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ANTI-SWING CONTROL OF MOBILE BOOM CRANE

ÖMER DUMLUPINAR

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Thesis Supervisor

Assist. Prof. Dr. Sina KUSEYRİ

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ÖMER DUMLUPINAR

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DANIŞMAN

Yrd. Doç. Dr. Sina KUSEYRİ

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GRADUATE STUDIES IN PURE AND APPLIED
SCIENCES**

Ömer DURLUPINAR, a Master of Science of Philosophy student of Marmara University Institute for Graduate Studies in Pure and Applied Sciences, defended his thesis entitled "Anti-swing Control of Mobile Boom Crane", on Jan 28, 2016 and has been found to be satisfactory by the jury members.

Jury Members

Assist. Prof. Dr. Sina KUSEYRİ (Advisor)

Marmara University (SIGN) 

Assist. Prof. Dr. Uğur TÜMERDEM (Jury Member)

Marmara Üniversitesi (SIGN) 

Assist. Prof. Dr. Berke GÜR (Jury Member)

Bahçeşehir Üniversitesi (SIGN) 

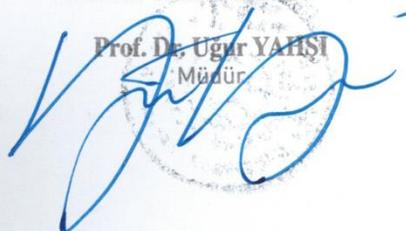
APPROVAL

Marmara University Institute for Graduate Studies in Pure and Applied Sciences Executive Committee approves that Ömer DURLUPINAR be granted the degree of Master of Science in Department of Mechanical Engineering 16.03.2016 (Resolution no: 2016/03-02).

Director of the Institute

Prof. Dr. Uğur YAHŞI
Müdür

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ÖZET

HAREKETLİ BOM VİNCİNİN SALINIMA KARŞI KONTROLÜ

Vinçlerin kullanımına, yüksek binaların inşası, değişik iş alanlarındaki fabrikalar ve tersaneler gibi çeşitli iş alanlarında her zaman ihtiyaç olmuştur. Üç tip vinç çeşidi vardır: Kızaklı Vinçler, Kule Vinçleri ve Bom Vinçleri. Bu tezde, hareketli bom vincinin fiziksel yapısı ve dinamik sistemi ile ilgili çalışmalar yapılmıştır.

İş sahasında güvenliği sağlamak ve işlem sürecini kısaltmak, vinç kontrolünü teşvik eden asıl unsurlardır. Bu tezin asıl amacı salınım açısını küçültecek bir kontrol sistemi oluşturmaktır. Bu amaca ulaşmak için hareketli bom vincinin dinamik denklemleri Lagrange metodu kullanarak elde edilmiştir ancak doğrusal olmayan hareket denklemleri bir çalışma noktasına göre doğrusallaştırılacak ve durum-uzay biçimine getirilecektir. Sonra, denetleyici doğrusal denklemler esas alınarak tasarlanacaktır.

Durum-uzay modelindeki değişkenler, denklemlerdeki değişkenlere göre belirlenerek durum-uzay modeli doğrusal denklemlere göre elde edilmiştir. Salınım açılarını sönümlemek ve modellenecek denetleyiciler ile karşılaştırmak için bir pasif sönümleyici modeli oluşturulmuştur. Lineer karesel denetleyici (LQR) ve gözlemci-temelli denetleyici, salınım açılarını küçültmek için tasarlandı. Hareketli mobil vincin dinamik sistemi çok-girişli çok-çıkışlı (MIMO) sistem olarak tanımlanır. İstenilen sistemin kutup konumlarına göre geri beslemeli denetleyici tasarlanmıştır. Kontrol yaklaşımındaki tüm adımlar bilgisayar temelli program kullanılarak gerçekleştirilmiştir.

Sistem çıkışı olarak iki farklı salınım açısı vardır. Sonuçlar salınım açılarının tasarlanan denetleyiciler sayesinde sönümlendiğini göstermiştir. Denetleyicilerin salınım açılarını düşürme zamanı, sadece pasif kontrol etkisi altında kalan sisteme göre çok daha iyi sonuçlar vermiştir.

ABSTRACT

ANTI-SWING CONTROL OF MOBILE BOOM CRANE

Cranes have always been required in the various work areas such as high-building construction, several kinds of factories, shipbuilding yard, etc. There are three basic types of cranes: Gantry Cranes, Tower Cranes and Boom Cranes. In this thesis, physical structure and dynamic system of a mobile boom crane are studied.

Increasing the safety and reducing the operation time are the main motivation factors behind the crane control. Here the main objective is to design a controller which reduces the swing angle. To achieve this, dynamic equations of a mobile boom crane are derived by using Lagrangian method. Naturally, the dynamics of the cranes are nonlinear, yet nonlinear equations of the motion will be linearized around an operating point and state-space form will be proposed. Then, controller is going to be designed based on the linear equations.

States are determined by the variables in the equations and state-space model for linear equations are constituted. Passive damping is modeled to damp the swing angles and compare the efficiency of controllers to be designed. Linear Quadratic Regulator (LQR) controller and observer-based feedback controller are designed to reduce the swing angle. The dynamic system of a mobile boom crane is described as a multi input-multi output (MIMO) system. Feedback controller is designed based on the desired system's poles location.

There are two different swing angles as outputs of the system. Results show that swing angles' oscillations are damped with help of the designed controllers. The damping times of swing angles in the effect of controllers are much better than the system which is in the effect of only passive control.

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1 INTRODUCTION

Gantry cranes, tower cranes, and boom cranes are main types of the cranes and they have been operated in many different workspaces such as heavy and dangerous material transferring, building constructions, etc [1, 2].

Firstly, gantry cranes, illustrated in Figure 1.1, are generally used in factories and shipyards. Their work spaces are limited in a horizontal path. Load can be moved forward and backward in this path, also up and down with help of the cable. Overall, a load in gantry cranes behave like a one-dimensional pendulum.

On the other hand, tower cranes given in Figure1.2 are mostly used in the construction sites of high buildings. A tower crane consists of 3 main parts: mast, forward jib and trolley. Mast and forward jib are fixed together perpendicularly and trolley could have a motion along the jib. The motion of trolley is similar to the motion on gantry crane but the rotational motion of mast on vertical axis, which is called slewing, makes difference.



Figure 1.1 Gantry Crane

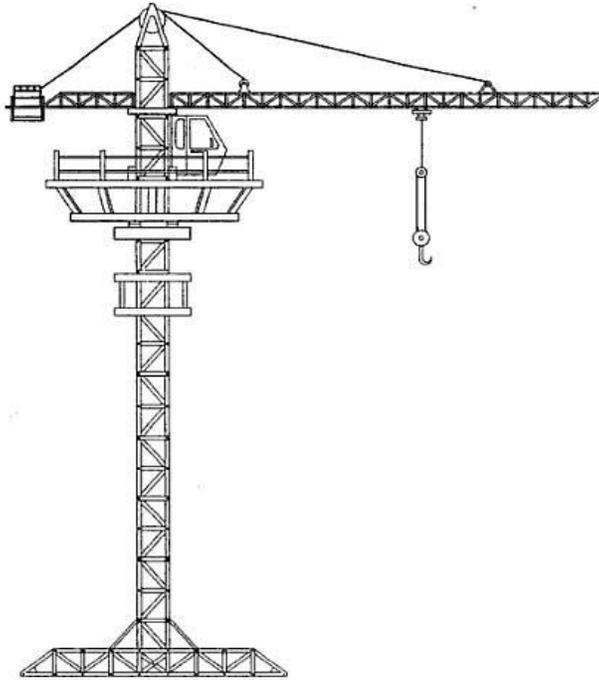


Figure 1.2 Tower Crane

Lastly, boom cranes are mainly used in the construction sites and shipyards as well. Boom cranes can be classified into two categories: stationary boom cranes and mobile boom cranes, see in Figure 1.3 and Figure 1.4, respectively. The difference between boom cranes and tower cranes is up and down motion of the boom, which is called luffing. Instead of the mast and the forward jib in tower cranes, they have only a boom. Dynamics of the boom and tower cranes are similar, yet it can be said former is more complicated.

As mentioned before, hazardous or very heavy materials are tend to be transported with cranes. This situation makes people more cautious about the safety of transportation procedure A swing in the cable that carries the load might create an undesired circumstance. All types of cranes are mostly operated by human beings. It is the operators responsibility to transform the load without swing. Swing motion is like pendulum oscillation. The operators should wait for a while to fade the swing away if the swing motion exceeds the limit. Prolongation of swing angles is not safe and also time consuming.



Figure 1.3 Stationary Boom Crane

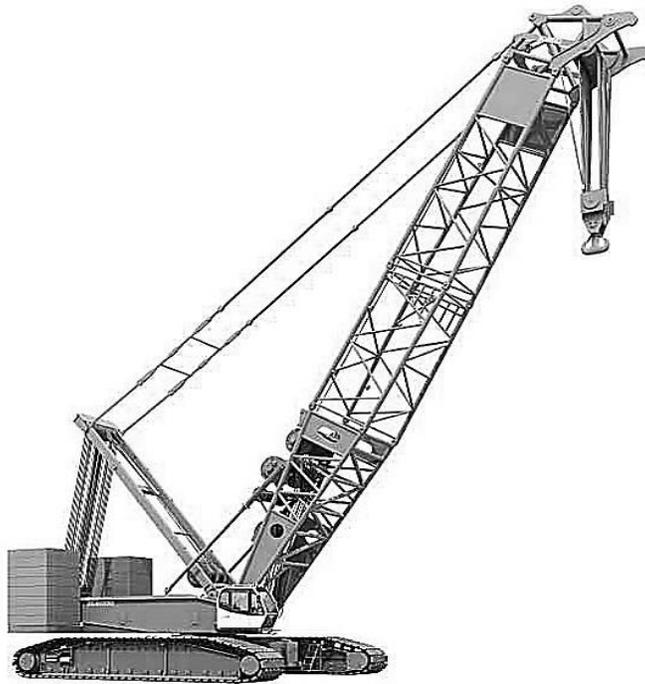


Figure 1.4 Mobile Boom Crane

In this thesis, controllers will be designed based on linearized equations of the mobile boom crane. Nonlinear equation of a mobile boom crane will be linearized, and controllers will be designed based on the linearized equations. The proposed controller will enable to operate the crane without any swing.

1.1 Literature Survey

Operating with minimum transportation time and swinging are desired crane utilization. The problem of transporting the load without swing has motivated many researchers to create control algorithms [1]. The operator has included to the control loop in respect of some researches, on the contrary, other researches claim that control loop has designed fully automated. As a first method, the operator included systems fed back the swing angles and its rate to be damped. [6]. This method increases the load transporting operation's time. A second way is adding a mechanical absorber to the structure of the crane [7]. Remarkable amount of power is required to be able to use this technique.

The fully automated systems can be built several techniques such as; generating trajectories, taking feedback of the swing angle or combining both these techniques in one. Generating trajectories transfer the load at the minimum time and swing. The acceleration of motion affects the swing of load, for this reason, many researchers have been focused on generating trajectories. The transfer time or the swing angle can be the objective function of this method [9, 10]. Input shaping is another method of generating trajectories. As input shaping method consists of acceleration pulses, it generated like that there is not remnant swing at the end of the transportation [11-13].

In the feedback control, a controller feeds back the position of trolley and swing angle of load. The gains of controller which changes the velocity of trolley are obtained by error signal [14-15]. This method means that, the system damping is firstly decreased to raise the speed, and then increased gradually. This reduces the transportation time. In another feedback controller, adaptive gains are calculated based on the pole-placement technique [16].

Defining the control algorithms with gain scheduling is better than previous methods because of the gains are fixed during the transfer operation. A linear feedback controller is designed by using full-state feedback at 1983 [18]. The gains are adjusted in keeping with to the length of cable. However, performance of the system will degrade when the cable length changes immoderately.

Another approach for control of cranes is designing two separate controllers. These feedback controllers are trajectory controller and swing controller mentioned before. Separation of these two control methods allows the researchers to design different trajectories considering different operating environment. Firstly, tracking controller is designed to get the load track a reference trajectory. Tracking feedback uses the position and the velocity of the trolley. Secondly, anti-swing controller controls the swing damping with the feedback of the swing angle and its rate. The position trajectory is generally based on the classical velocity pattern, which is obtained from open-loop optimal control or input shaping techniques. The tracking controller can be either a classical Proportional-Derivative (PD) controller [17, 19] or a Fuzzy Logic Controller (FLC) [20-22]. The anti-swing controller is also designed by different methods like delayed-position feedback or FLC [17, 19, 20, and 23]. In the design of controller, the effect of mass is generally omitted. When the mass of load is not counted in the design, and if very heavy load is used in the transfer operation, the controller design is failed [1].

1.2 Motivation and Objective

In crane control, many researchers focus on the control of the gantry cranes and tower cranes, however, there has been less effort to design controller for mobile boom cranes.

The main objective of this study is minimizing the swing angle of the load by designing practical controller. The swing, occurred while load transfer, must be reduced close to zero. The anti-swing controllers are designed to satisfy the work safety around the work environment and reducing the operation time.

1.3 Organization of Thesis

In Chapter 1, cranes are categorized and their characteristics are mentioned. Researches about crane control are examined and some of them are used in this thesis as references. Lastly, purpose of the thesis is indicated in Chapter 1, as well.

In Chapter 2, dynamic equations of mobile boom crane are obtained by using Lagrangian approach. Nonlinear dynamics of the system are linearized around the equilibrium point of the system. After linearization, system dynamics are shown in linear state-space form. In Chapter 3, a passive controller is designed as a friction force. LQR and observer-based feedback controller methods are designed as an active controller for the mobile boom crane.

Finally, in Chapter 4, the results obtained by both control methods are demonstrated and compared. In conclusion, the contribution of feedback control on the mobile boom crane model is examined and possible improvements that can contribute to future works are discussed.

2 MODELING

The dynamic equation of the motion of mobile boom crane is obtained by using Lagrangian approach. As shown in Figure 2.1, the mobile boom crane model is composed of three parts: Car, Boom and Load. The position vectors of these three parts are given as follow:

$$\begin{aligned}\bar{r}_L &= \{x + L \cos \alpha \sin \beta + l \cos \theta \sin \phi, y + L \cos \alpha \cos \beta - l \sin \theta, L \sin \alpha - l \cos \theta \cos \phi\} \\ \bar{r}_B &= \left\{x + \frac{1}{2} L \cos \alpha \sin \beta, y + \frac{1}{2} L \cos \alpha \cos \beta, \frac{1}{2} L \sin \alpha\right\} \\ \bar{r}_C &= \{x, y, 0\}\end{aligned}\tag{2.1}$$

where \bar{r}_L , \bar{r}_B and \bar{r}_C are the position vectors of load, boom, and car, respectively. First of all, it is assumed that all part of the crane is solid and cable length variations and rotational motion of boom about the x-axis are neglected.

The kinetic and potential energies of the system is given in equations below;

$$T = \frac{1}{2} M_1 \dot{\bar{r}}_L \cdot \dot{\bar{r}}_L + \frac{1}{2} M_2 \dot{\bar{r}}_B \cdot \dot{\bar{r}}_B + \frac{1}{2} M_3 \dot{\bar{r}}_C \cdot \dot{\bar{r}}_C + \frac{1}{2} J_o \dot{\beta}^2\tag{2.2}$$

$$V = M_1 g (L \sin \alpha - l \cos \theta \cos \phi) + \frac{1}{2} M_2 g L \sin \alpha - M_3 g z.\tag{2.3}$$

where, J_o is the moment of inertia of the boom about the z-axis. The velocities of these masses are calculated as follow;

$$\dot{\bar{r}} = \frac{d\bar{r}}{dt} + \bar{\omega} \times \bar{r}, \quad \bar{\omega} = \{0, 0, \dot{\beta}\}\tag{2.4}$$

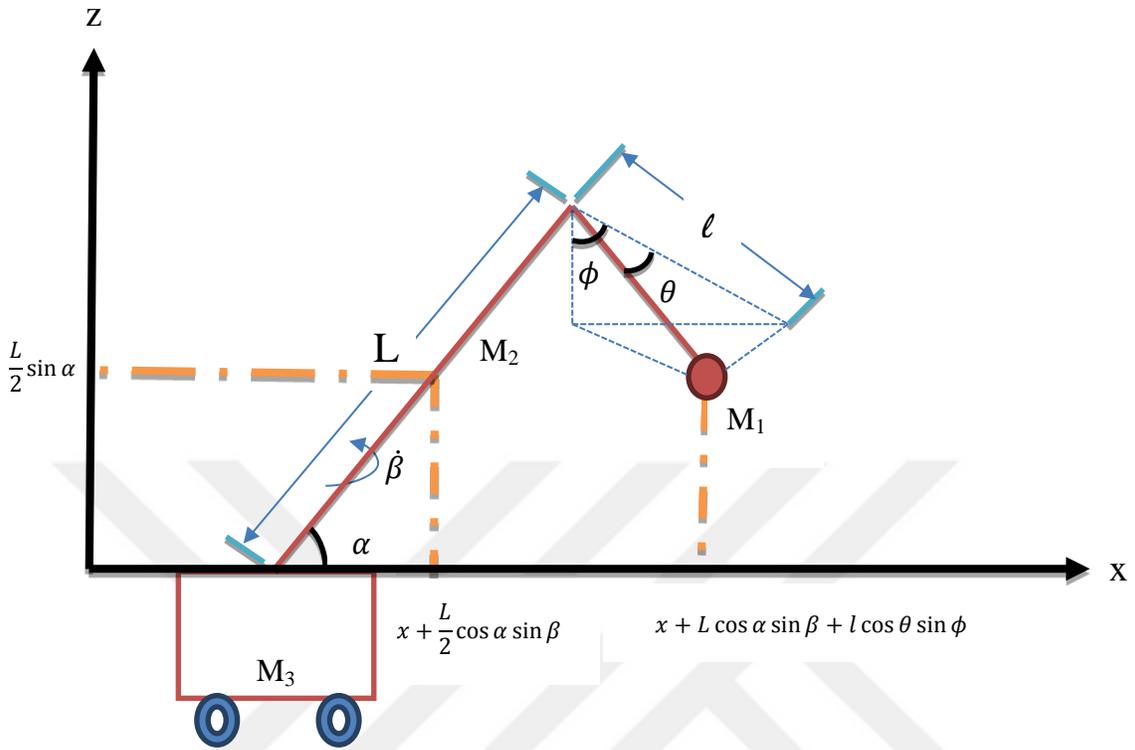


Figure 2.1 Mobile Boom Crane in 2-D (x-z axis)

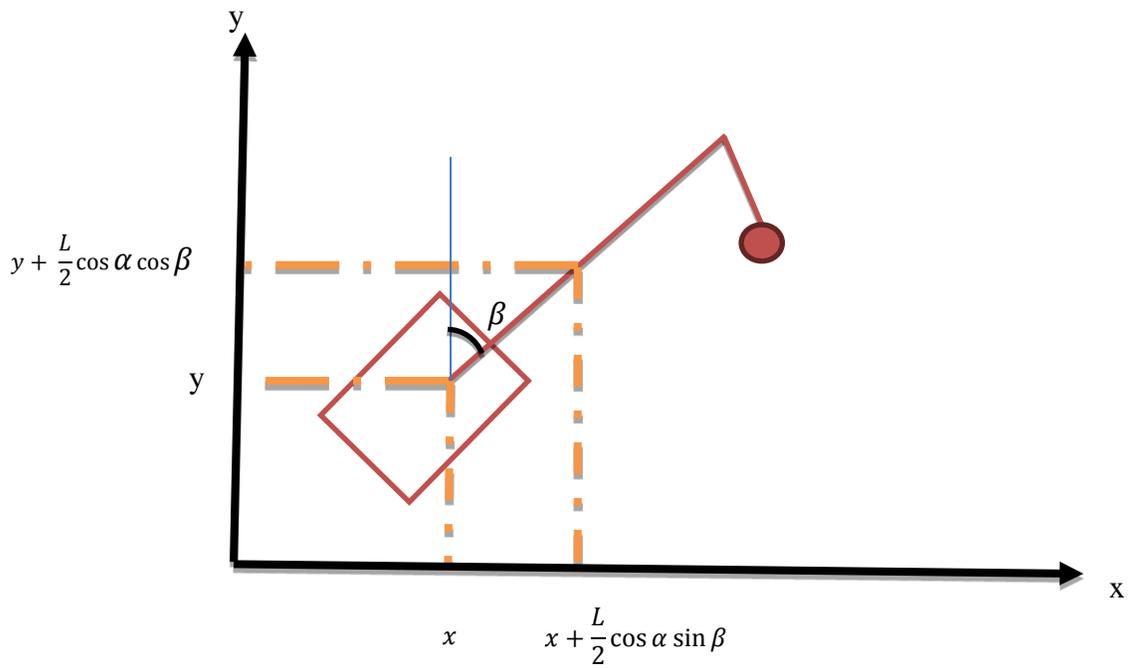


Figure 2.2 Mobile Boom Crane Model in 2-D(x-y axis)

In the following equations, it is shown how to obtain velocity of load.

$$\begin{aligned} \frac{d\vec{r}_L}{dt} = \{ \dot{x} + \dot{\beta}C \cos \beta - l\dot{\theta} \sin \theta \sin \phi + l\dot{\phi} \cos \theta \cos \phi, \dot{y} - C\dot{\beta} \sin \beta - l\dot{\theta} \cos \theta \\ , l\dot{\theta} \sin \theta \cos \phi + l\dot{\phi} \cos \theta \sin \theta \} \end{aligned} \quad (2.5)$$

$$\vec{\omega} \times \vec{r}_L = (-y\dot{\beta} - \dot{\beta}C \cos \beta + \dot{\beta}l \sin \theta)\vec{i} + (x\dot{\beta} + \dot{\beta}C \sin \beta + \dot{\beta}l \cos \theta \sin \phi)\vec{j} + 0\vec{k} \quad (2.6)$$

$$\begin{aligned} \vec{r}_L = \{ x - y\dot{\beta} + \dot{\beta}l \sin \theta - l\dot{\theta} \sin \theta \sin \phi + l\dot{\phi} \cos \theta \cos \phi, \\ \dot{y} - l\dot{\theta} \cos \theta + x\dot{\beta} + \dot{\beta}l \cos \theta \sin \phi, l\dot{\theta} \sin \theta \cos \phi + l\dot{\phi} \cos \theta \sin \phi \} \end{aligned} \quad (2.7)$$

where, $C = L \cos \alpha$.

As the same way of these calculations, velocities of the car and the boom can be derived similarly.

$$\begin{aligned} \vec{r}_L \cdot \vec{r}_L = \dot{x}^2 + y^2 \dot{\beta}^2 + \dot{\beta}^2 l^2 \sin^2 \theta + l^2 \dot{\theta}^2 + l^2 \dot{\phi}^2 \cos^2 \theta - 2xy\dot{\beta} + 2x\dot{\beta}l \sin \theta - 2lx\dot{\theta} \sin \theta \sin \phi \\ + 2lx\dot{\phi} \cos \theta \cos \phi - 2ly\dot{\beta}^2 \sin \theta + 2ly\dot{\beta}\dot{\theta} \sin \theta \sin \phi - 2ly\dot{\beta}\dot{\phi} \cos \theta \cos \phi \\ - 2l^2 \dot{\beta}\dot{\theta} \sin^2 \theta \sin \phi + 2l^2 \dot{\beta}\dot{\phi} \sin \theta \cos \theta \cos \phi + \dot{y}^2 + x^2 \dot{\beta}^2 + l^2 \dot{\beta}^2 \cos^2 \theta \sin^2 \phi \\ + 2xy\dot{\beta} - 2ly\dot{\theta} \cos \theta + 2ly\dot{\beta} \cos \theta \sin \phi - 2lx\dot{\beta}\dot{\theta} \cos \theta + 2lx\dot{\beta}^2 \cos \theta \sin \phi \\ - 2l^2 \dot{\beta}\dot{\theta} \cos^2 \theta \sin \phi \end{aligned} \quad (2.8)$$

$$\vec{r}_B \cdot \vec{r}_B = \dot{x}^2 + y^2 \dot{\beta}^2 - 2xy\dot{\beta} + \dot{y}^2 + x^2 \dot{\beta}^2 + 2xy\dot{\beta} \quad (2.9)$$

$$\vec{r}_C \cdot \vec{r}_C = \dot{x}^2 + y^2 \quad (2.10)$$

Lagrangian of the system is defined as:

$$\mathcal{L} = T - V \quad , \quad F_j = \frac{d}{dt} \left(\frac{\delta \mathcal{L}}{\delta \dot{q}_j} \right) - \frac{\delta \mathcal{L}}{\delta q_j} \quad , \quad j = 1, 2, 3, 4, 5 \quad (2.11)$$

where q is generalized coordinate. The generalized forces and generalized coordinates are defined as;

$$\bar{q} = \{x, y, \theta, \phi, \beta\} \quad , \quad \bar{F} = \{F_x, F_y, 0, 0, T_\beta\} \quad (2.12)$$

The Langrange equation (2.11) is applied for all generalized coordinates shown in Equation (2.12) and the equations of motion for mobile boom crane are obtained as;

$$\begin{aligned} F_x = & \ddot{x}(M_1 + M_2 + M_3) - 2\dot{y}\dot{\beta}(M_1 + M_2) - x\dot{\beta}^2(M_1 + M_2) - y\ddot{\beta}(M_1 + M_2) \\ & + M_1 l (\ddot{\beta} \sin \theta + 2\dot{\beta}\dot{\theta} \cos \theta - \ddot{\theta} \sin \theta \sin \phi - \dot{\theta}^2 \cos \theta \sin \phi - \dot{\theta}\dot{\phi} \sin \theta \cos \phi + \ddot{\phi} \cos \theta \cos \phi \\ & - \dot{\theta}\dot{\phi} \sin \theta \cos \phi - \dot{\phi}^2 \cos \theta \sin \phi - \dot{\beta}^2 \cos \theta \sin \phi) \end{aligned} \quad (2.13)$$

$$\begin{aligned} F_y = & \ddot{y}(M_1 + M_2 + M_3) + 2\dot{x}\dot{\beta}(M_1 + M_2) + x\ddot{\beta}(M_1 + M_2) - y\dot{\beta}^2(M_1 + M_2) \\ & + M_1 l (\dot{\theta}^2 \sin \theta - \ddot{\theta} \cos \theta + \ddot{\beta} \cos \theta \sin \phi - 2\dot{\beta}\dot{\theta} \sin \theta \sin \phi + 2\dot{\beta}\dot{\phi} \cos \theta \cos \phi + \dot{\beta}^2 \sin \theta) \end{aligned} \quad (2.14)$$

$$\begin{aligned} 0 = & M_1 (\ddot{\theta} l^2 - l\ddot{x} \sin \theta \sin \phi - l\dot{x}\dot{\theta} \cos \theta \sin \phi - l\dot{x}\dot{\phi} \sin \theta \cos \phi + l\dot{y}\dot{\beta} \sin \theta \sin \phi + l\dot{y}\dot{\beta} \sin \theta \sin \phi \\ & + l\dot{y}\dot{\beta}\dot{\theta} \cos \theta \sin \phi + l\dot{y}\dot{\beta}\dot{\phi} \sin \theta \cos \phi - l^2 \ddot{\beta} \sin \phi - l^2 \dot{\beta}\dot{\phi} \cos \phi - l\ddot{y} \cos \theta + l\dot{y}\dot{\theta} \sin \theta \\ & - l\dot{x}\dot{\beta} \cos \theta - l\dot{x}\ddot{\beta} \cos \theta + l\dot{x}\dot{y}\dot{\beta}\dot{\theta} \sin \theta - l^2 \dot{\beta}^2 \dot{\theta} \sin \theta \cos \theta + l^2 \dot{\phi}^2 \dot{\theta} \cos \theta \sin \theta - l\dot{x}\dot{\beta}\dot{\theta} \cos \theta \\ & - l\dot{x}\dot{\theta}^2 \cos \theta \sin \phi + l\dot{x}\dot{\theta}\dot{\phi} \sin \theta \cos \phi + l\dot{y}\dot{\beta}^2 \dot{\theta} \cos \theta - l\dot{y}\dot{\beta}\dot{\theta}^2 \cos \theta \sin \phi - l\dot{y}\dot{\beta}\dot{\theta}\dot{\phi} \sin \theta \cos \phi \\ & - l^2 \dot{\beta}\dot{\theta}\dot{\phi} \cos 2\theta \cos \phi + l^2 \dot{\beta}^2 \dot{\theta} \cos \theta \sin \theta \sin^2 \phi - l\dot{y}\dot{\theta}^2 \sin \theta + l\dot{y}\dot{\beta}\dot{\theta} \sin \theta \sin \phi - l\dot{x}\dot{\beta}\dot{\theta}^2 \sin \theta \\ & + l\dot{x}\dot{\beta}^2 \dot{\theta} \sin \theta \sin \phi + gl\dot{\theta} \sin \theta \cos \phi) \end{aligned}$$

(2.15)

$$\begin{aligned}
0 = & M_1(l^2\ddot{\phi}\cos^2\theta - 2l\dot{\phi}\dot{\theta}\cos\theta\sin\theta + l\ddot{x}\cos\theta\cos\phi - l\dot{x}\dot{\theta}\sin\theta\cos\phi - l\dot{x}\dot{\phi}\cos\theta\sin\theta \\
& - l\dot{y}\dot{\beta}\cos\theta\cos\phi - l\dot{y}\ddot{\beta}\cos\theta\cos\phi + l\dot{y}\dot{\beta}\dot{\theta}\sin\theta\cos\phi + l\dot{y}\dot{\beta}\dot{\phi}\cos\theta\sin\phi \\
& + l^2\ddot{\beta}\sin\theta\cos\theta\cos\phi + l^2\dot{\beta}\dot{\theta}\cos 2\theta\cos\phi - l^2\dot{\beta}\dot{\phi}\sin\theta\cos\theta\sin\phi + l\dot{x}\dot{\theta}\dot{\phi}\sin\theta\cos\phi \\
& + l\dot{x}\dot{\phi}^2\cos\theta\sin\phi - l\dot{y}\dot{\beta}\dot{\theta}\dot{\phi}\sin\theta\cos\phi - l\dot{y}\dot{\beta}\dot{\phi}^2\cos\theta\sin\phi + l^2\dot{\beta}\dot{\theta}\dot{\phi}\cos\phi \\
& + l^2\dot{\beta}\dot{\phi}^2\sin\theta\sin\theta\sin\phi - l^2\dot{\beta}^2\dot{\phi}\cos^2\theta\sin\phi\cos\phi - l\dot{y}\dot{\beta}\dot{\phi}\cos\theta\cos\phi \\
& - l\dot{x}\dot{\beta}^2\dot{\phi}\cos\theta\cos\phi + gl\dot{\phi}\cos\theta\sin\phi)
\end{aligned}$$

(2.16)

$$\begin{aligned}
T_\beta = & \ddot{\beta}[J_o + (x^2 + y^2)(M_1 + M_2) + M_1l(l\sin^2\theta + l\cos^2\theta\sin^2\phi - 2y\sin\theta + 2x\cos\theta\sin\phi)] \\
& + (M_1 + M_2)(2y\dot{y}\dot{\beta} - \ddot{x}y + 2x\dot{x}\dot{\beta} + x\ddot{y}) + M_1(2l^2\dot{\beta}\dot{\theta}\sin\theta\cos\theta + l\ddot{x}\sin\theta - 2\dot{y}\dot{\beta}\sin\theta \\
& - 2l\dot{y}\dot{\beta}\dot{\theta}\cos\theta + l\dot{y}\ddot{\theta}\sin\theta\sin\phi + l\dot{y}\dot{\theta}^2\cos\theta\sin\phi + 2l\dot{y}\dot{\theta}\dot{\phi}\sin\theta\cos\phi - l\dot{y}\ddot{\phi}\cos\theta\cos\phi \\
& + l\dot{y}\dot{\phi}^2\cos\theta\sin\phi - l^2\ddot{\theta}\sin\phi - l^2\dot{\theta}\dot{\phi}\cos\phi - l^2\dot{\phi}\dot{\theta}\sin\theta\cos\theta\cos\phi + l^2\dot{\theta}\dot{\phi}\cos 2\theta\cos\phi \\
& - l^2\dot{\phi}^2\sin\theta\cos\theta\sin\phi + 2l^2\dot{\beta}\dot{\theta}\sin\theta\cos\theta\sin^2\phi + 2l^2\dot{\beta}\dot{\phi}\cos^2\theta\sin\phi\cos\phi + l\dot{y}\cos\theta\sin\phi \\
& - l\dot{x}\ddot{\theta}\cos\theta + l\dot{x}\dot{\theta}^2\sin\theta + 2l\dot{x}\dot{\beta}\cos\theta\sin\phi - 2l\dot{x}\dot{\theta}\dot{\beta}\sin\theta\sin\phi + 2l\dot{x}\dot{\beta}\dot{\phi}\cos\theta\cos\phi)
\end{aligned}$$

(2.17)

Since the Equations (2.13)-(2.17) are nonlinear, they cannot be used for linear controller design. They can be linearized by some assumptions and reductions such as all swing angles assumed very small and the rates of change of the x , y and β are the same order of magnitude as the swing angles and their rates, and multiplications of rates of all states among each others are neglected.

$$F_x = \ddot{x}M_A + \ddot{\beta}(M_1l\theta - yM_D) - \ddot{\theta}M_1l\theta\phi + \ddot{\phi}M_1l \quad (2.18)$$

$$F_y = \ddot{y}M_A + \ddot{\beta}(xM_D + M_1l\phi) - \ddot{\theta}M_1l \quad (2.19)$$

$$0 = \ddot{\theta} + \frac{1}{l}(-\ddot{x}\theta\phi + \ddot{\beta}(y\theta\phi - l\phi - x) - \ddot{y} + g\dot{\theta}\theta) \quad (2.20)$$

$$0 = \ddot{\phi} + \frac{1}{l}(\ddot{x} + \ddot{\beta}(l\theta - y) + g\dot{\phi}\phi) \quad (2.21)$$

$$T_\beta = \ddot{\beta}[J_o + (x^2 + y^2)(M_1 + M_2) + M_1l(l\theta^2 + l\phi^2 - 2y\theta + 2x\phi)] \\ + (M_1 + M_2)(-\ddot{x}y + x\ddot{y}) + M_1(l\ddot{x}\theta + l_y\ddot{\theta}\theta\phi - l_y\ddot{\phi} - l^2\ddot{\theta}\phi - l^2\ddot{\phi}\theta + l\ddot{y}\phi - lx\ddot{\theta}) \quad (2.22)$$

where,

$$M_A = M_1 + M_2 + M_3$$

$$M_B = \frac{M_1}{M_1 + M_2 + M_3} \quad (2.23)$$

$$M_D = M_1 + M_2$$

The Equations (2.18)-(2.22) are still nonlinear to get a state-space representation for the linear dynamic system. The linearized equations can be obtained by the help of Jacobian method. Jacobian matrix is created in MATLAB that is shown in Appendix A.

$$\ddot{x} = \frac{\begin{pmatrix} F_x l - 2F_x M_B l + F_x M_B^2 l + M_D \ddot{\beta} l y - M_A M_B \ddot{\beta} l y - 2M_B M_D \ddot{\beta} l y + F_y M_B l \phi \theta + 2M_A M_B^2 \ddot{\beta} l y \\ -M_A M_B^3 \ddot{\beta} l y + M_B^2 M_D \ddot{\beta} l y + F_y M_B^2 l \phi \theta - 2M_A M_B^2 g l \phi \dot{\phi} + M_A M_B^3 g l \phi \dot{\phi} + M_A M_B \ddot{\beta} l^2 \phi^2 \theta \\ + M_A M_B g l \phi \dot{\phi} - 2M_A M_B^2 \ddot{\beta} l^2 \phi^2 \theta + M_A M_B^3 \ddot{\beta} l^2 \phi^2 \theta + M_A M_B \ddot{\beta} l \phi \theta x - M_B M_D \ddot{\beta} l \phi \theta x \\ M_A M_B^2 \ddot{\beta} l \phi^2 \theta^2 y - M_A M_B^2 \ddot{\beta} l \phi \theta x + M_B^2 M_D \ddot{\beta} l \phi \theta x - M_A M_B g l \phi \dot{\theta}^2 - M_A M_B \ddot{\beta} l \phi^2 \theta^2 y \\ + M_A M_B^2 g l \phi \dot{\theta}^2 \end{pmatrix}}{(M_A l (M_B - 1) (-M_B^2 + M_B \theta^2 \phi^2 + 2M_B - 1))} \quad (2.24)$$

$$\ddot{y} = \frac{\left(\begin{aligned} &F_y l - 2F_y M_B l + F_y M_B^2 l - M_D \ddot{\beta} l x - F_y M_B l^2 \phi^2 \theta^2 + 2M_A M_B \ddot{\beta} l x + 2M_B M_D \ddot{\beta} l x + F_x M_B l \phi \theta \\ &+ F_y M_B^2 l \phi^2 \theta^2 - 2M_A M_B^2 \ddot{\beta} l x + M_A M_B^3 \ddot{\beta} l x - M_B^2 M_D \ddot{\beta} l x - F_x M_B^2 l \phi \theta + 2M_A M_B^2 g l \dot{\theta} \theta \\ &- M_A M_B^3 g l \dot{\theta} \theta + M_A M_B^2 \ddot{\beta} l^2 \phi^3 \theta^2 - M_A M_B^3 \ddot{\beta} l^2 \phi^3 \theta^2 - M_A M_B g l \dot{\theta} \theta - M_A M_B \ddot{\beta} l \phi \theta y \\ &+ M_B M_D \ddot{\beta} l \phi \theta y - M_B^2 M_D \ddot{\beta} l \phi^2 \theta^2 x + M_A M_B^2 \ddot{\beta} l \phi \theta y - M_B^2 M_D \ddot{\beta} l \phi \theta y + M_A M_B^2 g l \phi^2 \dot{\phi} \theta \\ &- M_A M_B^3 g l \phi^2 \dot{\phi} \theta + M_B M_D \ddot{\beta} l \phi^2 \theta^2 x \end{aligned} \right)}{(M_A l (M_B - 1) (-M_B^2 + M_B \theta^2 \phi^2 + 2M_B - 1))} \quad (2.25)$$

$$\ddot{\theta} = \frac{\left(\begin{aligned} &x - F_y + M_D \ddot{\beta} - F_y M_B^2 + 2F_y M_B - M_A \ddot{\beta} x - F_x \phi \theta - M_A M_B^2 \ddot{\beta} x + M_B^2 M_D \ddot{\beta} x + 2M_A M_B \ddot{\beta} x \\ &- 2M_B M_D \ddot{\beta} x + F_x M_B \phi \theta - M_A \ddot{\beta} l \phi + M_A g \dot{\theta} \theta + 3M_A M_B \ddot{\beta} l \phi - 2M_A M_B g \dot{\theta} \theta + M_A \ddot{\beta} \phi \theta y \\ &- M_D \ddot{\beta} \phi \theta y - 3M_A M_B^2 \ddot{\beta} l \phi + M_A M_B^3 \ddot{\beta} l \phi + M_A M_B^2 g \dot{\theta} \theta - M_A M_B g \phi^2 \dot{\phi} \theta + M_A M_B^2 g \phi^2 \dot{\phi} \theta \\ &- M_A M_B \ddot{\beta} \phi \theta y + M_B M_D \ddot{\beta} \phi \theta y \end{aligned} \right)}{(M_A l (M_B - 1) (-M_B^2 + M_B \theta^2 \phi^2 + 2M_B - 1))} \quad (2.26)$$

$$\ddot{\phi} = \frac{\left(\begin{aligned} &-(F_x l - 2F_x M_B l + F_x M_B^2 l + M_D \ddot{\beta} l y - M_A M_B \ddot{\beta} l y - 2M_B M_D \ddot{\beta} l y + F_y M_B l \phi \theta + 2M_A M_B^2 \ddot{\beta} l y) \\ &- M_A M_B^3 \ddot{\beta} l y + M_B^2 M_D \ddot{\beta} l y + F_y M_B^2 l \phi \theta - 2M_A M_B^2 g l \phi \dot{\phi} + M_A M_B^3 g l \phi \dot{\phi} + M_A M_B \ddot{\beta} l^2 \phi^2 \theta \\ &+ M_A M_B g l \phi \dot{\phi} - 2M_A M_B^2 \ddot{\beta} l^2 \phi^2 \theta + M_A M_B^3 \ddot{\beta} l^2 \phi^2 \theta + M_A M_B \ddot{\beta} l \phi \theta x - M_B M_D \ddot{\beta} l \phi \theta x \\ &M_A M_B^2 \ddot{\beta} l \phi^2 \theta^2 y - M_A M_B^2 \ddot{\beta} l \phi \theta x + M_B^2 M_D \ddot{\beta} l \phi \theta x - M_A M_B g l \phi \dot{\theta} \theta^2 - M_A M_B \ddot{\beta} l \phi^2 \theta^2 y \\ &+ M_A M_B^2 g l \phi \dot{\theta} \theta^2 \end{aligned} \right) / l}{(M_A l (M_B - 1) (-M_B^2 + M_B \theta^2 \phi^2 + 2M_B - 1))} + \frac{y \ddot{\beta}}{l} - \ddot{\beta} \theta - \frac{g \phi \dot{\phi}}{l} \quad (2.27)$$

$$\ddot{\beta} = \frac{(T_\beta - (M_1 + M_2)(-\ddot{x}y + x\ddot{y}) - M_1(l\ddot{x}\theta + ly\ddot{\theta}\theta\phi - ly\ddot{\phi} - l^2\ddot{\theta}\phi - l^2\ddot{\phi}\theta + l\dot{y}\dot{\phi} - lx\ddot{\theta}))}{(J_o + (x^2 + y^2)(M_1 + M_2) + M_1l(l\theta^2 + l\phi^2 - 2y\theta + 2x\phi))} \quad (2.28)$$

Finally, linearized equations of accelerations have been obtained in (2.24)-(2.28). The values of the constants in these equations are given in Table 2.1.

A general state-space representation:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \quad (2.29)$$

State and input variables can be written as in Equations (2.30) and (2.31), respectively.

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \\ \theta \\ \dot{\theta} \\ \phi \\ \dot{\phi} \\ \beta \\ \dot{\beta} \end{bmatrix}, \quad \dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \\ \dot{x}_9 \\ \dot{x}_{10} \end{bmatrix} \quad (2.30)$$

$$u = \begin{bmatrix} F_x \\ F_y \\ T_\beta \end{bmatrix} \quad (2.31)$$

Table 2.1 Values of the Constants in A and B matrices

Constant	Description	Value
M_1	Mass of the Load	10000kg
M_2	Mass of the Boom	6000kg
M_3	Mass of the Car	60000kg
L	Length of the Boom	10m
l	Length of the Rope	3m
C	Constant	5
J_o	Moment of Inertia of Boom about z-axis	200000 kg.m ²
G	Gravitational Acceleration	9.81m/s ²

A and B matrices in numerical values are given as follow:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.088452693838637 & 0 & 1.506195177067986 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.575090188080921 & 0 & -0.019557682825702 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0.000006697391840 & 0 & 0 & 0 & -3.470624873029736 & 0 & 0.065209665329760 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.884762875153075 & 0 & -11.101008498253005 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0.973215568991712 & 0 & 0.215186678814981 & 0 & 0 & 0 \end{bmatrix}$$

(2.32)

$$B = \begin{bmatrix}
 0 & 0 & 0 \\
 0.000000117332517 & -0.016096254728309 & 0.000000838361622 \\
 0 & 0 & 0 \\
 -0.00000011733251 & 0.016096254728309 & -0.000000838361622 \\
 0 & 0 & 0 \\
 0.000001036843134 & -0.096032674986918 & 0.000008619583580 \\
 0 & 0 & 0 \\
 -0.000000575787669 & -0.046213815528985 & -0.000007395900504 \\
 0 & 0 & 0 \\
 0.000000707828724 & -0.138837847956841 & 0.000003963629755
 \end{bmatrix}$$

(2.33)

3 CONTROL APPROACH

3.1 Passive Control with Damping

Damping is modeled similar to the Coulomb friction on a spherical pendulum. Generally a frictional force goes against the direction of the motion, and has the opposite sign of the velocity[27]. General representation of the equation of the motion with Coulomb friction is:

$$\ddot{x} + b.\text{sgn}(\dot{x}) + f(x) = 0 \quad (3.1)$$

where, b is the coefficient of friction, $f(x)$ is called as 'restoring force', and $\text{sgn}(\dot{x})$ is the sign function which is;

$$\text{sgn}(\dot{x}) = \begin{cases} 1 & \text{for } \dot{x} > 0, \\ 0 & \text{for } \dot{x} = 0, \\ -1 & \text{for } \dot{x} < 0. \end{cases} \quad (3.2)$$

In this study, defined equations of motions are used for the swing angles. However, the coefficient of friction is needed to be included in the equations. The rope is chosen as ISO 4309:2010(steel wire rope) because, this type of ropes are used in almost all types of crane. In the sense of prolongation the life time of the rope and the boom, they are in use as lubricated. The coefficient of friction between the rope and the boom must be a combination of steel and steel with dynamic and lubricated, which is $\mu=0.15$.

Previously derived equations of motion and coefficient of friction are adequate to obtain new equations of motion for the swing angles.

$$\begin{aligned}
\ddot{\theta} = & \frac{1}{l} (\ddot{x} \sin \theta \sin \phi + \dot{x} \dot{\theta} \cos \theta \sin \phi + \dot{x} \dot{\phi} \sin \theta \cos \phi - \dot{y} \dot{\beta} \sin \theta \sin \phi - y \ddot{\beta} \sin \theta \sin \phi \\
& - y \dot{\beta} \dot{\theta} \cos \theta \sin \phi - y \dot{\beta} \dot{\phi} \sin \theta \cos \phi + \ddot{y} \cos \theta - \dot{y} \dot{\theta} \sin \theta + \dot{x} \dot{\beta} \cos \theta + x \ddot{\beta} \cos \theta \\
& - xy \dot{\beta} \dot{\theta} \sin \theta + \dot{x} \dot{\beta} \dot{\theta} \cos \theta + \dot{x} \dot{\theta}^2 \cos \theta \sin \phi - \dot{x} \dot{\phi} \dot{\theta} \sin \theta \cos \phi - y \dot{\beta}^2 \dot{\theta} \cos \theta \\
& + y \dot{\beta} \dot{\theta}^2 \cos \theta \sin \phi + y \dot{\beta} \dot{\theta} \dot{\phi} \sin \theta \cos \phi + \dot{y} \dot{\theta}^2 \sin \theta - \dot{y} \dot{\beta} \dot{\theta} \sin \theta \sin \phi \\
& + x \dot{\beta} \dot{\theta}^2 \sin \theta - x \dot{\beta}^2 \dot{\theta} \sin \theta \sin \phi - g \dot{\theta} \sin \theta \cos \phi) \\
& + \ddot{\beta} \sin \phi + \dot{\beta} \dot{\phi} \cos \phi + l \cos \theta \sin \phi g \mu \dot{\theta} + \dot{\beta}^2 \dot{\theta} \sin \theta \cos \theta - \dot{\phi}^2 \dot{\theta} \cos \theta \sin \theta + \dot{\beta} \dot{\theta} \dot{\phi} \cos 2\theta \cos \phi \\
& - \dot{\beta}^2 \dot{\theta} \cos \theta \sin \theta \sin^2 \phi
\end{aligned}
\tag{3.3}$$

$$\begin{aligned}
\ddot{\phi} = & (-2l \dot{\phi} \dot{\theta} \cos \theta \sin \theta + l \ddot{x} \cos \theta \cos \phi - l \dot{x} \dot{\theta} \sin \theta \cos \phi - l \dot{x} \dot{\phi} \cos \theta \sin \theta \\
& - l \dot{y} \dot{\beta} \cos \theta \cos \phi - l \dot{y} \ddot{\beta} \cos \theta \cos \phi + l \dot{y} \dot{\beta} \dot{\theta} \sin \theta \cos \phi + l \dot{y} \dot{\beta} \dot{\phi} \cos \theta \sin \phi \\
& + l^2 \ddot{\beta} \sin \theta \cos \theta \cos \phi + l^2 \dot{\beta} \dot{\theta} \cos 2\theta \cos \phi - l^2 \dot{\beta} \dot{\phi} \sin \theta \cos \theta \sin \phi + l \dot{x} \dot{\theta} \dot{\phi} \sin \theta \cos \phi \\
& + l \dot{x} \dot{\phi}^2 \cos \theta \sin \phi - l \dot{y} \dot{\beta} \dot{\theta} \dot{\phi} \sin \theta \cos \phi - l \dot{y} \dot{\beta} \dot{\phi}^2 \cos \theta \sin \phi + l^2 \dot{\beta} \dot{\theta} \dot{\phi} \cos \phi \\
& + l^2 \dot{\beta} \dot{\phi}^2 \sin \theta \sin \theta \sin \phi + l \cos \theta \sin \phi g \mu \dot{\theta} - l^2 \dot{\beta}^2 \dot{\phi} \cos^2 \theta \sin \phi \cos \phi - l \dot{y} \dot{\beta} \dot{\phi} \cos \theta \cos \phi \\
& - l \dot{x} \dot{\beta}^2 \dot{\phi} \cos \theta \cos \phi + g l \dot{\phi} \cos \theta \sin \phi) / l^2 \cos^2 \theta
\end{aligned}
\tag{3.4}$$

3.2 Feedback Controller Design

In this part, design of full-state feedback controller is examined by taking into account of the former obtained state-space form. The full-state feedback controller design depends on assigning a set of desired pole locations of the closed loop system.

Controllability must be discussed in the first place. Controllability defines whether it is possible to bring the states to the desired location with a designed input [24]. Controllability matrix is;

$$\mathcal{C} = [B \ AB \ A^2B \ \dots \ A^{n-1}B] \quad (3.5)$$

where A and B matrices are defined in (2.29) and n is the number of the state variables of the system. The system is controllable if and only if controllability matrix \mathcal{C} has full rank.

The control-law, K, of linear combination of state variables is shown as follow:

$$u = -Kx \quad (3.6)$$

where K is defined as:

$$K = [k_1 \ k_2 \ \dots \ k_n] \quad (3.7)$$

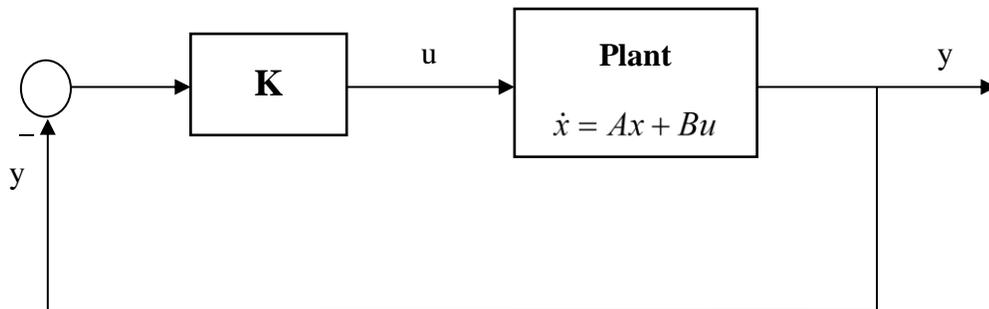


Figure 3.1 Feedback Control System Block Diagram

Linear Quadratic Regulator (LQR) control method can be used to obtain the control-law K . LQR is an effective design technique and it is essential of many control design approach [24]. The finite range, LQR is given by

$$J = \int_0^T (x' Q x + u' R u) dt \quad (3.8)$$

where Q and R are symmetric, positive (semi-) definite matrices. The controller output is

$$u = -R^{-1} B' P x \quad (3.9)$$

where, P is also symmetric and positive (semi-) definite matrix and its solution is given by

$$PA + A' P - Q + P B R^{-1} B' P = 0 \quad (3.10)$$

Equation (3.10) is called Ricatti equation. It is solvable if and only if the pair (A,B) is stabilizable and (Q,A) is detectable. In the design of LQR controller, all states of system must be fed back.

In many cases, all state variables cannot be measured. For these conditions, state variables could be reconstructed(estimated) by helping of an estimator. In the design of an estimator, concept of observability must be examined. A linear system is observable if system variables can be determined from the output data over finite time. Observability matrix is shown as:

$$\Theta = \begin{bmatrix} C \\ CA \\ CA^2 \\ \cdot \\ \cdot \\ \cdot \\ CA^{n-1} \end{bmatrix} \quad (3.11)$$

The system is observable if and only if rank of observability matrix Θ is equal to n .

Let \hat{x} be the estimate of the state, plant dynamics of the estimator would be,

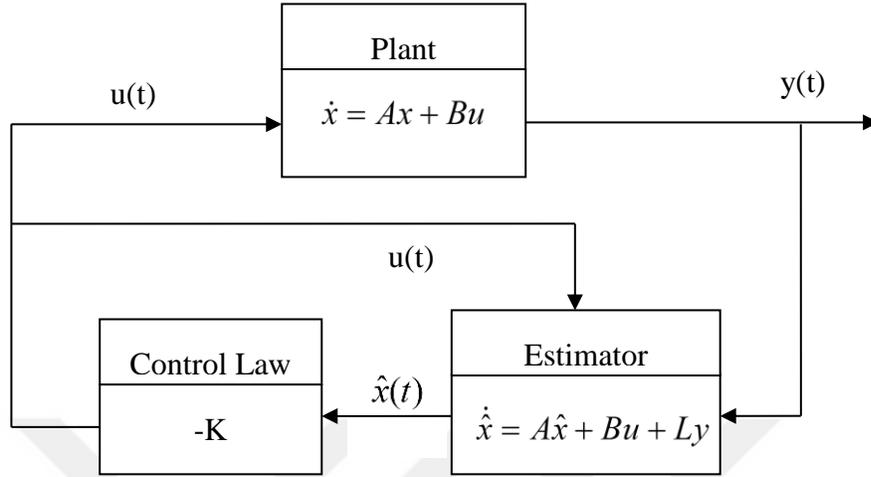


Figure 3.2 Observer-based Feedback Control System[25]

$$\dot{\hat{x}} = A\hat{x} + Bu \quad (3.12)$$

The estimator would be satisfactory if initial condition $x(0)$ can be obtained correctly and determine $\hat{x}(0)$ equal to $x(0)$ [25]. Observer-based controller feeds back the difference between the measured and the estimated output data and permanently corrects the model. Equation of observer, shown in Figure 3.2, is

$$\dot{\hat{x}} = A\hat{x} + Bu + Ly \quad (3.13)$$

where L is a proportional gain which defined as:

$$L = [l_1 \quad l_2 \quad \dots \quad l_n]^T \quad (3.14)$$

3.3 Observer-based Controller Design for the Mobile Boom Crane

An observer-based controller is also designed for the mobile boom crane. In general, measuring all state variables is difficult for two reasons: One of them is prohibitive cost of sensors and the other is that measuring all state variables physically may be impossible.

The previously derived dynamic equations of a mobile boom crane demonstrate that the plant of the system is a multi-input and multi output. State-space variables of mobile boom crane are defined in (2.29). The matrix A and matrix B is shown in (2.32) and (2.33) respectively. Output y can be defined as:

$$y = Cx + Du \quad (3.15)$$

Task of an estimator is to estimate the all states by using the information on few states. Independency among all ten state variables is examined in order to define C matrix. Since five position states are independent, they should be the inputs of the controller. Therefore, C and D matrices may be shown as;

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \text{ and } D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (3.16)$$

In the design of control-law for mobile boom crane, controllability and observability matrices were obtained by using MATLAB. They are given in detail in Appendix A. MATLAB code resulted controllability and observability matrices as full rank. In other words, the linearized system is controllable and observable. In the next step of controller design is arrangement of the suitable poles. There are two important factors to adjust the pole locations of closed-loop system such as: The settling time of the system should be acceptable and the input signals should have saturation constraints. The desired poles of the system can be seen in Appendix A. The linearized mobile boom crane system has 10 poles. All the poles of closed loop system are chosen on the real axis to damp the oscillatory response. Specific poles, which affect the swing angles directly, are chosen as dominant poles on the left half plane, hence they are the close to the imaginary axis.

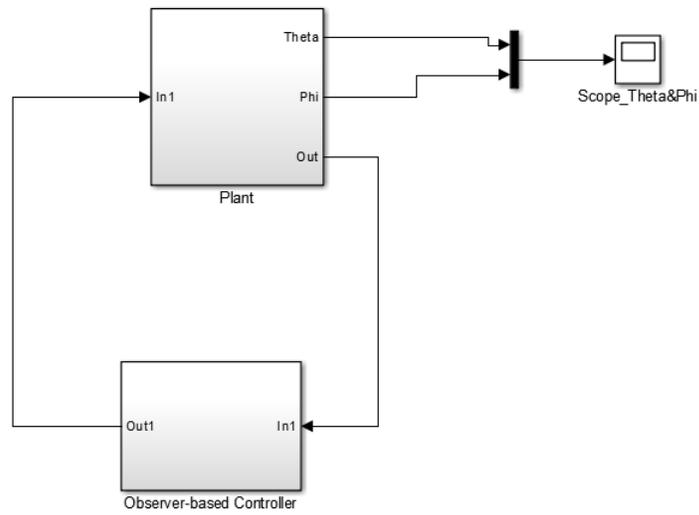


Figure 3.3 Observer-based Feedback Anti-swing Control Model in Simulink

When poles and matrices A and B are known, there are two way to obtain a control-law. The '*acker*' and '*place*' commands give the control-law. The '*acker*' command only works for the single input single output systems. Therefore, to obtain the control-law for mobile boom crane '*place*' command were used.

The selection procedure of the observer matrix L , defined in Equation (3.11), is similar to the selection of matrix K in controller design. Poles of the observer are generally chosen several times faster than the controller poles. After desired location of the estimator poles is specified, the L matrix is obtained in MATLAB. The observer-based controller is also simulated in Simulink shown in Figure 3.3 and the change of swing angles is given in Figure 3.4.

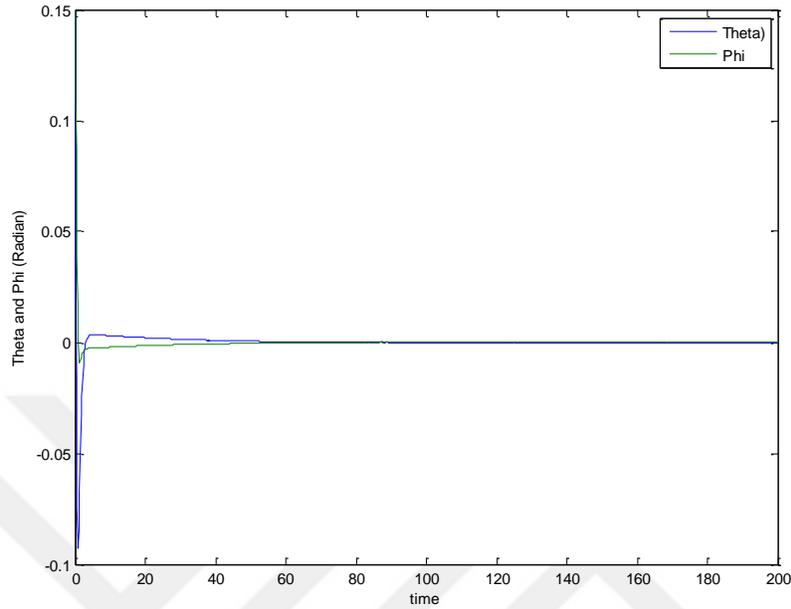


Figure 3.4 Time Response of Swing Angles to The Observer-based Feedback Controller

3.4 LQR Controller Design for the Mobile Boom Crane

In design of LQR controller, Q and R matrices are the controller design parameters. The Q matrix disciplines the state variables of x, the R matrix disciplines usage of control action u. A and B matrices are defined in (2.30) and constants of the system is given in Table3.1. LQR controller can be designed in computer software of MATLAB by the command of 'lqr'. However Q and R matrices should be chosen by the programmer. There are three basic way to decide Q and R matrices[28]. Firstly, Q matrix is chosen as the identity matrix,

$$Q = I, R = \rho I \quad (3.17)$$

where, ρ is the constant that should be varied to get a good response.

Second way of choosing matrices is getting them diagonal weights and chosen each elements of matrices to given equal effort for same error.

$$Q = \begin{bmatrix} q_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & q_n \end{bmatrix}, \quad R = \rho \begin{bmatrix} r_1 & \cdots \\ \vdots & \ddots & \vdots \\ \cdots & \cdots & r_n \end{bmatrix} \quad (3.18)$$

q_i 's and r_i 's could be chosen by the knowledge of desired error and ρ could be varied to adjust input-state balance. Last method of choosing weight matrices is trial and error technique [28]. Weight matrices are chosen for the linearized system of mobile boom crane by help of these three ways as follow:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 44444444 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 44444444 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.19)$$

$$R = 0.00001 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3.20)$$

Time response of closed loop system is shown in Figure 3.5.

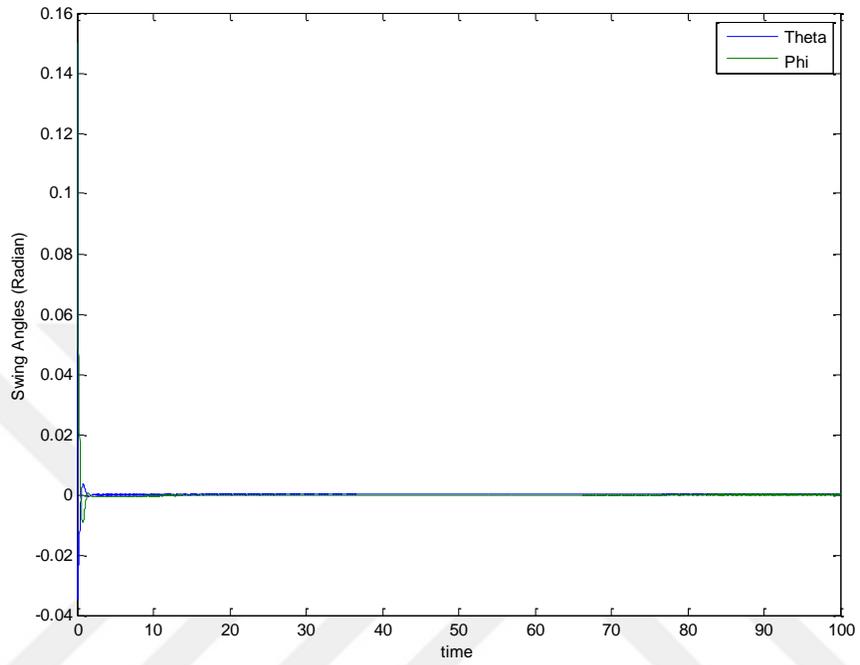


Figure 3.5 Time Response of Swing Angles to the LQR Controller

4 NUMERICAL RESULTS

In this chapter, the time responses of nonlinear system are examined. First of all, in the Figure 4.1, there is an oscillation motion of theta and phi angles which are output of the nonlinear system without any effect of active or passive control.

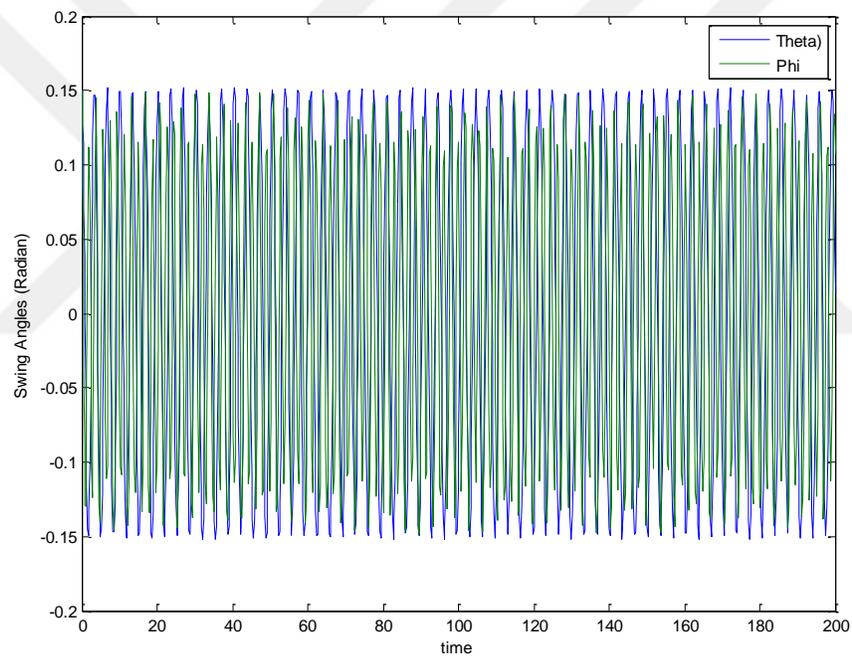


Figure 4.1 Time Responses of Swing Angles without Using Control

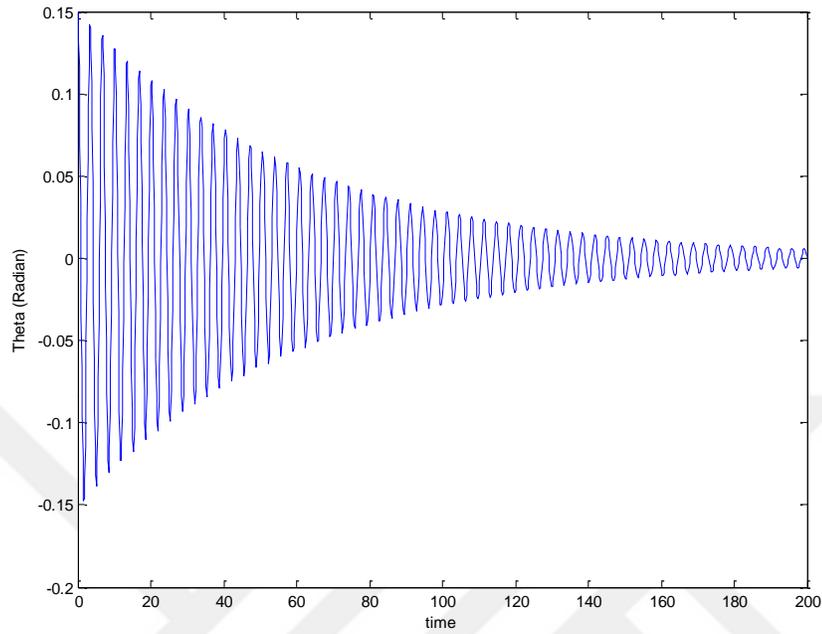


Figure 4.2 Time Response of Swing Angle Theta Using Passive Control

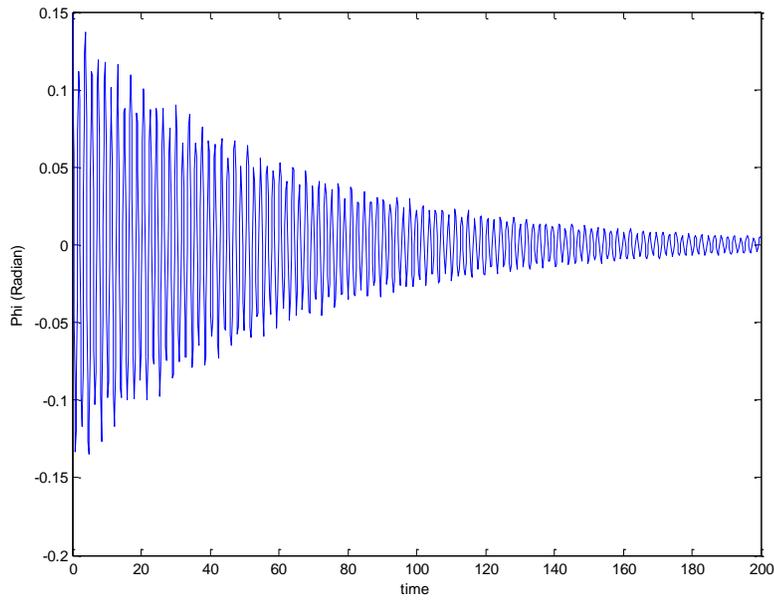


Figure 4.3 Time Response of Swing Angle Phi Using Passive Control

The effect of passive control on the swing angles, theta and phi, are shown in Figure 4.2 and Figure 4.3 respectively. The oscillation is declining steadily for both angles, but the oscillation is not over for a very long time. So requirement of a controller to reduce the swing angles in a shorter time is appeared by these figures.

Concordantly, the designed controllers are added to simulation of the system. The outputs of the system which controlled by observer-based controller, are shown in Figure 4.4 and Figure 4.5 respectively.

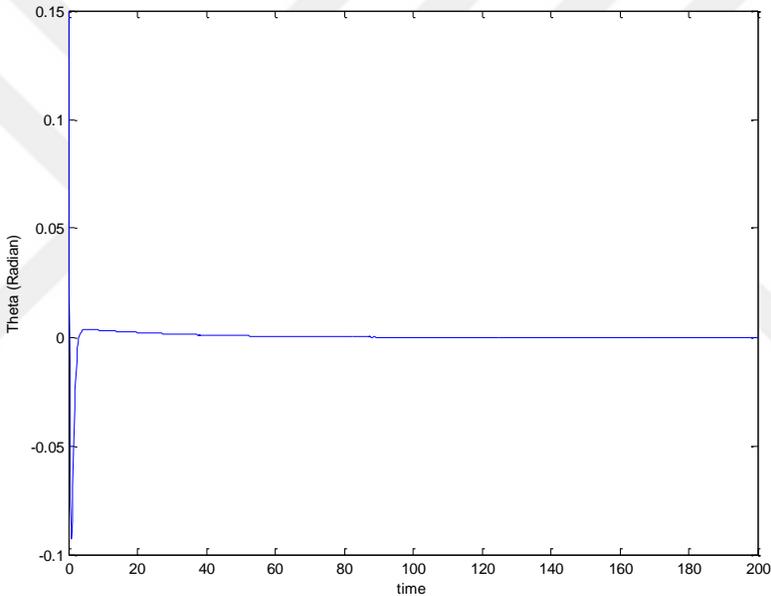


Figure 4.4 Time Response of Swing Angle Theta Controlled by Observer-based Feedback Controller

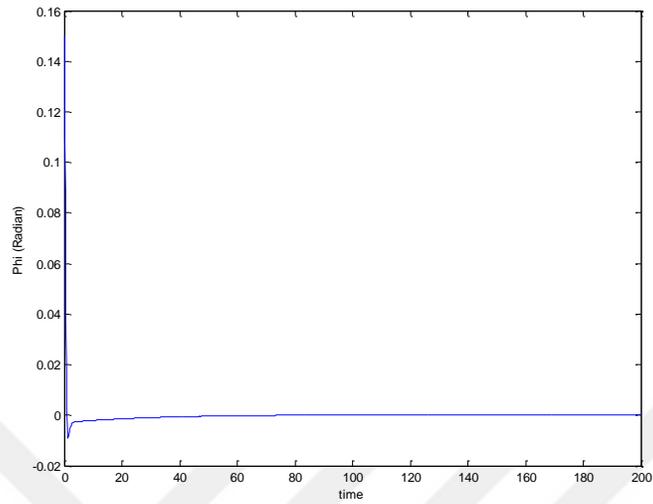


Figure 4.5 Time Response of Swing Angle Phi Controlled by Observer-based Feedback Controller

Controller outputs have very important effect to choose the poles of the closed loop system. Therefore these outputs should also be examined; see Figure 4.6.

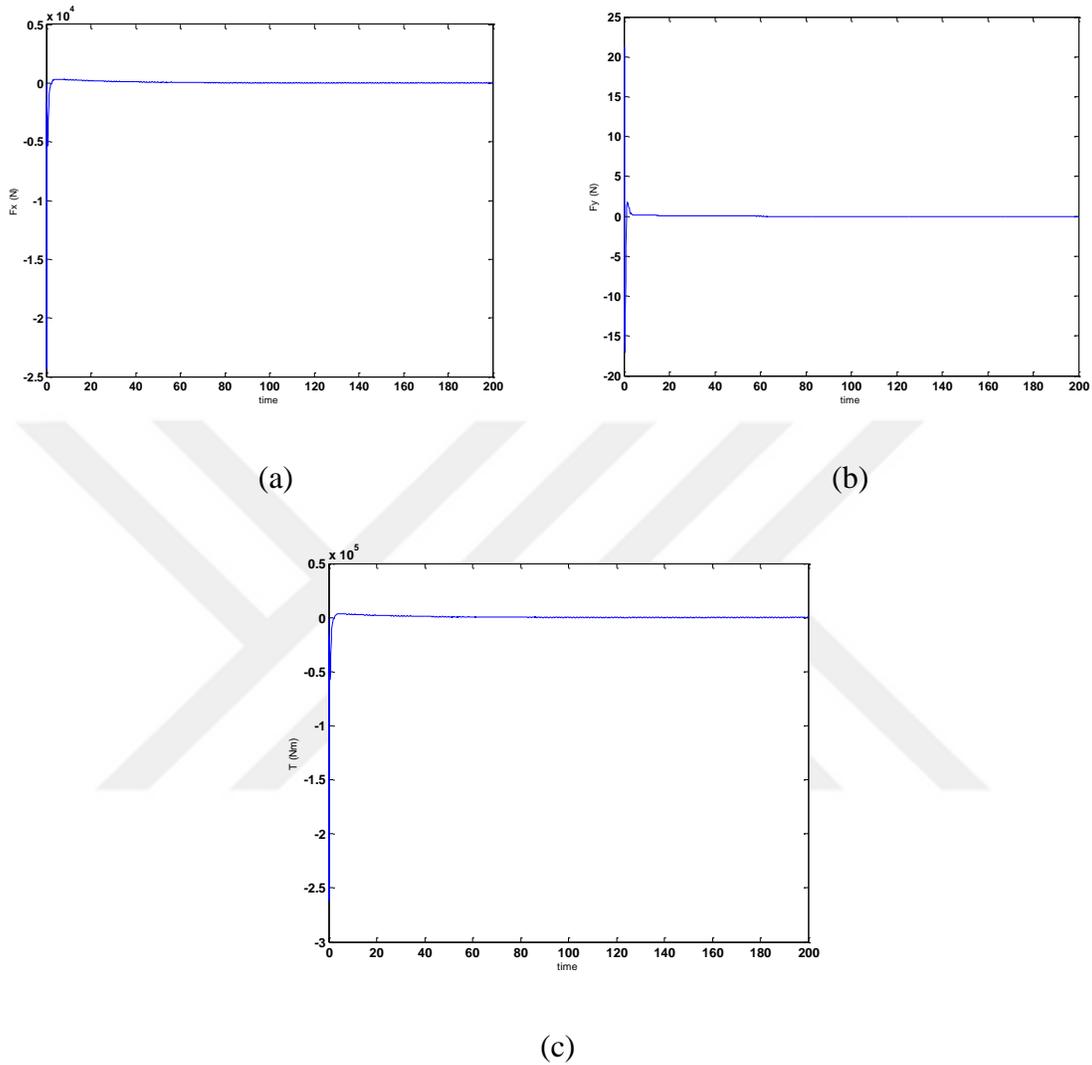


Figure 4.6 Observer-based Controller Outputs. (a) Force along the x-axis (b) Force along the y-axis (c) Torque about the z-axis

Despite of the time responses of swing angles in effect of observer-based controller is not seen satisfactory, comparison of the time responses to active and passive controls shows that the response to active controller is much better than the response of passive control.

The nonlinear model of mobile boom crane is also controlled by LQR controller. The weight matrices of LQR system are given in (3.19) and (3.20). The swing angles of the

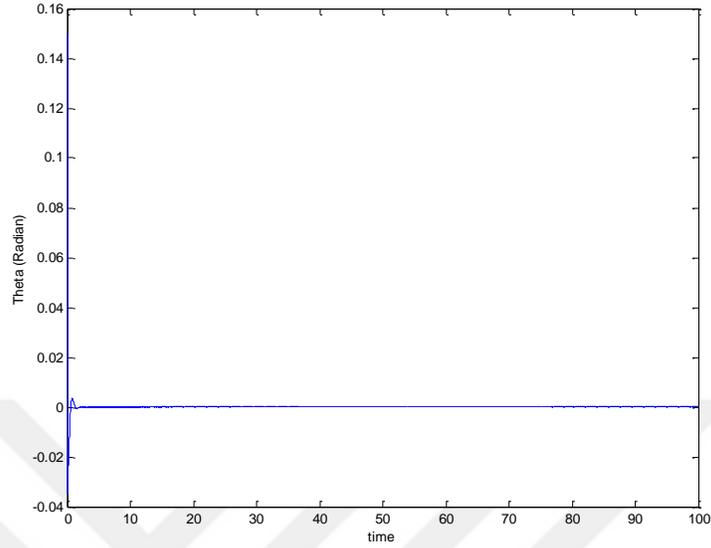


Figure 4.7 Time Response of Swing Angle Theta Controlled by LQR Controller

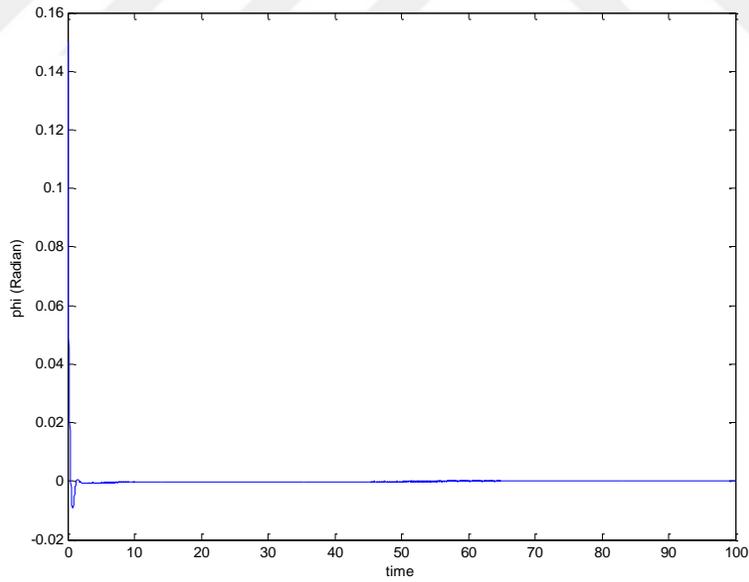
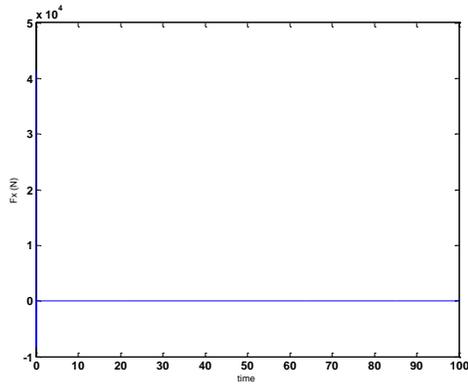
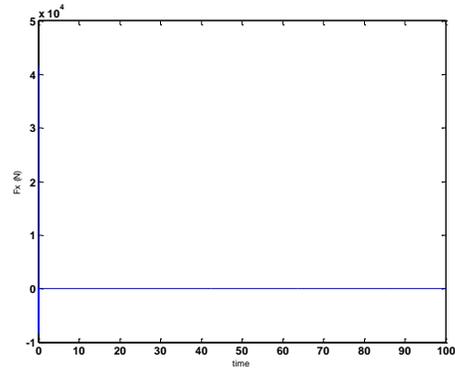


Figure 4.8 Time Response of Swing Angle Phi Controlled by LQR Controller

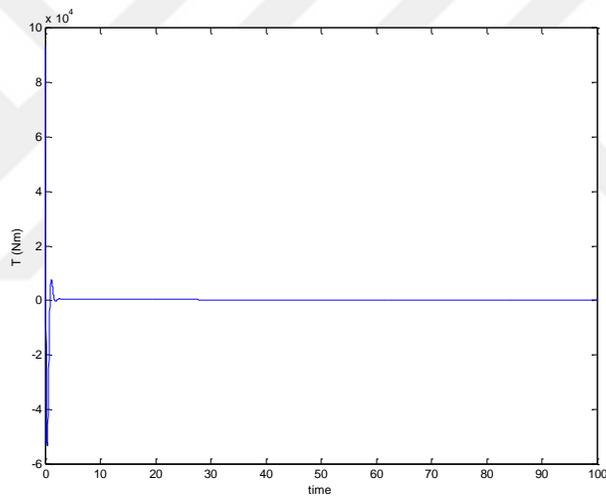
system, θ and ϕ are shown in Figures 4.7 and 4.8, respectively. The controller outputs of the system are shown in Figure 4.9.



(a)



(b)



(c)

Figure 4.9 LQR Controller Outputs. (a) Force along the x-axis (b) Force along the y-axis
(c) Torque about the z-axis

In Figures 4.10, 4.11 and 4.12 swing angles in effect of observer-based control, LQR control and passive control, respectively, are shown in the same graph.

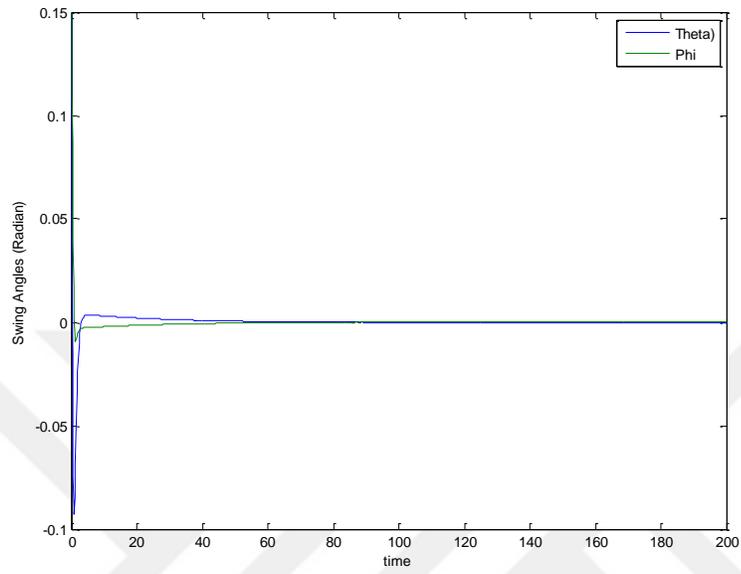


Figure 4.10 Time Responses of Swing Angles Using Observer-based Feedback Control

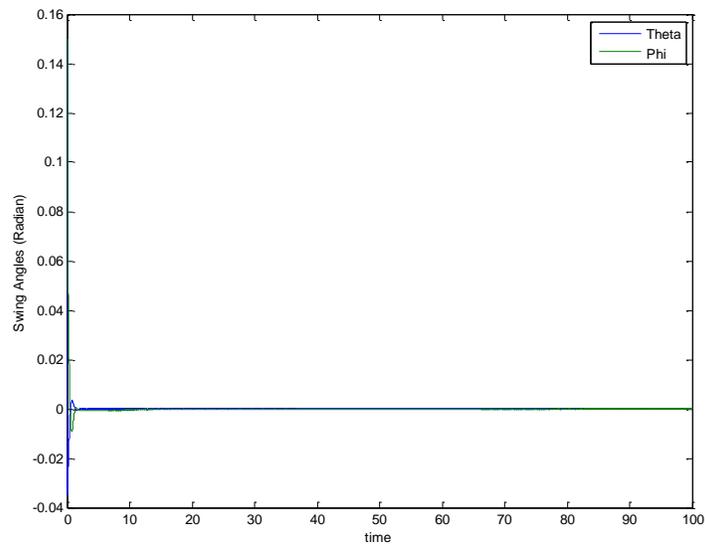


Figure 4.11 Time Responses of Swing Angles Using LQR Controller

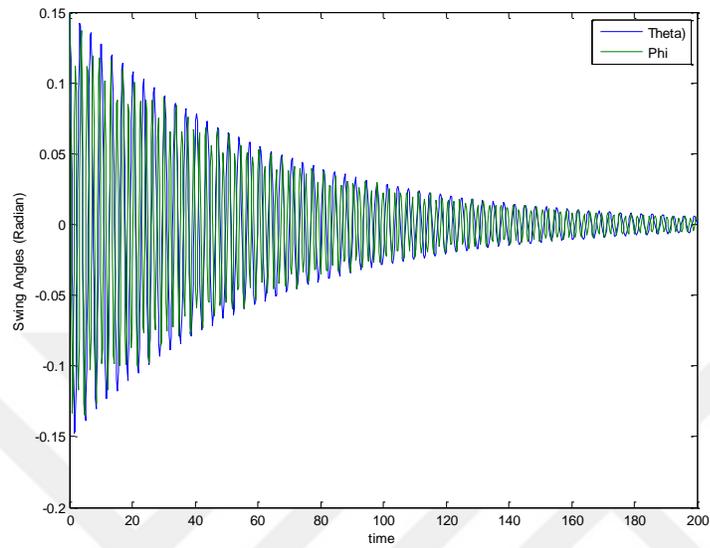


Figure 4.12 Time Responses of Swing Angles Using Passive Control

5 CONCLUSIONS AND FUTURE WORKS

5.1 Conclusions

The main objective of this thesis is designing a controller for mobile boom crane, which keeps the swing of the load small during the transfer operation. Nonlinear mathematical model of the boom crane is derived to achieve this objective. Mainly five variables are defined in this mathematical model where theta and phi are the swing angles. Lagrangian method is used for all variables to obtain the model. Nonlinear model is simulated in Simulink. Then, these nonlinear equations are linearized to be used in design of the linear controller. Observer-based controller and LQR controller are designed to achieve the aim. Damping of friction (passive control) is taken into account to compare the controller effect on the system. The coefficient of friction in the system is between the boom and the rope. Therefore, the friction force is damping the swing angles of the system.

In the design of the full-state feedback controller, the proper controller matrix has been found by using desired poles. All states of the system are fed back as controller input. F_x , F_y and T_β can be defined either controller output or system input. The similar procedure is performed in the design of the observer-based feedback controller. The proportional gains of the estimator are found by using desired poles locations. The state-space representation of the observer is constituted by the variables of the estimated states.

In conclusion, the system that controlled by LQR controller minimizes the swing angles in 3 seconds which is the fastest response among the controllers. However, to be able to constitute a closed loop system with LQR controller, all states must be available for control at all times and this situation is neither preferable nor feasible.

On the other hand, the system that controlled by observer-based controller is applicable for the real systems. The results of simulations show that the observer-based controller gets the swing angles close to zero approximately in 30 seconds. In spite of the fact that the time of

settling is relatively much for a system in effect of a controller, this time is almost 10 times less than the passive controller.

5.2 Future Works

In this study, five variables in the derivation of the equations of motion for the mobile crane were used. It is assumed that crane moves in x and y axes and has a rotational movement about the z axis. However, it is noted that cable length is assumed to be constant during the transfer operation. In further studies, it is an alternative choice to choose cable length as another variable of the system.

Additionally, a linear controller design approach can be implemented to minimize the swing of mobile boom crane is H_∞ robust control. Owing to the characteristics of H_∞ controller, the effects of disturbances, like wind, can be taken into consideration. Moreover the nonlinearities neglected in our study can be also taken into consideration as perturbation. By this way, it is possible to reach more realistic results.

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APPENDIX A

MATLAB COMMANDS

Values of constant in MATLAB editor are,

```
M1=10000           %mass of load
M2=6000            %mass of boom
M3=60000           %mass of car
L=10               %length of boom
l=3                %length of rope
alpha=pi/4;        %angle between boom and x-axis
C=L*cos(alpha)
g=9.81             %gravitational acceleration

MA=M1+M2+M3
MB=(M1+M2+M3)/M1
MD=M1+M2

Jo=200000          %moment of inertia of boom about
                   z-axis
```

Controllability and observability matrices are found by MATLAB by executing the commands below:

```
Co=ctrb(sys)
controllability=rank(Co)
Ob=obsv(sys)
observability=rank(Ob)
```

Then, in the command window of MATLAB, this result is observed:

```
co =
1.0e+02 *
Columns 1 through 6
0 0 0 0.000000001173325 -0.000160962547283 0.000000008383616
0.000000001173325 -0.000160962547283 0.000000008383616 0 0 0
0 0 0 -0.000000001173325 0.000160962547283 -0.000000008383616
-0.000000001173325 0.000160962547283 -0.000000008383616 0 0 0
0.000000010368431 -0.000960326749869 0.000000086195836 0 0 0
0.000000010368431 -0.000960326749869 0.000000086195836 0 0 0
-0.000000005757877 -0.000462138155290 -0.000000073959005 0 0 0
-0.000000005757877 -0.000462138155290 -0.000000073959005 0 0 0
0.000000007078287 -0.001388378479568 0.000000039636298 0 0 0
0.000000007078287 -0.001388378479568 0.000000039636298 0 0 0
Columns 7 through 12
0 0 0 -0.000000007755370 -0.000781013748628 -0.000000103772443
-0.000000007755370 -0.000781013748628 -0.000000103772443 0 0 0
0 0 0 -0.000000016218604 0.001521639592533 -0.000000134319748
-0.000000016218604 0.001521639592533 -0.000000134319748 0 0 0
0 0 0 -0.000000036360413 0.003302799107918 -0.000000303976310
-0.000000036360413 0.003302799107918 -0.000000303976310 0 0 0
0 0 0 0.000000073091841 0.004280538132939 0.000000897282419
0.000000073091841 0.004280538132939 0.000000897282419 0 0 0
0 0 0 0.000000008851700 -0.001034050919082 0.000000067972137
0.000000008851700 -0.001034050919082 0.000000067972137 0 0 0
```

Columns 13 through 18

0	0	0	0.000000106874402	0.006739467369391	0.000001324594928
0.000000106874402	0.006739467369391	0.000001324594928	0	0	0
0	0	0	0.000000055841422	-0.005285923875212	0.000000461241338
0.000000055841422	-0.005285923875212	0.000000461241338	0	0	0
0	0	0	0.000000130959699	-0.011183639044727	0.000001113499923
0.000000130959699	-0.011183639044727	0.000001113499923	0	0	0
0	0	0	-0.000000843563493	-0.044596096156076	-0.000010229686712
-0.000000843563493	-0.044596096156076	-0.000010229686712	0	0	0
0	0	0	-0.000000019658129	0.004135450297446	-0.000000102751254
-0.000000019658129	0.004135450297446	-0.000000102751254	0	0	0

Columns 19 through 24

0	0	0	-0.000001258987527	-0.068159647946766	-0.000015309412720
-0.000001258987527	-0.068159647946766	-0.000015309412720	0	0	0
0	0	0	-0.000000189775190	0.018487436430273	-0.000001553793835
-0.000000189775190	0.018487436430273	-0.000001553793835	0	0	0
0	0	0	-0.000000509521199	0.035906074197411	-0.000004531623846
-0.000000509521199	0.035906074197411	-0.000004531623846	0	0	0
0	0	0	0.000009480273790	0.485166773781616	0.000114545022514
0.000009480273790	0.485166773781616	0.000114545022514	0	0	0
0	0	0	-0.000000054071608	-0.020480577456251	-0.000001117616848
-0.000000054071608	-0.020480577456251	-0.000001117616848	0	0	0

Columns 25 through 30

0	0	0	0.000014234074137	0.733931843731436	0.000172126326131
0.000014234074137	0.733931843731436	0.000172126326131	0	0	0
0	0	0	0.000000617129653	-0.066044043040037	0.000004897481037
0.000000617129653	-0.066044043040037	0.000004897481037	0	0	0
0	0	0	0.000002386570859	-0.092978494763096	0.000023197111553
0.000002386570859	-0.092978494763096	0.000023197111553	0	0	0
0	0	0	-0.000105691405347	-5.354072117377354	-0.001275574680903
-0.000105691405347	-5.354072117377354	-0.001275574680903	0	0	0
0	0	0	0.000001544154668	0.139345777151737	0.000020238316089
0.000001544154668	0.139345777151737	0.000020238316089	0	0	0

controllability =

ob =

1.0e+04 *

Columns 1 through 6

0.0001000000000000	0	0	0	0	0
0	0	0.0001000000000000	0	0	0
0	0	0	0	0.0001000000000000	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0.0001000000000000	0	0	0	0
0	0	0	0.0001000000000000	0	0
0	0	0	0	0	0.0001000000000000
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0.00008845269384	0
0	0	0	0	-0.000157509018808	0
-0.000000000669739	0	0	0	-0.000347062487303	0
0	0	0	0	0.000088476287515	0
0	0	0	0	0.000097321556899	0
0	0	0	0	0	0.00008845269384
0	0	0	0	0	-0.000157509018808
0	-0.000000000669739	0	0	0	-0.000347062487303
0	0	0	0	0	0.000088476287515
0	0	0	0	0	0.000097321556899
-0.00000000059240	0	0	0	0.000102563945608	0
0.00000001054900	0	0	0	0.000544924327233	0
0.00000002324413	0	0	0	0.001210293150788	0
-0.00000000592560	0	0	0	-0.001289244023725	0
-0.00000000651801	0	0	0	-0.000318727697592	0
0	-0.00000000059240	0	0	0	0.000102563945608
0	0.00000001054900	0	0	0	0.000544924327233
0	0.00000002324413	0	0	0	0.001210293150788
0	-0.00000000592560	0	0	0	-0.001289244023725
0	-0.00000000651801	0	0	0	-0.000318727697592
-0.00000000686911	0	0	0	-0.001834799441077	0
-0.00000003649572	0	0	0	-0.001881106240806	0
-0.00000008105807	0	0	0	-0.004284545371007	0
0.00000008634572	0	0	0	0.015382731311565	0
0.00000002134644	0	0	0	0.000900447997743	0
0	-0.00000000686911	0	0	0	-0.001834799441077
0	-0.00000003649572	0	0	0	-0.001881106240806
0	-0.00000008105807	0	0	0	-0.004284545371007
0	0.00000008634572	0	0	0	0.015382731311565
0	0.00000002134644	0	0	0	0.000900447997743
0.000000012288371	0	0	0	0.022790416131672	0
0.000000012598506	0	0	0	0.006447694794276	0
0.000000028695279	0	0	0	0.015873164783297	0
-0.000000103024179	0	0	0	-0.174554637697200	0
-0.00000006030653	0	0	0	-0.000859627399077	0
0	0.000000012288371	0	0	0	0.022790416131672
0	0.000000012598506	0	0	0	0.006447694794276
0	0.000000028695279	0	0	0	0.015873164783297
0	-0.000000103024179	0	0	0	-0.174554637697200
0	-0.00000006030653	0	0	0	-0.000859627399077

Columns 7 through 10

0	0	0	0
0	0	0	0
0	0	0	0
0.0001000000000000		0	0
0	0	0.0001000000000000	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0.0001000000000000	0	0
0	0	0	0.0001000000000000
0.000150619517707	0	0	0
-0.000001955768283	0	0	0
0.000006520966533	0	0	0
-0.001110100849825	0	0	0
0.000021518667881	0	0	0
0	0.000150619517707	0	0
0	-0.000001955768283	0	0
0	0.000006520966533	0	0
0	-0.001110100849825	0	0
0	0.000021518667881	0	0
-0.001671451749010	0	0	0
0.000011439889923	0	0	0
-0.000095022142303	0	0	0
0.012329008476927	0	0	0
-0.000232532608869	0	0	0
0	-0.001671451749010	0	0
0	0.000011439889923	0	0
0	-0.000095022142303	0	0
0	0.012329008476927	0	0
0	-0.000232532608869	0	0
0.018561488141517	0	0	0
-0.000091458393355	0	0	0
0.001133767921560	0	0	0
-0.136948499941227	0	0	0
0.002560561358944	0	0	0
0	0.018561488141517	0	0
0	-0.000091458393355	0	0
0	0.001133767921560	0	0
0	-0.136948499941227	0	0
0	0.002560561358944	0	0
-0.206170885291322	0	0	0
0.000892608596496	0	0	0
-0.012865373310950	0	0	0
1.521269577436603	0	0	0
-0.028366092278160	0	0	0
0	-0.206170885291322	0	0
0	0.000892608596496	0	0
0	-0.012865373310950	0	0
0	1.521269577436603	0	0
0	-0.028366092278160	0	0

observability =

10

Commands to obtain Jacobian Matrix:

```
Syms M1 M2 M3 MA MB MD Fx Fy Tx xdot y ydot theta thetadot
    phi phidot beta betadot l g Jo
```

```
P1=MA*l*(1-MB)*(1-2*MB-theta^2*phi^2*MB+MB^2)
```

```
betadot=(T - M1*((1*theta*(Fx + Fy + Fx*MB^2 - 2*Fx*MB
    -Fy*MB - MA*MB*g*phi*phidot +
    MA*MB*g*thetadot*theta - MA*MB*phi*theta*y +
    MA*MB*g*phi^2*phidot*theta -
    MA*MB*g*phi^3*phidot*theta^2)))/(MA*(MB*phi^2*theta^
    2 - MB^2 + 2*MB - 1)) - (y*(Fx + Fy + Fx*MB^2 -
    2*Fx*MB - Fy*MB - MA*MB*g*phi*phidot +
    MA*MB*g*thetadot*theta - MA*MB*phi*theta*y +
    MA*MB*g*phi^2*phidot*theta -
    MA*MB*g*phi^3*phidot*theta^2))/(MA*(MB*phi^2*theta^
    2 - MB^2 + 2*MB - 1)) + (1*x*(Fy - Fy*MB +
    Fx*phi*theta + MA*g*thetadot*theta -
    MA*MB*g*thetadot*theta - MA*MB*phi*theta*y +
    MA*MB*g*phi^2*phidot*theta))/(MA*(MB*phi^2*theta^2
    - MB^2 + 2*MB - 1)) + (l^2*phi*(Fy - Fy*MB +
    Fx*phi*theta + MA*g*thetadot*theta -
    MA*MB*g*thetadot*theta - MA*MB*phi*theta*y +
    MA*MB*g*phi^2*phidot*theta))/(MA*(MB*phi^2*theta^2
    - MB^2 + 2*MB - 1)) - (l*phi*(Fy + Fy*MB^2 -
    2*Fy*MB - Fy*MA*MB^2*l - Fy*MB*phi^2*theta^2 +
    Fy*MA*MB*l + Fx*MB*l*phi*theta +
    MA*MB^2*g*l*thetadot*theta - MA*MB^2*l*phi*theta*y
    - MA*MB*g*l*thetadot*theta +
    MA*MB^2*g*l*phi^2*phidot*theta))/(MA*(MB*phi^2*thet
    a^2 - MB^2 + 2*MB - 1)) + (1*theta*(Fx + Fy +
    Fx*MB^2 - 2*Fx*MB - Fy*MB - MA*g*phidot*theta +
    MA*MB*g*phi*phidot + MA*MB*g*thetadot*theta -
    MA*MB*phi*theta*y - 2*MA*MB^2*g*phi*phidot +
    MA*MB^3*g*phi*phidot + MA*MB*g*phi^2*phidot*theta -
    MA*MB^2*g*phi^3*phidot*theta^2))/(MA*(MB -
    1)*(MB*phi^2*theta^2 - MB^2 + 2*MB - 1)) -
    (1*phi*theta*y*(Fy - Fy*MB + Fx*phi*theta +
    MA*g*thetadot*theta - MA*MB*g*thetadot*theta -
    MA*MB*phi*theta*y +
    MA*MB*g*phi^2*phidot*theta))/(MA*(MB*phi^2*theta^2
    - MB^2 + 2*MB - 1))) + MD*((x*(Fy + Fy*MB^2 -
    2*Fy*MB - Fy*MA*MB^2*l - Fy*MB*phi^2*theta^2 +
```

$$\begin{aligned}
& Fy*MA*MB*1 + Fx*MB*1*phi*theta + \\
& MA*MB^2*g*1*thetadot*theta - MA*MB^2*1*phi*theta*y \\
& - MA*MB*g*1*thetadot*theta + \\
& MA*MB^2*g*1*phi^2*phidot*theta)) / (MA*(MB*phi^2*theta \\
& a^2 - MB^2 + 2*MB - 1)) + (y*(Fx + Fy + Fx*MB^2 - \\
& 2*Fx*MB - Fy*MB - MA*g*phidot*theta + \\
& MA*MB*g*phi*phidot + MA*MB*g*thetadot*theta - \\
& MA*MB*phi*theta*y - 2*MA*MB^2*g*phi*phidot + \\
& MA*MB^3*g*phi*phidot + MA*MB*g*phi^2*phidot*theta - \\
& MA*MB^2*g*phi^3*phidot*theta^2)) / (MA*(MB - \\
& 1)*(MB*phi^2*theta^2 - MB^2 + 2*MB - \\
& 1))) / (((M1*((1*x*(MA*x - MD*x - MA*MB*x + MB*MD*x \\
& + MA*1*phi - MA*phi*theta*y + MD*phi*theta*y + \\
& MA*MB^2*1*phi - 2*MA*MB*1*phi + \\
& MA*MB*phi*theta*y)) / (MA*(MB*phi^2*theta^2 - MB^2 + \\
& 2*MB - 1)) - (1*phi*(2*MB*MD*x - MD*x - MB^2*MD*x + \\
& MA*MB*1^2*phi + 2*MA*MB^2*1*phi - MA*MB^3*1*phi - \\
& MA*MB^2*1*x + MB^2*MD*1*x - 2*MA*MB^2*1^2*phi + \\
& MA*MB^3*1^2*phi - MA*MB*1*phi + MA*MB*1*x - \\
& MB*MD*1*x + MB*MD*phi^2*theta^2*x + \\
& MA*MB^2*1*phi^3*theta^2 + MB^2*1*phi*theta*y - \\
& MA*MB*1*phi*theta*y + \\
& MB*MD*1*phi*theta*y)) / (MA*(MB*phi^2*theta^2 - MB^2 \\
& + 2*MB - 1)) - (y*(MD*y - MD*x - MA*MB*x + MB*MD*x \\
& + 2*MA*MB*y - 2*MB*MD*y + MA*1*phi + MA*1*theta - \\
& MA*MB^2*y + MB^2*MD*y - MA*phi*theta*y + \\
& MA*MB^2*1*phi + 3*MA*MB^2*1*theta - MA*MB^3*1*theta \\
& - 2*MA*MB*1*phi - 3*MA*MB*1*theta + \\
& MA*MB*phi^2*theta^2*y + \\
& MA*MB*phi*theta*y)) / (MA*(MB*phi^2*theta^2 - MB^2 + \\
& 2*MB - 1)) + (1^2*phi*(MA*x - MD*x - MA*MB*x + \\
& MB*MD*x + MA*1*phi - MA*phi*theta*y + \\
& MD*phi*theta*y + MA*MB^2*1*phi - 2*MA*MB*1*phi + \\
& MA*MB*phi*theta*y)) / (MA*(MB*phi^2*theta^2 - MB^2 + \\
& 2*MB - 1)) + (1*theta*(MD*y - MD*x - MA*MB*x + \\
& MB*MD*x + 2*MA*MB*y - 2*MB*MD*y + MA*1*phi + \\
& MA*1*theta - MA*MB^2*y + MB^2*MD*y - MA*phi*theta*y \\
& + MA*MB^2*1*phi + 3*MA*MB^2*1*theta - \\
& MA*MB^3*1*theta - 2*MA*MB*1*phi - 3*MA*MB*1*theta + \\
& MA*MB*phi^2*theta^2*y + \\
& MA*MB*phi*theta*y)) / (MA*(MB*phi^2*theta^2 - MB^2 + \\
& 2*MB - 1)) + (1*theta*(MA*x - MD*x + MD*y - MA*MB*x \\
& + MB*MD*x - MA*MB*y - 2*MB*MD*y + MA*1*phi + \\
& 2*MA*MB^2*y - MA*MB^3*y + MB^2*MD*y - \\
& MA*phi*theta*y + MA*MB^2*1*phi - 2*MA*MB*1*phi +
\end{aligned}$$

$$\begin{aligned}
& MA*MB*phi*theta*y + \\
& MA*MB^2*phi^2*theta^2*y) / (MA*(MB - \\
& 1)*(MB*phi^2*theta^2 - MB^2 + 2*MB - 1)) - \\
& (1*phi*theta*y*(MA*x - MD*x - MA*MB*x + MB*MD*x + \\
& MA*1*phi - MA*phi*theta*y + MD*phi*theta*y + \\
& MA*MB^2*1*phi - 2*MA*MB*1*phi + \\
& MA*MB*phi*theta*y) / (MA*(MB*phi^2*theta^2 - MB^2 + \\
& 2*MB - 1))) - MD*((x*(2*MB*MD*x - MD*x - MB^2*MD*x \\
& + MA*MB*1^2*phi + 2*MA*MB^2*1*phi - MA*MB^3*1*phi - \\
& MA*MB^2*1*x + MB^2*MD*1*x - 2*MA*MB^2*1^2*phi + \\
& MA*MB^3*1^2*phi - MA*MB*1*phi + MA*MB*1*x - \\
& MB*MD*1*x + MB*MD*phi^2*theta^2*x + \\
& MA*MB^2*1*phi^3*theta^2 + MB^2*1*phi*theta*y - \\
& MA*MB*1*phi*theta*y + \\
& MB*MD*1*phi*theta*y) / (MA*(MB*phi^2*theta^2 - MB^2 \\
& + 2*MB - 1)) + (y*(MA*x - MD*x + MD*y - MA*MB*x + \\
& MB*MD*x - MA*MB*y - 2*MB*MD*y + MA*1*phi + \\
& 2*MA*MB^2*y - MA*MB^3*y + MB^2*MD*y - \\
& MA*phi*theta*y + MA*MB^2*1*phi - 2*MA*MB*1*phi + \\
& MA*MB*phi*theta*y + \\
& MA*MB^2*phi^2*theta^2*y) / (MA*(MB - \\
& 1)*(MB*phi^2*theta^2 - MB^2 + 2*MB - 1))) / (Jo + \\
& MD*(x^2 + y^2) + M1*1*(1*phi^2 + 2*x*phi + \\
& 1*theta^2 - 2*y*theta)) + 1)*(Jo + MD*(x^2 + y^2) + \\
& M1*1*(1*phi^2 + 2*x*phi + 1*theta^2 - 2*y*theta))
\end{aligned}$$

pretty(betaddot)

$$x\dot{\dot{}} = (F_x*1 - 2*F_x*MB*1 + F_x*MB^2*1 + MD*y*betaddot*1 -$$

$$\begin{aligned}
& 2*MB*MD*y*betaddot*1 + MB^2*MD*y*betaddot*1 - \\
& MB*1*theta^2*phi^2*MA*y*betaddot + MB*theta*phi^2*MA* \\
& 1^2*betaddot - \\
& 2*MA*MB^2*1^2*betaddot*phi^2*theta + F_y*MB*theta*phi* \\
& 1 + F_y*1*MB^2*theta*phi - \\
& x*betaddot*MD*MB*theta*phi*1 + MB^2*x*betaddot*MD*the \\
& ta*phi*1 + MA*MB^3*1^2*betaddot*phi^2*theta + MA*MB*x*1 \\
& *betaddot*theta*phi - x*betaddot*1*MA*MB^2*theta*phi - \\
& g*thetadot*theta^2*MA*MB*phi*1 + g*thetadot*theta^2*M \\
& A*MB^2*phi*1 - \\
& MA*MB*y*betaddot*1 + 2*MA*MB^2*y*betaddot*1 + MA*MB^2*y \\
& *betaddot*theta^2*phi^2*1 - \\
& MA*MB^3*y*betaddot*1 + MA*MB*g*phidot*phi*1 -
\end{aligned}$$

$$2*MA*MB^2*g*phidot*phi^1+MA*MB^3*g*phidot*phi^1) / (P1)$$

pretty(xddot)

$$yddot=(Fy^1-Fy^1*MB-Fy*MB^1+Fy*MB^2^1-$$

$$Fy*theta^2*phi^2*MB^1+Fy*theta^2*phi^2*MB^2^1-x*betaddot*MD^1+2*x*betaddot*MD^1*MB-MB^2*MD*x*betaddot^1+x*betaddot*MB*MD*theta^2*phi^2^1-x*betaddot*MB^2*MD*theta^2*phi^2^1+Fx*MB*theta*phi^1-Fx*MB^2*theta*phi^1+MB*MD*y*betaddot*theta*phi^1-MB^2*MD*y*betaddot*theta*phi^1+MA*MB^2*g*phidot*theta*phi^2^1-MA*MB^3*g*phidot*theta*phi^2^1-MA*MB*y*betaddot*theta*phi^1+MA*MB^2*y*betaddot*theta*phi^1+MA*MB*x*betaddot^1-2*MA*MB^2*x*betaddot^1+x*betaddot*MA*MB^3^1-g*thetadot*theta*MA*MB^1+2*g*thetadot*theta*MA*MB^2^1-g*thetadot*theta*MA*MB^3^1+theta^2*phi^3*MA*MB^2^1^2*betaddot-theta^2*phi^3*MA*MB^3^1^2*betaddot) / (P1)$$

pretty(yddot)

$$thetaddot=(Fx*theta*phi+MD*y*betaddot*theta*phi+MA*MB*g*phido$$

$$t*theta*phi^2-MA*y*betaddot*theta*phi+MA^1*betaddot*phi-2*MA*MB^1*betaddot*phi+Fy-Fy*MB-x-betaddot*MD+MB*x*MD*betaddot+MA*MB^2^1*betaddot*phi+MA*x*betaddot-x*betaddot*MA*MB-g*theta*thetadot*MA+g*thetadot*theta*MA*MB-Fx*theta*phi*MB-MD*MB*y*betaddot*theta*phi-MA*MB^2*g*phidot*theta*phi^2+MA*MB*y*betaddot*theta*phi-MA*MB^1*betaddot*phi+2*MA*MB^2^1*betaddot*phi-Fy*MB+Fy*MB^2+x*betaddot*MD*MB-MB^2*MD*x*betaddot-MA*MB^3^1*betaddot*phi-MA*x*betaddot*MB+MB^2*MA*betaddot*x+g*thetadot*theta*MA*MB-g*thetadot*theta*MA*MB^2) / (P1)$$

pretty(thetaddot)

```

phiddot=(-Fx*1+2*Fx*MB*1-Fx*1*MB^2-
MD*y*betaddot*1+2*MB*MD*y*betaddot*1-
MB^2*MD*y*betaddot*1-
MA*MB*theta*phi^2*1^2*betaddot-
g*thetadot*theta^2*MA*MB^2*phi*1+2*MA*MB^2*1^2*beta
ddot*phi^2*theta-Fy*MB*theta*phi*1-
Fy*MB^2*theta*phi*1+x*betaddot*theta*phi*MB*MD*1-
x*betaddot*theta*phi*MB^2*MD*1-
MA*MB^3*1^2*betaddot*phi^2*theta-
MA*MB*x*betaddot*theta*phi*1+x*betaddot*MA*MB^2*the
ta*phi*1+g*thetadot*theta^2*MA*MB*phi*1+y*betaddot*
MA*1-2*y*betaddot*MA*MB*1+y*betaddot*MA*MB^2*1-
1^2*betaddot*theta*MA+3*1^2*betaddot*MA*MB*theta+1^
2*betaddot*theta^3*phi^2*MA*MB-
3*1^2*betaddot*theta*MA*MB^2-
MA*MB^2*theta^3*phi^2*1^2*betaddot+MA*MB^3*1^2*beta
ddot*theta-
g*phidot*phi*MA*1+2*g*phidot*phi*MA*MB*1+g*phidot*p
hi^3*theta^2*MA*MB*1-g*phidot*phi*MA*MB^2*1-
MA*MB^2*theta^2*phidot*phi^3*g*1)/(P1)

```

```
pretty(phiddot)
```

```

jcb=jacobian([xdot,xddot,ydot,yddot,thetadot,thetaddot,phidot
,phiddot,betadot,betaddot],[x,xdot,y,ydot,theta,the
tadot,phi,phidot,beta,betadot])

```

Closed loop system is obtained by state-space matrices and desired poles as follow:

```
sys = ss(A,B,CM,D)
tf(sys)
```

```
%Poles of the controller
pls=[-15;-16;-40;-41;-1.14;-2.14;-1.45;-2.45;-8;-8.1]
```

```
%Poles of the observer
plsob= [-45;-48;-120;-123;-3.42;-6.42;-4.35;-7.35;-24;
-24.3]
```

```
K=place(A,B,pls)
L=place(A',C',plsob)
OBC = ss(A-B*K-Lo*(CM-D*K),Lo,-K,0)
```

Output of the command above is:

```
OBC =

a =

      x1      x2      x3      x4      x5      x6      x7      x8      x9      x10
x1  -0.7279      1 -0.008697      0  0.04602      0  1.321e-14      0  2.354e-15      0
x2  -0.05715 -2.048  0.00386  0.584  0.4481 -0.3315  0.5745 -0.2932  0.0005004  0.5541
x3  -0.006808      0 -0.597      1 -0.07703      0 -1.903e-13      0 -3.795e-15      0
x4  0.006064  2.048 -0.05068 -0.584 -0.4559  0.3315 -0.5745  0.2932 -0.0005004 -0.5541
x5  0.05629      0 -0.07579      0 -0.9751      1  5.106e-14      0 -1.643e-14      0
x6  0.01029 -20.95 -0.02499  5.974  3.12 -3.04  0.05525 -2.572  0.005164  5.67
x7  8.341e-15      0 -1.546e-13      0  4.706e-14      0 -0.74      1  9.164e-18      0
x8  0.01559  17.79 -0.004722 -5.069 -0.93  1.927  10.75  1.388 -0.004467 -4.816
x9  1.519e-15      0 -2.719e-15      0 -8.479e-15      0  6.003e-18      0 -0.242      1
x10 -0.008447 -9.774  0.002662  2.789  2.947 -1.9  8.002 -1.786  0.001868  2.643

b = |
      u1      u2      u3      u4      u5
x1  0.7279  0.008697 -0.04602 -1.321e-14 -2.354e-15
x2  0.05537 -0.003308  0.06504  1.506 -8.352e-16
x3  0.006808  0.597  0.07703  1.903e-13  3.795e-15
x4  -0.004285  0.05013 -1.544 -0.01956  1.188e-15
x5  -0.05629  0.07579  0.9751 -5.106e-14  1.643e-14
x6  -0.02855  0.03061 -3.264  0.06521  5.738e-15
x7  -8.341e-15  1.546e-13 -4.706e-14  0.74 -9.164e-18
x8  -5.377e-15  4.964e-15  0.8848 -11.07 -5.176e-19
x9  -1.519e-15  2.719e-15  8.479e-15 -6.003e-18  0.242
x10 -3.134e-16  5.691e-16  0.9732  0.2152  0.00048

c =
      x1      x2      x3      x4      x5      x6      x7      x8      x9      x10
y1  -194.9 -2.229e+05  59.39  6.351e+04  2.024e+04 -2.71e+04 -8.604e+04 -2.099e+04  55.59  6.033e+04
y2  7.426e-05  0.9222 -0.0006551 -0.2877 -14.91  5.233 -84.46  6.321  0.0004163 -0.2322
y3  -2094 -2.394e+06  637.9  6.822e+05  2.174e+05 -2.911e+05 -9.242e+05 -2.255e+05  597.1  6.48e+05

d =
      u1  u2  u3  u4  u5
y1  0  0  0  0  0
y2  0  0  0  0  0
y3  0  0  0  0  0
```

Continuous-time state-space model.

APPENDIX B

REPRESENTATION OF NONLINEAR SYSTEM IN SIMULINK

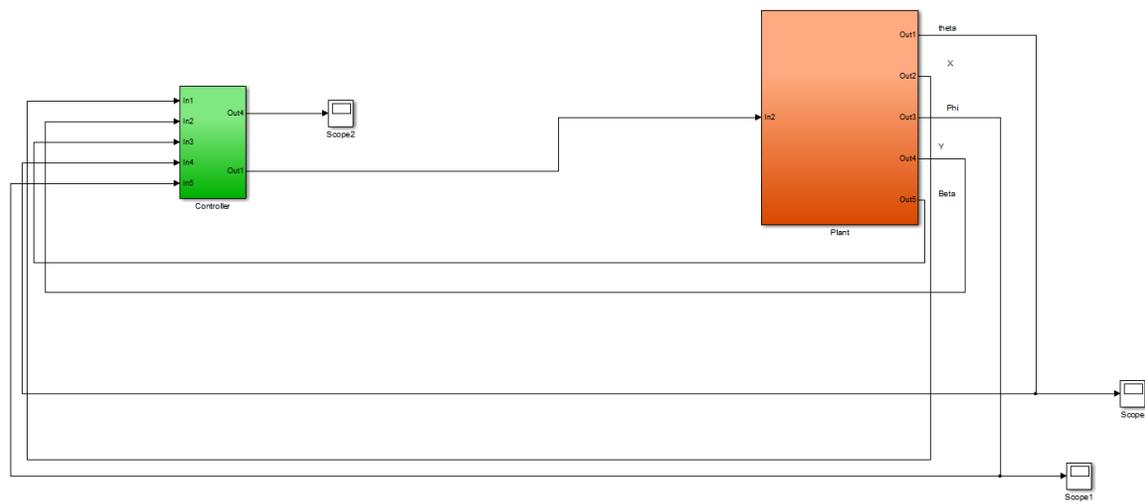


Figure B-1 Overall Appearance of Simulink Model of Nonlinear Equations

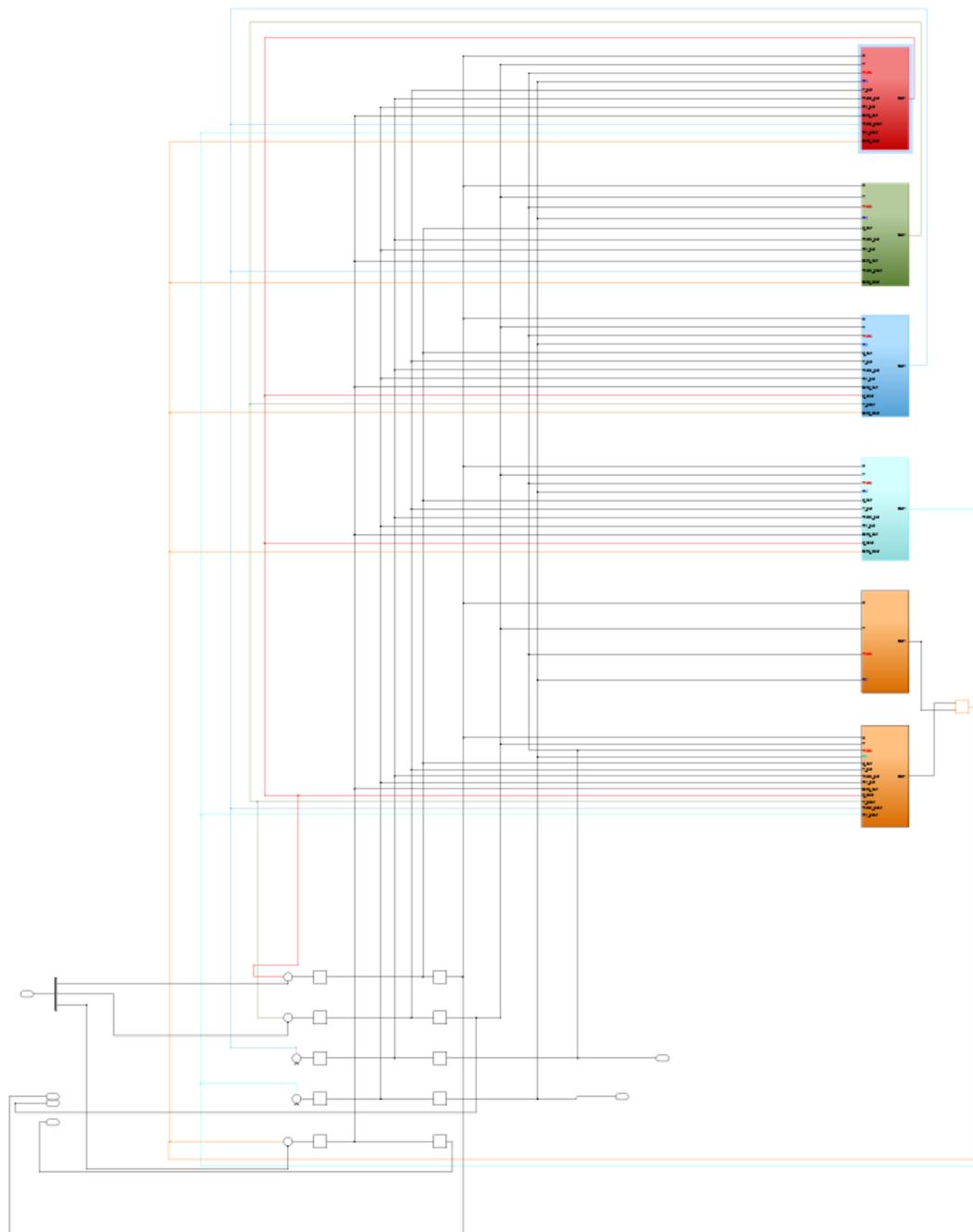


Figure B-2 Overall Appearance of Plant in Simulink Model of Nonlinear Equations

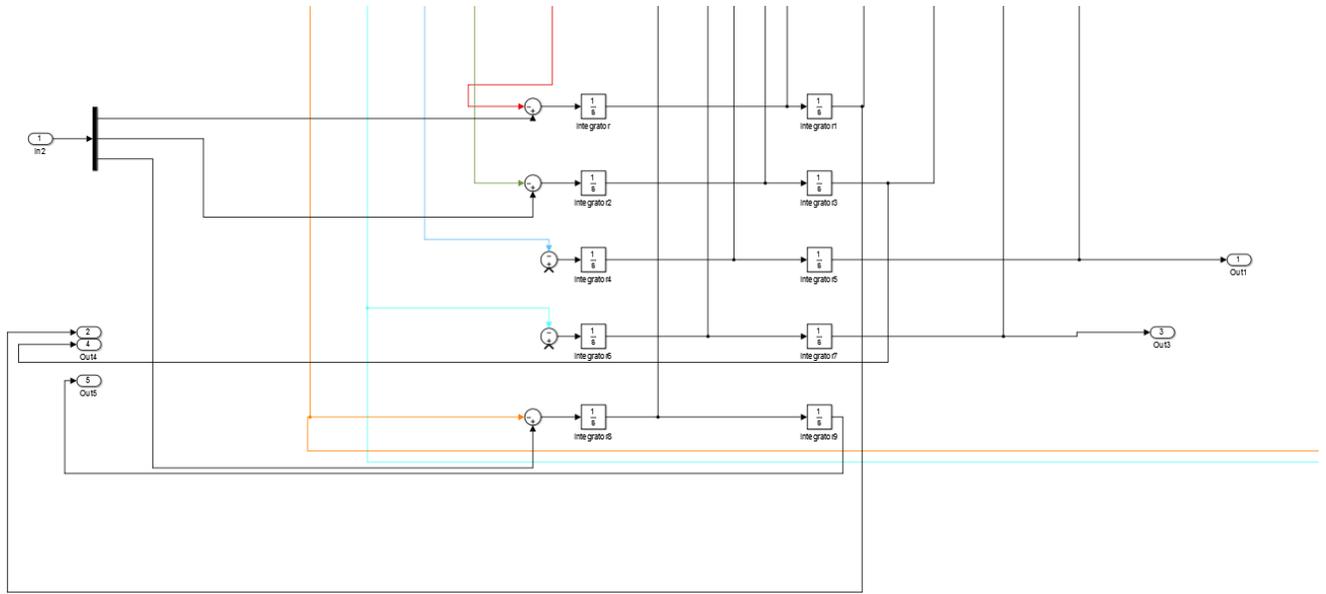


Figure B-3 Integrations of Variables in Equations

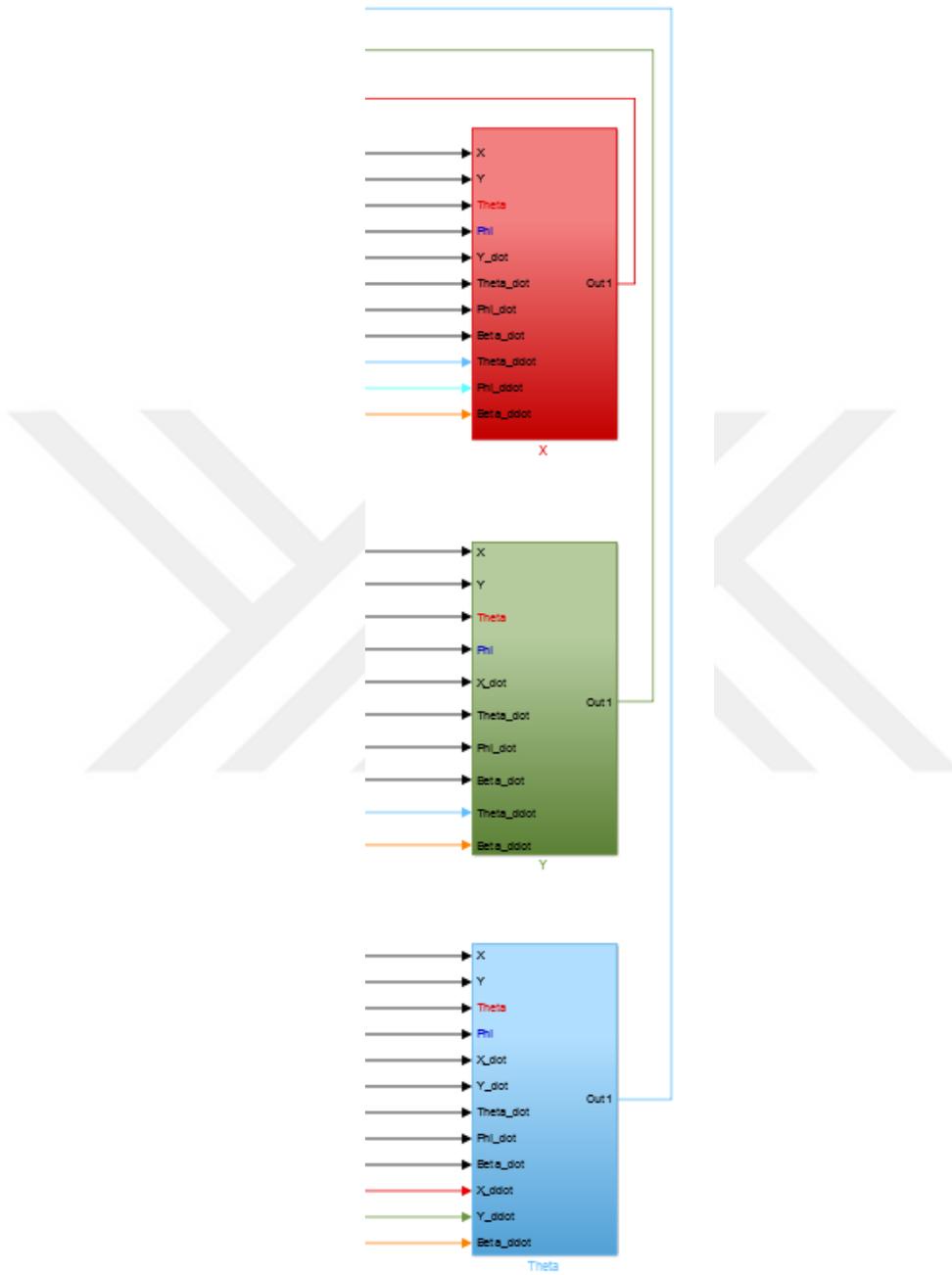


Figure B-4 Outlook to Subsystems of X, Y and θ in Plant

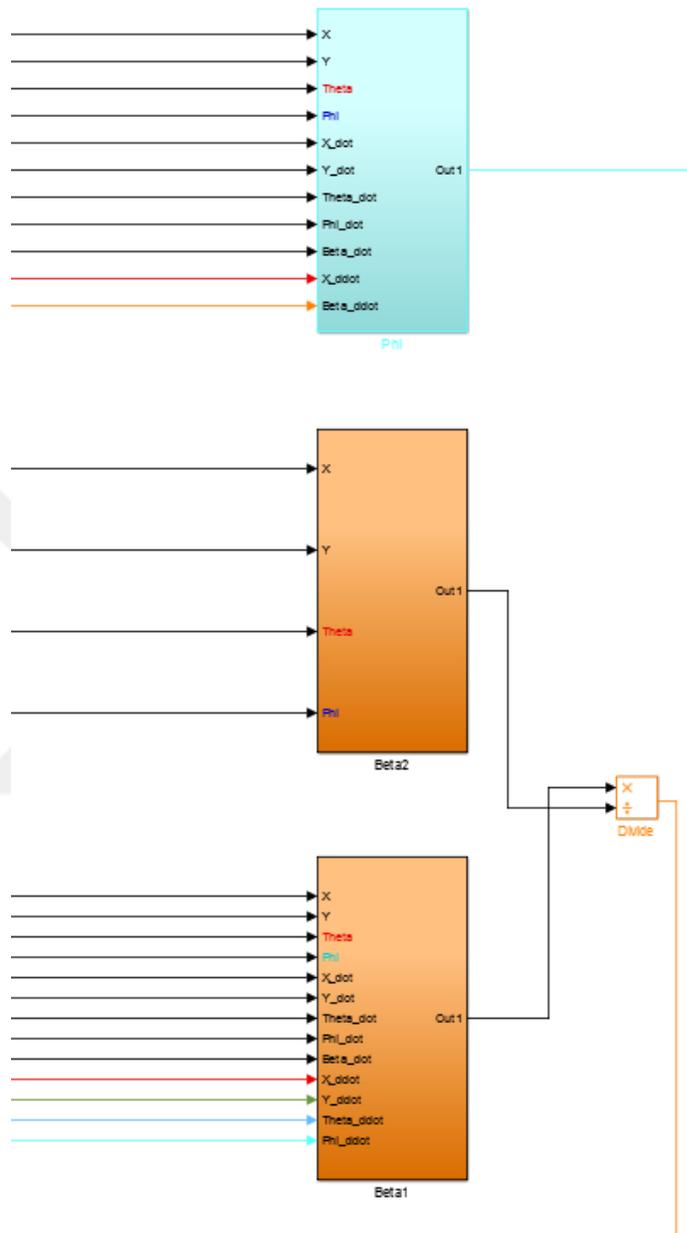


Figure B-5 Outlook to Subsystems of ϕ and β in Plant

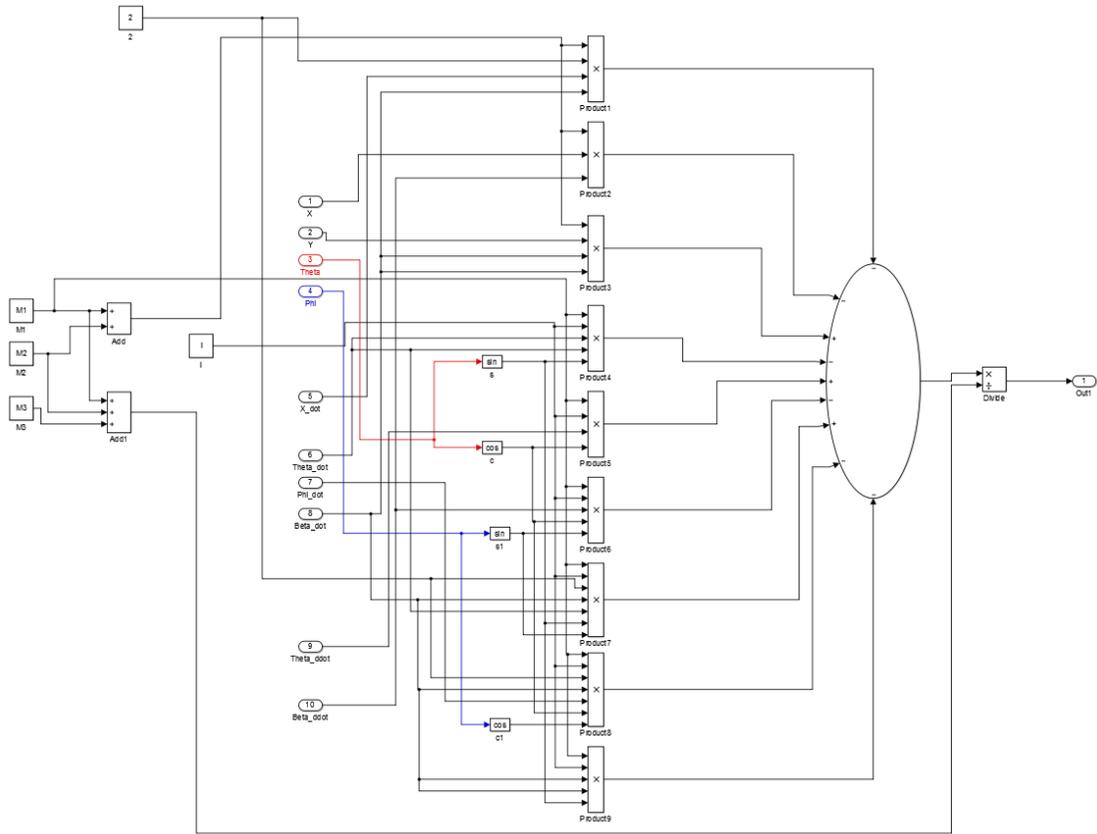


Figure B-6 Structure of Subsystem 'Y'

CURRICULUM VITAE

Personal details

Name : Ömer DUMLUPINAR
Date and Place of Birth : 15 November 1988, Erzincan, TURKEY
Nationality : Turkish
Telephone : +90-216-348 02 92/559
GSM : +90-554-465 01 99
E-mail : omerdumlupinar88@gmail.com,
omer.dumlupinar@marmara.edu.tr

Educational Background

2011 : BSc, Department of Mechanical Engineering,
Marmara University, Istanbul, TURKEY

Research Field

Feedback control, dynamics.