LOSS AVERSION AND REFUND POLICIES

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Loss Aversion and Refund Policies

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DECLARATION OF ORIGINALITY

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- this thesis contains no material that has been submitted or accepted for a degree or diploma in any other educational institution;
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ABSTRACT

Loss Aversion and Refund Policies

This thesis examines the relationship between loss aversion and refund policy decisions of firms. We consider a two-stage duopoly model with horizontal product differentiation, with and without loss aversion. Firms decide and announce their refund policies simultaneously in the first stage, and then in the second stage they compete in prices in a Hotelling type location model. Discretizing the refund policies as full refund and no refund policies, we analyze the equilibrium refund policies and the prices. On the buyers' side, they have incomplete information about their true valuation (location) of the product until they buy it, when they realize their true valuation (location). Buyers can return the product and buy from the other firm if the valuation mismatch is sufficiently high and there is an available refund option. We find that there is a price equilibrium for each possible refund policy combination in the absence of loss aversion. In the presence of loss aversion, we find that the set of equilibria can change with the degree of loss aversion, λ. Even though there is a price equilibrium for each possible refund policy combination with small λ, both firms adopting a no refund policy cannot be part of an equilibrium if λ is sufficiently high.

ÖZET

Kayıptan Kaçınma ve İade Politikaları

Bu tezde, kayıptan kaçınma ve firmaların iade politikası kararları arasındaki ilişkiyi inceliyoruz. Kayıptan kaçınmalı ve kaçınmasız modellerimizi kurarken yatay ürün farklılaşmasına sahip iki adımlı bir duopol modeli kullanıyoruz. İlk adımda firmalar iade politikalarına karar verirler ve aynı anda açıklarlar, ikinci adımda ise Hotelling benzeri bir lokasyon modelinde fiyatlar üzerinden rekabet ederler. İade politikalarını "tam iade" ve "hiç iade" olarak ayrıştırarak dengedeki fiyatları ve iade politikalarını analiz ediyoruz. Alıcılar ise ürünleri satın alana kadar ürünlerin kendileri içn gerçek değerleri hakkında eksik bilgiye sahiptirler ve ancak satın aldıklarında gerçek değerini öğrenirler. Bir alıcı için eğer ürünün gerçek değeri ile alıcının beklediği değer arasındaki fark çok ise ve firmalar tam iade hakkı tanıdılarsa, alıcı ürünü iade edip diğer firmadan satın alır. Kayıptan kaçınmanın bulunmadığı durumlarda her olası iade kombinasyonu için bir fiyat dengesi olduğunu, kayıptan kaçınmanın bulunduğu ortamlarda ise denge kümesinin kayıptan kaçınma katsayısına bağlı olduğunu buluyoruz. Kayıptan kaçınma katsayısı küçük iken her olası iade kombinasyonunda bir fiyat dengesi olmasına rağmen eğer kayıptan kaçınma katsayısı yeterince yüksek ise her iki firmanın da "hiç iade" politikasını seçtiği durum bir denge oluşturmamaktadır.

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TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

In markets with a large variety of substitutes, for buyers, the decision about which firm to buy from gets harder and harder. The reason for such buying decisions to become harder may be due to not only the increasing number of substitutes but also the fact that buyers are not fully informed about their exact tastes about the product characteristics they intend to buy. If a buyer is facing some uncertainty regarding her own tastes and preferences, then a purchase may result in a mismatch between the buyer’s ideal product and the actual product she buys. That is, a mismatch between the buyer’s expected valuation of the product she intends to buy and her actual valuation of the product, which she may learn after the purchase. Such a mismatch possibility may decrease the willingness to pay for buyers, especially when trying the product before the purchase is not possible. To avoid this problem, stores may offer after-sale services such as money-back guarantees, product exchanges, in-store gift cards, etc. In the period of the Covid pandemic, people tend to buy their needs online instead of visiting a store, and many online stores offer full refund due to the impossibility of trying the product before purchase. However, offering a full refund have some drawbacks for the seller. Returned products generally are sold at a lower price than the market price. Shipping costs and storing the refunded product are also other side effects of offering a refund. Hence, some sellers still tend not to use any refund policy. Therefore, in the context of uncertainty that buyers’ may have regarding their own valuations of a product, refund policies become highly relevant, both from sellers’ and buyers’ perspectives.

When a buyer faces some uncertainty regarding her own valuation for a product, there is also a highly relevant behavioral aspect, *loss aversion.* A buyer who does not know her exact valuation but has some idea in the form of some distribution over some range of possible values may form an expectation-based reference point prior to the purchase. Then, after the purchase, once the buyer learns her true valuation, she may compare it to her reference point, in which case, she may experience a loss or a gain, depending on whether the true valuation is larger than the reference point or not. When the true valuation turns out to be smaller than the valuation in the reference point, then the buyer feels an extra disutility. If the true valuation is larger than the reference point valuation, then there is an extra utility. However, a loss-averse buyer feels larger disutility from a loss than the utility she feels from the same sized gain. Therefore, in the context of oligopolistic competition with product differentiation, the sellers would have to optimize their refund policies considering the loss aversion of the buyers as well as the competition dynamics with the other firm. In this thesis, we address the question of how loss aversion of the buyers affects the oligopolistic competition and the equilibrium refund policies.

In order to see the effects of loss aversion on the equilibrium decisions of firms' refund policies, we analyze a dynamic duopoly model where two firms compete in prices in a horizontal product differentiation environment, which is modeled via a Hotelling type location model. Before a buyer decides which firm to buy from, she has incomplete information regarding her true valuation, which is her true location in the context of a location model. However, she knows that her true location is drawn from a subinterval, where the probability distribution of it is known to the buyer. Once she makes a purchase, she learns her true location, and then she decides whether to keep it or return it to the seller she bought from if she returns it, then she buys another unit from the other seller. If the seller she bought in the first place has a full refund policy, then she receives the entire price she initially paid. If there is no refund policy, she does not receive any money back. After the keep or return decisions are made and carried out, the profits of the firms and the payoffs of the buyers are realized. Assuming that buyers are loss averse á la Kahneman & Tversky (1979), and focusing on discrete refund policy decisions (either full refund or no refund), we characterize the equilibrium prices and refund policies.[1] We show that in an environment where buyers are loss neutral, all possible refund policy configurations constitute an equilibrium. However, when there is loss aversion, and it is strong enough, there is no equilibrium where both firms choose to announce no refund policy. Thus, under loss aversion, at least one firm offers a full refund policy, while without loss aversion, both firms offering no refund is an equilibrium.

In order to model product differentiation, we use Hotelling's linear city model.[2] To be more specific, there are two firms, which are located at the endpoints of a unit-length linear city from 0 to 1, depicted as $[0,1]$ interval. Buyers' true locations are in this $[0,1]$ interval. Although the products are the same, there is product differentiation which stems from the distance between a buyer's true location and the firm's location. Location can be interpreted in two ways:
(i) physical location of buyers and firms: a buyer who is at some distance from a firm has to incur a transportation cost, $t > 0$, per unit distance; (ii) product characteristics: if we take the $[0,1]$ interval as the characteristics space, then the location of a buyer represents the characteristics of her ideal product, and the distance between her true location (her ideal product) and the firm's location (characteristics of the firm's product) gives her disutility, which can also be captured by transportation cost per unit distance.

For every buyer, the valuation of the product is v, and it is sufficiently high so that each buyer buys exactly one unit from either one of two firms. If a buyer buys one unit from a firm and then decides to return it, that means she buys another unit from the other firm after returning the initial unit bought. Thus, there is full coverage of the market. Each buyer has incomplete information about her true location, and it is unknown until a purchase is made. However, each buyer has

[1] Kahneman & Tversky (1979) show that same-sized losses have a more significant impact on the value function than same-sized gains, hence loss-aversion. Also, see Kőszegi & Rabin (2006) for a general reference-dependent preferences model, where they show that gain-loss utility emerges when there is uncertainty and willingness to pay for a product increases as the expected probability of the purchase happening increases.

[2] See Hotelling (1929) for the canonical location model. For a circular city model, see Salop (1979)

a reference location, x_r, which is uniformly distributed over $[0,1]$ interval. A buyer who has a reference location knows that her true location is drawn from a uniform distribution over the range $[x_r - k, x_r + k]$. Here, k represents the accuracy or precision of the buyer's information about her true location. When $k = 0$, the buyer has complete information, and her true location is equal to the reference location; thus, she knows her true location. As it increases, ambiguity about her true location and the possibility of a mismatch with a firm increases. These parameters are common for both of our models, the benchmark model (without loss aversion) and the model with loss aversion. We model the price competition and the refund policy choices through a two-stage game played between the firms. In the first stage, both firms announce their refund policies that will apply to every unit they sell. Then, in the second stage, firms, after observing each other's refund policy, compete in prices in a simultaneous fashion. Then, the buyers, after observing both refund policies and the prices, decide which firm to buy from. After the purchase, buyers realize their true location, and depending on it, they keep the product or return it. Once a buyer returns a product, then she buys from the other firm.

In the model with loss aversion, for which we adopt the preferences with loss aversion modeled as in Kahneman & Tversky (1979), there is an additional gain/loss utility in addition to the intrinsic utility, and it is generated by comparing the location in the reference point and the true location. If a buyer buys from a firm located at y, then, once the true location is realized after the purchase, the buyer experiences a gain utility by an amount of location mismatch if actual location is closer to y than her reference location. The buyer, however, experiences a loss disutility by an amount λ times location mismatch, if reference location is closer to y than her actual location. Here, $\lambda > 1$ is the parameter that captures loss aversion, and a higher loss aversion coefficient implies stronger loss-aversion.

In both models, one without loss aversion and the other with loss aversion, we characterize the equilibrium prices and refund policies, assuming refund policies are binary: either no refund or full refund. Using symmetry, there are three possible refund policy combinations that can occur in the equilibrium: (i) both firms offering full-refund, (ii) both firms offering no-refund, and (iii) one firm offering full-refund and the other no-refund. We first fix a refund policy combination, and then analyze the price competition, and see if there is a pure strategy price equilibrium. To do this, we break the $[0,1]$ interval in terms of reference locations into several regions: a buyer, after buying from a firm, may keep the product for any true location realizations of hers, may return it for any true location realizations of hers, or may keep or return depending on her true location realization. These regions define a number of thresholds, and we analyze the game in the light of these thresholds. We find that when there is no loss aversion, there is an symmetric equilibrium for all three refund policy combinations, and each of the corresponding refund policy combinations constitutes an equilibrium. However, when we introduce loss aversion into the model, we find that it constitutes a symmetric price equilibrium for only the cases where both firms announce full-refund or only one firm announces full-refund. When both firms announce no-refund and there is loss aversion, symmetric price equilibrium exists only if the loss aversion coefficient is sufficiently low enough. Therefore, we find that in a world with strong enough loss aversion, adopting no refund policy by both firms cannot be equilibrium, in which case at least one of the firms offers full-refund in the equilibrium.

CHAPTER 2

LITERATURE REVIEW

The part of the literature that is relevant to our study consists of two strands and their intersection: the studies that deal with the competition when refund policies can be adopted, and the ones that focus on loss-averse buyers and the effects of loss-aversion on the firms' decisions. We briefly go over important and relevant papers in the literature.

The refund policy we consider in our model is a kind of repurchase agreement, where the seller commits to repurchase the product from the buyer at some rate of the original price. Che (1996) studies the refund policies in a monopoly setting, where buyers are risk-averse, and they learn their valuation after they make a purchase and experience the good. If the buyers are risk-averse enough, then it is optimal for the monopoly to adopt the return policy. The main tradeoff for buyers is that while the return policy can protect a buyer from ex-post loss, it makes the monopolist charge higher prices than she should.

There are also best price clauses that are studied in the literature. A "most favored customer"(MFC) clause defines a rebate on the original price for a customer if the seller sells her product at a lower price to other customers. A "meet or release"(MOR) clause defines the rebate on the original price for a customer if another seller sells her product at a lower price than the original price buyer paid. Schnitzer (1994) examines these two best price clauses in the context of competition and collusion. While MFC makes decreasing the price less appealing, MOR can counter price competition for the customers who want to buy their product as soon as possible. Butz (1990) studies these best price clauses and shows that a durable good monopolist can achieve the profit from rental agreements by using these best price clauses. Dana (1998) examines the advance-purchase discounts, which is a type of second-degree price discrimination. When there is uncertainty in buyers' demand and inventory cost is high, those buyers with low uncertainty in their demand and low valuations for the product tend to buy in advance, which is driven by the existence of buyers with high valuations and high uncertainty of demand. This result matches with the second-degree price discriminating monopoly even though there are many competitive firms.[1] Also, adopting a refund policy can increase selling in advance and decreases customers' risk. Xu et al. (2015) consider a model where buyers' valuations are realized only after the purchase and are also correlated with the return deadline. They show that the optimal refund amount is the salvage value of the product if the valuation of the product decreases over time and the equilibrium price under the refund policy is higher than the equilibrium price under the no refund policy. Also, a refund policy with a deadline is always better than no refund policy. Nasiry & Popescu (2012) show that when there is regret from the purchase because the valuation turned out to be lower than the price, the profits of the firms decrease. If the regret stems from a missed discount or the product being out of stock, then the profits may increase. Refund policies may, however, decrease the adverse effects on the profits of the firms. An experimental study, Wang (2009) shows that while full refund policies increase the likelihood of selling the product, the endowment effect of owning a product has a role in decreasing the likelihood of returning the product. Hence, offering a full refund policy increases net sales of a firm even though returning a product is costless for customers. Through a model of disappointment within the reference-dependent utility framework, Heiman et al. (2015) analyze the consumers' large premiums for return policies (money-back guarantees or MBG) and show how these premiums depend on return costs and the terms in the MBG contract. Using survey data, they show that an increase in the duration of the return period increases consumers' willingness to pay, and cash refunds are more preferred over in-store credits.

When buyers are loss-averse, there is a series of studies that analyze the effects of loss aversion on the competition when sellers may use certain selling strategies. Our study is closely related to this strand of the literature. Karle & Peitz

[1] For monopolist's optimal price discrimination when buyers are loss averse, see Carbajal & Ely (2016) and Hahn et al. (2018).

(2014) consider a duopolistic circular city model where some portion of the buyers are informed about their tastes while others are not and analyze the effect of loss-aversion on the market outcome. There is both price and match-value dimensions in their model, and they show that loss aversion is pro-competitive in the price dimension, and it is anti-competitive in the match-value dimension. In the asymmetric duopoly, the pro-competitive effect dominates the anti-competitive effect, and loss aversion has an overall pro-competitive effect. While they consider both informed and informed buyers, they focus on price competition without any refund policy possibility. We, however, consider refund policies as part of the sellers' strategies, while we assume all buyers are uninformed. Karle & Möller (2020) considers early and late purchase options for the loss-averse buyers, where the tradeoff is that a buyer can buy at a lower price early; however, a late purchase induces a better match value. They show that loss-aversion has an anti-competitive effect on prices and a pro-competitive effect on discounts. The main difference between their study and ours is that they analyze the effects of advance purchases comparing monopoly and duopoly, with and without loss aversion, but we analyze the market equilibrium when sellers may adopt refund policies, with loss aversion and without. Heidhues & Kőszegi (2008), in a circular city model where buyers are loss averse in both prices and match values, and the reference points are expectation based, show that a focal price equilibrium exists even if the firms have different cost distributions. Although sellers, who are competing in prices when buyers are loss averse, choose deterministic prices, Heidhues & Kőszegi (2014), show that monopoly may optimally introduce risk through a low and variable sales price and a high regular price. Picking a sale price from a distribution lower than the product value, a monopolist can induce the buyers to assign a positive probability of buying. Then, buyers may buy at a high regular fixed price, trying to avoid the possible loss from not buying. A similar intuition also holds when a seller sells two substitute goods; one good is used to create higher reference points through a low bargain price, and then charging a high price on the other good, which is purchased to avoid disappointment (Rosato (2016)). Piccolo & Pignataro (2018) analyze the effect of loss aversion on product experimentation and tacit collusion. They find that forbidding experimentation is better for firms' joint profit when the buyers are not too loss averse. When buyers are sufficiently loss averse, legalizing the experimentation is better for firms' joint profit. They also find that consumer surplus is maximized by banning experimentation in a static environment, and consumer surplus is maximized by allowing experimentation in a dynamic environment. Karle & Schumacher (2017) study the effect of advertising on monopoly's profit when buyers are loss-averse. They introduce the advertisements as a shift in the reference location of the buyers and show that monopolist chooses to provide incomplete information through the advertisements. If the monopolist has to choose between full information ads and no information ads, then choosing no information ads is better for the seller.

In a closely related study, Zhou (2011) analyzes the effects of reference dependence in a duopolistic price-competition model with horizontal product differentiation. Buyers lack information regarding the location of sellers or their own location, and they see the two products in a sequence, and a buyer takes the first product she sees as her reference point. They show that while the dominant firm chooses to randomize between high and low prices, the other firm chooses a constant medium price. Also, loss aversion in the price dimension increases competition, and loss aversion in the product dimension decreases competition. The main difference between this study and ours is that not only we introduce return policies, but also we consider each buyer having her own reference location drawn from a distribution. However, their loss aversion applies to only those buyers who buy from the second buyer they see. In our setting, loss aversion applies to any buyer who compares her reference location with her actual location, which is realized after the initial purchase.

CHAPTER 3

MODEL SETTING

We consider a two-stage duopoly model, where in the first stage, the firms announce their refund rates, and then in the second stage, refund rates being common knowledge, firms compete in prices in a horizontal product differentiation environment. The two firms, firm A and firm B, are located at the endpoints of a unit length linear city, represented by $[0,1]$ interval: Firm A is located at 0 and Firm B is located at 1. We denote firm i's location with y_i, for $i = A, B$. That is, $y_A = 0$ and $y_B = 1$. The firms produce and sell homogeneous products; however, the source of the product differentiation will be the transportation cost per distance each buyer needs to travel to the location of the firm from which the purchase is made. The valuation of any product is v for each buyer, and it is sufficiently high that all buyers buy one unit from one of the two firms. The transportation cost per distance is given by t.

Each buyer has incomplete information regarding their true location in the linear city $[0,1]$ in the following sense: Each buyer has a *reference location*, x_r, which is uniformly distributed over the interval $[0,1]$. A buyer with a reference point x_r does not know her true location x but knows that it is drawn from a uniform distribution over the range $[x_r - k, x_r + k]$, where $k > 0$. One interpretation for this modeling choice is that the buyer believes that her true location lies in the interval $[x_r - k, x_r + k]$, where the expected location is x_r (under the uniform distribution), which serves as her reference location. Note that $k > 0$ represents the precision of the buyer's information about her true location. When k is larger, the buyer's information about her true location is less precise. If $k = 0$, buyer's reference location is the true location, and she actually knows her true location. Since we assume $k > 0$, no buyer has complete information regarding their own true location before any purchase is made. Buyers learn their true locations only after they make a purchase from one of the firms.

We assume that the buyers with reference locations $x_r \in [0, k]$ and $x_r \in [1-k, 1]$ are loyal to the firm that is closest and always buy from that firm and never return, that is, all buyers with reference locations with $x_r \in [0, k]$ buy from Firm A, and all buyers with reference locations with $x_r \in [1-k, 1]$ buy from Firm B. We also assume that these buyers never return the product. Thus, we can ignore their effects on the equilibrium objects, as they will only alter the level of equilibrium profits, but not the equilibrium strategies.

Let the refund rates in the first stage be denoted by $R_i \in [0, 1]$, for $i = A, B$. A refund rate R_i indicates that in the case of a return of the product, firm i is obliged to pay back the buyer $(1 - R_i)P_i$. Thus, if $R_i = 0$, then there is full refund, and in case of a return of the product, the entire price P_i is paid back to the buyer. If $R_i = 1$, there is no refund at all. The prices in the second stage are denoted by P_i, for $i = A, B$. A buyer with a true location x, receives an ex-post utility of $v - tx - P_A$ if she buys from firm A at a price P_A, receives a utility of $v - t(1-x) - P_B$ if she buys from firm B at a price P_B. The terms tx and $t(1-x)$ represent the transportation costs this buyer needs to incur if she buys from firm A and firm B, respectively, and keeps it. For simplicity, we assumed that each product has zero marginal cost and zero salvage value.

Both firms announce their refund rates, R_i, simultaneously. These refund rates are observed publicly. Then, they announce their prices, P_i, simultaneously, and prices are publicly observed. Then, each buyer makes a decision regarding which firm to buy from, based on refund rates R_i, prices P_i, reference location x_r, transportation cost parameter t and *loss aversion* parameter λ. After the purchase, each buyer's true location x is realized. Once a buyer learns their own true location, then she can either keep the product or return it. If she returns it, then she receives the refund, if there is any, and she buys from the other firm. Thus, the timing of events is as follows:

Stage 1: Firms simultaneously announce their refund rates, R_A, and R_B, and these rates become common knowledge.
Stage 2: Firms simultaneously announce their prices, P_A and P_B, and these rates become common knowledge.
Stage 2.1: Each buyer decides which firm to buy from based on refund rates, prices and her reference location, x_r.
Stage 2.2: Each buyer learns her true location, x, drawn from $U[x_r - k, x_r + k]$, and then decides whether to keep it or return it.
Stage 2.3: Those buyers who return the product to firm i receive a refund $(1-R_i)P_i$, and then they buy from firm $-i$. All the payoffs and the profits are realized.

3.1 Loss Aversion

The overall utility of a loss averse buyer has two components. The first component is an intrinsic utility she receives from buying and consuming one unit of the product net of the price and transportation cost she pays, that is, $v - t|x - y_i| - P_i$ when she buys from firm i. The second component is a potential loss/gain utility based on the difference between her reference point utility and her actual intrinsic utility. We do not impose any kind of reference point in the price dimension, simply because the prices are announced before the buyers make any purchase. The reference point consists only of the reference location.

Suppose for a buyer with a reference location, x_r, who buys from firm i at price P_i, the true location is x, which is realized after the purchase. If x is closer to y_i than x_r is, then the buyer experiences a gain utility by an amount $|x_r - x|$. If x_r is closer to y_i than x is, then the buyer experiences a loss utility by an amount $\lambda|x_r - x|$, where $\lambda > 1$ is the parameter that captures loss aversion and known by both firms and buyers.

The utility of a loss-averse buyer with a reference location x_r, who buys from firm i, $i = A, B$, keeps the product, and realizes a true location x, is given as follows.

$$u(x_r,x,A)=\begin{cases} v-P_A-tx_r+t|x_r-x| & \text{if } x<x_r \\ v-P_A-tx_r-\lambda t|x_r-x| & \text{if } x>x_r \end{cases}$$

$$u(x_r,x,B)=\begin{cases} v-P_B-t(1-x_r)-\lambda t|x_r-x| & \text{if } x<x_r \\ v-P_B-t(1-x_r)+t|x_r-x| & \text{if } x>x_r \end{cases}$$

The utility of a loss averse buyer with a reference location x, who buys from firm i, $i=A,B$ and realizes a true location x_j is given as follows.

$$U(x_r,x,A)=\begin{cases} u(x_r,x,A) & \text{if } \textit{Keep} \\ u(x_r,x,B)-R_AP_A & \text{if } \textit{Return} \end{cases}$$

$$U(x_r,x,B)=\begin{cases} u(x_r,x,B) & \text{if } \textit{Keep} \\ u(x_r,x,A)-R_BP_B & \text{if } \textit{Return} \end{cases}$$

The expected utility of a loss averse buyer with a reference location x_r, who buys from firm i, $i=A,B$ before realizing her true location is given as follows.

$$EU(x_r,A)=\int_{x_r-k}^{x_r+k}\Big[Pr(Keep|x)u(x_r,x,A)+Pr(Return|x)[u(x_r,x,B)-R_AP_A]\Big]\frac{1}{2k}dx$$

$$EU(x_r,B)=\int_{x_r-k}^{x_r+k}\Big[Pr(Keep|x)u(x_r,x,B)+Pr(Return|x)[u(x_r,x,A)-R_BP_B]\Big]\frac{1}{2k}dx$$

CHAPTER 4

ANALYSIS

We first analyze a benchmark model. Since our focus is on the effect of loss aversion on the equilibrium prices and refund rates, we keep the information structure the same with our benchmark model. Thus, each buyer has a belief about her true location given by $U[x_r - k, x_r + k]$, where x_r is uniformly distributed over $[0, 1]$. However, there is no gain or loss utility based on the differences between the true/realized location and the expected location x.[1]

4.1 Model Analysis without Loss Aversion

As described in the model, each buyer has a belief about their true location, x, that the true location is drawn from a uniform distribution over $[x - k, x + k]$. The true location is drawn from this interval, and the buyer learns it, only after the buyer makes a purchase. Every buyer is associated with a single x value, where x is uniformly distributed over $[0, 1]$. The buyers with beliefs $x \in [0, k]$ and $x \in [1 - k, 1]$ are loyal to the firm that is closest and always buy from that firm and never return. Thus, we ignore their effects on the equilibrium objects, as they will only alter the level of equilibrium profits but not the equilibrium strategies. A representative buyer is shown in Figure 1.

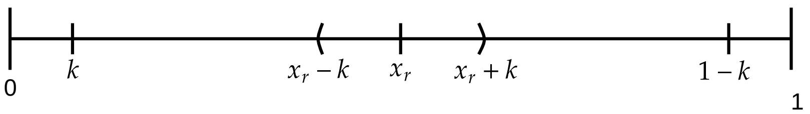

Figure 1. Representation of a buyer in the market

Once the refund rates and the prices are announced, each buyer has two options; Either buy from Firm A or Firm B. Once a buyer makes a purchase, she

[1]Alternatively, another benchmark model would have complete information for the buyers. When we introduce loss aversion, the overall effect on the equilibrium prices and refund rates would be a combination of both loss aversion and incomplete information. In that case, it would be hard to disentangle the effect of loss aversion and the effect of incomplete information.

learns her exact/true location. After the purchase and learning their own true location, buyers have two options again: They can either keep the product or return the product and buy from the other firm. When a buyer returns the product to Firm i, then Firm i refunds the buyer an amount $(1-R_i)P_i$.

Suppose that a buyer buys the product from firm A. After realizing her true location x, her utility is equal to $v - P_A - tx$ if she keeps the product. Note that this utility expression is decreasing in x. Her utility is equal to $v - P_B - R_A P_A - t(1-x)$ if she returns the product and buys from firm B. This utility expression is increasing in x. Thus, the location that equates these two utility expressions is the threshold location, $\hat{x}^A$, such that for all $x < \hat{x}^A$, the buyer keeps the product, and for all $x > \hat{x}^A$, she returns it and buys from firm B. We have $\hat{x}^A$ given by

$$\hat{x}^A = \frac{P_B - P_A + R_A P_A + t}{2t} \tag{4.1}$$

Note that if $\hat{x}^A > x_r + k$, then the buyer who buys from firm A, keeps the product for all possible true location values, $x \in [x_r - k, x_r + k]$. If $\hat{x}^A < x_r - k$, then the buyer who buys from firm A, returns the product for all possible true location values, $x \in [x_r - k, x_r + k]$. However, if $x_r - k \le \hat{x}^A \le x_r + k$, then the buyer who buys from firm A may keep or return depending on her true location. We define the threshold locations incorporating the above observation.

$$\hat{x}^{AK} = \hat{x}^A - k = \frac{P_B - P_A + P_A R_A + t - 2kt}{2t} \tag{4.2}$$

$$\hat{x}^{AR} = \hat{x}^A + k = \frac{P_B - P_A + P_A R_A + t + 2kt}{2t} \tag{4.3}$$

Thus, if $\hat{x}^{AK} > x_r$, then the buyer who buys from firm A, keeps the product for all possible true location values, $x \in [x_r - k, x_r + k]$. If $\hat{x}^{AR} < x_r$, then the buyer who buys from firm A, returns the product for all possible true location values, $x \in [x_r - k, x_r + k]$. If $\hat{x}^{AK} \le x_r \le \hat{x}^{AR}$, then the buyer who buys from firm A may keep or return depending on her true location. A buyer who buys from firm A is shown in Figure 2.

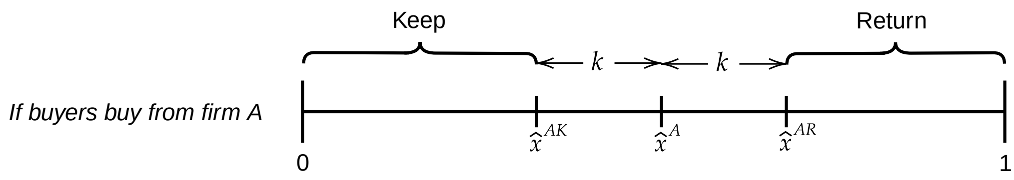

Figure 2. Buyers' decision based on their location for firm A

Suppose that a buyer buys the product from firm B instead of firm A. After realizing her true location x, her utility is equal to $v - P_B - t(1-x)$ if she keeps the product. Note that this utility expression is increasing in x. Her utility is equal to $v - P_A - R_B P_B - tx$ if she returns the product and buys from firm B. This utility expression is decreasing in x. Thus, the location that equates these two utility expressions is the threshold location, $\hat{x}^B$, such that for all $x < \hat{x}^B$, the buyer return the product, and for all $x > \hat{x}^B$, she keeps it and buys from firm A. We have $\hat{x}^B$ given by

$$\hat{x}^B = \frac{P_B - P_A - R_B P_B + t}{2t} \tag{4.4}$$

Note that if $\hat{x}^B < x_r - k$, then the buyer who buys from firm B, keeps the product for all possible true location values, $x \in [x_r - k, x_r + k]$. If $\hat{x}^B > x_r + k$, then the buyer who buys from firm B, returns the product for all possible true location values, $x \in [x_r - k, x_r + k]$. However, if $x_r - k \leq \hat{x}^B \leq x_r + k$, then the buyer who buys from firm B may keep or return depending on her true location. We define the threshold locations incorporating the above observation.

$$\hat{x}^{BK} = \hat{x}^B + k = \frac{P_B - P_A - R_B P_B + t + 2kt}{2t} \tag{4.5}$$

$$\hat{x}^{BR} = \hat{x}^B - k = \frac{P_B - P_A - R_B P_B + t - 2kt}{2t} \tag{4.6}$$

Thus, if $\hat{x}^{BK} > x_r$, then the buyer who buys from firm B, returns the product for all possible true location values, $x \in [x_r - k, x_r + k]$. If $\hat{x}^{BR} < x_r$, then the buyer who buys from firm B, keeps the product for all possible true location values,

$x \in [x_r - k, x_r + k]$. If $\hat{x}^{BR} \leq x_r \leq \hat{x}^{BK}$, then the buyer who buys from firm B may keep or return depending on her true location. A buyer who buys from firm B is shown in Figure 3.

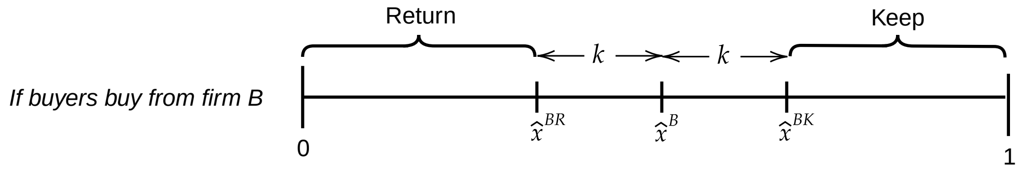

Figure 3. Buyers' decision based on their location for firm B

The expected utility of a buyer with a reference location x_r, who buys from firm i, $i = A, B$ can be described as following piecewise function before she realizes her true location x:

$$EU(x_r, A) = \begin{cases} v - P_A - t\int_{x_r-k}^{x_r+k} \frac{x}{2k} dx & \text{if } x_r \leq \hat{x}^{AK} \\ \frac{\hat{x}^A - x_r + k}{2k}[v - P_A - t\int_{x_r-k}^{\hat{x}^A} \frac{x}{\hat{x}^A - x_r + k} dx] & \text{if } \hat{x}^{AK} \leq x_r \leq \hat{x}^{AR} \\ \quad + \frac{x_r + k - \hat{x}^A}{2k}[v - P_B - R_A P_A - t\int_{\hat{x}^A}^{x_r+k} \frac{1-x}{x_r + k - \hat{x}^A} dx] & \\ v - P_B - R_A P_A - t\int_{x_r-k}^{x_r+k} \frac{1-x}{2k} dx & \text{if } x_r \geq \hat{x}^{AR} \end{cases}$$

$$EU(x_r, B) = \begin{cases} v - P_B - t\int_{x_r-k}^{x_r+k} \frac{1-x}{2k} dx & \text{if } x_r \geq \hat{x}^{BK} \\ \frac{x_r + k - \hat{x}^B}{2k}[v - P_B - t\int_{\hat{x}^B}^{x_r+k} \frac{1-x}{x_r + k - \hat{x}^B} dx] & \text{if } \hat{x}^{BR} \leq x_r \leq \hat{x}^{BK} \\ \quad + \frac{\hat{x}^B - x_r + k}{2k}[v - P_A - R_B P_B - t\int_{x_r-k}^{\hat{x}^B} \frac{x}{\hat{x}^B - x_r + k} dx] & \\ v - P_A - R_B P_B - t\int_{x_r-k}^{x_r+k} \frac{x}{2k} dx & \text{if } x_r \leq \hat{x}^{BR} \end{cases}$$

Both $EU(x_r, A)$ and $EU(x_r, B)$ are continuous utility functions on every point $x_r \in [0, 1]$.

We divide the entire market of $[0, 1]$ interval into five regions, defined by the four threshold values we find above. In each region, buyers know whether to keep or return the product depending on which firm they bought from. Note that these thresholds depend on the refund rates of the firms. We have the following implications in terms of the ordering of some of these thresholds:

$$
\begin{aligned}
&\hat{x}^{BK} - \hat{x}^{BR} = 2k \Rightarrow \hat{x}^{BK} > \hat{x}^{BR} \\
&\hat{x}^{AR} - \hat{x}^{AK} = 2k \Rightarrow \hat{x}^{AR} > \hat{x}^{AK} \\
&\hat{x}^{AK} - \hat{x}^{BR} = \frac{R_A P_A + R_B P_B}{2t} \Rightarrow \hat{x}^{AK} \geq \hat{x}^{BR} \\
&\hat{x}^{AR} - \hat{x}^{BK} = \frac{R_A P_A + R_B P_B}{2t} \Rightarrow \hat{x}^{AR} \geq \hat{x}^{BK}
\end{aligned} \tag{4.7}
$$

Above inequalities and threshold orders are valid under every possible refund rate pair, however, the order between $\hat{x}^{AK}$ and $\hat{x}^{BK}$ is ambiguous. This is because $\hat{x}^{BK} - \hat{x}^{AK} = \frac{4kt-(R_A P_A + R_B P_B)}{2t}$ and it can be negative or positive. Moreover, when $R_A = R_B = 0$, we face the equalities $\hat{x}^{AK} = \hat{x}^{BR}$ and $\hat{x}^{AR} = \hat{x}^{BK}$. Thus, there are three possible market outcomes with respect to equilibrium prices and refund rates: If either $R_A \neq 0$ or $R_B \neq 0$, then we have two subcases: $\hat{x}^{BK} > \hat{x}^{AK}$ or $\hat{x}^{AK} > \hat{x}^{BK}$, Case 1 and Case 2 below. If $R_A = R_B = 0$, we have the last case, Case 3 below. Each possible buyer distribution is shown in Figure 4, Figure 5 and Figure 6. These cases/regions and their respective threshold points are defined as follows:

Case 1: $\hat{x}^{BK} > \hat{x}^{AK}$	Case 2: $\hat{x}^{AK} > \hat{x}^{BK}$	Case 3: $R_A = R_B = 0$
$I_1 = [0, \hat{x}^{BR}]$	$I_1 = [0, \hat{x}^{BR}]$	$I_1 = [0, \hat{x}^{BR} = \hat{x}^{AK}]$
$I_2 = [\hat{x}^{BR}, \hat{x}^{AK}]$	$I_2 = [\hat{x}^{BR}, \hat{x}^{BK}]$	I_2 is not defined
$I_3 = [\hat{x}^{AK}, \hat{x}^{BK}]$	$I_3 = [\hat{x}^{BK}, \hat{x}^{AK}]$	$I_3 = [\hat{x}^{BR} = \hat{x}^{AK}, \hat{x}^{BK} = \hat{x}^{AR}]$
$I_4 = [\hat{x}^{BK}, \hat{x}^{AR}]$	$I_4 = [\hat{x}^{AK}, \hat{x}^{AR}]$	I_4 is not defined
$I_5 = [\hat{x}^{AR}, 1]$	$I_5 = [\hat{x}^{AR}, 1]$	$I_5 = [\hat{x}^{AR} = \hat{x}^{BK}, 1]$

We compare these regions in terms of expected utilities generated in each one of them by using $EU(x_r, A)$ and $EU(x_r, B)$ piecewise functions defined above. We use a subscript, l, to indicate Region l. Then, we write the expected utility of a buyer in Region l as follows $EU(x_r, A_l)$ and $EU(x_r, B_l)$, for $l = 1, ..., 5$. We also define the following difference:

$$\Delta_l(x_r) = EU(x_r, A_l) - EU(x_r, B_l)$$

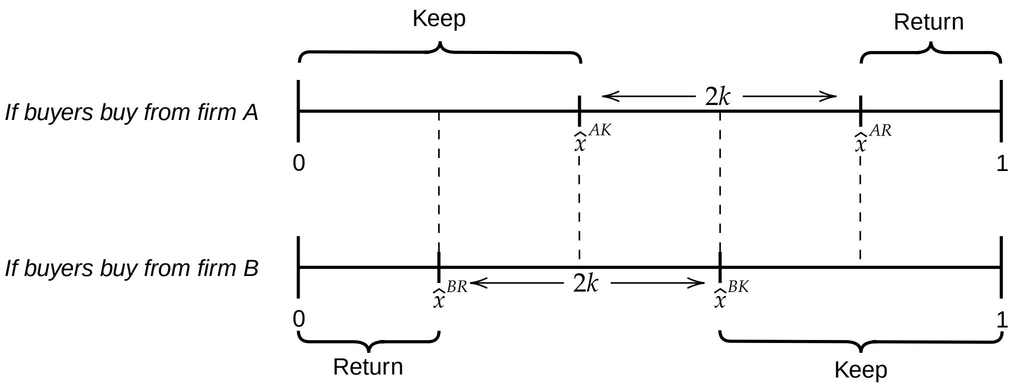

Figure 4. Buyers' decision when $\hat{x}^{BK} > \hat{x}^{AK}$

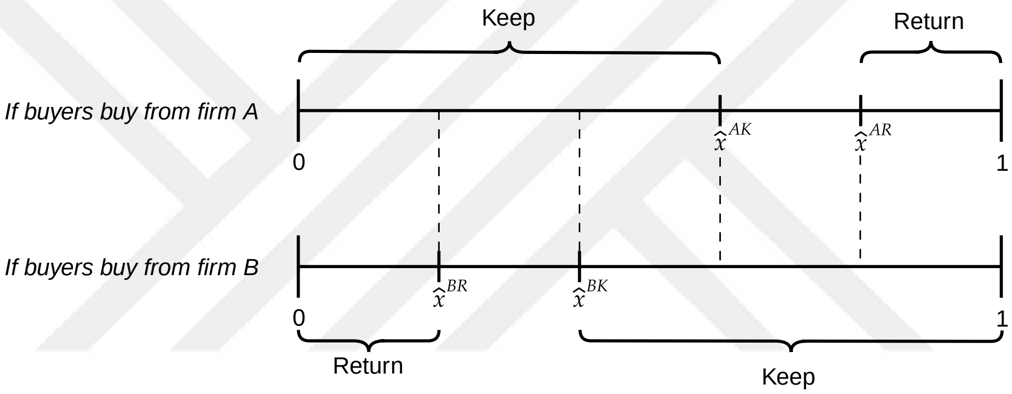

Figure 5. Buyers' decision when $\hat{x}^{AK} > \hat{x}^{BK}$

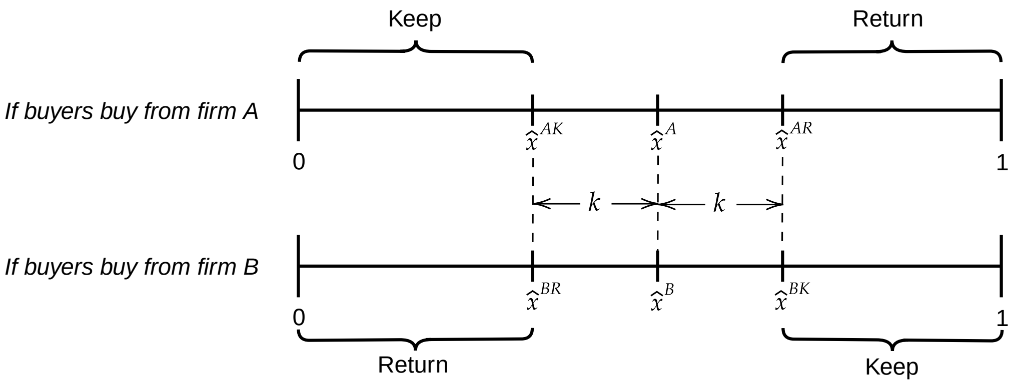

Figure 6. Buyers' decision when $R_A = R_B = 0$

If $\Delta_l(x_r) > 0$, then the buyer with reference location $x_r \in I_l$ buys from firm A, otherwise, buys from firm B. Now, to be able to carry out comparisons, we impose some structure regarding these thresholds which define the regions. First, we impose interior thresholds, that is, $\hat{x}^{BR}, \hat{x}^{BK}, \hat{x}^{AR}, \hat{x}^{AK} \in [0,1]$. We solve the equilibrium objects under this structure and then once we solve for the equilibrium, we will verify that this structure holds. Note that the difference between the thresholds are $\hat{x}^{AR} - \hat{x}^{AK} = \hat{x}^{BK} - \hat{x}^{BR} = 2k$. Thus, thresholds being interior necessarily implies that $k < \frac{1}{2}$. This is because otherwise, at least one of the thresholds would be outside the $[0,1]$ interval. Thus, we keep the following assumption for the rest of the analysis.

Assumption 1. $k < \frac{1}{2}$

Now, we can analyze each region one by one.
Region 1: $I_1 = [0, \hat{x}^{BR}]$. If a buyer's reference location, x_r, is between zero and $\hat{x}^{BR}$, then she keeps the product if she buys from firm A and returns the product if she buys from firm B. Hence, we can write her utility levels as

$$\begin{aligned} EU(x_r, A_1) &= v - P_A - tx_r \\ EU(x_r, B_1) &= v - P_A - R_B P_B - tx_r \end{aligned} \tag{4.8}$$

Region 2: $I_2 = [\hat{x}^{BR}, \hat{x}^{AK}]$ or $I_2 = [\hat{x}^{BR}, \hat{x}^{BK}]$ or I_2 is not defined depending on the case. Nonetheless, if a buyer is in Region 2, then she keeps the product if she buys from firm A. However, in this region, buyers who buy from firm B decide whether to keep or return after learning at their true location once they make the purchase. The utility levels are

$$\begin{aligned} EU(x_r, A_2) &= v - P_A - tx_r \\ EU(x_r, B_2) &= v - \frac{P_B + P_A + t + R_B P_B - kt}{2} + \frac{(P_A - P_B - t + R_B P_B + 2tx_r)^2}{8kt} \end{aligned} \tag{4.9}$$

Region 3: $I_3 = [\hat{x}^{AK}, \hat{x}^{BK}]$ or $I_3 = [\hat{x}^{BK}, \hat{x}^{AK}]$. If a buyer's location is in Region 3, there is two possible buyer decision depending on $\hat{x}^{AK}$ and $\hat{x}^{BK}$. Suppose

$\hat{x}^{BK} > \hat{x}^{AK}$, then each buyer in this region decides whether to keep or return after learning their true locations once they make the purchase. The utility levels are

$$EU(x_r, A_3) = v - \frac{P_B + P_A + t + R_A P_A - kt}{2} + \frac{(P_A - P_B - t - R_A P_A + 2tx_r)^2}{8kt}$$
$$EU(x_r, B_3) = v - \frac{P_B + P_A + t + R_B P_B - kt}{2} + \frac{(P_A - P_B - t + R_B P_B + 2tx_r)^2}{8kt} \tag{4.10}$$

Suppose $\hat{x}^{AK} > \hat{x}^{BK}$, then a buyer's decision is keeping the product for any of the firm as shown in Figure 5. The utility levels are

$$EU(x_r, A_3) = v - P_A - tx_r$$
$$EU(x_r, B_3) = v - P_B - t(1 - x_r) \tag{4.11}$$

Region 4: $I_4 = [\hat{x}^{BK}, \hat{x}^{AR}]$ or $I_4 = [\hat{x}^{AK}, \hat{x}^{AR}]$ or I_4 is not defined depending on the case. Nonetheless, if a buyer is in Region 4, then she keeps the product if she buys from firm B. However, in this region, buyers who buy from firm A decide whether to keep or return after learning their true location once they make the purchase. The utility levels are

$$EU(x_r, A_4) = v - \frac{P_B + P_A + t + R_A P_A - kt}{2} + \frac{(P_A - P_B - t - R_A P_A + 2tx_r)^2}{8kt}$$
$$EU(x_r, B_4) = v - P_B - t(1 - x_r) \tag{4.12}$$

Region 5: $I_5 = [\hat{x}^{AR}, 1]$. If a buyer's reference location, x_r, is between $\hat{x}^{AR}$ and one, then she keeps the product if she buys from firm B and returns the product if she buys from firm A. Hence, we can write her utility levels as

$$EU(x_r, A_5) = v - P_B - R_A P_A - t(1 - x_r)$$
$$EU(x_r, B_5) = v - P_B - t(1 - x_r) \tag{4.13}$$

Lemma 1. $\Delta_l(x_r)$ *is a continuous function on every point where* $x_r \in [0, 1]$.

Proof. See the Appendix. □

Lemma 2. $\Delta_l(x_r)$ *is a non-increasing function on every point where* $x_r \in [0,1]$.

Proof. See the Appendix. □

Lemma 3. *There exists a threshold point* $\tilde{x}_r$ *such that buyers on the left side of it buy from firm A, and those on the right side of it buy from firm B unless* $R_A = R_B = 0$.

Proof. Follows from Lemma 1 and Lemma 2. □

Results of above lemmas are shown in Figure 7. We can now start to analyze the effects of different refund rates in the market and their equilibrium price points.

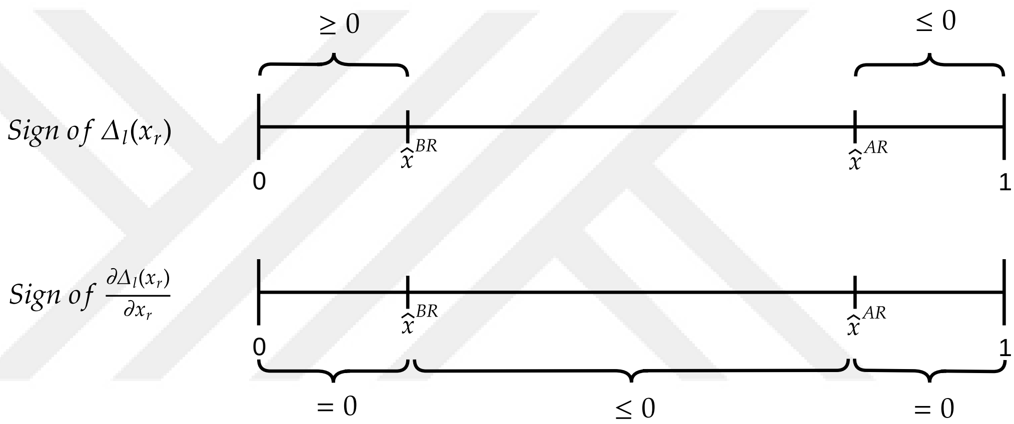

Figure 7. Signs of $\Delta_l(x_r)$ and $\dfrac{\partial \Delta_l(x_r)}{\partial x_r}$ for each region

4.1.1 Full Refund vs Full Refund

When both of firms adopt full refund policy , that is, $R_A = R_B = 0$, returning a product has no additional cost for the buyer, since the price paid is fully refunded. Inserting $R_A = R_B = 0$ into the threshold equations, we get

$$\hat{x}^{AK} = \hat{x}^{BR} = \frac{P_B - P_A + t - 2kt}{2t}$$
$$\hat{x}^{AR} = \hat{x}^{BK} = \frac{P_B - P_A + t + 2kt}{2t}$$
$$\hat{x}^{A} = \hat{x}^{B} = \frac{P_B - P_A + t}{2t}$$

When $R_A = R_B = 0$, we are in Case 3, and the market is split into only three regions as Figure 6, where $\hat{x}^{AK} = \hat{x}^{BR}$ and $\hat{x}^{AR} = \hat{x}^{BK}$, and Region 2 and Region 4 do not exist.

In each region, $l = 1,3,5$, buying from any firm results with the same expected utility $EU(x_r, A_l) = EU(x_r, B_l)$. Hence $\Delta_1(x_r) = \Delta_3(x_r) = \Delta_5(x_r) = 0$. When this happens, each buyer can choose Firm i with probability $\frac{1}{2}$. For Region 1, if a buyer buys from firm B, she returns the product, $x_r < \hat{x}^{BR}$, however if she buys from firm A, she keeps the product, $x_r < \hat{x}^{AK}$. Thus, after the return/keep decisions, every buyer in Region 1 ends up with the product from firm A. For Region 5, if a buyer buys from firm A, she returns the product, $x_r > \hat{x}^{AR}$, however if she buys from firm B, she keeps the product,$x_r > \hat{x}^{BK}$. Thus, after the return/keep decisions, every buyer in Region 5 ends up with the product from firm B. The realized demand of a firm is given by the set of buyers who either buy from this firm and keep the product or buy from other firm and return the product. Then, when the refund rates are $R_A = R_B = 0$, the expected profit levels as functions of the prices are expressed as

$$E\pi_A(P_A, P_B) = \hat{x}^{AK} P_A + \frac{1}{2}\int_{\hat{x}^{AK}}^{\hat{x}^{AR}} \int_{x_r-k}^{\hat{x}^{A}} \frac{P_A}{2k} dx\, dx_r + \frac{1}{2}\int_{\hat{x}^{BR}}^{\hat{x}^{BK}} \int_{x_r-k}^{\hat{x}^{B}} \frac{P_A}{2k} dx\, dx_r$$

$$E\pi_B(P_A, P_B) = (1-\hat{x}^{BK}) P_B + \frac{1}{2}\int_{\hat{x}^{AK}}^{\hat{x}^{AR}} \int_{\hat{x}^{A}}^{x_r+k} \frac{P_B}{2k} dx\, dx_r + \frac{1}{2}\int_{\hat{x}^{BR}}^{\hat{x}^{BK}} \int_{\hat{x}^{B}}^{x_r+k} \frac{P_B}{2k} dx\, dx_r \tag{4.14}$$

The best-response functions are then given by

$$\begin{aligned} \frac{\partial E\pi_A(P_A, P_B)}{\partial P_A} &= \frac{1}{2} - \frac{2P_A - P_B}{2t} = 0 \\ \frac{\partial E\pi_B(P_A, P_B)}{\partial P_B} &= \frac{1}{2} + \frac{P_A - 2P_B}{2t} = 0 \end{aligned} \tag{4.15}$$

Solving Equation 4.15, we find $P_A = P_B = t$. Inserting these prices into the equations for the thresholds, we get

$$\begin{aligned} \hat{x}^{AK} = \hat{x}^{BR} = \frac{1}{2} - k &\quad \in \quad [0,1] \\ \hat{x}^{AR} = \hat{x}^{BK} = \frac{1}{2} + k &\quad \in \quad [0,1] \\ \hat{x}^{A} = \hat{x}^{B} = \frac{1}{2} &\quad \in \quad [0,1] \end{aligned} \tag{4.16}$$

Note that these thresholds are all interior points in $[0,1]$. Also, the second order derivatives of the expected profit functions are negative.

$$\frac{\partial^2 E\pi_A(P_A, P_B)}{\partial P_A^2} = \frac{\partial^2 E\pi_B(P_A, P_B)}{\partial P_B^2} = -\frac{1}{t} \quad < \quad 0 \tag{4.17}$$

And the expected profit levels are

$$E\pi_A(P_A, P_B) = E\pi_B(P_A, P_B) = \frac{t}{2} \quad > \quad 0 \tag{4.18}$$

Thus, there is an equilibrium with $R_A = R_B = 0$, where the prices are $(P_A, P_B) = (t, t)$.

4.1.2 Full Refund vs No Refund

Only one of the two firms adopts a full refund policy and the other firm adopts no refund policy. Suppose $R_B = 0$ and $R_A = 1$.[2] For now we keep R_A in order to see the expressions in the equilibrium in terms of any $R_A > 0$. Inserting $R_B = 0$ into the threshold equations, we get

$$\begin{aligned} \hat{x}^{AK} = \frac{P_B - P_A + R_A P_A + t - 2kt}{2t} &\quad > \quad \hat{x}^{BR} = \frac{P_B - P_A + t - 2kt}{2t} \\ \hat{x}^{AR} = \frac{P_B - P_A + R_A P_A + t + 2kt}{2t} &\quad > \quad \hat{x}^{BK} = \frac{P_B - P_A + t + 2kt}{2t} \\ \hat{x}^{A} = \frac{P_B - P_A + R_A P_A + t}{2t} &\quad > \quad \hat{x}^{B} = \frac{P_B - P_A + t}{2t} \end{aligned}$$

[2] The other case with $R_A = 0$ and $R_B = 1$ yields the same results since the game is symmetric.

Unlike Section 4.1.1, in this possible equilibrium configuration, we cannot be sure about the relation between $\hat{x}^{AK}$ and $\hat{x}^{BK}$. Hence, we have to check for two cases, one with $\hat{x}^{BK} > \hat{x}^{AK}$, and the other with $\hat{x}^{AK} > \hat{x}^{BK}$.

Case 1: Let $\hat{x}^{BK} > \hat{x}^{AK}$. When $R_B = 0$, returning a product to firm B has no additional cost for the buyer, since the price paid is fully refunded. Inserting $R_B = 0$ into the equation Equation 4.8 and Equation 4.9, we get

$$\begin{aligned} \Delta_1(x_r) &= 0 \\ \Delta_2(x_r = \hat{x}^{BR}) &= 0 \\ \Delta_2(x_r = \hat{x}^{AK}) &= -\frac{(R_A P_A)^2}{8kt} \end{aligned} \tag{4.19}$$

Since we prove $\frac{\partial \Delta_l(x_r)}{\partial x_r} \leq 0$ where $l \in 2,3,4$ in Lemma 2, we know for sure $\tilde{x}_r = \hat{x}^{BR}$ by Equation 4.19. Hence, buyers with $x_r > \hat{x}^{BR}$ buy from firm B and buyers with $x_r \leq \hat{x}^{BR}$ buy either from firm A or firm B, each with the probability $\frac{1}{2}$. For Region 1, if a buyer buys from firm B, she returns the product, $x_r < \hat{x}^{BR}$, however if she buys from firm A, she keeps the product,$x_r < \hat{x}^{AK}$. Thus, after the return/keep decisions, every buyer in Region 1 ends up with the product from firm A. Therefore, buyers in Region 1 buy from A and the rest of them buy from firm B. Then, when the refund rates are R_A and $R_B = 0$, the expected profit levels as functions of the prices are expressed as

$$\begin{aligned} E\pi_A(P_A, P_B) &= \hat{x}^{BR} P_A + \int_{\hat{x}^{BR}}^{\hat{x}^{BK}} \int_{x_r - k}^{\hat{x}^{B}} \frac{P_A}{2k} dx\, dx_r \\ E\pi_B(P_A, P_B) &= (1 - \hat{x}^{BK}) P_B + \int_{\hat{x}^{BR}}^{\hat{x}^{BK}} \int_{\hat{x}^{B}}^{x_r + k} \frac{P_B}{2k} dx\, dx_r \end{aligned} \tag{4.20}$$

The best-response functions are then given by

$$\begin{aligned} \frac{\partial E\pi_A(P_A, P_B)}{\partial P_A} &= \frac{1}{2} - \frac{2P_A - P_B}{2t} \\ \frac{\partial E\pi_B(P_A, P_B)}{\partial P_B} &= \frac{1}{2} + \frac{P_A - 2P_B}{2t} \end{aligned} \tag{4.21}$$

Solving Equation 4.21, we find $P_A = P_B = t$ and inserting these prices into the equations for the thresholds,[3] we get

$$\frac{\partial^2 E\pi_A(P_A,P_B)}{\partial P_A^2} = \frac{\partial^2 E\pi_B(P_A,P_B)}{\partial P_B^2} = -\frac{1}{t} < 0$$

$$E\pi_A(P_A,P_B) = E\pi_B(P_A,P_B) = \frac{t}{2} > 0$$

and the thresholds are

$$\begin{aligned} \hat{x}^A &= \frac{1}{2} + \frac{R_A}{2} \in [0,1] & \hat{x}^B &= \frac{1}{2} \in [0,1] \\ \hat{x}^{BR} &= \frac{1}{2} - k \in [0,1] & \hat{x}^{BK} &= \frac{1}{2} + k \in [0,1] \\ \hat{x}^{AK} &= \frac{1}{2} - k + \frac{R_A}{2} \in [0,1] & \hat{x}^{AR} &= \frac{1}{2} + k + \frac{R_A}{2} \notin [0,1] \end{aligned} \tag{4.22}$$

With the result above, we see that one of our threshold points is not an interior point in the equilibrium: $\hat{x}^{AR} \notin [0,1]$. Hence, we have to move $\hat{x}^{AR}$ to outside of $[0,1]$ and recalculate firms' profit according to Figure 8.

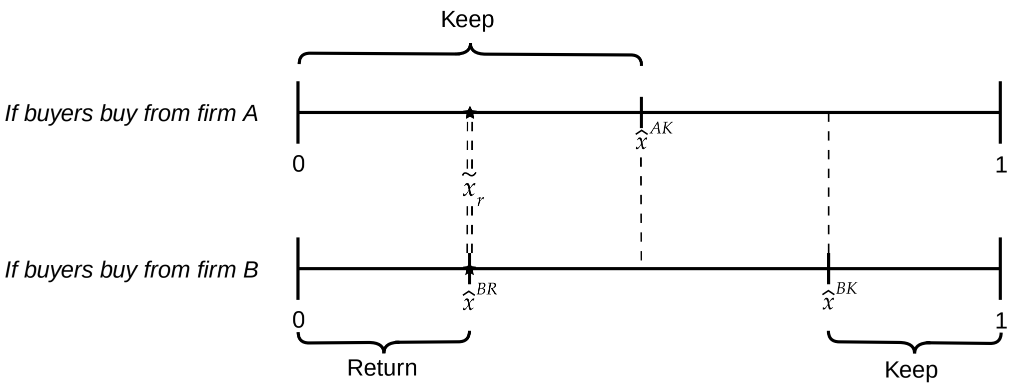

Figure 8. Reshaped market when $\hat{x}^{AR} \notin [0,1]$

When we recalculate firms' new profit according to Figure 8, expected profit equations of both firms remain the same because $\hat{x}^{AR}$ is not used in any of the expected profit equations. These results are valid for every $R_A \in (0,1]$ because any

[3] These prices do not depend on R_A. This is expected because when $R_B = 0$, all buyers except in Region 1 buy from firm B and get the full value of the money they paid if they refund. Buyers who buy from firm A are located only in Region 1 hence they keep the product and are not affected by R_A value.

$R_A > 0$ results with every buyer to buy from firm B except for those buyers who are located in Region 1. Buyers, who are in Region 1, buy the product from firm A and keep it; therefore, they are not affected by R_A. With the remodeling of the thresholds in the market, all the assumptions we make are satisfied with these results. Hence, there is no profitable price deviation and there is an equilibrium where $(P_A, P_B) = (t, t)$.

Case 2: Let $\hat{x}^{AK} > \hat{x}^{BK}$. Now, the thresholds and buyers' decisions change to the one depicted in Figure 5. Even with these changes, Equation 4.19 is still valid and $\tilde{x}_r = \hat{x}^{BR}$. Hence, buyers, who are on the left side of $\hat{x}^{BR}$, buy from firm A, and the others buy from firm B at the beginning. In Case 2, buyers who are in Region 3 now keep the product regardless of the firm from which the initial purchase is made. The only difference here is the start and end points of Region 3 and calculations of the expected profits result with the same equations as Equation 4.20. Hence, we find equilibrium price points same as Case 1 where $(P_A, P_B) = (t, t)$. Inserting this price pair we get the same results as in Equation 4.22 and Figure 8.

To sum up, when one of the firms offers full refund, and the other one offers no refund or less than full refund, there is an equilibrium independent from the comparison of $\hat{x}^{AK}$ and $\hat{x}^{BK}$.

4.1.3 No Refund vs No Refund

In this section, we consider the case where both firms announce their refund policies as no refund, that is, $R_A = R_B = 1$. Inserting $R_A = R_B = 1$ into the threshold equations, we get

$$\hat{x}^{AK} = \frac{P_B + t - 2kt}{2t} \quad > \quad \hat{x}^{BR} = \frac{-P_A + t - 2kt}{2t}$$
$$\hat{x}^{AR} = \frac{P_B + t + 2kt}{2t} \quad > \quad \hat{x}^{BK} = \frac{-P_A + t + 2kt}{2t}$$
$$\hat{x}^{A} = \frac{P_B + t}{2t} \quad > \quad \hat{x}^{B} = \frac{-P_A + t}{2t}$$

In this refund policy configuration, the threshold $\tilde{x}_r$ cannot be located as in the previous refund policy configurations. This threshold, $\tilde{x}_r$, can be any where in

$[\hat{x}^{BR}, \hat{x}^{AR}]$. Hence, there are three possible regions for possible equilibrium points: Region 2, 3 and 4. Because of the symmetry between Region 2 and Region 4, it is sufficient to check for only Regions 2 and 3. We proceed with checking whether $\tilde{x}_r$ can be in one of these two regions. However, the regions depend on the two possible cases: case with $\hat{x}^{BK} > \hat{x}^{AK}$, and case with $\hat{x}^{AK} > \hat{x}^{BK}$.

Case 1: Let $\hat{x}^{BK} > \hat{x}^{AK}$.

Case 1.1: $\tilde{x}_r \in [\hat{x}^{BR}, \hat{x}^{AK}]$: This means that $\tilde{x}_r$ is in Region 2 and it is equal to the point x_r where $\Delta_2(x_r) = 0$. When we solve it, there are two possible $\tilde{x}_r$ values.

$$\begin{aligned} x_{r_1} &= -\frac{P_A - t + 2kt - 2\sqrt{2P_B kt}}{2t} \\ x_{r_2} &= -\frac{P_A - t + 2kt + 2\sqrt{2P_B kt}}{2t} \end{aligned} \tag{4.23}$$

To check these two possible $\tilde{x}_r$ values, first we calculate profit equations for both firms. We use Figure 9 to calculate market share of both firms and find firms' profits via Equation 4.24.

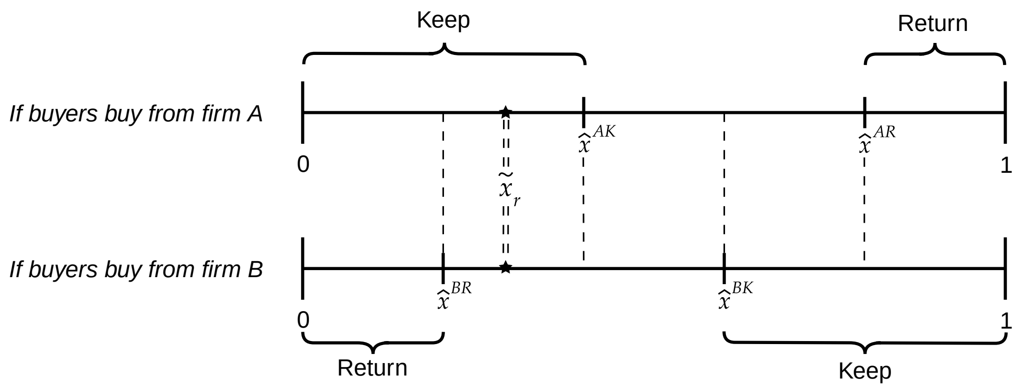

Figure 9. Market share if $\hat{x}^{BK} > \hat{x}^{AK}$ and $\tilde{x}_r$ in region 2

$$E\pi_A(P_A, P_B) = \tilde{x}_r P_A + \int_{\tilde{x}_r}^{\hat{x}^{BK}} \int_{x_r - k}^{\hat{x}^B} \frac{P_A}{2k} dx\, dx_r$$

$$\tag{4.24}$$

$$E\pi_B(P_A, P_B) = (1 - \hat{x}^{BK}) P_B + \int_{\tilde{x}_r}^{\hat{x}^{BK}} \int_{\hat{x}^B}^{x_r + k} \frac{P_B}{2k} dx\, dx_r + \int_{\tilde{x}_r}^{\hat{x}^{BK}} \int_{x_r - k}^{\hat{x}^B} \frac{P_B}{2k} dx\, dx_r$$

Note that $E\pi_B(P_A, P_B)$ is also equal to $(1 - \tilde{x}_r)P_B$ because Firm B does not give any money back to the buyers who decide to return, since $R_B = 1$. Now, we first try x_{r_1} to find an equilibrium by inserting it into Equation 4.24. When we take derivatives of Equation 4.24 with respect to each firm's price, we get

$$\begin{aligned} \frac{\partial E\pi_A(P_A,P_B)}{\partial P_A} &= \frac{1}{2} - \frac{2P_A - P_B}{2t} \\ \frac{\partial E\pi_B(P_A,P_B)}{\partial P_B} &= \frac{1}{2} + k + \frac{P_A - 3\sqrt{2P_B kt}}{2t} \end{aligned} \tag{4.25}$$

Equating the expressions in Equation 4.25 to zero, we find two possible (P_A, P_B) pairs:

$$\begin{aligned} P_{A_1} &= -t - 2kt - 3\sqrt{2kt}(\sqrt{t(14k-3)} - 3\sqrt{2kt}) \\ P_{B_1} &= -3t - 4kt - 6\sqrt{2kt}(\sqrt{t(14k-3)} - 3\sqrt{2kt}) \\ \\ P_{A_2} &= -t - 2kt + 3\sqrt{2kt}(\sqrt{t(14k-3)} + 3\sqrt{2kt}) \\ P_{B_2} &= -3t - 4kt + 6\sqrt{2kt}(\sqrt{t(14k-3)} + 3\sqrt{2kt}) \end{aligned} \tag{4.26}$$

To find a real price equilibrium, both of the prices should be real values. Hence, the expression $14k - 3$ in the root must be positive. Thus, the value of k should be greater than or equal to $\frac{3}{14}$. When we try (P_{A_1}, P_{B_1}) pair, $\hat{x}^B = \frac{3\sqrt{2k(14k-3)}}{2} - 8k + 1$. For the values $k \geq \frac{3}{14}$, $\hat{x}^B = \frac{3\sqrt{2k(14k-3)}}{2} - 8k + 1 < 0$. It is a contradiction to our assumptions and there is no equilibrium in this case. When we try (P_{A_2}, P_{B_2}) pair, $\hat{x}^B = -\frac{3\sqrt{2k(14k-3)}}{2} - 8k + 1$. For the values $k \geq \frac{3}{14}$, $\hat{x}^B = \frac{3\sqrt{2k(14k-3)}}{2} - 8k + 1 < 0$. It is a contradiction to our assumptions as well and there is no equilibrium in this case either.

When we check for x_{r_2}, we use Figure 9 and Equation 4.24 again because they are still valid. However, expanded version of Equation 4.24 changes since we try to find optimal prices for x_{r_2} instead of x_{r_1}. When we inserted x_{r_2} and take derivatives of firms' expected profits with respect to firms' prices, we find equations below.

$$\begin{aligned} \frac{\partial E\pi_A(P_A,P_B)}{\partial P_A} &= \frac{1}{2} - \frac{2P_A - P_B}{2t} \\ \frac{\partial E\pi_B(P_A,P_B)}{\partial P_B} &= \frac{1}{2} + k + \frac{P_A + 3\sqrt{2P_Bkt}}{2t} \quad > \quad 0 \end{aligned} \tag{4.27}$$

Since $\frac{\partial E\pi_B(P_A,P_B)}{\partial P_B} > 0$, we cannot find any (P_A, P_B) pair that makes $\frac{\partial E\pi_A(P_A,P_B)}{\partial P_A} = \frac{\partial E\pi_A(P_A,P_B)}{\partial P_A} = 0$. Hence, we can say that there is not any equilibrium where $\tilde{x}_r \in [\hat{x}^{BR}, \hat{x}^{AK}]$, which is Region 2. By a symmetric analysis, there is no equilibrium in Region 4 either.

Case 1.2: $\tilde{x}_r \in [\hat{x}^{AK}, \hat{x}^{BK}]$: This means that $\tilde{x}_r$ is in Region 3 and it is equal to the point x_r where $\Delta_3(x_r) = 0$. When we solve it, there is one possible $\tilde{x}_r$ value.

$$\tilde{x}_r = -\frac{2t(P_A + P_B) - 4kt(P_A - P_B) - (P_A^2 - P_B^2)}{4t(P_A + P_B)} \tag{4.28}$$

We calculate profit equations for both firms by inserting $\tilde{x}_r$ value we find in Equation 4.28. We use Figure 10 to calculate market shares of both firms and find firms' profits on Equation 4.29.

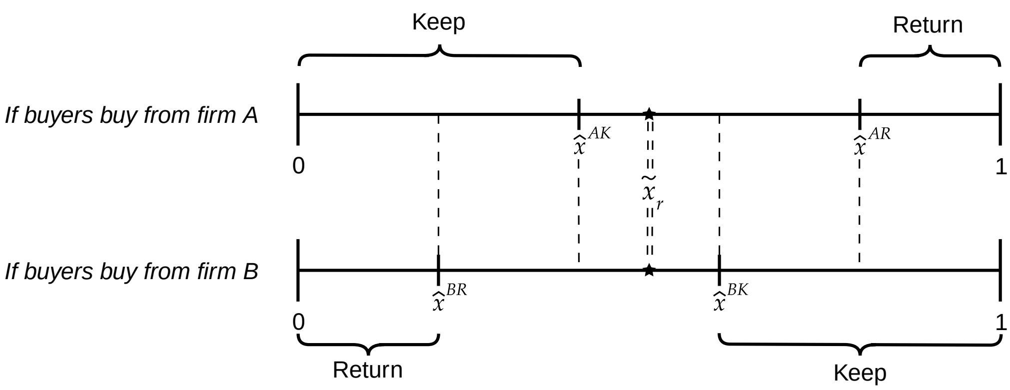

Figure 10. Market share if $\hat{x}^{BK} > \hat{x}^{AK}$ and $\tilde{x}_r$ in region 3

$$\begin{aligned} E\pi_A(P_A,P_B) &= \tilde{x}_r P_A + \int_{\tilde{x}_r}^{\hat{x}^{BK}} \int_{x_r - k}^{\hat{x}^B} \frac{P_A}{2k} dx\, dx_r \\ E\pi_B(P_A,P_B) &= (1-\tilde{x}_r)P_B + \int_{\hat{x}^{AK}}^{\tilde{x}_r} \int_{\hat{x}^A}^{x_r + k} \frac{P_B}{2k} dx\, dx_r \end{aligned} \tag{4.29}$$

We try x_r to find an equilibrium by inserting to Equation 4.29. When we take derivatives of Equation 4.29 with respect to each firm's price and equalize to zero, we find two possible (P_A, P_B) pairs:

$$\begin{aligned} P_{A_1} &= 3kt + t\sqrt{k(9k-4)} \\ P_{B_1} &= 3kt + t\sqrt{k(9k-4)} \\ \\ P_{A_2} &= 3kt - t\sqrt{k(9k-4)} \\ P_{B_2} &= 3kt - t\sqrt{k(9k-4)} \end{aligned} \tag{4.30}$$

Now, for a real valued price equilibrium, the value of k should be greater than or equal to $\frac{4}{9}$. When we insert (P_{A_1}, P_{B_1}) pair into our threshold values, we find:

$$\begin{aligned} \hat{x}^{AK} &= \frac{1}{2} + \frac{\sqrt{k}(\sqrt{9k-4}+\sqrt{k})}{2} \\ \hat{x}^{BK} &= \frac{1}{2} - \frac{\sqrt{k}(\sqrt{9k-4}+\sqrt{k})}{2} \\ \hat{x}^{AK} - \hat{x}^{BK} &= k + \sqrt{k(9k-4)} \end{aligned}$$

For every $k \geq \frac{4}{9}$, we have $k + \sqrt{k(9k-4)} > 0$, that is, $\hat{x}^{AK} > \hat{x}^{BK}$. However, this contradicts our assumption $\hat{x}^{AK} < \hat{x}^{BK}$. Now, when we insert (P_{A_2}, P_{B_2}) pair into our variable, we find:

$$\begin{aligned} \hat{x}^{AK} &= \frac{1}{2} - \frac{\sqrt{k}(\sqrt{9k-4}-\sqrt{k})}{2} \\ \hat{x}^{BK} &= \frac{1}{2} + \frac{\sqrt{k}(\sqrt{9k-4}-\sqrt{k})}{2} \\ \hat{x}^{AK} - \hat{x}^{BK} &= k - \sqrt{k(9k-4)} \end{aligned}$$

where $k - \sqrt{k(9k-4)} > 0$ is equivalent to $k > 9k-4$, that is, $k < \frac{1}{2}$. Thus, for every $\frac{1}{2} > k \geq \frac{4}{9}$, we have $\hat{x}^{AK} > \hat{x}^{BK}$. However, this contradicts our assumption $\hat{x}^{AK} < \hat{x}^{BK}$. Note that under our Assumption 1, we have $k < 1/2$, thus, values $k \geq 2$ are irrelevant. Hence, there is no equilibrium where $\tilde{x}_r \in [\hat{x}^{AK}, \hat{x}^{BK}]$, which is Region 3, in this case.

Case 2: Let $\hat{x}^{AK} > \hat{x}^{BK}$. In this case, the thresholds are as in Figure 5, and for $\hat{x}^{AK} > \hat{x}^{BK}$ to hold we need $P_A + P_B > 4kt$.

When we compare the regions in this case (with $\hat{x}^{AK} > \hat{x}^{BK}$) to the regions in the previous case (with $\hat{x}^{BK} > \hat{x}^{AK}$), only the buyers who have $x_r \in [\hat{x}^{BK}, \hat{x}^{AK}]$ may possibly alter their buying decisions. The expected utilities in this case are $EU(x_r, A_3)$ and $EU(x_r, B_3)$, given by the equations below:

$$
\begin{aligned}
EU(x_r, A_3) &= v - P_A - tx_r \\
EU(x_r, B_3) &= v - P_B - t(1 - x_r)
\end{aligned}
\tag{4.31}
$$

Now, we check the continuity of $\Delta_l(x_r)$ function with the new $EU(x_r, A_3)$ and $EU(x_r, B_3)$ functions.

$$
\begin{aligned}
EU(x_r = \hat{x}^{BK}, A_2) = EU(x_r = \hat{x}^{BK}, A_3) &= v - \frac{t}{2} - \frac{P_A}{2} - kt \\
EU(x_r = \hat{x}^{BK}, B_2) = EU(x_r = \hat{x}^{BK}, B_3) &= v - P_B - \frac{t}{2} - \frac{P_A}{2} + kt \\
\Delta_2(x_r = \hat{x}^{BK}) = \Delta_3(x_r = \hat{x}^{BK}) &= P_B - 2kt \\
\\
EU(x_r = \hat{x}^{AK}, A_3) = EU(x_r = \hat{x}^{AK}, A_4) &= v - P_A - \frac{t}{2} - \frac{P_B}{2} + kt \\
EU(x_r = \hat{x}^{AK}, B_3) = EU(x_r = \hat{x}^{AK}, B_4) &= v - \frac{t}{2} - \frac{P_B}{2} - kt \\
\Delta_3(x_r = \hat{x}^{AK}) = \Delta_4(x_r = \hat{x}^{AK}) &= 2kt - P_A
\end{aligned}
\tag{4.32}
$$

When we check the derivative of $\Delta_3(x_r)$ with respect to x_r, we find $\frac{\partial \Delta_3(x_r)}{\partial x_r} = -2t < 0$. Hence, we find the same specifications regarding $\Delta_l(x_r)$ function and its derivative function as in the previous case. Therefore, there are three possible regions, Region 2, 3 and 4, for possible equilibrium points. Because of the symmetry between Region 2 and Region 4, it is sufficient to check for only one of these regions, thus we check Regions 2 and 3.

Case 2.1: $\tilde{x}_r \in [\hat{x}^{BR}, \hat{x}^{BK}]$: This means that $\tilde{x}_r$ is in Region 2 and it is equal to the point x_r *where* $\Delta_2(x_r) = 0$. When we solve it, there are two possible $\tilde{x}_r$ values.

$$\begin{aligned} x_{r_1} &= -\frac{P_A - t + 2kt - 2\sqrt{2P_B kt}}{2t} \\ x_{r_2} &= -\frac{P_A - t + 2kt + 2\sqrt{2P_B kt}}{2t} \end{aligned} \tag{4.33}$$

To check these two possible $\tilde{x}_r$ values, first we calculate profit equations for both firms. We use Figure 11 to calculate market share of both firms and find firms' profits via Equation 4.34.

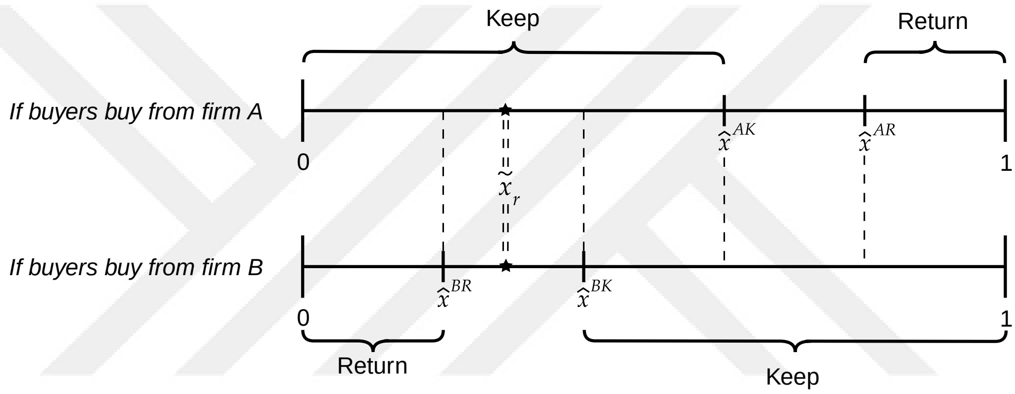

Figure 11. Market share if $\hat{x}^{AK} > \hat{x}^{BK}$ and $\tilde{x}_r$ in region 2

$$\begin{aligned} E\pi_A(P_A,P_B) &= \tilde{x}_r P_A + \int_{\tilde{x}_r}^{\hat{x}^{BK}} \int_{x_r-k}^{\hat{x}^{B}} \frac{P_A}{2k} dx\, dx_r \\ E\pi_B(P_A,P_B) &= (1-\hat{x}^{BK})P_B + \int_{\tilde{x}_r}^{\hat{x}^{BK}} \int_{\hat{x}^{B}}^{x_r+k} \frac{P_B}{2k} dx\, dx_r + \int_{\tilde{x}_r}^{\hat{x}^{BK}} \int_{x_r-k}^{\hat{x}^{B}} \frac{P_B}{2k} dx\, dx_r \end{aligned} \tag{4.34}$$

Note that $E\pi_B(P_A,P_B)$ is also equal to $(1-\tilde{x}_r)P_B$ because Firm B does not give any money back to the buyers who decide to return, since $R_B = 1$. Now, we first try x_{r_1} to find an equilibrium by inserting it into Equation 4.34. When we take derivatives of Equation 4.34 with respect to each firm's price, we get

$$\begin{aligned} \frac{\partial E\pi_A(P_A,P_B)}{\partial P_A} &= \frac{1}{2} - \frac{2P_A - P_B}{2t} \\ \frac{\partial E\pi_B(P_A,P_B)}{\partial P_B} &= \frac{1}{2} + k + \frac{P_A - 3\sqrt{2P_B kt}}{2t} \end{aligned} \tag{4.35}$$

Equating the expressions in Equation 4.35 to zero, we find two possible (P_A, P_B) pairs:

$$
\begin{aligned}
P_{A_1} &= -t - 2kt - 3\sqrt{2kt}(\sqrt{t(14k-3)} - 3\sqrt{2kt}) \\
P_{B_1} &= -3t - 4kt - 6\sqrt{2kt}(\sqrt{t(14k-3)} - 3\sqrt{2kt}) \\
\\
P_{A_2} &= -t - 2kt + 3\sqrt{2kt}(\sqrt{t(14k-3)} + 3\sqrt{2kt}) \\
P_{B_2} &= -3t - 4kt + 6\sqrt{2kt}(\sqrt{t(14k-3)} + 3\sqrt{2kt})
\end{aligned}
\tag{4.36}
$$

These price pairs are the same pairs as in Case 1.1, and we already showed that there is no such price pair as an equilibrium: for real valued prices we need $k \geq \frac{3}{14}$, however for both pairs of prices $\hat{x}_B < 0$.

When we check for x_{r_2}, we use Figure 11 and Equation 4.34 again because they are still valid. However, expanded version of Equation 4.34 changes since we try to find optimal prices for x_{r_2} instead of x_{r_1}. When we inserted x_{r_2} and take derivatives of firms' expected profits with respect to firms' prices, we find equations below.

$$
\begin{aligned}
\frac{\partial E\pi_A(P_A, P_B)}{\partial P_A} &= \frac{1}{2} - \frac{2P_A - P_B}{2t} \\
\frac{\partial E\pi_B(P_A, P_B)}{\partial P_B} &= \frac{1}{2} + k + \frac{P_A + 3\sqrt{2P_B kt}}{2t} \quad > \quad 0
\end{aligned}
\tag{4.37}
$$

Since $\frac{\partial E\pi_B(P_A,P_B)}{\partial P_B} \ > \ 0$, we cannot find any (P_A, P_B) pair that makes $\frac{\partial E\pi_A(P_A,P_B)}{\partial P_A} = \frac{\partial E\pi_A(P_A,P_B)}{\partial P_A} = 0$. Hence, we can say that there is not any equilibrium where $\tilde{x}_r \in [\hat{x}^{BR}, \hat{x}^{BK}]$, which is Region 2. By a symmetric analysis, there is no equilibrium in Region 4 either.

Case 2.2: $\tilde{x}_r \in [\hat{x}^{BK}, \hat{x}^{AK}]$: This means that $\tilde{x}_r$ is in Region 3 and it is equal to the point x_r where $\Delta_3(x_r) = 0$. When we solve it, there is one possible $\tilde{x}_r$ value.

$$
\tilde{x}_r = \frac{P_B - P_A + t}{2t} \tag{4.38}
$$

We calculate profit equations for both firms by inserting $\tilde{x}_r$ value we find in Equation 4.38. We use Figure 12 to calculate market shares of both firms and find firms' profits on

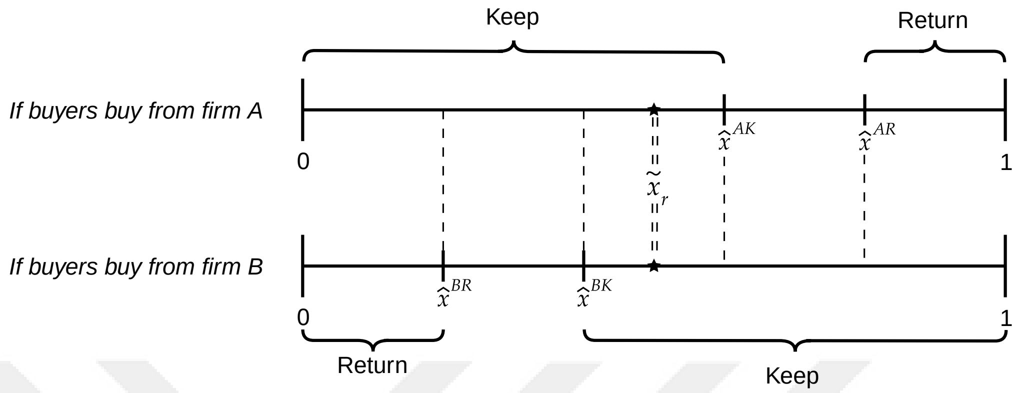

Figure 12. Market share if $\hat{x}^{AK} > \hat{x}^{BK}$ and $\tilde{x}_r$ in region 3

$$E\pi_A(P_A, P_B) = \tilde{x}_r P_A = \frac{P_A}{2} - \frac{P_A(P_A - P_B)}{2t}$$

$$E\pi_B(P_A, P_B) = (1 - \tilde{x}_r) P_B = \frac{P_B}{2} + \frac{P_B(P_A - P_B)}{2t} \tag{4.39}$$

We take the derivatives of Equation 4.39 with respect to each firm's price

$$\frac{\partial E\pi_A(P_A, P_B)}{\partial P_A} = \frac{1}{2} - \frac{2P_A - P_B}{2t}$$
$$\frac{\partial E\pi_B(P_A, P_B)}{\partial P_B} = \frac{1}{2} + \frac{P_A - 2P_B}{2t} \tag{4.40}$$

When we equalize Equation 4.40 to zero, we find $P_A = P_B = t$. Inserting these prices to equations we find results with:

$$\frac{\partial^2 E\pi_A(P_A, P_B)}{\partial P_A^2} = \frac{\partial^2 E\pi_B(P_A, P_B)}{\partial P_B^2} = -\frac{1}{t} < 0 \qquad E\pi_A(P_A, P_B) = E\pi_B(P_A, P_B) = \frac{t}{2} > 0$$

$$\hat{x}^A = 1 \in [0,1] \qquad \hat{x}^B = 0 \in [0,1]$$

$$\hat{x}^{BR} = -k \notin [0,1] \qquad \hat{x}^{BK} = k \in [0,1]$$

$$\hat{x}^{AK} = 1 - k \in [0,1] \qquad \hat{x}^{AR} = 1 + k \notin [0,1] \tag{4.41}$$

With the results above, we have $\hat{x}^{AR}, \hat{x}^{BR} \notin [0,1]$. Thus, two of our assumptions are not satisfied. We recalculate firms profit according to Figure 13 with the results we find on Equation 4.41.

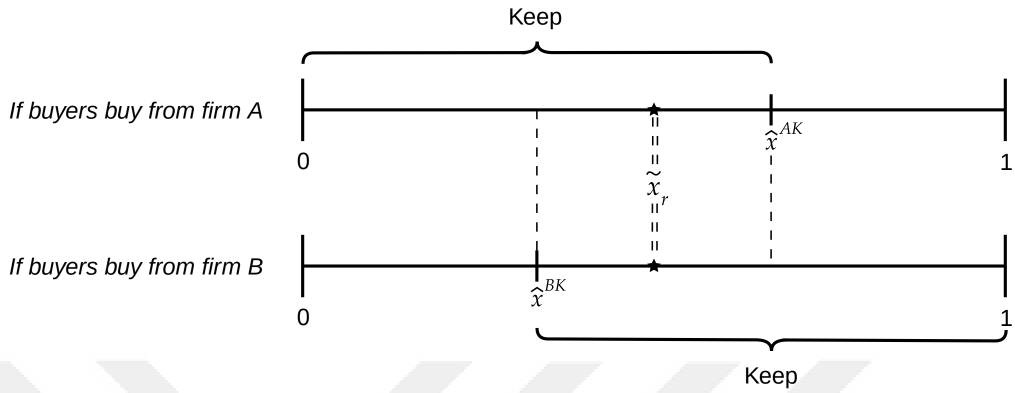

Figure 13. Reshaped market when $\hat{x}^{BR}, \hat{x}^{AR} \notin [0,1]$

When we combine Figure 13 and Equation 4.39, the expected profit equations of both firms remain the same because both $\hat{x}^{AR}$ and $\hat{x}^{BR}$ are not used in any of those equations. Checking the region of $\tilde{x}_r$, we get

$$\begin{gathered}\Delta_2(x_r = \hat{x}^{BK}) = \Delta_3(x_r = \hat{x}^{BK}) = t(1-2k) > 0 \\ \Delta_3(x_r = \hat{x}^{AK}) = \Delta_4(x_r = \hat{x}^{AK}) = t(2k-1) < 0 \\ \frac{\partial \Delta_3(x_r)}{\partial x_r} < 0\end{gathered} \tag{4.42}$$

Instead of five different regions, there are now three different regions, and it is the only difference in our calculations. Therefore, we find the same results, and all of the assumptions are satisfied. There is no profitable price deviation, thus there is an equilibrium point where $(P_A, P_B) = (t, t)$.

4.2 Model Analysis with Loss Aversion

The model with loss aversion has a similar setup to the benchmark model. As we described before, each buyer has a belief about their true location, x, that the true location is drawn from a uniform distribution over $[x_r - k, x_r + k]$. The true location is drawn from this interval, and the buyer learns it only after the buyer makes a purchase. Every buyer is associated with one x_r value, where x_r is uniformly distributed over $[0,1]$. The buyers with beliefs $x_r \in [0,k]$ and $x_r \in [1-k,1]$ are again loyal to the firm that is closest and always buys from that firm and never returns. Thus, we ignore their effects on the equilibrium objects, as they will only alter the level of equilibrium profits but not the equilibrium strategies, same as the benchmark model.

Suppose that a buyer buys the product from firm A. After realizing her true location x, her utility is equal to $v - P_A - tx - \lambda t(x - x_r)$ if she keeps the product. Note that this utility expression is decreasing in x. Her utility is equal to $v - P_B - R_A P_A - t(1-x) + t(x - x_r)$ if she returns the product and buys from firm B.[4] This utility expression is increasing in x. Thus, the location that equates these two utility expressions is the threshold location, $\hat{x}^A$, such that for all $x < \hat{x}^A$, the buyer keeps the product, and for all $x > \hat{x}^A$, she returns it and buys from firm B. We have $\hat{x}^A$ given by

$$\hat{x}^A = \frac{P_B - P_A + R_A P_A + t(\lambda x_r + (x_r + 1))}{t(\lambda + 3)} \tag{4.43}$$

Note that if $\hat{x}^A > x_r + k$, then the buyer who buys from firm A, keeps the product for all possible true location values, $x \in [x_r - k, x_r + k]$. If $\hat{x}^A < x_r - k$, then the buyer who buys from firm A, returns the product for all possible true location values, $x \in [x_r - k, x_r + k]$. However, if $x_r - k \leq \hat{x}^A \leq x_r + k$, then the buyer who buys from firm A may keep or return depending on her true location. Moreover, $\frac{\hat{x}^A}{x_r} = \frac{\lambda+1}{\lambda+3} < 1$. Hence increasing x_r results with respectively less increased $\hat{x}^A$ value and higher probability to refund the product if it is bought from firm A. We define the threshold locations incorporating the above observation.

$$\hat{x}^{AK} = \hat{x}^A - k = \frac{P_B - P_A + P_A R_A + t - kt(\lambda + 3)}{2t} \tag{4.44}$$

$$\hat{x}^{AR} = \hat{x}^A + k = \frac{P_B - P_A + P_A R_A + t + kt(\lambda + 3)}{2t} \tag{4.45}$$

[4] We assume that real location of buyer, x, is greater than x_r. However, assuming $x < x_r$ gives the same $\hat{x}^A$ value and does not have an effect on the future calculations.

Thus, if $\hat{x}^{AK} > x_r$, then the buyer who buys from firm A, keeps the product for all possible true location values, $x \in [x_r - k, x_r + k]$. If $\hat{x}^{AR} < x_r$, then the buyer who buys from firm A, returns the product for all possible true location values, $x \in [x_r - k, x_r + k]$. If $\hat{x}^{AK} \leq x_r \leq \hat{x}^{AR}$, then the buyer who buys from firm A may keep or return depending on her true location.

Suppose that a buyer buys the product from firm B instead of firm A. After realizing her true location x, her utility is equal to $v - P_B - t(1-x) - \lambda t(x_r - x)$ if she keeps the product. Note that this utility expression is increasing in x. Her utility is equal to $v - P_A - R_B P_B - tx + t(x_r - x)$ if she returns the product and buys from firm B.[5] This utility expression is decreasing in x. Thus, the location that equates these two utility expressions is the threshold location, $\hat{x}^B$, such that for all $x < \hat{x}^B$, the buyer return the product, and for all $x > \hat{x}^B$, she keeps it and buys from firm A. We have $\hat{x}^B$ given by

$$\hat{x}^B = \frac{P_B - P_A - R_B P_B + t(\lambda x_r + (x_r + 1))}{t(\lambda + 3)} \tag{4.46}$$

Note that if $\hat{x}^B < x_r - k$, then the buyer who buys from firm B, keeps the product for all possible true location values, $x \in [x_r - k, x_r + k]$. If $\hat{x}^B > x_r + k$, then the buyer who buys from firm B, returns the product for all possible true location values, $x \in [x_r - k, x_r + k]$. However, if $x_r - k \leq \hat{x}^B \leq x_r + k$, then the buyer who buys from firm B may keep or return depending on her true location. Moreover, $\frac{\hat{x}^B}{x_r} = \frac{\lambda+1}{\lambda+3} < 1$. Hence increasing x_r results with respectively less increased $\hat{x}^B$ value and higher probability to keep the product if it is bought from firm B. We define the threshold locations incorporating the above observation.

$$\hat{x}^{BK} = \hat{x}^B + k = \frac{P_B - P_A - R_B P_B + t + kt(\lambda + 3)}{2t} \tag{4.47}$$

$$\hat{x}^{BR} = \hat{x}^B - k = \frac{P_B - P_A - R_B P_B + t - kt(\lambda + 3)}{2t} \tag{4.48}$$

[5] We assume that real location of buyer, x, is lesser than x_r. However, assuming $x > x_r$ gives the same $\hat{x}^B$ value and does not have an effect on the future calculations.

Thus, if $\hat{x}^{BK} > x_r$, then the buyer who buys from firm B, returns the product for all possible true location values, $x \in [x_r - k, x_r + k]$. If $\hat{x}^{BR} < x_r$, then the buyer who buys from firm B, keeps the product for all possible true location values, $x \in [x_r - k, x_r + k]$. If $\hat{x}^{BR} \le x_r \le \hat{x}^{BK}$, then the buyer who buys from firm B may keep or return depending on her true location.

The expected utility of a buyer with a reference location x_r, who buys from firm i, $i = A, B$ can be described as following ex-ante piecewise function:

$$EU(x_r, A) = \begin{cases} v - P_A - t\int_{x_r-k}^{x_r+k} \frac{x}{2k}dx + t\int_{x_r-k}^{x_r} \frac{x_r-x}{2k}dx - \lambda t\int_{x_r}^{x_r+k} \frac{x-x_r}{2k}dx & \text{if } x_r \le \hat{x}^{AK} \\ \frac{\hat{x}^A - x_r + k}{2k}\Big[v - P_A - t\int_{x_r-k}^{\hat{x}^A} \frac{x}{\hat{x}^A - x_r + k}dx + t\int_{x_r-k}^{x_r} \frac{x_r - x}{\hat{x}^A - x_r + k}dx \\ -\lambda t\int_{x_r}^{\hat{x}^A} \frac{x - x_r}{\hat{x}^A - x_r + k}dx\Big] + \frac{x_r + k - \hat{x}^A}{2k}\Big[v - P_B - R_A P_A & \text{if } \hat{x}^{AK} \le x_r \le \hat{x}^{AR} \\ -t\int_{\hat{x}^A}^{x_r+k} \frac{1-x}{x_r + k - \hat{x}^A}dx + t\int_{\hat{x}^A}^{x_r+k} \frac{x - x_r}{x_r + k - \hat{x}^A}dx\Big] \\ v - P_B - R_A P_A - t\int_{x_r-k}^{x_r+k} \frac{1-x}{2k}dx + t\int_{x_r}^{x_r+k} \frac{x-x_r}{2k}dx & \text{if } x_r \ge \hat{x}^{AR} \\ -\lambda t\int_{x_r-k}^{x_r} \frac{x_r - x}{2k}dx \end{cases}$$

$$EU(x_r, B) = \begin{cases} v - P_B - t\int_{x_r-k}^{x_r+k} \frac{1-x}{2k}dx + t\int_{x_r}^{x_r+k} \frac{x-x_r}{2k}dx - \lambda t\int_{x_r-k}^{x_r} \frac{x_r-x}{2k}dx & \text{if } x_r \ge \hat{x}^{BK} \\ \frac{x_r + k - \hat{x}^B}{2k}\Big[v - P_B - t\int_{\hat{x}^B}^{x_r+k} \frac{1-x}{x_r + k - \hat{x}^B}dx - \lambda t\int_{\hat{x}^B}^{x_r} \frac{x_r - x}{x_r + k - \hat{x}^B}dx \\ +t\int_{x_r}^{x_r+k} \frac{x - x_r}{x_r + k - \hat{x}^B}dx\Big] + \frac{\hat{x}^B - x_r + k}{2k}\Big[v - P_A - R_B P_B & \text{if } \hat{x}^{BR} \le x_r \le \hat{x}^{BK} \\ -t\int_{x_r-k}^{\hat{x}^B} \frac{x}{\hat{x}^B - x_r + k}dx + t\int_{x_r-k}^{\hat{x}^B} \frac{x_r - x}{x_r + k - \hat{x}^B}dx\Big] \\ v - P_A - R_B P_B - t\int_{x_r-k}^{x_r+k} \frac{x}{2k}dx + t\int_{x_r-k}^{x_r} \frac{x_r-x}{2k}dx & \text{if } x_r \le \hat{x}^{BR} \\ -\lambda t\int_{x_r}^{x_r+k} \frac{x-x_r}{2k}dx \end{cases}$$

Both $EU(x_r, A)$ and $EU(x_r, B)$ are continuous utility functions on every point $x_r \in [0, 1]$.

We divide the entire market of $[0, 1]$ interval into five regions, same as the benchmark model, defined by the four threshold values we find earlier. In each region, buyers know whether to keep or return the product depending on which firm they bought from. We have the following implications in terms of ordering some of these thresholds:

$$\begin{aligned}
\hat{x}^{BK} - \hat{x}^{BR} &= k(\lambda+3) \Rightarrow \hat{x}^{BK} > \hat{x}^{BR} \\
\hat{x}^{AR} - \hat{x}^{AK} &= k(\lambda+3) \Rightarrow \hat{x}^{AR} > \hat{x}^{AK} \\
\hat{x}^{AK} - \hat{x}^{BR} &= \frac{R_A P_A + R_B P_B}{2t} \Rightarrow \hat{x}^{AK} \geq \hat{x}^{BR} \\
\hat{x}^{AR} - \hat{x}^{BK} &= \frac{R_A P_A + R_B P_B}{2t} \Rightarrow \hat{x}^{AR} \geq \hat{x}^{BK}
\end{aligned} \tag{4.49}$$

Above inequalities and threshold orders are valid under every possible refund rate pair, however, the order between $\hat{x}^{AK}$ and $\hat{x}^{BK}$ is ambiguous. This is because $\hat{x}^{BK} - \hat{x}^{AK} = \frac{2kt(\lambda+3)-(R_A P_A + R_B P_B)}{2t}$ and it can be negative or positive. Moreover, when $R_A = R_B = 0$, we face the equalities $\hat{x}^{AK} = \hat{x}^{BR}$ and $\hat{x}^{AR} = \hat{x}^{BK}$. Thus, there are three possible market outcomes with respect to equilibrium prices and refund rates: If either $R_A \neq 0$ or $R_B \neq 0$, then we have two subcases: $\hat{x}^{BK} > \hat{x}^{AK}$ or $\hat{x}^{AK} > \hat{x}^{BK}$, Case 1 and Case 2 below. If $R_A = R_B = 0$, we have the last case, Case 3 below. These cases/regions and their respective threshold points are defined as follows:

Case 1: $\hat{x}^{BK} > \hat{x}^{AK}$	Case 2: $\hat{x}^{AK} > \hat{x}^{BK}$	Case 3: $R_A = R_B = 0$
$I_1 = [0, \hat{x}^{BR}]$	$I_1 = [0, \hat{x}^{BR}]$	$I_1 = [0, \hat{x}^{BR} = \hat{x}^{AK}]$
$I_2 = [\hat{x}^{BR}, \hat{x}^{AK}]$	$I_2 = [\hat{x}^{BR}, \hat{x}^{BK}]$	I_2 is not defined
$I_3 = [\hat{x}^{AK}, \hat{x}^{BK}]$	$I_3 = [\hat{x}^{BK}, \hat{x}^{AK}]$	$I_3 = [\hat{x}^{BR} = \hat{x}^{AK}, \hat{x}^{BK} = \hat{x}^{AR}]$
$I_4 = [\hat{x}^{BK}, \hat{x}^{AR}]$	$I_4 = [\hat{x}^{AK}, \hat{x}^{AR}]$	I_4 is not defined
$I_5 = [\hat{x}^{AR}, 1]$	$I_5 = [\hat{x}^{AR}, 1]$	$I_5 = [\hat{x}^{AR} = \hat{x}^{BK}, 1]$

Now, to be able to carry out comparisons, we impose some structure regarding these thresholds, which define the regions. First, we impose interior thresholds, that is, $\hat{x}^{BR}, \hat{x}^{BK}, \hat{x}^{AR}, \hat{x}^{AK} \in [0,1]$. We solve the equilibrium objects under this structure, and then once we solve for the equilibrium, we will verify that this structure holds. Note that the difference between the thresholds are $\hat{x}^{AR} - \hat{x}^{AK} = \hat{x}^{BK} - \hat{x}^{BR} = k(\lambda+3)$. Thus, thresholds being interior necessarily implies that $k < \frac{1}{\lambda+3}$. This is because, otherwise, at least one of the thresholds would be outside the $[0,1]$ interval. Thus, we keep the following assumption for the rest of the analysis.

Assumption 2. $k < \frac{1}{\lambda+3}$

Now, we analyze each region one by one.

Region 1: $I_1 = [0, \hat{x}^{BR}]$. If a buyer's reference location, x_r, is between zero and $\hat{x}^{BR}$, then she keeps the product if she buys from firm A and returns the product if she buys from firm B. Hence, we can write her utility levels as

$$\begin{aligned} EU(x_r, A_1) &= v - P_A - tx_r + \frac{kt(1-\lambda)}{4} \\ EU(x_r, B_1) &= v - P_A - R_B P_B - tx_r + \frac{kt(1-\lambda)}{4} \end{aligned} \tag{4.50}$$

Region 2: $I_2 = [\hat{x}^{BR}, \hat{x}^{AK}]$ or $I_2 = [\hat{x}^{BR}, \hat{x}^{BK}]$ or I_2 is not defined depending on the case. Nonetheless, if a buyer is in Region 2, then she keeps the product if she buys from firm A. However, in this region, buyers who buy from firm B, decide whether to keep or return after learning at their true location once they make the purchase. The utility levels are

$$\begin{aligned} EU(x_r, A_2) &= v - P_A - tx_r + \frac{kt(1-\lambda)}{4} \\ EU(x_r, B_2) &= v - \frac{P_B + P_A + t + R_B P_B - 2kt}{2} + \frac{(P_A - P_B - t + R_B P_B + 2tx_r)^2}{4kt(\lambda+3)} \end{aligned} \tag{4.51}$$

Region 3: $I_3 = [\hat{x}^{AK}, \hat{x}^{BK}]$ or $I_3 = [\hat{x}^{BK}, \hat{x}^{AK}]$. If a buyer's location is in Region 3, there is two possible buyer decision depending on $\hat{x}^{AK}$ and $\hat{x}^{BK}$. Suppose $\hat{x}^{BK} > \hat{x}^{AK}$, then each buyer in this region decides whether to keep or return after learning their true locations once they make the purchase. The utility levels are

$$\begin{aligned} EU(x_r, A_3) &= v - \frac{P_B + P_A + t + R_A P_A - 2kt}{2} + \frac{(P_A - P_B - t - R_A P_A + 2tx_r)^2}{4kt(\lambda+3)} \\ EU(x_r, B_3) &= v - \frac{P_B + P_A + t + R_B P_B - 2kt}{2} + \frac{(P_A - P_B - t + R_B P_B + 2tx_r)^2}{4kt(\lambda+3)} \end{aligned} \tag{4.52}$$

Suppose $\hat{x}^{AK} > \hat{x}^{BK}$, then a buyer's decision is keeping the product for any of the firm as shown in Figure 5. The utility levels are

$$
\begin{aligned}
EU(x_r, A_3) &= v - P_A - tx_r + \frac{kt(1-\lambda)}{4} \\
EU(x_r, B_3) &= v - P_B - t(1-x_r) + \frac{kt(1-\lambda)}{4}
\end{aligned}
\tag{4.53}
$$

Region 4: $I_4 = [\hat{x}^{BK}, \hat{x}^{AR}]$ or $I_4 = [\hat{x}^{AK}, \hat{x}^{AR}]$ or I_4 is not defined depending on the case. Nonetheless, if a buyer is in Region 4, then she keeps the product if she buys from firm B. However, in this region, buyers who buy from firm A decide whether to keep or return after learning their true location once they make the purchase. The utility levels are

$$
\begin{aligned}
EU(x_r, A_4) &= v - \frac{P_B + P_A + t + R_A P_A - 2kt}{2} + \frac{(P_A - P_B - t - R_A P_A + 2tx_r)^2}{4kt(\lambda+3)} \\
EU(x_r, B_4) &= v - P_B - t(1-x_r) + \frac{kt(1-\lambda)}{4}
\end{aligned}
\tag{4.54}
$$

Region 5: $I_5 = [\hat{x}^{AR}, 1]$. If a buyer's reference location, x_r, is between $\hat{x}^{AR}$ and one, then she keeps the product if she buys from firm B and returns the product if she buys from firm A. Hence, we can write her utility levels as

$$
\begin{aligned}
EU(x_r, A_5) &= v - P_B - R_A P_A - t(1-x_r) + \frac{kt(1-\lambda)}{4} \\
EU(x_r, B_5) &= v - P_B - t(1-x_r) + \frac{kt(1-\lambda)}{4}
\end{aligned}
\tag{4.55}
$$

Lemma 4. $\Delta_l(x_r)$ *is a continuous function on every point where* $x_r \in [0,1]$.

Proof. The proof is similar to the proof of Lemma 1. See the Appendix for details.

□

Lemma 5. $\Delta_l(x_r)$ *is a non-increasing function on every point where* $x_r \in [0,1]$.

Proof. The proof is similar to the proof of Lemma 2. See the Appendix for details.

□

Lemma 6. *There exists a threshold point $\tilde{x}_r$ such that buyers on the left side of it buy from firm A, and those on the right side of it buy from firm B, unless $R_A = R_B = 0$.*

Proof. Follows from Lemma 4 and Lemma 5. □

We can now start to analyze the effects of different refund rates in the market and their equilibrium price points.

4.2.1 Full Refund vs Full Refund

When both of firms adopt full refund, that is, $R_A = R_B = 0$, returning a product has no additional cost for the buyer, since the price paid is fully refunded. Inserting $R_A = R_B = 0$ into the threshold equations, we get

$$\hat{x}^{AK} = \hat{x}^{BR} = \frac{P_B - P_A + t - kt(\lambda + 3)}{2t}$$
$$\hat{x}^{AR} = \hat{x}^{BK} = \frac{P_B - P_A + t + kt(\lambda + 3)}{2t}$$
$$\hat{x}^{A} = \hat{x}^{B} = \frac{P_B - P_A + t(\lambda x_r + x_r + 1)}{t(\lambda + 3)}$$

When $R_A = R_B = 0$, we are in Case 3, and the market is split into only three regions as Figure 6, where $\hat{x}^{AK} = \hat{x}^{BR}$ and $\hat{x}^{AR} = \hat{x}^{BK}$, and Region 2 and Region 4 do not exist.

In each region, $l = 1,3,5$, buying from any firm results with same expected utility $EU(x_r, A_l) = EU(x_r, B_l)$. Hence $\Delta_1(x_r) = \Delta_3(x_r) = \Delta_5(x_r) = 0$. When this happens, each buyer can choose Firm i with the probability $\frac{1}{2}$. For Region 1, if a buyer buys from firm B, she returns the product, $x_r < \hat{x}^{BR}$, however if she buys from firm A, she keeps the product,$x_r < \hat{x}^{AK}$. Thus, after the return/keep decisions, every buyer in Region 1 ends up with the product from firm A. For Region 5, if a buyer buys from firm A, she returns the product, $x_r > \hat{x}^{AR}$, however if she buys from firm B, she keeps the product,$x_r > \hat{x}^{BK}$. Thus, after the return/keep decisions, every buyer in Region 5 ends up with the product from firm B. The realized demand of a firm is given by the set of buyers who either buy from this firm and keep the product or buy from other firm and return the product. Then, when the refund rates are $R_A = R_B = 0$, the expected profit levels as functions of the prices are expressed as

$$E\pi_A(P_A,P_B) = \hat{x}^{AK}P_A + \frac{1}{2}\int_{\hat{x}^{AK}}^{\hat{x}^{AR}}\int_{x_r-k}^{\hat{x}^{A}} \frac{P_A}{2k}dx\,dx_r + \frac{1}{2}\int_{\hat{x}^{BR}}^{\hat{x}^{BK}}\int_{x_r-k}^{\hat{x}^{B}} \frac{P_A}{2k}dx\,dx_r$$

$$E\pi_B(P_A,P_B) = (1-\hat{x}^{BK})P_B + \frac{1}{2}\int_{\hat{x}^{AK}}^{\hat{x}^{AR}}\int_{\hat{x}^{A}}^{x_r+k} \frac{P_B}{2k}dx\,dx_r + \frac{1}{2}\int_{\hat{x}^{BR}}^{\hat{x}^{BK}}\int_{\hat{x}^{B}}^{x_r+k} \frac{P_B}{2k}dx\,dx_r \tag{4.56}$$

The best-response functions are then given by

$$\begin{aligned}\frac{\partial E\pi_A(P_A,P_B)}{\partial P_A} &= \frac{1}{2} - \frac{2P_A - P_B}{2t} = 0\\ \frac{\partial E\pi_B(P_A,P_B)}{\partial P_B} &= \frac{1}{2} + \frac{P_A - 2P_B}{2t} = 0\end{aligned} \tag{4.57}$$

Equation 4.57, we find $P_A = P_B = t$. Inserting these prices into the equations for the thresholds, we get

$$\begin{aligned}\hat{x}^{AK} = \hat{x}^{BR} &= \frac{1}{2} - \frac{k(\lambda+3)}{2} \quad \in \quad [0,1]\\ \hat{x}^{AR} = \hat{x}^{BK} &= \frac{1}{2} + \frac{k(\lambda+3)}{2} \quad \in \quad [0,1]\end{aligned} \tag{4.58}$$

Note that these thresholds are all interior points in $[0,1]$. Also, the second order derivatives of the expected profit functions are negative.

$$\frac{\partial^2 E\pi_A(P_A,P_B)}{\partial P_A^2} = \frac{\partial^2 E\pi_B(P_A,P_B)}{\partial P_B^2} = -\frac{1}{t} \quad < \quad 0 \tag{4.59}$$

And the expected profit levels are

$$E\pi_A(P_A,P_B) = E\pi_B(P_A,P_B) = \frac{t}{2} \quad > \quad 0 \tag{4.60}$$

Thus, there is an equilibrium with $R_A = R_B = 0$, where the prices are $(P_A, P_B) = (t,t)$.

4.2.2 Full Refund vs No Refund

Only one of the two firms adopts a full refund policy and the other firm adopts no refund policy. Suppose $R_B = 0$ and $R_A = 1$. For now we keep R_A in order to see the expressions in the equilibrium in terms of any $R_A > 0$. Inserting $R_B = 0$ into the threshold equations, we get

$$\hat{x}^{AK} = \frac{P_B - P_A + R_A P_A + t - kt(\lambda+3)}{2t} > \hat{x}^{BR} = \frac{P_B - P_A + t - kt(\lambda+3)}{2t}$$
$$\hat{x}^{AR} = \frac{P_B - P_A + R_A P_A + t + kt(\lambda+3)}{2t} > \hat{x}^{BK} = \frac{P_B - P_A + t + kt(\lambda+3)}{2t}$$
$$\hat{x}^{A} = \frac{P_B - P_A + R_A P_A + t(\lambda x_r + x_r + 1)}{t(\lambda+3)} > \hat{x}^{B} = \frac{P_B - P_A + t(\lambda x_r + x_r + 1)}{t(\lambda+3)}$$

Unlike Section 4.2.1, in this possible equilibrium configuration, we cannot be sure about the relation between $\hat{x}^{AK}$ and $\hat{x}^{BK}$. Hence, we have to check for two cases, one with $\hat{x}^{AK} > \hat{x}^{BK}$, and the other with $\hat{x}^{BK} > \hat{x}^{AK}$.

Case 1: Let $\hat{x}^{BK} > \hat{x}^{AK}$. When $R_B = 0$, returning a product to firm B has no additional cost for the buyer, since the price paid is fully refunded. Inserting $R_B = 0$ into the equation Equation 4.50 and Equation 4.51, we get

$$\begin{aligned} \Delta_1(x_r) &= 0 \\ \Delta_2(x_r = \hat{x}^{BR}) &= 0 \\ \Delta_2(x_r = \hat{x}^{AK}) &= -\frac{(R_A P_A)^2}{4kt(\lambda+3)} \end{aligned} \tag{4.61}$$

Since we prove $\frac{\partial \Delta_l(x_r)}{\partial x_r} \leq 0$ where $l \in 2,3,4$ in Lemma 5, we know for sure $\tilde{x}_r = \hat{x}^{BR}$ by Equation 4.61. Hence, buyers with $x_r > \hat{x}^{BR}$ buy from firm B and buyers with $x_r \leq \hat{x}^{BR}$ buy either from firm A or firm B, each with the probability $\frac{1}{2}$. For Region 1, if a buyer buys from firm B, she returns the product, $x_r < \hat{x}^{BR}$, however if she buys from firm A, she keeps the product,$x_r < \hat{x}^{AK}$. Thus, after the return/keep decisions, every buyer in Region 1 ends up with the product from firm A. Therefore, buyers in Region 1 buy from A and the rest of them buy from firm B. Then, when the refund rates are R_A and $R_B = 0$, the expected profit levels as functions of the prices are expressed as

$$E\pi_A(P_A,P_B) = \hat{x}^{BR}P_A + \int_{\hat{x}^{BR}}^{\hat{x}^{BK}} \int_{x_r-k}^{\hat{x}^{B}} \frac{P_A}{2k} dx\, dx_r$$

$$E\pi_B(P_A,P_B) = (1-\hat{x}^{BK})P_B + \int_{\hat{x}^{BR}}^{\hat{x}^{BK}} \int_{\hat{x}^{B}}^{x_r+k} \frac{P_B}{2k} dx\, dx_r \tag{4.62}$$

The best-response functions are then given by

$$\begin{aligned} \frac{\partial E\pi_A(P_A,P_B)}{\partial P_A} &= \frac{1}{2} - \frac{2P_A - P_B}{2t} \\ \frac{\partial E\pi_B(P_A,P_B)}{\partial P_B} &= \frac{1}{2} + \frac{P_A - 2P_B}{2t} \end{aligned} \tag{4.63}$$

Solving Equation 4.63, we find $P_A = P_B = t$ and inserting these prices into the equations for the thresholds, we get

$$\frac{\partial^2 E\pi_A(P_A,P_B)}{\partial P_A^2} = \frac{\partial^2 E\pi_B(P_A,P_B)}{\partial P_B^2} = -\frac{1}{t} < 0$$

$$E\pi_A(P_A,P_B) = E\pi_B(P_A,P_B) = \frac{t}{2} > 0$$

and the thresholds are

$$\begin{aligned} \hat{x}^{BR} &= \frac{1}{2} - \frac{k(\lambda+3)}{2} \in [0,1] & \hat{x}^{BK} &= \frac{1}{2} + \frac{k(\lambda+3)}{2} \in [0,1] \\ \hat{x}^{AK} &= \frac{1}{2} - \frac{k(\lambda+3)}{2} + \frac{R_A}{2} \in [0,1] & \hat{x}^{AR} &= \frac{1}{2} + \frac{k(\lambda+3)}{2} + \frac{R_A}{2} \notin [0,1] \end{aligned} \tag{4.64}$$

With the result above, we see that one of our threshold point is not an interior point in the equilibrium: $\hat{x}^{AR} \notin [0,1]$. Hence, we have to move $\hat{x}^{AR}$ to outside of $[0,1]$ and recalculate firms' profit according to Figure 8.

When we recalculate firms' new profit according to Figure 8, expected profit equations of both firms remain the same because $\hat{x}^{AR}$ is not used in any of the expected profit equations. These results are valid for every $R_A \in (0,1]$ because any $R_A > 0$ results with every buyer to buy from firm B except for those buyers who are located in Region 1. Buyers, who are in Region 1, buy the product from firm A and keep it; therefore, they are not affected by R_A. With the remodeling of the thresholds in the market, all the assumptions we make are satisfied with these results. Hence, there is no profitable price deviation and there is an equilibrium where $(P_A, P_B) = (t, t)$.

Case 2: Let $\hat{x}^{AK} > \hat{x}^{BK}$. Now, the thresholds and buyers' decisions change to the one depicted in Figure 5. Even with these changes, Equation 4.61 is still valid and $\tilde{x}_r = \hat{x}^{BR}$. Hence, buyers, who are on the left side of $\hat{x}^{BR}$, buy from firm A and the others buy from firm B at the beginning. In Case 2, buyers who are in Region 3 now keep the product regardless of the firm from which initial purchase is made. Only difference here is the start and end points of Region 3, and calculations of the expected profits result with same equations as Equation 4.62. Hence, we find equilibrium price points same as Case 1 where $(P_A, P_B) = (t, t)$. Inserting this price pair we get the same results as in Equation 4.64 and Figure 8.

To sum up, when one of the firms offers full refund, and the other one offers no refund or less than full refund, there is an equilibrium independent from the comparison of $\hat{x}^{AK}$ and $\hat{x}^{BK}$.

4.2.3 No Refund vs No Refund

In this section, we consider the case where both firms announce their refund policies as no refund, that is, $R_A = R_B = 1$. Inserting $R_A = R_B = 1$ into the threshold equations, we get

$$\hat{x}^{AK} = \frac{P_B + t - kt(\lambda+3)}{2t} \quad > \quad \hat{x}^{BR} = \frac{-P_A + t - kt(\lambda+3)}{2t}$$
$$\hat{x}^{AR} = \frac{P_B + t + kt(\lambda+3)}{2t} \quad > \quad \hat{x}^{BK} = \frac{-P_A + t + kt(\lambda+3)}{2t}$$
$$\hat{x}^{A} = \frac{P_B + t(\lambda x_r + x_r + 1)}{t(\lambda+3)} \quad > \quad \hat{x}^{B} = \frac{-P_A + t(\lambda x_r + x_r + 1)}{t(\lambda+3)}$$

In this refund policy configuration, the threshold $\tilde{x}_r$ cannot be located as in the previous refund policy configurations. This threshold, $\tilde{x}_r$, can be any where in $[\hat{x}^{BR}, \hat{x}^{AR}]$. Hence, there are three possible regions for possible equilibrium points: Region 2, 3 and 4. Because of the symmetry between Region 2 and Region 4, it is sufficient to check for only Regions 2 and 3. We proceed with checking whether $\tilde{x}_r$

can be in one of these two regions. However, the regions depend on the two possible cases: case with $\hat{x}^{BK} > \hat{x}^{AK}$, and case with $\hat{x}^{AK} > \hat{x}^{BK}$.

Case 1: Let $\hat{x}^{BK} > \hat{x}^{AK}$.

Case 1.1: $\tilde{x}_r \in [\hat{x}^{BR}, \hat{x}^{AK}]$: This means that $\tilde{x}_r$ is in Region 2 and it is equal to the point x_r where $\Delta_2(x_r) = 0$. When we solve it, there are two possible $\tilde{x}_r$ values.

$$\begin{aligned} x_{r_1} &= -\frac{P_A - t + 3kt + k\lambda t - 2\sqrt{P_B k t(\lambda+3)}}{2t} \\ x_{r_2} &= -\frac{P_A - t + 3kt + k\lambda t + 2\sqrt{P_B k t(\lambda+3)}}{2t} \end{aligned} \tag{4.65}$$

To check these two possible $\tilde{x}_r$ values, first we calculate profit equations for both firms. We use Figure 9 to calculate market share of both firms and find firms' profits via Equation 4.66.

$$\begin{aligned} E\pi_A(P_A, P_B) &= \tilde{x}_r P_A + \int_{\tilde{x}_r}^{\hat{x}^{BK}} \int_{x_r-k}^{\hat{x}^B} \frac{P_A}{2k} dx\, dx_r \\ E\pi_B(P_A, P_B) &= (1-\hat{x}^{BK})P_B + \int_{\tilde{x}_r}^{\hat{x}^{BK}} \int_{\hat{x}^B}^{x_r+k} \frac{P_B}{2k} dx\, dx_r + \int_{\tilde{x}_r}^{\hat{x}^{BK}} \int_{x_r-k}^{\hat{x}^B} \frac{P_B}{2k} dx\, dx_r \end{aligned} \tag{4.66}$$

Note that $E\pi_B(P_A, P_B)$ is also equal to $(1-\tilde{x}_r)P_B$ because Firm B does not give any money back to the buyers who decide to return, since $R_B = 1$. Now, we first try x_{r_1} to find an equilibrium by inserting it into Equation 4.66. When we take derivatives of Equation 4.66 with respect to each firm's price, we get

$$\begin{aligned} \frac{\partial E\pi_A(P_A, P_B)}{\partial P_A} &= \frac{1}{2} - \frac{2P_A - P_B}{2t} \\ \frac{\partial E\pi_B(P_A, P_B)}{\partial P_B} &= \frac{1}{2} + \frac{k(\lambda+3)}{2} + \frac{P_A - 3\sqrt{P_B k t(\lambda+3)}}{2t} \end{aligned} \tag{4.67}$$

Equating the expressions in Equation 4.67 to zero, we find two possible (P_A, P_B) pairs:

$$P_{A_1} = 24kt - t - 3t\sqrt{k(\lambda+3)(21k+7k\lambda-3)} + 8kt\lambda$$
$$P_{B_1} = 48kt - 3t - 6t\sqrt{k(\lambda+3)(21k+7k\lambda-3)} + 16kt\lambda$$

$$P_{A_2} = 24kt - t + 3t\sqrt{k(\lambda+3)(21k+7k\lambda-3)} + 8kt\lambda$$
$$P_{B_2} = 48kt - 3t + 6t\sqrt{k(\lambda+3)(21k+7k\lambda-3)} + 16kt\lambda \tag{4.68}$$

To find a real price equilibrium, both of the prices should be real values. Hence, the expression $21k + 7k\lambda - 3$ in the root must be positive. Thus, the value of $k(\lambda + 3)$ should be greater than or equal to $\frac{3}{7}$. When we try (P_{A_1}, P_{B_1}) pair, $\tilde{x}_r - \hat{x}^{BK} = \sqrt{(k(\lambda+3))}\sqrt{(16k(\lambda+3) - 6\sqrt{(k(\lambda+3))}\sqrt{(7k(\lambda+3)-3)} - 3)} - k(\lambda+3)$. For the values $k(\lambda+3) \geq \frac{3}{7}$, $\tilde{x}_r - \hat{x}^{BK} > 0$. That means $\tilde{x}_r$ cannot be in Region 2. It is a contradiction to our assumptions and there is no equilibrium in this case. When we try (P_{A_2}, P_{B_2}) pair, $\hat{x}^{BR}$ and $\hat{x}^{BK} < 0$ and $\hat{x}^{AR}$ and $\hat{x}^{KK} > 1$ for the values $k(\lambda + 3) \geq \frac{3}{7}$. It is a contradiction to our assumptions as well and there is no equilibrium in this case either.

When we check for x_{r_2}, we use Figure 9 and Equation 4.66 again because they are still valid. However, expanded version of Equation 4.66 changes since we try to find optimal prices for x_{r_2} instead of x_{r_1}. When we inserted x_{r_2} and take derivatives of firms' expected profits with respect to firms' prices, we find equations below.

$$\frac{\partial E\pi_A(P_A, P_B)}{\partial P_A} = \frac{1}{2} - \frac{2P_A - P_B}{2t}$$
$$\frac{\partial E\pi_B(P_A, P_B)}{\partial P_B} = \frac{1}{2} + \frac{k(\lambda+3)}{2} + \frac{P_A + 3\sqrt{P_B ktk(\lambda+3)}}{2t} \quad > \quad 0 \tag{4.69}$$

Since $\frac{\partial E\pi_B(P_A,P_B)}{\partial P_B} > 0$, we cannot find any (P_A, P_B) pair that makes $\frac{\partial E\pi_A(P_A,P_B)}{\partial P_A} = \frac{\partial E\pi_A(P_A,P_B)}{\partial P_A} = 0$. Hence, we can say that there is not any equilibrium where $\tilde{x}_r \in [\hat{x}^{BR}, \hat{x}^{AK}]$, which is Region 2. By a symmetric analysis, there is no equilibrium in Region 4 either.

Case 1.2: $\tilde{x}_r \in [\hat{x}^{AK}, \hat{x}^{BK}]$: This means that $\tilde{x}_r$ is in Region 3 and it is equal to the point x_r where $\Delta_3(x_r) = 0$. When we solve it, there is one possible $\tilde{x}_r$ value.

$$\tilde{x}_r = \frac{kP_B(\lambda+3)}{P_A+P_B} - \frac{P_A - P_B - 2t + 6kt + 2k\lambda t}{4t} \tag{4.70}$$

We calculate profit equations for both firms by inserting $\tilde{x}_r$ value we find in Equation 4.70. We use Figure 10 to calculate market shares of both firms and find firms' profits.

$$\begin{gathered} E\pi_A(P_A,P_B) = \tilde{x}_r P_A + \int_{\tilde{x}_r}^{\hat{x}^{BK}} \int_{x_r-k}^{\hat{x}^{B}} \frac{P_A}{2k} dx\, dx_r \\ E\pi_B(P_A,P_B) = (1-\tilde{x}_r)P_B + \int_{\hat{x}^{AK}}^{\tilde{x}_r} \int_{\hat{x}^{A}}^{x_r+k} \frac{P_B}{2k} dx\, dx_r \end{gathered} \tag{4.71}$$

We try x_r to find an equilibrium by inserting to Equation 4.71. When we take derivatives of Equation 4.71 with respect to each firm's price and equalize to zero, we find two possible (P_A, P_B) pairs:

$$\begin{gathered} P_{A_1} = \frac{9kt}{2} + \frac{t\sqrt{k(\lambda+3)(9k(\lambda+3)-8)}}{2} + \frac{3kt\lambda}{2} \\ P_{B_1} = \frac{9kt}{2} + \frac{t\sqrt{k(\lambda+3)(9k(\lambda+3)-8)}}{2} + \frac{3kt\lambda}{2} \\ \\ P_{A_2} = \frac{9kt}{2} - \frac{t\sqrt{k(\lambda+3)(9k(\lambda+3)-8)}}{2} + \frac{3kt\lambda}{2} \\ P_{B_2} = \frac{9kt}{2} - \frac{t\sqrt{k(\lambda+3)(9k(\lambda+3)-8)}}{2} + \frac{3kt\lambda}{2} \end{gathered} \tag{4.72}$$

Now, for a real valued price equilibrium, the value of $k(\lambda+3)$ should be greater than or equal to $\frac{8}{9}$. When we insert (P_{A_1}, P_{B_1}) pair into our threshold values, we find:

$$\begin{aligned} \hat{x}^{AK} &= \frac{1}{2} - \frac{\sqrt{k(\lambda+3)(9k(\lambda+3)-8)}}{4} + \frac{k(\lambda+3)}{4} \\ \hat{x}^{BK} &= \frac{1}{2} + \frac{\sqrt{k(\lambda+3)(9k(\lambda+3)-8)}}{4} - \frac{k(\lambda+3)}{4} \\ \hat{x}^{AK} - \hat{x}^{BK} &= \frac{k(\lambda+3)}{2} - \frac{\sqrt{k(\lambda+3)(9k(\lambda+3)-8)}}{2} \end{aligned}$$

For every $k(\lambda+3) \geq \frac{8}{9}$, we have $\hat{x}^{AK} > \hat{x}^{BK}$. However, this contradicts our assumption $\hat{x}^{AK} < \hat{x}^{BK}$. Now, when we insert (P_{A_2}, P_{B_2}) pair into our variable, we find:

$$\hat{x}^{AK} = \frac{1}{2} + \frac{\sqrt{k(\lambda+3)(9k(\lambda+3)-8)}}{4} + \frac{k(\lambda+3)}{4}$$
$$\hat{x}^{BK} = \frac{1}{2} - \frac{\sqrt{k(\lambda+3)(9k(\lambda+3)-8)}}{4} - \frac{k(\lambda+3)}{4}$$
$$\hat{x}^{AK} - \hat{x}^{BK} = \frac{k(\lambda+3)}{2} + \frac{\sqrt{k(\lambda+3)(9k(\lambda+3)-8)}}{2}$$

We have $\hat{x}^{AK} > \hat{x}^{BK}$. However, this contradicts our assumption $\hat{x}^{AK} < \hat{x}^{BK}$. Hence, there is no equilibrium where $\tilde{x}_r \in [\hat{x}^{AK}, \hat{x}^{BK}]$, which is Region 3, in this case.

Case 2: Let $\hat{x}^{AK} > \hat{x}^{BK}$. In this case, the thresholds are as in Figure 5, and for $\hat{x}^{AK} > \hat{x}^{BK}$ to hold we need $P_A + P_B > 2kt(\lambda+3)$.

When we compare the regions in this case (with $\hat{x}^{AK} > \hat{x}^{BK}$) to the regions in the previous case (with $\hat{x}^{BK} > \hat{x}^{AK}$), only the buyers who have $x_r \in [\hat{x}^{BK}, \hat{x}^{AK}]$ may possibly alter their buying decisions. The expected utilities in this case are $EU(x_r, A_3)$ and $EU(x_r, B_3)$, given by the equations below:

$$\begin{aligned} EU(x_r, A_3) &= v - P_A - tx_r + \frac{kt(1-\lambda)}{4} \\ EU(x_r, B_3) &= v - P_B - t(1-x_r) + \frac{kt(1-\lambda)}{4} \end{aligned} \tag{4.73}$$

Now, we check the continuity of $\Delta_l(x_r)$ function with the new $EU(x_r, A_3)$ and $EU(x_r, B_3)$ functions.

$$\begin{aligned} EU(x_r = \hat{x}^{BK}, A_2) = EU(x_r = \hat{x}^{BK}, A_3) &= v - \frac{t}{2} - \frac{P_A}{2} - \frac{kt(3\lambda+5)}{4} \\ EU(x_r = \hat{x}^{BK}, B_2) = EU(x_r = \hat{x}^{BK}, B_3) &= v - P_B - \frac{t}{2} - \frac{P_A}{2} + \frac{kt(\lambda+7)}{4} \\ \Delta_2(x_r = \hat{x}^{BK}) = \Delta_3(x_r = \hat{x}^{BK}) &= P_B - kt(\lambda+3) \\ \\ EU(x_r = \hat{x}^{AK}, A_3) = EU(x_r = \hat{x}^{AK}, A_4) &= v - P_A - \frac{t}{2} - \frac{P_B}{2} + \frac{kt(\lambda+7)}{4} \\ EU(x_r = \hat{x}^{AK}, B_3) = EU(x_r = \hat{x}^{AK}, B_4) &= v - \frac{t}{2} - \frac{P_B}{2} - \frac{kt(3\lambda+5)}{4} \\ \Delta_3(x_r = \hat{x}^{AK}) = \Delta_4(x_r = \hat{x}^{AK}) &= kt(\lambda+3) - P_A \end{aligned} \tag{4.74}$$

When we check the derivative of $\Delta_3(x_r)$ with respect to x_r, we find $\frac{\partial \Delta_3(x_r)}{\partial x_r} = -2t < 0$. Hence, we find the same specifications regarding $\Delta_l(x_r)$ function and its derivative function as in the previous case. Therefore, there are three possible regions, Region 2, 3 and 4, for possible equilibrium points. Because of the symmetry between Region 2 and Region 4, it is sufficient to check for only one of these regions, thus we check Region 2 and 3.

Case 2.1: $\tilde{x}_r \in [\hat{x}^{BR}, \hat{x}^{BK}]$: This means that $\tilde{x}_r$ is in Region 2 and it is equal to the point x_r where $\Delta_2(x_r) = 0$. When we solve it, there are two possible $\tilde{x}_r$ values.

$$
\begin{aligned}
x_{r_1} &= -\frac{P_A - t + 3kt + k\lambda t - 2\sqrt{P_B k t(\lambda+3)}}{2t} \\
x_{r_2} &= -\frac{P_A - t + 3kt + k\lambda t + 2\sqrt{P_B k t(\lambda+3)}}{2t}
\end{aligned}
\tag{4.75}
$$

To check these two possible $\tilde{x}_r$ values, first we calculate profit equations for both firms. We use Figure 11 to calculate market share of both firms and find firms' profits via Equation 4.76.

$$
\begin{aligned}
E\pi_A(P_A, P_B) &= \tilde{x}_r P_A + \int_{\tilde{x}_r}^{\hat{x}^{BK}} \int_{x_r - k}^{\hat{x}^{B}} \frac{P_A}{2k} dx\, dx_r \\
E\pi_B(P_A, P_B) &= (1 - \hat{x}^{BK}) P_B + \int_{\tilde{x}_r}^{\hat{x}^{BK}} \int_{\hat{x}^{B}}^{x_r + k} \frac{P_B}{2k} dx\, dx_r + \int_{\tilde{x}_r}^{\hat{x}^{BK}} \int_{x_r - k}^{\hat{x}^{B}} \frac{P_B}{2k} dx\, dx_r
\end{aligned}
\tag{4.76}
$$

Note that $E\pi_B(P_A, P_B)$ is also equal to $(1 - \tilde{x}_r)P_B$ because Firm B does not give any money back to the buyers who decide to return, since $R_B = 1$. Now, we first try x_{r_1} to find an equilibrium by inserting it into Equation 4.76. When we take derivatives of Equation 4.76 with respect to each firm's price, we get

$$
\begin{aligned}
\frac{\partial E\pi_A(P_A, P_B)}{\partial P_A} &= \frac{1}{2} - \frac{2P_A - P_B}{2t} \\
\frac{\partial E\pi_B(P_A, P_B)}{\partial P_B} &= \frac{1}{2} + \frac{k(\lambda+3)}{2} + \frac{P_A - 3\sqrt{P_B k t(\lambda+3)}}{2t}
\end{aligned}
\tag{4.77}
$$

Equating the expressions in Equation 4.77 to zero, we find two possible (P_A, P_B) pairs:

$$P_{A_1} = 24kt - t - 3t\sqrt{k(\lambda+3)(21k+7k\lambda-3)} + 8kt\lambda$$

$$P_{B_1} = 48kt - 3t - 6t\sqrt{k(\lambda+3)(21k+7k\lambda-3)} + 16kt\lambda$$

(4.78)

$$P_{A_2} = 24kt - t + 3t\sqrt{k(\lambda+3)(21k+7k\lambda-3)} + 8kt\lambda$$

$$P_{B_2} = 48kt - 3t + 6t\sqrt{k(\lambda+3)(21k+7k\lambda-3)} + 16kt\lambda$$

These price pairs are the same pairs as in Case 2.1, and we already showed that there is no such price pair as an equilibrium: for real valued prices we need $k(\lambda+3) \geq \frac{3}{7}$, however for (P_{A_1}, P_{B_1}) pair $\tilde{x}_r > \hat{x}^{BK}$ and for (P_{A_2}, P_{B_2}) pair $\hat{x}^{BR}$ and $\hat{x}^{BK} < 0$ and $\hat{x}^{AR}$ and $\hat{x}^{AK} > 1$.

When we check for x_{r_2}, we use Figure 11 and Equation 4.76 again because they are still valid. However, expanded version of Equation 4.76 changes since we try to find optimal prices for x_{r_2} instead of x_{r_1}. When we inserted x_{r_2} and take derivatives of firms' expected profits with respect to firms' prices, we find equations below.

$$\frac{\partial E\pi_A(P_A,P_B)}{\partial P_A} = \frac{1}{2} - \frac{2P_A - P_B}{2t}$$
$$\frac{\partial E\pi_B(P_A,P_B)}{\partial P_B} = \frac{1}{2} + \frac{k(\lambda+3)}{2} + \frac{P_A + 3\sqrt{P_B ktk(\lambda+3)}}{2t} \quad > \quad 0 \tag{4.79}$$

Since $\frac{\partial E\pi_B(P_A,P_B)}{\partial P_B} > 0$, we cannot find any (P_A, P_B) pair that makes $\frac{\partial E\pi_A(P_A,P_B)}{\partial P_A} = \frac{\partial E\pi_B(P_A,P_B)}{\partial P_B} = 0$. Hence, we can say that there is not any equilibrium where $\tilde{x}_r \in [\hat{x}^{BR}, \hat{x}^{AK}]$, which is Region 2. By a symmetric analysis, there is no equilibrium in Region 4 either.

Case 2.2: $\tilde{x}_r \in [\hat{x}^{BK}, \hat{x}^{AK}]$: This means that $\tilde{x}_r$ is in Region 3 and it is equal to the point x_r where $\Delta_3(x_r) = 0$. When we solve it, there is one possible $\tilde{x}_r$ value.

$$\tilde{x}_r = \frac{P_B - P_A + t}{2t} \tag{4.80}$$

We calculate profit equations for both firms by inserting $\tilde{x}_r$ value we find in Equation 4.80. We use Figure 12 to calculate market shares of both firms and find firms' profits on

$$\begin{aligned} E\pi_A(P_A, P_B) &= \tilde{x}_r P_A = \frac{P_A}{2} - \frac{P_A(P_A - P_B)}{2t} \\ E\pi_B(P_A, P_B) &= (1 - \tilde{x}_r) P_B = \frac{P_B}{2} + \frac{P_B(P_A - P_B)}{2t} \end{aligned} \tag{4.81}$$

We take the derivatives of Equation 4.81 with respect to each firm's price

$$\begin{aligned} \frac{\partial E\pi_A(P_A, P_B)}{\partial P_A} &= \frac{1}{2} - \frac{2P_A - P_B}{2t} \\ \frac{\partial E\pi_B(P_A, P_B)}{\partial P_B} &= \frac{1}{2} + \frac{P_A - 2P_B}{2t} \end{aligned} \tag{4.82}$$

When we equalize Equation 4.82 to zero, we find $P_A = P_B = t$. Inserting these prices to equations we find results with:

$$\begin{aligned} &\frac{\partial^2 E\pi_A(P_A, P_B)}{\partial P_A^2} = \frac{\partial^2 E\pi_B(P_A, P_B)}{\partial P_B^2} = -\frac{1}{t} < 0 && E\pi_A(P_A, P_B) = E\pi_B(P_A, P_B) = \frac{t}{2} > 0 \\ &\hat{x}^{BR} = -\frac{k(\lambda+3)}{2} \notin [0,1] && \hat{x}^{BK} = \frac{k(\lambda+3)}{2} \in [0,1] \\ &\hat{x}^{AK} = 1 - \frac{k(\lambda+3)}{2} \in [0,1] && \hat{x}^{AR} = 1 + \frac{k(\lambda+3)}{2} \notin [0,1] \end{aligned} \tag{4.83}$$

With the results above, we have $\hat{x}^{AR}, \hat{x}^{BR} \notin [0,1]$. Thus, two of our assumptions are not satisfied. We recalculate firms profit according to Figure 13 with the results we find on Equation 4.83 and the expected profit equations of both firms remain the same because both $\hat{x}^{AR}$ and $\hat{x}^{BR}$ are not used in any of those equations. Checking the region of $\tilde{x}_r$, we get

$$\begin{aligned} \Delta_2(x_r = \hat{x}^{BK}) &= \Delta_3(x_r = \hat{x}^{BK}) = t(1 - k(\lambda+3)) > 0 \\ \Delta_3(x_r = \hat{x}^{AK}) &= \Delta_4(x_r = \hat{x}^{AK}) = t(k(\lambda+3) - 1) < 0 \\ &\frac{\partial \Delta_3(x_r)}{\partial x_r} < 0 \\ \hat{x}^{AK} - \hat{x}^{BK} &= 1 - k(\lambda+3) \end{aligned} \tag{4.84}$$

Therefore, we find the same results and all of the assumptions are satisfied only if $k(\lambda+3) < 1$. Otherwise, $\hat{x}^{AK} < \hat{x}^{BK}$ and $\tilde{x}_r \notin [\hat{x}^{BK}, \hat{x}^{AK}]$ which is a contradiction. When $k(\lambda+3) < 1$, there is an equilibrium point where $(P_A, P_B) = (t, t)$.

4.3 Main Result

In this subsection, we provide a summary of our analysis and state our main result. Throughout our analysis, we focus on two models, one with loss aversion and another without loss aversion. In terms of refund policies, we consider discrete strategies: full-refund and no refund. By symmetry, there are three possible refund policy combinations in our duopoly setting with price competition. We show that there is always an equilibrium for all three refund policy combinations in an environment without loss aversion. When there is no loss aversion, all three refund policy combinations are part of an equilibrium. Thus, there is an equilibrium where both firms announce no refund policy. However, when buyers are loss averse, there are two possible equilibria in terms of refund policy combinations: either both firms announce full refund or one of them announces full refund while the other announces no refund. Thus, the refund policy combination (no refund, no refund) is no longer an equilibrium. Put differently, under loss aversion, at least one of the firms announces full refund policy. Now, we state this result formally.

Proposition 1. *When buyers are loss averse with* $\lambda > \frac{1}{k} - 3$*, if both firms adopt no-refund policies, then there is no price equilibrium.*

Thus, we have

Theorem 1. *When buyers are loss averse with* $\lambda > \frac{1}{k} - 3$*, no firm adopting full refund policy is not an equilibrium.*

Intuition for this result is that when buyers are loss averse, they are adversely affected by any possible mismatch (true location turns out to be worse than the reference location), which decreases the overall willingness to pay for the product. This, in turn, decreases the local monopoly power of the firms, driving down the firm profits. Thus, firms (at least one of them) announce full refund policy in the equilibrium so that the buyers' possible losses are diminished, and the firms restore their local monopoly power. When there is no loss aversion, firms can adopt no refund policies as the losses and gains are not affecting the buyers.

CHAPTER 5

DISCUSSION

5.1 Model Choice

While building the model to explain the relationship between loss aversion and refund policies, we tried several approaches before finalizing our model. At first, we tried to include a price dimension into the gain/loss utility in addition to the match/value dimension. It considerably complicated the calculation of the expected utilities of each buyer before calculating the equilibrium prices of both firms and comparing them with reference price values. Then, we omitted loss/gain utility in the price dimension in our model. This is not a strong simplification as it is usually the case that prices are announced by the firms and observed by the buyers, so there is no uncertainty regarding prices. The relevant uncertainty is when the buyer may not exactly know her tastes for the product; that is, she may not know her true location. Thus, we believe the match/value dimension is far more relevant in our setting.

Choosing the right reference points was another challenging topic for us. At first, we tried to build our model with discrete reference points. We tried a version of the model where all buyers have the same single reference location (we tried $\frac{1}{2}$). We also tried two discrete reference locations: half the population has $\frac{1}{4}$ and the other half has $\frac{3}{4}$ as their reference locations. It did not work, giving us either trivial solutions or no solutions. Hence, we imposed a distinct reference location, x_r, which is drawn from a uniform distribution over $[0,1]$ for each buyer and also added a stochastic distribution for the actual location from $[x_r - k, x_r + k]$ to avoid this problem. We believe this choice of modeling fits well with reality: buyers are heterogeneous in terms of their initial beliefs about their own tastes, and also each buyer is right about her own true taste up to the same degree, which is summarized by the parameter k.

During our research, another major difficulty we faced was the continuity of refund rates. At first, we tried to solve optimal refund rates from a continuous range of [0, 1] after calculating equilibrium prices. Changing the refund rates resulted in different model outputs such as the locations of $\hat{x}^{BK}$ and $\hat{x}^{AK}$. When the locations of threshold points are changed, it results in different expected profits for both firms and different optimal price values, which are used to find optimal refund rates. Therefore, we changed possible refund rates from a continuous range to a discrete range, simply full refund or no refund.

5.2 Future Work

We discuss some directions for future work in our framework. Our primary analysis focused on a duopoly price competition in a linear city model, and we placed each firm at the endpoints of the line, and locations choice was not part of the model. One can introduce location choice as a strategic variable and see how loss aversion affects the location equilibrium and the refund rates equilibrium.

Also, one can use a circular city model and place the firms on the circumference of a circle at equidistance from each other when firms enter the market. This makes it possible and easier to analyze the equilibrium number of firms and the welfare analysis in the price competition. A question one can ask would be whether loss aversion affects the equilibrium number of firms, with or without refund policies.

In our model, we assumed each buyer is uninformed: each buyer knows her true location only partially. We used a parameter k to represent the precision of buyers' about their actual location. Alternatively, one can consider a mixture of informed and uninformed buyers. The ratio of the informed buyers may have an additional role in the firms' refund policy equilibria as well as the price equilibria.

Another possible direction would be drawing buyers' actual locations from a normal distribution instead of uniform distribution. We used uniform distribution in our model for simplicity reasons, yet a normal distribution with the mean value of x_r can possibly be more realistic.

CHAPTER 6
CONCLUSION

Many firms allow the buyers to return the product even if the product is not defective. The reason may well be that the buyers are not certain regarding their valuation or taste about the product until they actually buy it. The sellers are providing the buyers with some sort of protection for the buyers against the possibility of a valuation mismatch while potentially increasing their sales. Once a purchase is made, the buyer realizes whether she likes the product or not and maybe decides whether to go for another seller's product. However, once a buyer faces some uncertainty regarding her own valuation, she may already form an expectation about her valuation and about her net utility from buying from a certain seller. In that case, it is natural to consider loss aversion within buyers' preferences. This thesis analyzes the firms' decisions of refund policies and their interaction with loss aversion in the context of duopolistic competition.

We use a dynamic Hotelling type location model where two firms compete in prices in a horizontal product differentiation environment. We use a reference-dependent model to capture buyers' preferences and incorporate loss aversion into the utility functions. Since buyers learn their exact valuation of the product only after they purchase the product, the possibility of a refund becomes important. Firms can choose either a full or no refund and announce it simultaneously. After the refund policies are publicly observed, they both announce their prices, and buyers decide which firm to buy from. Buyers can either keep or return the product while comparing their actual location and their reference location.

In our analysis, we investigate all possible refund policy combinations, whereby symmetry there are three possible ones. Both firms can adopt full refund, no refund, or one firm can adopt full refund while the other adopts no refund. We show that there is a symmetric price equilibrium for all three refund policy combinations in the absence of loss aversion. Hence, there is an equilibrium where both firms declare no refund policy. However, these three refund policies do not always constitute an equilibrium when buyers are loss-averse. If the loss aversion coefficient is sufficiently high, both firms adopting no refund policy is no longer an equilibrium. In other words, at least one of the firms should announce full refund policy under loss aversion.

To sum up, this thesis enlightens the relationship between loss aversion and refund policies. This study not only fills a theoretical gap in the literature on behavioral industrial organization but also provides a possible explanation why firms adopt full refund policies even when they do not have to, and explains to some extent the increase in the full refund policies offered by the sellers.

APPENDIX

PROOFS

Proof of Lemma 1

Proof. To prove this lemma, we need to show continuity of $EU(x_r, A)$ and $EU(x_r, B)$ piece-wise functions for each case of market outcome. In case of $R_A = R_B = 0$, $EU(x_r, A_l) = EU(x_r, B_l)$ for $l = \{1,3,5\}$ and $\Delta_l(x_r)$ is a continuous function. To check case 1 and case 2, we have to look for the points that change the buyer's decision as shown in Figure 4 and Figure 5. $\hat{x}^{BR}$ and $\hat{x}^{AR}$ points have same outcome on both cases however we have to check $\hat{x}^{AK}$ and $\hat{x}^{BK}$ points separately.

If the outcomes are the same for both cases:

$$EU(x_r = \hat{x}^{BR}, B_1) = EU(x_r = \hat{x}^{BR}, B_2) = v + kt - \frac{P_B + P_A + t + R_B P_B}{2}$$
$$EU(x_r = \hat{x}^{AR}, A_4) = EU(x_r = \hat{x}^{AR}, A_5) = v + kt - \frac{P_B + P_A + t + R_A P_A}{2}$$

If $\hat{x}^{BK} > \hat{x}^{AK}$:

$$EU(x_r = \hat{x}^{AK}, A_2) = EU(x_r = \hat{x}^{AK}, A_3) = v + kt - \frac{P_B + P_A + t + R_A P_A}{2} \quad (.1)$$
$$EU(x_r = \hat{x}^{BK}, B_3) = EU(x_r = \hat{x}^{BK}, B_4) = v + kt - \frac{P_B + P_A + t + R_B P_B}{2}$$

If $\hat{x}^{AK} > \hat{x}^{BK}$:

$$EU(x_r = \hat{x}^{BK}, B_2) = EU(x_r = \hat{x}^{BK}, B_3) = v - \frac{P_B}{2} - \frac{t}{2} - \frac{P_A}{2} + kt - \frac{R_B P_B}{2}$$
$$EU(x_r = \hat{x}^{AK}, A_3) = EU(x_r = \hat{x}^{AK}, A_4) = v - \frac{P_B}{2} - \frac{t}{2} - \frac{P_A}{2} + kt - \frac{R_A P_A}{2}$$

Hence, $\Delta_l(x_r)$ is a continuous function on every point where $x_r \in [0,1]$. □

Proof of Lemma 2

Proof. To prove this lemma, we need to show $\frac{\partial \Delta_l(x_r)}{\partial x_r} \leq 0$ on every point in the market. On case 3, $R_A = R_B = 0$, $\Delta_1(x_r) = \Delta_3(x_r) = \Delta_5(x_r) = 0$. Hence $\frac{\partial \Delta_l(x_r)}{\partial x_r} = 0$. In other cases, $\Delta_1(x_r) = R_B P_B$ and $\Delta_5(x_r) = -R_A P_A$. Therefore, $\frac{\partial \Delta_1(x_r)}{\partial x_r} = \frac{\partial \Delta_5(x_r)}{\partial x_r} = 0$. Moreover, $\frac{\partial \Delta_2(x_r)}{\partial x_r} = -t + \frac{P_B - P_A + t - R_B P_B - 2t x_r}{2k} \leq 0$. To see this, note that in this region, we have $x_r \geq \hat{x}^{BR} = \frac{P_B - P_A - R_B P_B + t - 2kt}{2t}$. That is, $2kt \geq P_B - P_A - R_B P_B + t - 2tx_r$ or $0 \geq -t + \frac{P_B - P_A - R_B P_B + t - 2tx_r}{2k} = \frac{\partial \Delta_2(x_r)}{\partial x_r}$. If $\hat{x}^{BK} > \hat{x}^{AK}$, then $\dfrac{\partial \Delta_3(x_r)}{\partial x_r} = -\dfrac{R_A P_A + R_B P_B}{2k} \leq 0$. If $\hat{x}^{AK} > \hat{x}^{BK}$, then $\dfrac{\partial \Delta_3(x_r)}{\partial x_r} = -2t < 0$. Last of all, $\dfrac{\partial \Delta_4(x_r)}{\partial x_r} = -t - \dfrac{P_B - P_A + t + R_A P_A - 2tx_r}{2k} \leq 0$. To see this, note that in this region, we have $x_r \leq \hat{x}^{AR} = \dfrac{P_B - P_A + R_A P_A + t + 2kt}{2t}$. That is, $-2kt \leq P_B - P_A + R_A P_A + t - 2tx_r$ or $0 \geq -t - \dfrac{P_B - P_A + R_A P_A + t - 2tx_r}{2k} = \dfrac{\partial \Delta_4(x_r)}{\partial x_r}$. Hence, $\Delta_l(x_r)$ is a non-increasing function on every point where $x_r \in [0,1]$. □

Proof of Lemma 4

Proof. To prove this lemma, we need to show continuity of $EU(x_r, A)$ and $EU(x_r, B)$ piece-wise functions for each case of market outcome. In case of $R_A = R_B = 0$, $EU(x_r, A_l) = EU(x_r, B_l)$ for $l = \{1,3,5\}$ and $\Delta_l(x_r)$ is a continuous function. To check case 1 and case 2, we have to look for the points that change the buyer's decision as shown in Figure 4 and Figure 5. $\hat{x}^{BR}$ and $\hat{x}^{AR}$ points have same outcome on both cases however we have to check $\hat{x}^{AK}$ and $\hat{x}^{BK}$ points separately.

If the outcomes are the same for both cases:

$$EU(x_r = \hat{x}^{BR}, B_1) = EU(x_r = \hat{x}^{BR}, B_2) = v + \frac{kt(\lambda+7)}{4} - \frac{P_B + P_A + t + R_B P_B}{2}$$
$$EU(x_r = \hat{x}^{AR}, A_4) = EU(x_r = \hat{x}^{AR}, A_5) = v + \frac{kt(\lambda+7)}{4} - \frac{P_B + P_A + t + R_A P_A}{2}$$

If $\hat{x}^{BK} > \hat{x}^{AK}$

$$EU(x_r = \hat{x}^{AK}, A_2) = EU(x_r = \hat{x}^{AK}, A_3) = v + \frac{kt(\lambda+7)}{4} - \frac{P_B + P_A + t + R_A P_A}{2} \quad (.2)$$
$$EU(x_r = \hat{x}^{BK}, B_3) = EU(x_r = \hat{x}^{BK}, B_4) = v + \frac{kt(\lambda+7)}{4} - \frac{P_B + P_A + t + R_B P_B}{2}$$

If $\hat{x}^{AK} > \hat{x}^{BK}$

$$EU(x_r = \hat{x}^{BK}, B_2) = EU(x_r = \hat{x}^{BK}, B_3) = v + \frac{kt(\lambda+7)}{4} - \frac{P_B + P_A + t + R_B P_B}{2}$$
$$EU(x_r = \hat{x}^{AK}, A_3) = EU(x_r = \hat{x}^{AK}, A_4) = v + \frac{kt(\lambda+7)}{4} - \frac{P_B + P_A + t + R_A P_A}{2}$$

Hence, $\Delta_l(x_r)$ is a continuous function on every point where $x_r \in [0,1]$. □

Proof of Lemma 5

Proof. To prove this lemma, we need to show $\frac{\partial \Delta_l(x_r)}{\partial x_r} \le 0$ on every point in the market. On case 3, $R_A = R_B = 0$, $\Delta_1(x_r) = \Delta_3(x_r) = \Delta_5(x_r) = 0$. Hence $\frac{\partial \Delta_l(x_r)}{\partial x_r} = 0$. In other cases, $\Delta_1(x_r) = R_B P_B$ and $\Delta_5(x_r) = -R_A P_A$. Therefore, $\frac{\partial \Delta_1(x_r)}{\partial x_r} = \frac{\partial \Delta_5(x_r)}{\partial x_r} = 0$. Moreover, $\frac{\partial \Delta_2(x_r)}{\partial x_r} = -t + \frac{P_B - P_A + t - R_B P_B - 2t x_r}{k(\lambda+3)} \le 0$. To see this, note that in this region, we have $x_r \ge \hat{x}^{BR} = \frac{P_B - P_A - R_B P_B + t - kt(\lambda+3)}{2t}$. That is, $kt(\lambda+3) \ge P_B - P_A - R_B P_B + t - 2t x_r$ or $0 \ge -t + \frac{P_B - P_A - R_B P_B + t - 2t x_r}{k(\lambda+3)} = \frac{\partial \Delta_2(x_r)}{\partial x_r}$. If $\hat{x}^{BK} > \hat{x}^{AK}$, then $\frac{\partial \Delta_3(x_r)}{\partial x_r} = -\frac{R_A P_A + R_B P_B}{k(\lambda+3)} \le 0$. If $\hat{x}^{AK} > \hat{x}^{BK}$, then $\frac{\partial \Delta_3(x_r)}{\partial x_r} = -2t < 0$. Last of all, $\frac{\partial \Delta_4(x_r)}{\partial x_r} = -t - \frac{P_B - P_A + t + R_A P_A - 2t x_r}{k(\lambda+3)} \le 0$. To see this, note that in this region, we have $x_r \le \hat{x}^{AR} = \frac{P_B - P_A + R_A P_A + t + kt(\lambda+3)}{2t}$. That is, $-kt(\lambda+3) \le P_B - P_A + R_A P_A + t - 2t x_r$ or $0 \ge -t - \frac{P_B - P_A + R_A P_A + t - 2t x_r}{k(\lambda+3)} = \frac{\partial \Delta_4(x_r)}{\partial x_r}$. Hence, $\Delta_l(x_r)$ is a non-increasing function on every point where $x_r \in [0,1]$. □

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