T.C.

GEBZE YÜKSEK TEKNOLOJİ ENSTİTÜSÜ

MÜHENDİSLİK VE FEN BİLİMLERİ

ENSTİTÜSÜ

EİNSTEİN ALAN DENKLEMLERİNE BİR ÇÖZÜM DENEMESİ

Meral ERTUĞRUL

YÜKSEK LİSANS TEZİ

MATEMATİK ANABİLİM DALI

GEBZE

2009

T.C.
GEBZE YÜKSEK TEKNOLOJİ ENSTİTÜSÜ
MÜHENDİSLİK VE FEN BİLİMLERİ
ENSTİTÜSÜ

EİNSTEİN ALAN DENKLEMLERİNE BİR ÇÖZÜM DENEMESİ

Meral ERTUĞRUL
YÜKSEK LİSANS TEZİ
MATEMATİK ANABİLİM DALI

TEZ DANIŞMANI
Yrd.Doç.Dr. Yücel ENGİNER

GEBZE
2009

YÜKSEK LİSANS TEZİ JÜRİ ONAY SAYFASI

G.Y.T.E. Sosyal Bilimler Enstitüsü Yönetim Kurulu'nun 15/06/2009 tarih ve 2009/13 sayılı kararıyla oluşturulan jüri tarafından 24/06/2009 tarihinde tez savunma sınavı yapılan Meral ERTUĞRUL'un tez çalışması Matematik Anabilim Dalında YÜKSEK LİSANS tezi olarak kabul edilmiştir.

JÜRİ

ÜYE
(TEZ DANIŞMANI) : Yrd.Doç. Dr. Yücel ENGİNER

ÜYE : Yrd. Doç. Dr. Coşkun YAKAR

ÜYE : Dr. Fatih DİKMEN

ONAY

G.Y.T.E. Sosyal Bilimler Enstitüsü Yönetim Kurulu'nun ………/………/20… tarih ve ……………/………… sayılı kararı.

İMZA/MÜHÜR

ÖZET

TEZ BAŞLIĞI: Einstein Alan Denklemlerine Bir Çözüm Denemesi

YAZAR ADI: Meral ERTUĞRUL

Bu tezde Einstein Alan Denklemlerine bir çözüm önerisi bulunmaya çalışıldı.Bu çalışma küresel simetrik bir uzayda yapıldı.Bu uzaydaki Enerji-Momentum Tensörünün ,madde-enerji yoğunluğu ve basınç terimi sıfırdan farklı kabul edildi.Bu tensörler Einstein Alan Denklemlerinde kullanılarak bu denklemlerin yaklaşık çözümü yapılmış ve metrik tensör bulunmuştur. Bu çalışmada diferansiyel denklemler için yaklaşık çözüm yöntemleri kullanılmıştır. Bu durumda bulunan uzay zamanının,madde-enerji dağılımını veren fonksiyona bağlı olduğunu gördük. Bulunan metrik tensörün De-Sitter metriği ile karşılaştırılması yapılmıştır.

SUMMARY

THESIS TİTLE : A Solution attempt to Einstein Field Equations

THESIS AUTHOR: Meral ERTUĞRUL

In this thesis, a solution for Einstein Field Equation is tried to be found. This research has been done at a spherically symmetric space. At this space, matter-energy intensity and pressure term of Energy- Momentum tensor have been accepted nonzero. Using these tensors in Einstein Field Equation, approximate solutions of these equations are done and metric tensor has been found. In this research approximate solution techniques are used for differential equations. We experienced that the space time in this condition depends on the function that shows the matter-energy distribution. the Metric is compared to De- Sitter's .

TEŞEKKÜR

Bu tez çalışmasında bilgisini ve yardımını benden esirgemeyen çok değerli danışmanım Yrd.Doç.Dr.Yücel ENGİNER'e ve Gebze Yüksek Teknoloji Enstitüsü Fen Fakültesi Matematik Bölümü'nün tüm öğretim elemanlarına teşekkür ederim.

Ayrıca çalışmalarımda beni destekleyen aileme de teşekkür ederim.

İÇİNDEKİLER DİZİNİ

Sayfa

1.GİRİŞ

1.1.Einstein Alan Denklemleri

Einstein alan denklemlerindeki g_{ab} bileşeni metrik tensördür.Metrik , uzaydaki iki nokta arasındaki en küçük yol olarak tanımlanır .Uzayda bu metrik

$$ds^2 = g_{ab} dx^a dx^b \tag{1.1}$$

ile ifade edilir.Burada a=b=0,1,2,3 olmak üzere ,söz konusu 4 boyutlu bu yapıda x^0 ıncı koordinat zamanı , x^1, x^2, x^3 koordinatları da uzayı oluşturur. Uzaysal kısım kartezyen,silindirik veya küresel koordinatlarda yazılabilir.Küresel koordinatlarda bu bileşenler (t, r, θ, ϕ) gibi ifade edilebilir. ds^2 hakkında birşeyler söyleyebilmek için g_{ab} metrik tensörünün yapısı bilinmelidir.

g_{ab} , Einstein Alan Denklemleri sayesinde çözülür. Einstein Alan Denklemleri

$$R_{ab} - \frac{1}{2} R g_{ab} = T_{ab} \tag{1.2}$$

şeklinde verilir.Bu konuyla ilgili detaylı çalışmalar Foster [1] , Ohanian H [2] , Wold [3] de bulunabilir.Bu eşitlikte
R_{ab} Ricci Tensörü , R Ricci Skaler Eğriliği , T_{ab} Enerji-Momentum Tensörüdür.

1.2.Einstein Alan Denklemlerinin Çözüm Algoritması

Çözüm için öncelikle g_{ab} ’ ye bir form önerilir. g^{ab} , g_{ab} nin ters matrisi olarak tanımlanır.

g^{ab} ve g_{ab} den faydalanılarak Christoffel Sembolleri elde edilir. Christoffel Sembolleri

$$\Gamma^{a}_{bc} = \frac{1}{2} g^{ad} \left[\partial_b g_{dc} + \partial_c g_{bd} - \partial_d g_{bc} \right] \quad 1.3$$

formundadır. (1.3) ile verilen formüller Riemann Eğrilik Tensörünü bulmakta kullanılır. Riemann Eğrilik Tensörü

$$R^{a}_{bcd} = \partial_c \Gamma^{a}_{bd} - \partial_d \Gamma^{a}_{bc} - \Gamma^{e}_{bd} \Gamma^{a}_{ec} - \Gamma^{e}_{bc} \Gamma^{a}_{ed} \quad 1.4$$

a ve c indisleri eşitlenerek Ricci tensörleri bulunur. Ricci tensörleri aşağıdaki gibidir.

$$R_{00} = R^{0}_{000} + R^{1}_{010} + R^{2}_{020} + R^{3}_{030}$$

$$R_{11} = R^{0}_{101} + R^{1}_{111} + R^{2}_{121} + R^{3}_{131}$$

$$R_{22} = R^{0}_{202} + R^{1}_{212} + R^{2}_{222} + R^{3}_{232}$$

$$R_{33} = R^{0}_{303} + R^{1}_{313} + R^{2}_{323} + R^{3}_{333}$$

$$R = g^{ab} . R_{ab} \quad 1.5$$

Formülü kullanılarak Ricci Skaler Eğriliği bulunur. Bulunan bilgiler $R_{ab} - \frac{1}{2} R g_{ab} = T_{ab}$ denkleminde yerine konularak ilgili diferansiyel denklem sistemi elde edilir. Teorik olarak bu denklemin çözümünde g_{ab} metriğinin bileşenleri bulunabilir.Bulunacak sonuçlar T_{ab} enerji-momentum tensörünün bileşenlerine bağlı olacaktır.

2.EİNSTEİN ALAN DENKLEMLERİNİN ÇÖZÜMÜ

2.1.Metrik Form

ds^2 metrik formunu küresel koordinatlara uygularsak $\left(0 < \theta < \pi\right)$ ve $\left(0 \le \phi \le 2\pi\right)$ olmak üzere aşağıdaki eşitlik elde edilir.

$$ds^2 = F\left(r\right)dt^2 + G(r)\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2\right) \tag{2.1}$$

Bu durumda metrik tensör g_{ab} aşağıdaki matris gibi ifade edilebilir.

$$g_{ab} = \begin{bmatrix} F(r) & 0 & 0 & 0 \\ 0 & G(r) & 0 & 0 \\ 0 & 0 & r^2 G(r) & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta G(r) \end{bmatrix} \tag{2.2}$$

Amaç F(r) ve G(r) yi bulmaktır.

2.2.Christoffel Sembolleri

g_{ab} metrik tensörünün tersi olarak g^{ab} yi tanımlamıştık. g^{ab} matrisi aşağıdaki gibidir.

$$g^{ab} = \begin{bmatrix} F^{-1}(r) & 0 & 0 & 0 \\ 0 & G^{-1}(r) & 0 & 0 \\ 0 & 0 & r^{-2} G^{-1}(r) & 0 \\ 0 & 0 & 0 & r^{-2} \sin^{-2} \theta G^{-1}(r) \end{bmatrix} \tag{2.3}$$

$g_{00} = F(r)$, $g_{11} = G(r)$, $g_{22} = r^2 G(r)$, $g_{33} = r^2 \sin^2 \theta G(r)$

$g^{00} = F^{-1}(r)$, $g^{11} = G^{-1}(r)$, $g^{22} = r^{-2} G^{-1}(r)$, $g^{33} = r^{-2} \sin^{-2} \theta G^{-1}(r)$ dir.

g_{ab} ve g^{ab} ler (1.3) denkleminde yerine yazılırsa aşağıdaki sonuçlar bulunur.

$\Gamma^0_{00} = 0$

$\Gamma^0_{10} = \frac{1}{2} F^{-1}(r) \frac{\partial}{\partial r} F(r)$

$\Gamma^0_{01} = \frac{1}{2} F^{-1}(r) \frac{\partial}{\partial r} F(r)$

$\Gamma^0_{11} = 0$

$\Gamma^0_{02} = 0$

$\Gamma^0_{12} = 0$

$\Gamma^0_{03} = 0$

$\Gamma^0_{13} = 0$

$$\Gamma^0_{20} = 0$$
$$\Gamma^0_{21} = 0$$
$$\Gamma^0_{22} = 0$$
$$\Gamma^0_{23} = 0$$
$$\Gamma^0_{30} = 0$$
$$\Gamma^0_{31} = 0$$
$$\Gamma^0_{32} = 0$$
$$\Gamma^0_{33} = 0$$
$$\Gamma^1_{00} = -\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} F(r)$$
$$\Gamma^1_{01} = 0$$
$$\Gamma^1_{02} = 0$$
$$\Gamma^1_{03} = 0$$
$$\Gamma^1_{10} = 0$$
$$\Gamma^1_{11} = \frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} G(r)$$
$$\Gamma^1_{12} = 0$$
$$\Gamma^1_{13} = 0$$
$$\Gamma^2_{00} = 0$$
$$\Gamma^2_{01} = 0$$
$$\Gamma^2_{02} = 0$$
$$\Gamma^2_{03} = 0$$
$$\Gamma^1_{20} = 0$$
$$\Gamma^1_{21} = 0$$
$$\Gamma^1_{22} = -\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} \left[r^2 G(r) \right]$$
$$\Gamma^1_{23} = 0$$
$$\Gamma^1_{30} = 0$$
$$\Gamma^1_{31} = 0$$
$$\Gamma^1_{32} = 0$$
$$\Gamma^1_{33} = -\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} \left[r^2 \sin^2\theta \, G(r) \right]$$
$$\Gamma^2_{20} = 0$$
$$\Gamma^2_{21} = \frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial r} \left[r^2 G(r) \right]$$
$$\Gamma^2_{22} = 0$$
$$\Gamma^2_{23} = 0$$
$$\Gamma^2_{10} = 0$$
$$\Gamma^2_{11} = 0$$
$$\Gamma^2_{12} = \frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial r} \left[r^2 G(r) \right]$$
$$\Gamma^2_{13} = 0$$
$$\Gamma^3_{00} = 0$$
$$\Gamma^3_{01} = 0$$
$$\Gamma^3_{02} = 0$$
$$\Gamma^3_{03} = 0$$
$$\Gamma^3_{10} = 0$$
$$\Gamma^3_{11} = 0$$
$$\Gamma^3_{12} = 0$$
$$\Gamma^2_{30} = 0$$
$$\Gamma^2_{31} = 0$$
$$\Gamma^2_{32} = 0$$
$$\Gamma^3_{13} = \frac{1}{2} r^{-2} \sin^{-2}\theta \, G^{-1}(r) \frac{\partial}{\partial r} \left[r^2 \sin^2\theta \, G(r) \right]$$
$$\Gamma^3_{20} = 0$$

$$\Gamma^3_{21} = 0$$

$$\Gamma^3_{22} = 0$$

$$\Gamma^3_{23} = \frac{1}{2}\, r^{-2} \sin^{-2}\theta\; G^{-1}(r) \frac{\partial}{\partial \theta}\left[r^2 \sin^2\theta\; G(r) \right]$$

$$\Gamma^3_{30} = 0$$

$$\Gamma^3_{31} = \frac{1}{2}\, r^{-2} \sin^{-2}\theta\; G^{-1}(r) \frac{\partial}{\partial r}\left[r^2 \sin^2\theta\; G(r) \right]$$

$$\Gamma^3_{32} = \frac{1}{2}\, r^{-2} \sin^{-2}\theta\; G^{-1}(r) \frac{\partial}{\partial \theta}\left[r^2 \sin^2\theta\; G(r) \right]$$

$$\Gamma^3_{33} = 0 \tag{2.4}$$

2.3.Rieman Eğrilik Tensörünün Bileşenlerinin Hesabı

(2.4) de bulunan christoffel sembolleri (1.4) denkleminde yerine yazılarak aşağıdaki riemann eğrilik tensörleri hesaplanır.

$$R^0_{101} = -\frac{\partial}{\partial r}\left[\frac{1}{2} F^{-1}(r)\, \frac{\partial}{\partial r} F(r) \right] + \left[\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} G(r) \right]\left[\frac{1}{2} F^{-1}(r)\, \frac{\partial}{\partial r} F(r) \right]$$
$$- \left[\frac{1}{2} F^{-1}(r)\, \frac{\partial}{\partial r} F(r) \right]\left[\frac{1}{2} F^{-1}(r)\, \frac{\partial}{\partial r} F(r) \right]$$

$$R^0_{110} = \frac{\partial}{\partial r}\left[\frac{1}{2} F^{-1}(r)\, \frac{\partial}{\partial r} F(r) \right] + \left[\frac{1}{2} F^{-1}(r)\, \frac{\partial}{\partial r} F(r) \right]\left[\frac{1}{2} F^{-1}(r)\, \frac{\partial}{\partial r} F(r) \right]$$
$$- \left[\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} G(r) \right]\left[\frac{1}{2} F^{-1}(r)\, \frac{\partial}{\partial r} F(r) \right]$$

$$R^0_{202} = \left[-\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r}\left[r^2 G(r) \right] \right]\left[\frac{1}{2} F^{-1}(r)\, \frac{\partial}{\partial r} F(r) \right]$$

$$R^0_{220} = -\left[-\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r}\left[r^2 G(r) \right] \right]\left[\frac{1}{2} F^{-1}(r)\, \frac{\partial}{\partial r} F(r) \right]$$

$$R^0_{303} = -\left[\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r}\left[r^2 \sin^2\theta\; G(r) \right] \right]\left[\frac{1}{2} F^{-1}(r)\, \frac{\partial}{\partial r} F(r) \right]$$

$$R^0_{330}=\left[\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, G(r)\right]\right]\left[\frac{1}{2}F^{-1}(r)\ \frac{\partial}{\partial r}\ F(r)\right]$$

$$R^1_{001}=-\frac{\partial}{\partial r}\left[-\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}F(r)\right]+\left[\frac{1}{2}F^{-1}(r)\ \frac{\partial}{\partial r}\ F(r)\right]\left[-\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}F(r)\right]$$
$$-\left[-\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}F(r)\right]\left[\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\ G(r)\right]$$

$$R^1_{010}=\frac{\partial}{\partial r}\left[-\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}F(r)\right]+\left[-\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}F(r)\right]\left[\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\ G(r)\right]$$
$$-\left[\frac{1}{2}F^{-1}(r)\ \frac{\partial}{\partial r}\ F(r)\right]\left[-\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}F(r)\right]$$

$$R^1_{212}=\frac{\partial}{\partial r}\left[-\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2G(r)\right]\right]+\left[-\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2G(r)\right]\right]\left[\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\ G(r)\right]$$
$$-\left[\frac{1}{2}\ r^{-2}\ G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\ G(r)\right]\right]\left[-\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2G(r)\right]\right]$$

$$R^1_{221}=-\frac{\partial}{\partial r}\left[-\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\ G(r)\right]\right]+\left[\frac{1}{2}\ r^{-2}\ G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\ G(r)\right]\right]\left[-\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2G(r)\right]\right]$$
$$-\left[-\frac{1}{2}\ G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\,G(r)\right]\right]\left[\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\ G(r)\right]$$

$$R^1_{313}=-\frac{\partial}{\partial r}\left[\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\ G(r)\right]\right]-\left[\frac{1}{2}\ G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\ G(r)\right]\right]\left[\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}G(r)\ \right]+$$
$$\left[\frac{1}{2}r^{-2}\sin^{-2}\theta\ G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\ G(r)\right]\right]\left[\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\ \left[r^2\sin^2\theta\ G(r)\right]\right]$$

$$R^1_{323}=-\frac{\partial}{\partial\theta}\left[\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, G(r)\right]\right]-\left[\frac{1}{2}\ r^{-2}\ G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right]\left[-\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2G(r)\right]\right]+$$
$$\left[\frac{1}{2}\ r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right]\left[\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, G(r)\right]\right]$$

$$R^1_{331} = \frac{\partial}{\partial r}\left[\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, G(r)\right]\right] - \left[\frac{1}{2}r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, G(r)\right]\right]\left[\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, G(r)\right]\right] +$$
$$\left[\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, G(r)\right]\right]\left[\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}G(r)\right]$$

$$R^1_{332} = \frac{\partial}{\partial\theta}\left[\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, G(r)\right]\right] - \left[\frac{1}{2}r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right]\left[\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, G(r)\right]\right] +$$
$$\left[\frac{1}{2}r^{-2}G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right]\left[-\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2 G(r)\right]\right]$$

$$R^2_{002} = -\left[-\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}F(r)\right]\left[\frac{1}{2}r^{-2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2 G(r)\right]\right]$$

$$R^2_{020} = \left[-\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}F(r)\right]\left[\frac{1}{2}r^{-2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2 G(r)\right]\right]$$

$$R^2_{103} = \left[\frac{1}{2}r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, G(r)\right]\right]\left[\frac{1}{2}r^{-2}G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right]$$

$$R^2_{112} = \frac{\partial}{\partial r}\left[\frac{1}{2}r^{-2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2 G(r)\right]\right] + \left[\frac{1}{2}r^{-2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2 G(r)\right]\right]\left[\frac{1}{2}r^{-2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2 G(r)\right]\right]$$
$$-\left[\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}G(r)\right]\left[\frac{1}{2}r^{-2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2 G(r)\right]\right]$$

$$R^2_{121} = -\frac{\partial}{\partial r}\left[\frac{1}{2}r^{-2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2 G(r)\right]\right] + \left[\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}G(r)\right]\left[\frac{1}{2}r^{-2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2 G(r)\right]\right]$$
$$-\left[\frac{1}{2}r^{-2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2 G(r)\right]\right]\left[\frac{1}{2}r^{-2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2 G(r)\right]\right]$$

$$R^2_{203} = \left[\frac{1}{2}r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right]\left[\frac{1}{2}r^{-2}G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right]$$

$$R^2_{020} = \left[-\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}F(r)\right]\left[\frac{1}{2}r^{-2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2 G(r)\right]\right]$$

$$R^{2}_{313} = -\frac{\partial}{\partial r}\left[\frac{1}{2}\, r^{-2}\, G^{-1}(r)\frac{\partial}{\partial \theta}\left[r^{2}\sin^{2}\theta\; G(r)\right]\right]$$
$$-\left[\frac{1}{2}\, r^{-2}\, G^{-1}(r)\frac{\partial}{\partial \theta}\left[r^{2}\sin^{2}\theta\; G(r)\right]\right]\left[\frac{1}{2}\, r^{-2}\, G^{-1}(r)\frac{\partial}{\partial r}\left[r^{2}\, G(r)\right]\right]$$
$$+\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\; G^{-1}(r)\frac{\partial}{\partial r}\left[r^{2}\sin^{2}\theta\; G(r)\right]\right]\left[\frac{1}{2}\, r^{-2}\, G^{-1}(r)\frac{\partial}{\partial \theta}\left[r^{2}\sin^{2}\theta\; G(r)\right]\right]$$

$$R^{2}_{323} = -\frac{\partial}{\partial \theta}\left[\frac{1}{2}\, r^{-2}\, G^{-1}(r)\frac{\partial}{\partial \theta}\left[r^{2}\sin^{2}\theta\; G(r)\right]\right]$$
$$-\left[\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^{2}\sin^{2}\theta\; G(r)\right]\right]\left[\frac{1}{2}\, r^{-2}\, G^{-1}(r)\frac{\partial}{\partial r}\left[r^{2}\, G(r)\right]\right]$$
$$+\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\; G^{-1}(r)\frac{\partial}{\partial \theta}\left[r^{2}\sin^{2}\theta\; G(r)\right]\right]\left[\frac{1}{2}\, r^{-2}\, G^{-1}(r)\frac{\partial}{\partial \theta}\left[r^{2}\sin^{2}\theta\; G(r)\right]\right]$$

$$R^{2}_{331} = -\frac{\partial}{\partial \theta}\left[\frac{1}{2}\, r^{-2}\, G^{-1}(r)\frac{\partial}{\partial \theta}\left[r^{2}\sin^{2}\theta\; G(r)\right]\right]$$
$$+\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\; G^{-1}(r)\frac{\partial}{\partial r}\left[r^{2}\sin^{2}\theta\; G(r)\right]\right]\left[\frac{1}{2}\, r^{-2}\, G^{-1}(r)\frac{\partial}{\partial \theta}\left[r^{2}\sin^{2}\theta\; G(r)\right]\right]$$
$$-\left[\frac{1}{2}\, r^{-2}\, G^{-1}(r)\frac{\partial}{\partial \theta}\left[r^{2}\sin^{2}\theta\; G(r)\right]\right]\left[\frac{1}{2}\, r^{-2}\, G^{-1}(r)\frac{\partial}{\partial r}\left[r^{2}\, G(r)\right]\right]$$

$$R^{2}_{332} = -\frac{\partial}{\partial \theta}\left[\frac{1}{2}\, r^{-2}\, G^{-1}(r)\frac{\partial}{\partial \theta}\left[r^{2}\sin^{2}\theta\; G(r)\right]\right]$$
$$+\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\; G^{-1}(r)\frac{\partial}{\partial \theta}\left[r^{2}\sin^{2}\theta\; G(r)\right]\right]\left[\frac{1}{2}\, r^{-2}\, G^{-1}(r)\frac{\partial}{\partial \theta}\left[r^{2}\sin^{2}\theta\; G(r)\right]\right]$$
$$-\left[\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^{2}\sin^{2}\theta\; G(r)\right]\right]\left[\frac{1}{2}\, r^{-2}\, G^{-1}(r)\frac{\partial}{\partial r}\left[r^{2}\, G(r)\right]\right]$$

$$R^{3}_{030} = \left[-\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}F(r)\right]\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\; G^{-1}(r)\frac{\partial}{\partial r}\left[r^{2}\sin^{2}\theta\; G(r)\right]\right]$$

$$R^{3}_{113} = \frac{\partial}{\partial r}\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\; G^{-1}(r)\frac{\partial}{\partial r}\left[r^{2}\sin^{2}\theta\; G(r)\right]\right]$$
$$+\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\; G^{-1}(r)\frac{\partial}{\partial r}\left[r^{2}\sin^{2}\theta\; G(r)\right]\right]\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\; G^{-1}(r)\frac{\partial}{\partial r}\left[r^{2}\sin^{2}\theta\; G(r)\right]\right]$$
$$-\left[\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\,G(r)\right]\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\; G^{-1}(r)\frac{\partial}{\partial r}\left[r^{2}\sin^{2}\theta\; G(r)\right]\right]$$

$$R^3_{123} = \frac{\partial}{\partial\theta}\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, G(r)\right]\right]$$
$$+\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, G(r)\right]\right]\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right]-$$
$$\left[\frac{1}{2}\, r^{-2}\, G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\, G(r)\right]\right]\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right]$$

$$R^3_{131} = -\frac{\partial}{\partial r}\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, G(r)\right]\right]$$
$$+\left[\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\, G(r)\right]\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, G(r)\right]\right]$$
$$-\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, G(r)\right]\right]\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, G(r)\right]\right]$$

$$R^3_{132} = -\frac{\partial}{\partial\theta}\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, G(r)\right]\right]$$
$$+\left[\frac{1}{2}\, r^{-2}\, G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\, G(r)\right]\right]\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right]$$
$$-\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, G(r)\right]\right]\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right]$$

$$R^3_{213} = \frac{\partial}{\partial r}\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right]$$
$$+\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right]\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, G(r)\right]\right]$$
$$-\left[\frac{1}{2}\, r^{-2}\, G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\, G(r)\right]\right]\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right]$$

$$R^3_{223} = \frac{\partial}{\partial\theta}\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right]$$
$$+\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right]\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right]$$
$$-\left[-\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2 G(r)\right]\right]\left[\frac{1}{2}\, r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, G(r)\right]\right]$$

$$R^3_{231}=-\frac{\partial}{\partial r}\left[\frac{1}{2}r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right]$$
$$+\left[\frac{1}{2}r^{-2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2 G(r)\right]\right]\left[\frac{1}{2}r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right]$$
$$-\left[\frac{1}{2}r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right]\left[\frac{1}{2}r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, G(r)\right]\right]$$

$$R^3_{232}=-\frac{\partial}{\partial\theta}\left[\frac{1}{2}r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right]$$
$$+\left[-\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}\left[r^2 G(r)\right]\right]\left[\frac{1}{2}r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, G(r)\right]\right]$$
$$-\left[\frac{1}{2}r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right]\left[\frac{1}{2}r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right]$$

$$R^3_{312}=\frac{\partial}{\partial r}\left[\frac{1}{2}r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right]$$
$$-\frac{\partial}{\partial\theta}\left[\frac{1}{2}r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, G(r)\right]\right]$$

$$R^3_{321}=\frac{\partial}{\partial\theta}\left[\frac{1}{2}r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, G(r)\right]\right]$$
$$-\frac{\partial}{\partial r}\left[\frac{1}{2}r^{-2}\sin^{-2}\theta\, G^{-1}(r)\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, G(r)\right]\right] \qquad 2.5$$

2.4.Ricci Tensörleri

Einstein Alan Denklemlerinin kesin çözümlerini bulmak zordur. $\varepsilon \ll 1$, $F(r)=c_0e^{ef}$, $G(r)=c_1e^{\varepsilon g}$ seçilerek, çözüm için yaklaşık bir metod uygulayacağız.

Kesin çözümler Stephani[5] den bakılabilir.

(2,5) de bulunan Riemann eğrilik tensörleri kullanarak aşağıdaki Ricci Tensörleri bulunur.

$$R_{00} = R_{000}^{0} + R_{010}^{1} + R_{020}^{2} + R_{030}^{3}$$

$$R_{00} = \begin{bmatrix} \frac{\partial}{\partial r}\left[-\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}c_0 e^{\varepsilon f(r)}\right] + \left[-\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}c_0 e^{\varepsilon f(r)}\right]\left[\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}c_1 e^{\varepsilon g(r)}\right] \\ -\left[\frac{1}{2}c_0^{-1}e^{-\varepsilon f(r)}\frac{\partial}{\partial r}c_0 e^{\varepsilon f(r)}\right]\left[-\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}c_0 e^{\varepsilon f(r)}\right] \end{bmatrix}$$

$$\left[\left[-\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}c_0 e^{\varepsilon f(r)}\right]\left[\frac{1}{2}r^{-2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\left[r^2 c_1 e^{\varepsilon g(r)}\right]\right]\right]$$

$$\left[\left[-\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}c_0 e^{\varepsilon f(r)}\right]\left[\frac{1}{2}r^{-2}\sin^{-2}\theta\, c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, c_1 e^{\varepsilon g(r)}\right]\right]\right]$$

$$\begin{aligned} R_{00} = {} & \frac{\partial}{\partial r}\left[-\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}c_0\varepsilon e^{\varepsilon f(r)}f'(r)\right] \\ & +\left[-\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}c_0\varepsilon e^{\varepsilon f(r)}f'(r)\right]\left[\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}c_1\varepsilon e^{\varepsilon g(r)}g'(r)\right] \\ & -\left[\frac{1}{2}c_0^{-1}e^{-\varepsilon f(r)}c_0\varepsilon e^{\varepsilon f(r)}f'(r)\right]\left[-\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}c_0\varepsilon e^{\varepsilon f(r)}f'(r)\right] \\ & +\left[-\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}c_0\varepsilon e^{\varepsilon f(r)}f'(r)\right]\left[\frac{1}{2}r^{-2}c_1^{-1}e^{-\varepsilon g(r)}c_1\left[2r\,e^{\varepsilon g(r)}+r^2\varepsilon\,e^{\varepsilon g(r)}g'(r)\right]\right] \\ & +\left[-\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}c_0\varepsilon e^{\varepsilon f(r)}f'(r)\right] \\ & \cdot\left[\frac{1}{2}r^{-2}\sin^{-2}\theta c_1^{-1}e^{-\varepsilon g(r)}\sin^2\theta c_1\frac{\partial}{\partial r}\left[2r\,e^{\varepsilon g(r)}+r^2\varepsilon\,e^{\varepsilon g(r)}g'(r)\right]\right] \end{aligned}$$

$$\begin{aligned} R_{00} = {} & -\frac{1}{2}c_1^{-1}c_0\varepsilon\left[\varepsilon e^{\varepsilon(f(r)-g(r))}\left(f(r)-g(r)\right)'f'(r)+e^{\varepsilon(f(r)-g(r))}f''(r)\right] \\ & +\left[-\frac{1}{2}c_1^{-1}c_0\varepsilon r^{-1}e^{\varepsilon(f(r)-g(r))}f'(r)\right] \\ & +\left[-\frac{1}{2}c_1^{-1}c_0\,r^{-1}\varepsilon e^{\varepsilon(f(r)-g(r))}f'(r)\right] \end{aligned}$$

$$R_{00} = -\frac{1}{2}c_1^{-1}c_0\varepsilon e^{\varepsilon(f(r)-g(r))}\left[f''(r)+2\,r^{-1}f'(r)\right]$$

$$R_{11} = R^{0}_{101} + R^{1}_{111} + R^{2}_{121} + R^{3}_{131}$$

$$R_{11} = \left[\begin{array}{l} -\frac{\partial}{\partial r}\left[\frac{1}{2}c_0^{-1}e^{-\varepsilon f(r)}\ \frac{\partial}{\partial r}c_0 e^{\varepsilon f(r)}\right] + \left[\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}c_1 e^{\varepsilon g(r)}\right]\left[\frac{1}{2}c_0^{-1}e^{-\varepsilon f(r)}\ \frac{\partial}{\partial r}c_0 e^{\varepsilon f(r)}\right] \\ -\left[\frac{1}{2}c_0^{-1}e^{-\varepsilon f(r)}\ \frac{\partial}{\partial r}c_0 e^{\varepsilon f(r)}\right]\left[\frac{1}{2}c_0^{-1}e^{-\varepsilon f(r)}\ \frac{\partial}{\partial r}c_0 e^{\varepsilon f(r)}\right] \end{array}\right]$$

$$\left[\begin{array}{l} -\frac{\partial}{\partial r}\left[\frac{1}{2}\ r^{-2}\ c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\left[r^2 c_1 e^{\varepsilon g(r)}\right]\right] + \left[\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}c_1 e^{\varepsilon g(r)}\right]\left[\frac{1}{2}\ r^{-2}\ c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\left[r^2\ c_1 e^{\varepsilon g(r)}\right]\right] \\ -\left[\frac{1}{2}\ r^{-2}\ c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\left[r^2\ c_1 e^{\varepsilon g(r)}\right]\right]\left[\frac{1}{2}\ r^{-2}\ c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\left[r^2\ c_1 e^{\varepsilon g(r)}\right]\right] \end{array}\right]$$

$$\left[\begin{array}{l} -\frac{\partial}{\partial r}\left[\frac{1}{2}\ r^{-2}\sin^{-2}\theta\ c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\left[r^2\sin^2\theta\ c_1 e^{\varepsilon g(r)}\right]\right] \\ +\left[\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}c_1 e^{\varepsilon g(r)}\right]\left[\frac{1}{2}\ r^{-2}\sin^{-2}\theta\ c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\left[r^2\sin^2\theta\ c_1 e^{\varepsilon g(r)}\right]\right] \\ -\left[\frac{1}{2}\ r^{-2}\sin^{-2}\theta\ c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\left[r^2\sin^2\theta\ c_1 e^{\varepsilon g(r)}\right]\right]\left[\frac{1}{2}\ r^{-2}\sin^{-2}\theta\ c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\left[r^2\sin^2\theta\ c_1 e^{\varepsilon g(r)}\right]\right] \end{array}\right]$$

$$R_{11} = \left[\begin{array}{l} -\frac{\partial}{\partial r}\left[\frac{1}{2}e^{-\varepsilon f(r)}\ \varepsilon e^{\varepsilon f(r)}f'(r)\right] + \left[\frac{1}{2}e^{-\varepsilon g(r)}\ \varepsilon e^{\varepsilon g(r)}g'(r)\right]\left[\frac{1}{2}e^{-\varepsilon f(r)}\ \ \varepsilon e^{\varepsilon f(r)}f'(r)\right] \\ -\left[\frac{1}{2}e^{-\varepsilon f(r)}\ \varepsilon e^{\varepsilon f(r)}f'(r)\right]\left[\frac{1}{2}e^{-\varepsilon f(r)}\varepsilon e^{\varepsilon f(r)}f'(r)\right] \end{array}\right]$$

$$+\left[\begin{array}{l} -\frac{\partial}{\partial r}\left[\frac{1}{2}\ r^{-2}e^{-\varepsilon g(r)}\left[2re^{\varepsilon g(r)} + r^2\varepsilon e^{\varepsilon g(r)}g'(r)\right]\right] \\ +\left[\frac{1}{2}e^{-\varepsilon g(r)}\ \varepsilon e^{\varepsilon g(r)}g'(r)\right]\left[\frac{1}{2}\ r^{-2}\ e^{-\varepsilon g(r)}\left[2re^{\varepsilon g(r)} + r^2\varepsilon e^{\varepsilon g(r)}g'(r)\right]\right] \\ -\left[\frac{1}{2}\ r^{-2}\ e^{-\varepsilon g(r)}\left[2re^{\varepsilon g(r)} + r^2\varepsilon e^{\varepsilon g(r)}g'(r)\right]\right]\left[\frac{1}{2}\ r^{-2}e^{-\varepsilon g(r)}\left[2re^{\varepsilon g(r)} + r^2\varepsilon e^{\varepsilon g(r)}g'(r)\right]\right] \end{array}\right]$$

$$+\left[\begin{array}{l} -\frac{\partial}{\partial r}\left[\frac{1}{2}\ r^{-2}\ e^{-\varepsilon g(r)}\left[2re^{\varepsilon g(r)} + r^2\varepsilon e^{\varepsilon g(r)}g'(r)\right]\right] \\ +\left[\frac{1}{2}e^{-\varepsilon g(r)}\ \ \varepsilon e^{\varepsilon g(r)}g'(r)\right]\left[\frac{1}{2}\ r^{-2}\ e^{-\varepsilon g(r)}\left[2re^{\varepsilon g(r)} + r^2\varepsilon e^{\varepsilon g(r)}g'(r)\right]\right] \\ -\left[\frac{1}{2}\ r^{-2}\ e^{-\varepsilon g(r)}\left[2re^{\varepsilon g(r)} + r^2\varepsilon e^{\varepsilon g(r)}g'(r)\right]\right]\left[\frac{1}{2}\ r^{-2}\ e^{-\varepsilon g(r)}\left[2re^{\varepsilon g(r)} + r^2\varepsilon e^{\varepsilon g(r)}g'(r)\right]\right] \end{array}\right]$$

$$R_{11} = -\frac{1}{2}\ \varepsilon f''(r) - \frac{\partial}{\partial r}\left[\ r^{-1} + \frac{1}{2}\ \varepsilon g'(r)\right] + \left[\frac{1}{2}r^{-1}\ \varepsilon g'(r)\right] - \left[r^{-2} + r^{-1}\varepsilon g'(r)\right]$$

$$-\frac{\partial}{\partial r}\left[\ r^{-1} + \frac{1}{2}\ \varepsilon g'(r)\right] + \left[\frac{1}{2}\ \ \varepsilon g'(r)\right]\left[\ r^{-1} + \frac{1}{2}\ \varepsilon g'(r)\right] - \left[r^{-2} + r^{-1}\varepsilon g'(r)\right]$$

$$R_{11}=-\frac{1}{2}\ \varepsilon f''(r)+r^{-2}-\frac{1}{2}\ \varepsilon g''(r)+\frac{1}{2}r^{-1}\ \varepsilon g'(r)-r^{-2}-r^{-1}\varepsilon g'(r)$$

$$+r^{-2}-\frac{1}{2}\ \varepsilon g''(r)+\frac{1}{2}\ r^{-1}\ \varepsilon g'(r)-r^{-2}-r^{-1}\varepsilon g'(r)$$

$$R_{11}=-\frac{1}{2}\ \varepsilon f''(r)-\varepsilon g''(r)-r^{-1}\varepsilon g'(r)$$

$$R_{22}=R^{0}_{202}+R^{1}_{212}+R^{2}_{222}+R^{3}_{232}$$

$$R_{22}=\left[-\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\left[r^2c_1e^{\varepsilon g(r)}\right]\right]\left[\frac{1}{2}c_0^{-1}e^{-\varepsilon f(r)}\ \frac{\partial}{\partial r}\ c_0e^{\varepsilon f(r)}\right]$$

$$+\frac{\partial}{\partial r}\left[-\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\left[r^2c_1e^{\varepsilon g(r)}\right]\right]$$

$$+\left[-\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\left[r^2c_1e^{\varepsilon g(r)}\right]\right]\left[\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\ c_1e^{\varepsilon g(r)}\right]$$

$$-\left[\frac{1}{2}\ r^{-2}\ c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\left[r^2\ c_1e^{\varepsilon g(r)}\right]\right]\left[-\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\left[r^2c_1e^{\varepsilon g(r)}\right]\right]$$

$$-\frac{\partial}{\partial\theta}\left[\frac{1}{2}\ r^{-2}\sin^{-2}\theta\ c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\ c_1e^{\varepsilon g(r)}\right]\right]$$

$$+\left[-\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\left[r^2c_1e^{\varepsilon g(r)}\right]\right]\left[\frac{1}{2}\ r^{-2}\sin^{-2}\theta\ c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\left[r^2\sin^2\theta\ c_1e^{\varepsilon g(r)}\right]\right]$$

$$-\left[\frac{1}{2}\ r^{-2}\sin^{-2}\theta\ c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\ c_1e^{\varepsilon g(r)}\right]\right]$$

$$.\left[\frac{1}{2}\ r^{-2}\sin^{-2}\theta\ c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\ c_1e^{\varepsilon g(r)}\right]\right]$$

$$R_{22}=\left[-\frac{1}{2}e^{-\varepsilon g(r)}\ \left[2re^{\varepsilon g(r)}+r^2\varepsilon e^{\varepsilon g(r)}g'(r)\right]\right]\left[\frac{1}{2}e^{-\varepsilon f(r)}\varepsilon\, e^{\varepsilon f(r)}f'(r)\right]$$

$$+\frac{\partial}{\partial r}\left[-\frac{1}{2}e^{-\varepsilon g(r)}\ \left[2re^{\varepsilon g(r)}+r^2\varepsilon e^{\varepsilon g(r)}g'(r)\right]\right]$$

$$+\left[-\frac{1}{2}e^{-\varepsilon g(r)}\ \left[2re^{\varepsilon g(r)}+r^2\varepsilon e^{\varepsilon g(r)}g'(r)\right]\right]\left[\frac{1}{2}e^{-\varepsilon g(r)}\ \varepsilon\, e^{\varepsilon g(r)}g'(r)\right]$$

$$-\left[\frac{1}{2}\ r^{-2}\ e^{-\varepsilon g(r)}\left[2re^{\varepsilon g(r)}+r^2\varepsilon e^{\varepsilon g(r)}g'(r)\right]\right]\left[-\frac{1}{2}e^{-\varepsilon g(r)}\left[2re^{\varepsilon g(r)}+r^2\varepsilon e^{\varepsilon g(r)}g'(r)\right]\right]$$

$$-\frac{\partial}{\partial\theta}\left[\frac{1}{2}\ \sin^{-2}\theta\ \ 2\sin\theta.\cos\theta\right]$$

$$+\left[-\frac{1}{2}e^{-\varepsilon g(r)}\left[2re^{\varepsilon g(r)}+r^2\varepsilon e^{\varepsilon g(r)}g'(r)\right]\right]\left[\frac{1}{2}\ r^{-2}\ e^{-\varepsilon g(r)}\left[2re^{\varepsilon g(r)}+r^2\varepsilon e^{\varepsilon g(r)}g'(r)\right]\right]$$

$$-\left[\frac{1}{2}\ \sin^{-2}\theta\ \ 2\sin\theta.\cos\theta\right]\left[\frac{1}{2}\ \sin^{-2}\theta\ \ 2\sin\theta.\cos\theta\right]$$

$$R_{22} = [-r]\left[\frac{1}{2}\varepsilon f'(r)\right] + \frac{\partial}{\partial r}\left[-r - \frac{1}{2}r^2\varepsilon g'(r)\right] + [-r]\left[\frac{1}{2}\varepsilon g'(r)\right]$$
$$+\left[1 + r\varepsilon g'(r)\right] - \frac{\partial}{\partial\theta}[\cot\theta] - \left[1 + r\varepsilon g'(r)\right] - \cot^2\theta$$

$$R_{22} = \left[-\frac{1}{2}\varepsilon r f'(r)\right] + \left[-1 - \frac{1}{2}\varepsilon\left[2rg'(r) + r^2 g''(r)\right]\right] - \left[\frac{1}{2}\varepsilon r g'(r)\right]$$
$$+\left[1 + r\varepsilon g'(r)\right] + \left[1 + \cot^2\theta\right] - \left[1 + r\varepsilon g'(r)\right] - \cot^2\theta$$

$$R_{22} = -\frac{1}{2}\varepsilon r f'(r) - \frac{3}{2}\varepsilon r g'(r) - \frac{1}{2}\varepsilon r^2 g''(r)$$

$$R_{33} = R^0_{303} + R^1_{313} + R^2_{323} + R^3_{333}$$

$$R_{33} = \left[-\left[\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, c_1 e^{\varepsilon g(r)}\right]\right]\left[\frac{1}{2}c_0^{-1}e^{-\varepsilon f(r)}\frac{\partial}{\partial r}c_0 e^{\varepsilon f(r)}\right]\right]$$
$$\left[\begin{array}{l} -\frac{\partial}{\partial r}\left[\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, c_1 e^{\varepsilon g(r)}\right]\right] \\ -\left[\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, c_1 e^{\varepsilon g(r)}\right]\right]\left[\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}c_1 e^{\varepsilon g(r)}\right] \\ +\left[\frac{1}{2}r^{-2}\sin^{-2}\theta\, c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, c_1 e^{\varepsilon g(r)}\right]\right]\left[\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, c_1 e^{\varepsilon g(r)}\right]\right] \end{array}\right]$$
$$\left[\begin{array}{l} -\frac{\partial}{\partial\theta}\left[\frac{1}{2}r^{-2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, c_1 e^{\varepsilon g(r)}\right]\right] \\ -\left[\frac{1}{2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\left[r^2\sin^2\theta\, c_1 e^{\varepsilon g(r)}\right]\right]\left[\frac{1}{2}r^{-2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial r}\left[r^2 c_1 e^{\varepsilon g(r)}\right]\right] \\ +\left[\frac{1}{2}r^{-2}\sin^{-2}\theta\, c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, c_1 e^{\varepsilon g(r)}\right]\right]\left[\frac{1}{2}r^{-2}c_1^{-1}e^{-\varepsilon g(r)}\frac{\partial}{\partial\theta}\left[r^2\sin^2\theta\, c_1 e^{\varepsilon g(r)}\right]\right] \end{array}\right]$$

$$R_{33} = \left[-\left[\frac{1}{2}e^{-\varepsilon g(r)}\sin^2\theta\left[2re^{\varepsilon g(r)} + r^2\varepsilon e^{\varepsilon g(r)}g'(r)\right]\right]\left[\frac{1}{2}e^{-\varepsilon f(r)}\varepsilon e^{\varepsilon f(r)}f'(r)\right]\right]$$
$$\left[\begin{array}{l} -\frac{\partial}{\partial r}\left[\frac{1}{2}e^{-\varepsilon g(r)}\sin^2\theta\left[2re^{\varepsilon g(r)} + r^2\varepsilon e^{\varepsilon g(r)}g'(r)\right]\right] \\ -\left[\frac{1}{2}e^{-\varepsilon g(r)}\sin^2\theta\left[2re^{\varepsilon g(r)} + r^2\varepsilon e^{\varepsilon g(r)}g'(r)\right]\right]\left[\frac{1}{2}e^{-\varepsilon g(r)}\varepsilon e^{\varepsilon g(r)}g'(r)\right] \\ +\left[\frac{1}{2}r^{-2}\sin^{-2}\theta\, e^{-\varepsilon g(r)}\sin^2\theta\left[2re^{\varepsilon g(r)} + r^2\varepsilon e^{\varepsilon g(r)}g'(r)\right]\right]\left[\frac{1}{2}e^{-\varepsilon g(r)}\sin^2\theta\left[2re^{\varepsilon g(r)} + r^2\varepsilon e^{\varepsilon g(r)}g'(r)\right]\right] \end{array}\right]$$
$$\left[\begin{array}{l} -\frac{\partial}{\partial\theta}[\sin\theta.\cos\theta] - \left[\frac{1}{2}e^{-\varepsilon g(r)}\sin^2\theta\left[2re^{\varepsilon g(r)} + r^2\varepsilon e^{\varepsilon g(r)}g'(r)\right]\right]\left[\frac{1}{2}r^{-2}e^{-\varepsilon g(r)}\left[2re^{\varepsilon g(r)} + r^2\varepsilon e^{\varepsilon g(r)}g'(r)\right]\right] \\ +\left[\sin^{-2}\theta\sin\theta.\cos\theta\right][\sin\theta.\cos\theta] \end{array}\right]$$

$$R_{33}=-\frac{1}{2}\sin^2\theta\, r\,\varepsilon f'(r)-\frac{\partial}{\partial r}\left[\sin^2\theta\left[r+\frac{1}{2}r^2\varepsilon g'(r)\right]\right]-\left[\frac{1}{2}\sin^2\theta\, r\,\varepsilon g'(r)\right]$$
$$+\left[r^{-1}+\frac{1}{2}\varepsilon g'(r)\right]\left[r\sin^2\theta+\frac{1}{2}\sin^2\theta r^2\varepsilon g'(r)\right]$$
$$-\cos 2\theta-\left[r\sin^2\theta+\frac{1}{2}\sin^2\theta r^2\varepsilon g'(r)\right]\left[r^{-1}+\frac{1}{2}\varepsilon g'(r)\right]+\cos^2\theta$$

$$R_{33}=-\frac{1}{2}\sin^2\theta\, r\,\varepsilon f'(r)-\sin^2\theta-r\sin^2\theta\varepsilon g'(r)-\frac{1}{2}r^2\sin^2\theta\varepsilon g''(r)-\frac{1}{2}\sin^2\theta\, r\,\varepsilon g'(r)$$
$$+\sin^2\theta+\sin^2\theta r\varepsilon g'(r)-\cos^2\theta+\sin^2\theta-\sin^2\theta-r\sin^2\theta\varepsilon g'(r)+\cos^2\theta$$

$$R_{33}=-\frac{1}{2}\sin^2\theta\, r\,\varepsilon f'(r)-\frac{1}{2}r^2\sin^2\theta\varepsilon g''(r)-\frac{3}{2}\sin^2\theta\, r\,\varepsilon g'(r) \qquad 2.6$$

2.5.Ricci Skaler Eğriliği

(1.5) formülünde (2.6) ve (2.3) de bulunan sonuçlar yerine konularak R Ricci Sakaler Eğriliği bulunur.

$$R=g^{bd}R_{bd}$$

$$g^{bd}=[g_{bd}]^{-1}$$

$$g^{bd}=\begin{bmatrix} g^{00} & 0 & 0 & 0\\ 0 & g^{11} & 0 & 0\\ 0 & 0 & g^{22} & 0\\ 0 & 0 & 0 & g^{33}\end{bmatrix}=\begin{bmatrix} F_{(r)}^{-1} & 0 & 0 & 0\\ 0 & G_{(r)}^{-1} & 0 & 0\\ 0 & 0 & r^{-2}G_{(r)}^{-1} & 0\\ 0 & 0 & 0 & r^{-2}\sin^{-2}\theta G_{(r)}^{-1}\end{bmatrix}$$

$$R_{bd}=\begin{bmatrix} R_{00} & 0 & 0 & 0\\ 0 & R_{11} & 0 & 0\\ 0 & 0 & R_{22} & 0\\ 0 & 0 & 0 & R_{33}\end{bmatrix}$$

$$R=g^{00}R_{00}+g^{11}R_{11}+g^{22}R_{22}+g^{33}R_{33}$$

$$R=\left[c_0^{-1}e^{-\varepsilon f(r)}\right]\left[-\frac{1}{2}c_1^{-1}c_0\varepsilon e^{\varepsilon(f(r)-g(r))}\left[f''(r)+2r^{-1}f'(r)\right]\right]$$
$$+\left[c_1^{-1}e^{-\varepsilon g(r)}\right]\left[-\frac{1}{2}\ \varepsilon f''(r)-\varepsilon g''(r)-r^{-1}\varepsilon g'(r)\right]$$
$$+\left[r^{-2}c_1^{-1}e^{-\varepsilon g(r)}\right]\left[-\frac{1}{2}\varepsilon r f'(r)-\frac{3}{2}\varepsilon r g'(r)-\frac{1}{2}\varepsilon r^2 g''(r)\right]$$
$$+\left[r^{-2}\sin^{-2}\theta c_1^{-1}e^{-\varepsilon g(r)}\right]\left[-\frac{1}{2}\sin^2\theta\, r\,\varepsilon f'(r)-\frac{1}{2}r^2\sin^2\theta\varepsilon g''(r)-\frac{3}{2}\sin^2\theta\, r\,\varepsilon g'(r)\right]$$

$$R=\left[-\frac{1}{2}c_1^{-1}\varepsilon e^{-\varepsilon g(r)}\right]\left[f''(r)+2r^{-1}f'(r)\right]+\left[-\frac{1}{2}c_1^{-1}\varepsilon e^{-\varepsilon g(r)}\right]\left[\ f''(r)+2g''(r)+2r^{-1}g'(r)\right]$$
$$+\left[-\frac{1}{2}c_1^{-1}\varepsilon e^{-\varepsilon g(r)}\right]\left[r^{-1}f'(r)+3r^{-1}g'(r)+g''(r)\right]$$
$$+\left[-\frac{1}{2}c_1^{-1}\varepsilon e^{-\varepsilon g(r)}\right]\left[r^{-1}f'(r)+g''(r)+3r^{-1}g'(r)\right]$$

$$R=\left[-\frac{1}{2}c_1^{-1}\varepsilon e^{-\varepsilon g(r)}\right]\left[2f''(r)+4r^{-1}f'(r)+4g''(r)+8r^{-1}g'(r)\right]$$

$$R=\left[-c_1^{-1}\varepsilon e^{-\varepsilon g(r)}\right]\left[f''(r)+2r^{-1}f'(r)+2g''(r)+4r^{-1}g'(r)\right] \qquad 2.7$$

2.6.Einstein Alan Denklemlerinin Diferansiyel Denklem Sistemi Olarak Oluşturulması

Bulunan sonuçlar (1,2) denkleminde yerine yazılırsa

$$R_{ab}-\frac{1}{2}R.g_{ab}=T_{ab}$$

$$R_{00}-\frac{1}{2}R.g_{00}=T_{00}$$

$$T_{00}=\left[-\frac{1}{2}c_1^{-1}c_0\varepsilon e^{\varepsilon(f(r)-g(r))}\right]\left[f''(r)+2r^{-1}f'(r)\right]$$
$$-\frac{1}{2}\left[-c_1^{-1}\varepsilon e^{-\varepsilon g(r)}\right]\left[f''(r)+2r^{-1}f'(r)+2g''(r)+4r^{-1}g'(r)\right].\left[c_0e^{\varepsilon f(r)}\right]$$

$$T_{00} = \left[-\frac{1}{2} c_1^{-1} c_0 \varepsilon e^{\varepsilon(f(r)-g(r))} \right] \left[f''(r) + 2r^{-1} f'(r) \right]$$
$$-\left[-\frac{1}{2} c_1^{-1} c_0 \varepsilon e^{\varepsilon(f(r)-g(r))} \right] \left[f''(r) + 2r^{-1} f'(r) + 2g''(r) + 4r^{-1} g'(r) \right]$$

$$T_{00} = \left[c_1^{-1} c_0 \varepsilon e^{\varepsilon(f(r)-g(r))} \right] \left[g''(r) + 2r^{-1} g'(r) \right] \qquad 2.8$$

$$R_{11} - \frac{1}{2} R.g_{11} = T_{11}$$

$$T_{11} = \left[-\frac{1}{2} \varepsilon f''(r) - \varepsilon g''(r) - r^{-1} \varepsilon g'(r) \right]$$
$$-\frac{1}{2} \left[-c_1^{-1} \varepsilon e^{-\varepsilon g(r)} \right] \left[f''(r) + 2r^{-1} f'(r) + 2g''(r) + 4r^{-1} g'(r) \right] . \left[c_1 e^{\varepsilon g(r)} \right]$$

$$T_{11} = \left[-\frac{1}{2} \varepsilon \right] \left[f''(r) + 2g''(r) + 2r^{-1} g'(r) \right]$$
$$+\left[-\frac{1}{2} \varepsilon \right] \left[-f''(r) - 2r^{-1} f'(r) - 2g''(r) - 4r^{-1} g'(r) \right]$$

$$T_{11} = \left[-\frac{1}{2} \varepsilon \right] \left[-2r^{-1} g'(r) - 2r^{-1} f'(r) \right]$$

$$T_{11} = \left[\varepsilon r^{-1} \right] \left[g'(r) + f'(r) \right] \qquad 2.9$$

$$R_{22} - \frac{1}{2} R.g_{22} = T_{22}$$

$$T_{22} = \left[-\frac{1}{2} \varepsilon r f'(r) - \frac{3}{2} \varepsilon r g'(r) - \frac{1}{2} \varepsilon r^2 g''(r) \right]$$
$$-\frac{1}{2} \left[\left[-c_1^{-1} \varepsilon e^{-\varepsilon g(r)} \right] \left[f''(r) + 2r^{-1} f'(r) + 2g''(r) + 4r^{-1} g'(r) \right] \right] . \left[c_1 r^2 e^{\varepsilon g(r)} \right]$$

$$T_{22} = \left[-\frac{1}{2} \varepsilon \right] \left[r f'(r) + 3r g'(r) + r^2 g''(r) \right]$$
$$+\left[\left[-\frac{1}{2} \varepsilon \right] \left[-r^2 f''(r) - 2r f'(r) - 2r^2 g''(r) - 4r g'(r) \right] \right]$$

$$T_{22} = \left[-\frac{1}{2} \varepsilon \right] \left[r f'(r) + 3r g'(r) + r^2 g''(r) - r^2 f''(r) - 2r f'(r) - 2r^2 g''(r) - 4r g'(r) \right]$$

$$T_{22} = \left[\frac{1}{2} r \varepsilon \right] \left[f'(r) + r f''(r) + g'(r) + r g''(r) \right] \qquad 2.10$$

$$R_{33}-\frac{1}{2}R.g_{33}=T_{33}$$

$$T_{33}=\left[-\frac{1}{2}\sin^2\theta\, r\,\varepsilon f'(r)-\frac{1}{2}r^2\sin^2\theta\varepsilon g''(r)-\frac{3}{2}\sin^2\theta\, r\,\varepsilon g'(r)\right]$$
$$-\frac{1}{2}\left[\left[-c_1^{-1}\varepsilon e^{-\varepsilon g(r)}\right]\left[f''(r)+2r^{-1}f'(r)+2g''(r)+4r^{-1}g'(r)\right]\right].\left[r^2\sin^2\theta\, c_1 e^{\varepsilon g(r)}\right]$$

$$T_{33}=\left[-\frac{1}{2}\sin^2\theta\, r\,\varepsilon f'(r)-\frac{1}{2}r^2\sin^2\theta\varepsilon g''(r)-\frac{3}{2}\sin^2\theta\, r\,\varepsilon g'(r)\right]$$
$$-\left[\left[-\frac{1}{2}c_1^{-1}r^2\sin^2\theta\, c_1 e^{\varepsilon g(r)}\varepsilon e^{-\varepsilon g(r)}\right]\left[f''(r)+2r^{-1}f'(r)+2g''(r)+4r^{-1}g'(r)\right]\right]$$

$$T_{33}=\left[-\frac{1}{2}r\,\varepsilon\sin^2\theta\right]\left[f'(r)+rg''(r)+3g'(r)\right]$$
$$-\left[\left[-\frac{1}{2}r\sin^2\theta\,\varepsilon\right]\left[rf''(r)+2f'(r)+2rg''(r)+4g'(r)\right]\right]$$

$$T_{33}=\left[-\frac{1}{2}r\,\varepsilon\sin^2\theta\right]\left[f'(r)+rg''(r)+3g'(r)-rf''(r)-2f'(r)-2rg''(r)-4g'(r)\right]$$

$$T_{33}=\left[\frac{1}{2}r\,\varepsilon\sin^2\theta\right]\left[rf''(r)+f'(r)+rg''(r)+g'(r)\right] \qquad 2.11$$

$R_{ab}-\frac{1}{2}R\,g_{ab}$ nin esas köşegen haricindeki elemanları sıfırdır.

3. DİFERANSİYEL DENKLEM SİSTEMİNİN ÇÖZÜMÜ

$$R_{00}-\frac{1}{2}R.g_{00}=T_{00}$$

$$R_{11}-\frac{1}{2}R.g_{11}=T_{11}$$

$$R_{22}-\frac{1}{2}R.g_{22}=T_{22}$$

$$R_{33}-\frac{1}{2}R.g_{33}=T_{33}$$

$$T_{00}=c_1^{-1}c_0\varepsilon\left\{1+\varepsilon f(r)-\varepsilon g(r)\right\}\left[g''(r)+2r^{-1}g'(r)\right]$$

$$T_{00}=c_1^{-1}c_0\varepsilon\left[g''(r)+2r^{-1}g'(r)\right]$$

$$T_{11}=\left[\varepsilon r^{-1}\right]\left[g'(r)+f'(r)\right]$$

$$T_{22}=\left[\frac{1}{2}r\varepsilon\right]\left[f'(r)+rf''(r)+g'(r)+rg''(r)\right]$$

$$T_{33}=\left[\frac{1}{2}r\varepsilon\sin^2\theta\right]\left[rf''(r)+f'(r)+rg''(r)+g'(r)\right]$$

$T_{22}=T_{33}=0$ *seçilir*

$$R_{33}-\frac{1}{2}R.g_{33}=\sin^2\theta\left[R_{22}-\frac{1}{2}R.g_{22}\right]=0$$

$\theta\neq 0$ *ve* $\theta\neq\pi$ *ve* $0<\theta<\pi$ *olduğundan* $\sin\theta\neq 0$ *olur*.

Bu nedenle $R_{22}-\frac{1}{2}R.g_{22}=0$ olur.

(2.10) ve (2.11) den

$$\left[\frac{1}{2}r\varepsilon\right]\left[f'(r)+rf''(r)+g'(r)+rg''(r)\right]=0$$

$$\left[f'(r)+rf''(r)+g'(r)+rg''(r)\right]=0 \qquad 3.1$$

4 denklem bu şekilde 3 denkleme indirgenir. $T_{00}=E(r)$, $T_{11}=P(r)$ seçilir.

$$\left[\varepsilon r^{-1}\right]\left[g'(r)+f'(r)\right]=P(r) \qquad 3.2$$

$$c_1^{-1}c_0\varepsilon\left[g''(r)+2r^{-1}g'(r)\right]=E(r) \qquad 3.3$$

(3.2) den $g'+f'=\dfrac{r}{\varepsilon}P$

$$f'=\frac{r}{\varepsilon}P-g' \qquad 3.4$$

r ye göre ikinci türev alınırsa $f''=\dfrac{1}{\varepsilon}P+\dfrac{r}{\varepsilon}P'-g''$ elde edilir.

f' *ve* f'' (3.1). denkleminde yerine yazılır.

$$\left[\frac{r}{\varepsilon}P-g'+r\frac{1}{\varepsilon}P+\frac{r^2}{\varepsilon}P'-r\,g''+g'+r\,g''\right]=0$$

$$\left[\frac{r}{\varepsilon}P+r\frac{1}{\varepsilon}P+\frac{r^2}{\varepsilon}P'\right]=0$$

$$\frac{2r}{\varepsilon}P+\frac{r^2}{\varepsilon}P'=0 \qquad 3.5$$

$$2P+r\,P'=0$$

$$2P+r\frac{dP}{dr}=0$$

$$r\frac{dP}{dr}=-2P \qquad 3.6$$

$$\frac{dP}{P}=-\frac{2}{r}dr$$

$$\int\frac{dP}{P}=-\int\frac{2}{r}dr$$

$$\ln P = -2\ln r + \ln k \qquad (k = sabit)$$

$$\ln \frac{P}{k} = \ln \frac{1}{r^2}$$

$$P = \frac{k}{r^2} \tag{3.7}$$

$$c_1^{-1} c_0 \varepsilon \left[g''(r) + 2r^{-1} g'(r) \right] = E(r)$$

$$g'' + 2r^{-1} g' = \frac{c_1}{c_0 \varepsilon} E$$

Bu denklemi çözmek için $g' = u$ dönüşümü yapılır.

$$u' + \frac{2}{r} u = \frac{c_1}{c_0 \varepsilon} E \tag{3.8}$$

Bu denklem h integrasyon çarpanı ile çarpılırsa

$$u'h + \frac{2}{r} h \; u = \frac{c_1}{c_0 \varepsilon} h \; E \tag{3.9}$$

$$h' = \frac{2}{r} h \qquad \text{olmalı} \tag{3.10}$$

$$\frac{h'}{h} = \frac{2}{r}$$

$$\int \frac{h'}{h} = \int \frac{2}{r}$$

$$\ln h = 2\ln r$$

$$h = r^2 \qquad \text{bulunur.} \tag{3.11}$$

$$u'h + \frac{2}{r} h \; u = \frac{c_1}{c_0 \varepsilon} h \, E \tag{3.12}$$

$$u'h + \frac{2}{r} h \; u = \frac{c_1}{c_0 \varepsilon} h \, E$$

$$u' r^2 + \frac{2}{r} r^2 \; u = \frac{c_1}{c_0 \varepsilon} r^2 \; E$$

$$\frac{d}{dr}\left(r^2 u\right) = \frac{c_1}{c_0 \varepsilon} r^2 \; E \tag{3.13}$$

$$\int d\left(r^2 u\right) = \int \frac{c_1}{c_0 \varepsilon} r^2 \; E \, dr \tag{3.14}$$

$$r^2 u = \frac{c_1}{c_0 \varepsilon} \int r^2 \; E \, dr + l \qquad \left(l = sabit\right) \tag{3.15}$$

$$u = \frac{c_1}{c_0 \varepsilon \, r^2} \int r^2 \; E \, dr + \frac{l}{r^2} \tag{3.16}$$

$$g' = \frac{dg}{dr} = \frac{c_1}{c_0 \varepsilon \, r^2} \int r^2 \; E \, dr + \frac{l}{r^2} \tag{3.17}$$

$$\int dg = \int \left[\frac{c_1}{c_0 \varepsilon \, r^2} \int r^2 \; E \, dr \right] dr + \int \frac{l}{r^2} dr + n \qquad \left(n = sabit\right) \tag{3.18}$$

$$g = \frac{c_1}{c_0 \varepsilon} \int \left[\frac{1}{r^2} \int r^2 \; E \, dr \right] dr + l \int \frac{1}{r^2} dr + n \tag{3.19}$$

$$f' = \frac{r}{\varepsilon} \frac{k}{r^2} - g' \tag{3.20}$$

$$f' = \frac{r}{\varepsilon}\frac{k}{r^2} - \left[\frac{c_1}{c_0\varepsilon\, r^2}\int r^2\ E\, dr + \frac{l}{r^2}\right] \tag{3.21}$$

$E = E_0$ Şeklinde bir sabit olarak seçilecektir.

$$f' = \frac{k}{r\varepsilon} - \frac{c_1}{c_0\varepsilon\, r^2}\int r^2\ E\, dr - \frac{l}{r^2} \tag{3.22}$$

$$f' = \frac{k}{r\varepsilon} - \frac{c_1}{c_0\varepsilon\, r^2}\int r^2\ E_0\, dr - \frac{l}{r^2} \tag{3.23}$$

$$f' = \frac{k}{r\varepsilon} - \frac{c_1}{c_0\varepsilon\, r^2} E_0 \frac{r^3}{3} - \frac{l}{r^2} \tag{3.24}$$

$$f' = \frac{k}{r\varepsilon} - \frac{c_1}{c_0\varepsilon}\frac{E_0}{3} r - \frac{l}{r^2}$$

$$f = \int \frac{k}{r\varepsilon} dr - \frac{c_1}{c_0\varepsilon}\frac{E_0}{3}\int r dr - \int \frac{l}{r^2} dr + m \tag{3.25}$$

$$f = \frac{k}{\varepsilon}\int \frac{dx}{x} + \frac{l}{r} - \frac{c_1}{c_0\varepsilon}\frac{E_0}{3}\frac{r^2}{2} + m \qquad \left(\frac{r}{B_0} = x\right) \qquad \left(dr = B_0 dx\right) \tag{3.26}$$

$$f = \frac{k}{\varepsilon}\ln x + \frac{l}{r} - \frac{c_1}{c_0\varepsilon}\frac{E_0}{3}\frac{r^2}{2} + m \tag{3.27}$$

$$f = \frac{k}{\varepsilon}\ln\frac{r}{B_0} + \frac{l}{r} - \frac{c_1 E_0}{6c_0\varepsilon} r^2 + m \qquad \left(l = m = 0\right) \quad \textit{seçilir} \tag{3.28}$$

$$f = \frac{k}{\varepsilon}\ln\frac{r}{B_0} - \frac{c_1 E_0}{6c_0\varepsilon} r^2 \tag{3.29}$$

$$F(r) = c_0 e^{\varepsilon f} \approx c_0\left(1+\varepsilon f\right) \tag{3.30}$$

(3.29) denklemi (3.30) da yerine yazılır.

$$F(r) = c_0\left[1+\varepsilon\left(\frac{k}{\varepsilon}\ln\frac{r}{B_0}-\frac{c_1E_0}{6c_0\varepsilon}r^2\right)\right] \tag{3.31}$$

$$F(r) = c_0 + c_0 k \ln\frac{r}{B_0}-\frac{c_1E_0}{6}r^2 \tag{3.32}$$

$$g = \frac{c_1}{c_0\varepsilon}\int\left[\frac{1}{r^2}\int r^2\ E\,dr\right]dr + 0 + n$$

$$g = \frac{c_1}{c_0\varepsilon}\int\left[\frac{1}{r^2}\int r^2\ E_0\,dr\right]dr + 0 + n \tag{3.33}$$

$$g = \frac{c_1E_0}{3c_0\varepsilon}\int r dr + 0 + n \tag{3.34}$$

$$g = \frac{c_1E_0}{3c_0\varepsilon}\frac{r^2}{2} + 0 + n \tag{3.35}$$

$$g = \frac{c_1E_0}{6c_0\varepsilon}r^2 + 0 + n \tag{3.36}$$

$$G(r) = c_1 e^{\varepsilon g} \approx c_1\left(1+\varepsilon g\right) \tag{3.37}$$

(3.36) denklemi (3.37) de yerine yazılır.

$$G(r) = c_1\left(1+\varepsilon\left(\frac{c_1E_0}{6c_0\varepsilon}r^2 + 0 + n\right)\right) \tag{3.38}$$

$$G(r)=c_1\left(1+\varepsilon\left(\frac{c_1E_0}{6c_0\varepsilon}r^2+n\right)\right) \qquad (n=0\ seçilir) \tag{3.39}$$

$$G(r)=c_1+\frac{c_1{}^2E_0}{6c_0}r^2 \tag{3.40}$$

(3.32) ve (3.40) fonksiyonları (2.1) denkleminde yerine yazılırsa aşağıdaki metrik bulunur.

$$ds^2=\left[c_0+c_0k\ln\frac{r}{B_0}-\frac{c_1E_0}{6}r^2\right]dt^2+\left[c_1+\frac{c_1{}^2E_0}{6c_0}r^2\right]\left[dr^2+r^2\,d\theta^2+r^2\sin^2\theta\ d\varphi^2\right] \tag{3.41}$$

Yukarıdaki eşitlikte metrikte$(c_0=1,\quad c_1=-1,\qquad seçilir)$ ve yerine yazılırsa aşağıdaki sonuç elde edilir.

$$ds^2=\left[1+\frac{E_0}{6}r^2+k\ln\frac{r}{B_0}\right]dt^2-\left[1-\frac{E_0}{6}r^2\right]\left[dr^2+r^2\,d\theta^2+r^2\sin^2\theta\ d\varphi^2\right] \tag{3.42}$$

Elde edilen bu sonuç uzay zamanının yapısını göstermektedir.Bu sonucun De-Sitter uzay zamanına benzediği görülmektedir.De-Sitter aşağıdaki gibidir.

$$ds^2=\left[1-\frac{\Lambda}{3}r^2\right]dt^2-\left[1-\frac{\Lambda}{3}r^2\right]^{-1}dr^2-r^2\,d\theta^2-r^2\sin^2\theta\ d\varphi^2 \tag{3.43}$$

De-Sitter uzay zamanı hakkında [7] ve [8] nolu referanslardan daha detaylı bilgi edinilebilir.

4. SONUÇLAR VE DEĞERLENDİRMLER

Bu çalışmada Einstein Alan Denklemlerine bir çözüm denemesi yapılmıştır.Enerji momentum tensörünün seçimine bağlı olarak uzay zamanın yapısını veren metrik bulunmuştur. Bu tezde $E(r) = E_0$ bir sabit olarak alınmıştır.İşlemlerin sonucunda

$$ds^2 = \left[1 + \frac{E_0}{6} r^2 + k \ln \frac{r}{B_0}\right] dt^2 - \left[1 - \frac{E_0}{6} r^2\right]\left[dr^2 + r^2\, d\theta^2 + r^2 \sin^2\theta\; d\varphi^2 \right]$$

bulunmuştur.

Burada $E_0 = -2\Lambda$ ve $2\Lambda \ll 1$ olarak alınırsa

$$ds^2 = \left[1 - \frac{\Lambda}{3} r^2 + k \ln \frac{r}{B_0}\right] dt^2 - \left[1 + \frac{\Lambda}{3} r^2\right]\left[dr^2 + r^2\, d\theta^2 + r^2 \sin^2\theta\; d\varphi^2 \right] \qquad 4.1$$

Şeklinde bulunur.

De-Sitter de $\left[1 - \frac{\Lambda}{3} r^2\right]^{-1} \approx \left[1 + \frac{\Lambda}{3} r^2\right]$ olarak yazılırsa yaklaşık De-Sitter aşağıdaki gibi olur.

$$ds^2 \approx \left[1 - \frac{\Lambda}{3} r^2\right] dt^2 - \left[1 + \frac{\Lambda}{3} r^2\right] dr^2 - r^2\, d\theta^2 - r^2 \sin^2\theta\; d\varphi^2$$

(4.1) sonucu ile De-Sitter in yaklaşık biçimini benzer olduğu görülür.

KAYNAKLAR

1. Foster J. And Nightingale J.D.,A Short Course in Genaral Relativity,Springer-Verlag, 1995

2. Ohanian H,Ruffini R,Gravitation and Spacetime,Norton, New York,Second Editon,1994

3. Wold RM, General Relativity, Universty of Chicago Pres, Chicago,1984

4.Misner CW, Thorne KS, Wheeler J.A. Gravitation,WH Freeman and Company,New York, 1973

5. Stephani, Hans; Kramer, Dietrich; MacCallum, Malcolm; Hoenselaers,Cornelius & Herlt,Eduard (2003). Exact Solutions of Einstein's Field Equations. Cambridge: Cambridge Universty Press

6. De Sitter,W. On Einstein theory of gravitation and its astronimical consequences Mon. Not. R. Astr. Soc. 78,3-28,(1917)

7. Sean M. Carroll, "The Cosmological Constant", Living Reviews. Relativity 4 ,(2001).

8 Realdi M and Peruzzi G. , Einstein, De-Sitter and the beginning of relativistic cosmology in 1917, Gen. Relativ. Gravit. 41,225-247, (2009)

ÖZGEÇMİŞ

22.06.1974 tarihinde Trabzon'da doğdu.İlk, orta ve lise öğrenimini aynı ilde tamamladı.1994 de Uludağ Üniversitesi Necatibey Eğitim Fakültesi Matematik Öğretmenliği bölümünden mezun oldu. 1994 yılında Mardin'e matematik öğretmeni olarak atandı. Halen Kocaeli ilinde Darıca Ülkün Yalçın Anadolu lisesinde matematik öğretmeni olarak görev yapmaktadır.