

ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL

**QUANTITATIVE ANALYSIS OF AIRCRAFT AERODYNAMIC DERIVATIVES
USING THE LEAST SQUARES METHOD IN A SIX DEGREES OF
FREEDOM FLIGHT SIMULATION ENVIRONMENT**

M.Sc. THESIS

Furkan ALTINIŞIK

Department of Aeronautical and Astronautical Engineering

Aeronautical and Astronautical Engineering Programme

AUGUST 2024

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**Furkan ALTINIŞIK
(511201165)**

Department of Aeronautical and Astronautical Engineering

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Thesis Advisor: Dr. Hayri ACAR

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İSTANBUL TEKNİK ÜNİVERSİTESİ ★ LİSANSÜSTÜ EĞİTİM ENSTİTÜSÜ

**UÇAK AERODİNAMİK TÜREVLERİNİN ALTI SERBESTLİK DERECELİ
UÇUŞ BENZETİM ORTAMINDA EN KÜÇÜK KARELER
YÖNTEMİ İLE KANTİTATİF ANALİZİ**

YÜKSEK LİSANS TEZİ

**Furkan ALTINIŞIK
(511201165)**

Uçak ve Uzay Mühendisliği Anabilim Dalı

Uçak ve Uzay Mühendisliği Programı

Tez Danışmanı: Dr. Hayri ACAR

AĞUSTOS 2024

Furkan ALTINIŞIK, a M.Sc. student of ITU Graduate School student ID 511201165 successfully defended the thesis entitled “QUANTITATIVE ANALYSIS OF AIRCRAFT AERODYNAMIC DERIVATIVES USING THE LEAST SQUARES METHOD IN A SIX DEGREES OF FREEDOM FLIGHT SIMULATION ENVIRONMENT”, which he/she prepared after fulfilling the requirements specified in the associated legislations, before the jury whose signatures are below.

Thesis Advisor : **Dr. Hayri ACAR**
Istanbul Technical University

Jury Members : **Dr. Barış BAŞPINAR**
Istanbul Technical University

Dr. Zafer ÖZNALBANT
Eskisehir Osmangazi University

.....

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Furkan ALTINIŞIK



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ABBREVIATIONS

A/C	: Aircraft
CFD	: Computational Fluid Dynamics
C.G.	: Center of Gravity
Der.	: Derivative
DOF	: Degrees of Freedom
EKF	: Extended Kalman Filter
Est.	: Estimation
Hz	: Hertz
KF	: Kalman Filtering
kHz	: Kilohertz
LS	: Least Squares
LSM	: Least Squares Method
MLE	: Maximum Likelihood Estimation
OLS	: Ordinary Least Squares
RLS	: Recursive Least Squares
UKF	: Unscented Kalman Filter



SYMBOLS

α	: Angle of Attack
β	: Angle of Sideslip
g	: Gravitational Acceleration
ρ	: Air Density
ϕ	: Bank Angle
θ	: Pitch Angle
ψ	: Heading Angle
ω	: Angular Velocity in Body Axis
$\delta_a, \delta_e, \delta_r$	: Control Surface Deflections
u, v, w	: Translational Velocities in Body Axis
p, q, r	: Rotational Velocities in Body Axis
b	: Wing Span
c	: Aerodynamic Chord
g	: Gravitational Acceleration
$\vec{F}_A$	: Aerodynamic Force Vector in Body Axis
$\vec{F}_G$	: Gravity Vector in Body Axis
$\vec{F}_T$	: Thrust Force Vector in Body Axis
$\vec{H}$	: Angular Momentum Vector
H_a	: Altitude of the Atmosphere
I	: Mass Moment of Inertia Matrix
I_{xx}	: Moment of Inertia Along the Body-x Axis
I_{yy}	: Moment of Inertia Along the Body-y Axis
I_{zz}	: Moment of Inertia Along the Body-z Axis
I_{xy}	: Product of Inertia on the Body xy Plane
I_{xz}	: Product of Inertia on the Body xz Plane
I_{yz}	: Product of Inertia on the Body yz Plane
L_a	: Lapse Rate of the Atmosphere
M	: Mach Number
$\vec{M}$	: Moment Vector
$\vec{M}_A$	: Aerodynamic Moment Vector in Body Axis
$M_{A/C}, m$	: Mass of the Aircraft
$\bar{q}$	: Dynamic Pressure
R_g	: Gas Constant
$\vec{r}_{dm}$	: Position Vector of Differential Mass in Body Axis
T	: Thrust Value in Body-x Axis
T_e	: Temperature of the Atmosphere
$\vec{V}_{dm}$	: Velocity of Differential Mass in Body Axis
$\vec{V}_T, \vec{V}_{wind}$	: Air Velocity in Wind Axis
$\vec{V}$	: Airspeed in Body Axis
n_x, n_y, n_z	: Load Factors in Body Axis
a_x, a_y, a_z	: Accelerations in Body Axis
P_{lm}	: Linear Momentum



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QUANTITATIVE ANALYSIS OF AIRCRAFT AERODYNAMIC DERIVATIVES USING THE LEAST SQUARES METHOD IN A SIX DEGREES OF FREEDOM FLIGHT SIMULATION ENVIRONMENT

SUMMARY

This thesis presents an in-depth analysis of aircraft aerodynamic derivatives using the Least Squares Method (LSM) within a six degrees of freedom (6-DOF) flight simulation environment. The primary objective is to evaluate and compare the performance of Ordinary Least Squares (OLS) and Recursive Least Squares (RLS) methods in estimating aerodynamic parameters under various flight conditions, including ideal, turbulent, and error-induced scenarios.

A detailed 6-DOF flight simulation model was developed using data from the SIAI Marchetti S211 aircraft. This model integrates various subsystems, including equations of motion, aerodynamics, engine dynamics, and atmospheric conditions.

The Newton-Raphson method was employed to maintain steady-state conditions, ensuring the aircraft's trim state was accurately represented. For solving the differential equations derived from the equations of motion, the Runge-Kutta method was chosen due to its robustness and accuracy in handling the nonlinearities associated with flight dynamics in the simulation model.

The aerodynamic forces and moments were linearized using the small disturbance theorem, which simplifies the complex nonlinear equations into a more manageable linear form. This linearization allowed for the formulation of force and moment coefficients as functions of aerodynamic derivatives. These derivatives, critical for understanding the aircraft's behavior, were estimated using both OLS and RLS methods.

Realistic flight data was simulated under various conditions, including ideal scenarios without any disturbances, scenarios with atmospheric turbulence, and scenarios with systematic sensor errors. The Dryden turbulence model was used to simulate realistic atmospheric disturbances, providing a continuous representation of turbulence that affects the aircraft during flight. Systematic sensor errors were introduced to understand their impact on the accuracy of parameter estimation.

The OLS method provided single-step parameter estimates by processing all data points simultaneously, making it straightforward and computationally efficient. In contrast, the RLS method updated parameter estimates incrementally as new data became available. This dynamic approach allowed the RLS method to adapt to changes over time, making it particularly suitable for real-time applications where system characteristics may vary.

Performance metrics such as the R^2 statistic and standard deviation were used to evaluate the estimation accuracy. These metrics provided quantitative measures of how

well the estimated parameters matched the true values, with the R^2 statistic indicating the proportion of variance explained by the model and the standard deviation providing a measure of the estimation precision.

The analysis revealed that both OLS and RLS methods produced accurate results under ideal and turbulent conditions. The presence of atmospheric turbulence did not significantly affect the estimation accuracy, as the average error introduced by the turbulence was zero. This robustness highlights the effectiveness of LSM in handling real-world flight data with environmental disturbances. However, when systematic sensor errors were introduced, both OLS and RLS methods showed biased estimation results. The bias was evident in the deviation of the estimated aerodynamic derivatives from their true values, underscoring the importance of accurate and error-free measurement data for reliable parameter estimation.

Further analysis demonstrated that increasing the sampling frequency improved the performance of the RLS method. At higher frequencies, such as 50 kHz, the RLS estimates converged more closely to the true values, even in the presence of systematic sensor errors. This improvement is attributed to the reduced information loss in higher frequency sampling, which captures more details and variations in the data that might be missed at lower frequencies. This finding suggests that higher sampling rates can effectively mitigate the adverse effects of sensor errors on parameter estimation.

The design of control surface inputs was identified as a crucial factor influencing the accuracy of aerodynamic parameter estimation. Optimal input design, which involves selecting appropriate control surface deflections, ensured accurate estimation results. Conversely, non-optimal inputs led to discrepancies between the estimated and true values. This emphasizes the need for carefully designed excitation maneuvers during flight tests to obtain reliable aerodynamic data.

The RLS method demonstrated particular advantages in dynamic environments due to its ability to update estimates in real-time. This adaptive capability allowed it to maintain accuracy even when the system characteristics changed over time. However, the OLS method exhibited slightly better performance at lower frequencies, showing less sensitivity to variations in sampling rates. Both methods showed distinct strengths, with OLS excelling in stable, low-frequency scenarios and RLS proving superior in dynamic, high-frequency conditions.

The theoretical expected value formulas for the parameter estimates were validated using the simulation model outputs. This validation confirmed the presence of bias when systematic errors were introduced and reinforced the high accuracy of estimates under both ideal and turbulent conditions.

In conclusion, this thesis provides a comprehensive evaluation of OLS and RLS methods for estimating aerodynamic derivatives in a 6-DOF flight simulation environment. The findings demonstrate the robustness of these methods under various flight conditions, highlight the impact of systematic sensor errors, and underscore the importance of optimal input design and high-frequency data sampling under linear database.

UÇAK AERODİNAMİK TÜREVLERİNİN ALTI SERBESTLİK DERECELİ UÇUŞ BENZETİM ORTAMINDA EN KÜÇÜK KARELER YÖNTEMİ İLE KANTİTATİF ANALİZİ

ÖZET

Bu tez çalışmasında, uçak aerodinamik türevlerinin altı serbestlik dereceli (6-DOF) bir uçuş simülasyon ortamında en küçük kareler yöntemi (LSM) kullanılarak kantitatif analizi yapılmıştır. Bu çalışmanın ana amacı, OLS ve RLS yöntemlerini kullanarak aerodinamik parametrelerin çeşitli uçuş koşullarında (ideal, türbülanslı ve hata içeren senaryolar) tahmin edilmesidir.

Çalışmanın bir diğer hedefi doğrusal aerodinamik veritabanı içeren model çıktılarıyla farklı koşullarda her iki yöntemin performansını değerlendirmek ve karşılaştırmaktır. Bu karşılaştırmada türbülans ve sensör hatası içeren koşullar uçağın gerçek uçuşta nasıl davrandığını daha iyi simüle edebilmek ve kestirim metodlarına etkisini araştırmak için oluşturulmuştur. Dolayısıyla çalışmada, uçak aerodinamik türevlerinin altı serbestlik dereceli bir uçuş benzetim ortamında en küçük kareler yöntemi ile nicel ve nitel analizi yapılmıştır. OLS ve RLS yöntemlerinin performansları, ideal ve türbülanslı modellerde karşılaştırılmış ve her iki yöntemin de doğruluk ve hassasiyet açısından güçlü yönleri belirlenmiştir. Sistemik sensör hatalarının kestirim sonuçlarına etkisi incelenmiş ve bu hataların minimize edilmesi ile daha doğru ve güvenilir sonuçlar elde edilmiştir.

Çalışmada, altı serbestlik dereceli uçuş simülasyon modeli, SIAI Marchetti S211 uçağının verileri kullanılarak geliştirilmiştir (referans değerler Roskam'dan alınmıştır). Veriler uçağın geometrik, kütle ve uçuş koşulu bilgilerini içermektedir. Bu model, hareket denklemleri, aerodinamik, motor dinamikleri ve atmosferik koşulları içeren çeşitli alt sistemleri entegre etmektedir. Newton-Raphson yöntemi, uçağın referans alınan verilerde belirtilen uçuş koşullarında trim durumunu korumak için kullanılmış ve Runge-Kutta yöntemi, hareket denklemlerinden türetilen diferansiyel denklemleri çözmek için seçilmiştir.

Doğrusal olmayan ve yüksek mertebeden türev içeren aerodinamik kuvvetler ve momentler, küçük bozuntular teoremi kullanılarak doğrusal hale getirilmiş ve bu doğrusal hale getirme işlemi, karmaşık doğrusal olmayan denklemleri daha yönetilebilir bir forma dönüştürmüştür. Bu sayede, kuvvet ve moment katsayıları, birinci dereceden aerodinamik türevlerin fonksiyonları olarak formüle edilmiştir. Bu türevler, uçağın davranışını anlamak için kritik öneme sahiptir ve hem OLS hem de RLS yöntemleri kullanılarak parametre kestirimleri yapılmıştır.

Oluşturulan altyapı ile model çıktıları gerçekçi uçuş verileri, ideal koşullar, atmosferik türbülans ve sistemik sensör hataları içeren çeşitli senaryolar altında simüle edilmiştir. Dryden türbülans modeli, sürekli atmosferik bozulmaları simüle etmek için

kullanılmış ve bu model, uçuş sırasında uçağı etkileyen türbülansın sürekli bir temsilini sağlamıştır. Belirli parametrelerin sistematik sensör hataları, parametre tahmininin doğruluğunu anlamak için modele dahil edilmiştir.

OLS yöntemi tüm veri noktalarını aynı anda işleyerek tek adımda çözülen parametre tahminleri sağlamış ve bu sayede basit ve hesaplama açısından verimli bir yöntem olmuştur. Buna karşılık RLS yöntemi yeni veriler geldikçe parametre tahminlerini artımlı olarak güncellemiştir. Bu dinamik yaklaşım RLS yönteminin zamanla değişikliklere uyum sağlamasına olanak tanımış ve bu sayede sistem özelliklerinin değişebileceği gerçek zamanlı uygulamalar için özellikle uygun hale gelmiştir. Fakat belirtilmelidir ki bu özelliğe karşın modelin veritabanında bulunan türev katsayıları sabit kalmış ve zamana göre dinamik değiştirilmemiştir. RLS yönteminin zamana göre tahmin kesinliğinin gerçek değeri sabit kalan aerodinamik türeve yakınsayarak zamana göre dinamik değiştiği gösterilmiştir. Dolayısıyla RLS yöntemi zamanla değişen ortamlarda belirgin avantajlar göstermiştir çünkü parametre tahminlerini gerçek zamanlı olarak güncelleme yeteneğine sahiptir. Bu uyarlanabilir yetenek sistem özellikleri değiştiğinde bile doğruluğu korumasına olanak tanır. Ancak OLS yöntemi daha düşük frekanslarda biraz daha iyi performans göstermiş ve örnekleme oranı varyasyonlarına karşı daha az duyarlılık göstermiştir. Her iki yöntem de farklı yönlerden güçlü yanlar göstermiş ve OLS daha kararlı ve düşük frekanslı senaryolarda üstünlük sağlarken, RLS dinamik ve yüksek frekanslı koşullarda daha üstün olmuştur. Dolayısıyla bu, OLS ve RLS yöntemlerinin aerodinamik türevlerin kestiriminde nasıl performans gösterdiğini ve her iki yöntemin de çeşitli uçuş koşullarında nasıl avantajlar sunduğunu göstermiştir. RLS yönteminin dinamik ortamlarda gerçek zamanlı uygulamalar için uygunluğunu ve yüksek örnekleme frekanslarında performansını artırma yeteneğini vurgulamıştır. OLS yöntemi ise daha sabit ve düşük frekanslı veri setlerinde daha iyi sonuçlar vermiştir.

Performans metrikleri olarak R^2 istatistiği ve standart sapma kullanılmıştır. Bu metrikler tahmin edilen parametrelerin gerçek değerlere ne kadar iyi uyduğuna dair nicel ölçüler sağlamış ve R^2 istatistiği model tarafından açıklanan varyans oranını, standart sapma ise tahminlerin hassasiyetini göstermiştir. Bu metrikler hem OLS and RLS yöntemleri için kullanılmış ve sonuçlar karşılaştırılmıştır.

Analizler hem OLS hem de RLS yöntemlerinin ideal ve türbülanslı koşullar altında yüksek doğrulukta sonuçlar verdiğini göstermiştir.

Atmosferik türbülansın varlığı ortalama hatanın sıfır olması nedeniyle tahmin doğruluğunu önemli ölçüde etkilememiştir. Bu durum LSM'nin çevresel bozulmalarla birlikte gerçek uçuş verilerini ele almadaki etkinliğini vurgulamaktadır. Ancak sistematik sensör hataları verildiğinde, hem OLS hem de RLS yöntemleri hatalı tahmin sonuçları göstermiştir. Bu hata tahmin edilen aerodinamik türevlerin gerçek değerlerden sapması olarak ortaya çıkmış ve güvenilir parametre tahminleri için doğru ve hatasız ölçüm verilerinin önemini vurgulamıştır. Sonuç olarak sistematik sensör hatalarının eklenmesi durumunda, her iki yöntemin de kestirim sonuçlarında bayaşıl sonuçlar elde edilmiştir. Ancak RLS yönteminin frekans artırılarak performansının iyileştirilebildiği ve daha doğru sonuçlar verdiği tespit edilmiştir. Sistematik sensör hataları ölçüm parametrelerine eklenmiş ve bu hataların kestirim sonuçlarına etkisi

incelenmiştir. Bu analizler sensör hatalarının aerodinamik türev kestirimlerine nasıl etki ettiğini ve bu hataların nasıl minimize edilebileceğini göstermiştir.

Örnekleme frekansının artırılması RLS yönteminin performansını önemli ölçüde iyileştirmiştir. Daha yüksek frekanslarda, örneğin 50 kHz'te, RLS tahminleri sistematik sensör hatalarının varlığında bile gerçek değerlere OLS'den daha yakın sonuçlar vermiştir. Bu iyileşme daha yüksek frekanslı örneklemenin bilgi kaybını azaltmasından kaynaklanmaktadır ve bu sayede verilerde kaçırılacak daha fazla ayrıntı ve değişikliği yakalamaktadır. Bu durum ayrıca kazanç matrisi formülündeki unutma faktörü parametresi açıklanarak da vurgulanmıştır. Bu bulgu yüksek örnekleme oranlarının sensör hatalarının parametre tahmini üzerindeki olumsuz etkilerini etkili bir şekilde azaltabileceğini göstermektedir. Bir başka deyişle örnekleme frekansının artırılması RLS yönteminin performansını olumlu yönde etkilemiş ve kestirim sonuçlarının gerçek değerlere yakınsadığı görülmüştür.

Frekansı 100 Hz ile yapılan analizlerde RLS yönteminin bazı türevlerde gerçek değerlerinden sapmalar gösterdiği belirlenmiştir. Ancak örnekleme frekansının 50 kHz'e çıkarılmasıyla birlikte, RLS yönteminin kestirim performansının arttığı ve kestirim değerlerinin gerçek değerlere yakınsadığı tespit edilmiştir. Dolayısıyla bu çalışmanın bir diğer önemli katkısı, RLS yönteminin dinamik ortamlarda ve yüksek örnekleme frekanslarında nasıl performans gösterdiğini ve sistematik sensör hatalarının etkilerini nasıl minimize edebileceğini göstermesidir. Ayrıca RLS yönteminin gerçek zamanlı uygulamalarda sistem özelliklerinin değişebileceği durumlarda nasıl avantajlar sunduğunu ve yüksek frekanslı veri setlerinde bilgi kaybını azaltma yeteneğini vurgulamaktadır.

Kontrol yüzeyi girişlerinin tasarımı aerodinamik parametre tahmininin doğruluğunu önemli ölçüde etkilemiştir. Optimum giriş tasarımı uygun kontrol yüzeyi açılarını seçmeyi içermiş ve doğru tahmin sonuçlarını garanti etmiştir. Bazı noktalarda RLS için sadece optimum girdi tasarımının yeterli olamayabileceği, ayrıca örnekleme frekansının da önemli olabileceği vurgulanmıştır. Dolayısıyla optimal olmayan girişler tahmin edilen ve gerçek değerler arasında farklılıklara yol açmıştır. Bu, güvenilir aerodinamik veriler elde etmek için uçuş testleri sırasında dikkatle tasarlanmış manevraların icra edilmesinde kontrol yüzeyi girdilerinin önemini vurgulamaktadır.

Parametre tahminleri için teorik beklenen değer formülleri simülasyon modeli çıktıları kullanılarak doğrulanmıştır. Bayaslı koşullar beklenen değer formülleri ile desteklenmiştir. Bu doğrulama, sistematik hatalar tanıtıldığında tahminlerdeki hatanın varlığını ve ideal ve türbülanslı koşullar altında tahminlerin yüksek doğruluğunu pekiştirmiştir.

Bu çalışmada elde edilen bulgular hem OLS hem de RLS yöntemlerinin aerodinamik parametre tahmininde, uygun koşullar altında, ne kadar etkili olduğunu göstermekte ve her iki yöntemin de çeşitli uçuş koşullarında nasıl avantajlar sunduğunu ortaya koymaktadır. Bu koşullar lineer aerodinamik veritabanı olan bir model için doğrulanmıştır. Önceden vurgulandığı gibi, OLS yöntemi daha sabit ve düşük frekanslı veri setlerinde daha iyi sonuçlar vermekte iken, RLS yöntemi dinamik ortamlarda ve yüksek frekanslı veri setlerinde üstün performans göstermektedir.

Çalışmanın son bölümünde yukarıda bahsedildiği gibi sistematik sensör hatalarının etkisi incelenmiş ve bu hataların nasıl minimize edilebileceği analiz edilmiştir. Sonuçlar sistematik hataların tahmin sonuçlarına önemli bir etkisi olduğunu ve bu hataların minimize edilmesi gerektiğini göstermiştir. Bu bulgular sensör hatalarının en küçük kareler yöntemi için aerodinamik türev tahminlerine önemli ölçüde etki ettiğini ve bu hataların nasıl minimize edilebileceğini anlamak için önemli bilgiler sağlamaktadır.

Kestirim tahminlerinde elde edilen bulgular, koşullar sağlandığında OLS ve RLS yöntemlerinin aerodinamik parametre tahmininde ne kadar etkili olduğunu göstermekte ve her iki yöntemin de çeşitli uçuş koşullarında nasıl avantajlar sunduğunu ortaya koymaktadır. Yüksek manevra kabiliyetine sahip ve doğrusal olmayan bir aerodinamik davranışa sahip bir uçak için her iki yöntem uygulamada zayıf kalabilir. Fakat her iki yöntem uçuş verilerindeki hatalar minimize edildiğinde güçlü ve hızlı bir tahmin aracı haline de gelebilir.

Sonuç olarak bu çalışmada uçak aerodinamik türevlerinin altı serbestlik dereceli bir uçuş benzetim ortamında en küçük kareler yöntemi ile kantitatif analizi yapılmıştır. OLS ve RLS yöntemlerinin performansları, ideal ve türbülanslı modellerde karşılaştırılmış ve her iki yöntemin de doğruluk ve hassasiyet açısından güçlü yönleri belirlenmiştir. Sistematik sensör hatalarının kestirim sonuçlarına etkisi incelenmiş ve bu hataların minimize edilmesi ile daha doğru ve güvenilir sonuçlar elde edilmiştir. Ayrıca bir diğer önemli sonuç olarak kazanç matrisindeki unutm faktörünün frekansın yüksek olması ile bağlantılı olduğu irdelenmiştir. Unutm faktörünün 1 olmasının örnekleme adım sayısı ile ilişkili olduğu vurgulanmıştır ve yüksek frekanslarda daha iyi sonuçlar alındığı görülmüştür.

1. INTRODUCTION

Measurements obtained from physical systems in real-world scenarios invariably display deviations from the theoretical or idealized values anticipated under controlled conditions. In this discussion, the term "physical system" refers to a system that requires comprehensive mathematical modeling for a precise description of its operational characteristics. Therefore, parameter estimation under realistic conditions is essential for accurately modeling physical systems, which includes the computation of aerodynamic derivatives. The creation of an extensive aerodynamic database is crucial, as it plays a vital role in determining the stability and control characteristics of an aircraft, thus emphasizing its importance in aerodynamic studies.

There are several techniques employed to generate aerodynamic data, which include Computational Fluid Dynamics (CFD) based numerical solutions and experimental methods conducted in wind tunnels. These methodologies are invaluable as they facilitate the direct measurement of forces and moments, which are critical components in the calculation of aerodynamic derivatives. While direct analysis of the center of pressure of airflow relative to a fixed reference point is not feasible in actual flight conditions, it becomes possible in controlled environments such as those offered by CFD simulations and wind tunnel experiments. In these settings, a reference point can be designated at a predetermined location on either scaled or full-size aircraft models, allowing for the systematic creation of accurate aerodynamic datasets.

For the extraction of precise aerodynamic data from genuine flight scenarios, it is imperative to implement parameter estimation techniques. Scholarly literature identifies a range of estimation methods, each necessitating the prior collection of specific flight data from the aircraft. This data could represent systems whose dynamics are either fully understood or remain unknown. A system where only the inputs and outputs are discernible, and the internal mechanics remain hidden, is typically classified as a "black box." Conversely, a "physics-based" open system

is characterized by known inputs, outputs, and transparent physical dynamics. This research is dedicated to the investigation of such an open system. In the first paragraph of this section, it is mentioned how well the dynamic system should be modeled under the appropriate conditions. It is also possible to evaluate simplified dynamic systems in terms of a comparison of two specific methods that are desired to be analyzed.

In particular, this thesis incorporates a sophisticated flight dynamics model with six degrees of freedom, specifically engineered to replicate real flight data with exceptional accuracy. The clarity and definition of the inputs and outputs of this model qualify it as an exemplary instance of a physics-based system. Essential data, including mass properties, geometric configurations, and aerodynamic data for this model, were sourced from the SIAI Marchetti S211 aircraft, as extensively documented in Roskam's definitive guide on airplane design [1]. This thorough methodological approach not only guarantees that the model accurately mirrors authentic flight conditions but also lays a robust foundation for a comprehensive analysis and deeper comprehension of aircraft dynamics under actual flight conditions. Moreover, such flight simulation models provide a controlled environment that is ideal for generating, analyzing, and validating data, along with their inputs and outputs, thus significantly enhancing the accuracy and reliability of the research conducted in aerodynamic studies. This controlled setting is instrumental in facilitating detailed examinations and reliable interpretations of the dynamic behaviors of aircraft, further solidifying the critical role of accurate data collection and analysis in the field of aerospace engineering.

A simple example of a physical system addressing a system with known dynamics is shown in Figure 1.1.

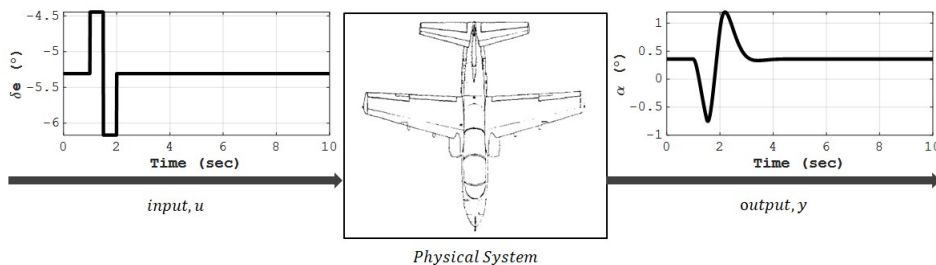


Figure 1.1 : Representation of Physical System with Known Dynamics

Within this framework, a flight simulation model with six degrees of freedom, featuring inputs and outputs as depicted in Figure 1.1, has been established. The numerical

values for the aerodynamic and engine-thrust derivatives utilized in the model are sourced from [1], as previously indicated. The corresponding flight simulation model is essentially comprised of blocks representing atmospheric conditions, aerodynamics, engine, and the equations of motion. The blocks of the flight simulation model with six degrees of freedom are outlined in Figure 1.2.

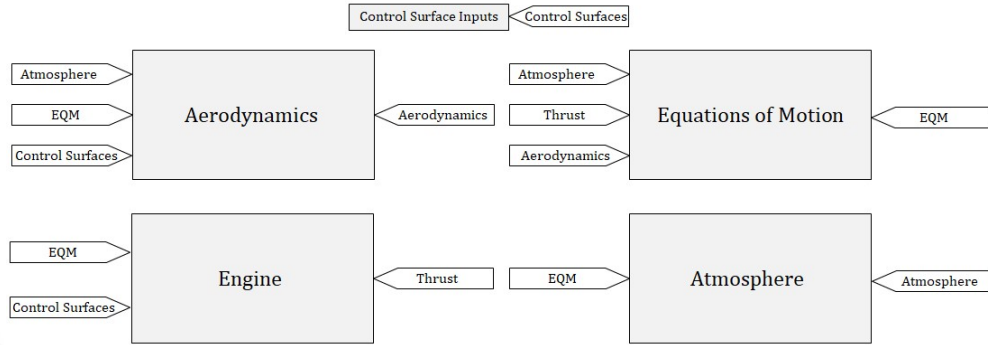


Figure 1.2 : The Main Blocks of Six Degrees of Freedom Flight Simulation Model

The diagram depicted in the provided figure outlines four principal components, representing a comprehensive mathematical model of a specific aircraft system that is characterized by six degrees of freedom. This model has been developed using classical Newtonian mechanics within the MATLAB-Simulink [2] environment. In the simulation model, the aircraft is configured to maintain a steady-state condition with the wings level, achieved through the application of the Newton-Raphson method for numerical optimization. This section builds on prior discussions where it was established that the aerodynamic derivatives crucial to the process of aerodynamic derivative estimation in the flight dynamics model are well-defined, with the entire database being organized around these derivatives.

For the purpose of numerically resolving the differential equations that form the basis of the motion equations in this model, the 4th order Runge-Kutta method was selected within the Simulink program for its accuracy and reliability. Additionally, the atmospheric block within the model includes an atmospheric turbulence model, specifically chosen as the continuous Dryden atmospheric turbulence model, which has been integrated directly within the Simulink framework. Detailed discussions on the choice of numerical methods for solving the differential equations and the

implementation details of the atmospheric turbulence model are not included in this document, as they utilize the foundational tools provided by Simulink.

Upon establishing the flight simulation infrastructure, it was assumed for the purposes of simulation that measurements within the model could be both flawless and flawed, thus simulations included the effects of atmospheric turbulence and sensor measurement inaccuracies as previously detailed. Following this setup, a comparative analysis was conducted between parameter estimates derived from two variations of the least squares method—termed "recursive" and "ordinary." These terms align with what is known in the field as the equation error method. The estimation of aerodynamic derivatives can be approached using various methodologies that are effective in both linear and nonlinear contexts, including the output error method, filter error method, equation error method, and contemporary artificial intelligence-based methods. It is noteworthy that the least squares method is considered a subset of the equation error method, often also referred to as regression analysis. This method is particularly effective in linear contexts but is adaptable enough to be applied in nonlinear settings where analyses may require iterative calculations instead of single-step processes.

In addition to these methodological explorations, error analyses were performed to evaluate the practical effectiveness of the parameter estimation methods employed. The R^2 statistic and the standard deviation were used as metrics to assess the performance of these methods within the context of the model. These metrics offered a quantitative evaluation of how closely the estimated parameters aligned with the true values. The R^2 statistic measured the percentage of variance the model could explain, while the standard deviation indicated the precision of the parameter estimates.

In summary, the parameter estimations of aerodynamic derivatives were conducted using a six degrees of freedom flight simulation model, which is linearly defined by the aerodynamic and thrust database through the application of both ordinary and recursive least squares methods. The ensuing sections of the study will provide a detailed exposition of the aircraft motion model, the aerodynamics, engine, and atmospheric models, and will clarify the trim algorithm. Additionally, the parameter estimation methods employed are thoroughly reviewed. Subsequent application studies explored

both ideal conditions and those involving turbulent distortions and systematic errors, with the final section of the study dedicated to discussing the results obtained from these various scenarios.

1.1 Purpose of Thesis

In a broad range of analytical scenarios, particularly in the design of control algorithms for aircraft, linear methodologies are frequently utilized to navigate away from the complex challenges posed by nonlinear dynamics. Within these contexts, considerable effort is dedicated to adjusting and refining the design parameters across different areas of aeronautical expertise to fit within a linearly acceptable realm. A prime example of such adaptation is the analysis of an aircraft's control and stability attributes at a specified trim point, which involves linearizing the system to simplify the computational processes. The results of these linearized analyses are crucial, as they provide a host of aerodynamic coefficients, for example C_{M_u} and C_{L_q} , which are integral to further studies and applications in aerospace engineering.

Within the sphere of these analyses, the least squares method has shown remarkable effectiveness when employed on datasets characterized by linear aerodynamic properties. When this method is executed with meticulous and precise separation of errors, it exhibits outstanding robustness in parameter estimation analysis. Nevertheless, the practical application of the least squares method using real flight data introduces several significant challenges.

One of the most important points of this thesis is to examine the power of simplified estimation analyses under appropriate conditions because the collection of real flight data frequently encounters obstacles such as logistical constraints, economic implications, and the inherent complexities of the experimental arrangements. These challenges are often exacerbated by issues related to the accessibility of the databases in which this data is stored or catalogued.

Addressing these challenges, focusing research within a simplified, linear aerodynamic framework not only alleviates the difficulties encountered in more complex and inaccessible nonlinear domains but also substantially enhances the analytical

capabilities. This focused approach allows for a more efficient and effective analysis, leveraging the strengths of the least squares method to yield insightful and actionable results. In the subsequent sections of this study, a detailed discussion will be presented on the specific conditions that make the least squares method an exceptionally powerful tool in the linear aerodynamics, demonstrating its capacity to deliver precise estimations and contribute significantly to the field.

1.2 Literature Review

The field of aerodynamics boasts an extensive array of methods for estimating aerodynamic derivatives, as extensively documented in existing scholarly literature. Notably, parameter estimation utilizing the least squares method ranks among the most venerable of these techniques, maintaining its relevance and applicability even in contemporary research. Within the broader framework of the least squares method, there exists a subdivision into various specialized sub-methods, each tailored to meet different analytical needs and scenarios. Some of these sub-methods are structured to provide quick, single-step solutions, offering straightforward results without iterative processes. In contrast, other sub-methodologies are designed to accommodate more complex, dynamic scenarios, providing solutions that evolve over time through recursive calculations and adjustments. This study aims to delve deeply into two primary variants of the least squares method: the ordinary least squares (OLS) method, which computes parameter estimates in a single computational step, and the recursive least squares (RLS) method, which continually updates its estimates to reflect new data and changes in dynamics through a recursive algorithm. By examining both methods, this study seeks to shed light on their operational mechanisms, the contexts in which each method excels, and how they can be effectively applied in aerodynamic studies to yield precise and reliable results. This exploration is intended to clarify the utility and distinctions of these methods, contributing valuable insights to the field of aerodynamic research.

Gary T. Chapman and Donn B. Kirk presents an innovative approach to determine aerodynamic coefficients from the observed motion of a body in flight, addressing the limitations of traditional methods mentioned in the paper [3]. Traditional techniques

rely on simplified equations of motion with assumptions like linear aerodynamics and constant roll rates, which are then fit to experimental data by adjusting coefficients. The new method proposed in this paper eliminates the need for these simplifications by using numerical solutions to the equations of motion, allowing for a more general analysis. The paper highlights the application of this method through four cases: (1) Linear aerodynamics, to replicate existing methods; (2) Nonlinear aerodynamics, through the analysis of ballistic-range flights of Gemini capsule models; (3) Variation of freestream density along a trajectory; and (4) Large-amplitude rolling motion of a trimmed Apollo Command Capsule model. The method involves a least-squares technique using differential corrections, requiring accurate partial derivatives of each dependent variable with respect to each unknown coefficient at every data point. The approach has proven successful, showing rapid convergence and accurate determination of aerodynamic parameters. The paper concludes that this new method offers a significant advancement in extracting aerodynamic coefficients from free-flight data, with potential applications in improving the design and analysis of flight vehicles.

Saderla, S., et al conducts a comprehensive aerodynamic analysis of a Cropped Delta Reflex Wing configuration-based Unmanned Aerial Vehicle [4]. Using full-scale static wind-tunnel tests followed by flight testing, the study aims to understand the UAV's longitudinal and lateral-directional aerodynamic characteristics. One of the key methods used for the analysis is least square method. Significant findings from the wind-tunnel tests and flight data analysis include the estimation of aerodynamic force and moment coefficients. These coefficients are crucial for understanding the UAV's performance, stability, and control characteristics. Jategaonkar compared the performance of different aerodynamic estimation approaches of the LSM using the real flight database of the ATTAS aircraft [5]. Afterwards, they provided external sensor measurement errors and sampled the effects of these errors on the estimation results. Morelli and Klein conducted LSM analyses using the flight test data of the NASA Twin Otter aircraft, researching the estimation results and errors through the data [6]. Eugene A. Morelli delves into various practical considerations associated with applying the equation-error approach for estimating aircraft dynamic model parameters [7]. Utilizing simulated flight data from an F-16 nonlinear simulation

enhanced with realistic noise sequences, the study illuminates challenges and solutions pertinent to parameter estimation techniques and outcomes, leveraging the known true parameter values from the simulation for validation. This comprehensive analysis, supported by simulated data and realistic noise models, provides valuable insights into the practical application of the equation-error method for aircraft dynamic model parameter estimation, highlighting the method's strengths and addressing potential pitfalls with practical solutions. Jared A. Grauer and Eugene A. Morelli discusses a real-time method for accurately determining the uncertainty levels of stability and control derivatives estimated using recursive least squares with time-domain data [8]. The method accounts for colored residuals, which are common in aircraft parameter estimation and can alter the predicted uncertainties. The approach is demonstrated through the analysis of both simulation data and flight test data for a subscale jet transport aircraft. The results indicate that the corrected uncertainties closely match the observed scatter in parameter estimates, offering a more accurate reflection than traditional uncertainty estimates that assume white residuals. The study shows only minor differences between batch and recursive estimates at the end of a maneuver, and also demonstrates that the autocorrelation can be effectively reduced to a minimal number of lags to decrease computation and memory storage without significantly impacting the accuracy of predicted uncertainty levels. Yvonne Ferrier, Nhan Nguyen, and Eric Ting from Stinger Ghaffarian Technologies, Inc., and NASA Ames Research Center explores the application of a real-time adaptive least-squares drag minimization algorithm on an aeroelastic model of a flexible-wing aircraft [9]. The model is based on the NASA Generic Transport Model (GTM) and incorporates the Variable Camber Continuous Trailing Edge Flap (VCCTEF) as an innovative aerodynamic control surface. The aim of the study is to demonstrate the potential of VCCTEF in improving the aerodynamic efficiency of transport aircraft by minimizing drag under various flight conditions. The drag minimization algorithm operates in real-time, adjusting the VCCTEF deflections to achieve minimum drag while maintaining an altitude-hold flight control mode at cruise conditions. The algorithm utilizes the Newton-Raphson method for optimization and identifies aerodynamic coefficient parameters in real-time using Recursive Least Squares (RLS). Kutluay, Umit, Mahmutyazicioglu, and Platin

focuses on the challenges and methodologies of applying the equation error method for aerodynamic parameter estimation in flight vehicles, specifically those not primarily designed for system identification or parameter estimation, like gliding flight vehicles [10]. The authors outline the inherent difficulties posed by such flight test data, including data collinearity due to the lack of designed excitation maneuvers and the presence of automatic flight control systems that maintain trimmed flight, thereby reducing the dynamics available for analysis. The paper discusses the implementation of the equation error method and stepwise regression for parameter estimation, highlighting the need for careful selection of regressors and the treatment of noise in the data. Eugene A. Morelli presents a method for creating global models of nonlinear aerodynamic coefficients using multivariate orthogonal functions derived from data [11]. The orthogonal nature of the modeling functions facilitates the determination of an adequate model structure and the corresponding parameter values, allowing for a straightforward decomposition of each orthogonal function into a series of ordinary polynomials in the independent variables. Morelli applies this technique to model the Z body axis aerodynamic force coefficient (C_z) of the F-18 High Angle of Attack Research Vehicle (HARV) across the entire subsonic flight envelope, using data from a tabular wind tunnel database. The model demonstrates good predictive capabilities and provides valuable insights into the aerodynamic behavior of the vehicle.

1.3 Hypothesis

The concept of an equilibrium point in aerodynamic studies refers to a condition where all forces and moments exerted on an aircraft are either non-existent or remain constant. The stability attributes of such an equilibrium point are intricately linked to the size and direction (sign) of the aircraft's aerodynamic derivatives. To ascertain these stability and control derivatives, one commonly employed technique involves the linearization of the aircraft's complex nonlinear equations through the application of the small disturbance theorem. This linearization process results in two sets of equations: longitudinal and lateral-directional, both of which contain vital stability and control derivatives.

Within the framework of a flight simulation model that operates with six degrees of freedom, the force and moment coefficients are constructed using a first-order Taylor series approximation, assigning distinct numerical values to each coefficient. These values represent the first-order derivatives of a specific aerodynamic coefficient, which are immediately multiplied by the parameters from which they were derived and then collectively summed. This algebraic addition produces a resultant force and moment coefficient. The precision in determining each derivative value within these coefficients is achieved through meticulous parameter estimation calculations utilizing the least squares method, also known in statistical terms as regression analysis.

It is crucial to provide the system with accurate and appropriate inputs to compute each derivative accurately. Failure to provide the correct inputs results in a discrepancy between the estimated results produced by the system's outputs and the actual numerical values of the derivatives.

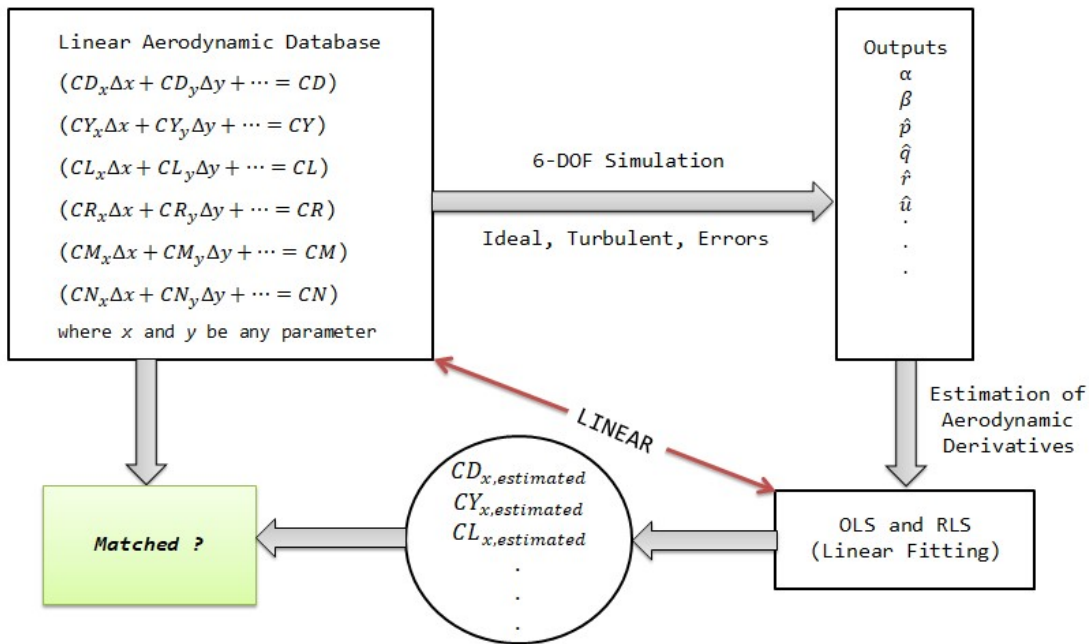


Figure 1.3 : A Brief Demonstration of Hypothesis Behind the Study

The Figure 1.3 illustrates a brief process for estimating aerodynamic derivatives, which are crucial parameters in understanding and predicting an aircraft's aerodynamic behavior. The process begins with a linear aerodynamic database that houses a set of derivatives, each representing the relationship between various aerodynamic forces and moments, such as lift, drag, and moments about different axes, with their

corresponding influencing parameters, such as angles of attack, sideslip angles, and control surface deflections. These coefficients are essential for characterizing the aerodynamic properties of an aircraft in a linearized manner, providing a foundational basis for the subsequent analysis.

These coefficients are then input into a six-degree-of-freedom (6-DOF) simulation, a sophisticated model that accurately represents the aircraft's movement in three-dimensional space. The simulation takes into account a range of flight conditions, including ideal scenarios, where the aircraft is assumed to be free from disturbances, as well as more complex situations involving turbulence and potential errors. The outputs of this simulation include a variety of dynamic parameters, such as angle of attack (α), sideslip angle (β), and non dimensional angular rates ($\hat{p}, \hat{q}, \hat{r}$), which reflect the aircraft's behavior in response to various inputs and environmental conditions.

The next step in the process involves the estimation of aerodynamic derivatives from these dynamic outputs. This is achieved using linear statistical techniques like Ordinary Least Squares (OLS) and Recursive Least Squares (RLS), both of which are methods of linear fitting. These techniques analyze the relationship between the output parameters from the 6-DOF simulation and the input coefficients, allowing for the estimation of new aerodynamic derivatives, such as estimated aerodynamic drag derivatives such as CD_u , aerodynamic side force derivatives such as CY_β , and aerodynamic lift derivatives such as CL_q . These estimated derivatives represent the aircraft's aerodynamic response characteristics under the given flight conditions.

Once these aerodynamic derivatives are estimated, they are compared to the original values contained in the linear aerodynamic database. This comparison is crucial as it determines whether the estimated derivatives accurately match the expected behavior derived from the initial database. If the estimated coefficients do not align well with the original values, it may indicate a need for further refinement in the modeling process or the estimation techniques used. This leads to an iterative feedback loop, where adjustments are made to improve the accuracy of the estimated aerodynamic derivatives.

In other words, the process involves algebraically and linearly summing the first-order aerodynamic derivatives of a specific coefficient. When correct inputs are utilized in conjunction with the dynamics of the flight simulation model, the aerodynamic derivative estimation values generated from the model's outputs will precisely match their true values. This precision assumes that no error-inducing scenarios are modeled in the outputs representing flight data, ensuring that the outputs of the flight simulation model are devoid of any factors that might introduce inaccuracies, such as sensor measurement errors or variations due to atmospheric turbulence. However, if atmospheric turbulence is considered in the model outputs, the flight data set diverges from the ideal state. Despite this, the accuracy of aerodynamic derivative estimations using this dataset remains high with the least squares method, because the average of any error introduced into the model outputs is zero. On the contrary, if constant sensor measurement errors with a non-zero average are introduced into the linear equations formulated by the least squares method, the outcomes of the aerodynamic estimations become biased, leading to deviations from their true values.

In conclusion, the findings from parameter estimation using the least squares method display a high degree of sensitivity to any errors with a non-zero mean or expected value. Such errors lead to significant deviations from the actual values of the aerodynamic derivatives. Conversely, when the errors present within the system are characterized as white noise, possessing an expected value of zero, they do not adversely impact the performance of the estimated values. Rather, these values are more likely to align closely with the true derivative values due to the nature of the error having no systematic bias. The entire set of calculations using the least squares method is derived from a data environment in which the flight simulation model is constructed on a linear basis, and where the aerodynamic derivatives are aggregated in a systematic manner. This specific configuration of the data environment plays a crucial role in ensuring the accuracy of the parameter estimations. Due to this structured and linear modeling of the flight simulation and the systematic approach to summing the aerodynamic derivatives, the results obtained from parameter estimations through the least squares method can be thoroughly assessed for their accuracy, robustness, and reliability.

2. MATHEMATICAL MODEL OF SIAI MARCHETTI S211

To conduct parameter estimation effectively, the development of a 'physics-based', dynamically informed model is pivotal for facilitating precise and reliable analysis of predictive outcomes. Aerodynamic parameter estimation hinges on the availability of data that mirrors actual flight conditions. As highlighted in earlier sections, acquiring datasets from real flights often presents challenges due to limitations like high costs. An alternative approach involves constructing a six degrees of freedom mathematical model, which can then be used to approximate the behavior observed under actual flight conditions. The outputs of this mathematical model can be tailored to simulate real flight parameters, including axial acceleration, velocity, and rotational turning rates, closely mirroring those recorded during actual flights.

A key distinction between real and simulated flights lies in the accessibility of data during simulations. In simulated environments, forces and moments are directly available for computational analysis, providing a clear advantage over real flight scenarios where measuring force and moment coefficients directly is typically unfeasible. Following this methodology, it is posited that the framework devised based on the first-order expansion of the Taylor series within the aerodynamic data accurately encapsulates the true aerodynamic characteristics of the aircraft. This approach underscores the value of sophisticated modeling in replicating and studying complex flight dynamics in a controlled and cost-effective manner.

2.1 Equations of Motion

The model featuring the equations of motion includes first-order differential equations that comprehensively capture the aircraft's six degrees of freedom orientation. This encompasses three translational movements (forward/backward, up/down, left/right) and three rotational movements (pitch, yaw, roll), and further integrates both kinematic equations, which describe motion without regard to forces, and navigation equations,

which are essential for determining the aircraft's position and trajectory. In this model, where the differential equations are meticulously derived using classical Newtonian mechanics, it is assumed that the aircraft remains rigid throughout its flight and maintains a constant mass, simplifying the analysis.

The motion equations of the aircraft are articulated in vector form, structured within the body axis system—a coordinate framework fixed to the moving body of the aircraft. These equations are constructed relative to the inertial reference axis, which serves as a stable, non-rotating frame of reference against which the aircraft's movements are measured. This setup allows for precise calculations and modeling of the aircraft's dynamics, enabling accurate predictions of its behavior under various flight conditions. As stated earlier, the motion equations, expressed in vector form in the body axis system and constructed relative to inertial reference axis, are shown below.

$$\vec{F}_A + \vec{F}_T + \vec{F}_G = m\dot{\vec{V}} + \vec{\omega} \times \vec{V}, \quad \left(m \left[\frac{d\vec{V}}{dt} \right]_{inertial}^{body} = \vec{F} \right) \quad (2.1)$$

$$\vec{M}_A = I\dot{\vec{\omega}} + \vec{\omega} \times I\vec{\omega}, \quad \left(\left[\frac{d\vec{H}}{dt} \right]_{inertial}^{body} = \vec{M} \right) \quad (2.2)$$

Where I represents the mass moment of inertia matrix, m the mass of the aircraft, $\vec{F}_A$ and $\vec{M}_A$ the forces and moments from aerodynamics, and $\vec{F}_T$ and $\vec{F}_G$ are the forces from thrust and gravity, respectively. Translational and rotational velocity vectors in the body axis are shown as $\vec{V} = [u \quad v \quad w]^T$ and $\vec{\omega} = [p \quad q \quad r]^T$, respectively.

Table 2.1 : Data of SIAI Marchetti S211 Airraft

Mass (m)	1814.4kg
I_{xx}	1084.7kg.m ²
I_{yy}	6507.9kg.m ²
I_{zz}	7050.3kg.m ²
I_{xz}	271.2kg.m ²
Wing Area (S)	12.6248m ²
Wing Span (b)	8.0162m
Mean Aerodynamic Chord ($\bar{c}$)	1.6459
True Velocity (V_T)	610 ft/sec ²
Mach	0.6
Altitude	25000ft

The information for the SIAI Marchetti S211 aircraft selected from Roskam [1] for the database to be used in the equation sets is shown in Table 2.1. The values given in the table is produced at an equilibrium point of the aircraft.

One of the purposes of the flight simulation model is also to accurately converge towards the given values to validate the sets of equations used in the model. The sub-blocks of equations of motion model and the flow diagram between the blocks are shown in Figure 2.1

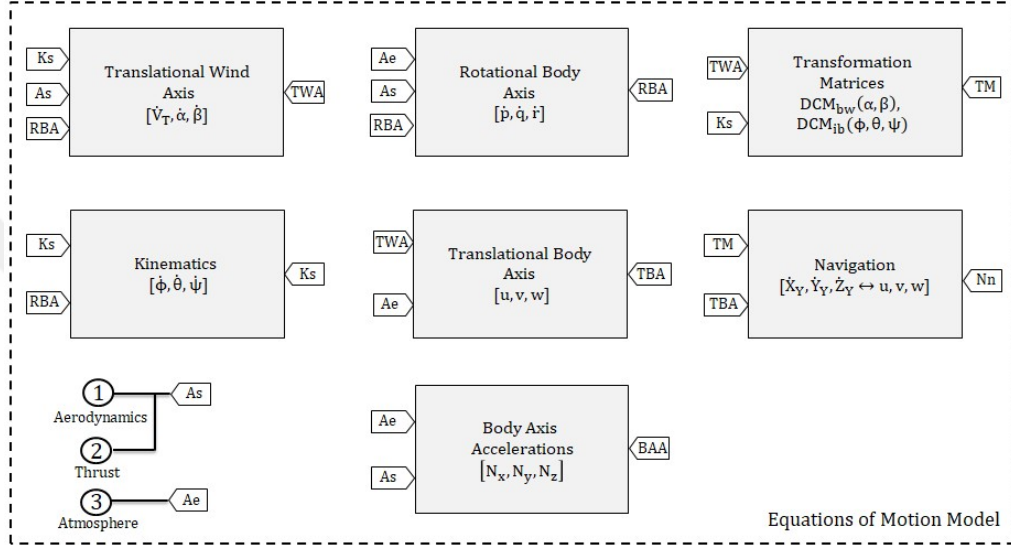


Figure 2.1 : Flow Diagram of the Equations of Motion Model

Each block is modeled internally with the sets of equations formulated in this section. As seen in the figure, the reference axis set for the equations of motion block is the wind axis, and the transition to the body axis is achieved by taking the integrals of the state variables in the wind axis.

A translational equation was given that is written in the body axis set and constructed with respect to the inertial reference axis. The angle of attack, sideslip angle, and true speed can be derived from the u , v , and w velocities contained within the linear velocity vector $\vec{V}$.

$$\alpha = \tan^{-1} \left(\frac{w}{u} \right), \quad \beta = \sin^{-1} \left(\frac{v}{|\vec{V}_T|} \right), \quad |\vec{V}_T| = V_T = \sqrt{u^2 + v^2 + w^2} \quad (2.3)$$

The derivative with respect to time can be taken for each equality in Equation 2.3.

$$\dot{\alpha} = \frac{u\dot{w} - w\dot{u}}{u^2 + w^2}, \quad \dot{\beta} = \frac{V_T\dot{v} - v\dot{V}_T}{V_T^2\sqrt{1 - (v/V_T)^2}}, \quad \dot{V}_T = \frac{1}{V_T}(u\dot{u} + v\dot{v} + w\dot{w}) \quad (2.4)$$

When the aircraft has both an angle of attack and a sideslip angle, the speed shown relative to the wind axis is $\vec{V}_T$, and it is referred to as the true speed. The reference axes used in the aircraft are depicted in Figure 2.2.

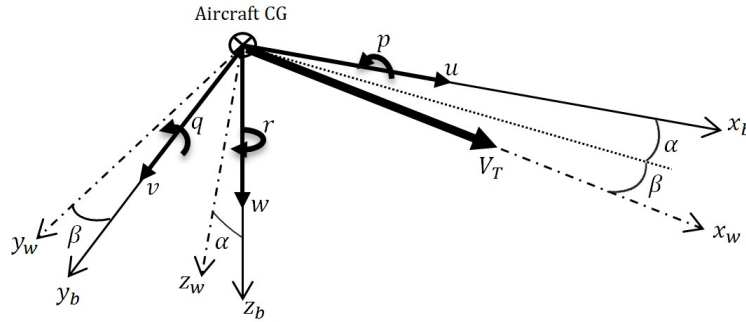


Figure 2.2 : Reference Axes Used in the Aircraft

True airspeed attached to the wind axis can be expressed in body axis.

$$u = V_T \cos \alpha \cos \beta, \quad v = V_T \sin \beta, \quad w = V_T \sin \alpha \cos \beta \quad (2.5)$$

The algebraic manipulation of Equation 2.1 can be expanded for three axes.

$$m\dot{u} = m(rv - qw) + \bar{q}SC_X - mg \sin \theta + T \quad (2.6)$$

$$m\dot{v} = m(pw - ru) + \bar{q}SC_Y + mg \cos \theta \sin \phi \quad (2.7)$$

$$m\dot{w} = m(qu - pv) + \bar{q}SC_Z + mg \cos \theta \cos \phi \quad (2.8)$$

By substituting Equations 2.6-2.8 into Equation 2.4 for $\dot{u}$, $\dot{v}$ and $\dot{w}$ and Equation 2.5 into Equation 2.4 for u , v and w , force equations in wind axis are derived (transferring from body force (C_X and C_Z) coefficients to wind (C_D and C_L) coefficients using Figure 2.2).

$$\dot{V}_T = -\frac{\bar{q}S}{m}C_D + \frac{T}{m} \cos \alpha \cos \beta + g(\cos \phi \cos \theta \sin \alpha \cos \beta + \sin \phi \cos \theta \sin \beta - \sin \theta \cos \alpha \cos \beta) \quad (2.9)$$

$$\dot{\alpha} = -\frac{\bar{q}S}{mV_T \cos \beta}C_L + q - \tan \beta(p \cos \alpha + r \sin \alpha) + \frac{g}{mV_T \cos \beta}(\cos \phi \cos \theta \cos \alpha + \sin \theta \sin \alpha) - \frac{T \sin \alpha}{mV_T \cos \beta} \quad (2.10)$$

$$\dot{\beta} = \frac{\bar{q}S}{mV_T}C_Y + p \sin \alpha - r \cos \alpha + \frac{g}{V_T} \cos \beta \sin \phi \cos \theta + \frac{\sin \beta}{V_T}(g \cos \alpha \sin \theta - g \sin \alpha \cos \phi \cos \theta) + \frac{\sin \beta}{V_T} \frac{T \cos \alpha}{m} \quad (2.11)$$

Equations 2.9, 2.10 and 2.11 are used in the Translational Wind Axis block shown in Figure 2.1.

For rotational motion, if selecting differential (small enough) mass, then the process begins by considering a small elemental mass dm , which is situated at a specific distance from the aircraft's center of gravity. This mass is assumed to be rotating around the center of gravity, exhibiting positive roll, pitch, and yaw rates, which are represented by P , Q , and R . The distance from the aircraft's center of gravity to this elemental mass is defined as a key parameter in the analysis. Figure 2.3 represents the differential mass locating about aircraft center of gravity.

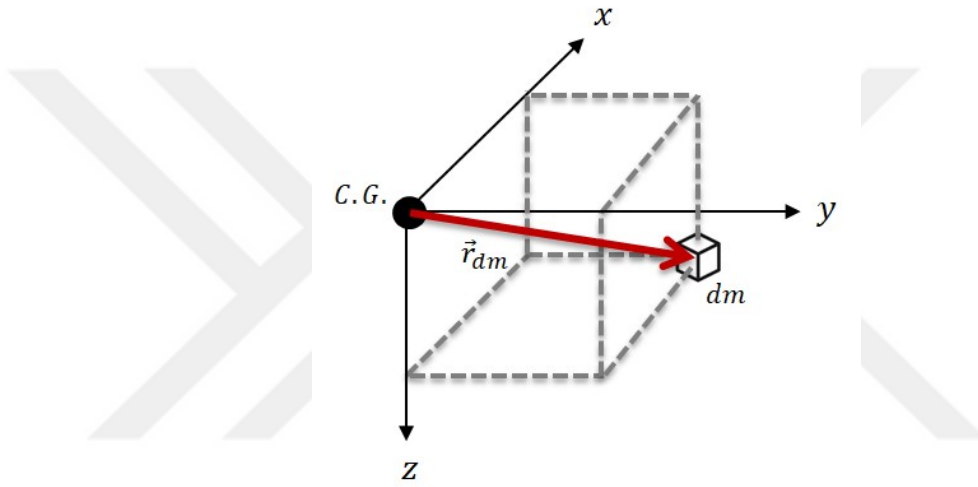


Figure 2.3 : Differential Mass in the Aircraft Body Axis

This setup allows for the examination of the rotational dynamics of the elemental mass in relation to the overall behavior of the aircraft [12].

$$\vec{r}_{dm} = x\hat{i} + y\hat{j} + z\hat{k} \quad (2.12)$$

The distances x , y and z from C.G. are defined in the aircraft body axis. Equation 2.2 can be constructed for dm and integrated for whole aircraft.

$$M_{A/C} = m = \int dm \quad (2.13)$$

An equation can be formulated to determine the velocity of the small mass, dm , which results exclusively from its rotation around the aircraft's center of gravity. In the process of developing the three force equations, the velocity associated with

the movement of the aircraft's center of gravity was also incorporated to ensure a comprehensive analysis. The velocity of the mass, $\vec{V}_{dm}$ relative to the center of gravity is then calculated using this derived expression. This approach allows for a more detailed understanding of the dynamic relationship between the small mass and the aircraft's overall rotation.

$$\vec{V}_{dm} = \left[\frac{d\vec{r}_{dm}}{dt} \right]_{body} + \vec{\omega}_{body} \times \vec{r}_{dm} \quad (2.14)$$

Since the aircraft is rigid body,

$$\left[\frac{d\vec{r}_{dm}}{dt} \right]_{body} = 0 \longrightarrow \vec{V}_{dm} = \vec{\omega}_{body} \times \vec{r}_{dm} \quad (2.15)$$

Equation 2.15 can be expanded as mathematical operation (cross product).

$$V_{dm} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ P & Q & R \\ x & y & z \end{vmatrix} \quad (2.16)$$

If the operation is yielded,

$$\vec{V}_{dm} = (Qz - Ry)\hat{i} + (Rx - Pz)\hat{j} + (Py - Qx)\hat{k} \quad (2.17)$$

Then the definition of angular momentum for differential mass can be mathematically expressed using linear momentum.

$$P_{lm} = dm\vec{V}_{dm} \quad (2.18)$$

Which P_{lm} defines linear momentum. Deriving angular momentum is cross product of linear momentum with position vector, $\vec{r}_{dm}$.

$$d\vec{H}_{dm} = \vec{r}_{dm} \times (dm\vec{V}_{dm}) \quad (2.19)$$

The cross product form of Equation 2.19 is,

$$d\vec{H}_{dm} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ (Qz - Ry) & (Rx - Pz) & (Py - Qx) \end{vmatrix} \quad (2.20)$$

After algebraic manipulation and integration, the right hand side of Equation 2.2 is easily carried out.

Rotational velocities on the aircraft body axis are represented as $\dot{p}$, $\dot{q}$ and $\dot{r}$. The first order differential equations for these rates are extracted from Equation 2.2.

$$\dot{p} = \frac{R + I_{yz}(q^2 - r^2) + I_{zx}(\dot{r} + pq) + I_{xy}(\dot{q} - rp) + (I_{yy} - I_{zz})qr}{I_{xx}} \quad (2.21)$$

$$\dot{q} = \frac{M + I_{zx}(r^2 - p^2) + I_{xy}(\dot{p} + qr) + I_{yz}(\dot{r} - pq) + (I_{zz} - I_{xx})rp}{I_{yy}} \quad (2.22)$$

$$\dot{r} = \frac{N + I_{xy}(p^2 - q^2) + I_{yz}(\dot{q} + rp) + I_{zx}(\dot{p} - qr) + (I_{xx} - I_{yy})pq}{I_{zz}} \quad (2.23)$$

Between Equation 2.21 and 2.23, the Rotational Body Axis block is being formulated. Here, the effect of the engine is ignored, and the product of inertia moments I_{xy} and I_{yz} are assumed to be zero. Additionally, R represents the rolling moment, M the pitching moment, and N the yawing moment.

The Euler block consists of kinematic equations and formulates the relationship between the derivatives of the Euler angles with respect to time and the angular rotation speeds in the body axis system.

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi / \cos \theta & \cos \phi / \cos \theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (2.24)$$

Equation 2.24 represents the Kinematics block seen in Figure 2.1. It explains the Euler rates and the rotational speeds in the body axis are not equal to each other and are connected by a transfer matrix.

From the true speed of the aircraft expressed in the wind axis, the speeds u , v , and w in the body axis can be obtained, as shown in Equation 2.5. To present the equations in a more general form, a transformation matrix between the wind and body axes can be established.

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \cos \alpha & -\cos \alpha \sin \beta & -\sin \alpha \\ \sin \beta & \cos \beta & 0 \\ \sin \alpha \cos \beta & -\sin \alpha \sin \beta & \cos \alpha \end{bmatrix} \begin{bmatrix} V_T \\ 0 \\ 0 \end{bmatrix} \quad (2.25)$$

The transformation matrix given in Equation 2.25 represents the $DCM_{bw}(\alpha, \beta)$ matrix in the Transformation Matrices block of the equations of motion model seen in Figure 2.1. In the same block, there is another transformation matrix, which is a transition matrix to the inertial reference system where Newton's equations can be written and

are applicable. It is also between body and inertial reference system.

$$\begin{bmatrix} \dot{X}_Y \\ \dot{Y}_Y \\ \dot{Z}_Y \end{bmatrix} = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad (2.26)$$

The transformation matrices in Equation 2.26 is modeled as $DCM_{ib}(\phi, \theta, \psi)$ in Transformation Matrices block. It is also modeled in Navigation block with Equation 2.26 itself. Another parameter in the body axis set is linear accelerations, which are expressed in terms of load factor, 'g'.

$$\frac{F_{A_x} + F_{T_x}}{mg} = n_x, \quad \frac{F_{A_y}}{mg} = n_y, \quad \frac{F_{A_z}}{mg} = n_z \quad (2.27)$$

These dimensionless acceleration parameters are assumed to be measurable through the aircraft's sensors.

Ultimately, the equations utilized within each block, as depicted in Figure 2.1, have been meticulously formulated to function according to the Equations of Motion model. The notations with dotted expressions above each parameter denote the time derivatives of those parameters (Newton's notation). By integrating these time derivatives, state variables are obtained, which are essential for describing the dynamics of the aircraft. Specifically, a complete representation of the aircraft's position and orientation in space requires the determination of 12 state variables. Consequently, the numerical resolution of these 12 first-order differential equations is achieved using the 4th order Runge-Kutta method. This method is highly regarded for its accuracy and efficiency in solving differential equations, providing detailed insights into the aircraft's attitude and behavior over time.

Furthermore, the underlying assumptions integrated into the model are pivotal, as they significantly influence the derived equations and the fidelity of the simulation. These assumptions are:

- The inertial reference axis system is considered to be fixed relative to the Earth. This assumption simplifies calculations by treating the Earth as a stable reference point, which is essential for flight dynamics.

- The aircraft is modeled as a rigid body, meaning that deformations or flexibilities in its structure under operational loads are neglected. This simplification is common in basic flight dynamics to focus on gross motion without the complexities of structural dynamics.
- The mass and mass distribution of the aircraft are assumed to be constant throughout the flight. This assumption excludes any changes due to fuel consumption or payload alterations, which might otherwise complicate the model.
- Symmetry about the xz plane is presumed, which implies that the aircraft's design and behavior are mirrored on either side of this plane, reducing the number of unique conditions that need to be considered.
- The curvature of the Earth is ignored, and a flat Earth model is used instead. While this may not perfectly represent long-distance flights, it greatly simplifies the mathematical modeling for most practical scenarios.
- The magnitude and direction of the gravitational acceleration are considered constant. This assumption negates the need to account for variations in gravity due to changes in altitude or geographic location.
- It is assumed that the thrust generated by the engines is directed solely along the x -axis of the aircraft's body axis and that it acts through the center of gravity. This assumption focuses on forward propulsion without the complications of thrust-induced moments or off-center force applications.

Each of these assumptions plays a crucial role in defining the scope and limitations of the model, shaping the simulation's accuracy and its applicability to real-world scenarios. By setting these specific conditions, such as assuming a rigid aircraft body or ignoring Earth's curvature, the model provides a simplified but precise framework that focuses on fundamental aerodynamic and flight dynamics principles. This deliberate simplification allows for a clear examination of core behaviors and interactions without the noise of more complex, variable factors that would require advanced computational resources and more sophisticated modeling techniques.

Furthermore, the assumptions collectively create a robust framework that supports the operational analysis of the aircraft under controlled, standard conditions. This standardized approach ensures consistency in simulations, which is crucial for comparative analyses and for validating theoretical predictions against empirical data. By operating within this well-defined framework, specific variables and effects can be isolated, thereby gaining deeper insights into the functioning and limitations of the aircraft.

2.2 Aerodynamics and Engine

The aerodynamic database plays a very important role in determining the stability and control characteristics of an aircraft. Different methods can be used to generate aerodynamic data. Examples include CFD-based numerical solutions and wind tunnel-based experimental solutions. As previously mentioned, there is no system in actual flight that can directly measure the forces and moments acting on an aircraft. Therefore, the accuracy of the aerodynamic database is of great importance. In this context, system definition and parameter estimation play a critical role in verifying the aerodynamic database. The database used in the model is linearly modeled with aerodynamic derivatives. It is assumed that these derivatives represent the aircraft's true flight characteristics in a perfect environment.

An aerodynamic coefficient depends on many parameters of an aircraft. One method is to relate the instantaneous changes on an aircraft to a function to represent force and moment coefficients. In this method, the instantaneous change on the aircraft can be considered as a small disturbance on the angular velocities, translational velocities, control deflections, and their time derivatives. In fact, this approach brings forth the small disturbance theorem to express the coefficients in terms of functions. With this approach, we can express the force and moment coefficients using the Taylor series expansions of small disturbances. Here, the Taylor series is applied at the equilibrium point, which is performed under the reference value conditions of the Marchetti aircraft

given in the Table 2.1.

$$C_Y = C_Y(u, \dot{u}, v, \dot{v}, w, \dot{w}, p, \dot{p}, q, \dot{q}, r, \dot{r}, \dots, \delta_e, \dot{\delta}_e, \delta_a, \dot{\delta}_a, \delta_r, \dot{\delta}_r) \quad (2.28)$$

$$= \frac{\partial C_Y}{\partial u} \Delta u + \frac{\partial C_Y}{\partial \dot{u}} \Delta \dot{u} + \dots + \dots + \frac{\partial C_Y}{\partial \dot{\delta}_r} \Delta \dot{\delta}_r + H.O.T.$$

Here, Y can be any parameter such as lift coefficient C_L , drag coefficient C_D or yawing moment coefficient C_N . The abbreviation 'H.O.T.' stands for 'higher order terms' and includes second and higher order derivatives. If the H.O.T. derivatives are also used in the equation, it becomes complex and turns into a nonlinear function. To avoid this complication, the instantaneous delta disturbances on the aircraft are considered small, and the numerical values of the higher order derivatives are negligible. Therefore, including the first-order significant parameters that will affect the force or moment coefficients becomes an effective and sufficient approach. Additionally, the significant parameters for each force and moment coefficient may vary. For example, while the pitch rate, q , is an effective variable for the lift coefficient, it is not an effective parameter for the rolling and yawing moment coefficients. Therefore, determining the effective parameters for each coefficient, in other words, identifying the effective aerodynamic and control derivatives, is important to reflect the characteristics of the aircraft.

$$C_D = C_{D_0} + C_{D_\alpha} \Delta \alpha + C_{D_u} \Delta \hat{u} + C_{D_{\delta_e}} \delta e \quad (2.29)$$

$$C_Y = C_{Y_\beta} \Delta \beta + C_{Y_p} \Delta \hat{p} + C_{Y_r} \Delta \hat{r} + C_{Y_{\delta_a}} \delta a + C_{Y_{\delta_r}} \delta r \quad (2.30)$$

$$C_L = C_{L_0} + C_{L_\alpha} \Delta \alpha + C_{L_u} \Delta \hat{u} + C_{L_{\dot{\alpha}}} \Delta \hat{\alpha} + C_{L_q} \Delta \hat{q} + C_{L_{\delta_e}} \delta e \quad (2.31)$$

$$C_R = C_{R_\beta} \Delta \beta + C_{R_p} \Delta \hat{p} + C_{R_r} \Delta \hat{r} + C_{R_{\delta_a}} \delta a + C_{R_{\delta_r}} \delta r \quad (2.32)$$

$$C_M = C_{M_0} + C_{M_\alpha} \Delta \alpha + C_{M_u} \Delta \hat{u} + C_{M_{\dot{\alpha}}} \Delta \hat{\alpha} + C_{M_q} \Delta \hat{q} + C_{M_{\delta_e}} \delta e \quad (2.33)$$

$$C_N = C_{N_\beta} \Delta \beta + C_{N_p} \Delta \hat{p} + C_{N_r} \Delta \hat{r} + C_{N_{\delta_a}} \delta a + C_{N_{\delta_r}} \delta r \quad (2.34)$$

The equations presented above employ a Taylor series approximation approach, in which higher-order terms are intentionally disregarded to simplify the model. This method is based on a linear assumption that is particularly applicable and valid when the perturbations in both the input and state vectors of the system are sufficiently small. Under such conditions, it is reasonable to assert that the magnitudes of higher-order

terms in the Taylor series expansion become progressively smaller and their impact on the system's behavior is minimal. Consequently, these higher-order terms are deemed insignificant relative to the first-order terms and can thus be safely neglected without compromising the accuracy of the model significantly. As a result of this approximation, the aerodynamic equations that are derived from this process are linear. These equations incorporate several parameters, which are denoted with a hat symbol. These hat-marked parameters are dimensionless, enhancing the universality and scalability of the model. Dimensionless parameters are crucial as they provide a means to compare aerodynamic forces and moments across different flight conditions and aircraft configurations without the need for recalibration for each specific scenario. They are represented in the equations as follows.

$$\Delta\hat{p} = \frac{\Delta pb}{2V_T} \quad \Delta\hat{q} = \frac{\Delta q\bar{c}}{2V_T} \quad \Delta\hat{r} = \frac{\Delta rb}{2V_T} \quad \Delta\hat{\alpha} = \frac{\Delta\dot{\alpha}\bar{c}}{2V_T} \quad \Delta\hat{u} = \frac{\Delta u}{V_T} \quad (2.35)$$

In this section of the analysis, V_T denotes the true airspeed of the aircraft, which is a crucial variable as it directly impacts the aerodynamic forces experienced during flight. The $\bar{c}$ represents the mean aerodynamic chord, an average distance across the wing's chord, which is a fundamental geometric parameter that influences the aerodynamic characteristics of the wing. The variable b stands for the wingspan, which is the distance from one wingtip to the other and is a critical factor in determining the aircraft's overall lift and stability. The parameters p, q, r are the angular velocities around the aircraft's principal axes, representing the axial rotation speeds within the body axis system. These rotations are pivotal in defining the aircraft's orientation and maneuverability.

In the mathematical representation of the flight dynamics, the responses of the angular velocity about the longitudinal axis q and the rate of change of the angle of attack $\dot{\alpha}$ over time are observed to be very similar, allowing for a unique approach in their treatment in the equations. Specifically, for the coefficient of lift C_L , the parameters can be combined as $C_{L_{\hat{q}}} = C_{L_{\alpha}} + C_{L_q}$. This summing approach is not only mathematically valid but also serves a practical purpose in the computational modeling process. By combining these terms, the model avoids algebraic looping, which can complicate the simulation and increase computational load. Algebraic looping occurs when outputs

of a set of equations are recursively fed back as inputs within the same timestep, which can lead to computational inefficiencies or errors.

Furthermore, the basic signal flow scheme of the aerodynamics model is meticulously outlined in Figure 2.4. This diagram serves as a crucial tool in visualizing and understanding the complex interactions and dependencies within the model. It provides a detailed visual representation of the pathways through which aerodynamic forces and moments are calculated and subsequently integrated into the broader flight dynamics model. By illustrating these processes, the diagram helps to clarify the intricate relationships between different aerodynamic parameters and how they influence one another throughout the operation of the aircraft.

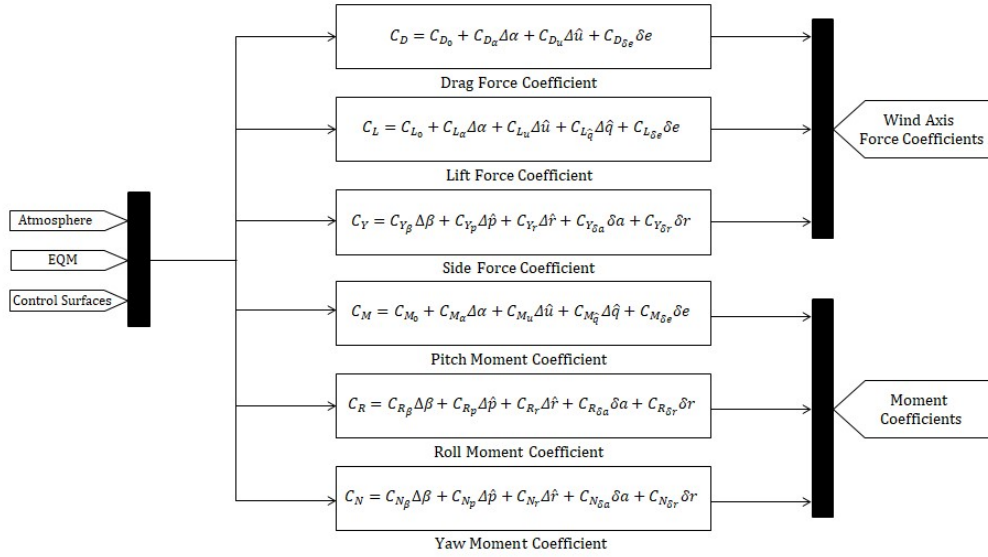


Figure 2.4 : Flow Diagram of the Aerodynamics Model

Where $\Delta\hat{q} = \Delta\hat{\alpha} + \hat{q}$. So, the time derivative of angle of attack in aerodynamics calculation has been extracted from formulation and merged with pitch rate.

The engine block within the simulation model has been developed using a linear approach to simplify the complex dynamics typically associated with engine performance. It operates under the assumption that the thrust vector generated by the engine aligns perfectly with the x-direction of the aircraft's body axis, which is the primary forward direction under standard aircraft coordinate definitions. Consequently, any lateral (y-direction) or vertical (z-direction) components of the thrust vector are considered negligible and are thus modeled as zero. This

simplification assumes that the engine's thrust is exclusively directed in the forward motion of the aircraft, which helps in reducing the complexity of the model and focuses the analysis on the primary thrust effects. This modeling approach is typical in preliminary design and analysis stages, where detailed characterization of off-axis thrust components may not be critical to the primary objectives of the simulation. The assumption of thrust alignment along the x-axis is practical for studying fundamental aerodynamic and propulsion interactions without the additional computational overhead of more complex vector components. In the sense of this, the formulation with given information becomes as follows.

$$C_T = C_{T_0} + (C_{T_u} \Delta \hat{u}) \delta_t \quad (2.36)$$

In the model, δ_t represents the thrust lever arm, a variable parameter that typically ranges between 0 and 1. This parameter is essential in determining the magnitude of the thrust force generated by the aircraft's engines, directly influencing the overall propulsion dynamics. The aircraft design assumes symmetry about the xz plane, which is a standard assumption in aerodynamics to simplify analysis and modeling. Due to this assumed symmetry, the initial values of the aerodynamic force and moment coefficients related to the lateral (y-axis) and directional (z-axis) stability and control of the aircraft are taken to be zero. This assumption is based on the premise that there should be no inherent side bias or directional preference when the aircraft is in a neutral, unbiased state.

In contrast, the coefficients in the longitudinal (x-axis) body axis, which pertain to the forward motion and primary aerodynamic forces acting on the aircraft, are assigned initial values. These include C_{L_0} , C_{T_0} and C_{M_0} , which represent the baseline or zero-angle coefficients of lift, thrust and pitching moment, respectively. C_{L_0} is critical as it provides a reference point for the lift generated under standard conditions, while C_{T_0} indicates the initial thrust coefficient, essential for understanding the propulsion capabilities of the aircraft at the start of flight dynamics analysis. These initial values are fundamental for calculating the lift, thrust and pitching moment that influence the aircraft's ascent and cruising phases, providing a baseline from which deviations due to changes in flight conditions or control inputs can be measured.

2.3 Atmosphere

The atmospheric block utilized in the model contains a comprehensive set of equations that are meticulously developed, taking the USA-International Standard Atmosphere [13]. This model is designed to accurately simulate a static atmosphere, meticulously demonstrating the variations in pressure, temperature, and density as a function of altitude. It is equipped with a robust infrastructure that is sensitive to and adjusts according to changes in altitude, ensuring precise modeling of atmospheric conditions. Furthermore, the input to this atmospheric model is primarily the altitude information, upon which it bases its calculations. Subsequently, the model outputs critical aerodynamic parameters such as the Mach number and dynamic pressure values, which are essential for understanding the behavior of objects moving through the atmosphere at high speeds.

The computational framework of this model is structured around two main types of altitude, each serving as a distinct benchmark for analysis. This dual approach allows for a more versatile application of the model across different atmospheric layers, enhancing the accuracy and reliability of the predictions made by the model.

The temperature variation within the Troposphere layer, located at the first altitude level of the atmosphere, ranging from 0 to 11 kilometers, is described in Equation 2.37. This layer represents the lowest segment of the Earth's atmosphere and exhibits a characteristic decrease in temperature with an increase in altitude.

$$T_e = T_0 - H_a L_a \quad (2.37)$$

Here, T_e represents the temperature, T_0 represents sea level temperature, and H_a denotes the altitude of the aircraft, and L_a signifies the lapse rate. This notation is essential in the context of atmospheric studies, where the lapse rate (L_a) refers to the rate at which atmospheric temperature decreases with an increase in altitude. With the calculated temperature T_e , ratio of specific heats γ and gas constant R_g , the speed of sound is used to calculate the Mach number.

$$a = \sqrt{\gamma R_g T_e} \quad (2.38)$$

Since the air density will vary according to the altitude of the aircraft, it will change according to temperature T_e , Lapse rate L_a , and the gas constant R_g .

$$\rho = \rho_0 \frac{(T_e/T_0)^{g/L_a R_g}}{(T_e/T_0)} \quad (2.39)$$

Where ρ_0 sea level density and g is gravitational acceleration.

If the aircraft exceeds an altitude of 11 km, which is considered the boundary of the troposphere, the atmospheric temperature is assumed to be constant. If the aircraft exceeds an altitude of 11 km, which is considered the boundary of the troposphere, the atmospheric temperature is assumed to be constant. Therefore, beyond a certain altitude, while the speed of sound remains constant, the air density is calculated as follows.

$$\rho = \bar{\rho} e^{(g/R_g T_e)(H_t - H_a)}, \quad \bar{\rho} = \rho_0 \bar{T}_e^{(g/L_a R_g)} / \bar{T}_e, \quad \bar{T}_e = T_e/T_0 \quad (2.40)$$

Here, the base height of the tropopause is denoted by H_t and is valued at 11 km. The maximum height of this layer is considered to be 20 km and it is a layer that separates the stratosphere and the troposphere. After calculating the density and speed of sound based on the aircraft's altitude, the Mach number and dynamic pressure can be calculated.

$$\bar{q} = \frac{1}{2} \rho V_T^2, \quad M = \frac{V_T}{a} \quad (2.41)$$

An integral component of the atmosphere block within the flight simulation model is the turbulence module, specifically utilizing the Dryden air turbulence model. This model is renowned for its application of Dryden's spectral definition to effectively simulate atmospheric turbulence in flight simulations. It achieves this by transforming band-limited white noise into realistic turbulence patterns through the use of carefully calibrated filters. These filters are designed to mimic the statistical characteristics of actual atmospheric turbulence as closely as possible, thus enhancing the realism of the simulation. The mathematical basis for this transformation process is detailed in the Dryden spectral definition, which includes complex mathematical expressions that govern how the turbulence interacts with an aircraft. These expressions are standardized and documented extensively in the military standards document

MIL-HDBK-1797. This document serves as a critical reference, providing the necessary theoretical and practical guidelines for implementing turbulence models in military and civilian aerospace applications. By adhering to these standards, the Dryden model ensures a high level of accuracy and reliability in the simulation of atmospheric disturbances, making it a staple in the field of aerospace simulation. A ready-made model from the MATLAB-Simulink program has been used to simulate the turbulence model. Figure 2.5 displays the inputs and outputs of the atmospheric turbulence model.

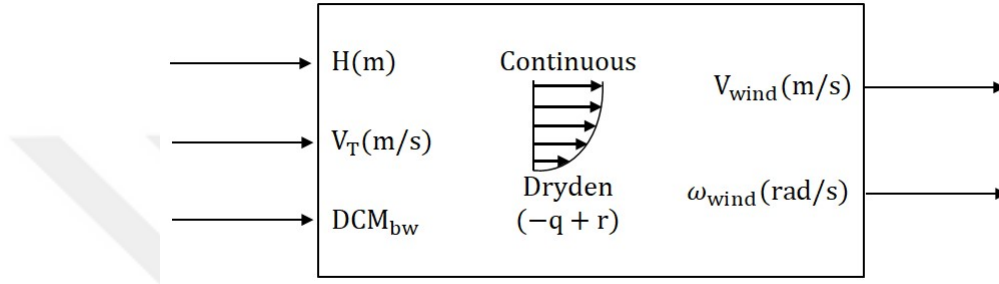


Figure 2.5 : Flow Diagram of Dryden Turbulence Model

As can be seen, the inputs to the block are altitude (modeled as H_a), true speed (V_T), and the transformation matrix from the wind frame to the body frame ($DCM_{bw}(\alpha, \beta)$). The outputs are the turbulences in the translational and rotational velocities in the body frame.

The mathematical formulation of the Dryden model is based on defining the turbulence components along longitudinal (u), lateral (v), and vertical (w) axes of the aircraft. The power spectral densities (PSD) for these components are given by specific formulas.

- Longitudinal turbulence (u-component):

$$\Phi_u(\omega) = \frac{\sigma_u^2 L_u}{\pi(1 + (\omega L_u / V_T)^2)} \quad (2.42)$$

where σ_u is the root-mean-square of the velocity, L_u is the scale length of turbulence, ω is the frequency, and V_T is the true airspeed.

- Lateral and vertical turbulence (v- and w-components):

$$\Phi_v(\omega) = \frac{\sigma_v^2 L_v}{\pi(1 + (\omega L_v/V_T)^2)} \quad (2.43)$$

$$\Phi_w(\omega) = \frac{\sigma_w^2 L_w}{\pi(1 + (\omega L_w/V_T)^2)} \quad (2.44)$$

The parameters σ_v , σ_w , L_v , L_w have similar interpretations as in the longitudinal case, tailored for lateral and vertical motions.

In MATLAB Simulink, the Dryden Turbulence Model is implemented as an integral component of the Aerospace Blockset, designed to enhance the realism of flight simulations by modeling atmospheric disturbances. This model accepts specific inputs such as altitude (H_a), which influences the turbulence intensity by reflecting varying atmospheric conditions at different heights; true airspeed (V_T), which determines the relative velocity of the aircraft against the air mass and thereby affects the turbulence scales encountered; and the attitude matrix $DCM_{bw}(\alpha, \beta)$, which translates the effect of wind turbulence from global-wind coordinates into aircraft body coordinates. Using these inputs, the model employs the well-established spectral densities previously discussed to compute the turbulence components.

Once calculated, these turbulence components are further processed to accurately align with the aircraft's motion through the atmosphere. This alignment involves transforming the linear and angular turbulence components from the wind reference frame to the aircraft body frame, ensuring that the simulated turbulence effects are both realistic and relevant to the actual aircraft dynamics. The outputs generated by this model are comprehensive, including both linear and angular perturbations. These outputs are critical for simulation trials as they provide detailed insights into how an aircraft might respond to various gust scenarios. The linear outputs mimic the direct aerodynamic forces acting on the aircraft, while the angular outputs simulate the induced rotational effects caused by asymmetric gust impacts.

This level of detailed simulation is crucial for the design and testing of aircraft, particularly in assessing the effectiveness of control systems under gusty conditions and improving the overall aerodynamic stability and performance.

3. TRIM ALGORITHM OF THE AIRCRAFT

Identifying the equilibrium point of the aircraft prior to initiating the simulation is crucial for ensuring that the simulation starts from a position that closely resembles a realistic flight condition. This step is fundamental because it sets the initial conditions from which all flight dynamics during the simulation will evolve. By applying the correct trim settings, the simulation enables the aircraft to maintain a balanced and stable flight profile throughout its operation. This is essential for the accuracy and reliability of the simulation results, as it represents the actual behavior of an aircraft under normal flying conditions. Therefore, establishing the wing level steady-state trim condition is a critical process that involves careful calibration of the parameters within the six degrees of freedom motion equations. This calibration ensures that the simulated aircraft behaves in a manner that is consistent with its real-world counterpart, maintaining stability and balance in its response to various flight conditions. By adjusting these parameters accurately, the simulation can effectively replicate the complex interplay of aerodynamic forces acting on the aircraft, providing valuable insights into its performance and handling characteristics under different scenarios.

The nonlinear system, which includes state variables that fundamentally determine aircraft behavior, can be succinctly represented using the following set of equations [14]:

$$\dot{x} = f(x, u), \quad y = g(x, u), \quad x(t_0) = x_0 \quad (3.1)$$

In this representation:

- $\dot{x}$ denotes the time derivative of the state vector x , which is a function of both the state x and the input u . This relationship, expressed as $f(x, u)$ captures the system's dynamics and describes how the state of the aircraft changes over time in response to various inputs and its current state.

- y represents the output vector, which is calculated through the function $g(x, u)$. This function maps the current state and input to the outputs, which are typically measurements or other observables relevant to the aircraft's operation, such as velocities, angles, or other sensor data.
- $x(t_0) = x_0$ establishes the initial condition of the state vector at time t_0 , setting the starting point for the system dynamics.

The dimensions of the vectors are specified as follows:

- x , the state vector, has dimensions $N_x \times 1$, where N_x is the number of state variables in the system. Each component of x corresponds to a specific state variable, such as position, velocity, or attitude angles, which are essential for describing the aircraft's condition at any given moment.
- u , the input vector, is of size $N_u \times 1$, with N_u representing the number of inputs to the system. These inputs could include control inputs like thrust levels, aileron angles, or other control surface settings, which directly influence the aircraft's flight path and attitude.
- y , the output vector, with dimensions $N_y \times 1$, where N_y is the number of outputs that the system produces. These outputs typically include the variables that are monitored to assess the performance and safety of the aircraft, such as altitude, speed, and other critical flight data.

Properly defining the state variables at the initial point as being at an equilibrium point is crucial for establishing a trim point for the aircraft. A trim point is defined as the condition where all forces and moments acting on the aircraft are balanced, resulting in no acceleration except that caused by gravity, thereby enabling steady flight. This precise adjustment of the state variables ensures that the aircraft maintains stable flight without requiring continuous control input adjustments.

In this study, efforts will be directed towards achieving convergence to the reference conditions outlined by Roskam for wing level steady flight conditions in which it is shown in Table 2.1. These reference conditions represent a standard or ideal set of

parameters that describe how the aircraft should behave when it is perfectly trimmed under normal operating circumstances. Converging to these conditions involves adjusting various control settings and configurations to match the theoretical models, which are based on extensive empirical data and aerodynamic principles. The goal is to ensure that the simulated or actual flight dynamics of the aircraft closely align with these established benchmarks, thereby verifying the aircraft's performance and handling characteristics under nominal flight conditions.

Let's now study in the analysis of linearized system dynamics, which are particularly relevant when examining the system's behavior near its equilibrium points. Linear models prove to be especially valid and effective within a limited range of small deviations around these points. The behavior of the dynamic system can be represented by equations linearized around an optimally chosen operational point. This linearization process simplifies the original nonlinear system equations, making them more tractable for analysis and control design. The resulting linear system equations can be expressed in state-space form as follows,

$$\dot{x} = A_{s_a}x + B_{s_b}u \quad (3.2)$$

$$y = C_{s_c}x + D_{s_d}u, \quad x(t_0) = x_0 \quad (3.3)$$

Equations 3.2 and 3.3 represent a linear system that has been perturbed by small numerical values at an equilibrium point. The subscript s denotes a wing level steady-state flight, where the freeze and free parameters in the model have been adjusted. The subscripts of s represent matrix dimensions such as $a = N_{\dot{x}} * N_x$, which means *rows x columns*. The linear system matrices A, B, C and D are as follows [14].

$$A_{s_a} = \frac{f(x + \zeta, u) - f(x - \zeta, u)}{2\epsilon}, \quad a = N_{\dot{x}} * N_x \quad (3.4)$$

$$B_{s_b} = \frac{f(x, u + \zeta) - f(x, u - \zeta)}{2\epsilon}, \quad b = N_{\dot{x}} * N_u \quad (3.5)$$

$$C_{s_c} = \frac{g(x + \zeta, u) - g(x - \zeta, u)}{2\epsilon}, \quad c = N_y * N_x \quad (3.6)$$

$$D_{s_d} = \frac{g(x, u + \zeta) - g(x, u - \zeta)}{2\epsilon}, \quad d = N_y * N_u \quad (3.7)$$

Where ε is small numeric perturbation value and ζ represents perturbation matrix which includes logical 0 and epsilon values such as,

$$\zeta = \begin{bmatrix} 0 \\ \varepsilon \\ 0 \\ 0 \\ \varepsilon \\ \cdot \\ \cdot \\ 0 \\ 0 \end{bmatrix}_{(N_x/N_{\dot{x}})*1} \quad (3.8)$$

These logical values are set with respect to freeze and free/float parameters which defines steady wing level flight. To more accurately approximate the steady-state condition of the aircraft to a realistic flight scenario, the gamma angle has not been ideally set to zero degrees. Instead, it has been assumed that the aircraft flies at a small but specific gamma angle. This small angle allows for a slight but measurable altitude change, which is still within acceptable limits for steady-state analysis. By introducing this non-zero gamma angle, the model better reflects the true dynamics and operational conditions of the aircraft. This approach ensures that the aircraft's altitude variation is minimized while maintaining a realistic representation of flight conditions.

In any mathematical system of equations, achieving a balance between the number of equations and the number of unknowns is fundamental to finding a unique and solvable solution. This principle is particularly critical in aircraft trim analysis. When the number of equations matches the number of unknowns, there is a possibility of finding a unique solution. This ensures that the aircraft's trim condition is well-defined, meaning there is one specific set of state variables and control inputs that balance all forces and moments, leading to steady flight.

If there are more unknowns than equations, the system is underdetermined, resulting in an infinite number of possible solutions. This ambiguity makes it impossible to pinpoint a specific trim condition. Conversely, if there are more equations than unknowns, the system is overdetermined, and it may not be possible to satisfy all the equations simultaneously, leading to no solution.

In linear algebra, a system of linear equations is represented as $Ax = b$, where A is a matrix of coefficients, x is the vector of unknowns, and b is the vector of constants. For a unique solution to exist, the matrix A must be square (i.e., the number of rows equals the number of columns), and it must be invertible. This condition is satisfied when the number of equations equals the number of unknowns.

In the context of nonlinear systems, similar principles apply when linearizing around an equilibrium point. The Jacobian matrix derived during the Newton-Raphson iterations must also be non-singular to facilitate convergence to a unique solution.

In aircraft trim analysis, the state variables (x) and control inputs (u) are adjusted to achieve equilibrium, where all forces and moments balance out. This equilibrium is defined by a set of nonlinear equations representing the aircraft's dynamics. The linearized form of these equations around a trim point simplifies the problem to solving a system of linear equations, where the derivatives of state variables ($\dot{x}$) and outputs (y) are matched with their respective equations.

3.1 Newton-Raphson Method: Mathematical Definition

The Newton-Raphson method is an iterative numerical technique used to find approximate solutions to systems of nonlinear equations. It is widely used in engineering and applied mathematics due to its simplicity and rapid convergence properties.

Consider a system of n nonlinear equations in n variables:

$$F(x) = \begin{bmatrix} f_1(x_1, x_2, x_3, \dots, x_n) \\ f_2(x_1, x_2, x_3, \dots, x_n) \\ \vdots \\ f_n(x_1, x_2, x_3, \dots, x_n) \end{bmatrix} \quad (3.9)$$

where $x = [x_1, x_2, x_3, \dots, x_n]^T$ is the vector of variables, and $F(x)$ represents the vector of nonlinear equations. The goal is to find the vector x such that $F(x) = 0$.

The Newton-Raphson method starts with initial guess $x^{(0)}$ for the solution and iteratively refines this guess using the following update rule:

$$x^{(k+1)} = x^{(k)} - J^{-1}(x^{(k)})F(x^{(k)}) \quad (3.10)$$

where $x^{(k)}$ is the current estimate of the solution at iteration k , and $J(x^{(k)})$ is the Jacobian matrix of $F(x)$ evaluated at $x^{(k)}$. The Jacobian matrix is defined as:

$$J(x) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & \cdots & \frac{\partial f_3}{\partial x_n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \frac{\partial f_n}{\partial x_3} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \quad (3.11)$$

The iteration continues until the changes in x between successive iterations are sufficiently small, indicating that convergence has been achieved. Mathematically, the convergence criteria can be expressed as:

$$\|x^{(k+1)} - x^{(k)}\| < \varepsilon \quad (3.12)$$

where ε is a small positive tolerance value. In practice, computing the inverse of the Jacobian matrix $J^{-1}(x^{(k)})$ can be computationally expensive. Instead, the update step is often formulated as solving a linear system:

$$J(x^{(k)})\Delta x^{(k)} = -F(x^{(k)}), \quad \Delta x^{(k)} = x^{(k+1)} - x^{(k)} \quad (3.13)$$

Thus, the iterative process outcomes,

$$x^{(k+1)} = x^{(k)} + \Delta x^{(k)} \quad (3.14)$$

This reformulation avoids the direct computation of the inverse Jacobian, making the Newton-Raphson method more efficient for large systems.

3.1.1 Application in aircraft trim analysis

In the context of aircraft trim analysis, the Newton-Raphson method is used to find the trim state where the aircraft is in equilibrium. The state variables and control inputs are iteratively adjusted to minimize the error between the desired and actual steady-state conditions.

Steps in Aircraft Trim Analysis

- Initial Guess: Start with an initial guess for the state variables and control inputs.
- Compute Residuals: Evaluate the nonlinear equations $F(x^{(k)})$ to compute the residuals.
- Compute Jacobian: Calculate the Jacobian matrix $J(x^{(k)})$ at the current guess.
- Solve Linear System: Solve the linear system to find the update Δx
- Update Guess: Update the state variables and control inputs.
- Check Convergence: Repeat the process until the residuals are within the specified tolerance.

The Newton-Raphson method provides a robust and efficient means of solving nonlinear systems of equations, making it well-suited for finding trim conditions in aircraft dynamics. By iteratively refining the state variables and control inputs, this method ensures that the aircraft achieves a steady-state flight condition where all forces and moments are balanced.

With the information of all the technical background provided above, it can be defined the float parameters in the state and input vectors specifically for the wing level steady state condition. These float parameters are critical for ensuring that the aircraft can be adjusted to maintain a stable and balanced flight. By allowing these parameters to vary, we can effectively analyze and simulate the aircraft's behavior as it seeks to achieve and maintain the steady state. The float parameters, which represent adjustable values

in the state and input vectors, help in fine-tuning the computational model to capture the dynamic responses of the aircraft. For the wing level steady state condition, these float parameters can be selected as follows, enabling a detailed and precise evaluation of the aircraft's performance and stability.

$$\begin{bmatrix} x_{float} \\ u_{float} \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \\ \phi \\ \theta \\ \delta_e \\ \delta_a \\ \delta_r \\ \delta_t \end{bmatrix} \quad (3.15)$$

where " δ " defines delta input parameters of the aircraft named "elevator", "aileron", "rudder", and "thrust". As seen, there are 8 float unknown variables and they should be converged iteratively by using freeze parameters specifically defined with wing level steady state flight.

$$\begin{bmatrix} \dot{x}_{freeze} \\ \dot{y}_{freeze} \end{bmatrix} = \begin{bmatrix} \dot{V}_T \\ \dot{\alpha} \\ \dot{\beta} \\ \dot{p} \\ \dot{q} \\ \dot{r} \\ n_y \\ \gamma \end{bmatrix} \quad (3.16)$$

Equation 3.16 defines freeze equation states balanced with unknown floats demonstrated in Equation 3.15. These settings implies that the steady-state conditions $\dot{p}, \dot{q}, \dot{r}$ require that the angular rates around the aircraft's roll, pitch, and yaw axes be either zero or constant (for wings level steady flight, it must be zero), as seen in scenarios like straight and level flight or coordinated steady turns. For these angular rates to remain unchanged, the aerodynamic moments generated by the control surfaces (ailerons, elevator, rudder) and the thrust moments produced by the engines must also be zero or constant, ensuring no rotational acceleration occurs.

Likewise, the conditions $\dot{V}_T, \dot{\alpha}, \dot{\beta}$ require the translational state variables—true airspeed (V_T), angle of attack α and sideslip angle β to be constant. This constancy is crucial for maintaining a steady flight path, whether in straight and level flight or during maneuvers like steady climbs or descents. To uphold these conditions, the aerodynamic

forces such as lift, drag, and side force must be balanced, ensuring they are either zero or constant. Any variation in these forces would lead to acceleration, disturbing the steady-state flight. These criteria ensure the aircraft is in dynamic equilibrium, with both forces and moments perfectly balanced, resulting in stable and predictable flight performance.

For every step in solving the system, the linearized equations of motion for the steady-state condition can be represented in matrix form using the Jacobian matrix, as shown below [14].

$$\begin{bmatrix} \dot{V}_T \\ \dot{\alpha} \\ \dot{\beta} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \dot{p} \\ \dot{q} \\ \dot{r} \\ \dot{N} \\ \dot{E} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} \frac{\partial V_T}{\partial V_T} & \frac{\partial V_T}{\partial \alpha} & \frac{\partial V_T}{\partial \beta} & \dots & \frac{\partial V_T}{\partial h} \\ \frac{\partial \alpha}{\partial V_T} & \frac{\partial \alpha}{\partial \alpha} & \frac{\partial \alpha}{\partial \beta} & \dots & \frac{\partial \alpha}{\partial h} \\ \frac{\partial \beta}{\partial V_T} & \frac{\partial \beta}{\partial \alpha} & \frac{\partial \beta}{\partial \beta} & \dots & \frac{\partial \beta}{\partial h} \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \frac{\partial h}{\partial V_T} & \frac{\partial h}{\partial \alpha} & \frac{\partial h}{\partial \beta} & \dots & \frac{\partial h}{\partial h} \end{bmatrix} \begin{bmatrix} V_T \\ \alpha \\ \beta \\ \phi \\ \theta \\ \psi \\ p \\ q \\ r \\ N \\ E \\ h \end{bmatrix} + \begin{bmatrix} \frac{\partial V_T}{\partial \delta_e} & \frac{\partial V_T}{\partial \delta_a} & \frac{\partial V_T}{\partial \delta_r} & \frac{\partial V_T}{\partial \delta_T} \\ \frac{\partial \alpha}{\partial \delta_e} & \frac{\partial \alpha}{\partial \delta_a} & \frac{\partial \alpha}{\partial \delta_r} & \frac{\partial \alpha}{\partial \delta_T} \\ \frac{\partial \beta}{\partial \delta_e} & \frac{\partial \beta}{\partial \delta_a} & \frac{\partial \beta}{\partial \delta_r} & \frac{\partial \beta}{\partial \delta_T} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \frac{\partial h}{\partial \delta_e} & \frac{\partial h}{\partial \delta_a} & \frac{\partial h}{\partial \delta_r} & \frac{\partial h}{\partial \delta_T} \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_a \\ \delta_r \\ \delta_T \end{bmatrix} \quad (3.17)$$

3.1.2 Definition of wing level steady flight

Wing level steady flight is a fundamental concept in aircraft dynamics, referring to a flight condition where the aircraft maintains a constant altitude and heading without any angular or linear accelerations. This state of equilibrium is critical for ensuring stable and predictable aircraft behavior, both in real-world operations and in simulation environments. The MATLAB-Simulink model is often employed to simulate and analyze such steady-state conditions due to its robust capabilities in handling complex dynamic systems.

In the MATLAB-Simulink model, the dynamics of the aircraft are represented by 12 state variables, which include parameters such as velocity, angles, and rates of change. These state variables are crucial for capturing the aircraft's behavior accurately. During each iteration of the simulation, the model is compiled and executed using specific MATLAB functions. The function "*feval(Model, [], X, U, 'outputs')*" evaluates the

outputs of the model given the current state variables and input variables, while `"feval(Model, [], X, U, 'derivs')"` computes the derivatives of the state variables, indicating how these states evolve over time. This iterative process allows the model to update the state and input variables continually, converging towards the desired steady-state condition.

To more accurately simulate real-world flight conditions, the gamma angle (γ) is not set to zero but is assigned a small value, as stated earlier. This adjustment reflects the slight variations and imperfections that occur in actual flight, thereby providing a more realistic representation of the aircraft's dynamics. In the given context, the gamma angle is set to $\gamma = -0.2884$ which helps simulate the non-ideal flight conditions typically encountered.

Achieving wing level steady flight requires both rotational and translational accelerations to be zero. This means the aircraft must fly with a constant velocity, angle of attack, and sideslip angle, while maintaining zero angular rates around its roll, pitch, and yaw axes. The specific conditions can be expressed as

$$\dot{V}_T, \dot{\alpha}, \dot{\beta}, \dot{p}, \dot{q}, \dot{r}, n_y = 0, \quad \gamma = -0.2884 \quad (3.18)$$

To enforce these steady-state conditions within the simulation, the state derivatives that are required to be constant are "frozen." This is achieved by setting these derivatives to logical "0" in the model. For example, Equation 3.18 defines these freeze state derivatives, ensuring they remain unchanged during the simulation. By freezing these state derivatives, the model simplifies the computational process, focusing on maintaining the critical parameters necessary for steady flight. This approach ensures the simulation accurately reflects the desired steady-state conditions, enabling detailed analysis of the aircraft's performance and stability.

Given with Equations 3.4 through 3.7, system equations are as followings.

$$\begin{bmatrix} \dot{x}_{freeze} \\ y_{freeze} \end{bmatrix} = \begin{bmatrix} A_{sa} & B_{sb} \\ C_{sc} & D_{sd} \end{bmatrix} \begin{bmatrix} x_{float} \\ u_{float} \end{bmatrix} \quad (3.19)$$

Up to now, all equations, definitions and logical parameters have been presented. To solve the system equations described in Equation 3.19, all parameters goes to left side

and are equated to zero.

$$F(\Gamma) = \begin{bmatrix} \dot{x}_{freeze} \\ y_{freeze} \end{bmatrix} - \begin{bmatrix} A_{sa} & B_{sb} \\ C_{sc} & D_{sd} \end{bmatrix} \begin{bmatrix} x_{float} \\ u_{float} \end{bmatrix} = 0 \quad (3.20)$$

This equation represents the steady-state condition where the system's freeze state derivatives and outputs are balanced by the float state and input variables. The function $F(\Gamma)$ encapsulates the residuals of the system's equations, which must be minimized to achieve equilibrium. The wing level steady-state equilibrium point or trim condition is found by applying a threshold value to the result in Equation 3.20 and is calculated using the Euclidean norm.

$$norm \left[\begin{bmatrix} \dot{V}_T \\ \dot{\alpha} \\ \dot{\beta} \\ \dot{p} \\ \dot{q} \\ \dot{r} \\ (n_{yu} - n_{y(ny=0)}) \\ (\gamma_u - \gamma_{(\gamma=\vartheta)}) \end{bmatrix} \right] = \left| \begin{bmatrix} \dot{V}_T \\ \dot{\alpha} \\ \dot{\beta} \\ \dot{p} \\ \dot{q} \\ \dot{r} \\ (n_{yu} - n_{y(ny=0)}) \\ (\gamma_u - \gamma_{(\gamma=\vartheta)}) \end{bmatrix} \right| < \varepsilon \quad (3.21)$$

The subscript u in the last two parameters of the matrix represents the current values of the system variables and ϑ is non-zero but small γ value. During each iteration, the algorithm measures the difference between these current values and the desired values for achieving the trim condition. This difference is critical for assessing how close the system is to reaching equilibrium. The tolerance value ε serves as a threshold, indicating the acceptable level of error. When the difference between the current values and the desired trim values falls below this threshold, the trim condition is considered to be met, signifying that the aircraft has reached a steady-state equilibrium.

At this point, the parameters within $x_{float(4 \times 1)}$ and $u_{float(4 \times 1)}$ which represent the float state and input variables respectively, have numerically converged. This convergence means that these variables have been adjusted to the precise values needed to maintain the steady flight condition. The iterative process ensures that the system continuously refines these parameters until the trim criteria are satisfied, leading to a stable and balanced flight configuration.

Finally, the flight simulation model has been established, and the infrastructure of the trim algorithm has been discussed. The model is ready to produce simulation outputs that will represent real-world flight data.

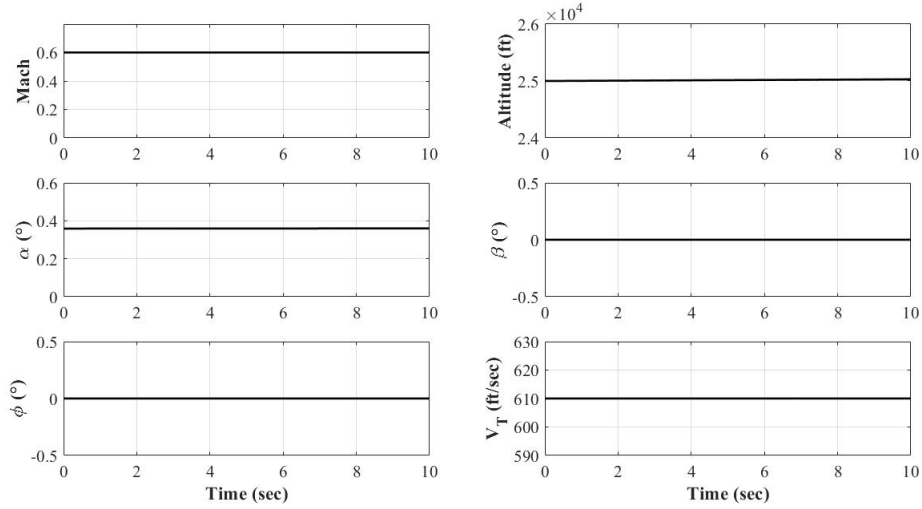


Figure 3.1 : Simulation Outputs for the Trim Point

The simulation outputs produced by finding the equilibrium point are shown in Figure 3.1. Using the established infrastructure, the flight conditions given in [1] and Table 2.1 have been achieved.

The other example simulation with control surface movement is also presented in Figure 3.2

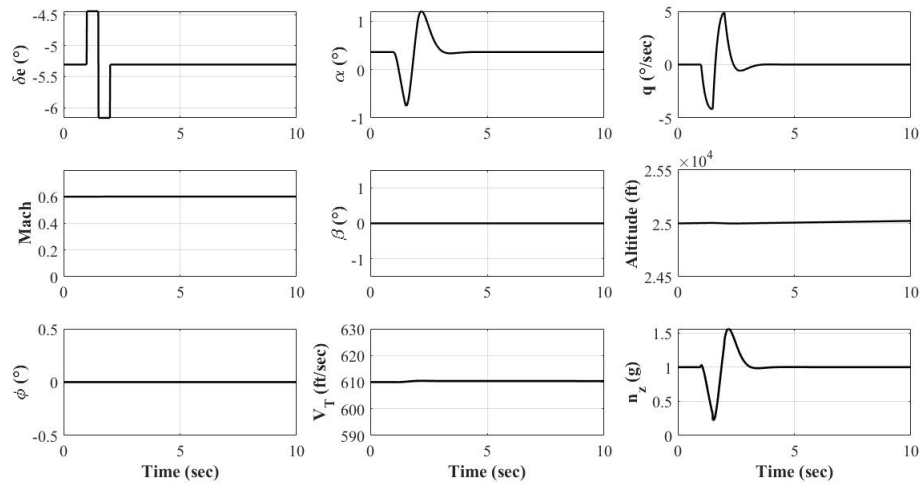


Figure 3.2 : Simulation Outputs with Elevator Doublet Around Trim Point

As seen in the figure, small elevator input has been given and the aircraft returns to its equilibrium point with very small altitude change.

4. PARAMETER ESTIMATION

Parameter estimation techniques are essential for accurately modeling aircraft dynamics. Various methods, including least squares (LS), maximum likelihood estimation (MLE), Kalman filtering (KF), and artificial intelligence-based methods, offer different strengths and sensitivities. Methods like ordinary least squares (OLS) are sensitive to measurement errors. The choice of method depends on the specific requirements of the estimation task, including the nature of the data, the presence of noise, and the need for real-time updating.

The least squares (LS) methods are among the most commonly used techniques for parameter estimation. These methods, also known as regression analysis, fall under the category of equation-error methods as they minimize a cost function directly defined in terms of the relationship between inputs and outputs. Ordinary Least Squares (OLS) minimizes the sum of the squared residuals between observed and predicted values, making it effective for linear models due to its simplicity and computational efficiency. Recursive Least Squares (RLS), an extension of OLS, updates parameter estimates as new data becomes available, making it particularly useful for real-time applications and adaptive filtering where parameters might change over time.

Maximum Likelihood Estimation (MLE) is a powerful technique that identifies parameter values maximizing the likelihood function given the observed data. MLE is highly effective in handling different types of data distributions and offers asymptotically unbiased and efficient estimates. This flexibility makes MLE a robust choice for various modeling scenarios in aircraft dynamics, where the underlying data distributions might be complex and varied.

Kalman filtering techniques, including the Extended Kalman Filter (EKF) and the Unscented Kalman Filter (UKF), are widely used for real-time parameter estimation and state tracking. EKF is used for nonlinear systems by linearizing the system around the current estimate, while UKF improves upon EKF by using a deterministic sampling

technique to handle nonlinearities more effectively without linearizing the system. These filters are particularly powerful for dynamic systems with continuously changing states and inputs, offering precise and adaptive estimation capabilities.

The output error method minimizes the difference between the observed output and the model output by adjusting the parameters. This method is particularly effective for dynamic systems where the output depends on past states and inputs. Similarly, the filter error method filters the error signal and adjusts the parameters to minimize the filtered error, reducing the impact of noise on parameter estimates. Both methods are widely used in control system design and analysis, where accurate modeling of the system's dynamics is crucial.

Artificial intelligence-based methods, such as neural networks and genetic algorithms (GAs), have also been employed in parameter estimation. Neural networks can approximate complex nonlinear relationships and are trained using data to estimate parameters. Genetic algorithms are optimization techniques that use principles of natural selection to find parameter values that minimize or maximize a given objective function. These methods are particularly useful for highly nonlinear systems and can handle large and complex datasets effectively.

4.1 Least Square Method

Least squares (LS) methods, often known as regression analysis, are a subset of techniques categorized as equation-error methods [5]. These methods aim to minimize a cost function, which is directly formulated based on the relationship between input and output variables. By minimizing this cost function, the LS methods effectively reduce the discrepancy between the observed data and the model's predictions. This process involves adjusting the model parameters to achieve the best fit, thereby ensuring that the equation representing the system's dynamics aligns as closely as possible with the actual data.

The image illustrates the concept of least squares (LS) regression, a technique commonly used in regression analysis to find the best-fitting line through a set of data points. This method minimizes the sum of the squares of the differences (errors)

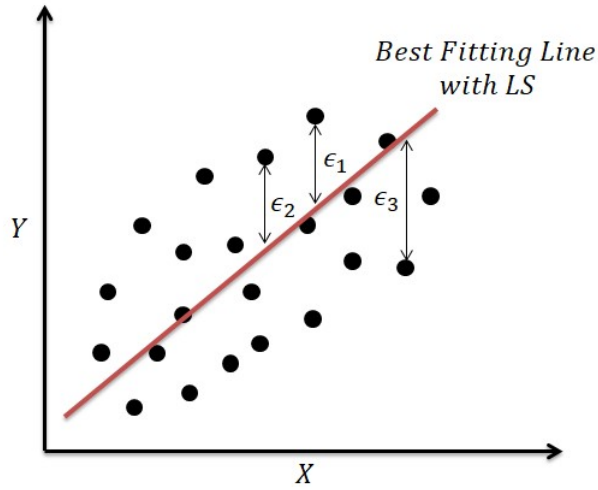


Figure 4.1 : The Best Fitting Line Demonstration for Example Data

between the observed values and the values predicted by the line. The black dots represent the observed data points, which consist of pairs of (X) (independent variable) and (Y) (dependent variable) values. The red line represents the line of best fit determined by the least squares method. This line is calculated to minimize the sum of the squared vertical distances (errors) from each data point to the line. The vertical arrows labeled $\epsilon_1, \epsilon_2, \epsilon_3$ denote the errors or residuals. Each error is the difference between the observed value Y and the value predicted by the best-fitting line at the corresponding X . In mathematical terms, for a given data point (X_i, Y_i) , the error ϵ_i is,

$$\epsilon_i = Y_i - \hat{Y}_i \quad (4.1)$$

where $\hat{Y}_i$ is the predicted value from the line. The goal of the least squares method is to find the line $\hat{Y} = a + bX$ that minimizes the sum of the squared errors (residuals). This is mathematically represented as:

$$\text{Minimize } S = \sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n (Y_i - (a + bX_i))^2 \quad (4.2)$$

where

- S is the sum of the squared errors.
- n is the number of data points.
- a is the y-intercept of the line
- b is the slope of the line.

By differentiating S with respect to a and b and setting the derivatives to zero, we obtain the normal equations, which can be solved to find the optimal values of a and b .

In this study, the OLS and RLS estimation methods will be examined using a 6 degrees of freedom model. Analyses will be conducted under conditions where there are no errors, such as sensor measurement errors or systematic errors, and the results of both methods will be compared and evaluated. Subsequently, the model described and established in the sections above will be refined to approximate real flight conditions by introducing inputs such as turbulence and systematic sensor errors. The results of both methods will then be compared and evaluated among themselves as well as against the ideal conditions previously examined. To support the results and analyses, visualizations and tables will be presented.

4.1.1 Ordinary least squares method

The Ordinary Least Squares (OLS) method is a fundamental statistical technique used for estimating the parameters of a linear regression model. It is widely used due to its simplicity and effectiveness in fitting a linear relationship between a dependent variable and one or more independent variables. The primary objective of OLS is to find the best-fitting line (or hyperplane in the case of multiple regressors) that minimizes the sum of the squared differences between the observed values and the values predicted by the linear model. These differences are called residuals. By minimizing the sum of the squared residuals, OLS ensures that the model predictions are as close as possible to the actual data points.

Consider a linear regression model with n observations and p predictors (independent variables):

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_p x_{ip} + \varepsilon_i, \quad i = 1, 2, 3, \dots, n \quad (4.3)$$

where

- y_i is the dependent variable.
- x_{ij} are independent variables
- $\beta_0, \beta_1, \dots, \beta_p$ are the parameters to be estimated.

- ε_i is the error term, assumed to be normally distributed with mean zero and constant variance.

In matrix notation, the equation set can be compactly written as:

$$y = X\beta + \varepsilon \quad (4.4)$$

where

- y is an $n \times 1$ vector of observations.
- X is an $n \times (p + 1)$ matrix predictors (including a column of ones for the intercept).
- β is a $(p + 1) \times 1$ vector of parameters.
- ε is an $n \times 1$ vector of errors.

The OLS estimator $\hat{\beta}$ is obtained by minimizing the sum of squared residuals.

$$S(\beta) = \sum_{i=1}^n \varepsilon_i^2 = \sum_{i=1}^n (y_i - x_i^T \beta)^2 \quad (4.5)$$

To find the parameter estimates that minimizes $S(\beta)$, the derivative of $S(\beta)$ is taken with respect to β and set it to zero.

$$\frac{\partial S(\beta)}{\partial \beta} = -2X^T(y - X\beta) = 0 \quad (4.6)$$

Solving for β , the normal equation be as:

$$X^T X \hat{\beta} = X^T y \quad (4.7)$$

The solution to these normal equations is the OLS estimator:

$$\hat{\beta} = (X^T X)^{-1} X^T y \quad (4.8)$$

The Equation 4.8 represents the least squares estimates of the parameter vector β . This equation is fundamental to the OLS method, providing the best linear unbiased estimates of the parameters by minimizing the sum of squared residuals.

The matrix $X^T X$ is referred to as the information matrix. This matrix is crucial because it encapsulates the information content in the data being analyzed. The information

matrix plays a significant role in parameter estimation as it influences the precision of the estimated parameters. In general, for data obtained from carefully conducted flight maneuvers, the matrix $X^T X$ is nonsingular, ensuring that the inverse exists and the parameter estimates are unique. Therefore, the assumption that $X^T X$ is nonsingular is crucial for the uniqueness of the OLS estimates. In practical applications, especially in the context of flight data, this assumption holds true due to the careful design of flight maneuvers aimed at covering a wide range of flight conditions. This ensures that the independent variables provide sufficient information to estimate the parameters uniquely.

The Ordinary Least Squares (OLS) estimator possesses several desirable properties under the Gauss-Markov assumptions, which include linearity, independence, homoscedasticity, and normality of the error terms. These properties are critical in ensuring the reliability and accuracy of the OLS method in parameter estimation.

The OLS estimator is unbiased, meaning that the expected value of the estimated parameters is equal to the true parameter values. So, By leveraging the properties of expectations and assuming that the errors have a mean of zero $E[\epsilon] = 0$, the expected value of the OLS estimator is shown to be equal to the true parameter values.

$$E[\hat{\beta}] = E[(X^T X)^{-1} X^T y] \quad (4.9)$$

$$= E[(X^T X)^{-1} X^T (X\beta + \epsilon)] \quad (4.10)$$

$$= E[(X^T X)^{-1} X^T X\beta] + E[(X^T X)^{-1} X^T X\epsilon] \quad (4.11)$$

$$= \beta + (X^T X)^{-1} X^T E[\epsilon] \quad (4.12)$$

$$= \beta \quad (4.13)$$

The unbiasedness property is a key feature of the Ordinary Least Squares (OLS) estimation method in regression analysis. It ensures that, on average, the OLS estimates do not systematically overestimate or underestimate the true values of the parameters being estimated. An estimator is considered unbiased if, over many samples, the average of the estimates equals the true parameter value. In other words, the estimator does not consistently miss the mark in one direction (either too high or too low). The unbiasedness of the OLS estimator relies on two main assumptions:

- **Zero Mean of Error Terms:** The errors in the regression model, which represent the difference between the observed values and the values predicted by the model, should have an average value of zero.
- **Uncorrelated Errors and Independent Variables:** The errors should not be related to the independent variables (the variables used to predict the dependent variable). This means the errors do not systematically increase or decrease with changes in the independent variables.

If the errors average out to zero, the model is not biased in one direction over another. This balance ensures that the predicted values are centered around the true values. When the errors are uncorrelated with the independent variables, the predictor variables are not influencing the errors. This separation means that the relationship identified between the predictor variables and the outcome variable is genuine and not distorted by the errors.

Unbiasedness means that, over many repeated samples, the OLS estimates will, on average, be correct. This provides confidence that the estimates are reliable representations of the true parameter values. The dependability property makes OLS a reliable method because it ensures that the estimation process does not introduce systematic errors into the results.

While unbiasedness guarantees that the estimates are correct on average, it does not mean that every estimate will be close to the true value. The variability of the estimates also affects their accuracy. The assumptions must hold true for the unbiasedness property to be valid. If the errors do not average out to zero or are correlated with the independent variables, the OLS estimates can be biased.

4.1.2 Recursive least squares method

The motion of a flight vehicle can be represented through well-defined equations of motion, with time-invariant parameters postulated to characterize aerodynamic effects. Traditional offline methods, often referred to as batch or post-flight analysis techniques, use all available data at once. Recursive methods, also known as sequential, adaptive, or real-time algorithms, process data point-by-point. These

methods update the parameter estimates as new data becomes available, making them suitable for online applications. RLS has been adapted for various applications, including flight control and system identification, due to its ability to handle time-varying parameters efficiently.

RLS methods are useful for onboard verification of recorded data, iterative experiment design, and online parameter estimation during flight. These methods approximate more elaborate nonrecursive methods. Challenges include the need for proper data management, handling noise sensitivity, and ensuring numerical stability during continuous parameter updates.

The RLS method is based on the least squares principle but adapted for recursive computation to handle new data incrementally. The RLS algorithm updates the parameter estimates and the covariance matrix incrementally as new data points become available. It uses a gain matrix K to adjust the parameter estimates based on new information. The RLS method starts with the initial parameter estimate $\hat{\beta}_0$ and initial covariance matrix $P(0)$. In the context of parameter estimation techniques used for aircraft dynamics, the Ordinary Least Squares (OLS) and Recursive Least Squares (RLS) methods treat the parameters related to stability and control derivatives differently. In the OLS method, the parameters within the β vector, which represent the stability and control derivatives, are assumed to be constant over the entire dataset of N samples. As stated earlier, the β vector contains unknown parameters that cannot be directly measured during flight. These parameters need to be estimated using the available data. In contrast, the RLS method allows the parameters within the β vector to change dynamically over the samples. This means that the stability and control derivatives are updated as new data becomes available, reflecting the time-varying nature of the aircraft's dynamics. The RLS method is designed to adapt to changing conditions in real-time, making it more suitable for scenarios where the aircraft's behavior is not constant.

In the RLS approach, the number of available samples at any given time is denoted by k . This represents the subset of the total data that is currently being used for the estimation process. In Equation 4.8, which originally described the estimation process

for N samples can be rewritten to apply to k samples.

$$\hat{\beta}(k) = (X_k^T X_k)^{-1} X_k^T Y_k \longrightarrow \hat{\beta}(k) = P(k) X_k^T Y_k \quad (4.14)$$

This reflects the iterative nature of RLS, where the parameter estimates are continuously updated as each new sample is processed. If it is assumed that one more sample is added to the available k samples, Equation 4.14 becomes as follows [5].

$$\hat{\beta}(k+1) = P(k+1) X_{k+1}^T Y_{k+1} \quad (4.15)$$

where

$$X_{k+1} = [X_k; x^T(k+1)], \quad Y_{k+1} = [Y_k; y(k+1)] \quad (4.16)$$

and

$$P(k+1) = (X_{k+1}^T X_{k+1})^{-1} \quad (4.17)$$

After the necessary algebraic manipulations, the $P(k+1)$ equation can be written as follows.

$$P(k+1) = P(k) - P(k)x(k+1) [x^T(k+1)P(k)x(k+1) + 1]^{-1} x^T(k+1)P(k) \quad (4.18)$$

In terms of K matrix, Equation 4.18 can be rewritten as follows.

$$P(k+1) = P(k) - K(k+1)x^T(k+1)P(k) \quad (4.19)$$

Here, $K(k+1)$ is referred to as the gain matrix.

$$K(k+1) = \frac{P(k)x(k+1)}{1 + x^T(k+1)P(k)x(k+1)} \quad (4.20)$$

Finally, if Equation 4.19 is substituted into Equation 4.15 and the necessary algebraic manipulations are performed, the recursive least squares estimation formula becomes as follows.

$$\hat{\beta}(k+1) = \hat{\beta}(k) + K(k+1)[y(k+1) - x^T(k+1)\hat{\beta}(k)] \quad (4.21)$$

Thus, all the necessary information to apply the recursive least squares method is shown in Equations 4.15 through 4.21. The sequence of applying the algorithm at each step is as follows:

1. Update the gain matrix (Equation 4.20)
2. Update the parameter estimation (Equation 4.21)
3. Update the covariance matrix of the parameter estimation error (Equation 4.19)

Like the OLS method, the RLS method requires certain assumptions for unbiasedness to hold:

- **Zero Mean of Error Terms:** The errors should have an average value of zero.
- **Uncorrelated Errors and Independent Variables:** The errors should not be correlated with the independent variables.

The RLS method updates parameter estimates incrementally as new data becomes available, unlike the OLS method, which uses all data points simultaneously. Despite this difference, the basic principles that ensure unbiasedness in OLS can also apply to RLS if the assumptions about the error terms hold true over time.

The assumptions must hold consistently over each time step for the RLS estimates to remain unbiased. This means that at each iteration, the new data point added must not introduce systematic bias. Also, The initial estimates and covariance matrix must be chosen appropriately. Poor initial estimates can affect the early iterations but will typically have less influence as more data is incorporated. The RLS method's ability to adapt to new data means it can handle changing conditions in the system being modeled. As long as the error assumptions hold for each new data point, the RLS estimates will remain unbiased.

- **Forgetting Factor:** In some variants of RLS, a forgetting factor is used to give more weight to recent data. The choice of this factor can influence the unbiasedness of the estimates, especially if it causes older, potentially biased data to be forgotten too quickly or too slowly. It is in the numerator section in Equation 4.20.

4.1.3 Performance metrics of the methods

The OLS estimator is the Best Linear Unbiased Estimator (BLUE), meaning it has the smallest variance among all linear unbiased estimators. This property is known as efficiency. Under the Gauss-Markov assumptions, no other linear unbiased estimator can have a smaller variance than the OLS estimator. This efficiency is expressed as:

$$\text{Var}(\hat{\beta}) = \rho^2 (X^T X)^{-1} \quad (4.22)$$

where ρ^2 is the variance of the error terms. The OLS estimator achieves the minimum possible variance, leading to more precise estimates. Besides, it can also be estimated with variance estimation. The estimate of the variance ρ^2 is given by:

$$\hat{\rho}^2 = \frac{1}{N - n_q} \sum_{k=1}^N [y(k) - X(k)\hat{\beta}]^2 \quad (4.23)$$

This formula adjusts for the degrees of freedom, providing an unbiased estimate of the error variance.

The parameter error covariance matrix P is defined as:

$$P = E[(\hat{\beta} - \beta)(\hat{\beta} - \beta)^T] \quad (4.24)$$

From Equation 4.23, the derivation shows that the estimated parameter covariance matrix is also equated as follows:

$$P = \hat{\rho}^2 (X^T X)^{-1} \quad (4.25)$$

The covariance matrix of the estimated parameters, denoted as P , quantifies the variability and uncertainty in the parameter estimates. To investigate the asymptotic consistency of the estimates with a large number of data samples, it is necessary to check the behavior of the covariance matrix. Asymptotic consistency implies that as N approaches infinity, the estimates converge to the true parameter values, and the covariance matrix should reflect this by approaching zero.

$$\lim_{N \rightarrow \infty} P = \lim_{N \rightarrow \infty} \frac{\hat{\rho}^2}{N} (X^T X)^{-1} = 0 \quad (4.26)$$

For each parameter estimate, the standard deviation σ is derived from the square root of the diagonal elements of the covariance matrix. It will be used as performance parameter to evaluate simulation outputs.

$$\sigma_j = \sqrt{P_{jj}}, \quad j = 1, 2, 3, \dots, n \quad (4.27)$$

The standard deviation σ_j is used as a performance parameter to evaluate the accuracy and reliability of the simulation outputs. Lower values of σ_j indicate higher precision in the parameter estimates, reflecting greater confidence in the simulation results.

The coefficient of determination, denoted as R^2 , is a statistical measure that represents the proportion of the variance in the dependent variable that is predictable from the independent variables. It provides a measure of how well the regression model explains the observed data.

$$R^2 = \frac{\sum_{k=1}^N (\hat{y}(k) - \bar{y})^2}{\sum_{k=1}^N (y(k) - \bar{y})^2} = \frac{\hat{\beta}^T X^T y - N\bar{y}^2}{y^T y - N\bar{y}^2} \quad (4.28)$$

where

- $\hat{y}(k)$ represents the predicted output (dependent variable) using the estimated model parameters. $\hat{\beta}$
- $y(k)$ represents the measured data samples.
- $\bar{y}$ is the mean of the observed data samples.
- $\hat{\beta}$ the least squares estimates of the parameter vector β .

The value of R^2 ranges between 0 and 1. An R^2 value of 1 indicates that the regression model perfectly explains the variability in the dependent variable, while a value of 0 indicates that the model does not explain any of the variability. It provides the proportion of the variation in the dependent variable y that is explained by the independent variables in the regression model. A higher R^2 value suggests a better fit of the model to the data. R^2 can be overestimated when the number of samples N is not large compared to n_q the number of independent variables. This overestimation occurs because R^2 tends to increase with the addition of more independent variables, even if some of them do not significantly contribute to the model.

5. IMPLEMENTATION

In the Ordinary Least Squares (OLS) method, the parameters are assumed to be constant over the entire dataset. This method processes all data points simultaneously, aggregating the information to provide a single, static estimate for each parameter. This approach is effective for systems where the relationships between variables are stable and do not change over time, as it captures an average effect based on the complete dataset.

Conversely, the Recursive Least Squares (RLS) method updates the parameter estimates incrementally as new data becomes available. This dynamic approach allows the parameters to change over time, adapting to variations in the system's characteristics. RLS is particularly well-suited for real-time applications where conditions are continually evolving and immediate updates are necessary. By incorporating new data points one at a time, RLS provides a flexible and responsive means of parameter estimation, ensuring that the model remains accurate as new information is obtained.

The equations formed with measurable variables play a crucial role in the estimation process for both OLS and RLS methods. These equations provide the necessary data to solve the least squares problems for each aerodynamic coefficient. By leveraging these measurable variables, the estimation techniques can accurately infer the aerodynamic parameters, which are essential for characterizing the aircraft's behavior.

As stated earlier, aerodynamic forces and moments on an aircraft cannot be directly measured due to the complexities involved in capturing these dynamic interactions in real-time. These complexities arise from the constantly changing airflow conditions around the aircraft, which are influenced by numerous factors such as speed, altitude, and maneuvering. However, other critical parameters can be accurately measured using advanced onboard sensors and instrumentation. These parameters include

angular velocities (pitch, roll, and yaw rates), linear velocities, control surface deflections (elevator, aileron, and rudder angles), angle of attack, and sideslip angle.

By leveraging these measurable variables obtained from flight data, it is possible to estimate the aerodynamic forces and moments indirectly. These estimations are achieved through sophisticated parameter estimation techniques, which utilize the measured data to infer the values of the unmeasured aerodynamic parameters. These techniques often involve complex mathematical models and algorithms that process the measured data to generate accurate estimates of the aerodynamic forces and moments.

$$\begin{aligned}
X_{C_D}(k) &= [1(k) \quad \alpha(k) \quad \hat{u}(k) \quad \delta_e(k)] \\
X_{C_Y}(k) &= [1(k) \quad \beta(k) \quad \hat{p}(k) \quad \hat{r}(k) \quad \delta_a(k) \quad \delta_r(k)] \\
X_{C_L}(k) &= [1(k) \quad \alpha(k) \quad \hat{q}(k) \quad \hat{u}(k) \quad \delta_e(k)] \\
X_{C_R}(k) &= [1(k) \quad \beta(k) \quad \hat{p}(k) \quad \hat{r}(k) \quad \delta_a(k) \quad \delta_r(k)] \\
X_{C_M}(k) &= [1(k) \quad \alpha(k) \quad \hat{q}(k) \quad \hat{u}(k) \quad \delta_e(k)] \\
X_{C_N}(k) &= [1(k) \quad \beta(k) \quad \hat{p}(k) \quad \hat{r}(k) \quad \delta_a(k) \quad \delta_r(k)]
\end{aligned} \tag{5.1}$$

Using the measurable parameters from flight data, it is possible to estimate the unmeasurable aerodynamic forces and moments. This involves creating relationships between the measurable and unmeasurable parameters using aerodynamic derivatives. The X vectors shown for each force and moment coefficient ($C_D, C_Y, C_L, C_R, C_M, C_N$) consist of independent measurement outputs. Each X vector is used to estimate the corresponding aerodynamic derivatives, which are not directly measurable. The approach leverages measurable flight parameters to estimate aerodynamic coefficients. This is crucial in flight dynamics and control, where accurate modeling of aerodynamic forces and moments is necessary for designing stable and efficient aircraft. So, aerodynamic derivatives, which describe how aerodynamic forces and moments change with respect to changes in flight parameters, are estimated using regression techniques. These derivatives are then used to construct models that predict the behavior of the aircraft under various flight conditions. The equations provided form the basis for regression analysis, where the goal is to determine the coefficients that best fit the observed data. By using the measurable parameters as predictors, the regression

model can estimate the aerodynamic derivatives that describe the unmeasured forces and moments.

The unknown aerodynamic derivatives for each X vector are as follows.

$$\begin{aligned}
 \beta_{C_D} &= [C_{D_0} \quad C_{D_\alpha} \quad C_{D_u} \quad C_{D_{\delta_e}}]^T \\
 \beta_{C_Y} &= [C_{Y_0} \quad C_{Y_\beta} \quad C_{Y_p} \quad C_{Y_r} \quad C_{Y_{\delta_a}} \quad C_{Y_{\delta_r}}]^T \\
 \beta_{C_L} &= [C_{L_0} \quad C_{L_\alpha} \quad C_{L_q} \quad C_{L_u} \quad C_{L_{\delta_e}}]^T \\
 \beta_{C_R} &= [C_{R_0} \quad C_{R_\beta} \quad C_{R_p} \quad C_{R_r} \quad C_{R_{\delta_a}} \quad C_{R_{\delta_r}}]^T \\
 \beta_{C_M} &= [C_{M_0} \quad C_{M_\alpha} \quad C_{M_q} \quad C_{M_u} \quad C_{M_{\delta_e}}]^T \\
 \beta_{C_N} &= [C_{N_0} \quad C_{N_\beta} \quad C_{N_p} \quad C_{N_r} \quad C_{N_{\delta_a}} \quad C_{N_{\delta_r}}]^T
 \end{aligned} \tag{5.2}$$

In the least squares method calculated for aerodynamic parameter estimation, dimensionless aerodynamic force and moment coefficients are used as the dependent variables. This method, which includes both Ordinary Least Squares (OLS) and Recursive Least Squares (RLS), solves a separate least squares problem for each force and moment coefficient.

The dependent variables in the least squares method are the dimensionless aerodynamic force and moment coefficients. These coefficients are critical for understanding the aerodynamic behavior of the aircraft. Each aerodynamic force and moment coefficient requires solving a distinct least squares estimation problem. This approach allows for precise estimation of each specific aerodynamic parameter. The forces and moments consist of linear and rotational parameters, representing the six degrees of freedom (6-DOF) dynamics of the aircraft. These parameters include:

- Linear parameters: Forces along the x, y, and z axes.
- Rotational parameters: Moments around the x, y, and z axes.

The sets of equations are formed using measurable variables such as angular velocities, linear velocities, control surface deflections, angle of attack, and sideslip angle. These

measurable variables are used to estimate the unknown aerodynamic derivatives.

$$\begin{aligned} C_X &= \frac{1}{\bar{q}S}(mgn_x - T) \\ C_Y &= \frac{1}{\bar{q}S}mgn_y \\ C_Z &= \frac{1}{\bar{q}S}mgn_z \end{aligned} \quad (5.3)$$

Force coefficients with measurable parameters are presented in Equation 5.3. The transformation equation of the force coefficients expressed in the body axis system to the wind axis system is also presented below.

$$\begin{bmatrix} C_D \\ C_Y \\ C_L \end{bmatrix} = [DCM_{bw}(\alpha, \beta)]^T \begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix}$$

The moment coefficient calculations are also provided below.

$$\begin{aligned} C_R &= \frac{1}{\bar{q}Sb}[I_{xx}\dot{p} - I_{xz}(pq + \dot{r}) + (I_{zz} - I_{yy})qr] \\ C_M &= \frac{1}{\bar{q}S\bar{c}}[I_{yy}\dot{q} + (I_{xx} - I_{zz})pr + I_{xz}(p^2 - r^2)] \\ C_N &= \frac{1}{\bar{q}Sb}[I_{zz}\dot{r} - I_{xz}(\dot{p} - qr) + (I_{yy} - I_{xx})pq] \end{aligned} \quad (5.4)$$

The force and moment coefficients mentioned above are derived from the measurement parameters that represent the flight data produced by the simulation outputs. These coefficients comes from undefined and unestimated aerodynamic derivatives. The measurement parameters from 6-DOF simulation output, such as angular velocities, linear velocities, control surface deflections, angle of attack, and sideslip angle, are used to calculate these coefficients accurately. These calculations ensure that the simulation outputs accurately reflect the actual flight conditions.

$$\Psi : [n_x, n_y, n_z, \dot{p}, \dot{q}, \dot{r}, p, q, r, I_{xx}, I_{yy}, I_{zz}, I_{xz}, \bar{q}, S, m, \bar{c}, b, g] \quad (5.5)$$

The above Ψ represents the variable parameters that can be measured and extracted from the flight data.

An important aspect in the estimation of aerodynamic derivatives is ensuring that the model receives appropriate control surface inputs. Providing suitable inputs is crucial for obtaining accurate estimates of the aerodynamic parameters. In Figure 5.1, the

parameter estimation results for the lift coefficient (C_L) are displayed, derived from a flight simulation conducted under ideal conditions. The simulation uses a randomly selected elevator input to demonstrate the effects on the lift coefficient. This example highlights how control surface inputs influence the accuracy and reliability of the parameter estimation process, ensuring that the model accurately reflects the aircraft's aerodynamic behavior.

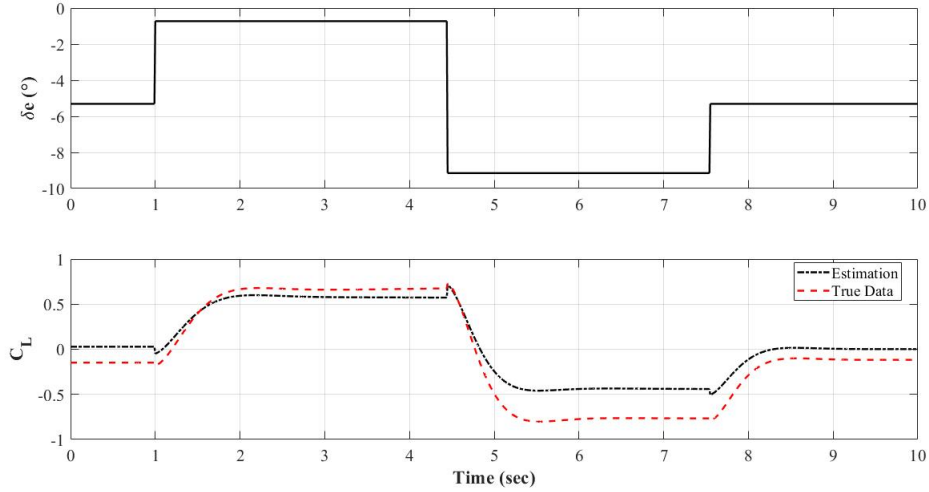


Figure 5.1 : Estimation of C_L with Non-Optimal Elevator Input

In the process of linearizing a system, the small values used to perturb the state variables must also be applied to the control surfaces. This approach is particularly important when creating a linear aerodynamic database for a specific flight condition. Providing appropriate control inputs is crucial for obtaining accurate and reliable parameter estimates. If excessively large and prolonged inputs are given, the least squares parameter estimation methods may fail to produce accurate results. Such inappropriate inputs can cause the aircraft, which is initially at an equilibrium point, to deviate from its steady-state condition, thereby disrupting the intended flight condition. This disruption underscores the importance of carefully designing control surface inputs. The design of control surface inputs plays a vital role in ensuring the accuracy of parameter estimation. Poorly designed inputs can lead to significant discrepancies between the actual values and the estimated values, indicating errors in the estimation process. Therefore, it is clearly evident that ensuring proper input design is essential

for maintaining the integrity of the flight simulation and achieving precise aerodynamic parameter estimation.

5.1 Evaluation of OLS Method

In this section, we will first present the Ordinary Least Squares (OLS) estimation results obtained under ideal conditions, where the model is assumed to be free of any noise. This will provide a clear understanding of the estimation accuracy in a perfect scenario. Specifically, we will compare the true values of the force and moment coefficients with the estimated values obtained through the OLS method. These comparisons are made by applying sufficiently small doublet inputs to the control surfaces in both the longitudinal and lateral-directional axes. The results of these comparisons are illustrated in Figures 5.2 and 5.3, respectively. Figure 5.2 shows the comparison for the longitudinal axis, while Figure 5.3 illustrates the comparison for the lateral-directional axis. These figures highlight the accuracy of the OLS method in estimating the aerodynamic coefficients under noise-free conditions, providing a baseline for further analysis.

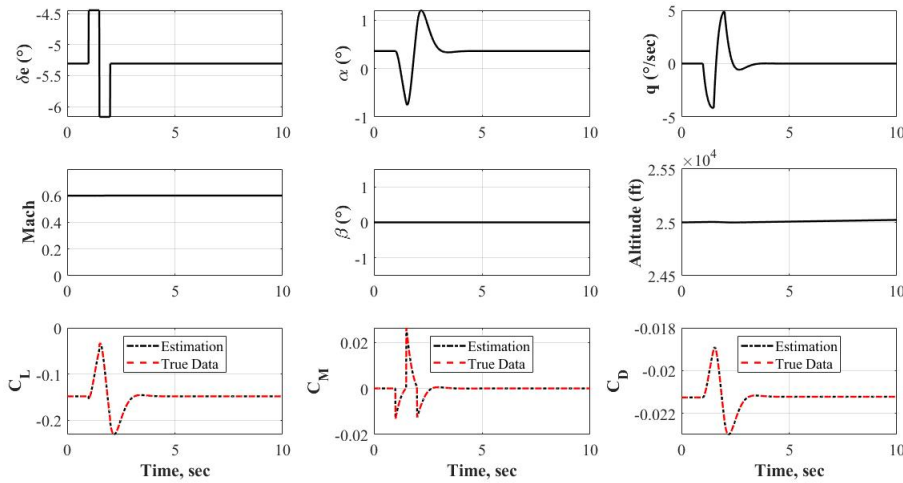


Figure 5.2 : Ideal Measurement Results in the Longitudinal Axis with OLS

The figure illustrates that the estimation results of the aerodynamic coefficients in the longitudinal axis closely match the true values. This alignment indicates the accuracy and reliability of the estimation process under ideal conditions. Furthermore, the significance of input design is evident when comparing Figure 5.1 and Figure 5.2.

These comparisons highlight how the choice and design of control surface inputs can impact the accuracy of the estimated aerodynamic coefficients.

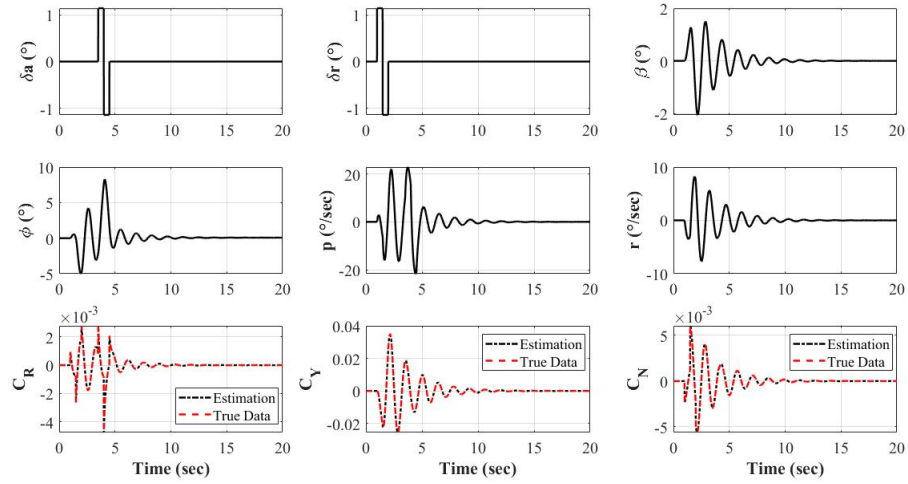


Figure 5.3 : Ideal Measurement Results in the Lateral-Directional Axis with OLS

Consequently, Table 5.1 demonstrates that with well-designed and appropriate control surface inputs, it is possible to achieve aerodynamic derivative estimates with very high accuracy. This underscores the importance of carefully selecting and implementing control surface inputs to ensure precise and accurate parameter estimation in aerodynamic studies. The table demonstrates that the Ordinary Least Squares (OLS)

Table 5.1 : Ideal Measurement Results of Aerodynamic Derivatives

Der.	True	Est.	R^2	σ	Der.	True	Est.	R^2	σ
C_{L_0}	0.149	0.149	1	3.63e-14	C_{R_0}	0	0	1	2.62e-20
C_{L_α}	5.5	5.5	1	3.16e-13	C_{R_β}	-0.11	-0.11	1	9.89e-18
C_{L_q}	14.2	14.2	1	2.24e-11	C_{R_p}	-0.39	-0.39	1	5.12e-17
C_{L_u}	0.084	0.084	1	4.92e-12	C_{R_r}	0.28	0.28	1	6.54e-17
$C_{L_{\delta_e}}$	0.38	0.38	1	3.86e-13	$C_{R_{\delta_a}}$	0.1	0.1	1	1.00e-17
C_{M_0}	-0.08	-0.08	1	2.46e-14	$C_{R_{\delta_r}}$	0.05	0.05	1	7.44e-18
C_{M_α}	-0.24	-0.24	1	2.14e-13	C_{Y_0}	0	0	1	2.44e-19
C_{M_q}	-27.3	-27.3	1	1.51e-11	C_{Y_β}	-1	-1	1	9.20e-17
C_{M_u}	0	0	1	3.33e-12	C_{Y_p}	-0.14	-0.14	1	4.77e-16
$C_{M_{\delta_e}}$	-0.88	-0.88	1	2.61e-13	C_{Y_r}	0.61	0.61	1	6.08e-16
C_{D_0}	0.0205	0.0205	1	3.86e-16	$C_{Y_{\delta_a}}$	0	0	1	9.37e-17
C_{D_α}	0.12	0.12	1	4.32e-15	$C_{Y_{\delta_r}}$	0.028	0.028	1	6.91e-17
C_{D_u}	0.05	0.05	1	8.82e-14	C_{N_0}	0	0	1	1.33e-17
$C_{D_{\delta_e}}$	0	0	1	4.27e-15	C_{N_β}	0.17	0.17	1	5.00e-15
$C_{N_{\delta_a}}$	-0.003	-0.003	1	5.10e-15	C_{N_p}	0.09	0.09	1	2.59e-14
$C_{N_{\delta_r}}$	-0.12	-0.12	1	3.76e-15	C_{N_r}	-0.26	-0.26	1	3.31e-14

estimation method yields exact results, as anticipated. This is evident from the fact that

for each estimated value, the R^2 statistic is 1, indicating perfect alignment with the true values. Additionally, the standard deviations of the estimates are very small, reflecting a high level of precision and minimal variability in the estimates. These performance indicators, including the high R^2 values and low standard deviations, strongly support the accuracy and reliability of the OLS estimation method. Consequently, the table confirms that the OLS method performs exceptionally well, providing highly accurate parameter estimates in this context.

In the second step, the Dryden continuous atmospheric turbulence block was activated in the flight simulation model, and the simulation was rerun. The control surface input values were kept the same. The atmospheric turbulence conditions are shown in Table 5.2.

Table 5.2 : Atmospheric Turbulence Conditions

Wind Speed	15 <i>m/sec</i>
Wind Direction	60°
Noise Sample Time	0.05 <i>sec</i>
Wing Span	16.2 <i>m</i>
Turbulence Intensity	(10 ⁻²)
Altitude	25000 <i>ft</i>

Figures 5.4 and 5.5 show the actual and estimated results in turbulent air conditions for the longitudinal and lateral-directional axes, respectively.

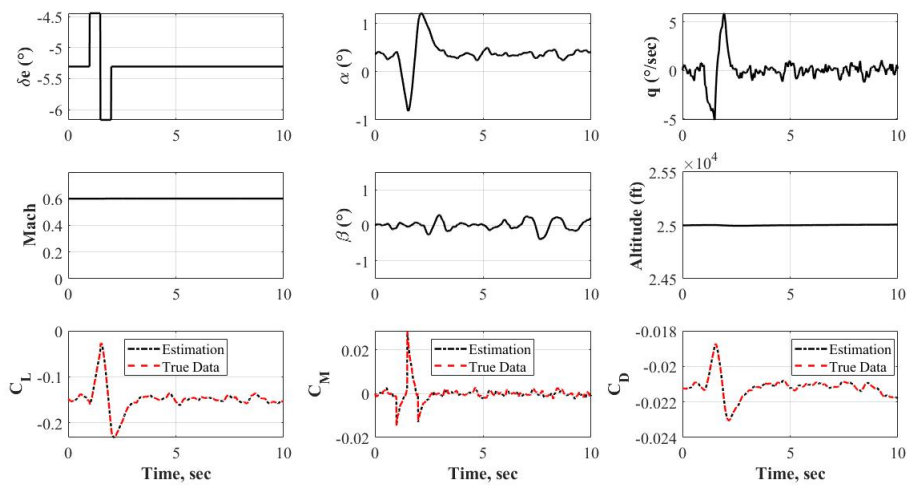


Figure 5.4 : Turbulent Measurement Results in the Longitudinal Axis with OLS

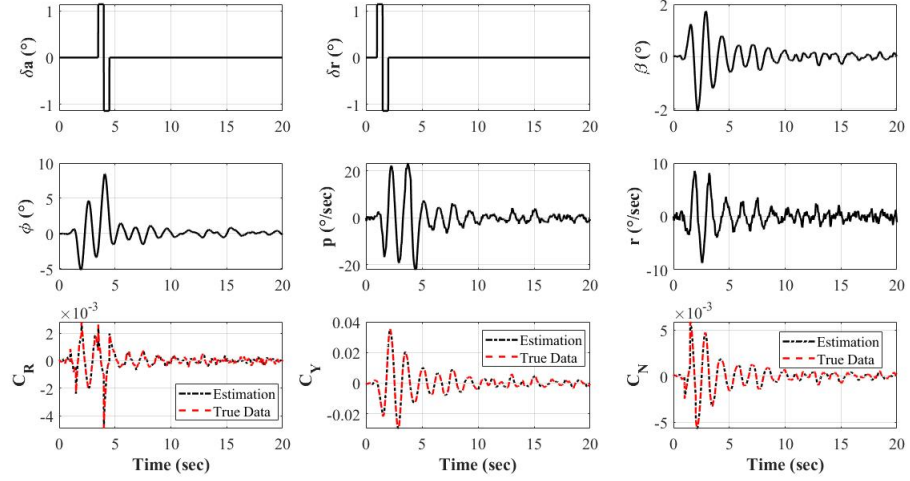


Figure 5.5 : Turbulent Measurement Results in the Lateral-Directional Axis with OLS

The results obtained for the actual and estimated values of the force and moment coefficients under conditions of active atmospheric turbulence clearly demonstrate that the Ordinary Least Squares (OLS) estimation method has once again yielded perfectly accurate results. The standard deviation values are remarkably close to those obtained under ideal conditions, indicating minimal deviation and high precision. Moreover, since all R^2 statistic values are equal to 1, this signifies perfect correlation and fit, further affirming the accuracy of the OLS method. Given this consistency in the results, there is no need to rewrite the values already presented in Table 5.1, as they effectively illustrate the reliability and robustness of the OLS estimation method under both ideal and turbulent conditions.

The reason why the turbulent model yields results that are very close to those of the ideal model is due to the nature of the generated atmospheric turbulence, which exhibits white noise (ϵ) properties. This type of noise has an expected value of zero ($E[\epsilon] = 0$), meaning that the random fluctuations introduced by the turbulence average out to zero over time. As a result, these fluctuations do not significantly affect the overall accuracy of the parameter estimates.

In this context, the expected value of Equation 4.8, which represents the parameter estimation process, can be expressed to reflect this inherent stability:

$$\begin{aligned} E[\hat{\beta}] &= E[(X^T X)^{-1} X^T Y] \\ &= E[(X^T X)^{-1} X^T (X\beta + \varepsilon)] = \beta + E[(X^T X)^{-1} X^T \varepsilon] \end{aligned} \quad (5.6)$$

The expected value of the aerodynamic derivative vector β , denoted as $E[\beta]$, is equal to β itself. This means that, on average, the estimated values of β will converge to the true values of the parameters.

$$E[\hat{\beta}] = \beta + (X^T X)^{-1} X^T E[\varepsilon], \quad E[\varepsilon] = 0, \longrightarrow E[\hat{\beta}] = \beta \quad (5.7)$$

At this stage, it is assumed that the measurement matrix X which contains the independent variables, is free from any errors. This assumption implies that the data used in the estimation process is both accurate and reliable, providing a solid foundation for the parameter estimation. Given this accurate and reliable data, the robustness of the Ordinary Least Squares (OLS) method is evident, even under conditions of turbulence. This robustness ensures that the estimates of the aerodynamic coefficients remain precise and reliable, despite the challenging conditions. Such performance demonstrates the OLS method's effectiveness in maintaining accuracy, highlighting its suitability for real-world applications where turbulent conditions are often encountered. This capability makes OLS a valuable tool in practical scenarios, ensuring consistent and trustworthy results in the presence of environmental variability.

In the third step, it is assumed that systematic sensor errors are present in the measurements. These erroneous measurement parameters are specifically considered to be in the sensors that measure linear accelerations, angular velocities, angle of attack, and sideslip angle within the aircraft's body axis system. The introduction of such errors can significantly impact the accuracy of aerodynamic parameter estimation, leading to potential deviations from the true values. To account for these inaccuracies, Δ error values of equal magnitude for the linear and rotational axes, as well as for the angle of attack and sideslip angle, are specified in the model. These error values are intentionally introduced to simulate the effects of systematic sensor inaccuracies, allowing for an assessment of how these inaccuracies influence the overall estimation

process. By understanding the impact of these errors, the robustness and resilience of the parameter estimation techniques can be evaluated, ensuring that the methods remain effective even when faced with imperfect measurement data.

$$\begin{aligned}\Delta a_x &= 1 \text{ m/sec}^2, & \Delta a_y &= 1 \text{ m/sec}^2, & \Delta a_z &= 1 \text{ m/sec}^2, & \Delta \alpha &= 1 \text{ deg} \\ \Delta p &= 1 \text{ deg/sec}, & \Delta q &= 1 \text{ deg/sec}, & \Delta r &= 1 \text{ deg/sec}, & \Delta \beta &= 1 \text{ deg}\end{aligned}\quad (5.8)$$

In Figures 5.6 and 5.7, sensor errors were introduced to the independent measurement parameters within the turbulent model, and the resulting aerodynamic coefficients were compared.

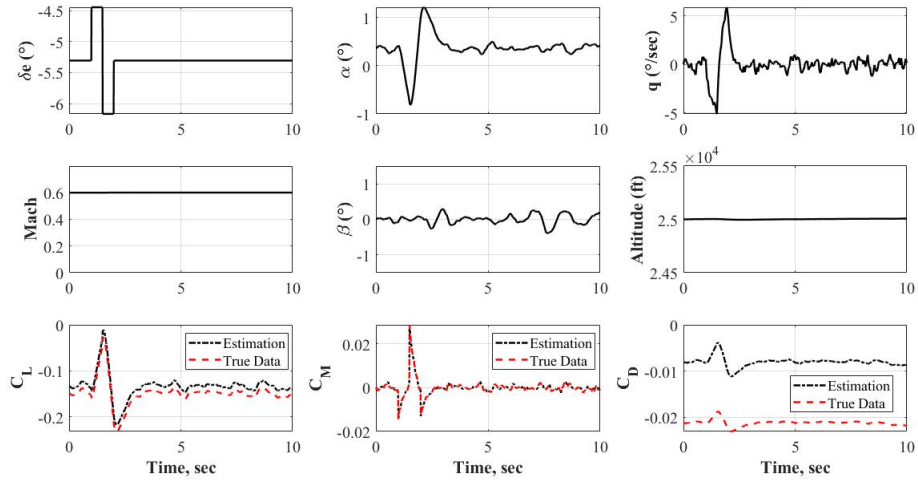


Figure 5.6 : Measurement Results with Turbulence and Sensor Errors in the Longitudinal Axis with OLS

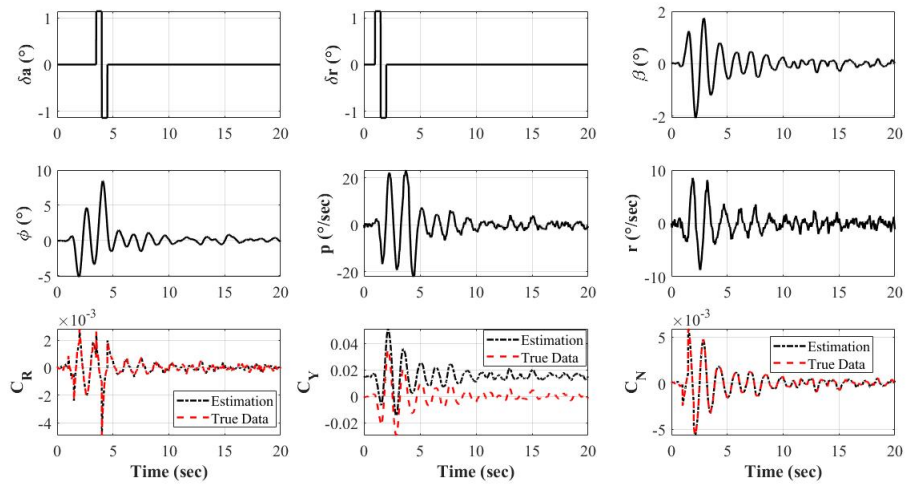


Figure 5.7 : Measurement Results with Turbulence and Sensor Errors in the Lateral-Directional Axis with OLS

This comparison was carried out to assess the impact of sensor inaccuracies on the estimated aerodynamic parameters. The comparisons were made with reference to the atmospheric turbulence model information detailed in Table 5.2 and Figure 2.5. These references provide the necessary context for understanding the conditions under which the comparisons were made, ensuring that the evaluation of the aerodynamic coefficients takes into account the specific characteristics of the turbulent environment and the introduced sensor errors. This approach allows for a comprehensive assessment of the robustness and accuracy of the parameter estimation methods in the presence of both turbulence and sensor inaccuracies.

Under conditions where sensor errors are also included, noticeable differences can be observed between the actual and estimated values, particularly in the force coefficients. The figures illustrate that the moment coefficients are less affected by sensor errors compared to the force coefficients. However, even if the differences between the actual and estimated values are not immediately apparent, the introduction of errors into the X vector, which contains the independent parameters, will still impact the aerodynamic derivative estimation results. In this context, the error-free X vector containing the measurement parameters can be expressed as follows.

$$X = X_m - \Delta X \quad (5.9)$$

Here, X_m represents the measured variables, while ΔX represents the systematic sensor measurement errors as defined in Equation 5.8. If Equation 4.4 is rewritten to account for atmospheric turbulence ε_t and systematic error ΔX , then

$$Y = (X_m - \Delta X)\beta + \varepsilon_t \quad (5.10)$$

Based on Equation 4.8, both sides of Equation 5.10 can be multiplied by $(X_m^T X_m)^{-1} X_m^T$.

$$(X_m^T X_m)^{-1} X_m^T Y = (X_m^T X_m)^{-1} X_m^T [(X_m - \Delta X)\beta + \varepsilon_t] \quad (5.11)$$

As seen in Equation 5.11, the left side is equal to the estimation vector $\hat{\beta}$. The right side of the equation can be algebraically rearranged as follows:

$$\begin{aligned} \hat{\beta} &= (X_m^T X_m)^{-1} X_m^T X_m \beta - (X_m^T X_m)^{-1} X_m^T \Delta X \beta + (X_m^T X_m)^{-1} X_m^T \varepsilon_t \\ &= \beta + (X_m^T X_m)^{-1} X_m^T (\varepsilon_t - \Delta X \beta) \end{aligned} \quad (5.12)$$

In Equation 5.12, the expected value of the atmospheric turbulence ε_t is zero ($E[\varepsilon_t] = 0$), and is independent of the β vector.

$$E[\hat{\beta}] = \beta - E[(X_m^T X_m)^{-1} X_m^T \Delta X] \beta \quad (5.13)$$

When comparing Equation 5.13 to Equation 5.7, it becomes evident that Equation 5.13 introduces bias. This bias results from the inclusion of systematic sensor errors in the X matrix, which contains the independent variables. Such errors directly impact the aerodynamic estimation values, leading to biased results in the estimation vector. As a result, even minor inaccuracies in the measurement parameters can cause significant deviations in the estimated aerodynamic derivatives. This underscores the critical importance of ensuring that measurement data is accurate and free from errors. Reliable and precise measurement data is essential for achieving accurate estimation results, as any systematic errors in the independent variables can substantially degrade the quality of the aerodynamic parameter estimates. Therefore, rigorous attention must be given to the calibration and validation of sensors to minimize systematic errors and ensure the integrity of the measurement data used in the estimation process.

As seen in Table 5.3, the standard deviations have increased compared to the ideal and turbulent conditions. According to the R^2 statistic, the drag coefficient is the parameter

Table 5.3 : OLS Results with Turbulence and Sensor Errors

Der.	True	Est.	R^2	σ	Der.	True	Est	R^2	σ
C_{L_0}	0.149	0.0333	0.7730	0.0158	C_{R_0}	0	0.002	1	1.34e-07
C_{L_α}	5.5	5.5133	0.7730	0.1240	C_{R_β}	-0.11	-0.11	1	6.26e-06
C_{L_q}	14.2	14.1813	0.7730	8.3116	C_{R_p}	-0.39	-0.39	1	2.56e-05
C_{L_u}	0.084	0.0829	0.7730	0.1557	C_{R_r}	0.28	0.28	1	3.65e-05
$C_{L_{\delta_e}}$	0.38	0.3791	0.7730	0.1582	$C_{R_{\delta_a}}$	0.1	0.1	1	5.75e-06
C_{M_0}	-0.08	-0.0735	1	1.04e-05	$C_{R_{\delta_r}}$	0.05	0.05	1	4.38e-06
C_{M_α}	-0.24	-0.24	1	8.18e-05	C_{Y_0}	0	0.0334	0.2207	0.0026
C_{M_q}	-27.3	-27.3	1	0.0055	C_{Y_β}	-1	-1.0131	0.2207	0.1219
C_{M_u}	0	0	1	1.03e-04	C_{Y_p}	-0.14	-0.1410	0.2207	0.4980
$C_{M_{\delta_e}}$	-0.88	-0.88	1	1.05e-04	C_{Y_r}	0.61	0.6085	0.2207	0.7099
C_{D_0}	0.0205	0.0034	0.0063	0.0084	$C_{Y_{\delta_a}}$	0	0	0.2207	0.1120
C_{D_α}	0.12	0.2072	0.0063	0.0847	$C_{Y_{\delta_r}}$	0.028	0.028	0.2207	0.0253
C_{D_u}	0.05	0.0517	0.0063	0.1345	C_{N_0}	0	-0.003	0.9871	1.97e-06
$C_{D_{\delta_e}}$	0	0.0025	0.0063	0.0878	C_{N_β}	0.17	0.17	0.9871	9.20e-05
$C_{N_{\delta_a}}$	-0.003	-0.003	0.9871	8.46e-05	C_{N_p}	0.09	0.0948	0.9871	3.76e-04
$C_{N_{\delta_r}}$	-0.12	-0.12	0.9871	6.44e-05	C_{N_r}	-0.26	-0.2596	0.9871	5.36e-04

most affected by the systematic error. This indicates that the drag coefficient's

estimation is significantly influenced by the presence of sensor inaccuracies. In contrast, the roll and pitch coefficients are the least affected by these systematic errors, demonstrating more robustness in their estimation. Consequently, as illustrated in Equation 5.13, the aerodynamic derivative estimates obtained using the Ordinary Least Squares (OLS) method exhibit biased results due to the introduction of systematic sensor errors. This highlights the importance of addressing sensor inaccuracies to ensure the reliability of aerodynamic parameter estimation.

5.2 Evaluation of RLS Method

In this section, where the aerodynamic derivative estimation results change dynamically, the steps performed in OLS will be followed. As an example of the estimation's dynamic change, Figure 5.8 shows how the aerodynamic derivative estimates of C_{D_u} change over time under turbulent conditions.

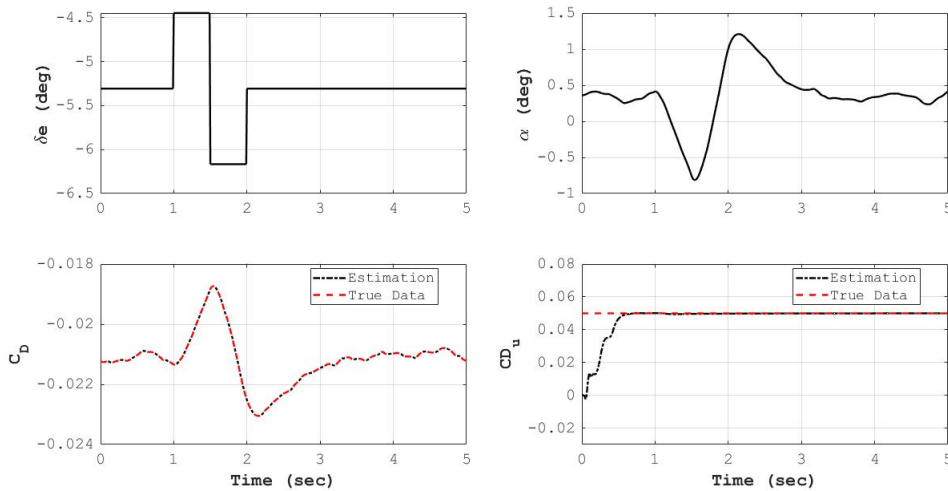


Figure 5.8 : An Example of Aerodynamic Derivative Estimation with RLS

As seen in the figure, the derivative estimation result converges to the true value in less than a second after a certain period. The elevator input is kept the same as in the OLS method.

In the previous section, it was concluded that there was no difference in the least squares estimation results between conditions with atmospheric turbulence and those with ideal measurements (Equation 5.7). This conclusion is also valid for RLS, and the results for the lateral-directional axes are shown in Figure 5.9. In the turbulent model

outputs shown in Figure 5.9, the intensity of the turbulence described in Table 5.2 has been increased ($10^{-2} \rightarrow 10^{-4}$). Additionally, the inputs used in the Ordinary Least Squares (OLS) method have been modified because differences were observed in the estimation results. The optimal inputs were determined through a process of trial and error, and the simulation outputs were generated as illustrated in Figure 5.9.

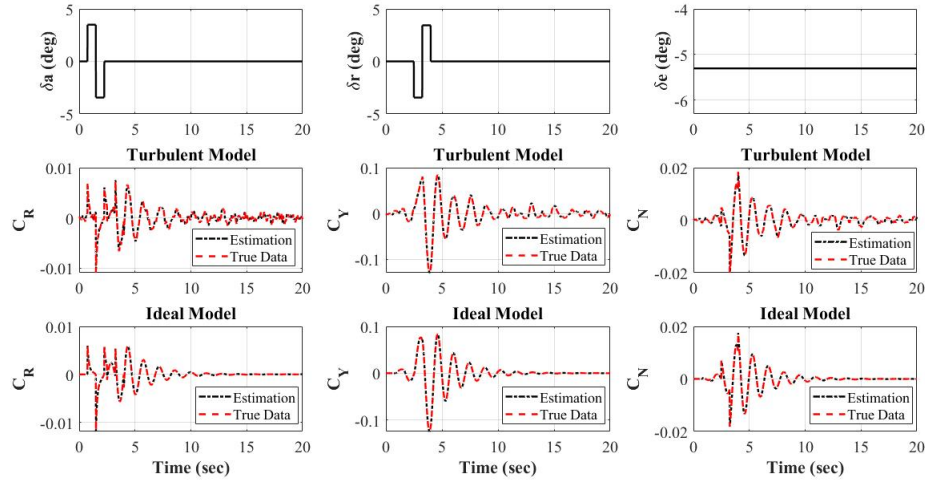


Figure 5.9 : RLS Estimation Performances under Ideal and Turbulent Conditions

Despite the increased intensity of atmospheric turbulence, the estimation values and the actual values of the force and moment coefficients align closely in both the ideal and turbulent models. This indicates that the method remains robust even under more severe turbulence conditions. The numerical values of the aerodynamic derivative estimation results are detailed in Table 5.4, further demonstrating the accuracy and reliability of the estimation process under varying conditions. In Table 5.4, K

Table 5.4 : Comparison of Estimation Results for RLS under Ideal and Turbulent Models

Der.	K	T	R^2	σ_K	σ_T	Der.	K	T	R^2	σ_K	σ_T
C_{N_0}	0	0	1	1.59e-09	1.42e-09	C_{R_0}	0	0	1	1.95e-09	1.75e-09
C_{N_β}	0.17	0.17	1	1.83e-07	1.49e-07	C_{R_β}	-0.11	-0.11	1	2.24e-07	1.84e-07
C_{N_p}	0.09	0.09	1	7.57e-07	6.33e-07	C_{R_p}	-0.39	-0.39	1	9.25e-07	7.82e-07
C_{N_r}	-0.26	-0.26	1	1.02e-06	8.21e-07	C_{R_r}	0.28	0.28	1	1.24e-06	1.01e-06
$C_{N_{\delta a}}$	-0.003	-0.003	1	1.74e-07	1.49e-07	$C_{R_{\delta a}}$	0.1	0.1	1	2.12e-07	1.83e-07
$C_{N_{\delta r}}$	-0.12	-0.12	1	1.36e-07	1.12e-07	$C_{R_{\delta r}}$	0.05	0.05	1	1.66e-07	1.38e-07
C_{Y_r}	0.61	0.61	1	2.32e-06	1.86e-06	C_{Y_0}	0	0	1	3.64e-09	3.22e-09
$C_{Y_{\delta a}}$	0	0	1	3.96e-07	3.37e-07	C_{Y_β}	-1	-1	1	4.18e-07	3.39e-07
$C_{Y_{\delta r}}$	0.028	0.028	1	3.10e-07	2.54e-07	C_{Y_p}	-0.14	-0.14	1	1.72e-06	1.44e-06

represents the ideal measurements, T represents the turbulent measurements, σ_K

represents the standard deviation of the ideal measurements, and σ_T represents the standard deviation of the turbulent measurements. The R^2 statistics are not given separately for each model because they are equal to 1 in both cases. The measurements were taken within the frequency range used in the Ordinary Least Squares (OLS) method, and the frequency value is 100 Hz. As shown in the table, the standard deviation values are similar for both the ideal and turbulent measurements, and the estimation results are perfectly accurate. The difference from the OLS method is that the standard deviation values are higher in the Recursive Least Squares (RLS) method. This indicates that at 100 Hz, the performance of OLS is better than that of RLS. Therefore, as mentioned in the OLS section, there is no need to present the estimation results of the two models in separate tables.

In the longitudinal axis analyses, the input design was not changed, and results were obtained at 100 Hz. The same inputs provided in the Ordinary Least Squares (OLS) method were used in this section as well. However, in the Recursive Least Squares (RLS) estimations, it was observed that some of the derivatives in the longitudinal axis differed from their true values. To investigate these differences, the sampling interval frequency was increased from 100 Hz to 50 kHz, and the results for a few samples are presented in Table 5.5.

Table 5.5 : Estimation Results with Different Frequencies

Derivative	100 Hz Samples	50 kHz Samples
C_{L_0}	0.144	0.149
C_{L_q}	10.4	14.2
C_{L_u}	0.069	0.084
$C_{L_{\delta_e}}$	0.33	0.38
C_{M_0}	-0.070	-0.080
C_{M_α}	-0.31	-0.24
C_{M_q}	-20.0	-27.3
C_{M_u}	0.0289	0
$C_{M_{\delta_e}}$	-0.78	-0.88

As seen in Table 5.5, when the sampling frequency is increased, the performance of the Recursive Least Squares (RLS) method improves, and the estimation results converge to the true values of the derivatives. An important reason for the convergence or divergence of RLS with respect to frequency is a coefficient called the forgetting

factor (λ), which was taken as 1 in the gain matrix formula given in Equation 4.20.

$$K(k+1) = \frac{P(k)x(k+1)}{\lambda + x^T(k+1)P(k)x(k+1)} \quad (5.14)$$

The forgetting factor λ , which appears in the denominator of the equation, is taken as a maximum value of 1. When λ is at its maximum value of 1, it implies that the method completely disregards the information from the previous step. Conversely, if the λ value is less than 1, it indicates that the estimation method takes into account the information from the previous steps in the data set. This consideration becomes particularly important in a data set with a frequency of 100 Hz. In such a scenario, completely ignoring the previous steps can significantly impact the value to which the method converges. Essentially, if the forgetting factor is set to 1, increasing the sampling frequency equates to increasing the number of data samples. This increase in sample size reduces the amount of 'forgotten' information loss in the intervals between samples. Consequently, with a higher sampling frequency, the method retains more information from the data set, leading to more accurate and reliable estimation results. Therefore, adjusting the forgetting factor and sampling frequency can play a crucial role in the performance of the estimation method. By carefully selecting these parameters, one can ensure that the method effectively balances between incorporating past information and adapting to new data, thereby enhancing the overall accuracy of the parameter estimation process.

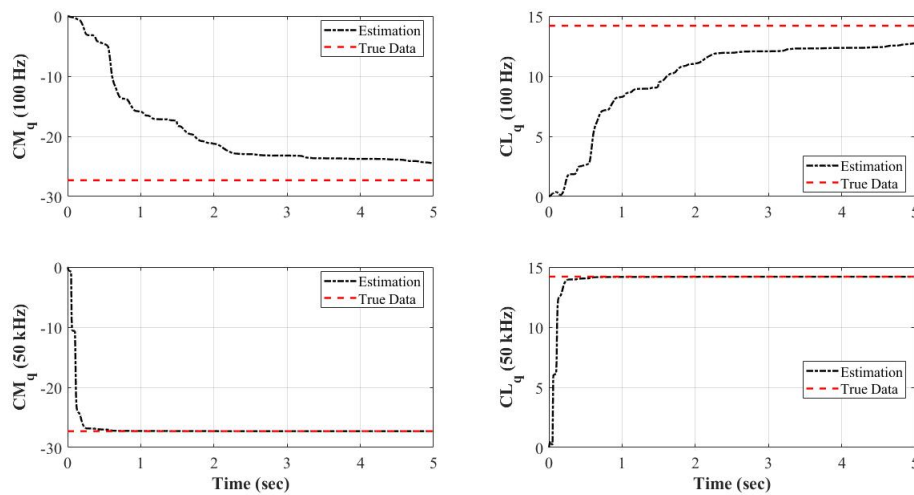


Figure 5.10 : RLS Estimation Performance at Different Frequencies

In Figure 5.10, the estimation performance of the Recursive Least Squares (RLS) method at 100 Hz and 50 kHz is shown specifically for C_{M_q} and C_{L_q} . The results show that for all estimated derivatives in the longitudinal axis that differ from their true values, the high-frequency measurements converge to their true values as shown in Figure 5.10. In contrast, the 100 Hz measurement results converge to a point different from their true values.

Up to this point, the RLS estimation results for both ideal and turbulent models have been shared. The discrepancies observed in the true values have been resolved by increasing the sampling frequency, and all estimation results have converged to the true values. In the third step, sensor measurement errors were added to the turbulent model, and the derivative estimation results are presented in Table 5.6.

Table 5.6 : RLS Results with Turbulence and Sensor Errors at 50 kHz

Der.	True	Est.	R^2	σ	Der.	True	Est.	R^2	σ
C_{L_0}	0.149	0.0333	0.8711	7.61e-04	C_{R_0}	0	0.002	1	9.03e-09
C_{L_α}	5.5	5.5134	0.8711	0.0058	C_{R_β}	-0.11	-0.11	1	4.17e-07
C_{L_q}	14.2	14.1659	0.8711	0.4178	C_{R_p}	-0.39	-0.39	1	1.71e-06
C_{L_u}	0.084	0.0829	0.8711	0.0156	C_{R_r}	0.28	0.28	1	2.48e-06
$C_{L_{\delta_e}}$	0.38	0.3789	0.8711	0.0076	$C_{R_{\delta_a}}$	0.1	0.1	1	3.72e-07
C_{M_0}	-0.08	-0.0734	1	5.22e-07	$C_{R_{\delta_r}}$	0.05	0.05	1	2.72e-07
C_{M_α}	-0.24	-0.24	1	3.99e-06	C_{Y_0}	0	0.0334	0.3573	0.0026
C_{M_q}	-27.3	-27.2	1	2.86e-04	C_{Y_β}	-1	-1.0130	0.3573	0.1219
C_{M_u}	0	0	1	1.07e-05	C_{Y_p}	-0.14	-0.1405	0.3573	0.4980
$C_{M_{\delta_e}}$	-0.88	-0.88	1	5.23e-06	C_{Y_r}	0.61	0.6087	0.3573	0.7099
C_{D_0}	0.0205	0.0034	0.012	0.0084	$C_{Y_{\delta_a}}$	0	0	0.3573	0.1120
C_{D_α}	0.12	0.2071	0.012	0.0847	$C_{Y_{\delta_r}}$	0.028	0.028	0.3573	0.0253
C_{D_u}	0.05	0.0509	0.012	0.1345	C_{N_0}	0	-0.003	0.9870	1.33e-07
$C_{D_{\delta_e}}$	0	0.0025	0.012	0.0878	C_{N_β}	0.17	0.17	0.9870	6.17e-06
$C_{N_{\delta_a}}$	-0.003	-0.003	0.9870	5.51e-06	C_{N_p}	0.09	0.0948	0.9870	2.54e-05
$C_{N_{\delta_r}}$	-0.12	-0.12	0.9870	4.03e-06	C_{N_r}	-0.26	-0.2597	0.9870	3.67e-05

When systematic sensor errors are introduced into the X vector, as illustrated in the table, a noticeable convergence in the estimation results of the Recursive Least Squares (RLS) method and the Ordinary Least Squares (OLS) method occurs. This convergence is clearly demonstrated in Table 5.3 and Table 5.6, where the results from both methods become strikingly similar. This similarity in outcomes suggests that the RLS and OLS methods are similarly influenced by the presence of systematic sensor errors, resulting in almost indistinguishable estimation outputs. The convergence

in results highlights that both methods respond to the systematic sensor errors in a comparable manner, ultimately leading to a reduction in their respective estimation accuracies.

Despite this, there are some critical distinctions to note between the two methods when dealing with data under these conditions. An increase in the sampling frequency reveals a significant advantage for the RLS method, which demonstrates superior performance compared to the OLS method. Specifically, the RLS method yields more optimistic results in terms of both standard deviation and R^2 statistics. This enhanced performance suggests that the RLS method is better equipped to handle higher-frequency data, where its dynamic updating capability allows for more precise and reliable parameter estimates. The ability of the RLS method to adapt more effectively to the high-frequency data underscores its robustness in such scenarios, where the precision of the estimates is crucial.

Moreover, the impact of the biased result described in Equation 5.13 is also apparent in the RLS estimation outputs. The bias introduced by the systematic sensor errors has a tangible effect on the accuracy of the estimated parameters, causing them to deviate from their true values. This bias is not exclusive to the RLS method; it similarly affects the OLS method. However, the RLS method's superior handling of high-frequency data helps to mitigate some of the negative effects of this bias. This suggests that, while both methods are adversely affected by systematic sensor errors, the RLS method's dynamic nature and adaptability provide it with an edge, particularly in scenarios involving high-frequency data sampling.

In conclusion, the introduction of systematic sensor errors affects both the RLS and OLS methods, leading to similar estimation results, as evidenced by the comparable outputs in Tables 5.3 and 5.6. However, the RLS method shows a clear advantage when the data sampling frequency is increased, offering more accurate and reliable estimates, despite the presence of bias introduced by the sensor errors. This makes the RLS method a more robust and effective choice for situations where high-frequency data is available, and precision is paramount.



6. CONCLUSIONS

In summary, The comparative analysis of OLS and RLS methods under various conditions (ideal, turbulent, and with systematic errors) highlighted the strengths and weaknesses of each method. The comprehensive analysis of aerodynamic parameter estimation using Ordinary Least Squares (OLS) and Recursive Least Squares (RLS) methods provided several key insights and findings. The study examined the performance of these methods under various conditions, including ideal, turbulent, and systematic error scenarios. The following points summarize the main conclusions drawn from this research:

6.1 Estimation of Aerodynamic Derivatives

In this study, a six degrees of freedom flight simulation model operating in a linear space was used to estimate aerodynamic derivatives. The model leveraged an aerodynamic database to simulate realistic flight conditions. Both Ordinary Least Squares (OLS) and Recursive Least Squares (RLS) methods were employed to perform the parameter estimations of the aerodynamic derivatives. The aerodynamic block within the model was designed to accurately reflect the true characteristics of the aircraft, ensuring that the estimation results could be compared against these known values for validation.

6.2 Performance in Ideal and Turbulent Models

The study involved a detailed analysis of the performance of both Ordinary Least Squares (OLS) and Recursive Least Squares (RLS) methods under ideal (error-free) and turbulent conditions. These conditions were chosen to evaluate the robustness and accuracy of the estimation methods across different scenarios that simulate real-world flight conditions. In the ideal model, which assumes perfect measurement conditions without any errors or disturbances, both OLS and RLS methods provided highly

accurate results. The estimation outputs closely matched the true values of the aerodynamic derivatives, demonstrating the efficacy of these methods in a controlled, error-free environment. When the analysis was extended to include turbulent conditions, where atmospheric disturbances are present, both methods still delivered accurate results. The turbulent model introduced realistic challenges by simulating the effects of atmospheric turbulence on the aircraft, thus testing the robustness of the estimation methods. The accurate results obtained under both ideal and turbulent conditions were supported and validated by the expected value formulas derived from the estimation equations. These formulas provided a theoretical foundation that confirmed the reliability of the estimation methods under different scenarios. Additionally, the study found that the introduction of white noise, characterized by an expected value of zero, into the estimation processes did not adversely affect the performance of either OLS or RLS methods. Both methods continued to yield results that converged closely to the true values, indicating their robustness against such noise.

6.3 Low-Frequency Analysis

When examining the performance of the methods at lower sampling frequencies, OLS was found to outperform RLS in both ideal and turbulent models. The analyses highlighted that the sampling frequency plays a critical role in the effectiveness of the RLS method. Specifically, lower frequencies resulted in less accurate RLS estimations compared to those achieved by OLS.

6.4 Importance of Input Design

The design of the input signals was identified as a crucial factor for the accuracy of both OLS and RLS estimation methods. For RLS, it was necessary to redesign the input shapes in the lateral-directional axes to ensure the estimation results were accurate and closely matched the true values. In the longitudinal axis, the input shapes were kept consistent for both OLS and RLS methods. However, discrepancies were observed in the RLS estimation results, which deviated from the true values. To address this issue, the sampling frequency was increased, resulting in improved performance of the

RLS method. With higher sampling frequencies, the RLS estimations converged more accurately to the true values.

6.5 Systematic Measurement Errors

The study also investigated the impact of systematic measurement errors (ΔX) on the estimation process. When these errors were introduced into the X vectors, both OLS and RLS methods produced biased outputs. To validate this observation, expected value formulas were developed by incorporating systematic errors into the X vector of the estimation equations. The model outputs, which included systematic measurement errors, were compared against these expected value formulas, confirming the presence of bias in the estimation results. The study's approach of comparing model outputs with expected value formulas provided a robust framework for identifying and understanding the sources of bias in the estimation process, offering valuable insights for improving the accuracy of aerodynamic parameter estimation in practical applications.

6.6 High-Frequency Analysis with Systematic Errors

The study explored the impact of high-frequency sampling on the performance of the Recursive Least Squares (RLS) method in the presence of systematic measurement errors. Systematic errors are persistent inaccuracies in sensor measurements that can significantly affect the accuracy of aerodynamic parameter estimation. It was found that when the sampling frequency was increased, the performance of the RLS method improved markedly. Specifically, at higher frequencies, such as 50 kHz, the RLS method produced more accurate and reliable estimation results despite the presence of systematic errors. This improvement is attributed to the fact that higher sampling frequencies reduce the interval between data points, which in turn minimizes the loss of information (forgetting factor). As a result, the estimation process becomes more robust, capturing more details and variations in the data that could otherwise be missed at lower frequencies. The study demonstrated that at high frequencies, the standard deviation and R^2 statistics for RLS showed better performance compared to the Ordinary Least Squares (OLS) method. This indicates that RLS is more

effective at handling high-frequency data, making it particularly suitable for real-time applications where rapid data acquisition and processing are critical. Furthermore, the high-frequency analysis revealed that the RLS method could mitigate the effects of systematic measurement errors more effectively than OLS. This is because the frequent updates in the RLS algorithm allow it to adapt quickly to changes in the data, correcting for systematic biases introduced by sensor errors. The enhanced performance of the RLS method at higher frequencies underscores the importance of considering the sampling frequency in the design and implementation of aerodynamic parameter estimation processes. It highlights that for applications requiring high accuracy and reliability, such as advanced flight control systems, increasing the sampling frequency can significantly improve the quality of the estimation results. The findings suggest that while both OLS and RLS methods are affected by systematic errors, the RLS method, with its adaptive capabilities and higher frequency sampling, offers a more resilient solution. This makes it a preferred choice for environments where measurement accuracy is critical, and sensor errors are a common challenge.

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CURRICULUM VITAE

Furkan Altınıřık:

EDUCATION:

- **B.Sc.:** 2015-2020, Istanbul Technical University, Faculty of Aeronautics and Astronautics, Astronautical Engineering
- **M.Sc.:** 2021-2024, Istanbul Technical University, Faculty of Aeronautics and Astronautics, Aeronautical and Astronautical Engineering

PROFESSIONAL EXPERIENCE AND REWARDS:

- 2021-Present, Flight Mechanics Engineer at Turkish Aerospace

PUBLICATIONS, PRESENTATIONS AND PATENTS ON THE THESIS:

- **Altınıřık F.**, Öznalbant Z., Acar, H., (2024). Uçak Aerodinamik Türevlerinin Altı Serbestlik Dereceli Uçuş Benzetim Ortamında En Küçük Kareler Yöntemi ile Kantitatif Analizi. *10. ULUSAL HAVACILIK ve UZAY KONFERANSI*, Eylül 18-20.