DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

RELIABILITY MODELING FOR THE PERFORMANCE OF SOLAR PHOTOVOLTAIC SYSTEMS

by
Melek ESEMEN

December, 2022
İZMİR

RELIABILITY MODELING FOR THE PERFORMANCE OF SOLAR PHOTOVOLTAIC SYSTEMS

A Thesis Submitted to the
Graduate School of Natural And Applied Sciences of Dokuz Eylül University
In Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in Statistics

by
Melek ESEMEN

December, 2022
İZMİR

Ph.D. THESIS EXAMINATION RESULT FORM

We have read the thesis entitled “**RELIABILITY MODELING FOR THE PERFORMANCE OF SOLAR PHOTOVOLTAIC SYSTEMS**” completed by **MELEK ESEMEN** under supervision of **PROF. DR. SELMA GÜRLER** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy.

....

Prof. Dr. Selma GÜRLER

Supervisor

....

Prof. Dr. Gözde Yazgı TÜTÜNCÜ

Thesis Committee Member

....

Assoc. Prof. Dr. İdil YAVUZ

Thesis Committee Member

....

Assoc. Prof. Dr. Funda İŞÇİOĞLU

Examining Committee Member

....

Assoc. Prof. Dr. Cemal Murat ÖZKUT

Examining Committee Member

Prof. Dr. Okan FISTIKOĞLU
Director
Graduate School of Natural and Applied Sciences

ACKNOWLEDGEMENTS

I would like to express my sincere gratitude and thanks to my advisor, Prof. Dr. Selma GÜRLER for her direction, support and encouragement. She always behaved in a friendly manner to me. I am honored to be her student.

Besides my advisor, I would like to thank Prof. Dr. Gözde Yazgı TÜTÜNCÜ and Assoc. Prof. Dr. İdil YAVUZ for their contributions and invaluable comments during my Ph.D. study. I also thank Prof. Dr. Serkan ERYILMAZ for the guidance and motivation for this study.

I would like to thank my academic sister, Büşra GÜNGÖR for her constructive comments, and helps. I would also like to thank my brother from another mother, Serdar GÜNGÖR for his social support. I also wish to thank Almila HACIOĞLU for her sisterhood. I would like to express my deepest longing to my dear deceased sister, Aslıhan ERDEM.

This thesis is dedicated to my deceased elder sister, Müesser ESEMEN. She is always in my heart. A very special appreciation goes to my parents Fatma - İlhan ESEMEN, and SİNANOĞLU family, my second family in İzmir for their support during my studies.

Finally, note that I am supported by the Scientific and Technological Research Council of Turkey TÜBİTAK) 2211/A National Ph.D. scholarship program and the Higher Education Council of Turkey (YÖK) 100/2000 Ph.D. scholarship program during the four years.

Melek ESEMEN

RELIABILITY MODELING FOR THE PERFORMANCE OF SOLAR PHOTOVOLTAIC SYSTEMS

ABSTRACT

Solar energy is one of the most widely used renewable energy sources. Photovoltaic systems directly convert solar energy into electricity with no carbon dioxide emission or any other air pollutants. The power generated by a photovoltaic system depends on the characteristics of solar irradiation and weather conditions. This stochastic nature of power systems prompts the researchers to use probabilistic and statistical techniques. In this thesis, we consider the distribution function of the power generated by a photovoltaic system to make predictions about its characteristics. We model the mean power generated by a photovoltaic system. We also consider that photovoltaic modules may have multistate working conditions and different performance levels depending on solar radiation. In this concept, we present a model for solar power systems with PV modules having various levels of operational performance and we develop a reliability model for the system's power regarding the threshold value that is the minimum required total performance level for the system. This model reflects the performance levels and the working probabilities of PV modules. The problem is evaluated under different conditions regarding the dependency of multistate PV modules. In addition, we provide the optimum number of photovoltaic modules that minimizes the total cost based on level of required total power production. For further analyses, we give real data applications to estimate the characteristics of power produced by the solar plant for a specific location in Izmir, Turkey. As the software programming tools, R v.1.2.5033 and Mathematica v.11.3 are used for the computations.

Keywords: Photovoltaic (PV) system, solar energy, distribution function, multistate system, optimization, reliability.

GÜNEŞ FOTOVOLTAİK SİSTEMLERİNİN PERFORMANSI İÇİN GÜVENİLİRLİK MODELLEMESİ

ÖZ

Güneş enerjisi en yaygın olarak kullanılan yenilebilir enerji kaynaklarından biridir. Fotovoltaik sistemler, karbondioksit emisyonu veya diğer hava kirleticileri olmadan güneş enerjisini doğrudan elektriğe dönüştürebilen sistemlerdir. Bir fotovoltaik sistem tarafından üretilen güç, güneş ışınımının özelliklerine ve hava koşullarına bağlıdır. Güç sistemlerinin bu stokastik doğası, araştırmacıları olasılıksal ve istatistiksel teknikleri kullanmaya yöneltmektedir. Bu tezde, bir fotovoltaik sistem tarafından üretilen gücün istatiksel özellikleri hakkında tahminlerde bulunmak için üretilen gücün dağılım fonksiyonu ele alınmıştır. Ardından bir fotovoltaik sistem tarafından üretilen ortalama gücü hesaplayan formül elde edilmiştir. Ayrıca, fotovoltaik modüllerin güneş ışınımına bağlı olarak çok durumlu çalışma koşullarına ve farklı performans seviyelerine sahip olabileceği üzerinde durulmuştur. Bu nedenle, çeşitli performans seviyelerine sahip PV modüllü güneş enerjisi sistemleri için bir model sunulmuş ve sistemden istenen minimum toplam performans seviyesi olan eşik değerine ilişkin sistem gücü için bir güvenilirlik modeli geliştirilmiştir. Bu model, PV modüllerinin performans seviyelerini ve modüllerin çalışma olasılıklarını yansıtmaktadır. Problem, çok durumlu PV modüllerinin farklı bağımlılık koşulları altında ele alınmıştır. Ayrıca, gerekli toplam güç üretimi düzeyine bağlı olarak toplam maliyeti en aza indiren optimum fotovoltaik modül sayısı belirlenmiştir. Daha ileri analiz için, İzmir, Türkiye'de belirli bir konum verileri kullanılarak güneş santrali tarafından üretilen gücün özelliklerini tahmin etmek için gerçek veri uygulaması gerçekleştirilmiştir. Yazılım programlama araçları olarak hesaplamalar için R v.1.2.5033 ve Mathematica v.11.3 kullanılmıştır.

Anahtar kelimeler: Fotovoltaik (FV) sistem, güneş enerjisi, dağılım fonksiyonu, çok durumlu sistem, optimizasyon, güvenilirlik.

CONTENTS

LIST OF FIGURES

Page

LIST OF TABLES

LIST OF SYMBOLS

GW	:	Gigawatt
W/m^2	:	Watt per square meter
G_{bi}	:	The global solar irradiation random variable (W/m^2)
G_s	:	The solar irradiation in a standard environment (W/m^2)
R_c	:	The certain irradiation point (W/m^2)
P_{sn}	:	The rated capacity of a PV module (W)
P_{PV}	:	The power generated by a PV module
$F_{G_{bi}}$	:	The cumulative distribution function (cdf) of G_{bi}
$f_{G_{bi}}$	:	The probability density function (pdf) of G_{bi}
P_T	:	The total power generated by the solar plant random variable
N	:	The number of PV module
$W_N(p_t)$	:	The cdf of power generated by a solar plant
$w_N(p_t)$	:	The pdf of power generated by a solar plant
$W_N(p_t;p)$	:	The cdf of power generated by a solar plant with N modules subject to failure
$w_N(p_t;p)$	:	The pdf of power generated by a solar plant with N modules subject to failure
μ_0	:	The mean power generated by a single PV module
μ_T	:	The mean power generated by the solar plant
n_1	:	The number of components of Type I
n_2	:	The number of components of Type II
t_j	:	The performance of components of Type I when they are in state j, $j = 1, 2, 3, 4.$
t_j^*	:	The performance of components of Type II when they are in state j, $j = 1, 2, 3, 4.$
p_j	:	The probability that a component of Type I is in state j, $j = 1, 2, 3, 4.$

p_j^*	: The probability that a component of Type II is in state j, $j = 1, 2, 3, 4$.
m	: The minimum required total performance level for the system
T_1	: The total performance of components of Type I
T_2	: The total performance of components of Type II
W	: Watt
kW/m^2	: Kilowatt per square meter
c_i	: The cost of i-th type of PV module
$n_{i,j}$	: The number of i-th type of module in j-th plant
r_0	: The given threshold reliability value
$n_{i,j}^{max}$	: The maximum number of i-th type of PV modules in the j-th plant

ABBREVIATIONS

IEA	: International Energy Agency
PV	: Photovoltaic
DEU	: Dokuz Eylul University
TMY	: Typical Meteorological Year

CHAPTER 1
INTRODUCTION

The energy need in the world has increased in recent years with the increase in the global population, the rapid development of technology, and industrialization. The fact that energy becomes a basic input in many areas such as electronic devices used in homes, factories, lighting, transportation, and heating increases energy consumption undoubtedly. A large part of the energy needed is provided by fossil resources. Due to the developing technology and the increase in the population, the consumption of electrical energy is rapidly increasing. To provide the increasing energy demand and to challenge climate change, energy production based on renewable energy sources attracts more attention than expected. In contrast to the rapid decline of fossil resources and their negative effects on the environment, renewable energy resources exist in nature, are environmentally friendly, unlimited, and free with no or little environmental impact resources. These reasons rapidly increase the importance and use of renewable energy sources. Renewable energy sources are energy sources such as solar, wind, hydro, geothermal, and biomass types. These are clean, self-existing, and inexhaustible resources in nature.

As reported by British Petroleum (2019), global energy consumption increased by 2.9% in 2018. This growth was the strongest one since 2010 and almost double the 10-year average. The growth of energy consumption was driven by natural gas, which contributed more than 40% of the increase. Apart from renewables, all fuels grew faster than their 10-year averages, though renewables still accounted for the second largest increment in energy growth. In parallel, carbon emissions grew at a rate of 2.0%, the fastest growth for seven years. According to British Petroleum (2021), due to the COVID-19 pandemic, there was the largest decline since World War II in primary energy consumption, which fell by 4.5% in 2020. The main reason for the drop in energy consumption is oil because there were lockdowns in the whole world and this situation decreased the demand for transportation. Depending on that, carbon emissions from energy use fell by 6.3%, the lowest level since 2011 and this was the largest decline since 1945 in primary energy consumption. The shares of primary energy consumption in the world for 2020 are given in Figure 1.1.

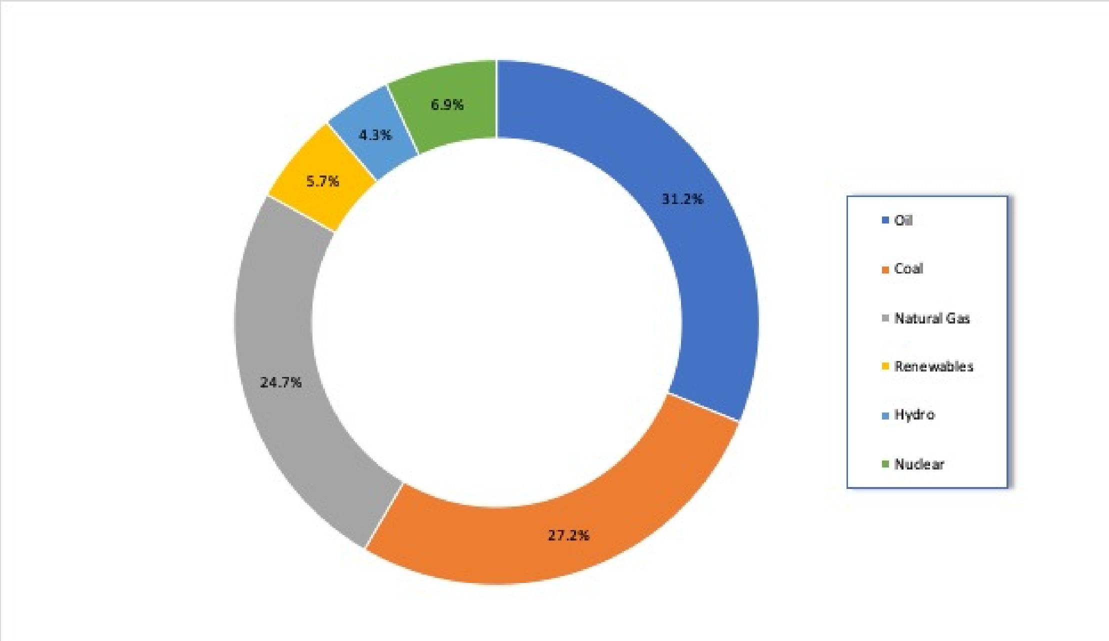

Figure 1.1 The shares of the primary global energy consumption (British Petroleum, 2021)

As seen in Figure 1.1, 90% of the energy need in the world is provided by fossil fuels and nuclear energy (British Petroleum, 2021). However, the amount of harmful gases in the atmosphere has increased greatly due to the use of fossil fuels. Besides the damage of them, they will be depleted in time. For these reasons, researchers have turned to renewable energy sources because these sources are more environmentally friendly and have no problems such as depletion of resources. According to the definition of the International Energy Agency (IEA), renewable energy is energy that can be obtained by natural processes that can be replenished continuously. Renewable energy sources can be found in nature in different forms such as; solar, wind, hydro, geothermal, biomass, and waves. International Energy Agency (2021) forecasts that annual additions to the world's renewable electricity capacity would be on the average around 305 GW between 2021 and 2026. According to the growth of renewables during the previous five years, this represents an acceleration of about 60%. There is also another annual record in 2021 for additions of renewable power capacity. Almost 290 GW of additional renewable energy will be installed in 2021, with solar PV accounting for more than half of that growth, followed by wind and hydropower. As one can see, solar energy is one of the rapidly growing renewable energy sources in recent.

1.1 The history of solar energy

Solar energy has been used for centuries for simple processes such as heating and obtaining hot water. However, for the first time, with the discovery of the PV effect of the sun by Becquerel (1839), it was possible to obtain electricity from solar energy. Then, Adams & Day (1877) observed the PV effect on solid selenium. The first PV cell was developed by Fritts (1883) with less than %1 efficiency of it. Ohl (1946) developed the silicon PV cell. Today's PV cell technology was determined by the Chapin et al. (1954) at Bell Laboratories on silicon PV cells. The increase in the search for alternative energy sources with the world oil crisis has also increased the interest in PV cells. Since this date, the production of PV cells has accelerated and research and development studies in this field have been given importance and cost was of secondary importance.

There are many beneficial aspects of solar energy. Solar energy is a clean and renewable energy source. It does not cause environmental pollution in its use and there is no depletion. It reduces electrical energy costs and electrical energy obtained from solar energy can be used in different sectors. Solar energy systems require low maintenance costs and operating costs are low. The longevity of solar energy systems can restore system installation costs. When necessary, the systems can be disassembled and transferred to the required areas. The system can be installed as much as needed energy. It ensures that the electrical energy needs can be met at affordable costs in places where there is no electricity network. It ensures that energy can be stored and used when necessary. Besides the positive aspects of the use of solar energy, there are also some disadvantages of the solar energy system: The system installation and space usage are high where it is installed. Energy production is affected by weather conditions. Energy storage is expensive and storage life is limited. The wastes generated in the manufacture of solar energy systems pollute the environment although it is clean energy in itself. It may not be sufficient for high energy demands. They are adversely affected by temperature and dust.

The working principle of PV cells is based on the PV effect and it is shown in Figure 1.2. When the photon in sunlight falls on PV cells, electrical voltage occurs at their ends. To convert the solar energy coming to the surface of the PV cell to electrical energy, PV cells must be produced from a semiconductor material that can absorb sunlight and allow the electrical charges to be separated from each other. Useful power output is obtained at the ends of the cells with the electron movements between the layers of PV cells and this semiconductor material.

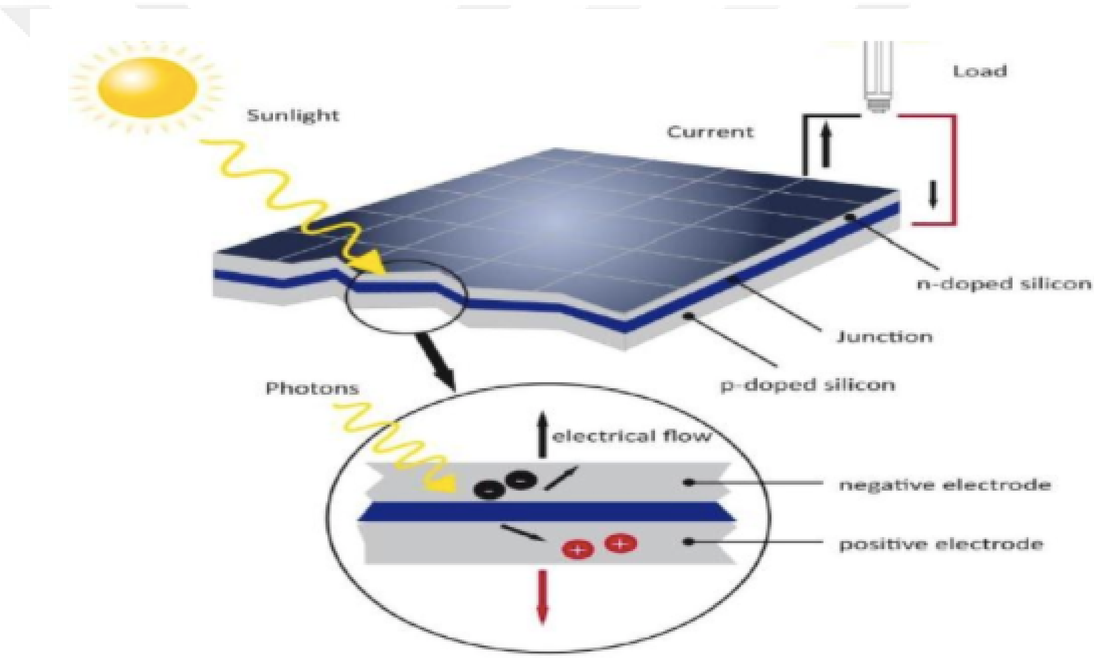

Figure 1.2 The working principle of a PV cell (Tobnaghi & Naderi, 2015)

PV cells are the basis of PV systems. If it is desired to increase the power output, PV modules are formed by connecting more than one PV cells on a surface in parallel or in series. As a result of connecting the PV modules to each other, more power output is obtained and this structure is called PV panel. The structure formed by combining the PV panels forms the PV array. Finally, the whole structure can be called as a PV system shown in Figure 1.3.

Energy produced by a PV system depends on conditions such as weather

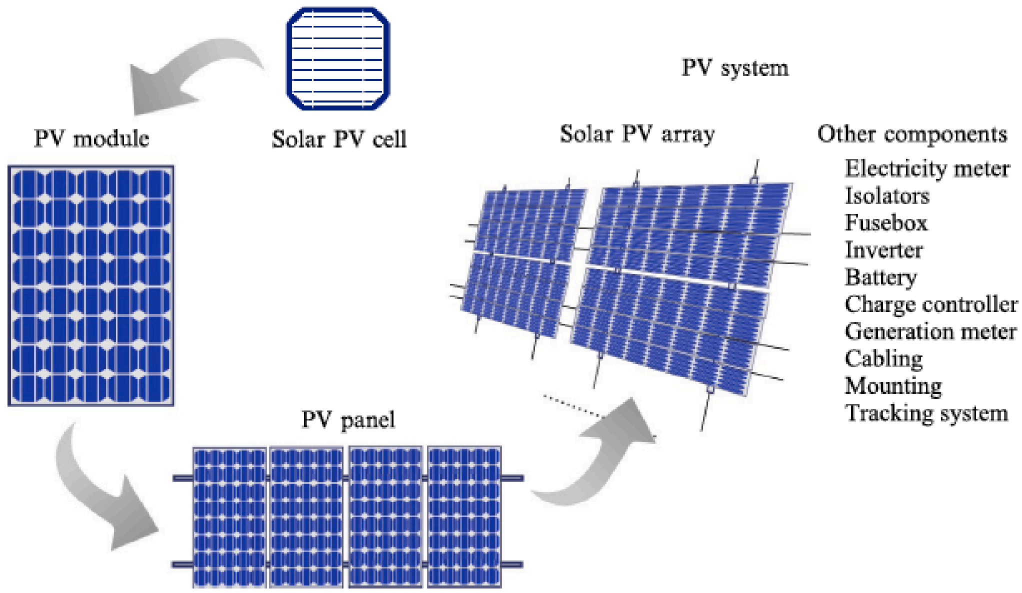

Figure 1.3 The typical PV system (Yang et al., 2018)

conditions, the angle of the sunlight, the surface temperature of the PV panel, and the level of solar irradiation. In this thesis, we focus on the solar irradiation effect on the generated power by PV modules and reliability modeling for the performance of PV systems. Since solar irradiation is a random variable, the power produced by a PV module is also a random variable. The stochastic nature of power output prompts the researchers to use probabilistic and statistical techniques. Many studies focused on the probabilistic modeling and especially reliability assessment of PV systems. The studies in the literature are focused on the reliability evaluation methods and reliability indices for a PV system and its components. Zhang et al. (2013) provided a review study for other reliability evaluation methods and reliability indices for PV systems. Alferidi & Karki (2017) also presented widely used reliability indices for the PV system. Moreover, there are some studies focused on the modeling of the power output of PV systems. Marwali et al. (1998) developed a power output model representing a relationship between the power output and the solar radiation. One can see the review study of Singh (2013) for further information on PV power output modeling. Also, for general information about PV systems, see the reference book of Patel (1999).

1.2 The motivation and the outline of the thesis

Most of the papers in the literature studied modeling the relationship between solar irradiation and power output and how to evaluate the reliability indices. In a power system, modeling the theoretical distribution of the power enables one to obtain all the characteristics and it is significant to make decisions in the planning and installation procedures of the power plant. There are few studies about the theoretical distribution of power generated by a renewable energy system. Tina et al. (2006) derived the probability density function (pdf) of power of a hybrid energy system consisting of wind turbine and PV modules. They used the convolution technique to obtain the pdf of power of the hybrid system with no consideration of reliability values of wind turbine and PV module but also estimate energy performance of the system by using the reliability index in energy literature, namely, energy expected not supplied. Eryilmaz & Devrim (2019) focused on the theoretical distribution of power by a wind plant considering the reliability of the wind turbine. In addition, they answer the question related to the minimum number of wind turbines that have to be constructed in a wind plant to get the minimum desired level of production capacity with an adequate level of probability. Kan et al. (2020) considered the dependence between turbine availability and the wind speed and derived the distribution of the wind farm power. Eryilmaz et al. (2021) studied the theoretical distribution of hybrid solar/wind power systems considering the reliability of components in the system. They used the cubic relationship between wind speed and the power output of the wind turbine and used a not more complicated power output model for the PV system rather than Tina et al. (2006)'s work.

As another approach in the context of power modeling, there are few studies related to energy systems with components having different performance levels. Eryilmaz (2018) considered a reliability model for a particular kind of multistate system and used the obtained model for calculating the power of the wind energy system. Also, the number of wind turbines to be installed in the wind plants is optimized by minimizing the total cost under the required total wind power. Devrim

& Eryilmaz (2021) defined a hybrid system consisting of a specified number of wind turbines and solar modules when assuming that the weights of wind turbines and solar modules correspond to the mean power produced by the turbine and by the module depending on the energy source conditions, respectively. They evaluated the performance of a hybrid energy system by using a weighted k-out-of-n reliability model in the study of Eryilmaz & Sarikaya (2014). They also calculated the importance of a single component in the renewable energy system. Eryilmaz & Kan (2020) modeled a wind power system including two wind farms using a weighted k-out-of-n system when the wind speeds of two farms are statistically dependent. They used two methods to model the dependency between wind speeds at two sites, namely, multivariate normal approximation and copulas-based methods. Larsen et al. (2020) defined a new multi-performance weighted multi-state K^{-}-out-of-n system and used this model for a power system that produces both electricity and heat. Eryilmaz & Ucum (2021) studied the lost capacity by the weighted k-out-of-n system upon failure. Particularly, they obtained the distribution of the lost capacity by the system and applied the model to a power system that consists of a specified number of generating units. They also stated that this quantity is significant to decide how much capacity will be suitable to renew the system at the time while the system does not work.

In this thesis, we are interested in how to derive the distribution of power generated by a solar plant taking into consideration the reliability of a PV module. The derived model enables us to estimate the plant power for a given solar irradiation distribution. Also, we can answer the interesting question which is related to the minimum number of PV modules that have to be installed in a plant for the required products with a given level of reliability. From another point of view, a PV system that consists of a large number of PV modules as main components is considered a multistate system. Multistate PV modules in a solar system and a power output model consisting of both linear and quadratic relationships between the solar radiation and the power are combined to evaluate the reliability of the system performance. In such a system, performance levels of PV modules at their states take place in a stochastic setting while the components are assumed different dependency conditions. The system consists of two types of PV modules which are categorized regarding their state probabilities and weights for performance. Thus, we obtain the reliability model for the system which is defined as the probability that the total power of the system is at least a given level, say m, under three cases related to the dependency of the components. We use the PV power output model given by Park et al. (2009) in Section 2.1 for the definition of the states of PV modules in our model. Additionally, we determine the optimum number of PV modules that will be installed on the plants based on the required power generation for a plant. We evaluate an optimization problem to find the optimal number of PV modules at each plant, minimizing the total cost based on the minimum required probability of total power generation of the two plants. Finally, we examine the effects of the changes in several parameter values on the optimal solution by sensitivity analysis.

In chapter two, some important concepts are introduced that will have used throughout the thesis. In the third chapter, the theoretical distribution of power generated by a solar plant is derived considering the PV modules subject to failure. Also, the mean solar plant power is estimated using the derived model. In addition, we focus on the minimum number of PV modules that have to be installed in a plant for the required products with a given level of reliability. In chapter four, the reliability of the performance of the PV system consisting of multistate PV modules is defined under different dependency conditions. The system is considered with two types of PV modules concerning their state probabilities and performances. In chapter five, three real data examples are presented to illustrate the theoretical finding of the thesis. Furthermore, the optimization problem is formulated to obtain the optimal number of PV modules in the solar plants by minimizing the total cost, and sensitivity analysis of the model is presented in 5.3. Chapter six presents the concluding remarks of the thesis.

CHAPTER 2
SOME IMPORTANT CONCEPTS ON PV POWER MODELING

In this chapter, we introduce some important concepts considered in the thesis. First, the power output model that will be used throughout the thesis is presented. Then, the commonly used distribution functions for the solar irradiation data and interested studies are mentioned. Finally, the reliability concepts for the system modeling are considered.

2.1 The PV power output model

In this section, we present the power output model that will be used throughout the thesis. Since solar irradiation is a random variable, the power produced by a PV module is also a random variable. Equation (2.1) represents the relationship between the solar radiation and the power output from a PV module (Park et al. (2009)).

$$P_{PV} = H(g_{bi}) = \begin{cases} P_{sn}\dfrac{g_{bi}^2}{G_s R_c} & , 0 \leq g_{bi} < R_c \\ \\ P_{sn}\dfrac{g_{bi}}{G_s} & , R_c \leq g_{bi} < G_s \\ \\ P_{sn} & , g_{bi} \geq G_s. \end{cases} \tag{2.1}$$

When the solar irradiation is between zero and a certain irradiation point, then the module generates power and there is a quadratic relationship between the solar radiation and the power. When the solar irradiation is between a certain irradiation point and solar irradiation in a standard environment, then the module generates power and there is a linear relationship between the solar radiation and the power. As it is seen in Figure Figure 2.1, if the solar irradiation is greater than the solar irradiation in a standard environment, then the module generates power at a constant rate P_{sn}.

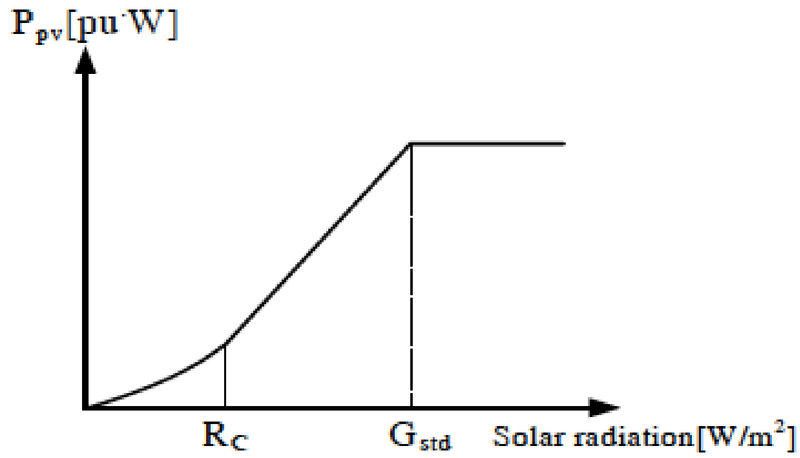

Figure 2.1 Power output of a PV module (Park et al., 2009)

Note that, this model assumes that all PV modules in a system are subject to the same solar irradiation and have the same power curve.

2.2 The probability distribution of the solar irradiation

For the statistical modeling of the solar irradiation data, the beta distribution function is commonly used. Salameh et al. (1995) compared the beta, log-normal and Weibull distributions for 30 years of solar irradiation data in Logan Airport, Boston. They divided the data into two different groups for different terms and observed that the Beta distribution was the best fit for most of the cases. Ettoumi et al. (2002) showed the statistical properties of solar data in a different location in Algeria using beta distributions. Atwa et al. (2009) used the beta distribution of modeling solar data in optimally allocating different types of renewable distributed generation units in the distribution system to minimize annual energy loss. Teng et al. (2012) also used beta distribution for solar irradiation data to study optimal charging/discharging scheduling of battery storage systems for interconnected PV generation systems. Moe & Lin (2018) modeled the solar irradiation data by beta distribution to obtain the PV power output in the Mandalay region. Singh & Fozdar (2020) focused on the double-sided optimum bidding strategy for the energy market considering wind and solar-based power generation. They used a probabilistic approach for modeling the uncertainty and their prediction errors in the cost function. They considered a beta distribution to model the solar radiation data. Yin et al. (2020) showed the effect of solar intermittency on future photovoltaic reliability using beta distribution.

The pdf and the cumulative distribution function (cdf) for beta distribution used throughout the application study are given respectively by

$$f(s;\alpha,\beta)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}s^{\alpha-1}(1-s)^{\beta-1}, \tag{2.2}$$

and

$$F(s;\alpha,\beta)=\frac{B(s;\alpha,\beta)}{B(\alpha,\beta)}=I_s(\alpha,\beta), \tag{2.3}$$

where $0\leq s\leq 1$ is the random solar irradiation (kW/m^2) and $\alpha,\beta\geq 0$ are the shape parameters.

When the solar irradiation is known as distributed by beta distribution in a region, then the average solar irradiation in the region is estimated by the mean formula of beta distribution given by

$$\frac{\hat{\alpha}}{\hat{\alpha}+\hat{\beta}}. \tag{2.4}$$

Weibull distribution can be also considered as an alternative distribution for solar irradiation data. Salameh et al. (1995) showed that the Weibull distribution was the second suitable distribution for solar radiation data. Afzaal et al. (2020) also used the Weibull distribution for the modeling of solar irradiation data in Islamabad, Pakistan. Kam et al. (2021) found that a global solar irradiation model provides an accurate estimation of PV power output considering the sizing of PV installation. They used Weibull distribution to model solar radiation data in different sites in France.

The pdf and the cdf for Weibull distribution used throughout the application study are given respectively by

$$f(s; b, a) = \frac{a}{b} \left(\frac{s}{b}\right)^{a-1} \mathrm{e}^{-(s/b)^a}, \tag{2.5}$$

and

$$F(s; b, a) = 1 - \mathrm{e}^{-(s/b)^a}, \tag{2.6}$$

where $s \geq 0$ is the random solar radiation (kW/m^2) and $a, b \geq 0$ are the shape and scale parameters, respectively.

When the solar irradiation is known as distributed by Weibull distribution in a region, then average solar irradiation in the region is estimated by the mean formula of Weibull distribution given by

$$b\Gamma(1 + \frac{1}{a}). \tag{2.7}$$

2.3 The reliability concepts for the system modeling

In general, reliability is defined as the probability that a system or a component will perform its intended functions satisfactorily for at least a given period under certain conditions. Probability theory has been used to analyze the reliability of the components, besides the reliability of the systems consisting of these components. The reliability of a system is a function of the reliability of its components as a system's performance is usually determined by the performance of its components. Time is a significant factor when defining the reliability of a component. Anybody always wants to know what will be the life of a device that is newly purchased. The lifetime of the devices can be defined as a random variable with a statistical distribution and related properties. The units of measurement for the lifetime may be hours, days, miles, cycles, etc. Let T be the continuous, non-negative random variable representing a device's lifetime with the pdf, $f(t)$ and the cdf, $F(t)$. Thus, the reliability of the device, denoted by $R(t)$, is defined as the probability that the device can last its intended functions beyond a specified period under operating conditions, given by

$$R(t) = P(T > t) = 1 - F(t) = \int_t^\infty f(x)dx.$$

It is clear that $R(0) = 1$ and $R(\infty) = 0$. Also, $R(t)$ is a non-increasing function of t.

The term *system* refers to a collection of components that perform a specific function (Kuo & Zuo (2003)). For example, a PV system can be called a system with the function of converting solar energy into electricity directly without carbon dioxide emission or any other air pollutants. Its major component includes PV modules for absorbing particles of light and emitting electrons and generating an electrical current. The performance of a system depends on the performance of the components of the system. In system reliability analysis, the relationship between components reliability and system reliability are considered. In most cases, the system and each component may be assumed that two possible states, working or failed. Let x_i represent the state of component i for $1 \leq i \leq n$ and

$$x_i = \begin{cases} 1, & \text{if } i\text{-th component works} \\ 0, & \text{if } i\text{-th component fails.} \end{cases} \tag{2.8}$$

Then, $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ is a vector that represents the states of all components. The state of the system is determined by the states of components, denoted by ϕ,

$$\phi = \begin{cases} 1, & \text{if the system works} \\ 0, & \text{if the system fails.} \end{cases} \tag{2.9}$$

The state of the system is a deterministic function of the states of the components and called it structure function of the system, written by

$$\phi = \phi(\mathbf{x}) = \phi(x_1, x_2, \ldots, x_n). \tag{2.10}$$

2.3.1 The k-out-of-n and the multistate k-out-of-n system models

One of the most emphasized systems in reliability problems is the k-out-of-n system model. Some examples of such fault-tolerant systems include the multi-engine system in a plane, the multi-display system in a simulator and the multi-transmitter system in a communication system Kuo & Zuo (2003). The k-out-of-n:G system is defined as the system having n components work (or is *Good*) if and only if at least k of the n components work. The k-out-of-$n : F$ system including n components fails if and only if at least k of the n components fail. Therefore, k-out-of-n:G system can be said equivalent to $(n-k+1)$-out-of-n:F system. The most well-known k-out-of-n systems are parallel and series systems. They are the special cases of the k-out-of-n systems. A parallel system is said to be equivalent to 1-out-of-n:G system and a series system is equivalent to n-out-of-n:G.

In a k-out-of-n:G system with independent and identical components, the number of working components follows the binomial distribution with parameters n and p. Thus, we have

$$P(\text{exactly } i \text{ components work}) = \binom{n}{i} p^i (1-p)^{n-i}, \quad i = 0, 1, \ldots, n. \tag{2.11}$$

The reliability of the system that is defined in previously can be written as:

$$R(k,n) = \sum_{i=k}^{n} \binom{n}{i} p^i (1-p)^{n-i}, \quad i = 0, 1, \ldots, n. \tag{2.12}$$

This model can be used if the components within a system have the same contribution to the performance of the system. So far, we have assumed that a system's components can only exist in one of two states: working or failed. This presumption causes the system's structure function to be a binary function of binary variables. As a result, binary system reliability models are used to describe the appropriate reliability models for systems. Practical reliability engineering has made extensive use of binary system reliability models. However, the binary assumption may not adequately represent all of the potential states that some engineering systems. As we defined earlier in the definition of reliability, to describe the satisfactory performance of a system, we can use more than two levels of performing, for example complete failure, partially working and perfect functioning. In multistate system models, both the system and the components may have more than two states. In other words, the system and its elements may take one of the $M+1$ possible states whose values can be $0, 1, 2, 3, ..., M$. Among these values, 0 represents the "complete failure,", M represents the "perfectly working", and values between 0 and M represent the different levels of operating status. Based on these assumptions, such a statistical model is a multistate discrete system model, since the states of each component and system can be expressed with integers. The assumptions of a multistate system are as follows:

- The state space of each component and system is $\{0, 1, 2, ..., M\}$.
- The states of all components are independent random variables.
- The state of the system is completely determined by the states of the components.
- A low state level indicates the worse or equal performance of the component or system.

Let $X_1, X_2, \ldots, X_n$ be the states of the components in the multistate system. Then the following notations can be seen in the multistate system:

- X_i: The state of the component i,
- $X_i = j$: The component i is in state j,
- $\phi(\mathbf{X})$: The state of the system which is also called as structure function of the system,
- $P(\phi(\mathbf{X}) = j)$: The probability of system is in state j.

It is possible to consider the multistate systems in two categories; simple multistate systems and generalized multistate systems. Simple multistate k-out-of-n systems must have the same system structure at each level. Simple multistate k-out-of-n:G systems must have at least k components in state j or above at any state j, for the system to be in state j or above. The performance of the system is represented by its state distribution. A state distribution function can be given in terms of reliability function, cdf, or pdf. The following notations may be seen in multistate systems:

- $p_{i,j}$: The probability that a component i is in state j,
- p_j: The probability that a component i is in state j when all components are independent and identical,
- $P_{i,j}$: The probability that a component i is in state j or above,
- $Q_{i,j} = 1 - P_{i,j}$: The probability that a component i is in state below j,
- $R_{s,j} = P(\phi(\mathbf{X}) \geq j)$: The probability of the system is in state j or above,
- $Q_{s,j} = 1 - R_{s,j}$: The probability of the system is in state below j,
- $R(k, n; j) = P(\phi(\mathbf{X}) \geq j)$: The probability that the k-out-of-n:G system being in state j or above,

- $r(k, n; j) = P(\phi(\mathbf{X}) = j)$: The probability that the k-out-of-n:G system being in state j.

The simple multistate k-out-of-n:G system model imposes the limit that the system structure at each system level must be the same. At various system levels, a multistate system may actually have different structures. As an example, consider a two-component system with three possible states. The system could be a 1-out-of-2:G structure at level 1; which means that for the system to be in state 1 or above, at least one component must be in state 1 or above. At level 2, it may have a 2-out-of-2:G structure, in which case the system must have at least two components in state 2 or above for it to be in that state. Such a k-out-of-n:G system model is more adaptable for modeling real-world applications. Huang et al. (2000) proposed a definition for the generalized multistate k-out-of-n:G system and presented the algorithm for the reliability evaluation in this model. In the Huang et al. (2000)'s definition; an n-component system is called a generalized multistate k-out-of-n:G system if $\phi(\mathbf{X}) \geq j$ $(1 \leq j \leq M)$ whenever there exists an integer value l $(j \geq l \geq M)$ such that at least k_l components are in state l or above. According to this definition, the structure of the multistate system may be different at different system levels. When k_j is constant, the system structure is the same for all system state levels. In this situation, the system is equal to a simple multistate k-out-of-n:G system. More detailed descriptions of multistate systems can be found in Kuo & Zuo (2003) and Lisnianski & Levitin (2003).

2.3.2 The weighted k-out-of-n system model

Besides, some systems can have components with different performance levels and these components affect the total performance of the system differently. In such a system, it will be more appropriate to characterize the system's reliability using the k-out-of-n model with components that have varying performance levels. One may consider the usual k-out-of-n model and the k-out-of-n model with components that having various performance levels (say weighted components) to better understand the differences between the two models. In the usual k-out-of-n model, the system with n components operates if at least k of n components works. Namely, only the number of working components affects the system's ability to function. On the other hand, k-out-of-n model with weighted components, each components have a weight $w_i > 0$ and the total weight of the components is $w = \sum_{i=1}^{n} w_i$. The system functions if and only if the total weight of working components is at least the specified value k. As k is a weight and different measuring unit, it can exceed n. In the literature, there are many studies related to the system with components having different performance levels. Wu & Chen (1994) proposed the weighted k-out-of-n model as a generalization of the k-out-of-n system and evaluated the reliability of this system by an efficient algorithm. When the weight of each component is one in the weighted k-out-of-n:G system, this system is said to be equivalent to the k-out-of-n:G system. Chen & Yang (2005) extended the weighted k-out-of-n system from one stage to a two-stage model with components in common. Li & Zuo (2008b) developed the notion of the binary-weighted k-out-of-n to multi-state weighted k-out-of-n model and proposed a recursive algorithm for evaluation of the system's reliability. Samaniego & Shaked (2008) discussed the structural properties of systems with weighted components and presented examples of how this system arises in real-world problems. Eryilmaz & Sarikaya (2014) studied binary systems that have two different types of components concerning their weights and reliabilities. Also, they found the optimal values of the number of components in each group under a minimum desired reliability value. Li et al. (2016) investigated the ordering property and reliability performance for the weighted k-out-of-n model with mutually dependent components. Cook (2018) considered weighted k-out-of-n system which has a required demand and determined the reliability of this system. Eryilmaz & Bozbulut (2019) studied the weighted k-out-of-n system having three-state components, namely, complete failure, partial working, and perfect functioning considering time dependency between the component in perfect functioning and partial working states. They also used recursive and non-recursive equations to evaluate the reliability of the system. Also see much more studies concerned with the weighted k-out-of-n model; Higashiyama (2001), Li & Zuo (2008a), Ding et al. (2010), Wang et al. (2012), Eryilmaz & Bozbulut (2014), Eryilmaz (2015), Gao et al. (2018), Zhuang et al. (2019), Sheng & Ke (2020), Mahmoudi et al. (2021) and Zhang (2021).

In this thesis, the reliability of a solar energy system consisting of multistate PV modules can be defined as the probability of the system performing satisfactorily a required level of power generation under specified conditions. If solar radiation exists and PV modules work properly, PV system produces power. Depending on the solar radiation, PV modules may be in different states as working, partially working and not working. Therefore, PV modules can be considered multistate components in terms of the contribution to the performance of the system. Here, the term *performance* may be more appropriate to express the power generated by PV modules. Thus, the reliability for performance of the PV system should be evaluated by taking into account both states of PV modules and the solar radiation random variable. In such a system, considering each PV module has a weight according to its state and the system has a minimum required power level, the reliability for performance of the PV system can be modeled by weighted k-out-of-n system.

CHAPTER 3
THEORETICAL DISTRIBUTION OF SOLAR PLANT POWER

In a power system, modeling the theoretical distribution of the power enables us to obtain all the characteristics and it is significant to make decisions in the planning and installation procedures of the power plant. In the setting of the PV power modeling, Marwali et al. (1998) developed a model representing a relationship between the power output and the solar irradiation. Park et al. (2009) and Choi et al. (2010) used this model for the reliability evaluation of a PV system and detailed how the distribution of solar irradiation is used in the reliability evaluation. These papers in the literature are related to the modeling of the relationship between solar irradiation and power output and how to evaluate the reliability indices. However, the amount of power generated by a PV system depends on both solar irradiation and reliability defined as the probability of PV modules performing their task in a certain period. Therefore, in modeling the power generation of the PV system, it is necessary to consider both solar irradiation, which is a source of randomness, and the working/failure state of the PV module. In this chapter, first, we derive the distribution of the power generated by a solar plant without considering the reliability of PV modules in the system. Then, in Section 3.1, the derivation of the theoretical distribution of solar plant power with PV modules subject to failure is considered. Additionally, we obtain the minimum number of PV modules that have to be installed in a plant for the required products with a given level of reliability.

Let P_T be the total power generated by a plant and G_{bi} be the solar irradiation random variables with the cdf $W_N(p_t)$ and $F_{G_{bi}}(g_{bi})$, respectively. Given the relationship between the solar irradiation and the power output of a PV module in Eq. (2.1), the total power by the plant that consists of N independent and identical PV modules can be defined as follows:

$$P_T = NP_{PV} = NH(g_{bi}). \tag{3.1}$$

Then, using the relation above, we have $P_{PV} = H(g_{bi}) = P_T/N$. For obtaining the cdf of the solar plant power, Eq. 2.1 can be re-expressed by the inverse power function for the defined regions given below:

$$H^{-1}(p_t/N) = \begin{cases} H_1^{-1}(p_t/N) = \left(\dfrac{p_t G_s R_c}{N P_{sn}}\right)^{1/2} & , 0 < p_t < N P_{PV_1}, \\ \\ H_2^{-1}(p_t/N) = \dfrac{p_t G_s}{N P_{sn}} & , N P_{PV_1} \leq p_t < N P_{sn}, \\ \\ \end{cases} \tag{3.2}$$

where $P_{PV_1} = P_{sn}\dfrac{g_{bi}^2}{G_s R_c}$, i.e., P_{PV_1} is the first part in Eq. (2.1). Therefore, using (2.1) and (3.2), the corresponding cdf of the power generated by the solar plant, $W_N(p_t)$, can be defined as the following piece wise function:

$$W_N(p_t) = \begin{cases} F_{G_{bi}}\left(\left(\dfrac{p_t G_s R_c}{N P_{sn}}\right)^{1/2}\right) & , 0 < p_t < N P_{PV_1}, \\ \\ F_{G_{bi}}\left(\dfrac{p_t G_s}{N P_{sn}}\right) - F_{G_{bi}}\left(R_c\right) & , N P_{PV_1} \leq p_t < N P_{sn}, \\ \\ 1 & , p_t = N P_{sn}. \end{cases} \tag{3.3}$$

For the derivation of the pdf of P_T using (3.3), we will need to use the Dirac delta function $\delta(.)$ to address the probabilities of solar power being exactly equal to $N P_{sn}$ (see for example, Eryilmaz & Devrim (2019)). For other values of P_T, the traditional change of variables method will be used. The corresponding pdf of P_T when $0 \leq p_t \leq N P_{sn}$ is defined as follows:

$$\begin{aligned} w_N(p_t) &= f_{G_{bi}}(H_1^{-1}(p_t/N))\frac{dH_1^{-1}(p_t/N)}{dp_t} + f_{G_{bi}}(H_2^{-1}(p_t/N))\frac{dH_2^{-1}(p_t/N)}{dp_t} \\ &+ (1 - F_{G_{bi}}(G_s))\delta(p_t - NP_{sn}), \end{aligned} \tag{3.4}$$

where $f_{G_{bi}}(.)$ is the pdf of the solar irradiation evaluated at $H_j^{-1}(p_t/N)$, $j = 1, 2$.

Notice that, we have

$$\frac{dH_1^{-1}(p_t/N)}{dp_t} = \frac{G_s R_c}{NP_{sn}} \left(\frac{p_T G_s R_c}{NP_{sn}}\right)^{-1/2} \tag{3.5}$$

and

$$\frac{dH_2^{-1}(p_t/N)}{dp_t} = \frac{G_s}{NP_{sn}}. \tag{3.6}$$

3.1 Theoretical distribution of solar plant power with PV modules subject to failure

The main purpose of this section is to derive the distribution of the power generated by a solar plant considering the reliability of PV modules. The total power of the solar plant is defined as $P_T = H(g_{bi}, \mathbf{X})$, where G_{bi} is the solar irradiation random variable and $\mathbf{X} = (X_1, X_2, \ldots, X_N)$ is a vector, which defines the state of PV modules, failure or working. Suppose that during a fixed period of time, i-th PV module works with probability p, that is $P\{X_i = 1\} = p$, $i = 1, \ldots, N$. Thus, the parameter p indicates the reliability of the PV module. Let us consider, a solar plant that consists of N independent and identical PV module is installed. Then the cumulative distribution function of the power generated by the solar plant can be calculated by:

$$W_N(p_t;p) = P\{P_T \le p_t\} = \sum_{i=0}^{N} \binom{N}{i} p^i (1-p)^{(N-i)} W_i(p_t), \qquad (3.7)$$

where

$$W_i(p_t) = \begin{cases} 0 & , p_t < 0 \\ K_{1,i}(p_t) & , 0 \le p_t < iP_{PV_1} \\ K_{2,i}(p_t) & , iP_{PV_1} \le p_t < iP_{sn} \\ 1 & , p_t \ge iP_{sn} \end{cases} \qquad (3.8)$$

with the functions defined from Eq. 3.3

$$K_{1,i}(p_t) = F_{G_{bi}}\left(\left(\frac{p_t G_s R_c}{iP_{sn}}\right)^{1/2}\right), \; K_{2,i}(p_t) = F_{G_{bi}}\left(\frac{p_t G_s}{iP_{sn}}\right) - F_{G_{bi}}(R_c). \qquad (3.9)$$

It is obvious that $W_0(p_t) = 0$ if $p_t < 0$ and $W_0(p_t) = 1$ if $p_t \ge 0$. Other details can be found in the Appendix given at the end.

For simplicity, the cdf of the power output by a single PV module can be written as follows:

$$W_1(p_t;p) = \begin{cases} 0 & , p_t < 0 \\ (1-p) + pK_{1,1}(p_t) & , 0 \le p_t < P_{PV_1} \\ (1-p) + pK_{2,1}(p_t) & , P_{PV_1} \le p_t < P_{sn} \\ 1 & , p_t \ge P_{sn}. \end{cases} \qquad (3.10)$$

Then the corresponding pdf which is obtained using (3.10) is given as below:

$$w_1(p_t;p) = pk_{1,1}(p_t) + pk_{2,1}(p_t) + w_1(0;p)\delta(p_t) + w_1(P_{sn};p)\delta(p_t - P_{sn}), \quad (3.11)$$

where $\delta(.)$ is the Dirac delta function, $k_{1,1}(p_t)$ and $k_{2,1}(p_t)$ are pdfs of $K_{1,1}(p_t)$ and $K_{2,1}(p_t)$, respectively. In equation (3.11), $w_1(0;p)$ defines the probability that no power is generated by the module. This case arises only when modules do not work. So, $w_1(0;p) = 1 - p$. Thus, (3.11) becomes

$$w_1(p_t;p) = pk_{1,1}(p_t) + pk_{2,1}(p_t) + (1-p)\delta(p_t) + p[1 - F_{G_{bi}}(G_s)]\delta(p_t - P_{sn}). \quad (3.12)$$

3.2 Mean power estimation and determination of the number of PV modules

Based on (3.12), the mean power generated by a single PV module is found as:

$$\mu_0 = p\left[\int_0^{P_{PV_1}} p_t k_{1,1}(p_t)dp_t + \int_{P_{PV_1}}^{P_{sn}} p_t k_{2,1}(p_t)dp_t + P_{sn}(1 - F_{G_{bi}}(G_s))\right]. \quad (3.13)$$

Thus, the mean power generated by the solar plant that consists of N identical PV modules is computed by $\mu_T = N\mu_0$.

3.3 Numerical Example

In this section, we present an application study using the information in Devrim & Eryilmaz (2021) which has evaluated the performance of a hybrid energy system that consists of a specified number of wind turbines and solar modules. They evaluated the performance of the hybrid system using the wind and solar data collected by the

Turkish State Meteorological Service (http://www.mgm.gov.tr) in Canakkale, Türkiye. Global total solar irradiation data (kwh/m^2) are measured by the meteorology station during daylight hours and its values (global total solar irradiation data/day length) were used considering only the daylight hours. For the application for our findings, we used the solar irradiation data for a month, March which has the beta distribution with the parameters $\alpha = 3.35600$ and $\beta = 2.66353$ estimated by the method of maximum likelihood Devrim & Eryilmaz (2021). The corresponding pdf and the cdf for beta distribution are given respectively by

$$f(s) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} s^{\alpha-1}(1-s)^{\beta-1}, \tag{3.14}$$

and

$$F(s) = \frac{B(s;\alpha,\beta)}{B(\alpha,\beta)} = I_s(\alpha,\beta), \tag{3.15}$$

where $0 \leq s \leq 1$ is the random solar irradiation (kW/m^2) and $\alpha, \beta \geq 0$ are the shape parameters.

All computations have been done using Mathematica v.11.3. Our assumptions throughout this section are as follows: the rated capacity of the PV module is $P_{sn} = 280\ W$, the solar irradiation in a standard environment is $G_s = 1000\ W/m^2$, and the certain irradiation point is $R_c = 150\ W/m^2$. Then, the cdf of the power generated by a single PV module can be expressed using (3.10):

$$W_1(p_t;p) = \begin{cases} 0 & , p_t < 0 \\ (1-p) + pK_{1,1}(p_t) & , 0 \leq p_t < 42 \\ (1-p) + pK_{2,1}(p_t) & , 42 \leq p_t < 280 \\ 1 & , p_t \geq 280. \end{cases} \tag{3.16}$$

Now, we can estimate the various characteristics of power by a PV module. For example, when we assume that the PV module is fully reliable (i.e. $p = 1$), the mean power generated by a single PV module can be computed using (3.13) as $\mu_0 = 156.03\ W$. When the PV module is subject to failure and its reliability is $p = 0.85$, then the mean power generated by a single PV module is computed as $\mu_0 = 132.626\ W$. Thus the mean power generated by the solar plant with identical $N = 10$ modules will be $\mu_T = N\mu_0 = 1326.26\ W$.

For the solar plant with N modules, we have obtained the cumulative probabilities for the power random variable. Table 3.1 gives the values of the cdf of power generated by a solar plant with N modules, $W_N(p_t; p)$ using (3.7) when the reliability of a PV module is $p = 0.95$. One can see from Table 3.1 that the probability of producing power at most $350\ W$ is 0.641710 in a solar plant that consists of two PV modules while the same probability is 0.061191 in a plant with five modules. Note that it also provides information about the reliability of the system (1-$W_N(p_t; p)$) which is the probability of providing the desired power level. This means that for the previous case that the probability of producing power more than $350\ W$ is 0.35829 (from $1 - 0.641710$) in a solar plant that consists of two PV modules while the same probability is 0.938809 (from $1 - 0.061191$) in a plant with five modules. It is clear that the reliability of the system to provide a certain power level increases as the number of PV modules increases.

In Figure 3.1, we plot $W_N(p_t; p)$ of the solar plant power (in Kw) when $N = 3$.

Table 3.1 Cumulative probabilities for the solar plant power when $p = 0.95$

$p_t\ (W)$	$N = 2$	$N = 3$	$N = 5$
50	0.008145	0.003294	0.001362
150	0.096887	0.018698	0.007851
280	0.430314	0.160665	0.025877
350	0.641710	0.286363	0.061191
450	0.898995	0.499691	0.136207
550	0.988842	0.705274	0.519548

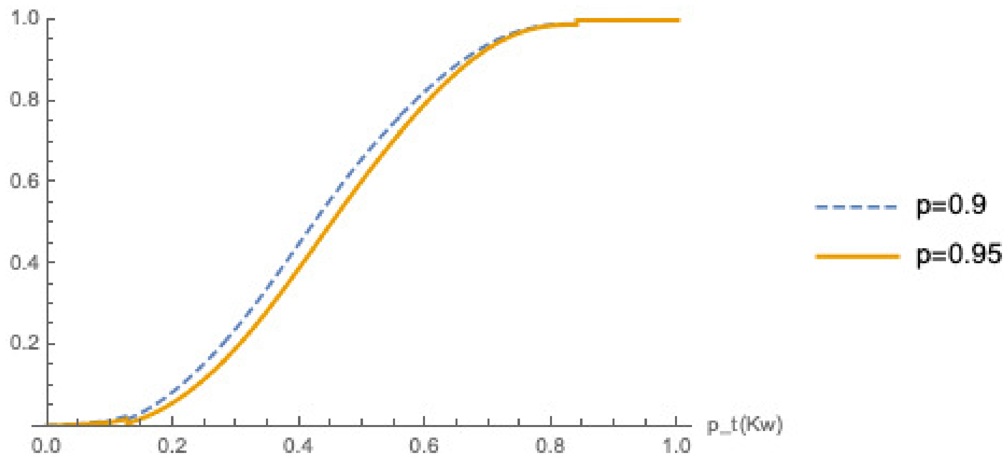

Figure 3.1 The cdf of the solar plant power when $N = 3$

3.4 Determination of the number of PV modules

Next, we consider the optimal number of PV modules for a solar plant subject to failure. We assume that the probability distribution is known for solar irradiation, hence we can answer the question about the minimum number of PV modules that have to be installed in a plant to generate the desired production p_{t_0} with a given threshold reliability value r_0. Now, it is enough to compute $W_N(p_{t_0}; p)$ to get the number of modules, N, considering the probability that

$$1 - W_N(p_{t_0}; p) = P\{P_T > p_{t_0}\} \geq r_0. \tag{3.17}$$

As shown previously from Table Table 3.1, when the desired level of power is $p_{t_0} = 150\ W$ with a given threshold reliability value $r_0 = 0.90$, the minimum number of PV modules that has to be installed in the plant can be obtained as $N = 2$.

As another way, we can consider the mean power generated by the plant in Eq. (3.13) to determine the minimum number of PV modules that has to be installed in the plant. The following inequality leads to answer the question considering the predetermined mean power generated by the plant, m_0:

$$\mu_T \geq m_0. \tag{3.18}$$

Recall that, when we assume the reliability of a module is $p = 0.85$, the mean power generated by the solar plant with identical $N = 10$ modules is $\mu_T = N\mu_0 = 1326.26\ W$. Now, if we suppose that the minimum level of mean power is $m_0 = 1300\ W$, then we can conclude that the minimum 10 PV modules have to be installed in the plant.

CHAPTER 4
RELIABILITY MODELING OF THE SOLAR PLANT POWER WITH MULTI-STATE PV MODULES

PV systems can be considered as multi-state systems. The system is consisting of a large number of PV modules as main components. PV modules are multi-state components in terms of their contribution to the performance of the system. Because, they can be in a specific state (working, partially working, and not working) depending on the solar radiation. Here, the term *performance* represents the power generated by PV modules. In such a system, the probability that the total power of the system is at least a given level, say m, can be defined as the reliability of the system.

4.1 Definitions

Consider a PV system that consists of $n = n_1 + n_2$ multistate components (PV modules) classified into two categories on the basis of their performance levels and state probabilities. According to the model in Eq. (2.1), we can define the states for each PV module as a complete failure (state 1), partial working (state 2, state 3), and perfect functioning (state 4) which are given in the followings:

State 1 : The PV module generates no power, i.e. there is no solar radiation or the PV module is broken down.

State 2 : The PV module generates power at a rate depending on the solar radiation in the interval $[0, R_c)$.

State 3: The PV module generates power at a rate depending on the solar radiation in the interval $[R_c, G_s)$.

State 4: The PV module generates power at the rated capacity.

Note that, the model assumes that all PV modules in a system are subject to the same solar radiation and have the same power curve. However, PV modules consisting of Type I and Type II have different performances as weights t_j and t_j^* according to their state, respectively. In this study we assume PV system with the total performance as a weight of components of Type I can be defined by the random variable

$$T_1 = \sum_{i=1}^{n_1} \sum_{j=1}^{4} t_j I(X_i = j), \tag{4.1}$$

where X_i represents the state of the i-th component and $I(E) = 1$, if E occurs, and $I(E) = 0$, if else. In a similar way, the total performance of the components for Type II is written by

$$T_2 = \sum_{i=1}^{n_2} \sum_{j=1}^{4} t_j^* I(X_i^* = j), \tag{4.2}$$

where X_i^* represents the state of the i-th component for Type II. Note that, the power is not generated when the components are in a complete failure state, $t_1 = t_1^* = 0$.

In this study, our objective is to obtain the reliability model as the probability that the PV system's performance is at least m threshold value, i.e. $P\{T_1 + T_2 \geq m\}$, using weighted k-out-of-n system. Reliability models are obtained under the three specified cases as explained below:

- Case A: All components are considered to be dependent.
- Case B: There are dependent components of the same type, but different types are independent.
- Case C: All components are considered to be independent.

4.2 Reliability models

In this section, we present the reliability models for multistate PV systems under the considerations of the defined cases above. As mentioned in the previous section, when PV modules work properly and solar radiation exists, the PV system produces power. Also, in our model, PV modules work in different states depending on solar radiation and this property affects the performance of the system. Therefore, we have to pay attention to the probabilities of being in different states, when we evaluate the reliability of the system.

Now, suppose a PV module is working correctly with probability p and it is down for a fixed period of time with probability $1-p$. Then the probabilities of being in states 1, 2, 3, and 4 for PV modules depending on the solar radiation can be computed by the followings, respectively:

$$\begin{aligned} p_1 &= 1-p, \\ p_2 &= pP\{0 \leq g_{bi} < R_c\} = p[F_{G_{bi}}(R_c)], \\ p_3 &= pP\{R_c \leq g_{bi} < G_s\} = p[F_{G_{bi}}(G_s) - F_{G_{bi}}(R_c))], \\ p_4 &= pP\{g_{bi} \geq G_s\} = p[1 - F_{G_{bi}}(G_s)]. \end{aligned} \tag{4.3}$$

Then, for Case A, the total performances of Type I and Type II components can be written as

$$T_1 = \sum_{j=1}^{4} t_j S_j^{(1)} \; and \; T_2 = \sum_{j=1}^{4} t_j^* S_j^{(2)}, \tag{4.4}$$

where $S_j^{(1)} = \sum_{i=1}^{n_1} I(X_i = j)$ and $S_j^{(2)} = \sum_{i=1}^{n_2} I(X_i^* = j)$, $j = 1, 2, 3, 4$. The joint probability mass function $P^A_{n_1,n_2}(c_2, c_3, c_4, d_2, d_3, d_4)$ of the random variables $S_j^{(1)}$ and $S_j^{(2)}$ can be written as follows:

$$P\{S_2^{(1)} = c_2, S_3^{(1)} = c_3, S_4^{(1)} = c_4, S_2^{(2)} = d_2, S_3^{(2)} = d_3, S_4^{(2)} = d_4\}$$

$$\begin{aligned} &= \binom{n_1}{c_2}\binom{n_1-c_2}{c_3}\binom{n_1-c_2-c_3}{c_4}\binom{n_2}{d_2}\binom{n_2-d_2}{d_3}\binom{n_2-d_2-d_3}{d_4} \\ &\quad P\{X_1 = 2, \ldots, X_{c_2} = 2, X_{c_2+1} = 3, \ldots, X_{c_2+c_3} = 3, \\ &\quad X_{c_2+c_3+1} = 4, \ldots, X_{c_2+c_3+c_4} = 4, X_{c_2+c_3+c_4+1} = 1, \ldots, X_{n_1} = 1, \\ &\quad X_1^* = 2, \ldots, X_{d_2}^* = 2, X_{d_2+1}^* = 3, \ldots, X_{d_2+d_3}^* = 3, X_{d_2+d_3+1}^* = 4, \ldots, \\ &\quad X_{d_2+d_3+d_4}^* = 4, X_{d_2+d_3+d_4+1}^* = 1, \ldots, X_{n_2}^* = 1\} \end{aligned} \tag{4.5}$$

for $0 \leq c_2 + c_3 + c_4 \leq n_1$ and $0 \leq d_2 + d_3 + d_4 \leq n_2$. Note that, since $S_2^{(1)} = c_2, S_3^{(1)} = c_3, S_4^{(1)} = c_4, S_2^{(2)} = d_2, S_3^{(2)} = d_3, S_4^{(2)} = d_4$ implies $S_1^{(1)} = n_1 - c_2 - c_3 - c_4$, $S_1^{(2)} = n_2 - d_2 - d_3 - d_4$ with probability 1, we do not need to write $S_1^{(1)}$ and $S_1^{(2)}$ in the left hand side of the Eq. 4.5. However, $S_1^{(1)}$ and $S_1^{(2)}$ are considered on the right-hand side of the equation. Thus, in the case of all components being dependent, the reliability of the PV system for a given level m is

$$\begin{aligned} &R_{n_1,n_2}^A(m) \\ &= P\{T_1 + T_2 \geq m\} \\ &= \sum_{\substack{c_2=0 \\ t_2c_2+t_3c_3+t_4c_4+t_2^*d_2+t_3^*d_3+t_4^*d_4 \geq m}}^{n_1} \sum_{c_3=0}^{n_1-c_2} \sum_{c_4=0}^{n_1-c_2-c_3} \sum_{d_2=0}^{n_2} \sum_{d_3=0}^{n_2-d_2} \sum_{d_4=0}^{n_2-d_2-d_3} P_{n_1,n_2}^A(c_2, c_3, c_4, d_2, d_3, d_4) \end{aligned} \tag{4.6}$$

For Case B, when different types of components are independent and there are dependent components of the same type, the joint probability mass function can be defined as

$P^B_{n_1,n_2}(c_2, c_3, c_4, d_2, d_3, d_4)$

$$= \binom{n_1}{c_2}\binom{n_1-c_2}{c_3}\binom{n_1-c_2-c_3}{c_4}\binom{n_2}{d_2}\binom{n_2-d_2}{d_3}\binom{n_2-d_2-d_3}{d_4}$$
$$P\{X_1 = 2, \ldots, X_{c_2} = 2, X_{c_2+1} = 3, \ldots, X_{c_2+c_3} = 3,$$
$$X_{c_2+c_3+1} = 4, \ldots, X_{c_2+c_3+c_4} = 4, X_{c_2+c_3+c_4+1} = 1, \ldots, X_{n_1} = 1\}$$
$$P\{X^*_1 = 2, \ldots, X^*_{d_2} = 2, X^*_{d_2+1} = 3, \ldots, X^*_{d_2+d_3} = 3, X^*_{d_2+d_3+1} = 4, \ldots,$$
$$X^*_{d_2+d_3+d_4} = 4, X^*_{d_2+d_3+d_4+1} = 1, \ldots, X^*_{n_2} = 1\} \quad (4.7)$$

The probability that the performance of the system is at least a given level m is

$R^B_{n_1,n_2}(m)$

$$= P\{T_1 + T_2 \geq m\}$$
$$= \sum_{\substack{c_2=0 \\ t_2c_2+t_3c_3+t_4c_4+t^*_2d_2+t^*_3d_3+t^*_4d_4 \geq m}}^{n_1} \sum_{c_3=0}^{n_1-c_2} \sum_{c_4=0}^{n_1-c_2-c_3} \sum_{d_2=0}^{n_2} \sum_{d_3=0}^{n_2-d_2} \sum_{d_4=0}^{n_2-d_2-d_3} P^B_{n_1,n_2}(c_2, c_3, c_4, d_2, d_3, d_4) \quad (4.8)$$

For the case C that is all components are independent, the joint probability mass function becomes

$P^C_{n_1,n_2}(c_2, c_3, c_4, d_2, d_3, d_4)$

$$= \binom{n_1}{c_2}\binom{n_1-c_2}{c_3}\binom{n_1-c_2-c_3}{c_4}\binom{n_2}{d_2}\binom{n_2-d_2}{d_3}\binom{n_2-d_2-d_3}{d_4}$$
$$p_2^{c_2} p_3^{c_3} p_4^{c_4} (1 - p_2 - p_3 - p_4)^{n_1-c_2-c_3-c_4}$$
$$(p^*_2)^{d_2} (p^*_3)^{d_3} (p^*_4)^{d_4} (1 - p^*_2 - p^*_3 - p^*_4)^{n_2-d_2-d_3-d_4} \quad (4.9)$$

and the reliability is

$$
\begin{aligned}
&R^C_{n_1,n_2}(m)\\
&\quad = P\{T_1+T_2\geq m\}\\
&\quad = \sum_{\substack{c_2=0 \\ t_2c_2+t_3c_3+t_4c_4+t_2^*d_2+t_3^*d_3+t_4^*d_4\geq m}}^{n_1}\sum_{c_3=0}^{n_1-c_2}\sum_{c_4=0}^{n_1-c_2-c_3}\sum_{d_2=0}^{n_2}\sum_{d_3=0}^{n_2-d_2}\sum_{d_4=0}^{n_2-d_2-d_3} P^C_{n_1,n_2}(c_2,c_3,c_4,d_2,d_3,d_4) \qquad (4.10)
\end{aligned}
$$

In order to make the probabilities clear, additional explanations are required for all cases given above. Recall that if a PV module operates right depending on solar radiation, then it generates power. When t_j denotes the PV module power produced in the states $j=1,2,3,4$, then it is obvious that $t_1=0$ and $t_4=P_{sn}$. Indeed, t_2 and t_3 indicate the mean power generated by a PV module when the solar radiation is in the interval $[0,R_c)$ and $[R_c,G_s)$, respectively. For state 2, the power can be written as

$$
t_2=\int_0^{R_c} P_{sn}\frac{g_{bi}^2}{G_sR_c}k_1(g_{bi})dg_{bi}, \qquad (4.11)
$$

where k_1 is the truncated pdf of the solar radiation on $(0,R_c)$ that is $k_1(g_{bi})=\dfrac{f(g_{bi})}{F(R_c)}$ and $k_1(g_{bi})=0$, otherwise. And for state 3, it can be written as

$$
t_3=\int_{R_c}^{G_s} P_{sn}\frac{g_{bi}}{G_s}k_2(g_{bi})dg_{bi}, \qquad (4.12)
$$

where k_2 is the truncated pdf of the solar radiation on (R_c,G_s) that is $k_2(g_{bi})=\dfrac{f(g_{bi})}{F(G_s)-F(R_c)}$ and $k_2(g_{bi})=0$, otherwise. The same formulas are used for t_j^*.

4.3 Numerical Example

Weibull distribution is also considered by Salameh et al. (1995) for solar radiation data. Thus, the results of this paper are also relevant to a region where the probability distribution of solar radiation is different from beta. The pdf and the cdf for Weibull distribution are given respectively by

$$f(s;\alpha,\beta) = \frac{\alpha}{\beta}\left(\frac{s}{\beta}\right)^{\alpha-1} \mathrm{e}^{-(s/\beta)^{\alpha}}, \tag{4.13}$$

and

$$F(s;\alpha,\beta) = 1 - \mathrm{e}^{-(s/\beta)^{\alpha}}, \tag{4.14}$$

where $s \geq 0$ is the random solar radiation (kW/m^2) and $\alpha, \beta \geq 0$ are the shape and scale parameters, respectively.

Our assumptions for this example are as follows: the rated capacities of the Type I and Type II PV modules are $P_{sn} = 25, 50\ W$, respectively. Also, the solar radiation in a standard environment is $G_s = 1000\ W/m^2$, and the certain irradiation point is $R_c = 150\ W/m^2$ for both types of PV modules. Suppose that all PV modules work properly with probability $p = 0.95$ for a set amount of time and that plant 1 consists of $n_1 = 3$ PV modules and plant 2 consists of $n_2 = 3$ PV modules. In addition, we assume that the solar radiation distributions in the regions of plants 1 and 2 are $Weibull(4.2, 1.3)$ and $Weibull(4.5, 0.8)$, respectively. Under these assumptions, the corresponding state probabilities for two types of PV modules are obtained by Eq. (4.3) given in Table 4.1.

Table 4.1 The corresponding state probabilities for PV modules when $p = 0.95$ under Weibull distribution

PV module	p_1	p_2	p_3	p_4
Type I	0.0500	0.0001	0.2684	0.6815
Type II	0.0500	0.0005	0.8875	0.0620

In Table 4.2, the performances of each type of PV modules are given using equations (4.11) and (4.12).

Table 4.2 Estimated performances for each state of PV modules (kW) when $p = 0.95$ under Weibull distribution

PV module	t_1	t_2	t_3	t_4
Type I	0.0000	0.0025	0.0198	0.0250
Type II	0.0000	0.0052	0.0353	0.0500

In Table 4.3, $R_{n_1,n_2}(m)$ for different m (W/m^2) values are given using Eq. 4.10. The reliability of a system under these conditions is calculated around 0.97 where m is 130. As one can see, the reliability value decreases when the threshold m value increased.

Table 4.3 Reliability values for the performance of PV system when $p = 0.95$ under Weibull distribution

m	80	90	100	130
$R(m)$	0.9997	0.9989	0.9979	0.9737
m	150	170	180	200
$R(m)$	0.8571	0.7320	0.3570	0.0088

For simplicity, we assumed that all PV modules are independent in above calculations and therefore we have used Eq. (4.10). If there is a dependency between PV modules, then the reliability equations in Case A and B should be used to compute reliability.

CHAPTER 5
REAL DATA APPLICATIONS

In this section, we give three real data examples for the illustration of theoretical findings in the previous Section 3.1 and Chapter 4.

Beta and Weibull distribution given in 2.2 are used for the solar radiation data in first and second example, respectively. Finally, the computations have been made using R v.1.2.5033 (data preparation) and Mathematica v.11.3 (calculations).

In order to make the analyzes, we used the data which were obtained from the PVGIS interactive tool (https://ec.europa.eu/jrc/en/pvgis) for Dokuz Eylul University (DEU), Faculty of Science ($38.367°$ Latitude, $27.202°$ Longitude) and Dokuz Eylul University (DEU), Faculty of Medicine ($38.396°$ Latitude, $27.030°$ Longitude), Izmir, Türkiye. PVGIS data is sourced from high-quality measuring sensors and publicly available for a long time period. The data planned to be used in the analysis are typical meteorological year (TMY) data of solar irradiation for certain months of the year, covering the period (2007-2016) for the given location. In this tool, global horizontal solar irradiation data (W/m^2) is measured during daylight hours. Since solar irradiation data is dependent on daylight hours, the solar irradiation data is divided by the day length to get the data more accurately, i.e., daily solar irradiation =(solar irradiation)/day length.

5.1 Solar Irradiation Data for the Faculty of Science

Let us consider a scenario that a decision maker wants to install a solar plant to supply the electric power for a specific location. In this case, the amount of solar irradiation in the region where the solar power plant is planned to be installed should be considered and the planning should be done accordingly. Since solar irradiation is a random variable, the probability distribution of solar irradiation data and some of its properties must be determined. Then, considering of the theoretical distribution of solar plant power taking into account the characteristics of PV modules subjected to failure will provide us more accurate results. We can also calculate the mean power produced by that solar plant and determine the minimum number of PV modules that should be installed with the probability of providing the least desired power level.

For this purpose, we used the theoretical findings in Section 2.12 and the data were obtained from the PVGIS interactive tool (https://ec.europa.eu/jrc/en/pvgis) for Dokuz Eylul University (DEU), Faculty of Science, Izmir, Türkiye (38.367° Latitude, 27.202° Longitude) as seen in Figure 5.1.

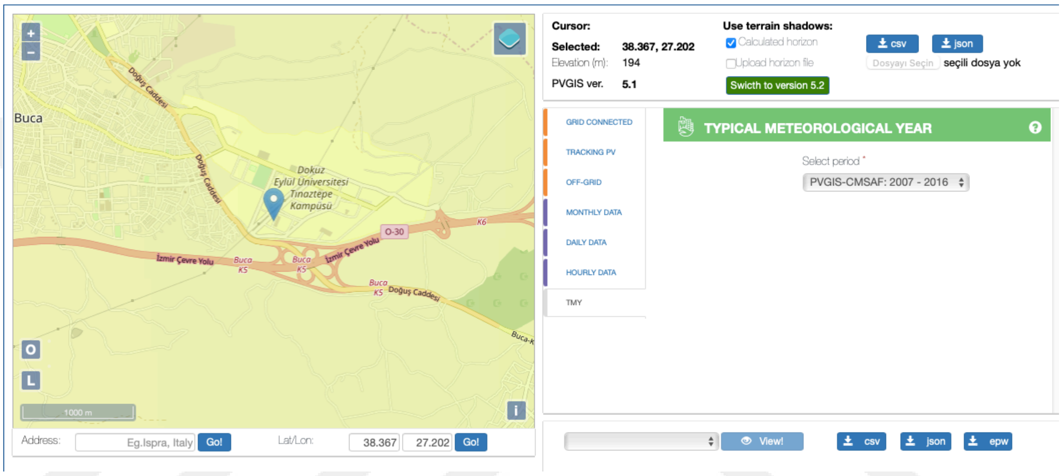

Figure 5.1 The coordinates of data for DEU, Faculty of Science

The data planned to be used in the analysis are typical meteorological year (TMY) data of solar irradiation for certain months of the year, covering the period (2007-2016) for the given location. The histogram of the daily average solar irradiation (kW/m^2) at Faculty of Science, DEU is shown in Figure 5.2.

We used beta distribution with the shape parameters $\alpha, \beta \geq 0$. Maximum likelihood estimates of the α and β parameters, Anderson-Darling (AD) goodness-of-fit test results and estimated mean solar irradiation are presented in Table 5.1. As seen in Table 5.1, all p-values are greater than the 0.05 significance level. Thus, the solar irradiation data for each month can be modeled with the beta distribution with $\hat{\alpha}$ and $\hat{\beta}$. The mean solar irradiation have been estimated by the formula $\frac{\hat{\alpha}}{\hat{\alpha}+\hat{\beta}}$.

Our assumptions for this application study are as follows: the rated capacity of the

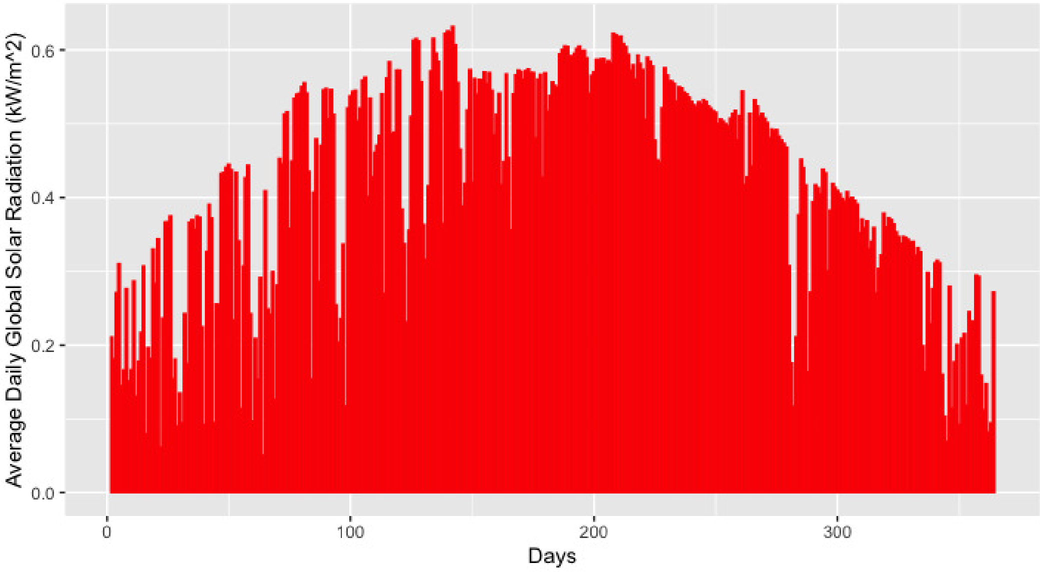

Figure 5.2 The average daily solar irradiation in the Faculty of Science, DEU

Table 5.1 Point estimates of parameters of the solar irradiation distribution and the p-values of AD test

Month	$\hat{\alpha}$	$\hat{\beta}$	Mean solar irradiation (kW/m^2)	p-value
January	4.20460	15.08123	0.218015	0.4269478
February	4.75891	10.16528	0.318872	0.5377671
March	2.93630	4.95088	0.372288	0.4880588
April	6.31650	7.49185	0.457441	0.5404154
May	9.31303	9.24445	0.501848	0.0545316
June	41.28674	36.67319	0.529589	0.7901044
July	286.60568	200.63740	0.588219	0.2779499
August	92.71945	76.53802	0.547801	0.2227270
September	150.33816	148.14115	0.503680	0.2008657
October	7.69043	12.56987	0.379581	0.1584497
November	101.04814	186.60547	0.351284	0.9247952
December	4.74844	18.63601	0.203060	0.5894238

PV module is $P_{sn} = 280\ W$, the solar irradiation in a standard environment is $G_s = 1000\ W/m^2$, and the certain irradiation point is $R_c = 150\ W/m^2$. Then, from (3.8) and (3.9), the cdf of the power generated by a single PV module can be written as:

$$W_1(p_t; p) = \begin{cases} 0 & , p_t < 0 \\ (1-p) + pK_{1,1}(p_t) & , 0 \leq p_t < 42 \\ (1-p) + pK_{2,1}(p_t) & , 42 \leq p_t < 280 \\ 1 & , p_t \geq 280. \end{cases} \tag{5.1}$$

In Table 5.2, the mean power generated by a single PV module for each month is given using Eq.(3.13) for two different working probability values of PV modules. Suppose that we are interested in the estimation of power generation for one of the hottest months in Izmir, Türkiye. The highest value of mean power generated by a single PV module is $164.701\ W$ on any day in July when a PV module is fully reliable i.e. $p = 1$. When the reliability of a module is $p = 0.90$, then the mean power generated by a single PV module is $148.231\ W$. Thus, the mean power generated by the solar plant with identical $N = 10$ modules will be $\mu_T = N\mu_0 = 1482.31\ W$ on any day in July.

Table 5.2 The mean power generated by a single PV module (kW) for Faculty of Science

Month	$\mu_0(p = 1)$	$\mu_0(p = 0.90)$
January	0.059384	0.053446
February	0.088923	0.080031
March	0.103718	0.093346
April	0.128064	0.115258
May	0.140517	0.126465
June	0.148285	0.133456
July	0.164701	0.148231
August	0.153384	0.138046
September	0.141030	0.126927
October	0.106252	0.095626
November	0.098360	0.088524
December	0.055004	0.049503

In Table 5.3, we compute the reliability values which are the probabilities that $N = 2, 3, 5$ PV modules produce power (W) more than the desired level p_t for any day in each month of the year. Notice that the cumulative probabilities for the solar plant power have been given in Table 3.1 as associated with this example. Now, Table 5.3 gives directly the values of $P\{P_T > p_t\} = 1 - P\{P_T \leq p_t\}$ computed using (3.7) when $p = 0.95$. One can see from Table 5.3 that the probability of producing power more than $150\ W$ is 0.49587 in the solar plant that consists of two PV modules on any day of January while the probability of the same event is 0.73553 on any day of March. Since the cdf value for $450\ W$ with two modules is equal to 1 for any day of July, the reliability value is zero and it is understood that the plant is producing power at full capacity. As expected, the reliability values are decreasing in the same number of PV modules when the level of producing power is increasing. Also, the reliability values are increasing at the same level of producing power when the number of PV modules is increasing for each month.

Table 5.3 Reliability values for the solar plant power in the Faculty of Science when $p = 0.95$

Month	$p_t\ (W)$	$N = 2$	$N = 3$	$N = 5$
January	50	0.87279773	0.92324444	0.96629137
	150	0.49587177	0.82566194	0.82773494
	280	0.22907338	0.34656114	0.73388012
	350	0.22515840	0.27121867	0.53995857
	450	0.22505981	0.24907878	0.36253593
	550	0.22505980	0.24759656	0.28379956
February	50	0.97129164	0.98707621	0.99559376
	150	0.64424454	0.91049890	0.96232469
	280	0.12108303	0.43701995	0.86328652
	350	0.06275846	0.23621126	0.70708014
	450	0.05575708	0.09739165	0.47035251
	550	0.05573480	0.06464965	0.27799325
March	50	0.95697446	0.97355519	0.98676550
	150	0.73553015	0.93111334	0.94538303
	280	0.27171928	0.59046444	0.89732560
	350	0.13643081	0.41172340	0.79018023
	450	0.07431179	0.22348795	0.62197153

Table 5.3 Continues

Month	p_t (W)	$N=2$	$N=3$	$N=5$
	550	0.07040449	0.12199851	0.45957904
April	50	0.99605601	0.99943995	0.99991146
	150	0.86845085	0.98045227	0.99793181
	280	0.33944666	0.75898485	0.97497138
	350	0.09821872	0.54307249	0.92610750
	450	0.00583100	0.24311000	0.79742204
	550	0.00361867	0.06277706	0.61124620
May	50	0.99741199	0.99986243	0.99999884
	150	0.92379620	0.99261178	0.99987423
	280	0.45731652	0.86609277	0.99428587
	350	0.12997272	0.67748134	0.97624138
	450	0.00207525	0.33213196	0.90265555
	550	0.00020411	0.07853838	0.75050012
June	50	0.99750000	0.99987500	0.99999969
	150	0.94608101	0.99601854	0.99998362
	280	0.63208767	0.95202323	0.99962785
	350	0.03963173	0.84397541	0.99840004
	450	0.00000003	0.39332311	0.98483311
	550	0.00000000	0.01014775	0.92152045
July	50	0.99750000	0.99987500	0.99999969
	150	0.99657451	0.99980559	0.99999940
	280	0.90245804	0.99274371	0.99996995
	350	0.04366631	0.86392495	0.99889646
	450	0.00000000	0.84902249	0.99863306
	550	0.00000000	0.00100809	0.97743097
August	50	0.99750000	0.99987500	0.99999969
	150	0.96192262	0.99720670	0.99998857

Table 5.3 Continues

Month	p_t (W)	$N=2$	$N=3$	$N=5$
	280	0.80678944	0.97839341	0.99985036
	350	0.01851752	0.85989335	0.99885854
	450	0.00000000	0.53628913	0.99080151
	550	0.00000000	0.00180955	0.96328590
September	50	0.99750000	0.99987500	0.99999969
	150	0.91524029	0.99370552	0.99997398
	280	0.49697651	0.93192148	0.99946310
	350	0.00000934	0.85631032	0.99881523
	450	0.00000000	0.11498115	0.98024309
	550	0.00000000	0.00000004	0.90985208
October	50	0.99506270	0.99927499	0.99990925
	150	0.78309823	0.96373211	0.99659021
	280	0.12631100	0.58516816	0.94875004
	350	0.01760336	0.31269268	0.85177244
	450	0.00635896	0.07251857	0.62864308
	550	0.00634429	0.01193143	0.37612146
November	50	0.99750000	0.99987500	0.99999969
	150	0.90159879	0.99261482	0.99996887
	280	0.00000016	0.63079048	0.99316280
	350	0.00000000	0.00968963	0.96086911
	450	0.00000000	0.00000000	0.67017221
	550	0.00000000	0.00000000	0.05532729
December	50	0.86022821	0.91847330	0.96654433
	150	0.46698223	0.80560171	0.80784432
	280	0.25558707	0.33977245	0.70367092
	350	0.25450371	0.29002290	0.50691622
	450	0.25449276	0.28027137	0.35230169

Table 5.3 Continues

Month	$p_t\ (W)$	$N = 2$	$N = 3$	$N = 5$
	550	0.25449276	0.27994465	0.29815173

Figure 5.3 is the cdf $P\{P_T \leq p_t\}$ graph that gives the probabilities of achieving the desired power levels when $N = 5$ for the solar plant power in June, one of the summer months. As expected, the solar plant power is increasing in p.

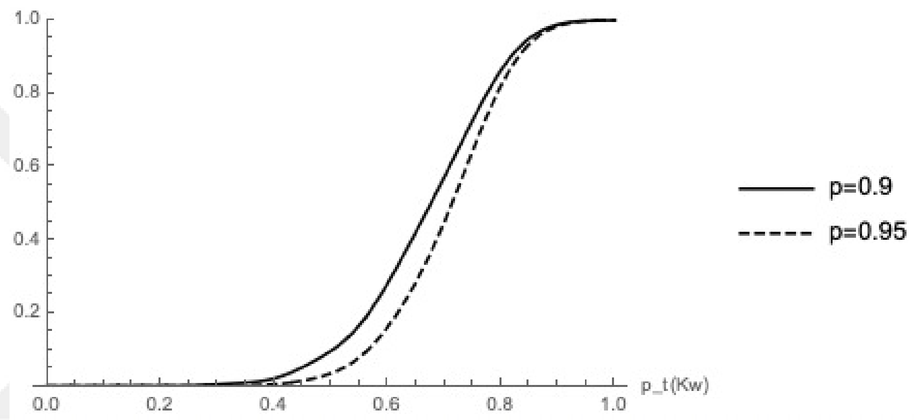

Figure 5.3 The cdf of the solar plant power in the Faculty of Science, in June when N= 5

5.1.1 Determination of the number of PV modules for the plant in the Faculty of Science

Next, we consider the optimal number of PV modules for a solar plant subject to failure. We assume that the probability distribution is known for solar irradiation, hence we can answer the question about the minimum number of PV modules that have to be installed in a plant to generate the desired production p_{t_0} with a given threshold reliability value r_0. Now, it is enough to compute $W_N(p_{t_0}; p)$ to get the number of modules, N, considering the probability that

$$1 - W_N(p_{t_0}; p) = P\{P_T > p_{t_0}\} \geq r_0. \tag{5.2}$$

For example, as shown in previously from Table 5.3, when the desired power level is $p_{t_0} = 450W$ with a given reliability value $r_0 = 0.95$ in any day of June, the minimum number of PV modules to be installed in the plant is found $N = 5$. Also, when the desired level of power is $p_{t_0} = 150W$ with a given reliability value $r_0 = 0.95$ in any day of October, there is no need to be installed $N = 5$ PV modules. Because, we provided inequality (5.2) with the probability is 0.96373 by $N = 3$ PV modules.

As another way, we can consider the mean power generated by the plant in Eq. (3.13) to determine the minimum number of PV modules that have to be installed in the plant. The following inequality leads to answer the question considering the predetermined mean power generated by the plant, m_0:

$$\mu_T \geq m_0. \tag{5.3}$$

where m_0 is the minimum level of mean power generated by the plant. Now, suppose that the minimum level of mean power is $m_0 = 1480\ W$ when $p = 0.90$ on any day in July. According to the approach given in Eq. (3.13), recall that the mean power generated by the solar plant was found that $\mu_0 = 148.231\ W$ on any day in July. Therefore, we conclude that the minimum $N = 10$ identical PV modules have to be installed in the plant for July to exceed $m_0 = 1480\ W$.

5.2 Solar Irradiation Data for the Faculty of Medicine

As a second example, we consider the same scenario as the previous one, but in a different place. It will be clear that even if a solar power plant uses the same type of modules, the generating capacity may vary due to differences in solar radiation data and its distribution in different locations. To present this situation, we used the theoretical results in section 3.1 and data were taken for Faculty of Medicine, DEU (38.395° Latitude, 27.035° Longitude) from PVGIS (https://ec.europa.eu/jrc/en/pvgis) as seen in Figure 5.4.

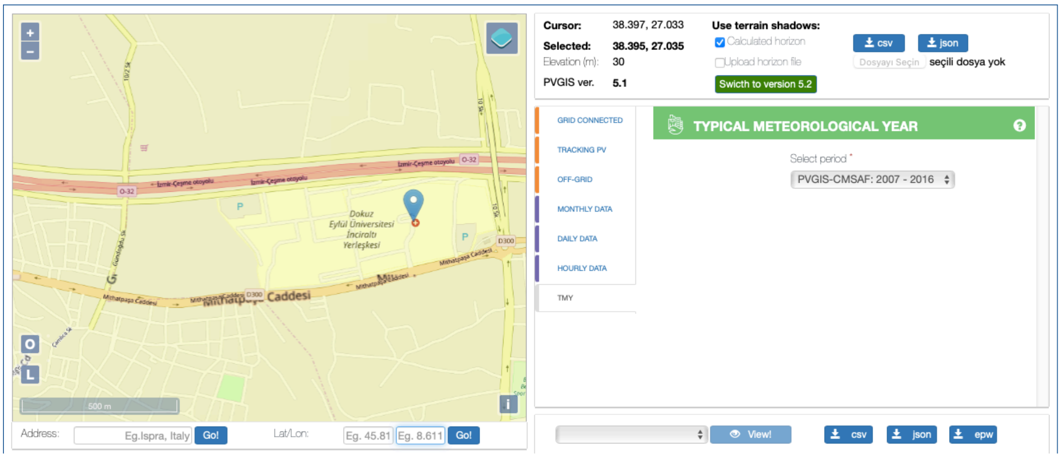

Figure 5.4 The coordinates of data for DEU, Faculty of Medicine

Similar as previous example, we organized the TMY data (2007-2016) for solar radiation data of specific months of the year for Faculty of Medicine of DEU. The histogram of the daily average solar irradiation in the Faculty of Medicine of DEU are figured out in Figure 5.5.

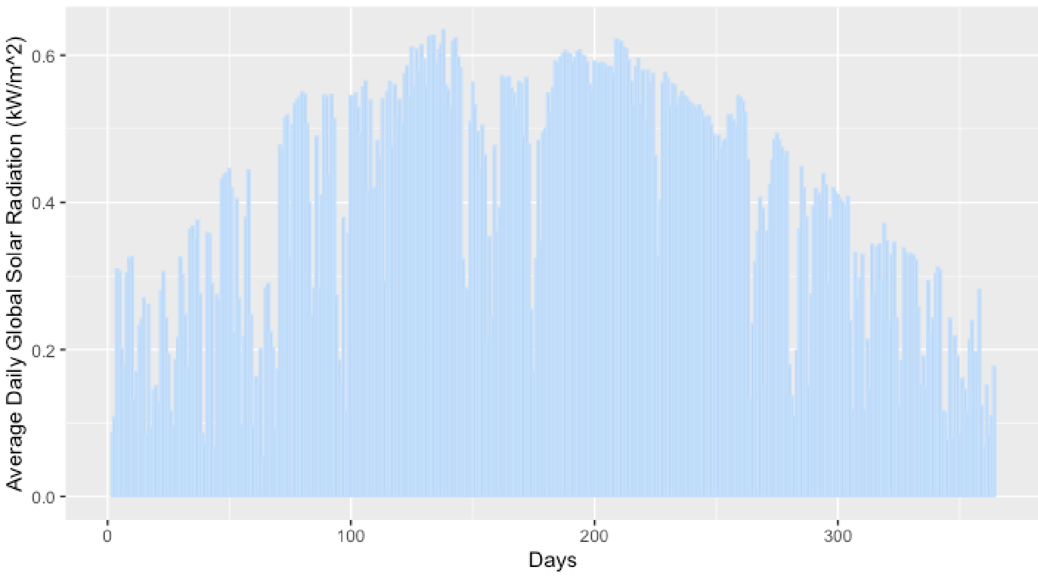

Figure 5.5 The average daily solar irradiation in the Faculty of Medicine of DEU

By using the approach of maximum likelihood, the Weibull distribution's parameters a and b have been estimated. Along with the estimated mean solar radiation, the maximum likelihood estimations of the parameters for the solar radiation distributions are shown in Table 5.4. The solar radiation data for each month may be modeled by the specified Weibull distribution with estimated values $\hat{a}$ and $\hat{b}$

because all p-values of the Anderson-Darling (AD) test are greater than the significance level 0.05 in the Table 5.4. In Table 5.4, the p-values of the AD test are also listed. The average solar radiation has been estimated using $b\Gamma(1+\frac{1}{a})$.

Table 5.4 Point estimates of the parameters of the solar radiation distribution in the Faculty of Medicine and the p-values of AD test

Month	$\hat{a}$	$\hat{b}$	Mean solar radiation (kW/m^2)	p-value
January	2.82701	0.23351	0.208006	0.69702676
February	2.88986	0.33513	0.298789	0.22563347
March	2.37800	0.40616	0.359995	0.62279887
April	4.72843	0.49360	0.451743	0.16964032
May	10.19518	0.58649	0.558412	0.07866271
June	5.11688	0.49732	0.457233	0.11692024
July	38.36983	0.59953	0.590904	0.19196749
August	15.96528	0.56002	0.541829	0.75034207
September	5.91824	0.48186	0.446691	0.06627745
October	4.60606	0.40826	0.373072	0.05002539
November	4.51817	0.30199	0.275652	0.34340416
December	2.67293	0.20761	0.184562	0.26593137

Assumptions for the application are as follows: the rated capacity of the PV module is $P_{sn} = 280\ W$, the solar irradiation in a standard environment is $G_s = 1000\ W/m^2$, and the certain irradiation point is $R_c = 150\ W/m^2$. Then, from (3.8) and (3.9), the cdf of the power generated by a single PV module is found as:

$$W_1(p_t;p) = \begin{cases} 0 & , p_t < 0 \\ (1-p) + pK_{1,1}(p_t) & , 0 \le p_t < 42 \\ (1-p) + pK_{2,1}(p_t) & , 42 \le p_t < 280 \\ 1 & , p_t \ge 280. \end{cases} \tag{5.4}$$

When the PV module is perfectly reliable (i.e. $p = 1$) and $p = 0.90$, the means power generated by a single PV module for each month are given in Table 5.5 using Eq. (3.13).

When the reliability of a module is $p = 0.90$, the highest value of mean power generated by a single PV module is $148.908W$ in July and the least value of mean power generated by a single PV module is $44.3836W$ in December. Thus, the mean power generated by the solar plant with identical $N = 10$ modules will be $\mu_T = N\mu_0 = 1489.08\ W$ in July and $\mu_T = N\mu_0 = 443.836\ W$ in December.

Table 5.5 The mean power generated by a single PV module (kW) in the Faculty of Medicine

Month	$\mu_0(p = 1)$	$\mu_0(p = 0.90)$
January	0.056580	0.050922
February	0.083057	0.074752
March	0.100186	0.090173
April	0.126470	0.113823
May	0.156355	0.140720
June	0.128015	0.115213
July	0.165453	0.148908
August	0.151712	0.136541
September	0.125069	0.112562
October	0.104408	0.093968
November	0.076962	0.069266
December	0.049315	0.044384

In Table 5.6, we compute the reliability values that are the probabilities of $N = 2, 3, 5$ PV modules produce power more than p_t Watt for each month of the year. Table 5.6, gives the values of $P\{P_T > p_t\} = 1 - P\{P_T \leq p_t\}$ computed using (3.7) when $p = 0.95$. One can see fromTable 5.6 that the probability of producing power more than $150\ W$ is 0.81652 in the solar plant that consists of three PV modules in January while the same probability is 0.90271 in a plant in February. Also, producing power of more than $550\ W$ is impossible in the solar plant that consists of three PV modules while the probability of the same event is 0.97731 in the plant with five modules in July. As expected, the reliability values are decreasing in the same number of PV modules when the level of producing power is increasing. Furthermore, the reliability values are increasing at the same level of producing power when the number of PV modules is increasing for each month.

Table 5.6 Reliability values for the solar plant power in the Faculty of Medicine when $p = 0.95$

Month	p_t (W)	$N = 2$	$N = 3$	$N = 5$
January	50	0.86971031	0.91767203	0.95942952
	150	0.45492716	0.81651749	0.82657490
	280	0.22475895	0.30270841	0.71661603
	350	0.22459384	0.25207641	0.49992624
	450	0.22459376	0.24707772	0.31640872
	550	0.22459376	0.24705314	0.25887261
February	50	0.95131720	0.97153805	0.98649557
	150	0.62981631	0.90271258	0.93691828
	280	0.12183560	0.41859789	0.85189158
	350	0.08633869	0.22427078	0.69053676
	450	0.08422683	0.11036924	0.45028431
	550	0.08422353	0.09348654	0.26222483
March	50	0.94898574	0.96657474	0.98178691
	150	0.72528702	0.92784123	0.93576539
	280	0.25582158	0.57387101	0.89235336
	350	0.13624383	0.39331039	0.78043517
	450	0.08633142	0.21380182	0.60546070
	550	0.08089582	0.12687051	0.43981750
April	50	0.99611932	0.99935512	0.99985505
	150	0.87907633	0.98366819	0.99801533
	280	0.31502604	0.78367085	0.98097830
	350	0.04585178	0.55728632	0.94216156
	450	0.00326673	0.20014293	0.82642719
	550	0.00322648	0.02265657	0.63164709
May	50	0.99749951	0.99987495	0.99999969
	150	0.96605173	0.99748964	0.99998924
	280	0.74142649	0.96589342	0.99965468

Table 5.6 Continues

Month	p_t (W)	$N = 2$	$N = 3$	$N = 5$
	350	0.13331945	0.85150129	0.99790098
	450	0.00000083	0.57630641	0.98587451
	550	0.00000083	0.03968219	0.93476446
June	50	0.99668501	0.99960011	0.99993198
	150	0.88939650	0.98644974	0.99882613
	280	0.32483816	0.80412373	0.98559097
	350	0.03802519	0.57977000	0.95329696
	450	0.00196407	0.20065869	0.84782982
	550	0.00195619	0.01657469	0.65594601
July	50	0.99750000	0.99987500	0.99999969
	150	0.99624280	0.99978071	0.99999929
	280	0.90164861	0.99262229	0.99996894
	350	0.00648784	0.85834743	0.99884997
	450	0.00000000	0.84602875	0.99855818
	550	0.00000000	0.00000000	0.97731119
August	50	0.99750000	0.99987500	0.99999969
	150	0.96055133	0.99710332	0.99998813
	280	0.76624140	0.97209457	0.99979368
	350	0.00281586	0.85019307	0.99863494
	450	0.00000000	0.52397598	0.98938495
	550	0.00000000	0.00000464	0.95119503
September	50	0.99713395	0.99977725	0.99998126
	150	0.89060039	0.98831466	0.99947079
	280	0.26090881	0.80580826	0.98949148
	350	0.00943427	0.56390539	0.96159273
	450	0.00090301	0.13286748	0.85550073
	550	0.00090301	0.00284299	0.64159664

Table 5.6 Continues

Month	$p_t\ (W)$	$N=2$	$N=3$	$N=5$
October	50	0.99364554	0.99836980	0.99956457
	150	0.79457367	0.96594610	0.99448812
	280	0.07982572	0.59919721	0.95326756
	350	0.00965742	0.29576634	0.86613573
	450	0.00892005	0.03582157	0.64591511
	550	0.00892005	0.00994016	0.36430891
November	50	0.98122954	0.99328340	0.99803402
	150	0.54585076	0.89828040	0.97675997
	280	0.03748083	0.22092916	0.84139532
	350	0.03742866	0.05301893	0.61058303
	450	0.03742866	0.04117292	0.25240842
	550	0.03742866	0.04117153	0.07054821
December	50	0.81301169	0.87478902	0.93485199
	150	0.46686118	0.80071709	0.75301255
	280	0.30922413	0.36486971	0.69561135
	350	0.30919876	0.34149174	0.50255632
	450	0.30919875	0.34012152	0.37420103
	550	0.30919875	0.34011863	0.34576376

In Figure 5.6, we plot the cdf $P\{P_T \leq p_t\}$ of the solar plant power in June when $N = 5$. As expected, the solar plant power is increasing in p.

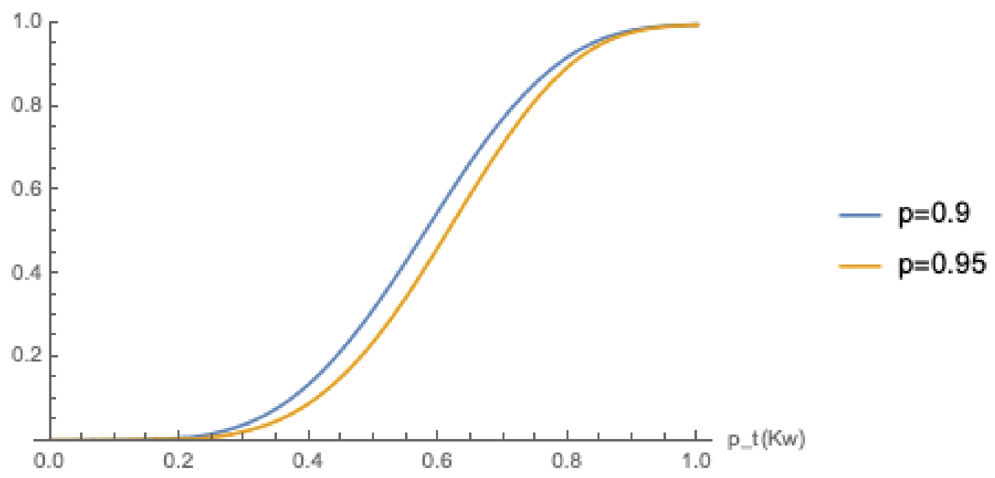

Figure 5.6 The cdf of the solar plant power in the Faculty of Medicine, in June when N= 5

5.2.1 Determination of the number of PV modules for the plant in the Faculty of Medicine

Furthermore, we could also answer the question about the minimum number of PV modules that have to be installed in a plant to generate the desired production p_{t_0} with a given reliability value r_0. The question could be answered considering the probability that

$$P\{P_T > p_{t_0}\} \geq r_0. \tag{5.5}$$

From Table 5.6, the minimum number of PV modules that has to be installed in the plant is found as $N = 5$ when the desired level of power is $p_{t_0} = 350$ with a given reliability value $r_0 = 0.95$ in June. Also, when the desired level of power is $p_{t_0} = 280$ with a given reliability value $r_0 = 0.95$ in May, there is no need to be installed $N = 5$ PV modules. Because, we provided inequality (5.2) with the probability is 0.96589 by $N = 3$ PV modules.

In another way, we need to consider the following inequality to answer the same question considering the required minimum level of mean power generated by the plant:

$$\mu_T \geq m_0, \tag{5.6}$$

where m_0 is the minimum level of mean power generated by the plant. Now, suppose that the minimum level of mean power is $m_0 = 1480\ W$ when $p = 0.90$ in July. According to the approach given in 3.13, we found that the mean power generated by the solar plant is $\mu_T = 148.908\ W$ in July. Therefore, the minimum $N = 10$ identical PV modules have to be installed in the plant for the months of July.

5.3 Determining the reliability values based on the solar irradiation data in the Faculty of Science and Medicine

In this case, we consider a solar energy system that would like to be installed to supply the electric power demand of two solar plants in two different regions. These regions may have different solar radiation and various types of PV modules may be preferred for installation in the regions. And decision-makers are concerned about the probability that the total power generated by two solar plants exceeds the desired level.

With this consideration, we will present some results using theoretical findings in section 4 under Case C. We will consider a solar energy system that consists of two solar plants located in two given different regions with different types of PV modules. Each solar plant consists of the same type of PV modules. The modules in the plants are assumed to work independently. Also, in clear, different regions are expected to have different solar radiation and correspondingly different solar radiation distributions. Finally, the probability that the power generated by two solar plants exceeds a given level will be evaluated for the given PV power model and solar radiation distributions.

The data that will be used to perform this analysis, were taken for the Faculty of Medicine of DEU, Balcova, Izmir (38.396° Latitude, 27.030° Longitude) and Faculty of Science of DEU, Buca, Izmir (38.367° Latitude, 27.202° Longitude) from PVGIS.

The TMY data (2007-2016) is organized for solar radiation data of one-year period for given locations. The data on solar radiation are divided by the length of the day in order to obtain data more precisely because the data on solar radiation depend on the length of the daylight hours, that is, daily solar irradiation=(solar radiation)/day length. The solar radiation in the Faculty of Medicine and Faculty of Science distributed as $Weibull(2.665028, 0.444578)$ and $Beta(3.6821, 5.2761)$ with the p-values are 0.4828 and 0.6372, respectively.

Assumptions for the application are as follows: the rated capacities of the Type I and Type II PV modules are $P_{sn} = 280, 170\ W$ and the solar irradiations in a standard environment are $G_s = 1000, 900\ W/m^2$, respectively. Also, the certain irradiation point is $R_c = 150\ W/m^2$ for both types. Assume that each type of PV module operates properly with probability $p = 0.97$ for a fixed period of time. Under these assumptions, the corresponding state probabilities are computed using (4.3) when the solar radiation in the Faculty of Medicine with the Type I of PV modules and Faculty of Science with the Type II of PV modules in Table 5.7.

Table 5.7 The corresponding state probabilities when $p = 0.97$

PV module	p_1	p_2	p_3	p_4
Type I	0.03000	0.05216	0.91768	0.00017
Type II	0.03000	0.03529	0.93452	0.00019

In Table 5.8, the power generated by PV modules are computed using equations (4.11) and (4.12) when the solar radiation in the Faculty of Medicine with the Type I of PV modules and Faculty of Science with the Type II of PV modules. Indeed, t_2 and t_3 state that the mean power generated by a PV module when the solar radiation in the interval $[0, R_c)$ and $[R_c, G_s)$, respectively.

Table 5.8 The power generated by a PV module (kW) when $p = 0.97$

PV module	t_1	t_2	t_3	t_4
Type I	0.00000	0.02381	0.11518	0.28000
Type II	0.00000	0.01748	0.07973	0.17000

In Table 5.9 - Table 5.11, the probability that total power generated by two solar

Table 5.9 Reliability values when $p = 0.97$ and $n_1 = n_2 = 4$

m	350	450	550	650	750	850
$R(m)$	0.9999	0.9987	0.9814	0.8918	0.5422	0.0010

Table 5.10 Reliability values when $p = 0.97$ and $n_1 = 4, n_2 = 3$

m	350	450	550	650	750	850
$R(m)$	0.9995	0.9916	0.9153	0.5800	0.0009	0.0004

Table 5.11 Reliability values when $p = 0.97$ and $n_1 = 3, n_2 = 4$

m	350	450	550	650	750	850
$R(m)$	0.9993	0.9879	0.8559	0.5906	0.0008	3.68964×10^{-7}

plants exceeds or equal to m , i.e. $R_{n_1,n_2}(m) = P\{T_1 + T_2 \geq m\}$ can be calculated using the data in Table 5.7 and Table 5.8 in Eq. 4.10 when plant in the Faculty of Medicine consists of $n_1 = (4, 4, 3)$ PV modules of Type 1 and plant in the Faculty of Science consists of $n_2 = (4, 3, 4)$ PV modules of Type 2 for different m (W/m^2) values, respectively. From Table 5.9 - Table 5.11 we can conclude that the highest reliability values for all levels of desired power of the system are in the case of $n_1 = 4, n_2 = 4$ when the working probabilities of modules for each type are $p = 0.97$. If less number of modules are desired to be installed, the case of $n_1 = 4, n_2 = 3$ can be selected under the assumption that the costs for each type of module are the same. Because this case has the highest reliability values for all levels of desired power of the system when the working probabilities of modules for each type is the same.

5.3.1 Optimization Problem: Determining the optimum number of PV modules by minimizing the total cost

It may be critical to understand the optimum number of PV modules to place in solar power plants in order to achieve the needed level of power production during a specific time period before the construction of solar power plants. Utilizing the minimal amount of PV modules necessary for a given power generating process can considerably lower the cost. We propose an optimization problem to determine the optimum number of Type I and Type II PV modules in each plant by reducing the overall cost subject to the minimum needed probability on the combined power output of the two plants. Let c_i represents the cost of i-th type of PV module and $n_{i,j}$ represents the number of i-th type of module in j-th plant. For given threshold reliability value $r_0 \in (0,1)$, desired level of power production m, and the maximum number of i-th type of PV modules that can be installed in the j-th plant $n_{i,j}^{max}$, the problem can be formulated as

$$\begin{aligned} \text{Minimize} \quad &: \quad c_i n_{i,j} \\ \text{Subject to} \quad &: \quad R_{n_1,n_2}(m) \geq r_0 \text{ and } 0 \leq n_{i,j} \leq n_{i,j}^{max}, \;\; i = 1,2 \text{ and } j = 1,2. \end{aligned} \quad (5.7)$$

The limitation on land size makes this restriction on the number of PV modules important. A lower bound on the probability of generating a power at or above m is provided by the constraint $R_{n_1,n_2}(m) \geq r_0$. Obviously, the choice of r_0's value must be large. Since n_1 and n_2 are integers, it is possible to find the optimum values by enumeration in the ranges of possible values. The optimum number of PV modules to be installed in the plants for various levels of required power production for various types of PV modules is calculated. The following cases will be taken into consideration:

- ***Case A***: PV modules of Type I are used in Plant 1, and the PV modules of Type II are used in Plant 2.
- ***Case B***: PV modules of Type I are used in Plant 2, and the PV modules of Type II are used in Plant 1.
- ***Case C***: PV modules of Type I and PV modules of Type II are used in Plant 1.
- ***Case D***: PV modules of Type I and PV modules of Type II are used in Plant 2.

Assumptions for the application are as follows: the rated capacities of the Type I and Type II PV modules are $P_{sn} = 400, 335\ W$, respectively. Also, the certain irradiation point is $R_c = 150\ W/m^2$ and the solar irradiation in a standard environment are $G_s = 1000\ W/m^2$ for both types. Assume that each type of PV module operates properly with probability $p = 0.97$ for a fixed period of time.

Since the same PV module model may produce a different amount of power in a different location, the cases A and B make difference. Table 5.12-Table 5.13 show that all possible values of total cost when $m = 600, c_1 = 1.3, r_0 = 0.95$ for Case A and Case B, respectively. The bold values are the optimal solution with respect to total cost with the optimum number of PV modules that have to be installed for the required given values.

Table 5.12 All possible values of total cost when $m = 600, c_1 = 1.3, c_2 = 1, r_0 = 0.95$ for Case A

$n^*_{2,2}$ \ $n^*_{1,1}$	0	1	2	3	4	5	6	7	8	9	10
0	-	-	-	-	-	-	7.8	9.1	10.4	11.7	13.0
1	-	-	-	-	-	7.5	8.8	10.1	11.4	12.7	14.0
2	-	-	-	**5.9**	7.2	8.5	9.8	11.1	12.4	13.7	15.0
3	-	-	-	6.9	8.2	9.5	10.8	12.1	13.4	14.7	16.0
4	-	-	6.6	7.9	9.2	10.5	11.8	13.1	14.4	15.7	17.0
5	-	6.3	7.6	8.9	10.2	11.5	12.8	14.1	15.4	16.7	18.0
6	6.0	7.3	8.6	9.9	11.2	12.5	13.8	15.1	16.4	17.7	19.0
7	7.0	8.3	9.6	10.9	12.2	13.5	14.8	16.1	17.4	18.7	20.0
8	8.0	9.3	10.6	11.9	13.2	14.5	15.8	17.1	18.4	19.7	21.0
9	9.0	10.3	11.6	12.9	14.2	15.5	16.8	18.1	19.4	20.7	22.0
10	10.0	11.3	12.6	13.9	15.2	16.5	17.8	19.1	20.4	21.7	23.0

Table 5.13 All possible values of total cost when $m = 600, c_1 = 1.3, c_2 = 1, r_0 = 0.95$ for Case B

$n^*_{1,2}$ \ $n^*_{2,1}$	0	1	2	3	4	5	6	7	8	9	10
0	-	-	-	-	-	-	6.0	7.0	8.0	9.0	10.0
1	-	-	-	-	-	6.3	7.3	8.3	9.3	10.3	11.3
2	-	-	-	**5.6**	6.6	7.6	8.6	9.6	10.6	11.6	12.6
3	-	-	-	6.9	7.9	8.9	9.9	10.9	11.9	12.9	13.9
4	-	-	7.2	8.2	9.2	10.2	11.2	12.2	13.2	14.2	15.2
5	-	7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5	15.5	16.5
6	7.8	8.8	9.8	10.8	11.8	12.8	13.8	14.8	15.8	16.8	17.8
7	9.1	10.1	11.1	12.1	13.1	14.1	15.1	16.1	17.1	18.1	19.1
8	10.4	11.4	12.4	13.4	14.4	15.4	16.4	17.4	18.4	19.4	20.4
9	11.7	12.7	13.7	14.7	15.7	16.7	17.7	18.7	19.7	20.7	21.7
10	13.0	14.0	15.0	16.0	17.0	18.0	19.0	20.0	21.0	22.0	23.0

Table 5.14-Table 5.21 include the optimal values of the number of PV modules that must be installed in the plants for all cases when $r_0 = 0.95$, $c_1 = 1.3, 1.4$, $n_{i,j}^{max} = 10$ where $i = 1, 2$ and $j = 1, 2$. As it is seen from Table 5.14-Table 5.17, in terms of minimum total cost, Case B should be preferred to Case A for the corresponding cost (c_1). An increase in the cost of Type I PV modules from $c_1 = 1.3$ to $c_1 = 1.4$ could not affect the optimal solutions for the number of PV modules for Case B, but the optimal number of PV modules for the level $m = 450, 600(W)$ changed in Case A. Clearly, it only increased the total cost for Case B. Even if $c_1 = 1.3$ Case A and $c_1 = 1.4$ in Case B are compared, Case B can be still preferred for all level of m.

Table 5.14 Optimal number of PV modules for the Case A when $c_1 = 1.3, c_2 = 1$

$m\ (W)$	$n_{1,1}^*$	$n_{2,2}^*$	$R_{n_{1,1}^*,n_{2,2}^*}(m)$	cost
450	3	1	0.9673	4.9
600	3	2	0.9513	5.9
750	1	6	0.9667	7.3
900	1	7	0.9827	8.3

Table 5.15 Optimal number of PV modules for the Case B when $c_1 = 1.3, c_2 = 1$

$m\ (W)$	$n_{2,1}^*$	$n_{1,2}^*$	$R_{n_{2,1}^*,n_{1,2}^*}(m)$	cost
450	4	0	0.9638	4.0
600	3	2	0.9513	5.6
750	7	0	0.9885	7.0
900	8	0	0.9825	8.0

Table 5.16 Optimal number of PV modules for the Case A when $c_1 = 1.4, c_2 = 1$

$m\ (W)$	$n_{1,1}^*$	$n_{2,2}^*$	$R_{n_{1,1}^*,n_{2,2}^*}(m)$	cost
450	0	5	0.9901	5.0
600	0	6	0.9607	6.0
750	1	6	0.9667	7.4
900	1	7	0.9827	8.4

From Table 5.18-Table 5.21, if we just wanted to install one plant to minimize additional costs that might arise in the future, then it would seem appropriate to install the solar plant close to plant 2, i.e. Faculty of Science, DEU for both level of c_1. Clearly, Case D should be preferred to Case C in terms of the minimum total cost. An

Table 5.17 Optimal number of PV modules for the Case B when $c_1 = 1.4, c_2 = 1$

$m\ (W)$	$n^*_{2,1}$	$n^*_{1,2}$	$R_{n^*_{2,1},n^*_{1,2}}(m)$	cost
450	4	0	0.9638	4.0
600	3	2	0.9513	5.8
750	7	0	0.9885	7.0
900	8	0	0.9825	8.0

increase in the cost of Type I PV modules from $c_1 = 1.3$ to $c_1 = 1.4$, the optimal number of PV modules changed for the level $m = 450, 750, 900(W)$ in Case C and the optimal number of PV modules changed only for the level $m = 600(W)$ in Case D. Even c_1 in Case C is cheaper, the case where the c_1 in Case D is higher may be still preferable since we have lower minimum total cost for all level of m.

Table 5.18 Optimal number of PV modules for the Case C when $c_1 = 1.3, c_2 = 1$ in the Faculty of Medicine, DEU

$m\ (W)$	$n^*_{1,1}$	$n^*_{2,1}$	$R_{n^*_{1,1},n^*_{2,1}}(m)$	cost
450	3	1	0.9638	4.9
600	1	5	0.9702	6.3
750	2	5	0.9502	7.6
900	3	5	0.9621	8.9

Table 5.19 Optimal number of PV modules for the Case D when $c_1 = 1.3, c_2 = 1$ in the Faculty of Science, DEU

$m\ (W)$	$n^*_{1,2}$	$n^*_{2,2}$	$R_{n^*_{1,2},n^*_{2,2}}(m)$	cost
450	1	3	0.9521	4.3
600	3	2	0.9627	5.9
750	1	6	0.9695	7.3
900	1	7	0.9612	8.3

Table 5.20 Optimal number of PV modules for the Case C when $c_1 = 1.4, c_2 = 1$ in the Faculty of Medicine, DEU

$m\ (W)$	$n^*_{1,1}$	$n^*_{2,1}$	$R_{n^*_{1,1},n^*_{2,1}}(m)$	cost
450	0	5	0.9640	5.0
600	1	5	0.9702	6.4
750	2	5	0.9502	7.8
900	0	9	0.9752	9.0

Table 5.21 Optimal number of PV modules for the Case D when $c_1 = 1.4, c_2 = 1$ in the Faculty of Science, DEU

m (W)	$n^*_{1,2}$	$n^*_{2,2}$	$R_{n^*_{1,2},n^*_{2,2}}(m)$	cost
450	1	3	0.9521	4.4
600	0	6	0.9607	6.0
750	1	6	0.9695	7.4
900	1	7	0.9612	8.4

5.3.2 Sensitivity Analysis

In this section, we give additional findings for sensitivity analysis of the model. Specifically, the impacts of changing the values of a number of parameter values on the optimal solution are investigated. The sensitivity analysis is based on Case A. From Table 5.22 - Table 5.24, we observed the effects of changing values of the cost of Type I PV module c_1, threshold reliability values r_0 and desired level of power m for Case A, respectively. In Table 5.22, it is seen that when $c_2 = 1$, increasing in c_1 only from 1 to 1.5 affects the number of Type I PV modules in plant 1, $n^*_{1,1}$, and Type II PV modules in plant 2, $n^*_{2,2}$, and depending on this change in optimal number of PV modules, the total cost and reliability also increase considering another parameters are constant and determined previously. Since the model aims to minimize the total cost, the decision was made to use only Type II PV modules in plant 2 while c_1 is increasing.

Table 5.22 Optimal $(n^*_{1,1}, n^*_{2,2})$ and total cost for changing values of c_1 when $m = 600, r_0 = 0.95$

c_1	1	1.5	2	2.5
$(n^*_{1,1}, n^*_{2,2})$	(3, 2)	(0, 6)	(0, 6)	(0, 6)
Total Cost	5.0	6.0	6.0	6.0
$R_{n^*_{1,1},n^*_{2,2}}(m)$	0.9513	0.9607	0.9607	0.9607

From Table 5.23, we can say that while $n^*_{2,2}$ is higher at lower threshold reliability values, r_0, it is necessary to use more $n^*_{1,1}$ to provide the optimization model as we increase r_0 from 0.80 to 0.95. When $r_0 = 0.80$, the model prefers more $n^*_{2,2}$ because it has lower cost.

For Table 5.24, we can say that after $m = 450$, the optimal total number of PV

Table 5.23 Optimal $(n^*_{1,1}, n^*_{2,2})$ and total cost for changing values of r_0 when $m = 600, c_1 = 1.3$

r_0	0.80	0.85	0.90	0.95
$(n^*_{1,1}, n^*_{2,2})$	(1, 4)	(2, 3)	(2, 3)	(3, 2)
Total Cost	5.3	5.6	5.6	5.9
$R_{n^*_{1,1},n^*_{2,2}}(m)$	0.8462	0.9103	0.9103	0.9513

modules increases continuously to provide the minimum required level of production, m, and $n^*_{2,2}$, that has lower cost, is preferred to keep the cost to a minimum.

Table 5.24 Optimal $(n^*_{1,1}, n^*_{2,2})$ and total cost for changing values of m when $r_0 = 0.95, c_1 = 1.3$

m	450	550	650	750	850	950
$(n^*_{1,1}, n^*_{2,2})$	(3, 1)	(0, 5)	(1, 5)	(1, 6)	(0, 8)	(0, 9)
Total Cost	4.9	5.0	6.3	7.4	8.0	9.0
$R_{n^*_{1,1},n^*_{2,2}}(m)$	0.9673	0.9627	0.9515	0.9667	0.9710	0.9827

In similar way, the effect of the changes in same parameters on the optimal solution are shown in Table 5.25 -Table 5.27 for Case B. Notice that $n^*_{2,1}$ represents the optimal number of Type II PV modules in plant 1 and $n^*_{1,2}$ represents the optimal number of Type I PV modules in plant 2 in here.

As different from Table 5.22, there are two optimal solutions for the case of $c_1 = 1.5$ in Table 5.25. Although the number of used module types remains the same after the $c_1 = 1.5$, reliability values are higher because they are used in different regions.

Table 5.25 Optimal $(n^*_{2,1}, n^*_{1,2})$ and total cost for changing values of c_1 when $m = 600, r_0 = 0.95$

c_1	1	1.5	2	2.5
$(n^*_{2,1}, n^*_{1,2})$	(3, 2)	(6,0)(3,2)	(6,0)	(6,0)
Total Cost	5.0	6.0	6.0	6.0
$R_{n^*_{2,1},n^*_{1,2}}(m)$	0.9513	0.9908(0.9513)	0.9908	0.9908

It is seen from Table 5.26, more $n^*_{2,1}$ are preferred in lower reliability threshold values. Although the total number remains the same for $r_0 = 0.95$, it is also preferred to use Type I PV modules in plant 2.

Finally, in Table 5.27, as expected, the total number of PV modules increases while

Table 5.26 Optimal $(n^*_{2,1}, n^*_{1,2})$ and total cost for changing values of r_0 when $m = 600, c_1 = 1.3$

r_0	0.80	0.85	0.90	0.95
$(n^*_{2,1}, n^*_{1,2})$	(5,0)	(5,0)	(5,0)	(3, 2)
Total Cost	5.0	5.0	5.0	5.6
$R_{n^*_{2,1}, n^*_{1,2}}(m)$	0.9430	0.9430	0.9430	0.9513

m is increasing. It is seen that the model is provided without using the Type I PV modules in plant 2, because c_1 is higher than c_2 and we aim to minimize the total cost.

Table 5.27 Optimal $(n^*_{2,1}, n^*_{1,2})$ and total cost for changing values of m when $r_0 = 0.95, c_1 = 1.3$

m	450	550	650	750	850	950
$(n^*_{2,1}, n^*_{1,2})$	(4,0)	(5,0)	(6,0)	(7,0)	(7,0)	(8,0)
Total Cost	4.0	5.0	6.0	7.0	7.0	8.0
$R_{n^*_{2,1}, n^*_{1,2}}(m)$	0.9638	0.9640	0.9908	0.9885	0.9726	0.9773

CHAPTER 6
CONCLUSION

Renewable energy sources exist in nature and are constantly renewing themselves without any production process. They are eco-friendly, unlimited, and free resources. One of the renewable energy sources is solar energy which is widely used in the world. Photovoltaic (PV) systems directly convert solar energy into electricity. When a solar plant is desired to be established to provide electric power, the amount of solar irradiation in the region where the solar power plant is planned to be installed should be considered and the planning should be done accordingly. Since solar irradiation is a random variable, the probability distribution of solar irradiation data and some of its properties must be determined. Then, considering the theoretical distribution of solar plant power taking into account the characteristics of PV modules subjected to failure will provide us with more accurate results. In this study, we derived the theoretical distribution of power generated by a solar plant with PV modules subject to failure, assuming all PV modules experience the same solar irradiation and have the same power curve. Besides, we calculated the mean power produced by that solar plant and determined the minimum number of PV modules that should be installed with the probability of providing the least desired power level. As another approach, the PV system has been considered a multistate system consisting of a large number of PV modules as main components. PV modules have been also considered multistate components in terms of their contribution to the performance of the system. Because, they can be in a specific state (working, partially working, and not working) depending on the solar radiation. Multistate PV modules in a solar system and power output model consisting of both linear and quadratic relationship between the solar radiation and the power are combined to evaluate the reliability for the system performance. We obtained the reliability models constructed for the defined three cases using the state probabilities of two types of multistate PV modules. Furthermore, we obtained the optimal number of PV modules that have to be installed in the plants considering minimum cost subject to the probability of producing the minimum required power level and we have given in further analyses. A real data study for the illustration of the theoretical results is presented assuming that the solar radiation distributions are beta and Weibull. In optimal power generation planning, the mean power of the solar plant has been concerned with the known PV module characteristics and the solar irradiation distribution of the region. Also, the minimum number of PV modules that have to be installed in a plant has been discussed for obtaining the required production level under the given threshold reliability value. Determining the number of PV modules will improve the performance of systems as it significantly increases the reliability of their power generation and prevents downtime. In addition, it has been seen from the numerical studies that the reliability value of the system varies depending on the solar radiation distribution, the working probability, and the type and number of PV modules. By using the results of this study, important decisions can be made before the solar plant construction in terms of both the number of PV modules and the provision of solar energy. In future work, the defined reliability models can be reconstructed for more than two types of PV modules. Also, the modeling of the power for PV systems in the dependency concept may be considered in more detail.

REFERENCES

Adams, W. G., & Day, R. E. (1877). V. the action of light on selenium. *Proceedings of the Royal Society of London*, *25*(171-178), 113–117.

Afzaal, M. U., Sajjad, I. A., Awan, A. B., Paracha, K. N., Khan, M. F. N., Bhatti, A. R., Zubair, M., Rehman, W. U., Amin, S., Haroon, S. S. et al. (2020). Probabilistic generation model of solar irradiance for grid connected photovoltaic systems using Weibull distribution. *Sustainability*, *12*(6), 2241.

Alferidi, A., & Karki, R. (2017). Development of probabilistic reliability models of photovoltaic system topologies for system adequacy evaluation. *Applied Sciences*, *7*(2), 176.

Atwa, Y., El-Saadany, E., Salama, M., & Seethapathy, R. (2009). Optimal renewable resources mix for distribution system energy loss minimization. *IEEE Transactions on Power Systems*, *25*(1), 360–370.

Becquerel, M. (1839). Mémoire sur les effets électriques produits sous l'influence des rayons solaires. *Comptes rendus hebdomadaires des séances de l'Académie des sciences*, *9*, 561–567.

British Petroleum (2019). Statistical review of world energy 2019. *Br. Pet*, (66), 1.

British Petroleum (2021). BP statistical review of world energy. *BP Plc, London, United Kingdom*, 14–16.

Chapin, D. M., Fuller, C. S., & Pearson, G. L. (1954). A new silicon $p-n$ junction photocell for converting solar radiation into electrical power. *Journal of applied physics*, *25*(5), 676–677.

Chen, Y., & Yang, Q. (2005). Reliability of two-stage weighted-k-out-of-n systems with components in common. *IEEE Transactions on Reliability*, *54*(3), 431–440.

Choi, J., Park, J., Shahidehpour, M., & Billinton, R. (2010). Assessment of CO2 reduction by renewable energy generators. In *2010 Innovative Smart Grid Technologies (ISGT)*, (1–5).

Cook, J. L. (2018). Reliability analysis of weighted k-out-of-n systems with variable demand. *Quality Engineering*, *30*(4), 687–693.

Devrim, Y., & Eryilmaz, S. (2021). Reliability-based evaluation of hybrid wind–solar energy system. *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, *235*(1), 136–143.

Ding, Y., Zuo, M. J., Lisnianski, A., & Li, W. (2010). A framework for reliability approximation of multi-state weighted k-out-of-n systems. *IEEE Transactions on reliability*, *59*(2), 297–308.

Eryilmaz, S. (2015). Mean time to failure of weighted k-out-of-n:G systems. *Communications in Statistics-Simulation and Computation*, *44*(10), 2705–2713.

Eryilmaz, S. (2018). Reliability analysis of multi-state system with three-state components and its application to wind energy. *Reliability Engineering & System Safety*, *172*, 58–63.

Eryilmaz, S., & Bozbulut, A. R. (2014). An algorithmic approach for the dynamic reliability analysis of non-repairable multi-state weighted k-out-of-n:G system. *Reliability Engineering & System Safety*, *131*, 61–65.

Eryilmaz, S., & Bozbulut, A. R. (2019). Reliability analysis of weighted-k-out-of-n system consisting of three-state components. *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, *233*(6), 972–977.

Eryilmaz, S., Bulanık, İ., & Devrim, Y. (2021). Reliability based modeling of hybrid solar/wind power system for long term performance assessment. *Reliability Engineering & System Safety*, *209*, 107478.

Eryilmaz, S., & Devrim, Y. (2019). Theoretical derivation of wind plant power distribution with the consideration of wind turbine reliability. *Reliability Engineering & System Safety*, *185*, 192–197.

Eryilmaz, S., & Kan, C. (2020). Reliability based modeling and analysis for a wind power system integrated by two wind farms considering wind speed dependence. *Reliability Engineering & System Safety*, *203*, 107077.

Eryilmaz, S., & Sarikaya, K. (2014). Modeling and analysis of weighted-k-out-of-n:G system consisting of two different types of components. *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, *228*(3), 265–271.

Eryilmaz, S., & Ucum, K. A. (2021). The lost capacity by the weighted k-out-of-n system upon system failure. *Reliability Engineering & System Safety*, *216*, 107914.

Ettoumi, F. Y., Mefti, A., Adane, A., & Bouroubi, M. (2002). Statistical analysis of solar measurements in Algeria using beta distributions. *Renewable Energy*, *26*(1), 47–67.

Fritts, C. E. (1883). On a new form of selenium cell, and some electrical discoveries made by its use. *American Journal of Science*, *3*(156), 465–472.

Gao, J., Yao, K., Zhou, J., & Ke, H. (2018). Reliability analysis of uncertain weighted k-out-of-n systems. *IEEE Transactions on Fuzzy Systems*, *26*(5), 2663–2671.

Higashiyama, Y. (2001). A factored reliability formula for weighted-k-out-of-n system. *Asia-Pacific Journal of Operational Research*, *18*(1), 61.

Huang, J., Zuo, M. J., & Wu, Y. (2000). Generalized multi-state k-out-of-n:G systems. *IEEE Transactions on reliability*, *49*(1), 105–111.

International Energy Agency (2021). *Renewables 2021 :Analysis and forecast to 2026*.

Kam, O. M., Noël, S., Ramenah, H., Kasser, P., & Tanougast, C. (2021). Comparative Weibull distribution methods for reliable global solar irradiance assessment in France areas. *Renewable Energy*, *165*, 194–210.

Kan, C., Devrim, Y., & Eryilmaz, S. (2020). On the theoretical distribution of the wind farm power when there is a correlation between wind speed and wind turbine availability. *Reliability Engineering & System Safety*, *203*, 107115.

Kuo, W., & Zuo, M. J. (2003). *Optimal reliability modeling: principles and applications*. John Wiley & Sons.

Larsen, E. M., Ding, Y., Li, Y. F., & Zio, E. (2020). Definitions of generalized multi-performance weighted multi-state K^{-}-out-of-n system and its reliability evaluations. *Reliability Engineering & System Safety*, *199*, 105876.

Li, W., & Zuo, M. J. (2008a). Optimal design of multi-state weighted k-out-of-n systems based on component design. *Reliability Engineering & System Safety*, *93*(11), 1673–1681.

Li, W., & Zuo, M. J. (2008b). Reliability evaluation of multi-state weighted k-out-of-n systems. *Reliability Engineering & System Safety*, *93*(1), 160–167.

Li, X., You, Y., & Fang, R. (2016). On weighted k-out-of-n systems with statistically dependent component lifetimes. *Probability in the Engineering and Informational Sciences*, *30*(4), 533–546.

Lisnianski, A., & Levitin, G. (2003). *Multi-state system reliability: Assessment, optimization and applications*. World scientific.

Mahmoudi, E., Meshkat, R. S., & Torabi, H. (2021). Copula-based reliability for weighted-k-out-of-n systems having randomly chosen components of m different types. *IEEE Transactions on Reliability*.

Marwali, M., Haili, M., Shahidehpour, S., & Abdul-Rahman, K. (1998). Short term generation scheduling in photovoltaic-utility grid with battery storage. *IEEE Transactions on Power Systems*, *13*(3), 1057–1062.

Moe, T. T., & Lin, K. M. (2018). Solar irradiance and power output modeling of photovoltaic module for reliability studies: case study of Mandalay Region. *ICSTI, Mandalay*, 1–6.

Ohl, R. S. (1946). Light-sensitive electric device. US Patent 2,402,662.

Park, J., Liang, W., Choi, J., El-Keib, A., Shahidehpour, M., & Billinton, R. (2009). A probabilistic reliability evaluation of a power system including solar/photovoltaic cell generator. In *2009 IEEE Power & Energy Society General Meeting*, (1–6). IEEE.

Patel, M. R. (1999). Wind and power solar systems. *Renewable Energy*, *30*.

Salameh, Z. M., Borowy, B. S., & Amin, A. R. (1995). Photovoltaic module-site matching based on the capacity factors. *IEEE Transactions on Energy conversion*, *10*(2), 326–332.

Samaniego, F. J., & Shaked, M. (2008). Systems with weighted components. *Statistics & Probability Letters*, *78*(6), 815–823.

Sheng, Y., & Ke, H. (2020). Reliability evaluation of uncertain k-out-of-n systems with multiple states. *Reliability Engineering & System Safety*, *195*, 106696.

Singh, G. K. (2013). Solar power generation by PV (photovoltaic) technology: A review. *Energy*, *53*, 1–13.

Singh, S., & Fozdar, M. (2020). Double-sided bidding strategy for power suppliers and large buyers with amalgamation of wind and solar based generation in a modern energy market. *IET Generation, Transmission & Distribution*, *14*(6), 1031–1041.

Teng, J.-H., Luan, S.-W., Lee, D.-J., & Huang, Y.-Q. (2012). Optimal charging/discharging scheduling of battery storage systems for distribution systems interconnected with sizeable PV generation systems. *IEEE Transactions on Power Systems*, *28*(2), 1425–1433.

Tina, G., Gagliano, S., & Raiti, S. (2006). Hybrid solar/wind power system probabilistic modelling for long-term performance assessment. *Solar energy*, *80*(5), 578–588.

Tobnaghi, D. M., & Naderi, D. (2015). The effect of solar radiation and temperature on solar cells performance. *Extensive journal of applied sciences*, *3*(2), 39–43.

Wang, Y., Li, L., Huang, S., & Chang, Q. (2012). Reliability and covariance estimation of weighted k-out-of-n multi-state systems. *European journal of operational research*, *221*(1), 138–147.

Wu, J.-S., & Chen, R.-J. (1994). An algorithm for computing the reliability of weighted-k-out-of-n systems. *IEEE Transactions on Reliability*, *43*(2), 327–328.

Yang, Y., Kim, K. A., & Ding, T. (2018). Modeling and control of PV systems. In *Control of power electronic converters and systems* (243–268). Elsevier.

Yin, J., Molini, A., & Porporato, A. (2020). Impacts of solar intermittency on future photovoltaic reliability. *Nature communications*, *11*(1), 1–9.

Zhang, P., Li, W., Li, S., Wang, Y., & Xiao, W. (2013). Reliability assessment of photovoltaic power systems: Review of current status and future perspectives. *Applied Energy*, *104*, 822–833.

Zhang, Y. (2021). Reliability analysis of randomly weighted k-out-of-n systems with heterogeneous components. *Reliability Engineering & System Safety*, *205*, 107184.

Zhuang, X., Yu, T., & Shen, L. (2019). On capacity evaluation for multi-state weighted k-out-of-n system. *Communications in Statistics-Simulation and Computation*, *48*(7), 2083–2098.

APPENDIX

Appendix 1: The cdf of power generated by solar plant

The cumulative distribution function in (3.7) can be expressed by conditioning on the number of working PV modules,

$$P\{P_T \le p_t\} = \sum_{i=0}^{N} P\{Y_N = i\} P\{P_T \le p_t | Y_N = i\},$$

where Y_N is the number of working modules. Then, we have,

$$P\{Y_N = i\} = \binom{N}{i} p^i (1-p)^{N-i}, \;\; i = 0, 1, \dots, N.$$

Let $W_i(p_t) = P\{P_T \le p_t | Y_N = i\}$ be the cdf of the power generated by the plant with i working modules. Obviously, $W_0(p_t) = 0$ if $p_t < 0$ and $W_0(p_t) = 1$, if $p_t \ge 0$. Using Eq. 2.1, we have

$$\begin{aligned} W_i(p_t) &= P\{iP_{sn}\frac{g_{bi}^2}{G_s R_c} \le p_t, g_{bi} < R_c\} + P\{iP_{sn}\frac{g_{bi}}{G_s} \le p_t, R_c \le g_{bi}\} \\ &= P\{g_{bi} \le \left(\frac{p_t G_s R_c}{iP_{sn}}\right)^{1/2}, g_{bi} < R_c\} + P\{R_c \le g_{bi} < \frac{p_t G_s}{iP_{sn}}\}. \end{aligned}$$

for $0 < p_t < iP_{sn}$. Thus, the first and second part in $W_i(p_t)$ express $K_{1,i}(p_t)$ and $K_{2,i}(p_t)$ in 3.9, respectively.

Hence, the explanations for Eq. (3.7) are completed.