

NETWORK DEPENDENT ALTRUISM AND ECONOMIC GROWTH

A Master's Thesis

by  
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Ankara  
July 2016





To my family

NETWORK DEPENDENT ALTRUISM AND ECONOMIC GROWTH

The Graduate School of Economics and Social Sciences  
of  
Bilkent University

by

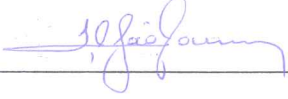
SEFANE ÇETİN

In Partial Fulfillment of the Requirements For the Degree of  
MASTER of ARTS

THE DEPARTMENT OF ECONOMICS  
İHSAN DOĞRAMACI BİLKENT UNIVERSITY  
ANKARA

July, 2016

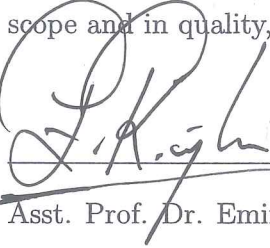
I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

  
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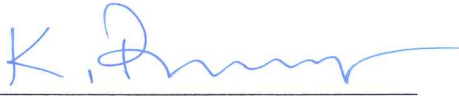
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## ABSTRACT

### NETWORK DEPENDENT ALTRUISM AND ECONOMIC GROWTH

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July 2016

This thesis studies reference dependent agents in a static network where the reference point is the average behaviour in one's neighbourhood. It shows that the economy grows at a constant rate on the balanced growth path while income inequality and the speed of convergence depend on the specific network structure. For particular networks, initial inequalities remain in the long-run. However, it is also possible that depending on the location of agents, a lagged agent may surpass the ones who had higher initial human capital than her and as a result, a society that has too disperse initial distribution may admit an equilibrium in which the wealth distribution converges towards equality in the long-run. In short, in an economy at which the production depends on human capital, income inequalities can be explained by taking the network structure of the economy into account.

*Keywords:* Altruism, Human Capital, Networks, Reference-Dependence.

## ÖZET

### AĞA BAĞLI FEDAKARLIK VE EKONOMİK BÜYÜME

Çetin, Sefane

Yüksek Lisans, İktisat Bölümü

Tez Danışmanı: Doç. Dr. H. Çağrı Sağlam

Temmuz 2016

Bu tez çalışması kişilerin çevrelerindeki statik ağı baz alan davranışlarını incelemektedir. Modelde, bir bireyin komşuluğundaki ortalama davranış bireyin referans noktasını oluşturmaktadır. Çalışmada ekonominin dengeli ve sabit bir hızla büyüdüğü, fakat gelir eşitsizliğinin ve yakınsama hızının ağın yapısına bağlı olduğu gösterilmiştir. Bazı ağlarda, ilk eşitsizlikler uzun vadede kalıcı olsa da düşük beşeri sermaye ile başlayan bir kişi komşularının yerleşimine bağlı olarak ekonomideki diğer bireyleri yakalayabilir. Bununla birlikte beşeri sermayeleri birbirinden çok farklı olan bireylerden oluşan bir toplum uzun vadede eşitliğe ulaşabilir. Sonuç olarak üretimin beşeri sermayeye bağlı olduğu bir toplumdaki gelir eşitsizlikleri toplumun ağ yapısı göz önünde bulundurularak açıklanabilir.

*Anahtar kelimeler:* Ağ, Beşeri Sermaye, Fedakarlık, Referans-Bağlılığı.

## ACKNOWLEDGEMENTS

I would first like to thank my thesis advisor Associate Professor H. Çağrı Sağlam for his continuous support and advice. His encouragement enhanced my concentration whenever I ran into a trouble. His patience and assistance helped me overcome many challenges and allowed me to complete this thesis.

I would also like to thank to Associate Professor Emin Karagözoğlu as the second reader of this thesis, and I am indebted to him for his very valuable comments on this thesis. I want to express my gratitude to Assistant Professor Ömer Kağan Parmaksız as an examining committee member.

Special thanks to all my precious friends, especially Berk İdem, Emre Karabulutluoğlu, Dođuhan Sündal, Gülayça Özcebe, Kerim Keskin, Merve Demirel, Nimet Kaya, Ömer Faruk Akbal, Sedat Ersoy and Toygar Kerman for their endless support throughout my studies and through the process of writing this thesis. I would not possibly be able to propound such a work without their presence.

Last but not the least; I would like to express the deepest appreciation to my family members, especially my mother for her moral support in my academic career.

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# CHAPTER I

## INTRODUCTION

What determines income inequality in a country? Why two similar agents end up having different locations in income distribution? Which policies should government adopt to establish equality among its citizens? Questions like these have been studied many times in the literature, yet we still don't have complete answers to them. Although, many theories seem to explain the reasons behind inequality, I believe that they all have a common deficiency in considering the structure of the society as a fully connected network. To address this deficiency, this paper presents an overlapping generations economy at which the accumulation of human capital depends on the location of agents and populated by heterogenous agents who care about consumption and human capital of their children relative to the average human capital of the neighbourhood they live in.

Human capital represents the ability to perform labor and promotes technological advances favouring growth. In many developed countries, production heavily depends on human capital. Hence, countries with a higher stock of human capital will experience higher economic growth. Although there are many theories on the accumulation of human capital, the very first channel to contribute one's human capital is the transfers of parental skills. Children inherit some skills and abilities from their parents through genetical transformation which implies that the child of a high skilled agent will tend to be high skilled as well. In addition to this, children improve their abilities by attending to formal education hence, the

quality of public schools carries an essential weight on children's human capital. Besides public schools, parents who want to improve their children's human capital more, engage in additional education investment by providing their children with further training. Along these channels, it is vital to account for one's local neighbourhood's effects. Considering the urban and rural areas in a country, we see that the development of human capital in these regions are particularly different. The high level of human capital in urban regions put an upward pressure on agents' education investment who locate in those regions which creates greater difference between the level of human capital accumulated in urban and rural areas. Furthermore, empirical evidence provided by Cavalcanti and Giannitsarou (2015) state that the average human capital of a node's neighbourhood amounts to 44% for the evolution of human capital, while 55% of it can be explained by parents human capital level. Hence, one's neighbourhood plays a crucial role on determining the level of human capital.

The effects of the structure of the neighbourhood is not only limited to the determination of education quality but also it plays an important role in shaping agents preferences. Friedman and Ostrov (2008) state that people care about their rank among others in terms of consumption. Considering the daily interactions of a household, one's neighbourhood can be influential on her decision making. Breitmöser and Tan (2013) tests a model of reference dependent altruism and shows that 55% of subjects use the opponent's payoffs as reference point suggesting that people have a tendency to base their actions according to their environment. Although people envy others' consumption, many of them behave altruistically so that they not only care about their own consumption but pay attention to their offsprings' consumption too. As a result of this, agents are assumed to be altruistic so that they care about investing in their children's education while at the same time they are reference dependent so that their decisions are affected by the society they live in.

The role of human capital on development has been addressed many times in the literature. For example, Lucas (1988), Barro (2001), Glomm and Ravikumar (1992), and Mankiw et al. (1992) highlight the importance of human capital accumulation

on productivity. Galor and Zeira (1993) explains the differences in human capital investment using wealth and income distributions across countries, while Banerjee and Newman (1993) attributes the dynamics of human capital investment to the market imperfections. The importance of education on the accumulation of human capital cannot be denied. Although public schools are the main source of education to build a country's human capital, many households do additional investments to the education of their child in order to provide them a prosperous future. Thereof, the channels to affect human capital accumulation can be summarized as follows; the transmission of parents' human capital, public schooling, additional education investments and local externalities. Mankiw et al. (1992) stresses the role of public schooling while Becker and Tomes (1986) and Durlauf et al. (1993) put emphasis on the share of parental transmissions on human capital accumulation. Additionally, Glomm (2004) and Lambrecht et al. (2005) express the effects of education spending on human capital accumulation. In addition to these channels, there is a need to address locational effects. Case and Katz (1991) emphasises significant neighbourhood peer effects based on a survey on low-income neighborhoods while Borjas (1995) provides empirical evidence about the effect of parent's ethnic neighbourhood on the skill level of children. Additionally, he suggests that there is a tendency of residing in a neighbourhood similar to one's characteristics. Furthermore, Durlauf (1996) explores how the distribution of income evolves depending on the neighbourhood choice resulting with economic stratification at which wealthier families choose to locate segregated from the poor ones. In light of these analyses, the network structure of a society is crucial in explaining the income inequality between agents and regions.<sup>1</sup>

Reference dependent preferences first modelled by Kahneman and Tversky (1979) under prospect theory. Afterwards, the trend becomes Catching-up-with-the-Joneses with Abel (1990). While Gali (1994) focuses on consumption externalities Ljungqvist and Uhlig (2000) considers a simple business cycle model. Moreover, Barro (1972) is the very first study to analyse reference dependence by introducing state dependent pricing. More recently, with Alvarez-Cuadrado et al. (2004) and Turnovsky and Monteiro (2007) this literature has been moved in the direction of the economy's transitional dynamics. People not only care about their consumption rela-

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<sup>1</sup>Datcher (1982), Corcoran et al. (1990) provide empirical evidence on the effect of neighbourhood characteristics determining income differences.

tive to their reference group, but also they care about their offspring. Some agents enjoy the state of giving which is called the joy of giving, Alvarez-Cuadrado and Long (2012) while some others derive utility from their offsprings' consumption or wealth which is called as family altruism, Lambrecht et al (2006) and Borissov (2013).<sup>2</sup>

Another body of literature that this study builds upon is networks. The structure and effects of networks have been studied many times in macroeconomics and financial markets. For example, Acemoglu et al. (2013) studies the economic forces shaping the relationship between the structure of the financial network and systemic risk via domino effects. Allen and Gale (2000) analyses how the banking system responds to contagion when banks are connected under different network structures and Serrano-Cinca (1996) applies a neural network to financial markets. Although, there are plenty of studies observing human capital dynamics and network effects separately, to my knowledge there are only a few works which combine these two. Fogli and Veldkamp (2012) explores how different social structures affect technology diffusion and thereby a country's rate of technological progress, Lindner and Strulik (2014) considers a mutual knowledge exchange network among countries and explains the diversity of growth performances while, Cavalcanti and Giannitsarou (2015) studies the dynamics of growth and inequality by embedding networks into an endogenous growth model. Along these lines of literature, this paper studies a simple model of human capital accumulation on a static network based on Cavalcanti and Giannitsarou (2015) with network dependent family altruism.

While this paper follows Cavalcanti and Giannitsarou (2015)'s framework, we add some new ingredients; locally determined price of education, parental transfers of human capital, heterogenous public schooling and reference dependence to their model. In Cavalcanti and Giannitsarou (2015), the poor can catch up with the rich and the economy exhibits equality if the network cohesion (a measure of how connected a network is) exceeds a threshold. In contrast, our model suggests that even at a fully connected network initial heterogeneities persist on the balanced

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<sup>2</sup>Andrew B. Abel (1988), Andreoni (1989) are some other examples of joy of giving kind of altruism.

growth path (BGP) implying that the poor may not catch up with the rich unless the public schooling quality is homogenous over all society. The remainder of this paper is organised as follows. Chapter 2 introduces the model. In chapter 3, the dynamics and stability of BGP is analysed and chapter 4 concludes.



## CHAPTER II

### THE MODEL

Production takes place in a centralised market. There is a continuum of identical firms with measure one. They produce the same consumption good. Output is produced by human capital,  $H_t$ , where  $H_t = \sum_{i=1}^N h_{it}$ . The production function is given by;

$$Y_t = H_t \tag{1}$$

The economy consists of  $N$  households. Each household lives for two periods and consists of one child and one adult. The household population is constant throughout time. The adult in node  $i$  at time  $t$  cares about current period consumption  $c_t^i$ , and the human capital of her child  $h_{t+1}^i$  relative to the average human capital of her neighbourhood  $\bar{h}_t^i$ . The more educated a household is the more productive she will be in the adulthood which will have a positive effect on her income. Agents are heterogenous in the sense that they have different initial human capital and it is strictly positive ( $h_0^i > 0$ ). The parent makes the decisions on behalf of the household by choosing consumption of household,  $c_t^i$  and investment in the education of her child,  $e_t^i$ . The investment in the education of the child is affected by the network neighbourhood the parent is located in. The network creates a positive externality as the future human capital is affected by the average human capital of the network neighbourhood. The parent dies when the first period ends and the child becomes the parent in period  $t+1$ . The lifetime utility for the household  $i$  born at period  $t$  is given by

$$\ln c_t^i + \eta \ln(h_{t+1}^i - \gamma \bar{h}_t^i) \tag{2}$$

where  $0 < \gamma \leq 1$  and  $0 < \eta < 1$ . The budget constraint of the household is given by;

$$c_t^i + p_t^i e_t^i = h_t^i \quad (3)$$

Moreover, the evolution of human capital is represented as

$$h_{t+1}^i = (1 - \delta)h_t^i + (\theta^i + e_t^i)\bar{h}_t^i \quad (4)$$

with  $\theta^i$  representing the level of public schooling at node  $i$  which is proportional to a node's initial human capital,  $h_0^i$  and constant throughout time. We assume that  $0 < \delta < 1$  exhibiting a diminishing transmission of parent's human capital. The level of human capital of the offspring depends on parental contribution, the level of public schooling and private investment which are affected by the local externality stemming from a node's neighbourhood (peer effects). If an agent chooses not to invest in private education, then the human capital of her offspring is formed by public schooling and the transmission of her own human capital. The problem of the household is then summarised by the following:

$$\max_{c_t^i, h_{t+1}^i} \ln c_t^i + \eta \ln(h_{t+1}^i - \gamma \bar{h}_t^i) \quad (5)$$

s.t.

$$c_t^i + p_t^i e_t^i = h_t^i \quad (6)$$

$$h_{t+1}^i = (1 - \delta)h_t^i + (\theta^i + e_t^i)\bar{h}_t^i \quad (7)$$

$$c_t^i, e_t^i \geq 0 \quad (8)$$

$$h_{i0} > 0, \text{ given} \quad (9)$$

The solution to this problem becomes:

$$e_t^i = \begin{cases} 0 & \frac{h_t^i}{p_t^i} \leq \frac{\theta^i - \gamma}{\eta + \delta - 1} \\ \frac{1}{(1+\eta)} \left[ (\eta + \delta - 1) \frac{h_t^i}{p_t^i} + \gamma - \theta^i \right] & \frac{h_t^i}{p_t^i} > \frac{\theta^i - \gamma}{\eta + \delta - 1} \end{cases}$$

$$c_t^i = \begin{cases} h_t^i & \frac{h_t^i}{p_t^i} \leq \frac{\theta^i - \gamma}{\eta + \delta - 1} \\ \frac{1}{(1+\eta)} \left[ (2 - \delta)h_t^i - (\gamma - \theta^i)p_t^i \right] & \frac{h_t^i}{p_t^i} > \frac{\theta^i - \gamma}{\eta + \delta - 1} \end{cases}$$

As it is seen by these equations, when the income of an agent relative to the price of private education is lower than a threshold, agents prefer not to invest in private education. The decision on private education depends on the difference between public schooling and reference dependence parameters. As the level of public schooling in one's network increases the agent becomes more reluctant on investing in private education and when the effect of reference dependence grows agents become more willing to spend for their children's' education, which creates positive externality.

## Network Structure

There are  $N$  heterogenous agents in the economy. Each node of the network consists of continuum of homogenous agents with measure one. The number of links and nodes are considered to remain the same over time. The network is undirected so that  $g_{ij} = g_{ji}$ , where  $g_{ij} \in \{0, 1\}$  represents the link between nodes  $i$  and  $j$ . If there is a link between  $i$  and  $j$  then  $g_{ij} = 1$ , otherwise 0. Moreover, it is assumed that  $g_{ii} = 1$ , which implies that when node  $i$  is isolated the average education investment of it's neighbourhood will be equal to it's own education investment. As a result, the network can be represented by an adjacency matrix  $G$ . The externality results from the average human capital of the network neighbourhood. Furthermore, the average human capital of node  $i$ 's neighbourhood is given by;

$$\bar{h}_t^i = \left( \frac{\sum_{j=1}^N g_{ij} h_t^j}{\sum_{j=1}^N g_{ij}} \right) \quad (10)$$

When node  $i$  has no neighbours,  $\bar{h}_t^i = h_t^i$ . If we have a complete network (i.e.  $g_{ij} = 1$  for all  $i, j$ ) the externality will turn out to be global.

**Definition 1.** *Given a network structure described by  $G$  and initial human capital levels of all households  $\{h_0^i\}_{i=1}^N$  with  $h_0^i > 0 \forall i = 1, 2, \dots, N$ , a competitive equilibrium at time  $t$  is a collection of households' allocations  $\{c_t^i, e_t^i\}_{i=1}^N$ , the price of*

education for each agent  $\{p_t^i\}_{i=1}^N$ , and human capital sequence  $\{h_t^i\}_{i=1}^N$  such that

a) the household chooses  $\{c_t^i, e_t^i\}$  to solve (5) subject to (6), (7), (8), and (9).

b) firms maximise profits and

c) the goods market clears, i.e.

$$\sum_{i=1}^N c_t^i + \sum_{i=1}^N p_t^i e_t^i = \sum_{i=1}^N h_t^i \quad (11)$$

## CHAPTER III

### DYNAMICS AND STABILITY

In a neighbourhood which is populated by high skilled agents, the price of education may be higher due to the fact that education is provided by teachers and it is higher to finance high skilled teachers. Moreover, heterogenous agents invest different amounts to educate their children affecting the average skill of the neighbourhood at each period. As a result, I assume that the price of private education is determined by the average income level of each neighbourhood, (i.e.  $p_t^i = \bar{h}_t^i$ ) which implies that nodes with wealthy neighbours face a high price of education.

Now,

$$e_t^i = \begin{cases} 0 & \frac{h_t^i}{\bar{h}_t^i} \leq \frac{\theta^i - \gamma}{\eta + \delta - 1} \\ \frac{1}{(1+\eta)} \left[ (\eta + \delta - 1) \frac{h_t^i}{\bar{h}_t^i} + \gamma - \theta^i \right] & \frac{h_t^i}{\bar{h}_t^i} > \frac{\theta^i - \gamma}{\eta + \delta - 1} \end{cases}$$

$$c_t^i = \begin{cases} h_t^i & \frac{h_t^i}{\bar{h}_t^i} \leq \frac{\theta^i - \gamma}{\eta + \delta - 1} \\ \frac{1}{(1+\eta)} \left[ (2 - \delta) h_t^i - (\gamma - \theta^i) \bar{h}_t^i \right] & \frac{h_t^i}{\bar{h}_t^i} > \frac{\theta^i - \gamma}{\eta + \delta - 1} \end{cases}$$

Moreover, the human capital of agent  $i$ 's child will be given by

$$h_{t+1}^i = \max \left\{ (1 - \delta) h_t^i + \theta^i \bar{h}_t^i, \frac{1}{(1 + \eta)} \left[ \eta (2 - \delta) h_t^i + (\eta \theta^i + \gamma) \bar{h}_t^i \right] \right\} \quad (12)$$

Thus, when a parent choose not to invest in the education of her child, the magnitude of the local (neighbourhood) externality on public schooling and skills inherited by the parent determine the evolution pattern of child's human capital while it is determined by the local externality stemming from reference dependent behaviour along with public schooling and inherited skills if parent invests in education. Since the price of education is relatively high for an agent who is surrounded by wealthy agents, she may choose not to invest in education rather benefit only from public schooling. Furthermore, the comparative statics of the evolution of human capital with respect to reference dependence and public schooling parameters are given as follows:

$$\frac{\partial h_{t+1}^i}{\partial \gamma} = \begin{cases} 0 & \frac{h_t^i}{\bar{h}_t^i} \leq \frac{\theta^i - \gamma}{\eta + \delta - 1} \\ \frac{1}{(1+\eta)} \bar{h}_t^i & \frac{h_t^i}{\bar{h}_t^i} > \frac{\theta^i - \gamma}{\eta + \delta - 1} \end{cases}$$

$$\frac{\partial h_{t+1}^i}{\partial \theta_i} = \begin{cases} \bar{h}_t^i & \frac{h_t^i}{\bar{h}_t^i} \leq \frac{\theta^i - \gamma}{\eta + \delta - 1} \\ \frac{\eta}{(1+\eta)} \bar{h}_t^i & \frac{h_t^i}{\bar{h}_t^i} > \frac{\theta^i - \gamma}{\eta + \delta - 1} \end{cases}$$

If  $\frac{h_t^i}{\bar{h}_t^i} \leq \frac{\theta^i - \gamma}{\eta + \delta - 1}$ , an agent's human capital is affected only by public schooling. When  $\frac{h_t^i}{\bar{h}_t^i} > \frac{\theta^i - \gamma}{\eta + \delta - 1}$ , reference dependence has higher effect than public schooling, which shows that agents doing relatively good compared to their neighbourhood care more about reference dependence. This result is in line with real life. Households show reference dependent behaviour or envy when they are above an income level while, the ones who earn below the subsistence level only spend their income on consumption to sustain their lives. Moreover, if the quality of public schools are high enough agents are more willing to spend their income on consumption instead of education investment. For tractability, let's define the growth factor of human capital of household  $i$  to be  $\phi_t^i := \frac{h_{t+1}^i}{h_t^i}$ . Now,

$$\phi_t^i = \max \left\{ (1 - \delta)h_t^i + \theta^i \frac{\bar{h}_t^i}{h_t^i}, \frac{1}{(1 + \eta)} \left[ \eta(2 - \delta) + (\eta\theta^i + \gamma) \frac{\bar{h}_t^i}{h_t^i} \right] \right\} \quad (13)$$

**Definition 2.** *On the BGP, the growth factor of agent  $i$  at period  $t + 1$  is equal to the growth factor of agent  $i$  at period  $t$ ,  $\forall i = 1, 2, \dots, N$ .*

**Proposition 1.** *Along the BGP, the growth factor of each agent is equal to the growth factor of her neighbourhood.*

*Proof.* Along the BGP,  $\phi_{t+1}^i = \phi_t^i = \phi^i \forall i = 1, 2, \dots, N$ .

$$\begin{aligned}
& \max \left\{ (1 - \delta)h_{t+1}^i + \theta^i \frac{\bar{h}_{t+1}^i}{h_{t+1}^i}, \frac{1}{(1 + \eta)} \left[ \eta(2 - \delta) + (\eta\theta^i + \gamma) \frac{\bar{h}_{t+1}^i}{h_{t+1}^i} \right] \right\} \\
& = \max \left\{ (1 - \delta)h_t^i + \theta^i \frac{\bar{h}_t^i}{h_t^i}, \frac{1}{(1 + \eta)} \left[ \eta(2 - \delta) + (\eta\theta^i + \gamma) \frac{\bar{h}_t^i}{h_t^i} \right] \right\} \\
& \quad \Rightarrow \frac{\bar{h}_{t+1}^i}{h_{t+1}^i} = \frac{\bar{h}_t^i}{h_t^i} \\
& \quad \Rightarrow \frac{\bar{h}_{t+1}^i}{\bar{h}_t^i} = \frac{h_{t+1}^i}{h_t^i} = 1 + g^i
\end{aligned} \tag{14}$$

□

It is quite trivial that the growth rate of one's human capital is constant only when the growth rate of human capital in her neighbourhood is constant. Moreover, on the BGP, each node grows in accordance with its' neighbourhood. This implies that when the economy is structured as a connected network, the growth rate of each node is the same while, having a clustered society implies that each cluster grows at a rate independent from another cluster. Assume that we have an economy consisting of two nodes; one rich and one poor. When there is no link between these nodes, each node will grow at a constant rate independent of each other (i.e. the rich node growing at a higher rate than the poor one). However, when these nodes are connected they will have the same growth rate on the BGP. Alternatively, we can consider a society consisting of various ethnic groups integrated in itself but having no connection with each other. This result suggests that each ethnic group will grow at a rate independent from one other but the members of each group will share the same growth rate of human capital explaining the different patterns of economic activity observed in blacks and whites, natives and immigrants, etc. In light of this analysis, it follows that the income gap between different nodes decreases as the structure of economy gets more unified.

**Proposition 2.** *Growth rate of human capital along the BGP is given by;*

$$g^i = \max \left\{ \theta^i \frac{\bar{h}_0^i}{h_0^i} - \delta, \frac{1}{(1+\eta)} \left[ (\eta\theta^i + \gamma) \frac{\bar{h}_0^i}{h_0^i} - (1 - \eta + \eta\delta) \right] \right\} \quad (15)$$

*Proof.* Suppose that  $\frac{h_t^i}{\bar{h}_t^i} \leq \frac{\theta^i - \gamma}{\eta + \delta - 1}$  then,  $h_{t+1}^i = (1 - \delta)h_t^i + \theta^i \bar{h}_t^i$

$$\begin{aligned} \Rightarrow h_0^i (1 + g^i)^{t+1} &= (1 - \delta)h_0^i (1 + g^i)^t + \theta^i \bar{h}_0^i (1 + g^i)^t \\ &\Rightarrow g^i = \theta^i \frac{\bar{h}_0^i}{h_0^i} - \delta \end{aligned}$$

For,  $\frac{h_t^i}{\bar{h}_t^i} > \frac{\theta^i - \gamma}{\eta + \delta - 1}$ ,  $h_{t+1}^i = \frac{1}{(1+\eta)} \left[ \eta(2 - \delta)h_t^i + (\eta\theta^i + \gamma)\bar{h}_t^i \right]$

$$\begin{aligned} \Rightarrow h_0^i (1 + g^i)^{t+1} &= \frac{1}{(1+\eta)} \left[ \eta(2 - \delta)h_0^i (1 + g^i)^t + (\eta\theta^i + \gamma)\bar{h}_0^i (1 + g^i)^t \right] \\ &\Rightarrow g^i = \frac{1}{(1+\eta)} \left[ (\eta\theta^i + \gamma) \frac{\bar{h}_0^i}{h_0^i} - (1 - \eta + \eta\delta) \right] \end{aligned}$$

□

The growth rate of an agent's human capital depends on initial heterogeneity and network structure along with public schooling and reference dependence. An agent doing relatively bad in her neighbourhood will grow at a higher rate whether she invests in education or not. However, when  $\gamma$  is sufficiently high the one who invests in education will grow faster. Hence, reference dependence gives an incentive for lagged agents to catch-up with their neighbours. Now let's define  $\hat{h}_t^i := \frac{h_t^i}{(1+g^i)^t}$ . Then,

$$\hat{h}_{t+1}^i = \begin{cases} h_0^i \frac{(1-\delta)\hat{h}_t^i + \theta^i \bar{h}_0^i}{(\theta^i \bar{h}_0^i + h_0^i)} & h_0^i \leq \frac{\theta^i - \gamma}{\eta + \delta - 1} \bar{h}_0^i \\ h_0^i \frac{\eta(2-\delta)\hat{h}_t^i + (\eta\theta^i + \gamma)\bar{h}_0^i}{\eta(2-\delta)h_0^i + (\eta\theta^i + \gamma)\bar{h}_0^i} & h_0^i > \frac{\theta^i - \gamma}{\eta + \delta - 1} \bar{h}_0^i \end{cases}$$

**Proposition 3.** *The BGP is characterised by the following first order difference*

equation:

$$\hat{h}_{t+1}^i = \begin{cases} h_0^i \frac{(1-\delta)\hat{h}_t^i + \theta^i \bar{h}_0^i}{(\theta^i h_0^i + h_0^i)} & h_0^i \leq \frac{\theta^i - \gamma}{\eta + \delta - 1} \bar{h}_0^i \\ h_0^i \frac{\eta(2-\delta)\hat{h}_t^i + (\eta\theta^i + \gamma)\bar{h}_0^i}{\eta(2-\delta)h_0^i + (\eta\theta^i + \gamma)\bar{h}_0^i} & h_0^i > \frac{\theta^i - \gamma}{\eta + \delta - 1} \bar{h}_0^i \end{cases}$$

The proof is given in the appendix.

**Proposition 4.** For each level of initial human capital  $h_0^i \forall i = 1, 2, \dots, N$ ,  $\hat{h}_{t+1}^i$  is at it's steady state which is given by:

$$\hat{h}^i = \begin{cases} \frac{\theta_i \bar{h}_0^i}{\theta_i h_0^i + \delta h_0^i} h_0^i & h_0^i \leq \frac{\theta^i - \gamma}{\eta + \delta - 1} \bar{h}_0^i \\ h_0^i & h_0^i > \frac{\theta^i - \gamma}{\eta + \delta - 1} \bar{h}_0^i \end{cases}$$

*Proof.* Since  $\frac{\partial \hat{h}_{t+1}^i}{\partial \hat{h}_t^i} > 0$  and  $\frac{\partial^2 \hat{h}_{t+1}^i}{\partial \hat{h}_t^i{}^2} = 0$ ,  $\hat{h}_{t+1}^i$  is a concave function of  $\hat{h}_t^i$ . Moreover, for any given  $h_0^i$  and  $\forall i = 1, 2, \dots, N$   $\lim_{t \rightarrow +\infty} \hat{h}_{t+1}^i \rightarrow M$  since  $\frac{(1-\delta)h_0^i}{(\theta^i h_0^i + h_0^i)} < 1$  and  $\frac{\eta(2-\delta)h_0^i}{\eta(2-\delta)h_0^i + (\eta\theta^i + \gamma)\bar{h}_0^i} < 1$ . Hence, the steady state of  $\hat{h}_{t+1}^i$  exists. At the steady state,  $\hat{h}_{t+1}^i = \hat{h}_t^i = \hat{h}^i$  which implies that

$$\hat{h}^i = \begin{cases} \frac{\theta_i \bar{h}_0^i}{\theta_i h_0^i + \delta h_0^i} h_0^i & h_0^i \leq \frac{\theta^i - \gamma}{\eta + \delta - 1} \bar{h}_0^i \\ h_0^i & h_0^i > \frac{\theta^i - \gamma}{\eta + \delta - 1} \bar{h}_0^i \end{cases}$$

□

**Proposition 5.** The BGP is stable for any network.

*Proof.* At the BGP, we have a system of N equations of the following form:

$$\hat{h}_{t+1}^i = \begin{cases} h_0^i \frac{(1-\delta)\hat{h}_t^i + \theta^i \bar{h}_0^i}{(\theta^i h_0^i + h_0^i)} & h_0^i \leq \frac{\theta^i - \gamma}{\eta + \delta - 1} \bar{h}_0^i \\ h_0^i \frac{\eta(2-\delta)\hat{h}_t^i + (\eta\theta^i + \gamma)\bar{h}_0^i}{\eta(2-\delta)h_0^i + (\eta\theta^i + \gamma)\bar{h}_0^i} & h_0^i > \frac{\theta^i - \gamma}{\eta + \delta - 1} \bar{h}_0^i \end{cases}$$

The eigenvalues of the Jacobian of this system are given by  $\frac{\partial \hat{h}_{t+1}^i}{\partial \hat{h}_t^i}$ ,  $\forall i = 1, 2, \dots, N$ .

$$\frac{\partial \hat{h}_{t+1}^i}{\partial \hat{h}_t^i} = \begin{cases} h_0^i \frac{(1-\delta)}{(\theta^i h_0^i + h_0^i)} & h_0^i \leq \frac{\theta^i - \gamma}{\eta + \delta - 1} \bar{h}_0^i \\ h_0^i \frac{\eta(2-\delta)}{\eta(2-\delta)h_0^i + (\eta\theta^i + \gamma)h_0^i} & h_0^i > \frac{\theta^i - \gamma}{\eta + \delta - 1} \bar{h}_0^i \end{cases}$$

Moreover, the system is stable since  $\frac{\partial \hat{h}_{t+1}^i}{\partial \hat{h}_t^i} < 1$  for any given  $h_0^i$  and  $\forall i = 1, 2, \dots, N$ . Hence, the BGP is stable for any type of network.  $\square$

No matter what the structure of the society is, the economy converges to the BGP and stays there forever. This result is in contrast with Cavalcanti and Giannitsarou (2015), they find out that the economy is at the steady state at which the society exhibits equality (i.e. each agent in the economy has the same constant level of income). However, in this framework the economy is on the BGP where any two nodes who are not connected grow at different rates which leads to persistent inequality. Additionally, the structure of the network affects the speed of convergence to the BGP and distribution of income at the equilibrium. A uniformly connected network converges to the BGP faster than a disperse network, implying that a stratified society requires a longer time to reach to the BGP. Moreover, income distributions of different network structures on the BGP will be different in the sense that connected neighbourhoods will benefit from local externality and will have similar growth dynamics while a disperse society may diverge in final income distribution. We can infer from this result that the sharp boundaries between urban and rural, rich and poor, black and white etc. create huge income gaps. In order to reach an egalitarian society it is crucial to eliminate the boundaries and establish a unified society.

**Lemma 6.** *Any two connected nodes have the same growth rate on the BGP.*

*Proof.* It follows from Proposition 1.  $\square$

As a result of reference dependence household's decision on education investment

is affected by their neighbours. Having a higher growing neighbour will trigger the agent's education spending and will result with an economy where connected agents reach the same human capital growth rate. Our model describes the following situation. Assume that the economy consists of a connected network where rich and poor nodes are connected in themselves and there is a group of nodes who act as a bridge between the rich and the poor, then on the BGP the wealth of nodes will be ranked as rich region the highest, the bridge having a middle income, and the poor region the lowest. Even if the nodes which lie on the bridge have lower initial human capital than the nodes in the poor region, the local externality will let them boost their human capital and move to a higher level than their initial ranks. Having a heterogenous society will cause initial skill inequalities to persist. However, subsidising public schools in poor regions may lead to a denser wealth distribution in the long-run.

**Proposition 7.** *Initial inequalities of human capital persist at complete networks.*

*Proof.* At a complete network  $\bar{h}_t^i = \bar{h}_t \forall i = 1, 2, \dots, N$  and hence,

$$h_{t+1}^i = \begin{cases} (1 - \delta)h_t^i + \theta^i \bar{h}_t & h_0^i \leq \frac{\theta^i - \gamma}{\eta + \delta - 1} \bar{h}_0 \\ \frac{1}{(1 + \eta)} \left[ \eta(2 - \delta)h_t^i + (\eta\theta^i + \gamma)\bar{h}_t \right] & h_0^i > \frac{\theta^i - \gamma}{\eta + \delta - 1} \bar{h}_0 \end{cases}$$

which implies that the difference among two individual's human capital is created by initial differences. □

Every node has the same reference point at a complete network. However, public schooling is associated with a node's initial human capital and stays constant through time resulting with the same heterogeneity among agents as the initial period. On the other hand, if we consider a discrete network where no agent is connected, we see that the one with highest initial human capital grows with the highest rate and the human capital gap among the rich and the poor widens. This provides an explanation to the question of why the blacks have lower income compared to whites. In general, whites have a higher stock of human capital and as a consequence of racism, blacks and whites do not interact much. As a result,

they encounter different patterns of growth and accounting for their initial heterogeneities mostly whites end up having higher income than blacks. Moreover, we face a similar situation among natives and immigrants in many countries. Immigrants who try to sustain their lives in another country usually face a low income growth compared to natives of that country. Such dynamics arise from the fact that immigrants having relatively low human capital have hard times to integrate in the native society. Since it is hard to integrate into the society if they do not share the same language and same culture, they live isolated leading to income gap between natives and immigrants. If the human capital of the immigrant is above the average of the natives, then immigrants tend to be adopted easily into the native society and share the same development patterns.

**Remark 8.** *On any connected network, every agent reaches the same level of human capital on the BGP as  $\theta_i \rightarrow \theta$ , establishing economy wide equality.*

If the society is homogenous in terms of public schooling so that every node has the same level of public school quality, the only force which creates heterogeneity will be initial heterogeneities and reference dependence associated with the structure of the network. When any two connected nodes receive the same level of public schooling, they exhibit equality in the long-run. This implies that no matter how segregated a society is they establish equality as long as they receive equal opportunities and they are connected.

## CHAPTER IV

### CONCLUSION

In this thesis, I have presented an OLG model with human capital where household's preferences are reference dependent and the reference point is determined by one's neighbourhood by extending the model presented by Cavalcanti and Giannitsarou (2015). The results are not in accordance with theirs. They show that the economy is at the steady state establishing overall equality in the society while I find out that the economy does not establish a steady state. Instead, the economy moves to a BGP where the connected agents grow at the same constant rate. The main result of this thesis is that on the BGP, the wealth distribution in the society depends on the specific network structure. As the society gets more connected they take advantage of local externality which puts pressure on agents to invest more in education. Although, agents having the same initial human capital level but different levels of public schooling may diverge in the long-run, a fully connected society establishes equality when public schooling is provided equally to different nodes. In this manner, these findings reveal that the structure of the network (society) seems to be an important determinant of income gap among different clusters of the society.

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## APPENDIX

*Proof of Proposition 3.3*

For,  $\frac{h_t^i}{\bar{h}_t^i} \leq \frac{\theta^i - \gamma}{\eta + \delta - 1}$ ,

$$\hat{h}_{t+1}^i = \frac{(1 - \delta)h_t^i + \theta^i \bar{h}_t^i}{(1 + g^i)^{t+1}} \quad (16)$$

$$\hat{h}_{t+1}^i = \frac{1}{(1 + g^i)} \left[ (1 - \delta)\hat{h}_t^i + \theta^i \bar{h}_0^i \right] \quad (17)$$

$$\hat{h}_{t+1}^i = h_0^i \frac{(1 - \delta)\hat{h}_t^i + \theta^i \bar{h}_0^i}{(\theta^i \bar{h}_0^i + h_0^i)} \quad (18)$$

For,  $\frac{h_t^i}{\bar{h}_t^i} > \frac{\theta^i - \gamma}{\eta + \delta - 1}$ ,

$$\hat{h}_{t+1}^i = \frac{1}{(1 + \eta)} \left[ \eta(2 - \delta) \frac{h_t^i}{(1 + g^i)^{t+1}} + (\eta\theta^i + \gamma) \frac{\bar{h}_t^i}{(1 + g^i)^{t+1}} \right] \quad (19)$$

$$\hat{h}_{t+1}^i = \frac{1}{1 + g^i} \frac{1}{(1 + \eta)} \left[ \eta(2 - \delta)\hat{h}_t^i + (\eta\theta^i + \gamma)\bar{h}_0^i \right] \quad (20)$$

$$\hat{h}_{t+1}^i = h_0^i \frac{\eta(2 - \delta)\hat{h}_t^i + (\eta\theta^i + \gamma)\bar{h}_0^i}{\eta(2 - \delta)h_0^i + (\eta\theta^i + \gamma)\bar{h}_0^i} \quad (21)$$