ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE ENGINEERING AND TECHNOLOGY

MULTI OBJECTIVE OPTIMIZATION OF STRUCTURES UNDER MULTIPLE LOADS USING SINGULAR VALUE DECOMPOSITION

Ph.D. THESIS

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Department of Mechanical Engineering

Mechanical Engineering Programme

JULY 2014

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<u>İSTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ</u>

ÇOK SAYIDA YÜKLERE MARUZ YAPILARIN TEKİL DEĞER AYRIŞTIRMASI İLE ÇOK AMAÇLI OPTİMİZASYONU

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TEMMUZ 2014

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Date of Submission : 25 June 2014
Date of Defense : 22 July 2014

To my family,

FOREWORD

This thesis is dedicated to my family, who gave me all the opportunities to accomplish all that I have and the drive to strive higher.

This thesis is individually dedicated to my mother Leyla Turan and to my father Mesut Ertan Turan, who have given me all the opportunities to have a better education and encouraged me to be independent and strong. Thank you for your guidance, encouragement, support and love over the years. To my loving wife Banu Turan, who has always been a big supporter to accomplish this thesis and always been there everytime I needed. Thank you for your patience, encouragement and love.

I wish sincerely to thank my supervisor Prof. Dr. Ata Muğan for his suggestions, encouragements and guidance in writing and approaching the different challenges during the thesis. He helped me to extent my academical borders and enriched my vision. Also I would like to thank Assist. Prof. Dr. Cengiz Baykasoğlu for all his encouragements and supports during this valuable journey.

I thank both of my thesis monitoring committee members, Prof. Dr. İbrahim Eksin and Assist. Prof. Dr. Zeki Yağız Bayraktaroğlu, for their guidance and direction.

Finally, I want to thank my colleagues in Ford Otomotiv Sanayi A.Ş. for their support in completion of this thesis.

June 2014

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ABBREVIATIONS

ACVM : Adjoint Complex Varible Method
AVM : Adjoint Varible Method
CA : Combined Approximations
CPU : Central Processing Unit
DE : Differential Evolution
DOF : Degree Of Freedom
DOM : Design Optimization Methodologies
DSA : Design Sensitivity Analysis
EC : Evolutionary Computation
ES : Evolution Strategies
ESLSO : Equivalent Static Loads Structural Optimization
FAST : Fourier Amplitute Sensitivity Test
FE : Finite Element
FEA : Finite Element Analysis
FEM : Finite Elements Method
FSCA : Frequency Shift Combined Approximations
GA : Genetic Algorithms
LASA : Local Analytical Sensitivity Analysis
LEAP : Large Admissible Perturbation
LESLP : Linearization Error Sequential Linear Programming
MDO : Multidisciplinary Design Optimization
MEMS : Micro Electro Mechanical Systems
MIMO : Multi Input Multi Output
MOO : Multi Objective Optimization
MOP : Multi Objective Programming
MTLA : Modified Thermal Load Approach
NBI : Normal Boundary Intersection
NVH : Noise Vibration Harshness
PCG : Preconditioned Conjugate Gradient
PSO : Particle Swarm Optimization
QFD : Quality Function Deployment
QP : Quadratic Programming
RMOL : Robust Multiobjective and Multi Level
SA : Sensitivity Analysis
SC : Sensitivity Coefficient
SLP : Sequential Linear Programming
SMW : Sherman-Morrison-Woodbury
SQP : Sequential Quadratic Programming
SSO : Sizing and Shape Optimization
SVD : Singular Value Decomposition
VDM : Virtual Distortion Method

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MULTIOBJECTIVE OPTIMIZATION OF STRUCTURES UNDER MULTIPLE LOADS USING SINGULAR VALUE DECOMPOSITION

SUMMARY

Aim of the optimization process is identifying the conditions giving the maximum or minimum value of a function. In system level optimization, objective function is the function of multiple variables. It is very common in engineering problems that there are multiple objective functions to be minimized simultaneously, that require special optimization methods.

In this thesis, optimization methods along with their applications, structural reanalysis and design sensitivity reanalysis in structural optimization are revisited. In addition, application and advantages of Singular Value Decomposition (SVD) to structural design optimization and Sherman-Morrison-Woodbury (SMW) formulas are investigated.

In optimization of a structure, if the structure is subject to simultaneous multiple load cases, combinations of load cases should be considered and worst possible load case should be investigated. However, implementation of conventional optimization approaches into these optimization problems may be impractical due to excessive Central Processing Unit (CPU) times since consideration of multiple load cases increases the associated computational load. This computational difficulty can be overcome by employing SVD to find the worst possible load case against which the structure should be optimized.

In this thesis, the SVD based optimization approach to optimization of a structure subject to simultaneous multiple load case is presented. Conventional optimization and SVD based optimization approaches are applied to a sample structure. It is shown that SVD based optimization approach has certain advantages over the conventional optimization techniques in the existence of simultaneous multiple load cases.

In structural optimization, sensitivity reanalysis of a structure subjected to modifications has a significant practical value. If the structural modifications result in low rank changes in the associated system matrices, the reanalysis of a structure could be completed with a computational load that is less than that of the complete analysis of the structure.

In this thesis, the SMW formulas along with the SVD are employed to compute the extremum sensitivity values and optimum perturbations of design variables such that desired changes in the responses are achieved, which are difficult to be obtained by using the response derivatives. Numerical examples are presented to show the advantages of the proposed approach. Accuracy of the solutions is checked analytically and comparisons between the CPU times of the SVD-based reanalysis and conventional optimization method are made, which show the advantage of the proposed SVD-based approach over conventional methods.

In the future, it can be extended such that the proposed structural and sensitivity reanalysis approach is embedded into the search algorithms in optimization problems to speed up the convergence of the optimization program.

ÇOK SAYIDA YÜKLERE MARUZ YAPILARIN TEKİL DEĞER AYRIŞTIRMASI İLE ÇOK AMAÇLI OPTİMİZASYONU

ÖZET

Optimizasyon işlemi, bir fonksiyonun maksimumunu veya minimumunu veren şartları belirlemek amacına yöneliktir. Küçük çaplı projelerde mühendislik zamanı ve maliyeti açısından optimizasyon uğraşısı verilmeyebilir. Bazen de çok karmaşık projeler için optimizasyon oldukça zor olabilir. Böyle hallerde alt sistemleri optimize etmek mümkündür. Ancak bu işlemin global minimumu vereceği şüphelidir. Optimizasyon işleminin başlangıcı, hangi kriterlerin optimize edileceği hususunda karar vermektir. Örneğin, bir uçak veya uzay aracında minimum ağırlık kriter olabilir. Minimum maliyet de en yaygın olarak seçilen kriterdir. Ayrıca, kısıt denklemleri de kriterler üzerine uygulanabilir.

Optimizasyon kavramıyla iki düşünce vurgulanmaktadır. Bunlardan birincisi alternatif sistemlerin karşılaştırılması, ikincisi ise tek bir sistemin içerisinde yapılan optimizasyondur veya. Tam bir optimizasyon, her bir sistemin alternatifin kendi içinde optimize edilmesi ve daha sonra optimize edilen tasarımların en iyisini seçme ile gerçekleşir. Sistem optimizasyonunda amaç fonksiyonu, genellikle birden fazla değişkenin fonksiyonudur. Bazı sistemler yüzlerce adet değişkene sahip olabilir. Bu durum detaylı optimizasyon yöntemleri gerektirir. Optimizasyon işleminde, matematik bağıntıları geliştirmek önemli bir çaba gerektirmekle beraber, bağ denklemlerini oluşturmak da ilave hesaplama yükü getirir.

Ondokuzuncu yüzyılın sonları, yirminci yüzyılın başlarında çok amaçlı optimizasyonun önemli temel konseptleri oluşturulmuş ve bu yöntemin matematiksel gelişimi yapılmıştır. Günümüzde genellikle çok amaçlı optimizasyon denilince Pareto çözüm akla gelir. Çok amaçlı programlama (ÇAP) problemleri tasarımcının elindeki bilgiyi nasıl yönetmek istediğine bağlı olarak (ki bu da hangi optimal çözümleri seçeceğine bağlıdır) sınıflandırılabilir. Çok amaçlı programlamayı çözecek üç tane genel yaklaşım vardır. Bunlar; Pareto-optimal set oluşturan yöntemler, Tercih temelli yöntemler ve Katılımlı yöntemlerdir.

Bir çok tasarım optimizasyon probleminde ilk önce ön yapı deplasman, frekans, gerilme v.b. performans fonksiyonunu bulmak için analiz programına girdi olarak verilir. Eğer yapının performansı yeterli bulunmazsa uygun bir yöntemle tasarım hassasiyeti hesaplanır. Tasarım hassasiyeti, sürekli hassasiyet analizi, sonlu farklar v.b. yöntemler ile hesaplanabilir. Tasarım hassasiyeti bilgisi kullanılarak, performans fonksiyonunun birinci derece veya ikinci derece yaklaşımları optimizasyon algoritmasına girilir. Optimizasyon algoritması yapının tepkisini iyileştirecek ve sınırlamaları gerçekleyecek tasarım değişkenlerini hesaplar. Bu çevrim, performans değerleri istenilen seviyeye gelinceye kadar devam ettirilir.

Yapısal optimizasyon problemleri, üç başlık altında sınıflandırılabilir. Bunlar; deliklerin, çubukların v.b'lerin yerlerini ve sayılarını hesaplayan yapısal topoloji optimizasyonu (boşluk açma) problemi, deliklerin veya çubuklardaki çözüm noktalarının en son yerini ve şeklini hesaplayan şekil optimizasyonu problemi ve kabukların kalınlığını, kirişlerin alanlarını v.b. hesaplayan boyut optimizasyon problemidir.

Topoloji optimizasyonu problemi, yüklerden ve sınır şartlarından başlayarak çözülür. Daha sonra en uygun topoloji yapısı, kaba bir yapı oluşturması beklenen tasarımcıya, üretilebilir, mantıklı bir tasarım oluşturması için verilir. Bu adım iyi bir mühendislik yaklaşımı ve tasarım yeteneği gerektirir. Bir sonraki adım, yapının şeklini (sınırlarını) optimum topoloji kullanarak optimize etmektir. Mevcut yöntemler, optimizasyon probleminde çap, uzunluk v.b. gibi global parametreleri kullanmaya izin vermektedir. Yapının tüm deliklerinin ve sınırlarının en son biçimi bu optimizasyon probleminin sonucunu belirler. Şekil optimize edildikten sonra son adım değişkenlerin boyutlarını (örneğin; kabuğun kalınlığını, kirişin boyunu v.b.) optimize etmektir. Üç boyutlu katı parçalarda bu son adıma ihtiyaç duyulmaz çünkü biçim optimizasyonu son ölçüleri verecektir. Tasarımcı sonuçtan memnun kalıncaya kadar yapısal optimizasyon işlemi devam ettirilir. Bazen bir iterasyon yeterli olmayabilir çünkü topolojinin çözümlenmesi yapıda optimum olmayan topolojiler ortaya çıkarabilir.Yapısal topoloji optimizasyonu süreksiz ve sürekli olmak üzere iki tip yapıya uygulanabilir.

Tez kapsamında, optimizasyon metotları uygulama alanları, yapısal tekrar analiz ve yapısal optimizasyonda tasarım hassasiyeti tekrar analizi ile ilgili çalışmalar hakkında bilgi verilmiştir. Ayrıca yapısal tasarım optimizasyonunda Tekil Değer Ayrıştırması (TDA) yönteminin kullanılmasının avantajlarından bahsedilmiştir.

Bir yapının optimizasyonunda eğer yapı aynı anda birden fazla yükleme koşuluna maruz kalıyorsa, farklı yükleme koşullarının kombinasyonları gözönüne alınarak mümkün olan en kötü yükleme koşulu incelenmelidir. Ne var ki, bu tip optimizasyon problemlerine konvansiyonel optimizasyon yaklaşımlarının uygulanması pratikte, çoklu yükleme koşullarının bilgisayar hesap yükünden dolayı çok fazla CPU zamanı harcanmasına sebep olmaktadır. Optimizasyon için gerekli olan en kötü yükleme koşulunun TDA ile ortaya konulması sayesinde, konvansiyonel yöntemde ortaya çıkan hesap yükü kolaylıkla aşılabilir.

Bu tezde, aynı anda farklı yükleme koşullarına maruz kalan bir yapının TDA temelli optimizasyon yaklaşımı ile optimizasyonu incelenmiştir. Ayrıca konvansiyonel optimizasyon ve TDA temelli optimizasyon yaklaşımı aynı örnek yapıya uygulanmıştır. Yapılan karşılaştırmalar ile eş zamanlı çoklu yükleme koşulları için TDA temelli optimizasyon yaklaşımının konvansiyonel optimizasyon tekniklerine göre avantajı ispat edilmiştir.

Bir yapıyı tasarlarken en önemli araçlardan biri de, yapısal sistemin parametrelerinin değişimine karşı tasarım kriterlerinin duyarlılığıdır. Tasarım duyarlılık analizi sistemin parametreleri arasındaki ilişkileri ve ölçülebilen bazı performans değerlerine karşı sistemin davranışını inceler. Tasarım duyarlılık analizi çalışmalarında, tasarım değişkenlerine karşı yapısal tepkinin hassasiyeti belirli kısıtlar altında ölçülebilen performans değerleri ile incelenir. Bu performans değerleri bazı matematiksel denklemler, özdeğer problemleri veya adi diferansiyel denklemler olabilir.

Duyarlılık analizinde genellikle deplasman, özdeğer, özvektör ve gerilme gibi parametreler kullanılır. Diğer taraftan, TDA temelli analizler de girdi-çıktı ilişkisi üzerine kurulu çalışmalar için bir hayli faydalıdır. Bir yapının tekil değerlerinin özel bir anlamı vardır, zira tekil değerlerin kareli ifadeleri girdi ve çıktı vektörleri arasındaki güç, enerji ve güç yoğunluğu oranlarını ifade eder.

Bu tezde, bir yapının tekil değerlerinin biçimlendirilmesinin, yapının tepkisinin biçimlendirilmesine denk olacağı ayrıca, tekil vektörlerin çıktıların girdiler ile ne tip bir ilişkide olduğunu gösterdiği belirtilmektedir. TDA'nın, zamana bağımlı ve zamandan bağımsız problemler için sonlu elemanlar yöntemi (SEY) denklemlerine uygulanabilir olduğu, buna bağlı olarak her bir sağ tekil vektörün, ilgili tekil değere eşit bir çıktı sağlamamız için yapıya hangi girdileri girmemiz gerektiğini ve sol tekil vektörün bu girdiye karşılık tepkinin, yapının farklı serbestlik derecelerinde nasıl dağıldığını gösterdiğinden bahsedilmektedir.

Bir yapının dizayn prosesi, sonunda kısıtları sağlayan en uygun çözüme ulaşılması beklenen çeşitli el veya bilgisayar dizayn iterasyonlarını içerir. Bu noktaya kadar belirli yük koşulları altında ve bazı alt parçalarda belirli değişiklikler yapılması halinde her bir dizayn iterasyonu için tüm sistemin yapısal ve duyarlılık analizini yapmak yaygındır. Ne var ki bu yaklaşım, ilgili yapısal matrisin sadece belirli bölümlerinin orjinal yapısal matristen farklı olduğu gerçeğini göz önüne almadığı için iterasyonlar esnasında analizlerin CPU zamanını gereksiz şekilde arttırır.

TDA'nın sistemlerin hızlı yapısal optimizasyonları, statik tekrar analizlerinin yapısal optimizasyonu, olasılık analizleri, yapısal durumunun gözlenmesi, duyarlılık analizi ve sistem tanımlaması için belirgin pratik değeri vardır.

Sherman-Morrison-Woodburry (SMW) formülleri ile ilgili çalışmanın amacı, ilgilenilen yapının matematik modelinin özellikle lineer denklem sistemi ile temsil edildiği sonlu elemanlar metotu benzeri hızlı ve etkili tekrar analiz teknikleri kullanılarak incelenmesidir. Her ne kadar yapısal tekrar analiz metotları altmış yıldan fazla bir süre mevcut olsalar bile, daha çok düşük dereceli modifikasyonları içeren yapısal tekrar analizler için etkilidirler.

SMW formülleri orjinal sistemin m farklı sağ taraflar ile çözümüne ve m modifikasyon derecesi olmak üzere, m. derece ilave sistemin çözümüne ihtiyaç duyar. Eğer m modifikasyon derecesi büyük ise, bu yaklaşım özellikle büyük yapılar ve çoklu yükleme koşulları için çok yavaştır.

Konvansiyonel tekrar analiz yaklaşımlarında ortak olarak kullanıldığı üzere, verilen yapının sistem matrisine ön pertürbasyonlar uygulanır. Bu pertürbasyonlar ya bir algoritma tarafından oluşturulur ya da tasarımcı tarafından kararlaştırılır.

Sonuçta, istenilen yapısal çıktı elde edilinceye kadar, özellikle büyük yapılar için (örneğin, m büyüktür) ve çoklu yükleme durumları olduğunda, yapının dizayn değişkenlerinin optimum pertürbasyonlarının hesaplanmasının pratik değeri vardır.

Yukarıdaki gerçeklerin ışığında bu çalışmada istenilen çıktı değişikliklerinin ve duyarlılık değerlerinin sağlanması için TDA kullanılmak suretiyle dizayn değişkenlerinin optimum pertürbasyonlarının verimli bir şekilde hesaplanması için bir yaklaşım geliştirilmiştir.

TDA'nın uygulandığı denklemler SMW formüllerine dayanmaktadır. Bu tezdeki sayısal örnekler, önerilen formülasyonların uygulanabilirliğini ve önerilen yaklaşımın avantajlarını göstermek için kullanılmıştır. SMW formüllerinin sistem matrislerinin TDA'sı ile birlikte, tasarım duyarlılıkları, dizayn değişkenlerinin optimum pertürbasyonlarının hesaplanması ve çıktının duyarlılığı üzerine daha fazla bilgi edinebilmek için kullanılabildiği gösterilmiştir.

Tez kapsamında TDA uygulanan SMW formülleri çıktılarda istenilen değişimler elde edilene kadar tasarım duyarlılık değerlerini ve dizayn değişkenlerinin optimum pertürbasyonlarını hesaplamak için kullanılmıştır. Ayrıca yapının ilgili tekil vektörlerinin dizayn değişkenlerinin optimum pertürbasyonlarını hesaplamak için kullanılabileceği gösterilmiştir. Ele alınan düzlemsel bir kafes sisteminin konvansiyonel çıktı türevleri hesaplanmış ve ilk önce yapının sonlu elemanlar modeli oluşturulup daha sonra Matlab'de geliştirilen programlar kullanılarak TDA temelli SMW formülleri ile tekrar analizi yapılmıştır. Önerilen formüllerin uygulanabilirliğini ve TDA temelli SMW formüllerinin konvansiyonel duyarlılık analizlerine göre avantajlarını göstermek için düzlemsel kafes sistemi üzerinden sayısal örnekler sunulmuştur. TDA temelli SMW formülleri kullanılarak yapılan tekrar analizin çıktı türevlerini kullanarak bir sistemin tasarım duyarlılığı ile ilgili daha fazla bilgi verdiği gösterilmiştir. Özellikle, sistem çıktısının en büyük ve en küçük duyarlılık değerlerini veren dizayn değişkenlerinin pertürbasyonları kolaylıkla hesaplanabilmektedir. Önerilen yaklaşımın verimliliğini gösteren CPU zamanları karşılaştırılmış ve çözümlerin doğruluğu analitik olarak kontrol edilmiştir.

Ayrıca tez kapsamında yapısal optimizasyon için iki örnek ele alınmış birincisinde ankastre mesnetle tek tarafından bağlı bir kiriş incelenmiştir. Yapı birçok bileşene ve herbir bileşen kalınlık, alan, uzunluk ve diğer geometrik ve malzeme parametreleri gibi optimize edilecek sayısız parametreye sahiptir. Her bir bileşen için uygulanan yükler altında optimum kesit yükseklikleri hesaplanmıştır. İkinci örnekte ise benzeri çalışma bir kamyon şasi sistemi için yapılmış ve konvansiyonel yöntemle CPU zamanları karşılaştırılmıştır.

1. INTRODUCTION

Aim of the optimization process is identifying the conditions giving the maximum or minimum value of a function. For complex systems, it is possible to divide the system into subsystems and optimize the sub-systems, but it may not give us the best possible solution all the time.

The start of an optimization process is to decide about the criteria to be optimized. For instance, minimum weight can be a criteria for an aircraft or an aerospace shuttle. Minimum cost is also a commonly chosen criteria for optimization problems.

Optimization process is known as Operations Research in industry. Various developments in operations research are achieved by the attempts due to optimizing mathematical models of the economical systems.

In an optimization process, it is mandatory to simulate the system's operational conditions for long service intervals since an optimum system under operation conditions may not give the optimum solution for different operation conditions.

There are two ideas to be emphasized with optimization term. One of them is the back-to-back comparison of alternative systems and the other is the optimization of just one system. A complete optimization is possible when every subsystem or alternative to be optimized in itself and then to choose the best option from the already optimized designs.

Optimization methods' power is coming from determining the best case without actually testing all possible cases by using mathematics. The development of searching the possible solution will therefore require some basic vector–matrix manipulations, a bit of linear algebra and calculus, and analysis. [1].

1.1 Purpose of Thesis

In this thesis, optimization methods along with their application areas, design sensitivity reanalysis in structural optimization and structural reanalysis review are revisited. In addition, applications of SVD along with SMW formulas to a structural reanalysis problem are investigated.

1.2 Literature Review

1.2.1 Optimization

The ever-increasing demand on engineers to lower production costs to withstand competition has prompted engineers to look for rigorous methods of decision making, such as optimization methods, to design and produce products both economically and efficiently. Optimization techniques, having reached a degree of maturity over the past several years, are being used in a wide spectrum of industries, including aerospace, automotive, chemical, electrical, and manufacturing industries. With rapidly advancing computer technology, computers are becoming more powerful, and correspondingly, the size and complexity of problems being solved using optimization techniques are also increasing. Optimization methods, coupled with modern tools of computer- aided design, are also being used to enhance the creative process of conceptual and detailed design of engineering systems. Essential proofs and explanations of the various techniques are given in a simple manner and new concepts are illustrated with the help of numerical examples [2-6].

1.2.2 Optimization methods

Optimization algorithms typically require the solution of many systems of linear equations. When large numbers of variables or constraints are present, these linear systems could account for much of the total computation time. Both direct and iterative equation solvers are needed in practice. Unfortunately, most of the off-the-shelf solvers are designed for single systems, whereas optimization problems require hundreds or thousands of systems. To avoid refactorization, or to speed the convergence of an iterative method Gill et al. [7] review various sparse matrices that arise in optimization and discuss compromises that are currently being made in dealing with them. Sequential quadratic programming (SQP) methods are the method of choice when solving small or medium-sized problems. Since they are complex methods, there are difficulties to adapt to solve large-scale problems. Murray [8] described some general ideas that may be used to resolve these difficulties. A number of SQP codes were written to solve specific applications and SQP methods using explicit second derivatives were proposed. Kanga et al. [9] proposed a robust optimization model to handle uncertainty during the process design stage, together with a decision-making procedure. They presented a comprehensive robust optimization model for process design problems based on a scenario-based approach, in conjunction with a decision-making procedure. Depending on the variable type (either scenario-dependent economic or technical), different robustness concepts can be introduced, considering economic and technical robustness measures as monotonic and even functions, respectively. Reliability-based Optimization is a most appropriate and an advantageous methodology for structural design. Its main feature is that it allows determining the best design solution (with respect to prescribed criteria) while explicitly considering the unavoidable effects of uncertainty. In general, the application of this methodology is numerically involved, as it implies the simultaneous solution of an optimization problem and the use of specialized algorithms for quantifying the effects of uncertainties. In view of this fact, several approaches have been developed in the literature for applying this methodology in problems of practical interest. Valdebenito and Schuëller [10] provided a survey on approaches for performing Reliability-based Optimization, with emphasis on the theoretical foundations and the main assumptions involved. Bukchin et al. [11] considered a facility design problem that consists of a system of assembly lines. To solve the facility design problem, they applied an efficiency frontier approach to analyze the trade-off between the facility area and transportation distance. On the other hand, Borgart and Stach [12] aimed at developing computational methods for form finding, optimization and production of complex geometry (spatial) structures, which should be elegant and constructible and that are easy to use for designers. In recent years the impact of various environmental conditions, either global aspects, such as climate change, or resulting local aspects, such as floodings, played an increasingly decisive role in the design of new buildings and structures. Understanding the interrelation between these impacts and the built environment is a major public and scientific interest. The increasing costs of energy, which are required for construction and maintaining the buildings, require optimized solutions. In their book, Chong and Zak give a broad knowledge of optimization theory and methods [13]. Bendsoe et al. [14] published a paper that shows the optimization methods for truss geometry and topology design.

Over the past two decades, there has been an increasing interest in using what has come to be called Evolutionary Computation (EC) in the analysis and optimization of structural systems. These methods include Genetic Algorithms (GA), Evolution Strategies (ES), Simulated Annealing and other stochastic based numerical methods. Each of these methods shares the drawback that they are very computationally intensive compared to deterministic methods. Furthermore, the computational burden can rapidly increase as the size of the analyzed structure increases. Borgard and Stach [15] investigated some fundamental principles concerning the logic of form optimization in nature in relation to special and physical constraints. One of the main aims of their study was to make a contribution towards a true and complete understanding of optimization processes in nature to establish, both by argument and evidence that the same optimization processes used in nature may also be used in architecture. Eschenauer and Grauer [16] examined how to put on coarse-grained parallelization and its implementation on clusters of workstations. Numerical tests and a special industrial application on an automotive wheel showed that cluster computing gave very promising results for the use of parallel and distributed solution strategies in optimization. Sequential approximate optimization is used to solve multibody optimum design problems [17]. In their book Belegundu and Chandrupatla reviewed the optimization concepts and applications in engineering [18]. Lai et al. [19] introduced a new method of dealing with optimization problems in quality function deployment (QFD) analysis. Yang et al. [20] also presented a QFD based optimization method, as an effort to reflect customer's preferences in making a trade-off between multiple objectives. Their approach can be applied to a variety of multi-criteria design problems where customer's preferences need to be considered.

SQP methods have proved highly effective for solving constrained optimization problems with smooth nonlinear functions in the objective and constraints. Gill et al. [21] have presented theoretical and practical details about an SQP algorithm for solving nonlinear programs with large numbers of constraints and variables, where the nonlinear functions are smooth and first derivatives are available. The algorithm minimizes a sequence of augmented Lagrangian functions, using a quadratic programming (QP) at each stage to predict the set of active constraints and to generate a search direction in both the primal and dual variables. Convergence is assured from arbitrary starting points. In their book Gen and Cheng [22] summarized the genetic algorithms (GA) and engineering design. Also Yang published the results on metaheuristic applications, which includes genetic optimization algorithms [192]. Polak [23] dealed with optimality conditions, algorithms, and discretization techniques for nonlinear programming, semi-infinitive optimization, and optimal control problems. Shape optimization problems has received a lot of attention in recent years, particularly in relation to a number of applications in physics and engineering that require a focus on shapes instead of parameters or functions. The goal of these applications is to deform and modify the admissible shapes in order to comply with a given cost function that needs to be optimized. In this respect, the problems are both classical (as the isoperimetric problem and the Newton problem of the ideal aerodynamical shape show) and modern (reflecting the many results obtained in the last few decades). The intriguing feature is that the competing objects are shapes, instead of functions, as it usually occurs in problems of the calculus of variations. This constraint often produces additional difficulties that lead to a lack of existence of a solution and to the introduction of suitable relaxed formulations of the problem. However, in certain limited cases an optimal solution exists, due to the special form of the cost functional and to the geometrical restrictions on the class of competing domains. Bucur and Buttazzo [24] collected the relevant lecture notes in their study. Kelley [25] studied various methods for unconstrained and bound constrained optimization by using algorithms written with MATLAB® codes. Zhou [26] presented a method to maximize the natural frequencies of vibration of truss-like continua with the constraint of material volume. A multiobjective design procedure indicates the optimum response of a structure and enables the designer to make a comparison among various possibilities. This obviously increases the computational effort as compared to a single objective design by necessitating repeated solutions. On the other hand, it puts the designer into a better position as a decision maker by showing the best capabilities of the structure under consideration. The final choice should be made by the designer after a study of the various alternatives.

1.2.3 Multiobjective optimization and pareto optimality

Adali [27] determined the pareto-optimal cross-sectional shapes for a clamped-hinged beam subjected to harmonic support motions with the objectives of minimizing the maximum deflection, maximum normal and shearing stresses and of maximizing the fundamental eigen frequency. The original design problem was transformed into a finite dimensional optimization problem by approximating the area function by linear splines, which was then solved by using the techniques of mathematical programming. The Pareto optimal solutions were computed by employing a scalar performance index, which was obtained by combining different criteria in a weighted sum. The results were presented in the form of optimal trade-off curves to assess the efficiencies of the designs with respect to different criteria. The design process of complex systems often resorts to solving an optimization problem, which involves different disciplines and where all design criteria have to be optimized simultaneously. Mathematically, this problem can be reduced to a vector optimization problem. The solution of this problem is not unique and is represented by a Pareto surface in the objective function space. Once a Pareto solution is obtained, it may be very useful for the decision-maker to be able to perform a quick local approximation in the vicinity of this Pareto solution for sensitivity analysis. Utyuzhnikov et al. [28] derived new linear and quadratic local approximations of the Pareto surface and compared to existing formulas. A method for detecting non-differentiable Pareto point was proposed and its limitations were pointed out in Christodoulou et al. [29] where a multi-objective identification method for structural model updating based on modal residuals was presented. Ghanmi et al. [30] presented a new approach to robust multi-objective and multi-level optimization (RMOL) of the design of complex mechanical structures. The optimization is at two levels: system and elements. At system-level, the robust multi-objective problem has four cost functions: on the one hand, the minimization of the global mass and displacement at a fixed point of the mechanical structure and on the other hand the maximization of both the robustness and displacement of the mass. At element-level, the robust multi-objective problem has two cost functions: minimization of the element mass and maximization of its robustness. System design is a complex task when design parameters have to satisfy a number of specifications and objectives, which often conflict with those of others. This challenging problem is called multi-objective optimization (MOO). The most common approximation consists of optimizing a single cost index with a weighted sum of objectives. However, once weights are chosen, the solution does not guarantee the best compromise among specifications, because there are an infinite number of solutions. A new approach can be stated, based on the designer's experience regarding the required specifications and the associated problems. This valuable information can be translated into preferences for design objectives, and will lead the search process to the best solution in terms of these preferences. Sanchis et al. [31] presented a preference method, which enumerates these a priori objective preferences and offers a problem formulation, which fits real-life engineering design. Procedure of weight selection is transformed into a selection of preference ranges, which have the same units as the objective functions. Thus the designer, after examining the results, may decide to explore other possibilities. As a result, a single objective is built automatically and no weight selection need to be performed. Utyuzhnikov et al. [32] presented a method for generating a well-distributed Pareto set in nonlinear multiobjective optimization. Figueira et al. [33] presented a multiple reference point approach for multi-objective optimization problems of discrete and combinatorial nature. To solve time-cost-quality tradeoff problems in construction, a Pareto multi-objective optimization approach was developed by Diao et al. [34]. Tappeta et al. [35] focused on Multidisciplinary Design and Optimization (MDO) of large scale systems that have multiple objective functions. Lindroth et al. [36] described a method for finding an optimal reduction of the set of objectives yielding a smaller problem whose Pareto optimal set w.r.t. a discrete subset of the decision space is as close as possible to that of the original set of objectives. Meza et al. [37] studied the evaluation of design concepts and the analysis of multiple Pareto fronts in multi-criteria decision-making using level diagrams. Multi-objective optimization algorithms can generate large sets of Pareto optimal (non-dominated) solutions. Identifying the best solutions across a very large number of Pareto optimal solutions can be a challenge. Therefore, it is useful for the decision-maker to be able to obtain a small set of preferred Pareto optimal solutions. Kao [38] analyzed a discrete optimization problem introduced to obtain optimal subsets of solutions from large sets of Pareto optimal solutions. It is useful with multi-objective optimization (MOO) to transform the objective functions such that they all have similar units and orders of magnitude. Marler and Arora evaluated various transformation methods using simple example problems [39]. Jia and Ierapetritou [40] proposed a multiobjective robust optimization model to deal with the problem of uncertainty in scheduling by considering the expected performance, model robustness and solution robustness. Normal boundary intersection (NBI) technique is utilized to solve the multiobjective model and successfully produce Pareto optimal surface that captures the trade-off among different objectives in the face of uncertainty. Hoogeveen [41] gave a survey of the most important results on multicriteria scheduling that have appeared in the literature. Also he provided an extensive introduction including two example problems and then discuss the relevant literature by paying special attention to the area of earliness-tardiness scheduling, scheduling with controllable processing times, simultaneous approximation, and new models. Egorov et al. [42] demonstrated the main capabilities of IOSO (Indirect Optimization based on Self-organization) technology algorithms, tools, and software, which can be used for the optimization of complex systems and objects. Ashby [43] reviewed methods of dealing with optimal selection of discrete entities to meet multiple objectives and adapted these methods to the specific case of material selection. Limbourg and Kochs [44] presented a novel way to tackle the optimization of system reliability. Feature modelling was applied to allow a very flexible formulation of the optimization problem. With feature models, the design space could be formulated and tailored rapidly to the user's needs. Georgiou et al. [45] applied a systematic methodology leading to the determination of the optimal values for the suspension damping and stiffness parameters of the two degrees of freedom quarter-car models moving over rough roads. Some of the models examined possessed passive suspension dampers with linear or dual rate characteristics. Car models with semi-active suspension systems were also considered. A popular method of "solving"multi-objective problems is to determine a Pareto optimal set or subset. However, this then requires the decision maker to select among this set of solutions, which is often large when there is more than two objective functions. Therefore, meaningful research has to be done to support the decision maker during this post-Pareto analysis phase. Taboada and Coit [46] reviewed two methods to prune the size of the Pareto optimal set. The combination of the two proposed methods could be ideally suited to address complex multi-objective optimization problems in which the Pareto optimal set is very large. For this type of problem, where the Pareto optimal set can contain thousands of solutions, the combination of the two pruning methods might be preferred. In such cases, pruning by using the non-numerical objective function ranking preferences method should be initially applied to obtain a Pareto subset that reflects the decision maker's objective function preference, and then, pruning by using data clustering can be applied to further reduce the size of the Pareto subset. Thus, the decision maker gets a smaller set of solutions to analyze in order to select one solution for implementation.

Many of the multi-objective optimization problems are often subject to parameters with uncertainties and noises. In such cases, to obtain robust solutions, small amounts of noise is added and evaluated with Monte Carlo simulation. Brik et al. [47] suggested a stochastic multi-objective optimization method, which takes into account uncertainties on the design parameters, for solving these types of multi-objective optimization problems. This methodology consists of increasing the objective function space with robustness functions in order to find robust and optimal solutions. The multi-objective optimization problem is solved with an evolutionary algorithm. A neural network is used to significantly reduce the computational time, in particular for the robustness function evaluations. Marler and Arora [48] presented a survey of current continuous nonlinear multiobjective optimization (MOO) concepts and methods. The methods are divided into three major categories: methods with a priori articulation of preferences, methods with a posteriori articulation of preferences, and methods with no articulation of preferences. Genetic algorithms are surveyed as well. Commentary is provided on three fronts, concerning the advantages and pitfalls of individual methods, the different classes of methods, and the field of MOO as a whole. The characteristics of the most significant methods are summarized. Conclusions are drawn that reflect often-neglected ideas and applicability to engineering problems. It is found that no single approach is superior. Rather, the selection of a specific method depends on the type of information that is provided in the problem, the user's preferences, the solution requirements and the availability of software. Suresh [49] demonstrated that optimal topologies for various volume fractions can be generated in a highly efficient manner, by directly tracing the Pareto-optimal curve. The three most significant contributions of the paper are as follows: (1) a theoretical framework for determining if a topology satisfies the necessary condition for local Pareto-optimality, (2) an efficient algorithm for tracing Pareto-optimal curves for compliance-related objectives, and (3) a compact

MATLAB® code for generating Pareto-optimal topologies. Mela and Koski [50] studied topology optimization of trusses under multiple loading conditions. Similarly, Aubin [189] studied a ten-member cantilever truss in order to compare the various optimal design methods.

1.2.4 Structural design optimization

Adeli [51] summarized advances in a number of fundamental areas of optimization with application in engineering design. Arora and Wang [52] reviewed alternative formulations for optimization and simulation of structural and mechanical systems and other related fields. Papadrakakis et al. [53] investigated the efficiency of various optimization methods based on mathematical programming and evolutionary algorithms for solving structural optimization problems under static and seismic loading conditions. The proposed hybrid optimization algorithms proved to be robust and efficient methods for structural optimization. Both combinations of genetic algorithms with successive quadratic programming and of evolution strategies manage to converge to better designs than those achieved by evolution strategies or successive quadratic programming alone at a reduced computational effort compared to the successive quadratic programming procedure. Saitou et al. [54] provided a bird-eye survey of the structural optimization, with a special emphasis on its relation to product development. Structural optimization procedures usually start from a given design topology and vary proportions or boundary shapes to achieve optimality under various constraints. Bremicker et al. [55] presented an approach for initiating formal structural optimization at an earlier stage, where the design topology is rigorously generated in addition to selecting shape and size dimensions. Park [56] proposed a method named as the Equivalent Static Loads method for nonlinear static response Structural Optimization (ESLSO). Ohsaki and Ikeda [57] studied optimization of geometrically nonlinear structures under stability constraint. Takezawa et al. [58] studied a new structural optimization method based on topology optimization techniques using frame elements where the cross-sectional properties can be treated as design variables. Vanderhlaat [59] presented a general design algorithm for the optimum geometry design of finite element structures where a reasonable initial geometry was specified.

Design of powertrain mounting bracket is always challenging in achieving good NVH characteristics, sound durability and simultaneously reduced weight. Depending on the design status, different schemes, i.e., size, topology and shape optimization, can be applied. Pan et al. [60] presented a case study of application of structural optimization in the design of a mount bracket. Firstly, both test and FEA (Finite Element Analysis) results expose the problems of initial design. Therefore, it is necessary to redesign the bracket. In topology optimization, design space and optimization parameters are defined with sufficient design freedom and time. Die direction and other manufacturability considerations for the casting components are vital. Shape optimization is then conducted to further decrease the weight and refine local weakness. Compared with original design of mount bracket, the mass on the final design is reduced. Final comparison in terms of weight and component performance illustrates that structural optimization techniques are effective to produce higher quality products at a lower cost. Haftka [61] compared three first-order and two second-order approximations for truss and composite laminate designs. The second-order approximations were found to be substantially more accurate for small changes in the design. Comparing the performance in optimization, very slow convergence was associated with the conservative-convex approximation. The second-order approximations did better than the first-order ones, but possibly not enough to compansate for their additional cost. Second order approximations were found to be more attractive for predicting the effect of change in problem parameters on the optimum design when the optimal design was not fully constrained. A variety of numerical methods was proposed in literature in purpose to deal with the complexity and nonlinearity of structural optimization problems. In practical design, sequential linear programming (SLP) is very popular because of its inherent simplicity and because linear solvers (e.g., Simplex) are easily available. However, SLP performance is sensitive to the definition of proper move limits for the design variables. Lamberti and Pappalettere [62] studied a new SLP algorithm that implements an advanced technique for defining the move limits. The linearization error sequential linear programming (LESLP) algorithm is formulated to overcome the traditional limitations of the SLP method. Optimum designs were found substantially insensitive to input parameters such as the initial value of the allowable linearization error and starting design point. Tsompanakis and Papadrakakis [63] presented a robust and efficient methodology for treating large-scale reliability-based structural optimization problems. The optimization is performed with evolutionary strategies, while the reliability analysis is carried out with the Monte Carlo simulation method incorporating the importance sampling technique to reduce the sample size. Efficient hybrid methods are implemented to solve the reanalysis-type problems that arise in the optimization phase with evolution strategies and in the reliability analysis with Monte Carlo simulations. These hybrid solution methods are based on the preconditioned conjugate gradient algorithm using efficient preconditioning schemes. Christensen and Klarbring [64] studied on three basic classes of geometrical optimization problems of mechanical structures, i.e., size, shape and topology optimization. Focus of their study was on concrete numerical solution methods for discrete and (finite element) discretized linear elastic structures. Rietz [65] considered weld optimization with stress constraints and thermal load. Sarma and Adeli [66] worked on the cost optimization of steel structures. Papadrakakis et al. [67] worked on the design optimization of steel structures. Schuttea and Haftka [68] proposed that a quasi-separable decomposition has as a technique, which can reduce the computational burden by performing most of the global search in low dimensional spaces separately for each subsystem. Locatelli [69] studied the multilevel structure of global optimization problems. Gobbi et al. [70] worked on the optimal design of ground vehicles and their subsystems, with particular reference to 'active' safety and comfort. The contribution of optimization has been essential to the more recent developments in design of new mechanical structures and materials. Dimitrovová and Rodrigues [71] applied the models of material and structural optimization to the design of passive vibration isolators. A computational tool to identify the optimal viscoelastic characteristics of a nonlinear one-dimensional isolator was developed. Markine et al. [72] studied on the optimization problem of the ride characteristics of a travelling truck. Under transition to market economy, the development of scientific concepts defining parametric optimization in automobile design is particularly important. Based on the theory of parametric optimization, the strategy of automobile industry development embracing the measures of environment protection, the needs of customers, the use of alternative power sources (e.g., electricity, hydrogen, biomass and sustainable energy sources) as well as control over exhausted burnt gases (i.e., zero toxicity automobile), a decrease in noise level and the use of intelligent transport systems (e.g., interactive data exchange, intelligent automobile) can be examined. Dyakov and Prentkovskis [73] studied on the optimization problems in designing automobiles. Farkas et al. [74] studied on the design and optimization of a vehicle bumper subsystem. Gholizade and Fattahi [75] proposed an efficient algorithm for optimal designing of truss structures by hybridizing the particle swarm optimization (PSO) and ant colony optimization (ACO) algorithms. Several optimization strategies for the structural design of wind turbine towers are developed and investigated through computer implementations by Hani M. Negm, Karam Y. Maalawi [76]. Borkowski [77] studied on the mathematical programming in structural mechanics. Arora [78, 79] covered several important topics in the subject of optimization of structural and mechanical systems. Burns [80] studied the important developments in structural optimization over the period 1972 to 2000 and included an extensive bibliography of books and research papers on the topic. Spillers and MacBain [81] worked on the structural optimization tools. Pope and Schmit [82] studied on the structural design applications of mathematical programming techniques.

1.2.5 Topology optimization

While compliance design for structures was the state-of-the-art in the early nineties, we see today that topology design is used for a broad range of structural problems (free and forced vibrations, buckling, snap, stress constraints, pressure loads, compliant mechanisms, material design, design of supports, crashworthiness, biomechanics etc.) with both linear and nonlinear analysis modelling. Moreover, new areas are today included in the problem types that can be handled, encompassing for example electrothermal actuators, MEMS, Stokes flow problems, piezoelectric transducers, electromagnetic, and band gap structures. Bendsoe and Sigmund [83] dealed with the topology design within the framework of searching for optimum "classical designs" made from isotropic materials, covering theory and computational procedures and describing the broad range of applications. They also studied on compliance design and on the use of composites and materials in the large structures for optimal structural design. Their work provided a unified presentation of methods for the optimal design of topology, shape and material for continuum and discrete structures. Yang and Chuang [84] formulated the topology design problem as a general optimization problem and solved by SLP. This approach is more general and provides an alternative to the homogenization method and the simultaneous analysis and design method. It has three major elements as follows: use the sequential linear programming method to solve a general nonlinear optimization problem, treat the material density as the design variable and use an emprical formula to penalize the intermediate density, use the adjoint variable method for the sensitivity analysis. It is shown that this method provides results comparable with those in the literature and it can be used for the weight reduction of structural components. Yang et al. [85] used the topology optimization for obtaining the best layout of vehicle structural components to achieve predetermined performance goals. Many vibration isolators are made of rubbers and they operate under small oscillatory load superimposed on large static deformation. Vibration isolators must have a certain degree of static stiffness in order to endure the static loading due to large gravitational and inertial forces. On the other hand, isolators must have a small dynamic stiffness in order to reduce the force transmission from vibrating systems to base structures. Therefore, both the static and dynamic behaviours of rubber should be simultaneously considered in the design process. The static behaviours of rubber under large and slow loads are generally treated with hyperelastic constitutive models. Rubber under fast dynamic loads can be modelled as a viscoelastic material. Lee and Youn [86] proposed a topology optimization approach for rubber vibration isolators under small oscillatory loads superimposed on large static deformation. Study considers both the static and dynamic performance for the structural stability and low transmissibility of the isolation systems. Andreassen et al. [87] presented an efficient 88-line MATLAB® code for topology optimization. Hsu and Hsu [88] presented a generalized topology optimization process and considered several fundamental issues on the quality of topology optimization results in order to achieve a clear topology optimization result. Swan and Rahmatalla [89] introduced a new methodology to solve large-size sparse systems in continuum topology optimization framework with relatively very low computational costs. Structural topology optimization was used to design structures subject to multiple kinds of physical phenomena such as static loads, free vibrations, forced vibrations, thermal loads, heat conduction, and many others. Structures of different configurations, i.e., trusses, beams, plates, shells and solids were designed using these techniques with great success. In addition, the sizes of the structures varied widely, from large airplanes and automobiles, to tiny micro electro mechanical systems (MEMS). Soto [90] reviewed the evolution of the subject of topology optimization of continuum structures since 1988. It is shown that structural topology optimization is a field that has many applications in industry. Fancello [91] studied an approach to the topology mass minimization of a body submitted to local material failure constraints, contact boundary conditions, and multiple load cases. Holmberg et al. [92] developed and evaluated a method for handling stress constraints in topology optimization. The stress constraints are used together with an objective function that minimizes mass or maximizes stiffness. They claim that it is not sufficient to optimize the structure for maximum stiffness and then continue with local shape optimization to remove stress concentrations. Instead, stress constraints to be considered from the very beginning. In general, optimizations subject to material failure constraints are difficult to solve because of the large number of nonlinear constraints that form highly nonlinear and discontinuous feasible regions. However, it is important to investigate these problems, since minimizing mass subject to failure constraints is the objective of many structural design problems. Lee et al. [93] studied a comparison of mass-constrained compliance minimization solutions and stress-constrained mass minimization solutions, with both fixed loading and design-dependent loading. The results are compared with those of compliance minimization problems for the same geometries and loading. Tsai and Cheng [94] proposed a technique for determining the material distribution of a structure to obtain desired eigenmode shapes for problems of maximizing the fundamental eigen frequency. Aside from maximizing the fundamental frequency, a method to modify existing eigenmodes to continuously evolve and assume the same shapes as the desired modes within the optimization process is proposed. Bruggi and Duysinx [95] dealed with a formulation for the topology optimization of elastic structures that aims at minimizing the structural weight subject to compliance and local stress constraints.

1.2.6 Shape optimization

The fundamental aim of shape optimization involves the optimal distribution of mass in space observing global and local design constraints. Mlcjnek and Schirrmacher [96] used energy approach for the evaluation of material properties. The application is extended to three-dimensional domains. Multiple loading cases are also considered. In many cases, structures optimized for multiple loads are more stable and robust than designs optimized for a single purpose. Since designs are usually required to perform in more than one environment, the ability to consider multiple load cases within the framework of shape optimization using a homogenization method should be added to the appeal of the strategy. Diaz and Bendsoe [97] presented a formulation for shape optimization of elastic structures subject to multiple load cases. The problem is solved using a homogenization method. Torigaki et al. [98] developed an optimization system by adopting a general purpose finite element analysis code. By developing the design optimization system that consists of modularized programs of optimization and homogenization with a general purpose finite element code, they made an effort to link up the optimization method using the homogenization method with complicated automotive designs. Component shape optimization normally requires a parameterized geometric representation or a generic model for the solid geometry, which evolves to an optimal design. Generic models for large-scale three-dimensional components are difficult to build. The difficulties result from the lack of robust automatic mesh generation and the availability of a parametric model. To remedy this problem, a basis function concept used in mathematics for representing an arbitrary function is employed for geometric representation of solids by Yang et al. [99]. Their approach does not require automatic mesh generation or parametric models for geometric representation and thus is suitable for large-scale complicated components. Numerical examples are used to demonstrate the applicability of this approach to realistic problems. Zhang [100] developed the modified thermal load approach for automatically generating basis vectors in structural shape optimization. Currently this approach is being used in automotive industry for vehicle weight reduction. Applications include body structures, chassis components and powertrain structures. The convergence characteristics and efficiency of the approach are demonstrated through numerical examples. The MSC/NASTRAN® is used for shape design (grid) sensitivity analysis. Uysal et al. [101] developed a finite element-based shape optimization program for three-dimensional shell structures and performed the shape optimization of shell structures. The shape optimization program is implemented by a job control language and a reliable finite element package program, i.e., ANSYS, is used for structural analysis. To achieve the shape optimization, different principles such as structural analysis, automatic mesh generation, sensitivity analysis and mathematical programming are inter-related. The objective was to minimize the weight of shell structure under constraints that are the maximum value of the von Mises stress in each element and move limits (extra constraint equations) for each design variable.

The design sensitivities are calculated using the finite difference method. The search for the final shape of a structure is performed using the linear programming technique. Arnout et al. [102] studied the parameter free approach by using the FE-based data as design variables, such as nodal coordinates and nodal thickness. During shape and thickness optimisation, this approach provides more design freedom for a limited modelling effort. Most shape optimization methods require parametric modeling and automatic mesh generation. The reduced basis method was introduced in shape optimization because it does not require the parametric modeling and auto-meshing. For this reason, it has found wide applications in the automotive industry. Zhang [103] studied the shape optimization capability in MSC/NASTRAN®. The Modified Thermal Load Approach (MTLA) for generation of shape basis vectors is described. A procedure is developed for generating and inputting these basis vectors to the MSC/NASTRAN®. The convergence characteristics and efficiency of incorporating MTLA in MSC/NASTRAN® optimization process are demonstrated through numerical examples. Haslinger and Makinen [104] studied an elementary mathematical introduction to Sizing and Shape optimization (SSO) problems by using the topics such as the existence of solutions, appropriate discretizations of problems, and convergence properties of discrete models. Also they dealed with modern computational aspects in shape optimization like sensitivity analysis and gradient, evolutionary, and stochastic type minimization methods, including methods of multiobjective optimization. They also presented nontrivial applications in various areas of industry such as contact stress minimization for elasto-plastic bodies, multidisciplinary optimization of an airfoil, and shape optimization of a tube. Azegami et al. [105] presented a numerical solution for shape optimization problems for link mechanisms, such as a piston-crank mechanism. Finally, they illustrated that reasonable shapes of links were obtained by their approach.

1.2.7 Design sensitivity analysis

Kleiber et al. [106] studied on solution tools for problems in mechanics, which have complex geometries, unilateral boundary conditions and complicated, highly nonlinear material behaviour. The influence of uncertainty among the characteristics of a problem was also addressed. This area of research is termed sensitivity analysis (SA) and examines the relationships between the parameters describing a system and its behaviour or response function. Yadava et al. [107] studied on the sensitivity derivatives of static responses. When studying a mathematical model it is not enough to compute individual solutions. It is equally important to determine systematically the influence of parameter variations on these solutions. The main task of sensitivity analysis is to identify critical parameter dependencies. Ostermann [108] reviewed the basic ideas of sensitivity analysis for deterministic models and emphasized the method of internal differentiation. Design optimisation methodologies (DOM) are showing in the last decade an impressive growth due to the ever increasing hardware capabilities and evolution of numerical methods. The use of the DOM has spread out in an experimental way, involving new application fields and leaving the aerospace field where they originated. One of the most interested industrial sectors is the automotive one. The design requirements of automotive industry are quite different from those of aerospace industry and the wide integration and employment of design optimisation methodologies are strictly connected to the possibility of satisfying them. The automotive industry is characterised by a strong competitiveness that leads to looking for technological improvement of its products. The design process of all products is continuously revised to achieve shorter time-to-market, higher total quality and lower costs. Chiandussi et al. [109] proposed a new approach to eliminate the verification phases and to unify the design process in one single phase where all disciplines involved in the definition and realisation of a certain product are considered together. Their method was to solve a multidisciplinary structural design optimisation problem involving linear and non-linear responses. The structure analysed in their study was the front control arm of a mid size commercial car. Lee and Lim [110] presented a method of direct differentiation for calculating the sensitivity coefficients in regard to the governing equation and the second-order perturbed equation. Static and dynamic response of random system including uncertainties for the random variable is calculated with the second-order perturbation method applied to the original governing equation. Zhang, [111] developed a tool to deal with sizing sensitivity analysis of linear and geometrically nonlinear problems. The capabilities of sensitivity analysis are developed as a general interface based on the ABAQUS code. In order to calculate the design sensitivities, Ghouali et al. [112] developed a new Local Analytical Sensitivity Analysis (LASA) under a rigorous mathematical basis by considering the highly nonlinear forging. Generally, the purpose of a sensitivity analysis is to determine which input parameters exert the most influence on model results. This information, in turn, allows unimportant parameters to be eliminated and provides direction for further research in order to reduce parameter uncertainties and increase model accuracy. Hamby [113] made a comprehensive review of more than a dozen sensitivity analysis methods of environmental models.

1.2.8 Structural design sensitivity analysis

Design sensitivity analysis of structures deals with the calculation of response derivatives with respect to design variables. These derivatives, called the sensitivity coefficients, are used in the solution of various problems. In design optimization, the sensitivity coefficients are often required to select a search direction. These coefficients are used also in generating approximations for the response of a modified system. In addition, the sensitivities are required for assessing the effects of uncertainties in structural properties on system response. Calculation of the sensitivities involves much computational effort, particularly in large structural systems with many design variables. As a result, there has been much interest in efficient procedures for calculating the sensitivity coefficients. Developments in methods for sensitivity analysis are discussed in many studies. Methods of sensitivity analysis for discretized systems can be divided into the following classes:

a. Finite-difference methods, which are easy to be implemented but might involve numerous repeated analysis and high computational cost, particularly in problems with many design or response variables. In addition, finite-difference approximations might have accuracy problems. The efficiency can be improved by using fast reanalysis techniques.

b. Analytical methods, which provide exact solutions but might not be easy to implement in some problems such as shape optimization.

c. "Semi-analytical" methods, which are based on a compromise between finite-difference methods and analytical methods. These methods use finite-difference evaluation of the right-hand-side vector. They are easy to implement but might provide inaccurate results.

In general, the following factors are considered in choosing a suitable sensitivity analysis method for a specific application: the accuracy of the calculations, the computational effort involved and the ease-of-implementation. The implementation effort is weighted against the performance of the algorithms as reflected in their computational efficiency and accuracy. The quality of the results and efficiency of the calculations are usually two conflicting factors. That is, higher accuracy is often achieved at the expense of more computational effort.

However, most approximations that are adequate for structural reanalysis are not sufficiently accurate for sensitivity analysis. In their study, Kirsch et al. [114] used approximate reanalysis to improve the efficiency of dynamic sensitivity analysis by finite-differences. Given the results of exact analysis for an initial design, the displacements for various modified designs are evaluated efficiently by the recently developed Combined Approximations (CA) approach. The sensitivity coefficients give engineers an important design tool for systematically improving design without using the time-consuming trial and error method. The multi-disciplinary considerations reduce the coordination between organizations and thus reduce the number of design cycles. Huang et al. [115] studied on a multi-disciplinary, sensitivity based design process. It provides a means for systematic weight reduction and quality improvement. Two design sensitivity analysis methods (i.e., the direct method and adjoint variable method) are presented and integrated into the design process based on their efficiency. The adjoint variable method is most useful in an early design stage where many design alternatives are tested. As for the direct method, it is more efficient when the number of design variables is small. This method is most suitable in the final, detailed design stage where all design constraints are considered and all important parameters are identified. The new design process was followed for the weight reduction of advanced truck frame designs. Dias and Pereira [116] presented an analytical sensitivity analysis methodology based on the direct differentiation method for rigid-flexible multibody systems. In the case of rigid-flexible multibody systems, it is found that numerical sensitivities may diverge from those obtained analytically. This fact is characteristic for the sensitivity analysis of periodic motions, when periods or frequencies of vibrations depend on design variables. Barbato and Conte [117] studied on the comparison of procedures for computing response sensitivities to material and discrete loading parameters for displacement-based and force-based materially non-linear by using finite element models of structural frame systems. Structural design sensitivity analysis concerns the relationship between design variables available to the design engineer and structural responses determined by the laws of mechanics.

The dependence of response measures such as displacement, stress, strain, natural frequency, buckling load, acoustic response, frequency response, noise-vibration-harshness (NVH), thermoelastic response and fatigue life on the material property, sizing, component shape, and configuration design variables is implicitly defined through the governing equations of structural mechanics. Choi and Kim [118] studied on first- and second-order design sensitivity analysis for static and dynamics responses of both linear and nonlinear structural systems, including elastoplastic and frictional contact problems. They presented design sensitivity analysis (DSA) theory and numerical implementation to create advanced design methodologies for mechanical systems and structural components, which will permit economical designs that are strong, stable, reliable, and have long service life; requiring highly sophisticated mathematics. Cacciola et al. [119] presented a method for the evaluation of response sensitivity of both classically and non-classically damped discrete linear structural systems under stochastic actions. Petrov [120] proposed a method to calculate, for a strongly nonlinear structure with friction contact interfaces, sensitivity of nonlinear forced response levels to variation of parameters of the friction contact interfaces, excitation forces and design parameters affecting dynamic properties of linear components of the assembled structure. The effectiveness of the method allows the first and second order sensitivity coefficients to be calculated simultaneously with the calculation of forced response without a significant increase of the computation effort. Cho and Jung [121] studied a continuum-based design sensitivity analysis (DSA) method for geometrically nonlinear systems with nonhomogeneous boundary conditions to topologically optimize the displacement–loaded nonlinear structures. Zhang and Domaszewski [122] presented a new efficient sensitivity analysis procedure for the optimization of shell structures without access to the finite element source code. The implementation is performed based on the ABAQUS® code. Feehery et al. [123] studied on a new algorithm and software for numerical sensitivity analysis of differential-algebraic equations. Kim et al. [124] developed a continuum-based configuration design sensitivity analysis method for dynamics of multibody systems. Haftka and Mroz [125] used the principle of virtual work to find the sensitivity derivatives of structural response with respect to stiffness parameters. In the sensitivity techniques, the adjoint variable method is quite popular because it reduces computation time and save computer resources. Commonly, the adjoint variable method employs exact analytical differentiation with respect to design variables. However, it can be cumbersome to precisely differentiate every given type of finite element. For improving this trouble, the numerical differentiation scheme can replace this exact manner of differentiation. Even though the numerical differentiation has some advantages, it suffers severely from in accuracy due to the perturbation size dilemma. Kima and Cho [126] employ a complex variable, which is not much influenced by the perturbation size. Then, the adjoint variable method combined with complex variables is applied to obtain the shape and size sensitivity for structural optimization. They provided a robust design sensitivity method by combining the adjoint variable method and the complex variable method (ACVM) in the eigenvalue problem. Adjoint variable method (AVM) is efficient and save computation time compared to other sensitivity schemes because it calculates the sensitivity values only in position that analyzer is willing to obtain. Moreover, once the adjoint variable is obtained, it can be successively or repeatedly used for the calculation of the sensitivity regardless of the design variable. Keulen et al. [127] reviewed options for structural design sensitivity analysis including global finite differences, continuum derivatives, discrete derivatives and computational or automated differentiation, in the context of accuracy and consistency, computational cost, and implementation options and effort. The global finite difference method is found to be the most convenient in implementation, but high cost and difficulty in finding appropriate perturbation size are disadvantages. The continuum method has advantages in theoretical soundness, low cost, consistency and possible different meshes for response and sensitivity. However, it requires more mathematical understanding. The discrete method has advantages in low cost and consistency, but has disadvantages in the requirement of the source code and dependence on perturbation size for the semi-analytical method. The computational derivative is found to be the most consistent among four methods. However, computational cost is usually higher than other methods and practical for small sized programs. The reliability and accuracy of parametric sensitivity results greatly depend on the perturbation scheme used to vary the parameter values and on the underlying assumptions about the model and/or the parameters. Sulieman et al. [128] conducted a comparison between three methods of parametric sensitivity in a multi-response nonlinear parameter estimation setting. The three methods investigated were as follows: conventional marginal sensitivity coefficient, profile-based parametric sensitivity measure and classical Fourier Amplitude Sensitivity Test (FAST). Noora et al. [129] studied a computational procedure for evaluating the sensitivity coefficients of porous viscoplastic solids under dynamic loading conditions. In their study, Schwarz and Ramm [130] considered the contribution of structural non-linearities like finite deformations, buckling or plasticity in the optimization process. Krishnakumar and Hoole [131] studied on a flexible parameterized mesh generator for optimization to model moving (i.e., optimized) shapes. The single algorithm presented covers various kinds of movement at once. The problem area of discontinuous objective functions was previously introduced and elastic deformation accompanied by a structural mapping was introduced to enforce the required rules with meshes, but the process is time consuming and involves repeatedly solving a larger structural problem rather than the immediate electromagnetic field problem at hand. The proposed and demonstrated mesh generator allows repeated solutions with iterated meshes so that it can be employed in a first-order optimization strategy exploiting its faster convergence rates. Kuo et al. [132] examined the relation between the vehicle body overall stiffness/strength characteristics and fatigue life. They also demonstrated how the MSC/NASTRAN® design sensitivity analysis capability can be employed to effectively identify design variables most affecting fatigue life through the body overall stiffness/strength evaluations and which lead to an improvement in fatigue life of a vehicle body structure when changed. The methods and concepts are demonstrated using a very simplified finite element model, which conceptually simulates a body structural system. Choi and Kim [133] worked on design sensitivity analysis of nonlinear structural systems using continuum design sensitivity analysis methods.

1.2.9 Structural design sensitivity analysis with SVD

The sensitivity and load matrices contain full sensitivity information, which can be analysed and interpreted using their SVDs. The most significant part of the input-output information can be identified by this technique. The information content of the decomposed sensitivities was demonstrated on the example of model reduction by Gerzen and Barthold [134]. They reduced model size down to five percent of all design variables and still had reasonable results. The SVD-based analysis is well suited to study the directional properties of inputs and outputs of a system. If an input is distributed in the direction of a right singular vector; the system response will be distributed to the system degrees of freedom in the direction of associated left singular vector with a gain that is equal to the corresponding singular value. Muğan [135] studied the input–output relationships of structures by using the SVD with an emphasis to localization and curve veering phenomena. As a result of singular-vector localization, the distribution of system's energy among different degrees of freedom changes drastically and abrupt changes in the outputs are observed in response to small changes in the input vector and the excitation frequency. It is shown that the power and energy transmission ratios between the input and output vectors in a system are bounded by the squares of the maximum and minimum singular values of the system, which do not change significantly as the number of oscillators increases for tuned systems. Additionally he showed that the first singular value has a special meaning since it is the largest system gain and corresponding right and left singular vectors give, respectively, the worst possible load case and the corresponding system response. While eigenvalue-based analysis give information about the resonance frequencies and vibration modes of a structure, singular values of the structure are related to the forced response characteristics and give the dynamic behavior in the frequency domain. Ersoy and Muğan [136] developed design sensitivity analysis based upon the singular value decomposition (SVD), which can be employed for static response, dynamic response and eigenvalue design sensitivity analysis of structures. The proposed sensitivity analysis was compared with the conventional techniques. For the static and dynamic response of a structure, it was shown that since the singular values σ_i determine displacement magnitudes in a structure, minimization of σ_1 is equivalent to minimization of the static and dynamic response magnitude of the structure. As the squares of singular values σ_i are directly related to power, energy and power spectral density ratios between the input and output vectors in a structure, shaping the singular values is the key to shaping the response of the structure. Furthermore, there they found that there was a close relationship between the σ_i loci and transfer function matrix components $| \sigma_{ij} |$ loci in the frequency domain. On the other hand, singular vectors are directly related to input–output directional relationships in a structure; they tell us how the outputs are related to the inputs, that needs to be further investigated in future studies as well. In comparison to eigenvalue design sensitivity analysis that is valid only at resonance frequencies, singular value based design sensitivity analysis yields more information than eigenvalue design sensitivity analysis and enables to study the dynamic behavior of a structure in frequency domain completely. In sum, they found out that SVD based design sensitivity analysis can give good insight into static and dynamic response of structures. In particular, it is computationally advantageous in case of multiple load cases and finding the worst case loading and sensitivity bounds of a structure. Since these calculations only require the smallest and largest singular values and corresponding singular vectors rather than all of them, it is computationally cheap to employ the SVD based sensitivity analysis.

Ersoy in his PhD thesis [218] used SVD for design sensitivity analysis of structures and developed a new method. He made comparison of the proposed sensitivity method with the conventional techniques. In the beginning of the thesis, basic ideas of the finite element structural analysis methods are presented then conventional methods such as design sensitivity analysis of static response, eigen values, and dynamic response are presented. Also SVD is used in finite element analysis for time-independent and time-dependent problems. It is shown how singular values can be used to give a frequency domain characterization for the limits to some appropriately defined gains. As the squares of the singular values are the bounds of power, energy and power spectral density ratios between the input and output vectors. Squares of the biggest and smallest singular values respectively σ_1^2 and σ_n^2 are shown as the average power ratio for periodic input signals and the energy ratio limit for non-periodic input signals according to Equations (4.4) and (4.5) and [218] page 26. As a structural analysis technique, the SVD is also applied to optimum laminate design problem of composites.

The difference of our thesis from the above study is that we have worked on the design sensitivity reanalysis and additionally we employed SVD with SMW formulas to make a reanalysis of the sample beam structure by using the SQP optimization algorithm for size optimization. In our thesis, fast reanalysis formulas based on SVD and its extension to optimum search directions in optimization algorithms are presented. In addition, we performed another numerical analysis on a truck chassis frame to prove the effectiveness of the SVD involvement in above mentioned optimization method as a comparison to the conventional SQP method.

1.2.10 Structural reanalysis

Kirsch et al. [137] developed a preconditioned conjugate gradient (PCG) method that is most suitable for reanalysis of structures. The method presented is easy to implement and can be used in a wide range of applications, including non-linear analysis and eigenvalue problems. Structural reanalysis aims to determine the variations in the displacement of a structure due to the addition or deletion of elements without solving the full degrees of freedom. The iterations change the design parameters at each step and utilize the factorization of stiffness matrix of initial design. Lee and Eun [138] studied a new reanalysis method to determine the additional forces that act on the initial structure and the displacements of the modified structure. It utilizes the compatibility conditions at the interfaces between the initial structure and the added or deleted members as static constraints, and applies the generalized inverse method to describe the static behavior of the constrained structure. Wu and Li [139] focused on static reanalysis of a structure with added DOFs where the nodes of the original structure form a subset of the nodes of the modified structure. The single step perturbation method is a recently developed structural dynamic modification technique. Ravi et al. [140] applied single step perturbation to complex structures. In vibration mode superposition analysis, the main computational effort is spent in the solution of the eigenproblem. In reanalysis procedures, this solution must be repeated for each change in the design. Kirsch and Bogomolni [141] showed in their study how the combined approximations (CA) method can be used to improve the efficiency of some common iterative procedures. Kirsch et al. [142] showed how the combined approximations approach, developed originally for linear static problems, can be used to obtain effective solutions of non-linear dynamic reanalysis problems. Wua et al. [143] focused on the reanalysis of structures with added degrees of freedom. Jang [144] presented a procedure to reanalyze a damaged structure using a finite-element force method of analysis. Perturbation analysis of constrained least-squares problems was adapted to handle reanalysis by the force method, and related theoretical and numerical results were presented. Kirsch and Papalambros [145] studied a unified approach for accurate approximations of displacements and displacement derivatives with respect to design variables. The solution procedure is based on the results of a single exact analysis at an initial design. Unlike common approximations of the structural response, the approach presented is not based on calculation of derivatives. Rather, approximations of displacements are used to evaluate displacement derivatives at various modified designs. Kirsch et al. [146] used the CA approach, developed originally for linear static reanalysis, can be used effectively for dynamic reanalysis. Most structural reanalysis methods developed in the past are suitable for the relatively simple case where the number of DOF’s is unchanged. Kirsch and Papalambros [147] presented the reanalysis approach suitable for problems where the number of DOF’s and the sizes of the stiffness matrix and the load vector are significantly changed. A unified approach for reanalysis of all types of topological modifications is presented.

Liu et al. [148] presented an approach for structural static reanalysis with unchanged number of degrees of freedom. They studied PCG method and a new preconditioner is constructed by updating the Cholesky factorization of the initial stiffness matrix with little cost. Reanalysis methods can be divided into two categories: approximate methods and direct methods.

Approximate reanalysis methods provide approximate solutions of the response of the modified structure using the information obtained during the full analysis of the original structure. These methods are applicable to modifications where the changes in design variables are small in magnitude, yet may significantly influence a large portion or the entire stiffness matrix. The approximate methods can be divided into the following four classes: local approximations, global approximations, CA and PCG approximations. The precondition technique is especially efficient in dealing with cases where small parts of elements are significantly modified while their major parts are slightly modified. In particular, when the number of the modified elements is small, a direct method can be established by utilizing the procedure of the construction of the preconditioner.

Direct methods give exact closed-form solutions and are suitable for cases where the changes in design variables are large in magnitude, yet only affect a relative small number of elements. Most of these methods update the inverse of the modified stiffness matrix using SMW formulae. Direct methods are inefficient when there are changes in many elements of the stiffness matrix. In our thesis, we used direct method and proved the efficiency even with many elements changes.

Zuo et al. [149] reviewed Fox and Kirsch’s static reanalysis methods and then presented a new hybrid Fox and Kirsch’s reduced basis method for structural static reanalysis. The hybrid method combines the merits of Fox’s polynomial fitting reanalysis and Kirsch’s combined approximations reanalysis and has the advantage of global-local approximation. For the large modification, the hybrid method generally has higher accuracy than Kirsch's method at the same computational cost. Moreover, the hybrid method does accelerate the process of structural optimization using genetic algorithm and slightly affect the accuracy of the optimal solutions. Hybrid method is a universal format of reduced basis and has the advantage of global-local approximation. The presented numerical results demonstrate that the hybrid method achieves the highest accuracy for large modification of structure. As a last step in the study, reanalysis methods are used to speed up GA-based structural optimization. In our thesis, we use the SVD with SMW formulas to speed up the reanalysis process by using the SQP algorithm.

Li et al. [150] compared several reanalysis methods for structural layout modifications with added degrees of freedom. These methods include the modified initial analysis methods, the modified initial analysis method with a scalar multiplier, and the PCG method. The high computational effort of solving multiple FE analysis is decreased by utilizing reanalysis procedures. Amir et al. [151] presented an efficient approach to robust topology optimization. While addressing two representative design problems, they also demonstrated that the benefits of applying a robust formulation can be achieved for a significantly reduced computational cost. Reanalysis is needed in many areas such as structural optimization, analysis of damaged structures, nonlinear analysis, probabilistic analysis, controlled structures, smart structures and adaptive structures. It is related to a wide range of applications in such fields as Aerospace Engineering, Civil Engineering, Mechanical Engineering and Naval Architecture. In a typical structural design process, the analysis must be repeated numerous times due to changes in the size of elements, the material properties, the geometry of the structure (coordinates of joints), the topology (number and orientation of elements and joints) and support conditions. The high computational cost involved in repeated analysis is one of the main obstacles in the solution of structural optimization problems, and only methods that do not involve much time consuming analysis are useful. In his book, Kirsch [152] deals with the problem of multiple repeated analysis (reanalysis) of structures that is common to numerous analysis and design tasks. In another book, Kirsch [153] summarized many years of research and developments on reanalysis of structures. For many years of development, the virtual distortion method (VDM) has proved to be a versatile reanalysis tool in various applications, including structures and truss-like systems.

Kołakowski et al. [154] presented a summary of principal achievements, demonstrating the capabilities of the VDM in both statics and dynamics, in linear and nonlinear analysis. Huang et al. [155] studied on a new modal reanalysis method for topological modifications of general finite element systems. Leu and Tsou [156] proposed a reduction method for the nonlinear dynamic analysis of framed structures. Redesign or inverse design is the process of generating a new optimal design, which satisfies performance specifications starting from a baseline design with undesirable performance. Koo and Bernitsas [157] studied the Large Admissible Perturbation (LEAP) methodology, which makes it possible to redesign a structure for large changes in performance objectives and redesign variables without trial and error or repetitive finite element analysis. Chen and Rong [158] presented an effective and efficient procedure for extracting vibration eigenpairs of topologically modified structures without resolving the new eigenproblem. Chen et al. [159] reviewed and compared five approximate methods for eigenvalue reanalysis of the modified structures. These are the second-order perturbation method, Bickford's method, Chen's method, and two hybrid methods called as, the Pade approximate method and the extended Kirsch method. Structural dynamic modification techniques can be defined as methods by which dynamic behavior of a structure is improved by predicting the modified behavior brought about by adding modifications like those of lumped masses, rigid links, dampers, beams etc. or by variations in the configuration parameters of the structure itself. The methods of structural dynamic modification, especially those with their roots in finite element models, are often described as reanalysis techniques. Trisovic et al. [160] presented the problem of improving dynamic characteristics of structures. New dynamic modification procedure was given as using distribution of potential and kinetic energy in every finite element for analysis. The main goal of dynamic modification is to increase natural frequencies and to increase the difference between them. Hea et al. [161] presented two new modal reanalysis methods for topological modification of undamped and damped structures. Those presentations are focused on the most challenging case of addition of joints, in which the structural model and the number of DOF are changed obviously. For reanalysis of real mode, because there simultaneously exist decoupling effect and coupling effect for eigenvector corresponding to the old DOF and the newly added DOF, mass orthogonality should be performed twice to obtain better eigenvalues and eigenvectors based on the result of improved dynamic condensation and the Kirsch approximation. As for reanalysis of complex mode, we can obtain good lower eigenpairs by Rayleigh-qotient inverse iteration based on the result of complex eigensubspace condensation. The availability of the above two newly presented reanalysis methods was proved by numerical examples. It is well known that finite element predictions are often called into question when they are in conflict with test results. The area known as model updating is concerned with the correction of finite element models by processing records of dynamic response from test structures. Model updating is a rapidly developing technology. Mottershead and Friswell [162] made a survey of model updating in structural dynamics. Trisovic et al. [163] dealed with the problem of improving dynamic characteristics of some structures. New dynamic modification procedure was also given as using distribution of potential and kinetic energy in every finite element for analysis. The main goal of their study on dynamic modification was again to increase natural frequencies and to increase the difference between them. The quality of the approximation and efficiency of calculations are usually two conflicting factors in selecting an approximate reanalysis model. This is also true in the approximate methods presented. Levy et al. [164] presented a solution procedure for reanalysis using a mixed exact-approximate approach. The procedure, which is based on the reduced basis method, uses an efficient exact reanalysis method to handle a limited number of dominant member area and topological changes. A modified initial design (MID), associated with which is a recalculated inverse of the modified stiffness matrix, is the starting point for the approximate technique. Han [165] studied on reanalysis using frequency response functions for correlating and updating dynamic systems. Arora [166] studied the response of the modified structures. Kassim et al. [167] reviewed the methods of static reanalysis of structures. Kirsch and Papalambros [168] studied a reanalysis method for highly nonlinear geometrical changes in structures. Kirsch [169] presented a unified reanalysis approach for structural analysis, design, and optimization that is based on the CA method. The method is suitable for various analysis models (linear, nonlinear, elastic, plastic, static, dynamic), different types of structures (trusses, frames, grillages, continuum structures), and all types of design variables (cross-sectional, material, geometrical, topological). The calculations are based on results of a single exact analysis. The computational effort found to be usually much smaller than that needed to carry out a complete analysis of modified designs. Accurate results were achieved by low-order approximations for significant changes in the design. Cacciola and Muscolino [170] studied on a procedure for determining the stationary first and second order response statistical moments of linear behaving modified systems under multi-correlated stationary Gaussian processes. Tao et al. [171] developed a new procedure for structural vibration (or eigenproblem) reanalysis based on iteration and inverse iteration method with frequency-shift and linear combination acceleration to reduce the high computational cost of structure reanalysis. With a suitable frequency-shift factor, the Frequency-Shift Combined Approximations (FSCA) method allows to calculate higher modes accurately. In lightweight structure design, vibration control is necessary to meet strict stability requirements and to improve the fatigue life of structural components. Due to ever-increasing demands on products, it is generally more convenient to include vibration prerequisites in a design process instead of using vibration control devices on fixed designs. One of the main difficulties associated to design optimization of complex and/or large structures is the numerous computationally demanding Finite Element (FE) calculations. Perdahcıoglu et al. [172] studied a strategy for efficient and accurate optimization of vibration characteristics of structures. Massa et al. [173] studied on the modal reanalysis of structures subjected to multiple modifications of various origins, which can greatly affect the mode shapes of these structures. Xu et al. [174] presented an adaptive reanalysis approach for GA structural optimization, extended from Kirsch's CA method. Haifeng et al. [175] studied on a preconditioned Richardson's iterative method for structural static reanalysis.

1.2.11 Sherman-Morrison-Woodbury (SMW)

Castillo et al. [176] studied on the problem of updating the inverse of a matrix. Several methods which allow calculating the inverse of a matrix when one or several rows (columns) are changed, or one or several rows and the same number of columns are added or removed were given. Riedel [177] studied a Sherman Morrison Woodbury (SMW) identity for rank augmenting matrices with application to centering. Kastner [178] studied on the SMW technique the "Add-On" method. The Sherman-Morrison-Woodbury formulas express the inverse of a matrix after a small rank perturbation to the inverse of the original matrix. The history of these formulas is presented by Hager [179] and various applications to statistics, networks, structural analysis, asymptotic analysis, optimization, and partial differential equations were discussed.

1.2.12 Structural reanalysis with SMW

Cha and Yoder [180] applied Sherman-Morrison-Woodbury formulas to analyze the free and forced responses of a linear structure carrying lumped elements.

1.2.13 Structural design sensitivity reanalysis

Reanalysis of static response and static response design sensitivity is of significance in the optimal design of structures. Liu and Chen [181] proposed a reanalysis method of static response and static response design sensitivity of locally repeateadly modified structures. By partitioning global stiffness matrix in terms of degrees of freedom affected directly by modification of structural parameters and matrix triangle factoring, the dimension of the equilibrium equation for reanalysis is decreased. Calculation of design sensitivities often involves much computational effort, particularly in large structural systems with many design variables. Approximation concepts, which are often used to reduce the computational cost involved in repeated analysis, are usually not sufficiently accurate for sensitivity analysis. Bogomolni et al. [182] used approximate reanalysis to improve the efficiency of dynamic sensitivity analysis. Using modal analysis, the response derivatives with respect to design variables are presented as a combination of sensitivities of the eigenvectors and the generalized displacements. A procedure intended to reduce the number of differential equations that must be solved during the solution process was proposed. Yang et al. [183] studied a modal reanalysis method for topological modifications of general finite element systems. In their method, all the three cases of the topological modifications, the number of DOFs is unchanged, decreased, and increased, was considered. One of the main obstacles in the solution of structural optimization problems is the need to repeat solutions of the analysis and sensitivity analysis equations. In large-scale structures, having complex analysis models, the computational effort may become prohibitive. To alleviate this difficulty, a general approach for repeated analysis and repeated sensitivity analysis, called combined approximations, was developed by Kirsch [184].

1.2.14 Design sensitivity reanalysis with SMW

It was summarized the various formulations and solution procedures for reanalysis and sensitivity reanalysis of linear, nonlinear, static and dynamic systems in Akgun et al. [185] where several exact fast static structural reanalysis techniques, introduced by researchers mostly for truss structures and some for frames and plate structures were reviewed. Their study showed that these methods are variants of the well-known SMW formulas for the update of the inverse of a matrix.

1.2.15 Thesis review

In this thesis, SVD based structural optimization is revisited. Differently from the studies presented in literature, during the multiobjective optimization process, SVD approach is applied to structural optimization and design sensitivity reanalysis for sample structures.

For structural optimization of a given structure, that is simultaneously loaded with multiple load cases, SVD is applied to the optimization problem and the outcome is compared to a conventional multiobjective optimization method. As a result, SVD is proved out to be a faster method for the CPU timing in comparison to the conventional method.

Secondly, SVD is applied to the SMW formulas used in a structural design reanalysis of a similarly loaded structure. Derived formulations allow fast reanalysis of structures, especially it is very advantageus in the existence of multiple load case. Results showed that SVD is much faster than the conventional design sensitivity reanalysis methods in terms of CPU timings.

Main advantage of the proposed method is that while the conventional methods are applying all the load cases following each other for each optimization iteration step, SVD can find the worst possible load case with the help of singular vectors and singular values. Therefore, it enables the optimization program to create the Pareto optimal sets faster than the conventional optimization methods and it saves CPU time.

For this reason SVD based optimization approach is found to be advantageous than the conventional optimization methods, which is the main contribution of this thesis.

As a further study; embedding the SVD based algorithm in a multi-objective optimization program for structural optimization and design sensitivity analysis will enable the user to create the Pareto optimum sets of the design parameters accurately in a shorter time. In addition, the derivations that enable one apply SVD based optimum parameter increment to optimum search direction calculation are also presented.

2. OPTIMIZATION METHODS

In system level optimization, objective function is generally the function of multiple variables. It is very common in engineering problems that there are multiple objective functions to be minimized simultaneously, that require special optimization methods. Besides, some constraints are also imposed on design variables.

2.1 Mathematical Definition of Optimization Problems

Mathematical definitions of optimization problems include the objective function and constraints. Lets show the so called objective function to be obtimized with "**y**". This "**y**" function is a function of so called independent variables x_1, x_2, x_n i.e., design variables. In this case, the objective function can be written as

$$\boldsymbol{y} = y(x_1, x_2, \ldots, x_n) \tag{2.1}$$

objective function can be more than one in quantity. For example

$$\boldsymbol{y} = \begin{cases} \sigma_{max}(Von\ Mises)\ or\ w_1(first\ natural\ frequency) \\ T_{\max}\ (maximum\ temperature) \\ W(weight) \end{cases}$$

Constraints exist in various events in physics.

Some of them are in equality form as follows

$$\phi_1 = \phi_1\,(x_1, x_2, \ldots\ldots, x_n) = 0 \tag{2.2}$$

$$\vdots \qquad \vdots$$

$$\phi_m = \phi_m\,(x_1, x_2, \ldots\ldots, x_n) = 0 \tag{2.3}$$

Some of them are in inequality form as follows

$$\varphi_1 = \varphi_1 (x_1, x_2, \ldots\ldots ,x_n) \le L_1 \quad \textbf{(2.4)}$$

$$\vdots \qquad \vdots \qquad\qquad\qquad \vdots$$

$$\varphi_i = \varphi_i (x_1, x_2, \ldots\ldots\ldots ,x_n) \le L_i \quad \textbf{(2.5)}$$

For example

$$\sigma_{\max} \le \sigma_{yield}$$

$$w_{\min} \le w_1$$

A constant number in an objective function will not affect the optimum independent variable values. As "a" is a constant number,

if $y = a + \mathbf{y} (x_1, x_2, \ldots\ldots ,x_n)$, then for the minimum value of y

$$\min [a + \mathbf{y} (x_1, x_2, \ldots\ldots ,x_n)] = a + \min [\mathbf{y} (x_1, x_2, \ldots\ldots ,x_n)] \quad \textbf{(2.6)}$$

can be written.

2.2 Calculation Methods

2.2.1 Lagrange multipliers

The fundamental principal of this optimization method is to use derivation process. Lagrange multipliers can be applied to the equality constrained problems. This method is not applied directly to the inequality constrained problems. Another difficulty in using this method is that it is mandatory that the derivatives for both objective and constraint functions should be calculated.

2.2.2 Dynamic programming

In fact, programming term means optimization. This optimization method is based on an optimum function determination rather than an optimum case point. Result of this and the rest of the optimization methods to be mentioned is the set of x_1, x_2,..., x_n

independent variable values which give the optimal value of the objective function y. Accordingly its result is a function, which is dependent on more than one variable.

2.2.3 Geometrical programming

This programming method optimizes the function, which is the sum of polynomials where variables are used in integer and non-integer exponential forms.

2.2.4 Linear programming

When Equations (2.1)–(2.5) become linear, then the optimization process is named as linear programming. Linear programming problem may be very big in size. Sometimes size of the optimization problem is so huge that it deals with thousands of variables.

2.3 Multi Objective Optimization

Real-world problems often have multiple conflicting objectives. For example, when purchasing computing equipments, we would usually like to have a high-performance system, but we also want to spend less money buying it. Obviously, in these problems, there is no single solution that is the best when measured by all objectives. These problems are examples of a special class of optimization problems called multi-objective optimization problems (MOPs). The question is what is an optimal solution for a multi-objective problem ?

With this definition of optimality, we usually find several trade-off solutions (called the Pareto optimal set to honor Vilfredo Pareto or Pareto optimal front (POF) for the plot of the vectors corresponding to these solutions). In that sense, the search for an optimal solution has fundamentally changed from what we see in the case of single-objective problems. The task of solving MOPs is called multi-objective optimization [217].

2.3.1 Pareto optimality

Through the end of nineteenth century and beginning of twentieth century basic and most important concepts of multi objective optimization is established by Edgeworth and Pareto. Canto also in the same time period studied the mathematical development of the methodology. Nowadays, when the multi objective optimization is mentioned, generally people think of the Pareto solution.

The optimization problem with a single objective discussed so far can be considered as a scalar optimization problem because the objective function always reaches a single global optimal value or a scalar. For multiobjective optimization, the multiple objective functions form a vector, and thus it is also called vector optimization.

Any multiobjective optimization problem can generally be written as

$$\underset{x \epsilon R^n}{minimize}\, \boldsymbol{f}(x) = \left[f_1(x), f_2(x), \dots, f_p(x)\right],$$

$$subject\ to \quad \boldsymbol{g}_j(x) \le 0, \quad j = 1,2,\dots, M, \tag{2.7}$$

$$\boldsymbol{g}_j(x) \le 0, \quad j = 1,2,\dots, M, \tag{2.8}$$

where $\boldsymbol{x} = (\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_n)^T$ is the vector of design variables.

The space $F = R^n$ spanned by the vectors of decision variables $\boldsymbol{x}$ is called the search space. The space $S = R^p$ formed by all the possible values of objective functions is called the solution space or objective space. Comparing with the single objective function whose solution space is (at most) R, the solution space for multiobjective optimization is considerably much larger. In addition, as we know that we are dealing with multiobjectives $\boldsymbol{f}(x) = [f_i]$, for simplicity, we can write f_i as $\boldsymbol{f}(x)$ without causing any confusion.

Multiobjective optimization problems, unlike a single objective optimization problem, do not necessarily have an optimal solution that minimizes all the multiobjective functions simultaneously. Often, different objectives may conflict each other and the optimal parameters of some objectives usually do not lead to optimality of other objectives (sometimes make them worse). For example, we want the first-class quality service on our holidays and at the same time we want to pay as little as possible. The high-quality service (one objective) will inevitably cost much more and this is in conflict with the other objective (to minimize cost).

Therefore, among these often conflicting objectives, we have to choose some tradeoff or a certain balance of objectives. If none of these are possible, we must choose a list of preferences so that which objectives should be achieved first. More importantly, we have to compare different objectives and make a compromise. This usually requires a formulation of a new evaluation modeling problem and one of the most popular approaches to such modeling is to find a scalar-valued function that represents a weighted combination or preference order of all objectives. Such a scalar function is often referred to as the preference function or utility function. A simple way to construct this scalar function is to use the weighted sum

$$\boldsymbol{u}\big(\boldsymbol{f}_1(\boldsymbol{x}), \dots, \boldsymbol{f}_p(\boldsymbol{x})\big) = \sum_{i=1}^{p} \alpha_i \boldsymbol{f}_i(\boldsymbol{x}) \quad \textbf{(2.9)}$$

where α_i are the weighting coefficients. For multiobjective optimization, we have to introduce some new concepts related to Pareto optimality.

A vector $\boldsymbol{u} = (\boldsymbol{u}_1, \dots, \boldsymbol{u}_n)^T \in F$, is said to dominate another vector $\boldsymbol{v} = (\boldsymbol{v}_1, \dots, \boldsymbol{v}_n)^T$ if and only if $\boldsymbol{u}_i \leq \boldsymbol{v}_i$ for $\forall i \in \{1, \dots, n\}$ and $\exists i \in \{1, \dots, n\}$: $\boldsymbol{u}_i < \boldsymbol{v}_i$. This 'partial less' or component-wise relationship is denoted by

$$\boldsymbol{u} \prec \boldsymbol{v}, \quad \textbf{(2.10)}$$

which is equivalent to

$$\forall i \in \{1, \dots, n\} : \boldsymbol{u}_i \leq \boldsymbol{v}_i \wedge \exists i \in \{1, \dots, n\}: \boldsymbol{u}_i < \boldsymbol{v}_i. \quad \textbf{(2.11)}$$

Here $\wedge$ means the logical 'and'. In other words, no component of $\boldsymbol{u}$ is larger than the corresponding component of $\boldsymbol{v}$, and at least one component is smaller. Similarly, we can define another dominance relationship $\preccurlyeq$ by

$$\boldsymbol{u} \preccurlyeq \boldsymbol{v} \Leftrightarrow \boldsymbol{u} \prec \boldsymbol{v} \vee \boldsymbol{u} = \boldsymbol{v}. \quad \textbf{(2.12)}$$

Here $\vee$ means 'or'. It is worth pointing out that for maximization problems, the dominance can be defined by replacing the symbol " $\prec$ " with the one “ $\succ$ ".

A point or a solution $\boldsymbol{x}_* \in R^n$ is called a Pareto optimal solution or noninferior solution to the optimization problem if there is no $\boldsymbol{x} \in R^n$ satisfying $\boldsymbol{f}_i(\boldsymbol{x}) \leq \boldsymbol{f}_i(\boldsymbol{x}_*), (i = 1,2, \dots, p)$. In other words, $\boldsymbol{x}_*$ is Pareto optimal if there exists no feasible vector (of decision variables in the search space) which would decrease some objectives without causing an increase in at least one other objective simultaneously. That is to say, optimal solutions are solutions which are not dominated by any other solutions. When mapping to objective vectors, they represent different trade-off between multiple objectives.

Furthermore, a point $x_* \in F$ is called a non-dominated solution if no solution can be found that dominates it. A vector is called ideal if it contains the decision variables that correspond to the optima of objectives when each objective is considered separately.

Unlike the single objective optimization with often a single optimal solution, multiobjective optimization will lead to a set of solutions, called the Pareto optimal set P^*, and the decision vectors x_* for this solution set are thus called non-dominated. That is to say, the set of optimal solutions in the decision space forms the Pareto (optimal) set. The image of this Pareto set in the objective or response space is called the Pareto front. In literature, the set x_*, in the decision space that corresponds to the Pareto optimal solutions is also called an efficient set. The set (or plot) of the objective functions of these non-dominated decision vectors in the Pareto optimal set forms the so-called Pareto front P or Pareto frontier.

Using the above notation, the Pareto front P can be defined as the set of non-dominated solutions so that

$$P = \{s \in S \mid \nexists s' \in S : s' \prec s\}, \tag{2.13}$$

or in term of the Pareto optimal set in the search space

$$P^* = \{x \in F \mid \nexists x' \in F : f(x') \prec f(x)\}. \tag{2.14}$$

The identification of the Pareto front is not an easy task, and it often requires a parametric analysis, say, by treating all but one objective, say, f_i, in a p-objective optimization problem so that f_i is a function of $f_1, \dots, f_{i-1}, f_{i+1}, \dots,$ and f_p. By maximizing the f_i when varying the values of the other $p - 1$ objectives so that the solutions will trace out the Pareto front [192].

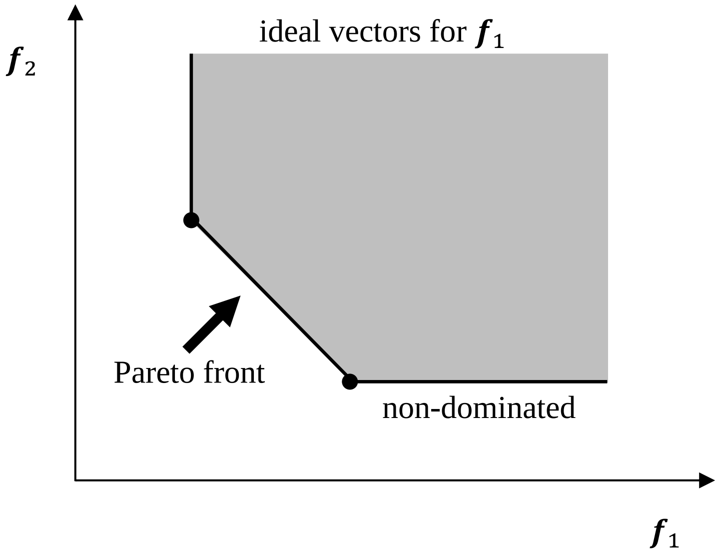


Figure 2.1 : Non-dominated set, Pareto front and ideal vectors in a minimization problem with two objectives $\boldsymbol{f_1}$ and $\boldsymbol{f_2}$.

As a summary, if there are no other solutions available to minimize an objective function without causing an increase on others, then this solution is a non-dominated or "Pareto Optimal" solution. If problem consists of two objective functions, then the result of optimization process is a Pareto Curve, which is formed by non-dominated solutions. Result of a triple objective function optimization problem is a Pareto Surface. Pareto optimization is also applicable for more than three objective functions consisting problems but direct visualization of the solution is not possible. During optimization in Pareto set creating methods, objective functions are treated independently from each other. Different objectives are defined with independent compatibility functions. Some methods do not consider the multi objectivity of the problem and treat one of the objective functions as a constraint function. Other methods like the summation method develop a new objective function by weighing different objective functions and summing them up. Pareto concept allows us to develop solution sets that we can make our own choices from it. Designer can choose one of the alternative optimal solutions after pareto optimal set is defined.

However, users practically need only one solution from the set of optimal trade-off solutions. Therefore, solving MOPs can be seen as the combination of both searching and decision-making [219]. In order to support this, there are four main approaches in the literature [220]. The first one does not use preference information (called no-preference). These methods solve a problem and give a solution directly to the decision maker. The second one is to find all possible solutions of the nondominated set and to then use the user preference to determine the most suitable one (called decision making after search, or posterior). Meanwhile, the third approach is to incorporate the use of preference before the optimization process; and hence it will result in only one solution at the end (called decision making before search, or priori). With this approach, the bias (from the user preference) is imposed all the time. The fourth approach (called decision making during search, or interactive) is to hybridize the second and third ones in which a human decision making is periodically used to refine the obtained trade-off solutions and thus to guide the search. In general, the second one is mostly preferred within the research community since it is less subjective than the other two.

2.3.2 No preference methods

Mathematically, in a k-objective optimization problem, a vector function $\vec{f}(\vec{x})$ of k objectives is defined as:

$$\vec{\boldsymbol{f}}(\vec{x}) = \begin{bmatrix} \boldsymbol{f}_1(\vec{x}) \\ \boldsymbol{f}_2(\vec{x}) \\ \cdots \\ \boldsymbol{f}_k(\vec{x}) \end{bmatrix} \qquad \textbf{(2.15)}$$

in which $\vec{x}$ is a vector of decision variables in the n-dimensional space R_n; n and k are not necessarily the same. A solution is assigned a vector $\vec{x}$ and therefore the corresponding objective vector, $\vec{\boldsymbol{f}}$. Therefore, a general MOP is defined as follows:

$$\min \boldsymbol{f}_i(\vec{x})|_{\vec{x}\in \mathrm{D}} \qquad \textbf{(2.16)}$$

where i = 1, 2, …, k and $D \in R_n$, called the feasible search region. All solutions (including optimal solutions) that belong to D are called feasible solutions.

In general, when dealing with MOPs, a solution x_1 is said to dominate x_2 if x_1 is better than x_2 when measured on all objectives. If x_1 does not dominate x_2 and x_2 also does not dominate x_1, they are said to be nondominated. If we use $\preccurlyeq$ between x_1 and x_2 as $x_1 \preccurlyeq x_2$ to denote that x_1 dominates x_2 and $\lhd$ between two scalars a and b, as $a \lhd b$ to denote that a is better than b (similarly, $a \rhd b$ to denote that a is worse than b, and a ⊀ b to denote that a is not worse than b), then the dominance concept is formally defined as follows.

For methods not using preference, the decision maker will receive the solution of the optimization process. They can make the choice to accept or reject it. For this, the no-preference methods are suitable in the case that the decision maker does not have specific assumptions on the solution. The method of global criterion [220], [221] can be used to demonstrate this class of methods.

For this method, the MOPs are transformed into single objective optimization problems by minimizing the distance between some reference points and the feasible objective region. In the simplest form (using L_p-metrics), the reference point is the ideal solution and the problem is represented as follows:

$$\min\left(\sum_{i=1}^{k}|\boldsymbol{f}_i(x) - z_i^*|^p\right)^{\frac{1}{p}} \qquad \textbf{(2.17)}$$

where z* is the ideal vector, and k is the number of objectives.

When p=1, it is called a Tchebycheff problem with a Tchebycheff metric and is presented as follows:

$$\min \max_{i=1,\ldots,k}|\boldsymbol{f}_i(x) - z_i^*| \qquad \textbf{(2.18)}$$

From the equation, one can see that the obtained solutions depend very much on the choice of the p’s value. Also, at the end the method will only give one solution to the decision maker.

2.3.3 Posteriori methods

For posteriori methods, the decision maker will be given a set of Pareto optimal solutions and the most suitable one will be selected based on the decision maker’s preference. Here, the two most popular approaches, weighted sum and ε-constraint, are summarized.

For the weighted-sum method, all the objectives are combined into a single objective by using a weight vector. The problem in Equation (2.16) is now transformed as in equation (2.19).

$$\min \boldsymbol{f}(\vec{x}) = w_1\boldsymbol{f}_1(\vec{x}) + w_2\boldsymbol{f}_2(\vec{x}) + \cdots + w_k\boldsymbol{f}_k(\vec{x}) | \vec{x} \in D \quad \textbf{(2.19)}$$

where i = 1, 2, …, k and D $\in R_n$.

The weight vector is usually normalized such that $\Sigma w_i = 1$. Figure 2.2 is used to demonstrate how the method works for problems with a 2D objective space. From equation (2.19) we can see that:

$$\boldsymbol{f}_2 = -\frac{w_1}{w_2}\boldsymbol{f}_1 + \frac{\boldsymbol{f}}{w_2}$$

This equation can be visualized as a straight line in the figure (the left one) with a slop of:

$$-\frac{w_1}{w_2}$$

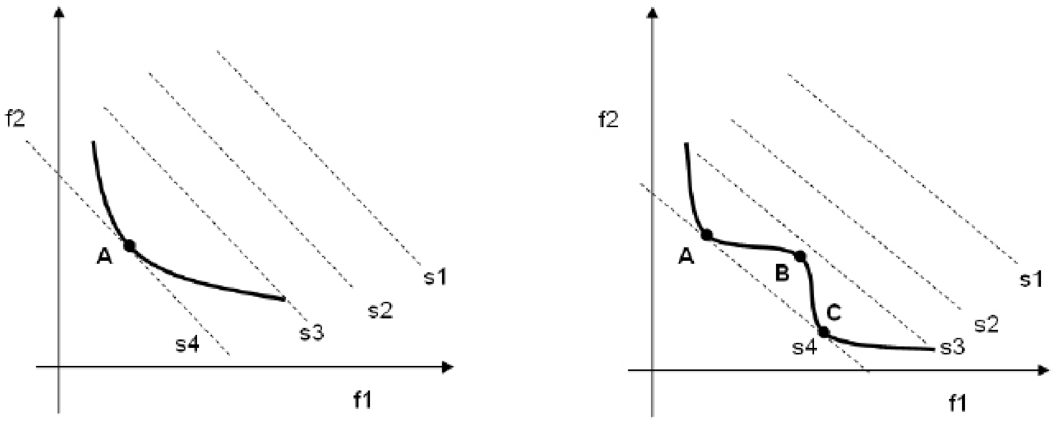
and an intercept of

$$\frac{\boldsymbol{f}}{w_2}$$

Therefore, when the optimization process is progressing, it is equivalent to moving the line towards the origin of the objective space until it reaches point A of the optimal set.

Although the weighted-sum method is simple and easy to use, there are two inherent problems. Firstly, there is the difficulty of selecting the weights in order to deal with scaling problems since the objectives usually have different magnitudes. Therefore, when combining them together, it is easy to cause biases when searching for tradeoff solutions. Secondly, the performance of the method is heavily dependent on the shape of the Pareto Optimal Front (POF). Consequently, it cannot find all the optimal solutions for problems that have a nonconvex POF. We can see this problem from Figure 2.2b where the optimization process will not reach any of the points of the Pareto set between A and C (such as B).

To overcome the difficulty of nonconvexity, the method of ε-constraint is introduced, where only one objective is optimized while the others are transformed as constraints. The problem in equation (2.16) is now transformed as in equation (2.20). Again, the problem is now transformed into a single objective one.



(a) **(b)**

Figure 2.2 : Demonstration of weighted-sum method in 2D objective space: Problem with convex POF on the left (a), and the one with non-convex POF on the right (b) [217].

$$\min f_j(\vec{x}) | \vec{x} \in \mathrm{D} \qquad \textbf{(2.20)}$$

subject to $\boldsymbol{f}_j(\vec{x}) \leq \varepsilon_i$ where i = 1, 2, …, k, i ≠ j and $D \in R_n$.

In this method, the ε vector is determined and uses the boundary (upper bound in the case of minimization) for all objectives i. For a given ε vector, this method will find an optimal solution by optimizing objective j. By changing ε, we will obtain a set of optimal solutions. Although, this method alleviates the difficulty of nonconvexity, it still has to face the problem of selecting appropriate values for the ε vector, since it can happen that for a given ε vector, there does not exist any feasible solution. An example is given in Figure 2.3 where ε_1 will give an optimal solution, while ε_2 will result in no solution at all.

2.3.4 Priori methods

For these methods, the decision maker must indicate the assumption about the preferences before the optimization process. Therefore, the issue is how to quantify the preference and incorporate it. Here, one obvious method is the weighted-sum where the weights can be used to represent the decision maker's preference.

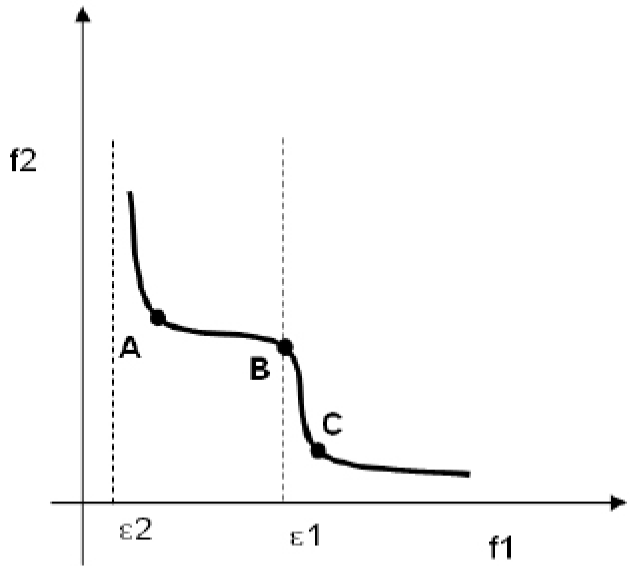


Figure 2.3 : Demonstration of the ε-constraint method in 2D objective space [217].

The optimization process is performed individually on each objective following the order of importance. After optimizing with the most important objective (the first objective), if only one solution is returned, it is the optimal solution. Otherwise, the optimization will continue with the second objective and with a new constraint on the obtained solutions from the first objective. This loop might continue to the last objective.

For the method of goal programming, aspiration levels of the objective functions will be specified by the decision maker. Optimizing the objective function with an aspiration level is seen as a goal to be achieved. In its simplest form, goal programming can be stated as here:

$$\min \sum_{i=1}^{k} |\boldsymbol{f}_i(x) - z_i| \quad \textbf{(2.21)}$$

where z is the vector indicating the aspiration levels. A more general formulation of this equation can be derived by replacing $|\boldsymbol{f}_i(x) - z_i|$ by $|\boldsymbol{f}_i(x) - z_i|^p$.

2.3.5 Interactive methods

Interactive methods allows the decision maker to interact with the optimization program (or an analyst). In general, the interaction is described step-by-step as follows [220]

• **Step 1:** Find an initial feasible solution,

• **Step 2:** Interact with the decision maker and

• **Step 3:** Obtain a new solution (or a set of new solutions). If the new solution (or one of them) or one of the previous solutions is acceptable to the decision maker, stop. Otherwise, go to Step 2.

With the interaction between the program and the decision maker, as indicated in [220], many weaknesses from the aforementioned approaches can be alleviated.

From these basic steps, it appears that the approach is very simple and practical. The preference is incorporated into the achievement function and therefore the problem becomes single objective. The perturbation of the reference point gives the decision maker more understanding of the Pareto optimal set.

2.4 Mathematical Definition of Constrained Optimization Problem

Initial step of an optimization process is to convert a physical case into a mathematical definition. Required equation set is in the form given by Equations (2.1) to (2.5). Since objective and constraint functions are definitive, optimization techniques can solve the associated problem. For instance, creating the objective function in a thermal system optimization problem is generally simple and mostly not an important job. Forming up the constraint equations are just the opposite.

Generally the below given procedure is followed to form up the constraint functions:

i. To identify all the constraints, e.g., capacity, temperature and pressure.

ii. To define the component characteristics and work substance properties by using governing equations.

iii. Mass and energy equilibrium definitions.

All above steps generally end up with an equation set that have more variables than existing in the objective function. Constraint equation set can eliminate those variables that do no exist in the objective function and reduce the overall variable quantity. Some optimization techniques allow those variables that do not exist in the objective function in their processes. Second and third steps in forming the constraint functions are similar to the simulation process.

2.4.1 Lagrange multipliers method

Classical optimization methods are based upon the principal of calculating and defining the optimal value of a function. Optimization by using calculation techniques is valid only when the objective function can be defined in derivative form and constraint functions are in equality form. When a few equality constraint functions exist, other calculation methods like linear and non-linear programming must be used. Besides, when the function is non-continuous and exists only at some special values of the parameters, these features prevent this method to be used.

Mathematical expression of an optimization problem with constraints is given below

$$\underset{x_i}{\text{Minimize}}\, y = y\,(x_1, x_2, \ldots\ldots, x_n) \tag{2.22}$$

$$\phi_1 = \phi_1\,(x_1, x_2, \ldots\ldots, x_n) = 0 \tag{2.23}$$

$$\vdots \qquad \vdots$$

$$\phi_m = \phi_m\,(x_1, x_2, \ldots\ldots, x_n) = 0 \tag{2.24}$$

Lagrange multipliers method shows that x_i will get the maximum value when it satisfies the following

$$\nabla y - \lambda_1 \nabla \phi_1 - \ldots - \lambda_m \nabla \phi_m = 0 \tag{2.25}$$

$$\phi_1 = \phi_1\,(x_1, x_2, \ldots\ldots, x_n) = 0 \tag{2.26}$$

$$\vdots \qquad \vdots$$

$$\phi_m = \phi_m\,(x_1, x_2, \ldots\ldots, x_n) = 0 \tag{2.27}$$

$$\nabla y = \frac{\partial y}{\partial x_1} i_1 + \frac{\partial y}{\partial x_2} i_2 + \ldots\ldots + \frac{\partial y}{\partial x_n} i_n \tag{2.28}$$

In Equation (2.25), λ_1, ……, λ_m constants are named as Lagrange multipliers. Those constants cannot be found without solving the relevant equations.

Sum of the terms on the left handside of the vectorial Equation (2.25) is zero. This means that the unit vector coefficients are equal to zero.

From Equation (2.25), we can demonstrate the following

$$\frac{\partial y}{\partial x_1} - \lambda_1 \frac{\partial \phi_1}{\partial x_1} - \ldots\ldots. - \lambda_m \frac{\partial \phi_m}{\partial x_1} = 0 \qquad \textbf{(2.29)}$$

$$\vdots$$

$$\frac{\partial y}{\partial x_n} - \lambda_1 \frac{\partial \phi_1}{\partial x_n} - \ldots\ldots. - \lambda_m \frac{\partial \phi_m}{\partial x_n} = 0 \qquad \textbf{(2.30)}$$

From Equation (2.29) to Equation (2.30) “n” quantity of scalar equations and from Equation (2.26) to Equation (2.27) “m” quantity of constraint equations together form up an equation set of “n+m” equations. Depending upon the solutions of those equations at the same quantity of unknowns $x_1, \ldots, x_n$ and λ_1, ……, λ_m parameters are found at their optimum values. These optimum values are used in Equation (2.28) and the optimal value y* of objective function y can be found.

Quantity of equality constraints (m) is always less than the number of variables (n). At the limit case where m=n, constraints will yield the values of the x_i independent variables and optimization process will not be possible anymore.

2.4.2 Unconstrained optimization

Lagrange multipliers’ equations can be applied to unconstrained optimization problems in the same way as it is applied to constrained optimization problems. Unconstrained oprimization is to be considered as a special case of the constrained optimization. If objective function y is a function of design variables $x_1, \ldots, x_n$,

$$y = y\,(x_1, x_2, \ldots\ldots, x_n) \qquad \textbf{(2.31)}$$

since ϕ constraints are zero, when the Lagrange multipliers equation is applied to Equation (2.31)

$$\nabla y = 0 \qquad \textbf{(2.32)}$$

or

$$\frac{\partial y}{\partial x_1} = 0, \frac{\partial y}{\partial x_2} = 0, \ldots\ldots, \frac{\partial y}{\partial x_n} = 0 \qquad \textbf{(2.33)}$$

can be written.

Where the derivatives become zero is called the critical point. This point can be either a maximum or a minimum. We investigate only one of those extremums, either maximum or minimum value. This means that the type of critical point should be defined. In order to do that, a seperate mathematical analysis might be necessary.

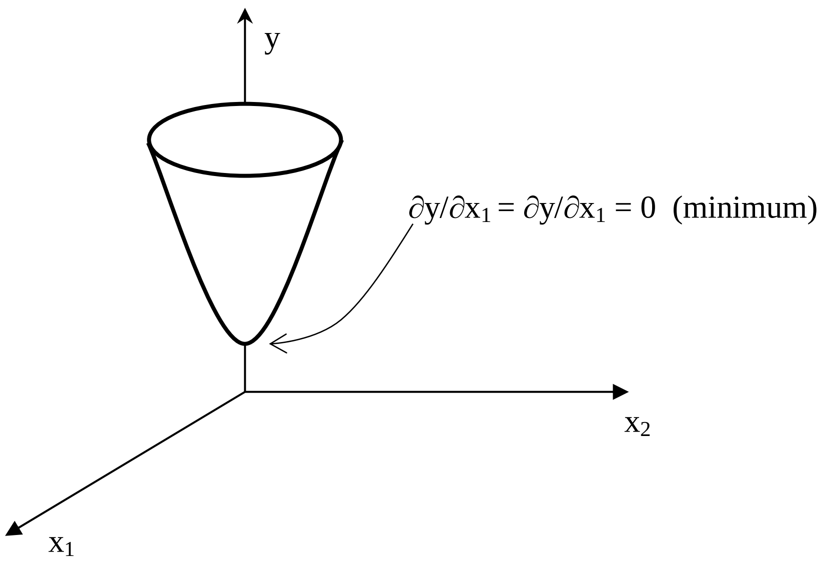


Figure 2.4 : Demonstration of the critical point.

Sometimes it is possible to convert a constrained problem into an unconstrained problem. In order to achieve that, solution values of the constraint equations can be placed inside the objective function and, by this way, constraints can be eliminated.

It is not possible for every case to convert a constraint equation into an unconstrained one all the time. Since it might not be possible to solve the objective function for its variables after locating the constraint functions inside. In this case, Lagrange multipliers technique given by Equation (2.25)-(2.27) should be used.

2.4.3 Sensitivity coefficient

In an optimization process, there is an additional rule defining the optimum value of the objective function and the optimality conditions, which is often used. After defining the optimum value and relevant conditions, it might be searched for the affects of a very small change on the constraints to the optimum value. For instance, if the (H) performance characteristic of one of the components in a hydraulic system is a constraint, searching the system capacity change due to an increase on this parameter is a sensitivity analysis.

Below expression shows the change of the optimum y* value in response to change in the value of H

$$SC = \frac{\partial\left(y^*\right)}{\partial H} \quad \textbf{(2.34)}$$

and the result is called as the sensitivity coefficient (SC).

In addition, this sensitivity coefficient is equal to the Lagrange multiplier, λ. This equality of SC to λ is not unique to this case but it is a common situation.

If more than one constraint exist, then different sensitivity coefficients can be found depending upon the changes on each constraint and can be shown as

$$SC_1 = \lambda_1, \ldots\ldots\ldots, SC_n = \lambda_n \quad \textbf{(2.35)}$$

3. STRUCTURAL DESIGN OPTIMIZATION

3.1 Structural Design Optimization Problem

Most of the design optimization problems can be solved by using the strategy shown in Figure 3.1. As a first step, the initial structure is used as an input to the analysis program to find performance indices such as the displacement, frequency, stress etc.

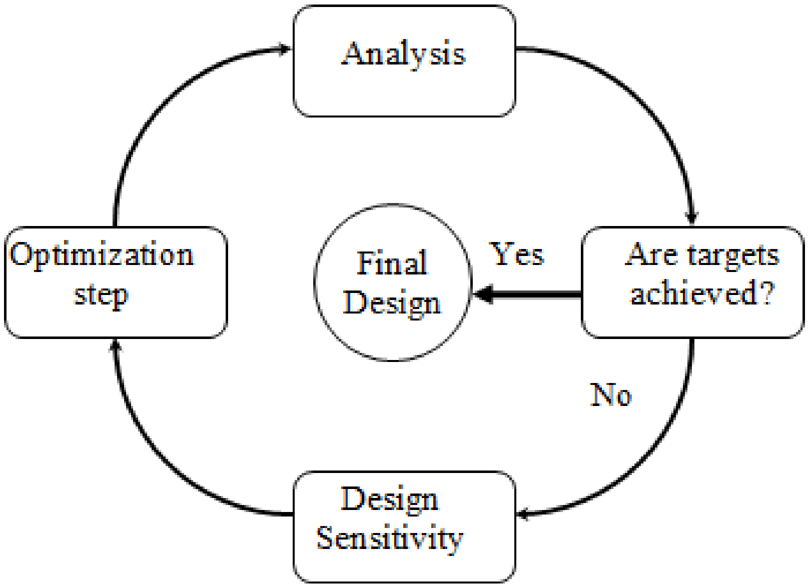


Figure 3.1 : Generic structural design optimization process.

If the performance of the structure is not found to be sufficient enough, then the design sensitivity is calculated with an appropriate method. Design sensitivity can be calculated by using several methods like continuous sensitivity analysis, finite differences etc. Using the design sensitivity information, first order or second order approaches to the performance function can be entered into the optimization algorithm. Optimization algorithm calculates the design variables, which will improve the structural response and satisfy the constraints. This cyclic process will continue until the performance values reach to the desired level.

Structural optimization problem consists of three different optimization problems (e.g., see Figure 3.2).

i. Structural topology optimization (opening holes) problem, that calculates the locations and quantities of holes, bars etc.,

ii. Shape optimization problem, that calculates the final location and shape of the holes and the nodes on the bars,

iii. Size optimization problem that calculates the thickness of the shells, cross sectional areas of the beams etc.

Below more in depth information is given about these three problems.

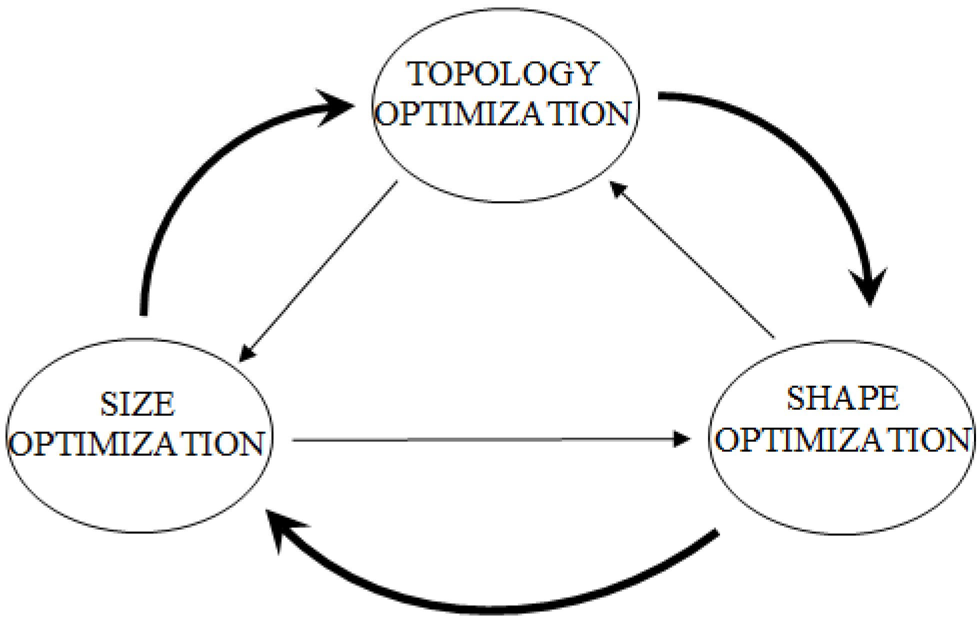


Figure 3.2 : Structural design optimization process.

3.2 Topology Optimization

Topology optimization problem is started by identifying the loads and constraints. Next, the most suitable topology is given to the designer to develop a rough, manufacturable and logical design. This step requires capable engineering approach and design skills.

3.3 Shape Optimization

A further step is to optimize the boundaries (shape) of the structure using optimum topology. Existing technology allows the usage of global parameters like diameter, length etc. in optimization problems. Final shape of all the holes and boundaries in the structure are defined as a result of this optimization problem.

3.4 Size Optimization

Final step after the shape optimization is to optimize the size of the variables (for example thickness of a shell, length of a beam etc.). This final step is not needed for solids since the shape optimization will supply the final dimensions.

Designer will continue performing this three step cyclic process until he or she is satisfied with the results of this optimization process. Sometimes one iteration may not be enough, which cause that, the solution for the topology might result as a non-optimum one.

These directions can be applied to the optimization problems as a first step and this will be a structural topology optimization.

4. DESIGN SENSITIVITY ANALYSIS

While designing a structure, one of the very important tools is the sensitivity of the cost function against the variations in the structural parameters. Design sensitivity analysis investigates the relationship between the system parameters and the response of the system against some of the measurable performance values. One can benefit from relevant studies in [195, 201, 202, 207, 208, 209, 210, 211, 212].

During the design sensitivity analysis, sensitivity of the structural response against the design variables can be investigated by some performance measures under some constraints. Those performance measures might be some mathematical equations, eigenvalue problems or ordinary differantial equations.

In sensitivity analysis, generally displacement, eigenvalue, eigenvector and stress like parameters are used. On the other hand, SVD based analysis is also highly valuable for input-output relationship related studies. Singular values of a structure has a special meaning since the squares of the singular values are the power, energy and power density ratio relations between the input and output vectors. As a result, adjusting singular values of a structure will cause the adjustment of the structural response. Besides the singular vector will show what kind of a relationship exists between the outputs and the inputs. Studying the input-output relationship in multivariable control systems, SVD is a sufficient and an efficient tool to be used, e.g., [195] and [200].

4.1 Properties of Singular Value Decomposition

Below SVD demonstration is based on studies [135, 190, 198]. Let's consider $\boldsymbol{A} \in C^{mxn}$ matrix, additionally $\boldsymbol{U} \in C^{mxm}$, $\boldsymbol{\Sigma} \in R^{mxn}$ and $\boldsymbol{V} \in C^{nxn}$ unit matrices are the SVD of $\boldsymbol{A}$. In that case, the matrix A can be written in below format;

$$\boldsymbol{A} = \boldsymbol{U\Sigma V}^{\mathrm{H}} \tag{4.1}$$

Here $\boldsymbol{U}$ and $\boldsymbol{V}$ columns are respectively $\boldsymbol{U} = [\boldsymbol{u}_1|\boldsymbol{u}_2|\ldots\ldots|\boldsymbol{u}_m]$, $\boldsymbol{V} = [\boldsymbol{v}_1|\boldsymbol{v}_2|\ldots\ldots|\boldsymbol{v}_n]$ left and right singular vectors and $\boldsymbol{V}^H$ is the conjugate transpose (Hermitian) of $\boldsymbol{V}$. If m=n

$\boldsymbol{\Sigma}$=Diag $\{\mu_1, \mu_2, \ldots\ldots\mu_m\}$

on the other hand, if m>n

$$\boldsymbol{\Sigma} = \begin{bmatrix} \Sigma_d \\ \boldsymbol{O}_{(m-n)\times n} \end{bmatrix} \quad \textbf{(4.2)}$$

if m<n

$$\boldsymbol{\Sigma} = \begin{bmatrix} \Sigma_d & \boldsymbol{O}_{(m-n)\times n} \end{bmatrix} \quad \textbf{(4.3)}$$

Here $\boldsymbol{\Sigma}_d = Diag\{\mu_1, \mu_2, \ldots\ldots\ldots\mu_p\}$, $p = \min(m,n)$, $\boldsymbol{O}_{i\times j} \in R^{i\times j}$ and those values are zero. Besides μ_i are the singular values of A. $\boldsymbol{u}_i$ and $\boldsymbol{v}_i$ are the orthonormal eigenvectors of $\boldsymbol{A}\boldsymbol{A}^H$ and $\boldsymbol{A}^H\boldsymbol{A}$. Then, we can write $\boldsymbol{U}\boldsymbol{U}^H = \boldsymbol{I}$ and

$$\boldsymbol{A}\boldsymbol{A}^H\boldsymbol{U} = \boldsymbol{U}\boldsymbol{\Sigma}^2 \quad \textbf{(4.4)}$$

$\boldsymbol{V}\boldsymbol{V}^H = \boldsymbol{I}$ and

$$\boldsymbol{A}^H\boldsymbol{A}\boldsymbol{V} = \boldsymbol{V}\boldsymbol{\Sigma}^2 \quad \textbf{(4.5)}$$

$\boldsymbol{I}$ is the identity matrix. In addition to this, for a square matrix $\boldsymbol{A}$, if $\boldsymbol{A} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^H$, then

$$\boldsymbol{A}^{-1} = \boldsymbol{V}\boldsymbol{\Sigma}^{-1}\boldsymbol{U}^H \quad \textbf{(4.6)}$$

Although the singular values of $\boldsymbol{A}$ are certainly identified, its singular vectors are infinite and for this reason they should be normalized. If $\boldsymbol{A} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^H$, then $\boldsymbol{U}' = \boldsymbol{U}\boldsymbol{e}^{j\theta}$, $\boldsymbol{V}' = \boldsymbol{V}\boldsymbol{e}^{j\theta}$ and j be the imaginary unit, $\boldsymbol{A} = \boldsymbol{U}'\boldsymbol{\Sigma}\boldsymbol{V}'^H$ for all θ is the SVD of $\boldsymbol{A}$. If $\boldsymbol{A} \in R^{mxn}$ matrix then all H superscripts will be converted to T.

4.2 Application of SVD to Finite Element Method Equations

SVD can be applied to the finite element method (FEM) equations for time dependent and time independent problems. For example, lets consider the below given time independent (static) linear equation system

$$\boldsymbol{Kz} = \boldsymbol{f} \tag{4.7}$$

where, $\boldsymbol{K} \in R^{n\times n}, \boldsymbol{z} \in R^{n}$ and $\boldsymbol{f} \in R^{n}$. From here, we can have the following result $\boldsymbol{z} = \boldsymbol{K}^{-1}\boldsymbol{f}$. Having $\boldsymbol{U} \in R^{n\times n}, \boldsymbol{\Sigma} \in R^{n\times n}$ and $\boldsymbol{V} \in R^{n\times n}$, let's consider that SVD of $\boldsymbol{K}^{-1}$ is existing so as to be calculated. Accordingly can be shown as $\boldsymbol{K}^{-1} = \boldsymbol{U\Sigma V}^{T}$.

In order to show that the system will give different responses to different inputs, SVD of its stiffness matrix is given in the diadic form below

$$\boldsymbol{K}^{-1} = \sum_{i=1}^{n} \mu_i \boldsymbol{u}_i \boldsymbol{v}_i^{T} \tag{4.8}$$

In physical systems, singular values are seperate from each other. If the force vector $\boldsymbol{f}$ is equal to the k^{th} right singular vector $\boldsymbol{f} = \boldsymbol{v}_k$

$$\boldsymbol{z} = \sum_{i=1}^{n} \mu_i \boldsymbol{u}_i \boldsymbol{v}_i^{T} \boldsymbol{v}_k \tag{4.9}$$

then cause v_i are orthonormal and $\boldsymbol{\delta_{ik}}$ is the Kronecker delta function $\boldsymbol{v}_i^{T}\boldsymbol{v}_k = \boldsymbol{\delta}_{ik}$, then we get

$$\boldsymbol{z} = \mu_k \boldsymbol{u}_k \tag{4.10}$$

$$\|\boldsymbol{z}\|_2 = \mu_k \tag{4.11}$$

Equation (4.10) shows when f is in the same direction with v_k, then the response vector $\boldsymbol{z}$ will be in the direction of $\boldsymbol{u}_k$ and Equation (4.11) shows that the gain of the response will be μ_k. Depending upon above relations, each right singular vector demonstrates which input values to be entered into the structure in order to have an equal response to the relevant singular value and left singular vector to show how the response to this input is being distributed to the different degree of freedoms of the structure [200].

We will use the below given structural dynamic matrix equation to apply SVD into the semi discrete equation system.

$$\boldsymbol{M}\ddot{\boldsymbol{z}} + \boldsymbol{C}\dot{\boldsymbol{z}} + \boldsymbol{K}\boldsymbol{z} = \boldsymbol{f} \qquad \textbf{(4.12)}$$

Here, mass matrix $\boldsymbol{M} \in R^{n \times n}$, viscous damping matrix $\boldsymbol{C} \in R^{n \times n}$, stiffness matrix $\boldsymbol{K} \in R^{n \times n}$, applied force vector $\boldsymbol{f} \in R^n$, displacement vector $\boldsymbol{z} \in R^n$, velocity vector $\dot{\boldsymbol{z}} \in R^n$ and acceleration vector $\ddot{\boldsymbol{z}} \in R^n$. Laplace transformation of Equation (4.12) is given below

$$\boldsymbol{D}(s) = \boldsymbol{G}(s)\boldsymbol{F}(s) = (\boldsymbol{M}s^2 + \boldsymbol{C}s + \boldsymbol{K})^{-1}\boldsymbol{F}(s) \qquad \textbf{(4.13)}$$

Here 's' is the complex Laplace transformation variable, $\boldsymbol{D}(s)$ and $\boldsymbol{F}(s)$ are in order the Laplace transformation of $\boldsymbol{z}(t)$ and $\boldsymbol{f}(t)$. Then, $\boldsymbol{G}(s)$ transformation matrix can be defined as given below

$$\boldsymbol{G}(s) = (\boldsymbol{M}s^2 + \boldsymbol{C}s + \boldsymbol{K})^{-1} \qquad \textbf{(4.14)}$$

The response of this system to the sinus input $\boldsymbol{f}(t) = \hat{\boldsymbol{f}}\sin(wt)$ that has the input frequency of "w", is the continuous linear $\boldsymbol{D}(jw)$.

$$\boldsymbol{D}(jw) = \boldsymbol{G}(jw)\hat{\boldsymbol{f}}, \qquad \textbf{(4.15)}$$

here $\hat{\boldsymbol{f}}$ is the input magnitude vector.

Magnitude of $\boldsymbol{D}_i(jw)$ in Equation (4.15) is the magnitude of the i^{th} element at the response vector $\boldsymbol{z}$, which is the displacement of one of the nodes. At the same time, phase of $\boldsymbol{D}_i(jw)$ is the phase angle between the i^{th} element of response $\boldsymbol{z}$ and input $\boldsymbol{f}$. Similar to the time independent case, if $\hat{\boldsymbol{f}}$ and $\boldsymbol{v}_k$ are at the same direction, response $\boldsymbol{D}(jw)$ is on the same direction as the gain value μ_k and $\boldsymbol{u}_k$. Singular vectors and singular values of $\boldsymbol{G}(jw)$ are the function of input frequency w, and if we would put them in order, we can write that, $\mu_1 \geq \mu_2 .... \geq \mu_n \geq 0$.

4.3 Sensitivity Analysis

Lets consider that equations of a system are written by using Laplace transformation variables like in Equation (4.13). In order to simplfy, we neglect the argument $s = jw$ and can write the SVD of G as $\boldsymbol{G} = \boldsymbol{U\Sigma V}^H$ for a harmonic input frequency w. Based on $\{\boldsymbol{v}_i\}$ input in every direction can be written as below

$$\boldsymbol{F} = \sum_{i=1}^{n} a_i \boldsymbol{v}_i \quad \textbf{(4.16)}$$

By using the orthonormal property of $\boldsymbol{v}_i$ we can calculate the a_i coeffiecients as written below

$$a_i = \langle \bar{\boldsymbol{v}}_i, \boldsymbol{F} \rangle \quad \textbf{(4.17)}$$

Here $\langle .,. \rangle$ L_2 shows internal multiplication and upper score shows the conjugance. Then, the response function $\boldsymbol{D}$ can be calculated as given below.

$$\boldsymbol{D} = \left(\sum_{i=1}^{n} \mu_i \boldsymbol{u}_i \boldsymbol{v}_i^H \right) \boldsymbol{F} = \sum_{i=1}^{n} a_i \mu_i \boldsymbol{u}_i \quad \textbf{(4.18)}$$

Besides, below equation will show the transformation function between the i^{th} output and j^{th} input.

$$\boldsymbol{G}_{ij} = \frac{\partial \boldsymbol{D}_i}{\partial \boldsymbol{F}_j} = \sum_{m=1}^{n} \mu_m \boldsymbol{u}_{m,i} \bar{\boldsymbol{v}}_{m,j} \quad \textbf{(4.19)}$$

Here $\boldsymbol{D}_i$ is the i^{th} value of $\boldsymbol{D}$, $\boldsymbol{u}_{m,i}$ is the i^{th} value of $\boldsymbol{u}_m$ and μ_m is the m^{th} singular value of $\boldsymbol{G}$. In addition, $\boldsymbol{G}_{ij}$ coefficient is a sensitivity value and shows what kind of an effect will take place in $\boldsymbol{D}_i$ due to the changes in $\boldsymbol{F}_j$.

5. SVD BASED SENSITIVIY ANALYSIS

Generally decision to be given upon which the most effective design change is based on that the design parameters having the most significant impact on the response function. Conventionally Equations (4.19) and (5.1) to be solved for each input frequency value, loading condition and design variable.

$$\left(\boldsymbol{M}s^2 + \boldsymbol{C}s + \boldsymbol{K}\right)\boldsymbol{z} = \boldsymbol{f}$$

If we take $s = jw$, we would get the below expression;

$$\left(-\boldsymbol{M}w^2 + jw\boldsymbol{C} + \boldsymbol{K}\right)\boldsymbol{z} = \boldsymbol{f} \qquad \textbf{(5.1)}$$

If we perturbate M, C, K, z and f once

$$\left[\left(\boldsymbol{M} + \boldsymbol{\Delta M}\right)s^2 + \left(\boldsymbol{C} + \boldsymbol{\Delta C}\right)s + \left(\boldsymbol{K} + \boldsymbol{\Delta K}\right)\right]\left(\boldsymbol{z} + \Delta \boldsymbol{z}\right) = \boldsymbol{f} + \boldsymbol{\Delta f}$$

we will get the above formulation. If we transform to frequency domain by writing $s = jw$ and if substitute this into above equation, we will get the below expression

$$\left[-\left(\boldsymbol{M} + \boldsymbol{\Delta M}\right)w^2 + \left(\boldsymbol{C} + \boldsymbol{\Delta C}\right)jw + \left(\boldsymbol{K} + \boldsymbol{\Delta K}\right)\right]\left(\boldsymbol{z} + \boldsymbol{\Delta z}\right) = \boldsymbol{f} + \boldsymbol{\Delta f}$$

If we sort this equation, we can get the following expression

$$\left(-\boldsymbol{M}w^2 + \boldsymbol{C}jw + \boldsymbol{K}\right)\left(\boldsymbol{z} + \boldsymbol{\Delta z}\right) + \left(-\boldsymbol{\Delta M}w^2 + \boldsymbol{\Delta C}jw + \boldsymbol{\Delta K}\right)\left(\boldsymbol{z} + \boldsymbol{\Delta z}\right) = \boldsymbol{f} + \boldsymbol{\Delta f}$$

$$\underbrace{\left(-\boldsymbol{M}w^2 + \boldsymbol{C}jw + \boldsymbol{K}\right).\boldsymbol{z}}_{f} + \left(-\boldsymbol{M}w^2 + \boldsymbol{C}jw + \boldsymbol{K}\right).\boldsymbol{\Delta z} + \left(-\boldsymbol{\Delta M}w^2 + \boldsymbol{\Delta C}jw + \boldsymbol{\Delta K}\right).\boldsymbol{z}$$
$$+ \underbrace{\left(-\boldsymbol{\Delta M}w^2 + \boldsymbol{\Delta C}jw + \boldsymbol{\Delta K}\right).\boldsymbol{\Delta z}}_{neglect} = \boldsymbol{f} + \boldsymbol{\Delta f}$$

By neglecting the second order terms and making above substitutions, we can find the following equation

$$\left(-w^2\boldsymbol{M} + jw\boldsymbol{C} + \boldsymbol{K}\right).\boldsymbol{\Delta z} = \boldsymbol{\Delta f} - \left(-w^2\boldsymbol{\Delta M} + jw\boldsymbol{\Delta C} + \boldsymbol{\Delta K}\right).\boldsymbol{z} = \hat{f} \qquad \textbf{(5.2)}$$

Here Δ sign shows the derivative with respect to variables ($\partial/\partial b_i$). In order to understand the most effective value of the design change on the response; Equations (5.1) and (5.2) should be solved for $\boldsymbol{\Delta M}, \boldsymbol{\Delta C}$ and $\boldsymbol{\Delta K}$ at the given load value by minimizing or maximizing $\boldsymbol{\Delta u}$. This will increase the calculation time and costs. By using SVD, solution of this problem is simplified as given below. Let $(\mu_1, \boldsymbol{u}_1, \boldsymbol{v}_1)$ and $(\mu_n, \boldsymbol{u}_n, \boldsymbol{v}_n)$ be the first and last singular values and singular vectors of the transformation matrix $\boldsymbol{G}$. Then, for input frequency w if the right hand side of Equation (5.2) comply with the below given equations

$$\boldsymbol{\Delta f} - (-w^2 \boldsymbol{\Delta M} + jw\boldsymbol{\Delta C} + \boldsymbol{\Delta K})\boldsymbol{z} = \boldsymbol{v}_1 \qquad \textbf{(5.3)}$$

we can write

$$\left\| \boldsymbol{\Delta f} - (-w^2 \boldsymbol{\Delta M} + jw\boldsymbol{\Delta C} + \boldsymbol{\Delta K})\boldsymbol{z} \right\|_2 = 1, \ \boldsymbol{\Delta z} = \mu_1 \boldsymbol{u}_1 \text{ ve } \left\| \boldsymbol{\Delta z} \right\|_2 = \sigma_1$$

or

$$\Delta f - (-w^2 \boldsymbol{\Delta M} + jw\boldsymbol{\Delta C} + \boldsymbol{\Delta K})\boldsymbol{z} = \boldsymbol{v}_n \qquad \textbf{(5.4)}$$

In that case, we can also write $\boldsymbol{\Delta z} = \mu_n \boldsymbol{u}_n$ and $\left\| \boldsymbol{\Delta z} \right\|_2 = \mu_n$. This means the maximization (or minimization) of the response sensitivity.

This result is very important since it denotes the “frequency response design sensitivity” of a structure (as a unique case, the statical response at $w = 0$). If we consider a truss system, the extremum sensitivity relations can easily be calculated by first and last singular values and corresponding singular vectors. For a given load condition and w frequency, Equations (5.3) and (5.4) should be solved for $\boldsymbol{\Delta M}, \boldsymbol{\Delta C}$ and $\boldsymbol{\Delta K}$ values to find the design variables that give the sensitivity relations. On the other hand, for given $\boldsymbol{\Delta M}, \boldsymbol{\Delta C}$ and $\boldsymbol{\Delta K}$ derivatives and w frequency, in order to find the loading conditions that give the sensitivity relations, firstly from Equations (5.3) and (5.4) corresponding $\boldsymbol{z}$ vectors are found. Then, Equation (5.1) is solved for $\boldsymbol{f}$ vector. In brief, SVD usage is simpler and faster in comparison to the direct calculation of sensitivity.

In this thesis, multiobjective optimization problem of dynamically loaded large scale structures is investigated. Structural system Equation (4.12) is formed by using FEM. If we show the system parameters that will be optimized by the parameter vector $\boldsymbol{x}$, $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ matrices of the structural system will be the functions of those parameters. This means, $\boldsymbol{M} = \boldsymbol{M}(x)$, $\boldsymbol{C} = \boldsymbol{C}(x)$ and $\boldsymbol{K} = \boldsymbol{K}(x)$. In this case, by representing the multiple loading cases, $\boldsymbol{f}$ force vector will have more than one force components.

If those forces are static (independent from time), then optimization process can be reduced down to an optimum topology/shape/size problem corresponding to a given load condition. This problem can be solved by using the optimization methods discussed previously.

If a sytem with multiple dynamic (time dependent) loading to be investigated, the most critical load condition that would be caused by the combinations of those forces within time should be found and the optimization should be performed accordingly. This critical load case can be found by using the SVD approach discussed above. The force distribution causing the most critical load case and system response (displacements) can be found by using the first singular vectors. In that case, regardless from the size of the structural system matrices, by using the selective eigenvalue solvers method, it is possible to calculate the solution set faster.

In this thesis, by using this approach, optimization of structures under multiple loads is investigated.

Point to be emphasized here is that the system matrices are $\boldsymbol{M} = \boldsymbol{M}(x)$, $\boldsymbol{C} = \boldsymbol{C}(x)$ and $\boldsymbol{K} = \boldsymbol{K}(x)$. Accordingly, it should be $\boldsymbol{v}_k = \boldsymbol{v}_k(x)$, $\boldsymbol{u}_k = \boldsymbol{u}_k(x)$ and $\mu_k = \mu_k(x)$ as a result of SVD of $\boldsymbol{K}(x)$. An SVD based optimization problem is a nonlinear optimization problem even for a linear system.

Analogy:

While $\boldsymbol{Kz} = \boldsymbol{f}$, we can write $\boldsymbol{z} = \boldsymbol{K}^{-1}\boldsymbol{f}$ and $\boldsymbol{K}^{-1} = \sum_{i=1}^{n} \boldsymbol{u}_i \mu_i \boldsymbol{v}_i^{H}$, if $\boldsymbol{f} = \boldsymbol{v}_k \Rightarrow \boldsymbol{z} = \mu_k \boldsymbol{u}_k$.

Since $\boldsymbol{v}_k$ and $\boldsymbol{u}_k$ are orthogonal unit vectors, maximum displacement vector direction will be on left hand side singular vector direction if we apply the force on the right hand side singular vector direction. When the maximum force is not in this $\boldsymbol{v}_k$

direction, again we will have the maximum displacement on the orthogonal vector direction.

6. STRUCTURAL OPTIMIZATION BY USING SVD

Lets consider an objective function $\psi(b)$ is given to be minimized. Accordingly, the derivative of $\psi(b)$ with respect to a design variable can be written as

$$\frac{d\psi(b)}{db} = \frac{\partial \psi}{\partial b} + \frac{\partial \psi}{\partial \mathbf{z}} \frac{\partial \mathbf{z}}{\partial b} \quad \textbf{(6.1)}$$

Here displacement vector $\boldsymbol{z}$, $\boldsymbol{K}(b)\boldsymbol{z}(b) = \boldsymbol{f}(b)$ can be found by using FEM equations. If we differentiate this equation with respect to a design derivative *b*, we get

$$\boldsymbol{K}\frac{\partial \mathbf{z}}{\partial b} + \frac{\partial \boldsymbol{K}}{\partial b}\mathbf{z} = \frac{\partial \boldsymbol{f}}{\partial b} \quad \textbf{(6.2)}$$

and from this equation we can find the below given expression,

$$\boldsymbol{K}\frac{\partial \mathbf{z}}{\partial b} = \frac{\partial \boldsymbol{f}}{\partial b} - \frac{\partial \boldsymbol{K}}{\partial b}\mathbf{z} \quad \textbf{(6.3)}$$

<u>Theorem 1:</u>

For a given matrix $\boldsymbol{A}$, $\mu_1(\boldsymbol{A}) = Maximum\{\|\boldsymbol{Ax}\|_2 : \boldsymbol{x} \in C^n, \|\boldsymbol{x}\|_2 = 1\}$ to be the largest singular value, and for a vector w, we have

$$\mu_1(\boldsymbol{A}) = \|\boldsymbol{Aw}\|_2 \quad \textbf{(6.4)}$$

Proof: given in [191] on page 145.

6.1 Optimization by Using Singular Directions for Multiple Loading Conditions

By using FEM, linear structural equations of a system can be defined as follows

$$Kz = f \qquad (6.5)$$

Here $K \in R^{nxn}$ is the global stiffness matrix, $z \in R^n$ is the unknown vector and $f \in R^n$ is the global load vector. In practice, more than one loading condition is commonly encountered. For example, loading conditions of a plane during a flight or an automobile during its drive on the road are not deterministic. Accordingly, all load conditions should be considered during investigation of the system response. For example, lets investigate the clamped-free beam under arbitrary loads.

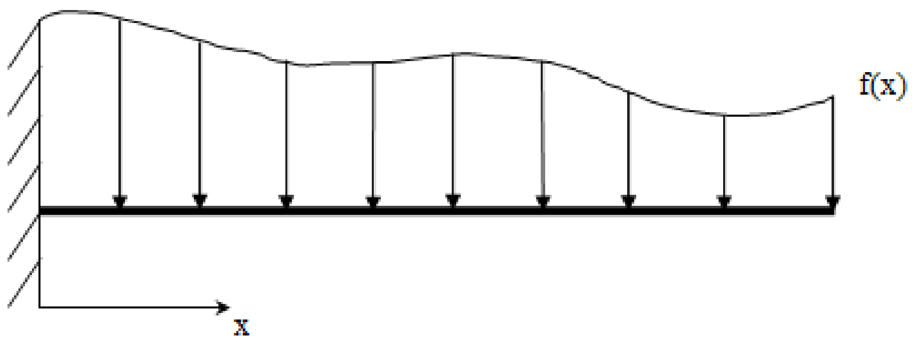


Figure 6.1 : A clamped-free beam under multiple loads.

Let's derive the mathematical model of the beam given in Figure 6.1 by using FEM. Accordingly, let's consider the discrete beam model formed by ten elements and eleven nodes shown below. Since the loads acting on the nodes of this beam can vary, there is an infinite number of loading combinations for the f_i loads acting on the nodes.

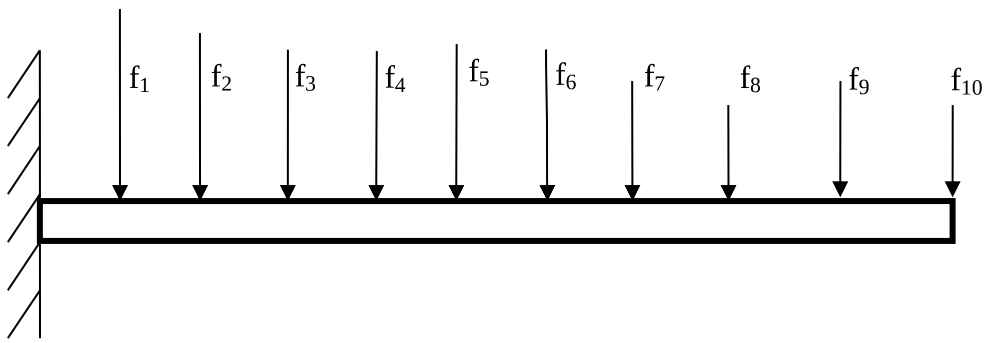


Figure 6.2 : Discrete beam model.

In this case, the worst case loading condition can be found with the below given theorem.

Theorem 2 : A system with a structural behaviour given by $\boldsymbol{Kz} = \boldsymbol{f}$. From Equations (4.8) to (4.11) and from Theorem 1, for the worst case loading condition, we can write $\boldsymbol{f} = \boldsymbol{v}_1\left(\boldsymbol{K}^{-1}\right)$ and in that case the system response will be $\boldsymbol{z} = \mu_1\left(\boldsymbol{K}^{-1}\right)\boldsymbol{u}_1\left(\boldsymbol{K}^{-1}\right)$. Here $\boldsymbol{u}_1\left(\boldsymbol{K}^{-1}\right)$ and $\boldsymbol{v}_1\left(\boldsymbol{K}^{-1}\right)$ are the first singular vectors of the matrix $\boldsymbol{K}^{-1}$.

Proof : Below properties can be used for the first singular value [191],

$$\mu_1(\boldsymbol{A}) = Maximum\left\{\|\boldsymbol{Ax}\|_2 : \boldsymbol{x} \in C^n, \|\boldsymbol{x}\|_2 = 1\right\} \quad \textbf{(6.6)}$$

For this reason, for the unit vector $\boldsymbol{w} \in C^n$, we can find $\mu_1(\boldsymbol{A}) = \|\boldsymbol{A}w\|_2$.

As a result, the worst possible loading condition is that the loads are distributed in the direction of the first right singular vector. In that case, multiple loading case optimization problem is converted into a singular value optimization problem.

6.2 Quadratic Optimization (QP) Problem

Let's consider a structural optimization problem where, $\psi(b)$ is the cost function, b_i is the design variable and $b = \lfloor b_1, b_2, ..., b_p \rfloor$ is the relevant vector, as follows

$$\underset{b}{Min}\,\psi(b) \quad \textbf{(6.7)}$$

while $h_i(b) = 0, i = 1, ..., p$ and $g_i(b) \le 0, i = 1, ..., m$. **(6.8)**

On the other hand, let b^k be the existing parameter vector and b^{k+1} be the next step's parameter vector. In that case, having $b^{k+1} = b^k + d$ where d is the parameter update at the k th step and if we use the first terms in Taylor series

$$\begin{aligned}
\psi(b^k + d) &= \psi(b^k) + \nabla \psi(b^k)^T d \\
h_i(b^k + d) &= h_i(b^k) + \nabla h_i(b^k)^T d = 0, i = 1, ..., p \\
g_i(b^k + d) &= g_i(b^k) + \nabla g_i(b^k)^T d = 0, i = 1, ..., m
\end{aligned} \quad \textbf{(6.9)}$$

we can get the above given equations. Then, the linearized optimization problem can be converted to the below given form.

6.2.1 Linearized optimization problem

$$\underset{b}{Min}\,\psi\left(b^k\right)+\nabla\psi\left(b^k\right)^T d \tag{6.10}$$

while

$$h_i\left(b^k\right)+\nabla h_i\left(b^k\right)^T d=0, i=1,......,p \tag{6.11}$$

$$g_i\left(b^k\right)+\nabla g_i\left(b^k\right)^T d=0\le 0, i=1,......,m \tag{6.12}$$

By neglecting the fixed term, $\psi\left(b^k\right)$ on the k th step, above given optimization problem can be solved by using SLP or Sequential Quadratic Programming (SQP) methods. From both of these methods SQP is preferred due to it’s faster convergence property [193]. Hence, the below given quadratic programming (QP) problem is formed.

$$\underset{b}{Min}\,\nabla\psi\left(b^k\right)^T d+\frac{1}{2}d^T d \tag{6.13}$$

Here we have

$$h_i\left(b^k\right)+\nabla h_i\left(b^k\right)^T d=0, i=1,......,p \tag{6.14}$$

$$g_i\left(b^k\right)+\nabla g_i\left(b^k\right)^T d=0\le 0, i=1,......,m \tag{6.15}$$

6.2.2 Dual problem

We convert this problem to the below given dual problem

$$\underset{b}{Min}\, c^T \lambda + \frac{1}{2}\lambda^T Q \lambda \qquad \textbf{(6.16)}$$

where λ is an n-tuple vector of optimization variables, c is an n-tuple vector containing coefficients of linear terms, and *nxn* matrix Q contains coefficients of square and mixed terms in the objective function. Here, $\lambda_i \geq 0, i = 1,2....,m$ and λ_i without limits, $i = m+1, m+2,........, m+p$ and Lagrange multiplier vector is given by

$$\lambda = \begin{Bmatrix} r \\ s \end{Bmatrix}, r = [r_1,........, r_m]^T \text{ and } s = [s_1,........, s_p]^T, \qquad \textbf{(6.17)}$$

Constraint vector :

$$R(b^k) = \begin{Bmatrix} g(b^k) \\ h(b^k) \end{Bmatrix} \qquad \textbf{(6.18)}$$

Jacobian of the constraint vector

$$J_R(b^k) = \begin{Bmatrix} J_g(b^k) \\ J_h(b^k) \end{Bmatrix}, \quad J_g(b^k) = \begin{Bmatrix} \nabla g_1(b^k)^T \\ \nabla g_2(b^k)^T \\ \vdots \\ \nabla g_m(b^k)^T \end{Bmatrix} \text{ and}$$

$$J_h(b^k) = \begin{Bmatrix} \nabla h_1(b^k)^T \\ \nabla h_2(b^k)^T \\ \vdots \\ \nabla h_p(b^k)^T \end{Bmatrix} \qquad \textbf{(6.19)}$$

Parameter update of d at the k th step

$$d = -\nabla \psi(b^k) - J_R(b^k)^T \lambda \qquad \textbf{(6.20)}$$

and

$$c = J_R\left(b^k\right)\nabla\psi\left(b^k\right) - R\left(b^k\right) \qquad \textbf{(6.21)}$$

Solution algorithm of the above problem is given in [193].

6.2.3 Implicit optimization problem

If there are more than one objective functions in an optimization problem, the optimization problem can typically be shown by

$$\underset{b}{Min}\,\psi_z(z) \text{ and } \psi_b(b) \qquad \textbf{(6.22)}$$

$$\text{while } \boldsymbol{K}(b)\boldsymbol{z}(b) = \boldsymbol{f}(b) \qquad \textbf{(6.23)}$$

$$h_i(z(b), b) = 0, i = 1, \ldots\ldots, p \qquad \textbf{(6.24)}$$

$$g_i(z(b), b) \leq 0, i = 1, \ldots\ldots, m \qquad \textbf{(6.25)}$$

Among the objective functions, $\psi_z(z)$ can consist of one or some of the factors like displacement, stress, natural frequency etc. and it is a function of $\boldsymbol{z}$ displacement vector. On the other hand, $\psi_b(b)$ is the function of weight and volume like b_i design variables only. The problem defined above is a multiobjective optimization problem.

Besides, since $\boldsymbol{K}(b)\boldsymbol{z}(b) = \boldsymbol{f}(b)$, in multiple loading cases, displacement vector is dependent on the loading condition and design variables [186]. This is shown as $\boldsymbol{z} = \boldsymbol{z}(\boldsymbol{f}, b)$.

6.2.4 A sample by using singular directions for multiple loading conditions

If $\psi_z(z)$, which is defining displacement, stress, natural frequency like objective functions in structural optimization problems, is assumed as convex, then by using Theorem 1, we can show $\psi_z(z)$ objective function for multiple load cases as given by

$$\underset{b}{Min}\,\psi_z(\boldsymbol{z}(\boldsymbol{f},b),b) = \underset{b}{Min}\,Max\,\psi_z\left(\boldsymbol{K}^{-1}(b)\boldsymbol{f}(b)\right) = \underset{f=\boldsymbol{v}_1(\boldsymbol{K}^{-1}(b))}{Min}\psi_z\left(\mu_1\left(\boldsymbol{K}^{-1}(b)\right)\boldsymbol{u}_1\left(\boldsymbol{K}^{-1}(b)\right)\right) \tag{6.26}$$

Accordingly, this is a singular value and singular vector optimization problem.

Example: A clamped-free beam is considered given in Figure 6.3 and an external load $\boldsymbol{f}$ is applied to the three nodes near the beam tip.

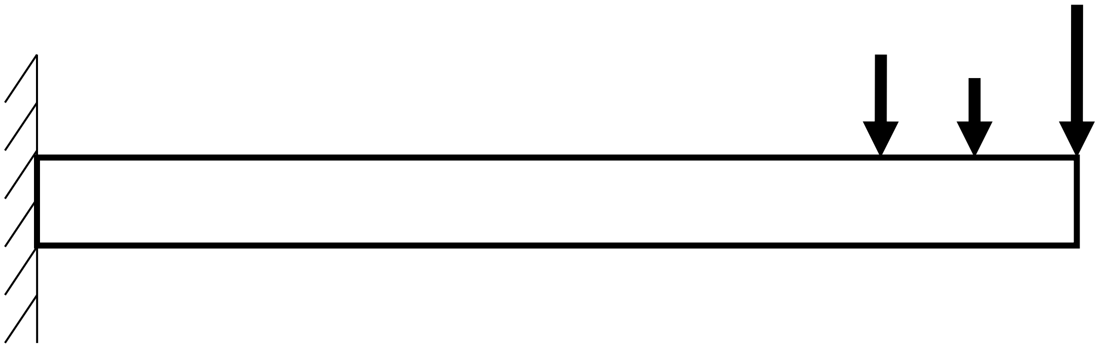


Figure 6.3 : Cantilever beam under a variable external load.

We divided this beam into 16 equal length elements and by running the optimization algorithm presented below, pareto sets of each cross section under variable singular loading are calculated. According to these pareto sets, we identified thickness values of each element. In this thesis, the comparison between the conventional approach and SVD method for minimum weight, and minimum displacement is performed by using the SQP method with the help of a Matlab® program.

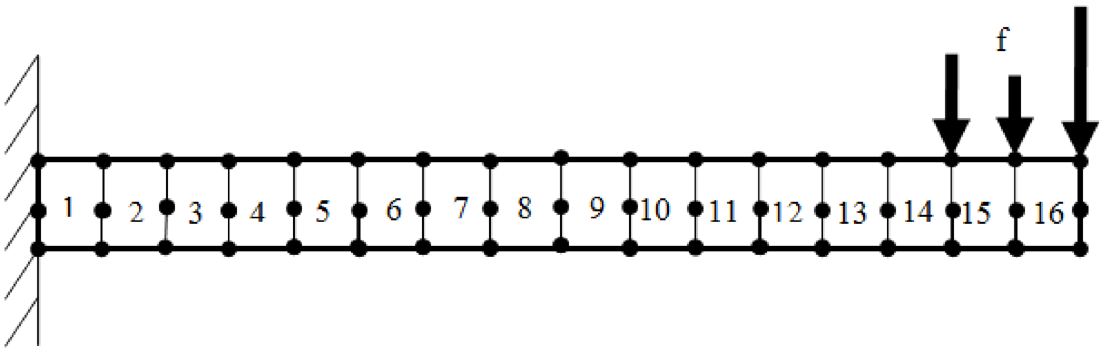


Figure 6.4 : A beam divided into 16 equal length elements.

The program consists of two sub-programmes. While the first program (fun) computes two different objective functions mathematically, second and the main program sends the calculated function values to fmincon function in Matlab® as an input and then form the Pareto optimum sets by using the SQP optimization method.

The main program calls fmincon function, which is set up by the "optimset", data vector defining optimization parameters. Fmincon calls fun sub-program at each iteration to find the optimum parameters that would minimize the cost function.

The main aim of the algorithm is to provide the Pareto optimum sets without any weighing factors to enable the user with making the choice of best solution freely according to his/her own needs. The methodology followed to find Pareto sets is given earlier. The program is summarized with the below given flow chart:

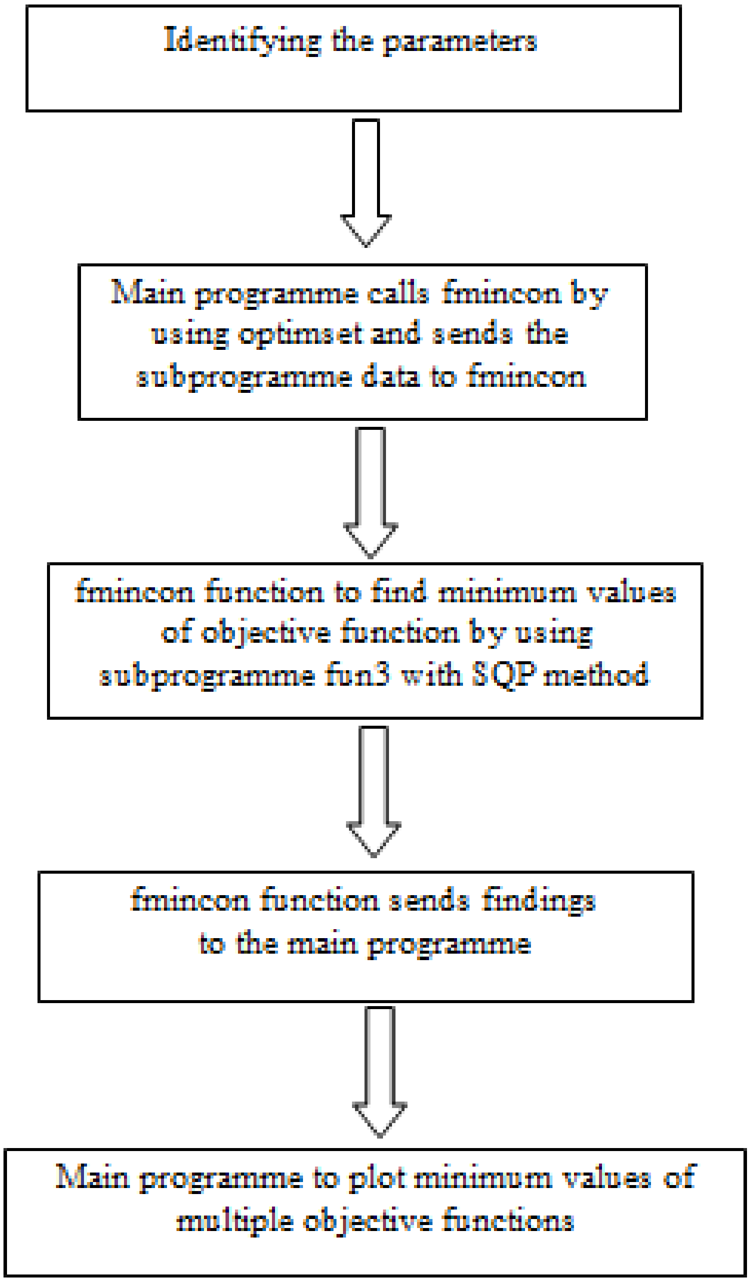
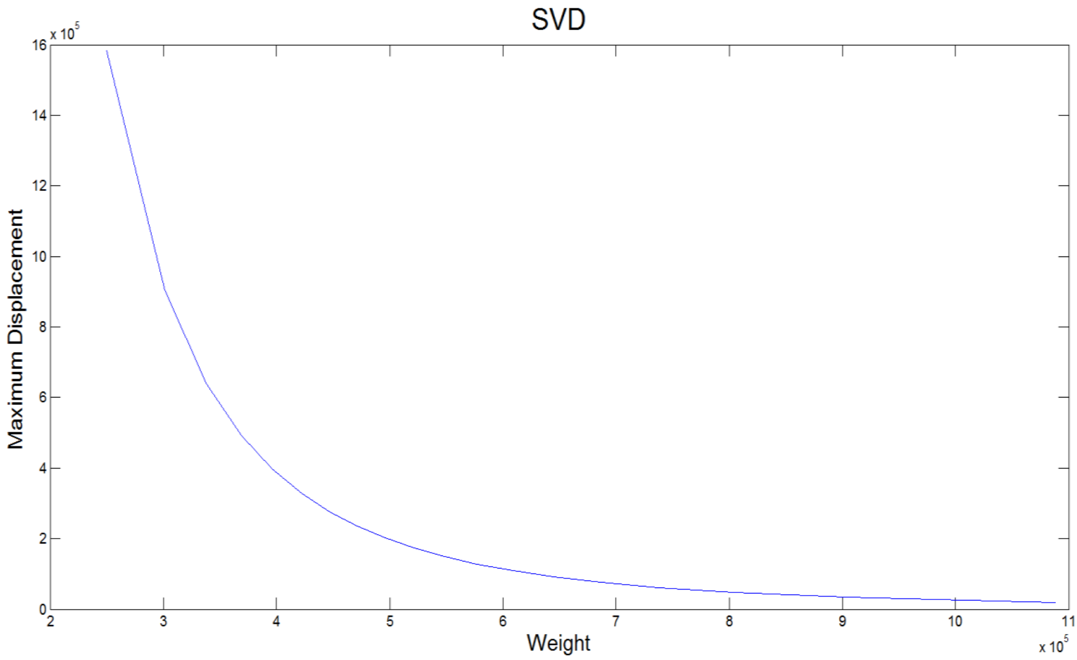
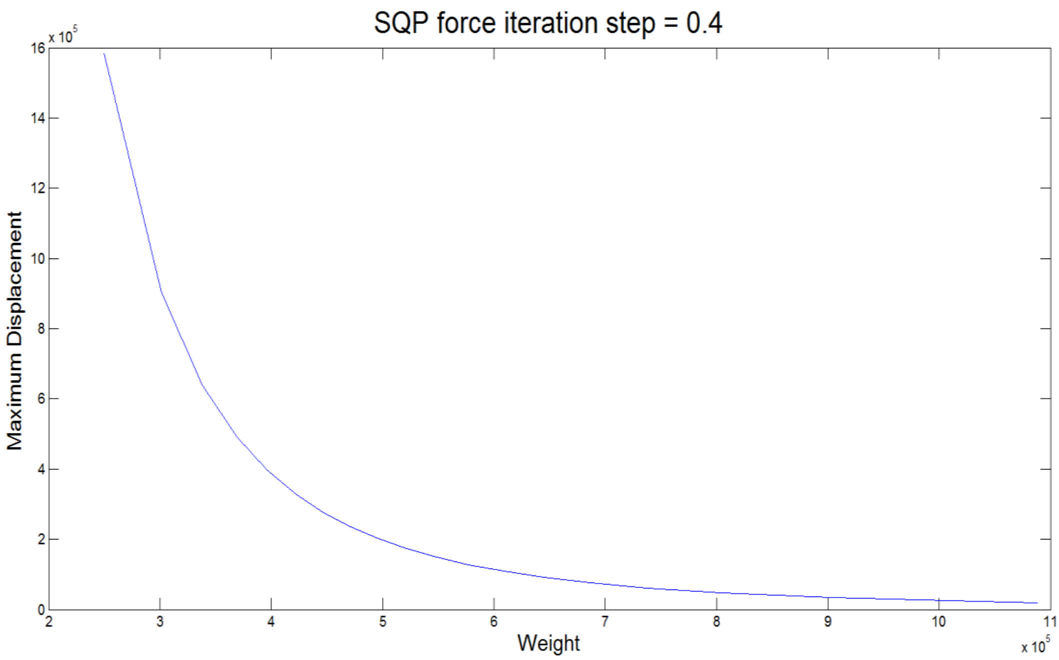


Figure 6.5 : MATLAB® program flow chart.

In Figure 6.6, we can see the objective functions' comparison according to the element numbers.



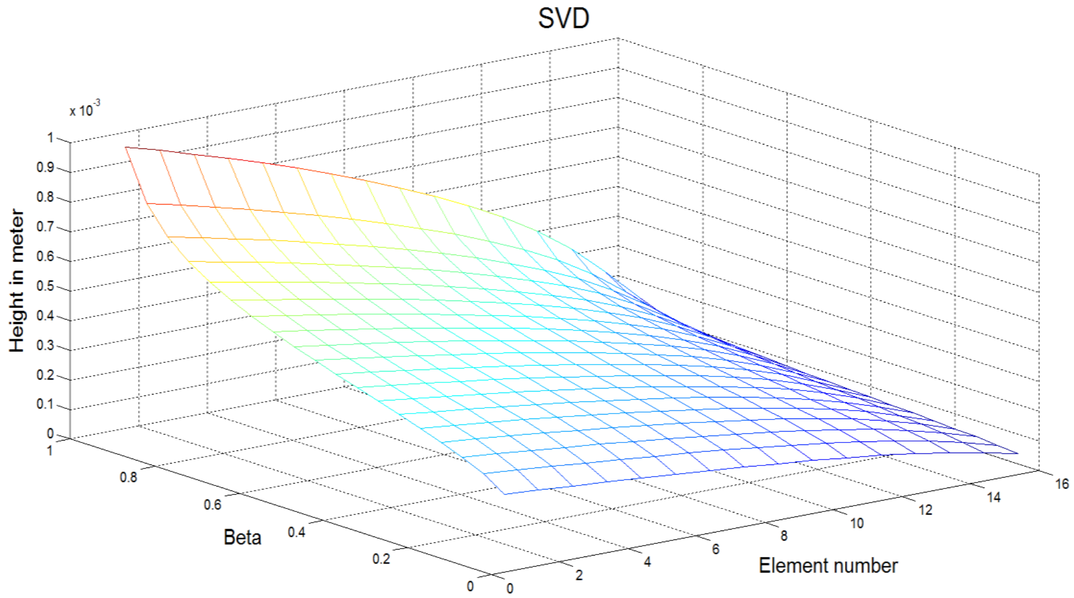
(a) : Weight vs. Maximum Displacement graph created by using SVD.



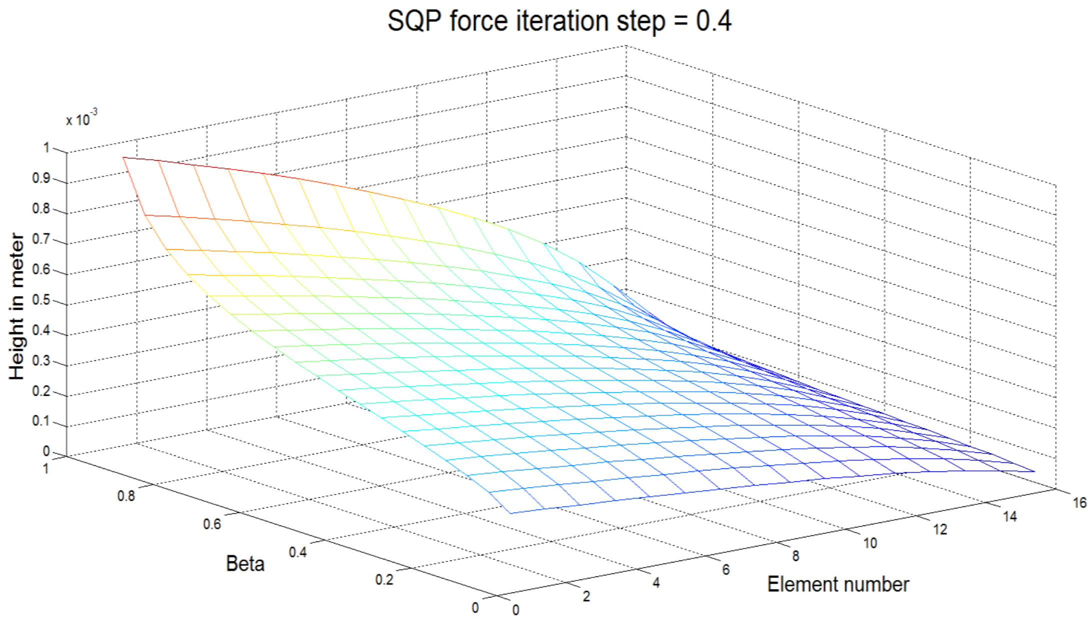
(b) : Weight vs. Maximum Displacement curve obtained by conventional method with 0.1.,0.2 and 0.4 force iteration steps.

Figure 6.6 : Comparison of SVD and conventional optimization solutions.

Finally in Figure 6.7, graphs those give the performances of objective functions in pareto optimum sets, are created to enable user choosing the most suitable solution.



(a) : Pareto optimum sets obtained by using SVD.



(b) : Pareto optimum sets obtained by conventional method with 0.1.,0.2 and 0.4 force iteration steps.

Figure 6.7 : Comparison of Pareto optimum sets obtained by the SVD and conventional method.

To show the efficiency of the proposed approach, CPU times of numerical solutions are presented in Table 6.1 that are obtained by using a computer having an Intel Core i7 CPU of 2.4 GHz. It is presented in Table 6.1 that computational cost of calculating the Pareto optimum sets with conventional method is much higher in comparison to the SVD based structural optimization.

CPU times for the conventional method with different external force value iteration steps are also given in Table 6.1. Although it is read as the higher the step value, the better the CPU timing is, in real life, the user needs to keep the iteration steps as small as possible to obtain the optimum results with the SQP. In SVD based algorithm, those force iteration steps are not applicable since the algorithm normalizes the force values with in itself already.

Table 6.1: CPU time comparison between the SVD and conventional method.

Method	Cost Function	CPU Time in Seconds
SVD	7.2597e+04	138.6755
Conventional with the step 0.1	7.2597e+04	355930.318
Conventional with the step 0.2	7.2597e+04	51312.50104
Conventional with the step 0.4	7.2597e+04	8286.783044

As a conclusion of this example, we can define SVD based structural optimization algorithm to be very fast answering algorithm in comparison to conventional approach in terms of CPU timing. While the largest singular value gives the worst case loading condition and the corresponding displacement and weight results in a relatively very short time, conventional approach should calculate the values of the objective functions for each external force iteration to find the optimum solutions. In sum, SVD is a powerful alternative tool to the conventional structural optimization methodologies.

6.3 Design Sensitivity Analysis by Using the Singular Vector Directions

Design sensitivity analysis definition is given by Equations (6.1) to (6.3)

$$\frac{d\psi}{db} = \frac{\partial \psi}{\partial b} + \frac{\partial \psi}{\partial \mathbf{z}} \frac{d\mathbf{z}}{db}$$

$$\boldsymbol{K}(b)\frac{d\boldsymbol{z}}{db} = \frac{\partial \boldsymbol{F}(b)}{\partial b} - \frac{\partial \boldsymbol{K}(b)}{\partial b}\boldsymbol{z}$$

Accordingly, to find the effect of design parameter b_i on the cost function $\psi = \psi(b, \boldsymbol{z}(b))$, a perturbation is given and value of the function $\psi = \psi(b, \boldsymbol{z}(b))$ is calculated at point $b_i + \Delta b_i$. In conventional design sensitivity analysis, by calculating the cost function variation corresponding to each b_i design variable perturbated with the same ratio (5% or 10% etc.), the most effective design variable on cost function is identified. On the other hand, if there are more than one design variable or multiple loading cases, it is generally very difficult to find the optimum perturbation amount since global stiffness matrix $\boldsymbol{K}$ is the non linear function of design variable b_i. For this purpose, singular value decomposition can be used as described below.

Derivation of displacement $d\boldsymbol{z}/db$ in Equation (6.1) for design sensitivity analysis is given below. Accordingly, we can write the below expression in line with Theorem 2.

When we write the Equation (6.3) with the approximate differentiation, we get the following expressions

$$\frac{\partial \boldsymbol{K}(b)}{\partial b} \cong \frac{\boldsymbol{K}(b+\Delta b) - \boldsymbol{K}(b)}{\Delta b} = \frac{\boldsymbol{\Delta K}}{\Delta b} \quad \textbf{(6.27)}$$

$$\frac{\partial \boldsymbol{F}(b)}{\partial b} \cong \frac{\boldsymbol{F}(b+\Delta b) - \boldsymbol{F}(b)}{\Delta b} = \frac{\boldsymbol{\Delta F}}{\Delta b} \quad \textbf{(6.28)}$$

$$\frac{d\boldsymbol{z}(b)}{db} \cong \frac{\boldsymbol{z}(b+\Delta b) - \boldsymbol{z}(b)}{\Delta b} = \frac{\boldsymbol{\Delta z}}{\Delta b} \quad \textbf{(6.29)}$$

As a result, we can write the below equation

$$\boldsymbol{K}(b)\Delta\boldsymbol{z} \cong \boldsymbol{\Delta F} - \boldsymbol{\Delta K z} \quad \textbf{(6.30)}$$

where

$$\boldsymbol{\Delta z} = \boldsymbol{K}(b)^{-1}(-\boldsymbol{\Delta K z} + \boldsymbol{\Delta F}) \qquad \textbf{(6.31)}$$

If

$$\boldsymbol{\Delta F} - \boldsymbol{\Delta K z} = \boldsymbol{v}_1\left(\boldsymbol{K}(b)^{-1}\right) \qquad \textbf{(6.32)}$$

then, we get the below equation

$$\boldsymbol{\Delta z} = \mu_1\left(\boldsymbol{K}(b)^{-1}\right)\boldsymbol{u}_1\left(\boldsymbol{K}(b)^{-1}\right) \qquad \textbf{(6.33)}$$

This is the maximum possible design sensitivity variation direction. In case of a design variable change in that direction as $b_i + \Delta b_i$, design sensitivity variation $d\psi/db = \partial\psi/\partial b + (\partial\psi/\partial \mathbf{z})(d\mathbf{z}/db)$ will get the maximum value. For this case, parameter change $b_i + \Delta b_i$ that would give the largest design sensitivity direction can be found by solving the below given optimization problem.

Problem:

$$\underset{\Delta b_i}{Min} \quad \left\| \boldsymbol{\Delta F} - \boldsymbol{\Delta K z} - v_1\left(\boldsymbol{K}(b)^{-1}\right) \right\|_2 \qquad \textbf{(6.34)}$$

such as,

$$\boldsymbol{\Delta K} = \boldsymbol{K}(b + \Delta b) - \boldsymbol{K}(b) \qquad \textbf{(6.35)}$$

$$\boldsymbol{\Delta F} = \boldsymbol{F}(b + \Delta b) - \boldsymbol{F}(b) \qquad \textbf{(6.36)}$$

Design parameter variation $b_i + \Delta b_i$ that will be found as the solution of this optimization problem will give us the best direction for the design variable change.

Solution Method:

If the constraints are defined as,

$$g_j(z) = 0 \tag{6.37}$$

$n_{ij} = \partial g_j / \partial z_i$ being the active constraint gradient, we can write

$$\boldsymbol{P} = \boldsymbol{I} - \left(\boldsymbol{N}^T \boldsymbol{N}\right)^{-1} \boldsymbol{N}^T \tag{6.38}$$

where $\boldsymbol{P}$ is the projection matrix, $\boldsymbol{N} = \lfloor n_{ij} \rfloor$, $\boldsymbol{I}$ is the identity matrix. This matrix projects a vector to the tangential sub-space of the active constraints. In that case, the best possible parameter update can be calculated as below

$$b_{i+1} = b_i + \Delta b \tag{6.39}$$

$$\Delta b = \boldsymbol{P} \frac{d\psi(b)}{db} \tag{6.40}$$

For proof please see [195] pages 169-171.

Example 2:

A truck chassis frame, having a 4500 mm length and a 708 mm width is considered as given below in Figure 6.8. At the rear of the frame, a 5 tonne load and at the front end of the frame a 1,5 tonne powerpack (engine & transmission) load are applied. Frame material ischosen to be St-37 steel. Its elasticity modulus is E=210000N/mm2 and its density is ρ=7,81x10-9 tonne / mm3. Leaf springs are used both at the front and at the rear suspension. North-south engine & transmission configuration is also used in the powerpack installation.

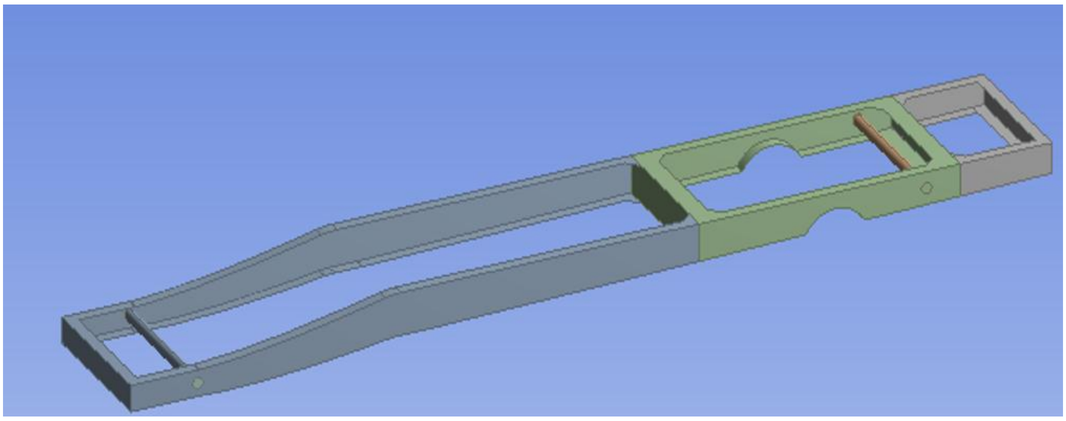


Figure 6.8 : Truck chassis frame.

The model of the truck chassis frame is meshed in Hypermesh software with linear quadrilateral shell meshes by using 1296 nodes, e.g., see Figure 6.9.

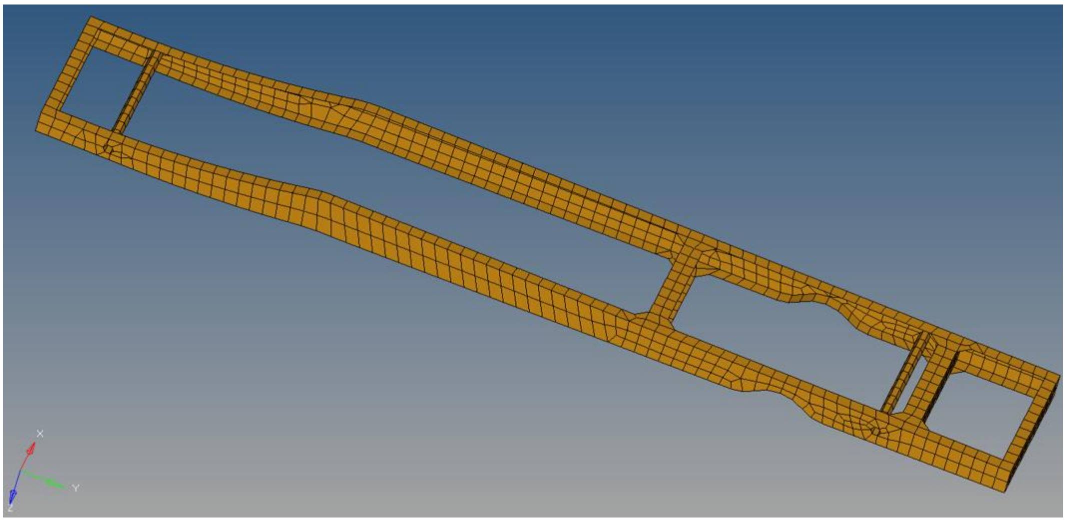


Figure 6.9 : Linear quad shell mesh is used for the model.

Then, the model is converted into a superelement in ANSYS, see Figure 6.10. The stiffness and mass matrices of the superelements are transferred into Matlab.

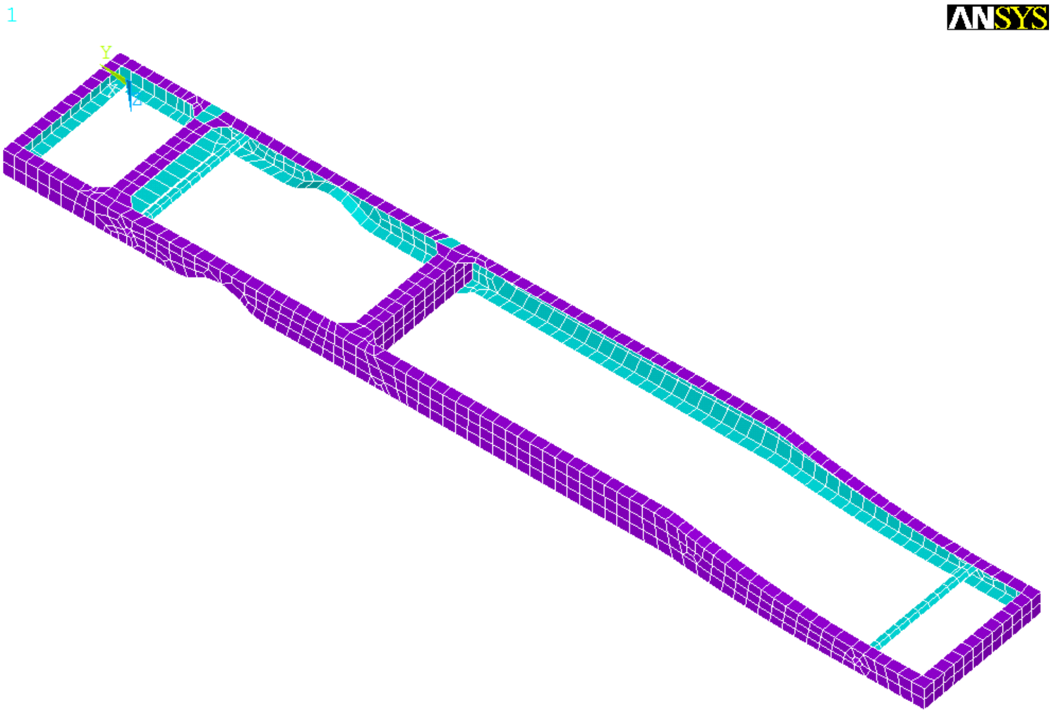


Figure 6.10 : The super element model created in ANSYS.

In Matlab, loads and constraints are applied on the corresponding nodes to the model. Loading is applied such that at the back of the frame, the load of 50000 N force is applied simulating the homogenous distribution and at the front of the frame the powerpack is mounted from four points, see Figures 6.11-6.13. As constraints, front and rear mounting points of leaf springs are chosen, see Figure 6.14.

Multiobjective optimization algorithm uses stiffness and mass matrices as input and calculates the optimum cost values driven by the optimum chassis frame thickness h_i which is used as the design parameter for the optimization algorithm. Similar to Example 1 chassis frame is also optimized under given load conditions. In order to simulate a real life engineering problem application design parameters were chosen such as to reflect the market usage. Accordingly the cost values through the frame thickness were optimized.

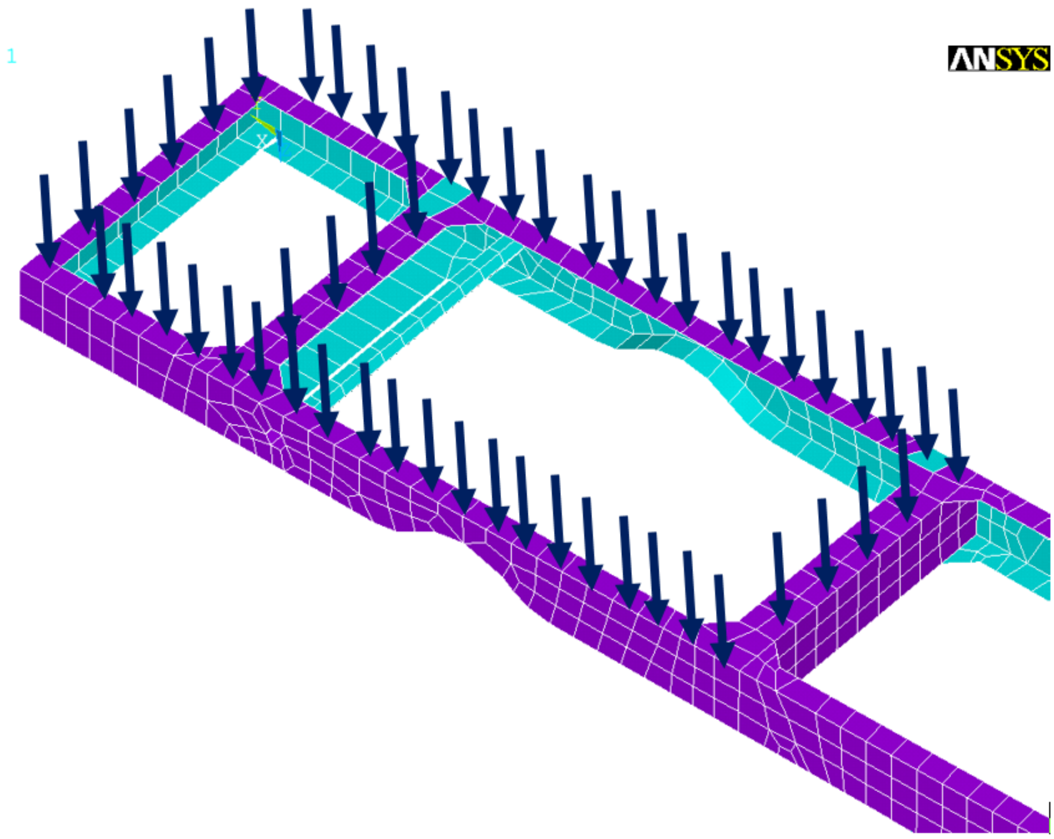


Figure 6.11 : Uniform distributed loading at the back of the chassis frame.

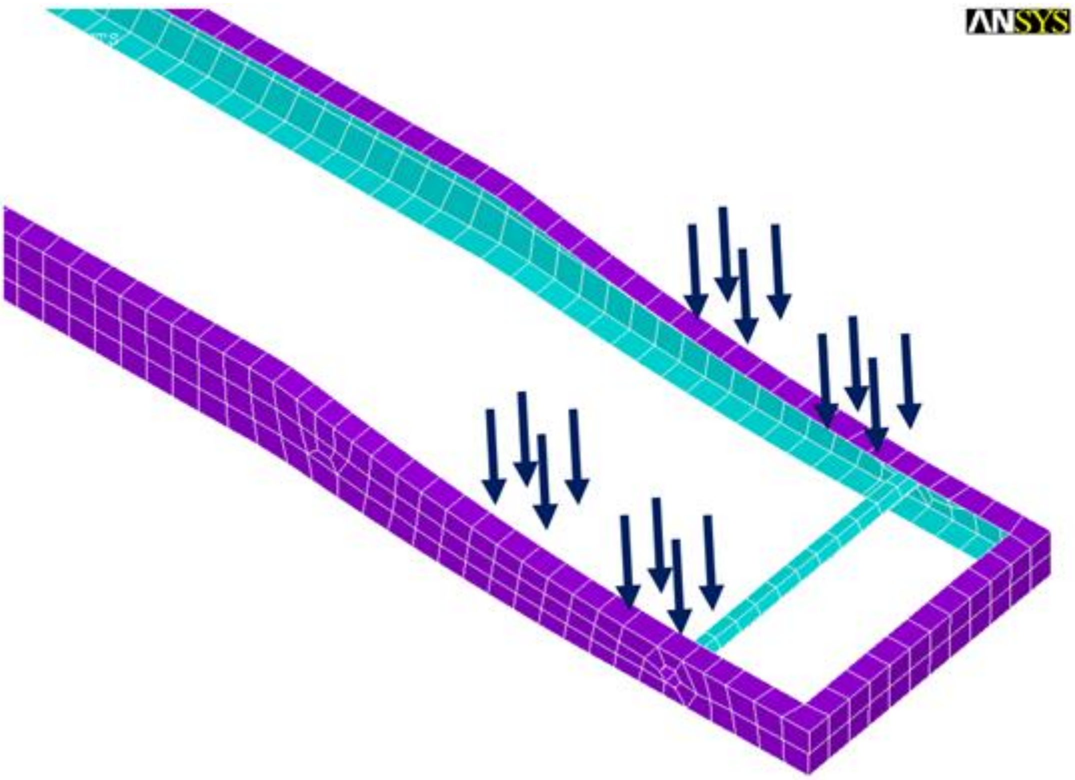


Figure 6.12 : Engine & Transmission loads are applied to the front of the frame where engine and transmission are mounted with brackets

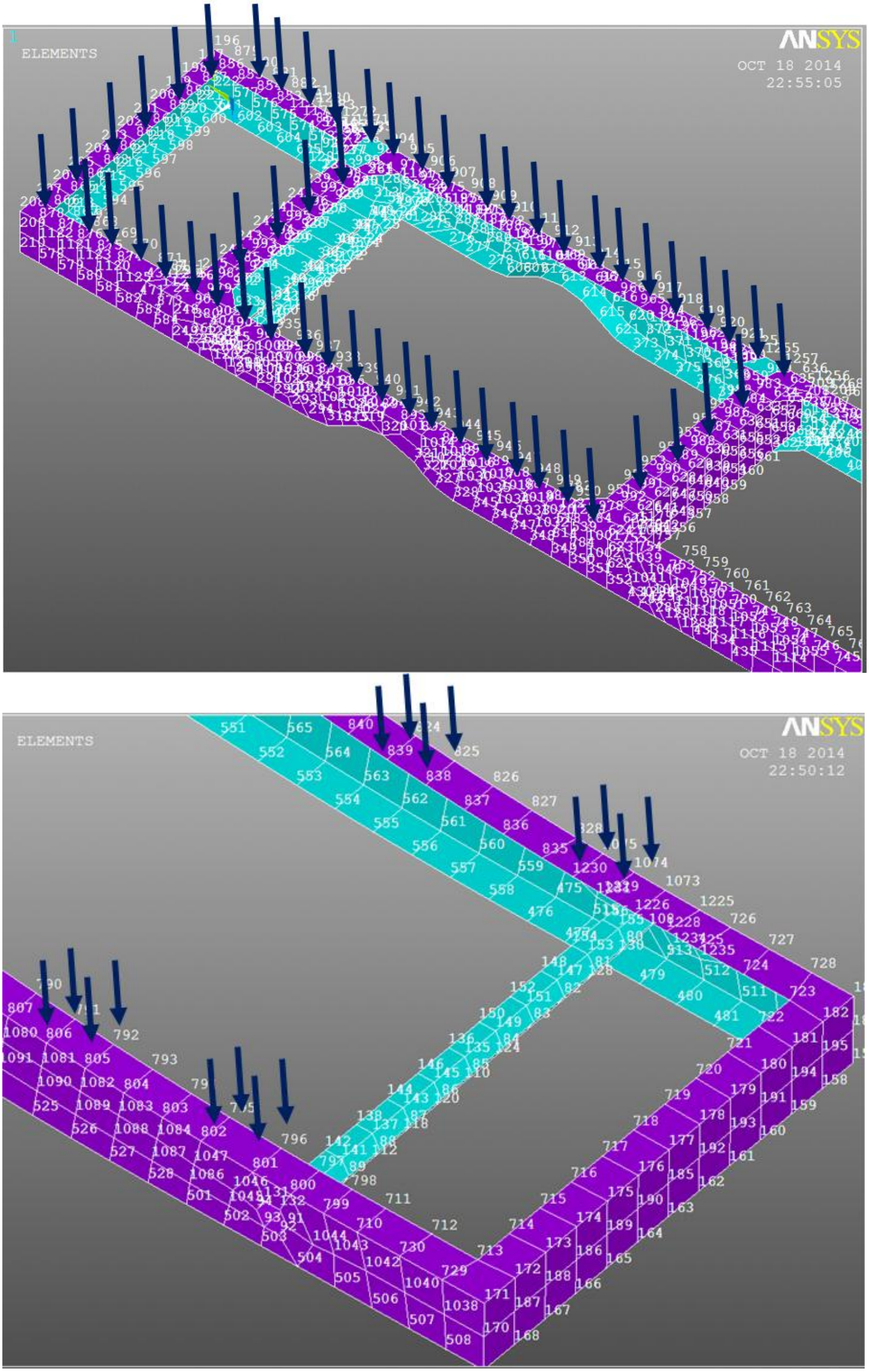
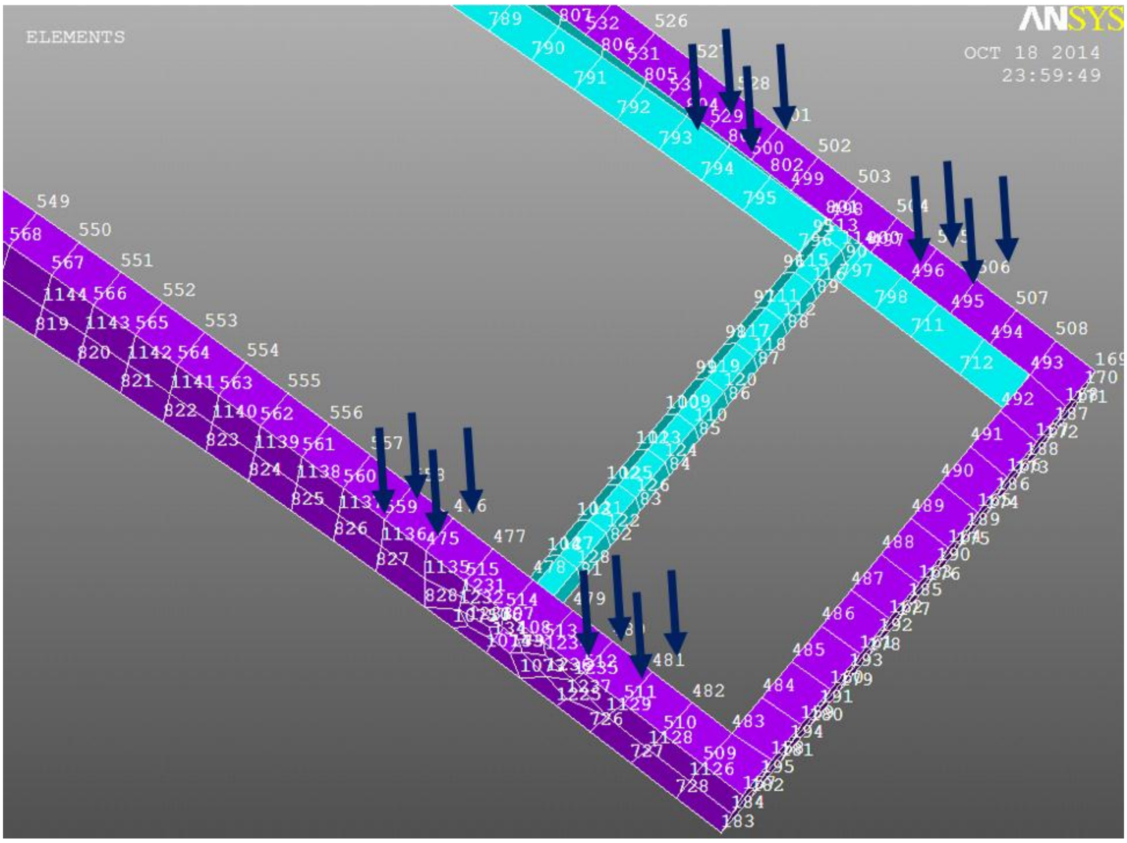


Figure 6.13 (a-b) : Loads are applied to the frame model through corresponding nodes.



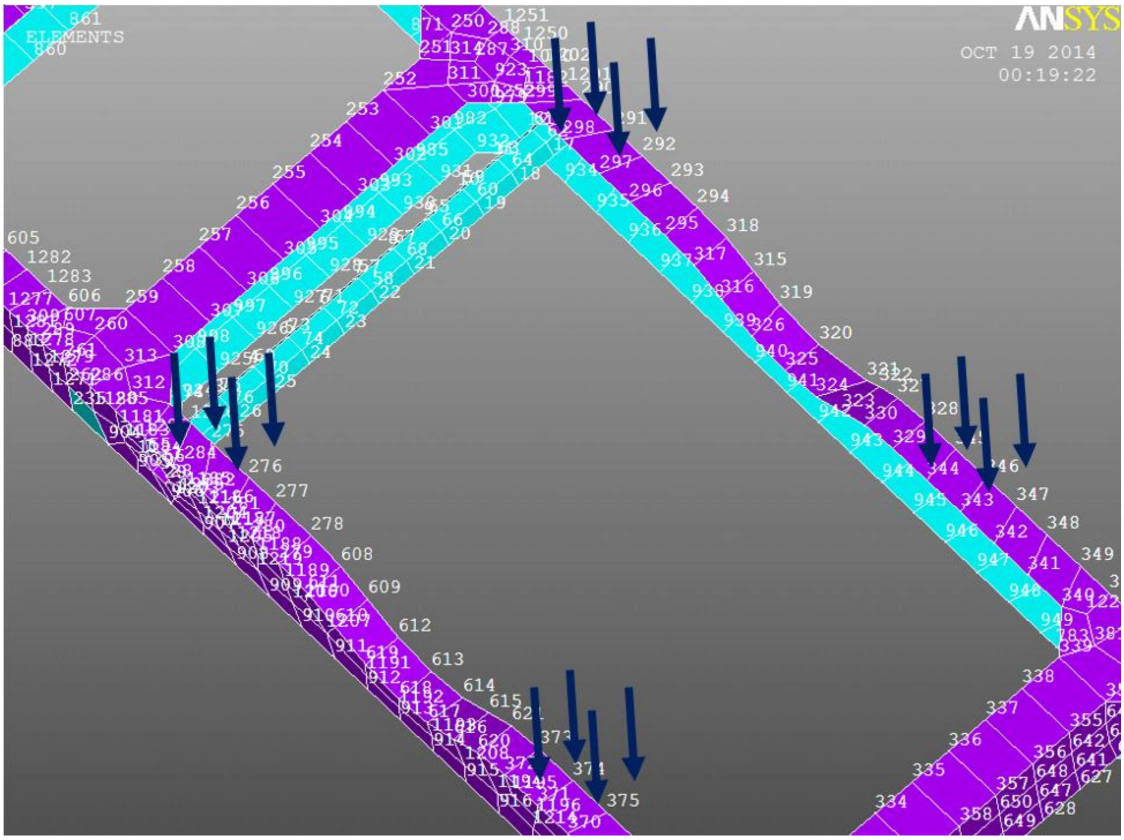


Figure 6.14 (a-b) : Model is constrained by Front & Rear Leaf Spring mounting points.

In this example, SVD based optimization approach is applied to find the Pareto optimal cross sectional thickness value of the chassis frame while minimizing the weight and maximum displacement of the frame by using the SQP method. As an outcome Pareto optimal solution sets are determined to enable the designer to choose for project demands. Following, the same procedure is repeated by using the conventional optimization formulation via SQP and Pareto optimal sets are calculated.

The weight of the chassis is a function of h and can be calculated accordingly. For the sake of simplicity, the weight is chosen to be equal to the magnitude of h for this application because the geometry of the frame, the length, the width and the density of the material are considered constant. For the exact weight values, one can multiply the outcome h value with the area and the density of the frame.

For this chassis frame application depending upon the maximum allowed deflection specification one can select the required weight/thickness rates accordingly from Figures 6.15-6.17. For instance, when we consider maximum allowed deflection as 1 mm then h=5.5 mm and its corresponding Weight and Maximum Displacement values will be the optimum solution for our application. Even if we further increase the thickness value, it will not improve the cost functions to satisfy the demand.

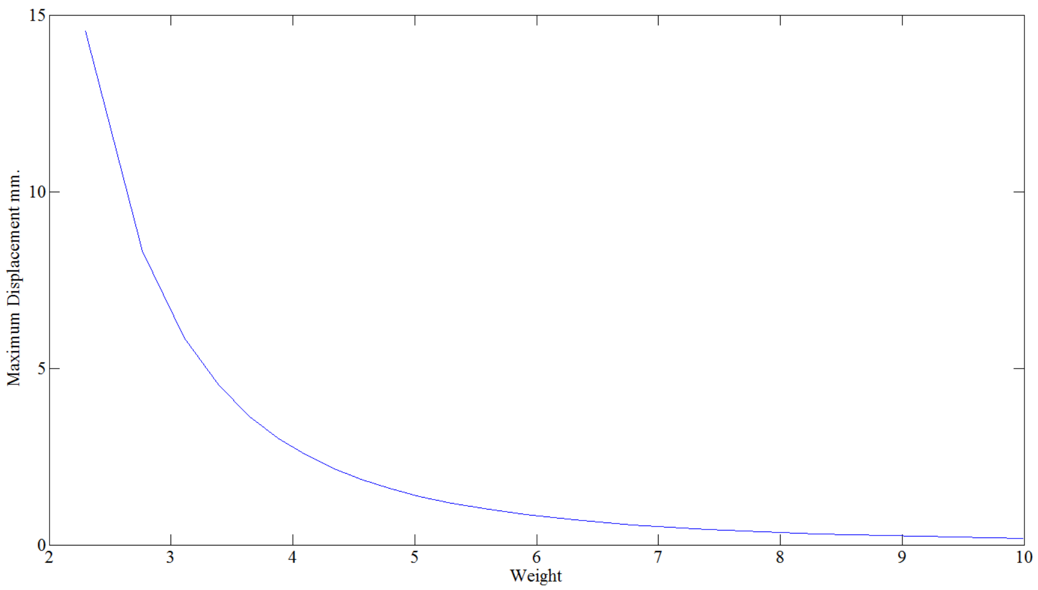


Figure 6.15 : Weight vs. Maximum Displacement graph created by using SVD with 19 beta values.

Benchmarking the real world usages for similar loading conditions, it can be seen that it is a common practice of automotive manufacturers to use chassis frame thicknesses between 5 to 7 mm.

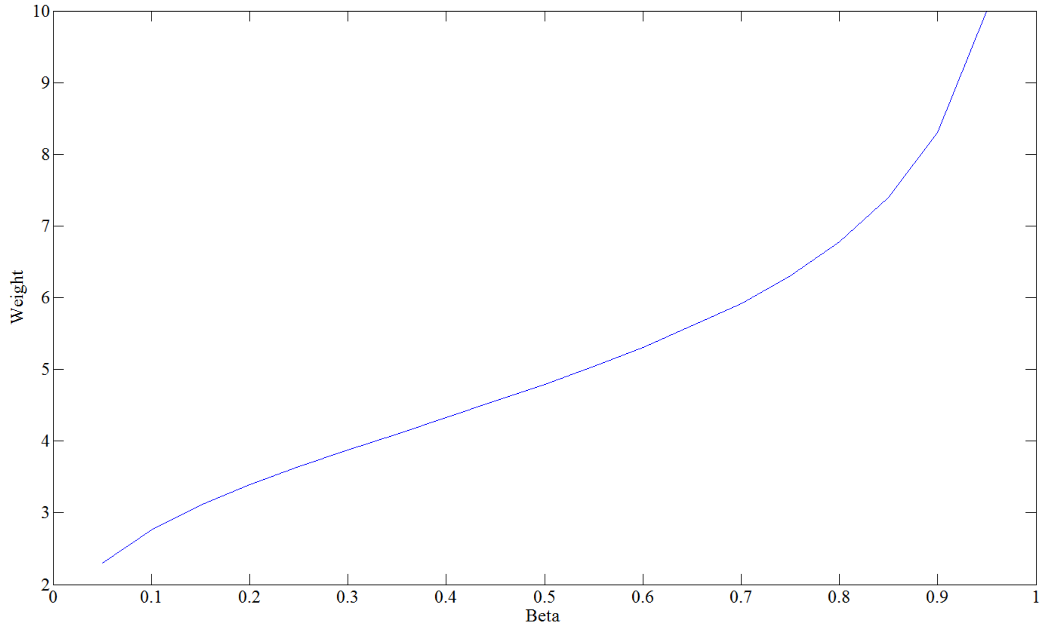


Figure 6.16 : Weight values calculated from 19 beta steps in Pareto optimal set calculations.

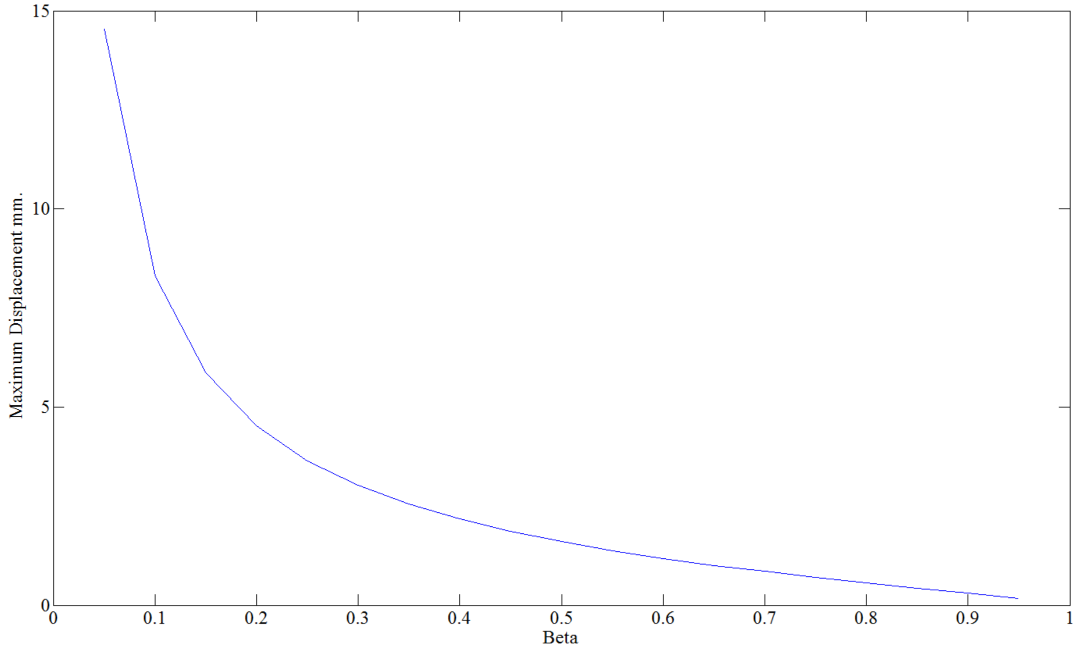


Figure 6.17 : Maximum displacement values calculated from 19 beta steps in Pareto optimal set calculations.

In order to make a comparison of SVD and conventional method same chassis frame example is also solved by conventional SQP algorithm. The output graphs from both SVD and conventional algorithm is given for three beta runs in Figures 6.18-6.19.

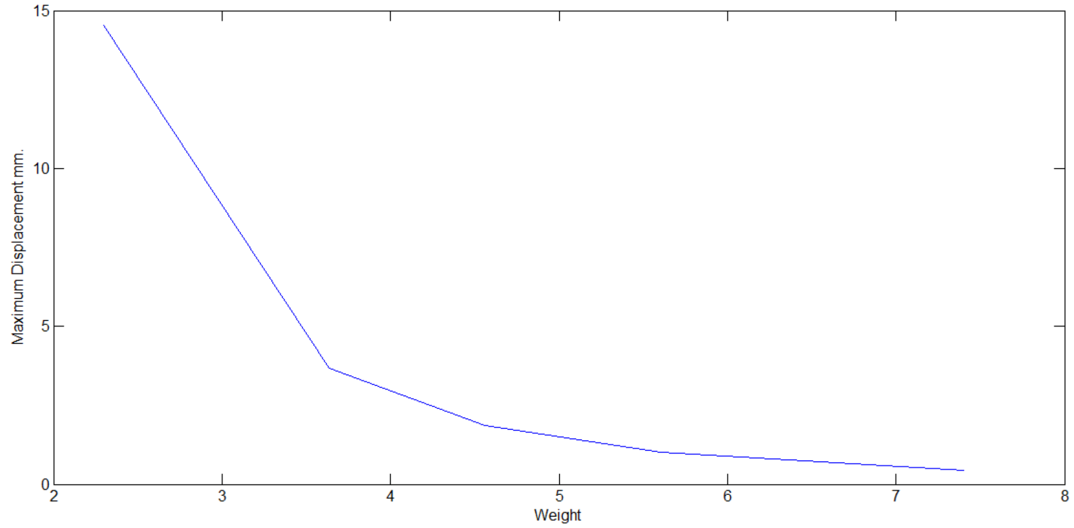


Figure 6.18 : Weight vs. Maximum Displacement graph by using SVD with 5 beta values in Pareto optimal set calculations.

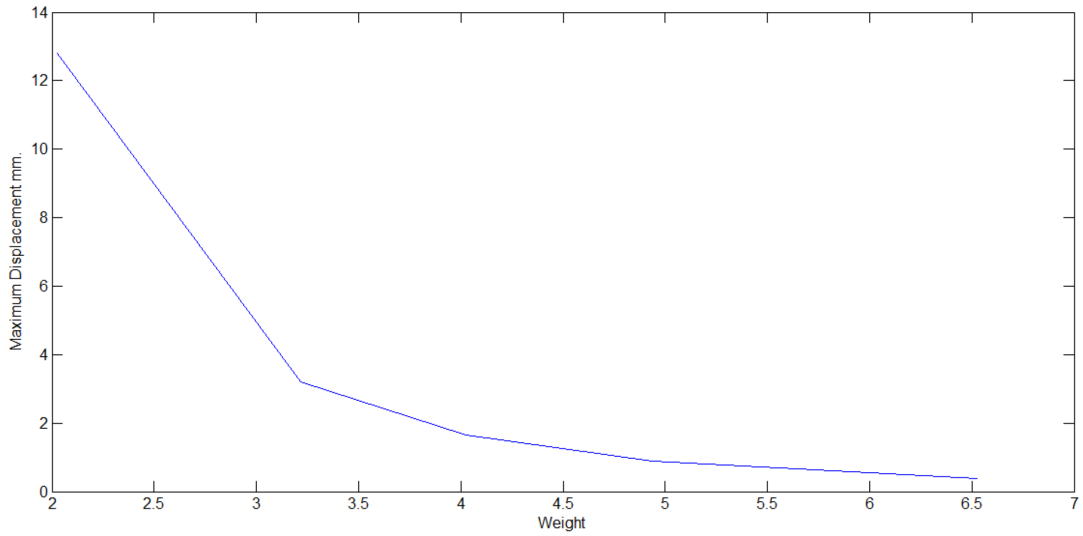


Figure 6.19 : Weight vs. Maximum Displacement graph by using conventional method with 5 beta values and 3 force iteration steps in Pareto optimal set calculations.

Comparison results are shown in Table 6.2. This example enables to have a better understanding of SVD performance for more complex problems than a simple constraint beam problem investigated in Example 1. In Table 6.2 it can be seen that,conventional method can reach to the solution with 5 beta values and 3 force iterations in 40.408 hours while SVD calculates the solution in 19.342 hours at the worst case loading conditions. That is the power of SVD that it can calculate the output according to the worst case loading condition but conventional method needs iteration between different loading conditions and delivers the result in the given resolution frame which has a direct impact on lengthening the calculation time. In order to increase the resolution one can increase the force iteration quantities where calculation will take much longer time as it can be seen from Table 6.2.

Hence, once more SVD proved itself to be a powerful tool in comparision to conventional method by being applied to a complex geometry that is being used in real life under different loading conditions.

Table 6.2: Intel Core i5 3.2 GHz CPU time comparison of SVD and conventional methods.

Method	Number of Beta values	Number of force iteration steps	3.2 GHz CPU Time in Seconds	3.2 GHz CPU CPU Time in Hours	RAM GB
SVD	3	-	42146.227616	11.707	8 GB
SVD	5	-	69632.030211	19.342	8 GB
SVD	10	-	152149.479840	42.364	8 GB
SVD	19	-	238572.916261	66.270	8 GB
Conventional	5	3	145470.437332	40.408	8 GB

7. STRUCTURAL AND SENSITIVITY REANALYSIS BASED ON SVD

7.1 History

Structural design process contains various manual or computerized design iterations to find an optimal solution that satisfies the constraints. For this reason, it is common to perform the structural and sensitivity analysis at each design iteration for the complete system having certain modifications at some components and under some load conditions; however, this approach unnecessarily increases the CPU times of the analysis since it does not consider the fact that only certain components of the associated structural matrices differ from the original structural matrices during the iterations. Thus, fast static reanalysis of systems has significant practical value in structural optimization, probabilistic analysis, structural health monitoring, sensitivity analysis and system identification.

There are many studies in literature about structural and sensitivity reanalysis. Reviews of reanalysis methods may be found in [166, 167, 196]. Additionally, a review of history of reanalysis techniques based on the SMW formulas are presented in [179] along with some applications. Moreover, it is shown in [185] that the solution of a system of linear equations associated with static reanalysis and having low-rank increments in the stiffness matrix are variants of the SMW formulas. Extension of low-cost linear reanalysis based on the SMW formulas to some nonlinear reanalysis problems is also presented in [185]. For the reanalysis of structures having geometrical changes, a solution procedure is presented in [168] that is based on the combined approximations method where the binomial series terms are used as basis vectors in reduced basis approximations. A unified approach to structural reanalysis, design and optimization is presented in [169] that is based on the combined approximations method. Efficient procedures for sensitivity analysis of large-scale structures are presented in [213] where approximation concepts based on explicit approximations of the response and the combined approximations are used to improve the efficiency. A general overview of the combined approximations method for reanalysis and sensitivity reanalysis of linear, nonlinear, static and dynamic systems is given in [184]. The reanalysis of linear structural systems subjected to stochastic inputs for both topological and non-topological structural modifications are studied in [170]. A procedure for structural vibration reanalysis is developed based on iteration and inverse iteration methods with frequency-shift and linear combination acceleration in [171]. By using a substructuring method, a reanalysis approach for efficient and accurate optimization of vibration characteristics of structures is presented in [172].

A reanalysis method based on modal analysis of the original structure, homotopy perturbation, projection techniques and Taylor series expansion is developed in [173]. In this paper, it is focused on the reanalysis of eigensolutions in a context of multiple modifications of various origins. The aim is to approximate the eigensolutions with a good level of precision and to reduce the global CPU time for a large set of modifications. Three methods discussed for reanalyzing perturbed eigenvalue problems, in a context of multiple modifications of various origins that are introduced into the mass and stiffness matrices. These methods are based on several concepts, namely homotopy perturbation, projection techniques and Taylor series expansion. The efficiency of the proposed methods in terms of precision and CPU time, according to the number of mode shapes, the order of truncation and the size of finite element models were compared. A reanalysis example of mode crossing/veering due to structural modification is also presented in this study. Three methods, which allow modifications of the mass and stiffness matrices studied are tested. The first one, the HP (Homotopy Perturbation) method, relies on the homotopy perturbation technique and expresses the perturbed eigensolutions in terms of high-order eigensolution perturbation. This domain of application is limited to modal problems in which the mode shapes do not change. The second one, the HPP (Homotopy Perturbation and Projection) method, uses the homotopy perturbation and projection techniques. It replaces the perturbed eigenvalue problem by a reduced eigenvalue problem. The reduced basis is built using the high-order eigensolution perturbation. This HPP method is efficient for the precise reanalysis of frequencies and eigenvectors with important modifications of the output solutions. Therefore, this method can be applied when only a few mode shapes are studied. The last one, the HPTP (Homotopy Perturbation, Projection and Taylor series expansion) method is the most general. This method is very efficient for the precise reanalysis of frequencies and eigenvectors, even if the mode shapes are greatly perturbed by the modifications.

The reanalysis of sensitivity to material characteristics of eigenvalues is studied in [203] that is based on the association of a homotopy transformation and perturbation method. An adaptive reanalysis method for genetic algorithm with application to fast truss optimization is given in [174], that is derived from the combined approximations method. In the work [194], a reanalysis procedure for load reconstruction and damage identification in structural health monitoring is presented which is based on the virtual distortion method. Based on the preconditioned Richardson's iterative method, an iterative method for structural static reanalysis is given in [175], which employs the potential energy function. A method for local sensitivity analysis based on the duality property of mathematical programming is developed and applied to regression and estimation problems in [214]. Based on the duality property, closed formulas to obtain all the local sensitivities (i.e., objective function, primal and dual variables) of a general nonlinear programming with respect to any parameter are presented in [204-205]. To perform a general analysis without assuming the existence of partial derivatives and without considering any active inequality constraints remain active, a perturbation approach is presented to yield all the local sensitivities of all variables at once in [206]. On the other hand, the design sensitivity analysis of structures based upon the SVD and Gateaux derivatives is presented in [136].

The reason to study the SMW formulas is to investigate a fast and efficient reanalysis technique in particular for the analysis of structures by using computational techniques such as the finite element method, where the mathematical model of the structure of interest is represented by a linear equation system [187,188]. The structure has many components and each component has numerous parameters to be optimized such as thickness, area, length and other geometric and material parameters. Altough the structural reanalysis methods have existed for more than sixty years, they are efficient for structural reanalysis involving low-rank modifications [185]. The SMW formulas require the solution of the original system with m different right-hand-sides and solution of an additional system of order m, where m is the rank of modification matrix [185]. If the rank of modification matrix *m* is large, this approach is very slow especially for large structures and in the existence of multiple load cases. In addition, it is commonly followed in conventional reanalysis approaches that a priori perturbations are introduced to system matrices of a given structure. These perturbations are either generated by an algorithm or determined by the designer. Thus, it is of practical value to calculate the optimum perturbations of design variables of a structure such that the desired changes in structural responses are achieved, in particular for large structures (i.e., *m* is large) and in the existence of multiple load cases. On the other hand, the SVD is implemented into multi-input-multi-output (MIMO) systems to investigate input-output relations of these systems, which has proven to be very useful and has given an insight into the system behavior [215]. Motivated by the advantages of the SVD to analyze MIMO systems, this thesis is initiated to implement the SVD into structural reanalysis problems.

In this thesis, due to above mentioned reasons, it is aimed to develop an approach to calculate optimum perturbations of design variables in an efficient manner where the SVD is employed to obtain the desired output changes and sensitivity values. Formulations in this section are based on the SMW formulas to which the SVD is applied. Numerical examples are presented to illustrate the applicability of proposed formulations and show the advantages of the proposed approach. Comparisons between the CPU times of the SVD-based approach and SQP method are made, which show the efficiency of the proposed approach. In addition, accuracy of the solutions are checked analytically. It is shown that the SMW formulas along with the SVD of the system matrices can be efficiently used to calculate the extremum sensitivities and optimum perturbations of design variables and to give more insight into the sensitivity of responses.

7.2 Sherman-Morrison-Woodbury Formulas

Consider a linear structure having *n* DOF whose governing equations are given by

$$\boldsymbol{K}\boldsymbol{d}_{\boldsymbol{0}} = \boldsymbol{f} \tag{7.1}$$

where, by the use of the finite element terminology, $\boldsymbol{K}$ is the global stiffness matrix, $\boldsymbol{d}_0$ is the displacement vector of the original structure and $\boldsymbol{f}$ is the global force vector. Suppose that there is a perturbation in the structure such that equations of the perturbed system are cast in the following form

$$(\boldsymbol{K} + \Delta\boldsymbol{K})\boldsymbol{d} = \boldsymbol{f} \tag{7.2}$$

where $\boldsymbol{d} = \boldsymbol{d_0} + \Delta\boldsymbol{d}$. The solution of Equation (7.2) can be obtained by the following Woodbury formula [179] giving the inverse of the modified stiffness matrix due to a rank-m modification matrix $\Delta\boldsymbol{K} = \boldsymbol{P}\boldsymbol{Q}^T$

$$(\boldsymbol{K} + \boldsymbol{P}\boldsymbol{Q}^T)^{-1} = \boldsymbol{K}^{-1} - \boldsymbol{K}^{-1}\boldsymbol{P}(1 + \boldsymbol{Q}^T\boldsymbol{K}^{-1}\boldsymbol{P})^{-1}\boldsymbol{Q}^T\boldsymbol{K}^{-1} \tag{7.3}$$

where $\boldsymbol{P}$ and $\boldsymbol{Q}$ are $n \times m$ *m*atrices. By postmultiplying the Equations (7.3) or (7.4) by the force vector, the SMW formulas can be applied to solve the response of the modified system whose computational load depends on the rank-m of the modification matrix $\Delta\boldsymbol{K}$. If $\boldsymbol{P}$ and $\boldsymbol{Q}$ are column vectors, then structural modification matrix is in the form of $\Delta\boldsymbol{K} = \boldsymbol{p}\boldsymbol{q}^T$ having rank-one. Computational cost of solving Equation (7.2) by typically using the Gaussian elimination method increases as the rank of $\Delta\boldsymbol{K}$ increases. For instance, in truss systems, the change of one truss element leads to a rank-one modification of $\boldsymbol{K}$. In addition, total change in *m* truss members results in a rank-*m* modification in the stiffness matrix that can be represented as a sum of rank-one modifications to *m* truss members, i.e. superposition principle [185]. Moreover, for a plane triangular element, if reanalysis is required, this may be viewed as three rank-one modifications [185].

Since the stiffness matrix $\boldsymbol{K}$ is usually symmetric, one can substitute $\boldsymbol{q} = \eta\boldsymbol{p}$ in Equation (7.3) where η is a scalar that may be +1 or -1 depending on the sign of the stiffness change. Thus, the SMW formulas [185] may be applied to the modified system by first solving the following linear equation system for the vector **r**

$$\boldsymbol{K}\mathbf{r} = \boldsymbol{p} \tag{7.4}$$

Then, the vector $\Delta\boldsymbol{d}$ can be calculated by

$$\Delta\boldsymbol{d} = \boldsymbol{d} - \boldsymbol{d}_0 = -\alpha\boldsymbol{r} \tag{7.5}$$

where α is the scalar given by

$$\alpha = \frac{\eta \boldsymbol{p}^T \boldsymbol{d}_0}{1 + \eta \boldsymbol{p}^T \boldsymbol{r}} \tag{7.6}$$

whose derivation can be found in [185]. In sum, the displacement vector $\boldsymbol{d}$ can be updated by the use of Equations (7.4) to (7.6).

7.3 Singular Value Decomposition

In this section, some of the basic properties of the SVD are revisited; for details, see [190] and [191]. Consider a matrix $\boldsymbol{A} \in \mathcal{C}^{n\times n}$, whose SVD is given by $\boldsymbol{A} = \boldsymbol{U\Sigma V}^H$, where $\boldsymbol{U} = [\boldsymbol{u}_1|\boldsymbol{u}_2|\cdots|\boldsymbol{u}_n] \in \mathcal{C}^{n\times n}, \boldsymbol{\Sigma} = \text{Diag}\{\sigma_1, \sigma_2, \dots, \sigma_n\} \in \mathcal{R}^{n\times n}$, $\boldsymbol{V} = [\boldsymbol{v}_1|\boldsymbol{v}_2|\cdots|\boldsymbol{v}_n] \in \mathcal{C}^{n\times n}$ and superposed H denotes the conjugate transpose. While σ_i are called the singular values of $\boldsymbol{A}$, $\boldsymbol{u}_i$ and $\boldsymbol{v}_i$ are the orthonormal vectors called the left and right singular vectors of $\boldsymbol{A}$, respectively. Note that singular values are ordered such that $\sigma_{i+1} \geq \sigma_i$, and σ_1 and σ_n are respectively the minimum and maximum singular values having special importance, since corresponding singular vectors of $\boldsymbol{u}_1$, $\boldsymbol{u}_n$, $\boldsymbol{v}_1$ and $\boldsymbol{v}_n$ are associated with input and output directions having extremum gain values [215].

Consider the following time-independent linear equation system

$$\boldsymbol{Kd} = \boldsymbol{f} \tag{7.7}$$

where $\boldsymbol{K} \in \mathcal{C}^{n\times n}$, $\boldsymbol{d} \in \mathcal{C}^n$ and $\boldsymbol{f} \in \mathcal{C}^n$. Then, $\boldsymbol{d} = \boldsymbol{K}^{-1}\boldsymbol{f}$. The SVDs of $\boldsymbol{K}^{-1}$ and $\boldsymbol{K}$ are denoted by by $\boldsymbol{K}^{-1} = \boldsymbol{U\Sigma V}^H$ and $\boldsymbol{K} = \boldsymbol{V\Sigma}^{-1}\boldsymbol{U}^H$. The SVD of $\boldsymbol{K}^{-1}$ may be written in the following dyadic form

$$\boldsymbol{K}^{-1} = \sum_{i=1}^{n} \sigma_i \boldsymbol{u}_i \boldsymbol{v}_i^H. \tag{7.8}$$

If the force vector $\boldsymbol{f} = \boldsymbol{v}_k$ is equal to the *k*th right singular vector, then Equation (7.8) yields

$$\boldsymbol{d} = \boldsymbol{K}^{-1}\boldsymbol{f} = \sum_{i=1}^{n} \sigma_i \boldsymbol{u}_i \boldsymbol{v}_i^H \, \boldsymbol{v}_k. \tag{7.9}$$

Since $\boldsymbol{v}_i$ are orthonormal, $\boldsymbol{v}_i^H \boldsymbol{v}_k = \delta_{ik}$ where δ_{ik} is the Kronecker delta function. Thus, we get $\boldsymbol{d} = \sigma_k \boldsymbol{u}_k$and $\|\boldsymbol{d}\|_2 = \sigma_k$. In sum, if $\boldsymbol{f}$ is in the direction of $\boldsymbol{v}_k$, then the displacement (or output) vector $\boldsymbol{d}$ is in the direction of $\boldsymbol{u}_k$ and the gain is equal to σ_k. Next, these derivations will be applied to structural sensitivity reanalysis of a structure by using the SMW formulas.

7.4 Reanalysis Based On Singular Value Decomposition

7.4.1 Structural perturbations in right singular vector directions

Consider a perturbation vector in the i^{th} right singular vector direction $\boldsymbol{p} = \boldsymbol{v}_i$ and η is given a priori; hence, we get $\Delta\boldsymbol{K} = \eta \boldsymbol{p}\boldsymbol{p}^T = \eta \boldsymbol{v}_i \boldsymbol{v}_i{}^T$ in Equation (7.2). Following the properties of the SVD given in Section 7.3, $\boldsymbol{K}\mathbf{r} = \boldsymbol{p} = \boldsymbol{v}_i$ holds true due to Equation (7.4) and we get $\mathbf{r} = \sigma_{\boldsymbol{i}} \boldsymbol{u}_i$. Subsequently, Equation (7.6) becomes

$$\alpha = \frac{\eta \boldsymbol{p}^T \boldsymbol{d}_0}{1 + \eta \boldsymbol{p}^T \boldsymbol{r}} = \frac{\eta \boldsymbol{v}_{\boldsymbol{i}}^{\boldsymbol{T}} \boldsymbol{d}_0}{1 + \eta \sigma_{\boldsymbol{i}} \boldsymbol{v}_{\boldsymbol{i}}^{\boldsymbol{T}} \boldsymbol{u}_i} \tag{7.10}$$

and $\Delta\boldsymbol{d} = -\alpha\mathbf{r} = -\alpha\sigma_{\boldsymbol{i}}\boldsymbol{u}_i$ holds due to Equation (7.5). In other words, the change in the displacement vector $\Delta\boldsymbol{d}$ is related to the singular values and vectors of the stiffness matrix of the original system $\boldsymbol{K}$. If the force vector is expressed as a combination of right singular vectors $\boldsymbol{f} = \sum_{i=1}^{n} \gamma_i \boldsymbol{v}_i$ where $\boldsymbol{\gamma} = \boldsymbol{V}^T \boldsymbol{f}$ due to the orthonormal property of $\boldsymbol{v}_i$, then the displacement vector of the original structure $\boldsymbol{d}_0$ is equal to the following

$$\boldsymbol{d}_0 = \sum_{j=1}^{n} \gamma_j \sigma_j \boldsymbol{u}_j \tag{7.11}$$

which follows from the properties of the SVD given in Section 7.3. On the other hand, if we expand the vector $\boldsymbol{d}_0$ in terms of right singular vectors as follows

$$\boldsymbol{d}_0 = \sum_{i=1}^{n} \beta_i \boldsymbol{v}_i \tag{7.12}$$

by equating the Equations (7.11) and (7.12) and by premultiplying both sides of them by the right singular vector $\boldsymbol{v}_{\boldsymbol{l}}^{\boldsymbol{T}}$, then we get the following

$$\beta_l = \sum_{j=1}^{n} \gamma_j \sigma_j \boldsymbol{v}_l^T \boldsymbol{u}_j \qquad \textbf{(7.13)}$$

Since $\boldsymbol{v}_l^T \boldsymbol{d}_0 = \beta_l$ holds due to the orthonormal property of singular vectors, Equation (7.10) becomes

$$\alpha = \frac{\eta \beta_i}{1 + \eta \sigma_i \boldsymbol{v}_i^T \boldsymbol{u}_i} \qquad \textbf{(7.14)}$$

which is of the perturbation vector $\boldsymbol{p} = \boldsymbol{v}_i$. It is pointed out in Section 7.3 that two extreme cases are the singular directions associated with the minimum and maximum singular values σ_1 and σ_n, which corresponds to the following two extremum cases for the changes in the displacement vector $\Delta \boldsymbol{d}$

If $\boldsymbol{p} = \boldsymbol{v}_1$,

$$\Delta \boldsymbol{d} = -\frac{\eta \beta_1 \sigma_1}{1 + \eta \sigma_1 \boldsymbol{v}_1^T \boldsymbol{u}_1} \boldsymbol{u}_1 \qquad \textbf{(7.15)}$$

If $\boldsymbol{p} = \boldsymbol{v}_n$

$$\Delta \boldsymbol{d} = -\frac{\eta \beta_n \sigma_n}{1 + \eta \sigma_n \boldsymbol{v}_n^T \boldsymbol{u}_n} \boldsymbol{u}_n \qquad \textbf{(7.16)}$$

that are obtained by using $\Delta \boldsymbol{d} = -\alpha \sigma_i \boldsymbol{u}_i$ and Equation (7.14). If $\boldsymbol{p}$ is a linear combination of right singular vectors $\boldsymbol{v}_i$, then the superposition principle is valid for above $\Delta \boldsymbol{d}$ expressions., e.g., see Section 7.6.

7.4.2 Directions of extremum changes in displacement vector

It is investigated in this section what the structural perturbation matrix $\Delta \boldsymbol{K} = \eta \boldsymbol{p}\boldsymbol{p}^T$ is for a given $\Delta \boldsymbol{d}$vector in Equation (7.2). Considering Equation (7.5) and assuming that the change in the displacement vector $\Delta \boldsymbol{d}$ is proportional to a left singular vector such as $\Delta \boldsymbol{d} = \sigma_i \boldsymbol{u}_i$, then Equation (7.5) becomes

$$\Delta \boldsymbol{d} = -\alpha \mathbf{r} = \sigma_i \boldsymbol{u}_i \qquad \textbf{(7.17)}$$

that gives

$$\mathbf{r} = \boldsymbol{u}_i \text{ and } \alpha = -\sigma_{\boldsymbol{i}} \tag{7.18}$$

Due to Equations (7.4) and (7.8), we get the perturbation vector $\boldsymbol{p}$ as follows.

$$\boldsymbol{p} = \boldsymbol{K}\mathbf{r} = \boldsymbol{K}\boldsymbol{u}_i = \sigma_{\boldsymbol{i}}^{-1}\boldsymbol{v}_i \tag{7.19}$$

By using $\alpha = -\sigma_{\boldsymbol{i}}$ in Equation (7.18), Equation (7.10) yields that

$$\alpha = -\sigma_{\boldsymbol{i}} = \frac{\eta \boldsymbol{p}^T \boldsymbol{d}_0}{1 + \eta \boldsymbol{p}^T \boldsymbol{r}} = \frac{\eta \sigma_{\boldsymbol{i}}^{-1} \boldsymbol{v}_{\boldsymbol{i}}^{\boldsymbol{T}} \boldsymbol{d}_0}{1 + \eta \sigma_{\boldsymbol{i}}^{-1} \boldsymbol{v}_{\boldsymbol{i}}^{\boldsymbol{T}} \boldsymbol{u}_i} = \frac{\eta \sigma_{\boldsymbol{i}}^{-1} \beta_i}{1 + \eta \sigma_{\boldsymbol{i}}^{-1} \boldsymbol{v}_{\boldsymbol{i}}^{\boldsymbol{T}} \boldsymbol{u}_i} \tag{7.20}$$

where $\boldsymbol{v}_l^{\boldsymbol{T}} \boldsymbol{d}_0 = \beta_l$ is employed in the last equality. By rearranging, we get

$$\eta = -\frac{\sigma_{\boldsymbol{i}}^2}{\beta_i + \sigma_{\boldsymbol{i}} \boldsymbol{v}_{\boldsymbol{i}}^{\boldsymbol{T}} \boldsymbol{u}_i} \tag{7.21}$$

In search of the extremum sensitivity cases, by assuming $\Delta\boldsymbol{d} = \sigma_{\boldsymbol{1}}\boldsymbol{u}_1$ and $\Delta\boldsymbol{d} = \sigma_{\boldsymbol{n}}\boldsymbol{u}_n$ in Equation (7.17) which are respectively the most and least sensitive directions, Equations (7.19) and (7.21) yield respectively the perturbation vectors $\boldsymbol{p}$ and parameters η that result in the most and least sensitivities in responses. If $\Delta\boldsymbol{d}$ is a linear combination of vectors of such as $\sigma_{\boldsymbol{i}}\boldsymbol{u}_i$, then the superposition principle is valid for $\boldsymbol{p}$ and η, e.g., see Section 7.6.

7.4.3 Perturbations for extremum changes in a subset of displacement vector

It has a practical value to obtain the perturbation vector $\boldsymbol{p}$ giving extremum changes in certain components of the displacement vector $\boldsymbol{d}$. Let $\widehat{\Delta\boldsymbol{d}}$ denote the subset of $\Delta\boldsymbol{d}$ that includes certain components of $\Delta\boldsymbol{d}$ defined by

$$\widehat{\Delta\boldsymbol{d}} = \boldsymbol{E}\,\Delta\boldsymbol{d} \tag{7.22}$$

where the transformation matrix $\boldsymbol{E}$ is defined such that

$$\boldsymbol{E} = \begin{bmatrix} \cdots \\ \boldsymbol{e}_j \\ \cdots \end{bmatrix} \tag{7.23}$$

and $\boldsymbol{e_j} = [0 \cdots 0 \quad 1 \quad 0 \cdots 0]$ whose j^{th} component is equal to 1 and all other components are zero. Subsequently,

$$\widehat{\boldsymbol{d}} = \boldsymbol{E}\boldsymbol{d} = \boldsymbol{E}\boldsymbol{K}^{-\mathbf{1}}\mathbf{f} \qquad \textbf{(7.24)}$$

Then, lets define $\widehat{\boldsymbol{K}} = \boldsymbol{E}\boldsymbol{K}^{-\mathbf{1}}$ that has the following SVD

$$\widehat{\boldsymbol{K}}^{-1} = \widehat{\boldsymbol{U}}\widehat{\boldsymbol{\Sigma}}\widehat{\boldsymbol{V}}^{H} \qquad \textbf{(7.25)}$$

Following Equation (7.17), the change in the subset of $\Delta\boldsymbol{d}$ is equal to

$$\widehat{\Delta\boldsymbol{d}} = \boldsymbol{E}\Delta\boldsymbol{d} = -\alpha\boldsymbol{E}\mathbf{r} = \hat{\sigma}_{\boldsymbol{i}}\widehat{\boldsymbol{u}}_i$$

that yields

$$\alpha = -\hat{\sigma}_{\boldsymbol{i}} \qquad \textbf{(7.26)}$$

On the other hand, we have

$$\boldsymbol{E}\mathbf{r} = \widehat{\boldsymbol{u}}_i \qquad \textbf{(7.27)}$$

Since $\boldsymbol{K}\mathbf{r} = \boldsymbol{p}$ and $\mathbf{r} = \boldsymbol{K}^{-1}\boldsymbol{p}$, Equation (7.27) becomes

$$\boldsymbol{E}\mathbf{r} = \boldsymbol{E}\boldsymbol{K}^{-1}\boldsymbol{p} = \widehat{\boldsymbol{u}}_i \qquad \textbf{(7.28)}$$

that gives an underdetermined equation system for the perturbation vector $\boldsymbol{p}$. Meanwhile, Equation (7.26) results in the following perturbation parameter η for a given $\widehat{\Delta\boldsymbol{d}}$

$$\eta = -\frac{\hat{\sigma}_{\boldsymbol{i}}^2}{\beta_i + \hat{\sigma}_{\boldsymbol{i}}\widehat{\boldsymbol{v}}_{\boldsymbol{i}}^{\boldsymbol{T}}\widehat{\boldsymbol{u}}_i} \qquad \textbf{(7.29)}$$

In sum, the underdetermined equation system defined by Equation (7.28) has to be solved to find the desired perturbation vector $\boldsymbol{p}$, which requires (for the existence of a solution) that the number of components of $\boldsymbol{p}$ to be perturbed is equal to the number of selected components of $\boldsymbol{d}$ (i.e., the size of $\widehat{\boldsymbol{d}}$). Consequently, we have flexibility in choosing the components of $\boldsymbol{p}$ to be perturbed.

7.5 Sensitivity Reanalysis

In structural design problems, some objective function(s) $\varphi(X)$ formulated to measure the performance of a structure has to be minimized such that a priori design requirements and constraints are satisfied, where X denotes design variables. Then, the objective of design sensitivity analysis is to find the dependence of objective function $\varphi(X)$ on design; namely, design sensitivity values $d\varphi(X)/dX$. Design sensitivity information is useful to carry out trade off analysis, to improve the design and to determine the most efficient design variable(s) X to achieve the desired change in objective function $\varphi(X)$.

Of special emphasis in this work is to calculate the perturbations in design variables that yield the extremum design sensitivity values for which the SVD is employed. To this end, the extremum values of response derivatives $d\varphi(X)/dX$ can be searched by using analytical and numerical methods existing in literature. By using the numerical derivative formulas [184], the response derivatives with respect to a design variable X can easily be calculated by giving a perturbation δX to a design variable X. Such a design of experimental approach is based on consideration of each design variable separately, which is computationally costly. On the other hand, it is shown in Section 7.6 that the proposed SVD-based sensitivity reanalysis can efficiently calculate the perturbations of design variables.

7.6 Numerical Examples

The analytical derivations presented in Sections 7.4 are applied to a planar truss system shown in Figure 7.1, where there are 25 truss elements and 28 degrees of freedom (DOF). For the sake of simplicity, it is selected that the magnitudes of applied forces are $F_1=F_2=1$, the cross-sectional areas of truss elements are constant $A_i=1$, the Young's modulus of truss element material is E=1 and the geometric parameter is L=1. The global DOF of the joints are denoted by z_i and the numbers of truss elements are written on the associated elements in Figure 7.1. Then, conventional sensitivity analysis, optimization runs and SVD-based reanalysis of the truss system shown in Figure 7.1 are completed by deriving the finite element model of the truss structure and using the programs developed in Matlab®.

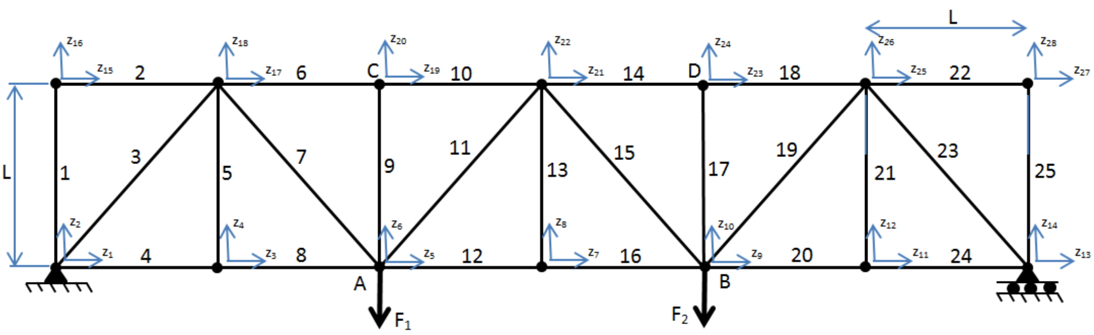


Figure 7.1 : Planar truss system.

First, design sensitivity analysis of the truss structure is studied. Suppose that design variables are the cross-sectional areas of truss members and the objective function is to reduce the deflections at joints A, B, C and D (i.e., z_6, z_{10}, z_{20} and z_{24}) in Figure 7.1. It is of interest to find the truss members having the greatest effect on the deflections of these joints. To this end, Table 7.1 lists the response derivatives of the vertical deflections at the selected joints of A, B, C and D in Figure 7.1, where 5% perturbation is introduced to cross-sectional areas of the truss members and the forward difference formula is employed [184]. The response derivatives listed in Table 7.1 are helpful if one wants to determine the most effective truss elements to be perturbed such that the desired response change is obtained at the DOF of interest in the structure. It is observed in Table 7.1 that the largest response derivative value of 2.5397 is obtained for the perturbations given to the 6th and 10th truss elements for the DOF of z_6 and z_{20}, while the same response derivative value is obtained for the perturbations given to the 14th and 18th truss elements for the DOF of z_{10} and z_{24}. It is also noteworthy that the 12th and 16th truss elements affect the DOF of z_6, z_{10}, z_{20} and z_{24} at the same amount having the response derivative value of 1.9048 which is relatively high in comparison with other response derivatives. Moreover, the 1st, 2nd, 5th, 9th, 11th, 13th, 15th, 17th, 21th, 22th and 25th truss elements have zero design sensitivity for the DOF of z_6, z_{10}, z_{20} and z_{24}.

In brief, in order to achieve the objective of reducing the deflections at joints A, B, C and D, the most effective way is to modify the cross-sectional areas of the 6th, 10th, 12th, 14th, 16th and 18th truss elements having relatively high response derivatives in comparison with other truss elements. On the other hand, it is difficult to determine the magnitudes of multiple perturbations in cross-sectional areas that yield the desired response changes. To this end, the derivations presented in Section 7.4 will be used. Note that when the formulations given in Section 7.4 are used, it is implicitly assumed that the objective function is to maximize (or minimize) the L_2-norm of $\Delta\boldsymbol{d}$ due to the underlying derivations based on the SVD.

Table 7.1 : Response derivatives for 5% perturbation in cross-sectional areas A_i.

Element Number	$\frac{dz_6}{dA}$	$\frac{dz_{10}}{dA}$	$\frac{dz_{20}}{dA}$	$\frac{dz_{24}}{dA}$
1	0	0	0	0
2	0	0	0	0
3	1.7958	8.9791e-1	1.7958	8.9791e-1
4	6.3492e-1	3.1746e-1	6.3492e-1	3.1746e-1
5	0	0	0	0
6	2.5397	1.2698	2.5397	1.2698
7	1.7958	8.9791e-1	1.7958	8.9791e-1
8	6.3492e-1	3.1746e-1	6.3492e-1	3.1746e-1
9	-7.8160e-13	-4.9738e-13	-8.5265e-13	-4.9738e-13
10	2.5397	1.2698	2.5397	1.2698
11	0	7.1054e-14	-1.4211e-13	7.1054e-14
12	1.9048	1.9048	1.9048	1.9048
13	9.9476e-13	9.9476e-13	9.9476e-13	9.9476e-13
14	1.2698	2.5397	1.2698	2.5397
15	-1.2790e-12	-1.1369e-12	-1.2790e-12	-1.2790e-12
16	1.9048	1.9048	1.9048	1.9048
17	-8.5265e-13	-9.9476e-13	-8.5265e-13	-9.9476e-13
18	1.2698	2.5397	1.2698	2.5397
19	8.9791e-1	1.7958	8.9791e-1	1.7958
20	3.1746e-1	6.3492e-1	3.1746e-1	6.3492e-1
21	1.4211e-13	3.5527e-13	2.1316e-13	3.5527e-13
22	0	0	0	0
23	8.9791e-1	1.7958	8.9791e-1	1.7958
24	3.1746e-1	6.3492e-1	3.1746e-1	6.3492e-1
25	0	0	0	0

Case 1:

Consider an optimization problem such that the objective is to find the perturbation vector $\boldsymbol{p}$ having unit length (i.e., $\|p\|_2 = 1$) that maximizes (or minimizes) the L_2-norm of $\Delta\boldsymbol{d}$. Suppose that the perturbation vector $\boldsymbol{p}$ is in the direction of a right singular vector $\boldsymbol{p} = \boldsymbol{v}_i$ that is examined in Section 7.4.1. Then, the changes in the displacement vector $\Delta\boldsymbol{d}$ are listed in Table 7.2 for the two extremum cases of $\boldsymbol{p} = \boldsymbol{v}_1$ and $\boldsymbol{p} = \boldsymbol{v}_{28}$ which correspond to the first (largest) and last (smallest) singular values of the matrix $\boldsymbol{K}^{-1}$. Note that it is selected as $\eta = 1$. The modification in the global stiffness matrix is defined by $\Delta\boldsymbol{K} = \eta\boldsymbol{p}\boldsymbol{p}^T$. Observe that there is a dramatic difference between the L_2-norms of the changes in the displacement vector such as

$\|\Delta\boldsymbol{d}\|_2 = 56.431$ for $\boldsymbol{p} = \boldsymbol{v}_1$ and $\|\Delta\boldsymbol{d}\|_2 = 1.0618 \times 10^{-4}$ for $\boldsymbol{p} = \boldsymbol{v}_{28}$, while $\boldsymbol{p}$ has unit length for both cases since $\|\boldsymbol{v}_1\|_2 = \|\boldsymbol{v}_{28}\|_2 = 1$. The ratio between L_2-norms of $\Delta\boldsymbol{d}$ corresponding to these two extreme cases is 531,465.43, which is dramatic. In other words, the modification matrix $\Delta\boldsymbol{K_1} = \boldsymbol{v}_1\boldsymbol{v}_1{}^T$ yields $\Delta\boldsymbol{d}$ whose L_2-norm is 531,465.43 times larger than that of the modification matrix $\Delta\boldsymbol{K_{28}} = \boldsymbol{v}_{28}\boldsymbol{v}_{28}{}^T$.

If the SQP method is employed to solve the above described optimization problem, it is given in Section 7.6.2 that total CPU time of the SVD-based computations is much lower than that of the SQP method based on sensitivity derivatives (e.g., see Table 7.5).

Table 7.2 : For the perturbation vector $\boldsymbol{p} = \boldsymbol{v}_i$, the change in displacement vector $\Delta\boldsymbol{d}$ for $\eta = 1$.

$\boldsymbol{v}_1$	$\boldsymbol{v}_{28}$	$\Delta\boldsymbol{d}$ if $\boldsymbol{p} = \boldsymbol{v}_1$ $\|\Delta\boldsymbol{d}\|_2 = 56.431$	$\Delta\boldsymbol{d}$ if $\boldsymbol{p} = \boldsymbol{v}_{28}$ $\|\Delta\boldsymbol{d}\|_2 = 1.0618 \times 10^{-4}$
2.4533e-18	1.3130e-16	0	1.0978e-20
-1.9626e-17	3.9589e-17	-1.1075e-15	1.0164e-20
2.9377e-2	-1.4814e-1	-1.6578	-1.5730e-5
-1.9796e-1	-1.3746e-2	1.1171e1	-1.4596e-6
5.8428e-2	3.8332e-1	-3.2972	4.0702e-5
-3.2074e-1	2.0425e-2	1.8100e1	2.1688e-6
9.9420e-2	-3.0812e-1	-5.6104	-3.2717e-5
-3.7298e-1	-1.0579e-3	2.1048e1	-1.1233e-7
1.3931e-1	4.1397e-1	-7.8613	4.3956e-5
-3.1277e-1	-1.7703e-2	1.7650e1	-1.8798e-6
1.5962e-1	-1.9333e-1	-9.0075	-2.0528e-5
-1.8358e-1	1.1742e-2	1.0360e1	1.2468e-6
1.7816e-1	8.6273e-2	-1.0054e1	9.1607e-6
0	1.4952e-16	1.1075e-15	1.1012e-20
1.5721e-1	6.9591e-2	-8.8717	7.3894e-6
0	4.0349e-17	-2.2151e-15	1.9444e-21
1.5547e-1	-2.4966e-1	-8.7732	-2.6510e-5
-1.9577e-1	4.9315e-2	1.1047e1	5.2364e-6
1.2787e-1	2.7158e-1	-7.2160	2.8837e-5
-3.2434e-1	-5.6933e-3	1.8303e1	-6.0453e-7
9.8857e-2	-4.5307e-1	-5.5786	-4.8109e-5
-3.6884e-1	3.7952e-3	2.0814e1	4.0299e-7
7.1554e-2	2.8503e-1	-4.0379	3.0266e-5
-3.1628e-1	4.9346e-3	1.7848e1	5.2397e-7
4.3456e-2	-2.8447e-1	-2.4523	-3.0206e-5
-1.8154e-1	-4.2126e-2	1.0245e1	-4.4730e-6
4.3944e-2	7.9293e-2	-2.4798	8.4196e-6
0	5.6454e-17	0	6.2661e-21

In brief, even if we introduce the same unit length perturbation vector $\boldsymbol{p} = \boldsymbol{v}_i$ to the structure, the displacement vector $\boldsymbol{d}$ may be affected in dramatically different ways.

Next, let's find the relation between the perturbations introduced to cross-sectional areas of truss members and their effect on the perturbation vector $\boldsymbol{p}$.. For instance, let's apply a 5% perturbation to the cross-sectional area of the 3rd truss element. Then, by using the finite element equations of the stiffness matrix $\boldsymbol{K}$ and the modification matrix of $\Delta\boldsymbol{K} = \eta\boldsymbol{p}\boldsymbol{p}^T$, one can find that

$$\boldsymbol{p} = [0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0.13296,0.13296,0,0,0,0,0,0,0,0,0,0,]^T \quad \textbf{(7.30)}$$

$$\eta = 1 \quad \textbf{(7.31)}$$

On the other hand, following the results in Section 7.4.1, if a perturbation vector $\boldsymbol{p}$ is a linear combination of $\boldsymbol{v}_i$ such that

$$\boldsymbol{p} = \sum_{i=1}^{28} a_i \boldsymbol{v}_i \quad \textbf{(7.32)}$$

where a_i are the coefficients that can be found by $\boldsymbol{a} = \boldsymbol{V}^T\boldsymbol{p}$ due to the orthonormal property of $\boldsymbol{v}_i$, then

$$\Delta\boldsymbol{d} = -\sum_{i=1}^{28} a_i \frac{\eta\beta_i\sigma_i}{1 + \eta\sigma_i\boldsymbol{v}_i^T\boldsymbol{u}_i}\boldsymbol{u}_i \quad \textbf{(7.33)}$$

Thus, it is shown that an arbitrary vector $\boldsymbol{p}$ can be generated by perturbing the cross-sectional areas appropriately and can be expressed as a linear combination of singular vectors $\boldsymbol{v}_i$ as defined in Equation (7.32).

Case 2:

Here, we will examine the inverse problem of Case 1. Consider the optimization problem whose objective is to find the perturbation vector $\boldsymbol{p}$ having unit length (i.e., $\|p\|_2 = 1$) for a given $\Delta\boldsymbol{d} = \sigma_i\boldsymbol{u}_i$. Given the largest and smallest possible changes in the displacement vector (i.e., $\Delta\boldsymbol{d} = \sigma_1\boldsymbol{u}_1$ and $\Delta\boldsymbol{d} = \sigma_{28}\boldsymbol{u}_{28}$, respectively), the corresponding perturbation vectors $\boldsymbol{p}$ and parameters η are given in Table 7.3

calculated by the formulas given in Section 7.4.2. It is noteworthy that while the L_2-norm of the perturbation vector is equal to unity $\|\boldsymbol{p}\|_2 = 1$ for both cases $\Delta\boldsymbol{d} = \sigma_1\boldsymbol{u}_1$ and $\Delta\boldsymbol{d} = \sigma_{28}\boldsymbol{u}_{28}$, L_2-norms of $\Delta\boldsymbol{d}$ vectors are dramatically different; namely, $\|\sigma_1\boldsymbol{u}_1\|_2 = 90.066$ and $\|\sigma_{28}\boldsymbol{u}_{28}\|_2 = 0.21798$ that yield the parameter values of $\eta_1 = -55.137$ and $\eta_{28} = -0.21858$, respectively. The ratio between L_2-norms of $\Delta\boldsymbol{d}$ vectors is 413.18 which is significant. In other words, the modification matrix $\Delta\boldsymbol{K}_{\boldsymbol{1}} = \eta_1\boldsymbol{v}_1\boldsymbol{v}_1{}^T$ yields $\Delta\boldsymbol{d}$ whose L_2-norm is 413.18 times larger than that of the modification matrix $\Delta\boldsymbol{K}_{\boldsymbol{28}} = \eta_{28}\boldsymbol{v}_{28}\boldsymbol{v}_{28}{}^T$. Note that if the SQP method is employed to solve the above defined optimization problem, total CPU time of the SQP method is in the same range as Case 1 which is presented in Table 7.5. In brief, the proposed formulation is considerably faster than the SQP method.

If the desired change in the displacement vector $\Delta\boldsymbol{d}$ is expressed as a linear combination of singular values and left singular vectors, then it is of the form

$$\Delta \mathrm{d} = -\sum_{\mathrm{i}=1}^{28} \mathrm{a_i}\sigma_\mathrm{i}\mathrm{u_i} \tag{7.34}$$

where $\mathrm{a_i}$ are given coefficients. Then, each term in Equation (7.34) originates from the perturbation vector defined by

$$\boldsymbol{p_i} = a_i\sigma_i^{-1}\boldsymbol{v}_i \tag{7.35}$$

and the parameter η_i is defined by Equation (7.21). Subsequently, the perturbation to the global stiffness matrix is obtained by superposition as follows

$$\Delta\boldsymbol{K} = \sum_{\boldsymbol{i}=\boldsymbol{1}}^{\boldsymbol{28}} \eta_i\boldsymbol{p_i}\boldsymbol{p_i^T} \tag{7.36}$$

Table 7.3 : For $\Delta\boldsymbol{d} = \sigma_1\boldsymbol{u}_1$ and $\Delta\boldsymbol{d} = \sigma_{28}\boldsymbol{u}_{28}$, perturbation vectors $\boldsymbol{p}$ and parameters η.

$\Delta\boldsymbol{d} = \sigma_1\boldsymbol{u}_1$ $\lVert\Delta\boldsymbol{d}\rVert_2 = 90.066$	$\Delta\boldsymbol{d} = \sigma_{28}\boldsymbol{u}_{28}$ $\lVert\Delta\boldsymbol{d}\rVert_2 = 0.21798$	$\boldsymbol{p}$ for $\Delta\boldsymbol{d} = \sigma_1\boldsymbol{u}_1$ $\eta = -55.137$	$\boldsymbol{p}$ for $\Delta\boldsymbol{d} = \sigma_{28}\boldsymbol{u}_{28}$ $\eta = -0.21858$
0	2.2537e-17	2.7238e-20	6.0235e-16
1.7677e-15	2.0866e-17	-2.1791e-19	1.8162e-16
2.6459	-3.2291e-2	3.2617e-4	-6.7960e-1
-1.7830e1	-2.9964e-3	-2.1980e-3	-6.3061e-2
5.2624	8.3556e-2	6.4872e-4	1.7585
-2.8888e1	4.4522e-3	-3.5611e-3	9.3700e-2
8.9544	-6.7165e-2	1.1039e-3	-1.4135
-3.3593e1	-2.3060e-4	-4.1412e-3	-4.8531e-3
1.2547e1	9.0237e-2	1.5467e-3	1.8991
-2.8170e1	-3.8589e-3	-3.4727e-3	-8.1214e-2
1.4376e1	-4.2141e-2	1.7722e-3	-8.8689e-1
-1.6535e1	2.5596e-3	-2.0383e-3	5.3868e-2
1.6046e1	1.8806e-2	1.9781e-3	3.9578e-1
-1.7677e-15	2.2606e-17	0	6.8593e-16
1.4160e1	1.5169e-2	1.7455e-3	3.1925e-1
3.5353e-15	3.9915e-18	0	1.8510e-16
1.4002e1	-5.4421e-2	1.7261e-3	-1.1453
-1.7632e1	1.0750e-2	-2.1736e-3	2.2623e-1
1.1517e1	5.9200e-2	1.4198e-3	1.2459
-2.9212e1	-1.2410e-3	-3.6011e-3	-2.6118e-2
8.9037	-9.8761e-2	1.0976e-3	-2.0785
-3.3220e1	8.2728e-4	-4.0952e-3	1.7411e-2
6.4446	6.2132e-2	7.9446e-4	1.3076
-2.8486e1	1.0756e-3	-3.5117e-3	2.2638e-2
3.9140	-6.2009e-2	4.8249e-4	-1.3050
-1.6351e1	-9.1826e-3	-2.0157e-3	-1.9325e-1
3.9579	1.7284e-2	4.8791e-4	3.6376e-1
0	1.2864e-17	0	2.5899e-16

Case 3:

In the optimization problem considered in Case 1, the whole DOF of the truss system are considered such that the desired changes in $\Delta\boldsymbol{d}$ are obtained by the perturbation vector $\boldsymbol{p}$ and parameter η. Suppose that we are interested in the perturbations of design variables that yield the most and least changes in certain DOF such as z_6, z_{10}, z_{20} and z_{24} (i.e., only four DOF in $\boldsymbol{d}$). By using the results presented in Section 7.4.3, the appropriate perturbation vectors $\boldsymbol{p}$ and values of parameters η are listed in Table 7.4 corresponding to the most and least changes in the L_2-norm of the displacement vector components z_6, z_{10}, z_{20} and z_{24}. It is selected arbitrarily that only the 6th,10th, 20th and 24th components of the perturbation vector $\boldsymbol{p}$ are allowed to be perturbed due to the reasoning explained in Section 7.4.3. Since the associated equation system is underdetermined, we have to choose the components of $\boldsymbol{p}$ to be perturbed whose number is equal to the number of selected DOF of the structure (i.e., only four components of $\boldsymbol{p}$). Therefore, the solutions presented in Table 7.4 are suboptimal.

Table 7.4 : For the perturbations in the 6th,10th,20th and 24th components of the vector $\boldsymbol{p}$, the vector $\boldsymbol{p}$ that yields extremum changes in the DOF of z_6, z_{10}, z_{20} and z_{24}.

Perturbed component	$\boldsymbol{p}$ for $\Delta\boldsymbol{d} = \sigma_1\boldsymbol{u}_1$ $\eta = 34.689$	p for $\Delta\boldsymbol{d} = \sigma_{28}\boldsymbol{u}_{28}$ $\eta = -0.34701$
p_6	-2.0513e-2	1.0597
p_{10}	-1.8004e-2	-8.8718e-1
p_{20}	-6.0313e-3	-8.8673e-1
p_{24}	-6.0155e-3	1.0983

7.6.1 Accuracy of solutions

In order to check the accuracy of numerical solutions obtained by the SVD-based reanalysis, the following analytical verification procedure is followed. If the largest or smallest possible change in the displacement vector $\Delta\boldsymbol{d}$ is achieved by the above procedures, then the following should hold

$$\frac{\partial\, \Delta\boldsymbol{d}}{\partial p_i} = 0, \text{ for all } p_i \qquad \textbf{(7.37)}$$

The largest and smallest possible changes in $\Delta\boldsymbol{d}$ presented in Tables 7.2 to 7.4 are checked by using the forward difference.

7.6.2 Comparison of CPU times

To show the efficiency of the proposed approach, CPU times of numerical solutions are presented in Table 7.5 that were obtained by using a computer having an Intel Core i7 CPU of 2.0 GHz. It is observed in Table 7.5 that computational cost of calculating the response derivatives is much more than that of SVD-based reanalysis. The main advantage of the SVD-based reanalysis approach is that they give the largest and smallest possible changes in the L_2-norm of $\Delta\boldsymbol{d}$ and the corresponding perturbation vector $\boldsymbol{p}$ at one run. With the help of an optimization algorithm, if response derivatives are used to find $\boldsymbol{p}$ that yield the largest or smallest possible changes in the L$_2$-norm of $\Delta\boldsymbol{d}$ along with the constraint of $\|\boldsymbol{p}\|_2 = 1$, its computational cost for a single optimization iteration is higher than that of the complete SVD-based reanalysis formulas.

Table 7.5 : CPU times of the calculations for the problem. For the SQP method to obtain maximum $\|\Delta\mathbf{d}\|_2$ provided that $\|\mathbf{p}\|_2 = 1$, initial parameter vectors are changed randomly.

Computations		Total CPU time (sec.)
Obtaining the static solution of the problem		0.02867
Calculation of response derivatives for all cross-sectional areas		0.2223
Calculation of results listed in Table 7.2 (Case 1)		0.0856
Calculation of results listed in Table 7.3 (Case 2)		0.11452
Calculation of results listed in Table 7.4 (Case 3)		0.12313
SQP solutions for Case 1	Number of runs is 10, Average Max $\|\Delta\mathbf{d}\|_2$=48.1262	0.7428
	Number of runs is 20, Average Max $\|\Delta\mathbf{d}\|_2$=48.3666	1.5052
	Number of runs is 50, Average Max $\|\Delta\mathbf{d}\|_2$=51.4852	3.6383
	Number of runs is 100, Average Max $\|\Delta\mathbf{d}\|_2$=53.0895	7.5734
	Number of runs is 1000, Average Max $\|\Delta\mathbf{d}\|_2$=54.5332	80.3955

Following, the problem in Case 1 whose solutions are listed in Table 7.2 (namely, the first and third columns of Table 7.2 corresponding to searching of $\boldsymbol{p}$ that yields the largest L$_2$-norm of $\Delta\boldsymbol{d}$ such that $\|\boldsymbol{p}\|_2 = 1$) is solved by using the SQP method to find the perturbation vector $\boldsymbol{p}$ that yields maximum $\|\Delta\boldsymbol{d}\|_2$ provided that $\|\boldsymbol{p}\|_2 = 1$. The SQP algorithm is run 5 times succeedingly by using 10, 20, 50, 100 and 1000 different random initial perturbation vector $\boldsymbol{p}$. Averages of CPU times and maximum values of $\|\Delta\boldsymbol{d}\|_2$ in these searches are also listed in Table 7.5. It is noteworthy that while the SVD-based reanalysis found that Max $\|\Delta\boldsymbol{d}\|_2 = 56.431$ (which is the upper bound that can be achieved) provided that $\|\boldsymbol{p}\|_2 = 1$, the SQP method found $\|\Delta\boldsymbol{d}\|_2$ values smaller than this upper bound and its best solution is $\|\Delta\boldsymbol{d}\|_2 = 55.2573$ after 6000 runs with different random initial parameter vectors $\boldsymbol{p}$ and its total CPU time is 4456.7 seconds. Total CPU times of the solutions obtained by the SQP method are larger by orders of magnitude than that of the SVD-based reanalysis which is 0.0856 sec for solving Case 1 (e.g., see Table 7.5). In sum, the SVD-based reanalysis approach superior to conventional reanalysis approaches based on response derivatives and the SQP method in terms of CPU time [216].

8. CONCLUSIONS

SVD based optimization approach is investigated in this thesis. There are three numerical examples given for comparisons with other methods, where SVD based optimization was found to be an extremely powerful methodology on the structural optimization and design sensitivity reanalysis of structures due to its fast and accurate solutions relative to conventional approaches.

In the first example, SVD is compared with conventional method by using a fixed-free beam under multiple loading conditions and the CPU timings are calculated.

In the second example similar study was applied to a chassis frame having a more complex geometry to simulate a real worl optimization problem. SVD method works for complex geometries such as a chassis frame with similar efficiency while the conventional SQP method will take too long time to calculate the optimum thickness for such a complex geometry like a truck chassis frame.

In the third example, comparison was made this time for the design sensitivity reanalysis. In all three examples, SVD proved out to be faster.

For the sensitivity reanalysis example, the Sherman-Morrison-Woodbury formulas to which the Singular Value Decomposition is applied are employed to compute the extremum sensitivity values and optimum perturbations of design variables such that the desired changes in responses are achieved. It is shown that the associated singular vectors of the structure can be used to calculate the optimum perturbations of design variables. Three cases were considered as follows:

1) If the perturbation vector $\boldsymbol{p}$ is in the direction of a right singular vector $\boldsymbol{p} = \boldsymbol{v}_i$, then the changes in the displacement vector $\Delta\boldsymbol{d}$ are calculated,

2) If one wants to obtain the largest and smallest possible changes in the vector $\Delta\boldsymbol{d}$, then the corresponding perturbation vectors $\boldsymbol{p}$ having unit length and parameters η are calculated,

3) If we are interested in the perturbations given to the design variables that yield the most and least changes in the DOF of interest, then the appropriate perturbation vectors $\boldsymbol{p}$ having unit length and parameters η are computed.

By considering a planar truss system, three optimization problems are defined in Section 7.6 whose conventional response derivatives are computed and the reanalysis employing the SMW formulas based on the SVD are completed by developing the FEM model of the truss structure and using the programs developed in Matlab®. Three numerical examples on the planar truss system are presented to show the applicability and advantages of the proposed SMW formulas based on the SVD over the conventional sensitivity analysis. It is shown that the reanalysis employing the SMW formulas based on the SVD gives more insight into the design sensitivity of a system which is difficult to be obtained by using the response derivatives. In particular, the perturbations of design variables that yield the largest and smallest sensitivity values can be computed easily. Accuracy of the solutions is checked analytically and comparisons between the CPU times of the SVD-based reanalysis and SQP method are made that show the advantage of the proposed approach over the conventional approaches. In the future, it will be studied that the proposed reanalysis approach is embedded into the search algorithms in optimization problems to speed up the convergence of the optimization algorithms and time dependent problems.

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CURRICULUM VITAE

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B.Sc.: Yıldız Technical University

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Professional Experience and Rewards:

Automotive Systems Product Development

3/1998 – Present Ford Otomotiv San. A.S Kocaeli/ TURKEY

Heavy Duty Truck Powertrain System Supervisor Dec.2013-Jul.2014 (8 months)

Responsible for the on going product development of Heavy Duty Truck Powertrain systems. Lead a team of 10 engineers. Team is responsible of design and release of Air Induction, Fuel, Cooling, Exhaust, SCR, Engine Mounts, Transmission & Shifter, Driveshaft, Clutch, and Rear Axle systems for weight, complexity and cost reduction and design quality improvements nearby the model year changes and mini launches.

Powertrain Engine Mount Systems & Cost Reduction Supervisor 2009-Dec.2013 (4 years)

Responsible of 15 different Ford vehicles' Mount systems Design & Release actions. Lead a team of 14 engineers. Mount systems cross carline Design & Release actions according to GPDS. Lead a team of 11 engineers. (at max.18 heads with fuel system zero AIMS & DURIS achieved in Cargo OPD within 4 months). Also responsible for Cost reduction design changes coordination for Powertrain systems. like Rear axle , Clutch , Transmission , Shifter systems , Air Intake , Exhaust and Fuel Systems with over 100 engineers. Besides responsible of Powertrain Installation Systems hybrid vehicle integrations. Knowledge of Ford PD tools like eFDVS, Integrator and processes like campaign prevention, fresh eyes, benchmarks. PTI coordination responsibility for OHSAS, PD-ISO9001 and VCA audits.

Vehicle Engineering Co-Leader 2006-2009 (2,5 years)

Transit connect engineering co-leader with 12 direct and 56 indirect reports. leading engineering activities like CMM(change management), Mini-Launches, sub-VQR (now VRT & VFG) and VQR. Responsible of Electrical , Chassis and Vehicle attributes sign-offs, concerns, alerts, customer satisfaction,warranty and TGW Roadmaps preparation as well as concurrence to Body and Powertrain systems design changes. Final approval before the design change is introduced to production to ensure robustness. Knowledge of Ford dealer systems network (eg. Early warning system, eTIS, QSF), manufacturing (eg. FTT, VRT, VFG) and quality systems (eg. BSAQ, AWS, Data cube) that would enable to develop operational management skills.

Chassis Systems Supervisor 2004-2006 (2 years)

Transit Connect and Transit commercial vehicles Plant Vehicle Team Chassis Supervisor. Responsible for Fuel , Brake , Steering , Suspension systems related customer warranty claims' solutions. Lead a team of 8 engineers. Performed supplier quality audit visits . Attended APQP reviews.

Vehicle Design Engineer 2002-2004 (2 years)

Responsible for Ford-Transit Connect Electrical, Powertrain and Chassis Systems as the leading Vehicle Engineer. Problem solving, quality improvements in design, new model year design analysis are the part of the daily tasks, that are being achieved in cooperation with the related design and manufacturing areas. This includes being responsible for the final design engineering check and approval before vehicle reaches to the end customer. Additionally responsible for Vehicle Dynamics Sign-offs for the design changes that have impacts on the performance and behavior of the product , which would also include the benchmarking process in comparison to competition.

Chassis System Engineer 1998-2002 June (4,5 years)

Steering System Engineer in Ford Transit Connect project. This task included; 1 year target setting for preliminary design studies in Dearborn-Michigan/ US and after the preliminary design stage, working with the objectives as a leading engineer to the suppliers an additional 1 year in Basildon/UK. Responsible for the design and release of the Hydraulic Power Assisted Steering System components and their weight, cost, timing, functionality data within the company discipline and by using the tools like AIMS, WERS, FDVS, DFMEA, DVP, and 8D, DoE, Robustness and Reliability Studies. Sufficient level of I-deas and Digital Buck software knowledge to follow up the design works is also available. Active role in Attribute and Confirmation Prototype builds. Capable of using NVH (Noise, Vibration & Harness), Vehicle Dynamics, Thermal Aero-Dynamics, Safety, Security and Durability attributes' feedback to the system and improve the design. Basic knowledge on the other Chassis Systems like suspension wheels &tires and brakes.

6/1996 – 7/1997 Gürdesan Gemi Makina Sanayi Kocaeli/ TURKEY

Project Engineer

Managerial Engineering at the design department of a ship decking and machinery equipment manufacturer. Hydraulic & Mechanical cranes, propellers and rudder design with the responsibilities of three designer colleagues tasks and the IT system operations within the design department.

4/1995 – 6/1996 Doruk Gemi A.S. Istanbul / TURKEY

Design Engineer

Design Engineering in ship construction area. Durability, Hydrodynamic, Hydrostatic and Stability calculations supported with by the help of AutoCAD design and drafting tools.

List of Publications and Patents:

- **Turan, A., Muğan, A.** (2013). Structural and Sensitivity Reanalysis Based on Singular Value Decomposition, *Structural and Multidisciplinary Optimization,* vol.48, pp. 327-337.