that there is no competition impact of parallel trade on prices.

In this section, we present a research proposal on a non-cooperative price bargaining model for pharmaceutical products between a health authority and a monopoly producer. We are going to investigate the model in details in our future research. The parties bargain according to the Rubinstein's sequential bargaining model. While Rubinstein [1982] assumes stationary preferences, they seem to be rather rare in real life situations (e.g., Cramton and Tracy [1994b]) and the necessity of using non-stationary preferences has been stressed in several works (see, e.g., Binmore [1987a], Coles and Muthoo [2003]).

Following the model by Jelovac [2010], we consider a monopolistic firm that produces a patented pharmaceutical product and a health authority, i.e., government. They negotiate the price of the the brand-name prescription drug. There is an existing price that has come up for renegotiation which specifies the price per unity of the drug. Two parties bargain over a discrete time and a potentially infinite horizon. They alternate in making offers of price for the prescription drug that the other party is free to accept or reject. An initial offer is made by the health authority. If the firm rejects the offer, then it is its turn to make a new offer. Upon either party's rejection of a proposed price, the health authority must decide whether to ban the drug from selling it in the domestic market, not to ban but also not to put it on the reimbursement list or to hold out and to put it on the list.

We use therefore Rubinstein's bargaining procedure (Rubinstein [1982]), but we generalize the model by using discount factors varying in time. The utility of each bargainer is given by the discounted sums of the payoffs from period 0 to infinity. More precisely, the utility of the health authority and of the firm is given by the discounted difference of consumer surplus and public expenses, and the discounted profit, respectively. We assume that the sequence of discount rates of a party can be arbitrary, with the only restriction that the infinite series that determines the utility for the given party must be convergent. Similarly to the wage bargaining analyzed in Ozkardas and Rusinowska [2014a], where the union can go on strike if an offer is rejected, in the pharmaceutical product model the health authority can make the banning decision. In order to analyze the price negotiation between a health authority and a monopolistic producer, we propose to apply our bargaining procedure to the model of Jelovac [2010].

Section 5.2.2 describes the pharmaceutical product price bargaining model with discount rates varying in time. Section 5.2.3 concerns the exogenous ban and reimbursement decisions of the health authority.

5.2.2 The model

We consider a price bargaining model of a pharmaceutical product between a health authority and a monopoly producer. The model is based on Jelovac [2010]. We assume that there is an existing price which has come up for renegotiation. Price of the drug is paid by the consumers and the health authority according to the degree of co-payment rate $\alpha \in [0, 1]$ where α is the proportion of the price paid by the consumer.

The demand function of the pharmaceutical product of the consumer is linear and it is equal to

$$q = a - \alpha p$$

where q is the demand of the consumers for the drug and p is the given price of this drug.

The objective of the health authority is to maximize the difference between the consumers surplus and public expenses. The public expenses of the health authority for the given drug is

$$PE(p) = (1-\alpha)\, pq = (1-\alpha)\, p\,(a - \alpha p)$$

and the consumers surplus is

$$CS(p) = I + \frac{1}{2}(a - \alpha p)^2$$

Hence, the objective function of the health authority is

$$OF(p) = \left(\frac{2\alpha - \alpha^2}{2}\right) p^2 - ap + \frac{a^2}{2} + I$$

and it has its minimum value at $\underline{p} = \frac{a}{\alpha(2-\alpha)}$.

On the other hand, the monopoly producer of the pharmaceutical product maximizes its profit, where the profit function is equal to

$$\Pi(p) = pq - F = ap - \alpha p^2 - F$$

F denotes the fix cost of the firm (R&D, advertising expenses, etc.) and we assume that there is no marginal cost of production. The monopolistic price of the drug is $p^M = \frac{a}{2\alpha}$.

Although $\underline{p} = \frac{a}{\alpha(2-\alpha)} > p^M = \frac{a}{2\alpha}$, we can assume that the monopolistic producer will not accept any price bigger than p^M. Hence, we can restrict our analysis to $p \in \left[0, p^M\right]$, where the objective function of the health authority is a decreasing function with the price of the pharmaceutical product. We have $OF(0) = I + \frac{a^2}{2}$ and $\Pi(0) = -F$ which gives us the maximum value for the health authority and the minimum value for the firm. On the other hand, $OF\left(p^M\right) = I + \frac{a^2(3\alpha-2)}{8\alpha}$ and $\Pi\left(p^M\right) = \frac{a^2}{4\alpha} - F$. We have therefore:

$$\text{for } p \in \left[0, p^M\right],\ OF(p) \in \left[I + \frac{a^2(3\alpha-2)}{8\alpha}, I + \frac{a^2}{2}\right] \text{ and } \Pi(p) \in \left[-F, \frac{a^2}{4\alpha} - F\right]$$

If the pharmaceutical product price is determined without any negotiation, then the firm sets the monopolistic price p^M to have the maximum profit. Without loss of generality, we can assume that the patients' surplus is greater or equal to the public expenses, and therefore for every $p \in \left[0, p^M\right]$, $I \geq \frac{a^2(2-3\alpha)}{8\alpha}$.

If the health authority uses different possible policies against the firm, then in particular it can ban the drug from the market or exclude it from the list of reimbursement. In case of the ban decision, the firm cannot sell its product on the market. Hence the profit and the public expenses will be $-F$ and $I + \frac{a^2}{2}$, respectively. On the other hand, if the health authority neither bans the drug nor puts it to the list of reimbursement, then the health authority will pay nothing for the drug which means that the co-payment rate will be $\alpha = 1$. In this case, the demand function of the consumers for this drug will be $q = a - p$, the objective function will be equal to $OF(p) = I + \frac{1}{2}(a-p)^2$ and the profit function will be $\Pi(p) = p(a-p)$. We get then $p^N = \frac{a}{2}$, $OF\left(p^N\right) = I + \frac{a^2}{8}$ and $\Pi\left(p^N\right) = \frac{a^2}{4} - F$.

The bargaining procedure between the health authority and the firm is the following. There is an existing price of the drug, here it is assumed p^M, and the parties negotiate for determining a price less then the monopolistic price. Both parties have complete information and they bargain sequentially over discrete time and a potentially infinite horizon. They make offers alternately and the other party is free to accept or to reject the offer. In case of a rejection, the health authority decides to make a sanction or not. It has two different sanctions: to ban the drug from the market by not allowing the firm to sell it, or to exclude the drug from the reimbursement list but to allow the firm to sell it in the market.

In the beginning of the bargaining, the health authority proposes $p_{h,0}$ to the firm. If the firm accepts the new price, then the agreement is reached and the payoffs for the health authority and the firm will be $OF\left(p_{h,0}\right)$ and $\Pi\left(p_{h,0}\right)$, respectively. If the firm rejects the offer, the health authority can either ban the prescription drug from the market and then the parties have payoffs $(I,0)$, or the health authority reimburses the drug with the existing price and the payoffs will be $\left(OF\left(p^{M}\right),\Pi\left(p^{M}\right)\right)$, or the health authority neither bans nor reimburses the prescription drug for this period and the payoffs will be $\left(OF\left(p^{N}\right),\Pi\left(p^{N}\right)\right)$. If there is no agreement in this period, then it is the firm's turn to make a new offer $p_{f,1}$ to the health authority in period 1. This procedure goes on until an agreement is reached, where $p_{h,2t}$ denotes the offer of the health authority made in an even-numbered period $2t$, and $p_{f,2t+1}$ denotes the offer of the firm made in an odd-numbered period $2t+1$.

We consider a bargaining in which preferences of the health authority and the firm are described by the sequence of discount factors varying in time, where $(\delta_{h,t})$ and $(\delta_{f,t})$ are the discount factors of the health authority and the firm in period $t \in \mathbb{N}$, respectively, and $\delta_{i,0}=1$, $0<\delta_{i,t}<1$ for $t \geq 1$ and $i=h,f$.

The result of the bargaining is either a pair (p,T), where p is the price agreed upon and $T \in \mathbb{N}$ is the number of proposals rejected during the bargaining, or a disagreement that gives the situation in which the parties never reach an agreement.

The utility of the result (p,T) for the health authority is equal to

$$U\left(p,T\right)=\sum_{t=0}^{\infty}\delta_{h}\left(t\right)u_{t}$$

where for each $t \in \mathbb{N}$

$$\delta_{h}\left(t\right):=\prod_{k=0}^{t}\delta_{h,k} \text{ and } \delta_{f}\left(t\right):=\prod_{k=0}^{t}\delta_{f,k}$$

and we have

$u_{t}=OF\left(p\right)=p^{2}\left(\frac{2\alpha-\alpha^{2}}{2}\right)-ap+\frac{a^{2}}{2}+I$ for each $t \geq T$, and if $T>0$ then for each

$0 \leq t < T$
$u_t = I + \frac{a^2}{2}$ if the health authority bans the prescription drug in period $t \in \mathbb{N}$,
$u_t = OF\left(p^N\right) = I + \frac{a^2}{8}$ if the health authority neither bans nor lists the prescription drug for reimbursement in period $t \in \mathbb{N}$,
$u_t = OF\left(p^M\right) = I + \frac{a^2(3\alpha-2)}{8\alpha}$ if the health authority holds out in period $t \in \mathbb{N}$.

The utility of the result (P, T) for the firm is equal to

$$V\left(p, T\right) = \sum_{t=0}^{\infty} \delta_f\left(t\right) v_t - F$$

where we have
$v_t = \Pi\left(p\right) = -\alpha p^2 + ap$ for each $t \geq T$, and if $T > 0$ then for each $0 \leq t < T$
$v_t = 0$ if the health authority bans the prescription drug in period $t \in \mathbb{N}$,
$v_t = \Pi\left(p^N\right) = \frac{a^2}{4}$ if the health authority neither bans not lists the prescription drug for reimbursement in period $t \in \mathbb{N}$,
$v_t = \Pi\left(p^M\right) = \frac{a^2}{4\alpha}$ if the health authority accepts to reimburse the prescription drug with the existing price in period $t \in \mathbb{N}$.

For simplicity we assume that the firm pays the cost F only once, in period 0. The disagreement is assumed to be the worst result both for the health authority and the firm.

We consider the family of strategies (s_h, s_f) where: in each period $2t$ the health authority proposes $p_{h,2t}$, in each period $2t+1$ it accepts an offer y of the firm if and only if $y \geq p_{f,2t+1}$; and in each period $2t+1$ the firm proposes $p_{f,2t+1}$, in each period $2t$ it accepts an offer x of the health authority if and only if $x \leq p_{h,2t+1}$. A strategy of the health authority specifies additionally the ban and reimbursement decision.

Furthermore, for every $t \in \mathbb{N}_+$

$$\Delta_h\left(t\right) := \frac{\sum_{k=t}^{\infty} \delta_h\left(t, k\right)}{1 + \sum_{k=t}^{\infty} \delta_h\left(t, k\right)} \text{ and } \Delta_f\left(t\right) := \frac{\sum_{k=t}^{\infty} \delta_f\left(t, k\right)}{1 + \sum_{k=t}^{\infty} \delta_f\left(t, k\right)}.$$

5.2.3 Exogenous ban and reimbursement decisions

Suppose that the health authority makes one of the three alternative decisions. Firstly, the health authority bans the prescription drug from the market in every disagreement period, i.e., it does not allow the firm to sell the drug. Secondly, the health authority does not ban it from the market but refuses to reimburse the drug by excluding it from the reimbursement list. In such a situation, patients need to pay the whole price ($\alpha = 1$) and this affects the demand function of the prescription drug. Lastly, the health authority is supposed to accept the monopoly price and reimbursement during the disagreement periods. For each of these three cases, we describe the infinite system of equations on the profits of the firm and the objective functions of the health authority. We leave the analysis of the systems and the more detailed study of the model for further research.

If we assume that the ban decision of the health authority is exogenously given and the health authority is supposed to ban the prescription drug in every period in which there is a disagreement, then for the analysis of the SPE of the form (s_h, s_f), we get the following infinite system of equations, for each $t \in \mathbb{N}$:

$$\Pi\left(p_{f,2t+1}\right) = \Pi\left(p_{h,2t+2}\right)\Delta_f\left(2t+2\right)$$

and

$$OF\left(p_{h,2t}\right) = \left(I + \frac{a^2}{2}\right)\left(1 - \Delta_h\left(2t+1\right)\right) + OF\left(p_{f,2t+1}\right)\Delta_h\left(2t+1\right)$$

After replacing the profit function of the firm and the objective function of the health authority by the corresponding formulas, we get the following infinite system of equations, for each $t \in \mathbb{N}$:

$$ap_{f,2t+1} - \alpha p_{f,2t+1}^2 = \left(ap_{h,2t+2} - \alpha p_{h,2t+2}^2\right)\Delta_f(2t+2)$$

and

$$\left(2\alpha - \alpha^2\right)p_{h,2t}^2 - 2ap_{h,2t} = \left(2\alpha - \alpha^2\right)\Delta_h(2t+1)p_{f,2t+1}^2 - 2a\Delta_h(2t+1)p_{f,2t+1}$$

Next, assume that the ban decision of the health authority is exogenously given but the health authority is supposed neither to ban the prescription drug nor to list it for reimbursement in every period in which there is a disagreement. Then we get the following infinite system of equations, for each $t \in \mathbb{N}$:

$$\Pi\left(p_{f,2t+1}\right) = \Pi\left(p^N\right)\left(1 - \Delta_f\left(2t+2\right)\right) + \Pi\left(p_{h,2t+2}\right)\Delta_f\left(2t+2\right)$$

and

$$OF\left(p_{h,2t}\right) = OF\left(p^N\right)\left(1 - \Delta_h\left(2t+1\right)\right) + OF\left(p_{f,2t+1}\right)\Delta_h\left(2t+1\right)$$

which after replacing the profit function and the objective function by the corresponding formulas leads to the following infinite system of equations, for each $t \in \mathbb{N}$:

$$ap_{f,2t+1} - \alpha p_{f,2t+1}^2 = \frac{a^2}{4}\left(I - \Delta_f(2t+2)\right) + \left(ap_{h,2t+2} - \alpha p_{h,2t+2}^2\right)\Delta_f(2t+2)$$

and

$$\left(2\alpha-\alpha^2\right)p_{h,2t}^2-2ap_{h,2t}a^2\left(1-\frac{5}{4}\Delta_h(2t+1)\right)=$$
$$\left(2\alpha-\alpha^2\right)\Delta_h(2t+1)p_{f,2t+1}^2-2a\Delta_h(2t+1)p_{f,2t+1}$$

In the third case, it is assumed that the ban decision of the health authority is exogenously given and the health authority is supposed to accept the monopolistic price and to make reimbursement in every period in which there is a disagreement. We have then, for each $t\in\mathbb{N}$

$$\Pi\left(p_{f,2t+1}\right)=\Pi\left(p^M\right)\left(1-\Delta_f\left(2t+2\right)\right)+\Pi\left(p_{h,2t+2}\right)\Delta_f\left(2t+2\right)$$

and

$$OF\left(p_{h,2t}\right)=OF\left(p^M\right)\left(1-\Delta_h\left(2t+1\right)\right)+OF\left(p_{f,2t+1}\right)\Delta_h\left(2t+1\right)$$

and hence, for each $t\in\mathbb{N}$

$$ap_{f,2t+1}-\alpha p_{f,2t+1}^2=\frac{a^2}{4\alpha}\left(I-\Delta_f(2t+2)\right)+\left(ap_{h,2t+2}-\alpha p_{h,2t+2}^2\right)\Delta_f(2t+2)$$

and

$$\left(2\alpha-\alpha^2\right)p_{h,2t}^2-2ap_{h,2t}+\frac{a^2(\alpha+2)}{4\alpha}\left(1-\Delta_h(2t+1)\right)=$$
$$\left(2\alpha-\alpha^2\right)\Delta_h(2t+1)p_{f,2t+1}^2-2a\Delta_h(2t+1)p_{f,2t+1}.$$

5.3 Concluding remarks

We applied the generalized wage bargaining model with varying discount rates to the important economic issues – price negotiation and pharmaceutical product price determination. Many of the previous studies in the literature on price negotiations focus on determining the reference points and did not reveal the optimal price between sellers and buyers. Although we made some restrictions in our model, we determined both the price level and the reference points that have impact on the price negotiation. We used complete information and sequential bargaining procedure where the preferences of the seller and the buyer vary in time. Using varying discount factors gives more possibilities for the characteristics of the parties and makes the model more realistic. Although preferences of the individuals may be constant while buying many consumption goods, for rare and/or privileged goods the parties' patience levels and preferences may vary during negotiations. Also some economic and social changes caused, for instance, by climate changes, epidemic increase, varying fashion requirements, make the preferences vary in time. Our generalized framework is therefore more suitable to model real-life situations.

Our results concern determining the unique SPE for no-delay strategies independent of the former history of the game and determining the equilibrium extreme payoffs of the seller and the buyer for the general case, i.e., without the restriction to no-delay strategies. It appears that the no-delay equilibrium strategy profiles support these extreme payoffs. Under equilibrium, neither the seller nor the buyer makes an unacceptable offers.

Furthermore, we presented our future research project in which we are going to investigate the sequential bargaining procedure in the model of Jelovac [2010]. Although the drug market is quite complex, applying our model to pharmaceutical price negotiations can help to get a deeper insight into such negotiations. In the pharmaceutical product market, there are two main parties that negotiate for the price: state or an agency that represents the state and a firm that produces the drug. Although the marginal cost of drug production is very low, R&D expenses are relatively high in comparison with the other markets. Most of the patented drugs are produced only by one firm that creates a monopole in the market. Considering discount rates varying in time is particularly important in the drug market, where the consumers' patience levels vary according to the urgency of their illnesses and the producers' patience levels vary according to the risk of losing the market despite the high R&D expenses.

Since the health authority has different types of sanctions to the firm for reducing the price, depending on the discount rates of the parties and the patients' co-payment rates, one can investigate the best strategy for the health authority to reduce the public expenses and to increase the patients' consumer surplus. On the other hand, the firm maximizes its profit according to the health authority's sanction decisions.

Chapter 6

Conclusions

The thesis provides the original contributions to the literature on wage bargaining by introducing discount factors varying in time to the union-firm wage bargaining models with different strike decisions of the union and with the lockout decision of the firm. The generalized framework models real life situations in a more accurate way and the results of the model give more insight into the collective wage negotiations.

First, in Chapter 2, we delivered an overview of different approaches to bargaining (the axiomatic approach initiated by Nash [1950] and the dynamic approach by Rubinstein [1982]) and the wage bargaining models investigated, e.g., in Fernandez and Glazer [1991], Haller and Holden [1990], Holden [1994], Houba and Wen [2008].

Secondly, in Chapter 3, we showed the importance of the generalized wage bargaining to model real life situations and investigated the wage bargaining with preferences varying in time. We analyzed the SPE in the union-firm wage bargaining and determined the SPE payoffs of the parties. First, we considered three games in this generalized setup, where the union strike decision is taken as exogenous: the case where the union is committed to strike in each period in which there is a disagreement, the case where the union is committed to strike only when its own offer is rejected, and the case where the union never strikes. We determined SPE for these games and compared the results among the three cases of the exogenous strike decisions. Afterwards, we investigated the general model with no assumption on the commitment to strike. We found SPE in which the strategies supporting the equilibria in the exogenous cases are combined with the minimum-wage strategies, provided that the union is sufficiently patient. We showed that if the firm is more patient than the union, then the firm is better off by playing the no-concession strategy, under which it rejects all offers and always makes an unacceptable offer. After determining the SPE of the general wage bargaining model, we generalized the method used in Houba and Wen [2008] and applied it to our model in order to find the supremum of the SPE payoffs of the union and the infimum of the SPE payoffs of the firm in the wage bargaining with discount rates varying in time. At the end of Chapter 3, we showed that there exist inefficient SPE in the model where the union strikes for uninterrupted T periods prior to reaching a final agreement.

In Chapter 4 we analyzed the extensions. First, we examined the union's hold out threats on wage determination, such as go-slow, with the parties' preferences varying in time. Then we considered a model in which the firm is allowed to engage in lockouts. In order to apply the go-slow strategies, we considered two different attitudes of the union, either hostile or altruistic, and we determined the SPE of wage bargaining depending on these attitudes. Next, we generalized the method used in Houba and Wen [2008] to the case when the strikes are not allowed and the union can threaten the firm with being on go-slow. We examined the game in which the firm can lock out the union. We determined the SPE payoff with an immediate agreement which yields the union a wage contract smaller than the existing wage contract. Under this equilibrium the firm always locks out the union after its own offer is rejected and holds out after rejecting the union's offer.

In Chapter 5, we applied the generalized wage bargaining model to real life problems such as price bargaining and presented our project on pharmaceutical product price negotiations. Firstly, we considered the price bargaining model in which there exists a monopolistic seller that sells a unique and indivisible good in a market with only one buyer. We determined the unique SPE of the model, when we restrict the analysis to history independent strategies with no delay. Then we relaxed the no-delay assumption and determined the highest equilibrium payoff of the seller and the lowest equilibrium payoff of the buyer for the general case. We showed that the no-delay equilibrium strategy profiles support these extreme payoffs. Finally, we propose to apply our generalized wage bargaining model with discount rates varying in time to the pharmaceutical product price negotiation. In this application, we consider the model in which the monopolistic drug producer and the heath authority bargain over the price of a patented drug. Differently from other studies, we again introduce discount factors varying in time to model the price determination. Health authority has an objective to increase the patients' surplus and to reduce public expenses, and in order to achieve its objective it uses several threats against the firm such as banning the drug or listing it out from the reimbursement.

Apart from the pharmaceutical product price negotiations, our future research agenda contains several more projects on wage bargaining. For further investigations of the bargaining model with varying discount rates, it could be of interest and importance to consider some other extensions and applications of this framework. While we considered a model with lockouts but with no strikes, we intend to examine a game in which both strikes of the union and the lockouts of the firm are allowed. Our conjecture is that it is possible to generate SPE in this game in which strikes alternate with lockouts before a final agreement. Furthermore, we could extend our model with go-slow option in which strikes are not allowed to the model in which the union can use both the go-slow and strikes threats. Also combining the lockouts, strikes and holdouts options in one model could lead to an interesting generalization of the models analyzed in this thesis.

Fernandez and Glazer [1991] mention multiple contract renegotiations as a possible extension of their model with constant discount rates. It would be interesting to investigate a similar extension of the model with discount rates varying in time and to allow for contracts that are repeatedly (potentially infinitely) renegotiated. For instance, one could suppose that contracts are periodically renegotiated every T periods after a contract has been established.

Several works concern the issues of bargaining power, both in the standard bargaining models and in the wage bargaining with constant discount rates. Since discount rates are usually crucial in determining bargaining power of parties, it would be important to study these issues in our framework with discount rates varying in time.

While we applied the generalized model to the price bargaining with one seller and one buyer, one could try to investigate a similar model with discount rates varying in time but with more than two parties.

One could also apply the model to political negotiations between governments or to negotiations on common usage of public goods. Also empirical studies could give better understanding of the wage determination in collective wage bargaining. Although the determination of varying discount rates in real life situations could be complicated, with a proper data set it might be possible to calculate exact bounds of wage levels.

Bibliography

Alhassan G Abdul-Muhmin. The effect of perceived seller reservation prices on buyers' bargaining behavior in a flexible-price market. *Journal of International Consumer Marketing*, 13(3):29–45, 2001.

Jayanta Bhattacharya and William B Vogt. A simple model of pharmaceutical price dynamics. *Journal of Law and Economics*, 46(2):599–626, 2003.

Kenneth G Binmore. *The Economics of Bargaining*, volume 1. Oxford Blackwell, 1987a.

Kenneth G Binmore. Perfect equilibria in bargaining models. In K. Binmore and P. Dasgupta, editors, *The Economics of Bargaining*, pages 77–105. Oxford: Blackwell, 1987b.

Kenneth G Binmore, Ariel Rubinstein, and Asher Wolinsky. The Nash bargaining solution in economic modelling. *The RAND Journal of Economics*, 17:176–188, 1986.

Kenneth G Binmore, Martin J Osborne, and Ariel Rubinstein. *Noncooperative Models of Bargaining.* Tel Aviv Univ., The Foerder Inst. for Economic Research, 1990.

Wilko Bolt. Striking for a bargain between two completely informed agents: Comment. *The American Economic Review*, 85(5):1344–1347, 1995.

Timothy N Cason, Daniel Friedman, and Garrett H Milam. Bargaining versus posted price competition in customer markets. *International Journal of Industrial Organization*, 21(2):223–251, 2003.

Melvyn G Coles and Abhinay Muthoo. Bargaining in a non-stationary environment. *Journal of Economic Theory*, 109(1):70–89, 2003.

Peter Cramton. Dynamic bargaining with transaction costs. *Management Science*, 37 (10):1221–1233, 1991.

Peter Cramton and Joseph Tracy. Strikes and holdouts in wage bargaining: theory and data. *The American Economic Review*, 82(1):100–121, 1992.

Peter Cramton and Joseph Tracy. The determinants of US labor disputes. *Journal of Labor Economics*, 12:180–209, 1994a.

Peter Cramton and Joseph Tracy. Wage bargaining with time-varying threats. *Journal of Labor Economics*, 12:594–617, 1994b.

Vincent P Crawford. A theory of disagreement in bargaining. *Econometrica*, 50:607–637, 1982.

Patricia M Danzon and Adrian Towse. Differential pricing for pharmaceuticals: reconciling access, R&D and patents. *International Journal of Health Care Finance and Economics*, 3(3):183–205, 2003.

Giuseppe De Marco and Jacqueline Morgan. Friendliness and reciprocity in equilibrium selection. *International Game Theory Review*, 10(01):53–72, 2008.

Giuseppe De Marco and Jacqueline Morgan. Altruistic behavior and correlated equilibrium selection. *International Game Theory Review*, 13(04):363–381, 2011.

David L Dickinson. Illustrated examples of the effects of risk preferences and expectations on bargaining outcomes. *The Journal of Economic Education*, 34(2):169–180, 2003.

Sara Fisher Ellison, Iain Cockburn, Zvi Griliches, and Jerry Hausman. Characteristics of demand for pharmaceutical products: an examination of four cephalosporins. *The RAND Journal of Economics*, pages 426–446, 1997.

Francesco Feri and Anita Gantner. Bargaining or searching for a better price? An experimental study. *Games and Economic Behavior*, 72(2):376–399, 2011.

Raquel Fernandez and Jacob Glazer. Striking for a bargain between two completely informed agents. *American Economic Review*, 81(1):240–52, 1991.

Javier Fernández-Blanco. A directed search model of intermediated trade. *European Economic Review*, 56:1481–1494, 2012.

Peter C Fishburn and Ariel Rubinstein. Time preferences. *International Economic Review*, 23:667–694, 1982.

Drew Fudenberg, David Levine, and Paul Ruud. Strike activity and wage settlements. *Unpublished. University of California, Berkeley, CA*, 1985.

Begoña Garcia Mariñoso, Izabela Jelovac, and Pau Olivella. External referencing and pharmaceutical price negotiation. *Health Economics*, 20(6):737–756, 2011.

Carmelo Giaccotto, Rexford E Santerre, and John A Vernon. Drug prices and research and development investment behavior in the pharmaceutical industry. *Journal of Law and Economics*, 48(1):195–214, 2005.

Faruk Gul and Hugo Sonnenschein. On delay in bargaining with one-sided uncertainty. *Econometrica*, 56(3):601–611, 1988.

Hans Haller and Steinar Holden. A letter to the editor on wage bargaining. *Journal of Economic Theory*, 52(1):232–236, 1990.

Olivier Hart. Bargaining and strikes. *The Quarterly Journal of Economics*, 104(1): 25–43, 1989.

P Jean-Jacques Herings and Arkadi Predtetchinski. Sequential share bargaining. *International Journal of Game Theory*, 41(2):301–323, 2012.

Steinar Holden. Bargaining and commitment in a permanent relationship. *Games and Economic Behavior*, 7(2):169–176, 1994.

Harold Houba and Quan Wen. Different time preferences and non-stationary contracts in negotiations. *Economics Letters*, 91(2):273–279, 2006.

Harold Houba and Quan Wen. On striking for a bargain between two completely informed agents. *Economic Theory*, 37(3):509–519, 2008.

Harold Houba and Quan Wen. Extreme equilibria in the negotiation model with different time preferences. *Games and Economic Behavior*, 73(2):507–516, 2011.

Harold Houba and Quan Wen. Backward induction and unacceptable offers. *Journal of Mathematical Economics*, 2014, Forthcoming.

Peter Ingram, David Metcalf, and Jonathan Wadsworth. Strike incidence in British manufacturing in the 1980s. *Industrial and Labor Relations Review*, pages 704–717, 1993.

Izabela Jelovac. On the relationship between the negociated price of pharmaceuticals and the patients' co-payment. In *Séminaire GAEL, Grenoble, 24 juin 2010*, 2010.

Daniel Kahneman. Reference points, anchors, norms, and mixed feelings. *Organizational Behavior and Human Decision Processes*, 51(2):296–312, 1992.

Daniel Kahneman and Amos Tversky. Prospect theory: An analysis of decision under risk. *Econometrica*, 47:263–291, 1979.

Panos Kanavos and Joan Costa-Font. Pharmaceutical parallel trade in Europe: stakeholder and competition effects. *Economic Policy*, 20(44):751–798, 2005.

John Kennan and Robert Wilson. Strategic bargaining models and interpretation of strike data. *Journal of Applied Econometrics*, 4(S1):S87–S130, 1989.

John Kennan and Robert Wilson. Bargaining with private information. *Journal of Economic Literature*, 31(1):45–104, 1993.

Henrik Kristensen and Tommy Gaerling. Determinants of buyers' aspiration and reservation price. *Journal of Economic Psychology*, 18(5):487–503, 1997.

Seungwoo Kwon, Byungjoon Yoo, Jinbae Kim, Wei Shang, and Gunwoong Lee. Reservation price reporting mechanisms for online negotiations. *Decision Support Systems*, 46(4):755–762, 2009.

Karl O Moene. Union's threats and wage determination. *The Economic Journal*, 98 (391):471–483, 1988.

Karl Ove Moene and Michael Wallerstein. Pay inequality. *Journal of Labor Economics*, 15(3):403–430, 1997.

Fiona M Scott Morton. Entry decisions in the generic pharmaceutical industry. *The RAND Journal of Economics*, pages 421–440, 1999.

Sonia Moulet and Juliette Rouchier. The influence of seller learning and time constraints on sequential bargaining in an artificial perishable goods market. *Journal of Economic Dynamics and Control*, 32(7):2322–2348, 2008.

Abhinay Muthoo. *Bargaining Theory with Applications*. Cambridge University Press, 1999.

John F Nash. The bargaining problem. *Econometrica*, 18:155–162, 1950.

Gregory B Northcraft and Margaret A Neale. Experts, amateurs, and real estate: An anchoring-and-adjustment perspective on property pricing decisions. *Organizational Behavior and Human Decision Processes*, 39(1):84–97, 1987.

Edward Oczkowski. An econometric analysis of the bilateral monopoly model. *Economic Modelling*, 16(1):53–69, 1999.

Martin J Osborne and Ariel Rubinstein. *Bargaining and Markets*, volume 34. Academic Press San Diego, 1990.

Ahmet Ozkardas and Agnieszka Rusinowska. An application of wage bargaining to price negotiation with discount factors varying in time. *Journal of Business Economics and Political Science*, 2(3):83–102, 2013.

Ahmet Ozkardas and Agnieszka Rusinowska. Wage bargaining with discount rates varying in time under different strike decisions. *RAIRO-Operations Research*, 48(3): 325–348, 2014a.

Ahmet Ozkardas and Agnieszka Rusinowska. Inefficient equilibria and lockouts in wage bargaining with discount rates varying in time. 2014b. Working Paper, Centre d'Economie de la Sorbonne.

Ahmet Ozkardas and Agnieszka Rusinowska. Holdout threats of the union during the wage bargaining. 2014c. Working Paper, Centre d'Economie de la Sorbonne.

Ahmet Ozkardas and Agnieszka Rusinowska. On equilibrium payoffs in wage bargaining with discount rates varying in time. *Economic Theory Bulletin*, Forthcoming.

Nina Pavcnik. Do pharmaceutical prices respond to potential patient out-of-pocket expenses? *RAND Journal of Economics*, pages 469–487, 2002.

W Duncan Reekie and DE Allen. Generic substitution in the UK pharmaceutical industry: a Markovian analysis. *Managerial and Decision Economics*, 6(2):93–101, 1985.

Alvin E Roth and John Henry Kagel. *The Handbook of Experimental Economics*, volume 1. Princeton University Press Princeton, 1995.

Ariel Rubinstein. Perfect equilibrium in a bargaining model. *Econometrica*, 50:97–109, 1982.

Ariel Rubinstein and Asher Wolinsky. Equilibrium in a market with sequential bargaining. *Econometrica*, 53(5):1133–1150, 1985.

Agnieszka Rusinowska. Bargaining problem with non-stationary preferences of the players. Ph.D. dissertation (written in Polish), 2000.

Agnieszka Rusinowska. On certain generalization of Rubinstein's bargaining model. In L. A. Petrosjan and V. V. Mazalov, editors, *Game Theory and Applications*, volume 8, pages 159–169. Nova Science Publishers, 2001.

Agnieszka Rusinowska. Refinements of Nash equilibria in view of jealous or friendly behavior of players. *International Game Theory Review*, 4(03):281–299, 2002a.

Agnieszka Rusinowska. Subgame perfect equilibria in model with bargaining costs varying in time. *Mathematical Methods of Operations Research*, 56(2):303–313, 2002b.

Agnieszka Rusinowska. Bargaining model with sequences of discount rates and bargaining costs. *International Game Theory Review*, 6(02):265–280, 2004.

Avner Shaked and John Sutton. Involuntary unemployment as a perfect equilibrium in a bargaining model. *Econometrica*, 52(6):1351–1364, 1984.

Joydeep Srivastava, Dipankar Chakravarti, and Amnon Rapoport. Price and margin negotiations in marketing channels: An experimental study of sequential bargaining under one-sided uncertainty and opportunity cost of delay. *Marketing Science*, 19(2): 163–184, 2000.

Daniel Trefler. Bargaining with asymmetric information in non-stationary markets. *Economic Theory*, 13(3):577–601, 1999.

Amos Tversky and Daniel Kahneman. Loss aversion in riskless choice: A reference-dependent model. *The Quarterly Journal of Economics*, 106(4):1039–1061, 1991.

Amos Tversky and Daniel Kahneman. Advances in prospect theory: cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, 5(4):297–323, 1992.

Dirk Van Poucke and Marc Buelens. Predicting the outcome of a two-party price negotiation: Contribution of reservation price, aspiration price and opening offer. *Journal of Economic Psychology*, 23(1):67–76, 2002.

Juan J Vidal-Puga. Delay in the alternating-offers model of bargaining. *International Journal of Game Theory*, 37(4):457–474, 2008.

John R Virts and J Fred Weston. Returns to research and development in the US pharmaceutical industry. *Managerial and Decision Economics*, 1(3):103–111, 1980.

Sally Blount White, Kathleen L Valley, Max H Bazerman, Margaret A Neale, et al. Alternative models of price behavior in dyadic negotiations: Market prices, reservation prices, and negotiator aspirations. *Organizational Behavior and Human Decision Processes*, 57:430–447, 1994.