APPLICATION OF A MULTI PERIOD MULTI RETAILER PRICE DECISION AND INVENTORY ALLOCATION MODEL

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ABSTRACT

APPLICATION OF A MULTI PERIOD MULTI RETAILER PRICE DECISION AND INVENTORY ALLOCATION MODEL

Effective dynamic pricing strategies and optimal inventory allocation have emerged as crucial drivers for operational success for vendor-retailer systems under supply chain constraints and economic uncertainties. Joint problems optimizing profit have been extensively studied in the literature while a gap remains for applications within a unified model with retailer specific warehouse to retailer lead times, lost sales costs as a cost component and inventory capacity levels. In this thesis, an existing multi period multi retailer profit optimization model is extended with these considerations. Real FMCG company data are calibrated to perform an industry application on the extended model leveraging their distributor sales channel's relevant practices and operational advantages for dynamic strategies. Empirical analysis using company data reveals that retailer capacity constraints act as a limiting factor under increasing transportation costs, with profit impacts ranging up to 1.01%, depending on supply chain cost ratios. The impact of fully dynamic pricing strategy on total profit is found to be up to 1.61%. Experimental results using synthetic data indicate that increased unit lost sales costs and longer lead times correlate to broader range of prices and elevated prices during initial frozen periods. Total profit is seen to be decreased under higher unit lost sales costs. Results indicate that straightforward comparison of total profit among different lead times is not straightforward due to complex trade-offs between model parameters. Nevertheless, insufficient initial retailer inventories result in lower total profit for longer lead times as longer lead times make it difficult for the model to compensate lost sales. Longer lead times require earlier allocation decisions, potentially impacting profitability in dynamic business environments. Model extensions significantly influence pricing and allocation decisions during frozen periods, suggesting that flexible systems can effectively compensate total profit through short-term adjustments rather than wholesale strategy revisions.

ÖZET

ÇOK PERİYOTLU ÇOK PERAKENDECİLİ FİYAT KARAR VE ENVANTER TAHSİS MODELİNİN UYGULAMASI

Tedarik zinciri kısıtlamaları ve ekonomik belirsizlikler altında, etkili dinamik fiyatlandırma stratejileri ve optimal envanter tahsisi, tedarikçi-perakendeci sistemleri için operasyonel başarının kritik belirleyicileri olarak öne çıkmıştır. Kâr optimizasyonu yapan ortak problemler literatürde kapsamlı bir şekilde incelenmiş olsa da perakende noktası bazlı teslim süreleri, kar fonksiyonundaki kayıp satış maliyetleri ve envanter kapasite seviyeleri içeren bütünleşik bir model ile uygulamalar için bir boşluk bulunmaktadır. Bu tezde, mevcut çok dönemli çok perakendecili kar optimizasyonu modeli bu hususlar dikkate alınarak genişletilmiştir. Dinamik stratejiler için distribütör satış kanalının operasyonel avantajlarından yararlanılarak genişletilmiş model üzerinde uygulamalı bir çalışma yapmak için gerçek hızlı tüketim şirketi verileri kalibre edilmiştir. Şirket verileri kullanılarak yapılan ampirik analiz, perakendeci kapasite kısıtlamalarının artan nakliye maliyetleri altında sınırlayıcı bir faktör olarak hareket ettiğini ve tedarik zinciri maliyet oranlarına bağlı olarak kâr etkilerinin 1.01%'e kadar ulaştığını ortaya koymaktadır. Tam dinamik fiyatlandırma stratejisinin toplam kâr üzerindeki etkisinin 1.61%'e kadar çıktığı tespit edilmiştir. Sentetik veri deneyleri, artan birim kayıp satış maliyetleri ve uzun teslim sürelerinin, başlangıçtaki donmuş dönemlerde daha geniş fiyat aralıkları ve yüksek fiyatlandırma ile ilişkili olduğunu göstermektedir. Toplam kârın, daha yüksek birim kayıp satış maliyetleri altında azaldığı görülmektedir. Sonuçlar, farklı teslim süreleri arasında toplam kârın karşılaştırılmasının model parametreleri nedeniyle doğrudan olamayacağını gösterirken, yetersiz başlangıç perakendeci stoklarının daha uzun teslim süreleri için daha düşük toplam kâra neden olduğunu göstermektedir. Uzun teslim süreleri erken tahsis kararları gerektirmekte ve dinamik iş ortamlarında kârlılığı etkileme potansiyeline sahiptir. Model uzantıları, donmuş dönemlerde fiyatlandırma ve tahsis kararlarını önemli ölçüde etkilemekte, bu da esnek sistemlerin toplam karı tüm stratejiyi revize etmek yerine kısa vadeli ayarlamalar yoluyla etkin bir şekilde telafi edebileceğini göstermektedir.

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a_{st}	Potential demand for retailer *s* in period *t*
b_{st}	Price elasticity of retailer *s* in period *t*
c_s	Inventory capacity of retailer *s*
cf_s	Critical fractile for retailer *s*
$D_{st}(P_t)$	Demand for the product at retailer *s* in period *t* with price P_t
h_0	Per period per unit inventory holding cost at the warehouse
h_s	Per period per unit inventory holding cost at retailer *s*
$I0_s$	Initial inventory level at retailer *s*
I_{st}	Inventory level at retailer *s* at the end of period *t*
l_s	Lead time from the warehouse to retailer *s*
ls_s	Per period per unit lost sales cost for retailer *s*
P_t	Selling price in period *t*
$\overline{p}_t$	Upper bound on price in period t
q	Initial inventory quantity at the warehouse
S	Set of retailers
T	Set of periods
U_{st}	Amount of products allocated to retailer *s* in period *t* which will arrive to retailer *s* in period (t+l_s)
w_{st}	Per unit transportation cost from the warehouse to retailer s in period *t*
WI_t	Inventory level at the warehouse at the end of period *t*
Y_{st}	Amount of products sold at retailer *s* in period *t*

LIST OF ACRONYMS/ABBREVIATIONS

DP	Dynamic Programming
EMPOP	Extended Multi Period Profit Optimization Problem
FMCG	Fast Moving Consumer Goods
LP	Linear Programming
MILP	Mixed Integer Linear Programming
MPOP	Multi Period Profit Optimization Problem
NLP	Nonlinear Programming
OWMR	One Warehouse and Multiple Retailers
QCP	Quadratically Constrained Problem
S&OP	Sales and Operations Planning
SCC	Supply Chain Costs
SP	Stochastic Programming
VMI	Vendor Managed Inventory

1. INTRODUCTION

In today's volatile retail environment, dynamic pricing and inventory allocation have become critical determinants of business success, especially as companies face unprecedented supply chain challenges and economic pressures. Dynamically adjusting the prices allows firms to adjust demand for their products which is highly required in competitive industries with a requirement of dynamic strategies such as activity promotions etc. Optimizing inventory levels in each stage of an end-to-end system became highly important due to increased supply chain costs (SCC) and high interest rates, with excess inventory becoming increasingly undesirable as companies prioritize cash liquidity. Rising SCC necessitate more frequent dynamic pricing adjustments, particularly in high-inflation economies, as exemplified by Turkey's recent economic climate. With the evolution of technology, information flow and data visibility in each stage of supply chain has increased rapidly. This enabled businesses to adapt more flexible systems for their supply chain and sales operations. These developments have contributed to an expansion in research focusing on joint pricing and allocation problems and their practical applications.

Dynamic pricing and allocation systems can be adapted more easily and efficiently thanks to the opportunity of instant data viewing of sales and inventory levels through information sharing. Important examples are present for joint supply chain partnerships between manufacturing companies and their retailer customers, such as the partnership between manufacturing P&G and Walmart [1]. Even though successful examples are valid, enhancement of joint systems remains an evolving challenge as companies follow the agenda of adaptation. Certain manufacturer-retail configurations present particularly suitable frameworks for efficient system adaptations. Mostly observed in fast moving consumer goods (FMCG) companies, distributors as sales channel also feature for such adaptation. As well as having different properties across industries and companies, some operational structures combine them as a direct extension of the company itself, directly positive affecting total revenue and profit coming from sales to their own retail customers and negative effect from inventory costs. Manufacturing company has the authority to execute inventory levels, orders and allocations for the distributors at these versions, which makes the system relevant for the discussed joint problem.

The motivation of this study comes from an FMCG company which has distributor sales channel setting with the properties mentioned. The company puts even higher emphasis to optimize their distributors channel inventory execution and sales operations in the recent times, driven by increasing sales volumes and continuous expansion of the distributor network. Each distributor has its own properties considering their locations, market profiles and price elasticities. Overall, the negative relation between prices and sales volume is clearly observable. The company has responsible people who monitor sales and inventory levels of the distributors and execute order fulfillment operations accordingly. Distributors' target sales volumes are assigned priorly based on growth volumes, historical sales data and supply constraints. Given the constraints on total available inventory across the whole sales channel, optimal allocation among distributors becomes critical for achieving demand balance and maximizing profit utilization from available inventory. A very recent study completed by Tunçinan et al. provides a total profit optimization problem that addresses this type of a problem under one retailer multi retailer (OWMR) settings, primarily focused on manufacturing companies and their own retailers [2]. The selected FMCG company shows relevant features with its distributor sales channel to be adapted to such a model for an industry application.

Industry applications of proposed mathematical models are considered to be more realistic and successful as they include the majority of the necessary constraints and parameters that may affect the decision variables and the objective function of the model. While existing studies, including the main motivation study mentioned above, have made significant contributions to dynamic pricing and allocation problems, there remains a gap in addressing other real-world constraints such as lost sales costs, positive shipment lead times, and retailer inventory capacities in an integrated model, particularly crucial in the FMCG sector. These three parameters are mostly observed to be ignored for the complexity they create in the mathematical models along with their in-effectiveness on the results for the objective of the corresponding studies. Still, there are also valid examples who contain these constraints in their model settings in separate ways.

FMCG sector is a highly competitive sector, and sales lost due to inventory shortages are considered to be beyond immediate single revenue impacts. Customers' future behaviors (order frequency, order volumes, promotional activity participation) along with their reliance on the manufacturing company heavily depends on the service provided to them. Consequently, sales volume targets across distribution channels are calibrated based on inventory availability during the planning period. Due to these reasons, addition of a unit lost sales cost to the model framework is highly required. As retailers are heterogenous both based on their locations and customer demand profiles, unit lost sales cost needs to be specific for each retailer. A total cost component needs to be added to the objective function of the model to reflect the negative impact of a sales lost. Lead times and retailer capacity parameters are ignored by even a higher majority of the studies in the literature. Still, in order to build realistic and comprehensive mathematical models, these two constraints are also needed to be added in the allocation and inventory level equations in the model formulation.

Calibration of real-company data for a relevant industry application is observed in many existing studies. Historical data usage is highly prevalent for building demand functions, deriving potential demand and price elasticity parameters by using past pricing and sales data. Companies also apply statistical regressions as a base input for their future sales forecasts, while modifying the raw outputs based on other parameters such as growth rate and market share. Certain parameters, including retailer holding costs and unit lost sales costs, require indirect derivation through alternative parameter inputs and formulations. Data generation process requires search of literature studies with relevant applications and deep knowledge for the practice of the operations for the related industry and company. Usage of real data in such applications is also relevant when it comes to assessing the results of the model. Real data can be more accurate for reflecting the ratios between parameters, cost increases and so on. Therefore, results may be more reliable when it comes to analyze the results and the behavior of the model. While this is valid, synthetic data generation remains highly necessary for examining the general behavior of models. Synthetically created data allows experimenters to build environments where the investigated parameters are observed while other parameters stay consistent. This allows efficient observations under different levels of a changing parameter.

This research study addresses these practical challenges by developing and performing applications of a multi period multi retailer profit optimization model. The study has three main objectives: extending an existing mathematical optimization model for joint dynamic pricing and allocation by incorporating realistic constraints and parameters including lost sales costs, positive lead times, and retailer capacity constraints, performing an industry application by calibrating real-company data to validate the model's practical applicability and effectiveness in a real-world FMCG setting, and analyzing the extended model's behavior under various parameter scenarios by using synthetic data instances. All results are aimed to be used as insights for the industry and future applications of the model.

The outline of the paper is as follows. Chapter 2 provides comprehensive literature review for the related topics. Chapter 3 first provides the details regarding on the experiment environment, then presents the model with assumptions and notations for its sets, parameters and decision variables. Chapter 4 first provides the details for data calibration process and application of the model, then presents the results discussion for both the industry application and the observations under synthetic data instances for model behavior and insights. Finally, Chapter 5 concludes the overall study and outlines possible future research recommendations.

2. LITERATURE REVIEW

Literature review is completed under four main titles for the related topics in this thesis, while some of the literature is found out to be covering multiple titles. Studies handling settings with lost sales costs, lead times and retailer capacities are searched in a detailed way due to the objective of this thesis. Additional importance is given to retail (and specifically FMCG) sector specific industry applications and their model settings. Literature review was also helpful to identify further topics for future work.

2.1. Dynamic Pricing

Dynamic pricing has always been a popular topic in literature for revenue management, operations management field and has wide range of sub-topics under its roof. Dynamic pricing due to competition or strategies under inventory capacitated scenarios are some examples. On the other hand, business challenges to apply full dynamic pricing has frequently been reflected in the literature. It is observed that predominantly complex mathematical models such as Nonlinear Programming (NLP), Stochastic Programming (SP), and Dynamic Programming (DP) are used. Linear Programming (LP) and Mixed Integer Linear Programming (MILP) models are often used as components of larger solution methods or in simplified scenarios. Gallego and Ryzin have one of the foundations for dynamic pricing with finite inventory using stochastic dynamic modelling [3]. Bitran and Caldentey have a highly comprehensive survey on dynamic pricing policies and their relation to revenue management [4]. They provide NLP models for both stochastic and deterministic price sensitive demand processes over a finite period of time for both multiple set of products and single type of product. Chen et al. point out static pricing policies occurring under operational constraints and improve a heuristic to address the challenges for dynamic pricing [5]. They require one single optimization at the beginning of the selling period for only small size of number of products with an SP model. Ma et al. provide an NLP while taking into account the consumer behavior while dynamically adjusting prices [6]. Lei et al. conduct a single-product revenue maximization problem with inventory constraints and for noisy demand function. However, they choose the price as non-parametric [7]. This study can be beneficial in some ways where there are more constraints on price selection due to business constraints. Zhang and Lu are among the researchers who indicate these constraints to adapt fully dynamic price adjustments [8]. They apply computational studies to assess dynamic pricing using NLP models.

Dynamic pricing can be an efficient lever for various strategies in FMCG (or overall retail) industry. Cohen et al. formulate a promotion optimization problem as an NLP while taking into account business promotion and pricing constraints [9]. They propose a general demand function which includes post-promotion sales affect that can be estimated from historic sales data. Historical sales data usage is also applied in this thesis to generate the linear demand functions for each retailer in each period. Raju et al. study the effect of brand loyalty and stated that promotion responses increase with less-loyal consumer profiles [10]. This result supports the significance of the topic of this thesis in terms of real-life applications of dynamic pricing. Caro and Gallien provide a study on mark-down pricing optimization problem that is a prevalent problem especially observed in fast fashion brands [11]. They formulate a MILP with approximations.

There are relevant studies completed which indicate the efficiency of linear models for dynamic pricing for revenue maximizing. Besbes and Zeevi consider a multiperiod single product pricing problem with an unknown demand curve with a simple linear model and showed its efficiency [12]. A good review paper for different type of demand functions including linear demand function is provided by Chenavaz et al [13]. Chen et al. also use stochastic approximation with linear demand in real-time pricing scenarios [5]. Li and Huh compare linear and logit demand models and show that linear demand can be a good approximation for more complex models [14]. Perakis and Sood highlight that linear demand models perform well in competitive, multi-period settings for perishable goods which is relevant for FMCG sector [15]. These studies were mostly searched in order to assess the effectiveness of the linear demand function selection on the results.

2.2. Joint Inventory and Pricing Problems

Federgruen and Heching have one of the oldest studies on simultaneous determination of pricing and inventory replenishment strategies in the face of demand uncertainty under single item and periodic review with price-dependent demand [16]. Their study is applicable to FMCG sector due to products requiring high frequency of replenishment. Chen and Simchi-Levi provide finite horizon, single product, periodic review model in which pricing and production/inventory decisions are made simultaneously [17]. Demand is dependent to price, there are both additive and multiplicative demand functions presented. Aydin and Porteus has set an assortment problem under Newsvendor setting for joint inventory level and pricing problem for multiple products [18]. Adida and Perakis show the effectiveness of linear demand on price in handling uncertainty for joint pricing and inventory control problems [19]. Chen and Simchi-Levi also provide a comprehensive survey on integrated production/inventory and pricing models with deterministic demand functions including linear demand function depending on price [20]. Yin and Rajaram consider the joint pricing and inventory control problem for a single product over a finite horizon and with periodic review [21]. Their model is applicable to seasonal products, changing economic conditions in a fluctuating demand environment which is relevant for FMCG sector. Chew et al. study a MILP for joint problem with time and price dependent deterministic demand [22]. Deng et al. consider a joint pricing and inventory problem with promotion constrains over a finite planning horizon for a single fastmoving consumer good under monopolistic environment [23]. Problem turns out to be a quadratic programming problem with reference-dependent demand function. Smith and Agraval present a joint inventory and pricing problem under inventory-dependent demand for the application of markdown pricing [24].

Another important point to stress is that most of the relevant and important literature assume the lead times in joint problems as zero, additional examples can be given as [25] and [26]. It is assumed to be like this due to complexity it creates in demand functions and its in-effectiveness on the corresponding literature's main study. However, for realistic applications as in this thesis, lead times need to be taken into account. Marand et al. consider a joint problem while taking positive lead times in an inventory system [27]. Chen et al. take into account with positive lead times for a single-stage inventory system however they show that their analysis can be extended to serial inventory systems [28]. Pang et al. also highlight that the majority of the literature assuming lead time as zero in joint problems and sets a dynamic pricing algorithm with positive lead times in set {0,1,2} [29].

2.3. Lost Sales Cost

Lost sales assumption is frequently observed in the inventory optimization, profit maximization and joint problems. However, not all studies include lost sale as part of the profit function as a cost component. One of the foundational studies is completed by Dada and Petruzzi where they extend the Newsvendor problem by inventory quantity and selling price are set simultaneously [30]. Literature is searched to investigate different type of problems that are related with lost sales. Güllü and Erkip have lost sales component differing in each period and same for all retailers for their two-echelon inventory optimization problem [31]. Dekker et al. propose an inventory model with several demand classes, prioritized according to importance and different critical levels for inventory policy [32]. Service level is presented as a constraint in their formulation for estimating penalty costs. Their study is a significant one for different lost sale costs being assigned to different retailers based on their prioritization. Jha and Shanker state that it is more complex to have a cost component in the profit function in OWMR settings, therefore they have service level constraints only as the representer of the issue [33]. Huh et al. study a single-product single-location inventory system under periodic review, where excess demand is lost, and the replenishment lead time is positive [34]. They investigate the results depending on the lost sales penalty cost/holding costs ratio. A very recent computational study done by Kouki et al. focus on a two-echelon system with network lost sales cost continuous review base-inventory policy [35]. They work with Poisson distributed uncertain demand. Bijvank and Johansen study a periodic review lost-sales model in terms of different lead times set [36].

Other main field for the lost sales studies is the pricing models to adjust the demands to mitigate lost sales, which also mostly include lost sales in the cost component of the profit function. Elmaghraby and Keskinocak provide a comprehensive survey on dynamic pricing strategies, including their use in managing demand when facing capacity constraints [37]. Bitran and Caldentey's study also touches the dynamic pricing policies from demand adjustment point of view [4]. Netessine and Shumsky study a yield management pricing problem with penalty costs [38]. Transchel and Minner study dynamic pricing under economic order decision setting to balance inventory costs and lost sales [39]. Chen et al. study periodic-review pricing and inventory control problem for a retailer with the cost component including the shortage costs [40]. They use a linear and price sensitive demand function which is important to show that lost sales studies can be completed under linear demand unlike the prevalently used function Poisson. Park et al. construct a MILP model for the vendor-managed inventory routing problem with lost sales, while maximizing the supply chain profit over a planning horizon under two-echelon setting [41]. Daniel and Chandrasekharan propose a genetic algorithm for the inventory routing problem with lost sales under a vendor-managed inventory strategy in a two-echelon supply chain [42].

2.4. OWMR Settings

It is observed that both dynamic pricing, or joint pricing and inventory problems can have different settings such as suppliers included, production capacities included or the presence of multiple vendors in a system. Multi echelon and one warehouse-multi retailer settings are often explored within supply chain management, operations planning, and economics literature due to their relevance in optimizing inventory and distribution strategies. Gallego and Simchi-Levi set an OWMR setting to discuss the direct shipment efficiency in overall inventory-routing strategies [43]. Axsäter and Marklund propose a continuous review for a two-echelon inventory system with one central warehouse and several nonidentical retailers [44]. Their policy allows to determine the expected total inventory holding and backorder costs for the entire system. Chen et al. have one of the fundamental and comprehensive studies addressing two-echelon distribution systems with a central vendor (supplier) and non-identical retailers for a single good [45]. They build and optimal strategy for maximizing total systemwide profits with discussing the impact of information sharing in a supply chain network. Güllü and Erkip handle the OWMR setting for optimal inventory allocation policies under cost components like shortage, shipment costs etc. [31]. Ryan et al. study a joint pricing and ordering problem in a similar setting with a single manufacturer and multiple retailers for a single good [46]. They choose linear demands as function of the prices for each retailer. Adeinat et al. propose a MILP model that maximizes the total profit per time unit for geographically dispersed retailers [47]. Differing than the assumptions in this thesis, their study allows for different retailers to have different pricing decisions. One of the comprehensive studies on OWMR settings which is also relevant to this paper’s application is completed by Lei et al. [48]. Their OWMR setting problem consists of a joint inventory and pricing problem with a one-time decision about the initial inventory on the inventory at the warehouse at the beginning of the selling season.

Their profit function also contains lost sales cost component; however, they assume the unit lost sales to be identical for all retailers which is one of the differing aspects of this paper's model.

Literature includes various studies including retailer capacities mostly as an extension in OWMR problems. Gayon et al. propose approximation algorithms for NP hard problem and extensions including capacity constraints at some retailers [49]. Hariga et al. set a vendor managed inventory system (VMI) which allows the supplier to place order on behalf of its customers and provide a cost-efficient heuristic and with storage capacity set before setting a contract with the retailer [50]. This way of work is exactly aligned with the company's way of work in this thesis. They highlight that greater profit is achieved when variability increased in retailers' demand and cost parameters. They extend their work by working on a similar VMI setting which allows the unequal frequency of shipments to retailers with again a storage capacity set as the maximum level of inventory in a retailer [51]. Ghiami et al. consider a two-echelon system with capacity constraints for both the warehouse and the retailer's warehouse but lets the retailer rent additional warehouse with additional cost [52]. This case is not applicable in this thesis' practice but can be an efficient perspective for settings that allow this choice. Federgruen et al. handle a two-echelon distribution system with capacity constraints on both the warehouse (depot) and the retailers [53]. They also consider specific lead times, but their construction of system is a two-stage system where the warehouse also gets to be replenished. Apornak sets a two-echelon supply chain with a single retailer with the considerations of capacity and lead time. He chooses the demand function as a linear price dependent function however he also adds the lead time as a factor to the demand function [54]. Ahmadi et al. worked on an OWMR setting where there are non-identical retailers with non-identical capacities under lost sale assumption [55]. Their demand function is normally distributed for all retailers. Their on-hand inventory at the beginning is chosen as a decision variable in each period which is not relevant with this thesis however retailer-based inventory capacities is a relevant point.

Table 2.1. Main studies on OWMR Settings.

Author	Model Method	Objective Function	Demand Function	Product (Single/ Multiple)	Lost Sales Cost Included	Positive Lead Times	Retailer Capacities
Lei et al. [48]	NLP	Maximize total profit	Price-dependent exponential	Single	Yes	No	No
Tunçinan et al. [2]	NLP,MILP	Maximize total profit	Price-dependent linear	Single	No	No	No
Chen et al. [40]	NLP	Maximize long-run average profit	General stochastic	Single	No	Yes	No
Chen and Simchi-Levi. [20]	NLP	Maximize total expected profit	Price-dependent linear	Multiple	No	No	No
Kouki et al. [35]	SP	Minimize total cost	Poisson	Single	Yes	Yes	No
Ahmadi et al. [55]	SP	Minimize total cost	Normal	Single	Yes	No	Yes
Federgruen et al. [53]	SP	Minimize expected total cost	Normal	Single	No	Yes	Yes
Aydin and Porteus [18]	NLP	Maximize expected profit	Price-dependent stochastic	Multiple	Yes	No	No
Gayon et al. [49]	MILP	Minimize total cost	Time-varying deterministic	Single	No	No	Yes
Cohen et al. [9]	NLP	Maximize profit from promotions	Price-dependent general non-linear	Multiple	No	No	No
Hariga et al. [51]	MILP	Minimize total cost	Time-varying deterministic	Single	No	Yes	Yes
Ghiami et al. [52]	NLP	Minimize total cost	Inventory-dependent linear	Single	No	No	Yes
Jha and Shanker. [33]	NLP	Minimize total cost	Normal	Single	No	Yes	No
Ryan et al. [46]	NLP	Maximize total profit	Linear	Single	No	No	No
Axsäter and Marklund [44]	SP	Minimize total cost	Poisson	Single	Yes	Yes	No
Chen et al. [45]	NLP	Maximize total profit	Stationary stochastic	Single	No	No	No
Gallego and Simchi-Levi. [43]	LP	Minimize total distribution costs	Stationary stochastic	Single	No	No	No
Besbes and Zeevi [12]	LP	Maximize revenue	Linear	Single	No	No	No
This Thesis	**NLP**	**Maximize total profit**	**Linear**	**Single**	**Yes**	**Yes**	**Yes**

Lead times can be assumed as zero in OWMR setting problems also but there are increasing number of examples with positive lead times, examples provided in the previous parts. Ganeshan has one of the foundational studies on a OWMR inventory optimization problem with positive lead times [56]. He considers three lead time components: a constant order processing time at each retailer, waiting time at the warehouse (if out of inventory), and the transit time from the warehouse to the retailer which is assumed to be a random variable in his study. This approach is quite realistic, distinguishing them from simple transportation durations that they are often conflated with in practice.

The literature review reveals several important gaps in existing literature that this thesis addresses. Properties of main studies on OWMR settings is summarized in Table 2.1. First, while OWMR settings have been extensively studied, most existing models either assume identical lost sales costs across retailers or handle lost sales through service level constraints rather than cost components. Second, though lead times are crucial in real-world applications, many joint pricing and inventory studies assume zero lead times in order to reduce model complexity. Third, while retailer capacity constraints have been considered in some OWMR studies, few works combine these constraints with both differentiated lost sales costs and positive lead times in a price-optimization context. The literature was helpful to both identify the gap and build an integrated model for an industry application. It was put out by the numerous studies that linear and price-sensitive demand function was a relevant choice for such application. Various studies on these topics provided insights for the application model setting that fits an FMCG company operations in the best way. Some literature discussed above was also the base for using the real company data for generating the variable functions and parameters.

3. MODEL FRAMEWORK

In this section, relevant operational practices of industry and the company are presented to provide a general framework for the model. Operations that are in the scope of the handled optimization problem, such as sales and logistics, are discussed in a more detailed way especially for the selected sales channel, stressing why this problem is applicable to the model in this thesis. The mathematical model is presented next explaining the model settings and this paper's extensions in a detailed way.

3.1. Business Environment

The joint pricing and allocation problem has applications across various retail settings, but its relevance and implementation vary significantly depending on the industry. The FMCG industry presents a particularly suitable context for this problem due to its unique characteristics: rapid inventory replenishments, complex distribution networks, and dynamic pricing requirements. In this sector, products are primarily sold through retail stores or e-commerce platforms rather than directly to consumers, necessitating efficient replenishment systems and strategic pricing decisions [23]. Retailer customers are mostly grouped among themselves considering their market dynamics, market sizes and consumer profiles. Examples can be given as Discounter sales channel which is defined as "retail format in which products are sold at prices that are in principle lower than an actual or supposed full retail price", or National Accounts which have multi-location, high volume, uniform facility clients. Distributor sales channel is one of the most prevalent sales and logistics models FMCG companies have, which build the indirect shipment logistics to their retailer customers.

FMCG companies exhibit diverse logistics operational models. Some operate all of their customer logistics operations by indirect shipments via distributor sales channels while others employ hybrid systems incorporating direct shipments to larger customers using rented transportation. Direct shipment can be misunderstood as the direct-to-consumer (DTC) way of operation, that has been growing in the couple of years. In Turkey, its examples can be seen via e-commerce retailers. Manufacturing companies can have their own sellers in these e-commerce platforms. They operate either in a separate and relatively smaller warehouses for this operation, or maybe from the same main warehouse where they store their finished goods. They directly get the order from the e-commerce platform and make the order ready and sent by the cargo shipments. This new-evolving trend is not in the scope of this thesis; however, it cannot be neglected as a new and hot topic way of operation. A basic figure can be seen in Figure 3.1 for a clear understanding for the structure of logistics and sales settings.

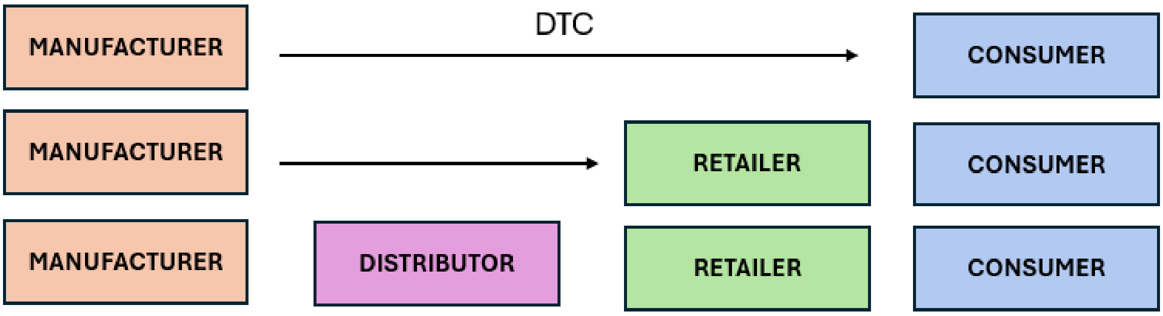


Figure 3.1. Manufacturer to consumer settings.

Price sensitivity holds particular significance in the FMCG sector due to extensive market competition. Examples can be given from the market in Turkey, where increasing private label sales volumes result in shifting consumer preferences toward lower-priced products and diminished brand loyalty. More adjustments on prices are needed under a less loyal profile of consumers [10]. Price elasticity demonstrates variance across sales channels, with higher sensitivity observed in Discounter and Distributor channels compared to National Accounts retailers. FMCG companies dynamically adjust their prices observing their competitors, the market conditions and their warehouse inventory levels. Pricing strategies adapt to inventory limitations, higher prices may be implemented during limited inventory availability to maximize profit, while periods of sufficient capacity may result in volume-driven strategies with lower pricing. One of sales and operations planning (S&OP) decisions' main input is the available inventory during a time period and how to use this volume in the most efficient way to optimize total profit. These features make the selected industry and company a relevant choice to adapt an optimization problem with finite initial inventory.

The industry application in this paper focuses on a multinational FMCG manufacturer for different type of FMCG products. From snacks to dips, cleaning supplies to ice-cream, deodorants to shampoo, it has around 30 different brands being sold in Turkey and 400 brands over the whole world. They have four different factories in Turkey for local production: one for Savory, two for ice-cream and one for home care and personal care products. The product that is selected for the industry application part of the thesis is one of the variants from bleach portfolio, which is locally produced and stored in the warehouse in Konya. The chosen product is a high-selling volume bleach product, requiring high replenishments to all retailer customers, including distributors sales channel.

The company operates through six distinct sales channels: National Accounts, Discounter Markets, Supermarkets, Cash&Carry, Health&Beauty and Distributors. All channels including Distributors have direct shipments by rented trucks from the warehouse while smaller retailers receive indirect shipment through distributors. The current practice for S&OP processes in the company is being done for both longer and shorter time periods. Considering the targeted growth rates, sales volume and turnover for each sales channel, sales volume and turnover targets build up a one, integrated sales volume for the planning horizon. Pricing choices maintain consistency within sales channels while allowing variation between different channels and across periods. There are two different discount effects: structural and variable. Structural discount is decided at the beginning of each year and is a constant discount rate which can be different for all sales channels. Variable discount differs in each time period, creates more space for dynamic pricing for changing trends in the market, competitor prices and strategic initiatives. Sales data clearly demonstrate price-demand relationships. S&OP cycle is highly dependent on the level of inventory that is foreseen to be available, with sales channel target volumes allocated as finite quantities during planning periods. Pricing decisions are directly affected from the available inventory, optimizing profit generation from available inventory. This operational reality is exemplified by recent instances of promotion cancellation and price-promotion agenda postponement due to inventory constraints. S&OP processes are completed distinctly for each product based on the inputs explained here. Therefore, building a system with a single product is relevant and can be adapted for different products for separately optimizing their prices, allocations and sales.

Distributors sales channel has more advantages than other sales channels for a joint and flexible dynamic pricing, allocation and sales strategies. Unlike other channels, the company partially operates the orders of distributors with a full visibility of their inventory levels and sales volumes. The company has the authority to allocate additional volumes or modify existing orders. Sales targets for each distributor for each product is being decided by the manufacturing company itself, while staying open the changes. Sales that a distributor makes to its small-sized market customers have a direct effect to the company's financials. Therefore, the distributors trade system can be considered as the main warehouse allocation decisions to each of the distributors, based on the demand coming from distributors' small-sized market retailer customers. That demand directly affects the distributors' sales and inventory levels, hence the allocations from the main warehouse. The unit selling price is the same to all the distributors since they are under the same sales channel. As discussed before, company adjusts their prices for all the sales channels, however, distributor sales channel has more flexibility in price adjustments due to integrated operations. An adequate volume (considering all constraints such as supply) is allocated to this whole sales channel for a planning period and it is set at the beginning of this period. Hence, it turns out to be an allocation problem to all distributors with finite initial inventory for a single product.

3.2. Problem Formulation and Mathematical Model

The existing profit optimization model of Tunçinan et al.'s study under OWMR setting is used as the base of the model in this thesis. This problem is a joint allocation, inventory and pricing problem under price-sensitive deterministic demand for a single product with finite initial inventory quantity in a multi period setting. The model's objective function maximizes total profit within the planning horizon by jointly deciding on the inventory allocations to each retailer, sales of each retailer in each period and prices in each period which is identical for all retailers. Model presented in this paper has three extensions added to the existing one: lost sales cost component in the objective function coming from unfulfilled demand, positive lead times from warehouse to retailers, and retailer capacity constraints for the selected product.

Tunçinan et al.'s model has the following summarized properties which apply to the extended model in this paper. There are equal discrete $|T|$ time periods and each period is denoted by $t \in T$ and $|S|$ retailers that are denoted with $s \in S$. Demand for the product at a retailer is a linear and strictly decreasing function depending on the selling price P_t, and given as $D_{st}(P_t)$ =a_{st}-b_{st} P_t where a_{st} > 0 and b_{st} > 0 denote potential demands and price elasticities respectively. The efficiency of linear demand function in this type of industry applications is studied in literature, one can see examples such as [12-15],[40]. An upper bound for price is imposed to the model in each period to prevent negative demands to be observed at any retailer in any time period, where $D_{st}(p)$ > 0 for $p \in [0, \overline{p}_t]$. h_0 and h_s denote the per period per unit warehouse and retailer holding costs respectively. Unit transportation cost from warehouse to retailers in period *t* is denoted with w_{st}. Since this is a joint allocation and pricing decision model, I_{st} is defined as the variable that represents the inventory level at each retailer at the end of each period t and U_{st} is defined as the decided inventory allocation amount from warehouse to each retailer in each period. To increase the profit, it may be chosen by the optimization model to partially fulfill the demand. Therefore, sales at each retailer in each period is defined as Y_{st}.

3.2.1. Extensions to the Multi Period Profit Optimization (MPOP) Model

The extensions change some of the assumptions in the existing MPOP model, below summarize the new assumptions and properties of the model in this thesis:

- Retailers may have initial inventory quantities depending on the lead time in this extended model, if the lead time is zero then no initial inventory is necessary. Initial inventories are denoted by $I0_s$ for each retailer *s* and lead times for each retailer are denoted with l_s for each retailer *s*. Replenishment decisions from warehouse to retailers are decided in the beginning of each period. An allocation decided in period *t* for a retailer *s* arrives to this retailer in period (t+l_s). Total available inventory to sell in the system becomes as (q+ $\sum_{s=1}^{S} I0_s$).
- The existing model assumes lost sales for unfulfilled demand, however no additional cost is incurred in the profit function. In this model, lost sales costs are not negligible and per unit lost sales costs are denoted with ls_s, specific for each retailer *s*. They are calculated by using the basic Newsvendor model formulation where it takes the critical fractiles for each retailer *s*, denoted with cf_s, and h_s as inputs, derived from

$$cf_s = \frac{ls_s}{h_s + ls_s}. \tag{3.1}$$

Per unit lost sales costs are identical for a retailer in all periods as retailer holding costs and critical fractiles are consistent.

- Total lost sales cost component is added to the objective function. Dekker et al. [32] uses an equation for penalty costs as

$$C_p(c, S) = \sum_{j=1}^{n} \pi_j \lambda_j (1 - \beta_j(c, S)), \tag{3.2}$$

and this is applied to the model in this thesis. The demand function is distributed with Poisson in their study, it can be seen that formulation takes the unfulfilled demand and multiplies it with the per unit lost sales cost which is π_j for all n demand classes j to find penalty costs. Same formula is adopted in this paper with linear demand function. Total lost sales cost is calculated as

$$\sum_{s=1}^{S} \sum_{t=1}^{T} ls_s((a_{st} - b_{st}P_t) - Y_{st}), \tag{3.3}$$

by multiplying unit lost sales cost of each retailer for each unit of unfulfilled demand.

- In this extended model, each distributor has an inventory capacity for the selected product. This allows the model to make allocation decisions in a more realistic fashion. Parameter c_s denotes the retailer capacities for retailer s. As observed in Hariga et al.'s studies, retailer capacities are handled to define the inventory levels rather than as a bound for allocation amounts [50,51]. This logic makes sense considering the operations of distributors where simultaneous sales and arrival of goods can occur that balance capacity, the company has the visibility of the inventory levels of the distributors and distributors are expected to keep in mind their capacities while ordering.
- One addition in the extended model for decision variable set is the warehouse inventory level at the end of each period t, denoted with WI_t. This represents the inventory level in the warehouse at the end of each period t after all the allocations are decided for all the retailers. This variable is used in the objective function for warehouse holding cost component, helped to simplify the objective function.

3.2.2. Mathematical Model

The extended multi period profit optimization model (EMPOP) is defined with its sets, parameters and variables:

Table 3.1. Decision variables.

Decision Variables	**Definition**
P_t	Selling price in period *t*
Y_{st}	Amount of products sold at retailer *s* in period *t*
I_{st}	Inventory level at retailer *s* at the end of period *t*
U_{st}	Amount of products allocated to retailer *s* in period *t* which will arrive to retailer s in period $(t + l_s)$
WI_t	Inventory level at the warehouse at the end of period *t*

Table 3.2. Sets and parameters.

Sets	**Definition**
s	Set of retailers in set *S*
t	Set of periods in set *T*
Parameters	**Definition**
$I0_s$	Initial inventory level at retailer *s*
l_s	Lead time from the warehouse to retailer *s*
c_s	Inventory capacity of retailer *s*
q	Initial inventory quantity at the warehouse
a_{st}	Potential demand for retailer *s* in period *t*
b_{st}	Price elasticity of retailer *s* in period *t*
h_s	Per period per unit inventory holding cost at retailer *s*
h_0	Per period per unit inventory holding cost at the warehouse
w_{st}	Per unit transportation cost from the warehouse to retailer *s* allocated in period *t*
ls_s	Per period per unit lost sales cost for retailer *s*
$\overline{p}_t$	Upper bound on price in period *t*

EMPOP:

$$\max \quad \sum_{s=1}^{S}\sum_{t=1}^{T}(P_tY_{st} - h_sI_{st} - w_{st}U_{st} + ls_sY_{st} - (a_{st} - b_{st}P_t)ls_s) - \sum_{t=1}^{T} h_0WI_t \tag{3.4}$$

$$I_{s,t} = I_{s,t-1} + U_{s,t-l_s} - Y_{s,t} \qquad s \in S,\ t = l_s+1,\dots,T \tag{3.5}$$

$$WI_t = WI_{t-1} - \sum_{s=1}^{S} U_{st} \qquad s \in S,\ t = 1,\dots,T \tag{3.6}$$

$$U_{s,t-l_s} = 0 \qquad s \in S,\ t = 0,\dots,l_s \tag{3.7}$$

$$WI_0 = q \tag{3.8}$$

$$I_{s0} = I0_s \qquad s \in S \tag{3.9}$$

$$I_{st} \le c_s \qquad s \in S,\ t \in T \tag{3.10}$$

$$\sum_{s=1}^{S}\sum_{t=1}^{T} U_{st} \le q \tag{3.11}$$

$$Y_{st} \le a_{st} - b_{st}P_t \qquad s \in S,\ t \in T \tag{3.12}$$

$$P_t \le \bar{p}_t \qquad t \in T \tag{3.13}$$

$$P_t, Y_{st}, I_{st}, U_{st} \ge 0 \qquad s \in S,\ t \in T \tag{3.14}$$

EMPOP objective function (3.4) maximizes total profit across the defined planning horizon. Revenue generation derives from total sales across all retailers. Costs have four different components: total transportation cost incurred from total allocation decided from warehouse to retailers, total warehouse holding cost arising from holding inventory at the warehouse, total retailer holding cost that comes from holding inventory in retailers for a period and finally the total lost sales cost resulting from unfulfilled demand at retailers. Total lost sales costs are added as the expression in (3.3), shown as its partially expanded form in the objective function.

Constraints (3.5) and (3.6) are the balance equations for retailer and warehouse inventory levels respectively in each time period. Retailer inventory balance equations reflect the lead time extension of the model with delayed product arrival from dispatch time by the retailer-specific lead time. Constraint (3.7) serves for the same extension and ensures that the first shipment arrives to a retailer only at $t > l_s$. Constraints (3.8) and (3.9) set initial retailer and warehouse inventory levels at time $t = 0$. Constraint (3.10) governs maximum product allocation from warehouse to retailers per period, considering retailer inventory levels to accommodate scenarios where warehouse shipments may exceed capacity but are balanced by sales at the retailers. Constraint (3.11) is the warehouse initial quantity restriction which ensures total allocation amount does not exceed the initial inventory quantity at the warehouse. Constraints (3.12) limit sales amount at a retailer in each time period by the observed demand which is a linear function dependent to the selling price at that period. Constraints (3.13) impose the upper bound on the selling price to prevent negative demands to be observed in retailers in any time period. Finally, constraints (3.14) are the non-negativity constraints for the decision variables of the model.

This mathematical model is an example NLP model due to multiplying two decision variables in the objective function. Model solution was facilitated through Gurobi solver version 11.0.3, which provides NLP optimization capabilities, enabling applications and results analysis.

4. COMPUTATIONAL EXPERIMENTS

This chapter explores the computational experiments and their findings. The first two sections present data calibration process using real company data and explain key considerations in translating the model to the chosen solver platform. The third section examines findings from the industry application, focusing on observations from the model extensions and the influence of dynamic pricing strategies on profit results. In the final section, different parameter settings are tested and evaluated using synthetic data, broadening the understanding of the model's potential. Throughout the analysis, several managerial insights are drawn from the results.

4.1. Data Calibration for Industry Application

Data analysis and calibration for the industry application is discussed in this part for all sets and parameters.

4.1.1. Period and Retailer Sets

24 periods are used for the industry application of the mathematical model, therefore set of periods is defined from 1 to 24. Each period represents weeks in this application, as distributors have average of weekly replenishment policy. There are 30 active distributors in the company for the selected product, therefore; retailer set is defined from 1 to 30. Distributors are denoted as D*s*, for example first retailer is denoted as D1. Months are denoted as M*t* in the rest of the paper, for example M1 denoting month 1.

4.1.2. Unit Price – Sales Volume

The relationship between unit sales price and sales volume is analyzed using six months of historical unit selling price-monthly sales volume data for each distributor. Cohen et al.'s study, who have other numerous of application studies under similar settings for dynamic pricing problems is a supporting example how to generate demand functions while working with real-company data [9]. It should be highlighted one more time that distributor channel is relevant for an application under this setting due to the customer profile with higher price elasticity. Distributor market customers are small-sized markets, and they show increased purchasing volumes at lower prices and may even prefer to maintain higher inventory levels for longer time periods. Higher prices have a significant negative effect on sales volume, clearly observed in the data.

Correlation coefficients between unit price and sales data for each distributor can be found in Table 4.1, along with potential demand and initial inventory quantities that will be discussed in the upcoming subsections. The average correlation coefficient across 30 distributors is -0.67, indicating a strong relationship between price and sales volume for this retailer group. The weakest coefficient is D22 which is -0.40, and the strongest is D1 with -0.95. It makes sense that the strength of the relation varies among distributors since they have geographical and market differences: D1 is located in Şanlıurfa, the customers may be more price sensitive, while D22 is in Kahramanmaraş and is a large monopolistic distributor in the area, its customers are less price sensitive, or the total sales volume stays similar throughout the months.

4.1.3. Price Elasticity

Price elasticity is constant in each of the time periods for a distributor with inspiration from several literature [2,4]. This parameter set is derived by applying linear regression to the historical unit price – sales volume data which is again a prevalent methodology. Monthly sales volumes are converted to weekly volumes and then used for the application. Table 4.1. shows the price elasticities assigned to each distributor thorough all periods.

4.1.4. Potential Demand

Potential demands of each distributor in each time period are derived from historical price-sales data and prior years' sales data, with several additional realistic factors incorporated after analysis. First, weekly splits are arranged as giving the final (fourth) week in each month an increase by 1.15 and decreasing with the same rate from the first week of each month. This comes from the operation dynamics of the company and the sales operations of the distributors. This pattern reflects typical distributor channel behavior, where sales tend to start slowly and accelerate toward month-end as customers' cash flow improves for debt payments and new orders. Additionally, distributors' lower priority in the company's channel hierarchy affects later-month inventory allocations.

It is also observed that even though the selling price in M4 is the lowest in the historic sales data, sales are lower for the majority of the distributors comparing to other months with higher unit price. On the contrary, even though the selling price is higher in M1, the sales volume is usually higher than M5 or M6. This comes from the seasonality effect. This product is not seasonal such as an ice cream product; however, the business dynamics always indicate higher sales volume in Quarter1 (M1-M2-M3) compared to Quarter 2 (M4-M5-M6), especially M4. Current forecasts and target volumes for the upcoming year confirm this trend. Based on these inputs, M1 and M3 potential demand data are increased with rate factor 1.05, while M4 is decreased with rate factor 1.10. The rates are standardized for all distributors.

4.1.5. Lead Times

First extension to the existing OWMR model is the positive lead times from warehouse to retailers. Distributors are clustered based on their location and their distances from the main warehouse. Lead time represents not only the direct shipping duration but also additional processing time for order routing. For example, for distributors located in Nevşehir, or Zonguldak, this preparation time takes longer than some others. In literature, there are valid examples for retail applications where additional components should be taken into account while assigning lead times [56]. The company has set of assigned average lead times considering several components. Table 4.2 shows the clustering for lead times. Lead times are chosen to be in the following set of options: {0,1,2}. The lead time extension is implemented efficiently by assigning lead times based on distributor locations and shipping routes. Distributors in close locations or on central shipping routes are assigned lead times as "0", while those in further locations and extended preparation time assigned lead times as "2". A recent study [29] validates this lead time set configuration as a logical approach for retail applications while maintaining the ability to observe its effects. It should be noted that distributors usually have weekly replenishments for the selected product, therefore lead times shorter than one week create no differential impact under model's weekly time framework. These operational characteristics provide logical justification for assigning zero lead times to such cases.

Table 4.1. Coefficients, price elasticities and initial inventory quantities of distributors.

Distributor	**Regression Coefficient**	**Price Elasticity**	**Initial Distributor Inventory**
D1	-0.95	136.70	6,000
D2	-0.74	364.90	0
D3	-0.50	161.30	4,000
D4	-0.95	134.78	12,000
D5	-0.65	207.50	8,500
D6	-0.46	148.77	8,000
D7	-0.80	25.32	3,000
D8	-0.62	99.30	2,000
D9	-0.57	122.01	2,000
D10	-0.62	154.32	2,500
D11	-0.76	42.90	1,500
D12	-0.79	64.70	14,000
D13	-0.65	157.14	0
D14	-0.61	75.85	7,000
D15	-0.74	155.39	7,500
D16	-0.65	31.93	1,000
D17	-0.75	277.49	8,000
D18	-0.62	144.09	3,000
D19	-0.71	55.40	2,500
D20	-0.74	77.60	6,000
D21	-0.72	59.15	2,000
D22	-0.40	60.72	5,000
D23	-0.47	146.5	18,000
D24	-0.74	62.32	0
D25	-0.56	98.88	10,000
D26	-0.68	62.70	7,500
D27	-0.67	154.18	4,000
D28	-0.65	97.90	400
D29	-0.61	196.50	0
D30	-0.60	186.29	0

Table 4.2. Lead time clustering.

Lead Time (Weeks)	Distributors
0	D2, D13, D24,D29,D30
1	D3,D8,D9,D10,D12,D14,D15,D17,D18,D26,D27
2	D1,D4,D5,D6,D7,D11,D16,D19,D20,D21,D22,D23,D25,D28

4.1.6. Unit Transportation Costs

The most updated closing inventory value from financial reports is used to calculate unit transportation costs. Raw & pack material costs, production costs, bought in productions, distribution and other supply chain costs such as write-offs build up the SCC of a product in the company. While not specifically observed in the literature, supply chain consulting companies' informative publications mostly indicate that the SCC composed almost 80% of the inventory value of a product, with 20% coming from other inventory and external costs, such as additional costs due to aging. Note that inventory value is converted to TL from EUR with the constant EUR/TL rate used in the company at that time.

An important input from the company responsible indicates that the majority of the SCC for the selected product comes from packaging material costs. Transportation costs are estimated at 7% of SCC, with customer transportation costs accounting for 50% of this portion. This made the average unit transportation cost from warehouse to distributor as 0.96 TL. In order to reflect the diversely located distributors, following set of values is used for this parameter in M1: {0,75,0.96,1.3}. The clustering for distributors is shown in Table 3.4. After setting these as M1 values, the costs are increased with rate 1.02 in each month to reflect real-life transportation and general SCC increases. An exception is decided for M3, where costs remain consistent with M2, allowing observation of a period without cost increases. Table 4.3 shows unit transportation costs clustering.

Table 4.3. Unit transportation cost clustering.

Transportation Cost per Unit (TL) in M1	Distributors
0.75	D3,D8,D9,D15,D19,D22,D23,D25,D26,D27
0.96	D2,D11,D13,D14,D17,D18,D20,D24,D28,D29,D30
1.30	D1,D4,D5,D6,D7,D10,D12,D16,D21

4.1.7. Inventory Holding Costs

Closing inventory value of one unit is again used to calculate both per period warehouse and distributor holding costs. Warehouse holding cost per piece per period is calculated as 0.278 TL with the company debt yearly interest ratio taken as 40% and multiplying this ratio with per piece inventory value, excluding per unit customer transportation costs. This is a fixed constant for all periods.

Retailer per unit per period holding costs are calculated including the customer transportation costs to the inventory value, resulting as 0.287 TL. Distributors incurring higher transportation costs are assigned to higher retailer holding costs within the following set: $\{0.279, 0.287, 0.32\}$. As anticipated, retailer holding costs consistently exceed warehouse holding costs.

4.1.8. Unit Lost Sales Cost & Critical Fractiles

The second extension to the optimization model is the lost sales cost component addition, calculated by using the basic Newsvendor formulation as its examples in literature can be found [48]. Per period per unit holding costs and critical fractiles for each distributor are used in the formulation, maintaining consistency in each period. Main input here is the critical fractiles for each distributor. These critical fractiles represent the targeted service levels for each distributor, decided based on their priority rankings defined in the company. Set containing three service levels are used: $\{0.75, 0.85, 0.95\}$. Distributors' service rates are aggregated to these three values considering their locations, sales volumes and priorities based on various internal factors. Table 4.4 shows the assigned service rates for each distributor. Depending on these critical fractiles and unit retailer holding costs, per unit lost sales cost for each retailer is generated. Prioritization of distributors is a crucial aspect of the company's inventory allocation strategy during capacity constraints, enhancing the model's practical applicability.

Table 4.4. Critical fractiles (target service levels) clustering.

Critical Fractiles (Service Levels)	**Distributors**
0.75	D7,D8,D9,D11,D12,D16,D19,D21,D22,D27,D28
0.85	D1,D3,D6,D10,D14,D17,D18,D20,D25,D26,D29
0.95	D2,D4,D5,D13,D15,D23,D24,D30

4.1.9. Warehouse Initial Inventory Quantity

Different aspects are interpreted while deciding on the initial inventory quantity at the warehouse, aiming to accurately reflect historical volumes and potential future targets. The S&OP reports target volumes are already production capacity constraints added volumes. After evaluating these mentioned data and reports, initial volume is decided to be 2,400,000 pieces, representing a realistic quantity for the six-month planning period. This quantity aligns with typical distributor sales channel allocations between periods M1 to M6. Monthly volume distribution remains flexible, allowing for optimal allocation and pricing decisions, including potential pre-builds from previous months. Hence, additional short-term volume constraints are not added to the model constraints.

4.1.10. Retailer Inventory Capacities

Each retailer has total inventory capacities in the measure of pallets, reflecting the multi-product nature of distributors who handle various HPC category brands from the company. Pallet sizes are standard but piece per pallet varies for each product. 10% of the total pallet capacity assumption is made for the selected product considering its volume in total sales of the distributors and market size along with the inputs coming from the company responsible. Capacity limits are established between 3,456 and 21,000 pieces. Retailer capacities are consistent in all periods for each distributor.

4.1.11. Initial Inventory Quantities at the Distributors

Initial inventory assignments to distributors are required as positive lead times are added to the model. Distributors with zero lead time received zero initial inventory. For the ones that have positive lead times, appropriate initial quantities are assigned based on their historical sales data. Initial inventory quantities make this case realistic and eliminates the lost sales cost that may be caused due to the initial allocation lead times. Assigned quantities can be found in Table 4.1.

4.2. Application of the EMPOP Model

All computations are carried out on a computer with Microsoft Windows 11 operating system. Coding and data insertion is completed through GAMS 47.4.0 and the model is solved by solver Gurobi with its latest version 11.0.3 at the time. Free academic licenses for both GAMS and Gurobi are used. Gurobi solved the model as a Qudratically Constrained Problem (QCP) while maximizing the objective function. Key components of the implementation are explained and presented in this section to illustrate how the theoretical model was translated into computational form.

- GAMS implementation required period set T to start from zero due to the initial inventory quantity assignments for retailers. Following this period set definition, several components needed specification for periods $t > 0$: price upper bounds in each period, inventory and warehouse balance equations and the allocation restriction equations based on lead times.
- It may be beneficial to note that upper bound prices in each period are calculated programmatically by using a_{st} and b_{st} rather than being input as pre-computed constants. Using a similar approach, unit lost sales costs for each retailer are calculated within the model, taking pre-determined critical fractiles and holding costs for each retailer as inputs.
- Positive lead times from main warehouse to retailers are reflected to the model by revising the allocation decision variable time index setting. This revision makes sure for a retailer that the ending inventory at the end of period t takes into account the left-over inventory from previous period, the new allocations decided in period *(t-ls)* but arrive at t and the total sales in period t. As discussed in the data calibration part, retailer capacities are used to restrict the final inventory at the end of each period for each retailer; therefore, inventory level decision variables are on the left side of the equations.

- A new decision variable for warehouse inventory level is introduced, enhancing both code efficiency and objective function simplicity. This definition facilitates easier tracking of end-of-period warehouse inventory levels during results analysis.

These constraints and equations can be found in Figure 4.1 as in their GAMS coding formats, following the order of the above explanations.

Lost sales cost component extension can be observed in the profit function of the model. The additional component in the profit function in the coding format is as follows:

```
-lostsale(s)*((a(s,t)-(b(s,t)*price(t))-sales(s,t)))
```

Finally, GAMS coding for the full profit function is as follows:

```
profit =e= sum((s,t),price(t)*sales(s,t)-h(s)*inv(s,t)-trans(s,t)*alloc(s,t)+
lostsale(s)*sales(s,t)-((a(s,t)-b(s,t)*price(t))*lostsale(s)))-sum(t$ord(t)>1),
h0*warehouse_inv(t))
```

```
upperbound(t)..

        upbound(t)$(ord(t)>1) = smin(s, a(s,t) / b(s,t))

invlevel(s,t)$(ord(t)> 1)..

        inv(s,t) =e= inv(s,t-1) + alloc(s,t-leadt(s)) - sales(s,t)

warehouse_balance(t)$(ord(t) > 1)..

        warehouse_inv(t) =e= warehouse_inv(t-1)- sum(s, alloc(s,t))

alleadtime(s,t)$(ord(t) <= leadt(s)+1)..

        alloc(s,t-leadt(s)) =e= 0

lostsale(s) = critfrac(s)*h(s)/(1-critfrac(s))

inv(s,t) =e= inv(s,t-1) + alloc(s,t-leadt(s)) - sales(s,t)

inv(s,t) =l= retcap(s)

sum(t$ord(t)>1), h0*warehouse_inv(t))
```

Figure 4.1. GAMS codes for key model constraints and equations.

4.3. Industry Application Results

All decision variables are briefly discussed in the following subsections for company application, along with the discussions for the extensions and dynamic pricing impacts.

4.3.1. Pricing Decisions

Price patterns reveal clear alignment with weekly demand splits, where the lowest prices are observed in the first weeks of each month, increasing towards the final week. Figure 4.2 shows the graph of the prices plotted for all periods. Prices are increased to adjust the demand to the desired level as there is an increasing trend in the potential demand within a month. Period between weeks 13-16 is the interval with lowest prices. As it can be recalled, that period has the lowest demand potential due to the nature of the business. These findings align with Tunçinan et al.'s insights regarding optimal price selection in the MPOP model, which highlight how price choices are adjusted based on potential demand in subsequent periods [2].

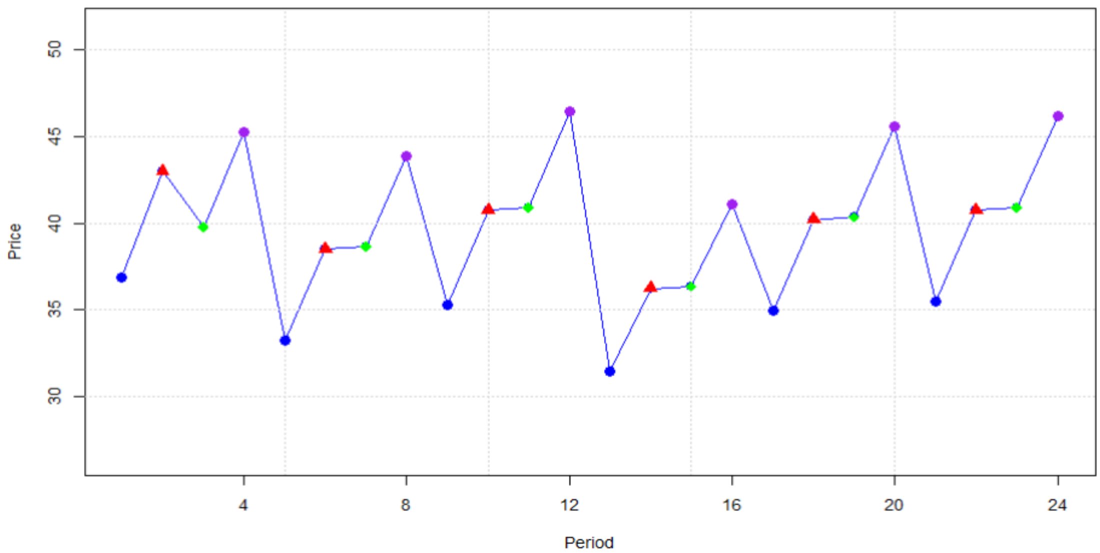


Figure 4.2. Prices during the planning period.

There is a slight upward trend in the prices that can be observed from the peak and the lowest prices each month. This trend is most clearly observed by comparing months with identical potential demands, which are M2, M5 and M6 or M1 and M3, discussed in data calibration part. In the industry, SCC typically show an upward trend in later periods due to inflation rates, particularly evident in Turkey's recent economic climate. In the data used, transportation data was increased regularly to reflect an increase. On the other hand, it can be more costly to sell a unit of product later since holding costs increase cumulatively each period. This is reflected in FMCG companies inventory value reporting cycles that the more inventory stays unsold, its value increases rapidly in each reporting period. The model's results emphasize the necessity of dynamic price adjustments to account for these increasing costs. It must be noted that the model leaves no inventory on hand at the warehouse. Therefore, these choices are under the scenario where there is enough potential demand of the distributors to liquidate the initially available inventory quantity.

4.3.2. Inventory Allocations

Figure 4.3 shows the total allocations decided in each period over all distributors, revealing a pattern of bulk allocations at specific times rather than balanced shipments throughout periods. This can be explained with the increasing trend of the transportation costs and the increasing nature of the potential demand data within a month. The model prioritizes larger, delayed shipments for minimizing transportation costs while taking into account the retailer inventory capacities and holding costs. Periods between 9-12 are different in terms of the transportation cost increase as discussed before. Due to this difference, it is seen that shipments are chosen to be made in a regular way rather than bulk loadings during periods 5-11. Transportation cost increase is out of equation during this period and holding costs are primarily considered while deciding on the allocations. Period 13 marks a transportation cost increase; therefore, the model waits until that time to make a bulk allocation decision.

The decreasing trend in allocations can be observed, driven by the increasing trend of SCC. It is better to sell earlier with lower SCC, hence more allocations occur during earlier periods and diminishing quantities while getting to the end of planning horizon. Increasing nature of the potential demands affects the choices as observed clearly in the graphic, however; periods between 5-12 indicate that potential demand is not the sole determining factor in allocation decisions.

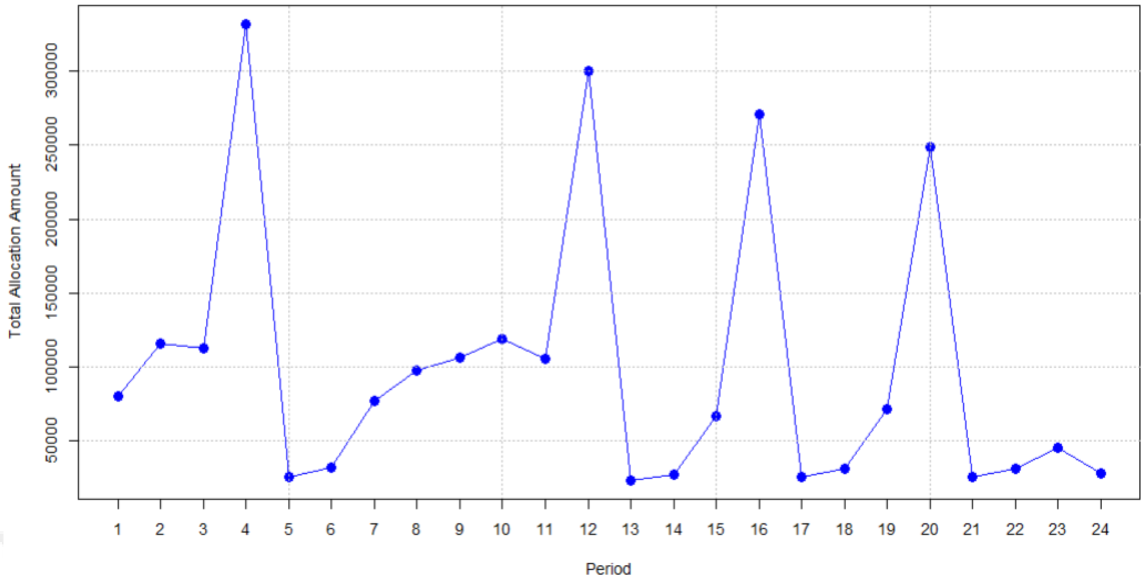


Figure 4.3. Total allocations during the planning period.

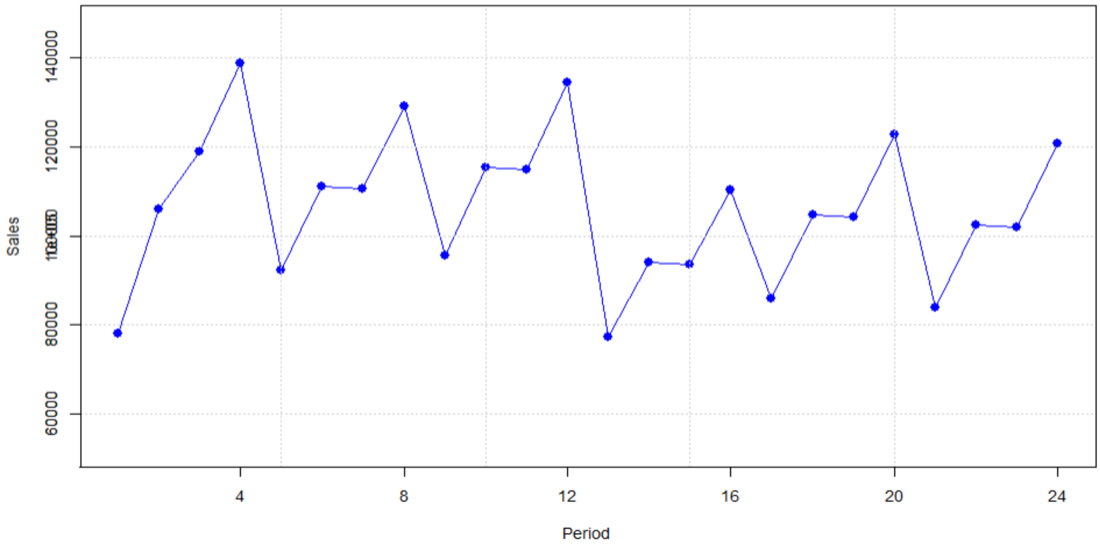


Figure 4.4. Total sales of all retailers during the planning periods.

4.3.3. Sales Decisions

Total sales volume is 2,560,000 pieces which is the total initial warehouse quantity and initial retailer inventory levels. Under these conditions, shipping and selling all available inventory is seen to be the most profitable strategy. Figure 4.4 shows the total sales in each period. As expected, the lowest sales occur during period 13-16, aligning with demand.

There is decrease in the peak sales within a month comparing the periods with same potential demands. It is better to sell earlier as possible since there is holding costs added in each period and transportation cost has an increasing trend. For instance, sales peak in M2 and subsequently decline in months M5 and M6. This pattern aligns with the industry's typical behavior, where the first quarter generally yields the highest sales volume for distributors of the company, validating this as an efficient strategy. Please note that the first period's anomaly in the graph come from the initial inventory assignments. However, this is relevant with the real-life cases observed in distributor sales channel. To give an example, during the distributors' inventory covers tend to be the lowest in the opening of a new year, leading high allocations in the prior periods to balance the inventory levels. This situation can be addressed by allocating additional inventory to distributors before the selling period ends.

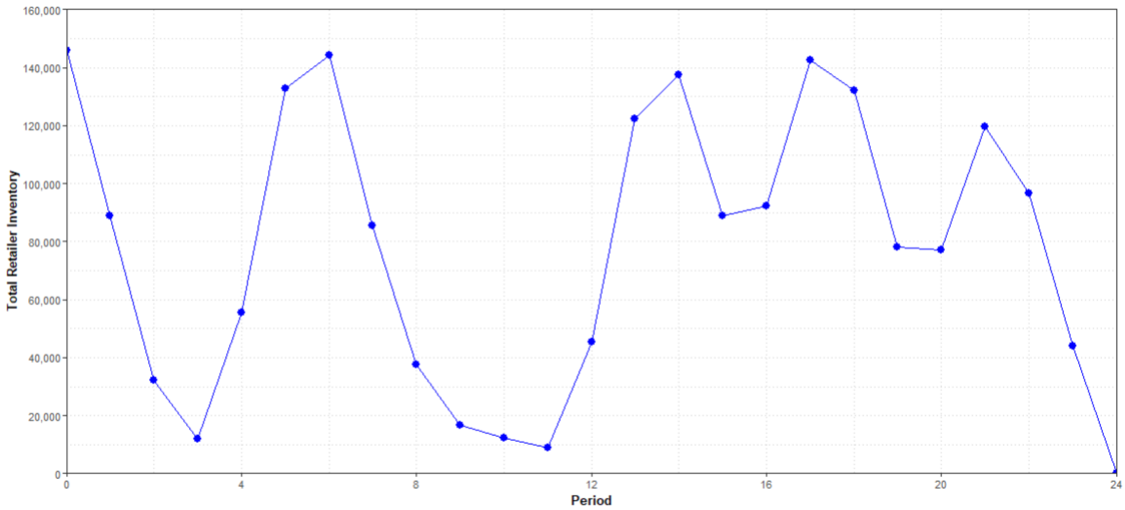


Figure 4.5. Total retailer inventory levels in each period.

4.3.4. Retailer Inventory Levels

The model chooses to keep inventory at the majority of the distributors except the ones with highest per unit holding costs. Even though all retailer holding costs are higher than warehouse holding cost, it can be beneficial to keep inventory in some periods for some distributors. Peak inventory levels remain within total retailer capacities. During the slowest demand period 13-16, distributors observed to have the highest inventory levels comparing, as can be found in Figure 4.5. Even though this time interval has the lowest demand potential, allocations were decided that can be recalled from Figure 4.2. This concludes that keeping inventory for several periods at the distributors can be more profitable due to the opportunity of shipping them with lower cost. At the end of the defined planning horizon, no inventory remains at any retailer location. This is expected since the leftovers are not added to the profit function as a cash input in the objective function.

4.3.5. Profit Impact of Retailer Capacity Constraints

Total profit is found to be 91,995,384.17 TL when retailer capacity constraints are removed from the model. This is a 0.12% increase comparing to the capacitated model. Unit transportation cost data are revised as the following to examine the impact of different rates: when monthly increase rate was adjusted from 0.02 to 0.10, the capacitated model's profit became 91,477,212.02 TL, while the un-capacitated model's profit became 91,722,333.43 TL, 0.26% increase in total profit. Further adjustment of the rate to 0.2 resulted in 90,124,331.32 TL for the capacitated model and 91,191,729.76 TL profit for the un-capacitated model, showing a 1.01% increase. It can be stated that retailer capacity constraints have a negative impact on total profit as they have a blocking effect on bulk shipments to retailers when transportation costs are increasing regularly, and the greatness of this impact depends on the ratio between transportation costs and holding costs. Steeper transportation cost increases lead to larger rate of profit improvements when capacity constraints are removed.

4.3.6. Observation of Positive Lead Times

Lead times affect the allocation decisions of the model. It was discussed before that model tends to make bulk allocations to minimize transportation costs. Figure 4.6 illustrates how distributors with positive lead times receive allocated inventory in advance to ensure timely availability, while distributors with zero lead time show no such allocation patterns. Bulk allocations are in the same periods for all lead time levels, right before transportation cost increases occur; distributors with positive lead times require additional inventory allocation ahead of these bulk timings in order to prevent lost sales. Prior allocation decisions have disadvantages considering the changing dynamics in potential demand or sales plans. Considering FMCG sector is a highly dynamic business, shorter lead times offer greater flexibility to respond to changing conditions.

Kindly observe that retailers with positive lead times have their final allocations prior, the latest period of allocations is period 23 for lead time 1 and period 22 for lead time 2. Allocations are not scheduled beyond these periods since later shipments would not be converted to sales within the defined planning period.

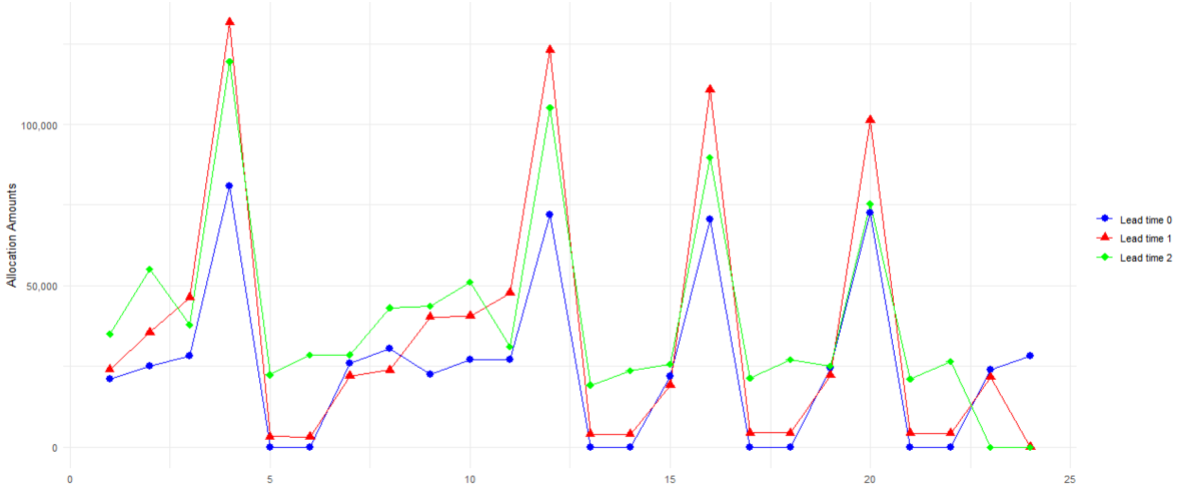


Figure 4.6. Total allocations for distributors aggregated over lead times.

4.3.7. Addition of Business Constraint & Static Pricing

Fully dynamic pricing is not that easy for companies to implement due to various reasons, a topic that has been extensively studied in literature [5],[8]. A monthly price consistency constraint was implemented to see the effects of having flexibility and more frequently update prices. This constraint aligns with the current practice of the selected company where the prices are usually adjusted monthly. Additionally, results under static pricing (single price in all periods) are analyzed as a third model. Results table 4.5 is presented along with unrestricted version for comparison of the three models. C1 represents total warehouse holding costs, C2 represents total shipment costs, C3 represents total retailer holding cost and C4 represents the total lost sales cost. The analysis shows that total profit decreases by 1.30% when moving from the unconstrained version to monthly adjusting, and by 1.61% when moving to static pricing, stressing the advantages of flexible dynamic pricing. While cost components showed minimal changes, total revenues decrease significantly. FMCG sector is a highly dynamic sector, along with the economic conditions private labels with lower prices has been growing rapidly, and with the observation the decrease of consumer's brand loyalty identities, well-known and rooted companies may need to adjust their prices than their current practice [10]. The results here support how dynamic pricing can be an effective strategy and make meaningful impact. CPU times of the models decrease from fully dynamic model to static pricing model, proving fully dynamic pricing is a more complex optimization problem.

Table 4.5. Overall cost and total profit results of different pricing models.

	C1 (TL)	C2 (TL)	C3 (TL)	C4 (TL)	Total Profit (TL)	CPU Time (seconds)
Full Dynamic Pricing	7,480,822.05	553,527.26	2,450,342.79	43,514.32	91,881,956.42	346.42
Constrained Dynamic Pricing	7,569,935.46	552,816.76	2,452,190.98	34,863.61	90,682,842.08	40.45
Static Pricing	7,559,810.20	554,313.14	2,451,863.19	44,086.28	90,408,579.60	4.05

4.4. Experiments with Synthetic Data

Synthetic data generation presents an efficient approach for understanding general model behavior across various parameter settings. In this section, data instances are generated to examine the effects of the extensions and observe EMPOP model behavior under different configurations.

The base parameters for the synthetic data are adopted from Tunçinan et al.'s study, where detailed information about existing parameters can be found. Unit transportation costs for each retailer is set to 0.7 and remains consistent through all periods. Per period unit holding cost is fixed at 0.07 for the warehouse and 0.21 for retailers, maintaining consistency throughout all periods. Similarly, price elasticity is constant and equal for all retailers, set to 7.5. It must be recalled that some of the company application's main insights are generated from SCC increases which is one of the main reasons for dynamic price adjustments [48]. SCC remain constant throughout all periods in this synthetic data. The discussion in this section focuses on observing the extensions of the EMPOP model. Retailer capacity constraints are out of context for this similar reason, with its discussion addressed in the industry application section.

Dataset for the extended parameters of the EMPOP which are lead time, unit lost sales cost and initial retailer inventories are generated for experimenting. Lead time levels are set to the following set, same as the industry application: $\{0,1,2\}$. Unit lost sales costs are set as the following by setting critical fractiles as the same in the company application: $\{0.63,1.19,3.99\}$. This leads to nine different combinations. Initial retailer inventories are generated with a systematic approach, first unconstrained optimal MPOP model sales during the frozen periods (equal to lead time) are recorded. $\frac{1}{5}$ of this optimal sales volume is set as the first level for *I0* and is denoted as $I0_1$, $\frac{1}{2}$ of the optimal is denoted as $I0_2$, optimal is denoted as $I0_3$ and finally $\frac{3}{2}$ of the optimal is set and indicated as $I0_4$. Observe that same level of *I0* results in varying quantity of total retailer inventories for different lead times (more inventory for longer lead time as it covers longer periods as frozen). This setting is chosen to eliminate an initial disadvantage on total profit for longer lead times. Four different levels are set for the warehouse initial quantity following the base study, which are denoted as q_1, q_2, q_3 and q_4 and represent $\frac{1}{3}$ of optimal total sales of MPOP, $\frac{2}{3}$ of optimal total sales of MPOP, optimal sales of MPOP and $\frac{4}{3}$ of optimal sales of MPOP respectively. Overall, there are 144 (*I0*, ls_s, l_s, *q*) combinations. Since *I0* assignments aren't needed when lead time is zero, the final count is reduced to 108 distinct runs. These scenarios are tested for both |*S*| =10 and |*S*| =20 and three types of potential demand data from Tunçinan et al.'s dataset: cyclical, decreasing, and constant trends. Total potential demand is the same for all different datasets under same |*S*|. The EMPOP model is run with 648 different configurations in total. Please note that initial retailer inventories are reduced from the initial warehouse inventory levels to prepare environments where the total inventories are equal for all models under same q. Incumbent gap from the optimal solution is set as 0.15% for the experiments under q_2 with l_s=0.63 due to long CPU times. This value is determined after several test runs.

Total number of retailers in the system affect total profit as total demand potential changes. As the number of retailers declines, total potential demand results in a proportional reduction. Consequently, this reduction results in a corresponding decrease in total profit, while maintaining consistent per-retailer profitability and pricing strategies throughout the planning period. Overall results can be observed in Table 4.6. The observed outcomes align with expectations, where reductions in retailer count and market potential demonstrate a proportional impact on overall system profitability.

Table. 4.6. Overall results of EMPOP model under different $|S|$ settings.

Number of Retailers	Avg. Total Profit	Avg. Per Period Per Retailer Profit	Avg. Min. Price	Avg. Max. Price
10	57,209.80	238.37	7.42	9.75
20	114,419.60	238.37	7.42	9.75

The results for $|S|$=20 are used in the rest of this chapter for further discussions. Each potential demand scenario is aggregated over 108 runs for the results of EMPOP model, following the previous explanation. Considering the fact that initial retailer inventories would create an advantage for longer lead times as there will be more amount of inventory without transportation costs, costs are incurred to them for the following experiments in this section. This addition solely affects total profit, as this component has a multiplication of two parameters (w_{st} and $I0_s$) and does not affect independent variables. This adjustment allows for a more efficient total profit comparison among different parameter settings and allows for more valuable managerial insights. Firstly, MPOP model results are presented in Table 4.7 to allow a clear basis for comparison with EMPOP model results.

Table 4.7. Overall total profit and pricing results of the MPOP model.

	Constant Demand Potential			Cyclical Demand Potential			Decreasing Demand Potential		
	Total Profit	Min. Price	Max. Price	Total Profit	Min. Price	Max. Price	Total Profit	Min. Price	Max. Price
MPOP	115,919.03	7.97	8.58	119,738.29	6.87	9.69	119,232.50	7.60	9.08

4.4.1. CPU Times

CPU times of the EMPOP model changing on the levels of the extended parameters are recorded and presented in Table 4.8. Lower unit lost sales cost results in higher CPU times. This may be a surprising conclusion since one may expect the extended parameters would increase the CPU time of the model, however a closer model setting to MPOP model results in higher CPU time. When unit lost sales costs are higher, avoiding lost sales becomes clearly preferable, simplifying the model's optimization process. On the other hand, as unit lost sales cost gets closer to other supply chain and holding costs, the model must evaluate more complex trade-offs. Lead times show a contrasting behavior, reflecting the additional complexity of managing longer planning horizons.

Table 4.8. Average CPU times of the EMPOP model.

Unit Lost Sales Cost	Avg. CPU Time (seconds)	Lead Time	Avg. CPU Time (seconds)
0.63	302.39	**0**	57.85
1.19	49.91	**1**	91.23
3.99	34.09	**2**	123.90

4.4.2. Total Profit and Pricing Decisions

EMPOP model significantly changes the prices during the initial frozen periods (one period for lead time one and two periods for lead time two), while maintaining consistent prices in other periods across different EMPOP configurations and in comparison, to the MPOP model. Figure 4.7 compares pricing decisions across EMPOP models with lead times of 1 (EMPOP1) and 2 (EMPOP2) against the MPOP model. Half of the total planning periods are shown in the graphs for a clear view, with rest of the periods showing similar trends for all potential demand scenarios. While both EMPOP settings demonstrate overall alignment with MPOP and among each other, significant price adjustments occur during initial frozen periods. Both lead time scenarios show higher prices in period, but EMPOP with lead time of one period closely align with MPOP model from period two and onwards. This pattern indicates that longer lead times require higher prices across a greater number of periods. This conclusion is valid for all potential demand sets.

EMPOP model with lead time zero results in identical pricing choices with MPOP for all demand potential scenarios, hence they are omitted from these figures. Expectedly, EMPOP model has lower average total profit due to integration of lost sales costs. These results depend on the total available inventory and potential demand data. It is observed that EMPOP model's average total profit is lower under q_1, however the same for all other q levels. This aligns with Tunçinan et al.'s experimental findings, which shows that the likelihood of lost sales behavior increases when total available inventory decreases [2]. With this paper's data, the model only shows lost sale behavior under q_1, therefore total profits and pricing choices are identical with MPOP for other q levels. Even for q_1, the pricing choices remain identical since the models select upper bounds as optimal prices in these cases.

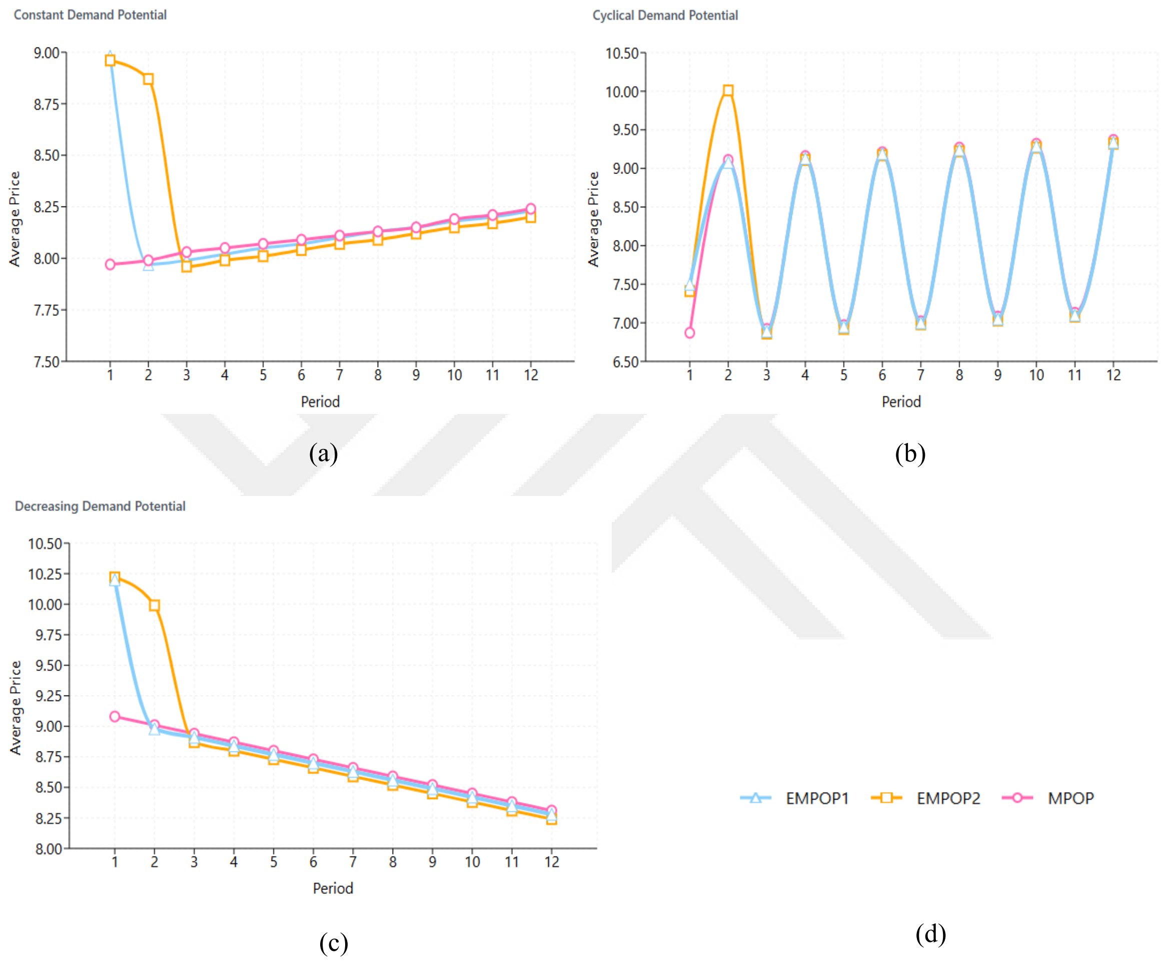


Figure 4.7. Average prices in each period under different lead times for constant (a), cyclical (b) and decreasing (c) demand potentials for models in (d).

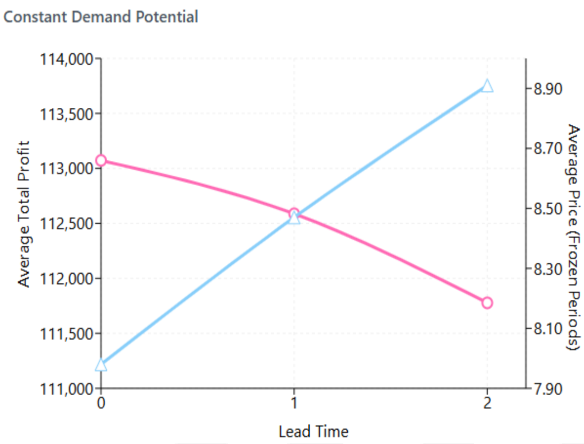
After observing EMPOP model significantly adjusts the prices during frozen periods while prices remain similar in the rest of the periods, metrics for comparing different levels of the new parameters are selected as overall average minimum and maximum prices, total profits and average prices during the periods one and two together as the longest lead time is set as two for the experiments. In the plots, this final metric is labeled as frozen period for simplicity and its relevancy in results' assessing is observed in Figure 4.7. Results can be found in Table 4.9 and Figure 4.8 for aggregating the runs over different lead times, including zero as a possible lead time.

Table 4.9. Overall maximum and minimum prices under different lead times.

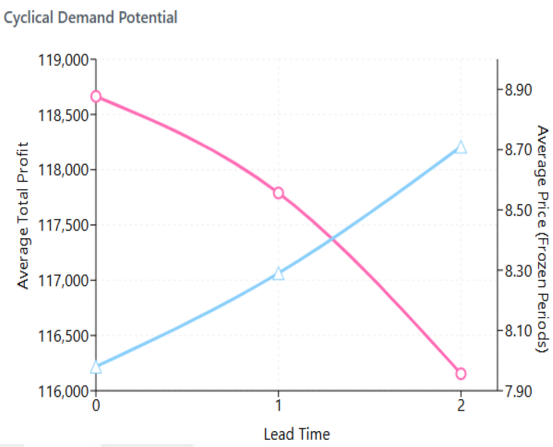
	Constant Demand Potential		**Cyclical Demand Potential**		**Decreasing Demand Potential**	
Lead Time	Avg. Min. Price	Avg. Max. Price	Avg. Min. Price	Avg. Max. Price	Avg. Min. Price	Avg. Max. Price
0	7.97	8.58	6.87	9.69	7.60	9.08
1	7.95	9.28	6.85	9.71	7.42	10.21
2	7.91	9.28	6.79	10.36	7.37	10.23

As seen in Figure 4.8, lead time zero has higher average total profit comparing to positive lead times for all demand potential data. In the plots, average total profit has a negative relationship also among the positive lead times. While these hold true for all configurations with insufficient initial retailer inventories ($I0_1$ and $I0_2$), other configurations ($I0_3$ and $I0_4$) result in higher total profit for longer lead times in the majority of the configurations, detailed results can be found in Appendix A. Under the same levels of q and $I0$, longer lead times require higher total initial retailer inventories based on the setting nature of the experiments. While more than required inventory may result as an excess inventory during initial periods and possess a disadvantage due to holding costs, total profits indicate that the advantage of starting with more inventory may also possess advantages. These advantages contain reduced main warehouse holding costs and increased total sales under some q levels. These complex results highlight that comparison among different configurations is not straightforward due to initial inventory assignments and results can vary depending on the ratios between model parameters. These findings are important to note for future applications of EMPOP model while setting the model and investigating its results.

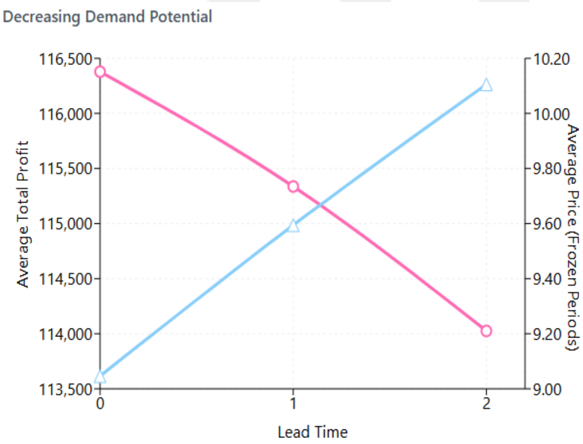
The negative relationship between lead times and total profit can be generalized for the cases with insufficient initial retailer inventories as a result of this paper's experimental settings. Looking at the subject from the real-life perspective, longer lead times may affect total profit suggesting they correspond to higher transportation costs, which is not always valid. In fact, companies sometimes choose shipping options with longer lead times to reduce their logistics costs. Therefore, this negative relationship can be sympathized considering the dynamic business environments. For instance, when a retailer suddenly needs more inventory upon a changing business circumstance, having insufficient inventory on hand becomes especially impactful in a negative way for retailers with longer lead times, proven by the experiments in this paper.



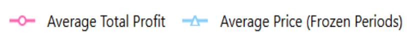
(a)



(b)



(c)



(d)

Figure 4.8. Change in the total profit and average price (in frozen periods) depending on lead times for constant (a), cyclical (b) and decreasing (c) demand potentials with labeling in (d).

Similar experiments are completed for analyzing how the unit lost sales costs affect the model results by aggregating EMPOP runs over different unit cost sales levels. Total profit of the EMPOP model decreases as the unit lost sales increases for all demand potential scenarios while average price in frozen periods increases, seen in Figure 4.9. Table 4.10 shows that the overall maximum prices have the tendency to increase as the unit lost sales cost increases. Similar to longer lead times, higher unit lost sales cost leads the model to choose a broader range of prices during the planning period. This finding has practical implications for businesses, it indicates that companies might benefit from more flexible pricing strategies for retailers with higher unit lost sales costs, which typically corresponds to retailers with higher service level targets. As there seems to be a close to linear negative relationship between unit lost sales cost and total profit for all potential demand types, the characteristics for price choices do not show such trend and vary among different demand data.

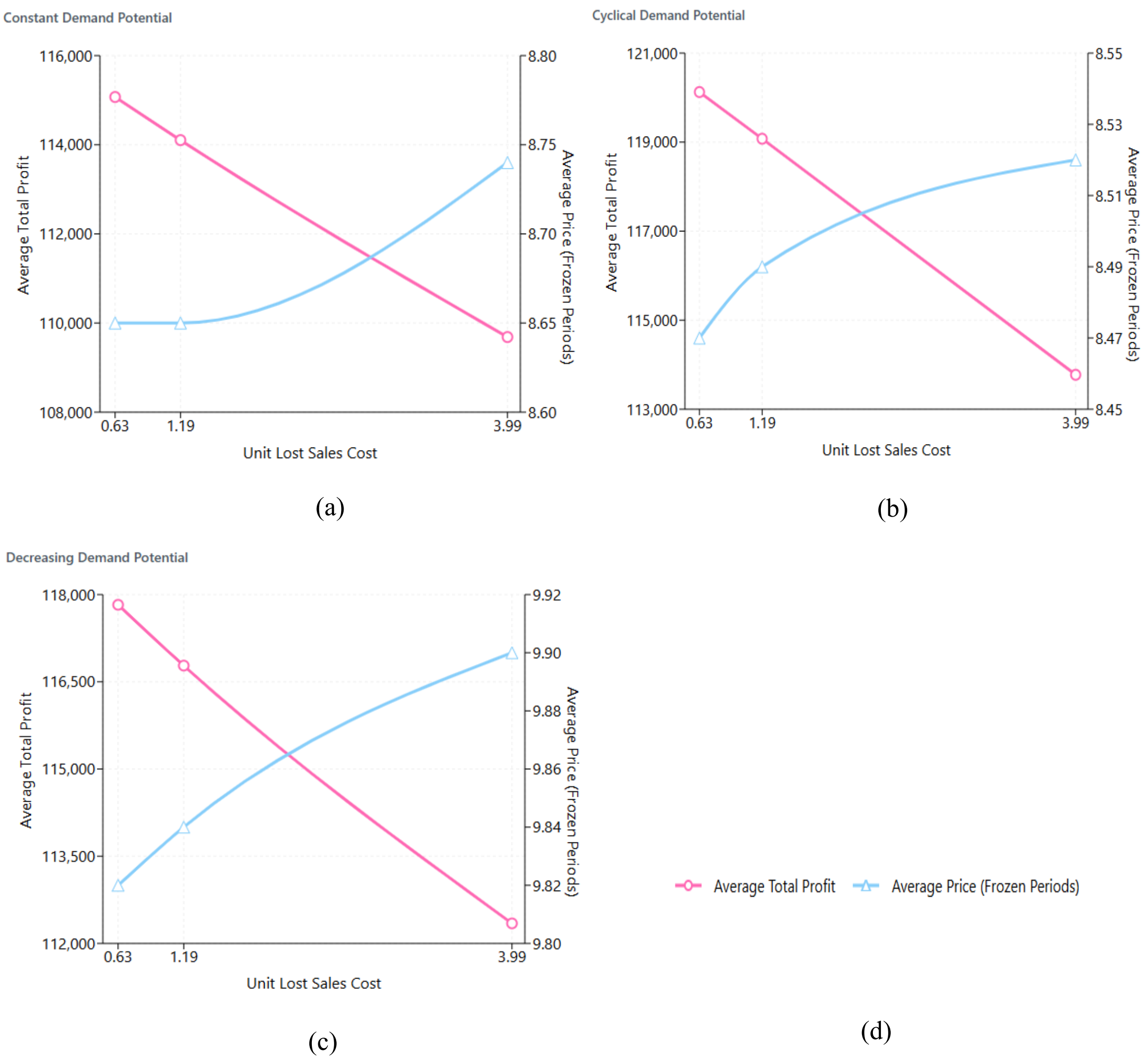


Figure 4.9. Change in the overall total profit and average price during first two periods for constant (a), cyclical (b) and decreasing (c) demand potentials with labeling in (d).

EMPOP model adjusts the prices during the frozen periods to decrease total lost sales cost. When unit lost sales cost is higher, the model responds by taking stronger measures to reduce demand, which leads to higher average prices during frozen periods and higher overall maximum prices. See that the results for all variables presented in the graphs and the table become closer to MPOP model as the unit lost sales cost decrease. This is an expected result since MPOP model has no lost sales cost component included in the objective function.

Table 4.10. Overall maximum and minimum prices for different unit lost sales costs.

	Constant Demand Potential		**Cyclical Demand Potential**		**Decreasing Demand Potential**	
Unit Lost Sales Cost	Avg. Min. Price	Avg. Max. Price	Avg. Min. Price	Avg. Max. Price	Avg. Min. Price	Avg. Max. Price
0.63	7.93	9.28	6.82	10.02	7.40	10.19
1.19	7.93	9.28	6.82	10.03	7.40	10.23
3.99	7.93	9.28	6.82	10.04	7.40	10.24

4.4.3. Inventory Allocation & Sales Decisions

The following figures present models' total inventory allocations aggregated over lead time levels, with separate graphs for each total inventory level (q_1-q_4) due to their direct influence on allocation quantities. Given that allocations are aggregated over also the initial retailer inventories, direct comparison of average allocation levels between EMPOP and MPOP models requires careful interpretations.

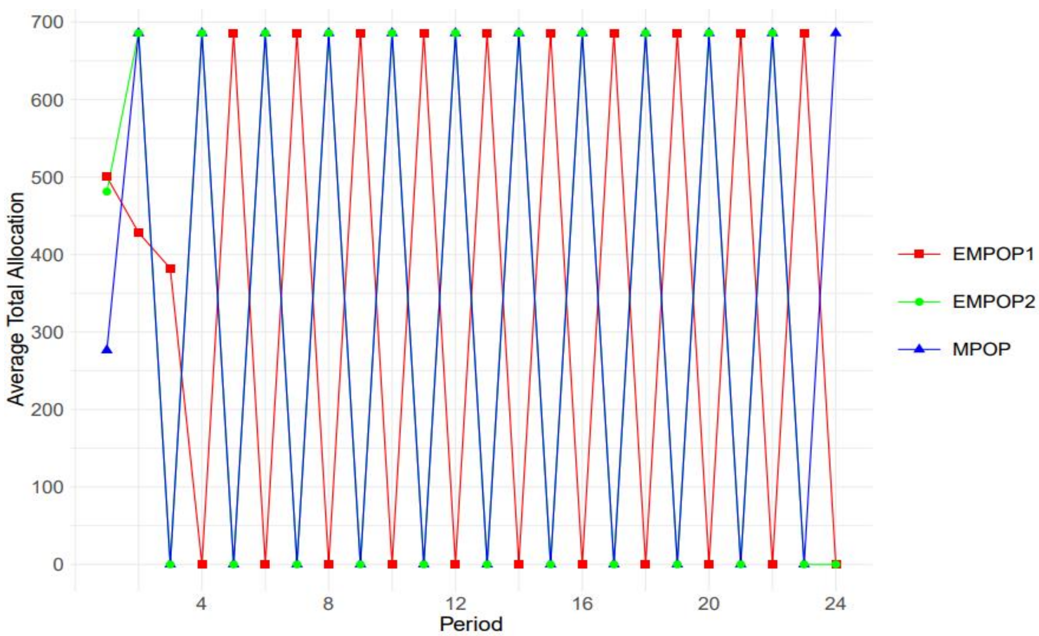


Figure 4.10. Average total allocation decisions of the models under q_1.

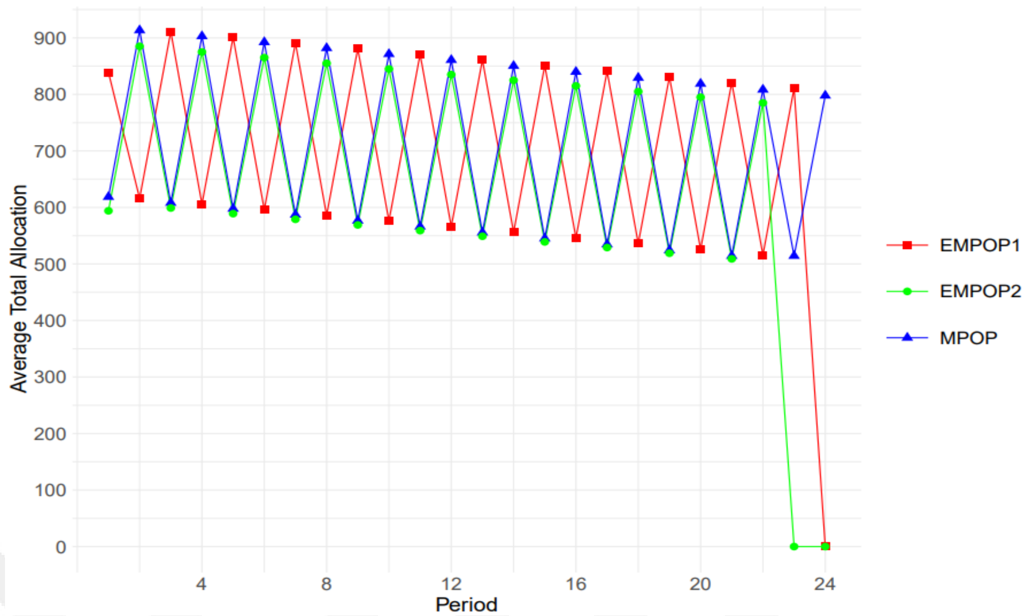


Figure 4.11. Average total allocation decisions of the models under q_2.

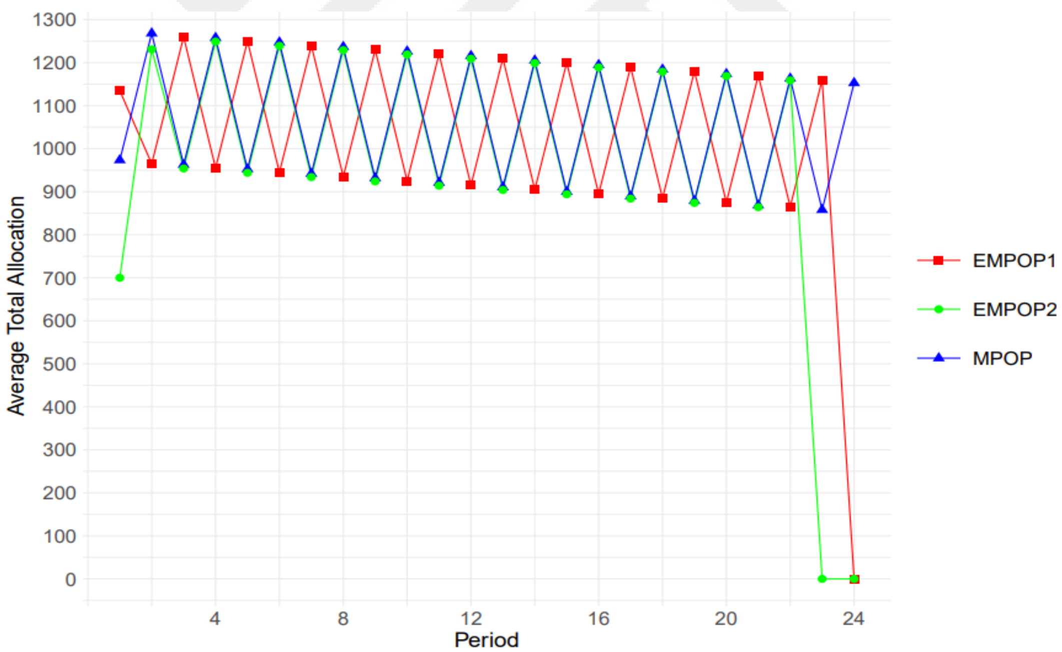


Figure 4.12. Average total allocation decisions of the models under q_3.

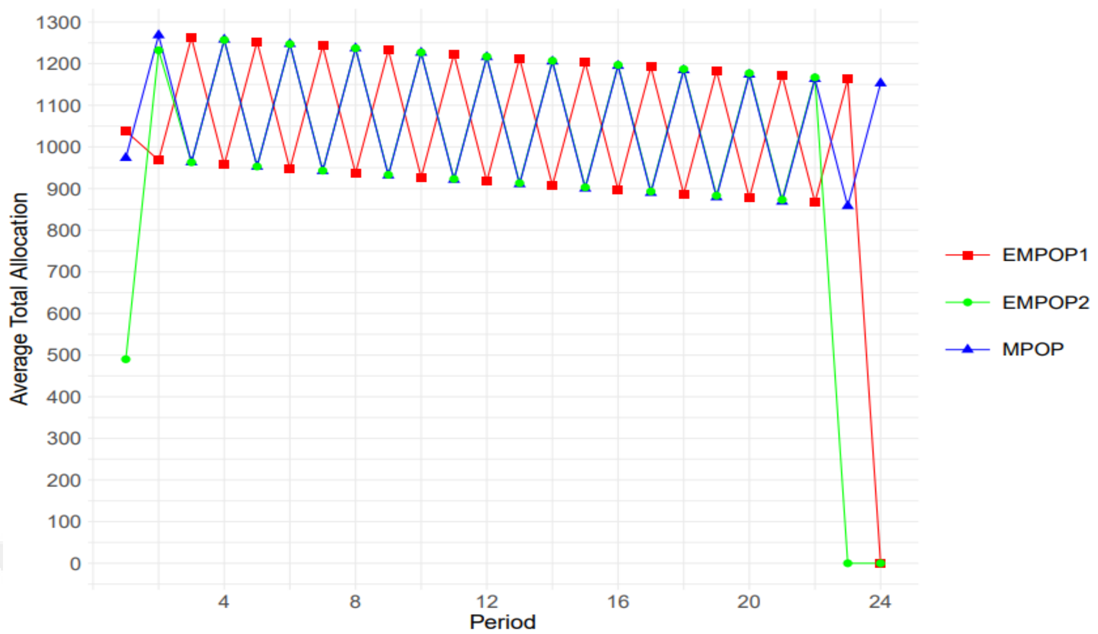


Figure 4.13. Average total allocation decisions of the models under q_4.

In scenarios with longer lead times and higher initial retailer inventory levels, EMPOP's total allocations seem to be lower than MPOP across all periods, as a result of higher initial retailer inventory levels. Lead time one scenarios provide the most meaningful comparison, revealing that EMPOP's average allocations slightly exceed MPOP during non-frozen periods. This pattern indicates the model's compensatory behavior, increasing allocations during available periods to achieve total allocations by the planning horizon's end. Figures 4.10-4.13 illustrate these patterns using cyclical potential demand data as an example.

As extensively analyzed by Tunçinan et al. [2], EMPOP models exhibit consistent behavior with total sales equaling total available inventory for q_1, q_2, and q_3, which remain at or below MPOP model optimal sales levels. Under these inventory levels, EMPOP total sales remain constant across all configurations. However, for q_4, which is above optimal total sales, sales fall below total available inventory, resulting in residual warehouse inventory at the planning horizon's end. EMPOP model configurations with different lead times demonstrate varying results at this point. Under $I0_1$ and $I0_2$ conditions, longer lead times correspond to reduced total sales compared to shorter lead times. When retailers experience insufficient inventories, extended lead times make potential sales loss compensation more challenging than with shorter lead times. This finding holds particular significance in business contexts where period-specific service levels should serve as crucial metrics, given these challenges of lost sales compensation. The selected company for the industry application has two distinct metrics to address the importance of on-time fulfillment: dispatch rate and on-time dispatch rate. While dispatch rate measures the fulfillment rate independent from demand’s timing, on-time dispatch rate considers a lost if a demand could not be fulfilled on time, even fulfilled later. The experiments in this paper support this type of distinct measurements and how companies should adapt their strategies giving on-time fulfillment required importance.

Analysis of allocation patterns reveal distinct timing differences across different lead time scenarios. Longer lead times cease allocations two periods before the horizon's end, while shorter lead times stop one period earlier. This pattern reflects the model's strategic prioritization of early allocations to ensure inventory availability during selling periods. In FMCG and other sectors where market conditions are highly dynamic, early decisions for allocation may carry out some risks. Longer lead times require earlier allocation decisions, reducing the level of operational flexibility and potentially impacting profitability when market conditions change instantly. This relationship between lead times and decision flexibility highlights a critical trade-off in supply chain management, where longer lead times may hinder the ability to adapt to dynamically changing conditions and plans.

The results of the model bring out another important insight for the industry. As seen both in the pricing and allocation graphs, the model optimizes total profit by adjusting these two variables significantly during the frozen periods, while maintaining consistent strategies across the broader planning horizon. This demonstrates that dynamic pricing and joint allocation decisions provide companies the flexibility to respond with short-term changes instead of revisions of their overall strategies. Rather than implementing wholesale changes to long-term strategy, businesses can use these decisions to make targeted adjustments during critical periods.

5. CONCLUSION

Existing multi period profit optimization model under one warehouse-multi retailer setting is extended with the addition of positive warehouse to retailer lead times, unit lost sales costs in the objective profit function and retailer inventory capacities. These extensions are implemented to enable a realistic industry application and validate model behavior across various parameter settings. The integration of these parameters addresses also a gap identified in relevant literature. The extended model named EMPOP is first studied by performing an industry application with calibrated real FMCG company data. Unit lost sales costs are chosen to be specific to each retailer and calculated by the Newsvendor formulation, retailer holding costs and critical fractiles (target service levels) taken as inputs in the formulation. Demand function is selected as a linear function, inspired from several literature, which is negatively dependent on the price. By adopting parameter calculations to a synthetic data, behavior of the model under different parameter settings with changing levels of unit lost sales costs and lead times are studied. Overall results also helped to generate managerial insights for the industry.

Model analysis reveals a pattern of bulk allocations under increasing unit transportation costs, with pricing and allocation decisions aligning with retailer potential demand. The retailer capacity constraints demonstrate a blocking effect under increasing unit transportation cost scenarios, with the ratio between transportation and other supply chain costs identified as key affecting parameter for the impact on the total profit impact. Business constraints are added to the model to see the significance of fully dynamic pricing strategy on the total profit restricting price adjustments to monthly intervals results in notable profit decreases from the un-capacitated model, with further profit reduction observed under static pricing where a single price is maintained across all periods. It is not easy to adapt fully dynamic pricing strategies for companies, while the chosen company and its distributors possess advantages for such adaptation.

EMPOP model is run under different parameter settings for further observations on model behavior. Synthetic data instances for the experiments are generated by assigning several levels of initial retailer inventory levels, lead times and unit lost sales costs. EMPOP model behavior is observed and compared with the MPOP model by selecting unit lost sales costs and lead times as independent parameters. Total profit of the EMPOP model decreases with increasing unit lost sales costs. Both longer lead times and higher unit lost sales costs expand the range of selected prices, suggesting potential benefits of flexible pricing strategies for companies. Longer lead times result in lower total profit for insufficient initial inventory levels. This suggests that longer lead times are riskier especially for companies with dynamic sales and business strategies. The complexity of the EMPOP model setting and interpretation is also discussed with the observation of model parameters' impacts on total profit. EMPOP model significantly adjusts the prices during the frozen periods, while maintaining alignment with different lead times and the MPOP model during other periods. This pattern extends to allocation decisions as well. These findings suggest that dynamic pricing and joint decision-making enable companies to address unexpected situations in shorter timeframes without comprehensive strategy revisions across the entire planning horizon. Longer lead times also indicate prior allocation decisions, again indicating inflexibility under changing conditions and possible negative impact on profit. The experiments with insufficient initial retailer inventories highlight the importance of selecting on-time fulfillment rate as a metric for companies, which is a valuable supporting insight.

For future work, it may be beneficial to use a different type of demand functions. Random demand selection and applying stochastic programming techniques for an NLP model would be worth for experimenting. Comprehensive survey by Huang et.al can be useful for observing additive, multiplicative or hybrid forms of price-dependent stochastic demand models [57]. Different demand function selection may be interesting to observe lost sales cost effects on the results. Poisson can be a good suggestion which is highly used in the existing literature. Several different business constraints can be added to observe the changes in the total profit and summarize the challenges for fully dynamic pricing, such as maximum number of adjustments, or a pre-determined range of prices. Different EMPOP model settings can be set to compare with MPOP model. One example can be instead of having equal total inventory to sell, total initial warehouse inventories can be set as equal and be observed if the results are in parallel with this paper. EMPOP model can be extended by allowing inventory flow between retailers, or by setting a multi-products joint profit optimization problem.

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APPENDIX A: DETAILED RESULTS OF EXPERIMENT INSTANCES

Raw results for the experiment configurations can be found in the following figures for each potential demand data:

Initial Retailer Inventory	Unit Lost Sales Cost	Lead Time	*q1* = 8,505			*q2* = 17,010			*q3* = 25,515			*q4* = 34,020		
			Min Price	Max Price	Total Profit	Min Price	Max Price	Total Profit	Min Price	Max Price	Total Profit	Min Price	Max Price	Total Profit
IO_1	3.99	0	10.00	10.00	51,055.12	8.87	9.67	131,188.00	6.51	7.32	136,398.00	6.51	7.32	121,514.00
IO_1	3.99	1	10.00	10.00	51,154.24	8.72	10.00	127,402.32	6.51	10.00	130,314.71	6.51	10.00	115,430.71
IO_1	3.99	2	10.00	10.00	51,247.05	8.57	10.00	123,396.38	6.51	10.00	124,140.72	6.51	10.00	109,256.72
IO_1	1.19	0	10.00	10.00	67,651.57	8.87	9.67	131,188.00	6.51	7.32	136,398.00	6.51	7.32	121,514.00
IO_1	1.19	1	10.00	10.00	67,660.02	8.72	10.00	128,662.32	6.51	10.00	131,423.71	6.51	10.00	116,540.71
IO_1	1.19	2	10.00	10.00	67,753.48	8.57	10.00	125,401.81	6.51	10.00	126,568.72	6.51	10.00	111,663.72
IO_1	0.63	0	10.00	10.00	70,861.10	8.87	9.67	131,188.00	6.51	7.32	136,398.00	6.51	7.32	121,514.00
IO_1	0.63	1	10.00	10.00	70,962.93	8.72	10.00	128,924.32	6.51	10.00	131,670.71	6.51	10.00	116,786.71
IO_1	0.63	2	10.00	10.00	71,054.48	8.57	10.00	126,787.38	6.51	10.00	127,104.72	6.51	10.00	112,186.72
IO_2	3.99	0	10.00	10.00	51,055.03	8.87	9.67	131,188.00	6.51	7.32	136,398.00	6.51	7.32	121,514.00
IO_2	3.99	1	10.00	10.00	51,293.28	8.79	10.00	129,677.81	6.51	10.00	134,836.78	6.51	10.00	119,952.78
IO_2	3.99	2	10.00	10.00	51,524.49	8.70	10.00	128,313.45	6.51	10.00	133,016.79	6.51	10.00	118,181.79
IO_2	1.19	0	10.00	10.00	67,651.59	8.87	9.67	131,188.00	6.51	7.32	136,398.00	6.51	7.32	121,514.00
IO_2	1.19	1	10.00	10.00	67,799.01	8.79	10.00	130,328.81	6.51	10.00	135,086.78	6.51	10.00	120,205.78
IO_2	1.19	2	10.00	10.00	68,030.72	8.70	10.00	128,676.45	6.51	10.00	133,577.79	6.51	10.00	118,694.79
IO_2	0.63	0	10.00	10.00	70,861.07	8.87	9.67	131,188.00	6.51	7.32	136,398.00	6.51	7.32	121,514.00
IO_2	0.63	1	10.00	10.00	71,100.48	8.79	10.00	130,458.81	6.51	10.00	135,306.78	6.51	10.00	120,356.78
IO_2	0.63	2	10.00	10.00	71,331.82	8.70	10.00	129,581.45	6.51	10.00	133,680.79	6.51	10.00	118,797.79
IO_3	3.99	0	10.00	10.00	51,055.50	8.87	9.67	131,188.00	6.51	7.32	136,398.00	6.51	7.32	121,514.00
IO_3	3.99	1	10.00	10.00	51,524.84	8.87	9.68	131,736.62	6.51	7.32	137,947.55	6.51	7.28	123,068.55
IO_3	3.99	2	10.00	10.00	51,825.39	8.84	9.68	133,033.90	6.54	7.31	139,182.58	6.51	7.25	124,312.58
IO_3	1.19	0	10.00	10.00	67,651.20	8.87	9.67	131,188.00	6.51	7.32	136,398.00	6.51	7.32	121,514.00
IO_3	1.19	1	10.00	10.00	68,030.01	8.87	9.68	132,217.62	6.51	7.32	137,943.55	6.51	7.28	123,068.55
IO_3	1.19	2	10.00	10.00	68,331.02	8.84	9.68	133,033.90	6.54	7.31	139,182.58	6.51	7.25	124,312.58
IO_3	0.63	0	10.00	10.00	70,861.60	8.87	9.67	131,188.00	6.51	7.32	136,398.00	6.51	7.32	121,514.00
IO_3	0.63	1	10.00	10.00	71,331.58	8.87	9.68	132,217.62	6.51	7.32	137,943.55	6.51	7.28	123,068.55
IO_3	0.63	2	10.00	10.00	71,633.05	8.84	9.68	133,033.90	6.54	7.31	139,182.58	6.51	7.25	124,312.58
IO_4	3.99	0	10.00	10.00	51,055.29	8.87	9.67	131,188.00	6.51	7.32	136,398.00	6.51	7.32	121,514.00
IO_4	3.99	1	10.00	10.00	51,419.26	8.81	9.68	132,083.42	6.44	7.32	137,752.33	6.51	7.28	122,873.33
IO_4	3.99	2	10.00	10.00	51,489.45	8.74	9.69	132,607.35	6.38	7.32	138,564.37	6.30	7.25	123,703.37
IO_4	1.19	0	10.00	10.00	67,651.36	8.87	9.67	131,188.00	6.51	7.32	136,398.00	6.51	7.32	121,514.00
IO_4	1.19	1	10.00	10.00	67,925.02	8.81	9.68	132,083.42	6.44	7.32	137,752.33	6.51	7.28	122,873.33
IO_4	1.19	2	10.00	10.00	67,190.82	8.74	9.69	132,607.35	6.38	7.32	138,564.37	6.30	7.25	123,703.37
IO_4	0.63	0	10.00	10.00	70,861.69	8.87	9.67	131,188.00	6.51	7.32	136,398.00	6.51	7.32	121,514.00
IO_4	0.63	1	10.00	10.00	71,226.50	8.81	9.68	132,083.42	6.44	7.32	137,752.33	6.51	7.28	122,873.33
IO_4	0.63	2	10.00	10.00	70,800.12	8.74	9.69	132,607.35	6.38	7.32	138,564.37	6.30	7.25	123,703.37

Figure A.1. Constant potential demand experiment results for $|S| = 20$.

Initial Retailer Inventory	Unit Lost Sales Cost	Lead Time	$q1$=8,505			$q2$=17,010			$q3$= 25,515			$q4$=34,020		
			Min Price	Max Price	Total Profit	Min Price	Max Price	Total Profit	Min Price	Max Price	Total Profit	Min Price	Max Price	Total Profit
IO_1	3.99	0	8.57	11.43	59,428.30	7.88	10.68	134,662.00	5.51	8.32	139,872.00	5.51	8.32	124,987.88
IO_1	3.99	1	8.57	11.43	59,298.00	7.80	10.53	132,748.39	5.55	8.57	135,580.56	5.55	8.57	120,695.74
IO_1	3.99	2	8.57	11.43	59,045.30	7.57	11.43	126,939.44	5.51	11.43	127,819.24	5.51	11.43	115,316.21
IO_1	1.19	0	8.57	11.43	75,934.30	7.88	10.68	134,662.00	5.51	8.32	139,872.00	5.51	8.32	124,987.88
IO_1	1.19	1	8.57	11.43	75,602.03	7.80	10.53	133,695.39	5.55	8.57	136,526.74	5.55	8.57	121,642.14
IO_1	1.19	2	8.57	11.43	75,550.30	7.57	11.43	129,486.78	5.51	11.43	130,200.21	5.51	11.43	117,606.21
IO_1	0.63	0	8.57	11.43	79,235.50	7.88	10.68	134,662.00	5.51	8.32	139,872.00	5.51	8.32	124,987.88
IO_1	0.63	1	8.57	11.43	79,077.90	7.80	10.53	133,885.39	5.55	8.28	136,730.02	5.55	8.28	122,176.74
IO_1	0.63	2	8.57	11.43	78,851.61	7.57	11.43	130,114.99	5.51	8.32	130,737.56	5.51	8.32	119,879.49
IO_2	3.99	0	8.57	11.43	59,428.30	7.88	10.68	134,662.00	5.51	8.32	139,872.00	5.51	8.32	124,987.88
IO_2	3.99	1	8.57	11.43	59,526.97	7.85	10.59	133,784.48	5.55	8.57	139,114.27	5.55	8.57	124,229.67
IO_2	3.99	2	8.57	11.43	59,832.50	7.69	11.43	131,989.79	5.50	11.43	136,695.53	5.50	11.43	121,810.53
IO_2	1.19	0	8.57	11.43	76,934.30	7.88	10.68	134,662.00	5.51	8.32	139,872.00	5.51	8.32	124,987.88
IO_2	1.19	1	8.57	11.43	75,505.44	7.85	10.59	134,187.48	5.55	8.57	139,320.19	5.55	8.57	124,436.33
IO_2	1.19	2	8.57	11.43	75,334.15	7.69	11.43	133,297.79	5.50	11.43	137,208.90	5.50	11.43	122,439.88
IO_2	0.63	0	8.57	11.43	79,235.50	7.88	10.68	134,662.00	5.51	8.32	139,872.00	5.51	8.32	124,987.88
IO_2	0.63	1	8.57	11.43	79,306.12	7.85	10.59	134,382.48	5.55	8.57	139,361.35	5.55	8.57	124,307.00
IO_2	0.63	2	8.57	11.43	79,639.18	7.69	11.43	133,877.71	5.50	11.43	137,311.64	5.50	11.43	123,203.53
IO_3	3.99	0	8.57	11.43	59,428.30	7.88	10.68	134,662.00	5.51	8.32	139,872.00	5.51	8.32	124,987.88
IO_3	3.99	1	8.57	11.43	69,510.60	7.88	10.68	135,722.31	5.51	8.32	141,453.28	5.51	8.28	126,573.56
IO_3	3.99	2	8.57	11.43	60,358.60	7.84	10.68	136,487.24	5.48	8.32	142,949.06	5.48	8.25	127,771.35
IO_3	1.19	0	8.57	11.43	75,934.30	7.88	10.68	134,662.00	5.51	8.32	139,872.00	5.51	8.32	124,987.88
IO_3	1.19	1	8.57	11.43	76,510.60	7.88	10.68	135,722.31	5.51	8.32	141,453.28	5.51	8.28	126,573.56
IO_3	1.19	2	8.57	11.43	76,864.20	7.84	10.68	136,486.59	5.48	8.32	142,949.06	5.48	8.25	127,771.35
IO_3	0.63	0	8.57	11.43	79,235.50	7.88	10.68	134,662.00	5.51	8.32	139,872.00	5.51	8.32	124,987.88
IO_3	0.63	1	8.57	11.43	79,811.50	7.88	10.68	135,722.31	5.51	8.32	141,453.28	5.51	8.28	126,573.56
IO_3	0.63	2	8.57	11.43	80,157.83	7.88	10.68	136,486.59	5.48	8.32	142,949.06	5.48	8.25	127,771.35
IO_4	3.99	0	8.57	11.43	59,428.30	7.88	10.68	134,662.00	5.51	8.32	139,872.00	5.51	8.32	124,987.88
IO_4	3.99	1	8.57	11.43	59,978.20	7.81	10.68	135,614.44	5.44	8.32	141,283.69	5.41	8.28	126,248.05
IO_4	3.99	2	8.57	11.43	60,076.66	7.69	10.70	136,044.17	5.32	8.33	141,998.30	5.24	8.25	127,141.60
IO_4	1.19	0	8.57	11.43	75,934.30	7.88	10.68	134,662.00	5.51	8.32	139,872.00	5.51	8.32	124,987.88
IO_4	1.19	1	8.57	11.43	76,481.80	7.81	10.68	135,614.44	5.44	8.32	141,283.69	5.41	8.28	126,248.05
IO_4	1.19	2	8.57	11.43	76,582.35	7.69	10.70	136,044.17	5.32	8.33	141,998.30	5.24	8.25	127,141.60
IO_4	0.63	0	8.57	11.43	79,235.50	7.88	10.68	134,662.00	5.51	8.32	139,872.00	5.51	8.32	124,987.88
IO_4	0.63	1	8.57	11.43	79,783.35	7.81	10.68	135,614.44	5.44	8.32	140,261.74	5.41	8.28	125,226.10
IO_4	0.63	2	8.57	11.43	79,880.17	7.69	10.70	136,044.17	5.32	8.33	141,998.30	5.24	8.25	124,788.19

Figure A.2. Cyclical potential demand experiment results for $|S|$=20.

Initial Retailer Inventory	Unit Lost Sales Cost	Lead Time	*q1* = 8,505			*q2* = 17,010			*q3* = 25,515			*q4* = 34,020		
			Min Price	Max Price	Total Profit	Min Price	Max Price	Total Profit	Min Price	Max Price	Total Profit	Min Price	Max Price	Total Profit
IO_1	3.99	0	8.57	11.43	57,248.00	8.57	9.88	133,541.00	6.32	7.51	138,752.00	6.32	7.51	123,868.00
IO_1	3.99	1.00	9	11.43	56,404.29	8.46	11.43	128,199.65	6.28	11.43	130,655.71	6.28	11.43	115,771.71
IO_1	3.99	2	8.57	11.43	55,590.31	8.23	11.43	122,706.47	6.24	11.43	122,666.98	6.24	11.43	107,783.98
IO_1	1.19	0	8.57	11.43	73,753.00	8.57	9.88	133,541.00	6.32	7.51	138,752.00	6.32	7.51	123,868.00
IO_1	1.19	1	8.57	11.43	72,910.20	8.46	11.43	129,631.80	6.28	11.35	131,927.71	6.28	11.35	117,044.71
IO_1	1.19	2	8.57	11.43	72,684.06	8.24	11.43	125,615.47	6.24	11.43	125,127.98	6.24	11.43	110,604.98
IO_1	0.63	0	8.57	11.43	77,055.00	8.57	9.88	133,541.00	6.32	7.51	138,752.00	6.32	7.51	123,868.00
IO_1	0.63	1	8.57	11.43	76,912.92	8.46	11.43	129,918.71	6.28	10.65	132,220.71	6.28	10.65	117,336.71
IO_1	0.63	2	8.57	11.43	76,700.04	8.24	11.43	126,586.47	6.24	11.43	126,088.98	6.24	11.43	111,215.98
IO_2	3.99	0	8.57	11.43	57,248.00	8.57	9.88	133,541.00	6.32	7.51	138,752.00	6.32	7.51	123,868.00
IO_2	3.99	1	8.57	11.43	56,807.39	8.54	11.43	131,286.46	6.28	11.43	136,281.28	6.28	11.43	121,397.28
IO_2	3.99	2	8.57	11.43	56,605.61	8.39	11.43	128,917.68	6.23	11.43	133,670.95	6.23	11.43	118,787.95
IO_2	1.19	0	8.57	11.43	73,753.00	8.57	9.88	133,541.00	6.32	7.51	138,752.00	6.32	7.51	123,868.00
IO_2	1.19	1	8.57	11.43	73,328.25	8.54	11.43	131,494.92	6.28	11.43	136,583.28	6.28	11.43	121,699.28
IO_2	1.19	2	8.57	11.43	73,111.61	8.39	11.43	130,317.68	6.23	11.43	134,276.95	6.23	11.43	119,392.95
IO_2	0.63	0	8.57	11.43	77,055.00	8.57	9.88	133,541.00	6.32	7.51	138,752.00	6.32	7.51	123,868.00
IO_2	0.63	1	8.57	11.43	76,358.76	8.54	11.43	131,584.64	6.28	11.43	137,480.28	6.28	11.43	121,862.28
IO_2	0.63	2	8.57	11.43	75,244.61	8.39	11.43	130,598.68	6.23	11.43	134,914.95	6.23	11.43	119,714.95
IO_3	3.99	0	8.57	11.43	57,248.00	8.57	9.88	133,541.00	6.32	7.51	138,752.00	6.32	7.51	123,868.00
IO_3	3.99	1	8.57	11.43	57,700.29	8.57	9.88	134,538.28	6.31	7.55	140,270.56	6.28	7.51	125,391.56
IO_3	3.99	2	8.57	11.43	57,962.22	8.57	9.86	135,287.37	6.31	7.49	141,440.91	6.24	7.49	126,572.91
IO_3	1.19	0	8.57	11.43	73,753.00	8.57	9.88	133,541.00	6.32	7.51	138,752.00	6.32	7.51	123,868.00
IO_3	1.19	1	8.57	11.43	74,201.54	8.57	9.88	134,538.28	6.31	7.55	140,270.56	6.28	7.51	125,391.56
IO_3	1.19	2	8.57	11.43	74,468.22	8.57	9.86	135,287.37	6.31	7.49	141,440.91	6.24	7.49	126,572.91
IO_3	0.63	0	8.57	11.43	77,055.00	8.57	9.88	133,541.00	6.32	7.51	138,752.00	6.32	7.51	123,868.00
IO_3	0.63	1	8.57	11.43	77,507.28	8.57	9.88	134,538.28	6.31	7.55	140,270.56	6.28	7.51	125,391.56
IO_3	0.63	2	8.57	11.43	77,863.22	8.57	9.86	135,287.37	6.31	7.49	141,440.91	6.24	7.49	126,572.91
IO_4	3.99	0	8.57	11.43	57,248.00	8.57	9.88	133,541.00	6.32	7.51	138,752.00	6.32	7.51	123,868.00
IO_4	3.99	1	8.57	11.43	57,580.64	8.57	9.83	134,378.92	6.32	7.46	140,498.83	6.28	7.46	125,062.83
IO_4	3.99	2	8.57	11.43	57,577.83	8.57	9.78	134,781.05	6.32	7.41	140,735.86	6.24	7.34	125,873.86
IO_4	1.19	0	8.57	11.43	73,753.00	8.57	9.88	133,541.00	6.32	7.51	138,752.00	6.32	7.51	123,868.00
IO_4	1.19	1	8.57	11.43	74,086.79	8.57	9.83	134,378.92	6.32	7.46	140,498.83	6.28	7.46	125,062.83
IO_4	1.19	2	8.57	11.43	74,083.83	8.57	9.78	134,781.05	6.32	7.41	140,735.86	6.24	7.34	125,873.86
IO_4	0.63	0	8.57	11.43	77,055.00	8.57	9.88	133,541.00	6.32	7.51	138,752.00	6.32	7.51	123,868.00
IO_4	0.63	1	8.57	11.43	77,385.09	8.57	9.83	134,378.92	6.32	7.46	140,498.83	6.28	7.46	125,062.83
IO_4	0.63	2	8.57	11.43	76,777.83	8.57	9.78	134,781.05	6.32	7.41	140,735.86	6.24	7.34	125,873.86

Figure A.3. Decreasing potential demand experiment results for $|S| = 20$.