DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

ON THE KINEMATIC MODELLING OF UNDULATING FIN RAY FOR FISH ROBOTS

by
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İZMİR

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M. Sc THESIS EXAMINATION RESULT FORM

We have read the thesis entitled **"ON THE KINEMATIC MODELLING OF UNDULATING FIN RAYS FOR FISH ROBOTS"** completed by **MELEK ERDOĞDU** under supervision of **ASSIST. PROF. DR. İLHAN KARAKILIÇ** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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Melek ERDOĞDU

ON THE KINEMATIC MODELLING OF UNDULATING FIN RAYS FOR FISH ROBOTS

ABSTRACT

The main purpose of this work is to investigate some differential geometric properties of undulating fin rays for a fish robot. We use a reference fin ray to get information about the motion of fish robots. The results show how the robotic fish is moving and the position of the robotic fish.

Keywords: biomimetics, fish robots, undulating robotic fin, curvature functions.

BALIK ROBOTLARININ YÜZGEÇ HAREKETİNİN KİNEMATİK MODELLEMESİ ÜZERİNE

ÖZ

Bu çalışmanın temel amacı balık robotlarının sırt yüzgeçlerinin bazı diferansiyel geometrik özelliklerini incelemektir. Bunun için referans bir sırt yüzgeci kullandık. Sonuçlar, bize balık robotun hareketi ve konumu hakkında bilgi veriyor.

Anahtar sözcükler: biomimetik, balık robotlar, dalgalanan robotik kılçıklar, eğrilik fonksiyonları.

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CHAPTER ONE
INTRODUCTION

1.1 Introduction

Biomimetics studies living organisms to derive new principles, theories and techonology and applies them to manmade devices. Biomimetic robots characterize their functions from animals such as fish, insects and birds. Fish are the research objects for underwater biomimetic robots. The reason for the focus in underwater biomimetics varies in different fields such as preserving the marine ecological environment, finding mineral resources in the deep of the oceans and discovering new kinds of organisms.

There are a lot of works on robotic fish. Wang, Shen & Hu (2006) studied on the kinematic modelling and dynamic analysis of long based undulation fin. Also Hu, Shen, Lin & Xu (2009) studied on biological inspirations, kinematic modelling, mechanism design and experiments on an undulating robotic fish inspired by “Gymnarchus Niloticus” which is a special kind of fish as seen in figure1.1. On the other hand, Low (2009) did a parametric study of modular undulating fin rays for fish robots.

Figure 1.1 Gymnarchus Niloticus.

As it is discussed in Wang, Shen & Hu (2006), the research in biology, mechanics and engineering areas are all interested in biofish robotic technologies. The researchers mimic the fish profile, locomotion and struction in their design of device for underwater propulsion as well as maneverability. Therefore Amiform fish

“Gymnarchus Niloticus” was studied to design biofish robots. While Amiform fish are swimming by undulating their long based dorsal fin, their body axis keep being straight in many cases. And they are able to swim as well as backwards as they do forwards by reserving the direction of wave from propogation on their long based dorsal fin as seen in the figure 1.2 (Hu, Shen & Low, 2009).

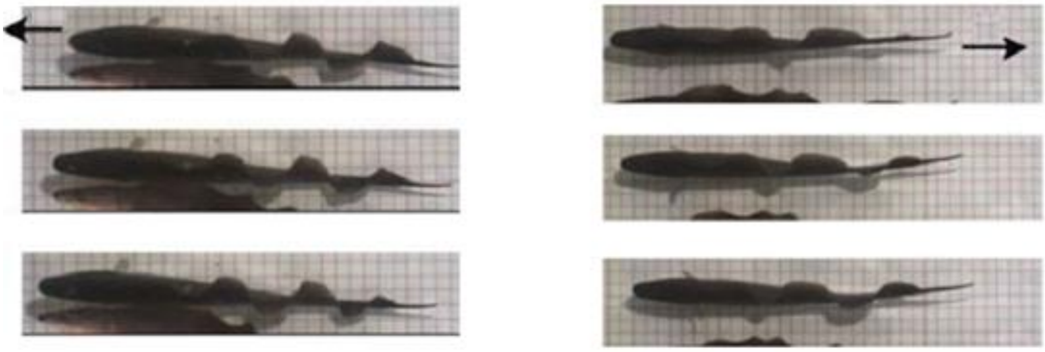

Figure 1.2 Forward and backward swimming of the fish “Gymnarchus Niloticus”.

In this work, we are concerned with the differential geometric properties and curvature theory of undulating fin ray of “ Gymnarchus Niloticus ” for fish robots. We have analyzed fish movement and the route of any fin ray $c(s_0, t)$ of fish robot when t is chosen as variable. Fortunately, the motion of reference fin ray $c(s_0, t)$ forms a ruled surface as the undulating fin ray of fish robot forms. The change in the movement of fish robot is related to the curvature and torsion of the spherical image curve of the reference fin ray. The change in the motion of selected reference fin ray gives us information about the movement of the fish. Therefore, we can have more information about the movement of fish robot if the number of selected reference fin ray is increased.

This work has been organised as follows;

In the first chapter, we study the kinematic modelling of fin rays for fish robot of “Gymnarchus Niloticus” (Hu, Shen, Lin & Xu, 2009).

In chapter two, we give curvature functions of a reference fin ray and depending on these functions, we find curvature and torsion of spherical image curve of the reference fin ray.

In chapter three, we studied on dual numbers. Dual spherical image of a reference fin ray was found for the aim of finding position of the fish robot.

In the last chapter, we concluded the study with the obtained results.

1.2 Kinematic Modelling of Fin Rays

In the study Hu, Shen, Lin & Xu (2009), undulating fin with a fixed baseline has been given in a ruled surface based kinematic model. Ruled surface is one of the typical surface in differential geometry. A ruled surface may be represented by a base curve and a rulling as follows;

$$x(u,v) = b(u) + v\,\delta(u)$$

where u and v are arbitrary real parameters. The line with direction $\delta(u)$ is the rulling and the curve $b(u)$ is the base curve of the ruled surface (O'Neill, 1997).

As it is discussed in the study Hu, Shen, Lin & Xu (2009), the undulating fin is represented as a ruled surface with fin ray as rulling and fin base as base curve. The ruled surface based model of long undulatory fin may be defined as

$$p(r,s,t) = b(s,t) + r\,d(s)c(s,t) \qquad 0 \leq s \leq L, \qquad 0 \leq r \leq 1$$

where $c(s,t)$ is time varying vector overlapping the fin ray at $x = s$, $b(s,t)$ is the fin base curve that can be described as the change of fin rays starting points along the fin base , and r is the normalized nondimensional parameter defining the ratio $|P_sP|$ to $|P_sP_e|$, in which $|P_sP|$ is the length from any point on the ruled surface to its corresponding start and $|P_sP_e|$ is the length of fin ray at $x = s$, L is the total lenght of fin ray as seen in figure 1.3.

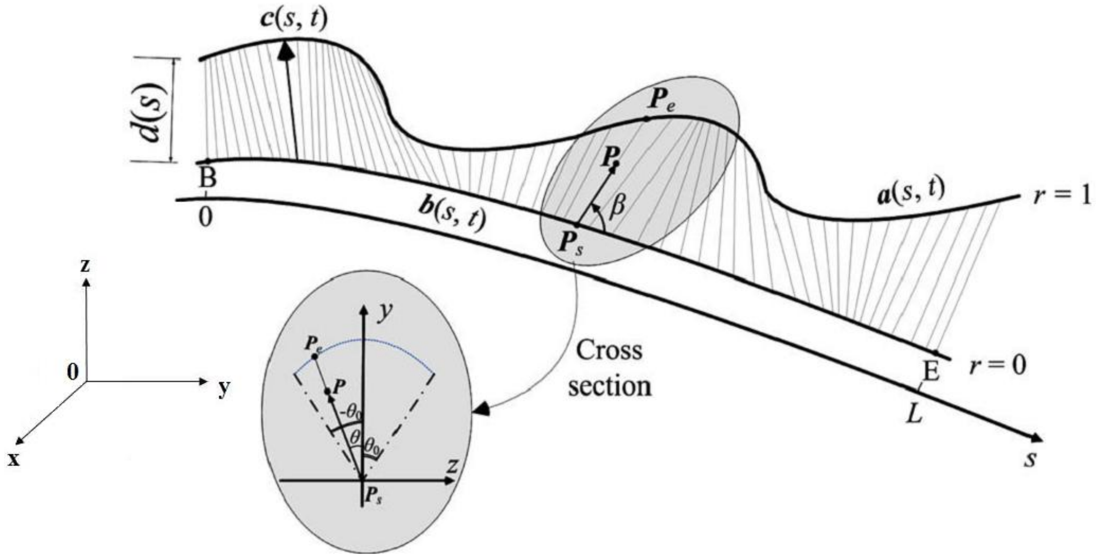

Figure 1.3 The ruled surface based model for undulating fin ray of Gymnarchus Niloticus.

According to biological observation, the undulation with the fin ray oscilatory angle; $\theta(s,t)$ has been defined as;

$$\theta(s,t) = \theta_0(s)\sin(\frac{2\pi}{\lambda}s + \frac{2\pi}{T}t + \emptyset)$$

where θ_0 is the maximum lateral angle, λ is the undulation wavelenght, T is the undulation cycle and $\emptyset$ is the original phase of the first fin ray. According to observations and analysis on the biological specimen, the inclined angle β is the direct factor to the asymetry. For further inspections see (Hu & others, 2009).

The base line of the ruled surface has been chosen in a simple form to simplify the calculation such as;

$$b(s,t) = \begin{pmatrix} s \\ 0 \\ 0 \end{pmatrix}, \qquad (1.2.1)$$

and the rulling $c(s,t)$ of the ruled surface in the figure 1.3 has been given as;

$$c(s,t) = \begin{pmatrix} \sin\theta(s,t)\cos\beta \\ \cos\theta(s,t) \\ \sin\theta\,(s,t)\sin\beta \end{pmatrix}. \qquad (1.2.2)$$

For application, β is assumed to be in the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. Therefore the ruled surface based model of fin ray can be defined as follows;

$$p(r,s,t)=\begin{pmatrix} s \\ 0 \\ 0 \end{pmatrix}+r\,d(s)\begin{pmatrix} \sin\theta(s,t)\cos\beta \\ \cos\theta(s,t) \\ \sin\theta\,(s,t)\sin\beta \end{pmatrix}. \qquad (1.2.3)$$

CHAPTER TWO

CURVATURE FUNCTIONS OF FIN RAYS

2.1 Curvature Functions of a Reference Fin Ray

In this section, we define the ruled surface $m(r,t)$ which is constructed by a reference fin ray $c(s_0,t)$ while the robotic fish is moving as in figure 2.1. We give the curvature functions of the ruled surface $m(r,t)$. The ruled surface $m(r,t)$ has the following parametrization;

$$m(r,t) = \gamma(t) + r\, c(s_0,t). \tag{2.1.1}$$

The regular curve $\gamma(t)$ can be taken as striction curve of $m(r,t)$.

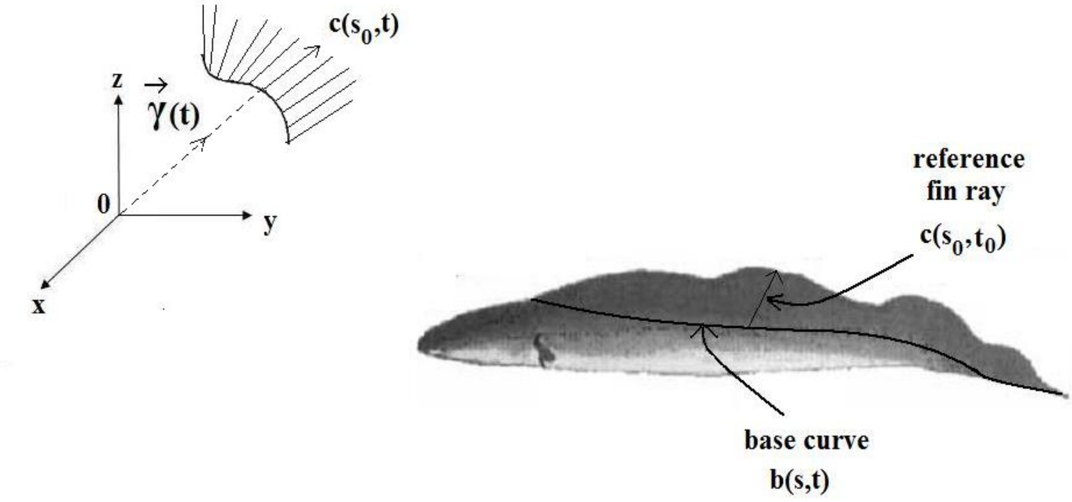

Figure 2.1 The ruled surface constructed by reference fin ray.

We will take an adapted frame field $\{E_1, E_2, E_3\}$ for our later purpose, where $\{E_1, E_2, E_3\}$ is obtained from the rulling $c(s_0,t)$.

The curvature functions of the ruled surface $m(r,t)$ can be interpreted as components of velocity of the Natural trihedron $\{\overrightarrow{E_1}, \overrightarrow{E_2}, \overrightarrow{E_3}\}$.

Let us take

$$\overrightarrow{E_1} = \vec{c}(s_0, t).$$

In terms of equation (1.2.1), this becomes

$$\overrightarrow{E_1} = \begin{pmatrix} \sin\theta(s_0, t)\cos\beta \\ \cos\theta(s_0, t) \\ \sin\theta\,(s_0, t)\sin\beta \end{pmatrix}. \tag{2.1.2}$$

E_2 is obtained as follows;

$$\overrightarrow{E_2} = \frac{\frac{d}{dt}\vec{c}(s_0,t)}{\left\|\frac{d}{dt}\vec{c}(s_0,t)\right\|}. \tag{2.1.3}$$

Taking derivative of equation (2.1.2) and substituting into equation (2.1.3) gives

$$\overrightarrow{E_2} = \frac{1}{\sqrt{\left(\frac{d\theta(s_0,t)}{dt}\right)^2 + (\sin\theta(s_0,t))^2\left(\frac{d\beta}{dt}\right)^2}} \begin{pmatrix} \cos\theta(s_0, t)\cos\beta\frac{d\theta(s_0,t)}{dt} - \sin\theta(s_0\,, t)\ \sin\beta\ \frac{d\beta}{dt} \\ -\sin\theta(s_0, t)\frac{d\theta(s_0, t)}{dt} \\ \cos\theta(s_0, t)\sin\beta\frac{d\theta(s_0, t)}{dt} + \ \sin\theta(s_0\,, t)\cos\beta\ \frac{d\beta}{dt} \end{pmatrix}. \tag{2.1.4}$$

We may define $\overrightarrow{E_3}$ by the cross product of $\overrightarrow{E_1}$ and $\overrightarrow{E_2}$.

$$\overrightarrow{E_3} = \overrightarrow{E_1} \times \overrightarrow{E_2}, \tag{2.1.5}$$

$$\overrightarrow{E_3} = \frac{1}{\sqrt{\left(\frac{d\theta(s_0,t)}{dt}\right)^2 + (\sin\theta(s_0,t))^2\left(\frac{d\beta}{dt}\right)^2}} \begin{pmatrix} \sin\beta\,\frac{d\theta(s_0,t)}{dt} + \sin\theta(s_0, t)\cos\theta(s_0, t)\cos\beta\,\frac{d\beta}{dt} \\ -\big(\sin\theta\big(s_0, t\big)\big)^2\,\frac{d\beta}{dt} \\ -\cos\beta\,\frac{d\theta(s_0,t)}{dt} + \sin\theta(s_0, t)\cos\theta(s_0, t)\sin\beta\frac{d\beta}{dt} \end{pmatrix}. \tag{2.1.6}$$

The Natural trihedron of the ruled surface $m(r, t)$ is located at the center of each rulling of $m(r, t)$. Let us take the striction curve $\vec{\gamma}(t)$ as

$$\vec{\gamma}(t) = \begin{pmatrix} \gamma_1(t) \\ \gamma_2(t) \\ \gamma_3(t) \end{pmatrix}. \tag{2.1.7}$$

Thus, we have the following differential formulas;

$$\frac{d}{dt}\vec{\gamma}(t) = \vec{\gamma}'(t) = \alpha\overrightarrow{E_1} + \mu\overrightarrow{E_2} + \xi\overrightarrow{E_3}. \tag{2.1.8}$$

The first coefficient of equation (2.1.8) can be found as follows;

$$\alpha = \frac{d}{dt}\vec{\gamma}(t)\overrightarrow{E_1},$$

$$\alpha = \sin\theta(s_0,t)\cos\beta\gamma_1'(t) + \cos\theta(s_0,t)\gamma_2'(t) + \sin\theta(s_0,t)\sin\beta\gamma_3'(t). \tag{2.1.9}$$

The second coefficient of the equation (2.1.8) is equal to zero since $\gamma(t)$ is the striction curve of $m(r,t)$

$$\mu = \vec{\gamma}'(t)\overrightarrow{E_2} = 0.$$

The last coefficient of equation (2.1.8) can be determined as follows;

$$\xi = \vec{\gamma}'(t)\overrightarrow{E_3},$$

$$\xi = \frac{1}{\sqrt{\left(\frac{d\theta(s_0,t)}{dt}\right)^2 + (\sin\theta(s_0,t))^2\left(\frac{d\beta}{dt}\right)^2}}\big(A_1(s_0,t)\gamma_1'(t) + A_2(s_0,t)\gamma_2'(t) + A_3(s_0,t)\gamma_3'(t)\big) \tag{2.1.10}$$

where $A_1(s_0,t) = \sin\beta\frac{d\theta(s_0,t)}{dt} + \sin\theta(s_0,t)\cos\theta(s_0,t)\cos\beta\frac{d\beta}{dt}$,

$A_2(s_0,t) = (\sin\theta(s_0,t))^2\frac{d\beta}{dt}$,

$A_3(s_0,t) = -\cos\beta\frac{d\theta(s_0,t)}{dt} + \sin\theta(s_0,t)\cos\theta(s_0,t)\frac{d\beta}{dt}$.

We will also differentiate the three unit vectors $\overrightarrow{E_1}$, $\overrightarrow{E_2}$, $\overrightarrow{E_3}$ to determine the first order angular variation of Natural trihedron. Taking derivative of equation (2.1.2) and substituting the equation (2.1.3) into the result gives

$$\frac{d}{dt}\overrightarrow{E_1} = \eta\,\overrightarrow{E_2} \tag{2.1.11}$$

where $\eta = \left\|\frac{d}{dt}\vec{c}(s_0,t)\right\| = \sqrt{\left(\frac{d\theta(s_0,t)}{dt}\right)^2 + (\sin\theta(s_0,t))^2\left(\frac{d\beta}{dt}\right)^2}.$ (2.1.12)

Orthogonal expansion of $\frac{d}{dt}\overrightarrow{E_2}$ yields;

$$\frac{d}{dt}\overrightarrow{E_2} = \zeta\overrightarrow{E_1} + \lambda\overrightarrow{E_2} + \rho\overrightarrow{E_3}. \tag{2.1.13}$$

Some simple computations on equation (2.1.13) yields;

$$\zeta = -\sqrt{\left(\frac{d\theta(s_0,t)}{dt}\right)^2 + (\sin\theta(s_0,t))^2\left(\frac{d\beta}{dt}\right)^2} = -\eta \tag{2.1.14}$$

$$\lambda = 0, \tag{2.1.15}$$

$$\rho = \frac{-\cos\theta(s_0,t)\,A(s_0,t) + \sin\theta(s_0,t)\,B(s_0,t)}{\eta^2} \tag{2.1.16}$$

where $A(s_0,t) = \frac{d\beta}{dt}\left(2\left(\frac{d\theta(s_0,t)}{dt}\right)^2 + (\sin\theta(s_0,t))^2\,(\frac{d\beta}{dt})^2\right),$

$$B(s_0,t) = \frac{d^2\theta(s_0,t)}{dt^2}\frac{d\beta}{dt} - \frac{d\theta(s_0,t)}{dt}\,\frac{d^2\beta}{dt^2}.$$

Finally, the first order derivative of E_3 can be found by taking derivative of equation (2.1.5)

$$\frac{d}{dt}\overrightarrow{E_3} = \frac{d}{dt}\overrightarrow{E_1} \times \overrightarrow{E_2} + \overrightarrow{E_1} \times \frac{d}{dt}\overrightarrow{E_2}.$$

Substituting equations (2.1.11) and (2.1.13) into above equation we get

$$\frac{d}{dt}\overrightarrow{E_3} = -\rho\overrightarrow{E_2}. \tag{2.1.17}$$

Thus, the first order angular variation of Natural trihedron of the ruled surface $m(r,t)$ can be expressed as follows;

$$\frac{d}{dt}\vec{\gamma}(t) = \alpha\overrightarrow{E_1} + \xi\overrightarrow{E_3},$$

$$\frac{d}{dt}\begin{bmatrix}\overrightarrow{E_1}\\ \overrightarrow{E_2}\\ \overrightarrow{E_3}\end{bmatrix} = \begin{bmatrix}0 & \eta & 0\\ -\eta & 0 & \rho\\ 0 & -\rho & 0\end{bmatrix}\begin{bmatrix}\overrightarrow{E_1}\\ \overrightarrow{E_2}\\ \overrightarrow{E_3}\end{bmatrix}$$

where α, ξ, η and ρ are given by equations (2.1.9), (2.1.10), (2.1.15) and (2.1.19), respectively. These four parameters are referred as curvature functions of the ruled surface $m(r,t)$.

2.2 Spherical Image Curve of a Reference Fin Ray

In this section we will give the curvature theory of spherical image curve

$$\sigma = \frac{\frac{d}{dt}\vec{c}(s_0,t)}{\left\|\frac{d}{dt}\vec{c}(s_0,t)\right\|} = \overrightarrow{E_2}$$

by using curvature functions which are obtained in previous section. The motion of σ represents the curving of the reference fin ray $c(s_0,t)$. The curvature and torsion of the arbitrary speed curve σ are

$$\kappa_\sigma = \frac{\|\sigma' \times \sigma''\|}{\|\sigma'\|^3},$$

$$\tau_\sigma = \frac{(\sigma' \times \sigma'')\sigma'''}{\|\sigma' \times \sigma''\|^2}$$

(where prime denotes derivative with respect to parameter t).

Taking derivatives of the curve $\sigma = \overrightarrow{E_2}$ by using equations (2.1.11), (2.1.13), (2.1.17) and substituting into above equations, we get

$$\kappa_\sigma = \sqrt{1 + \frac{(\eta\,\rho' - \eta'\rho)^2}{(\eta^2+\rho^2)^3}},$$

$$\tau_\sigma = \frac{(\eta^2+\rho^2)(\rho\eta'' + \eta\rho'') + (3\rho^2 - 3\eta^2)\eta'\rho' - 3\eta\rho((\rho')^2 - (\eta')^2)}{(\eta^2+\rho^2)^3 + (\eta\,\rho' - \eta'\rho)^2}$$

where η and ρ are curvature functions of the ruled surface $m(r,t)$ that are given by equations (2.1.12) and (2.1.16), respectively.

CHAPTER THREE

THE TRAJECTORY OF A FISH ROBOT

In this chapter, we study on the problem of finding position of a fish robot by using a reference fin ray. We use the Study mapping for this purpose. Firstly, we give some necessary information about the dual numbers. For details see Pottmann & Wallner (2001) and Fischer (1999).

3.1 The Dual Numbers

Definition 3.1.1 A dual number A can be defined as an ordered pair

$$A = (a, a^*) \tag{3.1.1}$$

of real numbers a and a^*.

The real numbers a and a^* of the expression (3.1.1) are called the real part and dual part of A, respectively. Simply, we can denote as follows;

$$ReA = a, \qquad DuA = a^*. \tag{3.1.2}$$

The set of dual numbers is denoted by D and

$$D = \{(a, a^*); a, a^* \in R\}. \tag{3.1.3}$$

The addition, $\oplus$, and multiplication, $\otimes$, can be defined on D. Let $A = (a, a^*)$ and $B = (b, b^*)$ be in D

$$A \oplus B = (a, a^*) \oplus (b, b^*) = (a + b, a^* + b^*), \tag{3.1.4}$$

$$A \otimes B = (a, a^*) \otimes (b, b^*) = (ab, ab^* + a^*b). \tag{3.1.5}$$

In addition, the dual number $(a, 0)$ is identified as a real number a. So, the set of dual numbers includes real numbers as a subset. Hence the operations defined by

(3.1.4) and (3.1.5) become the usual operations addition and multiplication when restricted to the real numbers.

The dual number $(0,1)$ is denoted by ε. So we may rewrite the expression (3.1.1) as follows;

$$A=(a,a^{*})=(a,0)\oplus(0,1)\otimes(a^{*},0)=a+\varepsilon a^{*}. \qquad (3.1.5)$$

Also, we can note that

$$\varepsilon^{2}=(0,1)\otimes(0,1)=(0,0).$$

That is $\varepsilon^{2}=0$. Hence, the dual number ε^{2} is called the zero element of D. And it is clear that $\varepsilon^{2}=\varepsilon^{3}=\cdots=\varepsilon^{n}=0$.

The set D is closed under the addition, $\oplus$. We may note that the addition, $\oplus$, is associative and commutative and every element A of D has an additive inverse $-A$. Hence $(D,\oplus)$ is an abelian group. The multiplication,$\otimes$, is closed on the set D, associative. The multiplication is distributive over the addition, that is, for all $A=(a,a^{*}),B=(b,b^{*}),C=(c,c^{*})$ in D

$$\begin{aligned}(A\oplus B)\otimes C&=\big((a,a^{*})\oplus(b,b^{*})\big)\otimes(c,c^{*})\\&=(a+b,a^{*}+b^{*})\otimes(c,c^{*})\\&=\big((a+b)c,(a^{*}+b^{*})c+(a+b)c^{*}\big)\\&=(ac+bc,a^{*}c+b^{*}c+ac^{*}+bc^{*})\\&=(ac,a^{*}c+ac^{*})\oplus(bc,b^{*}c+bc^{*})\\&=(A\otimes C)\oplus(B\otimes C).\end{aligned}$$

Similarly, the left distributive property $A\otimes(B\oplus C)=(A\otimes B)\oplus(A\otimes C)$ holds for all $A,B,C\in D$.

Hence $(D, \oplus, \otimes)$ is a ring. Moreover, the multiplication is commutative, that is,

$$A \otimes B = (a, a^*) \otimes (b, b^*) = (ab, ab^* + a^*b) = (ba, ba^* + b^*a) = B \otimes A$$

for all $A = (a, a^*), B = (b, b^*)$ in D.

We have also $(a, a^*) \otimes (1{,}0) = (1{,}0) \otimes (a, a^*) = (a, a^*)$ for all $A = (a, a^*)$ in D. This means $(1{,}0)$ is the identity element of D with respect to multiplication. Therefore, $(D, \oplus, \otimes)$ is a commutative ring with identity.

Definition 3.1.2 A dual vector $\vec{V}$ is defined by

$$\vec{V} = \vec{v} + \varepsilon \overrightarrow{v^*} \tag{3.1.6}$$

where $\vec{v}, \overrightarrow{v^*} \in R^3$, and the three dimensional dual space, D^3, is defined as

$$D^3 = \left\{ \vec{v} + \varepsilon \overrightarrow{v^*};\ \vec{v}, \overrightarrow{v^*} \in R^3 \right\}. \tag{3.1.7}$$

The standart algebraic properties for vectors in R^3 can also be defined in D^3. Let $\vec{V} = \vec{v} + \varepsilon \overrightarrow{v^*}, \vec{W} = \vec{w} + \varepsilon \overrightarrow{w^*} \in D^3$ and $A = (a, a^*) \in D$. The addition of dual vectors and multiplication of a dual vector by a dual number are defined as follows;

$$\begin{aligned} \vec{V} + \vec{W} &= \left(\vec{v} + \varepsilon \overrightarrow{v^*}\right) + \left(\vec{w} + \varepsilon \overrightarrow{w^*}\right) \\ &= (\vec{v} + \vec{w}) + \varepsilon(\overrightarrow{v^*} + \overrightarrow{w^*}). \end{aligned} \tag{3.1.8}$$

$$\begin{aligned} A \cdot \vec{V} &= (a, a^*) \cdot \left(\vec{v} + \varepsilon \overrightarrow{v^*}\right) \\ &= a\vec{v} + \varepsilon\, a\overrightarrow{v^*} + \varepsilon\, a^*\vec{v} + \varepsilon^2\, a^*\overrightarrow{v^*} \\ &= (a\vec{v}) + \varepsilon\left(a\overrightarrow{v^*} + \ a^*\vec{v}\right). \end{aligned} \tag{3.1.9}$$

Also, the dual scalar product of dual vectors is defined as follows;

$$\begin{aligned} \vec{V} \cdot \vec{W} &= \left(\vec{v} + \varepsilon \overrightarrow{v^*}\right) \cdot \left(\vec{w} + \varepsilon \overrightarrow{w^*}\right) \\ &= \vec{v} \cdot \vec{w} + \varepsilon\, \vec{v}\ \cdot \overrightarrow{w^*} + \varepsilon\, \overrightarrow{v^*} \cdot \vec{w}\ \varepsilon^2\, \overrightarrow{v^*} \cdot \overrightarrow{w^*} \\ &= (\vec{v} \cdot \vec{w}) + \varepsilon\left(\vec{v} \cdot \overrightarrow{w^*} + \overrightarrow{v^*} \cdot \vec{w}\right). \end{aligned} \tag{3.1.10}$$

The cross product of dual vectors is defined as follows;

$$\vec{V} \times \vec{W} = \left(\vec{v} + \varepsilon\overrightarrow{v^*}\right) \times \left(\vec{w} + \varepsilon\overrightarrow{w^*}\right)$$

$$= v \times w + \varepsilon\, v \times w^* + \varepsilon\, v^* \times w + \varepsilon^2\, v^* \times w^*$$

$$= (\vec{v} \times \vec{w}) + \varepsilon\left(\vec{v} \times \overrightarrow{w^*} + \overrightarrow{v^*} \times \vec{w}\right). \qquad (3.1.11)$$

For some nonzero $A, B \in D$, we have $A \otimes B = 0$ which means that A and B are zero divisors. (e.g., $2\varepsilon \otimes 3\varepsilon = 6\varepsilon^2 = 0$). Thus, D is not a field. On the other hand, the set D^3 satisfies all axioms of vector space, but its domain D is only a ring, it is not a field. This is why D^3 is a D- module. However, the elements of D^3 are also called vectors, the dual vectors.

Definition 3.1.3 The norm of a dual vector $\vec{V} = \vec{v} + \varepsilon\overrightarrow{v^*}$ is defined as follows;

$$\|\vec{V}\| = (\vec{V} \cdot \vec{V})^{\frac{1}{2}} = \left[(\vec{v} + \varepsilon\overrightarrow{v^*}) \cdot (\vec{v} + \varepsilon\overrightarrow{v^*})\right]^{\frac{1}{2}}$$

$$= \left(\vec{v} \cdot \vec{v} + \varepsilon\vec{v} \cdot \overrightarrow{v^*} + \varepsilon\, \overrightarrow{v^*} \cdot \vec{v} + \varepsilon^2\overrightarrow{v^*} \cdot \overrightarrow{v^*}\right)^{\frac{1}{2}}$$

$$= \left(\|\vec{v}\|^2 + 2\varepsilon\, \vec{v} \cdot \overrightarrow{v^*}\right)^{\frac{1}{2}}$$

$$= \|\vec{v}\| \left(1 + 2\varepsilon\, \frac{\vec{v} \cdot \overrightarrow{v^*}}{\|\vec{v}\|^2}\right)^{\frac{1}{2}}$$

$$= \|\vec{v}\| \left[\left(1 + \varepsilon\, \frac{\vec{v} \cdot \overrightarrow{v^*}}{\|\vec{v}\|^2}\right)^2\right]^{\frac{1}{2}}$$

$$= \|\vec{v}\| \left(1 + \varepsilon\, \frac{\vec{v} \cdot \overrightarrow{v^*}}{\|\vec{v}\|^2}\right)$$

$$= \left(\|\vec{v}\|, \frac{\vec{v} \cdot \overrightarrow{v^*}}{\|\vec{v}\|}\right). \qquad (3.1.12)$$

Definition 3.1.4 If the norm of a dual vector $\vec{V}$ is $(1, 0)$ then we call that $\vec{V}$ is a unit dual vector, that is

$$\|\vec{V}\| = \left(\|\vec{v}\|, \frac{\vec{v}\ \overrightarrow{v^*}}{\|\vec{v}\|}\right) = (1,0) \qquad (3.1.13)$$

which implies that $\|\vec{v}\| = 1$ and $\vec{v} \cdot \overrightarrow{v^*} = 0$.

Definition 3.1.5 The set of dual points

$$\left\{\vec{X} = \vec{x} + \varepsilon\overrightarrow{x^*};\ \|\vec{X}\| = 1, \vec{x}, \overrightarrow{x^*} \in R^3\right\} \qquad (3.1.14)$$

form a sphere which is called the dual unit sphere (D. U. S.) in D^3.

Definition 3.1.6 An oriented line ℓ is a line of R^3 which is defined with a point $p \in \ell$ and a unit vector $\vec{g}$ paralel to it, and the coordinates

$$\left(\vec{g}, \overrightarrow{g^*}\right) = (\vec{g}, \vec{p} \times \vec{g}) \qquad (3.1.15)$$

are called the Plücker coordinates of ℓ.

Since $\vec{g}$ is unit vector and $\overrightarrow{g^*} = \vec{p} \times \vec{g}$ is orthogonal to $\vec{g}$, we have

$$\|\vec{g}\| = 1 \text{ and } \vec{g} \cdot \overrightarrow{g^*} = 0. \qquad (3.1.16)$$

E. Study first combined the Plücker coordinates of a line into a dual vector by letting

$$\vec{G} = \vec{g} + \varepsilon\, \overrightarrow{g^*}\ . \qquad (3.1.17)$$

It is clear that $\|\vec{G}\| = \left(\|\vec{g}\|, \frac{\vec{g} \cdot \overrightarrow{g^*}}{\|\vec{g}\|}\right) = (1,0)$ by (3.1.16). Thus the dual vector $\vec{G}$ is unit. So, substituting the unit dual vector $\vec{G}$ at the center of D. U. S., $\vec{G}$ corresponds to the point $\left(\vec{g}, \overrightarrow{g^*}\right)$ on the D. U. S. Since the coordinates of dual point $\left(\vec{g}, \overrightarrow{g^*}\right)$ are Plücker coordinates of the oriented line ℓ, the oriented line corresponds to a dual point on the D. U. S. as seen in figure 3.1.

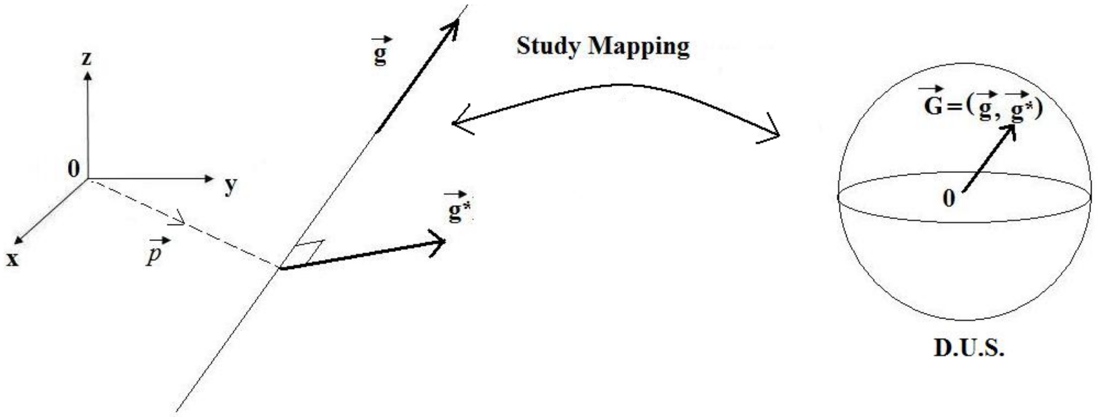

Figure 3.1 Plücker coordinates.

E. Study principle implies that dual points on the D. U. S. represents the straight lines in R^3 and vice versa. Hence we have the following theorem;

Theorem 3.1.1 (E. Study) There is one to one correspondence between the straight lines in R^3 and the dual points (not the pure dual points, $(0,\overrightarrow{g^*})$) of the D. U. S.

The motion of a point on the D. U. S. is the motion of dual unit vector oriented at the origin. If the motion on the D. U. S. is defined by the equation

$$\vec{X}(t)=\vec{x}(t)+\varepsilon\overrightarrow{x^*}(t) \tag{3.1.18}$$

then at each t, $\vec{X}(t)$ represents a straight line passing through the point $\vec{x}(t)\times\overrightarrow{x^*}(t)$ with direction $\vec{x}(t)$ by E. Study Theorem. The continuous change of the point $\vec{X}(t)$ on the D. U. S. draws a curve and this causes a continuous change of the represented straight line which is a ruled surface in R^3. This ruled surface has the following representation;

$$m(t,u)=\vec{x}(t)\times\overrightarrow{x^*}(t)+(\lambda+\mathrm{u})\vec{x}(t) \tag{3.1.19}$$

where λ is a real parameter.

On the other hand, the ruled surface with the following equation

$$m(t,u) = \vec{p}(t) + \mathrm{u}\,\vec{x}(t) \tag{3.1.20}$$

is represented by the curve

$$\vec{X}(t) = \vec{x}(t) + \varepsilon\vec{p}(t) \times \vec{x}(t) \tag{3.1.21}$$

on the D. U. S. as shown in figure 3.2.

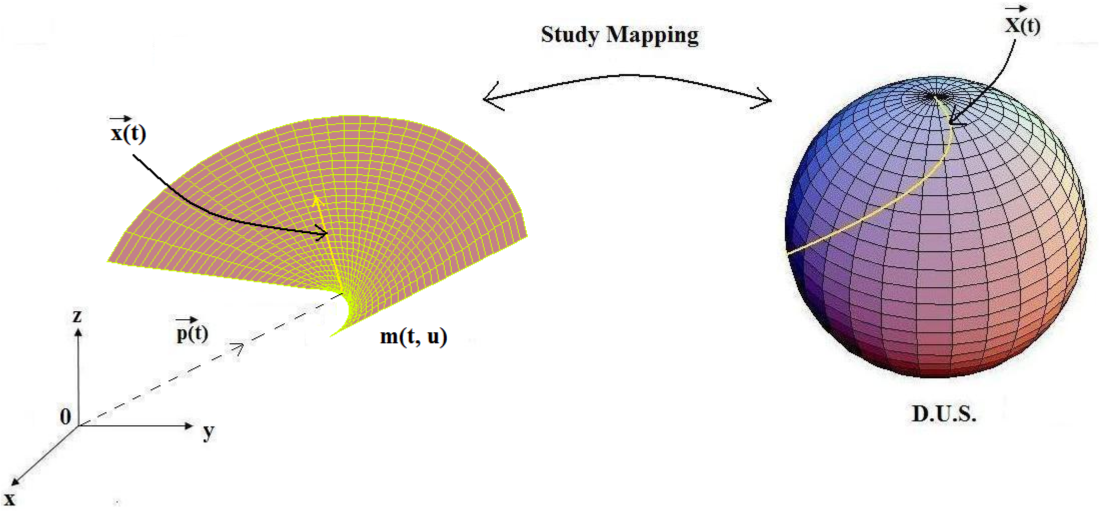

Figure 3.2 The motion of a point on the D. U. S.

3.2 The Dual Spherical Image of the Reference Fin Ray

We have considered the ruled surface $m(r,t) = \vec{\gamma}(t) + r\,\vec{c}(s_0,t)$ which is constructed by the reference fin ray $\vec{c}(s_0,t)$ while the fish robot is moving. This ruled surface represents the curve

$$\vec{X}(t) = \vec{c}(s_0,t) + \varepsilon\vec{\gamma}(t) \times \vec{c}(s_0,t) \tag{3.2.1}$$

on the D. U. S.

On the other hand, the curve $\vec{X}(t)=\vec{x}(t)+\varepsilon\overrightarrow{x^{*}}(t)$ on the D. U. S. represents a ruled surface

$$m(r,t)=\vec{x}(t)\times\overrightarrow{x^{*}}(t)+(\lambda+r)\,\vec{x}(t) \tag{3.2.2}$$

in R^3 which is obtained by the motion of reference fin ray $\vec{x}(t)$ on the base curve $\vec{x}(t)\times\overrightarrow{x^{*}}(t)$. Thus, for each fin ray of the fish robot there is one to one correspondence of a ruled surface and a dual curve on the D. U. S. Our aim is to use the D.U. S. as a radar. The movement of the fish robot implies a trajectory drawn by $\vec{c}(s_0,t)$. It is clear that this trajectory is a ruled surface. On the other hand this ruled surface corresponds a dual curve $\vec{X}(t)$ on the D. U. S. Hence the path of $\vec{X}(t)$ keeps some information about the movement of the fish robot. This is why we called the D. U. S. a kind of radar. In this part, we want to examine the trajectory of a reference fin ray $\vec{c}(s_0,t)$, hence the trajectory of the fish robot.

Let us consider an application of the results in this section. Let

$$\vec{G}(t)=\begin{pmatrix} t \\ t \\ \sqrt{1-2\,t^2} \end{pmatrix}+\varepsilon\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \qquad -\tfrac{1}{\sqrt{2}}<t<\tfrac{1}{\sqrt{2}}$$

be the curve on the D. U. S. drawn by a reference fin ray while the fish robot is moving. This curve correspondence to the ruled surface

$$m_1(\,t,u)=\begin{pmatrix} \sqrt{1-2\,t^2} \\ \sqrt{1-2\,t^2} \\ -2t \end{pmatrix}+u\begin{pmatrix} t \\ t \\ \sqrt{1-2\,t^2} \end{pmatrix}$$

which is a trejectory drawn by a reference fin ray. As seen in figure 3.3, this fin ray has the direction

$$\vec{\alpha}(t)=\begin{pmatrix} t \\ t \\ \sqrt{1-2\,t^2} \end{pmatrix}$$

and moves on the base curve

$$\vec{\gamma}(t) = \begin{pmatrix} \sqrt{1-2\,t^2} \\ \sqrt{1-2\,t^2} \\ -2t \end{pmatrix}.$$

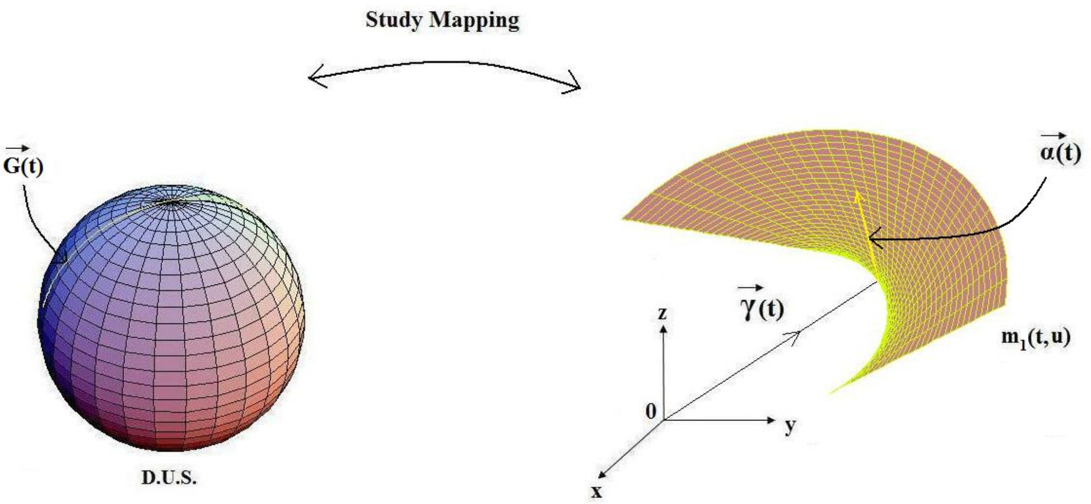

Figure 3.3 The trajectory of a fin ray corresponding to the given curve on the D.U.S.

This fin ray also has the lateral osculatory angle, θ, and inclined angle, β, with $\cos\theta = \cos\beta = 0$.

CHAPTER FOUR

CONCLUSIONS

In this study, a reference fin ray $\vec{c}(s_0, t)$ is used to investigate the bending of fish movement. For this purpose, the spherical image curve $\sigma = E_2$ is used to get information about the curving of $\vec{c}(s_0, t)$. The curvature and torsion of the curve σ are found in terms of curvature functions η and ρ. In other words, the curve σ is used to analyze the motion of a reference fin ray, hence the motion of robotic fish.

On the other hand, we use D.U.S. to find the position of fish robot. For this aim, we find the dual spherical image of the ruled surface $m(r, t)$ which is the curve $\overrightarrow{X(t)}$ on the D.U.S. Conversely, any curve on the D.U.S. corresponds to a ruled surface, which is constructed by a reference fin ray. Thus, we can examine the trajectory of $\vec{c}(s_0, t)$ by using D.U.S. That is why the D.U.S. can be used as a special kind of radar.

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