BRIBERY BETWEEN THE BIDDERS AND THE AUCTIONEER IN SEALED-BID AUCTIONS

A Dissertation

by

SEVKET ALPER KOC

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2002

Major Subject: Economics

ABSTRACT

Bribery Between the Bidders and the Auctioneer in Sealed-bid Auctions.

(August 2002)

Sevket Alper Koc, B.S., Middle East Technical University

Chair of Advisory Committee: Dr. William S. Neilson

Ordinarily in a first-price sealed-bid auction, all bidders submit their bids simultaneously, and the highest bidder receives the item and pays his bid. In this standard framework, there is no distinction between the seller and the auctioneer. Realistically, though, the seller must hire an auctioneer, leading to the possibility of corruption.

In this dissertation, corruption takes the following form. The auctioneer approaches the bidders and tells them if they pay a bribe of a certain amount and if they submit the highest bid, the auctioneer will change their bid so that they only have to pay the second-highest bid.

First, we study bribery between the auctioneer and bidders in a first-price auction and how it affects the bidding functions of the bidders and the welfare positions of the bidders, auctioneer and seller when it is common knowledge for bidders that the auctioneer is corrupt. We show that any bidder who pays the bribe bids his valuation and if a bidder does not pay the bribe and all the bidders with valuation less than his value do not pay the bribe, he bids his standard first-price bid.

Second, we analyze bribery between the auctioneer and the bidders in a second-price auction. This time the proposed corruption does not work because the bidders' dominant strategy is to bid their values whatever the other bidders do. We show that seller can avoid transfer to the auctioneer by simply demanding a second-price auction.

Third we study bribery between the auctioneer and bidders in a first-price auction in which there is a positive entry fee imposed by the seller. We show that another way for the seller to avoid the bribery is to use entry fee. The entry fee deters bribery in the sense that bidders with a smaller set of valuations pay the bribe.

ACKNOWLEDGMENTS

I am greatly indebted to my advisor William S. Neilson for sharing with me the problem, the road not taken, and the insights that constitute this work. His critical appraisal, friendship and continuous motivation have made this work possible. Every week he spent a long time with me and helped me identify and develop research ideas. I would like to thank Thomas Jeitschko for his comments and ideas, as well as the committee members Qi Li, John Van Huyck and Daren Cline. I would also like to thank my Graduate Council Representative Robin Autenrieth.

I appreciate Christi Ramirez's help which saved a lot of time with the many administrative problems. I also thank my colleague Wolfgang Kohler for his helpful comments on my dissertation.

To my parents, Niyazi and Zinet Koc, for their unconditional support, I will be always grateful.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER I

INTRODUCTION

Much has been written on the coalitions formed by bidders, usually called bidding rings or cartels[1]. These papers discuss models of single object first and second price sealed-bid auctions in which cooperative behavior between the bidders is permitted. In particular, the primary aim is to look at mechanisms that help to understand how cartels overcome the division of the spoils difficulties in the context of bidding at auctions. However, another important issue is corruption between the auctioneer and the bidders. This dissertation examines this corruption.

Ordinarily in a first-price sealed-bid auction, all bidders submit their bids simultaneously, and the highest bidder receives the item being auctioned off and pays his bid. In this standard framework, there is no distinction between the seller and the auctioneer. Realistically, though, the seller must hire an auctioneer, leading to the possibility of corruption.

In this dissertation, corruption takes the following form. The auctioneer approaches the bidders and tells them that if they pay a bribe of a certain amount and if they submit the highest bid, the auctioneer will change their bid so that they only have to pay the second-highest bid. One motivation for this form of corruption is its realism - auctioneers behave this way in some auctions for government food supply contracts in

---

This dissertation follows the style of the journal *Econometrica.*

[1] See, for example, Graham and Marshall (1987) and McAfee and McMillan (1992).

Turkey. A second motivation is tractability - the resulting mathematical model yields an analytic solution that can be used to discuss such issues as welfare effects.

Auctions have been analyzed extensively, beginning with the pioneering work of Vickrey (1961). Later studies examine some other issues in auction theory. Myerson (1981) and Riley and Samuelson (1981) study the issue of optimal auctions. Maskin and Riley (1984) and Matthews (1987) examine standard and optimal auctions for risk-averse bidders. Milgrom and Weber (1982) analyze a general symmetric auction environment that includes independent and affiliated types, and private or common values. Hendricks and Porter (1989), Graham and Marshall (1987), McAfee and McMillan (1992) and Cramton, Gibbons and Klemperer (1987) are about collusion between the bidders in auctions. And finally Ashenfelter (1989) and Porter (1995) are examples of empirical work on auctions.

Work on corruption on auctions has focused primarily on collusion among bidders. As mentioned earlier, Graham and Marshall (1987) and McAfee and McMillan (1992) are examples of theoretical works on this subject. They discuss models of single object first and second price sealed-bid auctions in which cooperative behavior between the bidders is permitted. Their primary aim is to analyze the mechanisms that help to understand how cartels overcome the division of the spoils difficulties in the context of bidding at auctions.

Little work has been done on collusion between individual bidders and the auctioneer. The working papers of Lengwiler and Wolfstetter (2000) and Burguet and

Perry (2000) are theoretical examples and the working paper of Allan Ingraham (2000) is an applied work on this topic.

Lengwiler and Wolfstetter (2000) propose a model of a coalition among the bidders and the auctioneer and examine how the anticipation of corruption affects the bidding, and how it changes the revenue ranking of typical auctions. Their corruption game is as follows: After bids have been submitted, the auctioneer reveals the second highest bid to the highest bidder in the first-price auction and he reveals the three highest bids to the highest and second highest bidder in the second-price auction. In the first case the auctioneer allows the highest bidder to lower his bid to the level of the second highest bid in exchange for a bribe. And in the second case the three parties agree on removing the second highest bid in exchange for side payments. They find that the bidders do not benefit from corruption in terms of equilibrium expected payoffs. The prospect of participating in a profitable corruption scheme induces them to raise their bids to such an extent that their entire surplus is competed away. They show that only the auctioneer benefits from it. The entire cost of corruption is borne by the seller.

Burguet and Perry (2000) consider a model of bribery in a sealed bid first-price procurement auction. In their model, there are only 2 bidders; one is an honest bidder and the other is a dishonest bidder. The auctioneer cannot award the contract at a price above the lowest bid at the auction, but in return for a bribe, he can award the contract to a dishonest bidder at the low bid of the honest bidder. They examine the equilibrium bidding functions of both bidders when it is common knowledge that the dishonest bidder can bribe the auctioneer. They find that the option of bribing and obtaining the contract after submitting a losing bid might be expected to induce less aggressive bidding by the dishonest bidder. And similarly, the honest bidder would bid more aggressively. Therefore bribery distorts the allocation of the contract in favor of the dishonest bidder.

Ingraham (2000) proposes a method of bidder-auctioneer cheating in sealed-bid auctions. Based on statistical properties of the bids, he develops a regression method for analyzing potential cheating of this type. He applies this regression specification to data from the New York City School Construction Authority auctions. And he finds evidence that there is cheating between the auctioneer and the bidders.

This dissertation is organized in three chapters. The second chapter studies bribery between the auctioneer and bidders in a first-price sealed-bid auction and how it affects the bidding functions of the bidders and the welfare positions of bidders, auctioneer and seller when it is common knowledge for bidders that the auctioneer is corrupt. The third chapter analyzes bribery between the auctioneer and the bidders in a second-price sealed-bid auction. And finally, the fourth chapter examines the differences between an entry fee and a bribe in a first-price sealed-bid auction and how bribery affects the bidding functions of the bidders and the welfare positions of bidders, auctioneer and seller when it is common knowledge for bidders that the auctioneer is corrupt.

This dissertation is different than the two theoretical papers listed above. First of all the bribe takes place before the bids are submitted rather than after the bids are submitted as in those papers. So, bidders decide whether or not to pay the bribe before submitting their bids. Consequently, all the bidders are involved in bribery process in our work. In Burguet and Perry (2000), however, only one of the bidders, that is dishonest, is involved in bribery. In Lengwiler and Wolfstteter (2000), only the highest bidder pays the bribe in first-price auction, and the highest and second highest bidder are involved in the bribery scheme in the second-price auction.

In chapter II, we study bribery between the auctioneer and the bidders in a first-price sealed-bid auction and how it affects the bidding functions of the bidders and the welfare positions of the bidders, auctioneer and seller when it is common knowledge for bidders that the auctioneer is corrupt. We show that, in the first-price sealed-bid auctions, the high type bidders i.e. the bidders who have valuations higher than some critical value pay a bribe to the auctioneer in exchange of some more information. The low type bidders do not pay the bribe because their valuations are not high enough to make a profit after paying the bribe.

It is interesting that we find that the high type bidders bid their own value as if they were in a second-price sealed-bid auction and the low type bidders still play the strategy that everyone plays in the only symmetric equilibrium of the standard first-price auction.

We show that in a first-price auction whether there is bribery or not, the bidder who has the highest value wins the auction. So the bribery equilibrium is Pareto efficient for the bidders: the highest value bidder gets the object.

Corruption has distributional effects in first-price sealed-bid auctions. Specifically, bidders' expected equilibrium payoffs are unaffected by the corruption. They are neither worse off nor better off in terms of the equilibrium expected payoffs.

However, there is a transfer of wealth from the seller to the auctioneer. All the gains of the auctioneer are borne by the seller as a loss; therefore the auction is still efficient.

In chapter III, we study bribery between the auctioneer and the bidders in a second-price sealed-bid auction. This time the proposed corruption doesn't work because the bidders' dominant strategy is to bid their values whatever the other bidders do. And they will pay the second highest bid anyway. Therefore, they would not pay the bribe to the auctioneer.

This chapter shows that revenue equivalence may break down with the introduction of corruption. We show that second price sealed-bid auction and first price sealed-bid auction may not give the same expected revenue to the seller, although the total revenue to both the seller and the auctioneer is the same in both auctions.

Chapter IV analyzes the differences between an entry fee and a bribe in a first-price sealed-bid auction; determining which is more favorable in terms of expected profit according to the bidders and the seller.

This time the seller imposes a positive entry fee on the bidders. We show that, in the first-price sealed-bid auction, bidders who have chosen to pay the entry fee and have valuations higher than some threshold value pay a bribe to the auctioneer, and bidders who have chosen to pay the entry fee and have valuations lower than that threshold value do not. In addition to that, bidders with enough low valuations do not pay the entry fee and, consequently, do not participate in the auction because they incur losses by entering the auction. Bidders who pay the bribe bid their own value as if they were in a second-price sealed-bid auction, and bidders who do not pay the bribe bid according to the standard equilibrium bid function from the first-price auction with entry fee.

This chapter is very interesting because we address the question of designing a bribe-proof mechanism. Can the seller design a mechanism by imposing an entry fee in which bidders have no incentive to accept to pay the bribe to auctioneer? Interestingly, we show that an entry fee partially deters bribery in the sense that bidders with a smaller set of valuations pay the bribe.

Throughout the dissertation we use sealed bid auctions because corruption cannot work in an open-bid auction since it lacks secrecy. It is also realistic to use sealed-bid auctions because open auctions may not be applicable if the bids are complicated documents as in the case of auctions of offshore oil rights.

We assume that the seller is passive, and therefore there is no issue of the detection and punishment of corruption. The seller is the principal and hires the auctioneer to run a sealed-bid auction. The only thing that the seller can do is he can set an entry fee or not. In addition to this, the seller's value of the object is zero.

CHAPTER II

BRIBERY BETWEEN THE BIDDERS AND THE AUCTIONEER IN FIRST-PRICE SELAED-BID AUCTIONS

2.1 Introduction

In this chapter we study bribery between the auctioneer and bidders in a first-price sealed-bid auction and how it affects the bidding functions of the bidders and the welfare positions of bidders, auctioneer and seller when it is common knowledge for bidders that the auctioneer is corrupt. We show that, in the first-price sealed-bid auctions bidders who have valuations higher than some critical value pay a bribe to the auctioneer, and bidders with low valuations do not. Bidders who pay the bribe bid their own value as if they were in a second-price sealed-bid auction, and bidders who do not pay the bribe bid according to the standard equilibrium bid function from the first-price auction. The resulting bid function for all bidders is increasing, and therefore the bidder with the highest value wins the auction, whether he pays the bribe or not, and the auction is efficient.

Corruption has distributional effects. Specifically, the bidders' expected equilibrium payoffs are unaffected by corruption. They are neither worse off nor better off in terms of the equilibrium expected payoffs. However, there is a transfer of wealth from the seller to the auctioneer. All the gains of the auctioneer are borne by the seller as a loss.

We proceed as follows. Section 2.2 presents the game and the notation. Section 2.3 examines the behavior of bidders, determining who pays the bribe and how they bid. Section 2.4 examines the auctioneer's behavior, characterizing the optimal bribe. Section 2.5 explores the welfare properties of the game in comparison to a first-price auction without corruption. Finally, Section 2.6 summarizes the results.

2.2 Structure of the Game

There is a seller of a single good who faces n risk neutral potential buyers. The seller has hired an auctioneer to run a sealed-bid first-price auction. In contrast to the standard first-price auction, the game is supplemented by corruption between the auctioneer and the bidders. The auctioneer approaches every bidder before the auction is held and tells them that if the bidder agrees to pay a bribe of α, and is the highest bidder, he pays the second-highest bid. Otherwise highest bidder pays his bid. Consequently, the game is 3-stage game. In the first stage the auctioneer sets α, in the second stage the bidders decide whether to pay α privately, and in the third stage the bidders choose their bids.

The bidders' valuations $v_1,...,v_n$ are independently and identically drawn from the distribution F with support $[0,1]$, with a density *f*, as in the standard symmetric private values model. We assume that the value of the object to the seller is zero and the reserve price is zero. There is no entry fee, making it optimal for all bidders to bid.

In a standard first-price or second-price sealed-bid auction a bidding strategy is a map $\beta^i : v_i \to b_i$. An equilibrium is a profile of strategies $(\beta^1,...,\beta^n)$ such that β^i is a best reply for i given the strategies of all other bidders. An equilibrium is symmetric if all bidders use the same strategy, $\beta^1 = ... = \beta^n$. We denote the equilibrium strategy of the symmetric equilibrium of the first price and second price auctions with β_1 and β_2, respectively.

As is well known, the unique symmetric equilibrium of the first-price auction is the profile of strategies $(\beta^1,...,\beta^n)$ such that all β^i's are equal and all β^i's are best responses for i given the strategies of all other bidders. This unique symmetric equilibrium strategy is given by,

$$\beta_1 = b_1(v_i) = v_i - \frac{1}{F^{n-1}(v_i)} \int_0^{v_i} F^{n-1}(y)dy. \tag{1}$$

Finally, the seller is passive in this game and we ignore issues related to the detection and punishment of corruption.

2.3 Bidder Behavior

In this section we analyze the behavior of bidders given the size of the bribe, α, set by the auctioneer. We want to find the bids of bidders who do and do not pay the bribe. If a bidder pays the bribe and is the highest bidder, he pays the second highest bid.

Therefore, after paying the bribe the bidder essentially participates in a second price auction, and his dominant strategy is to bid his valuation.

Proposition 1: Any bidder who pays the bribe bids his valuation, v_i .

If a bidder does not pay the bribe, if he wins he must pay his own bid. Consequently, and for the standard reasons, he pays less than his valuation. How much less depends on the behavior of other bidders. An immediate result follows if all bidders with lower valuations also decline the bribe.

Proposition 2: If bidder i does not pay the bribe and all the bidders with valuations below v_i do not pay the bribe, bidder i bids according to the function $b_1(v_i)$.

Proof: Let $b(v)$ denote the equilibrium bid function for bidders who choose not to pay the bribe. For the standard reasons, b is assumed to be increasing. By Proposition 1, all bidders who do pay the bribe bid their valuations, and $v \geq b(v)$ for all v . If bidder i does not pay the bribe, and all bidders with valuations below v_i also do not pay the bribe, bidder i only wins when his is the highest valuation. The theory of first price auctions then implies that, conditional on his own valuation being the highest, bidder i 's optimal bid is then $b_1(v_i)$. □

We now look for an equilibrium of the subgame that follows the auctioneer's choice of α. In particular, we look for an equilibrium in which bidders with high valuations pay the bribe, and bidders with low valuations do not. Let v^* denote the threshold valuation such that a bidder with valuation v^* is indifferent about paying the bribe, and, by hypothesis, all bidders with valuations above v^* pay the bribe and all those with valuations below v^* do not. A bidder with valuation v^* who pays the bribe only beats bidders with lower valuations, and earns expected surplus of $\int_0^{v*}[v^* - b_1(v)]dF^{n-1}(v) - \alpha$. A bidder with valuation v^* who does not pay the bribe earns expected surplus of $\int_0^{v^*}[v^* - b_1(v^*)]dF^{n-1}(v)$. The fact that the bidder is indifferent reduces to

$$\int_0^{v^*}\left[b_1\left(v^*\right) - b_1\left(v\right)\right]dF^{n-1}\left(v\right) = \alpha . \tag{2}$$

Thus, given α, v^* must satisfy the above equation.

A second interpretation of v^* arises from noticing that the first price bid function, $b_1(v)$, is the expected second-highest valuation conditional on v being the highest valuation. Consequently[2],

$$\int_0^{v^*} b_1(v^*)\,dF^{n-1}(v) = \int_0^{v^*} v\,dF^{n-1}(v)\,. \tag{3}$$

Using this fact, (2) can be re-written as

$$\int_0^{v^*} \left[v - b_1(v)\right] dF^{n-1}(v) = \alpha\,. \tag{4}$$

Equation (4) has a straightforward interpretation. Suppose that a bidder with valuation v_1 pays the bribe, bids v_1, and wins the auction, and that the second-highest bidder has valuation v. If the second-highest bidder paid the bribe, he bids v, and the winning bidder's surplus is $v_1 - v$. If the second-highest bidder did not pay the bribe, he bids $b_1(v)$, and the winning bidder's surplus is $v_1 - b_1(v) > v_1 - v$. There is a clear benefit when the second highest bidder does not pay the bribe. Now, note that revenue equivalence implies that a bidder's expected surplus from a second-price auction is identical to his expected surplus from a first-price auction. So, his expected surplus (gross of the bribe) is the same if he and everyone else pay the bribe or if he and everyone else do not pay the bribe. The benefit from the bribe, then, must come from the additional surplus from facing people who do not pay the bribe. This additional surplus is the quantity on the left-hand side of (4). The equation says that enough people must choose not to pay the bribe so that the additional surplus from paying the bribe exactly offsets the cost of the bribe.

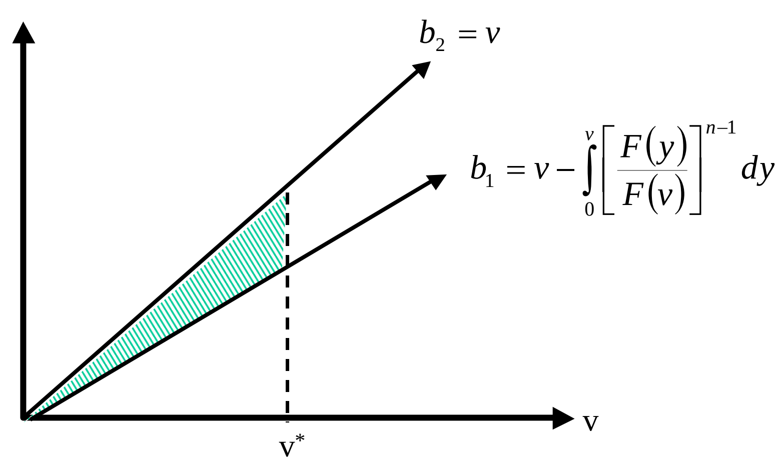

Figure 1- Illustration of the Bribe

Figure 1 shows this graphically. Bidders who pay the bribe bid according to the second-price auction bid function $b_2(v) = v$, and bidders who do not pay the bribe bid

---

[2] Note that

$$\int_0^{v^*} b_1(v^*) dF^{n-1}(v) = F^{n-1}(v^*)\left[v^* - \frac{1}{F^{n-1}(v^*)} \int_0^{v^*} F^{n-1}(v) dv \right] = v^* F^{n-1}(v^*) - \int_0^{v^*} F^{n-1}(v) dv = \int_0^{v^*} v dF^{n-1}(v),$$

where the last equality holds through integration by parts.

according to the first-price auction bid function $b_1(v)$. The left-hand term in equation (4) is the weighted area between these two functions over the interval [0,v^*), with the weights given by the distribution function $F^{n-1}(v)$, which is shown by the shaded area in the figure.

Theorem 1: Given the amount of the bribe α, and given that $\alpha < \int_0^1 [b_1(1) - b_1(v)]\, dF^{n-1}(v)$, then there exists a unique Bayesian-Nash equilibrium in which bidders with values in [0,v^*) do not pay the bribe and bidders with values in [v^*, 1] do pay the bribe, where v^* solves

$$\int_0^{v^*} [b_1(v^*) - b_1(v)]\, dF^{n-1}(v) = \alpha \tag{5}$$

and b_1 is the standard first-price auction bid function.

Proof: Consider bidder i with valuation v_i. Let $E[U_{ic}]$ denote his expected payoff if he pays the bribe (is corrupted) and let $E[U_{in}]$ denote if he does not. Assume that all other bidders behave as proposed.

Case 1: $v_i = v^*$. As discussed above, bidder i is exactly indifferent between paying the bribe and not paying it.

Case 2: $v_i < b_1(v^*)$**.** By Proposition 1 bidder i bids his value if he pays the bribe, and by Proposition 2 he bids $b_1(v_i)$ if he does not. If he does not pay the bribe, he beats bidders with valuations in the interval $[0,v_i)$, and if he does pay the bribe, since $v_i < b_1^{-1}(v_i) < v^*$, he beats bidders with valuations in $[0,b_1^{-1}(v_i))$. Consequently,

$$
\begin{aligned}
E[U_{ic}] - E[U_{in}] &= \int_0^{b_1^{-1}(v_i)} [v_i - b_1(v)]\, dF^{n-1}(v) - \alpha - \int_0^{v_i} [v_i - b_1(v_i)]\, dF^{n-1}(v) \\
&= \int_0^{v_i} [b_1(v_i) - b_1(v)]\, dF^{n-1}(v) + \int_{v_i}^{b_1^{-1}(v_i)} [v_i - b_1(v)]\, dF^{n-1}(v) \\
&\quad - \int_0^{v^*} \left[b_1(v^*) - b_1(v)\right] dF^{n-1}(v) \\
&< \int_0^{v_i} [b_1(v_i) - b_1(v)]\, dF^{n-1}(v) + \int_{v_i}^{b_1^{-1}(v_i)} \left[b_1(v^*) - b_1(v)\right] dF^{n-1}(v) \\
&\quad - \int_0^{v_i} \left[b_1(v^*) - b_1(v)\right] dF^{n-1}(v) - \int_{v_i}^{b_1^{-1}(v_i)} \left[b_1(v^*) - b_1(v)\right] dF^{n-1}(v) \\
&\quad - \int_{b_1^{-1}(v_i)}^{v^*} \left[b_1(v^*) - b_1(v)\right] dF^{n-1}(v) \\
&< 0
\end{aligned}
$$

Therefore bidder i doesn't pay the bribe.

Case 3: $b_1(v^*) \le v_i < v^*$**.** If bidder i pays the bribe, he bids his value, and bids higher than everyone else in $\left[0, v^*\right)$. If he does not pay the bribe, he bids $b_1(v_i)$.

$$
\begin{aligned}
E[U_{ic}]-E[U_{in}] &= \int_0^{v^*}[v_i - b_1(v)]\,dF^{n-1}(v) - \alpha \;-\; \int_0^{v_i}[v_i - b_1(v_i)]\,dF^{n-1}(v) \\
&= \int_0^{v^*}[v_i - b_1(v^*)]\,dF^{n-1}(v) - \int_0^{v_i}[v_i - b_1(v_i)]\,dF^{n-1}(v) \\
&= F^{n-1}(v^*)[v_i - b_1(v^*)] - \int_0^{v_i} F^{n-1}(v)\,dv \\
&= (v_i - v^*)F^{n-1}(v^*) + \int_0^{v^*} F^{n-1}(v)\,dv - \int_0^{v_i} F^{n-1}(v)\,dv \\
&= \int_{v_i}^{v^*} F^{n-1}(v)\,dv - \int_{v_i}^{v^*} F^{n-1}(v^*)\,dv \\
&< 0
\end{aligned}
$$

Consequently bidder i does not pay the bribe.

Case 4: $v_i > v^*$, and bidder i bids $b_i \in (v^*, v_i)$ if he does not pay the bribe.

$$
\begin{aligned}
E[U_{ic}]-E[U_{in}] &= \int_0^{v^*}[v_i - b_1(v)]\,dF^{n-1}(v) + \int_{v^*}^{v_i}[v_i - v]\,dF^{n-1}(v) - \alpha \\
&\quad - \int_0^{v^*}[v_i - b_i]\,dF^{n-1}(v) - \int_{v^*}^{b_i}[v_i - b_i]\,dF^{n-1}(v) \\
&= \int_0^{v^*}[b_i - b_1(v^*)]\,dF^{n-1}(v) + \int_{v^*}^{b_i}[b_i - v]\,dF^{n-1}(v) + \int_{b_i}^{v_i}[v_i - v]\,dF^{n-1}(v) \\
&> 0
\end{aligned}
$$

Consequently, bidder i pays the bribe.

Case 5: $v_i > v^*$**,** and bidder i bids $b_i \leq v^*$ if he does not pay the bribe. It is clear that he will not choose $b_i < b_1(v^*)$. Therefore he beats everyone with values in $[0, v^*)$.

$$E[U_{ic}] - E[U_{in}] = \int_0^{v^*} [v_i - b_1(v)]\, dF^{n-1}(v) + \int_{v^*}^{v_i} [v_i - v]\, dF^{n-1}(v) - \alpha$$

$$- \int_0^{v^*} [v_i - b_i]\, dF^{n-1}(v)$$

$$= \int_0^{v^*} \left[b_i - b_1(v^*)\right] dF^{n-1}(v) + \int_{v^*}^{v_i} [v_i - v]\, dF^{n-1}(v)$$

$$> 0$$

Consequently, bidder i pays the bribe.

This establishes that there is an equilibrium of the desired form. To establish uniqueness, note that since $b_1(v)$ is strictly increasing, $b_1(v^*) - b_1(v) > 0$ for all $v \in [0, v^*)$, so that $\int_0^{v^*} [b_1(v^*) - b_1(v)]\, dF^{n-1}(v)$ is strictly increasing in v^*. The right-hand side of for all left-hand side of (5) is constant, and so there can only be one value of v^* for which (5) is satisfied. أ

Informally, a bidder who draws a value less than v^* prefers not to pay the bribe because if he pays the bribe his surplus rises only by a fraction of the shaded region in Figure 1 but he must pay an amount equal to the entire shaded region, so the bribe makes him worse off. A bidder who draws a value higher than v^* prefers to pay the bribe because the extra surplus in the shaded region is exactly offset by the bribe, but if he does not pay the bribe he loses the auction to people with values lower than his.

The uniqueness of v^* for a given α, coupled with the fact that the left-hand side of (5) is strictly increasing in v^*, implies that there exists a strictly increasing function $v^*(\alpha)$ that describes the equilibrium threshold valuation as a function of the bribe.

2.4 Auctioneer Behavior

In the first period the auctioneer chooses the size of the bribe α that a bidder must pay in order to learn the second highest bid if he is the highest bidder. So, the auctioneer aims to maximize his expected revenue by choosing α. By Theorem 1, though, for any given α there is a unique threshold valuation v^* such that bidders with valuations above v^* pay the bribe and those with valuations below v^* do not. Because of the uniqueness, choosing α is the same as choosing v^*. Let

$$\alpha(v^*) = \int_0^{v^*} \left[b_1(v^*) - b_1(v)\right] dF^{n-1}(v) . \tag{6}$$

The auctioneer's expected revenue is given by

$$R(v^*) = n\left(1 - F(v^*)\right)\alpha(v^*) \tag{7}$$

where n is the number of bidders, $1-F(v^*)$ is the probability that a given bidder pays the bribe, and $\alpha(v^*)$ is the size of the bribe. Since choosing α is the same as choosing v^*, the auctioneer's problem is to choose v^* to maximize expected revenue.

It is apparent from (6) that $\alpha(v^*)$ is continuous since it is differentiable. As long as the distribution F of bidders' valuations is continuous, it follows that $R(v^*)$ is continuous, which establishes the next result.

Proposition 3: There exists an α that maximizes expected revenue, and the corresponding $v^* \in (0,1)$.

Proof: The problem of choosing α to maximize revenue is isomorphic to the problem of choosing $v^* \in [0,1]$ to maximize $R(v^*)$. Since the function is continuous on [0,1], it obtains a maximum. When $\alpha = 0, v^* = 0$ and $R(0) = 0$. Also, when $v^* = 1, 1-F(v^*) = 0,$ and $R(1) = 0$. Finally, since $R(v^*) > 0$ when $v \in (0,1)$, the result holds.□

For some distributions there is a unique α that maximizes the auctioneer's expected revenue. The uniform distribution is such an example. For the uniform distribution the revenue of the auctioneer is

$$R(v^*) = n(1-v^*)\alpha(v^*)$$

where

$$\alpha\left(v^*\right) = \left(v^*\right)^n \left(\frac{n-1}{n^2}\right).$$

So, the auctioneer’s problem becomes

$$\max_{\{v^*\}} n\left(1 - \mathrm{v}^*\right)\left(v^*\right)^n \left(\frac{n-1}{n^2}\right)$$

When we take the first order condition, we end up with

$$v^* = \frac{n}{n+1}$$

Surprisingly, this is the expected value of the highest value of the n bidders. Therefore, the auctioneer maximizes his bribe revenue by soliciting a bribe so large that only bidders whose valuations are above the expected highest valuation pay the bribe.

Another example is a triangular distribution with a density $f(v) = 2v$ and a cumulative distribution $F(v) = v^2$. For this distribution,

$$\alpha(v^*) = \frac{2n-2}{(2n-1)^2}(v^*)^{2n-1}$$

So, the auctioneer's problem becomes

$$\max_{\{v^*\}} n(1 - \mathrm{v}^*)(v^*)^{2n-1}\left(\frac{2n-2}{(2n-1)^2}\right)$$

Therefore,

$$v^* = \frac{2n-1}{2n}$$

This is smaller than the expected highest value, which is $\frac{2n}{2n+1}$.

2.5 Welfare Properties

We now turn to the welfare properties of the auction with bribery. We are interested in two issues. First, is the auction with bribery efficient; that is, does the bidder with the highest valuation get the object? Second, how do participants fare in comparison to a standard first-price auction without bribery? We begin with efficiency.

In general, an auction that awards the prize to the highest bidder is efficient if the bid function is increasing in the bidder's valuation. In the auction with bribery, the bid function can be written

$$b(v) = \begin{cases} b_1(v) & v < v^* \\ & if \\ v & v \geq v^* \end{cases} \tag{8}$$

where $b_1(v)$ is the standard first-price auction bid function, which is increasing. Since $b_1(v^*) < v^*$, the bid function $b(v)$ is increasing, and consequently the auction is efficient.

Proposition 4: The auction with bribery is efficient.

The next issue is a comparison with a first-price auction without bribery. Suppose that the auctioneer sets the bribe at α, and so, by Theorem 1, there exists a threshold valuation v^* such that bidders with valuations higher than v^* pay the bribe and those with valuations below v^* do not. Of course, if a bidder who does not pay the bribe wins the auction, no one else has paid a bribe either, and the outcome of the game is exactly the same as the outcome of the standard first-price auction without bribery. Furthermore, since only bidders with high valuations pay the bribe in equilibrium, no bidder loses to anyone who would not have beaten him in the standard first-price auction without bribery.

The interesting issue pertaining to the welfare of bidders involves bidders who pay the bribe. To that end, suppose that bidder i has valuation $v_i \geq v^*$, so that bidder i pays the bribe. His expected surplus is

$$E[U_{ic}] = \int_0^{v^*} [v_i - b_1(v)] dF^{n-1}(v) + \int_{v^*}^{v_i} [v_i - v] dF^{n-1}(v) - \alpha.$$

Using equation (4), this can be rewritten

$$E[U_{ic}] = \int_0^{v^*} [v_i - v] dF^{n-1}(v) + \int_{v^*}^{v_i} [v_i - v] dF^{n-1}(v) = \int_0^{v_i} [v_i - v] dF^{n-1}(v),$$

which is a bidder's expected surplus in a second-price auction. From revenue equivalence, however, we know that the expected surplus in a first-price auction and the expected surplus in a second-price auction are identical, and so bidders are indifferent between the auction with bribery and the auction without bribery. By Proposition 3, however, the auctioneer earns positive expected revenue from bribes. Because the auctioneer gains from the bribery but the bidders have the same expected surplus with and without bribery, it must be the case that the seller loses what the auctioneer gains. This proves the final proposition.

Proposition 5: In equilibrium, bribes are a transfer from the seller to the auctioneer.

In summary, although bribery changes the bid functions of some bidders, namely those with sufficiently high valuations, it has no effect on the final allocation of the prize or the welfare of the bidders. The bribes generate expected revenue for the auctioneer, and because bidders are not affected, the bribes also generate an expected loss to the seller compared to a first-price auction without bribery.

This analysis suggests that since bidders are not hurt by the corruption, but the seller is, it should be the seller who takes measures to fight corruption. No policy need be enacted to "protect" bidders from "unscrupulous" auctioneers.

2.6 Conclusion

In this paper we analyzed a model of bribery in sealed-bid first-price auctions. The bribery involves the auctioneer, who acts as an agent on behalf of the seller, and the bidders. Our results show that, given the size of the bribe set by the auctioneer, bidders with valuations above some threshold pay the bribe, while bidders with lower valuations do not. In equilibrium, bidders who pay the bribe bid their valuations while bidders who do not pay the bribe bid according to the standard first-price auction bid function. The auctioneer sets the bribe to trade off the amount collected from a bidder who pays the bribe and the number of bidders expected to pay it.

We also studied the welfare properties of the auction with bribery and show that it is efficient and, in equilibrium, bribes are a transfer from the seller to the auctioneer. Although bribery changes the bid functions of some bidders, namely those with sufficiently high valuations, it has no effect on the final allocation of the prize or the welfare of the bidders. The bribes generate expected revenue for the auctioneer, and because bidders are not affected, the bribes also generate an expected loss to the seller compared to a first-price auction without bribery.

CHAPTER III

BRIBERY BETWEEN THE BIDDERS AND THE AUCTIONEER IN SECOND-PRICE SEALED-BID AUCTIONS

3.1 Introduction

In second-price sealed-bid auctions collusion agreement between the bidders is easier to sustain than in first-price sealed-bid auctions. As Robinson (1985) shows, if there are no problems in coming to agreement among all bidders and abstracting from any concerns about detection, etc., the optimal agreement in a second-price auction is for the designated winner to bid infinitely high while all the other bidders bid zero. No other bidders have any incentive to cheat on this agreement. But in a first-price auction the bidders have to agree that the designated bidder bid a small amount while all the other ones bid zero. In this framework, most of the bidders then have a substantial incentive to cheat on the agreement.[3]

However, for the issue of corruption between the auctioneer and the bidders, the scenario is different. In this scenario, corruption takes the following form. The auctioneer approaches the bidders and tells them that if they pay a bribe of a certain amount and if they submit the highest bid, the auctioneer will change their bid so that they only have to pay the second-highest bid. But, in second price auctions bidders have dominant strategy.

---

[3] Milgrom (1987) develops a similar intuition to argue that repeated second-price auctions are more vulnerable to collusion than repeated first-price auctions.

They bid their values no matter what the other bidders do and pay the second highest bid anyway if they win. Hence, the sealed-bid second price auctions are not vulnerable to the proposed corruption scheme that involves the auctioneer and the winning bidder because they alone cannot change the price. They also need the collaboration of the second highest bidder to pull the price down to the third highest bid.[4] So, the bidders do not accept the offer made by the auctioneer, in other words they do not pay the bribe to the auctioneer. All they do is to play their dominant strategy and bid their value.

Due to the fact that the proposed corruption does not work in the second-price sealed-bid auctions the revenue equivalence theorem breaks down. The first-price and second-price auctions do not yield the same expected revenue to the seller.

The seller is passive; hence there is no issue of detection. This paper's main contribution is that the seller can avoid the transfer to the auctioneer by simply demanding a second-price auction instead of a first-price auction.

This paper is presented as follows: in section 3.2, we present the game and the notation. Section 3.3 examines the behavior of the bidders and the auctioneer. Section 3.4 characterizes the revenue equivalence theorem and shows how it breaks down. Section 3.5 concludes the discussion.

[4] See Lengwiler and Wolfstetter (2000).

3.2 Structure of the Game

There is a seller of a single good who faces n risk neutral potential buyers. The seller has hired an auctioneer to run a sealed-bid second-price auction. In contrast to the standard second-price auction, the game is supplemented by corruption between the auctioneer and the bidders. The auctioneer approaches every bidder before the auction is held and tells them that if the bidder agrees to pay a bribe of α, and is the highest bidder, he pays the second-highest bid. Consequently, the game is 3-stage game. In the first stage the auctioneer sets α, in the second stage the bidders decide whether to pay α privately, and in the third stage the bidders choose their bids.

The bidders' valuations $v_1,...,v_n$ are independently and identically drawn from the distribution F with support $[0,1]$, with a density *f*, as in the standard symmetric private values model. We assume that the value of the object to the seller is zero and the reserve price is zero. There is no entry fee, making it optimal for all bidders to bid.

As is well known, the unique symmetric equilibrium of the second-price auction is the profile of strategies $(\beta^1,...,\beta^n)$ such that all β^i's are equal and all β^i's are best responses for i given the strategies of all other bidders. This unique symmetric equilibrium strategy is given by,

$$b_2(v_i) = v_i. \tag{9}$$

Finally, the seller is passive in this game and we ignore issues related to the detection and punishment of corruption.

3.3 Bidders and Auctioneer Behavior

In this section we analyze the behavior of bidders given the size of the bribe, α, set by the auctioneer. We want to find the bids of bidders who do and do not pay the bribe. In a second-price auction no matter what the other bidders do, bidder i has a dominant strategy: he bids his valuation, v_i. And early in chapter 2, proposition 1 shows that any bidder who pays the bribe bids his valuation too. As a matter of fact a bidder, regardless of he pays the bribe or not, bids his valuation. Hence, he does not have incentive to collaborate with the auctioneer and as a result he will not pay any positive amount of bribe.

Theorem 1: Given the amount of the bribe α, there exists a unique equilibrium in which bidders with values in [0,1] do not pay the bribe and bid their valuation.

Proof: Bidders have dominant strategy in second-price auctions; they bid their valuation. Consider bidder i with valuation v_i. If he does not pay the bribe he bids v_i and if he wins he pays the second highest bid, which would be $v_{(2)}$. Then, his profit would be

$v_i - v_{(2)}$. If he pays the bribe he bids his value and if he wins he pays $v_{(2)}$. This tine his profit would be $v_i - v_{(2)} - \alpha$. As long as bribe is positive bidder i doesn't pay the bribe. أ

In the first period the auctioneer chooses the size of the bribe α that a bidder must pay in order to learn the second highest bid if he is the highest bidder. So, the auctioneer aims to maximize his expected revenue by choosing α. By Theorem 1, though, for any given α, bidders do not accept to pay the bribe to the auctioneer. Therefore, the auctioneer is indifferent about the size of the bribe.

3.4 Breakdown of Revenue Equivalence Theorem

As stated earlier the proposed corruption does not work in the second-price sealed-bid auctions and as a matter of fact, the revenue equivalence theorem breaks down. The first-price and second-price auctions do not yield the same expected revenue to the seller.

According to the revenue equivalence theorem, when each of a given number of risk neutral potential bidders of an object has a privately known value independently drawn from a common, strictly increasing distribution, then any auction mechanism in which (i) the highest value bidder always wins the auction, and (ii) any bidder with the lowest feasible value expects zero payoff, yields the same expected revenue to the seller and results in each bidder making the same expected payoff as a function of his value.[5]

In the second chapter we show that the first-price auction is still efficient and the auction awards the prize to the highest bidder and in equilibrium, bribes are a transfer from the seller to the auctioneer. Although bribery changes the bid functions of some bidders, namely those with sufficiently high valuations, it has no effect on the final allocation of the prize or the welfare of the bidders. From the standard auction theory we know that in the first-price auctions and second-price auctions the expected payoffs of the bidders are identical. As a result, bribery does not affect the expected payoffs of the bidders in two different auctions; they both yield the same expected payoffs to the bidders.

But in terms of the expected revenue of the seller, in the first-price auction with bribery it is the expected value of the second highest value minus the expected revenue of the auctioneer, which is

$$E\left(v_{(2)}\right) - n\left(1 - v^{*}\right)\alpha\left(v^{*}\right) \tag{10}$$

where the second term is the expected revenue of the auctioneer. We show that this expected revenue of the auctioneer is strictly positive. On the contrary, the seller's

[5] See Klemperer (1999)

revenue in the second-price auction with bribery is $E(v_{(2)})$. Hence, revenue of the seller is strictly greater in the second-price auction than in the first-price auction.

This is an important result that when we introduce the bribery into the model, the revenue equivalence theorem fails.

3.5 Conclusion

In this chapter we analyzed a model of bribery in sealed-bid second-price auctions. The bribery involves the auctioneer, who acts as an agent on behalf of the seller, and the bidders. Our results show that, given the size of the bribe set by the auctioneer, none of the bidders do pay the bribe and every bidder bid his valuation. This is because the bidders pay the second highest bid instead of their bids. There would be no advantage for them to pay the bribe. As a result, by requiring the auctioneer to run a second-price rather than a first-price auction, the seller can avoid the revenue loss caused by a corrupt auctioneer.

We also show that the revenue equivalence theorem breaks down when there is bribery issue. The first-price and second-price auctions do not yield the same expected revenue to the seller.

CHAPTER IV

BRIBERY AND ENTRY FEE IN FIRST-PRICE SEALED-BID AUCTIONS

4.1 Introduction

In this chapter we study bribery between the auctioneer and bidders in a first-price sealed-bid auction in which there is positive entry fee and how bribery affects the bidding functions of the bidders and the welfare positions of bidders, auctioneer and seller when it is common knowledge for bidders that the auctioneer is corrupt. We show that, in the first-price sealed-bid auctions bidders who have chosen to pay the entry fee and have valuations higher than some threshold value pay a bribe to the auctioneer, and bidders who have chosen to pay the entry fee and have valuations lower than that threshold value do not. In addition to that, bidders with enough low valuations do not pay the entry fee and, consequently, do not participate in the auction because they incur losses by entering the auction. Bidders who pay the bribe bid their own value as if they were in a second-price sealed-bid auction, and bidders who do not pay the bribe bid according to the standard equilibrium bid function from the first-price auction. The resulting bid function for all bidders is increasing, and therefore the bidder with the highest value wins the auction, whether he pays the bribe or not, and the auction is efficient.

In this chapter we also analyzes the differences between an entry fee and a bribe in a first-price sealed-bid auction; determining which is more favorable in terms of expected profit according to the bidders and the seller.

The seller is active in this case; he would choose an amount of entry fee that could eliminate the incentive for the high type bidders to cooperate with the auctioneer. However, we show that whatever amount of entry fee that he chooses, he would not able to eliminate the chance that the high type bidders cooperate with the auctioneer. He only reduces the probability that this could occur.

Corruption has distributional effects. Specifically, the bidders' expected equilibrium payoffs are unaffected by corruption. They are neither worse off nor better off in terms of the equilibrium expected payoffs. However, there is a transfer of wealth from the seller to the auctioneer. All the gains of the auctioneer are borne by the seller as a loss.

We proceed as follows. Section 4.2 presents the game and the notation. Section 4.3 examines the behavior of bidders, determining who pays the bribe and how they bid, examines the auctioneer's and the seller's behaviors, characterizing the optimal bribe and the optimal entry fee. Section 4.4 explores the welfare properties of the game in comparison to a first-price auction without corruption with entry fee. Section 4.5 explores how the critical value resulting from the entry fee affects the threshold value resulting from the bribery process in other words how the seller affects the auctioneer's behavior by imposing entry fee. Finally, Section 4.6 summarizes the results.

4.2 Structure of the Game

There is a seller of a single good who faces n risk neutral potential buyers. The seller has hired an auctioneer to run a sealed-bid first-price auction, and pays the auctioneer a fixed wage in exchange for his services. The bidders' valuations $v_1,...,v_n$ are independently and identically drawn from the distribution F with support $[0,1]$, with a density *f*, as in the standard symmetric private values model. We assume that the value of the object to the seller is zero and the reserve price is zero. But, the seller imposes a positive entry fee, c, on the bidders so that some of the bidders will not participate in the auction, in other words they will not bid. Low type bidders, i.e. the bidders who have valuations less than some critical value will not participate in the auction while high type bidders will.

For comparison purposes, we first analyze the game without bribery. This analysis is standard in the literature[6], but is repeated here to provide a benchmark. We conjecture an equilibrium in which all bidders with values less than a critical value v_0 not participating, and all other types bidding according to a strictly increasing function $b_1(\cdot)$. Therefore, in equilibrium, a bidder wins only if his value is the highest and greater than v_0. Hence, the equilibrium probability-of-winning function is

[6] See, for example, Matthews (1995)

$$Q_i(v) = \begin{cases} 0 & if \quad v < v_0 \\ F^{n-1}(v) & if \quad v \geq v_0 \end{cases} \tag{11}$$

The critical and the non-participating types have zero expected profit, implying that $EU_i = 0$ for $v \in [0, v_0]$. The expected profit for the high type bidders who do not pay bribes is given by[7]

$$EU_i = F^{n-1}(v)v - P(v) = \int_{v_0}^{v} F^{n-1}(y)\,dy \tag{12}$$

where $P(v)$ is the expected payment of type v bidder to the seller and is equal to

$$P(v) = c + F^{n-1}(v)b_1(v). \tag{13}$$

Solving these 3 equations yields the following unique symmetric equilibrium bid function:

---

[7] See Mathews (1995). The expected utility of bidder i in any equilibrium of any kind of auction depends only on his equilibrium probability-of-winning function, $Q_i(.)$, and the equilibrium profit of his lowest type. Here, his lowest type is v_0. And his expected profit is 0 when his value is v_0. Specifically,

$$EU(v_i) = EU(v_0) + \int_{v_0}^{v} Q_i(v)dv = \int_{v_0}^{v} F^{n-1}(v)dv$$

$$b_1(v) = \begin{cases} v - \int\limits_{v_0}^{v} \frac{F^{n-1}(y)}{F^{n-1}(v)} dy - \frac{c}{F^{n-1}(v)} & \text{for } v \geq v_0 \\ 0 \quad \text{for } v < v_0 \end{cases} \tag{14}$$

Bidder i with valuation v_0 is exactly indifferent between bidding and not bidding. Hence,

$$b_1(v_0) = 0 = v_0 - \frac{c}{F^{n-1}(v_0)}. \tag{15}$$

$b_1(v_0)$ equals zero because his probability winning function is $F^{n-1}(v_0)$ even if he bids less than $b_1(v_0)$. Since his bid does not affect his probability of winning function and since $b_1(v_0)$ is his optimal bid, it must be as low as possible, 0. This equation (15) uniquely determines the critical type v_0 as a function of the distribution of valuations F and the entry fee c.

Integrating equation (14) by parts yields the generalization

$$b_1(v) = \int\limits_{v_0}^{v} y \frac{(n-1)yF^{n-2}(y)f(y)}{F^{n-1}(v)} dy \tag{16}$$

Equation (16) shows that a bidder with valuation v bids the expected second highest value conditional on this value being less than v.

From the general auction theory[8], we know that the expected revenue of the seller is,

$$ER(S) = n\left[\int_{v_0}^{v}\left(v - \frac{1-F(v)}{f(v)}\right)F^{n-1}(v)f(v)\,dv\right] \tag{17}$$

The seller's problem is to maximize her expected revenue by choosing optimum level of the entry fee. The seller's choice of c affects her expected profit only in so for as it affects the critical type. Therefore, if v_0^* is the threshold type v_0 that maximizes (17), then any c satisfying $v_0(c) = v_0^*$ is optimal. So, the seller aims to maximize her expected revenue by choosing v_0. Hence, the first order condition is,

$$\frac{\partial ER(S)}{\partial v_0} = nF^{n-1}(v_0)f(v_0)\left(v_0 - \frac{1-F(v_0)}{f(v_0)}\right) = 0$$

It is apparent that $F(v)$ and $f(v)$ are positive for all v, therefore first order condition reduces to[9]

$$v_0 - \frac{1-F(v_0)}{f(v_0)} = 0 \tag{18}$$

[8] See, for example, Matthews (1995)

[9] See Riley and Samuelson (1981) for details. (Proposition 3)

And in general, for the distributions $F(.)$ usually employed in auction theory,

$$v-\frac{1-F(v)}{f(v)} \tag{19}$$

increases in v. If this is satisfied the solution will be unique. Even if (19) does not increase in v, the optimal v_0 satisfies (18), however, then (18) may have multiple solutions and only some of those solutions maximize the seller's profit.

An example illustrates the above results. For the uniform distribution the threshold value and the entry fee are, $v_0 = 1/2$ and $c = (1/2)^n$. The expected revenue of the seller is

$$\begin{aligned}
ER(S) &= n\left[\int_{1/2}^{1}\left(v-\frac{1-v}{v}\right)v^{n-1}dv\right] = n\left[\int_{1/2}^{1}(2v-1)v^{n-1}dv\right] \\
&= n\left[\frac{2}{n+1}-\frac{2(1/2)^{n+1}}{n+1}-\frac{1}{n}+\frac{(1/2)^{n}}{n}\right] = \frac{n-1}{n+1}+(1/2)^{n}\frac{1}{n+1} \\
&= E(v_{(2)})+(1/2)^{n}\frac{1}{n+1}
\end{aligned}$$

where $E(v_{(2)})$ is the expected value of the second highest value and this value is the seller's expected revenue when there is no entry fee. It is obvious that the expected revenue of the seller is strictly higher than in the case when there is no entry fee, since the second term is strictly positive.

Because the seller gains from the entry fee, it must be the case that the bidders lose what the seller gains. The expected utility of a low type bidder is zero and the high type bidder is,

$$EU_i = \int_{v_0}^{v_i} F^{n-1}(v)\,dv = \int_{1/2}^{v_i} F^{n-1}(v)\,dv = \frac{v_i}{n} - (1/2)^n$$

which is strictly less then his profit that would have been in the absence of the entry fee because the second term is strictly positive.

4.3 Equilibrium with Bribery

In this section, in contrast to the standard first-price auction, the game is supplemented by corruption between the auctioneer and the bidders. The auctioneer approaches every bidder before the auction is held and tells them that if the bidder agrees to pay a bribe of α, and if he is the highest bidder, he pays the second-highest bid. If the highest bidder did not pay the bribe, he pays his bid. Consequently, the game is a 4-stage game. In the first stage the seller sets c and the bidders with valuations higher than v_0 enter to the auction and pay the entry fee to the seller and the bidders with valuation less than v_0 do not participate in the auction. In the second stage the auctioneer sets α, in the third stage the bidders decide whether to pay α independently and simultaneously, and in the fourth stage the bidders choose their bids.

Bidders behavior: We analyze the behavior of bidders given the size of the bribe, α, set by the auctioneer. Specifically, we characterize the equilibrium of the subgame that follows the auctioneer's choice of α. To accomplish this, we look for an equilibrium in which bidders with high valuations pay the bribe, and bidders with low valuations do not.

The first task is to find the bids of bidders who do and do not pay the bribe. If a bidder pays the bribe and is the highest bidder, he pays the second highest bid. Therefore, after paying the bribe the bidder essentially participates in a second price auction, and his dominant strategy is to bid his valuation.

Proposition 1: Any bidder who pays the bribe bids his valuation, v_i.

If a bidder does not pay the bribe, if he wins he must pay his own bid. Consequently, and for the standard reasons, he bids less than his valuation. How much less depends on the behavior of other bidders. An immediate result follows if all bidders with lower valuations also decline the bribe.

Proposition 2: If bidder i does not pay the bribe and all the bidders with valuations below v_i do not pay the bribe, bidder i bids according to the function $b_1(v_i)$.

Proof: Let $b(v)$ denote the equilibrium bid function for bidders who choose not to pay the bribe. For the standard reasons, b is assumed to be increasing. By Proposition 1, all bidders who do pay the bribe bid their valuations, and $v \geq b(v)$ for all v. If bidder i does not pay the bribe, and all bidders with valuations below v_i also do not pay the bribe, bidder i only wins when his is the highest valuation. The theory of first price auctions then implies that, conditional on his own valuation being the highest, bidder i's optimal bid is then $b_1(v_i)$.□

Let v^* denote the threshold valuation such that a bidder with valuation v^* is indifferent about paying the bribe, and, by hypothesis, all bidders with valuations above v^* pay the bribe and all those with valuations below v^* do not. A bidder with valuation v^* who pays the bribe only beats bidders with lower valuations, and earns expected surplus of $\int_{v_0}^{v*}[v^* - b_1(v)]dF^{n-1}(v) - \alpha$. A bidder with valuation v^* who does not pay the bribe earns expected surplus of $\int_{v_0}^{v^*}[v^* - b_1(v^*)]dF^{n-1}(v)$. The fact that the bidder is indifferent reduces to

$$\int_{v_0}^{v^*}\left[b_1\left(v^*\right) - b_1\left(v\right)\right]dF^{n-1}\left(v\right) = \alpha . \tag{20}$$

A second interpretation of v^* arises from noticing that the first price bid function, $b_1(v)$, is the expected second-highest valuation conditional on v being the highest valuation. Consequently,

$$\int_{v_0}^{v^*} b_1(v^*)dF^{n-1}(v) = \int_{v_0}^{v^*} v dF^{n-1}(v) \tag{21}$$

Using this fact, (20) can be re-written as

$$\int_{v_0}^{v^*} [v - b_1(v)]\, dF^{n-1}(v) = \alpha \tag{22}$$

Equation (22) has a straightforward interpretation. Suppose that a bidder with valuation v_1 pays the bribe, bids v_1, and wins the auction, and that the second-highest bidder has valuation v. If the second-highest bidder paid the bribe, he bids v, and the winning bidder's surplus is $v_1 - v$. If the second-highest bidder did not pay the bribe, he bids $b_1(v)$, and the winning bidder's surplus is $v_1 - b_1(v) > v_1 - v$. There is a clear benefit when the second highest bidder does not pay the bribe. Now, note that revenue equivalence implies that a bidder's expected surplus from a second-price auction is identical to his expected surplus from a first-price auction. So, his expected surplus (gross of the bribe) is the same if he and everyone else pay the bribe or if he and everyone else do not pay the bribe. The benefit from the bribe, then, must come from the additional surplus from facing people who do not pay the bribe. This additional surplus is the quantity on the left-hand side of (22). The equation says that enough people must choose not to pay the bribe so that the additional surplus from paying the bribe exactly offsets the cost of the bribe.

Figure 2 shows this graphically. Bidders who pay the bribe bid according to the second-price auction bid function $b_2(v) = v$, and bidders who do not pay the bribe bid according to the first-price auction bid function $b_1(v)$. The left-hand term in equation (22) is the weighted area between these two functions over the interval $[v_0, v^*)$, with the weights given by the distribution function $F^{n-1}(v)$, which is shown by the shaded area in the figure.

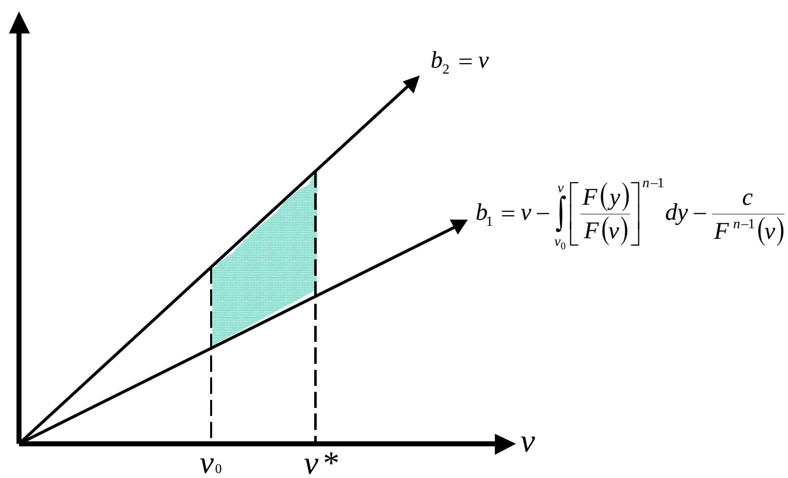

Figure 2 - Illustration of the Bribe with Entry Fee

Theorem 1: Given that $c < 1$ bidders with values $[v_0, 1]$ enter the auction, where v_0 solves $v_0 = \dfrac{c}{F^{n-1}(v_0)}$. Given that $\alpha < \int_{v_0}^{1} [b_1(1) - b_1(v)] \mathrm{d}F^{n-1}(\mathrm{v})$, then there exists a unique Bayesian-Nash equilibrium in which bidders with values in [v_0 ,v^*] do not pay the bribe and bidders with values in [v^*,1] do pay the bribe, where v^* solves

$$\int_{v_0}^{v^*} [b_1(v^*) - b_1(v)] dF^{n-1}(v) = \alpha \tag{23}$$

Proof: The first sentence is immediate from (12). Now **c**onsider bidder i with valuation v_i. Let $E[U_{ic}]$ denote his expected payoff if he pays the bribe (is corrupted) and let $E[U_{in}]$ denote if he does not. Assume that all other bidders behave as proposed.

Case 1: $v_i = v^*$. As discussed above, bidder i is exactly indifferent between paying the bribe and not paying it.

Case 2: $v_i < b_1(v^*)$**.** Bidder i bids his value if he pays the bribe, and he bids $b_1(v_i)$ if he does not. If he does not pay the bribe, he beats bidders with valuations in the interval $[v_0, v_i)$, and if he does pay the bribe, since $v_i < b_1^{-1}(v_i) < v^*$, he beats bidders with valuations in $[v_0, b_1(v_i))$. Consequently,

$$E[U_{ic}] - E[U_{in}] = \int_{v_0}^{b_1^{-1}(v_i)} [v_i - b_1(v)]\,dF^{n-1}(v) - \alpha \; - \int_{v_0}^{v_i} [v_i - b_1(v_i)]\,dF^{n-1}(v)$$

$$= \int_{v_0}^{v_i} [b_1(v_i) - b_1(v)]\,dF^{n-1}(v) + \int_{v_i}^{b_1^{-1}(v_i)} [v_i - b_1(v)]\,dF^{n-1}(v)$$

$$- \int_{v_0}^{v^*} \left[b_1(v^*) - b_1(v)\right] dF^{n-1}(v)$$

$$< \int_{v_0}^{v_i} [b_1(v_i) - b_1(v)]\,dF^{n-1}(v) + \int_{v_i}^{b_1^{-1}(v_i)} \left[b_1(v^*) - b_1(v)\right] dF^{n-1}(v)$$

$$- \int_{v_0}^{v_i} \left[b_1(v^*) - b_1(v)\right] dF^{n-1}(v) - \int_{v_i}^{b_1^{-1}(v_i)} \left[b_1(v^*) - b_1(v)\right] dF^{n-1}(v)$$

$$- \int_{b_1^{-1}(v_i)}^{v^*} \left[b_1(v^*) - b_1(v)\right] dF^{n-1}(v)$$

$$< 0$$

Therefore bidder i doesn't pay the bribe.

Case 3: $b_1(v^*) \le v_i < v^*$**.** If bidder i pays the bribe, he bids his value, and bids higher than everyone else in $\left[v_0, v^*\right)$. If he does not pay the bribe, he bids $b_1(v_i)$.

$$E[U_{ic}] - E[U_{in}] = \int_{v_0}^{v^*} [v_i - b_1(v)]\,dF^{n-1}(v) - \alpha \; - \int_{v_0}^{v_i} [v_i - b_1(v_i)]\,dF^{n-1}(v)$$

$$= \int_{v_0}^{v^*} \left[v_i - b_1(v^*)\right] dF^{n-1}(v) - \int_{v_0}^{v_i} [v_i - b_1(v_i)]\,dF^{n-1}(v)$$

By construction of $b_1(.)$, v_i maximizes $\int_{v_0}^{y} [v_i - b_1(y)]\,dF^{n-1}(v)$. So,

$$E[U_{ic}] - E[U_{in}] \le 0$$

Consequently bidder i does not pay the bribe.

Case 4: $v_i > v^*$, and bidder i bids $b_i \in (v^*, v_i)$ if he does not pay the bribe.

$$\begin{aligned}
E[U_{ic}] - E[U_{in}] &= \int_{v_0}^{v^*} [v_i - b_1(v)]\, dF^{n-1}(v) + \int_{v^*}^{v_i} [v_i - v]\, dF^{n-1}(v) - \alpha \\
&\quad - \int_{v_0}^{v^*} [v_i - b_i]\, dF^{n-1}(v) - \int_{v^*}^{b_i} [v_i - b_i]\, dF^{n-1}(v) \\
&= \int_{v_0}^{v^*} [b_i - b_1(v^*)]\, dF^{n-1}(v) + \int_{v^*}^{b_i} [b_i - v]\, dF^{n-1}(v) + \int_{b_i}^{v_i} [v_i - v]\, dF^{n-1}(v) \\
&> 0
\end{aligned}$$

Consequently, bidder i pays the bribe.

Case 5: $v_i > v^*$**,** and bidder i bids $b_i \le v^*$ if he does not pay the bribe. It is clear that he will not choose $b_i < b_1(v^*)$. Therefore he beats everyone with values in $[v_0, v^*)$.

$$\begin{aligned}
E[U_{ic}] - E[U_{in}] &= \int_{v_0}^{v^*} [v_i - b_1(v)]\, dF^{n-1}(v) + \int_{v^*}^{v_i} [v_i - v]\, dF^{n-1}(v) - \alpha \\
&\quad - \int_{v_0}^{v^*} [v_i - b_i]\, dF^{n-1}(v) \\
&= \int_{v_0}^{v^*} [b_i - b_1(v^*)]\, dF^{n-1}(v) + \int_{v^*}^{v_i} [v_i - v]\, dF^{n-1}(v) \\
&> 0
\end{aligned}$$

Consequently, bidder i pays the bribe.

This establishes that there is an equilibrium of the desired form. To establish uniqueness, note that since $b_1(v)$ is strictly increasing, $b_1(v^*)-b_1(v)>0$ for all $v\in[v_0,v^*)$, so that $\int_{v_0}^{v^*}[b_1(v^*)-b_1(v)]dF^{n-1}(v)$ is strictly increasing in v^*. The right-hand side of for all left-hand side of (23) is constant, and so there can only be one value of v^* for which (23) is satisfied. أ

Informally, a bidder who draws a value less than v^* prefers not to pay the bribe because if he pays the bribe his surplus rises only by a fraction of the shaded region in Figure 1 but he must pay an amount equal to the entire shaded region, so the bribe makes him worse off. A bidder who draws a value higher than v^* prefers to pay the bribe because the extra surplus in the shaded region is exactly offset by the bribe, but if he does not pay the bribe he loses the auction to people with values lower than his.

The uniqueness of v^* for a given α, coupled with the fact that the left-hand side of (23) is strictly increasing in v^*, implies that there exists a strictly increasing function $v^*(\alpha)$ that describes the equilibrium threshold valuation as a function of the bribe.

Auctioneer's behavior: In the second period the auctioneer chooses the size of the bribe α that a bidder must pay in order to learn the second highest bid if he is the highest bidder. So, the auctioneer aims to maximize his expected revenue by choosing α.

By Theorem 1, though, for any given α there is a unique threshold valuation v^* such that bidders with valuations above v^* pay the bribe and those with valuations below v^* do not. Because of the uniqueness, choosing α is the same as choosing v^*. Let

$$\alpha(v^*) = \int_{v_0}^{v^*} \left[b_1(v^*) - b_1(v)\right] dF^{n-1}(v) . \tag{24}$$

The auctioneer's expected revenue is given by

$$ER_A(v^*) = n(1 - F(v^*))\alpha(v^*) \tag{25}$$

where n is the number of bidders,[10] $1 - F(v^*)$ is the probability that a given bidder pays the bribe, and $\alpha(v^*)$ is the size of the bribe. Since choosing α is the same as choosing v^*, the auctioneer's problem is to choose v^* to maximize expected revenue.

It is apparent from (3.5) that $\alpha(v^*)$ is continuous since it is differentiable. As long as the distribution F of bidders' valuations is continuous, it follows that $ER_A(v^*)$ is continuous, which establishes the next result.

[10] n is the number of bidders including those who choose not to pay the entry fee.

Proposition 3: There exists an α that maximizes expected revenue, and the corresponding $v^* \in (v_0, 1)$.

Proof: The problem of choosing α to maximize revenue is isomorphic to the problem of choosing $v^* \in [v_0, 1]$ to maximize $ER_A(v^*)$. Since the function is continuous on $[v_0, 1]$, it obtains a maximum. When $\alpha = 0, v^* = v_0$ and $ER_A(0) = 0$. Also, when $v^* = 1, 1 - F(v^*) = 0$, and $ER_A(1) = 0$. Finally, since $ER_A(v^*) > 0$ when $v \in (v_0, 1)$, the result holds.□

Seller's behavior: In the first period the seller chooses the size of the entry fee c that a bidder must pay in order to enter to the auction. Because she knows that there is possibility of corruption between the bidders and the auctioneer, this time his objective function is different. His expected revenue is,

$$\begin{aligned} ER(S) &= n\left[\int_{v_0}^{v}\left(v - \frac{1-F(v)}{f(v)}\right)F^{n-1}(v)f(v)\,dv\right] - ER(A) \\ &= n\left[\int_{v_0}^{v}\left(v - \frac{1-F(v)}{f(v)}\right)F^{n-1}(v)f(v)\,dv\right] - n\left(1-F(v^*)\right)\alpha(v^*) \end{aligned} \qquad (26)$$

where

$$\alpha\left(v^*\right) = \left(n-1\right)\int_{v_0}^{v^*} \frac{f\left(v\right)}{F\left(v\right)} \int_{v_0}^{v} F^{n-1}\left(y\right) dy\, dv$$

Seller's objective is to maximize his expected revenue by choosing v_0. We wish to determine whether the existence of bribery induces the seller to raise or lower the entry fee. To this end, let $\hat{v}_0$ denote the optimal value of v_0 when there is no bribery, and let $v_0^{\ *}$ denote the optimal value of v_0 when there is bribery.

Proposition 4: $v_0^{\ *} > \hat{v}_0$

Proof: The derivative of expected revenue when there is bribery, evaluated at $\hat{v}_0$, is

$$\left.\frac{\partial ER\left(S\right)}{\partial v_0}\right|_{\hat{v}_0} = \frac{\partial}{\partial v}\left.\left[n\int_{v_0}^{v}\left(v - \frac{1-F\left(v\right)}{f\left(v\right)}\right)F^{n-1}\left(v\right)f\left(v\right)dv\right]\right|_{\hat{v}_0} - \frac{\partial}{\partial v}\left.\left[n\left(1-F\left(v^*\right)\right)\alpha\left(v^*\right)\right]\right|_{\hat{v}_0}$$

The first term is zero by the first order condition when there is no bribery. Since the second term in brackets is the auctioneer's expected revenue, by the envelope theorem the impact of changes in v^* caused by changes in v_0 can be ignored, and

$$\left.\frac{\partial ER\left(S\right)}{\partial v_0}\right|_{\hat{v}_0} = n\left(n-1\right)\left(1-F\left(v^*\right)\right)\int_{v_0}^{v^*}\frac{f\left(v\right)}{F\left(v\right)}F^{n-1}\left(v_0\right)dv$$

which is positive. □

4.4 Welfare Properties

We now turn to the welfare properties of the auction with bribery and entry fee. We are interested in two issues. First, is the auction with bribery efficient; that is, does the bidder with the highest valuation get the object? Second, how do participants fare in comparison to a standard first-price auction without bribery? We begin with efficiency.

Proposition 5: The auction with bribery is efficient as long as some bidder pays the entry fee.

The auction with bribery and an entry fee is not efficient if all the bidders have valuations less then v_0, because if the highest valuation is in $[0,v_0)$ the seller keeps the item even though a bidder values it more. In general, however, an auction that awards the prize to the highest bidder is efficient if the bid function is increasing in the bidder's valuation. In the auction with bribery, the bid function can be written

$$b(v)=\begin{cases} v & if \quad v \geq v^* \\ b_1(v) & if \quad v_0 \leq v < v^* \\ 0 & if \quad v < v_0 \end{cases} \tag{27}$$

where $b_1(v)$ is the standard first-price auction bid function with entry fee, which is increasing. Since $b_1(v^*) < v^*$, the bid function $b(v)$ is increasing, and consequently the auction is efficient if there exists at least one bidder having valuation higher than or equal to v_0.

The next issue is a comparison with a first-price auction with entry fee and without bribery after the seller imposes the entry fee. Suppose that the auctioneer sets the bribe at α, and so, by Theorem 1, there exists a threshold valuation v^* such that bidders with valuations higher than v^* pay the bribe and those with valuations below v^* do not. Of course, if a bidder who does not pay the bribe wins the auction, no one else has paid a bribe either, and the outcome of the game is exactly the same as the outcome of the standard first-price auction without bribery. Furthermore, since only bidders with high valuations pay the bribe in equilibrium, no bidder loses to anyone who would not have beaten him in the standard first-price auction without bribery.

The interesting issue pertaining to the welfare of bidders involves bidders who pay the bribe. To that end, suppose that bidder i has valuation $v_i \geq v^*$, so that bidder i pays the bribe. His expected surplus is

$$E[U_{ic}] = \int_{v_0}^{v^*} [v_i - b_1(v)] dF^{n-1}(v) + \int_{v^*}^{v_i} [v_i - v] dF^{n-1}(v) - \alpha.$$

Using equation (22), this can be rewritten

$$E[U_{ic}] = \int_{v_0}^{v^*} [v_i - v] dF^{n-1}(v) + \int_{v^*}^{v_i} [v_i - v] dF^{n-1}(v) = \int_{v_0}^{v_i} [v_i - v] dF^{n-1}(v),$$

which is a bidder's expected surplus in a second-price auction with entry fee. From revenue equivalence, however, we know that the expected surplus in a first-price auction and the expected surplus in a second-price auction are identical, and so bidders are indifferent between the auction with bribery and the auction without bribery. By Proposition 3, however, the auctioneer earns positive expected revenue from bribes. Because the auctioneer gains from the bribery but the bidders have the same expected surplus with and without bribery, it must be the case that the seller loses what the auctioneer gains. This proves the final proposition.

Proposition 6: In equilibrium, after the seller imposes entry fee, bribes are a transfer from the seller to the auctioneer.

In summary, although bribery changes the bid functions of some bidders, namely those with sufficiently high valuations, it has no effect on the final allocation of the prize or the welfare of the bidders. The bribes generate expected revenue for the auctioneer, and because bidders are not affected, the bribes also generate an expected loss to the seller compared to a first-price auction without bribery.

This analysis suggests that since bidders are not hurt by the corruption, but the seller is, it should be the seller who takes measures to fight corruption. No policy need be enacted to “protect” bidders from “unscrupulous” auctioneers.

4.5 Effects of Entry Fee on Bribery

It stands to reason that the existence of an entry fee would have an impact on the bribe set by the auctioneer. After all, the value to a bidder of paying the bribe arises when the second-highest bidder did not pay the bribe, and entry fees reduce the set of opponents for whom this matters. So, the entry fee may reduce the ability of the auctioneer to elicit bribes from the high-valuation bidders. The purpose of this section is to explore the impact that entry fees have on bribes.

We start with the revenue of the auctioneer when the entry fee is *c*, leading bidders with valuations in $[0,v_0)$ to choose not to participate in the auction:

$$ER_A\left(v^*\right) = n\left(1 - F\left(v^*\right)\right)\alpha\left(v^*\right) = n\left(1 - F\left(v^*\right)\right)\int_{v_0}^{v^*} \frac{f(v)}{F(v)}\left(\int_{v_0}^{v} F^{n-1}(y)\,dy\right)dv$$

The auctioneer’s problem is to maximize his expected revenue by choosing α. But, we have argued that the problem of choosing α to maximize revenue is isomorphic to the problem of choosing $v^* \in [v_0, 1]$ to maximize $ER_A\left(v^*\right)$. Hence we can use the first order condition,

$$\frac{\partial ER_A}{\partial v^*} = n\,(n-1)\left\{\left(1-F\left(v^*\right)\right)\frac{f\left(v^*\right)}{F\left(v^*\right)}\int_{v_0}^{v^*} F^{\,n-1}(y)\,dy - f\left(v^*\right)\int_{v_0}^{v^*}\frac{f(v)}{F(v)}\left(F^{\,n-1}(y)\,dy\right)dv\right\} = 0$$

The first order condition reduces to

$$\frac{1-F\left(v^*\right)}{F\left(v^*\right)}\int_{v_0}^{v^*} F^{\,n-1}(y)\,dy = \int_{v_0}^{v^*}\frac{f(v)}{F(v)}\int_{v_0}^{v} F^{\,n-1}(y)\,dy\,dv \qquad (28)$$

Unfortunately, further analysis of (28) does not yield tractable results. However, analysis under specific distributional assumptions is fruitful. For the uniform distribution equation (28) can be solved numerically. It reduces to

$$\frac{1-v^*}{v^*}\int_{1/2}^{v^*} y^{n-1}dy = \int_{1/2}^{v^*}\frac{1}{v}\int_{1/2}^{\mathrm{v}} y^{n-1}dy\,dv\,.$$

Integrating and rearranging yields

$$(1/2)^n \ln\left(\frac{v^*}{1/2}\right) = \left(\frac{1}{n}-\frac{1-v^*}{v^*}\right)\left[\left(v^*\right)^n - (1/2)^n\right].$$

TABLE 1

Comparison of v^* with and without entry fee

Number of bidders	v^* when there is entry fee	v^* when there is no entry fee
2	0.82651	0.66666
3	0.83757	0.75000
4	0.84915	0.80000
5	0.86091	0.83333
6	0.87243	0.85714
7	0.88334	0.87500
8	0.89336	0.88888
9	0.90237	0.90000
10	0.91033	0.90909
11	0.91731	0.91666
12	0.92340	0.92307
13	0.92874	0.92857
14	0.93341	0.93333
15	0.93754	0.93750
16	0.94119	0.94117
17	0.94445	0.94444
18	0.94737	0.94736
19	0.95000	0.95000
20	0.95238	0.95238
21	0.95454	0.95454
22	0.95652	0.95652
23	0.95833	0.95833
24	0.96000	0.96000
25	0.96153	0.96153
26	0.96296	0.96296
27	0.96428	0.96428
28	0.96551	0.96551
29	0.96666	0.96666
30	0.96774	0.96774

In Table 1 we solve v^* numerically for the number of bidders from 2 to 30. We then compare these results with those in which the seller does not impose an entry fee. From Chapter 2 we know that in the first-price auction without an entry fee v^* is $\frac{n}{n+1}$ for the uniform distribution, and these values are shown in the table. From Table 1 we see that v^* is larger in the entry fee case than in the no-entry fee case, although the two values converge after $n = 10$. So, at least for the uniform distribution, the existence of an entry fee deters bribery in the sense that bidders with a smaller set of valuations pay the bribe. It also shows that the effectiveness of an entry fee for deterring bribery diminishes as the number of bidders grows.

While the above result shows that an entry fee can deter bribery, it remains to be seen whether a higher entry fee serves as more of a deterrent. Since an increase in the entry fee leads to an increase in v_0, and since an increase in the bribe is equivalent to an increase in the threshold v^*, we can determine this effect by using the comparative statics derivative dv^* / dv_0.

Total differentiation of the first order condition yields,

$$
\begin{aligned}
&n(n-1)\left\{\left[1-F\left(v^*\right)\right]\frac{f\left(v^*\right)}{F\left(v^*\right)}(-1)F^{n-1}(v_0) - f\left(v^*\right)\left[\int_{v_0}^{v^*}\frac{f(v)}{F(v)}(-1)F^{n-1}(v_0)dv\right]\right\} \\
&+ n(n-1)\frac{dv^*}{dv_0}\left\{
\begin{aligned}
&- f^2\left(v^*\right)\frac{1}{F\left(v^*\right)}\int_{v_0}^{v^*}F^{n-1}(y)dy \\
&+\left[1-F\left(v^*\right)\right]\frac{f'\left(v^*\right)F\left(v^*\right)-f\left(v^*\right)}{F^2\left(v^*\right)}\int_{v_0}^{v^*}F^{n-1}(y)dy \\
&+\left[1-F\left(v^*\right)\right]\frac{f\left(v^*\right)}{F\left(v^*\right)}F^{n-1}\left(v^*\right) - f'\left(v^*\right)\int_{v_0}^{v^*}\frac{f(v)}{F(v)}\int_{v_0}^{v}F^{n-1}(y)dy\,dv \\
&- f\left(v^*\right)\frac{f\left(v^*\right)}{F\left(v^*\right)}\int_{v_0}^{v^*}F^{n-1}(y)dy
\end{aligned}
\right\} \\
&= 0
\end{aligned}
$$

The term in the big parenthesis is negative because of the second order condition for profit maximization by the auctioneer. So,

$$f\left(v^*\right)F^{n-1}\left(v_0\right)\left[\frac{1-F\left(v^*\right)}{F\left(v^*\right)}+\int_{v_0}^{v^*}\frac{f(v)}{F(v)}dv\right]=\frac{dv}{dv^*}\{+\} \tag{29}$$

where,

$$\int_{v_0}^{v^*}\frac{f(v)}{F(v)}dv=\ln\left(\frac{F\left(v^*\right)}{F\left(v_0\right)}\right)$$

As a result, we have the following conditions

$$If\quad\left\{\begin{matrix}\dfrac{1-F\left(v^*\right)}{F\left(v^*\right)}>\ln\left(\dfrac{F\left(v^*\right)}{F\left(v_0\right)}\right) & \Rightarrow\ \dfrac{dv^*}{dv_0}<0\\ \dfrac{1-F\left(v^*\right)}{F\left(v^*\right)}<\ln\left(\dfrac{F\left(v^*\right)}{F\left(v_0\right)}\right) & \Rightarrow\ \dfrac{dv^*}{dv_0}>0\end{matrix}\right\} \tag{30}$$

For high values of v^*, $\frac{1-F\left(v^*\right)}{F\left(v^*\right)}$ is very small so that $\frac{1-F\left(v^*\right)}{F\left(v^*\right)}<\ln\left(\frac{F\left(v^*\right)}{F\left(v_0\right)}\right)$ which leads to the expected result, $\frac{dv^*}{dv_0}>0$, so that increases in the entry fee lead to fewer bidders paying the bribe. To get an idea of how high v^* must be, if we set $F\left(v_0\right)=0.5$ we find that $dv^*/dv_0>0$ when $F\left(v^*\right)\geq 0.55$.

TABLE 2

Comparison of v^* with different values of entry fee

v^*				
Number of bidders	$v_0 = 0$	$v_0 = 0.3$	$v_0 = 0.5$	$v_0 = 0.8$
2	0.66666	0.75380	0.82651	0.93229
3	0.75000	0.78354	0.83757	0.93350
4	0.80000	0.81227	0.84915	0.93475
5	0.83333	0.83758	0.86091	0.93603
6	0.85714	0.85855	0.87243	0.93734
7	0.87500	0.87546	0.88334	0.93868
8	0.88888	0.88903	0.89336	0.94005
9	0.90000	0.90004	0.90237	0.94144
10	0.90909	0.90910	0.91033	0.94286
11	0.91666	0.91667	0.91731	0.94429
12	0.92307	0.92307	0.92340	0.94573
13	0.92857	0.92857	0.92874	0.94718
14	0.93333	0.93333	0.93341	0.94862
15	0.93750	0.93750	0.93754	0.95007
16	0.94117	0.94117	0.94119	0.95150
17	0.94444	0.94444	0.94445	0.95292
18	0.94736	0.94736	0.94737	0.95432
19	0.95000	0.95000	0.95000	0.95570
20	0.95238	0.95238	0.95238	0.95704
21	0.95454	0.95454	0.95454	0.95835
22	0.95652	0.95652	0.95652	0.95963
23	0.95833	0.95833	0.95833	0.96086
24	0.96000	0.96000	0.96000	0.96206
25	0.96153	0.96153	0.96153	0.96321
26	0.96296	0.96296	0.96296	0.96432
27	0.96428	0.96428	0.96428	0.96539
28	0.96551	0.96551	0.96551	0.96641
29	0.96666	0.96666	0.96666	0.96739
30	0.96774	0.96774	0.96774	0.96833

In the uniform distribution case, we get the expected result that increases in the entry fee lead to increases in the threshold valuation for paying the bribe. When the distribution of valuations is uniform, the optimal entry fee yields $F(v_0) = 0.5$. We also get $F\left(v^*\right) \geq 0.82$. For these values, $\frac{dv^*}{dv_0} > 0$ and hence, when the seller increases the entry fee, the threshold value for paying the bribe goes up. Table 2 shows the effects of increase in entry fee on the threshold valuation for paying the bribe. We start with no entry fee case and show that v^* increases when we increase the entry fee. We again use the uniform distribution with the number of bidders from 2 to 30.

We can also see the effects of an increase in v_0 on the expected revenue of the auctioneer. The expected revenue of the auctioneer is

$$ER(A) = n\left(1 - F\left(v^*\right)\right)(n-1)\int_{v_0}^{v^*} \frac{f(v)}{F(v)} \int_{v_0}^{v} F^{n-1}(y)\, dy\, dv = h\left(v_0, v^*\right)$$

To see the effects of v_0 we take the derivative of the above function with respect to v_0.

$$\frac{dER(A)}{dv_0} = \frac{dh}{dv_0} = h_1\left(v_0, v^*\right) + h_2\left(v_0, v^*\right)\frac{dv^*}{dv_0} \tag{31}$$

where $h_2\left(v_0, v^*\right)$ is zero by the envelope theorem. Hence,

$$\frac{dER(A)}{dv_0} = h_1\left(v_0, v^*\right) = n(n-1)\left(1 - F\left(v^*\right)\right)\int_{v_0}^{v^*} \frac{f(v)}{F(v)}(-1)F^{n-1}\left(v_0\right) dv < 0$$

As a result, it is clear that an increase in the entry fee leads to a decrease in the auctioneer's expected revenue.

Proposition 7: Increasing the entry fee reduces the auctioneer's expected revenue.

4.6 Conclusion

In this chapter we study bribery between the auctioneer and bidders in a first-price sealed-bid auction in which there is a positive entry fee, how bribery affects the bidding functions of the bidders, and the welfare positions of the bidders, auctioneer and seller when it is common knowledge that the auctioneer is corrupt. Our results show that the seller is more aggressive when there is the possibility of corruption, that is, he imposes an entry fee higher than he would have imposed in the absence of bribery. They also show that the threshold value that separates the bidders who pay the bribe from those who do not, increases when the seller imposes an entry fee on the bidders. The entry fee deters bribery in the sense that bidders with a smaller set of valuations pay the bribe. We also show that the effectiveness of an entry fee for deterring bribery diminishes as the number of bidders grows when the valuation distribution is uniform.

Our results also show that an entry fee not only can deter bribery, but also a higher entry fee serves as more of a deterrent. As a matter of fact, an increase in the entry fee leads to an increase in the threshold value, so that the ex ante probability of paying the bribe for a bidder decreases.

CHAPTER V

CONCLUSIONS

This dissertation examines corruption between the auctioneer and the bidders where corruption takes the following form. The auctioneer approaches the bidders and tells them that if they pay a bribe of a certain amount and if they submit the highest bid, the auctioneer will change their bid so that they only have to pay the second-highest bid. This is interesting because bidders decide whether or not to pay the bribe before submitting their bids, so that all bidders are involved in the bribery process.

In the second chapter we study bribery between the auctioneer and bidders in a first-price sealed-bid auction and how it affects the bidding functions of the bidders and the welfare positions of bidders, auctioneer and seller when it is common knowledge for bidders that the auctioneer is corrupt. Our results show that, given the size of the bribe set by the auctioneer, bidders with valuations above some threshold pay the bribe, while bidders with lower valuations do not. In equilibrium, bidders who pay the bribe bid their valuations while bidders who do not pay the bribe bid according to the standard first-price auction bid function. The auctioneer sets the bribe to trade off the amount collected from a bidder who pays the bribe and the number of bidders expected to pay it.

We also have shown the welfare properties of the auction with bribery and show that it is efficient and, in equilibrium, bribes are a transfer from the seller to the auctioneer. Although bribery changes the bid functions of some bidders, namely those with sufficiently high valuations, it has no effect on the final allocation of the prize or the welfare of the bidders. The bribes generate expected revenue for the auctioneer, and because bidders are not affected, the bribes also generate an expected loss to the seller compared to a first-price auction without bribery.

In chapter III, we study bribery between the auctioneer and the bidders in a second-price sealed-bid auction. This time the proposed corruption does not work because the bidders' dominant strategy is to bid their values whatever the other bidders do, and they will pay the second highest bid anyway. Therefore, they do not pay the bribe to the auctioneer. We have also shown that the revenue equivalence theorem breaks down when there is a bribery issue. The first-price and second-price auctions do not yield the same expected revenue to the seller. Chapter III's main contribution is that seller can avoid the transfer to the auctioneer by simply demanding a second-price auction instead of a first-price auction.

Chapter IV analyzes the differences between an entry fee and a bribe in a first-price sealed-bid auction; determining which is more favorable in terms of expected profit according to the bidders and the seller. We study bribery between the auctioneer and bidders in a first-price sealed-bid auction in which there is a positive entry fee, how bribery affects the bidding functions of the bidders, and the welfare positions of the bidders, auctioneer and seller when it is common knowledge for bidders that the auctioneer is corrupt. Our results show that, given the size of the bribe set by the auctioneer, bidders with valuations above some threshold pay the bribe, while bidders with lower valuations do not.

We show that the seller is more aggressive by imposing a higher amount of entry fee when there is possibility of bribery. We also show that an entry fee partially deters bribery in the sense that bidders with a smaller set of violations pay the bribe. We also show that the effectiveness of an entry fee for deterring bribery diminishes as the number of bidders grows. Another important result is that an entry fee not only can deter bribery, but also a higher entry fee serves as more of a deterrent. As a matter of fact an increase in the entry fee leads to an increase in the threshold value so that the ex ante probability of paying the bribe for a bidder decreases. Therefore the seller, by imposing a higher entry fee, can deter bribery to some degree, but there is always a positive probability that a bidder pays the bribe to the auctioneer as long as the entry fee is less than the highest valuation.

In future research we will carry out some empirical work on bribery issues between the auctioneer and the bidders in sealed-bid auctions. It will be interesting to apply our bribery model to government food supply contracts in Turkey.

REFERENCES

Ashenfelter, O. (1989): "How Auctions Work for Wine and Art," *Journal of Economic Perspectives,* 3, 23-36.

Burguet, R., and M. Perry (2000): "Bribery and Favoritism by Auctioneers in Sealed-Bid Auctions," Working paper, Rutgers University, New Brunswick, NJ.

Cramton, P., R. Gibbons, and P. D. Klemperer (1987): "Dissolving a Partnership Efficiently," *Econometrica,* 55, 615-632.

Graham, D. A., and R. C. Marshall (1987): "Collusive Bidder Behavior at Single Object Second-Price and English Auctions," *Journal of Political Economy,* 95, 1217-1239.

Hendricks, K., and R. H. Porter (1989): "Collusion in Auctions," *Annales d'Economic et de Statisque,* 15/16, 217-230.

Ingraham, A. (2000): "Testing for Cheating Between Bidders and Auctioneers in Sealed-Bid Auctions," Working paper, University of Maryland, College Park.

Klemperer, P. (1999): "Auction Theory: A Guide to the Literature," *Journal of Economic Surveys,* 13, 227-286.

Lengwiler, Y., and E. Wolfstetter (2000): "Auctions and Corruption," Working paper, Humboldt-Universit t zu Berlin, Berlin.

Matthews, A. S. (1995): "A Technical Primer on Auction Theory 1: Independent Private Values," Discussion paper No: 1096, Northwestern University, Evanston, IL.

Maskin, E., and J.G. Riley (1984): "Optimal Auctions with Risk Averse Buyers," *Econometrica,* 52, 1473-1518.

McAfee, R. P., and J. McMillan (1987): "Auctions and Bidding," *Journal of Economic Literature,* 25, 708-747.

Milgrom, P. R., and R. J. Weber (1982): "A Theory of Auctions and Competitive Bidding," *Econometrica,* 49**,** 1089-1122.

Milgrom, P. R. (1987): "Auction Theory," *Advances in Economic Theory: Fifth World Cogres.* Cambridge: Cambridge University Press.

Myerson, R. (1981): "Optimal Auction Design," *Mathematics of Operations Research,* 6, 58-73.

Porter, R. H. (1995): "The Role of Information in U.S. Offshore Oil and Gas Lease Auctions," *Econometrica*, 63, 1-28.

Riley, J. G., and W. F. Samuelson (1981): "Optimal Auctions," *American Economic Review,* 71, 381-392.

Robinson, M. S. (1985): "Collusion and the Choice of Auction," *Rand Journal of Economics,* 16, 141-145.

Vickrey, W. (1961): "Counterspeculation, Auctions, and Competitive Sealed Tenders," *Journal of Finance,* 16, 8-37.

VITA

Name: Sevket Alper Koc

Permanent Address: Cumhuriyet Mah. 1650 Sok.
Nesibedag Apt. B-Blok Kat:1 No:1
Mersin, Turkey

Education:

(January 1998 – August 2002)
Texas A&M University, College Station, Texas
Doctor of Philosophy in Economics

(September 1991 – August 1995)
Middle East Technical University, Ankara, Turkey
Bachelor of Science Degree in Economics

Work Experience:

(January 2002 – May 2002)
Texas A&M University, College Station, Texas; Economics Department
Research Assistant

(January 1997 – January 1998)
Kocaeli University, Kocaeli, Turkey; Economics Department
Teaching Assistant

(August 1996 – January 1997)
Aselsan, Ankara, Turkey
Export Analyst