ANALYSIS OF KAPPA MESON IN LIGHT CONE QCD SUM RULES

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ABSTRACT

ANALYSIS OF KAPPA MESON IN LIGHT CONE QCD SUM RULES

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In the present work some hadronic properties of the scalar κ meson are studied. Using the QCD sum rules approach, which is a nonperturbative method, the mass and the overlap amplitude of this meson are calculated. As well as the mass and the overlap amplitude, $\kappa \to K^+\pi^-$ decay is also studied. For this decay the coupling constant $g_{\kappa K^+\pi^-}$ is obtained using light cone QCD sum rules which is an extension of the QCD sum rules method. Moreover, the coupling constant is calculated using the experimental decay width and it is compared with the value obtained in light cone QCD sum rules approach. The result of the calculation of $g_{\kappa K^+\pi^-}$, the one obtained from light cone QCD sum rules approach, is also applied to acquire the $f_0 - \sigma$ scalar mixing angle, θ_s, using the ratio $g^2(\kappa \to K^+\pi^-)/g^2(\sigma \to \pi\pi)$ obtained from experimental decay width. The value of scalar mixing angle is also compared with its experimental results.

Keywords: QCD sum rules, κ meson, overlap amplitude, coupling constant,

scalar mixing angle.

ÖZ

IŞIK KONİSİ QCD TOPLAM KURALLARINDA KAPPA MEZONUNUN ANALİZİ

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Bu çalışmada skaler κ mezonunun bazı hadronik özellikleri incelendi. Perturbatif olmayan bir metod olan kuantum renk dinamiği toplam kuralları kullanılarak κ mezonunun kütlesi ve çakışma genliği hesaplandı. Bu hesapların yanında, $\kappa \to K^+\pi^-$ bozunumu incelendi. QCD toplam kurallarının bir uzantısı olan ışık konisi QCD toplam kuralları kullanılarak bu bozunumun kuplaj sabiti elde edildi. Bu kuplaj sabiti aynı zamanda bozunum genliğinin deneysel sonucu kullanılarak da hesaplandı ve ışık konisi QCD toplam kuralları yaklaşımıyla bulunan değeri ile karşılaştırıldı. Ayrıca ışık konisi QCD toplam kurallarından elde edilen kuplaj sabitinin değeri ve deneysel bozunum genliğinden hesaplanan $g^2(\kappa \to K^+\pi^-)/g^2(\sigma \to \pi\pi)$ oranı kullanılarak $f_0 - \sigma$ skaler karışım açısı, θ_s, hesaplandı ve deneylerden elde edilen sonucuyla karşılaştırıldı.

Anahtar Kelimeler: QCD toplam kuralları, κ mezonu, çakışma genliği, kuplaj sabiti, skaler karışım açısı.

To My Sister

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CHAPTER 1

INTRODUCTION

At the end of the classical era, a question has arisen "What holds the nucleus together?". There should be another force of which classical model has no explanation, that is more powerful than the electrical repulsion of protons. The physicists called this force as strong force. Together with this force there are four fundamental forces in nature, electromagnetic, gravitational, weak and strong. However, in contrary to electromagnetic and gravitational forces, we can not notice strong force. This is because of its short range which is about as much as the size of the nucleus while the ranges of gravitational and electromagnetic forces are infinite.

In 1934 the first theory of the strong force between nucleons was suggested by Yukawa. While he was trying to explain the force between protons and neutrons, he postulated the existence of a particle which explains the charge exchange between them. From the short range property of the strong force, he predicted the mass of this particle to be nearly 200 times that of the electron. Then in 1947, pion (π meson) which was identified with this particle was discovered by Powell and his colleagues in cosmic rays. By the time Yukawa indicated this particle, Quantum Choromodynamics (QCD) which is the theory of the strong interactions was not known. After QCD and quark model, one of the basis of QCD, had been established, internal structure of particles was understood.

According to quark model there are six quark flavors each of which carries three different colors; u, d, s, c, b, t and each of them has its antiquark. Quarks interact strongly and they all carry electric charge, i.e. u, c, t have an electric charge $+2/3$ and d, s, b have $-1/3$. Quarks have spin $1/2$ so they are fermions. They also carry other additional quantum numbers. Antiquarks have opposite quantum numbers to quarks.

Bound states of quarks form hadrons; mesons and baryons. According to quark model, mesons are the bound states of a quark and an antiquark, $q\overline{q}'$. The flavor of q and $\overline{q}'$ can be different. A $q\overline{q}'$ pair which carries orbital angular momentum L and spin angular momentum S has total angular momentum $J = L \oplus S$. Since quarks have the spin $S_q = 1/2$, a meson can take the spin angular momentum values $S = 0, 1$ where $S = 0$ expresses the spin singlet and $S = 1$ the spin triplet. Another property is parity in order to classify the meson since it can be classified in J^P. The parity for meson is $P = (-1)^L$ ($(-1)^L$ is due to the space part of the meson wave function, and the additional (-1) is due to the opposite intrinsic parities of fermions and antifermions.).

Table 1.1: The meson families with respect to quantum numbers.

$n^{2S+1}L_J$	L	S	J	$P = (-1)^{L+1}$	J^P	Family
1^1S_0	0	0	0	-	0^-	Pseudoscalar
1^3S_1	0	1	1	-	1^-	Vector
1^1P_1	1	0	1	+	1^+	Pseudovector
1^3P_0	1	1	0	+	0^+	Scalar
1^3P_1	1	1	1	+	1^+	Axialvector

As mesons can be classified in J^P, the kinds of mesons can be determined with respect to their states in terms of their radial excitation n, spin multiplicity $2S+1$, orbital angular momentum and total angular momentum, $n^{2S+1}L_J$. Table 1.1 shows the classification of mesons via their $n^{2S+1}L_J$ and J^P.

Isospin quantum number is conserved in all strong interactions and its third component is related to the charge of the particles. To distinguish each member of mesons in a family, the isospin and its third component should be assigned. In the quark family, u, d, s quarks are ultimately important since their masses are relatively small and therefore they form almost all of the hadrons making the universe. Among light quarks, u and d have isospin 1/2 and they constitute isospin doublet as do protons and neutrons. On the other hand the other quarks have isospin 0. They form the mesons as $I = 1$ isovector, $I = 0$ isoscalar and $I = 1/2$ isodoublet in $q\overline{q'}$ representation given in Table 1.2.

Table 1.2: Classification of mesons according to their isospins

		Mesons		
		$I = 1$	$I = 0$	$I = 1/2$
J^P	0^-	π	$\eta,\ \eta'$	$K(495)$
	1^-	ρ	$\omega,\ \phi$	$K^*(892)$
	1^+	b_1	$h_1,\ h_1'$	$K_1(1270)$
	0^+	$a_0(980)$	$\sigma(600)$ $f_0(980)$	$\kappa(800)$
		$a_0(1450)$	$f_0(1370)$ $f_0(1500)$ $f_0(1710)$	$K_0^*(1430)$

From the meson families, pseudoscalar, vector and pseudovector mesons are well established with theoretical calculations and experimental data analysis which indicate some hadronic properties of these mesons, such as decay width and mass. They can be predicted by QCD and can be considered as $q\overline{q'}$ state. On the other hand, identification of scalar mesons is very difficult. In the scalar family, $J^P = 0^+$, $a_0(980)$, $\sigma(600)$, $f_0(980)$ and $\kappa(800)$ or $K^*(800)$ are called light scalar mesons and the rest are called heavy scalar mesons. The light scalar mesons are extremely important in hadron physics, and in literature there are a lot of debates about them. First of all, masses of all light scalar mesons are expected to become larger since their quark configurations satisfy the $^3P_0$ quantum numbers. According to P-wave orbital excitation, their masses should be larger than 1 GeV [1, 2, 3] as heavy scalar mesons while according to experimental observations their masses are around $0.6 - 1.0$ GeV [4].

Moreover, as it can be seen from Table 1.3, $f_0(980)$ and $a_0(980)$ mesons have approximate masses, despite the fact that $a_0(980)$ meson consists of u and d quarks, while $f_0(980)$ meson has $s\bar{s}$ quark structure. Also the $\sigma(600)$ meson which is composed of u and d quarks, is lighter than $a_0(980)$ and that makes the

Table 1.3: Estimated mass and decay width values of light scalar masons

Meson	Mass (MeV)	Width (MeV)
$\sigma(600)$ or $f_0(600)$	$400 - 1200$	$600 - 1000$
$K_0^*(800)$ or $\kappa(800)$	672 ± 40	550 ± 34
$a_0(980)$	980 ± 20	$50 - 100$
$f_0(980)$	980 ± 10	$40 - 100$

light scalar mesons awkward [2].

These problems connected to the light scalar mesons bring about an idea. Jaffe first proposed that these light scalar mesons are the candidates of tetraquark which have the same quantum number with their $q\overline{q'}$ states and which express their mass spectrum [5]. However, there are a lot of discussions whether or not they have $q\overline{q'}$ structure [6], meson-meson molecular structure [7] or multiquark structure $(q\overline{q'})^2$ [5]. In addition to masses of these mesons, decay widths of them cause problems that are based on the couplings to decay channels. As an example, the coupling of decay produces large decay width for σ and κ mesons. With the idea of Jaffe they have become the subjects of many papers both theoretically and experimentally.

The possibility that the light scalar mesons have $(q\overline{q'})^2$ structure instead of $q\overline{q'}$ structure was studied in Ref.[8]. Ref.[9] gave a comparision about the ratios of coupling constants of $\kappa \to K\pi$ and $\sigma \to \pi\pi$ decays with respect to $q\overline{q'}$ state and four quark model of Jaffe. Considering two body decays of them, light scalar mesons were studied with respect to diquark- antidiquark states [10] and their coupling constants were also calculated in those papers. Taking into account the four quark structure, the study of these mesons were offered via QCD sum rules method [11, 12]. Masses and coupling constans of these mesons were calculated with this method. Their masses were also calculated with respect to relativistic quark model taking their structure as being diquark-antidiquark state [13]. In addition, $q\overline{q'}$ pair structure was also utilized to look into some properties, such as mass and decay width of these mesons [14, 15]. In Ref.[16], an analysis of masses of them was made taking into account this structure. Furthermore, using QCD sum rules approach, decay width constant and masses of light scalar mesons which have isospin $I = 1/2$ were calculated with respect to the their $q\overline{q'}$ state [17].

The presence of κ meson which is studied in this thesis has been reported with respect to the data of $E791$ [4, 18] and BES [19] collaborations on the observations in $D^+ \rightarrow K^-\pi^+\pi^+$ and $J/\psi \rightarrow K^+\pi^-K^+\pi^-$ decays. It exists in $K\pi$ scattering. In the literature there are lots of work about the elastic $K\pi$ scattering or the production. Cherry and Pennington showed that there is a pole at the region smaller than 0.83 GeV. The value of mass was not accurate but they ruled out the higher values of mass [20]. In addition, as in the Refs. [4, 21, 22], there are a lot of different mass and decay width results according to the experimental observations. Firstly, according to LASS collob-oration, when the data on $K\pi$ scattering were fitted there is no κ pole [21]. Secondly, according to the $E791$ the values of mass and decay width of κ meson are $m_\kappa = 797 \pm 19 \pm 43$ MeV and $\Gamma_\kappa = 410 \pm 43 \pm 87$ MeV with respect to the data on $D^+ \rightarrow K^-\pi^+\pi^+$ in the $K\pi$ scattering [4]. Thirdly, the BES colloboration find a pole at $m_\kappa - i\frac{\Gamma_\kappa}{2} = 760 \pm 20 \pm 40 - i(420 \pm 45 \pm 60)$ MeV with respect to the data on $J/\psi \rightarrow KK\pi$ in the $K\pi$ scattering [22]. Then Bugg made a combined fit to these data and he obtained the pole position as $m_\kappa - i\frac{\Gamma_\kappa}{2} = 663 \pm 8 \pm 34 - i(329 \pm 5 \pm 22)$ Mev [23]. In addition, accoring to particle data group, the pole position is given by $m_\kappa - i\frac{\Gamma_\kappa}{2} = 672 \pm 40 - i(550 \pm 34)$ MeV [24].

If the theoretical and experimental data support each other, the results of m_κ and Γ_κ are more reliable. In the literature, there are also theoretical calculations for the mass and the decay width of κ meson. In Ref.[3] using the experimental data and analycity of the amplitudes of two body scattering, the existence of this meson was shown and in comparison with the earlier experimental data, the value of mass was found lighter, $m_\kappa = 658 \pm 13$ MeV and in their paper they also found the value of decay width theoretically as $\Gamma_\kappa = 557 \pm 24$ MeV.

Moreover, the mass of the κ meson was calculated theoretically in Ref. [25] in the framework of QCD sum rules method, as it is in this thesis, and its value was found as $m_\kappa = 700 \pm 60$ MeV.

In this thesis, firstly the mass of κ meson and its overlap amplitude are calculated. Secondly, $\kappa K \pi$ vertex is studied and the coupling constant $g_{\kappa K^+ \pi^-}$, which arises as an exchange of κ meson in elastic $K\pi$ scattering and as a virtual κ meson state in J/ψ decay, is calculated using light cone QCD sum rules method. In these calculations,the quark structure of κ meson is considered as $q\overline{q'}$ state. Moreover in this work, we discuss the $f_0 - \sigma$ scalar mixing angle, θ_s, by considering the coupling constant, $g_{\sigma\pi\pi}$ calculated in light cone QCD sum rules. In Chapter 2, the traditional QCD sum rules and the light cone QCD sum rules are discussed. In Chapter 3, scalar κ meson is analysed in light cone QCD sum rules approach. The mass of κ meson and its overlap amplitude are calculated numerically. The numerical calculations of the coupling constant $g_{\kappa K \pi}$ and the scalar mixing angle, θ_s are discussed in Chapter 4. Finally, a conclusion is presented in Chapter 5.

CHAPTER 2

QCD SUM RULES

QCD sum rules method has been proposed in 1979 by Shifman, Vainshtein and Zakharov [26] for mesons. It has become the most useful method for calculating hadronic observable such as coupling constants, masses etc. Starting point of this method is asymptotic freedom property of QCD. Physical processes can be described by coupling constants that characterize the interactions and depend on the momentum transfer, Q^2. In Quantum Electrodynamics (QED), the effective coupling constant, α, is around $1/137$ at low momentum transfers and when momentum transfer increases, it increases slowly. However in QCD, the situation is different. In QCD, while at small momentum transfers the interactions between particles become strong, at large momentum transfers their interactions are weak. This is asymptotic freedom property of QCD.

The effective coupling constant in QCD, α_s, is measured from experiments as a function of energy given in Fig. 2.1. The experimental results are consistent with the prediction of QCD. The strength of the coupling decreases logarithmically with the increasing energy and it is defined as,

$$\alpha_s(Q^2) = \frac{g_s^2(Q^2)}{4\pi} \simeq \frac{1}{4\pi\beta_0 \ln(Q^2/\Lambda_{QCD}^2)}, \tag{2.1}$$

where β_0 is Gell-Mann-Low function constant and Λ_{QCD} is QCD scale parameter which has an important role in QCD. The interactions of quarks and gluons can be analyzed in two regions. When $Q^2 \gg \Lambda_{QCD}^2$, high momentum region

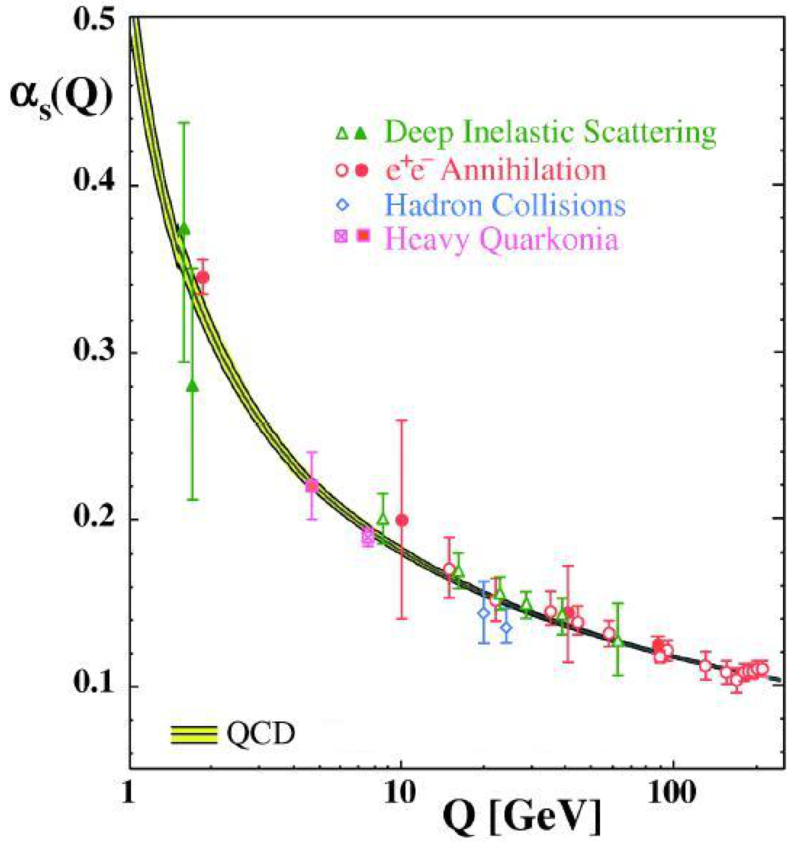

Figure 2.1: The dependency of QCD coupling constant on energy [27].

or at small distances, due to the asymptotic freedom property of QCD, quarks and gluons are nearly free. However, at long distances, in low momentum region where $Q^2 \sim \Lambda^2_{QCD}$ the interaction is strong and quarks starts to form hadrons. Due to confinement, nonperturbative effects are observed, and this means that perturbative calculations fail in these region. Therefore, a nonperturbative method is necessary for any calculation in this region one of which is the QCD sum rules method that is the method used in this work.

In this chapter, this method and one of its extensions, light cone QCD sum rules method, are described and a review of important features of them is given. The discussions follow closely Refs. [28, 29, 30].

2.1 Traditional QCD Sum Rules

The main idea of this method is to start from short distances where the perturbation theory is valid, and then to approach larger distances where the non-perturbative effects are observed step by step to gain information about the hadronic observable [31]. Therefore it can be said that the principle of this method is connection between high energies, quark gluon sector, and the low energies, hadron sector.

Hadrons, correspond to the low energies, are represented by their quantum numbers. In the application of this method time ordered correlation function of interpolating currents is taken into account. These interpolating currents are formed from quark fields, and have the same quantum numbers of the hadron they correspond to. Then Operator Product Expansion (OPE) is applied at $q^2 = -Q^2 \ll 0$ to obtain the theoretical side of the correlator. In traditional QCD sum rules, the expansion is performed with respect to operators which have the different mass dimension. On the other hand, inserting a complete set of hadronic states, the correlation function can be calculated and this side is known as the phenomenological part of the calculation of this correlator. Matching these two parts with the help of Borel transformation, QCD sum rules is obtained.

Two point and three point sum rules are the traditional QCD sum rules. With two point QCD sum rules, mass of the hadrons, transition amplitudes, branching ratios can be calculated. Using three point QCD sum rules, we can study matrix elements of transition operators. In this study, we use two point QCD sum rules to obtain the mass and overlap amplitude of κ meson. In order to use two point QCD sum rule which we use for mass calculation, first a two point correlator is necessary.

2.2 Two Point Correlation Function

QCD sum rules method is an advantageous method for nonperturbative approaches and it is based on QCD Lagrangian which has the form,

$$L_{QCD} = -\frac{1}{4}G^{\alpha}_{\mu\nu}G^{\alpha\mu\nu} + \sum_{q}\overline{\psi_q}(i\not{D} - m_q)\psi_q, \tag{2.2}$$

where $G^{\alpha}_{\mu\nu}$ is gluon field tensor, ψ_q is quark field with q representing the flavor of quarks, u, d, s, c, b, t and D is the covariant derivative. For the calculation containing nonperturbative effects one needs nonperturbative approaches one of which is the QCD sum rules that is the method used in this thesis. To apply this method the following correlation function, which is the main ingredient of the QCD sum rules method, is studied:

$$\Pi(q^2) = i\int d^4x e^{iq\cdot x}\langle 0|T\{j(x)\overline{j}(0)\}|0\rangle, \tag{2.3}$$

where $|\ 0\rangle$ is the nonperturbative vacuum state, T is the time ordering operator, q is the momentum of quarks and $j(x) = \overline{q_i}\Gamma q_j$ is the interpolating current which injects the quarks at the space time point x to the QCD vacuum. Quark currents have specific J, P quantum numbers which are chosen with respect to quantum numbers of hadrons under consideration. Choosing various structure of Γ, quantum numbers of this currents are determined. As an example, for scalar, pseudoscalar, vector and axialvector mesons, quark currents can be given respectively as [32];

$$\begin{aligned}
j^S &= \overline{q}_i q_j, & J^P &= 0^+\\
j^P &= i\overline{q_i}\gamma_5 q_j, & J^P &= 0^-\\
j^V_\mu &= \overline{q_i}\gamma_\mu q_j, & J^P &= 1^-\\
j^A_\mu &= \overline{q_i}\gamma_\mu\gamma_5 q_j, & J^P &= 1^+.
\end{aligned} \tag{2.4}$$

The correlation function, which is used to obtain the mass, transition amplitudes, branching ratios of hadrons, can be calculated in two different ways as it was mentioned previously. These two parts will be explained in detail in the following sections.

2.2.1 Phenomenological Side

The invariant amplitude, $\Pi(q^2)$, is a function of q^2. When $q^2 \equiv -Q^2$ is large and negative, the correlation function in Eq. (2.3), is related to only the short distance interactions where the perturbative effects are observed. In this region, the correlator can be calculated in connection with gluons and quarks. However, when q^2 gets close to positive values, long distance effects become important. In these regions interactions become stronger. This means that quarks start to create hadrons. For the phenemenological side of the calculation, we need a complete set of hadronic states that is inserted between two quark currents and which has the same quantum number with the interpolating current. This complete set is a unit operator for all hadronic states and is given as;

$$\begin{aligned} 1 &= |0\rangle\langle 0| + \sum_h \int \frac{d^4p_h}{(2\pi)^4} 2\pi\delta(p_h^2 - m_h^2)\Theta(p_h^0)|h(p_h)\rangle\langle h(p_h)| \\ &+ \text{higher Fock states}, \end{aligned} \tag{2.5}$$

where the summation is over all hadronic states which are composed by the quark currents. When we insert this complete set of hadronic states in the correlation function, we obtain the imaginary part of the phenomenological side of this correlator as;

$$\begin{aligned} 2Im\Pi(q^2) &= \sum_h \int \langle 0|j|h\rangle\langle h|\bar{j}|0\rangle d\tau_h (2\pi)^4\delta^4(q - p_h) \\ &= 2\pi f_h^2 \delta(q^2 - m_h^2) + 2\pi\rho^h(q^2)\Theta(q^2 - s_0^h), \end{aligned} \tag{2.6}$$

where $d\tau_h$ is over the phase space volume element of the hadron, m_h is the mass of the hadron which is represented by the quark current, f_h is defined as $\langle h|j(0)|0\rangle$ and it is the overlap amplitude for mesons. In Eq. (2.6) $\rho^h(q^2)$ represents the contribution of continuum and higher states that can be dealt with using quark hadron duality assumption.

Since Eq. (2.3) is an analytic function of q^2, we can use Cauchy Integral formula to be able to make a connection between the values of $\Pi(q^2)$ at positive values of q^2 and its values at negative values of q^2. For this connection one can use the contour that is shown in Fig. (2.2);

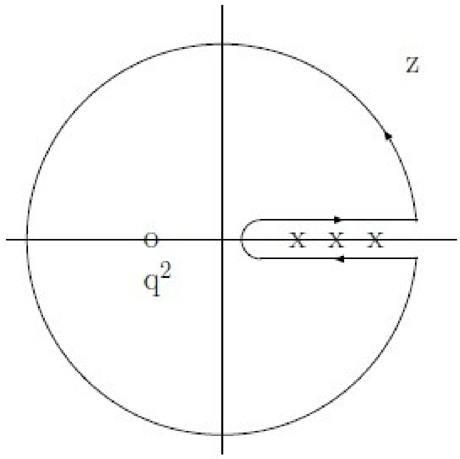

Figure 2.2: The complex z plane showing the contour that is used to calculate correlation function at the point q^2. Crosses represent hadronic thresholds at $q^2 > 0$.

According to this contour by the Cauchy formula the correlator for physically observed hadrons can be written as;

$$\begin{aligned}\Pi(q^2) &= \frac{1}{2\pi i}\oint_C dz \frac{\Pi(z)}{z-q^2} \\ &= \frac{1}{2\pi i}\oint_{|z|=R} dz \frac{\Pi(z)}{z-q^2} + \frac{1}{2\pi i}\int_{t_{min}}^{R} dz \frac{\Pi(z+i\epsilon)-\Pi(z-i\epsilon)}{z-q^2} \qquad (2.7)\end{aligned}$$

where R is the radius of contour and t_{min} is the threshold for real states. If we consider that the radius of contour is infinity, and so that $|z| \to \infty$, and if the

$\Pi(z)$ in the Eq. (2.7) goes to zero fast, the first term of the correlator vanishes. However, even if the first term does not vanish, the $1/(z-q^2)$ term can be expanded in terms of q^2/z and after a power of expansion, $\Pi(z)/z^n$ vanishes fast enough and the remaining terms which are the polynomial in q^2 are called subtraction terms. Therefore Eq. (2.7) becomes,

$$\Pi(q^2) = \frac{1}{2\pi i}\int_{t_{min}}^{\infty} dz \frac{\Pi(z+i\epsilon)-\Pi(z-i\epsilon)}{z-q^2} + \text{subtraction terms.} \tag{2.8}$$

For $q^2 < t_{min}$, the value of $\Pi(q^2)$ is real and according to Schwartz reflection principle, which is a theorem for complex analysis, for $q^2 > t_{min}$ the expression on the numerator of Eq. (2.8) takes the form $\Pi(z+i\epsilon)-\Pi(z-i\epsilon) = 2iIm\Pi(q^2)$. Therefore one obtains the dispersion relation as follows;

$$\Pi(q^2) = \frac{1}{\pi}\int_{t_{min}}^{\infty} ds \frac{Im\Pi(s)}{s-q^2} + \text{subtraction terms.} \tag{2.9}$$

The term $Im\Pi(s)/\pi$ is the spectral density, $\rho(s)$. Inserting spectral density in Eq. (2.9), and combining it with second part of the Eq. (2.6) the final form of physical side of the correlator is obtained;

$$\Pi^{phen}(q^2) = \frac{f_h^2}{m_h^2-q^2} + \int_{s_0^h}^{\infty} ds \frac{\rho^h(s)}{s-q^2} + \text{subtraction terms,} \tag{2.10}$$

where s_0^h is threshold for higher states.

2.2.2 Theoretical Side

The theoretical or QCD part of the correlation function can be calculated by applying Operator Product Expansion (OPE) in deep Euclidean region $Q^2 \gg \Lambda_{QCD}^2$. The main idea of OPE is to expand the time ordered product of two local operators, one of which is at point x and the other is at zero, in a set of local operators [28]. By the OPE which was developped by Wilson [33], the time ordered product of two local operators can be written as,

$$T\{A(x)B(0)\} = \sum_n C_n(x)O_n(0); \qquad x \to 0; \tag{2.11}$$

where C_n are the Wilson coefficient and O_n are the quark gluon field operators. Wilson coefficients contain the information about short distance effects and they can be calculated using perturbation theory. However only pure perturbative expansion is not valid because of nonperturbative effects in QCD. In QCD sum rules calculations, vacuum expectation values of operators (condensates) are used to parameterize the nonperturbative effects. Applying Fourier transformation the OPE correlation function can be obtained as,

$$\Pi^{OPE}(q^2) = \sum_n C_n(q^2)\langle O_n \rangle; \tag{2.12}$$

where $\langle O \rangle = \langle 0|O|0 \rangle$. The operators are classified with respect to both their dimension and Lorentz spin. For example, the wave function, ψ has the dimension 3/2, gluon field strength, $G_{\mu\nu}$ has 2 and ∇ operator has 1. With $n = 0$, the lowest dimension operator is unit operator related with perturbative contribution; $\langle 0|O_0|0 \rangle = 1$ so $\Pi^{pert}(q^2) = C_0(q^2)$. In perturbation theory, the vacuum condensates vanish except from the zero dimension. However, in QCD, the expectation values of operators are not zero for the higher dimensions since nature of vacuum changes with the nonperturbative effects. In this way, using OPE both perturbative and nonperturbative effects are taken into account, simultaneously.

Some examples of the field operators, which have spin 0, with different dimensions such as $3, 4, 5, 6$ are,

$$
\begin{aligned}
O_3 &= \overline{q}q \\
O_4 &= G^{\alpha}_{\mu\nu}G^{\alpha\mu\nu} \\
O_5 &= \overline{q}\sigma_{\mu\nu}G^{\alpha\mu\nu}t^{\alpha}q \\
O_6 &= f_{abc}G^{a\nu}_{\mu}G^{b\lambda}_{\nu}G^{c\mu}_{\lambda}...
\end{aligned} \tag{2.13}
$$

It is crucial that, for dimensions $n = 1, 2$ there are no colorless gauge and Lorentz invariant operators in QCD so, operators of dimension 1 or 2 do not appear in sum rules calculations. In addition, for dimensions $n > 6$, condensates play minor role so we can neglect them for most QCD sum rules calculations [29]. Calculating the correlation function using the OPE one obtains the spectral density which has the form, $\rho^{OPE}(q^2) = \frac{1}{\pi}Im\Pi^{OPE}(q^2)$, and obtains the theoretical part of the correlation function as;

$$
\Pi(q^2) = \int_0^{\infty} \frac{\rho^{OPE}(s)}{s - q^2} ds + \text{subtraction terms.} \tag{2.14}
$$

2.3 Borel Transformation and Derivation of QCD Sum Rules

Now, we have two representations for the correlator; one of them derived from phenomenological side in Sect. 2.2.1, $\Pi^{phen}(q^2)$, and the other from the theoretical side in Sect. 2.2.2, $\Pi^{OPE}(q^2)$. To obtain the sum rules, these two representations are equated;

$$
\begin{array}{ccc}
\Pi^{phen}(q^2) & = & \Pi^{OPE}(q^2) \\
\frac{f_h^2}{m_h^2 - q^2} + \int_{s_0^h}^{\infty} \frac{\rho^h(s)}{s-q^2} ds & = & \int_0^{\infty} \frac{\rho^{OPE}(s)}{s-q^2} ds \\
+\text{subtraction terms} & & +\text{subtraction terms}
\end{array} . \tag{2.15}
$$

However, in this equation there are undesired substraction terms and continuum contributions of higher energy terms. To get rid of the subtraction terms one can use a solution which is called Borel transformation and it is defined for a function $f(Q^2)$ as

$$B[f(M^2)] = \lim_{Q^2,n\to\infty} \frac{Q^{2n+1}}{n!}(-\frac{d}{dQ^2})^n f(Q^2), \qquad M^2 \equiv \frac{Q^2}{n} \tag{2.16}$$

where M^2 is Borel parameter. According to this transformation, whose details are given in Appendix B, subtraction terms become zero since Borel transformations eliminate any polynomial and the term $\frac{1}{s-q^2}$, where $q^2 = -Q^2$ transforms into $e^{-\frac{s}{M^2}}$ and that provides an exponential suppression of the continuum and the contribution of higher energy term. Finally the Eq. (2.15) becomes;

$$f_h^2 e^{-\frac{m_h^2}{M^2}} + \int_{s_0^h}^{\infty} \rho^h(s) e^{-\frac{s}{M^2}} ds = \int_0^{\infty} \rho^{OPE}(s) e^{-\frac{s}{M^2}} ds. \tag{2.17}$$

In the limit $Q^2 = -q^2 \to \infty$, where the correlator is completely perturbative, continuum term and contribution coming from higher states can be parameterized via the approximation given as,

$$\int_{s_0^h}^{\infty} \frac{\rho^h(s)}{s-q^2} ds = \int_{s_0}^{\infty} \frac{\rho^{OPE}(s)}{s-q^2} ds, \tag{2.18}$$

where s_0 is the continuum threshold and it is not necessary to be equal to s_0^h. Eq. (2.18) is called quark hadron duality assumption.

The OPE side of the Eq. (2.17) can be written in two separate parts as,

$$\begin{aligned} f_h^2 e^{-\frac{m_h^2}{M^2}} + \int_{s_0^h}^{\infty} \rho^h(s) e^{-\frac{s}{M^2}} ds &= \int_0^{s_0} \rho^{OPE}(s) e^{-\frac{s}{M^2}} ds \\ &+ \int_{s_0}^{\infty} \rho^{OPE}(s) e^{-\frac{s}{M^2}} ds. \end{aligned} \tag{2.19}$$

After performing Borel transformation on Eq. (2.18) and inserting the result in Eq. (2.19), the final form of the sum rules is obtained as;

$$f_h^2 e^{-\frac{m_h^2}{M^2}} = \int_0^{s_0} \rho^{OPE}(s) e^{-\frac{s}{M^2}} ds. \tag{2.20}$$

The M^2 appearing in Eq. (2.20) is the Borel parameter and it is an unphysical parameter. Therefore, it should not effect the value of the physical parameter that is obtained from QCD sum rules calculation. Hence, we should choose a region where the physical quantities do not depend on this unphysical quantity, M^2. The sum rule in Eq. (2.20) is not applicable if the parameter M^2 is too small. In the OPE side when we insert the vacuum condensates according to their dimensions to obtain the correlator that we write instead of $Im\Pi(s)/\pi = \rho^{OPE}(s)$, the vacuum condensates are proportional to $\frac{1}{(M^2)^n}$. So the contribution of higher dimensional condensates become too large to be neglected when M^2 is too small. Hence, it should be sufficiently large. On the other hand, this parameter should not be too large so that the contributions of higher states which are exponentially suppressed remains as a small part of the total integral [29]. So, one should choose an intermediate region where the physical quantities which are calculated by QCD sum rules do not depend on the Borel parameter, M^2. In addition in Eq. (2.20), the other unknown, s_0, different from Borel parameter, is not arbitrary. It is chosen with respect to the energy of the next excited state in the channel of quark current.

2.4 Light Cone QCD Sum Rules

One of the extensions of QCD sum rules is the light cone QCD sum rules. The starting point of this extension is the correlator which is the time ordered product of two quark currents that are sandwiched between an on mass shell state and vacuum. The structure of the correlation function in the light cone QCD sum rules is,

$$\Pi(q,p) = i \int d^4x e^{iq\cdot x} \langle \pi(p) | T\{j_1(x) j_2(0)\} | 0 \rangle. \tag{2.21}$$

In Ref. [29], as an example to calculate one of the contributions, u quark fields have been contracted to the correlator. For the free massless quark, the propagator is

$$iS_0(x,0) = \langle 0|T\{u(x)\overline{u}(0)\}|0\rangle = \frac{i\not{x}}{2\pi^2 x^4}, \tag{2.22}$$

where $\not{x} = x^\alpha \gamma_\alpha$. After inserting the propagator and using $\gamma_\mu\gamma_\alpha\gamma_\nu \rightarrow -i\epsilon_{\mu\alpha\nu\rho}\gamma^\rho\gamma_5 + \cdots$, the correlator takes the form;

$$\Pi(q,p) = -i\epsilon_{\mu\alpha\nu\rho}\int d^4x \frac{x^\alpha}{\pi^2 x^4} e^{iq\cdot x}\langle\pi(p)|\overline{u}(x)\gamma_\rho\gamma_5 u(0)|0\rangle. \tag{2.23}$$

To obtain the correlator, the matrix element in the correlator $\langle\pi(p)|\overline{u}(x)\gamma_\rho\gamma_5 u(0)|0\rangle$ has to be calculated. Considering the general form of this matrix element, $\langle\pi(p)|\overline{q}(x)\gamma_\rho\gamma_5 q(0)|0\rangle$, one can expand $q(x)$ around $x = 0$ as $q(x) = q(0) + x_\alpha\partial_\alpha q(x)|_{x=0} + \frac{1}{2}x_\alpha x_\beta\partial_\alpha\partial_\beta q(x)|_{x=0} + \cdots$. where Foch-Schwinger gauge, $x_\mu A^\mu = 0$ is used to write $\overleftarrow{\partial}\cdot x = \overleftarrow{D}\cdot x$. Therefore the expansion of this quark-antiquark operators around $x = 0$ become as,

$$\overline{q}(x)\gamma_\rho\gamma_5 q(0) = \sum_r \frac{1}{r!}\overline{q}(0)(\overleftarrow{D}\cdot x)^r\gamma_\rho\gamma_5 q(0) \tag{2.24}$$

and the matrix elements of them have general decomposition [29] as,

$$\begin{aligned} \langle\pi(p)|\overline{q}\overleftarrow{D}_{\alpha 1}\overleftarrow{D}_{\alpha 2}...\overleftarrow{D}_{\alpha r}\gamma_\rho\gamma_5 q|0\rangle &= (-i)^r p_{\alpha 1}p_{\alpha 2}...p_{\alpha r}p_\rho M_r \\ &+ (-i)^r g_{\alpha 1\alpha 2}p_{\alpha 3}...p_{\alpha r}p_\rho M_r' + \cdots. \end{aligned} \tag{2.25}$$

Here M_r and M_r' are the local operator matrix elements. After inserting Eq. (2.25) in correlator equation and integrating over x, the following equation is obtained,

$$\Pi(q,p) = \frac{1}{(-p^2)}\sum_{r=0}^{\infty}\left(\frac{2p\cdot q}{-p^2}\right)^r M_r + \frac{4}{(-p^2)^2}\sum_{r=2}^{\infty}\left(\frac{2p\cdot q}{-p^2}\right)^{r-2}\frac{1}{r(r-1)}M_r' + \cdots \quad (2.26)$$

Here, each term has $2p \cdot q/(-p^2) = [(p+q)^2 - p^2]/(-p^2)$ factor and this causes a problem. This constant is small when $(p+q)^2 \simeq p^2$ with $p \neq 0$. If not, infinite number of terms has to be kept in the expansion. The main idea of light cone QCD sum rules is to expand the matrix elements in the correlator for the small values of x^2. This is the reason that it is called as light cone QCD sum rules. In this method, unlike the traditional one, these matrix elements are expanded using the distribution amplitudes on light cone which have different twist. Twist is the difference between the dimension and the spin of the operator. For instance, if we consider the operator $\overline{q}q$, it is a three dimensional operator which carries spin 0 or 1. So, in Eq. (2.25) the lowest twist is 2 from the first term at the right hand side with the consideration of the twist 4 operators which appear after taking the derivative. For example, focusing only the lowest twist 2, the expansion of the matrix element in Eq. (2.23) at $x^2 = 0$ is,

$$\langle \pi(p)|\overline{u}(x)\gamma_\rho\gamma_5 u(0)|0\rangle_{x^2=0} = ip_\rho \frac{f_\pi}{\sqrt{2}}\int_0^1 du e^{iup\cdot x}\varphi_\pi(u), \quad (2.27)$$

where $\varphi_\pi(u)$ is the pion distribution amplitude, which describes pion momentum distributions, with twist 2 and u represents the momentum fraction carried by quarks, $0 < u < 1$. The distribution amplitudes are the nonperturbative inputs so their form should be determined from experiments or nonperturbative methods [34]. In this thesis, coupling constant of $\kappa \to K\pi$ decay is calculated using light cone QCD sum rules and the other distribution amplitudes of pion will be given in the calculation of this coupling constant.

CHAPTER 3

QCD SUM RULES FOR SCALAR KAPPA MESON

In this chapter we present the details of the calculations applied to obtain the mass, overlap amplitude of κ meson and coupling constant of $\kappa \to K\pi$ decay, $g_{\kappa K\pi}$. The mass and overlap amplitude of the κ meson is obtained using two point QCD sum rules. On the other hand, for the coupling constant, $g_{\kappa K\pi}$ the light cone QCD sum rules is used. In this chapter we also explain the calculation of coupling constant, $g_{\kappa K\pi}$ from its relation with the experimental decay width, Γ_κ.

3.1 Mass of Kappa Meson

This section is devoted to presenting the details of the calculations of the mass and the overlap amplitude of the κ meson. As discussed in Chapter 2, for mass calculations two point correlation function is used, which has the following form,

$$\Pi(p^2) = i \int d^4x e^{ip\cdot x} \langle 0|T\{j_\kappa(x) j_\kappa^\dagger(0)\}|0\rangle. \tag{3.1}$$

Firstly, we should get the physical side of the QCD sum rules. To carry it out, we insert the unit operator given in Eq. (2.5) into the correlation function. The overlap amplitude of kappa meson f_κ, is defined via the relation

$$f_\kappa = \langle 0|j_\kappa|\kappa\rangle. \tag{3.2}$$

As well as the mass of the κ meson the overlap amplitude f_κ will also be determined by the QCD sum rules for which there is no experimental information. Following the procedure given in Sect.2.2.1 and considering the Eq. (2.9) and Eq. (3.2), we obtain the following equation for the physical side of the correlator as

$$\Pi(p^2) = \frac{|f_\kappa|^2}{m_\kappa^2 - p^2} + \int_{s_0}^{\infty} \frac{ds\rho^h(s)}{s - p^2}. \tag{3.3}$$

As it was mentioned in the previous chapter, for the theoretical side of the correlator OPE is used. Since κ is a scalar meson, composed of s and d quark, the interpolating currents in Eq. (3.1) are;

$$j_\kappa(x) = \bar{s}(x)d(x) \tag{3.4}$$

$$j_\kappa^\dagger(0) = \bar{d}(0)s(0). \tag{3.5}$$

Using the Wick theorem the two point correlator can be written as;

$$\Pi(p^2) = -i\int d^4x e^{ip\cdot x} iS_{ab}^{s,\alpha\beta}(-x) iS_{ab}^{d,\beta\alpha}(x) \tag{3.6}$$

where $S_{ab}^{\alpha\beta}$ is the quark propagator which is defined as,

$$iS_{ab}^{q,\alpha\beta} \equiv \langle 0|T\{q_a^\alpha(x)\bar{q}_b^\beta(0)\}|0\rangle \tag{3.7}$$

and contains both the perturbative and nonperturbative contributions.

Since our propagator contains only light quarks; strange and down quarks, in our calculation we use light quark propagator and it is given explicitly up to dimension seven operators as [35];

$$
\begin{aligned}
iS_{ab}^{q,\alpha\beta} &\equiv \langle 0|T\{q_a^\alpha(x)\overline{q}_b^\beta(0)\}|0\rangle \\
&= \frac{i}{2\pi^2}\frac{1}{(x^2)^2}\delta_{ab}\not{x}^{\alpha\beta} - \frac{1}{12}\delta_{ab}\delta^{\alpha\beta}\langle\overline{q}q\rangle - \frac{1}{192}x^2\delta_{ab}\delta^{\alpha\beta}g\langle\overline{q}\sigma\cdot Gq\rangle \\
&+(-\frac{i}{32\pi^2})\frac{1}{x^2}g_sG_{\mu\nu}^At_{ab}^A(\not{x}\sigma^{\mu\nu}+\sigma^{\mu\nu}\not{x})^{\alpha\beta} \\
&+(-\frac{\pi^2}{3^32^7})x^4\delta_{ab}\delta^{\alpha\beta}\langle\overline{q}q\rangle\langle\frac{\alpha_s}{\pi}G^2\rangle + (-\frac{1}{4\pi^2})\frac{1}{x^2}\delta_{ab}\delta^{\alpha\beta}m_q \\
&+\frac{i}{48}\delta_{ab}m_q\langle\overline{q}q\rangle\not{x}^{\alpha\beta} + \frac{i}{3^22^7}x^2\delta_{ab}m_qg\langle\overline{q}\sigma\cdot Gq\rangle\not{x}^{\alpha\beta} \\
&+(-\frac{1}{32\pi^2})\left[\ln(-\frac{x^2\Lambda^2}{4})+2\gamma_{EM}\right]m_qg_sG_{\mu\nu}^At_{ab}^A(\sigma^{\mu\nu})^{\alpha\beta} \\
&+\frac{1}{3^22^5}x^2\delta_{ab}g_s^2\langle\overline{q}q\rangle^2\not{x}^{\alpha\beta} - \frac{1}{3^52^7}x^4\delta_{ab}\delta^{\alpha\beta}m_qg_s^2\langle\overline{q}q\rangle^2, \qquad (3.8)
\end{aligned}
$$

where α and β are spinor indices, a and b are color indices and γ_{EM} is Euler constant. Actually, this propagator is made up of three parts as it can be seen in the Fig.3.1. First one is the perturbative part of the propagator. The second part is the quark condensate part that reveals the interactions of quark fields which have the same flavor. The third part is gluonic part which reveals the interactions of quark fields with the gluon fields [28]. So the propagator can also be written as;

$$S^q(x) = S^f(x) + S^{q,cond}(x) + S^{gluon}(x). \qquad (3.9)$$

In Fig. 3.1 there are some diagrams of individual parts of the propagator. (a) shows the perturbative part of correlator which corresponds to the unit operator in OPE. The $\langle\overline{q}q\rangle$ operator of the correlator expresses the quark condensate part of propagator which is shown in (b). The first and the second diagrams of (b) correspond to the $\overline{s}s$ and $\overline{d}d$, respectively. In (c) there is one gluon and a quark condensate in the diagram and it is proportional to the contribution of

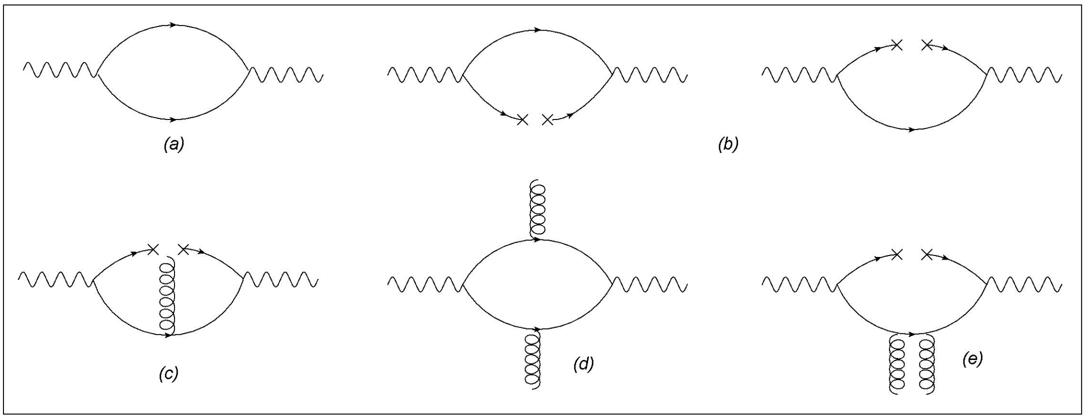

Figure 3.1: Feynman diagrams that contribute to Wilson Operator Product Expansion of the κ meson mass sum rules.

$g\langle\overline{q}\sigma Gq\rangle$ operator of the propagator. The $\langle\frac{\alpha_s}{\pi}G^2\rangle$ term represents the contribution coming from two gluons which can be shown as in (d). The final diagram, (e) is related with two gluons and a quark condensate contributions which is $\langle\overline{q}q\rangle\langle\frac{\alpha_s}{\pi}G^2\rangle$ term in the propagator. All of these diagrams are calculated in our work.

Now, one can calculate the perturbative part of the correlator, which is represented by Fig. 3.1(a). Inserting the first term of the propagator which is,

$$iS_{ab}^{\alpha\beta} = \frac{i}{2\pi^2}\frac{1}{(x^2)^2}\delta_{ab}\ \not{x}^{\alpha\beta} \tag{3.10}$$

into the Eq. (3.6) we get;

$$\Pi^{\alpha\beta,(a)}(p^2) = -i\int d^4x e^{ip\cdot x}\frac{(-i\ \not{x})^{\beta\alpha}}{2\pi^2(x^2)^2}\frac{(i\ \not{x})^{\alpha\beta}}{2\pi^2(x^2)^2}\delta^{ab}\delta^{ba}. \tag{3.11}$$

Here, $\delta^{ab}\delta^{ba}$ term arises due to the colors of quark and a and b are color indicies. Using the relation for the matrix multiplication $A^{\alpha\beta}B^{\beta\alpha} = Tr(AB)$ and taking the integration over x, for the perturbative part of correlation function we get;

$$\Pi^{(a)}(p^2) = -\frac{3}{8\pi^2} p^2 \ln(-p^2), \tag{3.12}$$

where the relation $Tr(\gamma^\mu \gamma^\nu) = 4g^{\mu\nu}$ is used. For $p^2 > 0$, the term $\ln(-p^2)$ is given by $\ln(-p^2) = \ln(p^2) - i\pi$ and via this relation we obtain the spectral density for the perturbative part as;

$$\rho^{(a)}(p^2) = \frac{1}{\pi} Im\Pi^{(a)}(p^2) = \frac{3}{8\pi^2} p^2. \tag{3.13}$$

Here $p^2 = -Q^2$ is the momentum transfer. Following the Borel transformation, if we carry out the continuum subtraction, the result for the perturbative term is obtained as;

$$\begin{aligned} & \int_0^\infty ds \rho^{(a)}(s) e^{-\frac{s}{M^2}} \\ = & \int_0^\infty ds \frac{3}{8\pi^2} s e^{-\frac{s}{M^2}} \quad \xrightarrow{\text{continuum subtraction}} \quad \int_0^{s_0} ds \frac{3}{8\pi^2} s e^{-\frac{s}{M^2}} \\ = & \frac{3}{8\pi^2} M^4 \left[1 - (1 + \frac{s_0}{M^2}) e^{-\frac{s_0}{M^2}} \right]. \end{aligned} \tag{3.14}$$

In order to obtain the other contributions which are represented by Fig 3.1 (b), (c), (d) and (e) one should insert the terms of the propagator including the condensates $\langle \overline{q} q \rangle$, $g\langle \overline{q} \sigma G q \rangle$ and $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ into the correlation function.

To obtain the quark condensate contributions, we use the parts of the propagator containing quark condensates which are given as,

$$iS_{ab}^{q,\alpha\beta} = \frac{i}{48} \delta_{ab} m_q \langle \overline{q} q \rangle \not{x}^{\alpha\beta}, \tag{3.15}$$

$$iS_{ab}^{q,\alpha\beta} = -\frac{1}{12} \delta_{ab} \delta^{\alpha\beta} \langle \overline{q} q \rangle. \tag{3.16}$$

If we place Eq. (3.15) for one of the propagator and Eq. (3.10) for the other in Eq. (3.6) the following relation is obtained,

$$\Pi_{ab}^{\alpha\beta,(b_1)}(p^2) = -i\int d^4x e^{ip\cdot x}\frac{-i}{48}\delta^{ba}m_s\langle\bar{s}s\rangle(\not{x})^{\beta\alpha}\frac{i}{2\pi^2}\frac{1}{(x^2)^2}\delta^{ab}(\not{x})^{\alpha\beta}. \quad (3.17)$$

Taking the trace and then evaluating the integral, Eq. (3.17) becomes;

$$\Pi^{(b_1)}(p^2) = -\frac{1}{2p^2}m_s\langle\bar{s}s\rangle. \quad (3.18)$$

To get spectral density from Eq. (3.18) one should use the identity,

$$\frac{1}{x \pm i\epsilon} = P\frac{1}{x} \mp i\pi\delta(x), \quad (3.19)$$

obtained in complex analysis. Here P stands for the principle value. Then the spectral density for this part is;

$$\rho^{(b_1)}(p^2) = \frac{1}{\pi}Im\Pi(p^2) = \frac{1}{2}m_s\langle\bar{s}s\rangle\delta(s). \quad (3.20)$$

After performing Borel transformation, the final form of the calculation for this part can be obtained as follows;

$$\int_0^{s_0} ds\rho^{(b_1)}(s)e^{\frac{-s}{M^2}} = \frac{1}{2}m_s\langle\bar{s}s\rangle. \quad (3.21)$$

For another similar contribution coming from quark condensate we use Eq. (3.16) with the following part of the propagator

$$iS_{ab}^{\alpha\beta} = -\frac{1}{4\pi^2}\frac{1}{x^2}\delta_{ab}\delta^{\alpha\beta}m_q \quad (3.22)$$

in place of the propagator given in Eq. (3.6). Then correlation function for this part becomes;

$$\Pi^{(b_2)}(p^2) = -i\int d^4x e^{ip\cdot x}\frac{-1}{4\pi^2}\frac{1}{x^2}\delta_{ab}\delta^{\alpha\beta}m_s\frac{-1}{12}\delta_{ba}\delta^{\alpha\beta}\langle\overline{d}d\rangle. \qquad (3.23)$$

If we apply the same procedure followed for the previous contributions, one can get the result of this term as;

$$\int_0^{s_0} ds\rho^{(b_2)}(s)e^{\frac{-s}{M^2}} = m_s\langle\overline{d}d\rangle. \qquad (3.24)$$

And finally the last quark condensate contribution is obtained when the part of the propagator,

$$iS_{ab}^{\alpha\beta} = -\frac{1}{32\pi^2}(\ln(-\frac{x^2\Lambda^2}{4}) + 2\gamma_{EM})m_q g_s G_{\mu\nu}^A t_{ab}^A(\sigma^{\mu\nu})^{\alpha\beta} \qquad (3.25)$$

is inserted for s quark propagator in Eq. (3.6). Here, the term which proportional to $2\gamma_{EM}$ gives zero after Borel transformation. The contribution of this calculation is as in the following equation,

$$\Pi^{(c)}(p^2) = i\int d^4x e^{ip\cdot x}m_s(-\frac{1}{32\pi^2})\ln(-\frac{x^2\Lambda^2}{4})\langle g_s\overline{d}\sigma^{\mu\nu}G_{\mu\nu}^A t^A d\rangle. \qquad (3.26)$$

In the above equation, the matrix element $\langle g_s\overline{d}\sigma^{\mu\nu}G_{\mu\nu}^A t^A d\rangle$ can be represented as,

$$\langle g_s\overline{d}\sigma^{\mu\nu}G_{\mu\nu}^A t^A d\rangle = m_0^2\langle\overline{d}d\rangle. \qquad (3.27)$$

As in the previous parts, if we continue to solve the Eq. (3.26), we get the contribution of this part as;

$$\int_0^{s_0} ds\rho^{(c)}(s)e^{-\frac{s}{M^2}} = \frac{1}{2M^2}m_s m_0^2\langle\bar{d}d\rangle. \qquad (3.28)$$

Our calculation contains also gluonic contributions which express the interactions of quarks with the gluon fields. One of them is two gluon emitted from different quark lines. If we use the fourth term of propagator, which is

$$iS_{ab}^{\alpha\beta} = -\frac{i}{32\pi^2}\frac{1}{x^2}g_s G_{\mu\nu}^A t_{ab}^A(\not{x}\sigma^{\mu\nu} + \sigma^{\mu\nu}\ \not{x})^{\alpha\beta} \qquad (3.29)$$

for both quark propagators of the correlation function, the following form is gained,

$$\begin{aligned}\Pi^{(d)}(p^2) &= -i\int d^4x e^{ip\cdot x}(-\frac{i}{32\pi^2})^2\frac{1}{(x^2)^2}g_s^2\langle G_{\mu\nu}^A G_{\sigma\rho}^B\rangle t_{ab}^A t_{ba}^B \\ &\quad \times(\not{x}\sigma^{\mu\nu} + \sigma^{\mu\nu}\ \not{x})^{\alpha\beta}(\not{x}\sigma^{\sigma\rho} + \sigma^{\sigma\rho}\ \not{x})^{\beta\alpha}. \qquad (3.30)\end{aligned}$$

The term $\langle G_{\mu\nu}^A G_{\sigma\rho}^B\rangle$ in this equation can be given as follows;

$$\langle G_{\mu\nu}^A G_{\sigma\rho}^B\rangle = \delta^{AB}\frac{1}{96}\langle G^2\rangle(g_{\mu\sigma}g_{\nu\rho} - g_{\mu\rho}g_{\nu\sigma}). \qquad (3.31)$$

Using Eq. (3.31) and taking the traces we obtain the correlation function for the gluonic contribution. Then if we continue with the same procedure with the other parts, the contribution of this part is obtained as;

$$\int_0^{s_0} ds\rho^{(d)}(s)e^{\frac{-s}{M^2}} = \frac{9}{64\pi^2}\langle g_s^2 G^2\rangle. \qquad (3.32)$$

Finally, there is gluonic contribution for which the two gluon is emitted from the same quark line. In this calculation, the contribution related with the gluonic condensate comes from strange quark and quark condensate comes from down quark. The term which we insert into the propagator of the strange quark for this part with two gluons is given in momentum space as [32],

$$iS_q = \frac{ig_s^2}{12}\langle G^2\rangle\delta_{ab}m_q\frac{p^2+m_q\not{p}}{(p^2-m_q^2)^4}. \tag{3.33}$$

If we use the Eq. (3.33) and the parts of the propagator for down quark which are,

$$iS_q = -\frac{1}{12}\delta_{ab}\delta^{\alpha\beta}\langle\overline{q}q\rangle, \tag{3.34}$$

$$iS_q = -\frac{1}{192}x^2\delta_{ab}\delta^{\alpha\beta}g_s\langle\overline{q}\sigma\cdot Gq\rangle, \tag{3.35}$$

respectively and apply the same procedure with the other parts, we obtained remaining contributions of the gluonic part as follows,

$$\int_0^{s_0} ds\rho^{(e)}(s)e^{\frac{-s}{M^2}} = \frac{1}{24M^4}\langle\overline{d}d\rangle\langle g_s^2G^2\rangle, \tag{3.36}$$

$$\int_0^{s_0} ds\rho^{(e)}(s)e^{\frac{-s}{M^2}} = \frac{1}{48M^6}\langle\overline{d}d\rangle m_0^2\langle g_s^2G^2\rangle. \tag{3.37}$$

The other terms in the propagator gives no contribution due to either the trace of odd product of gamma matrices or vanishes after Borel transformation. Now we can combine all the parts of theoretical side of the mass sum rule which are given in Eqs. (3.14), (3.21), (3.24), (3.28), (3.32), (3.36), (3.37), and match it with the physical side of the correlator and that leads us to the equation for the overlap amplitude f_κ of the quark current j_κ;

$$
\begin{aligned}
f(M^2) &= f_\kappa^2 e^{-\frac{m_\kappa^2}{M^2}} \\
&= \frac{3}{8\pi^2} M^4[1-(1+\frac{s_0}{M^2})e^{-\frac{s_0}{M^2}}] + \frac{9}{64\pi^2}\langle g_s^2 G^2\rangle \\
&+ m_s[\langle \bar{d}d\rangle + \frac{1}{2}\langle \bar{s}s\rangle + \frac{1}{2M^2}\langle \bar{d}d\rangle m_0^2 \\
&+ \frac{1}{24M^4}\langle \bar{d}d\rangle\langle g_s^2 G^2\rangle + \frac{1}{48M^6}\langle \bar{d}d\rangle m_0^2 \langle g_s^2 G^2\rangle].
\end{aligned} \tag{3.38}
$$

From the final result given in Eq. (3.38) we can also get the mass relation of the scalar κ meson with respect to the Borel mass as;

$$
m_\kappa^2 = M^4 \frac{d}{dM^2} \ln(f(M^2)). \tag{3.39}
$$

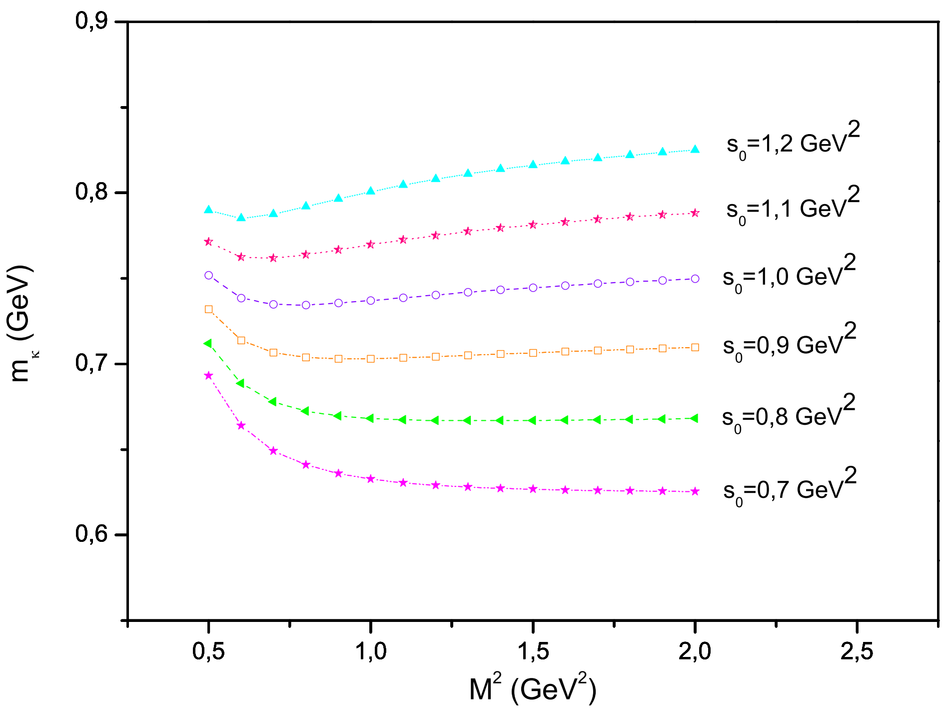

Figure 3.2: The dependence of mass of the κ meson on the Borel parameter for various s_0 values.

The above result is considered firstly for the mass of the κ scalar meson. For this purpose we plot the Fig. 3.2 which shows the mass of κ meson as a function of Borel parameter with respect to different values of threshold parameter. As it can be seen from the figure the most stable regions correspond to the threshold parameter, $s_0 = 0.8 - 1.0\ GeV^2$, and the Borel parameter interval $1.0 \leq M^2\ (GeV^2)\ \leq 2.0$. If we choose the Borel parameter from this interval as $M^2 = 1.5\ GeV^2$, we find mass of kappa meson $m_\kappa = 0.70 \pm 0.06\ GeV$. Fig.3.2 is also used to determine the most suitable value of threshold parameter s_0. If we choose the values of threshold parameter in this interval and if we use Eq. (3.38), we can calculate the overlap amplitude f_κ.

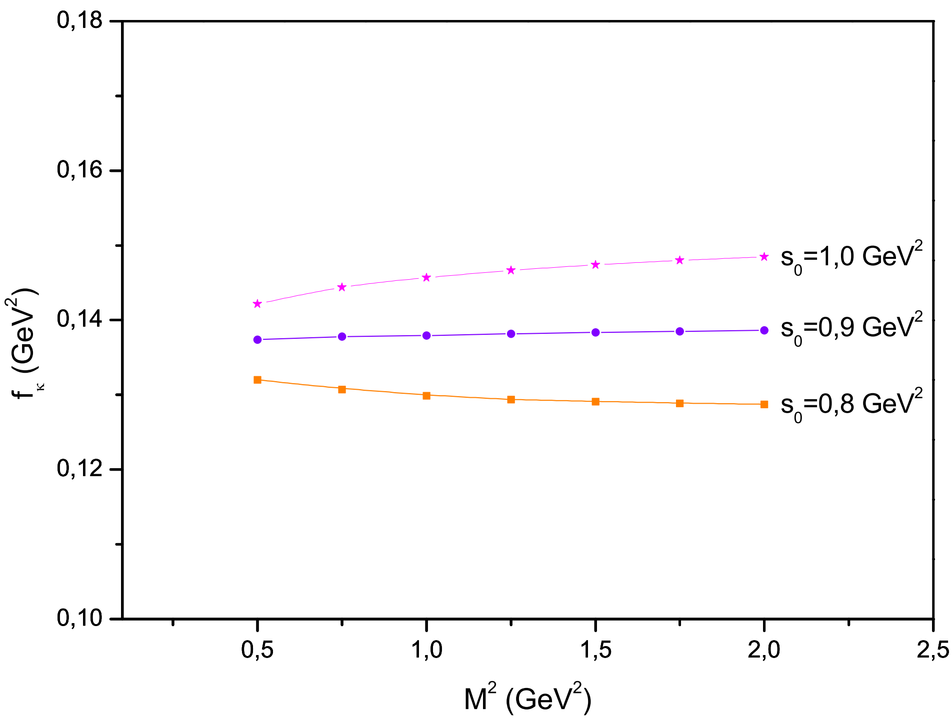

Figure 3.3: The dependence of overlap amplitude f_κ on the Borel parameter for various s_0 values.

Fig. 3.3 shows us the overlap amplitude as a function of Borel parameter with respect to the $s_0 = 0.8, 0.9$ and $1.0\ GeV^2$. From this figure the value of the overlap amplitude f_κ can be determined as $f_\kappa = 0.13 \pm 0.02\ GeV^2$. The errors in the m_κ and f_κ values come from the values of QCD parameters, and the uncertainties in the M^2 and s_0 values. For the calculation of m_κ and f_κ, we use the following input parameters [36],

$$\begin{aligned}
\langle g_s^2 G^2 \rangle &= 4\pi^2 \times (0.012 \pm 0.003) GeV^4 \\
m_s &= 0.15 \pm 0.02 \; GeV \\
\langle \overline{d} d \rangle &= -(0.240 \pm 0.010)^3 \; GeV^3 \\
\langle \overline{s} s \rangle &= 0.8 \langle \overline{d} d \rangle \\
m_0^2 &= 0.8 \; GeV^2
\end{aligned} \tag{3.40}$$

3.2 Coupling Constant $g_{\kappa K^+ \pi^-}$

In this section, calculation of the coupling constant $g_{\kappa K^+ \pi^-}$ of $\kappa \to K^+ \pi^-$ decay will be presented. In this thesis, we calculate the coupling constant using the light cone QCD sum rules and compare it with the value extracted from the experimental value of the κ decay width.

3.2.1 Coupling Constant $g_{\kappa K^+ \pi^-}$ in Light Cone QCD Sum Rules

To be able to use light cone QCD sum rules, as it was mentioned previously, first of all we need two point correlation function for this decay, that is;

$$\Pi_\mu(p+q, p) = i \int d^4x e^{ip \cdot x} \langle \pi^-(q) | T(j_{\mu,K}(x) j_\kappa^\dagger(0)) | 0 \rangle \tag{3.41}$$

where p and q are the four momentum of K^+ and π^- meson, respectively. In order to calculate this correlation function, first of all one should determine the interpolating quark currents. The interpolating current of the κ meson is given in Eq. (3.4) and the interpolating current of the K^+ meson, considering its quark structure, is chosen as $j_\mu^K = \overline{s} \gamma_\mu \gamma_5 u$. As it was mentioned in the previous chapter, the correlator in Eq.(3.41) can be calculated in two sides, which are called physical and theoretical sides. In order to obtain physical part of this correlator that represents the correlator in terms of hadronic degrees of freedom, one should insert a complete set of hadronic states. On the other hand, the correlator is represented in terms of quark gluon degrees of freedom in the second side of calculation which is the theoretical side of the correlation function. Then matching these two representations we can obtain the QCD sum rules to get the coupling constant $g_{\kappa K^+\pi^-}$.

For the physical side, we should insert a complete set of hadronic states that have the same quantum numbers with interpolating currents. We then obtain the correlation function as,

$$\begin{aligned}\Pi_\mu^{phen}(p+q,p) &= \frac{\langle 0 \mid j_\mu^K \mid K^+(p)\rangle\langle K^+(p)\pi^-(q) \mid \kappa(p+q)\rangle\langle\kappa|j_\kappa^\dagger \mid 0\rangle}{\left(p^2-m_{K^+}^2\right)\left[(p+q)^2-m_\kappa^2\right]} \\ &+ \int_{s_0} ds \int_{s_0'} ds' \, \frac{\rho_\mu^{cont}(s,s')}{[s-(p+q)^2](s'-p^2)}. \end{aligned} \tag{3.42}$$

If we use matrix elements in the correlator which are defined as,

$$\begin{aligned} < 0 \mid j_\mu^K \mid K^+(p) > &= i f_K p_\mu, \\ < K^+(p)\pi^-(q) \mid \kappa(p+q) > &= g_{\kappa K^+\pi^-}, \\ < \kappa(p+q) \mid j_\kappa \mid 0 > &= f_\kappa. \end{aligned} \tag{3.43}$$

The physical side of the correlator takes the following form,

$$\Pi_\mu^{phen}(p+q,p) = \frac{i g_{\kappa K^+\pi^-} \, f_K \, f_\kappa}{\left(p^2-m_{K^+}^2\right)\left[(p+q)^2-m_\kappa^2\right]} p_\mu + \ldots \,. \tag{3.44}$$

As it can be seen from Eq. (3.44) to obtain the coupling constant $g_{\kappa K^+\pi^-}$, we need the values of overlap amplitudes of κ and K mesons, f_κ and f_K. In this study we use the experimental value of f_K which is $f_K = (156.6 \pm\ 1 \pm 3.6)$ MeV [24, 37] and as for the f_κ, the result of the previous section $f_\kappa = 0.13 \pm 0.02\ GeV^2$ is employed.

Applying double Borel transformation with respect to $Q_1^2 = -(p+q)^2$ and $Q_2^2 = -p^2$ the final form of the physical side is acquired as,

$$\begin{aligned} \Pi^{phen} &= i g_{\kappa K^+ \pi^-} f_K f_\kappa e^{-m_\kappa/M_1^2} e^{-m_K/M_2^2} \\ &+ \int_{s_0} ds \int_{s_0'} ds' \rho(s,s') e^{-s/M_1^2} e^{-s'/M_2^2} \ . \end{aligned} \tag{3.45}$$

Here M_1^2 and M_2^2 are the Borel masses.

For the calculation of theoretical side, as in the mass calculation, we can use the propagator for light quarks. In order to do that, firstly one can write the correlation function, Eq. (3.41), inserting the interpolating quark currents as follows,

$$\begin{aligned} \Pi_\mu(p+q,p) &= i \int d^4x e^{ip\cdot x} \langle \pi^-(q) | \overline{s}(x) \gamma_\mu \gamma_5 u(x) \overline{d}(0) s(0) | 0 \rangle \\ &= i \int d^4x e^{ip\cdot x} \langle \pi^-(q) | \overline{d}^a_\alpha(0) s^a_\alpha(0) \overline{s}^b_\beta(x) (\gamma_\mu\gamma_5)_{\beta\delta} u^b_\delta(x) | 0 \rangle . \end{aligned} \tag{3.46}$$

Here, if we make the proper contractions and use $e^{-i\hat{p}\cdot x} q(x) e^{i\hat{p}\cdot x} = q(0)$ expression, Eq. (3.46) becomes,

$$\begin{aligned} \Pi_\mu(p+q,p) &= i \int d^4x e^{ip\cdot x} \langle \pi^-(q) | \overline{d}^a_\alpha(0) i S^{ab}_{\alpha\beta}(-x) (\gamma_\mu\gamma_5)_{\beta\delta} u^b_\delta(x) | 0 \rangle \\ &= i \int d^4x e^{i(p+q)\cdot x} \langle \pi^-(q) | \overline{d}^a_\alpha(-x) \\ &\times \ i S^{ab}_{\alpha\beta}(-x) (\gamma_\mu\gamma_5)_{\beta\delta} u^b_\delta(0) | 0 \rangle . \end{aligned} \tag{3.47}$$

In this part of our calculation, we use the following propagator given in Ref.[38],

$$
\begin{aligned}
iS_q(x,y) &= \frac{i(\not{x}-\not{y})}{2\pi^2(x-y)^4} - \frac{m_q}{4\pi^2(x-y)^2} \\
&- \frac{\langle \overline{q}q\rangle}{12}\left[1-\frac{im_q}{4}(\not{x}-\not{y})\right] \\
&- \frac{(x-y)^2}{192}m_0^2\langle \overline{q}q\rangle\left[1-\frac{im_q}{6}(\not{x}-\not{y})\right] \\
&- ig_s\int_0^1 du\left\{\frac{\not{x}-\not{y}}{16\pi^2(x-y)^2}G_{\mu\nu}(ux+\overline{u}y)\sigma^{\mu\nu}\right. \\
&- (ux^\mu+\overline{u}y^\mu)G_{\mu\nu}(ux+\overline{u}y)\gamma^\nu\frac{i}{4\pi^2(x-y)^2} \\
&- \left.\frac{im_q}{32\pi^2}G_{\mu\nu}(ux+\overline{u}y)\sigma^{\mu\nu}\left[\ln\left(\frac{-(x-y)^2\Lambda^2}{4}\right)+2\gamma_{EM}\right]\right\} \qquad (3.48)
\end{aligned}
$$

As in the mass calculation, firstly we calculate the contribution coming from the perturbative part of the propagator. To do that, if we insert the first term of the propagator into the correlation function we obtain;

$$
\Pi_{\mu,1}(p+q,p) = -i\int d^4x e^{i(p+q)\cdot x}\langle\pi^-(q)|\overline{d}(-x)\frac{ix_\lambda}{2\pi^2x^4}\gamma_\lambda\gamma_\mu\gamma_5 u(0)|0\rangle. \qquad (3.49)
$$

If we use the gamma matrices identity, $\gamma_\lambda\gamma_\mu\gamma_5 = g_{\lambda\mu}\gamma_5 - i\sigma_{\lambda\mu}\gamma_5$, our equation gains the form,

$$
\begin{aligned}
\Pi_{\mu,1}(p+q,p) &= \frac{1}{2\pi^2}\int d^4x\frac{e^{i(p+q)\cdot x}x_\lambda}{x^4} \\
&\times \langle\pi^-(q)|\overline{d}(-x)(g_{\lambda\mu}\gamma_5 - i\sigma_{\lambda\mu}\gamma_5)u(0)|0\rangle. \qquad (3.50)
\end{aligned}
$$

In order to continue to calculate the correlation function, we need the matrix elements $\langle\pi(q)|\overline{q}\Gamma_i q|0\rangle$ which can be represented in terms of the pion distribution amplitudes. In Ref. [39] they are given as,

$$
\langle\pi(p)|\overline{q}(x)i\gamma_5 q(0)|0\rangle = \mu_\pi\int_0^1 du e^{i\overline{u}p\cdot x}\varphi_p(u), \qquad (3.51)
$$

$$\begin{aligned}\langle \pi(p)|\overline{q}(x)\sigma_{\alpha\beta}\gamma_5 q(0)|0\rangle &= \frac{i}{6}\mu_\pi(1-\tilde{\mu}_\pi^2)(p_\alpha x_\beta - p_\beta x_\alpha) \\ &\times \int_0^1 du e^{i\overline{u}p\cdot x}\varphi_\sigma(u), \end{aligned} \tag{3.52}$$

where $\overline{u} = 1-u$, $\mu_\pi = f_\pi \frac{m_\pi^2}{m_u+m_d}$, $\tilde{\mu}_\pi = \frac{m_u+m_d}{m_\pi}$ and functions $\varphi_p(u)$ and $\varphi_\sigma(u)$ are functions of definite twist. In this work we have neglected the mass of light quarks so, $m_\pi^2 = 0$ and $\tilde{\mu}_\pi^2 = 0$. Then this part of correlator is obtained as follows,

$$\begin{aligned}\Pi_{\mu,1} &= -i\frac{f_\pi m_\pi^2}{m_u+m_d}\int_0^1 du\varphi_p(u)\frac{p'_\mu}{p'^2} \\ &+ \frac{i}{6}\frac{f_\pi m_\pi^2}{m_u+m_d}\int_0^1 du\varphi_\sigma(u)(\frac{q_\mu}{p'^2}+\frac{2q_\lambda p'_\lambda p'_\mu}{p'^4}), \end{aligned} \tag{3.53}$$

where $p' = p+q-\overline{u}q$. We can write as $p'^2 = p^2\overline{u}+(p+q)^2u = -Q_2^2\overline{u} - Q_1^2 u$ if we set $q^2 = 0$. If we apply Borel transformation with respect to $Q_1^2 = -(p+q)^2$ and $Q_2^2 = -p^2$, we then find the first contribution of the correlation function as,

$$\Pi_1 = i\mu_\pi M^2\left[\varphi_p(u_0)u_0 + \frac{1}{3}\varphi_\sigma(u_0) + \frac{1}{6}\varphi'_\sigma(u_0)\right], \tag{3.54}$$

where u_0 and M are the functions of the Borel masses M_1^2 and M_2^2, $u_0 = \frac{M_1^2}{M_1^2+M_2^2}$ and $M^2 = \frac{M_1^2 M_2^2}{M_1^2+M_2^2}$. To obtain the second part of the theoretical side we insert the second term of the propagator given in Eq. (3.48), into the correlator,

$$\Pi_{\mu,2}(p+q,p) = -i\int d^4x e^{i(p+q)\cdot x}\langle \pi^-(q)|\overline{d}(-x)\frac{m_s}{4\pi^2x^2}\gamma_\mu\gamma_5 u(0)|0\rangle. \tag{3.55}$$

The matrix element in the above equation is defined as [39],

$$\begin{aligned}\langle \pi^-(p)|\overline{q}(x)\gamma_\mu\gamma_5 q(0)|0\rangle &= -if_\pi p_\mu\int_0^1 du e^{i\overline{u}p\cdot x}\left[\varphi_\pi(u)+\frac{1}{16}m_\pi^2x^2\ \mathbb{A}(u)\right] \\ &- \frac{i}{2}f_\pi m_\pi^2\frac{x_\mu}{px}\int_0^1 du e^{i\overline{u}p\cdot x}\mathbb{B}(u). \end{aligned} \tag{3.56}$$

Here the terms proportional with $\mathbb{A}(u)$ and $\mathbb{B}(u)$ which are the functions of definite twist do not contribute the correlation function due to the vanishing integral and neglecting the light quark masses, $m_\pi^2 = 0$, respectively. By taking the integral over x, we get this part of the correlator as follows,

$$\Pi_{\mu,2} = i m_s f_\pi \int_0^1 du \varphi_\pi(u) \left(\frac{q_\mu}{p'^2}\right). \tag{3.57}$$

After performing Borel transformation, the second part of correlator then becomes,

$$\Pi_2 = -i m_s M^2 f_\pi \varphi_\pi(u_0). \tag{3.58}$$

The parts proportional with the quark condensate of the propagator do not contribute to the result because they vanish after the Borel transformation. Then the next part of the correlator obtained from the gluonic part of the propagator is,

$$\begin{aligned}
\Pi_{\mu,3}(p+q,p) &= \frac{i^2}{16\pi^2}\int_0^1 du \int d^4x \frac{e^{i(p+q)\cdot x}}{x^2} x_\lambda \\
&\times \langle \pi^-(q)|\overline{d}(-x) g_s \gamma_\lambda G_{\alpha\beta}(\overline{u}x)\sigma^{\alpha\beta}\gamma_\mu\gamma_5 u(0)|0\rangle \\
&- \frac{i^3}{4\pi^2}\int_0^1 \overline{u} du \int d^4x \frac{e^{i(p+q)\cdot x}}{x^2} x_\alpha \\
&\times \langle \pi^-(q)|\overline{d}(-x) g_s G_{\alpha\beta}(\overline{u}x)\gamma^\beta\gamma_\mu\gamma_5 u(0)|0\rangle. \qquad (3.59)
\end{aligned}$$

To calculate the above equation one should use the relations $\gamma_\lambda\sigma_{\alpha\beta} = i(g_{\lambda\alpha}\gamma_\beta - g_{\lambda\beta}\gamma_\alpha) + \epsilon_{\lambda\alpha\beta\delta}\gamma_\delta\gamma_5$ and $\gamma_\alpha\gamma_\beta = g_{\alpha\beta} - i\sigma_{\alpha\beta}$. Here the matrix element in terms of the distribution amplitude of pion is as follows [39],

$$
\begin{aligned}
\langle \pi^-(p)|\overline{q}(x)\sigma_{\mu\nu}\gamma_5 g_s G_{\alpha\beta}(vx)q(0)|0\rangle &= i\mu_\pi \Bigg[p_\alpha p_\mu \left(g_{\nu\beta} - \frac{1}{px}(p_\nu x_\beta + p_\beta x_\nu)\right) \\
&- p_\alpha p_\nu \left(g_{\mu\beta} - \frac{1}{px}(p_\mu x_\beta + p_\beta x_\mu)\right) \\
&- p_\beta p_\mu \left(g_{\nu\alpha} - \frac{1}{px}(p_\nu x_\alpha + p_\alpha x_\nu)\right) \\
&+ p_\beta p_\nu \left(g_{\mu\alpha} - \frac{1}{px}(p_\mu x_\alpha + p_\alpha x_\mu)\right)\Bigg] \\
&\times \int D\alpha e^{i(\alpha_{\overline{q}}+v\alpha_g)p\cdot x}\mathcal{T}(\alpha_i). \qquad (3.60)
\end{aligned}
$$

where the measure $D\alpha$ is given as

$$
\int D\alpha = \int_0^1 d\alpha_{\overline{q}} \int_0^1 d\alpha_q \int_0^1 d\alpha_g \delta(1-\alpha_{\overline{q}}-\alpha_q-\alpha_g) \qquad (3.61)
$$

After inserting this matrix element into the Eq. (3.59) and doing the mathematical calculations, the third part of the correlator is obtained as,

$$
\Pi_3(p+q,p) = 2i\mu_\pi \int_0^1 \overline{v}dv \int D_{\alpha_i}\mathcal{T}(\alpha_i)\frac{(p+q)^2-p^2}{p''^4}. \qquad (3.62)
$$

Here we use p'' for $p+q(1-\alpha_{\overline{q}}-\overline{v}\alpha_g)$. Then one should perform Borel transformation with respect to $Q_1^2=-(p+q)^2$ and $Q_2^2=-p^2$. After that, this part of the correlator becomes,

$$
\begin{aligned}
\Pi_3 &= 2iM^2\mu_\pi \int_0^1 d\alpha_q \int_0^1 d\alpha_{\overline{q}} \int_0^1 d\alpha_g \mathcal{T}(\alpha_i)(1-v) \\
&\times \delta'(\alpha_q+\alpha_g(1-v)-u_0). \qquad (3.63)
\end{aligned}
$$

Final procedure to obtain the theoretical part of the calculation is to add all parts of the correlator which are given by the Eqs. (3.54), (3.58) and (3.63):

$$\begin{aligned}
\Pi &= \Pi_1 + \Pi_2 + \Pi_3 \\
&= 2iM^2\mu_\pi \int_0^1 d\alpha_q \int_0^1 d\alpha_{\bar{q}} \int_0^1 d\alpha_g \mathcal{T}(\alpha_i)(1-v) \\
&\times \delta'(\alpha_q + \alpha_g(1-v) - u_0) - iM^2 f_\pi m_s \varphi_\pi(u_0) + iM^2\mu_\pi u_0 \varphi_p(u_0) \\
&+ iM^2\mu_\pi(1-\tilde{\mu}_\pi^2)\left[\frac{1}{3}\varphi_\sigma(u_0) + \frac{1}{6}u_0\varphi'_\sigma(u_0)\right]. \qquad (3.64)
\end{aligned}$$

At this stage, to attain the coupling constant $g_{\kappa K^+\pi^-}$ we match the Eqs. (3.45) and (3.64) with respect to the philosophy of QCD sum rules. The second part of the Eq. (3.45) shows us the continuum contribution. According to Refs. [40, 41], which suggest a prescription to subtract the contribution coming from the continuum, the Borel parameters can not be taken equal if they corresponds to channels that have different mass scales. Based on the observation that they are polynomials in $(1-u)$, the leading twist wave functions in our case; $\varphi_\pi(u)$ and $\varphi_p(u)$ can be written as,

$$\varphi_\pi(u) = \sum_{k=0}^{N} b_k(1-u)^k, \qquad \varphi_p(u) = \sum_{k=0}^{N} b'_k(1-u)^k \qquad (3.65)$$

to calculate their contribution in the duality region. Using the function,

$$f_N(x) = 1 - e^{-x}\sum_{k=0}^{N}\frac{x^k}{k!}, \qquad (3.66)$$

to subtract the contribution of continuum and higher states, the final form of the coupling constant is obtained as,

$$
\begin{aligned}
g_{\kappa K^+\pi^-} &= \frac{1}{f_K f_\kappa} e^{m_\kappa^2/M_1^2} e^{m_K^2/M_2^2} \Big\{ 2M^2 \mu_\pi f_0(s_0/M^2) \\
&\times \int_0^1 d\alpha_q \int_0^1 d\alpha_{\bar{q}} \int_0^1 d\alpha_g \mathcal{T}(\alpha_i)(1-v)\delta'(\alpha_q + \alpha_g(1-v) - u_0) \\
&- M^2 f_\pi m_s \sum_{k=0}^{N} b_k (\frac{M^2}{M_2^2})^k \Big[1 - e^{-A} \sum_{i=0}^{k} \frac{A^i}{i!} + e^{-A} \frac{M^2 m_\pi^2}{M_1^2 M_2^2} \frac{A^{k+1}}{(k+1)!} \Big] \\
&+ M^2 \mu_\pi u_0 \sum_{k=0}^{N} b'_k (\frac{M^2}{M_2^2})^k \Big[1 - e^{-A} \sum_{i=0}^{k} \frac{A^i}{i!} + e^{-A} \frac{M^2 m_\pi^2}{M_1^2 M_2^2} \frac{A^{k+1}}{(k+1)!} \Big] \\
&+ M^2 f_0(s_0/M^2) \mu_\pi (1 - \tilde{\mu}_\pi^2) \, [\frac{1}{3}\varphi_\sigma(u_0) + \frac{1}{6} u_0 \varphi'_\sigma(u_0)] \Big\} \,, \qquad (3.67)
\end{aligned}
$$

where $f_0(s_0/M^2) = 1 - e^{-s_0/M^2}$ and $A = s_0/M^2$. To analyze the sum rule numerically, $m_K = 0.494$ GeV and $f_K = (156.6 \pm 1 \pm 3.6)$ MeV [24, 37], $m_\kappa = (672 \pm 40)$ MeV [24], $f_\kappa = (0.13 \pm 0.02)$ GeV^2 , $f_\pi = 0.132$ GeV [40], and $m_s = 0.14$ GeV [42] values are used, and the pion wave functions existing in Eq. (3.67) are given in Appendix C.

3.2.2 Coupling Constant $g_{\kappa K^+\pi^-}$ from $\kappa \to K\pi$ Decay Width

The coupling constant $g_{\kappa K^+\pi^-}$, can also be extracted from the decay width of κ meson. In Feynman calculus, decay width can be expressed in terms of coupling constant. In order to calculate the decay width, the Golden rule can be used and according to the Golden rule when a particle decays into other particles,

$$
1 \;\to\; 2 + 3 + \cdots + n \qquad (3.68)
$$

the decay width is given as,

$$
\begin{aligned}
\Gamma &= \frac{1}{2m_1} \int |M|^2 (2\pi)^4 \delta^4(p_1 - p_2 - p_3 \cdots - p_n) \\
&\times \prod_{j=2}^{n} \frac{1}{2\sqrt{\vec{p}_j^{\,2} + m_j^2}} \frac{d^3\vec{p}_j}{(2\pi)^3}. \qquad (3.69)
\end{aligned}
$$

Here M is amplitude and it is calculated using Feynman rules, and p_i in the δ function are four momentum of particles. Using the Golden rule for this decay we obtain,

$$\Gamma(\kappa \to K\pi) = \frac{1}{32\pi^2 m_\kappa} \int |M|^2 \frac{\delta^4(p_\kappa - p_K - p_\pi)}{\sqrt{\vec{p}_K^2 + m_K^2}\sqrt{\vec{p}_\pi^2 + m_\pi^2}} d^3\vec{p}_K d^3\vec{p}_\pi. \tag{3.70}$$

The four dimensional delta function in the above equation can be separated as,

$$\delta^4(p_\kappa - p_K - p_\pi) = \delta(p_\kappa^0 - p_K^0 - p_\pi^0)\delta^3(\vec{p}_\kappa - \vec{p}_K - \vec{p}_\pi). \tag{3.71}$$

Here the κ meson is assumed to be at rest so $\vec{p}_\kappa = 0$ and $p_\kappa^0 = m_\kappa$. Therefore, Eq. (3.70) becomes,

$$\begin{aligned} \Gamma(\kappa \to K\pi) &= \frac{1}{32\pi^2 m_\kappa} \int |M|^2 \frac{\delta(m_\kappa - \sqrt{\vec{p}_K^2 - m_K^2} - \sqrt{\vec{p}_\pi^2 - m_\pi^2})}{\sqrt{\vec{p}_K + m_K^2}\sqrt{\vec{p}_\pi + m_\pi^2}} \\ &\times \delta^3(\vec{p}_K + \vec{p}_\pi) d^3\vec{p}_K d^3\vec{p}_\pi. \end{aligned} \tag{3.72}$$

According to the final delta function it is necessary to make the replacement, $\vec{p}_\pi \to -\vec{p}_K$. After this replacement and integration the decay width is obtained as,

$$\Gamma(\kappa \to K\pi) = \frac{|\vec{p}|}{8\pi m_\kappa^2 c}|M|^2. \tag{3.73}$$

Here $|\vec{p}|$ is the magnitude of the momentum of either of the outgoing particles in the rest frame of κ and is given as;

$$|\vec{p}| = \frac{1}{2m_\kappa}\sqrt{m_\kappa^4 + m_K^4 + m_\pi^4 - 2m_\kappa^2 m_K^2 - 2m_\kappa^2 m_\pi^2 - 2m_K^2 m_\pi^2}. \tag{3.74}$$

According to Feynman rules, to find the amplitude $-iM$, for each vertex the factor $-ig$ is written. If we draw the Feynman diagram of $\kappa \to K\pi$ decay, there is only one vertex hence propagators are not needed. The amplitude is $M = g_{\kappa K^+\pi^-}$. Then the decay width of $\kappa \to K\pi$ decay in terms of coupling constant is obtained as,

$$\Gamma(\kappa \to K^+\pi^-) = \frac{g^2_{\kappa K^+\pi^-}}{16\pi m^3_\kappa}\sqrt{\lambda(m^2_\kappa, m^2_K, m^2_\pi)} \qquad (3.75)$$

where $\lambda(m^2_\kappa, m^2_K, m^2_\pi) = m^4_\kappa + m^4_K + m^4_\pi - 2m^2_\kappa m^2_K - 2m^2_\kappa m^2_\pi - 2m^2_K m^2_\pi$. The coupling constant $g_{\kappa K\pi}$ can also be extracted from the decay width equation given in Eq. (3.75) via decay width value determined from experiments. The values of the coupling constant obtained from decay width calculation will be given and compared with the result obtained from light cone QCD sum rules in the next chapter.

CHAPTER 4

RESULTS AND COMPARISON

This chapter is devoted to the analysis of the coupling constant. In this chapter we also compare the values of the coupling constant obtained from light cone QCD sum rules and decay width calculation following the methods discussed in Chapter 3.

4.1 The variation of $g_{\kappa K^+\pi^-}$ via Borel Masses

In this thesis we obtained the mass and overlap amplitude of κ meson and its coupling constant in $\kappa \to K^+\pi^-$ decay. In order to determine the coupling constant we used light cone QCD sum rules with the help of Borel transformation and we obtained it as a function of Borel parameters and threshold parameter. However, as it was mentioned previously the Borel parameter is not a physical quantity and therefore our result should be independent of this quantity. To choose the stable region of the coupling constant with respect to the Borel parameters M_1^2 and M_2^2 we plot the Fig. 4.1 showing dependency of the coupling constant on these Borel masses. On the other hand the threshold parameter, s_0 is related to the energy of the next excited state and considering this property it is chosen in the region of $1.6 < s_0 < 2.2\ GeV^2$. In order to plot the Fig. 4.1, we used the value of the threshold parameter $s_0 = 2.0\ GeV^2$. Our figure depicts us that the coupling constant practically remains unchanged in the regions $1.0 < M_1^2 < 2.0\ GeV^2$ and $0.8 < M_2^2 < 1.2\ GeV^2$.

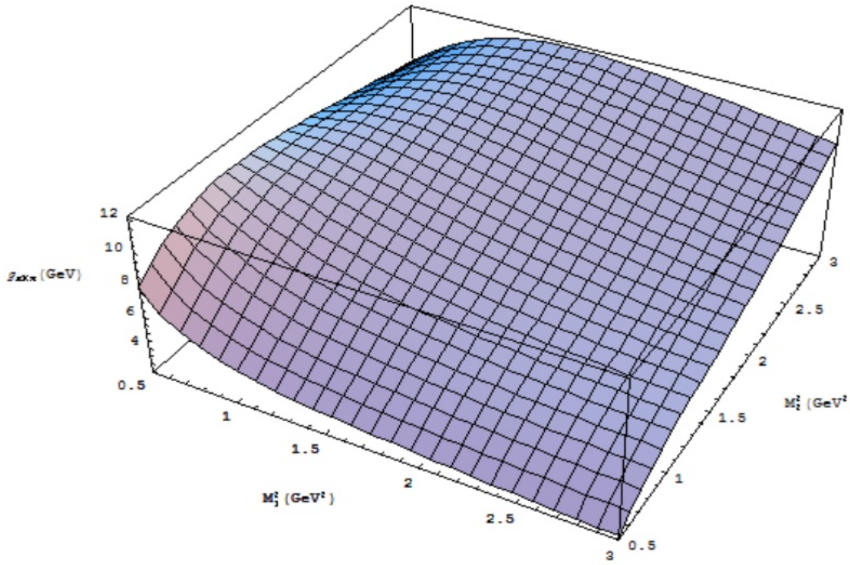

Figure 4.1: The dependence of coupling constant $g_{\kappa K^+\pi^-}$ on the Borel parameters, M_1^2 and M_2^2 for the threshold parameter $s_0 = 2.0\ GeV^2$.

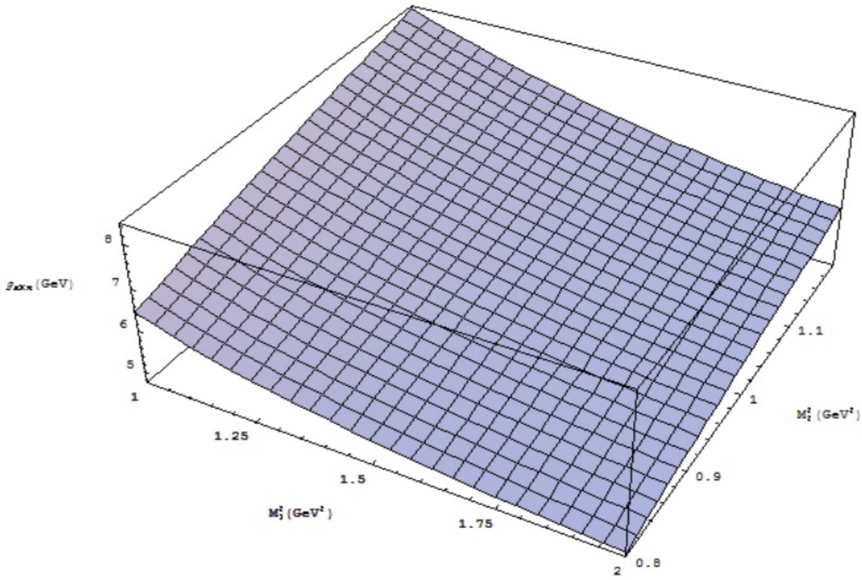

Figure 4.2: The stable region of the coupling constant $g_{\kappa K^+\pi^-}$ for the threshold parameter $s_0 = 2.0\ GeV^2$.

Fig. 4.2 shows the stability of the coupling constant, $g_{\kappa K^+\pi^-}$ in the chosen Borel mass regions given above.

4.2 The Dependence of $g_{\kappa K^+\pi^-}$ on the Threshold Parameter

As it was mentioned in the previous section, the threshold parameter is chosen in the region of $1.6 < s_0 < 2.2\ GeV^2$. To obtain the value of the coupling constant, we plot the dependence of $g_{\kappa K^+\pi^-}$ on the M_1^2 parameter for a fixed value of the parameter M_2^2. To observe the dependence of the coupling constant, $g_{\kappa K^+\pi^-}$, we choose several values of the M_2^2 from the interval given for it. Then, for a fixed value of $M_2^2 = 1.0\ GeV^2$, Fig. 4.3 is plotted which shows the dependence of the $g_{\kappa K^+\pi^-}$ on M_1^2 for the different values of threshold parameters $s_0 = 1.6,\ 1.8,\ 2.0,\ 2.2\ GeV^2$.

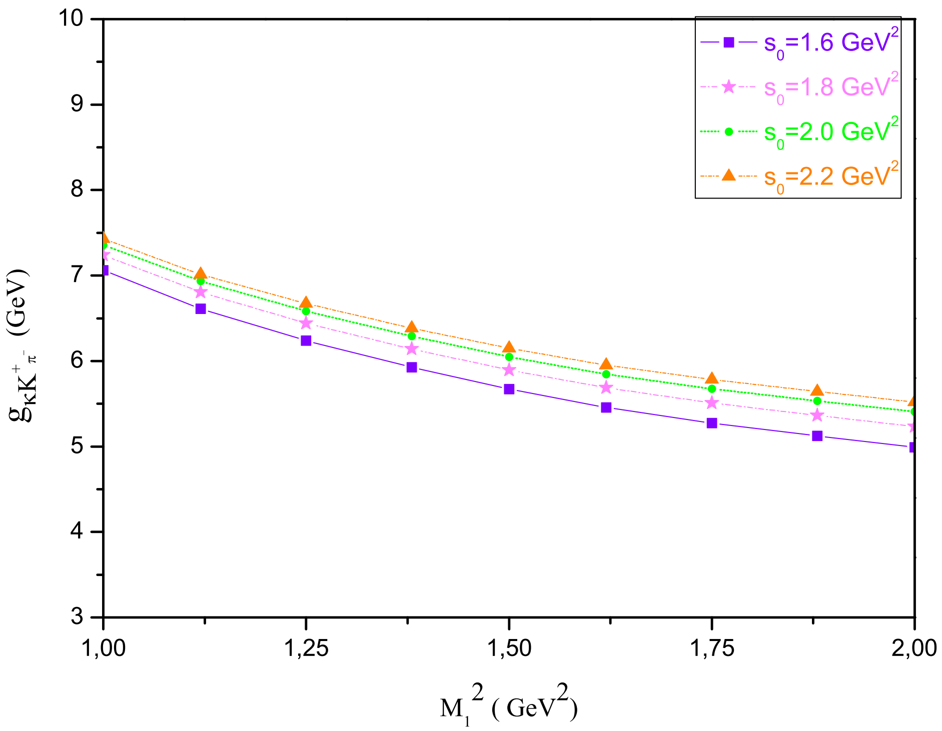

Figure 4.3: The dependence of coupling constant $g_{\kappa K^+\pi^-}$ on the Borel parameter M_1^2 for various values of the threshold parameter.

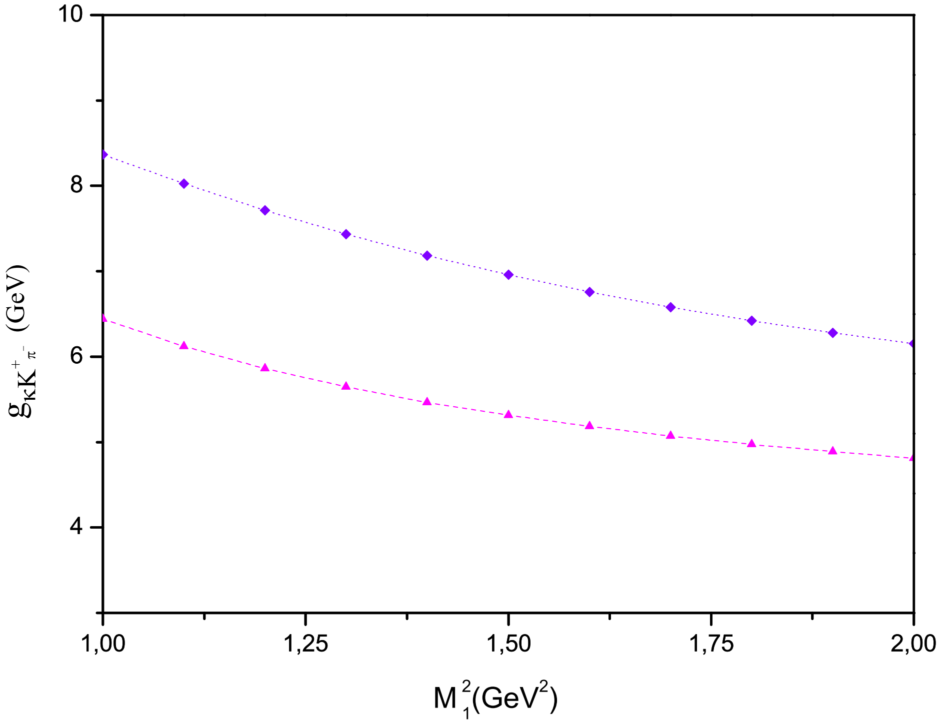

Figure 4.4: The dependence of coupling constant $g_{\kappa K^+\pi^-}$ on the Borel parameter M_1^2 for the limit values of the Borel parameter M_2^2 for the stability region of $g_{\kappa K^+\pi^-}$.

In Fig. 4.4 we use the limiting values of M_2^2, $M_2^2 = 0.8\,GeV^2$ and $M_2^2 = 1.2\,GeV^2$, to be able to obtain the variation of the coupling constant $g_{\kappa K^+\pi^-}$ with M_2^2. Considering both of the figures, Fig. 4.3 and Fig. 4.4, we estimate the value of the coupling constant as $g_{\kappa K^+\pi^-} = 6.0 \pm 1.0 GeV$. The error in this result arises due to variations in threshold parameter, and Borel parameters as well as the uncertainties of QCD parameters.

4.3 Comparison with Experimental Results

In Chapter 3, it was mentioned that the coupling constant $g_{\kappa K^+\pi^-}$ can be obtained from the decay width calculation of $\kappa \to K\pi$ decay. Using Eq. (3.75) the coupling constant is obtained in terms of the experimental decay width as,

$$g_{\kappa K^+\pi^-} = \left[16\pi m_\kappa^3 \sqrt{\lambda(m_\kappa^2, m_K^2, m_\pi^2)}\Gamma(\kappa \to K^+\pi^-)\right]^{1/2} \quad (4.1)$$

The particular decay mode of $\kappa \to K^+\pi^-$ is related to the total decay width as $\Gamma(\kappa \to K^+\pi^-) = \frac{2}{3}\Gamma(\kappa \to K\pi)$ [10]. Using a mean value of observed results of the decay width which is $\Gamma(\kappa \to K\pi) = 550 \pm 34\ MeV$, and the experimental value of the mass of κ meson, $m_\kappa = 672 \pm 40\ MeV$ [24] in Eq. (4.1) the coupling constant is extracted as $g_{\kappa K^+\pi^-} = 6.6 \pm 0.8\ GeV$. To obtain this result we use the masses for the K and π mesons as $m_K = 0.494\ GeV$ and $m_\pi = 0.135\ GeV$ [24]. The coupling constant $g_{\kappa K^+\pi^-}$ estimated from our calculations using light cone QCD sum rule is consistent with $g_{\kappa K^+\pi^-}$ obtained using the recent experimental results. In Ref.[10], $g_{\kappa K^+\pi^-}$ was also calculated using QCD sum rules approach considering the κ meson as a four quark state. In that paper, the structure of the κ meson is given as $\kappa = ud\overline{us}$ and the coupling constant of $\kappa \to K^+\pi^-$ decay was found $g_{\kappa K^+\pi^-} = 3.6 \pm 0.3\ GeV$. However this result is not consistent with the recent experimental data.

In this thesis $\sigma(600) - f_0(980)$ scalar mixing angle is also calculated using the coupling constant, $g_{\kappa K^+\pi^-}$, obtained from our calculation to comprehend its validity by comparing the recent value of this angle. The physical state of $\sigma(600)$ can be expressed as a combination of $s\overline{s}$ and $(u\overline{u} + d\overline{d})/\sqrt{2}$ isoscalar states [43] as,

$$\sigma = \cos\theta_s \frac{u\overline{u} + d\overline{d}}{\sqrt{2}} - \sin\theta_s s\overline{s} \quad (4.2)$$

Here θ_s is the mixing angle and in Ref.[43], it was estimated using light cone QCD sum rule. In that paper, coupling constant $g_{\sigma\pi\pi}$ was also calculated. Since there is no strange quark in the quark content of pion, the only contribution came from $J^S(x) = [\overline{u}(x)u(x) + \overline{d}(x)d(x)]/\sqrt{2}$ scalar current of σ. Hence, when there is no mixing, the coupling constant, $g'_{\sigma\pi\pi}$ is obtained. As a consequence of the sum rule, the coupling constant of $\sigma \to \pi\pi$ decay became $g_{\sigma\pi\pi} = \cos\theta_s g'_{\sigma\pi\pi}$.

The value of $g'_{\sigma\pi\pi}$ was estimated as $3.2 \leq g'_{\sigma\pi\pi} \leq 3.9\ GeV$ in the stability region. The ratio $g^2(\kappa \to K\pi)/g^2(\sigma \to \pi\pi)$ is given in terms of the mixing angle as,

$$\frac{g^2(\kappa \to K\pi)}{g^2(\sigma \to \pi\pi)} = \frac{g^2_{\kappa K\pi}}{\cos^2\theta_s g'^2_{\sigma\pi\pi}}, \tag{4.3}$$

The experimental decay widths for these transitions are $\Gamma_\sigma = (504 \pm 34)$ MeV and $\Gamma_\kappa = (550 \pm 34)$ MeV. Using decay width equations of the decays $\kappa \to K\pi$ and $\sigma \to \pi\pi$ the ratio is found to be $\frac{g^2(\kappa\to K\pi)}{g^2(\sigma\to\pi\pi)} = 4.4$. If we use this result in Eq. (4.3) with the values obtained from our light cone QCD sum rules calculation and the result of Ref.[43] given as $3.2 \leq g'_{\sigma\pi\pi} \leq 3.9$ GeV, the scalar mixing angle is found as $\theta_s = (35 \pm 15)^\circ$. Experimentally, this angle obtained from $J/\psi \to f_0(980)\phi$ and $J/\psi \to f_0(980)\omega$ decays is $\theta_s = (34 \pm 6)^\circ$ [44] and from $D_S^+ \to f_0(980)\pi^+$ and $D_S^+ \to \phi\pi^+$ decays is $35^\circ \leq \theta_s \leq 55^\circ$ [45]. Therefore it can be said that our estimated value is consistent with the experiment results.

CHAPTER 5

CONCLUSION

In the recent years, light scalar mesons have become subject of many papers both experimentally and theoretically. The reason is that there are some problems about our understanding of their structure. The comparison of experimental and theoretical results of hadronic properties of these mesons may provide required information to get rid of these issues.

In QCD, perturbative approaches fail owing to strong coupling and confinement effects so a nonperturbative approach is necessary to get information about hadronic observable. One of the nonperturbative approaches is QCD sum rules method.

κ meson is one of the light scalar mesons and in this thesis we investigated its hadronic properties, such as mass and coupling constant theoretically in the framework of QCD sum rules method.

First of all, the QCD sum rules method and one of its extensions, light cone QCD sum rules method were discussed. Secondly, using two point QCD sum rules, mass of scalar κ meson was calculated. For this calculation, we defined the correlator considering its structure as being $q\overline{q'}$ state. Its value was estimated as $m_\kappa = 0.70 \pm 0.06\ GeV$ consistent with the experimental results. In this work, the value of overlap amplitude of κ meson was also decided from the relation of overlap amplitude and the mass. The value of overlap amplitude was estimated as $f_\kappa = 0.13 \pm 0.02\ GeV$ determining the suitable values of threshold parameter in its interval.

The existence of scalar κ meson was first observed in $K\pi$ scattering by $E791$ collaboration. As well as the mass and the overlap amplitude, $\kappa K\pi$ vertex was studied in this thesis. We calculated the coupling constant of $\kappa \to K\pi$ decay using light cone QCD sum rules. The value of coupling constant, $g_{\kappa K\pi}$, was estimated as $g_{\kappa K\pi} = 6.0 \pm 1.0\ GeV$. Its value was also calculated using experimental decay width as $g_{\kappa K\pi} = 6.6 \pm 0.8\ GeV$. The coupling constant $g_{\kappa K\pi}$ estimated from our light cone QCD sum rules calculation is in agreement with $g_{\kappa K\pi}$ obtained using recent experimental results of decay width.

Finally, using the result of $g_{\kappa K\pi}$ calculation in light cone QCD sum rules, we obtained the $\sigma - f_0$ scalar mixing angle, θ_s. Its value was calculated as $\theta_s = (35 \pm 15)^\circ$. This angle is obtained from J/ψ and D_s^+ decays, experimentally as $\theta_s = (34 \pm 6)^\circ$ and $35^\circ \leq \theta_s \leq 55^\circ$, respectively. Comparing the value of $\sigma - f_0$ scalar mixing angle with its experimental results, it can be deduced that our results are acceptable.

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APPENDIX A

USEFUL INTEGRALS

The n dimensional Fourier transformation is defined as,

$$\int d^n x \frac{e^{ip\cdot x}}{(-x^2)^k} = -i\pi^2 \frac{\Gamma(\frac{n}{2}-k)}{\Gamma(k)} 2^{(n-2k)} (-\frac{1}{p^2})^{(\frac{n}{2}-k)}. \tag{A.1}$$

According to this transformation some useful integrals that are applicable for calculation of the correlator to obtain the QCD sum rules are given as follows [28],

$$i\int d^4 x e^{ip\cdot x} = 0,$$

$$i\int d^4 x e^{ip\cdot x} x^n = 0,$$

$$i\int d^4 x \frac{e^{ip\cdot x}}{x^2} = \frac{4\pi^2}{p^2},$$

$$i\int d^4 x \frac{e^{ip\cdot x}}{x^2} x_\alpha = 8i\pi^2 \frac{p_\alpha}{p^4},$$

$$i\int d^4 x \frac{e^{ip\cdot x}}{x^2} x_\alpha x_\beta = 8\pi^2 \Big[\frac{g_{\alpha\beta}p^2 - 4p_\alpha p_\beta}{p^6}\Big],$$

$$i\int d^4 x \frac{e^{ip\cdot x}}{x^4} = -\pi^2 \ln(-p^2),$$

$$i\int d^4x \frac{e^{ip\cdot x}}{x^4} x_\alpha = 2i\pi^2 \frac{p_\alpha}{p^2},$$

$$i\int d^4x \frac{e^{ip\cdot x}}{x^4} x_\alpha x_\beta = 2\pi^2 \left[\frac{g_{\alpha\beta}p^2 - 2p_\alpha p_\beta}{p^4}\right],$$

$$i\int d^4x \frac{e^{ip\cdot x}}{x^6} = \frac{\pi^2}{8} p^2 \ln(-p^2),$$

$$i\int d^4x \frac{e^{ip\cdot x}}{x^6} x_\alpha = -i\frac{\pi^2}{4} p_\alpha \left[\ln(-p^2) + 1\right],$$

$$i\int d^4x \frac{e^{ip\cdot x}}{x^6} x_\alpha x_\beta = -\frac{\pi^2}{4} \left[g_{\alpha\beta}[\ln(-p^2) + 1] + 2\frac{p_\alpha p_\beta}{p^2}\right],$$

$$i\int d^4x \frac{e^{ip\cdot x}}{x^8} = -\frac{\pi^2}{192} p^4 \ln(-p^2),$$

$$i\int d^4x \frac{e^{ip\cdot x}}{x^8} x_\alpha = i\frac{\pi^2}{96} p_\alpha p^2 \left[2\ln(-p^2) + 1\right],$$

$$i\int d^4x \frac{e^{ip\cdot x}}{x^8} x_\alpha x_\beta = \frac{\pi^2}{96} g_{\alpha\beta} p^2 \left[2\ln(-p^2) + 1\right] + 2p_\alpha p_\beta [2\ln(-p^2) + 3],$$

$$i\int d^4x e^{ip\cdot x} \ln\left(\frac{-x^2\Lambda^2}{4}\right) = -\frac{16\pi^2}{q^4}. \tag{A.2}$$

Note that not all the above integrals are convergent. For such cases, the above equations should be taken to mean that the imaginary parts of both sides of the equality are the same.

APPENDIX B

USEFUL BOREL TRANSFORMATIONS

In this appendix, the Borel transformation of some functions that are useful for calculation of QCD sum rules are given. The definition of Borel transformation is given in Eq. (2.16). According to this definition, the most commonly used Borel transformations in calculation of OPE side of the correlation function are given as [46],

$$\begin{aligned}
(Q^2)^n \ln Q^2 &\rightarrow (-1)^{n+1} n! (M^2)^{n+1}, \\
\alpha_s(Q^2)(Q^2)^n \ln Q^2 &\rightarrow (-1)^{n+1} n! \alpha_s(M^2)(M^2)^{n+1} + \cdots, \\
\frac{1}{(Q^2)^m} &\rightarrow \frac{1}{(m-1)!(M^2)^{m-1}}, \\
\frac{\alpha_s(Q^2)}{(Q^2)^m} &\rightarrow \frac{\alpha_s(M^2)}{(m-1)!(M^2)^{m-1}} + \cdots. \qquad \text{(B.1)}
\end{aligned}$$

where $m = 1, 2, 3, ...$, $n = 0, 1, 2, ...$ and ... expresses the corrections of higher order α_s.

Moreover, for the results of the integrals given in Appendix A, the following expressions in terms of Borel parameters are useful in correlation function calculations;

$$
\begin{aligned}
q^4 ln(-q^2) = Q^4 ln Q^2 &\quad\rightarrow\quad -2M^4 \ , \\
q^2 ln(-q^2) = -Q^2 ln Q^2 &\quad\rightarrow\quad -M^2 \ , \\
ln(-q^2) &\quad\rightarrow\quad -1 \ , \\
\frac{1}{q^2} = \frac{-1}{Q^2} &\quad\rightarrow\quad \frac{-1}{M^2} \ , \\
\frac{1}{q^2} &\quad\rightarrow\quad \frac{1}{M^4} \ , \\
\frac{1}{q^6} = \frac{-1}{Q^6} &\quad\rightarrow\quad \frac{-1}{2M^6} \ .
\end{aligned}
\tag{B.2}
$$

APPENDIX C

PION WAVE FUNCTIONS

This appendix gives the wave functions of pion, which we have used in the calculation of the coupling constant of $\kappa \to K\pi$ decay, $g_{\kappa K^+\pi^-}$. They are defined as [39],

$$\begin{aligned}
\varphi_\pi(u) &= 6u\bar{u}\left(1 + a_1^\pi C_1(2u-1) + a_2^\pi C_2^{\frac{3}{2}}(2u-1)\right), \\
\mathcal{T}(\alpha_i) &= 360\eta_3\alpha_{\bar{q}}\alpha_q\alpha_g^2\left(1 + w_3\frac{1}{2}(7\alpha_g - 3)\right), \\
\varphi_P(u) &= 1 + \left(30\eta_3 - \frac{5}{2}\mu_\pi^2\right)C_2^{\frac{1}{2}}(2u-1) \\
&+ \left(-3\eta_3 w_3 - \frac{27}{20}\mu_\pi^2 - \frac{81}{10}\mu_\pi^2 a_2^\pi\right)C_4^{\frac{1}{2}}(2u-1), \\
\varphi_\sigma(u) &= 6u\bar{u}\left[1 + \left(5\eta_3 - \frac{1}{2}\eta_3 w_3 - \frac{7}{20}\mu_\pi^2 - \frac{3}{5}\mu_\pi^2 a_2^\pi\right)C_2^{\frac{3}{2}}(2u-1)\right]. \quad (C.1)
\end{aligned}$$

Here $C_n^k(x)$s express the Gegenbauer polynomials and the constants, which are calculated considering renormalization scale $\mu = 1 \;\; GeV^2$, are given as $a_1^\pi = 0$, $a_2^\pi = 0.44$, $\eta_3 = 0.015$, and $w_3 = -3$.