

HUMAN TIBIAL BONE STRENGTH PREDICTION BY VIBRATION
ANALYSIS FOR DIAGNOSING PROGRESSING OSTEOPOROSIS

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ANALYSIS FOR DIAGNOSING PROGRESSING OSTEOPOROSIS**

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ABSTRACT

HUMAN TIBIAL BONE STRENGTH PREDICTION BY VIBRATION ANALYSIS FOR DIAGNOSING PROGRESSING OSTEOPOROSIS

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Osteoporosis is a metabolic bone disease that needs to be properly diagnosed. The current diagnosing procedure of osteoporosis is based on the mineral density of bones measured by common methods such as dual energy X-ray absorptiometry (DXA). However, due to the deficiencies and limitations of these common methods, investigations on the utilization of other non-invasive diagnosing methods have been executed. For instance, using vibration measurements seems to be a promising technique in diagnosing metabolic bone diseases such as osteoporosis and also in monitoring fracture healing. Throughout this study, bone structural modal parameters obtained from vibrations experiments with decreasing mineral density are examined and therefore, it is aimed to find a new approach to detect osteoporosis or progressing osteoporosis by investigating a relation between structural dynamic properties and mineral density of bone. The main advantage of this study is that loss factor, which is an inherit property

of bone, is investigated since in the previous studies mainly the changes in natural frequency of bones with the state of osteoporosis is examined.

In this thesis, both in vitro and in vivo experiments are carried out on human tibia specimens. The measured frequency response functions (FRFs) are analyzed using modal identification techniques to extract the modal parameters of the human tibia. The results obtained from in vitro experiments show that loss factor may be a powerful tool in diagnosing osteoporosis, however due to the difficulties encountered in the case of in vivo experiments makes the use of this parameter as a diagnosing tool difficult. It is also seen from in vivo experiments that there is a weak correlation between the natural frequencies of tibia and BMD measurements of patients. Therefore, in order to investigate the parameters affecting the natural frequencies of tibia, finite element (FE) model of human tibial bone is constructed. Using this FE model tibia, the effect of boundary conditions of experiments and geometry of the bone on natural frequencies of bone is examined. These analyses show that the effect of both boundary conditions and geometry of tibia is very high. Therefore, it is concluded that if the necessary conditions are satisfied, the using natural frequency information of tibia seems to be a possible and practical method that can be used to detect progressing osteoporosis. Also, using the FE model of tibia, the changes of natural frequencies of tibia with the variation in elastic modulus are investigated.

Keywords: Biomechanics, Diagnosing Osteoporosis, Tibia, Experimental Modal Analysis of Tibia.

ÖZ

İNSAN KEMİK MUKAVEMETİNİN İLERLEYEN OSTEOPOROZ TEŞHİSİ AMACIYLA TİTREŞİM ANALİZİ KULLANILARAK TAHMİN EDİLMESİ

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Osteoporoz zamanında teşhis edilmesi gereken metabolik bir kemik hastalığıdır. Günümüzde osteoporozun teşhisinde kemik mineral yoğunluğunu saptayacak yöntemler kullanılmaktadır. Fakat bu yöntemlerin yetersizliğinden dolayı, invazif olmayan yeni yöntemler geliştirilmesi için araştırmalar yürütülmektedir. Örneğin, titreşim testlerinin osteoporozun teşhisi ya da kırık iyileşme sürecinin izlenmesi amacıyla kullanılmasının umut vaat eden bir yöntem olduğu görünmektedir. Bu çalışma süresince titreşim testlerinden elde edilen verilerden yararlanarak çıkarılan kemiğin yapısal dinamik özellikleri incelenmiş ve bu nedenle osteoporozun ya da ilerlemekte olan osteoporozun teşhisinde kullanılması amacıyla kemiğin yapısal dinamik özellikleri ve kemiğin mineral yoğunluğu arasında bir karşılaştırma yaparak yeni bir yaklaşım geliştirilmiştir. Bu çalışmanın en büyük üstünlüğü, kemiğin karakteristik bir özelliği olan sönüm faktörünün incelenmesidir. Çünkü daha önceki çalışmalarda genel olarak sadece

osteoporozun durumuna baęlı olarak doęal frekansın deęiřimi incelenmiřtir.

Bu alıřma kapsamında, insan tibiası zerinde hem in vivo hem de in vitro kořullarda titreřim deneyleri yapılmıřtır. Bu deneylerden elde edilen frekans tepki fonksiyonları (FTP) incelenmiř ve modal analiz yntemleri ile kemięin yapısal dinamik zellikleri ıkarılmıřtır. In vitro kořullarda yapılan deney sonuları, snm faktrnn osteoporozun teřhisinde kullanılabilecek gl bir yntem olabileceęini gstermiřtir; fakat in vivo kořullarda yapılan deneyler bu belirtelerin teřhis amacıyla kullanılmasını zorlařtırdıęı gzlemlenmiřtir. In vivo alıřmalar sonucunda ayrıca kemięin doęal frekansı ile hastaların llen kemik mineral yoęunlukları arasında zayıf bir iliřki olduęu saptanmıřtır. Bu sebeple, kemięin doęal frekanslarını etkileyen faktrleri incelemek zere, insan tibiasının bir sonlu eleman modeli oluřturulmuřtur. Bu model yardımıyla titreřim deneylerindeki sınır kořullarının ve kemięin geometrisinin doęal frekans deęerleri zerindeki etkisi arařtırılmıřtır. Bu analizler sonucunda, hem sınır kořullarının hem de kemięin geometrisinin doęal frekans deęerlerini ok fazla etkiledięi grlmřtir. Bundan dolayı, gerekli kořullar saęlandıęında kemięin doęal frekans bilgisinin osteoporoz teřhisinde kullanılabilecek mmkn ve pratik bir metot olabileceęi sonucuna varılmıřtır. Ayrıca, tibia'nın sonlu eleman modelini kullanarak kemięin elastisite modl deęiřimiyle doęal frekansının nasıl deęiřtięi incelenmiřtir.

Anahtar kelimeler: Biyomekanik, Osteoporoz Teřhisi, Tibia, Tibianın Modal Titreřim Analizi.

To my family...

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LIST OF SYMBOLS

SYMBOLS

${}_sA_{jk}$	: s^{th} modal constant
${}_sB_{jk}$	: s^{th} complex constant term
D	: Diameter of modal circle
Ω	: Fixing frequency
$\Delta(\omega)$	: Inverse parameter
η_s	: s^{th} loss factor
α_{jk}	: Receptance
ω	: Frequency
ω_s	: s^{th} natural frequency
e_p	: Error function
ρ_{bone}	: Density
CT	: Data obtained from computer tomography scan
r	: Pearson product moment correlation coefficient
τ	: Kendall tau rank correlation coefficient
ρ	: Spearman's rank correlation coefficient
σ_x, σ_y	: Standard deviation of sample set of data
n_c, n_d	: Number of concordant and discordant pairs

CHAPTER 1

INTRODUCTION

The aim of this thesis is to obtain in vivo frequency response functions (FRFs) of human tibia and attempt to find a new approach to detect osteoporosis or progressing osteoporosis by making an analogy between structural dynamic properties and bone mineral density (BMD). Thus, an introduction to human bone structure for terminology purposes and osteoporosis will be given in the following sections.

1.1 On Human Bones

The skeletal system of human consists of various types of bones and connective tissues such as cartilage, ligaments ...etc. It supports the weight and forms the shape of the body. Besides these tasks, the skeletal system has other vital functions;

- i. Protection
- ii. Movement
- iii. Blood cell formation
- iv. Mineral and growth factor storage

Every bone in the skeletal system has two main components. The first one is the hard and dense outer shell which is called as the *compact or cortical bone*. The other one is the internal honeycomb of small needle-like part

called as the *spongy (cancellous) bone*. Figure 1.1 simply shows the basic structure of a long bone. The structure of long bones is nearly the same with each other. The diaphysis, the epiphyses, the metaphysis and the membranes are the basic elements. Diaphysis is the main (mid) section of a long bone and it is constructed mainly by a thick layer of compact bone. Epiphyses are the two ends of a long bone and membranes are the external surface covering the entire bone. The transition regions between the diaphysis and the epiphyses are called as metaphysis. The metaphysis is the location of higher metabolic activity in the bone[1].

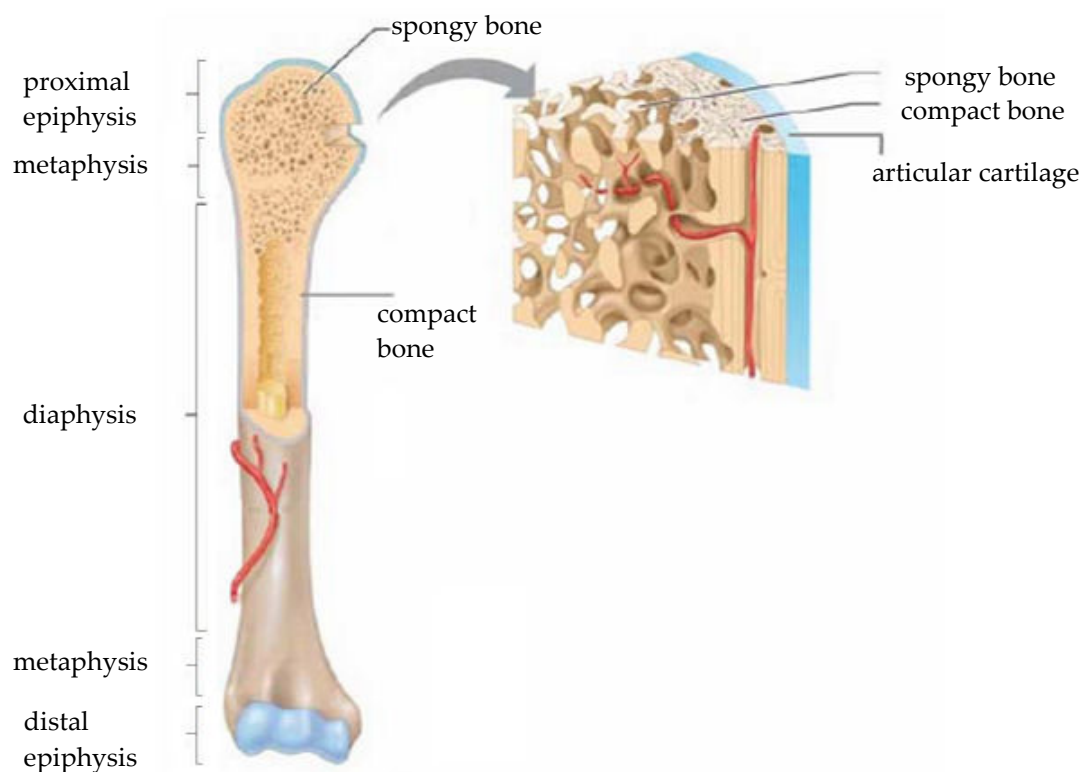


Figure 1.1. The structure of a long bone [1]

In this thesis, the study is focused on human tibial bone which is the larger of two bones below the knee connecting the ankle to the knee. In this study, selecting tibia as the bone of interest is due to several factors. However, being one of the most superficial bones in human body is the most important factor. Figure 1.2 shows the anterior views of the right tibia and fibula.

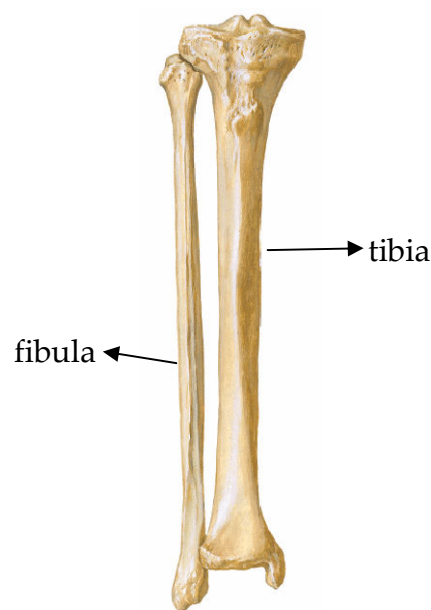


Figure 1.2. Anterior views of the right tibia and fibula [2]

1.2 On Osteoporosis

Osteoporosis is a silent disease characterized by low bone mass and a micro-architectural deterioration of bone tissue as can be seen in Figure 1.3, leading to enhanced bone fragility and consequent increase in fracture risk (World Health Organization, 1994) with high socio-economical impact on

the society [3]. According to the previous studies carried out, osteoporosis and associated fractures are a major public health concern and it is needed to diagnose this bone disease appropriately in order to prevent the outcomes of this illness such as decreased quality of life and mortality [3-5].

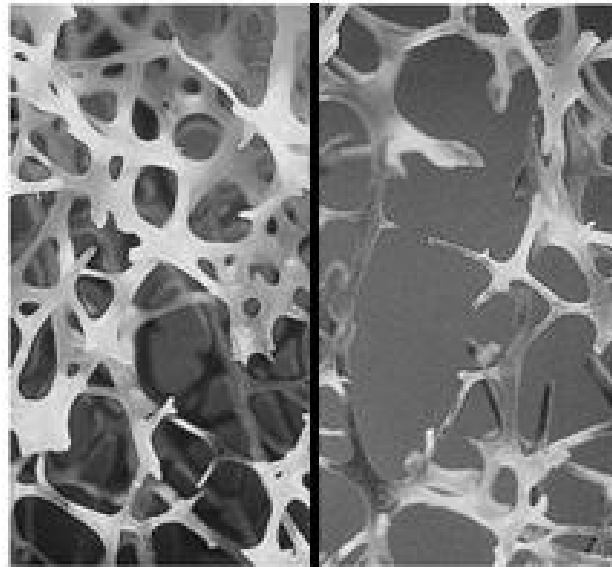


Figure 1.3 - Effect of osteoporosis on micro-architecture of bone

Left: Normal bone, Right: Osteoporotic bone

(International Osteoporosis Foundation - IFO)

An investigation performed by the *International Osteoporosis Foundation* (IFO) shows that every 30 seconds, a person in the European Union has an osteoporosis related fracture. Furthermore, according to a research performed in 2002 by National Osteoporosis Foundation, it was diagnosed that approximately 22 million women have low bone mass and 8 million postmenopausal women faces with osteoporosis in the United States.

Beside the health aspect, it is also inevitable that this bone disease has a brutal financial burden on economies of countries through work day losses as well as direct health care costs for medical services. For instance, in a study of Max et al. [6] about the burden of osteoporosis in California, it is stated that annual United States Government expenditure that is attributed to osteoporosis includes \$2.4 billion for direct health costs and over \$4 million for productivity losses during 1998. These investigations show the importance of this bone illness and how it affects the society in both health and economic aspects.

There are quite a few numbers of specialized tests for detecting osteoporosis such as dual energy X-ray absorptiometry (DXA), peripheral dual energy X-ray absorptiometry (pDXA), single energy X-ray absorptiometry (SXA), dual photon absorptiometry (DPA), single photon absorptiometry (SPA), quantitative computed tomography (QCT), and Quantitative Ultrasound (QUS), which are used to measure bone mineral density (BMD) of bone [6]. Among these measurements, DXA and QCT are the most common types that are used for diagnosis purposes. These numerical measurements can also be interpreted as Z or T scores which are the number of standard deviations (SD) of the individual's BMD scores that is away from the age-matched mean and young adult normal mean, respectively. According to World Health Organization, T scores above -1.0 SD are called normal, T scores between -1.0 SD and -2.5 SD are called osteopenia, and T scores lower than -2.5 SD are called osteoporosis, which are the degrees of the same bone disease.

Bone mineral density measured by DXA and/or QCT is a predictor of mechanical bone strength that has its limitations [4, 7-9]. For instance, as far as the strength of the bone is concerned, the organic part (collagen) and the micro-architecture of bone have as much importance as its mineral content that is not measured by these methods. Also, fragility fractures in normal and osteopenic but sometimes not in osteoporotic patients are seen in medical practice. As a result, the need for a non-invasive better diagnosing of bone strength increases gradually [5].

1.3 Literature Review

In recent years, due to the deficiencies of the common methods described in the previous section, there is a growing aim for implementing non-invasive diagnosing tools in the use of medical applications such as the diagnosing osteoporosis or detecting fracture healing. For instance, using the geometry of the bone in such a way that measuring the hip axis or width of femoral length [10] is a sample non-invasive method for detecting osteoporosis or monitoring the fracture healing period. Another method examined for these purposes is using ultrasonic guided waves [11-13]. However the most promising one is vibration analysis, mainly due to the ease of its application. Therefore, in the upcoming sections, a literature review on the use of vibration tests for both diagnosing osteoporosis or monitoring the fracture healing period will be given. The studies in the literature (e.g. see [4, 5, 14]) so far mainly focused on the natural frequency of the interested bone and the relationship of this parameter with mineral density of the bone. These can be grouped as experimental (e.g. see [14-16]) and computational/analytical studies (e.g. see [17-19]), although some of them

include both clusters. On the other hand, among the myriad of studies about diagnosing osteoporosis or monitoring the fracture healing period, the ones on tibia and only the most important ones of the rest will be discussed here since this research is centered on tibia.

1.3.1 Experimental Studies

The studies in literature related to diagnosing osteoporosis with alternative methods do not have too much history. Early investigations about this area started only a few decades ago. Jurist [14, 20], one of the pioneer researchers in this field, stated that mineral content of bone is not an adequate method of determining bone strength because even if there is a weak region in the bone, there is no certainty that its mineral should be low. In his research, he investigated the relationship between the fragility of bone and its modulus of elasticity. In the experiments carried out, he measured the resonant frequency of ulna using a two point impedance technique and calculated a parameter ' FL ', where F is the resonant frequency and L is the bone length, in order to compare it with the state of osteoporosis. He stated that this parameter has a potential value in diagnosing purposes. On the other hand, in a preliminary study of Christensen et al. [21], the resonance frequency of tibia was found by point impedance technique to minimize errors mentioned in Jurist's work. During that research, it was also observed that muscle tension and edema (abnormal accumulation of fluid beneath the skin), as well as the soft tissues, have considerable effect on the experimental data obtained.

In another study of Christensen et al. [15], different techniques (bone resonance analysis -BRA- and impulse frequency response -IFR-) for the assessment of stiffness of tibia were investigated with various boundary conditions. The IFR technique was found to be more appropriate since it is easier to perform and gives more accurate results for the bending modes. Later, in order to test the applicability and feasibility of the method, they performed a clinical study consisted of 51 persons [22]. From the in vivo experiments, they found frequency ranges for rigid and bending modes of tibia. However, no information was given about the BMD values of the participants and thus no comparison was made between the natural frequencies found and the BMD values.

In recent years, more clinical studies have been carried out with the conclusion that vibration analysis is a promising diagnosing tool in diagnosing osteoporosis or progressing osteoporosis and fracture healing period. For instance, Arpinar et al. [5] studied the correlation between natural frequency and BMD values of human tibial bone. In their study, they made the vibration measurements while the subjects were standing and compared the first natural frequency of bone with BMD measured by DXA. They stated that there is a correlation between these two parameters. One year later, a similar study was done by Özdurak et al. [4] on human radius of elderly man. Again, in this work, it is noted that there is a discrepancy between the first natural frequency and BMD values of the radius.

On the other hand, fracture healing process is also an important clinical phenomenon as diagnosing osteoporosis. The major dilemma in this

process is the healing time because the motion range of joints decreases if the patient is excessively immobilized whereas in early resumption, clinical healing does not occur and this leads to delayed union or even in re-fractures. Since time needed for radiological healing of bone is much more than the time for clinical healing, the common methods (X-ray methods) are insufficient. In 1991, in a study of Nowak et al. [23], one of the early investigations in this area, five dowel specimens with cuts having depths of $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ inches have been tested and considerable changes were noted in the first and second natural frequencies. Later, Akkus et al. [9, 24] performed an experimental study on rabbit tibia (in vitro) in order to see the effect of changes in mechanical stiffness of the callus (tissue that forms at fracture area) on the data obtained from vibration experiment during the fracture healing period. In their study, they found the mode shapes of fractured and intact rabbit tibiae and measured the BMD values correspondingly on specific days of healing process. They have observed that first mode shape of tibia seems to be very effective in tracking fracture healing. However they also noted that BMD measurements were not correlated with vibration transmission characteristics and mechanical stiffness obtained from three point bending measurements.

According to the previous research, one of the foremost difficulties encountered in in vivo experiments as mentioned earlier, arises due to the soft tissues (skin, muscles...) both overlying and underlying the bone. Studies in literature show that soft tissues have a considerable effect on the measured data. For example, in 1977 Saha and Lakes [25], investigated the effect of soft tissue on wave propagation and vibration tests and reported that significantly large errors were introduced due to the soft tissues. Also

based on the experiments he performed with different thickness of rubber layers to simulate the soft tissue, he concludes that as the thickness of the soft tissue increases, a broadening in input force spectrum occurs so that higher modes are not excited. Later, Cornelissen et al. [16] also investigated the structural dynamic properties of bones under the effect of soft tissues individually. In their study, they stated that the influence of muscle tissue is much higher than both the effect of the skin and the joints. However, as in previous studies, the deficiency of this research is that only the natural frequency is considered.

In later research about the influence of soft tissues, a set of solution methods were proposed in order to eliminate or minimize the influence of soft tissue on vibration measurements. For example, an attempt that was done by Soethoudt et al. [26] revealed that the preload that is applied by holding a small cylinder metal element manually pressed at the excitation point while holding the accelerometer during the excitation of the bone via impact hammer helps to decrease the damping effect of the skin and the underlying soft tissues.

1.3.2 Computational/Analytical Studies

Modeling human tibia is one of the important goals in order to better understand the dynamics and structure of the bone. Unlike experimental modal analysis, FE models analyses are not limited only to superficial bones. However, due to having a highly complex structure, it is quite difficult to develop a proper model of bone. Early models were based on dynamic analysis of continuous systems of beams and shafts. Hight et al.

[27] presented a model representing tibia as a beam element and for different boundary conditions and varying geometries, he predicted the natural frequencies of human tibia. On the other hand, the model suggested by Dhoerty et al. [17] and Jurist and Kianian [28] was formed by only circular solid shafts and hollow tube, respectively. However, these models were not sufficient to explain different transverse modes of bones since they are symmetrical about the diaphysis axis. Later, Collier et al. [29] came up with a model composed of a homogenous hollow triangular cross-section beam which is more suitable to simulate bone characteristics. Thus, in this model he was able to predict different transverse modes. In 1990 Thomsen [18] constructed a mathematical model of human tibia composed of non-uniform Timoshenko beam elements. In his model, he used isotropic materials both for the compact and cancellous parts of the bone, and he also included the bone marrow as a perfectly flexible material. On the other hand, some models presented used the wave propagation characteristics of bone as a diagnosing tool such that the bone was modeled as a thick walled cylindrical tube filled with fluid [30].

With the developments in finite element (FE) methods, more complex models of human bones have been analyzed. Especially for modeling computer tomography (CT) scan data have been used. For instance, Hobatho et al. [19, 31] developed a three dimensional finite element model of human tibia with isotropic material properties and investigated the effect on natural frequency due to changes in elastic modulus of bone parts (compact and cancellous). Later, they also prepared a model of human femur with the same procedure [32]. In 2002, Taylor et al. [33] constructed a finite element model of human femur with orthotropic bone elastic

constants. In their study, they stated that using FE models and modal testing together, it seems possible to obtain the orthotropic material properties of bone. In addition, in some studies FE analysis has been used to simulate the fracture healing period in human bones [34].

1.4 Objective

The main goal of this thesis is to find the applicability of a new approach to detect osteoporosis or progressing osteoporosis by looking for a relation between structural dynamic properties and bone mineral density, using the in vivo FRF data of human tibia obtained from vibration experiments. The motivation behind this study is that solely the change of natural frequency with progressing osteoporosis is investigated which is the most deficient part of the previous studies. However, natural frequencies of a structure depend on both boundary conditions and geometry. Therefore, this modal parameter may only be employed as an effective approach in the assessment of progressing osteoporosis, if these drawbacks mentioned are resolved, i.e. if an experimental set-up that prevents the change of boundary conditions of bone (i.e. all the tests are done with the same boundary conditions), is utilized and the measured natural frequencies are compared for a specific patient or patients having similar bone geometry. Therefore, it is more logical to investigate the change in 'loss factor', which is an inherent property of bone.

In this thesis, it is aimed to extract the loss factor of human tibial bone from in vivo and in vitro measurements and investigate the relation between this structural modal parameter and the osteoporosis, i.e. BMD values obtained

from DXA measurements. Moreover, the natural frequency of tibia is investigated through experimental methods and finite element (FE) model. Thus, in the view of natural frequency information of tibia, the change in its elastic modulus, which is another inherit property of bone, is investigated by using FE model. Also, the effect of boundary conditions on natural frequency of tibia is studied, again by using FE model. Thus, it is also intended to examine the applicability of the natural frequency approach in the diagnosis of osteoporosis or progressing osteoporosis.

1.5 Outline of the dissertation

The outline of the thesis is given below:

In Chapter 2, the background knowledge on modal analysis is given and modal identification methods are briefly discussed.

In Chapter 3, a preliminary experimental study in in vitro condition on human tibia obtained from fresh-frozen cadavers will be given. The structural modal parameters will be extracted by the modal identification methods and will be compared with respect to BMD values measured by DXA. Also, in this chapter, the effects of soft tissues are investigated.

In Chapter 4, in vivo experimentation of human tibia is discussed in detail. Firstly, the difficulties of in vivo experiments will be mentioned. Then FRFs obtained from these experiments will be identified and modal parameters of tibia will be extracted and compared among the osteoporotic, osteopenic and normal people.

In Chapter 5, finite element (FE) modeling of tibia is given and results obtained from the experiments discussed in Chapter 4 will be compared with the results of the FE model. Then, the effect of elastic modulus on natural frequency of tibia is investigated.

In the last chapter of the thesis, conclusions on this study will be made. Also suggestions on future work will be given.

CHAPTER 2

EXPERIMENTAL MODAL ANALYSIS

2.1 Introduction

In order to better understand the nature of a structure, there is always a need for experimental study. Therefore, modal analysis plays an important role as an engineering tool, especially for determining system's dynamic properties such as natural frequency, damping and mode shapes. In other words, modal analysis, or more accurately experimental modal analysis, is the field of measuring and analyzing the dynamic response of structures when excited by an input. On the other hand, this method is also used for optimizing and improving the dynamic characteristic of structures. Although, in many cases, computer modeling can be used as a powerful tool that serves for the purposes mentioned above, it is insufficient to identify specific dynamic characteristics of a structure such as non-linearity and damping [35].

Modal analysis is first used in 1940s, in aircraft industry to better understand these complex engineering structures. Until the last two decades ago, it had not shown much progress. However, with the development in computer technologies, especially in digital analyzers, the usage of this method gradually increased. Today, it has a diverse application area. For instance, besides mechanical and aeronautical

engineering applications, this method is also used in the areas of civil engineering, biomechanical, and transportation applications as well [36].

Modal analysis includes both theoretical and experimental methods. Theoretical modal analysis is a process to describe a system in terms of its modal parameters: natural frequency, loss factor and modal constant. On the other hand, experimental modal analysis also referred to as *modal testing*, focuses on obtaining modal model of a vibrating structure. In modal testing, the primary aim is to obtain frequency response functions (FRFs) which contain input – output relation of the measured system. These functions are calculated simply by taking the ratio of the output signal to the given input (force) signal. The second stage of the modal testing is constructing the modal model from measured experimental data through modal identification (modal extraction) techniques. These identification techniques are mainly based on curve fitting methods to the obtained data. Therefore, the measured FRF can be represented by a theoretical expression [37].

There are various methods and procedures employed for modal identification. Each of them has their own proper use. In other words, every method has advantages amongst others and has its specific limitations. However, the goal of all identification methods is to find appropriate coefficients of the theoretical expression that is constructed to represent the experimentally obtained FRF data.

Modal identification methods can be grouped in various ways: according to the domain (frequency or time domain) in which the identification

procedures are done, according to the frequency range considered, i.e. either only one mode or more modes will be extracted at each attempt (single degree of freedom methods -SDOF- or multi degree of freedom methods -MDOF-), or according to the number of FRFs included in the identification analysis (single FRF methods or multi FRF methods – Global or Polyreference methods). The selection of the appropriate method is extremely important. Although any method can be applied to a measured FRF data, it is highly recommended that a proper method should be utilized considering time and accuracy aspects.

Unfortunately, in practice, unexpected problems may arise inevitably; for instance, due to the modeling the damping effect or determining the model order of the investigated structure. In the former case, if the assumed damping characteristic does not match the real one, there occur significant errors in the extracted modal parameters of the vibrating structure. On the other hand, in the latter case, if the model order is not known in the frequency range of interest, besides the real modes it is possible to see some fictitious modes as well.

Furthermore, it is quite useful to check the measured FRF data before modal identification procedures. There are several visual checks that should be done whether the measured data is reliable or not [35].

In the upcoming sections, the modal identification methods employed in this thesis will be described in detail.

2.2 Modal Analysis Methods

In this study, in order to identify the modal parameters of the investigated structure, several different methods are used on measured FRF data. These can be grouped as SDOF and MDOF methods. SDOF methods are composed of *circle* and *line fit methods*. Also, *rational fraction polynomial method* is utilized as a MDOF method.

2.2.1 Single Degree of Freedom (SDOF) Methods

SDOF methods are the first type of modal analysis methods developed in the modal testing area. The basic philosophy underlying in the single degree of freedom modal analysis methods is that each mode in the frequency range of interest is examined separately and therefore, it is aimed to extract the modal parameters of the individual modes. For this reason, in the existence of close modes, it is not appropriate to use these methods.

The initial assumption, as in all the theories of SDOF methods, is quite simple. It is known that receptance of a structure can be expressed as,

$$\alpha_{jk}(\omega) = \sum_{s=1}^N \frac{{}_s A_{jk}}{\omega_s^2 - \omega^2 + i\eta_s \omega_s^2} \quad (2.1)$$

where ${}_s A_{jk}$ is the modal constant, ω_s is the s^{th} mode natural frequency, ω is the excitation frequency, η_s is the modal loss factor, j and k are the indices of the elements of the receptance matrix.

Equation (2.1) can also be expressed in the form [35];

$$\alpha_{jk}(\omega) = \frac{{}_r A_{jk}}{\omega_r^2 - \omega^2 + i\eta_r \omega_r^2} + \sum_{s=1, s \neq r}^N \frac{{}_s A_{jk}}{\omega_s^2 - \omega^2 + i\eta_s \omega_s^2} \quad (2.2)$$

The first part of the equation represents the mode of interest, 'r', and the second part represents the other modes. Therefore, if the modes of the structure are sufficiently distinct from each other, the first part becomes dominant in the vicinity of the mode 'r' and it can be said that the second part becomes independent of frequency, ω . Hence, equation (2.1) can be rewritten as,

$$\alpha_{jk}(\omega) = \frac{{}_r A_{jk}}{\omega_r^2 - \omega^2 + i\eta_r \omega_r^2} + {}_r B_{jk} \quad (2.3)$$

where ${}_r B_{jk}$ is a complex constant term representing the effect of remaining modes.

After introducing the basic theory of SDOF methods, in the following sections, the theory behind circle and line fit methods will be discussed briefly.

2.2.1.1 Circle Fit (O-Fit) Method

Circle fit method is the most widely used SDOF method. The analysis is performed using the *Nyquist* plot of the measured FRF data. Nyquist plot of SDOF systems is generally a circle-like curve. With the appropriate

parameter selection for the damping model (i.e. using structural damping model with receptance form of FRF data), these plots will be exact circles. This constitutes the basis of this method.

Considering equation (2.3), the modal constant term rA_{jk} can be treated as a shifting term and therefore, the fundamental equation of interest in this method can be expressed in the following form [35],

$$\alpha'_{jk}(\omega) = \frac{1}{\omega_r^2 \left[1 - \left(\frac{\omega}{\omega_r} \right)^2 + i\eta_r \right]} \quad (2.4)$$

The nyquist plot of equation (2.4) can be seen in Figure 2.1,

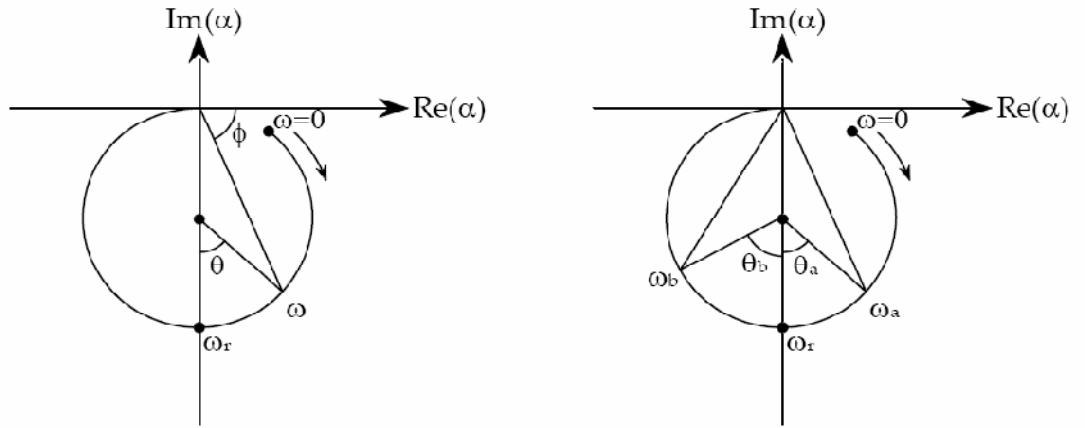


Figure 2.1 – Nyquist plot of modal circle (Properties of modal circle) [35]

Using Figure 2.1, the natural frequency, ω_r , can easily be found or it can be extracted by using the fact that ω_r is located at the maximum arc change point [35]. For any point on the Nyquist plot, the following relations can be written,

$$\text{Re}(\alpha) = \frac{1 - \left(\frac{\omega}{\omega_r}\right)^2}{\omega_r^2 \left[\left[1 - \left(\frac{\omega}{\omega_r}\right)^2 \right]^2 + \eta_r^2 \right]} \quad (2.5)$$

$$\text{Im}(\alpha) = \frac{\eta_r}{\omega_r^2 \left[\left[1 - \left(\frac{\omega}{\omega_r}\right)^2 \right]^2 + \eta_r^2 \right]} \quad (2.6)$$

The loss factor can be found by using Figure 2.1, and equations (2.5) and (2.6) as follows,

$$\tan\left(\frac{\theta_a}{2}\right) = \frac{|\text{Re}(\alpha(\omega_a))|}{|\text{Im}(\alpha(\omega_a))|} \quad (2.7)$$

$$\tan\left(\frac{\theta_b}{2}\right) = \frac{|\text{Re}(\alpha(\omega_b))|}{|\text{Im}(\alpha(\omega_b))|} \quad (2.8)$$

$$\eta_r = \frac{\omega_a^2 - \omega_b^2}{\omega_r^2} \frac{1}{\tan\left(\frac{\theta_a}{2}\right) + \tan\left(\frac{\theta_b}{2}\right)} \quad (2.9)$$

Lastly, after extracting natural frequency and loss factor, modal constant as the hindmost modal parameter needs to be identified. The diameter of the circle in the Nyquist plot can be expressed for the r^{th} mode as,

$${}_r D_{ij} = \frac{|{}_r A_{jk}|}{\omega_r^2 \eta_r} \quad (2.10)$$

It is obvious that once the Nyquist plot of the selected mode is drawn, using SDOF circle fit method the modal parameters can be extracted quite easily.

2.2.1.2 Line Fit (L - Fit) Method

Line fit method can be treated as a derivative of the circle fit method. In this case, instead of drawing the receptance of the measured FRF, the inverse of it is drawn so that for a SDOF system, a straight line is obtained for both imaginary and real parts. Therefore, rather than fitting a circle, by using this method, modal parameters are extracted by simply fitting straight lines to real and imaginary parts of measured FRFs.

Recall equation (2.3), since the last term, ${}_r B_{jk}$ in this equation contains the effect of the other modes on the mode of interest, a new FRF term, $\alpha''_{jk}(\omega)$, is defined such that [35],

$$\alpha''_{jk}(\omega) = \alpha_{jk}(\omega) - \alpha_{jk}(\Omega) \quad (2.11)$$

where $\alpha_{jk}(\Omega)$ and Ω are called as the fixing frequency term and fixing frequency, respectively. In this method, using equation (2.11), an inverse parameter $\Delta(\omega)$ is derived [35].

$$\Delta(\omega) = \frac{(\omega^2 - \Omega^2)}{\alpha_{jk}''(\omega)} \quad (2.12)$$

Substituting equation (2.11) into the inverse parameter equation,

$$\Delta(\omega) = \frac{(\omega^2 - \Omega^2)}{\left[\frac{{}_r A_{jk}}{\omega_r^2 - \omega^2 + i\eta_r \omega_r^2} + {}_r \mathcal{B}_{jk} \right] - \left[\frac{{}_r A_{jk}}{\omega_r^2 - \Omega^2 + i\eta_r \omega_r^2} + {}_r \mathcal{B}_{jk} \right]} \quad (2.13)$$

After some manipulations, equation (2.13) can be written in the following form,

$$\Delta(\omega) = \frac{(\omega_r^2 - \omega^2 + i\eta_r \omega_r^2)(\omega_r^2 - \Omega^2 + i\eta_r \omega_r^2)}{{}_r A_{jk}} \quad (2.14)$$

The modal constant can be written as,

$${}_r A_{jk} = a_m + ib_m \quad (2.15)$$

If the real and imaginary parts in equation (2.14) are explicitly written, the following linear functions of variable ω^2 are obtained.

$$\Delta(\omega) = \text{Re}(\Delta(\omega)) + i \text{Im}(\Delta(\omega)) \quad (2.16)$$

$$\text{Re}(\Delta(\omega)) = a_R \omega^2 + b_R \quad (2.17)$$

$$\text{Im}(\Delta(\omega)) = a_I \omega^2 + b_I \quad (2.18)$$

where, a_R and a_I are the slopes, b_R and b_I are the intercepts of equations (2.17) and (2.18). If these functions are drawn with respect to ω^2 , straight lines are obtained, both for the real and imaginary parts of the inverse parameter, $\Delta(\omega)$, whose slopes and intercepts are,

$$a_R = \frac{a_m}{a_m^2 + b_m^2} \Omega^2 - \frac{(a_m + b_m \eta_r)}{a_m^2 + b_m^2} \omega_r^2 \quad (2.19)$$

$$b_R = \frac{a_m (\omega_r^4 - \omega_r^2 \Omega^2 - \eta_r^2 \omega_r^4) + b_m \eta_r \omega_r^2 (2\omega_r^2 - \Omega^2)}{a_m^2 + b_m^2} \quad (2.20)$$

$$a_I = -\frac{b_m}{a_m^2 + b_m^2} \Omega^2 - \frac{(a_m \eta_r + b_m)}{a_m^2 + b_m^2} \omega_r^2 \quad (2.21)$$

$$b_I = \frac{a_m \eta_r \omega_r^2 (2\omega_r^2 - \Omega^2) + b_m (\omega_r^4 - \omega_r^2 \Omega^2 - \eta_r^2 \omega_r^4)}{a_m^2 + b_m^2} \quad (2.22)$$

Therefore, a series of straight lines are obtained by selecting frequencies, Ω , around the resonance (natural) frequency of the mode of interest. Figure 2.2 shows an example of the real and imaginary parts of the inverse parameter, $\Delta(\omega)$, obtained by selecting different frequencies around the natural frequency.

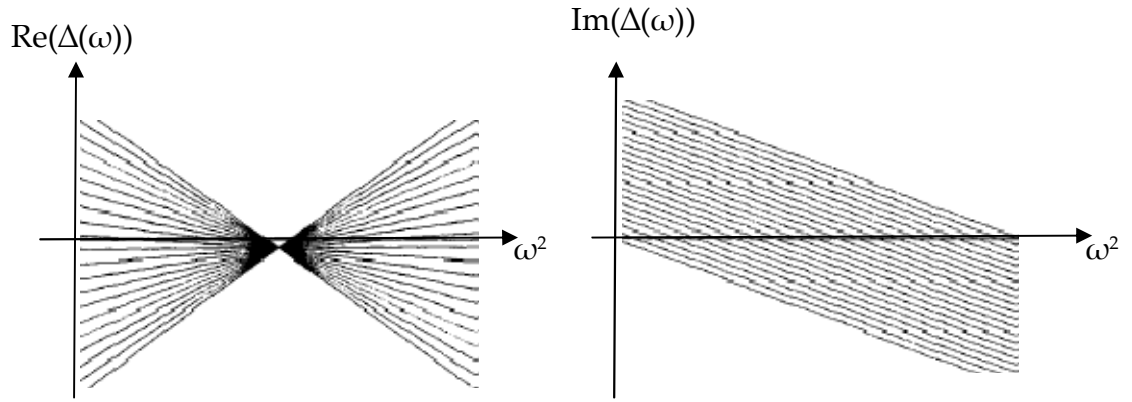


Figure 2.2 – Plots of the real and imaginary parts of $\Delta(\omega)$ [37]

Figure 2.3 shows the slopes of the straight lines plotted in Figure 2.2. As can be seen from equation (2.19) and equation (2.21), straight lines are obtained if these slopes are plotted against Ω^2 .

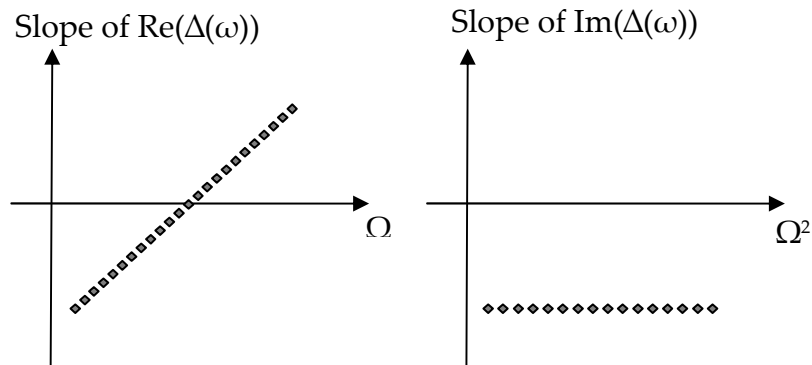


Figure 2.3 – Slopes of the lines in Figure 2.2 [37]

Using equations (2.19) – (2.22), the four unknown variables a_m , b_m , ω_r and η_r can be found. Therefore, the modal parameters of the system, A_r , ω_r and η_r , are extracted using the line fit method

2.2.2 Multi Degree of Freedom Methods

Although SDOF modal analysis methods are more commonly used, it is not possible to identify every system with these methods. In other words, there are some limitations for the utilization of SDOF methods. For instance, in the case of closely spaced modes where generally the modes of the system are not sufficiently separated or highly damped, the methods described in the previous section would be inappropriate to use. Therefore, for these cases some alternative methods were developed and they can be grouped as multi degree of freedom methods. In these methods, general philosophy is the extraction of the modal parameters of several modes simultaneously, instead of dealing with only one mode at each time as in the SDOF methods.

In this study, rational fraction polynomial method is used and therefore only the theory of this method will be given in the preceding section.

2.2.2.1 Rational Fraction Polynomial Method

In this approach, instead of using partial fraction form of the FRF, rational fraction form is used.

$$\alpha(\omega) = \frac{a_0 + a_1(i\omega) + a_2(i\omega)^2 + \dots + a_n(i\omega)^m}{b_0 + b_1(i\omega) + b_2(i\omega)^2 + \dots + b_n(i\omega)^n} = \frac{\sum_{k=0}^m a_k (i\omega)^k}{\sum_{k=0}^n b_k (i\omega)^k} \quad (2.23)$$

In curve fitting, the general aim is to minimize the difference between the analytical expression and the obtained data. Therefore, for a selected point, error between theoretically found data and analytically obtained data can be written as,

$$e_p = \alpha(\omega_p) - h_p \quad (2.24)$$

where h_p is a data point of the measured FRF. The error function can also be expressed in a more suitable form by substituting equation (2.23) into equation (2.24),

$$e'_p = \sum_{k=0}^m a_k (i\omega)^k - h_p \left[\sum_{k=0}^{n-1} b_k (i\omega)^k + (i\omega)^n \right] \quad (2.25)$$

Therefore, for all the data points (assuming there are P individual frequency values measured), an error function of the following form can be obtained.

$$\{E\} = \{e_1, e_2, e_3, e_4, \dots, e_P\}^T \quad (2.26)$$

Using equations (2.25) and (2.26), the error vector can be written as,

$$\{E\} = [U]\{A\} + [V]\{B\} - \{W\} \quad (2.27)$$

where,

$$[U] = \begin{bmatrix} 1 & i\omega_1 & \cdots & (i\omega_1)^m \\ 1 & i\omega_2 & \cdots & (i\omega_2)^m \\ \vdots & \vdots & & \vdots \\ 1 & i\omega_p & \cdots & (i\omega_p)^m \end{bmatrix} \quad (2.28)$$

$$[V] = \begin{bmatrix} h_1 & h_1(i\omega_1) & \cdots & h_1(i\omega_1)^{n-1} \\ h_2 & h_2(i\omega_2) & \cdots & h_2(i\omega_2)^{n-1} \\ \vdots & \vdots & & \vdots \\ h_p & h_p(i\omega_p) & \cdots & h_p(i\omega_p)^{n-1} \end{bmatrix} \quad (2.29)$$

$$[W] = \begin{bmatrix} h_1(i\omega_1)^n \\ h_2(i\omega_1)^n \\ \vdots \\ h_p(i\omega_p)^n \end{bmatrix} \quad (2.30)$$

$$\{A\} = \{a_0, a_1, \dots, a_m\}^T \quad (2.31)$$

$$\{B\} = \{b_0, b_1, \dots, b_{n-1}\}^T \quad (2.32)$$

In order to minimize the error, squared error criterion, which is expressed as,

$$J = \{E^*\}^T \{E\} \quad (2.33)$$

is used. This error equation is a function of both coefficients 'a' and 'b'. In order to minimize the error, its derivative with respect to coefficients 'a' and 'b' should be zero.

Taking the derivatives of the equation (2.33) with respect to the coefficients 'a' and 'b', and by rearranging them in a matrix form, the following equation can be obtained.

$$\begin{bmatrix} [U^*]^T [U] & -\text{Re}([P^*]^T [V]) \\ \left(-\text{Re}([U^*]^T [V])\right)^T & [V^*]^T [V] \end{bmatrix} \begin{Bmatrix} \{A\} \\ \{B\} \end{Bmatrix} = \begin{Bmatrix} \text{Re}([U^*]^T [W]) \\ \text{Re}([V^*]^T [W]) \end{Bmatrix} \quad (2.34)$$

Solving for the unknown coefficients, {A} and {B}, the rational fraction form of the FRF is obtained. Unlike the partial fraction form, these coefficients do not directly give the modal parameters of the system. Therefore, in order to identify the modal parameter information, poles and residues of the obtained rational fraction form are calculated as a further analysis. From these poles and residues information; natural frequency, damping ratio and modal constant information can be extracted easily. Detailed information about finding the modal parameters from the poles and residues information can be found in reference [38]. However, these equations are found to be ill-conditioned and thus they are not appropriate for digital computing. Therefore, equation (2.34) is re-formulated by the help of orthogonal functions. Since FRFs show *Hermitian* type of symmetry, orthogonal functions with the same characteristics should be used in the formulation. Hence, Forsythe Method is utilized for generating the orthogonal polynomials. Detailed information about these procedures can

be found in references [38, 39]. Then, $\Psi_{i,k'}$ be the generated orthogonal polynomials and following the same procedure described above, a modified and simple version of equation (2.34) can be obtained as [38],

$$\begin{bmatrix} [I] & -\text{Re}\left(2[U^*]^T[V]\right) \\ \left(-\text{Re}\left(2[U^*]^T[V]\right)\right)^T & [I] \end{bmatrix} \begin{Bmatrix} \{C\} \\ \{D\} \end{Bmatrix} = \begin{Bmatrix} \text{Re}\left([U^*]^T[W]\right) \\ \{0\} \end{Bmatrix} \quad (2.35)$$

where,

$$[U] = \begin{bmatrix} \Psi_{1,0} & \Psi_{1,1} & \cdots & \Psi_{1,m} \\ \Psi_{2,0} & \Psi_{2,1} & \cdots & \Psi_{2,m} \\ \vdots & \vdots & & \vdots \\ \Psi_{P,0} & \Psi_{P,1} & \cdots & \Psi_{P,m} \end{bmatrix} \quad (2.36)$$

$$[V] = \begin{bmatrix} h_1 \Psi_{1,0} & h_1 \Psi_{1,1} & \cdots & h_1 \Psi_{1,n-1} \\ h_2 \Psi_{2,0} & h_2 \Psi_{2,1} & \cdots & h_2 \Psi_{2,n-1} \\ \vdots & \vdots & & \vdots \\ h_P \Psi_{P,0} & h_P \Psi_{P,1} & \cdots & h_P \Psi_{P,n-1} \end{bmatrix} \quad (2.37)$$

$$[W] = \begin{bmatrix} h_1 \Psi_{1,n} \\ h_2 \Psi_{2,n} \\ \vdots \\ h_P \Psi_{P,n} \end{bmatrix} \quad (2.38)$$

$$\{C\} = \{c_0, c_1, \dots, c_m\}^T \quad (2.39)$$

$$\{D\} = \{d_0, d_1, \dots, d_{n-1}\}^T \quad (2.40)$$

Therefore, solving for the unknown coefficients, $\{C\}$ and $\{D\}$, of the orthogonal polynomials, the modal properties of the structure can be obtained by digital computing as described earlier.

CHAPTER 3

IN VITRO EXPERIMENTS

3.1 Extracting Modal Parameters of Human Tibia

For a detailed and immense analysis of human tibia, first it is required to acquire a basic idea about the dynamics of it. Therefore, before performing a large scale medical survey among patients, in this preliminary stage of the study, it is aimed to obtain modal parameters of human tibia via ex vivo (in vitro) modal tests and also analyze the effect of the soft tissues. For this purpose, a set of experiments were carried out on two fresh-frozen and one dry tibia. The specimens were obtained from and the measurements were made at Ankara University Faculty of Medicine, Department of Anatomy, Ankara.

3.1.1 Specimens

Two fresh tibia specimens obtained from above knee amputations of two different cadavers were used in order to extract structural modal parameters of human tibial bone in ex vivo condition. Both specimens were selected such that they show no history of bone diseases related to tibia. In order to prevent deterioration of the tissues and the tibia, both fresh tibia specimens were preserved at -20 °C after the amputation process.

3.1.2 Vibration Analysis

Analyzing the effect of the soft tissues on the overall (tibia-muscle-skin) system is an important goal in identifying the dynamics of the human tibia. Therefore, in order to analyze how the soft tissues affect the measurement data, a set of experiments under different conditions were carried out on these two fresh-frozen tibia specimens. These set of experiments are done in four different conditions and these conditions can be classified as,

- i. Intact (Case I)
- ii. After removal of the skin tissue around the excitation and response points (Case II)
- iii. After removal of the muscle tissue (Case III)
- iv. After exarticulation of the joints and detaching the fibula (Case IV)

Modal tests were carried out with a free-free boundary condition by placing the tibia on a soft rubber. Figure 3.1 and Figure 3.2, show exemplarily the experimental set-up for the Cases II and IV, respectively.

Excitation of each tibia specimen was performed at a specific point near to the middle section of the tibia (diaphysis) along the length of the bone by an impact hammer (Dytran Instruments, type 5800B3, S/N 4354, sensitivity=48.5 mV/lbf) and the response was recorded using a manually pressed accelerometer (Dytran Instruments, type 3035B, S/N 2436, sensitivity = 104 mV/g). Both signals were measured simultaneously with an instrument of Data Physics, "QUATTRO", to obtain the FRF data. The measurements were triggered by the input of the impact hammer's channel

and recorded only after at least five successive hits. Also, measurements were recorded in order to observe the first two modes (modes of interest) of the tibia in all cases described earlier. For this purpose the frequency range is taken as 0-800 Hz with a resolution of 1 Hz.

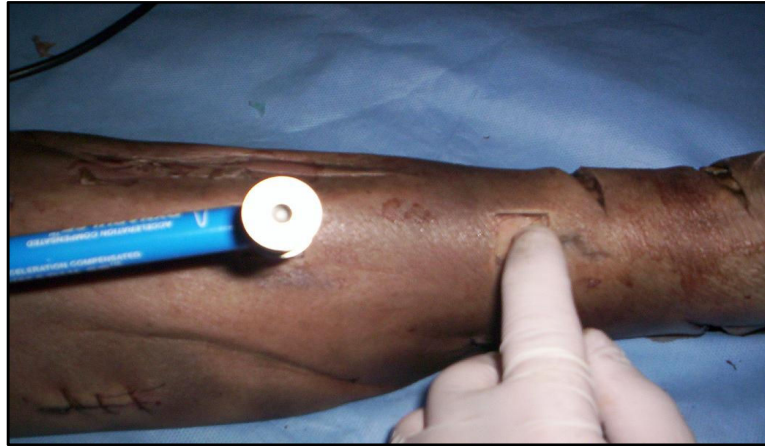


Figure 3.1 – Experimental set – up (Case II)



Figure 3.2 - Experimental set – up (Case IV)

The FRFs obtained through modal tests were analyzed by using the modal identification software MODENT® (Imperial College of Science, Technology and Medicine, London, UK, 1988). Circle and line fit methods were used for the identification of the modal parameters.

3.1.2.1 Effect of Soft Tissues

As indicated in the previous section, a four-stage experimental procedure was executed on the amputated fresh-frozen human tibia specimens. These experimental stages were extremely helpful in understanding the dynamics of the soft tissues and their effect of the overall FRF data obtained in in vivo condition. In Figure 3.3, the FRF plots of these experiments are given for the first tibia specimen.

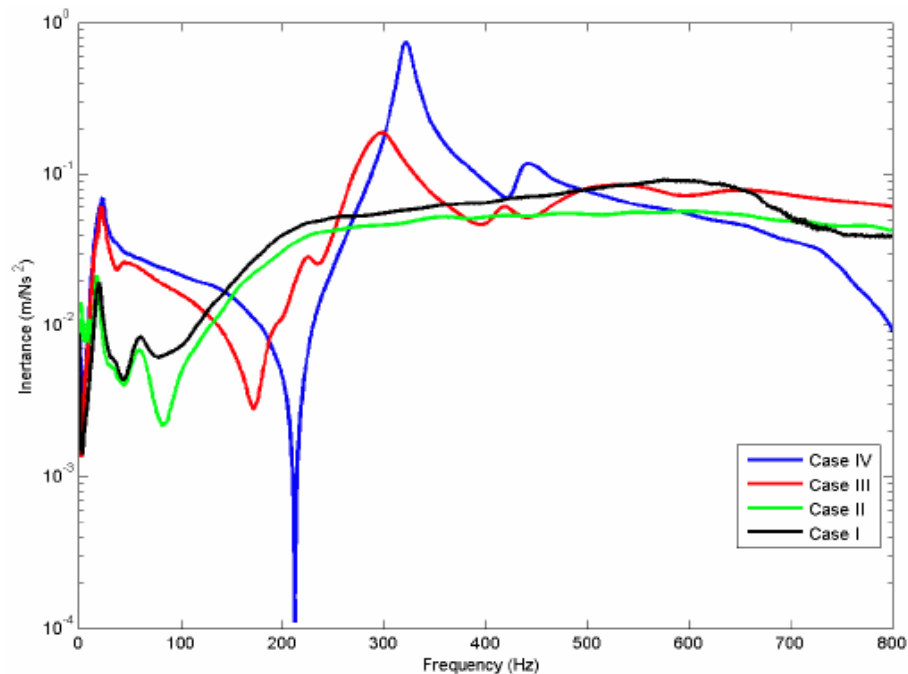


Figure 3.3 – Effect of the soft tissues on FRF of human tibia (tibia 1)

The effect of the soft tissues (skin, muscle, fibula and joints) can be examined individually. Unfortunately, due to the highly damped nature of the measured FRFs in the Cases I and II, it is not possible to perform modal identification procedures and obtain reliable and accurate modal parameters. More detailed information about in vivo modal parameter extraction will be given in Chapter IV. However, by examining Figure 3.3 only a general idea about the effects of skin and muscle tissues can be obtained.

As expected, after removal of the skin tissue, no significant changes occur in the FRF data obtained as shown in Figure 3.3 (Case I and Case II). It is due to the low mass and stiffness contribution to the overall system by the skin tissue. Therefore, its effect can be neglected.

From Figure 3.3, it can be easily deducted that muscle tissue has the highest effect on the measured FRF data. At first glance, it is seen that in Case III, the natural frequency values for the first two modes were highly shifted with respect to the values in Case II. Also from the shape of the plot around the resonance regions, it can be said that removing the muscle tissue decreases the overall damping values considerably. These findings can be explained by not only the additional heavy mass inserted to the system by the muscle group, but also the heavy damping introduced by the muscles.

In the case of the fibula and the joints, their effects can be interpreted more clearly after the modal identification procedure. After detaching the fibula, it is observed that the natural frequencies increase about 5-10 %. Also, the loss factors extracted decrease 60% for the first mode and 30% for the

second mode. One of the main reasons of these changes is the joints between the fibula and the tibia as well as the fibula itself.

As a conclusion, the existence of the soft tissues in the system makes the modal parameter extraction procedure difficult. In other words, even if there occurs a considerable change in the damping properties of the tibia itself, the soft tissues may suppress the effect of these changes to depict themselves in the modal loss factor of the overall (tibia-muscle-skin) system that are obtained from in vivo tests.

3.1.2.2 Modal Parameters of the Tibia

In this section, measurements that are done on two fresh-frozen tibiae (tibia 1 and tibia 2), after exarticulation of the joints and detaching the fibula, and one dry tibia (tibia 3) specimen are investigated. Before vibration tests, BMD values of all tibiae were found from DXA measurements and it is seen that both fresh-frozen tibiae have similar BMD values. For tibia 1 and 2, the FRFs obtained from the vibration experiments are given in Figure 3.4. Figure 3.5 shows the coherence plots of the measurements, indicating that experimental data obtained are reliable.

As mentioned earlier, the FRFs obtained were analyzed by using the modal identification software MODENT® (Imperial College of Science, Technology and Medicine, London, UK, 1988). Since it is seen in Figure 3.4 that the modes are sufficiently separated, in the modal identification procedure, circle and line fit methods are used.

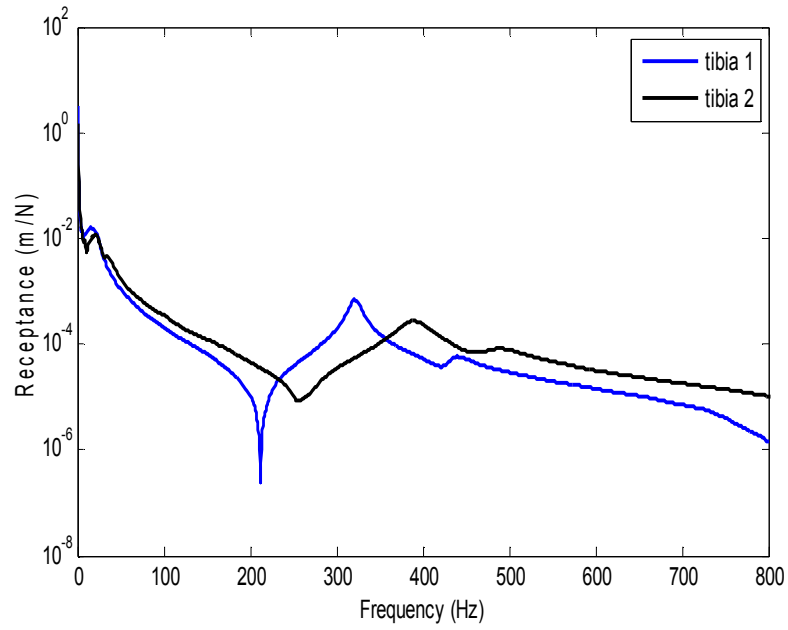


Figure 3.4 – FRF plots of the two fresh-frozen tibiae

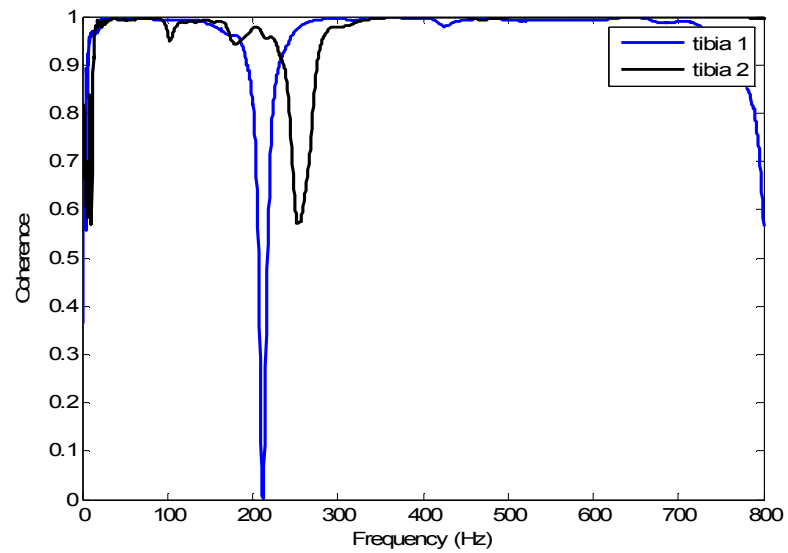


Figure 3.5 - Coherence plots of the FRF measurements of the two fresh-frozen tibiae

In a sample mode identification procedure, the screen is divided into four windows in which there are three plots and one table. Figure 3.6 shows the first plot which is on the upper left part of the modal identification screen. In this plot the inertance FRF measurements are shown since MODENT[®] uses this form of FRF measurement for all the modal identification procedure. MODENT[®] is a user interactive program; the modes to be identified must be selected by the user on this graph. After all the modes are identified, a regenerated FRF is drawn on the same plot as seen in Figure 3.6. The differences in the low and high frequency ranges are due to the out of range modes.

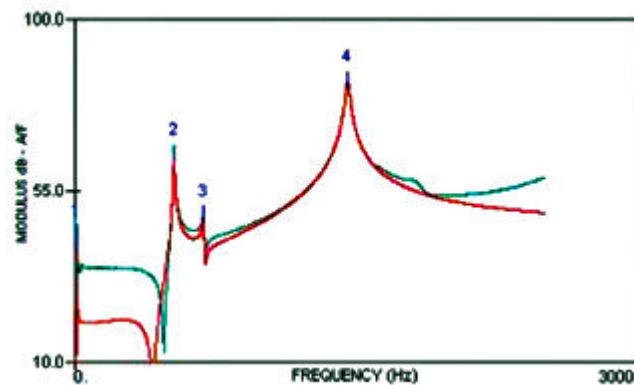


Figure 3.6 – Window (a): Inertance plot of the measurement

Experimental FRF: green curve

Regenerated FRF: red curve

Figure 3.7 shows the second plot on the screen which gives the Nyquist plot of the selected mode. In the Nyquist plot, MODENT[®] displays 21 FRF data points; one is the point that represents the selected resonance point and ten data points on each side of this selected point. At this step, user selects the

start and end points for an appropriate circle that fits best to this 21 FRF data points. Once the start and end points are selected, a regenerated Nyquist plot is drawn. As shown in Figure 3.7, the regenerated circle perfectly fits the data points, which is a good indication about the quality of the modal analysis.

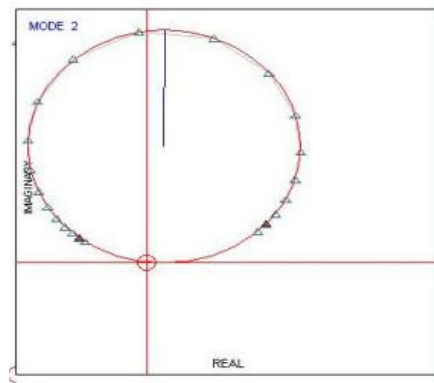


Figure 3.7 – Window (b): Nyquist plot of the selected mode

Figure 3.8 shows the third plot on the screen in which damping values are plotted as a three dimensional surface. This damping plot shows the variation of the damping values with respect to various frequency values. Having a flat surface, i.e. having almost the same damping value for any combination of frequencies implies that the system shows linear dynamic behavior.

Lastly, in the right lower corner of the screen, modal parameter information is given as a table. Apart from the identified modes, MODENT[®] does not give the information about the mass and stiffness residuals, i.e. the

contribution of the out of range modes. Instead of these residuals, two other modes are given; the lower one representing the lower out of range modes and the upper one representing the higher out of range modes.

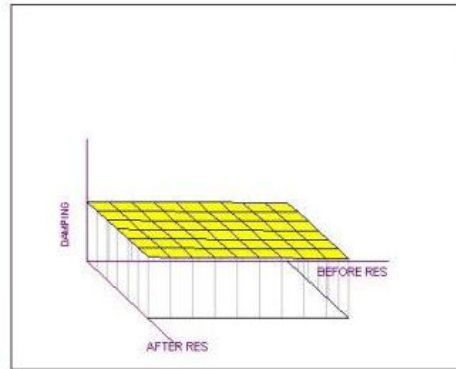


Figure 3.8 – Window (c): 3D damping plot of the selected mode

The modes of the measured FRFs are identified using an appropriate method: circle and line fit methods. The natural frequencies, the modal loss factors and the modal constants of the modes identified for the two fresh-frozen tibiae specimens are shown in Table 3.1 and Table 3.2, respectively. The rigid body modes due to free boundary conditions and the flexibility of soft rubber are not identified individually; instead, the modes representing the mass and the stiffness residuals are obtained as shown in Table 3.1 and Table 3.2.

Table 3.1 - Modal parameters of human fresh-frozen tibia (1st tibia specimen)

Mode	Frequency (Hz)	Loss Factor	Modal Constant	Phase	Method
0	0.0	0.001	0.879	21	Residual
1	321.3	0.040	1.320	-2	O-Fit
2	438.3	0.054	0.118	-44	L-Fit
3	1200	0.001	2.893	-5	Residual

Table 3.2 - Modal parameters of human fresh-frozen tibia (2nd tibia specimen)

Mode	Frequency (Hz)	Loss Factor	Modal Constant	Phase	Method
0	0.00	0.001	1.162	11	Residual
1	389.3	0.094	1.686	8	O-Fit
2	496.6	0.082	0.322	-75	L-Fit
3	1200	0.001	0.830	-63	Residual

By examining Figure 3.4 or Table 3.1 and Table 3.2, it can be said that there are some differences both in the natural frequencies and the loss factors of the two fresh-frozen tibiae specimens. The differences in the natural frequency values are expected since as mentioned in the previous chapters, the natural frequency is not a very suitable parameter for detecting osteoporosis or progressing osteoporosis since it is a susceptible modal parameter that can be affected by both the geometrical differences of the

systems and the boundary conditions in which the experiments are carried out. However, the modal loss factors were expected to be closer to each other since DXA measurements of these specimens showed that there is no considerable difference between the mineral contents of both fresh frozen tibiae (0.945 g/cm² and 0.917 g/cm² for the first tibia and second specimens). Therefore, these measurements may imply that not the only mineral content but also the collagen structure of bone affects the modal loss factors of the tibia. Yet, more specimens need to be tested to be able to generalize this conclusion.

On the other hand, in order to investigate and make a comparison of the modal parameters measured between the fresh-frozen and the dry tibiae, an identical experiment was carried out on a dry tibia. The modal parameters given in Table 3.3 are obtained.

Comparing the extracted modal parameters of the fresh-frozen and the dry tibiae specimens, two expected observations can be made about the natural frequencies and the loss factors. Firstly, it is seen that the natural frequencies of the dry tibia are higher than those of the fresh-frozen tibiae. Secondly, the modal loss factor of dry tibia is considerably smaller than those of the fresh-frozen ones. As mentioned, both results are expected since they are primarily related to the loss in mineral content and the collagen of the bone. However, the percentage change is much higher in the loss factor values of the tibia compared to the natural frequency values. Therefore, it can be concluded that loss factor of bones can be a strong indicator of the mineral content and the collagen of the bone.

Table 3.3 - Modal parameters of human dry tibia (3rd tibia specimen)

Mode	Frequency (Hz)	Loss Factor	Modal Constant	Phase	Method
0	0.00	0.001	45.276	0	Residual
1	526.1	0.011	18.328	-1	O-Fit
2	683.1	0.010	2.159	-173	O-Fit
3	1449.6	0.013	207.06	-178	O-Fit
4	3750.0	0.001	495.10	-5	Residual

Furthermore, having much larger percentage changes in the loss factors, compared to those in the natural frequencies when mineral content is reduced, is an encouraging result to believe that the damping measurements may be a promising technique in diagnosing osteoporosis or detecting progressing osteoporosis and monitoring the fracture healing period.

3.2 Modal Model of Human Tibia

After finding the modal parameters of the tibia both for the fresh-frozen and the dry conditions, mathematical models for human tibia can be obtained by using the extracted parameters presented in Table 3.1, Table 3.2 and Table 3.3. These mathematical (analytical) models, which may be referred as *regenerated* FRFs of tibia, can be formed by modal superposition of the modes identified and the modes representing the mass and stiffness residuals as follows,

$$\alpha_{jk}(\omega) = \sum_{s=1}^N \frac{s A_{jk}}{\omega_s^2 - \omega^2 + i\eta_s \omega_s^2} \quad (3.1)$$

In order to compare the results, the regenerated FRFs and the experimentally measured FRFs are plotted in Figure 3.9, Figure 3.10 and Figure 3.11, for the fresh-frozen tibia 1 and 2, and for the dry tibia, respectively.

From the regenerated FRF figures, it is clearly seen that the regenerated ones perfectly fit the experimental FRFs. Furthermore, similar measurements were made under different loading conditions and almost the same experimental FRFs were found. Therefore, it is concluded that human tibia shows linear dynamic behavior, at least in the range of forcing levels used in the experiments.

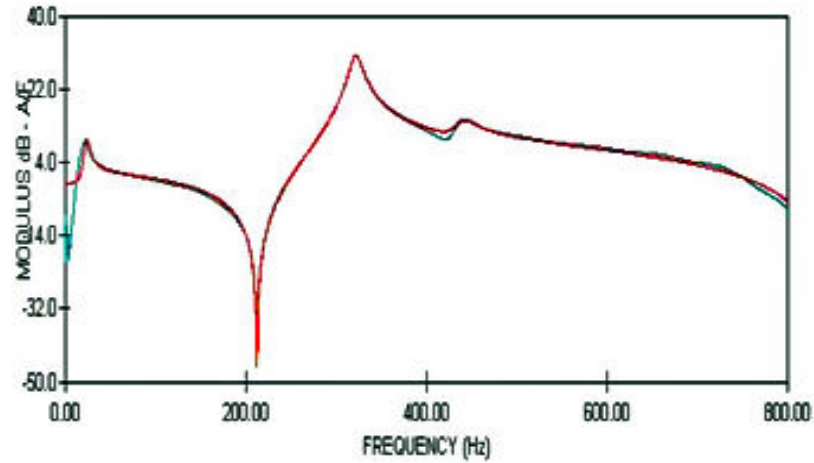


Figure 3.9 – Regenerated FRF of the first tibia specimen (tibia 1)

Experimental FRF: green curve

Regenerated FRF: red curve

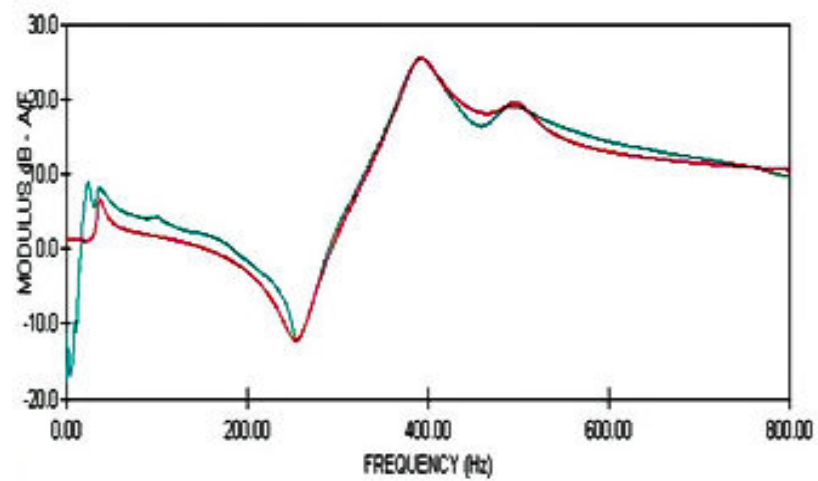


Figure 3.10 – Regenerated FRF of the second tibia specimen (tibia 2)

Experimental FRF: green curve

Regenerated FRF: red curve

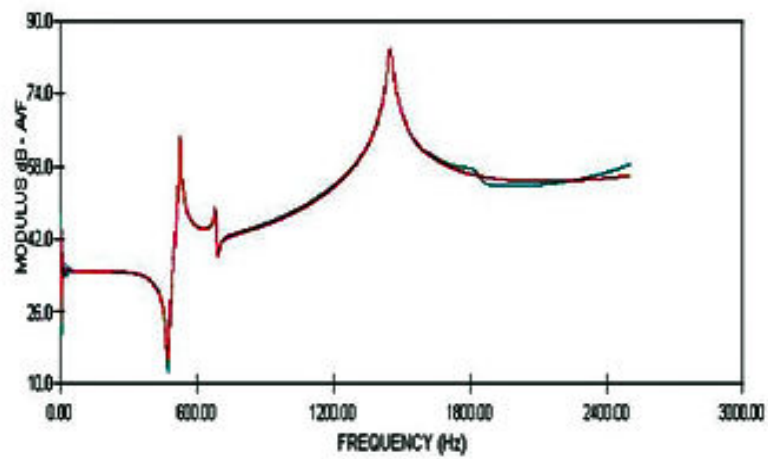


Figure 3.11 – Regenerated FRF of the first tibia specimen (tibia 3)

Experimental FRF: green curve

Regenerated FRF: red curve

CHAPTER 4

IN VIVO EXPERIMENTS

In the in vitro experiments section, it is shown that with decreasing mineral density of human tibia, which is known as the main indicator of osteoporosis, considerable changes occur in the modal parameters of tibia, especially in loss factor. Since the ultimate goal in this study is to perform in vivo test in order to diagnose osteoporosis or progressing osteoporosis, a large scale medical survey among patients was performed in this part of the study. The aim in these tests was to observe the change in the modal properties of tibia, as in the in vitro case, with BMD values, in other words the state of osteoporosis obtained from DXA measurements.

4.1 Materials and Methods

4.1.1 Subjects

This medical survey was performed at '70. Yıl Dinlenme ve Bakımevi' and 42 occupants of this facility volunteered to participate in this study. Information about the study was given and the possible side effects were explained to the participants, and a written consent was obtained from each participant. Since osteoporosis related diseases are seen mostly in females and one in three women aged over 50 years has this kind of disease [3], all of the subjects of this medical survey were chosen to be female and older than 50 years of age.

4.1.2 Bone Mineral Density Measurements

BMD measurements of all participants were performed at Middle East Technical University, Medical Center, Radio Diagnostic Unit using Lunar DPX densitometer (Lunar, Madison, WI). Both the measurements and the analysis were made by the same operator to prevent inter-operator technical errors. Patient positioning was carried out according to the instructions provided by the manufacturer and analyses of the measurements were conducted with manufacturer's software (Lunar version 4.6d software). BMD measurements of femur and spine were performed on 42 and 38 patients, respectively.

4.1.3 Modal Test Using Impact Hammer

Vibration measurements were made both on the left and right tibiae of the participants. Excitation of each tibia was performed at a specific point near to the middle section of the tibia (diaphysis) along the length of the bone by an impact hammer (Dytran Instruments, type 5800B3, S/N 4354, sensitivity = 48.5 mV/lbf) and the response was recorded using a manually pressed accelerometer (Dytran Instruments, type 3035B, S/N 2436, sensitivity = 104 mV/g). Both signals were measured simultaneously with an instrument of Data Physics, "QUATTRO", to obtain the FRF data.

The measurements were made at '70. Yıl Dinlenme ve Bakımevi', Ankara and the experiments were performed with free-free boundary conditions by placing the leg on a soft rubber in order to minimize the physical effects

coming from participants. Figure 4.1 shows the in vivo experimental arrangement used during the measurements.

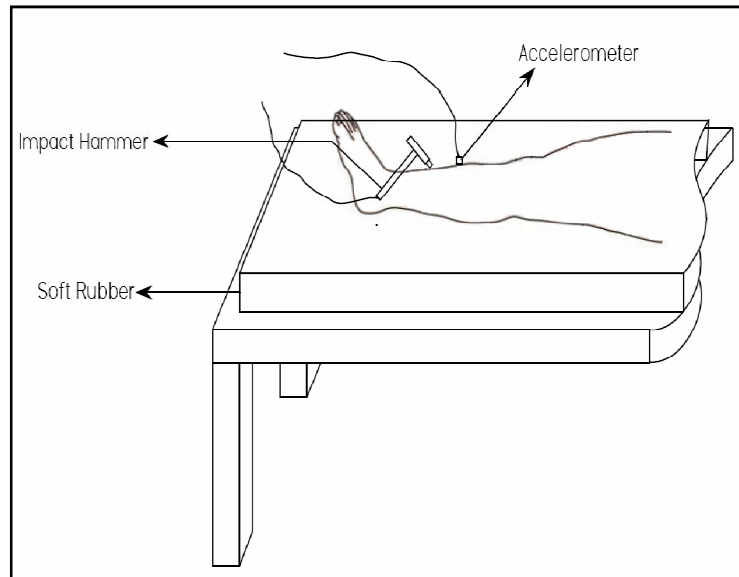


Figure 4.1 – In vivo experimental set - up

The measurements were carried out for a frequency range of 0-80 Hz with a resolution of 1 Hz, since from previous research it is known that modes of interest lie in this frequency range. The measurements were recorded only after at least five successive hits. Also, since the measurements were recorded for a finite length of time history, in the signal processing operations exponential windowing was used in order to minimize the effect of leakage. Also, in order to check the reliability of the experiments, all participants were tested three times monthly.

In this part of the study, SDOF modal identification methods are found to be inappropriate since the modes of interest are close to each other and

show highly damped behavior. Therefore, in order to extract modal parameters from the obtained FRF data for in vivo case, rational fraction polynomial (RFP) method, which was described in section 2.2.2.1, is utilized.

4.2 Extracting In Vivo Modal Parameters

A sample FRF plot obtained from the performed vibration tests and the corresponding coherence plot of this sample measurement, which indicates that experimental data obtained are reliable, are given in Figure 4.2 and Figure 4.3, respectively. As seen from these figures that the measured system is highly damped compared to most of the engineering cases encountered.

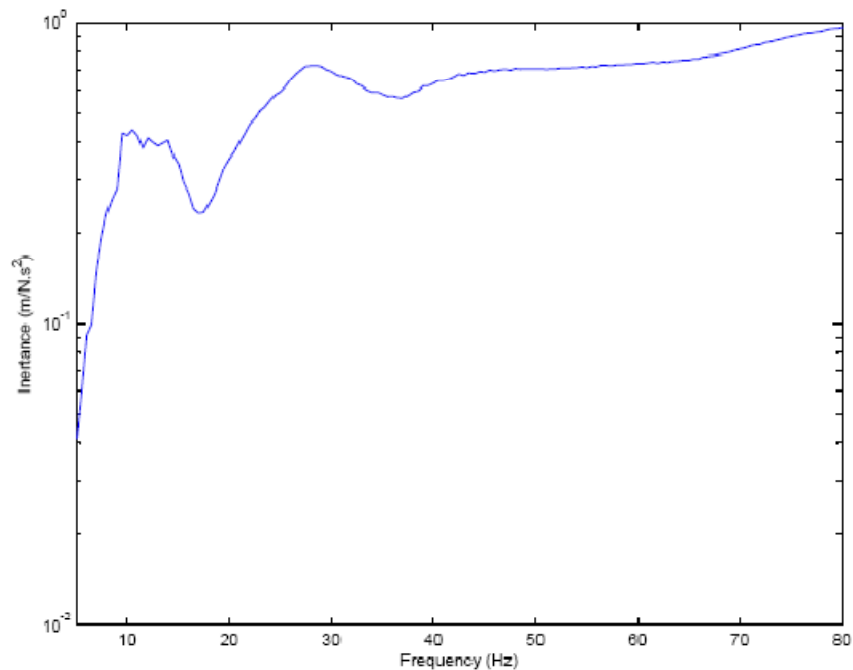


Figure 4.2 - A sample in vivo FRF plot of the tibia

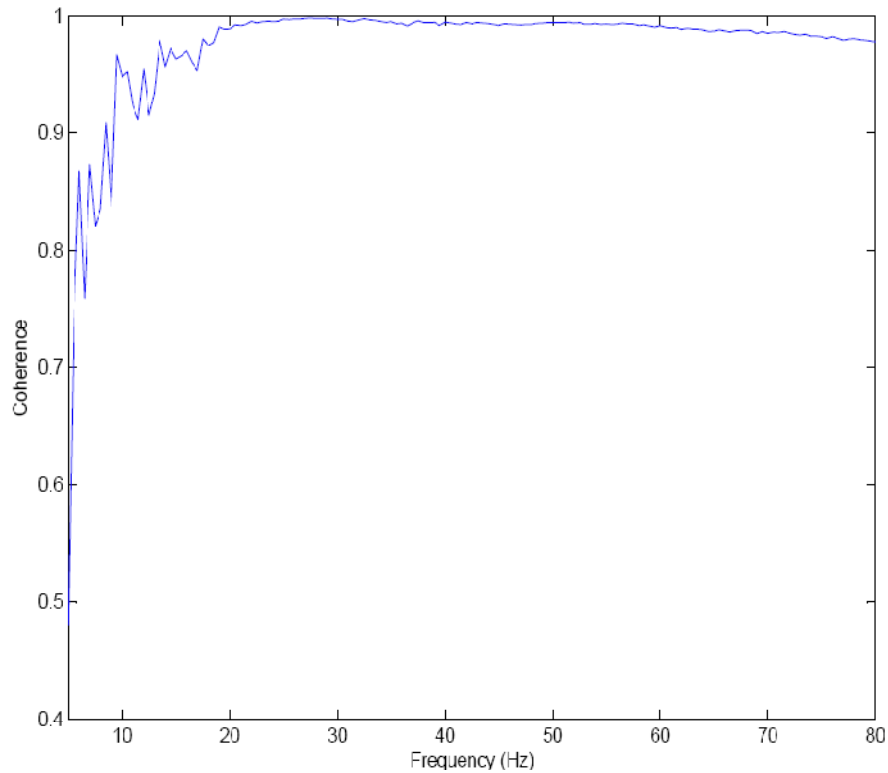


Figure 4.3 - Coherence plot of in vivo FRF measurement of the tibia

For this sample measurement, modal parameters were extracted using the RFP method. However, due to the high damping introduced to the system by the soft tissues, the results obtained from the modal identification procedures are not so unreliable. Therefore, for reliability purposes, the modal identification procedure were performed not only for a single frequency range but also for different frequency ranges and the changes in the modal parameters were investigated. Table 4.1 lists the modal parameters extracted by using various frequency ranges.

Table 4.1 - In vivo modal parameters of tibia for different frequency ranges

Frequency Range (Hz)	Mode	Natural Frequency (Hz)	Loss Factor
5-80	1	8.9	0.6876
	2	27.9	0.3980
5-90	1	9.7	0.6791
	2	27.6	0.3633
5-100	1	9.5	0.6603
	2	27.4	0.3409

The first mode identified is due to the skin tissue; therefore it is not appropriate to make conclusions using the modal data of this mode. However, investigating the second mode, it is seen that although the natural frequency values do not change significantly, the changes in loss factor cannot be disregarded.

As can be seen from Table 4.1, the percentage changes in the loss factor of the second mode are around 15% which implies that the systems shows a non linear damping behavior. However, in order to diagnose the osteoporosis or progressing osteoporosis by in vivo experiments, the modal parameters are needed to be identified with much more accuracy. In other words, due to the high damping introduced by the soft tissues, accurate and repeatable results are required in order to detect the contribution of the damping of the tibia to the overall damping measured. As discussed in the previous chapter, the loss factor of the tibia is found to be much less than the values obtained in in vivo case for overall system (tibia-muscle-skin). Hence, in order to observe a change in the damping of tibia that depicts

itself on the overall measured data, it is required to have a very high accuracy in parameters extracted from in vivo measurements.

Therefore, in this part of the study, only the natural frequencies extracted using RFP method are considered and compared with DXA measurement results. Table 4.2 shows the DXA measurement and natural frequency data for all the participants. For the intra observer reliability, results of two measurements that are carried out monthly for several patients are given in Table 4.3.

In order to better annotate the results, a statistical analysis was also performed using the Statistical Package for the Social Sciences (SPSS) 15.0 Software. Mean value and standard deviations of the experimental data were calculated. Parametric and non-parametric correlations (Pearson's correlation, Kendal's Tau, and Spearman's Rho - detailed information about these statistical analyses can be found in Appendix A) were calculated between the mechanical vibration analysis of the right and left tibiae and BMD measured by DXA of femur and spine. Furthermore, descriptive statistics of the participants, in other words the vibration analysis results of right and left tibiae, minimum and maximum BMD values and T scores of femur and spine and their mean values are presented in Table 4.3.

Table 4.2 – Result of DXA and vibration measurements of participants

Participant	DXA Measurement				Vibration Measurement	
	Spine		Femur		Natural Frequency (Hz)	
	BMD	T scores	BMD	T scores	Right Tibia	Left Tibia
1	0.669	-4.4	0.582	-3.5	28	31
2	0.929	-2.3	0.631	-2.9	22.5	28.3
3	0.877	-1.7	0.674	-2.6	26	27.5
4	0.92	-2.3	0.707	-2.4	22.5	26.5
5			0.707	-2.4	28.1	26.2
6	1.143	-0.1	0.704	-2.4	23.5	25.5
7	0.918	-2.4	0.742	-2.2	31.5	31.5
8	0.936	-2.2	0.742	-2.2	30	25
9	0.944	-2.1	0.738	-2.2	25.5	26
10	0.79	-3.4	0.763	-2	25.3	25.5
11	1.215	0.1	0.764	-2	21	28
12	1.259	0.5	0.754	-2	26.1	29
13	0.934	-2.2	0.727	-1.9	27.2	28.6
14	0.807	-3.3	0.781	-1.8	24	26.5
15			0.792	-1.7	25.5	25.3
16	0.967	-1.9	0.833	-1.4	28	29.5
17	1.012	-1.6	0.837	-1.4	38.5	27.3
18	1.099	-0.8	0.834	-1.4	30	34
19	1.182	-0.2	0.84	-1.3	27.4	23.3
20	1.196	0	0.852	-1.2	29.1	31.3
21	1.286	0.7	0.853	-1.2	29.5	29.5
22	0.761	-2.6	0.801	-1.2	23.2	22.5
23	0.946	-2.1	0.866	-1.1	30.5	31.5
24	1.131	-0.6	0.87	-1.1	27.9	28.7
25	1.059	-0.7	0.865	-1.1	28	29.5
26	1.524	2.7	0.879	-1	29.2	27
27	1.414	1.8	0.896	-0.9	27.8	33.5
28	1.287	0.7	0.902	-0.8	26.6	25.6
29			0.911	-0.7	27.5	28
30	1.246	0.4	0.933	-0.6	32.5	27.2
31	1.432	1.9	0.931	-0.6	28	29.9
32	0.991	-1.7	0.955	-0.4	34	27.5
33	1.117	-0.7	0.953	-0.4	34.5	29
34			0.952	-0.4	26.2	29.6
35	1.114	-0.7	0.966	-0.3	25.5	24
36	0.847	-2.9	0.979	-0.2	33.5	29
37	1.136	-0.32	0.973	-0.2	28	28.5
38	1.24	0.3	1.015	0.1	32	37
39	1	-1.7	0.962	0.3	35	28
40	1.313	0.9	1.042	0.3	30.5	30
41	1.279	0.7	1.117	1	31	33
42	1.109	-0.8	1.145	1.2	27	30.5

Table 4.3 - Result of DXA and vibration measurements of several participants for intra observer reliability

Participant	Vibration Measurements (Hz)			
	Measurement 1		Measurement 2	
	Right Tibia	Left Tibia	Right Tibia	Left Tibia
6	23.5	25.5	24.7	24.5
27	27.8	33.5	28.6	33.5
28	26.6	25.6	24.8	25.0
34	26.2	29.6	25.2	28.7

Table 4.4 - Descriptive statistics of participants (N_{total} = 42 participants)

	Min. Values	Max. Values	Mean Values
BMD values of Spine (g/cm²)	0.669	1.524	1.080
T values of Spine	-4.400	2.700	-0.922
BMD values of Femur (g/cm²)	0.582	1.145	0.852
T values of Femur	-3.500	2.000	-1.100
Natural Frequency of Right Tibia (Hz)	21.0	38.5	28.3
Natural Frequency of Left Tibia (Hz)	22.5	37.0	28.4

The participants in this medical survey were categorized into two groups according to their T scores, which show the state of the osteoporosis obtained from their BMD values. One of the groups consists of participants with osteopenia and osteoporosis whereas the other group includes only

normal (healthy) people. Figure 4.4 and Figure 4.5 shows the change of natural frequency with the health status of the participants according to the measurements obtained from each participant's femur.

In order to assess the extracted and measured data, both parametric and non-parametric statistical analysis were performed between the BMD and natural frequency values of tibia measured.

For linear dependence of variables, Pearson's correlation was applied and Pearson product-moment coefficient was found as 0.455 with a significance value of 0.01, and 0.327 with a significance value of 0.05 for the BMD values of femur and natural frequency values of right and left tibia, respectively.

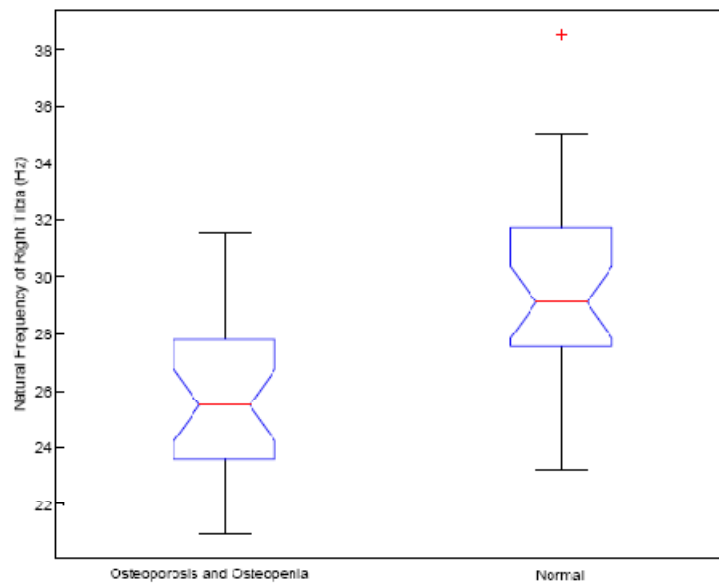


Figure 4.4 - Natural frequency (right tibia) vs. state of osteoporosis (femur)

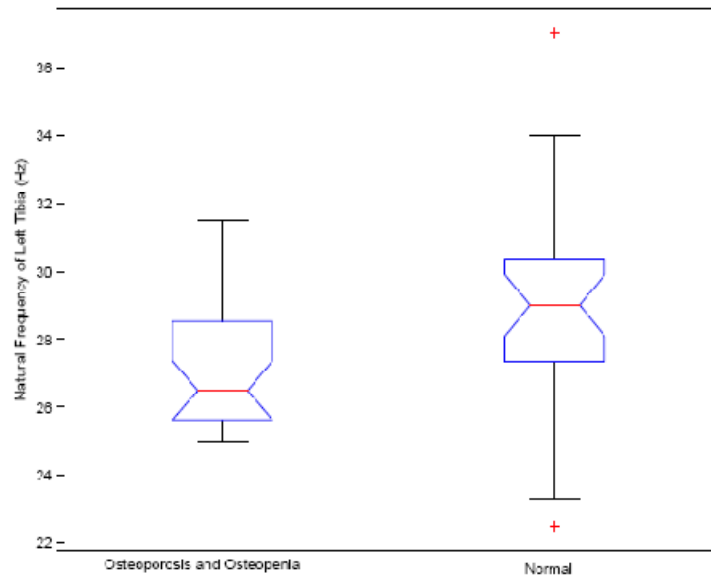


Figure 4.5 - Natural frequency (left tibia) vs. state of osteoporosis (femur)

For non-parametric analysis, in order to investigate the correspondence degree of the measured data Kendall's Tau, and in order to describe the relation of two variables by an arbitrary monotonic function Spearman's Rho correlations are used. Kendall's Tau rank correlation coefficient is obtained as 0.332 with a significance value of 0.01, and 0.222 with a significance value of 0.05 for the BMD values of femur and natural frequency values of right and left tibiae, respectively. Similarly, Spearman's rank correlation coefficient is found as 0.498 with a significance value of 0.01 and 0.331 with a significance value of 0.05 for the BMD values of femur and natural frequency of right and left tibiae, respectively.

Statistical analysis reveals that although there exists no correlation between the BMD values of spine and natural frequency values of tibiae, there is

both parametrical and non-parametrical correlation between the BMD values of femur and natural frequency values of right and left tibiae.

Figure 4.6 to Figure 4.9 show the change in natural frequency of the right and the left tibia with BMD measurements obtained from the femur and the spine.

As a conclusion, although the statistical analysis performed in this section suggests that there are both parametric and non-parametric correlations between the natural frequency values of tibia (both for the left and right tibiae) and BMD values of femur, these correlations are not very strong considering the correlation coefficients. In other words, the linear relationship of the variables is not so good. Furthermore, the box plots in Figure 4.4 and Figure 4.5 show that there is no definite discrimination between the values to diagnose the osteoporosis or progressing osteoporosis.

In a large portion of the studies in the literature, the main aim is to find a valid and an effective correlation between the natural frequencies and the BMD values of bone. Therefore, they mainly focus on the change of natural frequency with respect to the BMD values [5, 13, 17, 21, 27]. However, this study shows that natural frequency is not an appropriate parameter to diagnose osteoporosis, but it may be used to diagnose progressing osteoporosis by using an appropriate test set-up that provides the boundary conditions to be same for all experiments. Furthermore, these findings are in agreement with the results found in the study of Cornelissen et al [16]. In that study, the correlation between the natural frequencies of

the ulna and the degree of osteoporosis was also investigated and it was concluded that in the diagnosis of osteoporosis, the use of natural frequency alone has a limited value.

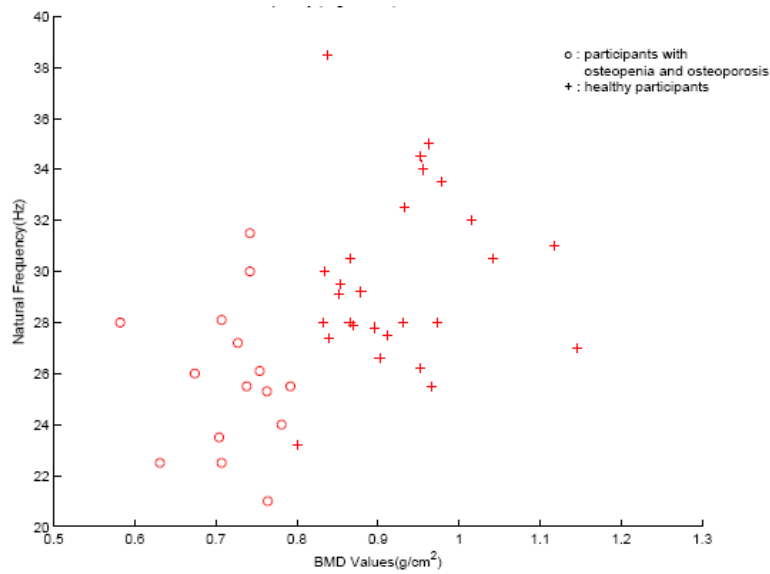


Figure 4.6 - Natural frequency (right tibia) vs. BMD values of the femur

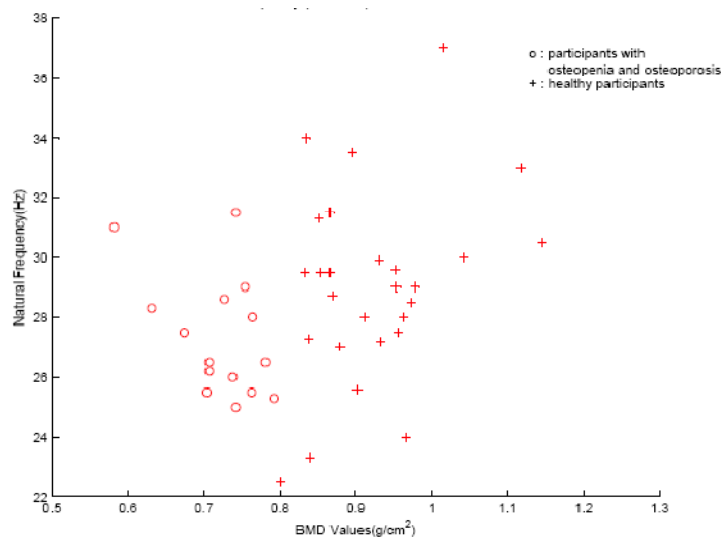


Figure 4.7 - Natural frequency (right tibia) vs. BMD values of the spine

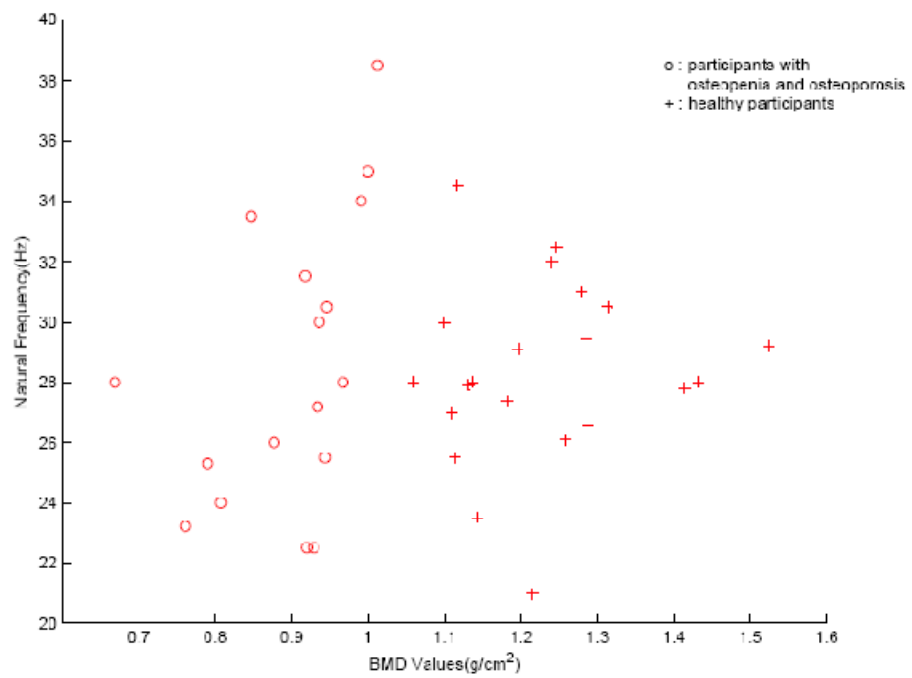


Figure 4.8 - Natural frequency (left tibia) vs. BMD values of the femur

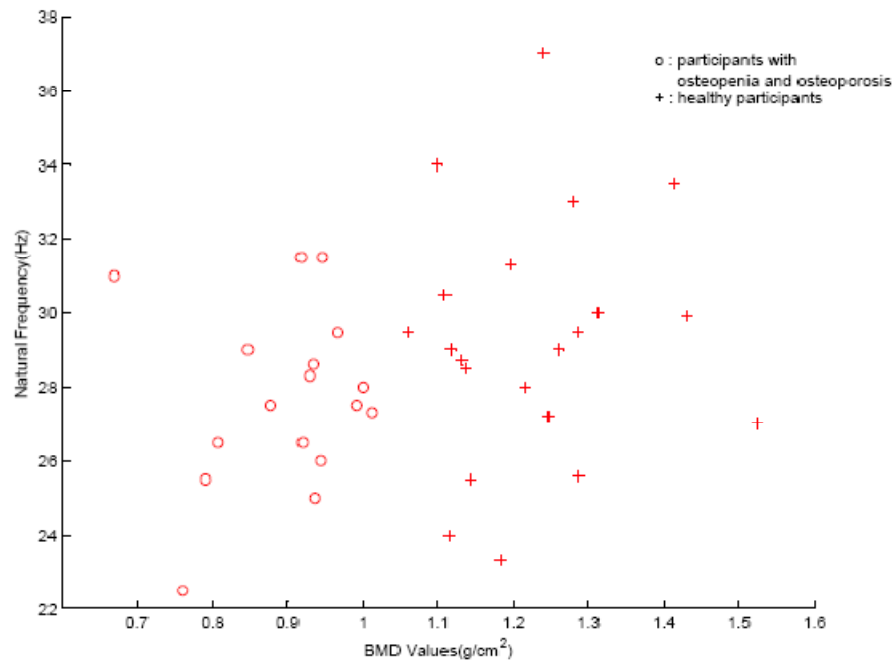


Figure 4.9 - Natural frequency (left tibia) vs. BMD values of the spine

However, since the natural frequencies of tibia depend on the geometry (thus the size) of the bone, more accurate results are expected if the extracted natural frequencies are normalized according to the length of tibia of each participant. Unfortunately, only the height information of the patients is available. Using this information approximate tibia lengths are obtained and the cluster plots given in Figure 4.6 to Figure 4.9 are plotted again. Since the formulations about the relation between body height and tibia length given in literature are not so accurate, again no definite discrimination between the natural frequencies corresponding to the state of osteoporosis exists. On the other hand, more realistic results are expected if the tibia length information is known.

On the other hand, it is seen in Chapter 3 that there is a considerable difference in the loss factors of fresh-frozen and dry tibiae which seems a strong parameters that can be used for diagnosing purposes. However, from the in vivo experiments it is seen that, its contribution to the overall damping of the system (tibia, fibula, muscles, skin and joints) is very little. Therefore, with the in vivo experimental methods used in this study, it does not seem practical to detect the changes in the loss factor of tibia from the extracted modal loss factor of the overall system. As a result, it can be concluded that due to the highly damped nature of the FRFs obtained from in vivo experiments, the extracted loss factors are not so reliable in diagnosing osteoporosis or progressing osteoporosis.

CHAPTER 5

THREE DIMENSIONAL MODEL OF HUMAN TIBIA

In Chapter 3, it is observed that loss factor which is an inherit property of bone can be used in identifying the condition of bone from the work carried out on fresh frozen tibiae. However, there are several difficulties in extracting the structural dynamic properties by in vivo modal analysis. For example, the damping introduced by the soft (skin and muscles) tissues makes the modal identification procedure highly difficult. Furthermore, this method can only be applied to superficial bones such as tibia and radius.

Finite element models (FEM) of bones have been widely used as a method to understand the dynamics of bones, since it can also be used for deep bones as well as the superficial ones. This method is widely used to investigate the stress and strain distributions of bone under different loading conditions or around the implants. Also, over the last two decades, as described in Chapter 1, these FEM of bones are used to determine the vibration behavior of bones.

In this study, the main objective is to construct a proper FEM of human tibial bone using the Computer Tomography (CT) scan data, that matches the experimental results obtained from modal analysis in Chapter 3. Furthermore, differences between using isotropic parameters and orthotropic parameters are investigated. Then, the influence of elastic

modulus, the size of the bone and the boundary conditions on the natural frequency of tibial bone are investigated.

5.1 Finite Element Modeling of Human Tibia

In order to construct FEM of human tibial bone, first of all CT scan images of tibia were taken. The CT scan images of a tibia specimen were taken with slice intervals of 0.6 millimeters. '*3D-Doctor Modeling and Measuring Software*' was used in order to obtain 3D model of the tibial bone from the slice images. A sample slice image that belongs to the middle section of the diaphysis of tibia is shown in Figure 5.1. From this image, boundaries for both cortical (hard outer layer of bones) and cancellous (spongy part of bones) sections were drawn. Therefore, a 3D model of a tibia specimen was developed by joining the boundary lines drawn for each slice image.

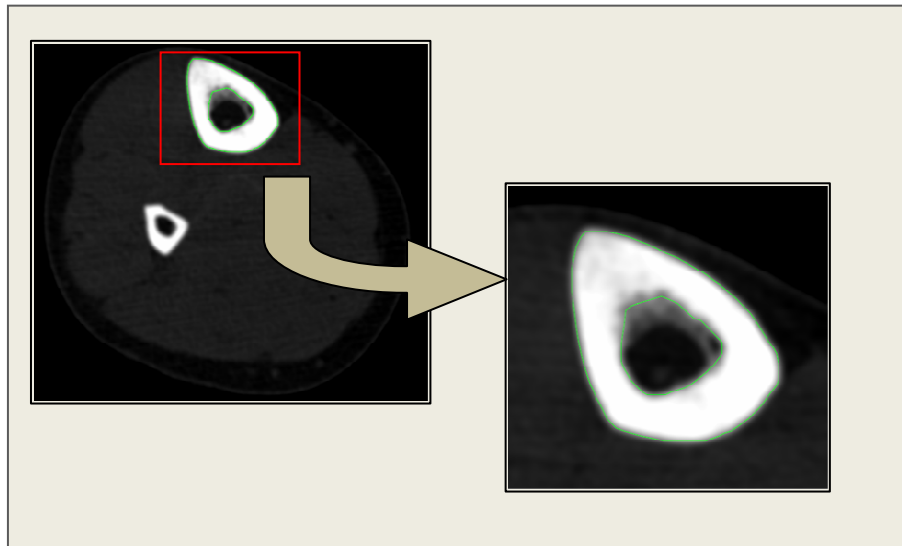
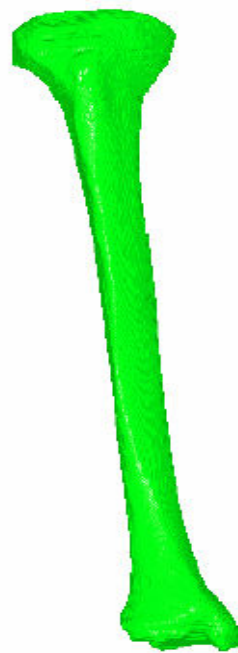


Figure 5.1 – A sample of slice image of CT scan of a tibia specimen

However, the constructed model using '*3D-Doctor Modeling and Measuring Software*' consisted of only the surfaces and was highly detailed and complex such that the complete system consists of 331116 triangles and 165564 nodes. Therefore, the developed 3D model should be simplified and smoothed for further analysis. For this purpose, '*Autodesk® 3Ds Max®*' software was employed. By using this software, the surface properties of the model were decreased by 99 % of its original properties. Figure 5.2 shows the complex and simplified model obtained from '*3D-Doctor Modeling and Measuring Software*' and '*Autodesk® 3Ds Max®*', respectively.



(a) Complex model



(b) Simplified model

Figure 5.2 – Complex and simplified models of human tibia

The main advantage of this simplification process is that the roughness of the obtained surfaces was resolved so that the analyses could be carried out more easily and in a short of time. The result of this process can also be seen clearly in Figure 5.2.

However, in order to obtain a proper FEM of human tibia, a solid model should be obtained from the simplified model formed of only surfaces. Therefore, following this simplification procedure, solid model was obtained using '*Mechanical Desktop 2005 Software®*' through surface stitching. As a result of these procedures, a FEM of human tibia was created from the raw CT scan images.

In the FE analysis '*MSC Patran®*' and '*MSC Nastran®*' software were employed. Figure 5.3 shows the meshed structure of tibia (composed of 10 node tetrahedral elements) used in this study. In the first step of this study, the analyses were conducted in two parts. The first one is carried out with isotropic bone parameters which imply that the material properties of bone are uniform in all directions. On the other hand, in the second part of the analyses orthotropic bone properties are used in which the material properties of bone depend on directions. Thereby, it is aimed to constitute a suitable model of human tibia and investigate how elastic modulus, size and boundary condition of bone affects the resonance frequencies.

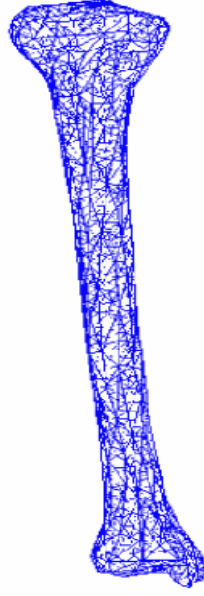


Figure 5.3 – 3D meshed model of human tibia

5.1.1 Isotropic Parameter Analysis

Although material properties of bone vary both in longitudinal, radial and circumferential directions, for simplicity isotropic material properties are used for the compact and spongy parts of the bone in this analysis. The elastic modulus values of tibia are selected within the limits specified in literature [33, 40, 41]. The density of bone is obtained from the CT scan data. Following relation is used for this purpose,

$$\rho_{bone} = (0.748 \times 10^{-3})CT + 1.136 \quad (5.1)$$

where CT is the average of data obtained from the CT scan of tibia and the constants in the given equation are related with the calibration of the computed tomography machine.

In Table 5.1, material properties used in the FE analysis are given. The elastic modules of compact and spongy parts of the tibia were selected such that the first two natural frequencies obtained from the experimental study (for tibia specimen 1) and from the FE analysis are identical.

Table 5.1 – Isotropic material properties of tibia used in the FEM analysis

Bone Properties	
Elastic Modulus of Compact Bone (GPa)	15
Elastic Modulus of Spongy Bone (GPa)	6
Density of Compact Bone (kg/m ³)	2.41E+03
Density of Spongy Bone (kg/m ³)	1.10E+03
Poisson's Ratio of Compact Bone	0.3
Poisson's Ratio of Spongy Bone	0.3

Figure 5.5 and Figure 5.6 show the mode shapes and natural frequencies for the first two modes obtained through 'MSC Patran and MSC Nastran' software. As seen from these figures both modes are bending modes and the natural frequencies of the first and second modes are 320 Hz and 413 Hz, respectively.

To verify the accuracy of the FEM of tibia, natural frequencies for the third mode obtained from in vitro experiments and FE analyses are compared. Therefore, in order to extract third natural frequency of tibia, FRFs measured for a frequency range of 0-1000 Hz was used. Figure 5.4 shows the FRF measurement of tibia specimen 1 for the frequency range specified. From the in vitro experiments and FE analysis, it was found that the natural frequencies for the third mode are 942 Hz and 921 Hz, respectively. As seen, the results found by FE analysis are in good agreement with those obtained from in vitro experiments.

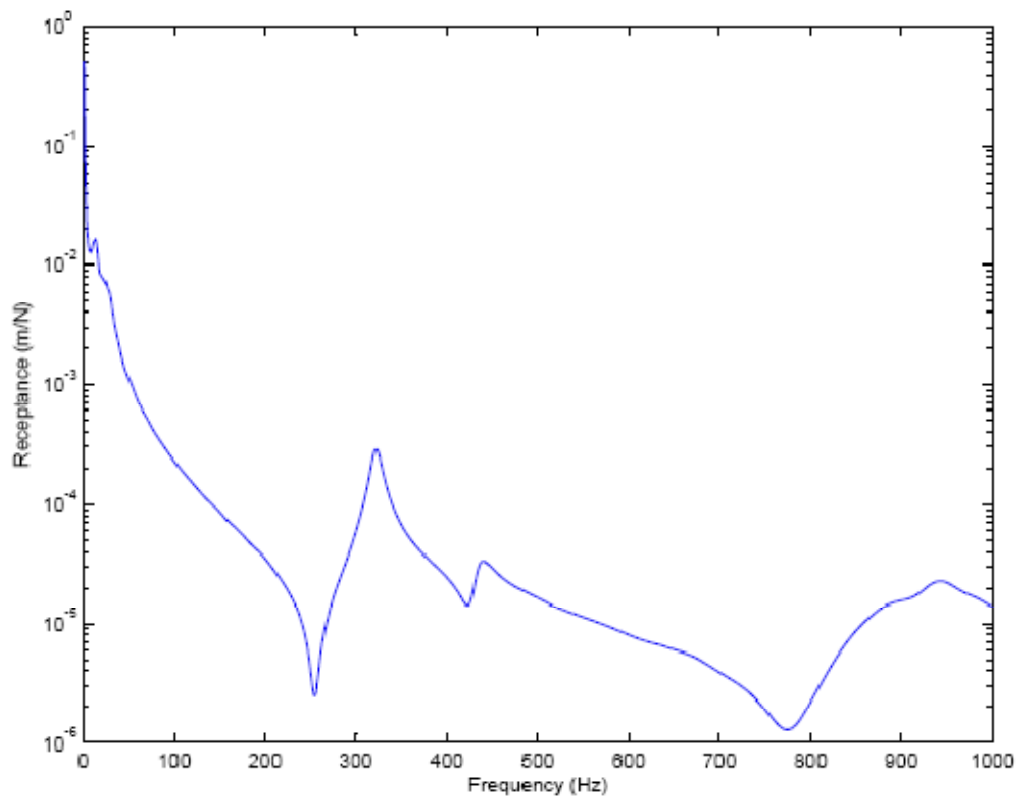


Figure 5.4 – FRF plot of tibia specimen 1

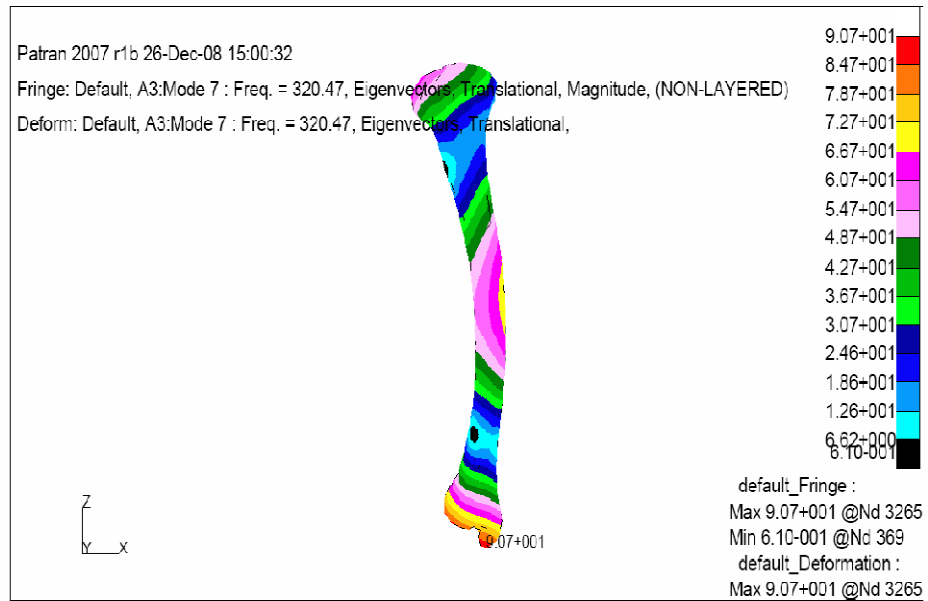


Figure 5.5 – First mode shape and natural frequency of tibia (isotropic case)

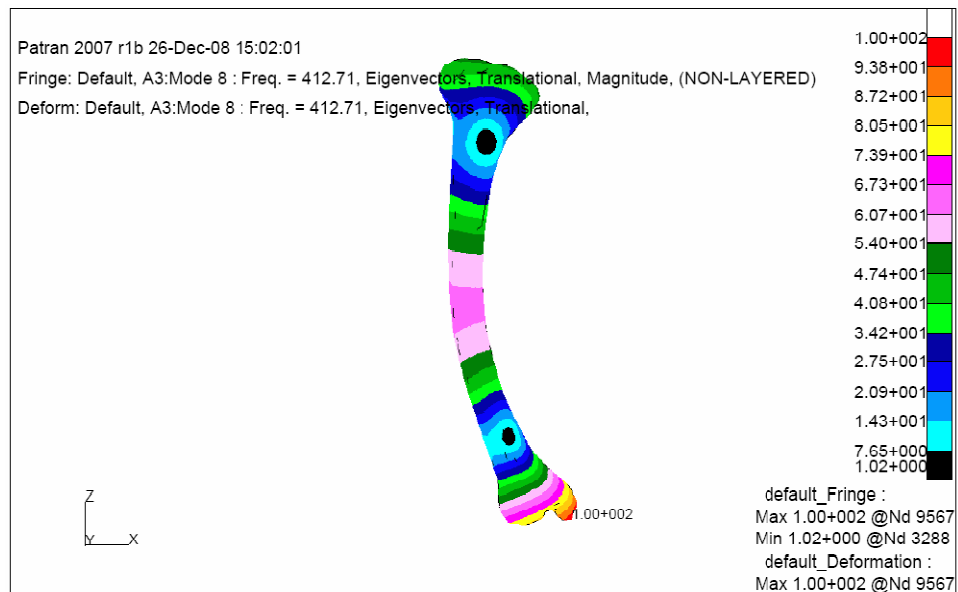


Figure 5.6 – Second mode shapes and natural frequency of tibia (isotropic case)

5.1.2 Orthotropic Parameter Analysis

As mentioned in the previous part, the material properties of bone change in radial, circumferential and longitudinal directions. Therefore, more realistic results are expected by using orthotropic material properties. Table 5.2 shows the properties used in the FE analysis. Axial, radial and circumferential directions are represented by subscripts '33', '11', and '22' respectively. As in the previous section, the density of the compact and spongy parts of the bone are found using equation (5.1).

Table 5.2 - Orthotropic material properties of tibia used in the FE analysis

Bone Properties	
E_{11} (GPa)	9
E_{22} (GPa)	10
E_{33} (GPa)	15.7
G_{12} (GPa)	2.2
G_{13} (GPa)	3.3
G_{23} (GPa)	4.1
ν_{12}	0.45
ν_{13}	0.17
ν_{23}	0.21
Density of Compact Bone (kg/m ³)	2.41E+03
Density of Spongy Bone (kg/m ³)	1.10E+03

Figure 5.7 and Figure 5.8 show the mode shapes and natural frequencies for the first two modes obtained through '*MSC Patran and MSC Nastran*' software. As seen from these figures both modes are bending bones and the natural frequencies of the first and second mode are 321 Hz and 409 Hz, respectively.

As in the isotropic materials case, natural frequencies for the third mode obtained from in vitro experiments and FE analyses are compared in order to verify the accuracy of the FEM model. It was extracted from in vitro experiments that the natural frequency for the third mode of tibia is 942 Hz. From the FE model using orthotropic materials properties, the natural frequency for the third mode of tibia was found as 944 Hz. As seen, the results found by FE analysis are in good agreement with those obtained from in vitro experiments. Therefore, it can be said that considering the scope of our analysis both methods give very close results to each other and are in good agreement with the experimentally obtained data.

However, if the analysis is carried out for a frequency range that is much higher (above 1 kHz) or if the purpose of the model is different (i.e. for calculating stress), since the stiffness matrix of the system is more realistic, the model using orthotropic properties is expected to give more accurate results and will be needed to be used. For example, some mode shifts are observed in the FE analysis in the isotropic case. Hence, considering both simplicity and accuracy aspects for the scope of this study, the model using isotropic material parameters, is found to be sufficiently satisfactory for further studies.

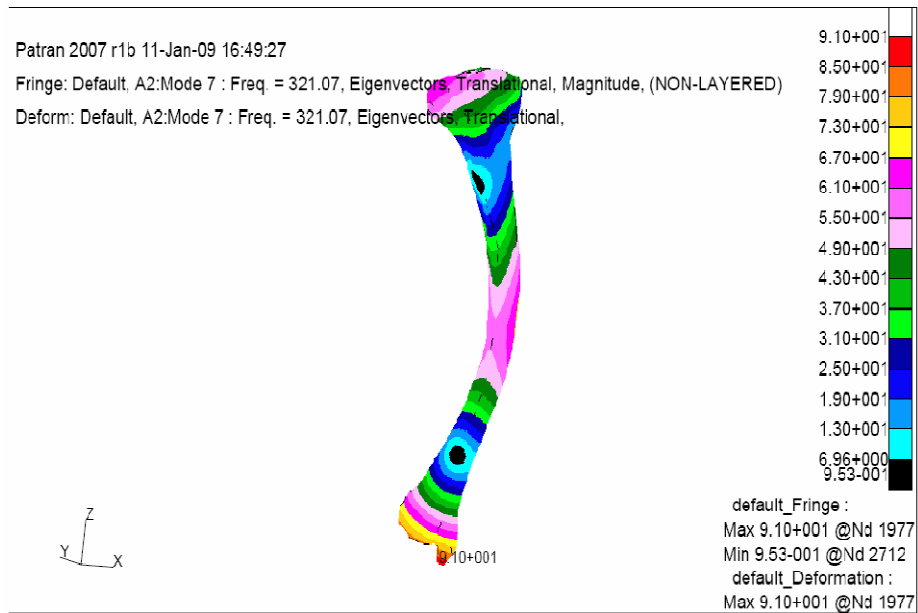


Figure 5.7 - First mode shape and natural frequency of tibia (orthotropic case)

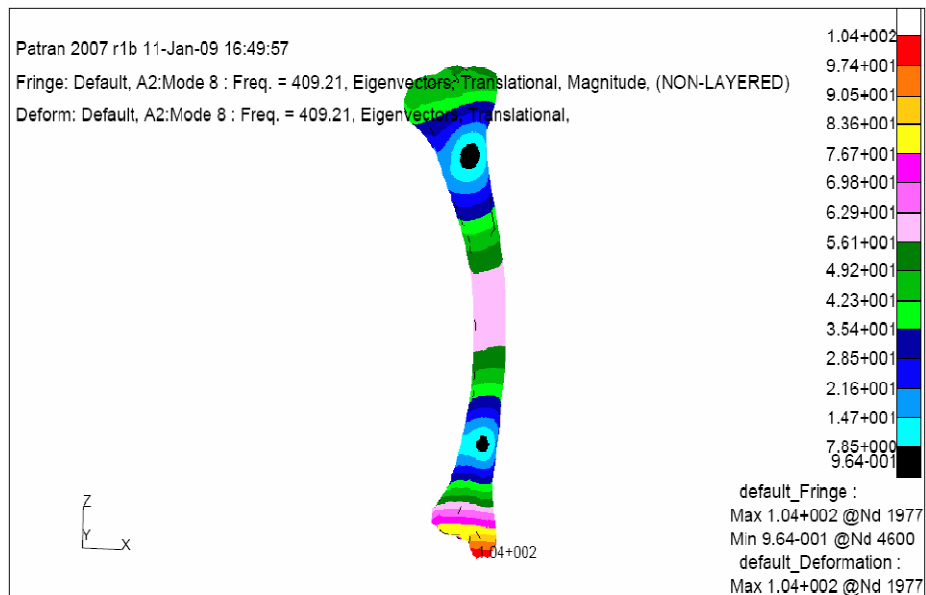


Figure 5.8 - Second mode shape and natural frequency of tibia (orthotropic case)

5.2 Effect of Elastic Modulus on Natural Frequency

In this stage of the study, it is aimed to observe the changes in natural frequency of tibial bone with the change in the elastic modulus of compact and spongy parts of the bone. For this purpose, firstly the effect in the change of the elastic modulus of the spongy part of the tibia on the natural frequency is investigated. The analyses are performed for 15 different cases. The unchanged material properties (elastic modulus of compact part of bone, density and Poisson's ratio of compact and spongy parts) are taken as given in Table 5.1. Table 5.3 shows the various cases considered and the corresponding natural frequencies found from the FE analysis.

Table 5.3 - Effect of elastic modulus of spongy part of tibia on natural frequencies of tibia

Cases	Elastic Modulus of Spongy Part of Tibia (MPa)	First Natural Frequency (Hz)	Second Natural Frequency (Hz)
1	300	319.6	411.5
2	350	319.7	411.8
3	400	319.9	412.0
4	450	320.0	412.2
5	500	320.2	412.3
6	550	320.3	412.5
7	600	320.5	412.7
8	650	320.6	412.9
9	700	320.7	413.1
10	750	320.9	413.2
11	800	321.0	413.4
12	850	321.1	413.6
13	900	321.3	413.7
14	950	321.4	413.9
15	1000	321.5	414.0

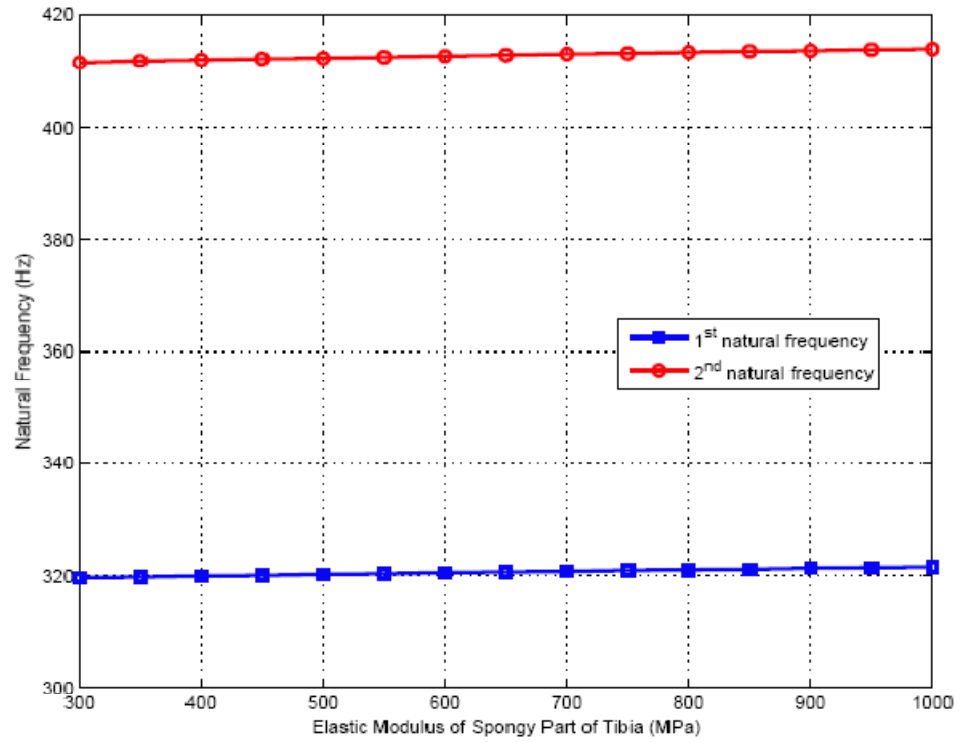


Figure 5.9 – Effect of elastic modulus of spongy part of tibia on natural frequencies of tibia

Secondly, the effect of changes in the elastic modulus of the compact part of the tibia on natural frequency is investigated. The analyses are performed for 31 different cases. The unchanged material properties (elastic modulus of compact part of bone, density and Poisson's ratio of compact and spongy parts) are taken as given in Table 5.1. Table 5.4 shows the various cases considered and the corresponding natural frequencies found from the FE analysis.

Table 5.4 - Effect of elastic modulus of compact part of tibia on natural frequencies of tibia

Cases	Elastic Modulus of Compact Part of Tibia (MPa)	First Natural Frequency (Hz)	Second Natural Frequency (Hz)
1	10000	262.3	337.8
2	10500	268.7	346.0
3	11000	274.9	354.1
4	11500	281.0	361.9
5	12000	287.0	369.6
6	12500	292.8	377.1
7	13000	298.6	384.5
8	13500	304.2	391.8
9	14000	309.7	398.9
10	14500	315.1	405.8
11	15000	320.5	412.7
12	15500	325.7	419.5
13	16000	330.9	426.1
14	16500	336.0	432.6
15	17000	341.0	439.1
16	17500	345.9	445.4
17	18000	350.7	451.7
18	18500	355.5	457.9
19	19000	360.3	464.0
20	19500	364.9	470.0
21	20000	369.5	475.9
22	20500	374.1	481.8
23	21000	378.6	487.6
24	21500	383.0	493.3
25	22000	387.4	498.9
26	22500	391.8	504.5
27	23000	396.1	510.1
28	23500	400.3	515.5
29	24000	404.5	521.0
30	24500	408.7	526.3
31	25000	412.8	531.6

Figure 5.10 shows the effect of elastic modulus of compact part of tibia on natural frequencies of tibia. For the given set of elastic modulus values, the total percentage change in elastic modulus of compact part of tibia is 100 %. This change corresponds to a change of 47 % both in the first and second natural frequencies of tibia. If the tibia is considered as a beam element composed of only compact part of tibia, i.e. if the spongy part of tibia is neglected, its natural frequency will be proportional with the square root of its elastic modulus since the length, mass and moment of inertia of tibia are constant in this analysis. Results found using this approximation for a few selected cases are listed in Table 5.5.

Table 5.5 - Effect of elastic modulus of compact part of tibia on natural frequencies of tibia (by approximation)

Cases	Elastic Modulus of Compact Part of Tibia (MPa)	First Natural Frequency (Hz)	Second Natural Frequency (Hz)
1	10000	261.7	337.0
5	12000	286.7	369.1
20	19500	365.4	470.6
31	25000	413.8	532.8

Therefore, two inferences can be made from these results. Firstly, the major changes in natural frequencies occur due to the changes in elastic modulus of compact (cortical) part of the tibia. Secondly, it can be said that the first two modes of tibia are affected in the same manner. Moreover, these changes can be represented by straight lines shown in Figure 5.10.

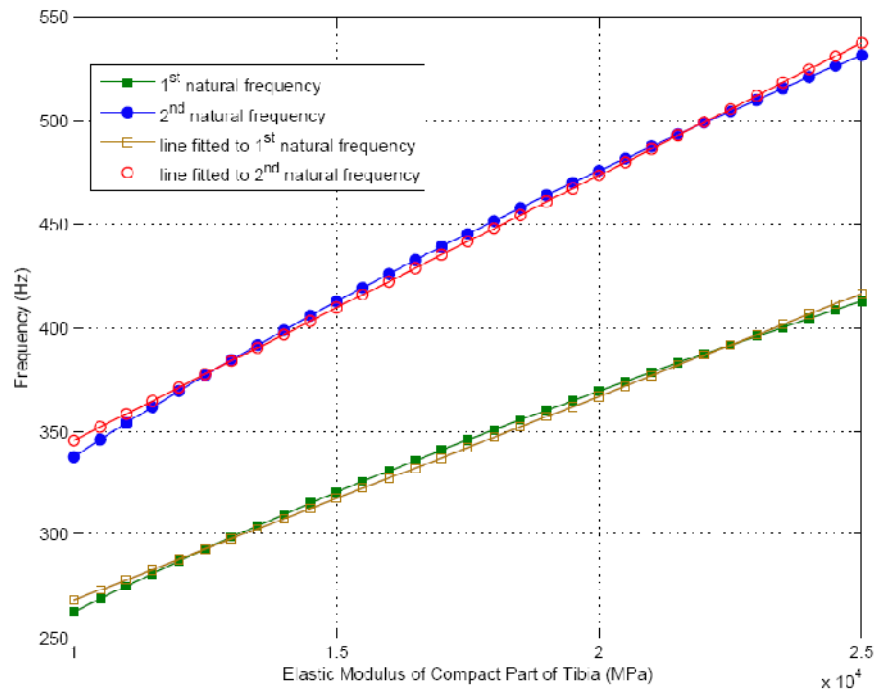


Figure 5.10 - Effect of elastic modulus of compact part of tibia on natural frequencies of tibia

5.3 Effect of Bone Size on Natural Frequency

In order to develop a new method to diagnose osteoporosis or progressing osteoporosis, it is needed that this new method must be applicable to all patients. However, there are a lot of diversities among people. For example, the bone size is one of the important factors to be kept in mind. Therefore, the next step of this study is to observe the effect of size of tibia on its natural frequencies.

The analyses were performed for 9 different cases with acceptable bone sizes. Material properties given in Table 5.1 were employed. Table 5.6

shows the various cases considered in the analyses and the corresponding natural frequencies found from the FE analyses.

Table 5.6 – Effect of size of tibia on natural frequencies of tibia

Size (Scale Factor)	Length (mm)	1 st Natural Frequency (Hz)	2 nd Natural Frequency (Hz)
1.20	414	267	344
1.15	397	279	359
1.10	380	291	375
1.05	362	305	393
1.00	345	320	413
0.95	328	337	434
0.90	311	356	459
0.85	293	397	511
0.80	276	401	516

The results show that the size of the tibia is an important parameter that affects its natural frequencies. As seen from Table 5.6, the size of tibia changes from 414 mm to 276 mm which corresponds to a change of 40 % in size and both the first and second natural frequencies of tibia are affected linearly. Figure 5.11 shows the effect of the size of tibia on natural frequencies of tibia more clearly.

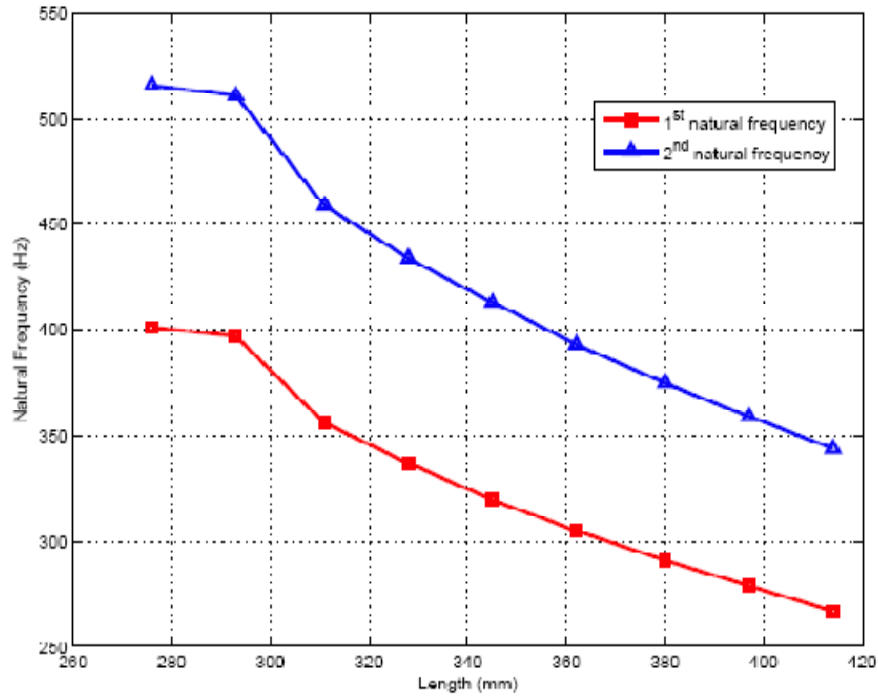


Figure 5.11 - Effect of size of tibia on natural frequencies of tibia

Therefore, it can be concluded that in the assessment of natural frequencies of tibia, it is not appropriate to make a comparison considering all patients. In fact, the analogy must be carried out for the patients having the same bone lengths. However, the change in natural frequencies of tibia can be an indicator for progressing osteoporosis for a specific patient. In other words, every patient must be evaluated individually if the natural frequency information of bones is used for diagnosis purposes.

5.4 Effect of Boundary Condition on Natural Frequency

In the previous part, it is denoted that diagnosis of progressing osteoporosis can be detected using the natural frequency information of bones only for a specific patient due to the size effect of bones. However,

boundary condition in which the experiments are carried out, is another important factor that needs to be considered as well, since the natural frequencies of a structure are highly sensitive to the boundary conditions. Therefore, in this last part of this FE study, the effect of boundary conditions on natural frequencies of tibia is investigated.

Up to this point, all the FE analyses performed are done with a free-free boundary condition. In order to see the effect of boundary condition, a sample analysis is performed where displacement of the proximal and distal epiphysis of tibia are fixed whereas the rotation of these parts are not restricted.

Figure 5.12 and Figure 5.13 show the first two the mode shapes and natural frequencies of the simply supported tibia obtained through '*MSC Patran and MSC Nastran*' software. As seen from these figures both modes are bending bones and the natural frequencies of the first and second modes are 681 Hz and 845 Hz, respectively. Table 5.7 shows the two cases considered in the analyses and the corresponding natural frequencies for the first two modes found from the FE analyses.

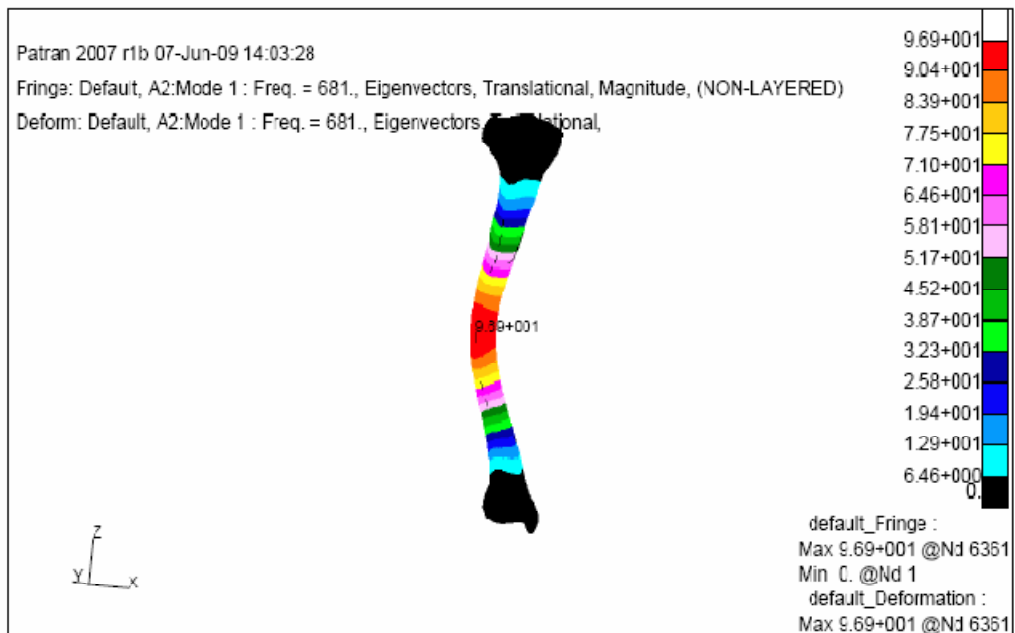


Figure 5.12 - First mode shape and natural frequency of tibia (simply supported case)

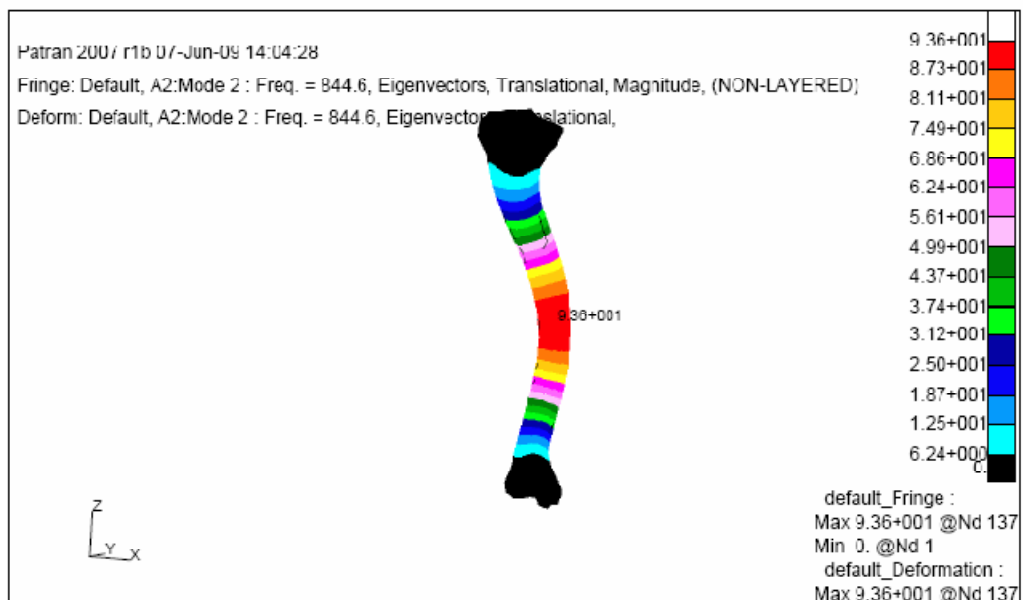


Figure 5.13 - Second mode shape and natural frequency of tibia (simply supported case)

Table 5.7 - Effect of boundary condition of tibia on natural frequencies of tibia

Boundary Condition	Natural Frequency (Hz)	
	1 st Mode	2 nd Mode
Free-Free	320	413
Simply Supported	681	845

It is obvious that the boundary condition, in which the experiments are carried out, has a significant effect on natural frequencies of tibia. Therefore, it can be concluded that the change in natural frequencies of tibia can be used as an indicator for progressing osteoporosis for a specific patient only if the experiments are carried out exactly with the same boundary conditions.

CHAPTER 6

DISCUSSION AND CONCLUSION

In this thesis, an alternative method for diagnosing osteoporosis or progressing osteoporosis is investigated by looking for a relation between the BMD values measured by DXA and the extracted structural modal parameters obtained from vibration experiments. For this purpose, firstly a preliminary in vitro study on human tibia obtained from fresh cadavers and then an in vivo study, a large scale medical survey among patients are performed. Also, as an alternative method, in order to see the effect of various parameters, such as elastic modulus of tibia, size of tibia and boundary condition in which the experiments are carried out, on structural modal parameters of tibia, FE analyses are employed.

6.1 Extracting Modal Parameters of Tibia

As mentioned in Chapter 1, previous researches in literature generally focus on the change of natural frequency alone in the diagnosis of osteoporosis [e.g. 4, 5, 21]. However, using natural frequency is not a convenient and easy approach for diagnosing purposes since there are various factors affecting this parameter such as the size and the geometry of the bone or the boundary conditions of the experiments. Therefore, it is aimed at the beginning of this research to find a new approach for diagnosing this bone disease by examining a more stable parameter such as loss factor. Since it is an inherit property of a bone, it is believed that

extracting loss factor of tibia, and studying the changes in this parameter may yield an alternative method to diagnose osteoporosis or progressing osteoporosis.

From the in vitro experiments, two important inferences can be derived. Firstly, although both fresh-frozen tibiae specimens have similar mineral content, the vibration experiments performed on them show that there is a considerable difference both in the extracted loss factors and natural frequencies of tibia. The difference in the natural frequency values between the specimens is expected due to the reasons mentioned. However, the difference between the loss factor values leads to the conclusions that bone mineral density measured by DXA is not the only parameter that affects structural modal parameters of tibia. Consequently, these measurements may imply that the collagen structure of bone also has a significant effect on modal loss factors of tibia. In other words, it can be said that bone mineral density measured by DXA seems to be an indefinite method to predict mechanical bone strength, as also stated in references [4, 7-9]. Secondly, comparing the results between the fresh frozen and dry tibiae, considerable differences are observed both in the natural frequencies and loss factors extracted. The dissimilarities both in BMD values and collagen structure of tibiae specimens constitute the differences in modal parameters of tibia. Furthermore, observing considerable low loss factor values for dry tibia specimen with respect to the fresh-frozen ones leads to the conclusion that loss factor of bones can be a strong indicator of the mineral content and the collagen of the bone. Yet, in order to generalize this conclusion, more tests should be performed on fresh-frozen tibiae.

In the second part of the study, a large scale medical survey is performed. From these in vivo experiments, several important inferences can be made. First of all, due to the effect of the soft tissues (skin and muscle tissues) surrounding the tibia, highly damped FRFs are measured. Consequently, even if a change occurs in the loss factor of tibia, it does not seem it is easy to detect this change; because the contribution of the loss factor of tibia to the overall damping of the system (skin, muscle, fibula and joints) is very little and the overall damping of the system cannot be identified very accurately. As a result, it can be said that the extracted loss factors from in vivo measurements are not so reliable in diagnosing osteoporosis and progressing osteoporosis due to the highly damped nature of the system.

Also, from the in vivo experiments, it is found that both parametric and non-parametric correlations exist between the BMD values of femur and natural frequencies of tibia. However, these correlations are weak and therefore have a limited value in generalized diagnosis of osteoporosis and progressing osteoporosis. Yet, the results obtained in Chapter 5 imply that if the boundary conditions of the experiments are held in the same way in each test, and the natural frequencies measured are compared for a specific patient or patients having similar bone geometry, natural frequency approach can be employed as an indicator for progressing osteoporosis.

6.2 The FE Model of Tibia

A three dimensional human tibial bone is modeled using isotropic and orthotropic bone material properties. Results obtained from both of the

approaches are in good agreement with the results obtained from experimental studies. The model using isotropic material properties is preferred for further analysis since it is easier to use.

In the first part of the FE analysis, the effects of elastic modulus of compact and spongy parts of tibia on its natural frequencies are investigated. It is observed that the major effect comes from the compact part of the bone whereas the changes in spongy part of tibia have negligible effect on the natural frequencies found. Also, results show that the tibia can be represented as a beam element consists of only compact part of tibia. In other words, it seems that from the natural frequency information obtained from vibration experiments, the elastic modulus of compact part of tibia can be predicted. Yet, more experiments should be carried out in order to generalize this conclusion.

In the second and third parts of the FE analysis, the factors (size and boundary condition) affecting the natural frequencies of tibia are examined. It is seen that both factors have high influence on the measured natural frequencies. Therefore, it is concluded that in the natural frequency approach, it is not logical to make general inferences about the state of osteoporosis of a patient as in the case of T and Z scores obtained from BMD information of bones. However, if the experiments are carried out exactly with the same boundary conditions and the results of patients are examined individually, the change in natural frequencies of tibia seems to be a possible and practical method that can be used to detect progressing osteoporosis

6.3 Suggestions on Future Work

In this thesis, it is aimed to perform experiments as much as possible in order to generalize the conclusions arrived in study. However, due to the difficulties in acquiring fresh tibia specimens and also the difficulties encountered in in vivo experiments (such as taking ethical permissions and voluntary participation form), the number of experiments performed is lower than that it is aimed. Hence, more experiments should be carried out for generalization of the same conclusions. Also, other experimental techniques should be employed.

Also, for the case of in vivo experiments, DXA measurements are made according to the common procedures (i.e. measurements are performed only for the spine and femur). Since the BMD of tibia is not measured, it is assumed that the mineral loss in tibia is similar to the loss in femur. However, more realistic results can be obtained by performing a large medical survey in which both DXA and vibration measurements are done on tibia. Also, since the FRFs obtained from in vivo experiments show nonlinear behavior, nonlinear modal identification procedures should be employed.

Lastly, in FE analysis, the geometrical differences of tibia are investigated only for different scales of the tibia modeled. However, constituting different FE models of tibia from several CT scan images obtained from different patients will improve the accuracy of the results.

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APPENDIX A

STATISTICAL ANALYSIS

A.1. Pearson Product Moment Correlation Coefficient

In statistics, Pearson correlation (sample correlation coefficient) is used to obtain the linear dependence between two variables. The Pearson correlation coefficient can be calculated using the following formula,

$$r = \frac{\sum_{i=1}^N (x_i - \bar{x})(y_i - \bar{y})}{(N-1)\sigma_x\sigma_y} \quad (\text{A.1. 1})$$

where $N, \bar{x}, \bar{y}, \sigma_x, \sigma_y$ are number of data points, mean value of sample x, mean value of sample y, standard deviation of sample x and standard deviation of sample y, respectively.

The Pearson correlation coefficient ranges from -1 to 1. The value of '-1' means that two variable are inversely linear whereas the value of '1' means that there is a perfect linear relationship between two variables. On the other hand, the value of '0' implies that there is no relationship between the selected two variables.

A.2. Kendall Tau Rank Correlation Coefficient

Kendall tau rank correlation coefficient is a non-parametric statistical analysis used in order to measure the degree of correspondence between

two rankings, i.e. two set of data x_i and y_i expressed as (x_i, y_i) . It can be calculated by the following equation,

$$\tau = \frac{n_c - n_d}{\frac{1}{2}N(N-1)} \quad (\text{A.2. 1})$$

where n_c , n_d and N are concordant pairs, discordant pairs and total number of pairs. Concordant pairs are pairs of data sets where,

$$\text{sgn}(x_1 - x_2) = \text{sgn}(y_1 - y_2) \quad (\text{A.2. 2})$$

Similarly, discordant pairs are pairs of data sets where,

$$\text{sgn}(x_1 - x_2) = -\text{sgn}(y_1 - y_2) \quad (\text{A.2. 3})$$

Again, Kendall tau rank correlation coefficient's value ranges from -1 to 1. The value of '-1' means that the two sets of data selected are completely in disagreement whereas the value '1' means that the selected two sets of data are perfectly in agreement with each other. On the other hand, the value of '0' implies that the two sets of data selected are completely unrelated.

A.3. Spearman's Rank Correlation Coefficient

Spearman's rank correlation coefficient is also a non-parametric correlation used in order to see how well an arbitrary function can be fitted between two variables. It can be calculated by the following formula,

$$\rho = \frac{N \left(\sum_{i=1}^N x_i y_i \right) - \left(\sum_{i=1}^N x_i \right) \left(\sum_{i=1}^N y_i \right)}{\sqrt{N \left(\sum_{i=1}^N x_i^2 \right) - \left(\sum_{i=1}^N x_i \right)^2} \sqrt{N \left(\sum_{i=1}^N y_i^2 \right) - \left(\sum_{i=1}^N y_i \right)^2}} \quad (\text{A.3. 1})$$

where N is the total number of data points. Again, Spearman's rank correlation coefficient's value obtained from equation (A.3.1) ranges from -1 to 1. The value of '-1' means that a perfect negative correlation exists whereas the value '1' means that a perfect positive correlation exists between the selected data sets. On the other hand, the value of '0' implies that the two sets of data selected are completely unrelated and thus no correlation exists between them.

APPENDIX B.

CONFERENCE PAPER

Proceedings of the 25th International Modal Analysis Conference, Orlando, Florida, February 19-22, 2007

Measuring Structural Dynamic Properties of Human Tibia by Modal Testing

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Nomenclature

α_{jk}	: receptance
sA_{jk}	: s^{th} modal constant
ω_s	: s^{th} natural frequency
ω	: frequency
η_s	: s^{th} loss factor

ABSTRACT

Identifying structural dynamic properties of human bones seems to be a promising technique in diagnosing metabolic bone diseases such as osteoporosis and in monitoring fracture healing. The major aim of this work is to observe the changes in the "loss factor" of human tibia with decreasing mineral density which is known as the main indicator of osteoporosis. In this preliminary stage of the study, the experiments are carried out in vitro, on tibia specimens obtained from fresh-frozen cadavers, as well as on dry tibia. The specimens that are placed on a soft rubber to have free-free boundary conditions are excited by an impact hammer and the response is measured using an accelerometer. The measured frequency response functions (FRFs) are analyzed using modal identification techniques to extract the modal parameters of the human tibia. It is observed that the regenerated FRFs from the modal model closely match with the experimentally measured ones, which is also an indication of the linear dynamic behavior of human tibia. Comparison of the modal parameters from fresh-frozen tibia with those obtained from dry tibia indicates that there is considerable change in the loss factor of tibia when it loses its mineral content and collagen. The results of a set of preliminary in vivo FRF measurements and the identified modal parameters are also presented in this study. It is concluded in this work that the modal loss factors identified from in vivo FRF measurements can be a good alternative in diagnosing progressing osteoporosis.

Keywords: biodynamic, tibia, modal identification of tibia, diagnosing osteoporosis

1. INTRODUCTION

Osteoporosis is a disease characterized by low bone mass and a micro-architectural deterioration of bone tissue, leading to enhanced bone fragility and a consequent increase in fracture risk (World Health Organization, 1994) with high socioeconomic impact on the society [1]. The common methods for detection of osteoporosis of the bone are dual energy X-ray absorptiometry (DXA) and quantitative computed tomography (QCT). According to World Health Organization T scores lower than -1.0 and -2.5 are called osteopenia and osteoporosis, respectively. Bone mineral density (BMD) measured by DXA and/or QCT is a predictor of mechanical bone

strength that has its limitations [2]. Fragility fractures in normal and osteopenic but sometimes not in osteoporotic patients are seen in medical practice. As a result, the need for better diagnosing of bone strength increases gradually [3].

Previous research [3-10] shows that osteoporosis could be diagnosed by vibration analysis, and as a result, this method is becoming an alternative diagnosis tool to identify the structure and strength of bones due to being noninvasive and precise. Earliest studies published by Perre et.al [7-10] mainly involve the assessment of natural frequencies and mode shapes of human bones (tibia and radius) both in vivo and in vitro using vibration and ultrasonic wave propagation analysis and finite element modeling. Results show that the natural frequencies of bones are decreasing with increasing level of osteoporosis, thus a correlation may exist and this can be used for the determination of bone strength.

Soft tissues and joints influence vibration measurements under in vivo conditions and interpretation of test results may be difficult. Perre et.al [8] investigated the effects of soft tissues on the structural dynamic characteristics of bones. Another attempt that has been done by Soethoudt et.al [11] reveals that the preload, that is applied by holding a small cylindrical metal element manually pressed at the excitation point while holding the accelerometer during the excitation of the bone via impact hammer, helps to decrease the damping effect of the skin and the underlying soft tissue. With vibration analysis of bone it is expected to have a better insight into the mechanical properties of bone in ex vivo and in vivo conditions.

The ultimate goal in this study is to obtain in vivo frequency response functions (FRFs) of human tibial bone and to develop a new approach to detect progressing osteoporosis by obtaining a correlation between structural dynamic properties and BMD measured by DXA and/or other properties of bone which causes osteoporosis. The most deficient part of the previous studies is that solely the change in natural frequency with progressing osteoporosis is investigated. However, natural frequencies of a structure depend on boundary conditions making it an inconvenient parameter for the assessment of osteoporosis, unless an experimental set up that prevents the change of boundary conditions of bone in each measurement is used. Therefore, it is believed that an alternative approach to detect progressing osteoporosis may be to investigate the change in "loss factor", which is an inherent property of bone.

In this preliminary stage of the study, the aim is first to determine the modal parameters of the tibia by ex vivo modal tests and then to carry out preliminary in vivo tests and thus to study the problems in monitoring the change in the loss factor of tibial bone from these measurements.

2. MODAL ANALYSIS OF TIBIA

2.1. Modal Parameters of Tibia

In order to obtain modal parameters of the tibia in vitro (ex vivo), a set of experiments are carried out on two specimens obtained through above knee amputations and show no history of diseases related to the tibia. The excitation is provided via an impact hammer (Dytran Instruments, type 5800B3, S/N 4354, sensitivity=48.5 mV/lbf) and the response was recorded using an accelerometer (Dytran Instruments, type 3035B, S/N=2436, sensitivity=104 mV/g). Both signals were measured simultaneously with an instrument of Data Physics, "QUATTRO", to obtain the FRF data.

The experiments are carried out with free-free boundary condition by placing tibia on a soft rubber as can be seen from Figure 1. Figure 2 shows the FRFs obtained for the two specimens and Figure 3 shows the coherence plots of the measurements which indicate that experimental data obtained are reliable.

The FRFs obtained were analyzed by using the modal identification software MODENT®. Circle and line fit methods are used for the identification of modal parameters. The natural frequencies, modal loss factors and modal constants of the modes identified for the two specimens are shown in Tables 1 and 2, respectively. The rigid body modes due to free boundary conditions and the flexibility of soft rubber are not identified and instead, mass and stiffness residuals are obtained as given in the tables. Although, the first modes are in the region of 300-400 Hz and the second ones are around 400-500 Hz, there are considerable differences between the natural frequencies of two specimens, as expected, since the two specimens are from two different cadavers and they do



Figure 1. Measurement of FRFs on fresh-frozen tibial bone

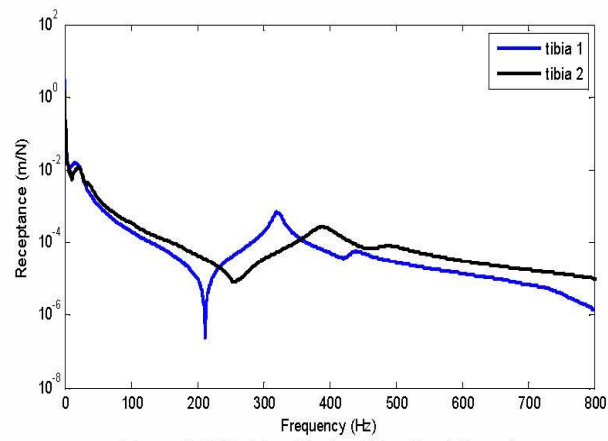


Figure 2. FRF plots of the two tibiae (fresh-frozen)

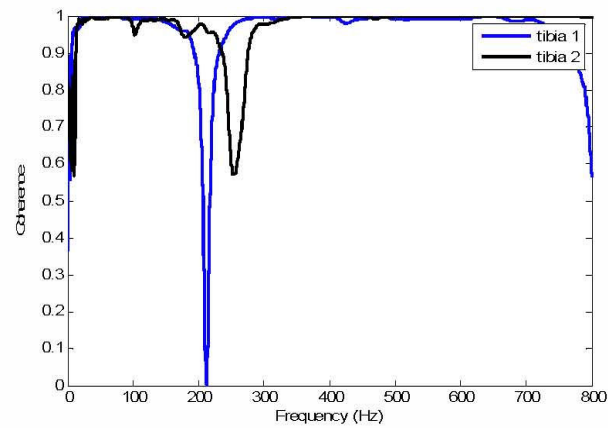


Figure 3. Coherence plots of the FRF measurements of the two tibiae (fresh-frozen)

not have identical geometries. However, the modal loss factors of the specimens were expected to be closer to each other. DXA tests showed that the mineral contents of both tibiae were not much different which implies that the differences between modal loss factors may only be partly due to the mineral content difference. Then it can be concluded that modal damping ratios of tibial bones are affected not only from the mineral content but also from the collagen which is the major organic component of the structure of bone. Yet, to be able to generalize this conclusion more specimens need to be tested.

Table 1. Modal properties of human fresh-frozen tibia (1)

Mode	Frequency (Hz)	Loss Factor	Modal Constant	Phase	Method
0	0.00	0.001	0.87932	21	Residual
1	321.27	0.040	1.3195	-2	0-Fit
2	438.34	0.054	0.11769	-44	L-Fit
3	1200	0.001	2.8932	-5	Residual

Table 2. Modal properties of human fresh-frozen tibia (2)

Mode	Frequency (Hz)	Loss Factor	Modal Constant	Phase	Method
0	0.00	0.001	1.1621	11	Residual
1	389.31	0.094	1.6864	8	0-Fit
2	496.59	0.082	0.32207	-75	L-Fit
3	1200	0.001	0.82952	-63	Residual

In order to study the modal parameters of a dry tibia, an identical experiment is carried out on a dry tibia (tibia 3). The modal properties obtained are given in Table 3.

Table 3. Modal properties of human tibia (dry)

Mode	Frequency (Hz)	Loss Factor	Modal Constant	Phase	Method
0	0.00	0.001	45.276	0	Residual
1	526.09	0.011	18.328	-1	0-Fit
2	683.12	0.010	2.1585	-173	0-Fit
3	1449.61	0.013	207.06	-178	0-Fit
4	3750.00	0.001	495.10	-5	Residual

Comparing the modal parameters of the fresh-frozen and dry tibia specimens, two expected observations can be made. Firstly, the natural frequencies of a dry tibia are higher, compared to those of fresh-frozen tibia. Secondly, the modal loss factor of dry tibia is considerably smaller than those of fresh-frozen ones. Both changes are primarily related to the loss in mineral content and collagen of the bone. Then it is concluded that modal loss factor data can be a strong indicator of the mineral content and collagen of bones. Furthermore, having much larger percentage changes in the loss factors, compared to those in natural frequencies when mineral content and collagen are reduced, was an encouraging preliminary result to believe that damping measurements may yield a promising technique in diagnosing progressing osteoporosis and monitoring fracture healing period.

2.2. Modal Model of Tibia

A mathematical model for human tibia can be obtained by using the modal data identified (Tables 1-3). Regenerated FRF of tibia can be obtained by modal superposition of the modes identified as follows:

$$\alpha_{jk}(\omega) = \sum_{n=1}^N \frac{sA_{jn}}{\omega_n^2 - \omega^2 + i\eta_n \omega_n^2} \quad (1)$$

The comparisons of the regenerated FRFs with experimentally measured FRFs are given in Figures 4 to 6, for fresh-frozen tibia 1 and 2, and for dry tibia, respectively.

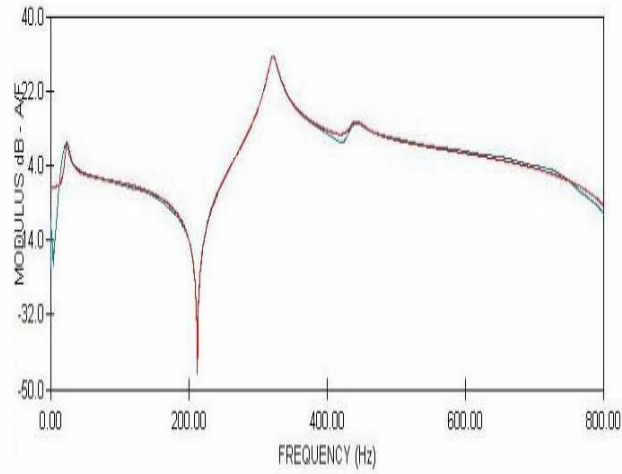


Figure 4. Regenerated FRF of First Test Specimen (fresh-frozen tibia 1)
Experimental FRF: green curve; Regenerated FRF: red curve

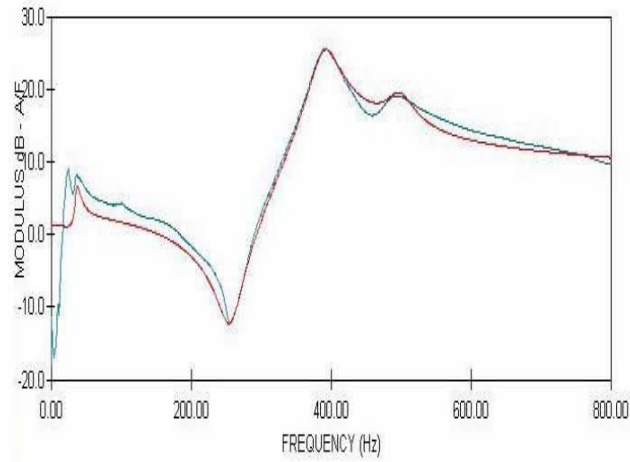


Figure 5. Regenerated FRF of Second Test Specimen (fresh-frozen tibia 2)
Experimental FRF: green curve; Regenerated FRF: red curve

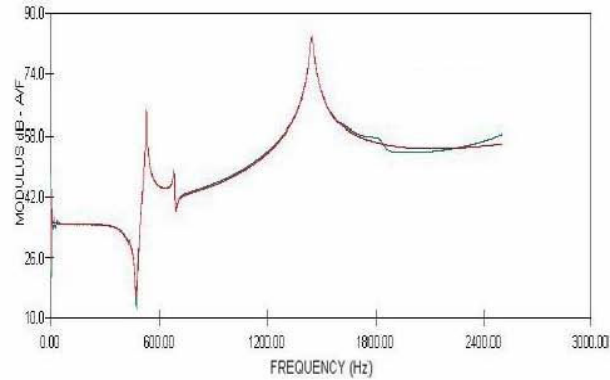


Figure 6. Regenerated FRF of Third Test Specimen (dry tibia)
Experimental FRF: green curve; Regenerated FRF: red curve

From Figures 4-6, it is clearly seen that the regenerated FRFs perfectly fit the experimental ones. Furthermore, obtaining almost the same experimental FRFs for different forcing levels indicate that tibial bone show linear dynamic behavior, at least in the range of forcing levels used in the experiments.

3. PRELIMINARY IN VIVO EXPERIMENTS

Since the ultimate goal in this study is to perform in vivo tests in order to diagnose bone strength, a set of preliminary in vivo experiments were carried out. The aim in these tests was to observe the differences between experimental FRFs obtained by ex vivo tests and in vivo ones, in other words, to investigate the effects of the skin, muscles, joints and fibula on the FRF plot of tibia.

As the boundary conditions during the experiment are not expected to affect the modal damping ratio, rather than using free end conditions, the motion of both ends of the lower part of the leg is prevented in order to have better measurements. The in vivo experimental instrumentation is the same as the one used in the ex vivo case. The measurements are taken by holding and pressing the accelerometer at the measurement point, and manually pressing a small cylindrical metal element at the excitation point.

The in vivo FRF measurement results are shown in Figure 7. As can be seen from this plot, due to the damping introduced by skin and muscles, the FRFs are heavily damped, and seem in agreement with similar measurements given in literature [3, 8].

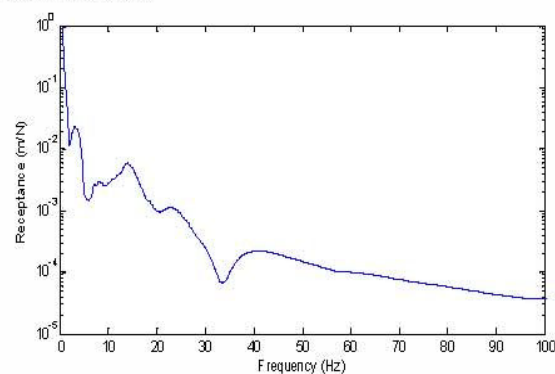


Figure 7. FRF plot of the tibia (in vivo)

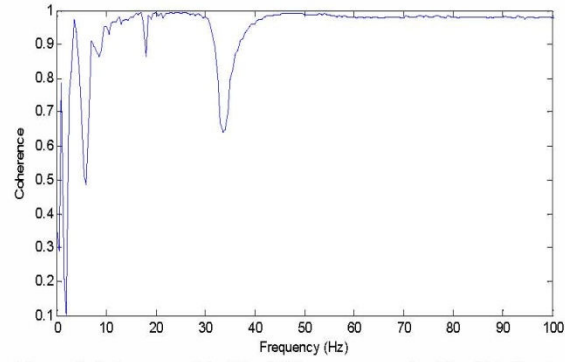


Figure 8. Coherence plot of the FRF measurements of the tibia (in vivo)

Plotting inertance graph instead of receptance, the modes can be seen more clearly. Figure 10 shows the inertance plot of the same experiment. Heavily damped nature of FRFs makes the modal identification difficult. Yet, the modal identification made by using MODENT® gave the modal loss factors around 0.2 for the first three modes as can be seen from Table 4. By using a simple theoretical discrete model, it is observed that even though the damping introduced by skin and muscles heavily dominate the modal loss factor, a change in the damping of the tibia will depict itself in the modal loss factor of the tibia-muscle-skin system that will be obtained from in vivo tests.

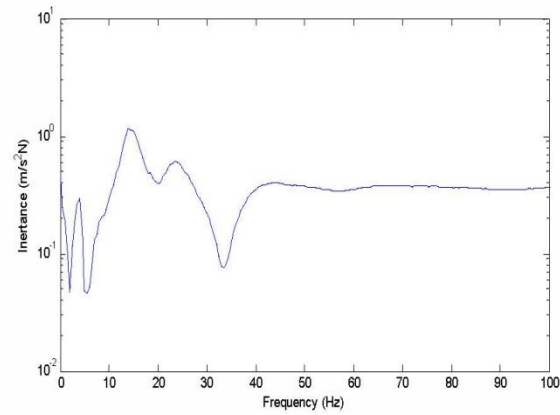


Figure 10. Inertance plot of the tibia (in vivo)

Table 4. Modal properties of human tibia in vivo

Mode	Frequency (Hz)	Loss Factor	Modal Constant	Phase	Method
0	0.00	0.001	0.16321	106	Residual
1	13.93	0.023	0.11837	-5	L-Fit
2	24.21	0.019	0.06736	-99	L-Fit
3	39.07	0.021	0.02897	34	L-Fit
4	150.00	0.100	0.33231	-117	Residual

4. CONCLUSIONS

In this paper the preliminary results of the study to diagnose osteoporosis from the in vivo frequency response function measurements of human tibial bone are presented. The previous studies using vibration measurements were generally based on the change of natural frequency with progressing osteoporosis to diagnose this metabolic bone disease. It is difficult to have success with such approaches since the natural frequencies measured do not depend only on the bone properties, but also on the geometry, dimensions and the boundary conditions during measurement. That makes the natural frequency an inconvenient parameter for the assessment of osteoporosis. However, damping measured as loss factor is an inherent property of bone, and therefore it is believed that it may be an alternative approach to detect progressing osteoporosis by studying the change in modal loss factor.

Firstly, experiments were carried out on two tibia specimens obtained from fresh-frozen cadavers with no history of metabolic bone disease or tibial fracture. The modal parameters identified from FRF measurements show that some differences can be observed both in the natural frequencies and loss factors of different specimens. It was expected to have differences in natural frequencies of two specimens, due to the reasons mentioned above. However, the modal loss factors of the specimens were expected to be closer to each other as DXA tests showed that the mineral contents of both tibiae were not much different. Then, these results may imply that modal loss factors of tibia are affected not only from the mineral content but also from the collagen which is the major organic component of the structure of bone. Yet, more specimens need to be tested to be able to generalize this conclusion.

Secondly, similar FRF experiments were carried out on a dry tibia, and the modal parameters were found to be considerably different from those of fresh-frozen tibia. These changes both in natural frequencies and loss factors are primarily related to the loss in mineral content and collagen of the bone. Having very large percentage changes in the loss factors when mineral content and collagen are reduced is an encouraging result to believe that damping measurements may yield a promising technique in diagnosing progressing osteoporosis and monitoring fracture healing period. However, the major difficulty in developing such a technique is observed in obtaining good in vivo FRF measurements from which reliable modal properties can be identified. This difficulty stems from the damping introduced by skin and muscles. Several different measurement techniques were tried to obtain good FRF measurements. Applying a preload by holding and pressing a small cylindrical metal element manually at the excitation point while holding the accelerometer during the excitation of the bone, as suggested in an earlier work [11], is found to be useful in decreasing the damping effect of the skin and the underlying soft tissue. The preliminary tests show that it is possible to identify modal loss factors from in vivo FRF measurements. When the modal loss factors in the order of 0.20 from the in vivo tests are considered in a purely theoretical model in which the modal damping of tibia itself is taken between 0.04 and 0.08, it is observed that a change in the damping of the tibia may still depict itself in the modal loss factor of the tibia-muscle-skin system that will be obtained from in vivo tests.

In conclusion, this study gives encouraging result to believe that damping measurements may be used in diagnosing progressing osteoporosis and monitoring fracture healing period. The study in progress includes in vivo tests on osteoporotic patients and comparing the results with those obtained from normal patients.

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