

ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL

**ADVANCED FUZZY SET EXTENSIONS AND NOVEL DECISION MAKING
MODELS**



Ph.D. THESIS

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Department of Industrial Engineering

Industrial Engineering Programme

JUNE 2025

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MODELLERİ**

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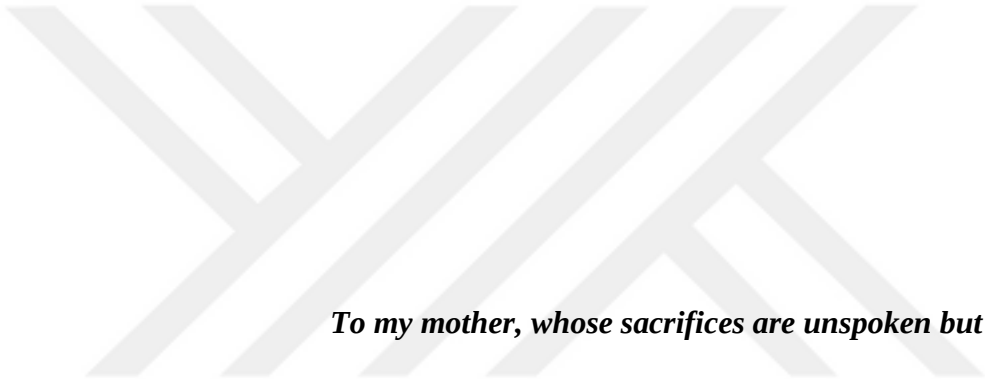
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To my mother, whose sacrifices are unspoken but deeply felt,



FOREWORD

When I look back on my academic journey, which I took hesitantly at first, I say, “Thank goodness.” This process taught me not only academic maturity, but also what I love, what I can believe in, and what idea I can fight for until the break of dawn. This journey was one of the greatest blessings of my life. This thesis is not only the result of an academic study; it is also a story of silently patient times, questioned meanings, inward reckonings, and sometimes just being able to say “keep going.” It bears the traces of an invisible labor, an invisible resistance, and unspoken thanks as much as the theories, models, and methods on the pages.

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ABBREVIATIONS

AHP	: Analytic Hierarchy Process
ANP	: Analytic Network Process
C-IFSs	: Circular Intuitionistic Fuzzy Sets
CINFUNs	: Continuous Intuitionistic Fuzzy Numbers
CINFUSs	: Continuous Intuitionistic Fuzzy Sets
CINFUWA	: Continuous Intuitionistic Fuzzy Weighted Arithmetic
CINFUWG	: Continuous Intuitionistic Fuzzy Weighted Geometric
CRITIC	: CRiteria Importance Through Intercriteria Correlation
CODAS	: COmbinative Distance-based ASsessment
CPFUNs	: Continuous Pythagorean Fuzzy Numbers
CPFUSs	: Continuous Pythagorean Fuzzy Sets
CPFUDWG	: Continuous Pythagorean Fuzzy Dombi Weighted Geometric
CPFUEWG	: Continuous Pythagorean Fuzzy Einstein Weighted Geometric
CPFUHWG	: Continuous Pythagorean Fuzzy Hamacher Weighted Geometric
CPFUWA	: Continuous Pythagorean Fuzzy Weighted Arithmetic
CPFUWG	: Continuous Pythagorean Fuzzy Weighted Geometric
DEVADA	: DEcision-making based on VAlue and Distance Analysis
DFSs	: Decomposed Fuzzy Sets
DPFSs	: Decomposed Pythagorean Fuzzy Sets
DPWAM	: Decomposed Pythagorean Weighted Arithmetic Mean
DPWGM	: Decomposed Pythagorean Weighted Geometric Mean
EDAS	: Evaluation based on Distance from Average Solution
IF	: Intuitionistic Fuzzy
IF-DEVADA	: Intuitionistic Fuzzy DEcision-making based on VAlue and Distance Analysis
IFSs	: Intuitionistic Fuzzy Sets
IFWA	: Intuitionistic Fuzzy Weighted Aritmetic
IFWG	: Intuitionistic Fuzzy Weighted Geometric
IOT	: Internet Of Things
IVIFSs	: Interval-Valued Intuitionistic Fuzzy Sets
IVIFWA	: Interval-Valued Intuitionistic Fuzzy Weighted Aritmetic

IVIFWG	: Interval-Valued Intuitionistic Fuzzy Weighted Geometric
MCDM	: Multi-Criteria Decision Making
MADM	: Multi-Attribute Decision Making
OFSs	: Ordinary Fuzzy Sets
PFSs	: Pythagorean Fuzzy Sets
q-ROFSs	: q-Rung Orthopair Fuzzy Sets
SFSs	: Spherical Fuzzy Sets
TOPSIS	: Technique for Order Preference by Similarity to Ideal Solution



SYMBOLS

μ	: Membership degree
μ^L, μ^U	: Lower and upper membership degree
μ_A^F, μ_A^D	: Functional point of view and dysfunctional point of view for membership degree
ϑ	: Nonmembership degree
ϑ^L, ϑ^U	: Lower and upper nonmembership degree
$\vartheta_A^F, \vartheta_A^D$	: Functional point of view and dysfunctional point of view for nonmembership degree
π	: Hesitancy degree
C_j	: j th criterion
A_i	: i th alternative
DM_k	: k th decision-maker
ω_j	: Weight of j th criterion
α	: Largeness or smallness of membership and nonmembership degrees
τ	: Continuous structure coefficient of membership and nonmembership degrees
λ	: Threshold value
$S(.)$	: Score function
$H(.)$	: Accuracy function
$Deff(.)$	: Deffuzification value
$\tilde{X}_k$	: Decision matrix of k th decision matrix
$\tilde{X}$	: Aggregated CINFU decision matrix
$\tilde{\mathfrak{X}}$	: Weighted CINFU decision matrix
$\Delta_{i,\tau}^*, \Delta_{i,\tau}^-$	: Separation measures
δ^*, δ^-	: Positive and negative ideal solutions
$\tilde{\Phi}_k$	: DPF criterion weight matrix for k th DM
$\tilde{\Phi}$	: Aggregated DPF criterion matrix
$\tilde{\Psi}_k$	: DPF decision matrix for k th DM
$\tilde{\Psi}$	: Aggregated DPF decision matrix



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ADVANCED FUZZY SET EXTENSIONS AND NOVEL DECISION MAKING MODELS

SUMMARY

Uncertainty is an inherent part of almost every decision-making process, especially in complex systems where information is incomplete, vague, or imprecise. Traditional mathematical models, which are largely based on binary logic and crisp data, often fall short in representing the subtleties and ambiguities that characterize real-world situations. In response to these limitations, Zadeh's introduction of fuzzy set theory in 1965 marked a foundational shift in how imprecision and vagueness are mathematically formalized. Fuzzy sets, by allowing partial membership, opened up new possibilities for modeling uncertainty in a more flexible and human-like manner. Since then, numerous extensions of fuzzy sets have been proposed, such as intuitionistic fuzzy sets, Pythagorean fuzzy sets, and q-rung orthopair fuzzy sets to better handle the increasing complexity and multidimensionality of uncertainty. These extensions offer more expressive frameworks that capture hesitation, disagreement, and detailed preference structures more effectively than classical models. However, despite these advancements, significant challenges remain. Existing models often fail to adequately address the levels of uncertainty and the nature of complex judgments. Moreover, the development of decision-making methods that are fully compatible with the latest fuzzy extensions is still limited, leaving a gap between theoretical progress and practical application.

Conventional fuzzy set frameworks, while effective in many contexts, often fall short in addressing continuous uncertainty, decomposability, or the increasing complexity of preference representation in MCDM tasks. This study presents a comprehensive and original contribution to the field of decision-making under uncertainty by developing novel fuzzy set extensions and proposing innovative multi-criteria decision-making (MCDM) methodologies. Responding to these methodological and conceptual gaps, this thesis proposes a series of novel fuzzy set extensions alongside new or adapted decision-making models, aiming to enhance both theoretical modeling and practical decision support. Motivated by the inherent vagueness, ambiguity, and complexity of real-world decision environments, the thesis handles key limitations in the representation and analysis of uncertainty, both from a theoretical and an applied perspective.

The first chapter introduces the Continuous Intuitionistic Fuzzy Sets (CINFUSs), an innovative extension of Atanassov's intuitionistic fuzzy sets. Unlike traditional models with discrete or static membership assignments, CINFUSs allow the membership and non-membership degrees to vary continuously within the intuitionistic constraint. This continuous formulation significantly enriches the expressive capacity of the fuzzy modeling framework by better accommodating the fluid nature of human judgment and uncertainty. In addition to offering a mathematically grounded structure, CINFUSs open the door to more flexible aggregation, comparison, and distance measures, which are critical in fine-tuned decision analysis.

Building upon this conceptual advancement, the Chapter 2 proposes the Decomposed Pythagorean Fuzzy Sets (DPFSs), which introduce a structural decomposition within the Pythagorean fuzzy framework. The DPFS model separates membership and non-membership components into optimistic and pessimistic to allow independent treatment and interpretation of each dimension. This decomposition enables more granular modeling of decision-makers' preferences and perceptions of uncertainty, particularly valuable in settings that require explanation, transparency, or sensitivity analysis. By disentangling the contributing factors of vagueness, the model not only facilitates interpretability but also improves the quality of aggregation and ranking mechanisms in MCDM processes.

The Chapter 3 presents the Continuous Pythagorean Fuzzy Sets (CPFUSs), which synthesizes the core idea of continuity (as introduced in CINFUSs) with the geometric and tolerant properties of the Pythagorean fuzzy set. The CPFUS framework is designed to reflect smooth variations in uncertainty while maintaining the mathematical relationship required by the Pythagorean condition. As a result, CPFUSs serve as a robust modeling environment for contexts that demand both expressive precision and a higher tolerance for hesitancy and ambiguity. This thesis enhances the theoretical diversity of fuzzy sets and offers a novel alternative to existing models in complex decision-making scenarios.

Focusing on methodological development from theoretical modelling, the Chapter 4 introduces a new MCDM method named IF-DEVADA (Intuitionistic Fuzzy DEcision-making based on VALue and Distance Analysis). This model is developed to address the limitations of static and single-perspective decision-making frameworks. IF-DEVADA integrates multiple distance-based evaluations under intuitionistic fuzzy conditions and accounts for dynamic and heterogeneous decision environments. It is particularly suitable for problems involving evolving preferences, multiple time periods, and group-based decisions. The method is validated through real-world applications that demonstrate its adaptability and analytical depth, marking it as a significant contribution to dynamic MCDM research.

The Chapter 5 applies and adapts the well-established TOPSIS method within the context of q -rung orthopair fuzzy sets (q -ROFS), one of the most flexible and recent extensions of fuzzy sets. The proposed q -ROFS-TOPSIS model extends traditional MCDM analysis by accommodating a higher capacity for uncertainty and contradiction in the input data, making it suitable for complex systems with incomplete or inconsistent information. By integrating q -ROFS into a classical yet powerful decision-making model, this study not only demonstrates the versatility of q -rung environments but also enhances the practical applicability of fuzzy sets in modern MCDM frameworks.

In its entirety, the thesis does not merely pursue technical novelty but strives to build a cohesive and future-oriented framework for modeling and managing uncertainty. Each chapter contributes a unique dimension, ranging from mathematical formulation to decision model construction, toward advancing the capabilities of both fuzzy set theory and decision-making science. The theoretical developments enhance how uncertainty is expressed, decomposed, and interpreted, while the methodological novelty provides robust tools for practical analysis across diverse application domains, including sustainability, policy prioritization, and strategic planning. By combining theoretical rigor with real-world applicability, this dissertation not only addresses current limitations in the literature but also lays the groundwork for future research in

fuzzy decision-making. Together, the five studies form a consistent and forward-looking framework that enhances how uncertainty is represented, interpreted, and used in analytical decision-making. The proposed models are designed to be extensible and adaptable, encouraging further exploration across areas where there is uncertainty, complexity, and multiple criteria.





GELİŞMİŞ BULANIK KÜME UZANTILARI VE YENİ KARAR VERME MODELLERİ

ÖZET

Belirsizlik, özellikle bilginin eksik, belirsiz veya kesin olmadığı karmaşık sistemlerde, neredeyse her karar verme sürecinin doğasında bulunan bir unsurdur. Geleneksel matematiksel modeller, büyük ölçüde ikili mantık ve kesin verilere dayanmakta olup, gerçek dünya koşullarını karakterize eden ince farkları ve belirsizlikleri temsil etmede yetersiz kalmaktadır. Bu sınırlamalara yanıt olarak, Zadeh'in 1965 yılında bulanık küme teorisini ortaya koyması, belirsizliğin matematiksel olarak nasıl kavramsallaştırıldığına dair köklü bir paradigma değişimini beraberinde getirmiştir. Bulanık kümeler, kısmi üyelik derecelerine izin vererek, geleneksel "ya vardır ya yoktur" yaklaşımının ötesine geçmekte ve insan düşüncesine daha yakın, esnek ve gerçekçi bir temsil sağlamaktadır.

Zadeh'in çalışmasını takiben, belirsizliklerin artan çok boyutluluğunu ve karmaşıklığını daha etkili biçimde temsil edebilmek amacıyla çeşitli bulanık küme uzantıları geliştirilmiştir. Bunlar arasında sezgisel bulanık kümeler, Pisagor bulanık kümeler ve q-rung ortopair bulanık kümeler gibi çok sayıda genişletilmiş bulanık küme yaklaşımı geliştirilmiştir. Bu uzantılar, sadece üyelik ve üyelik olmama derecelerini değil, aynı zamanda tereddüt, çelişki ve ayrıntılı tercih yapılarının daha etkili biçimde modellenmesine olanak sağlayarak klasik modellere göre daha güçlü bir ifade kapasitesi sunmaktadır. Ancak bu gelişmelere rağmen, mevcut modeller hâlâ belirsizlik düzeylerini ve karmaşık yargıların doğasını yeterince temsil edememektedir. yrica, güncel bulanık küme uzantılarıyla tam uyumlu, dinamik ve açıklanabilir karar verme yöntemlerinin sayısı da sınırlıdır. Bu durum, teorik gelişmelerle pratik uygulamalar arasındaki boşluğun hâlâ kapatılamamış olduğunu göstermektedir.

Klasik bulanık küme çerçeveleri birçok durumda etkili olsa da, sürekli belirsizliğin modellenmesi, ayrıştırılabilirlik ve tercih yapılarının artan karmaşıklığını ele alma konusunda yetersiz kalmaktadır. Bu çalışma, karar verme ve belirsizlik alanına özgün ve kapsamlı bir katkı sunarak yeni bulanık küme uzantıları geliştirmekte ve çok kriterli karar verme (ÇKKV) yöntemleri önermektedir. Bu tez, hem kavramsal hem de yönetsel boşluklara yanıt olarak; teorik modelleme ile pratik karar destek sistemlerini geliştirmeye yönelik yeni bulanık küme yapıları ve karar verme modelleri önermektedir. Gerçek dünya karar ortamlarının doğasında bulunan muğlaklık, belirsizlik ve karmaşıklıktan yola çıkan bu çalışma, belirsizliğin temsili ve analizine dair temel sınırlamaları hem teorik hem de uygulamalı bir bakış açısıyla ele almaktadır. Tez kapsamında, üç yeni bulanık küme uzantısı geliştirilmiş ve bu uzantılar çerçevesinde iki adet yenilikçi çok kriterli karar verme yöntemi önerilmiştir. Tezin temel amacı, karar ortamlarında karşılaşılan belirsizliklerin daha gerçekçi bir biçimde modellenmesi, açıklanması ve yönetilmesi için hem kavramsal hem de hesaplamaya dayalı güçlü çerçeveler sunmaktır.

Birinci bölümde, Atanassov'un sezgisel bulanık kümelerinin yenilikçi bir uzantısı olan Sürekli Sezgisel Bulanık Kümeler (Continuous Intuitionistic Fuzzy Sets - CINFUSs) tanıtılmıştır. Geleneksel modellerin aksine, bu yapı, üyelik ve üyelik dışı derecelerin sezgisel kısıtlara uygun şekilde sürekli değişmesine olanak tanımaktadır. Bu sürekli yapı, insan yargılarının akışkan doğasını daha iyi yansıtmakta ve bulanık modelleme çerçevesinin ifade kapasitesini önemli ölçüde artırmaktadır. Ayrıca bu yapı, karar analizinde kritik olan daha esnek toplulaştırma, karşılaştırma ve uzaklık ölçütlerinin geliştirilmesine de olanak sunmaktadır. Bu da karar analizinde daha hassas değerlendirmeler yapılmasını sağlamaktadır.

Bu kavramsal ilerlemenin üzerine inşa edilen ikinci bölümde, Ayrıştırılmış Pisagor Bulanık Kümeler (Decomposed Pythagorean Fuzzy Sets - DPFSs) önerilmiştir. Bu model, Pisagor bulanık çerçevesinde üyelik ve üyelik dışılık bileşenlerini iyimser ve kötümser olmak üzere ayrı ayrı yapılandırarak her boyutun bağımsız şekilde yorumlanmasını sağlamaktadır. Böylelikle karar vericilerin belirsizliğe dair algı ve tercihleri daha ayrıntılı şekilde modellenabilmekte, açıklanabilirlik ve duyarlılık analizleri kolaylaştırılmaktadır. Modelin sunduğu ayrıştırılmış yapı, hem yorumlamayı kolaylaştırmakta hem de ÇKKV süreçlerinde toplulaştırma ve sıralama kalitesini artırmaktadır.

Üçüncü bölümde, Sürekli Pisagor Bulanık Kümeler (Continuous Pythagorean Fuzzy Sets - CPFUSs) sunulmuştur. Bu model, CINFUS yapısındaki süreklilik fikrini, Pisagor bulanık kümelerinin geometrik ve toleranslı yapısıyla birleştirmektedir. CPFUS çerçevesi, belirsizlikteki yumuşak geçişleri yansıtırken Pisagor koşulunu da koruyarak, hem hassasiyet hem de belirsizliğe tolerans gereken karar ortamlarında güçlü bir modelleme alternatifi sunmaktadır. Bu tez, bulanık küme literatürüne teorik çeşitlilik katarak karmaşık karar verme bağlamlarında yeni bir modelleme seçeneği ortaya koymaktadır.

Teorik modellemekten yöntemsel geliştirmeye odaklanan dördüncü bölümde, IF-DEVADA (Intuitionistic Fuzzy DECision-making based on VALue and Distance Analysis) adlı yeni bir ÇKKV yöntemi önerilmiştir. Bu model, klasik ve durağan karar çerçevelerinin sınırlamalarını aşmak amacıyla geliştirilmiştir. IF-DEVADA yöntemi, sezgisel bulanık koşullar altında birden fazla uzaklık temelli değerlendirmeyi birleştirerek dinamik ve heterojen karar ortamlarını ele alır. Özellikle zaman içinde değişen tercihler, çok dönemli kararlar ve grup tabanlı karar süreçleri için uygundur. Gerçek dünya uygulamalarıyla doğrulanan yöntem, analitik derinliği ve uyarlanabilirliği ile dinamik ÇKKV araştırmalarına önemli katkı sunmaktadır.

Beşinci bölümde ise, en esnek ve güncel bulanık küme uzantılarından biri olan q-rung ortopair bulanık kümeler (q-ROFS) çerçevesinde klasik TOPSIS yönteminin uyarlanması gerçekleştirilmiştir. Önerilen q-ROFS-TOPSIS modeli, yüksek düzeyde belirsizlik ve çelişki içeren karar ortamlarında kullanılabilirliği artırmakta, eksik veya tutarsız bilgiye sahip karmaşık sistemlerde karar analizine katkı sağlamaktadır. Böylece hem q-rung yapılarının çok yönlülüğü ortaya konmakta hem de klasik karar modellerinin modern belirsizlik çerçevelerine entegrasyonu sağlanmaktadır.

Genel olarak bu tez, yalnızca teknik yenilikler sunmakla kalmayıp, aynı zamanda belirsizliğin modellenmesi ve yönetimi için bütüncül ve geleceğe dönük bir çerçeve inşa etmeyi hedeflemektedir. Her bölüm, teorik formülasyonlardan karar modeli inşasına kadar benzersiz katkılar sunmakta; belirsizliğin nasıl ifade edildiği, ayrıştırıldığı ve yorumlandığına dair önemli ilerlemeler sağlamaktadır. Teorik katkılar, belirsizlik modellemesindeki temsil gücünü artırırken; yöntemsel katkılar,

sürdürülebilirlik, politika önceliklendirmesi ve stratejik planlama gibi çeşitli uygulama alanlarında sağlam analiz araçları sunmaktadır.

Bu yönüyle tez, hem literatürdeki mevcut sınırlamaları aşmakta hem de bulanık karar verme alanında gelecek araştırmalara zemin hazırlamaktadır. Beş çalışma birlikte, belirsizliğin analitik karar süreçlerinde nasıl temsil edildiğini, yorumlandığını ve kullanıldığını geliştiren tutarlı ve yenilikçi bir yapı ortaya koymaktadır. Önerilen modeller, genişletilebilir ve uyarlanabilir şekilde tasarlanmış olup; belirsizlik, karmaşıklık ve çok kriterli yapıların olduğu her alanda yeni keşiflere olanak sunmaktadır.





1. INTRODUCTION

Decision-making lies at the very core of human activity. Whether in personal life, organizational strategy, public policy, or scientific inquiry, the act of choosing among alternatives is both inevitable and consequential. Yet, decisions are seldom made in ideal conditions. Instead, they often unfold in environments characterized by incomplete information, conflicting objectives, subjective judgments, and rapidly changing circumstances. In such environments, uncertainty becomes an inherent and unavoidable aspect of the decision-making process. Classical decision theories, rooted in the assumptions of rationality and complete information, provide classic mathematical structures for analyzing choices. However, real-world decision contexts frequently deviate from these idealized conditions. Decision-makers grapple with ambiguous data, imprecise evaluations, linguistic assessments, and human cognitive limitations. The complexity is further compounded by the dynamic interplay of multiple criteria, stakeholders with divergent perspectives, and the intrinsic fuzziness of concepts such as risk, preference, and satisfaction. There is a need for models that transcend simple binary logic and rigid classifications, that allow for more flexible handling of partial truths and varying degrees of belief, just as the human mind naturally works. An effective decision model should systematically incorporate uncertainty into its structure, allowing for flexible and realistic representations of knowledge and choice. Due to the failure of classical sets to adequately represent uncertainty, Fuzzy Sets (FSs), introduced by Lotfi A. Zadeh in 1965, have offered a new perspective, redefining the mathematical foundations of ambiguity, partial truth, and human-like reasoning. FSs have witnessed extensive developments through the introduction of various extensions, each aiming to capture deeper aspects of human reasoning under uncertainty. Among these extensions, Intuitionistic Fuzzy Sets (IFSs), Pythagorean Fuzzy Sets (PFSs), and their further generalizations have played a pivotal role, particularly in the fields of Multi-Criteria Decision-Making (MCDM) and artificial intelligence.

IFSs, introduced by Atanassov, have marked a significant progress in the field of fuzzy sets by including the degree of non-membership along with the degree of membership. Later, PFSs and q-rung orthopair fuzzy sets (q-ROFSs) have further generalized this structure, relaxing constraints and allowing greater modeling flexibility, especially in scenarios involving high degrees of uncertainty. Neutrosophic sets, picture fuzzy sets and spherical fuzzy sets continued to refine the representation of complex decision-making phenomena.

In parallel, the domain of MCDM has become increasingly significant due to the growing need for systematic and rational methods to evaluate multiple, often conflicting, criteria. Combining fuzzy set extensions with MCDM methodologies has proven to be a powerful strategy to deal with uncertainties in decision environments. The integration of fuzzy set extensions with MCDM techniques such as AHP, TOPSIS, CODAS and ELECTRE has not only enhanced the quality of decisions but also expanded the applicability of decision science to a broader range of real-world problems.

Despite the substantial advancements made in the field, several critical situations remain unaddressed, particularly when it comes to modeling uncertainty in a comprehensive and flexible manner. While some extensions manage to capture continuous uncertainty to a certain extent, they still face limitations in adequately decomposing complex membership structures or representing the ambiguity levels encountered in practice. These shortcomings hinder the full potential of fuzzy sets in providing effective decision-making tools for complex, uncertain environments. Moreover, despite the vast body of studies on MCDM, there remains a gap in the development of novel MCDM models that fully utilize both existing and new fuzzy set extensions. This gap limits the adaptability and robustness of decision-making models when applied to real-world problems, where uncertainty is a pervasive and dynamic factor. For this reason, the focus of the thesis is on the development of novel fuzzy set extensions that more effectively capture continuous uncertainty levels and allow for the decomposition of complex membership structures, and the model of novel MCDM models that integrate both existing and new fuzzy set extensions to address the unique needs of complex decision-making problems.

1.1 Purpose of Thesis

The primary purpose of this thesis is to advance the theoretical and methodological foundations of decision-making under uncertainty by addressing limitations within existing fuzzy set frameworks and multi-criteria decision-making (MCDM) methodologies. Despite notable developments in the field, traditional fuzzy set extensions often struggle to accurately capture the continuous, decomposed, and dynamic nature of uncertainty that characterizes real-world decision environments. Likewise, many MCDM approaches remain constrained in their capacity to exploit the full potential of emerging fuzzy methods, thereby limiting their applicability in increasingly complex and ambiguous environments. In this context, this thesis seeks to achieve two interrelated and complementary objectives. First, the thesis aims to develop advanced novel fuzzy set extensions- Continuous Intuitionistic Fuzzy Sets (CINFUS), Decomposed Pythagorean Fuzzy Sets (DPFS), and Continuous Pythagorean Fuzzy Sets (CPFUS) capable of more precisely modeling continuous uncertainty and decomposing complex membership structures. Each of the developed extensions handles critical limitations in existing models by enabling a more continuous, nuanced, and structurally decomposed modeling of uncertainty. Through these advanced extensions, the thesis significantly enhances the expressive power of fuzzy set theory, providing deeper and more flexible tools for capturing the complexities of human judgments and decision environments. Second, building on these theoretical advances, the thesis constructs novel MCDM methodologies specifically adapted to fuzzy set extensions proposed in the literature. A new decision-making method, IF-DEVADA (Intuitionistic Fuzzy multi-Distance based EVAluation for Aggregated Dynamic decision Analysis), is developed to handle decision problems characterized by intuitionistic fuzzy sets. Furthermore, a TOPSIS approach based on q-Rung Orthopair Fuzzy Sets, one of the recent fuzzy extensions in the literature, is proposed, demonstrating the adaptability of classical MCDM techniques to emerging fuzzy frameworks. These frameworks are to bridge the gap between theory and practice and offer decision-makers tools that are not only mathematically robust but also adaptable to the increasingly complex, ambiguous, and dynamic decision environments.

Beyond the methodological contributions, the thesis positions the studies within the broader area of decision science, highlighting the critical need to move beyond

traditional models towards more adaptive and flexible approaches. Through its dual focus on theoretical novelty and practical application, the thesis positions itself at the intersection of method development and real-world relevance, offering both scholars and practitioners a richer, more nuanced toolkit for navigating uncertainty.

Ultimately, the purpose of this thesis is not merely to introduce novelties. Rather, it aims to foster a deeper, systemic advancement in how uncertainty is modeled and managed in decision-making processes. In doing so, the thesis provides a versatile and forward-looking framework. This framework is designed to support both rigorous academic research and practical, real-world decision-making applications.

1.2 Impact

Various articles and proceedings based on the scope of this thesis are presented in the literature. The papers published within the scope of this thesis and submitted for publication are as follows:

1. Published Articles:

- Alkan, N. & Kahraman, C. (2021). Evaluation of government strategies against COVID-19 pandemic using q-rung orthopair fuzzy TOPSIS method. *Applied Soft Computing*, 110, 107653.
- Alkan, N. & Kahraman, C. (2022). An intuitionistic fuzzy multi-distance based evaluation for aggregated dynamic decision analysis (IF-DEVADA): Its application to waste disposal location selection. *Engineering Applications of Artificial Intelligence*, 111, 104809.
- Alkan, N. & Kahraman, C. (2023). Continuous intuitionistic fuzzy sets (CINFUS) and their AHP&TOPSIS extension: Research proposals evaluation for grant funding. *Applied Soft Computing*, 145, 110579.
- Alkan, N. & Kahraman, C. (2024). CODAS extension using novel decomposed Pythagorean fuzzy sets: Strategy selection for IOT based sustainable supply chain system. *Expert Systems with Applications*, 237, 121534.

- Alkan, N. & Kahraman, C. (2025). Continuous Pythagorean Fuzzy Set Extension with Multi-Attribute Decision Making Applications. *Informatica*, 1-43.

2. Proceedings:

- Alkan, N. & Kahraman, C. (2023). A novel continuous intuitionistic fuzzy CODAS method and its application to IOT service provider selection for sustainable supply chain. In *International Conference on Intelligent and Fuzzy Systems, INFUS 2023*, vol. 759 (pp. 371-381). Springer Cham.
- Alkan, N. & Kahraman, C. (2024). Continuous intuitionistic fuzzy CRITIC & WASPAS: Smart city evaluation. In *International Conference on Intelligent and Fuzzy Systems, INFUS 2024*, vol. 1088 (pp. 725-734). Springer Cham.

3. Under Review Articles:

- Alkan, N. Aydın, U., Menekşe, A. & Kahraman, C. Comparison of the Performances of Artificial Intelligence Bots Using Continuous Intuitionistic Fuzzy Evaluation based on Distance from Average Solution Method. *Engineering Applications of Artificial Intelligence (Under Review)*.
- Alkan, N. & Kahraman, C. Electric Vehicle Charging Station Location Selection using Continuous Intuitionistic Fuzzy Aggregation Operators. *Journal of Multiple-Valued Logic & Soft Computing (Under Review)*.

1.3 Contribution and Originality

This thesis makes a series of interrelated and substantial contributions to the fields of fuzzy set extensions and MCDM, specifically by proposing advanced novel fuzzy set extensions and designing new decision-making models that are theoretically grounded and practically applicable. These contributions are structured around five original studies, each of which addresses a specific gap in the literature and collectively aims to enhance our understanding of how uncertainty can be modeled, decomposed, and operationalized in decision-making processes.

The first study introduces the concept of Continuous Intuitionistic Fuzzy Sets (CINFUS), which represents a foundational extension of the classical intuitionistic fuzzy set framework. Traditional intuitionistic fuzzy sets can fail to capture the continuous nature of uncertainty, as they operate within a discrete structure. In response to this limitation, the developed CINFUSs enable a continuous representation of the membership and non-membership degrees, offering a richer and more flexible modeling framework for uncertainty. This development not only enhances the expressiveness of the intuitionistic structure but also aligns more closely with the way human judgment often unfolds along a continuity, particularly in complex and ambiguous decision contexts.

Building upon this direction, the second study proposes Decomposed Pythagorean Fuzzy Sets (DPFSs), a novel approach that enhances the interpretability of Pythagorean fuzzy sets. While Pythagorean fuzzy sets provide a larger uncertainty domain than intuitionistic fuzzy sets, they typically handle membership and non-membership information as a single, unified construct. In contrast, the DPFS framework decomposes the components, explicitly separating and modeling the membership and non-membership dimensions. This decomposition allows for more detailed analysis of uncertainty sources and significantly improves the transparency and tractability of decision analysis models using Pythagorean fuzzy sets.

The third study extends the logic of continuity to the Pythagorean fuzzy sets domain by proposing Continuous Pythagorean Fuzzy Sets (CPFS). Inspired by the theoretical motivations behind CIFS, this model brings a continuous structure to Pythagorean fuzzy sets, while maintaining the mathematical integrity of the Pythagorean constraint for both the membership and non-membership degrees. This continuous formulation offers a powerful new tool for modeling highly complex forms of uncertainty in decision-making scenarios that require more precise and sensitive input assessment. This study not only strengthens the theoretical underpinnings of Pythagorean fuzzy sets but also demonstrates their applicability in complex decision environments through illustrative applications.

Recognizing that theoretical advancements must be accompanied by robust methodological tools, the fourth study of this thesis introduces an original MCDM method named IF-DEVADA (Intuitionistic Fuzzy DEcision-making based on VAlue and Distance Analysis). Designed to work specifically within intuitionistic fuzzy

environments, IF-DEVADA presents a more comprehensive and adaptable approach to complex decision problems. IF-DEVADA integrates multiple distance-based evaluation criteria within a dynamic aggregation framework, enabling a more detail representation of decision-makers' preferences over time and across alternatives. Unlike many existing intuitionistic fuzzy MCDM models, which often rely on static or overly simplified structures, IF-DEVADA introduces a dynamic analysis mechanism that accounts for temporal variation in judgments. It also accommodates heterogeneous data and combines multiple distance measures to strengthen the robustness and sensitivity of the evaluation process. By doing so, it provides decision-makers with a more flexible and realistic tool for handling uncertainty in evolving environments.

Lastly, the fifth study bridges the gap between classical MCDM methods and advanced fuzzy environments by adapting the widely used TOPSIS method to the q-Rung Orthopair Fuzzy Set (q-ROFS) framework. q-ROFSs, as one of the most expressive fuzzy set extensions, allow for a broader domain of uncertainty representation. However, the integration of classical decision-making tools like TOPSIS into this new environment remains underexplored. By developing a TOPSIS model adapted to the q-ROFS framework, this study demonstrates how MCDM approaches can be redesigned to remain effective under more complex uncertainty structures. The proposed model is validated through practical application and showcases improved sensitivity, robustness, and interpretability.

Each proposed fuzzy set extension and decision-making model is validated through real-world case studies, illustrating both theoretical soundness and practical utility across different domains such as location selection, strategic prioritization, and proposal evaluation. Rather than focusing on narrow-scope novelties, the thesis builds a cohesive theoretical framework that systematically advances how uncertainty is represented and utilized in decision-making. This broader perspective strengthens the foundations of fuzzy set theory and contributes to the ongoing transformation of decision sciences.



2. CONTINUOUS INTUITIONISTIC FUZZY SETS (CINFUS) AND THEIR AHP & TOPSIS EXTENSION: RESEARCH PROPOSALS EVALUATION FOR GRANT FUNDING¹

After fuzzy set theory was introduced to the literature by Zadeh (1965) to handle uncertainties in complex problems, intuitionistic fuzzy sets (IFSs) have been introduced by Atanassov (1986) as an extension of Zadeh's ordinary fuzzy sets to better handle uncertainties. Then, many extensions of IFSs have been introduced to the literature by various researchers. Some of these fuzzy set extensions, such as interval-valued intuitionistic fuzzy sets (IVIFSs) (Atanassov & Gargov, 1989), Pythagorean fuzzy sets (PFSs) (Yager, 2013), Fermatian fuzzy sets (Senapati & Yager, 2020) and circular intuitionistic fuzzy sets (C-IFSs) (Atanassov, 2020) have taken place in the literature.

IFSs are represented by two parameters, membership and non-membership degrees satisfying the condition that the sum of the membership and non-membership degrees of each element in a fixed set of the universe is at most 1. IFSs and their direct extensions are generally expressed by discrete degrees of membership and non-membership. The meaning of membership and non-membership degrees is well-explained in Sevastjanov et al. (Sevastjanov et al., 2021a, 2021b). Although IFSs and their direct extensions have been used to tackle different real-life problems, they are often difficult to estimate the crisp values for membership and non-membership degrees (Alkan & Kahraman, 2022d). This situation, which is frequently encountered especially in decision-making processes, can cause uncertainty to not be fully expressed correctly, and this may lead to the inability to obtain accurate results (Alkan & Kahraman, 2022a). Linear intuitionistic fuzzy sets have been introduced by Li (2010) in the form of triangular fuzzy numbers. However, to the best knowledge of the authors, non-linear functions for membership and non-membership degrees have not been developed. These degrees must have a continuous structure in order to take into

¹ This chapter is based on the paper “Alkan, N., & Kahraman, C. (2023). Continuous intuitionistic fuzzy sets (CINFUS) and their AHP&TOPSIS extension: Research proposals evaluation for grant funding. *Applied Soft Computing*, 145, 110579.”

account the uncertainty at every point. This situation necessitates that all elements of the set constituting an event take values in a continuous range. The values of the elements of the set can have a symmetrical appearance represented by a bell-shaped curve. For this, the degrees of membership and non-membership, which have a bell-shaped structure, can be represented by a coefficient parameter in order to have continuous values. For a bell-shaped symmetrical structure, our assumption is that the coefficient should decrease the degrees of membership and non-membership as it moves away from the center. The degrees of membership and non-membership in the continuous structure to be obtained through the coefficient can be calculated with the function $\alpha\tau - \alpha\tau^2$. Thanks to this equation created by means of the coefficient τ , membership and non-membership degrees have a continuous structure. The coefficient τ takes its value between 0 and 1. At the same time, the value of α represents the largeness or smallness of the values that the membership functions will take in a continuous structure. Here, the smaller the value of α , the smaller the degree of membership and the greater the degree of non-membership, or vice versa. Therefore, continuous intuitionistic fuzzy sets (CINFUSs) are introduced in this study. CINFUSs are based on the fact that membership and non-membership degrees can be defined by a continuous structure by providing the following condition.

$$0 \leq \alpha\tau - \alpha\tau^2 + (4 - \alpha)\tau - (4 - \alpha)\tau^2 \leq 1 \quad (2.1)$$

or

$$0 \leq 4\tau - 4\tau^2 \leq 1 \quad (2.2)$$

CINFUSs are a generalization of IF theory developed with a concept similar to Gaussian fuzzy sets. With their flexible structures, CINFUSs can model decision maker's instability better than IF sets and add uncertainty to fuzzy membership and non-membership functions, as in Gaussian fuzzy sets. Likewise, CINFUSs, which extend the concept of Gaussian fuzzy sets and IF sets, combine the uncertainty and imprecision inherent in membership functions in problem modeling in a continuous way. This paper presents a new theoretical structure by taking advantages of Gaussian fuzzy sets and IFSs.

With the developments in technology, the importance of research projects is increasing day by day. However, many people need financial support to implement their research projects. Grants are the most preferred financial tool by researchers, and almost all researchers seek a grant for their research projects. However, it is extremely important to choose the most appropriate research proposal, as grant fundings are limited and highly demanded. For this, the research evaluation group must determine a detailed selection of criteria to make an excellent decision (Oztaysi et al., 2017). Besides, the evaluation of research proposals for grant fundings is a complex problem with imprecise information. Therefore, the selection of an appropriate project proposal requires an MCDM model to be considered in a fuzzy environment. For this, first of all, it is necessary to determine the importance weights of the criteria to be considered in the selection of a suitable research proposal, and then to rank the alternative projects. In this context, a CINFUS-based hybrid MCDM model is proposed to obtain the best and most appropriate proposal among various options by complex and uncertain decision-making models in this study. In the study, firstly, the fuzzy extension of the AHP method in the CINFUS environment is presented in order to obtain the criterion weights in the uncertainty environment. Then, the CINFUS-based TOPSIS method has been developed to rank alternative projects. The effectiveness of the developed CINFUS-based methodology is presented on a case study based on the evaluation of research projects for grant funding.

The remainder of this study is organized as follows. Section 2.1 includes the literature review on intuitionistic fuzzy sets and their extensions. Section 2.2 presents the basic principles of intuitionistic fuzzy sets (IFSs) and interval-valued intuitionistic fuzzy sets (IVIFSs). The theoretical principles and mathematical operators of CINFUS are given in Section 2.3. The extensions of CINFUSs to AHP and TOPSIS methods are given in Sections 2.4 and 2.5, respectively. In Section 2.6, the developed CINFU-based AHP and TOPSIS methods are applied to the evaluation of research project proposals for grant fundings, and extensive sensitivity analysis and comparative analysis are performed to demonstrate the validity of the results. Finally, the study ends in Section 2.7 with conclusions and recommendations for further research.

2.1 Literature Review on Intuitionistic Fuzzy Sets and Their Extensions

Atanassov (1986) introduced the idea of intuitionistic fuzzy sets as an extension of FSs to better handle uncertainty in data in real-life problems. IFSs represented by membership and non-membership degrees deal with uncertainties in data or due to incomplete information of decision-makers, but also reflect decision-makers' hesitations at the choice stages. After IFSs were introduced, new operations and models based on IFSs have been developed and frequently used by researchers. Atanassov (Atanassov, 1989, 1994), developed new operators, operations and their basic properties based on IFSs. Then, Atanassov et al. (2005) proposed an IFS-based multi-criteria decision-making process that includes many people and many measurement tools. Boran & Akay (2014) developed a biparametric similarity measure based on IFSs along with its proofs. Liu & Chen (2017) proposed intuitionistic fuzzy HM operators based on the Archimedean t-conorm and t-norm. In the study, intuitionistic fuzzy Archimedean Heronian aggregation operator and the intuitionistic fuzzy weight Archimedean Heronian aggregation (IFWAHA) operator have been developed by considering some properties and some special cases. Besides, a new multiple attribute group decision making method based on the proposed IFAHA and IFWAHA operators has been proposed in the study. Garg (2017) proposed weighted, ordered weighted and hybrid averaging aggregation operators based on Einstein operational laws. A novel intuitionistic fuzzy decision-making model based on developed operation laws has been applied in the study. Zhang & Xu (2015) developed a soft computing technique based on maximizing consensus and interval-valued intuitionistic fuzzy TOPSIS method. Then, Atanassov (1999) introduced second type of intuitionistic fuzzy sets (IFS2), a generalization of intuitionistic fuzzy sets and characterized by one degree of membership and one degree of non-membership. In IFS2s, the sum of the squares of the membership degree and non-membership degree must be less than or equal to one. After that, Yager (2013) called them as Pythagorean fuzzy sets (PFSs) represented with a larger area for membership degrees in 2013. Picture fuzzy sets characterized by the degrees of membership, non-membership, and hesitant independently for each element in a set have been introduced by Cường (2014). Along with these fuzzy sets, it has been suggested that the degree of hesitation should be assigned by the decision makers as well as the degrees of membership and non-membership, and thus it was recommended that the refused

degree should be calculated. Then, Yager (2017) introduced q-rung orthopair fuzzy sets (q-ROFSs) representing a general class of IFSs and PFSs. Q-ROFSs are characterized with both membership and non-membership degrees of an element in a fuzzy set and the sum of their q th power has to at most equal to 1. When $q = 3$ in q-ROFs, Senapati and Yager (2020) have considered as Fermatean fuzzy sets (FFSs) to q-ROFs. The extension of picture fuzzy sets was made together with spherical fuzzy sets developed by Senapati and Yager (2020). In SFSs, while the squared sum of membership, non-membership and hesitancy parameters can be between 0 and 1, each of them can be defined between 0 and 1 independently. After the SFSs, Mahmood et al. (2019) introduced T-spherical fuzzy sets where the sum of the t th powers of the membership, non-membership, and hesitancy degrees could be at most 1. Circular intuitionistic fuzzy sets (CIFSs) developed by Atanassov (2020) are a new extension of IFSs. These sets are defined as the sets where each element of the universe has a degree of membership and a degree of non-membership with a circle around them whose radius is r , satisfying that the sum of membership and non-membership degrees within this circle is at most equal to 1. Decomposed fuzzy sets (DFSs) is the newest fuzzy set extension recently introduced by Cebi et al. (2022) as an extension of IFSs. In DFSs, membership and non-membership degrees in IFSs are decomposed into optimistic and pessimistic.

2.2 Preliminaries: Intuitionistic Fuzzy Sets

In this section, intuitionistic fuzzy sets are briefly summarized as single-valued and interval-valued intuitionistic fuzzy sets, respectively.

2.2.1 Single-valued intuitionistic fuzzy sets

IFSs have been proposed by Atanassov in 1983 as an extension of ordinary fuzzy sets (Atanassov, 1986). IFSs are defined with a membership degree and a non-membership degree for each element in a fixed set that their sum is one or less than one.

Definition 2.1. Let X be a non-empty set. An IFS I in X is given by (Atanassov, 1986):

$$I = \{(x, \mu_I(x), \vartheta_I(x)) \mid x \in X\} \quad (2.3)$$

where the function $\mu_I: X \rightarrow [0,1]$ and $\vartheta_I: X \rightarrow [0,1]$ defines the membership and non-membership degrees of an element to the set I , respectively, with the condition that

$$0 \leq \mu_I(x) + \vartheta_I(x) \leq 1, \text{ for } \forall x \in X \quad (2.4)$$

The degree of hesitancy is obtained as follows:

$$\pi_I(x) = 1 - \mu_I(x) - \vartheta_I(x) \quad (2.5)$$

Definition 2.2. Let $\tilde{A} = (\mu_{\tilde{A}}, \vartheta_{\tilde{A}})$ and $\tilde{B} = (\mu_{\tilde{B}}, \vartheta_{\tilde{B}})$ be two IFS, then the mathematical operations on these two intuitionistic fuzzy numbers (IFN) are defined by Atanassov as follows (Atanassov, 1999):

$$\tilde{A} \oplus \tilde{B} = (\mu_{\tilde{A}} + \mu_{\tilde{B}} - \mu_{\tilde{A}}\mu_{\tilde{B}}, \vartheta_{\tilde{A}}\vartheta_{\tilde{B}}) \quad (2.6)$$

$$\tilde{A} \otimes \tilde{B} = (\mu_{\tilde{A}}\mu_{\tilde{B}}, \vartheta_{\tilde{A}} + \vartheta_{\tilde{B}} - \vartheta_{\tilde{A}}\vartheta_{\tilde{B}}) \quad (2.7)$$

$$\lambda \tilde{A} = (1 - (1 - \mu_{\tilde{A}})^\lambda, (\vartheta_{\tilde{A}})^\lambda) \quad (2.8)$$

$$\tilde{A}^\lambda = ((\mu_{\tilde{A}})^\lambda, 1 - (1 - \vartheta_{\tilde{A}})^\lambda) \quad (2.9)$$

Definition 2.3. Let $\tilde{A}_i = (\mu_{\tilde{A}_i}, \vartheta_{\tilde{A}_i}), (i = 1, 2, \dots, n)$ be a set of IFSs and $w = (w_1, w_2, \dots, w_n)^T$ be weight vector of $\tilde{A}_i$ with $\sum_{i=1}^n w_i = 1$, then an intuitionistic fuzzy weighted geometric (IFWG) the operator is a mapping $IFWG : \tilde{A}^n \rightarrow \tilde{A}$, where (Atanassov, 1999).

$$IFWG(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) = \left(\prod_{i=1}^n (\mu_{\tilde{A}_i})^{w_i}, 1 - \prod_{i=1}^n (1 - (\vartheta_{\tilde{A}_i})^{w_i}) \right) \quad (2.10)$$

Definition 2.4. Let $\tilde{A}_i = (\mu_{\tilde{A}_i}, \vartheta_{\tilde{A}_i}), (i = 1, 2, \dots, n)$ be a set of IFSs and $w = (w_1, w_2, \dots, w_n)^T$ be weight vector of $\tilde{A}_i$ with $\sum_{i=1}^n w_i = 1$, then an intuitionistic fuzzy weighted average (IFWA) the operator is a mapping $IFWA : \tilde{A}^n \rightarrow \tilde{A}$, where (Atanassov, 1999).

$$IFWA(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) = \left(1 - \prod_{i=1}^n (1 - (\mu_{\tilde{A}_i})^{w_i}), \prod_{i=1}^n (\vartheta_{\tilde{A}_i})^{w_i} \right) \quad (2.11)$$

Definition 2.5. Let $\tilde{A} = (\mu_{\tilde{A}}, \vartheta_{\tilde{A}})$ be an IFN, then the score function $S(\tilde{A})$ and accuracy function $H(\tilde{A})$ of $\tilde{A}$ can be defined as in Eqs. (2.12) and (2.13), respectively (Atanassov, 1999).

$$S(\tilde{A}) = \mu_{\tilde{A}} - \vartheta_{\tilde{A}} \quad (2.12)$$

$$H(\tilde{A}) = \mu_{\tilde{A}} + \vartheta_{\tilde{A}} \quad (2.13)$$

Definition 2.6. Let $\tilde{A} = (\mu_{\tilde{A}}, \vartheta_{\tilde{A}})$ and $\tilde{B} = (\mu_{\tilde{B}}, \vartheta_{\tilde{B}})$ be two IFNs. The distance between these two IFNs can be calculated by using normalized Euclidean distance as in Eq (2.14) (Szmidt & Kacprzyk, 2000).

$$d(A, B) = \sqrt{\frac{1}{2n} \sum_{i=1}^n (\mu_{A_i} - \mu_{B_i})^2 + (\vartheta_{A_i} - \vartheta_{B_i})^2 + (\pi_{A_i} - \pi_{B_i})^2} \quad (2.14)$$

2.2.2 Interval-valued intuitionistic fuzzy sets

As an extension of IFSs, IVIFSs have the ability to better represent DMs' hesitant and uncertain decisions with the help of membership definitions when evaluating criteria or alternatives.

Definition 2.7. Let X be a fixed set. An IVIFSs $\tilde{A}$ in X is an object having the form (Atanassov, 1999).

$$\tilde{A} = \{ \langle x, \mu_{\tilde{A}}(x), \vartheta_{\tilde{A}}(x) \rangle \mid x \in X \} \quad (2.15)$$

where $\mu_{\tilde{A}}(x) \subseteq [0, 1]$ and $\vartheta_{\tilde{A}}(x) \subseteq [0, 1]$ expresses the membership degree and non-membership degree of the element $x \in X$ to the set $\tilde{A}$, respectively. Also, $\mu_{\tilde{A}}(X)$ and $\vartheta_{\tilde{A}}(X)$ for each $x \in X$ are closed intervals and their lower and upper bounds are represented by $\mu_{\tilde{A}}^L(x)$, $\mu_{\tilde{A}}^U(x)$, $\vartheta_{\tilde{A}}^L(x)$, $\vartheta_{\tilde{A}}^U(x)$, respectively. Therefore, $\tilde{A}$ can also be defined as follows:

$$\mu_{\tilde{A}}(x) = [\mu_{\tilde{A}}^L(x), \mu_{\tilde{A}}^U(x)] \subseteq [0, 1] \quad (2.16)$$

$$\vartheta_{\tilde{A}}(x) = [\vartheta_{\tilde{A}}^L(x), \vartheta_{\tilde{A}}^U(x)] \subseteq [0, 1] \quad (2.17)$$

where the expression is subject to the condition $0 \leq \mu_{\tilde{A}}^U(x) + \vartheta_{\tilde{A}}^U(x) \leq 1$.

For every $x \in X$, $\pi_{\tilde{A}}(x) = [\pi_{\tilde{A}}^L(x), \pi_{\tilde{A}}^U(x)]$ is called the degree of hesitancy in IVIFSs, where $\pi_{\tilde{A}}^L(x) = 1 - \mu_{\tilde{A}}^U(x) - \vartheta_{\tilde{A}}^U(x)$ and $\pi_{\tilde{A}}^U(x) = 1 - \mu_{\tilde{A}}^L(x) - \vartheta_{\tilde{A}}^L(x)$.

Definition 2.8. Let $\tilde{A} = ([\mu_{\tilde{A}}^L, \mu_{\tilde{A}}^U], [\vartheta_{\tilde{A}}^L, \vartheta_{\tilde{A}}^U])$, $\tilde{A}_1 = ([\mu_{\tilde{A}_1}^L, \mu_{\tilde{A}_1}^U], [\vartheta_{\tilde{A}_1}^L, \vartheta_{\tilde{A}_1}^U])$ and $\tilde{A}_2 = ([\mu_{\tilde{A}_2}^L, \mu_{\tilde{A}_2}^U], [\vartheta_{\tilde{A}_2}^L, \vartheta_{\tilde{A}_2}^U])$ be three IVIFSs and $\lambda > 0$, then their operations are defined as follows (Atanassov, 1999):

$$\tilde{A}_1 \oplus \tilde{A}_2 = \left(\left[\begin{aligned} &(\mu_{\tilde{A}_1}^L) + (\mu_{\tilde{A}_2}^L) - (\mu_{\tilde{A}_1}^L)(\mu_{\tilde{A}_2}^L), \\ &(\mu_{\tilde{A}_1}^U) + (\mu_{\tilde{A}_2}^U) - (\mu_{\tilde{A}_1}^U)(\mu_{\tilde{A}_2}^U) \end{aligned} \right], [\vartheta_{\tilde{A}_1}^L \vartheta_{\tilde{A}_2}^L, \vartheta_{\tilde{A}_1}^U \vartheta_{\tilde{A}_2}^U] \right) \quad (2.18)$$

$$\tilde{A}_1 \otimes \tilde{A}_2 = \left([\mu_{\tilde{A}_1}^L \mu_{\tilde{A}_2}^L, \mu_{\tilde{A}_1}^U \mu_{\tilde{A}_2}^U], \left[\begin{aligned} &(\vartheta_{\tilde{A}_1}^L) + (\vartheta_{\tilde{A}_2}^L) - (\vartheta_{\tilde{A}_1}^L)(\vartheta_{\tilde{A}_2}^L), \\ &(\vartheta_{\tilde{A}_1}^U) + (\vartheta_{\tilde{A}_2}^U) - (\vartheta_{\tilde{A}_1}^U)(\vartheta_{\tilde{A}_2}^U) \end{aligned} \right] \right) \quad (2.19)$$

$$\lambda \tilde{A} = \left(\left[1 - (1 - (\mu_{\tilde{A}}^L))^\lambda, 1 - (1 - (\mu_{\tilde{A}}^U))^\lambda \right], [(\vartheta_{\tilde{A}}^L)^\lambda, (\vartheta_{\tilde{A}}^U)^\lambda] \right) \quad (2.20)$$

$$\tilde{A}^\lambda = \left([(\mu_{\tilde{A}}^L)^\lambda, (\mu_{\tilde{A}}^U)^\lambda], \left[1 - (1 - (\vartheta_{\tilde{A}}^L))^\lambda, 1 - (1 - (\vartheta_{\tilde{A}}^U))^\lambda \right] \right) \quad (2.21)$$

Definition 2.9. Let $\tilde{A}_i = ([\mu_{\tilde{A}_i}^L, \mu_{\tilde{A}_i}^U], [\vartheta_{\tilde{A}_i}^L, \vartheta_{\tilde{A}_i}^U])$ ($i = 1, 2, \dots, n$) be a set of IVIFSs and $w = (w_1, w_2, \dots, w_n)^T$ be weight vector of $\tilde{A}_i$ with $\sum_{i=1}^n w_i = 1$, then an interval-valued intuitionistic fuzzy weighted geometric (IVIFWG) the operator is a mapping $IVIFWG : \tilde{A}^n \rightarrow \tilde{A}$, where (Xu, 2007):

$$\begin{aligned} &IVIFWG(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) \\ &= \left(\left[\begin{aligned} &[\prod_{i=1}^n (\mu_{\tilde{A}_i}^L)^{w_i}, \prod_{i=1}^n (\mu_{\tilde{A}_i}^U)^{w_i}] \\ &[(1 - \prod_{i=1}^n (1 - (\vartheta_{\tilde{A}_i}^L)^{w_i}))^{w_i}, (1 - \prod_{i=1}^n (1 - (\vartheta_{\tilde{A}_i}^U)^{w_i}))^{w_i}] \end{aligned} \right] \right) \end{aligned} \quad (2.22)$$

Definition 2.10. Let $\tilde{A}_i = ([\mu_{\tilde{A}_i}^L, \mu_{\tilde{A}_i}^U], [\vartheta_{\tilde{A}_i}^L, \vartheta_{\tilde{A}_i}^U])$ ($i = 1, 2, \dots, n$) be a set of IVIFSs and $w = (w_1, w_2, \dots, w_n)^T$ be weight vector of $\tilde{A}_i$ with $\sum_{i=1}^n w_i = 1$, then an interval-valued intuitionistic fuzzy weighted average (IVIFWA) the operator is a mapping $IVIFWA : \tilde{A}^n \rightarrow \tilde{A}$, where (Xu, 2007):

$$\begin{aligned}
& IVIFWA(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) \\
&= \left(\left[\left(1 - \prod_{i=1}^n \left(1 - \left(\mu_{\tilde{A}_i}^L \right)^{w_i} \right) \right), \left(1 - \prod_{i=1}^n \left(1 - \left(\mu_{\tilde{A}_i}^U \right)^{w_i} \right) \right) \right] \right. \\
&\quad \left. \left[\prod_{i=1}^n \left(\vartheta_{\tilde{A}_i}^L \right)^{w_i}, \prod_{i=1}^n \left(\vartheta_{\tilde{A}_i}^U \right)^{w_i} \right] \right) \quad (2.23)
\end{aligned}$$

Definition 2.11. The score function $S(\tilde{A})$ of $\tilde{A}_i = \left([\mu_{\tilde{A}_i}^L, \mu_{\tilde{A}_i}^U], [\vartheta_{\tilde{A}_i}^L, \vartheta_{\tilde{A}_i}^U] \right)$ ($i = 1, 2, \dots, n$) is given as in Eq. (2.24) (J. Wu & Chiclana, 2014).

$$S(\tilde{A}) = \frac{\mu_{\tilde{A}}^L + \mu_{\tilde{A}}^U - \vartheta_{\tilde{A}}^L - \vartheta_{\tilde{A}}^U}{2} \quad (2.24)$$

Definition 2.12. Defuzzification of $\tilde{A}_i = \left([\mu_{\tilde{A}_i}^L, \mu_{\tilde{A}_i}^U], [\vartheta_{\tilde{A}_i}^L, \vartheta_{\tilde{A}_i}^U] \right)$ ($i = 1, 2, \dots, n$) is given as in Eq. (2.25).

$$\begin{aligned}
& Def(\tilde{A}_i) \\
&= \left(\frac{\mu_{\tilde{A}_i}^L + \mu_{\tilde{A}_i}^U + (1 - \vartheta_{\tilde{A}_i}^L) + (1 - \vartheta_{\tilde{A}_i}^U) + \mu_{\tilde{A}_i}^L \times \mu_{\tilde{A}_i}^U - \sqrt{(1 - \vartheta_{\tilde{A}_i}^L) \times (1 - \vartheta_{\tilde{A}_i}^U)}}{4} \right)^3 \quad (2.25)
\end{aligned}$$

2.3 Continuous Intuitionistic Fuzzy Sets (CINFUS)

This section includes the basic definitions related to CINFUSs. The basic operations of CINFUSs are mentioned as follows.

Definition 2.13. Let X be a fixed set. A CINFUS C in X is an object having the form:

$$C = \{ \langle x, F(\mu_C(x), \tau), F(\vartheta_C(x), \tau) \rangle \mid x \in X \} \quad (2.26)$$

where the function $F(\mu_C(x), \tau): X \rightarrow [0, 1]$ and $F(\vartheta_C(x), \tau): X \rightarrow [0, 1]$ denote continuous membership and non-membership degrees of x to the set C , respectively. Also, where $F(\mu_C(x), \tau) = \alpha\tau - \alpha\tau^2$ and $F(\vartheta_C(x), \tau) = (4 - \alpha)\tau - (4 - \alpha)\tau^2$. It satisfies the condition

$$\begin{aligned}
0 \leq F(\mu_C(x), \tau) + F(\vartheta_C(x), \tau) &\leq 1 \Rightarrow 0 \\
&\leq \alpha\tau - \alpha\tau^2 + (4 - \alpha)\tau - (4 - \alpha)\tau^2 \leq 1 \Rightarrow 0 \\
&\leq 4\tau - 4\tau^2 \leq 1, \quad \text{for } \forall x \in X
\end{aligned} \quad (2.27)$$

The degree of uncertainty in judgment is defined as $F(\pi, \tau) = 1 - \alpha\tau + \alpha\tau^2 - (4 - \alpha)\tau + (4 - \alpha)\tau^2 = 1 - 4(\tau - \tau^2)$. Figure 2.1 represents the membership, non-membership and hesitancy degrees of CINFUSs.

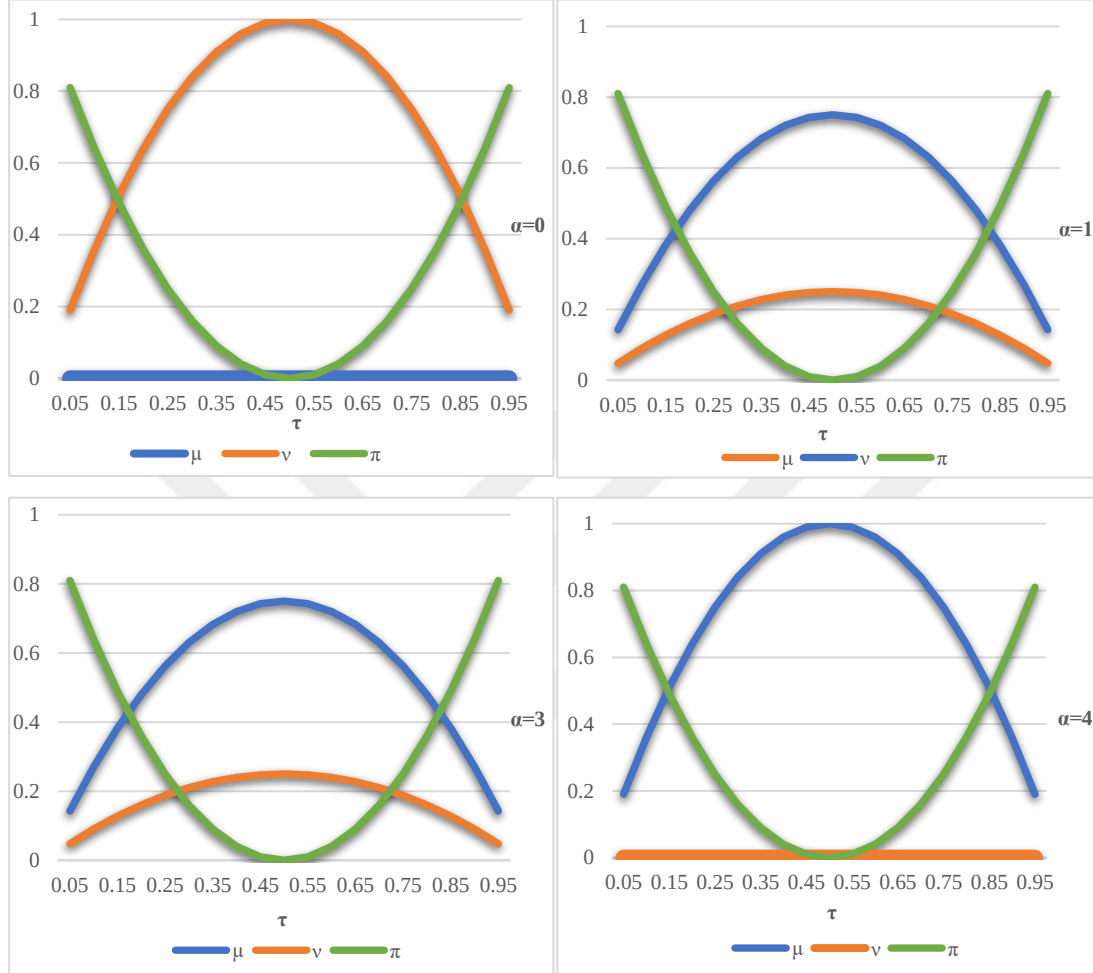


Figure 2.1 : Membership, non-membership and hesitancy degrees of CINFUSs for $\alpha=0$, $\alpha=1$, $\alpha=3$ and $\alpha=4$.

Definition 2.14. For every two $CINFUS_A = \langle (\alpha_A\tau - \alpha_A\tau^2), ((4 - \alpha_A)\tau - (4 - \alpha_A)\tau^2) \rangle$ and $CINFUS_B = \langle (\alpha_B\tau - \alpha_B\tau^2), ((4 - \alpha_B)\tau - (4 - \alpha_B)\tau^2) \rangle$, their basic operations are defined as follows:

$$\begin{aligned}
 CINFUS_A &\subset CINFUS_B \\
 &= \{(\alpha_A\tau - \alpha_A\tau^2), ((4 - \alpha_A)\tau - (4 - \alpha_A)\tau^2)\} \\
 &< (\alpha_B\tau - \alpha_B\tau^2), ((4 - \alpha_B)\tau - (4 - \alpha_B)\tau^2)\}
 \end{aligned} \tag{2.28}$$

and then,

$$\begin{aligned}
CINFUS_A &\subset CINFUS_B \\
&= \{(\alpha_A\tau - \alpha_A\tau^2) \\
&< (\alpha_B\tau - \alpha_B\tau^2), ((4 - \alpha_A)\tau - (4 - \alpha_A)\tau^2) \\
&> ((4 - \alpha_B)\tau - (4 - \alpha_B)\tau^2)\}
\end{aligned} \tag{2.29}$$

$$\begin{aligned}
CINFUS_A &= CINFUS_B \\
&\leftrightarrow \{(\alpha_A\tau - \alpha_A\tau^2), ((4 - \alpha_A)\tau - (4 - \alpha_A)\tau^2) \\
&= (\alpha_B\tau - \alpha_B\tau^2), ((4 - \alpha_B)\tau - (4 - \alpha_B)\tau^2)\}
\end{aligned} \tag{2.30}$$

and then,

$$\begin{aligned}
CINFUS_A &= CINFUS_B \\
&\leftrightarrow \{(\alpha_A\tau - \alpha_A\tau^2) \\
&= (\alpha_B\tau - \alpha_B\tau^2), ((4 - \alpha_A)\tau - (4 - \alpha_A)\tau^2) \\
&= ((4 - \alpha_B)\tau - (4 - \alpha_B)\tau^2)\}
\end{aligned} \tag{2.31}$$

$$\begin{aligned}
CINFUS_A \cup CINFUS_B \\
&= \{ \langle \max((\alpha_A\tau - \alpha_A\tau^2), (\alpha_B\tau \\
&- \alpha_B\tau^2)), \min(((4 - \alpha_A)\tau - (4 - \alpha_A)\tau^2), ((4 - \alpha_B)\tau \\
&- (4 - \alpha_B)\tau^2)) \rangle \}
\end{aligned} \tag{2.32}$$

$$\begin{aligned}
CINFUS_A \cap CINFUS_B \\
&= \{ \langle \min((\alpha_A\tau - \alpha_A\tau^2), (\alpha_B\tau \\
&- \alpha_B\tau^2)), \max(((4 - \alpha_A)\tau - (4 - \alpha_A)\tau^2), ((4 - \alpha_B)\tau \\
&- (4 - \alpha_B)\tau^2)) \rangle \}
\end{aligned} \tag{2.33}$$

Definition 2.15. Let $CINFUS_A = \langle (\alpha_A\tau - \alpha_A\tau^2), ((4 - \alpha_A)\tau - (4 - \alpha_A)\tau^2) \rangle$ and $CINFUS_B = \langle (\alpha_B\tau - \alpha_B\tau^2), ((4 - \alpha_B)\tau - (4 - \alpha_B)\tau^2) \rangle$ be three CINFUSs, then the mathematical operations on these two continuous intuitionistic fuzzy numbers (CINFUNs) are defined as follows:

$$\begin{aligned}
& CINFUS_A \oplus CINFUS_B \\
&= \langle ((\alpha_A + \alpha_B)\tau - (\alpha_A + \alpha_B)\tau^2 \\
&\quad - (\alpha_A\tau - \alpha_A\tau^2)(\alpha_B\tau - \alpha_B\tau^2)), ((4 - \alpha_A)\tau \\
&\quad - (4 - \alpha_A)\tau^2 ((4 - \alpha_B)\tau - (4 - \alpha_B)\tau^2)) \rangle
\end{aligned} \tag{2.34}$$

Moreover, the results of mathematical operations that consist of addition, multiplication, multiplication by and power of a scalar λ are shown in Figures 2.2, 2.3, 2.4, and 2.5, respectively. In Figure 2.2, $\alpha_A = 1$ and $\alpha_B = 2$ represent the magnitude rating of membership functions for $CINFUS_A$ and $CINFUS_B$, respectively. Also, the sum of the membership degrees, the sum of the degrees of non-membership of two CINFUNs and the corresponding degrees of membership, non-membership and hesitancy are shown in Figure 2.2, respectively.

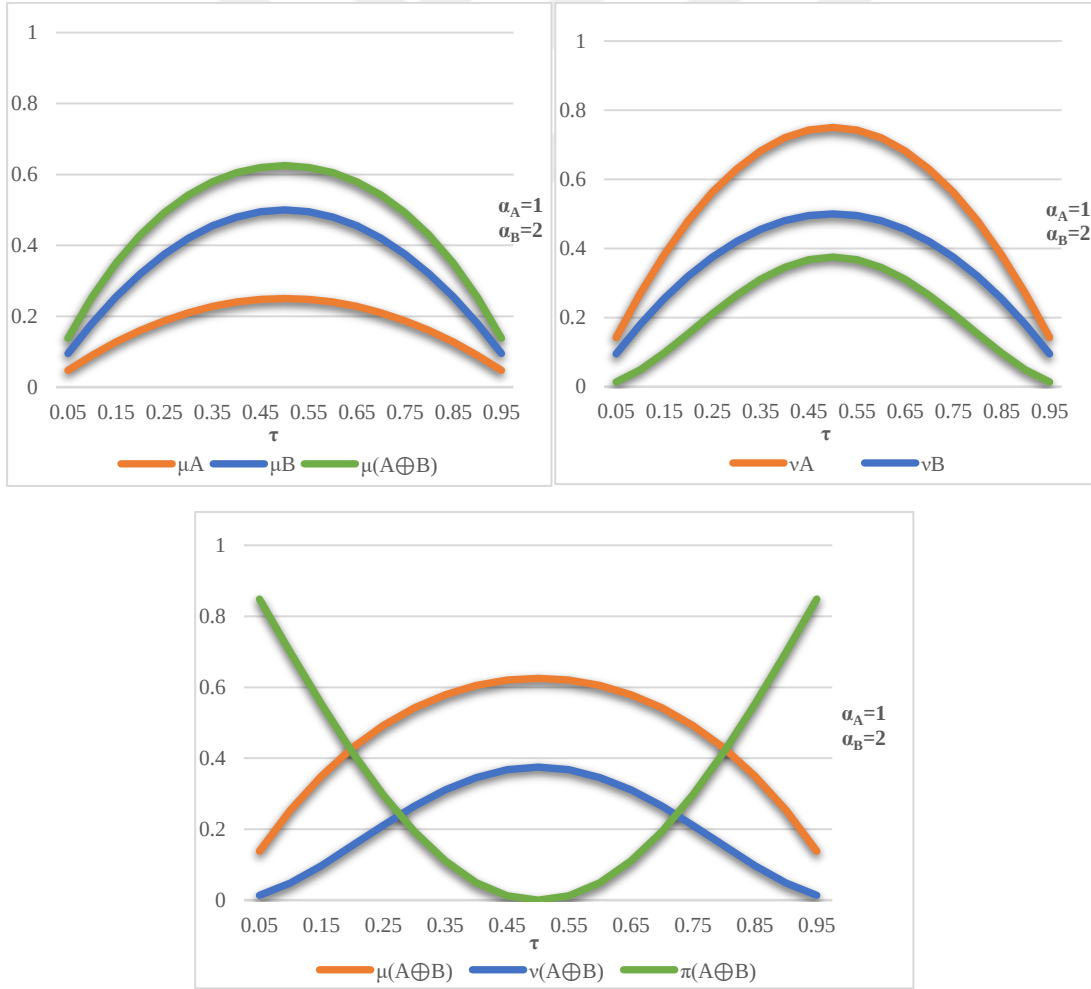


Figure 2.2 : Addition operation for two CINFUNs.

$$CINFUS_A \otimes CINFUS_B$$

$$\begin{aligned}
&= (\alpha_A \tau - \alpha_A \tau^2)(\alpha_B \tau - \alpha_B \tau^2), (4 - \alpha_A)\tau - (4 - \alpha_A)\tau^2 + (4 - \alpha_B)\tau \\
&- (4 - \alpha_B)\tau^2 - ((4 - \alpha_A)\tau - (4 - \alpha_A)\tau^2)((4 - \alpha_B)\tau - (4 - \alpha_B)\tau^2) \\
&= \langle ((\alpha_A + \alpha_B)(\tau - \tau^2)), ((8 - \alpha_A - \alpha_B)(\tau - \tau^2) \\
&- (4 - \alpha_A)(4 - \alpha_B)(\tau - \tau^2)^2) \rangle
\end{aligned}$$

(2.35)

In Figure 2.3, $\alpha_A = 3$ and $\alpha_B = 4$ represent the magnitude rating of membership functions for $CINFUS_A$ and $CINFUS_B$, respectively. Also, the product of the membership degrees, the product of the non-membership degrees of two CINFUNs and the corresponding degrees of membership, non-membership and hesitancy are shown in Figure 2.3, respectively.

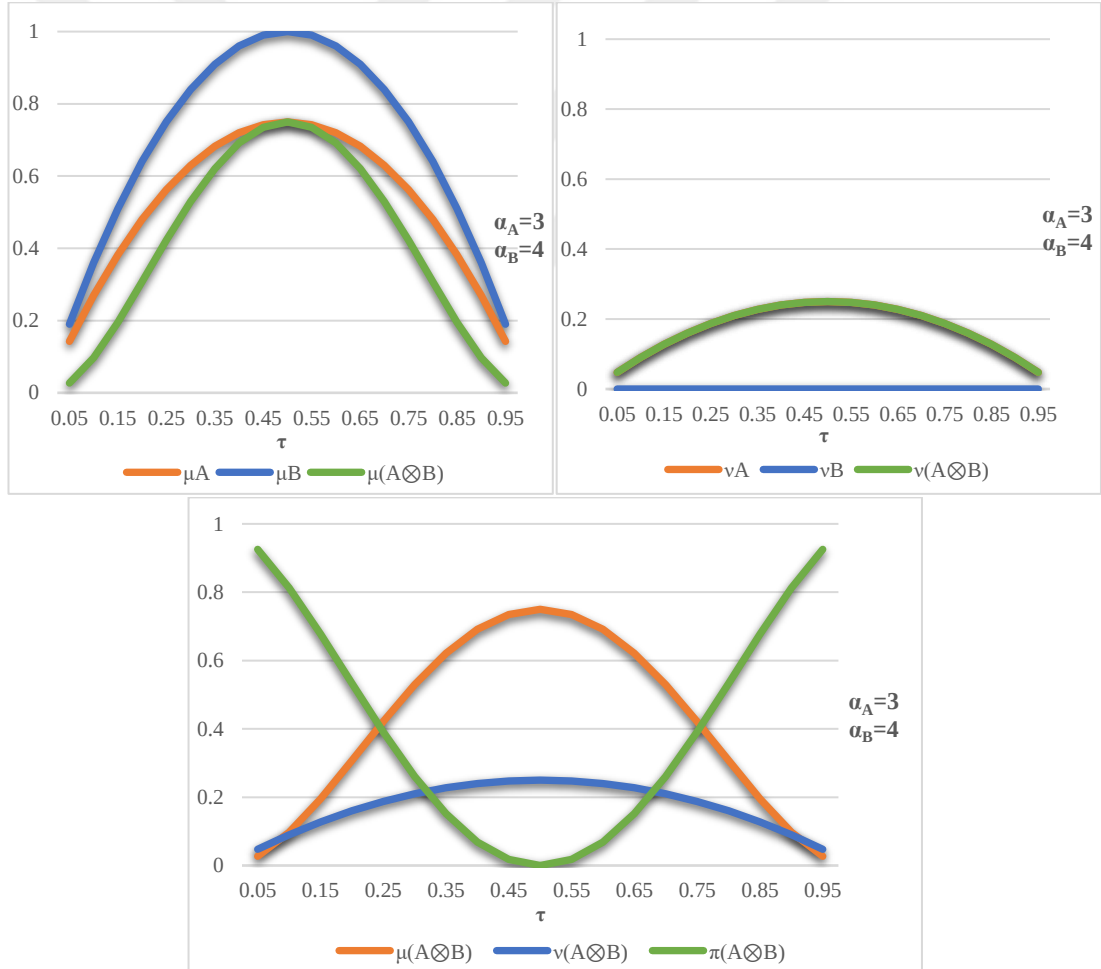


Figure 2.3 : Multiplication operation for two CINFUNs.

$$\lambda \times C - INFUS_A = \langle (1 - (1 - \alpha_A \tau + \alpha_A \tau^2)^\lambda), ((4 - \alpha_A)^\lambda (\tau - \tau^2)^\lambda) \rangle \quad (2.36)$$

In Figure 2.4, $\alpha_A = 1$ and $\lambda = 5$ represent the magnitude rating of membership functions and the value of scalar number for *CINFUS*. Also, the membership functions multiplied by the scalar number of CINFUN are presented in Figure 2.4.

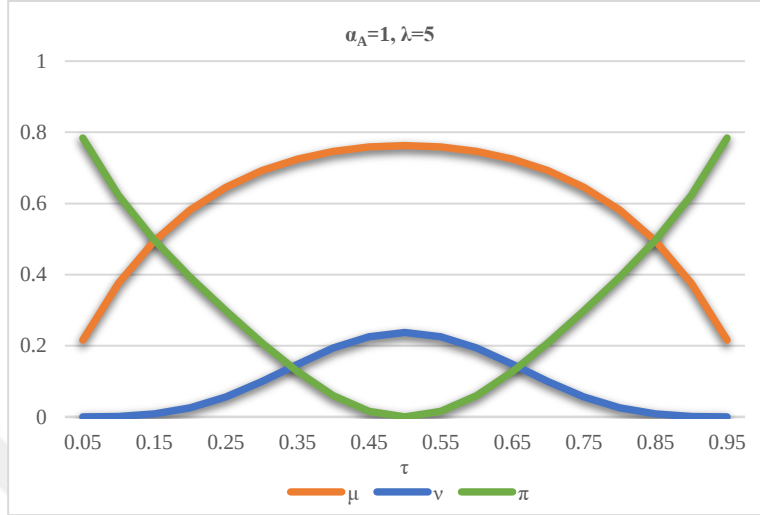


Figure 2.4 : Multiplication by a scalar of membership functions.

$$C - INFUS_A^\lambda = \langle (\alpha_A \tau - \alpha_A \tau^2)^\lambda, (1 - (1 - (4 - \alpha_A)(\tau - \tau^2))^\lambda) \rangle \quad (2.37)$$

In Figure 2.5, $\alpha_A = 3$ and $\lambda = 4$ represent the magnitude rating of membership functions and the value of scalar number for *CINFUS*. Also, the scalar power of CINFUN is presented in Figure 2.5.

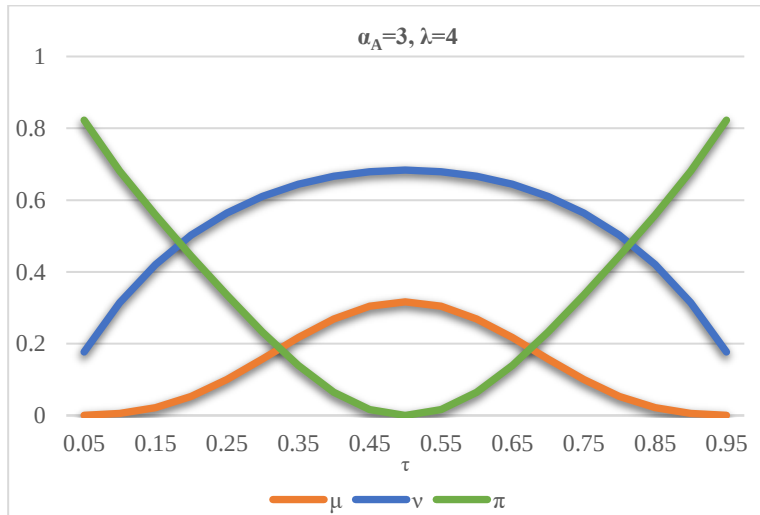


Figure 2.5 : Scalar number power of CINFUN.

Definition 2.16. Let $\tilde{A}_i = \langle (\alpha\tau - \alpha\tau^2)_i, ((4 - \alpha)\tau - (4 - \alpha)\tau^2)_i \rangle, (i = 1, 2, \dots, n)$ be a set of CINFUSs and $w = (w_1, w_2, \dots, w_n)^T$ be weight vector of $\tilde{A}_i$ with $\sum_{i=1}^n w_i = 1$, then a continuous intuitionistic fuzzy weighted geometric (CINFUWG) the operator is a mapping (CINFUWG) : $\tilde{A}^n \rightarrow \tilde{A}$, where

$$\begin{aligned} \text{CINFUWG}(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) \\ = \left(\prod_{i=1}^n (\alpha\tau - \alpha\tau^2)_i^{w_i}, 1 \right. \\ \left. - \prod_{i=1}^n \left(1 - ((4 - \alpha)\tau - (4 - \alpha)\tau^2)_i \right)^{w_i} \right) \end{aligned} \quad (2.38)$$

Definition 2.17. Let $\tilde{A}_i = \langle (\alpha\tau - \alpha\tau^2)_i, ((4 - \alpha)\tau - (4 - \alpha)\tau^2)_i \rangle, (i = 1, 2, \dots, n)$ be a set of CINFUSs and $w = (w_1, w_2, \dots, w_n)^T$ be weight vector of $\tilde{A}_i$ with $\sum_{i=1}^n w_i = 1$, then a continuous intuitionistic fuzzy weighted average (CINFUWA) the operator is a mapping CINFUWA : $\tilde{A}^n \rightarrow \tilde{A}$, where

$$\begin{aligned} \text{CINFUWA}(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) \\ = \left(1 - \prod_{i=1}^n (1 - (\alpha\tau - \alpha\tau^2)_i)^{w_i}, \right. \\ \left. \prod_{i=1}^n ((4 - \alpha)\tau - (4 - \alpha)\tau^2)_i^{w_i} \right) \end{aligned} \quad (2.39)$$

Definition 2.18. Let $\text{CINFUN} = \langle (\alpha\tau - \alpha\tau^2), ((4 - \alpha)\tau - (4 - \alpha)\tau^2) \rangle$ be an CINFUN, then the score function $S(\text{CINFUN})$ and accuracy function of CINFUN , $A(\text{CINFUN})$ can be defined as in Eqs. (2.40) and (2.41), respectively.

$$S(\text{CINFUN}) = \alpha(\tau - \tau^2) - (4 - \alpha)(\tau - \tau^2) = (2\alpha - 4)(\tau - \tau^2) \quad (2.40)$$

$$A(\text{CINFUN}) = \alpha(\tau - \tau^2) + (4 - \alpha)(\tau - \tau^2) = 4(\tau - \tau^2) \quad (2.41)$$

2.4 CINFU-AHP

In this section, the CINFU-AHP method is presented to prioritize the criteria. The steps of the developed method are given as follows:

The fuzzy pairwise comparison scale in Table 2.1 is used in the CINFU-AHP method.

Table 2.1 : Linguistic scale for criteria weighting.

Linguistic terms	CINFUNs	
	μ	ϑ
Absolutely less important than – (ALI)	$0\tau - 0\tau^2$	$4\tau - 4\tau^2$
Very strongly less important than – (VSLI)	$0.5\tau - 0.5\tau^2$	$3.5\tau - 3.5\tau^2$
Strongly Less important than – (StLI)	$\tau - \tau^2$	$3\tau - 3\tau^2$
Slightly less important than – (SILI)	$1.5\tau - 1.5\tau^2$	$2.5\tau - 2.5\tau^2$
Exactly equal importance – (EEI)	$2\tau - 2\tau^2$	$2\tau - 2\tau^2$
Slightly more important than – (SIMI)	$2.5\tau - 2.5\tau^2$	$1.5\tau - 1.5\tau^2$
Strongly more important than – (StMI)	$3\tau - 3\tau^2$	$1\tau - 1\tau^2$
Very strongly more important than – (VSMI)	$3.5\tau - 3.5\tau^2$	$0.5\tau - 0.5\tau^2$
Absolutely more important than – (AMI)	$4\tau - 4\tau^2$	$0\tau - 0\tau^2$

Step 1. Determine the alternatives, and relevant criteria to construct the framework. The set $A_i = \{A_1, A_2, \dots, A_m\}$ having $i = 1, 2, \dots, m$ alternatives, is evaluated by n decision criteria of set $C_j = \{C_1, C_2, \dots, C_n\}$, with $j = 1, 2, \dots, n$. Let $w = (w_1, w_2, \dots, w_n)$ be the vector set used to define the weights of criteria, where $w_j > 0$ and $\sum_{j=1}^n w_j = 1$. Decision-makers (DMs) are represented with $DM\ 1, DM\ 2, \dots, DM\ k$, who are experts in their fields to make judgments.

Step 2. Construct the linguistic pairwise comparison matrix as in Eq. (2.42), where two possible τ values are assigned by each expert. Substitute assigned the τ values ($0 < \tau < 1$) into the linguistic functions and construct the corresponding interval valued intuitionistic fuzzy numbers (IVIFNs) as in Eq. (2.43).

$$\tilde{\Psi}_k = \begin{matrix} C_1 \\ \vdots \\ C_n \end{matrix} \begin{bmatrix} \langle EEI; (0.5, 0.5) \rangle & \cdots & \langle \tilde{\psi}_{1nk}, (\tau_{1nk}, \tau_{2nk}) \rangle \\ \vdots & \ddots & \vdots \\ \langle \tilde{\psi}_{n1k}, (\tau_{n1k}, \tau_{n2k}) \rangle & \cdots & \langle EEI; (0.5, 0.5) \rangle \end{bmatrix} \quad (2.42)$$

$$\begin{aligned} & \tilde{\Psi}_{[k]} \\ &= \begin{matrix} C_1 \\ \vdots \\ C_n \end{matrix} \begin{bmatrix} \langle [\mu_{11k}^L, \mu_{11k}^U], [\vartheta_{11k}^L, \vartheta_{11k}^U] \rangle & \cdots & \langle [\mu_{1nk}^L, \mu_{1nk}^U], [\vartheta_{1nk}^L, \vartheta_{1nk}^U] \rangle \\ \vdots & \ddots & \vdots \\ \langle [\mu_{n1k}^L, \mu_{n1k}^U], [\vartheta_{n1k}^L, \vartheta_{n1k}^U] \rangle & \cdots & \langle [\mu_{nnk}^L, \mu_{nnk}^U], [\vartheta_{nnk}^L, \vartheta_{nnk}^U] \rangle \end{bmatrix} \end{aligned} \quad (2.43)$$

Step 3. Check the consistency of the IVIF pairwise comparison matrices of each DM. To measure the consistency of the DMs' judgments, IVIFNs are first clarified using Eq. (2.44). The crisp numbers obtained after defuzzification are matched with Saaty's scale, and then Saaty's classical consistency process is applied.

$$\begin{aligned}
& Def(\tilde{A}_i) \\
&= 9 \times \left(\frac{\mu_{\tilde{A}_i}^L + \mu_{\tilde{A}_i}^U + (1 - \vartheta_{\tilde{A}_i}^L) + (1 - \vartheta_{\tilde{A}_i}^U) + \mu_{\tilde{A}_i}^L \times \mu_{\tilde{A}_i}^U - \sqrt{(1 - \vartheta_{\tilde{A}_i}^L) \times (1 - \vartheta_{\tilde{A}_i}^U)}}{4} \right)^3
\end{aligned} \quad (2.44)$$

Step 4. Obtain the aggregated IVIF pairwise comparison matrix $\tilde{\Psi} = (\tilde{\psi}_{i'j})_{n \times n}$ by using *IVIFWG* aggregation operator given in Eq. (2.22). Aggregated IVIF pairwise comparison matrix $\tilde{\Psi} = (\tilde{\psi}_{i'j})_{n \times n}$ in which $\tilde{\psi}_{ij} = \langle [\mu_{i'j}^L, \mu_{i'j}^U], [\vartheta_{i'j}^L, \vartheta_{i'j}^U] \rangle$ represents the IVIFN for the comparison of criterion C_i and criterion C_j .

Step 5. Form the score judgment matrix $\tilde{P} = (\tilde{p}_{i'j})_{n \times n}$ by using Eq. (2.45).

$$\tilde{P} = \begin{bmatrix} (\mu_{11}^L - \vartheta_{11}^U, \mu_{11}^U - \vartheta_{11}^L) & \cdots & (\mu_{1n}^L - \vartheta_{1n}^U, \mu_{1n}^U - \vartheta_{1n}^L) \\ \vdots & \ddots & \vdots \\ (\mu_{n1}^L - \vartheta_{n1}^U, \mu_{n1}^U - \vartheta_{n1}^L) & \cdots & (\mu_{nn}^L - \vartheta_{nn}^U, \mu_{nn}^U - \vartheta_{nn}^L) \end{bmatrix} \quad (2.45)$$

Step 6. Calculate the interval multiplicative matrix $\tilde{\Xi} = (\tilde{\xi}_{i'j})_{n \times n}$ as in Eq. (2.46).

$$\begin{aligned}
\tilde{\Xi} &= \begin{bmatrix} (10^{\mu_{11}^L - \vartheta_{11}^U}, 10^{\mu_{11}^U - \vartheta_{11}^L}) & \cdots & (10^{\mu_{1n}^L - \vartheta_{1n}^U}, 10^{\mu_{1n}^U - \vartheta_{1n}^L}) \\ \vdots & \ddots & \vdots \\ (10^{\mu_{n1}^L - \vartheta_{n1}^U}, 10^{\mu_{n1}^U - \vartheta_{n1}^L}) & \cdots & (10^{\mu_{nn}^L - \vartheta_{nn}^U}, 10^{\mu_{nn}^U - \vartheta_{nn}^L}) \end{bmatrix} \\
&= \begin{bmatrix} (\xi_{11}^L, \xi_{11}^U) & \cdots & (\xi_{1n}^L, \xi_{1n}^U) \\ \vdots & \ddots & \vdots \\ (\xi_{n1}^L, \xi_{n1}^U) & \cdots & (\xi_{nn}^L, \xi_{nn}^U) \end{bmatrix}
\end{aligned} \quad (2.46)$$

Step 7. Determine the priority vector of the interval multiplicative matrix by calculating the interval $\tilde{\omega}_j$ of each criterion by using Eq. (2.47).

$$\tilde{\omega}_j = \left[\frac{\sum_{i=1}^n \xi_{i'j}^L}{\sum_{i=1}^n \sum_{j=1}^n \xi_{i'j}^U}, \frac{\sum_{j=1}^n \xi_{i'j}^U}{\sum_{i=1}^n \sum_{j=1}^n \xi_{i'j}^L} \right] = [\omega_j^L, \omega_j^U], j = 1, 2, \dots, n \quad (2.47)$$

Step 8. Construct the possibility degree matrix $\gamma = (\gamma_{i'j})_{n \times n}$ by comparing the priority vectors obtained in Step 7 through Eq. (2.48).

$$\gamma(\tilde{\omega}_{i'} > \tilde{\omega}_j) = \gamma_{i'j} = \frac{\max(0, \omega_{i'}^U - \omega_j^L) - \max(0, \omega_{i'}^L - \omega_j^U)}{(\omega_{i'}^U - \omega_j^L) + (\omega_{i'}^L - \omega_j^U)} \quad (2.48)$$

where $v_{i'j} \geq 0, v_{i'j} + v_{ji'} = 1, v_{i'i'} = 1$.

Step 9. Prioritize possibility degrees by using Eq. (2.49).

$$\rho_j = \frac{1}{n} \left[\sum_{j=1}^n v_{i'j} + \frac{n}{2} - 1 \right] \quad (2.49)$$

Step 10. Normalize the weights ρ_j and calculate the normalized crisp weights matrix w_j by using Eq. (2.50).

$$w_j = \frac{\rho_j}{\sum_{j=1}^n \rho_j} \quad (2.50)$$

Step 11. Repeat the steps 1-10 for the other pairwise comparison matrices which may be for the comparison of sub-criteria (if any) and for the comparison of alternatives.

Step 12. Combine the crisp weights of main criteria (w_j), sub-criteria ($w_{sub,j}$) and alternatives (w_i) in order to obtain the overall weights of the alternatives.

2.5 CINFU-TOPSIS

In this section, the TOPSIS method developed based on CINFUSs has been presented to evaluate the alternatives with respect to criteria. The steps of the developed method are given as follows:

Step 1. Determine the alternatives and relevant criteria to construct the framework of application. The set $A_i = \{A_1, A_2, \dots, A_m\}$ having $i = 1, 2, \dots, m$ alternatives, is evaluated by n decision criteria $C_j = \{C_1, C_2, \dots, C_n\}$, with $j = 1, 2, \dots, n$. Let $w = (w_1, w_2, \dots, w_n)$ be the vector set used to define the weights of criteria, where $w_j > 0$ and $\sum_{j=1}^n w_j = 1$. Decision-makers (DMs) are represented with $DM 1, DM 2, \dots, DM k$, who are experts in their fields to make judgments.

Step 2. Construct the continuous intuitionistic fuzzy (CINFU) decision matrices according to opinion of each DM by using the linguistic scale given in Table 2.2. The linguistic decision matrices are transformed into its corresponding continuous intuitionistic fuzzy numbers (CINFUNs). The CINFU decision matrix $\tilde{X}_k = (\tilde{x}_{ijk})_{n \times m}$ according to k th DM is shown in Table 2.3. Here, $\tilde{X}_k = (\tilde{x}_{ijk})_{n \times m}$ in which

$\tilde{x}_{ijk} = \left((\alpha\tau - \alpha\tau^2)_{ijk}, ((4 - \alpha)\tau - (4 - \alpha)\tau^2)_{ijk} \right)$ represents the CINFU value of alternative A_i with respect to criterion C_j of k th DM.

Table 2.2 : Linguistic scale for ratings of alternatives in terms of criteria.

Linguistic terms	CINFUN for alternatives	
	μ	ϑ
Absolutely Low Value – (ALV)	$0\tau - 0\tau^2$	$4\tau - 4\tau^2$
Very Low Value – (VLV)	$0.5\tau - 0.5\tau^2$	$3.5\tau - 3.5\tau^2$
Low Value – (LV)	$\tau - \tau^2$	$3\tau - 3\tau^2$
Slightly Low Value – (MLV)	$1.5\tau - 1.5\tau^2$	$2.5\tau - 2.5\tau^2$
Medium Value – (AV)	$2\tau - 2\tau^2$	$2\tau - 2\tau^2$
Slightly High Value – (MHV)	$2.5\tau - 2.5\tau^2$	$1.5\tau - 1.5\tau^2$
High Value – (HV)	$3\tau - 3\tau^2$	$1\tau - 1\tau^2$
Very High Value – (VHV)	$3.5\tau - 3.5\tau^2$	$0.5\tau - 0.5\tau^2$
Absolutely High Value – (AHV)	$4\tau - 4\tau^2$	$0\tau - 0\tau^2$

Table 2.3 : Decision matrix based on CINFUNs for k th DM.

Criteria	Alternatives		
	A_1	...	A_m
C_1	$\left((\alpha\tau - \alpha\tau^2)_{11k}, ((4 - \alpha)\tau - (4 - \alpha)\tau^2)_{11k} \right)$	...	$\left((\alpha\tau - \alpha\tau^2)_{1mk}, ((4 - \alpha)\tau - (4 - \alpha)\tau^2)_{1mk} \right)$
C_2	$\left((\alpha\tau - \alpha\tau^2)_{21k}, ((4 - \alpha)\tau - (4 - \alpha)\tau^2)_{21k} \right)$	...	$\left((\alpha\tau - \alpha\tau^2)_{2mk}, ((4 - \alpha)\tau - (4 - \alpha)\tau^2)_{2mk} \right)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
C_n	$\left((\alpha\tau - \alpha\tau^2)_{n1k}, ((4 - \alpha)\tau - (4 - \alpha)\tau^2)_{n1k} \right)$	...	$\left((\alpha\tau - \alpha\tau^2)_{nmk}, ((4 - \alpha)\tau - (4 - \alpha)\tau^2)_{nmk} \right)$

Step 3. Obtain the aggregated CINFU decision matrix. The decision matrices of DMs are aggregated in a single decision matrix by utilizing the *CINFUWG* aggregation operator given in Eq. (2.38). Aggregated CINFU decision matrix ($\tilde{X}$) is constructed according to each τ value, whose ranges from zero to one are determined by the expert as given in Table 2.4. Here, $\tilde{X} = (\tilde{x}_{ij})_{n \times m}$ in which $\tilde{x}_{ij} = (\mu_{ij,\tau}, \vartheta_{ij,\tau})$ indicates the aggregated CINFUN of i^{th} alternative according to j^{th} criterion.

Table 2.4 : Aggregated CINFU decision matrix.

Criteria	Alternatives		
	A_1		A_m
C_1	$(\mu_{11,\tau}, \vartheta_{11,\tau})$		$(\mu_{1m,\tau}, \vartheta_{1m,\tau})$
C_2	$(\mu_{21,\tau}, \vartheta_{21,\tau})$		$(\mu_{2m,\tau}, \vartheta_{2m,\tau})$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
C_n	$(\mu_{n1,\tau}, \vartheta_{n1,\tau})$		$(\mu_{nm,\tau}, \vartheta_{nm,\tau})$

Step 4. Obtain the continuous intuitionistic fuzzy (CINFU) weight vector of criteria from fuzzy AHP stage in Section 2.4.

Step 5. Obtain the weighted decision matrix. After the weights of criteria and the CINFU ratings of the alternatives are identified, the weighted CINFU decision matrix $\tilde{\mathfrak{S}} = (\tilde{\zeta}_{ij})_{n \times m}$ is obtained for each τ value by using Eq. (2.51).

$$\tilde{\mathfrak{S}} = |\tilde{\zeta}_{ij}| = |w_j \tilde{x}_{ij}| \quad (2.51)$$

where $w_j \tilde{x}_{ij} = (\mu_{\zeta_{ij}, \tau}, \vartheta_{\zeta_{ij}, \tau})$ shows the weighted CINFUN of i th alternative with respect to j th criterion.

Step 6. Determine the CINFU positive ideal solution (CINFU-PIS) and CINFU negative ideal solution (CINFU-NIS) based on the weighted CINFU decision matrix. After score values of each alternative with respect to criteria are calculated for each τ value by using Eq. (2.40), CINFU-PIS and CINFU-NIS are obtained for each τ value by Eq. (2.52) and Eq. (2.53), respectively.

$$\begin{aligned} CINFU - PIS = \delta^* = \{ \langle (\max_i \tilde{\zeta}_{ij} | j \in \mathcal{J}_1), (\min_i \tilde{\zeta}_{ij} | j \in \mathcal{J}_2) \rangle | j = \\ 1, 2, \dots, n \} = \{ \tilde{\zeta}_1^*, \tilde{\zeta}_2^*, \dots, \tilde{\zeta}_n^* \}, \text{ for } i = 1, 2, \dots, m \end{aligned} \quad (2.52)$$

$$\begin{aligned} CINFU - NIS = \delta^- = \{ \langle (\min_i \tilde{\zeta}_{ij} | j \in \mathcal{J}_1), (\max_i \tilde{\zeta}_{ij} | j \in \mathcal{J}_2) \rangle | j = \\ 1, 2, \dots, n \} = \{ \tilde{\zeta}_1^-, \tilde{\zeta}_2^-, \dots, \tilde{\zeta}_n^- \}, \text{ for } i = 1, 2, \dots, m \end{aligned} \quad (2.53)$$

where $\tilde{\zeta}_j^* = (\mu_{j, \tau}^*, \vartheta_{j, \tau}^*)$ is the maximum CINFUN with the highest score value among alternatives for j^{th} criterion and $\tilde{\zeta}_j^- = (\mu_{j, \tau}^-, \vartheta_{j, \tau}^-)$ is the minimum CINFUN with the lowest score value among alternatives for j^{th} criterion. Also, $\mathcal{J}_1, \mathcal{J}_2$ are benefit and cost criteria, respectively and $j = 1, 2, \dots, n$.

For simplicity, the intuitionistic fuzzy positive and negative ideal solutions can be determined as

$$\begin{aligned} CINFU - PIS = \delta^* = \{ \langle ((1, 0), \text{for benefit criteria}), ((0, 1), \\ \text{for cost criteria}) \rangle | \} = \{ \tilde{\zeta}_1^*, \tilde{\zeta}_2^*, \dots, \tilde{\zeta}_n^* \}, \text{ for } i = 1, 2, \dots, m \end{aligned} \quad (2.54)$$

$$\begin{aligned} CINFU - NIS = \delta^- = \{ \langle ((0, 1), \text{for benefit criteria}), ((1, 0), \\ \text{for cost criteria}) \rangle | \} = \{ \tilde{\zeta}_1^-, \tilde{\zeta}_2^-, \dots, \tilde{\zeta}_n^- \}, \text{ for } i = 1, 2, \dots, m \end{aligned} \quad (2.55)$$

In this paper, CINFU-PIS and CINFU-NIS in Eqs. (2.54) and (2.55) are used.

Step 7. Obtain the separation measures by calculating the distances for each alternative according to positive ideal solution δ^* and negative ideal solution δ^- . The separation measures of each alternative are calculated for each τ value based on the weighted CINFU decision matrix by using CINFU Euclidean distances given for positive and negative ideal solutions in Eq. (2.56) and Eq. (2.57), respectively.

$$\Delta_{i,\tau}^* = \sqrt{\frac{1}{2n} \sum_{j=1}^n \left((\mu_{ij,\tau} - 1)^2 + (\vartheta_{ij,\tau} - 0)^2 \right)} \quad \text{for } i = 1, 2, \dots, m \quad (2.56)$$

$$\Delta_{i,\tau}^- = \sqrt{\frac{1}{2n} \sum_{j=1}^n \left((\mu_{ij,\tau} - 0)^2 + (\vartheta_{ij,\tau} - 1)^2 \right)} \quad \text{for } i = 1, 2, \dots, m \quad (2.57)$$

Step 8. Calculate the relative closeness to ideal solution Y_i of each alternative for each τ value by using Eq. (2.58).

$$Y_{i,\tau} = \frac{\Delta_{i,\tau}^-}{\Delta_{i,\tau}^- + \Delta_{i,\tau}^*}, \quad \text{for } i = 1, 2, \dots, m \quad (2.58)$$

Step 9. Rank the alternatives according to final scores of $Y_{i,\tau}$. The best alternative(s) are selected based on the descending order of the relative closeness ($Y_{i,\tau}$) values for each τ value.

2.6 Application

In this section, we give the implementation steps of CINFU-AHP and CINFU-TOPSIS methods on research proposals evaluations for grant funding. The CINFU-AHP method developed to obtain the weights of the criteria is considered in the evaluation of research proposals for the grant funding, and then the CINFU-TOPSIS method developed to obtain the final scores and rankings of the alternative research proposals are used. Finally, comparison and sensitivity analysis are carried out. The flowchart of the developed methodology is given in Figure 2.6.

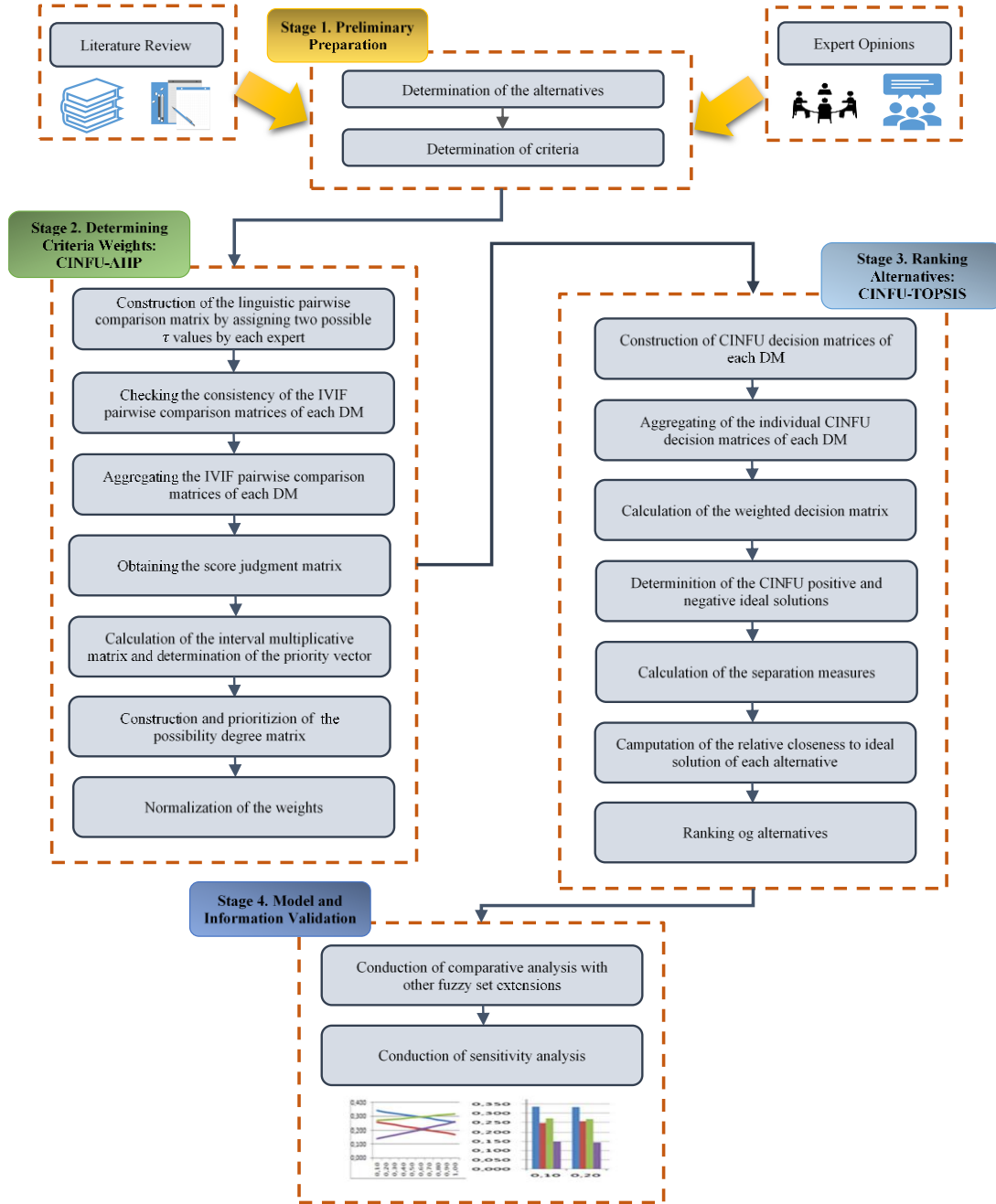


Figure 2.6 : Flowchart of the proposed approach.

2.6.1 Problem definition

Research project proposals should be evaluated with a detailed approach in order to provide financial support to research projects, which have a very important place for the development of certain technologies in a country. For financial support in research project proposals, the grant is the most preferred financial instrument since it is a non-refundable source of funding. However, since each financial provider has their own research priorities, the person or people requesting a grant should prepare their

research projects with great care and diligence. Besides, it is also extremely important to select the most appropriate research project proposal due to the large number of requests for limited grant funds. In order to make an excellent decision, many factors must be considered together by the group evaluating the projects. Therefore, at the stage of evaluation of research projects to be financially supported, the characteristics of the project proposals such as innovative, technical, economic and quality should be taken as a basis and these factors should be met in the best way. The fact that there are many factors that affect the selection of alternative research project proposals necessitates that all relevant factors be fulfilled in the most optimal way. Therefore, it is very difficult and complex to evaluate many conflicting factors simultaneously and to obtain an appropriate and correct outcome. Evaluation of research project proposal alternatives creates an MCDM problem as these criteria may contradict each other. Additionally, the linguistic evaluations including vagueness and impreciseness should be handled by a fuzzy MCDM method or methodology. Therefore, in this study, we present the newly developed CINFU-AHP and CINFU-TOPSIS methodology, which is capable of considering imprecise and uncertain evaluations in a continuous setting in the evaluation of research project proposal alternatives.

Because of the complexity of the decision to evaluate research project proposal alternatives, more than one decision maker has been involved in the implementation of the developed integrated methodology. The decision makers included in the study have been selected from among those who previously evaluated international research projects. All criteria affecting the research project proposals to be considered in the study have been determined based on the opinions of three decision-makers through professional communication and available literature. In this context, a hierarchical structure has been constructed as shown in Figure 2.7, taking into account 8 alternative project proposals and 10 criteria.

The criteria considered in the study can be briefly summarized as follows.

Innovativeness and quality (C1): This criterion indicates the importance, quality of the possible results to be obtained from the project and scientific novelty of the project. A project is expected to obtain significant results in its field of science. However, the development of different practices and standards in various fields is also important for a project. In addition, the originality of the project proposal also increases the chance

of a project being selected more than other projects (Chang et al., 2015; Eilat et al., 2008; Eisenreich et al., 2021; Kiliç & Kaya, 2015).

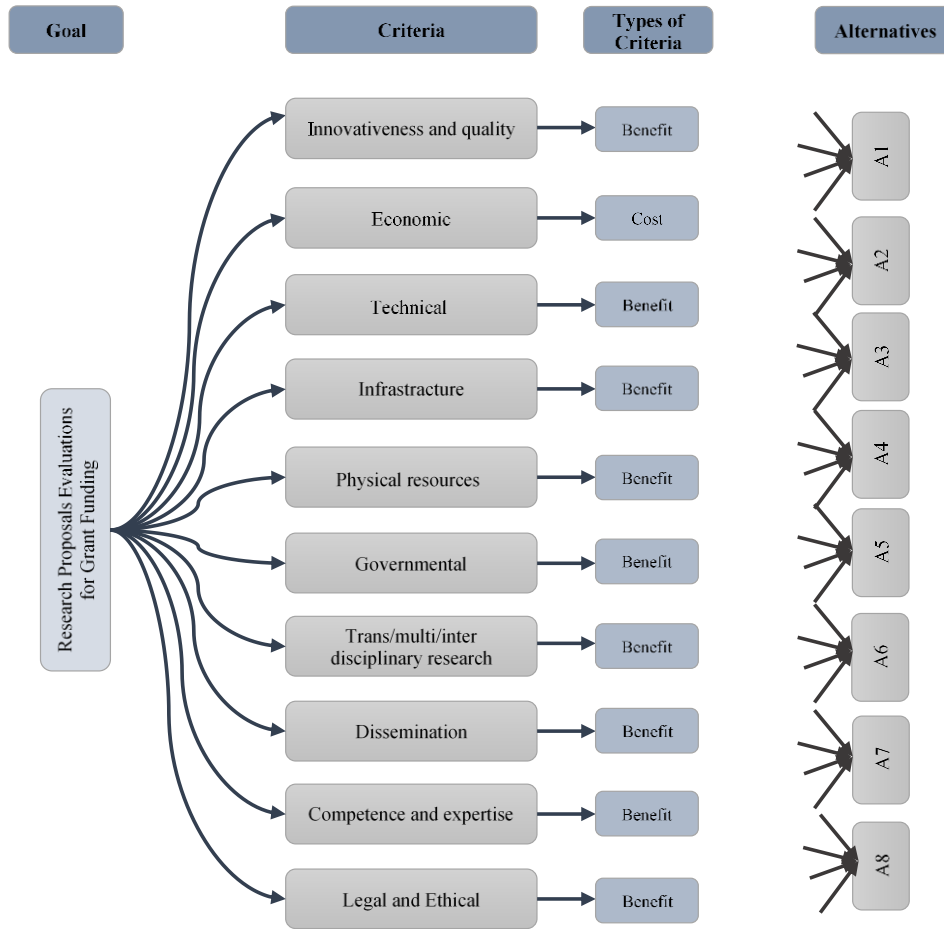


Figure 2.7 : Hierarchical structure of the problem.

Economic (C2): This criterion examines the economic feasibility of a proposed project. Uneconomical project proposals increase the likelihood of rejection by the project evaluation group (Chang et al., 2015; C. T. Chen & Hung, 2021; Eisenreich et al., 2021; Khalili-Damghani & Sadi-Nezhad, 2013; Kiliç & Kaya, 2015).

Technical (C3): This criterion evaluates the suitability of the proposed project in terms of technical feasibility. Projects that are technically appropriate, well-managed and including different technical types and features are necessary for the success of the person or people requesting grant support (Chang et al., 2015; Eilat et al., 2008; Kiliç & Kaya, 2015; Mohagheghi et al., 2020).

Infrastructure (C4): It includes the state of having the necessary research infrastructure for the implementation of the research project proposal. It is the general physical and organizational structure needed for researchers requesting grant support. It also defines

the elements of a framework that support the overall structure of the project proposal (Chen & Hung, 2021; Eilat et al., 2008; Kiliç & Kaya, 2015; Mohagheghi et al., 2020).

Physical resources (C5): This criterion defines the resources needed for daily use in the project, such as buildings and other machinery (Chang et al., 2015; C. T. Chen & Hung, 2021; Kiliç & Kaya, 2015; Mohagheghi et al., 2020) .

Governmental (C6): This criterion evaluates cooperation with the government to carry out a planned task and achieve common goals. A project in cooperation with government institutions is more preferred (Oztaysi et al., 2017) .

Trans/multi/inter disciplinary research (C7): This criterion describes the situation where different kinds of scientific disciplines are required for the implementation of a project. The multi/interdisciplinary coverage of the project area increases the selection chance of the project. Researchers aim to transfer knowledge from one discipline to another through interdisciplinary interaction (Eisenreich et al., 2021; Oztaysi et al., 2017).

Dissemination (C8): The dissemination effect of the project proposal is important for its success in the project area. With the dissemination, the person or people working in the project area are informed and it is a resource for the realization of similar successful projects. While evaluating the project proposals, the dissemination effect on the projects is taken into account (C. T. Chen & Hung, 2021; Khalili-Damghani & Sadi-Nezhad, 2013; Kiliç & Kaya, 2015; Oztaysi et al., 2017).

Competence and expertise (C9): The competence and expertise of researchers and research provide a framework for examining the effectiveness of the proposed study. It indicates the degree to which each applicant researcher meets the sufficiency of the research with the limited resources available. In addition, the publications and patents owned by the researchers increase the chances of the project being selected (C. T. Chen & Hung, 2021; Eisenreich et al., 2021; Kiliç & Kaya, 2015).

Legal and Ethical (C10): This criterion evaluates whether the project has been prepared in accordance with the rules set by the government. Projects are expected to comply with government rules (Chang et al., 2015; Eisenreich et al., 2021; Oztaysi et al., 2017).

The research proposals alternatives to be considered in the selection of the research proposal to be funded in the study have been determined as

- A1: Manufacture of a new wind turbine that can produce more electricity
- A2: Manufacture of rotating panels sensitive to the angle of the sun's rays that will produce the maximum electricity.
- A3: Construct highways for autonomous vehicles.
- A4: Usage of nuclear energy in space applications and nuclear rocket propulsion
- A5: Using blockchain technology for the information security of financial intuitions
- A6: Construction of garbage centers with automatic separation of solid waste
- A7: Application of Industry 4.0 principles in supply chain systems
- A8: Vertical farming for grain production without soil and sunlight

2.6.2 Problem solution

The weights of the criteria for alternative research proposals to be funded are determined by applying the following CINFU-AHP steps:

Step 1. After reviewing the literature in detail and taking the suggestions and opinions of the experts, the criteria and alternatives given in Section 2.6.1 are used in the selection of the best research proposal. Three decision-making experts, abbreviated as DM1, DM2, and DM3, have been selected from academics who are experts on multi-criteria decision-making in a fuzzy environment to evaluate the criteria and alternatives. The weights of these DMs with different levels of knowledge and experience are 0.35, 0.4 and 0.25, respectively.

Step 2. DMs evaluate the criteria based on the linguistic scale given in Table 2.1. DMs are let determine two different values of τ values between 0 and 1. The pairwise comparison matrices obtained from the linguistic evaluations of each DM and their τ values are given in Tables 2.5, 2.6, and 2.7, respectively. Then, the linguistic assessments of each DM are converted into corresponding CINFUNs based on their τ values. Interval-valued pairwise comparison matrix of the criteria constructed from DM1's corresponding CINFUNs are given in Table 2.8.

Table 2.5 : Linguistic pairwise comparison matrix for criteria with respect to DM1.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
C1	EEI;(0.5,0.5)	SIMI;(0.35,0.48)	EEI;(0.51,0.58)	VSMI;(0.13,0.41)	SIMI;(0.67,0.79)	VSMI;(0.03,0.08)	VSMI;(0.73,0.89)	SIMI;(0.71,0.83)	AMI;(0.41,0.5)	VSMI;(0.33,0.48)
C2	SILI;(0.43,0.78)	EEI;(0.5,0.5)	SILI;(0.22,0.35)	SIMI;(0.41,0.53)	SILI;(0.33,0.41)	SIMI;(0.41,0.58)	SIMI;(0.13,0.18)	EEI;(0.5,0.5)	SIMI;(0.8,0.88)	SIMI;(0.53,0.78)
C3	EEI;(0.6,0.9)	SIMI;(0.2,0.31)	EEI;(0.5,0.5)	StMI;(0.48,0.73)	EEI;(0.5,0.5)	StMI;(0.13,0.28)	StMI;(0.37,0.48)	SIMI;(0.66,0.79)	VSMI;(0.38,0.47)	StMI;(0.5,0.5)
C4	VSLI;(0.33,0.45)	SILI;(0.37,0.4)	StLI;(0.34,0.47)	EEI;(0.5,0.5)	StLI;(0.01,0.11)	EEI;(0.5,0.5)	EEI;(0.5,0.5)	SILI;(0.34,0.47)	StMI;(0.5,0.6)	SILI;(0.53,0.57)
C5	SILI;(0.21,0.35)	SIMI;(0.21,0.56)	EEI;(0.5,0.5)	StMI;(0.18,0.44)	EEI;(0.5,0.5)	SIMI;(0.18,0.22)	SIMI;(0.23,0.28)	SIMI;(0.38,0.43)	StMI;(0.42,0.59)	SIMI;(0.5,0.5)
C6	VSLI;(0.11,0.27)	SILI;(0.53,0.77)	StLI;(0.31,0.4)	EEI;(0.5,0.5)	SILI;(0.04,0.37)	EEI;(0.5,0.5)	EEI;(0.5,0.5)	SILI;(0.18,0.37)	SIMI;(0.21,0.34)	StLI;(0.31,0.35)
C7	VSLI;(0.1,0.3)	SILI;(0.6,0.7)	StLI;(0.05,0.15)	EEI;(0.5,0.5)	SILI;(0.32,0.48)	EEI;(0.5,0.5)	EEI;(0.5,0.5)	SILI;(0.35,0.45)	SIMI;(0.18,0.43)	StLI;(0.17,0.25)
C8	SILI;(0.05,0.44)	EEI;(0.5,0.5)	SILI;(0.27,0.34)	SIMI;(0.61,0.78)	SILI;(0.53,0.78)	SIMI;(0.31,0.48)	SIMI;(0.13,0.19)	EEI;(0.5,0.5)	StMI;(0.37,0.4)	SIMI;(0.4,0.5)
C9	ALI;(0.3,0.45)	SILI;(0.48,0.51)	VSLI;(0.05,0.15)	StLI;(0.29,0.52)	StLI;(0.66,0.71)	SILI;(0.45,0.57)	SILI;(0.33,0.49)	StLI;(0.25,0.5)	EEI;(0.5,0.5)	SIMI;(0.27,0.36)
C10	VSLI;(0.43,0.75)	SILI;(0.32,0.45)	StLI;(0.5,0.5)	SIMI;(0.66,0.81)	SILI;(0.5,0.5)	StMI;(0.81,0.95)	StMI;(0.03,0.18)	SILI;(0.53,0.77)	SILI;(0.45,0.55)	EEI;(0.5,0.5)

Table 2.6 : Linguistic pairwise comparison matrix for criteria with respect to DM2.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
C1	EEI;(0.5,0.5)	VSMI;(0.6,0.9)	SIMI;(0.21,0.35)	SIMI;(0.13,0.41)	SIMI;(0.42,0.59)	VSMI;(0.53,0.77)	StMI;(0.55,0.63)	StMI;(0.71,0.83)	AMI;(0.66,0.73)	StMI;(0.45,0.55)
C2	VSLI;(0.66,0.79)	EEI;(0.5,0.5)	StLI;(0.11,0.27)	EEI;(0.5,0.5)	SILI;(0.21,0.34)	EEI;(0.5,0.5)	SIMI;(0.2,0.43)	EEI;(0.5,0.5)	SIMI;(0.23,0.28)	SIMI;(0.52,0.66)
C3	SILI;(0.6,0.9)	StMI;(0.18,0.44)	EEI;(0.5,0.5)	SIMI;(0.33,0.45)	SIMI;(0.31,0.48)	StMI;(0.34,0.47)	SIMI;(0.54,0.63)	SIMI;(0.66,0.79)	VSMI;(0.13,0.45)	SIMI;(0.53,0.57)
C4	SILI;(0.2,0.31)	EEI;(0.5,0.5)	SILI;(0.18,0.44)	EEI;(0.5,0.5)	EEI;(0.5,0.5)	SIMI;(0.38,0.43)	EEI;(0.5,0.5)	SIMI;(0.34,0.47)	StMI;(0.24,0.54)	EEI;(0.5,0.5)
C5	SILI;(0.53,0.78)	SIMI;(0.29,0.52)	SILI;(0.01,0.11)	EEI;(0.5,0.5)	EEI;(0.5,0.5)	SIMI;(0.13,0.18)	EEI;(0.5,0.5)	SIMI;(0.38,0.43)	SIMI;(0.33,0.38)	SIMI;(0.21,0.31)
C6	VSLI;(0.66,0.71)	EEI;(0.5,0.5)	StLI;(0.53,0.78)	SILI;(0.11,0.27)	SILI;(0.04,0.37)	EEI;(0.5,0.5)	StLI;(0.71,0.78)	EEI;(0.18,0.37)	SIMI;(0.05,0.18)	SILI;(0.34,0.45)
C7	StLI;(0.22,0.35)	SILI;(0.6,0.7)	SILI;(0.66,0.71)	EEI;(0.5,0.5)	EEI;(0.5,0.5)	SIMI;(0.03,0.18)	EEI;(0.5,0.5)	SIMI;(0.35,0.45)	SIMI;(0.23,0.49)	SILI;(0.12,0.24)
C8	StLI;(0.18,0.44)	EEI;(0.5,0.5)	SILI;(0.73,0.89)	SILI;(0.66,0.79)	SILI;(0.41,0.58)	EEI;(0.5,0.5)	SILI;(0.02,0.18)	EEI;(0.5,0.5)	SIMI;(0.44,0.48)	SIMI;(0.06,0.19)
C9	ALI;(0.53,0.78)	SILI;(0.67,0.79)	VSLI;(0.13,0.18)	StLI;(0.34,0.47)	SILI;(0.1,0.3)	SILI;(0.18,0.37)	SILI;(0.32,0.44)	SILI;(0.25,0.5)	EEI;(0.5,0.5)	EEI;(0.5,0.5)
C10	StLI;(0.66,0.71)	SILI;(0.33,0.41)	SILI;(0.45,0.57)	EEI;(0.5,0.5)	SILI;(0.53,0.78)	SIMI;(0.53,0.77)	SIMI;(0.51,0.89)	SILI;(0.53,0.77)	EEI;(0.5,0.5)	EEI;(0.5,0.5)

Table 2.7 : Linguistic pairwise comparison matrix for criteria with respect to DM3.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
C1	EEI;(0.5,0.5)	SIMI;(0.21,0.56)	SIMI;(0.73,0.89)	VSMI;(0.73,0.89)	SIMI;(0.23,0.28)	SIMI;(0.23,0.28)	SIMI;(0.73,0.89)	StMI;(0.25,0.5)	StMI;(0.17,0.25)	StMI;(0.71,0.83)
C2	SILI;(0.35,0.48)	EEI;(0.5,0.5)	EEI;(0.5,0.5)	SIMI;(0.37,0.48)	EEI;(0.5,0.5)	SIMI;(0.27,0.36)	SIMI;(0.23,0.28)	SIMI;(0.33,0.48)	SIMI;(0.75,0.86)	EEI;(0.5,0.5)
C3	SILI;(0.34,0.47)	EEI;(0.5,0.5)	EEI;(0.5,0.5)	SIMI;(0.53,0.77)	EEI;(0.5,0.5)	StMI;(0.66,0.79)	StMI;(0.03,0.18)	StMI;(0.18,0.43)	SIMI;(0.23,0.28)	SIMI;(0.31,0.35)
C4	VSLI;(0.61,0.78)	SILI;(0.22,0.35)	SILI;(0.04,0.37)	EEI;(0.5,0.5)	EEI;(0.5,0.5)	EEI;(0.5,0.5)	SILI;(0.4,0.5)	SIMI;(0.27,0.36)	EEI;(0.5,0.5)	SILI;(0.17,0.25)
C5	SILI;(0.05,0.15)	EEI;(0.21,0.56)	EEI;(0.5,0.5)	EEI;(0.5,0.5)	EEI;(0.5,0.5)	SIMI;(0.42,0.59)	SIMI;(0.53,0.57)	StMI;(0.37,0.4)	SIMI;(0.25,0.5)	SIMI;(0.18,0.37)
C6	SILI;(0.67,0.79)	SILI;(0.33,0.41)	StLI;(0.66,0.71)	EEI;(0.5,0.5)	SILI;(0.53,0.57)	EEI;(0.5,0.5)	SILI;(0.5,0.5)	SIMI;(0.45,0.55)	SIMI;(0.53,0.77)	SILI;(0.8,0.88)
C7	SILI;(0.66,0.81)	SILI;(0.27,0.34)	StLI;(0.03,0.08)	SIMI;(0.18,0.22)	SILI;(0.71,0.83)	SIMI;(0.17,0.25)	EEI;(0.5,0.5)	EEI;(0.5,0.5)	SILI;(0.18,0.43)	SILI;(0.38,0.47)
C8	StLI;(0.05,0.44)	SILI;(0.4,0.45)	StLI;(0.45,0.57)	SILI;(0.71,0.83)	StLI;(0.53,0.78)	SILI;(0.31,0.48)	EEI;(0.5,0.5)	EEI;(0.5,0.5)	EEI;(0.5,0.5)	StLI;(0.33,0.49)
C9	StLI;(0.2,0.31)	SILI;(0.33,0.41)	SILI;(0.67,0.79)	EEI;(0.5,0.5)	SILI;(0.41,0.5)	SILI;(0.33,0.48)	SIMI;(0.38,0.47)	EEI;(0.5,0.5)	EEI;(0.5,0.5)	SILI;(0.03,0.18)
C10	StLI;(0.32,0.45)	EEI;(0.5,0.5)	SILI;(0.53,0.78)	SIMI;(0.53,0.77)	SILI;(0.45,0.57)	SIMI;(0.13,0.19)	SIMI;(0.21,0.34)	StMI;(0.31,0.35)	SIMI;(0.13,0.19)	EEI;(0.5,0.5)

Table 2.8 : IVIF pairwise comparison matrix for criteria with respect to DM1.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
C1	[[0.5,0.5], [0.5,0.5]]	[[0.569,0.624], [0.341,0.374]]	[[0.487,0.5], [0.487,0.5]]	[[0.396,0.847], [0.057,0.121]]	[[0.415,0.553], [0.249,0.332]]	[[0.102,0.258], [0.015,0.037]]	[[0.343,0.69], [0.049,0.099]]	[[0.353,0.515], [0.212,0.309]]	[[0.968,1], [0,0]]	[[0.774,0.874], [0.111,0.125]]
C2	[[0.257,0.368], [0.429,0.613]]	[[0.5,0.5], [0.5,0.5]]	[[0.257,0.341], [0.429,0.569]]	[[0.605,0.623], [0.363,0.374]]	[[0.332,0.363], [0.553,0.605]]	[[0.605,0.609], [0.363,0.365]]	[[0.283,0.369], [0.17,0.221]]	[[0.5,0.5], [0.5,0.5]]	[[0.264,0.4], [0.158,0.24]]	[[0.429,0.623], [0.257,0.374]]
C3	[[0.18,0.48], [0.18,0.48]]	[[0.4,0.535], [0.24,0.321]]	[[0.5,0.5], [0.5,0.5]]	[[0.591,0.749], [0.197,0.25]]	[[0.5,0.5], [0.5,0.5]]	[[0.339,0.605], [0.113,0.202]]	[[0.699,0.749], [0.233,0.25]]	[[0.415,0.561], [0.249,0.337]]	[[0.825,0.872], [0.118,0.125]]	[[0.75,0.75], [0.25,0.25]]
C4	[[0.111,0.124], [0.774,0.866]]	[[0.35,0.36], [0.583,0.6]]	[[0.224,0.249], [0.673,0.747]]	[[0.5,0.5], [0.5,0.5]]	[[0.01,0.098], [0.03,0.294]]	[[0.5,0.5], [0.5,0.5]]	[[0.5,0.5], [0.5,0.5]]	[[0.337,0.374], [0.561,0.623]]	[[0.72,0.75], [0.24,0.25]]	[[0.368,0.374], [0.613,0.623]]
C5	[[0.249,0.341], [0.415,0.569]]	[[0.415,0.616], [0.249,0.37]]	[[0.5,0.5], [0.5,0.5]]	[[0.443,0.739], [0.148,0.246]]	[[0.5,0.5], [0.5,0.5]]	[[0.369,0.429], [0.221,0.257]]	[[0.443,0.504], [0.266,0.302]]	[[0.589,0.613], [0.353,0.368]]	[[0.726,0.731], [0.242,0.244]]	[[0.625,0.625], [0.375,0.375]]
C6	[[0.049,0.099], [0.343,0.69]]	[[0.266,0.374], [0.443,0.623]]	[[0.214,0.24], [0.642,0.72]]	[[0.5,0.5], [0.5,0.5]]	[[0.058,0.35], [0.096,0.583]]	[[0.5,0.5], [0.5,0.5]]	[[0.5,0.5], [0.5,0.5]]	[[0.221,0.35], [0.369,0.583]]	[[0.415,0.561], [0.249,0.337]]	[[0.214,0.228], [0.642,0.683]]
C7	[[0.045,0.105], [0.315,0.735]]	[[0.315,0.36], [0.525,0.6]]	[[0.048,0.128], [0.143,0.383]]	[[0.5,0.5], [0.5,0.5]]	[[0.326,0.374], [0.544,0.624]]	[[0.5,0.5], [0.5,0.5]]	[[0.5,0.5], [0.5,0.5]]	[[0.341,0.371], [0.569,0.619]]	[[0.369,0.613], [0.221,0.368]]	[[0.141,0.188], [0.423,0.563]]
C8	[[0.071,0.37], [0.119,0.616]]	[[0.5,0.5], [0.5,0.5]]	[[0.296,0.337], [0.493,0.561]]	[[0.429,0.595], [0.257,0.357]]	[[0.257,0.374], [0.429,0.623]]	[[0.535,0.624], [0.321,0.374]]	[[0.283,0.385], [0.17,0.231]]	[[0.5,0.5], [0.5,0.5]]	[[0.699,0.72], [0.233,0.24]]	[[0.6,0.625], [0.36,0.375]]
C9	[[0,0], [0.84,0.99]]	[[0.374,0.375], [0.624,0.625]]	[[0.024,0.064], [0.166,0.446]]	[[0.206,0.25], [0.618,0.749]]	[[0.206,0.224], [0.618,0.673]]	[[0.368,0.371], [0.613,0.619]]	[[0.332,0.375], [0.553,0.625]]	[[0.188,0.25], [0.563,0.75]]	[[0.5,0.5], [0.5,0.5]]	[[0.493,0.576], [0.296,0.346]]
C10	[[0.094,0.123], [0.656,0.858]]	[[0.326,0.371], [0.544,0.619]]	[[0.25,0.25], [0.75,0.75]]	[[0.385,0.561], [0.231,0.337]]	[[0.375,0.375], [0.625,0.625]]	[[0.143,0.462], [0.048,0.154]]	[[0.087,0.443], [0.029,0.148]]	[[0.266,0.374], [0.443,0.623]]	[[0.371,0.371], [0.619,0.619]]	[[0.5,0.5], [0.5,0.5]]

For instance, the bold number $([0.257, 0.368], [0.429, 0.613])$ in C2-C1 comparison is calculated as follows: DM1 assigns SILI and determines the two τ values as $(0.43, 0.78)$. Using the $\tau = 0.43$ value, $\mu = 1.5\tau - 1.5\tau^2 = 1.5 \times 0.43 - 1.5 \times 0.43^2 = 0.368$ and $\vartheta = 2.5\tau - 2.5\tau^2 = 2.5 \times 0.43 - 2.5 \times 0.43^2 = 0.613$ and using the $\tau = 0.78$ value, $\mu = 1.5\tau - 1.5\tau^2 = 1.5 \times 0.78 - 1.5 \times 0.78^2 = 0.257$ and $\vartheta = 2.5\tau - 2.5\tau^2 = 2.5 \times 0.78 - 2.5 \times 0.78^2 = 0.429$.

Step 3. IVIFNs are defuzzified by using Eq. (2.44) and the consistency ratios of the interval valued pairwise comparison matrices of the DMs are calculated according to Saaty's classical consistency process using the linguistic scale given in Table 2.1.

Step 4. The interval valued pairwise comparison matrix of each DM is aggregated into a single pairwise comparison matrix by using IVIFWG operator given in Eq. (2.22) as in Table 2.9.

Step 5. The score judgment matrix based on the aggregated IVIF pairwise comparison matrix of the criteria is constructed as in Table 2.10 by using Eq. (2.45).

Table 2.9 : Aggregated IVIF pairwise comparison matrix for criteria.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
C1	([0.5,0.5], [0.5,0.5])	([0.387,0.757], [0.161,0.219])	([0.439,0.544], [0.343,0.402])	([0.318,0.68], [0.132,0.287])	([0.53,0.589], [0.325,0.354])	([0.329,0.569], [0.063,0.095])	([0.545,0.724], [0.173,0.198])	([0.397,0.58], [0.166,0.244])	([0.847,0.932], [0,0])	([0.753,0.786], [0.202,0.207])
C2	([0.123,0.17], [0.533,0.736])	([0.5,0.5], [0.5,0.5])	([0.137,0.239], [0.344,0.584])	([0.534,0.54], [0.456,0.459])	([0.275,0.346], [0.467,0.577])	([0.534,0.536], [0.456,0.456])	([0.354,0.513], [0.216,0.32])	([0.5,0.5], [0.5,0.5])	([0.369,0.465], [0.23,0.281])	([0.511,0.624], [0.31,0.374])
C3	([0.149,0.398], [0.21,0.562])	([0.427,0.66], [0.181,0.273])	([0.5,0.5], [0.5,0.5])	([0.566,0.661], [0.287,0.331])	([0.522,0.577], [0.39,0.422])	([0.53,0.694], [0.187,0.233])	([0.621,0.663], [0.311,0.332])	([0.415,0.561], [0.249,0.337])	([0.512,0.868], [0.078,0.124])	([0.658,0.665], [0.329,0.333])
C4	([0.183,0.23], [0.574,0.699])	([0.441,0.446], [0.531,0.538])	([0.222,0.322], [0.499,0.668])	([0.5,0.5], [0.5,0.5])	([0.127,0.283], [0.369,0.436])	([0.556,0.571], [0.409,0.418])	([0.5,0.5], [0.5,0.5])	([0.469,0.521], [0.426,0.475])	([0.602,0.747], [0.203,0.249])	([0.449,0.452], [0.543,0.547])
C5	([0.254,0.362], [0.424,0.605])	([0.477,0.621], [0.288,0.373])	([0.051,0.225], [0.228,0.346])	([0.479,0.573], [0.397,0.423])	([0.5,0.5], [0.5,0.5])	([0.31,0.389], [0.188,0.234])	([0.479,0.501], [0.428,0.438])	([0.589,0.613], [0.353,0.368])	([0.608,0.635], [0.302,0.317])	([0.479,0.565], [0.296,0.34])
C6	([0.079,0.107], [0.623,0.756])	([0.401,0.452], [0.481,0.547])	([0.185,0.246], [0.564,0.738])	([0.225,0.355], [0.346,0.495])	([0.058,0.35], [0.096,0.583])	([0.5,0.5], [0.5,0.5])	([0.325,0.366], [0.455,0.51])	([0.267,0.422], [0.322,0.51])	([0.184,0.427], [0.138,0.264])	([0.287,0.313], [0.591,0.642])
C7	([0.107,0.174], [0.453,0.702])	([0.315,0.36], [0.525,0.6])	([0.16,0.24], [0.408,0.505])	([0.5,0.5], [0.5,0.5])	([0.431,0.452], [0.516,0.547])	([0.143,0.41], [0.238,0.333])	([0.5,0.5], [0.5,0.5])	([0.476,0.517], [0.432,0.472])	([0.415,0.621], [0.25,0.372])	([0.152,0.24], [0.324,0.496])
C8	([0.114,0.284], [0.346,0.701])	([0.5,0.5], [0.5,0.5])	([0.188,0.309], [0.343,0.518])	([0.301,0.411], [0.364,0.498])	([0.322,0.368], [0.55,0.614])	([0.512,0.54], [0.443,0.459])	([0.065,0.269], [0.093,0.324])	([0.5,0.5], [0.5,0.5])	([0.644,0.656], [0.325,0.33])	([0.234,0.456], [0.192,0.285])
C9	([0,0], [0.752,0.995])	([0.287,0.346], [0.499,0.579])	([0.042,0.07], [0.324,0.493])	([0.218,0.249], [0.655,0.748])	([0.156,0.28], [0.395,0.583])	([0.264,0.357], [0.468,0.596])	([0.328,0.371], [0.547,0.619])	([0.244,0.325], [0.504,0.675])	([0.5,0.5], [0.5,0.5])	([0.497,0.525], [0.436,0.451])
C10	([0.156,0.182], [0.632,0.756])	([0.33,0.366], [0.55,0.61])	([0.321,0.323], [0.668,0.671])	([0.456,0.521], [0.419,0.448])	([0.294,0.374], [0.507,0.624])	([0.298,0.561], [0.196,0.304])	([0.171,0.554], [0.107,0.303])	([0.266,0.374], [0.443,0.623])	([0.451,0.451], [0.545,0.545])	([0.5,0.5], [0.5,0.5])

Table 2.10 : Score judgment matrix.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
C1	(0,0)	(0.168,0.596)	(0.037,0.201)	(0.031,0.549)	(0.176,0.264)	(0.235,0.506)	(0.346,0.551)	(0.154,0.413)	(0.847,0.932)	(0.547,0.584)
C2	(-0.613, 0.363)	(0,0)	(-0.446,-0.106)	(0.075,0.084)	(-0.302,-0.122)	(0.078,0.08)	(0.034,0.297)	(0,0)	(0.088,0.235)	(0.137,0.314)
C3	(-0.412,0.189)	(0.154,0.479)	(0,0)	(0.235,0.374)	(0.101,0.188)	(0.297,0.507)	(0.289,0.352)	(0.078,0.312)	(0.388,0.79)	(0.325,0.336)
C4	(-0.516,-0.344)	(-0.096,-0.085)	(-0.446,-0.177)	(0,0)	(-0.309,-0.087)	(0.139,0.162)	(0,0)	(-0.006,0.095)	(0.353,0.544)	(-0.098,-0.091)
C5	(-0.35,-0.062)	(0.105,0.333)	(-0.295,-0.003)	(0.056,0.176)	(0,0)	(0.076,0.201)	(0.041,0.073)	(0.221,0.259)	(0.291,0.334)	(0.138,0.269)
C6	(-0.677,-0.516)	(-0.146,-0.029)	(-0.553,-0.318)	(-0.27,0.009)	(-0.525,0.254)	(0,0)	(-0.185,-0.089)	(-0.243,0.1)	(-0.08,0.29)	(-0.355,-0.278)
C7	(-0.595,-0.279)	(-0.285,-0.165)	(-0.345,-0.168)	(0,0)	(-0.117,-0.064)	(-0.19,0.173)	(0,0)	(0.003,0.085)	(0.043,0.37)	(-0.344,-0.085)
C8	(-0.587,-0.062)	(0,0)	(-0.33,-0.034)	(-0.197,0.047)	(-0.292,-0.182)	(0.053,0.097)	(-0.259,0.176)	(0,0)	(0.314,0.331)	(-0.051,0.264)
C9	(-0.995,-0.752)	(-0.292,-0.153)	(-0.451,-0.254)	(-0.53,-0.405)	(-0.427,-0.115)	(-0.331,-0.111)	(-0.291,-0.176)	(-0.431,-0.178)	(0,0)	(0.047,0.089)
C10	(-0.599,-0.45)	(-0.28,-0.184)	(-0.35,-0.344)	(0.008,0.102)	(-0.33,-0.133)	(-0.006,0.365)	(-0.133,0.446)	(-0.357,-0.069)	(-0.095,-0.095)	(0,0)

Step 6. The interval multiplicative matrix is obtained based on the score judgment matrix by using Eq. (2.46) as given in Table 2.11.

Table 2.11 : Interval multiplicative matrix.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
C1	(1,1)	(1.473, 3.941)	(1.089, 1.588)	(1.075, 3.537)	(1.500, 1.835)	(1.717, 3.204)	(2.22, 3.552)	(1.424, 2.588)	(7.031, 8.554)	(3.521, 3.836)
C2	(0.244, 0.434)	(1,1)	(0.358, 0.784)	(1.19, 1.214)	(0.499, 0.756)	(1.197, 1.202)	(1.082, 1.982)	(1,1)	(1.225, 1.718)	(1.370, 2.059)
C3	(0.387, 1.544)	(1.426, 3.012)	(1,1)	(1.717, 2.367)	(1.261, 1.54)	(1.981, 3.212)	(1.946, 2.249)	(1.197, 2.052)	(2.442, 6.162)	(2.112, 2.167)
C4	(0.305, 0.453)	(0.801, 0.822)	(0.358, 0.665)	(1,1)	(0.491, 0.819)	(1.376, 1.451)	(1,1)	(0.986, 1.244)	(2.256, 3.498)	(0.798, 0.811)
C5	(0.446, 0.867)	(1.272, 2.152)	(0.506, 0.994)	(1.139, 1.500)	(1,1)	(1.192, 1.588)	(1.099, 1.184)	(1.665, 1.817)	(1.955, 2.156)	(1.375, 1.858)
C6	(0.211, 0.305)	(0.714, 0.935)	(0.28, 0.481)	(0.537, 1.021)	(0.298, 1.793)	(1,1)	(0.653, 0.814)	(0.571, 1.258)	(0.832, 1.948)	(0.441, 0.527)
C7	(0.254, 0.526)	(0.519, 0.684)	(0.452, 0.679)	(1,1)	(0.764, 0.863)	(0.645, 1.488)	(1,1)	(1.008, 1.217)	(1.104, 2.345)	(0.453, 0.823)
C8	(0.259, 0.867)	(1,1)	(0.468, 0.926)	(0.635, 1.114)	(0.510, 0.657)	(1.129, 1.25)	(0.551, 1.498)	(1,1)	(2.059, 2.144)	(0.890, 1.835)
C9	(0.101, 0.177)	(0.51, 0.704)	(0.354, 0.558)	(0.295, 0.393)	(0.374, 0.767)	(0.466, 0.774)	(0.512, 0.667)	(0.371, 0.663)	(1,1)	(1.114, 1.228)
C	(0.252, 0.355)	(0.525, 0.655)	(0.447, 0.452)	(1.019, 1.264)	(0.468, 0.736)	(0.985, 2.318)	(0.737, 2.796)	(0.439, 0.853)	(0.804, 0.804)	(1,1)

Step 7. The priority vector is determined by calculating the interval $\tilde{\omega}_j$ of each criterion based on the interval multiplicative matrix by using Eq. (2.47) as shown in Table 2.12.

Table 2.12 : Priority vector.

C1	[0.148,0.334]
C2	[0.061,0.121]
C3	[0.104,0.251]
C4	[0.063,0.117]
C5	[0.078,0.15]
C6	[0.037,0.1]
C7	[0.048,0.105]
C8	[0.057,0.122]
C9	[0.034,0.069]
C10	[0.045,0.112]

Step 8. The possibility degree matrix is calculated by comparing the priority vector based on Eq. (2.48) as given in Table 2.13.

Steps 9 and 10. After possibility degrees are prioritized by using Eq. (2.49), the obtained weights are normalized by using Eq. (2.50) as presented in Table 2.14.

Table 2.13 : Possibility degree matrix.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
C1	-	1.000	0.690	1.000	0.991	1.000	1.000	1.000	1.000	1.000
C2	0.000	-	0.082	0.511	0.324	0.684	0.622	0.512	0.921	0.602
C3	0.310	0.918	-	0.935	0.789	1.000	0.991	0.914	1.000	0.964
C4	0.000	0.489	0.065	-	0.307	0.681	0.616	0.502	0.932	0.597
C5	0.009	0.676	0.211	0.693	-	0.837	0.788	0.679	1.000	0.759
C6	0.000	0.316	0.000	0.319	0.163	-	0.431	0.337	0.675	0.427
C7	0.000	0.378	0.009	0.384	0.212	0.569	-	0.397	0.776	0.490
C8	0.000	0.488	0.086	0.498	0.321	0.663	0.603	-	0.881	0.586
C9	0.000	0.079	0.000	0.068	0.000	0.325	0.224	0.119	-	0.237
C10	0.000	0.398	0.036	0.403	0.241	0.573	0.510	0.414	0.763	-

Table 2.14 : Weights of criteria.

Criteria	Prioritized weights, ρ_j	Normalized weights, w_j
C1	1.268	0.148
C2	0.826	0.096
C3	1.182	0.138
C4	0.819	0.096
C5	1.033	0.121
C6	0.667	0.078
C7	0.721	0.084
C8	0.813	0.095
C9	0.505	0.059
C10	0.734	0.086

The final scores and rankings of the alternative research proposals for grant funding are determined by applying the steps of the developed new CINFU-TOPSIS method as follows:

Step 11. The DMs evaluate the alternatives with respect to the defined objectives and criteria based on the CINFU linguistic scale given in Table 2.2. The linguistic decision matrices created based on the evaluations of the DMs are given in Table 2.15. Then, the linguistic evaluations of each DM are transformed to their corresponding CINFUNs by utilizing the scale given in Table 2.2. CINFU decision matrix for DM1 has been presented in Table 2.16.

Table 2.15 : Linguistic decision matrix based on alternatives with respect to criteria for each DM.

	DM1							
	A1	A2	A3	A4	A5	A6	A7	A8
C1	LV	AHV	SHV	MV	HV	HV	SHV	VHV
C2	ALV	VHV	VHV	LV	MV	SHV	HV	SLV
C3	SHV	HV	SHV	MV	LV	VLV	MV	HV
C4	SHV	MV	LV	HV	SLV	MV	HV	HV
C5	MV	VLV	MV	SLV	SHV	SLV	VHV	ALV
C6	HV	LV	VLV	SLV	HV	AHV	ALV	MV
C7	LV	MV	HV	VLV	MV	VHV	MV	HV
C8	VLV	VLV	VHV	HV	LV	HV	MV	VLV
C9	VLV	HV	AHV	SHV	SLV	AHV	SLV	SHV
C10	VHV	VLV	MV	HV	LV	HV	SHV	VHV
	DM2							
	A1	A2	A3	A4	A5	A6	A7	A8
C1	MV	VHV	SLV	HV	MV	MV	HV	HV
C2	VLV	HV	AHV	VLV	SHV	MV	VHV	LV
C3	MV	HV	MV	HV	VLV	LV	SLV	VHV
C4	HV	SLV	VLV	VHV	LV	VHV	LV	HV
C5	HV	LV	VLV	MV	HV	LV	HV	SLV
C6	VHV	LV	LV	LV	VHV	HV	VLV	SHV
C7	VLV	SHV	VHV	LV	VHV	HV	SHV	SHV
C8	LV	SLV	HV	VHV	VLV	VHV	HV	LV
C9	LV	MV	LV	HV	ALV	SHV	MV	HV
C10	AHV	SLV	SLV	SHV	LV	VHV	HV	VHV
	DM3							
	A1	A2	A3	A4	A5	A6	A7	A8
C1	HV	HV	LV	VHV	HV	VHV	MV	MV
C2	LV	LV	VHV	ALV	HV	HV	AHV	SLV
C3	MV	VHV	SLV	VHV	LV	VLV	LV	HV
C4	VHV	LV	SLV	HV	SLV	HV	SLV	HV
C5	MV	MV	MV	HV	VHV	MV	VHV	SHV
C6	HV	SLV	VHV	SLV	HV	MV	VLV	MV
C7	LV	HV	HV	SLV	HV	SHV	VHV	MV
C8	VLV	LV	VHV	HV	ALV	HV	HV	VLV
C9	SLV	SLV	MV	HV	LV	HV	SHV	VHV
C10	VHV	SLV	MV	MV	VLV	SHV	VHV	HV

Table 2.16 : CINFU decision matrix for DM1.

DM1								
	A1	A2	A3	A4	A5	A6	A7	A8
C1	$(\tau - \tau^2,$ $3\tau - 3\tau^2)$	$(4\tau - 4\tau^2,$ $0\tau - 0\tau^2)$	$(2.5\tau - 2.5\tau^2,$ $1.5\tau - 1.5\tau^2)$	$(2\tau - 2\tau^2,$ $2\tau - 2\tau^2)$	$(3\tau - 3\tau^2,$ $1\tau - 1\tau^2)$	$(3\tau - 3\tau^2,$ $1\tau - 1\tau^2)$	$(2.5\tau - 2.5\tau^2,$ $1.5\tau - 1.5\tau^2)$	$(3.5\tau - 3.5\tau^2,$ $0.5\tau - 0.5\tau^2)$
C2	$(0\tau - 0\tau^2,$ $4\tau - 4\tau^2)$	$(3.5\tau - 3.5\tau^2,$ $0.5\tau - 0.5\tau^2)$	$(3.5\tau - 3.5\tau^2,$ $0.5\tau - 0.5\tau^2)$	$(\tau - \tau^2,$ $3\tau - 3\tau^2)$	$(2\tau - 2\tau^2,$ $2\tau - 2\tau^2)$	$(2.5\tau - 2.5\tau^2,$ $1.5\tau - 1.5\tau^2)$	$(3\tau - 3\tau^2,$ $1\tau - 1\tau^2)$	$(1.5\tau - 1.5\tau^2,$ $2.5\tau - 2.5\tau^2)$
C3	$(2.5\tau - 2.5\tau^2,$ $1.5\tau - 1.5\tau^2)$	$(3\tau - 3\tau^2,$ $1\tau - 1\tau^2)$	$(2.5\tau - 2.5\tau^2,$ $1.5\tau - 1.5\tau^2)$	$(2\tau - 2\tau^2,$ $2\tau - 2\tau^2)$	$(\tau - \tau^2,$ $3\tau - 3\tau^2)$	$(0.5\tau - 0.5\tau^2,$ $3.5\tau - 3.5\tau^2)$	$(2\tau - 2\tau^2,$ $2\tau - 2\tau^2)$	$(3\tau - 3\tau^2,$ $1\tau - 1\tau^2)$
C4	$(2.5\tau - 2.5\tau^2,$ $1.5\tau - 1.5\tau^2)$	$(2\tau - 2\tau^2,$ $2\tau - 2\tau^2)$	$(\tau - \tau^2,$ $3\tau - 3\tau^2)$	$(3\tau - 3\tau^2,$ $1\tau - 1\tau^2)$	$(1.5\tau - 1.5\tau^2,$ $2.5\tau - 2.5\tau^2)$	$(2\tau - 2\tau^2,$ $2\tau - 2\tau^2)$	$(3\tau - 3\tau^2,$ $1\tau - 1\tau^2)$	$(3\tau - 3\tau^2,$ $1\tau - 1\tau^2)$
C5	$(2\tau - 2\tau^2,$ $2\tau - 2\tau^2)$	$(0.5\tau - 0.5\tau^2,$ $3.5\tau - 3.5\tau^2)$	$(2\tau - 2\tau^2,$ $2\tau - 2\tau^2)$	$(1.5\tau - 1.5\tau^2,$ $2.5\tau - 2.5\tau^2)$	$(2.5\tau - 2.5\tau^2,$ $1.5\tau - 1.5\tau^2)$	$(1.5\tau - 1.5\tau^2,$ $2.5\tau - 2.5\tau^2)$	$(3.5\tau - 3.5\tau^2,$ $0.5\tau - 0.5\tau^2)$	$(0\tau - 0\tau^2,$ $4\tau - 4\tau^2)$
C6	$(3\tau - 3\tau^2,$ $1\tau - 1\tau^2)$	$(\tau - \tau^2,$ $3\tau - 3\tau^2)$	$(0.5\tau - 0.5\tau^2,$ $3.5\tau - 3.5\tau^2)$	$(1.5\tau - 1.5\tau^2,$ $2.5\tau - 2.5\tau^2)$	$(3\tau - 3\tau^2,$ $1\tau - 1\tau^2)$	$(4\tau - 4\tau^2,$ $0\tau - 0\tau^2)$	$(0\tau - 0\tau^2,$ $4\tau - 4\tau^2)$	$(2\tau - 2\tau^2,$ $2\tau - 2\tau^2)$
C7	$(1\tau - 1\tau^2,$ $3\tau - 3\tau^2)$	$(2\tau - 2\tau^2,$ $2\tau - 2\tau^2)$	$(3\tau - 3\tau^2,$ $1\tau - 1\tau^2)$	$(0.5\tau - 0.5\tau^2,$ $3.5\tau - 3.5\tau^2)$	$(2\tau - 2\tau^2,$ $2\tau - 2\tau^2)$	$(3.5\tau - 3.5\tau^2,$ $0.5\tau - 0.5\tau^2)$	$(2\tau - 2\tau^2,$ $2\tau - 2\tau^2)$	$(3\tau - 3\tau^2,$ $1\tau - 1\tau^2)$
C8	$(0.5\tau - 0.5\tau^2,$ $3.5\tau - 3.5\tau^2)$	$(0.5\tau - 0.5\tau^2,$ $3.5\tau - 3.5\tau^2)$	$(3.5\tau - 3.5\tau^2,$ $0.5\tau - 0.5\tau^2)$	$(3\tau - 3\tau^2,$ $1\tau - 1\tau^2)$	$(\tau - \tau^2,$ $3\tau - 3\tau^2)$	$(3\tau - 3\tau^2,$ $1\tau - 1\tau^2)$	$(2\tau - 2\tau^2,$ $2\tau - 2\tau^2)$	$(0.5\tau - 0.5\tau^2,$ $3.5\tau - 3.5\tau^2)$
C9	$(0.5\tau - 0.5\tau^2,$ $3.5\tau - 3.5\tau^2)$	$(3\tau - 3\tau^2,$ $1\tau - 1\tau^2)$	$(4\tau - 4\tau^2,$ $0\tau - 0\tau^2)$	$(2.5\tau - 2.5\tau^2,$ $1.5\tau - 1.5\tau^2)$	$(1.5\tau - 1.5\tau^2,$ $2.5\tau - 2.5\tau^2)$	$(4\tau - 4\tau^2,$ $0\tau - 0\tau^2)$	$(1.5\tau - 1.5\tau^2,$ $2.5\tau - 2.5\tau^2)$	$(2.5\tau - 2.5\tau^2,$ $1.5\tau - 1.5\tau^2)$
C10	$(3.5\tau - 3.5\tau^2,$ $0.5\tau - 0.5\tau^2)$	$(0.5\tau - 0.5\tau^2,$ $3.5\tau - 3.5\tau^2)$	$(2\tau - 2\tau^2,$ $2\tau - 2\tau^2)$	$(3\tau - 3\tau^2,$ $1\tau - 1\tau^2)$	$(\tau - \tau^2,$ $3\tau - 3\tau^2)$	$(3\tau - 3\tau^2,$ $1\tau - 1\tau^2)$	$(2.5\tau - 2.5\tau^2,$ $1.5\tau - 1.5\tau^2)$	$(3.5\tau - 3.5\tau^2,$ $0.5\tau - 0.5\tau^2)$

Step 12. The individual decision matrix of each DM is aggregated in a single decision matrix by using CINFUWG operator given in Eq. (2.38). The aggregated CINFU decision matrix of alternatives with respect to criteria is constructed for τ values between 0 and 1 with an increment of 0.1 as shown in Table 2.17.

Table 2.17 : Aggregated decision matrix for τ values.

	τ	A1	A2	A3	A4	A5	A6	A7	A8
C1	0.1	(0.156,0.192)	(0.318,0.041)	(0.146,0.207)	(0.243,0.112)	(0.23,0.127)	(0.239,0.117)	(0.229,0.129)	(0.257,0.098)
	0.2	(0.278,0.347)	(0.565,0.074)	(0.259,0.371)	(0.433,0.202)	(0.408,0.228)	(0.424,0.21)	(0.407,0.231)	(0.458,0.177)
	0.3	(0.365,0.465)	(0.741,0.098)	(0.34,0.493)	(0.568,0.269)	(0.536,0.302)	(0.557,0.28)	(0.534,0.304)	(0.601,0.236)
	0.4	(0.417,0.54)	(0.847,0.113)	(0.389,0.569)	(0.649,0.31)	(0.612,0.347)	(0.636,0.323)	(0.61,0.349)	(0.687,0.272)
	0.5	(0.434,0.566)	(0.882,0.118)	(0.405,0.595)	(0.676,0.324)	(0.638,0.362)	(0.663,0.337)	(0.636,0.364)	(0.715,0.285)
	0.6	(0.417,0.54)	(0.847,0.113)	(0.389,0.569)	(0.649,0.31)	(0.612,0.347)	(0.636,0.323)	(0.61,0.349)	(0.687,0.272)
	0.7	(0.365,0.465)	(0.741,0.098)	(0.34,0.493)	(0.568,0.269)	(0.536,0.302)	(0.557,0.28)	(0.534,0.304)	(0.601,0.236)
	0.8	(0.278,0.347)	(0.565,0.074)	(0.259,0.371)	(0.433,0.202)	(0.408,0.228)	(0.424,0.21)	(0.407,0.231)	(0.458,0.177)
	0.9	(0.156,0.192)	(0.318,0.041)	(0.146,0.207)	(0.243,0.112)	(0.23,0.127)	(0.239,0.117)	(0.229,0.129)	(0.257,0.098)
C2	0.1	(0,0.32)	(0.217,0.124)	(0.332,0.027)	(0,0.311)	(0.218,0.14)	(0.215,0.142)	(0.309,0.05)	(0.115,0.243)
	0.2	(0,0.572)	(0.385,0.231)	(0.591,0.049)	(0,0.556)	(0.387,0.25)	(0.383,0.255)	(0.549,0.09)	(0.204,0.433)
	0.3	(0,0.759)	(0.505,0.317)	(0.775,0.064)	(0,0.737)	(0.508,0.33)	(0.503,0.336)	(0.72,0.119)	(0.268,0.57)
	0.4	(0,0.887)	(0.577,0.377)	(0.886,0.074)	(0,0.862)	(0.581,0.379)	(0.574,0.385)	(0.823,0.137)	(0.306,0.653)
	0.5	(0,1)	(0.601,0.399)	(0.923,0.077)	(0,1)	(0.605,0.395)	(0.598,0.402)	(0.857,0.143)	(0.319,0.681)
	0.6	(0,0.887)	(0.577,0.377)	(0.886,0.074)	(0,0.862)	(0.581,0.379)	(0.574,0.385)	(0.823,0.137)	(0.306,0.653)
	0.7	(0,0.759)	(0.505,0.317)	(0.775,0.064)	(0,0.737)	(0.508,0.33)	(0.503,0.336)	(0.72,0.119)	(0.268,0.57)
	0.8	(0,0.572)	(0.385,0.231)	(0.591,0.049)	(0,0.556)	(0.387,0.25)	(0.383,0.255)	(0.549,0.09)	(0.204,0.433)
	0.9	(0,0.32)	(0.217,0.124)	(0.332,0.027)	(0,0.311)	(0.218,0.14)	(0.215,0.142)	(0.309,0.05)	(0.115,0.243)
C3	0.1	(0.195,0.165)	(0.281,0.079)	(0.181,0.176)	(0.243,0.112)	(0.068,0.288)	(0.059,0.297)	(0.135,0.221)	(0.287,0.072)
	0.2	(0.346,0.293)	(0.499,0.141)	(0.322,0.315)	(0.433,0.202)	(0.121,0.514)	(0.106,0.53)	(0.24,0.395)	(0.511,0.129)
	0.3	(0.454,0.385)	(0.655,0.185)	(0.423,0.415)	(0.568,0.269)	(0.159,0.676)	(0.139,0.697)	(0.315,0.521)	(0.67,0.17)
	0.4	(0.519,0.441)	(0.748,0.212)	(0.483,0.476)	(0.649,0.31)	(0.182,0.776)	(0.158,0.8)	(0.36,0.599)	(0.766,0.194)
	0.5	(0.541,0.459)	(0.779,0.221)	(0.503,0.497)	(0.676,0.324)	(0.189,0.811)	(0.165,0.835)	(0.375,0.625)	(0.798,0.202)

Table 2.17 (continued) : Aggregated decision matrix for τ values.

	τ	A1	A2	A3	A4	A5	A6	A7	A8
C3	0.6	(0.519,0.441)	(0.748,0.212)	(0.483,0.476)	(0.649,0.31)	(0.182,0.776)	(0.158,0.8)	(0.36,0.599)	(0.766,0.194)
	0.7	(0.454,0.385)	(0.655,0.185)	(0.423,0.415)	(0.568,0.269)	(0.159,0.676)	(0.139,0.697)	(0.315,0.521)	(0.67,0.17)
	0.8	(0.346,0.293)	(0.499,0.141)	(0.322,0.315)	(0.433,0.202)	(0.121,0.514)	(0.106,0.53)	(0.24,0.395)	(0.511,0.129)
	0.9	(0.195,0.165)	(0.281,0.079)	(0.181,0.176)	(0.243,0.112)	(0.068,0.288)	(0.059,0.297)	(0.135,0.221)	(0.287,0.072)
C4	0.1	(0.263,0.095)	(0.135,0.221)	(0.075,0.278)	(0.287,0.072)	(0.115,0.243)	(0.249,0.105)	(0.146,0.2)	(0.27,0.09)
	0.2	(0.468,0.17)	(0.24,0.395)	(0.134,0.496)	(0.511,0.129)	(0.204,0.433)	(0.443,0.191)	(0.26,0.363)	(0.48,0.16)
	0.3	(0.614,0.225)	(0.315,0.521)	(0.176,0.655)	(0.67,0.17)	(0.268,0.57)	(0.581,0.255)	(0.341,0.486)	(0.63,0.21)
	0.4	(0.702,0.258)	(0.36,0.599)	(0.201,0.755)	(0.766,0.194)	(0.306,0.653)	(0.664,0.294)	(0.39,0.566)	(0.72,0.24)
	0.5	(0.731,0.269)	(0.375,0.625)	(0.21,0.79)	(0.798,0.202)	(0.319,0.681)	(0.692,0.308)	(0.406,0.594)	(0.75,0.25)
	0.6	(0.702,0.258)	(0.36,0.599)	(0.201,0.755)	(0.766,0.194)	(0.306,0.653)	(0.664,0.294)	(0.39,0.566)	(0.72,0.24)
	0.7	(0.614,0.225)	(0.315,0.521)	(0.176,0.655)	(0.67,0.17)	(0.268,0.57)	(0.581,0.255)	(0.341,0.486)	(0.63,0.21)
	0.8	(0.468,0.17)	(0.24,0.395)	(0.134,0.496)	(0.511,0.129)	(0.204,0.433)	(0.443,0.191)	(0.26,0.363)	(0.48,0.16)
	0.9	(0.263,0.095)	(0.135,0.221)	(0.075,0.278)	(0.287,0.072)	(0.115,0.243)	(0.249,0.105)	(0.146,0.2)	(0.27,0.09)
C5	0.1	(0.212,0.145)	(0.084,0.265)	(0.103,0.237)	(0.287,0.072)	(0.263,0.095)	(0.123,0.233)	(0.296,0.063)	(0,0.255)
	0.2	(0.376,0.26)	(0.149,0.476)	(0.184,0.429)	(0.511,0.129)	(0.468,0.17)	(0.219,0.415)	(0.527,0.113)	(0,0.468)
	0.3	(0.494,0.344)	(0.196,0.632)	(0.241,0.576)	(0.67,0.17)	(0.614,0.225)	(0.288,0.548)	(0.691,0.149)	(0,0.644)
	0.4	(0.565,0.395)	(0.224,0.731)	(0.276,0.675)	(0.766,0.194)	(0.702,0.258)	(0.329,0.63)	(0.79,0.17)	(0,0.799)
	0.5	(0.588,0.412)	(0.233,0.767)	(0.287,0.713)	(0.798,0.202)	(0.731,0.269)	(0.343,0.657)	(0.823,0.177)	(0,1)
	0.6	(0.565,0.395)	(0.224,0.731)	(0.276,0.675)	(0.766,0.194)	(0.702,0.258)	(0.329,0.63)	(0.79,0.17)	(0,0.799)
	0.7	(0.494,0.344)	(0.196,0.632)	(0.241,0.576)	(0.67,0.17)	(0.614,0.225)	(0.288,0.548)	(0.691,0.149)	(0,0.644)
	0.8	(0.376,0.26)	(0.149,0.476)	(0.184,0.429)	(0.511,0.129)	(0.468,0.17)	(0.219,0.415)	(0.527,0.113)	(0,0.468)
	0.9	(0.212,0.145)	(0.084,0.265)	(0.103,0.237)	(0.287,0.072)	(0.263,0.095)	(0.123,0.233)	(0.296,0.063)	(0,0.255)

Table 2.17 (continued) : Aggregated decision matrix for τ values.

	τ	A1	A2	A3	A4	A5	A6	A7	A8
C6	0.1	(0.287,0.072)	(0.1,0.259)	(0.097,0.236)	(0.18,0.175)	(0.287,0.072)	(0.27,0.084)	(0,0.331)	(0.197,0.162)
	0.2	(0.511,0.129)	(0.177,0.461)	(0.172,0.434)	(0.32,0.314)	(0.511,0.129)	(0.48,0.153)	(0,0.59)	(0.35,0.289)
	0.3	(0.67,0.17)	(0.232,0.606)	(0.225,0.589)	(0.42,0.416)	(0.67,0.17)	(0.63,0.206)	(0,0.778)	(0.459,0.38)
	0.4	(0.766,0.194)	(0.266,0.694)	(0.258,0.694)	(0.48,0.478)	(0.766,0.194)	(0.72,0.239)	(0,0.902)	(0.525,0.435)
	0.5	(0.798,0.202)	(0.277,0.723)	(0.268,0.732)	(0.5,0.5)	(0.798,0.202)	(0.749,0.251)	(0,1)	(0.547,0.453)
	0.6	(0.766,0.194)	(0.266,0.694)	(0.258,0.694)	(0.48,0.478)	(0.766,0.194)	(0.72,0.239)	(0,0.902)	(0.525,0.435)
	0.7	(0.67,0.17)	(0.232,0.606)	(0.225,0.589)	(0.42,0.416)	(0.67,0.17)	(0.63,0.206)	(0,0.778)	(0.459,0.38)
	0.8	(0.511,0.129)	(0.177,0.461)	(0.172,0.434)	(0.32,0.314)	(0.511,0.129)	(0.48,0.153)	(0,0.59)	(0.35,0.289)
	0.9	(0.287,0.072)	(0.1,0.259)	(0.097,0.236)	(0.18,0.175)	(0.287,0.072)	(0.27,0.084)	(0,0.331)	(0.197,0.162)
C7	0.1	(0.068,0.288)	(0.218,0.14)	(0.287,0.072)	(0.078,0.275)	(0.249,0.105)	(0.272,0.086)	(0.226,0.13)	(0.227,0.131)
	0.2	(0.121,0.514)	(0.387,0.25)	(0.511,0.129)	(0.139,0.492)	(0.443,0.191)	(0.484,0.154)	(0.402,0.233)	(0.403,0.234)
	0.3	(0.159,0.676)	(0.508,0.33)	(0.67,0.17)	(0.182,0.65)	(0.581,0.255)	(0.635,0.204)	(0.528,0.309)	(0.529,0.309)
	0.4	(0.182,0.776)	(0.581,0.379)	(0.766,0.194)	(0.208,0.748)	(0.664,0.294)	(0.726,0.234)	(0.604,0.356)	(0.605,0.355)
	0.5	(0.189,0.811)	(0.605,0.395)	(0.798,0.202)	(0.217,0.783)	(0.692,0.308)	(0.756,0.244)	(0.629,0.371)	(0.63,0.37)
	0.6	(0.182,0.776)	(0.581,0.379)	(0.766,0.194)	(0.208,0.748)	(0.664,0.294)	(0.726,0.234)	(0.604,0.356)	(0.605,0.355)
	0.7	(0.159,0.676)	(0.508,0.33)	(0.67,0.17)	(0.182,0.65)	(0.581,0.255)	(0.635,0.204)	(0.528,0.309)	(0.529,0.309)
	0.8	(0.121,0.514)	(0.387,0.25)	(0.511,0.129)	(0.139,0.492)	(0.443,0.191)	(0.484,0.154)	(0.402,0.233)	(0.403,0.234)
	0.9	(0.068,0.288)	(0.218,0.14)	(0.287,0.072)	(0.078,0.275)	(0.249,0.105)	(0.272,0.086)	(0.226,0.13)	(0.227,0.131)
C8	0.1	(0.059,0.297)	(0.083,0.269)	(0.296,0.063)	(0.287,0.072)	(0,0.311)	(0.287,0.072)	(0.234,0.123)	(0.059,0.297)
	0.2	(0.106,0.53)	(0.148,0.481)	(0.527,0.113)	(0.511,0.129)	(0,0.556)	(0.511,0.129)	(0.416,0.22)	(0.106,0.53)
	0.3	(0.139,0.697)	(0.194,0.636)	(0.691,0.149)	(0.67,0.17)	(0,0.737)	(0.67,0.17)	(0.547,0.291)	(0.139,0.697)
	0.4	(0.158,0.8)	(0.221,0.735)	(0.79,0.17)	(0.766,0.194)	(0,0.862)	(0.766,0.194)	(0.625,0.335)	(0.158,0.8)
	0.5	(0.165,0.835)	(0.231,0.769)	(0.823,0.177)	(0.798,0.202)	(0,1)	(0.798,0.202)	(0.651,0.349)	(0.165,0.835)

Table 2.17 (continued) : Aggregated decision matrix for τ values.

	τ	A1	A2	A3	A4	A5	A6	A7	A8
C8	0.6	(0.158,0.8)	(0.221,0.735)	(0.79,0.17)	(0.766,0.194)	(0,0.862)	(0.766,0.194)	(0.625,0.335)	(0.158,0.8)
	0.7	(0.139,0.697)	(0.194,0.636)	(0.691,0.149)	(0.67,0.17)	(0,0.737)	(0.67,0.17)	(0.547,0.291)	(0.139,0.697)
	0.8	(0.106,0.53)	(0.148,0.481)	(0.527,0.113)	(0.511,0.129)	(0,0.556)	(0.511,0.129)	(0.416,0.22)	(0.106,0.53)
	0.9	(0.059,0.297)	(0.083,0.269)	(0.296,0.063)	(0.287,0.072)	(0,0.311)	(0.287,0.072)	(0.234,0.123)	(0.059,0.297)
C9	0.1	(0.078,0.275)	(0.193,0.161)	(0.174,0.161)	(0.253,0.106)	(0,0.293)	(0.278,0.078)	(0.172,0.185)	(0.263,0.095)
	0.2	(0.139,0.492)	(0.343,0.29)	(0.309,0.301)	(0.45,0.189)	(0,0.528)	(0.494,0.142)	(0.306,0.331)	(0.468,0.17)
	0.3	(0.182,0.65)	(0.45,0.385)	(0.406,0.414)	(0.591,0.248)	(0,0.711)	(0.648,0.19)	(0.402,0.436)	(0.614,0.225)
	0.4	(0.208,0.748)	(0.515,0.444)	(0.464,0.49)	(0.675,0.284)	(0,0.854)	(0.74,0.219)	(0.459,0.5)	(0.702,0.258)
	0.5	(0.217,0.783)	(0.536,0.464)	(0.483,0.517)	(0.704,0.296)	(0,1)	(0.771,0.229)	(0.478,0.522)	(0.731,0.269)
	0.6	(0.208,0.748)	(0.515,0.444)	(0.464,0.49)	(0.675,0.284)	(0,0.854)	(0.74,0.219)	(0.459,0.5)	(0.702,0.258)
	0.7	(0.182,0.65)	(0.45,0.385)	(0.406,0.414)	(0.591,0.248)	(0,0.711)	(0.648,0.19)	(0.402,0.436)	(0.614,0.225)
	0.8	(0.139,0.492)	(0.343,0.29)	(0.309,0.301)	(0.45,0.189)	(0,0.528)	(0.494,0.142)	(0.306,0.331)	(0.468,0.17)
	0.9	(0.078,0.275)	(0.193,0.161)	(0.174,0.161)	(0.253,0.106)	(0,0.293)	(0.278,0.078)	(0.172,0.185)	(0.263,0.095)
C10	0.1	(0.332,0.027)	(0.092,0.258)	(0.16,0.198)	(0.227,0.131)	(0.076,0.282)	(0.274,0.084)	(0.263,0.095)	(0.303,0.056)
	0.2	(0.591,0.049)	(0.163,0.462)	(0.285,0.353)	(0.403,0.234)	(0.135,0.501)	(0.488,0.15)	(0.468,0.17)	(0.539,0.101)
	0.3	(0.775,0.064)	(0.214,0.613)	(0.374,0.465)	(0.529,0.309)	(0.177,0.66)	(0.64,0.199)	(0.614,0.225)	(0.707,0.132)
	0.4	(0.886,0.074)	(0.245,0.71)	(0.428,0.532)	(0.605,0.355)	(0.202,0.757)	(0.732,0.228)	(0.702,0.258)	(0.808,0.152)
	0.5	(0.923,0.077)	(0.255,0.745)	(0.446,0.554)	(0.63,0.37)	(0.21,0.79)	(0.762,0.238)	(0.731,0.269)	(0.842,0.158)
	0.6	(0.886,0.074)	(0.245,0.71)	(0.428,0.532)	(0.605,0.355)	(0.202,0.757)	(0.732,0.228)	(0.702,0.258)	(0.808,0.152)
	0.7	(0.775,0.064)	(0.214,0.613)	(0.374,0.465)	(0.529,0.309)	(0.177,0.66)	(0.64,0.199)	(0.614,0.225)	(0.707,0.132)
	0.8	(0.591,0.049)	(0.163,0.462)	(0.285,0.353)	(0.403,0.234)	(0.135,0.501)	(0.488,0.15)	(0.468,0.17)	(0.539,0.101)
	0.9	(0.332,0.027)	(0.092,0.258)	(0.16,0.198)	(0.227,0.131)	(0.076,0.282)	(0.274,0.084)	(0.263,0.095)	(0.303,0.056)

Step 13. The CINFU weighted decision matrix is constructed for τ values by using Eq. (2.36) based on Tables 2.14 and 2.17 and is presented in Table 2.18.

Table 2.18 : CINFU weighted decision matrix for τ values.

	τ	A1	A2	A3	A4	A5	A6	A7	A8
C1	0.1	(0.025,0.783)	(0.055,0.624)	(0.023,0.792)	(0.04,0.723)	(0.038,0.737)	(0.04,0.727)	(0.038,0.738)	(0.043,0.709)
	0.2	(0.047,0.855)	(0.116,0.68)	(0.043,0.864)	(0.081,0.789)	(0.075,0.804)	(0.078,0.794)	(0.074,0.805)	(0.087,0.774)
	0.3	(0.065,0.893)	(0.181,0.709)	(0.06,0.901)	(0.117,0.823)	(0.107,0.838)	(0.113,0.828)	(0.107,0.839)	(0.127,0.808)
	0.4	(0.077,0.913)	(0.243,0.724)	(0.07,0.92)	(0.144,0.841)	(0.131,0.855)	(0.139,0.846)	(0.13,0.856)	(0.158,0.825)
	0.5	(0.081,0.919)	(0.271,0.729)	(0.074,0.926)	(0.154,0.846)	(0.14,0.86)	(0.149,0.851)	(0.139,0.861)	(0.17,0.83)
	0.6	(0.077,0.913)	(0.243,0.724)	(0.07,0.92)	(0.144,0.841)	(0.131,0.855)	(0.139,0.846)	(0.13,0.856)	(0.158,0.825)
	0.7	(0.065,0.893)	(0.181,0.709)	(0.06,0.901)	(0.117,0.823)	(0.107,0.838)	(0.113,0.828)	(0.107,0.839)	(0.127,0.808)
	0.8	(0.047,0.855)	(0.116,0.68)	(0.043,0.864)	(0.081,0.789)	(0.075,0.804)	(0.078,0.794)	(0.074,0.805)	(0.087,0.774)
	0.9	(0.025,0.783)	(0.055,0.624)	(0.023,0.792)	(0.04,0.723)	(0.038,0.737)	(0.04,0.727)	(0.038,0.738)	(0.043,0.709)
C2	0.1	(0,0.896)	(0.023,0.818)	(0.038,0.707)	(0,0.894)	(0.023,0.827)	(0.023,0.829)	(0.035,0.749)	(0.012,0.873)
	0.2	(0,0.948)	(0.046,0.868)	(0.083,0.747)	(0,0.945)	(0.046,0.875)	(0.045,0.876)	(0.074,0.793)	(0.022,0.923)
	0.3	(0,0.974)	(0.066,0.895)	(0.134,0.768)	(0,0.971)	(0.066,0.899)	(0.065,0.9)	(0.115,0.815)	(0.03,0.947)
	0.4	(0,0.988)	(0.08,0.91)	(0.189,0.778)	(0,0.986)	(0.08,0.911)	(0.079,0.912)	(0.154,0.826)	(0.035,0.96)
	0.5	(0,1)	(0.085,0.915)	(0.219,0.781)	(0,1)	(0.086,0.914)	(0.084,0.916)	(0.171,0.829)	(0.036,0.964)
	0.6	(0,0.988)	(0.08,0.91)	(0.189,0.778)	(0,0.986)	(0.08,0.911)	(0.079,0.912)	(0.154,0.826)	(0.035,0.96)
	0.7	(0,0.974)	(0.066,0.895)	(0.134,0.768)	(0,0.971)	(0.066,0.899)	(0.065,0.9)	(0.115,0.815)	(0.03,0.947)
	0.8	(0,0.948)	(0.046,0.868)	(0.083,0.747)	(0,0.945)	(0.046,0.875)	(0.045,0.876)	(0.074,0.793)	(0.022,0.923)
	0.9	(0,0.896)	(0.023,0.818)	(0.038,0.707)	(0,0.894)	(0.023,0.827)	(0.023,0.829)	(0.035,0.749)	(0.012,0.873)
C3	0.1	(0.029,0.78)	(0.044,0.704)	(0.027,0.787)	(0.038,0.739)	(0.01,0.842)	(0.008,0.846)	(0.02,0.812)	(0.046,0.696)
	0.2	(0.057,0.844)	(0.091,0.763)	(0.052,0.853)	(0.075,0.802)	(0.018,0.912)	(0.015,0.916)	(0.037,0.88)	(0.094,0.754)
	0.3	(0.08,0.877)	(0.136,0.792)	(0.073,0.886)	(0.109,0.834)	(0.024,0.947)	(0.02,0.951)	(0.051,0.914)	(0.142,0.783)

Table 2.18 (continued) : CINFU weighted decision matrix for τ values.

	τ	A1	A2	A3	A4	A5	A6	A7	A8
C3	0.4	(0.096,0.893)	(0.173,0.807)	(0.087,0.903)	(0.135,0.851)	(0.027,0.966)	(0.024,0.97)	(0.06,0.932)	(0.181,0.798)
	0.5	(0.102,0.898)	(0.188,0.812)	(0.092,0.908)	(0.144,0.856)	(0.029,0.971)	(0.025,0.975)	(0.063,0.937)	(0.198,0.802)
	0.6	(0.096,0.893)	(0.173,0.807)	(0.087,0.903)	(0.135,0.851)	(0.027,0.966)	(0.024,0.97)	(0.06,0.932)	(0.181,0.798)
	0.7	(0.08,0.877)	(0.136,0.792)	(0.073,0.886)	(0.109,0.834)	(0.024,0.947)	(0.02,0.951)	(0.051,0.914)	(0.142,0.783)
	0.8	(0.057,0.844)	(0.091,0.763)	(0.052,0.853)	(0.075,0.802)	(0.018,0.912)	(0.015,0.916)	(0.037,0.88)	(0.094,0.754)
	0.9	(0.029,0.78)	(0.044,0.704)	(0.027,0.787)	(0.038,0.739)	(0.01,0.842)	(0.008,0.846)	(0.02,0.812)	(0.046,0.696)
C4	0.1	(0.029,0.799)	(0.014,0.866)	(0.007,0.885)	(0.032,0.778)	(0.012,0.874)	(0.027,0.807)	(0.015,0.857)	(0.03,0.794)
	0.2	(0.059,0.844)	(0.026,0.915)	(0.014,0.935)	(0.066,0.822)	(0.022,0.923)	(0.054,0.854)	(0.028,0.908)	(0.061,0.839)
	0.3	(0.087,0.867)	(0.035,0.94)	(0.018,0.96)	(0.101,0.844)	(0.029,0.948)	(0.08,0.877)	(0.039,0.933)	(0.091,0.861)
	0.4	(0.109,0.878)	(0.042,0.952)	(0.021,0.974)	(0.13,0.855)	(0.034,0.96)	(0.099,0.89)	(0.046,0.947)	(0.115,0.872)
	0.5	(0.118,0.882)	(0.044,0.956)	(0.022,0.978)	(0.142,0.858)	(0.036,0.964)	(0.107,0.893)	(0.049,0.951)	(0.124,0.876)
	0.6	(0.109,0.878)	(0.042,0.952)	(0.021,0.974)	(0.13,0.855)	(0.034,0.96)	(0.099,0.89)	(0.046,0.947)	(0.115,0.872)
	0.7	(0.087,0.867)	(0.035,0.94)	(0.018,0.96)	(0.101,0.844)	(0.029,0.948)	(0.08,0.877)	(0.039,0.933)	(0.091,0.861)
	0.8	(0.059,0.844)	(0.026,0.915)	(0.014,0.935)	(0.066,0.822)	(0.022,0.923)	(0.054,0.854)	(0.028,0.908)	(0.061,0.839)
	0.9	(0.029,0.799)	(0.014,0.866)	(0.007,0.885)	(0.032,0.778)	(0.012,0.874)	(0.027,0.807)	(0.015,0.857)	(0.03,0.794)
C5	0.1	(0.028,0.792)	(0.011,0.852)	(0.013,0.841)	(0.04,0.728)	(0.036,0.753)	(0.016,0.031)	(0.041,0.008)	(0,0.035)
	0.2	(0.055,0.85)	(0.019,0.914)	(0.024,0.903)	(0.083,0.781)	(0.073,0.808)	(0.029,0.063)	(0.086,0.014)	(0,0.073)
	0.3	(0.079,0.879)	(0.026,0.946)	(0.033,0.936)	(0.125,0.807)	(0.109,0.835)	(0.04,0.091)	(0.132,0.019)	(0,0.117)
	0.4	(0.095,0.894)	(0.03,0.963)	(0.038,0.954)	(0.161,0.821)	(0.136,0.849)	(0.047,0.113)	(0.171,0.022)	(0,0.176)
	0.5	(0.101,0.899)	(0.032,0.968)	(0.04,0.96)	(0.175,0.825)	(0.147,0.853)	(0.049,0.121)	(0.188,0.023)	(0,1)
	0.6	(0.095,0.894)	(0.03,0.963)	(0.038,0.954)	(0.161,0.821)	(0.136,0.849)	(0.047,0.113)	(0.171,0.022)	(0,0.176)
	0.7	(0.079,0.879)	(0.026,0.946)	(0.033,0.936)	(0.125,0.807)	(0.109,0.835)	(0.04,0.091)	(0.132,0.019)	(0,0.117)
	0.8	(0.055,0.85)	(0.019,0.914)	(0.024,0.903)	(0.083,0.781)	(0.073,0.808)	(0.029,0.063)	(0.086,0.014)	(0,0.073)
	0.9	(0.028,0.792)	(0.011,0.852)	(0.013,0.841)	(0.04,0.728)	(0.036,0.753)	(0.016,0.031)	(0.041,0.008)	(0,0.035)

Table 2.18 (continued) : CINFU weighted decision matrix for τ values.

τ	A1	A2	A3	A4	A5	A6	A7	A8	
C6	0.1	(0.026,0.815)	(0.008,0.9)	(0.008,0.894)	(0.015,0.873)	(0.026,0.815)	(0.024,0.824)	(0,0.918)	(0.017,0.868)
	0.2	(0.054,0.853)	(0.015,0.942)	(0.015,0.937)	(0.03,0.914)	(0.054,0.853)	(0.05,0.864)	(0,0.96)	(0.033,0.908)
	0.3	(0.083,0.871)	(0.02,0.962)	(0.02,0.96)	(0.042,0.934)	(0.083,0.871)	(0.074,0.884)	(0,0.981)	(0.047,0.927)
	0.4	(0.107,0.88)	(0.024,0.972)	(0.023,0.972)	(0.05,0.944)	(0.107,0.88)	(0.094,0.895)	(0,0.992)	(0.056,0.937)
	0.5	(0.117,0.883)	(0.025,0.975)	(0.024,0.976)	(0.053,0.947)	(0.117,0.883)	(0.102,0.898)	(0,1)	(0.06,0.94)
	0.6	(0.107,0.88)	(0.024,0.972)	(0.023,0.972)	(0.05,0.944)	(0.107,0.88)	(0.094,0.895)	(0,0.992)	(0.056,0.937)
	0.7	(0.083,0.871)	(0.02,0.962)	(0.02,0.96)	(0.042,0.934)	(0.083,0.871)	(0.074,0.884)	(0,0.981)	(0.047,0.927)
	0.8	(0.054,0.853)	(0.015,0.942)	(0.015,0.937)	(0.03,0.914)	(0.054,0.853)	(0.05,0.864)	(0,0.96)	(0.033,0.908)
	0.9	(0.026,0.815)	(0.008,0.9)	(0.008,0.894)	(0.015,0.873)	(0.026,0.815)	(0.024,0.824)	(0,0.918)	(0.017,0.868)
C7	0.1	(0.006,0.901)	(0.02,0.848)	(0.028,0.802)	(0.007,0.897)	(0.024,0.827)	(0.026,0.813)	(0.021,0.842)	(0.021,0.843)
	0.2	(0.011,0.945)	(0.04,0.89)	(0.058,0.842)	(0.013,0.942)	(0.048,0.87)	(0.054,0.854)	(0.042,0.885)	(0.043,0.885)
	0.3	(0.014,0.968)	(0.058,0.911)	(0.089,0.861)	(0.017,0.964)	(0.071,0.891)	(0.081,0.875)	(0.061,0.906)	(0.061,0.906)
	0.4	(0.017,0.979)	(0.071,0.922)	(0.115,0.871)	(0.019,0.976)	(0.088,0.902)	(0.103,0.885)	(0.075,0.917)	(0.075,0.916)
	0.5	(0.018,0.982)	(0.075,0.925)	(0.126,0.874)	(0.02,0.98)	(0.094,0.906)	(0.112,0.888)	(0.08,0.92)	(0.08,0.92)
	0.6	(0.017,0.979)	(0.071,0.922)	(0.115,0.871)	(0.019,0.976)	(0.088,0.902)	(0.103,0.885)	(0.075,0.917)	(0.075,0.916)
	0.7	(0.014,0.968)	(0.058,0.911)	(0.089,0.861)	(0.017,0.964)	(0.071,0.891)	(0.081,0.875)	(0.061,0.906)	(0.061,0.906)
	0.8	(0.011,0.945)	(0.04,0.89)	(0.058,0.842)	(0.013,0.942)	(0.048,0.87)	(0.054,0.854)	(0.042,0.885)	(0.043,0.885)
	0.9	(0.006,0.901)	(0.02,0.848)	(0.028,0.802)	(0.007,0.897)	(0.024,0.827)	(0.026,0.813)	(0.021,0.842)	(0.021,0.843)
C8	0.1	(0.006,0.891)	(0.008,0.883)	(0.033,0.77)	(0.032,0.779)	(0,0.895)	(0.032,0.779)	(0.025,0.819)	(0.006,0.891)
	0.2	(0.011,0.941)	(0.015,0.933)	(0.068,0.813)	(0.066,0.823)	(0,0.946)	(0.066,0.823)	(0.05,0.866)	(0.011,0.941)
	0.3	(0.014,0.966)	(0.02,0.958)	(0.105,0.835)	(0.1,0.845)	(0,0.972)	(0.1,0.845)	(0.072,0.89)	(0.014,0.966)
	0.4	(0.016,0.979)	(0.023,0.971)	(0.137,0.845)	(0.129,0.856)	(0,0.986)	(0.129,0.856)	(0.089,0.901)	(0.016,0.979)
	0.5	(0.017,0.983)	(0.025,0.975)	(0.151,0.849)	(0.141,0.859)	(0,1)	(0.141,0.859)	(0.095,0.905)	(0.017,0.983)
	0.6	(0.016,0.979)	(0.023,0.971)	(0.137,0.845)	(0.129,0.856)	(0,0.986)	(0.129,0.856)	(0.089,0.901)	(0.016,0.979)
	0.7	(0.014,0.966)	(0.02,0.958)	(0.105,0.835)	(0.1,0.845)	(0,0.972)	(0.1,0.845)	(0.072,0.89)	(0.014,0.966)

Table 2.18 (continued) : CINFU weighted decision matrix for τ values.

	τ	A1	A2	A3	A4	A5	A6	A7	A8
C8	0.8	(0.011,0.941)	(0.015,0.933)	(0.068,0.813)	(0.066,0.823)	(0,0.946)	(0.066,0.823)	(0.05,0.866)	(0.011,0.941)
	0.9	(0.006,0.891)	(0.008,0.883)	(0.033,0.77)	(0.032,0.779)	(0,0.895)	(0.032,0.779)	(0.025,0.819)	(0.006,0.891)
C9	0.1	(0.005,0.927)	(0.013,0.898)	(0.011,0.898)	(0.017,0.876)	(0,0.93)	(0.019,0.861)	(0.011,0.905)	(0.018,0.871)
	0.2	(0.009,0.959)	(0.024,0.93)	(0.022,0.932)	(0.035,0.906)	(0,0.963)	(0.039,0.891)	(0.021,0.937)	(0.037,0.901)
	0.3	(0.012,0.975)	(0.035,0.945)	(0.03,0.949)	(0.051,0.921)	(0,0.98)	(0.06,0.907)	(0.03,0.952)	(0.055,0.916)
	0.4	(0.014,0.983)	(0.042,0.953)	(0.036,0.959)	(0.064,0.929)	(0,0.991)	(0.076,0.914)	(0.036,0.96)	(0.069,0.923)
	0.5	(0.014,0.986)	(0.044,0.956)	(0.038,0.962)	(0.069,0.931)	(0,1)	(0.083,0.917)	(0.038,0.962)	(0.075,0.925)
	0.6	(0.014,0.983)	(0.042,0.953)	(0.036,0.959)	(0.064,0.929)	(0,0.991)	(0.076,0.914)	(0.036,0.96)	(0.069,0.923)
	0.7	(0.012,0.975)	(0.035,0.945)	(0.03,0.949)	(0.051,0.921)	(0,0.98)	(0.06,0.907)	(0.03,0.952)	(0.055,0.916)
	0.8	(0.009,0.959)	(0.024,0.93)	(0.022,0.932)	(0.035,0.906)	(0,0.963)	(0.039,0.891)	(0.021,0.937)	(0.037,0.901)
	0.9	(0.005,0.927)	(0.013,0.898)	(0.011,0.898)	(0.017,0.876)	(0,0.93)	(0.019,0.861)	(0.011,0.905)	(0.018,0.871)
C10	0.1	(0.034,0.734)	(0.008,0.89)	(0.015,0.871)	(0.022,0.84)	(0.007,0.897)	(0.027,0.809)	(0.026,0.818)	(0.03,0.782)
	0.2	(0.074,0.772)	(0.015,0.936)	(0.028,0.915)	(0.043,0.883)	(0.012,0.943)	(0.056,0.85)	(0.053,0.859)	(0.064,0.821)
	0.3	(0.12,0.791)	(0.02,0.959)	(0.039,0.936)	(0.062,0.904)	(0.017,0.965)	(0.084,0.871)	(0.078,0.88)	(0.1,0.841)
	0.4	(0.17,0.8)	(0.024,0.971)	(0.047,0.947)	(0.076,0.915)	(0.019,0.976)	(0.107,0.881)	(0.099,0.89)	(0.132,0.851)
	0.5	(0.197,0.803)	(0.025,0.975)	(0.049,0.951)	(0.082,0.918)	(0.02,0.98)	(0.116,0.884)	(0.106,0.894)	(0.146,0.854)
	0.6	(0.17,0.8)	(0.024,0.971)	(0.047,0.947)	(0.076,0.915)	(0.019,0.976)	(0.107,0.881)	(0.099,0.89)	(0.132,0.851)
	0.7	(0.12,0.791)	(0.02,0.959)	(0.039,0.936)	(0.062,0.904)	(0.017,0.965)	(0.084,0.871)	(0.078,0.88)	(0.1,0.841)
	0.8	(0.074,0.772)	(0.015,0.936)	(0.028,0.915)	(0.043,0.883)	(0.012,0.943)	(0.056,0.85)	(0.053,0.859)	(0.064,0.821)
	0.9	(0.034,0.734)	(0.008,0.89)	(0.015,0.871)	(0.022,0.84)	(0.007,0.897)	(0.027,0.809)	(0.026,0.818)	(0.03,0.782)

Step 14. CINFU-PIS and CINFU-NIS are determined as in Eqs. (2.54) and (2.55), respectively.

Step 15. The separation measures of each alternative from CINFU-PIS and CINFU-NIS are calculated for each τ value by using Eqs. (2.56) and (2.57) based on Table 2.18 and are presented in Table 2.19.

Table 2.19 : Separation measures of each alternative for each τ value.

τ	$\Delta_{i,\tau}^*$							
	A1	A2	A3	A4	A5	A6	A7	A8
0.1	1.275	1.326	1.374	1.290	1.364	1.315	1.386	1.330
0.2	1.266	1.330	1.394	1.285	1.383	1.324	1.417	1.343
0.3	1.242	1.322	1.401	1.265	1.388	1.322	1.433	1.345
0.4	1.212	1.311	1.403	1.241	1.389	1.316	1.442	1.344
0.5	1.195	1.306	1.403	1.228	1.391	1.313	1.447	1.343
0.6	1.212	1.311	1.403	1.241	1.389	1.316	1.442	1.344
0.7	1.242	1.322	1.401	1.265	1.388	1.322	1.433	1.345
0.8	1.266	1.330	1.394	1.285	1.383	1.324	1.417	1.343
0.9	1.275	1.326	1.374	1.290	1.364	1.315	1.386	1.330
τ	$\Delta_{i,\tau}^-$							
	A1	A2	A3	A4	A5	A6	A7	A8
0.1	0.570	0.528	0.459	0.557	0.497	0.601	0.570	0.602
0.2	0.564	0.522	0.440	0.550	0.488	0.587	0.555	0.588
0.3	0.570	0.524	0.426	0.556	0.487	0.580	0.543	0.579
0.4	0.585	0.530	0.415	0.568	0.487	0.576	0.533	0.573
0.5	0.595	0.533	0.409	0.578	0.488	0.575	0.528	0.570
0.6	0.585	0.530	0.415	0.568	0.487	0.576	0.533	0.573
0.7	0.570	0.524	0.426	0.556	0.487	0.580	0.543	0.579
0.8	0.564	0.522	0.440	0.550	0.488	0.587	0.555	0.588
0.9	0.570	0.528	0.459	0.557	0.497	0.601	0.570	0.602

Step 16. By using Table 2.19, the relative closeness of each alternative is presented in Table 2.20 and Figure 2.8 illustrates the closeness coefficients to ideal solution with respect to τ values.

Table 2.20 : Relative closeness of alternatives for τ values.

τ	$\gamma_{i,\tau}$							
	A1	A2	A3	A4	A5	A6	A7	A8
0.1	0.570	0.528	0.459	0.557	0.497	0.601	0.570	0.602
0.2	0.564	0.522	0.440	0.550	0.488	0.587	0.555	0.588
0.3	0.570	0.524	0.426	0.556	0.487	0.580	0.543	0.579
0.4	0.585	0.530	0.415	0.568	0.487	0.576	0.533	0.573

Table 2.20 (continued): Relative closeness of alternatives for τ values.

τ	$r_{i,\tau}$							
	A1	A2	A3	A4	A5	A6	A7	A8
0.5	0.595	0.533	0.409	0.578	0.488	0.575	0.528	0.570
0.6	0.585	0.530	0.415	0.568	0.487	0.576	0.533	0.573
0.7	0.570	0.524	0.426	0.556	0.487	0.580	0.543	0.579
0.8	0.564	0.522	0.440	0.550	0.488	0.587	0.555	0.588
0.9	0.570	0.528	0.459	0.557	0.497	0.601	0.570	0.602

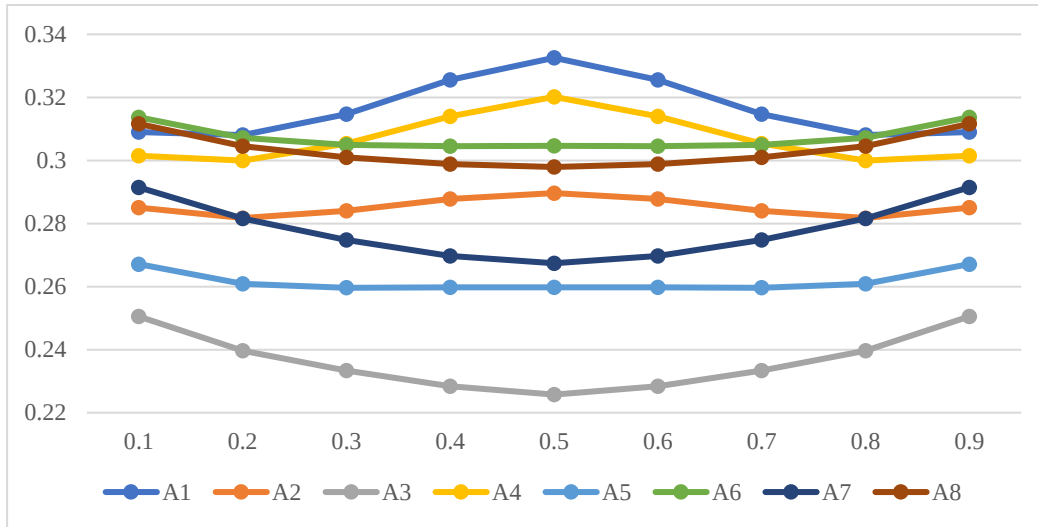
**Figure 2.8 :** Closeness coefficients to ideal solution with respect to τ values.

Figure 2.8 shows that the best alternative is A1, when $0.2 \leq \tau \leq 0.8$. In other τ values, the best alternative changes as A6. The worst three alternatives are A3, A5 and A7. The third rank is shared among A1, A6 and A8 depending on the different τ values.

Step 17. The alternatives according to final scores are ranked for τ values as given in Table 2.21 and Figure 2.9. The best alternative(s) are selected for τ values according to the descending order of final scores.

Table 2.21 : Rank of alternatives for τ values.

τ	A1	A2	A3	A4	A5	A6	A7	A8
0.1	3	6	8	4	7	1	5	2
0.2	1	5	8	4	7	2	6	3
0.3	1	5	8	2	7	3	6	4
0.4	1	5	8	2	7	3	6	4
0.5	1	5	8	2	7	3	6	4
0.6	1	5	8	2	7	3	6	4
0.7	1	5	8	2	7	3	6	4
0.8	1	5	8	4	7	2	6	3
0.9	3	6	8	4	7	1	5	2

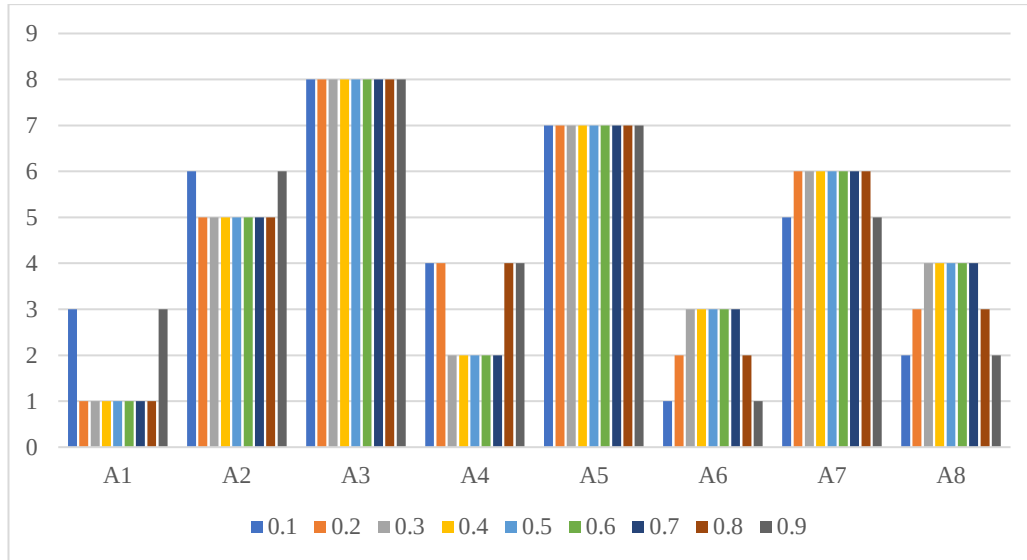


Figure 2.9 : Ranking results of research proposals for grant funding with respect to τ values.

2.6.3 Sensitivity analysis and validation of results

After the results are obtained in MCDM methods, the question arises whether small deviations in the values of the input parameters used change the results of the model and which results are obtained by applying other multi-criteria methods. Therefore, checking the robustness of the results in the developed model and analyzing the sensitivity to the changes in the input variables of the obtained results is a critical step for MCDM methods. In line with this scope, in this part of the study, sensitivity analysis and validity of the results are presented in three parts. In the first part, the sensitivity of the changes in criteria weights on ranking results is checked. In the second part, the sensitivity of the changes in the weights of the DMs to the ranking results is examined. In the third part, the validity of the results obtained from the developed CINFU-based MCDM model is checked with the ordinary fuzzy AHP&TOPSIS, IF-TOPSIS, IF-CODAS, IF-EDAS methodologies.

2.6.3.1 The effect of changes in criteria weights on ranking results

Since a small deviation in the weights of the criteria may cause a change in the ranking results of the alternatives, possible changes in the weights of the criteria need to be taken into account. For this purpose, in this section, the sensitivity analysis of the criterion weights on the final scores and ranking performance of the alternatives is examined. A sensitivity analysis consisting of 90 different scenarios has been conducted to analyze the effect of the change in weight of each criterion on the final

scores and ranking results of the alternatives. A total of 9 different changes have been applied to each criterion by increasing each criterion weight from 0.1 to 0.9. While the weight of each criterion was changed from 0.1 to 0.9 in each of the scenarios, the values of the remaining criteria weights in the scenarios have been proportionally arranged. The effects of the change in the weight of each criterion on the ranking results of the alternatives are shown in Figures 2.10-2.19, respectively.

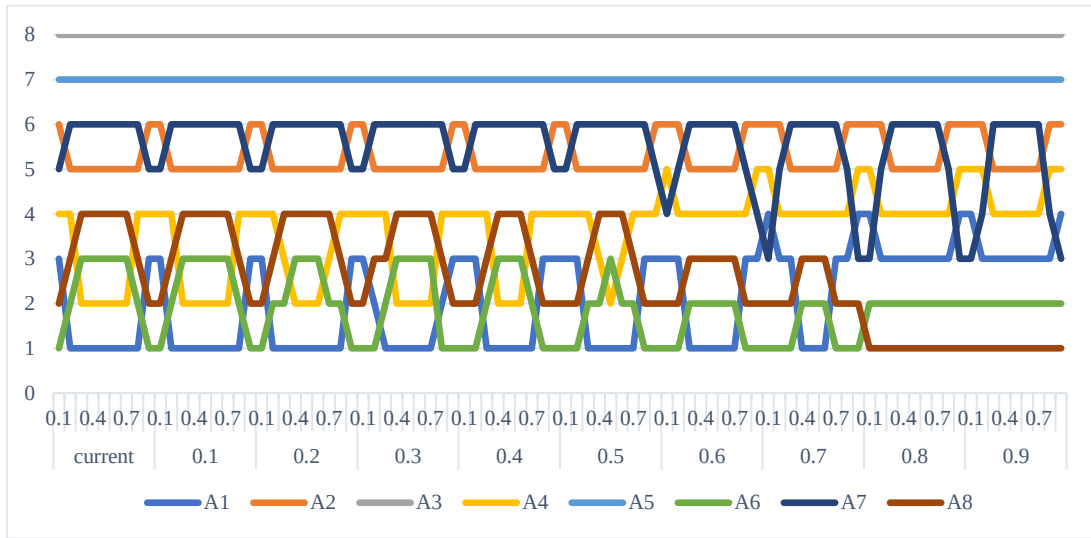


Figure 2.10 : Effect on the alternatives of the change in the weight of criterion 1.

At the relatively high values of the weight of the C1 criterion, alternative A1 is replaced by alternative A8 due to slight changes in the final scores of the most important alternative, A1, and it is seen that it is placed in the third or fourth place. Besides, it has been seen that the worst alternative A3 was not affected by the change in the weight of the C1 criterion and continued to be ranked as the worst alternative, again in the 8th rank. It has been observed that the rankings of the remaining alternatives changed with small changes in the relatively high values of the C1 criterion (see Figure 2.10).

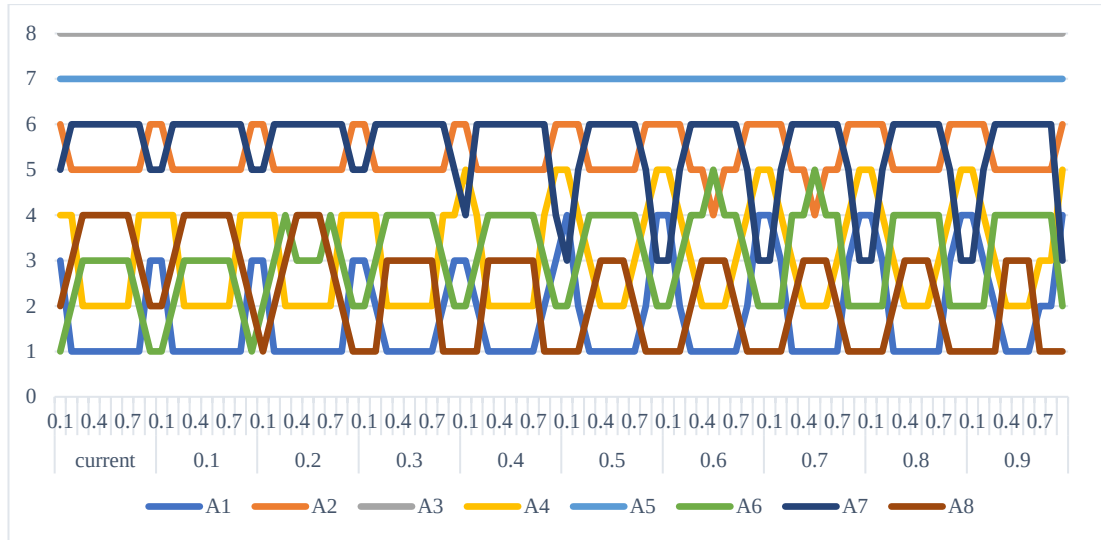


Figure 2.11 : Effect on the alternatives of the change in the weight of criterion 2.

According to Figure 2.11, when the effect of the change in the weight of the C2 criterion on the ranking results of the alternatives is examined, it is seen that the ranking of the best alternative A1 continues as the best alternative without changing in many τ values. It has been seen that the ranking of alternative A8 was better at relatively larger weights of the C2 criterion. Besides, it has been seen that the worst alternative A3 was not affected by the change in the weight of the C2 criterion and was ranked 8th as the worst alternative in all cases. It has been observed that the remaining alternatives caused a change in the final scores of the alternatives without causing a change in their ranking.

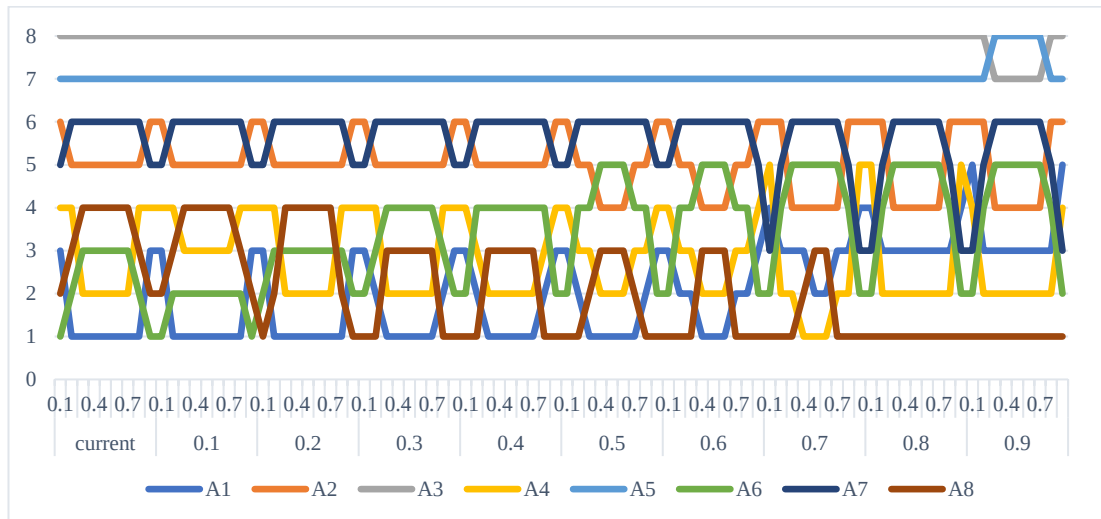


Figure 2.12 : Effect on the alternatives of the change in the weight of criterion 3.

According to Figure 2.12, when the effect of the change in the weight of the C3 criterion on the ranking results of the alternatives is examined, it is seen that the ranking of the first alternative A1 changes at the relatively high weights of the C3 criterion and ranks 3rd, 4th and 5th according to different τ values. In the higher weights of the C3 criterion, alternative A8 has been taken the first place. In the relatively higher weights of the C3 criterion, it has been seen that the A3 alternative was replaced by the A5 alternative, and the A5 alternative ranked 8th as the worst alternative. It has been observed that there was a change in the rankings as a result of small changes of the remaining other alternatives at the relatively high values of the C3 criterion.

Figure 2.13 : Effect on the alternatives of the change in the weight of criterion 4.

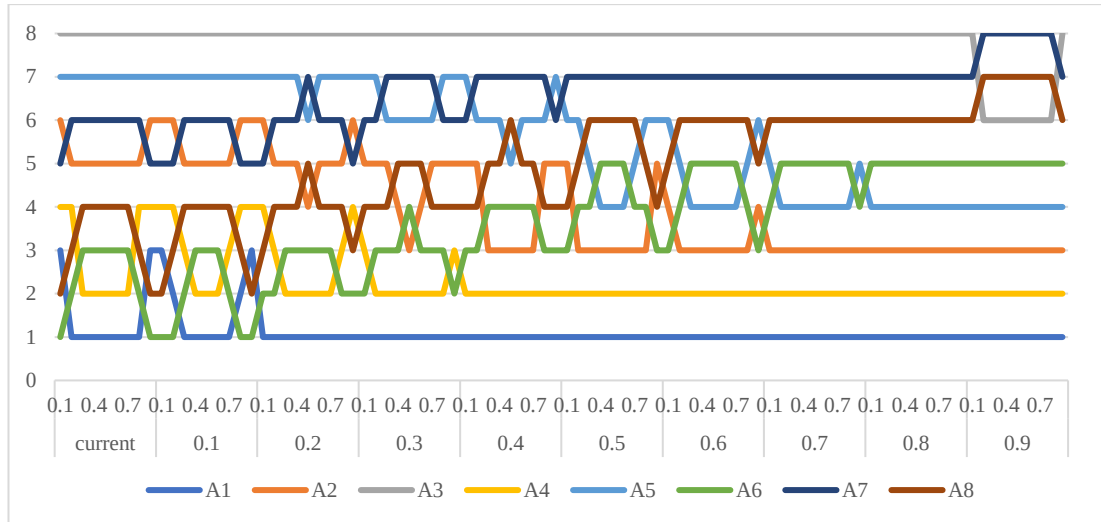


Figure 2.14 : Effect on the alternatives of the change in the weight of criterion 5.

The change in the C5 criterion weight did not cause a change in the ranking of the best alternative A1, and alternative A1 has again been in the first rank in all weight changes. Besides, when the weight of the C5 criterion is the highest, the worst alternative A3 ranks 6th, while alternative A7 takes the last place and becomes the worst alternative (see Figure 2.14).

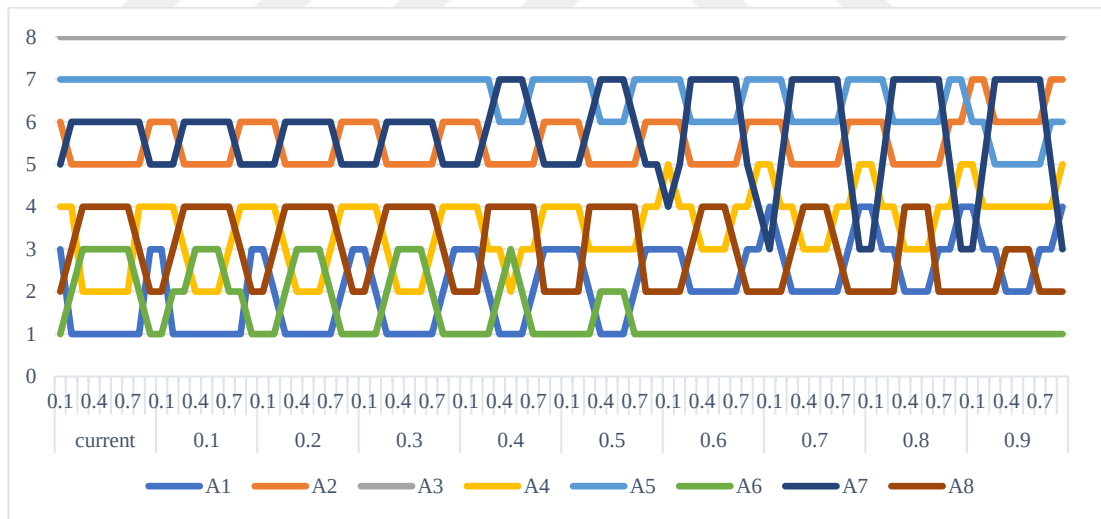


Figure 2.15 : Effect on the alternatives of the change in the weight of criterion 6.

According to Figure 2.15, alternative A1 maintains its position in the first rank in the relatively small weights of the C6 criterion, while alternative A6 is in the first rank in the relatively high weights of the C6 criterion. There has been no change in the ranking of the worst alternative A3 while a change has been caused in the ranking of the other alternatives.

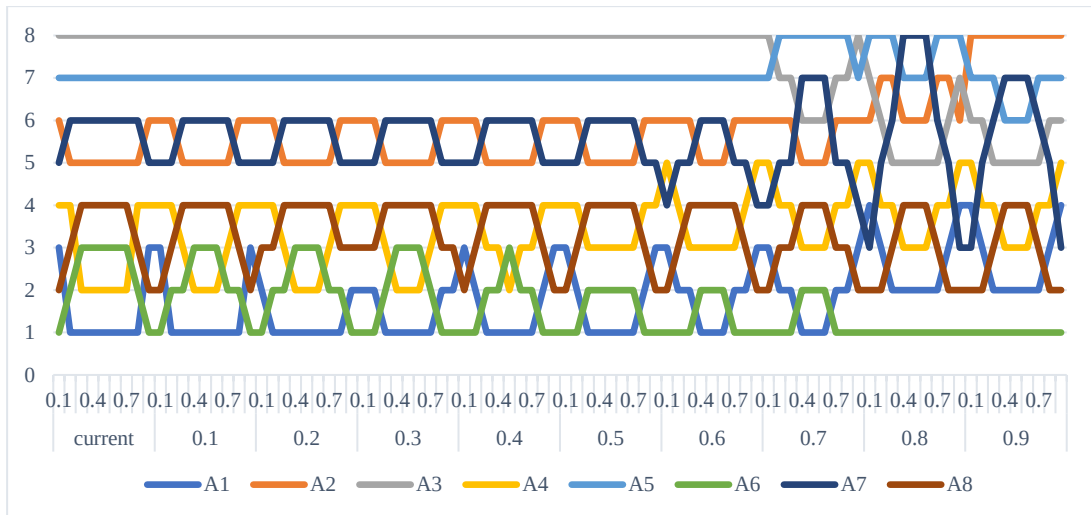


Figure 2.16 : Effect on the alternatives of the change in the weight of criterion 7.

According to Figure 2.16, alternative A1 maintains its position in the first rank in the relatively small weights of the C7 criterion, as in the C6 criterion, while alternative A6 is in the first rank in the relatively high weights of the C7 criterion.

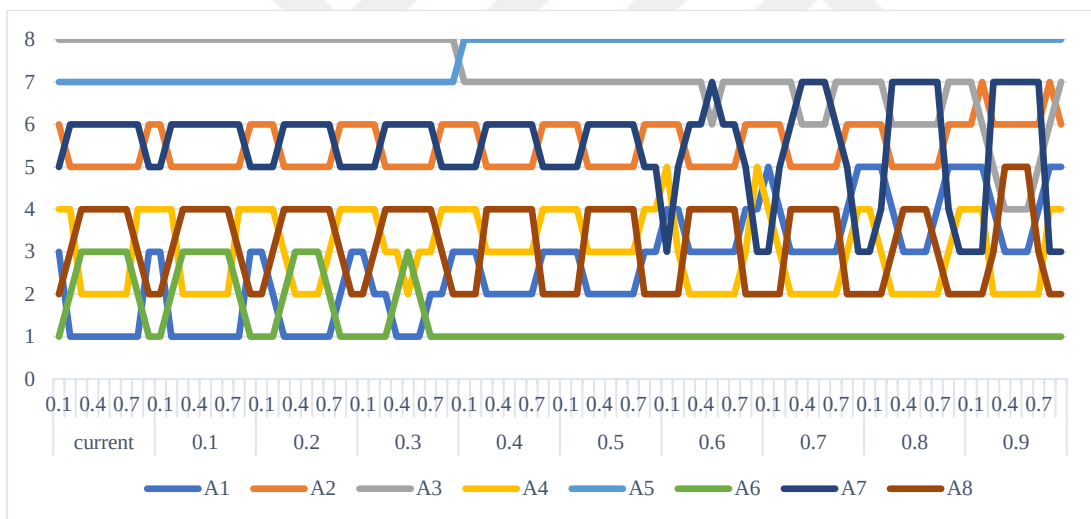


Figure 2.17 : Effect on the alternatives of the change in the weight of criterion 8.

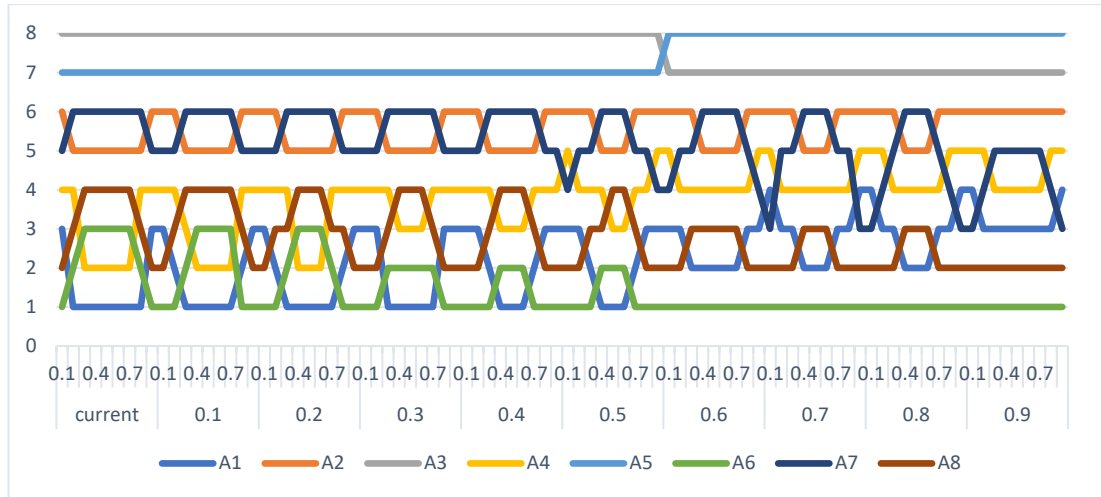


Figure 2.18 : Effect on the alternatives of the change in the weight of criterion 9.

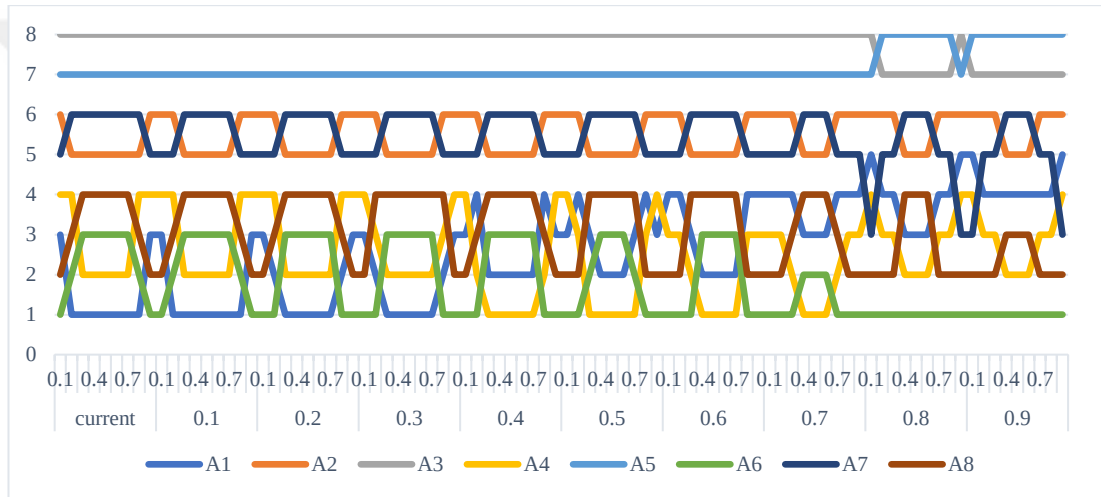


Figure 2.19 : Effect on the alternatives of the change in the weight of criterion 10.

According to Figures 2.17-2.19, the relatively small weights of the criteria C8, C9 and C10 did not cause a change in the order of the most important alternative A1, while the relatively high weights of the criteria C8, C9 and C10 have caused a change in the final scores enough to change the ranking of alternative A1 and have brought the alternative A6 to first place. In addition, the worst alternative A3 has changed its ranking in the relatively high weights of the C8, C9 and C10 criteria and is in the 7th rank. Changes in the rankings of the remaining alternatives have also been observed.

2.6.3.2 The effect of change in weights of DMs on results

The weights of DMs are of great importance on the results of fuzzy-based MCDM methods in which DMs are included. Therefore, in this section, a sensitivity analysis of the effect of different weighting coefficients of DMs on the final ranking result is

presented. A one-at-one sensitivity analysis consisting of 30 different scenarios has been conducted to analyze the effect of the change in weight of each DM on the ranking results. In the analysis, a total of 10 different changes have been applied in the weights of the three DMs included in the study, $\pm 5\%$, $\pm 10\%$, $\pm 15\%$, $\pm 20\%$ and $\pm 25\%$ and the ranking results have been analyzed in each change. In the scenarios, when the weight of each DM changes, the weights of the remaining DMs are proportionally regulated to meet the condition $\sum_{j=1}^n w_j = 1$. For example, if the initial weight of DM2 is reduced by 5%, its reduced value is 0.38. In this situation, the weight of DM1 is updated by using the following calculation $((1 - 0.38)/(1 - 0.4)) \times 0.35 = 0.362$. If the initial weight of DM2 is increased by 5%, its increased value is 0.42. In this situation, the weights of DM1 and DM3 is updated by using the same calculation. The changes in the weights of the DMs have been shown in Figures 2.20, 2.21 and 2.22, respectively.

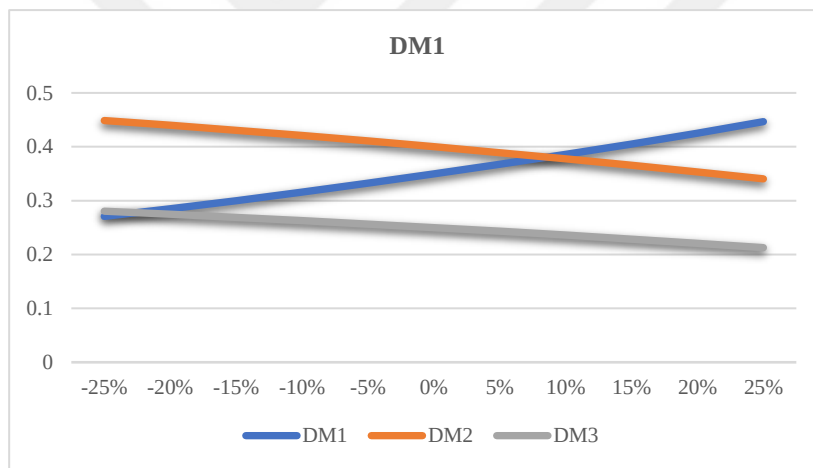


Figure 2.20 : Changes in weight of DM1.

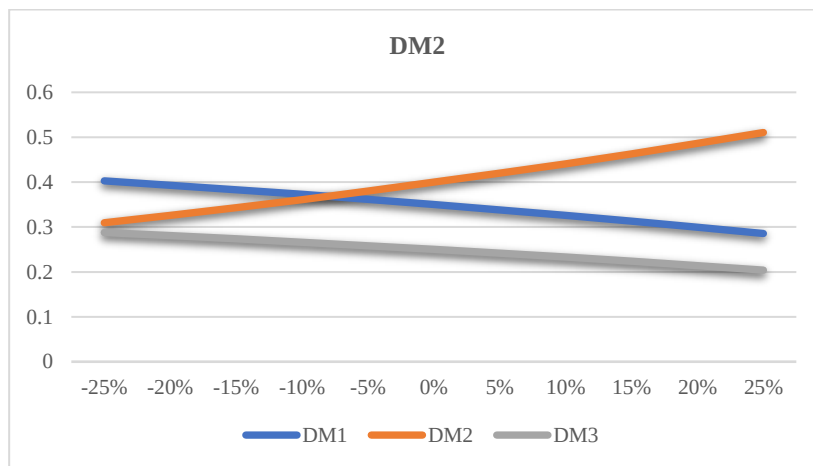


Figure 2.21 : Changes in weight of DM2.

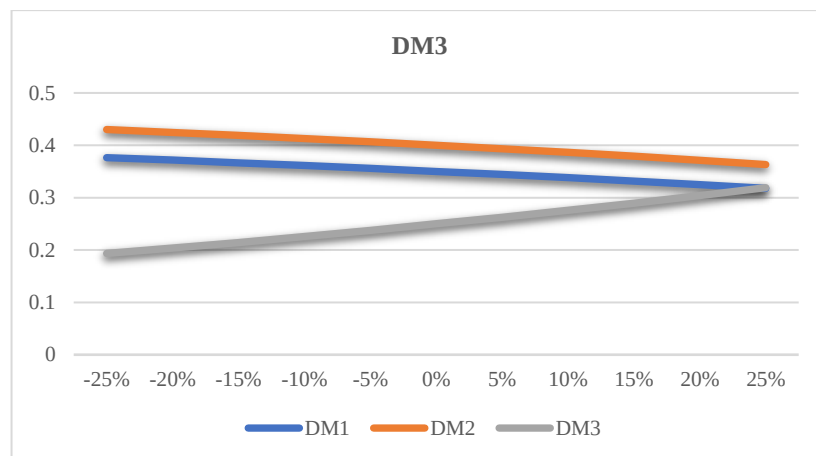


Figure 2.22 : Changes in weight of DM3.

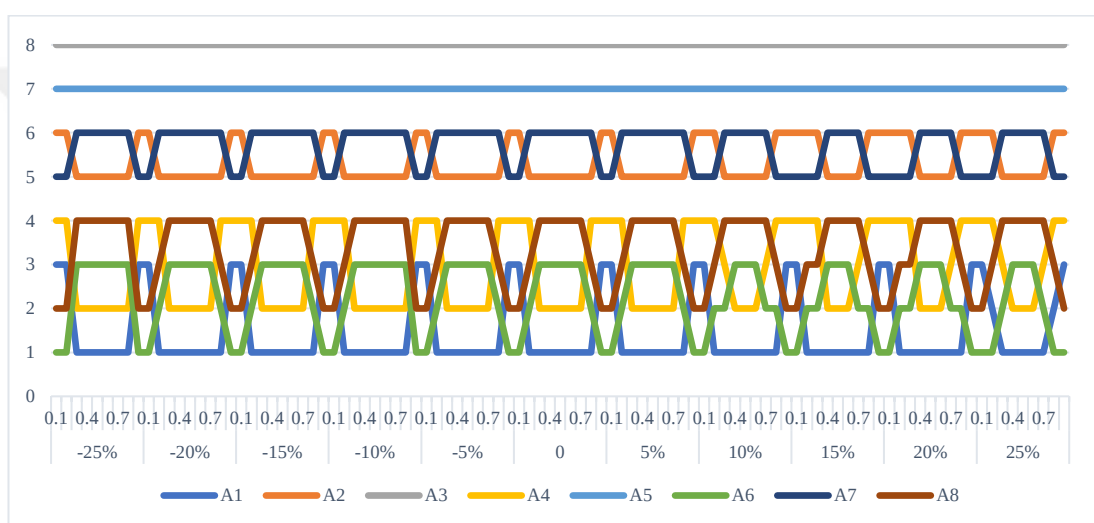


Figure 2.23 : Effects on the ranks of alternatives of DM1 weights.

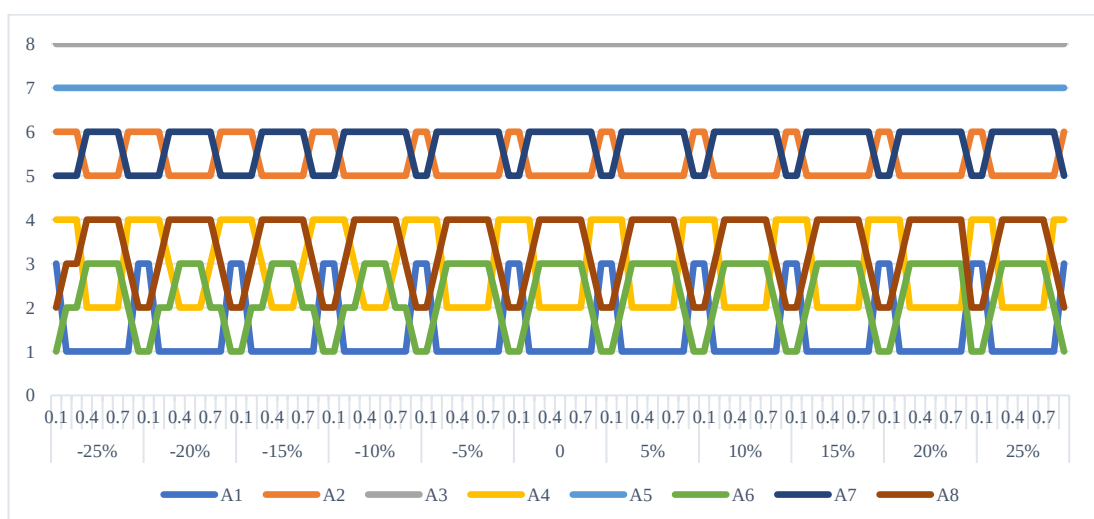


Figure 2.24 : Effects on the ranks of alternatives of DM2 weights.

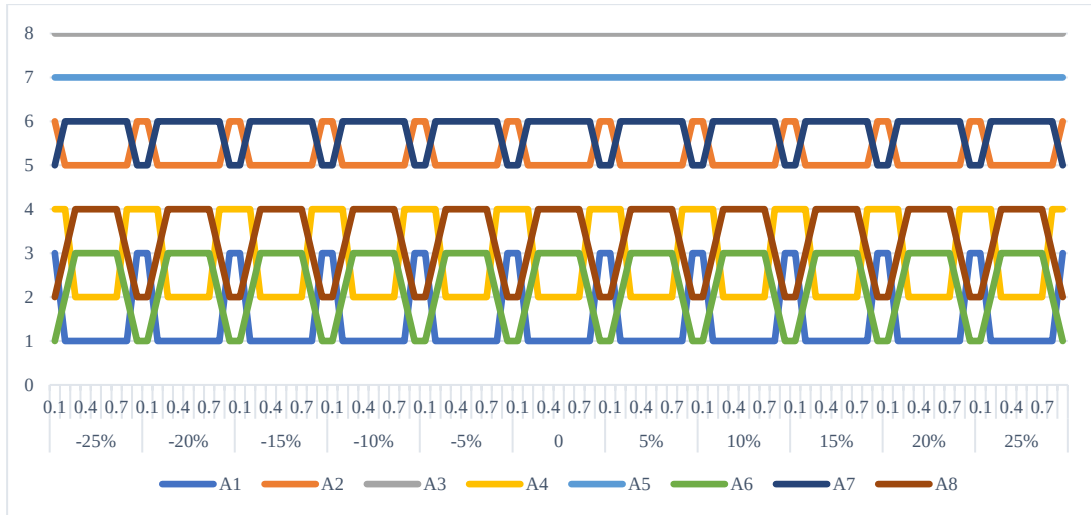


Figure 2.25 : Effects on the ranks of alternatives of DM3 weights.

According to Figure 2.23, when the effect of the change in DM1's weight on the rankings of the alternatives is examined, it is seen that alternative A1 takes the first place between the values of 0.2 and 0.8 τ in each scenario without changing its ranking, and takes the third place in the values of 0.1 and 0.9 τ without changing its ranking. In all scenarios, it has been seen that the worst alternative was in the 8th rank by preserving the A3 ranking, while it was in the 7th rank by maintaining its ranking in the alternative A5. There has been no change in the ranking of the remaining alternatives according to their τ values in all scenarios. When the effect of the change in the weights of DM2 and DM3 on the rankings of the alternatives is examined, the same situation has been occurred as seen in Figures 2.24 and 2.25. According to the obtained results, it has been seen that alternative A1 is the dominant alternative, although there are great changes in the weights of DMs. Based on the presented analysis, it can be concluded that the change in weights of DMs has no effect other than a very slight change in the final ranking results of the alternatives.

2.6.3.3 Comparative analysis with other MCDM methods

In this section, five different MCDM methods are applied to compare and validate the results of the developed CINFU-based AHP-TOPSIS model and show how reliable it is. On the problem of evaluation of research project proposals for grant fundings, fuzzy-TOPSIS and IFS-based TOPSIS, CODAS, VIKOR and EDAS methods in the literature have been compared. The evaluation scores and ranking results of the comparative analysis using the five MCDM methods are shown in Figure 2.26. According to the results obtained from different MCDM methods, it has been observed

that there were differences in the ranking order of the alternatives. According to the obtained results, it is seen that the dominant alternative obtained by TOPSIS, CODAS, VIKOR and EDAS methods is generally the same as the dominant alternative A1 obtained for different continuity points with the developed model. Besides, alternative A4, which is mostly in the second place for different continuity points in the developed method, is also in the second rank in other methods compared. In the study, for $\tau = 0.5$, the ranking results are illustrated in Figure 2.26.

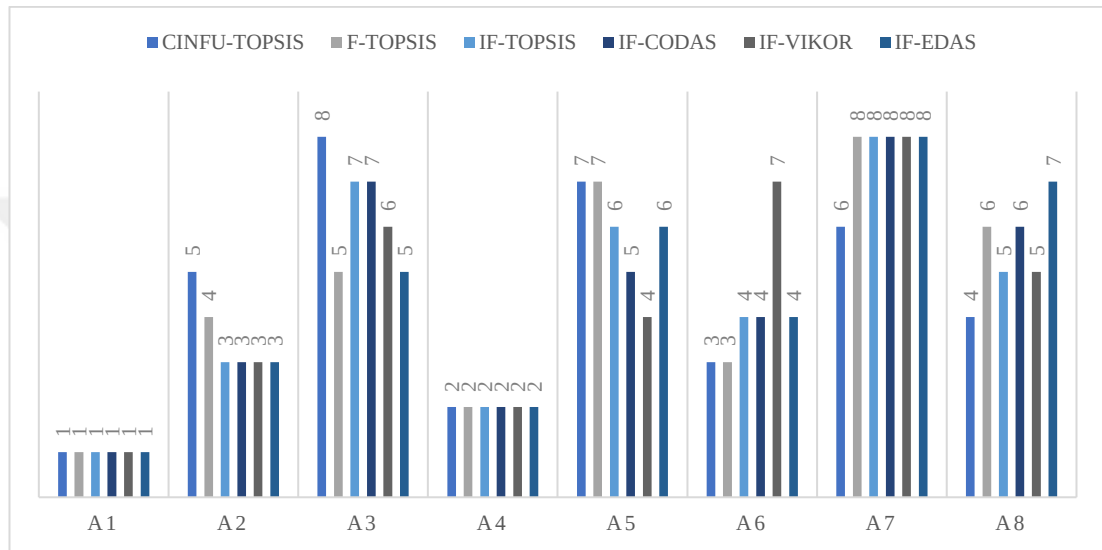


Figure 2.26 : Ranking results of the compared methods.

Spearman's rank correlation coefficient has been used to check whether there is a statistically significant correlation between the rankings obtained from the developed model and the compared models in the study. When the results obtained from Spearman's rank correlation coefficient are examined, the correlation value of 0.8 obtained from the correlation coefficient between the developed method and the F-TOPSIS and IF-CODAS methods, the correlation value of 0.9 obtained from the correlation coefficient between the developed method and the IF-TOPSIS method, and the correlation value of 0.7 obtained from the correlation coefficient between the developed method and the IF-EDAS method indicates that there is a strong relationship between the rankings, and finally, the correlation value of 0.6 obtained from the correlation coefficient between the developed method and the IF-VIKOR method shows that there is a very moderate relationship between the rankings. This indicates a strong correlation between the rankings in general and it can be concluded that the developed method is reliable by verifying its stability.

2.7 Conclusion

Since real-life decision-making problems bring many uncertainties along with their complex and contradictory features, the crisp values assigned by the decision makers are often insufficient to obtain accurate results and may cause errors. Uncertainties due to lack of information and inconsistencies among the decision-making group have required decision makers to make assessments in a wider area, and these assessments have needed to have the capacity to represent values in a continuous range that can take into account the uncertainty in every point. Therefore, the CINFUSs introduced in this study contribute to the decision makers' expression of their ideas in a more natural way in a continuous range. Especially, since CINFUSs have the ability to express information in a more flexible and continuous range, it provides a more powerful handling of problems in uncertain and ambiguous environments by expressing uncertainty accurately, thus contributing to more accurate management of the decision-making process. Based on this advantage, CINFUSs have been introduced in this study and the CINFUS-based AHP&TOPSIS methodology has been developed in order to better handle uncertainty. The developed methodology has been applied to determine the most suitable alternative research projects for grant funding.

By considering the importance of the comprehensive list of criteria, this study has aimed to provide grants to the right research projects by making more accurate and effective decisions for decision makers in the prioritization and selection of research project alternatives for grand funding. Eight research project alternatives are evaluated according to 10 criteria determined by extensive literature review and expert opinions. In the process of obtaining the criterion weights, which constitute the first step in prioritizing and selecting the alternatives, the CINFU-AHP method developed in order to obtain more accurate and reliable criterion weights within the scope of the study has been used. The findings show that the C1-innovativeness and quality criterion is the most important factor in evaluating research project alternatives, followed by the C3-Technical and C5-Physical resources criteria as the second and third most important factors, respectively. Then, the ranking and selection process of the alternatives, which is a critical and important issue for decision makers, has been determined by using the developed CINFUS-based TOPSIS method. According to the results of the developed methodology, it has been determined that the alternative A1-Manufacture of a new wind turbine that can produce more electricity is the most important project, followed

by A4-Usage of nuclear energy in space applications and nuclear rocket propulsion and A6-Construction of garbage centers with automatic separation of solid waste alternatives, respectively.

In the study, sensitivity analysis has been performed on the weights of the criteria and the weights of the decision makers, and the results of the developed methodology have been proven to be valid and reliable with slight differences according to different τ values. In addition, comparative analyzes have been made with F-TOPSIS, IF-TOPSIS, IF-CODAS, IF-VIKOR and IF-EDAS and the results have been compared. Alternative A1 has been determined as the most important alternative in all the compared methods, and it has been observed that there were no major differences in the ranking order for different tau values and similar results have been obtained. As a result of these 5 methods compared, it has been confirmed that the results obtained from the study were robust, and the results have been also proven with the high correlation obtained from the Spearman's rank correlation coefficient. It has also been proven to be a more powerful and reliable method than other popular methods.

The developed methods have been applied in the evaluation of research projects for grand funding and the advantages such as stability and precision have been utilized due to the better reflectivity of the uncertainty of the developed methods. The developed methodology, which is quite flexible, can be used for many problems other than the evaluation of research projects. The developed methods can also be applied to other real-world problems, as their benefits on research projects have proven to be best exploited.

The contributions and advantages of the study can be summarized as follows.

- In this study, a new fuzzy set extension CINFUS, which can take into account the uncertainty at each point and has a nonlinear continuous structure, has been introduced.
- CINFUSs, represented by second-order nonlinear functions and consisting of membership and non-membership degrees, have been used to develop a fuzzy AHP and TOPSIS methodology. With the developed methodology, it has been ensured that the best solutions are obtained between the contradictory and proportional criteria that need to be evaluated simultaneously in an uncertain environment.

- The study is also a milestone in showing how higher-order nonlinear functions can be used in intuitionistic fuzzy sets.
- The study presents a case study to prove the effectiveness, robustness and reliability of the presented methodology and to identify the most suitable research project alternative for grant funding.
- The CINFUS-based AHP-TOPSIS methodology, which has a flexible structure, is suitable for solving multi-criteria decision problems with more criteria and alternatives.
- The application of the CINFUS-based AHP-TOPSIS methodology to the problem of research project selection for grant funding has been tested by sensitivity and comparative analysis, and the obtained ranking has proven to be consistent and reliable. In this way, the potential and applicability of the developed CINFUS-based AHP-TOPSIS methodology has been fully demonstrated.
- This study provides an important contribution to the literature both for the problems within the scope of the evaluation of research projects and for the selection process in other real-life problems by developing a new evaluation model with the introduced new CINFUSs.

For future studies, CINFUSs can be extended to different MCDM methods. Various mathematical operators of CINFUS such as different aggregation operators, score functions can be developed. The developed methodology can be applied to various decision-making problems such as waste disposal site selection, sustainable policy selection, project selection, supplier selection and risk assessment. More specific and detailed criteria could be included for the selection of a suitable research project for grant funding.

3. CODAS EXTENSION USING NOVEL DECOMPOSED PYTHAGOREAN FUZZY SETS: STRATEGY SELECTION FOR IOT BASED SUSTAINABLE SUPPLY CHAIN SYSTEM²

In real life, imprecise human judgments and information involve uncertainty and ambiguity. To deal with the ambiguity inherent in ambiguous information, Zadeh (1965) introduced ordinary fuzzy sets (OFS), known as Type-1 fuzzy sets in 1965. Since ordinary fuzzy sets, which are only represented by membership degrees, cannot fully remove the uncertainty in real-world decision-making problems, various fuzzy set extensions have been developed by various researchers in the literature. Intuitionistic fuzzy sets (IFSs) introduced by Atanasov (1986) in 1983 as a direct extension of OFSs, have the ability to handle uncertain information more flexibly by being represented by the degree of membership and non-membership. Because of its advantage in modeling uncertain information, IFSs have received great attention from many researchers and have been used in various fields. Later, Atanassov and Gargov (1989) introduced IFS of the second type, in which the squared sum of membership and nonmembership degrees is equal to or less than one. This provided a larger domain area for assigning these degrees letting their sum can be larger than one. Then, Pythagorean fuzzy sets (PFSs), originally introduced by Atanassov and Gargov (1989), have been developed by Yager (2013) to provide a stronger description of uncertainty as a direct extension of IFSs. PFSs more effectively handle some facts, events, and related data that cannot be processed with other sets, such as OFSs and IFSs. Unlike OFSs and IFSs, PFSs are characterized by degrees of membership and non-membership, and the squared sum of these degrees must be at most one. With PFSs, data are more accurately reflected and uncertainties in real-life decision problems are better handled. While the uncertainties arising from the incomplete information of the decision-makers are addressed with PFSs, hesitations in the choices

² This chapter is based on the paper “Alkan, N., & Kahraman, C. (2024). CODAS extension using novel decomposed Pythagorean fuzzy sets: Strategy selection for IOT based sustainable supply chain system. *Expert Systems with Applications*, 237, 121534.”

of decision-makers can also be reflected. PFSs can handle uncertainty at a high level by providing missing components that are highly valuable in the decision-making process. After PFSs, many researchers have introduced many other extensions to the field of fuzzy sets and have been applied to many real-life problems. Complex Pythagorean fuzzy sets (CPFSs) are one of these extensions. A complex Pythagorean fuzzy set (CPFS) is an extension of a Pythagorean fuzzy set that is used to handle the vagueness with the degrees whose ranges are enlarged from real to complex subset with unit disc. The significance of CPFSs lies in the fact that they assigned membership grades from the unit circle in the plane, i.e., in the form of a complex number instead of $[0, 1]$ interval. Complex fuzzy sets cannot deal with information of yes and no type, while complex intuitionistic fuzzy sets work only for a limited range of values. To deal with these kinds of problems, the complex Pythagorean fuzzy set (CPFS) was introduced by Ullah et al. (2020). The latest extension of fuzzy sets has been with the decomposed fuzzy sets developed by Çebi et al. (2022) as a direct extension of IFSs. DFSs allow for more realistic and detailed modeling of uncertain information by handling human thoughts and perceptions from the functional point of view and the dysfunctional point of view. In DFSs, unlike IFSs, membership and non-membership degrees are handled from the functional point of view and dysfunctional point of view.

Today, the existence of many new fuzzy sets in the field of fuzzy set extensions has brought along the use of the developed fuzzy set extension with different methods and techniques, or the use of two different fuzzy set extensions together. For instance, Naz et al. (2023) proposed a multi-attribute group decision-making methodology based on 2-tuple linguistics under T-spherical fuzzy environment. In their study, they proposed some point aggregation operators based on 2-tuple linguistic T-spherical fuzzy numbers to handle all aspects of human views without any limitations. İlhan and Kutlu Gündoğdu (2023) used a MCDM methodology based on Hesitant Z-Fuzzy Linguistic Terms, using Hesitant fuzzy sets and Z-fuzzy linguistic terms together. In the study, the weights of the criteria have been obtained by the hesitant fuzzy Z-cognitive mapping method after the relations between the criteria have been determined by the hesitant fuzzy Z- Decision Making Trial and Evaluation Laboratory (DEMATEL) method. Ashraf et al. (2023) proposed a new generalized version of the q-linear Diophantine fuzzy sets called spherical q-linear Diophantine fuzzy sets. In the study,

they presented some novel operational laws of spherical q-linear Diophantine fuzzy sets. Ali and Naeem (2023b) proposed a MCDM model based on aczel–alsina aggregation operators by considering complex fuzzy sets and t-spherical fuzzy sets together. Alkan and Kahraman (2022b) developed Z-fuzzy Analytic Hierarchy Process (AHP) method by considering spherical fuzzy sets and Z-numbers together. Chander and Das (2023) proposed a decision-making approach based on hesitant t-spherical fuzzy linear regression model based on hesitant fuzzy sets and spherical fuzzy sets. In the study, they used the gradient descent technique to determine the optimal coefficients of the variables used in the regression model they proposed. Aydoğdu et al. (2023) developed a new TOPSIS method under the complex spherical fuzzy environment by considering spherical fuzzy sets together with complex fuzzy sets.

Fuzzy set extensions have the ability to handle uncertainty at a high level by providing missing information for today's real-life decision-making problems. Since real-life decision problems contain many proportional and contradictory qualities that require consideration of all criteria at the same time, the use of MCDM methods in which it is possible to evaluate many qualitative and quantitative factors in the decision-making process will enable more effective decisions to be managed. In MCDM methods, where more than one decision maker's views are included to handle complex decision problems, the fact that abstract criteria are more than concrete criteria causes decision-makers to carry out the evaluation process more difficult (Alkan & Kahraman, 2022a). There are several other MCDM models having different points of view to the solution of multi-criteria decision problems. Among these, we can count Stable Preference Ordering Towards Ideal Solution (SPOTIS) method (Bączkiewicz et al., 2021; Dezert et al., 2020; Kizielewicz et al., 2021, 2022; Shekhovtsov et al., 2022), Characteristic Objects Method (COMET) method (Faizi et al., 2017; Jankowski et al., 2017; Piegat & Sałabun, 2014; Sałabun, Karczmarczyk, & Wątróbski, 2019; Sałabun, Karczmarczyk, Wątróbski, et al., 2019; Watrobski et al., 2017; Wątróbski & Sałabun, 2016), Sequential Interactive Model For Urban Sustainability (SIMUS) method (Hontoria et al., 2021; Nigim et al., 2004; Stoilova & Munier, 2021) and Data vARIability Assessment Technique for Order of Preference by Similarity to Ideal Solution (DARIA-TOPSIS) method (Bączkiewicz & Wątróbski, 2022; Wątróbski et al., 2022). Different from these MCDM models, our model handles the uncertainty in the linguistic evaluations by using decomposed Pythagorean fuzzy CODAS method.

Besides, since the classical MCDM methods available in the literature are based on crisp numerical evaluations, they are insufficient due to incomplete information when considering the complex environment and real-life conditions. Therefore, methods should be modeled and developed to take into account the uncertainty in the data in calculations. Thus, the use of MCDM methods with fuzzy set extensions for today's real-life problems will provide more effective results (Ali, 2023a, 2023b; Ali et al., 2023; Ali & Garg, 2023; Ali & Naeem, 2023a, 2023c; Alkan & Kahraman, 2022d, 2023a; Faizi et al., 2021; Sajjad et al., 2022). In this context, the use of decomposed fuzzy sets together with PFSs will enable the ambiguous information encountered in real-life problems to be handled in a more realistic and detailed way and in a wider scope (Kahraman et al., 2023; Cebi et al., 2022; Cebi et al., 2023). Even though the name is similar to CPFSs, Decomposed Pythagorean fuzzy sets (DPFSs) take a completely different approach to membership function definition. The main purpose here is to ensure that the expert is consistent in assigning degrees of membership. It tries to do this by asking functional and dysfunctional questions. DPFSs are made up of two PFSs combined. As a result, complex Pythagorean fuzzy sets and decomposed Pythagorean fuzzy sets are fuzzy set extensions developed for completely different purposes.

As an example of DPFS, let's consider the proposition “it is quite sunny today” for Figure 3.1. The functional question is “*How sunny is it today?*” whereas the dysfunctional question is “*How cloudy is it today?*”. A consistent expert should assign a DPFS $\langle \mathcal{F}(0.90, 0.25), \mathcal{D}(0.25, 0.90) \rangle$.



Figure 3.1 : Proposition on today's weather.

The proposed DPFSs will be employed in the multi-criteria evaluation of sustainable digital technologies in this paper. There is a strong emphasis on sustainable digital technologies by governments, along with an interest in and importance in

sustainability. The use of IoT technologies for sustainability, especially in the supply chain, has had great importance among digital technologies, and this has made IoT technologies the technology promoted and developed by governments. This has also made IoT technology of critical importance in the field of supply chain management, which is often preferred by managers (Krishankumar & Ecer, 2023). With the increasing demand for IoT in the digitalized world, more complex supply chain problems have emerged and this has encouraged the development and implementation of new strategies in the development and implementation of a sustainable supply chain (Seker, 2022).

With the growing demand for the IoT, more complex supply chain issues have begun to emerge. This situation has necessitated the development and implementation of new strategies in the development and implementation of the IoT-based sustainable supply chain. In order to select the most suitable strategy among these strategies, which differ according to each sector, many factors must be evaluated together in line with the needs of each sector. It plays an important role in effectively determining which of the factors to be considered in the decision-making process, which factors will be excluded, or how effective the determined factors will be on the decision to be made. In order to consider all these criteria together, using MCDM methods will enable a more accurate strategy to be determined. However, since the choice of an IoT-based sustainable supply chain strategy brings with it uncertainties in the data, in this study, the developed decomposed Pythagorean fuzzy CODAS method, which has the ability to consider imprecise and uncertain assessments in more detail, is applied in a case study on the selection of an IoT-based sustainable supply chain strategy.

In this study, the development of decomposed Pythagorean fuzzy sets and their use together with MCDM methods will provide more consistent models with a larger domain to assign membership and non-membership degrees and thus obtaining more reliable results. The novelty of this paper is that functional and dysfunctional components of a DPFS provide a better model representing the thoughts in experts minds by preventing inconsistent judgments and invalid membership assignments. To the best knowledge of the authors, there is no approach handling the consistency concept by functional and dysfunctional questions in the extensions of ordinary fuzzy sets. In this context, DPFSs are developed for the first time in order to obtain the most appropriate result among various options in complex and uncertain decision-making

problems. In the study, DPFS-based MCDM model, namely DPF-CODAS method, is proposed to present the applicability and prove the validity of the developed DPFS after the introduction of DPFSs. The effectiveness of the developed DPF-CODAS method is presented with a case study on strategy selection for IoT-based sustainable supply chain. The paper is concluded by the limitations of the study and future research suggestions.

The remaining of this study is organized as follows. The basic principles and mathematical operations of decomposed Pythagorean fuzzy sets are presented with their proof in Section 3.1. The extensions of DPFSs to CODAS method are given in Sections 3.2. In Section 3.3, the developed DPF-CODAS method is applied to strategy selection for IoT-based sustainable supply chain systems, and comprehensive sensitivity analyses and comparative analysis are performed to demonstrate the validity of the results. Finally, the study ends with conclusions and recommendations for the future researches in Section 3.4.

3.1 Decomposed Pythagorean Fuzzy Sets

After the development of intuitionistic fuzzy sets with an innovation brought by Atanassov to membership functions, different extensions of IFSs have been introduced by many researchers. Decomposed fuzzy sets developed by Çebi et al. (2022), the latest new extension of intuitionistic fuzzy sets in the literature deal with membership and non-membership degrees from two different perspectives, the functional point of view and dysfunctional point of view, in order to measure the inconsistency in expert judgments.

Definition 3.1. A DFS $\tilde{A}$ in a non-empty set X is an object having the form (Çebi et al., 2022)

$$\tilde{A} = \{ \langle x, \mathcal{F}(\mu_{\tilde{A}}^{\mathcal{F}}(x), \vartheta_{\tilde{A}}^{\mathcal{F}}(x)), \mathcal{D}(\mu_{\tilde{A}}^{\mathcal{D}}(x), \vartheta_{\tilde{A}}^{\mathcal{D}}(x)) \rangle \mid x \in X \} \quad (3.1)$$

where the function $\mu_{\tilde{A}}^{\mathcal{F}}(x): X \rightarrow [0,1]$, $\vartheta_{\tilde{A}}^{\mathcal{F}}(x): X \rightarrow [0,1]$, $\mu_{\tilde{A}}^{\mathcal{D}}(x): X \rightarrow [0,1]$ and $\vartheta_{\tilde{A}}^{\mathcal{D}}(x): X \rightarrow [0,1]$ are the degrees of membership, non-membership of x to $\mathcal{F}$ and $\mathcal{D}$, respectively where $\mathcal{F}$ and $\mathcal{D}$ are functional point of view and dysfunctional point of view. DFSs must satisfy the following conditions

$$0 \leq \mu_A^{\mathcal{F}}(x) + \vartheta_A^{\mathcal{F}}(x) \leq 1 \quad (3.2)$$

$$0 \leq \mu_A^{\mathcal{D}}(x) + \vartheta_A^{\mathcal{D}}(x) \leq 1 \quad (3.3)$$

In this section, the basic principles and mathematical operations of single-valued Decomposed Pythagorean Fuzzy Sets (SV-DPFS), which consider both the properties of DFSs developed by Çebi et al. (2022) and the properties of Pythagorean fuzzy sets developed by Yager (2013), are introduced.

Definition 3.2. Let X be a universe of discourse. A decomposed Pythagorean fuzzy set (DPFS) $\tilde{A}$ in X is given by

$$\tilde{A} = \{ \langle x, \mathcal{F}(\mu_A^{\mathcal{F}}(x), \vartheta_A^{\mathcal{F}}(x)), \mathcal{D}(\mu_A^{\mathcal{D}}(x), \vartheta_A^{\mathcal{D}}(x)) \rangle \mid x \in X \} \quad (3.4)$$

where the function $\mu_A^{\mathcal{F}}(x): X \rightarrow [0,1]$, $\vartheta_A^{\mathcal{F}}(x): X \rightarrow [0,1]$, $\mu_A^{\mathcal{D}}(x): X \rightarrow [0,1]$ and $\vartheta_A^{\mathcal{D}}(x): X \rightarrow [0,1]$ are the degrees of membership, non-membership of x to $\mathcal{F}$ and $\mathcal{D}$, respectively, where $\mathcal{F}$ and $\mathcal{D}$ are functional point of view and dysfunctional point of view. DPFSs must satisfy the following conditions

$$0 \leq (\mu_A^{\mathcal{F}}(x))^2 + (\vartheta_A^{\mathcal{F}}(x))^2 \leq 1 \quad (3.5)$$

$$0 \leq (\mu_A^{\mathcal{D}}(x))^2 + (\vartheta_A^{\mathcal{D}}(x))^2 \leq 1 \quad (3.6)$$

$$0 \leq (\mu_A^{\mathcal{F}}(x))^2 + (\vartheta_A^{\mathcal{F}}(x))^2 + (\mu_A^{\mathcal{D}}(x))^2 + (\vartheta_A^{\mathcal{D}}(x))^2 \leq 2 \quad (3.7)$$

Definition 3.3. Let $\tilde{A} = \{ \mathcal{F}(\mu_A^{\mathcal{F}}, \vartheta_A^{\mathcal{F}}), \mathcal{D}(\mu_A^{\mathcal{D}}, \vartheta_A^{\mathcal{D}}) \}$, $\tilde{A}_1 = \{ \mathcal{F}(\mu_{A_1}^{\mathcal{F}}, \vartheta_{A_1}^{\mathcal{F}}), \mathcal{D}(\mu_{A_1}^{\mathcal{D}}, \vartheta_{A_1}^{\mathcal{D}}) \}$ and $\tilde{A}_2 = \{ \mathcal{F}(\mu_{A_2}^{\mathcal{F}}, \vartheta_{A_2}^{\mathcal{F}}), \mathcal{D}(\mu_{A_2}^{\mathcal{D}}, \vartheta_{A_2}^{\mathcal{D}}) \}$ be three DPFSs. The basic operators of DPFSs are defined as follows:

$$\begin{aligned} & \tilde{A}_1 \oplus \tilde{A}_2 \\ &= \left\{ \mathcal{F} \left(\sqrt{(\mu_{A_1}^{\mathcal{F}}(x))^2 + (\mu_{A_2}^{\mathcal{F}}(x))^2 - (\mu_{A_1}^{\mathcal{F}}(x))^2 (\mu_{A_2}^{\mathcal{F}}(x))^2}, \vartheta_{A_1}^{\mathcal{F}}(x) \vartheta_{A_2}^{\mathcal{F}}(x) \right), \right. \\ & \quad \left. \mathcal{D} \left(\sqrt{(\mu_{A_1}^{\mathcal{D}}(x))^2 + (\mu_{A_2}^{\mathcal{D}}(x))^2 - (\mu_{A_1}^{\mathcal{D}}(x))^2 (\mu_{A_2}^{\mathcal{D}}(x))^2}, \vartheta_{A_1}^{\mathcal{D}}(x) \vartheta_{A_2}^{\mathcal{D}}(x) \right) \right\} \end{aligned} \quad (3.8)$$

$$\begin{aligned}
& \tilde{A}_1 \otimes \tilde{A}_2 \\
&= \left\{ \begin{aligned} & \mathcal{F} \left(\mu_{\tilde{A}_1}^{\mathcal{F}}(x) \mu_{\tilde{A}_2}^{\mathcal{F}}(x), \sqrt{\left(\vartheta_{\tilde{A}_1}^{\mathcal{F}}(x) \right)^2 + \left(\vartheta_{\tilde{A}_2}^{\mathcal{F}}(x) \right)^2 - \left(\vartheta_{\tilde{A}_1}^{\mathcal{F}}(x) \right)^2 \left(\vartheta_{\tilde{A}_2}^{\mathcal{F}}(x) \right)^2} \right), \\ & \mathfrak{D} \left(\mu_{\tilde{A}_1}^{\mathfrak{D}}(x) \mu_{\tilde{A}_2}^{\mathfrak{D}}(x), \sqrt{\left(\vartheta_{\tilde{A}_1}^{\mathfrak{D}}(x) \right)^2 + \left(\vartheta_{\tilde{A}_2}^{\mathfrak{D}}(x) \right)^2 - \left(\vartheta_{\tilde{A}_1}^{\mathfrak{D}}(x) \right)^2 \left(\vartheta_{\tilde{A}_2}^{\mathfrak{D}}(x) \right)^2} \right) \end{aligned} \right\} \quad (3.9)
\end{aligned}$$

$$\begin{aligned}
& \lambda \tilde{A} \\
&= \left\{ \mathcal{F} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}}^{\mathcal{F}})^2 \right)^{\lambda}}, (\vartheta_{\tilde{A}}^{\mathcal{F}})^{\lambda} \right), \mathfrak{D} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}}^{\mathfrak{D}})^2 \right)^{\lambda}}, (\vartheta_{\tilde{A}}^{\mathfrak{D}})^{\lambda} \right) \right\} \quad (3.10)
\end{aligned}$$

$$\tilde{A}^{\lambda} = \left\{ \mathcal{F} \left((\mu_{\tilde{A}}^{\mathcal{F}})^{\lambda}, \sqrt{1 - \left(1 - (\vartheta_{\tilde{A}}^{\mathcal{F}})^2 \right)^{\lambda}} \right), \mathfrak{D} \left((\mu_{\tilde{A}}^{\mathfrak{D}})^{\lambda}, \sqrt{1 - \left(1 - (\vartheta_{\tilde{A}}^{\mathfrak{D}})^2 \right)^{\lambda}} \right) \right\} \quad (3.11)$$

Remark 3.3. Following equations present the special cases of λ and $\tilde{A}$.

a) If $\tilde{A} = \{\mathcal{F}(1,0), \mathfrak{D}(0,1)\}$ then,

$$\begin{aligned}
\lambda \tilde{A} &= \left\{ \mathcal{F} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}}^{\mathcal{F}})^2 \right)^{\lambda}}, (\vartheta_{\tilde{A}}^{\mathcal{F}})^{\lambda} \right), \mathfrak{D} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}}^{\mathfrak{D}})^2 \right)^{\lambda}}, (\vartheta_{\tilde{A}}^{\mathfrak{D}})^{\lambda} \right) \right\} = \\
& \{\mathcal{F}(1,0), \mathfrak{D}(0,1)\} \\
\tilde{A}^{\lambda} &= \left\{ \mathcal{F} \left((\mu_{\tilde{A}}^{\mathcal{F}})^{\lambda}, \sqrt{1 - \left(1 - (\vartheta_{\tilde{A}}^{\mathcal{F}})^2 \right)^{\lambda}} \right), \mathfrak{D} \left((\mu_{\tilde{A}}^{\mathfrak{D}})^{\lambda}, \sqrt{1 - \left(1 - (\vartheta_{\tilde{A}}^{\mathfrak{D}})^2 \right)^{\lambda}} \right) \right\} = \\
& \{\mathcal{F}(1,0), \mathfrak{D}(0,1)\}.
\end{aligned}$$

b) If $\tilde{A} = \{\mathcal{F}(0,1), \mathfrak{D}(1,0)\}$ then,

$$\begin{aligned}
\lambda \tilde{A} &= \left\{ \mathcal{F} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}}^{\mathcal{F}})^2 \right)^{\lambda}}, (\vartheta_{\tilde{A}}^{\mathcal{F}})^{\lambda} \right), \mathfrak{D} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}}^{\mathfrak{D}})^2 \right)^{\lambda}}, (\vartheta_{\tilde{A}}^{\mathfrak{D}})^{\lambda} \right) \right\} = \\
& \{\mathcal{F}(0,1), \mathfrak{D}(1,0)\} \\
\tilde{A}^{\lambda} &= \left\{ \mathcal{F} \left((\mu_{\tilde{A}}^{\mathcal{F}})^{\lambda}, \sqrt{1 - \left(1 - (\vartheta_{\tilde{A}}^{\mathcal{F}})^2 \right)^{\lambda}} \right), \mathfrak{D} \left((\mu_{\tilde{A}}^{\mathfrak{D}})^{\lambda}, \sqrt{1 - \left(1 - (\vartheta_{\tilde{A}}^{\mathfrak{D}})^2 \right)^{\lambda}} \right) \right\} = \\
& \{\mathcal{F}(0,1), \mathfrak{D}(1,0)\}.
\end{aligned}$$

c) If $\lambda \rightarrow 0$ and $0 < \mu_{\tilde{A}}^{\mathcal{F}}, \vartheta_{\tilde{A}}^{\mathcal{F}}, \mu_{\tilde{A}}^{\mathfrak{D}}, \vartheta_{\tilde{A}}^{\mathfrak{D}} < 1$ then,

$$\begin{aligned}
\lambda \tilde{A} &= \left\{ \mathcal{F} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}}^{\mathcal{F}})^2 \right)^{\lambda}}, (\vartheta_{\tilde{A}}^{\mathcal{F}})^{\lambda} \right), \mathfrak{D} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}}^{\mathfrak{D}})^2 \right)^{\lambda}}, (\vartheta_{\tilde{A}}^{\mathfrak{D}})^{\lambda} \right) \right\} \\
&= \{\mathcal{F}(0,1), \mathfrak{D}(0,1)\}
\end{aligned}$$

$$\tilde{A}^\lambda = \left\{ \mathcal{F} \left((\mu_{\tilde{A}}^{\mathcal{F}})^\lambda, \sqrt{1 - (1 - (\vartheta_{\tilde{A}}^{\mathcal{F}})^2)^\lambda} \right), D \left((\mu_{\tilde{A}}^{\mathcal{D}})^\lambda, \sqrt{1 - (1 - (\vartheta_{\tilde{A}}^{\mathcal{D}})^2)^\lambda} \right) \right\} = \{\mathcal{F}(1,0), D(1,0)\}.$$

d) If $\lambda \rightarrow \infty$ and $0 < \mu_{\tilde{A}}^{\mathcal{F}}, \vartheta_{\tilde{A}}^{\mathcal{F}}, \mu_{\tilde{A}}^{\mathcal{D}}, \vartheta_{\tilde{A}}^{\mathcal{D}} < 1$ then

$$\lambda \tilde{A} = \left\{ \mathcal{F} \left(\sqrt{1 - (1 - (\mu_{\tilde{A}}^{\mathcal{F}})^2)^\lambda}, (\vartheta_{\tilde{A}}^{\mathcal{F}})^\lambda \right), \mathcal{D} \left(\sqrt{1 - (1 - (\mu_{\tilde{A}}^{\mathcal{D}})^2)^\lambda}, (\vartheta_{\tilde{A}}^{\mathcal{D}})^\lambda \right) \right\} = \{\mathcal{F}(1,0), \mathcal{D}(1,0)\}$$

$$\tilde{A}^\lambda = \left\{ \mathcal{F} \left((\mu_{\tilde{A}}^{\mathcal{F}})^\lambda, \sqrt{1 - (1 - (\vartheta_{\tilde{A}}^{\mathcal{F}})^2)^\lambda} \right), \mathcal{D} \left((\mu_{\tilde{A}}^{\mathcal{D}})^\lambda, \sqrt{1 - (1 - (\vartheta_{\tilde{A}}^{\mathcal{D}})^2)^\lambda} \right) \right\} = \{\mathcal{F}(0,1), \mathcal{D}(0,1)\}$$

e) If $\lambda = 1$ then,

$$\begin{aligned} \lambda \tilde{A} &= \left\{ \mathcal{F} \left(\sqrt{1 - (1 - (\mu_{\tilde{A}}^{\mathcal{F}})^2)^\lambda}, (\vartheta_{\tilde{A}}^{\mathcal{F}})^\lambda \right), \mathcal{D} \left(\sqrt{1 - (1 - (\mu_{\tilde{A}}^{\mathcal{D}})^2)^\lambda}, (\vartheta_{\tilde{A}}^{\mathcal{D}})^\lambda \right) \right\} \\ &= \{\mathcal{F}(\mu_{\tilde{A}}^{\mathcal{F}}, \vartheta_{\tilde{A}}^{\mathcal{F}}), \mathcal{D}(\mu_{\tilde{A}}^{\mathcal{D}}, \vartheta_{\tilde{A}}^{\mathcal{D}})\} \end{aligned}$$

$$\begin{aligned} \tilde{A}^\lambda &= \left\{ \mathcal{F} \left((\mu_{\tilde{A}}^{\mathcal{F}})^\lambda, \sqrt{1 - (1 - (\vartheta_{\tilde{A}}^{\mathcal{F}})^2)^\lambda} \right), \mathcal{D} \left((\mu_{\tilde{A}}^{\mathcal{D}})^\lambda, \sqrt{1 - (1 - (\vartheta_{\tilde{A}}^{\mathcal{D}})^2)^\lambda} \right) \right\} \\ &= \{\mathcal{F}(\mu_{\tilde{A}}^{\mathcal{F}}, \vartheta_{\tilde{A}}^{\mathcal{F}}), \mathcal{D}(\mu_{\tilde{A}}^{\mathcal{D}}, \vartheta_{\tilde{A}}^{\mathcal{D}})\} \end{aligned}$$

i.e.

$$\lambda \tilde{A} = \tilde{A}$$

$$\tilde{A}^\lambda = \tilde{A}$$

Definition 3.4. Let $\lambda, \lambda_1, \lambda_2 \geq 0$, then

$$\tilde{A}_1 \oplus \tilde{A}_2 = \tilde{A}_2 \oplus \tilde{A}_1 \quad (3.12)$$

$$\tilde{A}_1 \otimes \tilde{A}_2 = \tilde{A}_2 \otimes \tilde{A}_1 \quad (3.13)$$

$$\lambda(\tilde{A}_1 \oplus \tilde{A}_2) = \lambda \tilde{A}_1 \oplus \lambda \tilde{A}_2 \quad (3.14)$$

$$(\tilde{A}_1 \otimes \tilde{A}_2)^\lambda = \tilde{A}_1^{\lambda} \otimes \tilde{A}_2^{\lambda} \quad (3.15)$$

$$\lambda_1 \tilde{A} \oplus \lambda_2 \tilde{A} = (\lambda_1 + \lambda_2) \tilde{A} \quad (3.16)$$

$$\tilde{A}^{\lambda_1} \otimes \tilde{A}^{\lambda_2} = \tilde{A}^{\lambda_1 + \lambda_2} \quad (3.17)$$

Proof 3.4. (i) by Eq. (3.8) of Definition 3.3, we have

$$\begin{aligned} \tilde{A}_1 \oplus \tilde{A}_2 &= \\ &\left\{ \mathcal{F} \left(\sqrt{\left(\mu_{\tilde{A}_1}^{\mathcal{F}}(x) \right)^2 + \left(\mu_{\tilde{A}_2}^{\mathcal{F}}(x) \right)^2 - \left(\mu_{\tilde{A}_1}^{\mathcal{F}}(x) \right)^2 \left(\mu_{\tilde{A}_2}^{\mathcal{F}}(x) \right)^2}, \vartheta_{\tilde{A}_1}^{\mathcal{F}}(x) \vartheta_{\tilde{A}_2}^{\mathcal{F}}(x) \right), \right. \\ &\left. \mathcal{D} \left(\sqrt{\left(\mu_{\tilde{A}_1}^{\mathcal{D}}(x) \right)^2 + \left(\mu_{\tilde{A}_2}^{\mathcal{D}}(x) \right)^2 - \left(\mu_{\tilde{A}_1}^{\mathcal{D}}(x) \right)^2 \left(\mu_{\tilde{A}_2}^{\mathcal{D}}(x) \right)^2}, \vartheta_{\tilde{A}_1}^{\mathcal{D}}(x) \vartheta_{\tilde{A}_2}^{\mathcal{D}}(x) \right) \right\} \\ \tilde{A}_2 \oplus \tilde{A}_1 &= \\ &\left\{ \mathcal{F} \left(\sqrt{\left(\mu_{\tilde{A}_2}^{\mathcal{F}}(x) \right)^2 + \left(\mu_{\tilde{A}_1}^{\mathcal{F}}(x) \right)^2 - \left(\mu_{\tilde{A}_2}^{\mathcal{F}}(x) \right)^2 \left(\mu_{\tilde{A}_1}^{\mathcal{F}}(x) \right)^2}, \vartheta_{\tilde{A}_2}^{\mathcal{F}}(x) \vartheta_{\tilde{A}_1}^{\mathcal{F}}(x) \right), \right\} \\ &\left\{ \mathcal{D} \left(\sqrt{\left(\mu_{\tilde{A}_2}^{\mathcal{D}}(x) \right)^2 + \left(\mu_{\tilde{A}_1}^{\mathcal{D}}(x) \right)^2 - \left(\mu_{\tilde{A}_2}^{\mathcal{D}}(x) \right)^2 \left(\mu_{\tilde{A}_1}^{\mathcal{D}}(x) \right)^2}, \vartheta_{\tilde{A}_2}^{\mathcal{D}}(x) \vartheta_{\tilde{A}_1}^{\mathcal{D}}(x) \right) \right\} \end{aligned}$$

and so, $\tilde{A}_1 \oplus \tilde{A}_2 = \tilde{A}_2 \oplus \tilde{A}_1$.

(ii) by Eq. (3.9) of Definition 3.3, we have

$$\begin{aligned} \tilde{A}_1 \otimes \tilde{A}_2 &= \\ &\left\{ \mathcal{F} \left(\mu_{\tilde{A}_1}^{\mathcal{F}}(x) \mu_{\tilde{A}_2}^{\mathcal{F}}(x), \sqrt{\left(\vartheta_{\tilde{A}_1}^{\mathcal{F}}(x) \right)^2 + \left(\vartheta_{\tilde{A}_2}^{\mathcal{F}}(x) \right)^2 - \left(\vartheta_{\tilde{A}_1}^{\mathcal{F}}(x) \right)^2 \left(\vartheta_{\tilde{A}_2}^{\mathcal{F}}(x) \right)^2} \right), \right\} \\ &\left\{ \mathcal{D} \left(\mu_{\tilde{A}_1}^{\mathcal{D}}(x) \mu_{\tilde{A}_2}^{\mathcal{D}}(x), \sqrt{\left(\vartheta_{\tilde{A}_1}^{\mathcal{D}}(x) \right)^2 + \left(\vartheta_{\tilde{A}_2}^{\mathcal{D}}(x) \right)^2 - \left(\vartheta_{\tilde{A}_1}^{\mathcal{D}}(x) \right)^2 \left(\vartheta_{\tilde{A}_2}^{\mathcal{D}}(x) \right)^2} \right) \right\} \end{aligned}$$

$$\begin{aligned} \tilde{A}_2 \otimes \tilde{A}_1 &= \\ &\left\{ \mathcal{F} \left(\mu_{\tilde{A}_2}^{\mathcal{F}}(x) \mu_{\tilde{A}_1}^{\mathcal{F}}(x), \sqrt{\left(\vartheta_{\tilde{A}_2}^{\mathcal{F}}(x) \right)^2 + \left(\vartheta_{\tilde{A}_1}^{\mathcal{F}}(x) \right)^2 - \left(\vartheta_{\tilde{A}_2}^{\mathcal{F}}(x) \right)^2 \left(\vartheta_{\tilde{A}_1}^{\mathcal{F}}(x) \right)^2} \right), \right\} \\ &\left\{ \mathcal{D} \left(\mu_{\tilde{A}_2}^{\mathcal{D}}(x) \mu_{\tilde{A}_1}^{\mathcal{D}}(x), \sqrt{\left(\vartheta_{\tilde{A}_2}^{\mathcal{D}}(x) \right)^2 + \left(\vartheta_{\tilde{A}_1}^{\mathcal{D}}(x) \right)^2 - \left(\vartheta_{\tilde{A}_2}^{\mathcal{D}}(x) \right)^2 \left(\vartheta_{\tilde{A}_1}^{\mathcal{D}}(x) \right)^2} \right) \right\} \end{aligned}$$

and so, $\tilde{A}_1 \otimes \tilde{A}_2 = \tilde{A}_2 \otimes \tilde{A}_1$.

(iii) by Eqs. (3.8) and (3.10) of Definition 3.3, we have

$$\begin{aligned}\lambda(\tilde{A}_1 \oplus \tilde{A}_2) &= \lambda \left\{ \begin{aligned} &\mathcal{F} \left(\sqrt{(\mu_{\tilde{A}_1}^{\mathcal{F}})^2 + (\mu_{\tilde{A}_2}^{\mathcal{F}})^2 - (\mu_{\tilde{A}_1}^{\mathcal{F}})^2 (\mu_{\tilde{A}_2}^{\mathcal{F}})^2}, \vartheta_{\tilde{A}_1}^{\mathcal{F}} \vartheta_{\tilde{A}_2}^{\mathcal{F}} \right), \\ &\mathfrak{D} \left(\sqrt{(\mu_{\tilde{A}_1}^{\mathfrak{D}})^2 + (\mu_{\tilde{A}_2}^{\mathfrak{D}})^2 - (\mu_{\tilde{A}_1}^{\mathfrak{D}})^2 (\mu_{\tilde{A}_2}^{\mathfrak{D}})^2}, \vartheta_{\tilde{A}_1}^{\mathfrak{D}} \vartheta_{\tilde{A}_2}^{\mathfrak{D}} \right) \end{aligned} \right\} \\ &= \left\{ \begin{aligned} &\mathcal{F} \left(\sqrt{1 - \left(1 - \left((\mu_{\tilde{A}_1}^{\mathcal{F}})^2 + (\mu_{\tilde{A}_2}^{\mathcal{F}})^2 - (\mu_{\tilde{A}_1}^{\mathcal{F}})^2 (\mu_{\tilde{A}_2}^{\mathcal{F}})^2\right)\right)^\lambda}, (\vartheta_{\tilde{A}_1}^{\mathcal{F}})^\lambda (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^\lambda \right), \\ &\mathfrak{D} \left(\sqrt{1 - \left(1 - \left((\mu_{\tilde{A}_1}^{\mathfrak{D}})^2 + (\mu_{\tilde{A}_2}^{\mathfrak{D}})^2 - (\mu_{\tilde{A}_1}^{\mathfrak{D}})^2 (\mu_{\tilde{A}_2}^{\mathfrak{D}})^2\right)\right)^\lambda}, (\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^\lambda (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^\lambda \right) \end{aligned} \right\}\end{aligned}$$

$$\begin{aligned}\lambda \tilde{A}_1 \oplus \lambda \tilde{A}_2 &= \left\{ \left(\mathcal{F} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}_1}^{\mathcal{F}})^2\right)^\lambda}, (\vartheta_{\tilde{A}_1}^{\mathcal{F}})^\lambda \right), \mathfrak{D} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}_1}^{\mathfrak{D}})^2\right)^\lambda}, (\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^\lambda \right) \right) \oplus \right. \\ &\quad \left. \left(\mathcal{F} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}_2}^{\mathcal{F}})^2\right)^\lambda}, (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^\lambda \right), \mathfrak{D} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}_2}^{\mathfrak{D}})^2\right)^\lambda}, (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^\lambda \right) \right) \right\} = \\ &\quad \left\{ \mathcal{F} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}_1}^{\mathcal{F}})^2\right)^\lambda + 1 - \left(1 - (\mu_{\tilde{A}_2}^{\mathcal{F}})^2\right)^\lambda - \left(1 - \left(1 - (\mu_{\tilde{A}_1}^{\mathcal{F}})^2\right)^\lambda\right) \left(1 - \left(1 - (\mu_{\tilde{A}_2}^{\mathcal{F}})^2\right)^\lambda\right)}, \right. \\ &\quad \left. (\vartheta_{\tilde{A}_1}^{\mathcal{F}})^\lambda (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^\lambda \right), \\ &\quad \mathfrak{D} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}_1}^{\mathfrak{D}})^2\right)^\lambda + 1 - \left(1 - (\mu_{\tilde{A}_2}^{\mathfrak{D}})^2\right)^\lambda - \left(1 - \left(1 - (\mu_{\tilde{A}_1}^{\mathfrak{D}})^2\right)^\lambda\right) \left(1 - \left(1 - (\mu_{\tilde{A}_2}^{\mathfrak{D}})^2\right)^\lambda\right)}, \right. \\ &\quad \left. (\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^\lambda (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^\lambda \right) \left. \right\} = \left\{ \mathcal{F} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}_1}^{\mathcal{F}})^2\right)^\lambda \left(1 - (\mu_{\tilde{A}_2}^{\mathcal{F}})^2\right)^\lambda}, (\vartheta_{\tilde{A}_1}^{\mathcal{F}})^\lambda (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^\lambda \right), \right. \\ &\quad \left. \mathfrak{D} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}_1}^{\mathfrak{D}})^2\right)^\lambda \left(1 - (\mu_{\tilde{A}_2}^{\mathfrak{D}})^2\right)^\lambda}, (\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^\lambda (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^\lambda \right) \right\} = \\ &\quad \left\{ \mathcal{F} \left(\sqrt{1 - \left(1 - \left((\mu_{\tilde{A}_1}^{\mathcal{F}})^2 + (\mu_{\tilde{A}_2}^{\mathcal{F}})^2 - (\mu_{\tilde{A}_1}^{\mathcal{F}})^2 (\mu_{\tilde{A}_2}^{\mathcal{F}})^2\right)\right)^\lambda}, (\vartheta_{\tilde{A}_1}^{\mathcal{F}})^\lambda (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^\lambda \right), \right. \\ &\quad \left. \mathfrak{D} \left(\sqrt{1 - \left(1 - \left((\mu_{\tilde{A}_1}^{\mathfrak{D}})^2 + (\mu_{\tilde{A}_2}^{\mathfrak{D}})^2 - (\mu_{\tilde{A}_1}^{\mathfrak{D}})^2 (\mu_{\tilde{A}_2}^{\mathfrak{D}})^2\right)\right)^\lambda}, (\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^\lambda (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^\lambda \right) \right\}\end{aligned}$$

and so, $\lambda(\tilde{A}_1 \oplus \tilde{A}_2) = \lambda \tilde{A}_1 \oplus \lambda \tilde{A}_2$.

(iv) by Eqs. (3.9) and (3.11) of Definition 3.3, we have

$$\begin{aligned}
& (\tilde{A}_1 \otimes \tilde{A}_2)^\lambda \\
&= \left\{ \mathcal{F} \left(\mu_{\tilde{A}_1}^{\mathcal{F}} \mu_{\tilde{A}_2}^{\mathcal{F}}, \sqrt{(\vartheta_{\tilde{A}_1}^{\mathcal{F}})^2 + (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^2 - (\vartheta_{\tilde{A}_1}^{\mathcal{F}})^2 (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^2} \right), \mathfrak{D} \left(\mu_{\tilde{A}_1}^{\mathfrak{D}} \mu_{\tilde{A}_2}^{\mathfrak{D}}, \sqrt{(\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^2 + (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^2 - (\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^2 (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^2} \right) \right\}^\lambda \\
&= \left\{ \mathcal{F} \left((\mu_{\tilde{A}_1}^{\mathcal{F}})^\lambda (\mu_{\tilde{A}_2}^{\mathcal{F}})^\lambda, \sqrt{1 - \left(1 - \left((\vartheta_{\tilde{A}_1}^{\mathcal{F}})^2 + (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^2 - (\vartheta_{\tilde{A}_1}^{\mathcal{F}})^2 (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^2 \right)^\lambda} \right), \right. \\
&\quad \left. \mathfrak{D} \left((\mu_{\tilde{A}_1}^{\mathfrak{D}})^\lambda (\mu_{\tilde{A}_2}^{\mathfrak{D}})^\lambda, \sqrt{1 - \left(1 - \left((\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^2 + (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^2 - (\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^2 (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^2 \right)^\lambda} \right) \right) \right\} \\
&\tilde{A}_1^\lambda \otimes \tilde{A}_2^\lambda = \left\{ \mathcal{F} \left((\mu_{\tilde{A}_1}^{\mathcal{F}})^\lambda, \sqrt{1 - \left(1 - (\vartheta_{\tilde{A}_1}^{\mathcal{F}})^2 \right)^\lambda} \right), \mathfrak{D} \left((\mu_{\tilde{A}_1}^{\mathfrak{D}})^\lambda, \sqrt{1 - \left(1 - (\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^2 \right)^\lambda} \right) \otimes \right. \\
&\quad \left. \mathcal{F} \left((\mu_{\tilde{A}_2}^{\mathcal{F}})^\lambda, \sqrt{1 - \left(1 - (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^2 \right)^\lambda} \right), \mathfrak{D} \left((\mu_{\tilde{A}_2}^{\mathfrak{D}})^\lambda, \sqrt{1 - \left(1 - (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^2 \right)^\lambda} \right) \right\} = \left\{ \mathcal{F} \left((\mu_{\tilde{A}_1}^{\mathcal{F}})^\lambda (\mu_{\tilde{A}_2}^{\mathcal{F}})^\lambda, \right. \right. \\
&\quad \left. \sqrt{1 - \left(1 - (\vartheta_{\tilde{A}_1}^{\mathcal{F}})^2 \right)^\lambda + 1 - \left(1 - (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^2 \right)^\lambda - \left(1 - \left(1 - (\vartheta_{\tilde{A}_1}^{\mathcal{F}})^2 \right)^\lambda \right) \left(1 - \left(1 - (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^2 \right)^\lambda \right)} \right. \\
&\quad \left. \mathfrak{D} \left((\mu_{\tilde{A}_1}^{\mathfrak{D}})^\lambda (\mu_{\tilde{A}_2}^{\mathfrak{D}})^\lambda, \right. \right. \\
&\quad \left. \sqrt{1 - \left(1 - (\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^2 \right)^\lambda + 1 - \left(1 - (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^2 \right)^\lambda - \left(1 - \left(1 - (\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^2 \right)^\lambda \right) \left(1 - \left(1 - (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^2 \right)^\lambda \right)} \right. \\
&\quad \left. \left\{ \mathcal{F} \left((\mu_{\tilde{A}_1}^{\mathcal{F}})^\lambda (\mu_{\tilde{A}_2}^{\mathcal{F}})^\lambda, \sqrt{1 - \left(1 - (\vartheta_{\tilde{A}_1}^{\mathcal{F}})^2 \right)^\lambda \left(1 - (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^2 \right)^\lambda} \right), \right. \right. \\
&\quad \left. \left. \mathfrak{D} \left((\mu_{\tilde{A}_1}^{\mathfrak{D}})^\lambda (\mu_{\tilde{A}_2}^{\mathfrak{D}})^\lambda, \sqrt{1 - \left(1 - (\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^2 \right)^\lambda \left(1 - (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^2 \right)^\lambda} \right) \right\} = \right. \\
&\quad \left. \left\{ \mathcal{F} \left((\mu_{\tilde{A}_1}^{\mathcal{F}})^\lambda (\mu_{\tilde{A}_2}^{\mathcal{F}})^\lambda, \sqrt{1 - \left(1 - \left((\vartheta_{\tilde{A}_1}^{\mathcal{F}})^2 + (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^2 - (\vartheta_{\tilde{A}_1}^{\mathcal{F}})^2 (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^2 \right)^\lambda} \right), \right. \right. \\
&\quad \left. \left. \mathfrak{D} \left((\mu_{\tilde{A}_1}^{\mathfrak{D}})^\lambda (\mu_{\tilde{A}_2}^{\mathfrak{D}})^\lambda, \sqrt{1 - \left(1 - \left((\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^2 + (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^2 - (\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^2 (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^2 \right)^\lambda} \right) \right) \right\} \\
&\text{and so, } (\tilde{A}_1 \otimes \tilde{A}_2)^\lambda = \tilde{A}_1^\lambda \otimes \tilde{A}_2^\lambda.
\end{aligned}$$

The other proofs of the equations in Definition 3.4 are omitted here since their proofs are trivial.

Definition 3.5. Let $\tilde{A}_i = \left\{ \mathcal{F}(\mu_{\tilde{A}_i}^{\mathcal{F}}, \vartheta_{\tilde{A}_i}^{\mathcal{F}}), \mathcal{D}(\mu_{\tilde{A}_i}^{\mathcal{D}}, \vartheta_{\tilde{A}_i}^{\mathcal{D}}) \right\}$, $(i = 1, 2, \dots, n)$ be a set of DPFSs and $\lambda_i = (\lambda_1, \lambda_2, \dots, \lambda_n)$ be weight vector of $\tilde{A}_i$ with $\sum_{i=1}^n \lambda_i = 1$ and $\lambda_i \in [0, 1]$, then a Decomposed Pythagorean Weighted Arithmetic Mean (DPWAM) operator is defined as

$$\begin{aligned} DPWAM(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) &= \lambda_1 \tilde{A}_1 \oplus \lambda_2 \tilde{A}_2 \oplus \dots \oplus \lambda_n \tilde{A}_n \\ &= \left\{ \mathcal{F} \left(\sqrt{1 - \prod_{i=1}^n \left(1 - (\mu_{\tilde{A}_i}^{\mathcal{F}})^2 \right)^{\lambda_i}}, \prod_{i=1}^n (\vartheta_{\tilde{A}_i}^{\mathcal{F}})^{\lambda_i} \right), \right. \\ &\quad \left. \mathcal{D} \left(\sqrt{1 - \prod_{i=1}^n \left(1 - (\mu_{\tilde{A}_i}^{\mathcal{D}})^2 \right)^{\lambda_i}}, \prod_{i=1}^n (\vartheta_{\tilde{A}_i}^{\mathcal{D}})^{\lambda_i} \right) \right\} \end{aligned} \quad (3.18)$$

Proof 3.5. We can prove Eq. (3.17) by using mathematical induction on n:

when n=2,

$$DPWAM(\tilde{A}_1, \tilde{A}_2) = \lambda_1 \tilde{A}_1 \oplus \lambda_2 \tilde{A}_2$$

According to Definition 3.3, we can see that both $\lambda_1 \tilde{A}_1$ and $\lambda_2 \tilde{A}_2$ are DPFSs, and the value of $\lambda_1 \tilde{A}_1 + \lambda_2 \tilde{A}_2$ is also a DPFSs. By the operational law (iv) in Definition 3.4, we have

$$\begin{aligned} \lambda_1 \tilde{A}_1 &= \left\{ \mathcal{F} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}_1}^{\mathcal{F}})^2 \right)^{\lambda_1}}, (\vartheta_{\tilde{A}_1}^{\mathcal{F}})^{\lambda_1} \right), \mathcal{D} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}_1}^{\mathcal{D}})^2 \right)^{\lambda_1}}, (\vartheta_{\tilde{A}_1}^{\mathcal{D}})^{\lambda_1} \right) \right\} \\ \lambda_2 \tilde{A}_2 &= \left\{ \mathcal{F} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}_2}^{\mathcal{F}})^2 \right)^{\lambda_2}}, (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^{\lambda_2} \right), \mathcal{D} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}_2}^{\mathcal{D}})^2 \right)^{\lambda_2}}, (\vartheta_{\tilde{A}_2}^{\mathcal{D}})^{\lambda_2} \right) \right\} \\ \lambda_1 \tilde{A}_1 \oplus \lambda_2 \tilde{A}_2 &= \left\{ \mathcal{F} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}_1}^{\mathcal{F}})^2 \right)^{\lambda_1}}, (\vartheta_{\tilde{A}_1}^{\mathcal{F}})^{\lambda_1} \right), \mathcal{D} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}_1}^{\mathcal{D}})^2 \right)^{\lambda_1}}, (\vartheta_{\tilde{A}_1}^{\mathcal{D}})^{\lambda_1} \right) \right\} \oplus \\ &\quad \left\{ \mathcal{F} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}_2}^{\mathcal{F}})^2 \right)^{\lambda_2}}, (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^{\lambda_2} \right), \mathcal{D} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}_2}^{\mathcal{D}})^2 \right)^{\lambda_2}}, (\vartheta_{\tilde{A}_2}^{\mathcal{D}})^{\lambda_2} \right) \right\} = \end{aligned}$$

$$\begin{aligned}
& \left\{ \mathcal{F} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}_1}^{\mathcal{F}})^2\right)^{\lambda_1} + 1 - \left(1 - (\mu_{\tilde{A}_2}^{\mathcal{F}})^2\right)^{\lambda_2} - \left(1 - \left(1 - (\mu_{\tilde{A}_1}^{\mathcal{F}})^2\right)^{\lambda_1}\right) \left(1 - \left(1 - (\mu_{\tilde{A}_2}^{\mathcal{F}})^2\right)^{\lambda_2}\right)}, \right. \\
& \left. (\vartheta_{\tilde{A}_1}^{\mathcal{F}})^{\lambda_1} (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^{\lambda_2} \right\}, \\
& \mathfrak{D} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}_1}^{\mathfrak{D}})^2\right)^{\lambda_1} + 1 - \left(1 - (\mu_{\tilde{A}_2}^{\mathfrak{D}})^2\right)^{\lambda_2} - \left(1 - \left(1 - (\mu_{\tilde{A}_1}^{\mathfrak{D}})^2\right)^{\lambda_1}\right) \left(1 - \left(1 - (\mu_{\tilde{A}_2}^{\mathfrak{D}})^2\right)^{\lambda_2}\right)}, \right. \\
& \left. (\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^{\lambda_1} (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^{\lambda_2} \right\} = \left\{ \mathcal{F} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}_1}^{\mathcal{F}})^2\right)^{\lambda_1} \left(1 - (\mu_{\tilde{A}_2}^{\mathcal{F}})^2\right)^{\lambda_2}}, (\vartheta_{\tilde{A}_1}^{\mathcal{F}})^{\lambda_1} (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^{\lambda_2} \right), \right. \\
& \left. \mathfrak{D} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}_1}^{\mathfrak{D}})^2\right)^{\lambda_1} \left(1 - (\mu_{\tilde{A}_2}^{\mathfrak{D}})^2\right)^{\lambda_2}}, (\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^{\lambda_1} (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^{\lambda_2} \right) \right\}
\end{aligned}$$

Thus, this result is true for $n = 2$. Assume that the result is true for $n = k$, Equation *DPWAM* holds, i.e.

$$\begin{aligned}
DPWAM(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_k) &= \lambda_1 \tilde{A}_1 \oplus \lambda_2 \tilde{A}_2 \oplus \dots \oplus \lambda_k \tilde{A}_k \\
&= \left\{ \mathcal{F} \left(\sqrt{1 - \prod_{i=1}^k \left(1 - (\mu_{\tilde{A}_i}^{\mathcal{F}})^2\right)^{w_i}}, \prod_{i=1}^k (\vartheta_{\tilde{A}_i}^{\mathcal{F}})^{w_i} \right), \right. \\
& \quad \left. \mathfrak{D} \left(\sqrt{1 - \prod_{i=1}^k \left(1 - (\mu_{\tilde{A}_i}^{\mathfrak{D}})^2\right)^{w_i}}, \prod_{i=1}^k (\vartheta_{\tilde{A}_i}^{\mathfrak{D}})^{w_i} \right) \right\}
\end{aligned}$$

Then, when $n = k + 1$,

$$\begin{aligned}
DPWAM(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_{k+1}) &= DPWAM(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_k) \oplus \lambda_{k+1} \tilde{A}_{k+1} = \\
& \left\{ \mathcal{F} \left(\sqrt{1 - \prod_{i=1}^k \left(1 - (\mu_{\tilde{A}_i}^{\mathcal{F}})^2\right)^{\lambda_i}}, \prod_{i=1}^k (\vartheta_{\tilde{A}_i}^{\mathcal{F}})^{\lambda_i} \right), \right. \\
& \quad \left. \mathfrak{D} \left(\sqrt{1 - \prod_{i=1}^k \left(1 - (\mu_{\tilde{A}_i}^{\mathfrak{D}})^2\right)^{\lambda_i}}, \prod_{i=1}^k (\vartheta_{\tilde{A}_i}^{\mathfrak{D}})^{\lambda_i} \right) \right\} \oplus \\
& \left\{ \mathcal{F} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}_{k+1}}^{\mathcal{F}})^2\right)^{\lambda_{k+1}}}, (\vartheta_{\tilde{A}_{k+1}}^{\mathcal{F}})^{\lambda_{k+1}} \right), \right. \\
& \quad \left. \mathfrak{D} \left(\sqrt{1 - \left(1 - (\mu_{\tilde{A}_{k+1}}^{\mathfrak{D}})^2\right)^{\lambda_{k+1}}}, (\vartheta_{\tilde{A}_{k+1}}^{\mathfrak{D}})^{\lambda_{k+1}} \right) \right\} =
\end{aligned}$$

$$\left\{ \mathcal{F} \left(\sqrt{1 - \prod_{i=1}^{k+1} \left(1 - (\mu_{\tilde{A}_i}^{\mathcal{F}})^2 \right)^{\lambda_i}}, \prod_{i=1}^{k+1} (\vartheta_{\tilde{A}_i}^{\mathcal{F}})^{\lambda_i} \right), \right. \\ \left. \mathcal{D} \left(\sqrt{1 - \prod_{i=1}^{k+1} \left(1 - (\mu_{\tilde{A}_i}^{\mathcal{D}})^2 \right)^{\lambda_i}}, \prod_{i=1}^{k+1} (\vartheta_{\tilde{A}_i}^{\mathcal{D}})^{\lambda_i} \right) \right\}$$

For $n = k + 1$, *DPWAM* operator also holds. *DPWAM* operator must be shown as a number of DPFS. As $\tilde{A}_i = \{\mathcal{F}(\mu_{\tilde{A}_i}^{\mathcal{F}}, \vartheta_{\tilde{A}_i}^{\mathcal{F}}), \mathcal{D}(\mu_{\tilde{A}_i}^{\mathcal{D}}, \vartheta_{\tilde{A}_i}^{\mathcal{D}})\}$ for all i is an DPFN, thus $0 \leq \mu_{\tilde{A}_i}^{\mathcal{F}}, \vartheta_{\tilde{A}_i}^{\mathcal{F}}, \mu_{\tilde{A}_i}^{\mathcal{D}}, \vartheta_{\tilde{A}_i}^{\mathcal{D}} \leq 1$ and $0 \leq (\mu_{\tilde{A}_i}^{\mathcal{F}})^2 + (\vartheta_{\tilde{A}_i}^{\mathcal{F}})^2 \leq 1, 0 \leq (\mu_{\tilde{A}_i}^{\mathcal{D}})^2 + (\vartheta_{\tilde{A}_i}^{\mathcal{D}})^2 \leq 1$. Thus,

$$0 \leq \prod_{i=1}^n \left(1 - (\mu_{\tilde{A}_i}^{\mathcal{F}})^2 \right)^{\lambda_i} \leq 1 \text{ and so, } 0 \leq \sqrt{1 - \prod_{i=1}^k \left(1 - (\mu_{\tilde{A}_i}^{\mathcal{F}})^2 \right)^{\lambda_i}} \leq 1 \text{ and } 0 \leq \prod_{i=1}^k (\vartheta_{\tilde{A}_i}^{\mathcal{F}})^{\lambda_i} \leq 1. \text{ Similarly, } 0 \leq \sqrt{1 - \prod_{i=1}^k \left(1 - (\mu_{\tilde{A}_i}^{\mathcal{D}})^2 \right)^{\lambda_i}} \leq 1 \text{ and } 0 \leq \prod_{i=1}^k (\vartheta_{\tilde{A}_i}^{\mathcal{D}})^{\lambda_i} \leq 1.$$

Hence, *DPWAM* is a DPFN and therefore, proof is completed.

Definition 3.6. Let $\tilde{A}_i = \{\mathcal{F}(\mu_{\tilde{A}_i}^{\mathcal{F}}, \vartheta_{\tilde{A}_i}^{\mathcal{F}}), \mathcal{D}(\mu_{\tilde{A}_i}^{\mathcal{D}}, \vartheta_{\tilde{A}_i}^{\mathcal{D}})\}$, $(i = 1, 2, \dots, n)$ be a set of DPFSs and $\lambda_i = (\lambda_1, \lambda_2, \dots, \lambda_n)$ be weight vector of $\tilde{A}_i$ with $\sum_{i=1}^n \lambda_i = 1$ and $\lambda_i \in [0, 1]$, then a Decomposed Pythagorean Weighted Geometric Mean (*DPWGM*) the operator is defined as

$$\begin{aligned} DPWGM(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) &= \lambda_1 \tilde{A}_1 \otimes \lambda_2 \tilde{A}_2 \otimes \dots \otimes \lambda_n \tilde{A}_n \\ &= \left\{ \mathcal{F} \left(\prod_{i=1}^n (\mu_{\tilde{A}_i}^{\mathcal{F}})^{\lambda_i}, \sqrt{1 - \prod_{i=1}^n \left(1 - (\vartheta_{\tilde{A}_i}^{\mathcal{F}})^2 \right)^{\lambda_i}} \right), \right. \\ &\quad \left. \mathcal{D} \left(\prod_{i=1}^n (\mu_{\tilde{A}_i}^{\mathcal{D}})^{\lambda_i}, \sqrt{1 - \prod_{i=1}^n \left(1 - (\vartheta_{\tilde{A}_i}^{\mathcal{D}})^2 \right)^{\lambda_i}} \right) \right\} \end{aligned} \quad (3.19)$$

Proof 3.6. The proof of this definition is similar to Definition 3.5, so we omit here.

3.2 Extension of CODAS Method by SV-DPFs

In this section of the study, the DPF-CODAS method is presented to rank the alternatives. The framework of the developed method is shown in the flowchart in Figure 3.2. The steps of the developed method are given as follows:

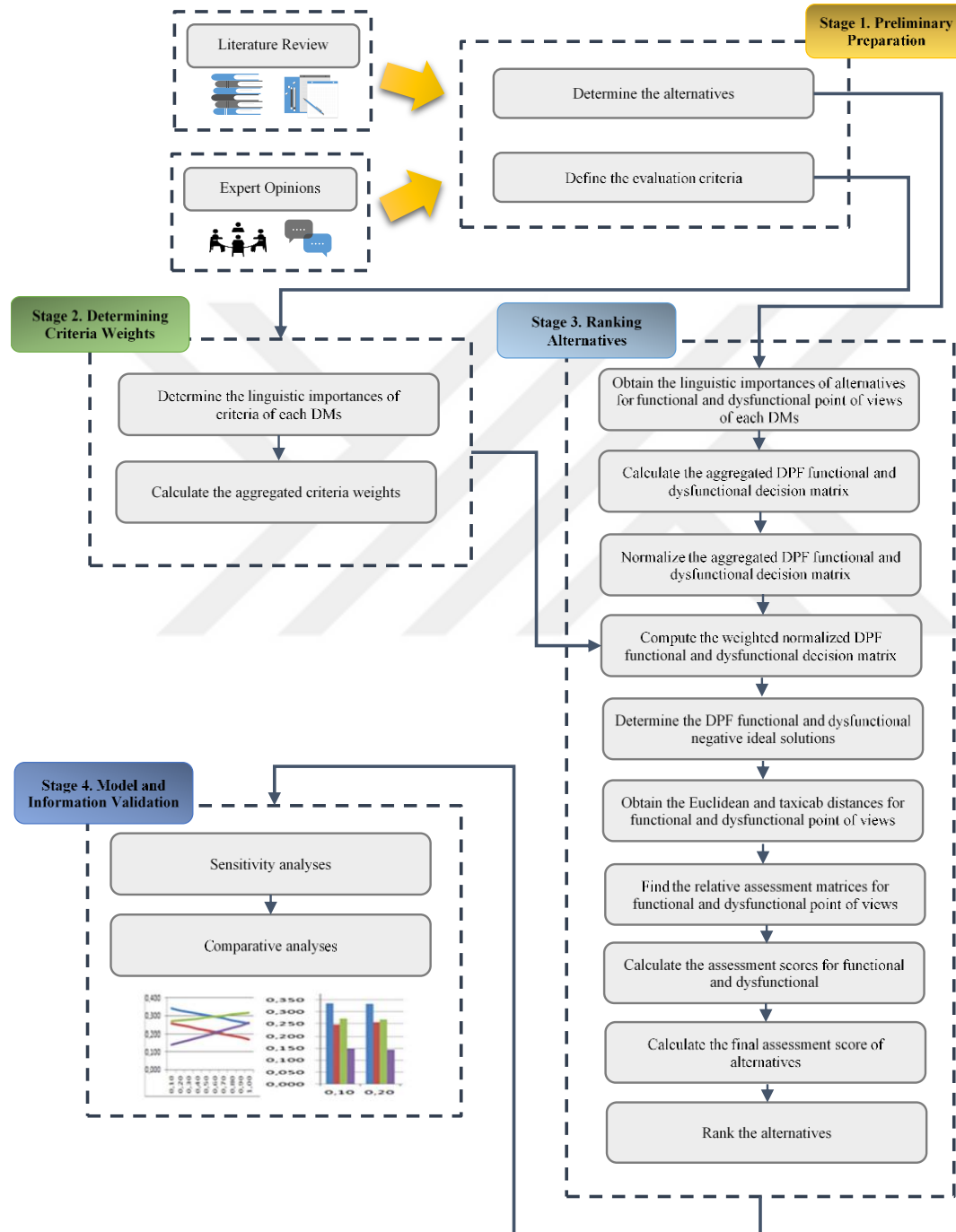


Figure 3.2 : Framework of the developed model.

Step 1. Determine the decision criteria, alternatives and the set of decision-makers (DMs) to structure the developed method. Suppose a set of alternatives is shown by $A_i = \{A_1, A_2, \dots, A_m\}$. Each alternative is evaluated based on the set of criteria denoted by $C_j = \{C_1, C_2, \dots, C_n\}$. Here, the weights of criteria are indicated with $\tilde{w}_j = (\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_n)$ where $\tilde{w}_j > 0$ and also, $i = 1, 2, \dots, m, j = 1, 2, \dots, n$. Suppose $D_1, D_2, \dots, D_K$ are set of DMs who are experts in their fields to make judgments.

Step 2. Collect the importance assessments for criteria and the decision matrices from each DM. The importance assessments and decision matrices are completed by using the linguistic terms given in Tables 3.1 and 3.2. A linguistic term for the functional question and another for the dysfunctional question are assigned in weighting criteria and constructing decision matrices. For instance, (HI, MHI) is one of the possible assignments.

Table 3.1 : DF linguistic scale for computing criteria weights.

Linguistic Terms	DPFSs for criteria	
	Functional	Dysfunctional
	(μ_C^F, ϑ_C^F)	(μ_C^D, ϑ_C^D)
Absolutely high importance (AHI)	(0.95,0.15)	(0.15,0.95)
Very high importance (VHI)	(0.85,0.25)	(0.25,0.85)
High importance (HI)	(0.75,0.35)	(0.35,0.75)
Medium high importance (MHI)	(0.65,0.45)	(0.45,0.65)
Average importance (AI)	(0.55,0.55)	(0.55,0.55)
Medium low importance (MLI)	(0.45,0.65)	(0.65,0.45)
Low importance (LI)	(0.35,0.75)	(0.75,0.35)
Very low importance (VLI)	(0.25,0.85)	(0.85,0.25)
Absolutely low importance (ALI)	(0.15,0.95)	(0.95,0.15)

Table 3.2 : DF linguistic scale for decision matrices.

Linguistic Terms	DPFSs for alternatives	
	Functional	Dysfunctional
	(μ_A^F, ϑ_A^F)	(μ_A^D, ϑ_A^D)
Absolutely high (AH)	(0.95,0.15)	(0.15,0.95)
Very high (VH)	(0.85,0.25)	(0.25,0.85)
High (H)	(0.75,0.35)	(0.35,0.75)
Medium high (MH)	(0.65,0.45)	(0.45,0.65)
Average (A)	(0.55,0.55)	(0.55,0.55)
Medium low (ML)	(0.45,0.65)	(0.65,0.45)
Low (L)	(0.35,0.75)	(0.75,0.35)
Very low (VL)	(0.25,0.85)	(0.85,0.25)
Absolutely low (AL)	(0.15,0.95)	(0.95,0.15)

After the linguistic variables determined for both criteria and alternatives are converted to DPFS values, DPF criteria weight matrix $\tilde{\Phi}_k = (\tilde{\varphi}_{jk})_{n \times 1}$ and DPF decision matrix $\tilde{\Psi}_k = (\tilde{\psi}_{ijk})_{n \times m}$ based on k^{th} DM is constructed as presented in Tables 3.3 and 3.4.

Table 3.3 : DPF weights vector for criteria from each DM, $k = 1, 2, \dots, K$.

Criteria	Weights
C_1	$\langle (\mu_{C_1k}^{\mathcal{F}}, \vartheta_{C_1k}^{\mathcal{F}}), (\mu_{C_1k}^{\mathcal{D}}, \vartheta_{C_1k}^{\mathcal{D}}) \rangle$
C_2	$\langle (\mu_{C_2k}^{\mathcal{F}}, \vartheta_{C_2k}^{\mathcal{F}}), (\mu_{C_2k}^{\mathcal{D}}, \vartheta_{C_2k}^{\mathcal{D}}) \rangle$
$\vdots$	$\vdots$
C_n	$\langle (\mu_{C_nk}^{\mathcal{F}}, \vartheta_{C_nk}^{\mathcal{F}}), (\mu_{C_nk}^{\mathcal{D}}, \vartheta_{C_nk}^{\mathcal{D}}) \rangle$

Table 3.4 : DPF decision matrix for alternatives from each DM, $k = 1, 2, \dots, K$.

Criteria	Alternatives				
	C_1	C_2	...	C_n	
A_1	$\langle (\mu_{A_11k}^{\mathcal{F}}, \vartheta_{A_11k}^{\mathcal{F}}), (\mu_{A_11k}^{\mathcal{D}}, \vartheta_{A_11k}^{\mathcal{D}}) \rangle$	$\langle (\mu_{A_12k}^{\mathcal{F}}, \vartheta_{A_12k}^{\mathcal{F}}), (\mu_{A_12k}^{\mathcal{D}}, \vartheta_{A_12k}^{\mathcal{D}}) \rangle$	...	$\langle (\mu_{A_1nk}^{\mathcal{F}}, \vartheta_{A_1nk}^{\mathcal{F}}), (\mu_{A_1nk}^{\mathcal{D}}, \vartheta_{A_1nk}^{\mathcal{D}}) \rangle$	
A_2	$\langle (\mu_{A_21k}^{\mathcal{F}}, \vartheta_{A_21k}^{\mathcal{F}}), (\mu_{A_21k}^{\mathcal{D}}, \vartheta_{A_21k}^{\mathcal{D}}) \rangle$	$\langle (\mu_{A_22k}^{\mathcal{F}}, \vartheta_{A_22k}^{\mathcal{F}}), (\mu_{A_22k}^{\mathcal{D}}, \vartheta_{A_22k}^{\mathcal{D}}) \rangle$	...	$\langle (\mu_{A_2nk}^{\mathcal{F}}, \vartheta_{A_2nk}^{\mathcal{F}}), (\mu_{A_2nk}^{\mathcal{D}}, \vartheta_{A_2nk}^{\mathcal{D}}) \rangle$	
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	
A_m	$\langle (\mu_{A_m1k}^{\mathcal{F}}, \vartheta_{A_m1k}^{\mathcal{F}}), (\mu_{A_m1k}^{\mathcal{D}}, \vartheta_{A_m1k}^{\mathcal{D}}) \rangle$	$\langle (\mu_{A_m2k}^{\mathcal{F}}, \vartheta_{A_m2k}^{\mathcal{F}}), (\mu_{A_m2k}^{\mathcal{D}}, \vartheta_{A_m2k}^{\mathcal{D}}) \rangle$	...	$\langle (\mu_{A_mnk}^{\mathcal{F}}, \vartheta_{A_mnk}^{\mathcal{F}}), (\mu_{A_mnk}^{\mathcal{D}}, \vartheta_{A_mnk}^{\mathcal{D}}) \rangle$	

Step 3. Obtain the aggregated weights for criteria by using DPFWG operator given in Eq. (3.19). The individual criteria weights matrices of DMs are aggregated in a single criteria weights matrix. Aggregated criteria weights matrix $\tilde{\Phi} = (\tilde{\varphi}_j)_{n \times 1}$ is constructed as given in Table 3.5. Here, $\tilde{\Phi} = (\tilde{\varphi}_j)_{n \times 1}$ in which $\tilde{\varphi}_j = \langle (\mu_{C_n}^{\mathcal{F}}, \vartheta_{C_n}^{\mathcal{F}}), (\mu_{C_n}^{\mathcal{D}}, \vartheta_{C_n}^{\mathcal{D}}) \rangle$ shows the aggregated DPF numbers for criteria according to j^{th} criterion.

Table 3.5 : DPF aggregated criteria weights vector.

Criteria	Weights
C_1	$\langle (\mu_{C_1}^{\mathcal{F}}, \vartheta_{C_1}^{\mathcal{F}}), (\mu_{C_1}^{\mathcal{D}}, \vartheta_{C_1}^{\mathcal{D}}) \rangle$
C_2	$\langle (\mu_{C_2}^{\mathcal{F}}, \vartheta_{C_2}^{\mathcal{F}}), (\mu_{C_2}^{\mathcal{D}}, \vartheta_{C_2}^{\mathcal{D}}) \rangle$
$\vdots$	$\vdots$
C_n	$\langle (\mu_{C_n}^{\mathcal{F}}, \vartheta_{C_n}^{\mathcal{F}}), (\mu_{C_n}^{\mathcal{D}}, \vartheta_{C_n}^{\mathcal{D}}) \rangle$

Step 4. Obtain the aggregated decision matrix $\tilde{\Psi} = (\tilde{\psi}_{ij})_{n \times m}$ by using DPFWG operator given in Eq. (3.19). The individual decision matrices of DMs are aggregated in a single decision matrix as given in Table 3.6. Here, $\tilde{\Psi} = (\tilde{\psi}_{ij})_{n \times m}$ in which $\tilde{\psi}_{ij} =$

$\langle(\mu_{A_{mn}}^{\mathcal{F}}, \vartheta_{A_{mn}}^{\mathcal{F}}), (\mu_{A_{mn}}^{\mathcal{D}}, \vartheta_{A_{mn}}^{\mathcal{D}})\rangle$ indicates the aggregated DPF numbers of i^{th} alternative according to j^{th} criterion.

Table 3.6 : DPF aggregated decision matrix.

Criteria	Alternatives			
	C_1	C_2	...	C_n
A_1	$\langle(\mu_{A_{11}}^{\mathcal{F}}, \vartheta_{A_{11}}^{\mathcal{F}}), (\mu_{A_{11}}^{\mathcal{D}}, \vartheta_{A_{11}}^{\mathcal{D}})\rangle$	$\langle(\mu_{A_{12}}^{\mathcal{F}}, \vartheta_{A_{12}}^{\mathcal{F}}), (\mu_{A_{12}}^{\mathcal{D}}, \vartheta_{A_{12}}^{\mathcal{D}})\rangle$	...	$\langle(\mu_{A_{1n}}^{\mathcal{F}}, \vartheta_{A_{1n}}^{\mathcal{F}}), (\mu_{A_{1n}}^{\mathcal{D}}, \vartheta_{A_{1n}}^{\mathcal{D}})\rangle$
A_2	$\langle(\mu_{A_{21}}^{\mathcal{F}}, \vartheta_{A_{21}}^{\mathcal{F}}), (\mu_{A_{21}}^{\mathcal{D}}, \vartheta_{A_{21}}^{\mathcal{D}})\rangle$	$\langle(\mu_{A_{22}}^{\mathcal{F}}, \vartheta_{A_{22}}^{\mathcal{F}}), (\mu_{A_{22}}^{\mathcal{D}}, \vartheta_{A_{22}}^{\mathcal{D}})\rangle$	...	$\langle(\mu_{A_{2n}}^{\mathcal{F}}, \vartheta_{A_{2n}}^{\mathcal{F}}), (\mu_{A_{2n}}^{\mathcal{D}}, \vartheta_{A_{2n}}^{\mathcal{D}})\rangle$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
A_m	$\langle(\mu_{A_{m1}}^{\mathcal{F}}, \vartheta_{A_{m1}}^{\mathcal{F}}), (\mu_{A_{m1}}^{\mathcal{D}}, \vartheta_{A_{m1}}^{\mathcal{D}})\rangle$	$\langle(\mu_{A_{m2}}^{\mathcal{F}}, \vartheta_{A_{m2}}^{\mathcal{F}}), (\mu_{A_{m2}}^{\mathcal{D}}, \vartheta_{A_{m2}}^{\mathcal{D}})\rangle$	...	$\langle(\mu_{A_{mn}}^{\mathcal{F}}, \vartheta_{A_{mn}}^{\mathcal{F}}), (\mu_{A_{mn}}^{\mathcal{D}}, \vartheta_{A_{mn}}^{\mathcal{D}})\rangle$

Step 5. Obtain the normalized aggregated decision matrix $\tilde{\Psi}^N = (\tilde{\psi}_{ij}^N)_{n \times m}$ by replacing the membership and non-membership degrees in each of the functional and dysfunctional values for cost criteria. It is not necessary to make this replacement for benefit criteria. The normalized values for decision matrix are calculated as given in Eq. (3.20).

$$\tilde{\Psi}^N = (\tilde{\psi}_{ij}^N) = \begin{cases} \langle(\mu_{A_{mn}}^{\mathcal{F}}, \vartheta_{A_{mn}}^{\mathcal{F}}), (\mu_{A_{mn}}^{\mathcal{D}}, \vartheta_{A_{mn}}^{\mathcal{D}})\rangle, & \text{for benefit criteria} \\ \langle(\mu_{A_{mn}}^{\mathcal{D}}, \vartheta_{A_{mn}}^{\mathcal{D}}), (\mu_{A_{mn}}^{\mathcal{F}}, \vartheta_{A_{mn}}^{\mathcal{F}})\rangle, & \text{for cost criteria} \end{cases} \quad (3.20)$$

Step 6. Calculate the weighted normalized aggregated decision matrix $\tilde{\Psi}_w^N = (\tilde{\psi}_{w,ij}^N)_{n \times m}$ by Eq. (3.21).

$$\tilde{\Psi}_w^N = (\tilde{\psi}_{w,ij}^N) = \tilde{w} \otimes \tilde{\Psi}^N \quad (3.21)$$

where $\tilde{\psi}_{w,ij}^N = \langle(\mu_{A_{mn,w}}^{\mathcal{F}}, \vartheta_{A_{mn,w}}^{\mathcal{F}}), (\mu_{A_{mn,w}}^{\mathcal{D}}, \vartheta_{A_{mn,w}}^{\mathcal{D}})\rangle$ shows the weighted normalized DPF decision matrix of i^{th} alternative with respect to j^{th} criterion.

Step 7. Determine the DPF functional negative ideal solution $(DPF\_NIS)_{\mathcal{F}}$ and DPF dysfunctional negative ideal solution $(DPF\_NIS)_{\mathcal{D}}$ based on the weighted normalized aggregated decision matrix by Eqs. (3.22) and (3.23).

$$(DPF\_NIS)_{\mathcal{F}} = \tilde{\delta}_j^{\mathcal{F}} = \min_i (\tilde{\psi}_{w,ij}^N)^{\mathcal{F}} \quad (3.22)$$

$$(DPF\_NIS)_{\mathcal{D}} = \tilde{\delta}_j^{\mathcal{D}} = \min_i (\tilde{\psi}_{w,ij}^N)^{\mathcal{D}} \quad (3.23)$$

where $\tilde{\delta}_j^{\mathcal{F}} = (\mu_{A_j,w}^{\mathcal{F}^-}, \vartheta_{A_j,w}^{\mathcal{F}^-})$ and $\tilde{\delta}_j^{\mathcal{D}} = (\mu_{A_j,w}^{\mathcal{D}^-}, \vartheta_{A_j,w}^{\mathcal{D}^-})$ are the minimum weighted normalized aggregated DPF functional and dysfunctional numbers with the lowest score value among alternatives for j^{th} criterion.

Step 8. Compute the Euclidean and taxicab distances of the alternatives to the DPF_NISs for functional and dysfunctional by using Eqs. (3.24)-(3.27), respectively.

$$\chi_i^{E^{\mathcal{F}}} = \sqrt{\frac{1}{2n} \sum_{j=1}^n \left(\left((\mu_{A_{ij},w}^{\mathcal{F}})^2 - (\mu_{A_j,w}^{\mathcal{F}^-})^2 \right)^2 + \left((\vartheta_{A_{ij},w}^{\mathcal{F}})^2 - (\vartheta_{A_j,w}^{\mathcal{F}^-})^2 \right)^2 + \left((\pi_{A_{ij},w}^{\mathcal{O}})^2 - (\pi_{A_j,w}^{\mathcal{O}^-})^2 \right)^2 \right)} \quad (3.24)$$

$$\chi_i^{E^{\mathcal{D}}} = \sqrt{\frac{1}{2n} \sum_{j=1}^n \left(\left((\mu_{A_{ij},w}^{\mathcal{D}})^2 - (\mu_{A_j,w}^{\mathcal{D}^-})^2 \right)^2 + \left((\vartheta_{A_{ij},w}^{\mathcal{D}})^2 - (\vartheta_{A_j,w}^{\mathcal{D}^-})^2 \right)^2 + \left((\pi_{A_{ij},w}^{\mathcal{P}})^2 - (\pi_{A_j,w}^{\mathcal{P}^-})^2 \right)^2 \right)} \quad (3.25)$$

$$\chi_i^{T^{\mathcal{F}}} = \sqrt{\frac{1}{2n} \sum_{j=1}^n \left(\left| (\mu_{A_{ij},w}^{\mathcal{F}})^2 - (\mu_{A_j,w}^{\mathcal{F}^-})^2 \right| + \left| (\vartheta_{A_{ij},w}^{\mathcal{F}})^2 - (\vartheta_{A_j,w}^{\mathcal{F}^-})^2 \right| + \left| (\pi_{A_{ij},w}^{\mathcal{O}})^2 - (\pi_{A_j,w}^{\mathcal{O}^-})^2 \right| \right)} \quad (3.26)$$

$$\chi_i^{T^{\mathcal{D}}} = \sqrt{\frac{1}{2n} \sum_{j=1}^n \left(\left| (\mu_{A_{ij},w}^{\mathcal{D}})^2 - (\mu_{A_j,w}^{\mathcal{D}^-})^2 \right| + \left| (\vartheta_{A_{ij},w}^{\mathcal{D}})^2 - (\vartheta_{A_j,w}^{\mathcal{D}^-})^2 \right| + \left| (\pi_{A_{ij},w}^{\mathcal{P}})^2 - (\pi_{A_j,w}^{\mathcal{P}^-})^2 \right| \right)} \quad (3.27)$$

Step 9. Determine the relative assessment matrices $\Xi^{\mathcal{F}} = (\xi_{ih}^{\mathcal{F}})_{m \times m}$ and $\Xi^{\mathcal{D}} = (\xi_{ih}^{\mathcal{D}})_{m \times m}$ by using Eqs. (3.28) and (3.29), respectively.

$$\xi_{ih}^{\mathcal{F}} = (\chi_i^{E^{\mathcal{F}}} - \chi_h^{E^{\mathcal{F}}}) + \left(\tau \left(\chi_i^{E^{\mathcal{F}}} - \chi_h^{E^{\mathcal{F}}} \right) \times \left(\chi_i^{T^{\mathcal{F}}} - \chi_h^{T^{\mathcal{F}}} \right) \right) \quad (3.28)$$

$$\xi_{ih}^{\mathcal{D}} = (\chi_i^{E^{\mathcal{D}}} - \chi_h^{E^{\mathcal{D}}}) + \left(\tau \left(\chi_i^{E^{\mathcal{D}}} - \chi_h^{E^{\mathcal{D}}} \right) \times \left(\chi_i^{T^{\mathcal{D}}} - \chi_h^{T^{\mathcal{D}}} \right) \right) \quad (3.29)$$

where $h = \{1, 2, \dots, n\}$. τ is a threshold function that is given in Eq. (3.30) and θ is a threshold parameter of this function which can be set by DMs.

$$\tau = \begin{cases} 1, & \text{if } |x| \geq \theta \\ 0, & \text{if } |x| < \theta \end{cases} \quad (3.30)$$

Step 10. Calculate the assessment scores of each alternative for functional point of view and dysfunctional point of view by using Eqs. (3.31) and (3.32).

$$\Omega_i^{\mathcal{F}} = \sum_{h=1}^m \xi_{ih}^{\mathcal{F}} \quad (3.31)$$

$$\Omega_i^{\mathcal{D}} = \sum_{h=1}^m \xi_{ih}^{\mathcal{D}} \quad (3.32)$$

Step 11. Obtain the final assessment score of each alternative by using Eq. (3.33).

$$\kappa_i = \frac{1}{2}(\Omega_i^{\mathcal{F}} + \Omega_i^{\mathcal{D}}) \quad (3.33)$$

Step 12. Rank the alternatives according to the decreasing values of final assessment scores (κ_i). The alternative with the highest κ_i is the best alternative.

A pseudo algorithm of the above-proposed method is presented in the following.

Algorithm: Pseudo Representation of DPF-CODAS Method

Input: n: number of criteria ($j = 1, 2, \dots, n$),
m: number of alternatives ($i = 1, 2, \dots, m$),
k: number of decision-makers ($k = 1, 2, \dots, K$),
 $\tilde{\psi}_{ijk}$ = evaluation of the m^{th} alternative with respect to the n^{th} criterion by k^{th} decision maker

Output: Rankings of alternatives

Begin:

For $k=1$ to K **do:**

Step 1: Assign the linguistic importance assessments for criteria based on functional and dysfunctional point of views (Based on Table 3.1)

Step 2: Convert linguistic terms to their corresponded DPF numbers for functional and dysfunctional point of views (Based on Table 3.1)

Step 3: Aggregate the decision makers for criteria weights based on functional and dysfunctional point of views by using DPFWG operator

End for

For $k=1$ to K **do:**

Step 4: Construct the linguistic decision matrix for functional and dysfunctional point of views (Based on Table 3.2)

Step 5: Convert linguistic evaluations to their corresponded DPF numbers for decision matrix based on functional and dysfunctional points of views (Based on Table 3.2)

Step 6: Aggregate the DPF functional and dysfunctional decision matrices by using DPFWG operator

End for

Step 7: Normalize the aggregated DPF functional and dysfunctional decision matrix (by using Eq. (3.20))

Step 8: Calculate the weighted normalized aggregated DPF functional and dysfunctional decision matrix (by using Eq. (3.21))

Step 9: Determine the DPF functional and dysfunctional negative ideal solutions (by using Eqs. (3.22) and (3.23))

Step 10: Calculate the Euclidean and taxicab distances of the alternatives to the DPF negative ideal solutions for functional and dysfunctional point of views (by using Eqs. (3.24)-(3.25))

Step 11: Determine the relative assessment matrices for functional and dysfunctional point of views (by using Eqs. (3.28) and (3.29))

Step 12: Compute the assessment scores of each alternative for functional point of view and dysfunctional point of view	(by using Eqs. (3.31) and (3.32))
Step 13: Find the final assessment score of each alternative	(by using Eq. (3.33))
Step 14: Rank the alternatives	
End	

3.3 Application

In this section, the developed method is applied to a strategy selection problem for IoT-based sustainable supply chain systems. First, the criteria and alternatives of the problem are presented in Section 3.3.1. Then, the solution of the problem is demonstrated in Section 3.3.2. Finally, the validation of results and sensitivity analysis are performed.

3.3.1 Criteria and alternatives

Today, environmental and economic problems that arise with rapid population growth, as well as poor governance and city planning problems, have caused the government to give the priority to them. In particular, the interest and importance of sustainable development goals has led to them being among the top issues addressed by many governments around the world. Along with the interest and importance in sustainability, governments have recently placed a strong emphasis on digital technologies to ensure sustainability in various fields. The use of IoT technologies, especially in order to ensure sustainability, has had great importance among digital technologies. These technologies have been recently used, preferred and developed by governments in various fields. With this, IoT technology has an importance that is frequently preferred by managers in the field of supply chain. IoT technology has had an important place in ensuring sustainability in processes and operations related to the supply chain, such as detecting anomalies in the supply chain system, optimizing the route on the supply chain network, and planning the supply chain system. However, the application of IoT to a sustainable supply chain has brought with it many challenges. With the increasing demand for IoT, more complex supply chain problems have started to emerge. This situation has also necessitated the development and implementation of new strategies in the development and implementation of the IoT-based sustainable supply chain. In order to choose the most appropriate strategy among these strategies, which may vary according to each sector, many factors should be considered together in line with the needs of each sector. It is critical to effectively

determine which factors will be taken into account, which will be excluded, or to what extent these factors will affect the decision to be taken, among the factors determined during the decision-making process. Reaching an appropriate and right conclusion by evaluating many conflicting factors requires the use of appropriate methods or techniques. Therefore, using multi-criteria decision-making methods to evaluate many conflicting factors together will enable a more appropriate and correct strategy to be determined. Besides, determining an appropriate strategy brings with it uncertainties in the data in the decision-making process. Therefore, in this study, the application of the newly developed decomposed fuzzy CODAS method, which has the ability to take into account imprecise and uncertain evaluations in the selection of an IoT-based sustainable supply chain strategy, is presented.

Five alternative strategies have been determined to select the most appropriate IoT-based sustainable strategy for a supply chain company operating in the manufacturing sector and 9 criteria have been identified to evaluate these alternatives. The hierarchical structure of the determined criteria and alternatives is presented in Figure 3.3. In addition, brief explanations of criteria and alternatives are as follows.

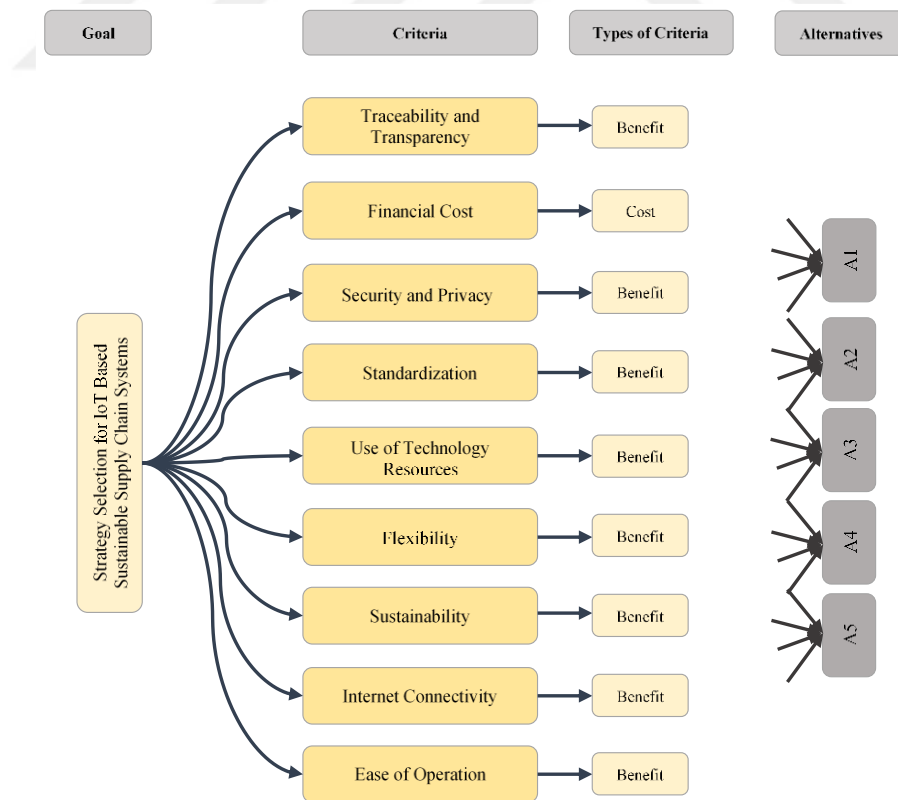


Figure 3.3 : Hierarchical structure of the problem.

Traceability and Transparency (C1): It means transparency in the operations and concepts of smart technologies and traceability into the identity, location and status of each node in the supply chain pipeline. With supply chain transparency and traceability, it contributes to streamlining transactions and maximizing sustainability in the supply chain (Torkayesh et al., 2022; Vimal et al., 2022).

Financial Cost (C2): It covers the costs including operations such as the implementation of smart technologies, training given to the employees, operation and maintenance. IoT technology chosen for supply chain management should provide cost-effective and affordable services (Krishankumar & Ecer, 2023; Seker, 2022; Torkayesh et al., 2022; Güneri & Deveci, 2023; Baranwal et al., 2020).

Security and Privacy (C3): Security and privacy issues may occur in communication between nodes in the supply chain. Cybersecurity vulnerabilities and data leaks should be kept to a minimum (Krishankumar & Ecer, 2023; Seker, 2022; Torkayesh et al., 2022; Vimal et al., 2022; Baranwal et al., 2020).

Standardization (C4): For the successful implementation of IoT, standards such as communication and privacy need to be set and implemented. It guarantees seamless integration between data and stakeholders (Seker, 2022; Li et al., 2015).

Use of Technology Resources (C5): Supporting the IoT-based supply chain system with cutting-edge technologies plays a critical role in achieving an optimized supply chain system along with the reduction in transportation costs and thus developing a better supply chain system (Vimal et al., 2022; Güneri & Deveci, 2023; Baranwal et al., 2020).

Flexibility (C6): With the IoT-based supply chain system, unexpected situations such as weather, various delays, possible accidents can be predicted and planned. Based on goods movements, managers can create and develop flexible contingency plans (Vimal et al., 2022).

Sustainability (C7): Some sustainable aspects such as economic, social and environmental aspects must be met by IoT-based supply chain systems. Therefore, the IoT technology to be considered in supply chain systems should include the cloud system, mobile application, appropriate sensors, GPS and ensure economic, social and environmental sustainability (Seker, 2022; Güneri & Deveci, 2023).

Internet Connectivity (C8): IoT adoption is hampered by electricity problems and insufficient internet access. it is therefore essential that all SC partners have an internet connection available (Seker, 2022; Torkayesh et al., 2022).

Ease of Operation (C9): Ease of operation has an important place for the sustainable and reliable operation of IoT technology in the supply chain system, as the high operational complexity can cause long downtimes, longer delivery times and increase in maintenance costs due to the increase in errors caused by human errors (Seker, 2022).

Dynamic Warehousing (A1): With dynamic warehousing, it is ensured that the inventory model and supply chain units are presented completely and accurately at any time for collaborating users. While traditional warehouses lag behind in keeping up with rapidly and volatile changing situations and conditions, dynamic storage offers instant and integrated information.

Mobile Application for Delivery Time Forecasting (A2): It provides an intelligent application with intelligent tracking and efficient distribution based on where the goods of all partners in the supply chain system are and when they will be delivered. It is compatible with IOS and Android mobile operating systems. With the use of the smart application by all supply chain partners, SMS notifications are sent their supply chain partners.

Radio Frequency Identification (RFID), geographic information system (GIS), and General Packet Radio Service (GPRS) Integration of Supply Chain System (A3): It enables the use of RFID, Global Positioning System (GPS), Global System for Mobile Communications (GSM) and GIS technologies as a web-based solution for real-time monitoring and management of the supply chain system. Thanks to the sensors placed on the vehicles, instant monitoring and tracking of the vehicles are reported. Depending on the condition of the vehicles on the road, an optimized road option is offered, saving fuel costs and preventing possible delays in delivery time.

Decentralization of Decision Making (A4): Decentralized decision-making is ensured thanks to the localized decision-making system based on big data from IoT modules. This makes it easier for all partners in the supply chain system to work in coordination on efficient actions. Teams are encouraged to make operational or strategic decisions without managerial approval. This distribution of decision-making gives team

members ownership of the decision-making process, which can increase engagement and productivity.

Smart Automation (A5): It performs all the tasks in the supply chain system without any human intervention. It enables the agility of the supply chain system to be further improved, helping teams adapt faster. It also brings many opportunities such as the development of improved delivery and distribution processes and the elimination of transportation costs

3.3.2 Solution using SV-DPF CODAS

In this section, the application steps of the developed DPF-CODAS method are presented for the evaluation of IoT based sustainable supply chain systems.

Step 1. For the solution of the considered problem, the five alternatives and nine criteria given in Section 3.3.1 have been determined based on literature review and expert opinions. Besides, a set of decision-makers $D_k (k = 1, 2, 3)$ is responsible for the evaluation of the determined alternatives and criteria. The weights of DMs are assigned as $D_k = \{0.35, 0.40, 0.25\}$ according to their knowledge and expertise in the selected problem.

Step 2. The linguistic evaluation results based on the DMs' judgments for the criteria and alternatives are given in Tables 3.7 and 3.8. Using Tables 3.1 and 3.2, the linguistic assessments presented in Tables 3.7 and 3.8 are converted to the corresponding DPF numbers as presented in Tables 3.9 and 3.10, respectively.

Table 3.7 : Linguistic criteria assessments of the decision-makers.

	DM1	DM2	DM3
C1	(HI, VHI)	(AHI, VHI)	(VHI, HI)
C2	(MHI, MHI)	(AI, MLI)	(HI, MHI)
C3	(AHI, VHI)	(VHI, HI)	(VHI, HI)
C4	(MLI, AI)	(LI, MLI)	(MHI, AI)
C5	(AHI, VHI)	(HI, VHI)	(HI, MHI)
C6	(VHI, HI)	(MHI, MHI)	(AI, MHI)
C7	(MHI, AI)	(HI, MHI)	(MHI, AI)
C8	(AI, MLI)	(MHI, AI)	(AI, MHI)
C9	(HI, MHI)	(MHI, MHI)	(MHI, HI)

Table 3.8 : Decision matrices from DMs.

DM1									
	C1	C2	C3	C4	C5	C6	C7	C8	C9
A1	(VH,AH)	(H,H)	(MH,H)	(H,MH)	(AH,AH)	(ML,A)	(VH,H)	(H,H)	(MH,H)
A2	(H,VH)	(A,MH)	(H,H)	(A,A)	(ML,ML)	(VL,L)	(H,MH)	(VH,VH)	(VH,VH)
A3	(AH,AH)	(VH,H)	(H,H)	(VH,VH)	(AH,AH)	(H,VH)	(VH,H)	(H,MH)	(AH,AH)
A4	(A,ML)	(VL,VL)	(VH,AH)	(VL,L)	(H,MH)	(MH,MH)	(A,ML)	(VL,VL)	(A,ML)
A5	(VL,VL)	(H,VH)	(H,MH)	(VH,H)	(VL,VL)	(L,VL)	(H,H)	(VH,H)	(VH,H)

DM2									
	C1	C2	C3	C4	C5	C6	C7	C8	C9
A1	(H, MH)	(VH, VH)	(H, H)	(MH, H)	(H, VH)	(MH, MH)	(H, H)	(MH, H)	(VH, H)
A2	(H, H)	(MH, A)	(MH, H)	(ML, A)	(A, MH)	(L, VL)	(MH, H)	(AH, VH)	(H, VH)
A3	(VH, H)	(ML, A)	(VL, L)	(VH, H)	(AH, VH)	(A, MH)	(H, H)	(MH, H)	(AH, VH)
A4	(MH, H)	(L, ML)	(MH, H)	(VH, VH)	(L, VL)	(H, VH)	(L, ML)	(VL, L)	(VL, L)
A5	(A, MH)	(VH, H)	(VH, VH)	(L, ML)	(VL, AL)	(VL, VL)	(H, MH)	(H, H)	(MH, H)

DM3									
	C1	C2	C3	C4	C5	C6	C7	C8	C9
A1	(AH, VH)	(VH, H)	(H, VH)	(VH, H)	(VH, AH)	(A, A)	(H, VH)	(MH, MH)	(H, MH)
A2	(H, H)	(MH, A)	(H, H)	(A, ML)	(A, ML)	(L, VL)	(MH, H)	(VH, VH)	(H, H)
A3	(AH, VH)	(VL, VL)	(H, MH)	(H, H)	(VH, AH)	(MH, H)	(H, VH)	(H, MH)	(AH, VH)
A4	(VL, VL)	(L, VL)	(L, L)	(VH, VH)	(A, ML)	(H, MH)	(ML, A)	(L, VL)	(ML, A)
A5	(ML, L)	(VH, H)	(MH, A)	(AL, VL)	(L, VL)	(AL, VL)	(VH, H)	(VH, VH)	(H, MH)

Table 3.9 : DF numbers of the linguistic assessments of the criteria.

	DM1	DM2	DM3
C1	$\langle(0.75,0.35),(0.85,0.25)\rangle$	$\langle(0.95,0.15),(0.85,0.25)\rangle$	$\langle(0.85,0.25),(0.75,0.35)\rangle$
C2	$\langle(0.65,0.45),(0.65,0.45)\rangle$	$\langle(0.55,0.55),(0.45,0.65)\rangle$	$\langle(0.75,0.35),(0.65,0.45)\rangle$
C3	$\langle(0.95,0.15),(0.85,0.25)\rangle$	$\langle(0.85,0.25),(0.75,0.35)\rangle$	$\langle(0.85,0.25),(0.75,0.35)\rangle$
C4	$\langle(0.45,0.65),(0.55,0.55)\rangle$	$\langle(0.35,0.75),(0.45,0.65)\rangle$	$\langle(0.65,0.45),(0.55,0.55)\rangle$
C5	$\langle(0.95,0.15),(0.85,0.25)\rangle$	$\langle(0.75,0.35),(0.85,0.25)\rangle$	$\langle(0.75,0.35),(0.65,0.45)\rangle$
C6	$\langle(0.85,0.25),(0.75,0.35)\rangle$	$\langle(0.65,0.45),(0.65,0.45)\rangle$	$\langle(0.55,0.55),(0.65,0.45)\rangle$
C7	$\langle(0.65,0.45),(0.55,0.55)\rangle$	$\langle(0.75,0.35),(0.65,0.45)\rangle$	$\langle(0.65,0.45),(0.55,0.55)\rangle$
C8	$\langle(0.55,0.55),(0.45,0.65)\rangle$	$\langle(0.65,0.45),(0.55,0.55)\rangle$	$\langle(0.55,0.55),(0.65,0.45)\rangle$
C9	$\langle(0.75,0.35),(0.65,0.45)\rangle$	$\langle(0.65,0.45),(0.65,0.45)\rangle$	$\langle(0.65,0.45),(0.75,0.35)\rangle$

Table 3.10 : DF decision matrices for each decision maker.

DM1									
	C1	C2	C3	C4	C5	C6	C7	C8	C9
A1	$\langle(0.85,0.25),(0.95,0.15)\rangle$	$\langle(0.75,0.35),(0.75,0.35)\rangle$	$\langle(0.65,0.45),(0.75,0.35)\rangle$	$\langle(0.75,0.35),(0.65,0.45)\rangle$	$\langle(0.95,0.15),(0.95,0.15)\rangle$	$\langle(0.45,0.65),(0.55,0.55)\rangle$	$\langle(0.85,0.25),(0.75,0.35)\rangle$	$\langle(0.75,0.35),(0.75,0.35)\rangle$	$\langle(0.65,0.45),(0.75,0.35)\rangle$
A2	$\langle(0.75,0.35),(0.85,0.25)\rangle$	$\langle(0.55,0.55),(0.65,0.45)\rangle$	$\langle(0.75,0.35),(0.75,0.35)\rangle$	$\langle(0.55,0.55),(0.55,0.55)\rangle$	$\langle(0.45,0.65),(0.45,0.65)\rangle$	$\langle(0.25,0.85),(0.35,0.75)\rangle$	$\langle(0.75,0.35),(0.65,0.45)\rangle$	$\langle(0.85,0.25),(0.85,0.25)\rangle$	$\langle(0.85,0.25),(0.85,0.25)\rangle$
A3	$\langle(0.95,0.15),(0.95,0.15)\rangle$	$\langle(0.85,0.25),(0.75,0.35)\rangle$	$\langle(0.75,0.35),(0.75,0.35)\rangle$	$\langle(0.85,0.25),(0.85,0.25)\rangle$	$\langle(0.95,0.15),(0.95,0.15)\rangle$	$\langle(0.75,0.35),(0.85,0.25)\rangle$	$\langle(0.85,0.25),(0.75,0.35)\rangle$	$\langle(0.75,0.35),(0.65,0.45)\rangle$	$\langle(0.95,0.15),(0.95,0.15)\rangle$
A4	$\langle(0.55,0.55),(0.45,0.65)\rangle$	$\langle(0.25,0.85),(0.25,0.85)\rangle$	$\langle(0.85,0.25),(0.95,0.15)\rangle$	$\langle(0.25,0.85),(0.35,0.75)\rangle$	$\langle(0.75,0.35),(0.65,0.45)\rangle$	$\langle(0.65,0.45),(0.65,0.45)\rangle$	$\langle(0.55,0.55),(0.45,0.65)\rangle$	$\langle(0.25,0.85),(0.25,0.85)\rangle$	$\langle(0.55,0.55),(0.45,0.65)\rangle$
A5	$\langle(0.25,0.85),(0.25,0.85)\rangle$	$\langle(0.75,0.35),(0.85,0.25)\rangle$	$\langle(0.75,0.35),(0.65,0.45)\rangle$	$\langle(0.85,0.25),(0.75,0.35)\rangle$	$\langle(0.25,0.85),(0.25,0.85)\rangle$	$\langle(0.35,0.75),(0.25,0.85)\rangle$	$\langle(0.75,0.35),(0.75,0.35)\rangle$	$\langle(0.85,0.25),(0.75,0.35)\rangle$	$\langle(0.85,0.25),(0.75,0.35)\rangle$

Table 3.10 (continued): DF decision matrices for each decision maker.

DM2									
	C1	C2	C3	C4	C5	C6	C7	C8	C9
A1	<(0.75,0.35), (0.65,0.45)>	<(0.85,0.25), (0.85,0.25)>	<(0.75,0.35), (0.75,0.35)>	<(0.65,0.45), (0.75,0.35)>	<(0.75,0.35), (0.85,0.25)>	<(0.65,0.45), (0.65,0.45)>	<(0.75,0.35), (0.75,0.35)>	<(0.65,0.45), (0.75,0.35)>	<(0.85,0.25), (0.75,0.35)>
A2	<(0.75,0.35), (0.75,0.35)>	<(0.65,0.45), (0.55,0.55)>	<(0.65,0.45), (0.75,0.35)>	<(0.45,0.65), (0.55,0.55)>	<(0.55,0.55), (0.65,0.45)>	<(0.35,0.75), (0.25,0.85)>	<(0.65,0.45), (0.75,0.35)>	<(0.95,0.15), (0.85,0.25)>	<(0.75,0.35), (0.85,0.25)>
A3	<(0.85,0.25), (0.75,0.35)>	<(0.45,0.65), (0.55,0.55)>	<(0.25,0.85), (0.35,0.75)>	<(0.85,0.25), (0.75,0.35)>	<(0.95,0.15), (0.85,0.25)>	<(0.55,0.55), (0.65,0.45)>	<(0.75,0.35), (0.75,0.35)>	<(0.65,0.45), (0.75,0.35)>	<(0.95,0.15), (0.85,0.25)>
A4	<(0.65,0.45), (0.75,0.35)>	<(0.35,0.75), (0.45,0.65)>	<(0.65,0.45), (0.75,0.35)>	<(0.85,0.25), (0.85,0.25)>	<(0.35,0.75), (0.25,0.85)>	<(0.75,0.35), (0.85,0.25)>	<(0.35,0.75), (0.45,0.65)>	<(0.25,0.85), (0.35,0.75)>	<(0.25,0.85), (0.35,0.75)>
A5	<(0.55,0.55), (0.65,0.45)>	<(0.85,0.25), (0.75,0.35)>	<(0.85,0.25), (0.85,0.25)>	<(0.35,0.75), (0.45,0.65)>	<(0.25,0.85), (0.15,0.95)>	<(0.25,0.85), (0.25,0.85)>	<(0.75,0.35), (0.65,0.45)>	<(0.75,0.35), (0.75,0.35)>	<(0.65,0.45), (0.75,0.35)>
DM3									
	C1	C2	C3	C4	C5	C6	C7	C8	C9
A1	<(0.95,0.15), (0.85,0.25)>	<(0.85,0.25), (0.75,0.35)>	<(0.75,0.35), (0.85,0.25)>	<(0.85,0.25), (0.75,0.35)>	<(0.85,0.25), (0.95,0.15)>	<(0.55,0.55), (0.55,0.55)>	<(0.75,0.35), (0.85,0.25)>	<(0.65,0.45), (0.65,0.45)>	<(0.75,0.35), (0.65,0.45)>
A2	<(0.75,0.35), (0.75,0.35)>	<(0.65,0.45), (0.55,0.55)>	<(0.75,0.35), (0.75,0.35)>	<(0.55,0.55), (0.45,0.65)>	<(0.55,0.55), (0.45,0.65)>	<(0.35,0.75), (0.25,0.85)>	<(0.65,0.45), (0.75,0.35)>	<(0.85,0.25), (0.85,0.25)>	<(0.75,0.35), (0.75,0.35)>
A3	<(0.95,0.15), (0.85,0.25)>	<(0.25,0.85), (0.25,0.85)>	<(0.75,0.35), (0.65,0.45)>	<(0.75,0.35), (0.75,0.35)>	<(0.85,0.25), (0.95,0.15)>	<(0.65,0.45), (0.75,0.35)>	<(0.75,0.35), (0.85,0.25)>	<(0.75,0.35), (0.65,0.45)>	<(0.95,0.15), (0.85,0.25)>
A4	<(0.25,0.85), (0.25,0.85)>	<(0.35,0.75), (0.25,0.85)>	<(0.35,0.75), (0.35,0.75)>	<(0.85,0.25), (0.85,0.25)>	<(0.55,0.55), (0.45,0.65)>	<(0.75,0.35), (0.65,0.45)>	<(0.45,0.65), (0.55,0.55)>	<(0.35,0.75), (0.25,0.85)>	<(0.45,0.65), (0.55,0.55)>
A5	<(0.45,0.65), (0.35,0.75)>	<(0.85,0.25), (0.75,0.35)>	<(0.65,0.45), (0.55,0.55)>	<(0.15,0.95), (0.25,0.85)>	<(0.35,0.75), (0.25,0.85)>	<(0.15,0.95), (0.25,0.85)>	<(0.85,0.25), (0.75,0.35)>	<(0.85,0.25), (0.85,0.25)>	<(0.75,0.35), (0.65,0.45)>

Step 3. The individual criteria weights matrix and decision matrices of DMs are aggregated in a single criteria weights matrix and a single decision matrix by using DPFWG operator given in Eq. (3.19) as presented in Table 3.11.

Table 3.11 : Aggregated criteria weights.

Criteria	Aggregated Criteria Weights
C1	<(0.85,0.26),(0.82,0.28)>
C2	<(0.63,0.48),(0.56,0.55)>
C3	<(0.88,0.22),(0.78,0.32)>
C4	<(0.45,0.66),(0.51,0.59)>
C5	<(0.81,0.3),(0.79,0.32)>
C6	<(0.68,0.43),(0.68,0.42)>
C7	<(0.69,0.41),(0.59,0.51)>
C8	<(0.59,0.51),(0.53,0.57)>
C9	<(0.68,0.42),(0.67,0.43)>

Table 3.12 : Aggregated decision matrix.

	C1	C2	C3	C4	C5	C6	C7	C8	C9
A1	<(0.83,0.28), (0.79,0.33)>	<(0.81,0.29), (0.79,0.31)>	<(0.71,0.39), (0.77,0.33)>	<(0.73,0.38), (0.71,0.39)>	<(0.84,0.27), (0.91,0.2)>	<(0.55,0.56), (0.59,0.51)>	<(0.78,0.32), (0.77,0.33)>	<(0.68,0.42), (0.72,0.38)>	<(0.75,0.36), (0.72,0.38)>
A2	<(0.75,0.35), (0.78,0.32)>	<(0.61,0.49), (0.58,0.52)>	<(0.71,0.39), (0.75,0.35)>	<(0.51,0.59), (0.52,0.58)>	<(0.51,0.59), (0.52,0.59)>	<(0.31,0.79), (0.28,0.82)>	<(0.68,0.42), (0.71,0.39)>	<(0.89,0.22), (0.85,0.25)>	<(0.78,0.32), (0.82,0.28)>
A3	<(0.91,0.2), (0.84,0.27)>	<(0.49,0.66), (0.5,0.63)>	<(0.48,0.67), (0.53,0.59)>	<(0.82,0.28), (0.78,0.32)>	<(0.92,0.18), (0.91,0.2)>	<(0.64,0.47), (0.74,0.37)>	<(0.78,0.32), (0.77,0.33)>	<(0.71,0.39), (0.69,0.41)>	<(0.95,0.15), (0.88,0.22)>
A4	<(0.48,0.64), (0.48,0.66)>	<(0.31,0.79), (0.32,0.79)>	<(0.61,0.52), (0.67,0.48)>	<(0.55,0.62), (0.62,0.53)>	<(0.51,0.61), (0.4,0.72)>	<(0.71,0.39), (0.72,0.39)>	<(0.44,0.67), (0.47,0.63)>	<(0.27,0.83), (0.29,0.82)>	<(0.38,0.73), (0.43,0.68)>
A5	<(0.4,0.72), (0.4,0.73)>	<(0.81,0.29), (0.78,0.32)>	<(0.76,0.35), (0.69,0.42)>	<(0.39,0.78), (0.46,0.67)>	<(0.27,0.83), (0.2,0.9)>	<(0.25,0.87), (0.25,0.85)>	<(0.77,0.33), (0.71,0.39)>	<(0.81,0.29), (0.77,0.33)>	<(0.74,0.37), (0.72,0.38)>

Step 4. The aggregated decision matrix is normalized by replacing the functional and dysfunctional values for cost criteria whereas benefit criteria are held fixed.

Table 3.13 : Normalized aggregated decision matrix.

	C1	C2	C3	C4	C5	C6	C7	C8	C9
A1	<(0.83,0.28), (0.79,0.33)>	<(0.79,0.31), (0.81,0.29)>	<(0.71,0.39), (0.77,0.33)>	<(0.73,0.38), (0.71,0.39)>	<(0.84,0.27), (0.91,0.2)>	<(0.55,0.56), (0.59,0.51)>	<(0.78,0.32), (0.77,0.33)>	<(0.42,0.68), (0.38,0.72)>	<(0.75,0.36), (0.72,0.38)>
A2	<(0.75,0.35), (0.78,0.32)>	<(0.58,0.52), (0.61,0.49)>	<(0.71,0.39), (0.75,0.35)>	<(0.51,0.59), (0.52,0.58)>	<(0.51,0.59), (0.52,0.59)>	<(0.31,0.79), (0.28,0.82)>	<(0.68,0.42), (0.71,0.39)>	<(0.22,0.89), (0.25,0.85)>	<(0.78,0.32), (0.82,0.28)>
A3	<(0.91,0.2), (0.84,0.27)>	<(0.5,0.63), (0.49,0.66)>	<(0.48,0.67), (0.53,0.59)>	<(0.82,0.28), (0.78,0.32)>	<(0.92,0.18), (0.91,0.2)>	<(0.64,0.47), (0.74,0.37)>	<(0.78,0.32), (0.77,0.33)>	<(0.39,0.71), (0.41,0.69)>	<(0.95,0.15), (0.88,0.22)>
A4	<(0.48,0.64), (0.48,0.66)>	<(0.32,0.79), (0.31,0.79)>	<(0.61,0.52), (0.67,0.48)>	<(0.55,0.62), (0.62,0.53)>	<(0.51,0.61), (0.4,0.72)>	<(0.71,0.39), (0.72,0.39)>	<(0.44,0.67), (0.47,0.63)>	<(0.83,0.27), (0.82,0.29)>	<(0.38,0.73), (0.43,0.68)>
A5	<(0.4,0.72), (0.4,0.73)>	<(0.78,0.32), (0.81,0.29)>	<(0.76,0.35), (0.69,0.42)>	<(0.39,0.78), (0.46,0.67)>	<(0.27,0.83), (0.2,0.9)>	<(0.25,0.87), (0.25,0.85)>	<(0.77,0.33), (0.71,0.39)>	<(0.29,0.81), (0.33,0.77)>	<(0.74,0.37), (0.72,0.38)>

Step 5. The weighted normalized aggregated decision matrix is calculated as in Table 3.14 by using Eq. (3.21) and Table 3.13.

Table 3.14 : Weighted normalized aggregated decision matrix.

	C1	C2	C3	C4	C5	C6	C7	C8	C9
A1	<(0.71,0.37), (0.65,0.42)>	<(0.5,0.55), (0.46,0.6)>	<(0.63,0.44), (0.61,0.45)>	<(0.33,0.72), (0.36,0.67)>	<(0.68,0.39), (0.72,0.37)>	<(0.38,0.66), (0.4,0.63)>	<(0.54,0.51), (0.46,0.59)>	<(0.25,0.78), (0.2,0.82)>	<(0.51,0.53), (0.49,0.55)>
A2	<(0.64,0.43), (0.65,0.41)>	<(0.37,0.66), (0.34,0.68)>	<(0.63,0.44), (0.59,0.46)>	<(0.23,0.8), (0.27,0.75)>	<(0.42,0.64), (0.41,0.64)>	<(0.21,0.83), (0.19,0.86)>	<(0.47,0.56), (0.42,0.61)>	<(0.13,0.92), (0.13,0.9)>	<(0.54,0.51), (0.55,0.5)>
A3	<(0.77,0.32), (0.69,0.38)>	<(0.32,0.73), (0.27,0.78)>	<(0.43,0.69), (0.42,0.65)>	<(0.37,0.7), (0.4,0.65)>	<(0.75,0.34), (0.72,0.37)>	<(0.44,0.6), (0.51,0.54)>	<(0.54,0.51), (0.46,0.59)>	<(0.23,0.8), (0.22,0.8)>	<(0.65,0.44), (0.6,0.47)>
A4	<(0.41,0.67), (0.39,0.69)>	<(0.2,0.84), (0.17,0.86)>	<(0.54,0.56), (0.53,0.56)>	<(0.25,0.81), (0.32,0.73)>	<(0.42,0.65), (0.32,0.75)>	<(0.49,0.55), (0.49,0.55)>	<(0.3,0.74), (0.28,0.74)>	<(0.49,0.56), (0.44,0.62)>	<(0.26,0.79), (0.29,0.75)>
A5	<(0.34,0.74), (0.33,0.75)>	<(0.49,0.55), (0.46,0.6)>	<(0.67,0.4), (0.54,0.51)>	<(0.17,0.88), (0.24,0.8)>	<(0.22,0.85), (0.16,0.91)>	<(0.17,0.89), (0.17,0.88)>	<(0.53,0.51), (0.42,0.62)>	<(0.17,0.86), (0.18,0.85)>	<(0.51,0.54), (0.49,0.55)>

Step 6. DPF functional and dysfunctional negative ideal solutions are obtained by the weighted normalized aggregated decision matrix as presented in Table 3.15 by using Eqs. (3.22) and (3.23).

Table 3.15 : Negative ideal solutions.

	Functional	Dysfunctional
C1	(0.34,0.74,0.58)	(0.33,0.75,0.57)
C2	(0.2,0.84,0.5)	(0.17,0.86,0.48)
C3	(0.43,0.69,0.59)	(0.42,0.65,0.64)
C4	(0.17,0.88,0.44)	(0.24,0.8,0.55)
C5	(0.22,0.85,0.48)	(0.16,0.91,0.37)
C6	(0.17,0.89,0.42)	(0.17,0.88,0.45)
C7	(0.3,0.74,0.61)	(0.28,0.74,0.61)
C8	(0.13,0.92,0.37)	(0.13,0.9,0.41)
C9	(0.26,0.79,0.56)	(0.29,0.75,0.6)

Step 7. The Euclidean and taxicab distances of alternatives are computed based on DPF functional and dysfunctional negative ideal solutions (NISs) as given in Table 3.16 by using Eqs. (3.24)-(3.27).

Table 3.16 : Euclidean and Taxicab distances.

	Euclidean Distances		Taxicab Distances	
	$\mathcal{F}$	$\mathcal{D}$	$\mathcal{F}$	$\mathcal{D}$
A1	0.326	0.474	0.591	0.563
A2	0.229	0.445	0.479	0.459
A3	0.338	0.572	0.569	0.545
A4	0.234	0.559	0.433	0.404
A5	0.201	0.360	0.399	0.342

Step 8. The relative assessment matrices are determined separately for functional and dysfunctional Euclidean and taxicab distances as given in Tables 3.17 and 3.18. The threshold parameter θ is 0.02 in this application

Table 3.17 : Relative assessment matrix based on functional distances.

	A1	A2	A3	A4	A5
A1	0.000	0.108	-0.012	0.107	0.149
A2	-0.086	0.000	-0.099	-0.004	0.030
A3	0.012	0.118	0.000	0.118	0.160
A4	-0.078	0.004	-0.090	0.000	0.034
A5	-0.101	-0.026	-0.114	-0.032	0.000

Table 3.18 : Relative assessment matrix based on dysfunctional distances.

	A1	A2	A3	A4	A5
A1	0.000	0.032	-0.100	-0.098	0.139
A2	-0.026	0.000	-0.116	-0.120	0.095
A3	0.096	0.138	0.000	0.013	0.255
A4	0.071	0.108	-0.013	0.000	0.211
A5	-0.089	-0.075	-0.169	-0.186	0.000

Step 9. After the assessment scores of each alternative are calculated as functional and dysfunctional scores, the final assessment scores are determined according to the arithmetic mean of the functional and dysfunctional assessment scores as in Table 3.19. When the alternatives are ranked according to the decreasing values of the arithmetic means, the order of ranking is obtained as $A3 > A1 > A4 > A2 > A5$.

Table 3.19 : Functional and dysfunctional assessment scores.

	Functional	Dysfunctional	Arithmetic Mean	Ranking
A1	0.353	-0.026	0.163	2
A2	-0.159	-0.167	-0.163	4
A3	0.408	0.502	0.455	1
A4	-0.130	0.377	0.124	3
A5	-0.272	-0.519	-0.396	5

3.3.3 Validation of results and sensitivity analysis

After the findings of MCDM approaches are acquired, the question of whether the study's input parameters affect the model's outcomes and what results can be produced by using alternative multi-criteria methods arises. The robustness of the results obtained from a proposed method, as well as its sensitivity to change in input variables which is an important step for MCDM methods, should be checked. According to this scope, this study's sensitivity analysis and results of validation are reported in three parts in this section. The impact of the criteria weights on the results is examined in the first part. The impact of changes in DMs' weights on the results has been discussed in the second part. The third part compares the proposed MCDM method's validation by employing other approaches requiring the same input data.

3.3.3.1 Effects of changes in criteria weights

Since a slight change in the weights of the criteria can cause a change in the rankings of the alternatives, a sensitivity analysis should be applied to observe if any change happens in the ranking. For this purpose, in this section, one-at-a-time sensitivity analysis is carried out by changing the weights of the criteria. The effects of criteria weights on the final scores and ranking results have been analyzed by using fuzzy linguistic terms ranging from *Absolutely high importance* to *Absolutely low importance* for functional and dysfunctional values. A total of 81 scenarios have been analyzed by creating new weight vectors for each criterion without changing the weights of other criteria. For example, when the weight of criterion C1 is changed from AHI to ALI without changing the weights of criteria C2-C9, the effect of criterion C1 on the score values of the alternatives is presented in Figure 3.4. The effects of the changes in other criteria are illustrated in Figures 3.4-3.12, respectively.

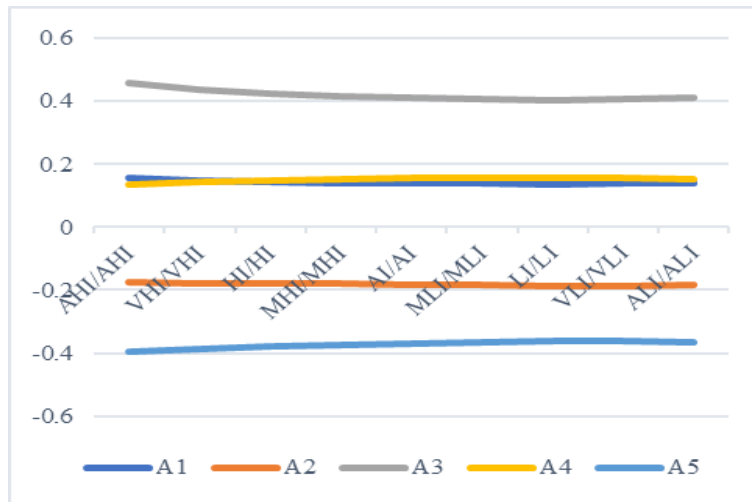


Figure 3.4 : Effect of the change in the weight of criterion C1.



Figure 3.5 : Effect of the change in the weight of criterion C2.

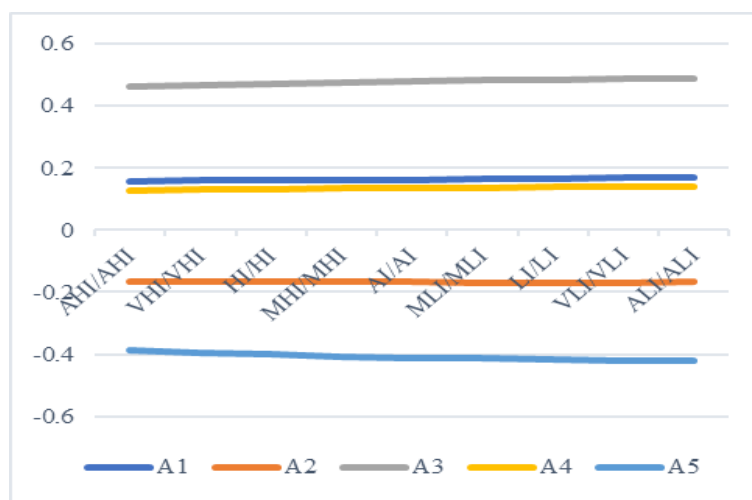


Figure 3.6 : Effect of the change in the weight of criterion C3.

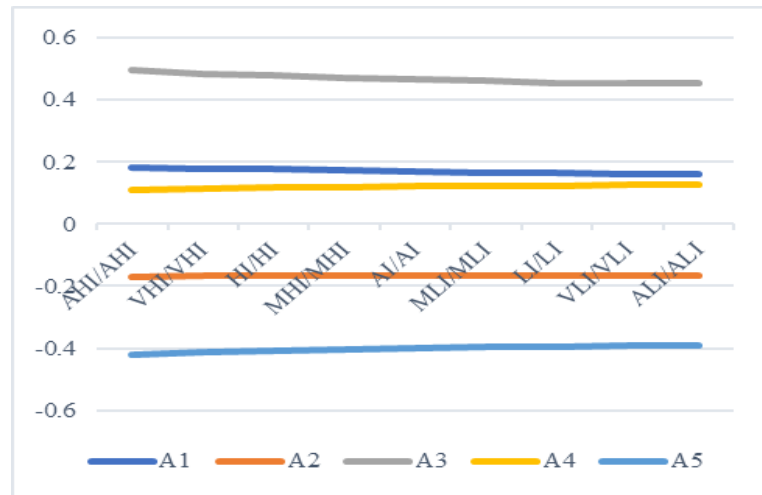


Figure 3.7 : Effect of the change in the weight of criterion C4.

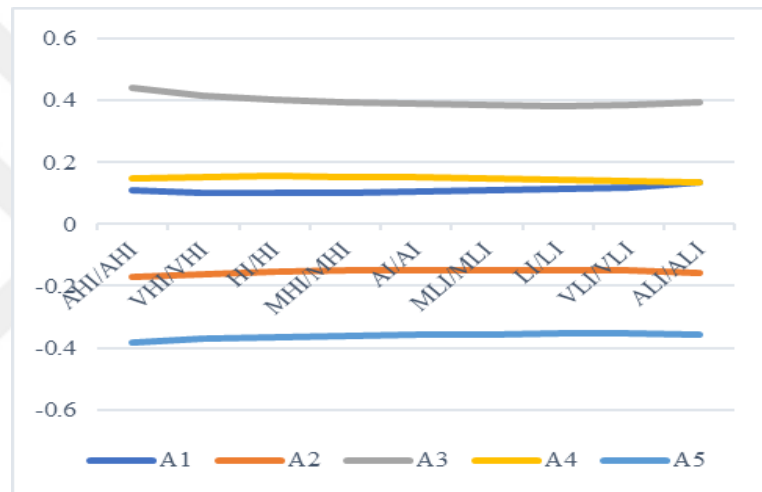


Figure 3.8 : Effect of the change in the weight of criterion C5.



Figure 3.9 : Effect of the change in the weight of criterion C6.



Figure 3.10 : Effect of the change in the weight of criterion C7.

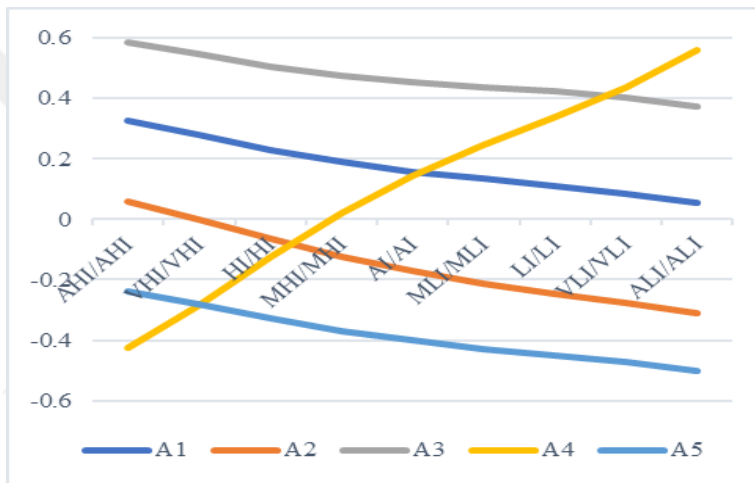


Figure 3.11 : Effect of the change in the weight of criterion C8.

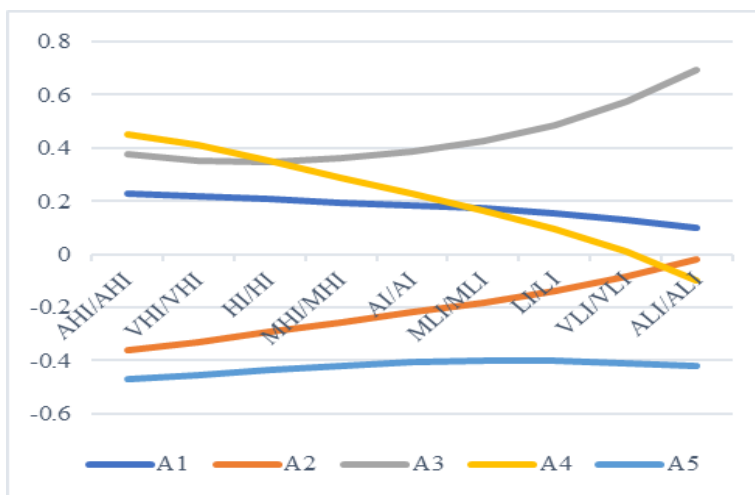


Figure 3.12 : Effect of the change in the weight of criterion C9.

According to the obtained results, it is seen that the changes in the weights of criteria C2, C3, C4, C5, C6 and C7 did not affect the ranking results of the alternatives while

the changes in the weights of criteria C1, C8 and C9 affected the ranking results. When the weight of criterion C1 was changed to AHI or VHI, it has been observed that the rankings of alternatives A1 and A4 were replaced by each other. When the weight of criterion C8 changes from AHI to ALI, the ranking of alternative A4 substantially changes from the last position to the first position. When the weight of criterion C9 is AHI or VHI, alternative A4 is in the first rank while the weight of criterion C9 is MHI or lower, alternative A3 is in the first rank.

3.3.3.2 Effect of changes in weights of DMs

The weights of DMs also have a significant impact on the outcomes of MCDM approaches. Therefore, a sensitivity analysis on the impact of changes in DMs weights is presented in this section. The impact of the changes in the weight of each DM on the ranking results has been examined using a one-at-one sensitivity analysis with 27 distinct scenarios. In the analysis, the effect of the changes in DMs weights on the ranking results has been examined by changing the weight of each DM from 0.1 to 0.9. When a DM's weight changes in the scenarios, the weights of the remaining DMs are correspondingly adjusted to satisfy the condition $\sum_{k=1}^K w_k = 1$. For instance, when we assign a weight of 0.3 to DM1, the other DMs will have weights of 0.35, which is obtained from $(1 - 0.3)/(3 - 1)$. The effects of the changes in the weights of the DMs on the results of the alternatives are presented in Figures 3.13-3.15.

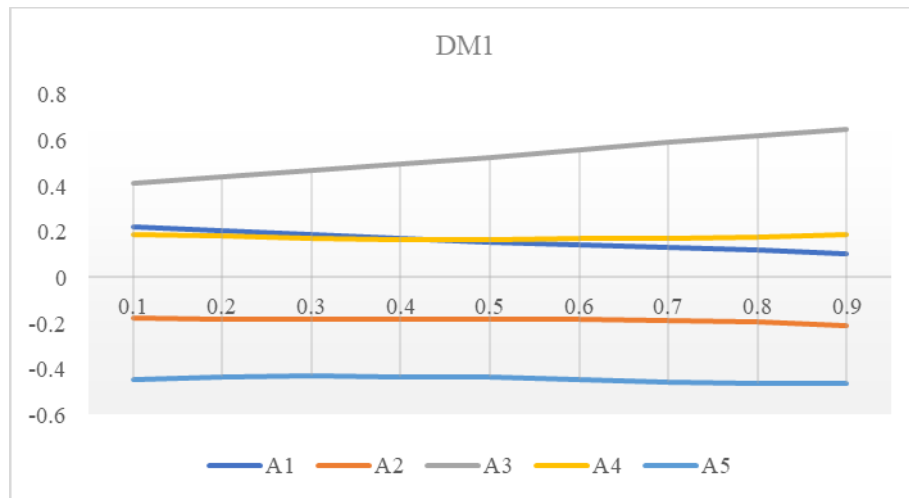


Figure 3.13 : Effect of changes in the weight of DM1.

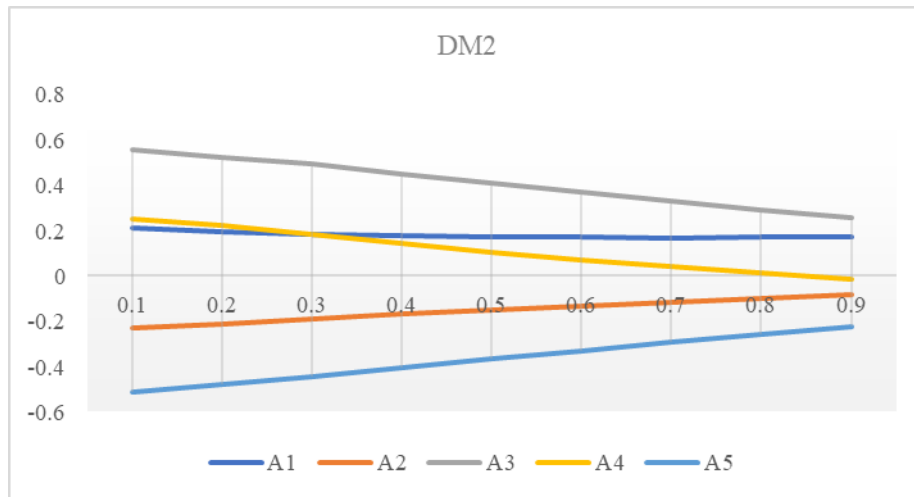


Figure 3.14 : Effect of changes in the weight of DM2.

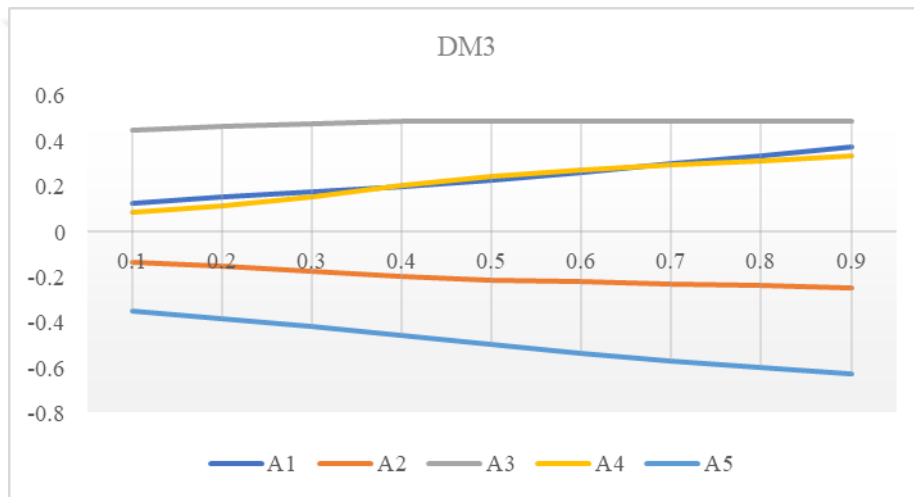


Figure 3.15 : Effect of changes in the weight of DM3.

When the effects of the changes in the weights of the DMs on the results are examined, it is seen that alternative A3 continued to be the dominant alternative in all scenarios while the ranking result of alternative A2 and alternative A5 did not change in any scenario. Besides, the change in the weights of DMs cause only the rankings of alternative A1 and alternative A4 to be swapped with each other.

3.3.3.3 Comparative analysis with other MCDM methods

In this section, the developed DPF-CODAS is compared and validated using three different MCDM techniques. For the strategy selection problem for IoT-based sustainable supply chain, decomposed fuzzy (DF)-TOPSIS, Pythagorean fuzzy (PF)-CODAS and PF-TOPSIS methods in the literature have been compared with the developed DPF-CODAS method. The ranking results of the comparative analysis

conducted by using three fuzzy-based MCDM methods have been shown in Figure 3.16. It has been observed that there are differences in the rankings of the alternatives according to the results obtained from different methods. It is seen that the dominant alternative obtained from both PF-CODAS, PF-TOPSIS methods and our proposed DPF-CODAS method is the same, which is A3. Besides, it has been determined that alternative A1, which is in the second rank in the developed method, is the same in PF-CODAS and PF-TOPSIS methods. It has been also seen that the ranking results obtained with the developed method are different from DF-TOPSIS method, which is another method compared. It has been determined that the dominant alternative A3 in the developed method was in the fourth rank in DF-TOPSIS method, and the most important alternative in DF-TOPSIS method was A4. Besides, it has been also observed that alternatives A4, A2 and A5, which were ranked in the third, fourth and fifth place, respectively, in the developed method were ranked differently in other methods.

DPF-CODAS has some advantages over DF-TOPSIS, PF-CODAS and PF-TOPSIS methods. The consistency of the experts is considered in DPF-CODAS and DF-TOPSIS whereas a larger domain is given to the experts by DPF-CODAS. The disadvantage of PF-CODAS and PF-TOPSIS methods is that the consistency is not considered in this method whereas a larger domain is presented to the experts.

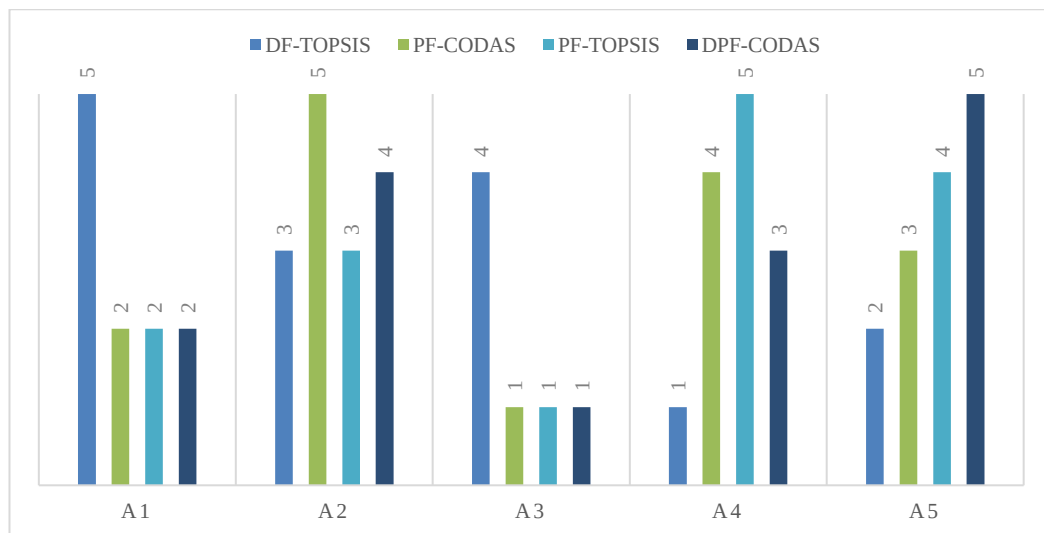


Figure 3.16 : Ranking results of the compared methods.

To determine whether there is a statistically significant correlation between the rankings handled from the proposed model and the compared models in the study, Spearman's rank correlation coefficient, weighted Spearman's rank correlation

coefficient and rank similarity coefficient have been used (Sałabun & Urbaniak, 2020; Shekhovtsov et al., 2020). The correlation results between the compared methods of the developed DPF-CODAS method have been presented in Figure 3.17. Spearman's rho correlation is estimated according to Eq. (3.34) to measure the intensity and direction (negative or positive) of the correlation between the ranking results with the developed method and the compared methods.

$$R = 1 - \frac{6 \sum_{i=1}^m d_i^2}{m(m^2 - 1)} \quad (3.34)$$

where d represents the difference of position of alternative i in methods and m represents the number of alternatives. When Spearman's rank correlation coefficient is examined, it has been determined that the relationships between the developed method and PF-CODAS, PF-TOPSIS methods were 0.7. It has been determined that there was a 0.6 correlation between the developed method and DF-TOPSIS method, and it has been seen that there was a moderate relationship between the rankings. It has been seen that the developed method had a strong relationship with the PF-CODAS and PF-TOPSIS methods, with the larger similarity between the rankings.

Higher positions in the rankings between the two methods compared in the weighted Spearman's rank correlation coefficient are more important. The significance weight is calculated for each comparison. Unlike the Spearman rank correlation coefficient, which examines whether the differences arise or not, it is the element that determines where the differences arise. The weighted Spearman's rank correlation coefficient is defined by Eq. (3.35).

$$r_w = 1 - \frac{6 \sum_{i=1}^m (x_i - y_i)((m - x_i + 1) + (m - y_i + 1))}{m^4 + m^3 - m^2 - m} \quad (3.35)$$

where m of the number of alternatives and the ranking results of each alternative in the two compared methods are x and y . When the weighted Spearman's rank correlation coefficient between the developed method and the compared DF-TOPSIS, PF-CODAS and PF-TOPSIS methods was examined, it has been determined that the relationship between them was strong with 0.736, 0.967 and 0.967, respectively.

The weight of a particular comparison between two methods in the rank similarity coefficient is determined by the importance of the position in the first rank used as the

reference rank during calculation. The rank similarity coefficient, which is an asymmetrical measure, is defined by Eq. (3.36).

$$WS = 1 - \sum_{i=1}^m 2^{-x_i} \frac{|x_i - y_i|}{\max((x_i - 1), (x_i - m))} \quad (3.36)$$

where m of the number of alternatives and the ranking results of each alternative in the two compared methods are x and y . It has been observed that the rank relations between the compared methods and the developed method with the rank similarity coefficient were 0.206 with DF-TOPSIS, 0.901 with PF-CODAS, and 0.846 with PF-TOPSIS. In the results obtained when the rank similarity coefficient method was used, it has been determined that the developed method in terms of ranking results had a strong correlation only with the PF-CODAS and PF-TOPSIS methods and a weak correlation with the DF-TOPSIS method.

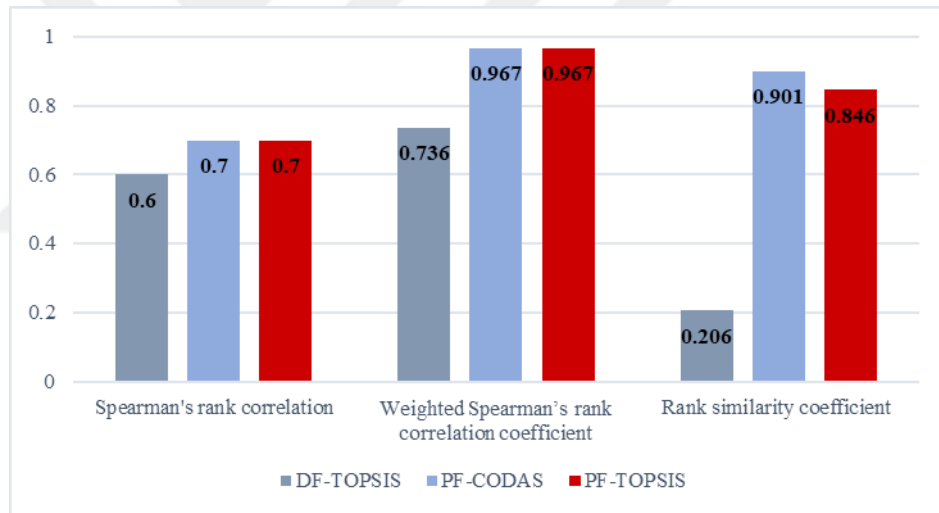


Figure 3.17 : Correlation results between the compared methods of the developed DPF-CODAS method.

3.4 Conclusions

When decision-makers assigned crisp values in the MCDM problems, it frequently causes insufficient evaluations since crisp values can not generally represent the uncertainty in human thoughts. Besides, the uncertainties caused by the lack of information and the inconsistencies between the decision groups require decision makers to make a wider range of assessments and for these assessments to take into account the uncertainty in more detail. Therefore, the DPFs introduced in this study

are more realistic and detailed modeling of uncertain information by addressing the ideas and perceptions of decision-makers from a functional and dysfunctional point of view. Especially since DPFs have the ability to express information in a more flexible and detailed way, they contribute to the better management of the decision-making process as well as to handle the problems more powerfully in uncertain and ambiguous environments. Based on this advantage, DPF-CODAS method has been developed in the study. The developed method has been applied to the strategy selection problem for an IoT-based sustainable supply chain system. This study has aimed to provide more consistent and effective decisions and make the right investments for decision-makers and policy-makers in the evaluation of sustainable IoT-based supply chain system strategies by taking into account the significance of an extensive list of criteria. Five strategies for sustainable IoT-based supply chain systems have been assessed with respect to 9 criteria identified by a comprehensive literature review and several expert opinions. By using the developed DPF-CODAS method, the ranking and selection of strategies for the IoT-based sustainable supply chain system, which is a significant issue for decision-makers and policy-makers, has been realized. According to the findings obtained from the developed method, A3-"RFID, GIS, and GPRS Integration of Supply Chain System" alternative is the most important strategy, followed by A1-Dynamic Warehousing, A4- Decentralization of Decision Making, A2-Mobile Application for Delivery Time Forecasting and finally A5 -Smart Automation alternatives.

In the study, it has been determined that the results obtained by the sensitivity analyses carried out on the criteria weights and DMs weights were robust and reliable. With this, in the comparison analysis using DF-TOPSIS, PF-CODAS and PF-TOPSIS methods, the ranking results obtained from the methods were compared and the differences obtained in the rankings were examined. In the obtained results, the most important alternative has been determined as A3 in the proposed method and in the PF-CODAS, PF-TOPSIS methods while the most important alternative in the DF-TOPSIS method has been determined as A4. Besides, it has been seen that the rankings obtained from the PF-CODAS and PF-TOPSIS methods were similar to the result of the developed method and there were slight differences in the ranking from the DF-TOPSIS method. The relationships in the ranking results obtained by Spearman's rank correlation coefficient, weighted Spearman's rank correlation coefficient and rank

similarity coefficient have been analyzed. According to the results obtained, it was determined that there was a moderate correlation between the developed method and the DF-TOPSIS method according to Spearman's rank correlation coefficient, while there was a strong and weak relationship in weighted Spearman's rank correlation coefficient and rank similarity coefficient, respectively. Besides, it has been determined that the developed DF-TOPSIS method has a strong relationship with PF-CODAS and PF-TOPSIS methods, according to all correlation elements. Thus, the developed method has proved the robustness of the results.

The developed method has been applied to strategy selection for IoT-based sustainable supply chain systems. Our proposed method which is quite flexible, can also be used in many problems outside the field of sustainable supply chain.

The contributions and advantages of the study can be summarized as follows.

- In this study, decomposed Pythagorean fuzzy sets have been developed for the first time, which deals with the properties of decomposed fuzzy sets and Pythagorean fuzzy sets together.
- The basic principles and mathematical operators of the developed fuzzy sets are presented with their proofs.
- In the study, a new extension of CODAS has been developed in the DPF environment.
- The study has presented an application on strategy selection for IoT-based sustainable supply chain systems to prove the effectiveness, robustness and reliability of the presented model.
- The DPF-CODAS method, which has a flexible structure, is suitable for solving multi-criteria decision problems with more criteria and alternatives.
- The application of the DPF-CODAS method to the strategy selection problem for IoT-based sustainable supply chain systems has been tested with sensitivity and comparative analyses and it has been shown that the obtained results are consistent and reliable and the proposed method can select the most appropriate strategy. Thus, the potential and applicability of the developed DPF- CODAS method have been fully demonstrated.
- This study has provided an important contribution to the literature both for IoT-based sustainable supply chain system problems and for the selection process

in other real-life problems by presenting a new evaluation model based on the MCDM method in the DPF environment.

For further studies, DPFSs can be extended to different MCDM methods. Various mathematical operators such as different addition operators and score functions of the introduced DPFS can be developed. The developed DPF-CODAS method can be applied to various decision-making problems such as sustainable policy selection, waste disposal site selection, renewable energy selection, electric vehicle selection, risk assessment and project selection. More specific and detailed criteria can be included in the strategy selection for an IoT-based sustainable supply chain system.



4. CONTINUOUS PYTHAGOREAN FUZZY SETS AND MULTI-ATTRIBUTE DECISION MAKING APPLICATIONS³

Uncertainty and fuzziness are an important part of complex decision-making problems, and solving such problems effectively plays an important role in various fields. In this context, fuzzy set theory has become a potent instrument for addressing uncertainty and improving decision-making processes. Traditional fuzzy sets (TFSs), which determine an element's degree of membership in a set, have been introduced by Zadeh (1965). Several fuzzy set extensions have been presented by different researchers in the literature in the ensuing periods because traditional fuzzy sets (TFSs) are limited to membership degrees and cannot address uncertainty in its entirety (Gündoğdu & Kahraman, 2019a; Senapati & Yager, 2020; Qahtan et al., 2023; Kou et al., 2024; Jan et al., 2023; Deveci et al., 2022; Alkan & Kahraman, 2023b). Intuitionistic fuzzy sets (IFSs), represented by degrees of membership and non-membership, have been developed by Atanassov (1986), enabling uncertain information to be handled more flexibly. IFSs have gained a lot of interest from researchers and have started to be employed in many different disciplines because of its advantage in representing uncertain information. As a direct extension of IFSs, Yager (2013) developed Pythagorean fuzzy sets (PFSs), which were initially presented by Atanassov as the second kind of IFS and which offer a more flexible representation of uncertainty. Some facts, events, and associated data that other sets like TFSs and IFSs cannot process are processed more effectively by PFSs. A fuzzy set can be discrete or continuous. In discrete fuzzy sets, elements are given as singletons whereas they are given with continuous membership functions in continuous fuzzy sets. Discrete Pythagorean fuzzy sets are composed of singletons including the membership and non-membership degrees of a certain x . Since singleton Pythagorean fuzzy sets have not yet been extended to continuous Pythagorean fuzzy sets in the literature, this paper introduces a continuous Pythagorean fuzzy set as the first time in the literature.

³ This chapter is based on the paper “Alkan, N. & Kahraman, C. (2025). Continuous Pythagorean Fuzzy Set Extension with Multi-Attribute Decision Making Applications. *Informatica*, 1-43.”

In PFSs represented with the membership and non-membership degrees, the sum of these degrees' squares must not exceed one. In this way, the uncertainties in decision problems can be better addressed with PFSs and the data can be reflected more accurately (Al-Samarraay et al., 2022). Even though a great deal of studies have been done on fuzzy sets beyond PFSs, it is still difficult to predict precise values for membership and non-membership degrees. This situation can lead to uncertainty not being fully defined and correct results not being obtained, especially in decision-making problems. The most recent innovation in the field of fuzzy set extension was made by Alkan and Kahraman (2023b) with CINFUS, where the degrees of membership and non-membership are defined by nonlinear functions, as a direct extension of IFS. In CINFUSs, uncertainty at every point can be taken into account with the continuous structure of membership and non-membership degrees (Alkan & Kahraman, 2023b; Altıntaş et al., 2024; Kizilkan Demir & Oztaysi, 2024; Alkan & Kahraman, 2024). Thus, degrees of membership and non-membership provide a continuous structure that changes according to how a decision-maker behaves to uncertainty. Due to these superior advantages of CINFUSs and the broader scope of PFS than IFSs, representing PFSs with a continuous structure, as in IFSs, will enable a broader and more accurate definition of uncertainty. Therefore, in this study, continuous Pythagorean fuzzy sets (CPFUSs) have been developed, which allow to express of fuzziness and uncertainties more accurately by extending PFSs to continuous value fields. By extending PFSs in a continuous context, CPFUSs will allow modeling of the degree to which elements are present in a continuous range of values. In this way, CPFUSs will enable real-life problems containing uncertain information to be addressed in a more realistic, detailed and broader context.

The innovative aspect of continuous Pythagorean fuzzy sets is that they represent the generalization of single-valued Pythagorean fuzzy sets. Each point on this generalized continuous function gives traditional Pythagorean fuzzy numbers available in the literature, consisting of single values of membership, non-membership and indecision parameters. In addition, another important innovative aspect is that it shows all possible Pythagorean fuzzy numbers that risk-averse, risk-taking or risk-neutral experts can assign on a unique continuous function.

In CPFUSs, which have a similar structure to CINFUSs, the values of the elements of a set have a symmetrical appearance, represented by a nonlinear curve. In order to

obtain a continuous function in CPFUSs, membership and non-membership degrees must be represented by a coefficient parameter. In CPFUSs, degrees of membership and non-membership are represented by the $\sqrt{\alpha\tau - \alpha\tau^2}$ function, ensuring the formation of a continuous structure. Thanks to the coefficient τ here, it is ensured that at each point, the degrees of membership and non-membership can account for the ambiguity. Thus, the coefficient τ is defined as the degree to which each element lies in the range of values between 0 and 1 and determines whether the decision maker is a risk taker, risk-averse, or risk-neutral. At the same time, the values of τ closer to the center of the functions indicates that the risk-averse decision maker is stronger, while the farther away from the center of the functions, the stronger the risk-taking decision maker is encountered. In a continuous structure, the magnitude of the values for membership and non-membership degrees between 0 and 1 is represented by the α value. The α value represents the magnitude or smallness of the values between 0 and 1 for membership and non-membership degrees in a continuous structure. In CPFUSs, the sum of the squares of the membership and non-membership degrees meets the condition $0 \leq (\sqrt{\alpha\tau - \alpha\tau^2})^2 + (\sqrt{(4-\alpha)\tau - (4-\alpha)\tau^2})^2 \leq 1$, provided that it is at most 1. With this condition, the continuous structure of CPFUSs in nonlinear form is ensured. Thus, with CPFUSs, the opinions of decision-makers can be taken into account continuously and more comprehensively.

In this study, it is aimed to construct CPFUSs and use them with MADM models to demonstrate their applicability. With CPFUSs, more consistent models will be created and therefore more reliable results will be obtained, thanks to the degrees of membership and non-membership that have a wider area of influence. One of the practical significances of the proposed CPFUSs is the more flexible representation of uncertainty, which provides greater flexibility in representing uncertainty compared to intuitionistic fuzzy sets. This enables more refined modeling of vague and ambiguous information, making CPFUSs suitable for complex decision-making scenarios. Another significance is the enhanced decision-making applications which allow decision-makers to capture more nuanced judgments, which can be applied in fields such as project evaluation, resource allocation, and supplier selection, where precise assessments of alternatives are crucial. Besides, advanced modeling of expert opinions is another significance of CPFUSs which makes them effective in situations where expert evaluations involve high levels of uncertainty or hesitation. CPFUSs can better

capture the hesitancy and conflicting opinions in expert assessments, leading to more accurate aggregation and consensus-building. In machine learning and data mining tasks such as clustering, classification, and pattern recognition, CPFUSs can enhance model performance by managing uncertain data more effectively. Moreover, CPFUSs facilitate more flexible aggregation of information by allowing the combination of different membership and non-membership values with minimal loss of information.

The novelty of this article offers models that are capable of representing the thoughts in the minds of experts in a wider scope and uncertainty at every point, by taking into account the behavior of experts towards risk. In this regard, after introducing the mathematical operations of the CPFUSs developed in the study, two different CPFUS-based MADM models have been proposed by considering the mathematical operations. The performance of the developed CPFUS-MADM models is presented first on the green supplier selection problem and then on the waste disposal site selection problem. To distinguish between the two MADM models and further demonstrate the applicability of CPFUSs, a model based on different aggregation operators introduced within the scope of the study has been used in the second MADM problem.

The rest of the study is structured as follows. The basic principles of CINFUSs are outlined in Section 4.1. Section 4.2 introduces the theoretical principles and mathematical operators of CPFUSs. In Section 4.3, the developed CPFUSs are implemented to green supplier selection and waste disposal site selection problems on two different MADM models, and sensitivity analysis and comparison analysis are conducted on each application model. Section 4.4 concludes the study with conclusions and suggestions for additional researches.

4.1 Continuous Intuitionistic Fuzzy Sets

There are some studies on the development of continuous fuzzy sets. Shahari & Rasmani (2020) investigate the potential use of continuous fuzzy sets to represent linguistic terms used in the fuzzy computing mapping process to investigate the consistency of the decision outcomes produced by the proposed approach. Wu et al. (2017) obtained some characterizations on the transitivity, mildly mixing property, a-transitivity, equicontinuity, uniform rigidity and proximality of Zadeh's extensions restricted on some invariant closed subsets of all upper semi-continuous fuzzy sets in

the level-wise metric. Rakus-Andersson (2010) explored continuous fuzzy numbers in the interval- and the α -cut forms to detect their similar nature. The study expands fuzzy probabilities of continuous fuzzy events in the form of continuous fuzzy sets. Alkan and Kahraman (2023b) developed continuous intuitionistic fuzzy sets (CINFUSs) to define the generalized IFSs.

This section discusses the fundamental operations of the CINFUSs. The detailed notions, basic operations and mathematical operations of the CINFUSs are mentioned as follows:

Definition 4.1. Let $\mathcal{X}$ be a universal set of discourse. A CINFUS $\mathbb{C}$ in $\mathcal{X}$ is defined as follows (Alkan & Kahraman, 2023b):

$$\mathbb{C} = \{ \langle x, F(\mu_{\mathbb{C}}(x), \tau), F(\vartheta_{\mathbb{C}}(x), \tau) \rangle \mid x \in \mathcal{X} \} \quad (4.1)$$

where the function $F(\mu_{\mathbb{C}}(x), \tau): X \rightarrow [0,1]$ and $F(\vartheta_{\mathbb{C}}(x), \tau): X \rightarrow [0,1]$ indicate continuous membership and non-membership degrees of the element $x \in \mathcal{X}$ to set $\mathbb{C}$, respectively. τ represents the expert's behavior against uncertainty. The closer the τ value gets to the center (0.5), the more risk-averse the expert will have, and the further it gets from the center (0.5), the more risk-taking the expert will be. Also, $F(\mu_{\mathbb{C}}(x), \tau) = \alpha\tau - \alpha\tau^2$ and $F(\vartheta_{\mathbb{C}}(x), \tau) = (4 - \alpha)\tau - (4 - \alpha)\tau^2$ satisfy the following condition $0 \leq F(\mu_{\mathbb{C}}(x), \tau) + F(\vartheta_{\mathbb{C}}(x), \tau) \leq 1 \Rightarrow 0 \leq \alpha\tau - \alpha\tau^2 + (4 - \alpha)\tau - (4 - \alpha)\tau^2 \leq 1 \Rightarrow 0 \leq 4\tau - 4\tau^2 \leq 1$ for every x .

The uncertainty degree in judgment is indicated as $F(\pi_{\mathbb{C}}(x), \tau) = 1 - \alpha\tau + \alpha\tau^2 - (4 - \alpha)\tau + (4 - \alpha)\tau^2 = 1 - 4(\tau - \tau^2)$. The membership, non-membership and hesitancy degrees of CINFUSs are presented for $\alpha = 1.5$ and $\alpha = 3.5$ in Fig. 4.1.

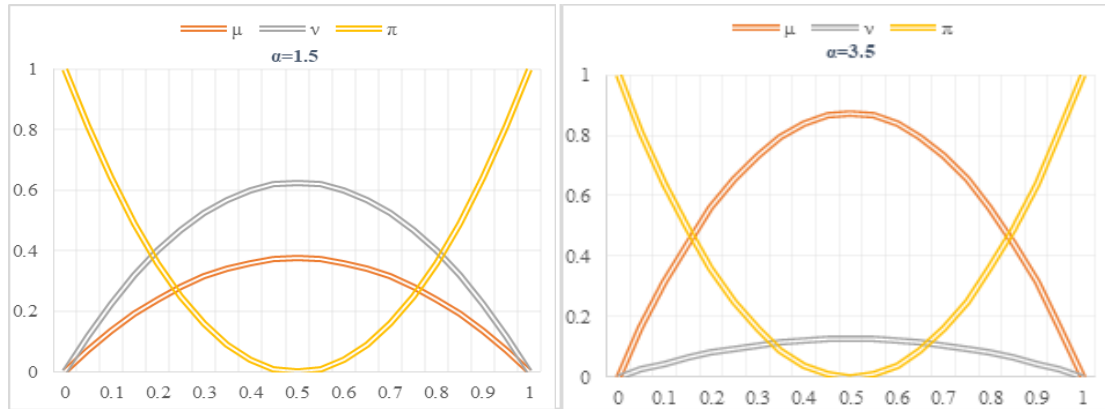


Figure 4.1 : Membership, non-membership and hesitancy degrees of CINFUSs for $\alpha=1.5$ and $\alpha=3.5$.

Definition 4.2. Let $CINFUS = \langle (\alpha\tau - \alpha\tau^2), ((4 - \alpha)\tau - (4 - \alpha)\tau^2) \rangle$, $CINFUS_A = \langle (\alpha_A\tau - \alpha_A\tau^2), ((4 - \alpha_A)\tau - (4 - \alpha_A)\tau^2) \rangle$ and $CINFUS_B = \langle (\alpha_B\tau - \alpha_B\tau^2), ((4 - \alpha_B)\tau - (4 - \alpha_B)\tau^2) \rangle$ be three CINFUSs, then the mathematical operations on these three continuous intuitionistic fuzzy numbers (CINFUNs) are expressed as follows (Alkan & Kahraman, 2023b):

$$\begin{aligned}
CINFUS_A \oplus CINFUS_B &= \langle ((\alpha_A + \alpha_B)(\tau - \tau^2) \\
&\quad - \alpha_A\alpha_B(\tau - \tau^2)^2), (((4 - \alpha_A)\tau \\
&\quad - (4 - \alpha_A)\tau^2)((4 - \alpha_B)\tau - (4 - \alpha_B)\tau^2)) \rangle
\end{aligned} \tag{4.2}$$

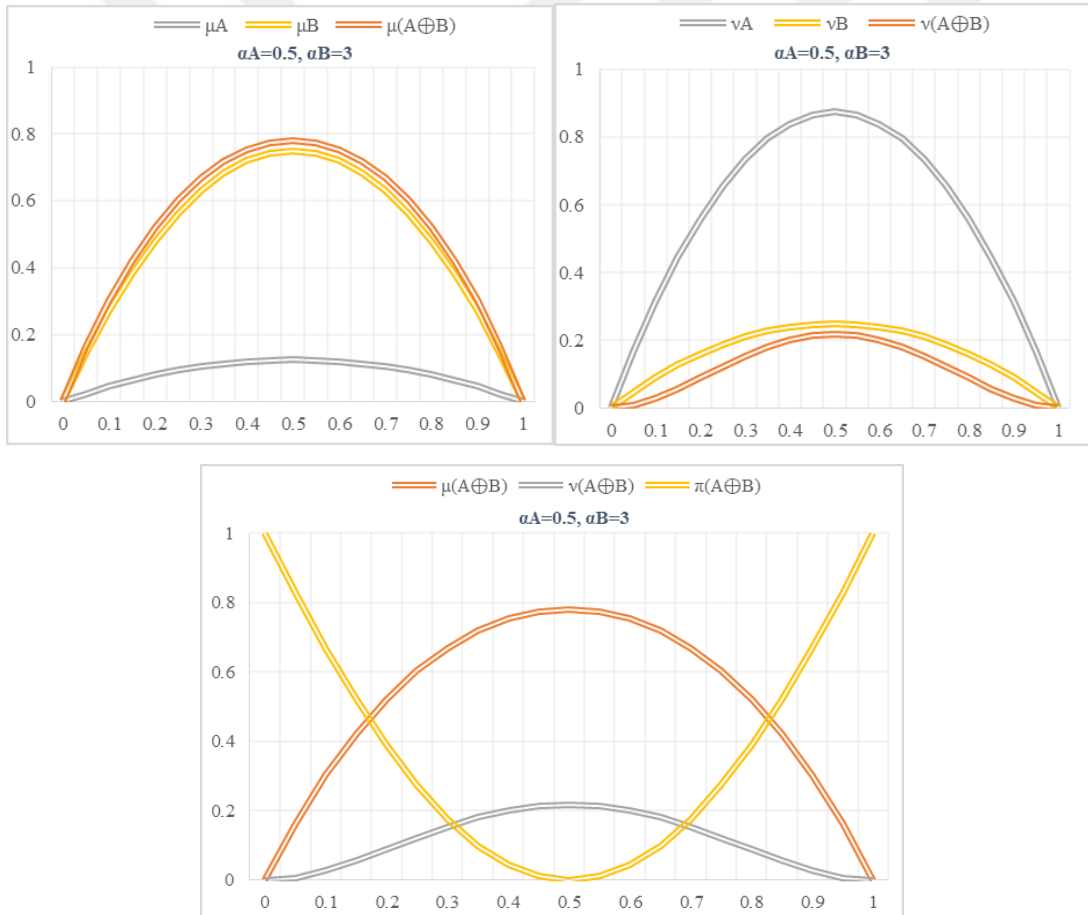


Figure 4.2 : Addition operation for two CINFUNs.

Moreover, the results of mathematical operations that consist of addition, multiplication, multiplication with a scalar and power are presented in Figs. 2, 3, 4, and 5, respectively. In Fig. 4.2, $\alpha_A = 0.5$ and $\alpha_B = 3$ represent the magnitude rating of membership functions for $CINFUS_A$ and $CINFUS_B$, respectively. The sum of the

membership degrees and non-membership degrees of two CINFUNs and the corresponding hesitancy degree are presented in Fig. 4.2.

$$CINFUS_A \otimes CINFUS_B$$

$$\begin{aligned}
&= \langle ((\alpha_A \tau - \alpha_A \tau^2)(\alpha_B \tau - \alpha_B \tau^2)), ((4 - \alpha_A)\tau - (4 - \alpha_A)\tau^2 \\
&+ (4 - \alpha_B)\tau - (4 - \alpha_B)\tau^2 \\
&- ((4 - \alpha_A)\tau - (4 - \alpha_A)\tau^2)((4 - \alpha_B)\tau - (4 - \alpha_B)\tau^2)) \rangle \\
&= \langle ((\alpha_A + \alpha_B)(\tau - \tau^2)), ((8 - \alpha_A - \alpha_B)(\tau - \tau^2) \\
&- (4 - \alpha_A)(4 - \alpha_B)(\tau - \tau^2)^2) \rangle
\end{aligned} \tag{4.3}$$

In Fig. 4.3, the magnitude ratings of membership functions for **CINFUS_A** and **CINFUS_B** are represented by $\alpha_A = 1$ and $\alpha_B = 2.5$, respectively. Also, the product of the membership degrees and the non-membership degrees of two CINFUNs and the corresponding hesitancy degree are presented in Fig. 4.3, respectively.

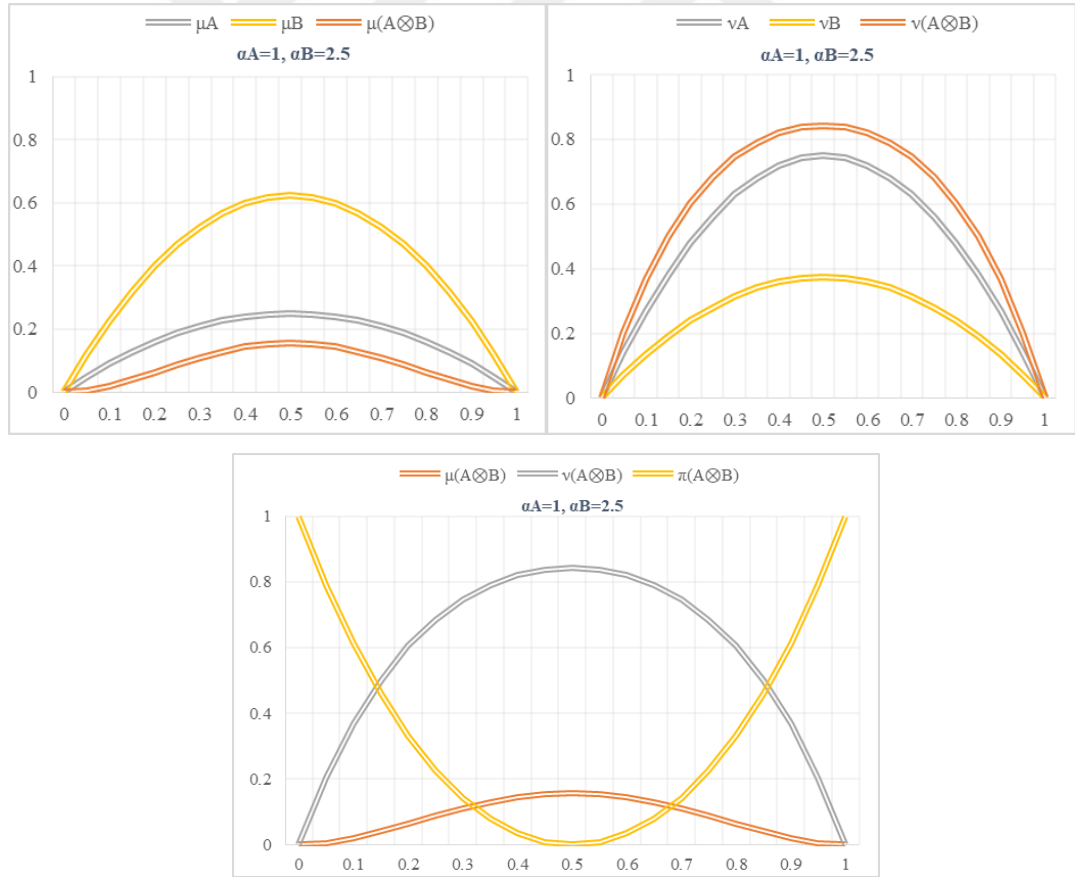


Figure 4.3 : Multiplication operation for two CINFUNs.

$$\lambda \times C - INFUS_A = \langle (1 - (1 - \alpha_A \tau + \alpha_A \tau^2)^\lambda), ((4 - \alpha_A)^\lambda (\tau - \tau^2)^\lambda) \rangle \tag{4.4}$$

In Fig. 4.4, $\alpha_A = 2$ and $\lambda = 4$ show the magnitude rating of membership functions and the value of the scalar number for *CINFUS*. Also, the membership, non-membership and hesitancy degrees multiplied by the λ scalar number of CINFUNs are shown in Fig. 4.4.

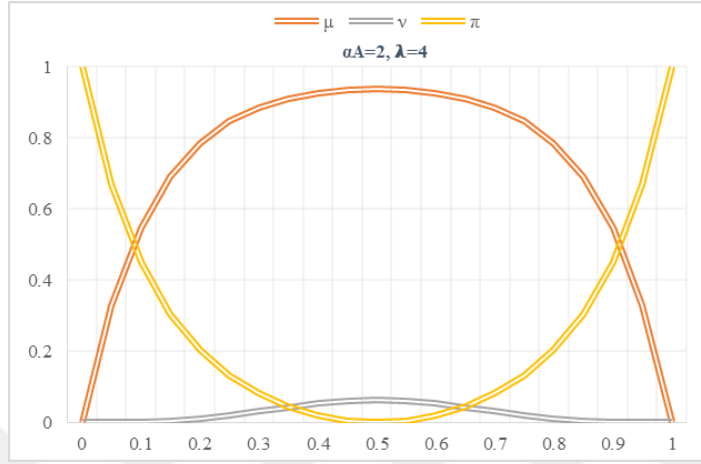


Figure 4.4 : Multiplication by scalar number of membership functions.

$$C - INFUS_A^\lambda = \langle (\alpha_A \tau - \alpha_A \tau^2)^\lambda, (1 - (1 - (4 - \alpha_A)(\tau - \tau^2))^\lambda) \rangle \quad (4.5)$$

In Fig. 4.5, $\alpha_A = 2.5$ and $\lambda = 2$ show the magnitude rating of membership functions and the value of the scalar number for *CINFUS*. Also, the scalar power of CINFUN is presented in Fig. 4.5.

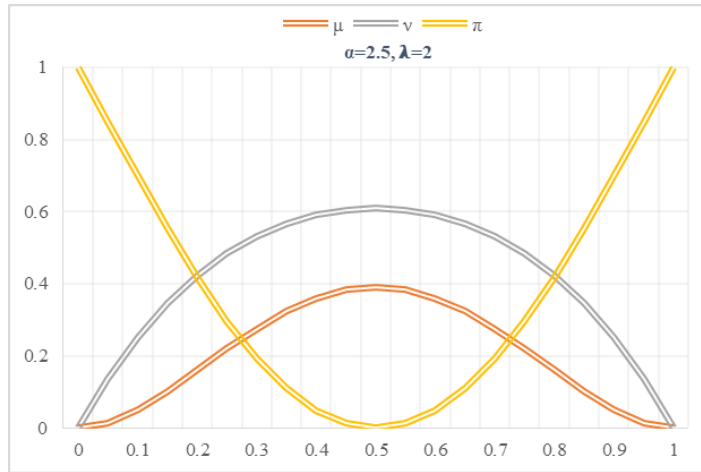


Figure 4.5 : Scalar number power of CINFUN.

Definition 4.3. Let $\tilde{A}_i = \langle (\alpha\tau - \alpha\tau^2)_i, ((4 - \alpha)\tau - (4 - \alpha)\tau^2)_i \rangle, (i = 1, 2, \dots, n)$ be a set of CINFUSs and $w = (w_1, w_2, \dots, w_n)^T$ be weight vector of $\tilde{A}_i$ with

$\sum_{i=1}^n w_i = 1$, then a continuous intuitionistic fuzzy weighted average (*CINFUWA*) is defined as follows (Alkan & Kahraman, 2023b):

$$\begin{aligned} CINFUWA(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) \\ = \left(1 - \prod_{i=1}^n (1 - (\alpha\tau - \alpha\tau^2)_i)^{w_i}, \right. \\ \left. \prod_{i=1}^n ((4 - \alpha)\tau - (4 - \alpha)\tau^2)_i^{w_i} \right) \end{aligned} \quad (4.6)$$

Definition 4.4. Let $\tilde{A}_i = \langle (\alpha\tau - \alpha\tau^2)_i, ((4 - \alpha)\tau - (4 - \alpha)\tau^2)_i \rangle, (i = 1, 2, \dots, n)$ be a set of CINFUSs and $w = (w_1, w_2, \dots, w_n)^T$ be weight vector of $\tilde{A}_i$ with $\sum_{i=1}^n w_i = 1$, then a continuous intuitionistic fuzzy weighted geometric (*CINFUWG*) is defined as follows (Alkan & Kahraman, 2023b):

$$\begin{aligned} CINFUWG(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) \\ = \left(\prod_{i=1}^n (\alpha\tau - \alpha\tau^2)_i^{w_i}, 1 \right. \\ \left. - \prod_{i=1}^n (1 - ((4 - \alpha)\tau - (4 - \alpha)\tau^2)_i)^{w_i} \right) \end{aligned} \quad (4.7)$$

Definition 4.5. Let $CINFUN = \langle (\alpha\tau - \alpha\tau^2), ((4 - \alpha)\tau - (4 - \alpha)\tau^2) \rangle$ be a CINFUN, then the score function $S(CINFUN)$ and accuracy function of $CINFUN$, $A(CINFUN)$ can be described as in Eqs. (8) and (9), respectively (Alkan & Kahraman, 2023b).

$$S(CINFUN) = \alpha(\tau - \tau^2) - (4 - \alpha)(\tau - \tau^2) = (2\alpha - 4)(\tau - \tau^2) \quad (4.8)$$

$$A(CINFUN) = \alpha(\tau - \tau^2) + (4 - \alpha)(\tau - \tau^2) = 4(\tau - \tau^2) \quad (4.9)$$

4.2 Continuous Pythagorean Fuzzy Sets

4.2.1 Arithmetic operations

This sub-section includes the fundamental conceptions related to CPFUSs. The basic operations of CPFUSs are defined as follows.

Definition 4.6. Let X be a finite universe of discourse. A CPFUS $\tilde{C}_p$ in X is an object having the form:

$$\tilde{C}_p = \{ \langle x, F(\mu_{\tilde{C}_p}(x), \tau), F(\vartheta_{\tilde{C}_p}(x), \tau) \rangle \mid x \in X \} \quad (4.10)$$

where the function $F(\mu_{\tilde{C}_p}(x), \tau): X \rightarrow [0, 1]$ and $F(\vartheta_{\tilde{C}_p}(x), \tau): X \rightarrow [0, 1]$ indicate continuous membership and non-membership degrees of x to the set $\tilde{C}_p$, respectively. Where $F(\mu_{\tilde{C}_p}(x), \tau) = \sqrt{\alpha\tau - \alpha\tau^2}$ and $F(\vartheta_{\tilde{C}_p}(x), \tau) = \sqrt{(4 - \alpha)\tau - (4 - \alpha)\tau^2}$. τ represents the decision maker's behavior against uncertainty. The closer the τ value is to the center (0.4 - 0.6), the more risk-averse the decision maker will be, and the further away from the center (0 - 0.2 or 0.8 - 1), the more risk-taking the decision maker will be. If the τ value is between 0.2 - 0.4 or 0.6 - 0.8, the decision maker exhibits a risk-neutral attitude. The type of risk attitude of the decision maker on CPFUSs is presented in Fig. 4.6. A CPFUS satisfies the condition

$$\begin{aligned} 0 \leq F\left(\left(\mu_{\tilde{C}_p}(x)\right)^2, \tau\right) + F\left(\left(\vartheta_{\tilde{C}_p}(x)\right)^2, \tau\right) &\leq 1 \Rightarrow 0 \\ &\leq \left(\sqrt{\alpha\tau - \alpha\tau^2}\right)^2 + \left(\sqrt{(4 - \alpha)\tau - (4 - \alpha)\tau^2}\right)^2 \\ &\leq 1, \text{ for } \forall x \in X \end{aligned} \quad (4.11)$$

The hesitancy is defined as $F(\pi, \tau) = \sqrt{1 - \left(\sqrt{\alpha\tau - \alpha\tau^2}\right)^2 - \left(\sqrt{(4 - \alpha)\tau - (4 - \alpha)\tau^2}\right)^2}$. If $\sqrt{\alpha\tau - \alpha\tau^2} = \mu' = L$ and $\sqrt{(4 - \alpha)\tau - (4 - \alpha)\tau^2} = \vartheta' = M$, then $F(\pi, \tau) = \sqrt{1 - L^2 - M^2} = \pi'$.

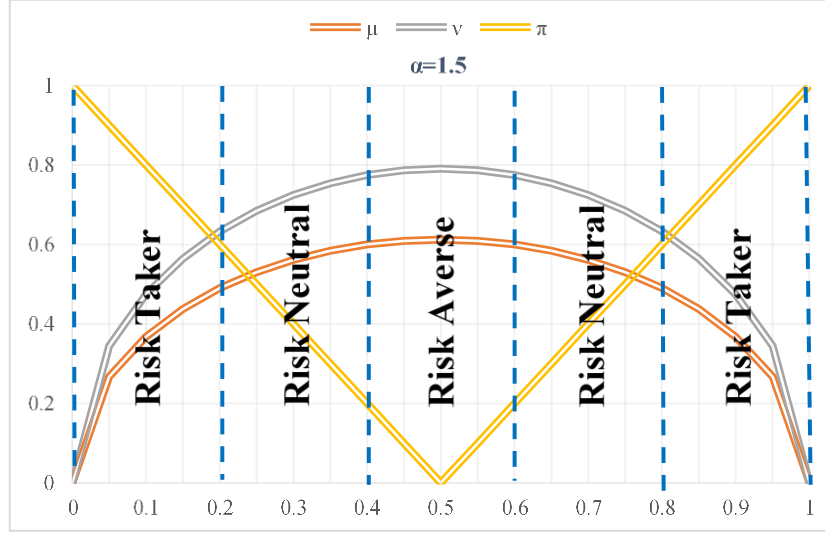


Figure 4.6 : Type of risk attitude of the decision maker on CPFUSs.

The membership, non-membership and hesitancy degrees of CPFUSs are represented for $\alpha=0.5$ and $\alpha=2.5$ as in Fig. 4.7.

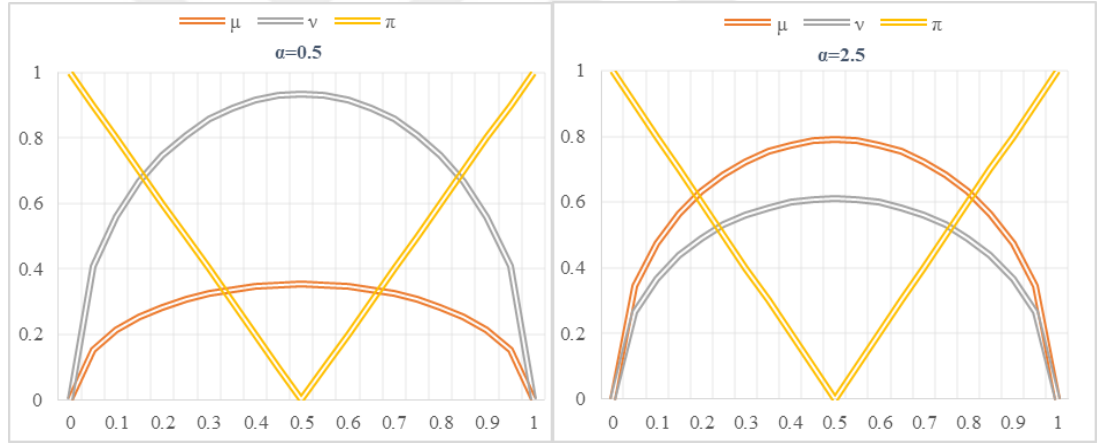


Figure 4.7 : Membership, non-membership and hesitancy degrees of CPFUSs for $\alpha=0.5$ and $\alpha=2.5$.

Definition 4.7. For every two $\tilde{Q}_m = (L_m, M_m)$ and $\tilde{Q}_n = (L_n, M_n)$, their basic operations are described as follows:

$$\tilde{Q}_m \subset \tilde{Q}_n = \{(L_m, M_m) < (L_n, M_n)\} \quad (4.12)$$

and thus,

$$\tilde{Q}_m \subset \tilde{Q}_n = \{L_m < L_n, M_m > M_n\} \quad (4.13)$$

$$\tilde{Q}_m = \tilde{Q}_n \leftrightarrow \{(L_m, M_m) = (L_n, M_n)\} \quad (4.14)$$

and thus,

$$\tilde{Q}_m = \tilde{Q}_n \leftrightarrow \{L_m = L_n, M_m = M_n\} \quad (4.15)$$

$$\tilde{Q}_m \cup \tilde{Q}_n = \{(max(L_m, L_n), min(M_m, M_n))\} \quad (4.16)$$

$$\tilde{Q}_m \cap \tilde{Q}_n = \{(min(L_m, L_n), max(M_m, M_n))\} \quad (4.17)$$

Definition 4.8. Let $\tilde{Q} = (L, M)$, $\tilde{Q}_m = (L_m, M_m)$ and $\tilde{Q}_n = (L_n, M_n)$ be three CPFUSs, then the mathematical operations on these three continuous Pythagorean fuzzy numbers (CPFUNs) are expressed as follows:

Addition operation:

$$\begin{aligned} \tilde{Q}_m \oplus \tilde{Q}_n &= \left(\sqrt{(\alpha_m + \alpha_n)(\tau - \tau^2) - \alpha_m \alpha_n (\tau - \tau^2)^2}, \right. \\ &\quad \left. \sqrt{((4 - \alpha_m)\tau - (4 - \alpha_m)\tau^2)((4 - \alpha_n)\tau - (4 - \alpha_n)\tau^2)} \right) \\ &= \left(\sqrt{(L_m)^2 + (L_n)^2 - (L_m)^2(L_n)^2}, \quad M_m M_n \right) \end{aligned} \quad (4.18)$$

The results of mathematical operations of CPFUSs are shown in Figs. 4.8, 4.9, 4.10, and 4.11, respectively. The membership and non-membership degrees of $\tilde{Q}_m$ and $\tilde{Q}_n$ CPFUNs and the sum of these two CPFUNs are presented for $\alpha_m = 1.5$ and $\alpha_n = 2.5$ in Fig. 4.8.

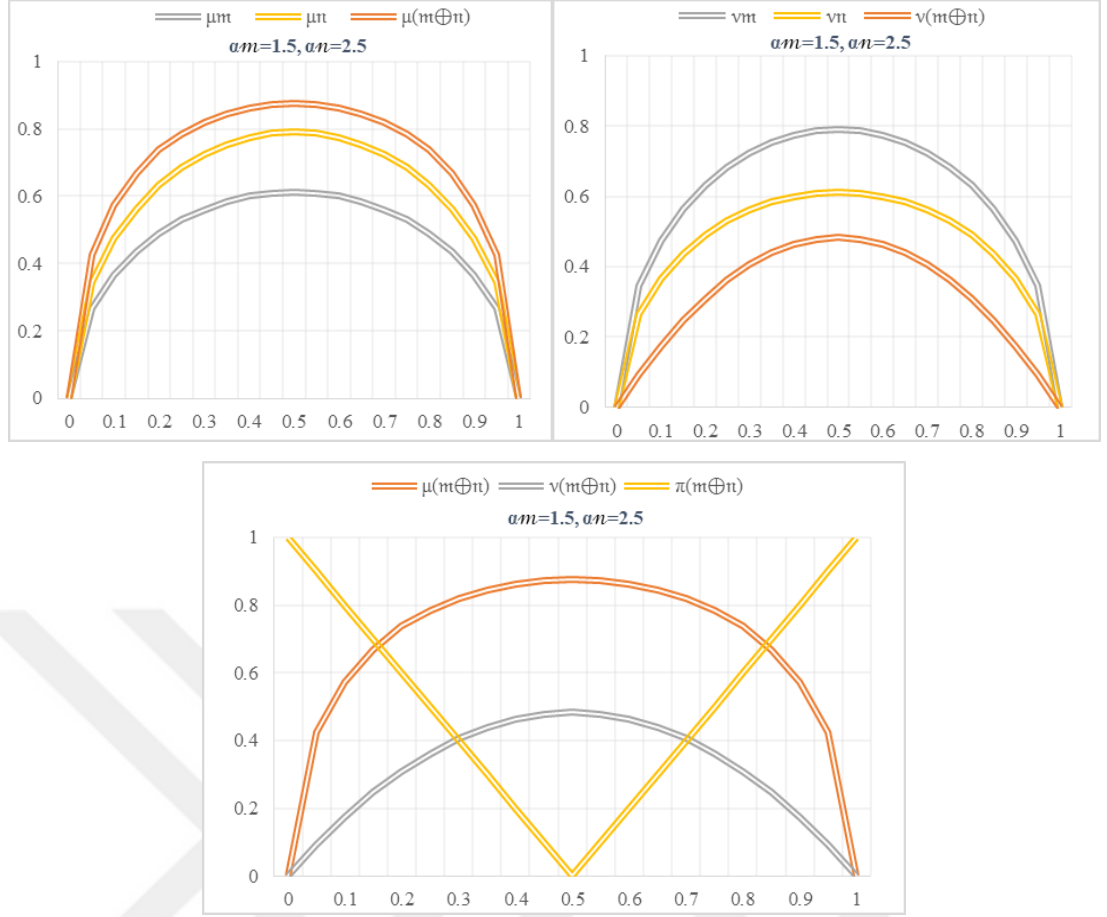


Figure 4.8 : Addition operation for two CPFUNs.

Multiplication operation:

$$\begin{aligned}
 & \tilde{Q}_m \otimes \tilde{Q}_n \\
 &= \left(\sqrt{\alpha_m \alpha_n (\tau - \tau^2)^2}, \right. \\
 & \quad \left. \sqrt{(8 - \alpha_m - \alpha_n)(\tau - \tau^2) - (4 - \alpha_m)(4 - \alpha_n)(\tau - \tau^2)^2} \right) \\
 &= \left(L_m L_n, \sqrt{(M_m)^2 + (M_n)^2 - (M_m)^2 (M_n)^2} \right)
 \end{aligned} \tag{4.19}$$

The membership and non-membership degrees of $\tilde{Q}_m$ and $\tilde{Q}_n$ CPFUNs for $\alpha_m = 3$ and $\alpha_n = 1.5$ and the product of these two CPFUNs are shown in Fig. 4.9.

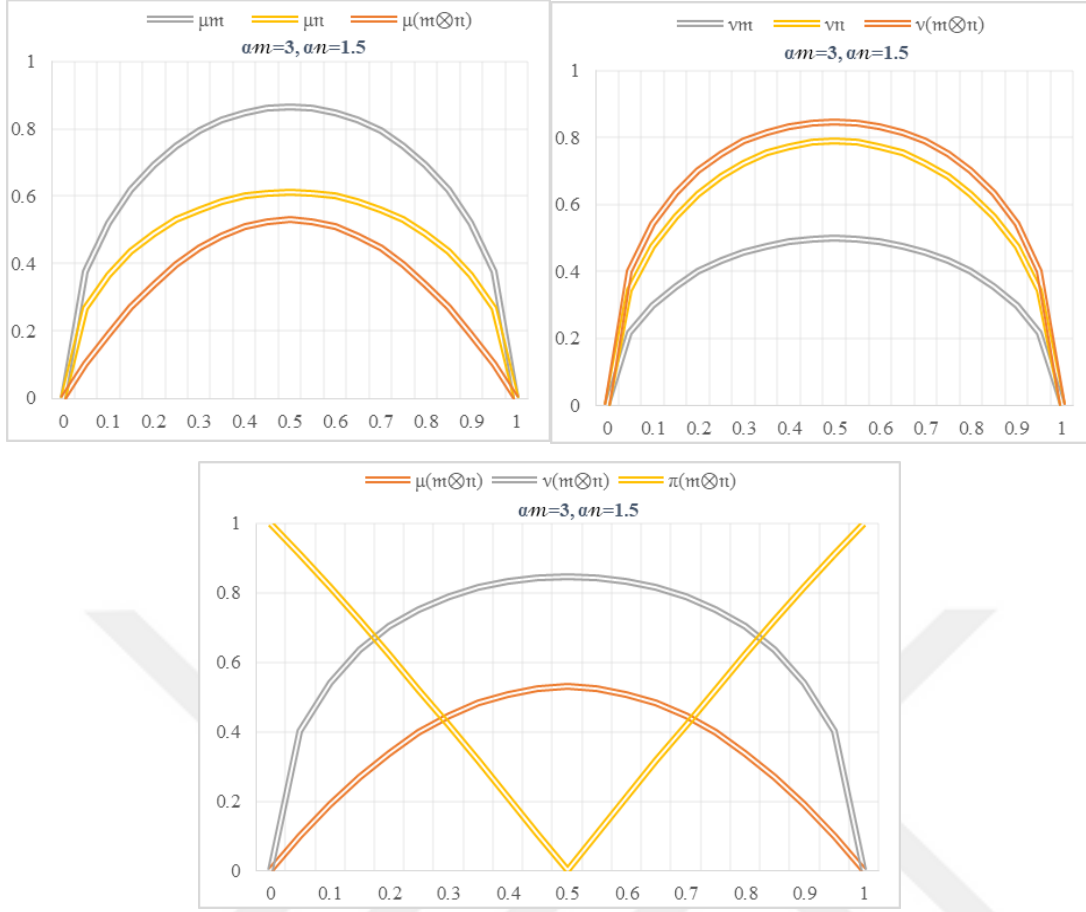


Figure 4.9 : Multiplication operation for two CPFUNs.

Multiplication by a Scalar:

$$\begin{aligned}
 \lambda \times \tilde{Q} &= \left(\sqrt{1 - (1 - \alpha\tau + \alpha\tau^2)^\lambda}, \left(\sqrt{((4 - \alpha)\tau - (4 - \alpha)\tau^2)} \right)^\lambda \right) \\
 &= \left(\sqrt{1 - (1 - (L)^2)^\lambda}, (M)^\lambda \right)
 \end{aligned} \tag{4.20}$$

In Fig. 4.10, the membership and non-membership degrees of a CPFUN for $\alpha = 0.5$ multiplied by the scalar number $\lambda = 3.5$ and the corresponding hesitation degree is shown.

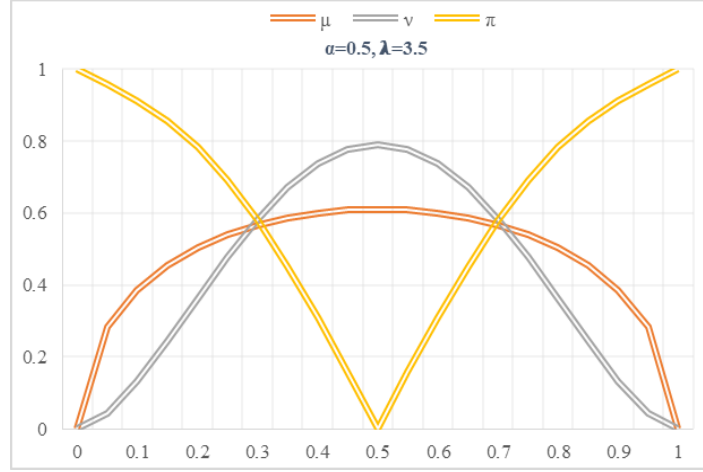


Figure 4.10 : Multiplication by a scalar of membership functions of a CPFUN.

Power operation:

$$\begin{aligned}
 (\tilde{Q})^\lambda &= \left(\left(\sqrt{(\alpha\tau - \alpha\tau^2)} \right)^\lambda, \sqrt{1 - (1 - (4 - \alpha)(\tau - \tau^2))^\lambda} \right) \\
 &= \left((L)^\lambda, \sqrt{1 - (1 - (M)^2)^\lambda} \right)
 \end{aligned} \tag{4.21}$$

The membership and non-membership degrees of a CPFUN for $\alpha = 2.5$ to the scalar number power $\lambda = 3$ and the corresponding degree of hesitation are presented in Fig. 4.11.

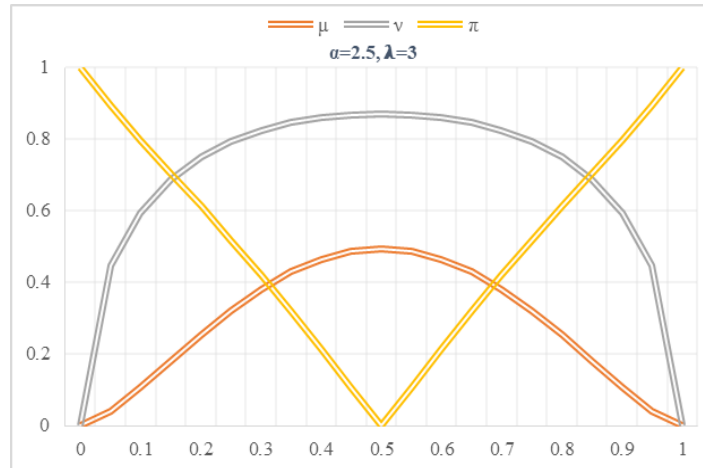


Figure 4.11 : Scalar number power of membership functions of a CPFUN.

4.2.2 Aggregation operators

In this sub-section, the continuous Pythagorean fuzzy weighted average aggregation operator and the continuous Pythagorean fuzzy weighted geometric aggregation operator of CPFUSs are given as follows.

Definition 4.9. Let $\tilde{\mathbb{C}}_{P_i} = \langle (L_i, M_i) \rangle, (i = 1, 2, \dots, n)$ be a set of CPFUSs and $\delta = (\delta_1, \delta_2, \dots, \delta_n)^T$ be weight vector of $\tilde{\mathbb{C}}_{P_i}$ with $\sum_{i=1}^n \delta_i = 1$, then a continuous Pythagorean fuzzy weighted average (CPFUWA) is expressed as in Eq. (4.22).

$$\begin{aligned}
& CPFUWA (\tilde{\mathbb{C}}_{P_1}, \tilde{\mathbb{C}}_{P_2}, \dots, \tilde{\mathbb{C}}_{P_n}) \\
&= \left(\sqrt{1 - \prod_{i=1}^n \left(1 - \left(\sqrt{(\alpha\tau - \alpha\tau^2)_i} \right)^2 \right)^{\delta_i}}, \prod_{i=1}^n \left(\sqrt{((4-\alpha)\tau - (4-\alpha)\tau^2)_i} \right)^{\delta_i} \right) \\
&= \left(\sqrt{1 - \prod_{i=1}^n (1 - (L_i)^2)^{\delta_i}}, \prod_{i=1}^n (M_i)^{\delta_i} \right)
\end{aligned} \tag{4.22}$$

Definition 4.10. Let $\tilde{\mathbb{C}}_{P_i} = \langle (L_i, M_i) \rangle, (i = 1, 2, \dots, n)$ be a set of CPFUSs and $\delta = (\delta_1, \delta_2, \dots, \delta_n)^T$ be weight vector of $\tilde{\mathbb{C}}_{P_i}$ with $\sum_{i=1}^n \delta_i = 1$, then a continuous Pythagorean fuzzy weighted geometric (CPFUWG) is expressed as in Eq. (4.23).

$$\begin{aligned}
& CPFUWG (\tilde{\mathbb{C}}_{P_1}, \tilde{\mathbb{C}}_{P_2}, \dots, \tilde{\mathbb{C}}_{P_n}) \\
&= \left(\prod_{i=1}^n \left(\sqrt{(\alpha\tau - \alpha\tau^2)_i} \right)^{\delta_i}, \sqrt{1 - \prod_{i=1}^n \left(1 - \left(\sqrt{((4-\alpha)\tau - (4-\alpha)\tau^2)_i} \right)^2 \right)^{\delta_i}} \right) \\
&= \left(\prod_{i=1}^n (L_i)^{\delta_i}, \sqrt{1 - \prod_{i=1}^n (1 - (M_i)^2)^{\delta_i}} \right)
\end{aligned} \tag{4.23}$$

Definition 4.11. Let $\tilde{\mathbb{C}}_{P_i} = \langle (L_i, M_i) \rangle, (i = 1, 2, \dots, n)$ be a set of CPFUSs and $\delta = (\delta_1, \delta_2, \dots, \delta_n)^T$ be weight vector of $\tilde{\mathbb{C}}_{P_i}$ with $\sum_{i=1}^n \delta_i = 1$, then a continuous Pythagorean fuzzy Dombi weighted geometric (CPFUDWG) is expressed as in Eq. (4.24).

$$CPFUDWG(\tilde{\mathbb{C}}_1, \tilde{\mathbb{C}}_2, \dots, \tilde{\mathbb{C}}_n)$$

$$= \left(\frac{1}{1 + \left\{ \sum_{i=1}^n \delta \left(\frac{1 - \sqrt{(\alpha\tau - \alpha\tau^2)_i}}{\sqrt{(\alpha\tau - \alpha\tau^2)_i}} \right)^\rho \right\}^{1/\rho}}, \right. \quad (4.24)$$

$$\left. 1 - \frac{1}{1 + \left\{ \sum_{i=1}^n \delta \left(\frac{\sqrt{((4-\alpha)\tau - (4-\alpha)\tau^2)_i}}{1 - \sqrt{((4-\alpha)\tau - (4-\alpha)\tau^2)_i}} \right)^\rho \right\}^{1/\rho}} \right)$$

Definition 4.12. Let $\tilde{\mathbb{C}}_{P_i} = \langle (L_i, M_i) \rangle$, $(i = 1, 2, \dots, n)$ be a set of CPFUSs and $\delta = (\delta_1, \delta_2, \dots, \delta_n)^T$ be weight vector of $\tilde{\mathbb{C}}_{P_i}$ with $\sum_{i=1}^n \delta_i = 1$, then a continuous Pythagorean fuzzy Einstein weighted geometric (*CPFUEWG*) is expressed as in Eq. (4.25).

$$CPFUEWG(\tilde{\mathbb{C}}_1, \tilde{\mathbb{C}}_2, \dots, \tilde{\mathbb{C}}_n)$$

$$= \left(\frac{2 \prod_{i=1}^n (\sqrt{(\alpha\tau - \alpha\tau^2)_i})^{\delta_i}}{\prod_{i=1}^n (2 - \sqrt{(\alpha\tau - \alpha\tau^2)_i})^{\delta_i} + \prod_{i=1}^n (\sqrt{(\alpha\tau - \alpha\tau^2)_i})^{\delta_i}}, \right. \quad (4.25)$$

$$\left. \frac{\prod_{i=1}^n \left(1 + \sqrt{((4-\alpha)\tau - (4-\alpha)\tau^2)_i} \right)^{\delta_i} - \prod_{i=1}^n \left(1 - \sqrt{((4-\alpha)\tau - (4-\alpha)\tau^2)_i} \right)^{\delta_i}}{\prod_{i=1}^n \left(1 + ((4-\alpha)\tau - (4-\alpha)\tau^2)_i \right)^{\delta_i} + \prod_{i=1}^n \left(1 - ((4-\alpha)\tau - (4-\alpha)\tau^2)_i \right)^{\delta_i}} \right)$$

Definition 4.13. Let $\tilde{\mathbb{C}}_{P_i} = \langle (L_i, M_i) \rangle$, $(i = 1, 2, \dots, n)$ be a set of CPFUSs and $\delta = (\delta_1, \delta_2, \dots, \delta_n)^T$ be weight vector of $\tilde{\mathbb{C}}_{P_i}$ with $\sum_{i=1}^n \delta_i = 1$, then a continuous Pythagorean fuzzy Hamacher weighted geometric (*CPFUHWG*) is expressed as in Eq. (4.26).

$$\begin{aligned}
& CPFUHWG(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) \\
&= \left(\frac{\varphi \prod_{i=1}^n (\sqrt{(\alpha\tau - \alpha\tau^2)_i})^{\delta_i}}{\prod_{i=1}^n (1 + (\varphi - 1)(1 - \sqrt{(\alpha\tau - \alpha\tau^2)_i})^{\delta_i}) + (\varphi - 1) \prod_{i=1}^n (\sqrt{(\alpha\tau - \alpha\tau^2)_i})^{\delta_i}}, \right. \\
&\quad \left. \frac{\prod_{i=1}^n (1 + (\varphi - 1)\sqrt{((4 - \alpha)\tau - (4 - \alpha)\tau^2)_i})^{\delta_i} - \prod_{i=1}^n (1 - \sqrt{((4 - \alpha)\tau - (4 - \alpha)\tau^2)_i})^{\delta_i}}{\prod_{i=1}^n (1 + (\varphi - 1)\sqrt{((4 - \alpha)\tau - (4 - \alpha)\tau^2)_i})^{\delta_i} + (\varphi - 1) \prod_{i=1}^n (1 - \sqrt{((4 - \alpha)\tau - (4 - \alpha)\tau^2)_i})^{\delta_i}} \right)
\end{aligned} \tag{4.26}$$

4.2.3 Score and accuracy functions

In the section, the score and accuracy functions of the continuous Pythagorean fuzzy sets are defined as follows.

Definition 4.14. Let $\mathfrak{H} = \langle (L, M) \rangle$ be an *CPFUN*. The following formulas can be indicated for the score function $\mathcal{S}(\mathfrak{H})$ and the accuracy function $\mathcal{A}(\mathfrak{H})$ of *CPFUNs* as in Eqs. (4.27) and (4.28), respectively.

$$\mathcal{S}(\mathfrak{H}) = (L)^2 - (M)^2 = 2\alpha\tau - 4\tau + 4\tau^2 - 2\alpha\tau^2 \tag{4.27}$$

where $\mathcal{S}(\mathfrak{H}) \in [-1, 1]$.

$$\mathcal{A}(\mathfrak{H}) = (L)^2 + (M)^2 = 4\tau - 4\tau^2 \tag{4.28}$$

where $\mathcal{A}(\mathfrak{H}) \in [0, 1]$.

Definition 4.15. Let $\mathfrak{H}_i = \langle (\sqrt{(\alpha\tau - \alpha\tau^2)_i}, \sqrt{((4 - \alpha)\tau - (4 - \alpha)\tau^2)_i}) \rangle = \langle (L_i, M_i) \rangle$ ($i = 1, 2$) be two *CPFUNs*, $\mathcal{S}(\mathfrak{H}_1)$ and $\mathcal{S}(\mathfrak{H}_2)$ be scores of $\mathfrak{H}_1$ and $\mathfrak{H}_2$, respectively, and $\mathcal{A}(\mathfrak{H}_1)$ and $\mathcal{A}(\mathfrak{H}_2)$ be the accuracy degrees of $\mathfrak{H}_1$ and $\mathfrak{H}_2$, respectively, then the comparative rules are as follows:

If $\mathcal{S}(\mathfrak{H}_1) < \mathcal{S}(\mathfrak{H}_2)$, then $\mathfrak{H}_1 < \mathfrak{H}_2$,

If $\mathcal{S}(\mathfrak{H}_1) > \mathcal{S}(\mathfrak{H}_2)$, then $\mathfrak{H}_1 > \mathfrak{H}_2$,

If $\mathcal{S}(\mathfrak{H}_1) = \mathcal{S}(\mathfrak{H}_2)$, then

- (1) If $\mathcal{A}(\mathfrak{H}_1) = \mathcal{A}(\mathfrak{H}_2)$, then $\mathfrak{H}_1 = \mathfrak{H}_2$;
- (2) If $\mathcal{A}(\mathfrak{H}_1) < \mathcal{A}(\mathfrak{H}_2)$, then $\mathfrak{H}_1 < \mathfrak{H}_2$;
- (3) If $\mathcal{A}(\mathfrak{H}_1) > \mathcal{A}(\mathfrak{H}_2)$, then $\mathfrak{H}_1 > \mathfrak{H}_2$;

4.2.4 Distance measurements

In this section, the Hamming and Euclidean distances of the continuous Pythagorean fuzzy sets are given as in Eqs. (4.29) and (4.30).

Definition 4.16. Let $\wp_i = \left(\sqrt{(\alpha\tau - \alpha\tau^2)_i}, \sqrt{((4 - \alpha)\tau - (4 - \alpha)\tau^2)_i} \right) = (L_i, M_i)$ ($i = 1, 2$) be two CPFUNs. Then, the Hamming and Euclidean distances between $\wp_1$ and $\wp_2$ are given as follows.

$$\mathfrak{D}_H(\wp_1, \wp_2) = \frac{1}{2} \left(\left| (L_{\wp_1})^2 - (L_{\wp_2})^2 \right| + \left| (M_{\wp_1})^2 - (M_{\wp_2})^2 \right| + \left| \left(\sqrt{1 - (L_{\wp_1})^2 - (M_{\wp_1})^2} \right)^2 - \left(\sqrt{1 - (L_{\wp_2})^2 - (M_{\wp_2})^2} \right)^2 \right| \right) \quad (4.29)$$

$$\mathfrak{D}_E(\wp_1, \wp_2) = \sqrt{\frac{1}{2} \left(\left((L_{\wp_1})^2 - (L_{\wp_2})^2 \right)^2 + \left((M_{\wp_1})^2 - (M_{\wp_2})^2 \right)^2 + \left(\left(\sqrt{1 - (L_{\wp_1})^2 - (M_{\wp_1})^2} \right)^2 - \left(\sqrt{1 - (L_{\wp_2})^2 - (M_{\wp_2})^2} \right)^2 \right)^2 \right)} \quad (4.30)$$

Definition 4.17. Let $\wp_i = \left(\sqrt{(\alpha\tau - \alpha\tau^2)_i}, \sqrt{((4 - \alpha)\tau - (4 - \alpha)\tau^2)_i} \right) = (L_i, M_i)$ ($i = 1, 2$) be two CPFUNs, then

1. $\mathfrak{D}_H(\wp_1, \wp_2) = \mathfrak{D}_H(\wp_2, \wp_1)$ and $\mathfrak{D}_E(\wp_1, \wp_2) = \mathfrak{D}_E(\wp_2, \wp_1)$.
2. $\mathfrak{D}_H(\wp_1, \wp_2) = 0$ and $\mathfrak{D}_E(\wp_1, \wp_2) = 0$ only if $\wp_1 = \wp_2$.
3. $0 \leq \mathfrak{D}_H(\wp_1, \wp_2) \leq 1$ and $0 \leq \mathfrak{D}_E(\wp_1, \wp_2) \leq 1$.

4.3 MADM Applications

In this section, we handle two MADM problems in which the proposed CPFUSs are utilized. The considered MADM problems are green supplier selection and waste disposal site selection. The flowchart showing the steps of the applications is presented in Fig. 4.12. The experts are three professors at the department of industrial engineering. The linguistic data are collected by a questionnaire from these professors. We first defined the linguistic terms such as Very Low, Low, High, and Very High, representing different levels of a particular attribute. Then, we converted the linguistic data to fuzzy numbers. Each linguistic term is converted into a fuzzy number based on the defined fuzzy sets. For example, a response of "Low Important" is translated into

$(\sqrt{\tau - \tau^2}, \sqrt{3\tau - 3\tau^2})$. Finally, we aggregated the fuzzy data to combine the assessments, creating a consensus or representative fuzzy score.

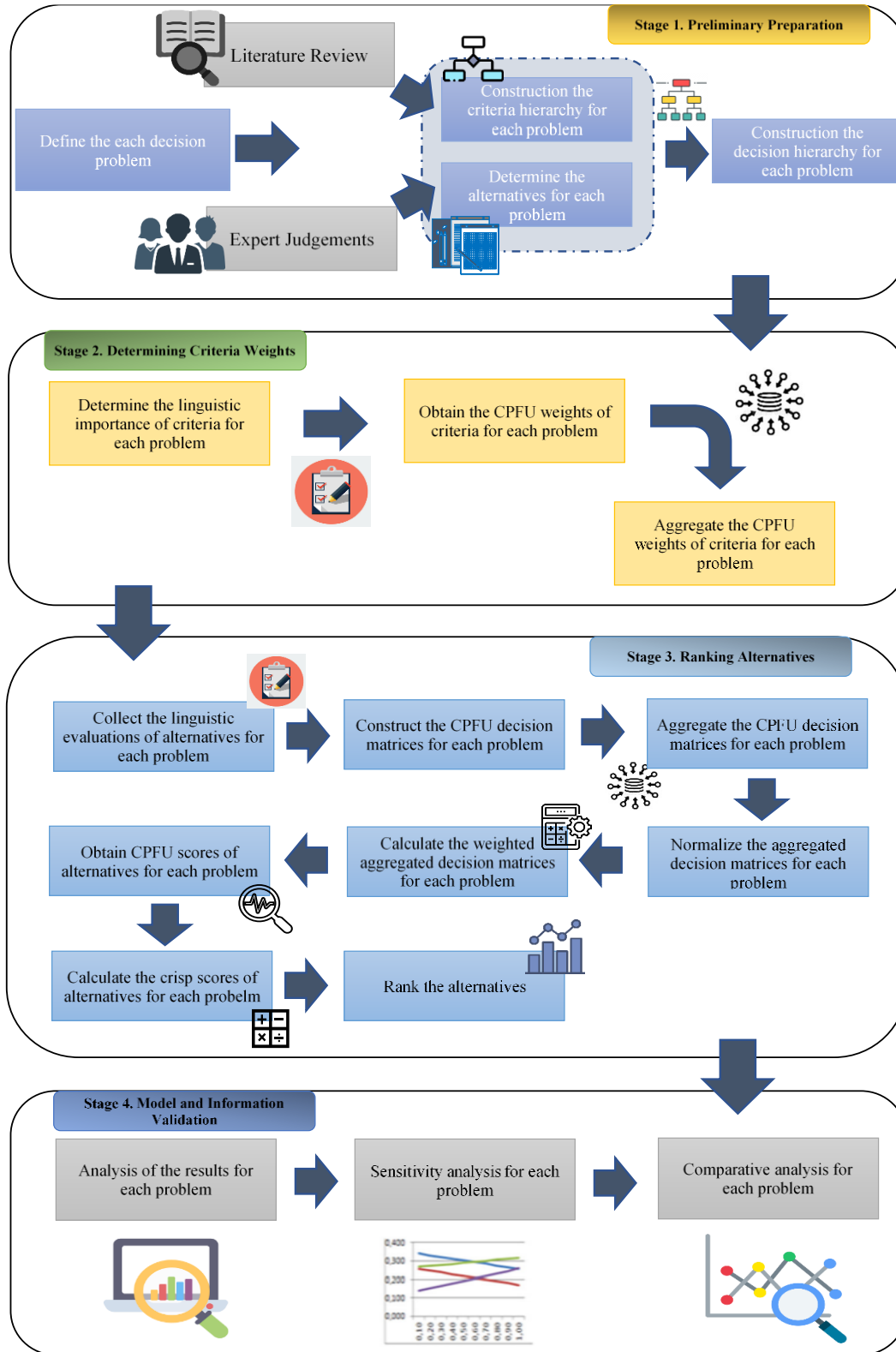


Figure 4.12 : Flowchart of the MADM applications.

4.3.1 Green supplier selection

Green supplier selection problem refers to a decision-making challenge faced by businesses and organizations when choosing suppliers for their products or services with a focus on environmental sustainability. In this context, "green" refers to environmentally friendly and socially responsible practices. This problem involves evaluating and selecting suppliers based on their commitment to and performance in areas such as environmental conservation, energy efficiency, waste reduction, and ethical labor practices (Kara et al., 2024; Saputro et al., 2024). Therefore, the existence of many criteria for choosing the best green supplier involves a complex problem that requires the optimal fulfillment of all relevant factors. Because there are multiple conflicting factors in the green supplier selection problem, the evaluation of green suppliers constitutes a multi-criteria decision-making (MCDM) problem (Wang et al., 2023). Besides, since today's situation and conditions bring uncertainty, addressing the green supplier selection problem with the fuzzy MCDM method or methodology provides more accurate results. Therefore, in this study, we first present a CPFU-based MCDM model that can take imprecise and uncertain evaluations into account more broadly and continuously in the evaluation of green supplier alternatives. The evaluation criteria are determined as C1-Reputation, C2-Saving in price, C3-Experience, C4-Environmental care and C5-Flexibility based on a comprehensive literature review. The criteria considered in the study can be briefly summarized as follows.

Reputation (C1): It focuses on how the supplier is perceived by stakeholders, including customers, competitors, regulatory bodies, and the general public, in relation to its commitment to sustainability (Gören, 2018; Goudarzi & Gholamian, 2024).

Saving in price (C2): Choosing a green supplier should be economically viable as well as environmental and social considerations. A lower price will provide more cost savings (Goudarzi & Gholamian, 2024; Ravi Ramakrishnan et al., 2020).

Experience (C3): It refers to the supplier's past performance and track record in implementing green and sustainable practices. This criterion evaluates a supplier's level of familiarity and success in incorporating environmentally friendly and socially responsible initiatives into their operations (Goudarzi & Gholamian, 2024; X. Wang et al., 2024).

Environmental care (C4): It is used to evaluate and select the suppliers according to their environmental performance and commitment to sustainable practices. This criterion focuses on assessing the effect of a supplier's operations on the environment and the extent to which they engage in eco-friendly initiatives (Goudarzi & Gholamian, 2024; Lo et al., 2018).

Flexibility (C5): It refers to a supplier's ability to adapt and respond to changes in environmental requirements, regulations, and market conditions. This criterion assesses how well a supplier can adjust its operations and practices to meet evolving sustainability standards, regulations, and any unforeseen challenges that may arise (Gören, 2018; Goudarzi & Gholamian, 2024; Ravi Ramakrishnan et al., 2020).

Implementing effective green supplier selection strategies can contribute to a company's overall sustainability goals and help meet the growing demand for environmentally responsible business practices. A medicine company is searching a supplier who cares environmental issues in the production activities. The medicine company has determined four green supplier alternatives. These alternatives have been evaluated by three experts, E1, E2 and E3 with respect to the considered criteria. The experts' weights are 0.25, 0.45 and 0.30, respectively. These experts are three professors at the department of industrial engineering.

Table 4.1 : Linguistic scale for criteria weighting based on CPFUNs.

Linguistic terms for criteria	Linguistic terms for alternatives	CPFUNs	
		μ	ϑ
Absolutely low important – (ALI)	Absolutely low performance – (ALP)	$\sqrt{0.25\tau - 0.25\tau^2}$	$\sqrt{3.75\tau - 3.75\tau^2}$
Very low important – (VLI)	Very low performance – (VLP)	$\sqrt{0.5\tau - 0.5\tau^2}$	$\sqrt{3.5\tau - 3.5\tau^2}$
Low important – (LI)	Low performance – (LP)	$\sqrt{\tau - \tau^2}$	$\sqrt{3\tau - 3\tau^2}$
Slightly low important – (SLI)	Slightly low performance – (SLP)	$\sqrt{1.5\tau - 1.5\tau^2}$	$\sqrt{2.5\tau - 2.5\tau^2}$
Medium important – (MI)	Medium performance – (MP)	$\sqrt{2\tau - 2\tau^2}$	$\sqrt{2\tau - 2\tau^2}$
Slightly high important – (SHI)	Slightly high performance – (SHP)	$\sqrt{2.5\tau - 2.5\tau^2}$	$\sqrt{1.5\tau - 1.5\tau^2}$
High important – (HI)	High performance – (HP)	$\sqrt{3\tau - 3\tau^2}$	$\sqrt{1\tau - 1\tau^2}$
Very high important – (VHI)	Very high performance – (VHP)	$\sqrt{3.5\tau - 3.5\tau^2}$	$\sqrt{0.5\tau - 0.5\tau^2}$
Absolutely high important – (AHI)	Absolutely high performance – (AHP)	$\sqrt{3.75\tau - 3.75\tau^2}$	$\sqrt{0.25\tau - 0.25\tau^2}$

After each expert evaluates the criteria as presented in Table 4.2 by utilizing the CPFU linguistic scale given in Table 4.1, the criteria weight matrix consisting of the linguistic evaluations of experts is transformed into the corresponding CPFUNs as presented in Table 4.3.

Table 4.2 : Linguistic assessments for weighting the criteria for green supplier selection.

	E1	E2	E3
C1	SLI	LI	VLI
C2	HI	VHI	HI
C3	HI	SHI	SHI
C4	VHI	HI	HI
C5	SLI	VLI	LI

Table 4.3 : Criteria weight matrix based on CPFUNs of each expert for green supplier selection.

	E1	E2	E3
C1	$(\sqrt{1.5\tau - 1.5\tau^2}, \sqrt{2.5\tau - 2.5\tau^2})$	$(\sqrt{1\tau - 1\tau^2}, \sqrt{3\tau - 3\tau^2})$	$(\sqrt{0.5\tau - 0.5\tau^2}, \sqrt{3.5\tau - 3.5\tau^2})$
C2	$(\sqrt{3\tau - 3\tau^2}, \sqrt{1\tau - 1\tau^2})$	$(\sqrt{3.5\tau - 3.5\tau^2}, \sqrt{0.5\tau - 0.5\tau^2})$	$(\sqrt{3\tau - 3\tau^2}, \sqrt{1\tau - 1\tau^2})$
C3	$(\sqrt{3\tau - 3\tau^2}, \sqrt{1\tau - 1\tau^2})$	$(\sqrt{2.5\tau - 2.5\tau^2}, \sqrt{1.5\tau - 1.5\tau^2})$	$(\sqrt{2.5\tau - 2.5\tau^2}, \sqrt{1.5\tau - 1.5\tau^2})$
C4	$(\sqrt{3.5\tau - 3.5\tau^2}, \sqrt{0.5\tau - 0.5\tau^2})$	$(\sqrt{3\tau - 3\tau^2}, \sqrt{1\tau - 1\tau^2})$	$(\sqrt{3\tau - 3\tau^2}, \sqrt{1\tau - 1\tau^2})$
C5	$(\sqrt{1.5\tau - 1.5\tau^2}, \sqrt{2.5\tau - 2.5\tau^2})$	$(\sqrt{0.5\tau - 0.5\tau^2}, \sqrt{3.5\tau - 3.5\tau^2})$	$(\sqrt{1\tau - 1\tau^2}, \sqrt{3\tau - 3\tau^2})$

The criteria weight matrix consisting of CPFUNs based on each expert is aggregated to obtain a single decision matrix considering the experts' risk behaviors by using Eq. (4.23) as in Table 4.4. Experts have determined their risk behaviors as risk taker, risk neutral and risk averse, respectively, based on their experience and knowledge in the field. Here, each expert has also determined the risk degree as 0.1, 0.36 and 0.42, respectively.

Table 4.4 : Aggregated criteria weights for green supplier selection.

C1	(0.408, 0.830)
C2	(0.772, 0.387)
C3	(0.696, 0.544)
C4	(0.760, 0.438)
C5	(0.387, 0.835)

The experts use the CPFU linguistic scale shown in Table 4.1 to assess the options in accordance with the stated goals and criteria. Experts' linguistic judgment matrices are built as shown in Table 4.5.

Table 4.5 : Linguistic decision matrix based on each expert for green supplier selection problem.

	E1					E2					E3				
	C1	C2	C3	C4	C5	C1	C2	C3	C4	C5	C1	C2	C3	C4	C5
A1	HP	HP	VHP	VHP	HP	HP	VHP	VHP	HP	HP	VHP	HP	HP	SHP	VHP
A2	SHP	MP	SHP	HP	HP	VHP	SHP	HP	SHP	HP	MP	SHP	VHP	HP	SHP
A3	VHP	AHP	HP	AHP	HP	HP	VHP	SHP	AHP	VHP	VHP	VHP	VHP	HP	AHP
A4	SHP	HP	HP	VHP	VHP	VHP	HP	HP	HP	VHP	HP	VHP	HP	SHP	VHP

After the linguistic decision matrices of experts are converted into the corresponding CPFUNs, each decision matrix is aggregated to obtain a single decision matrix by utilizing Eq. (4.23) as presented in Table 4.6. To obtain weighted aggregated decision matrix, the aggregated decision matrix is multiplied with aggregated criteria weight matrix by using Eq. (4.19) and the weighted aggregated decision matrix is computed as presented in Table 4.7.

Table 4.6 : Aggregated decision matrix for green supplier selection problem.

	C1	C2	C3	C4	C5
A1	(0.763, 0.406)	(0.772, 0.387)	(0.787, 0.374)	(0.739, 0.483)	(0.763, 0.406)
A2	(0.710, 0.503)	(0.662, 0.560)	(0.746, 0.419)	(0.715, 0.508)	(0.725, 0.493)
A3	(0.778, 0.394)	(0.812, 0.308)	(0.732, 0.473)	(0.806, 0.331)	(0.798, 0.305)
A4	(0.754, 0.401)	(0.746, 0.419)	(0.715, 0.508)	(0.739, 0.483)	(0.805, 0.317)

Table 4.7 : Weighted aggregated decision matrix for green supplier selection problem.

	C1	C2	C3	C4	C5
A1	(0.311, 0.860)	(0.596, 0.527)	(0.548, 0.628)	(0.562, 0.617)	(0.296, 0.865)
A2	(0.290, 0.876)	(0.511, 0.646)	(0.519, 0.648)	(0.544, 0.633)	(0.281, 0.878)
A3	(0.317, 0.858)	(0.627, 0.480)	(0.510, 0.673)	(0.612, 0.530)	(0.309, 0.852)
A4	(0.308, 0.859)	(0.575, 0.547)	(0.498, 0.691)	(0.562, 0.617)	(0.312, 0.853)

After the overall fuzzy score of each alternative is calculated with the weighted sum operation, the obtained CPFU scores are defuzzified by utilizing Eq. (4.27) and the crisp scores of the alternatives are identified as presented in Table 4.8.

Table 4.8 : Scores of alternatives and their rankings for green supplier selection.

	Overall CPFU scores	Crisp scores	Ranking
A1	(0.863,0.152)	0.722	2
A2	(0.824,0.204)	0.638	4
A3	(0.878,0.125)	0.756	1
A4	(0.856,0.161)	0.707	3

The alternatives are ranked by considering the obtained crisp scores as shown in Table 4.8 and Figure 4.13. According to the ranking results obtained, the ranking order of alternatives is determined as $A3 > A1 > A4 > A2$.

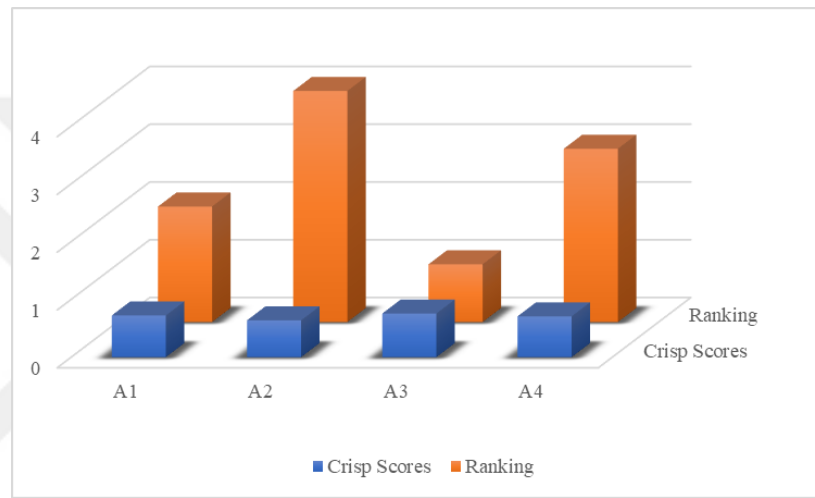


Figure 4.13 : Crisp scores and rankings of alternatives for green supplier selection.

4.3.2 Comparison with discrete IFS and discrete PFS

In this section, IFS-based and PFS-based MADM models are applied to compare and validate the results of the developed CPFU-MADM model. The comparative scores of the obtained results have been presented in Table 4.9 and Figure 4.14, respectively. According to the results of the compared model, the ranking order of the four alternatives is A3, A1, A4 and A2 in all models, respectively. Based on the results presented of the application of IFS-MADM (Alcantud, 2023) and PFS-MADM (Gao et al., 2018) models, Table 4.9 and Figure 4.14 clearly indicate that alternative A3 is the first ranked from the considered alternatives while alternative A1 obtains the second rank. Besides, the alternatives A4 and A2 are determined as the third and fourth-ranked alternatives, respectively.

Table 4.9 : Crisp scores of alternatives with respect to the compared decision models for green supplier selection.

	CPFU-MADM	IFS-MADM	PFS-MADM
A1	0.722	0.882	0.735
A2	0.638	0.814	0.618
A3	0.756	0.916	0.805
A4	0.707	0.864	0.686

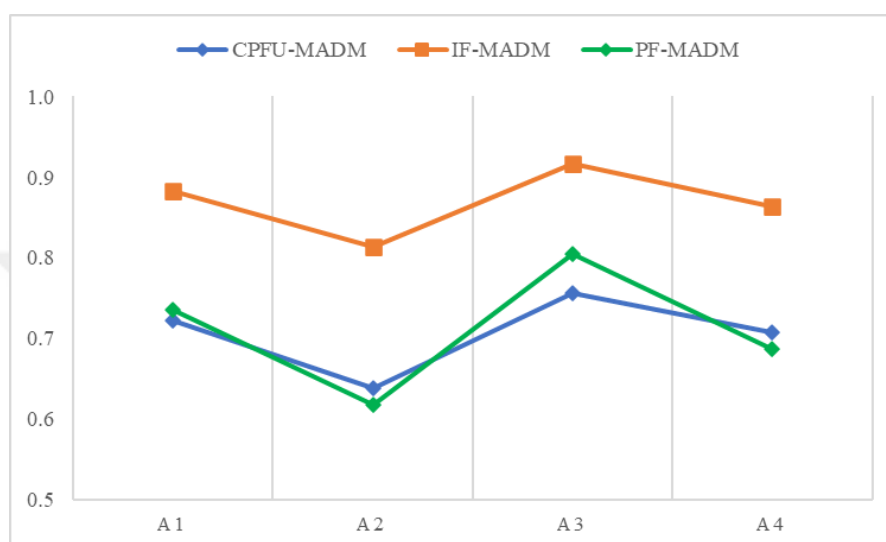


Figure 4.14 : Illustration of scores of alternatives for green supplier selection problem.

Uncertainty at every point can be taken into account with a continuous structure created thanks to the nonlinear functions used in the developed CPFU-based MADM model. Thanks to these structures of the CPFU-based MADM model, decision makers' ideas can be handled continuously and more comprehensively than IFSs and PFSs-based MADM models. Therefore, the developed CPFU-based MADM model has a higher degree of generalization than other IFSs and PFSs-based MADM models used for comparison.

4.3.3 Sensitivity analysis for green supplier selection problem

It is vital to determine whether variations in the input parameters alter the model results in order to assess the stability and robustness of the solutions found from MADM problems in the literature. A crucial stage in solving MADM problems is determining how sensitive the acquired results are to changes in the input variables and assessing how robust the model's conclusions are. Consistent with this scope, the sensitivity analysis of the findings is reported in two subsections in this section of the study. First,

through changes to the criteria's weight coefficients, the sensitivity of the CPFU-based MADM model to the ranking results has been examined. Then, the sensitivity of the change in experts' weights on the ranking results has been examined.

4.3.3.1 Sensitivity of the criteria weights

Following the determination of the criteria's weights through the use of the CPFUWG aggregation operator, the linguistic weight changes should be investigated to assess whether the modifications in criteria weights will result in an alteration of the alternatives' ranking. For this purpose, the linguistic weights of the criteria have undergone a one-to-one sensitivity analysis, and the effectiveness of the alternatives' ranking has been examined. It has been investigated how the final scores and ranking results of the alternatives are affected by the modification in the linguistic weight of each criterion. By changing the linguistic weight of each criterion from AHI to ALI, nine distinct modifications have been implemented for every criterion. The impact of altering the linguistic weights of the criterion on the alternatives' ranking results is presented in Fig. 4.15.

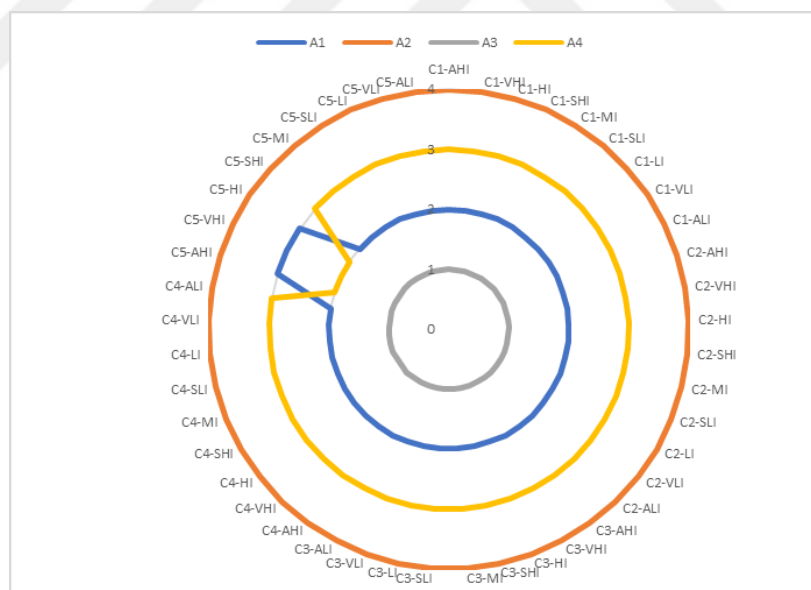


Figure 4.15 : Effects of the changes in the weights of the criteria for green supplier selection.

Changes in the linguistic weights of all criteria have a significant impact on the change in crisp scores of A_i alternatives ($i = 1, 2, 3, 4$). However, these changes do not lead to a significant change in the ranking of alternatives. The results show that while the ranking results of the alternatives are unaffected by changes in the weights of the C1, C2, C3, and C4 criteria, the ranking results of the alternatives are affected by changes

in the weight of the C5 criterion. It has been observed that when the weight of the C5 criterion is changed to AHI, VHI and HI, the rankings of the A1 and A4 alternatives are changed. It has been noted that alternative A3 keeps its dominant position by staying in the same order, whereas the last-ranked alternative, A2, stays the worst alternative in every situation. Only the weights of the C5 criterion have been shown to cause relatively minor variations in the ranks of the A1 and A4 alternatives. It can be concluded from the analysis that the final ranking results of the alternatives are not affected by the change in the weights of the criteria, save for a very tiny variation. This demonstrates that the developed model is robust, stable, and insensitive to changes in the criterion weights.

4.3.3.2 Sensitivity of the weights of experts

The weight of experts used in MADM problems can have a significant impact on the results. Therefore, in this part of the study, sensitivity analysis has been performed to investigate the impact of varying expert weights on the results for the green supplier selection problem. In the analysis, the weights of the three experts included in the green supplier selection problem were changed by $\pm 5\%$, $\pm 10\%$, $\pm 15\%$, $\pm 20\%$ and $\pm 25\%$, creating a total of 30 different scenarios consisting of 10 different scenarios for each expert and the ranking results were analyzed in each scenario. In the scenarios, the weights of the remaining experts are rearranged to satisfy the requirement $\sum_{j=1}^n w_j = 1$ when each expert's weight varies proportionately. The weight of each remaining expert is updated as follows. For example, for the green supplier selection problem, if the initial weight of E1 is increased by 5%, the increased weight is updated to 0.263. In this situation, the weight of E2 is updated using the formula $((1 - 0.263)/(1 - 0.25)) \times 0.45 = 0.443$. In this case, the same computation is applied to update the weight of E3. Changes in experts' weights are shown in Figs. 4.16, 4.17 and 4.18, respectively. The ranking results of the alternatives in each scenario are shown for all experts in Fig. 4.19. Based on the results obtained through each scenario, it has been noted that modifications to the expert weights do not impact the alternative rankings. Based on the 10 scenarios that were conducted for each expert, it has been concluded that alternative A3 is the best option and alternative A2 is the worst. This indicates that the MADM model, which was created using the CPFUWG operator as its basis, is reliable, stable, and insensitive to changes in the weights of experts.

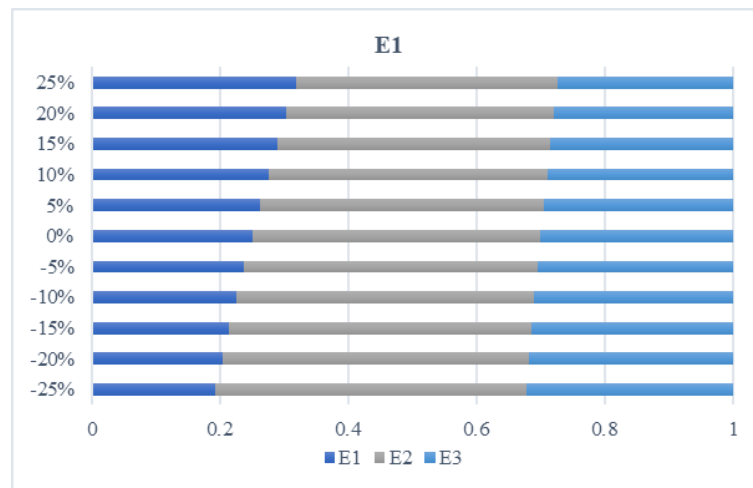


Figure 4.16 : Changes in the weight of E1 for green supplier selection.

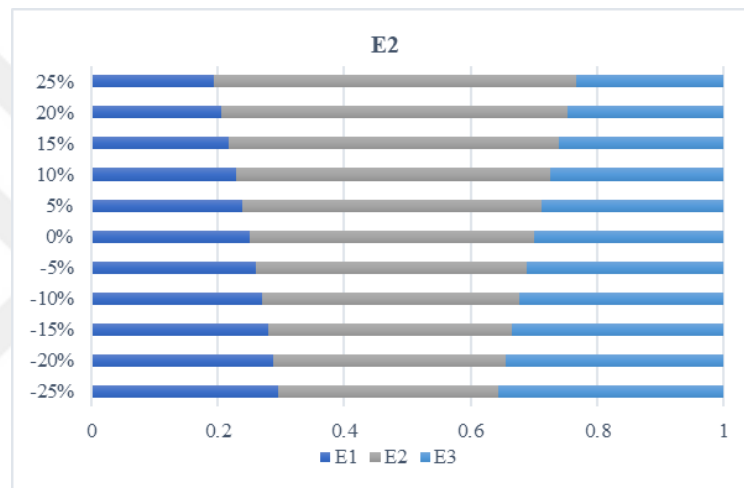


Figure 4.17 : Changes in the weight of E2 for green supplier selection.

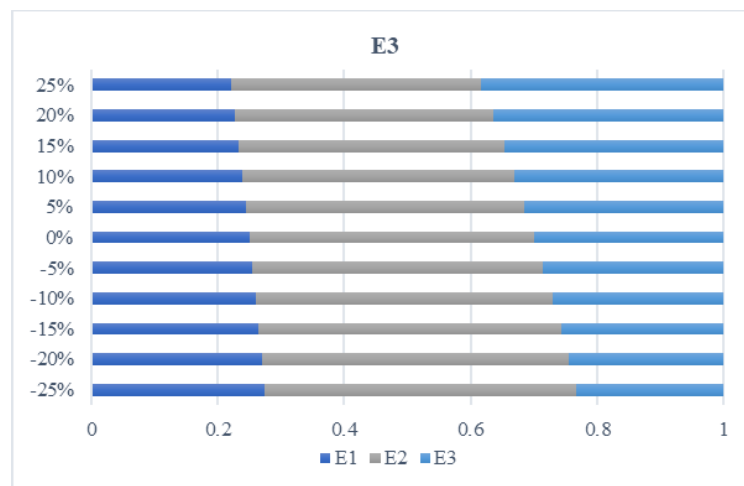


Figure 4.18 : Changes in the weight of E3 for green supplier selection.

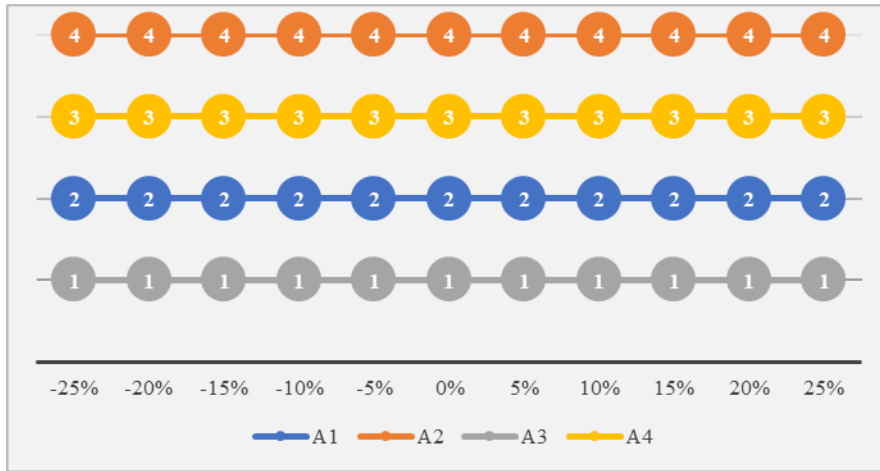


Figure 4.19 : Effect of changes in the weights of experts for green supplier selection.

4.3.4 Waste disposal site selection

The waste disposal site selection problem involves choosing optimal sites for the disposal of waste materials in a manner that is environmentally responsible, economically feasible, and socially acceptable. This problem is particularly relevant in waste management and urban planning, where decision-makers aim to determine suitable sites or facilities for the disposal of various types of waste generated by households, industries, and commercial activities. Therefore, determining regular waste disposal sites is of great importance in order for the infrastructure design in cities to be sustainable and cost-effective. The waste disposal site selection problem involves making decisions about the most suitable sites for waste disposal by considering multiple and often conflicting criteria. Therefore, the application of MCDM methods provides a systematic framework for assessing and prioritizing potential waste disposal sites based on a set of criteria, each reflecting different considerations and priorities (Alkan & Kahraman, 2022a). Thus, applying MCDM to the waste disposal site selection problem allows decision-makers to systematically evaluate various criteria, weigh their importance, and make informed choices that balance environmental, economic, and social considerations. Besides, it also provides a structured approach to managing the complexity of decision-making in waste management and disposal planning. In the second part of the study, we present a second CPFU-based MCDM model that is capable of taking imprecise and ambiguous statements more broadly and continuously into account in the evaluation of waste disposal site alternatives.

Istanbul municipality plans to build an additional waste disposal facility in one of the following alternatives which are A1- Şile-Kömürcüoda, A2- Silivri-Seymen, A3- Kemerburgaz- Odayeri and A4- Pendik-Kurtköy. The considered criteria in determining the new disposal facility are C1-ground and surface water, C2-land slope, C3-distance to protected area, C4-land and operation cost and C5-distance to roads. The criteria considered for the waste disposal site selection problem can be briefly summarized as follow.

Ground and surface water (C1): Waste disposal areas should be located as far away from water sources (lakes, ponds, rivers, etc.) as feasible in order to prevent polluting of surface waters. Planning for waste disposal sites should avoid having a detrimental influence on the area's surface water resources. However, various precautions need to be taken to prevent leachate, the liquid filtered from waste materials, from contaminating groundwater (Alkan & Kahraman, 2022a; Amiri, 2023; Gupta et al., 2021; Yousefi et al., 2018).

Land Slope (C2): Steep slopes are not suitable for landfill as they pose challenges such as increased construction costs, risks of soil erosion, limited accessibility and potential instability of infrastructure. The most appropriate waste disposal areas that balance operational efficiency, environmental sustainability and safety concerns should be determined by taking into account the land slope (Alkan & Kahraman, 2022a; Amiri, 2023; Gupta et al., 2021; Mishra & Rani, 2021; Yousefi et al., 2018).

Distance to protected area (C3): Potential waste disposal sites need to be established at a certain distance from designated protected areas such as national parks, wildlife reserves and conservation areas. Selection of potential waste disposal sites should aim to minimize the risk of environmental degradation, habitat destruction and biodiversity loss by avoiding areas that violate or negatively impact sensitive ecosystems (Alkan & Kahraman, 2022a; Amiri, 2023; Yousefi et al., 2018).

Land and operation cost (C4): It is important to identify sites that provide cost-effective solutions while ensuring efficient waste management practices. Land and operating costs include costs associated with purchasing or leasing land, site preparation, infrastructure development, ongoing operation and maintenance expenses, and any additional investment required to comply with environmental

regulations and safety standards (Alkan & Kahraman, 2022a; Amiri, 2023; Gupta et al., 2021; Mishra & Rani, 2021).

Distance to roads (C5): Potential waste disposal sites must be sufficiently far from existing transportation infrastructure such as highways, roads and transportation networks. Major roadways should be kept at a suitable distance from the waste disposal area, as sites located closer to roads offer advantages such as reduced transportation costs, shorter travel times and increased efficiency in waste management operations (Alkan & Kahraman, 2022a; Amiri, 2023; Gupta et al., 2021; Yousefi et al., 2018).

Possible waste disposal sites have been evaluated according to the criteria determined by three experts, E1, E2 and E3 which are weights of 0.35, 0.4 and 0.25, respectively. The criteria evaluations of each expert are expressed by utilizing the CPFU linguistic scale shown in Table 4.1, as in Table 4.10. The criteria weight matrices consisting of CPFUNs based on the experts' linguistic input are constructed as given in Table 4.11. Then, the aggregated decision matrices for each aggregation operator are calculated by considering the CPFU criteria weight matrices, weights of the experts and experts' risk behaviours with the help of each aggregation operator given in Eqs. (4.25), (4.26) and (4.27) as in Table 4.12. Experts have determined their risk behaviours as risk-neutral, risk-taker and risk-averse, respectively, based on their experience and knowledge in the waste disposal field. Here, each expert has also determined the risk degrees as 0.28, 0.08 and 0.45, respectively.

Table 4.10 : Linguistic assessments for weighting the criteria for waste disposal site selection problem.

	E1	E2	E3
C1	HI	VHI	VHI
C2	SHI	MI	SHI
C3	HI	HI	VHI
C4	SHI	HI	SHI
C5	LI	SLI	MI

Table 4.11 : Criteria weight matrix based on CPFUNs of each expert for waste disposal site selection problem.

	E1	E2	E3
C1	$(\sqrt{3\tau - 3\tau^2}, \sqrt{1\tau - 1\tau^2})$	$(\sqrt{3.5\tau - 3.5\tau^2}, \sqrt{0.5\tau - 0.5\tau^2})$	$(\sqrt{3.5\tau - 3.5\tau^2}, \sqrt{0.5\tau - 0.5\tau^2})$
C2	$(\sqrt{2.5\tau - 2.5\tau^2}, \sqrt{1.5\tau - 1.5\tau^2})$	$(\sqrt{2\tau - 2\tau^2}, \sqrt{2\tau - 2\tau^2})$	$(\sqrt{2.5\tau - 2.5\tau^2}, \sqrt{1.5\tau - 1.5\tau^2})$
C3	$(\sqrt{3\tau - 3\tau^2}, \sqrt{\tau - \tau^2})$	$(\sqrt{3\tau - 3\tau^2}, \sqrt{1\tau - 1\tau^2})$	$(\sqrt{3.5\tau - 3.5\tau^2}, \sqrt{0.5\tau - 0.5\tau^2})$
C4	$(\sqrt{2.5\tau - 2.5\tau^2}, \sqrt{1\tau - 1\tau^2})$	$(\sqrt{3\tau - 3\tau^2}, \sqrt{1\tau - 1\tau^2})$	$(\sqrt{2.5\tau - 2.5\tau^2}, \sqrt{1.5\tau - 1.5\tau^2})$
C5	$(\sqrt{1\tau - 1\tau^2}, \sqrt{3\tau - 3\tau^2})$	$(\sqrt{1.5\tau - 1.5\tau^2}, \sqrt{2.5\tau - 2.5\tau^2})$	$(\sqrt{2\tau - 2\tau^2}, \sqrt{2\tau - 2\tau^2})$

Table 4.12 : Aggregated criteria weight matrix based on each aggregation operator for waste disposal site selection problem.

	CPFUDWG	CPFUEWG	CPFUHWG
C1	(0.499, 0.349)	(0.700, 0.342)	(0.686, 0.345)
C2	(0.452, 0.519)	(0.581, 0.510)	(0.569, 0.513)
C3	(0.491, 0.367)	(0.682, 0.362)	(0.665, 0.364)
C4	(0.470, 0.499)	(0.625, 0.482)	(0.618, 0.488)
C5	(0.411, 0.682)	(0.451, 0.653)	(0.445, 0.662)

The experts evaluate the alternatives based on the determined objectives and criteria by using the CPFU linguistic scale presented in Table 4.1. The experts' linguistic evaluations are provided in Table 4.13. The CPFU decision matrices for waste disposal site selection problem are constructed by considering the experts' evaluations.

Table 4.13 : Linguistic decision matrix based on each expert for waste disposal site problem.

	E1					E2					E3				
	C1	C2	C3	C4	C5	C1	C2	C3	C4	C5	C1	C2	C3	C4	C5
A1	VLP	LP	HP	SLP	HP	LP	LP	HP	SLP	VHP	LP	SLP	SHP	LP	HP
A2	LP	LP	VHP	LP	VHP	LP	VLP	VHP	VLP	VHP	SLP	LP	VHP	VLP	VHP
A3	SHP	LP	HP	SLP	HP	LP	LP	HP	SLP	HP	SLP	LP	HP	SLP	HP
A4	MP	SLP	HP	MP	SHP	SLP	LP	SHP	LP	HP	LP	MP	HP	SLP	HP

After the linguistic decision matrix of each expert is converted into the corresponding CPFU values, the aggregated decision matrices are created by taking into account the experts' weight and risk behaviours towards the problem for each aggregation operator by using the aggregation operators given in Eqs. (4.24), (4.25) and (4.26) as in Tables 4.14, 4.15 and 4.16.

Table 4.14 : Aggregated decision matrix based on CPFUDWG operator for waste disposal site selection problem.

	C1	C2	C3	C4	C5
A1	(0.372, 0.791)	(0.392, 0.716)	(0.477, 0.460)	(0.409, 0.734)	(0.491, 0.396)
A2	(0.392, 0.716)	(0.362, 0.763)	(0.507, 0.287)	(0.355, 0.834)	(0.507, 0.287)
A3	(0.412, 0.634)	(0.387, 0.760)	(0.483, 0.410)	(0.416, 0.678)	(0.483, 0.410)
A4	(0.416, 0.715)	(0.405, 0.645)	(0.474, 0.425)	(0.407, 0.656)	(0.476, 0.458)

Table 4.15 : Aggregated decision matrix based on CPFUEWG operator for waste disposal site selection problem.

	C1	C2	C3	C4	C5
A1	(0.335, 0.750)	(0.400, 0.689)	(0.647, 0.443)	(0.441, 0.684)	(0.683, 0.385)
A2	(0.400, 0.689)	(0.331, 0.728)	(0.722, 0.285)	(0.303, 0.769)	(0.722, 0.285)
A3	(0.476, 0.602)	(0.379, 0.720)	(0.665, 0.403)	(0.465, 0.649)	(0.665, 0.403)
A4	(0.466, 0.658)	(0.449, 0.630)	(0.644, 0.421)	(0.455, 0.630)	(0.642, 0.446)

Table 4.16 : Aggregated decision matrix based on CPFUHWG operator for waste disposal site selection problem.

	C1	C2	C3	C4	C5
A1	(0.334, 0.761)	(0.396, 0.697)	(0.638, 0.449)	(0.438, 0.697)	(0.673, 0.390)
A2	(0.396, 0.697)	(0.328, 0.737)	(0.705, 0.286)	(0.301, 0.782)	(0.705, 0.286)
A3	(0.465, 0.611)	(0.377, 0.731)	(0.652, 0.406)	(0.461, 0.658)	(0.652, 0.406)
A4	(0.461, 0.672)	(0.441, 0.635)	(0.629, 0.423)	(0.447, 0.637)	(0.632, 0.451)

Generally, benefit and cost types are common types of attributes in MADM problems, where the cost type attributes need to be normalized. Therefore, the aggregated decision matrices created for each aggregation operator are normalized by replacing the membership and non-membership degrees for cost criteria to obtain the normalized aggregated decision matrices as in Tables 4.17, 4.18 and 4.19, respectively. It is not necessary to make this replacement for benefit criteria.

Table 4.17 : Normalized aggregated decision matrix based on CPFUDWG operator.

	C1	C2	C3	C4	C5
A1	(0.791, 0.372)	(0.716, 0.392)	(0.477, 0.46)	(0.734, 0.409)	(0.491, 0.396)
A2	(0.716, 0.392)	(0.763, 0.362)	(0.507, 0.287)	(0.834, 0.355)	(0.507, 0.287)
A3	(0.634, 0.412)	(0.760, 0.387)	(0.483, 0.410)	(0.678, 0.416)	(0.483, 0.410)
A4	(0.715, 0.416)	(0.645, 0.405)	(0.474, 0.425)	(0.656, 0.407)	(0.476, 0.458)

Table 4.18 : Normalized aggregated decision matrix based on CPFUEWG operator.

	C1	C2	C3	C4	C5
A1	(0.750, 0.335)	(0.689, 0.400)	(0.647, 0.443)	(0.684, 0.441)	(0.683, 0.385)
A2	(0.689, 0.400)	(0.728, 0.331)	(0.722, 0.285)	(0.769, 0.303)	(0.722, 0.285)
A3	(0.602, 0.476)	(0.720, 0.379)	(0.665, 0.403)	(0.649, 0.465)	(0.665, 0.403)
A4	(0.658, 0.466)	(0.630, 0.449)	(0.644, 0.421)	(0.630, 0.455)	(0.642, 0.446)

Table 4.19 : Normalized aggregated decision matrix based on CPFUHWG operator.

	C1	C2	C3	C4	C5
A1	(0.761, 0.334)	(0.697, 0.396)	(0.638, 0.449)	(0.697, 0.438)	(0.673, 0.390)
A2	(0.697, 0.396)	(0.737, 0.328)	(0.705, 0.286)	(0.782, 0.301)	(0.705, 0.286)
A3	(0.611, 0.465)	(0.731, 0.377)	(0.652, 0.406)	(0.658, 0.461)	(0.652, 0.406)
A4	(0.672, 0.461)	(0.635, 0.441)	(0.629, 0.423)	(0.637, 0.447)	(0.632, 0.451)

After that, to obtain weighted aggregated decision matrix, the normalized aggregated decision matrices are multiplied with aggregated criteria weight matrix by using Eq. (4.20) and the weighted aggregated decision matrices are determined as presented in Tables 4.20, 4.21 and 4.22, respectively.

Table 4.20 : Weighted aggregated decision matrix based on CPFUDWG operator.

	C1	C2	C3	C4	C5
A1	(0.395, 0.429)	(0.324, 0.533)	(0.234, 0.563)	(0.345, 0.531)	(0.201, 0.741)
A2	(0.358, 0.461)	(0.345, 0.493)	(0.249, 0.454)	(0.392, 0.449)	(0.208, 0.714)
A3	(0.316, 0.493)	(0.343, 0.513)	(0.237, 0.529)	(0.318, 0.554)	(0.198, 0.745)
A4	(0.357, 0.483)	(0.292, 0.567)	(0.233, 0.539)	(0.308, 0.554)	(0.195, 0.760)

Table 4.21 : Weighted aggregated decision matrix based on CPFUEWG operator.

	C1	C2	C3	C4	C5
A1	(0.525, 0.404)	(0.400, 0.545)	(0.441, 0.549)	(0.428, 0.564)	(0.308, 0.715)
A2	(0.482, 0.471)	(0.423, 0.482)	(0.492, 0.449)	(0.481, 0.432)	(0.325, 0.688)
A3	(0.422, 0.549)	(0.418, 0.518)	(0.453, 0.522)	(0.406, 0.592)	(0.300, 0.721)
A4	(0.461, 0.532)	(0.366, 0.599)	(0.439, 0.534)	(0.394, 0.589)	(0.290, 0.735)

Table 4.22 : Weighted aggregated decision matrix based on CPFUHWG operator.

	C1	C2	C3	C4	C5
A1	(0.522, 0.402)	(0.397, 0.541)	(0.424, 0.555)	(0.431, 0.561)	(0.300, 0.724)
A2	(0.478, 0.467)	(0.420, 0.477)	(0.468, 0.451)	(0.483, 0.428)	(0.314, 0.696)
A3	(0.419, 0.539)	(0.416, 0.514)	(0.434, 0.525)	(0.406, 0.590)	(0.290, 0.729)
A4	(0.461, 0.527)	(0.362, 0.593)	(0.418, 0.536)	(0.394, 0.585)	(0.281, 0.744)

Then, the overall CPFU score of each alternative is obtained by considering each aggregation operator by executing the weighted sum operation as presented in Table 4.23. The obtained CPFU scores are defuzzified by using Eq. (4.27) and the crisp scores and ranking of the alternatives are obtained by considering each aggregation operator as given in Table 4.24.

Table 4.23 : Overall CPFU scores of alternatives based on each aggregation operator for waste disposal site selection problem.

	CPFUDWG	CPFUEWG	CPFUHWG
A1	(0.629,0.051)	(0.799,0.049)	(0.792,0.049)
A2	(0.645,0.033)	(0.819,0.030)	(0.811,0.03)
A3	(0.594,0.055)	(0.768,0.063)	(0.76,0.063)
A4	(0.585,0.062)	(0.757,0.074)	(0.749,0.073)

Table 4.24 : Crisp scores and ranking of alternatives based on each aggregation operator for waste disposal site selection problem.

	CPFUDWG		CPFUEWG		CPFUHWG	
	Crisp score	Ranking	Crisp score	Ranking	Crisp score	Ranking
A1	0.393	2	0.635	2	0.625	2
A2	0.415	1	0.671	1	0.656	1
A3	0.350	3	0.586	3	0.573	3
A4	0.339	4	0.568	4	0.555	4

The alternatives are ranked in descending order based on the obtained crisp scores and the best alternative(s) is selected. The ranking of alternatives based on the obtained results is determined as $A2 > A1 > A3 > A4$ for all aggregation operators.

4.3.5 Comparison with IFS and PFS-based aggregation operators

In this section, different aggregation-based MADM models have been implemented to compare and verify the results of the developed aggregation-based CPFU-MADM models and to indicate how reliable it is. The comparative analysis has been conducted with intuitionistic fuzzy Dombi weighted geometric (IFDWG) (Seikh & Mandal, 2021), intuitionistic fuzzy Einstein weighted geometric (IFEWG) (Wang et al., 2023), intuitionistic fuzzy Hamacher weighted geometric (IFHWG) (Xia et al., 2012), Pythagorean fuzzy Dombi weighted geometric (PFDWG) (Khan et al., 2019), Pythagorean fuzzy Einstein weighted geometric (PFEWG) (Rahman et al., 2017) and Pythagorean fuzzy Hamacher weighted geometric (PFHWG) (Asif et al., 2025; S. J.

Wu & Wei, 2017)- based MADM models in the literature. The comparative scores and rankings of the obtained results are shown in Tables 4.25 and 4.26, respectively. The results show that there is no difference in the ranks of the alternatives since the ranking order of the alternatives obtained from the proposed approach is the same as the ranking order of the alternatives in all compared methods. Table 4.26 clearly indicates that in every situation, the alternative A2 is the best choice, and the alternative A4 is the worst.

Table 4.25 : Crisp scores of alternatives based on the compared models for waste disposal site selection problem.

	IFDWG	IFEWG	IFHWG	PFDWG	PFEWG	PFHWG
A1	-0.080	-0.249	-0.170	-0.001	-0.053	-0.017
A2	-0.018	-0.151	-0.085	0.038	0.023	0.051
A3	-0.104	-0.287	-0.203	-0.019	-0.087	-0.048
A4	-0.122	-0.318	-0.231	-0.032	-0.114	-0.073

Table 4.26 : Ranking results of alternatives based on the compared models for waste disposal site selection problem.

Aggregation operator	Ranking
CPFUDWG	A2>A1>A3>A4
CPFUEWG	A2>A1>A3>A4
CPFUHWG	A2>A1>A3>A4
IFDWG (Seikh & Mandal, 2021)	A2>A1>A3>A4
IFEWG (W. Wang & Liu, 2011)	A2>A1>A3>A4
IFHWG (Xia et al., 2012)	A2>A1>A3>A4
PFDWG (Khan et al., 2019)	A2>A1>A3>A4
PFEWG (Rahman et al., 2017)	A2>A1>A3>A4
PFHWG (S. J. Wu & Wei, 2017)	A2>A1>A3>A4

The results confirm the results obtained by using CPFU-based MADM models. Thus, the fact that the alternatives have the same ranking order indicates that the results are correlated with each other. Although all compared MADM models give the same results, the compared MADM models have some limitations according to the developed models. Uncertainty at every point can be taken into account with a continuous structure created thanks to the nonlinear functions used in the developed CPFU-based MADM models. In addition, other fuzzy-based models, with their linear structure, only address uncertainty at a certain point. Thanks to these structures of the

developed CPFU-based MADM models, they have the ability to reflect the ideas of decision-makers continuously and more comprehensively. Unlike other compared fuzzy-based MADM models, the developed CPFU-based MADM models have more flexibility. Flexibility is reflected in the ability to tune parameters in nonlinear functions that CPFU-based MADM models have. Moreover, in the developed CPFU-based MADM models, risk can be simulated during decision-making by changing the τ parameter, thanks to their changing structures according to the behavior of decision-makers towards uncertainty, whereas in other MADM models, it is not possible to evaluate the risk since the behavior of decision-makers towards uncertainty cannot be taken into account. It is possible to conclude that the proposed CPFU-based MADM models have a better degree of generalization than other fuzzy-based MADM models because of this property, which enables the models to be adjusted to the characteristics of real-world problems.

4.3.6 Sensitivity analysis for waste disposal site selection problem

Three subsections in this section of the study examine the sensitivity analysis of the findings from the waste disposal site selection problem. First, it is determined how sensitively the weight coefficients of the criterion change in relation to the ranking outcomes of the MADM models created using CPFU aggregation operators. Then, the analysis of the effect of the change in the developed CPFUDWG and CPFUHWG aggregation operators' ρ and φ parameters on the ranking results of the alternatives is examined. Finally, the sensitivity of the change in experts' weights to the ranking results are checked.

4.3.6.1 Sensitivity of the criteria weights for waste disposal site selection problem

For the waste disposal site selection problem, the weight of each criterion has been identified by utilizing CPFUDWG, CPFUEWG and CPFUHWG aggregation operators. By altering the linguistic weights of the criteria from AHI to ALI, the impact of each CPFU aggregation operator's changed criterion weight on the ranking results of the alternatives has been examined. A total of 45 different scenarios have been obtained by applying 9 different changes to each criterion, along with the change in the linguistic weight of each criterion. The impact on the alternatives' ranking outcomes of altering the linguistic weights of the criterion for each aggregation operator is presented in Figs. 4.20, 4.21 and 4.22, respectively.

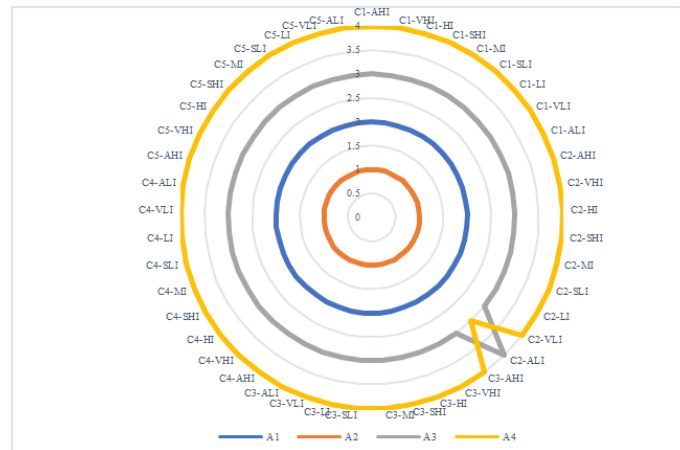


Figure 4.20 : Effects of the changes in the weights of the criteria based on CPUDWG operator for waste disposal site selection.

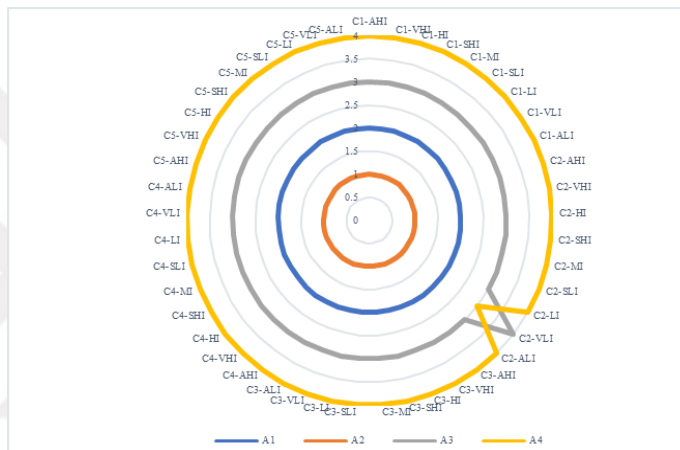


Figure 4.21 : Effects of the changes in the weights of the criteria based on CPUEWG operator for waste disposal site selection.

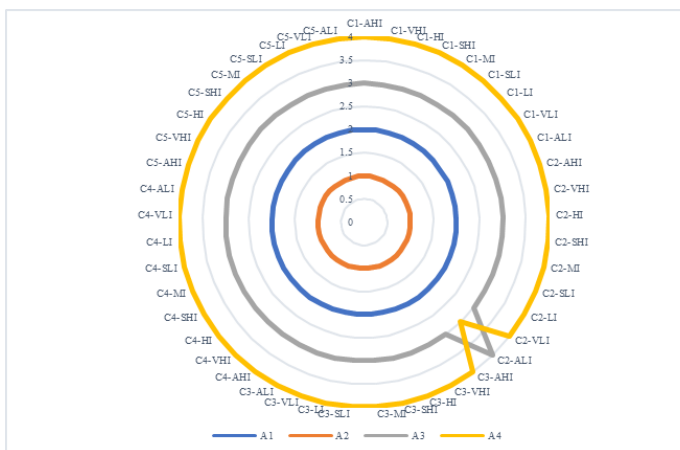


Figure 4.22 : Effects of the changes in the weights of the criteria based on CPUHWG operator for waste disposal site selection.

The findings of the sensitivity analyses, which were performed by using the criteria for every aggregation operator, showed that the alternatives' ranking results remained unchanged when the weights of the C1, C3, C4, and C5 criteria were changed. It has

been observed the crisp scores of the alternatives changed significantly when the linguistic weights of all the criteria changed, but the rankings of the alternatives did not change as a result of these alterations. The rankings of the A3 and A4 alternatives were found to change and take the fourth and third places, respectively, when the weight of the C2 criterion was altered to ALI. Based on the analysis, the initial ranking of alternatives A1 and A2 has been confirmed in all 45 scenarios. Therefore, the findings indicated that the only difference in the final ranking of the alternatives was a very tiny one caused by the change in the weights of the criterion. This further demonstrates the developed models' stability, robustness, and durability to changes in the criteria's weights.

For the validation of sensitivity analysis, we used Spearman's correlation coefficient. The correlation value of the sensitivity analysis results performed by changing the weights of the criteria was 0.8 for the green supplier selection problem. This value is a very high correlation coefficient and shows that their rankings have not changed significantly. The same correlation coefficient was calculated for the waste disposal site selection criteria and the Spearman correlation coefficient was calculated as 0.9. This value also shows that the order of importance of the criteria has not changed significantly.

4.3.6.2 Sensitivity of parameters ρ and φ

In the proposed MADM model based on CPFUDWG, CPFUEWG and CPFUHWG aggregation operators, changes in the ρ and φ parameters used in the CPFUDWG and CPFUHWG aggregation operators may cause changes in the ranking results of the alternatives. The values $\rho = 1$ and $\varphi = 1$ based on the condition that the values of the ρ and φ parameters are $\rho > 0$, $\varphi > 0$ have been adopted for the results obtained from the study. Therefore, in this part, the analysis of the effect of the change of ρ and φ parameters in the range $[1, 50]$ on the ranking results of the alternatives is presented. A sensitivity analysis with a total of 50 scenarios has been conducted for each parameter. In every successive scenario, the values of the parameters ρ and φ , which were initially set at one, have been increased by one. The impact of the variables ρ and φ on the variation of every option is shown in Figs. 4.23 and 4.24, respectively.



Figure 4.23 : Effects of parameter ρ on the scores of the alternatives.



Figure 4.24 : Effects of parameter ϕ on the scores of the alternatives.

The findings indicate that the alternatives' ranking is independent of the values selected for the ρ and ϕ parameters. As seen in Figs. 4.23 and 4.24, all alternatives had higher final score values, but the rankings remained unchanged as a result. In this context, it can be seen from the results that the alternative A2 is the dominant solution regardless of the parameters ρ and ϕ , and that the initial order $A2 > A3 > A1 > A4$ is verified for both aggregation operators. Therefore, based on the acquired values, it can be concluded that the initial solution is stable and reliable.

4.3.6.3 Sensitivity of the weights of experts for waste disposal site selection

In this part of the study, the same sensitivity analysis carried out in the previous application has been carried out on the weights of the three experts included in the waste disposal site selection problem. The changes in the weights of the experts, which are the same for all aggregation operators used in the study, are shown in Figs. 4.25, 4.26 and 4.27, respectively. It has been seen in each scenario that the ranking results

of the alternatives remained the same for all experts and all aggregation operators, and the results have been presented in Fig. 4.28.

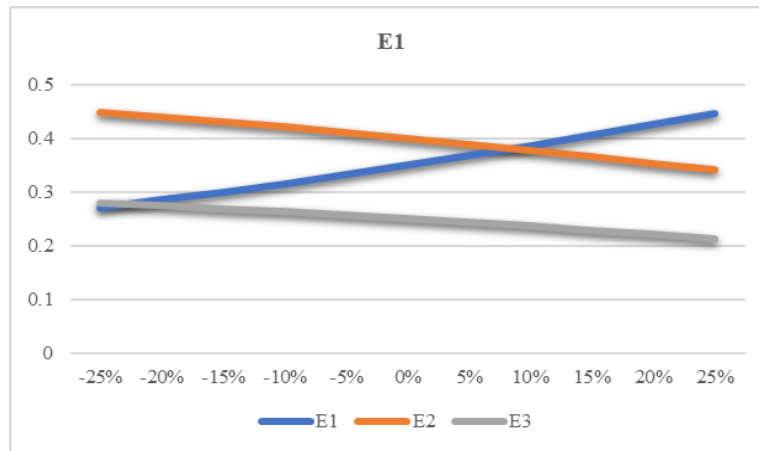


Figure 4.25 : Sensitivity of the weight of E1 for waste disposal site selection.

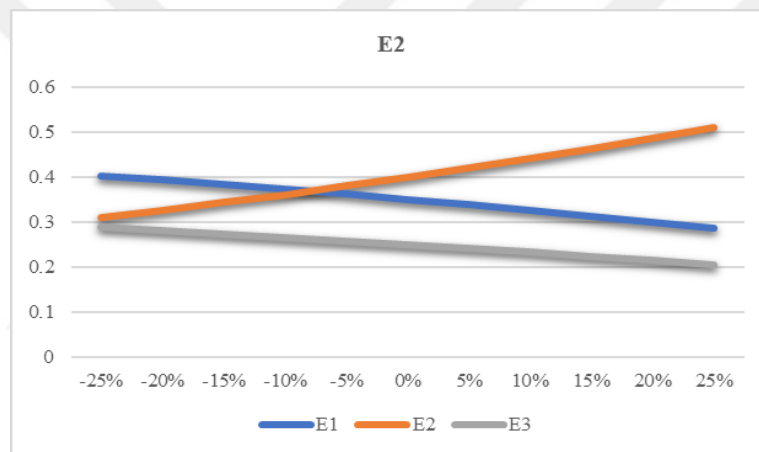


Figure 4.26 : Sensitivity of the weight of E2 for waste disposal site selection.

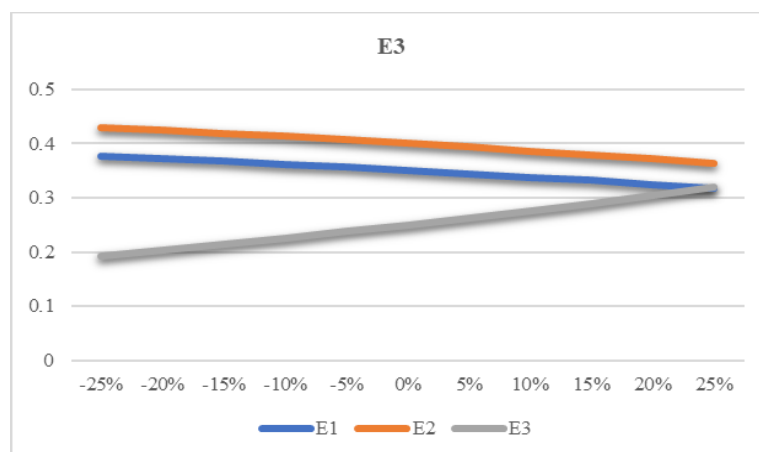


Figure 4.27 : Sensitivity of the weight of E3 for waste disposal site selection.

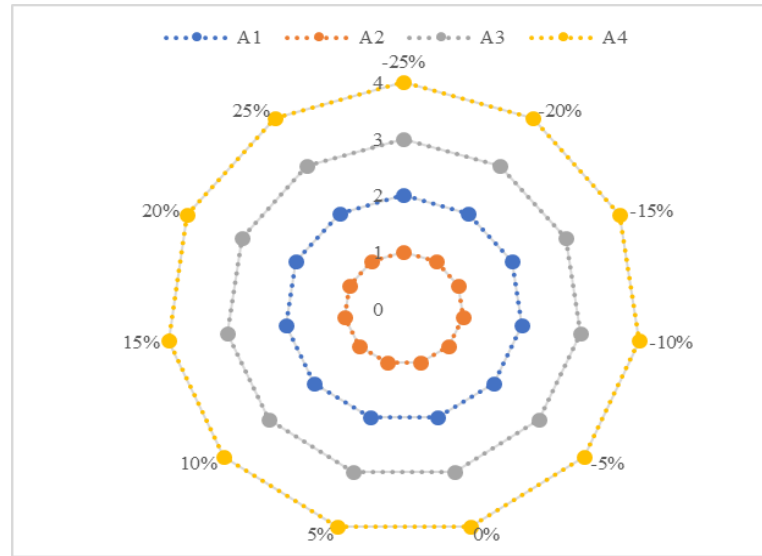


Figure 4.28 : Effect of changes in the weights of experts for waste disposal site selection.

According to the results obtained, it has been shown that the changes in the weights of the experts do not affect the ranking results of the alternatives for all MADM models depending the aggregation operators used in the study in each scenario. After ten scenarios were completed for each expert, it was found that alternative A2 was the most predominant and alternative A4 was the most inefficient. This demonstrates that no MADM model created using the aggregation operations employed in the study is impacted by changes in the expert weights and is stable and reliable.

4.3.7 Managerial implication

In environments where vagueness and impreciseness type uncertainty are experienced, multi-criteria decision making in companies must be a part of the decision support system. CPFUSs proposed for the Pythagorean fuzzy uncertainty environment, which emerged with a larger definition area given to intuitionistic fuzzy uncertainty, are sets that managers can use in any situation, regardless of their behavior towards risk. Risk-averse managers will be able to use the decision model by bringing the τ parameter, which represents expert's behavior against uncertainty, closer to 0.5 in CPFUS; risk-taking managers will be able to use the decision model by using values of the τ parameter close to 0 or 1. Managers using intermediate values will be risk-neutral managers. Several operators have been defined that can aggregate the opinions of different managers. Managers can choose the operator that gives the average that best represents their judgments. The proposed operators can be used not only for weighting the criteria but also for prioritizing the alternatives.

4.4 Conclusion

The complexity and contradictions included in real-world issues make MADM models more significant, but they also introduce ambiguity. Decision-makers' crisp values are insufficient to produce accurate findings in decision-making challenges due to uncertainties resulting from incomplete information and inconsistent decision-makers, which leads to inaccurate judgments. This has made it necessary for decision-makers' judgments to encompass a larger range and be able to describe values in a continuous range that can account for uncertainty at each point. In particular, CINFUSs have made it possible for IFSs to be utilized with continuous functions and to account for uncertainty at each point because of their superiority in modeling uncertainty information. Besides, the processing of specific facts, events, and related data that cannot be processed using IFSs can be done more efficiently by utilizing PFSs as a direct extension of IFSs to provide a more robust representation of uncertainty. Therefore, the CPFUSs presented in this study enable information modeling over a continuous range of values and contribute to a more comprehensive, accurate, and precise explanation of the uncertainty in decision-makers' opinions. Moreover, the developed CPFUSs are capable of handling degrees of uncertainty more sensitive and at the same time providing greater flexibility in the decision-making process. Based on this advantage, in this study, CPFUSs have been developed and focused on examining the usability of this new fuzzy set extension in MADM problems. In the study, after introducing the mathematical operations of CPFUSs, their applicability on different CPFU-MADM models based on mathematical operations has been demonstrated. While the first model developed based on the CPFUWG aggregation operator is applied on the green supplier selection problem, another model developed based on the CPFUDWG, CPFUEWG and CPFUHWG aggregation operators is presented on the waste disposal site selection problem.

First of all, by taking into account an extensive set of criteria established by thorough literature research and professional opinions in the field of green supplier selection, the study aims to identify the best green supplier. In this context, four green suppliers have been evaluated according to five criteria. The constructed model provided the following results: A3 is the best supplier, followed by A1, A4, and A2 alternatives, in that order. Then, sensitivity analyzes have been conducted on the linguistic weights of the criteria and the expert weights for the first CPFU-MADM model developed in the

study. The results of the first model created have been demonstrated to be robust and dependable and to be insensitive to modifications in the weights of the experts and criteria, based on the findings of both sensitivity analyses. Besides, the ranking results from the models have been compared, and the variations in the rankings have been investigated, in the comparative analysis carried out utilizing IF-based and PF-based MADM models. A3 has been identified to be the most significant alternative in both the developed and compared models, with A2 being the least desirable option. At the same time, it has been seen that the positions of the other options remain unchanged. Through a comparative analysis of these three distinct fuzzy-based models, the study's results have been shown to be consistent, dependable, and correlated with one another using the Spearman's rank correlation coefficient. According to the results obtained, although the identical ranking results are taken from the compared models, it has been concluded that the developed CPFU-based MADM models had a higher degree of generalization compared to other MADM models used for comparison.

Then, it has been intended to make the proper investments by creating more accurate and efficient selections for decision-makers in the prioritizing and selection of waste disposal site alternatives, taking into account the significance of the extensive list of criteria in the study. In this context, four waste disposal site alternatives have been evaluated according to five criteria through comprehensive literature review and expert opinions, with three distinct MADM models developed based on CPFUDWG, CPFUEWG and CPFUHWG aggregation operators. According to the results obtained from the second application, A2- Silivri-Seymen alternative has been identified as the best alternative site for MADM models developed based on all aggregation operators, while A4- Pendik-Kurtköy alternative has been determined as the worst alternative site. A1- Şile-Kömürcüoda and A3-Kemerburgaz- Odayeri alternatives were determined as the second and third alternative sites, respectively, for MADM models developed based on all aggregation operators. Sensitivity analyses have been performed on the linguistic weights of the criterion, expert weights, and ρ and ϕ parameters used in the CPFUDWG and CPFUHWG aggregation operators for the CPFU-MADM models built using aggregation operators. Based on the findings of the linguistic weight analysis of the criteria, it has been observed that the modifications made to the criteria weights for the MADM models created by using all aggregation operators had no effect on the ranking results of the alternatives—with the exception

of a minimal deviation—and that the alternatives maintained their original rankings. Then, through the application of sensitivity analysis to the expert weights, it has been shown that modifications to the expert weights did not affect the ranking outcomes of the alternatives across all scenarios and MADM models built using aggregation operators. Finally, the sensitivity study conducted on the ρ and ϕ parameters utilized in the CPFUDWG and CPFUHWG aggregation operators revealed an increase in all alternatives' final score values; however, this increase had no effect on the ranking. In this context, regardless of the ρ and ϕ parameters, the A2 alternative has been shown to be the dominating solution, confirming the original order for both aggregation operators. Based on the findings of sensitivity analysis, it has been established that the models constructed using aggregation operators are robust and reliable. Additionally, in the study, a comparison analysis has been carried out with models based on IF and PF based Dombi, Einstein and Hamacher aggregation operators. When the ranking results obtained from the comparison analysis were examined, it has been observed that the ranking results did not change in all compared models and the ranking of the alternatives was the same as the models developed based on aggregation operators. In this context, the consistency of the results of the MADM models developed based on aggregation operators have been verified and their validity has been proven. According to the results obtained, although the same ranking results are obtained from the compared models, it has been concluded that the developed CPFU-based MADM models had more flexibility and a higher degree of generalization, unlike the other compared IF and PF-based MADM models.

To further demonstrate the effectiveness and usability of the new CPFUSs introduced within the scope of the study, the developed models have been applied on two different decision problems: green supplier selection and waste disposal site selection. Their advantages, like stability and accuracy, have been benefited within the study's scope because the CPFUSs presented in the study and the models built within its scope better reflect uncertainty. The proposed models, which are quite flexible and have a high degree of generalization, can be implemented to distinct decision problems. Analyses and applications have shown that CPFUSs can be used successfully in various MADM models. CPFUSs show potential as a more effective and powerful tool in complex and real-world problems, especially in situations where uncertainty and ambiguity are intense. In this context, this study shows that the use of CPFUS in MADM models has

significant potential and that these new models can provide greater flexibility and precision in decision-making processes.

It also brings with it some limitations that should be taken into account when interpreting the results of its applications on the CPFUS and MADM models introduced within the scope of the study. The study has confirmed the effectiveness and robustness of MADM models developed based on CPFUSs and their aggregation operators. However, the mathematical complexity of CPFUSs and the calculation of CPFU Dombi, Einstein and Hamacher norms in MADM models may represent a kind of limitation for the wider application of this model. Especially in real-world decision problems where the problem size increases, more computing power may be required due to the complexity brought by mathematical operations. Besides, it may be necessary to determine appropriate parameters for the aggregation operators used in the CPFU-based MADM models developed within the scope of the study. Failure to set these parameters appropriately can have a negative effect on the effectiveness of the model. In addition, the use of the CPFUS may require certain choices and judgments to be made by the decision maker, which may lead to user bias. Additionally, the number of experts involved in the research is one of the study's weaknesses. For more complex decision-making problems where the problem size increases, evaluations may need to be carried out by considering a larger number of experts.

In future studies, it is recommended to develop continuous picture fuzzy sets or continuous spherical fuzzy sets or continuous neutrosophic sets or diophantine fuzzy sets (Kannan et al., 2025) in the same way. Besides, CPFUSs can be extended to various MCDM techniques or applied to various MADM models. It is possible to build a number of mathematical operations for the introduced CPFUSs, including score functions, various aggregation operators such as (Hussain & Ullah, 2024), (Imran et al., 2024) and similarity measurements. Additionally, for both application problems under study, a set of more specific and comprehensive criteria might be taken into account. In addition, a variety of real-world decision-making problems, including the assessment of sustainable supply chain systems, the selection of renewable energy sources, the selection of electric vehicle charging stations, and project selection, can be addressed by the MADM models that were created based on the presented CPFUSs and their aggregation operators.



5. AN INTUITIONISTIC FUZZY MULTI-DISTANCE BASED EVALUATION FOR AGGREGATED DYNAMIC DECISION ANALYSIS (IF-DEVADA): ITS APPLICATION TO WASTE DISPOSAL LOCATION SELECTION⁴

Many decision-making problems encountered in real-life problems contain many proportional and contradictory attributes, which it is necessary to consider all attributes simultaneously. To ensure that effective decisions are taken in the decision-making process, it is necessary to use various scientific methods, in which it is possible to evaluate many qualitative and quantitative factors (Erpolat Tasabat & Morais, 2019). Thanks to the developed scientific methods, it is aimed to find the most suitable solution by evaluating the existing alternatives in terms of many conflicting criteria in the decision-making process (Büyüközkan & Göçer, 2009). To overcome complex decision problems, MCDM methods are used as an effective method in which the opinions of more than one decision-maker are included (Alkan & Kahraman, 2021). Multi-criteria decision-making problems are the most important problem types, especially in which tangible or intangible criteria are involved. The fact that intangible criteria are more than tangible criteria in such problems causes a more difficult evaluation process for decision makers. Therefore, in such an environment, it is necessary for MCDM methods to determine the evaluation criteria and a number of possible alternatives, then collect appropriate information about the alternatives according to the criteria and evaluate them simultaneously for the purposes of the decision-makers (Karaşan & Kahraman, 2020). To overcome these difficulties, many models such as Analytic Hierarchy Process (AHP), Analytic Network Process (ANP), Višekriterijumsko kompromisno rangiranje (VIKOR), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), Measurement of alternatives and ranking according to COMpromise solution (MARCOS), Evaluation Based on Distance from Average Solution (EDAS), Combinative Distance-based Assessment

⁴ This chapter is based on the paper “Alkan, N., & Kahraman, C. (2022). An intuitionistic fuzzy multi-distance based evaluation for aggregated dynamic decision analysis (IF-DEVADA): Its application to waste disposal location selection. *Engineering Applications of Artificial Intelligence*, 111, 104809.”

(CODAS), Full Consistency Method (FUCOM), Elimination and Choice Translating Reality English (ELECTRE) have been developed and are still being developed today. In addition, MCDM methods are used in many fields such as energy, economic, industry and finance.

MCDM methods existing in the literature are based on pairwise comparison methods, distance-based methods, or outranking methods. In addition to the fact that the number of distance-based methods is higher in the literature, these methods are used more frequently than other methods because they rank the alternatives according to their closeness to the ideal solution using a distance measure. However, these methods use only a single type of distance measure, such as Euclidean, Hamming, cosine, which are the most commonly used in the literature, and various distance methods give different results in studies. Actually, their integration in a model can provide more accurate alternative rankings by yielding more accurate results.

One of the most used MCDM methods for weighting criteria is CRITIC method (Anilkumar et al., 2021; Gopal Naik et al., 2021; Goswami et al., 2021; Helmy et al., 2021; Jovčić & Průša, 2021; Kumaran, 2022; Lai & Liao, 2021; Mukhametzyanov, 2021; Y. Wu et al., 2021; Zafar et al., 2021). Unlike other criterion weighting methods such as AHP, ANP, BWM, SWARA and LBWA, the CRITIC method uses the entire decision matrix to find the criterion weights. The CRITIC method calculates a standard deviation value for each criterion and uses them to find the correlation of the criteria with other criteria. Thus, CRITIC method uses the objective weights of the criteria to reduce the subjective attribute of the decision process in an intuitionistic fuzzy environment of uncertainty.

Considering real-life conditions, information about the addressed problem is usually imprecise and cannot be described by crisp or deterministic models (Karaşan & Kahraman, 2020). Although decision-makers express their opinions with a crisp value in classical methods, when vague and ambiguous information is taken into account in decision making, these crisp values are often insufficient and inadequate for solving real decision-making problems, which causes errors in obtaining the right results (Büyüközkan & Göçer, 2021). Therefore, the vagueness and impreciseness of the data should be taken into account in the calculations and the methods should be modeled accordingly. To handle such type of vagueness, fuzzy distance measurements and decision-making methods should be used or developed. The fuzzy set theory has been

proposed by Zadeh (1965). Ordinary fuzzy sets are represented with a membership degree that has a value between zero and one (Precup et al., 2020; Yuhana et al., 2020). Different extensions of ordinary fuzzy sets such as type-2 fuzzy sets (Tolga et al., 2020; Zadeh, 1975), hesitant fuzzy sets (Torra, 2010), and intuitionistic fuzzy sets (IFSs) (Atanassov, 1986) have been proposed in the literature to obtain a clear solution for more complex decision-making problems. IFSs, which is one of the most widely used fuzzy sets in the literature, have the ability to handle uncertain information more flexibly and to cope with uncertainties better by using the degree of membership and degree of non-membership. IFSs not only deal with uncertainties arising from the incomplete knowledge of decision-makers but also reflect hesitations in decision makers' choices (Kaya et al., 2020). Through IFSs, data are more accurately reflected and uncertainties that may arise in the considered decision problem are better handled (Castillo & Melin, 2019a, 2019b; Martínez et al., 2019; Sotirov et al., 2018). In addition, in many complex MCDM problems, information and judgments about criteria and alternatives may change in the future. To take effective decisions by adapting to rapidly changing environmental conditions and manage decisions efficiently, the decision-making process should be handled with a dynamic structure and dynamic decision environment should be considered in the model. For this, flexible decision models are needed, including current and future information and judgments. However, most of the existing methods in the literature cannot include dynamic decision environment in the model and are insufficient to take into account the changes that may occur in a certain time period in the future. Since judgments on criteria and alternatives considered in classical MCDM methods are generally carried out only once, decision models are not created by considering a temporal or spatial thought. Especially since the values change over time, decision matrices created by classical methods will not have a fixed or static structure. This will result in limited validity of the outcome of the decision model and unrealistic decisions resulting from the ignorance of the predictions for the future. The proposed IF-DEVADA method presents a dynamic structure under uncertainty, which is a new contribution in MCDM area. The proposed method can use present and future information and combine different distance measures.

The importance of waste management is increasing day by day with the increasing amount of waste production due to the rapid growth in population, urbanization,

increasing product complexity, high material consumption, and changes in human lifestyle. Waste management includes both an important environmental task and various socio-cultural and economic issues. Therefore, it is necessary to plan and create a waste management system for environmental sustainability and better human health. Various techniques such as storage, heat treatment, biological treatment, and recycling are used for waste disposal. Among them, storage management is one of the most widely used methods because it is a simple and low-cost method. However, determining a suitable location for a landfill is a complex problem with imprecise information. Moreover, it is also necessary to consider conflicting criteria for the evaluation of alternative locations. Therefore, the selection of a suitable waste disposal location includes an MCDM model that should be considered in a fuzzy environment (Karasan et al., 2019; Özkan et al., 2020). For this, it is first necessary to determine the importance weights of the criteria to be considered in the selection of waste disposal location and then rank the alternatives. In this study, we first present the extension of the CRITIC method in the IFS environment to reduce the subjectivity of the decision-making process by determining the objective weights of the criteria in an environment of uncertainty.

The proposed methodology presents a dynamic approach to MCDM and benefits statistical data analysis of CRITIC method. This originality and contribution of the study constitutes the feature that distinguishes it from other MCDM methods. It observed that taking the changes that may occur over time as an expectation from the decision makers gives more realistic and more informed results. Incorporating a statistics-based weighting component into MCDM methodology causes more emphasis on data analysis. The proposed method is also capable of considering linguistic evaluations at different time points in an intuitionistic fuzzy environment. IFSs are the fuzzy set extension that received the least criticism compared to other extensions in the literature. For this reason, IFSs are preferred in this study. In addition, a new defuzzification operator for IFSs is developed in the study and give successful results.

To achieve the above-mentioned main purpose and fill the research gaps outlined, this study proposes methodology including a new IF-MCDM method, namely DEVADA and IF-CRITIC method. A case study on waste disposal location selection is presented to demonstrate the effectiveness of the developed IFS-based methodology. Based on

the above features, we can highlight the following advantages and innovations of the proposed methodology:

- The multi-criteria decision-making methodology developed in this study offers a new decision model that enables more reliable and effective decision-making by using uncertain and insufficient information in the decision-making process.
- In the first stage of the proposed multi-criteria decision-making methodology, a new extension of the CRITIC method, which uses the objective weights of the criteria to reduce the subjective attribute of the decision process in an intuitionistic fuzzy environment of uncertainty, is presented.
- The developed IF-DEVADA method handles uncertainty by enabling to express the opinions of decision-makers more flexibly.
- The proposed approach focuses on medium and long-term decisions by considering the temporal variation of the factors addressed in complex decision-making models, thus enabling more informed and accurate decisions to be made.
- It incorporates current and prospective information in decision-making problems and allows temporal variation to be taken into account.
- A new intuitionistic fuzzy (IF) defuzzification function is recommended.
- The feasibility and effectiveness of the developed multi-criteria decision-making methodology are demonstrated on a waste disposal location selection problem where the evaluations are open to temporal variations.
- A stronger multi-distance measurement is suggested, taking into account the integration of Euclidean and cosine distances, which are frequently used in the literature. Thanks to the multi-distance measurement, it is ensured that the distances of the alternatives to the ideal solutions are handled more accurately.

The remaining of this study is organized as follows. In Section 5.1, literature review on Distance-based MCDM methods based on IFSs, dynamic decision analysis approaches, and review of study in waste disposal location selection is conducted. In Section 5.2, the principles of IFSs are given, and IF distance measurements and IF aggregation operators are mentioned. The details of the proposed MCDM methods are presented in Section 5.3. In Section 5.4, the proposed MCDM methodology is applied

to the waste disposal location selection for Turkey and comprehensive sensitivity analysis and comparative analysis are conducted to demonstrate the validity of the results. Discussions on the study are presented in Section 5.5. Finally, the study ends with the conclusion and suggestions for further research in Section 5.6.

5.1 Literature Review

Our literature research is organized in three subsections so that the subject considered in the study can be better understood, its difference from other studies can be better revealed and its contributions can be addressed more clearly. The first subsection explores distance-based methods used with IFSs. The second subsection provides an overview of dynamic-based methods in the literature. The third subsection explores the MCDM methods used for waste disposal location selection.

5.1.1 Distance-based MCDM methods using IFSs

Many MCDM methods have been developed by various researchers for solving real-life decision problems. In MCDM methods, distance-based approaches are popular methods used to obtain the final ranking among alternatives. Distance-based MCDM methods used in the literature can be given as TOPSIS, CODAS, EDAS, Multi-attributive Border Approximation Area Comparison (MABAC), Preference Ranking On the Basis of Ideal-average Distance (PROBID). The applications of these methods in the literature are increasing day by day. In particular, these methods and their extensions with fuzzy sets are frequently used in various fields of application in the literature. Kaya et al. (2020) proposed an integrated IF MCDM methodology consists of AHP and TOPSIS based on IFSs to prioritize transition strategies for industry 4.0. Kahraman et al. (2017) presented the extension to IFSs of the classical EDAS method, which is based on the data belonging to membership, no-membership, and hesitance degrees. The developed method has been applied to evaluate the solid waste disposal site selection alternatives. Ashraf et al. (2021) developed a new distance measure for IFSs by using the generalized difference sequence spaces within p -summable intuitionistic fuzzy bounded variation. They applied the TOPSIS and GRA methods to the developed distance measure. To demonstrate the superiority of the proposed distance measurement, they show in detail the operation of the methods developed through examples. Chen et al. (2021) investigated the connection between

distance measure, similarity measure, and entropy measure based on intuitionistic hesitant fuzzy sets. They proved the superiority of the proposed distance measure with a pattern recognition example where it improves the numerical results obtained with the existing distance measures. Dülenci (2016) proposed a novel generalized distance measure for interval-valued intuitionistic fuzzy sets. The extension of the TOPSIS method to IFSs to calculate the separation measures of the proposed distance measure has been shown in the study. Mishra et al. (2020) developed new parametric intuitionistic fuzzy deviation measures and presented a new EDAS method approach by using their proposed deviation measures. Roy et al. (2019) proposed a new approach to solving MCDM problems with incomplete weight information by extending the CODAS method to IFSs. In the study, an optimization model based on intuitionistic fuzzy distance measurement has been presented to calculate the unknown weights of the evaluation criteria. Liang et al. (2019) introduced new distance measures for IFSs and extended the MABAC method to IFSs. Intuitionistic fuzzy Dombi generalized λ Shapley Choquet arithmetic mean operator and intuitionistic fuzzy Dombi generalized λ -Shapley Choquet geometric mean operator have been proposed to aggregate intuitionistic fuzzy information. Studies conducted in various fields on distance-based MCDM methods based on IFSs are presented in Table 5.1.

Table 5.1 : Studies on distance-based MCDM methods using IFSs.

Author(s)	Application area	Method(s)
Altan Koyuncu et al. (2021)	Industry 4.0 maturity model selection	IF-TOPSIS
Lu et al. (2021)	Chinese GVC investment project selection	AHP, TOPSIS
Suresh and Dillibabu (2021)	Software Risk Prediction	IF-TOPSIS, Fuzzy Decision Making Trial and Evaluation Laboratory (DEMATEL), Adaptive Neuro-Fuzzy Inference System (ANFIS)
Karaşan et al. (2020)	Location selection of electric vehicles charging stations	IF-DEMATEL, Interval-valued Intuitionistic Fuzzy (IVIF) AHP, IVIF-TOPSIS
Kilic and Yalcin (2020)	Green Supplier Selection	IF-TOPSIS, Two-Phase Fuzzy Goal Programming
Budak et al. (2020)	Real-time location systems selection for selection humanitarian relief logistics	IVIF DEMATEL, IVIF-ANP, IVIF-TOPSIS
Karagoz et al. (2020)	Authorized dismantling center location selection	IF-CODAS, IF- Weighted Aggregated Sum Product Assessment (WASPAS), IF-TOPSIS
Dogan et al. (2020)	Autonomous vehicle location selection	IVIF-AHP, IVIF-TOPSIS

Table 5.1 (continued) : Studies on distance-based MCDM methods using IFSs.

Author(s)	Application area	Method(s)
Chinram et al. (2021)	Hydropower plant selection	Intuitionistic Fuzzy Rough EDAS
Schitea et al. (2019)	Hydrogen mobility roll-up site selection	IF-WASPAS, IF- Complex Proportional Assessment (COPRAS), IF-EDAS
Bolturk and Kahraman (2018)	Wave energy facility location selection	IVIF-CODAS
Remadi and Frikha (2020)	Green material selection	Triangular intuitionistic fuzzy CODAS
Liu et al. (2019)	Healthcare risk analysis	IVIF-MABAC, Failure Mode Effect Analysis (FMEA)
Verma (2021)	Personnel selection	IF-Entropy, IF-MABAC
Alkan and Kahraman (2022c)	Pandemic hospital location selection	Circular IF-TOPSIS

5.1.2 Dynamic decision analysis approaches

Most of the studies conducted on classic MCDM methods neglect the effect of time on the importance of criteria and alternatives and evaluation data due to the changing conditions. However, these methods only reflect the current state of the problem. Therefore, in addition to the development of approaches that will enable MCDM problems encountered in real life to be handled dynamically, it should also be modeled in a way that reflects the uncertainty and hesitancy of decision-makers. In this context, many dynamic-based MCDM approaches based on IFSs have been presented by various researchers to address this problem in the literature. Fei and Feng (2021) proposed a new framework for dynamic MADM based on Dempster–Shafer theory in Pythagorean fuzzy environments. Tao et al. (2021) proposed a dynamic group MCDM approach to rank the alternatives based on the preference relationship by considering the alternative queuing method and intuitionistic fuzzy sets together. Baykasoğlu and Gölcük (2019) proposed a new dynamic MADM model based on fuzzy cognitive maps. In their study, by integrating the Jaya algorithm, which is a meta-heuristic optimization method, into fuzzy cognitive maps, they construct short, medium and long-term future decision matrices. Bali et al. (2015) presented a dynamic multi-attribute decision-making approach based on IFSs. They have integrated the AHP method and the dynamic evaluation by an intuitionistic fuzzy operator in the study. Hao et al. (2018) developed an IF Bayesian network based on dynamic structure. They proposed a conceptual framework for dynamic IF decision-making by using the developed technique and applied it for the mine emergency problem. Li et al. (2020) proposed a dynamic IF decision-making method based on prospect theory and VIKOR

method. Luo and Ren (2016) proposed a new dynamic IVIF decision-making method based on a similarity measure.

5.1.3 Review of study in waste disposal location selection

Many studies have published approaches based on MCDM methods conducted on waste disposal location selection in the literature. Mohammadi Seif Abad et al. (2021) presented an application on the integrated AHP- The Preference Ranking Organization METHod for Enrichment Evaluation (PROMETHEE) approach for municipal solid waste location selection. In the study, the determined criteria have been weighted using the AHP method. PROMETHEE method has been used for the selection of regions divided into 3 classes using Geographical Information System (GIS) software. Zarin et al. (2021) presented an integrated approach to suitable landfill site selection using GIS, AHP, and fuzzy weighted linear combination (WLC) techniques. In the study, 13 criteria have been determined and the alternatives have been prioritized according to these criteria. Çalış Boyacı et al. (2021) proposed a GIS-based hybrid hesitant fuzzy decision-making approach on site selection for waste vegetable oil and waste battery collection boxes. According to the opinions of decision makers, a total of 88 items according to 7 criteria and 15 alternatives have been determined. Rezaeisabzevar et al. (2020) presented a comprehensive review of landfill selection methods by emphasizing MCDM analysis. It has been shown that the most commonly used methods are WLC, ordinal weighted average (OWA), AHP, fuzzy AHP, TODIM (an acronym in Portuguese for Interactive and Multicriteria Decision Making), fuzzy TODIM, and ANP. Chabuk et al. (2019) presented landfill site selection based on 15 criteria. In the study, ten alternatives have been prioritized by using AHP and ratio scale weighting (RSW). Özkan et al. (2019) presented a state-of-the-art of recent papers in GIS-based MCDM modeling for landfill site suitability analysis. By examining a total of 106 studies published between 2005 and 2019, the studies have been classified according to GIS software, application area, uncertainty, MCDM techniques, cell sizes in GIS, and criteria. The results show that AHP and WLC are the most widely used MCDM methods for weighting the criteria and ranking the alternatives, respectively. Kalbar et al. (2012) develop a scenario-based multiple-attribute decision-making (MADM) methodology for the wastewater treatment alternative selection. The four most commonly used wastewater treatment technologies have been ranked according to the 6 scenarios determined. The scenarios

have been developed that capture the regional and local societal priorities of urban, suburban, and rural areas and translate them into the mathematical algorithm of the MADM methodology. Studies conducted for the waste disposal location selection based on uncertainty are as presented in Table 5.2.

Table 5.2 : Studies on the waste disposal location selection.

Author(s)	Method(s)	Uncertainty	Criteria	Study Origin
Özkan et al. (2020)	TOPSIS	Hesitant fuzzy linguistic term sets	Surface water, geology, groundwater, coastline, erosion, landslide, precipitation, temperature, earthquake, highways, residential areas, slope, elevation, aspect, land use	Turkey
Feyzi et al. (2019)	DEMATEL, ANP	Triangular fuzzy numbers	Elevation, slope, distance from rivers, groundwater depth, distance from the fault, solid texture, geology, distance from protected area, land use, distance from road, distance from electrical grid, distance from railway, distance from landfill, distance from tourist attraction distance from urban center	Iran
Liu et al. (2014)	VIKOR	Interval 2-tuple linguistic fuzzy sets	Adjacent land use, climate, road access, cost	Turkey
Wichapa and Khokhajaikiat (2017)	AHP, Goal programming	Fuzzy sets	Infrastructure, geological, environmental & social	Thailand
Gorsevski et al. (2012)	AHP, ordered weighted average (OWA)	Fuzzy sets	Slope, elevation, distance from rivers, distance from lakes, distance from springs, land use, hydrogeology, distance from faults, distance from urban and rural area, proximity to roads, proximity building materials, proximity dense population	Macedonia
Karasan et al. (2019)	AHP	Pythagorean fuzzy sets	Groundwater depth, land use, elevation and slope, odors nuisance, distance to urban settlements, public reaction, distance to historical and cultural sites, land cost, transportation cost, distance to road, distance to waste source, distance to airports	Turkey
Eghtesadifard et al. (2020)	DEMATEL, ANP, k-means clustering algorithm, MOORA, WASPAS, COPRAS	Fuzzy sets	Land price, distance from roads, distance from rivers, distance from lakes, distance from grasslands, distance from forest regions, distance from agricultural lands, climate conditions, distance from flood-prone areas, slope of land, terrain, AMSL, distance from residential areas	Iran
Soroudi et al. (2018)	DEMATEL, ANP, WLC	Fuzzy sets	Geology, aspect, slope, soil depth, erosion, distance from built-up area, distance from road, distance from fault, distance from infrastructure, distance from airport	Iran

5.2 Preliminaries

In this section, basic notions and operations of IFSs are briefly presented.

5.2.1 Intuitionistic fuzzy sets

IFSs, which are an extension of ordinary fuzzy sets have been introduced by Atanassov (1986). IFSs characterized by degrees of membership and non-membership allow to express the decision maker's opinions more flexibly. The sum of the membership and non-membership degrees of elements in an IFS is one or less than one as demonstrated in Definition 5.1. Thanks to these features, IFSs have been successfully applied to real-life problems in various fields (Alkan, 2021; Alkan & Kahraman, 2022c; Atanassov, 2017; Castillo et al., 2015; Chen & Huang, 2017; Karaşan et al., 2020; Kaya et al., 2020; Ren et al., 2016; Sotirov et al., 2016; Zeraatkar & Afsari, 2021).

Definition 5.1. Let X be a non-empty set. An IFS $\tilde{I}$ in X is given as in Eq. (5.1) (Atanassov, 1986).

$$\tilde{I} = \{(x, \mu_{\tilde{I}}(x), \vartheta_{\tilde{I}}(x)) | x \in X\} \quad (5.1)$$

where the function $\mu_{\tilde{I}}: X \rightarrow [0,1]$ and $\vartheta_{\tilde{I}}: X \rightarrow [0,1]$ represent the membership and non-membership degrees of an element to the sets $\tilde{I}$, respectively, and it satisfies the condition of

$$0 \leq \mu_{\tilde{I}}(x) + \vartheta_{\tilde{I}}(x) \leq 1 \quad \text{for } \forall x \in X \quad (5.2)$$

$\pi_{\tilde{I}}(x) = 1 - \mu_{\tilde{I}}(x) - \vartheta_{\tilde{I}}(x)$ represent the degree of hesitancy of x to $\tilde{I}$.

Definition 5.2. Let $\tilde{A} = (\mu_{\tilde{A}}, \vartheta_{\tilde{A}})$ and $\tilde{B} = (\mu_{\tilde{B}}, \vartheta_{\tilde{B}})$ be two intuitionistic fuzzy numbers (IFNs). Then some operations of two IFNs are defined as follows (Atanassov, 1986).

$$\tilde{A} \oplus \tilde{B} = (\mu_{\tilde{A}} + \mu_{\tilde{B}} - \mu_{\tilde{A}}\mu_{\tilde{B}}, \vartheta_{\tilde{A}}\vartheta_{\tilde{B}}) \quad (5.3)$$

$$\tilde{A} \otimes \tilde{B} = (\mu_{\tilde{A}}\mu_{\tilde{B}}, \vartheta_{\tilde{A}} + \vartheta_{\tilde{B}} - \vartheta_{\tilde{A}}\vartheta_{\tilde{B}}) \quad (5.4)$$

$$\lambda \tilde{A} = ((1 - (1 - \mu_{\tilde{A}})^\lambda), \vartheta_{\tilde{A}}^\lambda), \quad \lambda > 0 \quad (5.5)$$

$$\tilde{A}^\lambda = (\mu_{\tilde{A}}^\lambda, (1 - (1 - \vartheta_{\tilde{A}})^\lambda)), \quad \lambda > 0 \quad (5.6)$$

Definition 5.3. The subtraction $\ominus$ and division $\oslash$ operations for two IFNs $\tilde{A}$ and $\tilde{B}$ are defined as given in Eqs. (5.7) and (5.8) (Du, 2021).

$$\tilde{A} \ominus \tilde{B} = \begin{cases} \langle 0, \frac{\vartheta_{\tilde{A}}}{\vartheta_{\tilde{B}}} \rangle, & \text{if } \mu_{\tilde{A}} \leq \mu_{\tilde{B}}, \vartheta_{\tilde{A}} \leq \vartheta_{\tilde{B}} \\ \langle \frac{\mu_{\tilde{A}} - \mu_{\tilde{B}}}{1 - \mu_{\tilde{B}}}, \frac{\vartheta_{\tilde{A}}}{\vartheta_{\tilde{B}}} \rangle, & \text{if } 0 \leq \frac{\vartheta_{\tilde{A}}}{\vartheta_{\tilde{B}}} \leq \frac{1 - \mu_{\tilde{A}}}{1 - \mu_{\tilde{B}}} < 1 \\ \langle \frac{\mu_{\tilde{A}} - \mu_{\tilde{B}}}{1 - \mu_{\tilde{B}}}, \frac{1 - \mu_{\tilde{A}}}{1 - \mu_{\tilde{B}}} \rangle, & \text{if } \mu_{\tilde{A}} \geq \mu_{\tilde{B}}, \frac{1 - \mu_{\tilde{A}}}{1 - \mu_{\tilde{B}}} < \frac{\vartheta_{\tilde{A}}}{\vartheta_{\tilde{B}}} \\ \langle 0, 1 \rangle, & \text{if } \mu_{\tilde{A}} \leq \mu_{\tilde{B}}, \vartheta_{\tilde{A}} > \vartheta_{\tilde{B}} \end{cases} \quad (5.7)$$

$$\tilde{A} \oslash \tilde{B} = \begin{cases} \langle \frac{\mu_{\tilde{A}}}{\mu_{\tilde{B}}}, 0 \rangle, & \text{if } \mu_{\tilde{A}} \leq \mu_{\tilde{B}}, \vartheta_{\tilde{A}} \leq \vartheta_{\tilde{B}} \\ \langle \frac{\mu_{\tilde{A}}}{\mu_{\tilde{B}}}, \frac{\vartheta_{\tilde{A}} - \vartheta_{\tilde{B}}}{1 - \vartheta_{\tilde{B}}} \rangle, & \text{if } 0 \leq \frac{\mu_{\tilde{A}}}{\mu_{\tilde{B}}} \leq \frac{1 - \vartheta_{\tilde{A}}}{1 - \vartheta_{\tilde{B}}} < 1 \\ \langle \frac{1 - \vartheta_{\tilde{A}}}{1 - \vartheta_{\tilde{B}}}, \frac{\vartheta_{\tilde{A}} - \vartheta_{\tilde{B}}}{1 - \vartheta_{\tilde{B}}} \rangle, & \text{if } \frac{1 - \vartheta_{\tilde{A}}}{1 - \vartheta_{\tilde{B}}} < \frac{\mu_{\tilde{A}}}{\mu_{\tilde{B}}}, \vartheta_{\tilde{A}} > \vartheta_{\tilde{B}} \\ \langle 1, 0 \rangle, & \text{if } \mu_{\tilde{A}} > \mu_{\tilde{B}}, \vartheta_{\tilde{A}} \leq \vartheta_{\tilde{B}} \end{cases} \quad (5.8)$$

Definition 5.4. Let $\tilde{I} = (\mu_{\tilde{I}}, \vartheta_{\tilde{I}})$ be an IFN, then the score function $S(\tilde{I})$ and accuracy function $H(\tilde{I})$ of $\tilde{I}$ can be defined as in Eqs. (5.9) and (5.10), respectively (S. M. Chen & Tan, 1994).

$$S(\tilde{I}) = \mu_{\tilde{I}} - \vartheta_{\tilde{I}} \quad (5.9)$$

$$H(\tilde{I}) = \mu_{\tilde{I}} + \vartheta_{\tilde{I}} \quad (5.10)$$

The defuzzification process is used to obtain the net value from IFNs or to compare IFNs. However, many defuzzification functions developed in the literature include results with negative values and/or do not take into account the degree of hesitation (Chen & Tan, 1994; Liu & Wang, 2007; Wang et al., 2006). Besides, these defuzzification functions have shortcomings for some IFNs. Considering these drawbacks and shortcomings in the literature, a new defuzzification function has been developed as given in Definition 5.5.

Definition 5.5. Let $\tilde{A} = (\mu_{\tilde{A}}, \vartheta_{\tilde{A}})$ be an IFN, then the defuzzification function $\mathfrak{D}(\tilde{A})$ of this number is developed as in Eq. (5.11).

$$\mathfrak{D}(\tilde{A}) = \mu_{\tilde{A}} + \left(\frac{\mu_{\tilde{A}} \vartheta_{\tilde{A}} e^{(1 - \mu_{\tilde{A}} - \vartheta_{\tilde{A}})}}{2} \right)^2 \quad (5.11)$$

where $\mathfrak{D}(\tilde{A}) \in [0, 1]$. The larger the value of $\mathfrak{D}(\tilde{A})$, the larger the IFN $\tilde{A}$.

5.2.2 Intuitionistic fuzzy aggregations

Aggregation operators, which are used for different purposes in various studies, have an important place in the literature. They are frequently used for purposes such as aggregating decision matrices or ranking attributes, especially in MCDM methods. Many aggregation operators have been developed based on the weighted average (WA) operator, the ordered weighted average (OWA) operator, the weighted geometric (WG) operator, and the ordered weighted geometric (OWG) operator by various researchers in the literature. Xu (2007) developed aggregation operators such as intuitionistic fuzzy weighted averaging operator, intuitionistic fuzzy hybrid aggregation operator, and intuitionistic fuzzy ordered weighted averaging operator for aggregating intuitionistic fuzzy numbers. Xu and Yager (2006) introduced new geometric aggregation operators, such as the intuitionistic fuzzy weighted geometric (IFWG) operator, the intuitionistic fuzzy hybrid geometric (IFHG) operator, and the intuitionistic fuzzy ordered weighted geometric (IFOWG) operator. Rahman et al. (2021) proposed several Einstein hybrid aggregation operators based on confidence level, such as confidence intuitionistic fuzzy Einstein hybrid averaging operator, confidence intuitionistic fuzzy Einstein hybrid geometric operator, generalized confidence intuitionistic fuzzy Einstein hybrid averaging operator and generalized confidence intuitionistic fuzzy Einstein hybrid geometric operator. Dong and Geng (2021) developed new aggregation operators such as the trapezoid intuitionistic fuzzy linguistic Maclaurin symmetric mean (TIFLMSM) operator, trapezoid intuitionistic fuzzy linguistic generalized Maclaurin symmetric mean (TIFLGMSM) operator, trapezoid intuitionistic fuzzy linguistic weighted Maclaurin symmetric mean (TIFLWMSM) operator and trapezoid intuitionistic fuzzy linguistic weighted generalized Maclaurin symmetric mean (TI-FLWGMSM) operator. Liu et al. (2018) developed new operations such as the intuitionistic fuzzy weighted Dombi Bonferroni mean (IFWDBM) operator, intuitionistic fuzzy weighted Dombi geometric Bonferroni mean (IFWDGBM) operator, intuitionistic fuzzy Dombi Bonferroni mean (IFDBM) operator, intuitionistic fuzzy Dombi geometric Bonferroni mean (IFDGBM) operator. The most commonly used aggregation operators in the literature are as given in Definitions 5.6 and 5.7.

Definition 5.6. Let $\tilde{A}_i = (\mu_{\tilde{A}_i}, \vartheta_{\tilde{A}_i})$ ($i = 1, 2, \dots, n$) be a set of IFNs and $w = (w_1, w_2, \dots, w_n)$ be weight vector of $\tilde{A}_i$ with $\sum_{i=1}^n w_i$, then an intuitionistic fuzzy weighted average (IFWA) operator is (Xu, 2007):

$$IFWA(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) = \left(\left(1 - \prod_{i=1}^n (1 - \mu_{\tilde{A}_i})^{w_i} \right), \prod_{i=1}^n \vartheta_{\tilde{A}_i}^{w_i} \right) \quad (5.12)$$

Definition 5.7. Let $\tilde{A}_i = (\mu_{\tilde{A}_i}, \vartheta_{\tilde{A}_i})$ ($i = 1, 2, \dots, n$) be a set of IFNs and $w = (w_1, w_2, \dots, w_n)$ be weight vector of $\tilde{A}_i$ with $\sum_{i=1}^n w_i$, then an intuitionistic fuzzy weighted geometric (IFWG) operator is (Xu & Yager, 2006):

$$IFWG(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) = \left(\prod_{i=1}^n \mu_{\tilde{A}_i}^{w_i}, \left(1 - \prod_{i=1}^n (1 - \vartheta_{\tilde{A}_i})^{w_i} \right) \right) \quad (5.13)$$

5.3 Proposed Model

In real-world decision-making problems, classical MCDM methods perform selection among alternatives by considering only the current situation. However, in parallel with the rapidly changing environmental conditions, the judgments and opinions of decision-makers can also change, and therefore, the decision-making process must be handled in a dynamic structure to make effective decisions in a certain time period. The fact that the data used in decision making problems may change over time will cause the decision to be made now to be a wrong decision in the future. Therefore, there is a need for a dynamic decision-making model that can take changes in the data into account over time. The DEVADA method should be preferred because it has this feature. In addition, many decision-making environments contain imprecise, uncertain data and incomplete knowledge of decision-makers. Therefore, in this section, we propose a new dynamic MCDM method, namely IF-DEVADA to deal with multi-criteria decision-making problems and better address the uncertainties and hesitations of decision-makers. The developed method is based on a flexible decision-making model that incorporates current and future situations into decision-making process and takes into account uncertainty. Thanks to the developed dynamic model, changes that may occur at a certain time in the future are also taken into account in an environment of uncertainty. Anticipated future information about factors considered during an

application can be obtained using predictive models or can be estimated through linguistic expressions based on decision maker knowledge or other available sources. In this method, the ranking of alternatives is determined by considering two measures, Euclidean and cosine distances. Since the distances of each alternative from the positive and negative ideal solutions are calculated based on the Euclidean and cosine distances, a stronger multi-measurement unit is suggested. The degree of closeness of the combined Euclidean and cosine distances is determined by means of an expert-defined threshold parameter. Besides, we also present the extension of the CRITIC method with IFSs to reduce the subjective attribute of the decision process in an uncertain environment, taking into account the objective weights of the criteria. The framework of the proposed application is demonstrated in the flowchart given in Fig. 5.1. The steps of the developed application are detailed theoretically in the following steps.

Step 1. Determine the alternatives, and relevant criteria to construct the framework of the application. The set $A_i = \{A_1, A_2, \dots, A_m\}$ having $i = 1, 2, \dots, m$ alternatives, is assessed by n decision criteria of set $C_j = \{C_1, C_2, \dots, C_n\}$, with $j = 1, 2, \dots, n$. Let $w = (w_1, w_2, \dots, w_n)$ be the vector set used for defining the criteria weights, where $w_j > 0$ and $\sum_{j=1}^n w_j = 1$. Decision-makers are represented with $DM 1, DM 2, \dots, DM k$, who are experts in their fields to make judgments.

Step 2. Determine the IF weights of criteria for each decision maker using the scale given in Table 5.3. For weighting the criteria, various methods can be used such as AHP, ANP, BWM, FUCOM, and CRITIC. In this study, intuitionistic fuzzy CRITIC method has been developed for weighting the criteria. For this, the current linguistic decision matrices are constructed by using the linguistic scale given in Table 5.3 to evaluate according to decision-makers' opinions the alternatives in terms of criteria. After the current linguistic decision matrices are transformed its corresponding IFNs, the IF current decision matrix $\tilde{X}_k^C = (\tilde{x}_{ijk}^C)_{n \times m}$ based on $k^{th} DM$ is presented as given in Table 5.4. Here, $\tilde{X}_k^C = (\tilde{x}_{ijk}^C)_{n \times m}$ in which $\tilde{x}_{ijk}^C = (\mu_{ijk}^C, \vartheta_{ijk}^C)$ indicates the performance of alternative A_i in terms of criterion C_j of $k^{th} DM$.

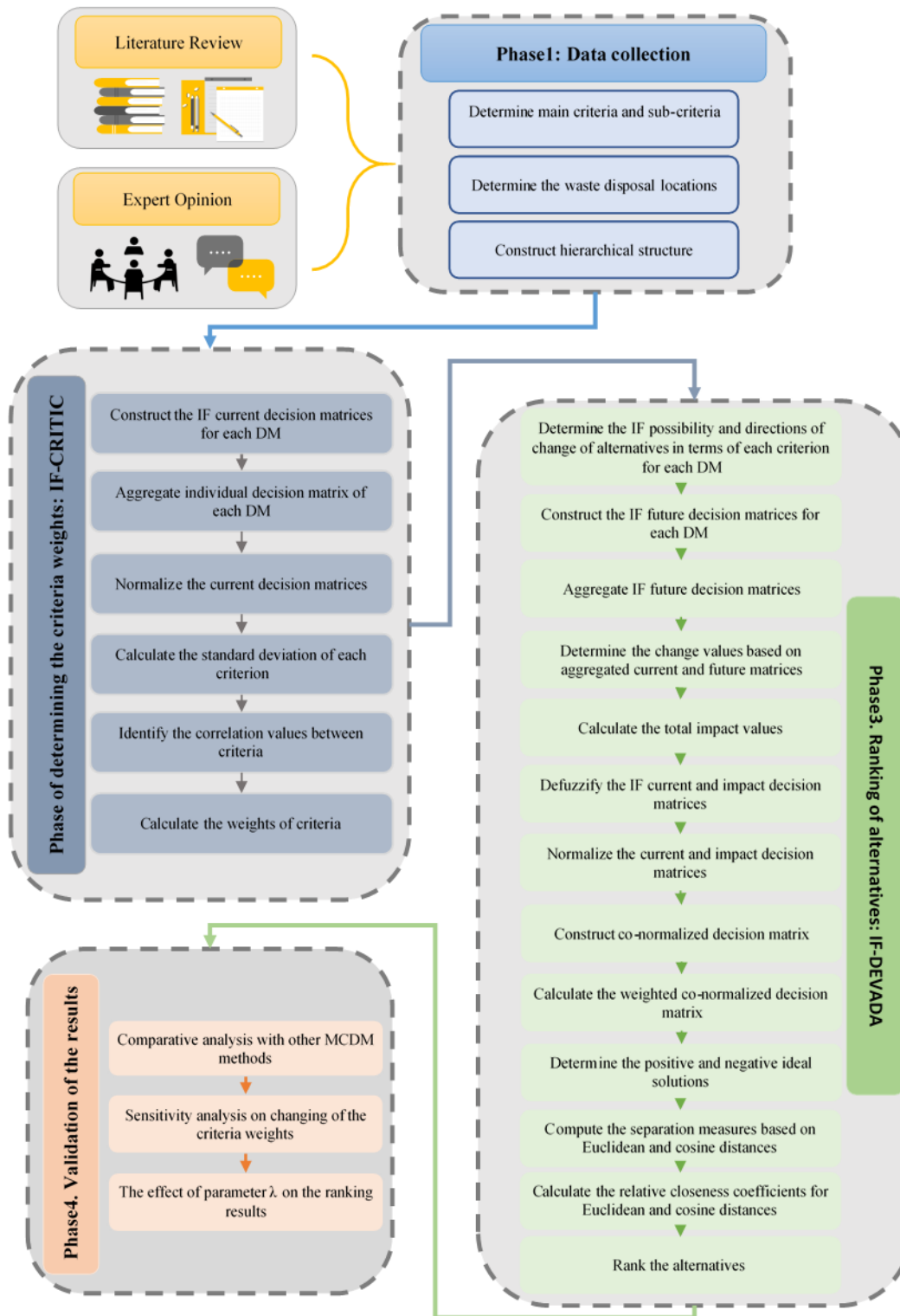


Figure 5.1 : Framework of the developed methodology.

Table 5.3 : Linguistic scale for alternative ratings.

Linguistic terms	IFN for alternatives	
	μ	ϑ
Absolutely Low Value – (ALV)	0.1	0.85
Very Low Value – (VLV)	0.2	0.75
Low Value – (LV)	0.3	0.65
Medium Low Value – (MLV)	0.4	0.55
Approximately Value – (AV)	0.5	0.45
Medium High Value – (MHV)	0.6	0.35
High Value – (HV)	0.7	0.25
Very High Value – (VHV)	0.8	0.15
Absolutely High Value – (AHV)	0.9	0.05

Table 5.4 : Current decision matrix based on IFN according to DM k .

Criteria	Alternatives			
	A_1	A_2		A_m
C_1	$(\mu_{11k}^C, \vartheta_{11k}^C)$	$(\mu_{12k}^C, \vartheta_{12k}^C)$		$(\mu_{1mk}^C, \vartheta_{1mk}^C)$
C_2	$(\mu_{21k}^C, \vartheta_{21k}^C)$	$(\mu_{22k}^C, \vartheta_{22k}^C)$		$(\mu_{2mk}^C, \vartheta_{2mk}^C)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
C_n	$(\mu_{n1k}^C, \vartheta_{n1k}^C)$	$(\mu_{n2k}^C, \vartheta_{n2k}^C)$		$(\mu_{nmk}^C, \vartheta_{nmk}^C)$

Step 3. Obtain the aggregated IF current decision matrix. The individual decision matrix of each DM is aggregated in a single decision matrix using *IFWG* given in Eq. (5.13). Aggregated IF current decision matrix $(\tilde{X}^C)$ is constructed as in Table 5.5. Here, $\tilde{X}^C = (\tilde{x}_{ij}^C)_{n \times m}$ in which $\tilde{x}_{ij}^C = (\mu_{ij}^C, \vartheta_{ij}^C)$ presents the aggregated IFN of i^{th} alternative with respect to j^{th} criterion.

Table 5.5 : Aggregated IF current decision matrix.

Criteria	Alternatives			
	A_1	A_2		A_m
C_1	$(\mu_{11}^C, \vartheta_{11}^C)$	$(\mu_{12}^C, \vartheta_{12}^C)$		$(\mu_{1m}^C, \vartheta_{1m}^C)$
C_2	$(\mu_{21}^C, \vartheta_{21}^C)$	$(\mu_{22}^C, \vartheta_{22}^C)$		$(\mu_{2m}^C, \vartheta_{2m}^C)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
C_n	$(\mu_{n1}^C, \vartheta_{n1}^C)$	$(\mu_{n2}^C, \vartheta_{n2}^C)$		$(\mu_{nm}^C, \vartheta_{nm}^C)$

Step 4. Determine the degrees of correlation between criteria. For this, the decision matrix is firstly defuzzified by using Eqs. (5.14) and (5.15) by considering the criteria type, which can be benefit type and cost type. Then, the standard deviation for each criterion is computed by using Eq. (5.16) and the correlation relationship of each criterion with other criteria is identified by using Eq. (5.17).

$$\varphi_{ij} = \frac{1 - \frac{\mu_{ij} \times \mu_- + \vartheta_{ij} \times \vartheta_- + \pi_{ij} \times \pi_-}{\mu_{ij}^2 \vee \mu_-^2 + \vartheta_{ij}^2 \vee \vartheta_-^2 + \pi_{ij}^2 \vee \pi_-^2}}{1 - \frac{\mu_- \times \mu_+ + \vartheta_- \times \vartheta_+ + \pi_- \times \pi_+}{\mu_-^2 \vee \mu_+^2 + \vartheta_-^2 \vee \vartheta_+^2 + \pi_-^2 \vee \pi_+^2}} \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n, \quad (5.14)$$

for positive attributes

$$\varphi_{ij} = \frac{1 - \frac{\mu_{ij} \times \mu_+ + \vartheta_{ij} \times \vartheta_+ + \pi_{ij} \times \pi_+}{\mu_{ij}^2 \vee \mu_+^2 + \vartheta_{ij}^2 \vee \vartheta_+^2 + \pi_{ij}^2 \vee \pi_+^2}}{1 - \frac{\mu_- \times \mu_+ + \vartheta_- \times \vartheta_+ + \pi_- \times \pi_+}{\mu_-^2 \vee \mu_+^2 + \vartheta_-^2 \vee \vartheta_+^2 + \pi_-^2 \vee \pi_+^2}} \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n, \quad (5.15)$$

for negative attributes

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^m (\varphi_{ij} - \bar{\varphi}_j)^2}{m}} \quad j = 1, 2, \dots, n \quad (5.16)$$

$$r_{jk} = \frac{\sum_{i=1}^m (\varphi_{ij} - \bar{\varphi}_j)(\varphi_{ik} - \bar{\varphi}_k)}{\sqrt{\sum_{i=1}^m (\varphi_{ij} - \bar{\varphi}_j)^2 \sum_{i=1}^m (\varphi_{ik} - \bar{\varphi}_k)^2}} \quad j = 1, 2, \dots, n; k = 1, 2, \dots, n \quad (5.17)$$

where μ_- and μ_+ are the lowest and highest degree of membership, respectively, ϑ_- and ϑ_+ are the lowest and highest degree of non-membership, respectively, and $\bar{\varphi}_j$ and $\bar{\varphi}_k$ are the average of the defuzzied values of the j th and k th criteria, respectively. The lowest and highest degrees of membership and non-membership are determined by using the score function given in Eq. (5.9).

Step 5. Calculate the criterion weights through the standard deviation of each criterion and its correlation between other criteria by using Eq. (5.18). After objective weights of criteria are normalized, the weight of each criterion is obtained by using Eq. (5.19).

$$C_j = \sigma_j \sum_{k=1}^n (1 - r_{jk}), \quad j = 1, 2, \dots, n \quad (5.18)$$

$$w_j = \frac{C_j}{\sum_{j=1}^n C_j} \quad (5.19)$$

Step 6. Determine the IF possibility of change in the future for alternatives with respect to each criterion depending on the direction of change for a given time period based on some additional research. The IF possibility of change in the future, either positive or negative, is determined by each DM using Table 5.6. The IF future decision matrix $\tilde{X}_k^F = (\tilde{x}_{ijk}^F)_{n \times m}$ based on k^{th} DM is constructed as given in Table 5.7 by considering the IF future change possibility of DM based on Eqs. (5.3) and (5.7). Here, $\tilde{X}_k^F =$

$(\tilde{x}_{ijk}^F)_{n \times m}$ in which $\tilde{x}_{ijk}^F = (\mu_{ijk}^F, \vartheta_{ijk}^F)$ indicates the performance of alternatives corresponding in the future.

Table 5.6 : Linguistic scale for alternative ratings.

Linguistic terms for positive direction	IFN for alternatives		Linguistic terms for negative direction
	μ	ϑ	
Positive Absolutely Low Value – (PALV)	0.1	0.85	Negative Absolutely Low Value – (NALV)
Positive Very Low Value – (PVLV)	0.2	0.75	Negative Very Low Value – (NVLV)
Positive Low Value – (PLV)	0.3	0.65	Negative Low Value – (NLV)
Positive Medium Low Value – (PMLV)	0.4	0.55	Negative Medium Low Value – (NMLV)
Positive Approximately Value – (PAV)	0.5	0.45	Negative Approximately Value – (NAV)
Positive Medium High Value – (PMHV)	0.6	0.35	Negative Medium High Value – (NMHV)
Positive High Value – (PHV)	0.7	0.25	Negative High Value – (NHV)
Positive Very High Value – (PVHV)	0.8	0.15	Negative Very High Value – (NVHV)
Positive Absolutely High Value – (PAHV)	0.9	0.05	Negative Absolutely High Value – (NAHV)

Table 5.7 : Future decision matrix based on IFN according to DM k .

Criteria	Alternatives			
	A_1	A_2	...	A_m
C_1	$(\mu_{11k}^F, \vartheta_{11k}^F)$	$(\mu_{12k}^F, \vartheta_{12k}^F)$	...	$(\mu_{1mk}^F, \vartheta_{1mk}^F)$
C_2	$(\mu_{21k}^F, \vartheta_{21k}^F)$	$(\mu_{22k}^F, \vartheta_{22k}^F)$	...	$(\mu_{2mk}^F, \vartheta_{2mk}^F)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
C_n	$(\mu_{n1k}^F, \vartheta_{n1k}^F)$	$(\mu_{n2k}^F, \vartheta_{n2k}^F)$	...	$(\mu_{nmk}^F, \vartheta_{nmk}^F)$

Step 7. Aggregate IF future decision matrices of each DM. The individual decision matrices of each DM are aggregated in a single decision matrix using *IFWG* given in Eq. (5.13). Aggregated IF future decision matrix $(\tilde{X}^F)$ is constructed as in Table 5.8. Here, $\tilde{X}^F = (\tilde{x}_{ij}^F)_{n \times m}$ in which $\tilde{x}_{ij}^F = (\mu_{ij}^F, \vartheta_{ij}^F)$ presents the aggregated IFN of i^{th} alternative with respect to j^{th} criterion.

Table 5.8 : Aggregated IF future decision matrix.

Criteria	Alternatives			
	A_1	A_2	...	A_m
C_1	$(\mu_{11}^F, \vartheta_{11}^F)$	$(\mu_{12}^F, \vartheta_{12}^F)$	...	$(\mu_{1m}^F, \vartheta_{1m}^F)$
C_2	$(\mu_{21}^F, \vartheta_{21}^F)$	$(\mu_{22}^F, \vartheta_{22}^F)$	...	$(\mu_{2m}^F, \vartheta_{2m}^F)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
C_n	$(\mu_{n1}^F, \vartheta_{n1}^F)$	$(\mu_{n2}^F, \vartheta_{n2}^F)$	...	$(\mu_{nm}^F, \vartheta_{nm}^F)$

Step 8. Determine the change that will occur in the given time period taking into account the constructed current and future decision matrices by using Eq. (5.20), which is adopted from Eq. (5.7), and create the future changes decision matrix $\tilde{S} = (\tilde{s}_{ij})$.

$$\tilde{s}_{ij} = \tilde{x}_{ij}^F \ominus \tilde{x}_{ij}^C = \begin{cases} \langle 0, \frac{\vartheta_{ij}^F}{\vartheta_{ij}^C} \rangle, & \mu_{ij}^F \leq \mu_{ij}^C, \vartheta_{ij}^F \leq \vartheta_{ij}^C \\ \langle \frac{\mu_{ij}^F - \mu_{ij}^C}{1 - \mu_{ij}^C}, \frac{\vartheta_{ij}^F}{\vartheta_{ij}^C} \rangle, & 0 \leq \frac{\vartheta_{ij}^F}{\vartheta_{ij}^C} \leq \frac{1 - \mu_{ij}^F}{1 - \mu_{ij}^C} < 1 \\ \langle \frac{\mu_{ij}^F - \mu_{ij}^C}{1 - \mu_{ij}^C}, \frac{1 - \mu_{ij}^F}{1 - \mu_{ij}^C} \rangle, & \mu_{ij}^F > \mu_{ij}^C, \frac{1 - \mu_{ij}^F}{1 - \mu_{ij}^C} < \frac{\vartheta_{ij}^F}{\vartheta_{ij}^C} \\ \langle 0, 1 \rangle, & \mu_{ij}^F \leq \mu_{ij}^C, \vartheta_{ij}^F > \vartheta_{ij}^C \end{cases} \quad (5.20)$$

Step 9. Calculate the total impact values that take into account the reflection according to the highest IF value of each criterion in the current decision matrix of each IF change and the position according to the IF maximum value of each criterion of each IF future value in the future decision matrix using Eq. (5.21). The IF total impact decision matrix $\tilde{\mathcal{T}} = (\tilde{t}_{ij})$ is obtained.

$$\tilde{t}_{ij} = (\tilde{s}_{ij} \oslash \max(\tilde{x}_{ij}^C)) \oplus (\tilde{x}_{ij}^F \oslash \max(\tilde{x}_{ij}^F)) \quad (5.21)$$

$i = 1, 2, \dots, m; j = 1, 2, \dots, n$

Step 10. Defuzzify the IF current and impact decision matrices by using Eqs. (5.22) and (5.23). Here, a novel defuzzification operator is developed by adopting from Eq. (5.11) to obtain the crisp values of IFNs.

$$\zeta_{ij}^C = \mu_{ij}^C + \left(\frac{\mu_{ij}^C \vartheta_{ij}^C e^{(1 - \mu_{ij}^C - \vartheta_{ij}^C)}}{2} \right)^2 \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n \quad (5.22)$$

$$\zeta_{ij}^t = \mu_{ij}^t + \left(\frac{\mu_{ij}^t \vartheta_{ij}^t e^{(1 - \mu_{ij}^t - \vartheta_{ij}^t)}}{2} \right)^2 \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n \quad (5.23)$$

Step 11. Calculate the normalized matrices $R^C = (r_{ij}^C)$ and $R^I = (r_{ij}^I)$ based on the defuzzified current (C) and impact (I) decision matrices, respectively. The normalized values r_{ij}^C and r_{ij}^I are obtained by using Eqs. (5.24) and (5.25) for the current decision matrix and Eqs. (5.26) and (5.27) for impact decision matrix, respectively, by considering the criteria type, which can be benefit type and cost type.

$$r_{ij}^C = \frac{\zeta_{ij}^C}{\max_j \zeta_{ij}^C}, \quad \text{for benefit criterion} \quad (5.24)$$

$$r_{ij}^C = 1 - \frac{z_{ij}^C}{\max_j z_{ij}^C}, \quad \text{for cost criterion} \quad (5.25)$$

$$r_{ij}^I = \frac{z_{ij}^t}{\max_j z_{ij}^t}, \quad \text{for benefit criterion} \quad (5.26)$$

$$r_{ij}^I = 1 - \frac{z_{ij}^t}{\max_j z_{ij}^t}, \quad \text{for cost criterion} \quad (5.27)$$

where $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$.

Step 12. Compute the co-normalized decision matrix $\Phi = (\varphi_{ij})$ that handles the current decision matrix and impact decision matrix together using Eq. (5.28).

$$\varphi_{ij} = \frac{r_{ij}^C + r_{ij}^I}{\max_i (r_{ij}^C + r_{ij}^I)} \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n \quad (5.28)$$

Step 13. Determine the weighted co-normalized decision matrix $\Psi = (\psi_{ij})$ using Eq. (5.29). The weighted co-normalized decision matrix is obtained by multiplying the co-normalized decision matrix with the criterion weights vector as given in Eq. (5.29).

$$\psi_{ij} = w_j \varphi_{ij} \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n \quad (5.29)$$

Step 14. Identify positive ideal solution (PIS) and negative ideal solution (NIS) by using Eqs. (5.30) and (5.31), respectively, based on the weighted co-normalized decision matrix.

$$PIS = S^* = \left\{ \left(\max_i \psi_{ij} \right) \middle| j = 1, 2, \dots, n \right\} = \{\psi_1^*, \psi_2^*, \dots, \psi_n^*\} \quad (5.30)$$

$$i = 1, 2, \dots, m$$

$$NIS = S^- = \left\{ \left(\min_i \psi_{ij} \right) \middle| j = 1, 2, \dots, n \right\} = \{\psi_1^-, \psi_2^-, \dots, \psi_n^-\} \quad (5.31)$$

$$i = 1, 2, \dots, m$$

Step 15. Obtain the separation measures by calculating the Euclidean and cosine distances for each alternative based on positive-ideal solution S^* and negative-ideal solution S^- . The separation measures of each alternative from the positive ideal

solution based on the Euclidean and cosine distances are computed by using Eqs. (5.32) and (5.33), respectively.

$$S_i^{E*} = \sqrt{\sum_{j=1}^n (\psi_{ij} - \psi_j^*)^2} \quad i = 1, 2, \dots, m \quad (5.32)$$

$$S_i^{C*} = 1 - \frac{\sum_{j=1}^n \psi_{ij} \cdot \psi_j^*}{\sqrt{\sum_{j=1}^n (\psi_{ij})^2} \sqrt{\sum_{j=1}^n (\psi_j^*)^2}} \quad i = 1, 2, \dots, m \quad (5.33)$$

The separation measures of each alternative from the negative ideal solution based on the Euclidean and cosine distances are calculated by using Eqs. (5.34) and (5.35), respectively.

$$S_i^{E-} = \sqrt{\sum_{j=1}^n (\psi_{ij} - \psi_j^-)^2} \quad i = 1, 2, \dots, m \quad (5.34)$$

$$S_i^{C-} = 1 - \frac{\sum_{j=1}^n \psi_{ij} \cdot \psi_j^-}{\sqrt{\sum_{j=1}^n (\psi_{ij})^2} \sqrt{\sum_{j=1}^n (\psi_j^-)^2}} \quad i = 1, 2, \dots, m \quad (5.35)$$

Step 16. Calculate the relative closeness coefficient (CC_i) of alternatives by using Eq. (5.36).

$$CC_i = \lambda \frac{S_i^{E-}}{(S_i^{E-} + S_i^{E*})} + (1 - \lambda) \frac{S_i^{C-}}{(S_i^{C-} + S_i^{C*})} = \lambda CC_i^E + (1 - \lambda) CC_i^C \quad (5.36)$$

$$i = 1, 2, \dots, m$$

where λ is a preference threshold parameter defining the preference weight of the expert in distance measures, and $0 \leq \lambda \leq 1$.

Step 17. Rank the alternatives from the one with the largest CC_i to the one with the smallest CC_i .

5.4 Application: Waste Disposal Location Selection

In this section, the developed methodology is shown on the waste disposal location selection problem for Istanbul. First, after presenting the definition of the problem, the criteria and alternatives of the problem are explained. Then, the solution of the problem

is shown on the developed methodology. Finally, comparison and sensitivity analysis are carried out.

5.4.1 Problem definition

The last and important step in waste management is the determination and selection of places that will ensure that wastes are disposed in the way that does not harm human health and the environment. However, many factors such as rapid population growth combined with poor governance and urban planning, environmental and economic impacts cause significant difficulties in the selection and management of waste disposal locations in developing countries. Today, rapid industrialization with sudden population growth, diversification and mismanagement of solid wastes, uncontrolled urbanization constitutes the main factors that increase solid waste production and consequently environmental degradation. Therefore, the regular storage of solid waste is an important method to prevent environmental pollution that may occur during the disposal of waste, and it constitutes the basic component of the solid waste management system. In addition, it is undoubtedly of great importance to determine the locations of regular waste disposal facilities for the infrastructure design of cities to be sustainable and economical. Waste disposal location selection must be based on many criteria such as transport distance of the waste, legal practices, environmental factors, size and availability of alternative land, transportation facility, population density, the structural condition of soil and ground, climatic conditions and meet these criteria in the best way at the same time (Karasan et al., 2019; Özkan et al., 2020). In terms of all these criteria, a suitable solid waste landfill should have the least impact on social, environmental, and economic conditions, comply with all legal regulations and be accepted by most of the society. The fact that there are many factors that are effective in the selection of a waste landfill site makes it necessary to meet the wishes of all relevant parties when deciding on the suitability of any area for the disposal facility. It is of critical importance to effectively determine which factors will be considered in the decision-making process, which factors will be excluded, or how much these factors will affect the decision to be made. Therefore, it is very difficult to evaluate many conflicting factors and come to an appropriate and correct conclusion. Since such criteria are also in conflict with each other, the selection of waste disposal location creates a multi-criteria decision-making problem. Moreover, changing conditions may render existing waste disposal locations unsuitable and thus, existing

waste disposal sites may no longer be preferred, and it may be necessary to find a new location. Therefore, in this study, we present the application of the new IF-CRITIC and IF-DEVADA methodology, which has the ability to take into account imprecise and uncertain assessments in waste disposal location selection.

Since 1994, several waste disposal facilities in Istanbul have been built and operated on the European and Asian sides. These facilities have been established in Kemerburgaz-Odayeri and Silivri-Seymen on the European side, and Şile-Kömürcüoda on the Asian side. Approximately 320 ha of land in Istanbul has been allocated to landfills for waste disposal. In Istanbul, approximately 17,800 tons of waste per day is disposed of with the regular landfill method. While 6,500 tons/day of this waste is sent to the European side, 11,300 tons/day is sent to the Asian side. The waste disposal process in landfills such as operation, additional site construction, monitoring and control of the site, collection-treatment of leachate and landfill gas management is carried out according to national and international standards. Possible problems arising from disposal processes are prevented thanks to the works such as the creation of an impermeable layer, the laying and compaction of wastes, the formation of an intermediate cover, the protection of ground and surface waters by collecting and treating leachate, and the collection and control of the resulting landfill gas carried out in the design and operation of the sanitary landfills (URL-1). With urbanization, the rapid growth of residential areas and the transportation of solid waste collected from the city directly to landfills is very costly and time-consuming. In order to both reduce these costs and save time, the concept of transfer stations, which act as intermediate processing centers, has emerged. Waste collected throughout Istanbul is taken from city centers and transported to landfills 24/7 thanks to transfer stations and waste transport trucks, reducing the traffic load and saving fuel, labor and time. There are eight transfer stations in Istanbul, namely Silivri, Halkalı, Yenibosna, Baruthane, Hekimbaşı, Küçükbakkalköy, Aydınlı, and Şile (URL-2). However, a new location needs to be determined due to the ever-increasing need for landfills with the increasing waste production. The examined alternative locations have been selected by authorized decision makers through professional communication. All criteria affecting the waste disposal location selection have been identified based on the decision-maker's opinion and available literature. To evaluate the waste disposal locations in Istanbul comprehensively, a hierarchical structure consisting of 3 main, 13 sub-criteria, and 5

alternatives is used by taking into account environmental, social, and economic factors. The possible waste disposal location alternatives for Istanbul have been determined as *A1-Arnavutköy*, *A2-Çatalca*, *A3-Şile*, *A4-Sultanbeyli*, and *A5-Ataşehir* and these alternatives have been shown on the map as given in Fig. 5.2. The proposed hierarchical structure of waste disposal location alternatives for Istanbul is presented in Fig. 5.3.

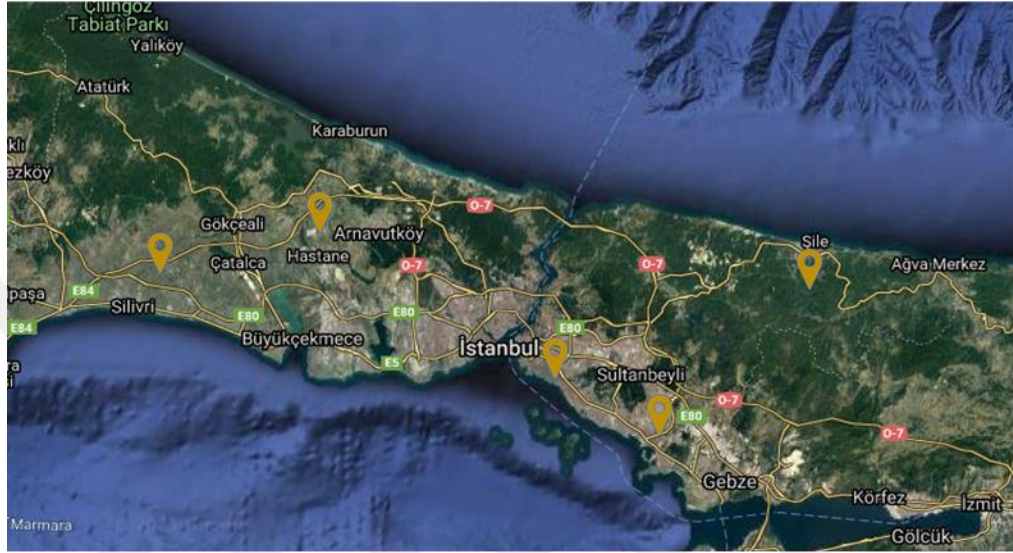


Figure 5.2 : Alternative waste disposal locations.

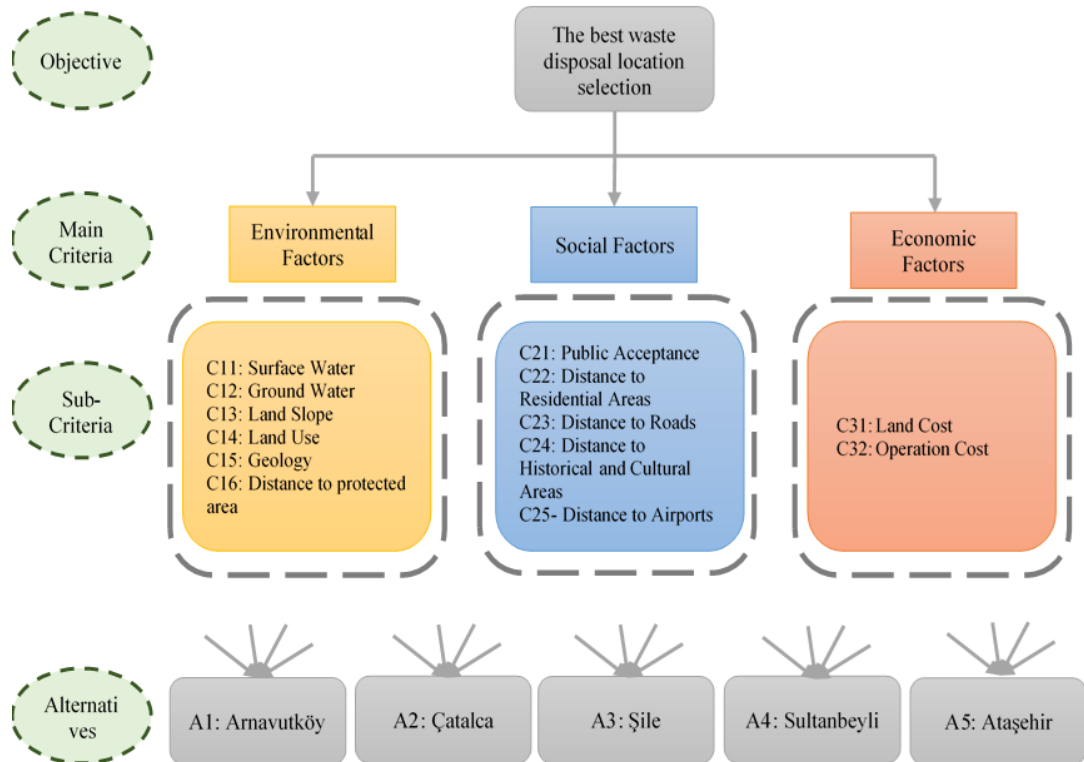


Figure 5.3 : Hierarchical structure of the problem.

The first step in the selection of waste disposal locations is to determine the weights of the main criteria and sub-criteria. For the stage of determining the weights of the criteria, we apply the IF-CRITIC method, where we extend the CRITIC method with IFs. We use the IF-CRITIC method because it eliminates the bias or subjectivity of the decision-makers in the weighting process. Then we apply the new IF-DEVADA method developed for the selection of the most suitable location. We propose the IF-DEVADA method as it takes into account ever-changing conditions such as waste disposal location selection.

The criteria used in the study can be briefly summarized as follow.

Environmental Factors

Surface Water: Since waste disposal areas have a significant impact on the pollution of surface waters, waste disposal areas should be as far away from water sources (lakes, ponds, rivers, etc.) as possible. Waste disposal areas should be planned in such a way that they do not adversely affect the surface water resources in the region (Bahrani et al., 2016; Özkan et al., 2020; Rezaeisabzevar et al., 2020; Şener & Şener, 2020; Şener et al., 2010).

Ground water: To prevent leachate from waste disposal areas from mixing with groundwater resources, a buffer zone should be created between the groundwater resource and the leachate produced and the waste disposal areas should be made impermeable (Karasan et al., 2019; Özkan et al., 2020; Rezaeisabzevar et al., 2020; Şener & Şener, 2020).

Land slope: The land slope is economically important for the construction of a waste disposal area. Construction in high-slope areas is uneconomical and requires a lot of excavation. Therefore, high-slope areas are not suitable for waste disposal areas (Bahrani et al., 2016; Karasan et al., 2019; Rezaeisabzevar et al., 2020; Şener & Şener, 2020; Şener et al., 2010).

Land use: Land use is an important factor for a sustainable environment. For land use, artificial surfaces, forests and natural areas, and wetlands are considered less suitable, while arid areas are considered the most suitable (Gorsevski et al., 2012; Karasan et al., 2019; Liu et al., 2018; Rezaeisabzevar et al., 2020; Şener & Şener, 2020; Şener et al., 2010).

Geology: Places with high potential for water adsorption of alluvium and limestone beds are not suitable for waste disposal areas. The soil permeability of the region where the waste disposal area will be established is an important criterion to be taken against possible construction faults and public opposition (Liu et al., 2018; Özkan et al., 2020; Şener et al., 2010).

Distance to protected area: Waste disposal areas are an important factor that should be established at a certain distance from protected areas such as national parks and wildlife habitats (Bahrani et al., 2016; Rezaeisabzevar et al., 2020; Şener & Şener, 2020; Şener et al., 2010).

Social Factors

Public acceptance: The establishment of a waste disposal area should be accepted and adopted by the public in order not to cause conflict among the public in the following processes (Banar et al., 2007; Karasan et al., 2019).

Distance from residential areas: Waste disposal areas should be located at a significant distance from residential areas to avoid negative effects such as odor, noise, aesthetic appearance, etc. (Gorsevski et al., 2012; Karasan et al., 2019; Liu et al., 2018; Rezaeisabzevar et al., 2020; Şener et al., 2010).

Distance to roads: A waste disposal area should be far enough away from the road so that it does not have a negative aesthetic effect on it. In addition, taking into account the economic factors, the waste disposal area that will increase the cost of the new access road construction should not be built too far from the main roads. Therefore, a waste disposal area should be located at a reasonable distance from major roads (Bahrani et al., 2016; Gorsevski et al., 2012; Karasan et al., 2019; Rezaeisabzevar et al., 2020; Şener et al., 2010).

Distance to historical and cultural areas: A waste disposal area should not be located in an area close to historical and cultural areas due to its negative effects such as aesthetics, odor, and noise (Bahrani et al., 2016; Karasan et al., 2019; Rezaeisabzevar et al., 2020).

Distance to airports: As waste disposal areas attract bird flocks such as seagulls, they pose potential risks for aviation security. Therefore, waste disposal must not be located near an airport and a buffer zone should be left (Karasan et al., 2019; Rezaeisabzevar et al., 2020).

Economic Factors

Land Cost: Land cost per unit area is an important factor in determining a suitable waste disposal area. Therefore, the cost of the land should be taken into account by decision-makers (Karasan et al., 2019; Liu et al., 2018)

Operation cost: The operation cost includes the costs affecting the construction and operation of the waste disposal to be installed. Construction and maintenance materials such as soil resources and clay materials that will be used throughout the life of the waste disposal area should be considered to prevent stabilization and proper drainage or water filtration (Gorsevski et al., 2012; Rezaeisabzevar et al., 2020).

5.4.2 Problem solution

In this section, we present the application steps of IF-CRITIC and IF-DEVADA methods for the evaluation of waste disposal locations in Istanbul. We will first present the steps for obtaining the criteria weights, and then the steps for obtaining the final scores and rankings of the alternatives.

The weights of the criteria considered in the evaluation of waste disposal locations are determined by applying the steps followed:

Step 1. After the literature is examined in detail, the criteria and alternatives given in Section 5.1 are used for the selection of the best waste disposal location. For the evaluation phase of waste disposal locations, a group of decision makers who will carry out the process is formed. The group of three decision makers, abbreviated as DM 1, DM 2, and DM 3, includes a lecturer who advises on waste management, a professor who carries out many projects and studies in the field of MCDM, and a consultant who advises on strategies and policies in the municipality. The weights of these decision-makers with different levels of experience are 0.3, 0.4, and 0.3, respectively.

Step 2. Using the intuitionistic fuzzy linguistic scale given in Table 5.3, the decision-maker group evaluates the alternatives in terms of the main criteria and the sub-criteria. A separate evaluation meeting was held with each decision maker. In order for the evaluations of the decision makers not to affect each other, the meetings were held one after the other on the same day. The current linguistic decision matrices created according to the evaluations of the decision makers are given in Tables 5.9 and 5.10,

respectively. The types of the criteria are also presented in Table 5.11. The linguistic evaluations of each DM are converted to their corresponding IFNs by using the scale given in Table 5.3. For instance, the current decision matrix based on the main criteria for each DM is presented in Table 5.12.

Table 5.9 : Current linguistic decision matrix based on main criteria for each DM.

	DM1			DM2			DM3		
	C1	C2	C3	C1	C2	C3	C1	C2	C3
A1	VHV	VHV	VHV	AHV	AHV	VHV	AHV	VHV	VHV
A2	HV	MHV	HV	HV	HV	HV	HV	HV	HV
A3	HV	MHV	HV	MHV	MHV	MHV	HV	MHV	HV
A4	MLV	AV	AV	AV	MHV	AV	AV	AV	MHV
A5	VHV	VHV	VHV	VHV	AHV	HV	AHV	VHV	VHV

Table 5.10 : Current linguistic decision matrix based on sub-criteria for each DM.

	DM1												
	C11	C12	C13	C14	C15	C16	C21	C22	C23	C24	C25	C31	C32
A1	VLV	MLV	AV	VHV	HV	VHV	MLV	VLV	VHV	AV	ALV	VLV	MLV
A2	MLV	LV	MHV	HV	HV	HV	MLV	LV	HV	AV	VLV	LV	LV
A3	AV	AV	MHV	HV	VHV	VHV	HV	VHV	LV	VHV	AHV	VLV	MLV
A4	LV	VLV	VLV	VLV	VLV	MLV	MHV	VLV	LV	VHV	AV	VHV	HV
A5	MHV	VHV	AV	AV	VHV	AV	LV	VLV	VHV	HV	MHV	VLV	MHV
	DM2												
	C11	C12	C13	C14	C15	C16	C21	C22	C23	C24	C25	C31	C32
A1	LV	HV	MHV	AHV	VHV	MHV	AV	VLV	VHV	HV	ALV	ALV	LV
A2	VLV	MHV	HV	HV	MHV	HV	AV	LV	HV	MLV	VLV	LV	LV
A3	HV	AV	AV	AHV	HV	AHV	HV	AHV	VLV	AHV	VHV	LV	HV
A4	MLV	VLV	LV	VLV	VLV	MHV	HV	ALV	MLV	HV	MLV	AHV	VHV
A5	AV	HV	MLV	MHV	HV	LV	VLV	ALV	VHV	MHV	AV	ALV	AV
	DM3												
	C11	C12	C13	C14	C15	C16	C21	C22	C23	C24	C25	C31	C32
A1	LV	AV	VHV	AHV	AHV	MHV	LV	ALV	AHV	AV	ALV	VLV	AV
A2	LV	AV	AHV	VHV	VHV	HV	MLV	VLV	HV	MHV	VLV	LV	AV
A3	AV	MHV	VHV	AHV	HV	VHV	HV	VHV	VLV	VHV	VHV	MLV	MHV
A4	AV	LV	VLV	LV	VLV	HV	MHV	LV	AV	HV	AV	VHV	VHV
A5	HV	AV	MHV	LV	AV	MLV	ALV	ALV	AHV	MLV	MLV	ALV	MHV

Table 5.11 : Types of criteria.

Criteria	Type	
	Cost	Benefit
C1. Environmental Factors		
C11- Surface Water		✓
C12- Ground Water		✓
C13- Land Slope	✓	
C14- Land Use		✓
C15- Geology	✓	
C16-Distance to protected area		✓
C2. Social Factors		
C21-Public Acceptance		✓
C22- Distance to Residential Areas		✓
C23- Distance to Roads		✓
C24- Distance to Historical and Cultural Areas		✓
C25- Distance to Airports		✓
C3. Economic Factors		
C31-Land Cost	✓	
C32-Operation Cost	✓	

Table 5.12 : Current decision matrix based on main criteria for each DM.

	DM1			DM2			DM3		
	C1	C2	C3	C1	C2	C3	C1	C2	C3
A1	(0.8,0.15)	(0.8,0.15)	(0.8,0.15)	(0.9,0.05)	(0.9,0.05)	(0.8,0.15)	(0.9,0.05)	(0.8,0.15)	(0.8,0.15)
A2	(0.7,0.25)	(0.6,0.35)	(0.7,0.25)	(0.7,0.25)	(0.7,0.25)	(0.7,0.25)	(0.7,0.25)	(0.7,0.25)	(0.7,0.25)
A3	(0.7,0.25)	(0.6,0.35)	(0.7,0.25)	(0.6,0.35)	(0.6,0.35)	(0.6,0.35)	(0.7,0.25)	(0.6,0.35)	(0.7,0.25)
A4	(0.4,0.55)	(0.5,0.45)	(0.5,0.45)	(0.5,0.45)	(0.6,0.35)	(0.5,0.45)	(0.5,0.45)	(0.5,0.45)	(0.6,0.35)
A5	(0.8,0.15)	(0.8,0.15)	(0.8,0.15)	(0.8,0.15)	(0.9,0.05)	(0.7,0.25)	(0.9,0.05)	(0.8,0.15)	(0.8,0.15)

Step 3. The individual decision matrix of each DM is aggregated in a single decision matrix by using *IFWG* given in Eq. (5.13). The aggregated IF current decision matrices based on the main criteria and the sub-criteria are constructed as given in Tables 5.13 and 5.14.

Table 5.13 : Aggregated current decision matrix based on the main criteria.

	C1	C2	C3
A1	(0.859, 0.091)	(0.829, 0.121)	(0.8, 0.15)
A2	(0.7, 0.25)	(0.658, 0.292)	(0.7, 0.25)
A3	(0.668, 0.282)	(0.6, 0.35)	(0.668, 0.282)
A4	(0.457, 0.492)	(0.528, 0.422)	(0.528, 0.422)
A5	(0.829, 0.121)	(0.829, 0.121)	(0.769, 0.181)

Table 5.14 : Aggregated current decision matrix based on the sub-criteria.

	C11	C12	C13	C14	C15	C16	
A1	(0.255,0.694)	(0.506,0.443)	(0.608,0.341)	(0.859,0.091)	(0.786,0.164)	(0.673,0.276)	
A2	(0.298,0.65)	(0.431,0.517)	(0.71,0.24)	(0.729,0.221)	(0.696,0.254)	(0.7,0.25)	
A3	(0.553,0.396)	(0.528,0.422)	(0.619,0.33)	(0.814,0.136)	(0.738,0.211)	(0.829,0.121)	
A4	(0.381,0.568)	(0.226,0.723)	(0.226,0.723)	(0.226,0.723)	(0.2,0.75)	(0.534,0.414)	
A5	(0.595,0.355)	(0.668,0.282)	(0.494,0.456)	(0.453,0.495)	(0.668,0.282)	(0.401,0.548)	
	C21	C22	C23	C24	C25	C31	C32
A1	(0.392,0.557)	(0.162,0.786)	(0.829,0.121)	(0.553,0.396)	(0.1,0.85)	(0.162,0.786)	(0.392,0.557)
A2	(0.428,0.522)	(0.266,0.684)	(0.7,0.25)	(0.494,0.456)	(0.2,0.75)	(0.3,0.65)	(0.35,0.599)
A3	(0.7,0.25)	(0.829,0.121)	(0.235,0.714)	(0.829,0.121)	(0.839,0.111)	(0.278,0.67)	(0.534,0.414)
A4	(0.628,0.321)	(0.183,0.763)	(0.381,0.568)	(0.738,0.211)	(0.468,0.482)	(0.829,0.121)	(0.758,0.192)
A5	(0.191,0.755)	(0.132,0.816)	(0.829,0.121)	(0.565,0.384)	(0.503,0.446)	(0.132,0.816)	(0.568,0.382)

Step 4. After each decision matrix is constructed based on the main and sub-criteria, decision matrices are normalized with respect to the criterion type, which can be benefit type or cost type by using Eqs. (5.14) and (5.15). Then, the standard deviation values of each criterion are obtained based on the normalized decision matrices as given in Tables 5.15 and 5.16, respectively.

Step 5. To determine the degree of relationship between each criterion, the obtained correlation values based on the main and sub-criteria are presented as seen in Tables 5.17 and 5.18, respectively.

Table 5.15 : Normalized decision matrix based on the main criteria.

	C1	C2	C3
A1	0.964	0.936	0
A2	0.687	0.494	0.340
A3	0.618	0.293	0.451
A4	0	0	0.925
A5	0.920	0.936	0.104
Standard Deviation	0.386	0.409	0.362

Table 5.16 : Normalized decision matrix based on the sub-criteria.

	C1					
	C11	C12	C13	C14	C15	C16
A1	0	0.623	0.233	0.976	0	0.677
A2	0.118	0.471	0	0.851	0.171	0.724
A3	0.734	0.663	0.207	0.937	0.089	0.916
A4	0.336	0	0.914	0	0.951	0.378
A5	0.815	0.894	0.478	0.452	0.225	0
Standard Deviation	0.3630	0.3327	0.3499	0.4155	0.3805	0.3576

	C2					C3	
	C21	C22	C23	C24	C25	C31	C32
A1	0.445	0.051	0.988	0.229	0	A1	0.967
A2	0.519	0.229	0.849	0	0.157	A2	0.827
A3	0.982	0.994	0	0.948	0.995	A3	0.852
A4	0.880	0.089	0.315	0.764	0.585	A4	0
A5	0	0	0.988	0.273	0.636	A5	0.994
Standard Deviation	0.390	0.412	0.447	0.397	0.3987	Standard Deviation	0.728

Table 5.17 : Correlation coefficients of the relationship between the main criteria.

	C1	C2	C3
C1	1	-0.850	0.717
C2		1	0.188
C3			1

Table 5.18 : Correlation coefficients of the relationship between the sub-criteria based on each main criterion.

	C1					
	C11	C12	C13	C14	C15	C16
C11	1	0.4317	0.2270	-0.2190	0.0245	-0.3753
C12		1	-0.5617	0.5952	-0.8299	-0.1229
C13			1	-0.9301	0.8803	-0.5883
C14				1	-0.9374	0.6756
C15					1	-0.3841
C16						1

	C2					C3	
	C21	C22	C23	C24	C25	C31	C32
C21	1	0.661	-0.884	0.734	0.374	C31	1
C22		1	-0.791	0.640	0.655	C32	1
C23			1	-0.921	-0.763		
C24				1	0.812		
C25					1		

In Tables 5.17 and 5.18, only one value is written because the reciprocal values in the pairwise comparison between the two criteria are the same.

Step 6. The final weights of the main criteria and the local weights of the sub-criteria are calculated by using Eq. (5.18) according to the standard deviation values and correlation coefficients as given in Table 5.19. The global weights of the sub-criteria are obtained by multiplying the weight of each sub-criterion by the weight of the main criterion to which it belongs as presented in Table 5.19. The final weights of main criteria and the global weights of the sub-criteria are shown in Figs. 5.4 and 5.5, respectively.

Table 5.19 : Weights of criteria.

Main Criteria	Weights	Sub-Criteria	Local Weights	Global Weights
C1	0.357	C11	0.142	0.051
		C12	0.145	0.052
		C13	0.166	0.059
		C14	0.192	0.069
		C15	0.189	0.068
		C16	0.165	0.059
C2	0.472	C21	0.153	0.072
		C22	0.147	0.070
		C23	0.415	0.196
		C24	0.137	0.065
		C25	0.147	0.069
C3	0.172	C31	0.519	0.089
		C32	0.481	0.083

According to the results obtained, the social factors are the most important main criterion with a weight of 0.472, followed by the environmental factors with a weight of 0.357, and the economic factors with a weight of 0.172 based on Table 5.18. As for the sub-criteria, the most important sub-criterion is the *C23- Distance to roads*, which belongs to the social factors main criterion with a weight of 0.196, followed by the second important sub-criterion is the *C31-Land cost*, which belongs to the economic factors main criterion with a weight of 0.089, followed by the third important sub-criterion is the *C32- Distance from residential areas*, which belongs to the economic factors main criterion with a weight of 0.083 and the sub-criterion having the least importance is *C11- Surface water*, which belongs to the environmental factors main criterion with a weight of 0.051.

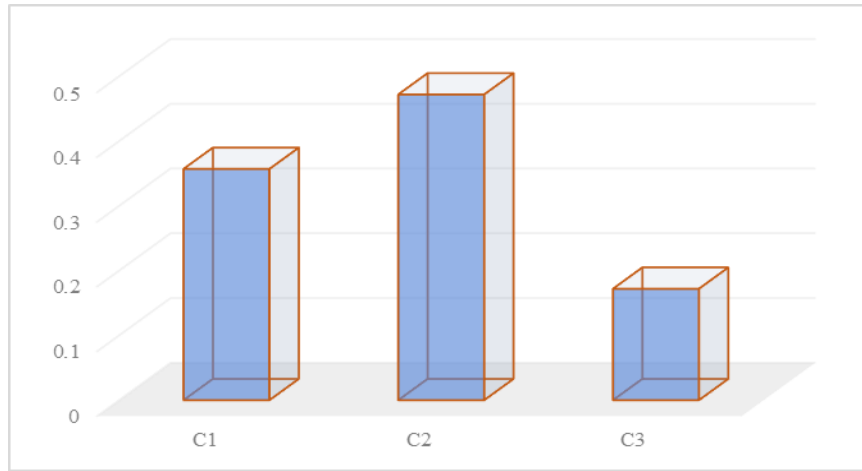


Figure 5.4 : Final weights of the main criteria.

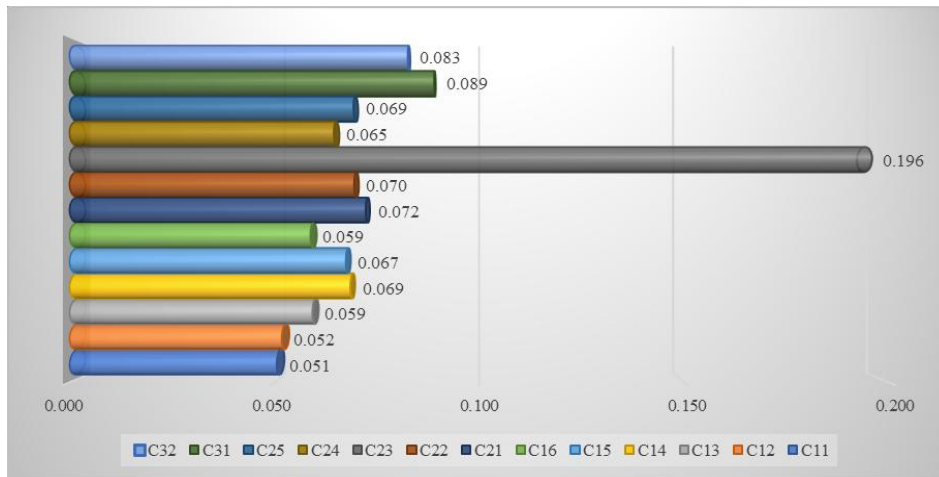


Figure 5.5 : Global weights of the sub-criteria.

After using the proposed IF-CRITIC method to obtain the criterion weights, we apply the developed IF-DEVADA method for the selection and ranking of waste disposal locations. Application solutions of IF- DEVADA method on the addressed problem are presented as follows.

Step 7. As a result of various research and decision makers' opinions, it is determined how the evaluations of waste disposal locations may change for a period of 3 years. The possibility and direction of changes for each DM are presented in Tables 5.20, 5.21, and 5.22, respectively. Here, the possibilities of change are determined using the IF linguistic scale given in Table 5.6. We used the same scale for change in positive and negative direction. Then, the future decision matrix based on IFNs for each DM is constructed by considering the possibility and direction of change. For instance, the future decision matrix for DM1 is obtained by using Eq. (5.20) as in Table 5.23

Table 5.20 : Future change possibility and direction of change for DM1.

	C11	Change	C12	Change	C13	Change	C14	Change	C15	Change	C16	Change		
A1	(0.3,0.65)	↑	(0.2,0.75)	↑	(0.4,0.55)	↑	(0.3,0.65)	↑	(0.4,0.55)	↑	(0.6,0.35)	↑		
A2	(0.3,0.65)	↓	(0.4,0.55)	↑	(0.3,0.65)	↓	(0.5,0.45)	↓	(0.3,0.65)	↓	(0.3,0.65)	↓		
A3	(0.2,0.75)	↑	(0.5,0.45)	↓	(0.7,0.25)	↓	(0.5,0.45)	↑	(0.3,0.65)	↓	(0.6,0.35)	↑		
A4	(0.5,0.45)	↑	(0.3,0.65)	↑	(0.3,0.65)	↓	(0.6,0.35)	↑	(0.3,0.65)	↑	(0.4,0.55)	↓		
A5	(0.5,0.45)	↑	(0.7,0.25)	↓	(0.5,0.45)	↑	(0.3,0.65)	↓	(0.2,0.75)	↓	(0.2,0.75)	↓		
	C21	Change	C22	Change	C23	Change	C24	Change	C25	Change	C31	Change	C32	Change
A1	(0.4,0.55)	↑	(0.4,0.55)	↓	(0.4,0.55)	↓	(0.3,0.65)	↑	(0.7,0.25)	↑	(0.3,0.65)	↑	(0.4,0.55)	↑
A2	(0.3,0.65)	↓	(0.3,0.65)	↑	(0.5,0.45)	↑	(0.2,0.75)	↑	(0.5,0.45)	↑	(0.2,0.75)	↑	(0.4,0.55)	↑
A3	(0.2,0.75)	↓	(0.3,0.65)	↓	(0.3,0.65)	↑	(0.3,0.65)	↓	(0.7,0.25)	↓	(0.2,0.75)	↑	(0.5,0.45)	↑
A4	(0.2,0.75)	↑	(0.6,0.35)	↑	(0.2,0.75)	↑	(0.6,0.35)	↑	(0.4,0.55)	↓	(0.3,0.65)	↓	(0.3,0.65)	↓
A5	(0.5,0.45)	↑	(0.3,0.65)	↑	(0.3,0.65)	↓	(0.6,0.35)	↑	(0.3,0.65)	↑	(0.3,0.65)	↓	(0.2,0.75)	↓

Table 5.21 : Future change possibility and direction of change for DM2.

	C11	Change	C12	Change	C13	Change	C14	Change	C15	Change	C16	Change		
A1	(0.3,0.65)	↓	(0.5,0.45)	↑	(0.4,0.55)	↑	(0.3,0.65)	↑	(0.3,0.65)	↓	(0.3,0.65)	↑		
A2	(0.3,0.65)	↓	(0.4,0.55)	↑	(0.5,0.45)	↓	(0.4,0.55)	↓	(0.3,0.65)	↑	(0.4,0.55)	↑		
A3	(0.4,0.55)	↑	(0.2,0.75)	↓	(0.5,0.45)	↑	(0.6,0.35)	↑	(0.5,0.45)	↑	(0.3,0.65)	↓		
A4	(0.4,0.55)	↓	(0.5,0.45)	↑	(0.2,0.75)	↑	(0.6,0.35)	↑	(0.6,0.35)	↑	(0.3,0.65)	↓		
A5	(0.4,0.55)	↑	(0.5,0.45)	↑	(0.4,0.55)	↑	(0.3,0.65)	↓	(0.2,0.75)	↑	(0.3,0.65)	↑		
	C21	Change	C22	Change	C23	Change	C24	Change	C25	Change	C31	Change	C32	Change
A1	(0.6,0.35)	↑	(0.4,0.55)	↓	(0.6,0.35)	↑	(0.5,0.45)	↑	(0.6,0.35)	↑	(0.4,0.55)	↑	(0.6,0.35)	↑
A2	(0.2,0.75)	↑	(0.4,0.55)	↓	(0.4,0.55)	↑	(0.3,0.65)	↑	(0.3,0.65)	↑	(0.3,0.65)	↓	(0.3,0.65)	↑
A3	(0.5,0.45)	↓	(0.3,0.65)	↓	(0.3,0.65)	↑	(0.6,0.35)	↓	(0.5,0.45)	↓	(0.3,0.65)	↑	(0.5,0.45)	↑
A4	(0.5,0.45)	↓	(0.4,0.55)	↑	(0.3,0.65)	↓	(0.5,0.45)	↓	(0.5,0.45)	↓	(0.5,0.45)	↓	(0.2,0.75)	↑
A5	(0.5,0.45)	↑	(0.7,0.25)	↑	(0.4,0.55)	↓	(0.2,0.75)	↓	(0.3,0.65)	↓	(0.6,0.35)	↑	(0.5,0.45)	↑

Table 5.22 : Future change possibility and direction of change for DM3.

	C11	Change	C12	Change	C13	Change	C14	Change	C15	Change	C16	Change		
A1	(0.2,0.75)	↓	(0.4,0.55)	↑	(0.2,0.75)	↓	(0.2,0.75)	↓	(0.2,0.75)	↓	(0.4,0.55)	↑		
A2	(0.3,0.65)	↓	(0.3,0.65)	↑	(0.6,0.35)	↓	(0.6,0.35)	↑	(0.2,0.75)	↓	(0.3,0.65)	↑		
A3	(0.3,0.65)	↑	(0.2,0.75)	↑	(0.2,0.75)	↓	(0.3,0.65)	↓	(0.3,0.65)	↑	(0.3,0.65)	↓		
A4	(0.3,0.65)	↓	(0.4,0.55)	↑	(0.3,0.65)	↑	(0.2,0.75)	↑	(0.3,0.65)	↑	(0.5,0.45)	↓		
A5	(0.2,0.75)	↑	(0.6,0.35)	↑	(0.1,0.85)	↑	(0.3,0.65)	↑	(0.4,0.55)	↑	(0.2,0.75)	↑		
	C21	Change	C22	Change	C23	Change	C24	Change	C25	Change	C31	Change	C32	Change
A1	(0.3,0.65)	↑	(0.6,0.35)	↑	(0.2,0.75)	↓	(0.7,0.25)	↑	(0.3,0.65)	↑	(0.3,0.65)	↑	(0.5,0.45)	↑
A2	(0.2,0.75)	↑	(0.3,0.65)	↑	(0.3,0.65)	↑	(0.5,0.45)	↑	(0.4,0.55)	↑	(0.2,0.75)	↓	(0.1,0.85)	↑
A3	(0.2,0.75)	↑	(0.2,0.75)	↓	(0.3,0.65)	↑	(0.2,0.75)	↓	(0.2,0.75)	↓	(0.4,0.55)	↑	(0.2,0.75)	↓
A4	(0.2,0.75)	↓	(0.8,0.15)	↑	(0.4,0.55)	↓	(0.5,0.45)	↓	(0.5,0.45)	↓	(0.5,0.45)	↑	(0.1,0.85)	↑
A5	(0.6,0.35)	↓	(0.5,0.45)	↑	(0.3,0.65)	↓	(0.1,0.85)	↓	(0.2,0.75)	↓	(0.3,0.65)	↑	(0.3,0.65)	↑

Table 5.23 : Future change decision matrix based on the sub-criteria for DM1.

	C11	C12	C13	C14	C15	C16	
A1	(0.44,0.488)	(0.52,0.413)	(0.7,0.248)	(0.714,0.231)	(0.82,0.138)	(0.92,0.053)	
A2	(0.143,0.846)	(0.58,0.358)	(0.429,0.538)	(0.85,0.113)	(0.571,0.385)	(0.571,0.385)	
A3	(0.6,0.338)	(0,1)	(0,1)	(0.85,0.113)	(0.714,0.231)	(0.5,0.429)	
A4	(0.65,0.293)	(0.44,0.488)	(0,1)	(0.68,0.263)	(0.44,0.488)	(0.64,0.303)	
A5	(0.8,0.158)	(0.333,0.6)	(0.75,0.203)	(0.65,0.293)	(0.75,0.2)	(0.375,0.6)	
	C21	C22	C23	C24	C25	C31	C32
A1	(0.64,0.303)	(0,1)	(0.667,0.273)	(0.286,0.692)	(0.73,0.213)	(0.44,0.488)	(0.64,0.303)
A2	(0.143,0.846)	(0.51,0.423)	(0.85,0.113)	(0.6,0.338)	(0.6,0.338)	(0.44,0.488)	(0.58,0.358)
A3	(0.625,0.333)	(0.714,0.231)	(0.51,0.423)	(0.714,0.231)	(0.667,0.2)	(0.36,0.563)	(0.7,0.248)
A4	(0.68,0.263)	(0.68,0.263)	(0.44,0.488)	(0.92,0.053)	(0.167,0.818)	(0.714,0.231)	(0.571,0.385)
A5	(0.65,0.293)	(0.44,0.488)	(0.714,0.231)	(0.88,0.088)	(0.72,0.228)	(0,1)	(0.5,0.467)

Step 8. The individual decision matrices of each DM are aggregated in a single decision matrix by using *IFWG* given in Eq. (5.13). Aggregated IF future decision matrix is as given in Table 5.24.

Table 5.24 : Aggregated IF future decision matrix.

	C11	C12	C13	C14	C15	C16	
A1	(0,1)	(0.659,0.284)	(0.732,0.217)	(0.802,0.139)	(0.802,0.147)	(0.807,0.151)	
A2	(0,1)	(0.651,0.292)	(0.497,0.451)	(0.861,0.103)	(0.665,0.287)	(0.702,0.253)	
A3	(0.675,0.269)	(0,1)	(0,1)	(0.821,0.111)	(0.776,0.176)	(0.654,0.279)	
A4	(0,1)	(0.525,0.408)	(0,1)	(0.597,0.339)	(0.501,0.428)	(0.493,0.462)	
A5	(0.757,0.194)	(0.574,0.365)	(0.682,0.263)	(0.623,0.317)	(0.696,0.256)	(0.454,0.499)	
	C21	C22	C23	C24	C25	C31	C32
A1	(0.639,0.302)	(0,1)	(0.797,0.151)	(0.438,0.528)	(0.572,0.358)	(0.446,0.482)	(0.695,0.251)
A2	(0.324,0.644)	(0,1)	(0.823,0.135)	(0.647,0.294)	(0.524,0.408)	(0,1)	(0.549,0.385)
A3	(0.58,0.374)	(0.766,0.178)	(0.467,0.462)	(0.736,0.196)	(0.756,0.162)	(0.475,0.453)	(0.671,0.287)
A4	(0.529,0.425)	(0.649,0.289)	(0.235,0.738)	(0.558,0.398)	(0,1)	(0.792,0.149)	(0.715,0.237)
A5	(0,1)	(0.548,0.383)	(0.739,0.201)	(0.555,0.416)	(0.397,0.574)	(0,1)	(0.63,0.328)

Step 9. The amount of change that will occur for the 3-year period determined by considering the aggregated decision matrix expected to occur in the future and the aggregated current decision matrix together is determined using Eq. (5.7). The future changes decision matrix is created as given in Table 5.25.

Table 5.25 : Future changes decision matrix.

	C11	C12	C13	C14	C15	C16	
A1	(0,1)	(0.309,0.641)	(0.317,0.637)	(0,1)	(0.077,0.894)	(0.41,0.545)	
A2	(0,1)	(0.387,0.564)	(0,1)	(0.488,0.464)	(0,1)	(0.006,0.994)	
A3	(0.273,0.678)	(0,1)	(0,1)	(0.037,0.82)	(0.142,0.834)	(0,1)	
A4	(0,1)	(0.386,0.563)	(0,1)	(0.479,0.468)	(0.377,0.571)	(0,1)	
A5	(0.4,0.548)	(0,1)	(0.371,0.576)	(0.311,0.639)	(0.084,0.911)	(0.088,0.911)	
	C21	C22	C23	C24	C25	C31	C32
A1	(0.406,0.543)	(0,1)	(0,1)	(0,1)	(0.525,0.421)	(0,0.613)	(0.499,0.451)
A2	(0,1)	(0,1)	(0.409,0.541)	(0.303,0.646)	(0.405,0.544)	(0,1)	(0.307,0.643)
A3	(0,1)	(0,1)	(0.303,0.648)	(0,1)	(0,1)	(0.273,0.676)	(0.293,0.692)
A4	(0,1)	(0.57,0.379)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)
A5	(0,1)	(0.479,0.47)	(0,1)	(0,1)	(0,1)	(0,1)	(0.143,0.857)

Step 10. Total impact values are determined, which consider the reflection according to the highest IF value among the alternatives of each criterion in the current decision matrix of each change value and the position according to the IF maximum value

among the alternatives of each criterion in the future decision matrix. The IF total impact decision matrix is obtained as in Table 5.26 by using Eq. (5.21).

Step 11. The IF current and impact decision matrices are defuzzified by using the developed novel defuzzification operator to obtain the crisp values of IFNs. The defuzzified current and impact decision matrices are presented in Tables 5.27 and 5.28, respectively.

Table 5.26 : IF total impact decision matrix.

	C11	C12	C13	C14	C15	C16	
A1	(0,1)	(0.464,0.5)	(0.447,0.522)	(0,1)	(0.098,0.873)	(0.495,0.482)	
A2	(0,1)	(0.573,0.4)	(0,1)	(0.569,0.41)	(0,1)	(0.006,0.994)	
A3	(0.409,0.547)	(0,1)	(0,1)	(0.041,0.804)	(0.175,0.808)	(0,1)	
A4	(0,1)	(0.46,0.497)	(0,1)	(0.387,0.569)	(0.3,0.656)	(0,1)	
A5	(0.672,0.3)	(0,1)	(0.487,0.475)	(0.262,0.698)	(0.093,0.907)	(0.057,0.94)	
	C21	C22	C23	C24	C25	C31	C32
A1	(0.58,0.391)	(0,1)	(0,1)	(0,1)	(0.474,0.5)	(0,0.732)	(0.64,0.334)
A2	(0,1)	(0,1)	(0.493,0.478)	(0.321,0.647)	(0.334,0.638)	(0,1)	(0.311,0.644)
A3	(0,1)	(0,1)	(0.207,0.751)	(0,1)	(0,1)	(0.197,0.763)	(0.356,0.644)
A4	(0,1)	(0.583,0.39)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)
A5	(0,1)	(0.413,0.548)	(0,1)	(0,1)	(0,1)	(0,1)	(0.156,0.844)

Table 5.27 : Defuzzied current decision matrix.

	C11	C12	C13	C14	C15	C16	C21	C22	C23	C24	C25	C31	C32
A1	0.121	0.389	0.514	0.81	0.728	0.594	0.257	0.052	0.777	0.447	0.021	0.053	0.257
A2	0.160	0.300	0.639	0.662	0.622	0.627	0.297	0.130	0.627	0.375	0.078	0.162	0.212
A3	0.447	0.416	0.528	0.760	0.673	0.777	0.627	0.777	0.105	0.777	0.788	0.142	0.424
A4	0.245	0.097	0.097	0.097	0.078	0.424	0.54	0.066	0.245	0.673	0.344	0.777	0.697
A5	0.498	0.587	0.375	0.326	0.587	0.267	0.071	0.036	0.777	0.461	0.386	0.035	0.465

Table 5.28 : Defuzzied impact decision matrix.

	C11	C12	C13	C14	C15	C16	C21	C22	C23	C24	C25	C31	C32
A1	0	0.341	0.322	0	0.02	0.38	0.486	0	0	0	0.355	0	0.561
A2	0	0.477	0	0.473	0	0.0001	0	0	0.378	0.184	0.197	0	0.173
A3	0.276	0	0	0.004	0.061	0	0	0	0.083	0	0	0.076	0.221
A4	0	0.336	0	0.252	0.162	0	0	0.49	0	0	0	0	0
A5	0.601	0	0.369	0.128	0.018	0.007	0	0.282	0	0	0	0	0.049

Step 12. The normalized decision matrices are calculated with respect to the criterion type, which can be benefit type and cost type based on the defuzzified current and impact decision matrices, respectively. The normalized values based on the current and impact decision matrices are obtained as in Tables 5.29 and 5.30, respectively.

Table 5.29 : Normalized current decision matrix.

	C11	C12	C13	C14	C15	C16	C21	C22	C23	C24	C25	C31	C32
A1	0.243	0.663	0.195	1	0	0.765	0.410	0.068	1	0.575	0.027	0.932	0.631
A2	0.321	0.511	0	0.817	0.146	0.807	0.474	0.168	0.807	0.482	0.099	0.792	0.696
A3	0.897	0.709	0.173	0.939	0.076	1	1	1	0.135	1	1	0.818	0.392
A4	0.492	0.166	0.848	0.120	0.893	0.545	0.860	0.085	0.315	0.86	0.436	0	0
A5	1	1	0.413	0.403	0.194	0.344	0.114	0.046	1	0.594	0.49	0.954	0.332

Table 5.30 : Normalized impact decision matrix.

	C11	C12	C13	C14	C15	C16	C21	C22	C23	C24	C25	C31	C32
A1	0	0.715	0.127	0	0.875	1	1	0	0	0	1	1	0
A2	0	1	1	1	1	0.0002	0	0	1	1	0.556	1	0.692
A3	0.459	0	1	0.008	0.624	0	0	0	0.220	0	0	0	0.606
A4	0	0.706	1	0.532	0	0	0	1	0	0	0	1	1
A5	1	0	0	0.270	0.888	0.019	0	0.576	0	0	0	1	0.912

Step 13. To consider the current and impact decision matrices together, the co-normalized decision matrix is computed by using Eq. (5.28). In this way, the impact of the change that will occur over time and the performance values in the current situation are considered together. The co-normalized decision matrix is presented in Table 5.31.

Table 5.31 : Co-normalized decision matrix.

	C11	C12	C13	C14	C15	C16	C21	C22	C23	C24	C25	C31	C32
A1	0.122	0.912	0.174	0.550	0.763	1	1	0.063	0.553	0.388	1	0.989	0.454
A2	0.161	1	0.541	1	1	0.457	0.336	0.155	1	1	0.638	0.917	1
A3	0.678	0.469	0.635	0.521	0.611	0.567	0.709	0.922	0.197	0.675	0.974	0.419	0.719
A4	0.246	0.577	1	0.359	0.779	0.309	0.610	1	0.175	0.584	0.425	0.512	0.72
A5	1	0.662	0.224	0.371	0.943	0.205	0.081	0.573	0.553	0.400	0.477	1	0.896

Step 14. The weighted co-normalized decision matrix is calculated based on the co-normalized decision matrix by considering the criterion weights obtained by the IF-CRITIC method. The weighted co-normalized decision matrix is obtained as given in Table 5.32.

Table 5.32 : Weighted co-normalized decision matrix.

	C11	C12	C13	C14	C15	C16	C21	C22	C23	C24	C25	C31	C32
A1	0.006	0.047	0.01	0.038	0.052	0.059	0.072	0.004	0.108	0.025	0.069	0.088	0.038
A2	0.008	0.052	0.032	0.069	0.068	0.027	0.024	0.011	0.196	0.065	0.044	0.082	0.083
A3	0.034	0.024	0.038	0.036	0.041	0.033	0.051	0.064	0.039	0.044	0.068	0.037	0.059
A4	0.012	0.03	0.059	0.025	0.053	0.018	0.044	0.07	0.034	0.038	0.03	0.046	0.06
A5	0.051	0.034	0.013	0.025	0.064	0.012	0.006	0.04	0.108	0.026	0.033	0.089	0.074

Step 15. Positive and negative ideal solutions based on the weighted co-normalized decision matrix are determined by using Eqs. (5.30) and (5.31). The PIS consists of the maximum values of the weighted decision matrix, while the NIS consists of the minimum values of the weighted decision matrix. Here, we used only one ideal solution for the different distance measures to calculate. Ideal solutions to be used for Euclidean distance and cosine distance measures are presented in Table 5.33.

Table 5.33 : Positive and negative ideal solutions for Euclidean and cosine distances.

	C11	C12	C13	C14	C15	C16	C21	C22	C23	C24	C25	C31	C32
PIS	0.051	0.052	0.059	0.069	0.068	0.059	0.072	0.07	0.196	0.065	0.069	0.089	0.083
NIS	0.006	0.024	0.010	0.025	0.041	0.012	0.006	0.004	0.034	0.025	0.03	0.037	0.038

Step 16. The separation measures of each alternative from PIS and NIS based on the Euclidean distance are identified as given in Table 5.34 by using Eqs. (5.32) and (5.34), respectively, while the separation measures based on the cosine distance are created as in Table 5.35 by using Eqs. (5.33) and (5.35), respectively.

Table 5.34 : Separation measures of the alternatives based on the Euclidean distance.

	A1	A2	A3	A4	A5
S_i^{E*}	0.145	0.100	0.181	0.194	0.150
S_i^{E-}	0.131	0.191	0.100	0.095	0.116

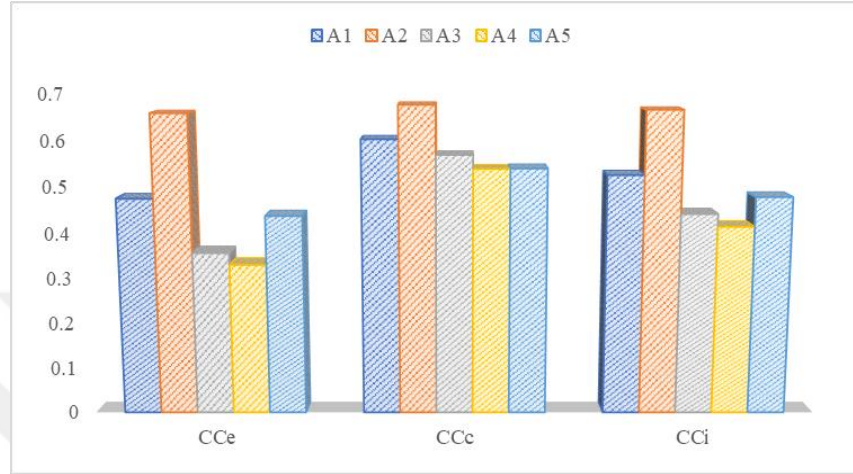
Table 5.35 : Separation measures of the alternatives based on the cosine distance.

	A1	A2	A3	A4	A5
S_i^{C*}	0.085	0.053	0.126	0.155	0.081
S_i^{C-}	0.129	0.108	0.165	0.18	0.095

Step 17. The relative closeness coefficient of each alternative is calculated based on the Euclidean and cosine distance measures by using Eq. (5.36). Here, we considered the weight of the Euclidean distance measure as 0.6 to give more weight to the Euclidean distance as a result of expert preference. The relative closeness coefficient and rank of each alternative are determined as in Table 5.36. Fig. 5.6 shows the relationship between alternatives on the horizontal axis and values of the CC_i^E , CC_i^C and CC_i on the vertical axis.

Table 5.36 : Relative closeness coefficients and ranks of the alternatives.

	A1	A2	A3	A4	A5
CC_i^E	0.474	0.656	0.356	0.330	0.436
CC_i^C	0.601	0.674	0.568	0.538	0.538
CC_i	0.525	0.663	0.441	0.413	0.477
Rank	2	1	4	5	3

**Figure 5.6 :** Final ranking of waste disposal locations.

Step 18. The relative closeness scores show that the ranking order of the alternatives is $A2 > A1 > A5 > A3 > A4$.

5.4.3 Validation of results and sensitivity analysis

When applying MCDM methods to real-world problems, it is necessary to check the robustness of the solutions obtained to verify the results. Since the validation phase of the results depends on the characteristics of the MCDM method used, different approaches have been adopted by various researchers in the literature to validate the results. In some studies, the results are compared with different MCDM methods for the validation phase, while in some studies the effect of changes in input parameters on the results has been analyzed (Büyüközkan & Göçer, 2009; Karagöz et al., 2021; Pamučar & Petrović, 2018; Stewart et al., 2013). Furthermore, some studies have also investigated the effect on the results of the change in criteria or decision-maker weights (Büyüközkan et al., 2019; Büyüközkan & Göçer, 2009; Pamučar & Petrović, 2018). Considering the various studies in the literature, the validation of the results and the robustness of the results in this study are presented in three sections. In the first part, the validation of the results obtained in the developed MCDM method is checked by

comparing it with other MCDM methods. In the second part, the effect of criterion weights on ranking results is analyzed. In the third part, the change of the λ parameter in the ranking results is analyzed to check whether the input parameters change the results of the model.

5.4.3.1 Comparative analysis with other MCDM methods

Various fuzzy MCDM methods and approaches have been considered by many researchers in the literature. In their studies, researchers aim to choose the right MCDM method by considering many factors such as the degree of accuracy, the calculation time, the knowledge and skill of the users on the subject, and the various assumptions that exist in the study. In this section, four different IF-based MCDM methods are applied to compare and validate the results of the developed IF-DEVADA method. The results of IF-DEVADA method are compared with IF-TOPSIS method proposed by Boran et al. (2009), IF-CODAS method proposed by Büyüközkan & Göçer (2020), and IF-EDAS method developed by Liang (2020), and MABAC method developed by Xue et al. (2016) which are distance-based methods. Since the current decision matrix and the future decision matrix are evaluated together in the developed method, it is aimed to observe the consequential differences that may arise by considering the current decision matrix in the comparison analysis carried out. The separation measures and closeness coefficients obtained from the IF-TOPSIS method based on current situation data are shown in Table 5.37. The relative assessment matrix and the evaluation scores of the candidates obtained from the IF-CODAS method are presented in Table 5.38. Weighted total positive and negative values, their normalized values, and evaluation scores obtained from the IF-EDAS method are presented in Table 5.39. The distances and total distances of the alternatives from the border approximate area based on the IF-MABAC method are as presented in Table 5.40. An overview of the comparative analysis conducted with four different IF-based MCDM methods is shown in Figure 5.7. Besides, the ranking results obtained for each of these methods are presented in Table 5.41.

Table 5.37 : Separation measures and closeness coefficients of the alternatives for IF-TOPSIS.

	A1	A2	A3	A4	A5
$d(A_i, A^*)$	0.327	0.225	0.296	0.238	0.327
$d(A_i, A^-)$	0.127	0.399	0.104	0.383	0.121
CC_i	0.279	0.640	0.259	0.616	0.270

Table 5.38 : CODAS relative assessments matrix.

	A1	A2	A3	A4	A5	H_i
A1	0	-0.736	0.0568	-0.709	0.018	-1.370
A2	0.736	0	0.826	0.0305	0.789	2.381
A3	-0.057	-0.826	0	-0.767	-0.039	-1.688
A4	0.709	-0.031	0.767	0	0.755	2.201
A5	-0.018	-0.789	0.039	-0.755	0	-1.523

Table 5.39 : Values of SP , SN , NSP , NSN , AS for EDAS.

	SP_i	SN_i	NSP_i	NSN_i	AS_i
A1	0.080	0.222	0.384	0.262	0.323
A2	0.026	0.174	0.127	0.422	0.275
A3	0.208	0.157	1	0.478	0.739
A4	0.133	0.301	0.638	0	0.319
A5	0.084	0.212	0.404	0.295	0.350

Table 5.40 : Distances and total distances of alternatives from the border approximate area.

	C11	C12	C13	C14	C15	C16	C21	C22	C23	C24	C25	C31	C32	S_i
A1	-0.012	0.005	-0.006	0.072	-0.012	0.008	-0.007	-0.010	0.150	-0.014	0.025	0.061	0.023	0.295
A2	-0.008	-0.003	-0.015	0.025	-0.005	0.013	-0.003	-0.001	0.058	-0.022	0.017	0.018	0.032	0.097
A3	-0.015	0.007	-0.007	0.052	-0.008	0.047	0.044	0.102	-0.108	0.050	0.096	0.023	-0.001	0.298
A4	-0.002	-0.019	0.046	-0.047	0.070	-0.013	0.029	-0.008	-0.069	0.021	0.012	-0.064	-0.026	-0.069
A5	0.020	0.026	0.005	-0.023	-0.002	-0.028	-0.028	-0.012	0.150	-0.012	0.016	0.075	-0.004	0.196

Table 5.41 : Ranking results for distance-based MCDM methods.

Method	Ranking
IF-DEVADA (our method)	A2, A1, A5, A3, A4
IF-TOPSIS (Boran et al., 2009)	A2, A4, A1, A5, A3
IF-CODAS (Büyüközkan & Göçer, 2020)	A2, A4, A1, A5, A3
IF-EDAS (Liang, 2020)	A3, A5, A1, A4, A2
IF-MABAC (Xue et al., 2016)	A3, A1, A5, A2, A4

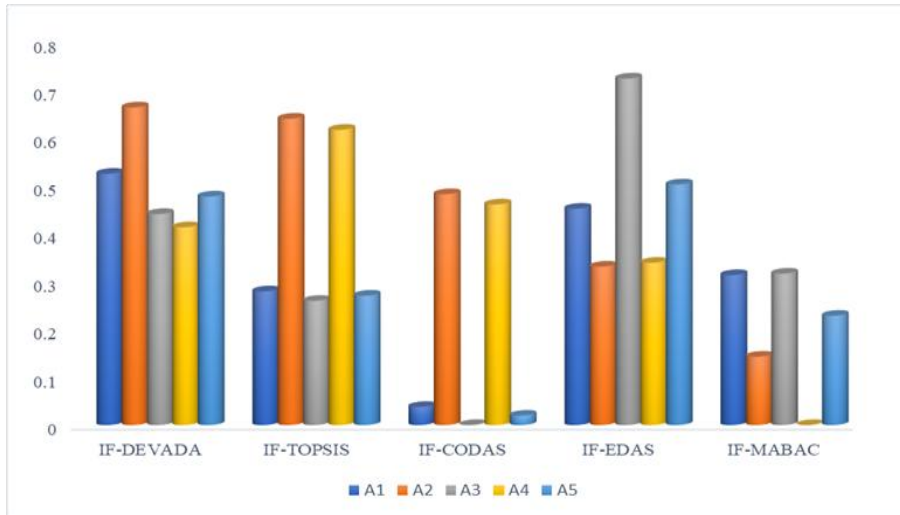


Figure 5.7 : Results on the evaluation scores of the distance-based MCDM methods.

As a result of these comparisons carried out with different MCDM methods, it has been observed that there are distinctive differences between the rankings. While all the compared methods take into account the current situation, our method obtains the ranking results by considering the combination of the current and possible future situations. According to the current situation, the best alternative location in IF-TOPSIS and IF-CODAS methods are determined as A2-Çatalca, giving the same result as our method. However, for the rest of the rankings, different results from our method have been obtained. Considering only the current situation in these methods have caused the ranking order to be different. In addition, since the score values of the first and second alternatives in IF-TOPSIS and IF-CODAS methods are very close to each other, it shows that this order can change if only the possible future situations are taken into account. Besides, in the other two methods (TOPSIS and CODAS methods), different ranking results are obtained. All methods used for determining the best waste disposal location have yielded ranking results based on the current data, ignoring the possible changes that will occur over time. Since IF-DEVADA method can handle possible future changes, it leads to more accurate and realistic results than other MCDM methods. Thus, the developed method can be efficiently used for perceptual and logical decision making by taking into account the current situation and the possible future changes for performance evaluation and selection of alternatives not only in waste disposal locations but also in all areas.

5.4.3.2 Sensitivity analysis on changing the weights of the criteria

In this section, a sensitivity analysis is conducted to check the robustness of the results from the developed model. For this, the effect of changing the weight coefficient of the most important sub-criterion (C23) on the ranking results is analyzed through different scenarios. We defined 20 scenarios based on 20 different weight coefficients by using Eq. (5.37).

$$w_{n\beta} = (1 - w_{n\alpha}) \frac{w_{\beta}}{(1 - w_n)} \quad (5.37)$$

where $w_{n\beta}$ represents the adjusted value of the criteria whose values of the weighted coefficient are lesser than the values of the weight of the most important criterion C23; $w_{n\alpha}$ states the reduced value of the criterion C23; w_{β} indicates the original value of the considered criterion and w_n defines the original value of criterion C23. In each scenario, the weight of the most important criterion C23 is reduced by 2%, while the values of the remaining criterion weights are proportionally regulated to meet the condition $\sum_{j=1}^n w_j = 1$. Changes in weight coefficients of the criteria are presented in Fig. 5.8, and their influence on the new rankings and relative closeness scores of alternatives are analyzed as shown in Figs. 5.9 and 5.10, respectively.

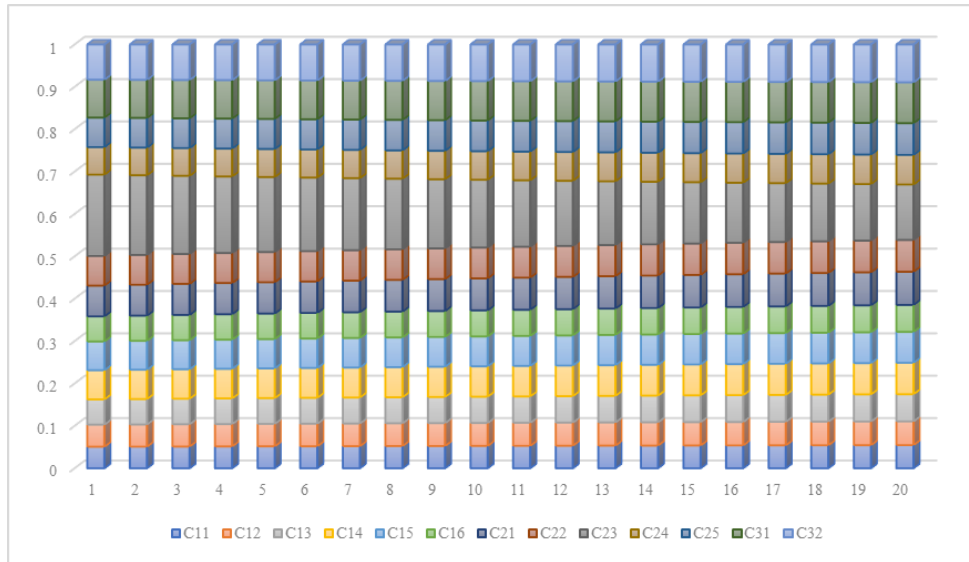


Figure 5.8 : Changes in the criterion weights through scenarios.

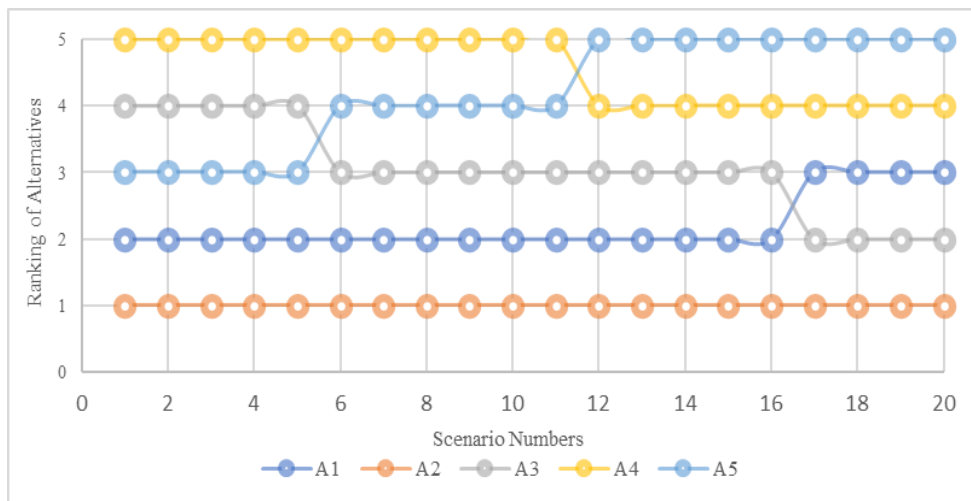


Figure 5.9 : Effects on the alternative ranks of change in the criterion weights.

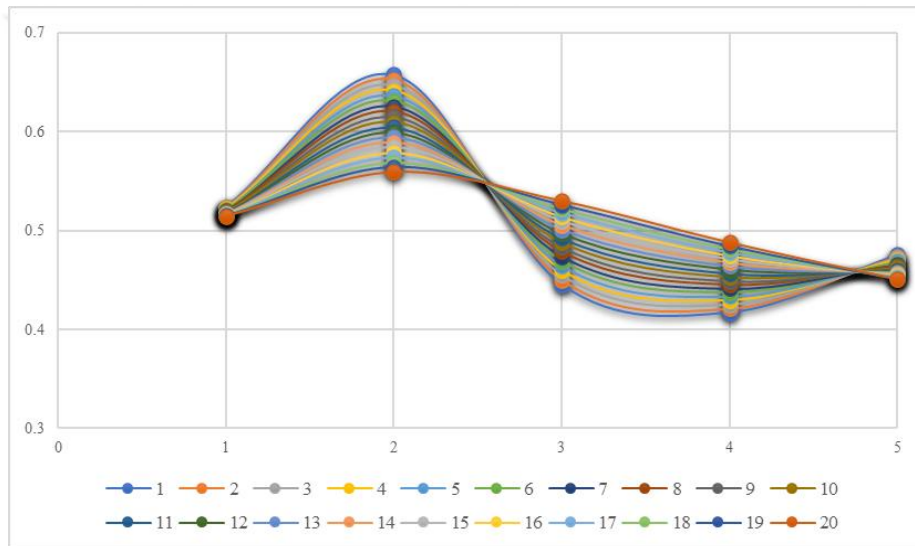


Figure 5.10 : Effects on the relative closeness scores of changes in the criterion weights.

Table 5.42 shows the robustness analysis results for the alternatives. A2 is the best alternative and 100% robust in the ranking of the alternatives. A1 and A3 share the second rank with the robustness percentages 80% and 20%, respectively. A1, A3, and A5 share the third rank with the robustness percentages 20%, 55%, and 25%, respectively. A3, A4, and A5 share the fourth rank with the robustness percentages 25%, 45%, and 30%, respectively. A4 and A5 share the fifth rank with the robustness percentages 55% and 45%, respectively.

Table 5.42 : Ranking results for distance-based MCDM methods.

Rank	Alternatives				
	A1	A2	A3	A4	A5
1	0	1	0	0	0
2	0.8	0	0.2	0	0
3	0.2	0	0.55	0	0.25
4	0	0	0.25	0.45	0.3
5	0	0	0	0.55	0.45

According to Fig. 5.9, when the effects of changes in criterion weights on alternative rankings are examined, it is seen that alternative A2 ranks first without changing its ranking throughout all scenarios. During the first 16 scenarios, alternative A1 is ranked second, keeping its rank, while it is ranked third in the remaining scenarios. Alternative A5, which is ranked third in the first five scenarios, is ranked fourth between the 6th and 11th scenarios and the fifth in the remaining scenarios. While the fourth best alternative, A3 keeps its rank in the first five scenarios, it is ranked third between the 6th and 16th scenarios and the second in the remaining scenarios. While the fifth best alternative, A4, keeps its rank in the first eleven scenarios, it is replaced by the A5 alternative for the remaining scenarios and ranked in fourth place. It has been observed that this type of change has occurred since the value of the most important criterion (C23) has decreased in the scenarios where the change is experienced. According to the results obtained, it has been seen that alternative A2 is the dominant alternative, despite the major changes in the most important criterion (C23). Based on the analysis presented, we can conclude that the change in weight value of the most important criterion, in general, may affect the final ranking results in multi-criteria models. In the analysis, changes in criterion weight led to minor changes in the rankings of alternatives excluding the dominant alternative, indicating that the solution obtained is stable and has a good advantage of the dominant alternative over the other alternatives. Then, in the study, Spearman's rank correlation coefficient has been used to check whether there is a significant statistical correlation between the first ranking and the rankings obtained through the scenarios. When the results obtained from Spearman's rank correlation coefficient are examined, it is seen that there is a very strong relationship between rankings with the value of 1 obtained from the correlation coefficient for the first five scenarios, a very strong correlation between the rankings with the value of 0.9 of the correlation coefficient for the sixth and eleventh scenarios, a strong correlation between the rankings with the value of 0.7 of the correlation

coefficient for the twelfth and sixteenth scenarios, and finally, a "moderate" correlation between the rankings with the 0.5 value of the correlation coefficient obtained only from the remaining four scenarios. Besides, the average value of the Spearman correlation coefficient through the scenarios is 0.795, which indicates that there is a very strong correlation between the rankings in general, and it can be concluded that the first solution is reliable by confirming its stability.

5.4.3.3 The effect of parameter λ on the ranking results

Since the determination of parameters is highly subjective, the ability to better reflect the preference in the presented different options of the decision-maker is not only a big advantage of the preference function but also a major disadvantage as a small change in values can lead to a possible change in the results (Ren et al., 2020). Therefore, in this section of the study, the effect of the change of the preference threshold parameter λ on the ranking results is analyzed. To comprehensively analyze the effect of the λ parameter on the relative closeness coefficient scores and ranking results of the alternatives in the proposed method, λ values have been changed between 0 and 1 with an increment of 0.1. The results of the changes in the preference threshold coefficient regarding the evaluation scores of the alternatives and the ranking results are shown in Figs. 5.11 and 5.12, respectively.

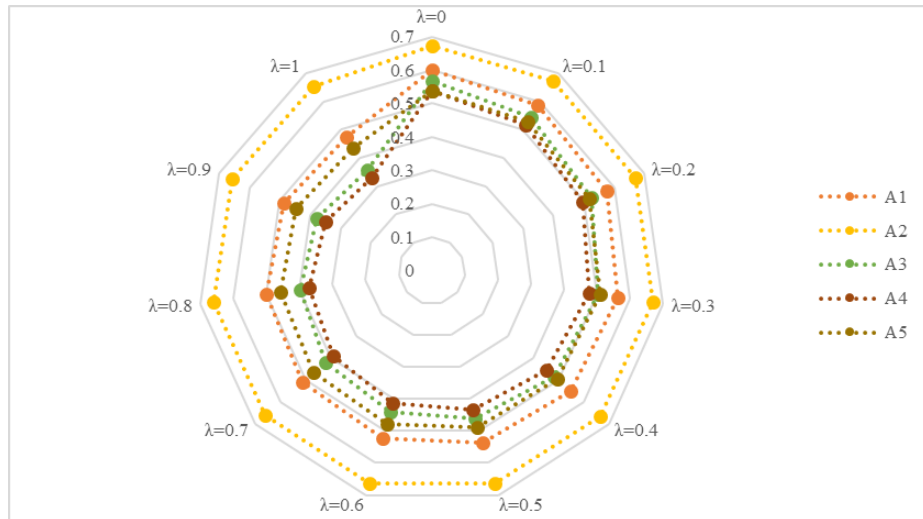


Figure 5.11 : Results on the alternative scores of the different preference thresholds.

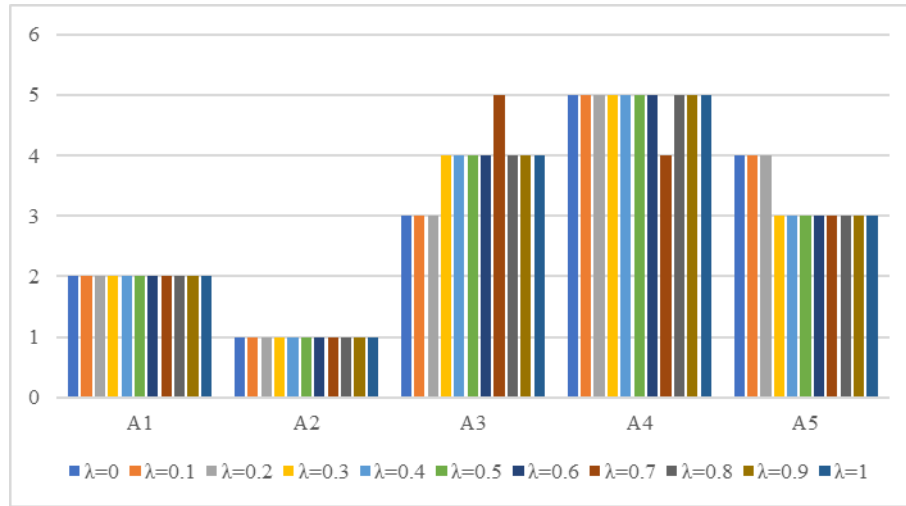


Figure 5.12 : Ranking results on alternatives of the different preference thresholds.

Based on the results obtained, we can conclude that the change of the λ parameter affects the change in the relative closeness coefficient values of the alternatives, but these changes are not sufficient to cause a large change in the ranking of the alternatives. It can be seen that the best alternative, A2, and the second-ranked alternative, A1, retain their rank in all cases. Changes in relative closeness scores are not sufficient to cause a large change in rankings, except for the first and second-ranked alternatives, as seen in Fig. 5.12. It should be emphasized that this situation is valid only for the example considered in this study and the attitude of the optimistic and pessimistic of the decision-maker is an important point. Considering the different values in the decision matrix, changes in the λ parameter can have a significant impact on the final results. Therefore, before making a final decision for such an analysis, the decision maker's attitude should be an indispensable step that must be carefully considered.

5.5 Discussion

- This study aims to show the superiority of the developed IF-DEVADA method. The remarkable advantage of the proposed method is that it provides a joint decision by considering current and future predictions together. Unlike the existing MCDM methods in the literature, it not only considers the current situation but also allows a decision to be made by taking into account the change predictions about the future.

- The proposed method determines the most suitable solution based on multiple distance measures, not depending on just one distance measure, unlike other distance-based methods. In our proposed strategy, we use a combination of Euclidean and cosine distances, which provide accurate results without relying on a single distance measurement unit.
- The proposed approach also utilizes the benefits of the CRITIC method, which provides the elimination of the subjective qualifications by taking into account the objective weights of the criteria, as well as the decision-making superiority of the DEVADA method in the evaluation of alternatives.
- An extension of the proposed approach with IFSs is presented to enable better handling of decision makers' assessments in an uncertain environment. The proposed approach is based on IFSs, enabling more reliable and effective decisions under uncertainty in a two dimensional space.
- The criteria of the evaluation framework in determining the waste disposal location are divided into 3 main criteria groups and a total of 13 sub-criteria are considered depending on these main criteria. While the most important main criterion is determined as environmental factors, the most important sub-criterion has been obtained as distance to roads.
- The evaluation framework offers benefits thanks to the CRITIC and DEVADA methods, which make it work well in an uncertain environment and under the presence of conflicting criteria. It is particularly useful for problems where considerations may vary, such as waste disposal location selection.
- The developed IF-based DEVADA method is compared with the distance-based methods developed in the IF environment in the literature. Since the distance-based methods in the literature only consider the current situation, differences are observed among the methods in terms of ranking results, as presented in Table 5.37. However, since there was no significant change in the future evaluations of alternative A2, which was ranked first in the developed approach, the same results have been obtained with the TOPSIS and CODAS methods.

To check the robustness of the developed IF-DEVADA method and to demonstrate its reliability, a sensitivity analysis is conducted on the change of criterion weights. The results of the sensitivity analysis have demonstrated the robustness and reliability of

the developed method and have also been validated using Spearman's rank correlation coefficient.

5.6 Conclusions and Limitations

The complex and contradictory features encountered in real-life problems have increased the importance of MCDM methods and provided many new methods and approaches to the literature in recent years. However, in many MCDM problems encountered, knowledge and judgment about alternatives vary depending on future trends. Since this situation may result in not taking correct and effective decisions on the problems to be addressed, the dynamic decision-making process should be included in MCDM problems. Furthermore, since real-life problems bring many uncertainties, the exact values assigned by the decision-makers are often insufficient to solve real decision-making problems, causing errors in obtaining correct results. Considering all these requirements in the literature, the primary aim of this study is to develop an IF-DEVADA method that takes into account possible future trends by adapting to rapidly changing situations and conditions and which will include a dynamic decision-making process. As a secondary aim, an IFS-based CRITIC method has been developed, which will ensure the elimination of subjective attributes by taking into account the objective weights of the criteria in an uncertain environment. An integrated IF-MCDM methodology based on CRITIC and DEVADA methods has been applied to waste disposal location selection with ever-changing situations and conditions such as industrialization, urbanization, and population growth. The application of the IF-DEVADA method is of great importance in prioritizing alternatives and making the right decisions, especially since the evaluation process of alternative locations includes many interrelated factors and temporal changes, as well as uncertainties arising from the incomplete information of decision-makers. The proposed study aims to make the right investments in the right locations by making more accurate and effective decisions for decision-makers and policymakers in the prioritization and selection of waste disposal location alternatives, taking into account the importance of a comprehensive list of qualitative and quantitative criteria. Five alternative locations determined within the scope of the study are evaluated according to 3 main criteria and 13 sub-criteria determined through extensive literature review and decision maker opinions. The process of weighting the criteria, which is the first

step in prioritizing and selecting the alternatives, has been determined by using the IF-CRITIC method, which provides the objective weights of the criteria. In the continuation of the study, the ranking and selection process of the alternatives, which constitute a critical and important issue for decision-makers and policymakers, have been determined using the IF-DEVADA method, which is the main purpose of this study. In the process of ranking and prioritizing the alternatives, a stronger multi-measurement unit has been used by considering the Euclidean and cosine distances, another important and powerful feature of the DEVADA method. According to the results obtained from the developed method, A2-Çatalca has been determined as the most important waste disposal location area for the province of Istanbul, followed by A1-Arnavutköy, A5-Ataşehir, A3-Şile, and A4-Sultanbeyli respectively.

In the study, sensitivity analysis has been conducted on different criterion weights and the results of the proposed approach have been seen to be robust and reliable, and its accuracy has been proven by Spearman's correlation coefficient. In addition, a sensitivity analysis has been also conducted on the λ parameter used in the multi-distance measurement unit. According to the results obtained, it has been seen that the change in the λ parameter has not been caused a significant change in the ranking and the most important alternative kept its ranking. In addition, comparative analyzes have been carried out with the distance-based methods IF-TOPSIS, IF-CODAS, IF-EDAS, and IF-MABAC and the results have been compared. In addition to the determination of A2 as the most important alternative in IF-DEVADA, IF-TOPSIS, and IF-CODAS methods, the same ranking results have been obtained in IF-TOPSIS and IF-CODAS methods. This situation has been caused by the fact that the most important alternative did not have enough difference to change its ranking in the prediction evaluations for the future. In other methods IF-EDAS and IF-MABAC, different ranking results have been obtained from the proposed methodology. As a result of all these analyses, it has been proven that the IF-DEVADA method produces consistent and robust results and is a more powerful and reliable method than other popular methods.

The developed methods have been applied to the waste disposal location selection and the advantages such as stability and precision of the developed methods have been utilized. The developed methods, which have a very flexible structure, can also be used for many problems in the field of location selection. Thus, with the importance given to urbanization in recent years, the contribution of municipalities to the city

administration can be increased by ensuring that decision-makers or policymakers make investments in the right location for the selection of waste disposal location, which includes constantly developing and changing conditions. The developed methods can be applied to other real-world problems as well, as it has been proven that the method's benefits on waste disposal location selection are best exploited.

The contributions and advantages of the study can be summarized as follows.

- Since the MCDM methods found in the literature take into account only the current situation and do not consider future trends, there is a need to develop new tools to make more efficient and reliable decisions. This study has developed a new decision methodology integrating IF-CRITIC and IF-DEVADA in order to obtain the best solutions among contradictory and proportional criteria that must be evaluated simultaneously in an uncertain environment.
- The IF-CRITIC method allows eliminating the subjective nature of the decision process by using the objective weights of the criteria.
- Since the developed IF-DEVADA method takes into account future trends as well as current evaluations, it enables the decision model to be handled in a dynamic structure.
- The developed methods have the ability to handle uncertain information more flexibly and better deal with uncertainties than ordinary fuzzy sets. Through IFs, not only are the uncertainties caused by the incomplete knowledge of the decision-makers to be dealt with but also the hesitations of the decision-makers in their choices are reflected.
- The IF-DEVADA method uses a stronger unit of measurement by considering two important units of distance measure together for the ranking and selection of alternatives. In the IF-DEVADA method, the obtained results are more logical because Euclidean and cosine distance measures are used together.
- IF-CRITIC and IF-DEVADA methods, which have a flexible structure, are suitable for solving multi-criteria decision models with more criteria and alternatives.
- The study presents a real case study to prove the effectiveness, robustness, and reliability of the presented method and to determine the most suitable waste disposal location for the province of Istanbul.

- The application of the developed methods to the waste disposal location selection problem has been tested with sensitivity and comparison analyzes and it has been proven that the alternatives are ranked correctly and the best one is selected.
- This study provides an important contribution to the literature for both the waste disposal location selection and the selection process in other real-life problems involving uncertainty, by extending the DEVADA method with IFSs.
- The developed method contributes to meet the needs of both decision-makers and policymakers in the field of waste management.

It has been seen that the proposed dynamic methodology can yield different results from other static MCDM methods. It has been observed that taking the changes that may occur over time as an expectation from the decision makers gives more realistic and more informed results. Incorporating a statistics-based weighting component into MCDM methodology caused more emphasis on data analysis and this reflected in the results.

The methodology we propose has some limitations, as in every proposed study. Interval-valued assessments cannot be used in the proposed model. The necessity of calculating the standard deviation and correlation coefficient in the CRITIC method requires leaving the fuzziness earlier. In addition, the fact that the subtraction and division operations in fuzzy set extensions are not well-defined causes the operations to be performed under several conditions. However, the slight sensitivity to the λ parameter brings a difficulty in adjusting the parameter value.

For further study, the developed method can be applied to various decision-making problems such as energy investment evaluation, supplier selection, project selection, and risk assessment. More specific and detailed criteria can be included in the selection of waste disposal site to improve the model and to make decisions more effective. In this study, only the Istanbul region has been considered. The selection process can be extended by considering a wider region of Turkey such as Marmara region or Aegean region. More decision makers with more experience and expertise in various fields could be included in the study. The developed methods can be extended by hesitant fuzzy sets, neutrosophic sets, Pythagorean fuzzy sets, picture fuzzy sets, spherical fuzzy sets, or q-rung fuzzy sets. In addition, the developed IF-DEVADA method can

be used with other criterion weighting methods such as AHP, ANP, BWM, FUCOM, and entropy-based approaches.





6. EVALUATION OF GOVERNMENT STRATEGIES AGAINST COVID-19 PANDEMIC USING Q-RUNG ORTHOPAIR FUZZY TOPSIS METHOD⁵

Coronaviruses are enveloped non-segmented positive-sense RNA viruses and broadly infected in mainly humans and other mammals. Although the coronavirus infections in most humans are mild, the epidemics of the two betacoronaviruses, severe acute respiratory syndrome coronavirus (SARS-CoV) and Middle East respiratory syndrome coronavirus (MERS-CoV) have resulted in numerous deaths (Huang et al., 2020). The high prevalence and wide distribution of coronaviruses have the potential to reveal newer and severe events (Huang et al., 2020; Zhu et al., 2020).

In December 2019, a series of patients with pneumonia of unexplained cause associated with the 2019 novel coronavirus (2019-nCoV) has emerged in Wuhan, China (Huang et al., 2020). The epidemic starting in China has been spread to other countries such as Italy, Japan, ABD, England, Spain, Turkey and many cases have been confirmed. Furthermore, this epidemic has resulted in the death of many cases worldwide. The COVID-19 virus, which continues to spread very easily and sustainably among people, has been described as a pandemic by the World Health Organization (WHO) on 11 March. Data published by WHO as of November 9, 2020, contains information that the epidemic has spread to 219 countries, there are more than 50 million confirmed cases and more than 1.2 million confirmed deaths globally (URL-3).

The number of studies on such a serious epidemic, the cause of which is not yet known and has no cure, has increased continuously in a short time. Ebrahim et al. (2020) proposed several strategies that could reduce the spread of COVID-19 over the community. They stressed that country-specific timing is a critical determinant to reduce the effects of the outbreak. Anderson et al. (2020) emphasized that various measures should be taken to improve the inevitable economic decline of governments

⁵ This chapter is based on the paper “Alkan, N., & Kahraman, C. (2021). Evaluation of government strategies against COVID-19 pandemic using q-rung orthopair fuzzy TOPSIS method. *Applied Soft Computing*, 110, 107653.”

due to the epidemic and what these measures could be. Wang et al. (2020) provided information on the protection of medical personnel in China and the reassignment of their medical resources. Marcel et al. (2020) explained the importance of testing, contact tracing, and self-isolation until vaccination or treatments are available to prevent the COVID-19 epidemic in Switzerland. Fang et al. (2020) simulated the spread dynamics of the COVID-19 epidemic and the effect of different control measures, conducted a sensitivity analysis to identify the key factor. There are several other important studies on modelling the spread of the COVID-19 pandemic (Carli et al., 2020; Giordano et al., 2020; Paré et al., 2020; Rahimi et al., 2023).

The COVID-19 epidemic has caused many effects such as economic, social, and psychological, in addition to its serious and great impact on health worldwide and this has directed the researchers to carry out more studies on this epidemic. 936 studies have been published in just three months according to the Scopus database and many reports are published daily by WHO. The rapid spread of the COVID-19 epidemic has prompted countries to adopt various strategies by directing to take many measures. For example, countries such as China, Italy have imposed the curfew, countries such as Turkey, France, and Belgium have imposed partial curfew, England has released its people to gain immunity. Due to the COVID-19 epidemic which negatively affects countries in all respects and becomes a major threat, it is necessary to determine the most effective strategy for governments by considering variety of criteria, however, there is no study in the literature that can help governments in this topic.

The assessment of governmental strategies involves several conflicting criteria such as cost minimization and digital technology purchase, which you must handle by a multiple criteria decision-making method. MCDM methods, an effective method in dealing with complex problems simultaneously evaluate various criteria by including the opinion of multiple decision-makers (DMs). Therefore, MCDM methods can be successfully used in determining the best strategy for governments. Dotoli et al. (2020) present a taxonomy on multi criteria group decision making under uncertainty. They perform a comparative analysis among some selected well-known MCDM techniques, namely the Analytic Hierarchy Process (AHP), the Preference Ranking Organization METHod for Enrichment of Evaluations (PROMETHEE), the Multi Attribute Utility Theory (MAUT), and the Data Envelopment Analysis (DEA) to show how they can properly support the specific decision-making process of Public

Procurement (PP) tenders. A comparison analysis is extended to the fuzzy counterparts of AHP and DEA, showing that these methods can be effectively applied to the PP sector under uncertainty. There are available numerous MCDM methods such as AHP (Saaty, 1980), Analytic Network Process (ANP) (Saaty, 2001), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) (Hwang & Yoon, 1981), Vise Kriterijumska Optimizacija I Kompromisno Resenje (VIKOR) (Opricovic, S. 1998), Elimination and Choice Translating Reality English (ELECTRE) (Roy, 1968) in the literature. TOPSIS method, which is a simple and useful MCDM method, is a distance-based method aiming to choose the best alternative with the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS) (Kutlu Gündoğdu & Kahraman, 2019b).

The choice of the model under uncertainty depends on availability of data. The data are obtained from the experts' knowledge based on a very short period of time since there is not yet much information about COVID-19 pandemic. If sufficient observations had been handled, stochastic modelling could be used. Although decision-makers express their opinions as a crisp value in the classical TOPSIS method, when uncertain and vague information takes into account in decision making, these crisp values are often insufficient and inadequate for the solution of real decision-making problems (Büyüközkan & Göçer, 2021). To handle such type of vagueness, Zadeh (1965) introduced the fuzzy set theory in 1965. Ordinary fuzzy sets represented with a membership function that has a value between zero and one have been developed several extensions of its to determine a clear solution in more complex decision-making problems (Büyüközkan & Göçer, 2021). After were introduced type-2 fuzzy sets by Zadeh (1975) in 1975, Atanassov (1986) extended the fuzzy sets into the intuitionistic fuzzy sets (IFSs) represented with membership and non-membership degrees including the hesitancy of decision-makers (Karaşan & Kahraman, 2018). Hesitant fuzzy sets (HFSs) introduced by Torra (2010) are the extension of fuzzy sets allowing a set of values that are many potential degrees of membership of an element (Karaşan & Kahraman, 2018). After intuitionistic type-2 fuzzy sets (IFS2) proposed by Atanassov and Gargov (1989), Yager (2013) called them as Pythagorean fuzzy sets (PFSs) in 2013. PFSs are characterized by a membership degree and a non-membership degree providing the condition that the square sum of membership and non-membership degrees is equal to one at most. Later, Yager (2017) introduced q-

q-rung orthopair fuzzy sets (q-ROFSs) representing a general class of IFSs and PFSs. q-ROFSs are characterized with both membership and non-membership degrees of an element in a fuzzy set and the sum of their q th power has to at most equal to 1. Yager expressed that as q increases the range of acceptable orthopairs increases and thus enables the experts more freedom than the other fuzzy set extensions in expressing their belief about membership grade (Yager, 2017). The recent extensions of ordinary fuzzy sets such as PFSs and Fermatean fuzzy sets (FFSs) are based on the certain powers of membership and non-membership degrees, which all aim at enlarging the domain area for the assignment of these degrees and almost have the same theoretical basis that is not so different. An inclusive fuzzy set extension of PFSs and FFSs is q-ROFSs, which is a generalization of intuitionistic fuzzy sets. Thus, we prefer q-ROFSs by considering the liberty of choosing the degree of power. This study aims at proposing a q-rung orthopair fuzzy (q-ROF) MCDM method for better handling of vagueness and imprecise information.

The contribution of our study is to develop a decision support system (DSS) for the assessment of governmental strategies against COVID-19 pandemic. In the developed DSS, the originalities of our study are as follows. It is known that in the literature there is no study evaluating the government strategies for a new type of coronavirus with a MCDM method. Due to this lack of literature, the main motivation of our study is to assess by using fuzzy-based MCDM method the strategies of governments for the COVID-19 epidemic, which is currently a major threat. The developed two methods are the linguistic q-ROF TOPSIS methods, based on the linguistic scales proposed by us in the literature, one scale for weighting the criteria and another one for the evaluations in the decision matrix. The weights are produced by an aggregation operator in the first method whereas we employ an entropy based weighted criteria in the second one. Comparisons with other q-ROF MCDM methods and intuitionistic fuzzy TOPSIS method are also presented for the validation of the proposed methods. A sensitivity analysis is conducted on the changes in the values of the q parameter to show and verify how the attitude of the decision maker has a change on the results. Additionally, we applied a scenario analysis to show the dynamic structure of our proposed methods on the considered problem.

In the study, q-ROFSs are used as it provides a stronger definition of uncertainty and thus more accurate management of the decision-making process by allowing decision

makers to make an assessment in a wider area due to the uncertainties that arise in the problems addressed and the lack of information and inconsistencies between expert groups. The criteria and alternatives to be used in the study are determined by examining the papers and reports in the literature and taking into account the expert opinions.

The rest of this paper is organized as follows. A literature review on COVID-19 and its MCDM methodologies and q-ROF MCDM methods is presented in Section 6.1. The preliminaries of IFSs, PFSs, and q-ROFSs, are given in Section 6.2. The details of the proposed approaches based on q-ROF TOPSIS are presented in Section 6.3. Application of the approaches, sensitivity analysis and comparative analysis are presented in Section 6.4. Finally, the study is concluded in Section 6.5.

6.1 Literature Review

Our literature review is divided into two subsections. In the first subsection of the “literature review”, MCDM methods on COVID-19 problems are summarized and in the second subsection q-ROFSs based MCDM methods are analyzed.

6.1.1 Literature review: MCDM on COVID-19 problems

COVID-19, caused by a new coronavirus, is a respiratory pandemic spreading from person to person. Although the number of studies conducted on this epidemic that spread rapidly in a very short time is very high, studies using MCDM methods are limited in the literature. These studies have been briefly summarized in the following.

Majumder et al. (2020) proposed a new TOPSIS approach in identifying the most important risk factor for COVID-19. In the proposed method, first time supremum metric for ideal solution is used instead of Euclidean distance. Mohammed et al. (2020) developed an integrated MCDM method to evaluate the different diagnostic models for COVID19 according to the determined criteria. In the proposed approach, the entropy method is used for determining the criteria weights and the TOPSIS method is used for the ranking of the COVID-19 diagnostic models. Yang et al. (2020) introduced a decision support algorithm based on the novel concept of the spherical normal fuzzy (SpNoF) set. In the study, the new score and accuracy function, the SpNoF Bonferroni mean operator and the weighted Bonferroni mean operator are developed. A multi-criteria decision-making method is established for antivirus mask

selection over the COVID-19 pandemic based on the proposed SpNoF operators. The sensitivity and comparative analyses are conducted to check the availability and superiority of the proposed method. Albahri et al. (2020) proposed an intelligence-integrated concept in the determination of the most suitable convalescent plasma and the prioritization of patients with COVID-19 for helping doctors hasten treatments. In the study, a new MCDM method called subjective and objective decision by opinion score method (SODOSM) is proposed, which consists of integrating several MCDM methods. Requia et al. (2020) presented a MCDM model based on the AHP method for ranking how each community attribute may influence the spread or control of COVID-19 at the municipal level. Albahri et al. (2020) developed an integrated AHP and VIKOR approach for evaluation and benchmarking AI techniques used in the classification of COVID-19 medical images.

6.1.2 Literature review: MCDM based on q-ROFSs

q-ROFSs have been used in MCDM methods recently. Studies applying a MCDM method based on q-ROFSs have been briefly analyzed in the following.

Banerjee et al. (2020) developed a closeness index-based q-ROF qualitative flexible (QUALIFLEX) methodology. Pinar and Boran (2020) introduced q-ROF TOPSIS and q-ROF ELECTRE based on the group decision-making approach for the supplier selection problem. Joshi and Gegov (2020) used them in a MCDM problem by proposing some q-ROF aggregation operators. Wang et al. (2020) introduced the q-ROF multi-attributive border approximation area comparison (MABAC) model to solve multiple attribute group decision making (MAGDM) problems. Liu and Wang (2018) proposed the q-ROF weighted averaging operator and the q-ROF weighted geometric operator to handle the multi-attribute decision making (MADM) problems under the fuzzy environment. Krishankumar et al. (2019) proposed the q-ROF complex proportional assessment (COPRAS) method for a renewable energy source prioritization problem. Wang et al. (2019) proposed a new q-ROF linguistic method to handle the supplier selection problem. Zeng et al. (2019) proposed a new hybrid q-ROF approach covering rough set and TOPSIS. Y. Xu et al. (2019) presented a new MADM approach by developing interval-valued q-rung dual hesitant fuzzy sets and also proposed new aggregation operators based on fuzzy sets developed. Deng et al. (2021) combined a q-ROFS with the multiplicative multi-objective optimization by

ratio analysis method and introduced an optimization-based consensus model. Li et al. (2018) proposed q-rung picture linguistic set (q-RPLS) to solve MAGDM problems and introduced new aggregation operators based on the proposed q-RPLS. Mi et al. (2019) introduced q-ROF VIKOR method integrated with the best worst method based on the proposed new score function. Wang and Li (2018) developed the interval-valued q-rung orthopair fuzzy (IVq-ROF) method. Liu et al. (2019) proposed IVq-ROF multi-objective optimization based on ratio analysis plus the full multiplicative form (IVq-ROF-MULTIMOORA) method. Darko and Liang (2020) introduced some q-ROF Hamacher aggregation operators and demonstrated their application by developing q-ROF Evaluation Based on Distance from Average Solution (EDAS) method.

6.2 Q- Rung Orthopair Fuzzy Sets: Preliminaries

In this section, basic notions and operations of IFSs, PFSs, and q-ROFSs are briefly reviewed.

6.2.1 Intuitionistic fuzzy sets

Intuitionistic fuzzy sets as an extension of the ordinal fuzzy set theory was introduced by Atanassov (1986). IFSs are characterized by membership degree and non-membership degree that their sum is one or less than one. It is demonstrated as given in Definition 6.1 (Atanassov, 1986).

Definition 6.1: Let X be a fixed set. An IFS $\tilde{I}$ in X is an object having the form given by:

$$\tilde{I} = \{(x, \mu_{\tilde{I}}(x), \vartheta_{\tilde{I}}(x)) \mid x \in X\} \quad (6.1)$$

where the function $\mu_{\tilde{I}}: X \rightarrow [0,1]$ and $\vartheta_{\tilde{I}}: X \rightarrow [0,1]$ defines the degree of membership and the degree of non-membership of an element to the sets $\tilde{I}$, respectively, with the condition that

$$0 \leq \mu_{\tilde{I}}(x) + \vartheta_{\tilde{I}}(x) \leq 1, \text{ for } \forall x \in X \quad (6.2)$$

The degree of hesitancy is calculated as follows:

$$\pi_I(x) = 1 - \mu_I(x) - \vartheta_I(x) \quad (6.3)$$

Definition 6.2: Let $\tilde{A} = (\mu_{\tilde{A}}, \vartheta_{\tilde{A}})$ and $\tilde{B} = (\mu_{\tilde{B}}, \vartheta_{\tilde{B}})$ be two intuitionistic fuzzy numbers (IFNs), then the addition and multiplication operations on these two IFNs were defined by Atanassov as follows:

$$\tilde{A} \oplus \tilde{B} = (\mu_{\tilde{A}} + \mu_{\tilde{B}} - \mu_{\tilde{A}}\mu_{\tilde{B}}, \vartheta_{\tilde{A}}\vartheta_{\tilde{B}}) \quad (6.4)$$

$$\tilde{A} \otimes \tilde{B} = (\mu_{\tilde{A}}\mu_{\tilde{B}}, \vartheta_{\tilde{A}} + \vartheta_{\tilde{B}} - \vartheta_{\tilde{A}}\vartheta_{\tilde{B}}) \quad (6.5)$$

6.2.2 Pythagorean fuzzy sets

Pythagorean fuzzy sets have been introduced by Yager (2013) as an extension of the intuitionistic fuzzy set. PFSs are characterized by two membership degrees named as membership and non-membership. In PFSs unlike IFSs, the sum of membership degree and non-membership degree assigned by decision-makers may be more than 1, but the sum of their squares has to be at most 1. PFSs are described as given in Definition 6.3.

Definition 6.3: Let X be a fixed set. A Pythagorean fuzzy set $\tilde{P}$ in X is an object having the form (Zhang & Xu, 2014):

$$\tilde{P} = \{\langle x, \mu_{\tilde{P}}(x), \vartheta_{\tilde{P}}(x) \rangle \mid x \in X\} \quad (6.6)$$

where the function $\mu_{\tilde{P}}: X \rightarrow [0,1]$ defines the degree of membership and $\vartheta_{\tilde{P}}: X \rightarrow [0,1]$ defines the degree of non-membership of the element $x \in X$ to $\tilde{P}$, respectively, and for every $x \in X$, it holds that:

$$0 \leq (\mu_{\tilde{P}}(x))^2 + (\vartheta_{\tilde{P}}(x))^2 \leq 1 \quad (6.7)$$

The degree of hesitancy is calculated as follows:

$$\pi_{\tilde{P}}(X) = \sqrt{1 - \mu_{\tilde{P}}(x)^2 - \vartheta_{\tilde{P}}(x)^2} \quad (6.8)$$

Definition 6.4: Let $\tilde{P}_1 = (\mu_{\tilde{P}_1}, \vartheta_{\tilde{P}_1})$ and $\tilde{P}_2 = (\mu_{\tilde{P}_2}, \vartheta_{\tilde{P}_2})$ be two Pythagorean fuzzy numbers (PFNs), then the operations of these two PFNs are defined as follows (Zhang & Xu, 2014):

$$\tilde{P}_1 \oplus \tilde{P}_2 = \left(\sqrt{\mu_{\tilde{P}_1}^2 + \mu_{\tilde{P}_2}^2 - \mu_{\tilde{P}_1}^2 \mu_{\tilde{P}_2}^2}, \vartheta_{\tilde{P}_1} \vartheta_{\tilde{P}_2} \right) \quad (6.9)$$

$$\tilde{P}_1 \otimes \tilde{P}_2 = \left(\mu_{\tilde{P}_1} \mu_{\tilde{P}_2}, \sqrt{\vartheta_{\tilde{P}_1}^2 + \vartheta_{\tilde{P}_2}^2 - \vartheta_{\tilde{P}_1}^2 \vartheta_{\tilde{P}_2}^2} \right) \quad (6.10)$$

6.2.3 Q- rung orthopair fuzzy sets

q-rung orthopair fuzzy sets introduced by Yager (2017) are represented with the degree of membership and non-membership. In q-ROFSs, the sum of the q th power of the membership and non-membership degrees must be at most equal to one (P. Liu & Wang, 2018). In Figure 6.1, it is easily observed that q ROFSs have an acceptable membership grade space larger than of IFNs and PFNs. q-ROFSs are described as demonstrate in Definition 6.5.

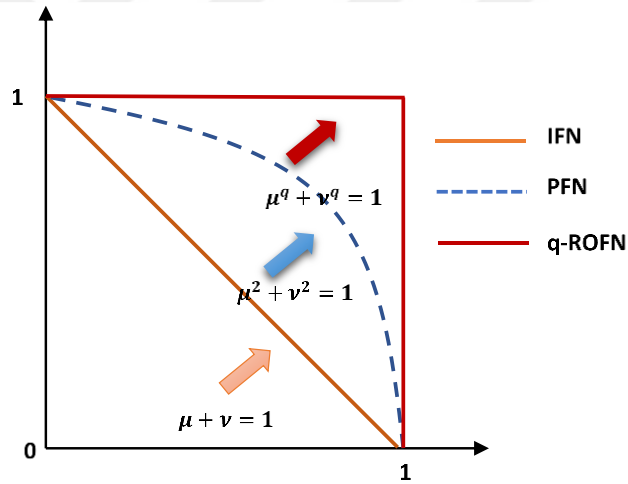


Figure 6.1 : Geometric space range of IFNs, PFNs, and q-ROFNs.

Definition 6.5: A q-ROFS $\tilde{Q}$ in a finite universe of discourse X is defined as follows by Yager (2017).

$$\tilde{Q} = \{(x, \mu_{\tilde{Q}}(x), \vartheta_{\tilde{Q}}(x)) | x \in X\} \quad (6.11)$$

where the function $\mu_{\tilde{Q}}: X \rightarrow [0,1]$ denotes the degree of membership and $\vartheta_{\tilde{Q}}: X \rightarrow [0,1]$ denotes the degree of non-membership of the element $x \in X$ to the set $\tilde{Q}$, respectively, with the condition that $0 \leq \mu_{\tilde{Q}}(x) + \vartheta_{\tilde{Q}}(x) \leq 1$, for every $x \in X$. The degree of indeterminacy is given as $\pi_{\tilde{Q}}(x) = \sqrt[q]{1 - \mu_{\tilde{Q}}(x)^q - \vartheta_{\tilde{Q}}(x)^q}$ (Ju et al., 2019)

Definition 6.6: Let $\tilde{Q} = (\mu_{\tilde{Q}}, \vartheta_{\tilde{Q}})$, $\tilde{Q}_1 = (\mu_{\tilde{Q}_1}, \vartheta_{\tilde{Q}_1})$ and $\tilde{Q}_2 = (\mu_{\tilde{Q}_2}, \vartheta_{\tilde{Q}_2})$ be three q-rung orthopair fuzzy numbers (q-ROFNs), then their operations can be defined as follows (P. Liu & Wang, 2018).

$$\tilde{Q}_1 \cap \tilde{Q}_2 = (\min\{\mu_{\tilde{Q}_1}, \mu_{\tilde{Q}_2}\}, \max\{\vartheta_{\tilde{Q}_1}, \vartheta_{\tilde{Q}_2}\}) \quad (6.12)$$

$$\tilde{Q}_1 \cup \tilde{Q}_2 = (\max\{\mu_{\tilde{Q}_1}, \mu_{\tilde{Q}_2}\}, \min\{\vartheta_{\tilde{Q}_1}, \vartheta_{\tilde{Q}_2}\}) \quad (6.13)$$

$$\tilde{Q}_1 \oplus \tilde{Q}_2 = \left((\mu_{\tilde{Q}_1}^q + \mu_{\tilde{Q}_2}^q - \mu_{\tilde{Q}_1}^q \mu_{\tilde{Q}_2}^q)^{1/q}, \vartheta_{\tilde{Q}_1} \vartheta_{\tilde{Q}_2} \right) \quad (6.14)$$

$$\tilde{Q}_1 \otimes \tilde{Q}_2 = \left(\mu_{\tilde{Q}_1} \mu_{\tilde{Q}_2}, (\vartheta_{\tilde{Q}_1}^q + \vartheta_{\tilde{Q}_2}^q - \vartheta_{\tilde{Q}_1}^q \vartheta_{\tilde{Q}_2}^q)^{1/q} \right) \quad (6.15)$$

$$\lambda \tilde{Q} = \left((1 - (1 - \mu_{\tilde{Q}}^q)^\lambda)^{1/q}, \vartheta_{\tilde{Q}}^\lambda \right), \quad \lambda > 0 \quad (6.16)$$

$$\tilde{Q}^\lambda = \left(\mu_{\tilde{Q}}^\lambda, (1 - (1 - \vartheta_{\tilde{Q}}^q)^\lambda)^{1/q} \right), \quad \lambda > 0 \quad (6.17)$$

Definition 6.7: Let $\tilde{Q} = (\mu_{\tilde{Q}}, \vartheta_{\tilde{Q}})$ be a q-ROFN, then the score function $S(\tilde{Q})$ and accuracy function $H(\tilde{Q})$ of $\tilde{Q}$ can be defined as in Eqs. (6.18) and (6.19), respectively (P. Liu & Wang, 2018).

$$S(\tilde{Q}) = \mu_{\tilde{Q}}^q - \vartheta_{\tilde{Q}}^q \quad (6.18)$$

$$H(\tilde{Q}) = \mu_{\tilde{Q}}^q + \vartheta_{\tilde{Q}}^q \quad (6.19)$$

Definition 6.8: Let $\tilde{Q}_i = (\mu_{\tilde{Q}_i}, \vartheta_{\tilde{Q}_i})$ ($i = 1, 2, \dots, n$) be a set of q-ROFNs and $w = (w_1, w_2, \dots, w_n)^T$ be the weight vector of $\tilde{Q}_i$ with $\sum_{i=1}^n w_i = 1$, then a q-rung orthopair fuzzy weighted average (q -ROFWA) operator is (P. Liu & Wang, 2018):

$$q\text{-ROFWA}(\tilde{Q}_1, \tilde{Q}_2, \dots, \tilde{Q}_n) = \left((1 - \prod_{i=1}^n (1 - \mu_{\tilde{Q}_i}^q)^{w_i})^{1/q}, \prod_{i=1}^n \vartheta_{\tilde{Q}_i}^{w_i} \right) \quad (6.20)$$

Definition 6.9: Let $\tilde{Q}_i = (\mu_{\tilde{Q}_i}, \vartheta_{\tilde{Q}_i})$ ($i = 1, 2, \dots, n$) be a set of q-ROFNs and $w = (w_1, w_2, \dots, w_n)^T$ be weight vector of $\tilde{Q}_i$ with $\sum_{i=1}^n w_i = 1$, then a q-rung orthopair fuzzy weighted geometric ($q - ROFWG$) operator is (P. Liu & Wang, 2018):

$$q - ROFWG (\tilde{Q}_1, \tilde{Q}_2, \dots, \tilde{Q}_n) = \left(\prod_{i=1}^n \mu_{\tilde{Q}_i}^{w_i}, \left(1 - \prod_{i=1}^n (1 - \vartheta_{\tilde{Q}_i}^q)^{w_i} \right)^{1/q} \right) \quad (6.21)$$

6.3 Proposed Methodologies

In this section, two different new approaches of q-ROF TOPSIS method have been propose to select the best COVID-19 strategy and rank strategies with their details. The framework of the proposed approaches has been given in the flowchart in Figure 6.2.

Method-I

The details of the first proposed method are as follows.

Step 1. Determine the alternatives, relevant criteria, and decision-makers (DMs) to construct the proposed approach. The set $A_i = \{A_1, A_2, \dots, A_m\}$ having $i = 1, 2, \dots, m$ alternatives, is assessed by n decision criteria of set $C_j = \{C_1, C_2, \dots, C_n\}$, with $j = 1, 2, \dots, n$. Let $w = (w_1, w_2, \dots, w_n)$ be the vector set used for defining the criteria weights, where $w_j > 0$ and $\sum_{j=1}^n w_j = 1$.

Step 2. Construct the q-ROF decision matrices (D_k) with respect to experts' opinions by using the scale given in Table 6.1. The q-ROF decision matrix based on Expert (k) to evaluate the alternatives in terms of each criterion is presented in Table 6.2. Here, $\tilde{D}_k = (\tilde{d}_{ijk})_{n \times m}$ in which $\tilde{d}_{ijk} = [\mu_{ijk}, \vartheta_{ijk}]$ is constructed by utilizing the linguistic terms scale given in Table 6.1. Accordingly, $\tilde{d}_{ijk}$ indicates the performance of alternative A_i in terms of criterion C_j of k^{th} expert.

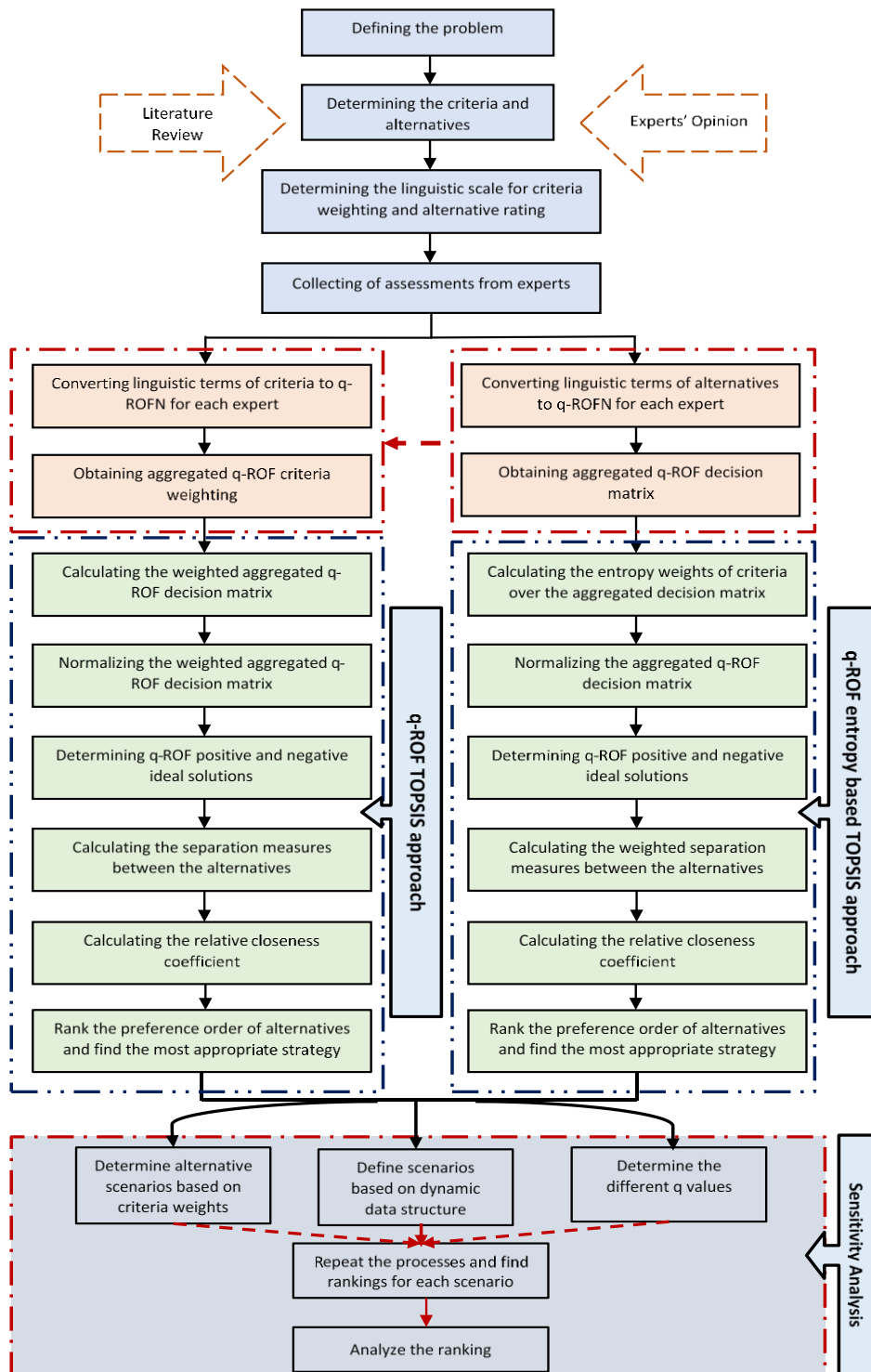


Figure 6.2 : Geometric space range of IFNs, PFNs, and q-ROFNs.

Table 6.1 : Linguistic scale for ratings of alternatives.

Linguistic terms	q-ROFN for alternatives	
	μ	ϑ
Certainly High Value – (CHV)	0.99	0.11
Very High Value – (VHV)	0.88	0.22
High Value – (HV)	0.77	0.33
Above Average Value – (AAV)	0.66	0.44
Average Value – (AV)	0.55	0.55
Under Average Value – (UAV)	0.44	0.66
Low Value – (LV)	0.33	0.77
Very Low Value – (VLV)	0.22	0.88
Certainly Low Value – (CLV)	0.11	0.99

Table 6.2 : The decision matrix based on q-rung orthopair fuzzy number with respect to expert k.

Criteria	Alternatives			
	A_1	A_2		A_m
C_1	$\langle [\mu_{11k}, \vartheta_{11k}] \rangle$	$\langle [\mu_{12k}, \vartheta_{12k}] \rangle$		$\langle [\mu_{1mk}, \vartheta_{1mk}] \rangle$
C_2	$\langle [\mu_{21k}, \vartheta_{21k}] \rangle$	$\langle [\mu_{22k}, \vartheta_{22k}] \rangle$		$\langle [\mu_{2mk}, \vartheta_{2mk}] \rangle$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
C_n	$\langle [\mu_{n1k}, \vartheta_{n1k}] \rangle$	$\langle [\mu_{n2k}, \vartheta_{n2k}] \rangle$		$\langle [\mu_{nmk}, \vartheta_{nmk}] \rangle$

Step 3. Compute the aggregated q-ROF decision matrix. The individual decision matrices are aggregated in one decision matrix using $q - ROFWG$ given in Eq. (6.21). Aggregated q-ROF decision matrix $(\tilde{D})$ is constructed as in Table 6.3. Here, $\tilde{D} = (\tilde{d}_{ij})_{n \times m}$ in which $\tilde{d}_{ij} = [\mu_{ij}, \vartheta_{ij}]$ is used to indicate the aggregated q-ROFN of i^{th} alternative with respect to j^{th} criterion.

Table 6.3 : Aggregated q-ROF decision matrix.

Criteria	Alternatives			
	A_1	A_2		A_m
C_1	$\langle [\mu_{11}, \vartheta_{11}] \rangle$	$\langle [\mu_{12}, \vartheta_{12}] \rangle$		$\langle [\mu_{1m}, \vartheta_{1m}] \rangle$
C_2	$\langle [\mu_{21}, \vartheta_{21}] \rangle$	$\langle [\mu_{22}, \vartheta_{22}] \rangle$		$\langle [\mu_{2m}, \vartheta_{2m}] \rangle$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
C_n	$\langle [\mu_{n1}, \vartheta_{n1}] \rangle$	$\langle [\mu_{n2}, \vartheta_{n2}] \rangle$		$\langle [\mu_{nm}, \vartheta_{nm}] \rangle$

Step 4. Determine the q-ROF weights of criteria for each expert $(\tilde{W})$ using the linguistic scale given in Table 6.4. Here, $\tilde{W}_k = (w_{jk})_{1 \times n}$ indicates q-ROFN of k^{th} expert with respect to j^{th} criterion.

Step 5. Aggregate the assessments on criteria weights. The individual assessments are aggregated in the criteria weight matrix $(\tilde{W})$ using $q - ROFWG$ given in Eq. (6.21). Here, $\tilde{W} = (w_j)_{1 \times n}$ shows the q-ROF weight of j^{th} criterion.

Table 6.4 : Linguistic scale for weighting of the criteria.

Linguistic terms	q-ROFN for criteria	
	μ	ϑ
Certainly High Importance – (CHI)	0.99	0.11
Very High Importance – (VHI)	0.88	0.22
High Importance – (HI)	0.77	0.33
Above Average Importance – (AAI)	0.66	0.44
Average Importance – (AI)	0.55	0.55
Under Average Importance – (UAI)	0.44	0.66
Low Importance – (LI)	0.33	0.77
Very Low Importance – (VLI)	0.22	0.88
Certainly Low Importance – (CLI)	0.11	0.99

Step 6. Calculate the weighted aggregated q-ROF decision matrix ($\tilde{R}$). After the weights of the criteria and the ratings of the alternatives are determined, the weighted aggregated q-ROF decision matrix ($\tilde{R}$) is obtained by multiplying the q-ROF weights vector (w_j) with the aggregated q-ROF decision matrix ($\tilde{D}$) by utilizing Eqs. (6.22) and (6.23) as follows:

$$\tilde{R} = (\tilde{r}_{ij})_{n \times m} \quad (6.22)$$

$$\tilde{r}_{ij} = w_j \otimes \tilde{d}_{ij} \quad (6.23)$$

where $\tilde{r}_{ij} = [\mu_{r_{ij}}, \vartheta_{r_{ij}}]$ shows the weighted aggregated q-ROFN of i^{th} alternative with respect to j^{th} criterion. Table 6.5 indicates the weighted aggregated q-ROF decision matrix.

Table 6.5 : Weighted aggregated q-ROF decision matrix.

Criteria	Alternatives			
	A_1	A_2		A_m
C_1	$\langle [\mu_{r_{11}}, \vartheta_{r_{11}}] \rangle$	$\langle [\mu_{r_{12}}, \vartheta_{r_{12}}] \rangle$		$\langle [\mu_{r_{1m}}, \vartheta_{r_{1m}}] \rangle$
C_2	$\langle [\mu_{r_{21}}, \vartheta_{r_{21}}] \rangle$	$\langle [\mu_{r_{22}}, \vartheta_{r_{22}}] \rangle$		$\langle [\mu_{r_{2m}}, \vartheta_{r_{2m}}] \rangle$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
C_n	$\langle [\mu_{r_{n1}}, \vartheta_{r_{n1}}] \rangle$	$\langle [\mu_{r_{n2}}, \vartheta_{r_{n2}}] \rangle$		$\langle [\mu_{r_{nm}}, \vartheta_{r_{nm}}] \rangle$

Step 7. Normalize the weighted aggregated q-ROF decision matrix. If the criterion is benefit type, then do nothing; if the criterion is a cost type, then the cost type criterion should be converted into a benefit type criterion. The normalized q-ROF decision matrix ($\tilde{A}$) is shown as follows.

$$\tilde{A} = \tilde{a}_{ij} = (\mu_{ij}, \vartheta_{ij}) = \begin{cases} (\mu_{ij}, \vartheta_{ij}), & \text{for benefit type } C_j \\ (\vartheta_{ij}, \mu_{ij}), & \text{for cost type } C_j \end{cases} \quad (6.24)$$

Step 8. Determine q-ROF positive ideal solution (q-ROFPIS) and q-ROF negative ideal solution (q-ROFNIS) given in Eqs. (6.25) and (6.27) by using score function and accuracy function based on the normalized decision matrix.

$$\tilde{A}_j^* = \max\{S(A_{1j}), S(A_{2j}), \dots, S(A_{mj})\} \quad j = 1, 2, \dots, n \quad (6.25)$$

or

$$\tilde{A}^* = \{\langle C_1, (\mu_1^*, \vartheta_1^*) \rangle, \langle C_2, (\mu_2^*, \vartheta_2^*) \rangle, \dots, \langle C_n, (\mu_n^*, \vartheta_n^*) \rangle\} \quad (6.26)$$

where $S(\tilde{A})$ is the score function of q-ROFN and (μ_j^*, ϑ_j^*) is the maximum q-ROFN with the highest score value among alternatives for j^{th} criterion. If the value $S(A_{1j}) = S(A_{2j}) = \dots = S(A_{mj})$, then the values of the accuracy function are compared.

$$\tilde{A}_j^- = \min\{S(A_{1j}), S(A_{2j}), \dots, S(A_{mj})\} \quad j = 1, 2, \dots, n \quad (6.27)$$

or

$$\tilde{A}^- = \{\langle C_1, (\mu_1^-, \vartheta_1^-) \rangle, \langle C_2, (\mu_2^-, \vartheta_2^-) \rangle, \dots, \langle C_n, (\mu_n^-, \vartheta_n^-) \rangle\} \quad (6.28)$$

where $S(\tilde{A})$ is the score function of q-ROFN and (μ_j^-, ϑ_j^-) is the minimum q-ROFN with the lowest score value among alternatives for j^{th} criterion. If the value $S(A_{1j}) = S(A_{2j}) = \dots = S(A_{mj})$, then the values of the accuracy function are compared.

Step 9. Obtain the separation measures by calculating the distances for each alternative according to positive-ideal ($\tilde{A}^*$) and negative-ideal solutions ($\tilde{A}^-$). Distance between each alternative, and positive-ideal solution ($\tilde{A}^*$) as well as negative-ideal solution ($\tilde{A}^-$) are calculated based on normalized Euclidean distance as in Eqs. (6.29) and (6.30), respectively.

$$d(A_i, A^*) = \left(\frac{1}{2n} \sum_{j=1}^n (|\mu_{ij}^q - (\mu_j^*)^q|^2 + |\vartheta_{ij}^q - (\vartheta_j^*)^q|^2) \right)^{1/2} \quad (6.29)$$

$$d(A_i, A^-) = \left(\frac{1}{2n} \sum_{j=1}^n \left(|\mu_{ij}^q - (\mu_j^-)^q|^2 + |\vartheta_{ij}^q - (\vartheta_j^-)^q|^2 \right) \right)^{1/2} \quad (6.30)$$

Step 10. Calculate the relative closeness coefficient (CC_i) of alternatives by using the following:

$$CC_i = \frac{d(A_i, A^-)}{d(A_i, A^-) + d(A_i, A^+)} \quad (6.31)$$

Step 11. Rank the alternatives according to final scores. The best alternative(s) are selected based on the descending order of the values of the relative closeness coefficient (CC_i).

Method-II

The second proposed method is constructed based on the entropy of q-ROFSs developed by Liu et al. (2018), and the TOPSIS method based on q-ROFSs developed by Liu et al. (2019). The steps of the proposed method are as follows.

Steps 1, 2, and 3: The first three steps contain the same steps as the first proposed method.

Step 4. Obtain the entropy values of each q-ROFN in the aggregated decision matrix by using the following:

$$KE_{q,ij}(x) = \frac{1}{\sqrt{2}} \sqrt{\left((\mu(x))^q \right)^2 + \left((\vartheta(x))^q \right)^2 + \left((\mu(x))^q + (\vartheta(x))^q \right)^2} \quad (6.32)$$

$$\begin{aligned} EN_{q,ij}(x) &= 1 - KE_{q,ij}(x) \\ &= 1 - \frac{1}{\sqrt{2}} \sqrt{\left((\mu(x))^q \right)^2 + \left((\vartheta(x))^q \right)^2 + \left((\mu(x))^q + (\vartheta(x))^q \right)^2} \end{aligned} \quad (6.33)$$

Step 5. Calculate the weights of criteria. The weights of criteria are calculated based on entropy values by using the following:

$$w_j = \frac{1 - \xi_j}{\sum_{j=1}^n (1 - \xi_j)}; \quad j = 1, 2, \dots, n \quad (6.34)$$

where $\xi_j = \frac{\sum_{i=1}^m EN_{q,ij}}{\sum_{i=1}^m \sum_{j=1}^n EN_{q,ij}}$ indicates the q-ROF entropy value.

Step 6. Normalize the q-ROF aggregated decision matrix. The aggregated decision matrix is normalized by transforming cost types of criteria to the benefit type as given in Eq. (24).

Step 7. Determine q-ROFPIS $\tilde{A}^* = \{\tilde{A}_1^*, \tilde{A}_2^*, \dots, \tilde{A}_n^*\}$ and q-ROFNIS $\tilde{A}^- = \{\tilde{A}_1^-, \tilde{A}_2^-, \dots, \tilde{A}_n^-\}$ calculated by the score and accuracy functions as given in Eqs. (18) and (6.19), respectively.

Step 8. Determine separation measures by calculating the weighted distances for each alternative according to positive-ideal ($\tilde{A}^*$) and negative-ideal solutions ($\tilde{A}^-$). Distance between each alternative, and positive-ideal solution ($\tilde{A}^*$) as well as negative-ideal solution ($\tilde{A}^-$) are calculated by utilizing Euclidian distance as given in Eqs. (6.35) and (6.36), respectively.

$$d(A_i, A^*) = \left(\frac{1}{2} \sum_{j=1}^n w_j \left(|\mu_{ij}^q - (\mu_j^*)^q|^2 + |\vartheta_{ij}^q - (\vartheta_j^*)^q|^2 \right) \right)^{1/2} \quad (6.35)$$

$$d(A_i, A^-) = \left(\frac{1}{2} \sum_{j=1}^n w_j \left(|\mu_{ij}^q - (\mu_j^-)^q|^2 + |\vartheta_{ij}^q - (\vartheta_j^-)^q|^2 \right) \right)^{1/2} \quad (6.36)$$

Step 9. Calculate the closeness coefficient (CC_i) of alternative $\tilde{A}_i$ by using Eq. (6.31) given in the first proposed method.

Step 10. Determine the optimal alternative(s) by ranking in descending order the alternative(s). In this step, we compare our two proposed methods to control its reliability and robustness.

6.4 Application

6.4.1 Problem definition

COVID-19, caused by a new coronavirus, is a respiratory pandemic spreading from person to person in many countries. This epidemic, which has various symptoms, is to continue its course according to the age of the persons and the diseases they have, and mortality rates are to vary depending on these situations. COVID-19 is to spread rapidly among people and cause serious effects if the necessary precautions are not taken. Therefore, the rapid spread of the COVID-19 epidemic has prompted countries to adopt various strategies by directing them to take many measures. Due to the

COVID-19 epidemic which negatively affects countries in all respects and becomes a major threat, it is necessary to determine the most effective strategy for governments by considering certain criteria. MCDM methods can be successfully used in determining the best strategy for governments. Therefore, in this section, the COVID-19 strategies of governments are evaluated by using the proposed new approaches and it is aimed to select the best strategy among several alternatives.

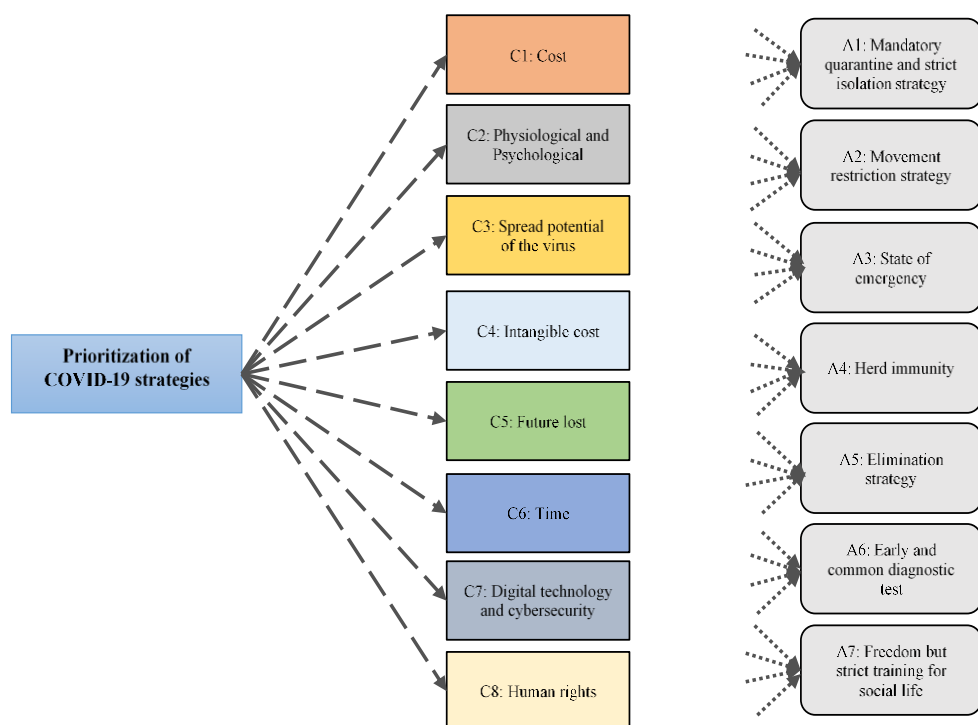


Figure 6.3 : Hierarchical structure of the problem.

In the group of decision makers consisting of three experts which are abbreviated as DM1, DM2, and DM3, there is a professor advising in the health sector, a lecturer advising on strategies and policies in the ministry of health, and a professor who has carried out many projects on multi-criteria decision making. The weights of these decision-makers who have different experience levels are 0.4, 0.3, and 0.3, respectively. The strategies applied by the governments and the criteria that affect these strategies are determined as 7 alternatives and 8 criteria as given in Figure 6.3 as a result of the expert opinions and evaluation of the studies in the literature.

6.4.2 COVID-19 strategies and selection criteria

Brief descriptions of the COVID-19 strategies considered are as follows:

Mandatory quarantine and strict isolation strategy (A1): This strategy includes stopping the social interactions of the whole society, tighter restrictions on travel or mandatory home lockdowns and self-isolation, and also applying rapid screening and contact tracing of infected individuals (URL-4, URL-5). The mandatory quarantine and strict isolation strategy have been implemented in China, Singapore, Japan, and Hong Kong (URL-5).

Movement restriction strategy (A2): Several different methods are preferred in this strategy. This strategy includes isolating suspicious cases in their own homes and quarantine their relatives, and removing and protecting the elderly and chronic ill people from social life in the risk group (URL-6). In this strategy, governments impose curfews either in cities with a high number of infections or at certain times of a day or at certain days of the week or based on risk groups. Due to COVID-19, several countries such as Turkey, Belgium, Spain, Ireland, Germany, France, Cyprus, Tunisia, and Libya have imposed various curfews (URL-7).

State of emergency (A3): The state of emergency stems from a government declaration in response to a state of emergency that poses a major threat to the country. A government can declare such a situation during an armed action against the state by civil unrest, a natural disaster, a medical epidemic, a financial or economic crisis (URL-8). Countries such as Finland, Hungary, Portugal, Romania, Bulgaria, and Spain have implemented the state of emergency because of COVID-19 (URL-9).

Herd immunity (A4): Herd immunity stems from the effects of individual immunity scaled to the level of the population. It means the indirect conservation from infection conferred to sensitive individuals when enough great ratio of immune individuals exists in a population. The aim is to provide that the virus spreads in a controlled manner. The herd immunity strategy has been implemented in the UK and Sweden (Kassem, 2020; URL-10).

Elimination strategy (A5): This strategy applied in New Zealand refers to reducing new cases in a defined geographical area, in this case New Zealand, to zero (or a very low defined target rate). The two primary aims of a COVID-19 elimination strategy are to eliminate transmission chains in New Zealand and to forbid the emergence of

new transmission chains caused by cases that arrive from outside the country. The basics of the strategy are based on border controls, robust case detection, surveillance, effective contact tracing, and strong community support of control measures (Baker et al., 2020).

Early and common diagnostic test (A6): By testing the vast population as free as possible in South Korea, after determining in which regions the disease is most concentrated, it has isolated the infected patients and quarantined them by monitoring their contact. In this way, there is no need to take measures such as curfew, quarantine, and restriction of movement. In this strategy, South Korea has disclosed real-time information about COVID-19 through special websites, mass media, telephone messages, and mobile applications (URL-5; URL-10).

Freedom but strict training for social life (A7): In this strategy, individuals can move freely, except for strict rules that must be followed in their social lives. If the mandatory rules are not followed, individuals are exposed to various financial penalties.

The criteria that are effective in prioritizing the strategies are as follows:

Cost (C1): Along with COVID-19, many sectors, from the agricultural sector to the manufacturing sector, could not perform their activities and suffered serious economic losses. Besides, the increase in consumption rate and decrease in productivity lead to serious material losses. Besides, the epidemic brings with it a decrease in growth rate, increase in the unemployment rate and inflation rate, the balance of payments and deterioration in budget balance, and external financing difficulties. Moreover, the tests used for the epidemic, treatment, therapy, medication, medical equipment, medical staff time, resources used for care, and access also lead to extra financial losses.

Physiological and Psychological (C2): The difficulties, risks, and uncertainties due to the coronavirus, which is declared as a pandemic worldwide, are to affect many people both physiologically and psychologically. With COVID-19, individuals feel seedy, bored, and tired, experience fear and anxiety about their health and the health of their loved ones, increase or decrease in eating and sleep patterns, and experience disconnection from social relationships, helplessness, sadness, anxiety, and anger. Besides, individuals are to experience difficulties in concentrating and gathering attention.

Spread potential of the virus (C3): The virus is transmitted very easily among people and the number of deaths caused by the virus is increasing day by day. Therefore, the potential for spreading the virus has an important effect on determining the most appropriate strategy. Governments should adopt a strategy to minimize disease spread.

Intangible cost (C6): Changing customer expectations with COVID-19 will lead to a reduction in customer satisfaction for many sectors and this will cause serious material losses depending on the loss of the customer. Besides, the loss of morale of employees and the decrease in the will to work will lead to a decrease in the productivity rate. COVID-19, which causes intangible costs like these, carries the risk of long-term financial losses and is, therefore, one of the important criteria to be considered in determining the most suitable strategy.

Future lost (C5): For a certain period of time, COVID-19 will cause employees to face a loss of skills and abilities, many industries not to be preferred by customers, and individuals to move away from social groups. Therefore, this criterion should be taken into consideration in determining the most appropriate strategy.

Time (C6): The least possible damage and overcoming of the epidemic is the primary target of all governments. Therefore, the time factor constitutes an important place in determining the most suitable strategy.

Digital technology and cybersecurity (C7): With digital technology and cybersecurity, another important criterion in determining the strategy, governments need to adopt the appropriate strategy where they can benefit more from technology. It is important to consider these two criteria together in determining the most appropriate strategy for the governments because of their potential to be exposed to too many cybersecurity attacks while making maximum use of technology.

Human rights (C8): The most appropriate strategy that requires human beings to be at the center should be determined against human rights violations experiencing in combating the epidemic.

6.4.3 Problem solution

For the first proposed q-ROF TOPSIS approach,

Step 1. An expert group of three decision-makers is defined in Sub-section 6.4.1, strategies and criteria are defined in Sub-section 6.4.2. The expert group evaluates strategies in line with the defined objectives and criteria.

Step 2. The linguistic decision matrix created based on the evaluations of experts using the scale in Table 6.1 is presented in Table 6.6. Then, the linguistic evaluations of DMs are converted to the corresponding q-ROFNs based on the scale in Table 6.1.

Step 3. Individual q-ROF decision matrices are aggregated to obtain the collective decision matrix using Eq. (6.21). In this process, the parameter determination stage requires scientific formulation or estimation. The parameter q can be given any value reflecting their optimistic and pessimistic attitudes by the group of experts. For the illustrative purpose, we present the results of $q = 5$ to provide a stronger definition of uncertainty, and to express information more flexibly, which indicates that the expert group has a relatively optimistic attitude. The aggregated q-ROF decision matrix is constructed as shown in Table 6.7.

Steps 4 and 5. The linguistic evaluations of the criteria assigned by DMs using the scale in Table 6.4 and their aggregated results based on q-ROFNs using Eq. (6.21) are shown in Table 6.8.

Table 6.6 : Weighted aggregated q-ROF decision matrix.

	DM1							DM2							DM3						
	A1	A2	A3	A4	A5	A6	A7	A1	A2	A3	A4	A5	A6	A7	A1	A2	A3	A4	A5	A6	A7
C1	VLV	AV	AV	VHV	AAV	VLV	LV	VLV	LV	LV	VHV	AAV	VLV	UAV	CLV	LV	LV	CHV	AV	VLV	AAV
C2	AAV	AAV	VHV	AAV	AV	UAV	VLV	LV	LV	LV	VHV	AAV	HV	LV	VLV	LV	UAV	VHV	AV	AAV	UAV
C3	VLV	LV	LV	VHV	AAV	CLV	VLV	CHV	VHV	VHV	VLV	HV	VHV	HV	VHV	HV	HV	VLV	AAV	HV	VHV
C4	AV	AAV	AV	VHV	UAV	VLV	LV	CLV	LV	UAV	VHV	AV	HV	LV	VLV	UAV	LV	CHV	UAV	HV	UAV
C5	CLV	LV	VLV	HV	AV	VLV	VLV	CLV	VLV	LV	VHV	AV	AV	AAV	CLV	LV	VLV	VHV	UAV	AV	AV
C6	VLV	LV	VLV	HV	AV	CLV	VLV	VHV	HV	HV	VLV	AAV	VHV	AV	VHV	HV	HV	VLV	AAV	HV	AV
C7	VHV	AAV	AAV	LV	AAV	CLV	VLV	VHV	HV	HV	VLV	AAV	VHV	LV	CHV	VHV	HV	VLV	AAV	VHV	VLV
C8	VHV	VHV	VHV	AAV	AV	VHV	HV	AAV	AAV	AV	HV	AAV	VHV	HV	HV	HV	AV	VHV	HV	VHV	AAV

Table 6.7 : Aggregated decision matrix.

	A1	A2	A3	A4	A5	A6	A7
C1	(0.179,0.947)	(0.405,0.717)	(0.405,0.717)	(0.912,0.205)	(0.625,0.485)	(0.22,0.88)	(0.443,0.691)
C2	(0.386,0.774)	(0.435,0.708)	(0.533,0.661)	(0.784,0.37)	(0.581,0.526)	(0.588,0.566)	(0.306,0.812)
C3	(0.524,0.763)	(0.571,0.654)	(0.571,0.654)	(0.383,0.816)	(0.691,0.418)	(0.368,0.931)	(0.486,0.764)
C4	(0.258,0.927)	(0.475,0.667)	(0.441,0.679)	(0.912,0.205)	(0.47,0.635)	(0.467,0.764)	(0.36,0.745)
C5	(0.11,0.99)	(0.292,0.815)	(0.248,0.857)	(0.834,0.285)	(0.514,0.593)	(0.381,0.776)	(0.403,0.772)
C6	(0.505,0.764)	(0.549,0.655)	(0.467,0.764)	(0.363,0.817)	(0.614,0.496)	(0.368,0.931)	(0.381,0.776)
C7	(0.912,0.205)	(0.754,0.38)	(0.724,0.389)	(0.259,0.848)	(0.66,0.44)	(0.383,0.931)	(0.248,0.857)
C8	(0.776,0.364)	(0.776,0.364)	(0.664,0.498)	(0.754,0.38)	(0.643,0.484)	(0.88,0.22)	(0.735,0.378)

Table 6.8 : Linguistic evaluations of criteria for each expert and aggregated criterion weights based on the first approach.

Criterion	DM1	DM2	DM3	Aggregated Results	Type	
					Cost	Benefit
C1	HI	VHI	CHI	(0.864,0.28)	✓	
C2	AAI	AAI	AI	(0.625,0.485)	✓	
C3	CHI	CHI	CHI	(0.99,0.11)	✓	
C4	HI	AI	HI	(0.696,0.448)	✓	
C5	VHI	AAI	VHI	(0.807,0.351)	✓	
C6	HI	VHI	VHI	(0.834,0.285)	✓	
C7	AI	AI	UAI	(0.514,0.593)		✓
C8	AAI	AI	UAI	(0.553,0.57)		✓

Step 6. The weighted aggregated q-ROF decision matrix is constructed by multiplying the q-ROF weights vector with the aggregated q-ROF decision matrix by using Eq. (6.15) as given in Table 6.9.

Table 6.9 : Weighted aggregated decision matrix based on the first approach.

	A1	A2	A3	A4	A5	A6	A7
C1	(0.154,0.947)	(0.35,0.718)	(0.35,0.718)	(0.788,0.291)	(0.54,0.491)	(0.19,0.88)	(0.383,0.692)
C2	(0.241,0.785)	(0.272,0.725)	(0.333,0.684)	(0.49,0.507)	(0.363,0.581)	(0.367,0.608)	(0.191,0.82)
C3	(0.518,0.763)	(0.565,0.654)	(0.565,0.654)	(0.379,0.816)	(0.684,0.418)	(0.364,0.931)	(0.481,0.764)
C4	(0.179,0.928)	(0.33,0.682)	(0.307,0.693)	(0.635,0.45)	(0.327,0.654)	(0.325,0.772)	(0.25,0.754)
C5	(0.089,0.99)	(0.236,0.817)	(0.201,0.858)	(0.673,0.373)	(0.415,0.601)	(0.308,0.778)	(0.325,0.774)
C6	(0.422,0.764)	(0.458,0.657)	(0.389,0.765)	(0.303,0.817)	(0.512,0.502)	(0.307,0.932)	(0.318,0.777)
C7	(0.469,0.594)	(0.388,0.605)	(0.372,0.606)	(0.133,0.863)	(0.339,0.616)	(0.197,0.937)	(0.128,0.871)
C8	(0.429,0.581)	(0.429,0.581)	(0.367,0.617)	(0.417,0.584)	(0.356,0.611)	(0.487,0.571)	(0.407,0.583)

Step 7. The weighted aggregated decision matrix is normalized by converting the cost type criterion to benefit type using Eq. (6.24) if the criterion is a cost type. The normalized decision matrix is as shown in Table 6.10.

Table 6.10 : Normalized decision matrix based on the first approach.

	A1	A2	A3	A4	A5	A6	A7
C1	(0.947,0.154)	(0.718,0.35)	(0.718,0.35)	(0.291,0.788)	(0.491,0.54)	(0.88,0.19)	(0.692,0.383)
C2	(0.785,0.241)	(0.725,0.272)	(0.684,0.333)	(0.507,0.49)	(0.581,0.363)	(0.608,0.367)	(0.82,0.191)
C3	(0.763,0.518)	(0.654,0.565)	(0.654,0.565)	(0.816,0.379)	(0.418,0.684)	(0.931,0.364)	(0.764,0.481)
C4	(0.928,0.179)	(0.682,0.33)	(0.693,0.307)	(0.45,0.635)	(0.654,0.327)	(0.772,0.325)	(0.754,0.25)
C5	(0.99,0.089)	(0.817,0.236)	(0.858,0.201)	(0.373,0.673)	(0.601,0.415)	(0.778,0.308)	(0.774,0.325)
C6	(0.764,0.422)	(0.657,0.458)	(0.765,0.389)	(0.817,0.303)	(0.502,0.512)	(0.932,0.307)	(0.777,0.318)
C7	(0.469,0.594)	(0.388,0.605)	(0.372,0.606)	(0.133,0.863)	(0.339,0.616)	(0.197,0.937)	(0.128,0.871)
C8	(0.429,0.581)	(0.429,0.581)	(0.367,0.617)	(0.417,0.584)	(0.356,0.611)	(0.487,0.571)	(0.407,0.583)

Step 8. After score values for each q-ROFN in the normalized decision matrix are calculated using Eq. (6.18), the positive ideal solution and negative ideal solutions are obtained by using Eqs. (6.25) and (6.27) as shown in Table 6.11.

Step 9. The separation measures for each alternative according to the positive-ideal and negative-ideal solutions are calculated using the Euclidean distance in Eqs. (6.29) and (6.30) as given in Table 6.12.

Step 10. The closeness coefficient of each alternative is calculated using Eq. (6.31) as shown in Table 6.13.

Step 11. The closeness coefficients indicate that the ranking order of the alternatives are $A1 > A6 > A3 > A2 > A7 > A5 > A4$.

Table 6.11 : Positive and negative ideal solutions based on the first approach.

Criterion	q-ROF positive ideal solution	q-ROF negative ideal solution
C1	(0.947,0.154)	(0.291,0.788)
C2	(0.82,0.191)	(0.507,0.49)
C3	(0.931,0.364)	(0.418,0.684)
C4	(0.928,0.179)	(0.45,0.635)
C5	(0.99,0.089)	(0.373,0.673)
C6	(0.932,0.307)	(0.502,0.512)
C7	(0.469,0.594)	(0.197,0.937)
C8	(0.487,0.571)	(0.367,0.617)

Table 6.12 : Separation measures of the alternatives based on the first approach.

	A1	A2	A3	A4	A5	A6	A7
$d(A_i, A^*)$	0.1573	0.32308	0.29854	0.39895	0.408	0.27079	0.31303
$d(A_i, A^-)$	0.40761	0.21887	0.23477	0.14011	0.17883	0.30502	0.18856

Table 6.13 : Closeness coefficient and ranks of the alternatives.

	A1	A2	A3	A4	A5	A6	A7
CC_i	0.7215	0.4039	0.4402	0.2599	0.3047	0.5297	0.3759
Rank	1	4	3	7	6	2	5

Now, we propose a new approach by applying different criteria weighting method. We compare the proposed two new approaches based on different criteria weighting methods on the results.

For the second proposed q -ROF entropy-based TOPSIS approach,

Step 1. To be valid and reliable of the results of the proposed methods, the same set of decision-makers, criteria and alternatives mentioned in the first proposed approach is used.

Steps 2 and 3. For this proposed approach, the scale given in Table 6.1 and the linguistic decision matrix in Table 6.6 created based on this scale are used. Besides, the aggregated decision matrix in Table 6.7, which is based on the linguistic decision matrices in the first proposed method, is used in the same way. The parameter q can be given any value by the group of experts. For the illustrative purpose, we present the results of $q=5$.

Steps 4 and 5. After the entropy values of each q -ROFN in the aggregated decision matrix are obtained using Eqs. (6.32) and (6.33), respectively, the weight of each criterion is calculated based on Entropy values using Eq. (6.34). The weights of criteria are as shown in Table 6.14.

Table 6.14 : Weights of criteria based on the second approach.

Criterion	Weight of criterion	Type	
		Cost	Benefit
C1	0.12628	✓	
C2	0.1205	✓	
C3	0.12452	✓	
C4	0.12479	✓	
C5	0.12723	✓	
C6	0.12404	✓	
C7	0.12797		✓
C8	0.12467		✓

Step 6. The aggregated decision matrix is normalized as given in Eq. (6.24). The normalized decision matrix is determined as in Table 6.15.

Step 7. After score values for each q -ROFN in the normalized decision matrix are calculated using Eq. (6.18), the positive ideal and negative ideal solutions are determined with the help of Eqs. (6.25) and (6.27) as shown in Table 6.16.

Step 8. Separation measures for each alternative based on the positive ideal and negative ideal solutions are calculated using the Euclidean distance given in Eqs. (6.29) and (6.30) as in Table 6.17.

Table 6.15 : Normalized decision matrix based on the second approach.

	A1	A2	A3	A4	A5	A6	A7
C1	(0.947,0.179)	(0.717,0.405)	(0.717,0.405)	(0.205,0.912)	(0.485,0.625)	(0.88,0.22)	(0.691,0.443)
C2	(0.774,0.386)	(0.708,0.435)	(0.661,0.533)	(0.37,0.784)	(0.526,0.581)	(0.566,0.588)	(0.812,0.306)
C3	(0.763,0.524)	(0.654,0.571)	(0.654,0.571)	(0.816,0.383)	(0.418,0.691)	(0.931,0.368)	(0.764,0.486)
C4	(0.927,0.258)	(0.667,0.475)	(0.679,0.441)	(0.205,0.912)	(0.635,0.47)	(0.764,0.467)	(0.745,0.36)
C5	(0.99,0.11)	(0.815,0.292)	(0.857,0.248)	(0.285,0.834)	(0.593,0.514)	(0.776,0.381)	(0.772,0.403)
C6	(0.764,0.505)	(0.655,0.549)	(0.764,0.467)	(0.817,0.363)	(0.496,0.614)	(0.931,0.368)	(0.776,0.381)
C7	(0.912,0.205)	(0.754,0.38)	(0.724,0.389)	(0.259,0.848)	(0.66,0.44)	(0.383,0.931)	(0.248,0.857)
C8	(0.776,0.364)	(0.776,0.364)	(0.664,0.498)	(0.754,0.38)	(0.643,0.484)	(0.88,0.22)	(0.735,0.378)

Table 6.16 : Positive and negative ideal solutions based on the second approach.

Criterion	q-ROF positive ideal solution	q-ROF negative ideal solution
C1	(0.947,0.179)	(0.205,0.912)
C2	(0.812,0.306)	(0.37,0.784)
C3	(0.931,0.368)	(0.418,0.691)
C4	(0.927,0.258)	(0.205,0.912)
C5	(0.99,0.11)	(0.285,0.834)
C6	(0.931,0.368)	(0.496,0.614)
C7	(0.912,0.205)	(0.383,0.931)
C8	(0.88,0.22)	(0.643,0.484)

Table 6.17 : Separation measures of the alternatives based on the second approach.

	A1	A2	A3	A4	A5	A6	A7
$d(A_i, A^*)$	0.16886	0.34544	0.33496	0.50209	0.44413	0.32309	0.36436
$d(A_i, A^-)$	0.50817	0.33643	0.34267	0.1478	0.29088	0.39784	0.30581

Step 9. The closeness coefficient (CC_i) of each alternative is determined using Eq. (6.31) as shown in Table 6.18.

Step 10. Taking into account the closeness values, ranks of the alternatives are as in Table 6.18. Results indicate that the ranking of alternatives are $A1 > A6 > A3 > A2 > A7 > A5 > A4$.

Table 6.18 : Closeness coefficient and ranks of the alternatives based on the second approach.

	A1	A2	A3	A4	A5	A6	A7
CC_i	0.75059	0.49339	0.50569	0.22742	0.39575	0.55184	0.45631
Rank	1	4	3	7	6	2	5

The results show that A1- *Mandatory quarantine and strict isolation strategy* is the most important strategy because of having a greater closeness coefficient for both the proposed approaches. Besides, it can be seen that the ranking of order for both the proposed approaches is the same in Tables 6.13 and 6.18. This can clearly show that the proposed methods are effective.

6.4.4 Sensitivity analysis

In this section, sensitivity analyses are performed to measure the robustness of the results of the proposed methods. For the proposed approaches based on q-ROF TOPSIS, the sensitivity analyses are first conducted to emphasize various scenarios in decision-makers' priorities on criterion weights that might alter the result of the proposed methods. In this analysis, we change the linguistic weights assigned by each decision-maker to a certain criterion, from CHI to CLI, respectively, while keeping the linguistic weights of other criteria constant. On the other hand, for the second proposed approach, q-ROF entropy-based TOPSIS, three different scenarios for each criterion are conducted. In this analysis, the entropy weight of each criterion is changed as 0.1, 0.5, and 0.9, respectively, while the others are proportionally kept constant. Using these new criteria weights, the closeness coefficients of alternatives are recalculated. The sum of the weights equals 1 in each case. For example, if the weight of C1 is changed to 0.1, the others are arranged as 0.129. Thus, the effects of the changes on alternative ranks are observed with both analyses. The outputs of each analysis are presented in Figures 6.4 and 6.5, respectively. In Figure 6.4, it can be seen that A1- *Mandatory quarantine and strict isolation strategy* is determined as the best alternative for strategy selection since it is superior to the six alternatives in all sensitivity analyses. This shows that our first proposed method's decisions are robust and effective. The changes in the ranks of alternatives occurred when the weight of C7 criterion changed. Besides, the closeness coefficients change for each different

criterion weight. This also proves that the first proposed approach is sensitive to the changes in the criteria' weights.

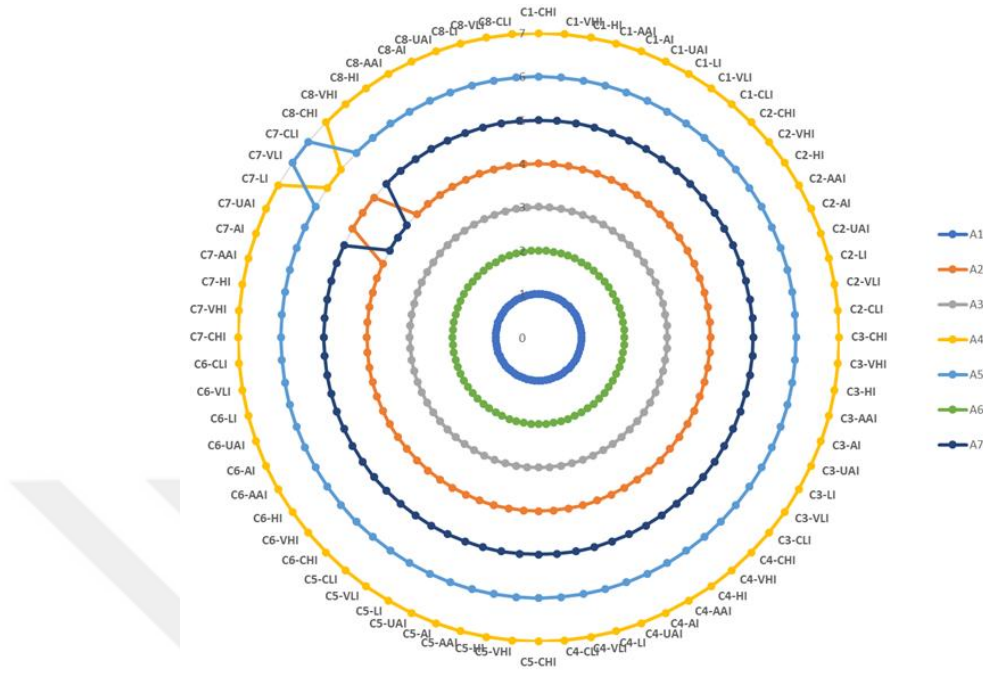


Figure 6.4 : Results of sensitivity analysis for the first proposed approach based on the criteria weights.

When the results are examined for the second proposed approach as seen in Figure 5, the changes made on *C3- Spread potential of the virus*, *C6- Time*, and *C8- Human rights* change the best alternative, unlike the others. When the entropy weight is increased to 0.5 for C3 criterion, and to 0.9 for C6 and C8, *A6- Early and common diagnostic test* is determined as the best alternative. C3, C6, and C8 are the most effective criteria for A6 since the value of C3, C6, and C8 criteria and the rank of A6 alternative increase. In all other cases, *A1- Mandatory quarantine and strict isolation strategy* is the best alternative for our second proposed approach. Besides, the other sensitivity analysis results can be summarized as follows:

When the entropy weight of C2 and C4 criteria are increased, they are the most effective criteria for A7 since the value of C2 and C4 criteria and the rank of A7 alternative increase. However, the order of A1 does not change.

When the entropy weight of C5 criterion is increased, it is the most effective criteria for A3 since the value of C5 criterion and the rank of A3 alternative increase. However, the order of A1 does not change.

When the entropy weight of C7 criterion is increased, it is the most effective criteria for A2 since the value of C7 criterion and the rank of A2 alternative increase. However, the order of A2 does not change.

These results show that our second method's decisions are robust, and results are sensitive, as well.

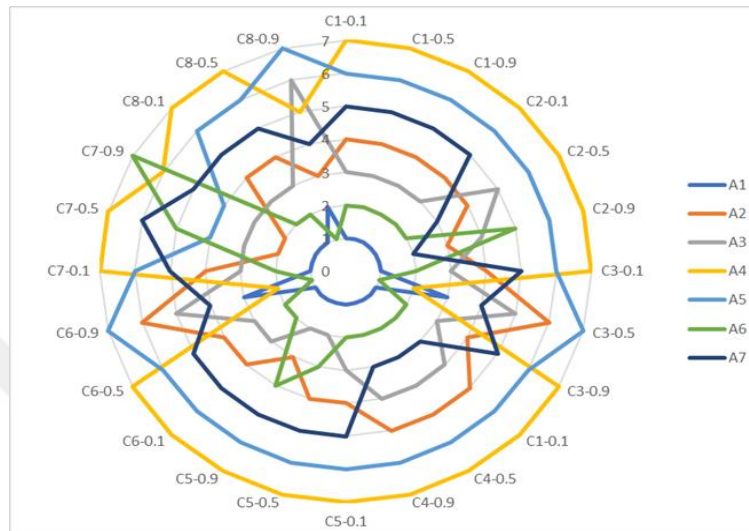


Figure 6.5 : Results of sensitivity analysis for the second proposed approach based on the criteria weights.

The proposed approaches allow decision-makers to expand their decision evaluation spaces based on the parameter q . The parameter q , which has a significant impact on the results, is very important for the proposed approaches. Therefore, to analyze the flexibility and sensitivity of the parameter q , we investigated the effect of different values of the parameter q on the decision results. For this, we recalculated the closeness coefficients for each alternative by reapplying the steps of the proposed approaches based on each change in the value of q . The ranking results of strategies for both proposed approaches are as given in Table 6.19. The changes of closeness coefficients values of strategies for the first and second proposed approaches are given in Figures 6.6 and 6.7, respectively.

In Table 6.19, Figure 6.8, and Figure 6.9 for both proposed approaches, it can be seen differences in the ranking order when the q parameter changes between 2 and 20. There have been minor changes in the ranking order for different values of parameter q . In both proposed approaches, when the q parameter is 2, i.e. PFSs, different ranking orders have been obtained. Furthermore, for the first proposed approach, when the q parameter is between 3 and 7, the ranking order is the same, while more different

ranking orders have been obtained for the q parameter values greater than 7. For the second proposed approach, different ranking orders have been also obtained at the relatively larger values of the q parameter. However, no matter how the parameter q changes, the alternative *A1- Mandatory quarantine and strict isolation strategy* has the biggest value of closeness coefficient in all cases for both approaches which means the alternative A1 is always the best choice. Besides, when the value of the parameter q is relatively small (from 2 to 5), the closeness coefficients are relatively larger and with the increase of q value (greater than 5), it has been observed that the closeness coefficients were generally smaller (see Figures 6.6 and 6.7). These results show that the q parameter has an influence on the decision results. Therefore, decision-makers should choose a suitable value of the q parameter according to their preferences

Table 6.19 : Ranking results for proposed approaches.

q	Ranking for the first proposed approach	Ranking for the second proposed approach
$q=2$	$A1 > A6 > A3 > A7 > A2 > A5 > A4$	$A1 > A3 > A2 > A6 > A7 > A5 > A4$
$q=3$	$A1 > A6 > A3 > A2 > A7 > A5 > A4$	$A1 > A6 > A3 > A2 > A7 > A5 > A4$
$q=4$	$A1 > A6 > A3 > A2 > A7 > A5 > A4$	$A1 > A6 > A3 > A2 > A7 > A5 > A4$
$q=5$	$A1 > A6 > A3 > A2 > A7 > A5 > A4$	$A1 > A6 > A3 > A2 > A7 > A5 > A4$
$q=6$	$A1 > A6 > A3 > A2 > A7 > A5 > A4$	$A1 > A6 > A3 > A2 > A7 > A5 > A4$
$q=7$	$A1 > A6 > A3 > A2 > A7 > A5 > A4$	$A1 > A6 > A3 > A2 > A7 > A5 > A4$
$q=8$	$A1 > A6 > A3 > A2 > A5 > A7 > A4$	$A1 > A6 > A3 > A2 > A7 > A5 > A4$
$q=8$	$A1 > A6 > A3 > A2 > A5 > A7 > A4$	$A1 > A6 > A3 > A2 > A7 > A5 > A4$
$q=9$	$A1 > A6 > A3 > A2 > A5 > A7 > A4$	$A1 > A6 > A3 > A2 > A7 > A5 > A4$
$q=10$	$A1 > A6 > A3 > A2 > A5 > A7 > A4$	$A1 > A6 > A3 > A2 > A7 > A5 > A4$
$q=15$	$A1 > A6 > A3 > A2 > A5 > A4 > A7$	$A1 > A6 > A3 > A2 > A5 > A7 > A4$
$q=20$	$A1 > A6 > A3 > A2 > A5 > A4 > A7$	$A1 > A6 > A3 > A2 > A5 > A7 > A4$

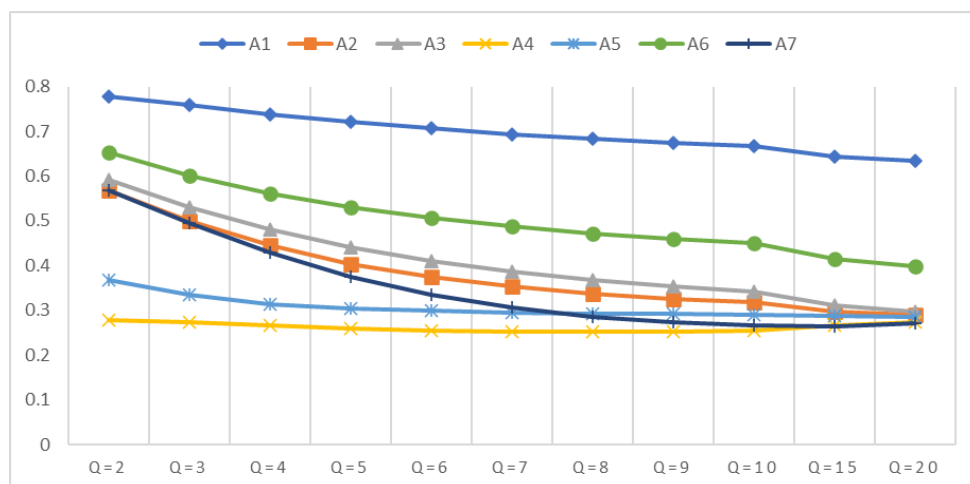


Figure 6.6 : Closeness coefficient values of alternatives based on the first proposed approach in different values of q .

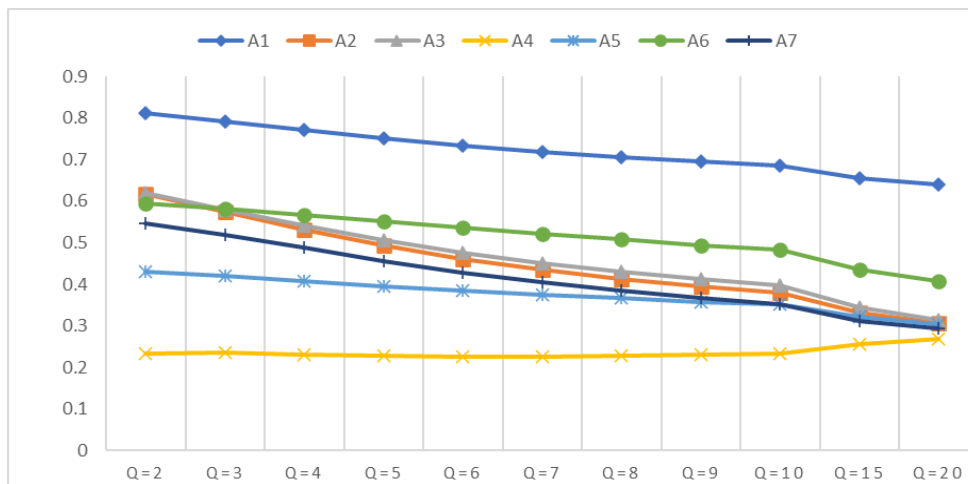


Figure 6.7 : Closeness coefficient values of alternatives based on the second proposed approach in different values of q .

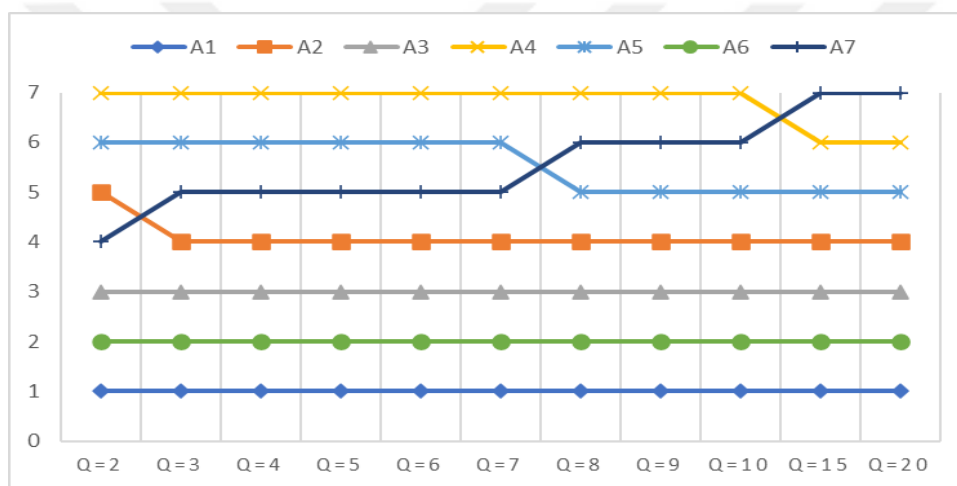


Figure 6.8 : Sensitivity analysis results of q parameter for the first proposed approach.

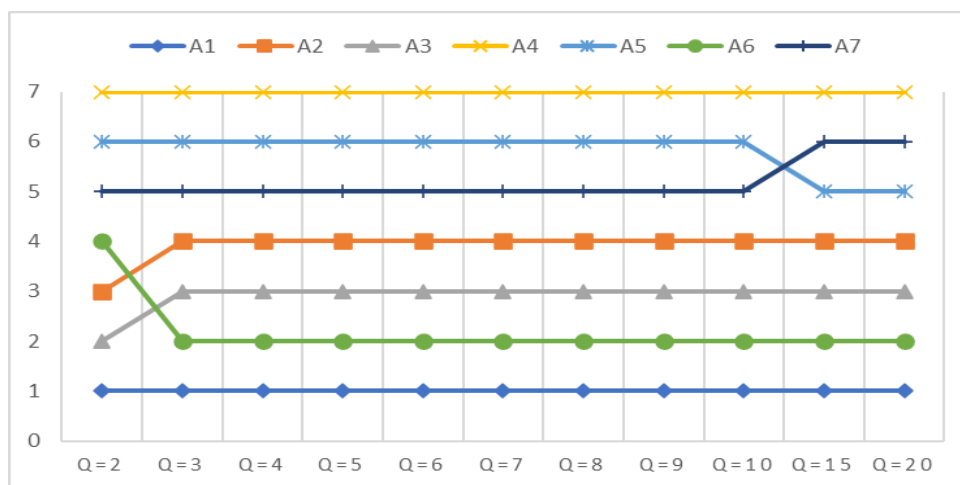


Figure 6.9 : Sensitivity analysis results of q parameter for the second proposed approach.

In the study, the evaluations obtained by experts in the current situation may vary according to the course of the pandemic, and this may cause a change in the rank of strategies that can be implemented. Therefore, to show the applicability of the developed methods, the evaluations considered need to be analyzed dynamically. In line with this scope, a scenario-based evaluation analysis is applied in the study. Since small changes in course of the pandemic did not cause a change in the results, experts were asked to make their evaluations by taking into account the worst and best cases in which the pandemic's course could occur. After obtaining the best and worst-case evaluations from the experts, the proposed methods have been rerun and the results obtained for both methods have been presented in Figures 6.10 and 6.11.

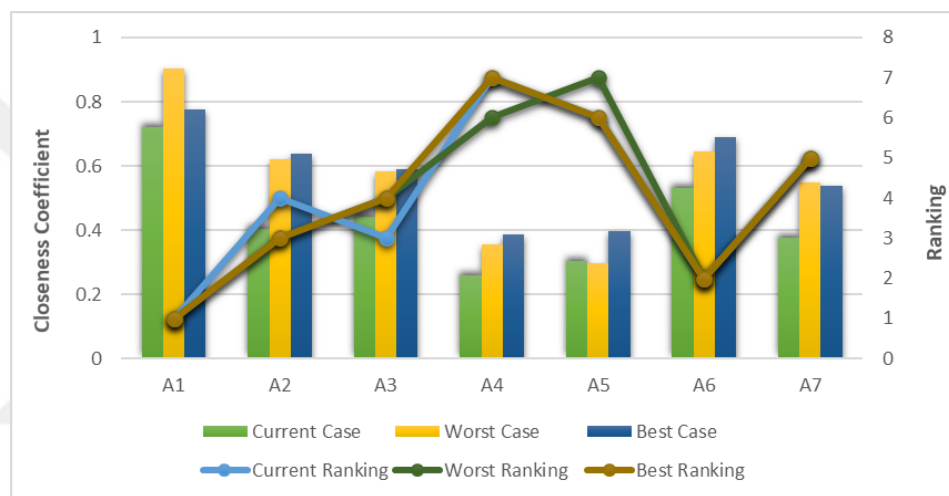


Figure 6.10 : Results of scenario analysis for the first proposed method.

According to the results obtained from Figure 6.10, when the effect of the change in expert evaluations in the worst case of the pandemic is analyzed, it is seen that the closeness coefficient scores vary without causing a big change in the ranking results. It has been observed that the alternatives *A1- Mandatory quarantine and strict isolation strategy* and *A6- Early and common diagnostic test* that should be considered as the first and second in the current situation are in the first and second rank in the worst case of the pandemic. According to the current situation, it is observed that the alternatives *A3- State of emergency* and *A2- Movement restriction strategy*, which ranked third and fourth, replaced each other, while the fifth-ranked alternative *A7- Freedom but strict training for social life* doesn't change. Besides, the alternatives *A5- Elimination strategy* and *A4- Herd immunity*, which rank sixth and seventh according to the current situation, are also observed to be replaced with each other. When considering the expert evaluations for the best case in which the course of pandemic

could happen, it is observed that the closeness coefficient scores vary without causing a big change in the ranking results. According to the results obtained, it is seen that the ranking of the best alternative *A1- Mandatory quarantine and strict isolation strategy* does not change again. When the rest of the ranking is examined, it is seen that only the alternatives *A3- State of emergency* and *A2- Movement restriction strategy* in the third and fourth ranks replace each other.

According to the scenario-based analysis results performed on the second proposed method as shown in Figure 6.11, it is observed that the changes in the expert evaluations for the worst case of the pandemic vary the closeness coefficient scores without causing a big change in the ranking results and it gives the same ranking results as the first method proposes. When considering the expert evaluations for the best case in which the course of pandemic could happen, it is seen that the closeness coefficient scores vary without causing a big change in the ranking results. It is observed that the *A1- Mandatory quarantine and strict isolation strategy* alternative, which should be considered first in the current situation, is also in the first place for the best situation in which the course of the pandemic could happen. According to the current situation, it is seen that the alternative *A6- Early and common diagnostic test* which is in the second place is ranked third, the alternative *A3- State of emergency* which is in the third place is ranked fourth, and the alternative *A2- Movement restriction strategy* which is in the fourth place is ranked second. It is observed that the rankings of the alternatives *A7- Freedom but strict training for social life*, *A5- Elimination strategy*, and *A4- Herd immunity* which are ranked fifth, sixth and seventh, respectively, does not change.

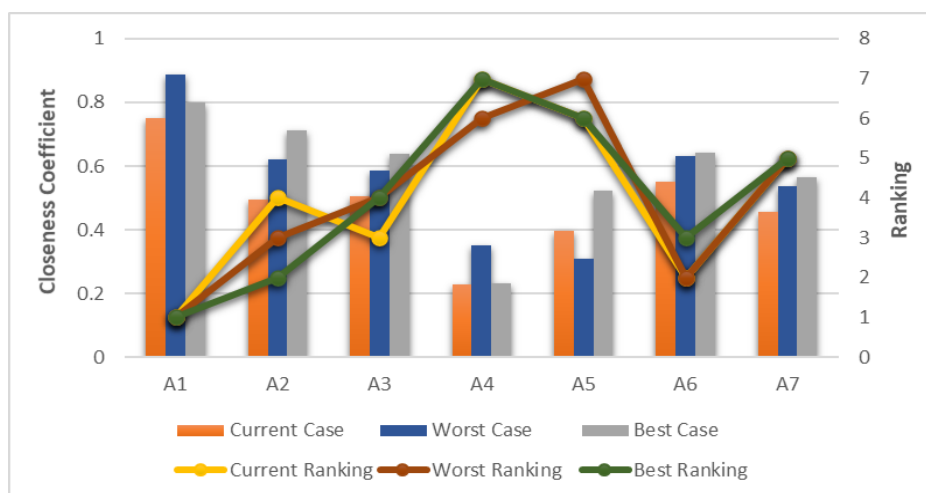


Figure 6.11 : Results of scenario analysis for the second proposed method.

As a conclusion, the scenario analysis applied on both methods change the final weights of the alternatives, but this change does not cause a major change in the final ranking order. It is seen that the strategy A1, which should be dealt with first in the methods proposed for all cases, does not change and still ranks first.

6.4.5 Comparative analysis

Various MCDM methods and approaches in fuzzy environment have been discussed by many researchers in the literature. Researchers tend to select the right MCDM method by considering many factors such as the required effort, desired accuracy, calculation time, users' skills and knowledge, and assumptions in their studies. In this study, TOPSIS method, which is a distance-based MCDM technique, is taken into consideration because of its simple and understandable content, strong computational ability and simple mathematical process with low calculation times, without needing high user skills and knowledge for the relationship between decision alternatives, as well as the ranking of the alternatives considering their relative proximity to the best solution.

In order to compare our proposed methods with other MCDM methods which are not based on distance measurement, we selected three different q-ROF-based MCDM methods to validate our results. The comparative analysis is conducted with q-ROF-PROMETHEE II (Z. Liu et al., 2020) with $q=5$, which is based on outranking relations and q-ROF-EDAS (Z. Li et al., 2020) with $q=5$, which is based on average solutions and q-ROF-TODIM (Krishankumar et al., 2021) with $q=5$, which is based on the prospect theory. The ranking results of the comparative analyses conducted by using q-ROF-based MCDM methods are shown in Figure 6.12. Based on the obtained results, it is seen that there are differences in the ranking order obtained between the proposed methods and other methods. However, it is seen that the rank of alternative A1 in all methods does not change, which is the first strategy to be considered.

In addition to comparisons with q-ROF MCDM methods, we compare our proposed q-ROF TOPSIS method with the intuitionistic fuzzy (IF) TOPSIS method proposed by Boran et al. (2009). The scales used for q-ROF TOPSIS are converted to intuitionistic fuzzy scale developed by Boran et al. (2009) for the comparison purpose. In the proposed scale, we modified the linguistic terms as given in Tables 6.20 and 6.21 and we used the same scale both in the ratings of alternatives and in the weighting of

criteria for a reliable comparison. We also used the same weights of decision-makers given in our proposed approach.

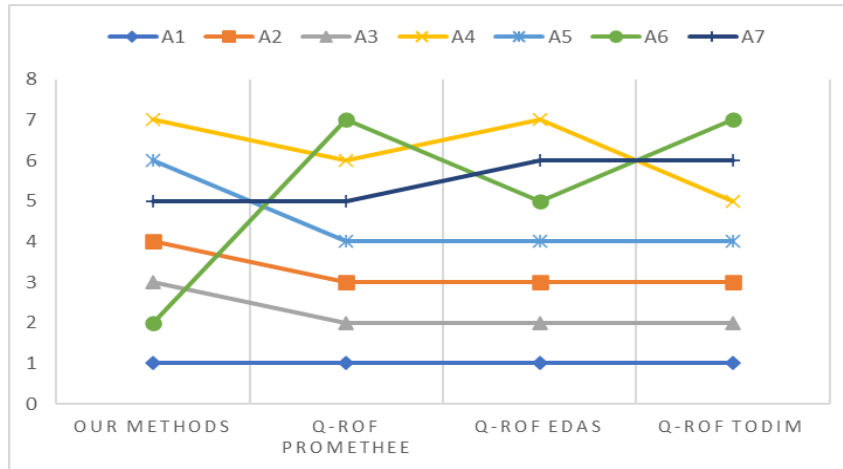


Figure 6.12 : Comparative analysis on q-ROF-based MCDM methods.

Table 6.20 : Linguistic scale for ratings of alternatives.

Linguistic terms	IF number for alternatives
	(μ, ϑ, π)
Very Very High – (VVH)	(0.9,0.1,0)
Very High – (VH)	(0.8,0.1,0.1)
High – (H)	(0.7,0.2,0.1)
Medium High – (MH)	(0.6,0.3,0.1)
Medium – (M)	(0.5,0.4,0.1)
Medium Low – (ML)	(0.4,0.5,0.1)
Low – (L)	(0.25,0.6,0.15)
Very Low – (VL)	(0.2,0.75,0.15)
Very Very Low – (VVL)	(0.1,0.9,0.1)

Table 6.21 : Linguistic scale for weighting of criteria.

Linguistic terms	IF number for alternatives
	(μ, ϑ, π)
Certainly High Importance - (CHI)	(0.9,0.1,0)
Very High Importance - (VHI)	(0.8,0.1,0.1)
High Importance - (HI)	(0.7,0.2,0.1)
Above Average Importance - (AAI)	(0.6,0.3,0.1)
Average Importance - (AI)	(0.5,0.4,0.1)
Under Average Importance - (UAI)	(0.4,0.5,0.1)
Low Importance - (LI)	(0.25,0.6,0.15)
Very Low Importance - (VLI)	(0.2,0.75,0.15)
Certainly Low Importance - (CLI)	(0.1,0.9,0.1)

The assessments of each decision-maker transformed from q-ROF values to IF values have been presented together with the aggregated criterion weights in Table 6.22.

Besides, the weighted aggregated decision matrix consisting of IF values are constructed as shown in Table 6.23.

Table 6.22 : IF values of criteria for each expert and aggregated IF criterion weights.

Criterion	DM1	DM2	DM3	Aggregated Values
C1	(0.7,0.2,0.1)	(0.8,0.1,0.1)	(0.9,0.1,0)	(0.809,0.132,0.059)
C2	(0.6,0.3,0.1)	(0.6,0.3,0.1)	(0.5,0.4,0.1)	(0.572,0.327,0.101)
C3	(0.9,0.1,0)	(0.9,0.1,0)	(0.9,0.1,0)	(0.9,0.1,0)
C4	(0.7,0.2,0.1)	(0.5,0.4,0.1)	(0.7,0.2,0.1)	(0.65,0.246,0.104)
C5	(0.8,0.1,0.1)	(0.6,0.3,0.1)	(0.8,0.1,0.1)	(0.754,0.139,0.107)
C6	(0.7,0.2,0.1)	(0.8,0.1,0.1)	(0.8,0.1,0.1)	(0.765,0.132,0.103)
C7	(0.5,0.4,0.1)	(0.5,0.4,0.1)	(0.4,0.5,0.1)	(0.472,0.428,0.1)
C8	(0.6,0.3,0.1)	(0.5,0.4,0.1)	(0.4,0.5,0.1)	(0.517,0.381,0.102)

Table 6.23 : Weighted aggregated decision matrix for IF values.

	A1	A2	A3	A4
C1	(0.081,0.82,0.1)	(0.1,0.293,0.575)	(0.575,0.132,0.293)	(0.293,0.575,0.132)
C2	(0.22,0.654,0.126)	(0.126,0.239,0.633)	(0.633,0.128,0.336)	(0.336,0.514,0.151)
C3	(0.633,0.301,0.065)	(0.065,0.555,0.327)	(0.327,0.118,0.555)	(0.555,0.327,0.118)
C4	(0.188,0.711,0.102)	(0.102,0.296,0.571)	(0.571,0.134,0.262)	(0.262,0.61,0.127)
C5	(0.075,0.914,0.011)	(0.011,0.157,0.691)	(0.691,0.152,0.111)	(0.111,0.743,0.146)
C6	(0.486,0.326,0.188)	(0.188,0.434,0.401)	(0.401,0.165,0.409)	(0.409,0.427,0.165)
C7	(0.395,0.485,0.12)	(0.12,0.331,0.537)	(0.537,0.132,0.313)	(0.313,0.562,0.125)
C8	(0.373,0.487,0.14)	(0.14,0.373,0.487)	(0.487,0.14,0.338)	(0.338,0.523,0.139)
	A5	A6	A7	
C1	(0.132,0.678,0.219)	(0.219,0.104,0.463)	(0.463,0.416,0.121)	
C2	(0.151,0.421,0.431)	(0.431,0.147,0.305)	(0.305,0.574,0.121)	
C3	(0.118,0.456,0.401)	(0.401,0.142,0.57)	(0.57,0.339,0.091)	
C4	(0.127,0.545,0.322)	(0.322,0.134,0.281)	(0.281,0.599,0.12)	
C5	(0.146,0.576,0.253)	(0.253,0.171,0.356)	(0.356,0.507,0.137)	
C6	(0.165,0.321,0.516)	(0.516,0.163,0.43)	(0.43,0.424,0.146)	
C7	(0.125,0.077,0.82)	(0.82,0.103,0.283)	(0.283,0.599,0.117)	
C8	(0.139,0.363,0.499)	(0.499,0.138,0.31)	(0.31,0.566,0.125)	

The positive ideal and negative ideal solutions consisting of IF values are determined as shown in Table 6.24. After separation measures for each alternative are calculated as given in Table 6.25, the closeness coefficient (CC_i) of each alternative is determined as shown in Table 6.26. Results of the IF-TOPSIS are as given in Table 6.26.

Table 6.24 : Positive and negative ideal solutions of the alternatives for IF-TOPSIS.

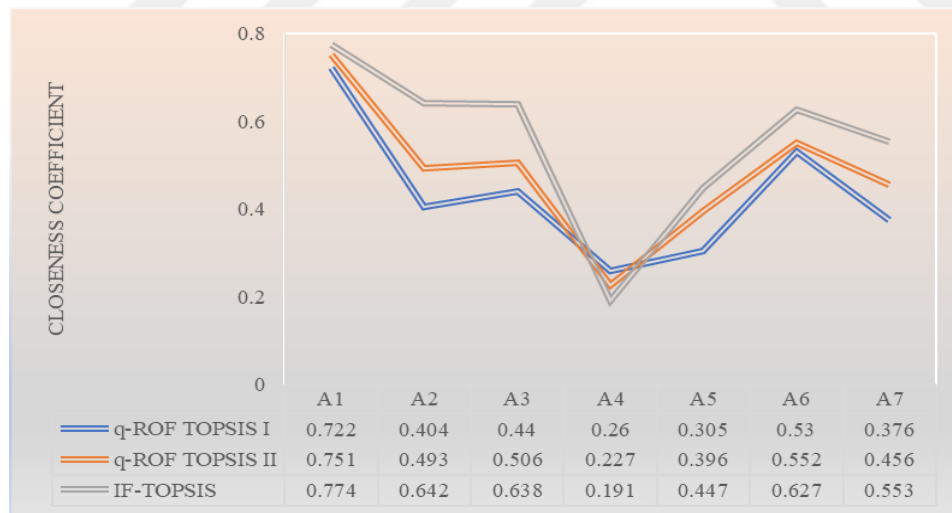
Criterion	IF positive ideal solution	IF negative ideal solution
C1	(0.081,0.82,0.1)	(0.678,0.219,0.104)
C2	(0.14,0.745,0.114)	(0.421,0.431,0.147)
C3	(0.456,0.401,0.142)	(0.633,0.301,0.065)
C4	(0.188,0.711,0.102)	(0.545,0.322,0.134)
C5	(0.075,0.914,0.011)	(0.576,0.253,0.171)
C6	(0.281,0.578,0.141)	(0.486,0.326,0.188)
C7	(0.395,0.485,0.12)	(0.07,0.829,0.101)
C8	(0.414,0.443,0.14)	(0.31,0.566,0.125)

Table 6.25 : Separation measures of the alternatives for IF-TOPSIS.

	A1	A2	A3	A4	A5	A6	A7
$d(A_i, A^+)$	0.104	0.141	0.147	0.364	0.219	0.159	0.194
$d(A_i, A^-)$	0.357	0.253	0.259	0.086	0.177	0.267	0.240

Table 6.26 : Closeness coefficient and ranks of the alternatives for IF-TOPSIS.

	A1	A2	A3	A4	A5	A6	A7
CC_i	0.774	0.642	0.638	0.191	0.447	0.627	0.553
Rank	1	2	3	7	6	4	5

**Figure 6.13** : Results of comparative analyses.

The results indicate that A1 is the best alternative in both q-ROF TOPSIS and IF-TOPSIS methodologies as shown in Figure 6.13 and the overall ranking obtained with IF-TOPSIS is A1, A2, A3, A6, A7, A5, and A4, respectively. Although these results give the same result with the best alternative determined by our proposed method, the ranking of the A2 and A6 alternatives have changed. Besides, it seems that the

differences between the weights of the alternatives are more distinct in our proposed method. This is indicative of the advantage of q-ROFS, which enables the uncertainties that arise in decision-making problems and the lack of information and inconsistencies between expert groups to be represented in a wider area.

6.5 Conclusion

COVID-19 is an epidemic that has not yet been approved, emerging in China and spreading all over the world. The severity of the epidemic increasing day by day has brought various difficulties to the governments. Although governments take various measures to prevent the spread of the epidemic in their communities, these measures are insufficient and therefore, it is necessary to ensure that the epidemic is managed most correctly with the most appropriate strategy. Almost all governments have adopted a strategy and implemented it in their communities. However, the implemented strategies have either failed to benefit many countries sufficiently or became harmful rather than beneficial. Some countries have been exposed to harmful effects socially or environmentally, while some countries have experienced harmful effects economically. Therefore, the strategies implemented by governments need to be evaluated and compared. At this point, the problem becomes an MCDM problem, where multiple alternatives should be evaluated under more than one criterion. However, due to uncertainties arising in the problem addressed and the lack of information and inconsistencies between expert groups, q-ROFSs that allow decision-makers to evaluate in a wider space should be addressed. Since q-ROFSs provide a stronger ability to contain broader information by expressing information more flexibly, it is more powerful to address problems in vague and uncertain environments. Particularly, using q-ROFSs in MCDM approaches, ambiguity and uncertainty are defined more strongly and thus the decision-making process can be managed more accurately. Based on this advantage, two different TOPSIS methods based on q-ROFSs have been proposed in this paper for the selection of the best strategy that can be implemented by governments. Alternatives and criteria have been determined in the light of the opinions of the experts and the information in the literature. According to the results of two different proposed approaches, *A1- Mandatory quarantine and strict isolation strategy* has been determined as the most important strategy that should be implemented by governments. On the other hand, the strategies that can be

implemented by the governments have been found to be A6, A3, A2, A7, A5, and A4, respectively, in order of importance in both proposed approaches.

In the study, sensitivity analysis has been conducted out on different criteria weights and q parameters. Sensitivity analysis conducted on criterion weights has shown that the results of the proposed approaches are robust and reliable. On the other hand, when the value of the q parameter changes, there have been minor differences in the ranking order for some values of q . However, the best strategy to be preferred by the governments in both approaches has not changed in all cases. In addition, in both proposed approaches, when the q parameter is 2, i.e. PFSs, different ranking orders have been obtained and this has indicated the real advantage of using q -ROFS in TOPSIS. Besides, it has been proven for both proposed approaches that decision-makers have a more optimistic attitude due to the higher closeness coefficients obtained for the relatively smaller values of the q parameter. It has been proven for both proposed approaches that decision-makers have a more pessimistic attitude due to the lower closeness coefficients obtained for relatively larger values of the q parameter. Furthermore, to show the applicability of the developed methods, the evaluations obtained by decision-makers have been analyzed dynamically. For this, a scenario-based evaluation analysis has been applied for changes that may occur in the course of the pandemic. According to the results, it has been observed that the best strategy was again *A1- Mandatory quarantine and strict isolation strategy* in cases where the pandemic's course is the best and the worst.

The comparative analyses have also been conducted with other q -ROF MCDM methods and the IF-TOPSIS method. It has been seen that the TOPSIS method compared with other MCDM techniques in fuzzy environment is a simple mathematical process with low computation time and strong computational capability. On the other hand, it has been revealed that our proposed approaches exhibit results involving more detailed and more comprehensive information in uncertain decision-making environments in the second comparison analysis.

For further study, triangular q -ROFSs, trapezoidal q -ROFSs, or interval-valued q -ROFSs instead of singleton q -ROFSs can be used in the proposed q -ROF TOPSIS methods. The proposed approaches can be extended to other types of ordinary fuzzy sets, such as type-2 fuzzy sets, hesitant fuzzy sets, and neutrosophic sets. Different

MCDM methods based on q-ROFSs can be developed to compare with our proposed approaches.





7. CONCLUSIONS AND RECOMMENDATIONS

Decision-making under uncertainty is a fundamental challenge across various disciplines, where available information is often incomplete, imprecise, or vague. Traditional models are frequently inadequate in capturing the complexity of such situations. In response, fuzzy set theory has emerged as a powerful tool for representing and managing uncertainty, enabling more flexible and realistic decision-making processes. Over time, numerous extensions of fuzzy sets—such as intuitionistic, Pythagorean, and q-rung orthopair fuzzy sets—have been developed to better handle the multifaceted nature of uncertainty. These theoretical advancements have been widely integrated into MCDM frameworks, offering more robust support for complex, real-world decisions. Despite this progress, current fuzzy frameworks and MCDM methods still face limitations in terms of continuity, decomposability, explainability, and real-world adaptability. Many fuzzy extensions remain discrete or rigid in structure, while several decision-making tools fail to account for dynamic, evolving, or interdependent uncertainty sources. Addressing these gaps requires not only methodological improvements but also a more in-depth re-investigation of how uncertainty is formulated mathematically and embedded in decision-making frameworks.

This thesis set out to explore and extend the theoretical and methodological frameworks of decision-making under uncertainty. In particular, the thesis focuses on developing new fuzzy set extensions and adapting or proposing MCDM methods that better match the complexities of real-world decision environments. Through five original research articles, the thesis contributes to the literature by presenting a coherent and forward-looking framework for uncertainty modelling, each addressing a specific theoretical gap or methodological limitation.

The first paper presented in Section 2 introduces the CINFUSs, which expands the classical intuitionistic fuzzy environment by allowing the membership and non-membership degrees to vary along continuous domains. This extension overcomes the rigidity of traditional discrete representations and offers a more flexible structure for capturing detailed uncertainties, especially in decision scenarios where input values

are inherently fluid or imprecise. CINFUSs lay a new mathematical foundation for continuous modeling within intuitionistic logic, bridging the gap between discrete fuzzy logic and continuous decision processes. In this study, the CINFUS-based AHP&TOPSIS methodology is developed in order to better handle uncertainty. Then, the developed methodology is applied to determine the most suitable alternative research projects for grant funding. In this context, the developed CINFU-AHP method is used in order to obtain more accurate and reliable criterion weights within the scope of the study. The ranking and selection process of the alternatives, which is a critical and important issue for decision makers, is determined by using the developed CINFUS-based TOPSIS method. Finally, the results are analyzed by applying both sensitivity and comparison analyses for verification and validation. The sensitivity analysis is performed on the weights of the criteria and the weights of the decision makers, and the results of the developed methodology have been proven to be valid and reliable with slight differences according to different τ values. In addition, comparative analyzes have been made with F-TOPSIS, IF-TOPSIS, IF-CODAS, IF-VIKOR and IF-EDAS and the results have been compared.

Building on the motivation for decomposability and interpretability, the second paper presented in Section 3 introduces the decomposed Pythagorean fuzzy sets. This developed fuzzy sets separate the complex membership structure of Pythagorean fuzzy sets into optimistic and pessimistic components. By enabling the separate examination of influence sources behind membership and non-membership, DPFSs enhance both the explainability and precision of fuzzy decision models. Based on this advantage, DPF-CODAS method is developed in the study. The developed method is applied to the strategy selection problem for an IoT-based sustainable supply chain system. By using the developed DPF-CODAS method, the ranking and selection of strategies for the IoT-based sustainable supply chain system, which is a significant issue for decision-makers and policy-makers, has been realized. Then, the results are analyzed by applying the sensitivity and comparison analyses for verification and validation of the developed method. The sensitivity analyses are carried out on the criteria weights and DMs weights. With this, in the comparison analysis using decomposed fuzzy-TOPSIS, Pythagorean fuzzy-CODAS and Pythagorean fuzzy-TOPSIS methods, the ranking results obtained from the methods are compared and the differences obtained in the rankings are examined.

The third paper extends the idea of continuity into the Pythagorean fuzzy domain by formulating the continuous Pythagorean fuzzy sets. The developed fuzzy sets provide a continuous formulation that allows decision makers to capture even small changes in their uncertainty while preserving the Pythagorean fuzzy set logic. CPFUSs enable modelling of information by handling a more comprehensive, accurate, and precise explanation of the uncertainty in decision-makers' opinions. Therefore, CPFUSs significantly increase the modeling capacity of complex uncertainty models encountered in decision-making problems. Based on this advantage, in the study, CPFUSs are developed and focused on examining the usability of this new fuzzy set extension in MADM problems. After introducing the mathematical operations of CPFUSs, their applicability to different CPFU-MADM models based on mathematical operations has been demonstrated. While the first model developed based on the CPFUWG aggregation operator is applied on the green supplier selection problem, another model developed based on CPFUDWG, CPFUEWG and CPFUHWG is presented on the waste disposal site selection problem. Then, the results are analyzed by applying the sensitivity and comparison analyses for verification and validation of both models. While the sensitivity analyses for the first CPFU-MADM model are conducted on the linguistic weights of the criteria and the expert weights in the study, the sensitivity analyses for the second CPFU-MADM model are performed on the linguistic weights of the criteria, expert weights, and ρ and ϕ parameters used in aggregation operators. Besides, the ranking results of the first model are compared, and the variations in the rankings are investigated, in the comparative analysis carried out utilizing IF-based and PF-based MADM models. The comparison analysis for the second model is carried out with models based on IF and PF-based Dombi, Einstein and Hamacher aggregation operators.

Because of the need for methodological tools compatible with these theoretical advances, the fourth paper introduces IF-DEVADA, a novel MCDM method adapted to the intuitionistic fuzzy environment. IF-DEVADA method incorporates dynamic aggregation, multiple distance measures, and time-variant decision structures, enabling a more realistic and flexible evaluation process. This method not only fills a methodological void in the dynamic decision-making literature but also demonstrates strong performance in managing evolving decision contexts characterized by ambiguity and change. The developed IF-DEVADA method includes a dynamic

decision-making process that takes into account possible future trends by adapting to rapidly changing situations and conditions. Besides, in the paper, an IFS-based CRITIC method is also developed, which will ensure the elimination of subjective attributes by taking into account the objective weights of the criteria in an uncertain environment. Then, an integrated IF-MCDM methodology is applied to waste disposal location selection with ever-changing situations and conditions such as industrialization, urbanization, and population growth. Finally, the results are analyzed by applying the sensitivity and comparison analyses for verification and validation of the developed method. The sensitivity analysis has been conducted on different criterion weights and on the λ parameter used in the multi-distance measurement unit. In addition, comparative analyses are carried out with the distance-based methods IF-TOPSIS, IF-CODAS, IF-EDAS, and IF-MABAC and the results have been compared. Finally, the fifth paper applies the q-rung orthopair fuzzy sets framework—one of the most expressive and recently developed fuzzy extensions—in constructing a robust adaptation of the TOPSIS method. The proposed q-ROF-TOPSIS approach strengthens the decision-maker's ability to manage extreme uncertainty and overlapping evaluations. By integrating the q-ROFS logic with a classical MCDM structure, this paper bridges the gap between advanced fuzzy sets and widely accepted decision analysis techniques. Based on this advantage, in the study, two different TOPSIS methods based on q-ROFSs are proposed for the selection of the best strategy that can be implemented by governments. Then, the results are analyzed by applying the sensitivity and comparison analyses for verification and validation of both models. The sensitivity analysis is conducted out on different criteria weights and q parameters. In addition, the comparative analyses are also conducted with other q-ROF MCDM methods- q-ROF PROMETHEE II, q-ROF-EDAS and q-ROF TODIM- and the IF-TOPSIS method. Furthermore, to show the applicability of the developed models in this paper, the evaluations obtained by decision-makers are analyzed dynamically with a scenario-based evaluation analysis on changes that may occur in the course of the pandemic.

Collectively, these five studies reveal several unifying themes. First, the studies demonstrate a consistent effort to enrich the expressiveness and adaptability of fuzzy models in capturing the multidimensional nature of uncertainty. Second, each study illustrates the connection of these fuzzy environments into well-structured, operational

MCDM models that are scalable and applicable in diverse domains. Third, each study emphasizes the importance of methodological coherence—ensuring that the form of uncertainty modeled is compatible with the structure of the decision-making technique employed.

Moreover, the integration of continuous and decomposed fuzzy set structures across multiple fuzzy paradigms (intuitionistic, Pythagorean, and q-rung orthopair) reveals a systematic attempt to bridge foundational theoretical gaps. This thesis does not merely propose novel constructs but interrelates them, allowing future researchers to trace a conceptual continuity from basic to advanced fuzzy models. In doing so, it contributes to the long-term unification of fuzzy set extensions under a broader theoretical structure.

Beyond their individual contributions, the studies in this thesis provide a cumulative advancement: a more comprehensive, flexible, and theoretically grounded framework for uncertainty-based decision-making. This integrated contribution not only bridges the gap between fuzzy sets and practical application but also opens new avenues for future research in both the extension of fuzzy models and the development of adaptive, domain-specific decision support systems.

The models and frameworks developed in this thesis are not limited to abstract theory, but also have direct implications for fields such as environmental policy, strategic management and technology assessment, where uncertainty and subjectivity dominate decision making. The adaptability of the proposed methods to dynamic, multi-criteria, and high-dimensional decision environments highlights their relevance and potential for real-world deployment.

The findings provide valuable insights into the theoretical and practical dimensions of the thesis, while also revealing important implications for future research. The first three studies emerge from a common conceptual motivation: traditional fuzzy set extensions, although useful, often have difficulty capturing the full spectrum of uncertainty, hesitation, and conflicting judgments encountered in real-world scenarios. The introduced CINFUSs and CPFUSs directly respond to this challenge by ensuring that the full range of values of membership and non-membership degrees are handled. Besides, Decomposed Pythagorean Fuzzy Sets (DPFS) offer a different yet complementary way to structurally separate the sources of uncertainty into their

components, thus improving interpretability without compromising model depth. Notably, the application of CINFUSs and CPFUSs in multi-criteria environments illustrates how these models can enhance decision granularity, particularly where expert judgments are detailed or contradictory. Importantly, the studies confirm that such modeling improves both the stability and responsiveness of decision outcomes, suggesting meaningful gains over existing fuzzy structures.

In the fourth article, the development of IF-DEVADA marks a conscious transition from theoretical modelling to methodological modelling. By incorporating a distance-based evaluation framework capable of handling intuitionistic fuzzy inputs across dynamic contexts, this method fills a critical gap in the MCDM literature. The validation of IF-DEVADA demonstrates its practical value, especially in settings characterized by temporal changes and the aggregation of different decision-maker opinions. Compared to distance-based MCDM models, IF-DEVADA exhibits greater adaptability and robustness, showing how methodological design can evolve in step with more sophisticated representations of uncertainty.

The final study, centered on q-rung orthopair fuzzy sets, reinforces the importance of aligning advanced fuzzy environments with established MCDM techniques. Rather than introducing a new fuzzy structure, this study focuses on the methodological adaptation of TOPSIS, thereby testing the compatibility of newer fuzzy frameworks with widely used decision models. The positive performance of the q-rung orthopair fuzzy TOPSIS model across multiple scenarios strengthens the case for its broader adoption, while also highlighting that theoretical novelty alone is insufficient without practical integration.

For further research, this thesis also offers opportunities to deepen the theoretical foundations of fuzzy decision making under uncertainty and to expand the practical relevance and usability of the developed models. Although IF-DEVADA introduces a dynamic dimension to MCDM, further exploration of real-time decision-making under evolving uncertainty is warranted. Future research may develop adaptive fuzzy decision systems capable of updating membership structures and preference weights in response to streaming data or shifting stakeholder priorities, offering potential applications in smart cities, logistics, and financial portfolio management. There is considerable potential for integrating the developed fuzzy extensions with other MCDM techniques or hybrid frameworks (e.g., AHP–VIKOR, BWM–PROMETHEE,

or Fuzzy DEMATEL). Future studies may propose novel hybrid structures that combine the strengths of different methods while being compatible with CINFUS, DPFS, CPFUS, or q-ROFS environments, allowing for more comprehensive and customized decision modeling. At a more foundational level, new forms of fuzzy set extensions could be developed that address other types of uncertainty such as hesitation, subjectivity, or ambiguity. Although this thesis has applied the proposed models in the contexts of sustainability, prioritization, and resource assessment, it also encourages their application to different application domains such as climate change policy, defense strategy, education planning, and ethical AI governance. Cross-domain testing will not only validate the generalizability of the models, but also highlight their practical limitations and needs for improvement. Additionally, future studies could explore the synergy between advanced fuzzy set extensions and machine learning algorithms. For instance, continuous fuzzy environments such as CINFUS and CPFUS could be employed to model uncertain inputs in supervised or unsupervised learning tasks, particularly in domains like medical diagnosis, risk forecasting, or recommendation systems. Embedding these fuzzy frameworks into learning pipelines may yield more robust and interpretable models, especially when data is imprecise or incomplete. To facilitate wider adoption, future research may focus on building open-source, modular software libraries that implement the proposed fuzzy extensions and decision models. A unified Python or R package with clear documentation, visualization modules, and decision templates would make the methodologies more accessible to researchers, practitioners, and policymakers.



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