ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL

ELECTROMAGNETIC SCATTERING FROM CONDUCTING SURFACES BY NYSTRÖM METHOD

M.Sc. THESIS

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İSTANBUL TEKNİK ÜNİVERSİTESİ ★ LİSANSÜSTÜ EĞİTİM ENSTİTÜSÜ

NYSTRÖM YÖNTEMİYLE İLETKEN YÜZEYLERDEN ELEKTROMANYETİK SAÇILMA

YüKSEK LİSANS TEZİ

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To my mother,

FOREWORD

Thank you to everyone who helped and inspired me in my Master's program. In particular, I would like to thank my advisor, Prof.Dr. Mehmet Çayören.
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July 2021 Faik Yaman
(Research Asistant)

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ABBREVIATIONS

App	: Appendix
CEM	: Computational Electromagnetics
EFIE	: Electric Field Integral Equation
Mom	: Method of Moments
NM	: Nyström Method
PEC	: Perfect Electric Conductor

SYMBOLS

$\vec{\mathbf{A}}$	: Magnetic Vector Potential
$\vec{\mathbf{B}}$	: Magnetic Flux Density
$\vec{\mathbf{D}}$	: Displacement Vector
$\boldsymbol{E}_x, \boldsymbol{E}_y, \boldsymbol{E}_z$	: Vector Components of Electric Field
f	: Frequency
$\boldsymbol{\varepsilon}$	: Dielectric Permittivity
g	: Green's Function of Free-space
$\boldsymbol{g}_{xx}$	: Second Derivative of Green's Function with respect to x-axis
$\vec{\mathbf{H}}$	: Magnetic Field
$\boldsymbol{J}_x, \boldsymbol{J}_y, \boldsymbol{J}_z$	: Vector Components of Surface Current Density
k	: Wave-number
$\boldsymbol{P_n(x)}$	: Legendre Polynomial Order of n
$\boldsymbol{\mu}$	: Magnetic Permeability
r	: Observation Point
$\boldsymbol{\phi}$	: Scalar Potential
r'	: Source Point
w	: Angular Frequency

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ELECTROMAGNETIC SCATTERING FROM CONDUCTING SURFACES BY NYSTRÖM METHOD

SUMMARY

The aim of this thesis is to calculate the scattered electric field from metal surfaces with the Nyström solution method. This topic is of interest to computational electromagnetic (CEM). CEM generally use Maxwell's Equations to calculate electric or magnetic fields. In CEM, Electric Field İntegral Equation (EFIE) is used to find scattered electric field and Magnetic Field İntegral Equation is used to find scattering magnetic field. These equations are derived from Maxwell's equations and Maxwell's equations can be written as differential form or integral form. CEM solves these equations by using numeric methods. The best known methods used by CEM are Moments Method, Finite Element Method, Finite Difference Time Domain Method. Finite Difference Time Domain Method solves differential equations and other methods solve integral equations. These methods have advantages and disadvantages while solving equations. Main advantage of using integrals equation is that if differential equations are used to solve electromagnetic problems , there is need to find scattering electric field of not interested observers. Because derivative value of a point depends on derivative values of near points. While using integral equation to solve electromagnetic problems, we do not need to know scattering electric fields of near points if integrals are convergence.This saves us time while coding if integral equations methods are used instead of differential equation to overcome electromagnetic problems.

In this study, Nyström Method (NM) was used to evaluate scattering electric field from metal surfaces. NM converts integral equations to series sum. The more folds the integral is, the more sum series are available. Nyström method is not new numeric method but it is new for CEM since 1990's. In NM, integral equations are solved and solution domain (scattered object) needs to meshes like other methods. These meshes can have different geometries like triangle, square, circle, cube and every meshes have equivalent number of sampling points and weight coefficients. Square meshes are selected to solve problem for this thesis.

Key of the NM is to apply quadrature rule on the integral operator. Quadrature rule turns integrals to series sum. In series sum, quadrature rule multiplies sampling points of integrand and weights coefficient of these sampling points. There are a lot of types quadrature rule such as Trapezoidal rule, Tanh-Sinh rule, Gauss-Kronrod rule, Gauss-Hermite rule, Gauss-Jacobi rule and Gauss-Legendre rule.

Gauss-Legendre quadrature rule is applied to solve EFIE in this thesis. In reality, this rule evaluates integrals which have lower boundary as -1 and upper boundary 1. For different boundaries, a simple transformation must be applied. Sampling points of rule is the roots of Legendre polynomials for integrals that have the boundary between -1 and 1. Weights coefficients also can be found from this polynomials.

Basic strategy to find scattered electric field is that first step is to find current density, that creates scattering electric field, is stimulated by a source. This source is generally plane wave source. Impedance matrix must be filled to find current density. EFIE is used while calculating impedance matrix. Impedance matrix is square matrix and gives relationship between incident electric field source (plane wave source) and current density. Impedance matrix can be thought that it gives relationship between every sampling point in terms of EFIE. İncident electric field source and current density matrix are column matrix. Current density can be determined with matrix operations.To calculate current density , there is need to take inverse of impedance matrix. Taking inverse of this matrix causes time consuming while coding.Even if there is a numerical method that solve this problem, while coding scattering electric field problem, we do not use these numeric aproaches. Second step is to find scattered electric field by using current density. To find scattering field, observation points must be defined. Same strategy in first step could be used to evaluate scattering field but, difference is that impedance matrix of second step is not square matrix, is rectangular matrix. Elements of this matrix shows relationship between sampling points and observation points. Scattering electric field is calculated with the matrix multiplication of current density matrix and impedance matrix. This was the strategy to evaluate scattering electric field.

Even if easy procedure, there is a problem because of quadrature rule. EFIE has green's function and second derivatives of green's function. In denominator of Green's function, there is a distance term whose symbol is R. Also, it is easily realized that second derivatives of green's function has R^3 term in denominator. These terms cause a singularity problem at the first stage of solution strategy because, the distance between same sampling points is zero. Singular terms are at the diagonals on impedance matrix. Gauss-Legendre quadrature rule is not valid for singular integrals. But, it is known that integral is defined compare to Cauchy principle value at the singular points although quadrature rule cannot handle it. To overcame singularity problem, there are a lot of solution strategies like Tanh-sinh quadrature rule, Radial angular-separation variable transform, Duffy's transform. In this study, Duffy's transform is applied to get rid of singularity problem. This transform is useful for triangular meshes. To calculate result of singular point, square meshes are divided to 4 triangular meshes. Result of one singular point is summation of result of 4 triangular meshes. Moreover, result of singular points have 2 terms. One of them is the result of Duffy's transformation and second term comes from limit of green's function when distance goes to 0. Total result which is diagonal element of impedance matrix equals summation of two terms. This singular point result is for green's function. To calculate results of derivatives of green's function, Duffy's transform can be used again. Different strategy is used to calculate results of derivatives. This was done with finite difference theorem in the thesis. Unlike the singular result of green's function, there is no contribution from limit for derivatives of green's function because this contribution is just number and derivative of this number is 0. It is easily understood that there is no singularity problem for non-diagonal terms of impedance matrix. Besides, there is no singularity problem while calculating scattered electric field from current density as observers are not on the scattered object.

In this thesis, first two chapter are about theory of scattering electric field with Nyström Method. The subject of last chapter is simulation results of scattering electric field from metallic squares and cubes. Simulation results are compared with the results of Feko program. Properties of simulations are given detail by detail like angle of propagation direction of plane wave, frequecy, shape of scattered object, mesh number, quadrature number. Simulation results are given in terms of phase results of scattering electric field and amplitude results of scattering electric field.

NYSTRÖM YÖNTEMİYLE
İLETKEN YÜZEYLERDEN
ELEKTROMANYETİK SAÇILMA

ÖZET

Bu tezin amacı metal yüzeylerden saçılan elektrik alanı Nyström çözüm yöntemini ile hesaplamaktır. Bu konu hesaplamalı elektromanyetik dalının (CEM) ilgilendiği bir konudur. Bu dal elektrik ve manyetik alanı hesaplamak için genellikle Maxwell's denklemlerinden türetilmiş olan elektrik alan integral denklemini ve manyetik alan integral denklemini kullanır. Bu denklemler diferansiyel formda da yazılabilir. CEM genellikle bu tarz denklemleri çözmek için sayısal çözüm yöntemlerini kullanır. En çok kullanılan çözüm yöntemleri ise Moment Metodu (Mom), Sonlu Elemanlar Metodu (FEM), Zaman Domeninde Sonlu Farklar Metodu (FDTD) olarak düşünülebilir.Bu yöntemlerden olan FDTD diferansiyel olarak çözüm önerirken, diğer yöntemler ise çözümü integral denklem olarak önerir. Bu yöntemlerin kendi aralarında avantajları ve dezavantajları da mevcuttur. İntegral denklemi kullanmanın ana avantajı, elektromanyetik problemleri çözmek için diferansiyel denklemler kullanılıyorsa, ilgilenmeyen gözlemcilerin saçılan elektrik alanını bulmaya ihtiyaç duyulmasıdır. Çünkü bir noktanın türev değeri, yakın noktaların türev değerlerine bağlıdır. Elektromanyetik problemleri çözmek için integral denklemi kullanırken, integraller yakınsaksa, yakın noktaların saçılan elektrik alanlarını bilmemize gerek yoktur. Bu, elektromanyetik problemlerin üstesinden gelmek için diferansiyel denklem yerine integral denklem yöntemlerinin kullanılması durumunda kodlama yaparken bize zaman kazandırır.

Bu tezde metal yüzeylerden saçılan elektrik alanı hesaplamak için Nyström metodu (NM) kullanılmış.NM integral denklemleri seri toplamına dönüştürür.İntegral kaç katlıysa o kadar sayıda toplam serisi mevcuttur. Bu metod yeni bir metod değildir fakat, bu metod 1990'lı yıllardan itibaren hesaplamalı elektromanyetik alanında kullanılmaktadır. NM çözümü yaparken integral denklemleri kullanır ve diğer yöntemlerde olduğu gibi elektrik alanın saçılacağı nesneyi küçük elemanlara böler. Elemeanlar kare, küp, daire gibi geometrik şekillerde olabilir. Problemi çözmek için bu tezde karesel elemanlar kullanılmıştır. Her eleman eşit sayıda örnekleme noktalarına ve ağırlık katsayılarına sahiptir.

NM'nin en kritik noktası kareleme kuralının integral operatörüne uygulanmasıdır. Kareleme kuralı integralleri seri toplamlarına dönüştürür ve bu seri toplamları integrand'ın örnekleme noktalarındaki değeri ile bu örnekleme noktalarına ait olan ağırlık katsayılarının çarpımından oluşur. Integraller için birçok kareleme kuralı yöntemi vardır ve bunlara örnek olarak Trapezoid kuralı, Tanh-Sinh kuralı,

Gauss-Kronrod kuralı, Gauss-Hermite kuralı, Gauss-Jacobi kuralı ve Gauss-Legendre kuralı verilebilir.

Bu tezde elektrik alan integral denklemini hesaplayabilmek için Gauss-Legendre kuralı kullanılmıştır. Bu kural limiti 1 ve +1 arasında olan integralleri hesaplar. Farklı bir aralıkta olan integralleri hesaplayabilmek için basit bir dönüşüm yapılır. Gauss-Legendre kuralında örnekleme noktaları Legendre polinomunun -1 ve +1 değerleri arasındaki kökleridir. Ağırlık katsayıları ise Legendre polinomu sayesinde rahatlıkla bulunabilir.

Saçılan elektrik alanı bulmak için 2 adım izlenir. İlk adımda saçılan alanı oluşturacak nesne bir kaynak tarafından uyarılır, bu kaynak genellikle düzlem dalgadır. İlk adımdaki amaç nesne üzerinde oluşan akım yoğunluğunu matriks işlemleri yardımıyla bulmaktır. Bu matriksler karesel matriks olan empedans matriksi, akım yoğunluğunu gösteren matriks ve kaynak tarafından nesneye gelen elektrik alanı gösteren sütun matriksleridir. Empedans matriksinin elemanları EFIE'ne göre oluşturulur ve her elemanı örnekleme noktalarının EFIE 'ne göre birbiri arasındaki ilişkiyi gösterir. Akım yoğunluğu ise empedans matriksinin tersinin gelen elektrik alan matriksinin çarpımıyla bulunur.Akım yoğunluğunu hesaplamak için empedans matrisinin tersini almak gerekir. Bu matrisin tersini almak kodlama yaparken zaman kaybına neden olmaktadır. Bu sorunu çözen sayısal bir yöntem olsa bile saçılma elektrik alan problemini kodlarken bu sayısal yaklaşımları kullanmıyoruz. İknci adımda ise amaç akım yoğunluğundan faydalanarak saçılan elektrik alanı bulmaktır. Bu adımdaki yöntem ile ilk adımdaki yöntem birbirine benzerdir fakat, burada empedans matriksinin karesel değildir. Bu adımda uzayda saçılan elektrik alanın hesaplanacağı gözlemci noktaları oluşturulur. Empedans matriksi ise bu gözlemci noktaları ile nesne üzerindeki örnekleme noktaları arasındaki ilişkiyi gösterir. Saçılan elektrik alan ise akım yoğunluğunu gösteren matrix ile empedans matriksinin çarpımıyla bulunur.

Saçılan elektrik alanı bulma işlemi kolay olmasına rağmen anlatılan ilk adımda bir tekillik problemi vardır. Bu problem EFIE denkleminin içerisindeki Green fonksiyonu ve bu fonksiyonun ikinci türevinden kaynaklanır. Çünkü, Green fonksiyonunun paydasında R ile gösterilen uzaklık terimi vardır. Green fonksiyonunun ikici türevinin paydasında ise R^3 terimi vardır. Empedans matriksinin diagonal elemanlarında (aynı noktaların etkileimini gösteren elemanlar) uzaklık 0 olacağı için bir tekillik oluşur. Bu tekillik problemi Gauss-Legendre kareleme kuralı ile çözülemez, bu kural tekilliği olmayan integraller için geçerlidir. Fakat, Cauchy Esas değerine göre EFIE 'nin tekil noktaları için de bir değeri vardır. Tekillik problemini çözmek için Tanh-sinh kareleme yöntemi, Radyal açısal-değişkenlere ayırma yöntemi, Duffy dönüşümü gibi yollar mevcuttur. Bu tezde Duffy dönüşümü tekilliği çözmek için kullanılmıştır. Bu dönüşüm üçgensel elemanlar için bir çözüm sunar. Kullandığımız karesel elemanları 4 tane üçgensel elemana bölerek sonucu bulabiliriz. Sonuç bu 4 üçgensel elemanın sonuçlarının toplamıdır. Ayrıca bu sonuca Green fonksiyonunun limit sıfıra giderken çıkan değeri de eklenmelidir. Bulunan toplam sonuç Green fonksiyonunun tekil olduğu noktadaki (empedans matriksinin diagonal elemanı) sonucudur. Green fonksiyonunun ikici türevinin tekilliğini hesaplayabilmek için de Duffy dönüşü kullanılabilinir . Ancak bunun yerine sonlu farklar yöntemi kullanılarak ikinci türevlerin tekillik değerleri hesaplanmıştır. Bu aşamada Green fonksiyonun ikinci türevinin limit 0'a giderken aldığı değer sıfır olacağı için bu hesaba gerek yoktur. Ayrıca şu da kolaylıkla anlaşılabilir ki, akım yoğunluğu hesaplanırken empedans matriksinin diagonel olmayan elemanlarında ve saçılan elektrik alan hesaplanırken empedans matriksinin tüm elemanlarında bir tekillik söz konusu değildir.

Bu tezde ilk iki ünitede saçılan elektrik alanın Nyström metodu ile nasıl bulunduğuna dair teoriye yer verilecektir. Son bölümde ise metalik kare ve küpler için saçılan elektrik alanların benzetim sonuçları üzerinde durulacaktır. Bu sonuçlar Feko benzetim programının sonuçları ile karşılaştırılmıştır.Simülasyonların özellikleri, düzlem dalganın yayılma yönünün açısı, frekans, saçılan cismin şekli, ağ sayısı, örnekleme sayısı gibi ayrıntılı olarak verilmiştir. Saçılan elektrik alanının faz sonuçları ve saçılan elektrik alanının genlik sonuçları açısından simülasyon sonuçları verilmiştir.

1. INTRODUCTION

The main idea of computational electromagnetic (CEM) problems is to solve Maxwell's equations by using integral equations or differential equations. Target of CEM deals with solution of electromagnetic problems like radiation and scattering. Thanks to improved computer technology, computational electromagnetics are developing day by day and application to new solution techniques will be easy to this research area. One of these solution techniques is Nyström Method (NM). NM considers Maxwell's Equations, that show solution of electromagnetic problems [1], in integral form.

1.1 Nyström Method

Nyström method is a numeric technique which finds the results of integrals in mathematics. NM assumes that every integral can be written as sum series [2]. Nyström method was formulated by E.J. Nyström in 1928 then it was improved in 1990.The method uses certain coefficients while doing this calculation. NM does not only use in mathematic. NM is used many applications of sciences that consider problems in integral form. One of them is electromagnetics. Maxwell's Equations solves most of electromagnetic problems like electromagnetic scattering. Maxwell's equations can be written integral form that is why NM is preferred this study. NM solved non-singular integral equations when it was formulated. This was the problem for electromagnetic because it has singular integral equation. Gedney overcame this condition with the formulation of locally-corrected Nyström method in 1990's.

1.2 Electromagnetic Scattering

The removal of the electromagnetic energy that comes to the object is electromagnetic scattering. Electromagnetic energy is transmitted with electromagnetic wave. The cause of electromagnetic scattering is the current that is created by electromagnetic wave on the object. With the knowledge of the scattering properties, progress has

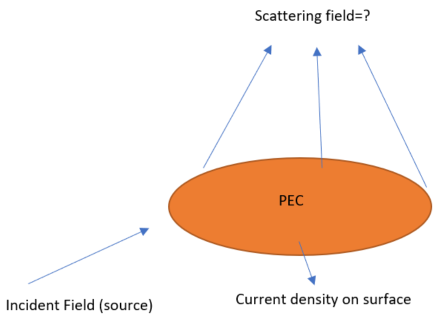

Figure 1.1 : Problem schema of thesis.

been made in many fields such as the military, industry and medicine. For example, detection of tumor or pulmonary edema is done more quickly and easily thanks to the developments in the field of electromagnetic scattering. These developments were also effective in the discovery of minerals..

1.3 Purpose of Thesis

The purpose of thesis is to find scattered electric field from metallic objects with Nyström Method. Metallic objects are called perfect electric conductor (PEC) in electromagnetics. NM does not solve Maxwell's Equation directly. NM solves Electric Field Integral Equation (EFIE) which derives from Maxwell's Equations. To find scattered electric field, the PEC object is exposed to the electromagnetic source that is generally plane-wave source. This source is called incident field in electromagnetics. After finding the current density, EFIE also calculates the scattered electric field using this current density. NM is applied to solve EFIE as numeric. NM converts integrals to finite series sum. NM does not need to basic functions unlike Method of Moments that is generally used to solve EFIE. Problem schema of thesis can be seen from figure 1.1 like:

1.4 Dissertation Outline

The remaining parts of the thesis are organized as follows: In chapter 2, it is formulated electromagnetic scattering theory with Maxwell's Equations and boundary conditions. We will also show derivation of the electric field integral equation. In chapter 3, we will demonstrate Nyström method to compute surface integrals. We will also talk about advantages and disadvantages of Nyström method according to Method of Moments. In chapter 4, we will highlight that how to apply Nyström method to EFIE to find scattering electric field. We will also show solution strategies of singularity problem in this chapter. In chapter 5, we will show simulation results of scattering electric field. These results will give together with results of Feko simulation program to compare them. In section 6, we will interpret simulation results, we will talk about future works to improve this study.

2. ELECTROMAGNETIC SCATTERING FROM PEC SURFACES

Electromagnetic wave is the wave that has electric and magnetic field. Electromagnetic scattering from surfaces means wave hits object then object scatters or delivers this wave in the space with different directions. The purpose of this project is to find what is the angular direction of scattering electric field and amplitude of scattering electric field. Also, reflection or transmission are a kind of scattering. Scattering from surfaces is one of the favorites of Computational Electromagnetics. In electromagnetics, problems solved with Maxwell's Equations. These equations can be considered as two forms which are differential form and integral form. Nyström method is a computational method to solve Maxwell's Equations in integral form by using quadrature rules. Integral equations in electromagnetics are known as Fredholm Integral Equations with first kind and second kind. Electromagnetic problems are usually solved depending on Electric Field Integral Equation and Magnetic Field Integral Equation. Electric Field Integral Equation shows relationship between electric field and surface current, this equation is first kind of Fredholm Equation. Magnetic Field Integral Equation shows relationship between magnetic field and surface current, this equation is second kind of Fredholm Equation. EFIE is more suitable than MFIE for problems with open geometry. In this thesis, scattering problem is solved with EFIE for PEC by using Nyström Method. In this chapter, firstly Maxwell's Equations are showed. Then it is told that how EFIE comes from Maxwell's Equations.

2.1 Maxwell's Equations

In this thesis, we study in frequency domain. There are four Maxwell's Equations in frequency domain which are:

$$\nabla \times \vec{E} = -iw\vec{B} \tag{2.1}$$

$$\nabla \times \vec{H} = iw\vec{D} + J \tag{2.2}$$

$$\nabla . \vec{B} = 0 \tag{2.3}$$

$$\nabla . \vec{D} = \rho_e \tag{2.4}$$

where, $\vec{E}$ is Electric Field, $\vec{B}$ is magnetic flux density, $\vec{H}$ is magnetic field, $\vec{D}$ is displacement vector (electric flux density), w is angular frequency, J is surface current density, ρ_e is volume charge density. It is well known that $\vec{D} = \varepsilon\vec{E}$ and $\vec{B} = \mu\vec{H}$ for isotropic mediums. $\nabla\times$ is the curl operator and $\nabla.$ is the divergence operator [3]. Equation 2.1 is known as Faraday's Law, equation 2.2 is known as Ampere's Law, equation 2.3 and equation 2.4 are knowns as Gauss's Law. These four equations can be considered as keys of electromagnetics. If one of the terms of Maxwell's Equation is known, others can be found easily. For Example, if electric field is calculated, it means magnetic field is clear when there is no charge density and current density.

2.2 Boundary Conditions

Boundary conditions are important not only electromagnetic problems but also differential and integral equations. They directly affect the solution. In CEM, EFIE and MFIE are written with boundary conditions. These conditions are different for dielectric materials and PECs. For interference of two material with the parameters ε_1, ε_2, μ_1 and μ_2 in electromagnetics, boundary conditions can be written as:

$$\vec{n}.(\vec{D}_1 - \vec{D}_2) = \rho_e \quad (2.5)$$

$$\vec{n}.(\vec{B}_1 - \vec{B}_2) = 0 \quad (2.6)$$

$$\vec{n}\, x\, (\vec{H}_1 - \vec{H}_2) = J_s \quad (2.7)$$

$$\vec{n}\, x\, (\vec{E}_1 - \vec{E}_2) = 0 \quad (2.8)$$

where $\vec{n}$ is the unit normal vector to boundary. These four equations are the for dielectric interference. The last two equation are generally used in electromagnetics. In this study, electromagnetic problem with PECs interference was considered. It is known that there is no field magnetic in the PEC objects ($\vec{E}_1 = \vec{H}_1 = 0$) and vectorial product of field and unit normal vector equal to tangential component of vector. So, conditions for PEC interference (boundary between PEC and dielectric) will be [4,5]:

$$\vec{t}.\vec{H}_2 = J_s \quad (2.9)$$

$$\vec{t}.\vec{E}_2 = 0 \quad (2.10)$$

2.3 Electric Field Integral Equation(EFIE)

EFIE is equation that relies on boundary condition of perfect electric conductor surface or dielectric surfaces in terms of electric field. In this part, EFIE is written according to PEC surface. EFIE can be written in time domain and frequency domain but, frequency domain was preferred. EFIE is valid for open and closed surfaces. Total tangential components of electric field on the surface of PEC must be 0. In CEM, there are two tangential components of electric field. One of them is the tangential component of incident electric field and second is tangential component of the scattering electric field. Compare to this idea, equation 2.10 can be written as different form like [6]:

$$\vec{E}_t^s(r = r_s) = -\vec{E}_t^i(r = r_s) \quad on \quad S \tag{2.11}$$

where S is conductor surface and r_s is any point on the surface of conductor, $\vec{E}_t^s$ is tangential component of scattering electric field and $\vec{E}_t^i$ is tangential component of incident electric field. In EFIE, problem can be considered as two steps. First step is to find surface current density on conductor. Second one is to calculate scattered field on space by using first step. Let's derive EFIE by using Maxwell's Equation. There are a lot of strategy to reach EFIE from Maxwell's Equations. It is started from magnetic vector potential $\vec{A}$ to derive EFIE. First of all, let's write relationship between magnetic flux density $\vec{B}$ and magnetic vector potential $\vec{A}$ [7]:

$$\nabla \times \vec{A} = \vec{B} \tag{2.12}$$

It is obvious from equation 2.1 and equation 2.12:

$$\nabla \times \vec{E} = -iw\nabla \times \vec{A} \tag{2.13}$$

$$\nabla \times (\vec{E} + iw\vec{A}) = 0 \tag{2.14}$$

$$\vec{E} + iw\vec{A} = -\nabla\phi \tag{2.15}$$

$$\vec{E} = -iw\vec{A} - \nabla\phi \tag{2.16}$$

Electric field in equation 2.10 is scattering electric field. To determine expressions of $\vec{A}$ and ϕ, combine equation 2.2 and Equation 2.12 [5, 8, 9]:

$$\nabla \times \nabla \times \vec{A} = \mu\vec{J} + iw\mu\varepsilon\vec{E} \tag{2.17}$$

From vector identity of curl of curl operator, equation 2.16 and equation 2.17 can be written as:

$$\nabla(\nabla.\tilde{\mathbf{A}}) - \nabla^2\tilde{\mathbf{A}} = \mu\tilde{\mathbf{J}} + \mathbf{iw}\mu\varepsilon(-\mathbf{iw}\tilde{\mathbf{A}} - \nabla\phi) \tag{2.18}$$

$$\nabla^2\tilde{\mathbf{A}} + \mathbf{k}^2\tilde{\mathbf{A}} = -\mu\tilde{\mathbf{J}} + \nabla(\mathbf{iw}\varepsilon\mu\phi + \nabla.\tilde{\mathbf{A}}) \tag{2.19}$$

where ϕ is scalar potential. Lorentz Gauge gives relationship between scalar potential ϕ and magnetic vector potential $\vec{A}$. Lorentz Gauge is:

$$\nabla^2\vec{A} + k^2\vec{A} = -\mu\vec{J} \tag{2.20}$$

From equation 2.19 and equation 2.20, it is clear that scalar potential ϕ is [8, 10]:

$$\phi = -\frac{\nabla.\vec{A}}{iw\varepsilon\mu} \tag{2.21}$$

Magnetic vector potential $\vec{A}$ can be found from Green function properties with Lorenz Gauge in integral form like:

$$A(\vec{r}) = \mu\int_S g(r,r')J(r')dS(r') \tag{2.22}$$

where $g(r,r')$ is free-space Green's function that solves scalar Helmholtz equation for dirac-delta source, r is the location of observer, r' is the location of source on the conducting surface. Scattering electric field could be found with equation 2.16, equation 2.21 and equation 2.22 as:

$$\vec{E}^s(r) = -iw\mu\int_S[1 + \frac{1}{k^2}\nabla\nabla.]g(r,r')J(r')dS(r') \tag{2.23}$$

where k is wave-number. If this equation is thought with boundary condition of PEC surfaces, new expression will be [11]:

$$-\vec{t}.iw\mu \int_S [1+\frac{1}{k^2}\nabla\nabla.]g(r,r')J(r')dS(r') = -\vec{t}.\vec{E}^i \tag{2.24}$$

where $\vec{t}$ shows tangential component of surface. This expression is Electric Field Integral Equation (EFIE). To solve EFIE easily, it is need to make expressions for equation. EFIE has different expression because of $\nabla\nabla.(g(r,r')J(r'))$ term. This term can be written as different expressions thanks to vector identity. With the vector identity there are four different forms of EFIE like:

$$\vec{t}.iw\mu \int_S [g(r,r').\vec{J}(r') + \frac{1}{k^2}g(r,r')\nabla'\nabla'.\vec{J}(r')]dS(r') = \vec{t}.\vec{E}^i \tag{2.25}$$

$$\vec{t}.iw\mu \int_S [g(r,r').\vec{J}(r') + \frac{1}{k^2}\nabla\nabla.(g(r,r')\vec{J}(r'))]dS(r') = \vec{t}.\vec{E}^i \tag{2.26}$$

$$\vec{t}.iw\mu \int_S [g(r,r').\vec{J}(r') + \frac{1}{k^2}\vec{J}(r').\nabla\nabla g(r,r')]dS(r') = \vec{t}.\vec{E}^i \tag{2.27}$$

$$\vec{t}.iw\mu \int_S [g(r,r').J(\vec{r}') + \frac{1}{k^2}(\nabla g(r,r'))\nabla'.\vec{J}(r')]dS(r') = \vec{t}.\vec{E}^i \tag{2.28}$$

These four equations can be used to solve EFIE. In this thesis Equation 2.27 is used. Let's expand our EFIE form.

$$\vec{t}.iw\mu \int_S g(r,r').J(r')dS(r') + \vec{t}.iw\mu \int_S \frac{1}{k^2}J(r').\nabla\nabla g(r,r')dS(r') = \vec{t}.\vec{E}^i \tag{2.29}$$

$$\vec{t}.iw\mu \int_S [1+\frac{1}{k^2}\nabla\nabla g(r,r')].J(r')dS(r') = \vec{t}.\vec{E}^i \tag{2.30}$$

$$g(r,r') = \frac{\exp(-jk|r-r'|)}{4\pi|r-r'|} \tag{2.31}$$

where $1+\frac{1}{k^2}\nabla\nabla g(r,r')$ is dyadic green's function for free-space [12]. EFIE are used to find surface current density $\vec{J}(r')$ and tangential component of scattering electric field $\vec{E}^s(r)$. Equation 2.30 is to find surface current density and integral is taken on the surface.

3. NYSTRÖM METHOD

Nyström method is invented 1930's. Nystörm method is a numeric method to solve integral equations with quadrature rules. In this study, NM is used to solve EFIE. Key idea of method is to discrete integration domain to parts or cells. NM handles problem part by part under the integral equation. Cells, that is discrete, have smaller dimensions because of good approximation. In shortly, quadrature rule is replaced integrals in NM. Integral equations and NM solve linear equation systems. Linear equations let electromagnetic problems to solve with matrix entries. It provides an easy understanding and implementation of problem solution.

3.1 Quadrature Rules

Quadrature rules are numerical method that calculates definite integrals in 1-D, 2-D or 3-D. Quadrature rules generally create sampling points and weight coefficients. Quadrature rule is important to Nyström method as sampling points of integrand is sampling points of quadrature points. There are many types of quadrature rules like Newton-Cotes, Mehler, Lobatto, Gaussian. Gaussian Quadrature rule is the best known quadrature rule between them. In this thesis, gaussian quadrature is applied to calculate integrals. In mathematics, Gaussian quadrature rule can be expressed as [13, 14]:

$$\int_{-1}^{1} f(x)dx = \sum_{i=1}^{n} w_i f(x_i) \tag{3.1}$$

where n is the number of sampling points, x_i is the roots of Legendre polynomials. Legendre polynomial of order n is:

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n \tag{3.2}$$

w_i is quadrature weight. Formulation of w_i is like:

$$w_i = \frac{2}{(1-x_i^2)(P_N'(x_i))^2} \tag{3.3}$$

where $P_N'(x_i)$ is the derivative of Legendre polynomial. This expression is valid for 1-d integrals and non-singular functions. This rule is known as Gauss-Legendre Quadrature rule. It evaluates integrals from -1 to 1. There is need to transform to calculate integrals from a to b, where a and b are reel numbers. In mathematics, Gauss-Legendre Quadrature form for arbitrary integral limits is:

$$\int_a^b f(x)dx = \frac{b-a}{2}\sum_{i=1}^{n} w_i f(\frac{b-a}{2}x_i + \frac{b+a}{2}) \tag{3.4}$$

In this thesis, it is worked with surface integrals so let's write double integral formula for Gauss-Legendre Quadrature rule:

$$\int_a^b \int_c^d f(x,y)dxdy = \frac{b-a}{2}.\frac{d-c}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij} f(\frac{b-a}{2}x_i + \frac{b+a}{2}, \frac{d-c}{2}y_i + \frac{d+c}{2}) \tag{3.5}$$

where x_i and y_i are sampling points, a and b are lower limit and upper limit of x boundary. c and d are lower limit and upper limit of y boundary. w_i is the quadrature weight for sampling point x_i, w_j is the quadrature weight for sampling point y_j where w_{ij} = w_i. w_j. Gauss-Legendre Quadrature rule is useful for non-integrals. There are also rules like double exponential quadrature rule which solves singular integrals but their implementations are harder [15]. In the table which is in next page shows x_i's and w_i's according to number of quadrature points n for Gauss-Legendre Quadrature rule in 1-D.

Table 3.1 : Sampling points and weights with respect to quadrature numbers.

n	x_i	w_i
1	0	2
2	$\pm\frac{1}{\sqrt{3}}$	1
3	0 $\pm\sqrt{\frac{3}{5}}$	8/9 5/9
4	±0.339981 ±0.861136	±0.652145 ±0.347855

Table (3.1) is to calculate one dimensional integrals. To calculate two dimensional integrals, sampling points of y_i will be same with sampling points of x_i. This stuation is valid for weights. But, difference between 1-d and 2-d integrals are that sampling point values (x_i,y_i) must be combined. For example, assume that 4-point quadrature rule are applied to calculate 2-d integral. It means 2-point quadrature rule is used two times. 4 sampling points will be $(-\sqrt{\frac{1}{3}},-\sqrt{\frac{1}{3}})$, $(-\sqrt{\frac{1}{3}},\sqrt{\frac{1}{3}})$,$(\sqrt{\frac{1}{3}},-\sqrt{\frac{1}{3}})$,$(\sqrt{\frac{1}{3}},\sqrt{\frac{1}{3}})$ in terms of (x_i,y_i) [16]. It was told that discretization is important factor for NM. Number of sampling points n is other factor that is effective on accuracy. Gauss legendre quadrature rule calculates one dimensional integrals with %100 accuracy if $m \leq 2n-1$. m is the order of integrand. For instance, $\int_{-1}^{1}(x^5+1)dx$ equals 2 compare to analytic solution. If 3-points quadrature rule is used, the result is also 2 with %100 accuracy. Error is bigger than 0 if number of quadrature point is less than 3.

3.2 Nystöm Method with Integral Equations

Integral equations show how to solve problems in mathematics and physic. In these types of equations, there is an integral kernel. It can be considered as variable function of equations. NM is applied to integral kernels. Other parameters are constant and they do not need to NM. Application of Nyström method is easy to integral equations as there is no need to know integral of function, it is just enough to know how to apply quadrature rule. Let's consider a physical problem that can be solved with integral equation.

$$\int_s k(x,x').u(x')dS' = \phi(x) \qquad x \in S \tag{3.6}$$

where $k(x,x')$ is the kernel of integral, $u(x')$ is the unknown function,$\phi(x)$ is the source function, x is the observation point and x' is the source point. In electromagnetic problems, $k(x,x')$ is the Green's Function, $u(x')$ is the current density, $\phi(x)$ is the incident electric field [16, 17]. Surface of solution domain is divided by meshes in NM. NM representation of integral equation can be written as:

$$\sum_{i=1}^{N}\sum_{j=1}^{Q_i} w_{ij}k(x,x'_{ij}).u(x'_{ij}) = \phi(x) \tag{3.7}$$

where Q_i is the number of the quadrature point at the ith element, N is the total mesh number, w_{ij} is the quadrature weight on jth point of the ith element. Integral equation can be solved with matrix approximation. The matrix of multiplication of first two terms $w_{ij}k(x,x'_{ij})$ is called impedance matrix, matrix of $u(x'_{ij})$ is unknown column matrix and shows unknown current densities on each sampling points, matrix of $\phi(x)$ is column matrix and shows scattering electric fields on each sampling points for electromagnetic scattering problems [16].In reality, the unknown column matrix can be $\phi(x)$ or $u(x_{ij})$ depending on the problem. This unknown column matrix is calculated with matrix operations. It will be explained in the next chapter.

3.3 Advantages and Disadvantages Compare to MoM

Nyström method have advantages and disadvantages compare to other numeric computational techniques. Method of Moments (Mom) is well-known numeric computational technique for integral equations in electromagnetics. Therefore, a comparison will be made between these two methods. Differences between them can be summarize as [16]:

- Mom has double-fold integrals when Nyström method has one-fold.
- Mom uses boundary condition for each mesh, Nyström method has boundary condition only sampling nodes.
- Basis and testing function of Mom is continuous, Nyström method has discontinuous basis and testing function like dirac delta function. It means representation of incident field is different for two methods.
- Increasing accuracy is easy for Nystörm Method, this can be done by using more meshes or more sampling points.
- Mom kernel has weakly singular ($1/Rtype$) though basis function, Nyström method has hyper-singular kernel ($\frac{1}{R^3}$) type.
- Mom has special basis function to eliminate hyper-singular kernel, Nystörm method has no chance to eliminate them.
- Nystörm method has more unknowns for same problem.
- Formulation of Nystörm method is simpler than Mom.

4. APPLICATION OF NYSTRÖM METHOD TO EFIE

Aim of EFIF is to find scattering electric field from surfaces. There are two steps to evaluate scattering field. First one is determining current density. Second step is determining scattering field with the help of current density. Interestingly, EFIE provides to solution of these two steps. In this thesis, EFIE is solved numerically with NM. It was told that NM transforms integral equation to sum series in the previous chapters. NM was valid for non-singular integral equations when the method was found, but electromagnetic integral equations have singular kernels due to the green function. To get rid of this situation, Locally-Corrected Nystörm method was defined by Gedney in 1990's. Though this new method, singular kernels can be calculated. Non-singular kernels are evaluated with quadrature rules. In reality, locally corrections are made for discrete samplings of surface current density. There is no need to use locally corrections to evaluate scattered fields from current density because green's function of scattered field is non-singular [18]. Using the quadrature rule will be enough to determine scattered field. In electromagnetics, scattering electric field can be expressed as [3]:

$$\vec{E}(r) = \int_{\Omega} G(\vec{r},\vec{r}').J(\vec{r}')dS' \tag{4.1}$$

where $G(\vec{r},\vec{r}')$ is Dyadic Green's function (kernel), $J(\vec{r}')$ is surface current density, $\vec{r}$ is observer location, $\vec{r}'$ is source location, Ω is the integration domain or surface of scattered object. If we assume that Ω domain has N meshes and each meshes has Q quadrature points. Scattering field can be written by using Nyström method as:

$$\vec{E}(r) = \sum_{n=1}^{N}\sum_{q=1}^{Q} w_q G(\vec{r},\vec{r_q}).J(\vec{r_q}) \tag{4.2}$$

In above, it is assumed that $\vec{r_q}$ is sampling points on the surface, $\vec{r}$ shows observation points whose incident fields are known. To find current densities on the sampling points, observation points will be sampling points. EFIE can be written again in terms of sampling points and observation points like:

$$\vec{E}(r_p) = \sum_{n=1}^{N} \sum_{q=1}^{Q} w_q G(\vec{r_p}, \vec{r_q}) J(\vec{r_q}) \tag{4.3}$$

where $\vec{E}(r_p)$ is incident field for observation points. This problem can be transformed to matrix form like:

$$[V] = [Z][I] \tag{4.4}$$

where $[Z]$ is known as impedance matrix with $2NQx2NQ$ dimension, $[Z]$ has multipication of quadrature weights and dyadic green's function. $[I]$ shows unknown current densities of sampling points ($J(\vec{r_q})$) with $2NQx1$ dimension, $[V]$ shows incident wave ($\vec{E}(r_p)$) on the observation points with $2NQx1$ dimension. The row number of matrix is $2NQ$ because EFIE deals with tangential components and there are two tangential component for surfaces even if they are arbitrary surface. $[I]$ matrix can be found easily with matrix operations as [18, 19]:

$$[Z]^{-1}[V] = [I] \tag{4.5}$$

Impedance matrix $[Z]$ has singularity because sampling points and observation points are same points. $[Z]^{-1}$ is inverse matrix of $[Z]$. Evaluation of $[Z]^{-1}$ can cause time consuming if mesh number and quadrature point are too big. To prevent time consuming, it is possible to use different numeric techniques to find $[Z]^{-1}$. By using equation 4.5, current density on sampling points are found. To find scattering field, equation 4.3 and equation 4.4 are used again. Difference is that $\vec{r_q}$ is observation point on free-space, $\vec{r_q}$ is not on the scattered object surface. $\vec{E}(r_p)$ is scattered electric field on observers. There is no singularity problem on impedance matrix $[Z]$. If observer number is M, matrix dimension of $[Z]$ is $3Mx2NQ$ and matrix dimension of $[V]$ will be $3Mx1$. Expressions have $3M$ term, this term can be considered to represent three-dimensional scattered electric field components. This was the introduction of Application of Nyström Method to EFIE. Now, let's make more expressions about calculation of singular and non-singular integrals and how to find scattering electric field by using matrix properties.

4.1 Non-Singular Element Calculation for EFIE

Non-singular Element means distance between two sampling points is not 0. This situation happens in the impedance matrix except diagonal elements while evaluating unkown surface current densities. When scattering electric field is calculated, all elements of the impedance matrix is non-singular as distance between sampling points, that is between space and surface of perfect electric conductor, cannot be 0. Impedance matrix is related with geometry of scattered object. EFIE has tangential components of surface of object. Assume that there is an arbitrary PEC surface or PEC object is at uv plane, n is the surface normal. Before the non-singular element calculations, let's expand dyadic green's function. We found dyadic green's function as $G(r,r') = [I + \frac{1}{k^2}\nabla\nabla g(r,r')]$, where I shows unit matrix, $\nabla\nabla g(r,r')$ is dyadic term. For $|r-r'| = R$, matrix form of dyadic green's function ($G(R)$) interms of tangential components will be:

$$G(R) = \begin{bmatrix} 1 + \frac{1}{k^2}\cdot g_{uu}(R) & \frac{1}{k^2}\cdot g_{uv}(R) \\ \\ \frac{1}{k^2}\cdot g_{vu}(R) & 1 + \frac{1}{k^2}\cdot g_{vv}(R) \end{bmatrix} \tag{4.6}$$

where $g_{uu}(R)$ means $\frac{d^2 g(R)}{du^2}$, scalar green's function is $g(R) = \frac{\exp(-jkR)}{4\pi R}$ and $R = \sqrt{u^2+v^2+n^2}$.It is easily understood that $g_{uv} = g_{vu}$. In equations above, u, v and n are difference between observer and source positions for u, v and n axis. For example, u=$u-u'$, u is u-axis of observer, u' is u-axis of source. Second derivatives of scalar green's function are like:

$$g_{uu}(R) = \frac{e^{-jk\sqrt{u^2+v^2+n^2}}}{4\pi} \quad [\quad \frac{3u^2}{(u^2+v^2+n^2)^{5/2}} - \frac{1}{(u^2+v^2+n^2)^{3/2}}$$

$$+\frac{3iku^2}{(u^2+v^2+n^2)^2} - \frac{ik}{(u^2+v^2+n^2)^1} - \frac{k^2u^2}{(u^2+v^2+n^2)^{3/2}} \quad] \tag{4.7}$$

$$g_{uv}(R) = \frac{e^{-jk\sqrt{u^2+v^2+n^2}}}{4\pi} \quad [\quad \frac{3uv}{(u^2+v^2+n^2)^{5/2}} + \frac{3ikuv}{(u^2+v^2+n^2)^2}$$

$$-\frac{k^2uv}{(u^2+v^2+n^2)^{3/2}} \quad] \tag{4.8}$$

$$g_{vv}(R) = \frac{e^{-jk\sqrt{u^2+v^2+n^2}}}{4\pi} \quad [\quad \frac{3v^2}{(u^2+v^2+n^2)^{5/2}} - \frac{1}{(u^2+v^2+n^2)^{3/2}}$$

$$\frac{3ikv^2}{(u^2+v^2+n^2)^2} - \frac{ik}{(u^2+v^2+n^2)^1} - \frac{k^2v^2}{(u^2+v^2+n^2)^{3/2}} \quad] \tag{4.9}$$

This was the calculation of non-singular terms for dyadic green's function. Expressions can be used easily with quadrature rules.

4.2 Singular Element Calculation for EFIE

As we mention last chapter, Nyström method has advantages but one of the major disadvantages of the Nyström method is the singularity problem. Singularity means there is a singular term, which makes denominator to 0, inside integral. When observation and source are in same point, singularity happens. In electromagnetic, causes of singularity is green's function and derivatives of green's function. Green's function has $1/R$ term, this type of singularity is weakly-singularity [20]. Second derivatives of Green's function have $1/R^3$ term, this type of singularity is hyper-singularity. These two types singularity is in EFIE. Moreover, first derivatives of green's function have $1/R^2$ term, this type of singularity is super-singularity. Super-singularity is in MFIE. To solve these singularity problems, there are a lot of solution strategies. Solution strategies can be considered as two kind which are numeric solutions and analytic solutions. Numeric solution techniques are generally related with quadrature rules. Numeric solutions firstly convert singular kernel to non-singular kernel by using special transforms. After transformation, non-singular kernel can be evaluated easily by using quadrature rules. One of the most used transformation methods are Duffy transformation and radial angular transformation. These transformations are useful to weakly-singular kernels. For hyper-singular kernels, techniques are need to more transformation. Numeric solution techniques suffer from quadrature rules as accuracy of techniques depends on number of quadrature points. While writing codes for numerical techniques, quadrature rules coded with loops. More loops are more running time. This is bad situation. In this thesis, our aim is to calculate scattered fields quickly. It is no need to quadrature rules to solve problem quickly. So, analytic solution techniques are useful instead of numerical solution to calculate singularities techniques. As everybody knows, analytical solution techniques give exact solution compared to numerical solution techniques. Unfortunately, there is limited research to analytical solution techniques for singular element calculations. Research are generally about numerical solution techniques. Fortunately, GOKHUN SELCUK shows analytical solution technique for singular elements in his doctorate thesis. In this study, Selcuk's approximation is used.

In this technique, it is thought that distance between source and observation goes to 0. Then, scalar green's function are separated to singular and non-singular parts like:

$$\lim_{R\to 0} g(R) = \lim_{R\to 0} \frac{e^{-jkR}}{4\pi R} = \lim_{R\to 0} \frac{1}{4\pi}\left[\quad \frac{e^{-jkR}-1}{R} + \frac{1}{R} \quad \right] \tag{4.10}$$

the first term of equation 4.10 is non-singular term, second term is singular term. With the L'Hospital's rule, first term can be calculate as [8, 19]:

$$\frac{1}{4\pi} \lim_{R\to 0} \frac{e^{-jkR}-1}{R} = \frac{-jk}{4\pi} \tag{4.11}$$

Result of non-singular term is just number so, this term can be used in quadrature rule while filling impedance matrix. Let's look at calculations of weakly-singular terms and hyper-singular terms.

4.2.1 Weakly-singular term calculation

Before begin to calculation ,let's remind EFIE:

$$\vec{t}.iw\mu \int_S g(r,r')J(r')dS(r) + \vec{t}\frac{1}{k^2}.iw\mu \int_S \nabla\nabla g(r,r').J(r')dS(r) = \vec{t}.\vec{E}^i \tag{4.12}$$

In above, first integral has weakly-singular term, second one has hyper-singular term. If constant terms ignore weakly-singular integral will be:

$$\lim_{r\to 0} \int_S g(r)J(r')dS(r) \tag{4.13}$$

EFIE is linear equation and surface current can be out of the integral. With the equation 4.11 and equation 4.13, integral will be form of:

$$\frac{1}{4\pi} J(r') \lim_{r\to 0} \int_S (-jk + \frac{1}{R}) dS(r) \tag{4.14}$$

In calculation, it is assumed that surface meshes are triangular and xy-plane. Singular point is at the origin like figure below.

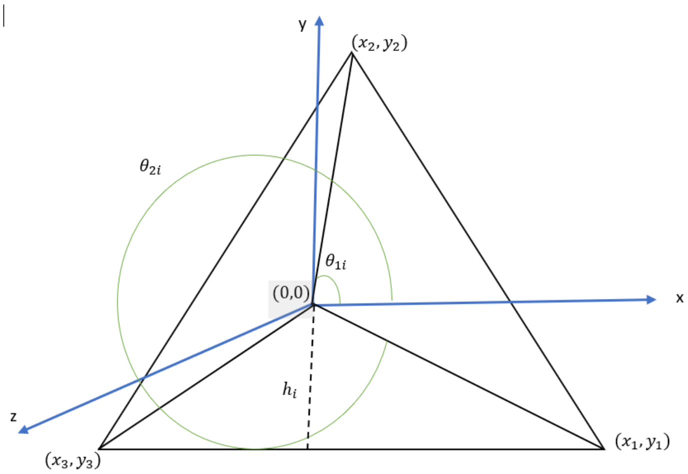

Figure 4.1 : Triangular mesh and singular point.

After Jacobian transform ($dS = RdRd\theta$), singular integral for one subtriangular can be written as [19]:

$$I_i = \int_{\theta_{1i}}^{\theta_{2i}} \int_0^{R_i(\theta)} \frac{1}{R} RdRd\theta \tag{4.15}$$

result is sum of 3 subtriangular.

$$I = \sum_{i=1}^{i=3} I_i \tag{4.16}$$

In figure 1, first subtriangular has vertices $(0,0)$, (x_1,y_1) and (x_2,y_2). Second subtriangular has vertices $(0,0)$, (x_2,y_2) and (x_3,y_3) and third has vertices $(0,0)$, (x_3,y_3) and (x_1,y_1). θ_{1i} is the angle between first edge of subtriangular and x-axis. θ_{2i} is the angle between second edge of subtriangular and x-axis. Expression of $R_i(\theta)$ is:

$$R_i(\theta) = \frac{h_i}{\cos(\theta - \theta_i)} \tag{4.17}$$

where h_i is the height of subtriangular edge that is not edge of other subtriangles. θ_i is the angle between height and x-axis. By evaluating integral in equation 4.17 compare to dR, new integral will be :

$$I_i = \int_{\theta_{1i}}^{\theta_{2i}} \frac{h_i}{\cos(\theta - \theta_i)} d\theta \tag{4.18}$$

with the integral table or knowing that $\int \frac{1}{cos(x)} dx = ln(|sec(x) + tan(x)|)$, result can be found as:

$$I_i = h_i \ \left[\ln(|\sec(\theta_{2i} - \theta_i) + \tan(\theta_{2i} - \theta_i)|) - \ln(|\sec(\theta_{1i} - \theta_i) + \tan(\theta_{1i} - \theta_i)|)\right] \tag{4.19}$$

This is the result of weakly-singular term calculation. It is no doubt that square mesh calculation can be made with same expression. For square mesh calculation, square must be divided 4 subtriangles and result is sum of results of these 4 subtriangulars.

$$I = \sum_{i=1}^{4} I_i \tag{4.20}$$

4.2.2 Hyper-singular element calculation for EFIE

Before begin to calculation, let's write hyper-singular term of EFIE :

$$\int_S \nabla\nabla g(r,r').J(r')dS(r) \tag{4.21}$$

surface current density $J(r')$ and double-gradient operator can be out of integral because of linearity, like weakly-singular term. By writing Green's function expansion for $r \to 0$, it will be:

$$J(r').\nabla\nabla \int_S (-jk + \frac{1}{R})dS(r) \tag{4.22}$$

$-jk$ term is a constant term, gradient of this term will be 0 and hyper-singular integral will be:

$$\nabla\nabla \int_S \frac{1}{R} dS(r) \tag{4.23}$$

There are two techniques to solve this hyper-singular integral. First technique is to use divergence theorem. Second technique is to use finite difference theorem. Now, let's talk about them:

Contour İntegration Method

The hyper-singular integral can be considered with divergence theorem as:

$$\lim_{\varepsilon \to 0} \int_S \nabla\nabla \frac{1}{\sqrt{R^2+\varepsilon^2}} dS = \lim_{\varepsilon \to 0} \int_\Gamma \vec{n}\nabla \frac{1}{\sqrt{R^2+\varepsilon^2}} dl' \tag{4.24}$$

where $\vec{n}$ is the normal unit vector, contour of the triangular mesh is Γ. As we know $\lim_{\varepsilon \to 0} \frac{1}{\sqrt{R^2+\varepsilon^2}} = \frac{1}{R}$ and from vector identities: $\nabla \frac{1}{R} = \frac{-\vec{a}_R}{R^2}$ [21]. Hyper-singular integral is:

$$\int_{\Gamma} \vec{n} \frac{-\vec{e_R}}{R^2} dl' \tag{4.25}$$

where $\vec{e_R}$ is the radial unit vector. Equation 4.25 is a dyadic term to calculate electric field in specified direction. For instance, electric field in $\vec{e_x}$ direction due to surface current density in $\vec{e_x}$ direction is calculated as:

$$I_{xx} = \int_{\Gamma} \vec{n}.\vec{e_x} \frac{-\vec{e_R}}{R_i^2(\theta)}.\vec{e_x} dl' \tag{4.26}$$

From figure 1 or from analytic geometry, dot products and dl can be explain as [19]:

$$\vec{e_R}.\vec{e_x} = \cos(\theta) \tag{4.27}$$

$$\vec{n_i}.\vec{e_x} = \cos(\theta_i) \tag{4.28}$$

$$\Gamma = \Gamma_1 + \Gamma_2 + \Gamma_3 \tag{4.29}$$

$$dl = \frac{r(\theta) d\theta}{\cos(\theta - \theta_i)} \tag{4.30}$$

Finally, dyadic terms will be calculated easily like:

$$I_{xx} = \sum_{i=1}^{3} \frac{\cos(\theta_i)}{h_i} (\sin(\theta_{2i}) - \sin(\theta_{1i})) \tag{4.31}$$

$$I_{xy} = \sum_{i=1}^{3} \frac{\cos(\theta_i)}{h_i} (\cos(\theta_{2i}) - \cos(\theta_{1i})) \tag{4.32}$$

$$I_{yx} = \sum_{i=1}^{3} \frac{\sin(\theta_i)}{h_i} (\sin(\theta_{2i}) - \sin(\theta_{1i})) \tag{4.33}$$

$$I_{yy} = \sum_{i=1}^{3} \frac{\sin(\theta_i)}{h_i} (\cos(\theta_{2i}) - \cos(\theta_{1i})) \tag{4.34}$$

Finite Difference Theorem Solution

Even if divergence theorem solution has good approximation and explicit formulations, finite difference solution is easy to understand and implement. Theorem uses derivative formulas of finite difference method. In mathematics, finite difference formulation is powerful method to calculate first and second derivatives of functions. Before using this solution for hyper-singular integrals, weakly-singular integral must be evaluated. Let's show hyper-singular integral with I^h and weakly-singular integral with I^w. Hyper-singular integral can be written in xy-plane as [22]:

$$I^h(x,y) = \nabla \int_{S'} \nabla' \frac{1}{\sqrt{(x-x')^2+(y-y')^2}} dx'dy' \tag{4.35}$$

where S' is the surface, unprimed variables means observation locations, primed variables means source locations. Integral I^h is dyadic with 4 terms which are $I^h_{xx}, I^h_{xy}, I^h_{yx}, I^h_{yy}$.For example I^h_{xx} is:

$$I^h_{xx}(x,y) = \frac{\partial}{\partial x} \int_{S'} \frac{\partial}{\partial x'} \frac{1}{\sqrt{(x-x')^2+(y-y')^2}} dx'dy' \tag{4.36}$$

with the property of $\frac{\partial}{\partial x'} = -\frac{\partial}{\partial x}$, hyper-singular integrals are like:

$$I^h_{xx}(x,y) = \frac{\partial}{\partial x}\frac{\partial}{\partial x} \int_{S'} \frac{-1}{\sqrt{(x-x')^2+(y-y')^2}} dx'dy' \tag{4.37}$$

$$I^h_{xy}(x,y) = \frac{\partial}{\partial x}\frac{\partial}{\partial y} \int_{S'} \frac{-1}{\sqrt{(x-x')^2+(y-y')^2}} dx'dy' \tag{4.38}$$

$$I^h_{yy}(x,y) = \frac{\partial}{\partial y}\frac{\partial}{\partial y} \int_{S'} \frac{-1}{\sqrt{(x-x')^2+(y-y')^2}} dx'dy' \tag{4.39}$$

Weakly-singular integral which must be calculated before is like:

$$I^w(x,y) = \int_{S'} \frac{-1}{\sqrt{(x-x')^2+(y-y')^2}} dx'dy' \tag{4.40}$$

By applying second order central difference formulation, hyper-singular integrals are found interms of weakly-singular integral. These are:

$$I_{xx}^{h}(x,y) = [\ \ I^{w}(x+\varepsilon,y) + I^{w}(x-\varepsilon,y) - 2I^{w}(x,y)\ \ ]/\varepsilon^{2} \tag{4.41}$$

$$I_{xx}^{h}(x,y) = [\ \ I^{w}(x,y+\varepsilon) + I^{w}(x,y-\varepsilon) - 2I^{w}(x,y)\ \ ]/\varepsilon^{2} \tag{4.42}$$

$$I_{xy}^{h}(x,y) = I_{yx}^{h}(x,y) = [\ \ I^{w}(x+\varepsilon,y+\varepsilon) + I^{w}(x-\varepsilon,y-\varepsilon)$$

$$-I^{w}(x+\varepsilon,y-\varepsilon) + I^{w}(x-\varepsilon,y+\varepsilon)\ \ ]/4\varepsilon^{2} \tag{4.43}$$

where ε is so small number. Results have good accuracy if ε is selected as 10^{-3} or 10^{-4}. There are 3 main error while calculating hyper-singular integrals. First one is errors of weakly-singulars integral. Second one is that second derivatives on the formulation of hyper-singular integrals have second order accuracy. Third one is roundoff error because of ε.

4.3 Evaluating Scattered Electric Field

In the last sections of this chapter, all integrals and matrix elements will be showed to evaluate scattering electric field. In this chapter, calculations steps of scattering electric field will be explained. There are two subsections on this chapter that are Calculation of surface current density and Calculation of scattering electric field.

4.3.1 Calculation of surface current density

Most important steps are to evaluate singular elements of matrixs. By taking inverse of impedance matrix, important contributions comes from diagonals (singular elements).These contributions are more important than non-singular elements. Let's define the calculation steps. Firstly, remind EFIE again.

$$\vec{t}.iw\mu \int_S [I + \frac{1}{k^2}\nabla\nabla] g(r,r').J(r')dS(r) = \vec{t}.\vec{E}_i(r) \tag{4.44}$$

where $\vec{t}$ is the tangential component of the perfect electric conductor surface. Scattering electric field can be solved after surface current density $J(r')$ is evaluated. Fortunately, EFIE is used to evaluate surface current density and scattering electric field. This quantity can be solved by using dyadic terms and matrix equation. Dyadic terms come from double gradient operator. By using cartesian coordinates and EFIE, matrix form of EFIE will be [16, 19, 23]:

$$iw\mu \begin{bmatrix} g(R) + \frac{1}{k^2}.g_{xx}(R) & \frac{1}{k^2}.g_{xy}(R) & \frac{1}{k^2}.g_{xz}(R) \\ \frac{1}{k^2}.g_{yx}(R) & g(R) + \frac{1}{k^2}.g_{yy}(R) & \frac{1}{k^2}.g_{yz}(R) \\ \frac{1}{k^2}.g_{zx}(R) & \frac{1}{k^2}.g_{zy}(R) & g(R) + \frac{1}{k^2}.g_{zz}(R) \end{bmatrix} \begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix} = \begin{bmatrix} E_x^i \\ E_y^i \\ E_z^i \end{bmatrix} \tag{4.45}$$

where E_x^i, E_y^i, E_z^i shows incident electric field components. First matrix in above is known as impedance matrix. Let's consider our problem in local coordinates that has u, v, n directions. Assume that there is a PEC object on uv-plane and incident electric field is plane-wave. Tangential components are $\vec{e_u}$ and $\vec{e_v}$. Because of the PEC surface, there will be no surface current density in $\vec{e_n}$ direction. Third row of the impedance matrix is 0 as $\vec{n}$ is not tangential component. Terms that have derivative of n is 0. New matrix equation will be [19]:

$$iw\mu \begin{bmatrix} g(R) + \frac{1}{k^2} \cdot g_{uu}(R) & \frac{1}{k^2} \cdot g_{uv}(R) \\ \frac{1}{k^2} \cdot g_{vu}(R) & g(R) + \frac{1}{k^2} \cdot g_{vv}(R) \end{bmatrix} \begin{bmatrix} J_u \\ J_v \end{bmatrix} = \begin{bmatrix} E_u^i \\ E_v^i \end{bmatrix} \tag{4.46}$$

If we assume there are N meshes and each mesh has Q quadrature points. Size of the impedance matrix is $2NQ$ x $2NQ$. Size of I matrix is $2Nq$ x 1. Every element of the impedance matrix is generally matrix which size is NQ x NQ. Every element of this matrix tell that how arbitrary two quadrature point affect each other. For example, element which is in 1.row and 2.column of matrix tell that what is the effect of 1. point on the 2.point. Let's expand the first element of impedance matrix like :

$$g(R) + \frac{1}{k^2} \cdot g_{uu}(R) = \int_S g(R) dS + \frac{1}{k^2} \int_S g_{uu}(R) dS \tag{4.47}$$

with the quadrature rule by using the equation 3.7, non-singular integrals are:

$$g(R) = \frac{S}{4} \sum_{i=1}^{NQ} \sum_{j=1}^{NQ} w_i g(r_i, r_j) \tag{4.48}$$

where S is the area of each mesh and $g(r_i, r_j) = \frac{\exp(-jk|r_i - r_j|)}{4\pi|r_i - r_j|}$. Second term will be form of:

$$\frac{1}{k^2} \cdot g_{uu}(R) = \frac{1}{k^2} \frac{S}{4} \sum_{i=1}^{NQ} \sum_{j=1}^{NQ} w_i g_{uu}(r_i, r_j) \tag{4.49}$$

Other 3 elements of singular part are written with same strategy. Let's expand singular term of first element(first term of impedance matrix on the equation 4.45).

$$g(u,v) = \frac{S}{4}\sum_{i=1}^{NQ}(-jk + I(u,v))w_i \tag{4.50}$$

$$g_{uu}(u,v) = \frac{S}{4}\sum_{i=1}^{NQ}(-jk + I^h_{uu}(u,v))w_i \tag{4.51}$$

The first singular element of impedance matrix with quadrature rule is:

$$g(u,v) + \frac{1}{k^2}g_{uu}(u,v) = \frac{S}{4}\sum_{i=1}^{NQ}(-jk + I(u,v) + \frac{1}{k^2}I^h_{uu}(u,v))w_i \tag{4.52}$$

I is the result of equation 4.40, I^h_{uu} is the result of equation 4.41. Other 3 elements of singular part are written with same strategy, by just considering u in terms of v. It is assumed plane wave has propagation in +n direction and polarization in +u direction with unit amplitude. Incident field of every sampling points are:

$$E^i(i) = \sum_{i=1}^{NQ}\vec{e}_u \exp(-jkn_i) \tag{4.53}$$

Incident field vector shows as V, surface current density vector shows as I. With the matrix equations, matrix form of surface current density $[I]$ is:

$$[V] = [Z][I] \tag{4.54}$$

$$[I] = [Z]^{-1}[V] \tag{4.55}$$

4.3.2 Calculation of scattering electric field

Last step is to find scattering field from surface current density. Aim is to find all components of scattering electric field. For this scenario, scattering electric field can have 3 components this is why third row of impedance matrix is not 0 but it is no need to calculate third column because there is no J_n component of current density. Dyadic matrix system can write as:

$$iw\mu \begin{bmatrix} g(R)+\frac{1}{k^2}\cdot g_{uu}(R) & \frac{1}{k^2}\cdot g_{uv}(R) \\ \frac{1}{k^2}\cdot g_{vu}(R) & g(R)+\frac{1}{k^2}\cdot g_{vv}(R) \\ \frac{1}{k^2}\cdot g_{nu}(R) & \frac{1}{k^2}\cdot g_{nv}(R) \end{bmatrix} \begin{bmatrix} J_u \\ \\ J_v \end{bmatrix} = \begin{bmatrix} E_u^s \\ E_v^s \\ E_n^s \end{bmatrix} \tag{4.56}$$

In this scenario, impedance matrix has no singular term because R is distance between surface and arbitrary point at free-space. With the help of quadrature rule, scattering field in all direction will be:

$$E_u^s(r) = iw\mu\frac{S}{4}\sum_{i=1}^{NQ} w_i[\ \ g(r',r)+\frac{1}{k^2}g_{uu}(r',r)\ \ ]J_u(i) + iw\mu\frac{S}{4}\sum_{i=1}^{NQ} w_i[\ \ +\frac{1}{k^2}g_{uv}(r',r)\ \ ]J_v(i) \tag{4.57}$$

$$E_v^s(r) = iw\mu\frac{S}{4}\sum_{i=1}^{NQ} w_i[\ \ \frac{1}{k^2}g_{vu}(r',r)\ \ ]J_u(i) + iw\mu\frac{S}{4}\sum_{i=1}^{NQ} w_i[\ \ g(r',r)+\frac{1}{k^2}g_{vv}(r',r)\ \ ]J_v(i) \tag{4.58}$$

$$E_n^s(r) = iw\mu\frac{S}{4}\sum_{i=1}^{NQ} w_i[\ \ \frac{1}{k^2}g_{nu}(r',r)\ \ ]J_u(i) + iw\mu\frac{S}{4}\sum_{i=1}^{NQ} w_i[\ \ \frac{1}{k^2}g_{nv}(r',r)\ \ ]J_v(i) \tag{4.59}$$

5. SIMULATION RESULTS

Scattering electric field from PEC are simulated with Python Programming Language. In Python Programming language, equations of previous chapter are coded. NumPy library was generally used to generate equations and matrix elements. Simulation results was compared with results of Feko Programming. Feko is one of the most known the electromagnetic simulation program. Comparisons were made using the amplitude and phase values of the scattering electric field. In simulations, plane wave is used as electromagnetic source. Mesh type are square mesh. To make a good comparison, mesh number, number of quadrature points for each mesh, frequency of plane-wave, propagation direction of plane wave and dimensions of PEC object were changed for each simulation. Angle of propagation direction of plane are selected as $0°$ and $45°$. Frequency are selected as 500 MHz,1GHz,3GHz and 10 GHz. Number of quadrature points for each mesh are selected as 9,16 and 25. Scattering electric field results can be considered as a function of these parameters while coding on Python. Simulation results are divided two parts according to shape of scattering PEC object. Two parts are simulation results for PEC square and simulation results for PEC cube. Simulation results of scattering electric field will be given component by component in terms of phase and amplitude values with Feko results. 6 simulation results will be showed this study. 4 of them are about PEC square and 2 of them are about PEC cube.

5.1 Simulation Results of PEC Square

PEC square is used as scattering object. There are four different experiment result for this part. Frequency and dimensions of PEC are same but polarization angles and plane of PEC are different in three of them. Last experiment result has different dimension of PEC and different frequencies compare to others. Let's show them.

5.1.1 Results of 1. simulation

PEC square has 5cm x 5cm dimension on xy-plane at z=0. Frequency is 1 GHz. Propagation direction of plane wave is +z direction. Plane wave has +x polarization. There are 21 observers, their x and y are coordinates are at 1m. Their z coordinates are between -1 m and 1m with 5 cm difference. Mesh number are 36 and all meshes are square. Every mesh has 9 sampling points. Feko uses 18 triangular meshes for this simulation. Simulation results of Feko and this thesis can be seen below.

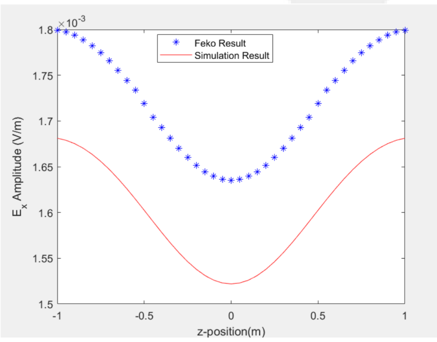

Figure 5.1 : E_x amplitude result of 1. simulation.

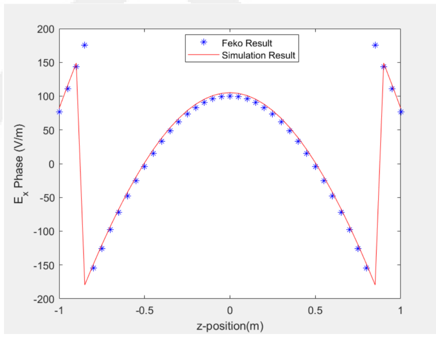

Figure 5.2 : E_x phase result of 1. simulation.

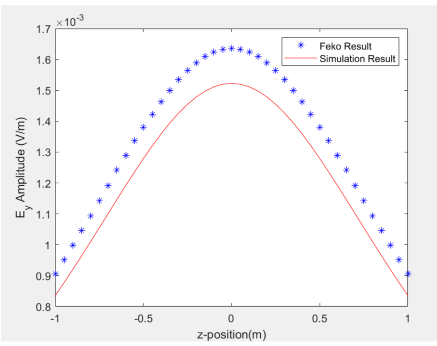

Figure 5.3 : E_y amplitude result of 1. simulation.

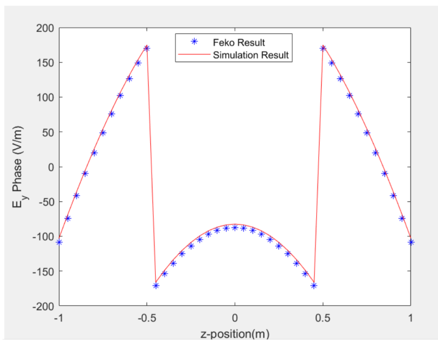

Figure 5.4 : E_y phase result of 1. simulation.

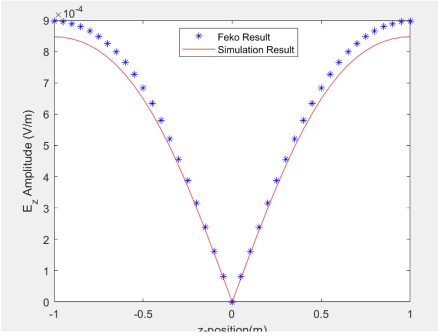

Figure 5.5 : E_z amplitude result of 1. simulation.

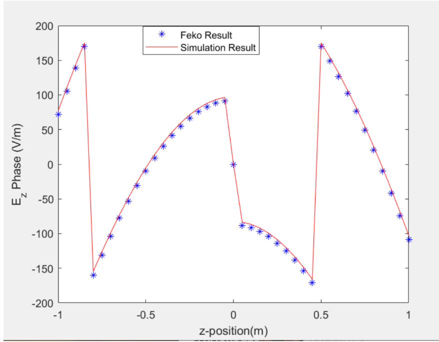

Figure 5.6 : E_z phase result of 1. simulation.

5.1.2 Results of 2. simulation

PEC square has 5cm x 5cm dimension on xy-plane at z=0. Frequency is 1 GHz. Propagation direction makes 45° angle with xz axis. There are 21 observers, their x and y are coordinates are at 1m. Their z coordinates are between -1 m and 1m with 5cm difference. Mesh number are 36 and all meshes are square. Every mesh has 9 sampling points. Feko uses 18 triangular meshes for this simulation. Simulation results of Feko and this thesis can be seen below.

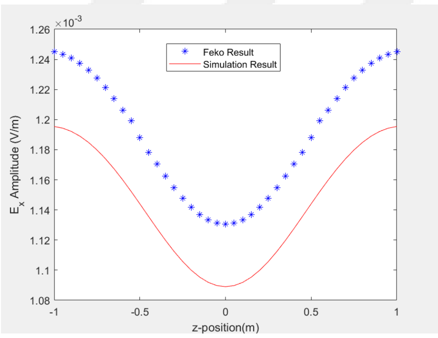

Figure 5.7 : E_x amplitude result of 2. simulation.

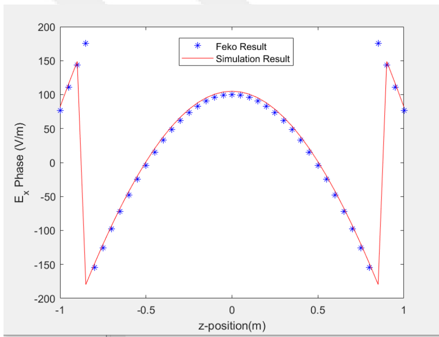

Figure 5.8 : E_x phase result of 2. simulation.

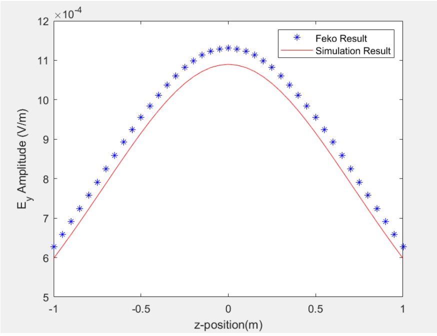

Figure 5.9 : E_y amplitude result of 2. simulation.

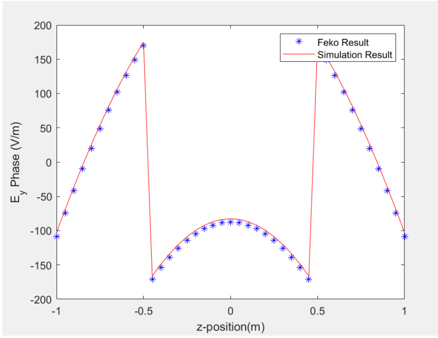

Figure 5.10 : E_y phase result of 2. simulation.

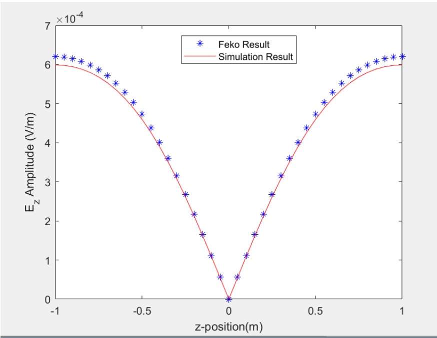

Figure 5.11 : E_z amplitude result of 2. simulation.

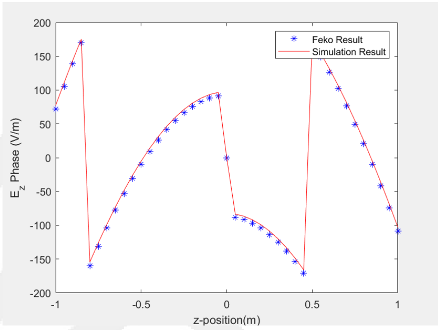

Figure 5.12 : E_z phase result of 2. simulation.

5.1.3 Results of 3. simulation

PEC square has 5cm x 5cm dimension on xy-plane at z=1. Frequency is 1 GHz. Propagation direction of plane wave makes 45° angle with xz axis. There are 21 observers, their x and y are coordinates are at 1m. Their z coordinates are between -1 m and 1m with 5 cm difference. Mesh number are 36 and all meshes are square. Every mesh has 9 sampling points. Feko uses 18 triangular meshes for this simulation. Simulation results of Feko and this thesis can be seen below.

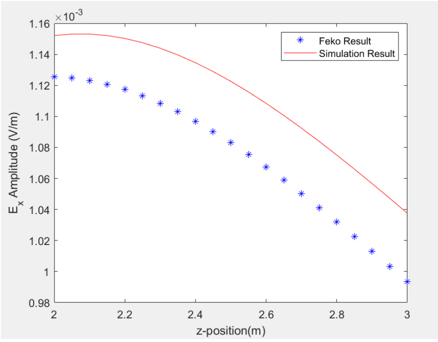

Figure 5.13 : E_x amplitude result of 3. simulation.

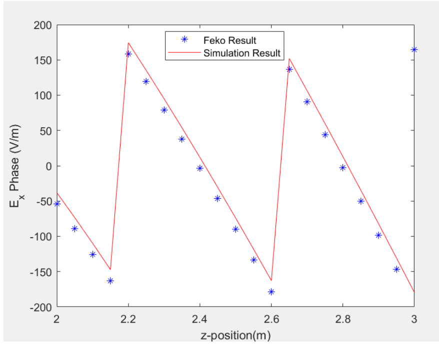

Figure 5.14 : E_x phase result of 3. simulation.

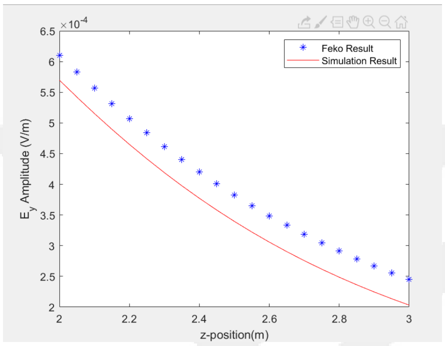

Figure 5.15 : E_y amplitude result of 3. simulation.

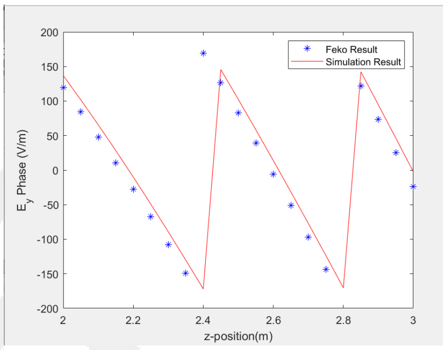

Figure 5.16 : E_y phase result of 3. simulation.

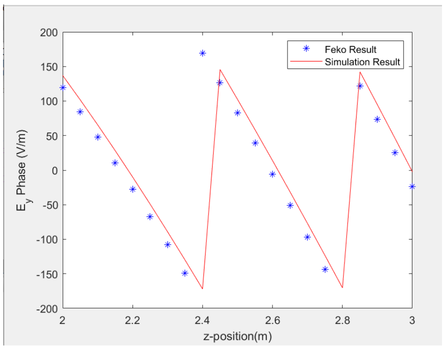

Figure 5.17 : E_z amplitude result of 3. simulation.

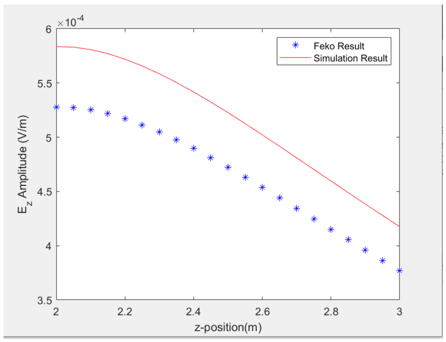

Figure 5.18 : E_z phase result of 3. simulation.

5.1.4 Results of 4. simulation

PEC square has 2cm x 2cm dimension on xy-plane at z=0. Frequency is 3 GHz. Propagation direction of plane wave makes 45° angle with xz axis. There are 21 observers, their x and y are coordinates are at 1m. Their z coordinates are between -1 m and 1m with 5cm difference. Mesh number are 16 and all meshes are square. Every mesh has 9 sampling points. Feko uses 26 triangular meshes for this simulation. Simulation results of Feko and this thesis can be seen below.

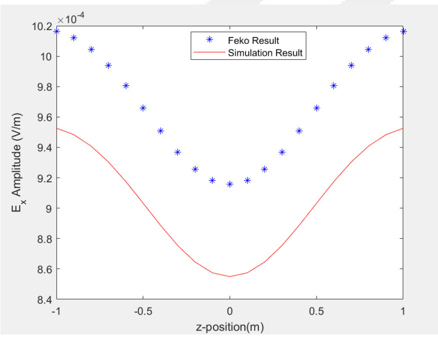

Figure 5.19 : E_x amplitude result of 4. simulation.

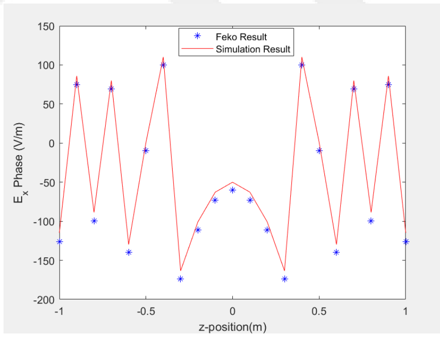

Figure 5.20 : E_x phase result of 4. simulation.

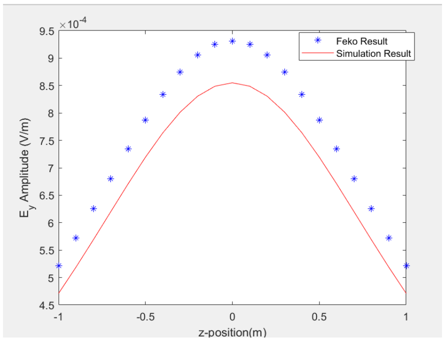

Figure 5.21 : E_y amplitude result of 4. simulation.

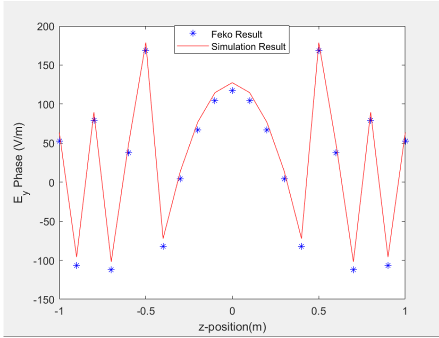

Figure 5.22 : E_y phase result of 4. simulation.

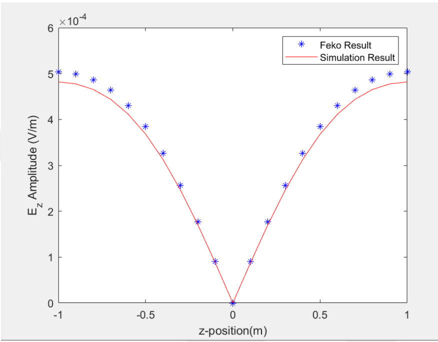

Figure 5.23 : E_z amplitude result of 4. simulation.

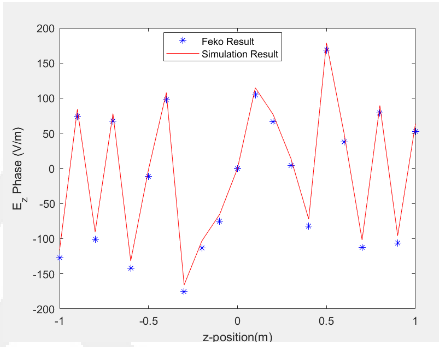

Figure 5.24 : E_z phase result of 4. simulation.

5.2 Simulation Results of PEC Cube

PEC cube is used as scattered object. There are 2 different experiment result for this part. Let's show them.

5.2.1 Results of 5. simulation

PEC cube has 10cm x 10cm x 10cm dimension. Center of PEC cube is at 5cm,5cm and 5cm on cartesian coordinate system. Frequency is 500 MHz. Propagation direction of plane wave makes 45° angle with xz axis. There are 21 observers, their x and y are coordinates are at 20cm. Their z coordinates are between -1 m and 1m with 5 cm difference. Mesh number are 150 and all meshes are square. Every mesh has 9 sampling points. Feko uses 108 triangular meshes for this simulation. Simulation results of Feko and this thesis can be seen below.

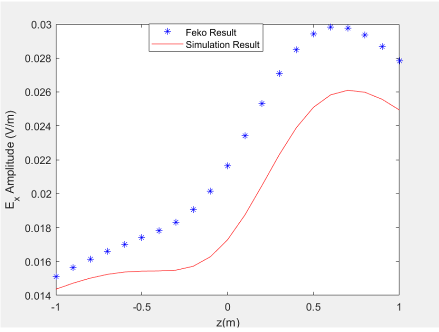

Figure 5.25 : E_x amplitude result of 5. simulation.

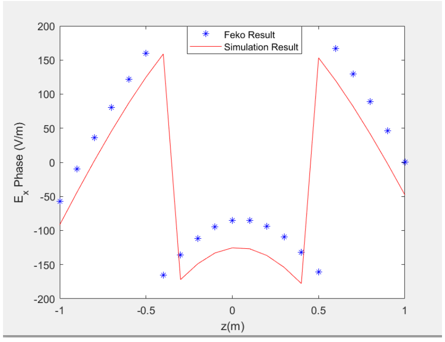

Figure 5.26 : E_x phase result of 5. simulation.

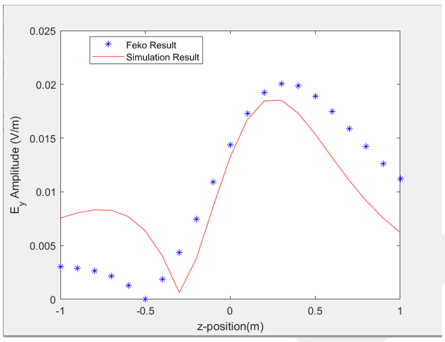

Figure 5.27 : E_y amplitude result of 5. simulation.

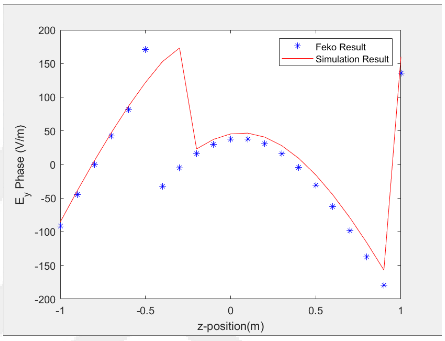

Figure 5.28 : E_y phase result of 5. simulation.

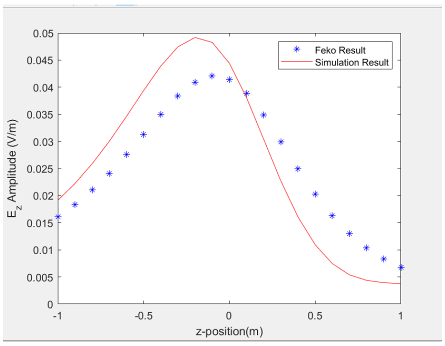

Figure 5.29 : E_z amplitude result of 5. simulation.

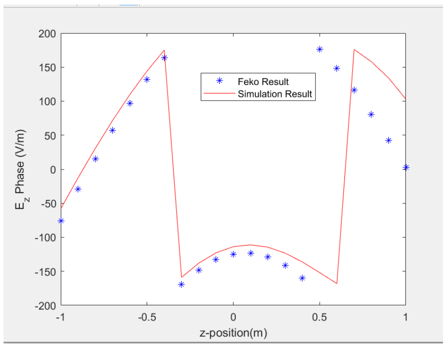

Figure 5.30 : E_z phase result of 5. simulation.

5.2.2 Results of 6. simulation

PEC cube has 1.5cm x 1.5cm x 1.5cm dimension. Center of PEC cube is at 0.75cm, 0.75cm and 0.75cm on cartesian coordinate system. Frequency is 3GHz. Propagation direction plane wave makes 45° angle with xz axis. There are 21 observers, their x and y are coordinates are at 20cm. Their z coordinates are between -1 m and 1m with 5 cm difference. Mesh number are 144 and all meshes are square. Every mesh has 9 sampling points. Feko uses 48 triangular meshes for this simulation. Simulation results of Feko and this thesis can be seen below.

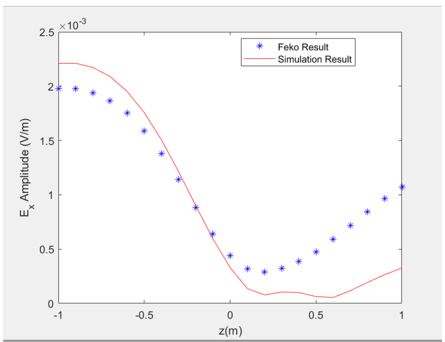

Figure 5.31 : E_x amplitude result of 6. simulation.

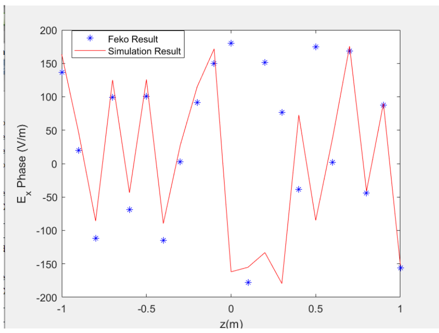

Figure 5.32 : E_x phase result of 6. simulation.

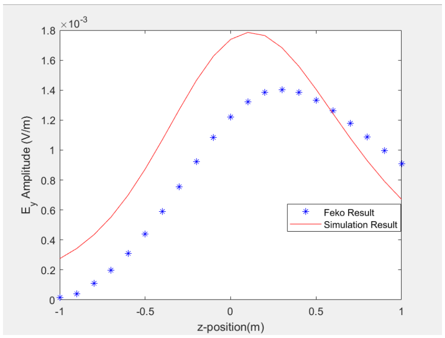

Figure 5.33 : E_y amplitude result of 6. simulation.

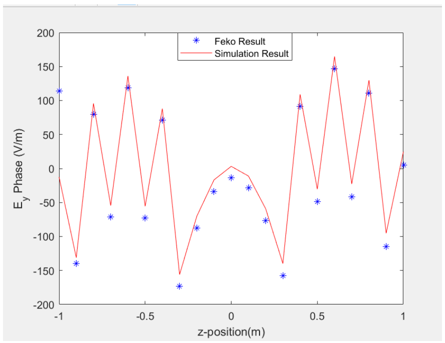

Figure 5.34 : E_y phase result of 6. simulation.

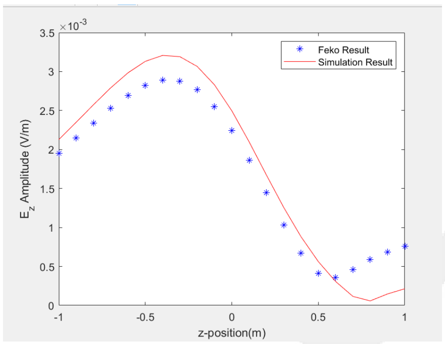

Figure 5.35 : E_z amplitude result of 6. simulation.

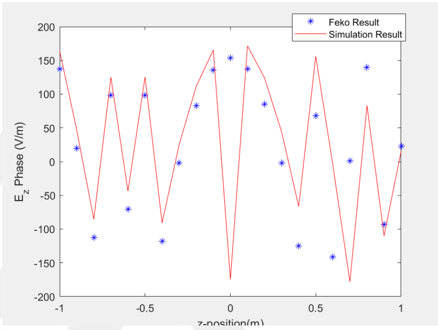

Figure 5.36 : E_z phase result of 6. simulation.

6. CONCLUSIONS AND RECOMMENDATIONS

In this work, EFIE is solved with NM and scattering electric field from conducting surfaces are found. Singularity of integrals is computed with Duffy's Transform and Finite Difference Theorem. Simulations demonstrate that there is a high accuracy for results of scattering electric field.

Integration of NM is easy for non-singular integral equations. Unfortunately, EFIE has singular kernel and solution of this problem is the hardest part of this thesis. There are many methods to overcome singular kernel but, most of them uses quadrature rules. Using quadrature rule is not acceptable, because quadrature rule means loops while coding its formulation. This causes time consuming and memory allocation. So, we need to analytic solution techniques like Duffy's transform to handle singularity. Even Duffy's transform is for triangular meshes, it can be implemented to square meshes. Duffy's transform was used for weakly singular terms. The finite difference theorem has been used for hypersingular terms even though it is a numerical method, as it is not more time consuming and memory allocation, unlike quadrature rules.

Mom is one of the most traditional solution techniques on electromagnetics. NM makes EFIE easier to understand than Mom. Surface current density is on the sampling points in NM. NM does not have basis functions unlike Mom. There is no divergence conforming in NM. So, NM has dyadic green's function. This situation cause that NM has more elements in impedance matrix and there is a additional hyper singular terms compare to Mom. It is clear that NM has more unknown terms according to Mom. Although there are disadvantages of NM, Mom uses more meshes or sampling points to increase accuracy.

We noticed that the increased number of sampling points for each mesh does not guarantee better accuracy. If number of sampling points increases, distance between sampling points goes to 0. This may create singular effect on non-singular matrix elements. Similar condition is valid for PEC cube. While increasing number of sampling points, distance between sampling points of adjacent surface can go 0. So, definition of singular term must be changed. Singular terms must not be just diagonal elements. Distance between sampling points should be important to accept matrix element as a singular.

It is also realized that when total sampling points number are same for two different simulation, their results will not be same because effect of non-singular terms, which are near to diagonal elements, are different. Also, contribution comes from singular elements are different because their mesh surfaces and their locations on the object surface are changed.

In the future, we plan to use high order scenario that can model arbitrary surfaces. With this scenario, we can find tangential components of arbitrary surface to apply EFIE. We also want to solve MFIE and Combined Field İntegral Equation that is a combination of EFIE and MFIE with high order scenario. Our last plan is to find S-parameters for multilayer structures by combining dyadic green's function and scattering fields.

REFERENCES

[1] **Harrington, R.F.** (1993). *Field computation by moment methods*, Wiley-IEEE Press.

[2] **Caliskan, F.** (2004). Electromagnetic Analysis of Planar Layered Structures, *Ph.D. Dissertation*, Georgia Institute of Technology.

[3] **Balanis, C.A.** (2012). *Advanced engineering electromagnetics*, John Wiley & Sons.

[4] **Shafieipour, M.** (2014). Efficient Error-Controllable High-Order Electromagnetic Modelling of Scattering on Electrically Large Targets with the Locally Corrected Nystrom Method.

[5] **Sheng, X.Q. and Song, W.** (2011). *Essentials of computational electromagnetics*, volume757, John Wiley & Sons.

[6] **Çayören, M.**, **Akduman, I.**, **Yapar, A. and Crocco, L.** (2007). A new algorithm for the shape reconstruction of perfectly conducting objects, *Inverse Problems*, *23*(3), 1087.

[7] **Sadiku, M.N.** (2000). *Numerical techniques in electromagnetics*, CRC press.

[8] **Gibson, W.C.** (2007). *The method of moments in electromagnetics*, Chapman and Hall/CRC.

[9] **Li, L.** (2016). Singularity Cancellation Transformations and Hierarchical Higher Order Basis Functions for the Hybrid Finite Element Boundary Integral Technique, *Ph.D. Dissertation*, Technische Universität München.

[10] **Rao, S.**, **Wilton, D. and Glisson, A.** (1982). Electromagnetic scattering by surfaces of arbitrary shape, *IEEE Transactions on antennas and propagation*, *30*(3), 409–418.

[11] **Kılınç, S.** (2010). Solution of electromagnetic scattering problems with the locally corrected Nyström method, *Ph.D. Dissertation*, bilkent university.

[12] **Zhang, X. and Liu, S.** (2005). Nystrom method for EM scattering of 3D conductor bodies, *2005 Asia-Pacific Microwave Conference Proceedings*, volume 5, IEEE, pp.4–pp.

[13] **Peterson, A.F. and Bibby, M.M.** (2009). An introduction to the locally-corrected Nyström method, *synthesis lectures on computational electromagnetics*, *4*(1), 1–115.

[14] **Burghignoli, P.**, **Di Nallo, C.**, **Frezza, F. and Galli, A.** (2003). A simple Nyström approach for the analysis of 3D arbitrarily shaped conducting and dielectric bodies, *International Journal of Numerical Modelling: Electronic Networks, Devices and Fields*, *16*(2), 179–194.

[15] **Takahasi, H. and Mori, M.** (1974). Double exponential formulas for numerical integration, *Publications of the Research Institute for Mathematical Sciences*, *9*(3), 721–741.

[16] **Tong, M.S. and Chew, W.C.** (2020). *The Nystrom Method in Electromagnetics*, John Wiley & Sons.

[17] **Tong, M.S. and Chew, W.C.** (2012). A novel meshless scheme for solving surface integral equations with flat integral domains, *IEEE transactions on antennas and propagation*, *60*(7), 3285–3293.

[18] **Gedney, S.D. and Young, J.C.**, (2014). The locally corrected Nyström method for electromagnetics, Computational Electromagnetics, Springer, pp.149–198.

[19] **Selçuk, G.** (2014). Application of nyström method for the solution of time domain electric field integral equation.

[20] **Lean, M.H. and Wexler, A.** (1985). Accurate numerical integration of singular boundary element kernels over boundaries with curvature, *International journal for numerical methods in engineering*, *21*(2), 211–228.

[21] **Canino, L.F.**, **Ottusch, J.J.**, **Stalzer, M.A.**, **Visher, J.L. and Wandzura, S.M.** (1998). Numerical solution of the Helmholtz equation in 2D and 3D using a high-order Nyström discretization, *Journal of computational physics*, *146*(2), 627–663.

[22] **Selcuk, G.**, **Demir, O. and Koc, S.S.** (2015). A novel numerical method for evaluation of hypersingular integrals in electromagnetics, *2015 9th International Conference on Electrical and Electronics Engineering (ELECO)*, IEEE, pp.294–297.

[23] **SHUNG, W.**, **WULEE, S.**, **LAW, C.L.**, **GA, D.** *et al.* (1980). SINGULARITY IN GREEN'S FUNCTION AND ITS NUMERICAL EVALUATION.

APPENDICES

APPENDIX A.1 : Example of Python code for electromagnetic scattering problem

APPENDIX A.1

In this part , we will show parts of Python code of the simulation result. Numpy library ,cmath and math asist us to solve scattering problem.The simulation is the 1. Simulation on the " Simulation Results" chapter. Simulation properties are that scattered PEC object is square with 5cm edge on the z=0 plane, polarization of plane wave is +x direction, propagation direction of plane wave is +z, amplitude of electric field of plane wave is 1 V/m. Frequency is 1 GHz, there are 21 observers , their x and y coordinates are at 1m, their z coordinates are between -1m and 1m with 5 cm difference, mesh number is 25 and all meshes are square, every mesh has 9 quadrature points.Function code of singularity problem for square mesh can be seen below.

```
def calculate_singular_element_weak(x_min,x_max,y_min,y_max,p_x,p_y):
    h_1=x_max-p_x
    h_2=y_max-p_y
    h_3=p_x-x_min
    h_4=p_y-y_min
    angle_h1=0
    angle_h2=np.pi/2
    angle_h3=np.pi
    angle_h4=3*np.pi/2
    angle_a=np.arctan(   (y_min-p_y)/(x_max-p_x)       )
    angle_b=np.arctan(   (y_max-p_y)/(x_max-p_x)       )
    angle_c=math.pi+ np.arctan(   (y_max-p_y)/(x_min-p_x)       )
    angle_d=-math.pi+np.arctan(   (y_min-p_y)/(x_min-p_x)       )
    res1=h_1* (  np.log( abs(   (1/(np.cos( angle_b-angle_h1 )))+ np.arctan ( angle_b-angle_h1 )        )              )-np.log(  abs(   (1/(np.cos( angle_a-angle_h1 )))+ np.arctan ( angle_a-angle_h1 )  ) )         )
    res2=h_2* (  np.log( abs(   (1/(np.cos( angle_c-angle_h2 )))+ np.arctan ( angle_c-angle_h2 )        )              )-np.log(  abs(   (1/(np.cos( angle_b-angle_h2 )))+ np.arctan ( angle_b-angle_h2 )  ) )         )
    res3=h_3* (  np.log( abs(   (1/(np.cos( angle_d-angle_h3 )))+ np.arctan ( angle_d-angle_h3 )        )              )-np.log(  abs(   (1/(np.cos( angle_c-angle_h3 )))+ np.arctan ( angle_c-angle_h3 )  ) )         )
    res4=h_4* (  np.log( abs(   (1/(np.cos( angle_a-angle_h4 )))+ np.arctan ( angle_a-angle_h4 )        )              )-np.log(  abs(   (1/(np.cos( angle_d-angle_h4 )))+ np.arctan ( angle_d-angle_h4 )  ) )         )
    return res1+res2+res3+res4
def calculate_singular_element_hypersingular_xx(x_min,x_max,y_min,y_max,p_x,p_y):
    eps=10**(-3)
    return -(calculate_singular_element_weak(x_min,x_max,y_min,y_max,p_x+eps,p_y)+calculate_singular_element_weak(x_min,x_max,y_min,y_max,p_x-eps,p_y)-2*calculate_singular_element_weak(x_min,x_max,y_min,y_max,p_x,p_y) )/(eps**2)
def calculate_singular_element_hypersingular_yy(x_min,x_max,y_min,y_max,p_x,p_y):
    eps=10**(-3)
    return -(calculate_singular_element_weak(x_min,x_max,y_min,y_max,p_x,p_y+eps)+calculate_singular_element_weak(x_min,x_max,y_min,y_max,p_x,p_y-eps)-2*calculate_singular_element_weak(x_min,x_max,y_min,y_max,p_x,p_y) )/(eps**2)
def calculate_singular_element_hypersingular_xy(x_min,x_max,y_min,y_max,p_x,p_y):
    eps=10**(-3)
    return -(calculate_singular_element_weak(x_min,x_max,y_min,y_max,p_x+eps,p_y-eps)+calculate_singular_element_weak(x_min,x_max,y_min,y_max,p_x-eps,p_y+eps)-calculate_singular_element_weak(x_min,x_max,y_min,y_max,p_x+eps,p_y+eps)\
        -calculate_singular_element_weak(x_min,x_max,y_min,y_max,p_x-eps,p_y-eps))/(4*eps**2)
```

Figure A.1 : Functions to solve singularity.

Function code of non-singular elements are:

```
def distance(x,y,z):
    return np.sqrt(x**2 + y**2 +z**2)

def scalar_green(x,y,z):
    return np.exp(-1j*self.k*distance(x,y,z)) / (4*math.pi*distance(x,y,z))

def g_xx(x,y,z):
    return scalar_green(x,y,z)*( 1 +    ( (distance(x,y,z)**(-4)) *(3*x**2 )/(self.k**2)   ) +   ( (distance(x,y,z)**(-3)) *(1j*3*x**2 )/(self.k) )  +  ( (-(distance(x,y,z)**(-2))  )/(self.k**2) )   +((- distance(x,y,z)**(-2)) *(x**2 )/(1) )     +     \
        +((- distance(x,y,z)**(-1)) *(1j )/(self.k) ) )
def g_xy(x,y,z):
    return scalar_green(x,y,z)*( 0 +    ( (distance(x,y,z)**(-4)) *(3*x*y )/(self.k**2)   ) +   ( (distance(x,y,z)**(-3)) *(1j*3*x*y )/(self.k) )  +  ( x*y*(-(distance(x,y,z)**(-2))  )/(1) )         )
def g_yx(x,y,z):
    return scalar_green(x,y,z)*( 0 +    ( (distance(x,y,z)**(-4)) *(3*x*y )/(self.k**2)   ) +   ( (distance(x,y,z)**(-3)) *(1j*3*x*y )/(self.k) )  +  ( x*y*(-(distance(x,y,z)**(-2))  )/(1) )         )
def g_zx(x,y,z):
    return scalar_green(x,y,z)*( 0 +    ( (distance(x,y,z)**(-4)) *(3*x*z )/(self.k**2)   ) +   ( (distance(x,y,z)**(-3)) *(1j*3*x*z )/(self.k) )  +  ( x*z*(-(distance(x,y,z)**(-2))  )/(1) )         )
def g_zy(x,y,z):
    return scalar_green(x,y,z)*( 0 +    ( (distance(x,y,z)**(-4)) *(3*y*z )/(self.k**2)   ) +   ( (distance(x,y,z)**(-3)) *(1j*3*y*z )/(self.k) )  +  ( y*z*(-(distance(x,y,z)**(-2))  )/(1) )         )
def g_yy(x,y,z):
    return scalar_green(x,y,z)*( 1 +    ( (distance(x,y,z)**(-4)) *(3*y**2 )/(self.k**2)   ) +   ( (distance(x,y,z)**(-3)) *(1j*3*y**2 )/(self.k) )  +  ( (-(distance(x,y,z)**(-2))  )/(self.k**2) )   +((- distance(x,y,z)**(-2)) *(y**2 )/(1) )     +     \
        +((- distance(x,y,z)**(-1)) *(1j )/(self.k) ) )
```

Figure A.2 : Functions for non-singular elements.

Codes of impedance matrix function and taking inverse of impedance matrix(with "np.linalg.inv") are:

```python
g_xx_impedance_matrix[i//p,i%p]=(-1j*2*math.pi*self.options['frequency']*self.options['mu'])*t*self.w_pq[(i%q)%q]*g_xx(x_difference[(i//p)*p + i%p],y_difference[(i//p)*p + i%p],z_difference)
g_xx_impedance_matrix[j//p,j%p]=(-1j*2*math.pi*self.options['frequency']*self.options['mu'])*(1/(4*math.pi))*t*self.w_pq[(j%q)%q]*(-1j*self.k +\
    calculate_singular_element_weak(self.mesh_points[(j%p)//q,0],self.mesh_points[(j%p)//q,1],self.mesh_points[(j%p)//q,2],self.mesh_points[(j%p)//q,3],self.x_pq[(j%p),0],self.y_pq[(j%p),0]) + \
        (1/((self.k)**2))* calculate_singular_element_hypersingular_xx(self.mesh_points[(j%p)//q,0],self.mesh_points[(j%p)//q,1],self.mesh_points[(j%p)//q,2],self.mesh_points[(j%p)//q,3],self.x_pq[j//p,0],self.y_pq[j//p,0])  )

g_xy_impedance_matrix[i//p,i%p]=(-1j*2*math.pi*self.options['frequency']*self.options['mu'])*t*self.w_pq[(i%q)%q]*g_xy(x_difference[(i//p)*p + i%p],y_difference[(i//p)*p + i%p],z_difference)
g_xy_impedance_matrix[j//p,j%p]=(-1j*2*math.pi*self.options['frequency']*self.options['mu'])*(1/(4*math.pi))*t*self.w_pq[(j%q)%q]*(  (1/self.k**2)\
    *  calculate_singular_element_hypersingular_xy(self.mesh_points[(j%p)//q,0],self.mesh_points[(j%p)//q,1],self.mesh_points[(j%p)//q,2],self.mesh_points[(j%p)//q,3],self.x_pq[j//p,0],self.y_pq[j//p,0])  )

g_yx_impedance_matrix[i//p,i%p]=(-1j*2*math.pi*self.options['frequency']*self.options['mu'])*t*self.w_pq[(i%q)%q]*g_xy(x_difference[(i//p)*p + i%p],y_difference[(i//p)*p + i%p],z_difference)
g_yx_impedance_matrix[j//p,j%p]=(-1j*2*math.pi*self.options['frequency']*self.options['mu'])*(1/(4*math.pi))*t*self.w_pq[(j%q)%q]*(  (1/self.k**2)\
    * calculate_singular_element_hypersingular_xy(self.mesh_points[(j%p)//q,0],self.mesh_points[(j%p)//q,1],self.mesh_points[(j%p)//q,2],self.mesh_points[(j%p)//q,3],self.x_pq[j//p,0],self.y_pq[j//p,0])  )

g_yy_impedance_matrix[i//p,i%p]=(-1j*2*math.pi*self.options['frequency']*self.options['mu'])*t*self.w_pq[(i%q)%q]*g_yy(x_difference[(i//p)*p + i%p],y_difference[(i//p)*p + i%p],z_difference)
g_yy_impedance_matrix[j//p,j%p]=(-1j*2*math.pi*self.options['frequency']*self.options['mu'])*(1/(4*math.pi))*self.w_pq[(j%q)%q]*t*(-1j*self.k \
    +calculate_singular_element_weak(self.mesh_points[(j%p)//q,0],self.mesh_points[(j%p)//q,1],self.mesh_points[(j%p)//q,2],self.mesh_points[(j%p)//q,3],self.x_pq[j//p,0],self.y_pq[j//p,0])+\
        (1/self.k**2)*calculate_singular_element_hypersingular_yy(self.mesh_points[(j%p)//q,0],self.mesh_points[(j%p)//q,1],self.mesh_points[(j%p)//q,2],self.mesh_points[(j%p)//q,3],self.x_pq[j//p,0],self.y_pq[j//p,0])  )


Z=np.zeros((2*len(g_xx_impedance_matrix),2*len(g_xx_impedance_matrix)),dtype=complex)

Z[0:len(g_xx_impedance_matrix),0:len(g_xx_impedance_matrix)]=g_xx_impedance_matrix[0:len(g_xx_impedance_matrix),0:len(g_xx_impedance_matrix)]

Z[0:len(g_xx_impedance_matrix),len(g_xx_impedance_matrix):2*len(g_xx_impedance_matrix)]=g_xy_impedance_matrix[0:len(g_xx_impedance_matrix),0:len(g_xx_impedance_matrix)]

Z[len(g_xx_impedance_matrix):2*len(g_xx_impedance_matrix),0:len(g_xx_impedance_matrix)]=g_yx_impedance_matrix[0:len(g_xx_impedance_matrix),0:len(g_xx_impedance_matrix)]

Z[len(g_xx_impedance_matrix):2*len(g_xx_impedance_matrix),len(g_xx_impedance_matrix):2*len(g_xx_impedance_matrix)]=g_yy_impedance_matrix[0:len(g_xx_impedance_matrix),0:len(g_xx_impedance_matrix)]

S=np.linalg.inv(Z)
```

Figure A.3 : Functions for impedance matrix.

In Figure A.3, "i" and "j" indexs shows all quadrature points.$self.w_pq$ shows quadrature weights.In figure A.4 which is below, there are codes of obervation coordinates ("obs"), incident electric field ("V"), non-singular impedance matrix ("g") , surface current density (J_x,J_y) and vector components of scattering electric field (Ex,Ey,Ez).

```python
obs_x=1
obs_y=1
source_z=0
V=np.zeros((2*len(self.vx),1),dtype=complex)
V[0:len(self.vx)]=1#V_x 'ler
V[len(self.vx):2*len(self.vx),0]=0#V_y'ler

J=np.dot(S,V)




np.savetxt('j.txt',J)
n=21
obs=np.zeros((n,3))

obs[:,2]=np.linspace(-1,1,n)

i=np.arange(len(self.x_pq)*len(obs))
i=np.asarray(i).ravel()

print(i//len(self.x_pq))
gxx=np.zeros((    len(obs),len(self.x_pq)   )  ,dtype=complex)
gxy=np.zeros((    len(obs),len(self.x_pq)   )  ,dtype=complex)
gyy=np.zeros((    len(obs),len(self.x_pq)   )  ,dtype=complex)
gzy=np.zeros((    len(obs),len(self.x_pq)   )  ,dtype=complex)
gzx=np.zeros((    len(obs),len(self.x_pq)   )  ,dtype=complex)
gxx[i//len(self.x_pq),i%len(self.x_pq)]=g_xx( obs_x-self.x_pq[i%len(self.x_pq),0],obs_y-self.y_pq[i%len(self.x_pq),0], obs[i//len(self.x_pq),2]  -source_z )*self.w_pq[(i%len(self.x_pq))   % q]
gxy[i//len(self.x_pq),i%len(self.x_pq)]=g_xy( obs_x-self.x_pq[i%len(self.x_pq),0],obs_y-self.y_pq[i%len(self.x_pq),0], obs[i//len(self.x_pq),2]  -source_z )*self.w_pq[(i%len(self.x_pq))   % q]
gyy[i//len(self.x_pq),i%len(self.x_pq)]=g_yy( obs_x-self.x_pq[i%len(self.x_pq),0],obs_y-self.y_pq[i%len(self.x_pq),0], obs[i//len(self.x_pq),2]  -source_z )*self.w_pq[(i%len(self.x_pq))   % q]
gzy[i//len(self.x_pq),i%len(self.x_pq)]=g_zy( obs_x-self.x_pq[i%len(self.x_pq),0],obs_y-self.y_pq[i%len(self.x_pq),0], obs[i//len(self.x_pq),2]  -source_z )*self.w_pq[(i%len(self.x_pq))   % q]
gzx[i//len(self.x_pq),i%len(self.x_pq)]=g_zx( obs_x-self.x_pq[i%len(self.x_pq),0],obs_y-self.y_pq[i%len(self.x_pq),0], obs[i//len(self.x_pq),2]  -source_z )*self.w_pq[(i%len(self.x_pq))   % q]

Jx=J[0:len(self.x_pq)]
Jy=J[len(self.x_pq):2*len(self.x_pq)]
Ex=(t)*(4*math.pi)*(-1j*2*math.pi*self.options['frequency']*self.options['mu'])*(np.dot(gxx,Jx) + np.dot(gxy,Jy))
Ey=(t)*(4*math.pi)*(-1j*2*math.pi*self.options['frequency']*self.options['mu'])*(np.dot(gxy,Jx) + np.dot(gyy,Jy))
Ez=(t)*(4*math.pi)*(-1j*2*math.pi*self.options['frequency']*self.options['mu'])*(np.dot(gzx,Jx) + np.dot(gzy,Jy))
```

Figure A.4 : Scattering electric field code.

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