

ANALYSIS AND DESIGN OF ROCKING STRUCTURES

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ABSTRACT

ANALYSIS AND DESIGN OF ROCKING STRUCTURES

Due to their high dynamic seismic resistance, ancient columns survived the severe ground shaking through the ages. The high seismic resistance of ancient rigid columns standing unrestrained on a rigid base is due to the fact that these columns were designed by ancient architects based on an articulated mechanism. Due to the rocking working principle, recentering of these free-standing structures is achieved with gravity. Therefore, these “unconventional” rocking structures behave quite differently from conventional structures and may exhibit superior stability during earthquakes. In this dissertation, the dynamic response of ancient columns that are allowed uplift and rock motion when subjected to ground excitation was studied.

This thesis aimed at investigating the dynamic rocking response of rigid blocks subjected to a variety of seismic excitations by performing numerical methods and contributing to the understanding of the fundamental dynamics of rocking systems. After deriving linear and nonlinear form of differential equations of motion about pivoting points, O or O' , a set of numerical approaches were employed for the evaluation of the dynamic response of the rigid blocks. The formulation of analytical equations and numerical solution of the rigid rocking blocks to trigonometric pulses and real earthquake records is achieved using a routine adopted to the numerical computing MATLAB package. The energy loss due to the impact between the rigid block and the base during the rocking movement is included in the program as the “coefficient of restitution (COR)” as suggested by Housner (1963).

Following chapters proceeds with examining the effect of changes in ground motion forms on overturning acceleration spectra (OAS). The influence of frequency and amplitude of the ground acceleration function on the overturning thresholds of rigid bodies were investigated. With the aim to see the scale effect on overturning curve, samples with different sizes and slenderness ratios were considered. Numerical results revealed that the rocking response of rigid bodies significantly alters with amplitude and frequency content of the ground acceleration, slenderness ratio as well as the size parameter.

ÖZET

SALLANAN RİJİT YAPILARIN DİNAMİK ANALİZİ VE TASARIMI

Antik sütunlar, yüksek deprem dayanıklılıkları nedeniyle çağlar boyunca şiddetli yer sarsıntılarına karşı dayanıklı olmuştur. Rijit bir taban üzerinde serbestçe duran antik rijit kolonların sismik dayanımının yüksek olması, antik kolonların o dönemin mimarları tarafından mafsallı bir mekanizmaya dayalı olarak tasarlanması gerçeğinden kaynaklanmaktadır. Salınım çalışma prensibi nedeniyle, bu serbest duran yapıların yeniden merkezlenmesi yerçekimi ile sağlanır. Dolayısıyla bu "alışılmışın dışında" sallanan yapılar, geleneksel yapılardan oldukça farklı davranır ve deprem sırasında üstün stabilite sergileyebilirler. Bu tezde, yer sarsıntısına maruz kaldıklarında yükselmeye ve sallanma hareketine izin verilen antik sütunların dinamik tepkisi incelenmiştir.

Bu tez, sayısal yöntemler kullanarak çeşitli sismik uyarılara maruz kalan rijit blokların dinamik davranışını araştırmayı ve sallanan sistemlerin temel dinamiklerinin anlaşılmasına katkıda bulunmayı amaçlamıştır. Sabitleme noktaları (O veya O') etrafında doğrusal ve doğrusal olmayan diferansiyel hareket denkleminin türetilmesinden sonra, rijit blokların dinamik davranışının değerlendirilmesi için bir dizi sayısal yaklaşım kullanıldı. Sallanan rijit blokların trigonometrik darbelere ve gerçek deprem kayıtlarına karşı analitik denklemlerinin formülasyonu ve sayısal çözümü, MATLAB bilgisayar programı kullanılarak elde edilmiştir. Sallanma hareketi sırasında rijit blok ile taban arasındaki etkileşimden kaynaklanan enerji kaybı, Housner (1963) tarafından önerilen "geri sıçrama katsayısı (COR)" olarak programda yer almaktadır.

Yer hareketi formlarındaki değişikliklerin devrilme ivme spektrumuna (OAS) etkisi, sonraki bölümlerde ele alınmıştır. Yer hareketinin frekans içeriği ve genliği, rijit cisimlerin devrilme eşiği üzerindeki etkilerinin incelendiği bir çalışma gerçekleştirildi. Farklı boyut ve narinlik oranlarına sahip örnekler, devrilme eğrisi üzerindeki etkisini görmek amacıyla dikkate alınmıştır. Sayısal sonuçlar, rijit cisimlerin sallanma tepkisinin, yer ivmesinin genlik ve frekans içeriği, narinlik oranı ve boyut parametresiyle önemli ölçüde değiştiğini göstermiştir.



Dedicated to my Mother, Sıdıka Zeynep SARIGÜL

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LIST OF SYMBOLS

B	Base of rigid block
b	Half-base of rigid block
H	Height of rigid block
h	Half-height of rigid block
M_o	Moment about pivoting point
A, B	Integration constants
R	Radial distance from the centre of rotation O to the centre of gravity
p	Frequency parameter of rigid block
I_o	Moment of Inertia about pivoting points (O or O')
Δt	time interval
m	Total mass of rigid block
W	Total weight of rigid block
g	Gravitational acceleration
a_p	the amplitude of acceleration pulse
V_p	the amplitude of velocity pulse
t_p	Duration of the pulse
a_o	Amplitude of harmonic base acceleration
t	Time
f	Frequency
a_g	Minimum ground acceleration to initiate rocking motion
$\ddot{u}_g(t)$	Ground motion acceleration
$y(t)$	State vector of system
$\dot{f}(t), \dot{y}(t)$	Time derivative of state vector
p_1	Coefficient
p_2	Coefficient
p_3	Coefficient
h	Time step size
k_1	Slope value at the beginning of the interval, using y
k_2	Slope value at the midpoint of the interval, using y and k_1
k_3	Slope value at the midpoint of the interval, using y and k_2

k_4	Slope value at the end of the interval, using y and k_3
S_v	Velocity response spectrum
O or O'	Pivoting points
$\theta(t)$	Angle of the rotation of rigid block
$\dot{\theta}(t)$	Angular velocity of rigid block
$\ddot{\theta}(t)$	Angular acceleration of rigid block
$sgn(\theta)$	Sign of angular displacement
θ_o	Initial rotation of rigid block
$\dot{\theta}_o$	Initial angular velocity of rigid block
$\ddot{\theta}_o$	Initial angular acceleration of rigid block
θ_i	Angular displacement of rigid block at time step i
θ_{i-1}	Angular displacement of rigid block at time step $i-1$
$\dot{\theta}_{i-1}$	Angular velocity of rigid block at time step $i-1$
$\ddot{\theta}_{i-1}$	Angular acceleration of rigid block at time step $i-1$
$\theta_C(t)$	General solution of homogeneous differential equation
$\theta_P(t)$	Particular solution of the equation of motion
α	Slenderness angle of rigid block
v	Coefficient of restitution
μ	Coefficient of friction
$\dot{\theta}_1$	Angular velocity before collision
$\dot{\theta}_2$	Angular velocity after collision
β	Newmark's method parameter
γ	Newmark's method parameter
ω_p	Angular frequency of the pulse
ω	Angular frequency
Ψ	Phase angle

LIST OF ABBREVIATIONS

2D	Two Dimensional
3D	Three Dimensional
AFAD	Disaster and Emergency Management Authority
CG	Centre of Gravity of Geometric Shape
CoAM	Conservation of Angular Momentum
COR	Coefficient of Restitution
EOM	Equation of Motion
FEM	Finite Element Methods
KE	Kinetic Energy of System
NOS	Normalized Overturning Spectrum
ODE	Ordinary Differential Equation
OS	Overturning Spectrum
PE	Potential Energy of System
PEER	Pacific Earthquake Engineering Research Center
PGA	Peak Ground Acceleration
PGV	Peak Ground Velocity
PGD	Peak Ground Displacement
RK4	Fourth-Order Runge Kutta Method
SDOF	Single Degree-of-Freedom
Tot_E	Total Energy of System
UDEC	The Universal Distinct Element Code

1. INTRODUCTION

Historical buildings and modern buildings can exhibit differences in their dynamic response to various external forces, including factors like seismic events, wind loads, vibrations, and other dynamic loadings. The difference between this dynamic behavior has attracted the attention of many researchers about how ancient structures, especially those located in seismically active regions, have survived to the present day, and as a result, the majority of studies have focused on this point and understanding it. Particularly, ancient columns, found in historical structures and monuments, have demonstrated remarkable resilience in withstanding severe earthquakes throughout history. Their ability to survive such seismic events can be attributed to their rocking working principle. Ancient builders designed structural systems fundamentally different from modern ones. Their structures were articulated mechanisms that displayed negative stiffness and low damping. The resilience of these "unconventional" articulated structures is remarkable, as they have survived multiple strong earthquakes over their 2.5 millennia existence [1]. Figure 1.1 and Figure 1.2 illustrate the ancient columns, including examples of both framed and single column configurations [2].



Figure 1.1. The ancient city of Laodicea North Agora in Laodicea on the Lycus –Denizli [2].



Figure 1.2. Ancient city of Diocaesarea-Olba, Mersin [2].

Understanding the dynamic rocking behavior of ancient columns is essential in structural engineering and preservation efforts to assess the stability, performance, and safety of historical structures. Engineers analyse factors affecting the dynamic response such as the shape, size, material properties, and the characteristics of dynamic loads on the rigid columns. As a result, understanding these geometric characteristics of block and the details of ground motion may provide a framework for analysing the behavior of rigid blocks. By studying the dynamic rocking motion of ancient columns, engineers can evaluate the risk of structural failure, develop retrofitting strategies to enhance structural resilience, and ensure the long-term preservation of these historical artifacts. This knowledge helps in safeguarding ancient columns and structures against potential damage or collapse due to dynamic forces, contributing to the conservation of our architectural heritage.

This dissertation concerns with the analysis of the free-standing ancient rigid columns resting on a rigid foundation are widely seen in historical sites. The primary objective of this

study was to explore the dynamic rocking characteristics of rigid blocks to a variety of seismic excitations by performing numerical solution methods. This research aimed to enhance comprehension of the contributing to the understanding of the fundamental dynamics of rocking systems. The numerical analyses were carried out for rigid rocking blocks under trigonometric pulses and real earthquake records.

Housner's significant contribution in 1963 played a pivotal role in pioneering research on the potential for overturning of rectangular solid blocks undergoing ground motions near the source [3]. He investigated how the details of ground motions, particularly those close to the seismic source, can lead to significant destructive forces that have the potential to overturn rigid blocks [3]. By considering ground motion characteristics such as the frequency content, amplitude, and duration of ground motions, Housner's research provided valuable insights into the mechanisms by which near-fault ground motions can cause structures to overturn, leading to advancements in this field [3]. Near-fault ground motions can be particularly destructive because these type of ground motions often have high levels of strong shaking with distinct pulses of energy caused by the rupture of the fault itself. In this dissertation, we expanded upon Housner's ground-breaking research to explore the potential for structures to overturn due to ground motion near the seismic source [3].

Housner introduced the overturning curve of a rectangular stand-alone block under the influence of a half sine pulse [3]. Similarly, Makris and Roussos [4] describe a set of cycloidal-shaped pulses that are suitable for characterizing near-fault ground motions. Makris and Vassiliou [5] demonstrated that the impact of a near-field seismic events, impulsive earthquake can be simulated effectively using a single rectangular pulse by selecting the appropriate pulse duration. The main motivation of this thesis stemmed from the necessity to understand the fundamental dynamics of rocking systems under pulse-like excitations.

This thesis aimed at developing rigorous and simplified numerical models of the problem to realistically simulate the behavior of free-standing rigid columns under a variety of ground excitation forms. The first part of this thesis is devoted to the investigating the rocking response

of rigid body cases under the trigonometric pulses. A set of trigonometric pulses involving one-cycle sine wave, half-cycle sine wave, rectangular constant acceleration pulse, Haar wavelet pulse, antisymmetric Ricker acceleration pulse and symmetric Ricker acceleration pulse, respectively were employed for the evaluation of the dynamic response of the rocking rigid columns. For each case, the influence of frequency content and amplitude of the ground acceleration function on the overturning thresholds of rigid bodies were examined. The second part of the chapter is devoted to the evaluation of overturning acceleration spectrums (OAS) of the variation of ground excitation forms. The influence of frequency and amplitude of the ground acceleration function on the overturning thresholds of rigid bodies were investigated. With the aim to see the scale effect on overturning curve, samples with different sizes and slenderness ratios were considered. Results revealed that the rocking response of rigid bodies significantly alters with amplitude and frequency content of the ground acceleration, slenderness ratio as well as the size parameter. Changes in size, and slenderness ratio are compared through the relevant plots and graphics.

Furthermore, a case of monolithic free-standing column involving the numerical modelling of the column of Temple of Olympian Zeus in Syracuse and evaluation of its response under devastating the El Centro earthquake (M_w 6.9) and 2023 Kahramanmaraş earthquake (M_w 7.7) was presented. The outcomes of this thesis indicate that the performance of the rigid bodies is highly sensitive to the variation of size (half-diagonal distance), R , and slenderness ratio, α , and details of ground motions.

Lastly, within the scope of this thesis, a case study was conducted on the Cathedral of the Annunciation, a structure that experienced severe damage during the catastrophic 2023 Pazarcık, Kahramanmaraş earthquake sequence. In order to understand the dynamic behavior of the Cathedral of the Annunciation during the earthquake, this research modeled and evaluated the front wall. This wall, which remained standing despite the seismic event, demonstrated the potential for rocking motion within its plane, a factor crucial to analyzing its structural response.

1.1. Objectives and Scope of the Study

The purpose of this doctoral dissertation research is to understand the dynamic rocking behavior of ancient columns which is essential in structural engineering and preservation efforts to assess the stability, performance, and safety of historical structures. Within the research, the dynamic response of the rocking rigid bodies was obtained through the examination of both the nonlinear and linear equations of motion in rocking structures employing two main numerical methodologies. The aforementioned equations are incorporated into MATLAB [6] and solved numerically.

The study revealed the sensitivity of the system's parameters, such as the dimensions and slenderness of the block. Moreover, this study indicated a correlation between the characteristics of ground motion and the likelihood of specific levels of rotation leading to overturning. An iterative-based numerical method for approximating solutions of nonlinear and linear equations was utilized for obtaining the rocking response of single rigid blocks. Newmark- β integration method with average acceleration was used for numerical integration and the 4th-5th order Runge-Kutta iterative procedure was used to solve the nonlinear and linear equations of motion. The study entails analyzing the free and forced rocking motion of a slender rigid rectangular block, taking into account varying block's geometric parameters and the different characteristics of ground motion. Numerical solutions of the linearized and nonlinear equations of motion utilizing Newmark- β and Runge-Kutta integration methods were compared to each other.

The dynamic behavior of solid blocks is highly sensitive to distinct long-period pulse, high-velocity pulses, and high level of accelerations that are typical characteristics of near-fault ground motion. Thus, in this research, near-fault ground motions were selected as the external seismic excitation instead of far-fault ground motions due to the presence of significant velocity pulses and high accelerations inherent in near-fault motions. Understanding how near-fault ground motion can induce the overturning of a rigid block is very crucial for evaluating the response to dynamic loading and seismic vulnerability of rocking structures to overturning. Consequently, to simulate the crucial attributes of near-fault earthquake records, pulse-like excitations were generated within the framework of this study. Subsequently, a majority of the

numerical analyses in this research focused on scenarios involving one-cycle sine wave, half-cycle sine wave, rectangular constant acceleration pulse, Haar wavelet pulse, antisymmetric Ricker acceleration pulse, and symmetric Ricker acceleration pulse, respectively.

Furthermore, a scenario was examined involving the numerical simulation of a monolithic stand-alone column representing the column of the Temple of Olympian Zeus in Syracuse. The study assessed its rocking behavior when subjected to the severe El Centro earthquake (M_w 6.9) and the 2023 Pazarcık, Kahramanmaraş earthquake (M_w 7.7). This thesis presents a case study of the Cathedral of the Annunciation, which sustained significant damage during the 2023 Pazarcık, Kahramanmaraş earthquake sequence. The front wall of the cathedral, which remained standing despite the earthquake and exhibited potential for rocking motion within its plane, was modeled and evaluated as part of this study.

This dissertation consists of a total of six chapters. The Chapter 2 provides a comprehensive literature review focusing on research and investigations related to evaluating the response of rocking rigid blocks under seismic excitations. The review encompasses the primary analytical, experimental, and numerical methodologies that have advanced this field. Early analytical investigations primarily put forth analytical formulations for predicting the oscillatory response of free-standing rigid blocks under simple harmonic base acceleration or various types of pulse-like excitations. In these analytical and experimental studies, the main objective was to understand the impact of block geometric properties and ground motion characteristics on the rocking behavior of the rigid block. Subsequent numerical investigations primarily concentrated on the dynamic response of historic structures (e.g., temples, columns, monuments, etc.) situated in earthquake-prone areas like Greece, Italy, and Turkey. The majority of experimental investigations carried out so far have aimed to validate the findings of analytical and numerical studies.

Within Chapter 3 of the thesis, details are provided on deriving the linear and nonlinear equations of motion that represent the rocking motion of an individual rigid block. The section provides a detailed information on the assumptions made in deriving the linearized form of the

equations of motion. Furthermore, the nonlinear and linear equations of motion with their analytical solutions were presented in this chapter.

In Chapter 4, all numerical analyses of both free and forced rocking motion were conducted for a particular slender rigid block using two integration techniques: Newmark- β method and the Runge-Kutta method. Initial parts of the chapter present the results of dynamic response in terms of angular displacement and angular velocity. For validation purpose, the plots and figures showing the rocking response results are graphically compared with each other. Therefore, through the comparison of numerical outcomes obtained from Newmark's time method and the Runge-Kutta method, it was understood which method yielded more accurate results. Second part of the chapter is devoted to the generation of overturning spectrums (OS). To achieve this, three rectangular rigid blocks having slenderness angle, $\alpha=10^\circ$, 20° and 30° were analysed under one-cycle sine, half-cycle sine wave, rectangular constant acceleration pulse, Haar wavelet, Antisymmetric Ricker pulse and Symmetric Ricker pulses, respectively. In the scope of the nonlinear approximation, the criteria for a rigid block to undergo overturning were defined, and the equation determining the minimal acceleration magnitude necessary to induce overturning was formulated. Following this, several cycloidal-shaped waveforms were utilized, and their similarity to recorded near-field seismic excitations was demonstrated. This chapter also includes the results obtained from analysis cases involving the numerical modeling of a monolithic stand-alone column symbolizing the column of the Temple of Olympian Zeus in Syracuse. All analyses were carried out to obtain the rocking response of monolithic ancient column under the severe El Centro earthquake (M_w 6.9) and the Pazarcık, Kahramanmaraş earthquake (M_w 7.7). The analyses proceed by integrating the E-W and N-S components of selected near-source acceleration records captured at station 3144, 4614, and NAR during Pazarcık, Kahramanmaraş earthquake (M_w 7.7). The potential for overturning associated with these motions was analysed using pulse-like excitation forms that can symbolize the reverse effects of near fault ground motion. The oscillatory response of ancient rigid column undergoing cycloidal pulses and near-fault ground motions was computed with a nonlinear formulation.

In Chapter 5, a case study was conducted within the scope of this thesis where the Cathedral of the Annunciation in İskenderun which suffered severe damage during the catastrophic 2023 Pazarcık, Kahramanmaraş earthquake sequence. The study focused on modeling and evaluating the front wall of the cathedral, which, despite experiencing the earthquake, survived with the potential for rocking motion within its plane. This wall was selected for analysis due to its unique behavior during the seismic event.

Chapter 6, the final chapter of the thesis, present the concluding remarks and main findings of this study along with discussions on future work.

2. LITERATURE SURVEY: DYNAMIC ROCKING RESPONSE OF RECTANGULAR RIGID BODIES UNDERGOING A VARIETY OF GROUND EXCITATION FORMS

2.1. Introduction

This chapter examines the existing literature and research conducted on the understating and description of the dynamic behavior of rocking rigid bodies experiencing numerous high-intensity seismic events. The dynamic behavior of rigid-block structures positioned on a firm foundation and subjected to ground excitation has been a subject of investigation over the last four decades. Rocking is perhaps the most commonly observed dynamic behavior of ancient structures such as Greek and Roman columns, obelisks, tombstones, monolithic statues, historic masonry walls and buildings subjected to seismic activities. Additionally, this type of motion is observed dynamic response of industrial and laboratory equipment such as concrete blocks used in nuclear reactors for radioactive shielding, heavy machinery and equipment installed in factories or laboratories, large precast concrete panels used in construction. Moreover, modern engineering structures such as seismic isolation systems using rocking mechanisms, precast concrete structures designed to exhibit controlled rocking during earthquakes can exhibit rocking motion when subjected to seismic excitations. Over the past two decades, there has been a growing emphasis among researchers on the study of the rocking of rigid blocks, encompassing both theoretical and experimental advancements as well as its applications in civil engineering.

Despite the lack of advanced computational tools and software, initial efforts in developing response evaluation methods were constrained to analytical and experimental models. Section 2.2 of this chapter provides a brief chronology of the analytical approaches developed for rocking structures under seismic excitations. Section 2.3 reviews the outstanding paper published by Housner [3]. Advancements in computer technology capable of accurately capturing complex responses have shifted research interest towards numerical models. These

numerical studies are discussed in Section 2.4. The final section (Section 2.5) of this chapter reviews experimental studies on this subject.

2.2. Early Studies based on Analytical Methods

Earliest analytical investigations into the dynamic behavior of stand-alone rigid blocks were presented by [7]. His study focused on the intensity of ground shaking that results in the tipping of a stand-alone column. Findings of Milne's study indicated that when the ground acceleration, $\ddot{u}_g$, surpasses the multiplication of gravitational acceleration, g , and the ratio of base and height (b/h) , the column overturns [7]. In other words, according to him, uplifting ground acceleration value of a rigid body is enough to overturn it [7]. However, one major drawback of his study is to use equivalent static analysis for a rocking rigid block to determine a minimum overturning acceleration amplitude value.

Another earliest study conducted by Kirkpatrick [8] aimed at finding main critical parameters that affect seismic resistance in columns prone to rocking. In his seminal paper, two critical parameters other than peak ground acceleration (PGA) of ground motion have great influence on the dynamic rocking response of a slender, stand-alone column. Findings of his study can be summarized as follows [8]:

- a) The first parameter: the size of column significantly affects the column's response to dynamic forces since it will be included in dynamic motion equation of a column via moment of inertia.
- b) The second parameter: time interval of ground motion.

After Milne's work [7], Kirkpatrick was successful in deriving the first spectrum showing the minimum accelerations necessary to topple a stand-alone column under harmonic base excitation and indicated that the influence of period of ground excitation on the level of acceleration which is necessary to topple a slender column cannot be underrated [8].

Following two great earthquakes in Japan, 1946 Nankai earthquake and 1949 Imaichi earthquakes, Ikegami and Kishinouye [9, 10] published two significant papers which unknowingly validate the theoretical findings of Kirkpatrick [8] on the response of a stand-alone column to dynamic loading. According to their field investigations after earthquakes, they were not able elucidate the factors contributing to the stability of gravestones (see Table 2.1) even if they had small value of static uplifting acceleration $(g b/h)$. In their 1950 study, Ikegami and Kishinouye noted the unexpected stability of tall, rectangular gravestones, suggesting that their large dimensions were responsible for preventing overturning [9, 10].

Table 2.1. The data obtained from field observations indicates the overturned and non-overturned grave-stones in Kainen and Kinomoto cities [10].

Place		Width	Height	α	a
Kainan-shi	Overtured	13 cm	43 cm	0.293	22.3 cm
	Non-overtured	24	115	0.218	55.3
Kinomoto-shi	Overtured	17	44	0.449	23.4
	Non-overtured	31	168	0.189	85.5

Thereafter, Housner [3] and Muto et al. [11] highlighted the fact that a simple static analysis for establishing overturning criteria under dynamic loading is not enough and requires a dynamic investigation. In his own words, Muto et al. [11] write “The actual seismic force application to the structure is not a statical force but a dynamical force. The overturning of large structures during even severe earthquakes are unheard of phenomena. Therefore, we found it advisable to make a dynamic study of the overturning problem so as to determine at what condition structures overturn and also to derive some useful data for aseismic design.”. Similarly, Housner [3] stating that the concept of using static lateral forces to represent earthquake effects is "misleading".

The concept of equivalent viscous damping was utilized by [12] to model the impact mechanism during rocking motion. They also developed computer programs for numerical analyses. In other words, Psycharis and Jennings [12] conducted a study on the uplift phenomenon of rigid bodies resting on viscoelastic foundations.

Spanos and Koh [13] investigated stand-alone blocks resting on rigid ground subjected to harmonic ground excitations. Their findings indicate that rocking rigid blocks exhibit greater resistance to toppling under high-frequency harmonic excitations compared to low-frequency harmonic excitations [13]. In an earlier study, Ishiyama [14] established the conditions for overturning by maintaining constant acceleration while varying excitation frequencies. Conversely, Spanos and Koh [13] delineated safe and unsafe regions on the excitation frequency versus excitation amplitude plane. Ishiyama [14] also proposed that both ground velocity and ground acceleration must be considered when assessing the dynamic response of rocking bodies.

The work conducted by Sinopoli [15] was among the pioneering endeavours developed to understand the problem of impact between rigid bodies through kinematic energy approach. In the analytical and numerical studies, monolithic columns and multi-block columns were considered. One of the main purpose of the paper was to show that, even if the pre-motion of the column is only rocking, after the impact, two different kinds of motion are possible [15]:

- Roto-translations of the rigid blocks,
- Slidings

For monolithic column, these two possible motions depend on the dimensional ratio between the height and the base diameter of the column. For a rocking column made of a number of stocky blocks, it depends on the same dimensional ratio [15].

Hogan [16, 17] is conducted an analytical study using the model of Housner [3] to reveal the behavior of a rigid block under horizontal simple harmonic forcing. Hogan [16,17] extended the study by further elucidating the mathematical framework of the issue through the introduction of orbital stability and Poincaré sections. Tso and Wong [18, 19] also investigated the steady-state dynamic behavior of rigid blocks through analytical and experimental methodologies. Their experimental research offered significant corroboration for the theoretical conclusions.

Rocking structures are widely recognized for their heightened susceptibility to prolonged ground motions, specifically due to the sustained presence of seismic pulses. To assess the safety of rocking blocks, Housner introduced the overturning curve, which plots acceleration as a function of duration, initially for a half-sine wave and a single rectangular pulse input [3]. Subsequent researchers expanded this concept to include other pulse shapes [5], [14], [20-28] and harmonic shaking [13], [29]. For complex signals, the overturning curve may exhibit bays and islands [5], [23-25], [27], [30]. Researchers conducted further studies on rocking behavior [31-39].

Researchers have conducted studies on overturning using synthetic earthquake records generated from white noise [3], [14], [24], [40, 41] and real earthquake records [5], [14], [24, 25], [42-44]. Makris and Vassiliou [5] demonstrated that the effect of a near-fault, pulse-like earthquake could be approximated by a single rectangular pulse input with an appropriately selected pulse interval. Ther and Kollár showed that the fullness of the replacement pulse and the presence of secondary pulses significantly impact the overturning curve [30]. The overturning curve can be extended to other signal waveforms [5], [23-27], [42].

2.3. An Examination of Housner's (1963) Groundbreaking Study on Rigid Block Response

Housner [3] was the first who comprehensively investigated the dynamic behavior of a single block exposed to a variety of seismic excitation. Figure 2.1 depicts the rectangular rigid block referenced by Housner [3]. Housner's ground-breaking publication in 1963 initiated a sequence of systematic investigations into the dynamic behavior and stability of structures prone to rocking [3]. These efforts culminated in the advancement of seismic isolation using rocking motion, an alternative solution for seismic resistance of tall, slender structures. Housner carried out analytical studies on the rocking dynamics of solid rectangular blocks based upon the three assumptions. The followings are the main assumptions of this study [3]:

- The rocking block and the ground are modeled as rigid,
- The coefficient of friction (COF) at the block-ground interface is high enough to preclude any slipping during motion,
- The collision between the ground and the block is modeled as perfectly inelastic, ensuring no bouncing occurs during rocking.

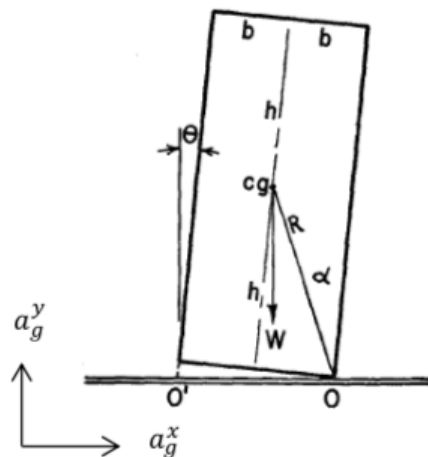


Figure 2.1. Housner's rocking block [3].

In his seminal paper, he firstly derived an equation of motion that describes the free-vibration of the rocking block. As shown in Figure 2.2, then he plotted a graph of free rocking equation of motion where the period of vibration, T , which is needed to complete one cycle of rocking motion plotted versus amplitude ratio θ_0/α and deduced that the period is strongly dependent upon the ratio of initial tilt angle, θ_0 , to slenderness angle, α , [3].

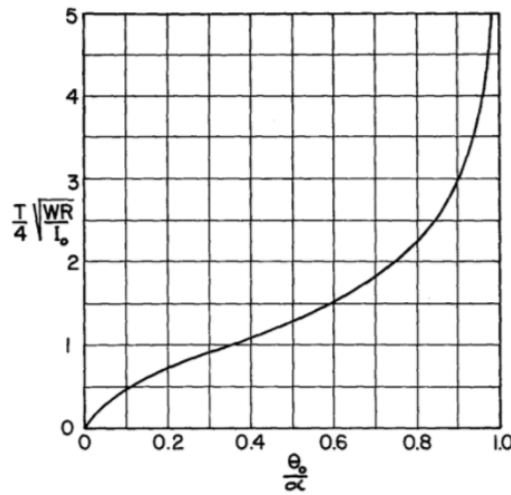


Figure 2.2. Period of the rocking block T with amplitude ratio (θ_0/α) plotted by Housner [3].

Housner [3] also considered that during the free rocking motion there is a dissipation of energy when the block impacted on the base. Accordingly, Housner [3] proposed the following geometry-dependent coefficient of restitution expression:

$$r = \left[1 - \frac{mR^2}{I_0} (1 - \cos(2\alpha)) \right]^2 \quad (2.1)$$

Additionally, Housner derived minimum overturning acceleration mathematical expressions for a rigid block subjected to three form of excitations such as constant acceleration, a single sine pulse and an earthquake type excitation, respectively [3].

According to Housner [3], the conditions for overturning under rectangular constant acceleration pulse of duration t_p , as illustrated in Figure 2.3, which can be expressed as follows:

$$\cosh \sqrt{WR/I_0} t_p = 1 + \frac{1}{2 \frac{a}{g\alpha} \left(\frac{a}{g\alpha} - 1 \right)} \quad (2.2)$$

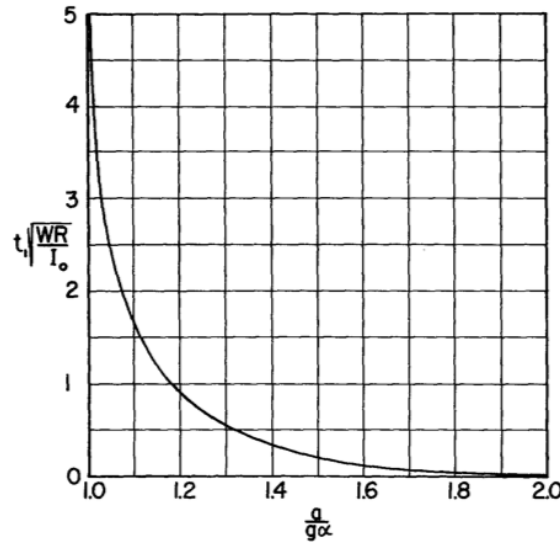


Figure 2.3. Overturning condition plotted as a function of constant acceleration a_p and duration of pulse t_p (or t_1) [3].

According to Housner [3], the conditions for overturning under a half-cycle sine pulse of duration t_p , as illustrated in Figure 2.4, can be written in the form

$$\frac{a_p}{\alpha g} \approx \sqrt{1 + \left(\frac{\omega_p}{p} \right)^2} \quad (2.3)$$

or

$$\frac{a_p}{\alpha g} \approx \sqrt{1 + \frac{I_0}{WR} \left(\frac{2\pi}{t_p} \right)^2} \quad (2.4)$$

Here, a_p refers to maximum acceleration amplitude will overturn the block. For large values of $\omega/p > 3$, the equation can be represented by

$$\frac{a_p}{\alpha g} \approx \frac{\omega_p}{p} \quad (2.5)$$

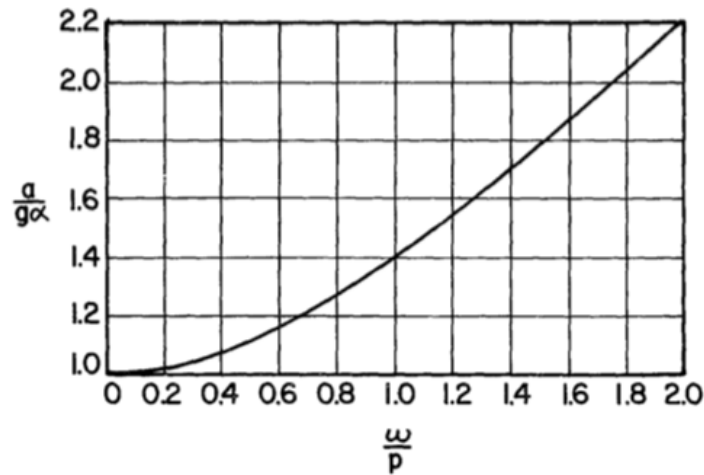


Figure 2.4. Overturning condition for a block under half-cycle sine pulse dependent on acceleration pulse a_p and frequency ratio $\left(\omega_p/p\right)$ [3].

Employing an energy-based method, Housner [3] put forth ingenious, rough analysis of overturning of blocks by earthquake ground motion. His approach assumes that the seismic stability of a rocking rigid block can be determined by comparing the energy input which is computed from the velocity response spectrum, S_v , of the earthquake ground motion and the energy necessary for the overturning of a rigid block [3]. The followings are the main assumptions of this study:

- The earthquake ground motion was idealized as white noise,
- Assuming the energy dissipation of rocking block is ignored,
- And linear form of equations of motion were used.

The overturning condition for a rigid block under earthquake like series of pulses can then be expressed in the form of

$$\frac{1}{2}WR\alpha^2 < \frac{1}{2}\frac{W}{g}\frac{MR^2}{I_0}S_v^2 \quad (2.6)$$

or

$$\alpha = \frac{S_v}{\sqrt{gR}}\sqrt{\frac{MR^2}{I_0}} \quad (2.7)$$

For tall, slender rocking rigid blocks, the above Equation (2.7) can be represented by

$$\alpha < \frac{S_v}{\sqrt{gR}} \quad (2.8)$$

If the value of slenderness angle, α , is greater than the right side of Equation (2.8), the probability of overturning the block is less than 50%. The outcomes of this Equation (2.8) offers some significant insight regarding the scale affect which makes the larger of two similar blocks having same slenderness ratio more stable than the smaller block during an earthquake.

2.4. Studies since 1963 based on Numerical Methods

In 1980, Yim, Chopra, & Penzien conducted a numerical study to determine the rocking dynamics of a solid block exposed to a several of the earthquake ground motions [20]. In their numerical analysis, a numerical procedure and computer program were developed to solve nonlinear equations of motion of rigid blocks resting on a rigid horizontal base. In their numerical analysis, a fourth order Runge-Kutta integration scheme was used. In order to prove the accuracy of the computer program, all the numerical results were compared with the analytical results for rigid blocks under single pulse excitation and compared with the experimental results of rigid block subjected to earthquake type excitations. As a result, accurate dynamic response of rigid blocks requires the utilization of high-order numerical integration scheme. In addition to previous studies, here, horizontal and vertical ground motions were considered as seismic excitations [20].

According to Yim et al. [20], the vertical component of seismic excitation significantly affects the rocking behavior of solid blocks. Yim et al. [20] deduced that the time-varying behavior of solid blocks is very sensitive to slight changes in geometric characteristic of rigid block, particularly in its size and slenderness ratio, and to properties of ground motion. Remarkable research conducted by Yim et al. [20] showed that after performing probabilistic analysis, the probability of a block exceeding any response level as well as the probability of overturning of the block increases due to increase in slenderness ratio (α) of the block, decrease in size (R) and increase in ground motion intensity measure, as illustrated in Figure 2.5.

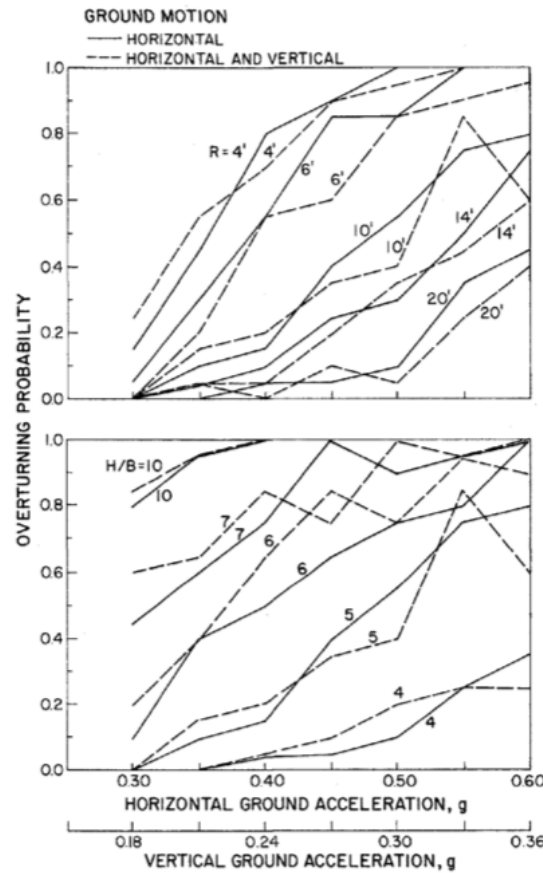


Figure 2.5. The probability of overturning of rigid blocks, all having same size parameter (R) but varying slenderness ratio (H/B) subjected to horizontal and vertical component of ground motion [20].

Numerous numerical and experimental studies have examined ancient structures such as temples, columns, and monuments, revealing their remarkable seismic resilience. This durability is crucial, particularly in Greece's seismically active regions, where these structures continue to stand today. In 2003, Psycharis, Lemos, Papastamatiou, Zambas, and Papantonopoulos conducted a detailed numerical investigation into the proposed restoration of the Parthenon Pronaos [45]. The study utilized the discrete element method (DEM) incorporating frictional joints to model the multi-element structure, including multi-drum columns and the epistyle. Their analysis primarily examined the deformations and failure modes of the multi-drum columns under seismic loading conditions [45]. Subsequent studies have further explored ancient columns or monuments, with a specific focus on their structural characteristics and historical significance [46-56].

Accurate fragility assessments, which are essential for estimating seismic vulnerability and informing decisions regarding earthquake-resistant design and retrofitting, necessitate a thorough understanding of rocking motion in columns. Numerous researchers have conducted studies in this field [55], [57-63].

Other researchers employed experimental tests alongside advanced Finite Element Methods (FEM) to examine the necessary criterion for overturning rigid blocks [23], [64-66]. While FEM can analyze challenges related to certain discontinuities, it is unsuitable for discontinuous systems characterized by continuously changing contact conditions among constituent bodies [67]. In contrast, Discrete Element Methods (DEM) were specifically developed for systems comprising distinct bodies capable of free movement and interaction through contact forces, offering automatic and efficient contact recognition.

Recent research utilizing licensed DEM software applications has demonstrated the effectiveness of DEM in analyzing masonry structures prone to joint failure as the primary collapse mode [45], [68-76]. Today, numerous computational tools based on DEM can be employed for seismic evaluation of masonry structures with UDEC [77] developed by Itasca

being the commonly utilized DEM code. Comprehensive and detailed reviews on the modeling of masonry structures can be found in the works of [47, 48], [72], [78-85].

2.5. Studies since 1963 based on Experimental Methods

Muto et al. [11] carried out study on dynamic rocking response of a column, comprising analytical and experimental analyses. Muto et al. [11] expanded upon the research initiated by Ikegami and Kishinouye [9-10], demonstrating that the time-varying response of a rocking column is influenced by negative stiffness. Consequently, its free-vibration behavior is not characterized by harmonic motion, but by hyperbolic sines and cosines.

Aslam, M., Godden, W.G., Scalise, D.T. [41] carried out an analytical and experimental investigation of dynamic rocking behavior of rectangular rigid bodies. One of the main motivation for this study was to establish safe design criteria for radiation shielding systems (see Figure 2.6) under strong motion earthquakes. Analytical parametric studies were made on the rocking dynamics of rigid blocks of varying sizes and aspect ratios, and different values of coefficient of restitution, under selected severe earthquake ground motions. In the experimental study, they performed a number of shaking table tests on concrete block stacks (see Figure 2.7) using horizontal and vertical component of ground motions. Experimental tests were conducted on a shaking table (at the University of California) using concrete blocks subjected to harmonic as well as to simulated earthquake notions [41].

Utilizing computational simulations, the dynamic response of rectangular blocks with diverse sizes and aspect ratios is examined under intense seismic events. The study investigates the influence of the coefficient of restitution (COR) and the vertical pre-stressing of blocks to the floor. This investigation primarily focuses on understanding the behavior of solid concrete block assemblies employed as radiation shields in particle accelerator facilities. Findings from the study highlight the critical need to prevent rocking in these systems due to the potential for overturning once rocking initiates [41].

The comparison of results of the study indicates a good agreement with small discrepancies exists between analytical and experimental results. The study also highlighted the tendency of toppling to slight changes in base geometry and coefficient of restitution as well as to the specific characteristics of the ground shaking. Additionally, the research identifies the significant influence of vertical earthquake components because downward earthquake acceleration may reduce the effective weight of the rigid block hence reducing the frictional force. Effective weight (W_e) of the block may be defined as [41]:

$$W_e = W \left(1 + \frac{a_g^y}{g} \right) \quad (2.9)$$

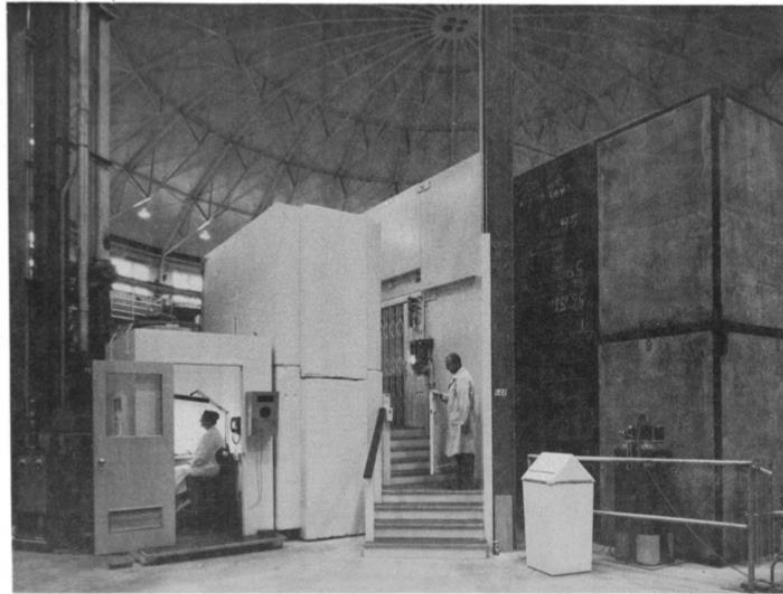


Figure 2.6. A typical radiation shielding system [41].

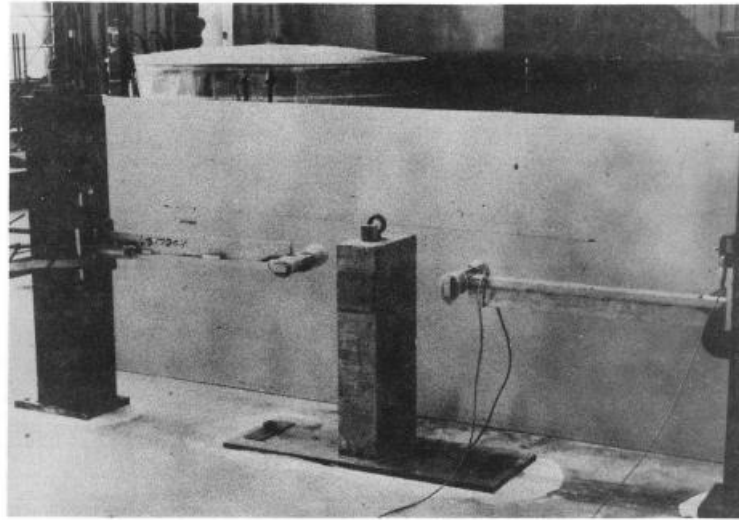


Figure 2.7. Test specimen of 30 in. x 6 in. concrete block stack [41].

The experimental studies performed by Priestly, Evison, and Carr [86] was intended to provide a means for estimating the possibility of foundation rocking of shear wall structures and developing an analysis procedure based upon elastic response spectrum. In the study, theoretical background developed by Housner [3] for free rocking rigid block was compared with the experimental results from a simple structural model with a number of different foundation conditions. The testing configuration (see Figure 2.8) subjected to free vibration, sinusoidal base acceleration and simulated seismic excitation. Further experimental investigations into rocking behavior were undertaken [87-91].

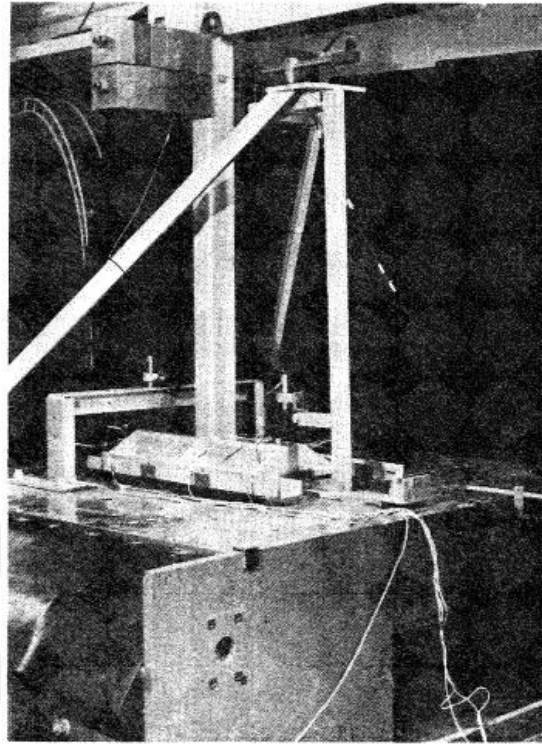


Figure 2.8. A simple one-sixth scale simulation of concrete masonry shear wall structure [86].

3. NUMERICAL MODELING

3.1. Introduction

This chapter explores detailed analytical formulations that describes the rocking behavior of a single solid block. To develop a thorough understanding of rocking behavior, this chapter first lays out the mathematical foundation for deriving both linear and nonlinear differential equations governing the motion of a stand-alone solid block. This chapter thoroughly discusses the assumptions underlying the linearized form of these equations. Additionally, both the nonlinear and linear equations of motion, along with their analytical solutions, are presented in detail.

The analytical solvability of the differential equations of motion for a rocking rigid block depends on the nature of the dynamic loading. In the absence of any dynamic force, the free vibration equation of motion can be solved analytically. Similarly, for a block undergoing constant acceleration from a state of rest, an analytical solution for the differential equation of motion is attainable. However, when the block experiences complex dynamic loading, such as real earthquake motion or impulsive forces characterized by pulse-like forms, analytical solutions become intractable. In such cases, numerical methods are employed to determine the block's rocking behavior, including its rotation, angular velocity, and angular acceleration at specific time intervals.

3.2. Concept of rocking rigid blocks

As a term, rocking motion refers to an oscillatory or swaying movement of a rigid block back and forth around the edge of its base. This type of motion is typically characterized by a repeated to-and-fro movement that can occur in various physical systems such as the rocking of a pendulum, rocking of an ancient column subjected to ground motion, concrete radiation shields, electrical transformers.

This dissertation deals with the two-dimensional (2D) rocking problem. Most of the mentioned studies in the section of literature review consider rocking problem as a planar (2-dimensional) problem. The analytical, numerical, and experimental studies on the dynamic response of three-dimensional (3D) rocking systems are very few. The 2D rocking problem involves analysing the rocking motion of a two-dimensional rigid body or structure around pivoting points, O or O'. In this problem, the object is free to rotate or rock back and forth in a plane.

3.3. Geometric Characteristics of Rocking Rigid Blocks

The dynamic response of free-standing rigid block significantly depends on its geometric characteristics. Semi-diagonal distance of the block, R , or size of the block calculated with Pythagorean theorem is a significant parameter which has most influence on the response of rigid blocks. Radial distance, R , connecting pivot points (O or O') to the geometric centre of the block makes an angle α with height of the block when the block is static. Slenderness ratio, α , is $\tan^{-1}(b/h)$. Slenderness ratio is another crucial geometric parameter that has an impact on the rocking response of the block. According to static moment equilibrium written in Eq. (2.1), the required minimum ground acceleration, a_{gmin} , to initiate rocking of the block is calculated with the product of mass by the slenderness ratio of the block.

3.4. Analytical Modeling of Rigid Blocks

The analytical modeling of rocking rigid blocks seeks to understand the complex dynamic behavior of these structures prone to rocking when exposed to external time-varying forces, such as earthquakes, pulse-like excitations. This study investigates the rocking behavior of a stand-alone block exposed to various loading conditions. Initially, the threshold acceleration amplitude necessary to initiate rocking motion is derived by applying static moment equilibrium about the point where it tips for a block at rest. Subsequently, the equation describing the free-vibration motion is formulated for a stand-alone block undergoing rocking motion. A closed-

form solution is derived for the equation governing free-vibration motion through analytical methods.

To further explore the dynamic response of the block, it is exposed to different forms of dynamic loading. The attainability of analytical solutions for the forced vibration equation of motion is contingent upon the specific form of dynamic loading applied.

3.4.1. Static Equilibrium Equation of Rigid Block

The single rigid block shown in Figure 3.1 is at rest. When it is subjected to static external loads, the “static” equation of motion of the following block can be written by equalizing the internal moment about pivot point, O that comes from the weight of the block to the external moment which is the product of the acceleration of the seismic excitation by the mass and half-height of the block.

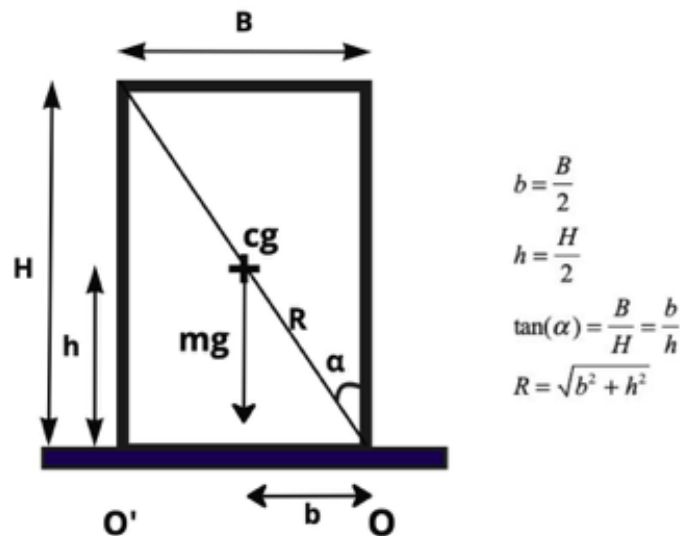


Figure 3.1. Single rigid block (rest state).

The following equation, called West's formula [7, 8], can be written taking into account “static” moment equilibrium of the column about the pivoting point O gives

$$\begin{aligned}\Sigma M_O &= 0 \\ mgb &= ma_g b \\ a_g &= g \frac{b}{h}\end{aligned}\tag{3.1}$$

where, a_g denote the amplitude of the seismic excitation, while b/h is slenderness ratio and g is gravitational acceleration, respectively.

If the amplitude of the ground excitation is greater or equal to the right side of the Equation (3.1) which is $g \frac{b}{h}$, the rigid block will uplift. It is thus seen that main geometric characteristics, including slenderness ratio $< b/h, \tan \alpha >$ and the size of the rigid block $< \sqrt{b^2 + h^2}, R >$ are critical parameters for defining overturning criteria of the block.

As mentioned in the previous paragraph, the free-standing rigid block will start to experience angular rotation, $\theta(t)$, when the base of the rigid block subjected to minimum required acceleration, $\ddot{u}_g$. Upon uplift has occurred, the block itself will exert additional restoring force which is rotational moment of inertia, $I_o \ddot{\theta}$ about the pivot point at the ground. As a term, rotational moment of inertia is a property of any rotating object with curvilinear motion. Rotational inertia or moment of inertia value measures the tendency of the mass moves along any curved path to resist to any changes in its state of rotation. As shown in Figure 3.2, as the mass of the block rotating about pivoting points, O or O', the block will exert rotational moment of inertia or additional restoring force which is always towards the direction of the equilibrium position.

3.4.2. Dynamic Equation of Motion of Free-Rocking Rigid Block

For a rigid block undergoing initial angular rotation, the static moment equilibrium about pivoting points which is written in Equation (3.1) is not valid. In this case, the dynamic moment equilibrium of the free rocking rigid block which experiencing rotation values gives

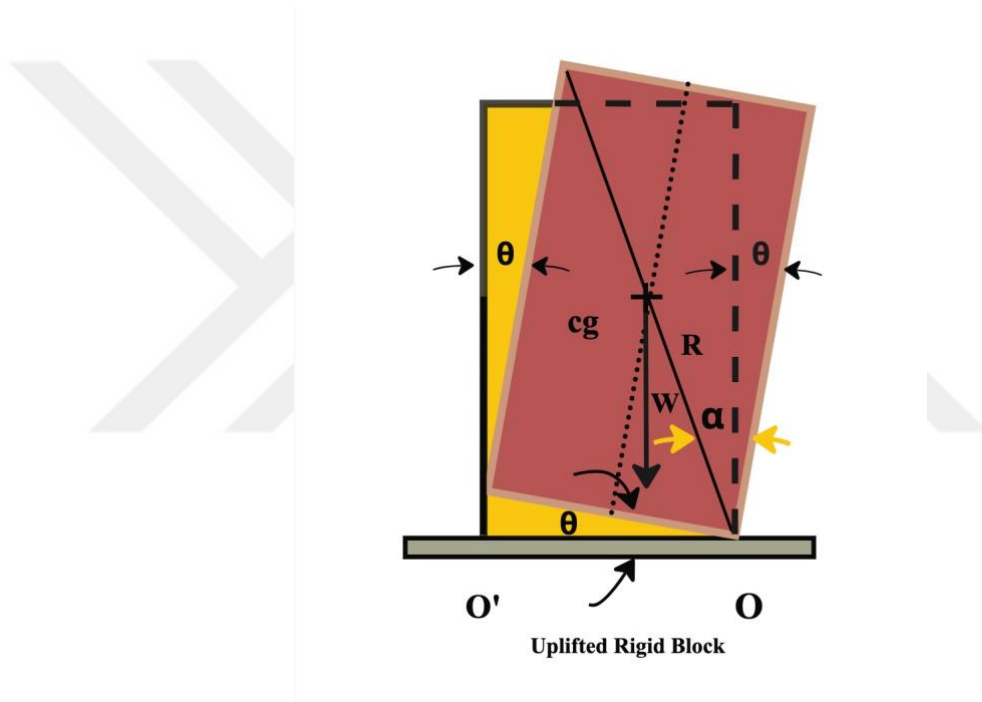


Figure 3.2. Uplifted rigid block with an initial displacement, $\theta_o(t)$.

$$I_o \ddot{\theta} + mgR \sin(\alpha - \theta) = 0 \quad (3.2)$$

Where I_o is the rotational moment of inertia and R is the size of the block. If we set frequency parameter, p , the Equation (3.2) can be further simplified as follows:

$$\ddot{\theta} + p^2 \sin(\alpha - \theta) = 0 \quad (3.3)$$

Where p is the frequency parameter and it equals to $\sqrt{\frac{mgR}{I_o}}$ mathematical expression.

Housner [3] accepted that the dynamic equation of motion for rigid blocks having an angle less

than 20° (for tall and slender blocks) can be further simplified ($\sin(\alpha - \theta) \approx \alpha - \theta$) and eventually becomes a 2nd order non-homogeneous linear differential equation:

$$\ddot{\theta} + p^2(\alpha - \theta) = 0 \quad (3.4)$$

$$\ddot{\theta} - p^2\theta = p^2\alpha \operatorname{sgn}(\theta)$$

As stated in Equation (3.4), this is a homogeneous second order differential equation with constant coefficients. Using initial conditions, including $\theta(t = 0) = \theta_o$ and $\dot{\theta}(t = 0) = 0$, the linearized form of the equation describing motion of free rocking stand-alone block can be easily solved. The full solution of the dynamic equation describing motion in Equation (3.4) includes the complementary solution, $\theta_c(t)$ and the particular solution, $\theta_p(t)$. The general solution of linearized homogeneous differential equation can be written as follows:

$$\theta_c(t) = Ae^{rt} \quad (3.5)$$

Since the roots of the auxiliary equation are same but opposite, the general solution of the differential equation given in Equation (3.4) is

$$\theta_c(t) = A \cosh pt + B \sinh pt \quad (3.6)$$

The particular solution of the differential equation of motion of free rocking solid block is $\theta_p(t) = \alpha$. By combining the general solution for homogeneous part and particular solution for non-homogeneous part, the full solution of linearized differential equation can be written as follows:

$$\theta(t) = (\theta_o - \alpha) \cosh pt + \alpha \quad (3.7)$$

3.4.3. Dynamic Equation of Motion of Forced-Rocking Rigid Block

In this dissertation, immense attention was given to the analysis of the rigid blocks which subjected to seismic excitation. In this case, depending on the form of seismic excitation, the differential equation of motion of the rigid block resting on the ground may be written by taking into consideration the sign of angular rotation, $\theta(t) < 0$ or $\theta(t) > 0$

$$I_o \ddot{\theta} + mgR \sin[\operatorname{sgn}(\theta)\alpha - \theta] = -m\ddot{u}_g R \cos[\operatorname{sgn}(\theta)\alpha - \theta] \quad (3.8)$$

In the above equation, $\theta(t)$ refers to the angular rotation, $\dot{\theta}(t)$ is the angular velocity and $\ddot{\theta}(t)$ is the angular acceleration term. Where $\text{sgn}(\theta)$ denotes sign function. When the block undergoes positive rotation value, $\theta(t) > 0$, the term $\text{sgn}(\theta)$ becomes positive, whereas the block undergoes negative rotation value, $\theta(t) < 0$, the term $\text{sgn}(\theta)$ becomes negative. Figure 3.3 illustrates a single rectangular block undergoing both positive and negative rotations.

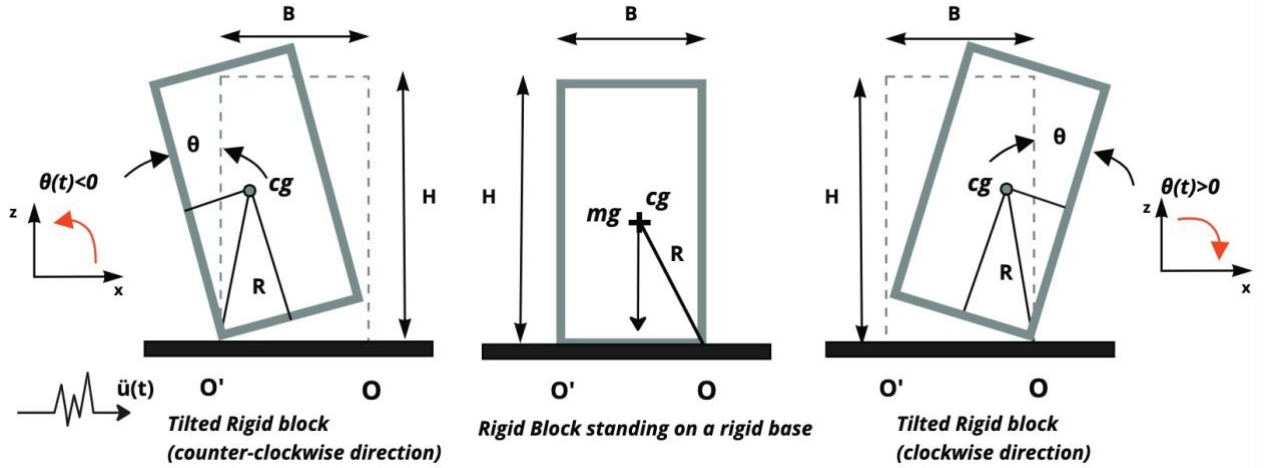


Figure 3.3. Rocking rigid block model experiencing positive and negative rotation.

3.4.4. Coefficient of Restitution for Rigid Body

During the rocking movement of a rigid block, it is assumed that there would be energy loss during an impact. The coefficient of restitution, v , describes the energy dissipation resulting from base and ground interaction. Mathematically, the coefficient of restitution can be expressed as the ratio of relative velocity after collision to the relative velocity before collision. It is alternatively referred to as the ratio of kinetic energy post and pre-impact. Using conservation of angular momentum (CoAM), the mathematical expressions about pivoting points O or O' just before collision and right after collision can be equalized as follows:

$$I_o \dot{\theta}_1 - 2mRb\dot{\theta}_1 \sin(\alpha) = I_o \dot{\theta}_2 \quad (3.9)$$

Here, $\dot{\theta}_1$ is the angular velocity before collision and $\dot{\theta}_2$ is the angular velocity immediately after collision.

$$v = \frac{\dot{\theta}_2^2}{\dot{\theta}_1^2} \quad (3.10)$$

It has a range between $0 \leq v \leq 1$. If the coefficient of restitution is 1 then it is perfectly elastic collision meaning no energy is lost during an impact. If the coefficient of restitution is 0, it is an inelastic impact.

$$v = \left[1 - \frac{3}{2} \sin(\alpha)^2 \right]^2 \quad (3.11)$$

3.5. Numerical Modeling of Rigid Blocks with Linear Formulation

The rocking dynamics of a stand-alone solid block under seismic event is governed by a non-homogeneous 2nd degree nonlinear differential equation with non-constant coefficients as written in Equation (3.8). Depending on the block slenderness and the angular rotation values, the dynamic equation of motion can turn into a simpler linearized differential equation and solution of the mathematically transformed equation of motion becomes easier to solve and not time consuming.

Assuming that the rigid block is undergoing small rotations, $\theta(t)$. As a result of this assumption, the simplification of trigonometric expressions $\sin(\text{sgn}(\theta)\alpha - \theta)$ and $\cos(\text{sgn}(\theta)\alpha - \theta)$ can be achieved using trigonometric identities for small rotations.

$$\sin(\text{sgn}(\theta)\alpha - \theta) = \text{sgn}(\theta) \sin(\alpha) \cos(\theta) - \text{sgn}(\theta) \cos(\alpha) \sin(\theta) \quad (3.12)$$

$$\cos(\text{sgn}(\theta)\alpha - \theta) = \text{sgn}(\theta) \cos(\alpha) \cos(\theta) + \text{sgn}(\theta) \sin(\alpha) \sin(\theta) \quad (3.13)$$

For a rigid block having small positive angular rotations, $\theta(t)$, the trigonometric terms in Equation (3.12) and Equation (3.13), such as $\sin(\theta)$ and $\cos(\theta)$ will be θ and 1, respectively.

$$\sin(\alpha - \theta) = \sin(\alpha) \cos(\theta) - \cos(\alpha) \sin(\theta) = \sin(\alpha) - \theta \cos(\alpha) \text{ for } \theta(t) > 0 \quad (3.14)$$

$$\cos(\alpha - \theta) = \cos(\alpha) \cos(\theta) + \sin(\alpha) \sin(\theta) = \cos(\alpha) + \theta \sin(\alpha) \text{ for } \theta(t) > 0 \quad (3.15)$$

Consider a single rigid block experiencing small negative angular rotations, $\theta(t)$, the two equations presented above can be expressed in the following form:

$$\sin(-\alpha - \theta) = -\sin(\alpha) \cos(\theta) - \cos(\alpha) \sin(\theta) = -\sin(\alpha) - \theta \cos(\alpha) \text{ if } \theta(t) < 0 \quad (3.16)$$

$$\cos(-\alpha - \theta) = \cos(\alpha) \cos(\theta) - \sin(\alpha) \sin(\theta) = \cos(\alpha) - \theta \sin(\alpha) \text{ if } \theta(t) < 0 \quad (3.17)$$

In order to obtain the generalized compacted form of dynamic rocking response of the rigid block, further simplification can be substituted in the following equation:

$$I_o \ddot{\theta} + mgR \operatorname{sgn}(\theta) [\sin(\alpha) - \theta \cos(\alpha)] = mR \ddot{u}_g \operatorname{sgn}(\theta) [\cos(\alpha) + \theta \sin(\alpha)] \quad (3.18)$$

Substitution of frequency parameter, $p = \frac{mgR}{I_o}$ into Equation (3.18) gives the following compacted form:

$$\ddot{\theta} + p^2 \operatorname{sgn}(\theta) [\sin(\alpha) - \theta \cos(\alpha)] = p^2 \frac{\ddot{u}_g}{g} \operatorname{sgn}(\theta) [\cos(\alpha) + \theta \sin(\alpha)] \quad (3.19)$$

As clearly seen from the equation derived above the assumption of small rotations is not sufficient to turn the Equation (3.19) into a linear form, thus further assumption can be made to derive a linearized equation of motion of rigid block for rocking motion. The assumption is that the angle of slenderness, α , is very small for tall and slender blocks. Housner [3] concluded that for tall, slender blocks having an angle α (measured in radians) less than 20° , the sine of the angle may be approximated by the value of the angle itself.

For small angle of slenderness and positive angular rotations, the Equation (3.19) is then

$$\ddot{\theta} + p^2 [\sin(\alpha) - \theta \cos(\alpha)] = p^2 \frac{\ddot{u}_g}{g} [\cos(\alpha) + \theta \sin(\alpha)] \quad (3.20)$$

$$\ddot{\theta} - p^2 \theta = -p^2 \alpha + p^2 \frac{\ddot{u}_g}{g} + p^2 \theta \alpha$$

In the equation, the term $p^2 \theta \alpha$ is approximately zero. The equation of motion is then

$$\ddot{\theta} - p^2 \theta = -p^2 \left(\alpha - \frac{\ddot{u}_g}{g} \right) \quad (3.21)$$

For small angle of slenderness and negative angular rotations, the equation of rocking motion can be written as follows

$$\ddot{\theta} + p^2[-\sin(\alpha) + \theta \cos(\alpha)] = p^2 \frac{\ddot{u}_g}{g} [-\cos(\alpha) - \theta \sin(\alpha)] \quad (3.22)$$

$$\ddot{\theta} + p^2\theta = p^2\alpha + p^2 \frac{\ddot{u}_g}{g} (-1 - \theta\alpha)$$

In the above equation, the term $\theta\alpha$ is approximately zero. The equation of motion is then

$$\ddot{\theta} + p^2\theta = -p^2 \left(\alpha - \frac{\ddot{u}_g}{g} \right) \quad (3.23)$$

As seen from Equation (3.21) and Equation (3.23) derived above a non-homogeneous 2nd degree linearized differential equation of a rigid block under seismic excitations may be written with a set of assumptions based on the geometric properties of the block.

For any sign of rotation, the linearized differential equation of a rigid block under seismic excitations can be written as:

$$\ddot{\theta}(t) = p^2 \left[-\text{asgn}[\theta(t)] + \theta(t) - \frac{\ddot{u}_g(t)}{g} \right] \quad (3.24)$$

In order to derive analytical solution of the non-homogeneous 2nd degree linearized differential equation of a rigid block, here, the base acceleration is taken as harmonic sinusoidal base excitation.

$$\ddot{\theta} + p^2\theta = -p^2 \left(\alpha - \frac{a_0}{g} \sin(\omega t) \right) \quad (3.25)$$

As stated in Equation (3.25), this is a non-homogeneous second order differential equation with constant coefficients. Using initial conditions, including $\theta(t = 0) = 0$ and $\dot{\theta}(t = 0) = 0$, the linearized form of the equation describing motion of free rocking solid block can be easily solved. The full analytical solution of the dynamic differential equation of motion in Equation (3.25) includes the complementary solution, $\theta_c(t)$ and the particular solution, $\theta_p(t)$. The general solution of linearized non-homogeneous differential equation is expressed as follows:

$$\theta_c(t) = Ae^{rt} \quad (3.26)$$

Since the roots of the auxiliary equation are same but opposite, the general solution of the differential equation given in Equation (3.25) is

$$\theta_c(t) = A \cosh pt + B \sinh pt \quad (3.27)$$

The particular solution of the equation of motion of free rocking rigid block is θ_p ,

$$\theta_p(t) = \alpha - \frac{a_0 p^2}{gx(\omega^2 + p^2)} \sin(\omega t) \quad (3.28)$$

By combining the general solution for homogeneous part and particular solution for non-homogeneous part, the full solution of linearized differential equation can be written as follows:

$$\theta(t) = A \cosh pt + B \sinh pt + \alpha - \frac{a_0 p^2}{gx(\omega^2 + p^2)} \sin(\omega t) \quad (3.29)$$

Using initial conditions, including $\theta(t = 0) = 0$ and $\dot{\theta}(t = 0) = 0$, the exact analytical solution of the linearized form of the equation of motion of forced-rocking stand-alone block can be expressed as follows:

$$\theta(t) = -\alpha \cosh pt + \frac{\omega p \frac{a_0}{g}}{\omega^2 + p^2} \sinh pt + \alpha - \frac{a_0 p^2}{gx(\omega^2 + p^2)} \sin(\omega t) \quad (3.30)$$

3.6. Numerical Modeling of Rigid Blocks with Nonlinear Formulation

This chapter examines in depth the nonlinear nature of the dynamic equation of motion of the individual block under horizontal base excitation. Apart from the numerical solution of the linearized form of non-homogeneous 2nd degree differential equation governing the motion of a rigid block, the numerical solution of the nonlinear aspects of the problem emerged as an appealing option. This approach allows for solving the dynamic equations of motion of the rigid block not only for slender blocks but also for non-slender blocks.

In an attempt to obtain the rocking behavior of a single rigid block under seismic excitation, conventional ordinary differential equation (ODE) solvers provided in MATLAB [6] are employed. The dynamic response can be modelled and calculated numerically using state-space representation. Here, the main target is to transform the differential equation of n-th order

into n differential equations of first order. The state vector can be used to determine the state form.

$$\{y(t)\} = \begin{pmatrix} \theta(t) \\ \dot{\theta}(t) \end{pmatrix} \quad (3.31)$$

Furthermore, the time derivative of state vector of the system can be written as:

$$\{\dot{y}(t)\} = \begin{pmatrix} \dot{\theta}(t) \\ \ddot{\theta}(t) \end{pmatrix} \quad (3.32)$$

With the given dynamic equation describing the motion of forced-rocking rigid block:

$$\ddot{\theta} + p^2 \operatorname{sgn}(\theta) [\sin(\alpha) - \theta \cos(\alpha)] = p^2 \frac{\ddot{u}_g}{g} \operatorname{sgn}(\theta) [\cos(\alpha) + \theta \sin(\alpha)] \quad (3.33)$$

We can solve Equation (3.33) for $\ddot{\theta}(t)$, giving us

$$\ddot{\theta}(t) = (-p^2) \left[\sin(\alpha \operatorname{sgn}[\theta(t)] - \theta(t)) + \frac{\ddot{u}_g(t)}{g} \cos(\alpha \operatorname{sgn}[\theta(t)] - \theta(t)) \right] \quad (3.34)$$

Inserting 3.32 into the derived state vector and replacing y and with the corresponding entries in the state vector results in the state form needed to solve the differential equation in MATLAB using the ode45-solver [6]:

$$f(t) = \{\dot{y}(t)\} = \begin{pmatrix} \dot{\theta}(t) \\ (-p^2) \left[\sin(\alpha \operatorname{sgn}[\theta(t)] - \theta(t)) + \frac{\ddot{u}_g(t)}{g} \cos(\alpha \operatorname{sgn}[\theta(t)] - \theta(t)) \right] \end{pmatrix} \quad (3.35)$$

In the case of tall and slender blocks (where the angle of slenderness, α , is very small) with the assumption of small rotations, the above state form can be turned into a linearized form, resulting in the simplification of Equation (3.32) to:

$$f(t) = \{\dot{y}(t)\} = \begin{pmatrix} \dot{\theta}(t) \\ (p^2) \left[-\alpha \operatorname{sgn}[\theta(t)] + \theta(t) - \frac{\ddot{u}_g}{g} \right] \end{pmatrix} \quad (3.36)$$

4. NUMERICAL ANALYSIS OF ROCKING RIGID BLOCKS

4.1 Introduction

The dissertation deals with analysis of the dynamic rocking behavior of rigid block undergoing three forms of seismic excitations, such as sinusoidal harmonic acceleration, pulse-like excitations and earthquake excitations. Hence, as described detailly in previous chapters, to simulate rocking behavior of single rigid blocks, MATLAB scripts were developed. The first part of the chapter presents the results of numerical solutions of equations of motion with linear formulation given by Equation (3.24). Numerical analyses for both free and forced rocking motion were performed on a specific slender rigid block employing two integration methods: the Newmark- β method and the Runge-Kutta method. For validation purposes, the plots and figures illustrating the rocking response results are graphically compared with one another.

The second part of the chapter is dedicated to the generation of overturning spectra (OS). To achieve this, three rectangular rigid blocks, each having slenderness angles of 10° , 20° , and 30° , were analyzed under various loading conditions, including one-cycle sine waves, half-cycle sine waves, rectangular constant acceleration pulses, Haar wavelets, antisymmetric Ricker pulses, and symmetric Ricker pulses. In the context of the nonlinear approximation, criteria for a single rigid block to undergo overturning were defined, and an equation determining the minimal acceleration magnitude necessary to induce overturning was formulated.

Following this, several cycloidal-shaped waveforms were used, and their similarity to recorded near-field seismic excitation was demonstrated. Further analysis focused on a monolithic, stand-alone column, a model representing the columns of the Temple of Olympian Zeus in Syracuse. To understand the potential for overturning during near-field seismic events, the study employed impulsive excitation forms that simulate the characteristic reverse effects observed in such ground motions. A nonlinear formulation was used to compute the oscillatory response of the solid column undergoing both cycloidal-shaped pulses and near-field ground motions.

4.2 General Information on Seismic Excitations

The 1940 El Centro earthquake ($M_w 6.9$), California, is of significant importance in the realm of earthquake engineering, particularly in structural design. This event has long served as a fundamental dataset in earthquake engineering studies. One key aspect that makes the El Centro earthquake a pivotal reference in this study is its early and detailed documentation of strong motion data. Subsequently, in this study, the 2023 Pazarcık, Kahramanmaraş ($M_w 7.7$) earthquake has also been considered. The rationale for incorporating the 2023 Pazarcık, Kahramanmaraş ($M_w 7.7$) earthquake data in this research lies in its complex ground motion characteristics, such as extended rupture lengths, strong near-field pulse, destructive aftershocks, and a broad spectrum of frequencies. Analyzing the data from the 2023 Pazarcık, Kahramanmaraş ($M_w 7.7$) earthquake contributes significantly to understanding of the fundamental dynamics of rocking systems.

4.3 Types and modeling of Pulse-like Excitations

The study of rocking rigid blocks subjected to pulse excitations has garnered significant interest among researchers. This is largely due to the fact that pulse excitations share similar characteristics with the ground motions observed during earthquakes. The study performed in this thesis for the cases involving one-cycle sine wave, half-cycle sine wave, rectangular constant acceleration pulse, Haar wavelet pulse, antisymmetric Ricker acceleration pulse and symmetric Ricker acceleration pulse, respectively. The Figure 4.1 below illustrates all pulse types utilized in the analysis.

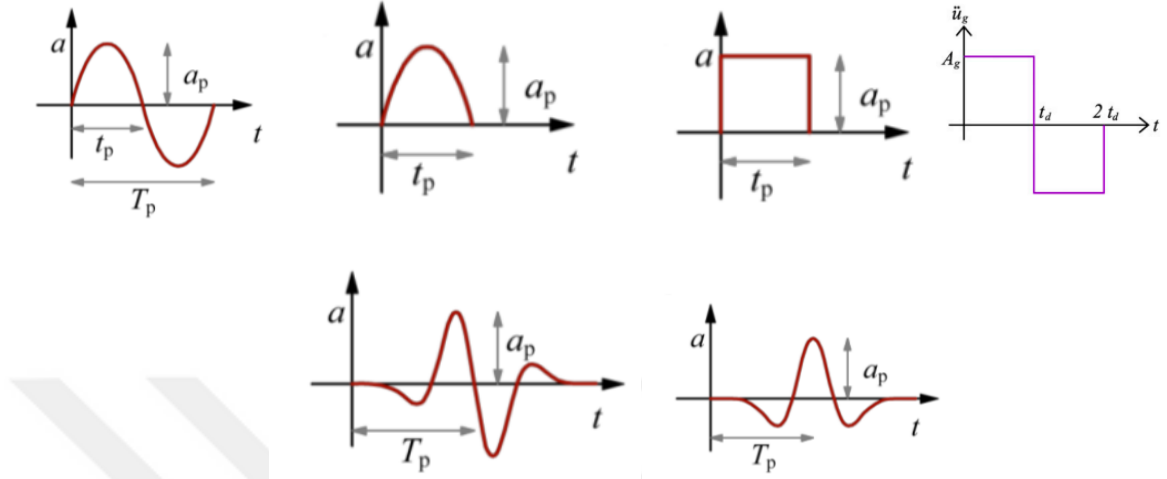


Figure 4.1. Various signal shapes employed in this study.

As mentioned in the literature review section, the concept of the overturning curve for a rectangular stand-alone block exposed to a half-sine pulse was pioneered by Housner [3]. When considering a particular block and a specific signal shape (e.g., one cycle sine pulse), the overturning curve can be defined as the boundary that delineates the safe and unsafe regions on the $a_p - t_p$ plane, where a_p represents the maximum pulse acceleration and t_p denotes the pulse duration. If the a_p and t_p parameters of a pulse fall on the left side of this curve, the block will not topple. Furthermore, if a_p is less than $a_{p,min}$ the block will not exhibit any rocking motion.

4.4. Numerical Analysis Results with Linear Approximation

This chapter of this dissertation involved dynamic response of rigid block subjected to many forms of excitations. The first phase of analysis involved the numerical solution of linearized equation of motion using Newmark- β method. Figures 4.3 through 4.22 show all the results are in terms of angular displacement and angular velocity. All values were computed numerically at each discrete time steps using Newmark's method of integration in MATLAB scripts.

The second phase of calculation involved numerical solution of linearized equation of motion using ode45 solver. The ode45 solver available in MATLAB was used for numerical integration of Equation (3.36). The ode45 solver based on an explicit Runge-Kutta integration method. In order to obtain realistic results in terms of angular displacement and angular velocity, all numerical analysis involved the energy dissipation at each impact using coefficient of restitution parameter, v . In both methods, the ODE of motion was integrated assuming the coefficient of restitution equals to

$$v = \left[1 - \frac{3}{2}\sin(\alpha)^2\right]^2 \quad (4.1)$$

Furthermore, the comparison of results showed that the numerical integration of linearized differential equation of motion using Newmark- β method is in good agreement with the numerical solution obtained from mathematical programming (MATLAB ode45) solver.

4.4.1. Solving Equation of Motion by Newmark- β and Runge-Kutta Method

In numerical analysis methods, instead of finding continuous functions of system responses, the values of functions are found at given time instants. The values of functions were calculated over the time interval $0, \Delta t, 2\Delta t, 3\Delta t, \dots$. In numerical methods, acceleration, velocity and displacement are generally first expressed approximately in terms of only one type of response (e.g. acceleration and velocity in terms of displacement). Then the equation of motion is defined over this unknown and since the t moment values are known, the solution or calculation is made for the next step ($t+1$). Therefore, these methods are also defined as incremental or time-stepping.

Newmark- β is a method of numerical integration [92]. This method uses a second order Taylor expansion for approximating dynamic response parameters such as velocities and accelerations. As a term, Taylor series is an expansion of a function $f(x)$ about a point and can be expressed as an infinite sum of function's derivatives at a single point. Hence, the response parameters including angular displacement, angular velocity and angular acceleration parameters are written as Taylor series expansions.

The general formula for Taylor series:

$$f(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2!}(x - a)^2 + \frac{f'''(a)}{3!}(x - a)^3 + \dots \quad (4.2)$$

For a single degree of freedom rocking rigid block, Taylor series can be used for incremental expression of angular displacement and angular velocity functions:

$$\theta(t + \Delta t) = \theta(t) + \dot{\theta}(t)\Delta t + \frac{\ddot{\theta}(t)}{2!}(\Delta t)^2 + \frac{\ddot{\theta}(t)}{3!}(\Delta t)^3 + \dots \quad (4.3)$$

$$\dot{\theta}(t + \Delta t) = \dot{\theta}(t) + \ddot{\theta}(t)\Delta t + \frac{\ddot{\theta}(t)}{2!}(\Delta t)^2 + \frac{\ddot{\theta}(t)}{3!}(\Delta t)^3 + \dots \quad (4.4)$$

In the Newmark method, fourth and higher derivative terms can be ignored:

$$\theta(t + \Delta t) = \theta(t) + \dot{\theta}(t)\Delta t + \frac{\ddot{\theta}(t)}{2!}(\Delta t)^2 + \frac{\ddot{\theta}(t)}{3!}(\Delta t)^3 \quad (4.5)$$

$$\dot{\theta}(t + \Delta t) = \dot{\theta}(t) + \ddot{\theta}(t)\Delta t + \frac{\ddot{\theta}(t)}{2!}(\Delta t)^2 \quad (4.6)$$

In the method, the contributions of the third derivative terms can be symbolized mathematically using the Newmark – beta coefficients, β and γ , respectively. The coefficients used in the Newmark beta method are in the range of $0 < \beta < 1$ and $0 < \gamma < 1$. In expression of angular displacement and angular velocity, the last third derivative of function can be written as:

$$\ddot{\theta}(t) = \frac{\ddot{\theta}(t+\Delta t) - \ddot{\theta}(t)}{\Delta t} \quad (4.7)$$

When the third derivative of the function substituted into Equation (4.5), and Equation (4.6), approximate expressions for angular displacement and angular velocity will be obtained.

$$\theta(t + \Delta t) = \theta(t) + \dot{\theta}(t)\Delta t + \left(1 - \frac{1}{2}\beta\right)\ddot{\theta}(t)(\Delta t)^2 + \beta\ddot{\theta}(t + \Delta t)(\Delta t)^2 \quad (4.8)$$

$$\dot{\theta}(t + \Delta t) = \dot{\theta}(t) + (1 - \gamma)\ddot{\theta}(t)\Delta t + \gamma\ddot{\theta}(t + \Delta t)(\Delta t) \quad (4.9)$$

Using Newmark's integration method, the angular acceleration at a time step (i+1) can be written as:

$$\ddot{\theta}(i + 1) = \left(\theta_{i+1} - \theta_i - \Delta t\dot{\theta}_i - \left(\frac{1}{2} - \beta\right)\Delta t^2\ddot{\theta}_i\right)\left(\frac{1}{\beta\Delta t^2}\right) \quad (4.10)$$

Here, angular acceleration term substituted into Equation (3.8) and then

$$I_o \frac{(\theta_{i+1} - \theta_i - \Delta t \dot{\theta}_i - (\frac{1}{2} - \beta) \Delta t^2 \ddot{\theta}_i)}{\beta \Delta t^2} - mgR\theta_{i+1} = -sgn(\theta_{i+1})mgR\alpha + mR\ddot{u}_{g,i+1} \quad (4.11)$$

In the above equation, rotation, angular velocity and angular acceleration at time step (i) is known parameters, whereas angular displacement at time step (i+1) is unknown parameter.

Another widely used and accepted method for the numerical solution of the first-order ordinary differential equations (ODEs) is Runge–Kutta method [93]. This alternative method results in higher accuracy without performing more operations.

Consider an ordinary differential equation as follows:

$$\begin{cases} \frac{dy}{dt} = y' = f(t, y) \\ y(t_o) = y_o \end{cases} \quad (4.12)$$

Here, y is an unknown equation of time, t and y_o is the initial value at zero time. Choose h > 1 to be the time step size and (t_i = t_o + h). The general formula for Runge-Kutta 4th order method (RK4) is given as:

$$\begin{cases} k_1 = hf(t_i, y_i) \\ k_2 = hf(t_i + h/2, y_i + k_1/2) \\ k_3 = hf(t_i + h/2, y_i + k_2/2) \\ k_4 = hf(t_i + h, y_i + k_3) \\ y_{i+1} = y_i + (k_1 + 2k_2 + 2k_3 + k_4)/6 \end{cases} \quad (4.13)$$

4.4.2. Geometric Characteristics of the Slender Block

Analysis of the numerical model was performed assuming frictional forces existing between two solid surfaces is quite high to prevent the occurrence of sliding. The study performed in this thesis for the cases involving linearized equation of motion. In order to convert the dynamic equation of motion of selected single rigid block model from nonlinear form to linear form, the geometric properties of the model were selected accordingly. According to Housner [3], in the case of slender blocks with an angle, α , less than about 20 degrees, the sine

of the difference of two angles can be approximated as the difference between the sines of the angles ($\sin(\alpha - \theta) \approx \alpha - \theta$). As a result, the nonlinear equation of motion can be further simplified into linearized form. Therefore, analysis of the numerical model was performed assuming the angle of slenderness α is less than 20° . Geometric characteristic values needed to write down linearized equation of motion for single rigid block such as base (B), height (H), half-diagonal (R), slenderness angle (α) are summarized in Table 4.1. Figure 4.2 shows the adopted model dimensions and configuration.

Table 4.1. Geometric characteristics of the slender block.

Specimen	B (m)	H (m)	$b = B/2$	$h = H/2$	R(m)	α (rad)	$\alpha(^{\circ})$	p (rad/sec)
N1	0.4m	1.6m	0.2m	0.8m	0.8246m	0.245	14.036	2.9871

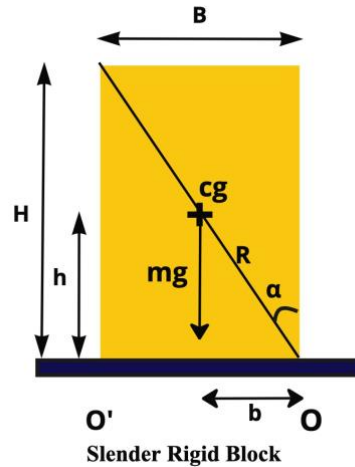


Figure 4.2. Rigid block model configuration used for analysis cases.

4.4.3. Dynamic Response of Free Rocking Motion

The analyses of free rocking motion were carried out for a slender rigid block having dimensions of $2b=0.4\text{m}$, $2h=1.6\text{m}$ in width and height, the angle of slenderness $\alpha = 0.245$ (rad), half-diagonal is $R=0.8246\text{m}$ and frequency parameter is $p = 2.9871$ rad/sec as

mentioned earlier in Table 4.1. The schematic of this rocking rigid model is shown in Figure 4.2. The Newmark- β numerical integration method was used to solve initial value problem. The Newmark time integration parameters assumed for this case are $\beta=0.25$ and $\gamma=0.5$. Numerical analysis for the rectangular rigid block with an initial tilt angle $\theta_0=0.24$ rad were carried out with time step size $dt=0.0015$ sec for 6 seconds considering solution accuracy and computational efficiency. The angular displacement and angular velocity for each time step are calculated using a MATLAB m-file. The plots presented in Figure 4.3 show the dynamic results in terms of rotation, angular velocity and total energy over time for an individual block for the first 6.0 seconds.

The plots presented in Figure 4.4 show the minimum amplitude of acceleration that is required to induce overturning of the rigid block for free vibration response, as defined by Equation (3.4), and a coefficient of restitution v equal to 0.91. As clearly seen from Figure 4.4, a rectangular rigid block having an initial tilt angle $\theta_0=0.25$ rad is highly critical for overturning.

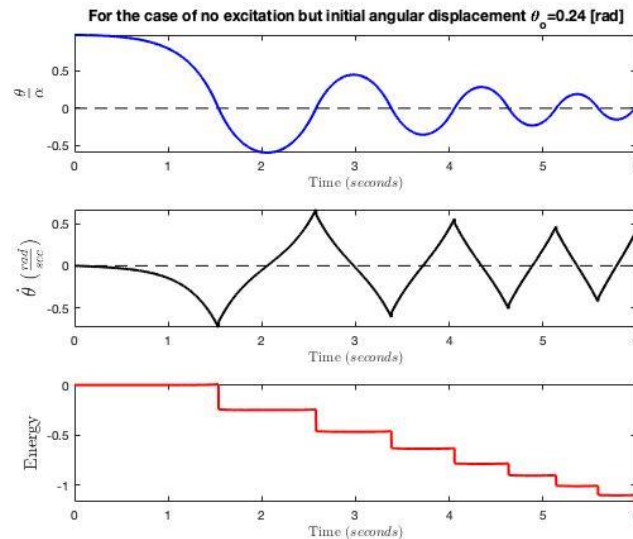


Figure 4.3. Time-domain histories of rotation, rotational velocity and total energy of a rigid block with initial, $\theta_0(t)$, displacement via Numerical Solution (Newmark- β) (No Overturning).

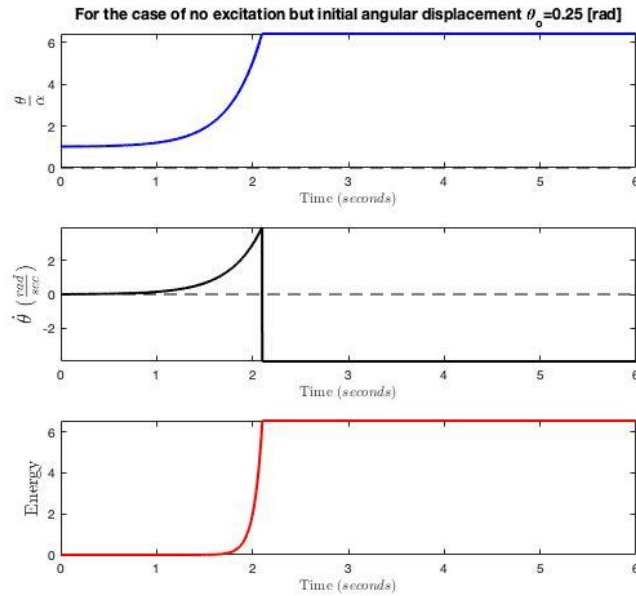


Figure 4.4. Time-domain histories of rotation, rotational velocity and total energy of a rigid block with initial, $\theta_o(t)$, displacement via Numerical Solution (Newmark- β) (Overturning).

4.4.4. Dynamic Response under the Sinusoidal and Cosine Harmonic Acceleration

Section 4.4.4 of this chapter presents results regarding two analyses of forced rocking motion for a slender rectangular rigid block having dimensions of $2b=0.4\text{m}$, $2h=1.6\text{m}$ in width and height. The angle of slenderness of the rectangular block is $\alpha = 0.245$ (rad), half-diagonal is $R=0.8246\text{m}$ and frequency parameter is $p = 2.9871$ rad/sec. The schematic of this rocking rigid model is shown in Figure 4.2. Numerical solutions for the rigid block subjected to harmonic base acceleration given by Equation (3.25) can be used to calculate linear dynamic response of the block. For all cases investigated herein in this chapter of dissertation the value of coefficient of restitution, v was taken to be equal to 0.91. This part of the study proceeds with dynamic results in terms of rotation, angular velocity and total energy time histories of single rocking rigid block when subjected to horizontal base acceleration function.

Numerical analysis for the rectangular rigid block under harmonic sinusoidal base acceleration were carried out with time step size $dt=0.0015$ sec for 6 sec. On the left Figure 4.5, Newmark- β numerical integration method was used for solving second-order differential equations (ODEs). On the right Figure 4.5, Runge-Kutta method was adopted for solving first-order differential equations (ODEs). Similar to analysis of free rocking motion, both methods were used to solve linearized form of the differential equations.

Following graphs were generated using a MATLAB m-file. The left graph pertains to the Newmark- β method. The right graph pertains to Runge-Kutta method. In general, results presented in Figure 4.5 indicate the maximum acceleration amplitude value that rectangular rigid block can resist on its base without overturning. In the Newmark graph, the second and third line shows angular displacement and angular velocity times histories, respectively. In the energy graph, the red line shows system energy according to Newmark- β solver. In the Runge-Kutta method, the second and third line presents angular displacement and angular velocity values at each time step.

On the left Figure 4.5, the threshold value for the acceleration is calculated as $a_p = 5.101$ m/sec² (0.52g) using numerical Newmark- β method and block does not overturn but simply swing until it attains a stable equilibrium, similarly on the right Figure 4.5, a_p is calculated as 5.101 m/sec² (0.52g) and block does not overturn for Runge-Kutta numerical algorithm. In addition, graphical comparison of dynamic response values given in Figures 4.5 left and right illustrates the both methods appear to be in good overall agreement, since during the first up to six rocking cycles the numerical responses match well.

In order to compare the response of a single solid block to sinusoidal base acceleration and the rocking dynamics of a solid block excited by one-cycle sine wave, another numerical analysis was performed using Newmark- β numerical approach. In Figures 4.6 and 4.7, the right graph presents the numerical outcomes under one-cycle sine wave. Figures 4.6 and 4.7 (left)

plot the dynamic response in terms of rotation, angular velocity and total energy time histories under harmonic sinusoidal acceleration. The analysis of dynamic responses experiencing harmonic sinusoidal base acceleration and one-cycle sine excitation reveals that a rigid block initiates its rocking motion upon the completion of the one-cycle sine pulse.

In practice, a block experiencing a half-sine pulse tends to tip over during its free vibration phase, rather than precisely at the moment when the pulse the pulse reaching the end. However, Housner [3] hypothesized that the necessary criterion for a block to overturn is that the angular displacement, θ , becomes equivalent to the slenderness angle, α , at the instant the half-sine waveform terminates. Building upon this principle, Housner [3] formulated an equation that gives the threshold acceleration amplitude necessary for overturning the block. Subsequent analysis indicates that Housner's solution [3] for the threshold acceleration amplitude necessary for overturning the block under a half-sine pulse is incorrect [4].

As illustrated in Figures 4.6 and 4.7, both the rotation and rotational velocity of the single rigid block persist beyond the expiration of the one-cycle sine pulse. It can therefore be concluded that, under the critical acceleration amplitude, the block overturns during its unforced oscillation phase rather than at the moment the one-cycle sine excitation ends.

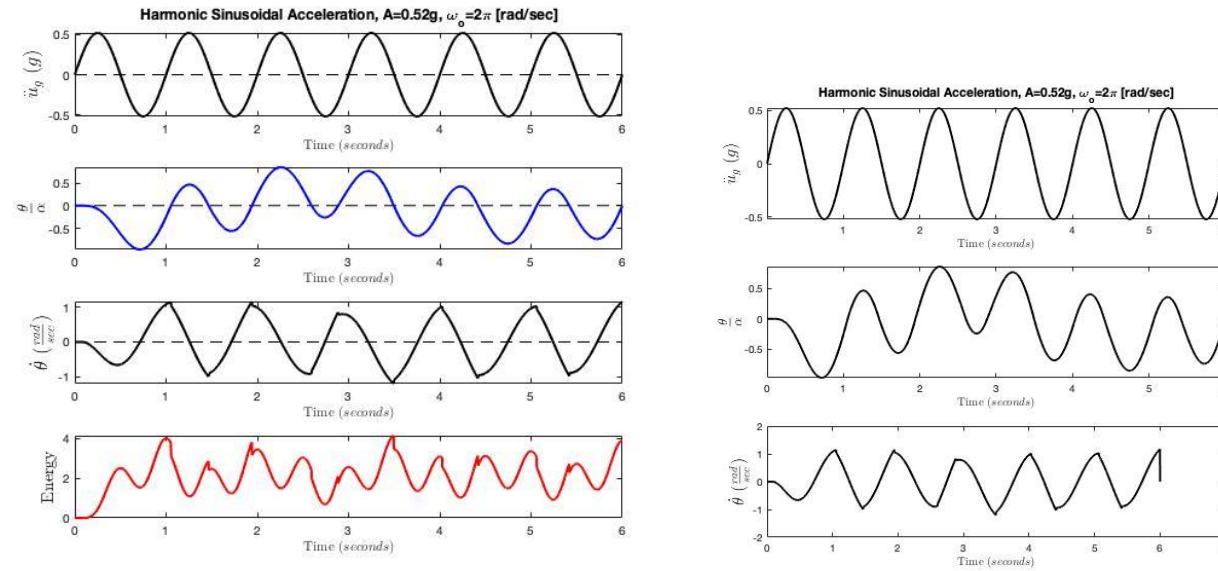


Figure 4.5. Time-domain histories of rotation and rotational velocity of a rigid block under sinusoidal harmonic base excitation Left: Numerical Solution (Newmark- β) Right: Numerical Solution (Runge-Kutta Method via ode45 solver) (No Overturning).

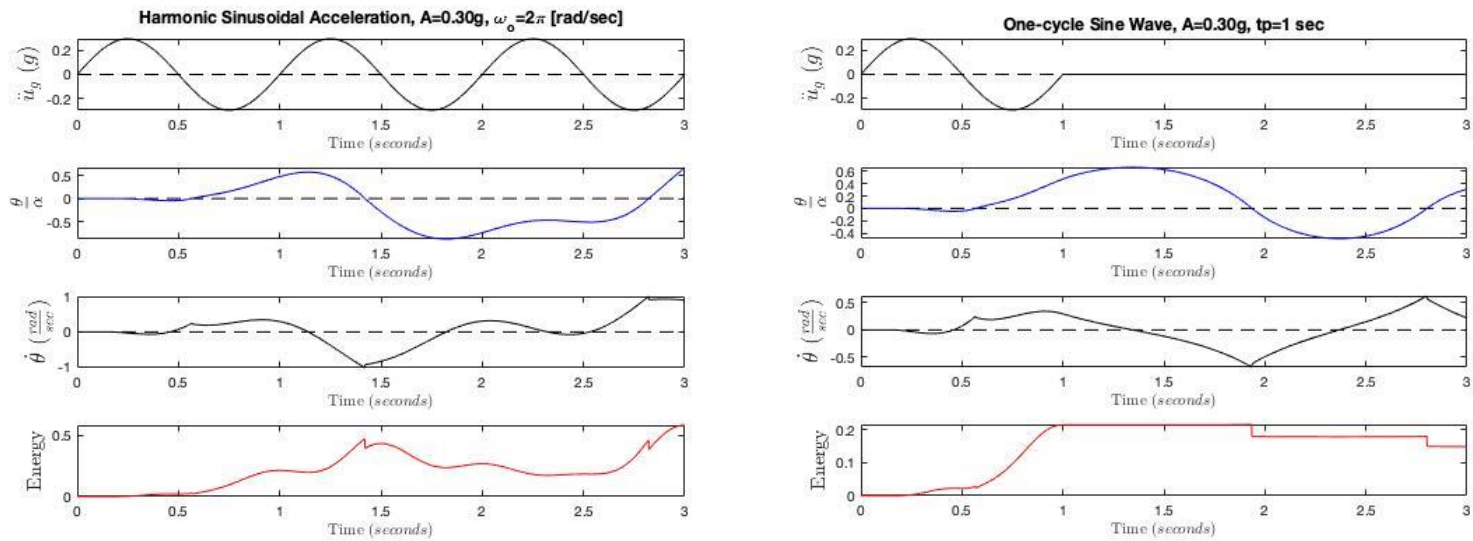


Figure 4.6. Time-domain histories of rotation and rotational velocity of a rigid block within the initial 3 secs under sinusoidal harmonic base excitation and one-cycle sine wave Left and Right: Numerical Solution (Newmark- β method).

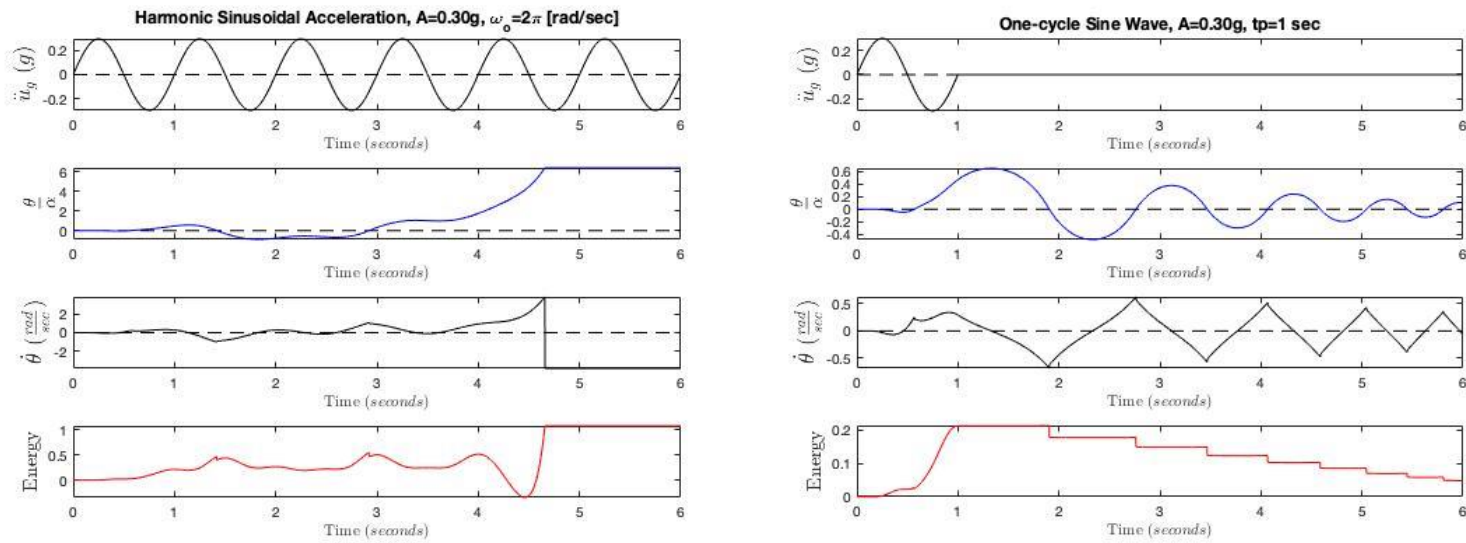


Figure 4.7. Time-domain histories of rotation and rotational velocity of a rigid block within the initial 6 secs under sinusoidal harmonic base excitation and one-cycle sine wave Left and Right: Numerical Solution (Newmark- β method).

Different from the previous analysis within the scope of this dissertation, results now presented in Figure 4.8 indicate that the threshold acceleration amplitude for overturning the rigid block for numerical approaches. On the left Figure 4.8, the magnitude of the acceleration pulse is calculated as $a_p = 5.297 \text{ m/sec}^2$ (0.54g) using Newmark- β numerical method and block overturns, similarly on the right Figure 4.8, a_p is calculated as 5.297 m/sec^2 (0.54g) and block overturns for Runge-Kutta numerical algorithm. As it can clearly be observed from the Figures the outcomes obtained from the two solvers are in very good agreement.

Similarly, plots of numerical results in Figures 4.9 and 4.10 were obtained for the rectangular rigid block when subjected to harmonic cosine wave. Analyses were carried out with time step size $dt=0.0015$ sec for the first 6 seconds. Similar to the harmonic sinusoidal acceleration, herein Newmark- β and Runge-Kutta solution approach were adopted as the numerical integration technique.

As shown in Figure 4.9, numerical results indicate the maximum acceleration amplitude values that rectangular rigid block can resist on its base without overturning. On the left Figure 4.9, the threshold acceleration pulse is calculated as $a_p = 2.698 \text{ m/sec}^2$ (0.274g) using numerical Newmark- β method and block does not overturn but simply swing until it attains a stable equilibrium, similarly on the right Figure 4.9, a_p is calculated as 2.747 m/sec^2 (0.280g) and block does not overturn for Runge-Kutta numerical algorithm.

Results presented in Figure 4.10 indicate that the threshold acceleration amplitude for overturning the rigid block for numerical approaches. On the left Figure 4.10, the acceleration pulse amplitude is calculated as $a_p = 2.698 \text{ m/sec}^2$ (0.275g) using numerical method and block overturns, similarly on the right Figure 4.10, a_p is calculated as 2.757 m/sec^2 (0.281g) and block overturns for numerical algorithm. Consequently, results obtained using two numerical methods with linear formulation can give identical outcomes.

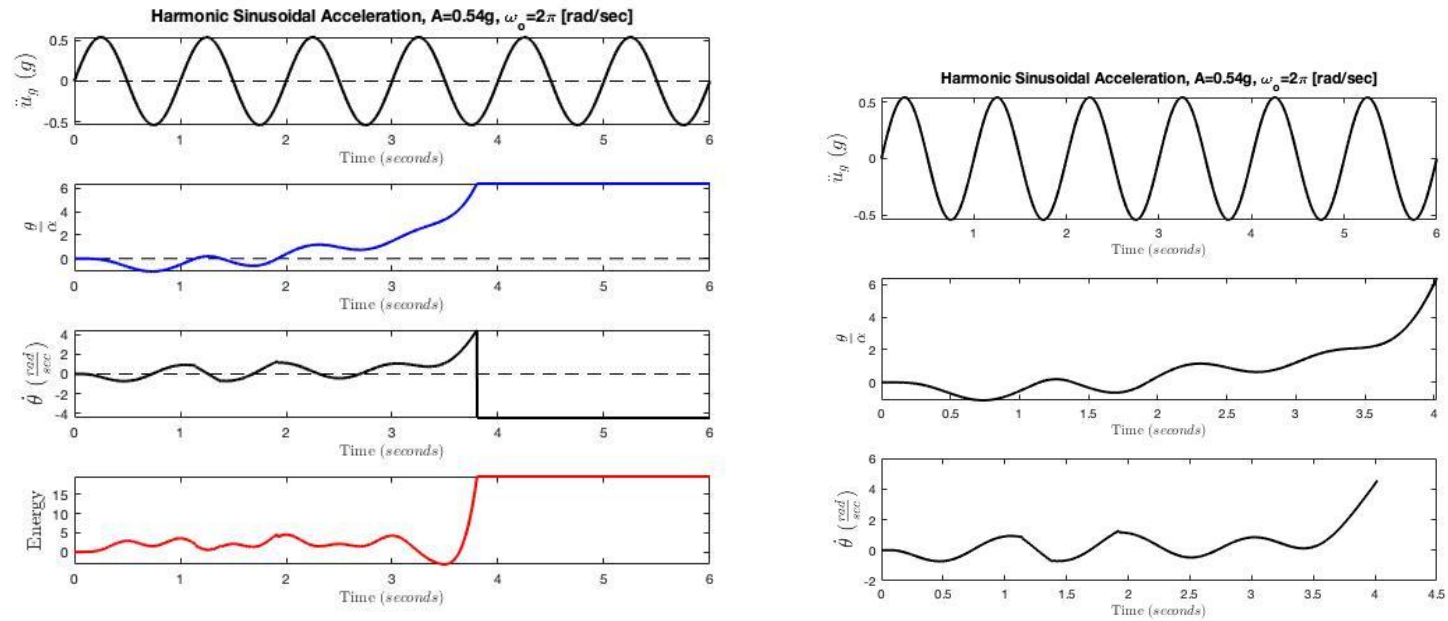


Figure 4.8. Time-domain histories of rotation and rotational velocity of a rigid block under sinusoidal harmonic base excitation Left: Numerical Solution (Newmark-β) Right: Numerical Solution (Runge-Kutta Method via ode45 solver) (Overturning).

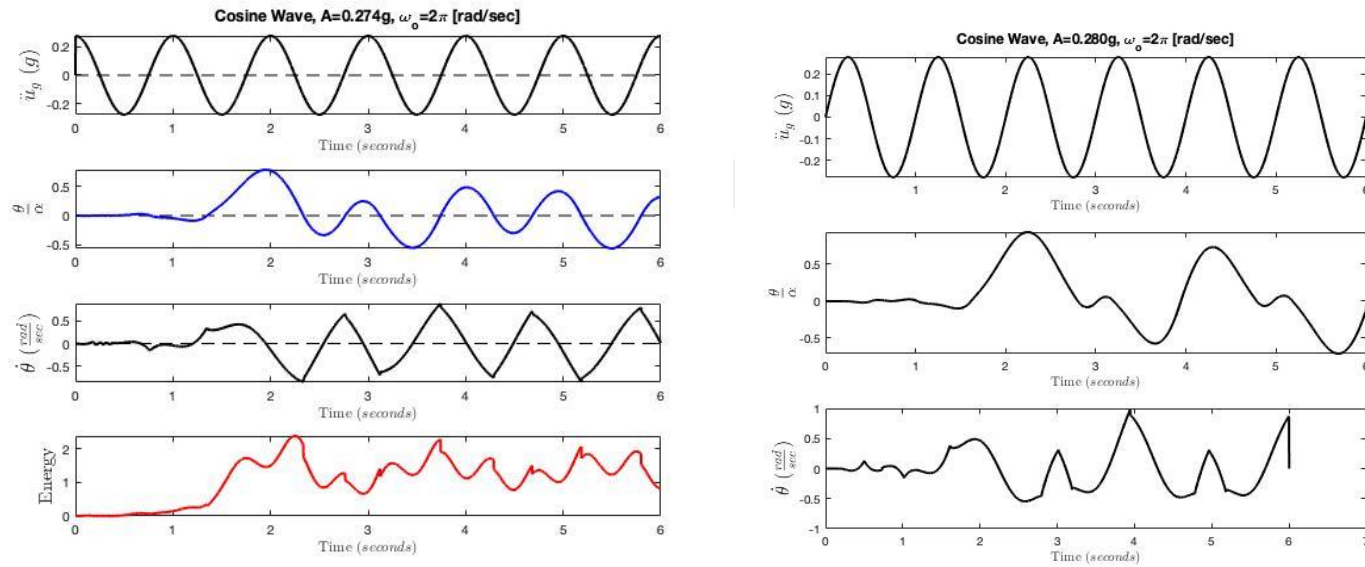


Figure 4.9. Time-domain histories of rotation and rotational velocity of a rigid block under cosine harmonic wave
 Left: Numerical Solution (Newmark- β) Right: Numerical Solution (Runge-Kutta Method via ode45 solver) (No Overturning).

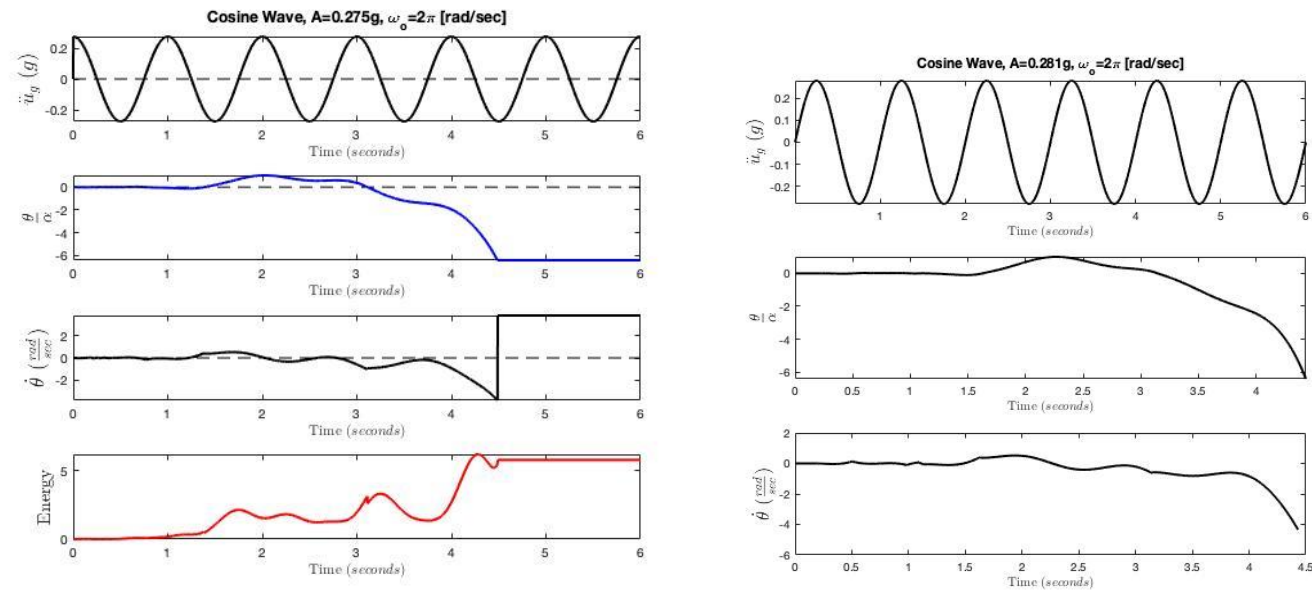


Figure 4.10. Time-domain histories of rotation and rotational velocity of a rigid block under cosine harmonic wave
 Left: Numerical Solution (Newmark- β) Right: Numerical Solution (Runge-Kutta Method via ode45 solver) (Overturning).

4.4.5. Dynamic Response to Pulse-like Excitations and Overturning Spectrum under Trigonometric Pulses

Numerical solutions for the rigid block subjected to pulse-like excitations given by Equation (3.35) and Equation (3.36) can be used to calculate nonlinear and linear dynamic response of the block. This part of the chapter proceeds with dynamic results in terms of rotation, angular velocity and total energy time histories of a single rigid block exposed to a variety of trigonometric pulses. Furthermore, the plots presented in Figure 4.11 through 4.22 show the threshold acceleration amplitude that is needed for overturning the rigid block subjected to pulse-like excitations. The study performed in this thesis for the cases involving one-cycle sine wave, half-cycle sine wave, rectangular constant acceleration pulse, Haar wavelet pulse, antisymmetric Ricker acceleration pulse and symmetric Ricker acceleration pulse, respectively.

Similar to the previous models, analyses were carried out for two cases, where in first case numerical results are obtained under one type of pulse-like excitation using Newmark- β integration method. Second case consists of the numerical solutions obtained from Runge-Kutta algorithm within the limits of linear approximation. To validate the accuracy of the path followed in the present study the same model was used for comparison of numerical solutions of linearized form of equation of motion in this dissertation.

4.4.5.1. Response to Full-cycle Sine Wave

Analysis carried out for a slender rectangular block having dimensions of $2b=0.4\text{m}$, $2h=1.6\text{m}$ in width and height. As mentioned earlier in Table 4.1, other geometric characteristics such as the angle of slenderness $\alpha = 0.245 \text{ (rad)}$, half-diagonal is $R=0.8246\text{m}$ and frequency parameter is $p = 2.9871 \text{ rad/sec}$. The schematic of this rocking rigid model is shown in Figure 4.2. Figure 4.11 and 4.12 present angular displacement, angular velocity and total energy time histories of a rectangular block under one-cycle sine wave. In addition, Figure 4.11 and 4.12

present the graphical comparison of two numerical methods namely Newmark time integration and Runge-Kutta method. Obtained results indicate that a good overall agreement appears to exist between the obtained result, even though values were not exactly the same.

Numerical results presented in Figure 4.11 indicate the maximum acceleration amplitude value that rectangular rigid block can resist on its base without overturning. On the left Figure 4.11, the threshold amplitude of acceleration is calculated as $a_p = 2.943 \text{ m/sec}^2$ (0.30g) using numerical Newmark- β method and block does not overturn but simply swing until it attains a stable equilibrium, similarly on the right Figure 4.11, a_p is calculated as 2.943 m/sec^2 (0.30g) and block does not overturn for Runge-Kutta numerical algorithm.

Results presented in Figure 4.12 indicate that the minimum necessary acceleration amplitude to topple the rigid block for analytical and numerical approaches. On the left Figure 4.12, the amplitude of acceleration is calculated as $a_p = 2.943 \text{ m/sec}^2$ (0.30g) using analytical method and block overturns, similarly on the right Figure 4.12, a_p is calculated as 2.943 m/sec^2 (0.30g) and block overturns for numerical algorithm. Results obtained using numerical method with linear formulation appear to be in good overall agreement with analytical results.

According to Nikos Makris and Yiannis Roussos [4], the alternative linear equation for calculating minimum required acceleration amplitude based on same range of frequency ratio ($\omega_p/p < 3.0$) can be written as follows:

$$\frac{a_p}{\alpha g} \approx 1 + \frac{1}{6} \frac{\omega_p}{p} \quad (4.14)$$

From Equation (4.14) under one-cycle sine wave the threshold acceleration amplitude required to topple the block is $a_p = 3.246 \text{ m/sec}^2$ (0.33g).

As an alternative to previous studies, the valid equation that gives the threshold acceleration amplitude necessary to topple a rectangular block under full-cycle sine wave was

derived. The numerical solver utilized in this dissertation for writing down nonlinear polynomial equation is ODE solvers in MATLAB. Among various ordinary differential equation (ODE) solvers which are provided by MATLAB the Ode45 which is based on Runge-Kutta formula was selected. In this study, Ode45 solver with tolerances in the order of 10^{-12} (RelTol) to 10^{-14} (AbsTol) for planar rocking problems was adopted to solve nonlinear form of equation of motion for a given block. In this study, a series of analyses were performed for a range of frequency ratio $\left(\frac{\omega_p}{p}\right)$. For each case, the necessary minimum acceleration amplitude values were calculated and the normalized overturning acceleration values of one-cycle sine pulse, $\left(\frac{a_p}{\alpha g}\right)$, was plotted against the frequency ratio, $\left(\frac{\omega_p}{p}\right)$. Using all the results obtained from analyses, a nonlinear polynomial equation in the form of $p(x)=p_1x^n + p_2x^{n-1}... + p_nx + p_{n+1}$ was derived and can be written as:

$$\frac{a_p}{\alpha g} \approx 0.05206 \left(\frac{\omega_p}{p}\right)^2 - 0.03013 \left(\frac{\omega_p}{p}\right) + 1.058 \quad (4.15)$$

From Equation (4.15) under one-cycle sine wave the threshold acceleration amplitude needed for overturning the block is $a_p = 2.943\text{m/sec}^2$ (0.30g).

Lastly, to prove the exactness of the proposed equation the threshold acceleration amplitude necessary to topple rigid block obtained at the end of the analysis was compared to previous equations proposed by researchers. It is seen that numerical results obtained from the proposed equation, under the assumptions of the nonlinear approximation, demonstrate good agreement with those reported in prior studies.

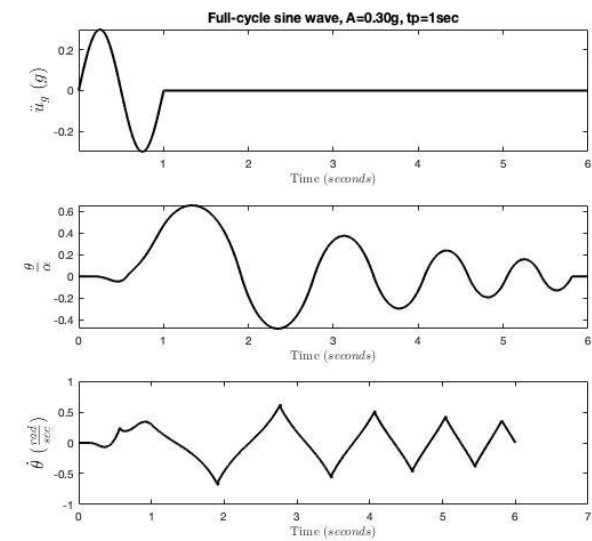
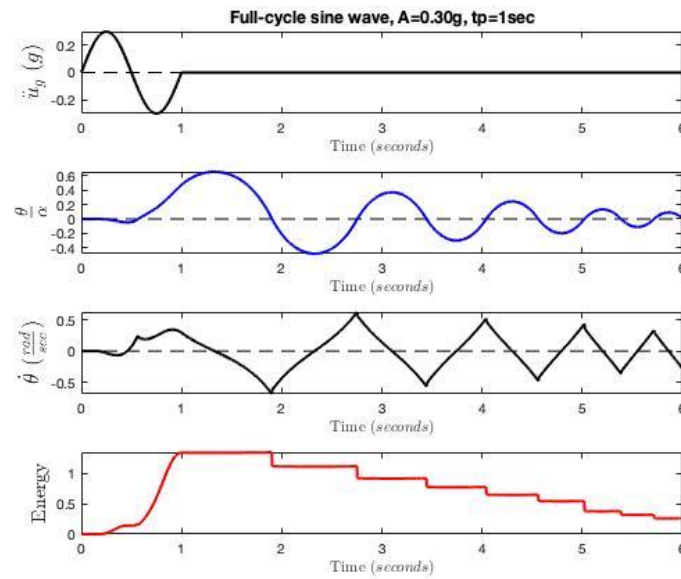


Figure 4.11. Time-domain histories of rotation and rotational velocity of a rigid block under one-cycle sine wave
 Left: Numerical Solution (Newmark- β) Right: Numerical Solution (Runge-Kutta Method via ode45 solver) (No Overturning).

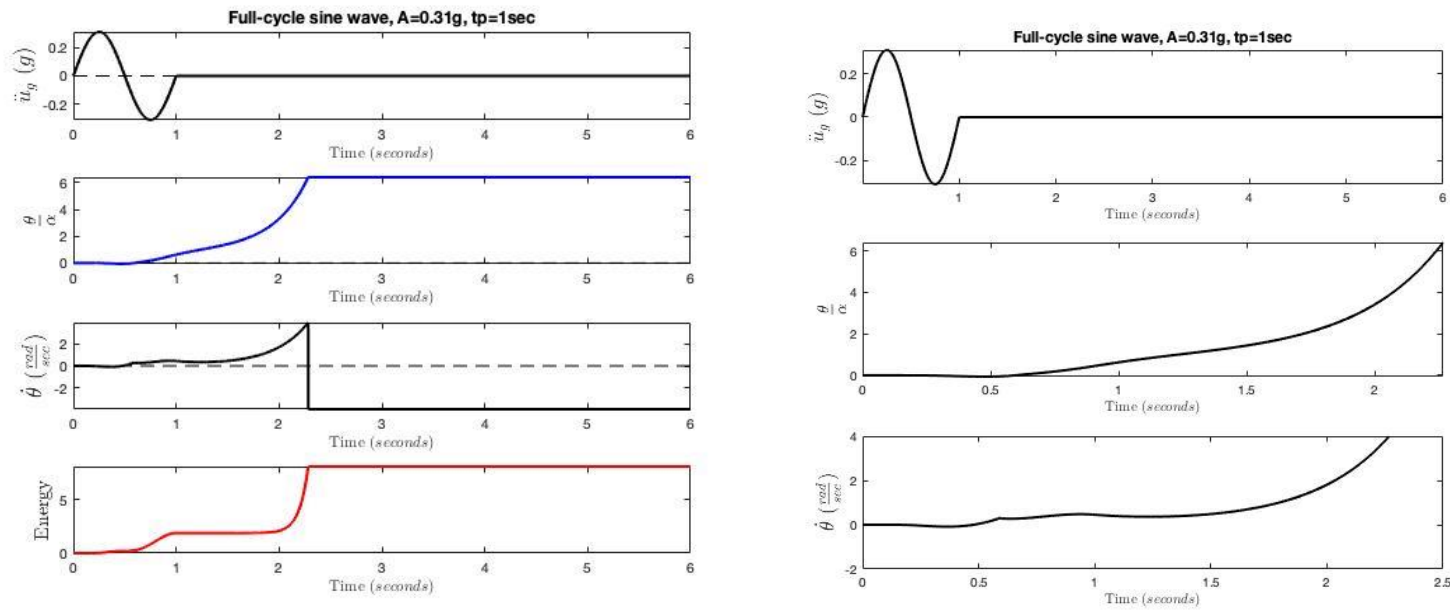


Figure 4.12. Time-domain histories of rotation and rotational velocity of a rigid block under one-cycle sine wave
 Left: Numerical Solution (Newmark-β) Right: Numerical Solution (Runge-Kutta Method via ode45 solver) (Overturning).

4.4.5.2. Response to Half-cycle Sine Wave

Analysis of the same numerical model was performed assuming a slender rectangular rigid block exposed to half-cycle sine pulse. Geometrical characterization parameters of this block (is the same model used for the rest of the cases investigated in this study) including width and height ($2b$ and $2h$), the angle of slenderness (α), half-diagonal (R), frequency parameter (p) are summarized in Table 4.1. Figures 4.13 and 4.14 plot the rotation, rotational velocity and total energy time histories of a stand-alone block undergoing half-cycle sine wave. The comparison of response quantities presented in Figures 4.13 and 4.14 indicate that a good overall agreement appears to exist between Newmark- β time integration and Runge-Kutta method, despite slight discrepancies in the values.

Numerical response quantities shown in Figure 4.13 indicate the maximum acceleration amplitude value that rectangular rigid block can withstand on its base without overturning. On the left Figure 4.13, the threshold acceleration pulse is calculated as $a_p = 4.415 \text{ m/sec}^2$ (0.45g) using numerical Newmark- β method and block does not overturn but simply swing until it attains a stable equilibrium, similarly on the right Figure 4.13, a_p is calculated as 4.415 m/sec^2 (0.45g) and block does not overturn for Runge-Kutta numerical algorithm.

In contrast, the obtained results presented in Figure 4.14 indicate the minimum acceleration amplitude necessary to induce the toppling of the rigid block for two numerical approaches. On the left Figure 4.14, the acceleration amplitude pulse is calculated as $a_p = 4.513 \text{ m/sec}^2$ (0.46g) using Newmark- β time integration method and block overturns, similarly on the right Figure 4.14, a_p is calculated as 4.513 m/sec^2 (0.46g) and block overturns for Runge-Kutta numerical algorithm. Results obtained using numerical methods with linear formulation appear to be in accordance with each other.

In Housner's paper [3], overturning equation by sinusoidal acceleration (half-cycle sine) for the range of frequency ratio ($\omega_p/p < 3.0$) can be written as follows:

$$\frac{a_p}{\alpha g} \approx \sqrt{1 + \left(\frac{\omega_p}{p}\right)^2} \quad (4.16)$$

or

$$\frac{a_p}{\alpha g} \approx \sqrt{1 + \frac{I_o}{WR} \left(\frac{2\pi}{T_g}\right)^2} \quad (4.17)$$

where a_p is the overturning amplitude of acceleration pulse and T_g is period of the ground acceleration. It is important to state the above relation was first derived by Kirkpatrick (1927) and it gives the required parameters which are required for overturning a rectangular rigid block under one-cycle sine pulse. According to Kirkpatrick (1927), the threshold acceleration amplitude required to topple the block is $a_p = 5.598 \text{ m/sec}^2$ (0.57g).

According to Nikos Makris and Yiannis Roussos [4], for a rigid block undergoing half-cycle sine wave, the alternative linear equation for calculating minimum overturning acceleration amplitude within the same range of frequency ratio ($\omega_p/p < 3.0$) is defined as follows:

$$\frac{a_p}{\alpha g} \approx 1 + \frac{1}{2} \frac{\omega_p}{p} \quad (4.18)$$

From Equation (4.18) under half-cycle sine wave the threshold acceleration amplitude necessary to induce the toppling of the rigid block is equal to $a_p = 4.931 \text{ m/sec}^2$ (0.50g).

In this dissertation, alternatively, the mathematical formula resulting in the minimum acceleration amplitude necessary to topple a rigid block under half-cycle sine wave was derived. Similarly, the numerical solver utilized in this dissertation for deriving nonlinear polynomial equation is Ode45 solver in MATLAB. In this case, a series of analyses were performed for a range of frequency ratio (ω_p/p). For each case, the required minimum acceleration amplitude values were determined. Subsequently, the normalized overturning acceleration values of half-

cycle sine pulse waveforms, $\left(\frac{a_p}{\alpha g}\right)$, was plotted against the frequency ratio, $\left(\frac{\omega_p}{p}\right)$. Based on the numerical results, a nonlinear polynomial equation in the form of $p(x)=p_1x^n + p_2x^{n-1}... + p_nx + p_{n+1}$ was derived. This equation can be expressed as:

$$\frac{a_p}{\alpha g} \approx 0.04568 \left(\frac{\omega_p}{p}\right)^2 + 0.3467 \left(\frac{\omega_p}{p}\right) + 0.9639 \quad (4.19)$$

According to Equation (4.19), under the influence of half-cycle sine wave, the threshold acceleration amplitude needed for overturning the block is $a_p = 4.513 \text{ m/sec}^2$ (0.46g).

To validate the accuracy of the proposed Equation (4.19), the threshold acceleration amplitude necessary to topple rigid block, as determined through the analysis, was compared to previously established equations from other researchers. The results obtained from the proposed equation, within the bounds of the nonlinear approximation, exhibit strong agreement with findings from prior studies.

Another important finding of the study indicates that the critical threshold for the minimum acceleration amplitude of the analysis model under one-cycle sine input is less than the acceleration amplitude value which is required to topple the same rectangular rigid block model under half-cycle sine wave. Changing excitation input from one-cycle sine to half-cycle sine wave was determined to play a significant role on the dynamic response of the analysis model. The minimum overturning acceleration amplitudes under half-cycle and one-cycle sine pulse for six different blocks having different slenderness, $\alpha=10^\circ$ (smaller block), 10° (larger block), 14° , 15° , 20° and 30° as shown in Table 4.3 were obtained in this study. With the assumption of linearized dynamic equation of motion of rigid block, the numerical outcomes were obtained via a numerical solver which is Ode45 in MATLAB. Comparison of the minimum required acceleration amplitude values obtained from numerical analyses indicated that under one-cycle sine wave the rectangular rigid block is more stable than the same analysis model excited by half-cycle sine pulse.

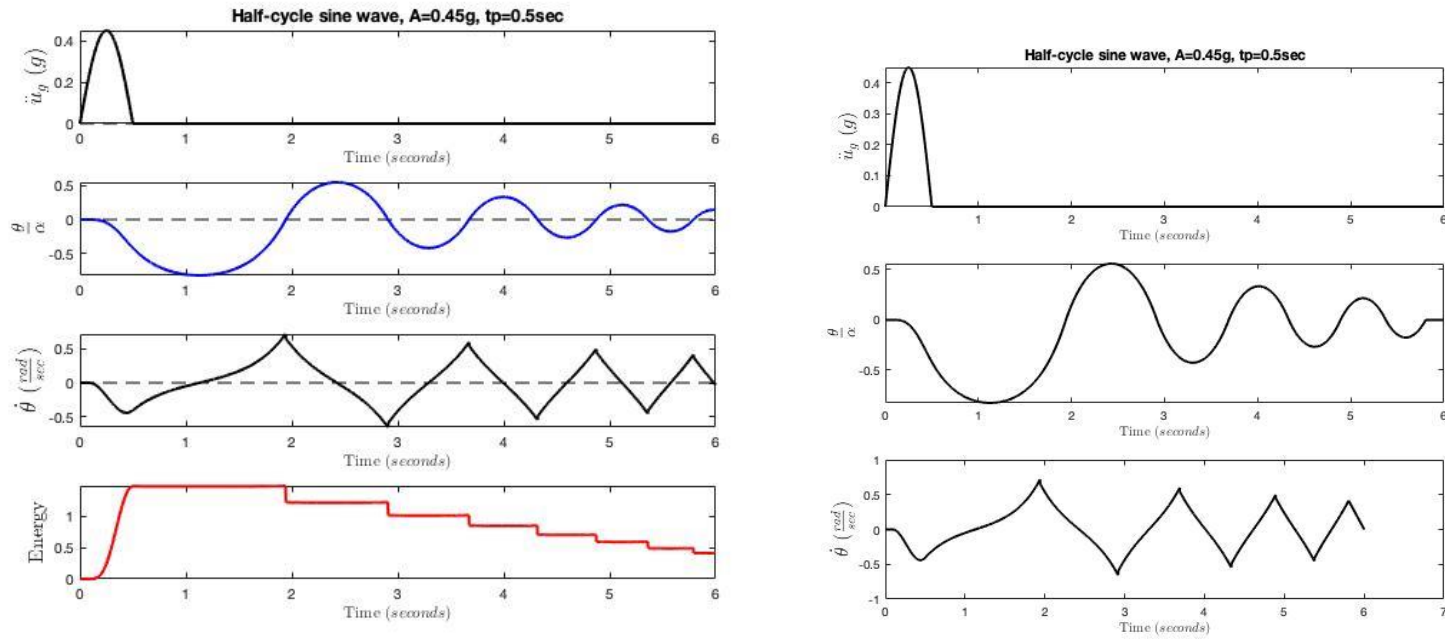


Figure 4.13. Time-domain histories of rotation and rotational velocity of a rigid block under half-cycle sine wave
 Left: Numerical Solution (Newmark- β) Right: Numerical Solution (Runge-Kutta Method via ode45 solver) (No Overturning).

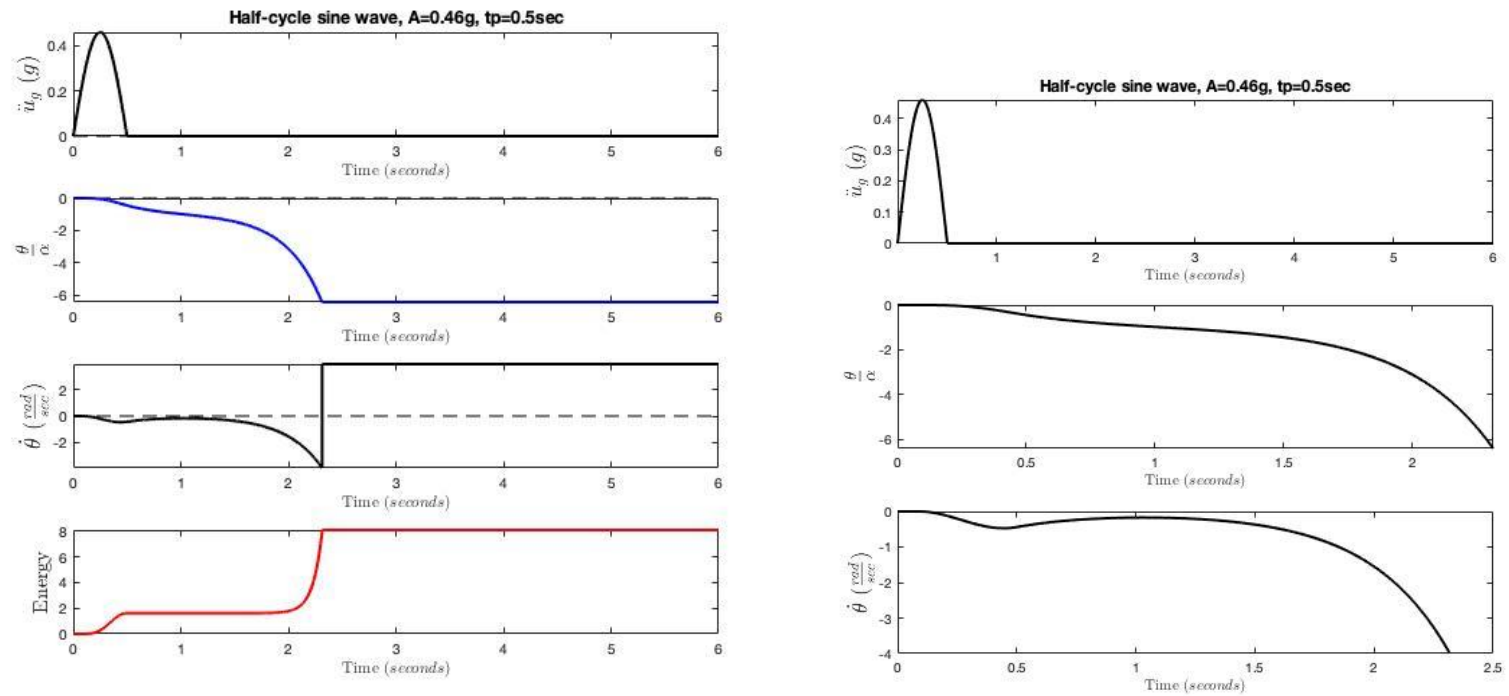


Figure 4.14. Time-domain histories of rotation and rotational velocity of a rigid block under half-cycle sine wave
 Left: Numerical Solution (Newmark-β) Right: Numerical Solution (Runge-Kutta Method via ode45 solver) (Overturning).

Table 4.2. Geometric characteristics of the slender blocks.

Blocks ID	B (m)	H (m)	$b = B/2$	$h = H/2$	R(m)	α (rad)	$\alpha(^{\circ})$	$p(\text{rad/sec})$
N1	0.3	1.7	0.15	0.85	0.8631	0.17467	10.008 (smaller)	2.9197
N2	0.9	5.1	0.45	2.55	2.5894	0.17467	10.008 (larger)	1.6856
N3	0.4	1.6	0.2	0.8	0.8246	0.24498	14.036	2.9871
N4	0.46	1.7	0.23	0.85	0.8806	0.26426	15.141	2.8905
N5	0.6	1.6	0.3	0.8	0.8544	0.35877	20.556	2.9345
N6	0.9	1.5	0.45	0.75	0.8746	0.54042	30.964	2.9004

Table 4.3. Overturning minimum acceleration amplitude under half-cycle and one-cycle sine pulses for blocks having different slenderness, α .

Blocks ID	R(m)	α (rad)	$\alpha(^{\circ})$	Half-cycle sine wave ($t_p=1\text{sec}$)	One-cycle sine wave ($t_p=1\text{sec}$)
N1	0.8631	0.17467	10.008 (smaller one)	3.2373 m/s ² (0.33g)	2.1582 m/s ² (0.22g)
N2	2.5894	0.17467	10.008 (larger one)	4.6107 m/s ² (0.47g)	2.7468 m/s ² (0.28g)
N3	0.8246	0.24498	14.036	4.513m/s ² (0.46g)	3.041m/s ² (0.31g)
N4	0.8806	0.26426	15.141	4.905m/s ² (0.50g)	3.3354m/s ² (0.34g)
N5	0.8544	0.35877	20.556	6.6708m/s ² (0.68g)	4.5126m/s ² (0.46g)
N6	0.8746	0.54042	30.964	10.791m/s ² (1.1g)	7.0632m/s ² (0.72g)

4.4.5.3. Response to Rectangular Constant Acceleration Pulse

Analysis of the same numerical model was performed assuming a slender rectangular rigid block exposed to rectangular constant acceleration pulse. Geometrical characterization parameters of this block (is the same model used for the rest of the cases investigated in this study) including width and height ($2b$ and $2h$), the angle of slenderness (α), half-diagonal (R), frequency parameter (p) are summarized in Table 4.1. Figures 4.15 and 4.16 plots the rotation, rotational velocity and total energy time histories of a rigid block undergoing rectangular constant acceleration pulse. The comparison of response quantities presented in Figures 4.15 and 4.16 indicate that a good overall agreement appears to exist between Newmark- β time integration and Runge-Kutta method, despite slight discrepancies in the values.

Numerical response quantities shown in Figure 4.15 indicate the maximum acceleration amplitude value that rectangular rigid block can withstand on its base without overturning. On the left Figure 4.15, the threshold acceleration pulse is calculated as $a_p = 4.611 \text{ m/sec}^2$ ($0.47g$) using numerical Newmark- β method and block does not overturn but simply swing until it attains a stable equilibrium, similarly on the right Figure 4.15, a_p is calculated as 4.513 m/sec^2 ($0.46g$) and block does not overturn for Runge-Kutta numerical algorithm.

In contrast, the obtained results presented in Figure 4.16 indicate the minimum acceleration amplitude necessary to induce the toppling of the rigid block for two numerical approaches. On the left Figure 4.16, the acceleration amplitude pulse is calculated as $a_p = 4.709 \text{ m/sec}^2$ ($0.48g$) using Newmark- β time integration method and block overturns, similarly on the right Figure 4.16, a_p is calculated as 4.611 m/sec^2 ($0.47g$) and block overturns for Runge-Kutta numerical algorithm. Results obtained using numerical methods with linear formulation appear to be in accordance with each other.

In this dissertation, alternatively, the mathematical formula resulting in the minimum acceleration amplitude necessary to topple a rigid block under rectangular constant acceleration pulse was derived. Similarly, the numerical solver utilized in this dissertation for deriving nonlinear polynomial equation is Ode45 solver in MATLAB. In this case, a series of analyses were performed for a range of frequency ratio (ω_p/p) . For each case, the required minimum acceleration amplitude values were determined. Subsequently, the normalized overturning acceleration values of half-cycle sine pulse waveforms, $(a_p/\alpha g)$, was plotted against the frequency ratio, (ω_p/p) . Based on the numerical results, a nonlinear exponential equation in the form of $f(x) = ae^{(-bx)} + c$ was derived. This equation can be expressed as:

$$\frac{a_p}{\alpha g} \approx 8.137e^{(-10.77t_p)} + 1.343 \quad (4.20)$$

From Equation (4.20) under rectangular constant acceleration pulse the threshold acceleration amplitude necessary to topple the block is $a_p = 4.513 \text{ m/sec}^2$ (0.46g).

To validate the accuracy of the proposed Equation (4.20), the threshold acceleration amplitude necessary to topple rigid block, as determined through the analysis, was compared to previously established equations from other researchers. The results obtained from the proposed equation, within the bounds of the nonlinear approximation, exhibit strong agreement with findings from prior studies.

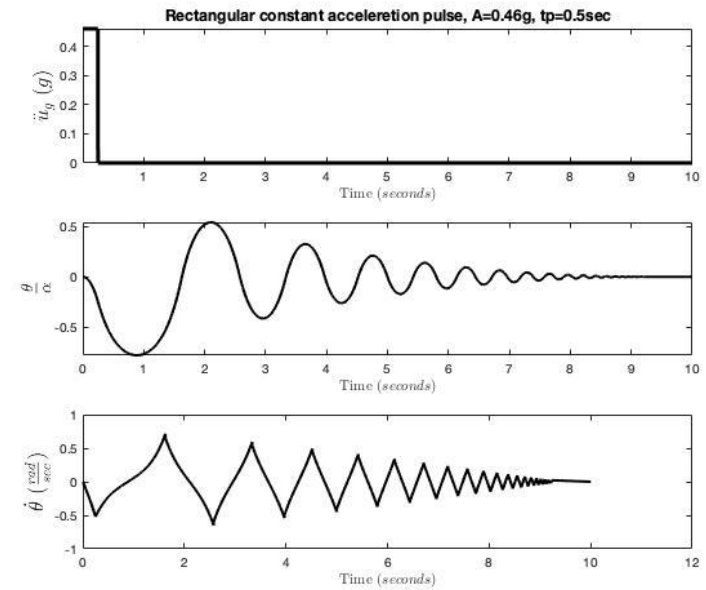
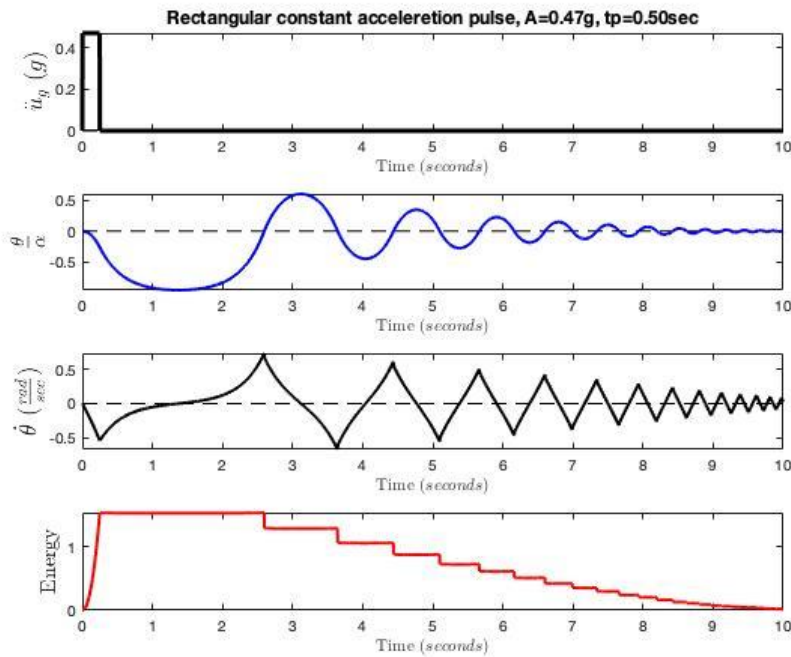


Figure 4.15. Time-domain histories of rotation and rotational velocity of a rigid block under rectangular constant acceleration pulse Left: Numerical Solution (Newmark- β) Right: Numerical Solution (Runge-Kutta Method via ode45 solver) (No Overturning).

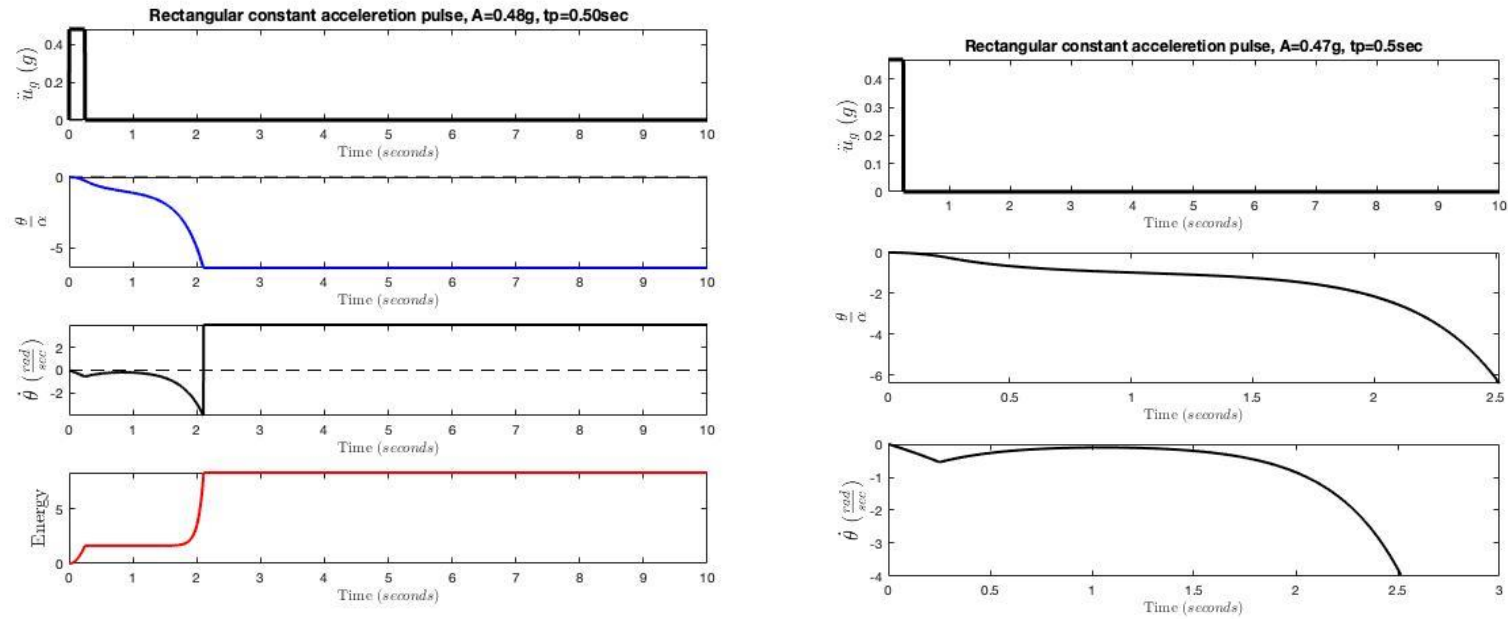


Figure 4.16. Time-domain histories of rotation and rotational velocity of a rigid block under rectangular constant acceleration pulse Left: Numerical Solution (Newmark- β) Right: Numerical Solution (Runge-Kutta Method via ode45 solver) (Overturning).

4.4.5.4. Response to Haar Wavelet Pulse

Analysis of the same numerical model was performed assuming a slender rectangular rigid block exposed to Haar wavelet pulse. Geometrical characterization parameters of this block (is the same model used for the rest of the cases investigated in this study) including width and height ($2b$ and $2h$), the angle of slenderness (α), half-diagonal (R), frequency parameter (p) are summarized in Table 4.1. Figures 4.17 and 4.18 plot the rotation, rotational velocity and total energy time histories of a rigid block undergoing Haar wavelet pulse. The comparison of response quantities presented in Figures 4.17 and 4.18 indicate that a good overall agreement appears to exist between Newmark- β time integration and Runge-Kutta method, despite slight discrepancies in the values.

Numerical response quantities shown in Figure 4.17 indicate the maximum acceleration amplitude value that rectangular rigid block can withstand on its base without overturning. On the left Figure 4.17, the threshold acceleration pulse is calculated as $a_p = 3.139 \text{ m/sec}^2$ ($0.32g$) using numerical Newmark- β method and block does not overturn but simply swing until it attains a stable equilibrium, similarly on the right Figure 4.17, a_p is calculated as 3.139 m/sec^2 ($0.32g$) and block does not overturn for Runge-Kutta numerical algorithm.

In contrast, the obtained results presented in Figure 4.18 indicate the minimum acceleration amplitude necessary to induce the toppling of the rigid block for two numerical approaches. On the left Figure 4.18, the amplitude of acceleration is calculated as $a_p = 3.237 \text{ m/sec}^2$ ($0.33g$) using Newmark- β time integration method and block overturns, similarly on the right Figure 4.18, a_p is calculated as 3.237 m/sec^2 ($0.33g$) and block overturns for Runge-Kutta numerical algorithm. Results obtained using numerical methods with linear formulation appear to be in accordance with each other.

In this dissertation, alternatively, the mathematical formula resulting in the minimum acceleration amplitude necessary to topple a rigid block under Haar wavelet pulse was derived. Similarly, the numerical solver utilized in this dissertation for deriving nonlinear polynomial equation is Ode45 solver in MATLAB. In this case, a series of analyses were performed for a range of frequency ratio (ω_p/p) . For each case, the required minimum acceleration amplitude values were determined. Subsequently, the normalized overturning acceleration values of Haar wavelet pulse waveforms, $(a_p/\alpha g)$, was plotted against the frequency ratio, (ω_p/p) . Based on the numerical results, a nonlinear exponential equation in the form of $f(x) = ae^{(-bx)} + c$ was derived. This equation can be expressed as:

$$\frac{a_p}{\alpha g} \approx 8.137e^{(-10.77t_p)} + 1.343 \quad (4.21)$$

From Equation (4.21) under Haar wavelet pulse the threshold acceleration amplitude necessary to topple the block is $a_p = 3.316 \text{ m/sec}^2$ (0.338g). In order to validate the accuracy of the proposed Equation (4.21), the threshold acceleration amplitude necessary to topple rigid block, as determined through the analysis, was compared to previously established equations from other researchers. The results obtained from the proposed equation, within the bounds of the nonlinear approximation, exhibit strong agreement with findings from prior studies.

Another important finding of the study indicates that the threshold acceleration amplitude for toppling of the analysis model under Haar wavelet input is less than the acceleration amplitude value which is necessary to topple the same rectangular rigid block model under rectangular acceleration pulse. The minimum overturning acceleration amplitudes under half-cycle and one-cycle sine pulse for six different blocks having different slenderness, $\alpha=10^\circ$ (smaller block), 10° (larger block), 14° , 15° , 20° and 30° as shown in Table 4.4 were obtained in this study. With the assumption of linearized dynamic equation of motion of rigid block, the numerical outcomes were obtained via a numerical solver which is Ode45 in MATLAB. Comparison of the minimum required acceleration amplitude values obtained from numerical analyses indicated that under Haar wavelet pulse the rectangular rigid block is more stable than the same analysis model excited by rectangular acceleration pulse.

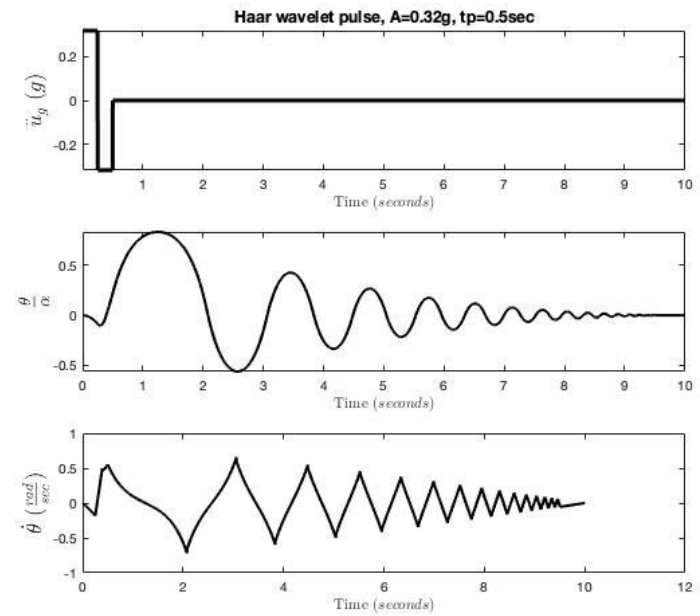
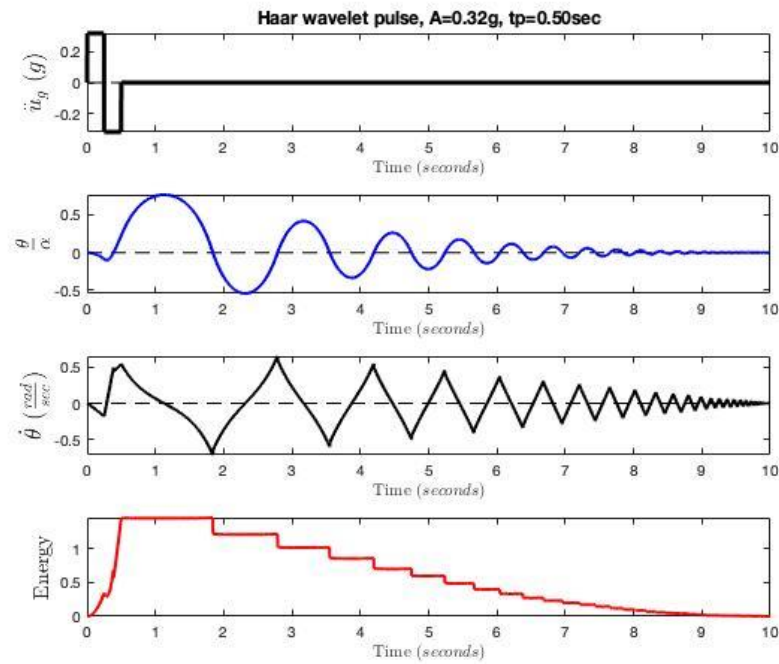


Figure 4.17. Time-domain histories of rotation and rotational velocity of a rigid block under Haar wavelet acceleration pulse
 Left: Numerical Solution (Newmark- β) Right: Numerical Solution (Runge-Kutta Method via ode45 solver) (No Overturning).

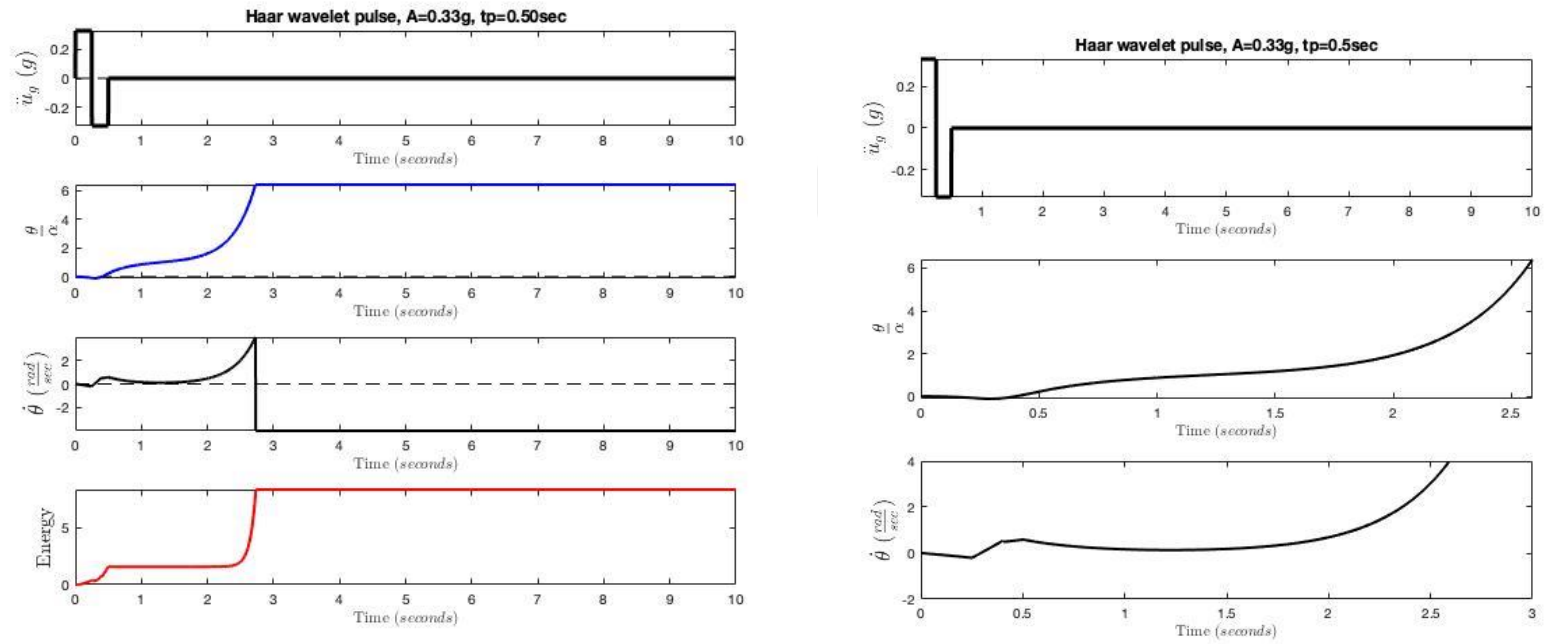


Figure 4.18. Time-domain histories of rotation and rotational velocity of a rigid block under Haar wavelet acceleration pulse
 Left: Numerical Solution (Newmark- β) Right: Numerical Solution (Runge-Kutta Method via ode45 solver) (Overturning).

Table 4.4. Overturning minimum acceleration amplitude under rectangular constant acceleration and Haar wavelet pulse for blocks having different slenderness, α .

Blocks ID	R(m)	α (rad)	α (°)	Rectangular constant acceleration pulse ($t_p=0.5\text{sec}$)	Haar wavelet ($t_p=0.5\text{sec}$)
N1	0.8631	0.17467	10.008 (smaller one)	3.3354 m/s ² (0.34g)	2.2563 m/s ² (0.23g)
N2	2.5894	0.17467	10.008 (larger one)	5.0031 m/s ² (0.51g)	7.4556 m/s ² (0.76g)
N3	0.8246	0.24498	14.036	4.6107m/s ² (0.47g)	3.2373m/s ² (0.33g)
N4	0.8806	0.26426	15.141	5.1012m/s ² (0.52g)	3.6297m/s ² (0.37g)
N5	0.8544	0.35877	20.556	6.867m/s ² (0.70g)	5.3955m/s ² (0.55g)
N6	0.8746	0.54042	30.964	10.791m/s ² (1.1g)	20.601m/s ² (2.1g)

4.4.5.5. Response to Antisymmetric Ricker Acceleration Pulse

Analysis of the same numerical model was performed assuming a slender rectangular rigid block exposed to antisymmetric Ricker acceleration pulse. Geometrical characterization parameters of this block (is the same model used for the rest of the cases investigated in this study) including width and height (2b and 2h), the angle of slenderness (α), half-diagonal (R), frequency parameter (p) are summarized in Table 4.1. Figure 4.19 and 4.20 plot the rotation, rotational velocity and total energy time histories of a rigid block undergoing antisymmetric Ricker acceleration pulse. The comparison of response quantities presented in Figures 4.19 and 4.20 indicate that a good overall agreement appears to exist between Newmark- β time integration and Runge-Kutta method, despite slight discrepancies in the values.

Numerical response quantities shown in Figure 4.19 indicate the maximum acceleration amplitude value that rectangular rigid block can withstand on its base without overturning. On the left Figure 4.19, the threshold acceleration pulse is calculated as $a_p = 8.338 \text{ m/sec}^2$ (0.85g) using numerical Newmark- β method and block does not overturn but simply swing until it

attains a stable equilibrium, similarly on the right Figure 4.19, a_p is calculated as 8.338 m/sec^2 (0.85g) and block does not overturn for Runge-Kutta numerical algorithm.

In contrast, the obtained results presented in Figure 4.20 indicate the minimum acceleration amplitude necessary to induce the toppling of the rigid block for two numerical approaches. On the left Figure 4.20, the amplitude of acceleration is calculated as $a_p = 8.437 \text{ m/sec}^2$ (0.86g) using Newmark- β time integration method and block overturns, similarly on the right Figure 4.20, a_p is calculated as 8.437 m/sec^2 (0.86g) and block overturns for Runge-Kutta numerical algorithm. Results obtained using numerical methods with linear formulation appear to be in accordance with each other. In this dissertation, alternatively, the mathematical formula resulting in the minimum acceleration amplitude necessary to topple a rigid block under antisymmetric Ricker acceleration pulse was derived. Similarly, the numerical solver utilized in this dissertation for deriving nonlinear polynomial equation is Ode45 solver in MATLAB. In this case, a series of analyses were performed for a range of frequency ratio (ω_p/p) . For each case, the required minimum acceleration amplitude values were determined. Subsequently, the normalized overturning acceleration values of antisymmetric Ricker acceleration pulse waveforms, $(a_p/\alpha g)$, was plotted against the frequency ratio, (ω_p/p) . Based on the numerical results, a nonlinear polynomial equation in the form of $p(x) = p_1x^n + p_2x^{n-1} \dots + p_nx + p_{n+1}$ was derived. This equation can be expressed as:

$$\frac{a_p}{\alpha g} \approx -4.864(t_p) + 7.513 \quad (4.22)$$

From Equation (4.22), for a rocking rigid block under antisymmetric Ricker pulse the threshold acceleration amplitude necessary to topple the block is $a_p = 8.705 \text{ m/sec}^2$ (0.88g). To validate the accuracy of the proposed Equation (4.22), the threshold acceleration amplitude necessary to topple rigid block, as determined through the analysis, was compared to previously established equations from other researchers. The results obtained from the proposed equation, within the bounds of the nonlinear approximation, exhibit strong agreement with findings from prior studies.

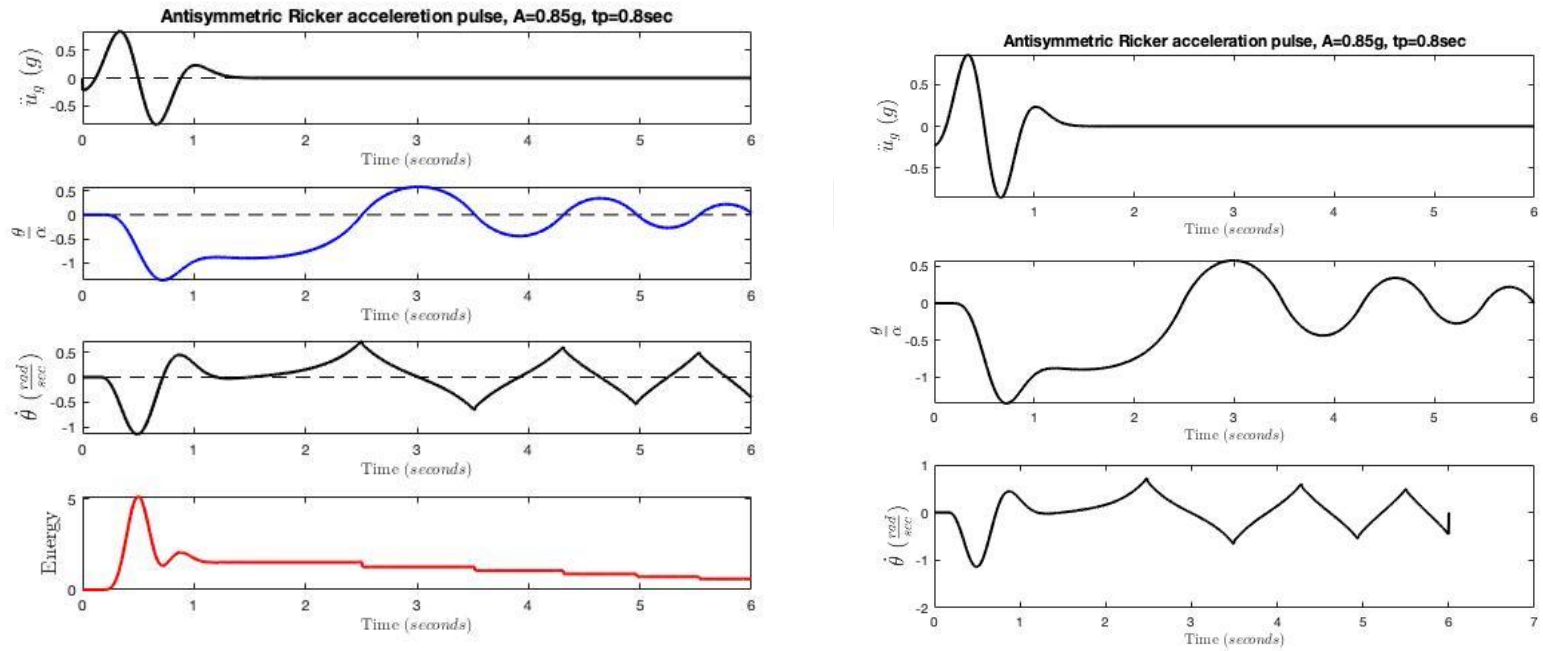


Figure 4.19. Time-domain histories of rotation and rotational velocity of a rigid block under Antisymmetric Ricker acceleration pulse Left: Numerical Solution (Newmark- β) Right: Numerical Solution (Runge-Kutta Method via ode45 solver) (No Overturning).

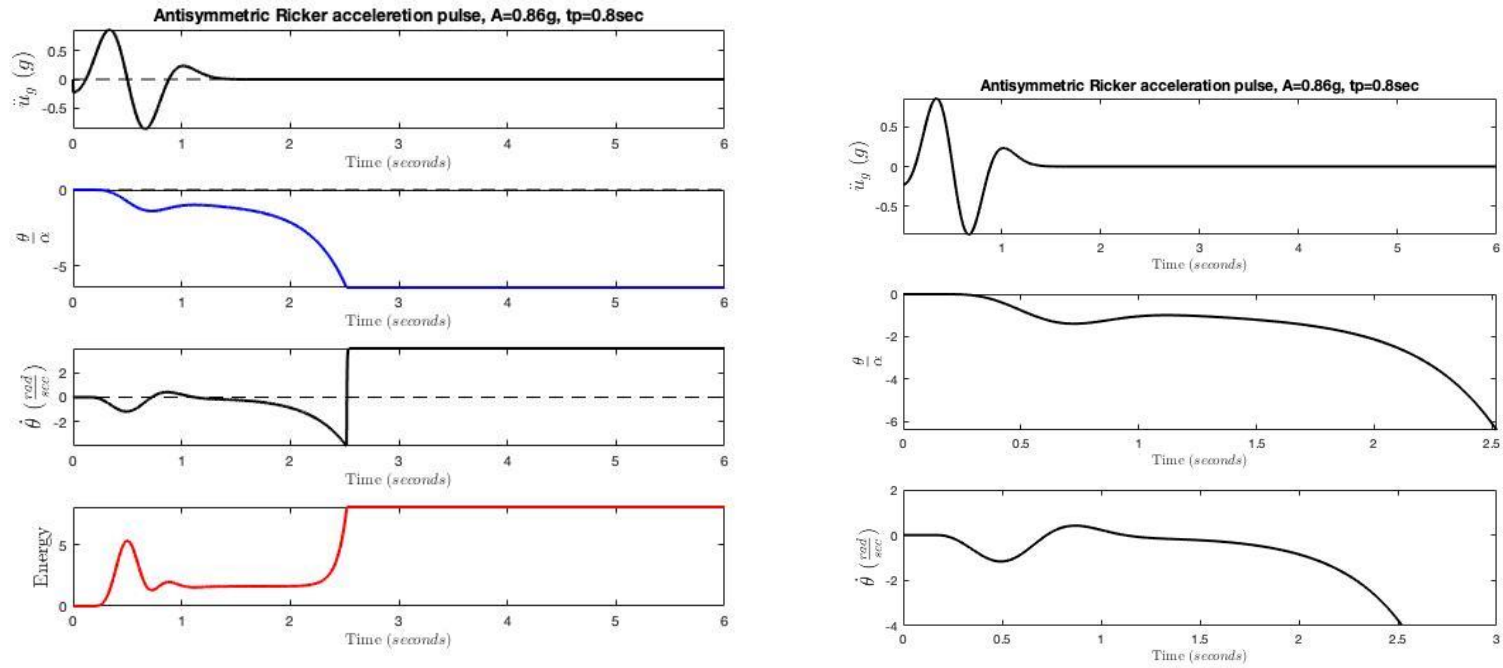


Figure 4.20. Time-domain histories of rotation and rotational velocity of a rigid block block under Antisymmetric Ricker acceleration pulse Left: Numerical Solution (Newmark- β) Right: Numerical Solution (Runge-Kutta Method via ode45 solver) (Overturning).

4.4.5.6. Response to Symmetric Ricker Acceleration Pulse

Analysis of the same numerical model was performed assuming a slender rectangular rigid block exposed to symmetric Ricker acceleration pulse. Geometrical characterization parameters of this block (is the same model used for the rest of the cases investigated in this study) including width and height ($2b$ and $2h$), the angle of slenderness (α), half-diagonal (R), frequency parameter (p) are summarized in Table 4.1. Figure 4.21 and 4.22 plot the rotation, rotational velocity and total energy time histories of a rigid block undergoing symmetric Ricker acceleration pulse. The comparison of response quantities presented in Figures 4.21 and 4.22 indicate that a good overall agreement appears to exist between Newmark- β time integration and Runge-Kutta method, despite slight discrepancies in the values.

Numerical response quantities shown in Figure 4.21 indicate the maximum acceleration amplitude value that rectangular rigid block can withstand on its base without overturning. On the left Figure 4.21, the threshold acceleration pulse is calculated as $a_p = 6.180 \text{ m/sec}^2$ (0.63g) using numerical Newmark- β method and block does not overturn but simply swing until it attains a stable equilibrium, similarly on the right Figure 4.21, a_p is calculated as 6.180 m/sec^2 (0.63g) and block does not overturn for Runge-Kutta numerical algorithm.

In contrast, the obtained results presented in Figure 4.22 indicate the minimum acceleration amplitude necessary to induce the toppling of the rigid block for two numerical approaches. On the left Figure 4.22, the amplitude of acceleration is calculated as $a_p = 6.278 \text{ m/sec}^2$ (0.64g) using Newmark- β time integration method and block overturns, similarly on the right Figure 4.22, a_p is calculated as 6.278 m/sec^2 (0.64g) and block overturns for Runge-Kutta numerical algorithm. Results obtained using numerical methods with linear formulation appear to be in accordance with each other.

In this dissertation, alternatively, the mathematical formula resulting in the minimum acceleration amplitude necessary to topple a rigid block under symmetric Ricker acceleration pulse was derived. Similarly, the numerical solver utilized in this dissertation for deriving nonlinear polynomial equation is Ode45 solver in MATLAB. In this case, a series of analyses were performed for a range of frequency ratio $\left(\frac{\omega_p}{p}\right)$. For each case, the required minimum acceleration amplitude values were determined. Subsequently, the normalized overturning acceleration values of symmetric Ricker acceleration pulse waveforms, $\left(\frac{a_p}{\alpha g}\right)$, was plotted against the frequency ratio, $\left(\frac{\omega_p}{p}\right)$. Based on the numerical results, a nonlinear polynomial equation in the form of $p(x)=p_1x^n + p_2x^{n-1} \dots + p_nx + p_{n+1}$ was derived. This equation can be expressed as:

$$\frac{a_p}{\alpha g} \approx 0.1498 \left(\frac{\omega_p}{p}\right)^2 + 0.01533 \left(\frac{\omega_p}{p}\right) + 1.047 \quad (4.23)$$

From Equation (4.23) under symmetric Ricker input the threshold for overturning acceleration amplitude is $a_p = 4.513 \text{ m/sec}^2$ (0.46g).

To validate the accuracy of the proposed Equation (4.23), the threshold acceleration amplitude necessary to topple rigid block, as determined through the analysis, was compared to previously established equations from other researchers. The results obtained from the proposed equation, within the bounds of the nonlinear approximation, exhibit strong agreement with findings from prior studies.

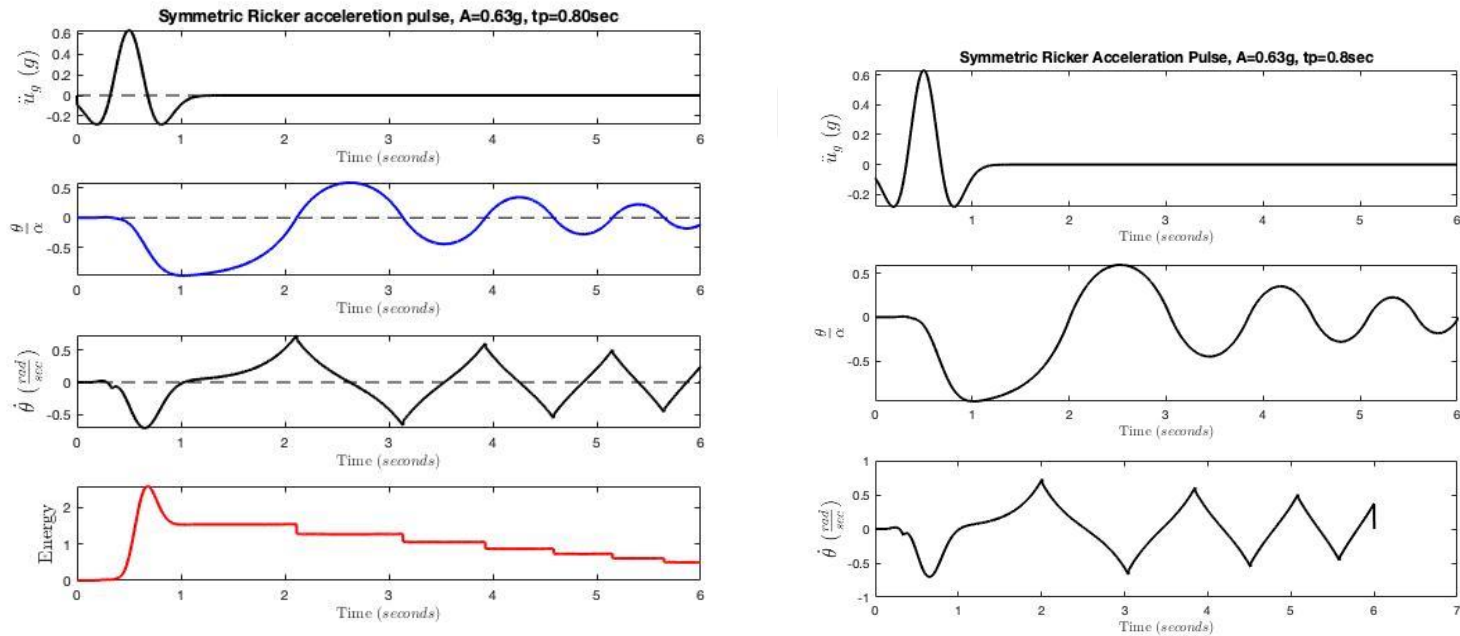


Figure 4.21. Time-domain histories of rotation and rotational velocity of a rigid block under Symmetric Ricker acceleration pulse Left: Numerical Solution (Newmark- β) Right: Numerical Solution (Runge-Kutta Method via ode45 solver) (No Overturning).

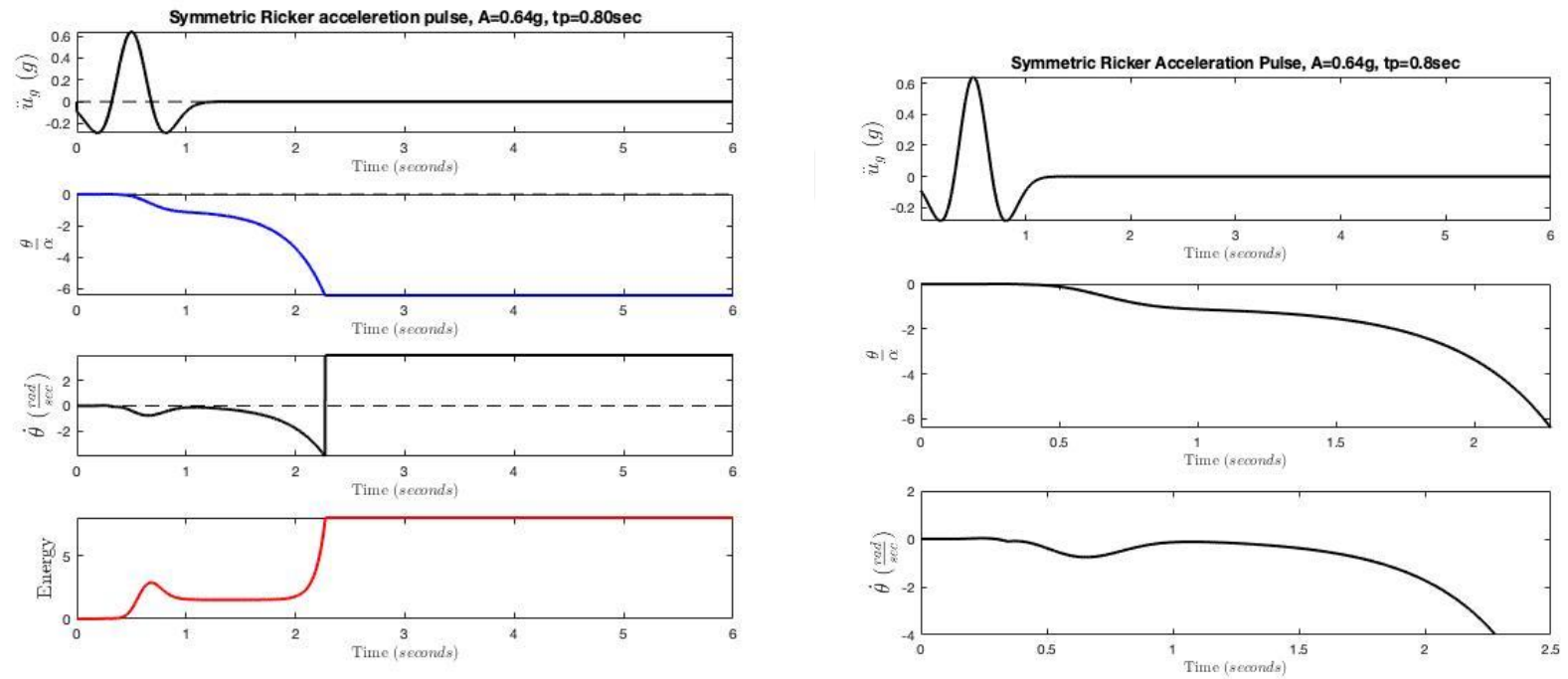


Figure 4.22. Time-domain histories of rotation and rotational velocity of a rigid block under Symmetric Ricker acceleration pulse Left: Numerical Solution (Newmark- β) Right: Numerical Solution (Runge-Kutta Method via ode45 solver) (Overturning).

4.4.5.7. Overturning Spectrum under Pulse-like Excitations

The section of this dissertation aimed at analysing the response of rectangular solid blocks exposed to a variety of pulse-like excitations and calculating the minimum required acceleration amplitude values leading to overturning of rigid block. For all cases studied in this thesis numerical results have been obtained in terms of normalized overturning acceleration of the pulse $(a_p/\alpha g)$ and the frequency ratio (ω_p/p) values that defines two main critical parameters of a pulse able to overturn a rectangular rigid block during analysis. Table 4.5 summarize all the geometric characteristics for each block ID including N1, N2, N3, N4, N5 and N6, respectively. As shown in Table 4.5, these geometric parameters have been employed to assess the dynamic performance of a rocking rigid block.

Table 4.5. Geometric characteristics of the slender blocks.

Specimens	B (m)	H (m)	$b = B/2$	$h = H/2$	R(m)	α (rad)	$\alpha(^{\circ})$	p (rad/sec)
N1	0.3	1.7	0.15	0.85	0.8631	0.17467	10.008 (smaller)	2.9197
N3	0.4	1.6	0.2	0.8	0.8246	0.24498	14.036	2.9871
N4	0.46	1.7	0.23	0.85	0.8806	0.26426	15.141	2.8905
N5	0.6	1.6	0.3	0.8	0.8544	0.35877	20.556	2.9345
N6	0.9	1.5	0.45	0.75	0.8746	0.54042	30.964	2.9004

Two series of analysis were carried out for each block under a specific type of pulse-like excitation. In the first part the nonlinear equation of motion of rocking rigid bodies under a pulse-like excitation was derived. Then, it is solved numerically via MATLAB programming. The results are presented in an overturning spectrum format for different characteristics of the rigid bodies. In the second part the linearized equation of motion of rocking rigid bodies under a pulse-like excitation was solved numerically. The results obtained from analysis were used to develop an overturning spectrum for different characteristics of the rigid bodies.

The first part of this section presents information regarding analyses outcomes for each block under one-cycle sine wave with amplitudes between αg and 0.35g with an increase in amplitude of 0.01g and frequencies between 0.1 and 4π with an increase in frequency of 0.25. At the end of the first analysis, an overturning acceleration spectrum (OAS) shown in Figure 4.23 was built using a numerical approach which is Runge-Kutta Method applied via ode45 solver in MATLAB. For the first case, this numerical integration technique was adopted to solve nonlinear form of dynamic equation of motion for each block ID. The second phase of the study yielded another overturning acceleration spectrum (OAS) shown in Figure 4.24 using Runge-Kutta numerical integration technique applied via ode45 solver. Differently from the first case, linearized form of equation of motion for each block specimen was solved numerically.

Three rectangular rigid blocks, with slenderness angles of 10° , 20° , and 30° , respectively, were analysed under a one-cycle sine wave (see Figure 4.23), assuming a nonlinear ordinary differential equation describing the block's dynamic motion. The equation of motion was solved numerically using a nonlinear state-space formulation and implemented in MATLAB.

Additionally, an overturning spectrum for a rigid block having slenderness angle, $\alpha=10^\circ$ was derived using an alternative linear equation for calculating minimum required acceleration amplitude proposed by Nikos Makris and Yiannis Roussos [4].

The validated expression in Equation (4.25) was subsequently utilized to generate the overturning acceleration amplitude spectrum (OAS) required to topple a block under a one-cycle sine wave. This equation was derived using data obtained from the nonlinear equation of motion for a rigid block with a slenderness angle of 10 subjected to a one-cycle sine wave. The data was initially plotted with linear scales on both axes. The plot indicates that a linear function is not an optimal fit for the data under a one-cycle sine wave, as the data points deviate from the Equation (4.14) proposed by Nikos Makris and Yiannis Roussos [4]. From the other possible functions, such as linear, exponential, logarithmic, reciprocal functions and power functions, only polynomial function (polynomial of second order) was a good choice to derive an

approximate equation for a rigid block under one-cycle sine wave. The polynomial function derived using MATLAB curve fitting interface have the following form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 \quad (4.24)$$

Here, the coefficients $a_n, a_{n-1}, \dots, a_1, a_0$ are real numbers, and n , which is a nonnegative integer, is the degree (order of the polynomial).

$$\frac{a_p}{\alpha g} \approx 0.05206 \left(\frac{\omega_p}{p} \right)^2 - 0.03013 \left(\frac{\omega_p}{p} \right) + 1.058 \quad (4.25)$$

The purple dashed line (polynomial of second order) shown in Figures 4.23 and 4.24 obtained from polynomial fitting method can give a good fit to all the data obtained from previous numerical analysis. Results obtained at the end of calculation procedures indicated that all the rocking rigid blocks having slenderness angle, α is less than 20 degrees passes through all the data points obtained from polynomial function (polynomial of second order). However, for cases with slenderness angle, α which is greater than 20 degrees exhibits a stiffening effect.

Another important point is that the comparison of the plots in Figure 4.23 and Figure 4.24 indicate that linearized and nonlinear numerical solution results in similar dynamic response for rectangular rigid blocks. But still, the numerical solution at each time step with the nonlinear approximation results in a high level of accuracy. Additionally, it is possible to get more precise results using small time steps leading to less numerical errors, however it requires more computational resources, as it involves solving a single set of nonlinear equations at each time step.

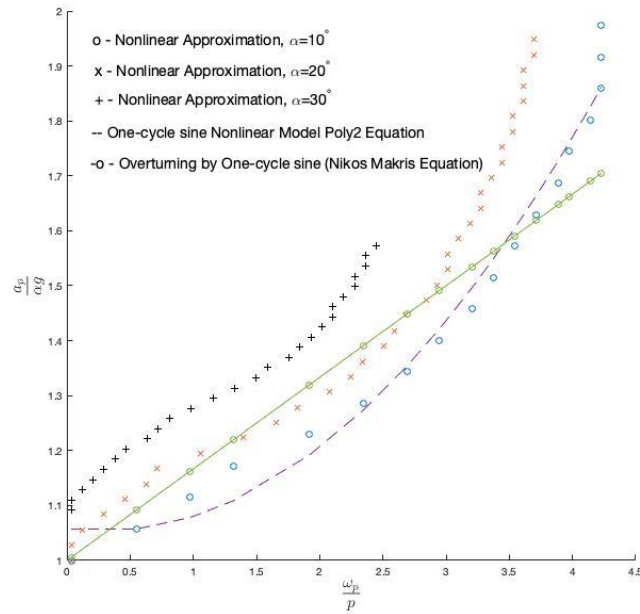


Figure 4.23. Spectrum of minimum acceleration of one-cycle sine wave required to overturn geometrically different rigid blocks ($\alpha=10^\circ$, 20° and 30° , respectively) with nonlinear formulation (Runge-Kutta Method via ode45 solver).

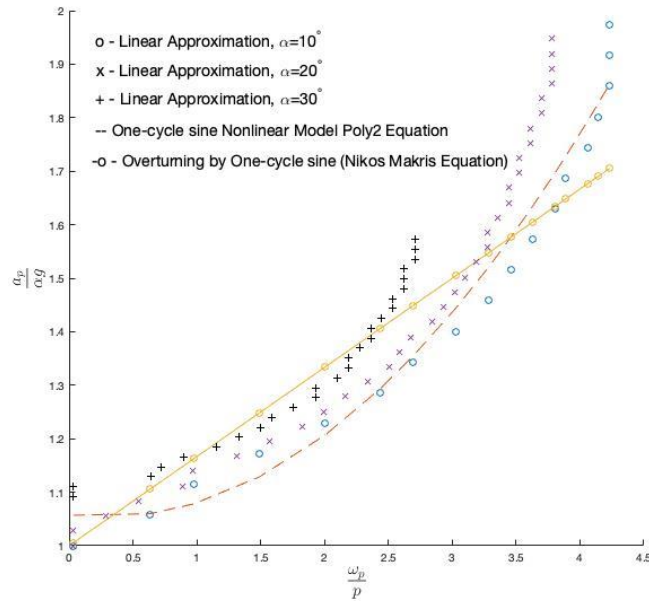


Figure 4.24. Spectrum of minimum acceleration of one-cycle sine wave required to overturn geometrically different rigid blocks ($\alpha=10^\circ$, 20° and 30° , respectively) with linear formulation (Runge-Kutta Method via ode45 solver).

The study's findings demonstrate that for non-slender blocks with slenderness angles exceeding 20° (e.g., 30°), the solution exhibits a stiffening effect for $\omega_p/p > 3, 2.5$. Conversely, for tall, slender blocks with slenderness angles less than 20 degrees, results reveal a near-linear correlation between threshold for overturning acceleration amplitude and frequency ω_p/p within the specified range of $\omega_p/p < 3.0$.

Furthermore, in order to validate the accuracy of Equation (4.25), a numerical analysis was conducted on six rectangular rigid blocks with slenderness ratios below 20 degrees. The analysis results, obtained using the numerical models detailed in Table 4.6, are presented in Figure 4.25. These six blocks, with slenderness angles of 4° , 6° , 8° , 10° , 12° , and 15° respectively, were subjected to a one-cycle sine wave excitation. The analysis assumed a nonlinear ordinary differential equation describing the block's dynamic motion, which was solved numerically using a nonlinear state-space formulation implemented in MATLAB.

Table 4.6. Dimensions of numerical models used in MATLAB script for validation purpose.

Specimens	B (m)	H (m)	$b = B/2$	$h = H/2$	R(m)	α (rad)	$\alpha(^{\circ})$	$p(\text{rad/sec})$
S1	0.10	1.4	0.05	0.7	0.7018	0.07131	4.086	2.9197
S2	0.15	1.4	0.075	0.7	0.7040	0.10674	6.116	2.9871
S3	0.20	1.4	0.10	0.7	0.7071	0.14189	8.1301	2.8905
S4	0.30	1.7	0.15	0.85	0.8631	0.17467	10.008	2.9197
S5	0.38	1.7	0.19	0.85	0.8746	0.21991	12.600	2.9004
S6	0.46	1.7	0.23	0.85	0.8806	0.26426	15.141	2.8905

In Figure 4.25, each plot showing the variation of minimum overturning acceleration amplitudes along the frequency parameter are compared with Equation (4.25) results derived under the section 4.4.5.7 of this chapter. Comparing of the obtained numerical results for each rectangular rigid body eventually led to the conclusion that derived polynomial Equation (4.25) results are in good agreement with numerical analysis results obtained from six rigid blocks,

therefore, the proposed approximate Equation (4.25) might be used for such cases having slenderness ratio less than 20° under one-cycle sine wave.

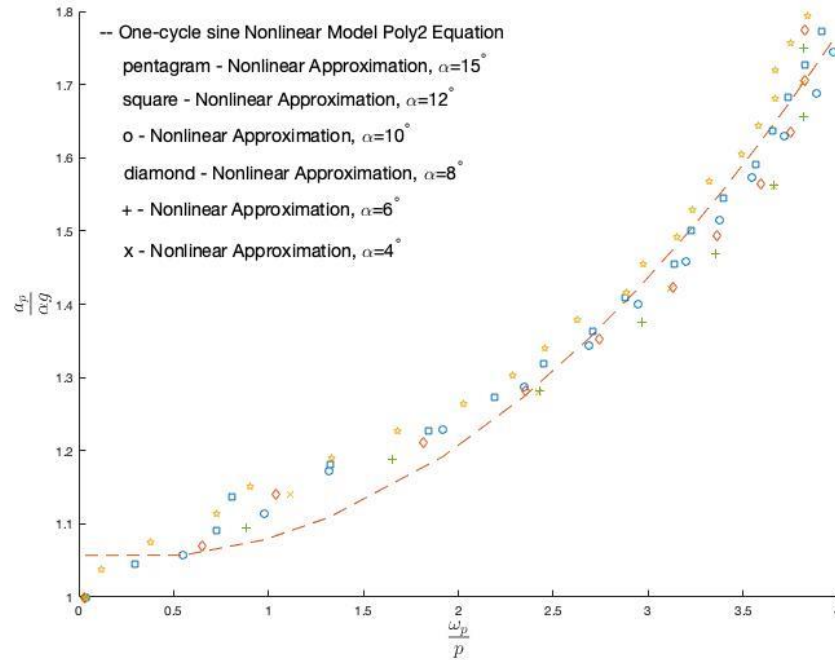


Figure 4.25. Spectrum of minimum acceleration of one-cycle sine wave required to overturn geometrically different rigid blocks with slenderness ratios less than 20 degrees ($\alpha=4^\circ, 6^\circ, 8^\circ, 10^\circ, 12^\circ$ and 15° , respectively) with nonlinear formulation (Runge-Kutta Method via ode45 solver).

The second part of this section presents information regarding analysis results for each block under half-cycle sine wave with amplitudes between αg and $0.35g$ with an increase in amplitude of $0.01g$ and frequencies between 0.1 and 4π with an increase in frequency of 0.25 . The first analysis, using a numerical Runge-Kutta approach implemented in MATLAB's ode45 solver, generated an overturning acceleration spectrum (OAS) for each block, as shown in Figure 4.26. This analysis employed the nonlinear form of the dynamic equation of motion. A second analysis, utilizing the same numerical approach but solving the linearized form of the equation of motion, resulted in another OAS for each block, depicted in Figure 4.27.

As shown in Figure 4.26, three rectangular rigid blocks having slenderness angle, $\alpha=10^\circ$, 20° and 30° , respectively were analysed under half-cycle sine wave based on the assumption that the ordinary differential equation which describes the dynamic equation of motion of the block is nonlinear. Using the nonlinear state space formulation, the EOM was solved at discrete time steps via MATLAB.

Later, the valid expression Equation (4.27) was used to generate the threshold overturning acceleration amplitude spectrum (OAS) required to topple a block under half-cycle sine wave. Using all data obtained from nonlinear equation describing the motion of a rigid block with a slenderness angle $\alpha=10^\circ$ subjected to half-cycle sine wave, the correct expression Equation (4.27) was yielded. The data was first plotted with linear scales on both axes. The plot indicated that a linear function was not the optimal fit, as the data points did not align with Housner's Equations (4.16) or (4.17). From the other possible functions, such as linear, exponential, logarithmic, reciprocal functions and power functions, only polynomial function (polynomial of second order) was a good choice to derive an approximate equation for a rigid block under one-cycle sine wave. The polynomial function derived using MATLAB curve fitting interface have the following form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 \quad (4.26)$$

Here, the coefficients $a_n, a_{n-1}, \dots, a_1, a_0$ are real numbers, and n , which is a nonnegative integer, is the degree (order of the polynomial).

$$\frac{a_p}{\alpha g} \approx 0.04568 \left(\frac{\omega_p}{p} \right)^2 + 0.3467 \left(\frac{\omega_p}{p} \right) + 0.9639 \quad (4.27)$$

The yellow dashed line (polynomial of second order) shown in Figures 4.26 and 4.27 obtained from polynomial fitting method can give a good fit to all the data obtained from previous numerical analysis. Results obtained at the end of calculation procedures indicated that all the rocking rigid blocks having slenderness angle, α is less than 20 degrees passes through all the data points obtained from polynomial function (polynomial of second order). However, for cases with slenderness angle, α which is greater than 20 degrees exhibits a stiffening effect.

Another important point is that the comparison of the plots in Figure 4.26 and Figure 4.27 indicate that linearized and nonlinear numerical solution results in similar dynamic response for rectangular rigid blocks. But still, the numerical solution at each time step with the nonlinear approximation results in a high level of accuracy. Moreover, it is possible to get more precise results using small time steps leading to less numerical errors, however it requires more computational resources, as it involves solving a single set of nonlinear equations at each time step.

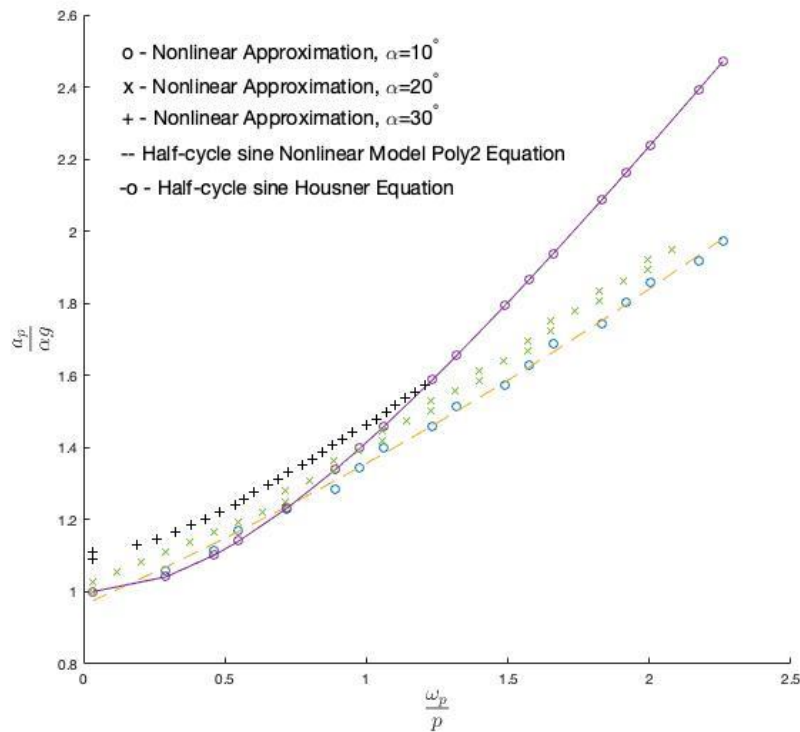


Figure 4.26. Spectrum of minimum acceleration of half-cycle sine wave required to overturn geometrically different rigid blocks ($\alpha=10^\circ$, 20° and 30° , respectively) with nonlinear formulation (Runge-Kutta Method via ode45 solver).

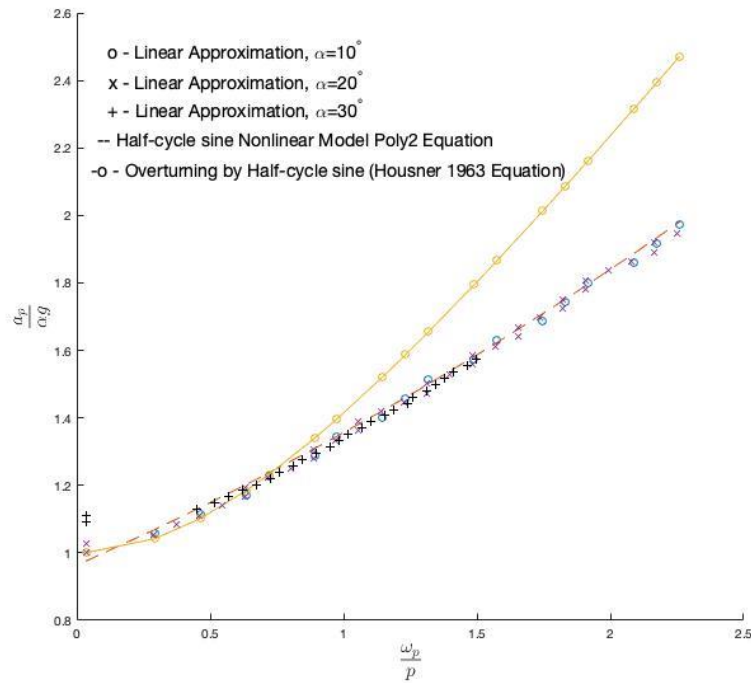


Figure 4.27. Spectrum of minimum acceleration of half-cycle sine wave required to overturn geometrically different rigid blocks ($\alpha=10^\circ$, 20° and 30° , respectively) with linear formulation (Runge-Kutta Method via ode45 solver).

In order to prove the accuracy of the derived Equation (4.27), numerical analysis was performed for six different rigid blocks having slenderness ratio less than 20 degrees. As shown in Table 4.6, six rectangular rigid blocks having slenderness angle, $\alpha=4^\circ$, 6° , 8° , 10° , 12° and 15° , respectively were analysed under half-cycle sine wave based on the assumption that the ordinary differential equation which describes the dynamic equation of motion of the block is nonlinear.

In Figure 4.28, each plot showing the variation of minimum overturning acceleration amplitudes along the frequency parameter are compared with the derived Equation (4.27) results under the section 4.4.5.7 of this chapter. Comparing of the obtained numerical results for each rectangular rigid body eventually led to the conclusion that derived polynomial Equation (4.27) results are in good agreement with numerical analysis results obtained from six rigid blocks,

therefore, the proposed approximate Equation (4.27) might be used for such cases having slenderness ratio less than 20° under half-cycle sine wave.

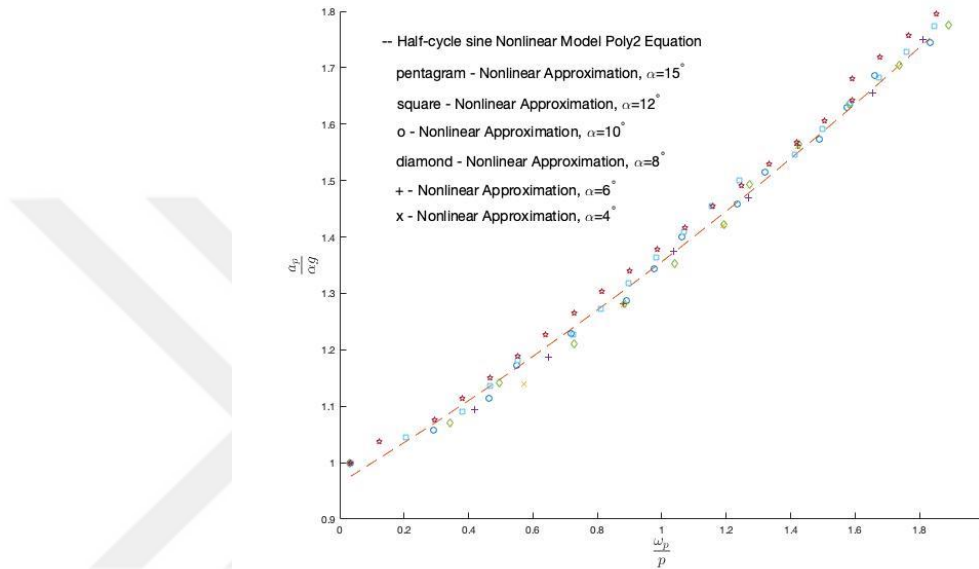


Figure 4.28. Spectrum of minimum acceleration of half-cycle sine wave required to overturn geometrically different rigid blocks with slenderness ratios less than 20 degrees ($\alpha=4^\circ, 6^\circ, 8^\circ, 10^\circ, 12^\circ$ and 15° , respectively) with nonlinear formulation (Runge-Kutta Method via ode45 solver).

The third part of this section presents information regarding analysis results for each block under rectangular constant acceleration pulse with amplitudes between 0.2g and 0.8g with an increase in amplitude of 0.01g and pulse duration between 0 and 0.70 sec with an increase in pulse duration of 0.005 s. At the end of the first analysis, an overturning acceleration spectrum (OAS) shown in Figure 4.29 was built using a numerical approach which is Runge-Kutta Method applied via ode45 solver in MATLAB. For the first case, this numerical integration technique was adopted to solve nonlinear form of dynamic equation of motion for each block ID. The second phase of the study yielded another overturning acceleration spectrum (OAS) shown in Figure 4.30 using Runge-Kutta numerical integration technique applied via ode45

solver. Differently from the first case, linearized form of equation of motion for each block specimen was solved numerically.

As shown in Figure 4.29, only rectangular rigid block having slenderness angle, $\alpha=10^\circ$ was analysed under rectangular constant acceleration pulse based on the assumption that the ordinary differential equation which describes the dynamic equation of motion of the block is nonlinear. Using the nonlinear state space formulation, the EOM was solved at discrete time steps via MATLAB.

Later, the correct expression Equation (4.29) was used to generate the threshold acceleration amplitude spectrum (OAS) necessary to topple the block excited by rectangular constant acceleration pulse. Using all data obtained from nonlinear equation describing the motion of a rigid block with a slenderness angle $\alpha=10^\circ$ subjected to rectangular constant acceleration pulse, the correct expression Equation (4.29) was yielded. The data was first plotted with linear scales on both axes. From the other possible functions, such as linear, polynomial, logarithmic, reciprocal functions and power functions, only exponential function was a good choice to derive an approximate equation for a rigid block under rectangular constant acceleration pulse. The exponential function derived using MATLAB curve fitting interface have the following form

$$f(x) = ae^{(-bx)} + c \quad (4.28)$$

Here, the coefficients a, b and c are real numbers.

$$\frac{a_p}{\alpha g} \approx 8.137e^{(-10.77t_p)} + 1.343 \quad (4.29)$$

The purple dashed line (exponential function) shown in Figures 4.29 and 4.30 obtained from curve fitting script can give a good fit to all the data obtained from previous numerical analysis. The plot shows that a straight line fits the data well, which is consistent with Housner's constant acceleration equation [3]. Results obtained at the end of calculation procedures indicated that the rocking rigid block having slenderness angle, α is less than 20 degrees passes

through all the data points obtained from exponential function. However, for cases with slenderness angle, α which is greater than 20 degrees exhibits a stiffening effect.

Another important point is that the comparison of the plots in Figure 4.29 and Figure 4.30 indicate that linearized and nonlinear numerical solution results in similar dynamic response for rectangular rigid blocks. But still, the numerical solution at each time step with the nonlinear approximation results in a high level of accuracy. Moreover, it is possible to get more precise results using small time steps leading to less numerical errors, however it requires more computational resources, as it involves solving a single set of nonlinear equations at each time step.

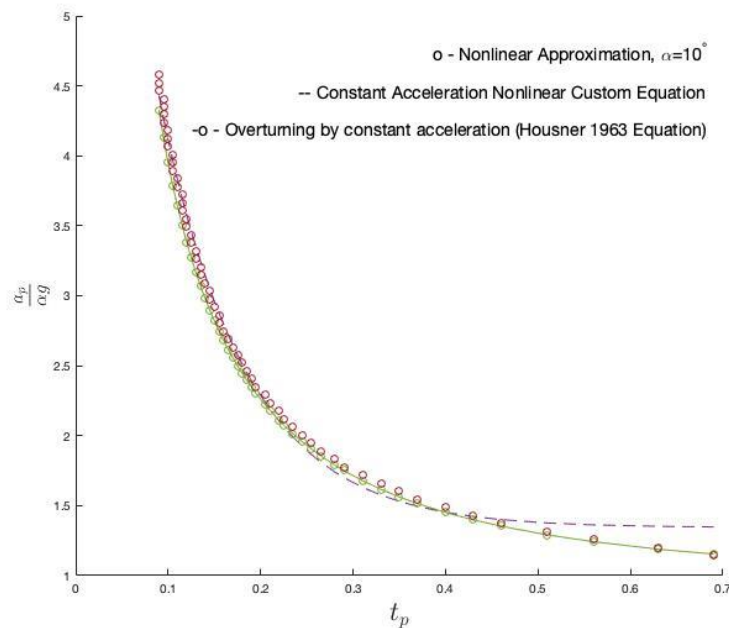


Figure 4.29. Spectrum of minimum acceleration of rectangular constant acceleration pulse required to overturn single rigid block ($\alpha=10^\circ$) with nonlinear formulation (Runge-Kutta Method via ode45 solver).

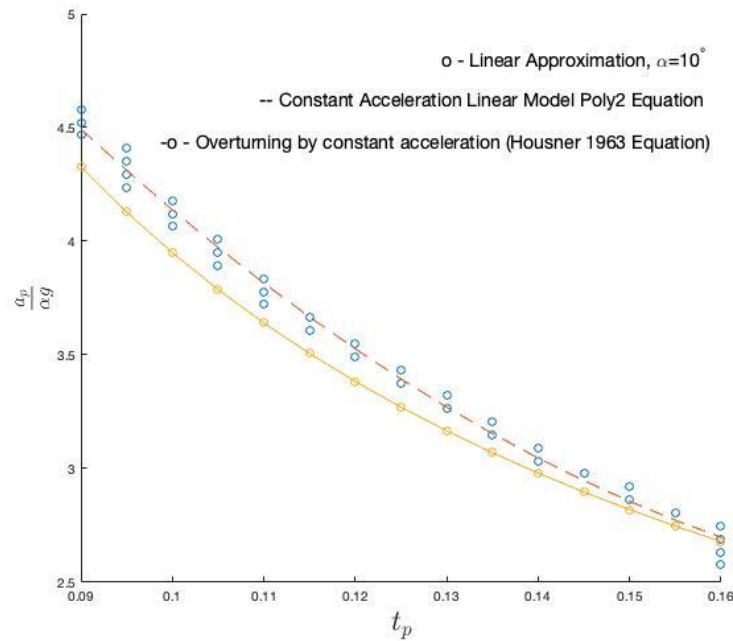


Figure 4.30. Spectrum of minimum acceleration of rectangular constant acceleration pulse required to overturn single rigid block ($\alpha=10^\circ$) with linear formulation (Runge-Kutta Method via ode45 solver).

This section presents the results of an analysis of a stand-alone block subjected to a Haar wavelet form, varying both amplitude (from 0.20g to 0.80g in 0.01g increments) and pulse duration (from 0 to 0.70 sec in 0.005 s increments). The analysis, using a numerical Runge-Kutta approach implemented in MATLAB's ode45 solver, generated an overturning acceleration spectrum (OAS) for the block, as shown in Figure 4.31. This analysis employed the nonlinear form of the dynamic equation of motion.

A single rectangular rigid block, with a slenderness angle of 10° , was analyzed under a Haar wavelet pulse, assuming a nonlinear ordinary differential equation describing the block's dynamic motion. The analysis, depicted in Figure 4.31, was conducted using a nonlinear state-space formulation, with the equation of motion solved numerically at discrete time steps via MATLAB.

Later, the valid expression Equation (4.31) was used to generate the minimum acceleration amplitude spectrum (OAS) necessary to topple the block excited by Haar wavelet pulse. Using all data obtained from nonlinear motion equation of a rigid block with a slenderness angle $\alpha=10^\circ$ subjected to Haar wavelet pulse, the correct expression Equation (4.31) was yielded. The data was first plotted with linear scales on both axes. From the other possible functions, such as linear, polynomial, logarithmic, reciprocal functions and power functions, only exponential function was a good choice to derive an approximate equation for a rigid block under Haar wavelet pulse. The exponential function derived using MATLAB curve fitting interface have the following form

$$f(x) = ae^{(-bx)} + c \quad (4.30)$$

Here, the coefficients a, b and c are real numbers.

$$\frac{a_p}{\alpha g} \approx 8.137e^{(-10.77t_p)} + 1.343 \quad (4.31)$$

The purple dashed line (exponential function) shown in Figure 4.31 obtained from curve fitting script can give a good fit to all the data obtained from previous numerical analysis. Results obtained at the end of calculation procedures indicated that all the rocking rigid blocks having slenderness angle, α is less than 20 degrees passes through all the data points obtained from exponential function. However, for cases with slenderness angle, α which is greater than 20 degrees exhibits a stiffening effect.

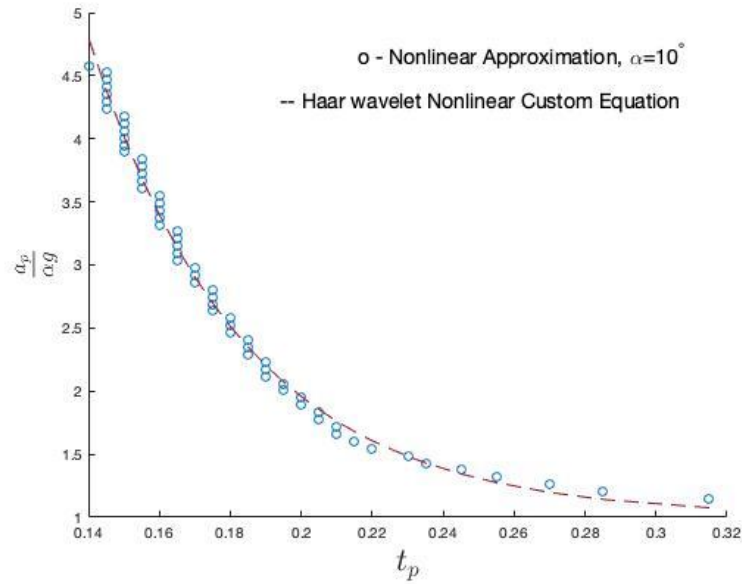


Figure 4.31. Spectrum of minimum acceleration of Haar wavelet pulse required to overturn single rigid block ($\alpha=10^\circ$) with nonlinear formulation (Runge-Kutta Method via ode45 solver).

The next section provides details on analysis findings for each block using an Antisymmetric Ricker acceleration pulse. The pulse ranges from αg to $0.70g$ with an increment of $0.01g$, and frequencies from 0.1 to 4π with a step of 0.05 . Following the initial analysis, an overturning acceleration spectrum (OAS) depicted in Figure 4.32 was constructed using the Runge-Kutta Method via the ode45 solver in MATLAB. In this first scenario, the Runge-Kutta Method was employed to address the nonlinear form of the dynamic equation of motion for each block ID. Subsequently, the second phase of the investigation produced another OAS represented in Figure 4.33, utilizing the Runge-Kutta numerical integration method with the ode45 solver. Unlike the first instance, a linearized version of the equation of motion for each block specimen was numerically solved in this case.

As shown in Figure 4.32, three rectangular rigid blocks having slenderness angle, $\alpha=10^\circ$, 20° and 30° , respectively were analysed under Antisymmetric Ricker acceleration pulse based on the assumption that the ordinary differential equation which describes the dynamic equation

of motion of the block is nonlinear. Using the nonlinear state space formulation, the EOM was solved at discrete time steps via MATLAB.

Later, the correct expression Equation (4.33) was used to generate the threshold acceleration amplitude spectrum (OAS) necessary to topple the block undergoing Antisymmetric Ricker acceleration wave. Using all data obtained from nonlinear equation describing the motion of a rigid block with a slenderness angle $\alpha=10^\circ$ subjected to Antisymmetric Ricker acceleration pulse, the correct expression Equation (4.33) was yielded. The data was first plotted with linear scales on both axes. From the other possible functions, such as linear, exponential, logarithmic, reciprocal functions and power functions, only polynomial function (polynomial of second order) was a good choice to derive an approximate equation for a rigid block under Antisymmetric Ricker acceleration pulse. The polynomial function derived using MATLAB curve fitting interface have the following form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 \quad (4.32)$$

Here, the coefficients $a_n, a_{n-1}, \dots, a_1, a_0$ are real numbers, and n , which is a nonnegative integer, is the degree (order of the polynomial).

$$\frac{a_p}{\alpha g} \approx 0.2102 \left(\frac{\omega_p}{p} \right)^2 - 0.6336 \left(\frac{\omega_p}{p} \right) + 1.683 \quad (4.33)$$

The red line (polynomial of second order) shown in Figure 4.32 obtained from polynomial fitting method can give a good fit to all the data obtained from previous numerical analysis. Results obtained at the end of calculation procedures indicated that all the rocking rigid blocks having slenderness angle, α is less than 20 degrees passes through all the data points obtained from polynomial function (polynomial of second order). However, for cases with slenderness angle, α which is greater than 20 degrees exhibits a stiffening effect. Another important point is that the comparison of the plots in Figures 4.32 and Figure 4.33 indicate that linearized and nonlinear numerical solution results in similar dynamic response for rectangular rigid blocks. But still, the numerical solution at each time step with the nonlinear approximation results in a high level of accuracy. Moreover, it is possible to get more precise results using small time steps leading to less numerical errors, however it requires more computational resources, as it involves solving a single set of nonlinear equations at each time step.

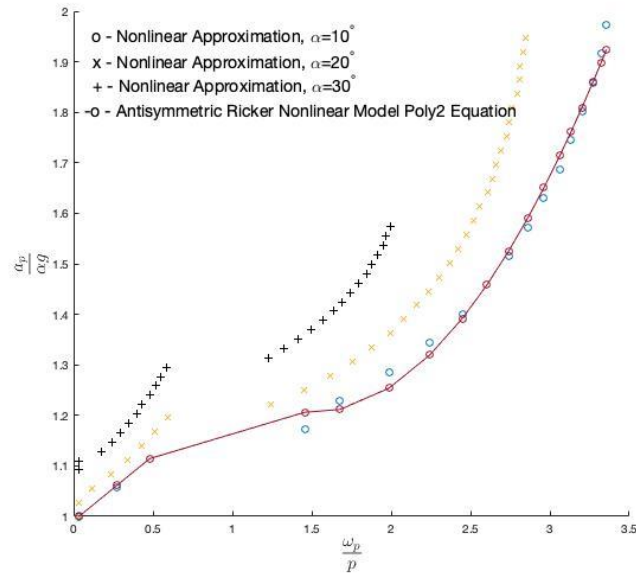


Figure 4.32. Spectrum of minimum acceleration of antisymmetric ricker acceleration pulse required to overturn geometrically different rigid blocks ($\alpha=10^\circ$, 20° and 30° , respectively) with nonlinear formulation (Runge-Kutta Method via ode45 solver).

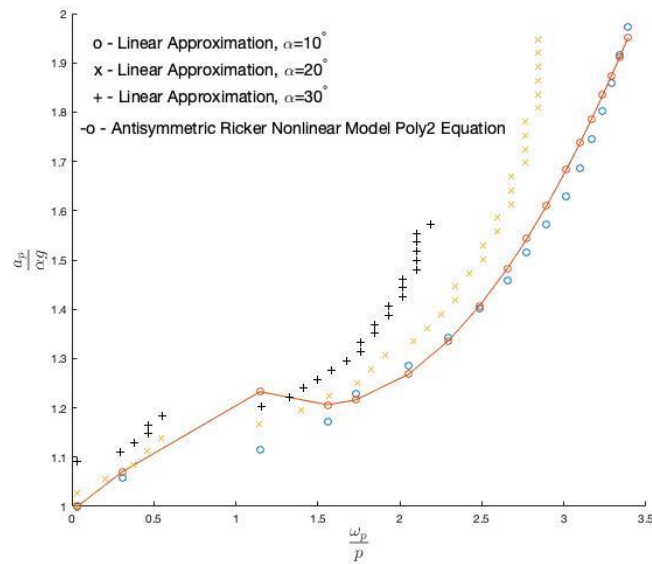


Figure 4.33. Spectrum of minimum acceleration of antisymmetric ricker acceleration pulse required to overturn geometrically different rigid blocks ($\alpha=10^\circ$, 20° and 30° , respectively) with linear formulation (Runge-Kutta Method via ode45 solver)

The last section presents information regarding analysis results for each block under Symmetric Ricker acceleration pulse with amplitudes between αg and $0.70g$ with an increase in amplitude of $0.01g$ and frequencies between 0.1 and 4π with an increase in frequency of 0.05 . At the end of the first analysis, an overturning acceleration spectrum (OAS) shown in Figure 4.34 was built using a numerical approach which is Runge-Kutta Method applied via ode45 solver in MATLAB. For the first case, this numerical integration technique was adopted to solve nonlinear form of dynamic equation of motion for each block ID. The second phase of the study yielded another overturning acceleration spectrum (OAS) shown in Figure 4.35 using Runge-Kutta numerical integration technique applied via ode45 solver. Differently from the first case, linearized form of equation of motion for each block specimen was solved numerically.

As shown in Figure 4.34, three rectangular rigid blocks having slenderness angle, $\alpha=10^\circ$, 20° and 30° , respectively were analysed under Symmetric Ricker acceleration pulse based on the assumption that the ordinary differential equation which describes the dynamic equation of motion of the block is nonlinear. Using the nonlinear state space formulation, the EOM was solved at discrete time steps via MATLAB.

Later, the correct expression Equation (4.35) was used to generate the minimum acceleration amplitude spectrum (OAS) necessary to topple the block undergoing Symmetric Ricker acceleration wave. Using all data obtained from nonlinear equation describing the motion of a rigid block with a slenderness angle $\alpha=10^\circ$ subjected to Symmetric Ricker acceleration pulse, the correct expression Equation (4.35) was yielded. The data was first plotted with linear scales on both axes. From the other possible functions, such as linear, exponential, logarithmic, reciprocal functions and power functions, only polynomial function (polynomial of second order) was a good choice to derive an approximate equation for a rigid block under.

Symmetric Ricker acceleration pulse. The polynomial function derived using MATLAB curve fitting interface have the following form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 \quad (4.34)$$

Here, the coefficients $a_n, a_{n-1}, \dots, a_1, a_0$ are real numbers, and n , which is a nonnegative integer, is the degree (order of the polynomial).

$$\frac{a_p}{\alpha g} \approx 0.1498 \left(\frac{\omega_p}{p} \right)^2 + 0.01533 \left(\frac{\omega_p}{p} \right) + 1.047 \quad (4.35)$$

The red line (polynomial of second order) shown in Figures 4.34 and 4.35 obtained from polynomial fitting method can give a good fit to all the data obtained from previous numerical analysis. Results obtained at the end of calculation procedures indicated that all the rocking rigid blocks having slenderness angle, α is less than 20 degrees passes through all the data points obtained from polynomial function (polynomial of second order). However, for cases with slenderness angle, α which is greater than 20 degrees exhibits a stiffening effect. Another important point is that the comparison of the plots in Figure 4.34 and Figure 4.35 indicate that linearized and nonlinear numerical solution results in similar dynamic response for rectangular rigid blocks. But still, the numerical solution at each time step with the nonlinear approximation results in a high level of accuracy. Moreover, it is possible to get more precise results using small time steps leading to less numerical errors, however it requires more computational resources, as it involves solving a single set of nonlinear equations at each time step.

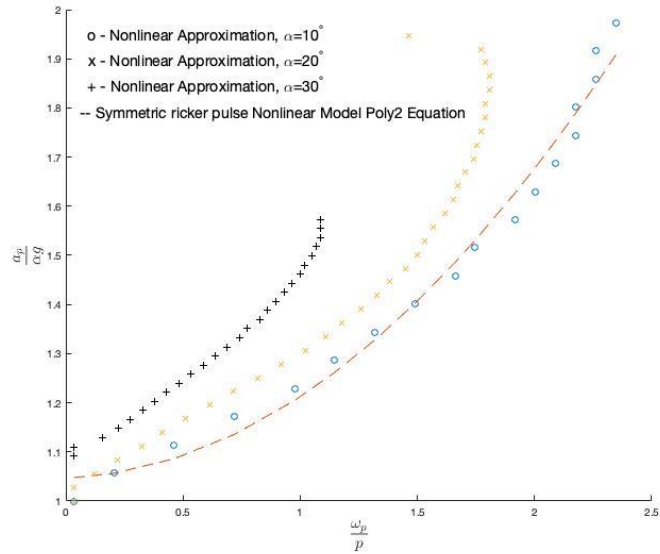


Figure 4.34. Spectrum of minimum acceleration of symmetric ricker acceleration pulse required to overturn geometrically different rigid blocks ($\alpha=10^\circ$, 20° and 30° , respectively) with nonlinear formulation (Runge-Kutta Method via ode45 solver).

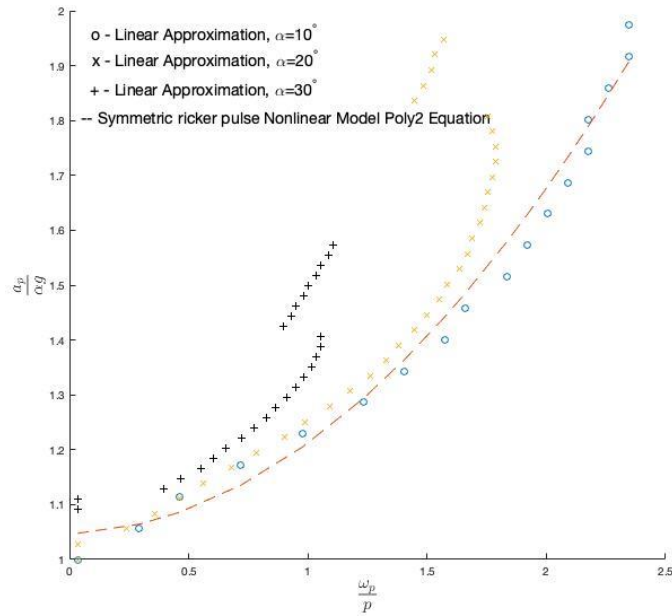


Figure 4.35. Spectrum of minimum acceleration of symmetric ricker acceleration pulse required to overturn geometrically different rigid blocks ($\alpha=10^\circ$, 20° and 30° , respectively) with linear formulation (Runge-Kutta Method via ode45 solver).

4.4.6. Nonlinear Dynamic Response to Earthquake Ground Motion

Equation (3.35) provides numerical solutions for a stand-alone block subjected to both real earthquake records and their pulse-like representations. These solutions can be used to calculate the block's nonlinear dynamic response. This section presents dynamic results for a single rigid block, including its rotation, angular velocity, and total energy time histories. These results are analyzed for both the El Centro 1940 earthquake (Mw 6.9) and the 2023 Pazarcık, Kahramanmaraş earthquake (Mw 7.7).

Records from stations closest to the seismic source of the 2023 Pazarcık, Kahramanmaraş earthquake (Mw 7.7) were selected to analyze the effect of near-field ground motion characteristics on the structures prone to rocking. These records, which exhibit distinct pulse characteristics (pulse duration (t_p) and velocity amplitude pulse (V_p)), were sourced from Xie et al. [95]. Using analytical formulations by Jacobsen and Ayre [94] and Makris [4], pulse-like excitations corresponding to these earthquake records were generated for use in the numerical study. These pulse-shaped excitations were implemented within a MATLAB framework.

4.4.6.1. Response to the El Centro earthquake (Mw 6.9)

The El Centro earthquake of 1940, also known as the Imperial Valley earthquake, occurred on May 19, 1940, in the southern region of California. It had a magnitude of 6.9 and caused significant damage in the town of El Centro and the surrounding areas. The earthquake resulted in the loss of life and destruction of buildings and infrastructure. It was one of the most powerful earthquakes to hit California in the 20th century and served as an important event for understanding seismic activity in the region. Here, the acceleration time-history used is based on the El Centro earthquake ground motion (also known as the Imperial Valley earthquake of 5/19/40) recorded at El Centro array 9, 180 in the Imperial Valley, California. This ground motion had a peak ground acceleration (PGA) of 0.348 g and a peak ground velocity (PGV) of 33.5 cm/s. In this study, ground motion data recorded at station El Centro Array #9 were downloaded from PEER page [97].

The numerical modeling utilized in the thesis incorporated several key features to enhance the accuracy and reliability of the simulations. One prominent feature was the use of real ancient column dimensions to capture the realistic dynamic behavior of model under seismic loading. This involved selecting dimensions that closely mirrored the size and slenderness of ancient columns, particularly focusing on columns sourced from the Temple of Olympian Zeus. The Temple of Olympian Zeus, also known as the Olympeion or Columns of the Olympian Zeus, is a colossal ruined temple in the center of Athens, Greece. The temple boasted massive columns, with some reaching a height of over 17 meters (56 feet) and about 2.3 meters (7.5 feet) in diameter at the base. Furthermore, the numerical analysis also involved the implementation of nonlinear form of dynamic equation of motion to accurately capture the dynamic response of the ancient column. By using appropriate integration methods, such as implicit or explicit time-stepping algorithms, the analysis successfully captured the nonlinear aspects of the rocking motion.

In this study, the rocking response of the ancient column has been computed numerically by solving the nonlinear equation of motion in MATLAB R2016, utilizing the 4th-5th order Runge-Kutta integration method (ode45).

Figure 4.36 illustrates the dimensions of the rocking column, along with the slenderness factor α , the semi-diagonal R , and the frequency parameter p . As illustrated in Figure 4.36, the columns of the Temple of Olympian Zeus stand at a height of 8 meters. Furthermore, these columns exhibit a slenderness of 0.165 radians and a half-diagonal distance of 4.026 meters.

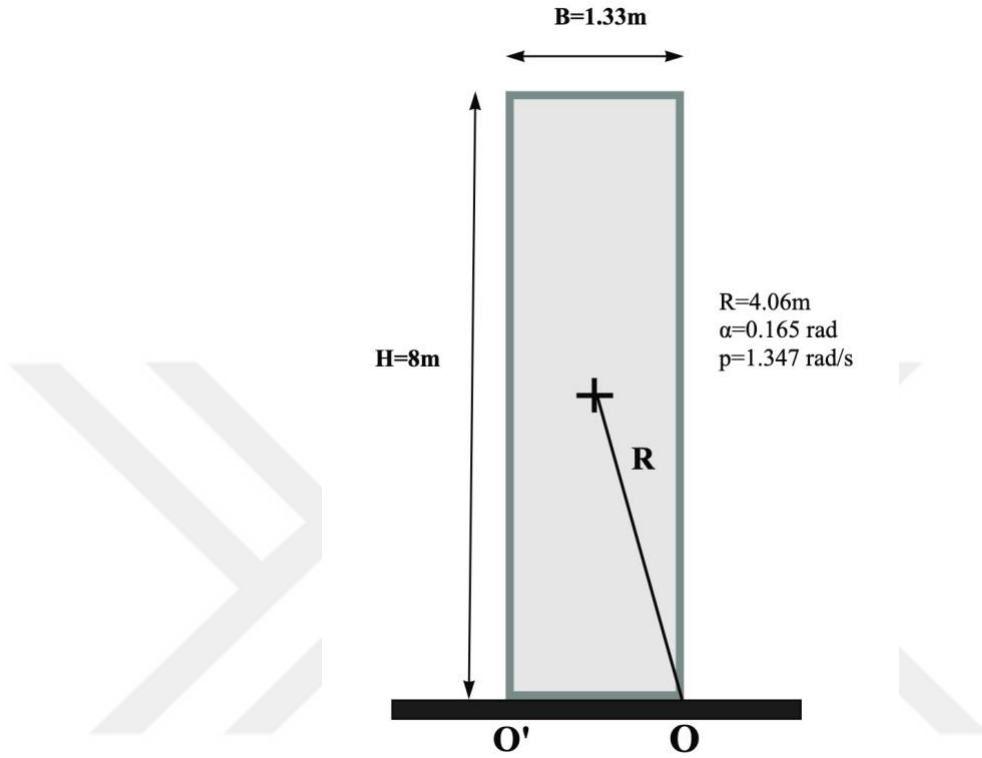


Figure 4.36. Rigid ancient column model configuration used for analysis cases.

In addition, the coefficient of restitution value provided in Equation (4.36) represents the maximum ν value for a block possessing a slenderness of α to experience rocking behavior. Throughout this study, all analyses are performed using this maximum coefficient of restitution value as specified in Equation (4.36). The restitution coefficient for the ancient column is determined to be $\nu = 0.96$.

$$\nu = \left[1 - \frac{3}{2} \sin(\alpha)^2 \right]^2 \quad (4.36)$$

The angular displacement and velocity time histories resulting from the nonlinear numerical analysis of the rocking motion of a slender rigid column are depicted in Figure 4.37. Figure 4.37 (left) illustrates the acceleration time history of 1940 El Centro earthquake, rotation and velocity time histories, respectively. Figure 4.37 (right) zooms into angular displacement of ancient column subjected to El Centro 1940 earthquake.

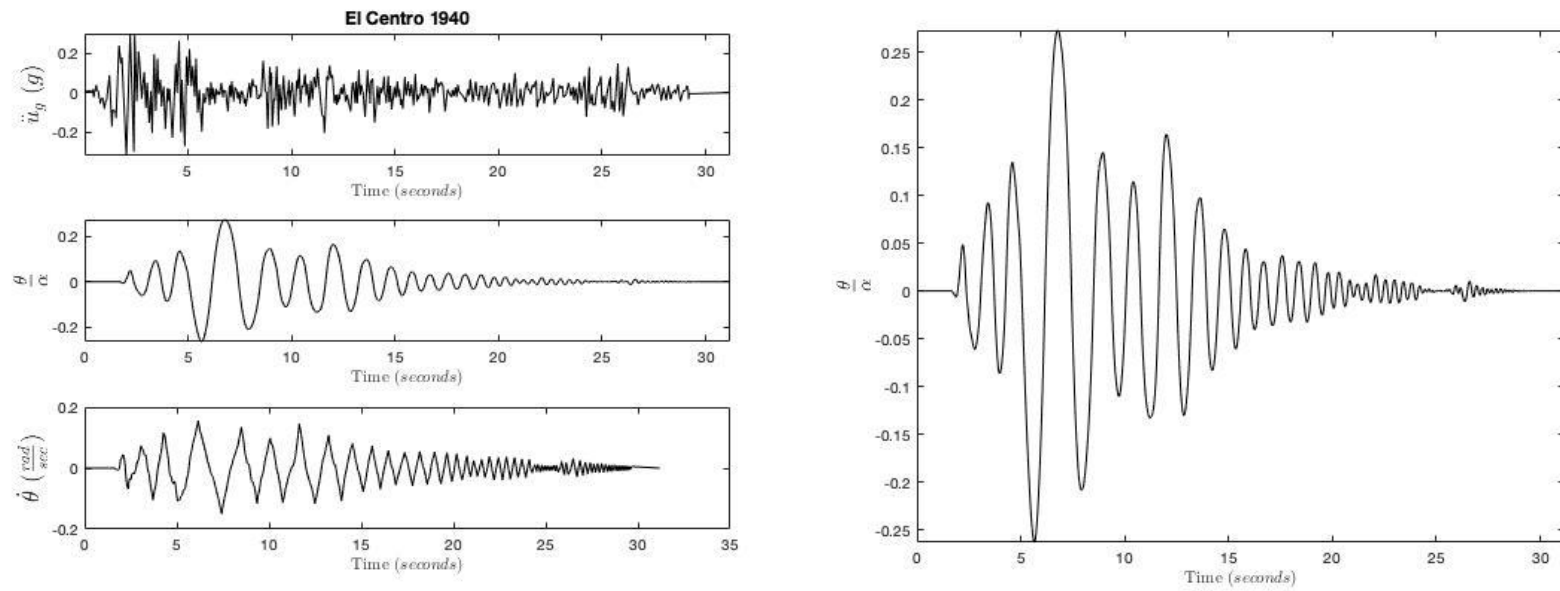


Figure 4.37. Time-domain histories of rotation and rotational velocity of a rigid column under El Centro 1940 earthquake (Nonlinear EOM) Right: Zoomed angular displacement plot in the same figure.

4.4.6.2. Response to the 2023 Pazarcık, Kahramanmaraş earthquake (M_w 7.7)

On February 6, 2023, a significant earthquake with a magnitude of M_w 7.7 struck southern Turkey at a depth of 8.6 km. The epicenter was identified at 37.288°N, 37.043°E, roughly 40 km northwest of Gaziantep. Approximately 9 hours later, another earthquake of M_w 7.6 occurred in Ekinözü, with its epicenter at 38.089°N, 37.239°E. The towns of Kahramanmaraş and Gaziantep, situated near the epicenters, endured severe damage, reaching a maximum intensity of XII on the Modified Mercalli Intensity Scale. This event is commonly known as the M_w 7.7 2023 Pazarcık (Kahramanmaraş) earthquake.

A total of 292 strong-motion stations operated by the Disaster and Emergency Management Authority of Turkey (AFAD) [98] successfully captured strong motion accelerations within a 436km of the causative fault. Station 3126 recorded the highest peak ground acceleration (PGA) of 1.23 g. Velocity waveforms from several stations in the near-source area displayed distinct pulse-like characteristics [96]. The purpose of this study was to examine how near-fault ground motions impact the dynamic response of rocking structures. To achieve this, the study focused on the investigation of three near fault ground motion records captured within a 5km proximity to the causative fault of the M_w 7.7 Kahramanmaraş-Pazarcık earthquake on February 6, 2023.

Near-fault ground motions exhibit a distinctive characteristic of large-velocity pulses, distinguishing them from far-field ground motions. These notable pulse-like motions can lead to significant irreversible deformation and an increased risk of structural failure [99, 100]. The observation of large velocity pulses traces back to the 1966 Parkfield earthquake in the U.S., the 1971 San Fernando earthquake, 1979 Imperial Valley earthquake, 1992 Landers earthquake, 1994 Northridge earthquake. Additionally, pulse-like ground motions were detected during the 6 February 2023 M_w 7.7 Pazarcık Earthquake (Kahramanmaraş, Türkiye).

Near-fault ground motion can be particularly dangerous for rigid blocks due to the presence of large-velocity pulses and high accelerations characteristic of such motions. In other

words, near-fault ground motions are characterized by prolonged strong pulses, which render them highly destructive to certain structures, notably free-standing tall slender rocking structures, due to the high velocities transferred to above-ground masses.

In this context, understanding how near-fault ground motion can lead to overturning of a rigid block is crucial for assessing the seismic vulnerability of rocking structures and implementing appropriate design and mitigation of the harmful effects of earthquake ground excitation to enhance their seismic resilience. In this study, the dynamic response of a single ancient column was examined under three different pulse-like excitations from near-fault earthquakes.

This phenomenon prompted Housner [3] to explore the potential risk for overturning from a near-fault earthquake on a rocking rigid block. The equations he derived for a stand-alone rigid block under a half sine pulse have been widely used by researchers in the field. But later studies demonstrated that Housner's analytical solution of the minimum acceleration amplitude required to cause the toppling of a rigid block under the influence of a half-sine waveform is overly cautious [3].

Based on the specific shape of the selected pulse, numerous numerical solutions and closed-form solutions have been suggested. A variety of shapes and enhancements have been created to model the primary pulse of near-fault ground motions, offering alternatives to the earthquake ground motion for examining the stability of blocks against overturning.

In 1997, Makris [4] introduced physically realizable cycloidal pulses of types A, B, and C_n . The time-dependent characteristics of acceleration, velocity, and displacement for type A and type B cycloidal-shaped pulses analytically expressed by (Jacobsen and Ayre [94]; Makris and Roussos [4]). While a type-A cycloidal-shaped waveform models a single forward motion, a type-B cycloidal-shaped waveform simulates a forward-and-backward motion. Conversely, a type- C_n pulse replicates a ground motion with n distinct cycles in its displacement profile. Time-domain histories of acceleration, velocity and displacement of a type A cycloidal-shaped pulse, as defined by Makris [4], are as follows:

$$\ddot{u}_g(t) = \omega_p \frac{v_p}{2} \sin(\omega_p t), 0 \leq t \leq T_p \quad (4.37)$$

$$\dot{u}_g(t) = \frac{v_p}{2} - \frac{v_p}{2} \cos(\omega_p t), 0 \leq t \leq T_p \quad (4.38)$$

$$u_g(t) = \frac{v_p}{2} t - \frac{v_p}{2\omega_p} \sin(\omega_p t), 0 \leq t \leq T_p \quad (4.39)$$

Time-dependent histories of acceleration, velocity and displacement of a type B cycloidal-shaped waveform, as defined by Makris [4], are as follows:

$$\ddot{u}_g(t) = \omega_p v_p \cos(\omega_p t), 0 \leq t \leq T_p \quad (4.40)$$

$$\dot{u}_g(t) = v_p \sin(\omega_p t), 0 \leq t \leq T_p \quad (4.41)$$

$$u_g(t) = \frac{v_p}{\omega_p} - \frac{v_p}{\omega_p} \cos(\omega_p t), 0 \leq t \leq T_p \quad (4.42)$$

This study utilized analytical formulas suggested by Makris [4] to model pulse-like excitations for the time-domain histories of acceleration, velocity, and displacement of type A and type B cycloidal-shaped pulses, respectively. Moreover, their similarity to the near-source ground motions recorded at Station 3144, Station 4615, and Station NAR during the Mw7.7 Pazarcık, Kahramanmaraş earthquake is illustrated. In this context, Table 4.7 summarizes the pulse characteristics for the three strong ground motion records captured at Station 3144, Station 4615 and Station NAR, respectively [95].

Table 4.7. Characteristics of three pulse-like recordings used in this study [95].

Station	Longitude (°E)	Latitude (°N)	Region	R _{rup} (km)	PGA (g)	PGV (m/s ²)	V _p (m/s)	t _p (s)
3144	36.485742	36.75691	Hassa, Hatay	1.3	0.78g	1.552	0.967	8.3
4615	37.13803	37.38676	Pazarcık, Kahramanmaraş	1.6	0.59g	1.628	0.786	5.7
NAR	37.1574	37.3919	Pazarcık, Kahramanmaraş	1.0	0.80g	1.261	0.734	4.7

As outlined in earlier sections, prior study involved numerical modeling of ancient column closely resembling the size and slenderness of columns from the Temple of Olympian Zeus that subjected to the 1940 El Centro earthquake. Similarly, the dimensions of this identical numerical model were used in the present study. Figure 4.38 illustrates the dynamic response of the ancient column under the devastating magnitude M_w 7.7 Kahramanmaraş earthquake. In this study, a nonlinear formulation was utilized to determine the dynamic response of the numerical model. As clearly seen from Figure 4.38, the nonlinear solution reveals that the ancient column had overturned under magnitude 7.7 Kahramanmaraş earthquake motion recorded at Station 3144, Hassa, Hatay.

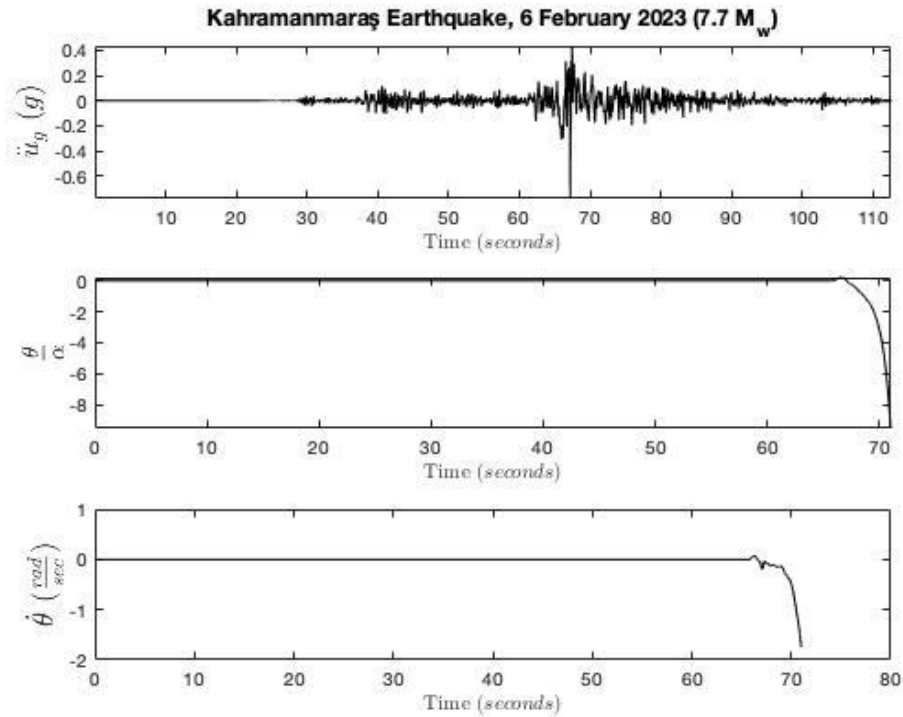


Figure 4.38. Time-domain histories of rotation and rotational velocity of a rigid column under 2023 Pazarcık, Kahramanmaraş earthquake (M_w 7.7) earthquake recorded at Station 3144 Hassa, Hatay (Overturned).

Figure 4.39 (left) depicts the time-domain histories of acceleration, velocity, and displacement of the February 6th, 2023 Pazarcık, Kahramanmaraş seismic event captured at seismic station 3144. This motion generated a forward-and-backward pulse, characterized by a duration of 8.3 seconds. Figure 4.39 (right) plots the time-domain histories of acceleration, velocity, and displacement of an inverse type-B cycloidal-shaped waveform expressed by (Jacobsen and Ayre [94]; Makris and Roussos [4]). In order to represent corresponding pulse, approximate values for the pulse length ($t_p = 8.3$ s) and velocity pulse amplitude ($V_p = 0.967$ m/s) were derived from the recorded motion at station 3144.

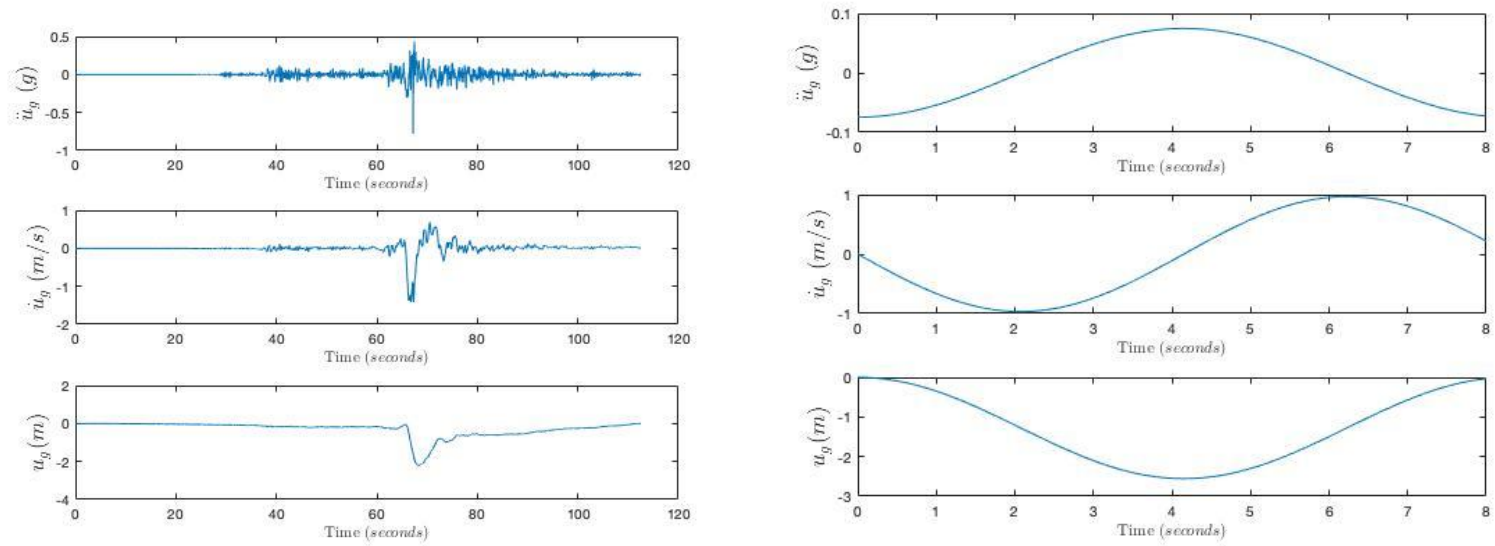


Figure 4.39. Left: Time domain acceleration, velocity and displacement recorded at Station 3144 Hassa, Hatay during the February 6th, 2023 Kahramanmaraş earthquake Right: an inverse cycloidal type-B pulse.

Another numerical analysis was conducted to see the dynamic response of the ancient column under an inverse type-B cycloidal pulse. Figure 4.40 indicates that the acceleration values obtained from type-B cycloidal pulse is not enough to initiate the rocking motion. For the numerical model to exhibit rocking motion under the influence of a B-type pulse, the input acceleration must be no less than $0.165g$. As a result, rocking motion is not observed in this analysis.

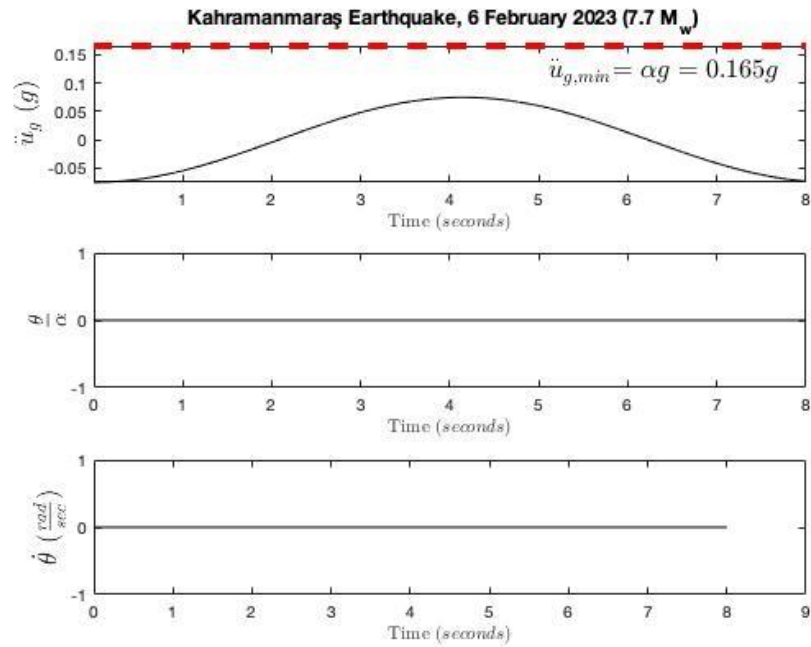


Figure 4.40. Time-domain histories of rotation and rotational velocity of a rigid column under an inverse cycloidal type-B pulse obtained from Station 3144 Hassa, Hatay (No Rocking motion).

Figure 4.41 depicts the dynamic response of the ancient column during the catastrophic M_w 7.7 Kahramanmaraş earthquake. Similar to prior studies, a nonlinear formulation was employed to calculate the dynamic response of the numerical model in this study. The nonlinear analysis presented in Figure 4.41 indicates that the ancient column experienced overturning due to the motion of the 2023 Pazarcık, Kahramanmaraş earthquake (M_w 7.7) recorded at Station 4615 in Pazarcık, Kahramanmaraş.

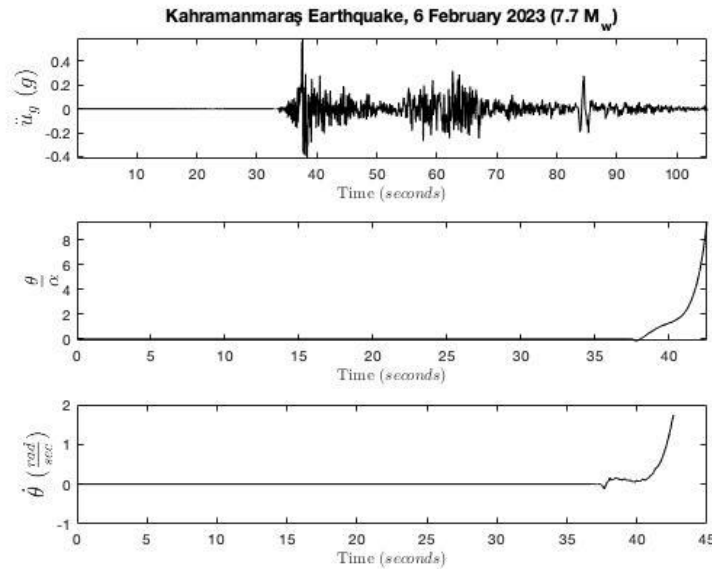


Figure 4.41. Time-domain histories of rotation and rotational velocity of a rigid column under 2023 Pazarcık, Kahramanmaraş earthquake (Mw 7.7) earthquake recorded at Station 4615 Pazarcık, Kahramanmaraş (Overturned).

On the left side of Figure 4.42, the time-domain histories of acceleration, velocity, and displacement histories from the February 6th, 2023 Pazarcık, Kahramanmaraş earthquake recorded at station 4615 are illustrated. On the right side of Figure 4.42, the acceleration, velocity, and displacement histories of an inverse type-B cycloidal-shaped pulse proposed by (Jacobsen and Ayre [94]; Makris and Roussos [4]) are presented. This motion generated a pulse characterized by a forward and backward oscillation, lasting approximately 5.7 seconds. In generating Figure 4.42 (right), approximate values of $T_p=5.7$ seconds and $V_p=0.786$ m/s were utilized for the pulse duration and velocity pulse amplitude observed in the recordings at station 4615 depicted on the left.

An additional numerical analysis was performed to observe the dynamic response of the ancient column when subjected to an inverse type-B cycloidal pulse. Figure 4.43 illustrates that the acceleration values derived from the type-B cycloidal pulse are insufficient to trigger rocking motion. To induce rocking motion for the numerical model under the influence of a B-type pulse, the input acceleration must be equal to or greater than $0.165g$. Consequently, rocking motion is not detected in this analysis.

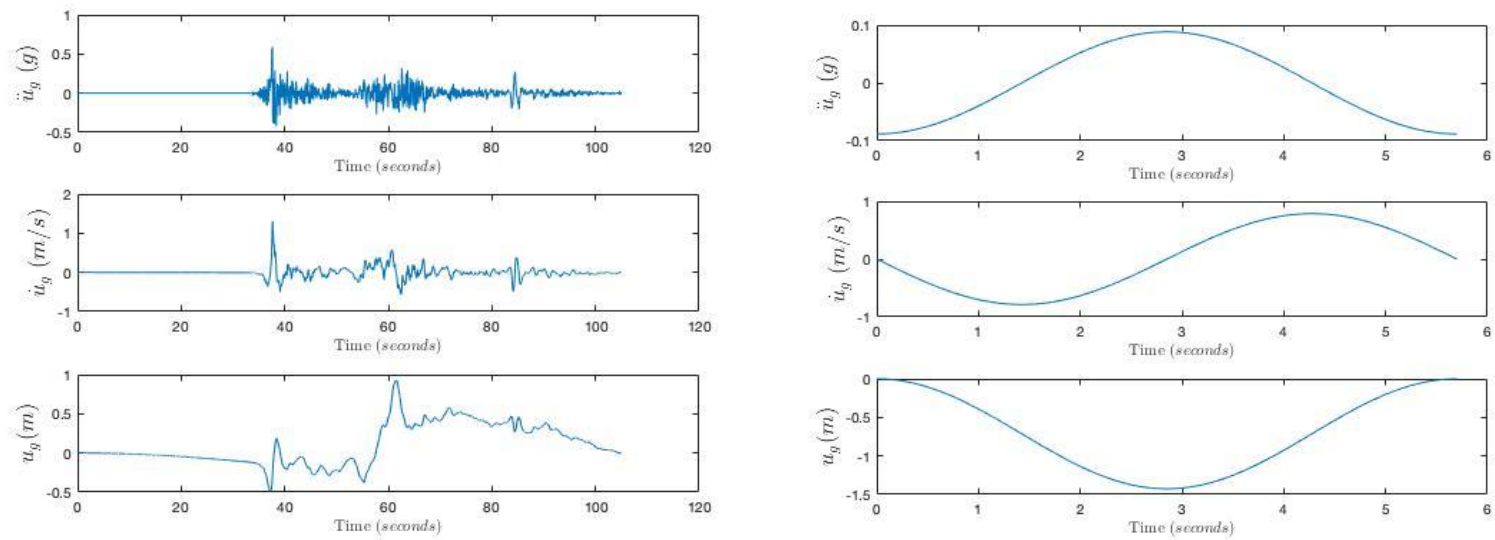


Figure 4.42. Left: Time-domain histories of acceleration, velocity and displacement recorded at Station 4615 Pazarcık, Kahramanmaraş during the February 6th, 2023 Kahramanmaraş earthquake Right: an inverse cycloidal type-B pulse.

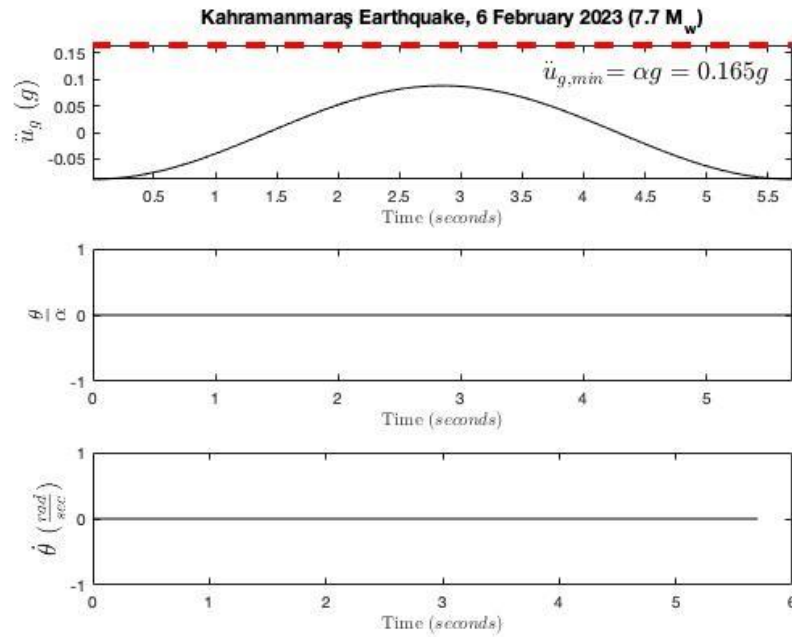


Figure 4.43. Time-domain histories of rotation and rotational velocity of a rigid column under an inverse cycloidal type-B pulse obtained from Station 4615 Pazarcık, Kahramanmaraş (No Rocking motion).

Figure 4.44 illustrates the dynamic behavior of the ancient column in response to the destructive 2023 Pazarcık, Kahramanmaraş earthquake (Mw 7.7). Similar to previous numerical analyses, a nonlinear approach was utilized to analyze the dynamic response of the numerical model in this investigation. The nonlinear assessment depicted in Figure 4.44 reveals that the ancient column undergoing overturning triggered by the 2023 Pazarcık, Kahramanmaraş earthquake (Mw 7.7) recorded at Station NAR in Pazarcık, Kahramanmaraş.

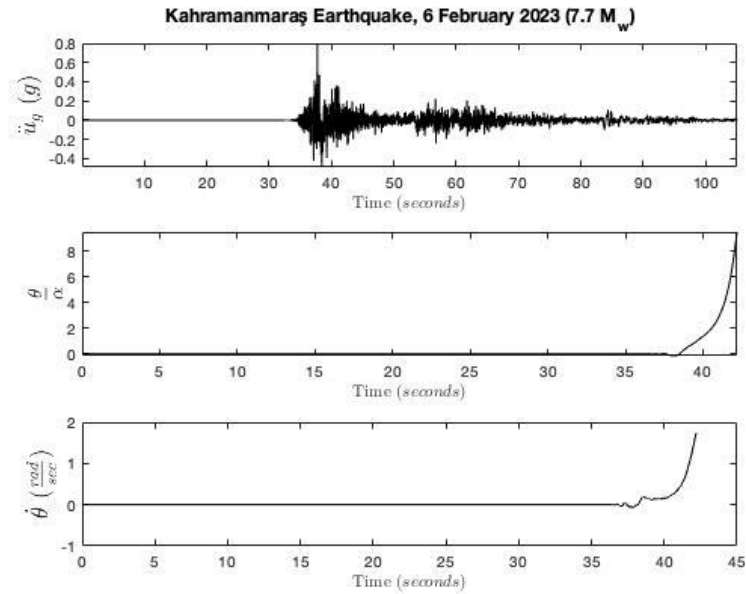


Figure 4.44. Time-domain histories of rotation and rotational velocity of a rigid column under 2023 Pazarcık, Kahramanmaraş earthquake (Mw 7.7) earthquake recorded at Station NAR Pazarcık, Kahramanmaraş (Overturned).

The left side of Figure 4.45 presents the records of acceleration, velocity, and displacement over time obtained from the February 6th, 2023 earthquake in Pazarcık, Kahramanmaraş, recorded at station NAR. On the right side of Figure 4.45, the acceleration, velocity, and displacement histories of a type-B cycloidal-shaped pulse proposed by Jacobsen and Ayre [94] and Makris [4] are depicted. This motion produced a back-and-forth pulse with a duration of approximately 5.7 seconds. To create Figure 4.45 (right), estimated values of $T_p=4.7$ seconds and $V_p=0.734$ m/s were utilized for the pulse duration and velocity pulse amplitude observed in the recordings at station NAR illustrated on the left.

An additional numerical analysis was conducted to examine the dynamic behavior of the ancient column under the type-B cycloidal-shaped waveform. As shown in Figure 4.46, the acceleration values obtained from the type-B cycloidal pulse were found to be insufficient to initiate rocking motion. To induce rocking behavior in the numerical model undergoing a B-type pulse, the input acceleration must equal or exceed 0.165g. Hence, rocking motion was not observed in this analysis

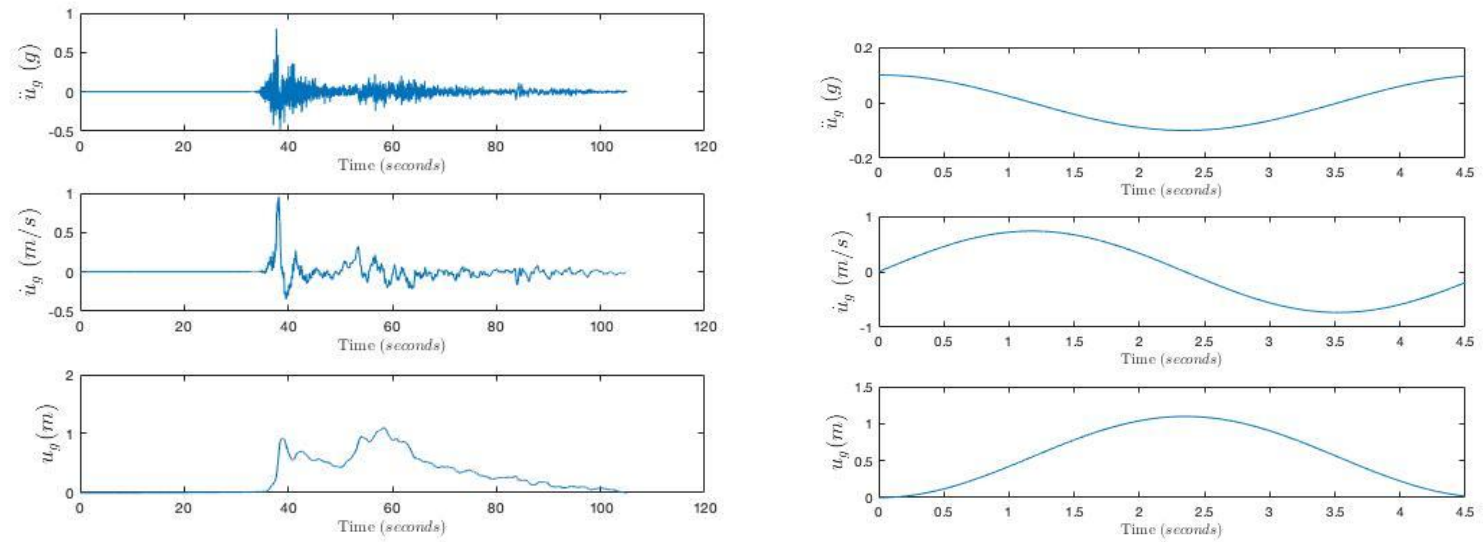


Figure 4.45. Left: Time-domain histories of acceleration, velocity and displacement recorded at Station NAR Pazarcık, Kahramanmaraş during the February 6th, 2023 Kahramanmaraş earthquake Right: a cycloidal-shape type-B pulse.

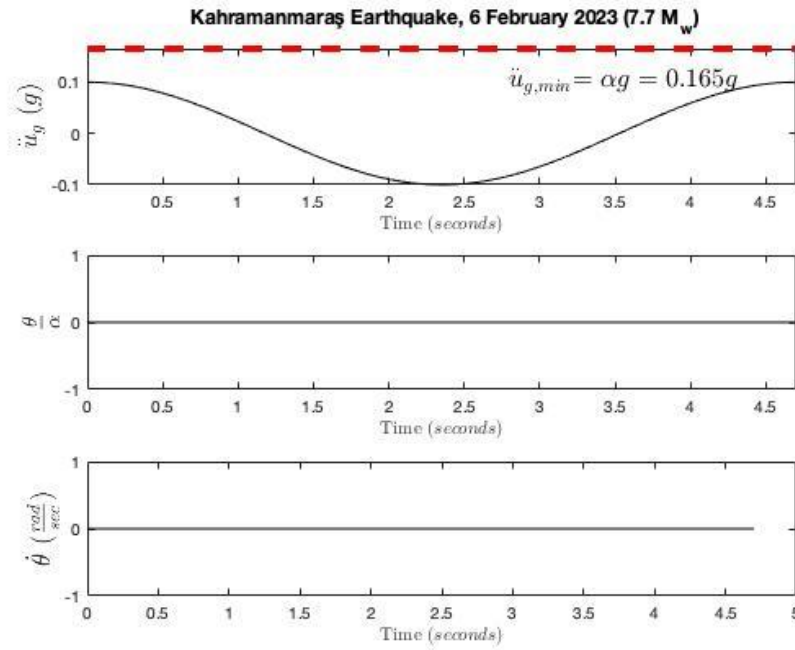


Figure 4.46. Time-domain histories of rotation and rotational velocity of a rigid column under a cycloidal type-B pulse obtained from Station NAR Pazarcık, Kahramanmaraş (No Rocking motion).

4.5. Numerical Results with Nonlinear Approximation

Numerical solutions derived from Equations (3.35) and (3.36) can be utilized to determine both the nonlinear and linear dynamic responses of a rigid block exposed to dynamic loading. This part of the chapter proceeds with dynamic results in terms of rotation and angular velocity time histories of a single solid block subjected to harmonic base acceleration. In addition, the plots presented in Figure 4.47 left and Figure 4.47 right show the minimum acceleration amplitude that do and do not overturn the rigid block subjected to harmonic base acceleration. Similar to the previous models, analyses were carried out for two cases, where in first case numerical results are obtained under harmonic base acceleration using 4th-5th order Runge-Kutta integration method within the limits of linear approximation. Second case consists of the numerical solutions obtained from Runge-Kutta algorithm within the bounds of nonlinear approximation. To validate the accuracy of the path followed in the present study the same model was used for comparison of numerical solutions of linearized form and nonlinear nature

of equation of motion in this dissertation. The subsequent section of this chapter presents dynamic results, including time histories of rotation, angular velocity, and total energy for a single rigid block subjected to a variety of trigonometric pulses. These results are further illustrated in Figure 4.48, where the left plot depicts the critical acceleration amplitude necessary to topple the block, while the right plot shows the acceleration amplitude that does not overturn the rigid block.

4.5.1. Nonlinear Dynamic Response under the Sinusoidal Harmonic Base Acceleration

Section 4.5.1 of this chapter compares the results of numerical analyses of forced rocking motion using both linear and nonlinear approximations. This study employs the same numerical model having dimensions of $2b=0.4\text{m}$, $2h=1.6\text{m}$ in width and height was used. The angle of slenderness of the rectangular block is $\alpha = 0.245 \text{ (rad)}$, half-diagonal is $R=0.8246\text{m}$ and frequency parameter is $p = 2.9871 \text{ rad/sec}$. Numerical approaches for the rigid block subjected to harmonic base acceleration was used to calculate linear and nonlinear dynamic response of the block. For all cases investigated herein in this chapter of dissertation the value of coefficient of restitution, v was taken to be equal to 0.91. This part of the study proceeds with dynamic results in terms of rotation and angular velocity time histories of single rocking stand-alone block when subjected to horizontal sinusoidal base acceleration.

In the Figure 4.47, 4th-5th order Runge-Kutta integration method (ode45 solver) was used for solving nonlinear and linear differential equations (ODEs). Numerical results presented in Figure 4.47 (left) indicate the maximum acceleration amplitude value that rectangular rigid block can resist on its base without overturning. On the left Figure 4.47, the threshold magnitude of acceleration pulse is calculated as $a_p = 5.101 \text{ m/sec}^2$ (0.52g) using numerical Runge-Kutta method and block does not overturn but simply swing until it attains a stable equilibrium. On the right Figure 4.47, results indicate that the critical acceleration amplitude to topple the rigid block. On the right side of the Figure 4.47, the peak of acceleration pulse is calculated as $a_p = 5.4 \text{ m/sec}^2$ (0.55g) using Runge-Kutta numerical method and block overturns. As it can clearly be observed from the Figure 4.47 (left and right) the nonlinear and linear outcomes obtained via the Runge-Kutta (ode45 solver) are in very good agreement.

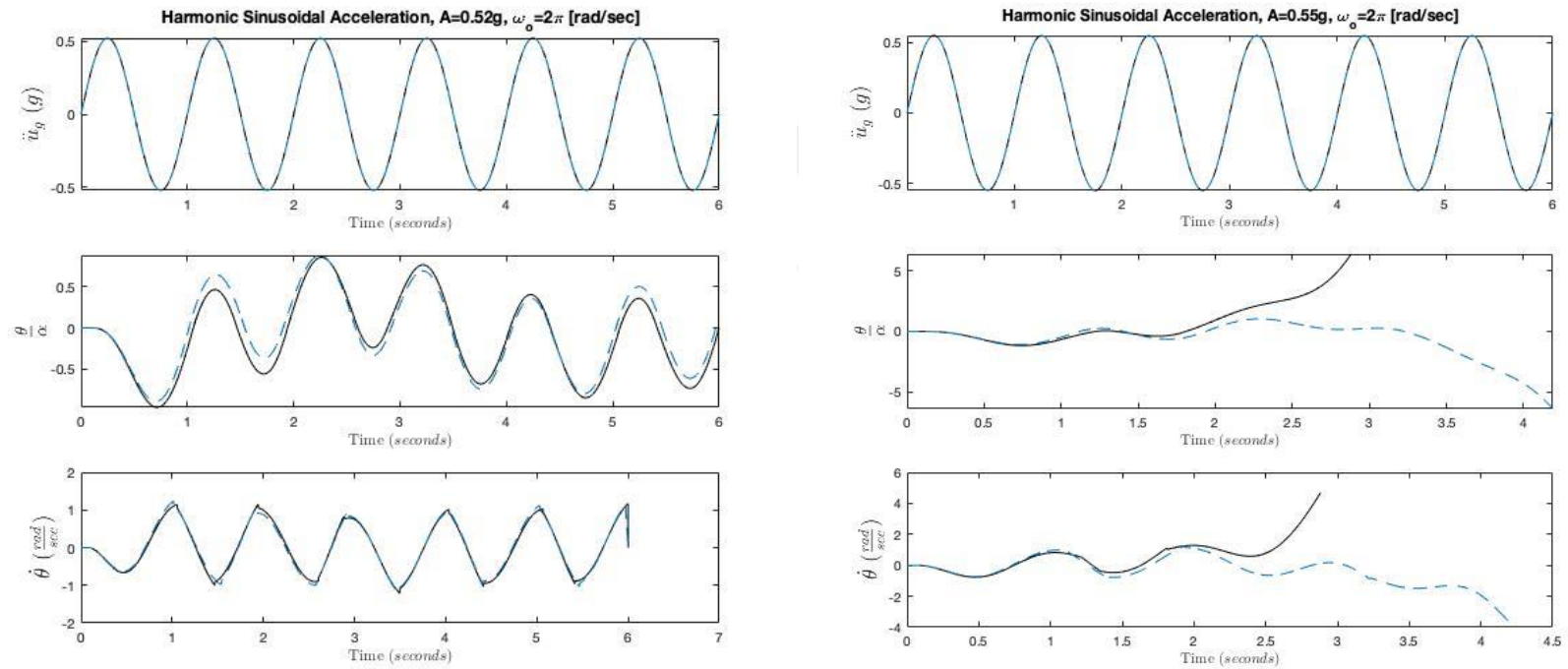


Figure 4.47. Time-domain histories of rotation and rotational velocity of a rigid block under harmonic sinusoidal acceleration
Solid line: Numerical solution with linear formulation. Dashed line: Numerical solution with nonlinear formulation. Left: No overturning Right: Overturning.

4.5.2. Nonlinear Dynamic Response and Overturning Spectrum under Pulse-like Excitations

Numerical solutions for the rigid block subjected to pulse-like excitations given by Equation (3.35) and Equation (3.36) can be used to calculate nonlinear and linear dynamic response of the block. This part of the chapter proceeds with dynamic results in terms of rotation, angular velocity and total energy time histories of a single rigid block undergoing a variety of trigonometric-shaped pulses. In addition, the plots presented in Figure 4.48 and Figure 4.49 show the minimum acceleration amplitude that do and do not overturn the rigid block subjected to pulse-like excitations. The study performed in this thesis for the cases involving one-cycle sine wave and half-cycle sine wave, respectively. Similar to the previous models, analyses were carried out for two cases, where in first case numerical results are obtained under one type of pulse-like excitation using 4th-5th order Runge-Kutta integration method within the limits of linear approximation. Second case consists of the numerical solutions obtained from Runge-Kutta algorithm within the limits of nonlinear approximation. To validate the accuracy of the path followed in the present study the same model was used for comparison of numerical solutions of linearized form and nonlinear nature of equation of motion in this dissertation.

4.5.2.1. Response to Full-cycle Sine Wave

Figure 4.48 presents time-domain histories of rotation and rotational velocity of a rigid block under one-cycle sine wave. The numerical solutions obtained for both the linear and nonlinear equations of motion demonstrate a degree of consistency. However, a comparison of these two solution sets reveals that the linear solution consistently produces more conservative, overestimated results compared to the nonlinear approximation. Numerical results presented in Figure 4.48 left indicate the maximum acceleration amplitude value that rectangular rigid block can resist on its base without overturning. On the left Figure 4.48, the peak amplitude of acceleration pulse is calculated as $a_p = 2.943 \text{ m/sec}^2$ (0.30g) using numerical Runge-Kutta method and block does not overturn.

Results presented in Figure 4.48 right indicate that the critical threshold for acceleration amplitude to topple the rigid block. On the right Figure 4.48, the peak of acceleration pulse is calculated as $a_p = 2.943 \text{ m/sec}^2$ (0.30g) and block overturns.



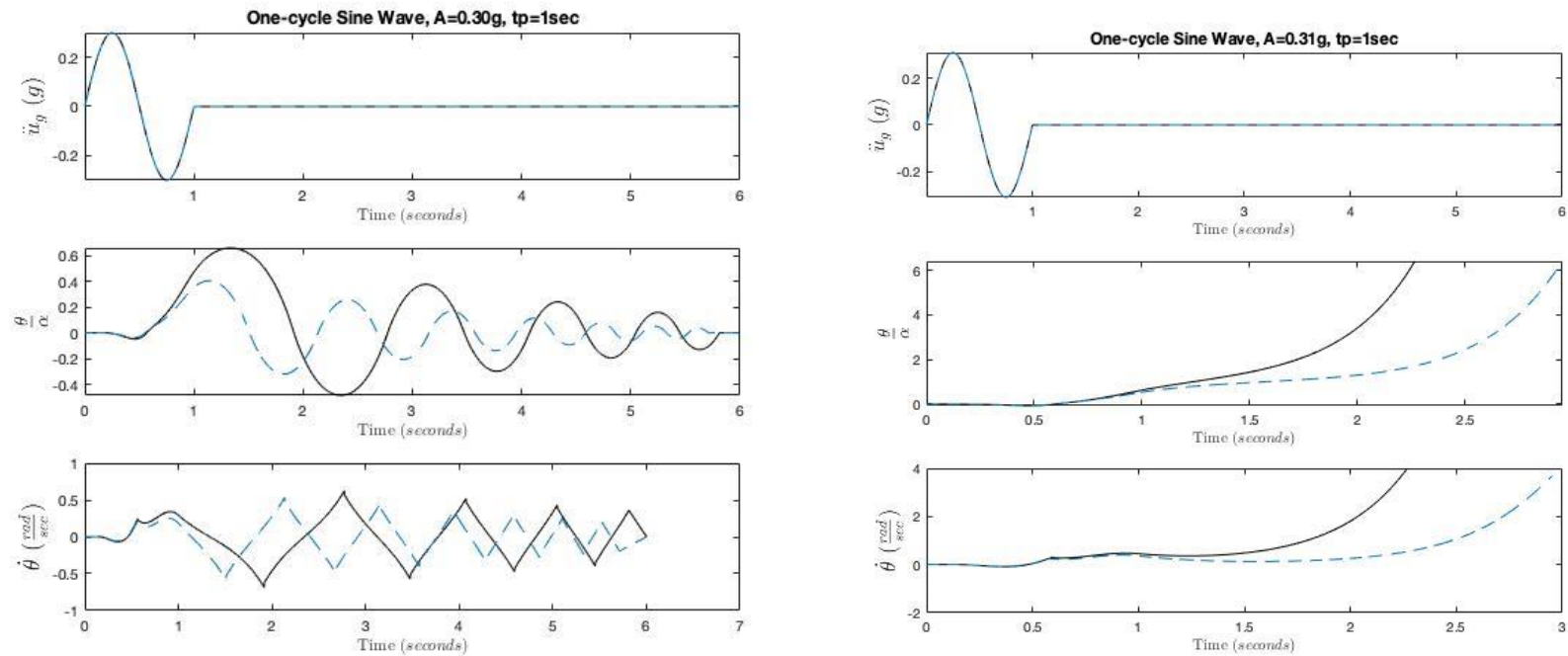


Figure 4.48. Time-domain histories of rotation and rotational velocity of a rigid block under one-cycle sine wave
Solid line: Numerical solution with linear formulation. Dashed line: Numerical solution with nonlinear formulation. Left: No overturning Right: Overturning.

4.5.2.2. Response to Half-cycle Sine Wave

Similar to the previous models, analyses were conducted for two scenarios. In the first scenario, numerical results were obtained under a half-cycle pulse excitation utilizing the 4th-5th order Runge-Kutta integration method within the framework of linear approximation. The second scenario involved obtaining numerical solutions using the Runge-Kutta algorithm within the context of nonlinear approximation. To validate the accuracy of the methodology employed in the present study, the same model was utilized for comparing the numerical solutions of the linearized form and the nonlinear nature of the equation of motion, as discussed in this dissertation. Figure 4.49 presents angular displacement and angular velocity time histories of a rigid block under half-cycle sine wave. Obtained results indicate that a good overall agreement appears to exist between the obtained result, even though values were not exactly the same. Numerical results presented in Figure 4.49 left indicate the maximum acceleration amplitude value that rectangular rigid block can resist on its base without overturning but simply swing until it attains a stable equilibrium. On the left Figure 4.49, the threshold magnitude of acceleration pulse is calculated as $a_p = 4.415 \text{ m/sec}^2$ (0.45g) using numerical Runge-Kutta method and block does not overturn. Results presented in Figure 4.49 right indicate that the required critical acceleration amplitude to topple the rigid block. On the right Figure 4.49, the magnitude of acceleration pulse is calculated as $a_p = 4.513 \text{ m/sec}^2$ (0.46g) and block overturns.

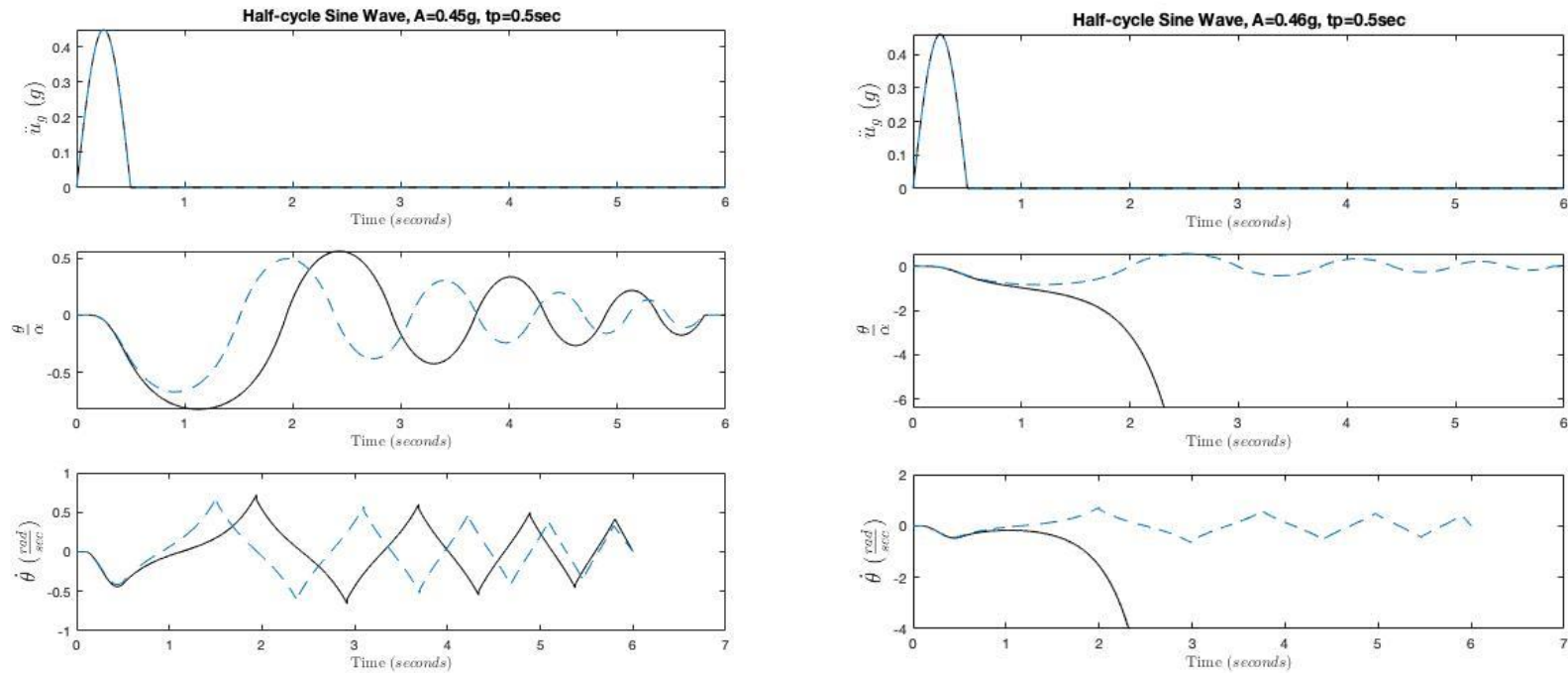


Figure 4.49. Time-domain histories of rotation and rotational velocity of a rigid block under half-cycle sine wave
Solid line: Numerical solution with linear formulation. Dashed line: Numerical solution with nonlinear formulation. Left: No overturning Right: Overturning.

5. CASE STUDY: CATHEDRAL OF THE ANNUNCIATION, İSKENDERUN

The Cathedral of the Annunciation also known as the Alexandrian Catholic Church is a historic Catholic cathedral located in Iskenderun, Hatay, Turkey. The Cathedral of the Annunciation in Iskenderun is positioned at a latitude of 36.587 and a longitude of 36.165. The cathedral served as a place of worship for the Catholic community in the region and was acknowledged for its intricate design and cultural significance.

The cathedral suffered severe structural damage following two earthquakes with a magnitude of Mw 7.7 (8.6 km depth) and Mw 7.6 (7.6 km depth) occurred in Pazarcık (Kahramanmaraş) and Elbistan (Kahramanmaraş). As a result, the cathedral experienced a collapse of both its roof and sections of its walls. The front wall, though severely damaged, managed to endure without collapsing entirely. Within this chapter, a numerical analysis was conducted on the aforementioned frontal wall of the Cathedral of the Annunciation.

This chapter presents a case study involving the numerical modelling of the front wall of the Cathedral of the Annunciation and evaluation of its response under cycloidal pulses motion used to simulate the effects of this Kahramanmaraş earthquake event. A pulse model proposed by Jacobsen and Ayre [94] and Makris [4] was used to represent near-fault ground motions. Additionally, this study employed a nonlinear formulation to calculate the rocking response of the front wall under near-source ground motions. Figure 5.1 exhibits an image of the Roman Catholic Church of the Annunciation prior to the earthquake, while Figure 5.2 and 5.3 present the church's appearance following the 2023 Kahramanmaraş earthquake sequence.



Figure 5.1. A view of the Roman Catholic Church of Annunciation, in Iskenderun, Turkey (before earthquake) [101].



Figure 5.2. The Cathedral of Iskenderun (Alexandretta) is in a state of ruin [101].



Figure 5.3. The Cathedral of Iskenderun (Alexandretta) destroyed by 7.7 magnitude earthquake [101].

Here, the analyses of rocking motion of front wall (in plane) were carried out for a numeric model having dimensions of $2b=0.43\text{m}$, $2h=6.02\text{m}$ in width and height, the angle of slenderness $\alpha = 0.0713 \text{ (rad)}$, half-diagonal is $R=3.0177\text{m}$ and frequency parameter is $p = 1.5615 \text{ rad/sec}$. Geometric parameters of this front wall including width and height ($2b$ and $2h$), the angle of slenderness (α), half-diagonal (R), frequency parameter (p) are summarized in Table 5.1.

Table 5.1. Geometric characteristics of the front wall of the Cathedral of the Annunciation (In Plane).

The front wall	B (m)	H (m)	$b = B/2$	$h = H/2$	R(m)	$\alpha \text{ (rad)}$
W1	0.43m	6.02m	0.215m	3.01m	3.0177m	0.0713

Figure 5.4 depicts the dynamic response of the ancient column during the catastrophic 2023 Pazarcık, Kahramanmaraş earthquake (Mw 7.7) earthquake. Similar to prior studies, a nonlinear formulation was employed to calculate the dynamic response of the numerical model in this study. The nonlinear analysis presented in Figure 5.4 indicates that the ancient column experienced overturning due to the motion of the 7.7 magnitude Kahramanmaraş earthquake recorded at Station 3116 in İskenderun, Hatay.

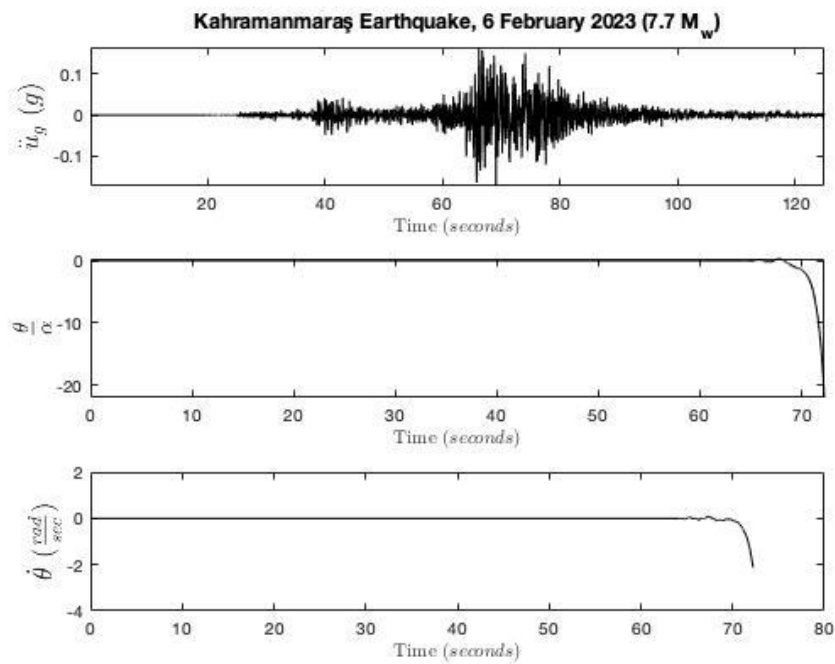


Figure 5.4. Time-domain histories of rotation and rotational velocity of the front wall under 2023 Pazarcık, Kahramanmaraş earthquake (Mw 7.7) recorded at Station 3116 İskenderun, Hatay (Overturned) (Nonlinear approximation).

This chapter presents a selected near-source ground motion recorded at Station 3116 İskenderun, Hatay and demonstrates its similarity to a physically cycloidal pulse type-B. For the strong ground motion record captured at Station 3116, the main characteristic values needed to define an inverse cycloidal pulse type-B such as V_p and t_p are summarized in Table 5.2 [95].

Table 5.2 Characteristics of one pulse-like excitation used in this study [95].

Station	Longitude (°E)	Latitude (°N)	Region	R_{rup} (km)	PGA (g)	PGV (m/s)	V_p (m/s)	t_p (s)
3116	36.6162	36.2066	İskenderun, Hatay	16.8	0.172g	0.525	0.29	14.5

On the left side of Figure 5.5, the East-West components of acceleration, velocity, and displacement records over time from the February 6th, 2023 Pazarcık, Kahramanmaraş earthquake recorded at station 3116 are illustrated. On the right side of Figure 5.5, the acceleration, velocity, and displacement histories of an inverse type-B cycloidal-shaped waveform proposed by (Jacobsen and Ayre [94]; Makris and Roussos [4]) are presented. The resulting pulse, characterized by a forward-and-backward oscillation, exhibited a duration of approximately 14.5 seconds. In order to create Figure 5.5 (right), approximate values of $T_p=14.5$ seconds and $V_p=0.29$ m/s were utilized for the pulse duration and velocity pulse amplitude observed in the recordings at station 3116 Hatay, İskenderun depicted on the left.

An additional numerical analysis was conducted to examine the dynamic behavior of the ancient column under the type-B cycloidal pulse. As shown in Figure 5.6, the acceleration values obtained from the type-B cycloidal pulse were found to be insufficient to initiate rocking motion. To induce rocking behavior in the numerical model undergoing a B-type pulse, the input acceleration must equal or exceed 0.165g. Hence, rocking motion was not observed in this analysis.

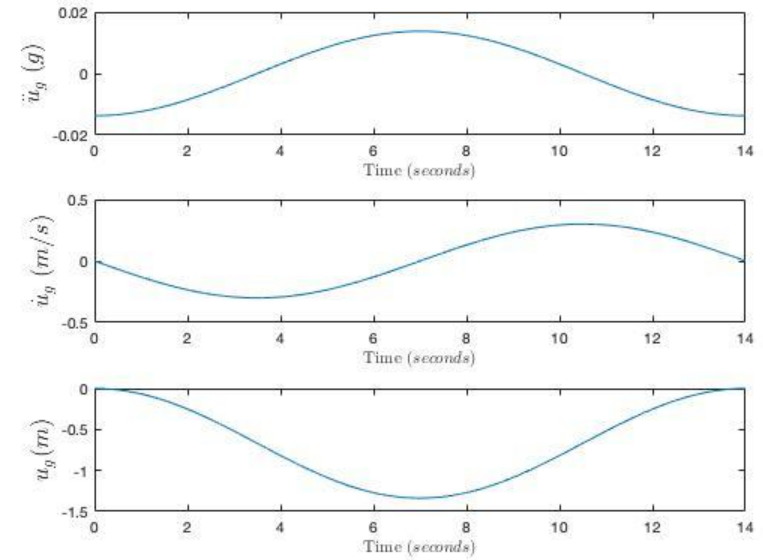
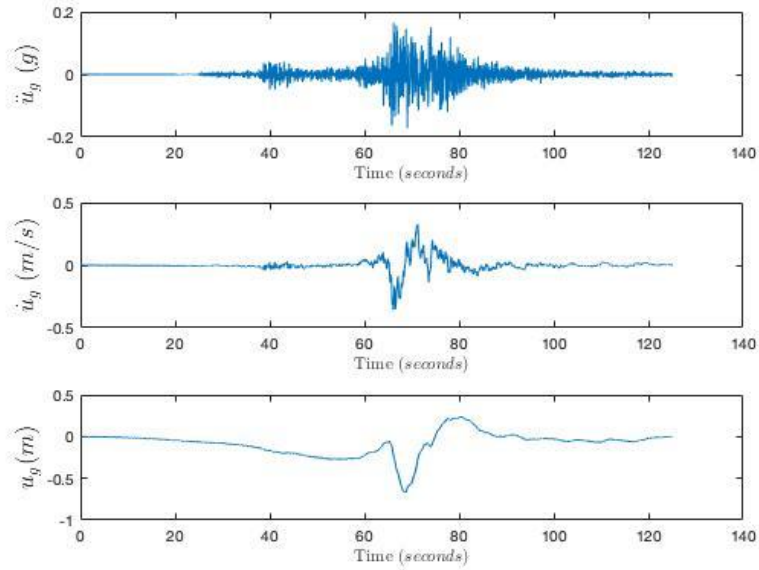


Figure 5.5. Left: Time-domain histories of acceleration, velocity and displacement recorded at Station 3116 İskenderun, Hatay during the February 6th, 2023 Kahramanmaraş earthquake Right: an inverse cycloidal type-B pulse.

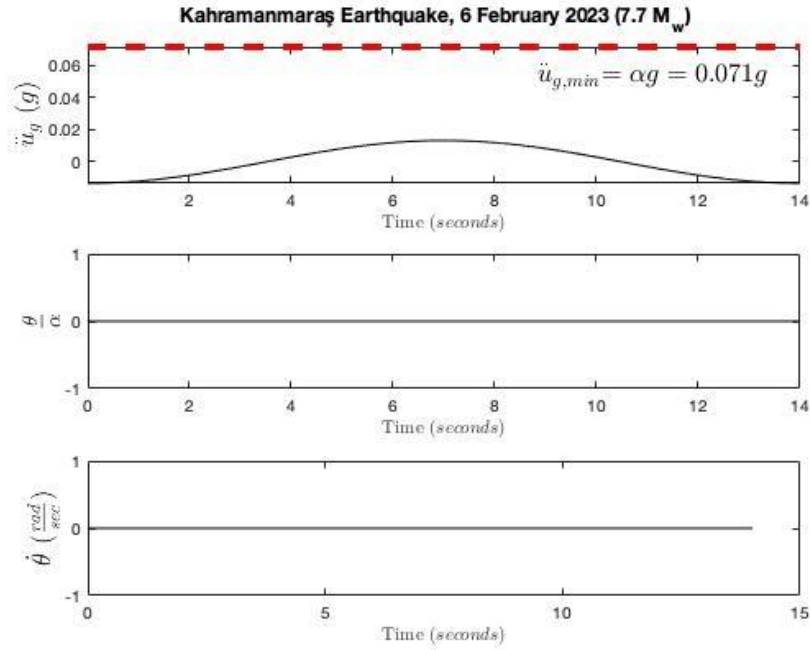


Figure 5.6. Time-domain histories of rotation and rotational velocity of the front wall under a cycloidal type-B pulse obtained from Station 3116 İskenderun, Hatay (No Rocking motion).

The numerical results obtained through analyses provided insight into why rocking motion is not observed while anticipating overturning under a long-duration pulse. The numerical results indicate that the maximum acceleration amplitude of the pulse is below the threshold, $\ddot{u}_g(t)$ necessary to initiate rocking motion. Consequently, the absence of rocking motion under this pulse condition is expected.

To evaluate the negative impact of pulse motion on the front wall of the Cathedral of the Annunciation exhibiting rocking motion in plane, the acceleration amplitude applied to the numerical model was increased and aligned closer to the minimum required acceleration amplitude, $\ddot{u}_g(t)$, for initiating rocking, without altering the pulse duration. Under these revised conditions, re-analysis of the numerical model revealed the presence of rocking motion as seen in Figure 5.7.

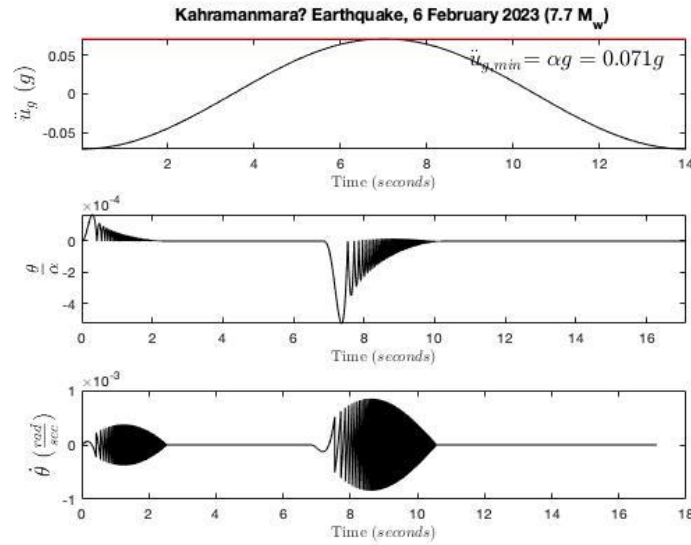


Figure 5.7. Time-domain histories of rotation and rotational velocity of the front wall under a cycloidal type-B pulse obtained from Station 3116 İskenderun, Hatay (Rocking motion).

Further extending the analysis, a new pulse form with the same duration but exceeding the minimum acceleration amplitude needed to trigger rocking motion was applied. Re-analysis under these conditions demonstrated that the pulse movement adversely affects a rocking front wall, indicating a potential risk of overturning as clearly seen in Figure 5.8.

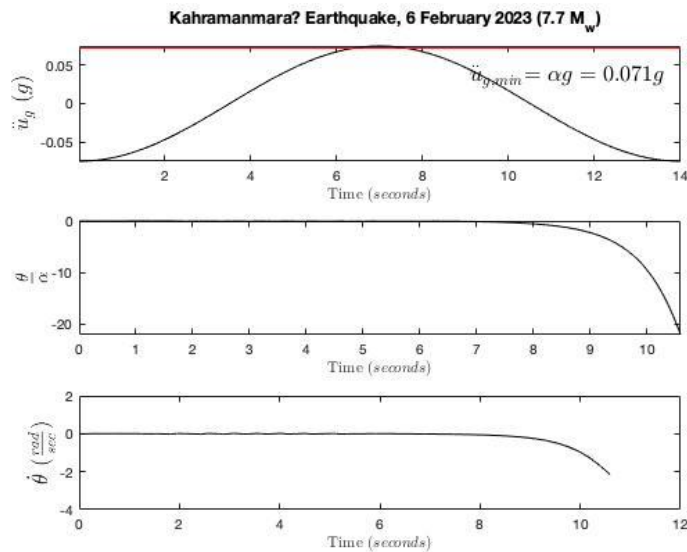


Figure 5.8. Time-domain histories of rotation and rotational velocity of the front wall under a cycloidal type-B pulse obtained from Station 3116 İskenderun, Hatay (Overturned).

6. CONCLUSIONS AND DISCUSSION

This dissertation focuses on the dynamic back-and-forth response of rigid blocks, commonly found in ancient structures such as free-standing columns resting on rigid foundations. The primary objective is to enhance our understanding of the fundamental dynamics of rocking systems by examining the response of these rigid blocks to various seismic excitations through numerical solutions. The research investigates the rocking behavior of rigid blocks excited by both trigonometric pulses and real earthquake records, providing insights into the complex interactions between seismic forces and structural stability.

This thesis presents a comprehensive study on the dynamic back-and-forth response of free-standing solid columns exposed to various seismic excitations. The research aimed to develop rigorous and simplified numerical models capable of realistically simulating the behavior of these structures under diverse excitation scenarios. The first part of the thesis focuses on investigating the rocking response of rigid bodies under a variety of trigonometric pulses. These pulses included one-cycle sine wave, half-cycle sine wave, rectangular constant acceleration pulse, Haar wavelet pulse, antisymmetric Ricker acceleration pulse, and symmetric Ricker acceleration pulse. The study examined the influence of frequency content and amplitude of the ground acceleration function on the overturning thresholds of rigid bodies for each pulse type. The second part of the thesis evaluates the overturning acceleration spectrums (OAS) for different ground excitation forms. The study explored the impact of frequency and amplitude of the ground acceleration function on the overturning thresholds of rigid bodies. To understand the influence of scale on the overturning curve, samples with varying sizes and slenderness ratios were analyzed. The results demonstrate a significant correlation between the rocking response of rigid bodies and the amplitude, frequency content, slenderness ratio, and size parameter of the ground acceleration. These findings are illustrated through comprehensive plots and graphics, highlighting the impact of each parameter on the structural behavior.

A scenario involving the numerical simulation of a monolithic free-standing column, representing the column of the Temple of Olympian Zeus in Syracuse, was investigated. The

study assessed the column's rocking dynamics when exposed to the severe El Centro earthquake (M_w 6.9) and the 2023 Pazarcık, Kahramanmaraş earthquake (M_w 7.7), with the aim of understanding the structural response under different seismic events. This thesis presents a case study of the Cathedral of the Annunciation, which suffered significant damage during the 2023 Pazarcık, Kahramanmaraş earthquake sequence. Despite the severe earthquake, the front wall of the cathedral remained standing, exhibiting potential for rocking motion within its plane. This study modeled and evaluated this wall to analyse its structural response. The primary conclusions drawn from this dissertation are as follows:

- The study reveals that the rocking behavior of rigid bodies is influenced by both geometric characteristics, such as slenderness ratio and size, and the characteristics of the ground acceleration function. Specifically, the frequency content and amplitude of the ground acceleration function have a significant impact on the overturning thresholds for different pulse types.
- While numerical results for both linear and nonlinear equations of motion show some level of agreement, the linear solution consistently overestimates the results compared to the nonlinear approximation. This finding highlights the importance of considering nonlinear effects for accurate analysis of dynamic responses.
- This research highlights the importance of excitation input type on the dynamic response of rocking systems. The study found that the analysis model's critical threshold for the acceleration amplitude under a one-cycle sine input was lower than that required under a half-cycle sine wave. This indicates that transitioning from a one-cycle sine wave to a half-cycle sine wave significantly impacts the model's susceptibility to toppling.
- The research findings indicate that the threshold acceleration amplitude necessary to topple the analysis model undergoing a Haar wavelet input was lower than the acceleration amplitude needed to topple the same rectangular rigid block model under a rectangular acceleration pulse.

- This study derived accurate mathematical expressions for generating the minimum acceleration amplitude spectrums (OAS) for a rigid block undergoing a variety of trigonometric-shaped pulses, including one-cycle sine wave, half-cycle sine wave, rectangular constant acceleration pulse, Haar wavelet pulse, antisymmetric Ricker acceleration pulse, and symmetric Ricker acceleration pulse. Analysis of rectangular rigid blocks with slenderness ratios less than 20° demonstrated a strong agreement between the results obtained from derived polynomial, exponential, and linear equations and those from numerical analysis.
- This dissertation provides further evidence to support the findings of Makris and Roussos [4], who demonstrated that Housner's equation [3] for the overturning threshold of rigid blocks is overly conservative. The numerical results obtained in this study, specifically for a rigid block exposed to a half-cycle sine wave, indicate that the threshold for toppling acceleration amplitude is lower than the values predicted by Housner's equation [3]. This finding reinforces the conclusion that Housner's equation overestimates the stability of rigid blocks under dynamic loading conditions [3].
- Additionally, Housner [3] derived the overturning condition for a rigid block based on the false assumption that the angular displacement equals to the block slenderness at the time when the half-sine pulse reaches its end. However, this study, in agreement with the findings of Makris and Roussos [4], demonstrates that a block actually overturns during its unforced oscillation phase, not at the exact moment the pulse ends. This research further supports the conclusion that the overturning threshold predicted by Housner's equation is overly conservative.
- The study demonstrated that under pulse-like excitations, rigid blocks did not exhibit rocking motion below the minimum required acceleration amplitude. This suggests that pulse-like excitations, under these specific conditions, do not have a discernible negative effect on rocking structures. To observe this negative effect, the pulse acceleration amplitude must be closer to the minimum necessary acceleration amplitude to induce

rocking motion. Under these conditions, it was observed that the rigid block, once initiated into rocking motion, overturned quickly. This highlights that pulse-like excitations, with their long durations and high velocities, are potentially dangerous for rocking structures. Consequently, the study concludes that all near-fault ground motions exhibiting pulse effects pose a significant risk to structures capable of rocking motion.

- The devastating 2023 Pazarcık, Kahramanmaraş earthquake (Mw 7.7) resulted in significant ground displacement, with satellite imagery, aerial photographs, and field surveys indicating a permanent displacement of 3–7 meters [102]. Interestingly, the displacement seismogram in Figure 5.5 exhibits a distinct pulse, suggesting a lack of fault displacement (fling step), extended analysis of near-source waveform data could reveal this crucial information. A future study focusing on fling-step recovery from near-source waveforms would be highly beneficial. Examining earthquake records from stations close to the fault could provide valuable insights into the actual fault displacement. The presence of a one-sided pulse in the velocity trace and an offset at the end of the displacement trace (fling-step) can be observed with meticulous data processing. By re-evaluating properly processed near-source strong motion data, one could investigate the inverse effect of the pulse excitation on the dynamic response of rocking structures, potentially leading to a better understanding of their behavior during earthquakes.

Future Studies:

- This foundational research on rigid blocks can be extended and applied to frame systems, providing a basis for more complex structural analyses.
- Parametric studies could be conducted to further explore the impact of different pulse-like forms on structural rocking dynamics.
- More studies might be done to understand how the connection between a rigid block and its base affects its dynamic response.
- This understanding of the dynamic rocking behavior of ancient columns might contribute to the development of seismic protection measures for historical structures,

particularly those located near fault lines and exhibiting a propensity for rocking movements.

- It would be beneficial to conduct a study examining how connectors between ancient columns and foundations influence the seismic stability of historical buildings.



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