EXPRESSION DYNAMICS OF MINORITY OPINION IN HOMOPHILY-ORIENTED NETWORKS

by

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"The world is a dynamic mess of jiggling things if you look at it right. And if you magnify, you will hardly see anything anymore because everything is jiggling in its own pattern, and there are a lot of little balls. It's lucky that we have such a large scale of view of everything, that we can see them as things, without having worry about these little atoms all the time."

ABSTRACT

EXPRESSION DYNAMICS OF MINORITY OPINION IN HOMOPHILY-ORIENTED NETWORKS

Minorities may remain silent for fear of social isolation. According to the spiral of silence theory, minority groups underestimate their numbers in society. This underestimation leads minorities to speak up less and other minorities to feel less supported, so minorities get caught in a spiral of silence. They can break out of this spiral by uniting with each other. People seek similarity in their relationships, and this is referred to as homophily in the literature. Under homophily, people can end their relationships with different people and establish new relationships with like-minded people. These dynamic networks are called homophily-oriented (H-O) networks. We investigate the expression dynamics of the minority opinion in H-O networks using the agent-based modeling approach, where we represent people as agents and the relationships between them as links. In a case where agents end their relationships with the opposing side but build relationships with anyone, we found that the minority can oppress the majority by distorting the perceptions of majority members because minorities increase their opinion expression probability by lowering their expectations in the first place. When agents connect directly with like-minded agents; they create echo chambers, and both majorities and minorities can be fully expressive. We then introduce new agent types into the model, representing three types of people: hardcore agents are always expressive minorities, loyal agents do not change their social circle but their expression decisions, and valiant agents are both expressive and loyal minorities. Minorities' loyalty to their social environment sabotages their expression in H-O networks but reduces echo chambers. Hardcore agents and valiant agents increase minority expression in static networks; in H-O networks, minority expression is already at its maximum, so they cannot increase it but valiant agents can decrease segregation in societies.

ÖZET

HOMOFİLİ YÖNELİMLİ AĞLARDA AZINLIK FİKRİNİN İFADE DİNAMİKLERİ

Azınlıklar yalnız kalma korkusuyla toplum içinde sessiz kalabilirler. Sessizlik sarmalı teorisine göre azınlık grupları toplumdaki sayılarının olduğundan düşük olduğunu sanma eğilimindededir. Bu da onları fikirlerini daha az ifade etmeye iter ve diğer azınlıklar fikirlerini olduğundan daha az destekleniyor gibi hissettirir ve azınlıklar sessizlik sarmalına kapılabilirler. Ancak benzerlerle bir olarak bu sarmaldan çıkabilirler. Bu benzerlik arayışı literatürde "homofili" olarak anılır. Homofili varken, insanlar karşıt görüşlülerle ilişkilerini sonlandırır veya benzer düşünen insanlarla yeni bağlar kurar. Bu dinamik ilişki ağına, homofili yönelimli (H-O) ağlar denir. İnsanları etmen ve aralarındaki ilişkileri bağlantı olarak temsil ettiğimiz etmen-tabanlı modelleme yaklaşımı ile H-O ağlarındaki azınlık görüşünün ifade dinamiklerini araştırıyoruz. Etmenlerin karşıt görüşlülerle ilişkisini sonlandırdığı ancak herhangi birine bağlandığı bir durumda, azınlıklar ilk etapta beklentilerini düşürerek ifade olasılıklarını arttırır ve bu da çoğunluk üyelerinin algılarını çarpıtarak azınlığın çoğunluğu ezebilmesini sağlar. Tüm etmenler benzerleriyle bağlantı kurduğunda, yankı odaları yaratır ancak herkes fikrini ifade edebilirler. Sonra modele üç farklı insanı temsil eden farklı etmenler ekliyoruz: fikirlerini her zaman ifade eden kararlı azınlıklar, sosyal çevrelerini değiştirmeyen ancak ifade tercihlerini değiştirebilen muhafazakar insanlar ve değişmeyen sosyalliğinde fikirlerini her zaman dile getiren azınlıklar olan dirençli insanlar. Azınlıkların sosyal çevrelerine olan bağlılıkları, H-O ağlarındaki ifadelerini sabote eder ama yankı odalarını azaltır. Kararlı etmenler ve dirençli etmenler, statik ağlarda azınlık ifadesini artırırlar. H-O ağlarında, azınlık ifadesi zaten maksimumdadır, bu yüzden onu artıramazlar, ancak dirençli etmenler toplumlardaki ayrışmayı da azaltabilirler.

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hd	Homophily Degree
hp	Hardcore Percentage
i	Agent
m	Minority Percentage
N	Population Size
nd	Average Node Degree
NE	Expressive Population Size
Nmajor	Majority Population Size
Nminor	Minority Population Size
r	Spatial Proximity
t	Time Step
α	Learning rate
β	Social parameter 1
γ	Social parameter 2
δ	Social parameter 3

LIST OF ACRONYMS/ABBREVIATIONS

ABM	Agent Based Modeling
EC	Echo Chamber
ECI	Echo Chamber Index
FPI	Friendship Paradox Index
H-O	Homophily-oriented
MI	Majority Illusion
R-D	Random-dynamic

1. INTRODUCTION

People prefer to live in and be a part of a society since societies provide major benefits such as food, security, shelter, and sociability. However, they are expected to abide by the rules of their society in order to fit in, or they choose the people they live with while excluding other. One of the most common reasons people become outcasts is their differences from the rest of society; this difference can originate from ethnicity, religion, appearance, or opinion. In this study, our main interest is in the differences of opinion among people and the effects of these differences on the dynamics of opinion expression and the segregation of society.

It is not easy to exclude people from society just because they have different opinions. This is because opinions can be hidden, unlike ethnicity and appearance. Therefore, people could choose not to express their opinions because they are afraid of isolation or they seek similar people to themselves to be able to express their opinions freely. Even if they have other options, the right to freedom of expression is one of the most important foundations of modern society and coexistence. It is protected by law, as stated in Article 19 of the Universal Declaration of Human Rights [1]:

> Everyone has the right to freedom of opinion and expression; this right includes freedom to hold opinions without interference and to seek, receive and impart information and ideas through any media and regardless of frontiers.

It is necessary to define the opinion discussed in this study to understand what opinion expression really means. The related studies go back to ancient Greece, where the studies of epistemology began. They studied opinion to separate it from knowledge. Plato, in his famous book “Republic” mentioned that opinion, in Greek doxa, is a way of thinking; and knowledge, “episteme” in Greek, is a tool to act wisely [2]. Aristotle explained that thought following a “truth” is knowledge, other types of thought are opinion [3]. Kant, on the other hand, discussed that there is an additional form of thought, and opinion is thought about a subject without being influenced by a fact or a subjective passion. If it is an objective fact, it is knowledge; and if there is a passion behind the thought, it is the person's belief [4]. While there are many different approaches to what an opinion is and what it represents in society, we will use Kant's approach.

So we know that opinions are subjective, naive and susceptible to bias, people might find it difficult to express their own opinions and prefer to hide them. As a result, it is observed that people with unsupported opinions become silent as Noelle-Neumann proposed in the theory of the spiral of silence [5].

The dynamics of opinion expression is a crucial element of modern societies, and it has long been studied in depth. Since the basis concerns politics, i.e. public opinion and democracies, the first studies of opinion dynamics date back to the first decades of the 20^{th} century [6]. However, it was not developed until the late 20^{th} century when definitions of minority and majority groups began to appear in the literature [7, 8]. In recent decades, physicists also began to study opinion dynamics using physics equations [9] and opinion dynamics has become an interdisciplinary field of research [10].

2. LITERATURE REVIEW

People have been using opinion expression to define themselves, communicate with each other, discuss topics, and progress in those topics throughout history. It is not common for people to want to express their opinions all the time but in this study, we are merely interested in the occasions where all people want to express their opinions.

We will use the voter model approach to opinion formation, which assumes a dichotomy of opinions, i.e., whether one is for or against something on certain issues. The voter model was first presented by Holley and Ligett in 1975 [11] and this model is studied in opinion dynamics [12]. It is also worth noting that when we speak of opinion expression, this means that people express their opinions in public and not in private conversations, so people have a reason to fear of isolation [13].

Opinion expression by people could create conflicts in society and as a result, it could force the minority in society to find different methods to fit into the group. The easiest method for minorities seems to be changing opinions and blending in. In the simulation model of Banisch et al. [9], all people express their opinions without fear of isolation in a static network. People are exposed to the opinion that is the majority in their neighborhood. After a long time, they change their opinion to fit in. As they confirm the same opinions and express their dissatisfaction with other opinions in this group, they exclude the opinions that clash with the opinion of the local majority, eventually forming echo chambers that lead society to segregation. Echo chambers are bubbles where people only hear the support of their own opinion and exclude the others in their network [14].

Because the local majority could be different from the global majority, changing opinions to fit in a group of people is not a valid solution. Moreover, it is claimed that opinions are subjective thoughts [15] and reflect a person's cumulative emotional history [16], so it is not easy to change them. As a result, people have to find other solutions to fit into society or to cope with the pressures they face. Some models propose that the actual opinion and the expressed opinions by people can differ [17, 18] so that they pretend to belong to the group. However, this is a way to silence oneself and stop expressing their real opinions and an example of people seeking comfort to get away from clashing with the majority. While it is a survival method, it also causes anxiety in people. Having conflicted opinions and not expressing them in social circles could cause people to have psychological disorders like cognitive dissonance [19]. Therefore, it is crucial to understand the conditions that silence people and how to overcome them.

Several topics under expression dynamic studies merit the study: the spiral of silence, public spheres, selective exposure and homophily, congregation and segregation, echo chambers, and majority illusion.

There are some cases where people feel alone with their opinions when they are not in reality. In the famous story "Emperor's New Clothes" [20], people remain silent because they believe that they are the only ones who cannot see the Emperor's new clothes and do not want to clash with the socially accepted norm. When just one of them speaks about the missing cloth, they all realize they are not alone and start pointing out the truth. This story is about the truth, not opinions. However, the motivations of people are the same: the silenced people fear both isolation and authority. Furthermore, people's behavior of seeking comfort is also similar.

People tend to overestimate the size of their group as we explore in detail in the section on majority illusions. They do this because they want to be a part of something big and feel supported [21]. However, in a society where the minority is greatly outnumbered in local communities, members of minority groups remain silent because people seek comfort and fear isolation and, they do not want to be outcasts in their society [5, 22, 23].

People do not speak when they believe they are in the minority, so the opinion they hold seems to have less support than it actually does.This lack of support for the opinion in question makes people who hold it feel even lonelier, which starts a trend of silence among all minorities. After a while, the entire group community falls into complete silence. This phenomenon is called the spiral of silence and was first studied by Elizabeth Noelle-Neuman in 1974 [5]. Spiral of Silence is an example of how local convergences become a global convergence theory that Axelrod formulated in 1997 [24].

In forming a spiral of silence, people scan the opinion climate, perceiving it with their "quasi-statistical senses" which is driven by their fear of isolation. People use their "quasi-statistical senses" or "quasi-statistical organs" to predict the proportions of minorities and majorities in a society [22]. Since they give people the illusion of statistical certainty in their predictions, but these are only a perception, they are called "quasi". When all people perceive the opinion climate, they make their willingness to express themselves dependent on it. While one group becomes the local majority, the local minority is no longer willing to speak out. When the proportion of minorities in society is small, it is statistically more likely that the local majority will be the global majority. As a result, the minority becomes less and less expressive and eventually falls into complete silence.

It is often studied in political science and Noelle-Neumann was herself a political scientist, but it has also become a relatively popular topic in opinion theory. Many studies revisit the theory of the spiral of silence [25, 26]; some expect debunked results but found even more concrete results [27].

One of the ways for minorities to deal with oppression is to reduce social pressure through group support [28]. The behavior of people seeking similar people to feel safe is called selective exposure or homophily [29]. People are likely to choose with whom they communicate and with whom they form relationships [30]. According to McPherson, homophily is inevitable in human life and influences the choice of partners, places of residence, and friendships [31].

Schelling [13] stated that homophily leads to separation and segregation of races and educational levels in daily life. Although there are political reasons for such segregation, homophily could be caused not only by systematic discrimination but also by cultural and genetic factors. Throughout history, similar groups have been observed to merge and create congregations and it is not possible to distinguish congregation and segregation from each other. We can say that echo chambers are reminiscent of Schelling's definition of the congregation.

The choice of seeking comfort rather than the truth or consensus could create confirmation bias [32]. People do not have to exclude or clash with the other side, society can be split into two different sides even if they only give each other positive feedback in a static social network [9]. Giving and hearing only positive feedback exclude trench warfare which is caused by the clashes of opinions and leads people with opposite ideas to more separation. However, when people do not argue with each other but tend to hear confirmations of their opinions, this would also lead to confirmation biases, therefore, echo chambers [14].

In societies where people interact and where there is more than one opinion on a single topic, segregation seems inevitable. There are several approaches to study echo chambers in society in terms of how and where they arise. Studies suggest that social media platforms are likely to create echo chambers [33] and societies may also create echo chambers depending on political views [34].

While the studies we mentioned were often conducted in the real world, there are also studies on online platforms [26]. In this day and age, people are no longer restricted by their geographical borders and can connect with people from different countries and backgrounds through the Internet. It was discussed that the Internet and social media will create public spheres [35, 36]. Public spheres are first introduced by Habermas and he defines them as the best form of democracy, where people come together and discusses their opinions without arguing [15].

Aside from the fact that not everyone can access the Internet, those who do have access use the Internet to search for opinions similar to their own and to find references that support their opinions [26, 37] and create echo chambers [38]. It is also argued that engaging with opposing opinions on the Internet could polarize people, similarly with trench warfare [39]. This shows us that the Internet has become another tool to divide and exclude people.

Moreover, with the development of technology, people can reach daily news through mass media in seconds. However, it is claimed that people prefer to remain silent when they are only oppressed by their social environment [24] and their opinion climate is mainly determined by the personal relationships and not by the facts or the news [5, 40], and a spiral of silence could form without media or Internet [41]. Therefore, in this study, we exclude the impact of mass media and the Internet on people.

Having dealt with the spiral of silence, selective exposure, and confirmation bias, we will now examine the notion of majority illusion. In elections, where people make their choices without any pressure from behind the curtain, people are often surprised that their party received this small percentage of the vote, even though their social circle is the supporters of their party. This is a consequence of the majority illusion. In contrast to the spiral of silence, there are conditions that make people believe they are in the majority even though they are in the minority and vice versa [21].

Majority illusion can also occur when minority groups behave like majority groups and do not feel threatened to express their opinions. This may lead to an oppressed majority since they may think that the minority group is the majority, and this illusion lead the majority group into silence [42].

To understand the impact of social pressure on minorities, Gaisbauer, Banisch and Olber [41] examine the conditions under which minorities are silenced. They assume that expression of opinion comes at a cost. The cost is determined by social factors; when the cost is low, people feel more comfortable expressing their opinions, and vice versa. They found that the proportion of minorities in society is also an important factor in the expression of minority opinions. The higher the proportion, the greater the number of minorities expressing themselves, regardless of the cost; however, minority opinions are suppressed at lower proportions at even low costs. Finally, they claimed that minority opinion expression increases under homophily at all costs and minority proportions.

Sohn and Geidner [43] study the influence of local perceptions on global recognition of the dominant group and suggests that network properties influence the formation of the spiral of silence. Network structures and sizes alter the number and distribution of opinions to which people are exposed, and thus their perceptions. They build a simulation model using an agent-based modeling approach in which they represent people as agents and relationships between them as links. They use thresholds to decide whether people express opinions or not, and thresholds are individual properties assigned to agents. They use them to measure the difference between the minority and majority groups under these conditions, and found that people often make mistakes when reading the opinion climate. This study argues that society tends to segregate when people only perceive their close relationships and make decisions in this regard, and that considering opinions in their social environment alone can lead people into a spiral of silence.

3. RESEARCH OBJECTIVE

The studies mentioned in the previous section show that in static networks when there is more than one opinion on a topic, the opinion of the minority is suppressed by the majority; and the spiral of silence can occur in a society where the media is excluded and people communicate with their friends and family only. It is also argued that the percentage of the minority is an important factor in the dynamics of minority opinion expression. We will study the dynamics of minority expression under the conditions that lead to the spiral of silence in homophily-oriented networks.

In this study, people can stay with the same people, change their social circle randomly, and they also have the opportunity to do the same under homophily. We refer to them as static networks, random-dynamic networks, and homophily-oriented networks, respectively. While the main aim is to investigate whether minority break their silence in homophily-oriented networks, we will compare the other network types to understand whether homophily-oriented networks make a significant difference in expression dynamics.

If minorities are more expressive in homophily-oriented networks, we will examine the conditions under which minorities express their opinions and what features of a homophily-oriented network produce the most expressive minorities. We are also interested in whether the occurrence of the spiral of silence correlates with the formation of echo chambers.

We want to know whether the expressive minority group can oppresses the majority group. Moreover, we are looking for a case where both the minority and the majority are fully expressive in a society without total segregation.

We know that people are endowed with different characteristics and that they can change the expression choices of themselves and those around them. We will investigate whether people with different characteristics prevent the spiral of silence in static networks. We will also examine whether these people with particular characteristics also change the dynamics of majority expression and the formation of echo chambers in homophily-oriented networks.

Finally, the research questions we intend to answer in this study are listed below:

- Can a minority group become more expressive under homophily?
- What properties of a homophily-oriented network cause the minority to be fully expressive?
- Is there a case where the majority group is silent while the minority group is expressive?
- Is it possible to have expressive people under homophily without complete segregation?
- If part of the minority group is always expressive, would this change the expressiveness of the majority group or the rest of the minority group?
- If some minorities cannot change their social circle in homophily-oriented networks, would the percentage of expressiveness of the minority group decrease?
- If some majorities cannot change their social circle in homophily-oriented networks, would the percentage of expressive minority decrease?
- If some minorities are always expressive while maintaining their social circles, how would they change the dynamics of minority expression?

4. METHODOLOGY

In answering the research questions we formulated in the previous section, we can use two different methodological approaches: the study of empirical data and the construction of a simulation model. Some studies work with empirical data from social networks such as Reddit and Twitter [44]. However, empirical data are associated with noise that needs to be reduced, and using simplified models allows us to change the conditions and network types, and freely analyze the results [45].

Among the simulation approaches, the agent-based modeling approach would be the most appropriate because we want to study the interaction between people and observe their interaction patterns [46]. ABM is based on individual decision makers called an "agent" [47]. Even small populations can develop complex dynamics within themselves, interacting with each other and creating unique patterns. With ABM, we can easily track these patterns and change the agent and network types as needed [45]. It is argued that ABM is also the most appropriate approach to explore the spiral of silence [24, 43].

We will create a model in which we represent people as agents and the relationships between them as links. We will assign them opinions that makes them a member of either minority and majority groups, and let them decide their expression of opinion in terms of their opinion climate; we will examine how the expression of opinion of minority and majority groups changes over time. The details of the model are explained in the next section.

5. MODEL DESCRIPTION

5.1. Model Overview

In this model there is an isolated society and by isolated society we mean that no one leaves this society and no new members join. We assume that there is a controversial issue in this society and everyone initially has an opinion, but not everyone expresses their opinion. Closely connected people can have different opinions because there is no opinion homophily in the network construction.

There are only two opposing opinions on the topic, there is no chance for a new opinion to spread in this society and one side always remains in the minority. The agents' decision to express their opinion depends on their perception of the opinion climate, and their decision will influence the opinion climate and indirectly the decisions of other agents. The opinions assigned to agents will be binary and free of passion. There are links between agents when they are neighbors, and neighbors represent all types of relationships a person can have: family, friends, and real neighbors.

In building the society, the main parameters are population size (N), average node degree (nd), minority percentage (m), and spatial proximity (r). We create agents and assign minority and majority opinions to the agents based on the minority percentage. We determine the values for these two parameters through experiments. The average node degree represents the average number of neighbors that agents can have and is assigned over the course of the experiments, taking into account population size and minority percentage. Finally, "spatial proximity" represents the area surrounding the agents. Agents create links with other agents in this area. We give the spatial proximity an average value with respect to the model space of the simulation program we use. It is seven and a pseudo-value since it is related with the simulation model but not the real world.

After creating the agents in population size, and determining what percentage of them are in the minority and what the average node degree is, we proceed to build the network. The agents establish links with other agents in their spatial proximity. The agents try to reach the average node degree while creating links and stop when they reach the average node degree value we set. Figure 5.1 shows the pseudo-code of the network construction.

```
Calculate Average Neighbors of Agents;
while Average Neighbors of Agents < nd do
    for agents = 1, 2, ..., N do
        if There are other agents in-radius r then
            Create a link with one of other agents in-radius r ;
            Calculate Average Neighbors of Agents;
        else
    end
end
```

Figure 5.1. Network construction algorithm.

Colors are assigned to agents based on their opinions and expressive choices. The minority group is blue and the majority group is red. When the agents are silent, their colors are pale blue and pale red, respectively.

The opinions in the neighborhood are the opinion climate of the agents. Each agent looks at the opinion climate and decides whether to express his opinion. The opinion climate influences opinion expression through a reward system. If agents share the same opinion with an expressive neighbor, the reward increases; if opinion of the expressive neighbors is different, it decreases.

The number of neighbors who both share and express the same opinion is called *Agreeable Neighbors*. The number of neighbors who are expressive but have an opposing opinion is called *Disagreeable Neighbors*.

The silent neighbors in both the minority and majority groups are neglected regardless of their opinion; the *Current Reward* is calculated as

$$Current\ Reward_{it} = \frac{Agreeable\ Neighbors_{it} - Disagreeable\ Neighbors_{it}}{Agreeable\ Neighbors_{it} + Disagreeable\ Neighbors_{it}}. \quad (5.1)$$

The indices i and t represent the agents and the time steps, respectively. The range of *Current Reward* is one to minus one. If all of an agent's expressive neighbors agree with the agent, its *Current Reward* is equal to one; and if they disagree, *Current Reward* is minus one. If there is an equality between the number of *Agreeable Neighbors* and *Disagreeable Neighbors* is equal, the agent's *Current Reward* is zero.

Agents do not change their opinion expression decision based on *Current Reward* alone because it is an instant reward in a specific time step. They calculate their *Expected Reward* while multiplying the *Current Reward* by their learning rate (α), as

$$Expected\ Reward_{it} = \alpha \times Current\ Reward_{it} + (1-\alpha) \times Expected\ Reward_{i(t-1)}. \quad (5.2)$$

The learning rate can take a value between zero and one. When it is zero, agents are not affected by the *Current Reward*, they become expressive or silent with their initial decisions. At a value of one, the agents' *Expected Reward* is equal to the *Current Reward*. Its range is between minus one and one, just like the range of *Current Reward*. This means that agents do not look at their past, but make their decisions immediately, which is not realistic. The calculation of *Expected Reward* is similar to reinforcement learning, and it is claimed that larger or smaller learning rate values would lead the model to inaccurate results but there is no uniform truth about determining the learning rate [48]. However, the commonly used value is 0.01 [49]. Therefore, we choose 0.01 as the learning rate. However, changing this rate would also be a topic of another study, as there is a study on the effects of laggard society on expression dynamics [50].

We use the *Expected Reward* value in calculating the *Expression Probability* as

$$Expression\ Probability_{it} = \frac{\beta}{\gamma + e^{(\delta \times Expected\ Reward_{it})}}. \tag{5.3}$$

The parameters we use to calculate *Expression Probability* change the slope of the curve and they are constant throughout the model. We take them as social properties so that all agents have the same value for each parameter, but the values for each parameter will be different. In the baseline; the value for β is one, for γ it is also one, and for δ it is minus five. With these values, the *Expression Probability* curve will be as shown in Figure 5.2: the x-axis represents *Expected Reward*, the y-axis represents *Expression Probability*.

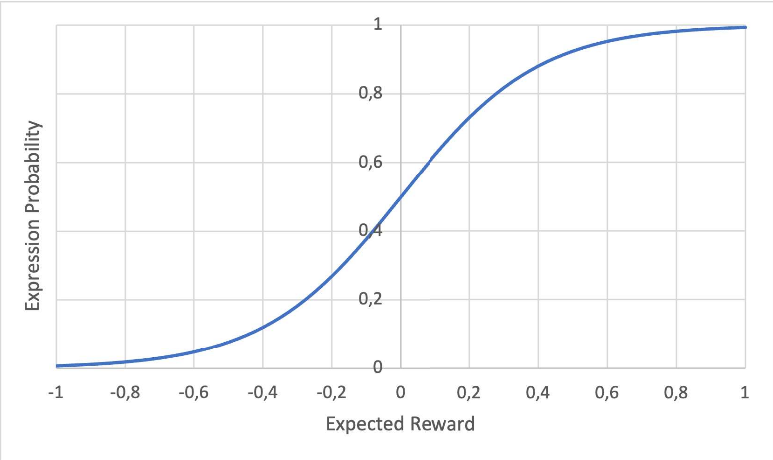

Figure 5.2. Expression probability graph.

In Figure 5.2, we see that as *Expected Reward* increases, the slope of the curve decreases for negative values of *Expected Reward* and increases for positive values of *Expected Reward*. When *Expected Reward* is equal to zero, *Expression Probability* is equal to 50%. It is important to note that we do not initially assign an expression decision to the agents, but rather they use the *Expression Probability* equation to make their initial expression decision. To give all agents a 50% chance of be expressive, we initially set the *Expected Reward* to zero.

In this setting of *Expression Probability*, society allows all people to express themselves, regardless of their expected reward, but in different probabilities. We see that the *Expression Probability* is lower than 50% for the negative values of *Expected Reward* and higher for the positive values. This shows us that in this society the *Expected Reward* is important and the *Expression Probability* is responsive to even a small change in the *Expected Reward* of the agents. In some societies, people would express themselves at all positive *Expected Reward* values, or they may not express themselves certain even when their *Expected Reward* is highest. For different values of these parameter sets, the society would respond to the agents' expression decisions, and the different types of societies are discussed in the Appendix A.

Not all agents are required to make opinion expression decision at each time step, but agents reconsider their decision if and only if;

- Their *Expected Reward* decreases and they are expressive,
- Their *Expected Reward* increases and they are silent.

```
for agents = 1, 2, ..., N do
    Calculate Current Reward(Eq.: 4.1);
    Calculate Expected Reward (Eq.: 4.2 );
    Calculate Expression Probability (Eq.: 4.3 );
end
for agents = 1, 2, ..., N do
    if Agent is silent then
        if Expected Reward_{i,j} > Expected Reward_{i,(j-1)} then
            Agent goes into expression decision process;
        else
    else
    end
    Agent is expressive if Expected Reward_{i,j} <
    Expected Reward_{i,(j-1)} then
        Agent goes into expression decision process;
    else
end
```

Figure 5.3. Opinion expression process algorithm.

We see the algorithm of the opinion expression process in Figure 5.3. It is followed by the expression decision process; the agents express their opinion with their *Expression Probability*. After, they adjust their colors accordingly. Agents perform the same process until the model reaches equilibrium.

5.2. Network Variations in the Model

Up to this point our network has been static, but in this section we move to a dynamic network. There are two different types of dynamic networks: random-dynamic networks and homophily-oriented networks. We will talk about the general rules and mention the differences between these two networks. We define two types of networks depending on whether or not they are under homophily, as shown in Table 5.1.

Each agent has an initial number of neighbors, which we call base sociability. Each agent tries to maintain its base sociability by creating links only when its current number of neighbors is less than its base sociability. This is because we want to keep the network type throughout the model. In this study, the network is built considering the average number of neighbors of the agents, and we want to keep this structure in dynamic networks as well, so that agents can only make connections when they cut one of their links. If they do not maintain their base sociability number, agents would make an excessive number of links under certain conditions and the network structure would be similar to random graphs. These agents may also create links with other agents that have fewer neighbors than their base sociability. We refer to agents that want to cut links or rewire as “initiator agents” to avoid confusion.

Table 5.1. Network classification.

	Random rewiring	Homophily in rewiring
Random link-cutting	R-D Network	H-O Network
Homophily in link-cutting	H-O Network	H-O Network

5.2.1. Link-Cutting Process

We classify the links as suitable for cutting and not suitable for cutting. In random link-cutting process, all agents volunteer to cut their links, and all their links are suitable for cutting. However, agents only cut one of their links in a single time step. Each initiator agent marks one of its suitable links as “cut”, but cannot cut all links labeled as “cut” at the same time. This would destroy the base network and result in two links per agent being cut since each link has two ends. Instead, we cut half of the links labeled as “cut” to cut one link per agent based on their cutting probability. The other half is not cut at this time step, but could be cut at later time steps. The agents who cut one of their links become initiator agents.

We assign cutting probabilities to the links. These probabilities are distributed among the links according to the normal distribution with a mean of 0.5 and a variation of 0.12 to prevent the agents from having extreme values in the cutting probability. If the probability is zero, these links are impossible to cut; if the probability is one, these links will be cut for sure.

In the link-cutting process under homophily, suitable links and initiator agents are selected by a set of rules. When agents want to change their environment, they compare their *Similarity Ratio* with the homophily degree. These agents could be both silent and expressive. The homophily degree represents the lowest percentage of similarity that agents seek in their neighborhood, and it is the same for all people. The *Similarity Ratio* represents the similarity in an agent’s neighborhood and is calculated for each agent as

$$Similarity\ Ratio_{it} = \frac{|Agreeable\ Neighbors_{it}|}{|CurrentNeighbors_{it}|}. \tag{5.4}$$

If their *Similarity Ratio* is lower than the homophily degree, agents are dissatisfied with their opinion climate. Thus, they check their neighborhood if they have suitable links for cutting. The agents they want to end their relationship with must be expressive and must hold different opinions.

Finally, if the agents are dissatisfied with their similarity ratio and their links are suitable for cutting, they label one of their suitable links as "cut". The rest of the link-cutting process is the same as the random link-cutting process. They cut half of the labeled links with the cutting probability and the agents become initiator agents. The initiator agents keep their other links. The process of link-cutting is summarized in Figure 5.4.

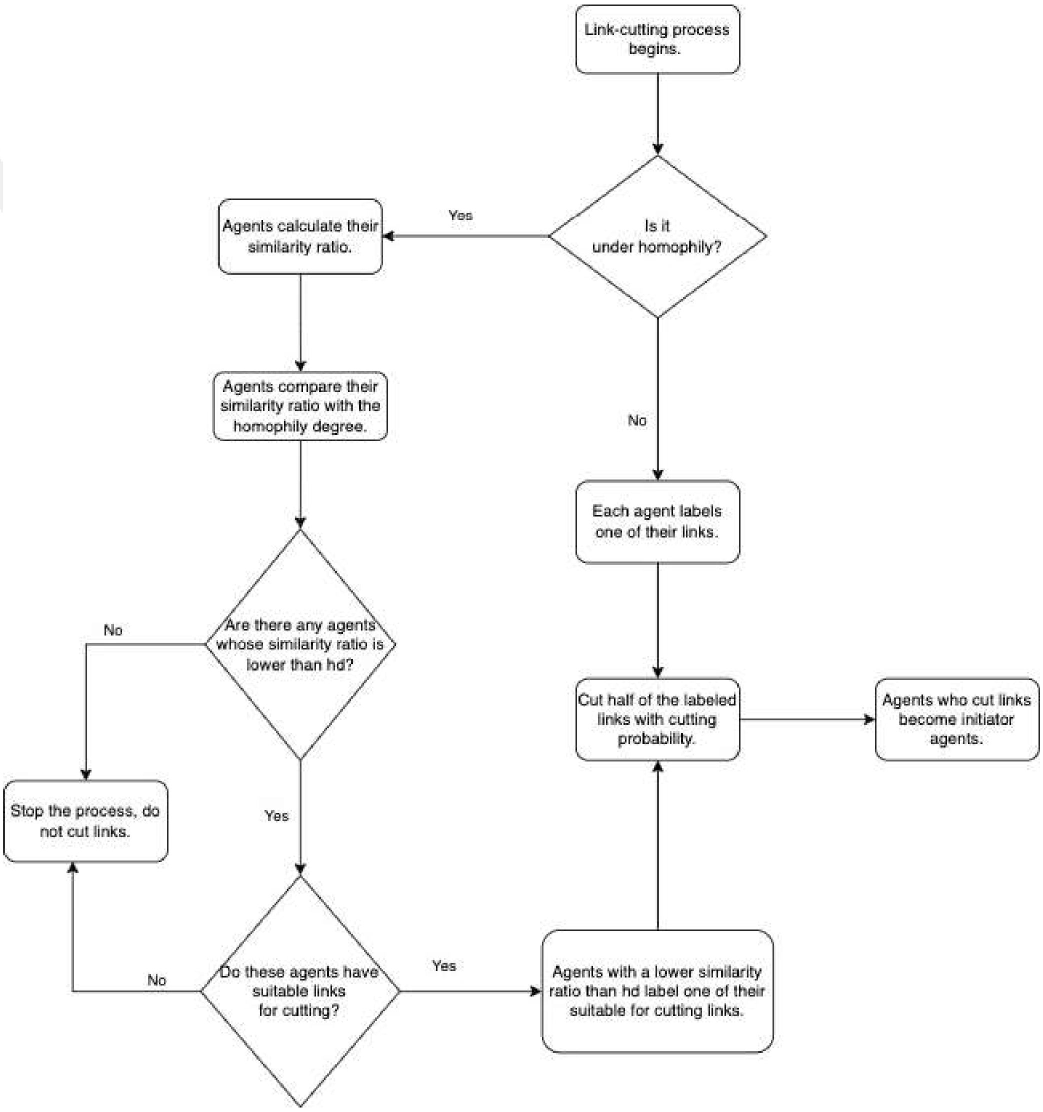

Figure 5.4. Flowchart of the link-cutting process.

5.2.2. Rewiring Process

After cutting links, agents begin to create links. Since agents want to maintain their base sociability, agents that have fewer neighbors than their base sociability are also the agents that want to rewire. We also refer to agents that want to rewire as "initiator agents". If the initiator agents cannot find agents to connect with at the time step they cut links, they continue their search in the next time steps until they reach their base sociability numbers. The process of creating a new link is based on only one rule: the initiator agents create a new link with one of the other agents that have fewer neighbors than their base sociability. In this way, no agent has a chance to exceed its base sociability. The agents cut one of their links and rewire in a cycle until no more links are cut or created, and eventually the model reaches equilibrium.

Agents can create links with other agents depending on the re-linking choices. If we assume that all agents can create links with any other agent, this is called a "random" re-linking choice. However, we can restrict the set of agents: agents can only make a new relationship with other agents if they are in the same spatial proximity, or they can only connect with their own neighbors' neighbors, in other words, their second-degree neighbors. Since there may or may not be homophily in rewiring and there are different choices, we can categorize the rules for rewiring as in Table 5.2. In total, initiator agents can establish links with other initiator agents according to six different re-linking rules. These re-linking rules are listed below.

- In *unbiased* re-linking rule, the initiator agent creates a link with one of the other agents who has fewer neighbors than their base sociability,
- In *geo-biased* re-linking rule, the initiator agent creates a link with one of the other agents who is in proximity and has fewer neighbors than their base sociability,
- In *second-degree* re-linking rule, the initiator agent creates a link with one of the other agents who is a neighbor of one of the initiator agent's neighbors and has fewer number of neighbors than their base sociability,
- In *opinion-biased* re-linking rule, the initiator agent creates a link with one of the

other agents who has the same opinion as the initiator agent and is expressive, and has fewer neighbors than their base sociability,

- In *geo-opinion-biased* re-linking rule, the initiator agent creates a link with one of the other agents who has the same opinion as the initiator agent and is expressive, is in the proximity, and has fewer neighbors than their base sociability,
- In *biased-second-degree* re-linking rule, the initiator agent creates a link with one of the other agents who has the same opinion as the initiator agent and is expressive, a neighbor is one of the initiator agent's neighbors and has fewer neighbors than its base sociability.

In determining spatial proximity, we use the radius constant (r) that we use in constructing networks. In other words, the rules *geo-biased* and *geo-opinion-biased* set an alternative network to the original network.

Table 5.2. Categorization of re-linking rules.

Agent Choice	Random Rewiring	Rewiring Homophily
Random	*unbiased*	*opinion-biased*
Proximate	*geo-biased*	*geo-opinion-biased*
Second-Degree	*second-degree*	*biased-second-degree*

5.3. Agent Variations in The Model

Agents have the same attributes in the base model, but we know that people have different personalities and different reactions to the fear of isolation. Agents could alter society's expressive dynamics by changing their attitudes and responses to the opinion climate. We introduce three different agent types into the model. Since our main concern is to analyze the expression dynamics of minority opinion, we will assign these different personalities to the minority group. In other words, these three different types of agents might just be from minority groups.

The first agent type is mentioned by Noelle-Neumann [5] and studied in [27, 51] as their hardcore personality could increase the percentage of opinion expressed by minorities and help other minorities break their spiral of silence. They are called hardcore agents and they express their opinions no matter what, so their learning value is zero. In homophily-oriented networks, they can still seek comfort from others by cutting their links with the opposite side and connecting with with the same opinionated agents.

While hardcore agents try to find a social circle they fit into, some agents are willing to adjust their expressions to fit their base social circle. These agents are called loyal agents, and they do not break their links, and also do not allow their neighbors to break their mutual links. They are not interested in rewiring, so the cutting probability of their links is zero.

On the other hand, there are agents who remain expressive in their social circle. They do not break their links or rewire because they do not try to fit into a social circle; they are already expressive with their permanent neighbors. These valiant agents are not affected by the opinion climate, so their learning value is zero. We make sure that the cutting probabilities of their links are zero. These valiant agents are basically a synthesis of hardcore and loyal agents.

5.4. Key Outputs

We look at some outputs to learn if agents are expressive and segregated. A spiral of silence can be observed when there are two groups, and one group thinks they are the minority in society and become silent after a while. This study is about the dynamics of minority opinion expression, but there are some cases where majorities behave like minorities, as well. We consider *Silent Minority Percentage* and *Silent Majority Percentage* as indicators of the spiral of silence.

The total size of the minorities is obtained by multiplying the minority percentage by the population size and it is constant throughout the model. However, their size could differ between different experiments. So we also took the total size of the minority as a parameters, as well.

We calculate the *Silent Minority Percentage* by dividing the number of silent minorities by the total size of the minority group ($Nminor$) in the model at each time step as

$$Silent\ Minority\ Percentage_t = \frac{|Silent\ Nmin_t|}{Nminor}. \tag{5.5}$$

Silent Majority Percentage could be used to see if the majority group is oppressed in the model. It is calculated by dividing the number of silent majorities by the total size of the majority group ($Nmajor$) in the model. This oppression can only occur if the majority group believes that the minority group's percentage is larger than the minority group's actual percentage. This is a consequence of the majority illusion. The total size of the majorities is also constant throughout the model. We calculate *Silent Majority Percentage* as

$$Silent\ Majority\ Percentage_t = \frac{|Silent\ Nmaj_t|}{Nmajor}. \tag{5.6}$$

We expect the *Silent Minority Percentage* to converge to at least 90% to observe a spiral of silence. In homophily-oriented networks, we expect the *Silent Minority Percentage* to decrease. To observe an oppressed majority, we expect the *Silent Majority Percentage* to be larger than the *Silent Minority Percentage*.

Finally, to see if there is a correlation between the expression dynamics and the formation of echo chambers we calculate the *Total Expressive Percentage* in the model as

$$Total\ Expressive\ Percentage_t = \frac{|NE_t|}{N}. \tag{5.7}$$

We look for majority illusion when *Percentage in Friendship Paradox* is more than 50% in the model and we will look if it is in the following section. There are different ways to measure majority illusion. We look at the local neighborhood of majority agents to measure it. The average perceived minority percentage is calculated separately for minority and majority groups. Each minority agent looks at the expressive ratio of minority agents in their neighborhood, and the average of them is called the perceived minority percentage by the minority. When majority agents do the same, it is called the perceived minority percentage by the majority. The number of majority agents whose perceived minority percentage is greater than the actual minority percentage is called *Percentage in Majority Illusion*.

We calculate how the perceived percentage of agents differs from the actual percentage of the minority because we want to know how the dynamics of opinion expression change depending on the opinion climate. It is calculated by looking at the individual perception of each agent and its discrepancy from the actual minority percentage. Finally, we calculate the average of these deviations. For the minority group, we call it *Deviation by Minority*, and for the majority group, we call it *Deviation by Majority*.

Echo chambers form when two opposing groups cluster within their group. It is important to know how many agents have accumulated in these echo chambers. We use *Percentage in Echo Chamber* to count agents belonging to an echo chamber in the model.To calculate *Percentage in Echo Chamber* we first compute *Echo Chamber Index* (ECI) only for expressive agents as

$$ECI_{it} = \frac{AgreeableNeighbors_{it}}{|Current\ Neighbors_{it}|}. \tag{5.8}$$

Total size of an agent's neighbors are represented as *Current Neighbors*. *Percentage in Echo Chamber* is calculated by dividing the number of expressive agents, at least half of whom agree and are expressive, by the total size of expressive agents (NE). We calculate *Percentage in Echo Chamber* as

$$Percentage\ in\ EC_t = \frac{|NE\ with\ (ECI \geq 0.50)_{it}|}{|NE_t|}. \tag{5.9}$$

When we observe both *Total Expressive Percentage* and *Percentage in Echo Chamber* output results converge to 100%; it means both groups are in echo chambers and society is segregated. When *Percentage in Echo Chamber* output results converge to 100%, *Silent Majority Percentage* converges to 0% and *Silent Minority Percentage* converges to 100%; we can say that there is a spiral of silence and majorities form echo chambers to oppress the minorities. When both *Total Expressive Percentage* and *Percentage in Echo Chamber* converges to 100%, we can say that both groups are in echo chambers and society is segregated.

In experiments, we may need to know the minority and majority percentages in echo chambers separately. So we define two more outputs: *Minority in Echo Chamber* and *Majority in Echo Chamber*. We calculate *Minority in Echo Chamber* as

$$Minority\ in\ EC_t = \frac{|Expressive\ Nminor\ with\ (ECI \geq 0.50)|}{|Expressive\ Nmin_t|} \tag{5.10}$$

and *Majority in Echo Chamber* as

$$Majority\ in\ EC_t = \frac{|Expressive\ Nmajor\ with\ (ECI \geq 0.50)|}{|Expressive\ Nmaj_t|}. \tag{5.11}$$

In conclusion, we can say that there are many different outputs when measuring outcomes. We summarize the primary outputs and their descriptions in Table 5.3.

Table 5.3. Primary outputs.

Name	Description
Silent Minority Percentage	Percentage of silent minority agents within minority group
Silent Majority Percentage	Percentage of silent majority agents within majority group
Total Expressive Percentage	Percentage of expressive agents in total
Percentage in Echo Chamber (Percentage in EC)	Percentage of expressive agents which are in an echo chamber
Percentage in Minority Illusion (Percentage in MI)	Percentage of majority agents which think their percentage is less than the actual percentage

5.5. Verification and Validation

We test the model and find that there are no running or syntax errors. We verify that the commands work properly using output plots, output results, and model visualizations.

In the model, the expressive minority group is blue and the expressive majority group is red. When they are silent, their color is pale blue and pale red, respectively. We look at the initial number of agents in these colors and compare them to the population size and the percentage of minorities. If the population size is 1000 and the percentage of minorities is 40%, we know that 600 agents belong to the majority and 400 agents belong to the minority. As mentioned earlier, we give them a 50% chance to express their opinion. We test the model to see if this is true. First, we see that in the model 201 agents from the minority group are silent and 298 agents from the majority group are silent. These numbers show us that almost half of the agents are initially silent, while the other half are expressive. Of the 1000 agents, 201 are pale blue, 298 are pale red, 199 are blue, and 302 are red. We see that the agents change their colors when they change their expression decisions.

In the model description, we claimed that agents in dynamic networks try to preserve their base sociability. To check the accuracy, we consider the number of agents whose current neighbors are equal to their base sociability. For Figure 5.3, we use a model with N = 500, m = 20% in the H-O network with *opinion-biased* re-linking rule and hd = 75%. This model reaches equilibrium before 100-time steps, so it is sufficient to show the time trajectory graph for 100-time steps only.

Figure 5.5 shows the number of agents that have the same number as their base sociability and fewer or more neighbors than their base sociability. We see that the majority of agents maintain their base sociability.

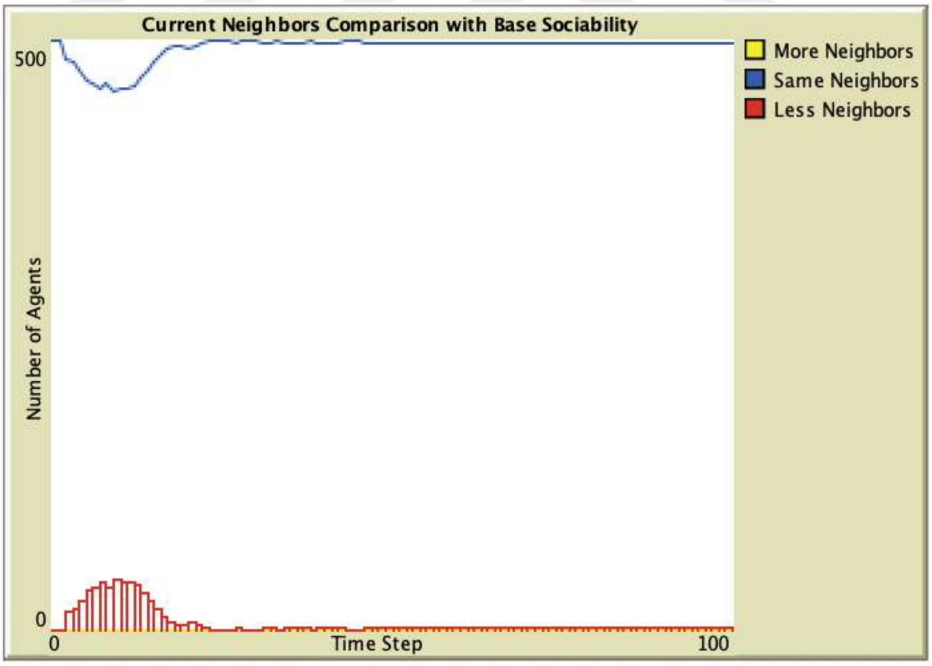

Figure 5.5. Comparison of current neighbors with base sociability.

Figure 5.5. also shows that if the initiator agents do not find other initiator agents to connect with, they cannot preserve their base sociability, but they continue their

search. In subsequent time steps, they may rewire to preserve their base sociability if they can find initiator agents. However, as we have claimed, in this model no agent has more neighbors than its base sociability in the model.

It has been argued that agents choose to become expressive through the influence of opinion climate in the formation of a spiral of silence [5]. In Figure 5.9, we run the model with m = 20% for 500-time steps; 250-time steps in the static network, and 250-time steps in the homophily-oriented network with *opinion-biased* re-linking rule with hd = 75%. We find that in the first 250 time steps, the minority's perceived minority percentage is 0% and they are in complete silence. However, their perception changes as they begin to cut their links with majority agents and create links with with minority agents; it increases to over 50%. While perception increases, the percentage of silent minorities decreases. Thus, we observe that the influence of the opinion climate changes the percentage of silent minorities in Figure 5.6.

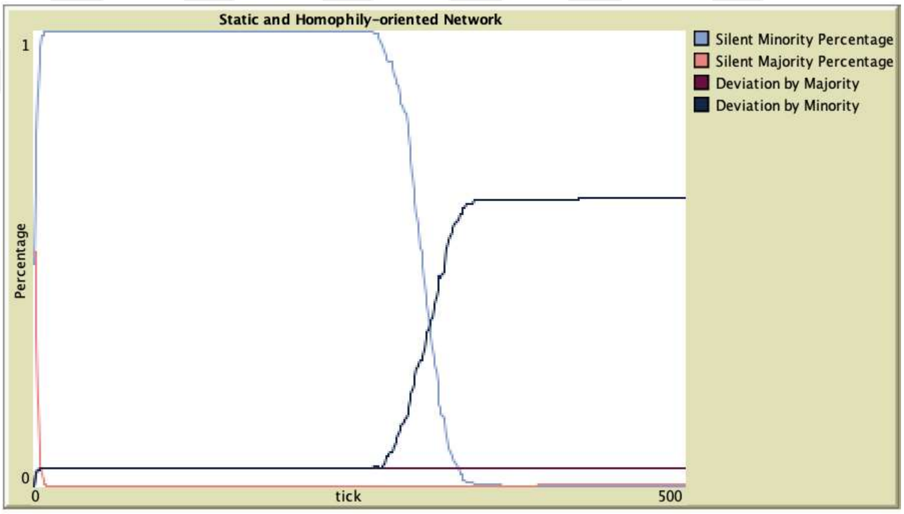

Figure 5.6. Expressiveness and opinion climate relation.

To observe majority illusion, it is claimed that we must observe the friendship paradox in the network [42]. The friendship paradox is a condition when an agent's number of neighbors is less than half the number of neighbors of its neighbors, and it is measured by $Friendship\ Paradox\ Index\ (FPI)$ as

$$Friendship\ Paradox\ Index_{it} = \frac{|Current\ Neighbors_{it}|}{|Average\ Number\ of\ Neihgbors\ of\ Neighbor_{it}|}. \tag{5.12}$$

and we calculate $Percentage\ in\ Friendship\ Paradox$ as

$$Percentage\ in\ Friendship\ Paradox_t = \frac{|N\ with\ (FPI \geq 0.50)_t|}{N}. \tag{5.13}$$

As we calculate it at each time step, we see that it is at least 80% in each of the models with any parameters in each experiment. If the majority of people find themselves in a friendship paradox, then it is sufficient for the majority illusion to occur [42]. Thus, we know that the circumstances are favorable for the majority illusion to occur.

6. BASE MODEL EXPERIMENTS AND RESULTS

This study is about the change in expression dynamics of minorities in different networks, mainly in homophily-oriented networks and our primary output is the silent minority percentage, as we stated before. However, we also want to know how the majority is affected by the minority expression dynamics and if these two groups can form echo chambers. Thus, the percentage in echo chamber and silent majority percentage are also considered our primary outputs. We will look at other outputs we have defined if they are necessary for the context.

The total population size in this model will vary between 500 and 1000 throughout the experiments to reduce the time complexity of the experiments.

We want to examine the echo chamber formation in the model with dynamic links, so we do not want to start with clustered local communities or disconnected neighborhoods. We would also like to distinguish between neighbors, neighbors of neighbors, and the agents near agents, since we have proposed different re-linking rules for these groups of agents and will use these rules in the experiments on homophily-oriented networks. When the clustering is high, we can find that the nearby agents and the neighbors of neighbors are the same agents; the same is true for the neighbors and the second-degree neighbors. So, we will determine the average node degree (nd) with respect to these issues. We will calculate the clustering coefficient for different population sizes (N) and determine which value we should use for the average node degree parameter. We determine five different values for the average node degree, which are shown in Table 6.1.

Table 6.1 shows that the clustering coefficient is at most 4.44% when the average node degree is two. This means that, on average, the agents' neighbors are neighbors with each other at a ratio of at most 4.44%. The difference between the clustering coefficient in different population sizes when the average node degree is equal to ten is too large, which could affect the consistency of the experiments. We now have three different options and select the average node degree of six. Due to time constraints, we will not conduct experiments with other average node degree values, therefore with other types of societies. This could be a topic of another study in the future.

Table 6.1. Clustering coefficient in different nd and N values.

	nd = 2	nd = 4	nd = 6	nd = 8	nd = 10
N = 500	4.44%	14.73%	23.66%	32.94%	42.22%
N =1000	1.61%	7.15%	11.24%	16.37%	19.94%

In all experiments, we get results from 50 repetitions. Some models reach equilibrium in 100-time steps and some require more than 750-time steps. Therefore, we will run each experiment for 1000-time steps. We are interested in the final results that each model gives after reaching equilibrium. We will talk about the time trajectory of the results if this is necessary for the explanation. We will also look at the variances of these 50 replications to see if the model gives stable results, if necessary. In initial time steps, agents in both groups express their opinion with a probability of 50%, as we mentioned before. So we expect them to start the experiments with 50% of silent percentage.

In the base model, there are three different types of networks: static, randomly dynamic (R-D), and homophily-oriented (H-O). We start with the static network to see how the percentage of the minority in the society and the population size of the society affect the dynamics of minority opinion expression.

After comparing the results of the static network with the R-D network, we will select the parameter sets that we will work with in H-O networks. We will study H-O networks in terms of different homophily types. Finally, we will compare the networks with the selected parameters with respect to the primary outputs.

6.1. Experiments on Static Networks

In studying the static networks, we start with two different population sizes (N) and four different minority percentage (m) values. We chose a population size of 500 and 1000 and a minority percentage of 10% to 40% with a discrepancy of 10%. Since the minority must always be less than half of the population, the highest value we use for the minority percentage will be 40%.

Table 6.2. Results in static networks.

	Silent Minority Percentage (%)		Percentage in EC (%)	
	N = 500	N = 1000	N = 500	N = 1000
m = 10%	100	100	88.69	88.47
m = 20%	99.68	99.94	73.29	73.28
m = 30%	96.74	99.79	55.09	54.93
m = 40%	81.63	91.46	36.96	36.14

Table 6.2 illustrates that both the silent minority percentage and the percentage in echo chamber decrease as the proportion of minorities in society increases, regardless of population size. However, even when the minority percentage is 40%, the percentage of the silent minority is above 80%.

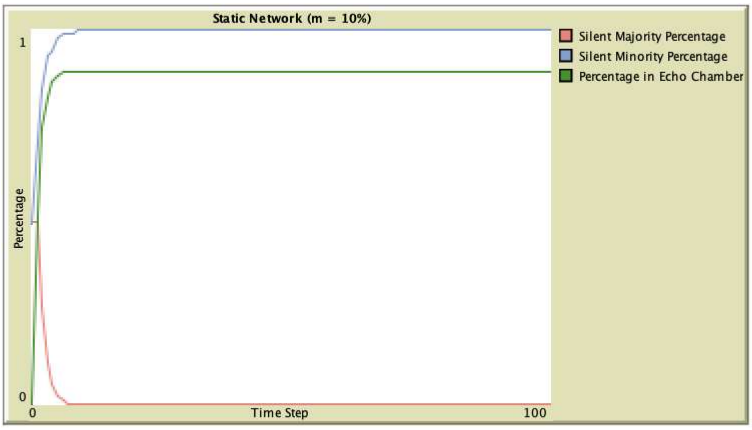

Figure 6.1. Silent percentages in static network when m = 10%.

Figure 6.1 and Figure 6.2 show that the percentage of the silent minority increases directly from 50% and the percentage of the silent majority decreases directly from 50%, and the discrepancy between the percentages of the silent minority and the silent majority decreases as the percentage of the minority increases. The percentage in echo chamber is not calculated in the initial state which is the reason why the result is zero. We observe that the percentage in echo chamber increases until equilibrium is reached.

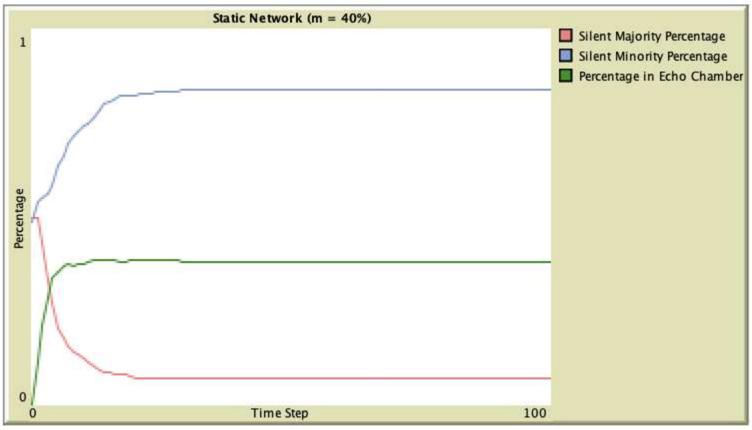

Figure 6.2. Silent percentages in static network when m = 40%.

We select the parameter sets that yield maximum, minimum, and average values with respect to the silent minority percentage in Table 6.3.

Table 6.3. Set of parameters that yields different levels of silent minority percentage.

Level	Parameter Set
Max (100%)	N = 500 and m = 10% N = 1000 and m = 10%
Avg (99.68%)	N = 500 and m = 20%
Min (81.63%)	N = 500 and m = 40%

In Table 6.3 we see that the maximum spiral of silence formation occurs when the minority percentage is lowest, and the minimum spiral of silence formation occurs when the minority percentage is highest. However, we can see that the maximum spiral of silence occurs at both values of the population size, but the minimum only occurs at a population size of 500. Before analyzing the differences between the population sizes, we test whether there is a statistically significant difference in key outputs between the minimum, average, and maximum levels in the static network. We choose the parameter set for the maximum level with 500 people and a minority percentage of 10% because we want to have the same population size for all levels. Therefore, the comparison of these levels also tests the impact of the minority percentage on the results.

We perform the two-tailed paired t-test when we examine whether there is a statistically significant difference in output between two parameters. Because we are interested in the differences between the models with the same outputs. In this study, all values in the tables are the average of the final results from 50 different replicates of the same model. When we compare the values, we mean that we are comparing the values of a sample of 50.

Table 6.4. Silent minority percentage(%) comparison between different levels.

Compared Levels	Compared Values (%)	p-value
m = 10% - m = 30%	81.63 - 96.74	$< .001$
m = 30% - m = 40%	96.74 - 100	$< .001$

In Table 6.4 and Table 6.5, we test the hypothesis that "there is no statistically significant difference between the compared values" with a confidence level of 0.99, and according to p-values we reject the hypothesis. In other words, there is a statistically significant difference in the silent minority percentage and the percentage in echo chamber between different minority percentages.

On the other hand, Table 6.5 shows that while the percentage of the silent minority decreases slightly, the percentage in the echo chamber decreases sharply as the percentage of the minority increases. To interpret it, we calculate the total expressive percentage. When we multiply the percentage of expressive minority and the majority by their total percentage, we find that 89.99% of the agents are expressive with 10% m, 79.97% with 20% m, 70.01 with 40% m, and finally 61.01% with 30% m. The decrease in total expressive percentage is small compared to the decrease in the percentage in the echo chamber. This means that people who express themselves are exposed not only to their own opinion, but also to the opposite opinion, since there are now expressive minorities in the model.

Table 6.5. Percentage in EC(%) comparison between different levels.

Compared Levels	Compared Values (%)	p-value
m = 10% - m = 30%	88.69 - 55.09	< .001
m = 30% - m = 40%	55.09 - 36.96	< .001

For the minority group, we do not see that their expressiveness does not increase the percentage in the echo chamber. So we can say that the increased expression of minority opinions does not increase the minority percentage in the echo chamber, but also decreases the majority percentage in the echo chamber. We look at the percentage of minorities and the majorities in the echo chamber separately to see if this is true. To examine how they change, it is sufficient to look at these values in the minimum, average, and maximum values from Table 6.3.

Table 6.6 shows us that our inference is correct and that the decrease in the percentage in EC is directly proportional to the decrease in the percentage of the majority in EC. The minority percentage in the echo chamber in the model is only 6%, even at its maximum level of expression. So the minority group are not clustered within themselves and when they are expressive, they cannot form echo chambers.

We found that the silent minority percentage is lower when the population size is 100 than when it is 500, when m = 30% and m = 40%. We perform a two-tailed t-test to determine if there is a statistically significant difference in the percentage of the silent minority between the different population sizes. We do not apply the t-test when the minority percentage is 10% because the average of the silent minority percentage is 100% with a variance of 0% for both population sizes.

Table 6.6. Percentage in EC(%) comparison between minority and majority.

	Percentage in EC(%)	Minority Percentage in EC (%)	Majority Percentage in EC(%)
m = 10%	88.69	0.00	98.54
m = 30%	55.09	1.03	78.26
m = 40%	36.96	6.27	57.43

According to the p-values in Table 6.7, there is a statistically significant difference in the silent minority percentage between N=500 and N=1000 when the minority percentage is 30% and 40%, but not when the minority percentage is 20%. This is due to the fact that the network density changes when the population size changes. When we increase the population size from 500 to 1000, the network density decreases from 0.1224 to 0.0606. When it decreases, the expression dynamics in the static networks changes, as it is claimed in [43]. As the minority percentage increases, we expect the results to differ in both populations, and with higher density, the variance of the results is likely to increase, as we see in Table 6.8.

Table 6.7. Silent minority percentage(%) comparison between different N values.

Parameter	Compared Values (%)	p-value
m = 20%	99.68 - 99.94	.228
m = 30%	96.74 - 99.79	$< .001$
m = 40%	81.63 - 91.46	$< .001$

As the minority percentage decreases the silent minority percentage increases for population sizes of 500 and 1000. We will continue our experiments with a selected population size because both yield similar results. There is a difference between population sizes with larger minority percentages, so we are looking at variances because we prefer a sample that produces results with less variance when we run the experiments.

Table 6.8. Variance(%) of sample set in silent minority percentage.

	N = 500	N =1000
m = 10%	0.00	0.00
m = 20%	0.02	< 0.001
m = 30%	0.09	< 0.001
m = 40%	0.84	0.80

The variance is zero for m = 10% in both population sizes, as we see in Table 6.8. For other minority percentages, the variance is smaller when the population size is 1000. Because of this small difference and the fact that we can work with lower variances, we choose a population size of 1000 for the base model. We will also look at the results when the population is 500 when we perform the analysis of the minimum, average, and maximum levels in selected networks.

6.2. Comparison of Static and Random-dynamic Networks

Up to this point, we have done all the experiments in static networks. We want to find out how the expression dynamics of minorities and the formation of echo chambers change when we allow agents to cut their existing links and connect to other agents randomly in Random-dynamic networks. We start with the comparison of static and R-D networks with respect to the silent minority percentage.

Figure 6.3 shows that the networks give different results when the percentage of the minority is 30% and 40%. We expected this because the links are cut randomly and the majority can become silent if they end up in a neighborhood whose local majority is the global minority. Thus, we expect to observe a majority illusion when the percentage of the minority is 30% and 40%. Therefore, we look at how the percentage in MI changes when we change the minority percentage.

Table 6.9 and Table 6.10 shows that when the minority percentage is at its lowest, the percentage of the majority group which is in a majority illusion is zero. However, as the silent minority percentage decreases, there are minority agents who get this illusion. This tells us that there is an inverse relationship between the percentage of the silent minority and the percentage of the majority illusion. Majority illusion means that part of the majority agents think they are the minority and in this case, there must be minority agents who think they are the majority. In R-D networks, randomly cutting links and rewiring would face off the majority agents to expressive minorities more. As the minority percentage increases, the number of expressive minorities also increases; their existence in the neighborhood could cause the majority to think minority percentage is more than it actually is. As we know perception has more impact than reality on expression decisions, we expect minority agents to express their opinions more when there is a majority illusion. We observe that R-D networks give a lower silent minority percentage and there are majority agents who are in this illusion except when the minority percentage equals 10%.

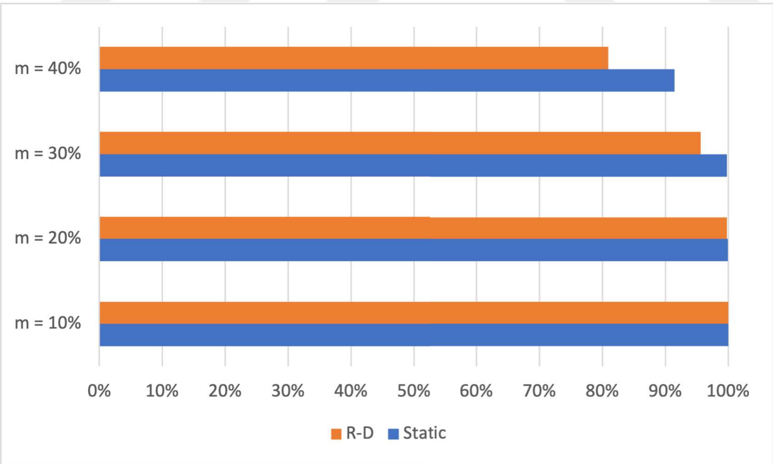

Figure 6.3. Silent minority percentage(%) comparison of static and R-D networks.

We look at the silent majority percentage to see if the percentage in majority illusion and the percentage of the silent majority are in a relationship. Table 6.9 and Table 6.10 show that the relationship of these two outputs are directly proportional in static and R-D networks.

We know that the model gives extreme values for the percentage of silent minorities at m = 10%, and we do not want to work with extreme cases. Since we want to distinguish the minority from the majority, we also exclude 40% of the minority percentage. Thus, we have two options that give similar results in the static network and slightly different results in R-D networks. For time efficiency, we choose one of them. From now on, we run all experiments with N = 100 and m = 20%, unless specified otherwise, and call it the base model. Table 6.11 shows that the other key outputs comparison when N = 1000 and m = 20%.

Table 6.9. Percentage in MI(%) comparison between static and R-D networks.

Network Type	m = 10%	m = 20%	m = 30%	m = 40%
Static	0.00	0.01	0.03	2.24
R-D	0.00	0.02	1.24	8.26

Table 6.10. Silent majority percentage(%) comparison of static and R-D networks.

Network Type	m = 10%	m = 20%	m = 30%	m = 40%
Static	0.01	0.05	0.08	4.02
R-D	0.05	0.23	4.21	18.47

Table 6.11 shows that the percentages of the silent minority are above 99%, and the percentages in the echo chamber are above at least 73% for each network. We apply a two-tailed t-test to test the hypothesis that “there is no statistically significant difference in outputs between static and R-D networks” with a 0.95 confidence level. We fail to reject the hypothesis according to p-values except for the percentage in echo chamber. While the difference is statistically significant, the values are not significant, so we neglect it.

Overall, we can say that in both static and R-D networks, the minority is oppressed by the majority, they fail to break the spiral of silence and an oppressed majority is not possible. While almost all majority group is expressive and almost all minority group is silent, the percentage in the echo chamber is only 73.28%. As we know, the percentage in echo chamber is calculated for expressive agents only and the expressive agents are from the majority group. When calculated, we see that 91.60% of the majority group in echo chamber is in the static network. If the same calculation is performed for R-D networks, a similar result is obtained. This tells us that the silent minority group is the majority in local neighborhoods of a few expressive majority agents. However these silent minorities are not in the same neighborhood so they cannot communicate with each other to form an echo chamber.

Table 6.11. Output results comparison between static and R-D networks.

Output	Static Network	R-D Network	p-values
Silent Minority Percentage	99.94	99.81	.027
Silent Majority Percentage	0.05	0.23	.004
Percentage in MI	0.01	0.02	.228
Percentage in EC	73.28	73.22	.335

We want to learn how silent minority expression changes when we give them a chance to nurture their relationships with other minority agents. Therefore, we look at the dynamics of the expression of minority opinions in homophily-oriented networks.

6.3. Experiments on Homophily-oriented Networks

After analysing the results in random-dynamic networks, we look at how homophily-oriented network changes the output results. We investigate whether the degree of homophily and the different re-linking rules make a difference in the results. There are six different re-linking rules in the homophily-oriented (H-O) networks.

To analyze the re-linking rules, we use the categorization of the rules based on the choice of agents they link to in Table 5.2, as we mentioned in the previous sections.

6.3.1. Analysis of the Homophily Degree and the Re-linking Rules

In this section, we will look at the impact of homophily degree and the re-linking rules in the results in general. Before analyzing them, recall that we refer agents who want to cut their links and rewire as initiator agents, establishing a new relationship can only be done between initiator agents, and the rules determine the initiator agents that connect to other initiator agents.

Initiator agents can choose any agents at random, they can choose one of the agents nearby, or they can choose one of their neighbors' neighbors. As we mentioned earlier, selecting an agent nearby means that we build an alternative neighborhood to the base neighborhood. We determine the nd value based on the re-linking rules. Our concern was to distinguish these rules that allow initiator agents to establish links with one of the agents from the cluster of neighbors of neighbors and the cluster of nearby agents from the neighbors of initiator agents. Figure 6.4 shows that both re-linking rules provide the initiator agents with different neighbors from their social circles.

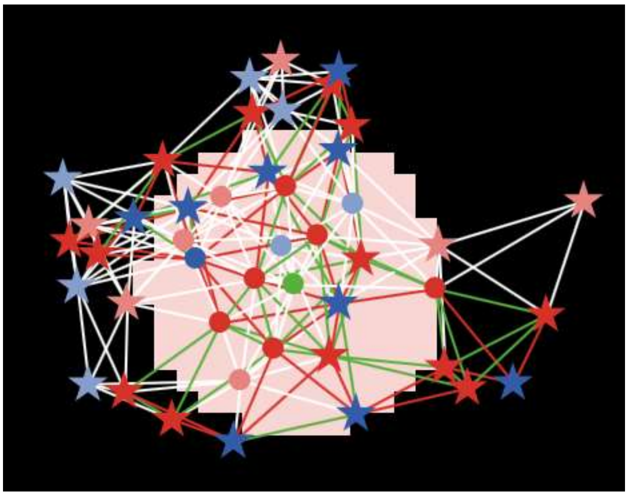

Figure 6.4. Model visual of different agent groups.

In Figure 6.4, we color the initiator agent green to distinguish it from the other agents. The pink-colored area represents the initiator agent's spatial proximity in which nearby agents are located, circle-shaped agents indicate the neighbors of the initiator agent, and the star-shaped agents represent the second-degree neighbors of the initiator agent. We see that these three groups overlap, but they are not the same. When the initiator agents are rewired according to these two rules, their neighborhoods do not remain the same.

The homophily degree represents at least what percentage of similarity an agent seeks in its “expressive” neighborhood and it is the same for all agents. We want to analyze it because the homophily degree (hd) determines which agents can cut their links in the network.

We begin with the analysis of the effects of homophily degree on the silent minority percentage. We take two values for the homophily degree to test whether it makes a difference in the model: 50% and 75%. If the homophily degree is 50%, it represents the agents who do not want to be in the minority in their neighborhood; and if it is 75%, it stands for the agents who certainly want to be the majority in their neighborhood.

Table 6.12 shows that a larger homophily degree gives a lower silent minority percentage in all re-linking rules except *second-degree* re-linking rule. We conduct a two-tailed t-test with the hypothesis “there is no statistically significant difference in silent minority percentage between the model with 50% hd and the model with 75% hd” for each re-linking rule with a confidence level of 0.99. According to p-values in Table 6.12, we fail to reject the hypothesis for *unbiased*, *geo-biased*, and *second-degree* re-linking rules and we reject it for the other re-linking rules.

Changing the homophily degree gives statistically significant different values for the re-linking rules with homophily. We detect the lowest silent percentage occurs when the homophily degree equals 75% with *opinion-biased* re-linking rule. The re-linking rule *opinion-biased* allows agents to create links with any expressive agent in the same opinion. Increasing the homophily degree makes more agents cut their links and as a result, there are more agents with fewer neighbors than their base sociability. So, more agents want to create links with the same opinion agents. However, when we look at the output results, we see that the difference is not significant so we neglect them even though it is statistically significant. Thus, we can say that change in homophily degree makes no significant difference in the silent minority percentage in any of the re-linking rules.

Table 6.12. Silent minority percentage (%) comparison of different hd values.

Re-linking Rules	hd = 50%	hd = 75%	p-value
unbiased	11.09	7.34	.422
geo-biased	0.54	0.44	.017
second-degree	61.79	64.87	.034
opinion-biased	0.36	0.05	< .001
$geo - opinion - biased$	0.53	0.28	< .001
biased-second-degree	0.52	0.27	< .001

Agents make connections to agents with the same opinions, and we know from Table 6.12 that minorities also become expressive. We expect the percentage in echo chamber to converge to 100% since agents are now both expressive and have links with similar agents. The percentage in echo chamber with the *opinion-biased* re-linking rule is 99.98% when hd equals 75%, as we can see in Table 6.13.

Table 6.13 shows that the percentage in echo chamber is lower when the homophily degree equals 50% for each re-linking rule. We test the hypothesis "there is no statistically significant difference in the percentage in echo chamber between the model with 50% hd and the model with 75% hd" for each re-linking rule with a confidence level of 0.99.

We reject the hypothesis for the re-linking rules when there is homophily in rewiring. Unlike the silent minority percentage test, we also reject this hypothesis for *geo-biased* re-linking rule. Thus, we can interpret that there is a statistically significant difference in the percentage in echo chamber between the model with 50% homophily degree and the model with 75% homophily degree for *geo-biased* re-linking rule. We can explain this difference by the nature of this re-linking rule; *geo-biased* re-linking rule allows initiator agents to connect with the others in a spatially close area only. While increasing the degree of homophily gives more agents the freedom to cut links with expressive majority agents, the number of initiator agents nearby remains the same. In other words, the number of disagreeable neighbors of agents decreases, while the number of agreeable neighbors remains the same as we increase the homophily degree. So that the percentage in echo chamber increases.

Table 6.13. Percentage in EC (%) comparison between different hd values.

Re-linking Rules	hd = 50%	hd = 75%	p-value
unbiased	67.66	68.50	.490
geo-biased	95.25	99.89	$<$ .001
second-degree	82.33	82.39	.850
opinion-biased	97.57	99.98	$<$ .001
geo-opinion-biased	96.21	99.92	$<$ .001
biased-second-degree	97.91	99.89	$<$ .001

The percentage in echo chamber converges to 100% when the silent minority percentage converges to 0%. However, if the silent minority percentage is greater than 60%, the percentage in echo chamber is still greater than 80%. Thus, we cannot speak of a correlation between these two results. Since the echo chambers are formed by the total expressive size of the population, we need the total expressive percentage to see if there is a correlation or not.

Figure 6.5 shows that silent majority percentage is significant in H-O networks only with the *unbiased* re-linking rule since it is over 37% in both hd values and it is greater than the silent minority percentage. We can say that minority oppress the majority. We look at the percentage in majority illusion to see if it gives a significant result and see that it is about 14% in both hd values. As these are significant results, it is lower than the silent majority percentage, this means that majority group has difficulty connecting with their own group even though they know they are still the majority, in *unbiased* re-linking rule. So the reason behind the silence of the majorities must be something different than the current state of the majorities. Because the percentage in the majority illusion shows the current perception, the silence is a cumulative result. To learn why majorities are oppressed by the minority, we will analyze the re-linking rule in detail in as we refer to it as H-O Type 1.

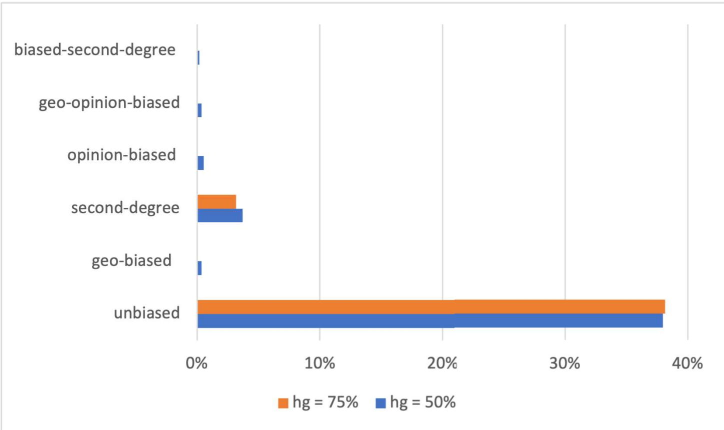

Figure 6.5. Silent Majority Percentage(%) comparison of different re-linking rules.

There are no significant difference in the silent minority percentage and the silent majority percentage between the hd values; we select one of them for the analysis, 75%. H-O Type 1 gives below 10% of minority and approximately 40% of majority as silent.

First of all, we will look at the time trajectory graph of H-O Type 1 to see how the minority oppresses the majority over time. The word “oppress” is used here to define the situation where the minority is more expressive than the majority in percentage terms. To speak of oppression, the minority does not have to be fully expressive, but we still expect at least half of the minority group to be expressive.

Figure 6.6 shows the time trajectory graph of the silent majority and the silent minority in H-O Type 1. Percentage of the silent minority increases while the silent majority percentage decreases in the first time steps, just as in static networks. Even though their behaviours in the first time steps show similarities with in the static networks, behind the scenes initiator agents cut one of their suitable links for cutting since this is a H-O network.

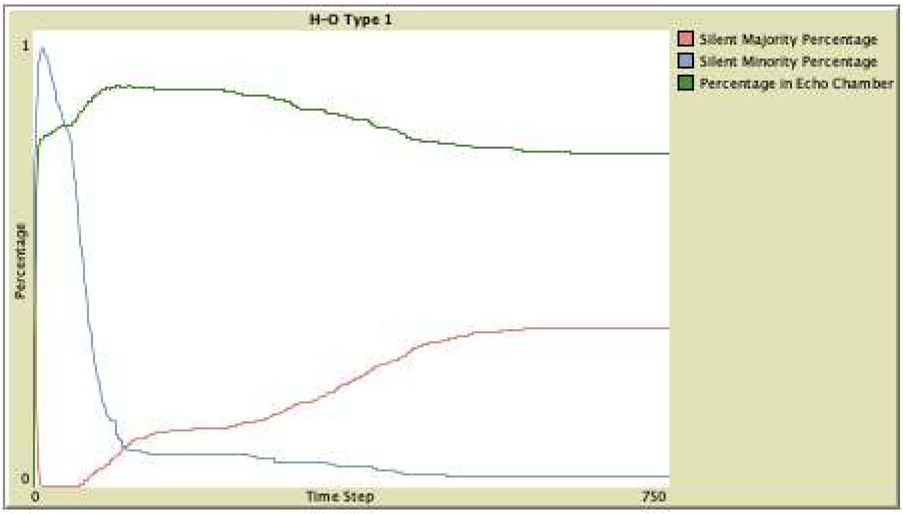

Figure 6.6. Time trajectory graph of H-O Type 1.

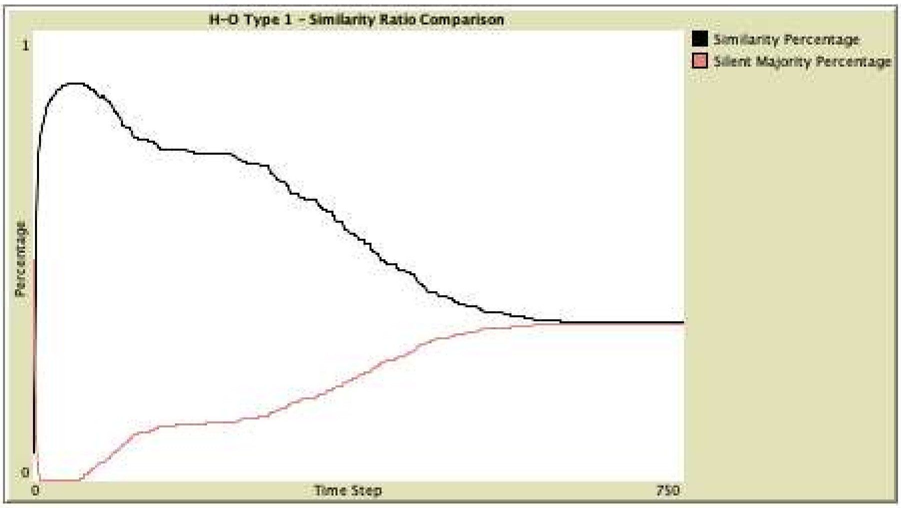

Figure 6.7. Comparison of similarity ratio satisfaction and becoming silent.

In the peak of the silent minority percentage, we see that almost 4% of the silent group is expressive. This would not cause the majority agents to become initiator agents. To become initiator agents, agents must both be dissatisfied with their similarity ratio, which means their similarity ratio is lower than the homophily degree, and their links must be suitable for cutting. Majorities are mostly satisfied with their similarity ratio as we see in Figure 6.7 and only 4% of the minorities expressive, so only a few majority members have limited number of suitable links for cutting. Unlike majorities, minority agents become initiator agents as we see in Figure 6.8 starting from the first time steps. So we know that the first links cut are the links between the silent minorities and the expressive majorities. When silent minorities cut their links, they can only connect with other initiators who are expressive majorities and the silent minorities. All silent minorities cut their links until they are satisfied with their similarity ratio or they have no suitable links for cutting.

Figure 6.8. Initiator agents population size.

Since the minorities are silent in the first steps and the majorities are expressive, the current rewards of the minorities in the first steps are negative. After cutting links with the majorities, who are all expressive, they become initiator agents. Since we know that only the expressive majorities and the silent minorities are initiator agents in the first steps, they could only create link with one of the majority agents or the silent minorities. When silent minorities connect with a majority agent, nothing changes and they remain silent. If they connect with another silent minority agent, they could become expressive. As we can see from Figure 6.9 and 6.10, a minority agent becomes expressive when the number of disagreeable neighbors decreases. This is because its expected reward was negative before it cut links. A current reward that is still negative but has a higher value than the expected reward increases the minority agent's expected reward and agents reconsider their decision to express when their expected reward increases. Thus, in this case, the minority becomes expressive. As we see in Figure 6.9, expected reward of this minority agents is still negative when it become expressive. However, after a while all majority neighbors of this minority agents become silent. As the all expressive neighbors of this agent are agreeable neighbors, this agent believe it is the majority in society. So we can talk about a majority illusion in this case.

At this point, we need to recall how agents make their expression decisions. Agents evaluate their expression decision precisely when they are expressive and their expression probability decreases or when they are silent and their expression probability increases. The change in the expression probability is caused by the change in current reward and expected reward. Once an agent is silent, it will continue to be silent if its current reward and expected reward are constant or decreasing; however, if the reward increases, it will reconsider its decision. The probability of speaking up is higher when the expected reward increases, so these agents are likely to speak up.

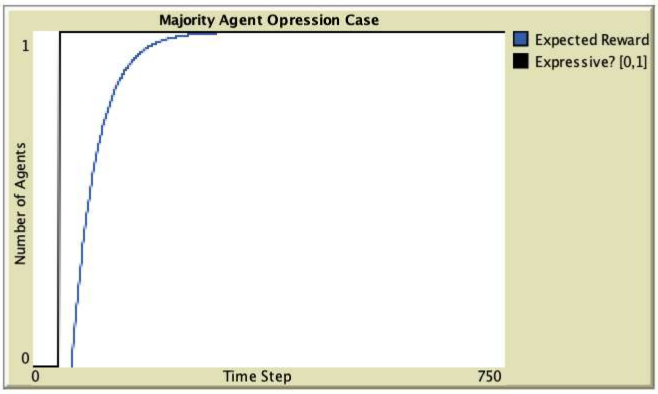

Figure 6.9. Expected reward and expressiveness of a minority agent.

When some minorities become expressive while reducing their disagreeable neighbors, they increase the reward for some minorities who are still silent and encourage them to speak out as well. While encouraging other minorities, we see that they also decrease the current rewards and the expected rewards of the majority agents. In Figure 6.11, we see how the expected reward of a majority agent decreases. As its expected reward decreases, the majority agent also reconsiders his decision to speak up and remains silent. As some majorities act similarly and remain silent, their silence also decreases the expected reward of some majorities and, as in a spiral of silence, part of the majority is silenced.

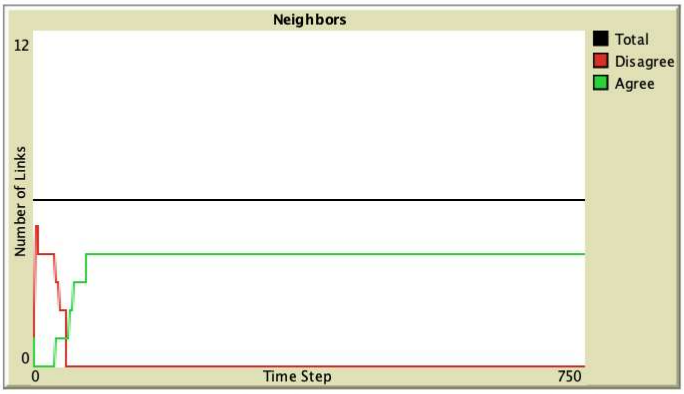

Figure 6.10. Expressive and total neighbors of a minority agent.

Now that majorities are no longer satisfied with their similarity ratio, they want to cut links. Since minorities are now expressive, they have suitable links to cut. When they cut links, the only initiator agents are the silent majorities and the expressive minorities, so they can make connections with one of these agents. They have to rewire at the time step they cut links due to the fact that the re-linking rule is *unbiased* and the probability of not finding a suitable agent in this re-linking rule is zero. So they have to cut their links just to connect to one of these agents. This does not increase the expected reward, but may actually decrease it.

Unlike the silent minority case in the first time steps, the expected reward of these silent majorities is non-negative. In the first steps, all majority agents were expressive, so their current rewards were one and their expected rewards increased sharply. Thus, creating a link with other agents of the silent majority would decrease their expected reward, since the current reward is still lower than the expected reward. Therefore, the majorities are trapped in a silence with a non-increasing expected reward and they have no chance to reconsider their decisions.

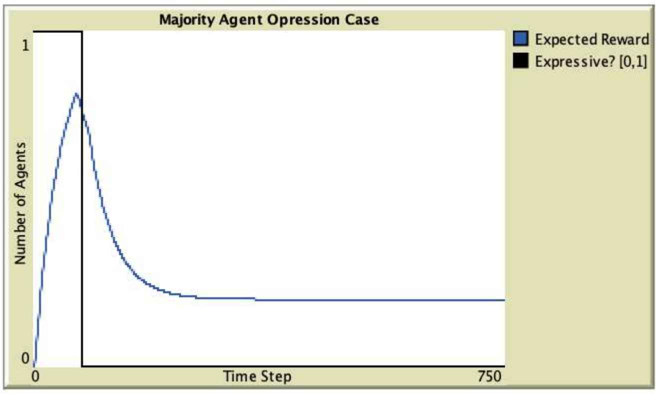

Figure 6.11. Expected reward and expressiveness of a majority agent.

Overall, majority agents start with the trust to their position in society and let the minority to change their social circles to find each other. However, minorities change the opinion climate perception of some majorities by reconsidering their decisions to become silent. Consequently, part of the majority is oppressed by the minority with a non-increasing expected reward, and cannot connect to expressive majorities because expressive majorities are still satisfied with their similarity ratios and do not become initiator agents.

The increase in the silent majority percentage until the model reaches equilibrium is similar to the formation of a spiral of silence of minority agents in a static network. The initial indifference of the majorities to their environment causes the minorities to become expressive. This triggers a trend that results in the majorities being silenced.

We have found that agents who are in the majority cannot be expressive because they are trapped in silence due to their non-increasing expected reward. With this statement, we claim that only a spark that would increase the expected reward of the majorities would change the situation. We make only one of the silent majority agents expressive by force after the model reaches equilibrium. This single change to the model leads to a cascade effect: all of the silent majority agents become expressive within a very short time and oppress the minority to lead them complete silence as we see in Figure 6.12.

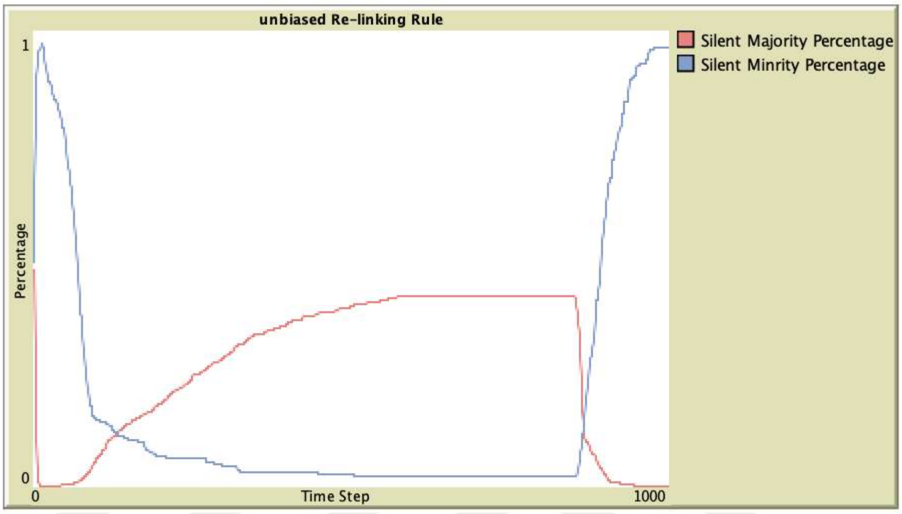

Figure 6.12. Cascade effect in majority group in H-O Type 1.

In summary, we can say that majority agents are oppressed by the minority in H-O Type 1. If an external force, as in Figure 6.12, makes only one of the majority agents expressive, this would change the current reward of all majority agents in a cascade. Thus, all majority agents become expressive in a very short time. This is the explanation for what is happening in Figure 6.12. We must note that this model would lead to completely different results for the silent percentages because of the presence of randomness in both network construction and rewiring. Thus, when we look at the results, we find that there are two outliers among the 50 replications that give the same results as the R-D network. When we remove the outliers from the data, we see that the percentage of the silent minority is on average 3.68% and the variance is 0.06%; and the percentage of the silent majority is on average 39.71 and the variance is 0.61%. These numbers are consistent with the equilibrium values in Figure 6.5.

Since this model leads to a majority oppressed by the minority, we also look at whether this is possible with different minority percentages. We analyze the model with a minority percentage between 5% and 50% in 5% increments when the population size is identical to the base model, 1000. At 50%, there is neither a minority nor a majority, but we will use these terms to avoid confusion.

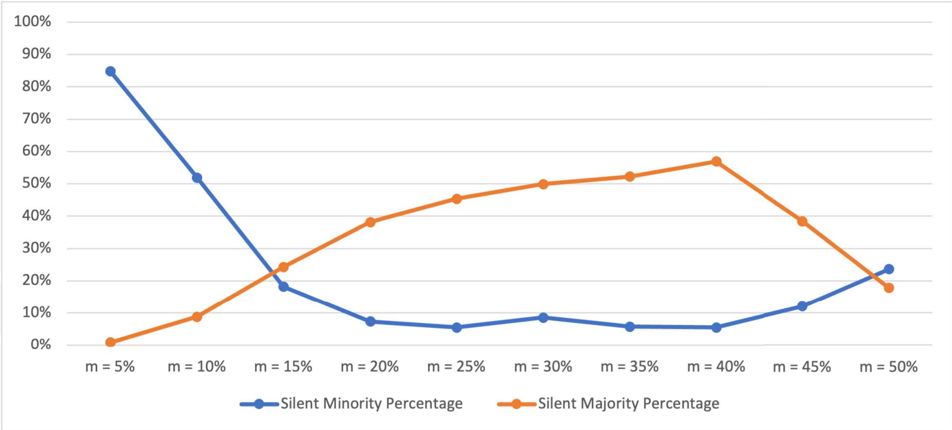

Figure 6.13. Silent percentages in H-O Type 1 in different m values.

Table 6.14 and Figure 6.13 show that there is a small difference between the percentage of the silent minority and the percentage of the silent majority when the minority percentage is 50%. When we perform a two-tailed t-test with a confidence level of 0.99 to see if this difference is significant, we find that the p-value is .3889. Thus, there is no statistically significant difference between the silent minority percentage and silent majority percentage when minority percentage is 50%. We perform the same test with minority percentage 15% and the p-value is .2964, so we can also say that there is no statistically significant difference between silent minority percentage and silent majority percentage when minority percentage is 15%. So we can say that these two percentages give a similar percentage of the minority group and the majority group; there is no oppression of any group, but they are still silent to the same extent. We can accept these two percentage as the turning points since we observe an oppression to minority below 15%, and an oppression to majority between 15% and 50%. As the minority and majority groups interchange above 50%, we do not interpret them to avoid confusion.

The number of oppressed majority increase between minority percentages of 25% and 40%. Figure 6.14 shows that the time trajectory graph when the minority percentage equals to 40% and we see similarities between Figure 6.6 and 6.11. When the minority percentage is 40%, the minority group still has the advantage of being the first initiator agents, but they are also more crowded now. As a result, they strongly oppress the majority and minorities are completely expressive.

Table 6.14. Outputs results comparison of different m values in H-O Type 1.

	Silent Minority Percentage (%)	Silent Majority Percentage (%)
m = 5%	84.80	0.91
m = 10%	51.94	8.76
m = 15%	18.19	24.33
m = 20%	7.34	38.14
m = 25%	5.50	45.38
m = 30%	8.56	49.92
m = 35%	5.74	52.26
m = 40%	5.51	56.89
m = 45%	11.98	38.45
m = 50%	23.63	17.82

Figure 6.15 shows us that minorities do not succeed in oppressing the majority in the smaller percentages of minority. Since they are outnumbered, and even if they are all expressive, it is hard to oppress majority since in some cases they cannot be local majorities with this percentage.

In Figure 6.16 model reaches an equilibrium where the both groups have the approximately same silent percentage. This shows us that one group cannot oppress the other if their percentage in society is equal. However, in their local communities they become local majorities so they can oppress the local minorities; as are results part of both groups are silent. The non-zero silent percentage of both groups support this and also support the study [24] suggest people make decisions while looking at their social circles only.

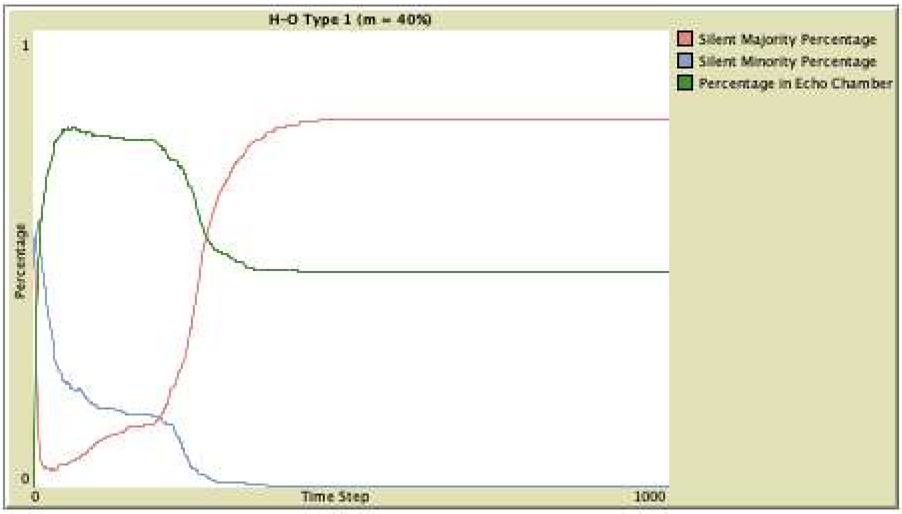

Figure 6.14. H-O Type 1 when m = 40%

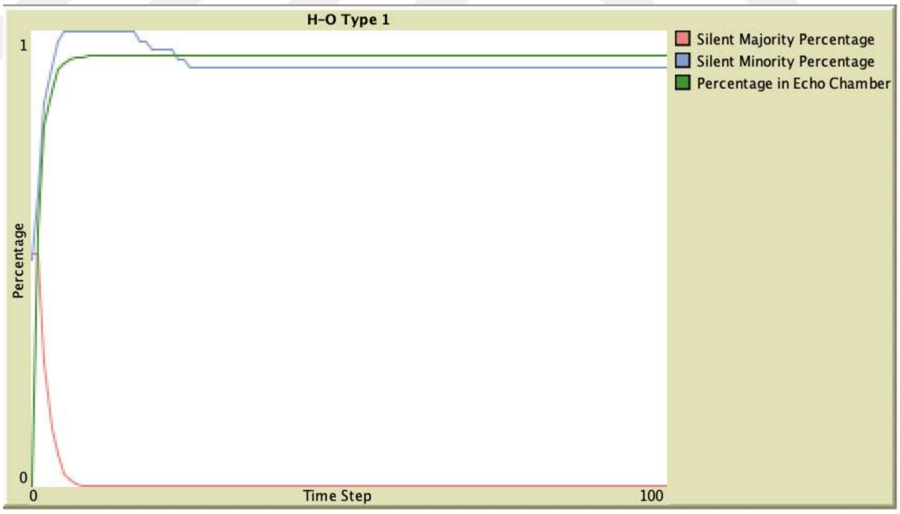

Figure 6.15. H-O Type 1 when m = 5%.

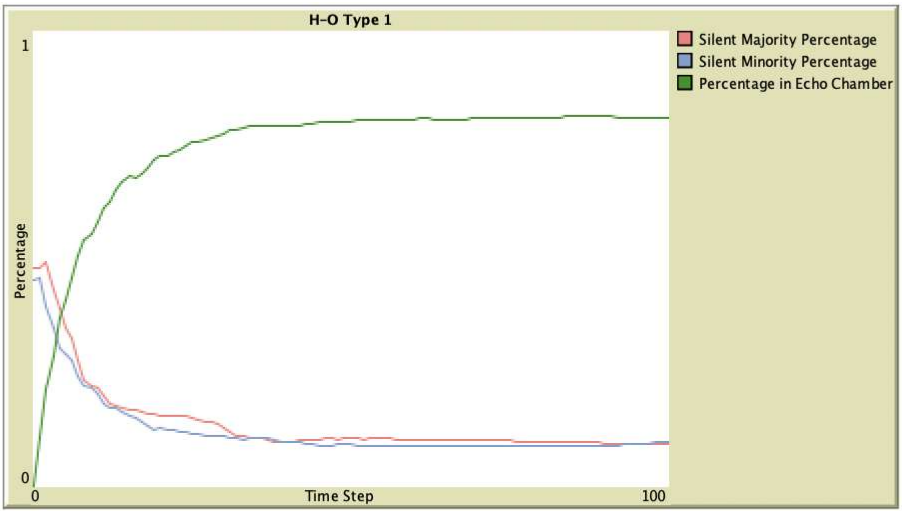

Figure 6.16. H-O Type 1 when m = 50%.

6.3.2. Comparison of Re-linking Rules

We will analyze whether the minorities are more successful in breaking spiral of silence and cluster within themselves more when there is homophily in rewiring. In H-O networks, we observe an increase in the silent minority percentage in the first steps. The reason is that the agents in the minority must first cut their links and then find each other. Therefore, the percentage of silent minority is about 50% in the beginning and increases to about 55% in the first time step. If there is homophily in the rewiring, both the silent minority percentage and the silent majority percentage fall toward zero. When there is no homophily in the rewiring, the percentages may behave different, as we see in the analysis of H-O Type 1.

When agents seek similarity in their new relationships, which means they rewire under homophily, there is a significant difference in the percentage of silent minority, regardless of the homophily degree. The only exception is the rule for re-linking where nearby agents are selected. As mentioned before, proximate re-linking rules are special cases since they limit the number of agents with which the initiator agent create link. The number of agents nearby with the same opinion and the total size of the agents nearby can be the same, so the introduction of homophily in rewiring would not effect the silent percentage of the minorities, contradictory to other re-linking rules as we see in Table 6.15.

Table 6.15. Silent minority percentage(%) comparison of different re-linking rules.

hd Values	Agent Selection	No Homophily in Rewiring	Homophily in Rewiring
hd = 50%	Random	16.61	0.36
	Proximate	0.54	0.53
	Second-Degree	61.79	0.52
hd = 75%	Random	4.73	0.05
	Proximate	0.44	0.28
	Second-Degree	64.87	0.27

For the other homophily-oriented re-linking rules, the number of agents they can create a link with is limited compared to the *opinion-biased* re-linking rule. It is expected that *opinion-biased* re-linking rule yields the lowest silent minority percentage among homophily-oriented re-linking rules; and also in H-O networks.

The largest difference in the percentage of the silent minority is between *second-degree* and *biased-second-degree*; they are 60.87% and 0.52%, respectively. Agents can connect with one of their neighbors of one of their neighbors in these re-linking rules. For each minority, it is difficult to find other minority agents among their neighbors of neighbors. To rewire, agents should break one of their links. If they miss other initiator minority agents while they are also the initiator agents themselves, they have to connect with one of the majority agents. Hence, they all fail to find each other to break the silence. As a result, they remain silent since they fear of isolation. If there is homophily in the rewiring, most of them can easily find each other and break the spiral of silence together. Breaking the spiral of silence is not a matter of each person speaking out on their own, the agents must act together to end their silence.

The maximum silent minority percentage occurs when there is *second-degree* re-linking rule in homophily-oriented network models, regardless of the homophily degree. We can say that the *second-degree* re-linking rule is the least successful homophily-oriented re-linking rule in terms of breaking spiral of silence. We compare the results of the *second-degree* rule in the H-O network with the R-D network, because we want to see if the least successful homophily-driven re-linking rule is still more successful in reducing the silent minority than R-D networks.

Table 6.16. Comparison of selected H-O network with R-D networks.

Model	Silent Minority Percentage (%)	Silent Majority Percentage (%)	Percentage in EC (%)
R-D Network	99.81	0.02	73.19
second-degree	64.87	3.18	82.39

According to Table 6.16, it is not necessary to perform the t-test because we see that there is a significant difference between the results of the R-D network and the H-O network with the *second-degree* re-linking rule. Since there is no homophily in rewiring when agents create links with others with *second-degree* re-linking rule. We can say that having homophily only in link-cutting process is enough for some minorities to become expressive. This is because they cut their links with the expressive majority agents and majority agents no longer oppress them, as is the case in R-D. Based on this result alone, we can say that minority expression is increasing under homophily.

As we know the percentages of the silent minority and the majority group now, we can calculate the total expressive percentage. The percentage in echo chambers is calculated for all expressive individuals combined. A high percentage in the echo chamber tells us that the echo chambers trap both the minority and majority groups in their own bubbles. We find that for any value of the degree of homophily, the correlation between the total expressive percentage and the percentage of echo chambers in H-O networks is greater than 0.999 in the base model.

6.3.3. Comparison of Homophily Types in H-O Networks

We find that if people cut their links under homophily but rewire randomly, it is enough to increase minority opinion expression. We want to know if it the same thing applies when people only rewire under homophily. So we will do a comparative analysis of these two different H-O networks. We found that the homophily degree makes no significant difference in silent minority percentage, we choose 75% as homophily degree value and do not make experiments with 50% hd.

Table 6.17. Output results in H-O networks without homophily in link-cutting.

Re-linking Rule	Silent Minority Percentage (%)	Silent Majority Percentage (%)	Percentage in EC (%)
opinion-biased	0.01	0.00	100
geo-opinion-biased	4.13	0.00	99.96
biased-second-degree	2.88	0.48	99.43

In Table 6.17, we see that the percentage in echo chamber converges to 100% when there is no homophily in link-cutting; this means that homophily degree has an impact on keeping different opinionated agents together. In addition, we see that the silent percentages are below 5%, regardless of the re-linking rule or group. In other words, the overall percentage of opinion expression is high and we can say that H-O networks without homophily in link-cutting is successful in breaking the silence of the minorities.

Table 6.18. Output results in H-O networks without homophily in rewiring.

Re-linking Rule	Silent Minority Percentage (%)	Silent Majority Percentage (%)	Percentage in EC (%)
unbiased	7.34	38.15	68.50
geo-biased	0.44	0.00	99.89
second-degree	64.87	3.18	82.39

Table 6.18 shows that the *geo-biased* re-linking rule gives most expressive minority percentage in the H-O network without homophily in rewiring. As in the *unbiased* re-linking rule, homophily in link-cutting makes the minority more advantageous than to the majority and can oppress majority.

When there is no homophily in link-cutting, each agent has an equal chance to cut one of its links since we do not select initiator agents. Since both groups can change neighbors simultaneously, we cannot expect the majority to remain silent. Overall, we can say homophily-oriented links without cutting homophily are more successful in creating an expressive environment, but they lead society to total segregation, regardless of the rules of re-linking.

6.4. Further Comparison of Selected Networks

Randomly dynamic networks have no homophily in both link-cutting and rewiring, so we compare the different homophily-oriented network types with R-D networks. We refer to *unbiased* re-linking rule when hd = 75% as H-O Type 1 in the previous section, we select other H-O Networks and name them as following the enumeration. We choose the *opinion-biased* re-linking rule among homophily-oriented network without link-cutting homophily and *opinion-biased* re-linking rule since they give the lowest silent minority percentage. We refer back to Table 5.1 and match it with our selected networks in Table 6.19. The details of the selected networks are summarized in the following list.

Table 6.19. Selected Network Classification.

	Random rewiring	Homophily in rewiring
Random link-cutting	R-D Network	H-O Type 2
Homophily in link-cutting	H-O Type 1	H-O Type 3

- R-D Network: There is no homophily in link-cutting and rewiring. We use the *unbiased* re-linking rule.

- H-O Type 1: There is homophily in link-cutting with hd = 75% but no homophily in rewiring. We use the *unbiased* re-linking rule.
- H-O Type 2: There is no homophily in link-cutting but homophily in rewiring. We use the *opinion-biased* re-linking rule.
- H-O Type 3: There is homophily in both link-cutting with hd = 75% and rewiring. We use the *opinion-biased* re-linking rule.

Table 6.20. Comparison of selected networks in the base model.

Network Type	Silent Minority Percentage (%)	Silent Majority Percentage (%)	Percentage in EC (%)
Static	99.94	0.05	73.28
R-D	99.76	0.23	73.22
H-O Type 1	7.34	38.15	68.50
H-O Type 2	0.01	0.00	100
H-O Type 3	0.05	0.00	99.98

In overview, we see that H-O Type 2 is the most successful network type in breaking spiral of silence but also in creating echo chambers. We can see that the society is segregated in two and none of them talk with each other while each group is totally expressive.

Figure 6.17 shows that there is a turning point in the percentage of silent minority in H-O type 2. Up to this point, the model gives results as in the static network, but thereafter the percentage of the silent minority drops to zero, while the percentage in echo chamber rises to one. This is because link-cutting and after finding similar agents takes time. Figure 6.18 illustrates that H-O Type 3 and H-O Type 2 have similar results in all three outputs.

In H-O Type 1, we find that 7.34% of the minority group is silent and 38.15% of the majority group is silent, and the percentage in echo chamber is the lowest and is 68.50%. In this network, it takes longer than in the others to reach equilibrium because the rewiring process is random. We have thoroughly analyzed H-O Type 1 in the previous sections, so we will not go into details here. The most important feature of H-O Type 1 is that it leads to suppression of the majority, the reasons for which we have already discussed.

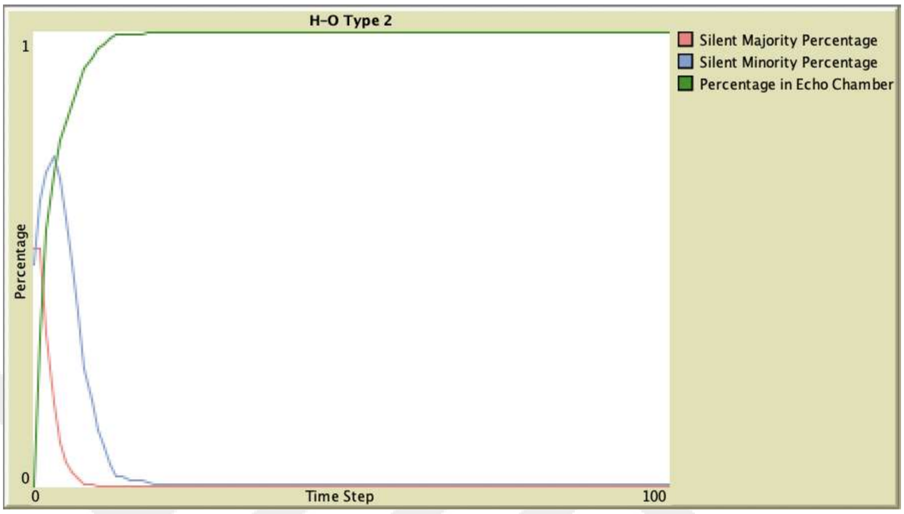

Figure 6.17. Time trajectory graph of H-O Type 2.

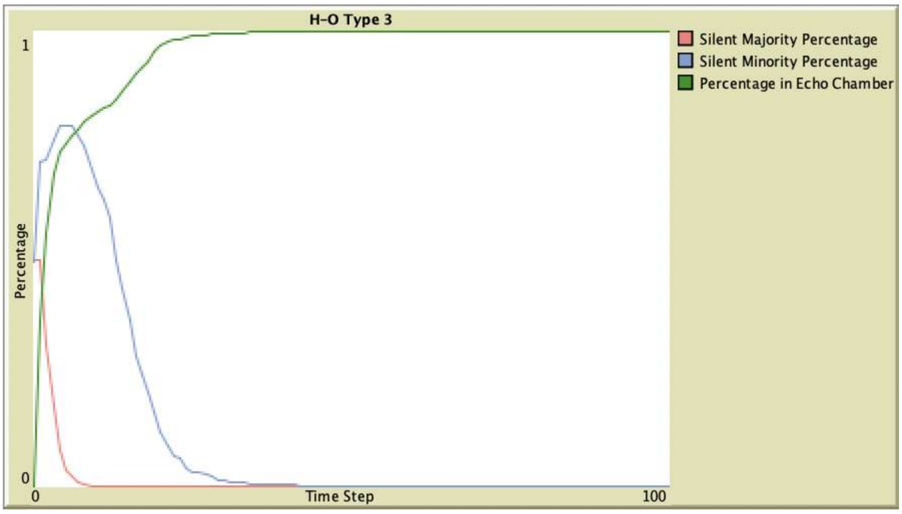

Figure 6.18. Time trajectory graph of H-O Type 3.

6.5. Minimum, Average and Maximum Level Analysis

We will check whether the parameters that yield the minimum, average, and maximum values in the static network also give the minimum, average, and maximum values in other selected networks. Table 6.19 summarizes the results in selected networks and different levels.

Static and randomly dynamic networks yield correlated results with static networks on all three outputs, and the percentage of silent minorities is at least 80% in both static and R-D networks. This changes in homophily-oriented networks and we observe full expression in both minority and majority. Table 6.21 illustrates that H-O Type 2 and H-O Type 3 lead to complete congregation and segregation in both the minority and majority, regardless of the minority percentage.

When the minority percentage is 30%, H-O Type 1 gives results similar to the base model. However, we see that a minority is more successful when it comes to expressing and oppressing the majority. We see that the percentage in the echo chamber decreases as the total expressive percentage also decreases.

On the other hand, if the percentage of the minority in H-O Type 1 is 10%, we see that the majority is not caught in a spiral of silence. This shows us that a lower minority percentage cannot suppress the majority, even if it is in an advantageous position with respect to the network type. When the minority percentage is 40%, the population size of the minority and the majority are close to each other. Thus, the advantage of being disadvantaged in the initial steps does not work very well for the minority group under these conditions. The minority group is still more expressive than the majority group in percentage terms, but we observe that the discrepancy between the percentages of the silent minority and the majority decreases when the percentage of the minority increases from 30% to 40%. This result is different from the results we obtain in the analysis of the H-O Type 1 section; it is caused by the differences in population size that affect network density as we discussed before. The oppression to majority in H-O Type 1 is related to the structure of the network, as mentioned before. Overall, we see that the only networks affected by the minority percentage change are static, R-D networks, and H-O Type 1.

Table 6.21. Output results comparison between different levels.

Output	Network Type	m = 10%	m = 30%	m = 40%
Silent Minority Percentage	Static	100	98.64	86.33
	R-D	100	99.55	92.78
	H-O Type 1	85.60	5.69	12.28
	H-O Type 2	0.00	0.00	0.00
	H-O Type 3	0.16	0.07	0.02
Silent Majority Percentage	Static	0.01	0.40	6.22
	R-D	0.00	0.36	6.53
	H-O Type 1	1.62	48.65	32.65
	H-O Type 2	0.00	0.00	0.00
	H-O Type 3	0.00	0.01	0.01
Percentage in EC	Static	88.47	55.03	36.42
	R-D	88.59	55.28	35,32
	H-O Type 1	89.32	58.10	65.62
	H-O Type 2	100	100	100
	H-O Type 3	99.94	99.96	99.96

7. EXTENDED MODEL EXPERIMENTS AND RESULTS

In this section, we will use the assumptions and definitions we used in the previous chapter. We will use the same networks that we selected in the experiments with the base model to compare the base model and the extended model.

We defined different agent characteristics in Section 5.3 and will conduct experiments with each type of agent to learn how they affect the results. As a reminder, there are three different types of agents. Hardcore agents are always expressive, loyal agents maintain their original links but can change their expressiveness, and valiant agents are always expressive and preserve their original links. We will work with selected networks: static, R-D, H-O Type 1, H-O Type 2, and H-O Type 3.

7.1. Experiments With Different Minority Characteristics

We start in this section with the experiments where only the minority group can consist of these types of agents because the dynamics of minority expression is our main concern. The population size is 1000 and the minority percentage is 20%. The experiments are conducted when half of the minority exhibit the characteristics, which means 100 agents from the minority, of the agent type in question. We refer to the minority agents that have the same characteristics as the base model as basic agents, as mentioned earlier. The silent basic percentage refers to the percentage of silent agents among the basic agents, and the expressive basic percentage indicates the percentage of expressive agents among the basic agents.

Table 7.1 summarizes the effects of different types of agents on expression and dynamics, as well as the percentage of agents in an echo chamber in different types of networks. We will analyze them separately and comparatively.

Table 7.1. Comparison of models with different minority types.

Model Type	Network Type	Silent Minority Percentage (%)	Silent Majority Percentage(%)	Percentage in EC(%)
Base Model	Static	99.94	0.05	73.28
	R-D	99.76	0.23	73.22
	H-O Type 1	7.34	38.14	68.50
	H-O Type 2	0.01	0.00	100
	H-O Type 3	0.05	0.00	99.98
Hardcore Minority Model	Static	49.04	1.26	72.93
	R-D	45.64	8.26	68.61
	H-O Type 1	15.66	6.98	84.42
	H-O Type 2	0.01	0.00	100
	H-O Type 3	0.02	0.00	99.98
Loyal Minority Model	Static	99.94	0.04	73.04
	R-D	99.55	00.39	73.14
	H-O Type 1	50.55	26.13	62.63
	H-O Type 2	47.04	1.25	88.24
	H-O Type 3	48.45	00.03	88.55
Valiant Minority Model	Static	48.90	1.36	72.95
	R-D	46.23	6.34	69.52
	H-O Type 1	13.38	25.20	64.79
	H-O Type 2	0.05	2.57	88.30
	H-O Type 3	0.00	0.56	88.89

We begin with the examining the effects of hardcore agents on the outcomes in the model. For each network type, we assume that at least half of the minority is always expressive since half of the agents are hardcore. Since they are always expressive, we will look at how the expression of other minority agents change throughout the model. In this case, silent basic percentage is important and we note that the number of hardcore agents and basic agents is equal and 100. In the base model, less than

0.1% of the basic agents are expressive. In hardcore minority model, almost 2% of the basic agents become expressive. This result as the hardcore agents influence the base agents' expressive decisions, but this influence is not necessarily large.

As for the silent majority percentage, we expect the majority to become oppressed by the hardcore minority, but only 8.26% of the majority is silent in the R-D network, compared to only 1.26% in static networks. We can say that there is a statistically significant difference in the percentage of silent majority between the base model and the hardcore minority model in both static and R-D networks, but we cannot speak of oppression because the minority is less expressive in percentage than the majority.

There is a difference in the silent minority percentage between the base model and the hardcore minority model, but it is insignificant. We look at how the percentage of hardcore agents changes the expression of basic minority agents; we define a parameter called the hardcore percentage (hp), which is the percentage of hardcore agents in the minority group. Table 7.2 shows the results of the model when the percentage of hardcore agents increases from 50% to 90%.

Table 7.2. Results comparison between different hp values.

	Silent Minority Percentage (%)	Expressive Basic Percentage (%)	Silent Majority Percentage(%)	Percentage in EC(%)
hp = 50%	49.04	1.92	1.26	72.93
hp = 60%	38.66	3.35	2.09	72.61
hp = 70%	28.79	4.03	2.73	72.41
hp = 80%	19.12	4.40	3.45	72.13
hp = 90%	9.40	6.00	4.15	72.16

In Table 7.1 and Table 7.2, we cannot see a significant difference between the percentage in echo chamber values in the static network between the base model and the model with hardcore minorities. This means that the existence of hardcore agents does not change the percentage in echo chamber in static networks.

Table 7.2 shows that the hardcore agents fail to turn the basic minority agents into expressive agents even when the percentage of hardcore agents is 90%; the percentage of the expressive basic minority is 6% while the number of basic agents is 20. Thus, it is not a significant percentage. As the number of hardcore agents increases, we see that the percentage of the silent majority increases. Even though the percentage of silent majority is still below 5%, it was below 0.1% in the base model. If there are hardcore agents in a society with a higher percentage of minorities, it may be possible to silence the majority. The oppression to the majority by hardcore minorities under these conditions could be a topic for another study.

Loyal agents are not like hardcore agents, they change their expressions to fit into their social circle. Since they are no different from basic agents except for their loyalty to their neighbors, we observe no difference in outcomes in static and R-D networks, as we can see in Figure 7.1.

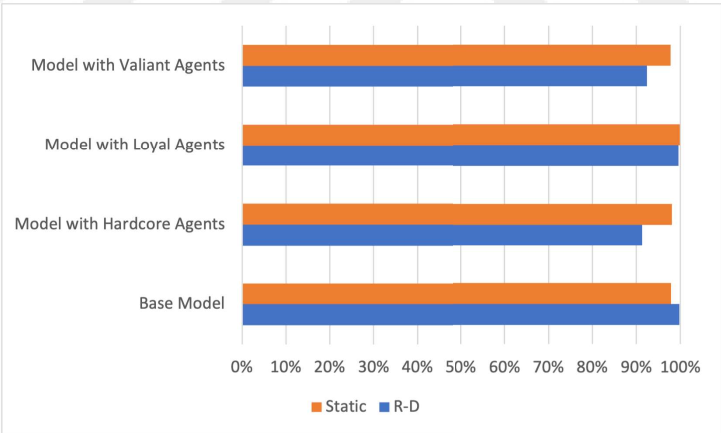

Figure 7.1. Silent basic percentage (%) comparison of different minority agents.

Loyalty to the neighbors makes no difference in static and R-D networks, so we observe no difference in the silent basic percentage between the hardcore minority model and the model with valiant minorities. There is also no significant difference between the percentage of silent majority and the percentage in echo chambers.

Valiant agents have both the characteristics of hardcore agents and loyal agents. This is because valiant agents are always expressive and do not change their social circle. As we mentioned earlier, loyalty does not make a significant difference in any of the results. We note that there is no significant difference between the models with the hardcore agents and the valiant agents in static and R-D networks; this supports that loyalty makes no difference in the results.

7.1.1. Hardcore Minority Model in H-O Networks

We know that H-O networks provide results in favor of minority expression. We would like to know how hardcore agents would change this situation.

H-O Type 2 and H-O Type 3 are extreme cases in the base model, and we get extreme results from these networks in the hardcore minority model. Figure 7.2 shows that it is similar to the base model, except that the silent minority percentage does not increase in the first time steps, but immediately decreases from 50% since hardcore agents cannot be silent in any case.

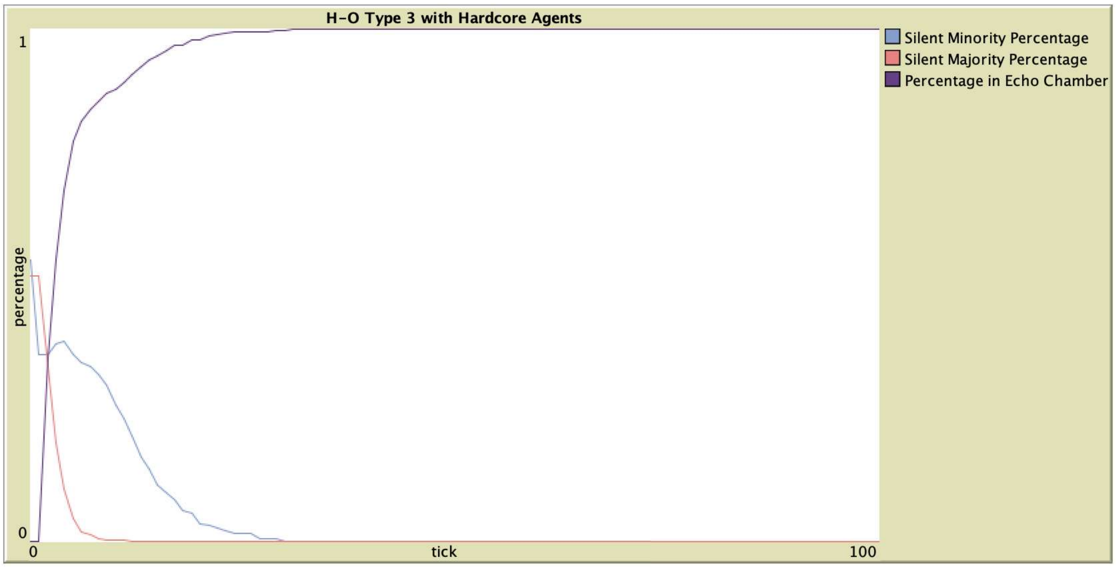

Figure 7.2. Time trajectory graph of hardcore minority model in H-O Type 3.

What is interesting here is the increase in the silent basic percentage and the decrease in the silent majority percentage in H-O Type 1. We expect these values to be just the opposite if we make half of the minority agents always expressive. However, H-O Type 1 is an exceptional network among the other selected networks. The existence of hardcore agents increases the number of majority agents who cut their links to maintain the similarity ratio at the beginning of the model and also presence of the hardcore agents do not allow the expected reward of the majorities to increase as in the base model. More majority agents cut their existing links with expressive minority agents. The similarity ratio of silent minority agents is not lower as in the base model because there are already expressive minority agents. As a result, fewer minority agents become initiator agents and so they cannot find each other as they could in the base model, and they cannot oppress the majority, as they could in the base model.

We also find that the percentage in echo chamber decreases in H-O Type 1. We can explain this decrease by the fact that the total expressive percentage decreases and the percentage in the echo chamber is calculated with expressive agents only.

7.1.2. Loyal Minority Model in H-O Networks

The silent minority percentage increases in all three homophily-oriented networks, because loyal agents refuse to cut links and create new links as they should in the homophily-oriented networks, they cannot connect with similar people.

H-O Type 2 always goes to the extreme in any experiment before, when we add the loyal agents, almost half of the minority group is silent. Only 0.1% of the basic minority group is silent; for the loyal minority group, the percentage is 94%. This means that almost all basic agents are expressive and almost all loyal agents are silent. These numbers tell us that agents who do not seek selective disclosure are destined to be silent even if they are in a homophily-oriented network, and the expression decision of the other half has no influence on the decision of the loyal minority group in the homophily-oriented network.

In H-O Type 3, agents can find similarities in opinion and select agents to connect with. By becoming loyal agents, they cannot find similar agents to connect with. The silent minority percentage has increased by over 900%, but we do not know whether this increase was caused by the silence of the loyal agents or the basic agents.

To understand this, we look at the results separately. 96% of loyal agents remain silent, while not even 1% of basic agents remain silent. These numbers also support the inference that loyal agents remain silent in H-O networks.

We have explained the oppression to majority in H-O Type 1 with the advantage that the minorities are the first initiator agents. However, with the loyal agents, their advantage is reduced because the loyal agents cannot cut their links. As a result, the minority becomes less expressive and we can no longer speak of oppressing the majority.

We observe a general decrease in the percentage in echo chambers when loyal agents are added to the model. This is to be expected as the total expressive percentage decreases. In the next sections, we will analyze the correlation between the total expressive percentage and the percentage in echo chamber.

7.1.3. Valiant Minority Model in H-O Networks

We will look at the results of introducing valiant agents that carry the properties of both loyal and hardcore agents. We will look at the results as we compare them with the loyal minority model and the hardcore minority model.

There is no significant difference between the hardcore minority model and the valiant minority model in terms of silent minority percentage. Valiant agents are already expressive loyal agents so their permanent social circle cannot silence them. Therefore, for hardcore agents, loyalty to neighbors does not affect the percentage of the silent minority.

There is a difference in the silent majority percentage between the hardcore and valiant models. We perform a two-tailed t-test to determine if this difference is statistically significant. Since the p-value for both H-O Type 2 and H-O Type 3 is < 000.1, we can say that the difference is statistically significant. However, since the differences are not significant, we can neglect them. This means that loyalty to neighbors does not affect the silent majority percentage when agents are always expressive.

We see that the silent minority percentage is 13.38% and the silent majority percentage is 25.20% in H-O Type 1 the valiant minority model. We find that the percentage of the silent minority is different than that of the hardcore minority model and this difference is not statistically significant as the p-value is .1492. We see that this model gives similar results in the percentage of silent majority as the loyal minority model. We look to see if there is a statistically significant difference. The p-value is .8464 and there is no statistically significant difference as we predicted.

In summary, the hardcore minority attribute is the driving force for the silent minority percentage and the loyalty attribute is the driving force for the silent majority percentage in homophily-oriented networks. So, valiant agents can both increase the expression and decrease the segregation level. Table 7.1 shows that the percentage in echo chamber decreases when there are valiant agents in H-O networks. This decrease can be explained by the links between minority and majority agents, which are not suitable for cutting.

When the total expressive percentage is above 99%, the percentage in echo chamber is above 99% in the base model and the model with hardcore agents in H-O Type 2 and H-O Type 3 networks. Although the percentage of expressiveness in the H-O Type 3 network is above 99%, percentage in echo chamber is below 90% in the valiant minority model.

The model with valiant minorities gives the lowest silent majority percentage, silent minority percentage, and percentage in echo chamber in H-O networks. The only model that gives the results of total expression without total segregation is the model with valiant agents in H-O Type 2 and H-O Type 3.

7.2. Loyal Majority Model Analysis

Majorities in static networks tend to be expressive, and giving them hardcore properties would not change that. However, if we gave the majority of loyal properties, we would get different results from the base model. We want to compare the majority and minority models by defining only 100 agents as loyal to the majority group. This is because we want to compare them when the average number of links of the loyal agents is the same. Since the loyal agents in static and R-D networks make no difference, we do not interpret them, but keep the results in the table to show that nothing changes.

Now that minorities are free to cut their links in H-O networks, they are more expressive in the loyal majority model than in the loyal minority model in Table 7.3. As we can see in Table 7.3, the silent minority percentage is lower than the silent majority percentage, so we can say that certain percentage of the majority is oppressed by the minority. However, since the difference between the percentages of the silent minority and the silent majority is small, we perform a two-tailed t-test to determine if this difference is statistically significant. We find that the p-value is .0952. Thus, there is no statistically significant difference between the silent minority percentage and the silent majority percentage. We cannot speak of oppression in this case. We look at the data to understand it and expect to see outliers. In Figure 7.3, there are no outliers, but the variance is very high, especially for the silent minority percentage. Figure 7.3 also shows that there are outcomes where the silent minority percentage converges to 100%, and the silent minority percentage to 0%. We look at the time trajectory graphs to observe how these results are produced by the model. There are two cases in the loyal majority model in H-O Type 1, which we refer to as loyal majority model case 1 and loyal majority model case 2.

Table 7.3. Comparison of loyal minorities and loyal majorities.

Model Type	Network Type	Silent Minority Percentage (%)	Silent Majority Percentage(%)	Percentage in EC(%)
Base Model	Static	99.94	0.05	73.28
	R-D	99.76	0.23	73.22
	H-O Type 1	7.34	38.14	68.50
	H-O Type 2	0.01	0.00	100
	H-O Type 3	0.05	0.00	99.98
Loyal Minority Model	Static	99.94	0.04	73.04
	R-D	99.55	00.39	73.14
	H-O Type 1	50.55	26.13	62.63
	H-O Type 2	47.04	1.25	88.24
	H-O Type 3	48.45	00.03	88.55
Loyal Majority Model	Static	99.91	0.04	73.35
	R-D	99.73	0.19	73.18
	H-O Type 1	29.78	33.25	64.35
	H-O Type 2	1.27	1.41	98.59
	H-O Type 3	0.36	1.50	98.63

In Figure 7.4, there is suppression of the majority by the minority as we see in the base model. We see that silent minority percentage increases in the first time steps and decreases in the following steps, just as in the base model, and the silent majority percentage increases at the beginning and decreases after a while: which leads to an oppressed majority.

Figure 7.5 illustrates the time trajectory graphs of the model with loyal majorities case 2. In the first few time steps, we can see a similarity between case 1 and case 2, but after a while, silent minority percentage remains the same instead of decreasing. We know that loyal agents do not cut their links, nor do they allow their neighbors to cut their mutual links. Since there must be minority agents who are neighbors of the loyal agents, these links between them would create a difficulty for minorities to find other minorities without homophily in rewiring. Thus, while silent majority percentage is at 0%, the silent minority percentage is over 75%; in this case, the minority is oppressed.

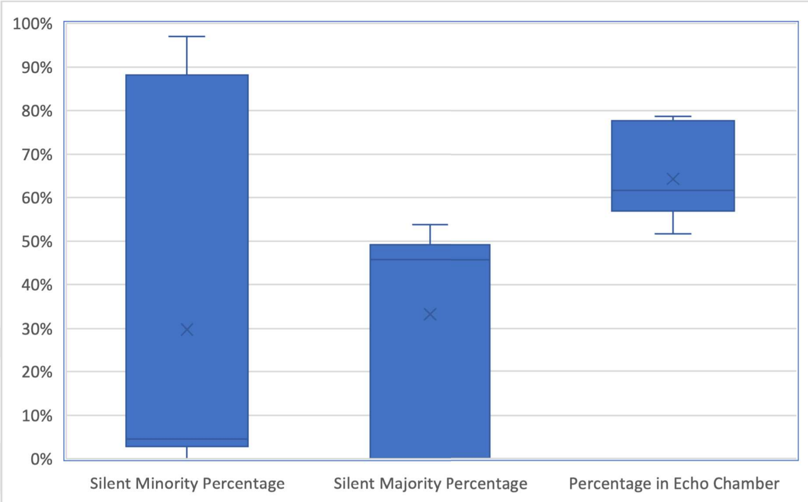

Figure 7.3. Box-plot graph of output results in H-O Type 1.

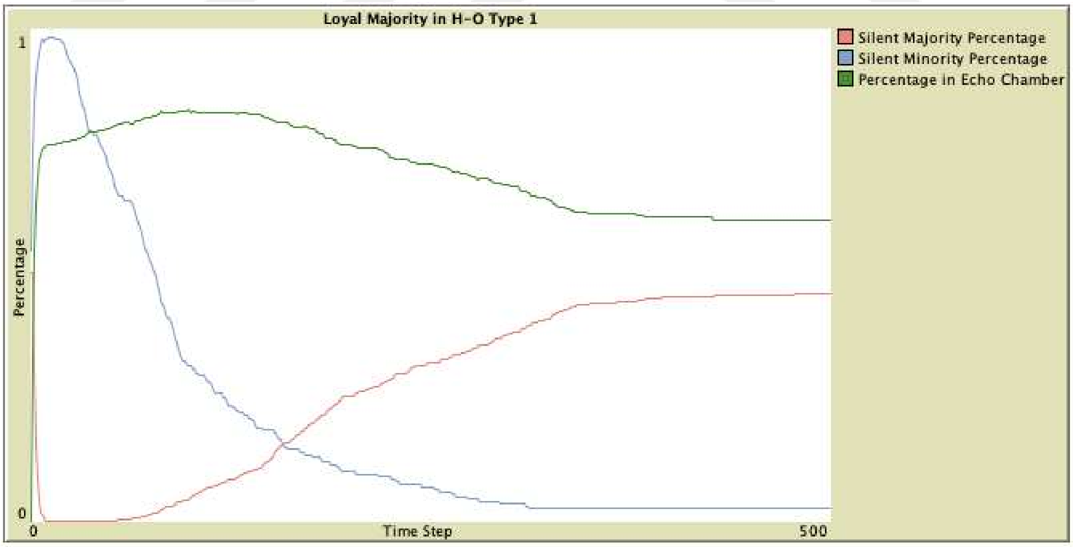

Figure 7.4. Time trajectory graph of loyal majority model case 1.

Consequently, the loyal majority model is not consistent in its results. Because links belong to loyal agents are assigned them in random and may or may not belong to minorities. If they also belong to the minority, we see the results of case 2; if the links, in general, do not belong to the minority, we can see the results of case 1. Finally, we know that this model leads to both oppressed minorities and oppressed majorities, but there is no case in which groups cannot oppress each other, as the average results suggest.

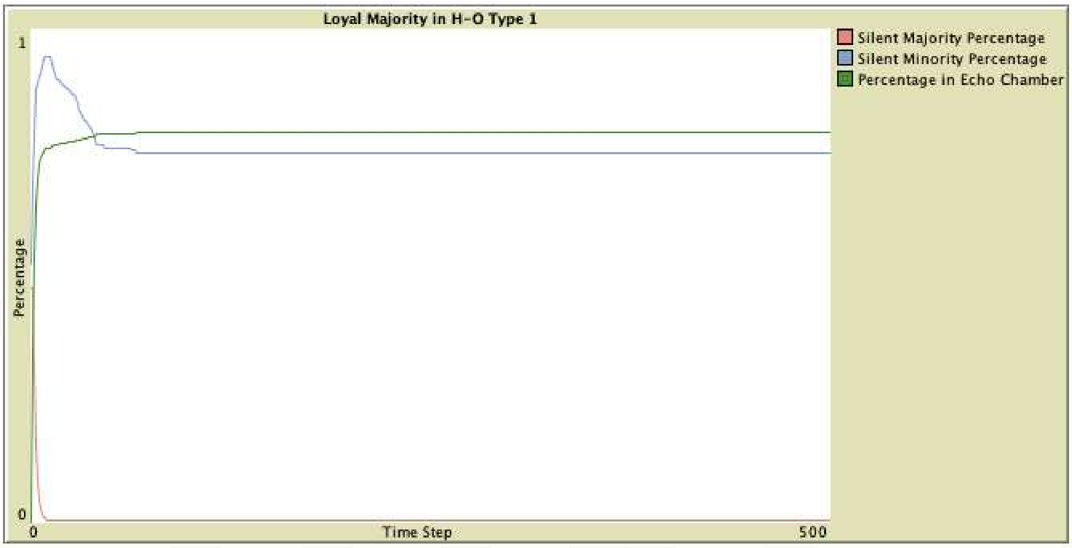

Figure 7.5. Time trajectory graph of loyal majority model case 2.

Finally, we will look at whether segregation increases as expression increases in models with different agent characteristics. We found that there is a relationship between the overall expression percentage and the percentage in the echo chamber in the base model. However, we see that the percentage in the echo chamber behaves differently when agents are loyal to the neighborhood. To understand the difference between the base model and the extended model on this point, we look at Table 7.4.

When agents are fully expressive, the percentage in the echo chamber converges to 100% even in the base model. If everyone is expressive and in echo chambers, people exclude different people and we can speak of segregation in that society. If all people are expressive but not all agents are in an echo chamber, some minorities and majorities in the same neighborhood talking to each other.

Table 7.4. Total expressive percentage(%) and percentage in EC(%) results.

Model Type	Network Type	Total Expressive Percentage(%)	Percentage in EC(%)
Base Model	Static	79.97	73.28
	R-D	79.86	73.22
	H-O Type 1	68.02	68.50
	H-O Type 2	100	100
	H-O Type 3	100	99.98
Hardcore Minority Model	Static	89.18	72.93
	R-D	84.27	68.61
	H-O Type 1	91.30	84.41
	H-O Type 2	99.99	100
	H-O Type 3	100	99.98
Loyal Minority Model	Static	79.98	73.04
	R-D	79.78	73.14
	H-O Type 1	68.99	62.63
	H-O Type 2	89.59	88.24
	H-O Type 3	90.29	88.55
Loyal Majority Model	Static	79.99	73.35
	R-D	79.91	73.18
	H-O Type 1	67.45	64.35
	H-O Type 2	98.62	98.59
	H-O Type 3	98.73	98.63
Valiant Minority Model	Static	89.13	72.95
	R-D	85.69	69.52
	H-O Type 1	77.16	64.79
	H-O Type 2	97.93	88.30
	H-O Type 3	99.55	88.89

In the base model, we see that the minority is suppressed by the majority and the minority is silent in static and R-D networks. While almost all agents of the majority are in an echo chamber, we know that there are some silent minorities in the neighborhood of the majority; so we can say that there are echo chambers in society, but not all agents are separated from each other. In H-O networks, we see that there is segregation in society because all agents are expressive and all are in an echo chamber.

As we see in Table 7.4, introducing hardcore minorities and loyal majorities to the model does not change it in H-O Type 2 and H-O Type 3. However, we see that there are still silent agents in H-O Type 1 when there are loyal majorities in the model; so we can say that most of the expressive agents are in echo chambers but the society is not segregated. When we introduce loyal minorities to the model, we see that both the expression and the number of agents in the echo chambers decrease. So we cannot talk about segregation at all.

Finally, when we introduce brave minorities into the model, compared to the baseline model, we see that expression increases in all networks and they make no significant difference in the number of agents in an echo chamber. In H-O networks, we see that the expression of both groups is similar to the base model, but we observe a decrease in the percentage in echo chambers. In other words: while in the base model, we observe segregation of society in H-O Type 2 and H-O Type 3, in the model with valiant agents society is not segregated. In this case, there are expressive minorities and majorities agents who talk to each other and stay in the same neighborhood.

8. DISCUSSION

We have conducted experiments in different networks with different parameters, and in this section, we summarize our main results. It is claimed that minority groups become more expressive and can break the spiral of silence when the minority percentage increases in a static network [41]. While we observe a decrease in the percentage of silent minorities, we cannot say that this breaks the silence because the silent minority percentage in our model is still above 70%. We also found that in our model, the complete silence of the minority group is certain when the proportion of minorities in static networks is less than 20%.

We do not observe a significant difference between static and R-D networks in terms of the percentage of silent minorities and the percentage in echo chambers. This is because the construction of the network was also randomized and randomly changing the links would not change anything about the network and thus the dynamics of minority expression. The only difference is in silent majority percentage since R-D networks yield a higher silent majority percentage. However, this percentage is small, and we cannot interpret it as an oppression, so we neglect it.

In homophily-oriented networks, there are different types of homophily. The effects of homophily in link-cutting and rewiring are analyzed separately and comparatively. When the re-linking rule is *second-degree* in an H-O network, we observe the highest silent minority percentage among all types of H-O networks. However, the percentage of silent minorities is still lower than for R-D networks.Thus, we can say that minority expression increases under homophily, as claimed in [41].

When there is no homophily in cutting links, but agents seek similarity in rewiring, i.e H-O Type 2, both minority and majority groups are expressive. This model means that agents can rebuild the network from scratch under homophily, and agents only make connections with similar agents. In other words, minority and majority groups form their echo chambers and exclude the other group from their social circles. Therefore, people tend to join within their group, and the percentage in the echo chamber is over 98%. Because there is a total expression for both groups and nearly all of the agents in an echo chamber, we can say that there is a segregation in this society. This means that nearly all agents only connect with like-minded people and do not have links with different people.

Minority and majority agents are completely expressive when homophily is present in both link-cutting and rewiring. This H-O network is referred to as H-O Type 3 and yields similar results to H-O Type 2. The difference between them is that the initiator agents are based on the homophily degree. The degree of homophily only affects the percentage in the echo chamber, but not the silent minority. In this case, there is no significant difference in the percentage in echo chamber between H-O Type 2 and H-O Type 3.

The most interesting result of our study is that the majority is suppressed by the minority in an H-O network, i.e., H-O Type 1, where there is no homophily in rewiring, but there is in link-cutting. This can be explained by the social dynamism of the minority agents. Minority agents change their social circles by cutting and rewiring links until they find a neighborhood where they can freely express their opinion and 75% of the neighborhood has the same opinion as them. Since their expected reward in the first steps was lower than zero, since almost all minorities were silent, even a small increase in their expected reward would encourage minorities to voice their opinions. The more minorities speak out, the expected reward of the other minorities start to increase. As a result, more and more minorities start to speak out. Meanwhile, both the current reward and the expected reward of the majorities decrease because there are now expressive minorities and silent majorities. The majorities have no chance to increase their reward, so they are stuck in silence with a non-increasing expected reward. In other words, the minorities change the opinion climate of the majority agents and distort their perception of the majority and the minority from the actual conditions in society. As a result, members of the majority group feel lonely because they believe that their opinions are not supported, when in fact they are the majority in global scale. In other words, they form a spiral of silence and find themselves in a minority illusion. The minority illusion is used in this context to explain that there are some majorities that believe they are the minority in society; meanwhile, we can observe that minorities believe they are the majority in society and we can speak of a majority illusion.

For the reasons we have explained, we can say that the minority illusion and the majority illusion are two concepts that are not mutually exclusive but mutually supportive. In this model, we find that the silent majority is waiting for just one person to express the opinion of the majority so that they too can express their opinion. If we make just one of them an expressive agent, all the agents of the silent majority begin to express themselves in a cascade. The percentage of the minority plays a crucial role in oppressing the majority in H-O Type 1. If the minority percentage is less than or equal to 10%, the important part of the majority is expressive and the minority cannot oppress the majority. This tells us that even though the minority is expressive and still has the social dynamism, they cannot oppress the majority since they are outnumbered.

If the percentage of minorities is 15% or both groups are equal in number, both groups are 80% expressive; so we can say that neither group can oppress the other, but there are still silent agents in both groups. Hence, local minorities and majorities oppress the other group in local neighborhoods. When the percentage of minority is between 20% and 45%, we can also observe an oppressed majority. The lowest percentage expression of majority occurs when the minority percentage is 40%, because in this percentage the social dynamism of minorities is still valid and their presence in the neighborhoods of majorities is also higher than in the other cases, because their number is higher.

Up to this point, all agents in the model were equivalent in their decision-making process regarding utterance and the processes of linking and rewiring; we will refer to this model as the base model and the agents as basic agents. We would like to know how the results in the static, R-D networks and the different H-O network types we mentioned earlier would change if we assign different characteristics to the agents. It is argued [5, 51] that "hardcore" individuals who are always expressive increase the expressiveness of the minority group. When we make half of the minorities "hardcore", we observe that the number of expressive minorities is increasing inherently. However, when we look at the percentages of minorities without this trait in the different minority percentages; we find that it is very low. So we found that hardcore people are expressive but they do not encourage other people to break their silence. We expect that minorities who are always expressive might oppress the majority, we do not observe it under these conditions, as well.

If we convert half of the minorities into loyal agents who cannot change their social circles but can change their expressions, nothing changes in the static and R-D networks. They were completely silent in these networks and their loyalty to their neighbors does not make them expressive. However, silent minority percentage decreases in H-O networks. We find that loyal minorities cannot be expressive even in H-O networks and their loyalty to neighbors does not affect the expressiveness of their basic neighbors. Basic minorities become expressive in H-O networks as they did in other models but loyal minorities remain silent.

We make the same number of agents loyal majorities this time. Loyalty to neighbors does not affect the expression decision of loyal majorities, these majorities are still expressive. This is because their number is large compared to the minority and the neighbors of the loyal agents are likely to be part of the majority group initially. Therefore, they are not particularly affected and give similar results to the base model in all networks except H-O Type 1. The loyal majority model can lead both the minority and majority groups to oppression in different cases, depending on the initial conditions. The reason why the model leads to these two contradictory results is the random selection of the loyal majorities and thus their neighbors.

If half of the minorities are loyal to their neighbors and also always expressive, they are valiant agents. The presence of valiant agents increases expressiveness and decreases segregation compared to their absence in the model. The only models that show almost complete expression and no complete segregation are the models with valiant agents in H-O Type 2 and H-O Type 3.

We know that the total expressive percentage and the percentage in echo chamber are highly correlated in the base models. However, introducing new agent characteristics into the model decreases the correlation between these two values in H-O networks, especially in Type 2 and H-O Type 3 because loyal agents decrease the percentage in the echo chamber independent of the expressive percentage.

9. CONCLUSION

In this study, we work with an isolated society which does not allow any people to leave or no more people to join. We determine the population size of this society with experiments. We do the experiments in this model while using a simulation model, and we use the agent-based modelling approach and represent the people and their relationships in the model. Increasing the population size changes the network structure and the expression dynamics as suggested in the literature [43].

As there are only two opinions in one topic in this society; there are only two opposing groups, as well. We determine one group to be minority and determine the minority percentage in society with experimentation as well. We found that minority become more expressive if the minority percentage increases as it was claimed [41] but minority cannot break their silence.

In this study, there are different types of homophily; such as ending relationships with opposing opinions and creating a new social circle based on similarity. They are all analyzed under the name of homophily-oriented networks.

Model results suggest that almost everyone become expressive when homophily prevails in building new relationships. However, as a price for their expressiveness, both groups form echo chambers. Echo chambers do not allow people to hear other opinions, which means that there is total segregation in these societies.

If there is homophily in ending the relationships, but people find new people with whom to connect randomly, the majority is oppressed by the minority. When we look at the percentages of minorities in society, we find that this oppression is significant when the proportion of minorities is not very small, i.e higher than 15%, or when the number of the two groups approaches each other, i.e less than or equal to 45%.

This oppression is caused by the fact that minorities have really low expected rewards and even a small improvement, like a slight increase in their expected reward, would make them reconsider their expression decision. This small increase occurs when the number of majority agents in their "expressive" neighbors decreases, but the number in their agreeable neighbors does not have to necessarily increase. So that, they become expressive and make some majorities silence in time. Majorities cannot increase their expected reward in a case where minorities act like majorities and cannot connect with expressive majorities, as well. It is important to note that this oppression towards majority is only caused since we limit agents with the constraints in their neighborhood size and their consideration of their expression decision with only change in their expected reward. If we change the rewiring process rules or the expected reward equation, this oppression would not be happen.

The oppression of the majority by the minority shows that the illusion of the majority and also the illusion of the minority is possible in this environment. As mentioned earlier, this society is a closed society in which each agent follows the same rules in making expressive choices, in link-cutting process and in rewiring process. Therefore, we cannot speak of an exogenous factor and none of the agents has a distinctive character that causes the minority to oppress the majority. As a result, we come to two main conclusions. The first is that the main factor that determines people's expressiveness is not the facts or the actual majority in society but people's perceptions, as claimed in the literature and our results support this. The second important finding is that minorities believe they are the majority in a closed society where there is no external force but only their own dynamics with the majority group and even oppress them.

In a case where minorities can end their relationships with opposing opinion holders but the people with whom they can connect are limited to neighbors of neighbors, they cannot create a comfortable space in which to express their opinions. However, minorities are also more expressive in this homophily-oriented network type than in the random- dynamic and static networks.

In societies, not all people have the same characteristics, the same resistance to public pressure, and the same fear of isolation. Some minorities express their opinions with determination, but their hardcore characteristics are not very helpful for other minorities to express their opinions in static networks. This is because they still seek comfort in their relationships, find each other to form their echo chambers, and become the local majority and exclude other people. Among these excluded people, there are some silent ones who believe they are alone. In homophily-oriented networks, we observe similar results as in the model without hardcore minorities. This means that some of the minorities, who are always expressive, do not affect expressiveness in homophily-oriented networks.

To encourage other silent minorities, expressive minorities should also maintain their social circle without hesitation. These courageous minorities who want to stay in their social circle while expressing their opinions are called valiant people. They encourage other minorities to express themselves without silencing the majority. When there are valiant minorities in society, all people become expressive. Moreover, valiant people become a bridge between the minority and majority groups; they reduce the level of separation in society.

Minorities who maintain social circles without having the determination to express themselves do not have much influence on minority expression. If the decision to speak out rests with them, loyal minorities cannot be expressive in homophily-oriented networks. In other words, minorities remain silent and their social circle is permanent under homophily. On the other hand, if some majorities are loyal to their social environment, we find that both the majorities and the minorities are fully expressive when there is homophily in rewiring. However, they are still oppressed by the minority when there is no homophily in rewiring, albeit with a lower probability. The interesting result is that the majority is equally likely to oppress the minority under these conditions. Thus, if a part of the majority does not transform its social circle under homophily in homophily-oriented networks, this would lead to the oppression of one of the groups.

In this study, we simplify society by assigning the same learning rate and social parameter values to all people, which changes the expression environment. The experiments and results can be applied to different types of societies. Moreover, we give limited individual characteristics to people. Analyzing the expression dynamics of giving different traits to different groups or introducing leaders who have more relationships than everyone else and give multiple rewards to their neighbors would be topics for other studies. We limit people to only two options, agree and disagree, and this is the main reason for the conflicts; and also to have two opinions, another opinion, which means two minority groups and one majority group in this society, would also be the subject of another study.

We assign links probabilities to cut to distinguish the relationships between people; some people can break their relationships with other easily, while some relationships, such as parents, children and siblings are not very easy to end and people have to sustain these relationships. However, we analyze the results at the people level, it means we did not look how these relationships and their strength affects the expression of minorities and the segregation in society. Classifying links as strong, medium, and weak based on their link strengths and analyzing their effects on expression and segregation would provide concrete results that support [52] and this could be a topic for another study. We also use a specific network for the experiments. Other network topologies such as Random Graph, Small World, and Barabasi-Albert would provide different and significant results and could be analyzed comparatively.

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APPENDIX A: SOCIETY PARAMETER ANALYSIS

The parameters we use to compute *Expression Probability* change the expressiveness of the agents. Since we have taken the same values for each parameter, these parameters actually determine the dynamics of the society in terms of expressiveness relative to *Expected Reward*. Thus, we will look at how the change in the parameters affects the relationship between *Expression Probability* and *Expected Reward*. Since these parameters denote the characteristics of societies, we will refer to them as social parameters.

We refer the parameters which we take to calculate in the base model as baseline values. We will change the values of these parameters one at a time, which means that the values of the other two will correspond to the values of the baseline. Changing the values of the social parameters may result in probabilities greater than one. Since this is not possible, we assume that they are equal to one and show them that way in Figure A.1, Figure A.2, and Figure A.3.

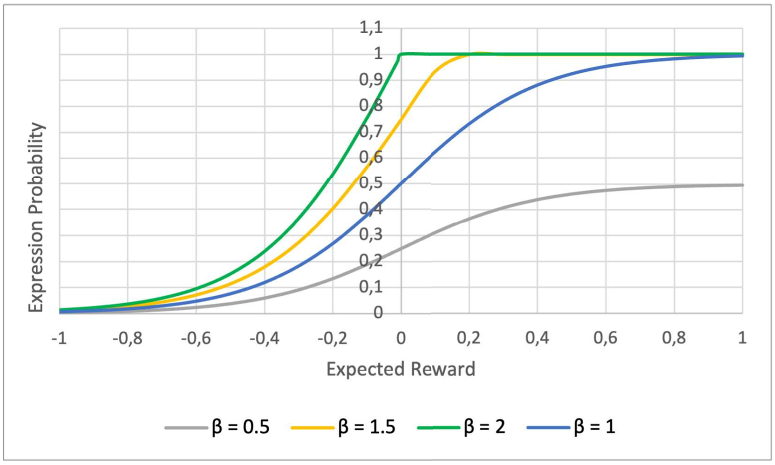

Figure A.1. Expression probability graph in different β values.

We start with β. Figure A.1 shows that as β increases, agents would be more likely to express themselves in the same rewards; when its value is two, all agents with non-negative *Expected Reward* express themselves with probability 100%. If we give β a value between zero and one, we find that the *Expression Probability* curve flattens. At a value of 0.5, the *Expression Probability* is 50% at the maximum *Expected Reward*; agents are likely to remain silent, regardless of their group.

In Figure A.2, we see that the curve flattens and the *Expression Probability* for all *Expected Reward* values decreases as the value of γ increases above one. The sensitivity of *Expression Probability* to *Expected Reward* decreases and the agents who are fully supported by their neighborhood express their opinion with a probability of 20%. Thus, agents tend to remain silent. When it decreases, agents with zero *Expected Reward* express themselves with a probability of over 60%, and most agents with positive *Expected Reward* express themselves with a probability of 100%. So we can say that people are expressive when γ is low, and are more likely to remain silent when γ increases.

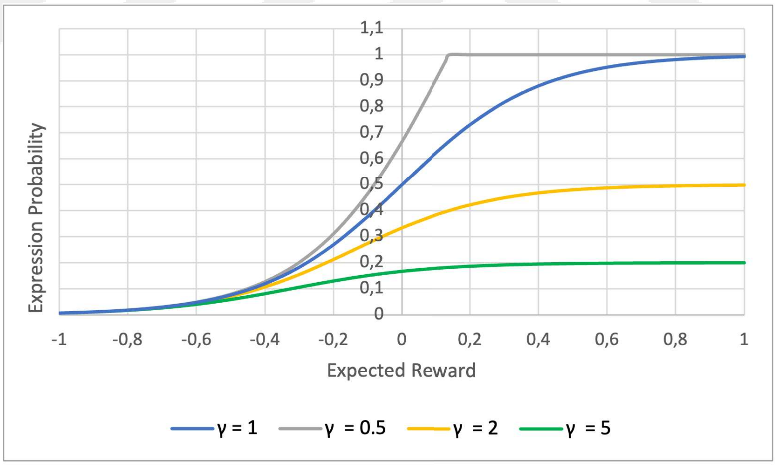

Figure A.2. Expression probability graph in different γ values.

Figure A.3 illustrates that the expression probability curve resembles a line as we increase the value of δ. As the value decreases, some agents express their opinions with a probability of 100% at different *Expected Reward* values and some agents certainly do not express their opinions. At low values such as -20, we observe that almost all agents with a positive *Expected Reward* express themselves and almost all agents with a negative *Expected Reward* remain silent; it resembles a threshold probability model where the threshold is zero. Agents with an *Expected Reward* of zero have 50% to speak up, regardless of the value of δ.

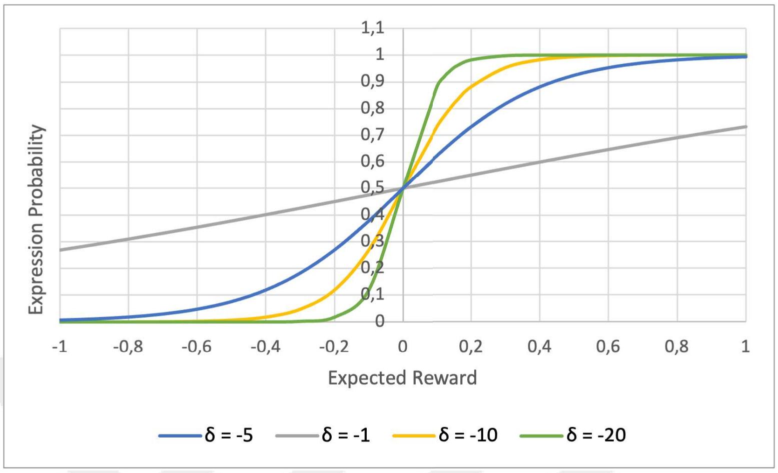

Figure A.3. Expression probability graph in different δ values.

These three parameters change the agents' opinion expression probabilities along with the the learning rate. We can call these four variables the environmental indicators that represent the relationship between society and individuals.

If we assume lower values than the baseline value for δ, we see that when people are the local minority, they remain silent, and when they are the local majority, they become expressive. So in this type of society, people must be the majority in their local neighborhood, otherwise they cannot be expressive. This means that in this society, there is no possibility for all people in society to express themselves and to get in contact with others.

At higher β values or lower γ values, local majorities in these societies are certainly more expressive and local minorities are encouraged to be more expressive. At lower β values or higher γvalues, even the local majorities express their opinions with a very low probability. Even if there is no authority in these societies, this would be a case for an invisible authoritarianism and would silence all people.