SOLUTION METHODS FOR PLANNING PROBLEMS IN WIRELESS MESH NETWORKS

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By
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August 2012

I certify that I have read this thesis and that in my opinion it is full adequate, in scope and in quality, as a dissertation for the degree of Master of Science.

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ABSTRACT

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IN WIRELESS MESH NETWORKS

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Wireless Mesh Networks (WMNs) consist of a finite number of radio nodes. A subset of these nodes, called gateways, has wired connection to the Internet and the non-gateway nodes transmit their traffic to a gateway node through the wireless media in a multi-hop fashion.

Wireless communication signals that propagate simultaneously within the same frequency band may interfere with one another at a receiving node and may therefore prevent successful transmission of data. In order to circumvent this problem, nodes on the network can be configured to receive and send signals in different time slots and through different frequency bands. Therefore, a transmission slot can be defined as a pair of a certain frequency band and a specific time slot. In addition, by adjusting the power level of a radio node, its transmission range can be modified.

Given a wireless mesh network with fixed node locations, demand rate at each node, and maximum power level for each node, we study the problem of carrying the traffic of each node to the Internet through the network. Our goal is to allocate capacities in proportion to the demand of each node in such a way that the minimum ratio is maximized. We propose a mixed integer linear programming (MILP) formulation to select a given number of gateway locations among the nodes in the network, to determine the routing of the traffic of each node through the gateway nodes, to assign transmission slots to each node in order to ensure no interference among wireless signals, and to determine the transmission power levels. In our study, we adopt the physical interference model, instead of the protocol interference, since this is more realistic.

Since MILP formulation becomes computationally inefficient for larger instances; we developed several different approaches. Then, we proposed a combinatorial optimization model which successfully solves most of the instances. We tested our models and methods in several data sets, and results are presented.

Keywords: Wireless Mesh Networks, Integer Programming, Gateway Selection, Routing, Transmission Slot Assignment.

ÖZET

ÇOKGEN BAĞLANTILI KABLOSUZ AĞLARIN PLANLANMA PROBLEMLERİ İÇİN ÇÖZÜM YAKLAŞIMLARI

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Kablosuz Çokgen Bağlantılı Ağlar (KÇBA) sonlu sayıda telsiz düğümünden oluşur. Bu düğümlerin ağ geçidi adı verilen bir alt kümesi İnternete doğrudan kablo ile bağlı iken, ağ geçidi olmayan noktalar trafiklerini kablosuz ortamda birbirleri üzerinden ağ geçidine aktarırlar.

Aynı frekans bandında aynı anda yayınlanan kablosuz iletişim sinyalleri alıcıda birbirine karışabilir ve bu sebeple başarılı veri aktarımını engelleyebilir. Bu problemi engellemek için ağdaki düğümler, farklı zaman dilimlerinde ve farklı frekanslar kullanacak şekilde ayarlanabilir. Ayrıca, bir telsiz düğümünün güç seviyesi ayarlanarak aktarım aralığı değiştirilebilir.

Bu çalışmada, verilen bir KÇBA için, düğümlerin lokasyonları, her düğümün talep oranı ve en yüksek güç seviyesi bilindiğinde, düğümlerin trafiklerinin İnternete nasıl taşınacağı problemine odaklandık. Amacımız, kapasiteleri her düğümün talebiyle orantılı bir şekilde ve en küçük oranı en büyükleyerek bölüştürmektir. Bu sebeple, verilen ağ düğümleri arasından ağ geçidi lokasyonlarını seçen, her nodun seçilen ağ geçitleri üstünden trafiklerini belirleyen, kablosuz sinyallerin karışmasını önleyecek şekilde veri aktarımının zaman aralıklarını belirleyen ve aktarımın güç seviyelerine karar veren bir Karışık Tam Sayılı Program (KTSP) önerdik. Bu çalışmada, protokol karışma modeli yerine daha gerçekçi olan fiziksel karışma modelini baz aldık.

İlk önerilen KTSP formulasyonu büyük örnekler için olurlu sonuçlar elde edemediğinden farklı yaklaşımlar üzerine yoğunlaştık. Önce bu model üzerine yapıla eklemelerle yeni yöntemler geliştirirken, sonrasında bir kombinatoryal optimizasyon modeli geliştirdik. Önerdiğimiz model ve yöntemleri farklı veri kümeleri üzerinde yapılan deneylerin sonuçları üzerinden karşılaştırdık. Kombinatoryal eniyileme modelinin diğerlerinden daha iyi sonuçlar verdiğini gözlemledik.

Anahtar Kelimeler: Kablosuz Çokgen Bağlantılı Ağlar, Tam Sayılı Programlama, Ağ Geçidi Seçimi, Rotalama, Aktarım Zaman Aralığı Ataması.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

Chapter 1

Introduction

A Wireless Mesh Network (WMN) is a wireless communication network in which the nodes are not directly connected to the Internet with wire but send their traffic to a router with wired connection on a multi-hop route. Since the required infra-structure is relatively simple, deployment of WMNs are cost efficient.

However, increasing the share of wireless communication in the transmission of the traffic on the network causes some extra problems. Simultaneous signals with the same frequency may cause interference. This fact may cause the transmission of signals to fail. Therefore, an efficiently working network can be obtained only with a priori planning. On a WMN, basically the route, schedule, gateway placement and power management decisions can be described as the planning problems.

In this study, those planning problems are addressed. The remaining of this thesis is organized as follows:

In the next chapter, features of the WMNs are discussed. Then in Chapter 3, a brief summary of the related literature is presented. The literature review is also followed by the problem definition, in same chapter. Then, in Chapter 4, an MILP model that addresses those mentioned planning problems jointly is proposed. Since this model is difficult to solve, in Chapter 5, some extensions for this MILP formulation are presented. In Chapter 6, a combinatorial optimization model is developed. Those models are tested on several network data and the results are analyzed in Chapter 7. Finally, in Chapter 8, the study is concluded with a brief summary and several suggestions for extensions and future work.

Chapter 2

Wireless Mesh Networks

2.1 Wireless Networks

Wireless communication technology is being widely used since the last decade of 20th century. The achievements in wireless technology gradually decrease the level of cabled infrastructure needed in communication networks. This fact can be observed in the progress of wireless communication protocols. There are mainly two wireless networking protocols defined by the Institute of Electrical and Electronics Engineering (IEEE). Those are; the older one, 802.11 Wireless Fidelity (Wi-Fi) and the relatively recent one, 802.16 Worldwide Interoperability for Microwave Access (Wi-Max). While Wi-Fi, which is commonly used in Wireless Local Area Networks (WLAN), is available to use in relatively small areas (30 - 100 m), Wi-Max can cover larger areas, up to 50 km. Therefore, by utilizing Wi-Max, Metropolitan Area Network (MAN) applications can be developed [1].

2.2 Wireless Mesh Networks

Wireless Mesh Networks (WMNs) are the family of wireless communication networks that is based upon a mesh topology. An illustration of mesh topology is provided in Figure 2.1. In a wireless network with that kind of a topology, mesh nodes in network serve as a host to their clients and as a router to other mesh nodes. The main difference between WMNs and the common wireless networks emerges from this second mission. Rather than being connected to the wired system directly, the traffic is transmitted to the node with wired connection, the gateway, on a route which may or may not consist of multiple hops. Therefore, higher coverage of area and clients can be attained with less wired infrastructure. This makes WMNs a relatively cost efficient alternative for wireless communication.

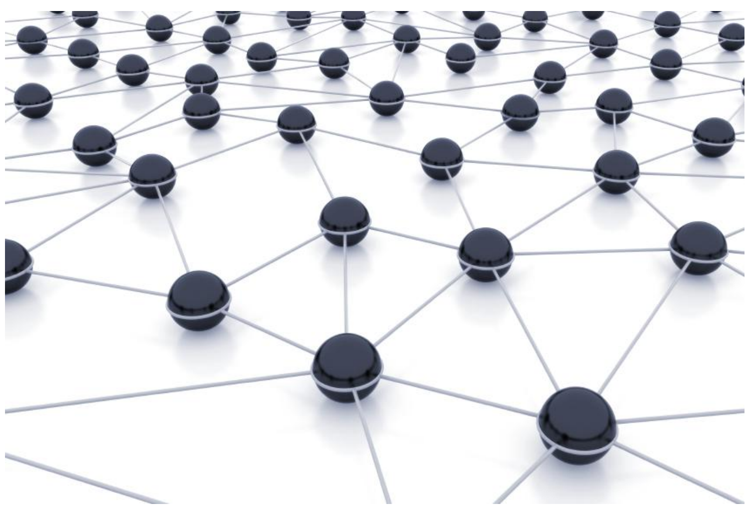

Figure 2-1 Mesh Topology

According to Akyildiz et al., a wireless mesh network consists of two types of nodes, mesh router (MR) and mesh client (MC). The mesh clients can be PCs, tablet computers, mobile phones [2]. A mesh client is the device that the final user utilizes, and can be mobile or static. On the other hand, mesh routers are statically placed, and as stated above, both provide service to clients, and transmit the traffic of other mesh routers. A typical wireless mesh network structure can be seen on Figure 2.2.

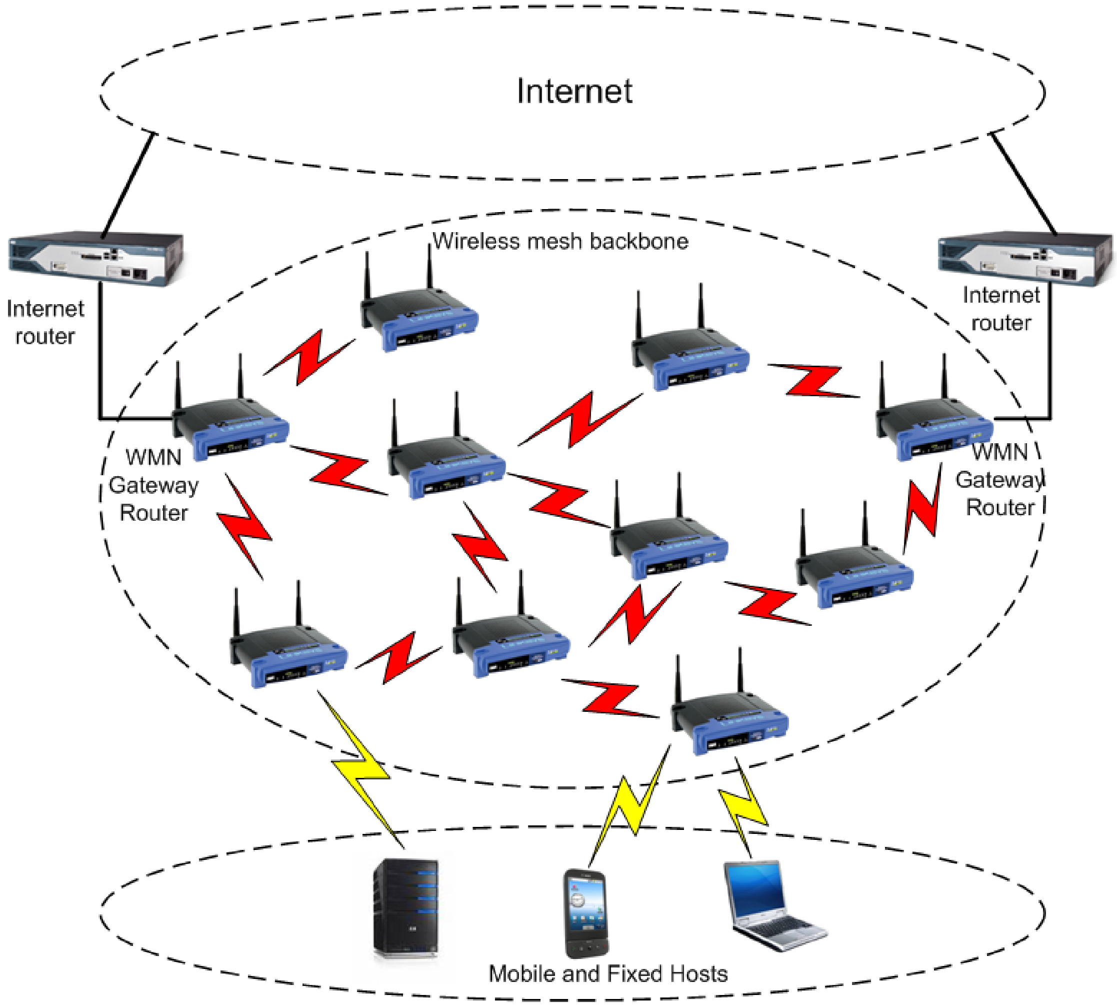

Figure 2-2 Wireless mesh network, routers, clients and gateways

The network topology is composed of mesh routers, nodes, and links between those nodes. The conditions affecting the existence of a link are discussed in the following part of this chapter. There is a certain capacity limiting the size of the data transmitted through a link.

2.3 Interference

Due to the orbicular propagation of wireless signals, when traffic is transmitted through a link, this may also affect remaining nodes as well. When a router sends signals to another router, those signals are also received by all routers within a certain radius from the sender router. If a router receives multiple signals at a time, none of them can be processed. Therefore, the transmission of data between two nodes is possible only if all of the other routers within a range of the destination node are silent during that time. Accordingly, the communication between two nodes may disable some other links. This phenomenon is called interference.

The interference issue is mathematically modeled by Gupta and Kumar with two different approaches [3]. Those are, protocol model and physical model respectively.

2.3.1 Protocol Model of Interference

Protocol interference model is a relatively simplistic way of modeling interference issue mathematically. In this model, transmission between two nodes, i and j, with positions X_i and X_j can be successfully performed if for any other node k with position X_k , which is also active simultaneously at the same frequency, the following inequality is satisfied.

$$\left|X_k - X_j\right| \geq (1 + \Delta)\left|X_i - X_j\right|. \tag{1}$$

In this inequality, $\Delta > 0$ corresponds to an auto-defensive guard like mechanism, “a guard zone” as stated by Gupta and Kumar that precludes any transmission from adjacent nodes.

2.3.2 Physical Model of Interference

As it can be figured out by the inequality (1), the protocol model for interference considers the affects of other nodes individually, rather than evaluating the total noise created by the whole on the network at that time and frequency. Additionally, the protocol model does not include the power level of the signal being transmitted. Combining those factors with the fact that the effect of distance on the signal noise is not linear; a more realistic, but a complex way of modeling interference can be obtained. Gupta and Kumar propose the following inequality for a transmission to be successfully received:

Let S be the set of nodes active at a certain frequency and time, let γ denote the ambient noise level, θ represent the Signal-to-Interference-Ratio (SIR), which is a threshold value affecting the reception of signal. Then, the signal sent from node, *i* to node *j*, with power P_i can be received if

$$\frac{\frac{P_i}{\left|X_i - X_j\right|^{\alpha}}}{\gamma + \sum_{\substack{k \in S \\ k \neq i}} \frac{P_k}{\left|X_k - X_j\right|^{\alpha}}} \geq \theta. \tag{2}$$

Here, α is the exponent by which the signal power decreases with distance. In other words, it can be said that, the signal received at the destination node is inversely proportional to the distance between the source and destination nodes to the power α. Therefore, the reciprocal of the α exponent of the distance between two nodes *i* and *j* is called the path loss between *i* and *j* and denoted by l_{ij} such that:

$$l_{ij} = \frac{1}{\left|X_i - X_j\right|^{\alpha}} \ . \tag{3}$$

Then, (2) can be rewritten in the following form:

$$\frac{P_i l_{ij}}{\gamma + \sum_{\substack{k \in S \\ k \neq i}} P_k l_{kj}} \geq \theta. \tag{4}$$

Then, assume that all nodes except *i* and *j* are inactive at some point in time. Then, the signal with power P_i can be transmitted between *i* and *j* if:

$$\frac{P_i l_{ij}}{\gamma} \geq \theta\ . \tag{5}$$

Therefore, denoting the maximum power that node *i* can transmit with P_i^{max} , transmission between *i* and *j* can occur only if

$$P_i^{max} l_{ij} \geq \theta\gamma\ . \tag{6}$$

Then, a link between two nodes exists if and only if (6) is satisfied. Defining

$g_{ij} = P_i^{max} l_{ij}$, we can rewrite (6) in the following form:

$$g_{ij} \geq \theta\gamma. \tag{7}$$

2.4 Access Schemes

Since interference allows only a limited part of the network to be active simultaneously, different approaches have been developed for planning the traffic in wireless communication networks. The traffic in WMNs can be sent using either decentralized contention based or centralized non-contention based schemes. In contention based schemes route or schedule are not a priori planned.

In non-contention based schemes, a central controller is required. In this study, we will focus on Frequency Based Multiple Access and Time Based Multiple Access which are both non-contention based schemes.

2.4.1 Time Division Multiple Access (TDMA)

TDMA is based on the logic that signals sent in different times do not create interference. Therefore, a unit time, called a frame, is divided into time slots and cyclically repeated. Interference is considered only within a time slot and if the number of time slots is greater than or equal to the number of nodes, a trivial schedule can be generated without considering the interference issue, by assigning each node a different time slot to transmit its traffic. Nodes can utilize all available frequencies in their assigned time slot.

2.4.2 Frequency Division Multiple Access (FDMA)

Interference can be avoided not only by dividing into slots but also sending signals on different frequency bands. Wireless mesh routers are equipped to transmit and receive signals from multiple channels. A node can send its traffic on the channels assigned to it without any time limitations.

2.4.3 Orthogonal Frequency Division Multiple Access (OFDMA)

OFDMA is basically the combination of FDMA and TDMA. Time frame is divided into slots and different frequencies can be used for transmission of different packages.

Chapter 3

Literature Review and Problem Definition

3.1 Review of the Related Literature on WMNs

WMNs have been a popular research subject in the last decade. Since setting up a WMN requires some critical and difficult decisions to be made, operations research methods have been used in some part of those studies, especially the ones aiming to solve those planning problems.

Gupta and Kumar [3] analyze the capacity of WMNs, and compare the difference between randomly placed nodes and optimally placed nodes. They also define two methods to model interference, since it directly affects the capacity of such networks. Klasing et al. [4] model the bandwidth allocation problem on radio networks as a multi-commodity flow problem, with a simplistic interference approach. In their study, they define capacities depending on the interference relations between links, while interference is defined as a binary relation considering the interaction between links in pairs, instead computing the cumulative effect on simultaneous traffic at any link. They name their formulation as Round Weighting Problem (RWP). To analyze the capacity in more detail, Caillouet et al.[5] provide optimization based methods to determine the theoretical capacity of a WMN, while working with a formulation analogous to the one in Klasing's study, in addition to providing a generic MILP model.

As the analysis by Gupta and Kumar [3] pioneers the research on capacity of WMNs, different methods and solutions have been developed to cope with the interference issue, besides analyzing the theoretical capacity. As an analogy to the access schemes described in Chapter 1, the solution methods to overcome the limitations of interference are mostly based on link scheduling or channel assignment.

Jain et al. [6] present methods on finding bounds on optimal throughput for a WMN, under the protocol interference model. They prove that the problem of finding the optimal throughput on a network under the protocol interference model is NP-Hard. On the other hand, Kodialam and Nandagopal [7] propose algorithms for link scheduling and channel assignment problem for both dynamic and static cases. Additionally, they propose a mathematical constraint satisfaction model to determine bounds on optimal solutions with a given objective.

Brar et al. [8], with the assumption of given traffic demands for links, propose a computationally efficient algorithm to determine the schedule under the physical interference model. Behzad and Rubin [9] present an MILP formulation which jointly addresses link scheduling and power management decisions, to determine the minimum schedule length. Since the model captures the power management decisions, the physical interference model is used. They define the problem they address as Integrated Link Scheduling and Power Control problem and give a proof that this problem is NP-

Complete. Capone and Carello [10] also provide an MILP formulation for the scheduling problem of a given set of links and minimum number of packets to be transmitted through each link. Power control and physical interference are included in the model, while the objective is to minimize the required number of time slots. They propose a column generation based solution approach for their formulation of the problem. Papadaki and Friderikos [11] propose a dynamic programming approach to schedule a given set of links, where each link has to be active for at least one time slot. Similar to Brar et al. and Behzad and Rubin's studies, Papadaki and Friderikos adopt the physical interference model in their study.

All of those studies mentioned about link scheduling start with a given set of links to be scheduled, *i.e.* the solution for routing part of the problem is taken as given, and they deal with the problem of scheduling a given route, under the interference constraints. Nevertheless, routing is also an important part of the problem, which directly effects the potential limitations for the interference.

Raniwala et al [12] addresses the channel assignment and routing problems together, under an IEEE 802.11 Wireless LAN setup. They provide an algorithm to obtain solutions. Additionally, they present a proof that channel assignment problem is NP Hard. Alicherry[13] et al. also address the channel assignment and routing problems jointly. They provide an MILP formulation, and present algorithms based upon the LP relaxation of the MILP. They assume that the multi path routing is possible and use the protocol interference model.

Capone et al. [14] handles the routing, scheduling and channel assignment problems together, and propose an MILP formulation. Similar to the approach in [10], they propose a column generation based solution methodology.

In contrast with the MILP formulations and network or graph theory based algorithms, meta-heuristic approaches to the planning problems of WMN are covered in only a few studies. One such example is conducted by Badia et al. [15], proposing a genetic algorithm to joint routing and link scheduling problem. Large instances for which MILP formulations fails to give a solution can be handled with their algorithm.

The discussion in this chapter up to now covers a brief part of the WMN literature, focusing on the studies on routing and scheduling methods. On the other hand, those studies do not provide the answer to another strategic question for WMNs. As, having less wired connection is a benefit of the WMN technology, appropriate choice of those cable connections, namely the gateways, plays a crucial role in the performance of the network. A significant part of the studies on WMNs are devoted to gateway selection methods.

Chandra et al. [16] provide an MILP formulation for the problem of minimum number of gateways, “Internet Transit Access Points” in the terminology they use, under quality of service constraints, while also showing that this problem is NP-Hard. Moreover, they propose a number of alternative algorithms for the same problem. Aoun et al. [17] address the same problem, the gateway placement decision is handled while quality of service constraints need to be satisfied. They provided a recursive dominating set algorithm to decide on minimum number of gateways. On the other hand, neither [16] nor [17] considers interference problem since routing and scheduling is not in the scope of those studies.

The recent studies on gateway placement, however, take possible routing alternatives and thus interference problem into account. Li et al. [18] address the gateway placement problem with the objective of throughput maximization, and interference issue is also embodied in the proposed ILP formulation. In this study, interference is considered with the protocol model. In their recent paper, Targon et al. [19] provide an ILP formulation to perform gateway placement that enables the network to satisfy the given demand with minimum deployment cost. The formulation proposed in this study captures the interference problem with the physical model. As interference is included, route and schedule are also within the outputs of the provided formulation. Zhou et al. [20] address the gateway placement problem with the objective of throughput maximization. In this study, an algorithm that decides on the gateway places is proposed. Additionally, a non-asymptotic analytical model is developed to determine the throughput depending on the given gateway placement decision.

The studies discussed so far address some part of the decision problems with a theoretical approach. On the other hand, there are already deployed wireless mesh networks, at least for experimental purposes. The outputs of an early example of a wireless mesh network deployed in Cambridge, Massachusetts are presented in Bicket et al [21].

As mentioned at the very first sentence of this chapter, WMNs have been studied in an extensive number of papers. The selection of studies presented here is mostly limited to the ones addressing planning problems. One may find more information about WMNs in the papers by Akyildiz et al [1] and Pathak and Dutta [22].

Here, a brief selection of studies on WMNs has been provided. In the literature, planning issues regarding WMNs have been analyzed separately. The parts of the problem are interrelated, and thus an integrated approach may be a good alternative, though creating computational complexities. Such a holistic approach is studied in Uzunlar's thesis [23]. In this study, an MILP formulation is provided, capturing gateway selection, routing, transmission slot assignment and power management. Also, a gateway selection heuristic is proposed.

3.2 Problem Definition

Centralized deployment of a WMN consists of a number of decisions to be made. At first, the places of the routers should be determined. This is followed by the choice of number and positions of the gateways. Then, the traffic among the network is planned. This includes routing, selection of the links to be used, and transmission slot assignment, by means of time slot and radio channel.

While planning the deployment of a WMN, either meeting the demand with minimum deployment cost, or providing the best service quality under a pre-determined budget can be aimed. The cost is directly related with the gateway decision; therefore, minimum cost is reached with the minimum number of gateways, whereas a pre-determined budget stands for the number of gateway points. In this study, the financial issues are not analyzed and the number of gateways is taken as a given parameter.

Deployment of a WMN includes routing and transmission slot assignment planning in two different levels. The first level is the transmission of traffic between MRs and the Internet, and the second level is the interaction between MCs and MRs. In this study, we address the planning of the traffic in the first level. For the second level, we simply assume that a MC is served by the nearest MR.

On a WMN, MRs transmit their traffic to the Internet on multi-hop routes. Therefore, each MR also transmits the traffic of other routers, for which they are on the specified route. Each MR, using this multi-hop mechanism, reaches the Internet through a gateway. The planning of this routing can be done in several different ways. A router can send its traffic via multiple paths or a single path. The routing on the network may or may not have a forest structure, which consists of a tree for each gateway point. To avoid increasing computational complexity, we limit our analysis on single path routing with forest structure. Therefore, division of traffic demand of a node is not allowed, and similarly, traffic passing through a node cannot be sent by dividing into multiple routes.

A significant part of the previous researches on WMNs adopts the protocol interference model that is described by Gupta and Kumar [3]. However, though this choice is better considering the computational complexity, the simplicity of that model may lead to an unrealistic representation of WMNs. Therefore, we adopt the physical interference model that is also defined in Gupta and Kumar's study.

For the transmission slots, the OFDMA scheme is adopted. Actually, TDMA, FDMA or OFDMA are indifferent by means of computation as long as the number of transmission slots is same. The number of transmission slots is the number of periods in a time frame for TDMA, number of radio channels available for FDMA, and their product for OFDMA. However, the number of radio channels that a router is equipped with is limited, and dividing a unit time into a large number of periods may diverge from reality since there may be, though negligibly little, some loss during the transition between periods. Therefore, OFDMA would be a realistic choice. The time frame is taken as 1 second. Therefore, the determined schedule is repeated every second. The traffic demand of routers and the transmission capacity of links are commonly denoted with the unit of megabits per second (Mbps), thus choosing the time frame of 1 second is convenient.

The power of transmitted signal directly affects the interference on network. For each transmission to be performed, the appropriate power level should be determined. Assuming that a router can change its power level a number of times in a second would not be realistic. Therefore, for each router, the power level is taken to be the same within the transmission slots. Any signal sent from this router is sent with this power level.

Our approach captures all of those sub-parts of the WMN planning problem jointly, and aims to maximize the quality of service. The minimum service level provided to a node is used as a measurement for quality of service, while service level of a node is defined as the provided traffic-demand ratio for each node. Then, the problem we address can be summarized as follows: Given the number and position of nodes, number of gateways to be placed, traffic demands for each node, which selection of the gateways, route, transmission slot assignment and associated power levels would maximize the minimum service level provided among routers?

Chapter 4

Model Formulation

In the previous chapter, a description of the problem which is aimed to be solved is presented. In this chapter, a mixed integer linear programming (MILP) formulation is proposed for the WMN planning problem.

A WMN can be modeled as a network, with nodes and arcs, where each MR corresponds to a node, and every possible connection forms an arc. The conditions under which signal transmission between two nodes are possible are given in equation (6).

4.1 Assumptions

In the MILP formulation, the following statements are assumed to hold.

- The locations of the MRs are known and static.
- The WMN is deployed in accordance with IEEE 802.16 WIMAX protocol.
- The MRs on the network are identical.

- Each MR sends its traffic to the Internet on a single path, where the union of those paths among routers forms a forest structure.
- The transmission slots are based on OFDMA scheme.
- Ambient noise power is static and equal at each point of the network.
- A time frame is 1 second.
- A router cannot switch between power levels within a time frame.

4.2 Sets and Parameters

- Let n be the number of nodes. Then, $N = \{1, \ldots, n\}$ denotes the set of all nodes.
- A denotes the set of arcs. Then, $A = \{(i, j) \in N^2 | (1.6) is\ satisfied\}$
- T denotes the number of time slots within a frame.
- K denotes the number of radio channels available at each router.
- The capacity of an arc is c Mbps.
- The wired connection at a gateway node has a transmission capacity of α Mbps.
- Number of gateways to be deployed is denoted by G.
- $d \in R_+^n$ is the vector of traffic demand for each node.
- $g \in R_+^{n \times n}$ is the matrix of the received strength of signal with maximum power between two nodes.
- γ denotes the ambient noise level. Ambient noise is the existing noise on the network without any signal.
- θ is the SINR threshold value. A signal is received if the ratio of its power to the cumulative noise on the network is higher than this value.

4.3 Decision Variables

$$x_{i,j}^{t,k} = \begin{cases} 1, & if\ the\ link\ (i,j)\ is\ active\ in\ transmission\ slot\ t\ and\ channel\ k \\ 0, & otherwise \end{cases}$$

$\forall\ (i,j) \in A, \forall t\ \in \{1, \dots, T\}, \forall k\ \in \{1, .., K\}$

$$y_i = \begin{cases} 1, & if\ node\ i\ is\ used\ as\ a\ gateway \\ 0, & otherwise \end{cases} \quad \forall i \in N$$

$$z_{i,j} = \begin{cases} 1, & if\ the\ link\ (i,j)\ is\ used\ for\ transmission \\ 0, & otherwise \end{cases} \quad \forall\ (i,j) \in A$$

$f_{i,j} = the\ maximum\ amount\ of\ flow\ carried\ on\ link(i,j) \quad \forall\ (i,j) \in A$

$P_i^{t,k} = the\ level\ of\ the\ transmitted\ power\ from\ node\ i\ in\ transmission\ slot\ t\ and\ channel\ k$

$\forall\ i \in N, \forall t\ \in \{1, \dots, T\}, \forall k\ \in \{1, .., K\}$

$\pi_i = the\ level\ of\ the\ power\ whenever\ node\ i\ transmits\ \ \forall i \in N$

$m = minimum\ service\ level$

4.4 Model Formulation

The developed model (*WMN1*) is quite similar to the one used in Uzunlar's thesis [23], with addition of channel assignments and some extra constraints for power control. Actually, the extra constraints are the expressions of the assumption that a router cannot switch between power levels within a second. The proposed model *WMN1* is as follows:

max $\quad m$

s.t.

$$f_{i,j} \leq \frac{c}{TK}\sum_{t=1}^{T}\sum_{k=1}^{K} x_{i,j}^{t,k}\ ,\ \forall (i,j) \in A \tag{8}$$

$$md_i + \sum_{s:(s,i)\in E} f_{s,i} \leq ay_i + \sum_{d:(i,d)\in E} f_{i,d} \quad ,\forall i \in N \tag{9}$$

$$md_i + \sum_{s:(s,i)\in E} f_{s,i} \geq \sum_{d:(i,d)\in E} f_{i,d} \quad ,\forall i \in N \tag{10}$$

$$\sum_{j:(i,j)\in E} z_{i,j} = (1 - y_i)\ ,\quad \forall i \in N \tag{11}$$

$$x_{i,j}^{t,k} \leq z_{i,j} \qquad ,\forall (i,j)\ \in A, \forall t\ \in \{1, \dots, T\}, \forall k\ \in \{1, \dots, K\} \tag{12}$$

$$\sum_{s:(s,i)\in E} x_{s,i}^{t,k} + \sum_{d:(i,d)\in E} x_{i,d}^{t,k} \leq 1 \quad ,\forall i \in N, \forall t\ \in \{1, \dots, T\}, \forall k\ \in \{1, .., K\} \tag{13}$$

$$P_i^{t,k} \leq \sum_{d:(i,d)\in E} x_{i,d}^{t,k} \qquad ,\forall i \in N, \forall t\ \in \{1, \dots, T\}, \forall k\ \in \{1, \dots, K\} \tag{14}$$

$$P_i^{t,k} \leq \pi_i \qquad ,\forall i \in N, \forall t\ \in \{1, \dots, T\}, \forall k\ \in \{1, \dots, K\} \tag{15}$$

$$P_i^{t,k} \geq \pi_i + \sum_{d:(i,d)\in E} x_{i,d}^{t,k} - 1 \quad ,\forall i \in N, \forall t\ \in \{1, \dots, T\}, \forall k\ \in \{1, \dots, K\} \tag{16}$$

$$P_i^{t,k} g_{i,j} + M_{i,j}\left(1 - x_{i,j}^{t,k}\right) \geq \theta\gamma + \sum_{\substack{s=1 \\ s\neq i,j}}^{n} \theta P_s^{t,k} g_{s,j}, \quad \forall (i,j)\ \in A, \forall t\ \in \{1, \dots, T\}, \forall k\ \in \{1, \dots, K\} \tag{17}$$

$$\sum_{i=1}^{N} y_i = G \tag{18}$$

$$x_{i,j}^{t,k}, y_i, z_{i,j} \in \{0,1\} \ , \forall i \in N, \forall t \ \in \{1, \dots, T\}, \forall k \ \in \{1, \dots, K\}, \forall (i,j) \ \in A \tag{19}$$

$$f_{i,j}, P_i^{t,k}, \pi_i, m \ \geq 0 \tag{20}$$

The $M_{i,j}$ value used in constraint (17) is calculated as in equation (21):

$$M_{i,j} = \theta\gamma + \sum_{\substack{s=1 \\ s\neq i,j}}^{n} \theta g_{s,j} \ , \forall (i,j) \ \in A \tag{21}$$

This model finds a gateway placement, route, transmission slot assignment and power levels for each transmission such that the minimum service level provided to any router is maximized.

Constraint sets (6)-(13) focus on the routing and scheduling decisions. The constraint sets (14)-(17) form the power management structure. The power management and routing & scheduling decisions are linked by the physical interference constraint, (17). And the last constraint states the condition of gateway selection.

The constraint (8) specifies the capacity of a link. At each time slot, $\frac{c}{TK}$ Mbps data can be transmitted, therefore, flow on a link is limited with the multiplication of this capacity with the number of transmission slots that there is traffic on that link.

The constraints (9) and (10) assure the balance of the flow and the capacity of wired transmission at a gateway node. The traffic sent from a node which is not a gateway is equal to the sum of incoming traffic and the provided service level times its demand. If it is a gateway node, then the difference between incoming and outgoing traffic cannot exceed the gateway capacity.

The constraint (11) forces every node to route its traffic, and also prevents a gateway from sending its traffic to any other node. Constraint (12) guarantees that transmission slots are assigned to only the links on the route. Therefore, this constraint forms the connection between *x* and *z* variables.

A node can either receive or send traffic at any time. This is imposed by constraint (13).

The constraint (14) imposes that power is assigned only to the active links at any transmission slot. Constraints (15) and (16) assure that the power level of a signal sent from a node is equal to the determined power level of this node. Constraint sets (14), (15) and (16) together, fix the power level of a link at a slot to 0, if that link is not active at any transmission slot. Otherwise, it is fixed to the π_i value.

The most complicated constraint at first sight seems to be the physical interference constraint, (17). If a link is active at any transmission slot, then this constraint enforces the physical interference condition stated in (2). Otherwise, the choice of $M_{i,j}$ ensures this inequality to be a tautology.

Constraint (18) fixes the number of gateways. Constraint sets (19) and (20) are integrality and non-negativity definitions of the decision variables.

This model is inadequate to solve planning problems. Therefore, a different approach, or a better model is needed. In the following chapter, extensions for this formulation are proposed. Then with those extensions, several methods are proposed. In the following chapters, also another MILP formulation will be developed for the same problem.

Chapter 5

Extensions for MILP Formulation

The *WMN1* formulation is inefficient to solve WMN planning problem. Therefore, it needed to be improved with cuts and other approaches. In this chapter, we propose 3 valid cuts for this formulation, and an additional IP based approach to cope with the difficulty created by interference problem.

5.1 Valid Inequalities

5.1.1 A Logical Cut

The current set of constraints enables, but does not force that every node is served. That is, the solutions with zero service level are in the feasible set. However, in the definition of problem, zero service level is not a desired solution. Then, those solutions can be prevented to be mathematically feasible with appropriate set of inequalities.

Consider following inequality:

$$\sum_{t=1}^{T}\sum_{k=1}^{K} x_{i,j}^{t,k} \geq z_{i,j} \qquad \forall (i,j) \in A \qquad (22)$$

Then, if a link is on the decided route, transmission slot assignment to this link is forced.

Lemma 5.1: Inequality (22) defines a set of valid inequalities on the set of optimal solutions of *WMN1*.

Proof: Let $z' \in \{0,1\}^{|A|}$, $y' \in \{0,1\}^{|N|}$ be two vectors satisfying (11) and (18) and all the remaining variables be equal to 0. Then, those variables form a feasible solution to WMN1. Since this selection of variables does not satisfy (22), this solution is cut by (22). On the other hand, consider an optimal solution. If there exists an optimal solution with positive service level, then for all nodes *i* that are not gateway, by constraint (9) and (10), outgoing flow should be positive. Then, by constraint (8), for some $j \in N$ and $t \in \{1, \ldots, T\}, k \in \{1, \ldots, K\}$, $x_{i,j}^{t,k}$ should be positive. Then by constraints (11) and (12), the corresponding $z_{i,j}$ value is 1, and *(i,j)* is the only link that is ever used, among the links with source *i*. Then for remaining links with source i, $z_{i,s}$ and $x_{i,s}^{t,k}$ is 0. Therefore, (22) is satisfied. Then (22) does not cut any optimal solutions. Therefore (22) is valid on the set of optimal solutions of WMN1.

5.1.2 Link Analysis for Physical Interference

Let $(i,j) \in A$ and $(s,d) \in A$. Then, assume that they are simultaneously active on the same channel with associated power levels $P_i^{t,k}$ and $P_s^{t,k}$. Then by constraint (17), the following two conditions hold:

$$P_i^{t,k} g_{i,j} \geq \theta\gamma + \theta P_s^{t,k} g_{s,j} \qquad (23)$$

$$P_s^{t,k} g_{s,d} \geq \theta\gamma + \theta P_i^{t,k} g_{i,d} \qquad (24)$$

Here, the cumulative noise term in (17) is omitted and this does not violate the inequality, since it is a positive term in the right hand side of the inequality.

Then, by multiplying (24) by $g_{s,d}$, we have:

$$P_i^{t,k} g_{i,j} g_{s,d} \geq \theta\gamma g_{s,d} + \theta\, P_s^{t,k}\, g_{s,j}\, g_{s,d} \tag{25}$$

Then, by substituting $P_s^{t,k} g_{s,d}$ with the right hand side value of (24) in (25), we have:

$$P_i^{t,k}\left(g_{i,j} g_{s,d} - \theta^2 g_{i,d} g_{s,j}\right) \geq \theta\gamma\left(g_{s,d} + \theta\, g_{s,j}\right) \tag{26}$$

Then, since $P_i^{t,k} \leq 1$

$$g_{i,j} g_{s,d} - \theta^2 g_{i,d} g_{s,j} \geq \theta\gamma\left(g_{s,d} + \theta\, g_{s,j}\right) \tag{27}$$

In the same way, multiplying (24) with $g_{i,j}$ and then substituting (23), we also have:

$$g_{i,j} g_{s,d} - \theta^2 g_{i,d} g_{s,j} \geq \theta\gamma\left(g_{i,j} + \theta\, g_{i,d}\right) \tag{28}$$

Therefore, we can say that, two links $(i, j) \in A$ and $(s, d) \in A$ can be simultaneously active if (27) and (28) holds.

Then, let us define a new set, $D \subset A^2$ such that

$$D = \left\{\left((i, j), (s, d)\right) \in A^2 \,\middle|\, \text{(27) or (28)does not hold}\right\}$$

Therefore, D is the set of link-pairs that cannot be active simultaneously. Then, the following set of the inequalities can be added without causing any feasible solution to become infeasible.

$$x_{i,j}^{t,k} + x_{s,d}^{t,k} \leq 1 \qquad \forall\left((i, j), (s, d)\right) \in D, \forall t, k \tag{29}$$

Lemma 5.2: The set of inequalities defined by (29) are valid for the set of optimal solutions of *WMN1*.

Proof: By the above derivation, set D is defined as the set of conflicting edge pairs. Therefore, there is no integer solution in which both of such edge pairs are active at the same transmission slot. Hence, the inequalities defined by (29) do not cut any integer solution. Thus, any optimal solution will satisfy (29)

■

In a similar fashion, such a relation can also be derived for link triples.

Then, let $(i,j), (s,d), (u,v) \in A$. Those three links can be active simultaneously if the following conditions are satisfied.

$$\begin{aligned} g_{i,j}g_{s,d}g_{u,v} - \theta^2\big(g_{i,j}g_{s,v}g_{i,j}g_{u,d} + g_{i,v}g_{s,d}g_{u,j} + g_{i,d}g_{s,j}g_{u,v}\big) \\ - \theta^3\big(g_{i,v}g_{s,j}g_{u,d} + g_{i,d}g_{s,v}g_{u,j}\big) \\ \geq g_{s,d}g_{u,v} - \theta^2 g_{s,v}g_{u,d} + \theta\big(g_{s,j}g_{u,v} + \vartheta g_{u,j}g_{s,v}\big) \\ + \theta\big(g_{u,j}g_{s,d} + \vartheta g_{u,d}g_{s,j}\big) \end{aligned} \tag{30}$$

$$\begin{aligned} g_{i,j}g_{s,d}g_{u,v} - \theta^2\big(g_{i,j}g_{s,v}g_{i,j}g_{u,d} + g_{i,v}g_{s,d}g_{u,j} + g_{i,d}g_{s,j}g_{u,v}\big) \\ - \theta^3\big(g_{i,v}g_{s,j}g_{u,d} + g_{i,d}g_{s,v}g_{u,j}\big) \\ \geq g_{i,j}g_{u,v} - \theta^2 g_{i,v}g_{u,j} + \theta\big(g_{i,d}g_{u,v} + \vartheta g_{u,d}g_{i,v}\big) \\ + \theta\big(g_{u,d}g_{s,d} + \vartheta g_{i,d}g_{s,v}\big) \end{aligned} \tag{31}$$

$$
\begin{aligned}
g_{i,j}g_{s,d}g_{u,v} - \theta^2\big(g_{i,j}g_{s,v}g_{i,j}g_{u,d} + \ & g_{i,v}g_{s,d}g_{u,j} + g_{i,d}g_{s,j}g_{u,v}\big) \\
& - \theta^3\big(g_{i,v}g_{s,j}g_{u,d} + g_{i,d}g_{s,v}g_{u,j}\big) \\
& \geq g_{s,d}g_{i,j} - \theta^2 g_{s,j}g_{i,d} + \theta\big(g_{s,v}g_{i,j} + \vartheta g_{i,v}g_{s,j}\big) \\
& + \theta\big(g_{i,v}g_{s,d} + \vartheta g_{i,d}g_{s,v}\big)
\end{aligned} \tag{32}
$$

Then, we can also define the set of conflicting triples:

$$C = \Big\{\big((i,j),(s,d),(u,v)\big) \in \ A^3 \ \Big|(34) \text{ or } (35)\text{or } (36) \text{ does not hold}\Big\}$$

Therefore, similarly, a valid inequality for link-triples can be added:

$$x_{i,j}^{t,k} + \ x_{s,d}^{t,k} + x_{u,v}^{t,k} \ \leq 2 \qquad \forall\big((i,j),(s,d)\big),(u,v) \in C, \forall t,k \tag{33}$$

5.2 Computing The Maximum Number of Edges That Can Be Simultaneously Active

If the maximum number of edges that can be simultaneously active, we can add this as a new constraint to the *WMN1* model. The formulation below gives the answer of this question:

Assume we have only the routing and power variables. Then, we can maximize the sum of routing variables subject to interference constraints. Additionally, the power variable p does not have time and channel indices, therefore it’s denoted with $p_i \ \forall \ i \in N$

$$\max \sum_{(i,j)\epsilon A} z_{i,j}$$

s.to

$$\sum_{(i,j)\in A} z_{i,j} + \sum_{(s,i)\in A} z_{j,i} \leq 1 \qquad \forall i \in N \tag{34}$$

$$p_i \leq \sum_{(i,j)\in A} z_{i,j} \qquad \forall i \in N \tag{35}$$

$$p_i g_{i,j} + M_{i,j}\left(1 - z_{i,j}\right) \geq \theta\gamma + \sum_{\substack{s=1 \\ s\neq i,j}}^{n} \theta p_s g_{s,j} \quad \forall (i,j) \in A \tag{36}$$

This model is named *MaxEdge* and gives the maximum number of edges that can be simultaneously active, and also a selection of this number of such edges.

5.3 Two Solution Methods

In addition to solving WMN1 directly, two additional methods are proposed using the above formulations. At first, it must be mentioned here that, from now on, *WMN1* refers to the generic MILP formulation with the addition of constraints (22) and (29). *WMNTriples* is obtained with the addition of (33) to *WMN1*.

Additionally, the *MaxEdge* model can be used to generate all non-conflicting subsets of 2^A. This can be obtained with iteratively solving *MaxEdge* with adding the constraint for the selected edges such that:

Let *w* be the objective value of *MaxEdge* at the last iteration. Let the selected edges be (i_α, j_α) for $\alpha \in \{1, \dots, w\}$

Then;

$$\sum_{\alpha \in \{1,\dots,w\}} z_{i_\alpha, j_\alpha} \leq \text{w-1} \tag{37}$$

Solving *MaxEdge* iteratively, until the obtained objective is 1 will generate the all non-conflicting subsets of *A*. Then, with this information, the required confliction constraints can be added to *WMN1* and thus the interference and power management constraints can be omitted. By this way, *WMNMaxEdge* is obtained.

If the maximum number of edges that can be simultaneously active is less than or equal to 3, than iteratively solving *MaxEdge* is unnecessary. Then, with w^* being the maximum number of edges that can be simultaneously active, we can add the following constraint:

$$\sum_{(i,j) \in A} z_{i,j} \leq w^* \tag{38}$$

With the addition of (38) to *WMNTriples* if $w^* = 3$, or to *WMN1* if $w^* \leq 2$ and omitting the interference and power management constraints *WMNMaxEdge* for this case is obtained.

Chapter 6

The Combinatorial Model

The set of all non-conflicting sets can be generated using *MaxEdge.* In our case, since the buffers and buffer capacities of the routers are not taken into account, *when* is not an important question, but *how many times* is the main question that we would like to answer for each possible transmission. Building on this idea, we have developed an MILP model, which allocates an available amount of transmission slots to non-conflicting sets. The single route assumption is preserved in this setup. Since *MaxEdge* model considers the physical interference while generating the sets, no additional power management or interference constraints are required in such an approach.

6.1 The Model

The *MaxEdge* model gives us the information of all non-conflicting subsets of *A.* Then, using this information, we can define the following set:

$$S = \{s \subseteq A | (i,j) \in A \ can \ be \ simultaneously \ active\}$$

Actually, if maximum number of edges to be opened is 1, then $S = A$. If it is 2, then $S = A \cup (A^2 \backslash D)$ and if it is 3, $S = A \cup (A^2 \backslash D) \cup (A^3 \backslash C)$. If it is more than 3, the *MaxEdge* model can be used to generate all other non-conflicting sets.

Then, let x_s denote the number of transmission slots that in which set *s* is active. (This variable is an integer variable, not a binary variable.)

Other variables are same as *WMN1*.

Then, the following model is proposed:

max m

s.to.

$$f_{i,j} \leq c/TK \sum_{s \in S:(i,j) \in s} x_s \qquad \forall (i,j) \in A \tag{39}$$

$$md_i + \sum_{(v,i) \in A} f_{v,i} \leq \sum_{(i,j) \in A} f_{i,j} + \mathrm{a}y_i \qquad \forall i \ \in N \tag{40}$$

$$md_i + \sum_{(v,i) \in A} f_{v,i} \geq \sum_{(i,j) \in A} f_{i,j} + \mathrm{a}y_i \qquad \forall i \ \in N \tag{41}$$

$$\sum_{(i,j) \in E} z_{i,j} = 1\text{-}y_i \qquad \forall i \ \in N \tag{42}$$

$$\sum_{s \in S:(i,j) \in s} x_s \leq \mathrm{TK}\, z_{i,j} \qquad \forall (i,j) \in A \tag{43}$$

$$\sum_{i \in N} y_i = g \qquad \forall i \ \in N \tag{44}$$

$$\sum_{s \in S} x_s = \mathrm{TK} \tag{45}$$

$$f_{i,j} \geq 0, \quad z_{i,j} \geq 0 \;\; \forall (i,j) \in A\,, \quad x_s \in \mathbb{Z}_+ \; \forall s \in S \tag{46}$$

This model decides the number of times that the edges of set *s* are active. The only constraint that can be considered as new is (43). This constraint ensures the connection between the set selection and routing. Additionally, (39) has a minor difference with (8), while having the same function. Finally, (45) states that the number of available transmission slots is TK.

6.2 A Logical Cut

The following inequality ensures that if an edge is on the routing forest, than there is at least one transmission slot in which a set including this edge is active:

$$\sum_{s \in S:(i,j) \in s} x_s \geq z_{i,j} \qquad \forall (i,j) \in A \tag{47}$$

The model with (51) is called *WMNComb.*

To sum up the last two chapters, *WMN*, *WMNTriples*, *WMNMaxEdge* and *WMNComb* models are proposed. Those models are tested with several data structures. The results are discussed in the next chapter.

Chapter 7

Computational Analysis

In the previous two chapters, a generic MILP formulation, an extension to this formulation and a combinatorial MILP formulation is proposed.. To analyze the affectivity of those methods, several computational experiments have been hold. In this chapter, an analysis of the results of those computational experiments will be shared.

In order to test the developed methods, five different network datasets have been used. Four of those data sets are generated according to the real population data of four districts in Turkey: Uskudar, Cankaya, Beyoglu and Fatih. The fifth one is a network on an 8x6 grid topology with randomly generated demand data.

7.1 Data Sets

The proposed methods are tested in five different networks.

Cankaya, Uskudar and 8x6 networks have grid topology while the nodes of Fatih and Beyoglu networks are placed depending on the neighborhoods.

For an illustration, the figures of Cankaya and Uskudar networks can be seen in Appendix 1A and 1B.

The node numbers for Uskudar, Cankaya, Beyoglu, Fatih and 8x6 are 15, 22, 24, 31 and 48 respectively. Therefore, Uskudar network can be considered as a small network, while Cankaya is mid-small, Beyoglu is medium, Fatih is mid-large and 8x6 is a large network. Actually, 8x6 network data is generated to challenge our methods with a very large and difficult problem.

7.2 Parameter Selection

The problem requires some more data parameters besides the place and demand of nodes to be set. SINR threshold value, ambient noise, the number of available channels in a router and possible number of time slots in a frame should be detected.

The SINR threshold value and ambient noise level are determined in order to satisfy connectivity in the network. It is assumed that each router can utilize 4 different radio channels where a time frame can be divided into 8 and 16 time slots. Those assumptions are chosen such that the technological capacities of the routers are not violated.

Since, in the assignment process, the number of transmission slots is the main factor that makes difference, the configurations are named according to the multiplication of time slots and number of channels. This multiplication is called TK number.

For deciding on the number of gateways, the *WMNComb* model is used with a different objective. With fixing the service level to 1, the total number of gateways is minimized and the 1 hr result of this model is used for each data as the number of gateways. Then, the required number of gateways satisfying demand for Uskudar, Cankaya, Beyoglu, Fatih and 8x6 are calculated as 2, 1, 2, 5 and 6 respectively.

The maximum number edges that can be simultaneously active for those networks are 1, 1, 2, 3 and 3 with the same order above. Note that, for the networks with this value less than 3, *WMNTriples* is same as *WMN*.

7.3 Computational Experiments

For each network, *WMN1, WMNTriples, WMNMaxEdge* and *WMNComb* methods are tested with TK values 32 and 64, with gateway numbers as mentioned above. In all of the computational experiments, the time limit is set as 1 hr.

For Uskudar data, all of the methods have found the same solutions. However, while *WMNComb* and *WMNMaxEdge* models prove the optimality of this solution, *WMN* cannot decrease the bound in 1 hour.

For all of the configurations described, the formulations has been solved with Gurobi 5.0.1 on a CPU Intel i7 860(8 threads, 8M cache, 2.8 GHz).

The results of those experiments can be seen in Table 5.1. The detailed results can be seen in Appendix 2.

For Cankaya network, with TK=32, the results are same as the Uskudar. The methods give the same solution, while *WMN* cannot prove its optimality in 1 hr. For TK=64, *WMNComb* and *WMNMaxEdge* have found the same solutions, which is slightly better than the solution of *WMN*, and proven as optimal.

For all of the cases in which the optimality of a solution is proven by both *WMNComb* and *WMNMaxEdge,* the CPU times of combinatorial model is less than *WMNMaxEdge.*

As the size of the network increases, the possibility of obtaining a proven optimal solution in 1-hr decreases. On the other hand, the difference between the performance of the methods become more obvious on the larger networks.

Table 7.1 The Experimental Results Summary

Configuration		WMNComb	WMNMaxEdge	WMNTriples	WMN
Uskudar	TK=32	1.07143	1.07143	1.07143	1.07143
Cankaya		1.25	1	1	1
Beyoglu		1.48515	1.48515	1.46648	1.46648
Fatih		1.25	0.9375	1	0.65217
8x6		1	0.53571	No Sol'n	0.57692
Uskudar	TK=64	1.18421	1.18421	1.18421	1.18421
Cankaya		1.25	1.21323	1.20535	1.20535
Beyoglu		1.75234	1.74419	1.64474	1.64474
Fatih		1.32353	0.57692	0.44776	0.9375
8x6		1.25	No Sol'n	No Sol'n	0.625

For example, in Beyoglu network with TK=32, *WMNComb* and *WMNMaxEdge* models find the same solution in 1-hr. However, the gap with the found best bounds are 1% for *WMNComb* and 25% for *WMNMaxEdge*. The solution of *WMN1* is worse than this solution. While TK=64, the solution found by *WMNComb* is better than the other methods, and it's also the proven optimal solution.

For the Fatih network, the difference between the *WMNComb* and the other methods increases. For TK=64, *WmnComb* finds more than twice of the solutions of other methods. Also in 8x6 network, *WmnComb's* solution is much better than the other methods. Actually, WMNTriples cannot find a solution for 8x6 in 1-hr, and also *WMNMaxEdge* fails to find a solution when TK=64. This is actually interesting, since *WMNMaxEdge* is expected to perform better than *WMN1*. However, in a large network like 8x6 grid network, the computation of non-conflicting triples cause extra complexity, and the number of constrains increase dramatically. This, most probably, is the cause why *WMNMaxEdge* fails to find a solution for this large network.

Another interesting point is that, with increasing TK value, *WMNComb* becomes slightly easier, while this change decreases the performance of other methods. The reason for that is the fact that, number of variables in *WMNComb* is independent of the other parameters except the number of non-conflicting sets. However, the number of variables increases in other methods when TK value is increased. For example, in 8x6 network, there are 3040 non-conflicting edge sets, including the single edges. The cardinality of edge set is 164. If we consider the *x* variables as analogous to each other between the two main MILP formulations, in *WMN1*, there are 5248 *x* variables when TK=32, and when TK duplicates, this value also duplicates. However, the number of *x* variables in *WMNComb* remains as 3040, and the limitations by the constraints relax slightly since the TK value increases.

To sum up, in all of the instances, *WmnComb* finds the best solution. The other methods could not beat combinatorial model in any of the instances. Even if same solutions could be reached, either solution times or the gaps indicate the superiority of combinatorial model.

Chapter 8

Conclusion

This study covers the planning problems of WMNs. WMNs provide fast and cost efficient network solutions. Since they do not require a complicated wired infra-structure, WMNs are easy to deploy.

In the first chapter, general characteristics of WMNs are defined and described. A brief summary of the literature about WMN planning problems is presented. Then our problem is defined as jointly making the routing, gateway selection, transmission slot assignment and power management decisions. In literature, most of the studies focus on only some part of those decisions but do not address them jointly.

For the solution of WMN Planning problem, at first, a generic MILP model, *WMN1*, is developed. Since this MILP formulation is computationally inefficient even in small instances, different approaches are proposed. First, two sets of constraints are added into *WMN1* to improve the performance of the model. Then, a third inequality is added, and with this addition, *WMNTriples* model is defined An easier MILP formulation is developed to check the maximum number of edges that can be simultaneously active, and generate all non-conflicting subsets of edge set A. By using this information, two different methods are proposed, *WMNMaxEdge* and *WMNComb.*

Our proposed methods are tested on 5 different networks. One of those networks is exteremely large and used for challenging our methods with such a case. The comparison of the experimental runs showed that the combinatorial model, *WMNComb,* performs better that the other models. In smaller instances, this can be observed in the solution times, and in larger instances, this fact is obvious on the 1-hr solutions of the methods. For small networks in which *WMN1* cannot prove the optimality of a solution in 1-hr, *WMNComb* solves to optimality in less than one second, and *WMNMaxEdge* solves the problem, though not as fast as *WMNComb.* When the data size become larger, *WMNMaxEdge* also become inefficient, even worse than *WMN*. However, in such cases, *WMNComb* gives much better solutions in 1-hr. Therefore, to conclude, *WMNComb* model is proposed as the main solution method presented in this thesis.

The *WMN1* model is actually not a new formulation for the literature. It is quite similar to Uzunlar's model in [23]. On the other hand, the solution methods based on this model, that is *WMNMaxEdge* is an important contribution of this thesis, along with the *MaxEdge* model. On the other hand, the main contribution of this work is the *WMNComb* model with its quite well performance in both small and large networks.

As a possible extension to this work, meta-heuristics for the WMN-Planning problem can be developed. As it is mentioned in Chapter 3, there are very few studies in this area. Additionally, considering the *WMNComb* model, column generation based approachs can be a decent extension to this thesis.

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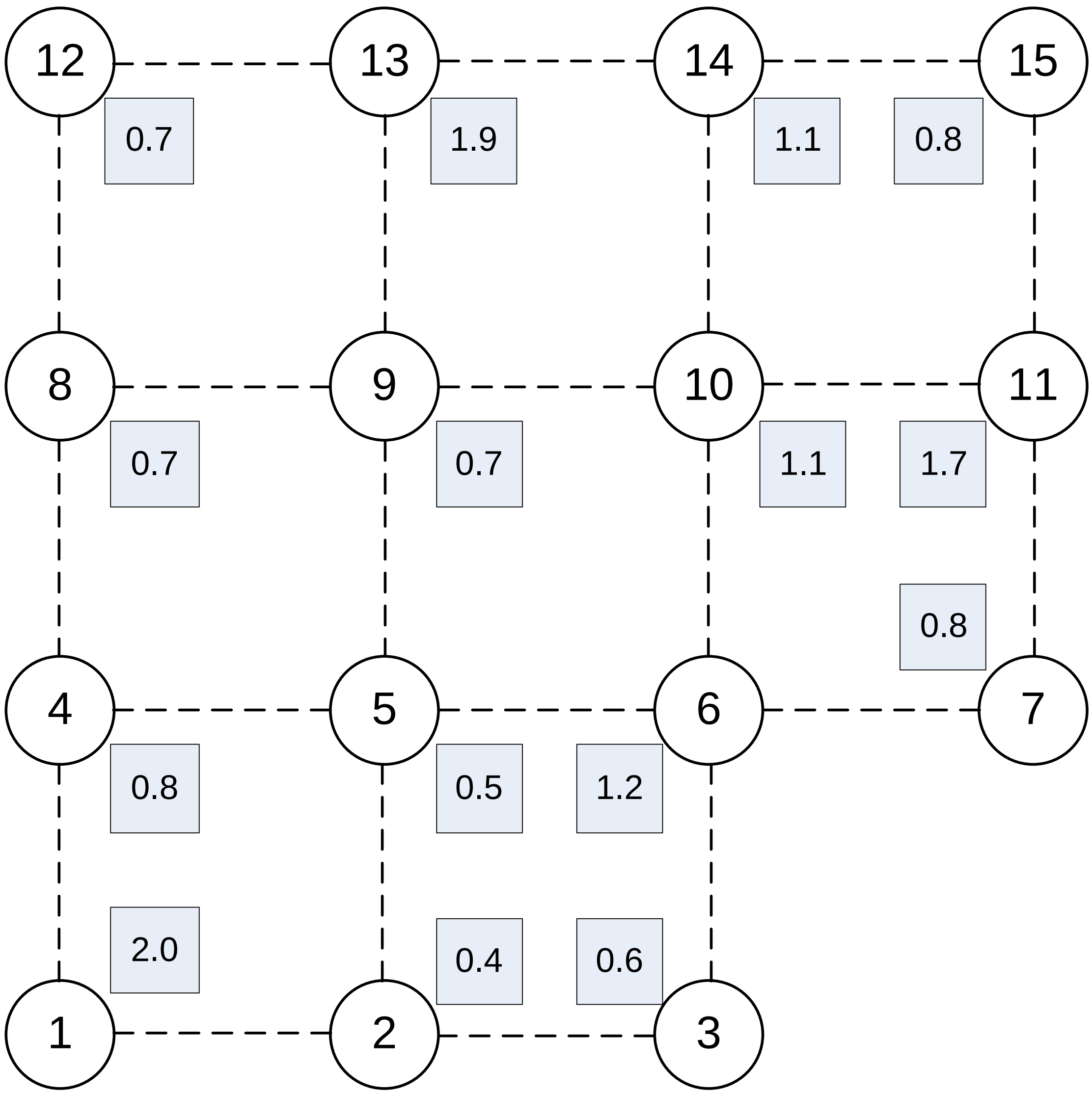
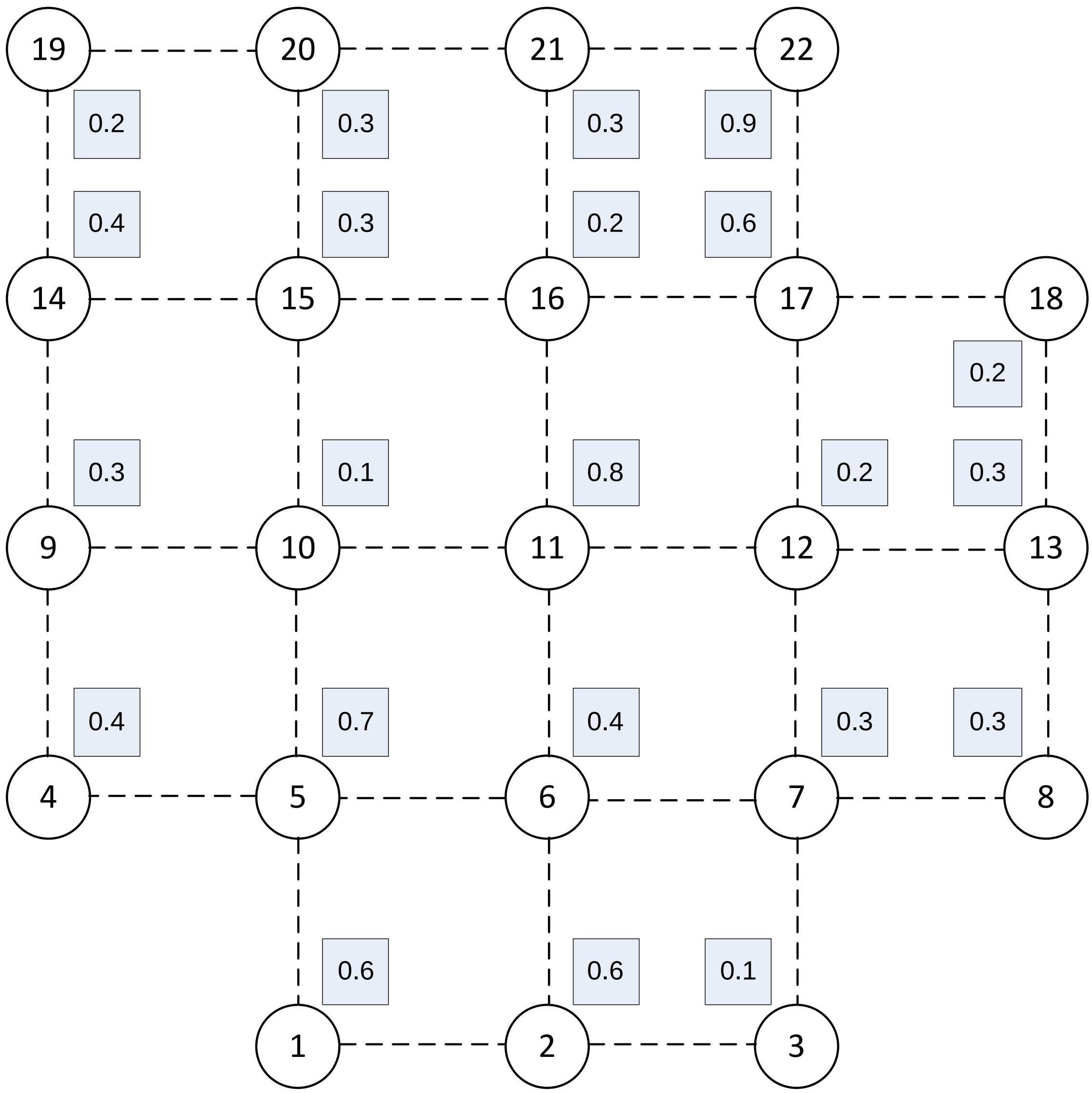
APPENDIX

APPENDIX 1A:Uskudar Network



The values on the nodes denote its demand.

APPENDIX 1B: Cankaya Network



The values on the nodes denote its demand.

APPENDIX 2: The Results of Computational Experiments

Configuration			WMNComb			
Data	TK	Gw	Sol'n	Bound	Gap	Time
Uskudar	32	2	1.07143	1.07143	0%	0
Çankaya	32	1	1	1	0%	1
Beyoğlu	32	2	1.48515	1.5	1%	3600
Fatih	32	5	1.25	1.33721	6.98%	3600
8x6	32	6	1	2.52699	152.70%	3600
Uskudar	64	2	1.18421	1.18421	0%	0
Çankaya	64	1	1.21323	1.21323	0%	3
Beyoğlu	64	2	1.75234	1.75234	0%	149
Fatih	64	5	1.32353	1.46242	10.49%	3600
8x6	64	6	1.25	2.53825	102.87%	3600

Configuration			WMNMaxEdge			
Data	TK	Gw	Sol'n	Bound	Gap	Time
Uskudar	32	2	1.07143	1.07143	0.00%	33
Çankaya	32	1	1	1	0.00%	137
Beyoğlu	32	2	1.48515	1.85524	24.92%	3600
Fatih	32	5	0.9375	4.39579	368.88%	3600
8x6	32	6	0.53571	6.64465	1140.34%	3600
Uskudar	64	2	1.18421	1.18421	0.00%	100
Çankaya	64	1	1.21323	1.21323	0.00%	1289
Beyoğlu	64	2	1.74419	2.08649	19.62%	3600
Fatih	64	5	0.57692	4.21238	630.14%	3600
8x6	64	6		No Sol'n		3600

Configuration			WMNTriples			
Data	**TK**	**Gw**	**Sol'n**	**Bound**	**Gap**	**Time**
Uskudar	32	2	1.07143	6	460.00%	3600
Çankaya	32	1	1	1.15772	15.77%	3600
Beyoğlu	32	2	1.46648	1.83349	25.03%	3600
Fatih	32	5	1	4.09174	309.17%	3600
8x6	32	6		No Sol'n		3600
Uskudar	64	2	1.18421	6	406.67%	3600
Çankaya	64	1	1.20535	1.29972	7.83%	3600
Beyoğlu	64	2	1.64474	4.45158	170.66%	3600
Fatih	64	5	0.44776	4.64022	936.32%	3600
8x6	64	6		No Sol'n		3600

Configuration			WMN			
Data	**TK**	**Gw**	**Sol'n**	**Bound**	**Gap**	**Time**
Uskudar	32	2	1.07143	6	460.00%	3600
Çankaya	32	1	1	1.15772	15.77%	3600
Beyoğlu	32	2	1.46648	1.83349	25.03%	3600
Fatih	32	5	0.65217	4.37085	570.20%	3600
8x6	32	6	0.57692	6.88335	1093.11%	3600
Uskudar	64	2	1.18421	6	406.67%	3600
Çankaya	64	1	1.20535	1.29972	7.83%	3600
Beyoğlu	64	2	1.64474	4.45158	170.66%	3600
Fatih	64	5	0.9375	3.82677	308.19%	3600
8x6	64	6	0.625	6.35579	916.92%	3600