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BENDING ANALYSIS OF CIRCULAR PLATES

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A thesis submitted by Ahmet KÖME in partial fulfilment of the requirements for the degree of MASTER OF SCIENCE is approved by the committee on 12.01.2022 in Department of Civil Engineering, Mechanics Program.

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Ahmet KÖME

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Dedicated to my father

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Ahmet KÖME

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LIST OF SYMBOLS

m_r, m_θ	Bending moments
D	Bending stiffness of the plate
w	Deflection
u, w	Displacement components
q, Q	Distributed load and nondimensional distributed load
q_f	Elastic foundation interface pressure
M_r, M_θ	Nondimensional bending moments
Ω, μ	Nondimensional maximum deflections
α	Nondimensional nonhomogeneous Winkler parameter
N_r, N_θ	Nondimensional normal forces
K_g, K_{gg}	Nondimensional Pasternak parameters
ξ	Nondimensional radial coordinate
K_w, K_{ww}	Nondimensional Winkler parameters
λ	Nonhomogeneous Winkler parameter
k_g	Pasternak parameter
n_r, n_θ	Normal forces
η	Number of terms in DTM
β, Γ	Orthotropy parameters
c	Parameter of thickness
h	Plate thickness
ν_r, ν_θ	Poisson's ratios
a, r	Radius of the plate, radial coordinate
ψ	Rotation
κ	Shear correction factor
q_r, Q_θ	Shear force, nondimensional shear force
G_{rz}	Shear modulus
$m_{r\theta}$	Torsional moment
k_w, k_3	Winkler parameter and nonlinear Winkler parameter
E_r, E_θ	Young's moduli for r and θ directions, respectively

LIST OF ABBREVIATIONS

BVP	Boundary Value Problem
(C), (S)	Clamped, Simply supported
CPT	Classical Plate Theory
DQM	Differential Quadrature Method
DTM	Differential Transform Method
DSC	Discrete Singular Convolution
FDM	Finite Difference Method
FEM	Finite Element Method
FSDT	First Order Shear Deformation Theory
FB	Forward-Backward Difference
FCB	Forward-Central-Backward Difference
FG	Functionally Graded
IVP	Initial Value Problem

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ABSTRACT

Bending Analysis of Circular Plates

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Department of Civil Engineering

Master of Science Thesis

Supervisor: Assoc. Prof. Dr. Murat ALTEKİN

Axisymmetric bending response of shear deformable circular plates under the action of uniform transverse pressure is investigated numerically. Isotropic, transversely isotropic, and orthotropic plates are studied in this computational study. It is assumed that the plate is resting on a four-parameter elastic foundation. The problem involves nonlinearity which arises from the nonlinear Winkler-type elastic foundation. The formulation is based on the first order shear deformation theory (FSDT). Cylindrical coordinate system is used in the analysis. A large number of numerical simulations are performed to study the effects of various parameters on the maximum deflection of circular plates. The solution is obtained by means of differential transform method (DTM), and finite difference method (FDM). Among several numerical solution methods such as FDM, differential quadrature method (DQM), and finite element method (FEM), DTM has been one of the recently developed numerical techniques in the solution of boundary value problems (BVPs), and initial value problems (IVPs). DTM provides a series expansion, and therefore, the accuracy of the results depends highly on the number of terms considered in the solution. The accuracy of the results obtained in the current study is validated through comparison study. The results reveal that the material properties have dominant effect on the bending behaviour of circular plates, and the influence of the elastic foundation should be rigorously examined.

Keywords: Plate, bending, FDM, DTM, elastic foundation

ÖZET

Dairesel Plakların Eğilme Analizi

Ahmet KÖME

İnşaat Mühendisliği Bölümü

Yüksek Lisans Tezi

Danışman: Doç. Dr. Murat ALTEKİN

Kayma şekil değiştirmesi yapabilen dairesel plakların dönel simetrik eğilmesi sayısal olarak analiz edilmektedir. Plak düzgün yayılı transvers basınç yükü etkisi altındadır. İzotrop, transvers izotrop ve ortotrop plaklar çalışılmaktadır. Plağın dört parametreli elastik zemine oturduğu varsayılmaktadır. Doğrusal olmayan Winkler zemini problemdeki doğrusal olmama durumunun nedenidir. Formülasyon birinci mertebe kayma deformasyon teorisine (FSDT) dayalıdır. Analizde silindirik koordinat sistemi kullanılmaktadır. Çeşitli parametrelerin maksimum çökme üzerindeki etkisini ortaya koyabilmek için çok sayıda sayısal simülasyon gerçekleştirilmektedir. Problemin çözümünde diferansiyel dönüşüm yöntemi (DTM) ve sonlu fark yöntemi (FDM) kullanılmaktadır. Sonlu fark yöntemi (FDM), diferansiyel kuadratur yöntemi (DQM) ve sonlu eleman yöntemi (FEM) gibi sayısal çözüm yöntemleri arasında DTM, yakın geçmişte ortaya çıkmış sayısal çözüm tekniklerinden biridir. DTM ile başlangıç değer problemleri (IVPs) ve sınır değer problemleri (BVPs) çözülebilmektedir. DTM, seri açılımı içerdiğinden çözümde kullanılan terim sayısı elde edilen sonuçların hassaslığı üzerinde büyük etkiye sahiptir. Karşılaştırma yoluyla bu çalışmada elde edilen sayısal sonuçların doğruluğu gösterilmektedir. Elde edilen sonuçlar, malzeme özelliklerinin dairesel plakların eğilme davranışı üzerinde baskın etkisi olduğunu ortaya koymaktadır. Elastik zemin etkisinin titizlikle araştırılması gerektiği de belirtilmektedir.

Anahtar Kelimeler: Plak, eğilme, FDM, DTM, elastik zemin

1

INTRODUCTION

Bending analysis of plates has been one of the leading research topics in applied mechanics. Different geometries, several plate models, and various mechanical properties have been extensively studied in the literature. Since material properties have importance in structural design, studying anisotropic plates is crucial.

Plate-type structures are generally in contact with elastic medium such as soil. Due to its practical importance, soil-structure interaction has received significant attention especially in the recent publications. Winkler and Pasternak have been two of the mostly used elastic foundation models (Basic information about these models is given in Appendix).

Generally it is difficult to find exact solutions for the governing equations of plates. Therefore, various numerical methods (e.g., FDM, FEM, Ritz, DQM, DTM, DSC) have been used in the studies (e.g. [1-52]).

1.1 Literature Review

Shear deformable plates have been thoroughly investigated for more than three decades (e.g., [1-8, 11, 15, 17-21, 31-34, 36-39].

Dumir and Shingal [1] studied geometrically nonlinear axisymmetric static and transient analysis of cylindrically orthotropic circular plates by means of orthogonal point collocation method. FEM based static and dynamic analyses of plates were made by Çalışkan [2], and Şamdan [3]. Belardi et al. [4] employed the Ritz method to examine composite circular plates. Han and Liew [5] employed the differential quadrature method (DQM) to investigate the axisymmetric bending analysis of plates under mechanical load. Civalek and Ersoy [6] presented a study on the bending and free vibration analysis of circular plates using singular convolution method. Li and Lee [7] analysed axisymmetric bending of moderately thick circular plate based on Mindlin plate theory. Aykılıç [8] examined the static and dynamic analysis of circular and elliptical plates resting on a two-parameter elastic foundation using the SAP2000 software.

Soil-plate interaction has been studied in the recent publications in which mostly one-parameter Winkler, or two-parameter Pasternak foundation models have been adopted (e.g., [9-21, 26-29, 31-35, 49-51]).

Nonlinear static analysis of orthotropic rectangular plates was presented by Dumir and Bhaskar [9]. Gupta et al. [10] employed the Ritz method for the buckling and vibration of orthotropic circular. Kutlu and Omurtag [11] examined the buckling analysis of moderately thick plates resting on orthotropic elastic foundation using FEM. Orthotropic plates on elastic foundation were studied by Zenkour [12]. Yu and Xu [13] studied the large deflection of a circular plate interacting with nonlinear elastic foundation. FEM was applied to the bending analysis of rectangular and circular plates resting on two-parameter elastic foundation by Buczkowski and Torbacki [14]. Altekin [15] investigated the geometrically nonlinear axisymmetric bending analysis of cylindrically orthotropic circular plates on three-parameter nonlinear elastic foundation by means of FDM, and DQM using seven unknowns at each grid point in the solution. Shariyat and Alipour [16] studied functionally graded (FG) circular plates on nonuniform elastic foundation. Kobayashi and Sonoda [17] worked on rectangular plates on Winkler foundation by using Mindlin plate theory. The Levy type single series forms were used to obtain solutions. Şahinkaya [18] investigated thick plates resting on elastic foundation regarding to Mindlin plate theory by employing FEM. Park and Choi [19] presented a study about bending, buckling and free vibration analyses of isotropic rectangular plates resting on Pasternak foundation within the framework of a simplified version of FSDT. Solutions of shear deformable plates on elastic foundation by means of DQM were presented by Liew et al. [20], and Han and Liew [21].

The differential transform method (DTM), which was employed to electric circuits for the first time by Zhou [22] in 1986, has been one of the widely employed solution methods used by several researchers in different field of engineering. Due to its simplicity and efficiency, DTM has been successfully applied in the recent computational papers (e.g., [22-35, 44-47]).

Yalçın et al. [23] employed DTM to analyse the free vibration of circular thin plates. Lal and Ahlawat [24] used DTM to examine the axisymmetric vibrations of FG circular plates. Large deflection of orthotropic rectangular thin plates was presented by Yeh et al. [25] by means of FDM, and DTM. Shariyat and Alipour [26] studied the vibration characteristics of circular FG thin plates on elastic foundations via DTM. Salawu et al. [27] investigated the free vibration analysis of a thin rectangular plate resting on elastic foundations using two-dimensional DTM. Kumar [28] applied DTM to analyse the free vibrations of isotropic rectangular plates resting on a Winkler foundation. Abbasi et al. [29] employed DTM to obtain the solution of bending analysis of FG circular plates resting on Winkler foundation. Mukhtar [30] analyzed the free vibration of orthotropic plates according to two-variable refined plate theory using differential transform and Taylor collocation methods. Alipour et al. [31]–[33] studied on free vibration of FG thick circular plates with different edge conditions resting on Winkler and Pasternak foundation. The DTM is applied to investigate the free vibration of the plate based on FSDT. Farhatnia et al. [34] worked on the thermal buckling analysis of a FG orthotropic circular plate resting on Pasternak foundation using DTM. Zhao and Jun [35] studied on free vibration and buckling analysis of FG thin rectangular plates resting on elastic foundation by means of DTM.

1.2 Objective of the Thesis

Axisymmetric bending response of circular plates on a four-parameter elastic foundation is investigated numerically on the basis of FSDT. Rather than performing a nonlinear analysis due to nonlinear plate equations, linear plate theory is adopted in this computational study in which nonlinearity is due to the elastic foundation. The problem is solved by means of DTM, and FDM. Isotropic, transversely isotropic, and orthotropic plates are examined. Since minimization of the maximum deflection is crucial in structural design, the unified effect of the material and the elastic foundation on deformation is studied. The influence of the boundary conditions on the results is discussed.

1.3 Hypothesis

Application of DTM to the bending analysis of shear deformable axisymmetric plates resting on four-parameter elastic foundation is an original contribution of the thesis. Nonlinearity is included in the formulation due to the nonlinear Winkler elastic foundation. Almost identical results are obtained by means of FDM using FB, and FCB formulations.

2

FIRST ORDER SHEAR DEFORMATION THEORY

2.1. Fundamental Concepts and Assumptions

Various theories have been proposed in the literature for the solution of plate bending problems. One of these theories is the Kirchhoff plate theory, which is used to solve thin plates. As the plate thickness increases, shear deformations occur in the thickness direction. Reissner [36] took these deformations into account by using Kirchhoff theory in his studies. Also, he took into consideration the normal stress in the direction perpendicular to the plate direction. Mindlin [37], on the other hand, proposed the Mindlin plate theory by ignoring the normal stress in the direction perpendicular to the plate plane, unlike Reissner plate theory.

In this study, Mindlin plate theory assumptions are used in order to obtain differential equations and moments.

The fundamental assumptions used in this theory are given below [38], [39].

-The material is homogeneous, isotropic and linearly elastic.

-The deflections are small relative to the plate thickness.

-Hooke's law applies to stress-strain relations.

-By plate definition, loads are perpendicular to the plate plane.

-The effects of normal stress (σ_z) and strain (ε_z) perpendicular to the midplane are neglected. In this case, the plate length change in the z direction is neglected. Thus, it is assumed that the plate thickness does not change during deformation.

-Plate doesn't have to be thin. It also allows the solution of thick plates.

-The Kirchhoff–Love hypothesis is not valid. Normals perpendicular to the midplane before strain are not normal to the midplane after strains. So the

Bernoulli Navier hypothesis is not valid. Thus, shear strains perpendicular to the plane are not neglected.

The plate under the action of uniform transverse pressure used in this study is shown in Figure 2.1.

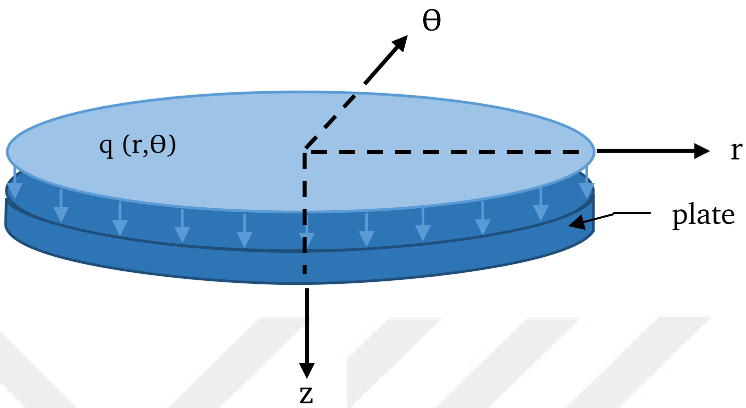

Figure 2.1 Schematic representation of a circular plate under the uniform transverse pressure

2.2. Basic Formulation for Cylindrically Orthotropic Axisymmetric Plates

A formulation derived in cylindrical coordinates involving cylindrical orthotropy is used in the study [40]. The equations of equilibrium for axisymmetric plates are given by [1], [40], [41].

$$\left(r\,n_r\right)' - n_\theta = 0 \tag{2.1}$$

$$\left(r\,m_r\right)' - m_\theta - r\,q_r = 0 \tag{2.2}$$

$$\left(rq_r\right)' + r\left(q - q_f\right) = 0 \tag{2.3}$$

The stress resultants, and the elastic foundation interface pressure are introduced by [1], [15], [16], [40], [41].

$$q_r = \kappa h G_{rz}\left(\psi + w'\right) \tag{2.4}$$

$$n_r = \frac{E_\theta h}{\left(\beta - {\nu_\theta}^2\right)}\left(u' + \nu_\theta \frac{u}{r}\right) \quad , \quad n_\theta = \frac{E_\theta h}{\left(\beta - {\nu_\theta}^2\right)}\left(\nu_\theta u' + \beta \frac{u}{r}\right) \tag{2.5}$$

$$m_r = D\left(\psi' + \nu_\theta \frac{\psi}{r}\right) \quad , \quad m_\theta = D\left(\nu_\theta \psi' + \beta \frac{\psi}{r}\right) \tag{2.6}$$

$$q_f = k_w\left(1 + \lambda r\right)w + k_3 w^3 - k_g w'' - k_g \frac{w'}{r} \tag{2.7}$$

Here, D indicates the flexural rigidity of the plate given by [1]

$$D = \frac{E_\theta h^3}{12\left(\beta - {\nu_\theta}^2\right)} \tag{2.8}$$

The elastic foundation interface pressure q_f is defined by the parameters k_w, k_g, k_3, and λ. The problem becomes nonlinear for the nonzero values of k_3.

2.3 Parameters and Nondimensional Variables

For convenience, the nondimensional variables and the parameters shown below are used in the study [1], [15], [42], [43].

$$u = hU \quad , \quad w = hW \quad , \quad w = \frac{qa^4}{D}\Omega \quad , \quad \frac{\mu}{\Omega} = 10^3 \tag{2.9}$$

$$n_r = E_\theta h N_r \quad , \quad n_\theta = E_\theta h N_\theta \quad , \quad q_r = E_\theta h Q_\theta \tag{2.10}$$

$$m_r = E_\theta h^2 M_r \quad , \quad m_\theta = E_\theta h^2 M_\theta \tag{2.11}$$

$$q = E_\theta Q \quad , \quad Q_c = Qc^4 \quad , \quad r = a\xi \quad , \quad c = \frac{a}{h} \tag{2.12}$$

$$k_w = \frac{E_\theta h^3}{a^4}K_w \quad , \quad k_g = \frac{E_\theta h^3}{a^2}K_g \quad , \quad k_3 = \frac{E_\theta h}{a^4}K_3 \tag{2.13}$$

$$k_w = \frac{D}{a^4}K_{ww} \quad , \quad k_g = \frac{D}{a^2}K_{gg} \quad , \quad \lambda = \frac{\alpha}{a} \tag{2.14}$$

$$\beta = \frac{\nu_\theta}{\nu_r} = \frac{E_\theta}{E_r} \quad , \quad \Gamma = \frac{G_{rz}}{E_r} \tag{2.15}$$

Using the nondimensional variables, and the parameters, Eqs. (2.1-2.3) can be written by

$$U' - \beta\frac{U}{\xi} + \xi U'' = 0 \tag{2.16}$$

$$\psi' - \beta\frac{\psi}{\xi}\psi\xi\psi'' - 12\kappa\frac{\Gamma}{\beta}c^2\left(\beta - \nu_\theta{}^2\right)\xi\psi - 12\kappa\frac{\Gamma}{\beta}c\left(\beta - \nu_\theta{}^2\right)\xi W' = 0 \tag{2.17}$$

$$\begin{aligned}&\kappa\frac{\Gamma}{\beta}\psi + \frac{\kappa}{c}\frac{\Gamma}{\beta}W' + \kappa\frac{\Gamma}{\beta}\xi\psi' + \frac{\kappa}{c}\frac{\Gamma}{\beta}\xi W'' + c\xi Q - \frac{K_w}{c^3}\xi W - \alpha\frac{K_w}{c^3}\xi^2 W\\ &-\frac{K_3}{c^3}\xi W^3 + \frac{K_g}{c^3}\xi W'' + \frac{K_g}{c^3}W' = 0\end{aligned} \tag{2.18}$$

where $(\ )' = \frac{d(\ )}{dr}$ for the dimensional equations such as Eqs. (2.1-2.7), and $(\ )' = \frac{d(\ )}{d\xi}$ for the nondimensional equations such as Eqs. (2.16-2.18).

2.4 Boundary Conditions

The boundary conditions, and the regularity conditions that are satisfied exactly in the solution are shown in Table 2.1.

Table 2.1 Boundary conditions and regularity conditions [15]

	r = a			r = 0		
	Boundary conditions			**Regularity conditions**		
(S)	w = 0,	u = 0,	$m_r = 0$	u = 0,	ψ = 0,	$q_r = 0$
(C)	w = 0,	u = 0,	ψ = 0	u = 0,	ψ = 0,	$q_r = 0$

3

METHOD OF SOLUTION

3.1. Differential Transform Method

Differential transform method (DTM) is a numerical method, which is based on series expansion. Using this method, linear or non-linear differential equations can be transformed into simple algebraic equations and these equations can easily be solved [44]. DTM was first proposed by Zhou [22] in 1986 and it was used to solve IVPs in electric circuits [45]. Then, Chen and Ho [46] developed the method and applied it to eigenvalue problems. Finally, Malik and Dang [47] carried out the method to continuous systems specific to vibration analysis of thin beams.

3.1.1. Mathematical Model

Differential transformation of function *y(x)* is defined as follows [23]:

$$Y(k)=\frac{1}{k!}\left[\frac{d^k y(x)}{dx^k}\right]_{x=x_0} \tag{3.1}$$

where *y(x)* is the original function and *Y(x)* is the transformed form. The differential inverse transform of *Y(x)* is defined as follows [23]:

$$y(x)=\sum_{k=0}^{\infty}(x-x_0)^k Y(k) \tag{3.2}$$

From equations (3.1) and (3.2), we obtain [23]

$$Y(k)=\sum_{k=0}^{\infty}\frac{(x-x_0)^k}{k!}\left[\frac{d^k y(x)}{dx^k}\right]_{x=x_0} \tag{3.3}$$

3.1.2. Theorems

Some of the essential theorems which are frequently used are given in Table 3.1.

Table 3.1 Fundamental theorems of DTM [23]

$f(r) = g(r) \pm h(r)$	$F[k] = G_{[k]} \pm H_{[k]}$
$f(r) = \lambda g(r)$	$F[k] = \lambda G_{[k]}$
$f(r) = g(r)h(r)$	$F[k] = \sum_{l=0}^{k} G_{[k-l]} H_{[k]}$
$f(r) = \frac{d^n g(r)}{dr^n}$	$F[k] = \frac{(k+n)!}{k!} G_{[k+n]}$
$f(r) = r^n$	$F[k] = \delta(k-n) = \begin{cases} 0 & if \quad k \neq n \\ 1 & if \quad k = n \end{cases}$

In the current study, the application of DTM to Eqs. (2.16-2.18) are presented as follows:

$$
\begin{aligned}
&\sum_{k_1=0}^{k} \delta(k_1-1)\frac{(k-k_1+1)!}{(k-k_1)!}\bar{U}(k-k_1+1) - \beta\bar{U}(k) \\
&+\sum_{k_1=0}^{k} \delta(k_1-2)\frac{(k-k_1+2)!}{(k-k_1)!}\bar{U}(k-k_1+2) = 0
\end{aligned}
\tag{3.4}
$$

$$
\begin{aligned}
&\sum_{k_1=0}^{k} \delta(k_1-1)\frac{(k-k_1+1)!}{(k-k_1)!}\bar{\psi}(k-k_1+1) - \beta\bar{\psi}(k) \\
&+\sum_{k_1=0}^{k} \delta(k_1-2)\frac{(k-k_1+2)!}{(k-k_1)!}\bar{\psi}(k-k_1+2) \\
&-12\kappa\frac{\Gamma}{\beta}c^2\left(\beta-{v_\theta}^2\right)\sum_{k_1=0}^{k} \delta(k_1-2)\bar{\psi}(k-k_1) \\
&-12\kappa\frac{\Gamma}{\beta}c\left(\beta-{v_\theta}^2\right)\sum_{k_1=0}^{k} \delta(k_1-2)\frac{(k-k_1+1)!}{(k-k_1)!}\bar{W}(k-k_1+1) = 0
\end{aligned}
\tag{3.5}
$$

$$
\begin{aligned}
&\kappa\frac{\Gamma}{\beta}\bar{\psi}(k)+\frac{\kappa}{c}\frac{\Gamma}{\beta}\frac{(k+1)!}{(k)!}\bar{W}(k+1)+\kappa\frac{\Gamma}{\beta}\sum_{k_1=0}^{k}\delta(k_1-1)\frac{(k-k_1+1)!}{(k-k_1)!}\bar{\psi}(k-k_1+1)\\
&+\frac{\kappa}{c}\frac{\Gamma}{\beta}\sum_{k_1=0}^{k}\delta(k_1-1)\frac{(k-k_1+2)!}{(k-k_1)!}\bar{W}(k-k_1+2)+cQ\delta(k-1)\\
&-\frac{K_w}{c^3}\sum_{k_1=0}^{k}\delta(k_1-1)\bar{W}(k-k_1)-\alpha\frac{K_w}{c^3}\sum_{k_1=0}^{k}\delta(k_1-2)\bar{W}(k-k_1)\\
&-\frac{K_3}{c^3}\sum_{k_3=0}^{k}\sum_{k_2=0}^{k_3}\sum_{k_1=0}^{k_2}\delta(k_1-1)\bar{W}(k_2-k_1)\bar{W}(k_3-k_2)\bar{W}(k-k_3)\\
&+\frac{K_g}{c^3}\sum_{k_1=0}^{k}\delta(k_1-1)\frac{(k-k_1+2)!}{(k-k_1)!}\bar{W}(k-k_1+2)+\frac{K_g}{c^3}\frac{(k+1)!}{(k)!}\bar{W}(k+1)=0
\end{aligned}
\tag{3.6}
$$

First, Eqs. (3.4-3.6) together with Table 2.1 are solved to find the values $\bar{W}(k),\bar{U}(k),\bar{\psi}(k)$ for $k \le k*$. The number of terms in DTM is defined by η where $\eta = 1 + k*$. Next, using the inverse transform introduced by

$$W = W(\xi) = \sum_{k=0}^{k*}\bar{W}(k)\xi^k \tag{3.7}$$

$$U = U(\xi) = \sum_{k=0}^{k*}\bar{U}(k)\xi^k \tag{3.8}$$

$$\psi = \psi(\xi) = \sum_{k=0}^{k*}\bar{\psi}(k)\xi^k \tag{3.9}$$

the expressions for the field variables *W, U,* and ψ are obtained.

3.2 Finite Difference Method

Finite difference method (FDM) is an old approximation method used for the solution of differential equations. The differential equations are replaced by algebraic equations on pointwise basis in the solution domain. Forward, central, and backward differences are used in this study.

Uniformly distributed N points are located along the radial coordinate in the domain [0 a]. The field variables *(W, U,* ψ*,* N_r*,* Q_r*,* M_r*)* are the unknowns in FDM. Using the forward, central, and the backward difference formulations given by [48].

$$f_i' \cong \frac{-3f_i + 4f_{i+1} - f_{i+2}}{2\Delta}, \qquad f_i' \cong \frac{-f_{i-1} + f_{i+1}}{2\Delta}, \qquad f_i' \cong \frac{f_{i-2} - 4f_{i-1} + 3f_i}{2\Delta} \tag{3.10}$$

Eqs. (2.1-2.6) are written in terms of the field variables, and the solution is obtained. Here, *Δ* indicates the step size in FDM, and the subscript *i* denotes the node number.

3.3 Newton-Raphson Method

Due to nonlinearity originating from the nonlinear Winkler model, nonlinear analysis is required. The Newton-Raphson method is used to solve the nonlinear equations.

4

ANALYSIS

The radius of the plate, and the shear correction factor in the computations are given by a=1 m, and κ=5/6, respectively.

4.1 Material Properties

The numerical values of the parameters β, Γ, and ν_θ shown in Table 4.1 are taken from Dumir and Shingal [1]. Unless otherwise stated ν_θ=0.25 is used in the numerical simulations.

Table 4.1 Material properties [1] and method of solution

β	Γ	ν_θ	Type of Material	Material Category	Method of Solution
1	0.4	0.25	Isotropic	M1	DTM
1	0.1	0.25	Transversely isotropic	M2	DTM
10	0.4	0.25	Orthotropic	M3	FDM

4.2 Convergence Study and Verification of the Algorithm

First, convergence studies are performed to determine (i) the number of terms in DTM, and (ii) the number of nodes in FDM (Table 4.2). The results reveal that k*=18, and N=71 are sufficient for admissible accuracy.

Table 4.2 Convergence study for the nondimensional maximum deflection of a circular plate on elastic foundation (K_w = 5, K_g = 1, K_3 =1, α = 0, Q_c = 12)

β	Γ	c	W: (S)	W: (C)	Method of Solution
1	0.4	100	1.2664	0.8658	DTM (k*=18)
1	0.4	100	1.2663	0.8657	DTM (k*=16)
1	0.4	100	1.2672	0.8656	DTM (k*=14)
10	0.4	100	1.2062	1.1016	FDM (N=71, FB)
10	0.4	100	1.2062	1.1016	FDM (N=66, FB)
10	0.4	100	1.2063	1.1017	FDM (N=61, FB)
10	0.4	100	1.2061	1.1014	FDM (N=71, FCB)
10	0.4	100	1.2061	1.1015	FDM (N=66, FCB)
10	0.4	100	1.2062	1.1017	FDM (N=61, FCB)

Next, the accuracy of the results obtained in the study is validated (Tables 4.3-4.6).

Table 4.3 Nondimensional maximum deflection μ_{max} of a homogeneous and isotropic (C) circular plate ($c = 100$, $(K_{ww}; K_3) = (200; 0)$, $Q_c = 12$, $\nu = 0.30$)

(C)	$K_{gg} = 3$	$K_{gg} = 28.9$	$K_{gg} = 300$	Method of Solution
μ_{max}	4.65	2.97	0.65	[42]
μ_{max}	4.6482	2.9723	0.6510	DTM (k*=18)

Table 4.4 Nondimensional maximum deflection of a circular plate ($K_w = 0$, $K_g = 0$, $K_3 = 0$, $\alpha = 0$, $Q_c = 12$)

β	Γ	c	Ω: (S)	Ω: (C)	W: (S)	W: (C)	Method of Solution
1	0.4	100	0.73836	0.17586			[1]
1	0.4	100	0.7384	0.1759	8.8603	2.1103	DTM (k*=18)
1	0.4	100				2.1103	[43]*
1	0.4	100			8.8335	2.1169	[SAP2000]
1	0.4	25	0.73948	0.17698			[1]
1	0.4	25	0.7395	0.1770	8.8738	2.1238	DTM (k*=18)
1	0.4	25				2.1238	[43]*
1	0.4	25			8.797125	2.122725	[SAP2000]
1	0.4	10	0.74578	0.18328			[1]
1	0.4	10	0.7458	0.1833	8.9494	2.1994	DTM (k*=18)
1	0.4	10				2.1994	[43]*
1	0.1	100	0.73858	0.17608			[1]
1	0.1	100	0.7386	0.1761	8.8630	2.1130	DTM (k*=18)
1	0.1	100				2.1130	[43]*
1	0.1	100			8.8362	2.1198	[SAP2000]
1	0.1	25	0.74308	0.18058			[1]
1	0.1	25	0.7431	0.1806	8.9170	2.1670	DTM (k*=18)
1	0.1	25				2.1670	[43]*
1	0.1	25			8.915775	2.174575	[SAP2000]
1	0.1	10	0.76828	0.20578			[1]
1	0.1	10	0.7683	0.2058	9.2194	2.4694	DTM (k*=18)
1	0.1	10				2.4694	[43]*
10	0.4	100	0.12632	0.05819			[1]
10	0.4	100	0.1263	0.0582	15.1593	6.9801	FDM (N=71, FB)
10	0.4	100	0.1263	0.0582	15.1576	6.9804	FDM (N=71, FCB)
10	0.4	100				6.9829	[43]*
10	0.4	25	0.12744	0.05932			[1]
10	0.4	25	0.1275	0.0593	15.2943	7.1151	FDM (N=71, FB)
10	0.4	25	0.1274	0.0593	15.2926	7.1154	FDM (N=71, FCB)
10	0.4	25				7.1179	[43]*
10	0.4	10	0.13374	0.06562			[1]
10	0.4	10	0.1338	0.0656	16.0503	7.8711	FDM (N=71, FB)
10	0.4	10	0.1337	0.0656	16.0486	7.8714	FDM (N=71, FCB)
10	0.4	10				7.8739	[43]*

Ω : Nondimensional maximum deflection [1]

[43]* These results are computed using the formula given in [43].

Table 4.5 Nondimensional maximum deflection of a circular plate on elastic foundation (K_w = 5, K_g = 0, K_3 = 0, α = 0, Q_c = 12)

β	Γ	c	W: (S)	W: (C)	Method of Solution
1	0.4	100	2.5306	1.3412	DTM (k*=18)
1	0.4	100	2.5261	1.3396	[SAP2000]
1	0.4	25	2.5308	1.3460	DTM (k*=18)
1	0.4	25	2.5243	1.345475	[SAP2000]
1	0.4	10	2.5315	1.3718	DTM (k*=18)
1	0.1	100	2.5306	1.3422	DTM (k*=18)
1	0.1	100	2.52510	1.3445	[SAP2000]
1	0.1	25	2.5312	1.3609	DTM (k*=18)
1	0.1	25	2.52850	1.360375	[SAP2000]
1	0.1	10	2.5340	1.4576	DTM (k*=18)
10	0.4	100	2.4338	2.1121	FDM (N=71, FB)
10	0.4	100	2.4331	2.1114	FDM (N=71, FCB)
10	0.4	25	2.4341	2.1211	FDM (N=71, FB)
10	0.4	25	2.4334	2.1204	FDM (N=71, FCB)
10	0.4	10	2.4354	2.1657	FDM (N=71, FB)
10	0.4	10	2.4348	2.1650	FDM (N=71, FCB)

Table 4.6 Nondimensional maximum deflection of a circular plate on elastic foundation (K_w = 5, K_g = 1, K_3 = 0, α = 0, Q_c = 12)

β	Γ	c	W: (S)	W: (C)	Method of Solution
1	0.4	100	1.3784	0.8868	DTM (k*=18)
1	0.4	100	1.3617	0.8811	[SAP2000]
1	0.4	25	1.3782	0.8894	DTM (k*=18)
1	0.4	25	1.3616	0.883625	[SAP2000]
1	0.4	10	1.3776	0.9035	DTM (k*=18)
1	0.1	100	1.3783	0.8873	DTM (k*=18)
1	0.1	100	1.3617	0.8816	[SAP2000]
1	0.1	25	1.3779	0.8975	DTM (k*=18)
1	0.1	25	1.361275	0.891525	[SAP2000]
1	0.1	10	1.3759	0.9487	DTM (k*=18)
10	0.4	100	1.3310	1.1832	FDM (N=71, FB)
10	0.4	100	1.3308	1.1830	FDM (N=71, FCB)
10	0.4	25	1.3343	1.1919	FDM (N=71, FB)
10	0.4	25	1.3341	1.1918	FDM (N=71, FCB)
10	0.4	10	1.3499	1.2318	FDM (N=71, FB)
10	0.4	10	1.3498	1.2317	FDM (N=71, FCB)

4.3 Numerical Examples

The effects of (i) the material properties, (ii) the parameter of thickness, and (iii) the parameters of elastic foundation on the maximum deflection are examined in several numerical examples (Tables 4.4-4.11). Deformed and undeformed plate configurations are shown in Figs. (4.1-4.2).

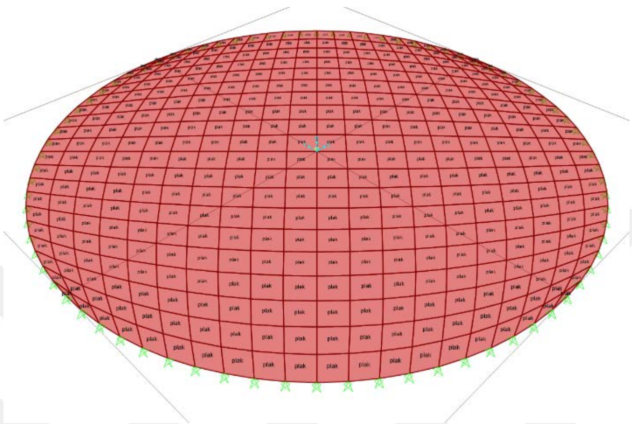

Figure 4.1 Undeformed simply supported plate configuration in SAP2000

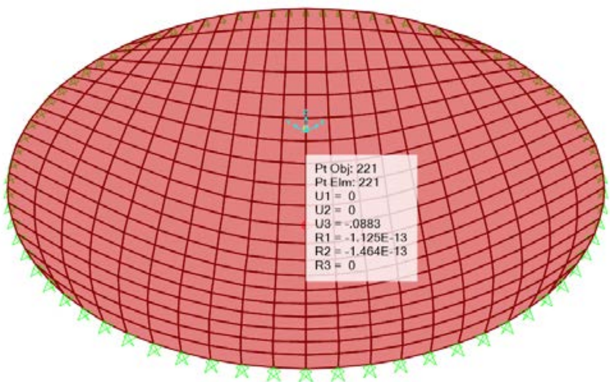

Figure 4.2 Deformed simply supported plate configuration in SAP2000

Table 4.7 Nondimensional maximum deflection of a circular plate on elastic foundation ($K_w = 5$, $K_g = 0$, $K_3 = 0$, $Q_c = 12$)

β	Γ	c	α	W: (S)	W: (C)	Method of Solution
1	0.4	100	0.050	2.4926	1.3328	DTM (k*=18)
1	0.4	100	0.025	2.5115	1.3370	DTM (k*=18)
1	0.4	10	0.050	2.4934	1.3629	DTM (k*=18)
1	0.4	10	0.025	2.5123	1.3673	DTM (k*=18)
1	0.1	100	0.050	2.4926	1.3337	DTM (k*=18)
1	0.1	100	0.025	2.5115	1.3379	DTM (k*=18)
1	0.1	10	0.050	2.4957	1.4471	DTM (k*=18)
1	0.1	10	0.025	2.5147	1.4523	DTM (k*=18)
10	0.4	100	0.050	2.3897	2.0836	FDM (N=71, FB)
10	0.4	100	0.050	2.3891	2.0829	FDM (N=71, FCB)
10	0.4	100	0.025	2.4115	2.0978	FDM (N=71, FB)
10	0.4	100	0.025	2.4109	2.0971	FDM (N=71, FCB)
10	0.4	10	0.050	2.3922	2.1359	FDM (N=71, FB)
10	0.4	10	0.050	2.3916	2.1353	FDM (N=71, FCB)
10	0.4	10	0.025	2.4135	2.1507	FDM (N=71, FB)
10	0.4	10	0.025	2.4130	2.1501	FDM (N=71, FCB)

Table 4.8 Nondimensional maximum deflection of a circular plate on elastic foundation ($K_w = 5$, $K_g = 1$, $K_3 = 0$, $Q_c = 12$)

β	Γ	c	α	W: (S)	W: (C)	Method of Solution
1	0.4	100	0.050	1.3670	0.8829	DTM (k*=18)
1	0.4	100	0.025	1.3726	0.8848	DTM (k*=18)
1	0.1	100	0.050	1.3670	0.8834	DTM (k*=18)
1	0.1	100	0.025	1.3726	0.8854	DTM (k*=18)
10	0.4	100	0.050	1.3191	1.1745	FDM (N=71, FB)
10	0.4	100	0.050	1.3190	1.1744	FDM (N=71, FCB)
10	0.4	100	0.025	1.3250	1.1788	FDM (N=71, FB)
10	0.4	100	0.025	1.3249	1.1787	FDM (N=71, FCB)

Table 4.9 Nondimensional maximum deflection of a circular plate on elastic foundation ($K_w = 5$, $K_g = 0$, $K_3 = 1$, $\alpha = 0$, $Q_c = 12$)

β	Γ	c	W: (S)	W: (C)	Method of Solution
1	0.4	100	1.8709	1.2509	DTM (k*=18)
1	0.4	25	1.8702	1.2542	DTM (k*=18)
1	0.4	10	1.8664	1.2723	DTM (k*=18)
1	0.1	100	1.8708	1.2516	DTM (k*=18)
1	0.1	25	1.8680	1.2647	DTM (k*=18)
1	0.1	10	1.8554	1.3287	DTM (k*=18)
10	0.4	100	1.6611	1.5964	FDM (N=71, FB)
10	0.4	100	1.6608	1.5959	FDM (N=71, FCB)
10	0.4	25	1.6594	1.5975	FDM (N=71, FB)
10	0.4	25	1.6590	1.5970	FDM (N=71, FCB)
10	0.4	10	1.6504	1.6019	FDM (N=71, FB)
10	0.4	10	1.6501	1.6015	FDM (N=71, FCB)

Table 4.10 Nondimensional maximum deflection of a circular plate on elastic foundation (K_w = 5, K_g = 1, K_3 =1, α = 0, Q_c = 12)

β	Γ	c	W: (S)	W: (C)	Method of Solution
1	0.4	100	1.2664	0.8658	DTM (k*=18)
1	0.4	25	1.2662	0.8681	DTM (k*=18)
1	0.4	10	1.2650	0.8805	DTM (k*=18)
1	0.1	100	1.2663	0.8663	DTM (k*=18)
1	0.1	25	1.2655	0.8753	DTM (k*=18)
1	0.1	10	1.2614	0.9198	DTM (k*=18)
10	0.4	100	1.2062	1.1016	FDM (N=71, FB)
10	0.4	100	1.2061	1.1014	FDM (N=71, FCB)
10	0.4	25	1.2085	1.1082	FDM (N=71, FB)
10	0.4	25	1.2084	1.1081	FDM (N=71, FCB)
10	0.4	10	1.2194	1.1380	FDM (N=71, FB)
10	0.4	10	1.2193	1.1378	FDM (N=71, FCB)

Table 4.11 Nondimensional maximum deflection of a circular plate on elastic foundation (K_w = 5, K_g = 1, K_3 =1, Q_c = 12)

β	Γ	c	α	W: (S)	W: (C)	Method of Solution
1	0.4	100	0.050	1.2581	0.8623	DTM (k*=18)
1	0.4	100	0.025	1.2622	0.8641	DTM (k*=18)
1	0.4	10	0.050	1.2567	0.8769	DTM (k*=18)
1	0.4	10	0.025	1.2608	0.8787	DTM (k*=18)
1	0.1	100	0.050	1.2580	0.8627	DTM (k*=18)
1	0.1	100	0.025	1.2622	0.8645	DTM (k*=18)
1	0.1	10	0.050	1.2531	0.9156	DTM (k*=18)
1	0.1	10	0.025	1.2572	0.9177	DTM (k*=18)
10	0.4	100	0.050	1.1979	1.0949	FDM (N=71, FB)
10	0.4	100	0.050	1.1978	1.0948	FDM (N=71, FCB)
10	0.4	100	0.025	1.2020	1.0982	FDM (N=71, FB)
10	0.4	100	0.025	1.2019	1.0981	FDM (N=71, FCB)
10	0.4	10	0.050	1.2113	1.1311	FDM (N=71, FB)
10	0.4	10	0.050	1.2112	1.1310	FDM (N=71, FCB)
10	0.4	10	0.025	1.2154	1.1345	FDM (N=71, FB)
10	0.4	10	0.025	1.2153	1.1344	FDM (N=71, FCB)

Bending solutions of circular plates are given in Tables 4.4-4.11. The effects of the elastic foundation are investigated in Tables 4.5-4.11. First, the effect of Winkler is studied (Table 4.5). Next, the influence of Pasternak is examined (Table 4.6). The effect of nonhomogeneous Winkler is presented in Table 4.7. A three-parameter elastic foundation including nonhomogeneous Winkler is introduced in Table 4.8. The solution due to nonlinear Winkler is shown in Table 4.9. A three parameter nonlinear elastic foundation is introduced in Table 4.10. Finally, a four-parameter nonlinear elastic foundation is presented in Table 4.11.

4.4 Numerical Results

A large variety of numerical simulations are made to highlight the influence of (i) the elastic foundation, (ii) the material properties, and (iii) the boundary conditions on the central deflection (Tables 4.4-4.11).

- The solutions reveal that compared to M2, and M3, M1 minimizes the maximum deflection of the plate in case of no elastic foundation (Table 4.4).
- The unified effects of the boundary conditions and the material properties are dominant in minimizing the maximum deflection if there is elastic foundation (Table 4.5). For example, M3, and M1 minimize the central deflection for (S), and (C) plates, respectively. This statement holds for Winkler-, nonhomogeneous Winkler-, and Pasternak-type elastic foundations (Tables 4.5-4.8). For the nonlinear elastic foundations the same statement holds (Tables 4.9-4.11).
- The results corresponding to M1, and M2 are very close to each other with increasing value of c.
- The effect of the nonhomogeneous Winkler on the results does not seem to be negligible.
- The influence of the nonlinearity depends highly on the combined effects of the boundary conditions and the material category (Tables 4.5, 4.9-4.10).

5

RESULTS AND DISCUSSIONS

Axisymmetric bending analysis of circular plates under uniformly distributed transverse pressure is made numerically within the framework of FSDT. The elastic medium which is interacting with the plate is modelled by a four-parameter elastic foundation. Cylindrical coordinate system is used in the formulation. Three types of material categories are examined: (i) Homogeneous and isotropic, (ii) Transversely isotropic, and (iii) Cylindrically orthotropic. The solution is obtained by means of DTM, and FDM. The formulation includes three field variables for DTM, and six field variables for FDM. Apart from an exceptional case, the problem is linear. The exceptional case is that nonlinear analysis is required due to the nonlinear Winkler-type elastic foundation. Newton-Raphson method is employed to solve the nonlinear equations. Simply supported (S), and clamped (C) plates are examined in the study.

A large number of numerical simulations are made to investigate the effects of several parameters (e.g., parameter of thickness, material properties, boundary conditions, and parameters of the elastic foundation) on the maximum deflection. The unified influence of the boundary conditions and the nonhomogeneous Winkler-type elastic foundation on the solution is highlighted. The results reveal that the material properties have dominant effect in minimizing the maximum deflection.

For future work nonlinear plate equations may be considered for better understanding the bending behaviour of plates if large deflection analysis is required.

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A

EFFECTS OF ELASTIC FOUNDATION

Determination of the ground effect is difficult due to the complex nature of the elastic soil environment. For this reason, some idealized soil models have been used to solve the problems of plate resting on elastic foundation.

A1. Winkler Model

One of the one-parameter soil models is Winkler model which was first proposed by Winkler [49] in 1867. According to his proposal, the deflection, *w*, of the soil environment at a point on the surface is directly proportional to the stress, q , applied at the point. Also, the deflection is not affected by stresses at other points. In this case, the soil is considered as a medium consisting of infinitely close and independent springs. A plate schema resting on a Winkler model is shown in Figure A.1.

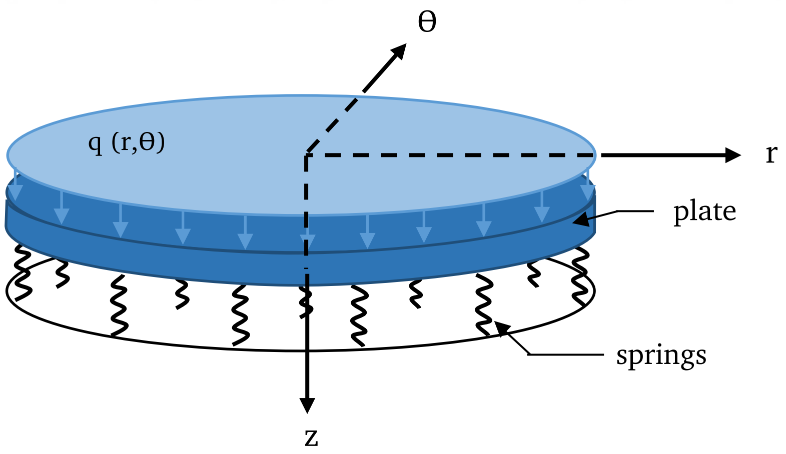

Figure A.1 A plate on Winkler model

Eq. (A.1) represents the response of the Winkler model:

$$q(r,\theta) = k_w(r,\theta) \tag{A.1}$$

where k_w is the elastic spring coefficient and expresses the reaction force with units of stress per unit length.

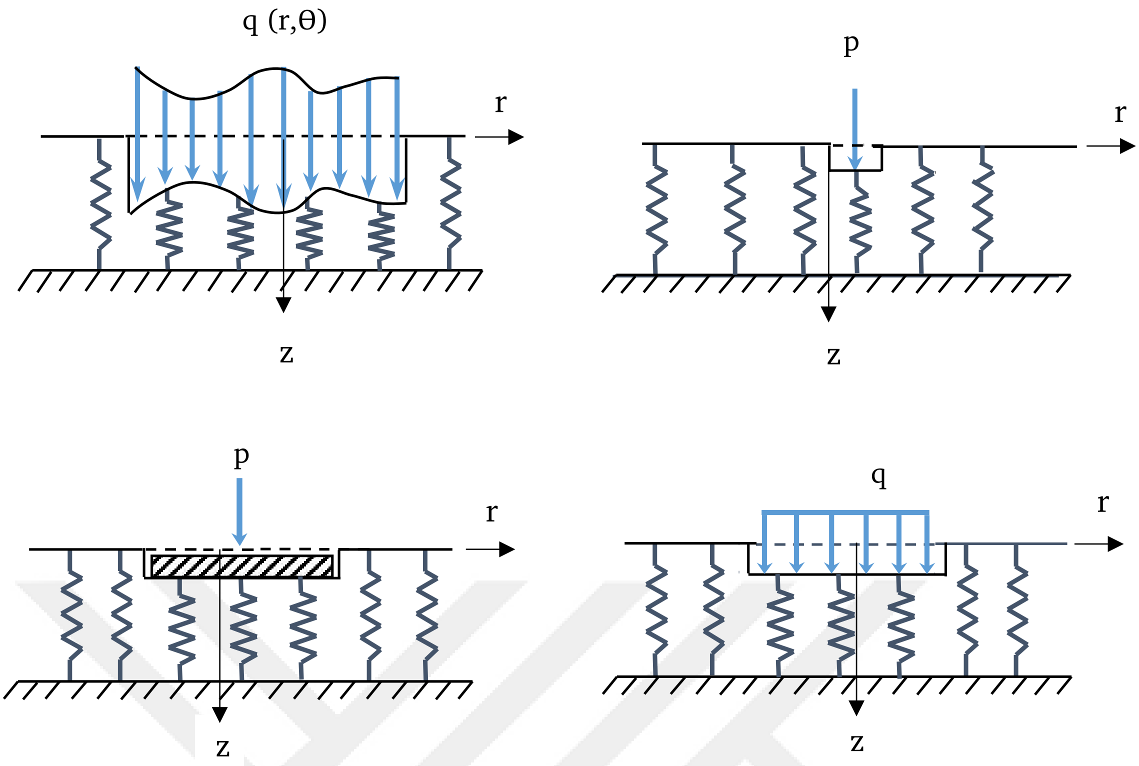

Figure A.2 Displacements of the Winkler model with various load types [50]

Displacements in elastic soil for 4 different loading conditions are shown schematically in Figure A.2. It can be said that this model has a handicap because the effects of springs on each other are not taken into account. Although this soil model does not fully reflect the reality, it is the most used soil model because of its simplicity and it provides an adequate approximation for many soil problems.

A2. Pasternak Model

Since the horizontal interaction between the springs in the Winkler model is ignored, two-parameter soil models have been developed to consider this interaction. One of the two-parameter soil models is Pasternak model [51]. In this model, it is assumed that there is a shear layer consisting of incompressible vertical members that deform only in transverse shear. This shear layer is assumed to be connected to the springs (Figure A.3).

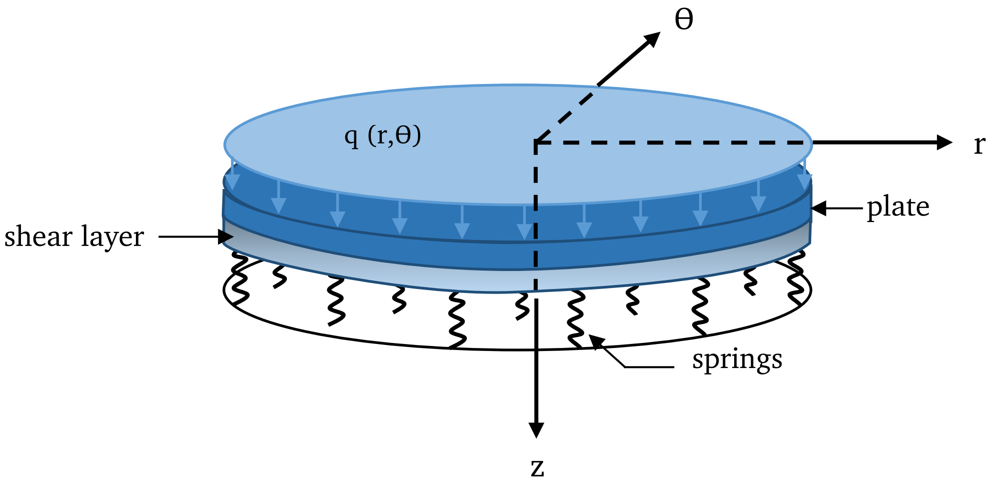

Figure A.3 A plate on Pasternak model

The ground response function (A.2) for the Pasternak model is as follows

$$q(r,\theta) = k_w(r,\theta) - k_g \nabla^2 w(r,\theta) \tag{A.2}$$

where k_g is the shear modulus of elastic foundation.

PUBLICATIONS FROM THE THESIS

Conference Presentations

1. M. Altekin and A. Köme, ‘’Axisymmetric Bending Analysis of Circular Plates on Elastic Foundation”, in Proceedings of the *4th International E-Conference on Mathematical Advances and Applications*, pp. 40, 29 May 2021.

Projects

1. Bending Analysis of Circular Plates, Assoc. Prof. Dr. Murat ALTEKİN, FYL-2021-4183, 12.03.2021-12.01.2022, Researcher.