ESTIMATION OF STEADY-STATE TEMPERATURE DISTRIBUTION IN POWER TRANSFORMER BY USING FINITE DIFFERENCE METHOD

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Approval of the thesis:

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ABSTRACT

ESTIMATION OF STEADY-STATE TEMPERATURE DISTRIBUTION IN POWER TRANSFORMER BY USING FINITE DIFFERENCE METHOD

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Estimating the temperature distribution in transformer components in the design stage and during the operation is crucial since temperatures above the thermal limits of these components might seriously damage them. Thermal models are used to predict this vital information prior to actual operations. In this study, a two-dimensional, steady-state model based on the Finite Difference Method (FDM) is proposed to estimate the temperature distribution in the three-phase, SF_6 gas insulated-cooled power transformer. The model can predict the temperature distribution at the specific discredited locations in the transformer successfully. This study also compares predicted temperatures of the model proposed in this study with the results of the previous study which is based on Finite Element Method (FEM) and the results of the research performed by the designers of the transformer. The results

show that FDM model proposed in this study can be used to estimate the temperature distribution in the transformer with an acceptable accuracy and can be an alternative of the previous study which is based on FEM.

Keywords: heat transfer, finite difference method, transformer, SF_6 gas

ÖZ

GÜÇ TRANSFORMATÖRÜNÜN İÇİNDEKİ KARARLI HAL SICAKLIK DAĞILIMININ SONLU FARKLAR YÖNTEMİYLE TAHMİNİ

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Tasarım aşamasında ve operasyon sırasında transformatörün içindeki sıcaklık dağılımını tahmin etmek önemlidir çünkü ısıl limitlerin üzerindeki sıcaklıklar parçalara ciddi biçimde zarar verebilir. Isıl modeller bu hayati bilgiyi gerçek operasyonlardan önce tahmin etmek için kullanılırlar. Bu çalışmada üç-faz, SF_6 gaz izoleli-soğutmalı güç transformatöründeki sıcaklık dağılımını tahmin etmek için iki boyutlu, Sonlu Farklar Yöntemini temel alan, kararlı hal modeli önerilmiştir. Model, transformatörün içindeki ayrıştırılmış belirli bölgelerdeki sıcaklık dağılımını tahmin edebilir. Bu çalışma ayrıca burada önerilen modelin tahmin ettiği sıcaklıkları bundan önce yapılan, Sonlu Elemanlar Yöntemini temel alan modelin sonuçlarıyla ve transformatörün tasarımcılarının yaptığı araştırmayla kıyaslamaktadır. Sonuçlar, Sonlu Farklar Yöntemini temel alan, burada önerilen modelin transformatörün

içindeki sıcaklık dağılımını kabul edilebilir doğrulukta tahmin edebildiğini ve bir önceki çalışma olan Sonlu Elemanlar Yöntemini temel alan modele alternatif olabileceğini göstermektedir.

Anahtar Kelimeler: ısı iletimi, sonlu farklar yöntemi, transformatör, SF_6 gazı

To My Dear Family

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LIST OF ABBREVIATIONS

2-D : Two-Dimensional
3-D : Three-Dimensional
Eqn : Equation
FDM : Finite Difference Method
FEM : Finite Element Method
GFAN : Gas-forced, Air-natural
GNAN: Gas-natural, Air-natural
GFAF : Gas-forced, Air-forced
GR : Grashof Number
NU : Nusselt Number
OOP : Object-Oriented Programming
PDE : Partial Differential Equation
PR : Prandtl Number
RA : Rayleigh Number
RE : Reynolds Number
SF6 : Sulfur Hexafluoride

CHAPTER 1

INTRODUCTION

Power transformers have an important role in power transmission and distribution systems. Transformers perform crucial conversion processes by operating upward and downward and thus enabling effective transmission and distribution of electricity. Since high voltage is preferred in the transmission process, the voltage is stepped up by transformers prior to the transmission process. Conversely, clients often use voltage in low ranges. As a result, at the terminal of the transmission and distribution systems the need of stepping down the voltage occurs, which is also enabled by transformers.

In an idealized model, voltage conversion, upward or downward, occurs in the transformer without any loss of electrical energy. However, in the real model, during the voltage conversion, certain part of the electrical energy is converted to other type of energies such as thermal energy. These unintended conversions are called “losses”. Losses have to be taken into account not only for the undesirable reduction in total electrical energy but also for the negative consequence of generating thermal energy that might increase the temperature of the transformer components, especially the core and the windings. This temperature increase might result in degradation of the transformer’s performance or

its components could be damaged. As a result, the transformer might be out of service, which is not desired.

For the reasons stated above, predicting the temperature distribution in the transformer for different loading and ambient conditions would be highly beneficial and essential in design and operation stages.

1.1 AIM OF THE THESIS

In this thesis, the aim is to develop a two-dimensional model that estimates the steady-state temperature distribution in power transformer for design and operational purposes. There are some feasible methods like Finite Element Method (FEM) and Finite Difference Method (FDM) that can be used to formulate heat transfer. In this study, the model is based on FDM. FDM gives chance to predict the temperatures of the specific discredited locations in the transformer. The heat transfer mechanism in the model depends on conduction mode in homogeneous and between two solid media, convection mode on the boundaries between the solid and fluid media. In addition, making comparison is also aimed between the result of the model proposed in this thesis and result of the previous model [1] which is based on FEM and the data given by the designers of the transformer [16].

There are previous studies in literature about the temperature distribution in a power transformer depending on numerical techniques. A model was proposed in [2] that calculate the average temperatures of the components in the transformer by using energy balance method. In

the model, the transformer was assumed to be composed of three major components. These are the core and winding assembly, the SF_6 gas and the tank. Differential equations were obtained by using energy balance between the three major components and these were solved using numerical technique. After that, another model based on FEM [1] was proposed by the same researchers. FEM divides the model into meshes; thus, prediction of the temperature of the specific descritized location is possible which is similar to FDM approach.

1.2 ORGANIZATION OF THE THESIS

The rest of the thesis is organized as follows: Chapter 2 presents a background on heat transfer, finite difference method and root-finding algorithms. Chapter 3 presents the problem definition of the heat transfer in SF6 Gas cooled-insulated Power Transformer and the solution formula of the heat transfer problem. Chapter 4 presents the details of the program developed for predicting the temperature distribution in the transformer. Chapter 5 presents the results of the simulation studies of the transformer for different ambient and loading conditions and conclusions and suggestions for future study. Finally, in the Appendices, detailed information is given on the thermo physical properties of air and SF6 gas as well as the functions that are used in the program developed in this study in order to calculate value these properties for different temperatures.

CHAPTER 2

THEORY OF HEAT TRANSFER AND NUMERICAL SOLUTION METHODS

2.1 CONCEPT OF HEAT TRANSFER

Heat transfer or heat is the energy in transit due to temperature difference. The science of heat transfer deals with the analysis of the rate of heat transfer taking place in a system. Whenever there is a temperature difference in a medium or between media, heat transfer occurs in order to reach a thermal equilibrium [3] [4] [5].

The quantity of transfer energy during heat transfer can not be measured directly. However, it can be related to a measurable quantity: temperature. Thus, the knowledge of temperature distribution of a system becomes important. Once the temperature distribution is obtained, heat flux can also be determined [3].

Determination of the temperature distribution is essential since it affects the design and performance of most of the systems. Therefore, many branches of science have interests to solve the heat transfer problem in order to determine temperature distribution.

In the heat transfer studies, process of thermal energy transfer is considered in three distinct modes: conduction, convection and radiation.

2.1.1 CONDUCTION

Conduction is the mode of transfer in which energy exchange takes place from the region of high temperature to that of low temperature by kinetic motion or direct impact of molecules. In other words, conduction can be described as the transfer of energy from more energetic to the less energetic particles of a substance due to interactions between particles [3] [4].

The law of heat conduction reveals after the use of Joseph Fourier, a French Mathematician and physicist, in his analytic theory of heat. This law is given as for the heat flow in x direction

$$Q_x = -kA\frac{dT}{dx} \qquad \text{(Eqn. 2.1)}$$

where Q_x is the rate of heat flow through area A in the positive x direction. *k* is the thermal conductivity of the material and it is a positive quantity [3]. The minus sign is a consequence of the fact that heat is transferred in the direction of decreasing temperature. Therefore, Q_x becomes negative because of the minus sign in the equation 2.1. There is also another related term, called heat flux which can be expressed as the heat in transit per unit area and it can be expressed as

$$q_x = \frac{Q_x}{A} = -k\frac{dT}{dx} \qquad \text{(Eqn. 2.2)}$$

Thermal conductivity needs extra attention since it affects the heat conduction process directly and changes with the change in temperature and depends on material properties. Thermal conductivity provides an indication of the rate at which energy is transferred by diffusion process [4]. Thermal conductivity of the solid is larger than that of liquid which is larger than that of gas. Materials which have good electrical conductivity are also good at thermal conductivity because of the free electrons in their molecular structure.

In order to obtain the one-dimensional heat conduction equation, a volume element having thickness of Δx and having an area of A normal to the x axis is considered. Energy balance for this volume element is stated as [4]

$$\begin{pmatrix} Net\ rate\ of \\ heat\ gain\ by \\ conduction \end{pmatrix} + \begin{pmatrix} Rate\ of \\ energy \\ generation \end{pmatrix} = \begin{pmatrix} Rate\ of \\ increase\ of \\ internal\ enrgy \end{pmatrix} \qquad \text{(Eqn. 2.3)}$$

One-dimensional heat conduction equation is

$$\frac{1}{A}\frac{\partial}{\partial x}\left(Ak\frac{\partial T(x,t)}{\partial x}\right) + g = \rho c_p \frac{\partial T(x,t)}{\partial t} \qquad \text{(Eqn. 2.4)}$$

In addition, if the area A does not vary with x, the equation can be simplified as

$$\frac{\partial}{\partial x}(k\frac{\partial T(x,t)}{\partial x})+g=\rho c_p\frac{\partial T(x,t)}{\partial t} \qquad \text{(Eqn. 2.5)}$$

where

c_p = specific heat of material, J/(kg x ^oC)

g = energy generation rate per unit volume, W/m^3

t = time, s

ρ = density of material, kg/m^3

Note that, on the left-hand side, first term of the equation 2.5 represents the net rate of heat gain by conduction. Second term represents the rate of energy generation. Right-hand side of the same equation represents rate of increase of internal energy. It is worth to mention that this part of the equation is time dependent. However, If the right-hand side of the equation 2.5 is taken as 0, the heat conduction equation is called as “steady-state” equation since the temperature does not change in time If not, it is called “transient” equation.

It is important to emphasize that equation 2.5 is developed for the rectangular coordinates in one-dimension (x-axis). Equation can be reformed for the other coordinate systems such as cylindrical coordinates or spherical coordinates.

Once the one-dimensional, time-dependent heat conduction equation is obtained, it is easy to develop two-dimensional or three-dimensional heat conduction equation by using the same approach in two or three directions. For the 3-D rectangular coordinate system (x, y, z), the heat conduction equation becomes [4]

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{1}{k}g=\rho c_p\frac{\partial T(x,t)}{\partial t} \qquad \text{(Eqn. 2.6)}$$

where T is function of x, y, z and t.

To find the temperature distribution in a medium, heat equation must be solved in an appropriate way. To solve the heat equation properly, physical conditions existing on the boundaries must be specified. In addition, the initial condition must also be specified when the equation is time-dependent [3]. There are three types of boundary condition: prescribed boundary condition, prescribed heat flux boundary condition and convective/radiative boundary condition.

For the prescribed boundary condition, the distribution of the temperature at the boundary surface may be specified as a function of position and time. This kind of boundary condition is commonly called as Dirichlet condition or boundary condition of first kind [5]. Figure 2.1 shows the prescribed temperature boundary condition.

For the prescribed heat flux boundary condition, the rate of heat supply to a boundary surface is specified which is also called as Neumann condition, or a boundary condition of the second kind [5]. A special case of prescribed heat flux condition is adiabatic boundary which means that the boundary is perfectly insulated. Figure 2.2 shows the prescribed heat flux boundary condition

Other boundary conditions, convective and radiative, are going to be discussed in Section 2.1.2 and 2.1.3.

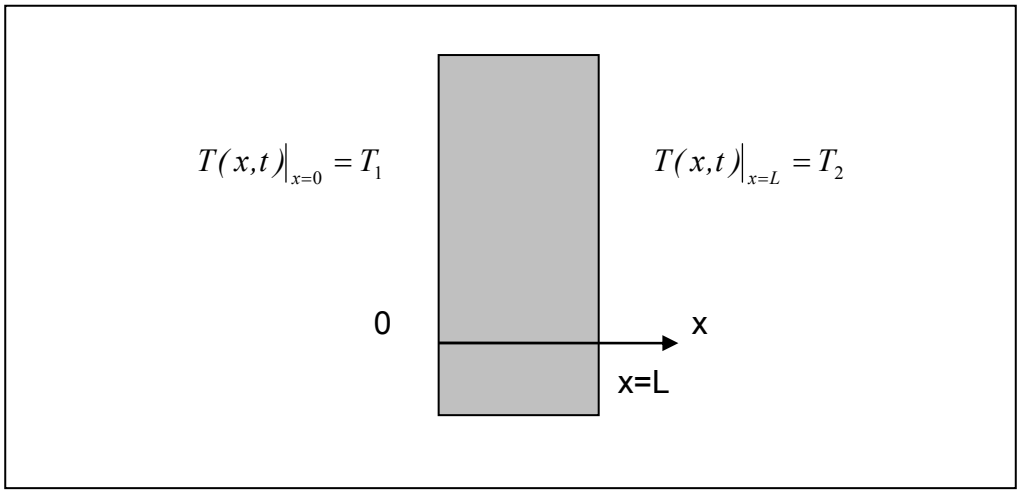

Figure 2.1 – Prescribed temperature boundary condition

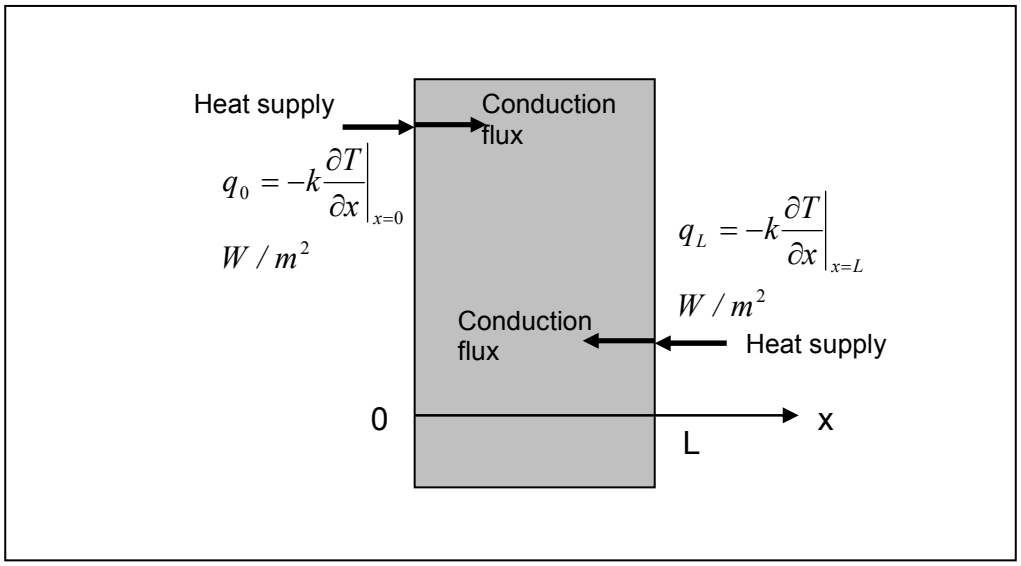

Figure 2.2 - Prescribed heat flux boundary condition

2.1.2 CONVECTION

When fluid flows over a solid body or inside a duct and if there is temperature difference between solid and fluid media, a heat transfer takes places as a result of the motion of the fluid relative to the surface. This mode of heat transfer is called convection [3]. In fact, convection mode of heat transfer is composed of two mechanisms: random molecular motion (diffusion) and bulk or macroscopic motion. Random molecular motion dominates near the surface where fluid velocity is close to zero. On the other hand, bulk motion becomes important when moving away from the surface [4].

Convection can be classified according to the nature of the fluid flow. If the flow of the fluid is obtained artificially, for instance with a pump or a fan or the wind, the convection is called forced convection. On the other hand, If the fluid motion is set up by buoyancy effects resulting from density difference caused by temperature difference in the fluid, the heat transfer is called natural or free convection [3].

Heat flux comprised by convection heat transfer mode is given as

$$q = h(T_s - T_f) \qquad \text{(Eqn. 2.7)}$$

where h is the heat transfer coefficient for convective heat transfer mode, T_s represents the surface temperature and T_f symbolizes the fluid temperature. Note that for a hot surface, heat flux q is positive and it is negative when fluid is hotter than the surface for this form of the equation 2.7.

Calculating the heat transfer coefficient, h, is a complicated process. For different geometries, different formulas are used. For this thesis only flat plate surfaces (horizontal and vertical, not inclined) are considered. Therefore, formulas that are related to flat plate surfaces are given only. For more information, any heat transfer book can be used as resource for the researcher.

Heat transfer coefficient depends on some parameters. It is given that

$$h = \frac{k}{L} NU \qquad \text{(Eqn. 2.8)}$$

where k is thermal conductivity of the medium, L is the dimension of the surface that convection heat transfer occurs and NU is the Nusselt number.

Nusselt number (NU) is a dimensionless parameter. Dimensionless parameters are used when calculating heat transfer coefficient. Since these parameters represent only the ratio of quantities, they have no unit [6]. In convection mode of heat transfer, there are other dimensionless parameters other than NU, such as Prandtl number (PR), Grashof number (GR), Rayleigh number (RA) and Reynolds number (RE).

NU can be interpreted as the ratio of heat transfer by convection to conduction across the fluid layer. If the value of NU is unity, it can be interpreted as there is no convective heat transfer. On the other hand, larger value for NU implies that heat transfer mostly depends on convection [3].

In order to obtain reliable heat transfer relations for free convection, experimental data are often desirable. There are some empirical correlations for NU given in [3]

For free convection, NU is calculated in different ways whether the object or plate is located horizontally or vertically. For a vertical plate where the temperature is uniform, NU is calculated by using the equation for correlating free convection on a vertical plane as shown in [1] and [3]

$$NU^{\frac{1}{2}} = 0.825 + \frac{0.387RA^{\frac{1}{6}}}{\left[1 + \left(0.492/PR\right)^{\frac{9}{16}}\right]^{\frac{8}{27}}} \qquad \text{(Eqn. 2.9)}$$

For horizontal hot plate, facing up or cold plate facing down, NU is calculated by using the correlation as shown in [1] and [3]

$$NU = 0.14\left(GR * PR\right)^{\frac{1}{3}} \qquad \text{(Eqn. 2.10)}$$

For horizontal hot plate, facing down or cold plate facing up, NU is calculated by using the correlation as shown in [1] and [3]

$$NU = 0.27\left(GR * PR\right)^{\frac{1}{4}} \qquad \text{(Eqn. 2.11)}$$

The calculation of the GR, PR and RA can be found in [3] or any heat transfer analysis book in the literature.

On the other hand, for forced convection, NU is calculated whether the flow of the fluid laminar or turbulent. If the RE is less than 5 x 10^5, the flow is laminar and NU is calculated as given in [1] and [3]

$$NU = 0.678 * PR^{\frac{1}{3}} * RE^{\frac{1}{2}} \quad \text{(Eqn. 2.12)}$$

If RE is greater than 5 x 10^5, then the flow is turbulent and the NU becomes shown in [1] and [3]

$$NU = 0.029 * PR^{0.43} * RE^{0.8} \quad \text{(Eqn. 2.13)}$$

There are different formulations for NU given in the books for different cases. Geometry and location of the material decides the equation of the NU, in fact. For some geometry that are used widely, like plates, ducks, tube, circular or non-circular cylinder, equations are developed with the help of empirical data. In order to find more information about NU, books related to heat transfer such as [3] and [4] can be used.

2.1.3 RADIATION

All substances continuously emit electromagnetic radiation due to the molecular and atomic agitation associated with the internal energy of the material. The emitted radiation energy can be in the range of radio waves to cosmic rays. However, thermal radiation is only related to radiation detected as heat or light [7].

For conduction and convection mode of heat transfer, heat transfer depends on the temperature difference. For thermal radiation, mechanism is different since the thermal radiation between two bodies depends on temperature difference in the power range of 4 or 5 [7].

Black body, which is commonly used in order to explain the thermal radiation parameter, is defined as an ideal body that allows all the incident radiation to pass into it (no reflected energy) and absorbs all the incident radiation (no transmitted energy). Hence, black body is called as perfect absorber. Due to this fact, it can be used as a standard that the real materials can be compared. Moreover, black body emits the maximum radiation energy and can also be used as a reference for the real materials to be compared [7].

Temperature dependence of the thermal radiation of the black body is exhibited by Stefan-Boltzmann law. According to the law, heat flux of the black body due to thermal radiation is

$$q = \sigma T^4 \qquad \text{(Eqn. 2.14)}$$

where T is the surface temperature of the black body and σ is the Stefan-Boltzmann constant. Note that heat flux given in equation 2.14 is the part that contains only the emitted heat flux. However, at the same time, black body receives and absorbs thermal energy from another black body surrounds the first one. Therefore, a new concept, net heat flux, must be defined: Net heat flux is the difference of the emitted and absorbed heat fluxes and given as

$$q_{net} = q_{emit} - q_{absorb} = \sigma T^4 - \sigma T^4{}_e \qquad \text{(Eqn. 2.15)}$$

where T_e is the temperature of the black body surrounds the small one.

Up to now, only the black body, which is theoretical concept, is considered. However, real materials do not emit the full amount of their radiative fluxes. Instead, they radiate only a portion of it. The ratio of this emitted flux to black body emitted flux is called emissivity. "ε" symbolizes the emissivity of the material. Materials except black body, which are called as "gray", have the emissivity less than 1 and greater than zero. Similarly, real materials are not able to absorb all the received flux just like black body does. Absorptivity is the ratio of the energy incident on a body that is absorbed by the body to the black body absorption. "α" symbolizes the absorptivity of the material.

After these definitions, net flux for real materials become

$$q_{net} = q_{emit} - q_{absorb} = \varepsilon\sigma T_1^{\,4} - \alpha\sigma T_2^{\,4} \qquad \text{(Eqn. 2.16)}$$

where T_1 is the surface temperature of the material, T_2 is the surroundings temperature, σ is the Stefan-Boltzmann constant which is 5.6704×10^{-8} ($J \times m^{-2} \times s^{-1} \times K^{-4}$) and "$\varepsilon$" is the emissivity and "α" is the absorptivity of the surface.

Diffuse-gray is an important concept of the thermal radiation since most of the calculations related to radiation mode of heat transfer are made with the assumption that the surfaces are diffuse-gray. When a surface is diffuse-gray, emissivity and absorptivity of the material do not depend on wavelength or direction of the radiation, but depend on temperature. In addition, for the diffuse-gray surfaces [7]

$$\alpha(T) = \varepsilon(T) \qquad \text{(Eqn. 2.17)}$$

Therefore, net heat flux of the thermal radiation of the surface is calculated as [35], [36]

$$q = \sigma\varepsilon\left(T_1^4 - T_2^4\right) \qquad \text{(Eqn. 2.18)}$$

2.2 FINITE DIFFERENCE METHOD

Numerical methods are very useful for solving partial differential equations when the problems could not be solved by exact analysis techniques because of some reasons such as complex geometries and complicated boundary conditions. In addition to this, development in computer technology encourages researchers to use numerical methods instead of solving the problems with analytic methods [8].

For solving the partial differential equations especially related to heat and mass transfer, finite difference method (FDM) and finite element method (FEM) are widely used [8]. Each method has its advantages and disadvantages. FDM is easy to learn, easy to implement and requires less computational work compared to FEM. On the other hand, using FDM is restricted to the simple geometries and the accuracy of the FDM should be examined because of the truncation error caused by Taylor's series expansion while FEM has flexibility to be used for the irregular geometries and truncation error resulting from Taylor's series expansion does not appear in the formulation [8] [9].

2.2.1 DEFINITION OF FINITE DIFFERENCE METHOD

When a differential equation is solved analytically over a region subject to specified boundary condition, the resulting solution satisfies the differential equation at every point in the region. However, if the solution is so complex or there is no analytic solution, numerical techniques could be used to solve the problem [8].

Finite difference method (FDM) is a numerical method that approximates the solution of the differential equation by approximating the derivates with finite differences [10]. When FDM approach is used, the problem is discredited so that values of the dependent variables are considered only at a finite number of nodal points instead of every point in the region [8].

Two approaches are used to discredited the derivatives in partial differential equations: Taylor Series Expansion and Control Volume Approach.

Taylor Series is a representation of the function as an infinite sum of terms calculated from the values of its derivatives at a single point. Taylor Series Expansion of a function f(x) about a point x_0 in the forward direction is given as [8]

$$f(x_0+\Delta x)=f(x_0)+\frac{df}{dx}\Big|_0 \Delta x+\frac{d^2f}{d^2x}\Big|_0 \frac{(\Delta x)^2}{2!}+\frac{d^3f}{d^3x}\Big|_0 \frac{(\Delta x)^3}{3!}+\ldots$$

(Eqn. 2.19)

By rearranging the equation 2.19, forward finite difference approximations for the first derivative become [8]

$$\frac{df}{dx}\Big|_0 = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} + O(\Delta x) \qquad \text{(Eqn. 2.20)}$$

where $O(\Delta x)$ is called as the truncation error and obtained by using equations 2.19 and 2.20 as

$$O(\Delta x) = \frac{d^2 f}{d^2 x}\Big|_0 \frac{(\Delta x)^2}{2!} + \frac{d^3 f}{d^3 x}\Big|_0 \frac{(\Delta x)^3}{3!} + \ldots \qquad \text{(Eqn. 2.21)}$$

Backward and central finite difference approximations are also used. Taylor Series Expansion of a function f(x) about a point x_0 in the backward direction is given as [8]

$$f(x_0 - \Delta x) = f(x_0) - \frac{df}{dx}\Big|_0 \Delta x + \frac{d^2 f}{d^2 x}\Big|_0 \frac{(\Delta x)^2}{2!} - \frac{d^3 f}{d^3 x}\Big|_0 \frac{(\Delta x)^3}{3!} + \ldots$$

(Eqn. 2.22)

By rearranging the equation 2.22, backward finite difference approximations for the first derivative become [8]

$$\frac{df}{dx}\Big|_0 = \frac{f(x_0) - f(x_0 - \Delta x)}{\Delta x} + O(\Delta x) \qquad \text{(Eqn. 2.23)}$$

and central finite difference approximations are obtained by subtracting equation 2.19 from equation 2.22 and rearranging the equation for derivative of function f(x)

$$\frac{df}{dx}\Big|_0 = \frac{f(x_0 + \Delta x) - f(x_0 - \Delta x)}{2\Delta x} + O(\Delta x)^2 \qquad \text{(Eqn. 2.24)}$$

It is important to notice that truncation error for central finite difference approximation for the first derivative, $O(\Delta x)^2$, is different than one related to forward and backward finite different approximation for the first derivative such that

$$O(\Delta x)^2 = \frac{d^3 f}{d^3 x}\Big|_0 \frac{(\Delta x)^2}{6} + \frac{d^5 f}{d^5 x}\Big|_0 \frac{(\Delta x)^5}{120} + \ldots \qquad \text{(Eqn. 2.25)}$$

Truncation error results from neglecting the higher order derivative terms in Taylor Series Expansion. The truncation error identifies the difference between the exact solution of a differential equation and its approximate numerical solution without the round-off error which is caused by rounding-off during the computation [8]. Equation 2.26 shows the definition of the truncation error.

$$\left(\begin{array}{l}\text{Exact solution}\\ \text{of PDE}\end{array}\right) - \left(\begin{array}{l}\text{Solution of finite}\\ \text{difference equation}\\ \text{without the}\\ \text{round - off error}\end{array}\right) = \left(\begin{array}{l}\textit{Truncation}\\ \textit{error}\end{array}\right) \qquad \text{(Eqn. 2.26)}$$

On the other hand, Control Volume Approach could be an alternative for Taylor Series Expansion for discrediting the derivatives of PDE.

In Control Volume approach, finite difference equations are developed by constraining the partial differential equation to a finite volume [8]. Control Volume approach is based on the conservation of specific

physical quantity such as mass, momentum or thermal energy. It is also known as Finite Volume approach. The approach employs numerical balances of a conserved variable over small control volumes [11]. If the formulation over the volume is appropriate, numerical solution will satisfy the conservation principle globally [11].

Implementing the Control Volume Approach could be divided into small steps. In the first step, equation in integral form considering the conservation principle is stated. The next step is dividing the volume into non-overlapping control volumes. After the control volumes are defined, the integral conservation statement is applied to a typical volume [11]. If the unit length in the depth dimension is assumed, integrals could be over in the surface due to the unit length of the depth. A typical two dimensional region divided by using control volume approach is shown in Figure 2.3. Finally, numerical approximation can be constructed that satisfies conservation principle locally [11].

A discrete analysis based on the control volume approach can often be constructed directly from a conservation statement, provided only that the quantity that are conserved are known [11].

The control volume approach for the development of finite difference equations has many advantages since it is applicable to multidimensional problems, complicated boundary conditions and situations that need variable mesh size or physical conditions. On the other hand, there is a trade off: determination of the error occurred while using control volume approach is difficult compared to determination of error occurred in Taylor series expansion [8].

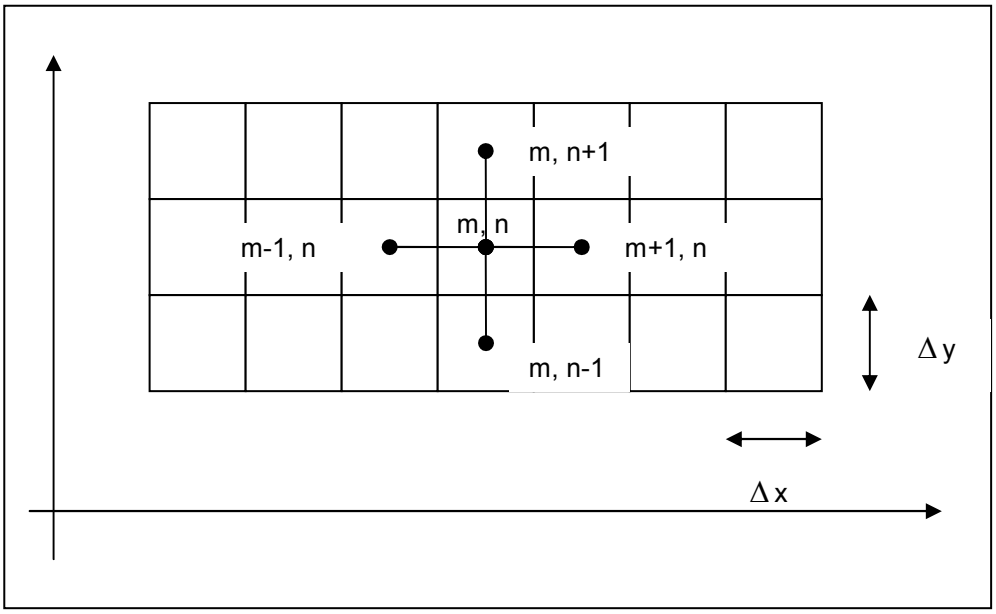

Figure 2.3- Two dimensional region divided by using control volume approach

2.2.2 FINITE DIFFERENCE METHOD IN HEAT TRANSFER

In order to develop the finite difference equation of heat transfer, a incremental volume element is considered. The steady-state energy balance equation for this volume can be declared as [3]

$$\begin{pmatrix}\text{Rate of heat}\\ \text{entering the}\\ \text{volume element}\end{pmatrix} + \begin{pmatrix}\text{Rate of energy}\\ \text{generation within}\\ \text{the volume element}\end{pmatrix} = 0 \qquad \text{(Eqn. 2.27)}$$

The dimensions of the incremental volume element are Δx, Δy and Δz. By taking the Δz as unit dimension, i.e. 1, the incremental volume can be considered as incremental area. As an example, an incremental area with its adjacent incremental areas is Figure 2.4

In Figure 2.4, $T_{i,j}$ denotes the node temperatures. By writing the appropriate mathematical expressions, equation 2.27 takes the form [3]

$$(Ak)_{m-1,m}\frac{T_{m-1,n}-T_{m,n}}{\Delta x}+(Ak)_{m+1,m}\frac{T_{m+1,n}-T_{m,n}}{\Delta x}+(Ak)_{n-1,n}\frac{T_{m,n-1}-T_{m,n}}{\Delta y}$$
$$+(Ak)_{n+1,n}\frac{T_{m,n+1}-T_{m,n}}{\Delta y}+A_m\Delta x g_m=0$$

(Eqn. 2.28)

where k is the thermal conductivity of the material, A is the cross sectional area of the element normal to the x axis. Equation 2.28 is the two-dimensional, steady-state heat conduction equation with energy generation in rectangular coordinates [3 which is formed by writing the energy balance equation between the each adjacent node to the center node. In addition, energy generation is also taken into account, if exists.

Heat transfer occurs not only in the inside of the material, but also over boundaries in convection, radiation. Therefore, boundary conditions are also important to analyze the heat transfer. There are some boundary conditions like prescribed temperature, prescribed heat flux, convection and radiation in the literature [3].

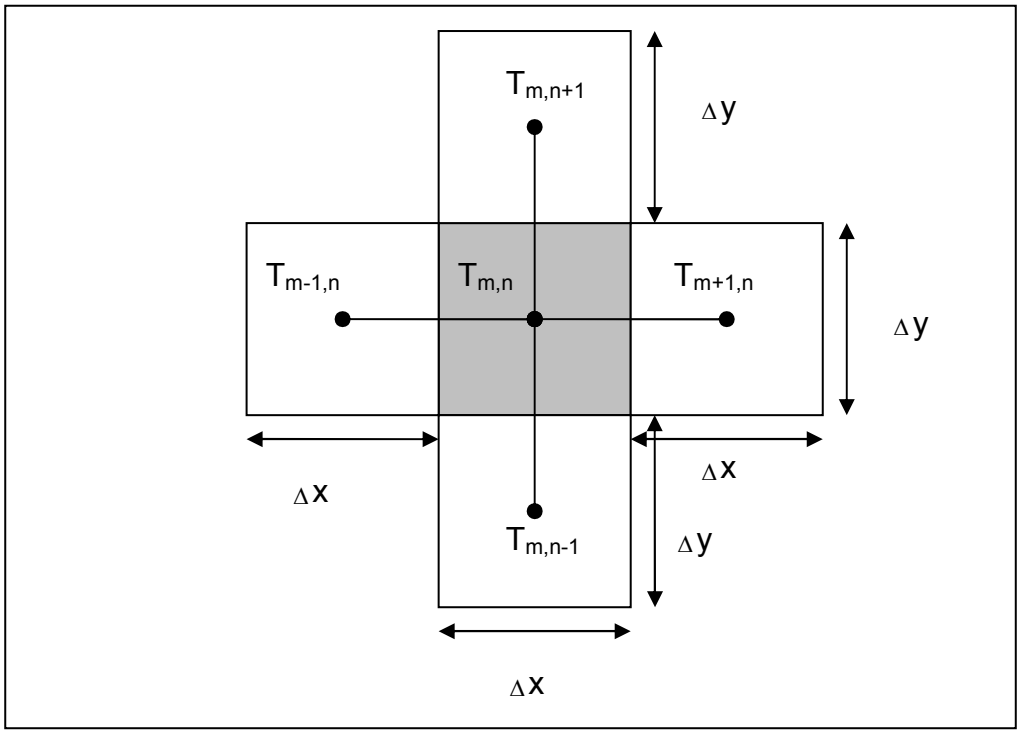

Figure 2.4- Incremental area with its adjacent incremental areas

Prescribed temperature is nothing but the specified temperature value at the boundary. Similarly, prescribed heat flux is the heat flux that its value takes constant value or function. If the specified heat flux occurs on the boundary, the energy balance equation for the differential volume element at the boundary becomes [3]

$$\begin{pmatrix} Rate\ of\ heat \\ supply\ through\ the \\ boundary\ surface \end{pmatrix} + \begin{pmatrix} Rate\ of\ heat \\ entering\ by \\ conduction \end{pmatrix} + \begin{pmatrix} Rate\ of\ energy \\ generation\ within \\ the\ volume\ element \end{pmatrix} = 0$$

(Eqn. 2.29)

Prescribed heat flux can be included to the equation 2.28 such as

$$q_0 A_m + (Ak)_{m-1,m} \frac{T_{m-1,n} - T_{m,n}}{\Delta x} + ...+ = 0 \qquad \text{(Eqn. 2.30)}$$

First element of the equation 2.30 exist duo to the prescribed heat flux. q_0 represents the prescribed heat flux.

When convective boundary condition is considered, energy balance equation can be stated as [3]

$$\begin{pmatrix} Rate\ of\ heat \\ gain\ through\ the \\ surface\ by\ convection \end{pmatrix} + \begin{pmatrix} Rate\ of\ heat \\ entering\ by \\ conduction \end{pmatrix} + \begin{pmatrix} Rate\ of\ energy \\ generation\ within \\ the\ volume\ element \end{pmatrix} = 0$$

(Eqn. 2.31)

Convective boundary condition can be included to the equation 2.28 such as

$$hA_{m+1,m}(T_{m+1,n} - T_{m,n}) + (Ak)_{m-1,m} \frac{T_{m-1,n} - T_{m,n}}{\Delta x} + ...+ = 0$$

(Eqn. 2.32)

Similarly, energy balance equation must be modified in such a way when radiation boundary condition occurs in the heat transfer problem.

$$\begin{pmatrix} Rate\ of\ heat \\ gain\ through\ the \\ surface\ by\ radiation \end{pmatrix} + \begin{pmatrix} Rate\ of\ heat \\ entering\ by \\ conduction \end{pmatrix} + \begin{pmatrix} Rate\ of\ energy \\ generation\ within \\ the\ volume\ element \end{pmatrix} = 0$$

(Eqn. 2.33)

Finally, radiation boundary condition can be included to the equation 2.28 such as

$$A_{m+1,m}\varepsilon\sigma(T^4{}_{m+1,n} - T^4{}_{m,n}) + (Ak)_{m-1,m}\frac{T_{m-1,n} - T_{m,n}}{\Delta x} + \ldots + = 0$$

(Eqn. 2.34)

where "ε" is the emissivity of the boundary surface and "σ" is the Boltzmann constant.

2.3 ROOT FINDING ALGORITMHS

Generally, one of the important information about a function f(x) is the root of the function. Root of the function f is nothing but the x value that

$$f(x) = 0 \qquad \text{(Eqn. 2.35)}$$

Methods that are used for finding the roots of the functions are called "root finding methods". For most of the functions, especially which are nonlinear, it is impossible to find the root analytically. Instead, numerical methods are used. Numerical methods are classified as direct methods and iterative methods. Direct methods are those which can be completed in a predetermined finite number of steps [12]. On the other hand, iterative methods run until a convergence criterion is met. Thus, with iterative methods, roots that are found at the end of the process are the approximate values and therefore, contain some error parts.

Most well-known root finding algorithms are Bisection method, Newton method and Secant method. Both methods are based on iterative approach. Bisection method is the simplest method and most robust method under some conditions. One of the important requirement for this method is that function f must be continuous. In addition to its simplicity and robustness, Bisection method does not depend on derivative of the function. Although this method is reliable, it converges slowly [13].

Bisection method requires two initial points, a and b, where f(a) and f(b) have opposite signs which means the root of the function must lie in the interval formed by two initial points for a continuous function. Thus the convergence of the function is guaranteed.

$$x_0 = \frac{a+b}{2} \qquad \text{(Eqn. 2.36)}$$

The method divides the interval into two parts by computing midpoint of the interval. If c is not the root, there are two other possibilities: f(a) and $f(x_0)$ have opposite signs or f(b) and $f(x_0)$ have opposite signs. Interval that contains the root is selected. After that, midpoint is refreshed and the same procedures are repeated. In this way, the interval that might contain the root of function reduces in width 50% at each step [14]. On the other hand, if the root of the function does not exist in the selected interval, a precaution must be taken in order to save the algorithm form an endless iteration. Limiting number of the iteration might be a precaution. Figure 2.5 shows Bisection method graphically.

Although Bisection method is robust and easy to implement, other methods, such as Newton methods, could be more efficient if the total number of iteration is considered.

Newton's method is popular and has many variations. It is based on the principle of successive linearization, a technique in which the nonlinear problem is replaced by a succession of linear problems whose solutions converge to the solution of the nonlinear problem [12]. Figure 2.6 shows the Newton's method graphically.

Newton's method is based from the first order Taylor expansion. Since the first order Taylor expansion approximate the function f as

$$f(x) = f(x_n) + f'(x_n)(x - x_n) \quad \text{(Eqn. 2.37)}$$

and

$$f(x) = 0 \quad \text{(Eqn. 2.38)}$$

where x is the root of function f. After rearranging the equation 2.37 and equation 2.38

$$0 = f(x_n) + f'(x_n)(x - x_n) \quad \text{(Eqn. 2.39)}$$

Therefore, x becomes

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad \text{(Eqn. 2.40)}$$

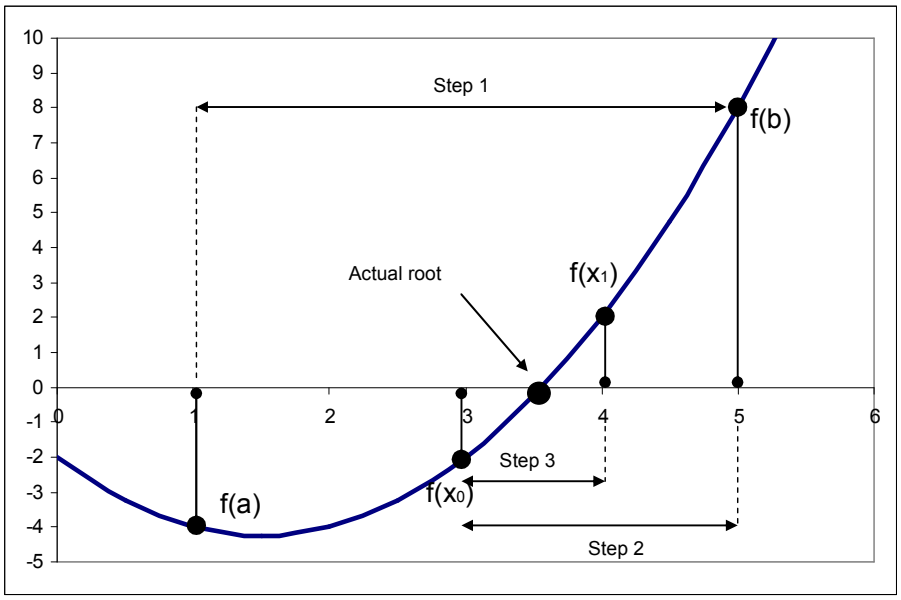

Figure 2.5 - Bisection method

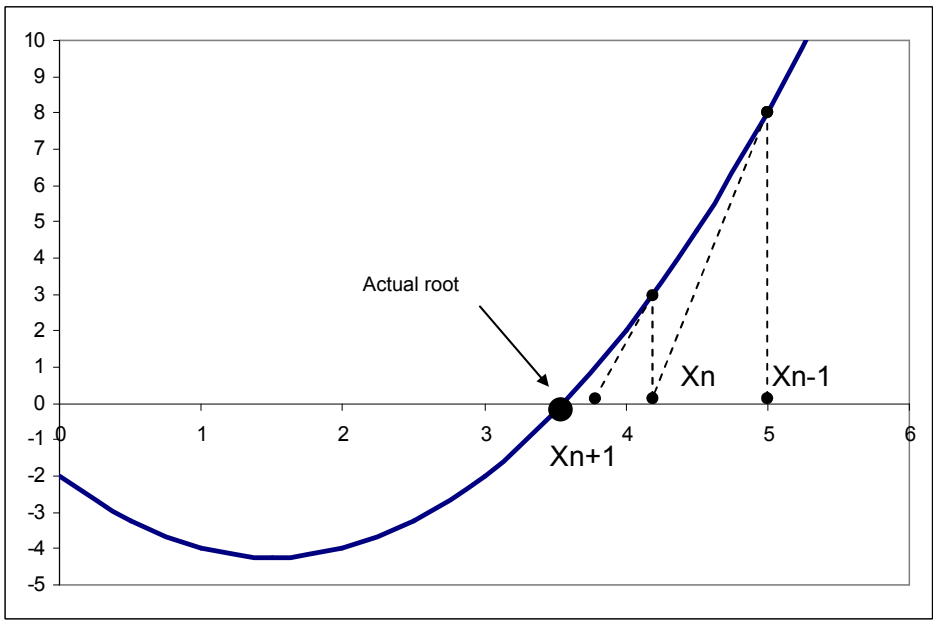

Figure 2.6 - Newton's method

The main algorithm of the Newton's method starts with setting n=1. Creating an initial guess is necessary to start the procedure. Second step is computing the x for the next step as declared in equation 2.40. After solving the equation obtained, if the absolute value of the difference between previous value and the new value of the x is smaller than a certain amount, convergence is obtained and the result is the new value of the x. Otherwise, if the absolute value of the difference between previous value and the new value of the x is not smaller than a certain amount, setting n=n+1 is required. As a result, new value of the x, if the x is not the root, becomes previous value of and will use for the calculating the new value of the x in the next iteration. After that, re-computing the x for the next step as declared in equation 2.40 with new values is required. The algorithm ends when the absolute value of the difference between previous value and the new value of the x is smaller than a certain amount and result is the new value of x. The flow chart of the Newton's method is shown in Figure 2.7.

The main advantage of the Newton's method over Bisection method is that Newton's method convergences very fast compared to the Bisection method. This observation is true especially when the initial point is chosen close to the root of the function. However, there is constrain on the function in order to use Newton's method: function must be differentiable in the interval and in addition, the derivative of the function must be supplied [12].

Despite its advantage, there are some disadvantages of using the Newton's method. One of them is that global convergence may fail in some cases. For instance, if f'(x)=0 or in other words, algorithm hits a local minima or maxima point, then the next step can not be calculated.

Thus, there is no guarantee of converge for the Newton method. The method may not always convergence or it may convergence to the desired root if more than one root exists. Initial guess is very critical on the success of convergence [12].

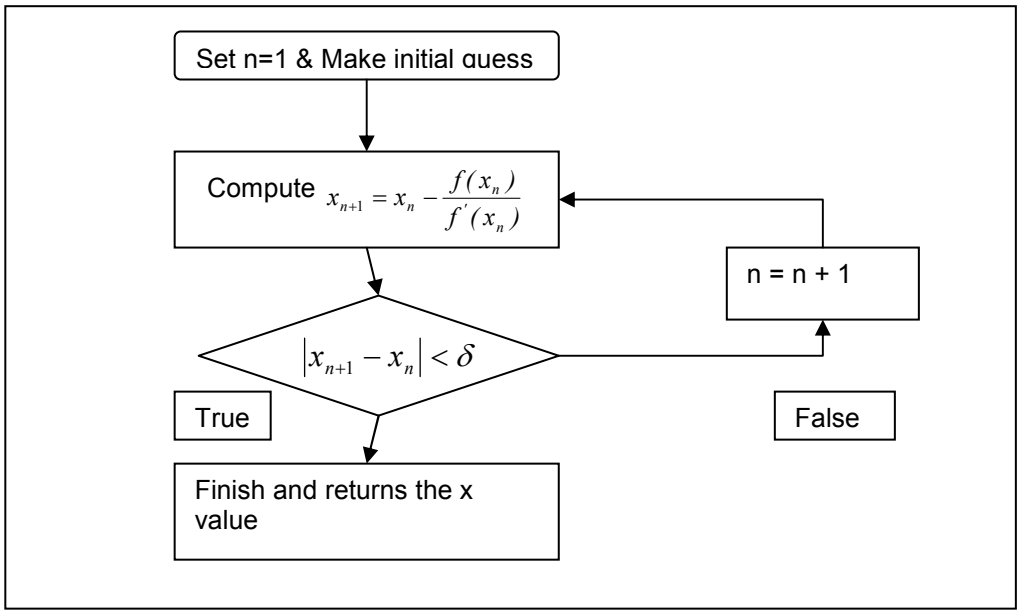

Figure 2.7 - Flow chart of the Newton's method

In addition, obtaining the derivative of the function f is not always easy even if the function f is differentiable. Replacing differentiation with finite difference approximations results a new method, called Secant method.

Finite Difference method approximates the differentiation such that

$$f'(x_n) = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}} \qquad \text{(Eqn. 2.41)}$$

and if the equation 2.40 is reformed by using equation 2.41, result becomes

$$x_{n+1} = x_n - \frac{f(x_n) * (x_n - x_{n-1})}{f(x_n) - f(x_{n-1})} \qquad \text{(Eqn. 2.42)}$$

Equation 2.42 is the definition of Secant step for the root finding algorithm. The difference in requirement between the Newton's method and Secant method is that Secant method needs the values of function f at the two previous iterations. Root finding algorithm for Secant method is the same as Newton's method with an exception: equation 2.42 is used instead of equation 2.40. Figure 2.8 shows the Secant method graphically.

When the convergence rate is considered, Secant method is faster than the Bisection method and it is slower than the Newton's method. However, since Newton's method requires the evaluation of both function f and its derivative at every step, Secant method may well be faster in practice [15].

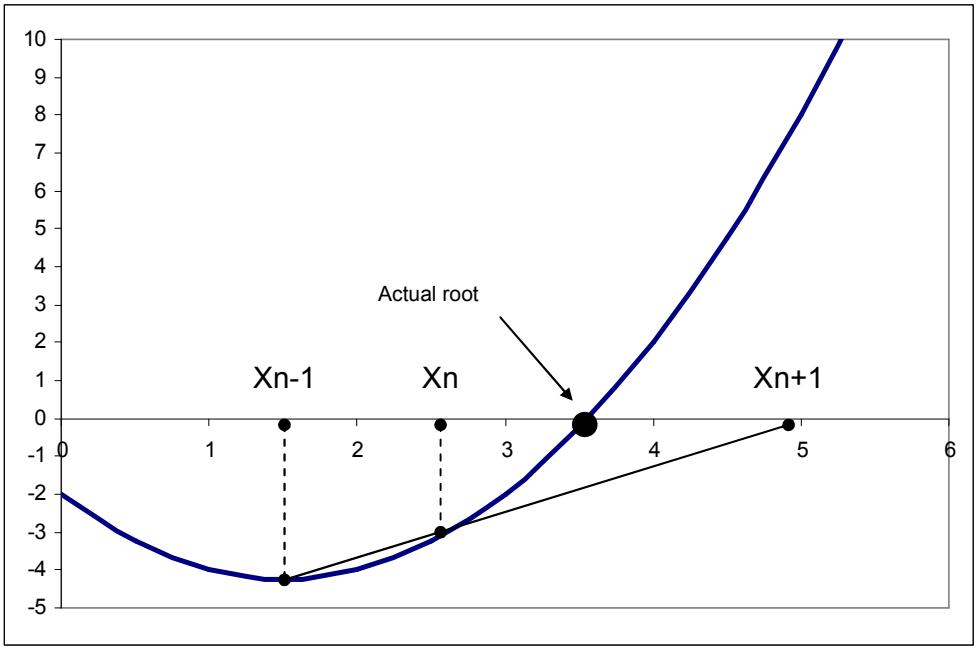

Figure 2.8 – Secant method

CHAPTER 3

TRANSFORMER MODELLING OF TEMPERATURE DISTRIBUTION

3.1 PROBLEM

The main problem in this thesis is determining the temperature distribution in a transformer. In order to obtain it, boundary conditions must be decided, thermo physical properties of the materials must be known and finally heat transfer equation must be obtained. Even though, these steps are done successfully, the results can not be obtained easily solving the heat transfer equations by using analytic methods is not easy. Because of this reason, most of the heat transfer problems are solved with numerical methods. The model proposed here is based on the FDM which is a kind of numerical method and the obtained heat transfer equations are solved with Secant method. Detailed information about FDM and Secant method were given in Section 2.2.2 and 2.3, respectively.

The transformer used in this thesis is a 3-phase, SF_6 gas insulated-cooled power transformer. Ratings and other necessary information about the transformer are given in Section 3.2.2.

3.2 MODELLING

3.2.1 ASSUMPTIONS

Assumptions are made to simplify computations in a study. In this study, some assumptions are also made, few of which are the common with the previous study [1].

The most important one is that it is assumed that temperature distribution in the axis of dept does not vary with the variation of position. By assuming it, model can be considered as 2-D instead of 3-D which decreases the computation effort and time significantly. Another assumption is that heat generated per unit volume per unit time in the core and windings is uniformly distributed. In addition, the dielectric loss on the winding insulation is not taken into account since it is significantly small compared to the copper loss [1].

The idea that transformer can be seen as symmetric from the y-axis located at the center of transformer as seen in Figure 3.1 gives the opportunity to work on only one half of the transformer. As a result, the model only focuses on the right half of the transformer since left half will be the same as the right half. This simplification results in decrease in computation time.

The previous study [1] used the assumption that the bottom of the transformer was totally insulated and no heat transfer occurred from there. This idea has been replaced with the new assumption that the bottom of the transformer is semi-insulated such that air temperature at the bottom is 30°C greater than the ambient temperature. By assuming

that, the model offers a limited heat transfer from the bottom of the transformer which is in fact more realistic than the previous model.

3.2.2 PROPERTIES OF THE TRANSFORMER USED

Gas insulated-cooled transformers are important because they can be located in the densely populated urban areas since the gas used to insulate and to cool the transformer is non-flammable and non-explosive [16]. On the other hand, the conventional oil-immersed power transformers have a risk such as burning since oil used in the transformer is a flammable material.

The SF_6 gas insulated-cooled power transformer used in this thesis was first produced by Mitsubishi Electric Corporation, Japan in 1990. In 1993, designers of the transformer published a paper about the design principles and performance of the transformer [16].

SF_6 is a colorless, non-toxic and non-flammable gas that is used to insulate the high voltage equipment such as circuit breakers, switchgear and transformers. The main reason of using SF_6 gas in high voltage equipments is that it has much higher dielectric strength than air or dry nitrogen [17]. Using SF_6 gas in transformers results in two benefits: Insulation and cooling. Therefore, adaptation of SF_6 gas alone both for these purposes reduces the total cost and maintenance of the transformer compared with the one with the liquid-cooled [16].

The ratings and the features of the transformer are given in Table 3.1. Knowing the dimensions of the transformers is necessary since they are used in forming the incremental volume/area in FDM formulation. The dimensions of the core and windings are given in Figure 3.1. Note that the units are given in meter.

The losses occurring in the core and the windings are given in [16]. According to [16]

- No load loss = 14 kW
- Load loss at 75°C, 30 MVA base (275/11 kV tap) = 162 kW

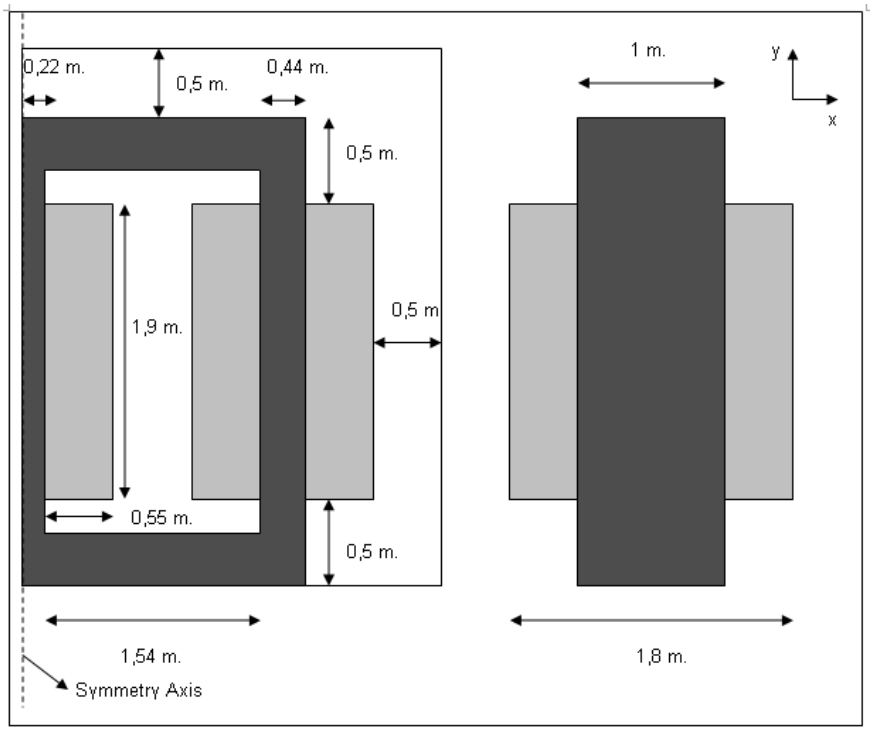

Figure 3.1- Cross section of the core and winding of the transformer from the front view and side view

Table 3.1- Ratings and features of the transformer used in model

Phase		Three
Capacity		15 / 30 / (33,75)
Cooling type		GNAN / GFAN / (GFAN)
Primary voltage **&** **Insulation Level (BIL)**		288.75 kV ~ 275 kV ~ 206.25 kV (19 taps) 1050 kV
Secondary voltage & **Insulation Level (BIL)**		11 kV 95 kV
Frequency		50 Hz
Thermal insulation class		E (Coil temperature limit is 75 °C by resistance method)
Gas pressure (Main tank)		240 kPa at 20 °C
Sound level		55 dB at 2 m.
Dimensions	**Inside the tank**	W 2.3 x L 6.5 x 3.4 m
	Overall	W 4.5 x L 12.4 x 5.75 m (Including sound proof walls)
Weight of core and coil		45000kg
Total weight		106000 kg (Including sound proof walls)

Calculating the losses in terms of W/m^3 is necessary since it is used during the computation in this unit for the core and windings. In order to simplify the calculation, the core is assumed to be in the rectangular shape and winding is assumed to be in the circular shape at the end but rectangular shaped in the middle as seen in Figure 3.3. With the help of

these assumptions and the dimensions given in Figure 3.1 and Figure 3.2, losses are calculated as 2120 W/m^3 for core and 4300 W/m^3 for windings.

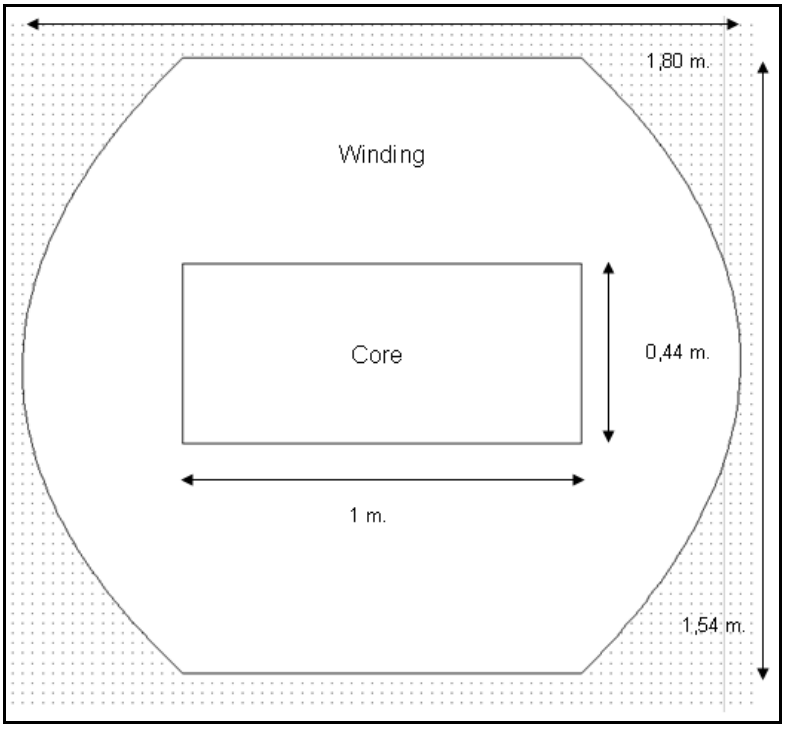

Figure 3.2- Cross section of the single core and winding of the transformer from the top view

3.2.3 HEAT TRANSFER MECHANISM

The model proposed in this thesis is based on FDM. The main idea in FDM is to divide the volume into incremental parts and realize the energy balance in these elements. In addition, under some constrains

and assumptions such as no variation of the temperature occurs with the variation of position in the axis of depth, incremental volume can be considered as incremental areas and the model becomes 2-D. This simplification results in a decrease in the computation effort.

The transformer that is divided into incremental areas is shown in Figure 3.3. Shapes of the incremental areas are square and the side lengths of them are chosen as 0.11 meter since this number is a divisor of most of the dimensions of the transformer.

Heat transfer mechanism is based on five stages: The first stage is the transfer of the heat from inner part of core and windings to their surface. At this stage, heat is transferred by conduction. Second stage is the transfer of the heat from the surface of the core and winding to the gas by convection. Third stage is the transfer of the heat in the SF_6 gas by conduction and from the SF_6 gas to inner surface of the tank by convection. Forth stage is the transmission of the heat in the tank walls by conduction. Finally, the heat is dissipated from the outer surface of the tank by convection [1].

FDM is mainly based on conduction; convection is considered as the boundary condition only. If the incremental area is inside the medium, in other words, it is surrounded by the same type of incremental areas which means there is no boundary condition, heat transfer mode is conduction. There is another scenario that can be possible for conduction. Conduction can also occur between two different types of solids.

Thermal contact resistance or thermal contact conductance concept appears here since the two media may have different thermal conductivity properties. Thermal contact conductance is the ability to conduct heat between two different bodies [18]. It is very difficult to obtain the value of thermal contact conductance since it depends on some parameters which are difficult to obtain such as contact pressure, surface roughness/flatness, surface deformations and surface cleanness. Because of this reason, thermal contact conductance is assumed not to appear on the boundary in this model and it is assumed as all heat flux appearing on the surface of the one solid material is conducted to the other solid material. This mode of conduction occurs between core and winding and core and tank.

Conduction mode of heat transfer is the base of the FDM since other modes are evaluated as boundary conditions. Details of formulation of the conduction mode of heat transfer based FDM were given in Section 2.2.

Conduction mode of heat transfer occurs in the core, windings, SF_6 gas and in the tank walls. For different materials, k has different values. For windings (copper), k is taken as 395 W/m K [19]. Although, there is variation with temperature variations, the value does not change significantly. As a result, the value of k is assumed to be constant. Similarly for core (iron), k is taken as 50 W/m K [19] and for tank (steel), k is taken as 45 W/m K [19]. However, for SF_6 gas, thermal conductivity changes significantly with the change in temperature. For this reason, it is taken as not a constant value but a function which depends on temperature. Figure A.1 in Appendix A.1 shows the variation of the thermal conductivity of SF_6 gas with temperature [20].

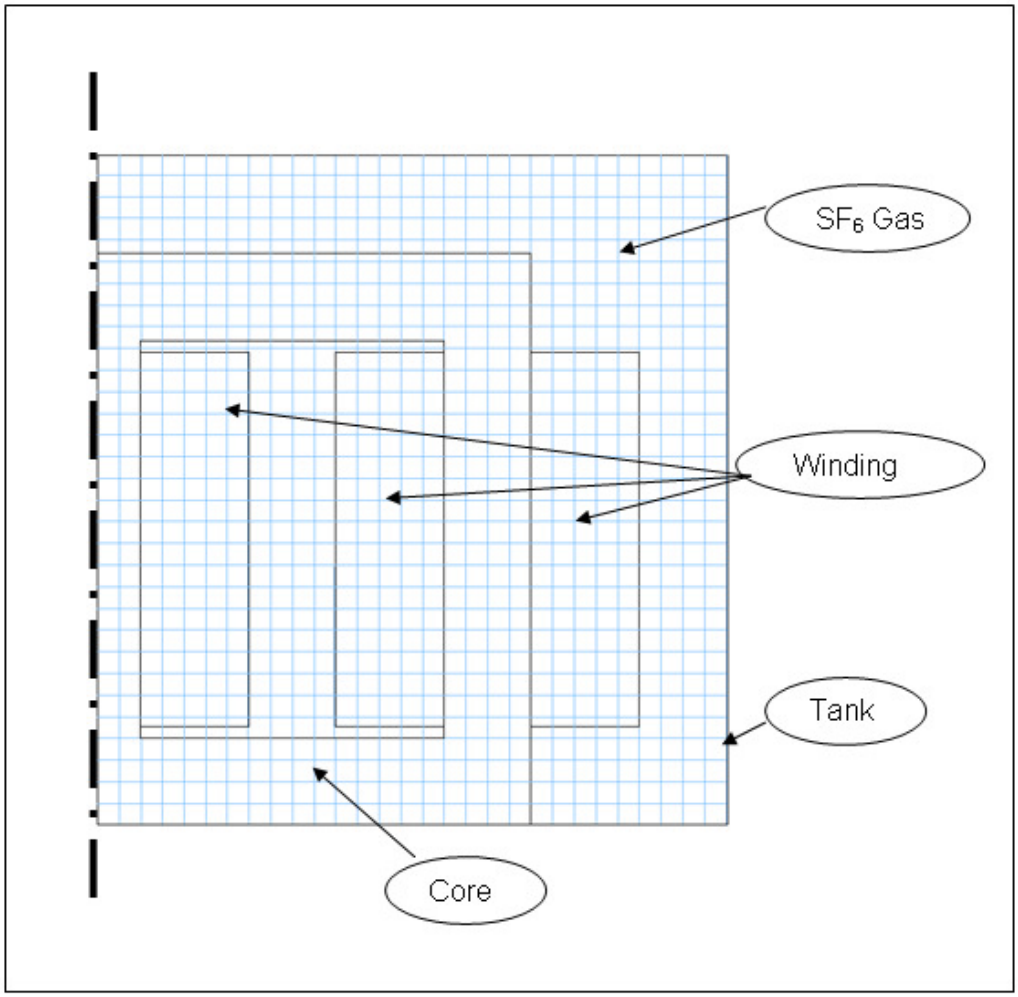

Figure 3.3- Node distribution over the transformer.

If the right adjacent node in Figure 3.5 is also solid, but composed of a different material, q becomes

$$q = \frac{(T_{m+1,n} - T_{m,n})}{\frac{l/2}{k_{m+1,n}A} + \frac{l/2}{k_{m,n}A}} \qquad \text{(Eqn. 3.1)}$$

In order to define the convective boundary condition, type of convection must be defined. The cooling system of the transformer is defined as gas natural-air-natural (GNAN) or gas forced-air natural (GFAN). Convective boundary condition inside the tank is defined as forced convection and boundary condition outside the tank as natural convection (GFAN) over the fifty percent of loading conditions by the manufacturer [16]. The average heat transfer coefficient inside the tank is calculated as 55 W/m^2 K and given in [16]. In order to obtain this value, ventilation speed of the SF_6 gas inside the tank is taken as 5 m/s.

For the forced convection of SF_6 gas, Nusselt number (NU) is calculated according to the equations 2.12 and 2.13. In order to calculate the Nusselt number, Reynolds number (RE) and Prandtl number (PR) must be calculated first. PR and RE are functions of temperature. They are re-calculated before every use. Calculation methods of the RE and PR are given in Appendix A.2.

For the natural convection of air which occurs between the outside wall of the tank and air, Nusselt number (NU) is calculated according to the equation 2.9, 2.10 and 2.11. Thus, Rayleigh number (RA) and Prandtl number (PR) must be calculated to calculate Nusselt number (NU). Prandtl number of air does not vary significantly with the temperature variation and because of this reason, it is taken as a constant with the value of 0.713 [21]. Calculation method of RA and GR are given in Appendix B. Details of formulation of the convection mode of heat transfer based FDM were given in Section 2.2.

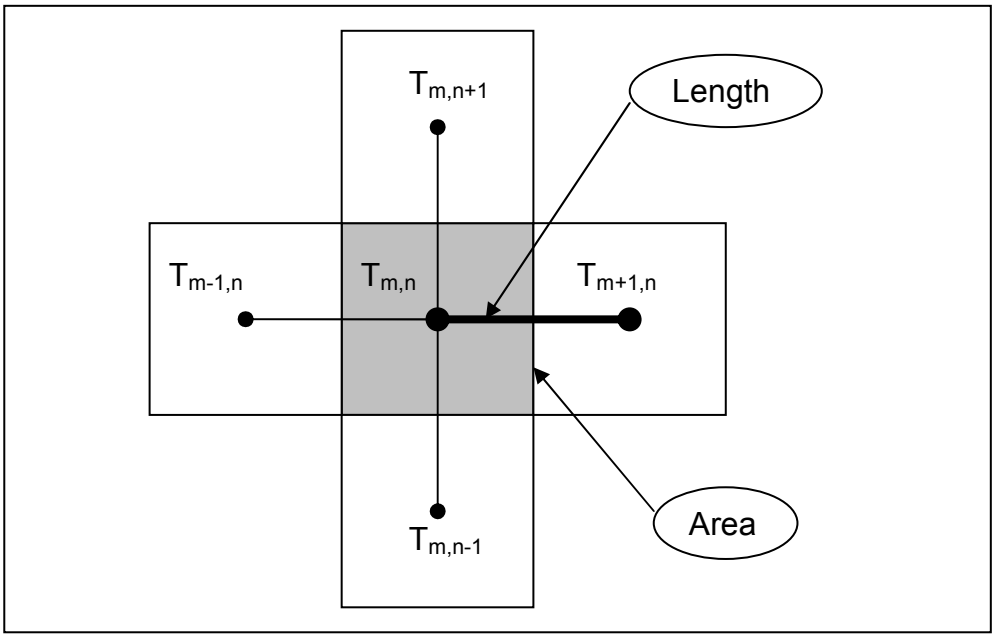

Figure 3.5- Adjacent nodes in FDM

Radiation mode of heat transfer which is a boundary condition might occur on the surface of core, windings and the tank. Radiation is mostly temperature dependent in infrared wavelength.

However, when the radiation boundary condition is taken into account, not exactly but roughly, the results show that there is a small variation in the temperature distribution in the transformer. For instance, difference between results of the simulation studies that radiation boundary condition is taken into account and ignored is 0.5°C on the tank and 1°C on the core and winding. Therefore, it is concluded that radiation mode of heat transfer as boundary condition has very small effect on the temperature distribution and because of this reason it is ignored in this study.

3.2.4 SOLUTION OF HEAT TRANSFER EQUATION

In the previous section, method of obtaining the approximated heat transfer equation is summarized. Obtaining the equation for a node is nothing but writing the heat transfer equations for each adjacent node and summing them and equating them to zero for steady state solutions. If the heat generation exists in the node, it must be also taken into account and be added to the equation.

After obtaining the equations, they are solved with Secant method for new values of node temperatures with the help of their previous values since they are taken as initial points.

Since the method depends on the iteration principle, in each iteration, 990 of node temperatures are refreshed one by one. After the convergence criterion is accomplished, final node temperatures are returned as steady-state node temperatures for a given load and ambient conditions. Details of the program will be given in Chapter 4 and details of the Secant method was presented in “Root Finding Algorithms”, in Section 2.3.

It is worth to mention that before using the Secant method in the program for this study, the implemented function for Secant method is checked by solving some known equations with this function and comparing the results with the real results in order to assure that implemented function works properly.

CHAPTER 4

PROGRAM DEVELOPED

In this thesis, a computer program is developed in order to estimate the steady state temperature distribution in the power transformer. The program is developed in C# programming language by using Microsoft® Visual C # 2005 Express Editions. Object-Oriented Programming (OOP) paradigm is used. OOP is based on the objects and their interactions to develop programs [22]. OOP is chosen since transformer is divided into incremental areas due to FDM approach for the solution of the heat transfer problem and each of them could be thought as an *object*. In addition, heat transfer could be thought of as the interaction between the incremental areas of the transformer. Each incremental area has its specific values such as temperature and thermal conductivity due to being composed of different materials. With the help of this approach, temperature of each node could be monitored individually

4.1 CLASS STRUCTURE OF THE PROGRAM

In OOP, classes are the templates for creating the object in a way that created object has the same field, in other words, properties and the same methods [23] and [24]. Class structure is the base of the objects and therefore of the OOP.

In the program, one main class, *node* is used. *Node* class is implemented since the model consists of the incremental areas and there is one node in the center of each incremental area. Each node has the fields such that temperature, previous temperatures, thermal conductivity, dimensions, heat transfer coefficient and, if exists, losses. These properties may be different for each node. In fact, this is the main comfort that OOP gives the developer to construct the program.

The transformer is composed of four different types of materials: *Core*, *winding*, *fluid* that surrounds the core and winding and *outersurface (tank)*. As a result, four classes are implemented for the purpose of simulating the materials. The objects created by using these classes should have the same fields with different values. For instance, an object created from *core* class has the thermal conductivity about 50 Watt/meter Kelvin. On the contrary, object created from *winding* class has thermal conductivity of 395 Watt/meter Kelvin. Because of this requirement, a common property of the OOP is used: Inheritance. Subclasses could be created by using the inheritance properties of OOP languages. Subclasses are the specialized versions of the main class [25]. In the program developed for this study, *node* class is considered as main class and *core, winding, fluid* and *outersurface* classes are subclasses of this class. Class diagram of the program is given in figure 4.1

Class diagram is the static structure diagram that describes the structure of the system by showing the system classes, attributes and their relationships [26]. In Figure 4.1, rectangular boxes symbolize

classes and the arrows start from subclasses and end in main classes symbolize generalization operation, i.e. *inheritance*.

4.2 ALGORITHM

In numerical analysis, problems could be solved in two different ways: Direct way and iterative way. In the program developed for this study, an iterative approach is applied. Iterative algorithm attempts to solve the problem by finding the successive approximation to the solution where it starts from an initial guess and ends when a certain convergence condition is achieved [13].

Program starts with creation of the objects. As mentioned in Section 4.1, there are four types of objects created by using the related classes and their types are decided according to their positions. Model is composed of incremental areas. In fact, each of them is an object and these are located in a way that whole incremental areas form a matrix structure.

At the end of the object creation part, initial value of temperature field of the objects is assigned. The part of the program up to this point executes only once, at the beginning of the execution.

After creating the object, calculation of the temperature part starts. This calculation process occurs iteratively and the process ends when the convergence condition is satisfied which is in fact the convergence of each node individually. There are 3 loops that are located one within the

other. The inner loop is a "*for"* loop and its aim is to seek the nodes in a row. The middle loop is also a "*for"* loop and its aim is to seek the rows in the matrix. The outer loop is a *"while"* loop. The two inner "*for"* loops execute while number of the convergence nodes is not equal to the total number of the nodes in the matrix.

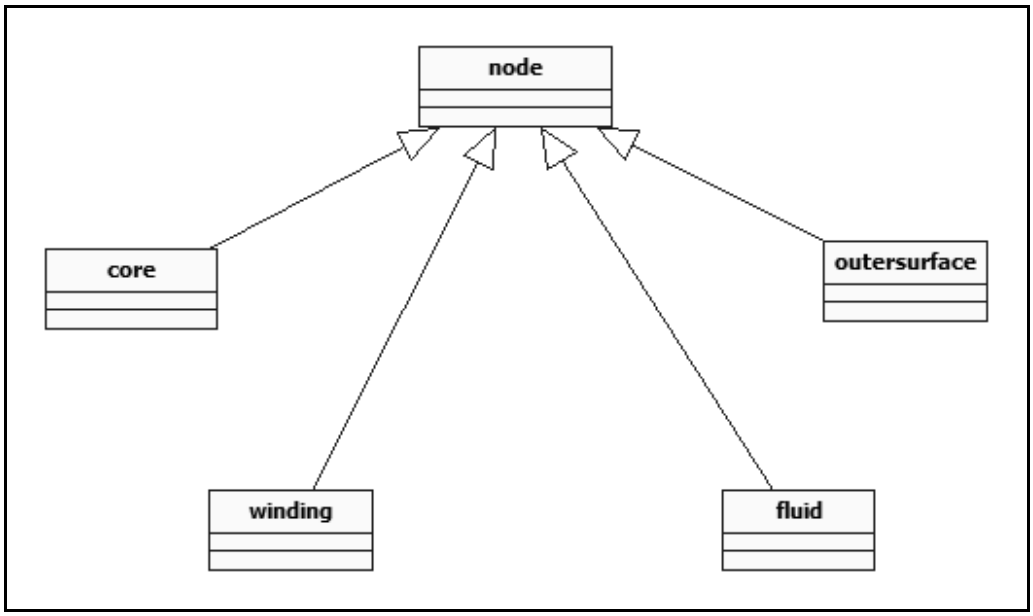

Figure 4.1- Class diagram of the program

In order to understand the algorithm properly, an example can be beneficial. Let *i* be the node number in the *j*th row of the matrix. The algorithm flows such that *i*th node type is checked. According to its type, related function that calculates the energy balance equation is called. There are four different functions and each function calculates the

energy balance equation of the related objects that are created from four different classes and returns the coefficients of the energy balance equations in an array. After composing the energy balance equation, the function that is used for root finding purpose, based on the Secant Method, is called in order to calculate the temperature of the node since the root of the energy balance equation gives the new value of the temperature of the node for the given initial value which is, in fact, the previous value of the temperature of the node.

Root finding method is a numerical method that calculates the value of *x* such that [28]

$$f(x) = 0 \qquad \text{(Eqn. 4.1)}$$

There are few very well-known root finding algorithms such as Newton method, Secant method and Bisection method. More information about root finding algorithms was given in Section 2.3.

After the new value of the temperature of the node is obtained, it is compared with the previous value. Absolute value of the difference of the two temperatures is compared with a constant, ε, in order to decide that the node temperature is converged or not. The Boolean field of the object "converge" is assigned as "*true*" if the temperature of the node converges. Otherwise, it remains *false*.

Temperature of each node is calculated in this way and determined whether it converged or not. At the end of the matrix, the number of the nodes that converge is checked. If the number of the nodes that converge is not equal to the total number of the nodes inside the model,

then the program starts to a new iteration to calculate the temperatures of the nodes from the beginning of the matrix and resets the Boolean field “converge” of all objects and the number of the nodes that converged. The program ends when all nodes in the model converge.

Finally, after the convergence is satisfied for main function, i.e. all nodes in the matrix structure converged, temperature of the each node is written in the output file. Output file type is chosen as an Microsoft Office Excel file since each node in the model can be represented as single cell on the same sheet of the Excel file, which allows the temperatures of the nodes to be read easily. Figure 4.2 shows the general flow chart of the program.

4.3 FUNCTIONS

In the program, developed in this study, five important functions are used. Four of them are used to obtain the energy balance for the related four different types of nodes whereas the last one is used to solve the calculated energy balance equations. The functions that calculate the energy balance equations are calEqnOuterSurface, calEqnFluid, calEqnCore and calEqnWinding. As mentioned in Section 4.1, there are four different classes which results in 4 different types of objects whose properties are different from each other. Although, these four functions are very similar since the main aim for developing these are the same: Calculating the energy balance equation of the node for the condition of iteration and returning the coefficients of it in an array,

they are decomposed by the fact that each type of object has different properties that is used while calculating the energy balance equations.

Functions, calEqnFluid, calEqnCore and calEqnWinding, have the same algorithm: Check the adjacent nodes, nodes next to the right, up, left and down. If the adjacent node is the same type for a single direction, only conduction mode of heat transfer occurs and the energy balance equation is formed in this way, like given in equation 2.28 Otherwise, if the adjacent node is different, the boundary condition should be considered. As a result, convective heat transfer mode should be taken into account and the heat transfer equation is in the form of equation 2.32.

At the beginning of the function, array that contains the coefficients of the energy balance equation is reset for safety during the calculation. After that, type of right adjacent node is checked. If the types of two nodes are the same, or both nodes are solid types, core and winding for instance, the equation is formed in the way of conduction heat transfer mode only. During the forming of the equation, for the fluids, thermal conductivity, k, changes significantly with the changes in temperature. Because of this, thermal conductivity of the fluid, i.e. SF_6 gas, is calculated every time before it is used.

The function calculates the thermal conductivity of SF6 gas is given in Appendix A-1. For other thermal conductivities used in the model, iron thermal conductivity, copper thermal conductivity and outer surface material thermal conductivity are assumed constant.

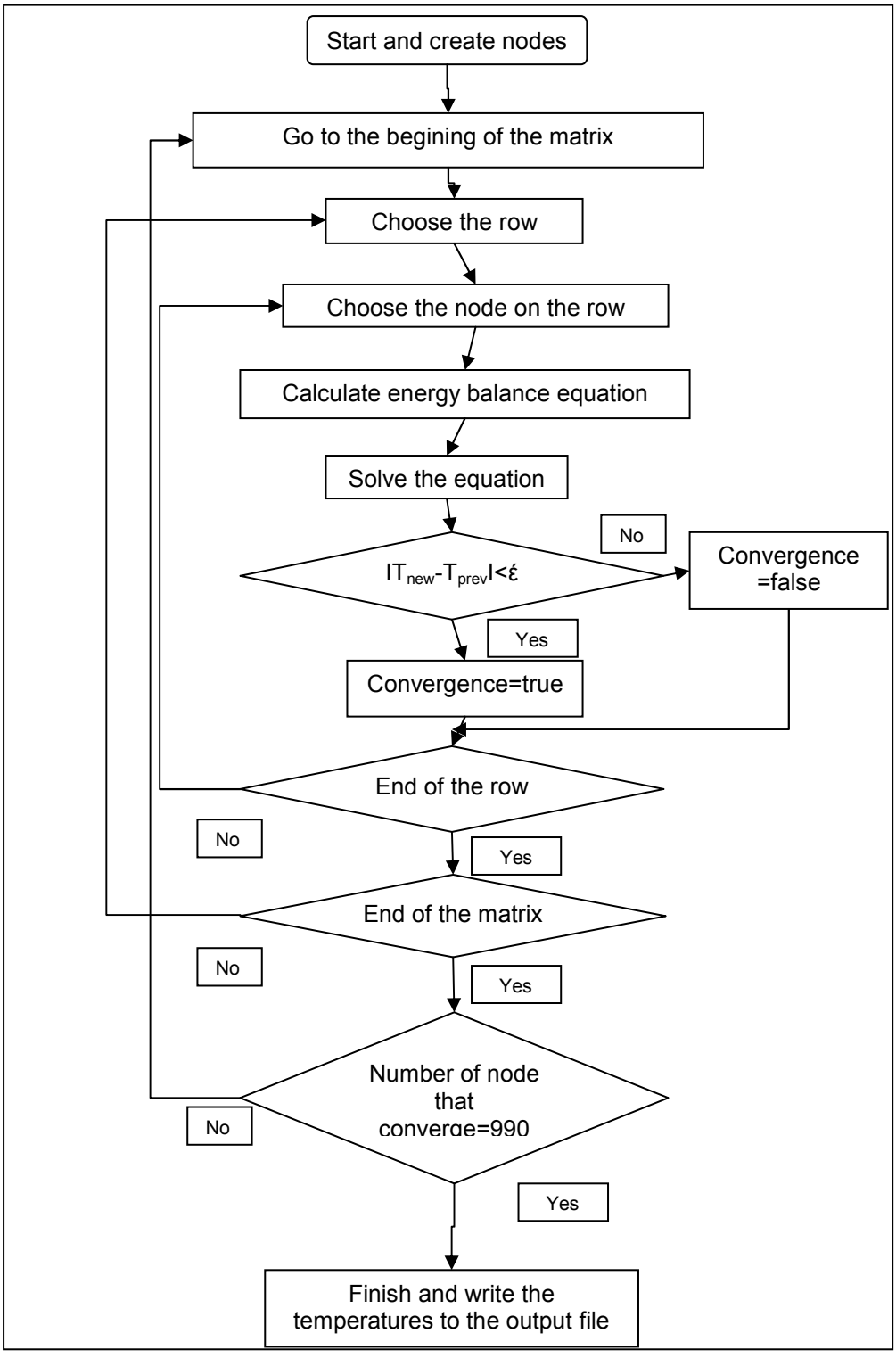

Figure 4.2 - Flow chart of the program

If one of the two adjacent nodes is solid, in other words core, winding or outer surface material and the other node is fluid; there is a boundary. Due to this boundary, convection should be considered for the heat transfer process. For the convection, heat transfer coefficient, h, is calculated every time before it is used since it changes with the changes in temperature for the fluids. Details of calculating the heat transfer coefficient is given Section 2.1.2.

After creating the energy balance equation as a result of the interaction of the node with the adjacent one, located right side, functions repeat this process for the other three directions, up, left and down and add the coefficients of the energy balance equations that are calculated to the related elements of the array. Finally, functions return this array in order to calculate the new temperature of the node.

The function that forms the energy balance equation for the outer surface material is calEqnOuterSurface. This function is also very similar to the three functions explained above. However, the difference is that adjacent nodes for the tank is known due to the geometry of it. For the upper surface, right and left adjacent are also the outer surface nodes. Up adjacent is air and down adjacent is fluid node. Similarly, for the side and bottom surface, up and down adjacent nodes are also known. Due to this geometric simplicity, creating energy balance equation for a node is much simpler than the other three functions. At the end of the function, array that contains the coefficients of the energy balance equation is returned in order to calculate the temperature of the node as done in the three other functions.

There is another main function that is used for solving the energy balance equation created by the calEqnOuterSurface, calEqnFluid, calEqnCore and calEqnWinding functions. Equation is solved in an iterative way by using Secant method. Initial guess is taken as the temperature value of the node calculated previously. The function returns a value and this value is assigned to the new temperature value of the related node.

The five functions explained above, are the main functions used in the program. Dividing the program into five main function decreases the effort to implement the program since implementing the small steps needs less effort and time compared to the implementing the whole program. In addition, dividing the program into small steps gives chance to developers for increasing the modularity, using the part of the program in other programs, which is desired.

CHAPTER 5

RESULTS AND CONCLUSIONS

5.1 VERIFICATION OF THE MODEL

The model developed in this thesis is checked by comparing the results obtained with the data given by the designers of the transformer [16] and the results of the previous study which is based on FEM [1]. The steady-state hottest winding temperature is given as 84.5°C by the designers when the transformer is loaded at its rated value and 13°C ambient temperature [16]. It was predicted as 89.5°C in the previous study [1] for the same ambient temperature and the loading condition. It is predicted as 84°C in this study and the temperature distribution in the transformer under conditions stated is given in Figure 5.1. In addition, for the 112 percent of load and at the same ambient temperature, the steady-state hottest winding temperature is given as 100°C by the designers [16]. It was calculated as 105°C in the previous study [1] and it is predicted as 113°C in this study. Considering these results, it can be concluded that at the rated load, the hottest winding temperature of the transformer predicted in this study is much closer to the designers' data.

5.2 OBSERVATIONS

The results for different loading and ambient conditions and the interpretations and observations on these are given in this chapter.

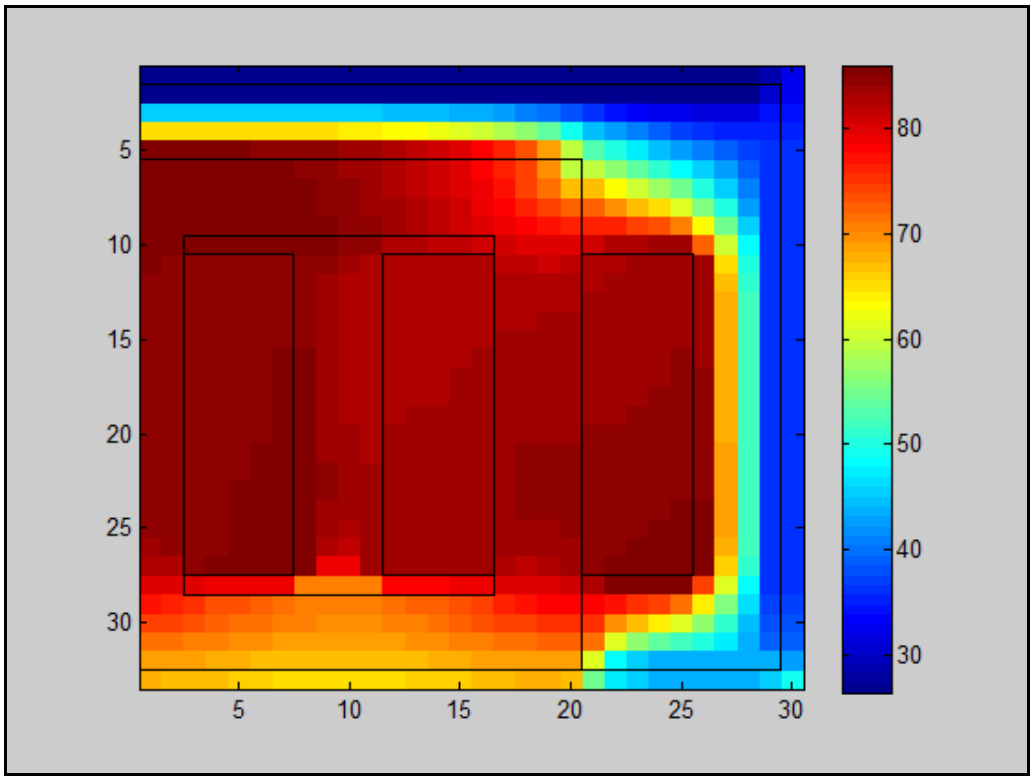

Figure 5.1- Temperature distribution in the transformer for full load and 13 °C ambient temperature.

The color mapping method is used to show the temperature distribution. This method decides the colors of the related location, i.e. the incremental area need in FDM, according to the function that takes the

temperature value of the node as input and returns the related color for this value. Color mapping method is used by most commercial analysis programs as well as MATLAB ©. In this study, MATLAB © is used to show the temperature distribution in the transformer.

As seen in Figure 5.1, the hottest parts of the transformer are the windings as expected due to the heat generated by the copper losses and the hottest winding is the one located on the symmetry axis on which there is no heat transfer. Temperatures of the nodes on the core close to the symmetry axis becoming hotter. The bottom of the transformer is assumed as semi-insulated as explained in Chapter 3, whereas heat transfer into ambient is still remarkable. Therefore, the temperatures at the bottom of the transformer tank vary in the range of 60-70°C and the temperatures at the bottom of the core are in the range of 65-75°C. Temperatures at the bottom of the core are low compared to the ones in the top side. It can be interpreted as the conduction occurred between the core and the tank is dominant compared to the heat convection occurring between the top of the core and the SF_6 gas. Finally, temperature distribution is the SF_6 gas is as expected since temperatures of the nodes closer to the core and winding are high and temperatures of the nodes closer to the tank are low.

It is also observed that the temperature of the node on the upper right corner of the core is lower than the temperatures value of the other nodes. At this corner node, the heat transfer to the gas medium is through two incremental surfaces whereas it is through one surface for the other nodes on the core boundary. However, no significant temperature decline in the corner of the windings is observed since the

loss per incremental area in the winding is almost 6 times greater than the loss that occurs in the core.

5.3 RESULTS FOR DIFFERENT AMBIENT TEMPERATURES AND LOADING CONDITIONS

Population on the earth is concentrated especially in the middle of the hemispheres since the climate in these places is much more suitable for living. Most of the European countries, most part of the United States and Turkey exist in this range. One of the most apparent properties of the climate of this middle range is that there are significant temperature differences between the seasons and the months. In addition, if the locations of the cities are far away from the coast, temperature differences between day and night in these places become larger. Ankara, the capital city of Turkey, can be a good example for this situation since the differences between the measured lowest temperatures and the highest temperatures can be in the range of 50°C [27]. Due to this variation in the ambient temperature, the devices whose performances depend mostly on ambient conditions, like transformers, are affected significantly. In this section, temperature distribution in the transformer for different loading conditions and different extreme ambient temperatures measured in Ankara is examined. The highest and the lowest temperatures measured in Ankara between the years 1975 and 2007 by months are given in Table 5.1.

Table 5.1- The highest and the lowest temperatures observed in Ankara in the range of 1975-2007 by months

Months	**The Lowest Temperature Value Measured Between 1975-2007 (°C)**	**The Highest Temperature Value Measured Between 1975-2007 (°C)**
January	-21.2	16.6
February	-21.5	19.9
March	-19.2	25.7
April	-6.7	30.3
May	-1.6	33
June	5	37
July	6.2	40.8
August	7.2	39
September	2.8	36
October	-3.4	32.2
November	-8.8	24.4
December	-14.6	18

Temperature distribution in the transformer for two extreme ambient temperatures, the lowest value (-21.5°C, February) and the highest value (40.8°C, July) when the transformer is loaded at 100 percent of its rated value are given in Figure 5.2 and Figure 5.3, respectively.

For the lowest ambient temperature, temperature distribution is such that the temperatures of side and top walls of the tank are around 0°C, whereas bottom of the tank is around 30°C since there is a direct contact between bottom of the core and bottom of the tank. Temperatures of the nodes in the core vary in accordance with the location of the node on the core since nodes on the left upper side and the left side are hotter then the nodes on bottom. For the winding, the highest temperature predicted is around 73 °C.

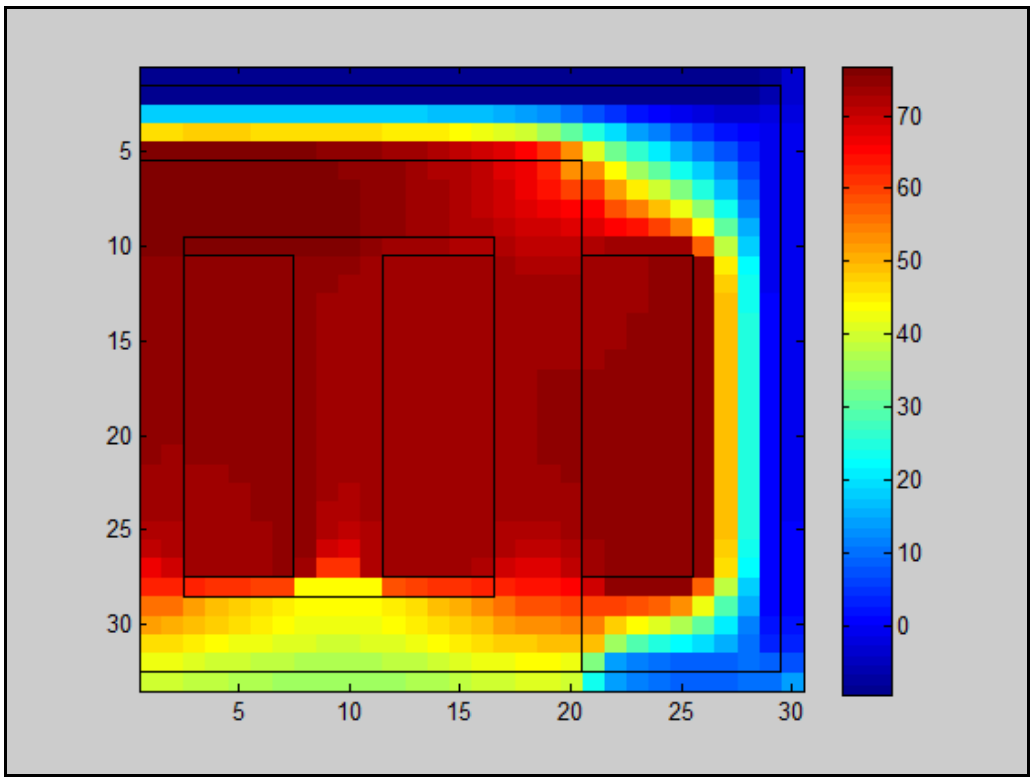

Figure 5.2- Temperature distribution in the transformer for full load and the lowest ambient temperature measured in Ankara (February)

For highest ambient temperature, 40.8°C, temperatures of the nodes on top of the tank are around the 45°C, whereas, temperatures of the nodes on side of it are close to 60°C. In addition, at the bottom of the tank, temperatures are close to 80°C because of the assumption that bottom side of the tank is semi-insulated. The highest gas temperature is around 80°C and the hottest winding temperature is 92°C. Temperature distribution in the transformer is shown in Figure 5.3

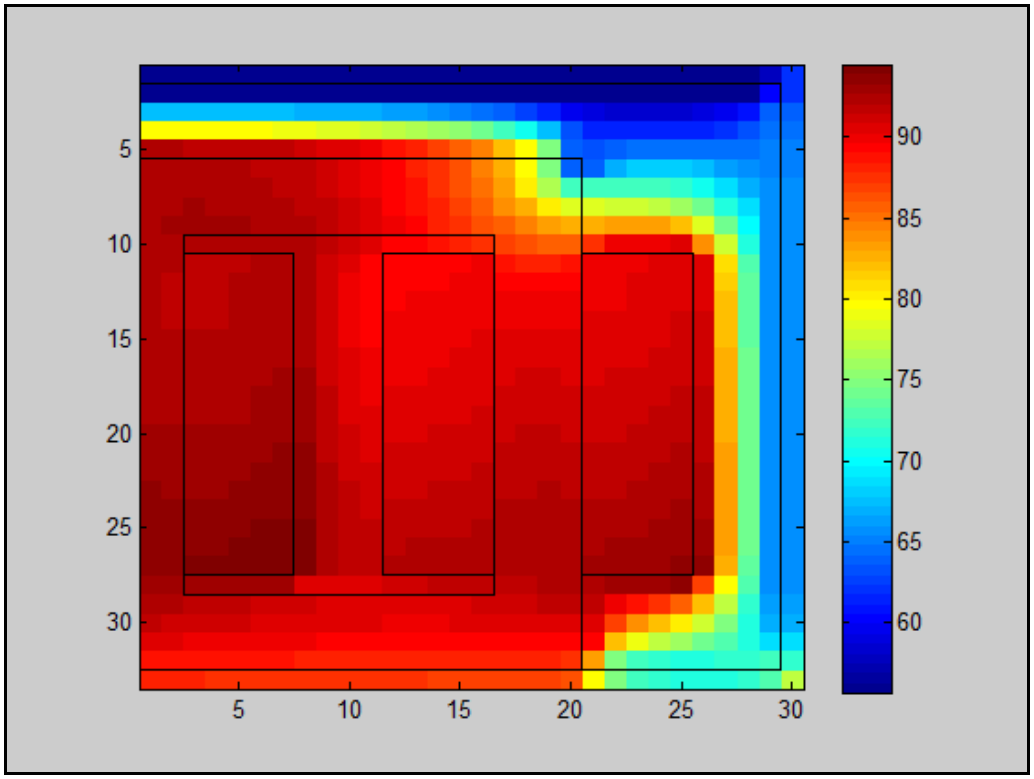

Figure 5.3- Temperature distribution in the transformer for full load condition and the highest ambient temperature measured in Ankara (July)

The hottest winding temperatures for the 100 percent of rated load and the extreme ambient temperatures by months are computed and given in Figure 5.4. When it is examined, the hottest winding temperature according to the lowest and the highest ambient temperatures changes significantly. The lowest and the highest values are 73°C and 92°C, respectively.

The hottest winding temperatures for the highest ambient temperatures of all months excluding the winter season are found just above the allowable maximum temperature on the winding, i.e. 88°C, by the designers [16]. In order to prevent this situation, some precaution should be taken such as decreasing the loading of the transformer.

It is also important to examine the variations of the hottest winding temperatures for different loading conditions at the ambient temperatures of 13°C and 40.8°C. It is known that loss occurring on the winding is called copper loss and it results from the resistance of the copper. Copper loss is calculated as given

$$W_{loss} = I^2 R \qquad \text{(Eqn. 5.1)}$$

It can be concluded that copper loss changes exponentially with the linear change of loading. In Table 5.2, copper losses and related loading ratios are given.

In Figure 5.5, copper losses versus hottest winding temperature are given for the ambient temperatures of 13°C and 40.8°C. When the copper loss is in the range between 60 and 75 percent of its rated value, the rate of rise of the hottest winding temperatures is small, even it is almost constant for 40.8°C ambient. In this range, the differences

between the two hottest winding temperatures for the same loading conditions and different ambient temperatures are around 20°C. It can be interpreted that the transformer loading is less effective compared to ambient temperature on the hottest winding temperature. In the range between 80 and 100 percent of loss, the rate of rise of the hottest winding temperatures is, however, significantly high, especially after the 85 percent. In addition, the difference between the hottest winding temperatures for two different ambient conditions decreases as the transformer loading increases. For this range, it is concluded that loading is dominant compared to the ambient temperature in determining the hottest winding temperatures.

Table 5.2- Cooper losses according to the ratio of transformer loadings

Transformer Loading (%)	**Copper Loss (%)**
0,77	0,60
0,81	0,65
0,84	0,70
0,87	0,75
0,89	0,80
0,92	0,85
0,95	0,90
0,97	0,95
1,00	1,00

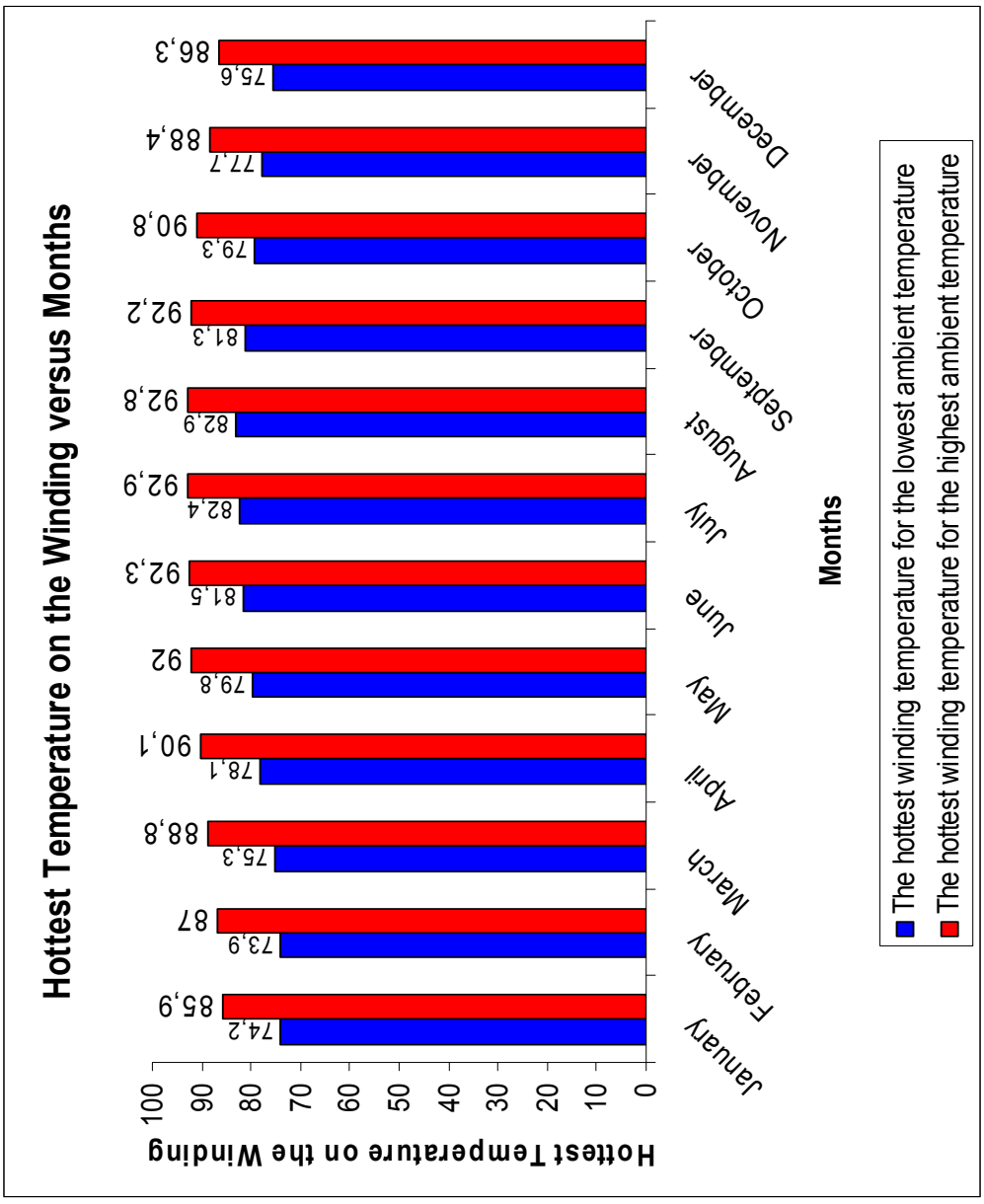

Figure 5.4- Predicted hottest winding temperature value of the transformer by months

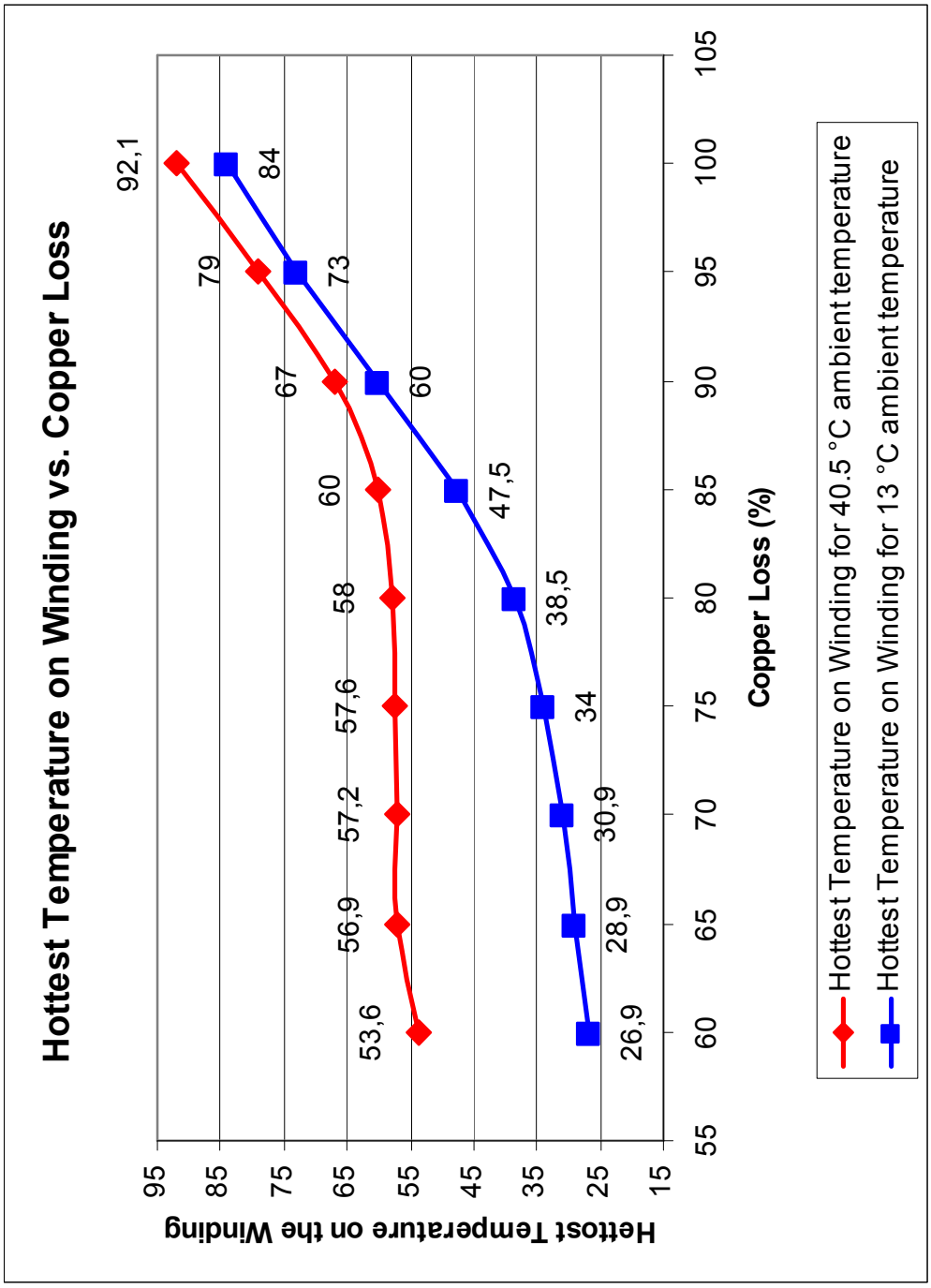

Figure 5.5- Hottest winding temperature for different loading conditions for the ambient temperatures 13 °C and 40.8 °C

5.4 CONCLUSIONS

Power transformers are one of the vital components for the transmission and distribution systems. Being out of service of a power transformer might cause serious problems for transmission and distribution systems which is an undesirable situation. Precautions should be taken to prevent these situations in design stage.

One important factor that causes problems in power transformers is the heat produced in their core and windings. Produced heat beyond the thermal limits in these components might result in degradation in transformer performance or, even worse, might cause serious permanent damage. Predicting the temperature distribution in the transformer under different load and ambient conditions is important to prevent serious damage and lengthen the service life of the transformer. Different models based on various approaches have been developed for the purpose of predicting temperature distribution in the transformer.

In this study, a 2-D, FDM based model that predicts the steady-state temperature distribution in a transformer is proposed. According to finite difference approach, the whole cross-sectional area is divided into incremental areas and for each, energy balance equation is written in terms of the temperatures of the nodes which are located at the center of incremental areas. Secant method is used to solve the energy balance equations obtained. Thermal model used in this study is based on the conduction mode of heat transfer in the solid media and gas media, the convection mode of heat transfer at the boundaries between

the solid and gas media. A computer program is developed in Microsoft Visual C# platform to predict the temperature distribution in a three-phase power transformer which is rated 30 MVA and SF_6 gas insulated-cooled. The results obtained are compared with the data released by the designers of the transformer [16] and also compared with the results of a previous study [1] which is based on the FEM. The results show that, the hottest winding temperature is closer to the designers' data within 1 percent for the 100 percent of rated load. Under the loaded condition, for the 112 percent of rated load, the hottest winding temperature is 13 percent higher than the designers' data. These differences are 6 percent higher in [1].

There are some remarkable observations about the results. One of them is that ambient temperature affects the temperature distribution in the transformer as expected. Another observation is that when copper loss is in the range of 60 and 75 of its rated value, the variation of the maximum temperature of the winding is almost constant as seen in Figure 5.5. The different ambient temperatures result in difference between the hottest winding temperatures obtained for the same loads in this range. For this range, it can be seen that ambient temperature is dominant in determining the maximum temperatures of the winding. On the other hand, the rate of rise of maximum temperatures is considerably high when the copper loss is in the range of 80 and 100 percent. Difference between the maximum temperatures on the winding becomes closer for different ambient temperatures in this range. Therefore, it can be concluded that loading is dominant compared with ambient temperature when it is closer to the rated value.

In conclusion, as demonstrated in this study, a 2-D model that estimates the steady-state temperature distribution in a transformer, based on FDM is an acceptable model. This study can be further developed by using a 3-D steady-state model to improve the accuracy. Power transformers might be subjected to short circuit currents. Although these currents exist for short durations, they affect the temperature distribution in transformers. Thus, estimation of the temperature distribution under short circuit conditions can also be considered as a further study. Finally, although the effect of radiation boundary condition is small compared with convection boundary condition in determining the temperature distribution in the transformer, it might be taken into account and calculations related to radiation is done with the help of concept “view factor”.

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APPENDIX A

THERMO PHYSICAL PROPERTIES OF SULFUR HEXAFLUORIDE (*SF_6*) GAS

A.1-THERMAL CONDUCTIVITY OF SULFUR HEXAFLUORIDE (SF_6) GAS

Thermal conductivity of the gases changes with temperatures. Table A.1 shows the value of the thermal conductivity of the SF_6 gas [20].

Table A.1- Thermal conductivity of SF_6 gas

Temperature [^oC]	Thermal Conductivity x 10^4 [W/cm K]
0	1.0
25	1.3
100	1.9
200	2.5
300	3.1
400	3.6
500	4.1

The unit given here is W/cm K but in the formulation used in the model requires the thermal conductivity in unit of W/m K. As a result, the values calculated are converted to the desired unit.

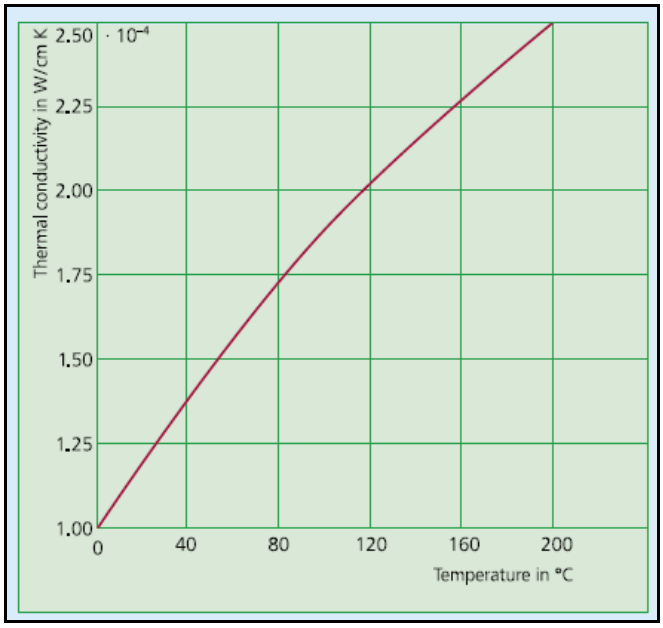

Figure A.1- Thermal conductivity of SF_6 gas with different temperatures

The function used to calculate the thermal conductivity of SF_6 gas for specific temperature in the program is given in Figure A.2. It is important to mention that the function calKSF6 takes the temperature in Kelvin and it calculates the new value of thermal conductivity. However, if the temperature is below 0^oC, the function returns a constant value, 0.01 W/m K. Thermal conductivity is re-calculated with the previous temperature before it is used in the equation.

```
public static double calKSF6(double temp)
{
    double cond = 0;

    if (temp < 273)
    {
        cond = 0.01;
    }

    else
    {
        cond = (1 + (0.0094 * (temp - 273))) / 100;

    }
    return cond;
}
```

Figure A.2- Function used for calculating the thermal conductivity of SF_6 gas for a given temperature

A.2-CALCULATING THE PRANDTL NUMBER (PR) AND REYNOLDS NUMBER (RE) FOR SF_6 GAS

In order to calculate PR and RE, first of all, viscosity and kinematic viscosity must be calculated. Viscosity is the measure o the resistance of a fluid which is being deformed by either shear stress or extensional stress [28]. Kinematic viscosity is however he ratio of the viscous force to the inertial force, the latter characterized by the fluid density. Kinematic viscosity is defined as [28]

$$\nu = \frac{\mu}{\rho} \qquad \text{(Eqn. A.1)}$$

where ν is the kinematic viscosity, μ is the dynamic viscosity and ρ is the fluid density. Viscosity of the SF_6 gas is given in [20] and it is

linearized as given in Figure A.3. Note that function viscosity takes the temperature and returns the viscosity of SF_6 gas for this temperature.

```
        public static double viscosity(double temp)
        {
            double v = 0;

            if (temp < 273)
            {
                v = 0.0000141;
            }

            else if ((temp >= 273) && (temp < 298))
            {
                v = 0.0000141 + (((0.0000153 - 0.0000141) /
25) * (temp - 273));
            }

            else if ((temp >= 298) && (temp < 373))
            {
                v = 0.0000153 + (((0.0000186 - 0.0000153) /
75) * (temp - 298));
            }

            else
            {
                v = 0.0000186 + (((0.0000228 - 0.0000186) /
100) * (temp - 373));
            }
            return v;
        }
```

Figure A.3- Function used for calculating viscosity of SF_6 gas for a given temperature

Kinematic viscosity is calculated according to the equation A.1. Pressure of the SF_6 gas is given in [16] as 240 kilo Pascal. This pressure value is very close to 2.2 atmospheric pressure. Density of the

SF_6 gas at 20^{o}C is 2.2 atmospheric pressure is given as 13.73 kg/m^3 [29] Therefore density of the SF_6 gas at 240 kilo Pascal pressure is taken as 13.73 kg/m^3.

Prandtl number (PR) is the ratio of kinematic viscosity and thermal diffusitivity. PR is given in [30] for different temperature values. When it is linearized, the result is a function that takes temperature and returns the PR value for that temperature. The function is given in Figure A.4.

Reynolds number is a dimensionless number that gives a measure of the ratio of inertial forces to viscous forces and consequently quantifies the relative importance of these two types of forces for given flow conditions [31]. It is calculated for a flat plate as

$$RE = \frac{VL}{\mu} \qquad \text{(Eqn. A.2)}$$

where RE represents the Reynolds number, V represent the velocity of the fluid, L represents the length of the object that the flow is going through or around and μ represents the kinematic viscosity of the fluid. In order to calculate the RE, kinematic viscosity must be calculated first. As seen in the function calculates the RE given below, kinematic viscosity function is called with the temperature parameter and returns kinematic viscosity number. This number is used for calculating the RE for obtaining the energy balance equation which is developed for getting the new value of the temperature of the node. Note that function given below takes the dimension and temperature values as input parameters and returns the value of the RE for these input parameters.

```
public static double prandtlNumber(double temp)
        {
            double pN = 0;

            if (temp < 273)
            {
                pN = 0.8296;
            }
            else if ((temp >= 273) && (temp < 293))
//0-20 derece arası
            {
                pN = 0.8296 - (((0.8296 - 0.7972) / 20) *
(temp - 273));
            }
            else if ((temp >= 293) && (temp < 313))
//20-40 derece arası
            {
                pN = 0.7972 - (((0.7972 - 0.7780) / 20) *
(temp - 293));
            }
            else if ((temp >= 313) && (temp < 333))
//40-60 derece arası
            {
                pN = 0.7780 - (((0.7780 - 0.7751) / 20) *
(temp - 313));
            }
            else if ((temp >= 333) && (temp < 353))
//60-80 derece arası
            {
                pN = 0.7751 - (((0.7751 - 0.7728) / 20) *
(temp - 333));
            }
            else if ((temp >= 353) && (temp < 373))
//80-100 derece arası
            {
                pN = 0.7728 - (((0.7728 - 0.7710) / 20) *
(temp - 353));}
            else
            {
                pN = 0.7710 - (((0.7728 - 0.7678) / 50) *
(temp - 373));
            }
            return pN;
        }
```

Figure A.4- Function used for calculating Prandtl Number of SF_6 gas for a given temperature

```
        public static double reynoldsNumber(double T, double
dim)
        {
            double rN = 0;
            double kinematicVis = kinematicViscosity(T);
            rN = SF6Velocity * dim / kinematicVis;
            return rN;
        }
```

Figure A.5- Function used for calculating Reynolds Number of SF_6 gas for a given temperature

APPENDIX B

THERMO PHYSICAL PROPERTIES OF AIR

In order to calculate Nusselt number for natural convection of air, PR, RA and GR must be calculated. In the model, it is assumed that PR does not change significantly; therefore it is taken as 0.713, in other words, as a constant value [21].

GR approximates the ratio of the buoyancy to viscous force acting on a fluid [32]. It is calculated as

$$GR = \frac{g\beta(T_1 - T_2)L^3}{\nu^2} \qquad \text{(Eqn. B.1)}$$

where g is the gravity which is approximately 10 m/s^2, β is the volumetric thermal expansion coefficient (equal to approximately 1/T, for ideal fluids, where T is absolute temperature), T's are temperatures of surface and fluid, L is the length of the surface and ν is the kinematic viscosity of the fluid. Note that, kinematic viscosity of air is necessary to calculate the GR. Kinematic viscosity of the air was obtained form [33]. Figure B.1 shows the kinematic viscosity of air versus temperature distribution graph.

Although, the air temperature does not change for a single convergence, the program was used for different air temperature values; as a result, kinematic viscosity is calculated once at the beginning of the program.

RA is defined as the product of the Grashof number, which describes the relationship between buoyancy and viscosity within a fluid, and the Prandtl number, which describes the relationship between momentum diffusivity and thermal diffusivity. Hence the Rayleigh number itself may also be viewed as the ratio of buoyancy forces and (the product of) thermal and momentum diffusivities [34]. RA is calculated as

$$RA = GRxPR \qquad \text{(Equation B.2)}$$

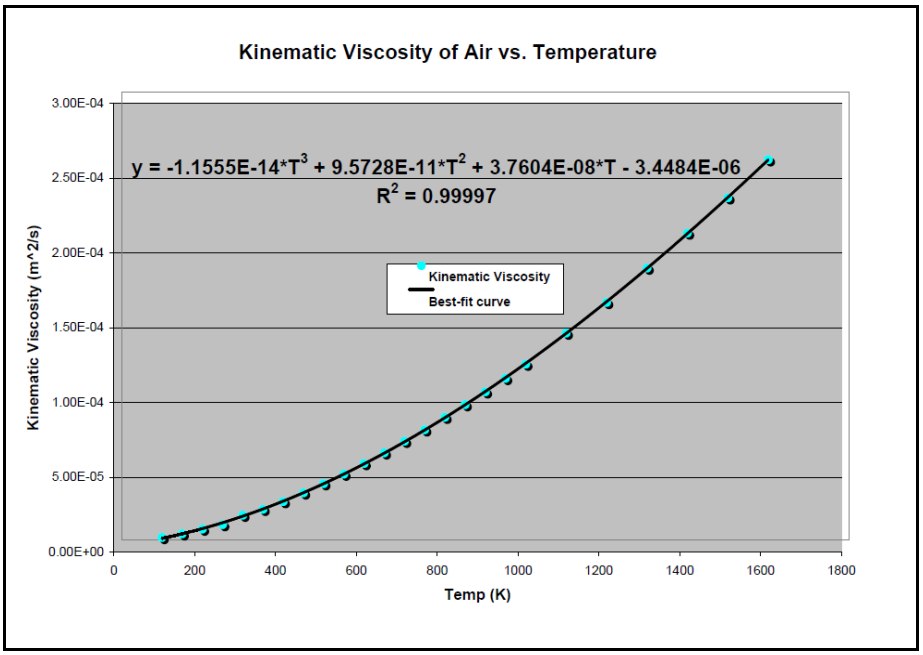

Figure B.1- Kinematic viscosity of air versus temperature