

COMPETITION OF MANUFACTURERS FOR RETAILER SHELF SPACE

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ABSTRACT

COMPETITION OF MANUFACTURERS FOR RETAILER SHELF SPACE

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In this thesis, we consider two partially substitutable products sold through a two-echelon supply chain, consisting of two suppliers and one retailer. We assume a random aggregate demand that is split between the two products, so the products have separated random demand as long as both are available. In case of a stock-out in one of the products, that product's demand can spill over to the competing product, which causes a correlation between the final demand of the products and their stocking quantities. The retailer decides order quantities by considering the prices and the spillover rates of the products. The suppliers play a Stackelberg game on price to increase the retailer's order quantities. We analyze the equilibrium pricing decision of the suppliers and ordering decision of the retailer in a three stage dynamic game for a single period. We investigate the effect of spillover rate on pricing decision of the suppliers, the effect of spillover rates on the inefficiency caused by decentralization in the supply chain and the effect of disregarding the spillover rates by one of the suppliers or all parties in the chain.

Keywords: Supplier competition, Stackelberg game, Spillover effect, Substitution

ÖZ

ÜRETİCİLERİN PERAKENDE SATIŞ NOKTASINDAKİ RAF YERLERİ İÇİN REKABETİ

DİLER, BETÜL

Yüksek Lisans, Endüstri Mühendisliği Bölümü

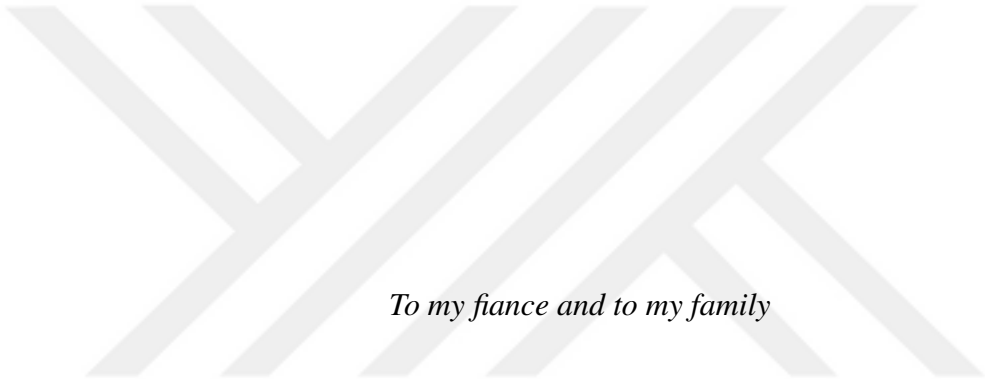
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Bu çalışma ile, ürün ikame oranlarının üreticilerin toptan fiyat kararları ve perakendecinin sipariş miktarları üzerindeki etkisi incelenmiştir. Birbiri yerine ikame edilebilir ürünler üreten iki üretici ve bu ürünlerin satıldığı bir perakendeciden oluşan, iki kademeli bir tedarik zinciri ele alınmıştır. Pazarda, bu iki ürün için ortak rassal bir talep olduğu, ve bu toplam talebin üreticilerin pazar payları oranında ürünlere paylaştırıldığı kabul edilmiştir. Bir ürünün stoğu tükendiğinde, ilk tercihi olan ürünü bulamayan tüketicinin belirli bir oranla rakip ürünü satın aldığı kabul edilmiştir. Bu durumda, perakendecinin sipariş miktarlarını, ürünler arasındaki ikame oranı ve ürünlerin toptan satış fiyatları yönlendirmektedir. Perakendeci ve üreticilerin kararları bir oyun modeli ile incelenmiştir. Oyunda, öncelikle ilk üretici, kendi ürünü için toptan satış fiyatını belirler, sonrasında, lider üreticinin toptan satış fiyatını izleyen rakip üretici kendi ürünü için toptan satış fiyatını belirler, son olarak perakendeci, maksimum karı elde edeceği sipariş miktarlarını belirler. Bu model ile, üreticilerin ürünleri arasındaki ikame edilebilirlik durumunun fiyatlandırma ve sipariş miktarı kararları üzerindeki etkisi, tedarik zinciri verimliliği üzerindeki etkisi ve üretici/perakendecinin bu ikame oranlarını görmezden gelmesinin yaratacağı etkiler araştırılmıştır.

Anahtar Kelimeler: Oyun teorisi, Üretici rekabeti, Ürün ikame oranları



To my fiancé and to my family

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CHAPTER 1

INTRODUCTION

Nowadays, we observe increasing product variety across all product categories. This is attributable to developed technology, high consumption rates, and intense competition in markets. Manufacturers always work to develop new products. When a new product is produced and introduced, thanks to effective communication channels, a demand emerges immediately for the product. If the product turns out profitable, other manufacturers quickly produce similar ones and put into service for consumers' tastes. So, in a short time, there arises a wide range of similar products, with different brands and flavors, for the consumption by people. Thus for a single category a high variety of products is available on shelves.

Retailers use different category management or assortment strategies in order to handle this wide variety of products. Firms compete on prices and quantities in order to survive in this highly competitive market. In this business environment, there is both horizontal and vertical competition between firms. A supplier competes with other suppliers in order to take part on the shelves and to have higher profits.

For a consumer product category like soft drinks, dairy products and a bar of bitter chocolate, etc. the substitutability between products is high. For instance, consider the market demand for soft drinks. It is reasonable to assume that there is an aggregate demand for soft drinks over which brands in the category (like Pepsico and Coca Cola) have a known size/share. Each customer who goes to the shelf has a first choice brand in mind and buys the product of that brand if available. However, if that product is not available, either a lost-sale occurs or the customer can buy the competing product with another brand if it is available. When a consumer switches to the

competitor's product when his first choice is not available on shelf, we say demand for the first product "spills over" to the competitor's product.

In such a competitive business environment, a supplier cannot win by concentrating only on his own products, he should be aware of the other products in the market/in that category, he should also evaluate their demands, costs, prices and spillover rates. Similarly, retailers cannot be successful by only concentrating on individual products while setting the order quantities. They should consider all products in a category altogether and analyze substitutability of the competing products; i.e., spillover rates. Monitoring spillover/substitution rates of the market helps the retailers and the suppliers to understand the market behaviors.

Our main motivation to conduct this study is to analyze the effects of spillover on competition of suppliers for retailer's order quantities. We study a category of consumer goods that consists of two partially substitutable products from two suppliers. The suppliers sell their products through a common retailer. In the market, there is an aggregate random demand for the entire category. The aggregate demand is shared by the two products according to the market sizes of the suppliers. Therefore, if both products are available, the sales of these products depends on the market shares of the suppliers. However, in a case of a stock-out in one of the products, the customer may choose to buy the competing product with a known probability which is referred to as spillover rate. The spillover probability of the customers causes a correlation between the final demand of products and their stocking quantities. The product with higher retailer margin results with higher profit for the retailer so, in the case of a spillover, the retailer overstocks the product with higher margin and understocks the other one in order to maximize her own profit. The suppliers can motivate the retailer order quantities by changing their wholesale prices. Therefore, these two suppliers compete on price for the retailer order quantities.

In this setting, to investigate the effect of supplier competition on the equilibrium pricing decisions of the suppliers and ordering decisions of the retailer, we consider a supply chain that consists of two suppliers and one retailer. We analyze the competition in this chain with a single period dynamic game of three stages. In the first stage, the first supplier sets his wholesale price. Then, in the second stage, the second

supplier sets his wholesale price after observing the leader's wholesale price. Finally, in the third stage, the retailer determines order quantities for both products, by considering wholesale prices and retail prices, aggregate demand of the category, market shares of the suppliers, and spillover rates.

After analyzing the aforementioned game for the suppliers' and the retailer's decisions, we conduct numerical experiments in order to analyze the changes in equilibrium with respect to spillover rates and spillover information. Our main research objectives are to investigate; k

- the effect of substitutability (i.e., spillover effect) on pricing decisions of the suppliers
- the effect of substitutability on the inefficiency in the total supply chain due to decentralization
- the effect of disregarding the spillover rates by one of the suppliers or all parties in the game

Our major findings can be summarized as follows. We find that the retailer's order decision depends on the relative profitability of the products; i.e., the ratio of wholesale price to the retail price. If this ratio is the same for both products, the retailer's order quantities are equal to the Newsvendor order quantities. Otherwise, the product with lower ratio (i.e. higher profitability) is favored from the retailer's perspective.

Through our numerical experiments with identical suppliers, we detected that there is a second mover advantage. That is, determining wholesale price after the other supplier is beneficial. In terms of the effects of the spillover rates, we observe that the retailer's profit increases but the suppliers' profits decrease in spillover rates mainly because the price competition between the suppliers intensifies with substitutability.

We observe that, the inefficiency due to decentralization in the supply chain decreases with spillover rates. As spillover increases, suppliers' competition strengthens which in turn mitigates the double marginalization in the chain. In fact, when products are perfect substitutes (the spillover rates are equal to 1), supplier competition turns into a Bertrand competition with zero profits for each.

We also compare the profits under the cases when the spillover rates are ignored with the case when the spillover rates are taken into account by all parties. The suppliers prefer an environment where all parties ignore spillover. We observe that, the suppliers' profits are higher but the retailer's profit is lower when all of the players in the game ignores the spillover rates. When the spillover rates are disregarded, the products have separated demands and there is virtually no competition between the suppliers. On the other hand, if only one of the suppliers disregards the spillover rates, a loss is observed in that supplier's profit which causes a gain in the competing supplier's profit; but a loss or gain may be observed in the retailer's profit depending on the spillover rates.

The rest of the thesis is organized as follows; in the next chapter, we review the related literature. Then, in Chapter 3 the model description and analysis are provided. In Chapter 4, the game between players and the players' best responses are described and characterized, and analyzed further with numerical studies. Finally, in Chapter 5, the conclusion of the study and the summary of the insights are pointed out.

CHAPTER 2

LITERATURE REVIEW

In this chapter, we review the literature related to our work. We group and organize the related literature under separate groups. In Section 2.1, we consider studies related to category and assortment management. In the following sections, we consider firstly, the studies related to limited shelf space constraint, next the studies related to supply chain coordination with contracts, then studies related to inventory-dependent demand, and finally studies about stock-out-based demand substitution.

2.1 Category and Assortment Management

The studies in this section analyze the supply chain competition with category and/or assortment management strategies. In our setting, however, there is no category/assortment management strategy adopted by the retailer and the products in the assortment are certain and known by all players of the game.

Heese and Martinez-de-Albeniz [10] derive and compare the equilibrium pricing and assortment strategies in an environment with two suppliers and one retailer. The suppliers sell partially substitutable products with random demand through a single retailer. They characterize the effect of retailers' assortment strategies on manufacturer competition. In their setting; firstly, the retailer determines an assortment strategy. The retailer's alternative strategies are (i) announcing all the assortment strategy so the products in the assortment are certain and known by the manufacturers, (ii) announcing only the number of products in the assortment, (iii) no pre-announcement. Then the manufacturer charges a margin for each product sold. Retailer also deter-

mines a unit margin for each product included in the assortment. Their model under the first scenario, where the retailer announces all her assortment strategy is similar to our study, since we also assume the category consists of two known products. They find that the dominant strategy for the retailer is to announce only the number of products in her assortment but not its composition. By doing this, the retailer forces manufacturers to quote lower margins to guarantee their participation.

Aydin and Heese [2] investigate a setting with a retailer and N manufacturers. The retailer negotiates with manufacturers over the split of profits from an assortment. In the event that the retailer and manufacturer cannot agree on how to split the profit from the assortment, the negotiation breaks down. The retailer then chooses the best assortment from the remaining $N-1$ manufacturers. The authors find that when a manufacturer differentiates its product, such improvements not only benefit the retailer but might even benefit the competing manufacturers. In this thesis, however, we analyze a supply chain with two suppliers, and we focus on the pricing and ordering behavior after the assortment is already determined.

Kurtulus and Toktay [11] investigate the effect of category captainship, where one of the manufacturers is assigned as "captain" by the retailer and all retail prices for the products in that category are determined by the captain instead of the retailer. In order to analyze the effect of category captainship, they compare two settings; the retailer determines retail prices and the captain manufacturer determines retail prices for all products in that category. In the first setting, the retailer determines shelf space for the category, then the manufacturers simultaneously determine their wholesale prices, and finally the retailer determines retail prices to maximize the total category profit subject to the shelf space constraint. In the second setting, again firstly the retailer determines the self space for the category, then only the non-captain manufacturer sets its wholesale price and finally the category captain sets retail prices in order to maximize his and the retailer's aggregate profit. As a result of their analyses, they state that, although the category captainship is criticized for being disadvantageous to the non-captain manufacturer, when the products are differentiated, category captainship may benefit also the non-captain manufacturer. Their first setting is similar with our study, but we assume no shelf space constraint, work with exogenous retail prices but demand is random and there exists spillover between products.

2.2 Limited Shelf Space

The following studies analyze the supply chain competition under a limited shelf space.

Martinez de Albeniz and Roels [16] study the competition among multiple manufacturers for the shelf space of a retailer. The retailer determines its shelf space allocation between the products. They model two different Stackelberg game settings under shelf-space-dependent deterministic demand. In the first case, the suppliers simultaneously set the retail and wholesale prices and then the retailer chooses its shelf space allocation. In the second case, suppliers simultaneously set only their wholesale prices and then the retailer determines its shelf space allocation with the retail price of the products. Based on the determined shelf space allocations and retail prices demand occurs. By comparing these two models, they characterize the inefficiencies caused by suboptimal pricing decision and the inefficiency that can be caused by suboptimal shelf space allocation. The authors state that, the inefficiency induced by suboptimal shelf space allocation is very small compared with the inefficiency caused by double marginalization. In our model, as in Martinez de Albeniz and Roels [16], the suppliers determine their wholesale prices; however, they do not set the prices simultaneously, instead they set the prices in a sequential manner. Furthermore, in our setting, the retail prices are given and known, and the retailer determines the order quantities instead of retail prices and shelf spaces. While they focus on the shelf space effects, we consider an environment with random demand and do not consider the shelf space constraint. Instead, we focus on the effects of spillover rates on the supply chain competition.

Herran et al. [8] analyze a supply chain with two competing manufacturers and one retailer. Firstly, manufacturers simultaneously determine their wholesale prices then the retailer sets the retail prices and shelf spaces for the two products. They analyze the impact of manufacturer's wholesale prices on the retailer's shelf space and pricing decisions under a deterministic demand assumption. They find the equilibrium shelf space allocation and assess its difference from the equilibrium that optimizes only the retailer's performance. They also analyze the driving forces behind the equilibrium shelf space allocation. They observe that, the lower the unit cost and/or the greater

the price elasticity, the greater the shelf space allocated to that brand; and, the higher the shelf-space elasticity, the lower the wholesale prices and the profits of all channel members.

Berman and Perry [4] analyze the shelf space allocation and inventory decisions for two products. They assume the product demand is random and is influenced by both shelf space and the inventory level. In our model also, demand is random but not shelf-space-dependent. They suggest two heuristic approaches for retailers to determine shelf space allocation and inventory leveling. In the first heuristic they disregard the shelf space effect, and in the second heuristic they disregard the inventory level effect. Then by comparing the results of both heuristics, they state that, disregarding the shelf-space-dependence of the demand is less harmless than disregarding the inventory level dependence. We also analyze the optimal order quantities of the retailer, but under the spillover effect. We analyze the supplier price equilibrium whereas they focus on the retailer's shelf space and inventory decisions.

2.3 Supply Chain Coordination with Contracts

The papers under this section analyze the supplier competition in the supply chain and coordinating contracts.

Shang et al. [17] analyze the information sharing strategies of a retailer with her manufacturers. They consider a supply chain with two competing manufacturers selling substitutable products through a common retailer. In this setting, firstly, before any demand signal is available, retailer and manufacturers contract on information sharing. Then if a contract is signed, retailer shares the demand signal with the manufacturers. Accordingly, manufacturers simultaneously determine the wholesale prices and the retailer determines the retail price for both products. Each manufacturer supplies his product to the retailer as much as the realized demand, so there is no demand uncertainty at the time of production. They use a linear demand function that depends on retail price and competition intensity. In this setting, the authors characterize the conditions that the retailer shares information with none, one or both of the manufacturers and analyze the effects of information sharing on strategic decisions of the

firms. They argue that, information contracting always benefits the retailer.

Cachon and K  k [9] study a case where a retailer sells two substitutable products supplied by two competing manufacturers. Manufacturers simultaneously announce payment schemes and contract offers to the retailer. Then the retailer chooses the prices, which in return determine the product demand, since they assume a price dependent linear demand function. By this study they try to find the changes in the contracting process when manufacturers compete. They analyze the wholesale contract, quantity discount contract and two-part tariff contract. As a result of their study, they point out that unlike in a supply chain with a single manufacturer, a manufacturer must leave the retailer with some profit to induce the retailer to carry the manufacturer's product. They also state that, the manufacturers may induce the retailer to accept sophisticated contracts when substitutability is low. Their results reinforce the significance of investing to differentiate their products. Our study, as in this work, focuses on the effect of substitution on the equilibrium decisions; but in our setting, the retailer determines the order quantities instead of retail prices.

2.4 Inventory-Dependent Demand

The papers in this group study problem settings where the demand increases with stocking quantity. In this respect, they are similar to the stocking quantity effect we capture through spillovers.

Yang and Zhang [18] and Martinez and Gonzalez [1] study optimal stocking levels of a single product for a multi period problem. They assume the rate at which an item is sold depends on its inventory level through stochastic differential equations. Although these cannot be solved explicitly, when the demand rate is linear in the stocking level, they provide closed-form formulas for the inventory path. Then they also find the optimal starting inventory levels and product sales duration. They state that inventory levels are highly dependent on the sensitivity of demand to inventory level, which lead to very high inventory levels when this sensitivity is high. In [18] also, they state that inventory-dependent-demand strengthens overstocking risk.

Balakrishnan et al. [3] consider and analyze the effect of stocking decisions on prof-

itability. They assume that, the demand of products increase with the inventory levels. They firstly characterize the products that require early replenishment, in terms of their costs, revenues and demand parameters. Then they compare the performance of the optimal policy with the performance of the traditional models. The authors state that, when inventory stimulates demand, applying the standard economic order quantity (EOQ) model yields an suboptimal order quantity. The profit-maximizing policy orders larger quantities, and can replenish inventory even before on-hand stock is fully depleted. They suggest that for products with moderate to high levels of both demand stimulation and batching economies, firms can increase profits by adopting the "stack them high, let 'em fly" philosophy, implemented via the optimal policy.

Netessine and Rudi [14] analyze the optimal stocking levels for two substitutable products with inventory level dependent random demand. They analyze the problem both under competitive and non-competitive scenarios, in the competitive case order quantities are determined by separated competitive decision makers and in the non-competitive case the central decision maker determines the order quantities. McGillivray and Silver [13] analyze the same problem in a non-competitive setting and Parlar [15] analyze the competitive case. Netessine and Rudi [14] find that competition increases stocking levels. Their studies under non-competitive environment (where there is a single decision maker to determine optimal stocking levels) are similar to the retailer's order decisions in our study. But they analyze the effect of competition on inventory levels under inventory-dependent demand but we analyze the effects of spillover rates under stock-out-based demand substitution. Moreover, they focus only on order decision of the retailer who is the single decision maker in their setting; whereas, we focus on the equilibrium in the supply chain that comprises of two manufacturers and one retailer.

2.5 Stock-Out-Based Demand Substitution

The following papers, as in our study, assume a stock-out-based demand substitution.

Boyaci [6] studies a multi-channel distribution system. In their model, the manufacturer sells his product through an independent retailer as well as through his wholly-

owned channel. Each channel has a local stochastic demand; manufacturer and retailer stock the product to satisfy their own customers' demand. In the event of a stock-out in a channel, spill-over of customers may occur. By focusing on the stocking levels of the manufacturer's wholly-owned channel and the retailer channel, the author investigates coordination mechanisms, and shows that most of the well-known, simple contracts fail to achieve coordination in this problem. We also analyze the effects of the spillover rates on the equilibrium decisions. But in our case, we assume a single channel (retailer) but two competing products on that channel and focus on the equilibrium strategies in the chain.

Ülkü and Yılmaz [12] analyze the optimal order decision of the retailer and also analyze the effects of channel coordination contracts. They also consider a two level supply chain with two substitutable products. The demand for the two products is random and independent as long as both are available. If a product stocks out, some of its demand is transferred to the available one with a known probability which creates a dependence on the amount of purchased items. Differently from our study, they assume a single manufacturer that produces two substitutable products and they take the wholesale prices as given. They also consider buy-back contracts and state that full return with partial credits contracts allow supply chain coordination under substitution.

Chiang [7] analyzes a two channel supply chain with a manufacturer and a retailer. The manufacturer distributes the product to customers through its own channel and through an independent retailer. The supplier and the retailer simultaneously choose their stock levels facing stochastic demand model, and the customers dynamically substitute between the two channels in the event of a stock-out. The paper analyzes the impact of customers' stock-out-based substitution on the product availability and the channel efficiency of a dual-channel supply chain. Our study is similar to his work in terms of the demand spillover assumption. He also analyze the effect of the spillover rates on equilibrium. But he analyzes a supply chain with a single product of a single manufacturer but with two different channels, which is different from our study. As a result of their study, they state that the effect of the stock-out-based demand substitution may increase or decrease the efficiency of a decentralized supply chain; however in our setting, we find that the spillover effect always increase the

efficiency of a decentralized supply chain. In addition, he analyzes the independent supplier-retailer case also under Stackelberg game setting and state that the channel efficiency of the dual channel distribution can be improved by the emergence of Stackelberg leadership from either the supplier or the retailer.

Borin et al. [5] analyze the optimal assortment and shelf space decisions for the retailer in a multi product category under a shelf space constraint determined by the retailer. They assume each product has separate deterministic demand with a shelf space elasticity but in the case of a stock-out, a proportion of the customers can switch to other available products. They analyze different shelf stocking rules and algorithms. They find that retailers who ignore assortment/stock-out effects may be losing substantial amounts of sales. They concentrate on the quantity decision for the single/multiple products from the retailer's perspective who is the single decision maker in this setting.

CHAPTER 3

MODEL DESCRIPTION AND ANALYSIS

In this study, we consider a two echelon supply chain consisting of two suppliers and a retailer in a single-period setting. The suppliers (S_1 and S_2) sell substitutable products to the same retailer (R). Both suppliers and the retailer are profit maximizers. The suppliers compete on wholesale prices to capture more of the retailer's demand in a Stackelberg setting. Then, the retailer, based on the wholesale and retail prices, determines the order quantities from both suppliers. Thus we consider the ordering decision of the retailer together with the pricing decisions of the suppliers.

We consider an environment where the two products are very similar, and the aggregate demand for these products is modeled by a random variable which follows a known continuous distribution. Each supplier's share in the aggregate demand is deterministic and known. If the first choice of a customer is not available, he/she will switch to the other product with a certain probability, if the other product is available.

The notation we use is provided in Table 3.1.

Our modeling assumptions are listed below:

1. Suppliers produce on order, so their production quantities are equal to quantities ordered by the retailer.
2. Production costs (c_1, c_2) and retail prices (p_1, p_2) are exogenous and both products are profitable; i.e., $0 < c_1 < p_1$ and $0 < c_2 < p_2$.
3. Since the products are very similar, there is not a significant difference between retail prices, p_1 and p_2 . Therefore, $(p_1 - p_2\alpha_1)$ and $(p_2 - p_1\alpha_2)$ values are always

Table 3.1: Notation

Decision variables	Description
w_1	wholesale price of supplier 1
w_2	wholesale price of supplier 2
Q_1	order quantity of retailer for product 1
Q_2	order quantity of retailer for product 2
Parameters	Description
p_1	retail price of supplier 1's product
p_2	retail price of supplier 2's product
X	aggregate demand for product 1 and product 2
$f(x)$	probability density function of total random demand X
$F(x)$	cumulative distribution function of total random demand X
β	product 1's share in the aggregate demand
$\tilde{D}_1$	original demand for product 1 (βX)
$\tilde{D}_2$	original demand for product 2 ($(1 - \beta)X$)
α_1	spillover rate from product 1 to product 2
α_2	spillover rate from product 2 to product 1
D_1	demand for product 1 ($D_1 = \tilde{D}_1 + \alpha_2(\tilde{D}_2 - Q_2)^+$)
D_2	demand for product 2 ($D_2 = \tilde{D}_2 + \alpha_1(\tilde{D}_1 - Q_1)^+$)
c_1	unit production cost of product 1
c_2	unit production cost of product 2

nonnegative. Therefore, when both of the products are available on retailer shelves, selling one unit of product 1 is more profitable than substituting it with product 2; similarly selling one unit of product 2 is more profitable than substituting it with product 1.

4. Leftovers are salvaged at no cost or revenue.

In this environment, we formulate the ordering and pricing decisions as a three-stage dynamic game. In the first stage, supplier 1, who is the game leader, sets his wholesale price (w_1). Then, in the second stage, supplier 2, who is the follower, sets his wholesale price with respect to leader's wholesale price, ($w_2(w_1)$). In the third and the last stage of the game, the retailer determines its order quantities (Q_1, Q_2) with respect to wholesale prices set by the suppliers. Finally, market demand for the products realizes, and corresponding revenue of the retailer is realized.

We will use backward induction to characterize the subgame perfect equilibrium of the game. We start with the third stage to characterize the retailer's ordering decision

(Section 3.1) and then we move on to the first and second stages to characterize the pricing decisions of suppliers (Section 3.2).

3.1 Third Stage: Retailer's Ordering Decision

After the wholesale prices w_1 and w_2 are determined by the suppliers, the retailer determines order quantities Q_1 and Q_2 in order to maximize her own profit function. Given the demand realization, the profit function of the retailer is characterized as

$$\pi_R(Q_1, Q_2 | X = x) = -w_1 Q_1 - w_2 Q_2 + \begin{cases} p_1 \beta x + p_2 (1 - \beta)x, & \text{if } \beta x \leq Q_1, (1 - \beta)x \leq Q_2 \\ p_2 Q_2 + p_1 \beta x + p_1 \min(\alpha_2((1 - \beta)x - Q_2), Q_1 - \beta x), & \text{if } \beta x \leq Q_1, (1 - \beta)x > Q_2 \\ p_1 Q_1 + p_2 (1 - \beta)x + p_2 \min(\alpha_1(\beta x - Q_1), Q_2 - (1 - \beta)x), & \text{if } \beta x > Q_1, (1 - \beta)x \leq Q_2 \\ p_1 Q_1 + p_2 Q_2, & \text{if } \beta x > Q_1, (1 - \beta)x > Q_2 \end{cases}$$

Note that the retailer's profit function depends on the relationship between Q_1 and Q_2 . If $\frac{Q_1}{\beta} \leq \frac{Q_2}{1-\beta}$, the second expression in the row is infeasible and the third expression in the row is infeasible otherwise. That is, if $\frac{Q_1}{\beta} \leq \frac{Q_2}{1-\beta}$ the retailer's realized profit simplifies into:

$$\pi_{R1}(Q_1, Q_2 | X = x) = -w_1 Q_1 - w_2 Q_2 + \begin{cases} p_1 \beta x + p_2 (1 - \beta)x, & \text{if } x \leq \frac{Q_1}{\beta} \\ p_1 Q_1 + p_2 (1 - \beta)x + p_2 \min(\alpha_1(\beta x - Q_1), Q_2 - (1 - \beta)x), & \text{if } \frac{Q_1}{\beta} < x \leq \frac{Q_2}{1-\beta} \\ p_1 Q_1 + p_2 Q_2, & \text{if } x > \frac{Q_2}{1-\beta} \end{cases}$$

On the other hand, if $\frac{Q_1}{\beta} > \frac{Q_2}{1-\beta}$, the retailer's profit becomes:

$$\pi_{R2}(Q_1, Q_2 | X = x) = -w_1 Q_1 - w_2 Q_2 + \begin{cases} p_1 \beta x + p_2 (1 - \beta)x, & \text{if } x \leq \frac{Q_2}{1-\beta} \\ p_2 Q_2 + p_1 \beta x + p_1 \min(\alpha_2((1 - \beta)x - Q_2), Q_1 - \beta x), & \text{if } \frac{Q_2}{1-\beta} < x \leq \frac{Q_1}{\beta} \\ p_1 Q_1 + p_2 Q_2, & \text{if } x > \frac{Q_1}{\beta} \end{cases}$$

As a result, the retailer's expected profit, π_R is characterized as:

$$\pi_R(Q_1, Q_2) = \begin{cases} \pi_{R1}(Q_1, Q_2), & \text{if } \frac{Q_1}{\beta} \leq \frac{Q_2}{1-\beta} \\ \pi_{R2}(Q_1, Q_2), & \text{if } \frac{Q_1}{\beta} > \frac{Q_2}{1-\beta} \end{cases}$$

where expected profit if $\frac{Q_1}{\beta} \leq \frac{Q_2}{1-\beta}$, π_{R1} is characterized as:

$$\begin{aligned} \pi_{R1}(Q_1, Q_2) &= -w_1 Q_1 - w_2 Q_2 + (p_1 \beta + p_2 (1 - \beta)) \int_0^{\frac{Q_1}{\beta}} x f(x) dx + p_1 Q_1 (F(\frac{Q_2}{1-\beta}) - F(\frac{Q_1}{\beta})) \\ &\quad + p_2 \int_{\frac{Q_1}{\beta}}^{\frac{Q_2 + \alpha_1 Q_1}{(1-\beta) + \alpha_1 \beta}} (1 - \beta)x + \alpha_1 (\beta x - Q_1) f(x) dx + p_2 \int_{\frac{Q_2 + \alpha_1 Q_1}{(1-\beta) + \alpha_1 \beta}}^{\frac{Q_2}{1-\beta}} Q_2 f(x) dx \\ &\quad + (p_1 Q_1 + p_2 Q_2) \int_{\frac{Q_2}{1-\beta}}^{\infty} f(x) dx \\ &= -w_1 Q_1 - w_2 Q_2 + p_1 \beta \int_0^{\frac{Q_1}{\beta}} x f(x) dx + p_2 (1 - \beta) \int_0^{\frac{Q_2 + \alpha_1 Q_1}{(1-\beta) + \alpha_1 \beta}} x f(x) dx \\ &\quad + p_2 \alpha_1 \beta \int_{\frac{Q_1}{\beta}}^{\frac{Q_2 + \alpha_1 Q_1}{(1-\beta) + \alpha_1 \beta}} x f(x) dx - p_2 \alpha_1 Q_1 (F(\frac{Q_2 + \alpha_1 Q_1}{(1-\beta) + \alpha_1 \beta}) - F(\frac{Q_1}{\beta})) \\ &\quad + p_1 Q_1 - p_1 Q_1 F(\frac{Q_1}{\beta}) + p_2 Q_2 (1 - F(\frac{Q_2 + \alpha_1 Q_1}{(1-\beta) + \alpha_1 \beta})) \end{aligned}$$

and expected profit if $\frac{Q_1}{\beta} > \frac{Q_2}{1-\beta}$, π_{R2} is characterized as:

$$\begin{aligned} \pi_{R2}(Q_1, Q_2) &= -w_1 Q_1 - w_2 Q_2 + (p_1 \beta + p_2 (1 - \beta)) \int_0^{\frac{Q_2}{1-\beta}} x f(x) dx + p_2 Q_2 (F(\frac{Q_1}{\beta}) - F(\frac{Q_2}{1-\beta})) \\ &\quad + p_1 \int_{\frac{Q_2}{1-\beta}}^{\frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2 (1-\beta)}} \beta x + \alpha_2 ((1 - \beta)x - Q_2) f(x) dx \\ &\quad + p_1 \int_{\frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2 (1-\beta)}}^{\frac{Q_1}{\beta}} Q_1 f(x) dx + (p_1 Q_1 + p_2 Q_2) \int_{\frac{Q_1}{\beta}}^{\infty} f(x) dx \\ &= -w_1 Q_1 - w_2 Q_2 + p_1 \beta \int_0^{\frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2 (1-\beta)}} x f(x) dx + p_2 (1 - \beta) \int_0^{\frac{Q_2}{1-\beta}} x f(x) dx \\ &\quad + p_1 \alpha_2 (1 - \beta) \int_{\frac{Q_2}{1-\beta}}^{\frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2 (1-\beta)}} x f(x) dx - p_1 \alpha_2 Q_2 (F(\frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2 (1-\beta)}) - F(\frac{Q_2}{1-\beta})) \\ &\quad + p_2 Q_2 - p_2 Q_2 F(\frac{Q_2}{1-\beta}) + p_1 Q_1 (1 - F(\frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2 (1-\beta)})) \end{aligned}$$

Lemma 3.1.1 *Retailer profit function, π_R , is continuous at $\frac{Q_1}{\beta} = \frac{Q_2}{1-\beta}$.*

Proof of Lemma 3.1.1 is provided in Appendix A.

We take first and second derivatives of these profit functions and firstly analyze the concavity of the profit functions and then we find the optimal order quantities by equating first derivatives to zero.

Lemma 3.1.2 *π_{R1} and π_{R2} are jointly concave in Q_1 and Q_2 .*

Proof of Lemma 3.1.2 is provided in Appendix B.

Lemma 3.1.3 *The unconstrained maximizers Q_{11}^* and Q_{12}^* of π_{R1} are given as:*

$$Q_{11}^* = \beta \max(0, F^{-1}(\frac{p_1 - w_1 - \alpha_1(p_2 - w_2)}{p_1 - \alpha_1 p_2}))$$

$$Q_{12}^* = F^{-1}(\frac{p_2 - w_2}{p_2}) (1 - \beta + \alpha_1 \beta) - \alpha_1 \beta \max(0, F^{-1}(\frac{p_1 - w_1 - \alpha_1(p_2 - w_2)}{p_1 - \alpha_1 p_2}))$$

Proof of Lemma 3.1.3 is provided in Appendix C.

Lemma 3.1.4 *The unconstrained maximizers Q_{21}^* and Q_{22}^* of π_{R2} are given as:*

$$Q_{21}^* = F^{-1}(\frac{p_1 - w_1}{p_1}) (\beta + \alpha_2(1 - \beta)) - \alpha_2 \max(0, (1 - \beta) F^{-1}(\frac{p_2 - w_2 - \alpha_2(p_1 - w_1)}{p_2 - \alpha_2 p_1}))$$

$$Q_{22}^* = \max(0, (1 - \beta) F^{-1}(\frac{p_2 - w_2 - \alpha_2(p_1 - w_1)}{p_2 - \alpha_2 p_1}))$$

The calculations for the proof of Lemma 3.1.4 is similar with the proof of Lemma 3.1.3 and given in Appendix D.

Proposition 3.1.5 *Given the wholesale prices of the suppliers, the optimal order quantities of the retailer is given by,*

$$(Q_1^*, Q_2^*) = \begin{cases} (Q_{11}^*, Q_{12}^*), & \text{if } \frac{w_1}{p_1} \geq \frac{w_2}{p_2} \\ (Q_{21}^*, Q_{22}^*), & \text{if } \frac{w_1}{p_1} < \frac{w_2}{p_2} \end{cases},$$

where

$$Q_{11}^* = \max(0, \beta F^{-1}(\frac{p_1 - w_1 - \alpha_1(p_2 - w_2)}{p_1 - \alpha_1 p_2}))$$

$$Q_{12}^* = F^{-1}(\frac{p_2 - w_2}{p_2}) (1 - \beta + \alpha_1 \beta) - \alpha_1 \max(0, \beta F^{-1}(\frac{p_1 - w_1 - \alpha_1(p_2 - w_2)}{p_1 - \alpha_1 p_2})),$$

and

$$Q_{22}^* = \max(0, (1 - \beta) F^{-1}(\frac{p_2 - w_2 - \alpha_2(p_1 - w_1)}{p_2 - \alpha_2 p_1}))$$

$$Q_{21}^* = F^{-1}(\frac{p_1 - w_1}{p_1}) (\beta + \alpha_2(1 - \beta)) - \alpha_2 \max(0, (1 - \beta) F^{-1}(\frac{p_2 - w_2 - \alpha_2(p_1 - w_1)}{p_2 - \alpha_2 p_1})).$$

Calculations for the proof of 3.1.5 is provided in Appendix E.

Corollary 3.1.6 *Optimal order decisions that maximize π_{R1} is equal to the optimal order decisions that maximize π_{R2} if $\frac{w_1}{p_1} = \frac{w_2}{p_2}$.*

Proof of Corollary 3.1.6 is provided in Appendix F.

Corollary 3.1.7 *When $w_1 \geq p_1 - \alpha_1(p_2 - w_2)$ the optimal order quantities of the retailer are $Q_1^* = 0$ and $Q_2^* = F^{-1}(\frac{p_2 - w_2}{p_2}) (1 - \beta + \alpha_1 \beta)$.*

Proof of Corollary 3.1.7 is provided in Appendix G.

Inequality $w_1 \geq p_1 - \alpha_1(p_2 - w_2)$ corresponds to $\alpha_1(p_2 - w_2) \geq p_1 - w_1$. The inequality means, the retailer margin from product 1 is smaller than or equal to the retailer margin obtained when she substitutes product 1 with product 2. Therefore, for large values of w_1 , that are $w_1 \geq p_1 - \alpha_1(p_2 - w_2)$, retailer's optimal decision is to place no order for product 1 and substituting it with product 2. The retailer place an order for product 2 as much as $Q_2^* = F^{-1}(\frac{p_2 - w_2}{p_2}) (1 - \beta + \alpha_1 \beta)$, which is equal to the sum of expected demand of product 2 (that is $F^{-1}(\frac{p_2 - w_2}{p_2}) (1 - \beta)$) and expected spillovered demand from product 1 to product 2 (that is $\alpha_1 F^{-1}(\frac{p_2 - w_2}{p_2}) \beta$).

Corollary 3.1.8 *While $w_1 \leq p_1 - \frac{(p_2 - w_2)}{\alpha_2}$, the optimal order decision of the retailer is: $Q_2^* = 0$ and $Q_1^* = F^{-1}(\frac{p_1 - w_1}{p_1}) (\beta + \alpha_2(1 - \beta))$*

Proof of Corollary 3.1.8 is provided in Appendix H.

As being similar with the case when $w_1 \geq p_1 - \alpha_1(p_2 - w_2)$; when $w_1 \leq p_1 - \frac{(p_2 - w_2)}{\alpha_2}$ (corresponds to $p_2 - w_2 \leq \alpha_2(p_1 - w_1)$), the retailer's optimal decision is to place no order for product 2 and to place an order for product 1 as much as $Q_1^* = F^{-1}\left(\frac{p_1 - w_1}{p_1}\right) (\beta + \alpha_2(1 - \beta))$ which is sum of the expected demand for product 1 and expected spillovered demand from product 2 to product 1.

The optimal decisions of the retailer is summarized in Table 3.2:

Table 3.2: Retailer's order quantity decisions

Cases	Conditions	$Q_1^*(w_1, w_2)$	$Q_2^*(w_1, w_2)$
Case 1	$w_1 \leq p_1 - \frac{(p_2 - w_2)}{\alpha_2}$	$F^{-1}\left(\frac{p_1 - w_1}{p_1}\right)(\beta + \alpha_2(1 - \beta))$	0
Case 2	$\frac{w_1}{p_1} < \frac{w_2}{p_2}$	$F^{-1}\left(\frac{p_1 - w_1}{p_1}\right)(\beta + \alpha_2(1 - \beta)) - \alpha_2 Q_2$	$F^{-1}\left(\frac{p_2 - w_2 - \alpha_2(p_1 - w_1)}{p_2 - \alpha_2 p_1}\right)(1 - \beta)$
Case 3	$\frac{w_1}{p_1} = \frac{w_2}{p_2}$	$F^{-1}\left(\frac{p_1 - w_1}{p_1}\right)\beta$	$F^{-1}\left(\frac{p_2 - w_2}{p_2}\right)(1 - \beta)$
Case 4	$\frac{w_1}{p_1} > \frac{w_2}{p_2}$	$F^{-1}\left(\frac{p_1 - w_1 - \alpha_1(p_2 - w_2)}{p_1 - \alpha_1 p_2}\right)\beta$	$F^{-1}\left(\frac{p_2 - w_2}{p_2}\right)(1 - \beta + \alpha_1\beta) - \alpha_1 Q_1$
Case 5	$w_1 \geq p_1 - \alpha_1(p_2 - w_2)$	0	$F^{-1}\left(\frac{p_2 - w_2}{p_2}\right)(1 - \beta + \alpha_1\beta)$

Corollary 3.1.9 When $\frac{w_1}{p_1} \leq \frac{w_2}{p_2}$ retailer's order quantities, Q_1^* and Q_2^* are not dependent on α_1 ; i.e., the spillover rate from product 1 to product 2. Similarly, when $\frac{w_1}{p_1} \geq \frac{w_2}{p_2}$ retailer's order quantities, Q_1^* and Q_2^* are not dependent on α_2 ; i.e., the spillover rate of from product 2 to product 1.

Corollary 3.1.10 $Q_1^*(w_1, w_2)$ is decreasing in w_1 and $Q_2^*(w_1, w_2)$ is increasing in w_1 .

Proof of Corollary 3.1.10 is provided in Appendix I.

We can have the following observations on the optimal order decision of the retailer.

Product margins and spillover rates drive the retailer's order decisions. When w_1 is low enough, when $w_1 \leq p_1 - \frac{(p_2 - w_2)}{\alpha_2}$, the retailer places no order for product 2 because retailer's margin from product 1 is very high compared to her margin from product 2. Instead of selling product 2, selling an additional product 1 from the spillover demand

is more profitable for the retailer. Therefore, when $w_1 \leq p_1 - \frac{(p_2 - w_2)}{\alpha_2}$, supplier 2 is out of the market and from this point on we refer to this case as "Case 1: Supplier 2 is out of the market ". As w_1 increases, Q_1 decreases and Q_2 increases. Similarly, when w_2 is low enough, when $w_1 \geq p_1 - \alpha_1(p_2 - w_2)$, the retailer places no order for product 1; and from this point on we refer to this case as "Case 5: Supplier 1 is out of the market ".

When $\frac{w_1}{p_1} = \frac{w_2}{p_2}$, spillover rates (α_1 and α_2) have no effect on the optimal order decisions of the retailer, Q_1^* and Q_2^* . When $\frac{w_1}{p_1} = \frac{w_2}{p_2}$, which corresponds to $\frac{p_2 - w_2}{p_2} = \frac{p_1 - w_1}{p_1}$, so we can write optimal order decision of the retailer as $Q_1^* = \beta F^{-1}\left(\frac{p_1 - w_1}{p_1}\right) = \beta F^{-1}\left(\frac{p_2 - w_2}{p_2}\right)$, and $Q_2^* = (1 - \beta)F^{-1}\left(\frac{p_2 - w_2}{p_2}\right) = (1 - \beta)F^{-1}\left(\frac{p_1 - w_1}{p_1}\right)$. It is seen that α_1 and α_2 has no effect on retailer's order decision.

When $\frac{w_1}{p_1} = \frac{w_2}{p_2}$, retailer's total order quantity for these two products is equal to the Newsvendor order quantity for aggregate demand of these products. Total order quantity of retailer can be defined as, $Q^* = Q_1^* + Q_2^* = F^{-1}\left(\frac{p_1 - w_1}{p_1}\right) = F^{-1}\left(\frac{p_2 - w_2}{p_2}\right)$, which is equal to the Newsvendor order quantity: $Q^* = F^{-1}\left(\frac{\text{underagecost}}{\text{underagecost} + \text{overagecost}}\right)$ where underage cost is $p_1 - w_1$ or $p_2 - w_2$ and overage cost is w_1 or w_2 . And the share of a product in the total order quantity is proportional with that product's original demand rate in the total expected demand. It is seen that, $\frac{Q_1^*}{Q^*} = \beta$ and $\frac{Q_2^*}{Q^*} = 1 - \beta$, which means that the total order quantity is partitioned to product 1 and product 2 according to their original demand rates in the market; $Q_1^* = \beta Q^*$ and $Q_2^* = (1 - \beta)Q^*$. From this point on we refer to the cases where $\frac{w_1}{p_1} = \frac{w_2}{p_2}$ as "Case 3: Newsvendor case".

When $\frac{w_1}{p_1} < \frac{w_2}{p_2}$, the retailer's margin from product 1 is larger compared to her margin from product 2. Which means, selling one unit from product 1 is more profitable for her than selling one unit from product 2. Therefore, to give higher order from product 1 by considering expected spillover demand from product 2 to product 1 is profitable for the retailer. Under this case, Supplier 1 is the favorable supplier from the retailer's perspective and from this point on we refer to the case where $\frac{w_1}{p_1} < \frac{w_2}{p_2}$ as "Case 2: Supplier 1 is favored ". Similarly, when $\frac{w_1}{p_1} > \frac{w_2}{p_2}$ the retailer increases the order quantity from product 2, by considering α_1 . So supplier 2 is favored from the retailer's perspective and from this point on we refer to the case where $\frac{w_1}{p_1} > \frac{w_2}{p_2}$ as "Case 4: Supplier 2 is favored ".

3.2 Second stage: Supplier 2's Wholesale Price Decision

In this stage, we analyze the suppliers' wholesale price decisions. We assume that supplier 1 is the leader and supplier 2 is the follower. We model the resulting environment as a Stackelberg game. We start our analysis with the follower's problem, which is to maximize its profit,

$$\pi_2(w_1, w_2) = Q_2(w_2 - c_2), \quad (3.1)$$

given w_1 . Although the profit expression in Equation 3.2 looks simple, its analysis is cumbersome due to the nature of the optimal order quantity (see Table 3.2). To illustrate, we next assume that the aggregate demand is uniformly distributed between $[0, T]$ and analyze the behavior of the supplier profits.

Next, we characterize supplier 2's profit function $\pi_2(w_1^*, w_2^*, Q_1^*(w_1, w_2), Q_2^*(w_1, w_2))$ for uniform demand distribution.

Proposition 3.2.1

When $\alpha_1\beta(p_2 - \alpha_2p_1) \leq \alpha_2(1 - \beta)(p_1 - \alpha_1p_2)$ supplier 2's profit function is unimodal.

Proof of Proposition 3.2.1 is provided in Appendix J.

If the condition in Proposition 3.2.1 is not satisfied, the expected profit of the second supplier can be bimodal, where one of the local maximums locates at the left hand side of the point $w_2 = p_2 \frac{w_1}{p_1}$ and the other one locates at the right hand side of that point.

We can see the possible bimodal behavior of the profit function on a numerical example. Under the parameter set given in Table 3.3, the condition $\alpha_1\beta(p_2 - \alpha_2p_1) \leq \alpha_2(1 - \beta)(p_1 - \alpha_1p_2)$ is not satisfied and supplier 2's profit function is bimodal as it is seen in Figure 3.1. Therefore, in such cases, we cannot characterize the best response of supplier 2.

When we solve supplier 2's best response problem, we get the following graph for supplier 2's profit function with respect to w_2 ,

Table 3.3: Parameters for bimodal supplier profit functions

c_1	c_2	p_1	p_2	β	α_1	α_2	demand	w_1
3	3	10	10	0.5	0.8	0.2	U(0,100)	5

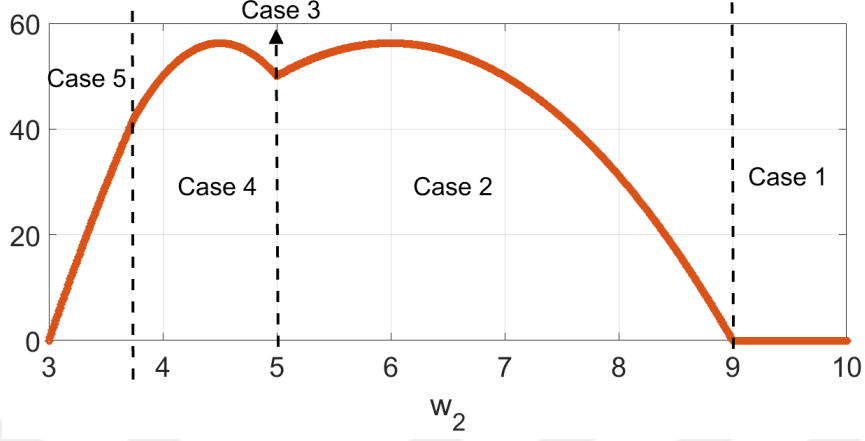


Figure 3.1: Supplier 2's profit function wrt w_2 , for $w_1 = 5$

Corollary 3.2.2 When $\alpha_1\beta(p_2 - \alpha_2p_1) \leq \alpha_2(1 - \beta)(p_1 - \alpha_1p_2)$, the best response function for supplier 2, w_2^* , is given by:

$$w_2^*(w_1) = \begin{cases} \frac{p_2+c_2}{2}, & \text{if } D_2 < w_1 \\ p_2 - \frac{p_1-w_1}{\alpha_1}, & \text{if } C_2 < w_1 \leq D_2 \\ \frac{p_2+c_2}{2} - \frac{\alpha_1(\beta)(p_1-w_1)p_2}{2((1-\beta)(p_1-\alpha_1p_2)+\alpha_1\beta p_1)}, & \text{if } A_2 < w_1 \leq C_2 \\ p_2 \frac{w_1}{p_1}, & \text{if } B_2 < w_1 \leq A_2 \\ \frac{p_2-\alpha_2(p_1-w_1)+c_2}{2}, & \text{if } w_1 \leq B_2 \end{cases}$$

where,

$$A_2 = \frac{p_1}{p_2} \left(\frac{(1-\beta)(p_1-\alpha_1p_2)(p_2-c_2)}{2(1-\beta)(p_1-\alpha_1p_2)+\alpha_1(\beta)p_1} + c_2 \right)$$

$$B_2 = \frac{p_1(p_2-\alpha_2p_1+c_2)}{2p_2-\alpha_2p_1}$$

$$C_2 = p_1 - \frac{\alpha_1(p_2-c_2)((1-\beta)(p_1-\alpha_1p_2)+\alpha_1(\beta)p_1)}{2((1-\beta)(p_1-\alpha_1p_2)+\alpha_1(\beta)p_2)-\alpha_1\alpha_1(\beta)p_2}$$

$$D_2 = p_1 - \frac{\alpha_1(p_2-c_2)}{2}$$

Proof of Corollary 3.2.2 is provided in Appendix K.

The supplier profit functions are not unimodal under all conditions. Therefore, we cannot characterize the suppliers' best response for the general case and so we cannot solve the Stackelberg equilibrium analytically. In the following section, we will continue further analyses with numerical experiments.

CHAPTER 4

NUMERICAL STUDY

In this section, we provide our results from numerical experiments. We conduct numerical experiments in order to analyze the suppliers' best response decisions in the first and second stages of the game. We analyze the changes in the equilibrium decisions with respect to changes in the parameters, and we search for the answers of our research questions. We start our analyses from the second stage of the game; i.e., the follower supplier's best response to a given wholesale price of the leader supplier, w_1 . Then in the consequent sections, we move on to the first stage and analyze the leader supplier's price decisions and the subgame perfect equilibrium of the game. Throughout this section, our base parameter set is as given in Table 4.1, we assume that demand is uniformly distributed over $[0,100]$. We analyze the changes in the best response decisions and profit functions by changing one parameter at a time and taking others given as in Table 4.1.

Table 4.1: Base Model Parameters

c_1	c_2	p_1	p_2	β	α_1	α_2	demand	w_1
3	3	10	10	0.5	0.5	0.5	U(0,100)	5.5

In this setting, under uniform demand assumption, retailer's optimal orders are characterized as in Table 4.2:

4.1 Follower Behavior Analyses

In this part, we conduct a numerical study in order to analyze the behavior of the follower's profit function. We analyze the changes in the follower's profit function,

Table 4.2: Retailer's ordering decisions under uniform demand

Cases	Conditions	Q_1^*	Q_2^*
Case 1	$w_2 \geq p_2 - \alpha_2(p_1 - w_1)$	$100 \left(\frac{p_1 - w_1}{p_1} \right) (\beta + \alpha_2(1 - \beta))$	0
Case 2	$\frac{w_1}{p_1} < \frac{w_2}{p_2}$	$100 \left(\frac{p_1 - w_1}{p_1} \right) (\beta + \alpha_2(1 - \beta)) - \alpha_2 Q_2$	$(1 - \beta) 100 \left(\frac{p_2 - w_2 - \alpha_2(p_1 - w_1)}{p_2 - \alpha_2 p_1} \right)$
Case 3	$\frac{w_1}{p_1} = \frac{w_2}{p_2}$	$100 \left(\frac{p_1 - w_1}{p_1} \right) \beta$	$100 \left(\frac{p_2 - w_2}{p_2} \right) (1 - \beta)$
Case 4	$\frac{w_1}{p_1} > \frac{w_2}{p_2}$	$\beta 100 \left(\frac{p_1 - w_1 - \alpha_1(p_2 - w_2)}{p_1 - \alpha_1 p_2} \right)$	$100 \left(\frac{p_2 - w_2}{p_2} \right) (1 - \beta + \alpha_1 \beta) - \alpha_1 Q_1$
Case 5	$w_2 \leq p_2 - \frac{(p_1 - w_1)}{\alpha_1}$	0	$100 \left(\frac{p_2 - w_2}{p_2} \right) (1 - \beta + \alpha_1 \beta)$

$\pi_2(w_1, w_2)$, with respect to spillover rates, α_1 and α_2 ; leader's wholesale price, w_1 ; production cost, c_2 ; retail price, p_2 ; and market size, β ; by changing one parameter at a time. We take w_1 as given and equal to 5.5 as a base value, and other parameters at base parameter values. We solve the follower's problem by full enumeration of w_2 , by changing it's value from 3 to 10 with increment size 0.01.

We start our analysis by varying c_2 , which is the production cost for product 2. We plot $\pi_2(w_1 = 5.5, w_2)$ for c_2 values 2.5, 3 and 3.5 in Figure 4.1. The optimal points are marked on the figures, X corresponds to w_2 and Y corresponds to $\pi_2(w_1, w_2)$.

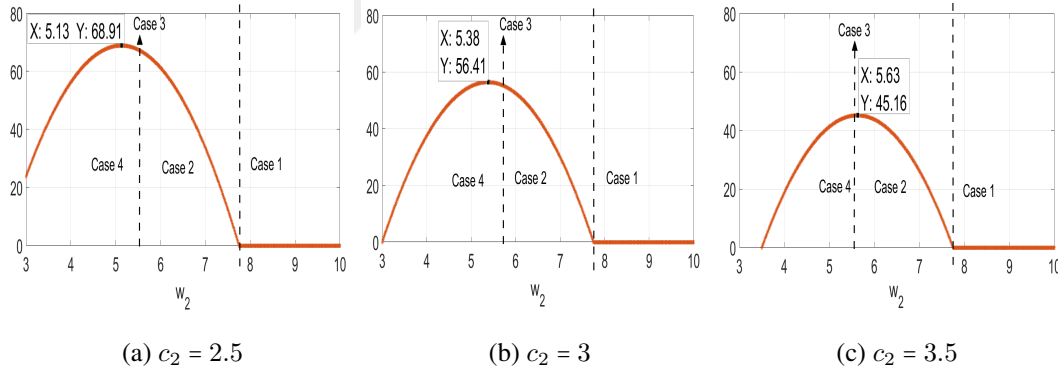


Figure 4.1: Follower profit wrt w_2 , for different values of c_2

As seen in Figure 4.1, an increase in c_2 increases his best response value, w_2^* and decreases the follower's profit. The profit function of the follower is $\pi_2 = Q_2 (w_2 - c_2)$; so an increase in c_2 decreases the follower's margin and correspondingly decreases his profit. In order to conserve his margin, the follower wants to increase w_2^* , however, Q_2^* is decreasing in w_2 . Thus, the follower cannot recover all the loss in his margin. As seen in the examples, w_2^* is 5.13, 5.538 and 5.63 at c_2 values 2.5, 3 and 3.5,

respectively; corresponding margin values are 2.63, 2.38 and 2.33 and corresponding Q_2^* values are 26.91, 23.70 and 19.38, respectively. As a result of decreased follower margin and decreased order quantity, the follower's profit decreases in c_2 .

Next we analyze the changes in the follower's profit function, $\pi_2(w_1 = 5.5, w_2)$, with respect to p_2 which is retail price for the follower's product (product 2). Figure 4.2 shows the follower's profit function for p_2 values of 8, 10 and 12. These values of p_2 , with the base parameter values, satisfy our assumptions $p_1 - \alpha_1 p_2 \geq 0$ and $p_2 - \alpha_2 p_1 \geq 0$. The optimal points are marked on the figures, X corresponds to w_2 and Y corresponds to $\pi_2(w_1, w_2)$.

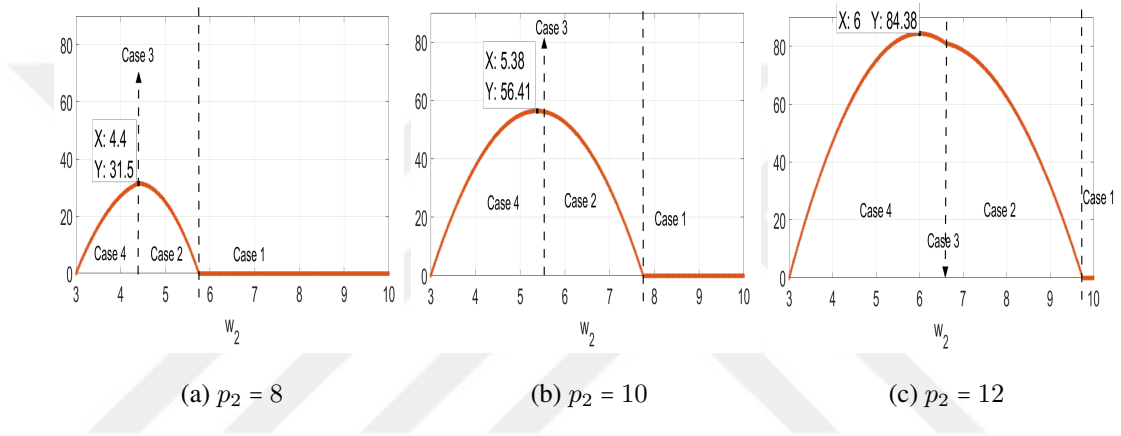


Figure 4.2: Follower profit wrt w_2 , for different values of p_2

As it is seen in Figure 4.2, the follower's profit and his best response value, w_2^* is increasing in p_2 . Note that for an increase in p_2 , both the follower's margin and the retailer's margin increase, and the increase in the retailer's margin is higher than the increase in the follower's margin. (Compare 0.98 vs 1.02 when p_2 increases from 8 to 10, and 0.62 vs 1.36 when it increases from 10 to 12.) The increase in the retailer's margin increases the quantity ordered from the follower (as it is seen from Table 4.2, Q_2^* increases in p_2). As a result of both the increase in follower margin and the increase in order quantity, the follower's profit increases in p_2 .

Next we analyze the changes in the follower's profit function with respect to β , which is the market share of product 1 in the total aggregate demand for the category. We analyze $\pi_2(w_1 = 5.5, w_2)$ for β values 0.25, 0.5 and 0.75, in Figure 4.3. The op-

timal points are marked on the figures, X corresponds to w_2 and Y corresponds to $\pi_2(w_1, w_2)$.

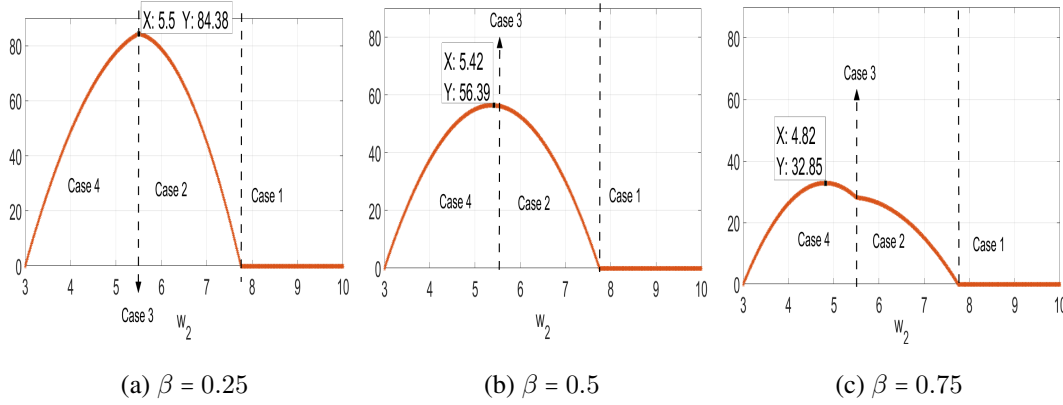


Figure 4.3: Follower profit wrt w_2 , for different values of β

As it is seen in Figure 4.3, as β increases, the follower's profit and w_2^* decreases. The reason is, as seen in Table 4.2, an increase in β increases the demand potential of product 1 and consequently the retailer's order quantity increases and that of product 2 decreases. Moreover, since Q_2^* increases when w_2 decreases, the follower decreases w_2^* as β increases, in order to recover the decrease in Q_2 caused by increased β . Remember from Corollary 3.1.9, if $\frac{w_2}{p_2} < \frac{w_1}{p_1}$ supplier 2 is favored, so the follower sets w_2 below w_1 to benefit from the spillover rate from product 1 to product 2.

Next we analyze the changes in the follower's profit function with respect to w_1 , which is the wholesale price for product 1. By taking other parameters at the base parameter values, we plot $\pi_2(w_1 = 4.5, w_2)$, $\pi_2(w_1 = 5.5, w_2)$ and $\pi_2(w_1 = 6.5, w_2)$ in Figure 4.4. The optimal points are marked on the figures, X corresponds to w_2 and Y corresponds to $\pi_2(w_1, w_2)$.

As it is seen in Figure 4.4, as w_1 increases, the follower's profit and $w_2^*(w_1)$ increases. Note that, the increase in $w_2^*(w_1)$ is smaller than the corresponding increase in w_1 . When w_1 increases from 4.5 to 5.5, $w_2^*(w_1)$ increases from 5.12 to 5.38.

When $w_1 = 4.5$, the follower sets $w_2^*(w_1)$ larger than w_1 , the retailer's optimal order decisions occurs in the region of Case 2 (Supplier 1 is favored) (as described in Table 4.2) so the retailer's order quantity from product 1 is larger than order quantity of

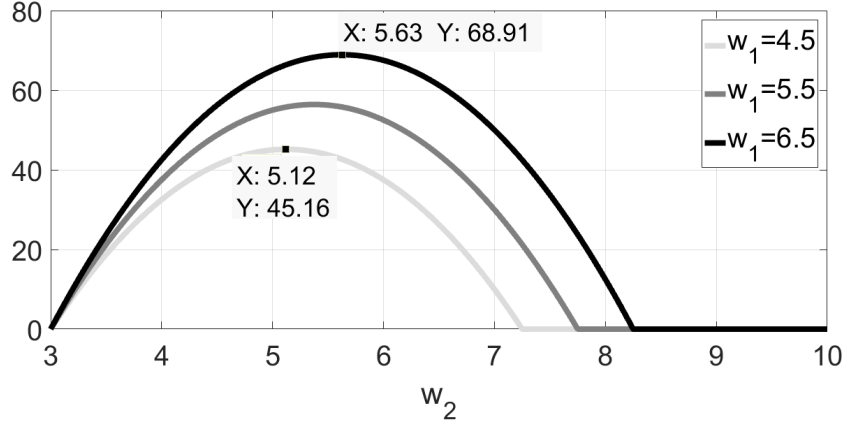


Figure 4.4: Follower profit wrt w_2 , for different values of w_1

product 2. The follower does not choose to set a smaller wholesale price to switch to the Case 4 region where his product is favorable, in order to conserve his margin and profit. But when w_1 increases to 5.5, the follower increases $w_2^*(w_1)$ and he can set a smaller wholesale price than the leader's, $w_2^*(w_1) = 5.38$. The retailer's optimal order decisions occurs in the Case 4 region (the region where $\frac{w_2}{p_2} < \frac{w_1}{p_1}$) and the retailer's order quantity is larger from product 2 than that of product 1. As a result, the follower's profit increases. Similarly, when w_1 increases to 6.5, the follower again increases his margin by setting his wholesale price to 5.63, and stays in the region of Case 4. In fact, when we solve the problem for many other values of w_1 , we observe that there is a threshold value (which is 5.33 for this parameter set), when the leader's wholesale price, w_1 , is smaller than this threshold value then the follower determines $w_2^*(w_1)$ larger than w_1 and equilibrium occurs in the region of Case 2 and when w_1 is larger than this threshold value, the follower sets $w_2^*(w_1)$ as smaller than w_1 and the equilibrium occurs in the region of Case 4 and when w_1 is equal to this threshold value, $w_2^*(w_1) = w_1$ and equilibrium occurs in the region of Case 3 (the Newsvendor case).

Next we analyze the changes in the follower profit function with respect to α_1 , which is the spillover rate from product 1 to product 2. In Figure 4.5 below, we plot $\pi_2(w_1 = 5.5, w_2)$ for α_1 values 0.25 and 0.5 and 0.75.

As it is seen from Table 4.2, and as stated in Corollary 3.1.9, α_1 has an effect on the retailer's order decisions only if $\frac{w_2}{p_2} < \frac{w_1}{p_1}$ (resulting in Case 4:Supplier 2 is favored).

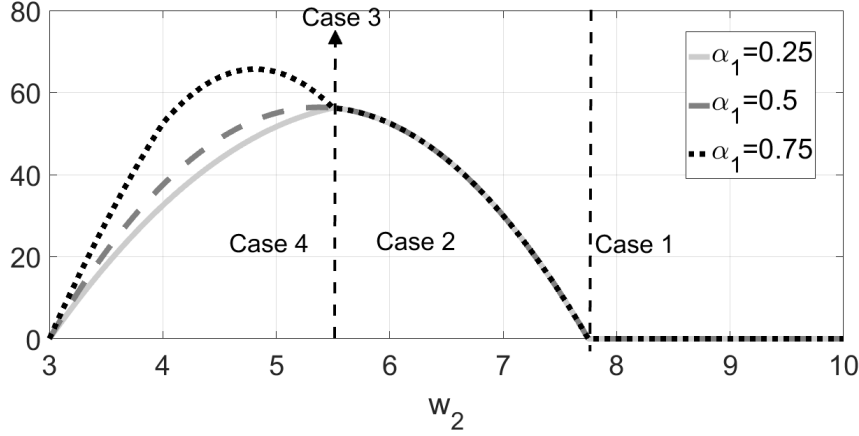


Figure 4.5: Follower profit wrt w_2 , for different values of α_1

Therefore, as seen in Figure 4.5, in the region of Case 2, the follower's profit is the same for all values of α_1 ; but in the region of Case 4, it increases in α_1 . Hence, as α_1 increases the expected spillover demand from product 1 to product 2 increases so Q_2^* increases. When we solve the problem for the many other values of α_1 we observe that there is a threshold value (which is 0.44 for our base parameter set) and for all values of α_1 that are smaller than this threshold value, the equilibrium occurs at the region of Case 3 and $w_2^* = w_1$ (which is 5.5 for the base parameter set). But when α_1 is larger than this value, the equilibrium occurs at the region of Case 2 and $w_2^* < w_1$ and it decreases in α_1 . Mainly because, Q_2^* is higher for smaller values of w_2 , and it is more profitable for the follower to decrease his margin in order to leverage his spillover advantage due to increasing α_1 .

And here, we analyze the changes in the follower profit function with respect to α_2 , which is the spillover rate from product 2 to product 1. In Figure 4.6, we plot $\pi_2(w_1 = 5.5, w_2)$ for α_2 values 0.25 and 0.5 and 0.75.

As it is seen from Table 4.2, and as stated in Corollary 3.1.9, α_2 has an effect on the retailer's order decisions only if $\frac{w_2}{p_2} > \frac{w_1}{p_1}$ (resulting in Case 2: Supplier 1 is favored). Therefore, as seen in Figure 4.6, the follower's profit is the same for all values of α_2 in the region of Case 4. But in the region of Case 2, it is decreasing in α_2 . When we solve the problem for many different cases, we observed that, for α_2 values that are smaller than a threshold value (which is 0.38 for the base parameter set), the equilibrium occurs at the region of Case 2 and $w_2^*(w_1)$ is larger than w_1 and decreases in α_2 but

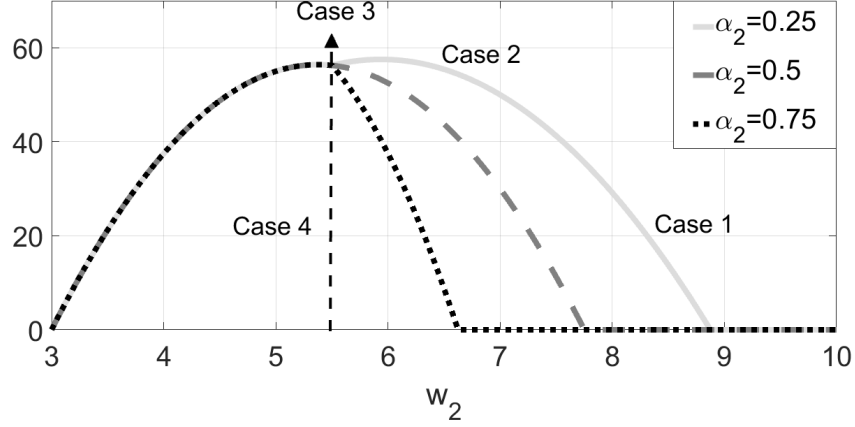


Figure 4.6: Follower profit wrt w_2 , for different values of α_2

when α_2 is larger than this threshold value the equilibrium occurs at the region of Case 4 and $w_2^*(w_1)$ is stable (at 5.38) in α_2 . Which shows that, for the values of α_2 that are smaller than or equal to this threshold value (which is 0.38 for the base parameter set), it is profitable for the follower to set w_2 larger than w_1 . Because the expected spillover demand from product 2 to product 1 is small and thus the retailer will not substitute much to product 1. So the follower chooses a higher wholesale price by accepting a smaller order quantity. But for values of α_2 greater than this threshold value, it is preferable to set smaller w_2 for the follower, in order to avoid the big loss in order quantity if he prices otherwise. This threshold value of α_2 is equal to 0.38 when α_1 is 0.5 but it is smaller for higher values of α_1 and it is higher for the smaller values of α_1 . For example when $\alpha_1 = 0.6$ threshold α_2 value is 0.23 and when $\alpha_1 = 0.4$ threshold α_2 value is 0.44.

4.2 The Leader's Price Decision and The Equilibrium Analyses

In this section we move on to the analysis of the leader's price decision by taking into account the follower supplier's price decision and the retailer's order quantity, and we characterize the consequent equilibrium. We find the equilibrium by using our theoretical results in Section 3.1 for the retailer's decisions and then full enumeration of w_1 and w_2 . We solve the problem by changing values of w_1 and w_2 from 3 to 10 (from production cost to retail price) with increment size 0.01. Firstly, we consider identical suppliers and we use the values given in Table 4.3 as base model parameters.

The equilibrium with these parameters is given in Table 4.4 below.

Table 4.3: Base Model Parameters

c_1	c_2	p_1	p_2	β	α_1	α_2	$demand$
3	3	10	10	0.5	0.5	0.5	U(0;100)

Table 4.4: Equilibrium at base model parameters

w_1^*	w_2^*	Q_1^*	Q_2^*	π_1^*	π_2^*	π_R^*
5.5	5.38	21.9	23.7	54.75	56.406	104.02

As seen in Table 4.4, in case of identical suppliers, there is a second mover (supplier 2) advantage. Supplier 2 captures this advantage by setting a lower price than the leader's. For the identical suppliers, the equilibrium occurs at the region of Case 4 where the retailer's order quantity is larger for the follower's product (all other symmetric games that we have analyzed resulted the same way). The leader can increase the order quantity for his product by setting a smaller w_1 and switching to the region of Case 2. However, the leader knows that, if he decreases w_1 the follower also will decrease w_2 in order to sustain the relationship $\frac{w_2}{p_2} < \frac{w_1}{p_1}$ (where equilibrium occurs at the region of Case 4). As we analyzed in Section 4.1, the follower gives up pricing below w_1 when w_1 is smaller than or equal to a threshold value, which is 5.33 for the base parameter set. When w_1 smaller than this threshold value, the best responses follow the result in the region of Case 2 but for the greater values of w_1 the equilibrium occurs at the region of Case 4. Under this parameter set, the region of Case 2 results less profit for both of the suppliers; since the increase in the leader's order quantity cannot recover the decrease in his margin. Therefore, lowering w_1 would only erode the leader's margin without making up in his profit. To illustrate, if the leader sets $w_1 = 5$, then the follower's best response is $w_2^*(w_1) = 5.25$, which results in higher order quantity for the leader, $Q_1^*(5, 5.25) = 26.25$, $Q_2^*(5, 5.25) = 22.5$ but less profit for both of the suppliers, $\pi_1 = 52.5$ and $\pi_2 = 50.625$ compared to their profits at equilibrium given in Table 4.4. Therefore, in the symmetric case, leader sets $w_1^* = 5.5$ and accepts to be the unfavored supplier.

Next, we analyze the effects of parameters on equilibrium decisions and profits. We also provide the centralized system solution as a benchmark. We further analyze the effects of parameters both on the centralized solution and on the inefficiency caused

by decentralization.

4.2.1 Equilibrium changes in supplier costs

In this section, we analyze the changes in equilibrium, when we change the suppliers' costs while we use the base values for all other parameters.

If the suppliers' costs are differentiated, then the low cost supplier sets his wholesale price below the other's wholesale price. If the leader has the lower cost, he can set a low wholesale price by conserving his margin and profit. Then, the follower cannot match such a low price, and prefers to conserve his margin. Consequently, the retailer gives larger orders from the leader's product, and the leader has a higher profit than the follower. As seen in Table 4.5, the follower, when he has the cost advantage can set more aggressive prices, which pushes the leader to even higher prices. Compare $c_1 = 2, c_2 = 3$ vs $c_1 = 3, c_2 = 2$; in the first case the leader set his wholesale price to 4.99 but in the second case the follower set lower wholesale price which is 4.84.

Table 4.5: Equilibrium decision and profit changes in supplier costs

c_1	c_2	w_1^D	w_2^D	Q_1^D	Q_2^D	π_1^D	π_2^D	π_R^D	π_T^D	Q_1^C	Q_2^C	π_T^C
2	3	4.99	5.25	26.35	22.45	78.79	50.51	119.33	248.62	45	30	285
3	3	5.50	5.38	21.90	23.70	54.75	56.41	104.02	215.18	35	35	245
4	3	5.99	5.50	17.60	24.95	35.02	62.38	91.43	188.82	25	40	215
5	3	6.50	5.63	13.15	26.20	19.73	68.91	80.26	168.89	15	45	195
3	2	5.35	4.84	20.70	28.35	48.65	80.51	121.27	250.43	30	45	285
3	3	5.50	5.38	21.90	23.70	54.75	56.41	104.02	215.18	35	35	245
3	4	5.63	5.91	23.25	19.05	61.15	36.39	89.76	187.29	40	25	215
3	5	5.82	6.46	24.10	14.50	67.96	21.17	76.03	165.17	45	15	195

When the cost of the leader or the follower increases, the wholesale prices of both suppliers increase. Because the high-cost supplier cannot be as competitive anymore, and increases his wholesale price in order to protect his margin. His competitor, facing less pressure from the competitor, also increases his wholesale price, since he can easily sustain a price below the other's. It is also observed that, the follower's order quantity increases in the leader's cost. Although w_1^* and w_2^* both increase in c_1 , supplier 2 becomes even more preferable over supplier 1, Q_1^* decreases and Q_2^* increases. The reason is that, when c_1 increases, w_1^* increases but Q_1^* decreases, and some of this decreased quantity (depending on the spillover rate from product 1 to

product 2) is added to the order quantity for product 2, so Q_2^* increases. The other way around is also true; the leader's order quantity increases in the follower's cost.

When a supplier's cost increases, the total quantity ordered by the retailer decreases because of increasing wholesale prices. Therefore, the retailer's profit decreases in supplier costs. As a result of aggressive pricing of the follower, the retailer's profit decreases more in c_2 compared to decrease in c_1 . So, it can be said that, the retailer would prefer the leader to have the cost advantage, rather than the follower to have the cost advantage.

For the centralized case, total order quantity and supply chain profit decreases in supplier costs. Since the product with lower cost is more profitable for the supply chain, when one of the supplier's cost increases, the order quantity of that supplier's product decreases. The order quantity of the competing product will increase to account for the spillover. Under imperfect substitution ($\alpha_1 < 1$ and $\alpha_2 < 1$), total order quantity decreases in supplier costs. As it is seen in Table 4.5, when c_1 increases from 2 to 3, Q_1^C decreases from 45 to 35. Since spillover rate α_1 is 0.5, Q_2^C increases by 5, whereas Q_1^C decreases by 10; so total order quantity decreases from 75 to 70.

We analyze the inefficiency caused by decentralization by calculating the ratio of the decentralized total supply chain profit over the centralized supply chain profit. As it is seen in Table 4.6, there is not a significant change in inefficiency in c_1 , the inefficiency can increase or decrease in c_1 , but the inefficiency decreases in c_2 .

Table 4.6: Supply chain inefficiency changes in supplier costs

c_1	c_2	π_T^D	π_T^C	π_T^D/π_T^C
2	3	248.62	285	0.872
3	3	215.18	245	0.878
4	3	188.82	215	0.878
5	3	168.89	195	0.866
3	2	250.43	285	0.879
3	3	215.18	245	0.878
3	4	187.29	215	0.871
3	5	165.17	195	0.847

4.2.2 Equilibrium changes in retail prices

In this section, we discuss the changes in equilibrium when we change the retail prices.

When the retail prices are differentiated, the supplier with the lower retail price has lower profit. And the profit of a supplier increases in his own retail price. The leader's profit decreases in the follower's retail prices, but the follower's profit may increase or decrease in the leader's retail price. As seen in Table 4.7, total decentralized and centralized supply chain profits increase in retail prices.

Table 4.7: Equilibrium decision and profit changes in retail prices

p_1	p_2	w_1^D	w_2^D	Q_1^D	Q_2^D	π_1^D	π_2^D	π_R^D	π_T^D	Q_1^C	Q_2^C	π_T^C
8	10	4.28	5.57	24.2	21.4	30.93	55.04	92.39	178.36	25.0	40.0	202.5
10	10	5.50	5.38	21.9	23.7	54.75	56.41	104.02	215.18	35.0	35.0	245.0
12	10	6.57	5.46	22.6	22.8	80.58	56.00	112.96	249.54	40.6	31.3	292.2
14	10	7.76	5.53	22.3	22.4	105.91	56.67	119.48	282.07	46.4	25.0	342.9
10	8	5.97	4.77	20.1	20.2	59.77	35.79	73.21	168.76	40.0	25.0	202.5
10	10	5.50	5.38	21.9	23.7	54.75	56.41	104.02	215.18	35.0	35.0	245.0
10	12	5.03	6.26	25.7	23.3	52.07	75.80	130.47	258.33	31.3	40.6	292.2
10	14	4.73	7.18	27.9	23.3	48.27	97.19	152.80	298.25	25.0	46.4	342.9

In order to understand the changes, we solve the problem by changing the value of p_1 from 8 to 12 with an increment size of 0.1. When we solve the problem for many different cases, we observe a threshold value of p_1 (which is 9.9 for the base parameter set). As it is seen in Figure 4.7; when p_1 is smaller than this threshold value, the equilibrium occurs in the region of Case 2 (Supplier 1 is favored). Just after this threshold value the equilibrium switches to the region of Case 4 (Supplier 2 is favored), and as a result of the bimodal nature of the leader's profit function, an immediate increase in w_1 and correspondingly an immediate decrease in Q_1^* is observed, which causes an increase in π_1 .

Next, in order to understand the changes, we solve the problem by changing the value of p_2 from 8 to 12 with an increment size of 0.1. We observe a threshold value of p_2 (which is 10.2 for the base parameter set). As seen in Figure 4.8, when p_2 is smaller than this threshold value, equilibrium occurs in the region of Case 4 (Supplier 2 is favored). Just after this threshold value equilibrium switches to the region of Case 2 (Supplier 1 is favored), and as a result of the bimodal nature of the follower's profit

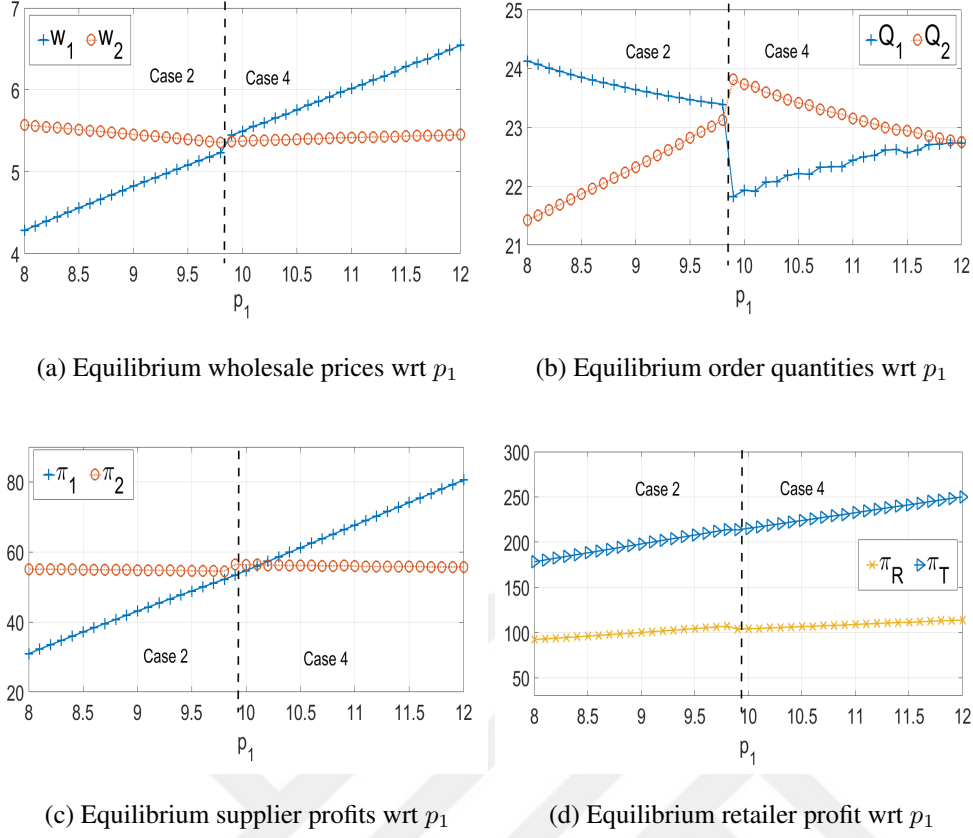


Figure 4.7: Equilibrium decisions and profits changes in p_1 for decentralized supply chain

function, an immediate decrease in w_2 and correspondingly an immediate increase in Q_2^* is observed, which causes a decrease in π_2 .

Under differentiated prices, the retailer prefer the follower to have the higher retail price, since in that case the retailer's profit is higher (compare π_R values for the cases $(p_1 = 12, p_2 = 10)$ vs $(p_1 = 10, p_2 = 12)$).

For the centralized case, the order quantity of a product increases in its own retail price but decrease in the competing product's retail price. However, the same trend is not true for the decentralized case. Since, in the decentralized case, the suppliers react to against the changing retail prices in order to sustain or increase their own profits by changing their wholesale prices. For example, compare the decentralized vs centralized order quantities for both products at rows $(p_1 = 8, p_2 = 10)$ vs $(p_1 = 10, p_2 = 10)$. When p_1 increased from 8 to 10; the centralized order quantity for

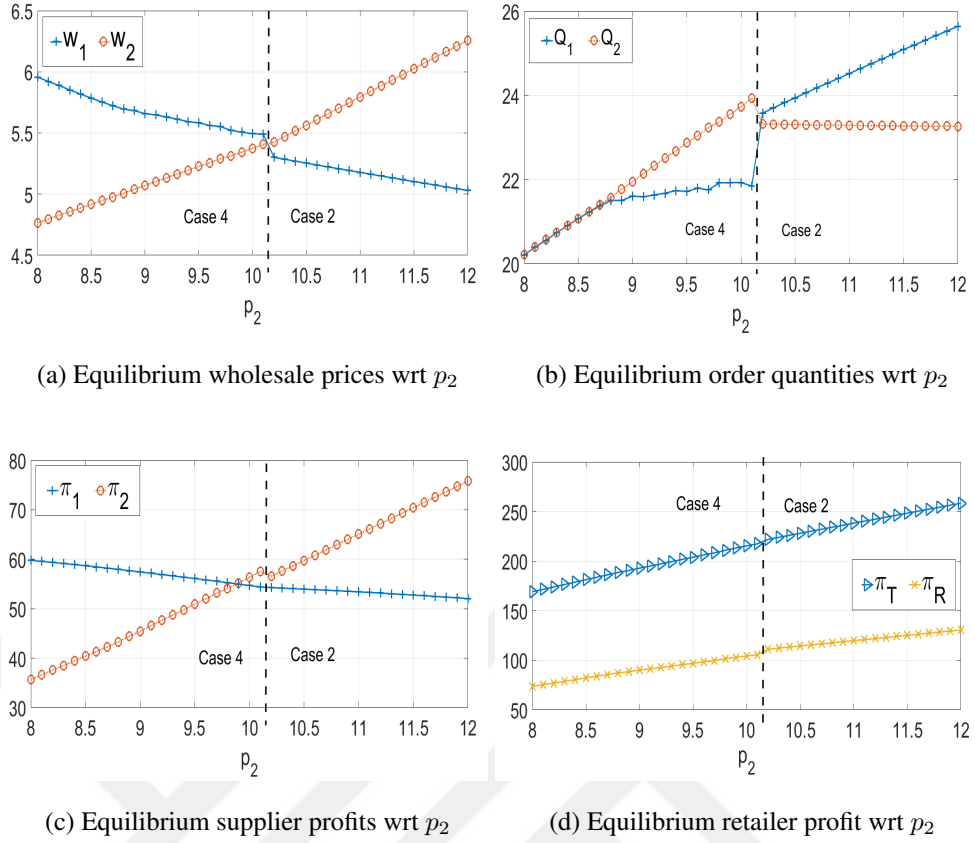


Figure 4.8: Equilibrium decisions and profits changes in p_2 for decentralized supply chain

product 1 increases and that for product 2 decreases; however, the decentralized order quantities behave just the opposite, the order quantity for product 1 decreases but that for product 2 increases. Thus when $p_1 = 8$ the equilibrium occurs at the region of Case 2 (Supplier 1 is favored), but when p_1 increases to 10, the follower sets a smaller wholesale price than the leader, and the equilibrium switches to the region of Case 4 (Supplier 2 is favored).

We analyze the inefficiency caused by decentralization by calculating the ratio of the decentralized total supply chain profit over the centralized supply chain profit. As it is seen in Table 4.8, the inefficiency increases with p_1 but there is not a significant change in inefficiency with p_2 , the inefficiency can increase or decrease in p_2 .

Table 4.8: Supply chain inefficiency changes in retail prices

p_1	p_2	π_T^D	π_T^C	π_T^D/π_T^C
8	10	178.36	202.5	0.881
10	10	215.18	245	0.878
12	10	249.54	292.2	0.854
14	10	282.07	342.9	0.823
10	8	168.76	202.5	0.833
10	10	215.18	245	0.878
10	12	258.33	292.2	0.884
10	14	298.25	342.9	0.870

4.2.3 Equilibrium changes in spillover rates

In this section, we discuss the equilibrium behavior with respect to the spillover rate from one supplier to the other one.

We analyze the spillover effects on equilibrium decisions and profits. Firstly by changing α_1 , secondly by changing α_2 and then by changing them together, from 0 to 1 with an increment size 0.25. The results for the examples with different spillover rates are given in Table 4.9 and 4.10. Then in order to gain a better insight we plot equilibrium prices, quantities and profits with respect to α_1 in Figures 4.9 and with respect to α_2 in Figure 4.10; for the spillover rates changing from 0.1 to 0.9 with an increment size 0.05. The results in the tables and the figures show that in equilibrium, the product that is highly substitutable ends up having a lower wholesale price. The supplier with higher spillover rate, by setting his wholesale price lower, avoids being the "unfavorable" supplier and suffering from a big drop in his order quantity.

Firstly, we analyze the equilibrium for the changing values of α_1 . As seen in Table 4.9 and in Figure 4.9, for the base parameter set, we observed that when $\alpha_1 \leq 0.15$, the equilibrium occurs at the region of Case 3 (the Newsvendor case) where $w_2^*(w_1) = w_1^*$. For the larger values of α_1 , when $\alpha_1 < \alpha_2$, equilibrium occurs at the region of Case 4 but when $\alpha_2 < \alpha_1$ equilibrium switches to the region of Case 2.

When $\alpha_1 < \alpha_2$, for the leader, being "unfavorable" does not present a huge loss in order quantity and he can set a higher wholesale price to increase his margin and profit. And when $\alpha_1 < 0.15$, for the follower also, being "favorable" is not attractive

since the expected spillover from product 1 to product 2 is very small. Therefore, the follower sets his wholesale price equal to the leader's. However when α_1 increases, being the "favorable" supplier is more profitable for the follower because he cannot run the risk of any spillover from product 2 to product 1, and he sets a wholesale price below the leader's, as seen in Table 4.9 and in Figure 4.10. When $\alpha_1 = \alpha_2$, by using the second mover advantage, the follower again sets a lower price than the leader, and becomes the "favored" supplier. But when $\alpha_1 > \alpha_2$, the leader's potential loss in order quantity due to spillovers increases and cannot run the risk of pricing higher than the follower anymore. The leader sets aggressively low wholesale price to be the "favored" supplier.

Table 4.9: Equilibrium decision and profit changes in spillover rates

α_1	α_2	w_1^D	w_2^D	Q_1^D	Q_2^D	π_1^D	π_2^D	π_R^D	π_T^D	Q_1^C	Q_2^C	π_T^C
0	0.5	6.50	6.50	17.5	17.5	61.25	61.25	61.25	183.75	35	35	245
0.25	0.5	6.04	6.01	19.8	20.0	60.04	60.20	79.00	199.25	35	35	245
0.5	0.5	5.50	5.38	21.9	23.7	54.75	56.41	104.02	215.18	35	35	245
0.75	0.5	4.82	5.20	27.8	22.1	50.60	48.62	125.04	224.26	35	35	245
1	0.5	3.55	4.89	38.9	18.9	21.42	35.63	173.78	230.82	35	35	245
0.5	0	5.89	6.50	20.6	17.5	59.39	61.25	72.86	193.49	35	35	245
0.5	0.25	5.65	5.96	22.3	19.7	59.01	58.26	88.19	205.46	35	35	245
0.5	0.5	5.50	5.38	21.9	23.7	54.75	56.41	104.02	215.18	35	35	245
0.5	0.75	5.50	5.38	21.9	23.7	54.75	56.41	104.02	215.18	35	35	245
0.5	1	5.50	5.38	21.9	23.7	54.75	56.41	104.02	215.18	70	0	245

As it is seen in Table 4.9, when the leader's spillover rate is zero, both suppliers set their wholesale prices as if they are the sole supplier to the retailer. There is no risk of a spillover from product 1 to product 2, so the leader can set a high price. The follower sets his wholesale price equal to the leader's price in order to avoid a spillover from product 2 to product 1, and because underpricing will not help at all due to zero spillover from the leader's demand. If the leader decreases w_1 in order to gain the advantage of that spillover, the follower also decreases w_2 in order to avoid that. Therefore, the leader does not lower his price, and equilibrium occurs at Case 3 (the Newsvendor case).

When the follower's spillover rate is zero, the follower sets his wholesale price to 6.5, but the leader choose a lower price, (which is 5.89 for the base parameter set). Because there is a risk of spillover from product 1 to product 2, and if the leader sets a higher price, the follower can take advantage by setting a lower wholesale price than

the leader. So the leader cannot set a price as high as the follower.

Note that, as it seen in Figure 4.9, both suppliers' equilibrium prices decrease in α_1 . Similarly, as it is seen in Figure 4.10, supplier equilibrium prices also decrease in α_2 . However, when $\alpha_2 > \alpha_1$, the equilibrium is stable in α_2 , the follower sets his price to 5.38, and the leader sets his price to 5.5. As a result of the second mover advantage in this game, the follower conserves his profit despite the high spillover rates. Whereas, the leader's loss in profit increases with his spillover rate.

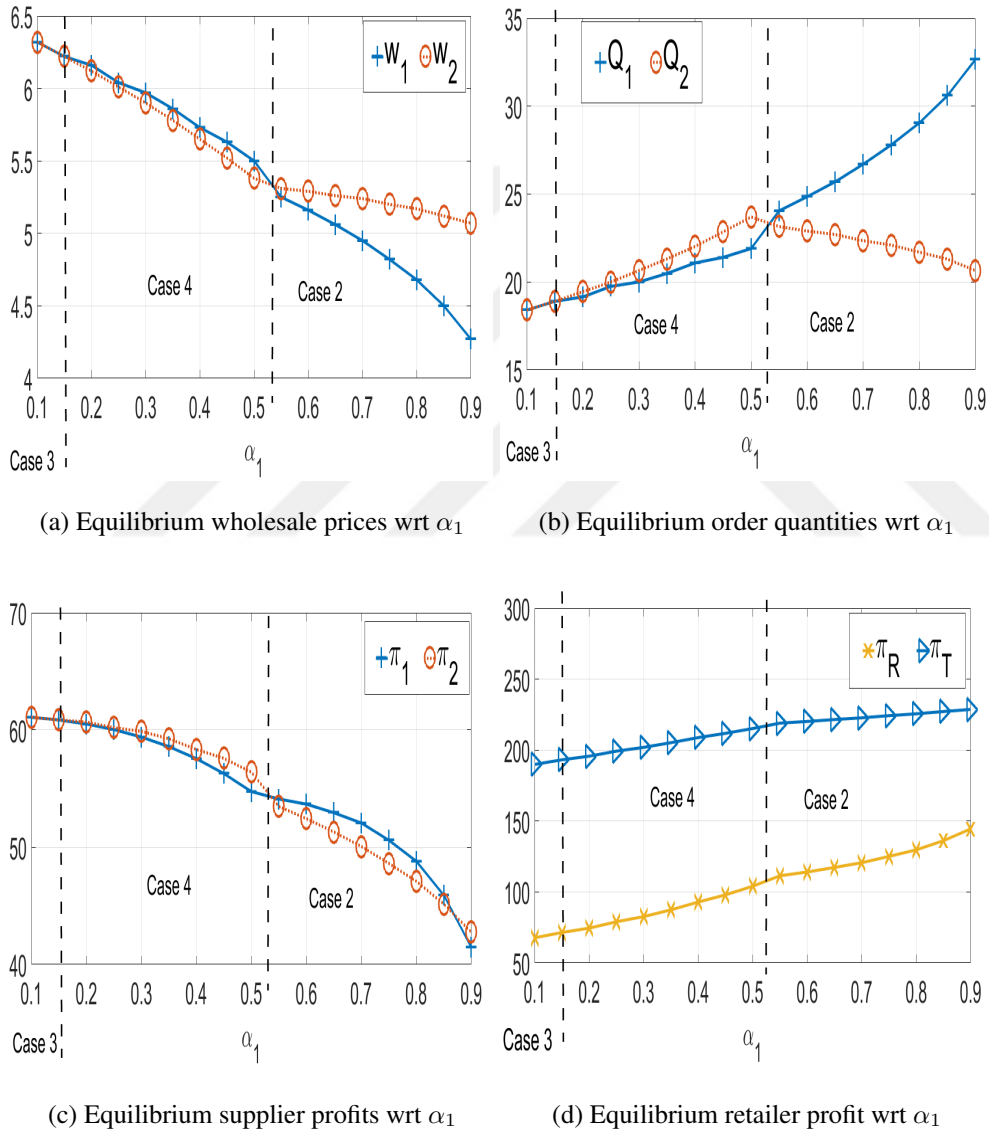
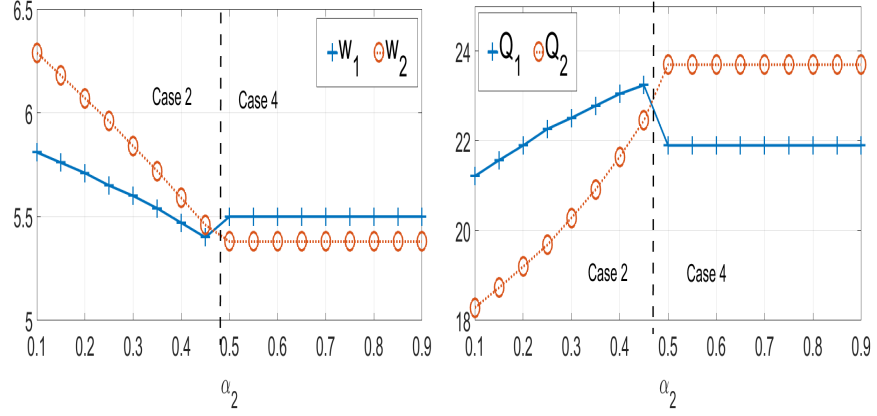
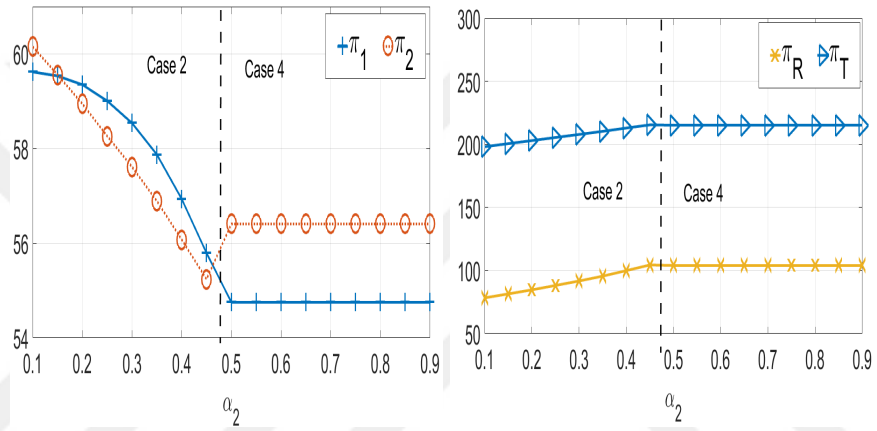


Figure 4.9: Equilibrium decisions and profits changes in α_1 for decentralized supply chain



(a) Equilibrium wholesale prices wrt α_2 (b) Equilibrium order quantities wrt α_2



(c) Equilibrium supplier profits wrt α_2 (d) Equilibrium retailer profit wrt α_2

Figure 4.10: Equilibrium decisions and profits changes in α_2 for decentralized supply chain

If the supplier's spillover rates are equal and increases together, as given in Table 4.10, the follower always sets $w_2^*(w_1)$ smaller than w_1^* . Competition is very high for the larger values of spillover rates, indeed perfect competition is observed between the suppliers when the spillover rates are 1; i.e., the price competition boils down to Bertrand competition and both suppliers make zero profit.

When we compare the retailer profits under different spillover rates, in Tables 4.9, 4.10 and in Figures 4.9 and 4.10, it is seen that, from the retailer's point of view, higher spillover rates in the market are preferable. The reason is that, supplier competition intensifies with higher spillover rates. This intense competition results in lower

Table 4.10: Equilibrium decision and profit changes in spillover rates

α_1	α_2	w_1^D	w_2^D	Q_1^D	Q_2^D	π_1^D	π_2^D	π_R^D	π_T^D	Q_1^C	Q_2^C	π_T^C
0	0	6.50	6.50	17.5	17.5	61.3	61.3	61.3	183.8	35	35	245
0.25	0.25	6.04	6.01	19.8	20.0	60.0	60.2	79.0	199.3	35	35	245
0.5	0.5	5.50	5.38	21.9	23.7	54.8	56.4	104.0	215.2	35	35	245
0.75	0.75	4.71	4.52	23.6	30.25	40.4	46.0	145.3	231.6	35	35	245
1	1	3.01	3.01	34.95	34.95	0.4	0.4	244.3	244.998	35	35	245

wholesale prices and consequently lower supplier profits, but higher order quantities, higher retailer profit and a higher total supply chain profit. In such a competitive market, the retailer prefers to have these two competing brands together in her shelves. However, high spillover rates are not profitable for suppliers; therefore suppliers would prefer to differentiate their products to prevent spillover.

When we analyze the centralized supply chain profits under different spillover rates, for the centralized supply chain, spillover rates have no effect on ordering decisions. As it is seen in Tables 4.9 and 4.10, the most profitable decision is giving order for both products as if they are independent products (with no spillovers). When suppliers' spillover rates are 1, any order scheme that adds up to 70 is optimal for the system.

Table 4.11: Supply chain inefficiency changes in spillover rates

α_1	α_2	π_T^D	π_T^C	π_T^D/π_T^C
0	0.5	183.75	245	0.75
0.25	0.5	199.25	245	0.81
0.5	0.5	215.18	245	0.88
0.75	0.5	224.26	245	0.92
1	0.5	230.82	245	0.94
0.5	0	193.49	245	0.79
0.5	0.25	205.46	245	0.84
0.5	0.5	215.18	245	0.88
0.5	0.75	215.18	245	0.88
0.5	1	215.18	245	0.88
0	0	183.8	245	0.75
0.25	0.25	199.3	245	0.81
0.5	0.5	215.2	245	0.88
0.75	0.75	231.6	245	0.95
1	1	244.998	245	1.00

We analyze the inefficiency caused by decentralization by calculating the ratio of the

decentralized total supply chain profit over the centralized supply chain profit. As it is seen in Table 4.11, the inefficiency in the chain due to decentralization decreases in spillover rates. Because, in the decentralized supply chain, as spillover rates increase price competition between suppliers increases. This results in lower wholesale prices and consequently lower supplier profits, but higher order quantities, higher retailer profit and higher total supply chain profit. As also seen in Tables 4.9 and 4.10, increased spillover rates intensifies price competition and mitigates the double marginalization; consequently increases the order quantities and the total decentralized supply chain profit. As a result, the inefficiency caused by decentralization decreases. In fact, when the products are perfect substitutes, inefficiency vanishes.

4.2.4 Equilibrium changes in market share of suppliers

In this section, we discuss the changes in equilibrium when we change β , which is the market share of the leader (the market share of the follower is automatically equal to $1 - \beta$).

In this analysis, we cannot observe a monotonic trend in prices with respect to β , we discuss our observations here. As observed in Table 4.12, for the small values of β (in the range of 0.1 and 0.2), being in the region of Case 2 is not only profitable for the leader but also profitable for the follower. Because, follower's market share is high and having a larger margin recovers the loss in order quantity caused by expected spillover from product 2 to product 1. Similarly for the higher values of β , 0.8 and 0.9 in Table 4.12, being in the region of Case 4 is more profitable for both of the suppliers.

However, for the intermediate values of β , both suppliers want to be "favored supplier" to obtain spillover demand, by setting his wholesale price below the competing supplier's price. For β values smaller than 0.5, the follower sets w_2^* equal to w_1^* . Because, for the follower, higher margin is more profitable than the increase in order quantity caused by expected spillover. So the equilibrium occurs at the region of Case 3. However, when β reaches to 0.5, the expected spillover demand from product 1 to product 2 increases and the follower sets a lower wholesale price. For the higher values of β , the leader's loss from spillover increases, and both suppliers set more

aggressive prices.

Table 4.12: Equilibrium decision and profit changes in market share of suppliers

β	w_1^D	w_2^D	Q_1^D	Q_2^D	π_1^D	π_2^D	π_R^D	π_T^D	Q_1^C	Q_2^C	π_T^C
0.1	4.47	5.12	11.38	38.07	16.73	80.71	124.36	221.79	7	63	245
0.2	4.75	5.19	14.02	34.96	24.54	76.56	120.88	221.98	14	56	245
0.3	5.88	5.88	12.36	28.84	35.59	83.06	84.87	203.53	21	49	245
0.4	5.63	5.63	17.48	26.22	45.97	68.96	95.48	210.42	28	42	245
0.5	5.5	5.38	21.9	23.7	54.75	56.41	104.02	215.18	35	35	245
0.6	5.16	5.29	29.56	18.32	63.85	41.95	114.68	220.48	42	28	245
0.7	4.95	5.24	36.22	13.41	70.63	30.04	123.37	224.04	49	21	245
0.8	5.39	4.66	31.04	16.52	74.19	27.42	115.66	217.26	56	14	245
0.9	5.39	4.43	32.85	14.21	78.51	20.32	115.29	214.13	63	7	245

It is observed that the profit of the leader increases in β ; however, the profits of the follower and the retailer do not change monotonically.

The change in market size has no effect on centralized equilibrium profits, since the centralized order decisions changes with respect to the changes in the market size. We cannot observe any monotonic change in the inefficiency caused by decentralization with respect to market share.

In the next section, we discuss the effect of disregarding spillover rates, under different settings.

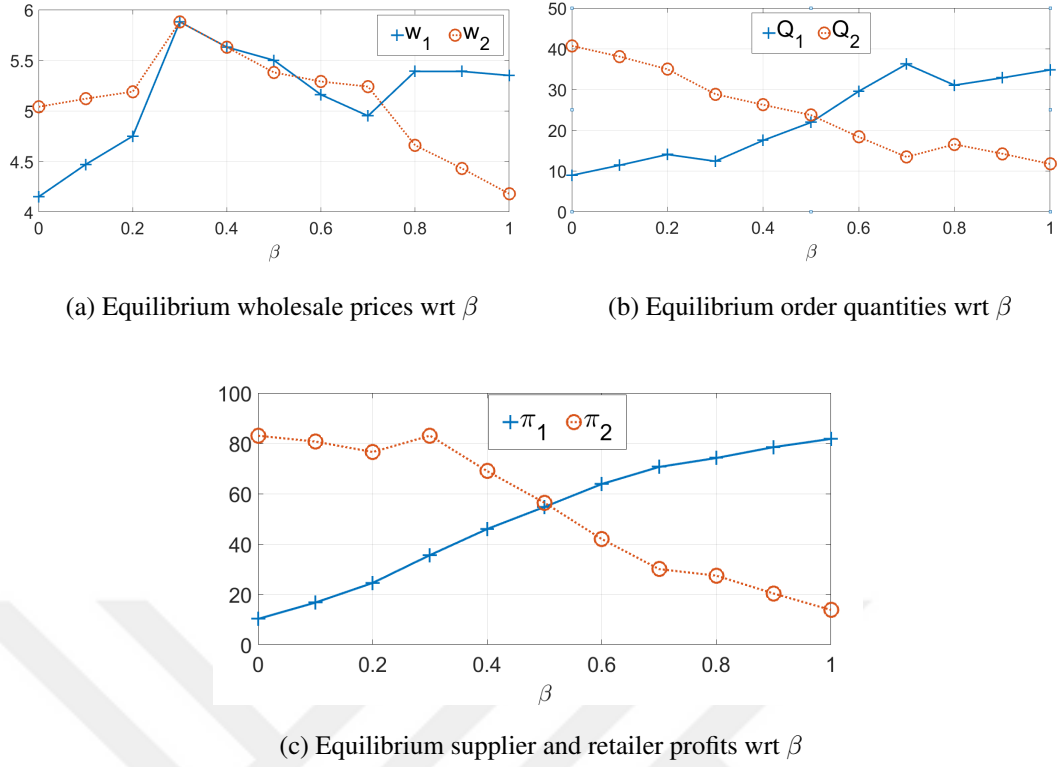


Figure 4.11: Equilibrium decisions and profits changes in β for decentralized supply chain

4.3 The Effect of Disregarding the Spillover Rates

In this section, we analyzed the suppliers' and the retailer's profits in equilibrium firstly for the case when all of the players, the suppliers and the retailer, ignore the spillover rates while making their decisions. Then, we consider the case where only one of the suppliers, first the leader then the follower, ignores the spillover rate, but the competing supplier and the retailer take into account substitution in their strategies.

4.3.1 Everybody ignores the spillover rates

In this case we analyze the supply chain profits when all players disregard the spillover rates. We solve for the retailer's order quantity decision and for the supplier's pricing decisions as if there is no spillover rate. Then we calculate the expected retailer profit by taking the spillover rates into account. We take spillover values as changing from

0.1 to 0.9 with increment size 0.1 and we take other parameters at base parameter values. For all spillover rates, the equilibrium decisions and profits are as given in Table 4.13. (Superscript E indicates the case "Everybody ignores".)

Table 4.13: Equilibrium decisions and profits when spillover rates are disregarded

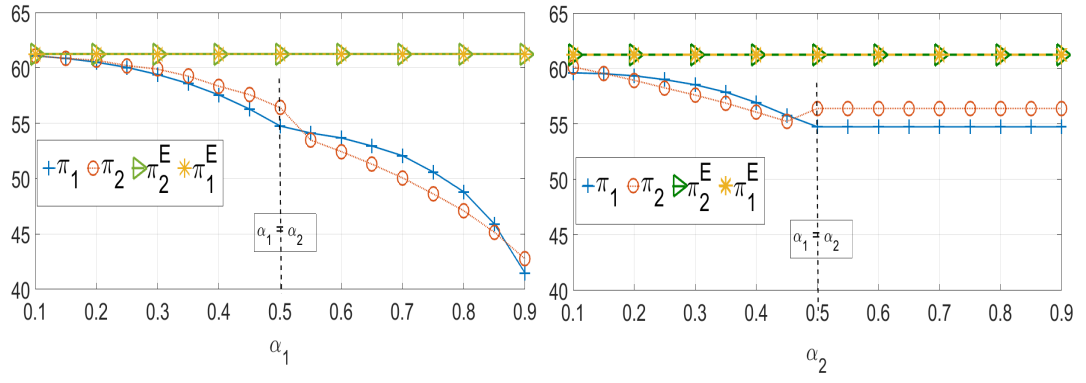
w_1^E	w_2^E	Q_1^E	Q_2^E	π_1^E	π_2^E	π_R^E	π_T^E
6.5	6.5	17.5	17.5	61.25	61.25	61.25	183.75

As it is seen, when all players ignore the spillover rates, the retailer sets the order quantities as if each supplier is her sole supplier. Her expected profit with these order quantities remains the same as spillover rates change. Because, in our model the aggregate demand is based on a single random variable and the expected aggregate demand is shared by the products according to given market shares of the suppliers, which are β and $1-\beta$. Thus, the retailer either understocks or overstocks both products for any demand realization, and spillover cannot take effect.

Figure 4.12 compares the profits when spillover rates are disregarded, to the case where the spillover rates are not taken into account by all parties.

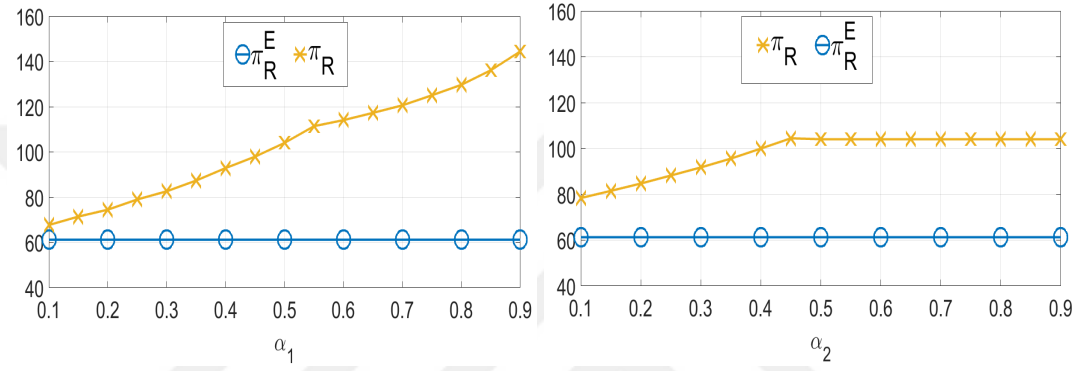
Ignoring the spillover rates favors the suppliers. Both suppliers have higher profits when everybody ignores the spillover rates. When the spillover rates are taken into account, the competition intensity between the suppliers increases which produces lower prices and lower profits for both. The supplier's loss caused by the spillover increases with α_1 . It increases also with α_2 when $\alpha_1 > \alpha_2$ but when $\alpha_1 < \alpha_2$ the suppliers' loss is constant in α_2 . Under this parameter set α_2 has no effect on ordering decision of the retailer.

On the other hand, when spillover rates are disregarded, the retailer's profit and total supply chain profit is lower. Spillover rates strengthen the retailer through the intense supplier competition and decrease the double marginalization in the supply chain. Thus, disregarding the spillover rates is in favor of suppliers but it is to the detriment of the retailer and the total supply chain performance.



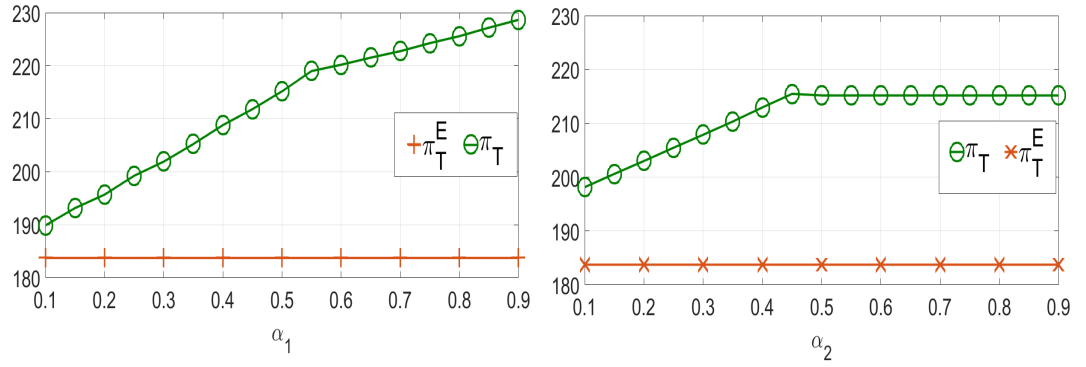
(a) Suppliers' profits in α_1

(b) Suppliers' profits in α_2



(c) Retailer's profit in α_1

(d) Retailer's profit in α_2



(e) Total supply chain profit in α_1

(f) Total supply chain profit in α_2

Figure 4.12: The effect of disregarding spillover rates on equilibrium profits for changing in α_1 and α_2 values

4.3.2 The leader ignores the spillover rates

In this section, we analyze the supply chain profits when the leader (supplier 1) disregards the spillover rates. We solve the problem for the leader's pricing decision as if there is no spillover rate, but we solve the follower's pricing decision and the retailer's order quantity decision by considering the spillover rates. In the examples, we use different spillover values and we take other parameters at base parameter values. For different spillover rates, the equilibrium decisions and profits are summarized in Table 4.14. (Superscript L indicated the case "The leader ignores").

Table 4.14: Equilibrium decision and profits when the leader ignores the spillover rates

α_1	α_2	w_1^L	w_2^L	Q_1^L	Q_2^L	π_1^L	π_2^L	π_R^L	π_T^L
0.1	0.5	6.50	6.33	17.41	18.44	60.92	61.42	64.31	186.64
0.3	0.5	6.50	5.98	16.39	21.21	57.35	63.22	71.32	191.88
0.5	0.5	6.50	5.63	13.15	26.20	46.03	68.91	80.26	195.19
0.7	0.5	6.50	5.28	3.27	37.83	11.43	86.26	95.00	192.70
0.9	0.5	6.50	6.11	0.00	36.96	0.00	114.93	71.88	186.81
0.5	0.1	6.50	5.63	13.15	26.20	46.03	68.91	80.26	195.19
0.5	0.3	6.50	5.63	13.15	26.20	46.03	68.91	80.26	195.19
0.5	0.5	6.50	5.63	13.15	26.20	46.03	68.91	80.26	195.19
0.5	0.7	6.50	5.63	13.15	26.20	46.03	68.91	80.26	195.19
0.5	0.9	6.50	5.63	13.15	26.20	46.03	68.91	80.26	195.19

Since the leader decides his wholesale price without considering the spillover rates, under any spillover rate he determines the same wholesale price, which is 6.5. In this case, the second mover advantage is even stronger. Since the leader sets a high wholesale price, given the leader's wholesale price, the follower enjoys a high wholesale price and still remains the "favorable" supplier. As the spillover rate from product 1 to product 2 increases, the follower's order quantity increases and the leader's order quantity decreases. When $\alpha_1 < 0.8$, the follower's wholesale price decreases in α_1 (in order to increase Q_2^* more with the increasing α_1), the follower's and the retailer's profits and the total supply chain profit increases in α_1 . But when $\alpha_1 \geq 0.8$, the equilibrium occurs at the region of Case 5 (Supplier 1 is out of the market). Since the leader is out of the market, the competition between suppliers vanishes and the follower has the power to increase his profit more. When $\alpha_1 \geq 0.8$, the follower's

wholesale price increases in α_1 , because, as seen in Table 4.2, the condition that makes the region of Case 5 feasible is $w_2 \leq p_2 - \frac{(p_1 - w_1)}{\alpha_1}$. And the value of w_2 that satisfy this condition increases with α_1 . Thus, the follower's profit increases but the retailer's and the total chain profit decreases in α_1 .

On the other hand, as α_2 changes, the follower's pricing decision and the retailer's order quantities do not change. As seen in Figure 4.13, α_2 has an effect only in the region of Case 2 (where $\frac{w_2}{p_2} > \frac{w_1}{p_1}$), but the leader prefers to set a price in the other region. The optimal point is marked on the figure, X corresponds to w_2 and Y corresponds to $\pi_2(w_1, w_2)$. (This is due to the high value of w_1 which is 6.5; for the lower of w_1 , an equilibrium on the region of Case 2 is possible.)

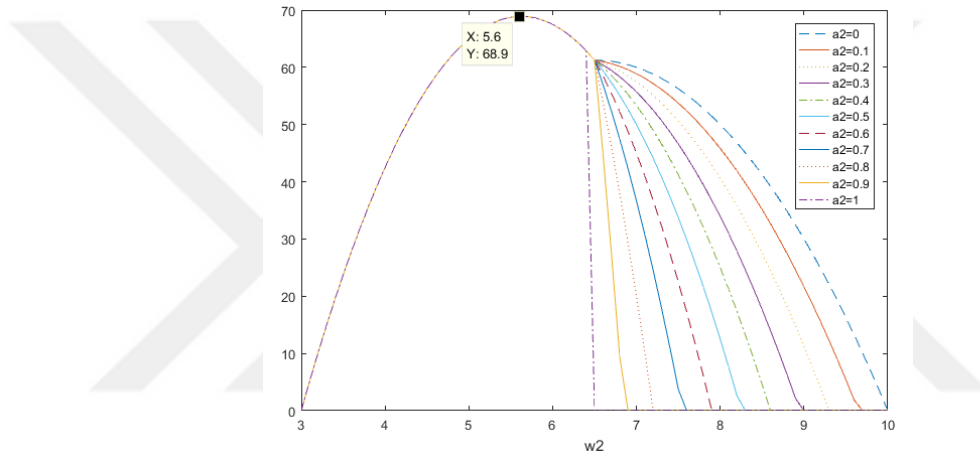
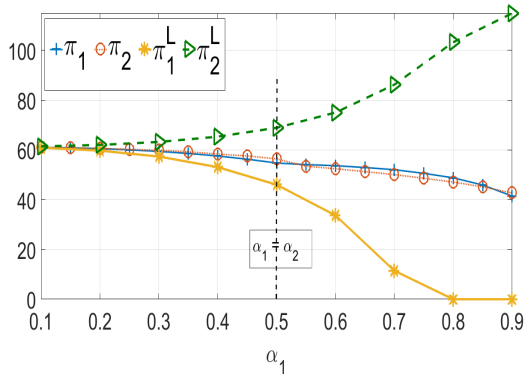
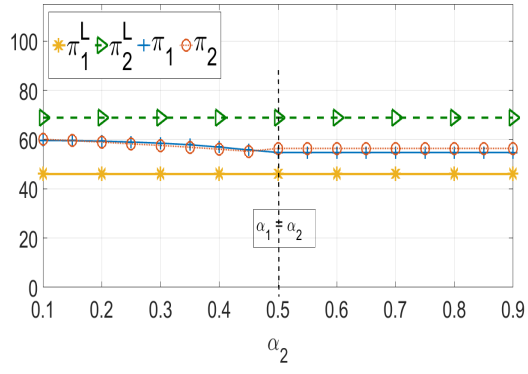


Figure 4.13: Follower' profits in w_2 for different α_2 values

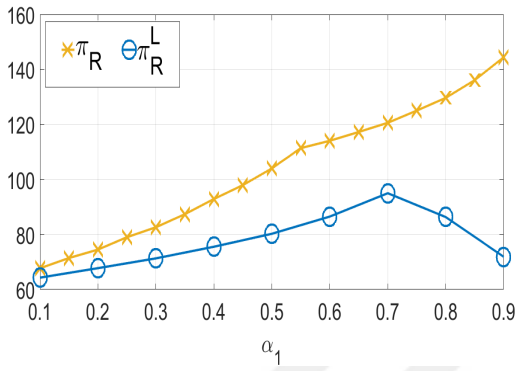
Next, we compare the profits when the leader disregards the spillover rates, with the base case when the spillover rates are taken into account by all parties, in Figure 4.14.



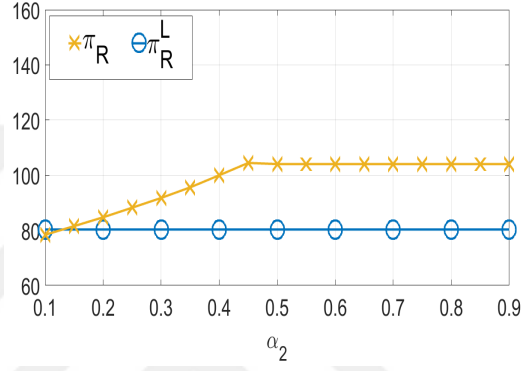
(a) Suppliers' profits in α_1



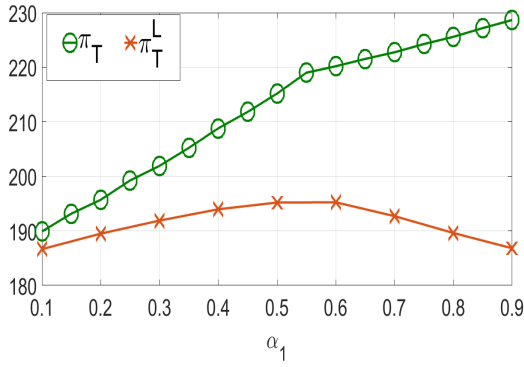
(b) Suppliers' profits in α_2



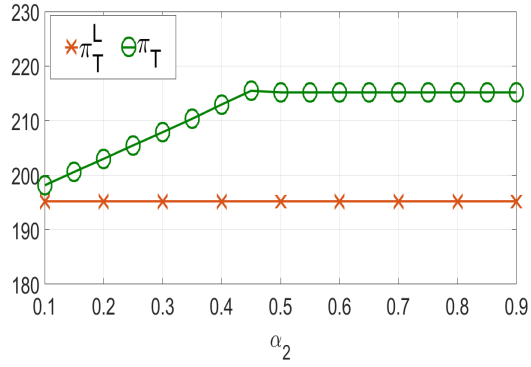
(c) Retailer's profit in α_1



(d) Retailer's profit in α_2



(e) Total supply chain profit in α_1



(f) Total supply chain profit in α_2

Figure 4.14: The effect of leader's disregarding of spillover rates on profits

For any spillover rate, disregarding the spillover rates causes the leader to lose, when compared to the base case. The leader's loss from disregarding the spillover rates increases in α_1 , and consequently the follower's gain increases. The retailer's profit and the total supply chain profit are lower when the leader ignores the spillover rates,

compared to the case where nobody ignores the spillover rates. This is mainly due to the weakened supplier competition and the strengthened double marginalization effect in the chain.

On the other hand, the leader's loss from disregarding the spillover rates decreases in α_2 , the follower's gain increases in α_2 . When the leader ignores the spillover rates, the competition between the suppliers is very weakened. This is an advantage for the follower but it is to the detriment of the retailer and of the total supply chain performance.

4.3.3 The follower ignores the spillover rates

In this third case, we analyze the supply chain profits when the follower (supplier 2) disregards the spillover rates. The leader does not know that the follower is unaware of the spillover rates. We solve the problem for the follower's pricing decision as if there is no spillover rate, but we solve the leader's pricing decision and the retailer's order quantity decision by considering the spillover rates. We assume the leader expects the follower to take into account substitution in his decisions as well. We take spillover values changing as in Table 4.15 and other parameters at base parameter values. For the selected combination of parameters, the equilibrium decisions and profits are summarized in Table 4.15. (Superscript F indicates the case "The follower ignores").

Table 4.15: Equilibrium decision and profits when the follower ignores the spillover rates

α_1	α_2	w_1^F	w_2^F	Q_1^F	Q_2^F	π_1^F	π_2^F	π_R^F	π_T^F
0.10	0.50	6.30	6.50	19.50	16.50	64.35	57.75	64.95	187.05
0.30	0.50	5.90	6.50	23.50	14.50	68.15	50.75	73.55	192.45
0.50	0.50	5.40	6.50	28.50	12.00	68.40	42.00	86.55	196.95
0.70	0.50	4.90	6.50	33.50	9.50	63.65	33.25	102.05	198.95
0.90	0.50	4.20	6.50	40.50	6.00	48.60	21.00	127.95	197.55
0.50	0.10	5.80	6.50	21.39	17.11	59.89	59.89	74.86	194.64
0.50	0.30	5.50	6.50	24.64	15.36	61.61	53.75	82.32	197.68
0.50	0.50	5.40	6.50	28.50	12.00	68.40	42.00	86.55	196.95
0.50	0.70	5.40	6.50	35.83	4.67	86.00	16.33	90.58	192.92
0.50	0.90	5.40	6.50	43.70	0.00	104.88	0.00	100.51	205.39

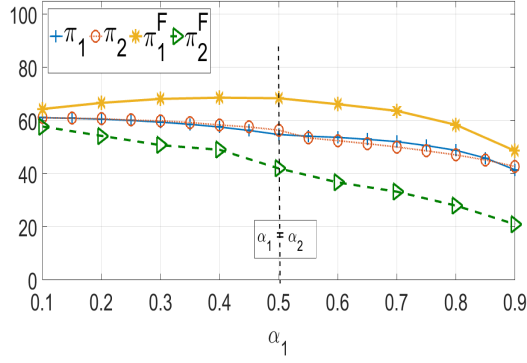
Since the follower decides his wholesale price without considering the spillover rates, he determines always the same wholesale price, which is 6.5. Since the leader sets his wholesale price expecting the follower to set w_2 by considering the spillover rates, the leader sets the same wholesale prices as in the base case. w_1^* is always smaller than 6.5, the leader stays in the region of Case 2 where $Q_1^* > Q_2^*$.

As the spillover rate from product 1 to product 2 increases, the leader's wholesale price decreases (expecting an aggressive price from the follower) so his order quantity increases in α_1 . The leader's profit increases in α_1 when $\alpha_1 < 0.5$ but it decreases in α_1 for higher values of α_1 , since w_1^* decreases sharply for the values $\alpha_1 > 0.5$. The follower's order quantity and the profit decreases, but the retailer's profit and total supply chain profit increases in α_1 .

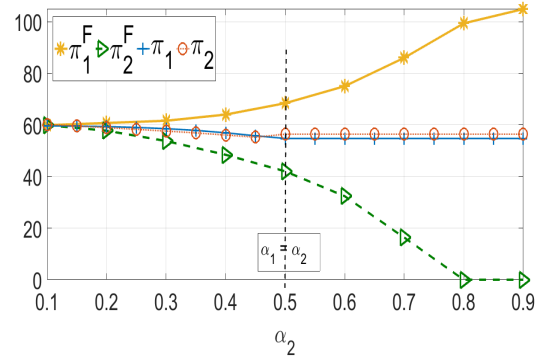
When $\alpha_2 < 0.5$, the leader's wholesale price decreases in α_2 but the leader's order quantity and profit increases in α_2 , but when $\alpha_2 \geq 0.5$, w_1^* does not change in α_2 but Q_2^* and π_2 decrease in α_2 . For all values of α_2 , the retailer profit and total supply chain profit increase in α_2 . The follower's order quantity and the profit decrease in α_2 , even when $\alpha_2 \geq 0.8$. The retailer only orders from the leader supplier a quantity that includes the expected spillover from the follower supplier. So the leader's profit increases more.

Next, we compared the profits when the follower disregards the spillover rates, with the profits when the spillover rates are not disregarded, in Figure 4.15.

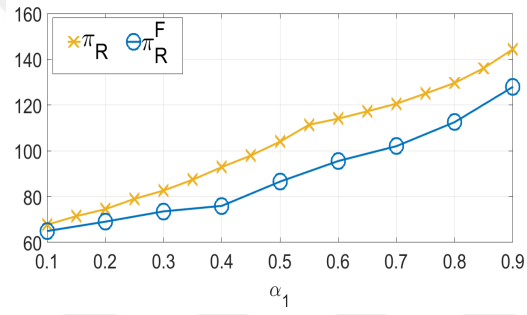
Disregarding the spillover rates causes the follower to have a drop in profit for any spillover rate. When compared to the base case, the follower's loss from disregarding substitution increases in spillover rates, the leader's gain increases in α_2 but it is nearly stable in α_1 . The retailer's profit and total supply chain profit is lower when the follower ignores the spillover rates when compared to the base case. When the follower ignores the spillover rates, the competition between the suppliers is weak. This becomes an advantage for the leader but (except the cases where $\alpha_2 < 0.3$) it is to the detriment of the retailer and of the total supply chain performance.



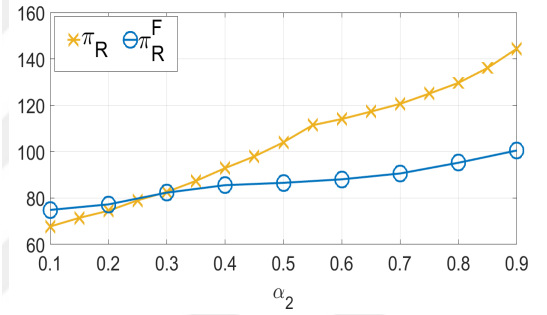
(a) Suppliers' profits in α_1



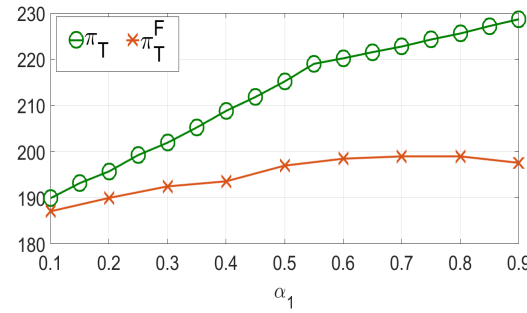
(b) Suppliers' profits in α_2



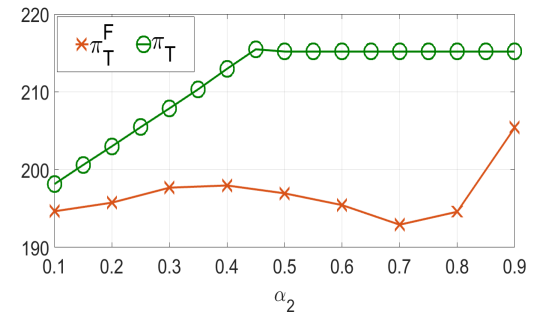
(c) Retailer's profit in α_1



(d) Retailer's profit in α_2



(e) Total supply chain profit in α_1



(f) Total supply chain profit in α_2

Figure 4.15: The effect of disregarding spillover rates on equilibrium profits for changing in α_1 and α_2 values



CHAPTER 5

CONCLUSION

In this thesis, we study two partially substitutable products sold through a two-echelon supply chain, consisting of two suppliers and one retailer. We assume a random aggregate demand which is shared between the products according to the known market sizes of the suppliers. Therefore, the products have separated random demand as long as both are available. However, in a case of a stock-out in one of the products, that product's demand can spill over the competing product, which causes a correlation between the final demand of the products and their stocking quantities. These two suppliers compete on price for the retailer order quantities. Under this setting, we investigate the effect of supplier competition on the equilibrium pricing decisions of the suppliers and ordering decisions of the retailer. We analyze the problem with a single period Stackelberg game model. We provide the retailer's and the suppliers' expected profit functions and analyze their optimal decisions. Then, we conduct a numerical study to gain better insights about the equilibrium and its behavior. We investigate the effect of spillover rate on pricing decision of the suppliers, the effect of spillover rates on the inefficiency in total supply chain profit and the effect of disregarding the spillover rates by one of the suppliers or all parties in the chain.

As a result of our analysis we obtain the following results. Firstly, when we characterize the retailer's optimal order decisions, we find that the optimal order decision of the retailer depends on the relative profitability of the products; i.e., the ratio of wholesale price to the retail price. If these values for both products are equal, the retailer's order quantities are equal to the Newsvendor order quantities. Otherwise, the product with lower profitability factor is favored from the retailer's perspective.

Secondly, when we solve the problem for identical suppliers, we observed a second mover advantage. Next, when we analyze the problem for the different spillover rates, it is observed that, the retailer's profit increase in spillover rates, but the suppliers' profits decrease in spillover rates because the price competition between the suppliers intensify in spillover rates.

Then, we compare the centralized and decentralized supply chain profits for different spillover rates. We observed that, the inefficiency caused by decentralization in the supply chain decreases in spillover rates. As spillover increases supplier competition strengthens which in turn mitigates the double marginalization in the chain. In fact, when the products are perfect substitutes suppliers competition turns into a Bertrand competition with zero profit for each.

Finally we investigate the cases when the spillover rates are ignored by all players, then only by the leader and only by the follower. We compare the profits under these cases with the case when the spillover rates are taken into account by all parties. We observe that, when the spillover rates are disregarded by all players, the suppliers' profits are higher but the retailer's profit is lower compared to regarded case, for any spillover rate. Which shows that, spill over rates are an advantage for the retailer. On the other hand, if only one of the suppliers disregards the spillover rates, compared to case when the spillover rates are regarded, a loss is observed in that supplier's profit which causes a gain in the competing supplier's profit; but either a loss or a gain can be observed in retailer's profit depending on the spillover rates.

Our research can be extended by a shelf space constraint determined by the retailer. Together with the spillover rates, we expect the shelf space constraint stimulates the supplier competition. Additionally, it would be interesting to investigate this problem with a category management strategy adopted by the retailer, where the retailer assigns one of the suppliers as category leader and that supplier determines the equilibrium order quantities for both products.

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APPENDIX A

PROOF OF LEMMA 3.1.1

Retailer profit function, π_R , is continuous at $\frac{Q_1}{\beta} = \frac{Q_2}{1-\beta}$. Because the retailer profit function is characterized as,

$$\pi_R(Q_1, Q_2) = \begin{cases} \pi_{R1}(Q_1, Q_2), & \text{if } \frac{Q_1}{\beta} \leq \frac{Q_2}{1-\beta} \\ \pi_{R2}(Q_1, Q_2), & \text{if } \frac{Q_1}{\beta} > \frac{Q_2}{1-\beta} \end{cases}$$

and if $\frac{Q_1}{\beta} = \frac{Q_2}{1-\beta}$, π_{R1} is equal to π_{R2} :

$$\pi_{R1}(Q_1, Q_2|D = x) = \pi_{R2}(Q_1, Q_2|D = x) =$$

$$-w_1 Q_1 - w_2 Q_2 + \begin{cases} p_1 \beta x + p_2 (1 - \beta)x, & \text{if } x \leq \frac{Q_1}{\beta} \\ p_1 Q_1 + p_2 Q_2, & \text{if } x > \frac{Q_1}{\beta} \end{cases}$$



APPENDIX B

PROOF OF LEMMA 3.1.2

The first derivatives of π_{R1} with respect to Q_1 and Q_2 are given by:

$$\begin{aligned}
 \frac{\partial \pi_{R1}}{\partial Q_1} &= p_1 - w_1 + p_1 \frac{Q_1}{\beta} f\left(\frac{Q_1}{\beta}\right) + p_2 \alpha_1 \frac{Q_2 + \alpha_1 Q_1}{1 - \beta + \alpha_1 \beta} f\left(\frac{Q_2 + \alpha_1 Q_1}{1 - \beta + \alpha_1 \beta}\right) \\
 &\quad - p_2 \alpha_1 \frac{Q_1}{\beta} f\left(\frac{Q_1}{\beta}\right) + p_2 \alpha_1 \frac{Q_1}{\beta} f\left(\frac{Q_1}{\beta}\right) - p_2 \alpha_1 F\left(\frac{Q_2 + \alpha_1 Q_1}{(1 - \beta) + \alpha_1 \beta}\right) \\
 &\quad + p_2 \alpha_1 F\left(\frac{Q_1}{\beta}\right) - p_2 \alpha_1 \frac{Q_2 + \alpha_1 Q_1}{1 - \beta + \alpha_1 \beta} f\left(\frac{Q_2 + \alpha_1 Q_1}{1 - \beta + \alpha_1 \beta}\right) \\
 &\quad - p_1 F\left(\frac{Q_1}{\beta}\right) - p_1 Q_1 \frac{1}{\beta} f\left(\frac{Q_1}{\beta}\right) \\
 &= p_1 - w_1 - p_2 \alpha_1 F\left(\frac{Q_2 + \alpha_1 Q_1}{(1 - \beta) + \alpha_1 \beta}\right) \\
 &\quad + p_2 \alpha_1 F\left(\frac{Q_1}{\beta}\right) - p_1 F\left(\frac{Q_1}{\beta}\right)
 \end{aligned} \tag{B.1}$$

$$\begin{aligned}
 \frac{\partial \pi_{R1}}{\partial Q_2} &= -w_2 + p_2(1 - \beta) \frac{1}{1 - \beta + \alpha_1 \beta} \frac{Q_2 + \alpha_1 Q_1}{1 - \beta + \alpha_1 \beta} f\left(\frac{Q_2 + \alpha_1 Q_1}{1 - \beta + \alpha_1 \beta}\right) \\
 &\quad + p_2 \alpha_1 \beta \frac{1}{1 - \beta + \alpha_1 \beta} \frac{Q_2 + \alpha_1 Q_1}{1 - \beta + \alpha_1 \beta} f\left(\frac{Q_2 + \alpha_1 Q_1}{1 - \beta + \alpha_1 \beta}\right) \\
 &\quad - p_2 \alpha_1 Q_1 \frac{1}{1 - \beta + \alpha_1 \beta} f\left(\frac{Q_2 + \alpha_1 Q_1}{1 - \beta + \alpha_1 \beta}\right) + p_2 - p_2 F\left(\frac{Q_2 + \alpha_1 Q_1}{1 - \beta + \alpha_1 \beta}\right) \\
 &\quad - p_2 Q_2 f\left(\frac{Q_2 + \alpha_1 Q_1}{1 - \beta + \alpha_1 \beta}\right) \\
 &= p_2 - w_2 - p_2 F\left(\frac{Q_2 + \alpha_1 Q_1}{1 - \beta + \alpha_1 \beta}\right)
 \end{aligned} \tag{B.2}$$

Second order partial derivatives of π_{R1} are given as

$$\frac{\partial^2 \pi_{R1}}{\partial Q_1^2} = -p_2 \alpha_1 \frac{\alpha_1}{1 - \beta + \alpha_1 \beta} f\left(\frac{Q_2 + \alpha_1 Q_1}{1 - \beta + \alpha_1 \beta}\right) - \frac{1}{\beta} f\left(\frac{Q_1}{\beta}\right) (p_1 - p_2 \alpha_1) < 0,$$

and

$$\frac{\partial^2 \pi_{R1}}{\partial Q_2^2} = -p_2 \frac{1}{1 - \beta + \alpha_1 \beta} f\left(\frac{Q_2 + \alpha_1 Q_1}{1 - \beta + \alpha_1 \beta}\right) < 0.$$

Cross-partial derivative of profit function with respect to Q_1 and Q_2

$$\frac{\partial^2 \pi_{R1}}{\partial Q_1 \partial Q_2} = -p_2 \frac{\alpha_1}{1 - \beta + \alpha_1 \beta} f\left(\frac{Q_2 + \alpha_1 Q_1}{1 - \beta + \alpha_1 \beta}\right) < 0,$$

The determinant of the Hessian matrix is given as

$$\frac{\partial^2 \pi_{R1}}{\partial Q_1^2} \frac{\partial^2 \pi_{R1}}{\partial Q_2^2} - \left(\frac{\partial^2 \pi_{R1}}{\partial Q_1 \partial Q_2}\right)^2 = (p_1 - p_2 \alpha_1) p_2 \frac{1}{\beta} f\left(\frac{Q_1}{\beta}\right) \frac{1}{1 - \beta + \alpha_1 \beta} f\left(\frac{Q_2 + \alpha_1 Q_1}{1 - \beta + \alpha_1 \beta}\right) > 0.$$

Hence, π_{R1} is jointly concave in Q_1 and Q_2 .

The first derivatives of π_{R2} with respect to Q_1 and Q_2 are given by:

$$\begin{aligned} \frac{\partial \pi_{R2}}{\partial Q_1} &= -w_1 + p_1 \beta \frac{1}{\beta + \alpha_2(1 - \beta)} \frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2(1 - \beta)} f\left(\frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2(1 - \beta)}\right) \\ &\quad + p_1 \alpha_2 (1 - \beta) \frac{1}{\beta + \alpha_2(1 - \beta)} \frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2(1 - \beta)} \\ &\quad - p_1 \alpha_2 Q_2 \frac{1}{\beta + \alpha_2(1 - \beta)} f\left(\frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2(1 - \beta)}\right) + p_1 - p_1 F\left(\frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2(1 - \beta)}\right) \\ &\quad - p_1 Q_1 \frac{1}{\beta + \alpha_2(1 - \beta)} f\left(\frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2(1 - \beta)}\right) \\ &= p_1 - w_1 - p_1 F\left(\frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2(1 - \beta)}\right) \end{aligned} \quad (\text{B.3})$$

$$\begin{aligned} \frac{\partial \pi_{R2}}{\partial Q_2} &= -w_2 + p_1 \beta \frac{\alpha_2}{\beta + \alpha_2(1 - \beta)} \frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2(1 - \beta)} f\left(\frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2(1 - \beta)}\right) + p_2 \frac{Q_2}{1 - \beta} f\left(\frac{Q_2}{1 - \beta}\right) \\ &\quad + p_1 \alpha_2 (1 - \beta) \frac{\alpha_2}{\beta + \alpha_2(1 - \beta)} \frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2(1 - \beta)} f\left(\frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2(1 - \beta)}\right) \\ &\quad - p_1 \alpha_2 \frac{Q_2}{1 - \beta} f\left(\frac{Q_2}{1 - \beta}\right) - p_1 \alpha_2 F\left(\frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2(1 - \beta)}\right) \\ &\quad - p_1 \alpha_2 Q_2 \frac{\alpha_2}{\beta + \alpha_2(1 - \beta)} f\left(\frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2(1 - \beta)}\right) + p_1 \alpha_2 F\left(\frac{Q_2}{1 - \beta}\right) \\ &\quad + p_1 \alpha_2 Q_2 \frac{1}{1 - \beta} f\left(\frac{Q_2}{1 - \beta}\right) + p_2 - p_2 F\left(\frac{Q_2}{1 - \beta}\right) - p_2 Q_2 \frac{1}{1 - \beta} f\left(\frac{Q_2}{1 - \beta}\right) \\ &\quad - p_1 Q_1 \frac{\alpha_2}{\beta + \alpha_2(1 - \beta)} f\left(\frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2(1 - \beta)}\right) \\ &= p_2 - w_2 - p_1 \alpha_2 F\left(\frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2(1 - \beta)}\right) + p_1 \alpha_2 F\left(\frac{Q_2}{1 - \beta}\right) - p_2 F\left(\frac{Q_2}{1 - \beta}\right) \end{aligned} \quad (\text{B.4})$$

Second order partial derivatives of π_{R2} are given as,

$$\frac{\partial^2 \pi_{R2}}{\partial Q_1^2} = -p_1 \frac{1}{\beta + \alpha_2(1 - \beta)} f\left(\frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2(1 - \beta)}\right) < 0,$$

and

$$\frac{\partial^2 \pi_{R2}}{\partial Q_2^2} = -p_1 \alpha_2 \frac{\alpha_2}{\beta + \alpha_2(1 - \beta)} f\left(\frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2(1 - \beta)}\right) - \frac{1}{1 - \beta} f\left(\frac{Q_2}{1 - \beta}\right) (p_2 - p_1 \alpha_2) < 0.$$

Cross-partial derivative of profit function with respect to Q_1 and Q_2 :

$$\frac{\partial^2 \pi_{R2}}{\partial Q_1 \partial Q_2} = -p_1 \frac{\alpha_2}{\beta + \alpha_2(1 - \beta)} f\left(\frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2(1 - \beta)}\right) < 0,$$

π_{R2} , the determinant of the Hessian matrix is given as

$$\frac{\partial^2 \pi_{R2}}{\partial Q_1^2} \frac{\partial^2 \pi_{R2}}{\partial Q_2^2} - \left(\frac{\partial^2 \pi_{R2}}{\partial Q_1 \partial Q_2}\right)^2 = (p_2 - p_1 \alpha_2) p_1 \frac{1}{1 - \beta} f\left(\frac{Q_2}{1 - \beta}\right) \frac{1}{\beta + \alpha_2(1 - \beta)} f\left(\frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2(1 - \beta)}\right) > 0$$

Hence, π_{R2} is jointly concave in Q_1 and Q_2 .



APPENDIX C

PROOF OF LEMMA 3.1.3

π_{R1} is jointly concave in Q_1 and Q_2 , so we can find the optimal Q_1 and Q_2 from the first order conditions.

$$\frac{\partial \pi_{R1}}{\partial Q_1} = p_1 - w_1 - \alpha_1 p_2 F\left(\frac{Q_2 + \alpha_1 Q_1}{(1 - \beta) + \alpha_1 \beta}\right) + \alpha_1 p_2 F\left(\frac{Q_1}{\beta}\right) - p_1 F\left(\frac{Q_1}{\beta}\right) = 0 \quad (\text{C.1})$$

$$\frac{\partial \pi_{R1}}{\partial Q_2} = p_2 - w_2 - p_2 F\left(\frac{Q_2 + \alpha_1 Q_1}{1 - \beta + \alpha_1 \beta}\right) = 0 \quad (\text{C.2})$$

From Equation (C.2) we get:

$$F\left(\frac{Q_2 + \alpha_1 Q_1}{1 - \beta + \alpha_1 \beta}\right) = \frac{p_2 - w_2}{p_2} \quad (\text{C.3})$$

by inserting Equation (C.3), into Equation (C.1) we get:

$$F\left(\frac{Q_1}{\beta}\right) = \frac{p_1 - w_1 - \alpha_1(p_2 - w_2)}{p_1 - \alpha_1 p_2} \quad (\text{C.4})$$

From Equation (C.4):

$$Q_{11}^* = \begin{cases} 0, & \text{if } p_1 - w_1 - \alpha_1(p_2 - w_2) < 0 \\ \beta F^{-1}\left(\frac{p_1 - w_1 - \alpha_1(p_2 - w_2)}{p_1 - \alpha_1 p_2}\right), & \text{if otherwise} \end{cases}$$

From Equation (C.3):

$$\begin{aligned} Q_{12}^* &= F^{-1}\left(\frac{p_2 - w_2}{p_2}\right) (1 - \beta + \alpha_1 \beta) - \alpha_1 Q_1 \\ &= F^{-1}\left(\frac{p_2 - w_2}{p_2}\right) (1 - \beta + \alpha_1 \beta) - \alpha_1 \max\left(0, \beta F^{-1}\left(\frac{p_1 - w_1 - \alpha_1(p_2 - w_2)}{p_1 - \alpha_1 p_2}\right)\right) \end{aligned}$$



APPENDIX D

PROOF OF LEMMA 3.1.4

π_{R2} is jointly concave, so we can find the profit maximizing Q_1 and Q_2 values using the first order conditions.

$$\frac{\partial \pi_{R2}}{\partial Q_1} = p_1 - w_1 - p_1 F\left(\frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2(1 - \beta)}\right) \quad (\text{D.1})$$

$$\frac{\partial \pi_{R2}}{\partial Q_2} = p_2 - w_2 - p_1 \alpha_2 F\left(\frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2(1 - \beta)}\right) + p_1 \alpha_2 F\left(\frac{Q_2}{1 - \beta}\right) - p_2 F\left(\frac{Q_2}{1 - \beta}\right) \quad (\text{D.2})$$

From Equation (D.1) we get:

$$F\left(\frac{Q_1 + \alpha_2 Q_2}{\beta + \alpha_2(1 - \beta)}\right) = \frac{p_1 - w_1}{p_1} \quad (\text{D.3})$$

by inserting Equation (D.3) into Equation (D.2) we get:

$$F\left(\frac{Q_2}{1 - \beta}\right) = \frac{p_2 - w_2 - \alpha_2(p_1 - w_1)}{p_2 - \alpha_2 p_1} \quad (\text{D.4})$$

From equations (D.3) and (D.4):

$$Q_{22}^* = \begin{cases} 0, & \text{if } p_2 - w_2 - \alpha_2(p_1 - w_1) < 0 \\ (1 - \beta) F^{-1}\left(\frac{p_2 - w_2 - \alpha_2(p_1 - w_1)}{p_2 - \alpha_2 p_1}\right), & \text{if otherwise} \end{cases}$$

$$\begin{aligned} Q_{21}^* &= F^{-1}\left(\frac{p_1 - w_1}{p_1}\right) (\beta + \alpha_2(1 - \beta)) - \alpha_2 Q_2 \\ &= F^{-1}\left(\frac{p_1 - w_1}{p_1}\right) (\beta + \alpha_2(1 - \beta)) - \alpha_2 \max\left(0, (1 - \beta) F^{-1}\left(\frac{p_2 - w_2 - \alpha_2(p_1 - w_1)}{p_2 - \alpha_2 p_1}\right)\right) \end{aligned}$$



APPENDIX E

PROOF OF PROPOSITION 3.1.5

We have characterized the retailer's profit function, π_R , as,

$$\pi_R(Q_1, Q_2) = \begin{cases} \pi_{R1}(Q_1, Q_2), & \text{if } \frac{Q_1}{\beta} \leq \frac{Q_2}{1-\beta} \\ \pi_{R2}(Q_1, Q_2), & \text{if } \frac{Q_1}{\beta} > \frac{Q_2}{1-\beta} \end{cases},$$

where Q_{11}^* and Q_{12}^* are unconstrained maximizers for π_{R1} and Q_{21}^* and Q_{22}^* are unconstrained maximizers for π_{R2} , we can write optimal order decision of the retailer as,

$$(Q_1^*, Q_2^*) = \begin{cases} (Q_{11}^*, Q_{12}^*), & \text{if } \frac{Q_1}{\beta} \leq \frac{Q_2}{1-\beta} \\ (Q_{21}^*, Q_{22}^*), & \text{if } \frac{Q_1}{\beta} > \frac{Q_2}{1-\beta} \end{cases}.$$

When we insert Q_{11}^* and Q_{12}^* in to $\frac{Q_1}{\beta} \leq \frac{Q_2}{1-\beta}$, it is seen that, it holds only if $\frac{w_1}{p_1} \geq \frac{w_2}{p_2}$ is true:

$$\begin{aligned} \frac{Q_{11}^*}{\beta} &\leq \frac{Q_{12}^*}{1-\beta} \\ \frac{Q_{11}^*}{\beta} &\leq \frac{1}{1-\beta} F^{-1}\left(\frac{p_2 - w_2}{p_2}\right) (1 - \beta + \alpha_1 \beta) - \frac{1}{1-\beta} \alpha_1 Q_{11}^* \\ Q_{11}^* \left(\frac{1}{\beta} + \frac{\alpha_1}{1-\beta}\right) &\leq \frac{1}{1-\beta} F^{-1}\left(\frac{p_2 - w_2}{p_2}\right) (1 - \beta + \alpha_1 \beta) \\ \frac{Q_{11}^*}{\beta} \frac{1 - \beta + \alpha_1 \beta}{1 - \beta} &\leq \frac{1 - \beta + \alpha_1 \beta}{1 - \beta} F^{-1}\left(\frac{p_2 - w_2}{p_2}\right) \\ \frac{Q_{11}^*}{\beta} &\leq F^{-1}\left(\frac{p_2 - w_2}{p_2}\right) \end{aligned}$$

$$\begin{aligned}
F^{-1}\left(\frac{p_1 - w_1 - \alpha_1(p_2 - w_2)}{p_1 - \alpha_1 p_2}\right) &\leq F^{-1}\left(\frac{p_2 - w_2}{p_2}\right) \\
\left(\frac{p_1 - w_1 - \alpha_1(p_2 - w_2)}{p_1 - \alpha_1 p_2}\right) &\leq \left(\frac{p_2 - w_2}{p_2}\right) \\
p_1 w_2 &\leq p_2 w_1 \\
\frac{w_1}{p_1} &\geq \frac{w_2}{p_2}
\end{aligned}$$

Similarly, when we insert Q_{21}^* and Q_{22}^* in to $\frac{Q_1}{\beta} > \frac{Q_2}{1-\beta}$, it is seen that, it holds only if

$\frac{w_1}{p_1} < \frac{w_2}{p_2}$ is true:

$$\begin{aligned}
\frac{Q_{21}^*}{\beta} &> \frac{Q_{22}^*}{1-\beta} \\
\frac{Q_{22}^*}{1-\beta} &< \frac{1}{\beta} F^{-1}\left(\frac{p_1 - w_1}{p_1}\right) (\beta + \alpha_2(1-\beta)) - \frac{1}{\beta} \alpha_2 Q_2^* \\
Q_{22}^* \left(\frac{1}{1-\beta} + \frac{\alpha_2}{\beta}\right) &< \frac{1}{\beta} F^{-1}\left(\frac{p_1 - w_1}{p_1}\right) (\beta + \alpha_2(1-\beta)) \\
\frac{Q_{22}^*}{1-\beta} \frac{\beta + \alpha_2(1-\beta)}{\beta} &< \frac{(\beta + \alpha_2(1-\beta))}{\beta} F^{-1}\left(\frac{p_1 - w_1}{p_1}\right) \\
\frac{Q_{22}^*}{1-\beta} &< F^{-1}\left(\frac{p_1 - w_1}{p_1}\right) \\
F^{-1}\left(\frac{p_2 - w_2 - \alpha_2(p_1 - w_1)}{p_2 - \alpha_2 p_1}\right) &< F^{-1}\left(\frac{p_1 - w_1}{p_1}\right) \\
\frac{p_2 - w_2 - \alpha_2(p_1 - w_1)}{p_2 - \alpha_2 p_1} &< \frac{p_1 - w_1}{p_1} \\
p_2 w_1 &< p_1 w_2 \\
\frac{w_1}{p_1} &< \frac{w_2}{p_2}
\end{aligned}$$

Hence, when $\frac{w_1}{p_1} \geq \frac{w_2}{p_2}$, the equilibrium point realizes at Q_{11}^* and Q_{12}^* else, since the retailer profit function is continuous and jointly concave in Q_1 and Q_2 , the equilibrium point realizes at region $\frac{w_1}{p_1} < \frac{w_2}{p_2}$, where Q_{21}^* and Q_{22}^* are feasible.

APPENDIX F

PROOF OF COROLLARY 3.1.6

By taking $\frac{w_1}{p_1} = \frac{w_2}{p_2}$ we show that Q_{11}^* and Q_{12}^* , which are the unconstrained maximizers of π_{R1} , are equal to Q_{21}^* and Q_{22}^* , which are the unconstrained maximizers of π_{R2} , respectively:

when $\frac{w_1}{p_1} = \frac{w_2}{p_2}$ then $w_2 = p_2 \frac{w_1}{p_1}$ and corresponding Q_{11}^* , Q_{21}^* , Q_{12}^* and Q_{22}^* values are as in the following:

$$\begin{aligned}
 Q_{11}^* &= \beta F^{-1}\left(\frac{p_1 - w_1 - \alpha_1(p_2 - w_2)}{p_1 - \alpha_1 p_2}\right) \\
 &= \beta F^{-1}\left(\frac{p_1 - w_1}{p_1}\right), \\
 Q_{21}^* &= F^{-1}\left(\frac{p_1 - w_1}{p_1}\right) (\beta + \alpha_2(1 - \beta)) - \alpha_2(1 - \beta) F^{-1}\left(\frac{p_2 - w_2 - \alpha_2(p_1 - w_1)}{p_2 - \alpha_2 p_1}\right) \\
 &= F^{-1}\left(\frac{p_1 - w_1}{p_1}\right) (\beta + \alpha_2(1 - \beta)) - \alpha_2(1 - \beta) F^{-1}\left(\frac{p_1 - w_1}{p_1}\right) \\
 &= \beta F^{-1}\left(\frac{p_1 - w_1}{p_1}\right),
 \end{aligned}$$

it is seen that $Q_{11}^* = Q_{21}^*$;

$$\begin{aligned}
 Q_{12}^* &= F^{-1}\left(\frac{p_2 - w_2}{p_2}\right) (1 - \beta + \alpha_1 \beta) - \alpha_1 \beta F^{-1}\left(\frac{p_2 - w_2}{p_2}\right) \\
 &= (1 - \beta) F^{-1}\left(\frac{p_2 - w_2}{p_2}\right), \\
 Q_{22}^* &= (1 - \beta) F^{-1}\left(\frac{p_1 - w_1}{p_1}\right) \\
 &= (1 - \beta) F^{-1}\left(\frac{p_2 - w_2}{p_2}\right),
 \end{aligned}$$

and it is also seen that $Q_{12}^* = Q_{22}^*$, which means $\pi_R = \pi_{R1} = \pi_{R2}$ at this point.



APPENDIX G

PROOF OF COROLLARY 3.1.7

When $\frac{w_1}{p_1} \geq \frac{w_2}{p_2}$, the optimal order decision of the retailer is characterized as:

$$Q_1^* = \max(0, \beta F^{-1}(\frac{p_1 - w_1 - \alpha_1(p_2 - w_2)}{p_1 - \alpha_1 p_2}))$$

$$Q_2^* = F^{-1}(\frac{p_2 - w_2}{p_2}) (1 - \beta + \alpha_1 \beta) - \alpha_1 \max(0, \beta F^{-1}(\frac{p_1 - w_1 - \alpha_1(p_2 - w_2)}{p_1 - \alpha_1 p_2}))$$

It is seen that, Q_1^* is equal to zero if:

$$F^{-1}(\frac{p_1 - w_1 - \alpha_1(p_2 - w_2)}{p_1 - \alpha_1 p_2}) \leq 0 \Leftrightarrow$$

$$\frac{p_1 - w_1 - \alpha_1(p_2 - w_2)}{p_1 - \alpha_1 p_2} \leq 0 \tag{G.1}$$

Since, we assume that $(p_1 - \alpha_1 p_2)$ is always positive; Equation (G.1) is less than or equal to zero only if:

$$p_1 - w_1 - \alpha_1(p_2 - w_2) \leq 0$$

$$w_1 \geq p_1 - \alpha_1(p_2 - w_2)$$



APPENDIX H

PROOF OF COROLLARY 3.1.8

When $\frac{w_1}{p_1} < \frac{w_2}{p_2}$, the optimal order decision of the retailer is characterized as:

$$Q_2^* = \max(0, (1 - \beta) F^{-1}\left(\frac{p_2 - w_2 - \alpha_2(p_1 - w_1)}{p_2 - \alpha_2 p_1}\right))$$

$$Q_1^* = F^{-1}\left(\frac{p_1 - w_1}{p_1}\right) (\beta + \alpha_2(1 - \beta)) - \alpha_2 \max(0, (1 - \beta) F^{-1}\left(\frac{p_2 - w_2 - \alpha_2(p_1 - w_1)}{p_2 - \alpha_2 p_1}\right))$$

It is seen that Q_2^* is equal to zero if:

$$F^{-1}\left(\frac{p_2 - w_2 - \alpha_2(p_1 - w_1)}{p_2 - \alpha_2 p_1}\right) \leq 0$$

and it is equal zero if:

$$\frac{p_2 - w_2 - \alpha_2(p_1 - w_1)}{p_2 - \alpha_2 p_1} \leq 0 \tag{H.1}$$

since we assume $(p_2 - \alpha_2 p_1) > 0$, Equation (H.1) is less than or equal to zero only if:

$$p_2 - w_2 - \alpha_2(p_1 - w_1) \leq 0$$

$$w_1 \leq p_1 - \frac{(p_2 - w_2)}{\alpha_2}$$



APPENDIX I

PROOF OF COROLLARY 3.1.10

When $\frac{w_1}{p_1} \geq \frac{w_2}{p_2}$, $Q_1 = \beta F^{-1}\left(\frac{p_1 - w_1 - \alpha_1(p_2 - w_2)}{p_1 - \alpha_1 p_2}\right)$. The expression can be rewritten as $F\left(\frac{Q_1}{\beta}\right) = \frac{p_1 - w_1 - \alpha_1(p_2 - w_2)}{p_1 - \alpha_1 p_2}$. Taking the derivatives of both sides with respect to w_1 , we get

$$\begin{aligned} \frac{\partial Q_1(w_1)}{\partial w_1} \frac{1}{\beta} f\left(\frac{Q_1(w_1)}{\beta}\right) &= -\frac{1}{p_1 - \alpha_1 p_2} \\ \frac{\partial Q_1(w_1)}{\partial w_1} &= -\frac{\beta}{p_1 - \alpha_1 p_2} \frac{1}{f\left(\frac{Q_1(w_1)}{\beta}\right)} < 0 \end{aligned} \quad (\text{I.1})$$

The inequality in I.1 follows since we assume that $(p_1 - \alpha_1 p_2)$ is always positive. Hence, when $\frac{w_1}{p_1} \geq \frac{w_2}{p_2}$, $Q_1(w_1)$ is decreasing in w_1 .

When $\frac{w_1}{p_1} \geq \frac{w_2}{p_2}$, $Q_2 = F^{-1}\left(\frac{p_2 - w_2}{p_2}\right) (1 - \beta + \alpha_1 \beta) - \alpha_1 Q_1$. The expression can be rewritten as $F\left(\frac{Q_2(w_1) + \alpha_1 Q_1(w_1)}{1 - \beta + \alpha_1 \beta}\right) = \frac{p_2 - w_2}{p_2}$. Taking the derivatives of both sides with respect to w_1 , we get

$$\begin{aligned} \left(\frac{\partial Q_2(w_1)}{\partial w_1} + \alpha_1 \frac{\partial Q_1(w_1)}{\partial w_1}\right) \frac{1}{1 - \beta + \alpha_1 \beta} f\left(\frac{Q_2(w_1) + \alpha_1 Q_1(w_1)}{1 - \beta + \alpha_1 \beta}\right) &= 0 \\ \frac{\partial Q_2(w_1)}{\partial w_1} &= -\alpha_1 \frac{\partial Q_1(w_1)}{\partial w_1} > 0 \end{aligned} \quad (\text{I.2})$$

Inequality I.2 follows from Inequality I.1.

When $\frac{w_1}{p_1} < \frac{w_2}{p_2}$, $Q_2^* = \max(0, (1 - \beta) F^{-1}\left(\frac{p_2 - w_2 - \alpha_2(p_1 - w_1)}{p_2 - \alpha_2 p_1}\right))$ and can be rewritten as $F\left(\frac{Q_2(w_1)}{1 - \beta}\right) = \frac{p_2 - w_2 - \alpha_2(p_1 - w_1)}{p_2 - \alpha_2 p_1}$. Taking the derivatives of both sides with respect to w_1 , we get

$$\begin{aligned} \frac{\partial Q_2(w_1)}{\partial w_1} \frac{1}{1 - \beta} f\left(\frac{Q_2(w_1)}{1 - \beta}\right) &= \frac{\alpha_2}{p_2 - \alpha_2 p_1} \\ \frac{\partial Q_2(w_1)}{\partial w_1} &= \frac{(1 - \beta)\alpha_2}{p_2 - \alpha_2 p_1} \frac{1}{f\left(\frac{Q_2(w_1)}{1 - \beta}\right)} > 0 \end{aligned} \quad (\text{I.3})$$

The inequality in I.3 follows since $(p_2 - \alpha_2 p_1)$ is always positive. Hence, when $\frac{w_1}{p_1} < \frac{w_2}{p_2}$, $Q_2^*(w_1)$ is increasing in w_1 .

When $\frac{w_1}{p_1} < \frac{w_2}{p_2}$, $Q_1^* = F^{-1}(\frac{p_1 - w_1}{p_1}) (\beta + \alpha_2(1 - \beta)) - \alpha_2 Q_2^*$ and can be rewritten as $\partial(F(\frac{Q_1^*(w_1) + \alpha_2 Q_2^*(w_1)}{\beta + \alpha_2(1 - \beta)})) = \partial(\frac{p_1 - w_1}{p_1})$. By taking the derivatives of both sides with respect to w_1 , we get

$$\begin{aligned} \left(\frac{\partial Q_1^*(w_1)}{\partial w_1} + \alpha_2 \frac{\partial Q_2^*(w_1)}{\partial w_1} \right) \frac{1}{\beta + \alpha_2(1 - \beta)} f\left(\frac{Q_1^*(w_1) + \alpha_2 Q_2^*(w_1)}{\beta + \alpha_2(1 - \beta)}\right) &= -\frac{1}{p_1} \\ \left(\frac{\partial Q_1^*(w_1)}{\partial w_1} + \alpha_2 \frac{\partial Q_2^*(w_1)}{\partial w_1} \right) &= -\frac{\beta + \alpha_2(1 - \beta)}{p_1} \frac{1}{f\left(\frac{Q_1^*(w_1) + \alpha_2 Q_2^*(w_1)}{\beta + \alpha_2(1 - \beta)}\right)} \\ \frac{\partial Q_1^*(w_1)}{\partial w_1} &= -\alpha_2 \frac{\partial Q_2^*(w_1)}{\partial w_1} - \frac{\beta + \alpha_2(1 - \beta)}{p_1} \frac{1}{f\left(\frac{Q_1^*(w_1) + \alpha_2 Q_2^*(w_1)}{\beta + \alpha_2(1 - \beta)}\right)} < 0 \end{aligned} \quad (\text{I.4})$$

Inequality I.4 follows from Inequality I.3.

These calculations show that, always, Q_1^* is decreasing and Q_2^* is increasing in w_1 .

APPENDIX J

PROOF OF PROPOSITION 3.2.1

The profit function of supplier 2 is equal to $\pi_2 = Q_2(w_2 - c_2)$, in order to analyze unimodality of the profit function we analyze the first and second order conditions of supplier 2's profit function:

$$\frac{\partial \pi_2(w_2)}{\partial w_2} = \frac{\partial Q_2(w_1, w_2)}{\partial w_2} (w_2 - c_2) + Q_2(w_1, w_2)$$

We can summarize the retailer's best response for product 2; i.e., Q_2^* as follows:

$$Q_2^*(w_1, w_2) = \begin{cases} F^{-1}\left(\frac{p_2 - w_2}{p_2}\right) (1 - \beta + \alpha_1 \beta) & \text{if } w_2 \leq p_2 - \frac{(p_1 - w_1)}{\alpha_1} \\ \\ F^{-1}\left(\frac{p_2 - w_2}{p_2}\right) (1 - \beta + \alpha_1 \beta) \\ -\alpha_1 \beta F^{-1}\left(\frac{p_1 - w_1 - \alpha_1 (p_2 - w_2)}{p_1 - \alpha_1 p_2}\right) & \text{if } p_2 - \frac{(p_1 - w_1)}{\alpha_1} < w_2 < p_2 \frac{w_1}{p_1} \\ \\ (1 - \beta) F^{-1}\left(\frac{p_2 - w_2 - \alpha_2 (p_1 - w_1)}{p_2 - \alpha_2 p_1}\right) & \text{if } p_2 \frac{w_1}{p_1} \leq w_2 < p_2 - \alpha_2 (p_1 - w_1) \\ 0 & \text{if } w_2 \geq p_2 - \alpha_2 (p_1 - w_1), \end{cases}$$

Then we have

$$\frac{Q_2^*(w_1, w_2)}{\partial w_2} = \begin{cases} -\frac{(1-\beta)+\alpha_1(\beta)}{p_2} \frac{1}{f\left(\frac{Q_2(w_2)}{(1-\beta)+\alpha_1(\beta)}\right)} & \text{if } w_2 \leq p_2 - \frac{(p_1 - w_1)}{\alpha_1} \\ \\ -\frac{(1-\beta)+\alpha_1(\beta)}{p_2} \frac{1}{f\left(\frac{Q_2(w_2)+\alpha_1 Q_1(w_2)}{(1-\beta)+\alpha_1(\beta)}\right)} \\ -\frac{\alpha_1 \alpha_1(\beta)}{p_1 - \alpha_1 p_2} \frac{1}{f\left(\frac{Q_1(w_2)}{\beta}\right)} & \text{if } p_2 - \frac{(p_1 - w_1)}{\alpha_1} < w_2 < p_2 \frac{w_1}{p_1} \\ \\ -\frac{(1-\beta)}{p_2 - \alpha_2 p_1} \frac{1}{f\left(\frac{Q_2(w_2)}{(1-\beta)}\right)} & \text{if } p_2 \frac{w_1}{p_1} \leq w_2 < p_2 - \alpha_2 (p_1 - w_1) \\ 0 & \text{if } w_2 \geq p_2 - \alpha_2 (p_1 - w_1), \end{cases}$$

and corresponding first order conditions of supplier 2's profit function can be written as

$$\frac{\partial \pi_2(w_2)}{\partial w_2} = \begin{cases} -\frac{1-\beta+\alpha_1\beta}{p_2} \frac{1}{f(\frac{Q_2(w_2)}{(1-\beta)+\alpha_1\beta})} (w_2 - c_2) \\ + F^{-1}\left(\frac{p_2-w_2}{p_2}\right) (1 - \beta + \alpha_1\beta) & \text{if } w_2 \leq p_2 - \frac{p_1-w_1}{\alpha_1} \\ \\ -\frac{1-\beta+\alpha_1\beta}{p_2} \frac{1}{f(\frac{Q_2(w_2)+\alpha_1 Q_1(w_2)}{1-\beta+\alpha_1\beta})} (w_2 - c_2) \\ -\frac{\alpha_1\alpha_1\beta}{p_1-\alpha_1 p_2} \frac{1}{f(\frac{Q_1(w_2)}{\beta})} (w_2 - c_2) \\ + F^{-1}\left(\frac{p_2-w_2}{p_2}\right) (1 - \beta + \alpha_1\beta) \\ -\alpha_1\beta F^{-1}\left(\frac{p_1-w_1-\alpha_1(p_2-w_2)}{p_1-\alpha_1 p_2}\right) & \text{if } p_2 - \frac{(p_1-w_1)}{\alpha_1} < w_2 < p_2 \frac{w_1}{p_1} \\ \\ \left(\frac{-(1-\beta)}{p_2-\alpha_2 p_1} \frac{1}{f(\frac{Q_2(w_2)}{(1-\beta)})}\right) (w_2 - c_2) \\ + (1 - \beta) F^{-1}\left(\frac{p_2-w_2-\alpha_2(p_1-w_1)}{p_2-\alpha_2 p_1}\right) & \text{if } p_2 \frac{w_1}{p_1} \leq w_2 < p_2 - \alpha_2(p_1 - w_1) \\ \\ 0 & \text{if } w_2 \geq p_2 - \alpha_2(p_1 - w_1), \end{cases}$$

under uniform demand, these first order conditions can be written as follows

$$\frac{\partial \pi_2(w_2)}{\partial w_2} = \begin{cases} -\frac{1-\beta+\alpha_1\beta}{p_2} T(w_2 - c_2) \\ + T\left(\frac{p_2-w_2}{p_2}\right) (1 - \beta + \alpha_1\beta) & \text{if } w_2 \leq p_2 - \frac{p_1-w_1}{\alpha_1} \\ \\ \left(-\frac{1-\beta+\alpha_1\beta}{p_2} - \frac{\alpha_1\alpha_1\beta}{p_1-\alpha_1 p_2}\right) T(w_2 - c_2) \\ + T\left(\frac{p_2-w_2}{p_2}\right) ((1 - \beta) + \alpha_1(\beta)) \\ -\alpha_1\beta T\left(\frac{p_1-w_1-\alpha_1(p_2-w_2)}{p_1-\alpha_1 p_2}\right) & \text{if } p_2 - \frac{(p_1-w_1)}{\alpha_1} < w_2 < p_2 \frac{w_1}{p_1} \\ \\ \frac{-(1-\beta)}{p_2-\alpha_2 p_1} T(w_2 - c_2) \\ + (1 - \beta) T\left(\frac{p_2-w_2-\alpha_2(p_1-w_1)}{p_2-\alpha_2 p_1}\right) & \text{if } p_2 \frac{w_1}{p_1} \leq w_2 < p_2 - \alpha_2(p_1 - w_1) \\ \\ 0 & \text{if } w_2 \geq p_2 - \alpha_2(p_1 - w_1), \end{cases}$$

then corresponding second order conditions of supplier 2's profit function can be written as

$$\frac{\partial^2 \pi_2(w_2)}{\partial w_2^2} = \begin{cases} -T \frac{1-\beta+\alpha_1\beta}{p_2} - T \frac{1-\beta+\alpha_1\beta}{p_2} & \text{if } w_2 \leq p_2 - \frac{p_1-w_1}{\alpha_1} \\ \left(-\frac{1-\beta+\alpha_1\beta}{p_2} - \frac{\alpha_1\alpha_1\beta}{p_1-\alpha_1p_2} \right) T & \text{if } p_2 - \frac{(p_1-w_1)}{\alpha_1} < w_2 < p_2 \frac{w_1}{p_1} \\ -T \frac{1-\beta+\alpha_1\beta}{p_2} - \alpha_1\beta T \frac{\alpha_1}{p_1-\alpha_1p_2} & \\ \frac{-(1-\beta)}{p_2-\alpha_2p_1} T - (1-\beta) T \left(\frac{1}{p_2-\alpha_2p_1} \right) & \text{if } p_2 \frac{w_1}{p_1} \leq w_2 < p_2 - \alpha_2(p_1 - w_1) \\ 0 & \text{if } w_2 \geq p_2 - \alpha_2(p_1 - w_1). \end{cases}$$

As it is seen, the second order conditions are negative, $\frac{\partial^2 \pi_2(w_2)}{\partial w_2^2} < 0$, which means each piece of supplier 2's profit function is concave, and there is a unique optimal for each of the pieces. Note that, $\frac{\partial \pi_2(w_2)}{\partial w_2}$ function has three breakpoints. In the following parts (i)-(ii) and (iii), we analyze the first derivatives at these transition points, in order to characterize the behavior of $\pi_2(w_1, w_2, Q_1^*(w_1, w_2), Q_2^*(w_1, w_2))$, just before and after the transition points. Since all of the pieces are concave, it is sure that if the profit function starts to decrease at any point belong to a piece it continues to decrease up to end of that piece. Therefore, by analyzing the limits at the transition points we look for whether or not while a piece of the profit function is decreasing the next piece can start to increase.

(i) The first transition point of $\pi_2(w_2)$ is at $w_2 = p_2 - \frac{(p_1-w_1)}{\alpha_1}$. Firstly, we analyze the left and the right limits of the profit function at this point.

The left limit, $\lim_{w_2 \rightarrow (p_2 - \frac{(p_1-w_1)}{\alpha_1})^-} \frac{\partial \pi_2(w_2)}{\partial w_2}$ is calculated as

$$\begin{aligned} F^{-1}\left(\frac{p_1-w_1}{\alpha_1 p_2}\right) \left((1-\beta) + \alpha_1(\beta) \right) - \frac{(1-\beta)+\alpha_1(\beta)}{p_2} \frac{1}{f(F^{-1}(\frac{p_1-w_1}{\alpha_1 p_2}))} \left(p_2 - \frac{p_1-w_1}{\alpha_1} - c_2 \right) \\ T \left(\frac{p_1-w_1}{\alpha_1 p_2} \right) \left((1-\beta) + \alpha_1(\beta) \right) - \frac{(1-\beta)+\alpha_1(\beta)}{p_2} T \left(p_2 - \frac{p_1-w_1}{\alpha_1} - c_2 \right) \end{aligned} \quad (J.1)$$

Equation (J.1), $\lim_{w_2 \rightarrow (p_2 - \frac{(p_1-w_1)}{\alpha_1})^-} \frac{\partial \pi_2(w_2)}{\partial w_2}$ is negative if and only if

$$\begin{aligned} T \left(\frac{p_1-w_1}{\alpha_1 p_2} \right) \left((1-\beta) + \alpha_1(\beta) \right) &<? \frac{(1-\beta)+\alpha_1(\beta)}{p_2} T \left(p_2 - \frac{p_1-w_1}{\alpha_1} - c_2 \right) <=> \\ \left(\frac{p_1-w_1}{\alpha_1} \right) &<? \left(p_2 - \frac{p_1-w_1}{\alpha_1} - c_2 \right) <=> \\ w_1 &> p_1 - \frac{\alpha_1(p_2-c_2)}{2} \end{aligned}$$

From this point on, we refer to $p_1 - \frac{\alpha_1(p_2 - c_2)}{2}$ as D_2 . As a result, when $w_1 > D_2$, π_2 is decreasing in w_2 , when w_2 approaches $w_2 = p_2 - \frac{(p_1 - w_1)}{\alpha_1}$ from the left. On the other hand, if $w_1 < D_2$, then π_2 is increasing in w_2 .

Then, the right limit, $\lim_{w_2=(p_2-\frac{(p_1-w_1)}{\alpha_1})^+} \frac{\partial \pi_2(w_2)}{\partial w_2}$ is calculated as

$$\begin{aligned} & F^{-1}\left(\frac{p_1-w_1}{\alpha_1 p_2}\right) (1 - \beta + \alpha_1 \beta) - \frac{1-\beta+\alpha_1\beta}{p_2} \frac{(p_2 - \frac{p_1-w_1}{\alpha_1} - c_2)}{f(F^{-1}(\frac{p_1-w_1}{\alpha_1 p_2} + \frac{\beta F^{-1}(0)}{1-\beta+\alpha_1\beta}))} - \frac{\alpha_1 \alpha_1 \beta}{p_1 - \alpha_1 p_2} \frac{(p_2 - \frac{p_1-w_1}{\alpha_1} - c_2)}{f(F^{-1}(0))} \\ & F^{-1}\left(\frac{p_1-w_1}{\alpha_1 p_2}\right) (1 - \beta + \alpha_1 \beta) - \frac{1-\beta+\alpha_1\beta}{p_2} \frac{(p_2 - \frac{p_1-w_1}{\alpha_1} - c_2)}{f(F^{-1}(\frac{p_1-w_1}{\alpha_1 p_2}))} - \frac{\alpha_1 \alpha_1 \beta}{p_1 - \alpha_1 p_2} \frac{(p_2 - \frac{p_1-w_1}{\alpha_1} - c_2)}{f(F^{-1}(0))} \\ & T\left(\frac{p_1-w_1}{\alpha_1 p_2}\right) (1 - \beta + \alpha_1 \beta) - \left(\frac{1-\beta+\alpha_1\beta}{p_2} + \frac{\alpha_1^2 \beta}{p_1 - \alpha_1 p_2}\right) T\left(p_2 - \frac{p_1-w_1}{\alpha_1} - c_2\right) \end{aligned} \quad (J.2)$$

It is seen that:

$$\begin{aligned} \lim_{w_2 \rightarrow (p_2 - \frac{(p_1-w_1)}{\alpha_1})^+} \frac{\partial \pi_2(w_2)}{\partial w_2} &= \left(\lim_{w_2 \rightarrow (p_2 - \frac{(p_1-w_1)}{\alpha_1})^-} \frac{\partial \pi_2(w_2)}{\partial w_2} \right) \\ &\quad - T \frac{\alpha_1^2 \beta}{p_1 - \alpha_1 p_2} \left(p_2 - \frac{p_1-w_1}{\alpha_1} - c_2\right) \end{aligned}$$

It is straightforward to show that $(p_2 - \frac{p_1-w_1}{\alpha_1} - c_2) > 0$ when $w_1 > D_2$. Hence, Equation (J.2) < Equation (J.1), which means when π_2 is decreasing at the left hand side of the point $w_2 = p_2 - \frac{(p_1-w_1)}{\alpha_1}$, it continues to decrease at the right hand side of that point.

When $w_1 > D_2$, the profit function is decreasing at the both sides of the point $w_2 = p_2 - \frac{(p_1-w_1)}{\alpha_1}$. However, when $w_1 < D_2$, the profit function is increasing at the left hand side of that point and if and only if $\lim_{w_2=(p_2-\frac{(p_1-w_1)}{\alpha_1})^+} \geq 0$ it continues to increase at the right hand side, else it starts to decrease at the left hand side of the point $w_2 = p_2 - \frac{(p_1-w_1)}{\alpha_1}$.

$\lim_{w_2=(p_2-\frac{(p_1-w_1)}{\alpha_1})^+}$ is negative iff

$$\begin{aligned} \left(\frac{p_1-w_1}{\alpha_1 p_2}\right) ((1 - \beta) + \alpha_1 (\beta)) &<? \frac{(1-\beta)+\alpha_1(\beta)}{p_2} \left(p_2 - \frac{p_1-w_1}{\alpha_1} - c_2\right) + \frac{\alpha_1 \alpha_1 (\beta)}{p_1 - \alpha_1 p_2} \left(p_2 - \frac{p_1-w_1}{\alpha_1} - c_2\right) <=> \\ w_1 &> p_1 - \frac{\alpha_1(p_2 - c_2)((1-\beta)(p_1 - \alpha_1 p_2) + \alpha_1(\beta)p_1)}{2((1-\beta)(p_1 - \alpha_1 p_2) + \alpha_1(\beta)p_2) - \alpha_1 \alpha_1 (\beta)p_2} \end{aligned}$$

From this point on, we refer to $p_1 - \frac{\alpha_1(p_2 - c_2)((1-\beta)(p_1 - \alpha_1 p_2) + \alpha_1(\beta)p_1)}{2((1-\beta)(p_1 - \alpha_1 p_2) + \alpha_1(\beta)p_2) - \alpha_1 \alpha_1 (\beta)p_2}$ as C_2 . As a result, when $w_1 > C_2$ the profit function is increasing when w_2 approaches to point $w_2 = p_2 - \frac{(p_1-w_1)}{\alpha_1}$ from the left but it is decreasing when w_2 approached that point

from the right. However, if $w_1 < C_2$ the profit function is increasing at both sides of that point.

The behavior of the supplier 2's profit function before and after the point $w_2 = p_2 - \frac{(p_1 - w_1)}{\alpha_1}$ can be summarized as follows:

$\lim_{w_2 = (p_2 - \frac{(p_1 - w_1)}{\alpha_1})^-}$	$\lim_{w_2 = (p_2 - \frac{(p_1 - w_1)}{\alpha_1})^+}$	conditions
-	-	$w_1 > D_2 > C_2$
+	-	$D_2 > w_1 > C_2$
-	+	not feasible
+	+	$D_2 > C_2 > w_1$

(ii) Next we analyze the behavior of profit functions before and after the point $w_2 =$

$p_2 \frac{w_1}{p_1}$. The left limit, $\lim_{w_2 \rightarrow (p_2 \frac{w_1}{p_1})^-} \frac{\partial \pi_2(w_2)}{\partial w_2}$ is calculated as

$$F^{-1}\left(\frac{p_1 - w_1}{p_1}\right) (1 - \beta) - \left(\frac{(1 - \beta) + \alpha_1(\beta)}{p_2} + \frac{\alpha_1 \alpha_1(\beta)}{p_1 - \alpha_1 p_2}\right) \frac{(p_2 \frac{w_1}{p_1} - c_2)}{f(F^{-1}(\frac{p_1 - w_1}{p_1}))} \quad (\text{J.3})$$

Equation J.3, $\lim_{w_2 \rightarrow (p_2 \frac{w_1}{p_1})^-} \frac{\partial \pi_2(w_2)}{\partial w_2}$ is negative iff

$$\begin{aligned} F^{-1}\left(\frac{p_1 - w_1}{p_1}\right) f(F^{-1}(\frac{p_1 - w_1}{p_1})) &<? \left(\frac{(1-\beta)+\alpha_1(\beta)}{p_2} + \frac{\alpha_1 \alpha_1(\beta)}{p_1 - \alpha_1 p_2}\right) <=> \\ &\left(p_2 \frac{w_1}{p_1} - c_2\right) \frac{2}{(1-\beta)} \\ T\left(\frac{p_1 - w_1}{p_1}\right) \frac{2}{T} &<? \left(\frac{(1-\beta)(p_1 - \alpha_1 p_2) + \alpha_1(\beta)p_1}{p_1 - \alpha_1 p_2}\right) <=> \\ &\left(p_2 \frac{w_1}{p_1} - c_2\right) \left(\frac{2}{(1-\beta)}\right) \left(\frac{2}{p_2}\right) \\ \frac{p_1 - w_1}{p_1} &<? \left(\frac{(1-\beta)(p_1 - \alpha_1 p_2) + \alpha_1(\beta)p_1}{p_1 - \alpha_1 p_2}\right) <=> \\ &\left(p_2 \frac{w_1}{p_1} - c_2\right) \left(\frac{2}{(1-\beta)}\right) \left(\frac{2}{p_2}\right) \\ w_1 &> \frac{p_1}{p_2} \left(\frac{(1-\beta)(p_1 - \alpha_1 p_2)(p_2 - c_2)}{2(1-\beta)(p_1 - \alpha_1 p_2) + \alpha_1(\beta)p_1} + c_2\right) \end{aligned}$$

From this point on we refer to $\frac{p_1}{p_2} \left(\frac{(1-\beta)(p_1 - \alpha_1 p_2)(p_2 - c_2)}{2(1-\beta)(p_1 - \alpha_1 p_2) + \alpha_1(\beta)p_1} + c_2\right)$ as A_2 . When $w_1 > A_2$, $\pi_2(w_2)$ is decreasing, when w_2 approaches to point $w_2 = p_2 \frac{w_1}{p_1}$ from the left.

The right limit, $\lim_{w_2 \rightarrow (p_2 \frac{w_1}{p_1})^+} \frac{\partial \pi_2(w_2)}{\partial w_2}$ is calculated as

$$\frac{\partial \pi(w_2)}{\partial w_2} = (1 - \beta) F^{-1}\left(\frac{p_2 - p_2 \frac{w_1}{p_1} - \alpha_2(p_1 - w_1)}{p_2 - \alpha_2 p_1}\right) - \left(\frac{(1 - \beta)}{p_2 - \alpha_2 p_1} \frac{(p_2 \frac{w_1}{p_1} - c_2)}{f\left(\frac{Q_2(w_2)}{(1 - \beta)}\right)}\right) \quad (\text{J.4})$$

Equation J.4, $\lim_{w_2 \rightarrow (p_2 \frac{w_1}{p_1})^+} \frac{\partial \pi_2(w_2)}{\partial w_2}$, is negative iff

$$\begin{aligned} F^{-1}\left(\frac{p_2 - p_2 \frac{w_1}{p_1} - \alpha_2(p_1 - w_1)}{p_2 - \alpha_2 p_1}\right) f\left(\frac{Q_2(w_2)}{(1 - \beta)}\right) &<? \left(\frac{p_2 \frac{w_1}{p_1} - c_2}{p_2 - \alpha_2 p_1}\right) <=> \\ T\left(\frac{p_2 - p_2 \frac{w_1}{p_1} - \alpha_2(p_1 - w_1)}{p_2 - \alpha_2 p_1}\right) \frac{1}{T} &<? \left(\frac{p_2 \frac{w_1}{p_1} - c_2}{p_2 - \alpha_2 p_1}\right) <=> \\ p_2 - p_2 \frac{w_1}{p_1} - \alpha_2(p_1 - w_1) &<? p_2 \frac{w_1}{p_1} - c_2 <=> \\ w_1 &> \frac{p_1(p_2 - \alpha_2 p_1 + c_2)}{2p_2 - \alpha_2 p_1} \end{aligned}$$

From this point on we refer to $\frac{p_1(p_2 - \alpha_2 p_1 + c_2)}{2p_2 - \alpha_2 p_1}$ as B_2 . When $w_1 > B_2$, $\pi_2(w_2)$ is decreasing just after the point $w_2 = p_2 \frac{w_1}{p_1}$.

The behavior of the supplier 2's profit function, before and after the point $w_2 = p_2 \frac{w_1}{p_1}$ can be summarized as in the following:

$\lim_{w_2 \rightarrow (p_2 \frac{w_1}{p_1})^-} \frac{\partial \pi_2(w_2)}{\partial w_2}$	$\lim_{w_2 \rightarrow (p_2 \frac{w_1}{p_1})^+} \frac{\partial \pi_2(w_2)}{\partial w_2}$	conditions
-	-	$w_1 > \max(A_2, B_2)$
+	-	$A_2 > w_1 > B_2$
-	+	$B_2 > w_1 > A_2$
+	+	$\min(A_2, B_2) > w_1$

While the profit function is decreasing at the left hand side of the point $w_2 = p_2 \frac{w_1}{p_1}$, it continues to decrease at the right hand side only if $B_2 \leq A_2$ which simplifies into:

$$\begin{aligned} \frac{p_1}{p_2} \left(\frac{(1 - \beta)(p_1 - \alpha_1 p_2)(p_2 - c_2)}{2(1 - \beta)(p_1 - \alpha_1 p_2) + \alpha_1(\beta)p_1} + c_2 \right) &\geq \frac{p_1(p_2 - \alpha_2 p_1 + c_2)}{2p_2 - \alpha_2 p_1} \\ \alpha_1 \beta (p_2 - \alpha_2 p_1) &\leq \alpha_2 (1 - \beta)(p_1 - \alpha_1 p_2) \end{aligned}$$

Which is guarantied by the condition in Equation 3.2.1.

(iii) The last transition point of $\pi_2(w_2)$ is $p_2 - \alpha_2(p_1 - w_1)$. When w_2 approaches to this point from the left hand side, derivative of the profit function is as follows

$$\begin{aligned}
\lim_{w_2 \rightarrow (p_2 - \alpha_2(p_1 - w_1))^-} \frac{\partial \pi_2(w_2)}{\partial w_2} &= \left(-\frac{(1-\beta)}{p_2 - \alpha_2 p_1} \frac{1}{f\left(\frac{Q_2(w_2)}{(1-\beta)}\right)} \right) (w_2 - c_2) \\
&\quad + (1 - \beta) F^{-1}\left(\frac{p_2 - w_2 - \alpha_2(p_1 - w_1)}{p_2 - \alpha_2 p_1}\right) \\
&= -\frac{1-\beta}{p_2 - \alpha_2 p_1} T(w_2 - c_2) + (1 - \beta) T\left(\frac{p_2 - w_2 - \alpha_2(p_1 - w_1)}{p_2 - \alpha_2 p_1}\right) \\
&= -\frac{1-\beta}{p_2 - \alpha_2 p_1} T(p_2 - \alpha_2(p_1 - w_1) - c_2) + 0
\end{aligned}$$

It is seen that, it is always negative just before this point, and we know that after this point, $\pi_2(w_2)$ is equal to zero. As a result of these calculations, the behavior of the profit function can be summarized as in the following

Conditions	transition point 1	transition point 2	transition point 3
$D_2 < w_1$	(-, -)	(-, -)	(-, 0)
$C_2 < w_1 \leq D_2$	(+, -)	(-, -)	(-, 0)
$A_2 < w_1 \leq C_2$	(+, +)	(-, -)	(-, 0)
$B_2 < w_1 \leq A_2$	(+, +)	(+, -)	(-, 0)
$w_1 \leq B_2$	(+, +)	(+, +)	(-, 0)

Therefore, it is true that, π_2 is a unimodal function, under condition $\alpha_1 \beta (p_2 - \alpha_2 p_1) \leq \alpha_2 (1 - \beta) (p_1 - \alpha_1 p_2)$.



APPENDIX K

PROOF OF COROLLARY 3.2.2

In the Proof of Proposition 3.2.1, we have characterized the behavior of supplier 2's profit function. And in the following parts (i)-(ii)-(iii)-(iv) and (v), since all pieces of the profit function are concave, we find the optimal $w_2^*(w_1)$ values by equating corresponding derivatives to zero.

$$\frac{\partial \pi_2(w_2)}{\partial w_2} = \begin{cases} -\frac{1-\beta+\alpha_1\beta}{p_2} T(w_2 - c_2) + T\left(\frac{p_2-w_2}{p_2}\right)(1-\beta+\alpha_1\beta) & \text{if } w_2 \leq p_2 - \frac{p_1-w_1}{\alpha_1} \\ \left(-\frac{1-\beta+\alpha_1\beta}{p_2} - \frac{\alpha_1\alpha_1\beta}{p_1-\alpha_1p_2}\right) T(w_2 - c_2) + T\left(\frac{p_2-w_2}{p_2}\right)((1-\beta)+\alpha_1(\beta)) - \alpha_1\beta T\left(\frac{p_1-w_1-\alpha_1(p_2-w_2)}{p_1-\alpha_1p_2}\right) & \text{if } p_2 - \frac{(p_1-w_1)}{\alpha_1} < w_2 < p_2 \frac{w_1}{p_1} \\ \frac{-(1-\beta)}{p_2-\alpha_2p_1} T(w_2 - c_2) + (1-\beta) T\left(\frac{p_2-w_2-\alpha_2(p_1-w_1)}{p_2-\alpha_2p_1}\right) & \text{if } p_2 \frac{w_1}{p_1} \leq w_2 < p_2 - \alpha_2(p_1 - w_1) \\ 0 & \text{if } w_2 \geq p_2 - \alpha_2(p_1 - w_1), \end{cases}$$

(i). When $w_1 > D_2$, the profit function is decreasing before and after the point $w_2 = p_2 - \frac{p_1-w_1}{\alpha_1}$. The optimal point realizes before this point and found by equating the first derivative to zero:

$$\begin{aligned} \left(-\frac{(1-\beta)+\alpha_1(\beta)}{p_2} T\right)(w_2 - c_2) + T\left(\frac{p_2-w_2}{p_2}\right)((1-\beta)+\alpha_1\beta) &= 0 \\ \left(\frac{(1-\beta)+\alpha_1(\beta)}{p_2}\right)(w_2 - c_2) &= \left(\frac{p_2-w_2}{p_2}\right)((1-\beta)+\alpha_1\beta) \end{aligned}$$

$$p_2 - w_2 = w_2 - c_2$$

$$w_2^* = \frac{p_2 + c_2}{2}$$

It is straightforward to see $w_2^* = \frac{p_2 + c_2}{2} \leq p_2 - \frac{p_1 - w_1}{\alpha_1}$ by plugging D_2^+ into w_1 .

(ii). When $C_2 < w_1 \leq D_2$, the profit function is increasing before the point $w_2 = p_2 - \frac{p_1 - w_1}{\alpha_1}$ but decreasing after that point. So the optimal point realizes at that point:

$$w_2^* = p_2 - \frac{p_1 - w_1}{\alpha_1}$$

(iii). When $A_2 < w_1 \leq C_2$, the profit function is decreasing before and after the point $w_2 = p_2 \frac{w_1}{p_1}$. The optimal point realizes before this point, where the following derivative is equal to zero:

$$\begin{aligned} & \left(-\frac{1-\beta+\alpha_1\beta}{p_2} - \frac{\alpha_1^2\beta}{p_1-\alpha_1p_2} \right) T(w_2 - c_2) + T\left(\frac{p_2-w_2}{p_2}\right) (1-\beta+\alpha_1\beta) - \alpha_1\beta T\left(\frac{p_1-w_1-\alpha_1(p_2-w_2)}{p_1-\alpha_1p_2}\right) = 0 \\ & \left(\frac{(1-\beta)+\alpha_1(\beta)}{p_2} + \frac{\alpha_1\alpha_1(\beta)}{p_1-\alpha_1p_2} \right) (w_2 - c_2) = \left(\frac{p_2-w_2}{p_2} \right) ((1-\beta) + \alpha_1(\beta)) - \alpha_1\beta \left(\frac{p_1-w_1-\alpha_1(p_2-w_2)}{p_1-\alpha_1p_2} \right) \\ & w_2^* = \frac{p_2+c_2}{2} - \frac{\alpha_1(\beta)(p_1-w_1)p_2}{2((1-\beta)(p_1-\alpha_1p_2)+\alpha_1\beta p_1)} \end{aligned}$$

By plugging A_2^+ into w_1 , it is seen that $w_2^* = \frac{p_2+c_2}{2} - \frac{\alpha_1(\beta)(p_1-w_1)p_2}{2((1-\beta)(p_1-\alpha_1p_2)+\alpha_1\beta p_1)} \leq p_2 \frac{w_1}{p_1}$.

(iv). When $B_2 < w_1 \leq A_2$, the profit function is increasing before the point $w_2 = p_2 \frac{w_1}{p_1}$ but it is decreasing after that point. Therefore, the optimal point realizes at that point:

$$w_2^* = p_2 \frac{w_1}{p_1}$$

(v). When $w_1 \leq B_2$, the profit function is increasing before and after the point $w_2 = p_2 \frac{w_1}{p_1}$. So the optimal point realizes after that point, where the following derivative is equal to zero:

$$\begin{aligned} & (1-\beta) T\left(\frac{p_2-w_2-\alpha_2(p_1-w_1)}{p_2-\alpha_2p_1}\right) - \left(\frac{(1-\beta)(w_2-c_2)T}{p_2-\alpha_2p_1}\right) = 0 \\ & \frac{p_2-w_2-\alpha_2(p_1-w_1)}{p_2-\alpha_2p_1} = \frac{(w_2-c_2)}{p_2-\alpha_2p_1} \\ & w_2^* = \frac{p_2-\alpha_2(p_1-w_1)+c_2}{2} \end{aligned}$$

When we plug B_2^- into w_1 , it can be seen that $w_2 = \frac{p_2-\alpha_2(p_1-w_1)+c_2}{2} \geq p_2 \frac{w_1}{p_1}$

Supplier 2's pricing decision is characterized and there is a unique optimal for supplier 2 under the condition $\alpha_1\beta(p_2 - \alpha_2p_1) \leq \alpha_2(1-\beta)(p_1 - \alpha_1p_2)$.