T.C.
İSTANBUL KÜLTÜR UNIVERSITY
INSTITUTE OF GRADUATE STUDIES

TIME–COST TRADE-OFF ANALYSIS IN A CONSTRUCTION PROJECT USING MIXED INTEGER LINEAR PROGRAMMING: A CASE STUDY

Master of Science Thesis

Abdulrhman NAWAWRAH

Department: Industrial Engineering

Programme: Engineering Management

Supervisor: Prof. Dr. Murat ERMİŞ

APRIL 2022

T.C.

İSTANBUL KÜLTÜR UNIVERSITY

INSTITUTE OF GRADUATE STUDIES

TIME–COST TRADE-OFF ANALYSIS IN A CONSTRUCTION PROJECT USING MIXED INTEGER LINEAR PROGRAMMING: A CASE STUDY

Master of Science Thesis

Abdulrhman NAWAWRAH

1900001264

Department: Industrial Engineering

Programme: Engineering Management

Date of Defense Examination:	**01/04/2022**
Supervisor:	**Prof. Dr. Murat ERMİŞ**
Members of Examining Committee:	**Prof. Dr. Ö.Koray ŞAHİNGÖZ** **Assist. Prof. Dr. İ.Ethem TARHAN**

APRIL 2022

ACKNOWLEDGEMENT

I would like to show my appreciation to everyone who made it possible for me to accomplish this study.

To my thesis supervisor have my deep gratitude. Prof. Dr. Murat Ermiş for his support, advice, criticism, and insight throughout the study. I would also express my gratitude to İstanbul Kültür University for all the support given me during my studies and up until the completion of my thesis. I want to thank all of my friends and colleagues for their constant support and encouragement throughout the writing of my thesis. I would also thank my family for their unwavering support, and I believe that my mother's and father's prayers were God's hand in guiding me through my studies.

To my dear Brother Eng. Mohammed Nawawrah

Who supported me to the first step in my studies

Rest in Peace

Date: 01/04/2022 Abdulrhman Nawawrah

TABLE OF CONTENTS

ACKNOWLEDGEMENT .... **iii**

TABLE OF CONTENTS .... **vi**

LIST OF TABLES .... **vi**

LIST OF FIGURES .... **vii**

LIST OF SYMBOLS .... **viii**

ÖZET .... **ix**

ABSTRACT .... **x**

1. INTRODUCTION **1**

2. LITERATURE REVIEW **6**

2.1. Scheduling Using the CPM and PERT Methods **6**

2.2. Emerging TCTP Technique to Address the Time-Cost Trade-off Problem **8**

2.3. Using Mixed Integer Linear TCTP in Project Scheduling **10**

3. METHODOLOGY **13**

3.1. CPM and PERT **13**

3.1.1. Work Breakdown Structure (WBS) 14

3.1.2. Critical Path Method (CPM) 14

3.1.3. Program Evaluation and Review Technology (PERT) 14

3.1.4. Budget 14

3.2. Techniques and Tools **14**

3.2.1. Forward Pass 15

3.2.2. Backward Pass 15

3.3. Mixed Integer Linear Programming Model **18**

4. IMPLEMENTATION AND RESULTS **22**

4.1. Work Breakdown Structure (WBS) **22**

4.2. Calculating Critical Path with CPM **22**

4.3. Calculating critical path by using PERT 25
4.4. Risk Analysis by Monte Carlo Simulation 27
4.5. Using Mixed Integer Linear Programming to Expedite the Project .. 28
4.5.1. Problem Formulation 30
4.6. Analysis and Discussion 34
5. CONCLUSION 40
REFERENCES 42

LIST OF TABLES

Table 4.1: Work Breakdown Structure (WBS) 23

Table 4.2: Activity analysis of the project using PERT 26

Table 4.3: Results of Monte Carlo simulation 27

Table 4.4: The data of the project case to crashing 29

Table 4.5: Mixed integer linear programming results from Excel Solver 38

LIST OF FIGURES

Figure 3.1: The network diagram model in CPM 15

Figure 3.2: Direct and indirect cost curve of the activity... 17

Figure 3.3: Linear relationship time and cost trade-off... 18

Figure 3.4: Flowchart of the solution approach 19

Figure 4.1: Calculation of critical path in CPM diagram... 24

Figure 4.2: Results of Monte Carlo simulation.. 27

Figure 4.3: Objective function and constraints in Excel Solver 33

Figure 4.4: Excel sheet contains the data from the tables and the optimal values for the solution 35

Figure 4.5: Normal and Expedited project execution schedule 39

LIST OF SYMBOLS

WBS	Work Breakdown Structure
CPM	Critical Path Method
PERT	Program Evaluation and Review Technique
AoN	Activity on Node
ES	Earlies Start for the activity
EF	Earliest Finish for the activity
LF	Latest Finish for the activity
LS	Latest Start for the activity
S	Slack time
T_e	Expected duration in PERT
a	Optimistic estimate in PERT
m	Most likely estimate in PERT
b	Pessimistic estimate in PERT
σ^2	Standard variance
N_c	Normal cost (direct cost)
X_i	Number of days for crashing
i	Set of all possible activity can be crashing
A_c	Cost of crashing per unit time
Y_c	Indirect cost per unit time
T_t	Total duration of project (total time of critical path)
P	Probabilities for the project time
pdf	Probability density function
cdf	Cumulative density function

Üniversite	**: İstanbul Kültür Üniversitesi**
Enstitü	**: Lisansüstü Eğitim Enstitüsü**
Anabilim Dalı	**: Endüstri Mühendisliği**
Programı	**: Mühendislik Yönetimi**
Tez Danışmanı	**: Prof. Dr. Murat Ermiş**
Tez Türü ve Tarihi	**: Yüksek Lisans – Nisan 2022**

ÖZET

İNŞAAT PROJELERİNİN ÇİZELGELENMESİNDE ZAMAN-MALİYET ÖDÜNLEŞME PROBLEMLERİNİN KARIŞIK-TAMSAYI PROGRAMLAMA İLE OPTİMİZE EDİLMESİ: BİR VAKA ÇALIŞMASI

Abdulrhman Nawawrah

Projenin kesin teslim tarihi, bütçe, vb. kısıtlar altında inşaat projelerinin çizelgelenmesinde zaman ve maliyet arasındaki ödünleşim sorunu projeyi planlamanın karmaşıklığı ve zorluğu nedeniyle birçok araştırmacı tarafından üzerinde çalışılan en önemli problemlerden birisidir. Bu nedenle, TCTP'deki çoğu çalışma, projeyi bütçelenen maliyeti aşmayacak şekilde zamanında tamamlamayı hedefler. Bu amaca ancak toplam proje süresini kısaltmak için bazı faaliyetler hızlandırılarak ulaşılabilir. Bu da faaliyetlerin maliyetine yansır ve projenin toplam maliyetini artırır. En son inşaat teknolojilerindeki gelişmelerin bir sonucu olarak proje yönetimi daha karmaşık hale geldiğinden, araştırmacılar proje kısıtlamalarıyla başa çıkmak ve üstesinden gelmek için eksiksiz ve uygun bir matematiksel model geliştirmeye çalışmaktadırlar.

Bu tez, mühendislerin ve proje yöneticilerinin projeden mümkün olan en kısa sürede ve mümkün olan en düşük maliyetle en iyi getiriyi elde etmelerini sağlayacak çizelgenin oluşturulması için uygun ve pratik bir çözüm yaklaşımı sunmaktadır. Faaliyetlerin kritik yolunu belirlemek için CPM ve PERT yöntemleri kullanılmış ve ardından sonuçların geçerliliğini sağlamak için Monte Carlo simülasyonu kullanılmıştır. Daha sonra, TCTPS problemi için türetilmiş karışık tamsayılı doğrusal programlama modeli, kaynakların mevcut olduğu ve sınırlı olmadığı varsayılarak MS Excel Çözücü kullanılarak çözülmüştür. Geliştirilen yaklaşım, gerçek dünyadaki bir uygulamaya, yani lüks villa inşaat projesine uygulandı ve zaman-maliyet dengesi sorununu çözmeye yardımcı olacak önerilerde bulunduk.

Anahtar Kelimeler: Zaman-maliyet ödünleşmesi, inşaat projesi çizelgeleme, kritik yol yöntemi, Microsoft Excel yazılımı, PERT, karışık tamsayılı doğrusal programlama

University	**: İstanbul Kültür University**
Institute	**: Institute of Graduate Studies**
Department	**: Industrial Engineering**
Program	**: Engineering Project Management**
Supervisor	**: Prof. Dr. Murat Ermiş**
Degree Awarded and Date	**: Master of Science – April 2022**

ABSTRACT

TIME–COST TRADE-OFF ANALYSIS IN A CONSTRUCTION PROJECT USING MIXED INTEGER LINEAR PROGRAMMING: A CASE STUDY

Abdulrhman Nawawrah

The issue of the trade-off between time and cost when scheduling building projects under some constraints such as the final project delivery date, the budget allocated to the project, etc. is one of the most important problems that have been analyzed by a lot of research. Therefore, most studies in TCTP target to finish the project on time without exceeding the budget. This goal can only be reached by speeding up some activities to shorten the overall project duration. This, in turn, is reflected in the cost of the activities, increasing the total cost of the project. As project management has become more complex because of advances and capabilities in the latest building technologies, researchers have sought to develop a complete and flexible mathematical scheduling model to deal with project constraints.

This thesis presents a convenient and practical solution approach for the creation of the schedule, that will enable engineers and project managers to get the best return on the project in the shortest possible time and at the lowest possible cost. The critical path of the activities was determined using CPM and PERT methodologies, and the findings were validated using Monte Carlo simulation. The TCTPS problem is solved using MS Excel Solver, utilizing the resulting mixed-integer linear programming approach, assuming resources are available and unlimited. The developed approach was applied to a real-world application, namely the luxury villa construction project.

Keywords: Time–cost trade-off, project scheduling, critical path method, PERT, mixed integer linear programming.

1. INTRODUCTION

Often construction projects require clear planning due to their complexity and overlapping activities, which makes decision-makers need to reduce the number of alternatives and choose the best one. Multiple planning techniques are a suitable solution for deciding between multiple alternatives and goals that are difficult to measure. In decision-making, it is not preferable to rely on an alternative that meets only one criterion. This explains necessity of using multi-criteria evaluation programs to reach the best solution that helps decision makers (DMs) to make their decisions. The performance and cost of building projects are directly influenced by proper planning at the design stage (Mela et al., 2012).

In recent times, the presence of large and complex projects in infrastructure, urban cities, and sustainable projects has increased the need for successful project management that defines the relationships and roles of stakeholders in managing and implementing the project and accordingly. A plan must be developed to employ labor, equipment, administrative cadres, and others to obtain the best effect in time, cost, and effort of projects. In these projects, it becomes difficult to review all of the details and adapt them in order to make the best decisions, necessitating the existence of mechanisms as well as development of theories and a system that aids in making appropriate decisions (Koo et al., 2015).

The project management tools and techniques help in effectively planning projects and their activities but the completing the project at the specified time and cost still extremely difficult for project managers. There are still many possible mistakes that lead to complications in implementing these plans or failure, such as lack of factors analysis. That expected risk stems from the mistakes of technicians, workers, or managers (labor-force) factors, the internal or external environment factors, or the use of one technique in planning or one alternative that does not meet all the required standards. These and other elements are more essential in deciding the project's success than failure since they impact the duration of project operations, which increases the project's time, cost, or both (Karabulut, 2017).

Modern construction projects necessitate the integration of planning, management, evaluation, and the adoption of appropriate procedures to ensure the best project implementation and results that meet stakeholders' expectations. Construction Management (CM) is defined by the Construction Management Association of America (CMAA) as a professional service that manages numerous needs such as cost, time, environment, safety and quality using effective management strategies. It comprises a set of phases that start with project planning and keep going through the operation and maintenance phases. CM is described by the holder as a management system meant to assure the effective completion of a project (CMAA ,1999).

Each project has a goal that must be accomplished, and therefore there are several steps that must be followed to implement the project in a manner that guarantees the achievement of its objectives. In construction projects, the various activities need to be presented in order of size from highest to lowest, a schematic network drawn for all activities, and an estimate of the least possible time to finish the project. There are reference limits such as the time allowed for the submit of the project and the allowable cost. These reference limits are commonly determined as the budget allotted, quality and scheduling, to obtain the optimal cost and time for the project. So, the project has three important dimensions to achieve the goal, which are the project schedule, costs within the budget limits, and the assumed quality requirements. Therefore, time and costs are the variables that must be optimized (Lujić et al., 2019).

Many smart systems and approaches have been developed in many studies and projects to assist site engineers and managers in meeting their commitments and project deadlines (Chen et al., 2013; Naticchia et al., 2019). One of the most important of these aspects is time management and schedule preparation, which ensures the completion of the project in a timely and cost-effective manner allocated to the project (Faghihi et al., 2014). This is largely determined by the organization of construction activities, including defining their start and end dates, the mechanisms used in their execution, and determining the necessary expenditures for each activity. As a result, the TCTP was coined (Liu et al., 2020).

The time-cost trade-off is a great competitive advantage for firms that can lower cost and time at the same time in the original TCTP efficiency. It is one of the most important components of construction management and the most vital phase in project scheduling (Cheng & Tran, 2014). What can shorten the time of the project by expediting certain of its activities with a cost? In terms of practical applicability, the original TCTP still confronts several obstacles when it comes to real-world case studies. The great majority of modifications throughout the years have dealt with possible shortcuts and their implications on project length, but activity extension is equally essential in decreasing project cost, particularly in the case of TCTP difficulties connected with recurrent projects (Tran et al., 2019).

A lot of work has been done on various methodologies such as Scheduling, Goal Programming, Critical Path Method (CPM), Project Evaluation & Review Technique (PERT), Fuzzy Goal Programming and dynamic programming in project planning, estimating project completion time, and optimizing time activities (Bintang M. R.et al., 2019).

The project can be managed by using the available multi-criteria tools and techniques to accomplish the task in the best way within the specified time and at the lowest possible cost. For appropriate project planning and scheduling of major projects, both the CPM and the PERT are utilized. Some issues have been solved successfully using CPM and PERT techniques in planning construction projects, analyzing, controlling, and scheduling, and because PERT and CPM are network-based technologies, they help in monitoring the project and following it up to complete it on time (Bintang M. R.et al., 2019).

The CPM and PERT are tools for project management and planning technique. It defines activities as critical or non-critical to identify the critical path and act on it to reduce project time. The CPM is useful for projects whose activities overlap and have complex relationships and it defines the activities that fall on the critical path. These operations must be executed on time and without delay for the project to be completed on time. The action in the crucial route is dependent on the completion of the activity before it is on the same path. Some projects may have many critical paths. CPM enables project workers to better monitor work progress and the completion of tasks required for job progress.

PERT is clearly used for tracking large projects. PERT charts represent each task as an arrow. The duration of each activity is represented as a square containing four-time estimates: optimistic, pessimistic, likely, and expected. PERT and CPM are likewise interchangeable. However, businesses often prefer to deal with the critical path method because it reduces the cost and provides quality assurance for each activity before going to the next. While PERT is often used in research and development and planning studies, it is also employed in other contexts, while industrial and construction projects prefer to work with both systems.

CPM and PERT have proven to be successful in scheduling projects when time and cost limits are not specified. However, the project is frequently restricted to a specific delivery time and the total cost that cannot be exceeded. For this reason, to obtain an effective practical method that practitioners can rely on to fulfill the project's deadline at the lowest possible cost, many researchers try to improve these methods (Hegazy & Menesi, 2012).

After determining the initial CPM schedule, the analytical capabilities and computation efficiency of CPM must be improved by employing additional approaches individually. Two key supplemental strategies for CPM scheduling are the Time-Cost Trade-off Problem (TCTP) and Constrained Resource Scheduling (CRS) (Tiwari S. & Johari S. 2015).

The project is made up of several activities with varying normal duration that are scheduled based on their relationships one with another which is known as precedence constraints, and the project's end time. This is the shortest feasible time to accomplish the project with all its activities based on precedence constraints. To meet a specific project deadline, the manager may have to shorten the duration of some activities by increasing employment, equipment, and machines, or training the crews involved in this activity. This modification results a new duration of the activity and a different cost. This option demands a cost-time trade-off, also determining which activities can be expedited to make certain that the project is completed in the quickest way possible and for the lowest costs (Hochbaum, 2016).

In this thesis and after reviewing the literature, it was determined that, due to the complex nature of the TCTP problem, the proposed algorithms and applicable methods are somewhat complex and difficult to deal with in practice and for realistic use, and their applications are limited to the development of research and studies. However, it can be improved and modified to make it more useable and manageable in the real life, particularly for small and medium projects. As a result, the goal of this study is to provide a mixed integer linear programming technique for finding easier and faster solutions to the TCTP issue that are also accurate. The following are the thesis key contributions:

- To develop a method that produces reasonably accurate results quickly and easily that can be utilized in the real world to help with project scheduling within the constraints of activities, time, and cost.

- The strategy in this thesis is to design a flexible model that can be readily adjusted and utilized by combining the linear programming model with introducing some 0-1 binary variables after applying scheduling techniques such as CPM and PERT.

- The approach is tested in a real-world case. The findings and the possibility of this strategy contributing to the project's schedule are confirmed. The results are significantly beneficial.

- We have come up with a few recommendations. The most notable of it is that this technique can be coded and integrated into one of the popular scheduling programs, or it may be used as a standalone executable one.

The following is how this thesis is structured. Section 2 examines the relevant literature, focusing on CPM, PERT, and TCTP, as well as the evolution of these theories. Section 3 describes how we can connect these theories to create a practical way to use them on the site. In Section 4, we apply these approaches to a construction project example, identifying the critical path, activities that can be accelerated, and the time and cost trade-off for that project. Section 5 discusses the efficacy of this approach and our findings. the references we used in this thesis are listed in Section 6.

2. LITERATURE REVIEW

Project management is optimal planning that uses available tools and resources to achieve predetermined goals according to given constraints (Yıldız, 2015). The activities are linked to each other by specific relationships in projects (Rençber, 2011). Project management is concerned with completing investment projects at the lowest possible cost, using available resources in the best possible way, and keeping in mind predetermined objectives. Therefore, project management aims to avoid wasting resources and thus avoid wasting time or exceeding the given budget (Sarıca, 2006, p. 19).

Undoubtedly, investment projects have a great contribution to the economies of developing countries. However, it is imperative that the available resources are used optimally in these investments (Nahmias et al., 2019). Failure to implement these projects with a good and understandable plan will result in poor quality outputs, even if the project is successfully completed. This will damage the project due to lost time and increased cost.

The management of construction projects requires full control over the planning, implementation, and progress steps (Prascevic & Prascevic, 2017). Integration, monitoring, and control of project stakeholders and evaluation and selection of options in ensuring customer satisfaction are the cornerstones of construction project management (Walker 1989).

2.1. Scheduling Using the CPM and PERT Methods

Appropriate scheduling of all project operations is critical for the project to be accomplished in a timely and cost-effective manner (Biswas et al., 2016). To achieve this result, strategic planning, customer focus, employee engagement and training are used to solve TCTP (Karabulut, 2017).

The initial applications of modern project management date back to the Industrial Revolution (Kır, 2007). In view of the limitations of the technologies that were available at that time, modern techniques in project management started to appear slowly in the twentieth century. Henry GANTT created the GANTT diagram in 1917, which is being used today for project management, that clearly shows the path of the project (Kır, 2007). After World War II, companies began to look for more practical ways in project management to cope with their challenges and rapid development. In this regard, CPM and PERT techniques have been developed, which are formulas developed from the GANTT chart (Özkan, Ş., 2004). PERT was first used in the development of the Polaris missile designed by the US Navy in 1958 (Yıldız, 2015). PERT methodology has been used in the management of major projects such as the Keban Dam and the Bosphorus Bridge in Turkey (Rençber, 2011, p. 31).

Another project planning method CPM was developed by DuPont and UNIVAC in the late 1950s for scheduling maintenance in chemical plants and has now become a frequently used project planning technique in the construction industry. CPM is a method that uses network analysis to improve project cost and project completion time by trying to shorten the time of critical path activities (Bintang M. R. et al., 2019). DuPont and Remington Rand developed a planning system with the CPM method for facility maintenance and general repair in the chemical industry and was used for the first time in a chemical plant construction project in 1958 (Yıldız, 2015). However, there are also many projects, such as scientific research and development studies, where the expected duration of action is uncertain or cannot be ignored due to their nature (Lujić et al., 2019).

There are two basic approaches to project network planning: Activity on Arrow (AoA) and Activity on Node (AoN) (Hochbaum, 2016). Initially, arrows were used to indicate activities in CPM. Activities were connected via nodes, and it was called Arc on Activity (AoA). In the 1960s, John W. Fondal, a professor at Stanford University, created a new notation to describe activities at CPM. Fondal suggested assigning activities to square nodes. The nodes were linked with arrows pointing to the sequence. The AoN method quickly became the industry standard (Kielmas, Maria, 2019).

There are many research and studies on CPM and PERT project planning techniques. Project management was carried out and project completion times were calculated using these techniques in the Olympic ice rink construction project (Sarıca, 2006), Marmaray Üsküdar tunnel construction project (Karadeniz, 2007), and natural gas installation construction project (Rençber, 2011).

2.2. Emerging TCTP Technique to Address the Time-Cost Trade-off Problem

TCTP method emerged when CPM and PERT could not create a project schedule with many variables and were limited to projects with fixed duration and cost. The aim of this technique is to shorten the critical path to achieve the project's actual objective at the lowest possible cost (Chassiakos et al., 2005). TCTP analysis is a valuable management tool that can be used to accelerate the project to compensate for delays and minimize cost/time issues. Project duration can be shortened by speeding up activities in the critical path of the project by using additional resources and equipment or by working overtime. As a result, the TCTP algorithm finds the most cost-effective critical activity that needs a faster (but more expensive) construction approach. This increases direct expenses (materials, labor, and equipment, etc.) that can be compensated by reducing indirect costs and completing the job in less time (Gould, 2005).

There is a tremendous amount of research being done in the field of TCTP. Shouman et al. (1991) used mixed integer linear programming and CPM to create a framework for natural gas projects. The goal of this study is to use the crashing concept to achieve the lowest total cost. It was claimed that approximately 41 percent of the works in the construction sector between 1990 and 2002 were completed on time (Tiwari, Sharma, 2020). For TCTP, a new approach was proposed combining integer linear programming (Liu et al., 1995). Several scholars have used dynamic programming to reach a compromise between two key parts of the project (De et al., 1995).

Recently, several metaheuristic optimizations approaches, including genetic algorithms (GA), evolutionary algorithms (EA), particle swarm optimization (PSO), and others, have been applied to project scheduling problems. Feng et al. (1997) used evolutionary algorithms to build time-cost trade-off situations. In a building construction project, researchers developed a machine learning and genetic algorithm-based system (MLGAS), which surpassed nonlinear TCTP (Li et al., 1999). In 2003, Poonambalam et al. (2003) used genetic algorithms to address sequencing challenges in mixed-model assembly lines in the industrial arena and found that GA outperformed. A multi-objective time-cost-quality trade-off problem was also solved using the evolutionary algorithm (Shahsavari Poura et al., 2013; Azaron et al., 2006).

A multi-objective optimal problem with four objective functions was used to organize the cost-benefit trade-off problem, and it was solved using a genetic algorithm. Several academics have developed and used a method based on evolutionary algorithms, which is hybridized with some other solution methodologies, for solving discrete TCTP problems (Biswas et al., 2016). Several scholars have utilized fuzzy logic to solve issues with uncertainty and applied fuzzy logic theory to examine the elements influencing project quality (Shahsavari Poura et al., 2013). In another study, it was recommended to use ANN together with MOGA (multi-objective genetic algorithm) to improve the project schedule in nonlinear TCTP (Pathak, B.K. et al. (2008). In 2009, Chen et al. optimized discounted cash flows using an ant colony optimization technique in his project scheduling. Zeinalzadeh (2011) demonstrated a mathematical solution for reducing the overall cost of a building construction project using MILP-Lingo12. However, because the complexity of project scheduling problems is non-polynomial time hardness (NP-hard) (De et al., 1997), most of the solution efforts for this type of problem have been associated with meta-heuristic approaches.

Genetic Algorithms (GAs) are the most used to solve DCOP as shown in the literature (Hegazy, T. 1999; Sönmez and Bettemir, 2012). Also, Wuliang & Chengen (2009) proposed an efficient GA for TCTP with multimode resource limitations. In addition, Zhang et al. (2013) introduced GA-based optimization models for DCOP in iterative projects that consider resource continuity and various logical sequences of operations for various units. Besides GAs, numerous additional meta-heuristic techniques for TCTP solution including ant colony optimization (ACO) (Chen et al., 2009), shuffled frog-leaping algorithm (SFLA) (Elbeltagi et al., 2007), particle swarm optimization (PSO) (Aminbakhsh & Sonmez, 2016), variable neighborhood search and taboo search (He et al., 2017) developed. It has been claimed that only a few approaches based on PSO, and GA can find near-optimal TCTPs in large projects within reasonable computation time. However, their success is strongly dependent on the design factors involved in these algorithms, such as mutation of the GA and crossover rate, inertia weight, cognitive component, and social factor. When these methods are used in construction scheduling, it is also necessary to develop strategies to tune the design parameters to obtain good solutions for different projects with various activity networks. In conclusion, it is clear that large-scale TCTPs have not been adequately researched and current approaches are still constrained by the need for parameter tuning when faced with one-of-a-kind building projects (Aminbakhsh & Sönmez, 2016).

Other research has been done on the use of PERT networks to solve the multi-objective allocation of resources issue using optimal control theory (Azaron & Tavakkoli-Moghaddam, 2006). A multi-objective TCTP-based novel analytical methodology was developed that uses the time discretization process to obtain the lowest total direct cost, the lowest average of the project delivery date, and the lowest variance to end the project (Azaron et al., 2007). The Pareto front was used to solve a new style of Time-Cost Efficiency in a non-deterministic scenario using a hybrid approach based on fuzzy logic and GA (Eshtehardi et al., 2008). Recently, researchers proposed using a Radial Basis Function (RBF) neural network that also considers risk to solve the multi-objective time-cost trade-off problem in dynamic PERT networks (C. Li & Wang, 2009).

The TCTP in strategic planning and some study findings are also discussed. For example, in a TCTP, a hypothetical loss of quality penalty was considered (Kim et al., 2012). Choi & Kwak (2012) built a decision-making model for balancing time and cost incentive structures. Yang (2011) also provided a stochastic TCTP allocation strategy that focuses on correlation and stochastic dominance.

2.3. Using Mixed Integer Linear TCTP in Project Scheduling

In the various time–cost scenarios, the TCTP has been widely researched in situations where activities can only be expedited at specified different stages and at a stated cost for every period level. As a result, the cost function is no longer convex, and the problem is now NP-hardness. Each action in the linear TCTP has a constant expediting cost per unit. The convex cost variable, which is computed as the double-convex flow of the least expensive network in polynomial time, is also solved in polynomial time using linear programming (Ahuja et al., 2004). The linear time-cost trade-off issue is an essential simplification of discrete variants of the problem, such as the TCTP when resources are limited, in addition to having numerous direct applications. For example, in (Skutella M.1998), linear TCTP was employed as an approximation algorithm for discrete TCTP. Researchers have been exploring the linear TCTP issue and approach for solving it for more than five decades. Garcia-Nieves et al. (2018) presented a successful integer linear programming model to solve TCTP with constrained resources (RCPSP-RA) considering continuity limitations, different execution modes within the same activity, and acceleration process. Likewise, Reda (1990) pioneered the use TCTP in projects that include repeated activities and applied a mathematical methodology based on linear programming in planning recurring work to reduce project costs. The model takes into consideration continuity limitations, finish-to-start activity connections, and constant production rates. Senouci and Eldin (1996) introduced a non-linear dynamic project scheduling approach based on dynamic programming, which incorporates a better TCTP analysis. The method was able to handle a wide range of staff formations, production rates, and lag times, as well as linear, non-linear, and discontinuous activity time cost variables. to achieve a deadline. Zou et al. (2018) proposed a mixed integer linear programming approach that uses linear optimization to predict the number of personnel required per activity. The first model considered continuous activities, stable production rates, and cost analyses. Furthermore, the previous model's scope was expanded by integrating a bi-objective optimization that allowed for discontinuous activity execution (Zou et al., 2018).

Despite ongoing improvements in repeating activities planning tools, none of these models provide a robust or accurate linear programming mathematical formulation for TCTP that appropriately combines as many linear scheduling aspects as possible (García-Nieves et al., 2019). TCTP has been extensively investigated in the literature as previously shown. Mixed-integer linear programming (MILP) can be used to determine the optimal crashing schedule when the crashing cost function is linear.

In the cases of discrete, convex, and concave functions, these strategies are applied for TCTPs which having discrete cost functions in particular, various heuristics are available which have proven to be extremely NP-hard to establish an ideal crashing schedule.

Monte Carlo (MC) simulation was first used by van Slyke in 1963 to reduce these risks. It's a popular tool for evaluating the resulting schedule. We did a review of the related literature to explain the significance of this study. Adaptive solutions are further differentiated if a crashing policy requires time information for all activities at the start of a project or only uses the current state that is gradually determined as the project progresses.

In this study, a simple and practical approach that practitioners can apply in the real world will be developed, combining the critical path method with mixed integer linear programming, using basic computer applications such as Excel and MS Project.

3. METHODOLOGY

3.1. CPM and PERT

Usually, the activities and tasks in construction projects are not fixed in terms of time, due to the difference in their durations from one project to another in relation to the characteristics of the project, its location, and the details of these activities, in this study data were taken from a contracting company for a residential villa project. Data consist of each activity and what it depends on from the activities that precede it (predecessor of activity).

Project scheduling is based on three objectives, schedule, budget, and scope, as shown in this detail:

1. Scope
   Product Breakdown Structure (PBS)
   Work Breakdown Structure (WBS)
2. Schedule
   Milestone Plan
   Critical Path Method (CPM)
   Program Evaluation and Review Technique (PERT)
3. Budget
   Resource Plan
   Cost Plan
   Payment Plan

What defines these three goals are the main constraints, time, cost, and in these projects, it's not possible to focus on one goal without the other.

The case was dealt with in this research utilizing a defined approach to identify the three objectives, using CPM / PERT methodologies, which would be used after describing the constraints that were considered.

3.1.1. Work Breakdown Structure (WBS)

Project tasks are divided into fundamental activities in this way to clearly identify the project's scope, as it is considered a straightforward and convenient method of analysing and simplifying it in project management.

3.1.2. Critical Path Method (CPM)

To assess which activities are on the critical route and have an impact on the project's life cycle, the critical path technique was utilized. The critical path technique is a well-known and widely used strategy for planning and scheduling projects, particularly those with a large number of activities.

3.1.3. Program Evaluation and Review Technology (PERT)

PERT is a method and technique like Critical Path. Using PERT can define which tracks and activities should be minimized to reduce the life of the project as a whole. In this tool the project flow plan is improved by allocating time to activities so that the project timing becomes it looks ideal. then determines the PERT-based average time. The average lifetime of the event is calculated by dividing the sum of the optimistic, 4 times the most probable duration, and the pessimistic duration, by 6.

3.1.4. Budget

The budget was estimated according to the engineers' experience in the project, historical data, and using common contract pricing.

3.2. Techniques and Tools

As mentioned above, we will deal with CPM / PERT techniques in the case study, and this is a brief explanation of these tools and their mechanism of work. There are key steps that we work on in CPM / PERT: First, we define the project tasks and then break them down into initial activities and define the relationship between each activity (previous and next), draw a network diagram for the project using AoN form like given in Figure 3.1, taking into account the relationships that govern activities with each other and estimate time and costs of each activities, time requirements and activities costs are estimated, then we determine the earliest time for the start of the activity and the last time for the completion of the activity, and accordingly the critical path is determined (Yıldız, 2015).

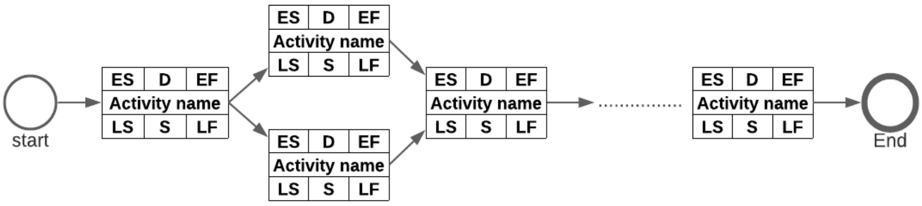

Figure 3.1: The network diagram model in CPM

3.2.1. Forward Pass

Earlies start time (ES) for the activity is calculated as:

$$ES = \max(EF) \tag{3.1}$$

where, EF is maximum Earliest Finish time from previous activities.

Earliest Finish time (EF) for the activity calculated as:

$$EF = ES + D \tag{3.2}$$

while D is approximate time frame for the activity.

These calculations are completed for all activities in the project from beginning to end, to determine the earliest possible start and finish times, as well as the duration of each project activity.

3.2.2. Backward Pass

Then, to establish the important activities and critical pathways in the project, the computations are returned from the end to the beginning to compute Latest Finish time (LF), Latest Start time (LS), and Slack time(S).

Latest finish time (LF) for the activity calculated as:

$$LF = \min(LS) \tag{3.3}$$

where, LS is minimum latest start time from successor activities.

Latest start time (LS) for the activity calculated as:

$$LS = LF - D \tag{3.4}$$

where, LF is latest finish time for this activity and D is estimated duration for it.

Slack time(S) calculation to determine critical path activities:

$$S = LS - ES \tag{3.5}$$

Activities with zero slack time represent critical path activities. Now the beta distribution is used to find out how long each activity takes in the network diagram and is calculated as:

$$T_e = \frac{a+4m+b}{6} \tag{3.6}$$

where T_e stands for the expected return, the optimistic estimate is a, the most likely estimate is m, and the pessimistic estimate is b.

This approach used in PERT, to reduce the time required to do some activities, in order to obtain the completion time of the project and make it more appropriate and near ideal. After preparing the CPM chart and calculating all the activities' times as shown above, standard variances are calculated for each activity in order to be able to determine the reliability or uncertainty of the entire project:

$$\sigma^2 = \frac{(a-b)^2}{6} \tag{3.7}$$

where a would be the optimistic time of activity and b would be the pessimistic time of activity. The total of the variations for all critical activities along the longest critical path determines the project's variance:

$$\sigma_P^2 = \sum(variances\ of\ activities\ on\ critical\ path) \tag{3.8}$$

Then, the standard deviation of the project is equal to the square root of the sum of the variances of these activities:

$$\sigma_P = \sqrt{\sigma_P^2} \tag{3.9}$$

After clarifying the CPM, PERT, and MC simulation, the next section will be on the results and analysis for applying these techniques.

Construction projects need a substantial amount of time and effort, as well as the use of multiple resources and a significant amount of investment. As a result, one of the primary objectives of the project scheduling is to produce a variety of plans and possibilities from which to pick the best while keeping in mind the existing time restrictions and the project completion time given. one specific aim is to optimize the project's length, creating a strategy that results in the lowest total project expenses.

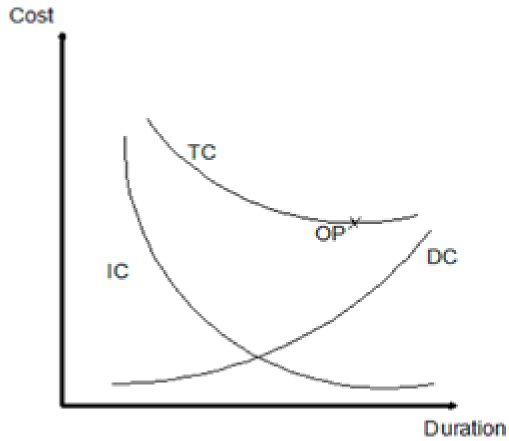

Figure 3.2: Direct and indirect cost curve of the activity

Part of this effort entails defining a project's time cost curve in Figure 3.2. It shows the relationship between the direct / indirect cost and duration, which represent the expenses of the project at various time points. Considering the connections between the activities and their sequence, as well as the fact that each action may be performed in a variety of ways, each with its own duration and expense, the goal is to discover the best implementation solution for each activity so that the project may be completed on time and on budget (Chassiakos et al., 2005).

The project cost-time curve demonstrates that direct costs begin high and then decrease over time, while indirect costs begin low and then increase over time. The best cost of the project is indicated by the optimal point on the curve as shown in Figure 3.2. However, as the total cost of implementing the activity, we find the linear relationship between the cost of the activity and the duration of its implementation, as shown in the Figure 3.3 and accordingly, we use mixed integer linear programming to trade-off between cost and time.

3.3. Mixed Integer Linear Programming Model

The CPM network and project's time–cost linkages are transformed into constraints and objective functions via mathematical approaches utilized in construction management literature. For measuring TCTP in a project, the ILP-Solver approach has proven to be effective. The mixed integer linear programming (ILP) technique is employed to address the TCTP in this study because of its easy-to-use Microsoft Excel solver (Tiwari and Johari, 2015).

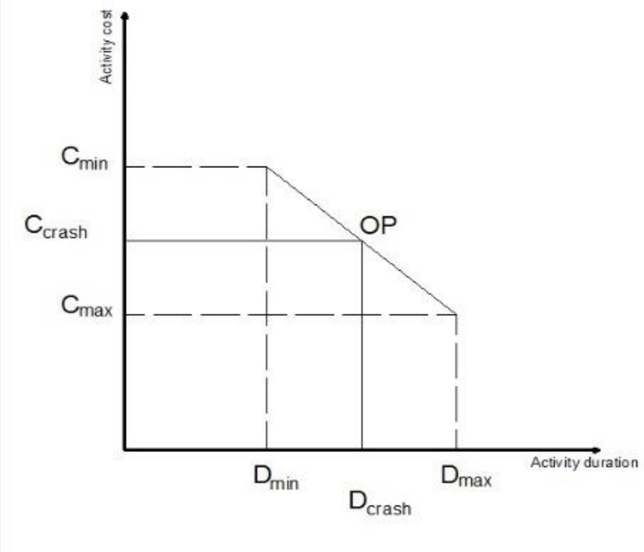

Figure 3.3: Linear relationship time and cost trade-off

As shown in Figure 3.3, the mixed-integer linear programming approach seeks for the best solution and the best trade-off between getting the earliest time at the lowest cost. Here it is assumed that the relationship is linear in the total cost change over time of the activity in the project, as mentioned earlier, in order to trade-off between them in an easy, fast and practical way. This approach gives us the earlier time with the lowest cost by accelerating critical activities using materials, equipment or labors contribute to the normal cost of activity, and in the solution proposed in this thesis. The CPM approach may be used to determine the project's critical path, and then apply the restrictions and conditions to these activities using mixed integer linear programming MILP to expedite and to reduce the project's overall time, then reorganize the activities using the PERT method once again, and if a new critical path or new critical activities appear, we speed up them again until we get the optimal solution to expedite the project, as shown in the project flowchart process in Figure 3.4.

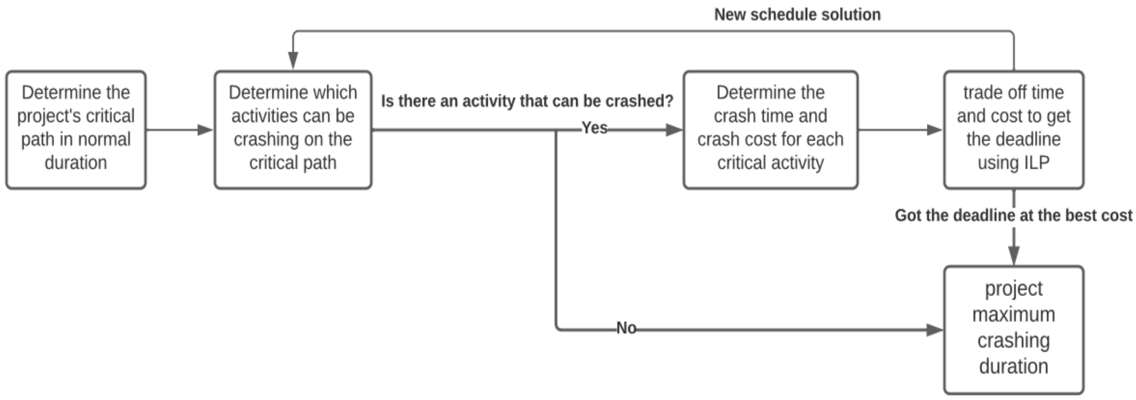

Figure 3.4: Flowchart of the solution approach

In order to be comfortable, practical, and successful in realistic life, we make the following assumptions in our solution:

- Any modification, acceleration, or cost reduction of one activity has no effect on the cost or time of any other activity.
- The project's equipment and resources are unlimited and available at any time.
- Any change in the activity's cost and duration is deemed a linear variable and may be dealt with as such.

These assumptions are, in fact, close to reality, but they may alter and place external constraints on the project. Here in this approach does not address external constraints, and it is up to the project manager to introduce additional constraints to the project and deal with them.

In general, mixed integer linear programming is developed and represented in two parts:

i) The objective function that attempts to minimize time while lowering cost.
ii) The trade-off between them to produce the optimal solution, as previously indicated.

This is to ensure that the sequence of activities is logical and as required by the project, with no conflict between them, where precedence constraints and financial constraints are dealt with, precedence relationships are used to model FS (finish-to-start), SS (start-to-start), and FF (finish-to-finish), and another relationship is SF (start-to-finish) However, this connection is uncommon in construction projects and is more common in chemical processes where the activity of one reaction continues and does not finish until the beginning of another reaction activity. These restrictions and linkages may be accompanied by time delays or overlap between activities (referred to as lags or leads), where any logical constraints imposed by the projects must be considered.

Financial constraints are established by the direct cost, the indirect cost, material penalties for late completion, incentives (rewards) for early completion, and so on.

<u>Problem Formulation:</u>

Define the decision variable and we need to minimize cost by:

$$\min \sum_i (A_c X_i + F_c Y_i + R_c T_i + N_c) \tag{3.10}$$

where:

A_c = Cost of crashing per unit time

X_i = Number of days can be crashing

$Y_i = \begin{cases} 1, & \text{if the crashing is selected for activity } i \\ 0, & \text{otherwise} \end{cases}$

i = Set of all possible activity can be crashed.

So, if $X_i \geq 0$ then Y_i = 1, and if $X_i = 0$ then $Y_i = 0$

F_c = Fixed cost for each crashing action (and paying for one time at the beginning of crashing).

R_c = Indirect cost per unit time.

T_i = Total duration of project (total time of critical path).

N_c = Normal cost (direct cost). It is calculated by adding the normal costs of each activity before crashing.

The cost of crashing is represented by the first and second term of the objective function to take the lowest possible value, the third term represents indirect cost of the project.

The precedence's of the model are given by the following relationships:

- Finish to Start (FS):

$$ES_{i+1} - EF_i \geq 0 \tag{3.11}$$

- Finish to Start with lag (FS+ d):

$$ES_{i+1} - EF_i \geq d \tag{3.12}$$

- Finish to Start with lead (FS− d):

$$ES_{i+1} - EF_i \geq -d \tag{3.13}$$

- Start to Start (SS):

$$ES_{i+1} - ES_i \geq 0 \tag{3.14}$$

- Start to Start with lag (SS+ d):

$$ES_{i+1} - ES_i \geq d \tag{3.15}$$

- Finish to Finish (FF):

$$EF_{i+1} - EF_i \geq 0 \tag{3.16}$$

- Finish to Finish with lag (FF+ d):

$$EF_{i+1} - EF_i \geq d \tag{3.17}$$

4. IMPLEMENTATION AND RESULTS

In this section, we review the data and analyses obtained from applying CPM/PERT techniques to the residential villa project, as we shall explain later, the outcomes of which will be used in the mixed integer linear programming modelling later to expedite the project.

We will work to establish which activities we will crashed in order to accomplish the desired target based on the constraints and conditions. The approach will be applied to the case study with the necessary values and computations, and the results will be analysed and explained at the end of this section.

4.1. Work Breakdown Structure (WBS)

The Work Breakdown Structure (WBS) and the relationships between each activity and the activities before and after it are shown in Table 4.1. The WBS helps identify activities that are critical to completing the project and cannot be delayed. We will also use Monte Carlo simulations to see how likely it is that the project will be completed given the risks involved.

4.2. Calculating Critical Path with CPM

There must be one starting activity, where the Earliest Start time (ES) is zero and equals Latest Start time (LS), and one ending activity, where the Earliest Finish time (EF) equals Latest Finish time (LF) and represents the time for the entire project, which was equal in this project 160 days before crashing.

The network in Figure 4.1 shows the critical activities in the project, which if they are late for the specified time cause a delay in the time of the project as a whole, and the critical path is {(1.1), (1.2), (1.3), (1.4), (1.5), (1.6), (2.1), (3.1), (4.1), (5), (6), (7), (9)}. Note that, Earliest Start time (ES), and Earliest Finish time (EF) has been calculated by Forward Pass, then Latest Start time (LS) and Latest Finish time (LF) has been calculated by Backward Pass for each activity, individually.

Table 4.1: Work Breakdown Structure (WBS)

Activity Num	Activity name	Predecessor	Description	Duration (days)
1	1.1	-	Design	9
2	1.2	1.1	Project Permission	6
3	1.3	1.2,1.1	Site and Earth Work	11
4	1.4	1.3	Foundation	9
5	1.5	1.4	Isolation and backfill	2
6	1.6	1.5	Ground Slab	8
7	2.1	1.6	First Floor Work co.	24
8	2.2	2.1	First floor cleaning	2
9	2.3	2.2	Partition Wall 1st floor	7
10	2.4	2.3	Electrical and mechanical works 1st	4
11	2.5	2.4,2.3	Plaster Work 1st floor	6
12	2.6	2.5,2.4	Paint Work 1st floor	8
13	3.1	2.1	Second Floor Work co.	23
14	3.2	3.1	Second floor cleaning	2
15	3.3	3.2	Partition Wall 2nd floor	7
16	3.4	3.3	Electrical and mechanical works 2nd	4
17	3.5	3.4,3.3	Plaster Work 2nd floor	6
18	3.6	3.5,3.4	Paint Work 2nd floor	8
19	4.1	3.1	Roof Work	23
20	4.2	4.1	Roof floor cleaning	2
21	4.3	4.2	Partition Wall roof floor	5
22	4.4	4.3	Electrical and mechanical works roof	3
23	4.5	4.4,4.3	Plaster Work roof floor	4
24	4.6	4.5,4.4,3.6	Paint Work roof floor	6
25	5	4.1	Installation	6
26	6	5	Stonework	22
27	7	6,4.6,2.6	Carpentry	12
28	8	4.1	Site finish work	8
29	9	8,7,6	End and Delivery	5

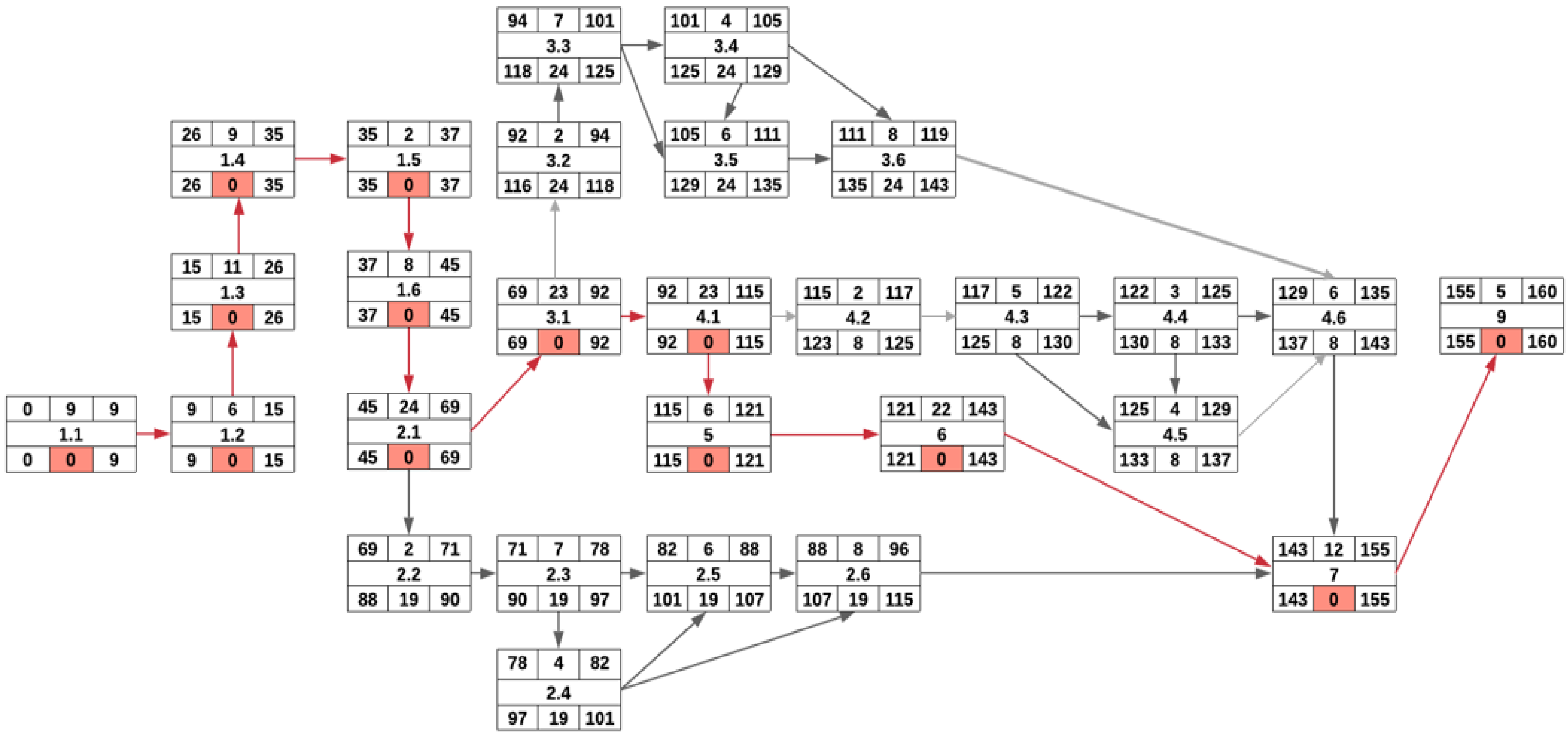

Figure 4.1: Calculation of critical path in CPM diagram

4.3. Calculating critical path by using PERT

The Table 4.2 shows PERT calculations for the project activities, as each activity has Optimistic Time (*a*), Most Likely Time (*m*), and finally Pessimistic Time (*b*), and based on these three times the average time for each activity allowed for the activity period is calculated, and the standard deviation and activity variances were calculated, to determine the critical path that was identical with the CPM and the lifetime of the project was the same as 160.

Table 4.2: Activity analysis of the project using PERT

Act Num	Act name	Predecessor	Critical Path	a	m	b	D	ES	EF	LS	LF	S	σ^2	σ
1	1.1	-	Yes	7	9	13	9	0	9	0	9	0	6.00	2.45
2	1.2	1.1	Yes	3	6	8	6	9	15	9	15	0	4.17	2.04
3	1.3	1.2,1.1	Yes	9	11	14	11	15	26	15	26	0	4.17	2.04
4	1.4	1.3	Yes	7	9	12	9	26	35	26	35	0	4.17	2.04
5	1.5	1.4	Yes	1	2	5	2	35	37	35	37	0	2.67	1.63
6	1.6	1.5	Yes	6	8	9	8	37	45	37	45	0	1.50	1.22
7	2.1	1.6	Yes	21	24	29	24	45	69	45	69	0	10.67	3.27
8	2.2	2.1	No	1	2	4	2	69	71	88	90	19	1.50	1.22
9	2.3	2.2	No	5	7	10	7	71	78	90	97	19	4.17	2.04
10	2.4	2.3	No	2	4	7	4	78	82	97	101	19	4.17	2.04
11	2.5	2.4,2.3	No	5	6	8	6	82	88	101	107	19	1.50	1.22
12	2.6	2.5,2.4	No	5	8	12	8	88	96	107	115	19	8.17	2.86
13	3.1	2.1	Yes	21	23	26	23	69	92	69	92	0	4.17	2.04
14	3.2	3.1	No	1	2	3	2	92	94	116	118	24	0.67	0.82
15	3.3	3.2	No	5	7	10	7	94	101	118	125	24	4.17	2.04
16	3.4	3.3	No	2	4	6	4	101	105	125	129	24	2.67	1.63
17	3.5	3.4,3.3	No	5	6	9	6	105	111	129	135	24	2.67	1.63
18	3.6	3.5,3.4	No	5	8	12	8	111	119	135	143	24	8.17	2.86
19	4.1	3.1	Yes	20	23	28	23	92	115	92	115	0	10.67	3.27
20	4.2	4.1	No	1	2	3	2	115	117	123	125	8	0.67	0.82
21	4.3	4.2	No	4	5	8	5	117	122	125	130	8	2.67	1.63
22	4.4	4.3	No	1	3	4	3	122	125	130	133	8	1.50	1.22
23	4.5	4.4,4.3	No	2	4	7	4	125	129	133	137	8	4.17	2.04
24	4.6	4.5,4.4,3.6	No	4	6	7	6	129	135	137	143	8	1.50	1.22
25	5	4.1	Yes	3	6	8	6	115	121	115	121	0	4.17	2.04
26	6	5	Yes	19	22	27	22	121	143	121	143	0	10.67	3.27
27	7	6,4.6,2.6	Yes	10	12	16	12	143	155	143	155	0	6.00	2.45
28	8	4.1	No	6	8	12	8	115	123	147	155	32	6.00	2.45
29	9	8,7,6	Yes	3	5	7	5	155	160	155	160	0	2.67	1.63
Project Completion Time = 160 Days														
Critical Path = (1.1), (1.2), (1.3), (1.4), (1.5), (1.6), (2.1), (3.1), (4.1), (5), (6), (7), (9)														

4.4. Risk Analysis by Monte Carlo Simulation

Here we specify the potential risks to complete the project on time, and as a precautionary (*P*) measure it's in the following Table 4.3 show analyzing the risks of the project. To assess the likely possibilities during the project time, the probability density function (*pdf*) and the cumulative density function (*cdf*) were utilized.

Table 4.3: Results of Monte Carlo simulation

Duration	P	PDF	CDF
122	2	0.02%	0.02%
129	8	0.08%	0.10%
136	77	0.76%	0.86%
143	466	4.62%	5.49%
150	947	9.40%	14.88%
157	1725	17.11%	32.00%
164	2865	28.43%	60.42%
171	2237	22.19%	82.62%
178	1015	10.07%	92.69%
185	684	6.79%	99.47%
192	53	0.53%	100.00%

The graph in Figure 4.2 shows the probability of completing the project is in 157 days is 32%, and in 164 days as stated in CPM and PERT calculations 60%, in 171 days is 83%, and in 185 days approximately 100% according to the results of the Monte Carlo simulation. It is noted that these results are more realistic than CPM and PERT because they take potential risks taken into consideration.

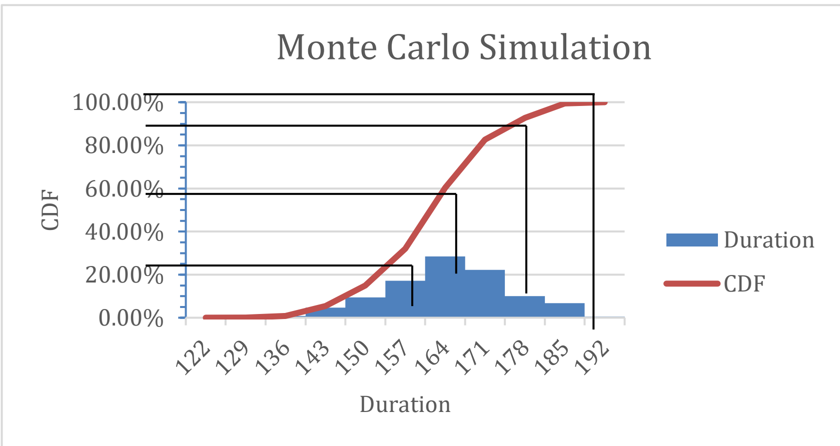

Figure 4.2: Results of Monte Carlo simulation

4.5. Using Mixed Integer Linear Programming to Expedite the Project

It is common to use project scheduling techniques in infrastructure projects and construction projects in general, and residential construction is one of these projects, and the residential villa project and its concurrent activities are a case that we will study in this thesis and apply the mixed integer linear programming technique to accelerate.

We have reached a critical path in this project by CPM and PERT methods, and it would normally take 160 days to complete the tasks and activities of the residential villa, with a 40% chance of missing the deadline according to the Monte Carlo simulation method, but what if the owner asked the manager to finish the project before this date?

As shown in data Table 4.4, the project consists of 29 activities, which link precedence relationships illustrated by the Predecessor column. For each activity, there is a certain period that can be crashing for a cost, where the possible crash duration shows the number of days that can be reduced so that the activity ends early, the cost per day crashing shows cost of this acceleration each day is in thousands of dollars, while the normal cost displays the cost of each activity prior to the decrease, and the last column indicates if the activity on the project's critical path, as indicated before in the CPM technique.

But what if it is required to complete the project in 120 days advance at the lowest feasible cost, with indirect charges of $1250 each day in the project.

This project is subject to a few simple constraints, which are as follows:

1) The project's life <= 120 days (deadline).
2) The number of days that can be saved in each activity should be <= the possible crash duration in each activity as given in the Table 4.4.
3) The resources and equipment utilized to execute each activity are not restricted.
4) The project's cost should not exceed one million dollars.

Table 4.4: The data of the project case to crashing

Act Num	Act Name	Predecessor	Description	Duration (days)	Possible Crash Duration	Normal Cost ($\times 10^3$)	Fixed Cost for Crashing	Cost/day Crashing ($\times 10^3$)	Critical Path
1	1.1	-	Design	9	3	6	0.25	0.5	Yes
2	1.2	1.1	Project Permission	6	2	8	0	1.75	Yes
3	1.3	1.2,1.1	Site and Earth Work	11	4	7	2	1	Yes
4	1.4	1.3	Foundation	9	3	16	1	1.5	Yes
5	1.5	1.4	Isolation and backfill	2	1	2	1	1	Yes
6	1.6	1.5	Ground Slab	8	2	19	1.5	2	Yes
7	2.1	1.6	First Floor Work co.	24	9	95	5	3	Yes
8	2.2	2.1	First floor cleaning	2	1	1	0.25	1	No
9	2.3	2.2	Partition Wall 1st floor	7	2	5	1.5	1	No
10	2.4	2.3	Electrical and mechanical works 1st	4	1	14	1	2	No
11	2.5	2.4,2.3	Plaster Work 1st floor	6	2	6	2	2	No
12	2.6	2.5,2.4	Paint Work 1st floor	8	3	10	1	1.3	No
13	3.1	2.1	Second Floor Work co.	23	6	85	5	3	Yes
14	3.2	3.1	Second floor cleaning	2	1	1	0.25	1	No
15	3.3	3.2	Partition Wall 2nd floor	7	2	4.5	1.5	1	No
16	3.4	3.3	Electrical and mechanical works 2nd	4	1	12	1	2	No
17	3.5	3.4,3.3	Plaster Work 2nd floor	6	2	5	2	2	No
18	3.6	3.5,3.4	Paint Work 2nd floor	8	3	8	1	1	No
19	4.1	3.1	Roof Work	23	9	60	5	4	Yes
20	4.2	4.1	roof floor cleaning	2	1	1	0.25	1	No
21	4.3	4.2	Partition Wall roof floor	5	2	3	1.5	1	No
22	4.4	4.3	Electrical and mechanical works roof	3	1	8	1	1.5	No
23	4.5	4.4,4.3	Plaster Work roof floor	4	1	4	2	1	No
24	4.6	4.5,4.4,3.6	Paint Work roof floor	6	2	8	1	1.5	No
25	5	4.1	Installation	6	4	5	2	2	Yes
26	6	5	Stonework	22	5	150	3	3.5	Yes
27	7	6,4.6,2.6	Carpentry and aluminum	12	2	110	2.5	2.5	Yes
28	8	4.1	site finish work	8	3	65	1.5	3	No
29	9	8,7,6	End and Delivery	5	0	4	0	2	Yes
					78	722.5			
Project Completion Time = 160 Days	Deadline = 120 Days	Indirect cost = 1250$ per day							
Critical Path = (1.1), (1.2), (1.3), (1.4), (1.5), (1.6), (2.1), (3.1), (4.1), (5), (6), (7), (9)									

4.5.1. Problem Formulation

In this example, reducing the objective function of the ILP model indicates the direct, indirect, and breakdown cost of the project and is represented by Equation 3.10.

The cost of crashing for the critical path activities {(1.1), (1.2), (1.3), (1.4), (1.5), (1.6), (2.1), (3.1), (4.1), (5), (6), (7), and (9)} added to the normal cost of each activity, in addition to the indirect cost for each day in the project.

The project's objective function now is:

$$\min z = \sum\big(0.5X_{1,1}+0.25Y_{1,1} + 1.75X_{1,2}+0Y_{1,2} + 1X_{1,3} + 2Y_{1,3} + 1.5X_{1,4}+Y_{1,4} + 1X_{1,5}+Y_{1,5} + 2X_{1,6}+1.5Y_{1,6} + 3X_{2,1}+5Y_{2,1} + 3X_{3,1}+5Y_{3,1} + 3X_{4,1}+5Y_{4,1} + 2X_5+2Y_5 + 3.5X_6+3Y_6 + 2.5X_7 + 2Y_7 + 2X_9+0Y_9\big) + 1.25\,T_{\mathrm{i}} + 722.5 \quad (4.18)$$

In modelling precedence constraints in this project, we note that all relationships between critical path activities are Finish-to-Start (FS) relationships. This means that

$ES_{i+1} - EF_i \geq 0$

When $EF = ES + d$ from Equation 3.2

Then $EF_{i+1} - EF_i - d_{i+1} \geq 0$

In order to ensure that the equations are connected, we add the function of days that can be broken into the equation and make sure that the activities do not overlap with each other when solving, so the equation becomes as follows:

$$EF_{i+1} - EF_i + X_{i+1} \geq d_{i+1} \quad (4.19)$$

while EF_{i+1} is maximum earliest finish time from the next activity, EF_i is maximum earliest finish time from the previous activity, d_{i+1} is estimated duration for the next activity, and X_{i+1} is number of days can be crashing.

As a result, each two activities on the critical path that have a precedence relationship must be connected in one equation to ensure that the activity which required to start when the finish of the activity that precedes it does not begin before the completion of the previous activity.

Here, in the equations below, the relationships between each activity and the one following it are given as constraints:

$$EF_{1,1} - EF_{start} + X_{1,1} \geq 9 \tag{4.20}$$

$$EF_{1,2} - EF_{1,1} + X_{1,2} \geq 6 \tag{4.21}$$

$$EF_{1,3} - EF_{1,1} + X_{1,3} \geq 11 \tag{4.22}$$

$$EF_{1,3} - EF_{1,2} + X_{1,3} \geq 11 \tag{4.23}$$

$$EF_{1,4} - EF_{1,3} + X_{1,4} \geq 9 \tag{4.24}$$

$$EF_{1,5} - EF_{1,4} + X_{1,5} \geq 2 \tag{4.25}$$

$$EF_{1,6} - EF_{1,5} + X_{1,6} \geq 8 \tag{4.26}$$

$$EF_{2,1} - EF_{1,6} + X_{2,1} \geq 24 \tag{4.27}$$

$$EF_{3,1} - EF_{2,1} + X_{3,1} \geq 24 \tag{4.28}$$

$$EF_{4,1} - EF_{3,1} + X_{4,1} \geq 23 \tag{4.29}$$

$$EF_{5} - EF_{4,1} + X_{5} \geq 6 \tag{4.30}$$

$$EF_{6} - EF_{5} + X_{6} \geq 22 \tag{4.31}$$

$$EF_{7} - EF_{6} + X_{7} \geq 12 \tag{4.32}$$

$$EF_{9} - EF_{6} + X_{9} \geq 5 \tag{4.33}$$

$$EF_{9} - EF_{7} + X_{9} \geq 5 \tag{4.34}$$

To ensure that the project is completed within the 120-day period, we set another requirement for the last activity that the Earliest Finish Time be less than the deadline.

$$EF_{9} \leq 120 \tag{4.35}$$

To satisfy the criterion that the number of days permitted for crashing cannot be exceeded:

$$X_{1,1} \leq 3y_{1,1} \quad (4.36)$$

$$X_{1,2} \leq 2y_{1,2} \quad (4.37)$$

$$X_{1,3} \leq 4y_{1,3} \quad (4.38)$$

$$X_{1,4} \leq 3y_{1,4} \quad (4.39)$$

$$X_{1,5} \leq 1y_{1,5} \quad (4.40)$$

$$X_{1,6} \leq 2y_{1,6} \quad (4.41)$$

$$X_{2,1} \leq 6y_{2,1} \quad (4.42)$$

$$X_{3,1} \leq 6y_{3,1} \quad (4.43)$$

$$X_{4,1} \leq 9y_{4,1} \quad (4.44)$$

$$X_5 \leq 4y_5 \quad (4.45)$$

$$X_6 \leq 5y_6 \quad (4.46)$$

$$X_7 \leq 2y_7 \quad (4.47)$$

$$X_9 = 0y_9 \quad (4.48)$$

Fulfil the condition that the cost is not allowed to exceed one million dollars:

$$0.5X_{1,1}+0.25Y_{1,1} + 1.75X_{1,2}+0Y_{1,2} + 1X_{1,3} + 2Y_{1,3} + 1.5X_{1,4}+Y_{1,4} + 1X_{1,5}+Y_{1,5} + 2X_{1,6}+1.5Y_{1,6} + 3X_{2,1}+5Y_{2,1} + 3X_{3,1}+5Y_{3,1} + 3X_{4,1}+5Y_{4,1} + 2X_5+2Y_5 + 3.5X_6+3Y_6 + 2.5X_7 + 2Y_7 + 2X_9+0Y_9 + 1.25T_{\mathrm{i}} \leq 1000 - 722.5 \quad (4.49)$$

so that the sum of the direct cost and the indirect cost in addition to the cost of crashing is less than one million dollars.

It is also necessary to add a condition that ensures that all results of the variables are an integer and another condition that ensures that they do not have a negative value:

$$Y_i \in (0,1) \tag{4.50}$$

$$X_i, EF_i \text{ integer} \tag{4.51}$$

If it weren't for the condition that X_i must be an integer and Y_i must be 0 or 1. This would clearly be a mixed-integer linear programming problem. It is simple to prove that the above constraints are adequate to ensure that we select the best solutions and activities that can be crashed, without the need for extra constraints.

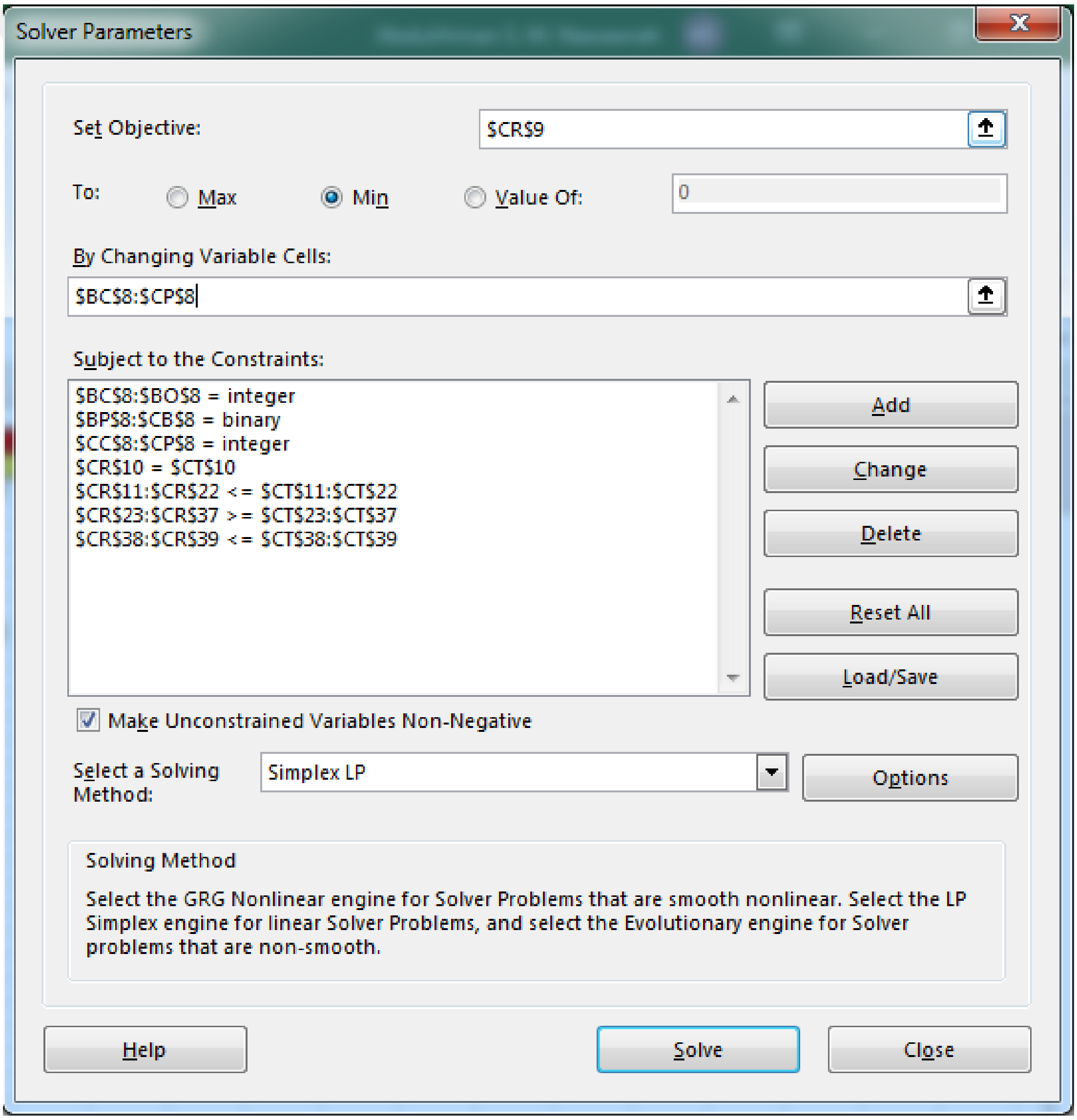

Figure 4.3: Objective function and constraints in Excel Solver

Figure 4.3 shows how to enter the objective function and constraint statements into Excel Solver to apply mixed integer linear programming to get the best outcome in the shortest amount of time at the lowest feasible cost.

In Figure 4.4, row 2 represents the decision variables in the solution and their addresses, and the rest of the rows represent the constraints, and the solution targets cell AC3, which represents the lowest cost of the crashing costs and the indirect costs of the project, and we excluded the direct costs of each activity as a fixed.

4.6. Analysis and Discussion

Table 4.5 displays the results of expediting the project using mixed integer linear programming, as obtained by Excel Solver. It appears in the beginning according to columns (CW to DC) activities data, when column (CZ) shows whether the activity is on the critical path or not before acceleration and crashing, we get the total direct cost of all activities from collecting normal costs, which was $722,500, the possible crashing duration of each activity and his fixed and direct cost appears in column (DD to DF), and in column (DG) crashing duration for each activity, which shows the number of days that reduced for each activity, and this period may be result of an increase in workers or equipment and so on, such as increasing one backfill machine and two workers to speed up the activity from two days to one day at an additional cost of $1,000 per day, we fill in the next column (DH), which shows the cost of crashing for each accelerated activity, and from the sum of this column we obtain the cost of crashing the project which was $95,500, We calculated slack using the PERT technique again in columns from (DI to DM), based on column (DN) that displays the new durations of activities after crashing, and in the last column the critical activities were selected, and their total is the length of the project. The revised deadline was in the range of 120 days.

After speeding up, the total cost of the project is = The direct cost of all activities + the indirect cost per day + the crash cost

	BB	BC	BD	BE	BF	BG	BH	BI	BJ	BK	BL	BM	BN	BO	BP	BQ	BR	BS	BT	BU	BV	BW	BX	BY	BZ	CA	CB
6																											
7		X1.1	X1.2	X1.3	X1.4	X1.5	X1.6	X2.1	X3.1	X4.1	X5	X6	X7	X9	Y1.1	Y1.2	Y1.3	Y1.4	Y1.5	Y1.6	Y2.1	Y3.1	Y4.1	Y5	Y6	Y7	Y9
8		3	2	4	3	1	2	6	6	7	4	0	2	0	1	1	1	1	1	1	1	1	1	1	0	1	0
9		0.5	1.75	1	1.5	1	2	3	3	4	2	3.5	2.5	2	0.25	0	2	1	1	1.5	5	5	5	2	3	2	0
10														1													0
11		1													-3												
12			1													-2											
13				1													-4										
14					1													-3									
15						1													-1								
16							1													-2							
17								1													-6						
18									1													-6					
19										1													-9				
20											1													-4			
21												1													-5		
22													1													-2	
23		1																									
24			1																								
25				1																							
26				1																							
27					1																						
28						1																					
29							1																				
30								1																			
31									1																		
32										1																	
33											1																
34												1															
35													1														
36														1													
37														1													
38																											
39		0.5	1.75	1	1.5	1	2	3	3	4	2	3.5	2.5	2	0.25	0	2	1	1	1.5	5	5	5	2	3	2	0
40																											

Figure 4.4: Excel sheet contains the data from the tables and the optimal values for the solution

BP	BQ	BR	BS	BT	BU	BV	BW	BX	BY	BZ	CA	CB	CC	CD	CE	CF	CG	CH	CI	CJ	CK	CL	CM	CN	CO	CP	CQ	CR	CS	CT	CU
Y1.1	Y1.2	Y1.3	Y1.4	Y1.5	Y1.6	Y2.1	Y3.1	Y4.1	Y5	Y6	Y7	Y9	A0	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13					
1	1	1	1	1	1	1	1	1	1	0	1	0	0	6	10	17	23	24	30	48	65	81	83	105	115	120					
0.25	0	2	1	1	1.5	5	5	5	2	3	2	0														1.25		270.25			
												0																0	=	0	
-3																												0	<=	0	
	-2																											0	<=	0	
		-4																										0	<=	0	
			-3																									0	<=	0	
				-1																								0	<=	0	
					-2																							0	<=	0	
						-6																						0	<=	0	
							-6																					0	<=	0	
								-9																				-2	<=	0	
									-4																			0	<=	0	
										-5																		0	<=	0	
											-2																	0	<=	0	
													-1	1														9	>=	9	
														-1	1													6	>=	6	
														-1		1												15	>=	11	
															-1	1												11	>=	11	
																-1	1											9	>=	9	
																	-1	1										2	>=	2	
																		-1	1									8	>=	8	
																			-1	1								24	>=	24	
																				-1	1							23	>=	23	
																					-1	1						23	>=	23	
																						-1	1					6	>=	6	
																							-1	1				22	>=	22	
																								-1	1			12	>=	12	
																								-1		1		15	>=	5	
																									-1	1		5	>=	5	
																										1		120	<=	120	
0.25	0	2	1	1	1.5	5	5	5	2	3	2	0														1.25		270.25	<=	277.5	

Figure 4.4: (Continued)

The total of the values in (∑ column DB) is the direct cost. The indirect cost is calculated by multiplying the duration of the project after speeding up (∑ column DO) by the indirect cost per day ($1250 per day). The crash cost is equal to the total of all activities' days which crashed multiplied the crash cost for each activity adds fixed cost for each activity crashed = SUM (column DH) + Fixed cost for crashed activity (column DE). Again, using Equation 3.10:

$$\min \sum_{i} (A_c X_i + F_c Y_i + R_c T_i + Nc) = (95.5) + (24.75) + (1.25 * 120) + (722.5)$$

Total cost after crashing is $992,750.

Figure 4.4 presents the Gantt chart for the normal and expedited durations, it clearly shows the reducing of the project delivery time from 160 days to 120 days by speeding up some activities on the critical path.

Table 4.5: Mixed integer linear programming results from Excel Solver

	CW	CX	CY	CZ	DA	DB	DC	DD	DE	DF	DG	DH	DI	DJ	DK	DL	DM	DN	DO
1																			
2	Dead line =	120																	
3	Total cost =	992.75	thousand dollars																
4																			
5	Act Num	Act name	Predecessor	Critical Path	normal Duration	normal cost $*10^{-3}$	crash duration	possible crash duration	Fixed cost for crashing	cost/day crashing $*10^{-3}$	crashing duration	Direct carsh cost $*10^{-3}$	ES	EF	LS	LF	S	new activity duration	total time of project (CP)
6	1	1.1	-	TRUE	9	6	6	3	0.25	0.5	3	1.5	0	6	0	6	0	6	6
7	2	1.2	1.1	TRUE	6	8	4	2	0	1.75	2	3.5	6	10	6	10	0	4	4
8	3	1.3	1.2,1.1	TRUE	11	7	7	4	2	1	4	4	10	17	10	17	0	7	7
9	4	1.4	1.3	TRUE	9	16	6	3	1	1.5	3	4.5	17	23	17	23	0	6	6
10	5	1.5	1.4	TRUE	2	2	1	1	1	1	1	1	23	24	23	24	0	1	1
11	6	1.6	1.5	TRUE	8	19	6	2	1.5	2	2	4	24	30	24	30	0	6	6
12	7	2.1	1.6	TRUE	24	95	15	9	5	3	6	18	30	48	30	48	0	18	18
13	8	2.2	2.1	FALSE	2	1	1	1	0.25	1	0	0	48	50	78	80	30	2	0
14	9	2.3	2.2	FALSE	7	5	5	2	1.5	1	0	0	50	57	80	87	30	7	0
15	10	2.4	2.3	FALSE	4	14	3	1	1	2	0	0	57	61	87	91	30	4	0
16	11	2.5	2.4,2.3	FALSE	6	6	4	2	2	2	0	0	61	67	91	97	30	6	0
17	12	2.6	2.5,2.4	FALSE	8	10	5	3	1	1.3	0	0	67	75	97	105	30	8	0
18	13	3.1	2.1	TRUE	23	85	17	6	5	3	6	18	48	65	48	65	0	17	17
19	14	3.2	3.1	FALSE	2	1	1	1	0.25	1	0	0	65	67	72	74	7	2	0
20	15	3.3	3.2	FALSE	7	4.5	5	2	1.5	1	0	0	67	74	74	81	7	7	0
21	16	3.4	3.3	FALSE	4	12	3	1	1	2	0	0	74	78	81	85	7	4	0
22	17	3.5	3.4,3.3	FALSE	6	5	4	2	2	2	0	0	78	84	85	91	7	6	0
23	18	3.6	3.5,3.4	FALSE	8	8	5	3	1	1	0	0	84	92	91	99	7	8	0
24	19	4.1	3.1	TRUE	23	60	15	9	5	4	7	28	65	81	65	81	0	16	16
25	20	4.2	4.1	FALSE	2	1	1	1	0.25	1	0	0	81	83	85	87	4	2	0
26	21	4.3	4.2	FALSE	5	3	3	2	1.5	1	0	0	83	88	87	92	4	5	0
27	22	4.4	4.3	FALSE	3	8	2	1	1	1.5	0	0	88	91	92	95	4	3	0
28	23	4.5	4.4,4.3	FALSE	4	4	3	1	2	1	0	0	91	95	95	99	4	4	0
29	24	4.6	4.5,4.4,3.6	FALSE	6	8	4	2	1	1.5	0	0	95	101	99	105	4	6	0
30	25	5	4.1	TRUE	6	5	2	4	2	2	4	8	81	83	81	83	0	2	2
31	26	6	5	TRUE	22	150	17	5	3	3.5	0	0	83	105	83	105	0	22	22
32	27	7	6,4.6,2.6	TRUE	12	110	10	2	2	2.5	2	5	105	115	105	115	0	10	10
33	28	8	4.1	FALSE	8	65	5	3	1.5	3	0	0	81	89	107	115	26	8	0
34	29	9	8,7,6	TRUE	5	4	5	0	0	2	0	0	115	120	115	120	0	5	5
35						722.5		78	24.75		40	95.5							120
36	Project Completion Time = 120 Days																		
37	Critical Path = (1.1),(1.2),(1.3),(1.4),(1.5),(1.6),(2.1),(3.1),(4.1),(5),(6),(7),(9)																		

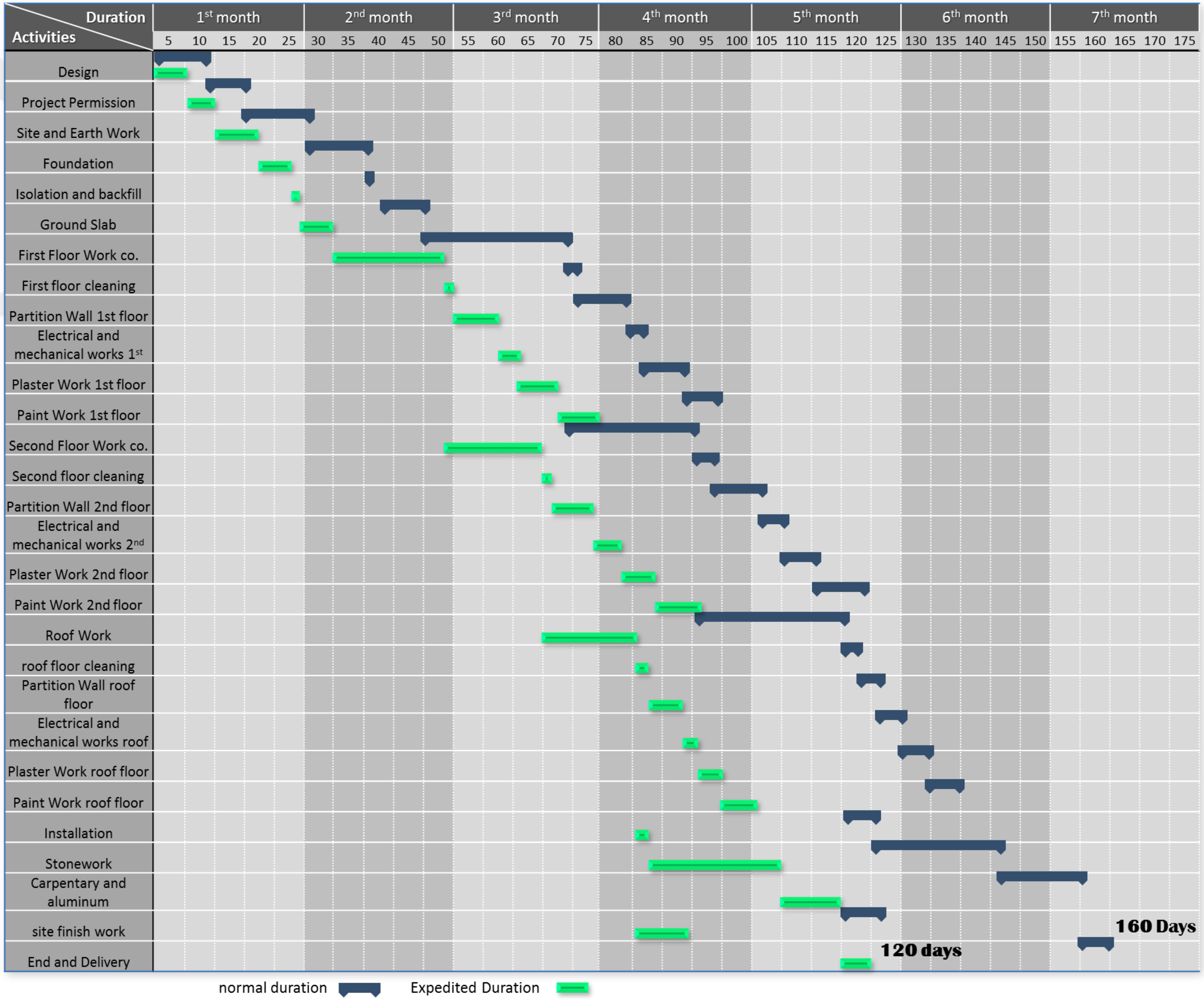

Figure 4.5: Normal and Expedited project execution schedule

5. CONCLUSION

This work aims to achieve a quick and simple method that is easy to deal with and implement for practitioners in the field or students in the classroom to schedule small and medium projects, using the critical path method CPM or PERT to determine the project finishing time and the critical activities in the project. Then use integer linear programming to determine which critical activities can be crashed at the lowest possible cost.

When dealing with Excel Solver to compare between time and cost through integer linear programming, you can add or modify any new constraints, and deal dynamically with any variables that may appear in the critical path, the cost before the crash in the case we used was $922,500, and the project length was 160 days. Considering all of the activities that may be decreased and estimating the cost of the activities that had to be lowered in order to meet the project's new and required deadline of 120 days, the total cost of crashing the project was $992,750. This indicates that the project was sped up by 40 days from 78 days were available to crashing and the cost of crashing was $120,250.

It should be noted that the number of days that can be crashed is 78 days as shown in the example and if all the activities are expedite in these days, the life of the project will be approximately 105 days with a crashed cost equal to 190,500$ and a total cost of 1,044,250$. Thus, it would be a bad choice if we crashed all activities and shortening the life of the project to 105 days. Here highlights the role of this approach in achieving the best cost in the least possible period and choosing the critical activities to crash them.

The risk analysis using Monte Carlo simulations revealed that the chances of completing the project in 120 days are less than 2%. However, the mixed integer linear programming method and this approach assume that the resources in the project are unlimited, which opens the way for research to developing a mixed integer linear programming approach and making it more practical and accurate, also can be recommended to coded and created an algorithm from this method, to be used in computer programs that deal with project scheduling, such as MS Project or Primavera, where it can be utilized in learning because it is simple to understand and deal with.

REFERENCES

Ahuja, R. K., Hochbaum, D. S., & Orlin, J. B. (2004). A cut-based algorithm for the nonlinear dual of the minimum cost network flow problem. *Algorithmica*, *39*, 189–208. https://doi.org/10.1007/s00453-004-1085-2

Aminbakhsh, S., & Sonmez, R. (2016). Discrete particle swarm optimization method for the large-scale discrete time-cost trade-off problem. *Expert Systems with Applications*, *51*, 177–185. https://doi.org/10.1016/j.eswa.2015.12.041

Azaron, A., Katagiri, H., & Sakawa, M. (2007). Time-cost trade-off via optimal control theory in markov PERT networks. *Annals of Operations Research*, *150*(1), 47–64. https://doi.org/10.1007/s10479-006-0149-x

Azaron, A., & Tavakkoli-Moghaddam, R. (2006). A multi-objective resource allocation problem in dynamic PERT networks. *Applied Mathematics and Computation*, *181*(1), 163–174. https://doi.org/10.1016/j.amc.2006.01.027

Biswas, S. K., Karmaker, C., Biswas, T., Biswas, S. K., Karmaker, C. L., & Biswas, T. K. (2016). Time-cost trade-off analysis in a construction project problem: case study. *International Journal of Computational Engineering Research*, *6*(10), 32-38.

Bintang M. R., Sungkono K. R. & Sarno R., (2019). Time and cost optimization in feasibility test of CCTV project using CPM and PERT, 2019 International Conference on Information and Communications Technology (ICOIACT), Yogyakarta, Indonesia, 24-25 July, IEEE. dol: 10.1109/ICOIACT46704.2019.8938466

Chassiakos, A. P., Asce, A. M., & Sakellaropoulos, S. P. (2005). Time-cost optimization of construction projects with generalized activity constraints. *Journal of Construction Engineering and Management, 131*(10), 1115-1124. https://doi.org/10.1061/ASCE0733-93642005131:101115

Cheng, M. Y., & Tran, D. H. (2014). Two-phase differential evolution for the multiobjective optimization of time-cost tradeoffs in resource-constrained construction projects. *IEEE Transactions on Engineering Management*, *61*(3), 450–461. https://doi.org/10.1109/TEM.2014.2327512

Chen, S. M., Griffis, F. H., Chen, P. H., & Chang, L. M. (2013). A framework for an automated and integrated project scheduling and management system. *Automation in Construction*, *35*, 89–110. https://doi.org/10.1016/j.autcon.2013.04.002

Choi, K., & Kwak, Y. H. (2012). Decision support model for incentives/disincentives time-cost tradeoff. *Automation in Construction*, *21*(1), 219–228. https://doi.org/10.1016/j.autcon.2011.06.006

Construction Management Association of America (CMAA), (1999). Standard construction management services and practice. *3rd ed. Construction Management Association of America.*

De, P., Dunne, E. J., Ghosh, J. B., & Wells, C. E. (1995). The discrete time-cost tradeoff problem revisited. *European Journal of Operational Research* (Vol. 81). 225-238

De, P., Dunne, E. J., Ghosh, J. B., & Wells, C. E. (1997). Complexity of the discrete time-cost tradeoff problem for project networks. *Operations Research*, *45*(2), 302–306. https://doi.org/10.1287/opre.45.2.302

Elbeltagi, E., Hegazy, T., & Grierson, D. (2007). A modified shuffled frog-leaping optimization algorithm: Applications to project management. *Structure and Infrastructure Engineering*, *3*(1), 53–60. https://doi.org/10.1080/15732470500254535

Eshtehardian, E., Afshar, A., & Abbasnia, R. (2008). Time-Cost Optimization: Using GA and fuzzy sets theory for uncertainties in cost. *Construction Management and Economics*, *26*(7), 679–691. https://doi.org/10.1080/01446190802036128

Faghihi, V., Reinschmidt, K. F., & Kang, J. H. (2014). Construction Scheduling using genetic algorithm based on building information model. *Expert Systems with Applications, 41*(16), 7565–7578. https://doi.org/10.1016/j.eswa.2014.05.047

Feng C., L. Liu, and S. A. Burns. (1997) "Using Genetic Algorithms to solve construction time-cost trade-off Problems," *J. Comput. Civil Eng.*, vol. 11, pp. 184–189, 1997.

García-Nieves, J. D., Ponz-Tienda, J. L., Ospina-Alvarado, A., & Bonilla-Palacios, M. (2019). Multipurpose linear Programming Optimization model for repetitive activities scheduling in construction projects. *Automation in Construction*, *105*. https://doi.org/10.1016/j.autcon.2019.03.020

García-Nieves, J. D., Ponz-Tienda, J. L., Salcedo-Bernal, A., & Pellicer, E. (2018). The Multimode Resource-Constrained project scheduling problem for repetitive activities in construction projects. *Computer-Aided Civil and Infrastructure Engineering*, *33*(8), 655–671. https://doi.org/10.1111/mice.12356

Gould, F. E. (2005). Managing the construction process: Estimating, scheduling and project control, Pearson Education, Upper Saddle River, NJ.

Hegazy, T. (1999). Optimization of Construction time-cost trade-off analysis using genetic algorithms. *Canadian Journal of Civil Engineering*, 26(6), 685–697. doi:10. 1139/l99-031.

Hegazy, T., & Menesi, W. (2012). Heuristic Method for satisfying both deadlines and resource constraints. *Journal of Construction Engineering and Management*, *138*(6), 688–696. https://doi.org/10.1061/(asce)co.1943-7862.0000483

He, Z., He, H., Liu, R., & Wang, N. (2017). Variable neighbourhood search and tabu search for a discrete time/cost trade-off problem to minimize the maximal cash flow gap. *Computers and Operations Research*, *78*, 564–577. https://doi.org/10.1016/j.cor.2016.07.013

Hochbaum, D. S. (2016). A polynomial time repeated cuts algorithm for the time cost tradeoff problem: The linear and convex crashing cost deadline problem. *Computers and Industrial Engineering*, *95*, 64–71. https://doi.org/10.1016/j.cie.2016.02.018

Karabulut, M. (2017). Application of Monte Carlo simulation and PERT/CPM techniques in planning of construction projects: A Case Study. *Periodicals of Engineering and Natural Sciences*, *5*(3), 408–420. https://doi.org/10.21533/pen.v5i3.152

Karadeniz, C. Ö. (2006). PERT-CPM ile Proje Planlama, Değerlendirme ve Bir İşletme Uygulaması, Yüksek Lisans Tezi, Marmara Üniversitesi Sosyal Bilimler Enstitüsü, İstanbul.

Kielmas, Maria. "History of the Critical Path Method." Houston Chronicle, 2019, smallbusiness.chron.com/history-critical-path-method-55917.html. Accessed 9 Sept. 2019.

Kim, J. Y., Kang, C. W., & Hwang, I. K. (2012). A practical approach to project scheduling: Considering the potential quality loss cost in the time-cost tradeoff problem. *International Journal of Project Management*, *30*(2), 264–272. https://doi.org/10.1016/j.ijproman.2011.05.004

Kır, E., (2007), *Yazılım Sektöründe Proje Yönetimi*, Yüksek Lisans Tezi, Kadir Has Üniversitesi, Sosyal Bilimler Enstitüsü.

Koo, C., Hong, T., & Kim, S. (2015). An Integrated Multi-Objective Optimization model for solving the construction time-cost trade-off problem. *Journal of Civil Engineering and Management*, *21*(3), 323–333. https://doi.org/10.3846/13923730.2013.802733

Li, C., & Wang, K. (2009). The risk element transmission theory research of Multi-Objective Risk-Time-Cost Trade-off. *Computers and Mathematics with Applications*, *57*(11–12), 1792–1799. https://doi.org/10.1016/j.camwa.2008.10.051

Li, H., Cao, J.-N., & Love, P. E. D. (1999.). Using Machine Learning and GA to solve time-cost trade-off problems. *Journal of Construction Engineering and Management.* Vol. 125, No. 5. ISSN 0733-9634/99/0005-0347– 0353

Liu, D., Li, H., Wang, H., Qi, C., & Rose, T. (2020). Discrete symbiotic organisms search method for solving large-scale time-cost trade-off problem in construction scheduling. *Expert Systems with Applications*, *148*. https://doi.org/10.1016/j.eswa.2020.113230

Lujić, R., Barković, D., & Jukić, J. (2019). Minimizing the pessimistic time of activity in overhaul project. *Tehnicki Vjesnik*, *26*(2), 391–397. https://doi.org/10.17559/TV-20180410114808

Mela, K., Tiainen, T., & Heinisuo, M. (2012). Comparative study of multiple criteria Decision-Making methods for building design. *Advanced Engineering Informatics*, *26*(4), 716–726. https://doi.org/10.1016/j.aei.2012.03.001

Nahmias S., Cheng Y. (2009), *Production and Operations Analysis* (6th ed.). Boston Burr Ridge, IL Dubuque, IA Madison, Wl New York San Francisco St. Louis Bangkok Bogota Caracas Kuala Lumpur Lisbon London Madrid Mexico City Milan Montreal New Delhi Santiago Seoul Singapore Sydney Taipei Toronto: *Me Graw Hill*.

Naticchia, B., Carbonari, A., Vaccarini, M., & Giorgi, R. (2019). Holonic execution system for real-time construction management. *Automation in Construction*, *104*, 179–196. https://doi.org/10.1016/j.autcon.2019.04.018

Özkan, Ş., (2004). Yöneylem Araştırması Kantitatif Karar Teknikleri, Ankara: *Nobel Yayın Dağıtım, Selçuk Üniversitesi Sosyal Bilimler Meslek, Vol 8* (1-2), pp, 189 – 212.

Pathak, B.K., Srivastava, S., and Srivastava, K., (2008). Neural network embedded with multi-objective genetic algorithm to solve nonlinear time cost trade-off problem of project scheduling. *Journal of scientific and industrial research, 67, 124-131*.

Poonambalam, S.G., Aravindan, P., and SubhaRao, M., (2003). Genetic algorithms for sequencing problems in mixed model assembly lines. *Computers and Industrial Engineering 45, 669-690.* doi:10.1016/j.cie.2003.09.001

Prascevic, N., & Prascevic, Z. (2017). Application of fuzzy AHP for ranking and selection of alternatives in construction project management. *Journal of Civil Engineering and Management*, *23*(8), 1123–1135. https://doi.org/10.3846/13923730.2017.1388278

Reda R.,(1990). RPM: repetitive project modeling, *Journal of Construction Engineering and Management. 116*(2), 316–330. https://doi.org/10.1061/(ASCE)0733-9364(1990)116:2(316).

Rençber, B. A. (2011). Proje Yönetiminde PERT Tekniği ve Bir Uygulama, *Gazi Üniversitesi Endüstriyel Sanatlar Eğitim Fakültesi Dergisi, Vol.27*, p31.

Sarıca, İ. (2006). *CPM ve PERT Teknikleri ile Proje Planlama ve İşletmelerde Uygulama*, Yüksek Lisans Tezi, Uludağ Üniversitesi Sosyal Bilimler Enstitüsü, Bursa, (pp.19-29).

Senouci A., Eldin N.(1996), Dynamic programing approach to scheduling nonserial linear projects, *Journal in Computing in Civil Engeneering.* 10,106–114. https:// doi.org/10.1061/(ASCE)0887-3801(1996)10:2(106).

Shahsavari Poura, N., Tavakkoli-Moghaddam, R., & Asadi, H. (2013). Optimizing a multi-objectives flow shop scheduling problem by a novel genetic algorithm. *International Journal of Industrial Engineering Computations*, *4*(3), 345–354. https://doi.org/10.5267/j.ijiec.2013.03.008

Shouman, M. A., Abu El-Nour, A. and Elmehalawi, E., (1991). Scheduling natural gas projects in CAIRO using CPM and time cost tradeoff. *Alexandria Engineering Journal*, *30*(2), 157-166.).

Skutella, M. (1998). *Approximation and randomization in scheduling*. Ph.D. Thesis, Berlin, Germany: Technische Universität Berlin, Fachbereich Mathematik

Sonmez, R., & Bettemir, Ö. H. (2012). A hybrid genetic algorithm for the discrete time-cost trade-off problem. *Expert Systems with Applications*, *39*(13), 11428–11434. https://doi.org/10.1016/j.eswa.2012.04.019

Tiwari, Sharma, (2020). Literature Review: project time-cost trade-off in construction projects. *International Journal of Engineering and Technical Research (IJETR)* ISSN: 2321-0869 (O) 2454-4698 (P), Volume-10, Issue-3.

Tiwari S., & Johari, S. (2015). Project Scheduling by Integration of time cost trade-off and constrained resource scheduling. *Journal of The Institution of Engineers (India): Series A*, *96*(1), 37–46. https://doi.org/10.1007/s40030-014-0099-2

Tran, D. H., Chou, J. S., & Luong, D. L. (2019). Multi-objective symbiotic organisms' optimization for making time-cost tradeoffs in repetitive project scheduling problem. *Journal of Civil Engineering and Management*, *25*(4), 322–339. https://doi.org/10.3846/jcem.2019.9681

van Slyke, R. M. (1963). Letter to the Editor—Monte Carlo methods and the PERT problem. *Operations Research*, *11*(5), 839–860. https://doi.org/10.1287/opre.11.5.839

Walker, A. (1989). *Project management in construction*. 2nd edition BSP Professional Books.

Wuliang, P., & Chengen, W. (2009). A multi-mode resource-constrained discrete time-cost tradeoff problem and its genetic algorithm-based solution. *International Journal of Project Management*, *27*(6), 600–609. https://doi.org/10.1016/j.ijproman.2008.10.009

Yang, I. T. (2011). Stochastic time-cost tradeoff analysis: A distribution-free approach with focus on correlation and stochastic dominance. *Automation in Construction*, *20*(7), 916–926. https://doi.org/10.1016/j.autcon.2011.03.008

Yıldız, A. (2015). Analysis of Oil Well Drilling Costs in Different Locations with PERT-CPM project planning techniques. In *Social Sciences Research Journal* (Vol. 4, Issue 2).

Zeinalzadeh, A., (2011). An application of mathematical model to time cost tradeoff problem. *Australian Journal of Basic and Applied Sciences, 5(7), 208-214.*

Zhang, L. H., Huang, Q., Fan, X. S., Wu, H. Y., Yang, J., & Feng, A. N. (2013). Clinicopathological significance of SIRT1 and p300/CBP expression in gastroesophageal junction (GEJ) cancer and the correlation with E-cadherin and MLH1. *Pathology Research and Practice*, *209*(10), 611–617. https://doi.org/10.1016/j.prp.2013.03.012

Zou, X., Zhang, L., & Zhang, Q. (2018). A Biobjective Optimization Model for deadline satisfaction in line-of-balance scheduling with work interruptions consideration. *Mathematical Problems in Engineering*, *2018*. https://doi.org/10.1155/2018/6534021