ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE ENGINEERING AND TECHNOLOGY

AN INTUITIONISTIC FUZZY RULE-BASED APPROACH TO FMEA

M.Sc. THESIS

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ISTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ

HATA TÜRLERİ VE ETKİLERİ ANALİZİNE KURAL TABANLI SEZGİSEL BULANIK MANTIK YAKLAŞIMI

YÜKSEK LİSANS TEZİ

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Muhammet Nur Seyda KARATEPE, a M.Sc. student of İTU Graduate School of Science Engineering and Technology student ID 507141216 successfully defended the thesis/dissertation entitled "AN INTUITIONISTIC FUZZY RULE-BASED APPROACH TO FMEA", which he prepared after fulfilling the requirements specified in the associated legislations, before the jury whose signatures are below.

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Dedicated to my Father Hamza, my Mother Aynur, and to my spouse to be Kübra,
also special thanks to my cat Gölge, my brother Burak and my sister Zehra,

FOREWORD

I had outstanding experience at my master programme at İstanbul Technical University. I am simply grateful to my advisor Assoc. Prof. Dr. Umut ASAN. Your valuable lessons and directions guided me to complete my research. Thank you for believing in me.

May 2019

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Industrial Engineer

TABLE OF CONTENTS

Page

ABBREVIATIONS

AI	**:** Artificial Intelligence
D	**:** Detection
FRB	**:** Fuzzy Rule Based
FRBS	: Fuzzy Rule Based System
FMEA	**:** Failure Mode and Effects Analysis
FS	**:** Fuzzy Sets
H	**:** High
HW	**:** Hardware
IFRB	**:** Intuitionistic Fuzzy Rule Based
IFRBS	: Intuitionistic Fuzzy Rule Based System
IFS	**:** Intuitionistic Fuzzy Sets
L	**:** Low
M	**:** Medium
MCDM	: Multi-Criteria Decision Making
O	**:** Occurrence
RPN	**:** Risk Priority Number
S	**:** Severity
SW	**:** Software

SYMBOLS

$\boldsymbol{\mu}$	**:** Degree of Membership
$\boldsymbol{v}$	**:** Degree of Non-Membership
$\boldsymbol{\pi}$	**:** Hesitancy Index, Degree of Uncertainty

LIST OF TABLES

Page

LIST OF FIGURES

Page

AN INTUITIONISTIC FUZZY RULE-BASED APPROACH TO FMEA

SUMMARY

With the advancing technology, systems are getting more complex. Regardless of the industry, failures will hinder the demanded performance criteria and operations and cease the required functionality. Failure Mode and Effects Analysis (FMEA) is developed in order to evaluate, assess and analyze potential failures in the system and in time it has become state of the art methodology, which is now vastly used in most of the industries. FMEA considers three important factors, which are the severity of the failure, likelihood of detection and probability of occurrence for each potential failure mode. Even though traditional FMEA is being used extensively in many areas for evaluating failure modes and risks, FMEA usually involves human judgments and linguistic expression in real life combining with lack of information and imprecise data available in the systems, which in return creates uncertainty and vagueness in the system. Using traditional risk priority number (RPN) calculation method with crisp values cannot represent the uncertainty in risk assessment. In addition to that, several other shortcomings of the traditional FMEA is reported by several studies in the literature. To deal with these problems, an intuitionistic fuzzy rule-based approach (IFRB) for assessing risk prioritization in FMEA with Takagi Sugeno method is proposed in this thesis. IFRB takes account of non-membership degree as well as membership degree which in allows considering a degree of hesitancy to represent indeterminacy and uncertainty in a more consistent and comprehensive way. IFRB has three basic components. These components are; fuzzification of Severity (S), Detection (D) and Occurrence (O) inputs, rule-based inference engine and defuzzification to risk priority number replacing traditional multiplication of S, D and O factors. The proposed approach mimics how FMEA experts express their knowledge, experiences, and behavior in real life. Result of this study showed that an intuitionistic fuzzy rule-based approach to FMEA can solve several critiques, it can put a better overall performance and represent ambiguity and uncertainty in a more extensive and compatible way compared to traditional RPN method and classical fuzzy inference system.

HATA TÜRLERİ VE ETKİLERİ ANALİZİNE KURAL TABANLI SEZGİSEL BULANIK MANTIK YAKLAŞIMI

ÖZET

Gelişen teknoloji ile sistemler daha karmaşık hale geliyor. Hangi sektörde olursa olsun istenen performans ölçütlerini ve operasyonlarını engelleyecek ve gerekli işlevselliği durduracak hatalar ve türevleri buluncaktır. Hata Türü ve Etkileri Analizi (HTEA), sistemdeki potansiyel hataları değerlendirmek ve analiz etmek için geliştirilmiştir ve zamanla, çoğu endüstri tarafından sıklıkla kullanılan yöntem haline gelmiştir.

Hata Türü, bir sistemin çalışma davranışı ve müşterileri üzerinde etkisi olan olası bir hata veya hata türevidir. Etki analizi, bu başarısızlıkların sonuçlarını ve etkilerini inceler. Hata Türü ve Etkileri Analizi, her bir potansiyel hata türü için üç önemli faktörü göz önünde bulundurur. Bunlar, hatanın etkisi ve şiddeti (Ş), hatanın tespit edilebilirliği (T) ve hatanın meydana gelme olasılığıdır (O). Bu faktörlerin ilgili uzmanlar tarafından atanan dereceleri kullanılarak her potansiyel hata türü için Risk Öncelik Puanı (RÖP) hesaplanır ve bu puan potansiyel hata türlerinin ne kadar risk teşkil edildiğini belirlemek ve hata türlerini kendi aralarında önceliklendirmek için kullanılır. RÖP'nın yüksekliği hata türünün riskinin yüksekliğini gösterir.

Hata Türü ve Etkileri Analizinin temelleri 1940'larda ABD savunma endüstrisi tarafından sistem ve ekipman hata türlerini değerlendirmek için atılmış ve Amerikan ordusu bu tekniği ordu standartlarına MIL-STD-1629 prosedürü ile eklemiştir Daha sonra NASA'da (The National Aeronautics and Space Administration), 1963'te ve 1973'te Apollo'yu geliştirirken uzay sistemlerini olduğundan daha güvenilir hale getirebilmek için Hata Türü ve Etkileri Analizini kullanmıştır. 1970'lerde Ford Motor Company, otomotiv endüstrisinde Hata Türü ve Etkileri Analizi tekniğinin uygulanmasına öncülük etmiştir. Bu metodoloji sayesinde medikal, havacılık, savunma vb. endüstriler de kendi güvenilirlik standartlarını geliştirmiş ve methodun evrenselleşmesini sağlamıştır.

Hata Türü ve Etkileri Analizi, Sistem Hata Türü ve Etkileri Analizi, Tasarım Hata Türü ve Etkileri Analizi, Süreç Hata Türü ve Etkileri Analizi ve Hizmet Hata Türü ve Etkileri Analizi olmak üzere dört tipte sınıflandırılabilir.

Günümüz dünyasında HTEA, kuruluşlar için kalitelerini, güvenilirliklerini ve emniyetlerini artırmaya yönelik bir çok avantaj sağlar. Bu avantajlar aşağıda ki gibi sıralanabilir;

- Bir ürünün veya hizmetin her bir alt sistemi üzerinde farklı bilgi ve disipline sahip farklı uzmanlar arasındaki iletişimi arttırır. Müşterilerin ihtiyaçlarını analiz etmek ve tanımlamak için çeşitli bilgilere sahip tedarikçilerin bir araya getirilmesini sağlar.
- Her türlü uzman yardımıyla, karmaşık sistem hatalarına daha basit çözümler bulabilir.

- Tüm olası hata türlerini, bunların ilişkilerini, sebeplerini ve tüm sistem üzerindeki etkilerini ele almak için çok etkili bir araçtır.
- Gerçekçi ve güvenilir hata türlerinin tanımlanması, güvenilir hizmet ve ürünlerin oluşturululmasını, garanti ve geliştirme süreçlerinin iyileşmesini sağlar. Böylece artan kalite ile müşteri memnuniyeti de arttırılır.
- Anlamayı kolaylaştırması ve görselleştirmesi sayesinde verimli araçtır.
- Kullanıcı odaklı olduğu için etkili bir eğitim aracıdır.

Buna ek olarak, literatürdeki çeşitli çalışmalarda geleneksel Hata Türü ve Etkileri Analizi için bazı eksiklikler dezavantajlar mevcuttur. Bu çalışmanın odaklandığı sorunlar aşağıdaki gibi sıralanabilir.

- Geleneksel HTEA’da, her hata türü için Ş, T ve O faktörleri uzmanlar tarafından doğal insan davranışları, düşünme biçimi ve sözsel ifadeleştirmesi ile derecelendirilir ve bu sistemde belirsizlik oluşturur. Sözsel ifadeler ile derecelendirilmesine rağmen net matematiksel değerleri olan geleneksel Risk Öncelik Puanı hesaplama yöntemini kullanmak, risk değerlendirme sürecinde oluşan belirsizliği temsil edemez. Diğer yandan bu aşamada uzmanlar tam, yeterli, doğru ve kesin bilgiye de genellikle sahip değildir, bu da bir kararsızlık ve tereddüt oluşturur. Bu noktada, Hata Türü ve Etkileri Analizinin uzmanların kararsızlığını ve sistemde ki belirsizliği hesaba katmadan mevcut ve olası hata türleri hakkında risk ve kalite analizi yapmaya çalıştığını belirtmekte fayda var. Tam, yeterli ve kesin bilgiyi elde edebilmek bazen mümkün olsada, bunu başarabilmek çok fazla kaynak gerektirir. Bu kaynaklar sadece maliyet ve zaman değil aynı zamanda işlem kabiliyeti ve donanım yeterliliği ile de sınırlıdır. Ve çoğu zaman, tam ve kesin bilgiye sahip olmanın getireceği katma değer harcanacak kaynak için yeterli değildir.
- Her faktörü basitçe çarpmak, bu faktörler arasındaki bağımlılığı görmezden gelir. Ş, T ve O farklı ortamlarda farklı önem veya ağırlıklara sahip olabilir.
- İlk bakışta, RÖP'ün 1'den 1000'e kadar bütün değerleri alabildiği zannedilebilir. Ancak, 1 ila 10 arasında sadece tam sayı değerleri alabilen 3 tam sayının, Ş, T ve O’nun, çarpımı, 1000 değerden yalnızca 120sini üretebiliyor. Bu, kalan 880 değerinin ölçekte üretilemediğini gösterir. RÖP değerleri sürekli değildir. Alınabilen değerlerin %94’ü 500’ün altındadır. Ve alınabilen değerlerin ortalaması zannedilebileceği gibi 500 değil, 166’dır.
- Ş, T ve O’nun çarpımı ile üretilebilen 120 RÖP değerinden sadece 6 tanesi Ş, T ve O faktörlerinin tek bir çarpım kombinasyonu ile üretilebilir. Kalan 114 değer, en az Ş, T ve O faktörlerinin 3 farklı kombinasyonu ile üretilebiliyor. Bu, her bir faktörün farklı etkilerinin göz ardı edilmesine yol açar. Örnek olarak, 60 değeri, Ş, T ve O faktörlerinin 24 farklı kombinasyonu ile hesaplanabilir.
- Normalde, sıralı ölçekler arasında matematiksel işlemler gerçekleştirilemez, çünkü sıralı ölçekteki her bir sıra arasındaki fark miktarı subjektiftir. Bu fark ölçülemez ve her bir sıra arasındaki fark aynı değildir. Ancak geleneksel HTEA’da Ş, T ve O faktörleri sıralı ölçekler kullanıyor ve bu 3 sıralı ölçekle RÖP’ı matematiksel işlemlerle hesaplanıyor.

- Ayrıca, çarpma işleminin kendisi Ş, T ve O değerlerinde artış ve azalmaya karşı çok hassastır. Ayrıca, potansiyel RÖP değerlerinin sadece % 6'sı ile 500'ün üzerindedir (normalde 500, 1 ila 1000 arasında medyandır). Bu yüzden faktörler arasındaki ilişkiyi göstermek için çarpımın kullanılması sorgulanmalıdır.
- Ş, T ve O faktörleri arasında dolaylı bir ilişkiyi olabilir ve geleneksel HTEA bunu görmezden gelir.

Bu tezde, Hata Türü ve Etkileri Analizinde ki risk öncelik puanı hesaplamasına bir sezgisel bulanık mantık yaklaşımı öneriliyor. Önerilen yaklaşım, HTEA uzmanlarının bilgilerini, deneyimlerini ve davranışlarını gerçek hayatta nasıl ifade ettiklerini taklit etmektedir. HTEA'da Kural Tabanlı Sezgisel Bulanık Mantık yaklaşımı, normal bulanık mantıktan farklı olarak elemanın bir kümeye ait olma derecesini hesaba kattığı gibi bir elemanın kümeye ait olmama derecesini de hesaba kattığından, bir tereddüt derecesi hesaplar. Bu sayede belirsizliği, kesin olmayan, yanlış ve eksik bilgiyi daha tutarlı ve kapsamlı bir şekilde temsil edebilir. Sezgisel Bulanık Mantığın üç temel bileşeni vardır. Bunlar Bulanıklaştırma, kural tabanlı anlamlaştırma ve durulaştırmadır. 1 ile 10 arasında doğal sayı değeri alabilen Ş, T ve O'nun çarpımı yerine bulanıklaştırma, kural tabanlı anlamlaştırma ve durulaştırma sayesinde yukarıda bahsedilen diğer dezavantajlar kaldırılmış olur.

Bulanıklaştırma işlemi, ait olma ve ait olmama fonksiyonlarını kullanarak kullanıcının bilgilerini, deneyimini ve gözlemlenen verilerini bulanık hale çevirir. Bu aşamada Ş, T ve O'ya ait olma ve ait olmama fonksiyonlarının ve buna bağlı dilsel değişkenlerin tanımlanması gerekir. Bu fonksiyonlar sayesinde Ş, T ve O'ya atanan net değerler bulanıklaştırılır.

Sonrasında anlamlaştırma aşamasında kullanılacak olan "Eğer-O zaman" anlaştırma kuralları, dilsel değişkenlere göre türetilebilir, dolayısıyla, sistemin girdileri ve çıktıları arasındaki ilişki belirlenir. "Eğer-O zaman" kuralı "Eğer x1 A ise, o zaman x2 B'dir" olarak örneklenebilir. Buna göre, A ve B dilsel değişkenlerdir ve "Eğer x1 A ise" şart ve "O zaman x2 B'dir" sonuçtur. Dolayısıyla, bulanık bir kural, bir koşulu ve bununla ilgili sonucu içerir. HTEA için Ş, T ve O girdi, RÖP çıktısıdır. Anlamlaştırma kurallarından elde edilen çıktılar seçilecek methodalarla birleştirilir ve bir çıktı elde edilir.

Anlamlaştırma aşamasında türetilen sonuçlar hala bulanık bir değerdir. Net bir çıktı elde etmek için durulaştırılması gerekir. Bu amaç için, çıktıların ait olma fonksiyonu ve ait olmama fonksiyonu tersten kullanılır. Bu işleme durulaştırma denir. Bu işlem sonrasında net bir RÖP çıktısı elde edilir.

Bu yöntemle HTEA daha iyi bir genel performans sergileyecek ve uyumlu bir şekilde bilgi eksikliği, bilgi kısıtı, belirsizlik ve kararsızlıktan kaynaklanan problemleri çözecektir.

1. INRODUCTION

Having problems in systems is inevitable regardless of the area, region or industry. And companies have to evaluate the potential risks which threaten the system's quality and performance. However, there is an effective tool which is so-called "Failure mode, effects, and analysis" (FMEA) to analyze these failures and to minimize possible risks.

FMEA is one of the systematic techniques to detect and prioritize possible failures of a system or product. By this analysis, all failure modes (FM) are specified, verified and classified based on their significance and related risks on the performance of the system will be defined. Besides, causes and future effects of each FM are evaluated to take preventive actions and precautions, that help to improve the quality of the whole system.

The failure mode (FM) is a possible mode of a system's errors or failures having an impact on a system's working behavior and thereby on the customers. Effect analysis is the study to examine the outcomes and impacts of these failures. Risk priority number (RPN) is used to determine how much risk is posed by potential failure modes by considering severity, occurrence and detection factors of each FM (Chang et al., 2009).

First developments on FMEA date back to USA defense industry in the 1949s for evaluating their system and equipment failures. NASA used FMEA while developing Apollo to have reliable space systems in 1963 then in 1973, the American army used this technique in the army standard ''MIL-STD-1629: procedures for performing a failure mode effects and criticality analysis''. Ford Motor Company also pioneered practicing the FMEA technique in the automotive industry in the 1970s. Via this methodology, various industries including medical, aerospace, defense, etc. developed their own reliability standards. In today's technology, FMEA became a universal state of art methodology in different manufacturing and service industries (Chang et al., 1999; Asan and Soyer, 2015; Chang et al., 2009).

FMEA can be classified into four types which are System FMEA, Design FMEA, Process FMEA, and Service FMEA. The descriptions and general attributes of each are explained below.

(i) System FMEA is an analysis of failure modes at the system level, consisting of several subsystems. By this:

- open points and related functions on the entire system such as integration of subsystems, interactions of each subsystem, system reliability, the performance of the entire system
- interactions between the environment and human,
- working behaviors and interactions with other external systems

can be analyzed (Asan and Soyer, 2015).

(ii) Design FMEA is an analysis of failure modes at the design stages of the products. In other words, it focuses on the early detection of failures while the product is first designed. In this way, design related deficiencies are specified and it ensures the quality of the product in terms of its reliability and safety standards at the early design process.

(iii) Process FMEA is an analysis of possible failure modes related to process deficiencies. This concept is used to detect and analyze the manufacturing and assembly related failures of a product.

(iv) Service FMEA is an analysis of available services before the product delivery to the customers. Its main focus is to analyze and detect the deficiencies because of process related or system related failures. Its goal is to maximize customer satisfaction by predetermined failure modes.

In this study, application and case study is based on System FMEA and Process FMEA in a manufacturing and assembly line with several processes and subsystems which involves also human interaction.

Today, FMEA is a widely used methodology that has many upsides for organizations, having a deep focus on increasing their quality, reliability, and safety. These advantages can be listed as follows (Asan and Soyer, 2015).

- It increases the communication between experts who have different knowledge and expertise on each subsystem of a product or service. It provides a common platform for experts having various knowhow to analyze and identify the needs of customers.
- With the help of various experts, complex system failures can find simpler solutions.
- It is a very efficient tool to address all possible failure modes, their relationship, their causes and their impacts on the entire system.
- The identification of realistic and reliable failure modes creates reliable services and products, improve the warranty and development process. Thus, it increases customer satisfaction with the increased quality of the products and services.
- It is a user-oriented tool which enables efficient visualization for its users and managers. Also, it is an effective training tool.

Despite the given advantages above, several disadvantages of the FMEA methodology reported in the literature. For further details on the disadvantages of the FMEA, please refer to Section 2.2. Risk assessment in FMEA is subjective and done by an individual expert or group decision of several experts. Experts use their expertise, perspective, concerns, and available knowledge in order to assess potential risks. Like natural language in real life, human expressions (e.g. experts assessments) usually cannot be classified to completely false or true. Usually, human expressions contain a degree of true or false. For example question of "how hot is the coffee" can have many answers like "somehow hot or nearly hot" which cannot be considered as completely hot. In other words, "somehow hot" expression has a degree of truth of being hot. Consequently, these linguistic variables cannot be converted into binary values 0 and 1 and cannot be computed with mathematical equations. Fuzzy sets are designed to overcome this issue by representing linguistic variables with a degree of truth. Just like the above examples, fuzzy sets (hot) represent the elements (somehow hot) with a degree of membership. In other words, an element is a member of a fuzzy set with a grade on a closed interval [0,1] instead of either being a member of the set (true) or not being a member of the set (false). L. A. Zadeh introduced this methodology in 1965.

Fuzzy sets are based on the analysis of analog inputs having continuous membership values to the sets. In other words, logic is not based on specific and precise values or numbers, but it is based on the fuzziness of the inputs. At this point, it is worth to mention that the word “fuzzy” refers to uncertainty or gray area.

A Fuzzy controller makes use of fuzzy sets in the control systems by implementing empirical rules. So a fuzzy controller computes with linguistic rules and human-like intelligence rather than using mathematical calculations. Fuzzy logic is the system to generate a fuzzy conclusion from logical rules with fuzzy controls, which has fuzzy premises and fuzzy output (Aguila et al., 2016; Aguila et al., 2017; Akram et al., 2014; Zadeh, 1965; Zadeh, 1975; Ross, 2009; Butt and Akram, 2016).

Fuzzy logic is used in various disciplines like engineering, signal processing, social sciences, graph theory, robotics computer networks, medical sciences, etc. It can be used where

- computing with equations are complex
- inputs are represented by linguistic variables but not by precise numbers
- finding precise, and complete data is resource consuming or not possible
- desired output can tolerate a degree of uncertainty
- decisions can be based on linguistic models mimicking human behavior and thinking (Akram et al., 2014; Akram et al., 2013).

Later in 1983, Atanassov introduced intuitionistic fuzzy sets as an extension to fuzzy sets and as a higher order fuzzy set, enabled representing uncertainty and vagueness of the system in a more compatible and extensive way. In intuitionistic fuzzy sets, each element has the degree of membership as well as the degree of non-membership. Accordingly, when the sum of the degree of membership and degree of non-membership is less than 1, the difference represents the degree of uncertainty of the element. Degree of uncertainty can be also defined as hesitancy or indeterminacy of an element belonging to the fuzzy set (Akram et al., 2014; Akram et al., 2013).

Since intuitionistic fuzzy sets are higher order fuzzy sets, their implementation brings complexity on some resources including time, volume, or memory size by including a non-membership function. Also maintaining intuitionistic fuzzy sets is harder due to the same complexity. Nevertheless, if the necessary resources difference between intuitionistic fuzzy sets and fuzzy sets are insignificant then intuitionistic Fuzzy sets can give better results by representing uncertainty in the system, handling imprecise and vague concepts appropriately (Akram et al., 2014; Akram et al., 2013; Butt and Akram, 2016; Aguila et al., 2016).

1.1 Problem Definition

Even though traditional FMEA is used extensively in many areas for evaluating failure modes and risks which usually involves human judgments and linguistic expression, it is unable to handle uncertainty and vagueness in the system. Today, with advanced SW, sensor, internet, data solutions and multilayered HW-SW integrated subsystems, industries have to face, develop and maintain very complex systems. As a result, industries usually have inadequate and imprecise information. On the other hand, getting complete and precise information is a tradeoff between resources and added value. Resources are not limited by only cost and time, but also technology like processing capability and HW competence. At this point, it is worth to mention that FMEA attempts to make risk and quality analysis without having sufficient or certain information about the system and its possible failure modes at the present and future (Chang et al., 2009; Akram et al., 2014; Butt and Akram, 2016). In order to deal with uncertainty and vagueness, new modeling techniques were introduced such as Fuzzy Logic. For further details on methodologies dealing with uncertainty in FMEA please refer to Section 2.3.

Detecting and giving priority to any failure mode and the analysis on them generally brings to cope with insufficient and unclear information. In addition, it requires analyzing the subjective ideas of the specialists. The reason to have such uncertain, subjective and ambiguous information can be as the followings: (i) lack of information, knowledge or education or insufficient processing competencies, (ii) implicit evaluation based on varied ranking standards, i.e. with the change in conditions, (iii) partial and subjective competence.

It is strange enough that even if there is adequate knowledge in a group of experts, the third point above can be still valid. The reason for those decisions in FMEA studies are the results of ideas collected by experts with different backgrounds and subjective evaluations. (Chin et al., 2009b; Song et al., 2014).

Therefore, it is clear that having the correct evaluations including risk factors is not possible because of the above reasons. On the other hand, if only accuracy based methods are used, then the vague factors will of course ignored. At this point, it is worth to mention the drawbacks of the traditional FMEA (Asan and Soyer, 2015):

i. If there is no sufficient data, experts could not analyze it because the distribution cannot be exactly determined in a very small domain.

ii. Experts cannot assess when there is carelessness or negligence in any case.

iii. Experts can only be able to consider consistent data, they cannot consider and integrate various other data at the same time.

iv. Experts cannot consider whether the given information is relying on any hypothesis.

v. Experts cannot give importance or trust in a subjective analysis of the specialists.

vi. Experts cannot consider different personal ideas from many specialists from different disciplines.

Therefore, Experts cannot analyze any mixed information including risk factors, possibilities, personal ideas, hypothesis, etc.

As a result, prioritizing and identifying possible failure modes and effects in the risk assessment process requires dealing with vague or unclear information and subjective evaluations (Asan and Soyer, 2015).

This study proposes an intuitionistic fuzzy rule-based approach which can be represented by a fuzzification, inference, and defuzzification with a linear combination of two classical fuzzy sets which are membership and non-membership functions with Takagi Sugano formula.

1.2 Purpose of the Thesis

The goal of this thesis is sufficiently representing the uncertainty and vagueness in the risk assessment, mimicking human thinking by fully integrating a degree of hesitancy into FMEA. The second goal is overcoming shortcomings and disadvantages of traditional FMEA methodology specified in Section 2.2 by using newly proposed IFRB and comparing IFRB to FRB. Finally yet importantly there are other

Intuitionistic Fuzzy set-based or hybrid, integrated approaches to FMEA, but none of these studies are based on Intuitionistic Fuzzy rule-based approach.

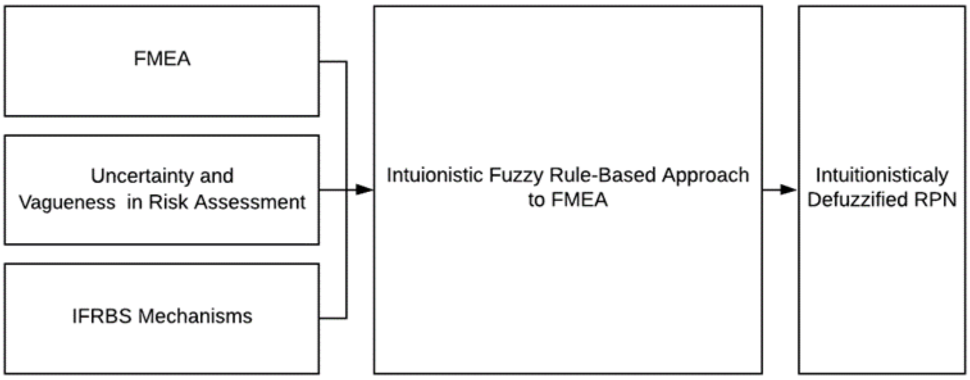


Figure 1.1 : Basic Methodology.

The traditional FMEA inputs and ratings will be integrated into the proposed method by IFRBS mechanisms and considering uncertainty in the system. Finally, an Intuitionistically defuzzified RPN output will be given. The related basic methodology can be found in Figure 1.1.

1.3 Organization of the Thesis

This study consists of the following sections: Chapter 2 presents FMEA, its concepts, and criticisms from literature about the classical FMEA approach. In this context, concepts like risk priority number (RPN), failure modes, calculation of risk factor, the dependency between failure modes will be explained. In addition, a literature review is available for the methodologies dealing with uncertainty in FMEA. Chapter 3 introduces the Fuzzy Logic and intuitionistic fuzzy logic by the introduction of fuzzy sets and intuitionistic fuzzy sets. Later on inference systems including fuzzification, inference engine and defuzzification techniques are explained to model the inputs to outputs. Chapter 4 explains the proposed IFRB model for FMEA the detailed methodology and algorithm. Chapter 5 presents all details of the application, including data collection, analysis, results and comparisons between IFRB, FRB and Traditional RPN Method. Chapter 6 contains general discussions on the proposed method and future research directions.

2. FAILURE MODE AND EFFECTS ANALYSIS

In this section, FMEA concepts and definitions will be introduced, and then criticism on the traditional FMEA will be listed from literature survey. It will be closed with an overview of methods dealing with uncertainty under FMEA.

2.1 Concepts and Definitions

FM are potential errors or failures which are terminating systems operations or preventing necessary functions within expected performance required by the system (ISO/IEC-15026-1, 2013). Therefore, each potential failure mode needs to be evaluated based on prioritization because of the limited resources to solve or prevent the failure or error in the system. In order to prioritize, effects analysis is done for each failure modes to study and examine the outcomes and impacts of these failure modes. Risk priority number (RPN) is used to define how much risk is posed by potential failure modes under effect analysis than in order to prioritize potential risks. RPN evaluates the amount of risk by each failure mode based on the following factors: (i) Severity of outcome (S), (ii) its probability of occurrence (O), (iii) the likelihood of its detection afterward (D). For each failure mode, S, O and D factors are assessed by experts and assigned an integer number from 1 to 10, as explained in Tables 2.1, 2.2 and 2.3. Then, RPN is calculated with a multiplication of the S, O and D factors, at the end RPN will be in the range from 1 to 1000. In equation form;

$$RPN = O \times S \times D \qquad (2.1)$$

Accordingly, higher RPN value of an FM implies that it has a higher potential risk and it needs to be evaluated with higher priority. (Chang et al., 2009; Asan and Soyer, 2015; Bozdağ et al., 2015; Meraj and Farhad, 2015; Liu et al., 2013)

Table 2.1 : Severity scale for a FM.

Rating	Effect	Severity of Effect
10	Hazardous without warning	Very high severity ranking when a potential FM effects safe system operation without warning
9	Hazardous with warning	Very high severity ranking when a potential FM effects safe system operation with warning
8	Very high	System inoperable with destructive failure without compromising safety
7	High	System inoperable with equipment damage
6	Moderate	System inoperable with minor damage
5	Low	System inoperable without damage
4	Very low	System operable with significant degradation of performance
3	Minor	System operable with some degradation of performance
2	Very minor	System operable with minimal interference
1	None	No effect

Table 2.2 : Probability of occurrence scale for a FM.

Rating	Probability of occurrence	Failure probability
10	Extremely high: failure is almost inevitable	>1 in 2
9	Very high	1 in 3
8	Repeated failures	1 in 8
7	High	1 in 20
6	Moderately high	1 in 80
5	Moderate	1 in 400
4	Relatively low	1 in 2000
3	Low	1 in 15,000
2	Remote	1 in 150,000
1	Nearly impossible	<1 in 1,500,000

Table 2.3 : Likelihood of detection scale for a FM.

Rating	Detection	Detection
10	Absolute uncertainty	Potential cause/mechanism and subsequent FM cannot be detected
9	Very remote	Very remote chance of detecting potential cause/mechanism and subsequent FM
8	Remote	Remote chance of detecting potential cause/mechanism and subsequent FM
7	Very low	Very low chance of detecting potential cause/mechanism and subsequent FM
6	Low	Low chance of detecting potential cause/mechanism and subsequent FM
5	Moderate	Moderate chance of detecting potential cause/mechanism and subsequent FM
4	Moderately high	Moderately high chance of detecting potential cause/mechanism and subsequent FM
3	High	High chance of detecting potential cause/mechanism and subsequent FM
2	Very high	Very high chance of detecting potential cause/mechanism and subsequent FM
1	Almost certain	Potential cause/mechanism and subsequent FM will be detected

2.2 Criticism on Traditional FMEA

Traditional FMEA is simple to use and holds several advantages. However, at the same time, it represents multiple weaknesses and problems. A brief summary of shortcomings can be found below from several studies in the literature, which enlighten those weaknesses. (Asan and Soyer, 2015; Bozdağ et al., 2015; Chang et al., 2009; Meraj and Farhad, 2015). This study focuses on the first 8 bullets given below:

- In FMEA, ratings are subjective and risk assessment has uncertainty. While assigning S, O and D ratings, It is nearly impossible to have complete and precise information for intangible quantities and linguistic expressions.

- Simply multiplying each factor ignores the relativity between those factors. S, O, and D may have different importance or weights in different environments.
- Every environment may represent different challenges to improve S, O and D factors of FMs. Difficulty, feasibility, and complexity of the further improvements on FMs are not considered in FMEA. For example, factors that require fewer challenges may carry higher importance in a system.
- At first glance, it can be seen as RPN takes values starting from 1 to 1000 which represents the scale. But the multiplication of S, O, and D which are three integer values from 1 to 10, can only generate 120 unique values out of 1000 values in scale. Which shows that the remaining 880 values cannot be generated in this way. RPN values are not continuous. It can mislead to think that the average of RPN values is 500 and approximately half of the RPN values are higher than 500 (assumed median). But actually, the average of the RPN values are 166, only 6% percent of the RPN values are above 500.
- Only six RPN values out of 120 possible values can be generated with a unique combination of S, O and D ratings. Remaining 114 Values can be generated with at least 3 different combinations of S, O, and D. Which leads to ignoring different effects of each factor. As an example, RPN value 60 can be calculated by 24 different combinations of S, O and D factors. The entire purpose of generating RPN values is prioritizing FMs and risks then preventing most important FMs with limited resources. However, RPN values which can be calculated with different sets of S, O, and D cannot show the differences in relevant risk implications for each set.
- Normally mathematical operations cannot be performed between ordinal scales since the amount of difference between each rank in the ordinal scale is subjective which means that it can not be measured and each difference is not identical. But in traditional FMEA, S, O, and D are using ordinal scales and RPN values are calculated with mathematical operations with these 3 ordinal scales.
- Also, multiplications itself is very sensitive to increase and decrease in the S, O and D values. Which also can be seen by only 6% of the potential RPN values is above 500 (which is normally median in a 1 to 1000 scale). So using

multiplication is also questionable for representing the relationship between factors.

- There could be an indirect relationship between S, O and D factors although traditional FMEA ignores it.
- There could be dependencies between FMs and those relationships are also ignored with traditional FMEA:
- The conversion method is different for O factor and S, D factors. There is a non-linear relationship between O ratings and O's probability scale and there is a linear relationship with D, S and their probability scale.

There may be other significant factors for assessing FMs like costs, mass production, quality, etc. Traditional FMEA only considers 3 factors while ignoring other potential factors.

2.3 Methods Developed for Dealing with Uncertainty in FMEA

According to Liu et al. (2013), majority of methods developed for dealing uncertainty in FMEA can be classified according to their methodology basis (Refer to Figure 2.1), which is mathematical programming, rule based artificial intelligence, Multi-Criteria Decision Making (MCDM). The most used method of all is fuzzy rule-based AI and then MCDM. Although MCDM has the capability to solve problems related to modeling by weighting, scaling, aggregation, and uncertainties in the assigning rating and subjectiveness, Fuzzy Rule-based AI is preferred. Because firstly, risk assessment can be customized according to different perspectives, secondly it can handle uncertainty and incomplete information in a steady way, and lastly, it can represent the relationship between factors in a more flexible and practical way. However, Fuzzy Rule-Based AI has also some shortcomings. Creating competent and therefore complex IF-THEN rules can be found inefficient and can create unnecessary cost. Otherwise, if rules are not competent inferences can be inaccurate and biased. Also to keep rules competent with the changes in the system, product, etc, these rules have to be maintained and updated from time to time. Other than rules, membership functions are also hard to generate and maintain. Among implemented Fuzzy Sets and its extensions, IFS is being benefitted increasingly in recent years in FMEA because of its extensive and compatible way of handling uncertainty (Asan and Soyer, 2015).

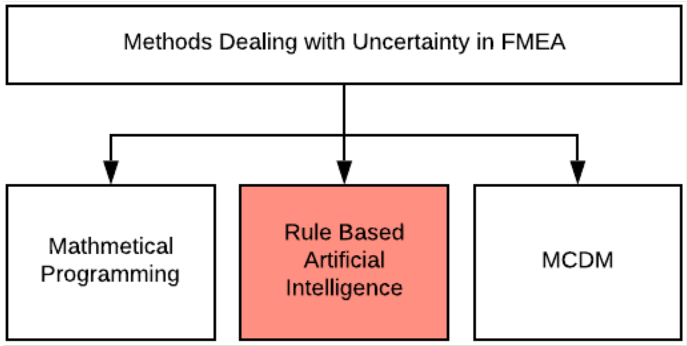


Figure 2.1 : Classification for Methods Developed for Dealing with Uncertainty in FMEA.

Modeling the vagueness and ambiguous data is an outstanding feature of fuzzy theory. The vagueness here may be caused by insufficient data because of the restricted human capabilities and information. (Liu and Lin 2010) There will be, of course, some ambiguous input of the system. At the same time, the system being considered may have some complicated parts, which therefore highly increase the fuzziness of the FMEA. As a solution to these problems, fuzzy set theory has been introduced to express all of these fuzzy factors. In 1995, Bowles and Pelaez used the S, O and D as risk elements in FMEA. Then, failure modes of these risk factors are evaluated by using verbal factors. Then, degree of membership of FMs is determined. After that, inputs of the fuzzy system are assessed via logical operator. The last step is defuzzification of the conclusions and determination of the risk range of each failure modes. Another example is the study of Liu et al. (2012) Who used linguistic variables to rank the risk of each S,O, D. Based on that, he developed a new methodology to represent the risk amount of failure modes. Another study is improved by Lin et al. (2014) to have an assessment on people's trustworthiness on the medical appliance. In that assessment, again, fuzzy linguistic and fuzzy logic is applied to cope with the personal comments of different specialists considering important factors which affects the reliability of FMEA experts. All of these examples demonstrates that fuzzy set theory have more upsides than the classical FMEA methods. These upsides can be summarized as the follows (Asan and Soyer, 2015):

1) both numerical and qualitative features of data can be used and analyzed.
2) as shown in the above examples, amount of risk in each failure mode can be evaluated via verbal concepts.
3) S,O,D may be assessed together via logical operators easily.

3. INTUITIONISTIC FUZZY INFERENCE SYSTEM

In this section, firstly fuzzy sets will be introduced as a generalization of the classical sets theory, i.e. sets with crisp boundaries. In addition, intuitionistic fuzzy sets will be introduced as a higher order of fuzzy sets with a comparison with fuzzy sets. Thenceforward fuzzy inference system and intuitionistic fuzzy inference systems will be introduced for formulating, modeling and mapping inputs to outputs with fuzzy logic.

3.1 Fuzzy Sets

In 1965, Zadeh proposed a theory to represent linguistic variables in a mathematical mean named as Fuzzy Sets Theory, which is a generalization of the classical, crisp sets. The Fuzzy Set theory is developed in order to deal with incomplete information and linguistic modeling, wherein real-world problems, available information and system usually contain vague concept and incomplete information. Due to the limitation of human knowledge, noises and disturbances are usually found in available data (Liu and Lin 2010). With the limitation of human perception, it is nearly impossible to obtain precise and certain data or obtaining precise and certain data is too costly and time consuming for achieving confident results. In the classical set theory, an element either fully belongs to one set or the element doesn’t belong to the set at all. Nevertheless, in fuzzy set theory, each element is belonging to sets is defined as membership values within a continuous interval from 0 to 1 (Meraj and Farhad, 2015; Asan and Soyer, 2015).

3.2 Intuitionistic Fuzzy Sets

Regardless of usage of fuzzy sets for imprecise and vague concepts in many areas now, fuzzy sets are not able to handle every type of uncertainty appropriately (Butt and Akram, 2016; Aguila et al., 2016). To address this problem, several extensions of fuzzy sets were proposed in literature known as type-2 fuzzy sets. The first extension is proposed by L. A. Zadeh in 1975 and other approaches followed afterward. Among these, In 1983 and 1986 Atanassov proposed another extension to handle uncertainty and hesitation with a higher level including an intuitive approach(Butt and Akram, 2016; Asan and Soyer, 2015).

In fuzzy set theory, an element belongs to a set with membership value between 0 and 1. In other words, each element doesn't belong to a set with a non-membership value and that value can be found by subtracting the membership value of the same set from 1. Consequently, the sum of the membership value and the non-membership value of an element is equal to 1.

This is not the case in intuitionistic fuzzy set theory. the sum of the membership value (μ) and the non-membership value (ν) of an element can be equal to or lower than 1. Whenever it is lower than 1, the remaining difference represents the uncertainty of belonging or not belonging to the set. This uncertainty or indeterminacy is identified as the intuitionistic fuzzy index (π)" which can represent expressions like "food is slightly sweet for me but still I am not sure". For that reason, intuitionistic fuzzy set theory can handle uncertainty and imprecision in a way which is consistent and more comprehensive (Aguila et al., 2017; Asan and Soyer, 2015).

Extensions of fuzzy sets like intuitionistic fuzzy sets are implemented lately in many areas such as software controls, robotics, pattern recognition, social sciences, and others (Butt and Akram, 2016). Moreover, several studies revealed improved performance in the system with fuzzification of input parameters and designing a rule-based inference system while representing a further degree of uncertainty, which mimics the behavior of human way of thinking.

However using higher order fuzzy sets makes the processing and modeling much more complex, then only if the resources such as time, processing capability etc. are not a limitation, higher-order fuzzy sets can produce better solutions (Aguila et al., 2017; Akram et al., 2014; Akram et al., 2013).

According to Atanassov (1986), in a universe U, intuitionistic fuzzy set A is defined as follows:

$$A = \{(u, \mu_A(u), v_A(u)) | u \in U\} \; or \; (A = (\mu_A, v_A)) \tag{3.1}$$

where the function $\mu_A: U \rightarrow [0,1]$ defined the degree of membership and function $v_A: U \rightarrow [0,1]$ defines degree of non-membership of the element U to A. Where the functions should meet the following criteria;

$$0 \leq \mu_A(u) + v_A(u) \leq 1 \ \ (\forall u \in U) \tag{3.2}$$

For each intuitionistic fuzzy set A in universe U,

$$\pi_A(u) = (1 - \mu_A(u) - v_A(u)) \tag{3.3}$$

represents the intuitionistic fuzzy index of element U to A. Intuitionistic fuzzy index is also called as degree of uncertainty or degree of hesitancy. Therefore intuitionistic fuzzy index enables representing real world problems and imperfect knowledge (Dongfeng et al., 2012; Akram et al., 2014).

3.3 Fuzzy Rule-Based System

First of all, fuzzy control is intended to imitate human behavior while taking control action to decide on any vagueness (Butt and Akram, 2016).

People use intangible words to explain abstract or vague concepts, actually, by using the fuzzy premises. To illustrate this point, “good enough”, “clear enough”, “very successful” are some expressions bringing vague meanings. Actually, these expressions may be the results of some data, however, that data may also be not sufficient. Or, data can have some noisy effects itself. At this point, fuzzy sets are being developed to model and analyze such kind of data (Aguila et al., 2017; Butt and Akram, 2016). FRBS is a linguistic IF-THEN rule-based decision modeling and control system using the input data and generating the output by fuzzification, fuzzy inference engine and defuzzification as shown in Figure 3.1 (Garg et al., 2013). But still, there may be some cases where even FRBS is not able to model the fuzzy data because of the dominant amount of uncertain data. However, the fuzzy set theory is being developed more to model such noisy information (Aguila et al., 2017; Butt and Akram, 2016).

In fuzzy theory, FRBS is a very famous concept that is using fuzzy rules. Commonly, it is used in areas like decision systems, automatic control or data analytics. FRB includes the followings: (i) a fuzzy rule, (ii) inference methodology that drives the conclusion, (iii) the results after the inferences based on the fuzzy rules. FRBS inputs may be fuzzy or crisp, however, its outputs are fuzzy. Fuzzy logic is a way of expressing of heuristic/vague concepts as an algorithm. Most famous algorithms are called as “Mamdani Model” by Mamdani and Assilian (1975) and “Sugeno Model” by Sugeno (1985). Mamdani and Assilian (1975) used some logical conjunctions like “and, or” to express some conditions together. In this way, the outputs are shaped via the integration of various conditions. Hence, their method while creating a reasoning method is to compose different conditions via some logical connectors. And Mamdani and Sugeno model shares a similar fuzzification and fuzzy operator methods. The main difference of Sugeno model is that output membership functions are being either linear or constant.

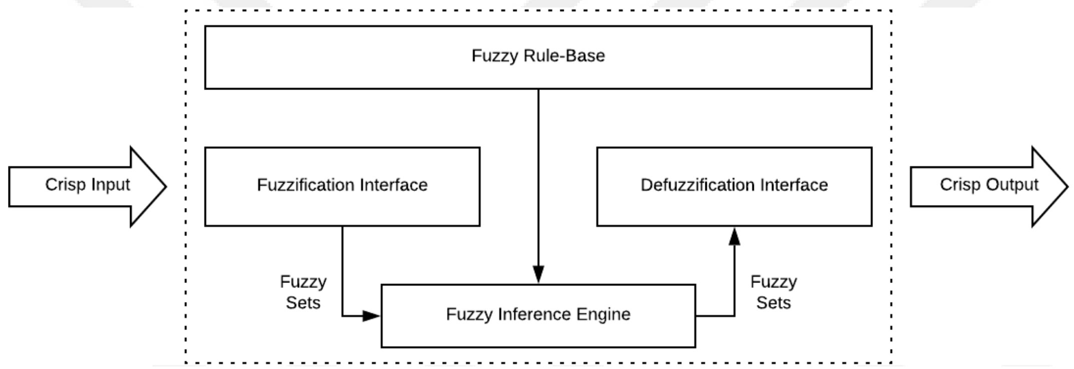


Figure 3.1 : Fuzzy Rule Based System

Main components and steps of a fuzzy inference system are given below (Meraj and Farhad, 2015):

1) Construction of FRB mechanisms
    a) Specifying Linguistic Input Variables and Output Variables
    b) Specifying Membership Functions Functions for Each Variable
    c) Specifying Rules
    d) Specifying Fuzzification, Inference Engine and Defuzzification mechanisms
2) Fuzzification
    a) Calculating Membership values from crisp values of each input variable
3) Inference Engine

a) Implementing FRB Rules for membership values and converting fuzzified input values accordingly to output fuzzy sets

b) Combining the Results of Each Rule using determined algorithm

4) Defuzzification

a) Converting Inferred fuzzy output data to crisp (non-fuzzy) value with chosen methodology.

Fuzzification process translates user's knowledge, experience and observed data into fuzzy sets by membership functions (MF). Membership functions can be determined by two ways; (i) by the utiliser, (ii) by machine learning. There are various forms of membership functions (MF), like z-shape, Gaussian, triangular, trapezoidal, linear, etc. In this thesis, triangular MF is used.

After the fuzzification, the inference engine can be derived according to membership functions. Hence, the relation between the inputs and outputs of the system is determined with inference rules which are so-called "implication functions". These functions express the "IF-THEN" statements of fuzzy control. "IF-THEN" rule can be exemplified as "If x_1 is A, then x_2 is B". According to that, A and B are linguistic variable. Also, "x_1 is A" is a condition and "x_2 is B" is a result statement. is a result statement. Hence, a fuzzy rule includes a condition and its corresponding result. As a summary, a fuzzy rule may be in the form of "If the condition, then the result". Thus, if the condition is satisfied, then the result is obtained according to the IF-THEN rule (Akram et al., 2014; Butt and Akram, 2016).

The conclusions derived from the inference engine is still a fuzzy value. It needs to be defuzzified to obtain a crisp output. For this goal, the membership function of the output value is used reversely. This process is called defuzzification. There are different methods for defuzzification such as center of gravity, center of gravity for singletons, center of area, left most maximum, and rightmost maximum.

3.4 Intuitionistic Fuzzy Rule-Based System

Both IFRB and FRB follows similar steps in methodology. The main difference is IFRB takes non-membership degree also in to account in creating the degree of uncertainty or hesitancy margin. Accordingly, we can propose that the linear combination of non-membership degree based fuzzy system and membership degree based fuzzy system can produce an intuitionistic fuzzy rule-based system (Akram et al., 2014; Butt and Akram, 2016; Castillo and Melin, 2003; Castillo et al., 2006). In that case, the following equation will apply:

$$IFS = (1 - \pi)FS_{\mu} + \pi FS_{v} \quad (3.4)$$

Main components and steps of an Intuitionistic fuzzy inference system are as below (Akram et al., 2014; Butt and Akram, 2016; Dongfeng et al., 2011):

1. Construction of IFRB mechanisms
    a. Specifying Linguistic Input Variables and Output Variables
    b. Specifying Membership Functions and Non-Membership Functions for Each Variable
    c. Specifying Rules
    d. Specifying Fuzzification, Inference Engine and Defuzzification mechanisms
2. Fuzzification
    a. Calculating Membership values from crisp values of each input variable
    b. Calculating Non-Membership values from crisp values of each input variable
3. Inference Engine
    a. Implementing IFRB Rules for membership and non-membership values and converting fuzzified input values accordingly to output fuzzy sets
    b. Combining the Results of Each Rule using determined algorithm
4. Defuzzification
    a. Converting Inferred fuzzy output data to crisp (non-fuzzy) value with determined methodology.

Further details of each step can be found in Section 4.

4. PROPOSED IFRB METHOD FOR FMEA

After listing the potential failure modes, causes and effects, IFRBS will be initiated by taking S, O, and D crisp inputs. By using defined membership and non-membership factors S, O and D crisp values will be fuzzified separately. S, O and D membership degrees will be inferred with determined implication functions in inference engine. Same will be applied separately to non-membership degrees of S, O and D. Once all rules are inferred for both membership and non-membership values, the intuitionistic defuzzification process will start and defuzzification will be done using membership and non-membership degrees. Finally producing a crisp output value for IFRB RPN which will be further used for ranking the FMs. The related structure is shown in Figure 4.1.

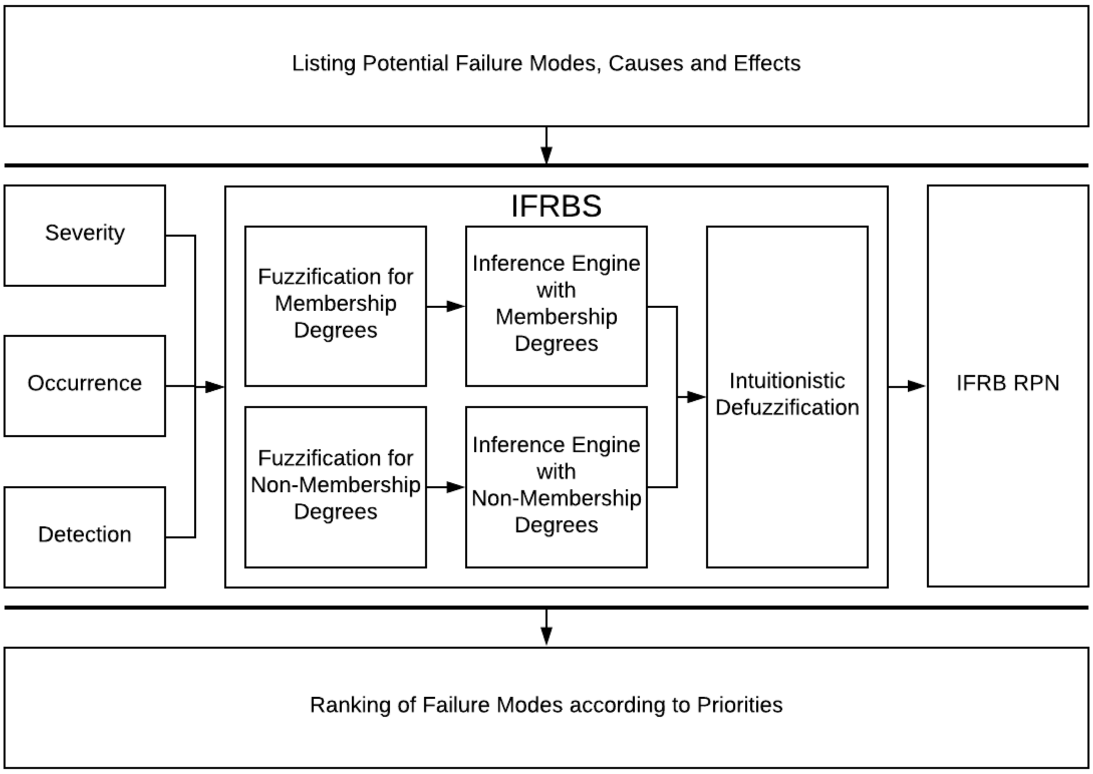


Figure 4.1 : Proposed IFRB model for FMEA.

Steps of the proposed IFRB method for FMEA is summarized below:

1. Listing Potential Failure Modes, Causes and Effects
2. Construction of IFRB mechanisms for FMEA
    a. Crisp Inputs: Severity (S), Occurrence (O), Detecetability
    b. Crisp Output: RPN
    c. Specifying Linguistic Input Variables and Output Variables
    d. Specifying Membership Functions and Non-Membership Functions for Each Variable
    e. Specifying Rules
    f. Specifying Fuzzification, Inference Engine and Defuzzification mechanisms
3. Fuzzification
    a. Calculating Membership values from crisp values of each input variable
    b. Calculating Non-Membership values from crisp values of each input variable
4. Inference Engine
    a. Implementing IFRB Rules for membership and non-membership values and converting fuzzified input values accordingly to output fuzzy sets
    b. Combining the Results of Each Rule using determined algorithm
        i. Min Method for “AND” in membership rules
        ii. Max Method for “OR” in membership rules
        iii. Max Method for “AND” in non-membership rules
        iv. Min Method for “OR” in non-membership Rules
5. Defuzzification
    a. Converting Inferred fuzzy output data to crisp (non-fuzzy) value with with Takagi Sugeno method.

b. Averaging Activated Defuzzified Rules

4.1 Listing Potential Failure Modes, Causes and Effects

In this step, all potential risks and failure modes are listed by experts in order to understand type of failures in the system, the sources and root causes of these failures and how these failures occur.

4.2 Construction of IFRB mechanisms for FMEA

As previously mentioned there are three inputs in FMEA which are Severity (S), Occurrence (O) and Detection (D). The linguistic variables and corresponding membership and non-membership functions needs to be defined for each input. It can be found with expert's knowledge, decision, and perspective, or it can be designed with machine learning methodologies (Meraj and Farhad, 2015). In our study membership and non-membership functions are triangular but there are several different membership functions which can be used for linguistic variables.

According to linguistic variables, a set of IF-THEN rules needs to be defined for S, O and D inputs with relation to output. As explained before fuzzy IF-THEN rules for FMEA can be expressed by a form like "if Severity is A and Occurrence is A and Detection is A than RPN is B".

4.3 Fuzzification

The crisp inputs for the factors S, O and D are intuitionistically fuzzified in this step. For each factor, the specified membership and non-membership functions of linguistic variables are used. And as an end result, fuzzified S, O and D values will be found for each defined membership and non-membership functions.

4.4 Inference Engine

All of the defined IF-THEN rules will applied with fuzzified input values, S, O and D considering both membership, and as well as non-membership functions by Mamdani inference method (1974). As an output, A fuzzy output, RPN value, will be inferred and calculated for each rule with fuzzified membership values. Also, a second fuzzy

RPN value will be inferred and calculated for each rule with fuzzified non-membership values.

4.5 Defuzzification

Multiple methodologies are available in the literature but this study will implement Takagi Sugeno formula (Leekwijck and Kerre, 1999) for defuzzification. As explained earlier, the defuzzification step combines inferred output of each rule and convert it to a crisp output which in this case, the final output is RPN. Takagi Sugeno Equation which derived as a linear combination of membership and non-membership degrees can be found as below:

$$x = \frac{\sum_{j=1}^{M} x^j((1-\pi_{A^j})\mu_{A^j} + v_{A^j}\pi_{A^j})}{\sum_{j=1}^{M}((1-\pi_{A^j})\mu_{A^j} + v_{A^j}\pi_{A^j})} \quad (4.1)$$

Where x is as an element in an intuitionistic fuzzy set of A, in this case, fuzzy set of RPN. x^j denotes the jth sample space in RPN. And μ_A and ν_A are the degree of membership, and non-membership of x in A. π_A is the degree of hesitancy.

Inferred fuzzy RPN value from inference engine for membership degrees will be the maximum value of μ_{A^j} can take. In other words, it will act as a maximum cut for the RPN output membership function.

Inferred fuzzy RPN value from inference engine for non-membership degrees will be minimum value of v_{A^j} can take. In other words, it will act as a minimum cut for the RPN output non-membership function.

5. APPLICATION

5.1 Problem Case

The proposed method applied at a highly complex and sophisticated production line of diesel injectors. Each component is going through several processes like handling, welding, cutting, cleaning while assembling other sub-parts and frames which needs high accuracy and precision in serial production. Also each process stream data through programmable logic controllers and sensors to data analytics platform in order to improve the system and avoid each of potential failures by Industry 4.0 solutions such as event management, machine learning, preventive maintenance, smart supply management, and other sub solutions. FMEA is highly regarded and maintained over the years and implemented with several quality experts. The FMEA experts stated that FMs are getting harder to manage and maintain with increasing complexity and vagueness in the system. Since lots of FMs generate the same results, sometimes there are uncertainties while assigning rates to each factor. And in some processes and lines, some factors can carry higher risks than other factors but these effects are ignored in traditional FMEA.

5.2 Data Collection

The input data set (Refer to Table 5.1), 14 FMs and S, O and D ratings used in this study are taken from a diesel injector production plant. For better understanding, FMs are sorted by descending order to traditional RPN and named accordingly, i.e. FM1 has the highest traditional RPN and FM14 has the lowest. Still, some FMs which shares the same RPN value named randomly according to their rank, i.e. FM4 and FM5 produce the same RPN. Each of these ratings is assigned by FMEA experts with group decision and each of the FMs listed has a development track, preventive measures, and core problem definitions. The same production line also utilizes advanced Industry 4.0 methodologies which produce very high production quality indicators.

Table 5.1 : Input Data set for Failure Modes and Assigned S, O and D Ratings.

Failure Modes	Severity	Occurrence	Detection
FM1	6	7	8
FM2	9	4	7
FM3	10	4	6
FM4	9	2	8
FM5	6	6	4
FM6	10	2	7
FM7	10	2	6
FM8	4	5	6
FM9	10	2	5
FM10	10	1	9
FM11	9	2	5
FM12	6	2	5
FM13	9	2	3
FM14	6	2	3

Severity, Occurrence and Detection inputs will be fuzzified intuitionistically. The membership and non-membership functions are adopted from a study by Akram et al. (2014) on intuitionistic fuzzy logic control, since defining membership and non-membership functions of factors are not in the focus of this study.

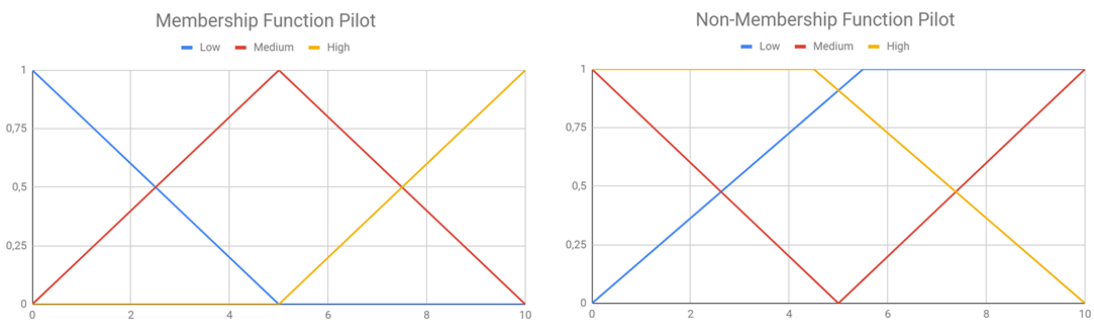


Figure 5.1 : Membership and Non-Membership function graphics of S, O, D and RPN.

Linguistic variables of fuzzy sets are defined as Low, Medium and High. S, O, D and RPN membership and non-membership functions pilots for linguistic variables can be found in Figure 5.1.

Mathematical expressions of the membership and non-membership functions of S, O, D and RPN can be found below:

$$\{ x \mid x \in \mathbb{Z},\ 1 \leq x \leq 10 \}$$

$$\mu_{low}(x) = \begin{cases} \dfrac{5-x}{5-0} & x < 5 \\ 0 & else \end{cases} \qquad v_{low}(x) = \begin{cases} 1 & x > 5 \\ 1 - \left[\dfrac{5{,}5-x}{5.5}\right] & else \end{cases}$$

$$\mu_{med}(x) = \begin{cases} 1 - \left|\dfrac{5-x}{5}\right| & else \\ 1 & x = 5 \end{cases} \qquad v_{med}(x) = \begin{cases} \left|\dfrac{5-x}{5}\right| & else \\ 0 & x = 5 \end{cases} \tag{5.1}$$

$$\mu_{high}(x) = \begin{cases} \dfrac{x-5}{5} & x > 5 \\ 0 & else \end{cases} \qquad v_{high}(x) = \begin{cases} \dfrac{10-x}{5.5} & x > 5 \\ 1 & else \end{cases}$$

The IF-THEN rules for inference are studied and defined with the expert who applies preventive measures by Industry 4.0 methodologies and designs the FMs used in the input data set. All possible combination of Low, Medium and High variables for S, O and D factors are used in the inference engine. Each IF-THEN rule statements are specifically tailor-made by senior FMEA expert of the line according to the need of the system considering the importance of each factor in the overall system. Rules for this study can be found below:

- Rule 1: If Severity is Low AND Occurence is Low AND Detection is Low than RPN is Low
- Rule 2: If Severity is Low AND Occurence is Medium AND Detection is Low than RPN is Low
- Rule 3: If Severity is Low AND Occurence is High AND Detection is Low than RPN is Low

- Rule 4: If Severity is Low AND Occurence is Low AND Detection is Medium than RPN is Low
- Rule 5: If Severity is Low AND Occurence is Medium AND Detection is Medium than RPN is Medium
- Rule 6: If Severity is Low AND Occurence is High AND Detection is Medium than RPN is Medium
- Rule 7: If Severity is Low AND Occurence is Low AND Detection is High than RPN is Low
- Rule 8: If Severity is Low AND Occurence is Medium AND Detection is High than RPN is Medium
- Rule 9: If Severity is Low AND Occurence is High AND Detection is High than RPN is Medium
- Rule 10: If Severity is Medium AND Occurence is Low AND Detection is Low than RPN is Low
- Rule 11: If Severity is Medium AND Occurence is Medium AND Detection is Low than RPN is Medium
- Rule 12: If Severity is Medium AND Occurence is High AND Detection is Low than RPN is High
- Rule 13: If Severity is Medium AND Occurence is Low AND Detection is Medium than RPN is Low
- Rule 14: If Severity is Medium AND Occurence is Medium AND Detection is Medium than RPN is Medium
- Rule 15: If Severity is Medium AND Occurence is High AND Detection is Medium than RPN is High
- Rule 16: If Severity is Medium AND Occurence is Low AND Detection is High than RPN is Medium
- Rule 17: If Severity is Medium AND Occurence is Medium AND Detection is High than RPN is Medium

- Rule 18: If Severity is Medium AND Occurence is High AND Detection is High than RPN is High
- Rule 19: If Severity is High AND Occurence is Low AND Detection is Low than RPN is Medium
- Rule 20: If Severity is High AND Occurence is Medium AND Detection is Low than RPN is High
- Rule 21: If Severity is High AND Occurence is High AND Detection is Low than RPN is High
- Rule 22: If Severity is High AND Occurence is Low AND Detection is Medium than RPN is Medium
- Rule 23: If Severity is High AND Occurence is Medium AND Detection is Medium than RPN is High
- Rule 24: If Severity is High AND Occurence is High AND Detection is Medium than RPN is High
- Rule 25: If Severity is High AND Occurence is Low AND Detection is High than RPN is Medium
- Rule 26: If Severity is High AND Occurence is Medium AND Detection is High than RPN is High
- Rule 27: If Severity is High AND Occurence is High AND Detection is High than RPN is High.

5.3 Analysis

5.3.1 Analysis of traditional FMEA

Results of the traditional FMEA method by $RPN = O \times S \times D$ formula and ranking for FMs can be found in Table 5.2. Average rank method is used for tied ranks.

Table 5.2 : RPN Results and Rankings of FMs by Traditional FMEA Method. (*Normalized).

Failure Modes	Severity	Occurrence	Detection	RPN	RPN Ranking
FM1	6	7	8	336	1
FM2	9	4	7	252	2
FM3	10	4	6	240	3
FM4	9	2	8	144	4.5
FM5	6	6	4	144	4.5
FM6	10	2	7	140	6
FM7	10	2	6	120	7.5
FM8	4	5	6	120	7.5
FM9	10	2	5	100	9
FM10	10	1	9	90	10.5
FM11	9	2	5	90	10.5
FM12	6	2	5	60	12
FM13	9	2	3	54	13
FM14	6	2	3	36	14

5.3.2 Analysis of FRBS FMEA

For comparison with IFRBS, FRBS was performed in Matlab Fuzzy Logic Toolbox with defined membership functions and rules. Screenshots from Matlab Fuzzy Logic Toolbox related to the calculation can be found in Figure 5.2, Figure 5.3 and Figure 5.4.

FRB RPNs can be found in Table 5.3, in given table FMs are sorted by descending order and average rank method is used for tied ranks. Also for a better comparison with other methods, FRB RPN values are normalized and then multiplied by 1000 for rescaling.

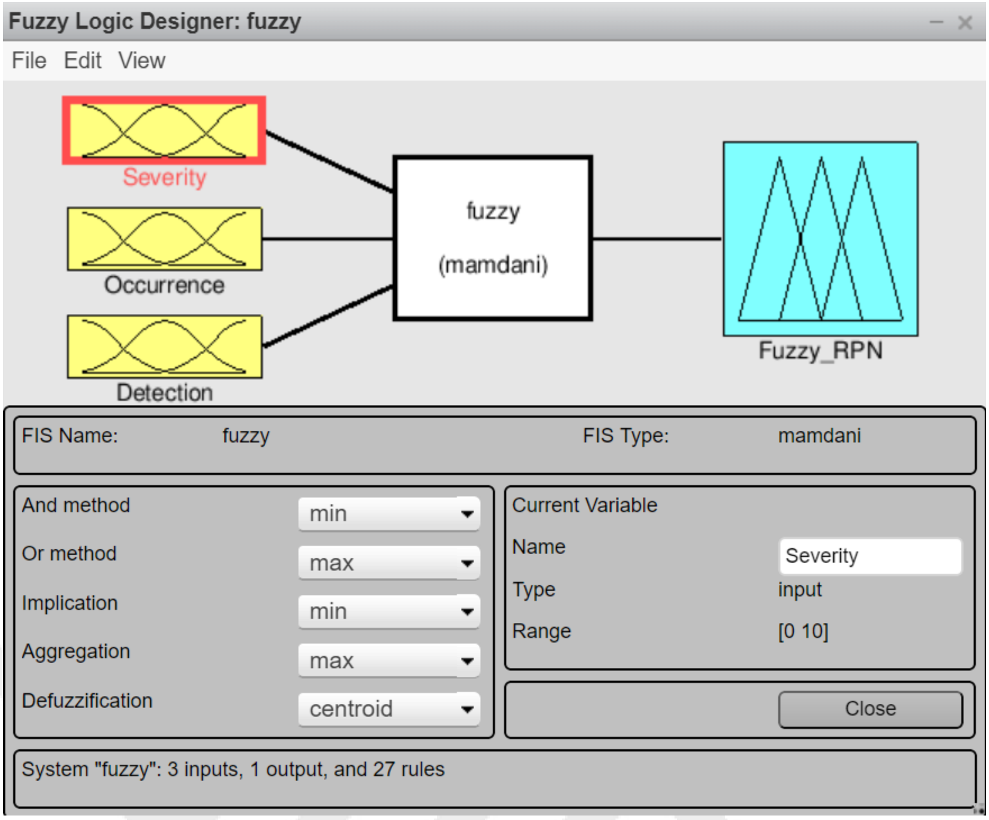


Figure 5.2 : Fuzzy Logic Designer / Fuzzy Properties from MATLAB for FRB FMEA.

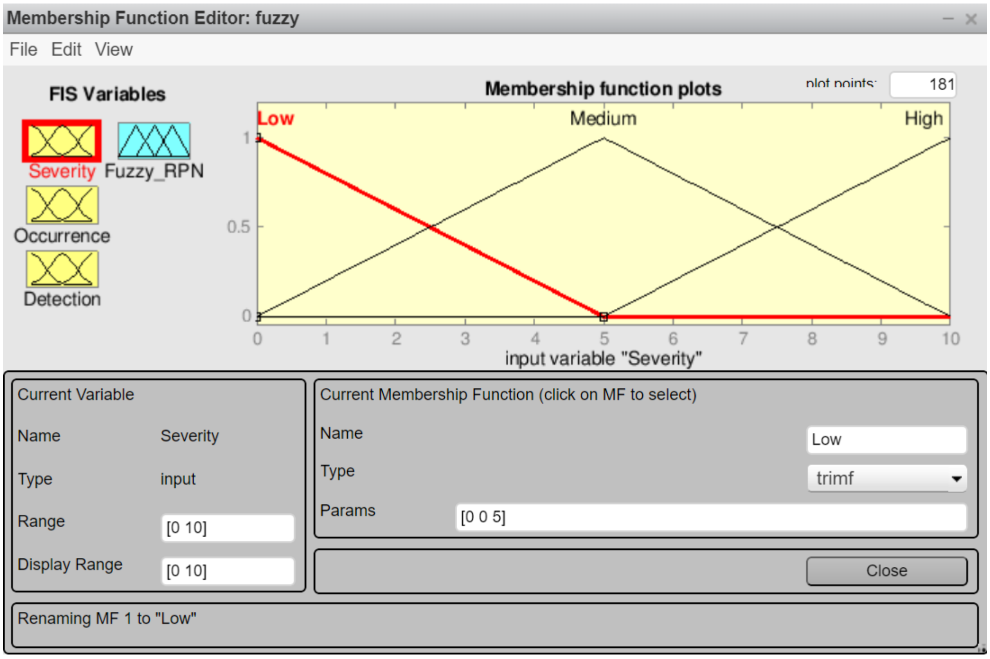


Figure 5.3 : Severity Membership Function Editor from MATLAB for FRB FMEA.

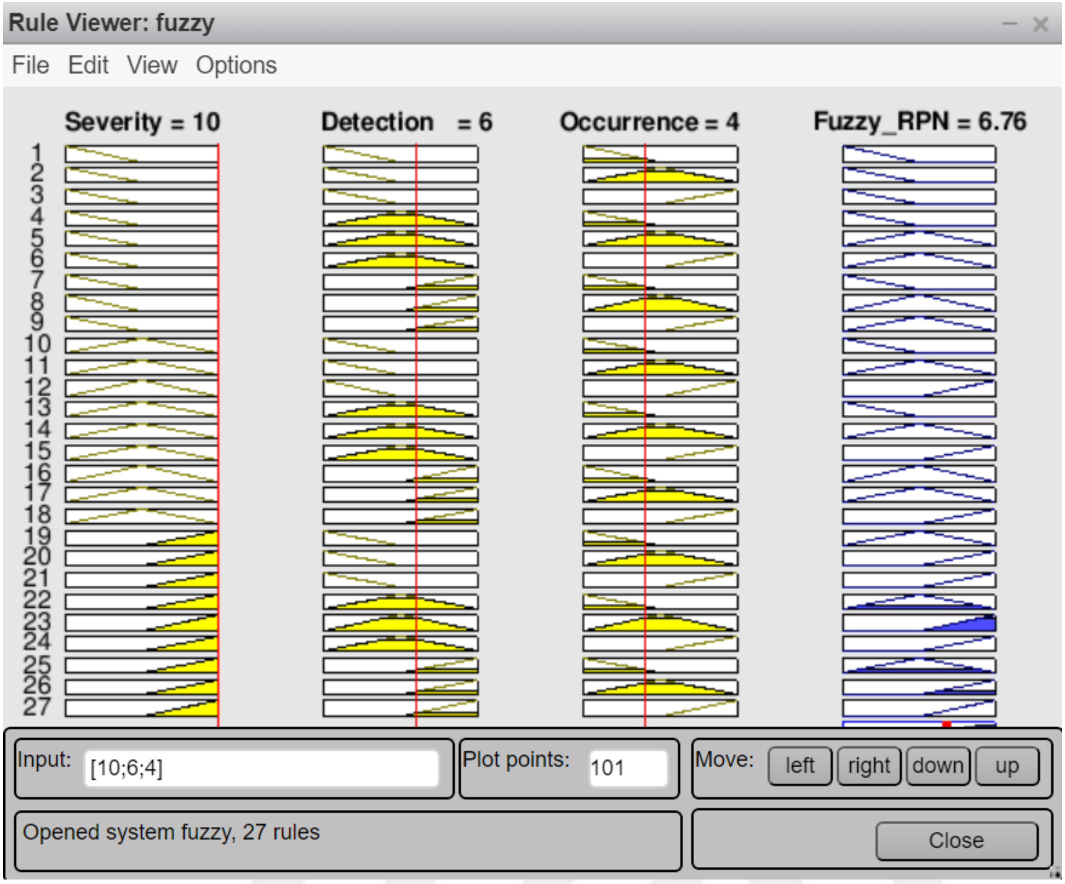


Figure 5.4 : Rule Viewer and Fuzzy RPN calculation of FM3 from MATLAB for FRB FMEA.

Related RPN normalization formula can be found below:

$$RPN_{Normalized} = \frac{RPN - RPN_{Min}}{RPN_{Max} - RPN_{Min}} \tag{5.2}$$

Where FRB RPN_{Min} is 324 which is result of an FM when S, O and D is 1. And FRB RPN_{Max} is 837 which is result of an FM when S, O and D is 10.

5.3.3 Analysis of IFRBS FMEA

In this section FM3 Intuitionistic Fuzzy RPN with Severity Rating of 10, Occurence Rating of 4 and Detection Rating of 6 will be calculated by the proposed method.

5.3.3.1 Analysis of IFRBS FMEA

By membership and non-membership functions given in Section 5.2, we execute crisp inputs of S=10, O=4, and D=6.

Table 5.3 : RPN Results and Rankings of FMs by Fuzzy Inference System. (*Normalized).

Failure Modes	Severity	Occurrence	Detection	F.RPN	F.RPN*	Rank
FM3	10	4	6	676	686.2	1
FM2	9	4	7	631	598.4	2
FM1	6	7	8	540	421.1	4.5
FM6	10	2	7	540	421.1	4.5
FM7	10	2	6	540	421.1	4.5
FM9	10	2	5	540	421.1	4.5
FM4	9	2	8	528	397.7	8
FM11	9	2	5	528	397.7	8
FM13	9	2	3	528	397.7	8
FM5	6	6	4	511	364.5	10.5
FM10	10	1	9	511	364.5	10.5
FM8	4	5	6	500	343.1	12
FM12	6	2	5	424	194.9	13.5
FM14	6	2	3	424	194.9	13.5

The intuitionisticaly fuzzified membership and non-membership values of S (10), O (4) and D (6) of FM3 are found as below in order to Small, Medium and High.

$$\mu_S(10) = \{0, 0, 1\} \quad ; \quad v_S(10) = \{1, 1, 0\}$$

$$\mu_O(4) = \{0.2, 0.8, 0\} \;\; ; \;\; v_O(4) = \{0.73, 0.2, 1\}$$

$$\mu_D(6) = \{0, 0.8, 0.2\} \;\; ; \;\; v_D(6) = \{1, 0.2, 0.73\}$$

Example calculation for $\mu_S(10)$ and $v_S(10)$ can be found below:

$$\mu_{low}(10) = \begin{cases} \dfrac{5-x}{5-0} & x < 5 \\ 0 & else \end{cases} = 0 \qquad v_{low}(10) = \begin{cases} 1 & x > 5 \\ \dfrac{x-5}{5-0} & else \end{cases} = 1$$

$$\mu_{med}(10) = \begin{cases} 1 - \left|\frac{5-10}{5}\right| & else \\ 1 & x = 5 \end{cases} = 0 \quad v_{med}(10) = \begin{cases} \left|\frac{5-10}{5}\right| & else \\ 0 & x = 5 \end{cases} = 1$$

$$\mu_{high}(10) = \begin{cases} \frac{10-5}{5-0} & x > 5 \\ 0 & else \end{cases} = 1 \qquad v_{high}(10) = \begin{cases} \frac{10-10}{11/2} & x > 5 \\ 1 & else \end{cases} = 0$$

5.3.3.2 Inference engine

After inuitionistic fuzzification is done for each S, O and D input factors, inference engine is triggered. Intuionistic Fuzzy RPN is found by Min Method for "AND" in each membership rule and Max Method for "AND" in each non-membership rule.

- Rule 1: If Severity is Low AND Occurence is Low AND Detection is Low than RPN is Low

When rule 1 is triggered, inferred RPN membership value can be calculated by <u>min</u> operator as follow:

S Low $\wedge$ O Low $\wedge$ D Low = 0 $\wedge$ 0.2 $\wedge$ 0 = 0

When rule 22 is triggered, RPN non-membership value can be calculated by max operator as follow:

S Low $\vee$ O Low $\vee$ D Low = 1 $\vee$ 0.73 $\vee$ 1 = 1

- Rule 22: If Severity is High AND Occurence is Low AND Detection is Medium than RPN is Medium

When rule 22 is triggered, inferred RPN membership value can be calculated by min operator as follow:

S High $\wedge$ O Low $\wedge$ D Medium = 1 $\wedge$ 0.2 $\wedge$ 0.8 = 0.2

When rule 22 is triggered, RPN non-membership value can be calculated by max operator as follow:

S High $\vee$ O Low $\vee$ O Medium = 0 $\vee$ 0.73 $\vee$ 0.2 = 0.73

When the same procedure is followed by all rules. Corresponding FM3 inferred membership and non-membership values can be found as given in Table 5.4 and Table 5.5 respectively.

Table 5.4 : Corresponding Membership Values of FM3 for Each Rule. *Low (L), Medium (M), High (H).

Rule #	S	O	D	IF RPN
1	0 (L)	0,2 (L)	0 (L)	0 (L)
2	0 (L)	0,8 (L)	0 (M)	0 (L)
3	0 (L)	0 (L)	0 (H)	0 (L)
4	0 (L)	0,2 (M)	0,8 (L)	0 (L)
5	0 (L)	0,8 (M)	0,8 (M)	0 (M)
6	0 (L)	0 (M)	0,8 (H)	0 (M)
7	0 (L)	0,2 (H)	0,2 (L)	0 (L)
8	0 (L)	0,8 (H)	0,2 (M)	0 (M)
9	0 (L)	0 (H)	0,2 (H)	0 (M)
10	0 (M)	0,2 (L)	0 (L)	0 (L)
11	0 (M)	0,8 (L)	0 (M)	0 (M)
12	0 (M)	0 (L)	0 (H)	0 (H)
13	0 (M)	0,2 (M)	0,8 (L)	0 (L)
14	0 (M)	0,8 (M)	0,8 (M)	0 (M)
15	0 (M)	0 (M)	0,8 (H)	0 (H)
16	0 (M)	0,2 (H)	0,2 (L)	0 (M)
17	0 (M)	0,8 (H)	0,2 (M)	0 (M)
18	0 (M)	0 (H)	0,2 (H)	0 (H)
19	1 (H)	0,2 (L)	0 (L)	0 (M)
20	1 (H)	0,8 (L)	0 (M)	0 (H)
21	1 (H)	0 (L)	0 (H)	0 (H)
22	1 (H)	0,2 (M)	0,8 (L)	0,2 (M)
23	1 (H)	0,8 (M)	0,8 (M)	0,8 (H)
24	1 (H)	0 (M)	0,8 (H)	0 (H)
25	1 (H)	0,2 (H)	0,2 (L)	0,2 (M)
26	1 (H)	0,8 (H)	0,2 (M)	0,2 (H)
27	1 (H)	0 (H)	0,2 (H)	0 (H)

5.3.3.3 Defuzzification

As explained in Section 4.5 the inferred fuzzy output data of each rule, RPN, will be converted to crisp (non-fuzzy) value with equation 4.1 using membership and non-membership function of RPN.

With this method rule 22, 23, 25 and 26 is activated (meaning that for membership output is bigger than “0”).

In the case of Rule 22, RPN will be Medium, inferred μ value is 0.2 and inferred v value is 0.73. With equation 5.1 following calculations and statements can be done.

$$0.2 = \mu_{Med}(x) = 1 - \left|\frac{5 - x}{5 - 0}\right|$$

Table 5.5 : Corresponding Non-Membership Values of FM3 for Each Rule. *Low (L), Medium (M), High (H).

Rule	S	O	D	IF RPN
1	1 (L)	0,73 (L)	1 (L)	1 (L)
2	1 (L)	0,2 (L)	1 (M)	1 (L)
3	1 (L)	1 (L)	1 (H)	1 (L)
4	1 (L)	0,73 (M)	0,2 (L)	1 (L)
5	1 (L)	0,2 (M)	0,2 (M)	1 (M)
6	1 (L)	1 (M)	0,2 (H)	1 (M)
7	1 (L)	0,73 (H)	0,73 (L)	1 (L)
8	1 (L)	0,2 (H)	0,73 (M)	1 (M)
9	1 (L)	1 (H)	0,73 (H)	1 (M)
10	1 (M)	0,73 (L)	1 (L)	1 (L)
11	1 (M)	0,2 (L)	1 (M)	1 (M)
12	1 (M)	1 (L)	1 (H)	1 (H)
13	1 (M)	0,73 (M)	0,2 (L)	1 (L)
14	1 (M)	0,2 (M)	0,2 (M)	1 (M)
15	1 (M)	1 (M)	0,2 (H)	1 (H)
16	1 (M)	0,73 (H)	0,73 (L)	1 (M)
17	1 (M)	0,2 (H)	0,73 (M)	1 (M)
18	1 (M)	1 (H)	0,73 (H)	1 (H)
19	0 (H)	0,73 (L)	1 (L)	1 (M)
20	0 (H)	0,2 (L)	1 (M)	1 (H)
21	0 (H)	1 (L)	1 (H)	1 (H)
22	0 (H)	0,73 (M)	0,2 (L)	0,73 (M)
23	0 (H)	0,2 (M)	0,2 (M)	0,2 (H)
24	0 (H)	1 (M)	0,2 (H)	1 (H)
25	0 (H)	0,73 (H)	0,73 (L)	0,73 (M)
26	0 (H)	0,2 (H)	0,73 (M)	0,73 (H)
27	0 (H)	1 (H)	0,73 (H)	1 (H)

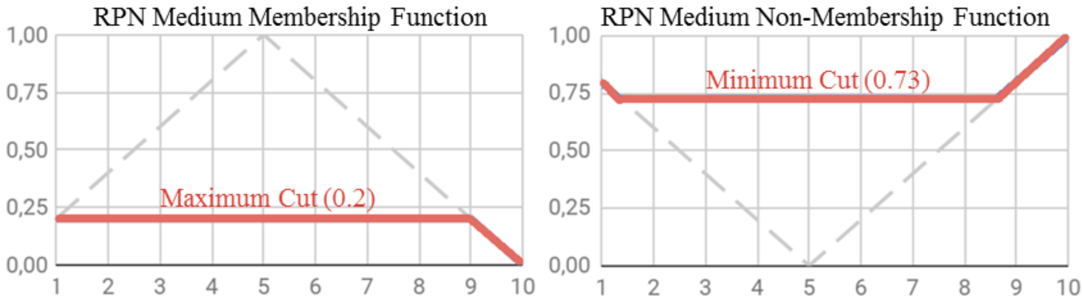
Then $x = 1$ or $x = 9$ which means between 1 and 9 μ can take maximum value of 0.2.

$$0.73 = v_{Med}(x) = \left|\frac{5 - x}{5 - 0}\right|$$

Then $x = 1.35$ or $x = 8.65$ which means between 1.35 and 8.65 v can take minumum value of 0.73.

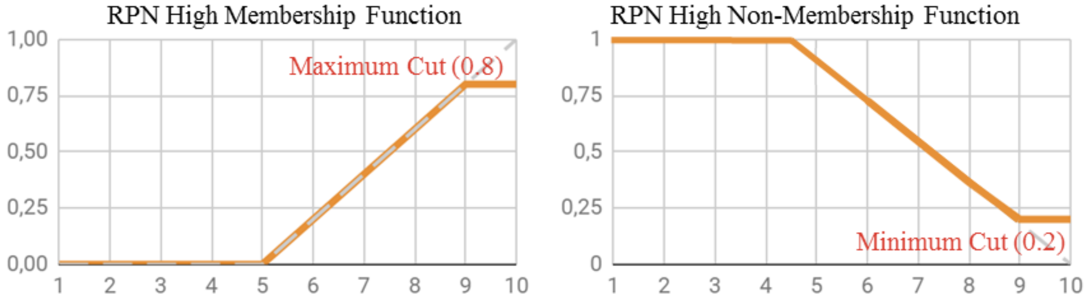
Same process can be done for all rules to find out maxiumum value which μ can take and minimum value which v can take.

Defuzzification implementation for Rule 22, 23, 25 and 26 can be found in Figure 5.5, Figure 5.6, Figure 5.7 and Figure 5.8 respectively.



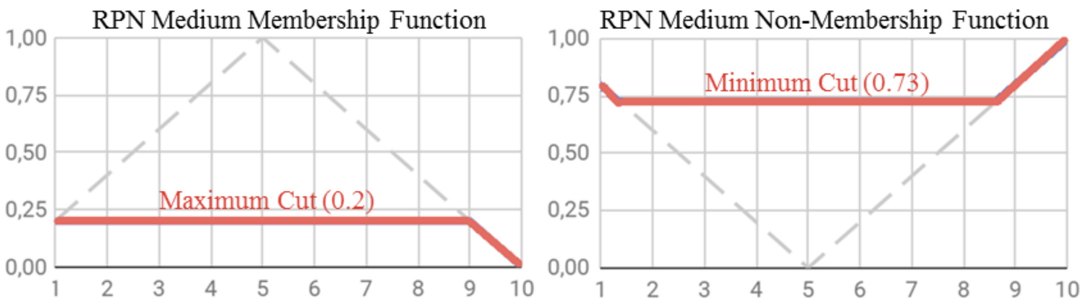
x	μ_x	ν_x	π_x	$X = (1 - \pi_x) * \mu_x$	$Y = \nu_x * \pi_x$	X+Y	x*(X+Y)
1	0.20	0.80	0.00	0.20	0.00	0.20	0.20
2	0.20	0.73	0.07	0.19	0.05	0.24	0.48
3	0.20	0.73	0.07	0.19	0.05	0.24	0.72
4	0.20	0.73	0.07	0.19	0.05	0.24	0.95
5	0.20	0.73	0.07	0.19	0.05	0.24	1.19
6	0.20	0.73	0.07	0.19	0.05	0.24	1.43
7	0.20	0.73	0.07	0.19	0.05	0.24	1.67
8	0.20	0.73	0.07	0.19	0.05	0.24	1.91
9	0.20	0.80	0.00	0.20	0.00	0.20	1.80
10	0.00	1.00	0.00	0.00	0.00	0.00	0.00
					Total:	2.07	10.34
					(x*(X+Y))/(X+Y)=	**5**	

Figure 5.5 : Defuzzification of Rule 22 with Takagi Sugeno.



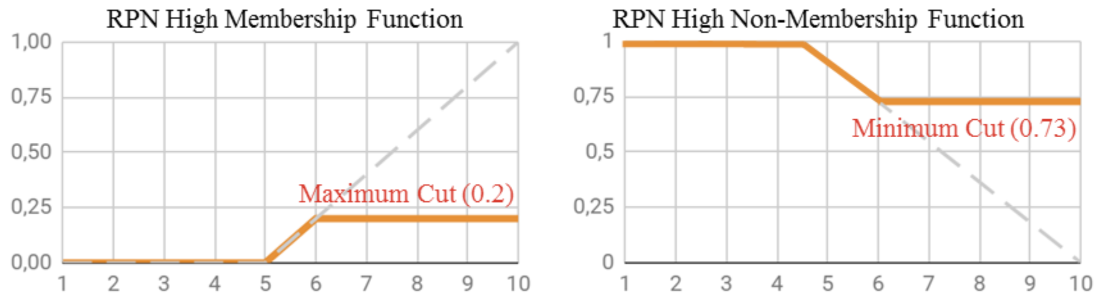
x	μ_x	ν_x	π_x	$X = (1 - \pi_x) * \mu_x$	$Y = \nu_x * \pi_x$	X+Y	x*(X+Y)
5	0.00	0.91	0.09	0.00	0.08	0.08	0.41
6	0.20	0.73	0.07	0.19	0.05	0.24	1.43
7	0.40	0.55	0.05	0.38	0.03	0.41	2.86
8	0.60	0.36	0.04	0.58	0.01	0.59	4.73
9	0.80	0.20	0.00	0.80	0.00	0.80	7.20
10	0.80	0.20	0.00	0.80	0.00	0.80	8.00
					Total:	2,.92	24.63
					(x*(X+Y))/(X+Y)=	**8.43**	

Figure 5.6 : Defuzzification of Rule 23 with Takagi Sugeno.



x	μ_x	ν_x	π_x	$X = (1 - \pi_x) * \mu_x$	$Y = \nu_x * \pi_x$	X+Y	x*(X+Y)
1	1	0.20	0.80	0.00	0.20	0.00	0.20
2	2	0.20	0.73	0.07	0.19	0.05	0.24
3	3	0.20	0.73	0.07	0.19	0.05	0.24
4	4	0.20	0.73	0.07	0.19	0.05	0.24
5	5	0.20	0.73	0.07	0.19	0.05	0.24
6	6	0.20	0.73	0.07	0.19	0.05	0.24
7	7	0.20	0.73	0.07	0.19	0.05	0.24
8	8	0.20	0.73	0.07	0.19	0.05	0.24
9	9	0.20	0.80	0.00	0.20	0.00	0.20
10	10	0.00	1.00	0.00	0.00	0.00	0.00
					Total:	2.07	10.34
					(x*(X+Y))/(X+Y)=	**5**	

Figure 5.7 : Defuzzification of Rule 25 with Takagi Sugeno.



x	μ_x	ν_x	π_x	$X = (1 - \pi_x) * \mu_x$	$Y = \nu_x * \pi_x$	**X+Y**	**x*(X+Y)**
5	0.00	0.91	0.09	0.00	0.08	0.08	0.41
6	0.20	0.73	0.07	0.19	0.05	0.24	1.43
7	0.20	0.73	0.07	0.19	0.05	0.24	1.67
8	0.20	0.73	0.07	0.19	0.05	0.24	1.91
9	0.20	0.73	0.07	0.19	0.,05	0.24	2.15
10	0.20	0.73	0.07	0.19	0.05	0.24	2.38
					Total:	1.27	9.95
					(x*(X+Y))/(X+Y)=	**7.8**	

Figure 5.8 : Defuzzification of Rule 26 with Takagi Sugeno.

By averaging 4 activated outcomes of rule 22, 23, 25 and 26 outcome RPN can be found as:

$$\frac{5+8.43+5+7.81}{4}=6.56$$

If the same procedure is followed for all of the FMs presented in this study, IFRB RPN values can be found as in Table 5.10. In the table, FMs are sorted by descending order to IFRB RPN and average rank method is used for tied ranks. Also for a better comparison with other methods, IFRB RPN values are normalized with Equation 5.2 and then multiplied by 1000 for rescaling, where IFRB RPN_{Min} is 351.2 which is IFRB RPN output of an FM when S, O and D is 1 and IFRB RPN_{Max} is 853.3 which is IFRB RPN output of an FM when S, O and D is 10.

Table 5.6 : RPN Results and Rankings of FMs by Intuionistic Fuzzy Inference System. (*Normalized).

Failure Modes	Severity	Occurrence	Detection	IF.RPN	IF RPN*	Rank
FM1	6	7	8	716.4	427.9	1
FM5	6	6	4	710.4	420.9	2
FM3	10	4	6	656.0	357.2	3
FM6	10	2	7	652.0	352.4	4
FM7	10	2	6	646.1	345.6	5
FM10	10	1	9	638.6	336.7	6
FM9	10	2	5	634.0	331.4	7
FM2	9	4	7	550.6	233.6	8
FM4	9	2	8	547.7	230.2	9
FM11	9	2	5	531.5	211.2	10
FM12	6	2	5	525.7	204.5	11
FM13	9	2	3	519.3	197.0	12
FM14	6	2	3	504.5	179.6	13
FM8	4	5	6	491.7	164.6	14

5.4 Comparison and Results

RPN calculations by traditional FMEA method, FRB method and IFRB method can be found in Table 5.11.

Table 5.7 : RPN Results and Ranking Comparisons by Traditional, FRB and IFRB methodologies. (*Normalized).

FMs	RPN	RPN Rank	FRB RPN*	FRB RPN Rank	IFRB RPN*	IFRB RPN Rank
FM1	336	1	421,1	4,5	427,9	1
FM2	252	2	598,4	2	233,6	8
FM3	240	3	686,2	1	357,2	3
FM4	144	4,5	364,5	10,5	420,9	2
FM5	144	4,5	397,7	8	230,2	9
FM6	140	6	421,1	4,5	352,4	4
FM7	120	7,5	421,1	4,5	345,6	5
FM8	120	7,5	343,1	12	164,6	14
FM9	100	9	421,1	4,5	331,4	7
FM10	90	10,5	364,5	10,5	336,7	6
FM11	90	10,5	397,7	8	211,2	10
FM12	60	12	194,9	13,5	204,5	11
FM13	54	13	397,7	8	197,0	12
FM14	36	14	194,9	13,5	179,6	13

Where different sets produced the same results in traditional FMEA, IFRB FMEA produced different results. For example, FM4 & FM5, FM7 & FM8, and FM10 & FM11 have the same traditional RPN values but different IFRB RPN values. IFRBS produced 14 different RPN value while traditional FMEA produced 11 different RPN values. In return, FRBS gave only 7 unique RPN value in the given set, in other words, produced same FRB RPN values with a more set of combination of factors, i.e. FM1, FM6, FM7 and FM9 have different traditional and IFRB RPN values but the same FRB RPN values.

By using spearman rank correlation method, the association between traditional RPN, FRB RPN, and IFRB RPN also found statistically significant. Between traditional RPN and FRBS RPN, rs found as 0.663. Between traditional RPN and IFRBS RPN, rs found as 0.686. Between FRB RPN and IFRBS RPN, rs found as 0.59.

6. CONCLUSIONS

6.1 General Discussion

FMEA usually involves human judgments and linguistic expression in real life combined with lack of information and imprecise data available in the systems. Using traditional RPN calculation method with crisp values cannot sufficiently represent the uncertainty in the risk assessment. In this study, intuitionistic fuzzy rule-baed approach is proposed to represent uncertainty, ambiguity, and vagueness in FMEA risk assessment. To enable the calculations of the related steps and formulations, a simulation is developed in Excel.

Each FM has produced different unique RPN values in the given data set by IFRB FMEA method while traditional FMEA and FRB FMEA has produced less.

Current rule based approach doesn’t ignore the relativity between the S, O and D factors. Especially due to Severity has a more impact compared to the other two factors from the rule-base perspective, FM6, FM7, FM9 and FM10 have a higher priority compared to their earlier ranking. Importance of factors can further differentiated with tailor-made and different S, O and D membership and non-membership functions.

From RPN scale perspective IFRB FMEA produced 14 unique values and the normalized results show that values are much more equally distributed on the scale 1-1000. From this perspective, FRB FMEA has given worse performance compared to with only 7 unique variables.

Results showes that intuitionistic fuzzy logic in FMEA provides a more diversified ranking, by integrating a degree of hesitancy it is able to represent uncertainty in the system.

This study’s main contribution to literature is development of a FMEA methodology which integrates a degree of hesitancy with Fuzzy rule based artifical intelligence. The proposed method has been able to solve also several shortcomings presented in the Section 2.2. Lastly, it provides a more flexibility in order to assess risks and can represent the confidence of expert by giving a degree of hesitancy. IFRB provides a structure which is more resembling the human nature of thinking and expression which FMEA factor evaluation was already based on.

Also as presented earlier while having advantages, IFRB FMEA carries some disadvanteges like the higher complexity, consuming more resources and harder maintenance.

6.2 Future Research Directions

This study can be further extended to generate more complex and appropriate membership and non-membership functions which can resolve problems from Section 2.2 which are not adressed in this study and provide a better understanding of IFRB FMEA. In addition, used functions are in triangular form, it can be further improved to involve other types of functions. Furthermore, the number of rules can be reduced to increase the efficiency of the system. Lastly, all combinations of factors can be investigated further in order to understand the effect on the scale fully.

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