A WAY TO COMPARE MATHEMATICS TEACHER CANDIDATES' MATHEMATICAL KNOWLEDGE FOR TEACHING: TEDS-M RELEASED TESTS

by

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ABSTRACT

A WAY TO COMPARE MATHEMATICS TEACHER CANDIDATES' MATHEMATICAL KNOWLEDGE FOR TEACHING: TEDS-M RELEASED TESTS

This study aimed to investigate mathematics teacher candidates' mathematical knowledge for teaching (MKT) by using Mathematics Knowledge Instrument for Pre-service Primary and Secondary Mathematics Teachers which were Turkish translated versions of TEDS-M Primary and Secondary Released Items. The sample of the study was comprised of three groups of teacher candidates from two universities in Istanbul: primary mathematics education, secondary mathematics education and mathematics departments. The data gathered from both freshman and senior students. Firstly, this study aimed to examine differences in MKT of teacher candidates at the beginning and at the end of the undergraduate education. For all three departments separately, senior students had statistically significant higher scores than freshman students. The second purpose of the study was to discover the differences between secondary mathematics education and mathematics departments in teacher candidates' MKT acquired while studying in these departments. While there was no significant difference between freshman secondary mathematics education and freshman mathematics students, senior secondary mathematics education students had statistically significant higher MKT scores than senior mathematics students. Lastly, this study was also aimed to conduct international comparisons. In primary level, for 32 of 34 items, the correct response percentages of senior students from primary mathematics education were higher than international average. In secondary level, for 28 of 31 items, senior students from secondary mathematics education or mathematics departments were above the international average. The common content area of items that both primary and secondary mathematics teacher candidates were below the international average was "data".

ÖZET

MATEMATİK ÖĞRETMEN ADAYLARININ ÖĞRETİM İÇİN MATEMATİKSEL BİLGİLERİNİ KARŞILAŞTIRMA YOLU: TEDS-M YAYIMLANMIŞ TESTLERİ

Bu çalışma, matematik öğretmen adaylarının öğretim için matematiksel bilgilerini (ÖMB) araştırmayı amaçlamıştır. Bu amaç için, TEDS-M yayımlanmış İlköğretim ve Ortaöğretim ölçeklerinin Türkçe çevrileri olan İlköğretim ve Ortaöğretim Matematik Öğretmen Adayları için Matematik Bilgisi Ölçekleri kullanılmıştır. Çalışmanın örneklemini, İstanbul'daki iki üniversitenin ilköğretim ve ortaöğretim matematik öğretmenliği ile matematik bölümlerinde okuyan birinci ve son sınıf öğretmen adayları oluşturmaktadır. İlk olarak, öğretmen adaylarının bölümlerine başladıkları ve bitirdikleri zamanki ÖMB' lerindeki farlılıkları incelenmiştir. Ayrı ayrı her üç bölüm için, son sınıf öğrencileri, birinci sınıf öğrencilerinden istatistiksel olarak anlamlı daha yüksek puanlar almışlardır. Bu çalışmanın ikinci amacı da ortaöğretim matematik öğretmenliği ve matematik bölümlerinde okuyan öğretmen adaylarının, bu bölümlerde okurken edindikleri ÖMB'lerindeki faklılıkları da keşfetmektir. Birinci sınıfta okuyan ortaöğretim matematik öğretmenliği ve matematik öğrencileri arasında anlamlı bir fark bulunmazken, son sınıfta okuyan ortaöğretim matematik öğretmenliği öğrencileri, son sınıfta okuyan matematik öğrencilerinden istatiksel olarak anlamlı daha yüksek puan almışlardır. Son olarak, bu çalışma uluslararası bir karşılaştırma yapmayı da amaçlamıştır. İlköğretim düzeyinde, 34 maddeden 32'sinde ilköğretim matematik öğretmenliği son sınıf öğrencilerinin doğru cevap yüzdeleri uluslarası ortalamadan daha yüksektir. Ortaöğretim düzeyinde de, 31 maddeden 28'inde ortaöğretim matematik öğretmenliği ya da matematik bölümü son sınıf öğrencileri, uluslarası ortalamanın üzerinde yer almaktadır. Hem ilköğretim hem de ortaöğretim matematik öğretmen adaylarının, uluslararası ortalamanın altında kaldıkları maddelerin ortak alanın da "veri" olduğu tespit edilmiştir.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ACRONYMS/ABBREVIATIONS

MATH	Mathematics Major
MCK	Mathematics Content Knowledge
MKT	Mathematical Knowledge for Teaching
MKI-P	Mathematics Knowledge Instrument for Preservice Primary Mathematics Teachers
MKI-S	Mathematics Knowledge Instrument for Preservice Secondary Mathematics Teachers
MPCK	Mathematics Pedagogical Content Knowledge
PCK	Pedagogical Content Knowledge
PMATH	Primary Mathematics Education
SMATH	Secondary Mathematics Education
SMK	Subject Matter Knowledge
TEDS-M	Teacher Education and Development Study in Mathematics
TIMSS	Trends in International Mathematics and Science Study

1. INTRODUCTION

It is indisputable that teaching is an important profession and teachers doing an important job. There is a consensus that teachers have a strategic importance among the other components (students and curriculum) of education, because the effectiveness of educational system depend on teachers' success (Safran, Kan, Üstündağ, Birbudak, & Yıldırım, 2014). Therefore, teacher training should be considered attentively and being a teacher should have some requirements.

When we look at the Turkey's history of teacher training, it is obviously seen that there is not a consistent teacher training policy has been pursued (Ozoğlu, Gür, & Çelik, 2010). After universities had been responsible for teacher training in 1982, the length of elementary school teacher education programs was determined as 2 years and for the primary and secondary school teacher education programs the length of study was determined as 4 years. From 1982 to now, Council of Higher Education –CHE (Yükseköğretim Kurulu-YÖK) in Turkey made four important reforms. Firstly, in 1989, the length of study in elementary school teacher education programs was increased to 4 years. Second reform was in 1997; secondary school teacher education programs became integrated BS and MS programs and students would study 5 years rather than 4 years. In addition to length of study, pedagogy courses changed and the programs focused on more practice. As a third reform, in 2006-2007 academic year, the content of teaching related courses changed for both primary and secondary school teacher education programs. While these changes were actualizing in teacher education programs, there were also some other approaches to train more teachers in order to overcome teacher shortage. For example, in addition to teacher education programs, there were also non-thesis master's programs and teaching certificate programs which train graduates of other faculties to become teachers. Lastly in 2010, CHE bestowed graduates of faculty of art and sciences a right to be teachers after completing teaching certificate programs. Even today, there are new developments for teacher training and teacher education programs.

At the end of the 2012-2013 academic year, CHE declared that there would not be any student quota for secondary school teacher education programs in the 2013-2014 academic year (Anadolu Ajansı, 2013). The teachers need in secondary schools would be met by graduates of faculty of art and sciences. However, in the past few days (spring term of the 2013-2014 academic year) CHE announced a new development in its website: secondary school teacher education programs will admit new students in 2014-2015 academic year and the period of study will decrease to 4 years and additionally teaching certificate programs will also continue (Yükseköğretim Kurulu, 2014).

Consequently, with the final reform, it is seen that there is still inconsistency in how teachers will be trained and who will be teachers. Therefore, today, there is not a unique way to be a teacher in Turkey. In order to be a teacher, it is necessary to graduate from either a teacher education program or a faculty of art and science department by completing a teaching certificate program. In other words, for teaching mathematics specifically, graduates of primary mathematics teacher education programs, secondary mathematics teacher education programs, and undergraduate programs in mathematics departments have a chance to be mathematics teachers formally. It might be expected that teacher candidates are trained differently since the content and requirements of teacher education programs and undergraduate program in mathematics are different than each other.

Primary mathematics teacher education programs include 50-60% content knowledge and skills, 25-30% professional teaching knowledge and skills, and 15-20% general knowledge courses (Yükseköğretim Kurulu, 2007). In secondary mathematics teacher education programs, students are required to complete 50% content knowledge and skills, 30% professional teaching knowledge and skills, and 20% general knowledge courses (Orta Doğu Teknik Üniversitesi Eğitim Fakültesi, 2013). Since the mathematical need for primary and secondary mathematics teachers are not the same (Zazkis & Leikin, 2010), the content of the programs differ from each other. However, it is obvious that the focus of both teacher education programs is to train mathematics teachers. On the other hand, mathematics departments' undergraduate education programs include 70% content knowledge and 30% general knowledge courses. Mathematics students take many advanced mathematics courses but they are not required to take any courses related with teaching. Before being mathematics teachers, they would make up these deficiencies with the teaching certificate programs.

In Turkey, recent changes, especially since 2010, show that teacher training policies are based on employment policies rather than training high quality teachers (Özoğlu *et al.*, 2010). Although there was a teacher shortage problem before, today there is a teacher surplus problem. Today, being a teacher in Turkey requires overcoming some difficulties because of the supply-demand disequilibrium. Teacher candidates are in a big competition to be assigned as teachers to public schools. Graduation from teacher education programs or completing teaching certificate programs is not enough; they must take the high-stake national Public Personnel Selection Examination-PPSE (Kamu Personeli Seçme Sınavı-KPSS) to be teacher in public schools.

In June 2013, minister of education declared that the number of teacher candidates who expected to be assigned was 296.500 at that time (El, 2013). Each year besides approximately 33.700 number of students graduate from faculties of education, there are approximately 39.300 number of graduates from other faculties who took the certificate to teach (MEB, 2011). Each year the number of candidates who expected to be assigned increases gradually since only a few of them can be assigned in public schools. According to data from the Ministry of National Education-MoNE (Milli Eğitim Bakanlığı-MEB), in February 2014 total number of teacher candidate who applied to MoNE and took the exam is 61.033 but the total number of assigned ones is 9.375. It means just 15 % of teacher candidates took the opportunity of being teachers (MEB, 2014). Among those 3.214 of them applied for being secondary mathematics teachers and only 7 % of them were assigned. For primary level, the percentage is more optimistic, 749 teacher candidates were assigned among 1.528 applicants. Indeed, assigned teachers had to score minimum 87 points over 100 from PPSE. So, it may be said that the purpose of the exam is to select teacher candidates according to PPSE scores.

On the other hand, international and national studies showed that measuring teacher quality is a complex phenomenon (Wilson, 2007). As an indication of teacher quality, teacher knowledge is mostly tried to be measured. It is stated that the subject matter knowledge and pedagogical content knowledge are essential components of teacher knowledge (Ball & McDiarmid, 1990).

Particularly in mathematics education, there are several studies and research projects that are aimed to develop instruments to measure teachers' mathematics content knowledge for teaching (Ball, Thames, & Phelps, 2008; Krauss, Baumert, & Blum, 2008; Tatto *et al.*, 2008). Among them The Teacher Education and Development Study in Mathematics (TEDS-M) stands out as being designed for an international and comparative study for both primary and secondary preservice mathematics teachers by Tatto and her colleagues. The primary and secondary instruments that developed for TEDS-M cover the content and cognitive domains of primary and secondary level mathematics. The items were designed to measure mathematics content knowledge and mathematics pedagogical content knowledge. Tatto and her colleagues did not develop the instruments country based but they designed for them international usage and national adaptations.

The main purpose of the current study is to investigate the knowledge of mathematics content and pedagogical content knowledge of teacher candidates by comparing them both according to their years of study in university and the departments that they are studying in. For this purpose, TEDS-M measures were most appropriate measures because these measures cover the content and cognitive domains of primary and secondary level mathematics separately. However, Turkey was not one of the participant countries of TEDS-M, so the instrument was neither adapted in Turkish nor applied in Turkey. As a result of this study, these measures will also be translated in Turkish and adapted to be use in Turkish.

2. REVIEW OF THE LITERATURE

More than 30 years of teacher education, particularly mathematics teachers' preparation has been an issue among the research field of mathematics education. Recently, under this area the issue of mathematics teachers' knowledge, its development, measuring, and its effects on teaching and learning settings have been studied. Researchers have defined and described teacher knowledge by developing models and investigating teachers' mathematical knowledge. Within the current study the review of the literature related to mathematics teachers' knowledge will be presented by focusing on the teacher knowledge approaches, related studies and instruments that were designed to measure mathematics teachers' knowledge.

2.1. Teachers' Knowledge

Researchers who have studied teachers' knowledge tried to answer such questions: "What do teachers need to know?", "What are the components of their knowledge?", "What they should learn, know and understand?", "How should they learn?" It is commonsense that teacher knowledge is not monolithic (Franke & Fennema, 1992) but it is multidimensional (Even & Tirosh, 2002).

Up to the present, teacher knowledge and its components has been described and modeled in different ways by different researchers who either developed their own model (Shulman, 1986; Ball *et al.*, 2008) or revised the existing models and interpreted them into their setting (Franke & Fennema, 1992; Tatto *et al.*, 2008). Today it is possible to say that many teacher knowledge approaches have been influenced by the Shulman's (1986) model of teachers' knowledge. Shulman made an important contribution to the research area by describing teachers' content knowledge and categorizing its components. As Petrou and Goulding (2011) stated, in Shulman model, the most influential categories was the new concept of Pedagogical Content Knowledge (PCK). Shulman (1986) described PCK as: "that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding" (p. 9). After him, other researchers have attempted to develop their own models by building on his description of teacher knowledge. So, Shulman's teacher knowledge model should be considered in detail.

In their research project called Knowledge Growth in Teaching, Shulman and his colleagues investigated the components of teachers' content knowledge through the following question: "what are the domains and categories of content knowledge in the minds of teachers?" According to him, there are three categories of content knowledge: subject matter knowledge (SMK), pedagogical content knowledge (PCK), and curricular knowledge (CK) (Shulman, 1986).

Subject Matter Knowledge was defined as "the amount and organization of knowledge per se in the mind of the teacher" (Shulman, 1986, p. 9). This is not only knowing facts, procedures or concepts of a field but SMK also requires deep understanding of them and knowing the underlying reasons of facts, procedures and concepts. Shulman explains this feature of SMK as saying that "the teacher need not only understand that something so, the teacher must further understand why it is so" (p. 9).

On the other hand, Pedagogical Content Knowledge includes knowledge of "the ways of subject that make it comprehensible to others" (p.9). This means that the teacher need to know using representations, illustrations, analogies and demonstrations and also giving examples and explaining concepts in order to make them understandable. For that purpose, teachers should also be aware of preconceptions and misconceptions of students.

The last category of teacher knowledge that Shulman developed is Curricular Knowledge. It consists of the knowledge of instructional material, and the lateral and vertical aspect of curriculum. He stated that "I would expect a professional teacher to be familiar with the curriculum materials under study by his or her students in other subjects they are studying at the same time." (p. 10).

Shulman's conceptualization of teachers' knowledge leaded up research about teacher knowledge by the categorization of SMK, PCK and CK. His categorization was beyond the specific content area and it was like an umbrella categorization for any field of education. On the other hand, it was criticized to consider teacher knowledge as static knowledge (Petrou & Goulding, 2011) although it is changing and developing constantly (Franke & Fennema, 1992). Either in order to determine different requirements of teachers' knowledge for the specific content area or emphasize its dynamic nature, different models and conceptualizations were developed and discussed for mathematics teachers' knowledge (Ernest, 1989; Franke & Fennema, 1992; Usiskin, 2001; Ball, 2003; Ball *et al.*, 2008; Tatto *et al.*, 2008; Krauss, Brunner, Kunter, & Baumert, 2008; Zazkis & Leikin, 2010; Petrou & Goulding, 2011).

2.2. Mathematics Teachers' Knowledge

Shulman's conceptualization of teachers' knowledge provided a basis for research in the field of mathematics education. Researchers have been studied mathematics teachers' knowledge in different perspectives. They conceptualized mathematics teachers' knowledge by describing its components (Franke & Fennema, 1992; Ball, 2003; Döhrmann, Kaiser, & Blömeke, 2012) and several researches were conducted to investigate this knowledge. For the investigation researchers tried to develop a rigorous instrument which measure mathematics teachers' knowledge (Hill, Rowan, & Ball, 2005; Krauss, Brunner, *et al.*, 2008; Tatto *et al.*, 2008). The literature related with these issues will be summarized in following subsections.

2.2.1. Conceptualizing Mathematics Teachers' Knowledge

2.2.1.1. Mathematical Knowledge for Teaching. When Shulman's model of teacher knowledge is transferred to mathematics education, it can be seen that there exists a promising and widely used model of mathematical knowledge for teaching. This is the Mathematical Knowledge for Teaching (MKT) model which has been developed by Ball and her colleagues (2008) within the Learning Mathematics for Teaching (LMT) project. In 2000, the project started to investigate mathematical knowledge needed for teaching, and examine development of such knowledge. Furthermore, researchers developed items to measure knowledge required for teaching (Hill, Schilling, & Ball, 2004). In other words the purpose of the project was twofold: developing a model and developing an instrument for teachers' mathematical knowledge for teaching.

The MKT model of teacher content knowledge is based on Shulman's categorization and its developers expressed that MKT model is a refinement to Shulman's categories of SMK and PCK. Ball and her colleagues subdivided these two into six categories: common content knowledge (CCK), specialized content knowledge (SCK) and horizon content knowledge (HCK) that are under the SMK; knowledge of content and students (KCS), knowledge of content and teaching (KCT) and knowledge of curriculum as sub-domains of PCK. Figure 2.2 shows the MKT model which is the refinement of Shulman's categorization.

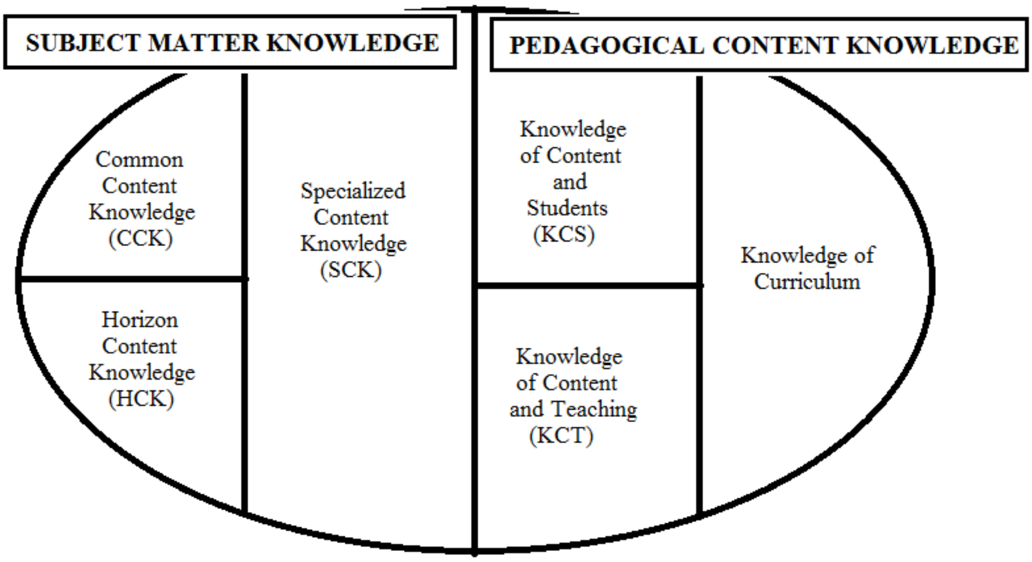


Figure 2.1. Mathematical knowledge for teaching.
(Ball *et al.*, 2008, p. 403)

CCK includes knowing and being able to do the mathematics that any well-educated person is expected to know. For a teacher, it requires being aware of the mathematical correctness and appropriateness of students' answer and definitions in textbooks. It is defined as "the knowledge teachers need in order to be able to do the work that they are assigning their students" (Ball *et al.*, 2008, p. 398). CCK consists of knowing what and knowing how.

From the same point of view, it can be said that SCK consist of knowing how and knowing why. A teacher has to know more than factual and procedural knowledge; why procedures work, how mathematical facts and claims are justified, and how to derive formulas (Ball, 2003). SCK is defined as the knowledge unique for teachers not the other professions. For example an engineer or a physicist use mathematical facts, procedures, rules and formulas but they do not need to know more. In addition, as a part of SCK "knowing mathematics sufficiently for teaching requires being able to unpack ideas and make them accessible as they are first encountered by learner" (Ball, 2003, p. 4) and in fact this characteristic differentiates mathematics teachers from the mathematicians. Furthermore, SCK does not require knowledge of students or teaching. Also, it should be noted that teachers may never directly teach their SCK to students.

Lastly, under SMK there is a component of horizon content knowledge (HCK). Ball and her colleagues (2008) described it as "an awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (p. 403). It requires seeing whole picture of a puzzle by knowing where each piece should be placed. Therefore HCK can be considered as knowing what and knowing what is beyond.

As Shulman (1986) stated, SMK is necessary but not sufficient knowledge for teaching, so Ball also examined PCK through for teaching mathematics. Shulman (1986) stressed PCK is "amalgam of content and pedagogy" and Ball (2003) defined the subcategories of PCK by reconsidering Shulman's idea. The first subcategory is knowledge of content and students (KCS) that requires knowing about how students learn mathematics. Teachers need to be aware of students' preconceptions and misconceptions as well as they need to be able to use interesting and motivating examples for students. These types of tasks require interaction between specific mathematical understanding and acquaintance with students and their mathematical thinking (Ball *et al.*, 2008).

The second one is knowledge of content and teaching (KCT) that includes both knowing about mathematics and knowing about teaching it. KCT requires knowledge of instructional designs and teaching strategies that interact with mathematical knowledge. Teachers need to decide about examples and models they will use in order to represent a topic or to develop the mathematical concepts (Ball *et al.*, 2008). The last category, the knowledge of curriculum, is similar to curricular knowledge in Shulman's model. Knowledge of curriculum includes both knowledge of mathematics and knowledge of curriculum and interactions with each other.

Although MKT model has been used widely in the field of mathematics education, there are some criticisms about it. Because it was developed by considering elementary and middle school mathematics teachers but not secondary, generally the criticisms are about the secondary mathematics teachers' mathematical needs that are deficient in the MKT model. Mathematics that a secondary teachers need to know is at the much higher level than elementary teachers since "the higher the level taught, the more the teacher needs to know" (Usiskin, 2001, p. 86). Therefore a secondary mathematics teacher needs to know great deal of mathematics (Usiskin, 2001) and needs to be competent in mathematics more than one level above that at which they teach (Peterson, 1998).

On the other hand MKT model is criticized because of the borders between its subcategories are becoming vague. Petrou and Goulding (2011) asserted that the distinction between the SCK and PCK in MKT model is not clear. Their claim is based on the definition of the concept of SCK that Ball and her colleagues (2008) stated. Petrou and Goulding (2011, p. 17) presented their argumentation as follows:

> "This concept is central in the conceptualization of teachers' mathematical knowledge and is defined as the mathematical knowledge that is used in classroom settings and needed by teachers in order to teach effectively. [...] After all, PCK is also uniquely needed by teachers and is used in classroom settings."

Due to such criticisms, there were studies which investigated adaptation of MKT model in secondary setting or developed new models. Furthermore, Krauss and his colleagues (2008) criticized the MKT model since its components could not be clearly described for secondary mathematics teachers. They constructed and implemented an instrument that is Cognitively Activating Instruction (COACTIV) instrument to measure pedagogical content knowledge and content knowledge of secondary mathematics teachers. Based on their findings, they claimed that the distinction between CCK and SCK may be less useful for the instruments addressing secondary level because others who are not teachers may not be able to solve the items of these instruments (Krauss, Baumert, & Blum, 2008). However, in elementary level instruments, items that measure CCK may be solved by others who are not teachers but SCK is unique knowledge for teacher and these characteristics differentiate SCK from CCK. For secondary level, this distinction is unclear since both CCK and SCK require deep collage level mathematics knowledge for teaching secondary school mathematics (Zazkis & Leikin, 2010).

Therefore, it is claimed that components of MKT do not meet the mathematical need for secondary mathematics teachers and the description of SCK should be refined for secondary mathematics teachers (Zazkis & Leikin, 2010). On the purpose of achieving SCK for teaching mathematics at the secondary level Zazkis and Leikin (2010) define Advanced Mathematical Knowledge (AMK) as knowledge of subject matter acquired during undergraduate studies at colleges or universities. They claim that AMK is necessary condition for identification of secondary teachers' mathematics knowledge but they are aware of insufficiency of it.

As a consequence, MKT model is criticized due to the vagueness of differences between its subcategories and not addressing secondary mathematics teaching. However, MKT model is widely used and internationally accepted model in the field of teaching and indeed there is not much criticism about the definition of the MKT. On the other hand, since MKT model is built on Shulman's teacher knowledge model which was criticized considering teacher knowledge as static knowledge (Petrou & Goulding, 2011), MKT model may also be criticized in the same way. Therefore, from the different perspectives (Franke & Fennema, 1992; Thompson, 1992; Richardson, 1996; Tatto *et al.*, 2008), the dynamic nature of mathematics teachers' knowledge should be considered for the conceptualization of mathematics teachers' knowledge.

2.2.1.2. Dynamic Nature of Mathematics Teachers' Knowledge. Franke and Fennema (1992) argued that teachers' knowledge is not static on the contrary it is changing and developing constantly. Based on this point of view they modeled the interactive and dynamic nature of teacher knowledge. As seen in Figure 2.2 the model consists of knowledge of content of mathematics, knowledge of pedagogy, knowledge of students' cognition and teachers' belief. In this model each component is in the context and "within a given context, teachers' knowledge of content interacts with knowledge of pedagogy and students' cognitions and combines with beliefs to create a unique set of knowledge that drives classroom behavior" (p. 162). With this model, as Franke and Fennema (1992) said, they tried to pointed out that some component of teacher knowledge evolve through teaching and how teachers use it must change as the context in which they work changes.

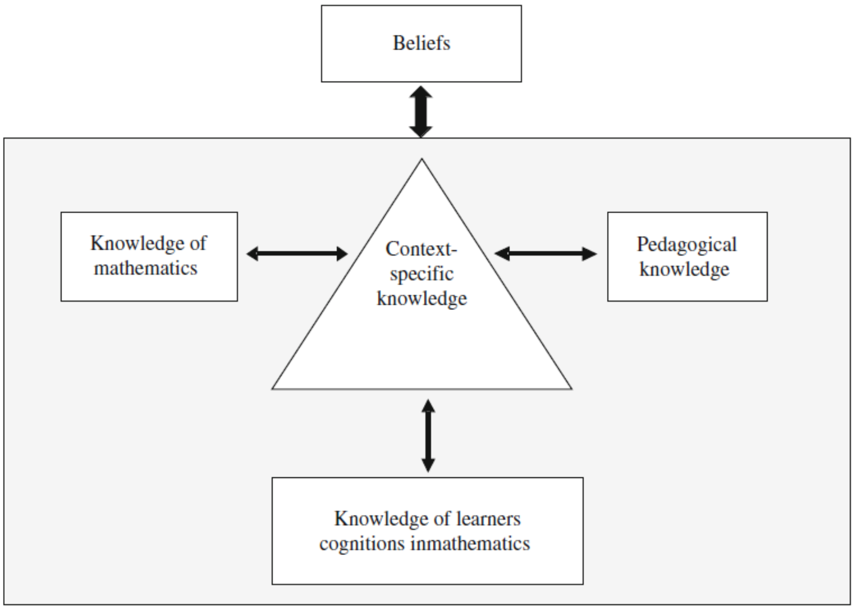


Figure 2.2. Teacher knowledge: developing in context.

(Franke & Fennema, 1992, p. 162)

The dynamic interaction between components of mathematics teachers' knowledge and belief were also discussed by Ernest (1989) and he claimed that it was the lack of Shulman's conceptualization. Moreover, he pointed out the necessity of model of the different types of knowledge, beliefs and attitudes of the mathematics teacher, and their relationship with practice (Ernest, 1989). MKT model did not meet this need and not fill the gap of dynamic nature of teacher knowledge model. Therefore, researchers developed their own model while examining teacher knowledge in this point of view (Tatto *et al.*, 2008; Döhrmann *et al.*, 2012).

Teacher Education and Development Study in Mathematics (TEDS-M), in 2008, aimed to investigate how teachers are prepared to teach mathematics both in primary and secondary level from a broader perspective by considering dynamic nature of mathematics teachers' knowledge. This study focused on the relationship between teacher education policy and practice and preservice teachers' readiness to teach mathematics (Tatto *et al.*, 2008). Therefore there were several variables to investigate and to examine its interrelationships: characteristics of future teachers, characteristics of future educators, characteristics of teacher education program, future teacher knowledge and future teachers' beliefs.

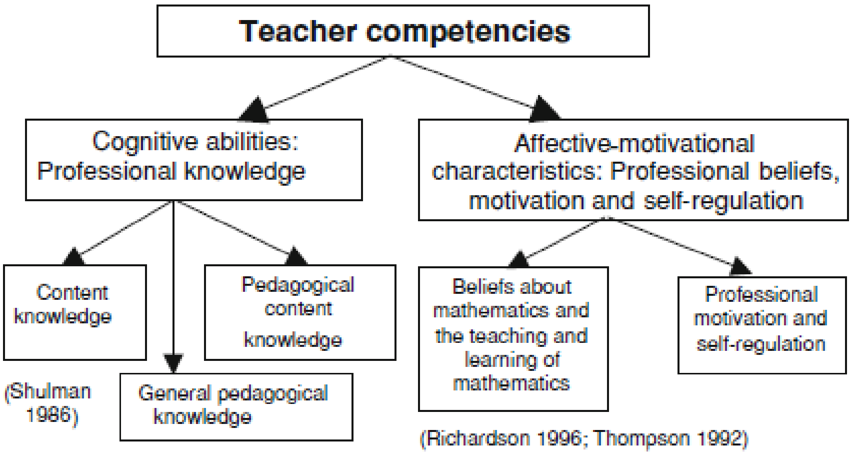


Figure 2.3. Conceptual model of teachers' professional competencies. (Döhrmann *et al.*, 2012, p. 327)

For the investigation of these variables, particularly to measure effectiveness of mathematics teacher education programs, TEDS-M developed a conceptual model of mathematics teachers' professional competencies (Döhrmann et al., 2012). In this conceptual model (Figure 2.3), teacher competencies categorized under two: (i) cognitive abilities and (ii) affective-motivational characteristics. The category of cognitive abilities, based on Shulman's categorization, includes three components of professional knowledge: content knowledge, general pedagogical knowledge and pedagogical content knowledge. The other category of affective-motivational characteristics consists of two components related with professional beliefs, motivation and self-regulation. First component is belief about mathematics, and the teaching and learning of it. Second one is professional development and self-regulation. These two components are based on the concept of teachers' belief as discussed by Richardson (1996) and Thompson (1992).

To sum up, researchers developed models in order to describe mathematics teachers' knowledge. It can be said that these models present a bigger picture of mathematics teachers' knowledge by considering its nature and identifying its components. After the conceptualization, theoretical frameworks allow researchers to study on investigation and development of teachers' mathematics knowledge for teaching.

2.2.2. Investigating and Developing Mathematics Teachers' Knowledge

Teachers' mathematics content and pedagogical content knowledge for teaching mathematics develop in undergraduate years for preservice teachers. When they become teachers, these knowledge continue to develop through teaching. However, it should be noted that foundations of teachers' knowledge are laid in teacher preparation programs. If the stronger these foundations laid, the better teachers can construct new knowledge in practice (Ball, Lubienski, & Mewborn, 2001). Therefore, how preservice teachers developed their knowledge in their undergraduate education is important because they will construct new knowledge on their situated knowledge.

During teacher education programs, preservice mathematics teachers have chances to improve both their SMK and PCK. Mathematics teachers need to have specific knowledge which is unique for teachers (Ball *et al.*, 2008). The mathematics content and mathematics pedagogical content knowledge that teachers should acquire may be defined as teachers' mathematics. Usiskin (2001) defines teachers' mathematics as a substantial body of mathematics as a consequence of teaching situations and he identifies the components of teachers' mathematics that shows what teachers need to know. According to him teacher mathematics includes having following competencies (Usiskin, 2001, p. 96).

- Explanation of new ideas.
- Alternate definitions and their consequences.
- The wide range of applications of the mathematical ideas being taught.
- Responses to questions that learners have about what they are learning.
- Why concepts arose and how they have changed over time.
- How problems and proofs can be extended and generalized.
- How ideas studied in school relate the ideas students may encounter in later mathematics study.

Similarly, Ball and her colleagues (2008) determine mathematical tasks of teaching which involves an uncanny kind of unpacking of mathematics. According to them these tasks are special works of teachers routinely done and these works are unique for teachers. Some of the mathematical tasks of teaching are as follows p. 400]ball2008 (Ball *et al.*, 2008, p. 400):

- Presenting mathematical ideas.
- Choosing and developing usable definitions.
- Responding to students why questions.
- Finding an example to make a specific mathematical point.
- Giving or evaluating mathematical explanations.
- Linking representations to underlying ideas and to other representations.
- Modifying tasks to be either easier or harder.
- Selecting representations for particular purposes.
- Evaluating the plausibility of students' claims.

These components of teachers' mathematics and mathematical tasks for teaching describe what mathematics teachers need to know and also consist of some specific skills and knowledge teaching mathematics. So, for the investigation of a particular aspect of teachers' knowledge these competencies and tasks may guide research studies.

As it is stated by Usiskin (2001) and Ball *et al.* (2008), mathematics teachers not only need to know alternate definitions but also able to choose and develop usable definitions. Leikin and Zazkis (2010) consider the idea that being able to exemplify and define mathematical concepts is a fundamental component of teachers' content knowledge. Therefore, they attempted to explore and develop preservice mathematics teachers' knowledge of definitions for mathematical concepts in various content areas of mathematics. Another important component of mathematics teachers' knowledge is using multiple representations that teachers need to know (Shulman, 1986). Studies shows that lack of knowledge of different representations cause difficulties in understanding some mathematical concepts (Sevimli & Delice, 2011) and multiple representations help to comprehend them (Durakkaya, Şenel, *et al.*, 2011).

By considering specific skills and competencies, it is possible to investigate and develop teachers' SMK and PCK. For example, Ubuz and Yayan (2010) tried to investigate primary teachers' SMK by using open ended questions which includes several tasks from the domain of decimals. After the investigation, they pointed out the necessity to enhance primary teachers' SMK in undergraduate years. As a matter of fact, teachers' SMK mostly develops in their undergraduate years with the courses that they have taken as preservice teachers. Mathematics courses provide them to improve their advanced mathematical skills and knowledge but this is not sufficient to enhance teachers' SMK. A teacher needs to unpack the mathematical knowledge in order to make it teachable and understandable for students (Aslan-Tutak, 2012). At this point, content courses help preservice teachers to enhance their SMK by using several mathematical practices (Aslan-Tutak & Ertas, 2013). Aslan-Tutak and Ertas (2013) stated that during content course not only their SMK is improved but also a component of preservice techers' PCK, knowledge of content and teaching, is affected by mathematical practices. Moreover, Charalambous, Hill and Ball (2011) points out the effect of methods courses on preservice teachers' knowledge. In their study, they examined some cases of preservice elementary teachers during the method courses and they found a strong link between preservice teachers' subject matter knowledge and their performance in providing explanations.

Researchers are strongly agreed that even though knowing mathematics is necessary for teaching mathematics, there are other important aspects of teaching which are required for effective mathematics instruction (Shulman, 1986; Ball *et al.*, 2008; Zazkis & Leikin, 2010; Bukova-Güzel, Kula, Uğurel, & Özgür, 2010). The components of dynamic teacher knowledge, subject matter knowledge-SMK, pedagogical content knowledge-PCK, beliefs and practice, interact with each other (Franke & Fennema, 1992; Walshaw, 2012; Türnüklü, 2005; Neubrand, Seago, Agudelo-Valderrama, DeBlois, & Leikin, 2009). On the basis of this idea, because of the interaction, researchers tried to investigate not just PCK but also mathematics teachers' knowledge as a whole either considering components of SMK and PCK at the same time (Hacıömeroğlu, 2009; Tekin-Sitrava & Işıksal-Bostan, 2013; Baş *et al.*, 2013). Alternatively, there are studies focusing on SMK and belief component (Ubuz, Özdil, & Eryılmaz Çevirgen, 2013).

PCK has a special importance because it is influenced by all other components; SMK, practice and belief. Therefore, PCK has multidimensional nature. Some researchers argue that this complex nature makes it difficult to investigate PCK by using efficient measures (Wilson, 2007). So, PCK may be studied by focusing its specific components as researchers described (Shulman, 1986; Ball *et al.*, 2008; Usiskin, 2001; Krauss, Brunner, Kunter, & Baumert, 2008). Yeşildere and Akkoç (2012) aimed to investigate elementary mathematics teachers' PCK by focusing on two components of PCK: knowledge of students understanding and difficulties, and knowledge of topic-specific strategies and representations. Studies shows that preservice teachers have difficulties in identifying the source of students' mistakes and misconceptions and how to deal with them (Kılıç, 2010; Bingölbali, Akkoç, Ozmantar, & Demir, 2011; Durakkaya, Aksu, et al., 2011). Another PCK components such as curriculum knowledge (Baştürk & Dönmez, 2011) and knowledge of different teaching methods (Dönmez & Baştürk, 2010) were also studied with preservice teachers. Even if preservice teachers do not have their own students, their knowledge of students studied within the teaching practicum courses (Baki, Baki, & Arslan, 2011). Preservice teachers' competencies were examined within the teaching experience course regarding their performances in field experience school (Kayhan & Argün, 2009).

Field experience and practicum courses provide preservice teachers an opportunity to experience real classrooms but these are limited for them in order to link SMK with teaching and instruction (Morris, Hiebert, & Spitzer, 2009). It is clear that teachers' instruction is affected from their content knowledge (Franke & Fennema, 1992). Based on their subject matter and pedagogical content knowledge, teachers design mathematics that will be done in class and students will interact and think about. The knowledge and competencies that preservice teachers obtained during teacher education programs may provide a basis for the development of teachers' knowledge with the content, methods and practicum courses (Ball *et al.*, 2001). However, there are a few studies that consider the effectiveness of teacher education programs by comparing different programs in international level (Schmidt *et al.*, 2007; Tatto *et al.*, 2008). In national level, for example in Turkey, both the graduates of mathematics teacher education programs and undergraduate program in mathematics departments have chance to be mathematics teachers. Therefore, researchers discuss the effectiveness of different programs through being teachers having intended knowledge and competencies (Safran *et al.*, 2014). Safran and his colleagues aimed to compare performance of teacher candidates graduating from mathematics teacher education programs and those graduating from mathematics department in terms of their scores of the Public Personnel Selection Examination-PPSE content test (Safran *et al.*, 2014). Their study showed that graduates of mathematics teacher education programs performed better in PPSE. However, teachers will continue to learn from their teaching experience in addition to theoretical background they possess when they graduate from teacher education programs. So, classroom interactions play an important role in developing teachers' knowledge and competencies.

Due to the dynamic nature of teacher knowledge, teachers can develop their mathematics knowledge through classroom interactions with students (Franke & Fennema, 1992). In addition to this, they develop new mathematical knowledge during the all stage of their work: planning, performing, and analyzing lesson (Leikin & Zazkis, 2007). Mathematics that students do in class may be a source of teachers' mathematical knowledge. Teachers can learn new solutions, methods, properties and questions by listening to their students (Watson, n.d.). Moreover students' mistakes and misconceptions can contribute teachers' knowledge in practice. When observing students' mistakes during instructional interactions with them, teachers search for new explanations or clarifications in order to correct students' understanding (Leikin & Zazkis, 2007).

Furthermore, research shows that the positive relationships between teachers' knowledge and students' learning (Peterson, 1998) and teachers' mathematical knowledge for teaching may be an indicator for students' achievement (Hill, Rowan, & Ball, 2005). As Franke and Fennema (1992) stated not only studies on mathematics teaching but also studies on other disciplines support this claim. While studying on teacher knowledge, the purpose is not to prepare teachers who know more mathematics, the main focus and the goal is to improve students' learning (Ball, 2003). In others words, researchers tried to explore teachers knowledge for effective teaching and learning. Even though it may seem clear that knowledgeable teacher will do a better job in effective instruction, in order to investigate such relationship, it is necessary to measure teachers' knowledge. But, it is not easy to measure such a complex concept (Wilson, 2007), as discussed in teacher knowledge models. In the next part of the literature review, some examples of instrument development studies to measure teachers' mathematics knowledge will be discussed.

2.2.3. Measuring Mathematics Teachers' Knowledge

After the result of the international tests of Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA) were examined, many countries with low students' performances initiated further investigation on reasons of the low student scores (Schmidt *et al.*, 2007; Krauss, Neubrand, Blum, & Baumert, 2008). In order to improve students mathematics, the effective mathematics teaching in classrooms is required (NCTM, 2000). A fundamental element of effective mathematics teaching is teacher knowledge (Walshaw, 2012). Therefore, students' low scores may be related with their teachers' mathematical knowledge. However, since there are few instruments to measure teachers' knowledge directly, questions about the role of teachers' knowledge on teaching and learning remain empirically unanswered (Krauss, Neubrand,*et al.*, 2008).

For many years researchers work on identifying teachers' mathematics knowledge for teaching and how it develops but until 2004 they had not tried to measure this knowledge in a rigorous way (Hill *et al.*, 2004). In the past studies, teachers' subject matter knowledge was defined with the number of courses taken in collage or with teachers' scores on standardized tests. Similarly Franke and Fennema (1992) pointed out that in studies teacher knowledge was defined as the number of university level mathematics courses completed. However, number of mathematics courses does not represent teachers' subject matter knowledge (Even, 1993). Therefore with development of models to understand of the nature of teacher knowledge, it was necessary to measure teacher knowledge from these perspectives. Although developing scalable efficient measures for content knowledge for teaching is difficult (Wilson, 2007), researchers tired to develop rigorous, effective and valid instruments to measure mathematics teachers' knowledge (Hill *et al.*, 2004; Krauss, Brunner, *et al.*, 2008; Tatto *et al.*, 2008). Within this section of the literature review, three instruments; MKT measure, COAVTIV measure and TEDS-M measure will be discussed by considering comparing their conceptual and structural characteristics.

2.2.3.1. Mathematical Knowledge For Teaching-MKT. In Learning Mathematics for Teaching (LMT) project, researchers wanted to develop measures of the mathematical knowledge used in teaching (Schilling & Hill, 2007). For the purpose of exploring teachers' mathematical knowledge for teaching, they developed both Mathematical Knowledge for Teaching (MKT) model and MKT measures. Up to the project, there was no rigorous instrument that measures teachers' mathematical content knowledge (Hill *et al.*, 2004).

Regarding to elementary and middle school levels, MKT measure includes three mathematics content area: (i) number and operations, (ii) patterns, functions, and algebra, (iii) geometry. Also this instrument addresses three domains of teacher knowledge: (i) content knowledge, (ii) knowledge of content and student (iii) knowledge of content and teaching. All of the items are multiple choice and based on classroom scenarios. In Appendix A there are three examples from MKT released items from three different domains of teacher knowledge.

The MKT measure is one of the first example that deal with teachers' content knowledge as a whole. It is still widely used in order to investigate preservice or inservice mathematics' teachers' knowledge (Hill *et al.*, 2005; Delaney, Ball, Schilling, & Zopf, 2008; Aslan-Tutak, 2009; Çopur Gençtürk, 2012). Researchers either used MKT Measure by adapting their own setting or developed new measures for different purposes based on this measure. MKT measure was designed for specifically one country, the United States, not for international usage. However it was adapted to use in some other countries such as Ireland, Norway, Indonesia, Korea and Ghana (Delaney *et al.*, 2008; Ng, Mosvold, & Fauskanger, 2012; Kwon, Thames, & Pang, 2012; Cole, 2011). Besides being a culture based instrument, the MKT measure was also developed by considering elementary school teachers, not secondary. For both secondary teachers and international purposes new instruments were developed by other research groups.

2.2.3.2. Cognitively Activating Instruction-COACTIV. Studies which are conducted with elementary school teachers consider lower levels of specific expertise according to secondary school teachers, so it is needed to investigate how differs secondary school teachers' knowledge (Krauss, Brunner, *et al.*, 2008). In Germany, researchers of the COACTIV project developed a test to conceptualize and assess teacher competencies by considering secondary level teaching (Krauss, Brunner, *et al.*, 2008).

The project was conducted with a sample of teachers whose classes participated in the PISA 2003/04 in Germany. One of the purposes of the project is to investigate the relationship between students' performance in PISA and teachers' competencies.

While students' achievements were measured in PISA, their teachers' knowledge measured within the COACTIV project (Krauss, Baumert, & Blum, 2008). In the COACTIV project secondary school mathematics teachers' pedagogical content knowledge (PCK) and content knowledge (CK) were assessed. PCK and CK were distinguished by following the Shulman's categorization (Krauss, Neubrand, *et al.*, 2008).

The instrument consists of both PCK test and CK test to examine the connectedness of two knowledge categories in teachers who have different mathematical expertise (Krauss, Brunner, *et al.*, 2008). PCK was examined in three subscales: tasks, students and instruction.

Krauss *et al.* (2008) defined content knowledge as in-depth background knowledge on the contents of the secondary level mathematics curriculum. CK test consists of items that cover relevant content areas in order to measure teachers' conceptual or procedural skills (Krauss, Brunner, *et al.*, 2008). Different than MKT measure all of the PCK and CK items are open-ended format in the COACTIV measure.

Similar to MKT measure, COACTIV measure is also country specific, not international. However, unlikely MKT measure, the COACTIV measure is not widely used. Researchers released and shared only four sample of questions from The COACTIV measure (Appendix B).

2.2.3.3. Teacher Education and Development Study in Mathematics-TEDS-M. TEDS-M is one of the studies of The International Association for the Evaluation of Educational Achievement (IEA). IEA studies generally focus on student achievement and the factors related with it from an international perspectives. For years, Trends in International Mathematics and Science Study-TIMSS is conducted by IEA. Differences in students' achievement level in TIMSS encouraged researchers to study teacher education from an international perspective in order to investigate how school mathematics curriculum and mathematics teaching quality differ across countries. They assumed that since teacher education is a part of schooling, it should be related with teachers' and students' knowledge (Tatto *et al.*, 2008). However the main goal was not to look into the relationships between teachers' professional knowledge and students' achievement (Blömeke & Delaney, 2012). The focus of the study is on preservice teachers; their preparation, knowledge and beliefs.

There were three components that were studied as different parts of the TEDS-M related to primary and secondary mathematics teachers: (i) teacher education policy, schooling, and social contexts at the national level, (ii) teacher education routes, institutions, programs, standards, and expectations for teacher learning, (iii) the mathematics and related teaching knowledge of future mathematics teachers. For the investigation of the last component, "TEDS-M research team developed some items, and solicited items from the Knowing Mathematics for Teaching Algebra Project at Michigan State University (2006), the Learning Mathematics for Teaching Project at the University of Michigan (2006), from researchers in ACER in Australia, and from the countries participating in TEDS-M"(Tatto, Rodriguez, Reckase, Rowley, & Lu, 2014, p. 11). Finally, they designed primary and secondary level instruments which include items that measure mathematics content knowledge (MCK) and mathematics and pedagogical content knowledge (MPCK) (Appendix C and Appendix D).

The MCK test includes four content areas of mathematics (number, algebra, geometry and data) and three cognitive dimensions (knowing, applying and reasoning). The MPCK test consists of two parts: knowledge of curricula and planning, and interactive knowledge about how to enact mathematics for teaching and learning. These MCK and MPCK items are not disjoint since "it is impossible to construct disjoint sub-domains because the solution of an item in the domain MPCK generally requires MCK" (Döhrmann, Kaiser & Blömeke, 2012, p. 336). All of the MCK and MPCK items designed in three different formats: Multiple Choice, Complex Multiple Choice, and Open Constructed Response.

"TEDS-M is the first cross-national study to provide data on the knowledge that future primary and secondary school teachers acquire during their mathematics teacher education" (Tatto *et al.*, 2012, p.17). 17 countries participated in the TEDS-M study so the instrument was not developed country based but it was designed for international usage and national adaptations.

In this chapter, the literature related to mathematics teachers' knowledge was shared by focusing on its conceptualization, investigation and development. Finally, three measures of mathematics teachers' knowledge were discussed. The literature showed that for more than 30 years, teacher knowledge has been one of the mostly discussed issues all over the world as well as in Turkey. In Turkey, for the investigations on development of teachers' mathematics knowledge for teaching, researchers generally discussed only some components of the knowledge by focusing on a specific mathematics subject. Accordingly, the instruments are generally content specific and refer to a part of teachers' knowledge like subject matter knowledge, pedagogical content knowledge or their specialized components. It can be said that many of the studies were conducted in a small scale. Therefore, in Turkish setting, it is necessary to study preservice teachers' knowledge in a broader scale by measuring it in a rigorous way.

3. SIGNIFICANCE OF THE STUDY

Teachers' content knowledge affects their instruction (Franke & Fennema, 1992) and also it can be said that it has an impact on students' learning and achievement (Peterson, 1998; Hill *et al.*, 2005). In order to investigate the role of teachers' knowledge on teaching and learning, it should be measured with instruments which are parallel to teachers' mathematics knowledge that they use in classrooms. However, in international area, a few instruments were developed to measure teachers' mathematics content knowledge for teaching (Ball *et al.*, 2008; Krauss, Baumert, & Blum, 2008; Tatto *et al.*, 2008). Among these, the instrument of Teacher Education and Development Study in Mathematics (TEDS-M) designed for multicultural purposes within the cross-national comparative study so it allows national adaptation and implementation in other countries.

In Turkey, students' scores in international studies; Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA) were very low compared to other countries. Students' low performances in mathematics may indicate many problems in educational setting but one of them may be related with mathematics teachers' knowledge. The knowledge and competencies that a teacher has, is lying at the core of the effective teaching (Walshaw, 2012).

> Students learn mathematics through the experiences that teachers provide. Thus, students' understanding of mathematics, their ability to use it to solve problems, and their confidence in, and disposition toward, mathematics are all shaped by the teaching they encounter in school. The improvement of mathematics for all students requires effective mathematics teaching in all classrooms (NCTM, 2000, p. 16-17).

Foundations of teachers' content knowledge are laid in teacher preparation programs and after graduation teachers start to teach with their knowledge that is attained in undergraduate years. Therefore, it is important to investigate how knowledgeable teachers are, before they start to teach. In Turkey, the Student Selection and Placement Center administers an exam, Public Personnel Selection Examination-PPSE (Kamu

Personel Seçme Sınavı-KPSS), which is used for assigning teachers to public schools. PPSE is a high stake test, so while effectiveness of the test was considered, teacher educators has been discussing measuring teacher knowledge.

In international research area, TEDS-M researchers developed TEDS-M tests to measure future primary and secondary mathematics teachers' mathematical knowledge for teaching which includes mathematics content knowledge and mathematics pedagogical content knowledge. In Turkey, there are three groups of mathematics teacher candidates in terms of the departments that they graduate. Graduates of primary mathematics education, secondary mathematics education and mathematics departments have a chance to be mathematics teachers. Therefore, this study aims to investigate mathematics teacher candidates' mathematics content and pedagogical content knowledge by using the Turkish translated versions of TEDS-M Primary and Secondary Released Items. The instruments are Mathematics Knowledge Instruments for Preservice Primary Mathematics Teachers (MKI-P) and Mathematics Knowledge Instruments for Preservice Secondary Mathematics Teachers (MKI-S).

4. STATEMENT OF THE PROBLEM

The current study strives to investigate mathematical knowledge for teaching (MKT) of preservice mathematics teachers. In Turkey, the graduates of undergraduate programs in three departments, primary mathematics education (PMATH), secondary mathematics education (SMATH) and mathematics (MATH), have a chance to be mathematics teachers. Therefore, this study aims to examine the knowledge of university students who were studying PMATH, SMATH and MATH.

Furthermore, since this study is a cross-sectional study it is not possible to measure participants' progresses in different times but it allows comparing different population groups at a single point in time. So, in this study, freshman and senior students' MKT was compared in order to investigate the difference in students' knowledge between the time they started to study and the time they graduated. In addition to this, senior students MKT was compared according to international data from Teacher Education and Development Study in Mathematics (TEDS-M) released items.

Another aim of the study is to compare the knowledge of students who were studying secondary mathematics education and who were studying mathematics. Graduates of these two departments have a chance to be a mathematics teacher at secondary level but the undergraduate education programs are notably different. Mathematics program does not consist of any pedagogy or education courses but more advanced mathematics courses than secondary mathematics education program.

To sum up the purpose of the present study is to investigate and compare the MKT of six groups of participants: freshman-PMATH, freshman-SMATH, freshman-MATH, senior-PMATH, senior-SMATH and senior-MATH. In accordance with this purpose two instruments were used: Mathematics Knowledge Instrument for Preservice Primary Mathematics Teachers (MKI-P) and the Mathematics Knowledge Instrument for Preservice Secondary Mathematics Teachers (MKI-S) which are Turkish translated versions of TEDS-M primary and secondary released items.

4.1. Variables

The dependent variable of the study is mathematical knowledge for teaching (MKT) whereas the independent variables are department and status. The descriptions and operational definitions of the variables are stated in following subsections.

4.1.1. Mathematical Knowledge for Teaching (MKT)

In this study, MKT was operationalized in terms of test scores obtained from Mathematics Knowledge Instruments: MKI-P and MKI-S. In the literature teachers' content knowledge is defined with the components of Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK) (Shulman, 1986). In the field of mathematics education, teachers' MKT is also identified with the components SMK and PCK by specifying subcategories for each (Ball *et al.*, 2008). SMK includes Common Content Knowledge , Specialized Content Knowledge and Knowledge at the Mathematical Horizon. PCK consists of Knowledge of Content and Students, Knowledge of Content and Teaching and Knowledge of Curriculum.

Starting from the idea of TEDS-M (Tatto *et al.*, 2008) within the scope of the current study, MKT variable was studied through two variables; Mathematics Content Knowledge (MCK) and Mathematics Pedagogical Content Knowledge (MPCK). The present study considers the MKT in total and also the components, MCK and MPCK, separately.

(i) *Mathematics Content Knowledge (MCK)*: This variable was operationalized in terms of test scores obtained from MCK items that were specified in the instruments. For this study, MCK can be described as which spans Common Content Knowledge and Specialized Content Knowledge. Also, "MCK includes not only basic factual knowledge of mathematics but also the conceptual knowledge of structuring and organizing principles of mathematics as a discipline" p. 327]doh2012 (Döhrmann *et al.*, 2012, p. 327).

(ii) *Mathematics Pedagogical Content Knowledge (MPCK)*: This variable was operationalized in terms of test scores obtained from MPCK items that were specified in the instruments. MPCK can be described as which cover Knowledge of Content and Students and Knowledge of Content and Teaching. So it includes being aware of students' ideas and representations also it is the knowledge about how to present essential mathematical concepts and how to make them understandable for students who may have some learning difficulties.

4.1.2. Department

The first independent variable is the department that participants were enrolled. There are three different categories for this variable: primary mathematics education (PMATH), secondary mathematics education (SMATH) and mathematics (MATH) departments.

4.1.3. Status

The second independent variable, status, indicates the participants' year of study at their universities. There are two different categories for this variable: freshman who is in the first year of study and senior who is in the last year of study.

4.2. Research Questions

The research questions of the study include comparisons of different groups according to teacher candidates' status and departments. Figure 4.1 presents a visual representation of these comparisons. There are seven research questions that this study focuses on and they are stated as follows:

(i) Is there any statistically significant difference between MKT of senior and freshman students from primary mathematics education department as measured by MKI-P?
    - Is there any statistically significant difference between MCK of senior and freshman students from primary mathematics education department as mea-

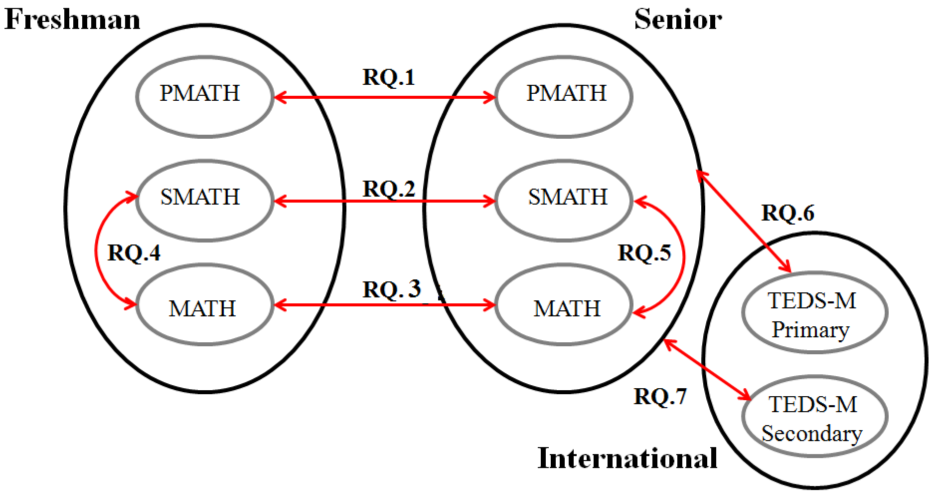


Figure 4.1. Research Questions.

sured by MKI-P?

- Is there any statistically significant difference between MPCK of senior and freshman students from primary mathematics education department as measured by MKI-P?

(ii) Is there any statistically significant difference between MKT of senior and freshman students from secondary mathematics education department as measured by MKI-S?

  - Is there any statistically significant difference between MCK of senior and freshman students from secondary mathematics education department as measured by MKI-S?
  - Is there any statistically significant difference between MPCK of senior and freshman students from secondary mathematics education department as measured by MKI-S?

(iii) Is there any statistically significant difference between MKT of senior and freshman students from mathematics department as measured by MKI-S?

  - Is there any statistically significant difference between MCK of senior and freshman students from mathematics department as measured by MKI-S?
  - Is there any statistically significant difference between MPCK of senior and freshman students from mathematics department as measured by MKI-S?

(iv) Is there any statistically significant difference between MKT of freshman students from mathematics department and secondary mathematics education department as measured by MKI-S?
  - Is there any statistically significant difference between MCK of freshman students from mathematics department and secondary mathematics education department as measured by MKI-S?
  - Is there any statistically significant difference between MPCK of freshman students from mathematics department and secondary mathematics education department as measured by MKI-S?

(v) Is there any statistically significant difference between MKT of senior students from mathematics department and secondary mathematics education department as measured by MKI-S?
  - Is there any statistically significant difference between MCK of senior students from mathematics department and secondary mathematics education department as measured by MKI-S?
  - Is there any statistically significant difference between MPCK of senior students from mathematics department and secondary mathematics education department as measured by MKI-S?

(vi) How does primary mathematics education senior students' scores differ according to international data from TEDS-M Primary Released Items?

(vii) How does secondary mathematics education and mathematics senior students' scores differ according to international data from TEDS-M Secondary Released Items?

4.3. Research Hypothesis

In the current study, the null hypotheses are stated as follows:

(i) There is no statistically significant difference between senior and freshman students from primary mathematics education department in their MKT.
  - There is no statistically significant difference between senior and freshman students from primary mathematics education department in their MCK.

  - There is no statistically significant difference between senior and freshman students from primary mathematics education department in their MPCK.

(ii) There is no statistically significant difference between senior and freshman students from secondary mathematics education department in their MKT.

  - There is no statistically significant difference between senior and freshman students from secondary mathematics education department in their MCK.
  - There is no statistically significant difference between senior and freshman students from secondary mathematics education department in their MPCK.

(iii) There is no statistically significant difference between senior and freshman students from mathematics department in their MKT.

  - There is no statistically significant difference between senior and freshman students from mathematics department in their MCK.
  - There is no statistically significant difference between senior and freshman students from mathematics department in their MPCK.

(iv) There is no statistically significant difference between freshman students from mathematics and secondary mathematics department in their MKT.

  - There is no statistically significant difference between freshman students from mathematics and secondary mathematics department in their MCK.
  - There is no statistically significant difference between freshman students from mathematics and secondary mathematics department in their MPCK.

(v) There is no statistically significant difference between senior students from mathematics and secondary mathematics department in their MKT.

  - There is no statistically significant difference between senior students from mathematics and secondary mathematics department in their MCK.
  - There is no statistically significant difference between senior students from mathematics and secondary mathematics department in their MPCK.

5. METHODS

The current study aims to investigate mathematical knowledge for teaching (MKT) of teacher candidates from primary education (PMATH), secondary education (SMATH) and mathematics (MATH) departments. The data was collected from two different universities that are the only ones which have these three departments together in Istanbul. Since data was collected from selected participants at a single point in time, cross-sectional survey design was used (Gay, Mills, & Airasian, 2011). Also this study can be defined as a descriptive comparative study because it is aimed to examine differences between groups in MKT without trying to infer causes of differences (Lodico, Spaulding, & Voegtle, 2011). Participants of the study are comprised of two status groups (freshman and senior) from each three departments. The data gathered from all participants by the Mathematics Knowledge Instrument for Preservice Primary Mathematics Teachers (MKI-P) and the Mathematics Knowledge Instrument for Preservice Secondary Mathematics Teachers (MKI-S). These instruments comprised of Turkish adapted *Teacher Education and Development Study in Mathematics (TEDS-M)* released items which consist of mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK) items for preservice primary and secondary mathematics teachers.

5.1. Sampling and Participants

The participants of this study are senior and freshman university students who were studying PMATH, SMATH or MATH in two public universities in Istanbul. The target population is senior and freshman university students of those three departments in Istanbul. All universities in Istanbul, which has all three departments, were included in sample. Based on departments' quota declared by Student Selection and Placement Center, the approximate number of target population was 600.

The instruments were given to all students who enrolled to last semester methods course of PMATH department, students who enrolled to last semester methods course of SMATH department, students who enrolled last year required mathematics course of MATH department and students who enrolled first year mathematics courses. Students were asked to voluntarily participate to the study and totally 360 students formed the sample.

The sample consisted of 270 female and 90 male students aged between 17 and 34 years (M = 20.50, SD = 2.35). Moreover, 215 freshman and 145 senior students were participated in this study. There were 140 participants from PMATH department who will teach in grades 4-8, 74 from SMATH department who will teach 8-12 and 146 from MATH department. The detailed differentiation of participants according to their departments, status and universities are demonstrated in Table 5.1.

Table 5.1. Number of Participants.

	PMATH		SMATH		MATH		
	Freshman	Senior	Freshman	Senior	Freshman	Senior	**Total**
University A	31	16	27	25	32	15	146
University B	59	34	0	22	66	33	214
Total	90	50	27	47	98	48	360

As seen in Table 5.1 the Freshman-SMATH cell is zero for University B because the , Council of Higher Education –CHE (Yükseköğretim Kurulu-YÖK) declared that there would not be any student quota for secondary school education departments in the 2013-2014 academic year (Anadolu Ajansı, 2013). However, University A had freshman SMATH students since there was an English preparation year before starting to study in department. Therefore, the data from freshman students who were studying SMATH was gathered from just University A.

Moreover, all participants of the study were asked to explain whether they had an informal teaching experience like tutoring or teaching in cram schools. Table 5.2 consists of teaching experience percentages of participants among their groups.

Table 5.2. Informal Teaching Experience Percentages of Participants.

Department	**Status**	**No Experience**	**Experience**
PMATH	**Freshman**	97	3
	Senior	28	78
SMATH	**Freshman**	70	30
	Senior	24	76
MATH	**Freshman**	85	15
	Senior	30	70

5.2. Instruments

The main purpose of this study is to investigate the mathematical content and pedagogical content knowledge of mathematics teacher candidates. In accordance with this purpose, Mathematics Knowledge Instruments for Primary (MKI-P) and Mathematics Knowledge Instruments for Secondary (MKI-S) were used (Appendix E and Appendix F). These instruments are Turkish translated versions of TEDS-M Primary and Secondary Instruments (Appendix C and Appendix D) which were developed to measure pre-service teachers' knowledge of mathematics content and pedagogical content for teaching at the end of their teacher training.

These instruments consist of mathematical content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK) items. These items were desveloped by TEDS-M researchers considering the framework of Trends in International Mathematics and Science Study (TIMSS) 2007 (Tatto *et al.*, 2008). MCK items comprised of four content areas: number, algebra, geometry and data, whereas it has three cognitive dimensions: knowing, applying, and reasoning. These content areas and cognitive domains were adapted from TIMSS 2007 framework. Furthermore, MPCK items consist of two parts: knowledge of curricula planning and interactive knowledge about how to enact mathematics for teaching and learning. These were aligned with PCK domains in literature. Furthermore, three different items format were used: Multiple Choice

(MC), Complex Multiple Choice (CMC), and Open Constructed Response (CR).

Table 5.3. MCK Primary Items.

Cognitive Domain	Content Domain				
	Algebra	**Geometry**	**Number**	**Data**	**Total**
Knowing	7	3	5	-	15
Applying	3	3	1	1	8
Reasoning	-	-	-	1	1
Total	10	6	6	2	24

MKI-P includes 24 MCK and 10 MPCK items. Table 5.3 demonstrate the distribution of 24 MCK items according to content and cognitive domains. The division of 10 MPCK items in terms of curriculum and planning and enacting dimensions as well as four content domains can be seen in Table 5.4.

Table 5.4. MPCK Primary Items.

	Content Domain				
	Algebra	**Geometry**	**Number**	**Data**	**Total**
Curriculum & Planning	1	2	2	1	6
Enacting	1	-	2	1	4
Total	2	2	4	2	10

Similarly, MKI-S contains 23 MCK and 9 MPCK items. Table 5.5 illustrate the distribution of 23 MCK items according to content and cognitive domains. The division of 9 MPCK items in terms of curriculum & planning and enacting dimensions as well as four content domains can be seen in Table 5.6.

In this study, MKI-P was administered to participants who were studying primary mathematics education to teach at grade levels 4 to 8. MKI-S was given two groups

Table 5.5. MCK Secondary Items.

Cognitive Domain	Content Domain				
	Algebra	**Geometry**	**Number**	**Data**	**Total**
Knowing	-	2	4	-	6
Applying	5	4	-	1	10
Reasoning	2	1	4	-	7
Total	7	7	8	1	23

Table 5.6. MPCK Secondary Items.

	Content Domain				
	Algebra	**Geometry**	**Number**	**Data**	**Total**
Curriculum & Planning	4	-	-	-	4
Enacting	1	-	3	1	5
Total	5	0	3	1	9

of participants: who were studying secondary mathematics education or mathematics. Both groups of graduates have chance to teach at grade levels 9 to 12.

For the development of TEDS-M items, item response theory (IRT) was used (Tatto *et al.*, 2014). TEDS-M test developers shared the reliabilities for MCK and MPCK tests for the primary and secondary international samples based on IRT estimates. The reliability estimates of the complete tests are .83 for primary MCK, .66 for primary MPCK, .91 for secondary MCK and .72 for secondary MPCK (Tatto *et al.*, 2014). For the sample of the current study the reliability coefficients for MKI-P and MKI-S were also calculated. The reliability coefficient alpha values calculated as .48 for MKI-P items and .63 for MKI-S items. These are not as high as original complete test.

5.2.1. Translation of TEDS-M Released Tests

Internationally accepted study TIMMS was conducted in different countries with different languages; different cultures and different education systems. In order to provide equivalence of national versions, translations and national adaptation procedures were strict and structured with guidelines. Similarly, teacher education and work of teaching differ from one country to other, so measurement of teacher knowledge need to be sensitive to these differences (Delaney *et al.*, 2008). For the international study TEDS-M, researchers used translation and adaptation guidelines of TIMSS (Tatto *et al.*, 2008). These guidelines were also followed for this current study.

TEDS-M Primary and Secondary Instruments translated in Turkish before using to collect data for the present study. Turkish translated versions of instruments were named as MKI-P and MKI-S respectively. The method which was used while translating for each of two instruments consists of three phases. Firstly, items translated in Turkish by the researcher who is fluent in English. The translated documents were reviewed by a mathematics educator who is expert in the content area and fluent in English, a three-year experienced mathematics teacher who is fluent in English and a professional translator. According to their reviews and comments, all of the items checked in detail and the revisions of translation completed.

At the second phase, the original tests were administered a group of preservice mathematics teachers who are native in Turkish and fluent in English. The same group took the translated versions of tests 3 weeks apart. In the first implementation 41 students from primary education and 29 students from secondary education completed the tests in the course that they have taken. For the second implementation, researcher went to same courses and administered the translated versions of tests to students who were at class in that day. 35 translated primary test and 20 translated secondary test returned. Among these, the number of students who completed both original and translated versions of test is 31 for primary and 16 for secondary. Although 31 numbers of students completed both versions of primary items, scores of 24 students compared since 7 of them left blank most of the questions.

After scoring, the correlation coefficient of participants' total points that they got from English and Turkish versions were calculated by using software of Statistical Package for Social Sciences (SPSS). Since the number of participants was low for both secondary ($n = 16$) and primary ($n = 24$) groups, Spearman correlations were used. The Spearman correlation results show that two versions of primary instruments are moderately correlated ($r(22) = .48$, $p <.05$) and two versions of secondary instrument also moderately correlated ($r(14) = .54$, $p <.05$). As seen in results, both of correlations coefficients are between .40 and .60 and the effect sizes are described as moderate (Cohen, 1988).

At the last phase, the method of back translation was used to check the quality of translation and to investigate linguistic or conceptual errors in translation. Also it was used to consider particular attention to sensitive translation problems across cultural correspondence of the two versions. This process includes translation of Turkish versions of MKI-P and MKI-S back to original language English. A graduate student, who has mathematics degree from the US and is doing master in mathematics education, translated MKI-S back into English. For the primary level instrument, MKI-P was translated back into English by a PhD student from mathematics education who is research assistant in primary mathematics education. Both for MKI-P and MKI-S, two versions were compared item by item. It was determined that some items in Turkish translations include extra information or phrases different than the original one. These differences were determined and corrected. Despite all of the reviews and corrections, after the first implementation, it was realized that one item under 9th question in MKI-S was ill-translated. For the other implementations it was corrected but the item 9c was not included data analysis. The original TEDS-M Primary and Secondary Instruments and their translated versions, MKI-P and MKI-S, can be seen in Appendix C, D, E and F.

5.3. Data Collection

The data was collected from participants at a single point in two different time periods. The same data collection procedure was followed for both University A and University B. Instruments administered to senior students during the last two weeks of spring semester of 2012-2013 academic year and the data was gathered from freshman students during the first two weeks of fall term of 2013-2014 academic year.

In the first phase, end of spring semester, MKI-P was given the senior students who were enrolled last semester teaching methods course in PMATH department during their class time. MKI-S administered to senior students who were studying SMATH or MATH. Students who were studying SMATH and enrolled last semester teaching method course took the MKI-S during the class time. For the MATH students, in order to reach all senior students MATH department's last semester course "Real Analysis" was chosen. In University A, the instructor of the course did not give permission to collect data in the class time so an e-mail was sent all students who were in the course list. The MKI-S instrument was given students who replied the e-mail. Since the classes had finished early in University B, the MKI-S instrument was given to MATH students on their final exam day.

In the second phase, at the beginning of the 2013-2014 academic year, the instruments, MKI-P and MKI-S, were administered freshman students who were in their first year of studying SMATH, PMATH and MATH. In University A, the data gathered from these students during the class time of a mathematics course that is first semester required course for students who are studying SMATH, PMATH and MATH. In University B, freshman students who were studying PMATH and MATH took the MKI-P and MKI-S in different time during their class time of different courses. There were no freshman SMATH students in University B so there is no data for this group of students.

After data collection, open ended items were scored according to scoring guide of TEDS-M Primary and Secondary Instruments. Moreover another rater scored 20 % of data which were randomly selected regarding all 12 groups that were classified according to their universities (University A and University B), departments (PMATH, SMATH and MATH) and status (freshman and senior). The answers of 28 participants who took MKI-P and 44 participants who took MKI-S were scored by a graduate student in mathematics education. Pairwise comparisons among raters were examined and the level of agreement was calculated. Two raters came to 96.75 % agreement for 11 MKI-P open ended items and 97.92 % agreement for 8 MKI-S open ended items.

5.4. Data Analysis

According to independent variables status and department, there are 6 different groups: Freshman-PMATH, Senior-PMATH, Freshman-SMATH, Senior-SMATH, Freshman-MATH, Senior-MATH. For each groups descriptive statistics were calculated separately for scores obtained from MKI-P and MKI-S. Total scores that participants got from these tests were called as MKT score. Since these tests consist of MCK and MPCK items, in addition to total scores their MCK and MPCK scores were calculated.

Associated with research questions, not only descriptive statistics of scores was calculated but also groups' scores were compared by using appropriate statistical testing methods. Firstly, the groups of participants who had different status were compared. Among the participants from PMATH departments, freshman and senior participants' scores were compared according to their MKT, MCK and MPCK scores. Similarly, SMATH freshman and senior students' MKT, MCK and MPCK scores were compared. Finally, MKT MCK and MPCK scores of freshman and senior participants from MATH departments were compared.

Secondly, the groups of participants who were studying in different departments were compared. As it is stated in literature, the mathematical needs for primary and secondary are different than each other and so, in this study different instruments were used for primary and secondary mathematics teacher candidates. Therefore, participants from PMATH departments were not compared with the other participants from SMATH and MATH departments. The departmental comparison was made between only SMATH and MATH departments. Among freshman, participants from SMATH and MATH departments were compared according to their MKT, MCK and MPCK scores. Similarly, senior SMATH and MATH students' MKT, MCK and MPCK scores were also compared.

In all cases, two groups of participants' mean scores were compared by using software of Statistical Package for Social Sciences (SPSS). If all of the assumptions were met the independent sample t-test were used. If one of the assumptions were violated non-parametric Mann-Whitney U test were used. The cases that the distributions of scores are significantly deviated from normal distribution (the normality assumption was violated) and so non-parametric Mann-Whitney U test was used, are stated as follows: (i) among PMATH participants, MCK and MPCK mean scores' comparisons of freshman and senior, (ii) among SMATH participants, MPCK mean scores' comparisons of freshman and senior, (iii) among MATH participants, MPCK mean scores' comparisons of freshman and senior, (iv) among freshman participants, MPCK mean scores' comparisons of SMATH and MATH, (v) among senior participants, MPCK mean scores' comparisons of SMATH and MATH.

6. RESULTS

In this section, the findings of the present research will be presented. The results of data analysis will be presented in two sections considering primary mathematics teaching and secondary mathematics teaching separately. These sections include both the descriptive (mean, standard deviation, and range) and comparative statistics (comparisons regarding department and status) associated with research questions. Moreover, item statistics; percentages of right and wrong answers of participants will be presented.

6.1. Primary Mathematics Teaching

The Mathematics Knowledge Instrument for Preservice Primary Mathematics Teachers (MKI-S) was given two groups of participants who were classified according to their year of study (status) in Primary Mathematics Education (PMATH) Departments of two universities (Freshman-PMATH and Senior-PMATH). Totally 140 PMATH students participated in this study (90 freshmen and 50 senior). The mean, standard deviation, minimum and maximum scores were calculated for the participants' test scores obtained from MKI-P ($M = 29.93$ and $SD = 3.71$). The maximum total score for MKI-P is 43 points but the calculated maximum total score is 38 and the minimum score is 18 in this study. The distribution of total score of all PMATH students (freshman and senior) were illustrated in Figure 6.1.

MKI-P consists of 34 Mathematical Knowledge for Teaching (MKT) items in total and 24 of them are Mathematics Content Knowledge (MCK) and 10 of them are Mathematics Pedagogical Content Knowledge (MPCK) items. While total maximum point for MCK items is 26, it is 17 for MPCK items. MKT scores, MCK scores and MPCK scores were computed and analyzed separately as stated in research questions. Following subsection includes these analyses with comparing two groups which were classified according to their status.

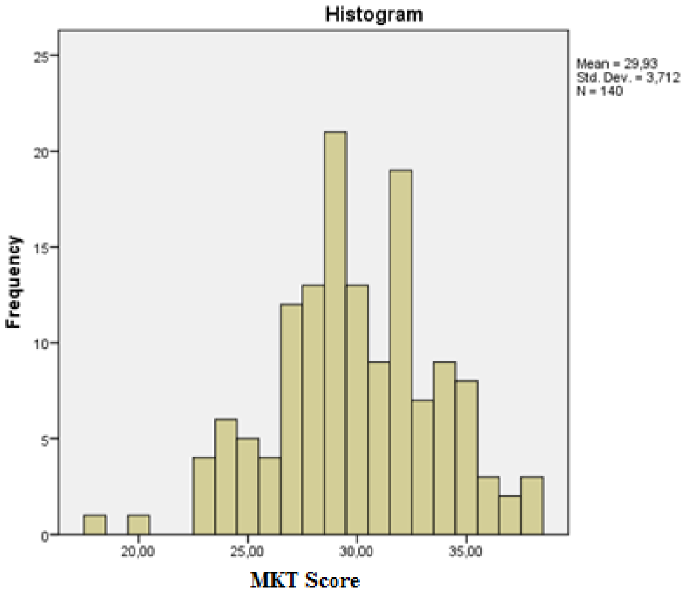


Figure 6.1. Distribution of MKT Scores from MKI-P.

6.1.1. Comparison according to Status

Associated with research questions, MKT, MCK and MPCK mean scores of participants who were studying PMATH were compared as a function of their status: freshman or senior. Since the current study aims to investigate whether there is a difference in MKT between senior and freshman students, their scores were compared by using appropriate statistical testing methods (independent sample t-test or non-parametric Mann-Whitney U test).

The data gathered from 90 freshman and 50 senior students who were from PMATH department in two universities. Mean scores, standard deviations, minimum and maximum scores of freshman and senior students separately stated in Table 6.1.

The statistical test results show that there is a significant difference between freshman and senior students in MKT, MCK and MPCK scores. The t-test results indicate that seniors' MKT score significantly 3.16 points higher than freshmen's MKT score, t (138) = 5.27, p <.001. Moreover, the values of Cohen's d is .92 which indicates

Table 6.1. Means, Standard Deviations, Minimum and Maximum Scores of MKI-P.

		M	***SD***	**Min.**	**Max.**
MKT	**Freshman**	28.80	3.35	18	38
	Senior	31.96	3.49	23	38
MCK	**Freshman**	21.83	2.28	15	26
	Senior	23.22	1.93	18	26
MPCK	**Freshman**	6.97	2.07	2	13
	Senior	8.74	2.47	4	14

the strong effect (Cohen, 1988). According to Mann-Withney U test results, senior students have significantly higher MCK ($Z = 3.70$, $p <.001$) and MPCK ($Z = 4.08$, $p <.001$) scores than freshman students.

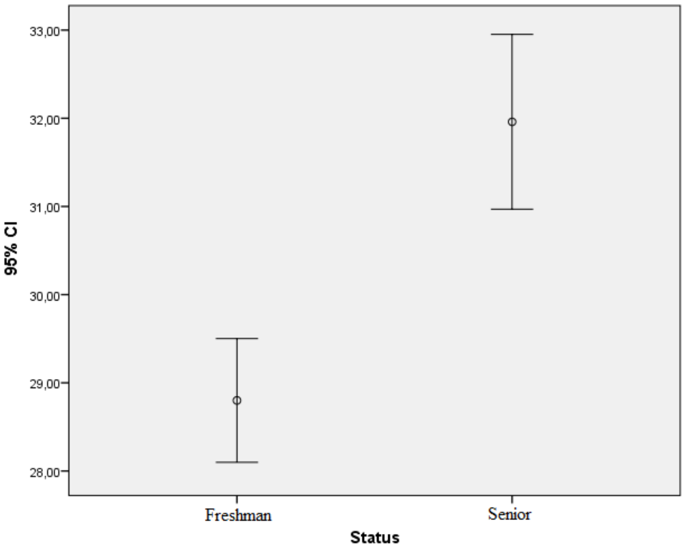


Figure 6.2. Confidence Intervals of MKT Scores for Freshmen and Seniors in MKI-P.

Moreover in Figure 6.2 Error Bar Chart presents graphical representation of the variability of data by comparing confidence intervals (95%) of freshmen and seniors. As the chart in Figure 6.2 confidence intervals for freshmen (95% CI [28.10, 29.50]) and seniors (95% CI [30.97, 32.95]) do not overlap, this would suggest that there is a real difference between the population means. Also there is a gap between the upper bound of CI for freshmen (29.50) and the lower bound of CI for seniors (30.97).

6.1.2. Comparison of Primary Teacher Candidates to International Results

In this section item by item comparisons will be presented according the correct response percentages of senior PMATH students. Table 6.2 not only includes correct response percentages of each item in MKI-P for the sample of this current study but also consists of the international averages that were obtained from Teacher Education and Development Study in Mathematics (TEDS-M).

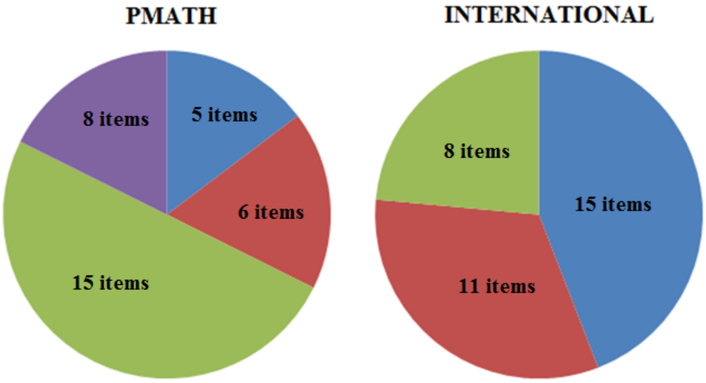


Figure 6.3. Pie Charts of Item Distributions according to Correct Response Percentages of PMATH and International Primary.

The data indicates that for 32 items over 34, the correct response percentage of PMATH students is higher than international average. For items 14 and 17b, while the international averages are more than 65 %, less than 50 % of PMATH students answered these items correctly. Moreover, less than 60 % of PMATH students gave correct answer for 5 items (Item 2, 7b, 14, 17b, 23). More than 60 % but less than 80 % answered 6 items correctly (Item 3d, 6b, 7a, 10b, 18a, 21). For 23 items, more than 80 % of them gave correct answer and among these 23 items, all of them answered 8 items correctly (Item 3c, 6a, 10a, 15a, 16, 18b, 18d, 20). Figure 6.3 demonstrates this distribution via pie chart.

Table 6.2. Correct Response Percentages of MKI-P Items.

	PMATH	**International**
Item 1	88	28
Item 2	50	28
Item 3a	98	81
Item 3b	96	86
Item 3c	100	92
Item 3d	70	64
Item 4	96	67
Item 5	86	61
Item 6a	100	78
Item 6b	68	54
Item 7a	60	32
Item 7b	46	32
Item 8	90	82
Item 9	98	54
Item 10a	100	78
Item 10b	68	52
Item 11	96	49
Item 12	86	38
Item 13	82	60
Item 14	46	67
Item 15a	100	56
Item 15b	98	51
Item 16	100	85
Item 17a	94	85
Item 17b	44	74
Item 18a	78	74
Item 18b	100	89
Item 18c	88	69
Item 18d	100	42
Item 19	96	97
Item 20	100	74
Item 21	76	33
Item 22	80	38
Item 23	54	48

6.2. Secondary Mathematics Teaching

The Mathematics Knowledge Instrument for Preservice Secondary Mathematics Teachers (MKI-S) was given four different groups of participants (Freshman-MATH, Freshman-SMATH, Senior-MATH, and Senior-SMATH). These groups were determined according to their departments (SMATH or MATH) that participants were studying in and their years of study (status: freshman or senior). Totally 220 participants took the MKI-S and mean, standard deviation, minimum and maximum scores were calculated ($n = 120$, $M = 22.88$ and $SD = 4.36$). Below Figure illustrates the distribution of MKT score of all participants who took MKI-S (Figure 6.4).

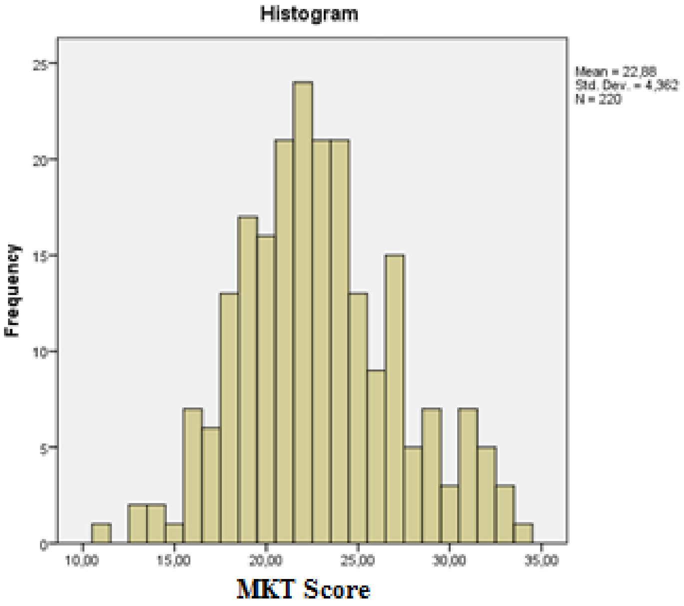


Figure 6.4. Distribution of MKT Scores from MKI-S.

MKI-S consists of 32 MKT items in total and 23 of them are MCK and 9 of them are MPCK items. The maximum total score that could be obtained from MKI-S is 35 points which is sum of 27 points from MCK items and 8 points from MPCK items. MKT scores, MCK and MPCK scores were computed and analyzed separately as stated in research questions. Following subsections include these analyses with comparing different groups which were classified according to their status and departments.

At the time of data collection, in University B, there were no freshman students who were studying SMATH so the comparisons for Freshman-SMATH students were conducted for only University A. These are the comparison between Freshman-SMATH and Senior-SMATH and also the comparison between Freshman-SMATH and Freshman-MATH.

6.2.1. Comparison according to Status

Associated with research questions, mathematical knowledge for teaching (MKT), mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK) mean scores of participants were compared according to their years of study (status); freshman or senior. In order to investigate whether there is a significant difference in Mathematical Knowledge for Teaching between freshman and senior students, their scores were compared by using appropriate statistical testing methods (independent sample t-test or non-parametric Mann-Whitney U test).

Both freshman and senior participants who are from two different departments, SMATH or MATH, took MKI-S. The comparison based on their status conducted by considering SMATH and MATH departments separately. Therefore, the results will be presented as comparison according to status among SMATH and among MATH in the following subsections.

6.2.1.1. Comparison according to Status among SMATH Department. The data, that was gathered from 27 freshman and 25 senior students who were studying SMATH in University A, were analyzed. Descriptive statistics are given in Table 6.3.

The statistical tests results indicate that there are significant differences in all MKT, MCK and MPCK scores between freshman and senior students who were studying SMATH. Independent sample t-test results indicate that senior students have significantly 6.84 points higher MKT mean score (t (50) = 8.68, p <.001) than freshman students and Cohen's d = 2.41 shows strongly large effect (Cohen, 1988). Also, senior

Table 6.3. Means, Standard Deviations, Minimum and Maximum Scores of MKI-S for Freshman and Senior SMATH.

		n	***M***	***SD***	**Min.**	**Max.**
MKT	**Freshman**	27	22.04	2.72	17.00	27.00
	Senior	25	28.88	2.96	22.00	34.00
MCK	**Freshman**	27	16.33	2.35	11.00	21.00
	Senior	25	22.40	2.53	16.00	26.00
MPCK	**Freshman**	27	5.70	1.07	3.00	7.00
	Senior	25	6.48	1.08	4.00	8.00

students have significantly 6.07 points higher MCK mean score (t (50) = 8.95, p <.001) than freshman students and Cohen's d = 2.49 indicates strongly large effect (Cohen, 1988). Moreover according to Mann-Withney U test there is a significant difference in MPCK mean ranks between groups: seniors are significantly higher than freshmen, Z = 2.53, p <.05.

Furthermore, in Figure 6.5 Error Bar Chart presents graphical representation of the variability of data by comparing confidence intervals (95%) of freshmen and seniors. As the chart in Figure 6.5 confidence intervals for freshmen (95% CI [20.96, 23.11]) and seniors (95% CI [27.65, 30.10]) do not overlap, this would suggest that there is a real difference between the population means. Also there is a gap between the upper bound of CI for freshmen (23.11) and the lower bound of CI for seniors (27.65).

6.2.1.2. Comparison according to Status among MATH Department. The data that was gathered from 98 freshman and 48 senior students who were studying MATH, was analyzed. Descriptive statistics are given in Table 6.4.

The statistical tests results showed significant differences in all MKT, MCK and MPCK scores between freshman and senior students who were studying MATH. The

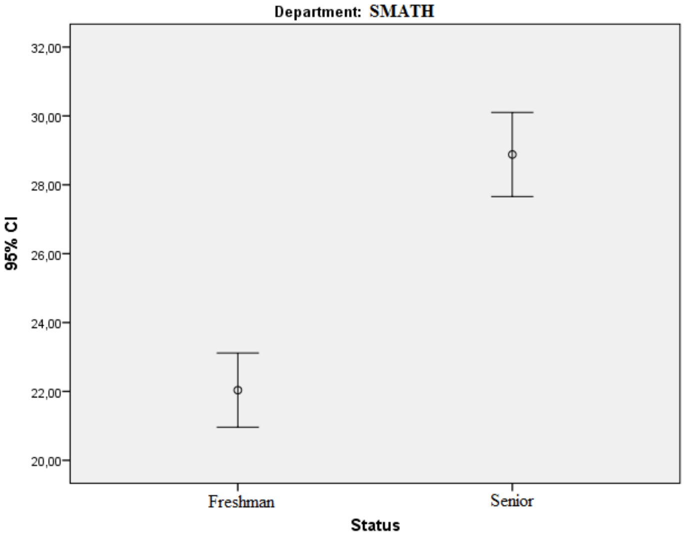


Figure 6.5. Confidence Intervals of MKT Scores for Freshmen and Seniors SMATH in MKI-S.

results of independent sample t-tests indicate that the MKT mean score of senior students is significantly 2.72 points higher than the MKT mean score of freshman students, t (144) = 4.14, p <.001 with Cohen's d = .69 medium effect (Cohen, 1988). Also, the t-test result shows that senior students have significantly 2.23 points higher scores than freshman students, t (144) = 3.82, p <.001 with Cohen's d = .65 medium effect (Cohen, 1988). Moreover according to Mann-Whitney U test there is a significant difference in MPCK mean ranks between groups. Seniors were significantly higher than freshmen, Z = 2.57, p <.01.

Moreover in Figure 6.6 Error Bar Chart presents graphical representation of the variability of data by comparing confidence intervals (95%) of freshmen and seniors. As the chart in Figure 6.6 confidence intervals for freshmen (95% CI [20.16, 21.55]) and seniors (95% CI [22.34, 24.91]) do not overlap, this would suggest that there is a real difference between the population means. Also there is a gap between the upper bound of CI for freshmen (21.55) and the lower bound of CI for seniors (24.91).

Table 6.4. Means, Standard Deviations, Minimum and Maximum Scores of MKI-S for Freshman and Senior MATH.

		n	***M***	***SD***	**Min.**	**Max.**
MKT	**Freshman**	98	20.86	3.45	11.00	29.00
	Senior	48	23.63	4.42	16.00	33.00
MCK	**Freshman**	98	15.27	3.05	7.00	22.00
	Senior	48	17.50	3.80	10.00	26.00
MPCK	**Freshman**	98	5.59	1.11	3.00	8.00
	Senior	48	6.13	1.35	3.00	8.00

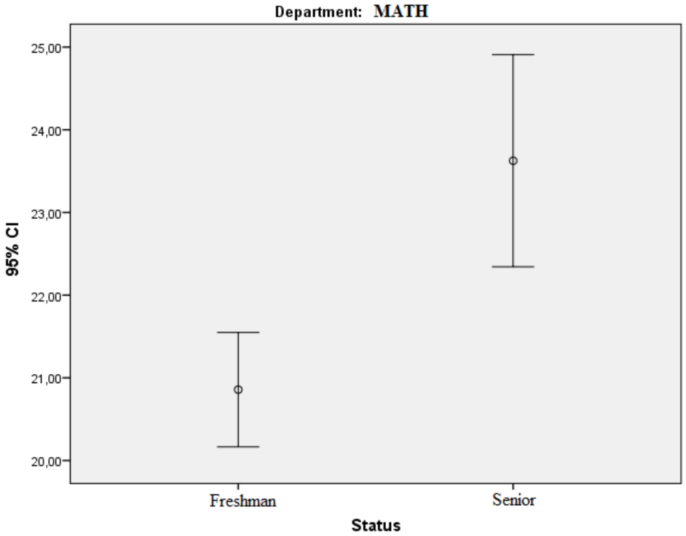


Figure 6.6. Confidence Intervals of MKT Scores for Freshmen and Seniors MATH in MKI-S.

6.2.2. Comparison according to Departments

Associated with research questions, MKT, MCK and MPCK mean scores of participants were compared according to their department that they were studying MATH or SMATH. As literature said, the mathematical needs for primary and secondary are different than each other and in this study also different instruments were used for primary and secondary mathematics teacher candidates. Therefore, participants from PMATH departments were not compared with the other participants from SMATH and MATH departments. The departmental comparison was made between only SMATH and MATH departments. In order to investigate whether there is a significant difference in MKT between students from MATH and SMATH departments, the appropriate statistical analyses were conducted (independent sample t-test or non-parametric Mann-Whitney U test).

The comparison based on participants' departments conducted for freshman and senior groups separately. Therefore, the results will be presented as comparison according to department among freshman students and among senior students in the following subsections.

6.2.2.1. Comparison according to Departments among Freshman Students. The data, that was gathered from 32 MATH and 27 SMATH students who were freshman in University A, was analyzed. Descriptive statistics are given in Table 6.5.

The independent sample t-test results indicates that there is no significant difference in neither MKT score (t (57) = .35, p = .73) nor MCK score (t (57) = .38, p = .71) between freshman students who were studying MATH or SMATH. Moreover, according to Mann-Whitney U test, among freshman students, there is no significant group difference in their MPCK mean ranks between MATH and SMATH students, Z = -.10, p = .92.

Moreover in Figure 6.7 Error Bar Chart presents graphical representation of the

Table 6.5. Means, Standard Deviations, Minimum and Maximum Scores of MKI-S for Freshman MATH and SMATH.

		n	*M*	*SD*	Min.	Max.
MKT	**MATH**	32	22.34	3.79	11.00	29.00
	SMATH	27	22.04	2.72	17.00	27.00
MCK	**MATH**	32	16.63	3.34	7.00	22.00
	SMATH	27	16.33	2.35	11.00	21.00
MPCK	**MATH**	32	5.72	.99	4.00	8.00
	SMATH	27	5.70	1.07	3.00	7.00

variability of data by comparing confidence intervals (95%) of freshmen MATH and SMATH students. As the chart in Figure 6.7 there is a substantial overlap between confidence intervals for MATH (95% CI [20.98, 23.71]) and SMATH (95% CI [20.96, 23.11]). So, it seems likely that there is no real difference in population.

6.2.2.2. Comparison according to Departments among Senior Students. The data, which was gathered from 48 MATH and 47 SMATH students who were senior, was analyzed. Descriptive statistics are given in Table 6.6.

The results of t-tests indicate that the MKT mean score of students from SMATH department is significantly 3.2 points higher than those from MATH department, t (93) = 3.72, p <.001 and Cohen's d = .76 with the marginal large effect size (Cohen, 1988). Also, the t-test result shows that SMATH students have significantly 2.96 points higher MCK scores than MATH students, t (93) = 4.00, p <.001 Cohen's d = .82 with the large effect size (Cohen, 1988). Moreover according to Mann-Whitney U test there is no significant difference in MPCK mean ranks between students who were studying MATH and those who were studying SMATH, Z = -1.00, p >.05.

Moreover in Figure 6.8 Error Bar Chart presents graphical representation of the

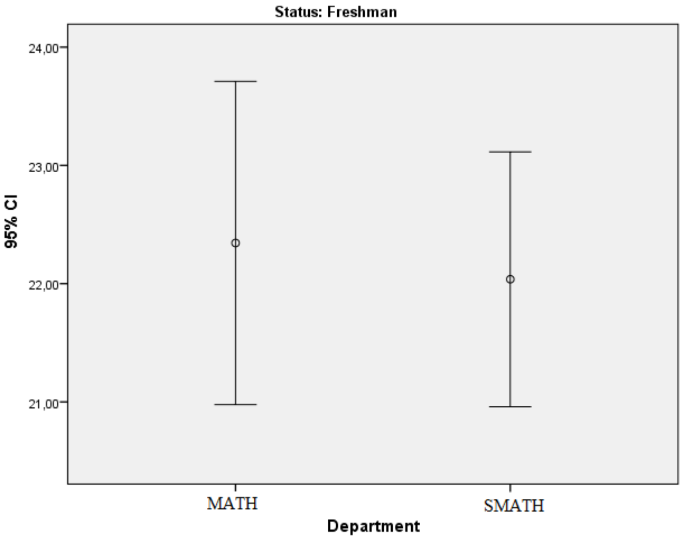


Figure 6.7. Confidence Intervals of MKT Scores for Freshmen MATH and SMATH in MKI-S.

Table 6.6. Means, Standard Deviations, Minimum and Maximum Scores of MKI-S for Senior MATH and SMATH.

		n	M	SD	Min.	Max.
MKT	**MATH**	48	23.63	4.42	16.00	33.00
	SMATH	47	26.83	3.96	18.00	34.00
MCK	**MATH**	48	17.50	3.80	10.00	26.00
	SMATH	47	20.45	3.35	14.00	26.00
MPCK	**MATH**	48	6.13	1.35	3.00	8.00
	SMATH	47	6.38	1.19	4.00	8.00

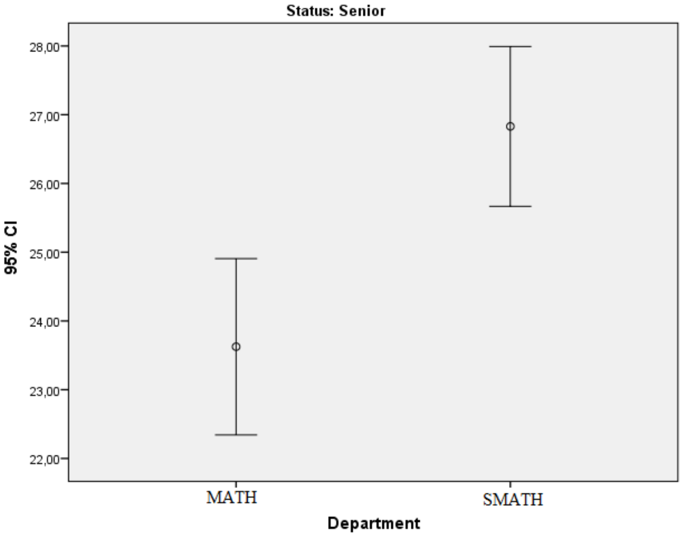


Figure 6.8. Confidence Intervals of MKT Scores for Senior MATH and SMATH in MKI-S.

variability in data by comparing confidence intervals (95%) of senior students from MATH and SMATH departments. As the chart in Figure 6.6 confidence intervals for MATH (95% CI [22.34, 24.91]) and SMATH (95% CI [25.67, 27.99]) do not overlap, this would suggest that there is a real difference between the population means. Also there is a gap between the upper bound of CI for MATH students (24.91) and the lower bound of CI for SMATH students (25.67).

6.2.3. Comparison of Secondary Teacher Candidates to International Results

In this section item by item comparisons will be presented according the correct response percentages of senior students who were studying MATH or SMATH. Table 6.7 not only includes correct response percentages of each item in MKI-S for the sample of this current study and also consists of the international averages that were obtained from Teacher Education and Development Study in Mathematics (TEDS-M).

The data indicates that for 28 items over 31, the correct response percentage of

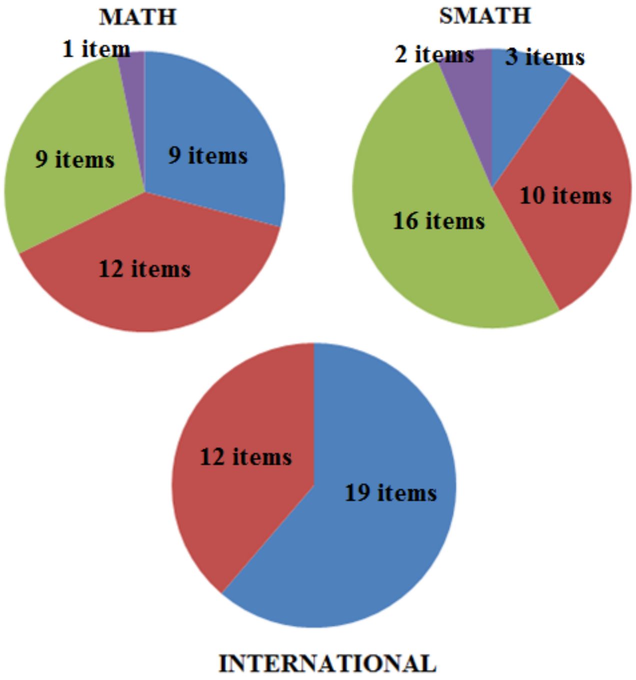


Figure 6.9. Pie Charts of Item Distributions according to Correct Response Percentages of MATH, SMATH and International.

SMATH or MATH students is higher than international average. For the items 7a, 12b and 13c, the international average is more than the percentage of MATH and SAMTH students answered these items correctly. Moreover, while less than 60 % of MATH students gave correct answer for 9 items (Item 1b, 2c, 7a, 7b, 7c, 8, 9d, 10d, 13c), less than 60 % of SMATH this is for 3 items (Item 7a, 8, 13c) . Whereas more than 60 % but less than 80 % of MATH students answered 12 items correctly, this for 10 items for SMATH students. For 10 items, more than 80 % of MATH students gave correct answer and among these 10 items, for only 1 item all of them gave the correct answer (Item 1a1). For SMATH students, more than 80 % of them answered 18 items correctly and among these 18 items, 100 % of them answered 2 items correctly (Item 1a1, 4). Figure 6.9 demonstrates these distributions via pie charts.

Table 6.7. Correct Response Percentages of MKI-S Items.

	MATH	**SMATH**	**International**
Item 1a1	100	100	72
Item 1a2	94	98	50
Item 1b	52	72	50
Item 2a	83	94	44
Item 2b	92	89	54
Item 2c	52	62	37
Item 3	75	91	53
Item 4	98	100	57
Item 5a	65	81	53
Item 5b	65	83	51
Item 6a	98	94	75
Item 6b	75	87	46
Item 6c	85	77	60
Item 7a	42	32	41
Item 7b	50	68	39
Item 7c	48	75	60
Item 8	31	43	19
Item 9a	90	87	78
Item 9b	96	87	78
Item 9d	56	70	64
Item 10a	60	83	46
Item 10b	75	77	63
Item 10c	69	96	58
Item 10d	52	66	54
Item 11	75	85	35
Item 12a	88	83	71
Item 12b	60	64	69
Item 13a	75	85	70
Item 13b	63	87	61
Item 13c	40	32	53
Item 14	65	77	21

7. DISCUSSION AND CONCLUSION

The current study was designed and conducted for the following major goals. This study aimed to compare mathematical knowledge for teaching (MKT) of teacher candidates who were in their first year (freshman) and who were in their last year (senior) of study in their undergraduate education. Another purpose of the study was to compare teacher candidates according to their departments that they were studying in (primary primary mathematics education-PMATH, secondary mathematics education-SMATH or mathematics-MATH). In order to identify the MKT of the participants, Mathematics Knowledge Instrument for Preservice Primary Mathematics Teachers (MKI-P) and Mathematics Knowledge Instrument for Preservice Secondary Mathematics Teachers (MKI-S) were used. These instruments were Turkish translated versions of Teacher Education and Development Study in Mathematics (TEDS-M) released items. TEDS-M was an international study which was conducted in 2008 with 17 different countries. TEDS-M research group shared item by item correct response percentages from international data. This study was used these percentages to compare participants' MKT descriptively with international averages.

In this chapter, the conclusion will be discussed within five sections. In first two sections, teacher candidates' characteristics and their preparation for being mathematics teachers during their undergraduate education will be discussed. The differences between participants who have just started their undergraduate education and who are about to finish will be considered. Moreover, in addition to teacher education programs of primary mathematics education and secondary mathematics education, mathematics department whose graduates have a chance being mathematics teachers will be discussed in these sections. Latter two sections consist of international comparison of the results of the study with TEDS-M and TIMSS results. Lastly, limitations of the study and the suggestions for further research will be mentioned in the last section.

7.1. At the Beginning of being a Mathematics Teacher

In Turkey, since 1982, universities have been training teachers within the faculty of education. However, even after 1982, these faculties were not the unique source of teachers (different opportunities for other faculties' graduates). Today, as it was before, not only graduates from faculties of education become teachers but also graduates from faculty of arts and sciences have a chance to be teachers if they complete teaching certificate program. Therefore, in order to be mathematics teacher there are two ways. Someone can be a mathematics teacher after graduating from one of the mathematics teacher education programs (primary mathematics education for primary school or secondary mathematics education for secondary school) or after graduating from mathematics department and then completing a certification program for teaching.

The participants of current study were potential teacher candidates. They were students from primary mathematics education, secondary mathematics education or mathematics departments in two universities in Istanbul. These students were placed these departments of the universities by the Student Selection and Placement Center-SSPC (Öğrenci Seçme ve Yerleştirme Merkezi-ÖSYM) based on their scores of university entrance exam and their preferences. According to data from SSPC in 2012, these two universities' primary mathematics education departments were ranked in the first 5 among 69, secondary mathematics education departments were ranked in the first 2 among 22 and mathematics departments were ranked in the first 11 among 121 (evening education programs and programs with scholarship of the same university are included).

In this study, firstly freshman students were examined according to their MKT scores. Since the mathematical needs for primary and secondary level and this study was used two different instrument for primary and secondary levels, the departmental comparison was made only between participants from secondary mathematics education and mathematics departments. So, freshman secondary mathematics education and freshman mathematics students in University A were compared according to their performances in MKI-S. (It should be noted that there was not any freshman secondary mathematics education student in University B so the data from only University A were used to compare freshman secondary mathematics education and mathematics students.) The statistical test results indicated that there is no significant difference between freshman secondary mathematics education and freshman mathematics students in terms of MCK, MPCK and MKT scores.

On the other hand, among senior students, the differences in MCK and MKT scores between mathematics and secondary mathematics education students were highly significant. These results show that while there was no departmental difference between freshman students, there was a significant departmental difference between senior students. So, it may be inferred that the departmental difference between senior secondary mathematics education and senior mathematics students may be because of the education of different undergraduate programs. Therefore, the effects of each of these programs on participants' performances in MKI-P and MKI-S should be considered.

7.2. Mathematics Teacher Training From Freshman to Senior Years

In order to compare effect of undergraduate programs on teacher candidates' MKT, MKT scores of all three departments' freshman and senior students were compared. The statistical analysis showed that for all groups of participants from three departments (primary mathematics education, secondary mathematics education and mathematics) there were significant differences between freshman and senior students in their MCK, MPCK and MKT scores. The undergraduate education in these departments may contribute to improve MKT (MCK and MPCK) of teacher candidates. In order to explain departments' effect on MKT, curriculum of departments were examined.

During the teacher education programs in primary mathematics education and secondary mathematics education departments, students take several theoretical and methodological courses which provide them to improve their knowledge of mathematics and knowledge of teaching. primary mathematics education departments' teacher education programs include 50-60% content knowledge and skills, 25-30% professional teaching knowledge and skills, and 15-20% general knowledge courses (Yüksek Öğretim Kurumu (YÖK), 2007). In secondary mathematics education departments students are required to complete 50% content knowledge and skills, 30% professional teaching knowledge and skills, and 20% general knowledge courses (ODTÜ, 2013). Therefore, for participants from primary mathematics education and secondary mathematics education departments the difference in MCK, MPCK and MKT scores between freshman and senior students may be explained by teacher education programs. However, mathematics departments' undergraduate education programs include 70% content knowledge and 30% general knowledge courses. Mathematics students take many advanced mathematics courses but they are not required to take any courses related with teaching. Therefore, for the participants from mathematics department, it is difficult to explain the significant difference between freshman and senior students in MPCK scores by the undergraduate education program. In order to explain this, it should be considered how PCK develops and how it is measured and assessed.

7.2.1. Development of PCK of Teacher Candidates

Firstly, examining the definition of PCK and how teachers' develops this knowledge may be explanatory while discussing the results of the study. Even though, teacher education programs are the most influential factors that affect PCK of preservice teachers, there are other factors when the nature of PCK is considered. PCK includes knowledge of "the ways of subject that make it comprehensible to others" (Shulman, 1986, p. 9). This means that a teacher needs to know using appropriate representations, illustrations, analogies, demonstrations and explaining concepts in order to make them understandable. So, teaching experiences play an important role in teachers' PCK (Ball *et al.*, 2008). Because of this, teacher education programs include many teaching experience (eg., field experience, practicum) opportunities to preservice teachers. Furthermore, the participants from mathematics department had informal teaching experience (eg., tutoring and teaching in cram schools) which may explain the higher MPCK scores of senior students comparing to freshman students. All participants of the study were asked to explain whether they had an informal teaching experience. It should be noted that 70 percentages of senior mathematics students stated that they had teaching experiences. Also, teaching experience percentages were 76 for secondary mathematics education senior students and 78 for primary mathematics education senior students. In other words, mathematics students had informal teaching experiences during their undergraduate education as much as primary mathematics education and secondary mathematics education students.

Secondly, measuring and assessing PCK is another issue which should be considered by focusing on its nature in order to explain the results of the study. The results showed no significant difference between MPCK scores of senior mathematics and secondary mathematics education students. As a matter of fact, it is difficult to explain this result, since mathematics students were not required to take any teaching related courses. However, they take many advanced mathematics courses and they develop their Advanced Mathematical Knowledge which is defined as knowledge of the subject matter acquired during undergraduate education at universities (Zazkis & Leikin, 2010). It should be noted that Advanced Mathematical Knowledge is necessary but not sufficient condition for achieving the specialized knowledge for teaching in secondary level (Zazkis & Leikin, 2010). According to literature PCK may be conceptualized not only as knowledge of students' thinking and conceptions, but also knowledge of explanations, representations and alternative definitions of mathematical concepts, and knowledge of multiple solutions to mathematical tasks (Shulman, 1986; Ball *et al.*, 2008; Krauss, Baumert, & Blum, 2008). As it is seen, in PCK's multidimensional nature, deep mathematical knowledge plays an important role because it can provide teachers to use effective explanations, representations and alternative definitions. Moreover, due to this complex nature of PCK, developing scalable efficient measures for content knowledge for teaching is very difficult (Wilson, 2007).

For example, one of the MPCK questions in MKI-S (question 9) asks to determine if knowledge is needed to prove the quadratic formula. This question requires being able to prove quadratic formula first and then differentiate which knowledge is needed. It measures knowledge of content and teaching but firstly deep mathematical knowledge is necessary to answer this question correctly. Actually, if one can prove

Table 7.1. MPCK Items of MKI-S with the Correct Response Percentages of Senior MATH and SMATH Students.

	Content Domain	**PCK Sub-Domain**	**Label**	**Correct Response Percentage**	
				MATH	**SMATH**
Item 1b	Algebra	Enacting	Analyze why one word problem is more difficult than another.	52	72
Item 6a	Number	Enacting	Determine whether student's response is a valid proof.	98	94
Item 6b	Number	Enacting		75	87
Item 6c	Number	Enacting		85	77
Item 9a	Algebra	Curriculum and Planning	Determine if knowledge is needed to prove the quadratic formula.	90	87
Item 9b	Algebra	Curriculum and Planning		96	87
Item 9d	Algebra	Curriculum and Planning		56	70
Item 12b	Data	Enacting	Explain student's thinking about histogram.	60	64

the quadratic formula, there is nothing to know anything else to determine which knowledge is needed to prove. So, it is not easy to differentiate and measure this kind of knowledge and skills. Difficulty in measuring PCK may explain the unexpected result that there is no difference in MPCK scores between senior mathematics and secondary mathematics education students. Moreover, this study used the instrument MKI-S which tried to measure MPCK with a few items (8 items under 4 questions). Therefore limited domains of PCK were able to be measured with this instrument. Table 7.1 lists the all MPCK items in MKI-S by giving information about items' content domain and PCK sub-domain and labels which show intended ability to answer correctly. Also the correct response percentages of senior mathematics and senior secondary mathematics education students were stated in Table 7.1. For example, Items 9a and 9b were answered correctly by the more percent of senior mathematics students than senior secondary mathematics education students. However, for Item 1b which requires analyzing why one word problem is more difficult than another, correct response percentage of secondary mathematics education students is much better than mathematics students. Senior mathematics and secondary mathematics education students' different reactions to different questions may be based on their knowledge and skills that obtained during undergraduate education. Therefore, in order to investigate the nature of the difference, how teacher candidates improve MKT during undergraduate education should be considered.

7.2.2. Development of MKT of Teacher Candidates

The results showed that senior students have better MKT compared to freshman students for all departments (primary mathematics education, secondary mathematics education and mathematics). Figure 7.1 illustrates the distinction between such differences by considering error bars which show confidence intervals of MKT scores for primary mathematics education, secondary mathematics education and mathematics students respectively. The gap between upper bound of confidence interval for freshman scores and lower bound of confidence interval for senior scores is greater in primary mathematics education and secondary mathematics education departments compared to mathematics department. This explains that whereas there was no significant difference between freshman secondary mathematics education and mathematics students' MKT scores, the highly significant difference occurred when senior students were compared. The most serious difference between secondary mathematics education and mathematics senior students is undergraduate education that they were about to finish. Therefore, the difference in MKT scores may be explained by comparing the knowledge and skills that students acquired while studying secondary mathematics education or MATH.

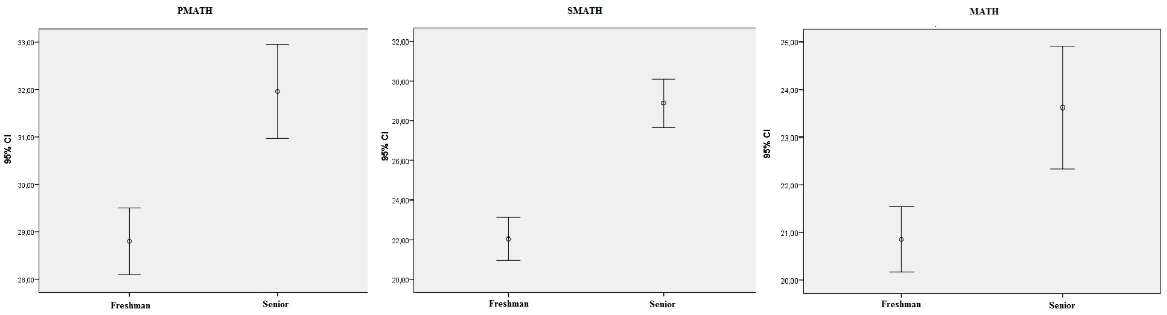


Figure 7.1. Confidence Intervals of MKT Scores of Freshman and Senior Students in PMATH, SMATH and MATH Departments.

Mathematics students take more advanced mathematics courses than secondary mathematics education students. On the other hand, not mathematics but secondary mathematics education students take teaching methods, general pedagogy and practicum courses. In teaching methods and practicum courses they discuss and experience school mathematics that they will teach. Therefore, as teacher candidates, they have a chance to reconsider and relearn school mathematics. Teaching related courses provide preservice teachers an opportunity to unpack their knowledge and ask why and how questions on high school mathematics topics. However, mathematics students move beyond high school mathematics and study mathematics in advanced level. They do not go back to high school mathematics unless you need to study with a high school student (informal teaching experience). These kinds of differences between education programs may make a highly significant difference in MKT scores between senior secondary mathematics education and mathematics students.

Furthermore, it can be said that the results of Public Personnel Selection Examination-PPSE (Kamu Personel Seçme Sınavı-KPSS) in 2013 supports the results of the current study. KPSS is required for being a teacher in public schools. For mathematics teacher candidates, the exam consists of three parts: general knowledge test, general pedagogical test and mathematics content knowledge test. Mathematics content knowledge test includes items that are aimed to measure knowledge of mathematics and knowledge of pedagogy for teaching mathematics. According to content knowledge test results, there was a significant difference between graduates from secondary mathematics education departments ($n = 2201$, $M = 63.27$) and graduates from mathematics departments ($n = 11434$, $M = 61.08$) and the mean score of test takers who are from secondary mathematics education was higher ($t = 4.50$, $p < .001$) than MATH. So it may indicated that teacher candidates who graduated from secondary mathematics education departments have intended qualifications more than graduates of mathematics departments (Safran *et al.*, 2014). This PPSE data gathered from all regions of the Turkey supports what the current study investigated.

7.3. International Comparison of Teacher Candidates' MKT

The current study compared MKT of teacher candidates according to both their status and departments at the national level. In a larger scope, where the Turkish mathematics teacher candidates take place at the international level, is another issue which should be considered. TEDS-M reported the results of the study based on the data from 17 countries and the data shows international average of item by item correct responses percentages among the all participants of TEDS-M study. This data gives us a chance to compare the international data with the present study. Below charts demonstrate the difference in performances of participants for MKI-P (Figure 7.2) and MKI-S (Figure 7.3) with the international average in TEDS-M items.

When the chart in Figure 7.2 for primary is considered, it is obviously seen that primary mathematics education senior students' performances better than the international average performances in nearly all items. However international averages are much better for Item 14 and Item 17b. Both of the items are MPCK items (enacting and curriculum &planning respectively) and their content domain is “data”. Item 14 requires finding the similarities and differences in data presentation and Item 17b requires determining difficulty with a data representation problem. This might be due to the fact that they had limited learning experience with Data Display, Data Interpretation or Data Analysis concepts. When the participants of the study were in middle or high school, these concepts were not included in the mathematics curriculum. Moreover it may be showed that during their undergraduate education in primary mathematics education department, it was not provided students with opportunities to improve their statistics and data skills.

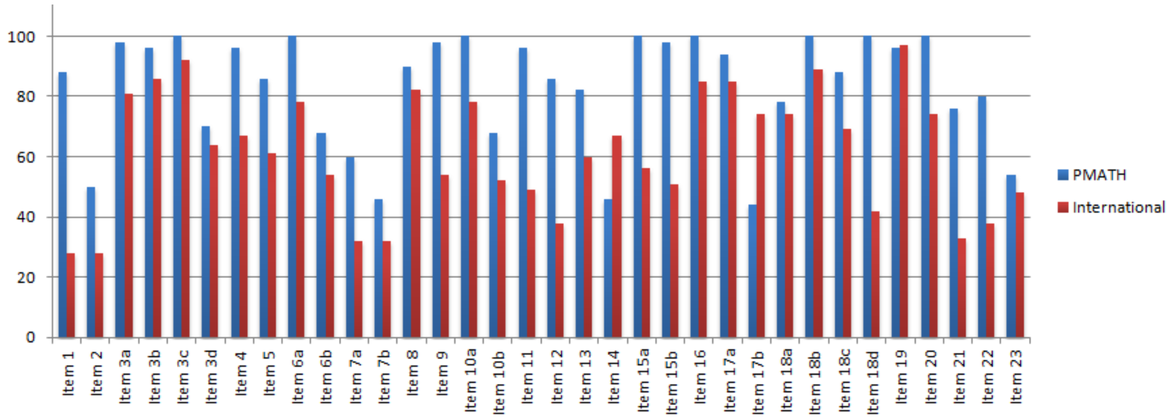


Figure 7.2. Chart of MKI-P Items’ Correct Response Percentages.

In Figure 7.3, the chart presents the comparative data of correct response percentages of senior MATH, senior secondary mathematics education students with international average. By looking this data it may be easily said that international average performances are lower than participants both from mathematics and secondary mathematics education departments. There are only three items that the international average is better than the others. First one is Item 7a which is a MCK item (algebra & applying) and requires determining whether a situation can be modeled by an exponential function. The situation was that “The height h of a ball t seconds after it is thrown into the air”. Second one is Item 12b which is a MPCK item (data-enacting) and requires explaining student's thinking about histogram. The last one is Item 13c which is a MCK item and requires correcting students' answers about lines of symmetry in a rhombus.

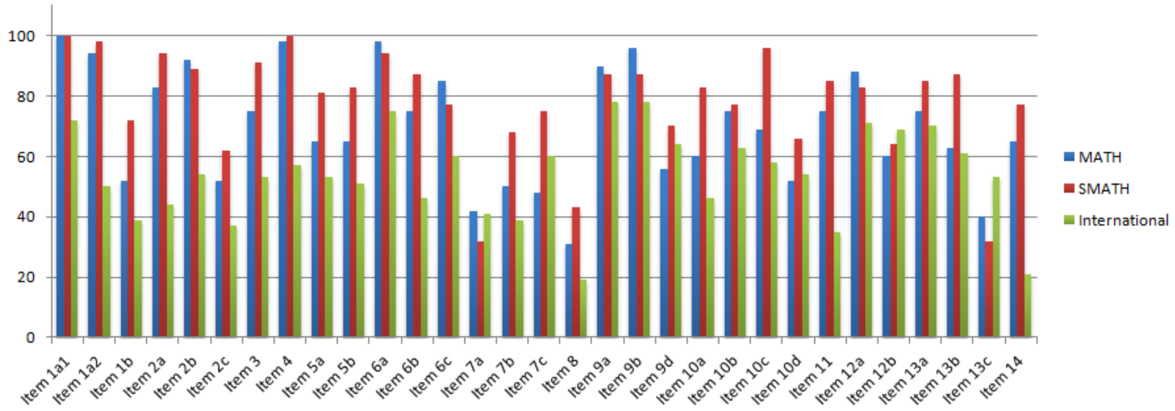


Figure 7.3. Chart of MKI-S Items' Correct Response Percentages.

The results of the study show that both primary and secondary preservice teachers are not good at the content domain "data". Furthermore, according to TIMSS 2011 report, in Turkey, mathematics teachers of eight graders thought that they were less qualified in the content domain of data and chance compared to other content domains (Yıldırım, Yıldırım, Ceylan, & Yetişir, 2013).

The reason of this insufficiency in the content domain "data" may be seen more obviously after curriculum analysis. For example, in the 2009 Turkish Primary Education Mathematics Curriculum, there was an only one objective in the 6th grade addresses this content domain: generate problems for research and collect data (Milli Eğitim Bakanlığı Talim ve Terbiye Kurulu Başkanlığı, 2009). In 2013, with the reform in mathematics curriculum, this situation changed and the number of objectives in primary education mathematics curriculum increased to twelve from one. In 6th grade, there are 3 objectives for generating research problems and data collection and 3 for data data analysis; in the 7th grade thare are 4 objectives for data processing and 2 for data analysis (Milli Eğitim Bakanlığı Talim ve Terbiye Kurulu Başkanlığı, 2013).

7.4. Limitations and Suggestion for Further Research

In this section, the limitations of the current study and suggestions for further research will be presented. This study was conducted under certain circumstances so that it consists of some limitations. First of all, this study is limited to two public universities in Istanbul. According SSPC data in 2013, there are 62 primary mathematics education, 23 secondary mathematics education and 187 mathematics departments in Turkey. Therefore, the results may not be generalized to all students of these departments in Turkey. Moreover, these two universities, that were the sample of the study, are among the most preferred 11 universities of Turkey for primary mathematics education, secondary mathematics education and mathematics departments. In order to be able to generalize these results to all Turkish teacher candidates, this study could be conducted in a larger scope.

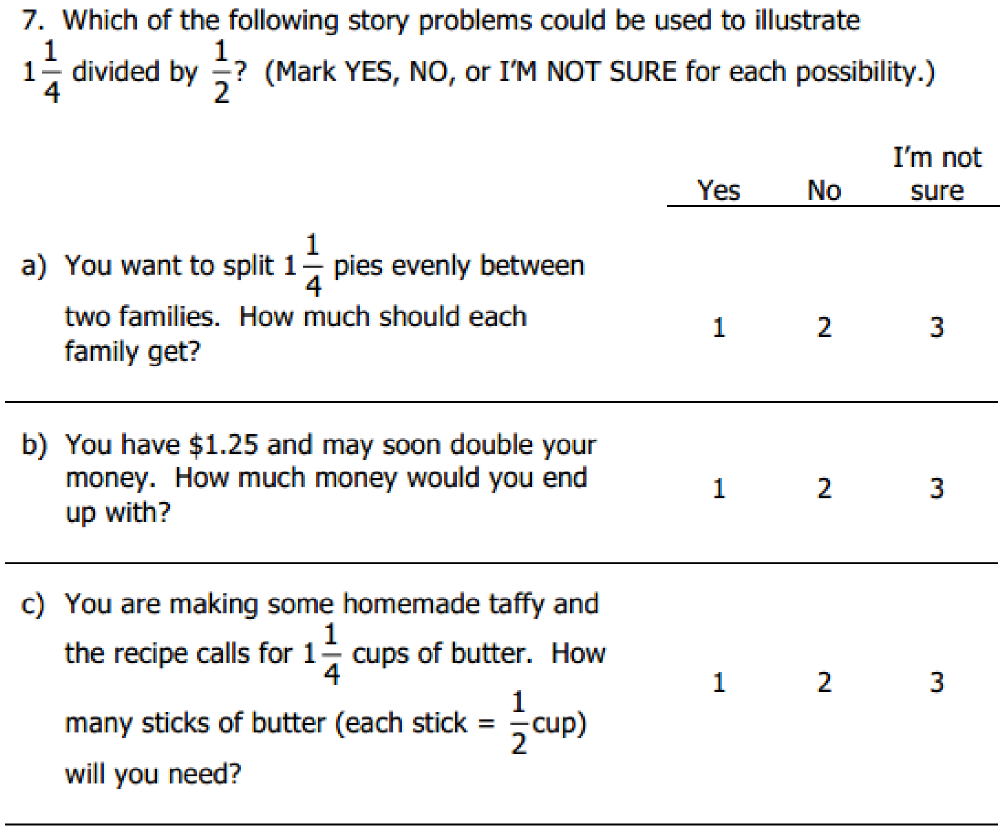
The results of the current study were compared to international results of the TEDS-M according to correct response percentages of Turkish tranlated versions of TEDS-M Primary/Secondary Items and the original tests. These comparisons showed that the average performances of participants of the current study were better than international average. When it is considered that teachers' mathematical knowledge is related with student's mathematics achievement (Hill *et al.*, 2005), it may be expected that Turkish students should be also higher than the international average of students. However, according to TIMSS results Turkey was below the average, even took place near the bottom. This may be because of that there were huge differences between schools in TIMSS Turkey but for this study two high ranked universities were sampled. When TIMSS 2011 results are examined on school bases it can be seen that there is a huge differences between the best and the worst in TIMSS mathematics score: the best score was 752 (more than the best country Korea, 613) and the worst score was 289 (less than the worst country Ghana, 331) (Yıldırım *et al.*, 2013). However, within this study two of the best 11 universities were considered. Therefore, in order to investigate the relationship between teachers' MKT and students' achievement in Turkey, a large scope study may be conducted.

If long-term comparison is not possible, cross-sectional methods are used to infer about differences between groups. In this study, it was not practical to follow freshman students to their senior years, so cross-sectional method was used to investigate difference between freshman and senior students' MKT acquired while studying in primary mathematics education, secondary mathematics education and mathematics departments. Therefore, a longitudinal study may be conducted to examine the differences of undergraduate education programs in teacher candidates' MKT.

The last but not the least, this study could not observe how teacher candidates would use their knowledge in real classrooms. In Turkey teacher education programs are mainly focused on theoretical knowledge, paying less attention to practical knowledge to be obtained later on the job. not only what teachers know but also how teachers are able to use mathematical knowledge in the course of their work (Ball *et al.*, 2001) and how much they transfer their knowledge and skills to their classrooms (D. Cohen, Raudenbush, & Ball, 2003) is important. Therefore, a further research may focus on observing participants in their real classrooms and investigating how they are using their MKT.

APPENDIX A: MKT Measure Sample Items

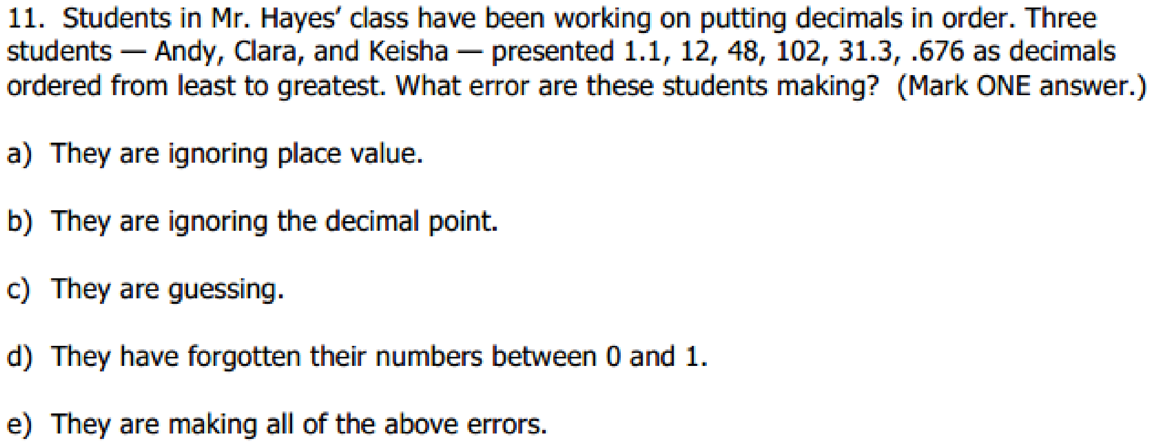
ELEMENTARY CONTENT KNOWLEDGE ITEM SAMPLE



7. Which of the following story problems could be used to illustrate $1\frac{1}{4}$ divided by $\frac{1}{2}$? (Mark YES, NO, or I'M NOT SURE for each possibility.)

	Yes	No	I'm not sure
a) You want to split $1\frac{1}{4}$ pies evenly between two families. How much should each family get?	1	2	3
b) You have $1.25 and may soon double your money. How much money would you end up with?	1	2	3
c) You are making some homemade taffy and the recipe calls for $1\frac{1}{4}$ cups of butter. How many sticks of butter (each stick = $\frac{1}{2}$cup) will you need?	1	2	3

KNOWLEDGE OF CONTENT AND STUDENTS ITEM SAMPLE



11. Students in Mr. Hayes' class have been working on putting decimals in order. Three students — Andy, Clara, and Keisha — presented 1.1, 12, 48, 102, 31.3, .676 as decimals ordered from least to greatest. What error are these students making? (Mark ONE answer.)

a) They are ignoring place value.

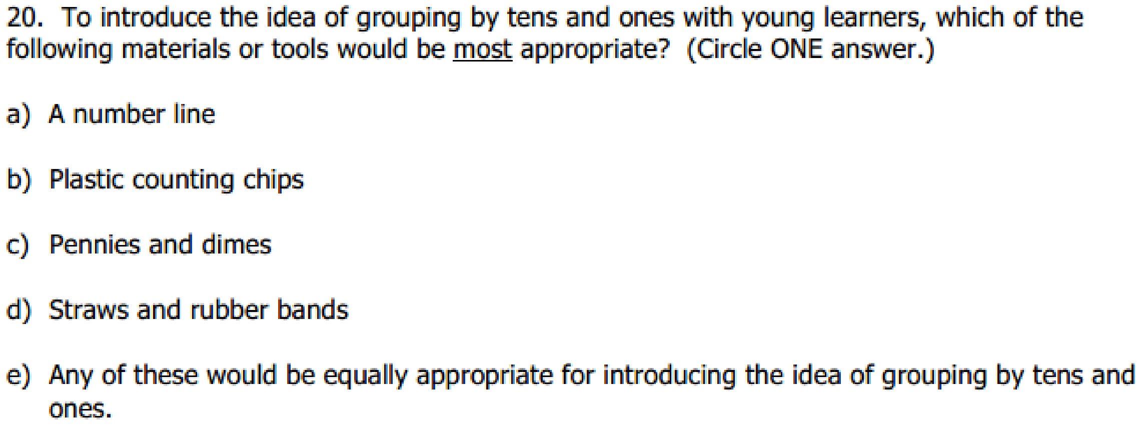
b) They are ignoring the decimal point.

c) They are guessing.

d) They have forgotten their numbers between 0 and 1.

e) They are making all of the above errors.

KNOWLEDGE OF CONTENT AND TEACHING ITEM SAMPLE



20. To introduce the idea of grouping by tens and ones with young learners, which of the following materials or tools would be <u>most</u> appropriate? (Circle ONE answer.)

a) A number line

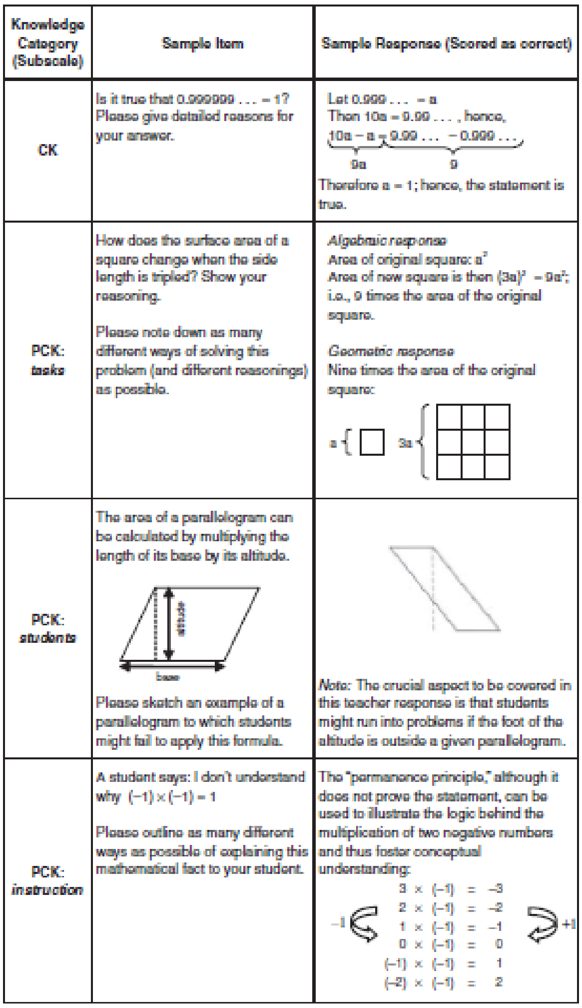
b) Plastic counting chips

c) Pennies and dimes

d) Straws and rubber bands

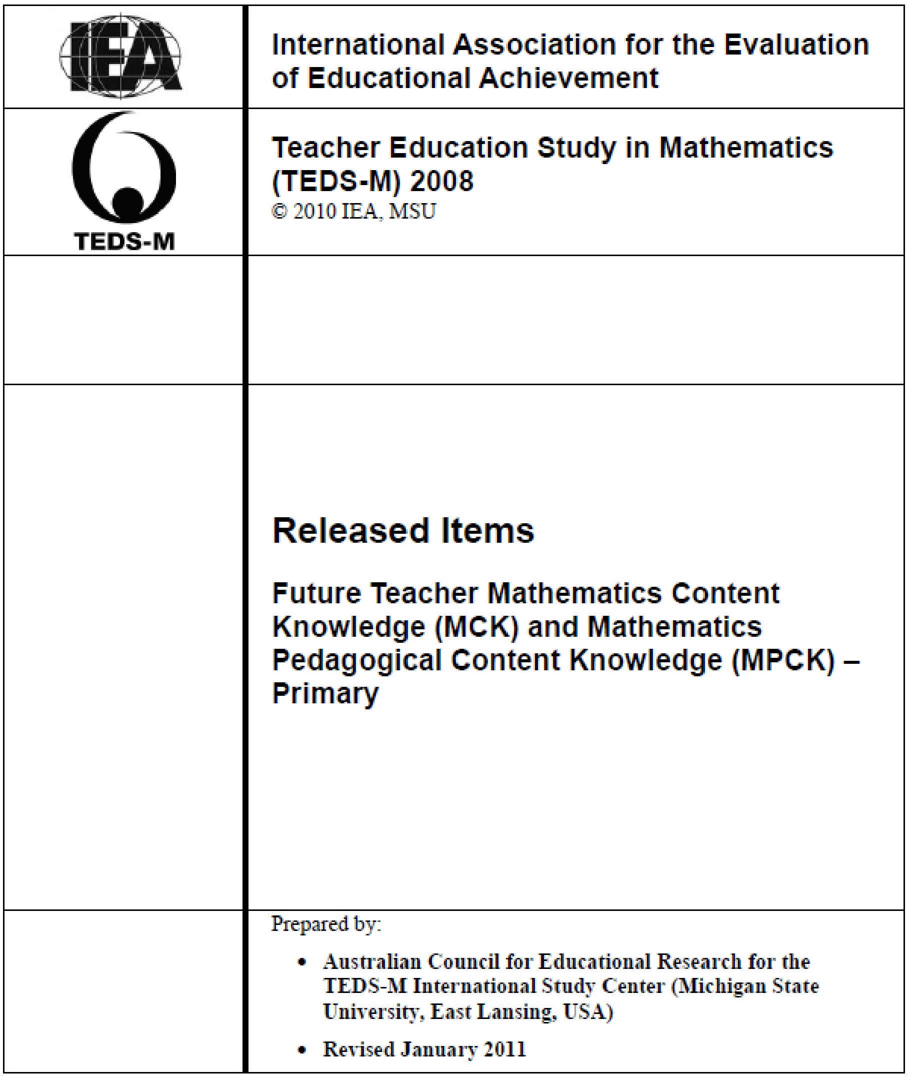
e) Any of these would be equally appropriate for introducing the idea of grouping by tens and ones.

APPENDIX B: COACTIV Measure Released Items



Knowledge Category (Subscale)	Sample Item	Sample Response (Scored as correct)
CK	Is it true that 0.999999 . . . = 1? Please give detailed reasons for your answer.	Let 0.999 . . . = a Then 10a = 9.99 . . . , hence, $\underbrace{10a - a}_{9a} = \underbrace{9.99\ldots - 0.999\ldots}_{9}$ Therefore a = 1; hence, the statement is true.
PCK: ***tasks***	How does the surface area of a square change when the side length is tripled? Show your reasoning. Please note down as many different ways of solving this problem (and different reasonings) as possible.	*Algebraic response* Area of original square: a^2 Area of new square is then $(3a)^2 = 9a^2$; i.e., 9 times the area of the original square. *Geometric response* Nine times the area of the original square:
PCK: ***students***	The area of a parallelogram can be calculated by multiplying the length of its base by its altitude. Please sketch an example of a parallelogram to which students might fail to apply this formula.	*Note:* The crucial aspect to be covered in this teacher response is that students might run into problems if the foot of the altitude is outside a given parallelogram.
PCK: ***instruction***	A student says: I don't understand why $(-1) \times (-1) = 1$ Please outline as many different ways as possible of explaining this mathematical fact to your student.	The "permanence principle," although it does not prove the statement, can be used to illustrate the logic behind the multiplication of two negative numbers and thus foster conceptual understanding: $3 \times (-1) = -3$ $2 \times (-1) = -2$ $1 \times (-1) = -1$ $0 \times (-1) = 0$ $(-1) \times (-1) = 1$ $(-2) \times (-1) = 2$ −1 +1

APPENDIX C: TEDS-M Primary Released Items



TEDS-M

International Association for the Evaluation of Educational Achievement

Teacher Education Study in Mathematics (TEDS-M) 2008

Released Items

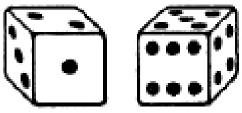
Future Teacher Mathematics Content Knowledge (MCK) and Mathematics Pedagogical Content Knowledge (MPCK) – Primary

Prepared by:

- Australian Council for Educational Research for the TEDS-M International Study Center (Michigan State University, East Lansing, USA)
- Revised January 2011

ID: MFC106	MS Booklet: PM1, PM5	MS Block: B1PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Data		Sub-domain: Applying	

Two fair six-sided number cubes are thrown in a probability game and the two numbers at the top are recorded.



[Josie] wins if the difference between the two numbers is 0, 1 or 2.
[Farid] wins if the difference between the two numbers is 3, 4 or 5.

The students discuss whether the game is fair.

Which of the following statements is correct?

Check one box.

A. Both have an equal chance of winning. ☐$_1$

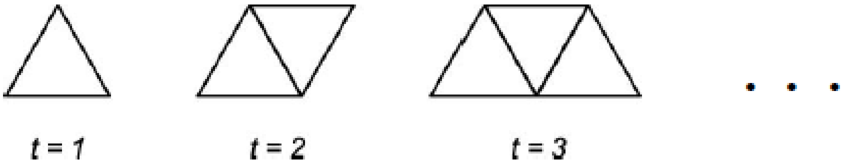
B. [Josie] has the greater chance of winning. ☐$_2$

C. [Farid] has the greater chance of winning. ☐$_3$

D. As the game involves number cubes, it's not possible to say who has the greater chance of winning. ☐$_4$

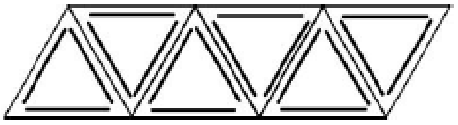
ID: MFC108	MS Booklet: PM1, PM5	MS Block: B1PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MPCK	Content Domain: Algebra		Sub-domain: Enacting	

[Amy] is building a sequence of geometric figures with toothpicks by following the pattern shown below. Each new figure has one extra triangle.
Variable *t* denotes the position of a figure in the sequence.



In finding a mathematical description of the pattern, [Amy] explains her thinking by saying:

I use three sticks for each triangle.



Then I see that I am counting one stick twice for each triangle, except the last one, so I have to remove those.

Variable *n* represents the total number of toothpicks used in a figure.

Which of the equations below best represent [Amy's] statement in algebraic notation?

Check one box.

A. $n = 2t + 1$ ☐$_1$

B. $n = 2(t + 1) - 1$ ☐$_2$

C. $n = 3t - (t - 1)$ ☐$_3$

D. $n = 3t + 1 - \mathrm{t}$ ☐$_4$

ID: MFC202A MFC202B MFC202C MFC202D	MS Booklet: PM1, PM2	MS Block: B2PM	Item Format: CMC	Max Points: 4
Knowledge Dimension: MCK	Content Domain: Algebra		Sub-domain: Knowing	

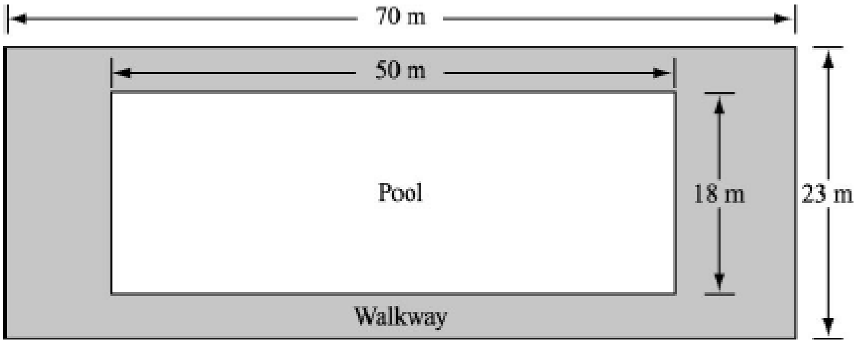
Indicate whether each of the following statements is true for the set of all whole numbers a, b and c greater than zero.

Check one box in each row.

		True	Not True
A.	$a - b = b - a$	☐$_1$	☐$_2$
B.	$a \div b = b \div a$	☐$_1$	☐$_2$
C.	$(a + b) + c = a + (b + c)$	☐$_1$	☐$_2$
D.	$(a - b) - c = a - (b - c)$	☐$_1$	☐$_2$

ID: MFC203	MS Booklet: PM1, PM2	MS Block: B2PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Geometry		Sub-domain: Applying	

A rectangular-shaped swimming pool has a paved walkway (shaded) around it as shown.



not to scale

What is the area of the walkway?

Check one box.

A. 100 m^2 ☐$_1$

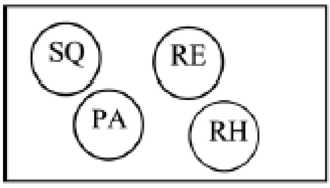
B. 161 m^2 ☐$_2$

C. 710 m^2 ☐$_3$

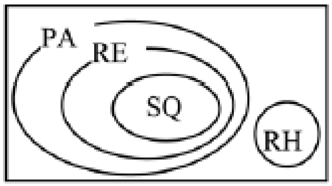
D. 1610 m^2 ☐$_4$

ID: MFC204	MS Booklet: PM1, PM2	MS Block: B2PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Geometry		Sub-domain: Knowing	

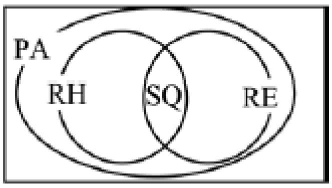
Three students have drawn the following Venn diagrams showing the relationships between four quadrilaterals:
Rectangles (RE), Parallelograms (PA), Rhombuses (RH), and Squares (SQ).



[Tian]



[Rini]



[Mia]

Which student's diagram is correct?

Check one box.

A. [Tian] ☐$_1$

B. [Rini] ☐$_2$

C. [Mia] ☐$_3$

ID: MFC206A	MS Booklet: PM1, PM2	MS Block: B2PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Number		Sub-domain: Applying	

(a) A machine uses 2.4 litres of fuel for every 30 hours of operation.
How many litres of fuel will the machine use in 100 hours if it continues to use fuel at the same rate?

Check one box.

A. 7.2 ☐₁

B. 8.0 ☐₂

C. 8.4 ☐₃

D. 9.6 ☐₄

ID: MFC206B	MS Booklet: PM1, PM2	MS Block: B2PM	Item Format: CR	Max Points: 1
Knowledge Dimension: MPCK	Content Domain: Number		Sub-domain: Planning	

(b) Create a different problem of the same type as the problem in (a) (same processes/operations) that is **EASIER** for <primary> children to solve.

ID: MFC208A	MS Booklet: PM1, PM2	MS Block: B2PM	Item Format: CR	Max Points: 2
Knowledge Dimension: MPCK	Content Domain: Number		Sub-domain: Enacting	

[Jeremy] notices that when he enters 0.2 × 6 into a calculator his answer is smaller than 6, and when he enters 6 ÷ 0.2 he gets a number greater than 6. He is puzzled by this, and asks his teacher for a new calculator!

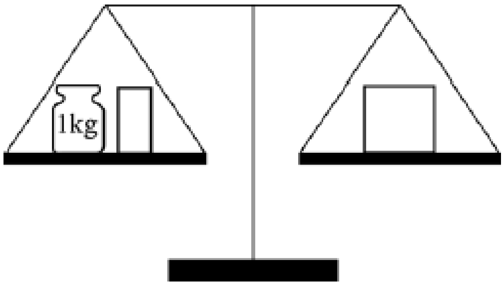
(a) What is [Jeremy's] most likely misconception?

ID: MFC208B	MS Booklet: PM1, PM2	MS Block: B2PM	Item Format: CR	Max Points: 2
Knowledge Dimension: MPCK	Content Domain: Number		Sub-domain: Enacting	

(b) Draw a visual representation that the teacher could use to model 0.2 × 6 to help [Jeremy] understand **WHY** the answer is what it is?

ID: MFC303	MS Booklet: PM2, PM3	MS Block: B3PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Algebra		Sub-domain: Applying	

The objects on the scale make it balance exactly. On the left pan there is a 1 kg mass and half a brick. On the right pan there is one whole brick.



What is the mass of one whole brick?

Check <u>one</u> box.

A. 0.5 kg ☐1

B. 1 kg ☐2

C. 2 kg ☐3

D. 3 kg ☐4

ID: MFC304	MS Booklet: PM2, PM3	MS Block: B3PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Number		Sub-domain: Knowing	

How many decimal numbers are there between 0.20 and 0.30?

Check <u>one</u> box.

A. 9 ☐$_1$

B. 10 ☐$_2$

C. 99 ☐$_3$

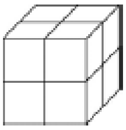
D. An infinite number ☐$_4$

ID: MFC307A	MS Booklet: PM2, PM3	MS Block: B3PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Geometry		Sub-domain: Knowing	

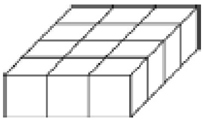
The following problem was given to <primary school> children.

All the small blocks are the same size. Which stack of blocks has a different volume from the others?

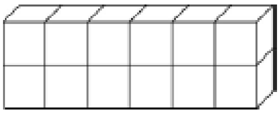
A.



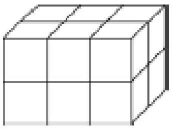
B.



C.



D.



(a) What is the correct answer to this question?

Check one box.

A. Stack A ☐ 1

B. Stack B ☐ 2

C. Stack C ☐ 3

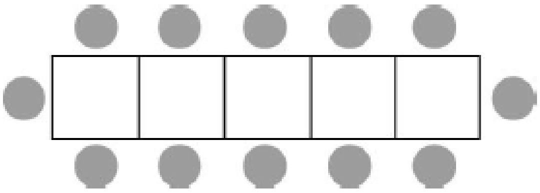
D. Stack D ☐ 4

ID: MFC307B	MS Booklet: PM2, PM3	MS Block: B3PM	Item Format: CR	Max Points: 2
Knowledge Dimension: MPCK	Content Domain: Geometry		Sub-domain: Curriculum/Planning	

(b) How could the question above be rewritten so that it assesses the same skills but **WITHOUT** using the word **VOLUME**?

ID: MFC308	MS Booklet: PM2, PM3	MS Block: B3PM	Item Format: CR	Max Points: 2
Knowledge Dimension: MCK	Content Domain: Algebra		Sub-domain: Applying	

A square table can seat four people, one on each side. When 5 square tables are placed side by side, as shown below, 12 people can sit around them, 5 on each side and 2 on the ends.

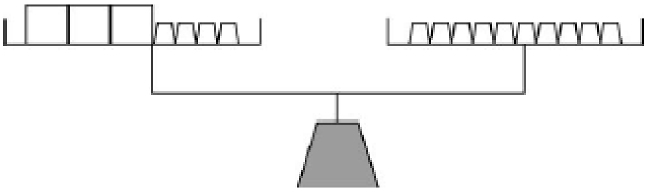


How many people can sit around n square tables when they are placed side by side?

Write your answer to the problem in terms of n.

ID: MFC312	MS Booklet: PM2, PM3	MS Block: B3PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MPCK	Content Domain: Algebra		Sub-domain: Curriculum/Planning	

If B represents the weight (in grams) of each box, ☐, pictured below, and ⏢ represents a one-gram weight, the equation $3B + 4 = 10$ can be pictured by the pan balance shown below.



An inequality such as $3B + 4 < 10$ or $3B + 4 > 10$ would show one side of the pan balance lower than the other.

Ms. [Clarke] is preparing to teach a unit on solving linear equations and inequalities.

If X represents the weight of a given box, which of the following sentences can **NOT BE REPRESENTED** by a pan balance?

Check one box.

A. $13 = 4X + 5$ ☐1

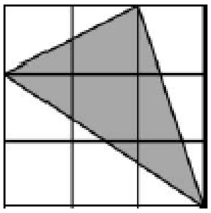
B. $3X + 10 = 4$ ☐2

C. $3X + 3 = 2X + 15$ ☐3

D. $9 + 6X < 21$ ☐4

ID: MFC408	MS Booklet: PM3, PM4	MS Block: B4PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Geometry		Sub-domain: Applying	

The area of each small square is 1 cm^2.



What is the area of the shaded triangle in cm^2?

Check one box.

A. 3.5 cm^2 ☐$_1$

B. 4 cm^2 ☐$_2$

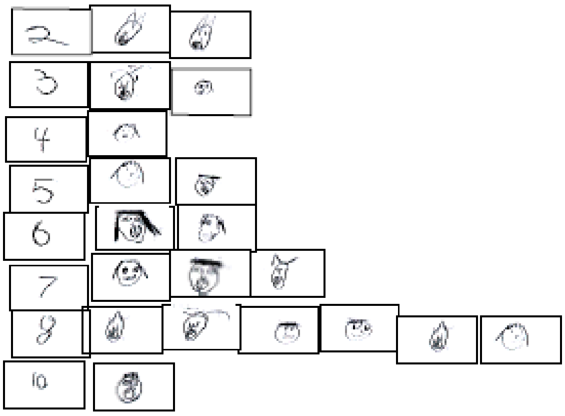
C. 4.5 cm^2 ☐$_3$

D. 5 cm^2 ☐$_4$

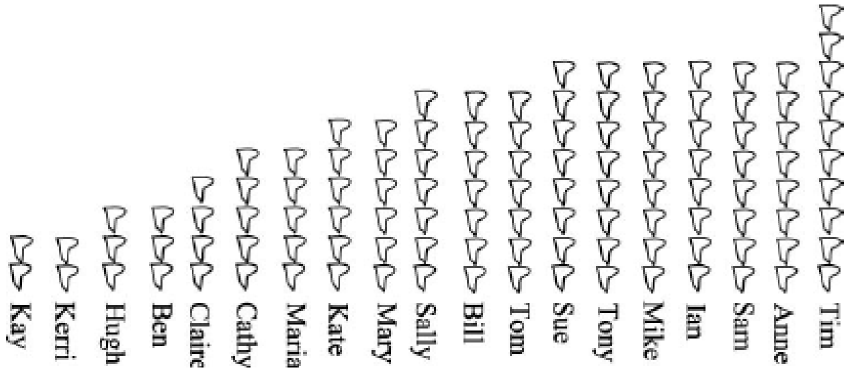
ID: MFC410	MS Booklet: PM3, PM4	MS Block: B4PM	Item Format: CR	Max Points: 2
Knowledge Dimension: MPCK	Content Domain: Data		Sub-domain: Enacting	

Imagine that two <primary> students in the same class have created the following representations to show the number of teeth lost by their classmates.[3]

[Mary] drew pictures of her classmates on cards to make this graph.



[Sally] cut out pictures of teeth to make this graph.



From a data presentation point of view, how are the representations alike and how are they different?

Alike:

Different:

---

[3] This item was used with permission of the author, Dr. Maria Alejandra Sorto, and is based on her Ph. D. dissertation, *Prospective middle school teachers' knowledge about data analysis and its application to teaching*, completed in 2004 at Michigan State University.

ID: MFC412A MFC412B	MS Booklet: PM3, PM4	MS Block: B4PM	Item Format: MC	Max Points: 2
Knowledge Dimension: MCK	Content Domain: Algebra		Sub-domain: Knowing	

[Sam] wanted to find three consecutive **EVEN** numbers that add up to 84.
He wrote the equation $k + (k + 2) + (k + 4) = 84$.

(a) What does the letter k represent?

Check one box.

A. The least of the three even numbers. ☐$_1$

B. The middle even number. ☐$_2$

C. The greatest of the three even numbers. ☐$_3$

D. The average of the three even numbers. ☐$_4$

Which of the following expressions could represent the sum of three consecutive **ODD** numbers?

Check one box.

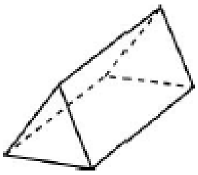
A. $m + (m + 1) + (m + 3)$ ☐$_1$

B. $m + (m + 2) + (m + 4)$ ☐$_2$

C. $m + (m + 3) + (m + 5)$ ☐$_3$

D. $m + (m + 4) + (m + 6)$ ☐$_4$

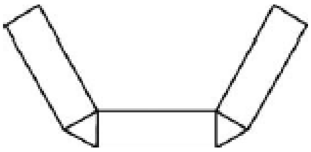
ID: MFC501	MS Booklet: PM4, PM5	MS Block: B5PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Geometry		Sub-domain: Knowing	



Which of the following could be folded to make a shape like the 3-D figure above?

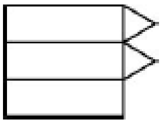
Check one box.

A.



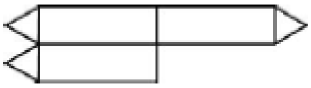
☐1

B.



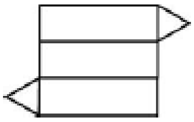
☐2

C.



☐3

D.

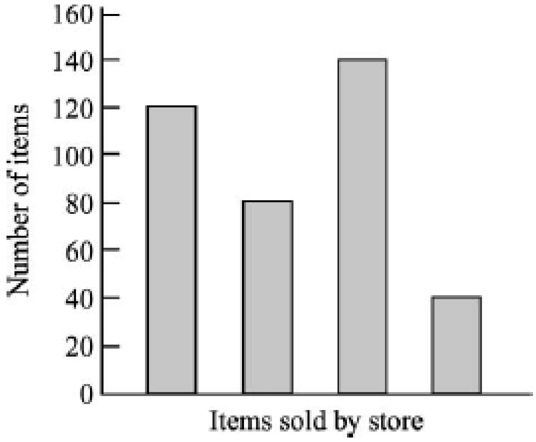


☐4

ID: MFC502A	MS Booklet: PM4, PM5	MS Block: B5PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Data		Sub-domain: Reasoning	

The following problem was given to children in <primary> school.

The graph shows the number of pens, pencils, rulers and erasers sold by a store in one week.



The names of the items are missing from the graph. Pens were the item most often sold. Fewer erasers than any other item were sold. More pencils than rulers were sold.

(a) How many pencils were sold?

Check one box.

A. 40 ☐1

B. 80 ☐2

C. 120 ☐3

D. 140 ☐4

ID: MFC502B	MS Booklet: PM4, PM5	MS Block: B5PM	Item Format: CR	Max Points: 2
Knowledge Dimension: MPCK	Content Domain: Data		Sub-domain: Curriculum/Planning	

(b) Some <primary> students would experience difficulty with a problem of this type. What is the main difficulty you would expect? Explain clearly with reference to the problem.

ID: MFC503A MFC503B MFC503C MFC503D	MS Booklet: PM4, PM5	MS Block: B5PM	Item Format: CMC	Max Points: 4
Knowledge Dimension: MCK	Content Domain: Number		Sub-domain: Knowing	

Indicate for each number whether it is rational or irrational.

Check one box in each row.

		Rational	**Irrational**
A.	π	☐$_1$	☐$_2$
B.	2	☐$_1$	☐$_2$
C.	$\sqrt{49}$	☐$_1$	☐$_2$
D.	$-\frac{3}{2}$	☐$_1$	☐$_2$

ID: MFC505	MS Booklet: PM4, PM5	MS Block: B5PM	Item Format: CR	Max Points: 2
Knowledge Dimension: MPCK	Content Domain: Number		Sub-domain: Curriculum/Planning	

A <Grade 1> teacher asks her students to solve the following four story problems, in any way they like, including using materials if they wish.

Problem 1:	[Jose] has 3 packets of stickers. There are 6 stickers in each pack. How many stickers does [Jose] have altogether?
Problem 2:	[Jorgen] had 5 fish in his tank. He was given 7 more for his birthday. How many fish did he have then?
Problem 3:	[John] had some toy cars. He lost 7 toy cars. Now he has 4 cars left. How many toy cars did [John] have before he lost any?
Problem 4:	[Marcy] had 13 balloons. 5 balloons popped. How many balloons did she have left?

The teacher notices that two of the problems are more difficult for her children than the other two.

Identify the **TWO** problems which are likely to be more **DIFFICULT** to solve for <Grade 1> children.

Problem _______ and Problem _______

ID: MFC508	MS Booklet: PM4, PM5	MS Block: B5PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Algebra		Sub-domain: Applying	

Matchsticks are arranged as shown in the figures.

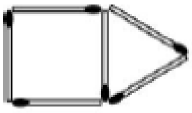


Figure 1

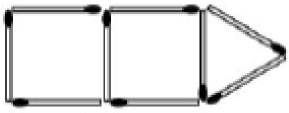


Figure 2

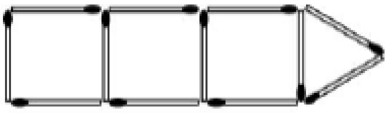


Figure 3

If the pattern is continued, how many matchsticks would be used to make Figure 10?

Check one box.

A. 30 ☐$_1$

B. 33 ☐$_2$

C. 36 ☐$_3$

D. 39 ☐$_4$

E. 42 ☐$_5$

ID: MFC509	MS Booklet: PM4, PM5	MS Block: B5PM	Item Format: CR	Max Points: 2
Knowledge Dimension: MCK	Content Domain: Algebra		Sub-domain: Knowing	

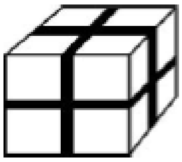
Students who had been studying algebra were asked the following question:

> For any number n, which is larger, $2n$ or $n + 2$?

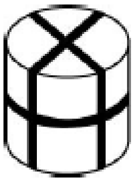
Give the answer and show your reasoning or working.

ID: MFC511	MS Booklet: PM4, PM5	MS Block: B5PM	Item Format: CR	Max Points: 2
Knowledge Dimension: MCK	Content Domain: Geometry		Sub-domain: Applying	

Two gift boxes wrapped with ribbon are shown below. Box A is a cube of side-length 10 cm. Box B is a cylinder with height and diameter 10 cm each.



A



B

Which box needs the longer ribbon? ________

Explain how you arrived at your answer

ID: MFC513	MS Booklet: PM4, PM5	MS Block: B5PM	Item Format: CR	Max Points: 2
Knowledge Dimension: MPCK	Content Domain: Geometry		Sub-domain: Curriculum/Planning	

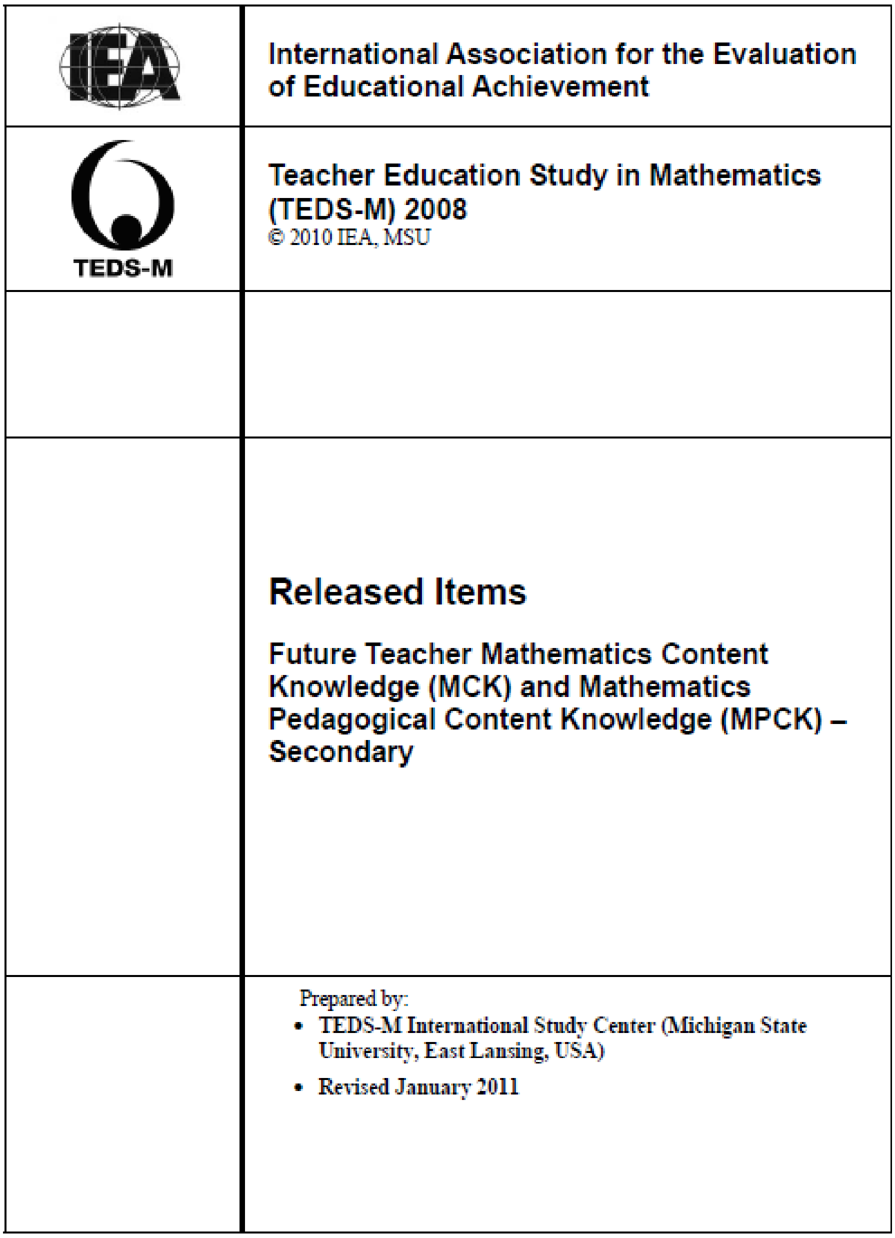
When teaching children about length measurement for the first time, Mrs. [Ho] prefers to begin by having the children measure the width of their book using paper clips, then again using pencils.

Give **TWO** reasons she could have for preferring to do this rather than simply teaching the children how to use a ruler?

Reason 1:

Reason 2:

APPENDIX D: TEDS-M Secondary Released Items



International Association for the Evaluation of Educational Achievement

TEDS-M

Teacher Education Study in Mathematics (TEDS-M) 2008

Released Items

Future Teacher Mathematics Content Knowledge (MCK) and Mathematics Pedagogical Content Knowledge (MPCK) – Secondary

Prepared by:
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- Revised January 2011

ID: MFC604A1 MFC604A2	MS Booklet: SM1, SM3	MS Block: B1SM	Item Format: CR	Max Points: 2
Knowledge Dimension: MCK	Content Domain: Algebra		Sub-domain: Applying	

The following problems appear in a mathematics textbook for <lower secondary school>.

1. [Peter], [David], and [James] play a game with marbles. They have 198 marbles altogether. [Peter] has 6 times as many marbles as [David], and [James] has 2 times as many marbles as [David]. How many marbles does each boy have?

2. Three children [Wendy], [Joyce] and [Gabriela] have 198 zeds altogether. [Wendy] has 6 times as much money as [Joyce], and 3 times as much as [Gabriela]. How many zeds does each child have?

(a) Solve each problem.

Solution to Problem 1:

Solution to Problem 2:

ID: MFC604B	MS Booklet: SM1, SM3	MS Block: B1SM	Item Format: CR	Max Points: 1
Knowledge Dimension: MPCK	Content Domain: Algebra		Sub-domain: Enacting	

(b) Typically Problem 2 is more difficult than Problem 1 for <lower secondary> students. Give one reason that might account for the difference in difficulty level.

ID: MFC610A MFC610C MFC610D	MS Booklet: SM1, SM3	MS Block: B1SM	Item Format: CMC	Max Points: 3
Knowledge Dimension: MCK	Content Domain: Number		Sub-domain: Knowing	

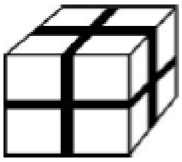
Determine whether each of the following is an irrational number always, sometimes or never.

Check one box in each row.

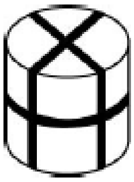
		Always	**Sometimes**	**Never**
A.	The result of dividing the circumference of a circle by its diameter.	☐$_1$	☐$_2$	☐$_3$
C.	The diagonal of a square with side of length 1.	☐$_1$	☐$_2$	☐$_3$
D.	Result of dividing 22 by 7.	☐$_1$	☐$_2$	☐$_3$

ID: MFC511	MS Booklet: PM4, PM5	MS Block: B5PM	Item Format: CR	Max Points: 2
Knowledge Dimension: MCK	Content Domain: Geometry		Sub-domain: Applying	

Two gift boxes wrapped with ribbon are shown below. Box A is a cube of side-length 10 cm. Box B is a cylinder with height and diameter 10 cm each.



A



B

Which box needs the longer ribbon? ________

Explain how you arrived at your answer

ID: MFC704	MS Booklet: SM1, SM2	MS Block: B2SM	Item Format: CR	Max Points: 2
Knowledge Dimension: MCK	Content Domain: Geometry		Sub-domain: Applying	

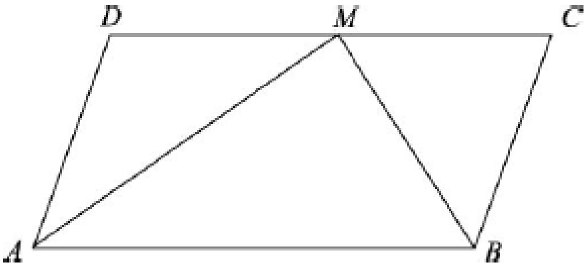
On the figure, $ABCD$ is a parallelogram, $\angle BAD = 60^0$, AM and BM are angle bisectors of angles BAD and ABC respectively. If the perimeter of $ABCD$ is 6 cm, find the sides of triangle ABM.

Write your answers on the lines below.

$AB =$ _______cm

$AM =$ _______cm

$BM =$ _______cm



ID: MFC705A MFC705B	MS Booklet: SM1, SM2	MS Block: B2SM	Item Format: CMC	Max Points: 2
Knowledge Dimension: MCK	Content Domain: Geometry		Sub-domain: Knowing	

We know that there is only one point on the real line that satisfies the equation $3x = 6$, namely $x = 2$.

Suppose now that we consider this same equation in the plane, with coordinates x and y, and then in space with coordinates x, y, and z. What does the set of points that satisfy the equation $3x = 6$ look like in these settings?

Check one box in each row.

	One point	**One line**	**One plane**	**Other**
A. The solution to $3x = 6$ in the plane	☐$_1$	☐$_2$	☐$_3$	☐$_4$
B. The solution to $3x = 6$ in space	☐$_1$	☐$_2$	☐$_3$	☐$_4$

ID: MFC709A MFC709B MFC709C	MS Booklet: SM1, SM2	MS Block: B2SM	Item Format: CMC	Max Points: 3
Knowledge Dimension: MPCK	Content Domain: Number		Sub-domain: Enacting	

Some <lower secondary school> students were asked to prove the following statement:

> When you multiply 3 consecutive natural numbers, the product is a multiple of 6.

Below are three responses.

[Kate's] answer

A multiple of 6 must have factors of 3 and 2.
If you have three consecutive numbers, one will be a multiple of 3.

Also, at least one number will be even and all even numbers are multiples of 2.

If you multiply the three consecutive numbers together the answer must have at least one factor of 3 and one factor of 2.

[Leon's] answer

$1 \times 2 \times 3 = 6$

$2 \times 3 \times 4 = 24 = 6 \times 4$

$4 \times 5 \times 6 = 120 = 6 \times 20$

$6 \times 7 \times 8 = 336 = 6 \times 56$

[Maria's] answer

n is any whole number

$$n \times (n + 1) \times (n + 2) = (n^2 + n) \times (n + 2)$$

$$= n^3 + n^2 + 2n^2 + 2n$$

Canceling the n's gives $1 + 1 + 2 + 2 = 6$

Determine whether each proof is valid.

Check one box in each row.

		Valid	Not valid
A.	[Kate's] proof	☐$_1$	☐$_2$
B.	[Leon's] proof	☐$_1$	☐$_2$
C.	[Maria's] proof	☐$_1$	☐$_2$

ID: MFC710A MFC710B MFC710C	MS Booklet: SM1, SM2	MS Block: B2SM	Item Format: CMC	Max Points: 3
Knowledge Dimension: MCK	Content Domain: Algebra		Sub-domain: Applying	

Indicate whether each of the following situations can be modeled by an exponential function.

Check one box in each row.

		Yes	No
A.	The height *h* of a ball *t* seconds after it is thrown into the air.	☐$_1$	☐$_2$
B.	The amount of money *A* in a bank after *w* weeks, if each week *d* zeds are put in the bank.	☐$_1$	☐$_2$
C.	The value *V* of a car after *t* years if it depreciates *d* % per year.	☐$_1$	☐$_2$

ID: MFC711	MS Booklet: SM1, SM2	MS Block: B2SM	Item Format: CR	Max Points: 2
Knowledge Dimension: MCK	Content Domain: Algebra		Sub-domain: Reasoning	

Prove the following statement:

If the graphs of linear functions

$$f(x) = ax + b \text{ and } g(x) = cx + d$$

intersect at a point P on the x-axis, the graph of their sum function

$$(f + g)(x)$$

must also go through P.

ID: MFC712A MFC712B MFC712C MFC712D	MS Booklet: SM1, SM2	MS Block: B2SM	Item Format: CMC	Max Points: 4
Knowledge Dimension: MPCK	Content Domain: Algebra		Sub-domain: Planning	

A mathematics teacher wants to show some <lower secondary school> students how to prove the quadratic formula.

Determine whether each of the following types of knowledge is needed in order to understand a proof of this result.

Check one box in each row.

		Needed	**Not needed**
A.	How to solve linear equations.	☐$_1$	☐$_2$
B.	How to solve equations of the form $x^2 = k$, where $k > 0$.	☐$_1$	☐$_2$
C.	How to complete the square of a trinomial.	☐$_1$	☐$_2$
D.	How to add and subtract complex numbers.	☐$_1$	☐$_2$

ID: MFC802A MFC802B MFC802C MFC802D	MS Booklet: SM2, SM3	MS Block: B3SM	Item Format: CMC	Max Points: 4
Knowledge Dimension: MCK	Content Domain: Number		Sub-domain: Reasoning	

You have to prove the following statement:

If the square of any natural number is divided by 3, then the remainder is only 0 or 1.

State whether each of the following approaches is a mathematically correct proof.

Check one box in each row.

Yes **No**

A. Use the following table:

Number	1	2	3	4	5	6	7	8	9	10
Square	1	4	9	16	25	36	49	64	81	100
Remainder when divided by 3	1	1	0	1	1	0	1	1	0	1

☐$_1$ ☐$_2$

B. Demonstrate that $(3n)^2$ is divisible by 3 and for all other numbers, $(3n \pm 1)^2 = 9n^2 \pm 6n + 1$ which always has a remainder of 1 once it has been divided by 3. ☐$_1$ ☐$_2$

C. Choose a natural number *n*, find its square n^2, and then check whether the statement is true or not. ☐$_1$ ☐$_2$

D. Check the statement for the first several prime numbers and then draw a conclusion based on the Fundamental Theorem of Arithmetic. ☐$_1$ ☐$_2$

ID: MFC804	MS Booklet: SM2, SM3	MS Block: B3SM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Number		Sub-domain: Knowing	

A class has 10 students. If at one time, 2 students are to be chosen, and another time 8 students are to be chosen from the class, which of the following statements is true?

Check <u>one</u> box.

A. There are more ways to choose 2 students than 8 students from the class. ☐1

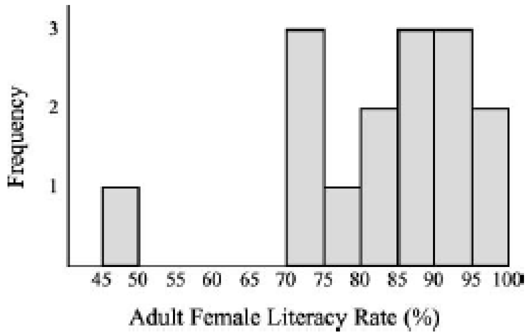
B. There are more ways to choose 8 students than 2 students from the class. ☐2

C. The number of ways to choose 2 students equals the number of ways to choose 8 students. ☐3

D. It is not possible to determine which selection has more possibilities. ☐4

ID: MFC806A	MS Booklet: SM2, SM3	MS Block: B3SM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Data		Sub-domain: Applying	

The following graph gives information about the adult female literacy rates in Central and South American countries.[3]



Suppose you ask your students to tell you how many countries are represented in the graph. One student says, "There are 7 countries represented."

Check one box.

Right **Wrong**

a) Is the student right or wrong? ☐ 1 ☐ 2

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[3] This item is copyright 2004 by Maria Alejandra Sorto as part of her Ph. D. dissertation *Prospective Middle School Teachers' Knowledge about Data Analysis and its Application to Teaching* at Michigan State University. It is used with her permission

with her permission

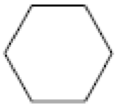
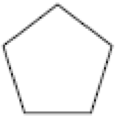
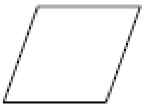
ID: MFC806B	MS Booklet: SM2, SM3	MS Block: B3SM	Item Format: CR	Max Points: 1
Knowledge Dimension: MPCK	Content Domain: Data		Sub-domain: Enacting	

b) In your opinion, what was the student thinking in order to arrive at that conclusion?

ID: MFC808A MFC808B MFC808C	MS Booklet: SM2, SM3	MS Block: B3SM	Item Format: CMC	Max Points: 3
Knowledge Dimension: MCK	Content Domain: Geometry		Sub-domain: Applying	

Your students have been working on symmetry. They were given the task below requiring them to decide the number of lines of symmetry for three different shapes.

Answers of [Sam] and [Michael] are shown in the table. Correct the answers of each student by checking correct or incorrect.

		Students and their answers about the number of the lines of symmetry	
Shape	**Shape name**	**[Sam]**	**[Michael]**
	regular hexagon	6 ☐$_1$ Correct ☐$_2$ Incorrect	12 ☐$_1$ Correct ☐$_2$ Incorrect
	regular pentagon	5 ☐$_1$ Correct ☐$_2$ Incorrect	10 ☐$_1$ Correct ☐$_2$ Incorrect
	rhombus	4 ☐$_1$ Correct ☐$_2$ Incorrect	2 ☐$_1$ Correct ☐$_2$ Incorrect

Note: This CMC question originally was considered as six items. After psychometric analysis, it was recoded as three items and scored as follows.

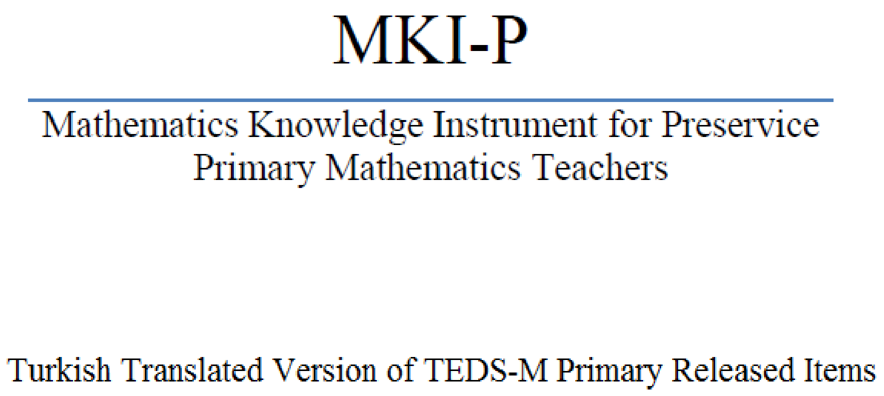
MFC808A: Score 1 if answers of both Sam and Michael are correctly checked (1 and 2); otherwise, score 0.
MFC808B: Score 1 if answers of both Sam and Michael are correctly checked (1 and 2); otherwise, score 0.
MFC808C: Score 1 if answers of both Sam and Michael are correctly checked (2 and 1); otherwise, score 0.

ID: MFC814	MS Booklet: SM2, SM3	MS Block: B3SM	Item Format: CR	Max Points: 2
Knowledge Dimension: MCK	Content Domain: Algebra		Sub-domain: Reasoning	

Let $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ and $B = \begin{bmatrix} t & u \\ v & w \end{bmatrix}$. Then $A \otimes B$ is defined to be $\begin{bmatrix} pt & qu \\ rv & sw \end{bmatrix}$.

Is it true that if $A \otimes B = \mathrm{O}$, then either $A = \mathrm{O}$ or $B = \mathrm{O}$ (where O represents the zero matrix)? Justify your answer.

APPENDIX E: Mathematics Knowledge Instrument for Preservice Primary Mathematics Teachers (MKI-P)

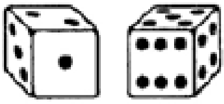


MKI-P

Mathematics Knowledge Instrument for Preservice
Primary Mathematics Teachers

Turkish Translated Version of TEDS-M Primary Released Items

1.MFC106
Bir olasılık oyununda 2 tane 6 yüzlü hilesiz zar atılıyor ve zarın üst yüzüne gelen sayılara bakılıyor.



Eğer 2 sayı arasındaki fark 0, 1 ya da 2 olursa Caner kazanıyor.
Eğer 2 sayı arasındaki fark 3, 4 ya da 5 ise Fatih kazanıyor.

Öğrenciler bu oyununun adil olup olmadığını tartışıyorlar.
Bu durumda aşağıdakilerden hangisi doğru olur?

Yalnız <u>bir</u> kutucuk işaretleyiniz.

A. Her ikisinin de kazanma şansı eşittir. ☐1

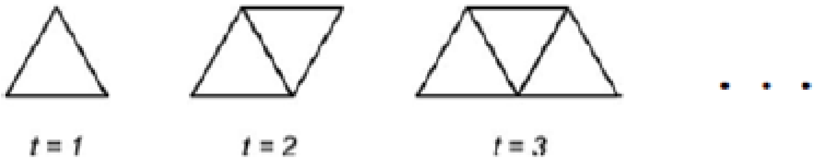
B. Caner'in kazanma şansı daha yüksektir. ☐2

C. Fatih'in kazanma şansı daha yüksektir. ☐3

D. Zarlarla oynanan bir oyun olduğundan, kimin daha yüksek kazanma şansı olduğunu söylemek mümkün değildir. ☐4

2. MFC108

Aslı, kürdanları kullanarak her yeni adımda üçgen sayısını bir arttıracak şekilde aşağıdaki örüntüyü oluşturuyor. Örüntüde her adımın sırasını da t harfiyle belirtiyor.



Bu örüntüye matematiksel bir açıklama getirirken, Aslı kendi düşüncesini şöyle açıklıyor:

Her üçgen için 3 kürdan kullandım.

Sonra gördüm ki sonuncusu dışındaki her üçgen için bir kürdanı iki defa sayıyorum. Bu yüzden onları çıkartmak zorundayım.

n değişkeni şekildeki toplam kürdan sayısını belirttiğine göre aşağıdaki eşitliklerden hangisi Aslı'nın açıklamasının cebirsel gösterimini en iyi şekilde ifade eder?

Yalnız <u>bir</u> kutucuk işaretleyiniz.

A. $n = 2t + 1$ ☐

B. $n = 2(t + 1) - 1$ ☐

C. $n = 3t - (t - 1)$ ☐

D. $n = 3t + 1 - t$ ☐

3. MFC202

a,b ve c sıfırdan büyük tam sayılar olduklarına göre aşağıdaki ifadelerin her birinin doğru olup olmadığını belirtiniz.

__Her__ satır için __bir__ kutucuk işaretleyiniz.

		Doğru	**Doğru Değil**
A.	$a - b = b - a$	☐$_1$	☐$_2$
B.	$a \div b = b \div a$	☐$_1$	☐$_2$
C.	$(a + b) + c = a + (b + c)$	☐$_1$	☐$_2$
D.	$(a - b) - c = a - (b - c)$	☐$_1$	☐$_2$

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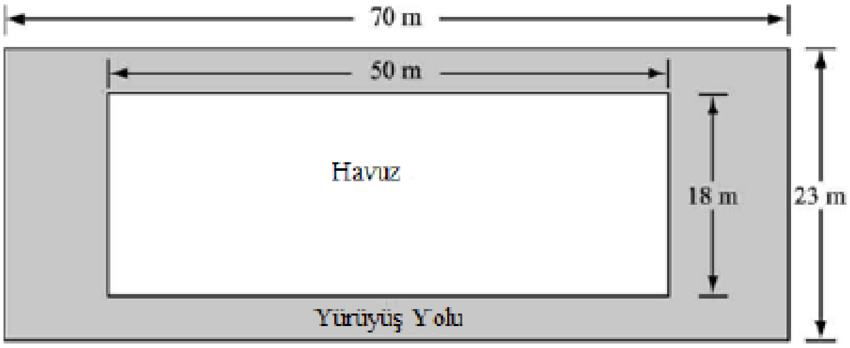
4. MFC203

Dikdörtgen şeklindeki bir yüzme havuzunun etrafı şekilde görüldüğü gibi yürüyüş yolu yapmak için asfaltlanıyor (taralı kısım).



Yürüyüş yolunun alanını bulunuz.

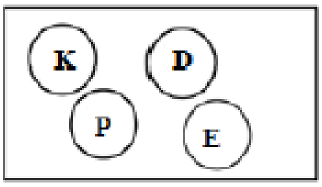
Yalnız __bir__ kutucuk işaretleyiniz.

A.	$100\ m^2$	☐$_1$
B.	$161\ m^2$	☐$_2$
C.	$710\ m^2$	☐$_3$
D.	$1610\ m^2$	☐$_4$

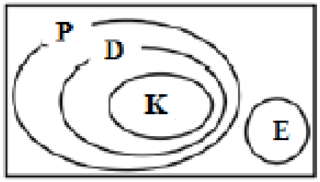
5. MFC204

3 öğrenci belirtilen dörtgenler arasındaki ilişkiyi göstermek için aşağıdaki Venn şemalarını çiziyor.

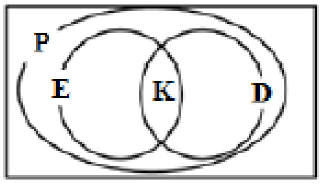
Dikdörtgen(D), Paralelkenar(P), Eşkenar Dörtgen(E) ve Kare(K).



Taner



Aslı



Melda

Hangi öğrencinin çizdiği şema doğrudur.

Yalnız bir kutucuk işaretleyiniz.

A. Taner ☐$_1$

B. Aslı ☐$_2$

C. Melda ☐$_3$

6. MFC206

(a) Bir makine çalışırken her 30 saat için 2,4 litre mazot tüketiyor. Eğer bu makine aynı oranda mazot tüketmeye devam ederse, 100 saat içinde kaç litre mazot tüketir?

Yalnız <u>bir</u> kutucuk işaretleyiniz.

A. 7,2 ☐

B. 8,0 ☐

C. 8,4 ☐

D. 9,6 ☐

(b) Yukarıdaki (a şıkkındaki) problemle aynı türde (aynı işlem ve adımlar), ilkokul öğrencilerinin **DAHA KOLAY** çözebileceği başka bir problem yazınız.

7. MFC208

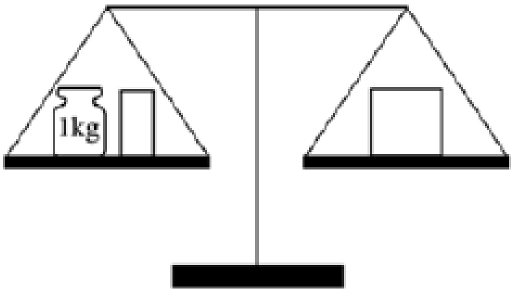
Ceren hesap makinesinde *0,2 x 6* işlemini yapınca cevabın 6'dan küçük olduğunu görüyor. Hesap makinesine *6 ÷ 0,2* işlemini yazınca da bu sefer 6'dan daha büyük bir sayı elde ediyor. Bu durum Ceren'in kafasını epey karıştırıyor ve öğretmeninden yeni bir hesap makinesi istiyor.

(a) Ceren'in kavram yanılgısı büyük ihtimalle ne olabilir?

(b) Cevapların **NEDEN** olması gerektiği gibi olduğunu Ceren'in anlamasına yardımcı olmak amacıyla, öğretmen *0,2 x 6* işlemini modellemek için nasıl bir gösterim şekli kullanabilir? Lütfen çiziniz.

8. MFC303

Şekilde eşit kollu terazideki cisimler dengededir. Terazinin sol kefesinde 1 kg'lık ağırlık ile bir yarım tuğla vardır. Sağ kesesinde de bir bütün tuğla bulunmaktadır.



Bu durumda bir bütün tuğlanın ağırlığı ne kadardır?

Yalnız <u>bir</u> kutucuk işaretleyiniz.

A. 0,5 kg ☐$_1$

B. 1 kg ☐$_2$

C. 2 kg ☐$_3$

D. 3 kg ☐$_4$

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9. MFC304

0,20 ile 0,30 sayıları arasında kaç ondalık sayı vardır?

Yalnız <u>bir</u> kutucuk işaretleyiniz.

A. 9 ☐$_1$

B. 10 ☐$_2$

C. 99 ☐$_3$

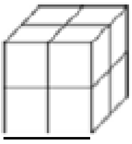
D. Sonsuz sayıda ☐$_4$

10. MFC307

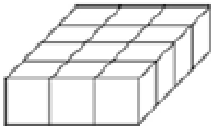
Aşağıdaki soru ilkokul öğrencilerine veriliyor.

Küçük kutuların hepsi aynı büyüklüktedir. Bu kutulardan oluşturulan aşağıdaki bloklardan hangisinin hacmi diğerlerinden farklıdır?

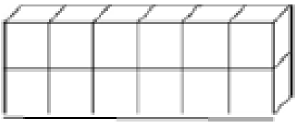
A.



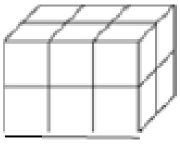
B.



C.



D.



(a) Bu sorunun doğru cevabı nedir?

Yalnız bir kutucuk işaretleyiniz.

A. Blok A ☐1

B. Blok B ☐2

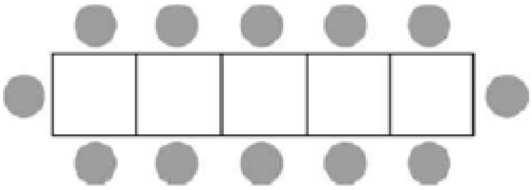
C. Blok C ☐3

D. Blok D ☐4

(b) Yukarıdaki soru tekrar yazılacak olsa, öğrencilerin aynı beceresini ölçecek şekilde ama "HACİM" sözcüğünü KULLANMADAN nasıl yazılırdı?

11. MFC308

Bir kare masaya her kenarda bir kişi olacak şekilde dört kişi oturabiliyor. Aşağıda görüldüğü şekilde 5 tane kare masa yan yana diziliyor. Kenarlara 5'er kişi, uçlarda da 2 kişi olmak üzere karelerin etrafına 12 kişi oturabiliyor.

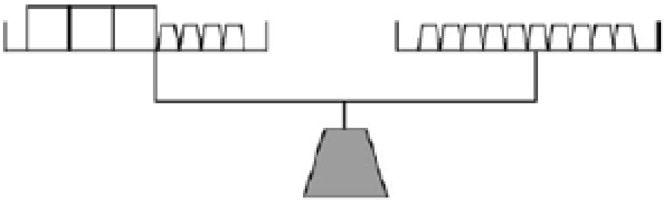


n tane masa yan yana dizilirse bu masaların etrafına kaç kişi oturabilir?

Bu sorunun cevabını *n* cinsinden yazınız.

12. MFC312

Aşağıdaki düzenekte, ⏢ şeklindeki cisimler 1 gramlık ağırlıkları temsil ediyor. □ şeklindeki her bir kutunun ağılığı (gram olarak) da "*K*" harfi ile ifade edilirse, $3K+4=10$ denklemi aşağıdaki gibi bir eşit kollu bir terazi ile resmedilebilir.



$3K+4 < 10$ ya da $3K+4 > 10$ gibi eşitsizlik durumları da, bir kefesi diğerinde daha aşağıda duran bir eşit kollu teraziyle gösterilebilir.

Canan öğretmen birinci dereceden denklemelerin ve eşitsizliklerin çözümü üzerine bir ünite planı hazırlıyor.

X verilen bir kutunun ağırlığını ifade ettiğine göre, aşağıdakilerden hangisi bir eşit kollu teraziyle **GÖSTERİLEMEZ.**

Yalnız <u>bir</u> kutucuk işaretleyiniz.

A. $13 = 4X + 5$ ☐

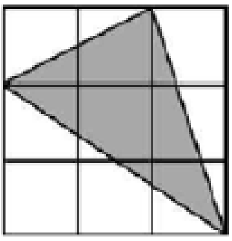
B. $3X + 10 = 4$ ☐

C. $3X + 3 = 2X + 15$ ☐

D. $9 + 6X < 21$ ☐

13. MFC408

Şekildeki her bir küçük karenin alanı 1cm^2'dir.



Bu durumda taralı üçgenin alanı kaç cm^2'dir?

Yalnız <u>bir</u> kutucuk işaretleyiniz.

A. 3,5 cm^2 ☐$_1$

B. 4 cm^2 ☐$_2$

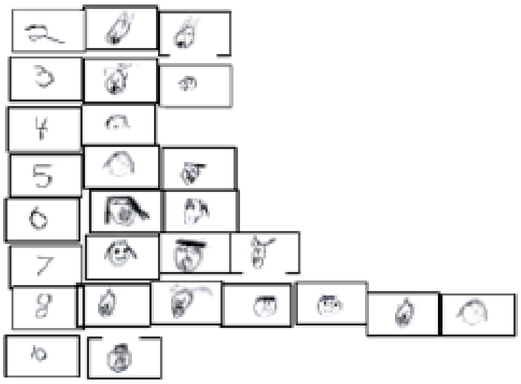
C. 4,5 cm^2 ☐$_3$

D. 5 cm^2 ☐$_4$

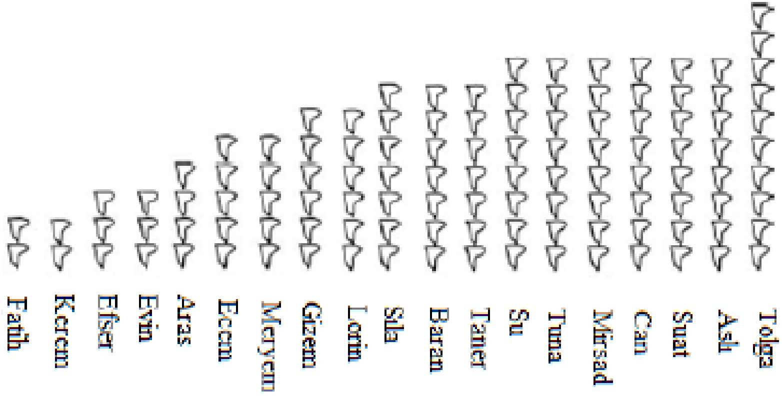
14. MFC410

Bir sınıftaki iki ilkokul öğrencisinin, sınıf arkadaşlarının düşürdükleri dişleri göstermek için aşağıdaki görselleri hazırladıklarını farz edin.

Meltem bu grafiği hazırlamak için sınıf arkadaşlarının resimlerini kartlara çiziyor.



Seda bu grafiği hazırlamak için diş şeklinde kağıtlar kesiyor.



Veri sunumu açısından, bu iki gösterim nasıl benzerlikler ve farklılıklar gösterir?

Benzerlikler:

Farklılıklar:

15. MFC412

Sinan, toplamları 84 olacak şekilde ardışık üç **ÇİFT** sayı bulmak istiyor. Bunun için şu denklemi yazıyor: $k + (k+2) + (k+4) = 84$

(a) *k* harfi burada neyi ifade etmektedir?

Yalnız bir kutucuk işaretleyiniz.

A. Bu üç çift sayının en küçüğünü. ☐1

B. Ortadaki çift sayıyı. ☐2

C. Bu üç çift sayının en büyüğünü. ☐3

D. Bu üç çift sayının aritmetik ortalamasını. ☐4

(b) Aşağıdakilerden ifadelerden hangisi ardışık üç **TEK** sayının toplamını gösterir?

Yalnız bir kutucuk işaretleyiniz.

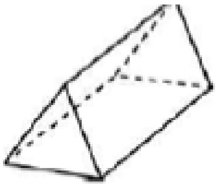
A. $m + (m + 1) + (m + 3)$ ☐1

B. $m + (m + 2) + (m + 4)$ ☐2

C. $m + (m + 3) + (m + 5)$ ☐3

D. $m + (m + 4) + (m + 6)$ ☐4

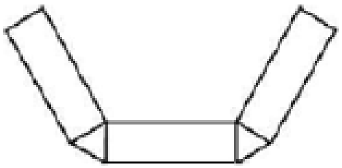
16. MFC501



Aşağıdakilerden hangisi katlandığında yukarıdaki üç boyutlu şekil elde edilebilir?

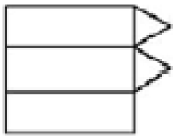
Yalnız <u>bir</u> kutucuk işaretleyiniz.

A.



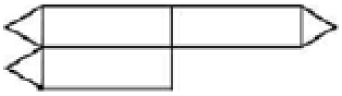
☐1

B.



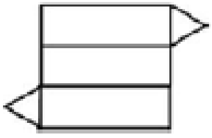
☐2

C.



☐3

D.

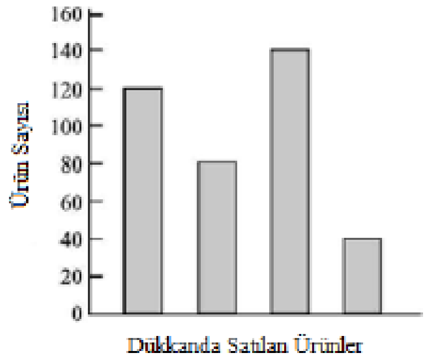


☐4

17. MFC502

Aşağıdaki problem ilkokul öğrencilerine veriliyor.

Aşağıdaki grafikte bir dükkânda bir haftada satılan kurşun kalem, tükenmez kalem, cetvel ve silgi sayıları gösterilmektedir.



Ürünlerin isimleri grafikte yer almıyor. Dükkânda en çok tükenmez kalem satılmış. Silgi diğerlerine göre en az satılan ürün olmuş. Kurşun kalem satışı da cetvel satışından daha fazlaymış.

(a) Kaç tane kurşun kalem satılmıştır?

Yalnız <u>bir</u> kutucuk işaretleyiniz.

A. 40 ☐$_1$

B. 80 ☐$_2$

C. 120 ☐$_3$

D. 140 ☐$_4$

(b) Bazı ilkokul öğrencileri bu tarz bir soruda zorluk çekebililer. Sizce bu soruda, öğrencilerin asıl zorlanacakları şey ne olurdu? Soruyla ilişkilendirerek açıklayınız.

18. MFC503

Aşağıdaki sayıların rasyonel mi irrasyonel mi olduğunu kutucukları işaretleyerek belirtiniz.

Her satır için bir kutucuk işaretleyiniz.

		Rasyonel	İrrasyonel
A.	π	☐1	☐2
B.	2	☐1	☐2
C.	$\sqrt{49}$	☐1	☐2
D.	$-\frac{3}{2}$	☐1	☐2

---

19. MFC505

Bir 1.sınıf öğretmeni öğrencilerinden aşağıdaki 4 problemi, istedikleri yoldan ve isterlerse araç gereç de kullanarak çözmelerini istiyor.

Problem 1: Can'ın 3 paket çıkartması vardır. Her pakette 6 tane çıkartma olduğuna göre Can'ın bütün çıkartmalarının sayısı kaçtır?

Problem 2: Evrim'in akvaryumda 5 tane balığı vardı. Doğum gününde Evrim'e 7 tane daha balık hediye edildi. Bu durumda Evrim'in kaç tane balığı oldu?

Problem 3: Murat'ın oyuncak arabaları vardı. Murat oyuncak arabalarının 7 tanesini kaybetti ve geriye 4 tane kaldı. Murat'ın kaybetmeden önce kaç tane oyuncak arabası vardı?

Problem 4: Mine'nin 13 tane balonu vardı. Balonlardan 5 tanesi patladı. Mine'nin geriye kaç tane balonu kalmıştır?

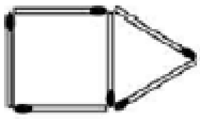
Öğretmen bu problemlerden ikisinin öğrencileri için diğer ikisine göre daha zor olduğunu fark ediyor.

Hangi **İKİ** problemin 1. sınıf öğrencilerinin çözmesi için daha **ZOR** olabileceğini belirtiniz.

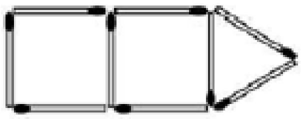
Soru_________ve Soru_________

20. MFC204

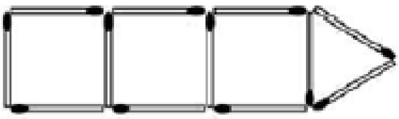
Kibrit çöpleri aşağıdaki şekillerde görüldüğü gibi diziliyor.



Şekil 1



Şekil 2



Şekil 3

Örüntü bu şekilde devam ederse, Şekil 10'u yapmak için kaç tane kibrit çöpü kullanılır?

Yalnız bir kutucuk işaretleyiniz.

A. 30 ☐$_1$
B. 33 ☐$_2$
C. 36 ☐$_3$
D. 39 ☐$_4$
E. 42 ☐$_5$

---

21. MFC509

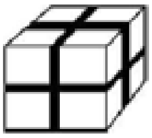
Cebir öğrenen öğrencilere aşağıdaki soru soruluyor:

> Herhangi bir n sayısı için, $2n$ mi yoksa $n+2$ mi daha büyüktür?

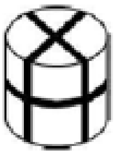
Cevabınızı yazınız ve mantıksal açıklamanızı ya da işlemlerinizi gösteriniz.

22. MFC511

İki hediye paketi aşağıdaki gibi kurdelelerle paketleniyor. A paketi bir kenarı 10 cm olan bir küp, B paketi ise yüksekliği ve çapı 10 cm olan bir silindirdir.



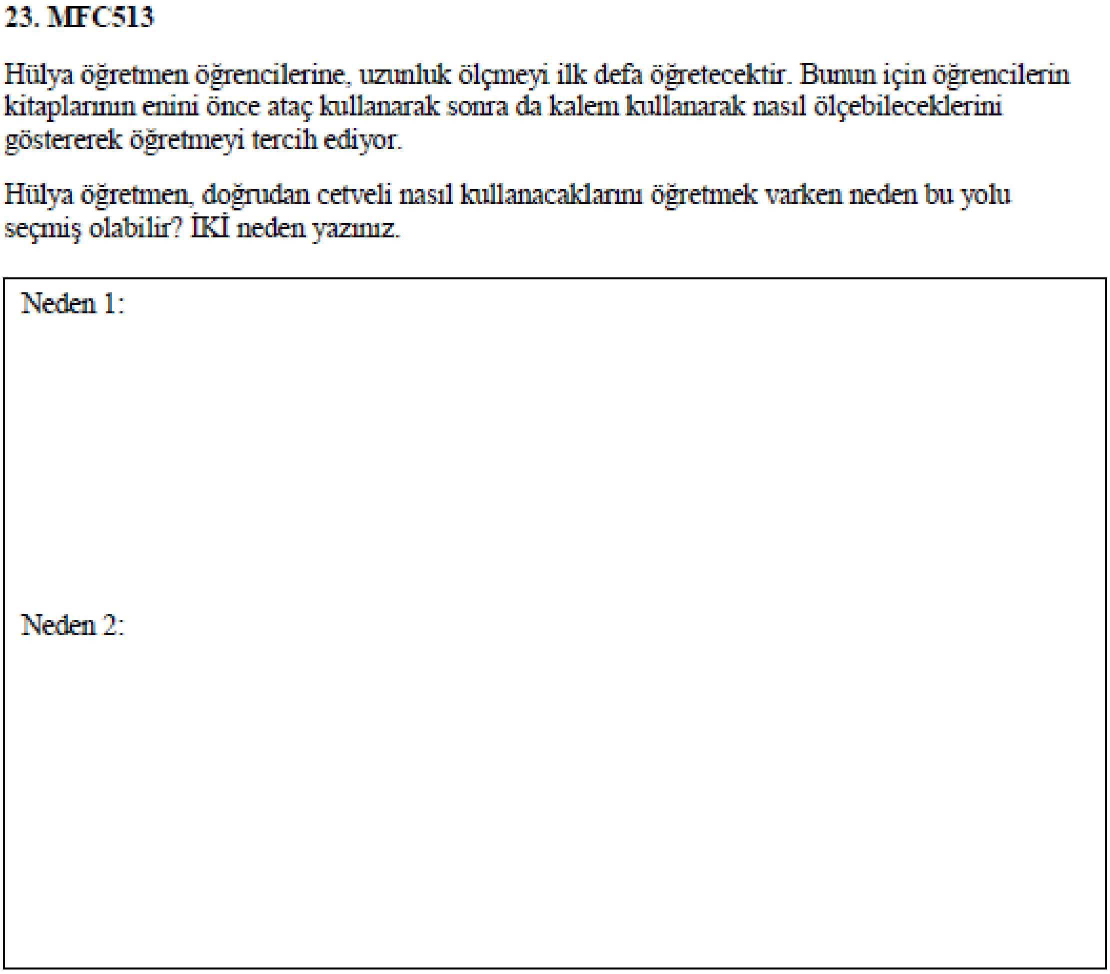
A



B

Hangi paket için daha uzun kurdeleye ihtiyaç vardır? ________

Bu sonuca nasıl ulaştığınızı açıklayınız.



23. MFC513

Hülya öğretmen öğrencilerine, uzunluk ölçmeyi ilk defa öğretecektir. Bunun için öğrencilerin kitaplarının enini önce ataç kullanarak sonra da kalem kullanarak nasıl ölçebileceklerini göstererek öğretmeyi tercih ediyor.

Hülya öğretmen, doğrudan cetveli nasıl kullanacaklarını öğretmek varken neden bu yolu seçmiş olabilir? İKİ neden yazınız.

Neden 1:

Neden 2:

APPENDIX F: Mathematics Knowledge Instrument for Preservice Secondary Mathematics Teachers (MKI-S)

MKI-S

Mathematics Knowledge Instrument for Preservice Secondary Mathematics Teachers

Turkish Translated Version of TEDS-M Secondary Released Items

1. MFC604

Bir ortaokul matematik kitabında aşağıdaki problemler yer alıyor.

1. Barkın, Deniz ve Engin bilyelerle bir oyun oynuyorlar. Toplam 198 tane bilyeleri vardır. Barkın'ın bilyeleri Deniz'in bilyelerini 6 katı kadar ve Engin'in bilyeleri Deniz'in bilyelerinin 2 katı kadardır. Barkın, Deniz ve Engin'in kaçar bilyesi vardır?
2. Didem, Sinan ve Berna'nın toplamda 198 tane jetonları vardır. Didem'in jetonları Sinan'ın jetonlarının 6 katı kadar, Berna'nın jetonlarının da 3 katı kadardır. Didem, Sinan ve Berna'nın kaçar jetonları vardır?

(a) İki problemi de çözünüz.

1.Problemin Çözümü:

2.Problemin Çözümü:

(b) Ortaokul öğrencileri genellikle 2. problemi yapmakta, 1. probleme göre daha çok zorlanırlar. Problemlerin zorluk seviyelerindeki bu farklılığı açıklayacak bir neden yazınız.

---

2. MFC610

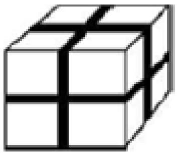
Aşağıdaki ifadelerin her zaman, bazen, hiçbir zaman irrasyonel sayı olup olmadığını belirtiniz.

Her satır için bir kutucuk işaretleyiniz.

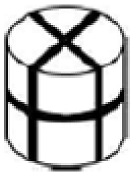
		Her zaman	**Bazen**	**Hiçbir zaman**
A.	Çemberin çevresinin, çapına bölümünden elde edilen sayı.	☐1	☐2	☐3
B.	Bir kenar uzunluğu 1 birim olan karenin köşegeni.	☐1	☐2	☐3
C.	22'nin 7'ye bölümünden çıkan sonuç.	☐1	☐2	☐3

3. MFC511

İki hediye paketi kurdeleyle aşağıda gösterildiği gibi paketleniyor. A paketi bir kenarı 10 cm olan bir küp, B paketi ise yüksekliği ve çapı 10 cm olan bir silindirdir.



A



B

Hangi paket için daha uzun kurdeleye ihtiyaç vardır? ________

Bu sonuca nasıl ulaştığınızı açıklayınız.

4. MFC704

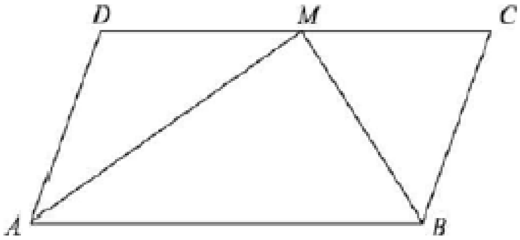
Şekilde $ABCD$ bir paralelkenardır. $\widehat{BAD} = 60°$, AM ve BM sırasıyla $\widehat{BAD}$ ve $\widehat{ABC}$ açılarının açıortaylarıdır. $ABCD$ paralelkenarının çevresi 6 cm olduğuna göre ABM üçgeninin tüm kenarlarını bulunuz.

Cevapları aşağıdaki boşluklara yazınız.

AB=_______ cm

AM=_______ cm

BM=_______ cm



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5. MFC705

Bir doğru üzerinde $3x = 6$ denklemini sağlayan sadece bir nokta olduğunu biliyoruz ve o da $x = 2$'dir

Aynı denklemi, x ve y koordinatları ile düzlem üzerinde, sonra da x,y ve z koordinatlarıyla uzayda düşünelim. Bu durumlarda $3x = 6$ denklemini sağlayan noktalar kümesinin görüntüsü nedir?

<u>Her</u> satır için <u>bir</u> kutucuk işaretleyiniz.

		Bir Nokta	**Bir Doğru**	**Bir Düzlem**	**Diğer**
A.	Düzlemde $3x = 6$'ın çözümü.	☐1	☐2	☐3	☐4
B.	Uzayda $3x = 6$'ın çözümü.	☐1	☐2	☐3	☐4

6. MFC705

Ortaokul öğrencilerinden aşağıdaki ifadeyi ispatlamaları isteniyor:

> 3 ardışık doğal sayının çarpımı her zaman 6'nın katıdır.

Buna karşılık 3 öğrencinin verdiği cevaplar aşağıdaki gibidir:

Pelin'in Cevabı

6'nın katı olan bir sayının çarpanları arasında 3 ve 2 olmak zorundadır. Eğer 3 ardışık sayı varsa bunlardan bir tanesi her zaman 3'ün katı olacaktır.

Ayrıca, sayılardan en az bir tanesi de çift olacaktır ve tüm çift sayılar da 2'nin katıdır.

Bu nedenle 3 ardışık sayı çarpıldığında çıkan sonucun en az bir tane 3 çarpanı ve bir tane de 2 çarpanı olmak zorundadır.

Bora'nın Cevabı

$1 \times 2 \times 3 = 6$

$2 \times 3 \times 4 = 24 = 6 \times 4$

$4 \times 5 \times 6 = 120 = 6 \times 20$

$6 \times 7 \times 8 = 336 = 6 \times 56$

Emel'in Cevabı

n bir tam sayı'dır

$$\begin{aligned} n \times (n+1) \times (n+2) &= (n^2 + n) \times (n+2) \\ &= n^3 + n^2 + 2n^2 + 2n \end{aligned}$$

n'leri götürürsek, $1 + 1 + 2 + 2 = 6$ olur.

Bu ispatların geçerli olup olmadığını belirtiniz.

__Her__ satır için __bir__ kutucuk işaretleyiniz.

		Geçerli	**Geçerli Değil**
A.	Pelin'in cevabı	☐$_1$	☐$_2$
B.	Bora'nın cevabı	☐$_1$	☐$_2$
C.	Emel'in cevabı	☐$_1$	☐$_2$

7. MFC710

Aşağıda verilen durumların her birinin üstel bir fonksiyonla modellenmesinin mümkün olup olmayacağını belirtiniz.

<u>Her</u> satır için <u>bir</u> kutucuk işaretleyiniz.

		Evet	**Hayır**
A.	Havaya atılan bir topun t saniye sonraki yüksekliği h.	☐$_1$	☐$_2$
B.	Her hafta d miktar para yatırılan bir banka hesabındaki, h hafta sonraki para miktarı A.	☐$_1$	☐$_2$
C.	Her yıl % d değer kaybeden bir arabanın t yıl sonraki değeri V.	☐$_1$	☐$_2$

---

8. MFC711

Aşağıdaki ifadeyi ispatlayınız.

Eğer $f(x) = ax + b$ ve $g(x) = cx + d$ doğrusal fonksiyonlarının grafikleri x ekseni üzerinde bir P noktasında kesişiyorsa, bu iki fonksiyonun toplam fonksiyonunun $(f + g)(x)$ grafiği aynı P noktasından geçer.

9. MFC712

Bir matematik öğretmeni ortaokul öğrencilerine ikinci dereceden denklemlerin köklerini veren formülün ispatını göstermek istiyor.
Aşağıdaki bilgilerden hangileri bu ispatı anlayabilmek için gereklidir? Belirtiniz.

Her satır için bir kutucuk işaretleyiniz.

		Gerekli	**Gerekli Değil**
A.	Doğrusal denklemlerin nasıl çözüleceğini bilmek.	☐$_1$	☐$_2$
B.	$k > 0$ için $x^2 = k$ şeklindeki denklemlerin nasıl çözüleceğini bilmek.	☐$_1$	☐$_2$
C.	Üç terimli bir ifadeyi, tam kareye nasıl dönüştüreceğini bilmek.	☐$_1$	☐$_2$
D.	Karmaşık sayılarla toplama çıkarma işlemlerinin nasıl yapılacağını bilmek.	☐$_1$	☐$_2$

10. MFC802

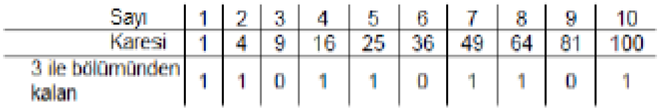
Aşağıdaki ifadeyi ispatlamanız gerekiyor:

Herhangi bir doğal sayının karesinin 3 ile bölümünden kalan sadece 0 ya da 1 olabilir.

Aşağıda verilen yaklaşımların her birinin matematiksel olarak doğru bir ispat olup olmadığını belirtiniz.

Her satır için bir kutucuk işaretleyiniz.

		Evet	**Hayır**
A.	Aşağıdaki tabloyu kullanmak:	☐1	☐2



Sayı	1	2	3	4	5	6	7	8	9	10
Karesi	1	4	9	16	25	36	49	64	81	100
3 ile bölümünden kalan	1	1	0	1	1	0	1	1	0	1

		Evet	**Hayır**
B.	$(3n)^2$'nin *3* ile bölünebilir olduğunu ve diğer bütün sayılar için, $(3n \pm 1)^2 = 9n^2 \pm 6n + 1$ ifadesinin 3 ile bölümünden kalanın her zaman 1 olacağını göstermek.	☐1	☐2
C.	Bir n doğal sayısı seçmek ve onun karesi n^2'yi bulup, ifadenin doğru olup olmadığını kontrol etmek.	☐1	☐2
D.	İlk bir kaç asal sayı için ifadeyi kontrol etmek ve sonra aritmetiğin temel teoremini kullanarak bir sonuca varmak.	☐1	☐2

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11. MFC804

10 öğrencinin olduğu bir sınıftan bir gün 2 öğrenci seçiliyor, aynı sınıftan başka bir gün ise 8 öğrenci seçiliyor. Bu durumda aşağıdaki ifadelerden hangisi doğrudur?

Yalnız bir kutucuk işaretleyiniz.

A. Sınıftan 2 öğrenci seçmenin yolu, 8 öğrenci seçmekten daha fazladır. ☐1

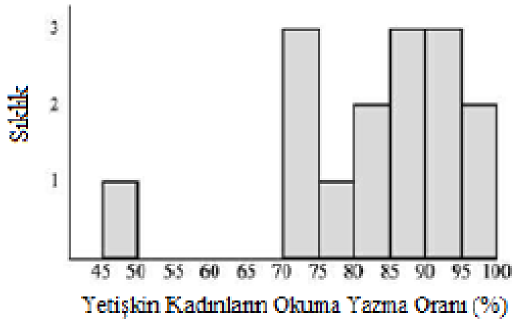
B. Sınıftan 8 öğrenci seçmenin yolu, 2 öğrenci seçmekten daha fazladır. ☐2

C. Sınıftan 2 öğrenci seçmenin yoluyla 8 öğrenci seçmenin yolu eşittir. ☐3

D. Hangi seçimde daha çok ihtimal olduğunu belirlemek mümkün değildir. ☐4

12. MFC806

Aşağıdaki grafik Güney ve Orta Amerika'daki ülkelerde yaşayan yetişkin kadınların okuma yazma oranları ile ilgili bilgi içermektedir.



Öğrencilerinize bu grafikte kaç ülkenin gösterildiğini sorduğunuzu düşünün. Bir öğrenci "7 ülke gösteriliyor" diye cevap veriyor.

Yalnız <u>bir</u> kutucuk işaretleyiniz.

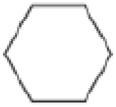
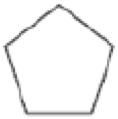
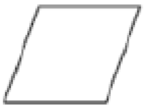
	Doğru	**Yanlış**
a) Öğrencinin verdiği cevap doğru mudur?	☐1	☐2

b) Sizce, öğrenci nasıl düşündüğü için bu sonuca varmıştır?

13. MFC808

Öğrencilerinizle simetri konusunu işliyorsunuz. Verilen bir etkinlikte öğrencilerden aşağıdaki üç farklı şekil için simetri eksenleri sayılarını belirtmeleri isteniyor.

Suat'ın ve Melike'nin cevapları aşağıdaki tabloda gösterilmektedir. İki öğrencinin de cevaplarının doğru olup olmadığını ayrı kutucukları işaretleyerek değerlendiriniz.

		Öğrenciler ve şekillerin simetri eksenleri sayısı için verdikleri cevapları	
Şekil	**Şeklin İsmi**	**Suat**	**Melike**
	düzgün altıgen	6 ☐$_1$ Doğru ☐$_2$ Yanlış	12 ☐$_1$ Doğru ☐$_2$ Yanlış
	düzgün beşgen	5 ☐$_1$ Doğru ☐$_2$ Yanlış	10 ☐$_1$ Doğru ☐$_2$ Yanlış
	eşkenar dörtgen	4 ☐$_1$ Doğru ☐$_2$ Yanlış	2 ☐$_1$ Doğru ☐$_2$ Yanlış

14. MFC814

$A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ ve $B = \begin{bmatrix} t & u \\ v & w \end{bmatrix}$ veriliyor. $A \otimes B$ işlemi $\begin{bmatrix} pt & qu \\ rv & sw \end{bmatrix}$ olarak tanımlanıyor.

Buna göre aşağıdaki ifade doğru mudur?

$A \otimes B = \mathrm{O}$ ise ya $A = \mathrm{O}$ ya da $B = \mathrm{O}$'dır. (O sıfır matris'ini ifade ediyor)

Açıklayınız.

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