



YAŞAR UNIVERSITY

GRADUATE SCHOOL

PHD THESIS

**A STUDY ON MODELING AND SOLUTION
METHODS OF RESOURCE CONSTRAINED
PROJECT SCHEDULING PROBLEMS**

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ABSTRACT

A STUDY ON MODELING AND SOLUTION METHODS OF RESOURCE CONSTRAINED PROJECT SCHEDULING PROBLEMS

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Effective management and scheduling of projects with limited resources has become critical in today's competitive environment, where organizations face pressures to deliver projects on time, within budget, and with optimal resource utilization. The Resource-Constrained Project Scheduling Problem (RCPSp) offers a powerful framework to address these challenges.

This thesis aims to fill key gaps identified in the existing RCPSp literature, particularly regarding the application of continuous-time mathematical models. A comprehensive literature review revealed a notable scarcity of studies employing continuous-time approaches, especially for Multi-Mode Resource-Constrained Project Scheduling Problems (MRCPSps). To address this gap, a novel continuous-time mathematical model was developed and validated using benchmark problems. The results demonstrate its effectiveness in handling complex scheduling scenarios, offering a more flexible and scalable alternative to traditional discrete-time models and other continuous-time-based models in the literature.

Additionally, the thesis explores solution methods that extend beyond conventional approaches, culminating in the development of an original heuristic solution method. This method significantly enhances computational efficiency while providing effective solutions. Furthermore, a new benchmark problem library was created to address the limitations of existing datasets, providing a valuable resource for future research in RCPSp and related fields.

Keywords: resource-constrained project scheduling, project management, mathematical modeling, heuristics, test problem library.

ÖZ

KAYNAK KISITLI PROJE ÇİZELGELEME PROBLEMLERİNİN MODELLENMESİ VE ÇÖZÜM YÖNTEMLERİ ÜZERİNE ÇALIŞMA

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Sınırlı kaynaklarla projelerin etkin yönetimi ve zamanlanması, günümüzün rekabetçi ortamında, organizasyonların projeleri zamanında, bütçe dahilinde ve kaynakları en iyi şekilde kullanarak teslim etme baskısı altında giderek daha kritik hale gelmiştir. Kaynak Kısıtlı Proje Zamanlama Problemi (RCPS), bu zorlukların üstesinden gelmek için güçlü bir çerçeve sunar.

Bu tez, mevcut RCPS literatüründe belirlenen önemli boşlukları doldurmayı, özellikle sürekli zaman matematiksel modellerinin uygulanmasıyla ilgili eksiklikleri gidermeyi amaçlamaktadır. Kapsamlı bir literatür incelemesi, özellikle Çok Modlu Kaynak Kısıtlı Proje Zamanlama Problemleri (MRCPS) için sürekli zaman yaklaşımlarını kullanan çalışmaların dikkate değer bir şekilde az olduğunu ortaya koymuştur. Bu boşluğu gidermek için, yenilikçi bir sürekli zaman matematiksel modeli geliştirilmiş ve bu model, kıyaslama problemleri kullanılarak doğrulanmıştır. Sonuçlar, modelin karmaşık çizelgeleme senaryolarındaki etkinliğini göstermekte ve ayrık zaman modellerine ve literatürdeki diğer sürekli zaman temelli modellere göre daha esnek ve ölçeklenebilir bir alternatif sunduğunu ortaya koymaktadır.

Ayrıca, tez geleneksel yaklaşımların ötesine geçen çözüm yöntemlerini de araştırmakta ve bu çalışmalar, özgün bir sezgisel çözüm yöntemi geliştirilmesiyle sonuçlanmaktadır. Bu yöntem, hesaplama verimliliğini önemli ölçüde artırırken etkili çözümler sunmaktadır. Ayrıca, mevcut veri setlerinin sınırlamalarını gidermek için yeni bir kıyaslama problem kütüphanesi oluşturulmuştur ve bu kütüphane, gelecekteki RCPS ve ilgili alanlardaki araştırmalar için değerli bir kaynak sağlamaktadır.

Anahtar Kelimeler: kaynak kısıtlı proje zamanlama, proje yönetimi, matematiksel modelleme, sezgisel yöntemler, test problem kütüphanesi

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Güler Öztürk Görgülü

İzmir, 2024

TEXT OF OATH

I declare and honestly confirm that my study, titled “A STUDY ON MODELING AND SOLUTION METHODS OF RESOURCE CONSTRAINED PROJECT SCHEDULING PROBLEMS” and presented as a PhD Thesis, has been written without applying any assistance inconsistent with scientific ethics and traditions. I declare, to the best of my knowledge and belief, that all content and ideas drawn directly or indirectly from external sources are indicated in the text and listed in the list of references.

Güler Öztürk Görgülü

14.08.2024



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SYMBOLS AND ABBREVIATIONS

SYMBOLS:

α	Alfa
β	Beta
ε	Epsilon

ABBREVIATIONS:

CT	Continuous Time
CPM	Critical Path Method
DT	Discrete Time
ILP	Integer Linear Programming
MILP	Mixed-Integer Linear Programming
MM	Multi-Mode
MRCPSP	Multi – Mode Resource Constrained Project Scheduling Problem
NON-RE	Non-Renewable
NPV	Net Present Value
PERT	Program Evaluation and Review Technique
PSPLIB	Project Scheduling Problems Library
R&D	Research and Development
REN	Renewable
RCPSP	Resource Constrained Project Scheduling Problem
SM	Single Mode

1. CHAPTER: INTRODUCTION

Project scheduling is a key aspect of the project management process, emphasizing the critical importance of defining the initiation and completion timelines of project activities, along with the strategic allocation of necessary resources for their execution. This complex undertaking is essential for ensuring project activities are sequenced and timed effectively, adhering to resource constraints and precedence relations to guarantee timely project completion. This challenge is recognized in the field as the Resource-Constrained Project Scheduling Problem (RCPSP), a topic that has garnered considerable notice across various industries, including software development, construction engineering, and the project management segments of research and development (R&D) departments. The difficulty of solving RCPSP, given its requirement to balance activity scheduling with limited resources, has made it an attractive area of research, although existing models often fail to fully address the wide range of scheduling issues that emerge in diverse operational contexts.

Since the mid-20th century, the Critical Path Method (CPM) by Kelley, J. E. et al.(1958) has emerged as a prevalent technique for modeling and controlling projects within defined assumptions and boundaries, identifying critical paths where delays in any activity could lead to overall project delays. Following CPM, the Program Evaluation and Review Technique (PERT) by Clark C.E. (1958) offers a technique to calculate the probability of finishing a project by a certain duration or date associated with a specific probability level.

However, both CPM and PERT, along with Gantt charts, do not consider resource constraints. This "time-only" analysis presents significant limitations, as it fails to account for resource conflicts that arise when multiple activities compete for the same scarce resources. Such oversights highlight the inadequacy of these methods in addressing the complexities of real-life project scheduling, where the trade-off between activity duration and resource availability becomes crucial.

In the late 1980s, there was increasing interest in scheduling algorithms that accounted for resource constraints, acknowledging the intricate realities of project management that necessitate a balance between time scheduling and resource allocation. This evolution in scheduling methodology underscores the emergence of the RCPSP has become a well-established issue in the field, attracting extensive research efforts from domains such as operations research and construction management.

1.1. Real-Life Importance of the Problem

Real-life projects are much more comprehensive and extensive. Also, in real-life problems, the number of activities is much higher. Additionally, while standard RCPSP models assume fixed activity durations, real-life project problems often involve different durations. If an activity is performed in a single way, which is determined by a fixed resource requirement and fixed duration, it is termed “single mode”. However, in real-life cases, different execution “modes” may be defined for an activity such that each mode matches a different resource requirement and activity duration. The problem is known as the multi-mode RCPSP (MRCPSP) when several modes are available for selecting an activity. Each mode matches a particular resource/duration and (or) cost/ duration trade-off selection for the activity under review. Blazewicz J. et al. (1983) stated that RCPSP is an NP-hard problem, and hence the MRCPSP is also NP-hard. This complexity sets limitations on the optimal solution methods for the problem, allowing optimal solutions to be applied only to smaller projects. Research within MRCPSP has focused on meta-heuristics and priority-based heuristics, which do not ensure an optimal solution. Kolisch R. et al. (1999) stated that the effectiveness of these algorithms is debatable, and as the difficulty of the network enlarges, the performance of these algorithms significantly diminishes. Furthermore, the performance of widely used popular software packages in resource allocation is arguably inadequate and requires enhancement. One of the objectives of this thesis is to develop an efficient algorithm to obtain near-optimum or optimum solutions to the RCPSP.

1.2. Scope and Limitations of the Thesis

RCPSP originates from the job-shop scheduling problem in operations research and is designed to handle various scheduling challenges where resources are limited. This study focuses on the RCPSP and its variants, especially on the MRCPSP.

For this research, it is assumed that the activities cannot be paused or preempted once they have started, and each is characterized by resource demands and non-negative durations. All the parameters within this study are considered deterministic, implying a stable structure for the project scheduling.

The durations of activities are treated as discrete values. Renewable and nonrenewable resources are both considered. Although there are models in which multiple projects are considered, only a single project was focused on in our study. Although there are some other performance measures for project scheduling, makespan minimization is considered in this thesis.

The structure of the thesis is outlined as follows: Chapter Two offers a detailed overview of the definition of the problem. Chapter Three summarizes the present literature on RCPSP & MRCPSP and states the motivation of the thesis. In Chapter Four, an original mathematical model has been proposed and defined in detail. Chapter Five is dedicated to the computational comparisons. The following chapter introduces an original heuristic solution developed specifically for this thesis, with a thorough examination of the algorithms involved and their testing outcomes. The final chapter concludes the thesis, summarizing key findings and implications of the research.

2. CHAPTER: RESOURCE CONSTRAINED PROJECT SCHEDULING PROBLEM

RCPSP considers a project with a set of activities defined as $I = \{1, 2, 3, \dots, i, \dots, j, \dots, I\}$. The duration of an activity (i) is indicated by (d_i). Due to technological requirements, certain precedence relationships exist between activities. A set, P_{prec} , includes the precedence relations. This set is composed of activity pairs (i, j) indicating that activity (i) is the predecessor of activity (j). Each activity requires specific amounts of resources to be carried out. The problem is to find the best sequencing and scheduling of activities while adhering to precedence constraints and resource limitations. The objective function is usually explained as minimization of makespan although some other objectives may be used.

The characteristics of the objective function and different environments of activities and resources may lead to numerous variations and extensions of the basic RCPSP. Those factors are discussed below.

2.1. Resources

The status of the resources may differ in different environments. If a resource can be reused after each time period, then it is called a *renewable* resource. It means a renewable resource is accessible for each period. Specific examples are equipment, machines, laboratories, and workforce.

Nonrenewable resources are finite throughout the entire planning horizon, without any limitations within each specific period. A typical case is the financial budget allocated for a project.

Doubly constrained resources are restricted both on a per-period basis and over the entire planning horizon. A typical example is budget constraints, which limit the total capital available for the entire project while also restricting its use during each period. Another instance is limiting resource utilization within a specific portion of the planning horizon. An example would be a monthly planning horizon where workers'

weekly working hours, rather than daily hours, are restricted by their employment contract.

2.2. Activities

If an activity is carried out in a single manner, it is defined by a set of specific resource requirements and duration, it is termed single mode.

On the contrary, when various modes may be chosen for an activity, it is termed a multi-mode activity. For example, the problem environment may allow defining different durations for an activity depending on the amount of the resource allocated. To rephrase, activities can be performed in various execution modes. Table 2.1. denotes an imaginary case in which there are different durations for the activities if different amounts of resources are allocated.

Table 2.1. Durations of Activities in Different Modes Related to Amount of Resource Allocation

Activity	Amount of Resource Allocated									
	1	2	3	4	5	6	7	8	9	10
A								2	1	
B			1							
C		3	2	1						
D						4	2	2	1	
E						2	2	1		
F				3	1					
G				1						
H						2				
K				1						
P				3	2	1				

Source: Author

To avoid irrelevant entries, various cells are left blank intentionally. The first row indicates that activity “A” requires a minimum of 8 units of resource to be performed and the corresponding duration is 2 time units. However, its duration decreases to 1 unit if the amount of resource allocation is 9 units. This means that activity A has two execution modes. However, the duration of activity “B” consumes a minimum of 3 units of resource and its duration does not change with the additional resource allocation. For activity “P”, there are three execution modes. The first mode represents a duration of 3 time units with minimal 4 units of resource. The second and third modes represent additional amounts of resource allocations, which lead gradual decrease in duration.

The description of multi-mode activities given above is based on a renewable resource. In some other environments, it may require to define multi-mode activities based on non-renewable resources.

It is clear that the existence of multi-mode activities complicates the representation and hence the solution of the problem. Because you need to decide simultaneously on both the specific mode and starting time of each activity.

Notice that, for multi-mode activities described above, we may define an empirical discrete function for each activity that regulates the link between duration and amount of resource allocation. However, in some other project environments, that function may be a continuous one. In other words, the resource may be continuously divisible.

There is another distinguishing factor related to activities. The project environment may require us to classify activities as either preemptive or non-preemptive. Preemptive activities can be interrupted while in progress whereas non-preemptive activities cannot be interrupted once it starts.

2.3. Performance Measures & Objective Functions

Makespan minimization is the most used goal in project scheduling. The makespan represents the duration from the beginning to the completion of the project. Since the project usually starts at $t = 0$, minimizing the makespan means reducing the longest completion time of all activities.

Other performance metrics related to time include minimizing the flow time of activities or, if due dates are specified, minimizing the delays.

Net present value(NPV) maximization: When a project involves substantial cash flows, such as expenses for progress payments and starting activities for completing parts of the project, the NPV criterion becomes a more suitable indicator of project performance. This measure produces a cost-critical path and activity schedule, as opposed to the time-critical path and schedule determined by the makespan objective.

Quality maximization: The quality of activities and projects may be used as a performance measure of the schedule. Icmeli-Tukel O. et al.(1997) were pioneers in incorporating the objective of quality into project scheduling.

2.4. Single Project vs Multiple Projects

So far, the descriptions and discussions are related to the “single project” case. However, the modeling perspective is still valid in the case of “multi-projects” since it is possible to bind or integrate multiple projects artificially into a single “super” project. Resources are shared from a common pool. With this approach, multiple projects can be defined and modeled using scheduling models and methods discussed for single project cases.

However, there may be some environments in which multiple projects should be treated independently instead of integrating them into a single project. For example, individual projects may have their resources although there is a common pool of global shared resources. In another case, the finish date or due date of each individual project may be an important factor in scheduling. Even in such cases, basic issues are still valid although it requires modifications in modeling such as formulating relevant constraints and organizing the objective function accordingly in order to represent the requirements of those cases.

2.5. Time Domain: Discrete vs Continuous

In the vast majority of the studies in literature, researchers use models in which the time horizon is discrete, i.e. represented by a number of discrete time intervals and they are called “discrete-time-based models”. Such models are compact, easy to understand, and relatively flexible to accommodate side constraints.

On the other hand, there are certain disadvantages of discrete-time-based models. For example, by definition, using a discrete approximation of the time horizon results in

suboptimal solutions. Moreover, as the number of activities grows and the time horizon expands, the model should incorporate a significant number of binary variables.

Consequently, efforts in the past decade have focused on developing models based on continuous-time representations. This approach is reported to offer the potential for creating more accurate and efficient models. However, there is still a limited number of studies on “continuous-time-based” models.



3. CHAPTER: LITERATURE REVIEW AND MOTIVATION

3.1. Literature Review for Basic Structure of the Problem

A comprehensive search has been conducted in the literature to identify as many relevant articles as possible. An article database has been developed to make an analysis of the literature related to deterministic RCPSP. To formulate the list of articles in the database, first, the “survey” papers have been investigated. Among various surveys, the five most cited articles have been chosen for further studies. Those surveys are shown below in Table 3.1.

Table 3.1. Chosen Survey Articles

Year of the Survey	Number of citations	Topic	Author
	Scopus		
1995	285	<i>A Survey on the Resource-Constrained Project Scheduling Problem</i>	Ozdamar, L. et al. (1995)
1998	473	<i>Resource-Constrained Project Scheduling: A Survey of Recent Developments</i>	Herroelen, W. et al. (1997)
2001	395	<i>An integrated survey of deterministic project scheduling</i>	Kolisch, R, et al. (2001)
2010	775	<i>A survey of variants and extensions of the resource-constrained project scheduling problem</i>	Hartmann, S., et al.(2010)
2022	112	<i>An updated survey of variants and extensions of the resource-constrained project scheduling problem</i>	Hartmann, S., et al.(2022)

Source: Author

Survey papers have been reviewed thoroughly and all the papers in their reference lists have been examined. If any paper is related to deterministic RCPSP, it is imported to the article database of this study. The list of the database contains repeating entries due to the papers included in more than one survey. Such entries have been eliminated to get a database with unique entries.

There are only two articles included in all of the surveys. The first one is by Pritsker A. A. et al. (1969), in which the first mathematical model is formulated for RCPSP. The other one is by Demeulemeester E. L. et al. (1996) which presents the first successful solution method (a branch and bound algorithm) for the problem involving with and without preemptive activities.

Once the database was established, then the articles in the database were examined to identify their distinguishing features based on the following factors.

- Is the article concerned with multi-mode activities or single-mode activities?
- Is the time domain discrete or continuous?
- Are the resources renewable, or non-renewable?

The database is classified and grouped depending on the questions above and its summary is presented in Table 3.2 below. It contains the following abbreviations: “DT” stands for Discrete-Time, “CT” stands for Continuous-Time, “REN” stands for Renewable Resource, “NON-REN” stands for Non-Renewable Resource, and “P” is the abbreviation of Preemptive. Additionally, the articles which are shown with * in this table are listed in Appendix 1.

Table 3.2. Clusters of Articles by Distinguishing Features

Resources	Single Mode		Multi-Mode	
	DT	CT	DT	CT
REN	Afshar Nadjafi, B. et al.(2017) Afshar-Nadjafi, B. et al.(2013) Agarwal, A. et al.(2011) Alvarez-Valdes, R. et al.(1989) Anthonisse, J. M. et al.(1988) Artigues, C. et al.(2003) Baar, T. et al.(1998) Balas, E. (1971) Balas, E. et al. (1996) Ballestín, F. (2007) Ballestín, F. et al. (2006)	Achuthan, N. et al.(2001) Demeulemeester, E. L. et al.(1996)P Elmaghraby, S.E. (1995) Franck, B. et al.(2001)P Icmeli, O. ET AL.(1996) Jia, Q. et al.(2013) Jozefowska, J. et al.(2000) Koné, O. (2012)	Afshar-Nadjafi, B. et al.(2012) Ahn, T. et al.(1995) Ahn, T. et al.(1998) Alcaraz, J. et al.(2003) Artigues, C. et al.(2000) Alvarez-Valdes, R. et al.(1989) Balouka, N. et al.(2016) Barrios, A. et al.(2009) Basnet, C. (2016) Bellenguez, O. et al.(2005) Bianco, L. et al.(1998)	Almeida, B. F.et al.(2016) Castro, P. M. et al.(2006) Ozturk and Oner(2020) Waligora, G. (2008)

	Ballestín, F. et al.(2008)P Baptiste, P. et al.(1999) Baroum, S. et al.(1999) Baroum, S. (1992) Baroum, S. et al.(1989) Bartusch, M. et al.(1988) Belhe, U. et al.(1995) Pritsker, A.A.B. et al.(1969) *and 230 others	Koné, O. et al.(2011) Koné, O. et al.(2013) Kopanos, G.M. et al.(2014) Kumar, N. et al.(2015) Liu, Z. et al.(2015) Naber, A. (2017) Palpant, M. et al. (2004) Rihm, T. et al.(2021) Rom, W.O. et al.(2002)	Boctor, F. F. Boctor, F. F. (1993) .Boctor, F. F. (1996) Bouleimen, K. et al.(2003) Brucker, P. et al.(2001)P Buddhakulsomsiria, J. et al.(2007)P Cesta, A. et al.(2002) *and 65 others	
REN + NON REN	Agrawal, M. K. et al.(1996) Akkan, C. et al.(2005) Chaleshtarti, A.S. et al.(2014) Nudtasomboon, N. et al.(1997) * and 7 others	Kyriakidis, T. et al.(2012)	Azizoglu, M. et al.(2015), Bilolikar, V. S. et al.(2016) Brucker, P. et al.(2003) Coelho, J. et al.(2011) *and 5 others	Chakraborty, R. K. et al.(2014) Gnägi, M. et al.(2018) Gnägi, M. et al.(2019) Kyriakidis, T. et al.(2012) Zapata, J. C. et al.(2008)

Source: Author

When Table 3.2. is examined, some observations may be articulated as follows.

- majority of the studies involve in single project with renewable resources and single-mode activities,
- relatively very few articles concerned with developing continuous time-based models,
- preemptive activities did not attract much interest and hence there are not many articles concerning preemption.

3.2. Literature Review for Solution Methods

RCPSP is classified as strongly NP-hard Blazewicz J. et al. (1983). Although mathematical models have been proposed for the problem, unfortunately, it is possible to solve it optimally only for small-scale projects. Due to this limitation, numerous researchers have turned their attention toward other solution methods such as developing heuristics and employing meta-heuristics.

3.2.1. Mathematical Models

Numerous mathematical models have been suggested to solve RCPSPs. Based on their time representation, those models can be categorized into two primary categories, continuous-time, and discrete-time models. In discrete-time models proposed by Talbot F.B. et al. (1982) and Pritsker A.A.B. et al., (1969), the time horizon is discrete

i.e., it is shown by a series of discrete-time intervals. Although discrete-time-based models are relatively flexible and simple, there are some disadvantages as well. For example, as the number of activities grows and the time horizon stretches, a large number of binary variables should be included in the model since a decision variable is associated with each activity - time unit couple. Therefore, efforts have been undertaken to develop models using continuous-time representations. This approach is reported to offer the potential for developing more effective models.

A novel approach is presented by Koné O. et al. (2011) and Artigues C. et al. (2013) for continuous-time modeling, in which the time representation is developed on the notion of "events." This approach enables the definition of a continuous-time domain instead of numerous discrete time intervals, significantly reducing the number of decision variables. Using this "event" approach, they proposed two distinct models: which are termed "On/Off Event (OOE)" and "Start/End Event (SEE)" model.

An additional continuous-time-based approach has been suggested by Artigues C. et.al. (2003), in which a "flow-based" formulation is utilized. It utilizes the network flow concept, where resources are transferred between activities. Both event-based and flow-based models provide novel approaches. However, they are designed to solve single-mode RCPSPs and they can't be used to solve MRCPSP.

There are relatively few studies that propose mathematical models to consider MRCPSPs. One of the models is presented by Kyriakidis T.S. et al., (2012) and Kopanos G.M. et al., (2014) which depends on the resource task network (RTN) representation, a method utilized in process scheduling problems. It is a continuous-time-based MILP model that can be used to solve both RCPSPs and MRCPSPs.

An alternative formulation is presented by Chakraborty et al., (2014), that have developed two event-based models for multi-mode RCPSP inspired by the proposed event-based models in Koné O. et al., (2011).

Another alternative continuous-time modeling approach has been suggested by Gnägi et al., (2019), which uses the resource assignment concept, and hence it is called the "assignment-based" approach. In this approach, two types of binary variables are utilized to formulate the resource capacity constraints. First, each unit of each renewable resource is associated with each activity. They are called the assignment variables and they represent which individual units of renewable resources are used

for the execution of each activity. The second type of variable, known as the sequencing variable, determines the sequence in which pairs of activities are assigned to the same renewable resource unit. Using that approach, two MILP models are proposed to solve single and multi-mode problems.

In this study, we have also proposed another two MILP models in Ozturk and Oner (2020) to solve MRCPPs. The formulations of the models were based on the flow and event-based approaches. Both models were justified and tested using the benchmark problem instances presented in Kolisch et al., (1995), It is reported that the event-based model performs considerably poorly concerning solution times when it is compared to the flow-based model.

3.2.2. Solution Methods Other Than Mathematical Models

Another query has been made in order to show the groupings of the articles depending on the solution methods. Table 3.3. below is a summary report in which articles are grouped by solution methods.

Table 3.3. Articles Tabulated by Solution Methods

Solution Approach	Description	References	Remarks
Heuristics	Finding a near-optimal solution	Ahn, T. et al.(1998), Akkan, C. et al.(2005), Almeida, B. F. et al.(2016), Alvarez-Valdes, R. et al.(1989) , Alvarez-Valdes, R. et al.(2006), Alvarez-Valdes, R. et al.(2008), Anthonisse, J. M. et al.(1988), Alvarez-Valdes, R. et al.(1989), Ballestín, F. et al.(2006), Ballestín, F. et al.(2008) , Baroum, S. et al.(1989), Bartels, J. H. et al.(2009), Basnet, C. (2016), Belhe, U. et al.(1995), Bell, C. E. et al.(1991) Bellenguez, O. et al. (2005), Beşikci, U. et al.(2015) , Bey, R. B. et al.(1981), Bianco,L. et al.(1998), Bock, D. B. et al.(1990), Boctor, F. F. (1990), Boctor, F. F. (1993), Boctor, F. F. (1996), Bomsdorf, F. et al.(2008), Bowers, J. A. (2000), Brinkmann, K. et al.(1996), Browning, T. R. et al.(2006), Brucker, P. (2002), Brucker, P. et al.(2001), Buddhakulsomsiria, J. et al.(2007), Cavalcante, C. C. B. et al.(2001) , Cesta, A. et al.(2002), Chakraborty, R. K. et al.(2019),Chen, Z.J. (2010), Chen, J. et al.(2009) , Chiu, H. N. et al. (2002) , Cooper, D. F. (1976), Daniels, R. L. et al.(1994), Davis, E. W. Kumanan, S. et al.(2006), Kuster, J. et al.(2006), Lawrence, S. R. et al.(1993), Leachman, R.C. et al.(1990), Lee, J.K. et al.(1996), , Li, K. Y. et al.(1992), Liu, Z. et al.(2015), Lova, A. et al.(2000), Maniezzo, al.(1997), Thesen, A. (1976), Thomas, P. R. et al.(1997), Tormos, P. et al.(2001), Tormos, P. et al.(2003), Tsubakitani, S. et al.(1990), Tukel, O. I. et al.(2001), Ulusoy, G. et al.(1989), Ulusoy, G. et al.(1994) Valls, V. et al.(2005), Vanden Berghe, G. et al.(2011), Vanhoucke, M. et al.(2001), Vos, S. et al.(2007), Wang, L. et al.(2012), Whitehouse, G. E. et al.(1979) , Zhan, J. (1994), Zheng, X. et al.(2015), Zhu, D. et al.(1997)	The limitations of mathematical models in solving complex and large RCPSPs have prompted the creation of heuristic and metaheuristic approaches.

Solution Approach	Description	References	Remarks
Meta Heuristics	Genetic algorithm (GA), Tabu Search (TS), Simulated Annealing(SA) Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO), Differential Evolution (DE) and hybrid	<p>(GA): Afshar-Nadjafi, B. et al.(2017), Afshar-Nadjafi, B. et al.(2012), Alcaraz, J. et al.(2003), Ballestín, F. (2007), Balouka, N. et al.(2016), Barrios, A. et al.(2009), Beşikci, U. et al.(2015), Delgoshaei, A. et al. (2016), Ghoddousi, P. et al.(2013), Gholizadeh-Tayyar, S. et al.(2016), Goncalves, J.F. et al.(2008), Goncalves, J.F. et al.(2011), Goncharov, E. et al.(2017), Goncharov, E. (2023), Hartmann, S. (1998), Hartmann, S. (1999), Hartmann, S. (2001), Kohlmorgen, U. et al.(1999), Kumanan, S. et al.(2006), Leon, V.J. et al.(1995), Mendes, J.J. et al.(2009), Mori, M. et al.(1997), Najafi, A.A. et al.(2006), Ozdamar, L. (1999), Padman, R. et al.(1997), Peteghem, V. V. et al.(2010), Proon, S. et al.(2011), Ranjbar, M. et al.(2007), Ranjbar, M. et al.(2008), Sebt, M. et al.(2015), Servranckx, T. et al.(2024), Shadrokh, S. et al.(2007), Shtub, A. et al.(1996), Tseng, L. Y. et al.(2009), Ulusoy, G. et al.(2000), Ulusoy, G. et al.(2000), Valls, V. et al.(2008), Zamani, R. (2017), Zhu, J. et al.(2011)</p> <p>TS: Artigues, C. et al.(2003), Baar, T. et al.(1998), Bukata, L. et al(2013), , Bukata, L. et al(2015), Dai, H. et al.(2018), Drezet, L.E. et al.(2008), Icmeli, O. et al.(1994), Klein, R. (2000b), Mika, M. et al.(2005), Mika, M. et al.(2008), Nonobe, K. et al.(2002), de Reyck, B. et al.(1998), Servranckx, T. et al.(2019), Thomas, P. R. et al.(1998), Waligora, G. (2008), Zhu, D. et al.(1999)</p> <p>SA: Boctor, F. F. (1996), Bouleimen, K. et al.(2003), Cho, J. H. et al.(1997), Etkar, R. et al.(1997), Jozefowska, J. et al.(2000), Jozefowska, J. et al.(2001), Mausser, H.E. et al.(1996) Mika, M. et al.(2005), Shtub, A. et al.(1996)</p> <p>PSO: Chen, R.M. (2011), Fahmy, A. et al.(2014), Jarboui, B. et al.(2008), Khalilzadeh, M. et al.(2012), Koulinas, G. et al.(2014), Kumar, N. et al.(2015), Zhang, H. et al.(2006), Zhang, H. et al.(2006)</p> <p>ACO: Chiang, C.W. et al.(2008), Fink, A. et al.(2012), Li, H. et al.(2013), Thiruvady, D. et al.(2013)</p> <p>DE: Afshar-Nadjafi, B. et al.(2013), Chen, M. et al.(2015), Damak, N. et al.(2009) , Lorenzoni, L.L. et al.(2006), Rahimi, A. et al.(2013)</p> <p>Hybrid: GA&PSO: Sebt, M. et al.(2016) GA&SA: Bettemir, O. H. et al.(2014), Bilolikar, V. S. et al.(2016), Sonmez, R. et al.(2012) Greedy&GA: Delgoshaei, A. et al.(2015) GA& Neural Network: Agarwal, A. et al.(2011) DE&Greedy: Cheng, M.Y. et al.(2016) Myszkowski, P.B. et al.(2018) ACO&Scatter Search: Chen, W. et al.(2010) Evolutionary Algorithm&GA Elloumi, S. et al.(2010) Scatter Search&Electromagnetism Debels, D. et al.(2006)</p>	

Source: Author

Table 3.3 of the literature review examines a range of solution techniques proposed for solving the RCPSP. Genetic algorithms are one of the most widely studied methods as can be seen from the table.

In addition to these methods, “matheuristics” is also being used. Matheuristic is an optimization approach that combines mathematical programming techniques (such as mathematical modeling and optimization) with heuristic methods. This type of method leverages the precision and accuracy of mathematical programming with the speed and flexibility of heuristic methods to solve complex and large-scale problems.

Lodi A. et al. (2010) suggest that a significant question is whether a reliable Integer Linear Programming (ILP) model can be developed for a scheduling problem. Alternatively, researchers might need to consider extending the problem model in various ways, such as through hybridization. Also, Zhu G. et al. (2006) offer a cutting-edge solution for PSPLIB instances, utilizing a Branch-and-Cut scheme. This approach also incorporates hybridization with a genetic algorithm. Their technique leverages the mathematical formulation of the MRCPSPP while simultaneously employing heuristic and metaheuristic techniques to explore the problem's characteristics.

Matheuristics, as highlighted by Boschetti M.A. et al. (2009), are gaining significant focus in the literature. These methods aim to develop algorithms that efficiently solve optimization problems by integrating mathematical programming with (meta)heuristic methods. Matheuristics are defined as hybrid algorithms that combine elements of (meta)heuristics and mathematical programming.

Research on “matheuristics” for project scheduling problems is limited. Gerhards P. et al. (2017) presented a matheuristic that combines an adaptive large-neighborhood search algorithm with mixed-integer programming to address the MRCPSPP, achieving impressive results on the MMLIB+ dataset. Fernandes G.A. et al. (2021) suggested a local branching mathematical strategy for solving the MRCPSPP and validated the method's effectiveness through numerical experiments.

Toffolo T. A. M. et al. (2016) concentrate on the multimode resource-constrained multi-project scheduling problem (MRCMPSP), using a combination of integer

programming and heuristic methods to handle multiple projects and various execution modes. Their approach involves generating feasible schedules through a series of IP-based components, aiming for high-quality solutions in a competitive context.

Until this point, a comprehensive and in-depth literature review has been meticulously conducted. In the subsequent section, we will focus exclusively on MRCPSPs and provide a detailed exploration of various variants and extensions.

3.3. Exclusive Literature Review for MRCPSPs.

Extensions and variants of the RCPSP have been progressed to handle the limitations of the original model and to better show the complex realities of project management. These adjustments are crucial to adapting to diverse project constraints, duration, and resource dynamics, establishing more realistic and practical scheduling solutions.

In the categorization of RCPSP variants and extensions, resources, and activities are the two dimensions used to structure different scheduling problems. This categorization approach is used because it provides a systematic way of the vast array of scheduling challenges, distinguishing between the nuances of resource allocation and task execution. Classifying the problems based on resources and activities helps in developing and applying more aimed and effective scheduling methods that are tailored to specific constraints and requirements of different projects.

3.3.1. Generalized Activity Concepts.

Preemptive Scheduling:

In the original RCPSP, each task must be finished in a single, uninterrupted way. Once a task is started, it must be carried out continuously until finished. The studies by Slowinski R. et al. (1994) and Weglarz J. et al. (1977) show the procedures to get optimal solutions for scenarios involving continuous processing times for various activities.

On the other hand, pre-emptive scheduling allows for a more flexible approach where tasks can be paused and resumed at a later point. This flexibility is especially beneficial in project environments that are subject to frequent changes. The study presented by Slowinski R. et al. (1981) was the pioneer in introducing the concept of pre-emption. Dhib C. et al. (2014) specifically focus on the requirement that a resource allocated to

an activity must remain consistent to complete it post-pre-emption, aiming to limit the disruption pre-emption. On the contrary, Afshar-Nadjafi B. et al. (2014), Turkgenç A. et al. (2019), and Peteghem, V.V. et al. (2010) allow activities to be paused at any moment and resumed later without any extra cost. This relaxation of constraints aims to enhance the optimal makespan by extending the solution space, a contrast to Dhib, C. (2014) 's focus on resource consistency post-pre-emption.

Study of Cheng, J. et al. (2015), distinguish between activities that can be paused due to temporary resource insufficiency and those that cannot. This approach addresses how resource availability affects project scheduling, considering pre-emptive scenarios in project scheduling problems. Besides that, it is contextualized within a framework that includes specified release times and deadlines for each activity.

Moukrim A. et al. (2015) focus on a variant of the RCPSPs in which the activities can be interrupted freely, without any restrictions.

The studies stated above are included in the Hartmann S. et al. (2022) survey. Below, additional significant and relevant articles from the literature that were not covered in their paper are considered.

In the study by Delgoshaei A. et al. (2016), the authors inspected the effect of using pre-emptive resources on the NPV in the context of RCPSP. Their focus is on understanding how pre-emptive resources can influence the financial outcomes of project scheduling, specifically in terms of maximizing the NPV.

Han Y. et al. (2011), address the concept of pre-emption in RCPSP by focusing on activity splitting. They explore how allowing activity splitting in a pre-emptive scenario can reduce the total makespan of a project compared to a non-pre-emptive approach where activity splitting is not permitted. Their analysis is illustrated through a 10-activity network, comparing the project timelines in both pre-emptive and non-pre-emptive scenarios.

In the study by Ma W. et al. (2015), the authors explore pre-emption in project scheduling from another perspective. This model enables activities to be split into multiple sub-activities. Each activity is performed with a unit duration, with no extra cost. The target is to ensure that these sub-activities remain consistent in their execution modes throughout the project. While Ma W. et al. (2015) suggest a special

model for splitting activities without extra costs, Tavana M. et al. (2014) take a unique approach to handling pre-emption in RCPSP. They evaluate different assumptions for activity interruptions due to factors such as resource restrictions or technical problems. They focus on reducing the number of pre-emptions for each activity to suggest that the time between pre-emption and restart should be minimized and decrease the risk of restarting from the beginning. Additionally, their model enables activities to be restarted in several modes post-pre-emption, which can lead to executing parts of an activity in several execution modes.

Adding to these different perspectives and methodologies, in the context of pre-emptive MRCPSPP, Hosseini Z. et al. (2014) suggest a mathematical model that shows the complexity of incorporating pre-emption into MRCPSPP. It examines the impact of pre-emption on the finishing times of activities, provided they fall within their latest and earliest periods. Also targeting to maximize the NPV and minimize tardiness.

Time-Dependent Resource Request

In the original RCPSP, each activity, labeled as 'j', uniformly requires a fixed amount of a specific resource, 'k', denoted as r_{jk} . This implies that the resource requirement remains consistent throughout the activity's processing time. However, in a variation of this model, the resource requirements are dynamic, changing over time. This means for activity j, the resource requirement for resource k varies in each time period t, represented as r_{jkt} .

While traditional project scheduling models often assume a constant resource usage throughout an activity, it's feasible to adjust this resource usage during the activity's execution. This modification has to respect specific boundaries: there are maximum and minimum bounds for resource consumption and an essential minimum time lag between any changes in this usage. A project scheduling approach is suggested by Fündeling C.-U. et al. (2010), where the total resource demand is known, however, its distribution over the project's timeline is determined by the schedule. This approach guarantees that the total resource demand is encountered by the activity's end, selecting a flexible resource profile that considers both upper and lower bounds on resource consumption and a minimum time lag between consecutive resource consumption changes.

That idea is enlarged by Hosseinian A.H. et al. (2019) by evaluating the resource requirements that vary with time, considering multiple skills. Another layer of complexity is added by them to the scheduling problem, as it considers controlling resources that have several requirements over time and are also skill-specific.

An advanced version of the classic project scheduling problem is examined by Hartmann S. et al. (2012) which involves changes in demands and resource availability for each activity over several time periods. This method admits that resources needed for activities in a project can be different, and the amount of resources available can also vary with time. This approach provides a more realistic and dynamic view of RCPSP.

There are other studies in this context that are not included in the Hartmann S. et al. (2022) survey. For example, Zimmermann A. et al. (2017) demonstrate the application of the variable resource requirement model in practical scheduling scenarios. They approach the scheduling of candidates undertaking various tasks as a modified version of RCPSP (MRCPSP). This modification includes a single resource whose requirement changes over time and incorporates mode assignment constraints.

It is worth noting that there are only a limited number of studies about the problems in which the resource requirements are dynamic, changing over time. There aren't even mathematical models except the one proposed by Fündeling C.-U. et al. (2010). This area presents a promising opportunity for further research.

Setup Times:

In certain instances, a resource (for example, a machine) needs to be readied before an activity can commence. The duration required for this preparation is referred to as setup time.

The paper by Quintanilla S. et al. (2012) offers a distinctive approach to setup times in project scheduling by incorporating generalized precedence relationships (GPRs) in multi-mode project scheduling. This method focuses on the complexities of task sequencing and resource allocation, considering both time and work constraints in GPRs, thereby providing a more nuanced understanding and modeling of setup times in project management.

This study is the only reference featured in the Hartmann survey in the context of setup times. However, the following articles also offer contributions to the analysis of setup times in MRCPSPs.

The MRCPSP with schedule-dependent setup times, as Mika M. et al. (2006) primarily discusses the incorporation and modeling of setup times, emphasizing their critical role in preparing resources for specific activities. It investigates several types of setup times, such as sequence-dependent and sequence-independent, and provides a novel category, schedule-dependent setup times. The aim is to consider factors like multiple resource requests, resource constraints, and activity precedence, to optimize resource utilization and project timelines by integrating these setup times into project scheduling models.

However, the complexity of setup times when resources are moved between different locations for several activities, with the setup time fluctuating based on these locations is highlighted by Mika M. et al. (2006). A more dynamic and practical model of project scheduling, reflecting the fluctuating time requirements for resource preparation based on their allocation over time is presented by this method.

The more diverse and complex nature of setup operations is analyzed by Mika M. et al. (2012), covering their costs, and combining these considerations into MRCPSP models. This method provides more understanding of setup times, showing their fluctuation based on factors such as activity sequences and resource characteristics.

An additional variation is discovered by Kadri et al. (2014), the MRCPSP with sequence-dependent transfer times. In this approach, transfer times are incurred while carrying resource units across multiple locations, with the transfer duration changing based on the locations of the activities. However, with the multi-site RCPSP, in this situation, the locations for carrying out the activities are supposed to be predetermined.

A novel approach is presented by Bigler T. et al. (2022) to multi-site RCPSP, emphasizing transportation times between sites. Differing from earlier models, this paper introduces a continuous-time model that accurately accounts for the movement of resources between multiple locations, a remarkable advancement in considering setup times in project scheduling. This method provides for a more realistic and dynamic representation of resource allocation and scheduling across several locations, focusing on a gap in previous research.

Tradeoff Problems:

The concept of trade-off primarily involves balancing several project constraints, such as cost, time, and availability of resources, across different modes of operation in MRCPSPs.

The aim is on the discrete time-cost trade-off problem (DTCTP), particularly addressing the budget version in the paper by Hazır Ö. et al. (2010). This method considers minimizing project completion time while obeying budget constraints and standard precedence conditions, providing a novel perspective on time management and resource allocation in RCPSP. The novel perspective stays in balancing project duration with budget constraints. This method remarks optimizing project scheduling by considering both time and cost, an important aspect in project management where resources are limited and efficient use of both budget and time is crucial.

The trade-off in project management between increasing resource usage and reducing activity duration is focused by Colak E. et al. (2014). It provides a MIP model and heuristic approaches for optimizing resource allocation within project deadlines. The aim is to balance resource investment versus time savings to accomplish efficient project completion.

Conversely, a complex project scheduling problem concentrating on the trade-off between cost, time and quality is suggested by Khalili-Damghani K. et al. (2015). It presents a novel mixed-integer programming approach and compares it with other methods to optimize these conflicting objectives in multi-mode project environments.

Optimizing project scheduling and staffing under constraints are concentrated by Van Den Eeckhout M. et al. (2019). It presents a heuristic method that considers trade-offs between resource demand and project duration, particularly balancing project deadlines and personnel costs. The research is remarkable for its integrated approach to managing resource constraints and time in project management, mainly in contexts with scheduling requirements and complex staffing.

There are also other remarkable researches that are not referred to in Hartmann S. et al. (2010)'s survey. For instance, a cost-minimization model for project scheduling that considers discrete trade-offs in cost, quality, time and risk is presented by Polancos R.V. et al. (2023). It applies integer linear programming to assign resources by skill

set, focusing on cost efficiency. In contrast, a comprehensive framework for project selection and evaluation, integrating multiple criteria like strategic, financial and risk factors is offered by Tavana M. et al. (2014). It shows a hybrid multi-criteria decision-making approach, distinguished by its wider focus on project selection rather than particular scheduling and resource allocation.

Wuliang P. et al. (2009) present a new multi-mode resource-constrained discrete time-cost trade-off model (MRC-DTCTP). This model combines the general DTCTP and the MRCPSP, focusing on renewable resources like manpower. The authors developed a genetic algorithm to solve this model, demonstrating its effectiveness through comparative analysis with an exact algorithm.

3.3.2. Generalized Resource Concepts.

Non-Renewable and Partially Renewable Resources:

It is generally assumed in RCPSPs, that the resources are fully available in each period. However, non-renewable refers to resources that are limited over the complete planning horizon and cannot be replenished once used. A typical example is a project's capital budget.

Hartmann S. et al. (2022) features only one study in this context, which is presented by Tao S. et al. (2018). That article describes non-renewable resources, such as materials and energy. They are recognized as crucial and limited elements within the context of MRCPSPs.

There are also other relevant studies as well. For example, in Gerhards P. (2020), Ghoddousi P. et al. (2013), and Azizoglu M. et al. (2015), the focus is on the allocation and consumption of non-renewable resources for specific activities within a project. This approach views non-renewable resources, such as budgets or rare materials, as elements that are consumed by individual project activities, and these resources are useful for modeling outsourced activities to external contractors. Notably, Gerhards P. (2020) aggregates non-renewable resources into a single one.

Partially renewable resources are a type of resource that can replenish themselves, but only during certain periods or under specific conditions. They are available in varying capacities during these distinct time frames. This concept combines elements of both non-renewable and renewable resources. Like renewable resources, they can

regenerate, but this regeneration is limited or conditional, similar to non-renewable resources.

Partially renewable resource constraints were initially introduced by Böttcher J. et al. (1999), treating the conventional non-renewable and renewable constraints as specific cases.

Cumulative Resources:

Neumann K. et al. (2006) introduced the concept of cumulative resources, integrating them into an RCPSP that includes both maximal and minimal time lags. Their study involves cumulative resources, which experience depletion and replenishment over time. These resources can be thought of as the inventory levels in a storage facility with a limited capacity. The range of this inventory is defined by two limits: it cannot fall below a certain safety stock level, and it cannot exceed the storage facility's maximum capacity.

Bartels J.-H. et al. (2009) applied cumulative resources in an MRCPSP with maximal and minimal time lags, focusing on a project involving testing and engineering activities in the automotive industry. A test device is modeled as a cumulative resource, as it can be constructed, utilized, and ultimately destroyed in a crash test, thereby depleting its capacity until the project's completion.

Bartels J.-H. et al. (2010) set a limit on the maximum availability of cumulative resources in an MRCPSP, a decision driven by the finite capacities of inventories. Kyriakidis T.S. et al. (2012) adopted a similar stance but further considered a specific network structure. This structure places additional constraints on the required activities.

Hurink J.L. et al. (2011) research explored a unique scenario in which subsets of activities utilize the capacity of cumulative resources. When any activity in such a subset begins for the first time, it occupies a predetermined number of capacity units specific to that subset. These units remain occupied until the final activity in the subset is completed. Moreover, there's an added stipulation that the capacity units are sequentially numbered, necessitating that each subset is allocated consecutively numbered capacity units.

Multi Skilled

The Multi-Skill Resource-Constrained Project Scheduling Problem (MS-RCPSP) in a multi-mode setting is specifically suitable in complex projects where resources are the workers and they have several skill sets. The difficulty of the problem is enhanced notably with the integration of multi-modes and multi-skills, as it inserts layers of decision-making regarding mode selection, resource allocation, and scheduling, all while obeying constraints such as resource availability, deadlines, and activity dependencies. This makes it a complex realistic model for many real-world RCPSPs.

On the other hand, the impracticality of using the MRCPSPP for multiple skills as a result of the excessive number of modes it generates is emphasized by Bellenguez O. et al. (2007). In MRCPSPP, each mode matches a diverse subset of staff members whose skills correspond to an activity's needs, leading to an expansion of modes as the number of feasible staff combinations increases.

Heimerl C. et al. (2009) have discussed a model for staffing and scheduling multiple projects involving a multi-skilled workforce. It addresses the problem of allocating multi-skilled external and internal human resources to IT projects, intending to decrease labor costs. The model is an MIP with a tight LP-bound, focusing on efficiently allocating human resources based on their specific skills and efficiencies.

Continuous Resources:

In traditional RCPSPs, resources are typically available only in discrete quantities, akin to manpower and machines. However, Waligóra G. et al. (2010) discuss discrete-continuous project scheduling problems aimed at minimizing the makespan, focusing on projects requiring both discrete and one specific continuous, renewable resource. It presents both exact and heuristic methods for allocating continuous resources, explores the concept of resource discretization, and examines a special case involving identical processing rate functions.

In the Hartmann S. et al. (2010) survey, only the aforementioned article is mentioned under this topic. In the subsequent sections, papers not covered in Hartmann's survey are explained.

Weglarz J. et al. (1977) in their study, expanded the concept of renewable resources to include those that can be divided continuously. This approach is particularly beneficial

for resources such as energy or raw materials like liquids, which can be measured and used in varying amounts rather than in fixed, indivisible units.

Józefowska J. et al. (2000), introduced a novel method where continuous resource allocations were transformed into discrete units. This process effectively converted the continuous model into a more traditional discrete MRCPSP, but in a form that excludes non-renewable resources. The variety of modes available for different tasks within this model is directly influenced by the granularity of the discretization, meaning the total count of discrete resource allotments that are defined.

Further Resource Constraints:

Fu F. et al. (2014) and Zoraghi N. et al. (2017) expanded the MRCPSP by introducing a new category of resources termed materials. In this context, materials are utilized by various activities, akin to non-renewable resources, emphasizing their consumption aspect in the project scheduling framework.

Poppenborg J. et al. (2016) delve into the complexities of resource management in hospital evacuations, specifically addressing resources that are blocked by ongoing activities until the next resource becomes available to continue the process.

The three papers mentioned above are included in Hartmann's survey. However, the following papers, relevant to the same topic, are not covered in the Hartmann S. et al. (2022) article.

Gerhards P. (2020), specifically addresses nuclear dismantling with a detailed method combining programming and metaheuristics to optimize resource use, emphasizing cost and efficiency in this high-stakes field. Hübner F. et al. (2021), however, offer a wider perspective on project scheduling challenges and strategies, including but not focused solely on nuclear dismantling, highlighting general issues in managing large-scale projects.

3.4. Motivation of the Thesis

The effective management and scheduling of projects, particularly those with limited resources, has become increasingly critical in today's competitive and resource-scarce environment. As organizations across industries face mounting pressure to deliver projects within budget, on time, and with optimal resource utilization, the challenge of

scheduling tasks while considering constraints such as limited manpower, materials, and finances has never been more relevant.

RCPSP stands at the intersection of operations research, project management, and optimization, offering a robust framework for addressing these challenges. Despite significant advancements in the field, many practical issues remain unresolved, particularly when dealing with complex, real-world scenarios where resources are not only scarce but also interdependent and subject to varying levels of availability over time.

The motivation for this thesis is to identify and address the gaps in the existing RCPSP literature through focused research. A thorough literature review revealed a notable scarcity of studies in mathematical models employing continuous-time approaches. To bridge this gap, the first goal was to investigate developing a mathematical model using the continuous-time approach, especially for MRCPSPs. As a result of this thesis, we have managed to develop a novel continuous-time mathematical model, which is rigorously validated using benchmark problems.

The solution methods other than mathematical models are also thoroughly investigated. Therefore, another intention of this research was to explore solution methods beyond traditional mathematical modeling and to develop an original heuristic solution method for contributing to the literature.

Another key motivation is to examine the benchmark libraries utilized to test and evaluate the existing mathematical models and solution methods. As a result of this thesis, a new benchmark library has been generated to advance the field further.

4. PROBLEM DEFINITION & MODEL FORMULATIONS

4.1. Problem Definition and Basic Existing Formulations

RCPSP involves a project with a set of activities defined as $I = \{1, 2, 3, \dots, i, \dots, j, \dots, I\}$. The duration of an activity (i) is represented by (d_i). There are precedence relations between activities. A set, P_{prec} , includes the precedence relations. This set is composed of activity pairs (i, j) indicating that activity (i) is the predecessor of activity (j). Certain amounts of resources are required for each activity to be performed. The problem is to find the best sequencing and timing of activities without violating resource limitations and precedence constraints. The objective function is defined as the minimization of makespan.

The problem becomes less challenging if we disregard the resource constraints, as the CPM technique can then provide quick solutions. Blazewicz J. et al.(1983) demonstrated that when resource constraints are introduced, the problem becomes NP-hard.

The initial mathematical model for RCPSP was introduced by Pritsker A.A. et al.(1969). It may be defined as follows. Let I be the set of all actual activities; $I = \{i = 0, 1, \dots, I+1\}$, where 0 and $I+1$ are dummy activities representing the initiation and completion of the project, respectively. On the other hand, let's define R as the set of resources; $R = \{r = 1, 2 \dots R\}$. The details of the model are given below.

Indices/Sets

I	set of activities, $i = 0 \dots I+1$
T	set of discrete time periods, $t = 0 \dots T$
R	set of renewable resources, $r = 1 \dots R$

Parameters:

K_{rt}	Capacity of resource at time t
U_{ir}	Resource usage of activity i for resource r
d_i	Duration of activity i

ES_i, LS_i	Earliest & latest starting time for activity i . Each parameter here is an input and determined by preprocessing.
P_{prec}	The precedence set. It contains the precedence relations composed of activity pairs (i, j)

Decision Variables:

$$x_{it} = \begin{cases} 1 & \text{if activity } i \text{ starts at time } t \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Minimize } \sum_{t=ES_i}^{LS_i} tx_{(I+1),t} \quad (1)$$

Constraints:

$$\sum_{t=ES_i}^{LS_i} x_{it} = 1 \quad \forall i \in I \cup \{I+1\} \quad (2)$$

$$\sum_{t=ES_j}^{LS_j} tx_{jt} \geq \sum_{t=ES_i}^{LS_i} tx_{it} + d_i \quad \forall (i,j) \in P_{prec} \quad (3)$$

$$\sum_{i=1}^I U_{ir} \sum_{q=\max(ES_i, t-d_i+1)}^{\min(LS_i, t)} x_{iq} \leq K_{rt} \quad \forall r \in R \text{ and } \forall t \in T \quad (4)$$

$$x_{00} = 1 \quad (5)$$

$$x_{it} = 0 \quad \forall i \in I \cup \{I+1\}, \quad t \in T \setminus \{ES_i, LS_i\} \quad (6)$$

$$x_{it} \in \{0,1\} \quad \forall i \in I \cup \{I+1\}, \quad \forall t \in \{ES_i, LS_i\} \quad (7)$$

Equation (1) specifies the project's objective function, which is to minimize the makespan, or equivalently, to minimize the start time of the final dummy job. Constraint (2) requires that each activity be executed exactly once. Constraint (3) maintains the precedence relationships between activities. Meanwhile, constraint set (4) enforces the resource capacity constraints. Finally, constraint (6) ensures that the start time of any activity occurs between its earliest and latest start time. The earliest start time of each activity can simply be set to zero. However, to minimize the number of decision variables, they can be determined by *forward pass* calculations if the classical CPM technique is used without considering resource constraints. The latest

start times may be set equal to T to be safe. However, they may be determined by *backward pass* calculations if the project duration is set to T , which is chosen to be sufficiently large. Otherwise, the problem may become infeasible.

Sample Problem and Solution:

Assume that there is a project involving 10 activities, with the precedence relationships depicted in Figure 4.1. The diagram uses AON representation and the activities are shown in boxes and identified with capital letters. The arcs denote the precedence relations. Numerical values on the top of the boxes show the durations of the activities.

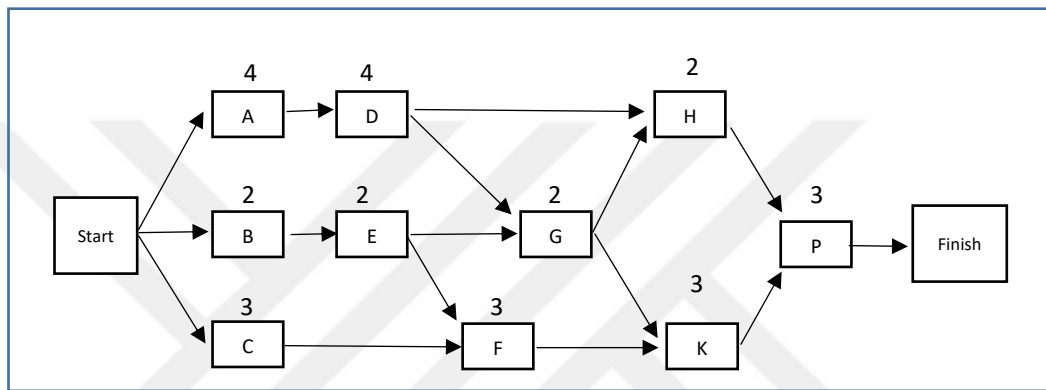


Figure 4.1. Precedence Relations of Sample Project

Source: Author

The scheduling horizon is $T=25$ discrete time units. Assume that there are two different resources $R=2$ and the capacity of each resource is equal to 10 for each period. On the other hand, the resource requirements for the activities are provided in Table 4.1.

Table 4.1. Resource Requirements of Activities

	Activities											
	dummy	A	B	C	D	E	F	G	H	K	P	dummy
RESOURCE-1	0	8	3	2	6	6	4	4	6	4	4	0
RESOURCE-2	0	2	4	7	7	1	2	5	3	4	3	0

Source: Author

In order to show the impact of the resource constraints, the problem is first solved using the CPM technique without considering resource constraints. After that, the formulation defined above is used to solve the problem including all constraints. The Gantt charts of the corresponding solutions are given in Figure 4.2. and Figure 4.3. respectively.

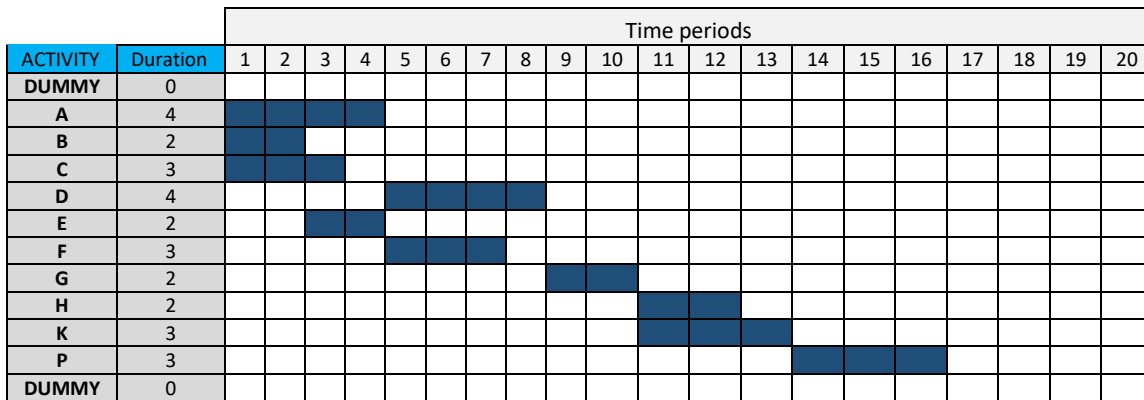


Figure 4.2. Gantt Chart of the Project without Resource Constraints

Source: Author

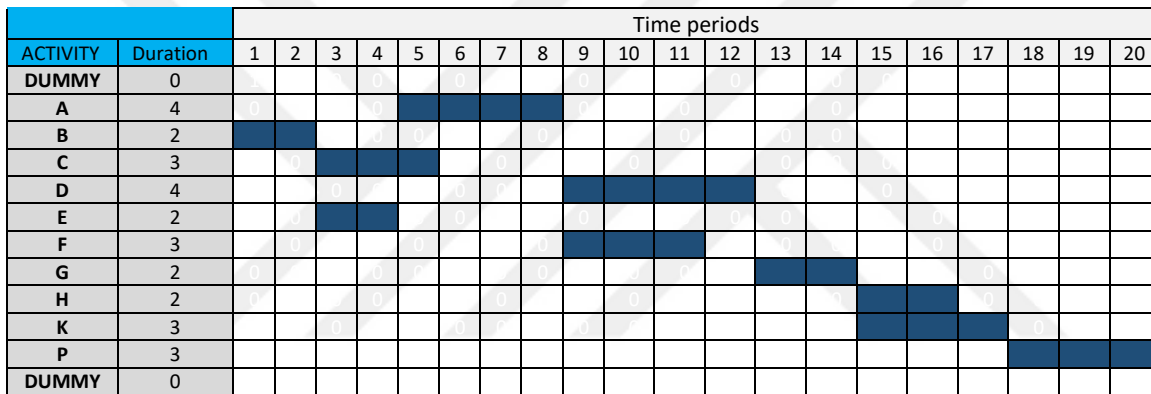


Figure 4.3. Gantt Chart of the Project Including Resource Constraints

Source: Author

Notice that when resource constraints are inserted, some activities are not allowed to be performed simultaneously although it is viable concerning precedence constraints. Hence, the project takes longer time with resource constraints. Corresponding resource allocations are given in Figure 4.4. and Figure 4.5. respectively.

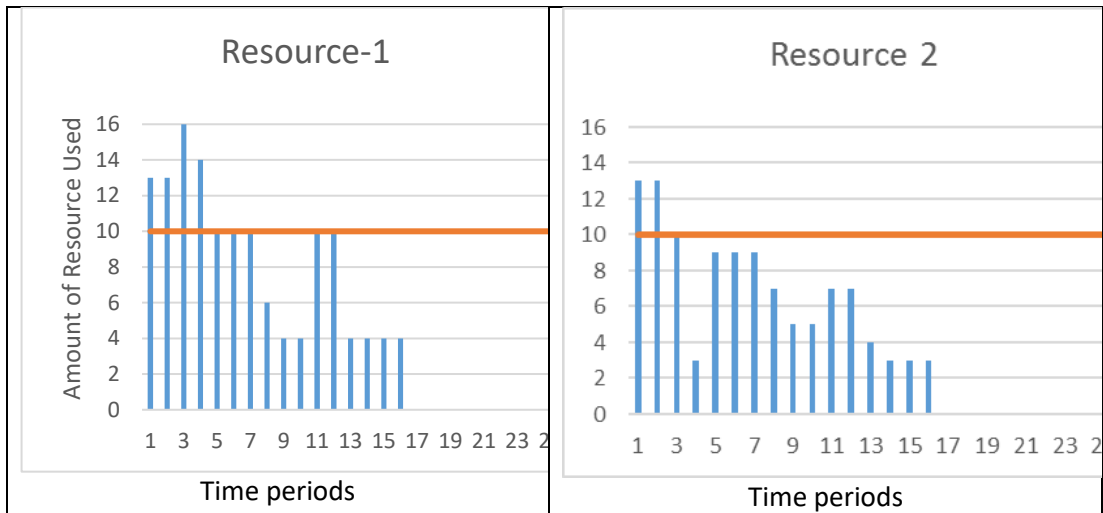


Figure 4.4. Allocations of Resources by Time (Resource constraints are not applied)

Source: Author

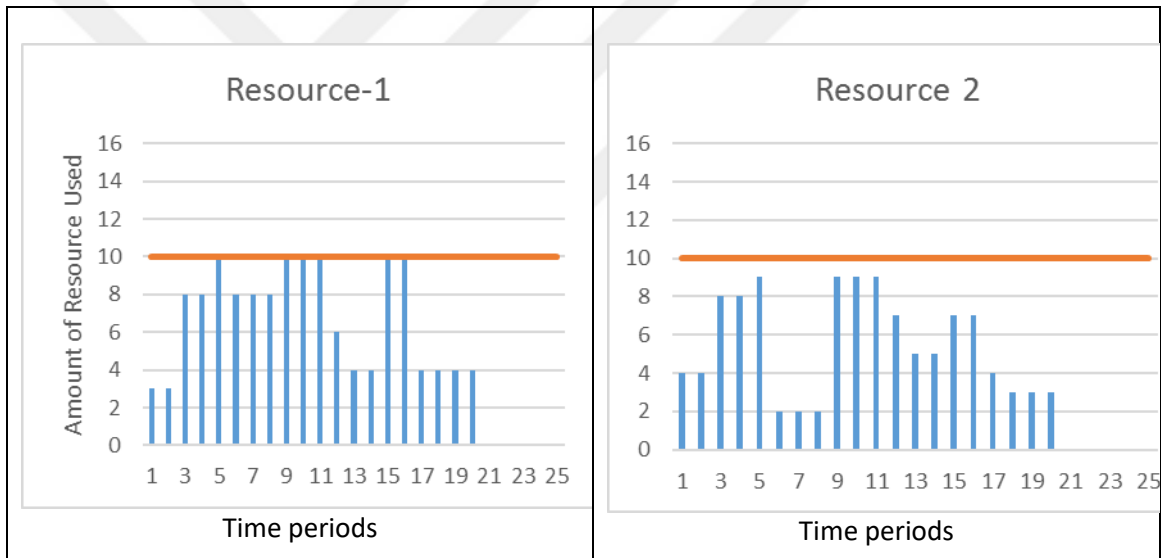


Figure 4.5. Allocations of Resources by Time (Resource constraints are applied)

Source: Author

Following Pritsker, several alternative DT models have been proposed such as Talbot F.B. et al. (1982), Hartmann, S. et al. (2010) and Kolisch, R. et al. (2001). Talbot F.B. et al. (1982) paper presents methods for optimizing nonpreemptive RCPSP by balancing time and resource trade-offs while selecting the best execution modes for each task.

In the vast majority of the studies in literature, researchers use models in which the time horizon is discrete, i.e. shown by a number of discrete time intervals. Such models are called “discrete-time-based models”.

It is seen that the discrete-time-based model is smaller concerning the number of variables and constraints and hence is more efficient when used for small-size sample problems. On the other hand, there are some disadvantages of discrete-time-based models. For example, a discrete approximation of the time horizon leads to suboptimal solutions by definition. Moreover, large numbers of binary variables should be included in the model as the number of activities increases and the time horizon stretches.

For that reason, efforts have been made to develop models based on continuous-time representations, especially in the past two decades. It is reported that this approach provides an opportunity for creating more precise and effective models. However, there are still relatively few studies on “continuous-time-based” models.

4.1.1. Single-Mode Continuous Time Models

Koné O. et al.(2011) presented two ”event-based” formulations that are inspired by the works of Castro P. M. et al.(2006) and Schwindt C. et al.(2000) on short-term batch scheduling. Both formulations consider single-project and single-mode activities. These formulations are called “Start/End” and “On-Off” event-based models respectively.

Single-mode Start/End Event-Based Model

Koné O. et al. (2011) proposed a model in which time representation is built on the notion of “events”. An event occurs whenever an activity starts or ends. Since we consider “Finish-To-Start” precedence relationships and an activity starts immediately when its predecessor ends, the start time of an activity is either 0 or coincides with the end time of some other activity. Consequently, the number of events may be restricted to the number of activities plus one. A set of events E can be defined such as $E = \{0, 1, 2, 3, \dots, I\}$ where I is the number of activities. The details of the model is as follows.

Indices/Sets

I set of activities, $i = 1 \dots I$

R set of renewable resources, $k = 1 \dots R$

E set of events, $e = 0 \dots I$

Parameters:

B_k Capacity of resource k

b_{ik} Resource usage of activity i for resource k

d_i Duration of activity i

P_{prec} The precedence set. It contains the precedence relations composed of activity pairs (i, j)

Decision Variables:

$$x_{ie} = \begin{cases} 1, & \text{if activity } i \text{ starts at event } e \\ 0, & \text{otherwise} \end{cases}$$

$$y_{ie} = \begin{cases} 1, & \text{if activity } i \text{ ends at event } e \\ 0, & \text{otherwise} \end{cases}$$

t_e : time of event e ,

Events are enumerated such that $t_0 \leq t_1 \leq t_2 \leq \dots \leq t_I$

r_{ek} : amount of resource k required immediately after event e

$$\text{Minimize} = t_I \quad (8)$$

Subject to,

$$t_0 = 0 \quad (9)$$

$$t_f \geq t_e + d_i x_{ie} - d_i(1 - y_{if}) \quad \forall (e, f) \in E^2, f > e, \quad \forall i \in I \quad (10)$$

$$t_{e+1} \geq t_e \quad \forall e \in E, \quad e < I \quad (11)$$

$$\sum_{e \in E} x_{ie} = 1 \quad \forall i \in I \quad (12)$$

$$\sum_{e \in E} y_{ie} = 1 \quad \forall i \in I \quad (13)$$

$$\sum_{e'=e}^I y_{ie'} + \sum_{e'=0}^{e-1} x_{je'} \leq 1 \quad \forall (i, j) \in P_{\text{prec}}, \quad \forall e \in E \quad (14)$$

$$\sum_{v=0}^e y_{iv} + \sum_{v=e}^I x_{iv} \leq 1 \quad \forall i \in I, \quad \forall e \in E \quad (15)$$

$$r_{0k} = \sum_{i \in I} b_{ik} x_{i0} \quad \forall k \in R \quad (16)$$

$$r_{ek} = r_{(e-1)k} + \sum_{i \in I} b_{ik} x_{ie} - \sum_{i \in I} b_{ik} y_{ie} \quad \forall e \in E, e \geq 1, k \in R \quad (17)$$

$$r_{ek} \leq B_k \quad \forall e \in E, k \in R \quad (18)$$

The objective function (8) aims to minimize the total project duration. Constraint (9) requires event 0 to start at time 0. Inequalities (10) guarantee that if activity i starts at event e and ends at event f (i.e., $x_{ie} = 1$ and $y_{if} = 1$), then $t_f \geq t_e + d_i$. Any alternative set of values for x_{ie} and y_{if} yield either, $t_f \geq t_e$ or $t_f \geq t_e - d_i$, which are redundant with constraint (11), which orders the events. Constraints (12) and (13) require that a start event and an end event occur only once. Constraint (14) ensures that the precedence relationships between activities are maintained. If (i,j) represents a precedence relation and activity i ends at or after event e , then j cannot start before event e . Constraint (16) releases the total resource demands of the activities that start at event 0. Constraint (17) is resource conservation constraints. More precisely, for each resource k , its demand immediately following event e (r_{ek}) is equal to its demand immediately following the previous event " $e - 1$ " ($r_{e-1,k}$), plus the demand required by the activities that start at event e , minus the demand needed by the activities that end at event e . Constraint (18) restricts the resource demand at each event to match the available resource capacity.

Unfortunately, the model was not correct. A set of necessary constraints representing the relative positioning of end and start events of activities was unintentionally omitted in the paper. Artigues, C. et al.(2013) published a paper two years later to make a correction and proposed inserting the Constraint (15).

Single-Mode On/Off Event-Based Model

On/Off based model is a variant of the Start/End Event Based that employs a single type of binary variable per event. In this model, the number of events precisely matches the number of activities I .

Indices/Sets

- I set of activities, $i = 1 \dots I$
- R set of renewable resources, $k = 1 \dots R$

E set of events, $e = 1 \dots I$

Parameters:

B_k Capacity of resource k

b_{ik} Resource usage of activity i for resource k

p_i Duration of activity i

P_{prec} The precedence set. It contains the precedence relations composed of activity pairs (i, j)

Decision Variables:

$$z_{ie} = \begin{cases} 1, & \text{if activity } i \text{ starts at event } e \text{ or still being processed immediately after event } e \\ 0, & \text{otherwise} \end{cases}$$

t_e : time of event e ,

Events are enumerated such that $t_0 \leq t_1 \leq t_2 \leq \dots \leq t_I$

$$\text{Minimize} = C_{\max} \quad (19)$$

Subject to,

$$C_{\max} \geq t_e + (z_{ie} - z_{i(e-1)}) p_i \quad \forall e \in E, \forall i \in I \quad (20)$$

$$t_0 = 0 \quad (21)$$

$$t_f \geq t_e + ((z_{ie} - z_{i(e-1)}) - (z_{if} - z_{i(f-1)}) - 1) p_i \quad \forall (e, f, i) \in E^2, f > e \neq 0, \quad (22)$$

$$t_{e+1} \geq t_e \quad \forall e \neq I - 1 \in E, \quad (23)$$

$$\sum_{e'=0}^{e-1} z_{ie'} \leq e(1 - (z_{ie} - z_{i(e-1)})) \quad \forall e \neq 0 \in E \quad (24)$$

$$\sum_{e'=e}^{I-1} z_{ie'} \leq (n - e)(1 + (z_{ie} - z_{i(e-1)})) \quad \forall e \neq 0 \in E \quad (25)$$

$$\sum_{e \in E} z_{ie} \geq 1 \quad \forall i \in I \quad (26)$$

$$z_{ie} + \sum_{e'=0}^e z_{je'} \leq 1 + (1 - z_{ie})e \quad \forall e \in E, \quad \forall (i, j) \in P_{\text{prec}} \quad (27)$$

$$\sum_{i=0}^{I-1} b_{ik} z_{ie} \leq B_k \quad \forall e \in E, \forall k \in R \quad (28)$$

$$t_e \geq 0 \quad \forall e \in E \quad (29)$$

$$z_{ie} \in \{0,1\} \quad \forall i \in I, \forall e \in E \quad (30)$$

The objective function (19) aims to minimize project duration. Constraint (20) links the makespan with events. Constraints (21) and (23) impose the event sequencing. Constraint (22) links the binary optimization variables z_{ie} to the continuous optimization variables t_e . They also guarantee that, if activity i starts at event e and ends at event f , then the time difference between events f and e is at least the processing time of activity i ($t_f \geq t_e + p_i$). Constraints (24) and (25) ensure non-preemption. Constraint (26) ensures that each activity is executed at least once throughout the project. Constraint (27) describes precedence constraints. Constraint(28) is the resource constraint limiting the total demand of activities in process at each event.

On/Off event-based model and Start/End event-based model were introduced by Koné O. et al. (2011). Unfortunately, both models have mistakes. Start/End event-based has a missing constraint which is representing the relative positioning of start and end events of activities. Artigues C. et al. (2013) published a paper two years later to make correction. It is interesting to note that the article containing the incomplete model is one of the most cited papers since 2011 while its follow-up article has been cited only a few times since it was published in 2013.

On/Off based model is also not correct due to the missing link between time and events. Koné O. et al.(2013) published a paper two years later to make an extension that takes into account storage resources. In this article, they used again On/off event-based formulation but they changed Constraint (24) and (25) signs from \geq to \leq without any mention of that in the article. Also, there is another formulation written by Koné O. et al.(2010). All these three articles have the same formulation but some constraints are changed. We have tried all of them and also tried our formulation but unfortunately, the model does not work due to the missing relation between time and events. In order to explain better, let us use create below example.

Sample Problem and Solution:

Assume that there is a project that considers 10 activities and the precedence relations are given as shown in Figure 4.6. The diagram uses AON representation and the activities are represented in boxes and identified with capital letters. The arcs denote

the precedence relations. Numerical values on the top of the boxes show the durations of the activities.

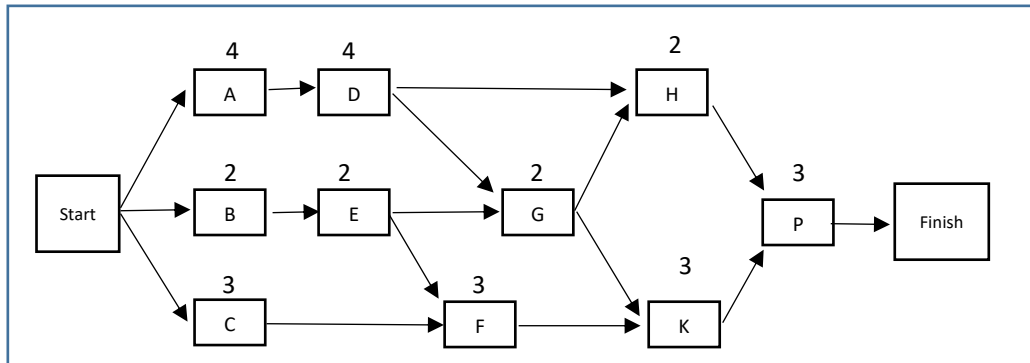


Figure 4.6. Precedence Relations of Sample Project

Source: Author

Assume that there are two different resources $R=2$ and the capacity of each resource is equal to 10 for each period. On the other hand, the resource requirements for the activities are specified in Table 4.2. below.

Table 4.2. Resource Requirements of Activities

	Activities											
	dummy	A	B	C	D	E	F	G	H	K	P	dummy
RESOURCE-1	0	8	3	2	6	6	4	4	6	4	4	0
RESOURCE-2	0	2	4	7	7	1	2	5	3	4	3	0

Source: Author

After that, the On/Off Event-based formulation defined above is used to solve the problem. The Gantt charts of the corresponding solutions are given in Figure 4.7.

ACTIVITY	Duration	Event											
		0	1	2	3	4	5	6	7	8	9	10	11
Time of Event		0	0	2	4	4	4	4	8	10	13	16	
DUMMY	0												
A	4												
B	2												
C	3												
D	4												
E	2												
F	3												
G	2												
H	2												
K	3												
P	3												
DUMMY	0												

Figure 4.7. Gantt Chart of the Project Based on Events.

Source: Author

Constraints (20) and (22) link the makespan with the events and provide the link between the binary optimization variables z_{ie} to the continuous optimization variables t_e , and guarantees that the processing time of an activity is equal to the processing time of this activity does not work correctly.

Event	Resource1	Resource2
0	10	9
1	3	4
2	6	1
3	0	0
4	0	0
5	0	0
6	10	9
7	4	5
8	10	7
9	4	3

Figure 4.8. Relation Between Events and Resource Constraints

Source: Author

As we can see from Figure 4.8., the value of Resource 1 is 10 at Event 0, and the value of Resource 1 is 3 at Event 1. Since both events start at time 0, it extends the capacity limit which becomes 13 for Resource 1. Since the link between times and events is not working on the model it does not consider the capacity constraint on time. The same logic is valid for Resource 2 and other resources that have the same conflict.

Another original approach in continuous time modeling was proposed by Artigues C. et al.(2003). It is called the “flow-based” formulation. It uses the network flow concept in which resources flow (transferred) between activities that correspond to nodes in the network. This formulation also deals with single-project and single-mode activities.

Single-Mode Flow-Based Model:

Indices/Sets

- A** set of real activities, $i = 1, 2, \dots, n$
- I** set of activities, $i = 0, 1, 2, \dots, n, n+1$ (0 and $n+1$ are the dummy activities)
- R** set of renewable resources, $k = 1 \dots R$

Parameters:

- B_k Availability of resource k
- b_{ik} the amount of resource k used per time period during the execution of activity i . ($b_{0k} = B_k$, $b_{n+1,k} = B_k$)

d_i	Duration of activity i
E	The precedence set. It contains the precedence relations composed of activity pairs (i, j)

Decision Variables:

$$x_{ij} = \begin{cases} 1, & \text{if activity } i \text{ is processed before activity } j \\ 0, & \text{otherwise} \end{cases}$$

f_{ijk} : the quantity of resource k that is transferred from activity i (at the end of its processing) to activity j (at the start of its processing).

S_i : Starting time of activity i

$$\text{Minimize} = S_{n+1} \quad (31)$$

Subject to,

$$x_{ij} + x_{ji} \leq 1 \quad \forall (i, j) \in (I)^2, i < j, \quad (32)$$

$$x_{ik} \geq x_{ij} + x_{jk} - 1 \quad \forall (i, j, k) \in (I)^3 \quad (33)$$

$$S_j - S_i \geq -M + (p_i + M)x_{ij} \quad \forall (i, j) \in (I)^2 \quad (34)$$

$$f_{ijk} \leq \min(b_{ik}, b_{jk}) x_{ij} \quad \forall (i, j) \in I, \forall k \in R \quad (35)$$

$$\sum_{j \in A \cup \{0, n+1\}} f_{ijk} = b_{ik} \quad \forall i \in A, \forall k \in R \quad (36)$$

$$\sum_{i \in A \cup \{0, n+1\}} f_{ijk} = b_{jk} \quad \forall j \in A, \forall k \in R \quad (37)$$

$$x_{ij} = 1 \quad \forall (i, j) \in E \quad (38)$$

$$f_{ijk} \geq 0 \quad \forall (i, j) \in (I)^2, \forall k \in R \quad (39)$$

$$f_{(n+1)0k} = B_k \quad \forall k \in R \quad (40)$$

$$S_i \geq 0 \quad \forall i \in I \quad (41)$$

$$x_{ij} \in \{0, 1\} \quad \forall (i, j) \in (I)^2 \quad (42)$$

M is a large number that can be set to any valid upper bound of the makespan. The objective function (31) aims to minimize the project duration. Constraint (32) specifies that for two distinct activities, either j precedes i or i precedes j , or j and i are processed concurrently. Constraint (33) expresses the transitivity of the precedence relations. Constraint (35) connects x_{ij} variables and flow variables. If i precedes j , the maximum

flow sent from i to j is set to $\min\{b_i, b_j\}$ while if i does not precede j the flow must be zero. Constraints (36) and (37) are resource flow conservation constraints. Constraint (38) ensures the preexisting precedence constraints. Constraint (39) provides the conservation of the flow.

Table 4.3. given below presents a comparison of continuous time and discrete time-based models. Assume that there is a project which consists of 10 activities. The scheduling horizon is $T=25$ discrete time units. Also, assume that there are two different resources $R=2$ and the capacity of each resource is equal to 10 for each period.

Table 4.3. Comparison of Continuous Time and Discrete Time Models

	Number of Binary Variables		Number of Cont. Variables	Number of Constraints
	At most	Actual		
Disc. Time Model Formulation(1)-(7)	$T*(I+2)$ $25*12=300$	$\sum_{i=1}^{I+1}(LS_i - ES_i)$ $= 96$	None	$ P_{prec} + (T + 1)R + (I + 1)$ $=79$
Cont. Time Model Formulation(8)-(18)	$2I^2 + 2I$ $2*100+2*10$ $=220$	$2I^2 + 2I$ $2*100+2*10$ $=220$	$(I+1)*(R+1)$ $=33$	$(1/2)I^3 + I^2 + (3 + P_{prec} + R)I + P_{prec} + R + 1$ $= 867$

Source: Author

It is observed that the discrete time-based model has fewer variables and constraints, making it more efficient for solving our sample problem. However, as the number of activities grows and the time horizon extends, continuous time-based models would be more efficient and provide more accurate results by definition. Therefore, that approach still promises potential.

4.1.2. Multi-Mode Continuous Time Models

Among the few studies, Kyriakidis et al. (2012) and Kopanos G.M. (2014) developed continuous time-based MILP models for deterministic single and multi-mode RCPSP with both non-renewable and renewable resources. Their modeling approach utilizes the RTN representation, a network-based technique applied in process scheduling problems. Additionally, Sabzheparvar M. et al. (2008) proposed a continuous time formulation for the MM-RCPSP with generalized precedence relations, where the

minimal or maximal time lags between pairs of activities can vary depending on the selected modes.

A conference paper is presented by Chakraborty R.K. et al. (2014), who have proposed two event-based models for multi-mode RCPSP inspired by the event-based models proposed by Kone O. et al. (2011) for single-mode RCPSP. The subsequent section provides details of those two event-based models, named Start/End Event and On/Off Event.

Multi-mode Start/End Event-Based Model

Indices/Sets

I	set of activities, $i = 0 \dots I+1$
R	set of renewable resources, $k = 1 \dots R$
E	set of events, $e = 0 \dots I$
M	set of modes, $m = 1, 2, \dots, M$

Parameters:

B_k	Capacity of resource k
b_{imk}	Resource usage of activity i in mode m for resource k
d_{im}	Duration of activity i in mode m
P_{prec}	The precedence set. It contains the precedence relations composed of activity pairs (i, j)

Decision Variables:

$$x_{ime} = \begin{cases} 1, & \text{if activity } i \text{ starts at event } e \text{ at mode } m \\ 0, & \text{otherwise} \end{cases}$$

$$y_{ime} = \begin{cases} 1, & \text{if activity } i \text{ ends at event } e \text{ at mode } m \\ 0, & \text{otherwise} \end{cases}$$

t_e : time of event e ,

Events are enumerated such that $t_0 \leq t_1 \leq t_2 \leq \dots \leq t_I$

r_{ek} : amount of resource k required immediately after event e at mode m

$$\text{Minimize} = t_I \tag{43}$$

Subject to,

$$t_0 = 0 \tag{44}$$

$$t_f \geq t_e + d_{im}x_{ime} - d_{im}(1 - y_{imf}) \quad \forall (e, f) \in E^2, f > e, \forall i \in I, \forall m \in M \quad (45)$$

$$t_{e+1} \geq t_e \quad \forall e \in E, \quad e < I \quad (46)$$

$$\sum_{m=1}^M \sum_{e \in E} x_{ime} = 1 \quad \forall i \in I \quad (47)$$

$$\sum_{m=1}^M \sum_{e \in E} y_{ime} = 1 \quad \forall i \in I \quad (48)$$

$$\sum_{m=1}^M \sum_{e'=e}^I y_{ime'} + \sum_{m=1}^M \sum_{e'=0}^{e-1} x_{jme'} \leq 1 \quad \forall (i, j) \in P_{prec}, \quad \forall e \in E \quad (49)$$

$$\sum_{m=1}^M \sum_{v=0}^e y_{imv} + \sum_{m=1}^M \sum_{v=e}^I x_{imv} \leq 1 \quad \forall i \in I, \forall e \in E, \forall m \in M \quad (50)$$

$$r_{1k} = \sum_{m=1}^M \sum_{i \in I} b_{imk} x_{im1} \quad \forall k \in R \quad (51)$$

$$r_{ek} = r_{e-1,k} + \sum_{m=1}^M \sum_{i \in I} b_{imk} x_{ime} - \sum_{m=1}^M \sum_{i \in I} b_{imk} y_{ime} \quad \forall e \in E, e \geq 1, k \in R \quad (52)$$

$$r_{ek} \leq B_k \quad \forall e \in E, \quad k \in R \quad (53)$$

$$x_{ime} \in \{0,1\} \quad \forall i \in I, \forall e \in E, \forall m \in M \quad (54)$$

$$y_{ime} \in \{0,1\} \quad \forall i \in I, \forall e \in E, \forall m \in M \quad (55)$$

The objective function (43) is to minimize project duration. Constraint (44) forces event 0 to start at time 0. Inequalities (45) guarantee that if activity i starts at event e and ends at event f (i.e., $x_{ime} = 1$ and $y_{imf} = 1$), then $t_f \geq t_e + d_{im}$. Any other set of values for x_{ime} and y_{imf} yield either, $t_f \geq t_e$ or $t_f \geq t_e - d_{im}$, which are redundant with constraint (39), which orders the events. Constraints (47) and (48) require that a start event and an end event occur only once. Constraint (49) checks that the precedence relations between activities are not violated. If (i, j) represents a precedence relation and activity i ends at or after event e , then j cannot start before event e . Constraint(50) represents the relative positioning of the start and end events of activities. Unfortunately, Chakraborty R.K. et al.(2014) forgot to add this constraint to their model. We added this constraint to the model and then it worked properly. Constraint (51) releases the total resource demands of the activities that start at event

0. Constraint (52) is resource conservation constraints. Constraint (53) restricts the resource demand at each event to the available resource capacity.

Multi-mode On/Off Event-Based Model

Indices/Sets

I	set of activities, $i = 1 \dots I$
R	set of renewable resources, $k = 1 \dots R$
E	set of events, $e = 1 \dots I$
M	set of modes, $m = 1, 2, \dots, M$

Parameters:

B_k	Capacity of resource k
b_{imk}	Resource usage of activity i in mode m for resource k
p_{im}	Duration of activity i in mode m
P_{prec}	The precedence set. It contains the precedence relations composed of activity pairs (i, j)

Decision Variables:

$$x_{ime} = \begin{cases} 1, & \text{if activity } i \text{ starts at event } e \text{ at mode } m \text{ or still being processed immediately after event } e \\ 0, & \text{otherwise} \end{cases}$$

t_e : time of event e ,
Events are enumerated such that $t_0 \leq t_1 \leq t_2 \leq \dots \leq t_I$

r_{ek} : amount of resource k required immediately after event e

$$\text{Minimize} = C_{max} \quad (56)$$

Subject to,

$$C_{max} \geq t_e + (x_{ime} - x_{im(e-1)}) p_{im} \quad \forall e \in E, \forall i \in I, \forall m \in M \quad (57)$$

$$t_0 = 0 \quad (58)$$

$$t_f \geq t_e + ((x_{ime} - x_{im(e-1)}) - (x_{imf} - x_{im(f-1)}) - 1) p_{im} \quad \forall (e, f, i) \in E^2, f > e \neq 0, \forall m \in M \quad (59)$$

$$t_{e+1} \geq t_e \quad \forall e \neq I - 1 \in E, \quad (60)$$

$$\sum_{m=1}^M \sum_{e'=0}^{e-1} x_{ime'} \leq e(1 - (x_{ime} - x_{im(e-1)})) \quad \forall (i, j) \in P_{prec}, \forall e \in E \quad (61)$$

$$\sum_{m=1}^M \sum_{e'=e}^I x_{ime'} \leq (n - e)(1 + (x_{ime} - x_{im(e-1)})) \quad \forall (i, j) \in P_{prec}, \forall e \in E \quad (62)$$

$$\sum_{m=1}^M \sum_{e \in E} x_{ime} \geq 1 \quad \forall i \in I \quad (63)$$

$$x_{ime} + \sum_{m=1}^M \sum_{e'=0}^e x_{jme'} \leq 1 + (1 - x_{ime})e \quad \forall e \in E, \quad \forall (i, j) \in P_{\text{prec}} \quad (64)$$

$$\sum_{m=1}^M \sum_{i=0}^{l-1} b_{imk} x_{ime} \leq B_k \quad \forall e \in E, \quad \forall k \in R \quad (65)$$

$$t_e \geq 0 \quad \forall e \in E \quad (66)$$

$$x_{ime} \in \{0,1\} \quad \forall i \in I, \forall e \in E, \forall m \in M \quad (67)$$

The objective function (56) aims to minimize the project duration. Constraint (57) links the makespan with events. Constraints (58) and (60) ensure the event sequencing. Constraint (59) links the binary optimization variables z_{ie} to the continuous optimization variables t_e . They also guarantee that, if activity i starts at event e and ends at event f , then the time difference between events f and e is at least the processing time of activity i . Constraints (61) and (62) ensure non-preemption. Constraint (63) ensures that each activity is processed at least once during the project. Constraint (64) describes precedence constraints. Constraint(65) is the resource constraint limiting the total demand of activities in process at each event. Since the single-mode model does not work due to the missing link between time and event, the multi-mode model also does not work properly.

Continuous Time Assignment-Based Model

Another original modeling approach for MRCPSP which uses continuous time representation has been proposed by Gnagi M. et al. (2019). It is called the “assignment-based” formulation. The approach is based on the resource-assignment, mode selection, and sequencing variables. Based on the same concept, they defined four variants of the model which they call MCTAB, MCTABO, MCTAB-Extended, and MCTABO-Extended. They claim that their MILP model delivers the best performance (in solution time and quality of objective function) when compared to other MILP models in the literature. However, there is a drawback in their model since the renewable resources are represented as discrete units and each discrete unit of each

resource is mapped with a binary variable. Therefore when the number and the capacities (amount) of resources increase, then the number of binary variables increases as well. The details of the models are as follows.

Indices/Sets

- I** set of activities, $i = 0, 1, 2, \dots, n, n+1$ (0 and $n+1$ are the dummy activities)
- R** set of renewable resources, $k = 1 \dots R$
- M** set of modes, $m = 1, 2, \dots, M$
- N** set of renewable resources, $k = 1 \dots N$
- E** The precedence set. It contains the precedence relations composed of activity pairs (i, j)
- TE** set of all pairs of activities that cannot be executed in parallel due to their precedence relations.

Parameters:

- R_k : Capacity of renewable resource k
- W_k : Capacity of non-renewable resource k
- r_{ikm} : requirement of activity i of renewable resource k per period when executed in mode m
- w_{ikm} : requirement of activity i of non-renewable resource k per period when executed in mode m
- p_{im} : duration of activity i if it is executed on mode m
- p_i^{max} : maximum duration of activity $i \in I$
- T: Planning horizon. Calculated as the sum of the maximum durations of activities.
- ES_i : Earliest start time of activity $i \in I$
- LS_i : Latest start time of activity $i \in I$

MCTAB(Assignment based model without auxiliary variables)

Decision Variables:

$$y_{ij} = \begin{cases} 1, & \text{if activity } i \text{ must be completed before the start of activity } j \\ 0, & \text{otherwise} \end{cases}$$

S_i : Starting time of activity i

$$r_{ik} = \begin{cases} 1, & \text{if activity } i \text{ is assigned to unit } l \text{ of renewable resource } k \\ 0, & \text{otherwise} \end{cases}$$

$$x_{im} = \begin{cases} 1, & \text{if activity } i \text{ is processed on mode } m \\ 0, & \text{otherwise} \end{cases}$$

$$\mathbf{Minimize} = S_{n+1} \quad (68)$$

Subject to,

$$r_{ik}^l + r_{jk}^l \leq 1 + y_{ij} + y_{ji} \quad \forall (i,j) \in (I), i < j, (i,j) \notin TE, k \in R; l = 1, \dots, Rk \quad (69)$$

$$\sum_{l=1}^{Rk} r_{ik}^l = \sum_{m \in Mi} r_{ikm} x_{im} \quad (i \in I; k \in R) \quad (70)$$

$$\sum_{i \in I} \sum_{m \in Mi} w_{ikm} x_{im} \leq W_k \quad (k \in N) \quad (71)$$

$$S_i + \sum_{m \in Mi} p_{im} x_{im} \leq S_j \quad ((i,j) \in E) \quad (72)$$

$$S_i + \sum_{m \in Mi} p_{im} x_{im} \leq S_j + (LS_i + p_i^{max} - ES_j) * (1 - y_{ij}) \quad (i,j) \in I: i \neq j, (i,j) \notin TE \quad (73)$$

$$\sum_{m \in Mi} x_{im} = 1 \quad (i \in I) \quad (74)$$

$$S_i \geq 0 \quad \forall i \in I \quad (75)$$

$$y_{ij} \in \{0,1\} \quad (i,j) \in I: i \neq j, (i,j) \notin TE \quad (76)$$

$$x_{im} \in \{0,1\} \quad \forall i \in I, \forall m \in M \quad (77)$$

$$r_{ik}^l \in \{0,1\} \quad i \in I, k \in R; l \in \{1, \dots, Rk\} \quad (78)$$

The objective function (68) aims to minimize project duration. Constraint (69) links the resource assignment variables to the sequencing variables. Constraint (70) determines the amount of renewable resource units is equal to the number required by activity in its selected execution mode. Constraint (71) ensures that the capacities of non-renewable resources are not exceeded. Constraint (72) expresses the transitivity of the precedence relations. Constraint (73) links the sequencing variable with the start time of variables. Constraint (74) enforces selecting only one executing mode for the activities.

MCTABO(Assignment-based model with auxiliary variables)

The MCTABO model is founded on five distinct types of variables, which are;

S_i = Start time of activity i

$y_{ij} = 1$ if activity i starts before or at the same time as activity j , 0 activity j starts before or at the same time as activity i

$r_{ik}^l = 1$ if activity i is assigned to unit l of renewable resource k , 0 otherwise

$z_{ij} = 1$ if activities i and j use the same renewable resource unit, 0 otherwise

$x_{im} = 1$ if activity i is executed in mode m , 0 otherwise

The difference between MCTAB and MCTABO is the following; Instead of constraint (69), constraint (79) is used. Constraint (79) associates the resource-assignment variables with the auxiliary resource-overlap variables. When two activities, i and j , are assigned to the same resource unit, the related auxiliary resource-overlap variable is constrained to be one.

$$r_{ik}^l + r_{jk}^l \leq 1 + y_{ij} + y_{ji} \quad \forall (i, j) \in (I), i < j, (i, j) \notin TE, k \in R; l = 1, \dots, Rk \quad (69)$$

$$r_{ik}^l + r_{jk}^l \leq 1 + z_{ij} \quad \forall (i, j) \in (I), i < j, (i, j) \notin TE, k \in R; l = 1, \dots, Rk \quad (79)$$

Constraint (73) is replaced by constraint (80) and (81). The purpose of these constraints is to connect the sequencing variables, start-time variables, and auxiliary resource-overlap variables. If two activities j and i are assigned to the same resource unit, i.e., $z_{ij} = 1$, then these activities must be scheduled sequentially; then, either activity j must be completed before the start of activity i , i.e, $y_{ij} = 0$ or activity i must be completed before the start of activity j , i.e, $y_{ij} = 1$. If there is no resource overlap between the activities i and j , i.e., $z_{ij} = 0$, then either activity i must start before or at the same time as activity j , i.e, $y_{ij} = 1$, or activity j must start before or at the same time as activity i , i.e, $y_{ij} = 0$.

$$S_i + \sum_{m \in M_i} p_{im} x_{im} \leq S_j + (LS_i + p_i^{max} - ES_j) * (1 - y_{ij}) \quad (i, j) \in I: i \neq j, (i, j) \notin TE \quad (73)$$

$$S_i + \sum_{m \in M_i} p_{im} x_{im} - p_j^{max} (1 - z_{ij}) \leq S_j + (LS_i + p_i^{max} - ES_j) * (1 - y_{ij}) \quad (i, j) \in I: i < j, (i, j) \notin TE \quad (80)$$

$$S_j + \sum_{m \in M_j} p_{jm} x_{jm} - p_j^{max} (1 - z_{ij}) \leq S_j + (LS_j + p_j^{max} - ES_i) * y_{ij} \quad (i, j) \in I: i < j, (i, j) \notin TE \quad (81)$$

$$z_{ij} \in \{0,1\} \quad (i, j) \in I: i < j, (i, j) \notin TE \quad (82)$$

MCTAB extended and MCTABO extended models

The MCTABO extended and MCTAB extended models are expected to enhance model performance by removing certain symmetric solutions from the search space and by incorporating additional redundant constraints. These constraints explicitly enforce the sequential scheduling of pairs or triples of activities that cannot be processed concurrently due to resource constraints.

For each renewable resource $k \in R$, an activity i_k^* is selected with the largest minimum requirement $r_{i_k^*}^{min} = \min_{m \in M_i} \{r_{ikm}\}$ and constraint (83) is added to MCTABO and MCTAB models. For each renewable resource $k \in R$ these constraints allocate the first $r_{i_k^*,k}^{min}$ renewable resource units to the execution of activity i_k^* .

$$r_{i_k^*,k}^l = 1 \quad (k \in R, l \in \{1, \dots, r_{i_k^*,k}^{min}\}) \quad (83)$$

All pairs of activity mode combinations $((i, m_i), (j, m_j))$ with $i, j \in I$, $m_i \in M_i$ and $m_j \in M_j$ are analyzed for that the requirement for some renewable resource exceeds the resource capacity, $r_{ikm_i} + r_{jkm_j} > R_k$ for some renewable resource $k \in R$; I^2 contain all the pairs of activity mode combinations with $i < j$, excluding all pairs of activity mode combinations in TE (set of all pairs of activities that cannot be executed in parallel due to their precedence relations).

In each feasible solution, at least one unit of resource k is to be assigned to both activities i and j , if activity i is executed in modes m_i and activity j is executed in modes m_j . Thus, activities i and j must be scheduled sequentially; therefore redundant constraints (84 - 85) are added. Constraint (84) is added to MCTAB model and constraint (85) is added to MCTABO model.

$$y_{ij} + y_{ji} \geq 1 + x_{im_i} + x_{jm_j} - 1 \quad (((i, m_i), (j, m_j)) \in I^2), \quad (84)$$

$$z_{ij} \geq x_{im_i} + x_{jm_j} - 1 \quad (((i, m_i), (j, m_j)) \in I^2), \quad (85)$$

Also all triples of activity mode combinations $((i, m_i), (j, m_j), (h, m_h))$, are analyzed where $i, j, h \in I$, $m_i \in M_i$, $m_j \in M_j$, $m_h \in M_h$ with $r_{ikm_i} + r_{jkm_j} + r_{hkm_h} > R_k$ for some renewable resource $k \in R$, set I^3 contain all triples of activity-mode combinations with $i < j < h$ excluding all pairs of activity-mode combinations in TE.

Redundant constraints (86 and 87) are added to MCTAB and MCTABO respectively.

$$y_{ij} + y_{ji} + y_{ih} + y_{hi} + y_{jh} + y_{hj} \geq x_{im_i} + x_{jm_j} + x_{hm_h} - 2$$

$$(((i, m_i), (j, m_j), (h, m_h)) \in I^3), \quad (86)$$

$$z_{ij} + z_{ih} + z_{jh} \geq x_{im_i} + x_{jm_j} + x_{hm_h} - 2(((i, m_i), (j, m_j), (h, m_h)) \in I^3), \quad (87)$$

4.2. A Novel Continuous Time MILP Model for MRCPSP

In this thesis, a novel continuous-time MILP model has been developed for RCPSP in which activities can be conducted in different (multi) modes. The model has been verified by solving benchmark problems in the literature. A conference paper is accepted and presented by Ozturk and Oner (2020) at the 2020 9th International Conference on Industrial Technology and Management Conference (ICITM) hosted by Oxford University (11-13 February 2020).

The formulation is inspired by the flow-based model developed by Artigues C. et al.(2003). It uses a network flow concept in which resources flow (transferred) between activities that correspond to nodes in the network. Artigues' model considers a single mode of activities. We have extended the idea for multi-mode activities. Details of the model are given below:

Indices/Sets

A	set of real activities, $i = 1, 2, \dots, n$
I	set of activities, $i = 0, 1, 2, \dots, n, n+1$ (0 and n+1 are the dummy activities)
R	set of renewable resources, $k = 1 \dots R$
M	set of modes, $m = 1, 2, \dots, M$
N	set of non-renewable resources, $k = 1 \dots N$

Parameters:

B_k : Availability of renewable resource k

- W_k : Capacity of non- renewable resource k
 b_{ikm} : the amount of renewable resource k used by activity i on mode m
 w_{ikm} : the amount of non-renewable resource k used by activity i on mode m
 d_{im} : Duration of activity i if it is executed on mode m
 E :The precedence set. It contains the precedence relations composed of activity pairs (i,j)

Decision Variables:

$$x_{ij} = \begin{cases} 1, & \text{if activity } i \text{ is processed before activity } j \\ 0, & \text{otherwise} \end{cases}$$

f_{ijk} = quantity of resource k that is transferred from activity i to activity j

S_i : Starting time of activity i

$$y_{im} = \begin{cases} 1, & \text{if activity } i \text{ is processed on mode } m \\ 0, & \text{otherwise} \end{cases}$$

p_i : The duration of activity i in the optimal solution. It is determined when the optimal activity mode is decided. It should take one of the d_{im} values.

z_{ij} : auxiliary variable that is used for avoiding non-negativity. $z_{ij} = x_{ij} * p_i$

u_{ik} : The amount of resource k used by activity i in the optimal solution. It is determined when the optimal activity mode is decided. It should take one of the b_{ikm} values.

r_{ijk} : auxiliary variable that is used for avoiding non-negativity. $r_{ijk} = x_{ij} * u_{ik}$

$$\text{Minimize} = S_{n+1} \tag{88}$$

Subject to,

$$x_{ij} + x_{ji} \leq 1 \quad \forall (i,j) \in (I)^2, i < j \tag{89}$$

$$x_{ig} \geq x_{ij} + x_{jg} - 1 \quad \forall (i,j,g) \in (I)^3 \tag{90}$$

$$S_j - S_i \geq -M + z_{ij} + M * x_{ij} \quad \forall (i,j) \in (I)^2 \tag{91}$$

$$z_{ij} \leq M * x_{ij} \quad \forall (i,j) \in (I)^2 \tag{92}$$

$$z_{ij} \leq p_i \quad \forall (i,j) \in (I)^2 \tag{93}$$

$$z_{ij} \geq p_i - (1 - x_{ij}) * M \quad \forall (i,j) \in (I)^2 \tag{94}$$

$$p_i = \sum_{m=1}^M d_{im} y_{im} \quad \forall i \in I \quad (95)$$

$$f_{ijk} \leq r_{ijk} \quad \forall k \in R, \forall (i, j) \in (I)^2, j > 0, i < n + 1 \quad (96)$$

$$r_{ijk} \leq M * x_{ij} \quad \forall k \in R, \forall (i, j) \in (I)^2 \quad (97)$$

$$r_{ijk} \leq u_{ik} \quad \forall k \in R, \forall (i, j) \in (I)^2 \quad (98)$$

$$r_{ijk} \geq u_{ik} - (1 - x_{ij}) * M \quad \forall k \in R, \forall (i, j) \in (I)^2 \quad (99)$$

$$u_{ik} = \sum_{m=1}^M b_{ikm} y_{im} \quad \forall i \in I, \forall k \in R \quad (100)$$

$$\sum_{j \in A \cup \{0, n+1\}} f_{ijk} = u_{ik} \quad \forall i \in A, \forall k \in R \quad (101)$$

$$\sum_{j \in A \cup \{0, n+1\}} f_{ijk} = B_k \quad \forall i \in \{0, n+1\}, \forall k \in R \quad (102)$$

$$\sum_{i \in A \cup \{0, n+1\}} f_{ijk} = u_{jk} \quad \forall j \in A, \forall k \in R \quad (103)$$

$$\sum_{i \in A \cup \{0, n+1\}} f_{ijk} = B_k \quad \forall j \in \{0, n+1\}, \forall k \in R \quad (104)$$

$$f_{(n+1)0k} = B_k \quad \forall k \in R \quad (105)$$

$$x_{ij} = 1 \quad \forall (i, j) \in E \quad (106)$$

$$\sum_{m=1}^M y_{im} = 1 \quad \forall i \in I \quad (107)$$

$$\sum_{i \in I} \sum_{m \in M_i} w_{ikm} y_{im} \leq W_k \quad (\in N) \quad (108)$$

$$f_{ijk} \geq 0 \quad \forall (i, j) \in (I)^2, \forall k \in R \quad (109)$$

$$S_i \geq 0 \quad \forall i \in I \quad (110)$$

$$p_i \geq 0 \quad \forall i \in I \quad (111)$$

$$z_{ij} \geq 0 \quad \forall (i, j) \in (I)^2 \quad (112)$$

$$u_{ik} \geq 0 \quad \forall i \in I, \forall k \in R \quad (113)$$

$$r_{ijk} \geq 0 \quad \forall (i, j) \in (I)^2, \quad \forall k \in R \quad (114)$$

$$x_{ij} \in \{0,1\} \quad \forall (i, j) \in (I)^2 \quad (115)$$

$$y_{im} \in \{0,1\} \quad \forall i \in I, \quad \forall m \in M \quad (116)$$

The precedence relations are defined by a set E of index pairs such that $(i, j) \in E$ means that activity i must precede activity j . It is assumed that given a precedence activity-on-node graph $G(V, E)$ whose nodes correspond to activities $V = A \cup \{0, n+1\}$, and arcs correspond to precedence relations where 0 and $n+1$ are starting and finishing dummy activities respectively.

M is a large number. The objective function (88) aims to minimize project duration. Constraint (89) specifies that for two distinct activities, either j precedes i , or i precedes j , or i and j are processed simultaneously. Constraint (90) expresses the transitivity of the precedence relations. Constraint (91) links the start time of j and i w.r.t. variable x_{ij} and z_{ij} . Constraints (92) – (94) are a set of constraints used to linearize the nonlinear relation of $(z_{ij} = x_{ij} * p_i)$. When $x_{ij} = 1$ (i precedes j) then $z_{ij} = p_i$ and, in that situation, Constraint (91) becomes active to enforce $S_j \geq S_i + p_i$. If $x_{ij} = 0$ then Constraint (91) is always satisfied. Constraint (95) determines the duration of activities depending on the optimal execution mode. Constraint (96) links the flow variable f_{ijk} with the variable r_{ijk} and hence with the variable x_{ij} since $r_{ijk} = x_{ij} * u_{ik}$. However it is a non-linear equation, therefore Constraints (97) – (99) are inserted to linearize that relation. When $x_{ij} = 1$ (i precedes j) then Constraints (97) – (99) ensures $r_{ijk} = u_{ik}$ and, in that case, the maximum resource flow sent from i to j is set u_{ik} which is the amount of resource k used by the activity i . On the other hand, if i does not precede j , then the flow f_{ijk} must be zero. Constraint (100) determines the amount of resources used by activities depending on the optimal execution mode. Constraint (101) and Constraint (103) are resource conservation constraints. Meanwhile, (102) and (104) impose resource availability constraints and Constraint (105) ensures the conservation of the flow. Constraint (106) sets the pre-existing precedence constraints. Constraint (107) enforces selecting only one executing mode for the activities. Constraint (108) guarantees that the capacities of non-renewable resources are not exceeded.

The model is verified by solving numerous problems, especially those taken from the PSPLIB. We conducted experiments to compare the proposed model with the best continuous-time MILP model in the literature Gnagi M. et al.(2019).

4.3. Benchmark Problem Libraries

There are various problem libraries available for verifying and evaluating the RCPSP and MRCPSP models. These libraries provide a standardized set of test instances that researchers use to evaluate and examine the performance of various scheduling algorithms. Researchers can download the benchmark sets required for evaluating their algorithms. Additionally, they can add new results to libraries. One of the most widely used among these libraries is PSPLIB. It was published by Kolisch, R. et al. (1996). PSPLIB consists of benchmark sets for multi-mode and single-mode RCPSPs.

Some studies have created variants of the PSPLIB library by modifying the problem instances in this library. For instance, Gnagi, M. et al. (2019) created a set of instances using PSPLIB benchmark instances by applying the following method. This approach involves two parameters, α and β . Assume that there are N activities in a project. They began by randomly choosing α activities out of the total N activities. For each chosen activity, they increased the duration by a factor of β plus δ , where δ is a random variable uniformly distributed between zero and one. They then rounded these modified durations to the nearest whole number. They used $\alpha = 10$ and $\beta = 25$. This process yielded a set of instances with tasks having durations that vary from one to as much as 260-time units. Originally, the duration of activities in PSPLIB ranged between 1 and 10 time units. The resulting new sets are called D20 and D30 respectively. In this regard, we consider the sets D20 and D30 to form a new library that is called D-PSPLIB. This library is used primarily to show the superiority of continuous-time-based models, especially the models in Gnägi M. et al. (2019), over the discrete-time models such as those given in Pritsker A.A.B. (1969) and Talbot F.B. et al. (1982).

Another library used for RCPSP is the Boctor F. F. (1993). The Boctor dataset, as proposed by Boctor in 1993, comprises two subsets. It includes a set of 50 activities (Boctor50) and another of 100 activities (Boctor100). Each project within this dataset utilizes one, two, or four types of renewable resources and designates one, two, or four

execution modes per activity. Additionally, the Boctor dataset is available for download from the PSPLIB server, a prominent resource for project scheduling problems.

To address the limitations found in the PSPLIB and Boctor datasets, Van Peteghem V. et al. (2014) defined a new problem library called MMLIB that includes projects in which the activities have multiple modes of execution. The instances were created using the RanGen generator that was developed by Vanhoucke, M. et al.(2008). The properties of the library are explained as follows. Diversity in the generated instances regarding project characteristics (the order strength, OS, is defined as the number of precedence relations divided by the theoretical maximum number of precedence relations.) and resource characteristics (Resource Strength RS and Resource Factor RF) is ensured. Secondly, each instance is guaranteed to possess at least one viable solution. Thirdly, no modes are dismissible in the dataset. Considerations for both non-renewable and renewable resources are incorporated.

4.4. A New Library for RCPSP

The test instances in the literature are mentioned comprehensively in section 4.3. However, there was no instance set highlighting the impact of scaling the amount of resource capacities and requirements. Therefore, we defined a new problem library RD-PSPLIB that has two new sets R20 and R30, which are derived from D20 and D30 respectively. The new problem instances have the same input values and parameters as in the respective counterparts in D-PSPLIB; the difference is the capacity of resources and resource requirements of activities. The resource requirements of activities and the capacities of resources are scaled (increased) simultaneously using a procedure similar to that described above. In this procedure, all of the activities in a problem instance are subject to scaling instead of partial selection i.e., $\alpha = N$ where N is the number of activities in the problem. For each activity, the resource requirement for each mode is multiplied by $\beta + \varepsilon$, where ε is a uniformly distributed random number between zero and one. The parameter $\beta = 25$. That process results in two new sets of instances consisting of activities with resource requirements ranging from 0 to up to 260 time units. Originally, the resource requirements of activities in PSPLIB and DPSPLIB were ranging between 0 and 10 units. To avoid unfeasibility, the resource capacities are also scaled by multiplying with the factor $\beta + \varepsilon$.

5. COMPUTATIONAL RESULTS

To verify our models, the benchmark problem library PSPLIB (see section 4.3) is used. PSPLIB presents a set of benchmark instances for the evaluation of solution procedures for RCPSPs & MRCSPs. We have used the test instances from PSPLIB with 20 and 30 activities. We have observed that our models solve the test instances optimally, or at least, find a feasible solution to the problem within a limited CPU time.

For comparing the performance of our models with the existing models, we have chosen the four mathematical models developed by Gnägi M. et al., (2019). Because they examined, evaluated, and compared all other known models. They reported that their models are the best-performing models in the literature. For this reason, we will compare the performance of our model with the models MCTAB, MCTABO, MCTAB-Extended, and MCTABO-Extended models (See 4.1.2, Continuous Time Assignment-Based Model).

The problem sets used for comparison come from various sources. The first group of the sets is selected from PSPLIB. The library contains different sets of problems for the standard RCPSP and its variants. We consider the problem sets that have multi-mode activities. There are two sets of such problems that are called “J20” and “J30” in which the number of activities is 20 and 30 respectively. The set J20 includes 554 problem instances whereas J30 includes 562 instances. In each problem instance, there are two distinct renewable and non-renewable resources, and all activities have three separate modes defined.

The second group of problem sets which are defined by Gnägi M. et al., (2019), includes two sets of problems which are called “D20” and “D30” (See Section 4.3). They are derived from J20 and J30 such that they have the same parameters and input values except for the duration of activities. Basically, the durations of some activities are scaled (increased) in a process discussed and described in Koné O. et al., (2011).

Moreover, we used our new problem library RD-PSPLIB which has two new sets R20 and R30, which are derived from D20 and D30 respectively (See 4.4).

All models are coded in the latest version of LINGO optimization software and solved in a computer with an Intel Core i9 processor, 32.0 GB RAM and a 3.10 GHz CPU.

The following benchmark problem sets have been used to test the models. J20 has 554 problem instances and J30 has 640 problem instances. A total of 3348 problem instances have been solved, consisting of below three libraries containing this instances with 3×554 and 3×562 .

- a. PSPLIB - Kolisch R. et al.(1996a)
- b. D20 & D30 - Gnägi M et al.(2019)
- c. R20 & R30 - Our proposed library

Since the problems are considered to be large size, it takes time to solve them. Therefore, a time limit of 900 seconds is set for each instance. The following performance metrics are used to compare the models.

Feas (%): The percentage of instances where a feasible schedule was found within the 900-second time limit.

Opt (%): The percentage of instances for which a feasible schedule has been found and proven optimal within 900 seconds.

GapLB (%): Average relative deviation between the objective function value of the best schedule obtained by the solver (OFV) and the lower bound returned by the solver (LB), calculated as $(OFV-LB)/LB$.

GapCP (%): Average relative deviation between OFV and the critical-path-based lower bound(CP), calculated as $(OFV-CP)/CP$.

GapBEST (%): Average relative deviation between OFV and the objective function value of the best schedule (BEST) found among those found with all tested models, calculated as $(OFV-BEST)/BEST$.

Cons: Average number of constraints.

Vars: Average number of variables.

The outcomes of the comparative analysis are displayed in the following tables. Bolded values represent the best outcomes across the five models. Table 5.1. and Table 5.2. show the comparisons for the problem instances in PSPLIB and D-PSPLIB libraries respectively. It is evident that the proposed model generally performs better. However, notice that the constraints and numbers of variables in MCTAB and MCTABO models are much less than in the proposed model.

Table 5.1. Comparative Analysis for PSPLIB problem library.

Set	Model	Feas(%)	Opt(%)	Best(%)	Gap LB (%)	Gap CP (%)	Gap Best (%)	#Cons	#Total Vars
J20	MCTAB	100 %	45 %	50 %	58 %	72 %	36 %	4311	1127
	MCTABO	100 %	40 %	43 %	52 %	66 %	32 %	4318	1414
	MCTAB - Extended	100 %	51 %	55 %	53 %	69 %	34 %	4810	1513
	MCTABO-Extended	100 %	43 %	47 %	47 %	62 %	29 %	4817	1998
	Proposed Model	100 %	53 %	87 %	19%	28 %	3 %	16780	3016
J30	MCTAB	100 %	23 %	33 %	84 %	95 %	26 %	15174	4160
	MCTABO	100 %	18 %	24 %	88 %	100 %	30 %	15048	4540
	MCTAB - Extended	100 %	23 %	28 %	104 %	118 %	40 %	16851	4608
	MCTABO-Extended	100 %	18 %	22 %	99 %	112 %	38 %	16913	5650
	Proposed Model	88 %	40 %	85%	27 %	39 %	4 %	45685	6308

Source: Author

Table 5.2. Comparative Analysis for D-PSPLIB problem library.

Set	Model	Feas(%)	Opt(%)	Best(%)	Gap LB (%)	Gap CP (%)	Gap Best (%)	#Cons	#Total Vars
D20	MCTAB	100 %	52 %	55 %	39 %	49 %	30 %	4311	1227
	MCTABO	100 %	50 %	53 %	39 %	48 %	29 %	4311	1412
	MCTAB - Extended	100 %	58 %	60 %	38 %	50 %	31 %	4810	1513
	MCTABO-Extended	100 %	50 %	53 %	40 %	50 %	32 %	4810	1997
	Proposed Model	100 %	67 %	90 %	10 %	15 %	1 %	16780	3016
D30	MCTAB	100 %	34 %	42 %	74 %	79 %	28 %	15044	4074
	MCTABO	100 %	27 %	34 %	77 %	82 %	31 %	15035	4535
	MCTAB - Extended	100 %	32 %	36 %	94 %	102 %	42 %	16815	4595
	MCTABO-Extended	100 %	27 %	27 %	140 %	150 %	66 %	16815	5601
	Proposed Model	91 %	51 %	80 %	33 %	36 %	6 %	45685	6308

Source: Author

When the models are compared using the problem instances from the RD-PSPLIB library (See Table 5.3), the performances of MCTAB & MCTABO and their extensions are even decreased significantly compared to the proposed model. Notice that the number of variables and constraints of those models has increased approximately 20-fold on average. It is due to the modeling approach since they use binary decision variables, which associate each unit of each renewable resource to each activity. Therefore, the number of constraints and decision variables increases quickly

as the resource requirements of the activities and the capacities of the resources increase. It seems a serious drawback for those models. The proposed model demonstrates superior performance to the rival models in every performance metric. Notice that constraints and the number of variables in the proposed model remain the same for all problem libraries.

Table 5.3. Comparative Analysis for RD-PSPLIB problem library.

Set	Model	Feas(%)	Opt(%)	Best(%)	Gap LB (%)	Gap CP (%)	Gap Best (%)	#Cons	#Total Vars
R20	MCTAB	92 %	17 %	8 %	125 %	165 %	127 %	103148	24723
	MCTABO	97 %	23 %	5 %	135 %	189 %	147 %	103148	24907
	MCTAB - Extended	87 %	13 %	9 %	116 %	156 %	119 %	103697	24674
	MCTABO-Extended	81 %	6 %	6 %	214 %	283 %	228 %	103697	25158
	Proposed Model	100 %	65 %	99 %	12 %	17 %	0 %	16780	3016
R30	MCTAB	11 %	2 %	13 %	139 %	154 %	103 %	329058	45355
	MCTABO	12 %	1 %	13 %	351 %	359 %	248 %	328594	45914
	MCTAB - Extended	6 %	2 %	14 %	91 %	97 %	65 %	330459	45617
	MCTABO-Extended	6 %	2 %	14 %	209 %	269 %	184 %	331420	46462
	Proposed Model	88 %	52 %	100 %	27 %	30 %	0 %	45685	6308

Source: Author

Table 5.4. Average Solution Times (secs)

Set	Model	All Instances			Optimal Only Solutions		
		PSPLIB	D- PSPLIB	RD- PSPLIB	PSPLIB	D- PSPLIB	RD- PSPLIB
J20	MCTAB	540	460	811	109	56	323
	MCTABO	587	482	776	113	57	355
	MCTAB - Extended	480	411	831	73	62	336
	MCTABO-Extended	557	471	862	92	41	268
	Proposed Model	453	313	342	54	35	46
J30	MCTAB	718	623	888	100	69	516
	MCTABO	759	671	895	96	47	357
	MCTAB - Extended	723	642	888	119	84	401
	MCTABO-Extended	760	685	893	99	86	528
	Proposed Model	607	478	521	147	160	229

Source: Author

Finally, Table 5.4. shows the average solution times in seconds. The solution time of a problem instance is regarded as 900 seconds if the optimal solution is not attained earlier. The first part in Table 5.4., labeled as “All instances” shows the average solution times of all instances even though some of them are not solved optimally. In the second part, the average solution times are shown if we consider only the problem instances that are solved optimally.



6. CHAPTER: AN ORIGINAL HEURISTIC for RCPSP

In practice, meta-heuristic and heuristic approaches are typically employed for solving large-scale problems due to their ability to provide results within acceptable solution times, albeit without guaranteeing optimality. Numerous meta-heuristic and heuristic methodologies have been developed to address RCPSPs.

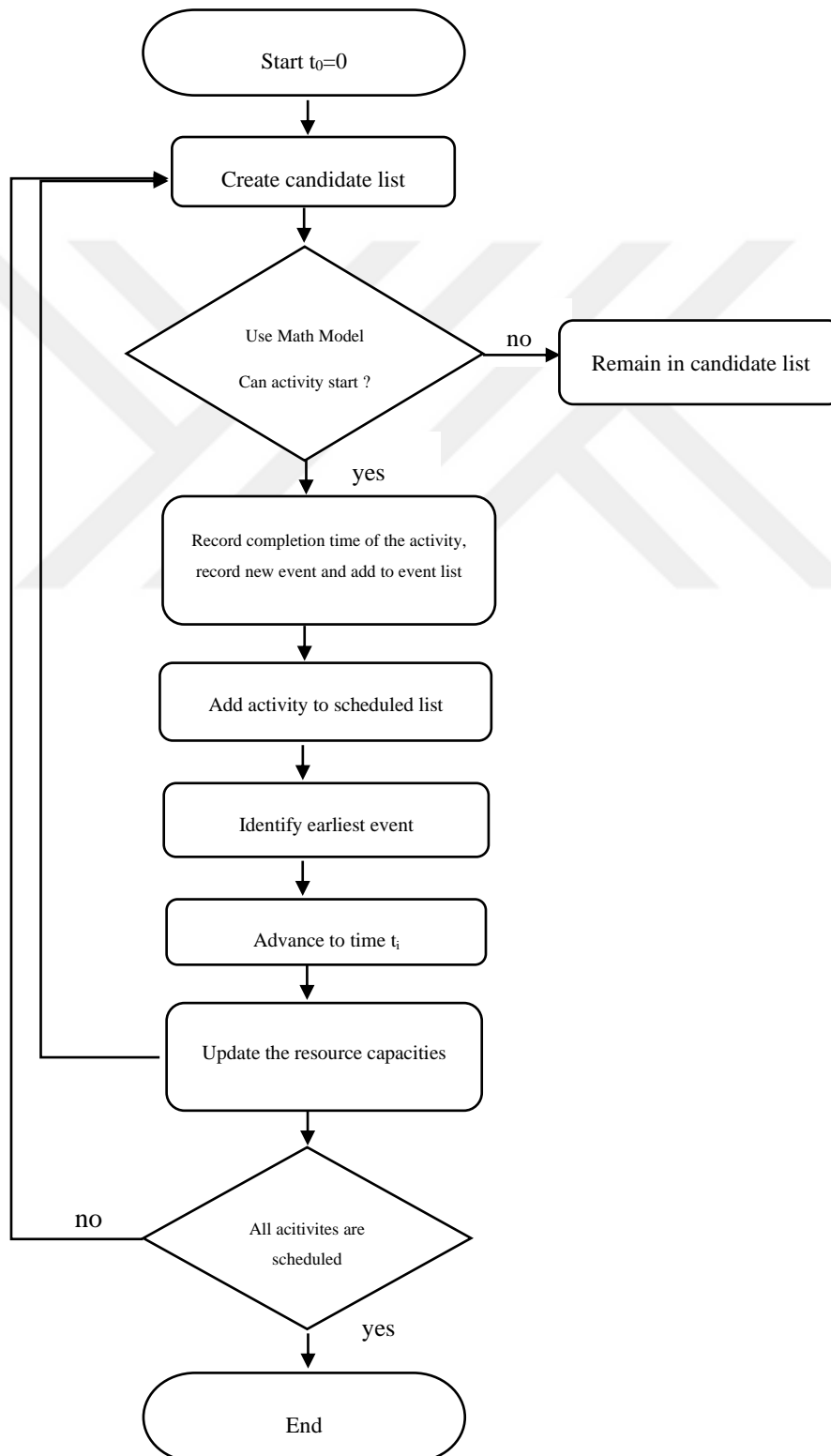
The proposed heuristic method uses the “events” in the project. Each activity in the project has a corresponding start event and end event. These events denote the beginning and completion of the task, respectively. An event list refers to a data structure that captures the changes or events that occur during the execution of a project.

At the beginning of the project, the event list has a single entry which denotes the beginning of the project and the time is set to zero, $t_0=0$. If we consider the priority relationships, there are few (a limited number of) “candidate” activities that can start at that moment. The start of some or all may depend on the availability of resources. We can use a math model for the optimal allocation of resources to decide which activities may start at that moment and which activities should be delayed due to resource constraints. Once the math model decides the activities to start at that moment, the completion times of those activities are recorded as new events and inserted in the event list. The activities that are not chosen (and hence delayed) remain in the candidate activity list to compete in the next stages. Scheduled activities are kept in a separate list.

In the next step, the event list is sorted by the occurrence time in ascending order, and the earliest event is identified. The time is advanced to that time as in the discrete event simulation clock mechanism. Let t_1 denote the time at that moment. At least one of the scheduled activities should be completed at time t_1 . The following parameters are updated at this stage.

- 1- Identify the new candidate activities to be scheduled at time t_1 by considering the successors of the activities completed at time t_1
- 2- Update the resource capacities (some scheduled activities may not be completed yet at time t_1 , and hence some portions of the resources may not be available)

The flow chart of the method is shown below.



The resource allocation model is used again to decide which activities to be scheduled at time t_1 . The solution defines new scheduled activities that generate new events and candidate activities. The algorithm terminates once all activities have been scheduled. The structure of the resource allocation problem may vary. We will define five different structures below. Let's define the common parameters and variables first.

Indices/Sets

C	set of candidate activities at an event, $i = 1..n$
R	set of renewable resources, $k = 1..R$
E	set of events, $e = 0..I$

Parameters:

B_k	Capacity of resource k at an event
a_{ik}	Resource usage of activity i for resource k
d_i	Duration of activity i

Decision Variables:

$$x_i = \left\{ \begin{array}{l} 1, \text{ if activity } i \text{ starts} \\ 0, \text{ otherwise} \end{array} \right\} \quad i \in C$$

Resource Allocation Model-1

In this approach, we first solve the original problem without resource constraints. The CPM may be used to find the optimal solution and to identify the critical activities as well as the floats of each activity. Let CPM indicate the solution (makespan) and f_i indicate the floats of activities. ($i \in C$)

$$\text{Max } Z = \sum_i^C ((CPM - f_i) * x_i)$$

Subject to,

$$\sum_i^C a_{ik} * x_i \leq B_k \quad \forall k \in R$$

$$x_i \in \{0,1\} \quad \forall i \in C$$

This model prioritizes activities with smaller float values. As a result, we ensure that critical activities are scheduled as early as possible. Float values are revised and updated at each event.

Resource Allocation Model-2

This approach is similar to resource allocation model-1 above, but it also considers the duration of the activities as well.

$$\text{Max } Z = \sum_i ((CPM - f_i) * x_i + (\sum_j d_j) * x_i) \quad j \in C, i \neq j$$

Subject to,

$$\sum_i^C a_{ik} * x_i \leq B_k \quad \forall k \in R$$

$$x_i \in \{0,1\} \quad \forall i \in C$$

For the objective function coefficients of each activity, we add the sum of the durations of other candidate activities. Therefore, if the float values are equal, instead of making a random selection, we give priority to the activity with a shorter duration.

Resource Allocation Model-3

In this approach, a different performance measure is used for resource allocation. For each activity, an imaginary rectangle is defined for each resource. The length and height of the rectangle correspond to the duration and resource usage of the activity respectively. The areas of those rectangles are incorporated in the objective function coefficients.

$$\text{Max } Z = \sum_i^C (\sum_k a_{ik} * d_i) * x_i$$

Subject to,

$$\sum_i^C a_{ik} * x_i \leq B_k \quad \forall k \in R$$

$$x_i \in \{0,1\} \quad \forall i \in C$$

The idea is to minimize the unused portions of the resources, i.e., to maximize the utilization of the resources at each event.

Resource Allocation Model-4

In this approach, we use the goal programming approach to define the resource allocation problem. Let M be a big number.

$$\text{Minimize} = \sum_k Slack_k + \sum_k M * Surplus_k$$

Subject to,

$$(\sum_i a_{ik}) * x_i + Slack_k - Surplus_k = B_k \quad \forall k \in R$$

$$x_i \in \{0,1\} \quad \forall i \in C$$

Various numerical experiments have been conducted to evaluate the proposed solution frameworks. These experiments consist of 6, 7, 8, 9 and 10 activities. These approaches exhibit relative superiority over each other depending on the specific problem. Some approaches yield optimal solutions for certain problems, while others excel in solving different problems optimally.

Sample Problem and Solution

Assume that there is a project with 7 activities and the precedence relations are given as shown in Figure 6.1. The diagram uses AON representation and the activities are represented in boxes and identified with capital letters. The arcs denote the precedence relations.

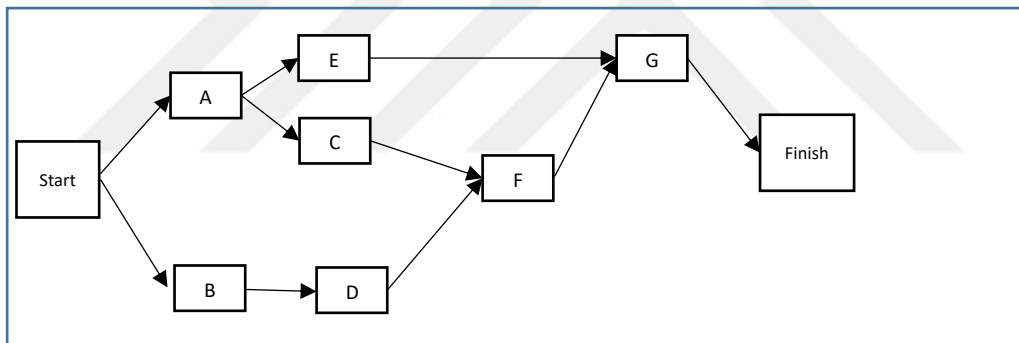


Figure 6.1. Precedence Relations of Sample Project

Source: Author

Assume that there are two different resources $R=2$ the capacity of resource1 is equal to 3 and Resource-2 is equal to 4 for each period. Table 6.1. describes the duration of the activities.

Table 6.1. Durations of Activities

DUMMY	A	B	C	D	E	F	G	DUMMY
0	2	1	1	1	3	2	1	0

Source: Author

Table 6.2. and Table 6.3. represent the resource usage of each activity.

Table 6.2. Resource1 Usage of Activities

RES USAGE	RES USAGE	RES USAGE	RES USAGE	RES USAGE	RES USAGE	RES USAGE	RES USAGE	RES USAGE
DUMMY	A	B	C	D	E	F	G	DUMMY
0	3	3	2	1	2	0	3	0

Source: Author

Table 6.3. Resource2 Usage of Activities

RES USAGE	RES USAGE	RES USAGE	RES USAGE	RES USAGE	RES USAGE	RES USAGE	RES USAGE	RES USAGE
DUMMY	A	B	C	D	E	F	G	DUMMY
0	2	2	4	3	1	2	2	0

Source: Author

The starting point of our solution method is at $t=0$. A list called the "candidate activities" is created. This array holds the activities that can be scheduled at that event moment, taking into account the priority relationships. A and B are added to the list of candidate activities at time=0. Once the candidate events are determined, the resource allocation model takes over and decides which candidate events are to be scheduled at that moment. For the sample problem above, let's use "the resource allocation model-4" to solve the problem. The model can be written as follows.

$$\text{Minimize} = Slack_1 + Slack_2 + M * Surplus_1 + M * Surplus_2$$

Subject to,

$$3x_A + 3x_B + Slack_1 - Surplus_1 = 3$$

$$2x_A + 2x_B + Slack_1 - Surplus_1 = 4$$

$$x_A, x_B \text{ binary}$$

The optimal solution is $(x_A = 1, x_B = 0)$. There is also an alternative optimal solution such as $(x_A = 0, x_B = 1)$. It means that either only activity A or only activity B can be scheduled at time $t=0$. Assume that activity A is chosen at random. Immediately, the finishing time of activity A is calculated and inserted in the event list. Therefore, the event list has now an entry with time $t_1 = t_0 + d_A = 2$ since the duration of activity A is 2. Meanwhile, activity B is delayed and it is kept in the candidate activity list.

The next step is to determine the earliest time in the event list. It is time $t_1=2$ and corresponds to the completion time of activity A. The time is advanced to time $t_1=2$.

In the next step, the new candidate events are determined. The successors of activity A may be candidate events provided that the precedence relationships are not violated. In our sample problem, the new candidate events are C and E. They are inserted in the candidate activity list and therefore the list contains the activities B, C and E.

At this point, the resource allocation model takes over again to make the decision. The model would be as follows.

$$\text{Minimize} = Slack_1 + Slack_2 + M * Surplus_1 + M * Surplus_2$$

Subject to,

$$3x_B + 2x_C + 2x_E + Slack_1 - Surplus_1 = 3$$

$$2x_B + 4x_C + 1x_E + Slack_1 - Surplus_1 = 4$$

x_B, x_C, x_E binary

The optimal solution is $(x_B = 0, x_C = 1, x_E = 0)$. Therefore, the finishing time of activity C is calculated and inserted in the event list. The event list has now a new entry with time $t_2 = t_1 + d_C = 2 + 1 = 3$ since the duration of activity C is 1. Meanwhile, the activity B and E are delayed and they are kept in the candidate activity list.

The algorithm iterates in this loop until all activities are scheduled. Finally, the solution is given as presented in Figure 6.2.

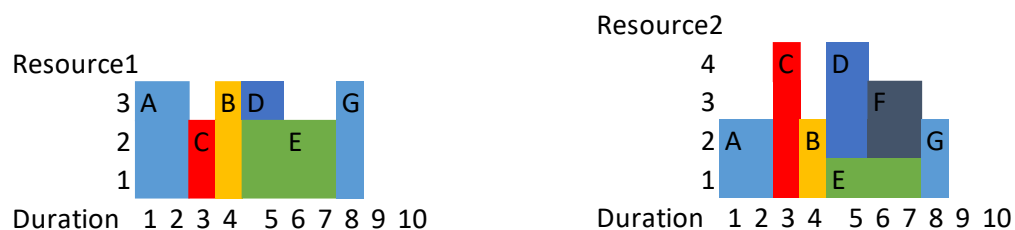


Figure 6.2. Optimal Solution

Source: Author

Resource Allocation Model-5

In this approach, we merge Model 4 and Model 3.

$$\text{Min} \sum_k \text{Slack}_k + \sum_k M * \text{Surplus}_k - \sum_{m=1}^M \sum_i^C \left(\sum_k a_{imk} * d_{im} \right) * x_{im} + \sum_{m=1}^M \sum_i^C \sum_k g_{imk} * x_{im}$$

Subject to,

$$\sum_{m=1}^M \sum_i^C a_{imk} * x_{im} + \text{Slack}_k - \text{Surplus}_k = B_k \quad \forall k \in R$$

$$\sum_{m=1}^M \sum_i^C g_{imk} * x_{im} \leq G_k \quad \forall k \in P \text{ (non - renewable resources) and } G_k \text{ is the remaning capacity}$$

$$\sum_{m=1}^M x_{im} \leq 1, \quad \forall i \in C$$

$$x_i \in \{0,1\} \quad \forall i \in C$$

For the evaluation of the proposed solution frameworks, various numerical experiments have been conducted. Our proposed solution method is verified by solving numerous problems, taken from PSPLIB. All models are coded in the latest version of VBA and solved in a computer with Intel Core i9 processor, 32.0 GB RAM and a 3.10 GHz CPU. There are many instances of RCPSP listed in PSPLIB. We have chosen the sets of the single-mode problems in which the number of activities are 30, 60 and 90 respectively. Each set contain 480 different problems. In all problems, there are four distinct renewable resources. Table 6.4. shown the comperative analysis of the proposed heuristic method. The solution method we proposed was able to find the optimal value in 25% of the 480 instances in the J30 library and a feasible solution in 100% of them. Additionally, we reported deviations from the optimal value. For J30, the deviations from the optimal value were 8.7 % when we included the optimal solutions, and 11.6 % when we excluded the optimal solutions.

Table 6.4. Comperative Analysis for PSPLIB

Measures	J30	J60	J90
Opt(%)	25 %	24 %	24 %
Feas(%)	100 %	100%	100 %
Gap (%) including optimal solutions	8.7 %	9.4 %	8.1 %
Gap (%) excluding optimal solutions	11.6 %	12.5 %	11.6 %

Source: Author

7. CHAPTER: CONCLUSION & FUTURE WORK

In this thesis, the Resource-Constrained Project Scheduling Problem (RCPSP) was examined with a particular emphasis on multi-mode problems. To support this effort, an extensive and comprehensive literature review was conducted, systematically classifying the relevant studies based on specific characteristics. This review not only provided a structured overview of the field but also identified gaps in the existing literature, helping to shape the direction of our research. The literature study, in itself, serves as a valuable reference point that will guide and inspire future research efforts in this area. In fact, we prepared a paper on this topic, which was presented at the ICDAM (International Conference on Data Analysis and Management) in London in June 2024 and it is expected to be published soon.

In response to the identified gaps, two novel continuous-time Mixed Integer Linear Programming (MILP) models were developed, incorporating event and resource-flow-based concepts. These models capture the dynamic nature of project scheduling more accurately by allowing for continuous time variables, eliminating the reliance on discrete intervals. This results in a more flexible and scalable solution approach, particularly beneficial for large and complex projects. The proposed models were thoroughly tested against benchmark problems from established problem libraries in the literature. Especially the flow-based model proved its effectiveness and computational efficiency. The results demonstrated that this flow-based model outperformed the other known MILP models in the literature, offering better performance in terms of both solution quality and computational efficiency.

In addition to testing our models on existing benchmark problems, a new set of problem instances was created. These instances were designed to emphasize the advantages of the proposed continuous-time model, particularly in handling scenarios with larger resource requirements and capacities. To the best of our knowledge, this thesis is the first in the literature to present such a library specifically focused on large

resource requirements and capacities. This novel contribution offers a dedicated framework for addressing challenges related to high-demand resources, filling a critical gap in the existing body of research.

Beyond the development of new models, this thesis also explored solution methods beyond conventional mathematical programming. Specifically, an original heuristic method was proposed, leveraging events within the project structure. In this method, each project activity is represented by two events: a start event and an end event. By organizing these events into an event list, the heuristic effectively manages and tracks the sequence of activities throughout the project's execution. This event-based approach provides a flexible and adaptive solution framework, capable of handling dynamic changes within the scheduling process. The heuristic approach was shown to be particularly effective in reducing computational time while maintaining effective solutions, making it a valuable alternative to purely mathematical models.

Looking ahead, there are several promising avenues for future research. One of the key areas is the further enhancement of the proposed models through the incorporation of improved lower bounds (LBs). These enhancements could lead to even more efficient solution methods, capable of handling larger and more complex problem instances. Additionally, further exploration of continuous-time modeling approaches could yield new insights and methods, potentially leading to more effective solutions for RCPSP. The integration of “matheuristics”, which combines the strengths of mathematical programming and heuristic methods, also represents a promising direction for future work, as it could enhance the performance of existing models across a wide range of problem domain.

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APPENDICES

APPENDIX 1. COMPLETE LIST OF ARTICLES by DISTINGUISHING FEATURES

	Single Mode	Multi Mode
	DT	DT
REN	<p>Afshar-Nadjafi, B. et al.(2017), Afshar-Nadjafi, B. et al.(2013), Agarwal, A. et al.(2011), Alvarez-Valdes, R. et al.(1989), Anthonisse, J. M. et al. (1988), Artigues, C. et al. (2003), Baar, T. et al. (1998), Balas, E. (1971), Balas, E. (1996) , Ballestín, F.(2007), Ballestín, F. et al.(2006), Ballestín, F. et al.(2008)P, Baptiste, P. et al. (1999), Baroum, S. (1992), Baroum, S. et al.(1989), Baroum, S. et al. (1999) , Bartusch, M et al. (1988), Belhe, U. et al.(1995), Bell, C. E. et al.(1991), Bell, C. E. et al.(1990) , Berthold, T. et al.(2010), Bertsimas, D. et al.(2013), Bettemir, O. H. et al.(2014), Bey, R. B. et al.(1981), Bianco, L. et al.(2011), Bianco, L. et al.(2011), Bianco, L. et al.(2012), Bianco, L. et al.(1999)P, Bibiks, K. et al.(2018), Blazewicz, J. et al.(1983)P, Boctor, F. F. (1990), Boctor, F. F. (1996), Bomsdorf, F. et al.(2008), Bowers, J. A. (2000), Brinkmann, K. et al. (1996), Brucker, P. (2002) P, Brucker, P. et al. (1999), Brucker, P. et al. (2000)P, Brucker, P. (1999) et al.(1998), Bukata, L. et al.(2013), Bukata, L. et al.(2015), Cavalcante, C. C. B. et al.(2001), Chen, R.M. (2011), Chen, Z.J. (2010), Chen, W. et al.(2010), Cho, J. H. et al.(1997), Christofides, N. et al.(1987), Cooper, D.F. (1976), Damay, J. et al. (2007)P, Davies, E. M. (1973), Davis, E. W. (1975), Davis, E. W. et al.(1971)P, Davis, E. W. et al.(1975), Davis, K. R. et al.(1992), Dayanand, N. et al.(1997), Dayanand, N. et al.(2001), De, P. et al.(1995), De, P. et al.(1997), Debels, D.P et al.(2006), Debels, D.P et al.(2008), Deckro, R. F. et al.(1989), Deckro, R. F. et al.(1990), Delgoshaei, A. et al.(2015), Demeulemeester, E. L. et al.(1992), Demeulemeester, E. L. et al. (1995), Demeulemeester, E.L. et al. (1996), Demeulemeester, E. L. et al. (1996), Demeulemeester, E. L. et al. (1997), Demeulemeester, E. L. et al. (1997), Demeulemeester, E. L. et al. (1997), Demeulemeester, E. L. et al. (1994), Dobson, G. et al.(1989), Dodin, B. et al.(1989), Dorndorf, U. et al.(1999), Dorndorf, U. et al.(1999), Drexl, A. (1991), Drexl, A. et al. (2001), Elmaghaby, S.E. et al.(1990), Elsayed, E.A. (1982), Erenguc, S.S. et al.(1999), Erschler, J. et al.(1989), Etgar, R. et al.(1997), Fahmy, A. et al.(2014), Fleszar, K. et al.(2004), Franck, B. et al.(1996), Franck, B. et al.(1997), Goncalves, J.F. et al.(2011), Goncharov, E. et al.(2017), Gu, Z. et al.(1998), Hartmann, S. (1999), Hartmann, S. (2012), Hartmann, S. et al.(2000), Hastings, A.J. (1972), Heilmann, R. (1997), Herroelen, W.S. et al.(1997), Icmeli, O. et al. (1994), Icmeli, O. et al. (1996a), Icmeli, O. et al. (1996b), Icmeli-Tukel, O. et al.(1997), Icmeli, O. et al.(1998), Johnson, T.J.R. (1967), Jozefowska, J. et al.(2000), Kaplan, L. (1988)P, Karabati, S. et al.(1993), Khattab, M. et al.(1990), Khattab, M. et al.(1991), Kimms, A. (2001),</p>	<p>Afshar-Nadjafi, B. et al. (2012), Ahn, T. et al.(1995), Ahn, T. et al.(1998), Alcaraz, J. et al.(2003), Artigues, C. et al.(2000), Ballestín, F. et al.(2011), Balouka, N. et al.(2016), Barrios, A. et al.(2009), Basnet, C. (2016), Bellenguez, O. et al.(2007), Bianco, L. et al.(1998), Boctor, F. F. (1993), Boctor, F. F. (1996), Bouleimen, K. et al.(2003), Brucker, P. et al.(2001)P, Brucker, P. et al.(1999), Brucker, P. et al.(2003), Buddhakulsomsiria, J. et al.(2006)P, Buddhakulsomsiria, J. et al.(2007)P, Burgelman, J. et al.(2018), Cesta, A. et al.(2002), Chen, M. et al.(2015), Cheng, J. et al.(2015), Chiang, C.W. et al.(2008), Dai, H. et al.(2018), Damak, N. et al.(2009), Delgoshaei, A. et al.(2016)P, Demeulemeester, E.L. et al.(2000), Drexl, A. et al.(1993), Drezet, L.E. et al.(2008)P, Elsayed, S. et al.(2017), Erenguc, S.S. et al.(2001), Franck, B. et al.(1997), Fu, F. et al.(2011), Ghoddousi, P. et al.(2013), Hartmann, S. (2001), Hartmann, S. et al.(1998), Heilmann, R. (2001) , Heilmann, R. (1997) Jarboui, B. et al.(2008), Jozefowska, J. et al.(2001), Khalilzadeh, M. et al.(2012), Kolisch, R. et al.(1997), Kyriakidis, T.S. et al.(2012), Lorenzoni, L.L. et al.(2006), Maghsoudlou, H. et al.(2016), Maghsoudlou, H. et al.(2017), Maniezzo, V. et al.(1999), Messelis, T. et al.(2014), Mika, M. et al.(2005), Mika, M. et al.(2008), Mori, M. et al.(1997), Myszkowski, P.B. et al.(2018), Nabipoor Afruzi, E. et al.(2014), Néron, E. (2002), Nonobe, K. et al.(2002), Ozdamar, L. (1999), Pérez, Á. Et al.(2014), Pesch, E. (1999), Peteghem, V.V. et al.(2010)P, Rahimi, A. et al.(2013), de Reyck, B. et al.(1999), Sabzehparvar, M. et al. (2008), Salewski, F. et al.(1997), Sebt, M. et al.(2015), Sebt, M. et al.(2016), Sprecher, A. (1994), Sprecher, A. (2000), Sprecher, A. et al. (1998a), Sprecher, A. et al. (1998b) , Sprecher, A. et al. (1997), Tseng, L. Y. et al.(2009), Ulusoy, G. et al.(2001) Vanden Berghe, G. et al.(2011), Wang, L. et al.(2012), Zhang, H. et al.(2006)</p>

	<p>Kis, T. (2005), Kis, T. (2006), Klein, R. (1998), Klein, R. (2000a), Klein, R. (2000b), Klein, R. et al.(1998a), Klein, R. et al.(1998b), Klimek, M. et al.(2011), Kochetov, Y. et al.(2003), Kohlmorgen, U. et al.(1999), Kolisch, R. (1996a), Kolisch, R. (1996b), Kolisch, R. et al.(1996a), Kolisch, R. et al.(1999), Kolisch, R. et al.(2006), Kopanos, G.M. et al.(2014), Koulinas, G. et al.(2014)P, Kreter, S. et al.(2016), Kruger, D. et al.(2009), Kuster, J. et al. (2006), Kuster, J. et al.(2008a), Lee, J.K. et al.(1996), Leon, V.J. et al.(1995), Mausser, H.E. et al.(1996), Mendes, J.J. et al.(2009), Mika, M. et al.(2006), Mingozzi, A. et al.(1998), Mohring, R.H. (1984), Mohring, R.H. et al.(1995), Mohring, R.H. et al.(1998), Mohring, R.H. et al.(2003), Naber, A. et al.(2014), Najafī, B.A. et al.(2008)P, Najafī, A.A. et al.(2006), Naphade, K.S. et al.(1997), Nazareth, T. et al.(1999), Neumann, K. et al.(1995), Neumann, K. et al.(1999), Neumann, K. et al.(2000), Neumann, K. et al.(2002), Neumann, K. et al.(2002), Neumann, K. et al.(2006), Nkasu, M.M. et al.(1997), Nonobe, K. et al.(2006), Norbis, M. et al.(1988), Nubel, H. (2001), Oguz, O. et al.(1994), Ozdamar, L. et al.(1993), Ozdamar, L. et al.(1996), Ozdamar, L. et al.(1998), Padman, R. (1991), Padman, R. et al.(1993), Padman, R. et al.(1997), Padman, R. et al.(1997), Padman, R. et al.(2006), Pan, N. et al.(2011), Paraskevopoulos, D.C. et al.(2012), Paraskevopoulos, D.C. et al.(2016), Patterson, J.H. et al. (1974), Patterson, J.H. (1976), Patterson, J.H. et al.(1976) Patterson, J.H. (1984), Paul, S.K. et al.(2014), Petrovic, R. (1968), Polancos, R. V. et al.(2023), Pollack-Johnson, B. (1995), Proon, S. et al.(2011), Radermacher, F.J. (1985/86), Ranjbar, M. et al.(2007), Ranjbar, M. et al.(2008), Ranjbar, M. et al.(2009), de Reyck, B. et al.(1995), de Reyck, B. et al.(1998), de Reyck, B. et al.(1998), Reyck, B. et al. (1998), Reyck, B. et al. (1999), Russell, R.A. (1986), Sampson, S.E. et al.(1993), Sankaran, J.K. et al.(1999), Schutt, A. et al.(2013), Schwindt, C. (1996), Schwindt, C. (2000), Shadrokh, S. et al.(2007), Shewchuk, J. P. et al.(1995), Shrage, L. (1970), Shtub, A. et al.(1996), Simpson, W. P. et al.(1996)P, Skutella, M. (1998), Slowinski, R. et al.(1978), Smith-Daniels, D. E. et al.(1987), Smith-Daniels, D. E. et al.(1996), Sonmez, R. et al.(2012), Sprecher, A. et al.(1995), Stadler, H. (2005), Stinson, J. P. (1976), Stinson, J. P. et al.(1978), Sung, C. S. et al.(1994), Sung, C. S. et al.(1997), Talbot, F. B. et al.(1978), Thesen, A. (1976), Thesen, A. (1977), Thomas, P. R. et al.(1997), Thomas, P. R. et al.(1998),Thiruvady, D. et al.(2013), Tormos, P. et al.(2003), Tukul, O. I. et al.(2001), Ulusoy, G. et al.(2000), Ulusoy, G. et al.(1989), Ulusoy, G. et al.(1994),Valls, V. et al.(2005)P, Valls, V. et al.(2008), Vanhoucke, M. (2006), Vanhoucke, M. (2006), Vanhoucke, M. et al.(2001), Vanhoucke, M. et al.(2001), Vanhoucke, M. et al.(2001), Vanhoucke, M. et al.(2001), Vanhoucke, M. et al.(2001), Vanhoucke, M. et al.(2001), Vanhoucke, M. et al.(2001), Vanhoucke, M. et al.(2001), Vanhoucke, M. et al.(2001), Vanhoucke, M. et al.(2001), Vanhoucke, M. et al.(2001), Weglarz, J. et al.(1977), Whitehouse, G. E. et al.(1979), Willis, R. J. et al.(1976), Yamashita, D. S. et al.(2007), Yang, K. K. et al.(1993), Yau, C. et al.(1990), Zamani, R. (2013), Zamani, R. (2017), Zhan, J. (1994), Zheng, X. et al.(2015), Zhu, D. et al.(1997), Zhu, D. et al.(1997), Zhu, D. et al.(1997)</p>	
REN + NON REN	<p>Agrawal, M. K. et al.(1996), Akkan, C. et al.(2005), Chaleshtarti, A.S. et al. (2014), Demeulemeester, E.L. et al.(1998), Dorndorf, U. et al.(1999), Gargeya, V.B. et al.(1996), Leachman, R.C. et al.(1990), Nudtasomboon, N. et al.(1997), Ozdamar, L. et al.(1996), de Reyck, B. et al.(1996), Viana, A. et al.(2000)</p>	<p>Azizoglu, M. et al.(2015), Bilollikar, V. S. et al.(2016), Brucker, P. et al.(2003),Coelho, J. et al.(2011), Gerhards, P. (2020), Kadri, R. et al.(2014), Li, H. et al.(2013), Li, Y. et al.(2011), Sebt, M. et al.(2015)P</p>