NEW TRADE MODELS WITH DIFFERENT DISTRIBUTIONS

A Master's Thesis

by
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Ankara
September 2013

I dedicate this thesis to my family and my husband, Hüseyin, for their constant support and unconditional love. I love you all dearly.

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Graduate School of Economics and Social Sciences
of
İhsan Doğramacı Bilkent University

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HÜSNİYE BURÇİN İKİZLER

In Partial Fulfillment of the Requirements For the Degree
of
MASTER OF ARTS

in

THE DEPARTMENT OF
ECONOMICS
İHSAN DOĞRAMACI BILKENT UNIVERSITY
ANKARA

SEPTEMBER 2013

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

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ABSTRACT

NEW TRADE MODELS WITH DIFFERENT DISTRIBUTIONS

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September 2013

In this thesis, we estimate the Ricardian trade model of Eaton and Kortum (2002) using a different extreme value distribution for the productivity distributions of countries. Due to its analytical convenience, it is now a common tradition in international trade literature to assume that the distribution of productivities follows a Fréchet distribution, which is the case for Eaton and Kortum (2002) model as well. However, recent studies have shown that the estimation results are sensitive to this parametrization. In view of this, we estimate the Eaton and Kortum (2002) model where Weibull distribution is used for the distribution of productivities and show that estimated results change when we use another distribution.

Keywords: Trade, technology, geography, welfare gains, extreme value distributions, productivity.

ÖZET

FARKLI DAĞILIMLARLA YENİ TİCARET MODELLERİ

İKİZLER, Hüsniye Burçin

Yüksek Lisans, Ekonomi Bölümü

Tez Yöneticisi: Yard. Doç. Ayşe Özgür Pehlivan

Eylül 2013

Bu tezde, Eaton ve Kortum (2002)'nin Ricardocu ticaret modelini tahmin ederken ülkelerin verimlilik dağılımını göstermede Fréchet dağılımını almak yerine diğer uç değer dağılımı olan Weibull dağılımını kullandık. Eaton ve Kortum (2002) modelinde de olduğu gibi, analitik kolaylık nedeniyle, verimlilik dağılımının Fréchet dağılımını izlediğini varsaymak artık uluslararası ticaret literatüründe yaygın bir gelenek olmuştur. Ancak, son çalışmalar tahmin sonuçlarının bu parametrelemeye duyarlı olduğunu göstermiştir. Bu görüş karşısında, Eaton ve Kortum (2002) modeli verimlilik dağılımı için Weibull uç değer dağılımını kullanarak tahmin yaptık ve farklı dağılım kullanıldığında tahmin edilen değerlerin değiştiğini gösterdik.

Anahtar Kelimeler: Ticaret, teknoloji, coğrafya, refah artışı, uç değer dağılımları, verimlilik.

ACKNOWLEDGEMENTS

I would like to express the deepest appreciation to my academic advisor Assist. Prof. Ayşe Özgür Pehlivan for all I have learned from her and for her continuous help and support in all stages of this thesis. She rigorously taught me how to become a good researcher and assistant, and how to minimize my mistakes while I am learning from them. One simply could not wish for a better or friendlier advisor. My sincere thanks also go to my committee, Assoc. Prof. Fatma Taşkın and Assist. Prof. Nil İpek Şirikçi for their comments and helpful suggestions during my thesis defense. Their guidance has served me well and I owe them my heartfelt appreciation. I also like to make special mention of my second reader Assoc. Prof. Fatma Taşkın for reading my reports, commenting on my views and helping me understand and enrich my ideas. I would also like to express my gratitude to Bilkent University Graduate School of Economics and Social Sciences for their financial support.

My colleagues, Abdülkadir, Ayşegül, Cihan, Davut, Elif, Emir, Giray, Oğuz, Şiva, have all extended their support in a very special way, and I gained lots from them, through their personal and scholarly interactions, their suggestions at various points of my research. I also acknowledge my old pal Ender Uludağ for his well wishes.

More importantly, none of this would have been possible without the love and patience of my family. I would like to acknowledge the people who mean world to me, my parents Necmettin Yılmaz and Betül Yılmaz, my sister Burcu Yılmaz Kahraman and her husband Emre Kahraman and my brother

Yasin Yılmaz. Thank you mom, dad, sister and brother for showing faith in me and giving me liberty to choose what I desired.

Lastly, and most importantly, I am very much indebted to my husband Hüseyin İkizler who supported me in every possible way to see the completion of this work as well as his wonderful family who all have been supportive and caring.

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CHAPTER 1

INTRODUCTION

In their seminal paper, Eaton and Kortum (2002) introduce a Ricardian model of international trade where they incorporate geographical features and technological differences across countries in order to explain bilateral trade flows. They estimate their models using bilateral trade data. In their model, productivities are assumed to come from a Fréchet distribution. They estimate welfare gains and conduct counterfactuals using this distribution. When we look at the recent trade literature we also see the tradition of using either Fréchet or Pareto distribution to represent productivities in these new trade models, by which we mean the Ricardian and heterogeneous firm models of trade. Recent studies, however, show that these estimations depend highly on this parametrization. According to Arkolakis, Costinot and Rodriguez-Clare (2012), welfare gains depend on two statistics and one of them is related to a single parameter of this productivity distribution. Also, Simonovska and Waugh (2011), while using a richer price data set compared to Eaton and Kortum (2002), show that welfare gains are very sensitive to the estimates of this parameter. In view of these recent studies, it seems to be an important question how the estimates of welfare gains change if we use different types of distributions instead of Fréchet distribution. In our paper, we estimate Eaton and Kortum (2002) model using another extreme value distribution which is Weibull. We solve and estimate the model using this distribution and compare the results with Eaton and Kortum (2002).

Eaton and Kortum (2002) is an extension of Dornbusch, Fischer and Samuelson (1977) Ricardian trade model. Dornbusch, Fischer and Samuelson (1977) uses only two countries like home and foreign whereas Eaton and Kortum (2002) extend their model to multiple countries. In Dornbusch, Fischer and Samuelson (1977), since there are only two countries, they can rank the relative efficiencies from home's perspective. [1] Relative wages in these countries will determine the cutoff point in the ranking, where home will produce the goods the left of the cutoff and foreign will produce to the right of it. However, when there are N countries and for these N countries there is no such natural ordering. In order to handle this problem, Eaton and Kortum (2002) introduce the probabilistic representation of efficiencies/productivities for each country. In Eaton and Kortum (2002), countries are assumed to draw their productivities from a country specific productivity distribution which is assumed to be Fréchet.

$$F_i(z) = e^{-T_i z^{-\Theta}} \quad where\ T_i > 0\ and\ \Theta > 1. \tag{1.2}$$

In this distribution, z is the efficiency parameter, Θ is the comparative advantage and T_i is the country i's absolute advantage. The fact that Dornbusch, Fischer and Samuelson model was for only two countries which introduced an important hurdle in terms of taking Dornbusch, Fischer and Samuelson Ricardian model to bilateral trade data as well and bilateral trade data obviously consists of more than two countries. By extending Dornbusch, Fischer

[1] For the sake of simplicity assume there are n goods. In Dornbusch, Fischer and Samuelson (1977) and Eaton and Kortum (2002) there is the continuum of goods assumption, however, the argument will follow for the continuum of goods case as well.

$$\frac{a_F^1}{a_H^1} > \frac{a_F^2}{a_H^2} > \frac{a_F^3}{a_H^3} > \cdots > \frac{a_F^{n-2}}{a_H^{n-2}} > \frac{a_F^{n-1}}{a_H^{n-1}} > \frac{a_F^n}{a_H^n} \tag{1.1}$$

a^j is the amount of labor needed to produce one unit of good j.

and Samuelson to N countries Eaton and Kortum (2002) is able to take Dornbusch, Fischer and Samuelson Ricardian trade model to data.

After Eaton and Kortum (2002), many studies (Bernard, A. B., Eaton, J., Jenson, J. B., & Kortum, S. (2003), Eaton, J., Kortum, S., & Kramarz, F. (2008), etc.) have been using this framework, however, recent studies such as Arkolakis, Costinot and Rodriguez-Clare (2012) and Simonovska and Waugh (2011) have started to question/reexamine the welfare gains implications. Simonovska and Waugh (2011) apply Eaton Kortum (2002) estimator to a new disaggregate price and trade flow data for 123 countries in 2004. They use richer data set from Eaton and Kortum (2002). Price data is taken from EIU Worldwide Cost of Living Survey and it has 111 tradable goods for each country instead 50 as in Eaton and Kortum (2002). They calculate Θ roughly 4.12 which is approximately 50% less than 8.28 which belongs to Eaton and Kortum (2002). Simonovska and Waugh (2011) state that this difference doubles the welfare gains from international trade. Arkolakis, Costinot and Rodriguez-Clare (2012) claim that welfare contribution is depends only on two parameters. One of these parameters is about the share of expenditure on domestic goods. Another one, the most important one for us, is Θ which is related to the productivity distributions.

In Pehlivan and Vuong (2013), they also consider an Eaton-Kortum model with dropping the Fréchet distribution assumption and estimate those distributions nonparametrically from the data. However, they face different problems due to the nonparametric estimation and due to the fact that they use disaggregated data. However, here, we use exactly the same aggregated data that Eaton and Kortum (2002) use and believe that in order to understand how sensitive current results of the literature to the choice of distribution using another extreme value distribution like Weibull and Gumbel and estimating the results accordingly will provide important insights.

In this study, we work the Eaton-Kortum model for another specific distribution. We want to show how the estimates of the welfare gains change when we take different productivity distributions. Since Fréchet is an extreme value distribution, we replicate the model with another extreme value distribution which is Weibull for the productivity.

$$F_i(z) = 1 - e^{-\left(\frac{z}{\nu_i}\right)^{\alpha}} \tag{1.3}$$

where $\nu_i > 0$, $z > 0$ and $\alpha > 0$.

We do not have closed form solutions with Weibull distribution. For this reason, we use different numerical methods to estimate the parameters of the model compared to Eaton and Kortum (2002). For computational simplicity, we assume that labor is the only factor of production. However, in order to compare our results with Eaton and Kortum (2002), we reestimate Eaton and Kortum (2002) with both assuming that labor is the only factor of production and using our numerical methods. We find actually different estimates but the ordering of the countries according to the estimated absolute advantage parameters do not change much. Nevertheless, estimated comparative advantage parameters are very different when we compare these two productivity distributions. According to Eaton and Kortum (2002), comparative advantage exerts a force in favor of trade while geographic barriers put one against it. This means that welfare gains are effected more by comparative advantage. As a consequence, considering different distributions might provide further insights to gains from trade estimates.

The thesis proceeds as follows. Chapter 2 introduces the model. Chapter 3 contains the data and the empirical application. In chapter 4, we report and interpret the empirical findings. Chapter 5 concludes. The appendices contains the data sets and codes.

CHAPTER 2

THE MODEL

2.1 Eaton and Kortum (2002) with Weibull

There exist a continuum of goods $j \in [0,1]$ and N countries. Country i's efficiency in producing good j is denoted by $z_i(j)$. Cost of a bundle of inputs of country i is c_i. For geographical barriers, we use Samuelson's iceberg trade costs assumption as in Eaton and Kortum (2002).According to this assumption, d_{ni} units need to be produced in country i in order to deliver one unit of good from country i to n. We assume that $d_{ni} > 1$ when $n \neq i$, d_{ni}=1 when n= i. For any countries i, k and n, $d_{ni} \leq d_{nk}d_{ki}$ must be satisfied because of no cross border arbitrage. Cost of delivering one unit of good j produced in country i to n is: $\dfrac{c_i d_{ni}}{z_i(j)}$
Assume that there exist perfect competition and constant returns to scale. Price offered by country i to country n to supply one unit of good j is:

$$P_{ni}(j) = \frac{c_i d_{ni}}{z_i(j)} \tag{2.1}$$

Buyers in country n can look for the best deal among all countries and they will pay (winning price):

$$P_n(j) = \min\{P_{ni}(j) : \; i = 1, 2, \ldots, N\} \tag{2.2}$$

Buyers value goods according to the CES utility function:

$$U = \Big[\int_0^1 Q(j)^{\frac{\sigma-1}{\sigma}} d_j\Big]^{\frac{\sigma}{\sigma-1}} \tag{2.3}$$

The exact price index for the CES objective function (2.3) is

$$\mathcal{P}_n = \Big[\int_0^1 P_n(j)^{1-\sigma} d_j\Big]^{\frac{1}{1-\sigma}} \tag{2.4}$$

For each good j, productivity, $z_i(j)$, is assumed to be drawn from the following Weibull distribution for any j:

$$F_i(z) = 1 - e^{-\left(\frac{z}{\nu_i}\right)^{\alpha}} \tag{2.5}$$

where $\nu_i > 0$, $z > 0$ and $\alpha > 0$. In Eaton and Kortum (2002), these productivities are assumed to come from Fréchet distribution:

$$F_i(z) = e^{-T_i z^{-\Theta}} \quad \textit{where } T_i > 0 \textit{ and } \Theta > 1. \tag{2.6}$$

T_i is the location parameter and Θ is the shape parameter. Eaton and Kortum (2002) interprets T_i's as the absolute advantage parameter which shows the state of technology of country i. They interpret Θ as the comparative advantage parameter when Θ is low it shows high variation/heterogeneity across efficiencies indicating stronger force in favor of trade. Distribution of $P_{ni}(j)$, we call them the price offered by country i to country n for good j, are:

$$G_{ni}(p) = e^{-\left(\frac{c_i d_{ni}}{p\nu_i}\right)^{\alpha}} \tag{2.7}$$

Distribution of $P_n(j)$, which is the winning price in country n:

$$G_n(p) = 1 - \prod_{i=1}^{N}\left(1 - e^{-\left(\frac{c_i d_{ni}}{p\nu_i}\right)^{\alpha}}\right) \tag{2.8}$$

Define Π_{ni} as the probability that country i provides a good at the lowest price in country n.

$$\Pi_{ni} = Pr[P_{ni}(j) \leq \min\{P_{ns}(j) : s \neq i\}]; \tag{2.9}$$

In Eaton and Kortum (2002), they can get a closed form of Π_{ni} :

$$\Pi_{ni} = \int_0^\infty \prod_{s \neq i} \left[e^{-T_s \left(\frac{c_s d_{ns}}{p}\right)^{-\Theta}} \right] d\left(1 - e^{-T_i \left(\frac{c_i d_{ni}}{p}\right)^{-\Theta}}\right) = \frac{T_i (c_i d_{ni})^{-\Theta}}{\Phi_n} \tag{2.10}$$

where $\Phi_n = \sum_{i=1}^{N} T_i (c_i d_{ni})^{-\Theta}$. However, using Weibull distribution the integral above becomes:

$$\Pi_{ni} = \int_0^\infty \prod_{s \neq i} \left[1 - e^{-\left(\frac{c_s d_{ns}}{p \nu_s}\right)^{\alpha}} \right] d\left(e^{-\left(\frac{c_i d_{ni}}{p \nu_i}\right)^{\alpha}} \right) \tag{2.11}$$

which does not simplify in our case.

In addition, in Eaton and Kortum (2002) the distribution of the price of a good that country n actually buys from any country i, $G_{n|i \; is \; winner}(p|i \; is \; winner)$, is the same as the unconditional one which is $G_n(p)$. So, conditioning on source does not make a difference in terms of the price distribution. However, with Weibull distribution, this is no longer the case. Price of a good that country n actually buys from any country i has the distribution:

$$\begin{aligned} G_{n|i \; is \; winner}(p|i \; is \; winner) &= Pr[P_{ni} \leq p | \text{n buys from country i}] \quad (2.12) \\ &= \frac{\int_0^p \prod_{s \neq i}(1 - G_{ns}(q)) dG_{ni}(q)}{\int_0^\infty \prod_{s \neq i}(1 - G_{ns}(p)) dG_{ni}(p)} \end{aligned}$$

where $G_{ni}(p) = e^{-\left(\frac{c_i d_{ni}}{p \nu_i}\right)^{\alpha}}$. Due to the continuum of goods assumption and law of large numbers, we have the following:

$$\frac{X_{ni}}{X_n} = \frac{E[X_{ni}(j)]}{E[X_n(j)]} \tag{2.13}$$

where X_{ni} is the expenditure of country n on goods coming from i and X_n is the total spending of country n. In Eaton and Kortum (2002), since there is the Fréchet distribution the right hand side of (2.13) is also equal to the probability that country i provides a good at the lowest price in country n. Hence, we have:

$$\frac{X_{ni}}{X_n} = \frac{E[X_{ni}(j)]}{E[X_n(j)]} = \Pi_{ni} \tag{2.14}$$

This provides a link between $\frac{X_{ni}}{X_n}$, which can be obtained from the data, and their model since Π_{ni} contains the parameters of their model, i.e.

$$\frac{X_{ni}}{X_n} = \Pi_{ni} = \frac{T_i(c_i d_{ni})^{-\Theta}}{\Phi_n} \tag{2.15}$$

where $\Phi_n = \sum_{i=1}^{N} T_i(c_i d_{ni})^{-\Theta}$. There is no such relation when Weibull distribution is used. In other words, second equality in (2.14) does not hold for the Weibull case. We have the following expressions instead:

$$\frac{E[X_{ni}(j)]}{E[X_n(j)]} = \frac{\left[\int_0^\infty Q_n(j)P_n(j)d(G_{n|i\ is\ winner}(p|i\ is\ winner))\right]\Pi_{ni}}{\int_0^\infty Q_n(j)P_n(j)dG_n(p)} \tag{2.16}$$

Then, using (2.13) and (2.16) the equation becomes:

$$\frac{X_{ni}}{X_n} = \frac{[\int_0^\infty Q_n(j)P_n(j)d(G_{n|i\ is\ winner}(p|i\ is\ winner))]\Pi_{ni}}{[\int_0^\infty Q_n(j)P_n(j)dG_n(p)]} \tag{2.17}$$

Recall the demands for the CES objective function $Q_n(j)$:

$$Q_n(j) = \frac{P_n(j)^{-\sigma}Y_n}{\int_0^1 P_n(s)^{1-\sigma}ds} \tag{2.18}$$

When we use (2.18), we get:

$$\frac{X_{ni}}{X_n} = \frac{\left[\int_0^\infty \frac{P_n(j)^{1-\sigma}Y_n}{\int_0^1 P_n(s)^{1-\sigma}ds} d\left(\frac{\int_0^{P_n(j)} \prod_{s\neq i}(1-G_{ns}(q))dG_{ni}(q)}{\int_0^\infty \prod_{s\neq i}(1-G_{ns}(p))dG_{ni}(p)}\right)\right]\left[\int_0^\infty \prod_{s\neq i}[1-G_{ns}(p)]d(G_{ni}(p))\right]}{\left[\int_0^\infty \frac{P_n(j)^{1-\sigma}Y_n}{\int_0^1 P_n(s)^{1-\sigma}ds} d(G_n(p))\right]} \tag{2.19}$$

Notice that the result of an integral from zero to infinity is the number. This means that this integral can be taken out of the integral as a constant number. As a result, this integral can be simplified. Likewise, the result of an integral from zero to 1 is a number. This means that this integral can be taken out of the integral as a constant number. Then, (2.19) simplifies to:

$$\frac{X_{ni}}{X_n} = \frac{\int_0^\infty P_n(j)^{1-\sigma} d\left[\int_0^{P_n(j)} \prod_{s\neq i}(1-G_{ns}(q))dG_{ni}(q)\right]}{\int_0^\infty P_n(j)^{1-\sigma} d(G_n(p))} \tag{2.20}$$

The open form of (2.20) becomes:

$$\frac{X_{ni}}{X_n} = \frac{\int_0^\infty P_n(j)^{1-\sigma} d\left[\int_0^{P_n(j)} \prod_{s\neq i}(1-e^{-\left(\frac{c_s d_{ns}}{q\nu_s}\right)^\alpha}) d(e^{-\left(\frac{c_i d_{ni}}{q\nu_i}\right)^\alpha})\right]}{\int_0^\infty P_n(j)^{1-\sigma} d\left[1-\prod_{i=1}^N (1-e^{-\left(\frac{c_i d_{ni}}{P_n(j)\nu_i}\right)^\alpha})\right]} \tag{2.21}$$

We apply Leibniz Integral Rule to the (2.21).[1] Then, we get the equation which forms the basis of our estimation for Weibull distribution:

$$\frac{X_{ni}}{X_n} = \frac{\int_0^\infty P_n(j)^{1-\sigma}\left[\prod_{s\neq i} 1-e^{-\left(\frac{c_s d_{ns}}{P_n(j)\nu_s}\right)^\alpha}\right] e^{-\left(\frac{c_i d_{ni}}{P_n(j)\nu_i}\right)^\alpha} \left(\frac{c_i d_{ni}}{\nu_i}\right)^\alpha \alpha P_n(j)^{-\alpha-1} d(P_n(j))}{\int_0^\infty P_n(j)^{1-\sigma} d\left[1-\prod_{i=1}^N (1-e^{-\left(\frac{c_i d_{ni}}{P_n(j)\nu_i}\right)^\alpha})\right]} \tag{2.22}$$

As we can see from (2.22), in the Weibull case we can not obtain a simple relation as in Eaton and Kortum (2002). On the contrary, Fréchet case has a well closed form and $\frac{X_{ni}/X_n}{X_{ii}/X_i}$ can be expressed as below which is used in the

[1]Leibniz Integral Rule: $\frac{d}{dx}(\int_{f_1(x)}^{f_2(x)} g(t)dt) = g[f_2(x)]f_2'(x) - g[f_1(x)]f_1'(x)$

estimation of Θ.

$$\frac{X_{ni}/X_n}{X_{ii}/X_i} = \left(\frac{p_i d_{ni}}{p_n}\right)^{-\Theta} \tag{2.23}$$

We can interpret the left hand side of (2.23) as how involved country i is in trade. When country i makes too much trade, X_{ii}/X_i becomes lower and X_{ni}/X_n becomes higher. As a result, the left hand side of (2.23) will be higher when country i makes too much trade.

In Eaton and Kortum (2002), the cost of an input bundle in country i is taken as $c_i = w_i^{\beta}\mathcal{P}_i^{1-\beta}$ where w_i is the wage in country i and $\mathcal{P}_i$ is the exact price index in country i, used as a price index for intermediate goods. Again with Fréchet assumption and using the cost form above, they obtain the following relation which is actually the structural equation they used to estimate parameters of interest.

$$\frac{X'_{ni}}{X'_{nn}} = (\frac{T_i}{T_n})^{1/\beta}(\frac{w_i d_{ni}}{w_n})^{-\Theta} \tag{2.24}$$

where $X'_{ni} = \dfrac{X_{ni}}{(X_i/X_{ii})^{(1-\beta)/\beta}}$ and T_i is the country i's absolute advantage. They take the logarithm of the above equation, they get

$$ln(\frac{X'_{ni}}{X'_{nn}}) = -\Theta ln(d_{ni}) + (1/\beta)ln(\frac{T_i}{T_n}) - \Theta ln(\frac{w_i}{w_n}) \tag{2.25}$$

Eaton and Kortum (2002) define $S_i = (1/\beta)ln(T_i) - \Theta ln(w_i)$ and then above equation is simplified to

$$ln(\frac{X'_{ni}}{X'_{nn}}) = -\Theta ln(d_{ni}) + S_i - S_n \tag{2.26}$$

and they obtain the estimates of parameters of interest.

However, we do not have a closed form for exact price index with Weibull. The exact price index will depend on prices of each good in this country. Also, c_i depends on prices of each good in country i. In consequence, $G_{ni}(p)$ will be depend on the prices of each good for country i and also $G_n(p)$ will be depend on the prices of each good for every country. Unlike Eaton and Kortum (2002), we do not have a closed form for exact price index which will create further computational difficulties. For this reason, we deal with the case where labor is the only factor of production, i.e. $c_i = w_i$. To compare the estimates of the welfare gains of both our case and Eaton and Kortum (2002) case, we have to consider $c_i = w_i$ for Eaton and Kortum (2002) as well (obviously it is a special case with $\beta = 1$). We use the following equation in our estimation.

$$\frac{X_{ni}}{X_{nn}} = \frac{X_{ni}/X_n}{X_{nn}/X_n} = \frac{T_i}{T_n}\left(\frac{w_i d_{ni}}{w_n}\right)^{-\Theta} \tag{2.27}$$

Taking the logarithm of (2.27), we get

$$ln(\frac{X_{ni}}{X_{nn}}) = ln(\frac{T_i}{T_n}) - \Theta ln(\frac{w_i}{w_n}) - \Theta ln(d_{ni}) \tag{2.28}$$

Following the terminology of Eaton and Kortum (2002), we define $S_i = ln(T_i) - \Theta ln(w_i)$. Then, (2.28) becomes

$$ln(\frac{X_{ni}}{X_{nn}}) = -\Theta ln(d_{ni}) + S_i - S_n \tag{2.29}$$

Above equation forms the basis of estimation when we consider $c_i = w_i$ for Eaton and Kortum (2002) case.

2.2 About Distributions

In this part, we would like to provide an overview of those three extreme value distributions and how they are linked. In their seminal paper, Eaton and Kortum (2002) use the Fréchet distribution for the distribution of productivities (z).

$$F_i(z) = e^{-T_i z^{-\Theta}} \quad where\ T_i > 0\ and\ \Theta > 1. \tag{2.30}$$

In this distribution, Θ represents the comparative advantage and T_i represents the country i's absolute advantage. Fréchet distribution has infinite mean when $\Theta < 1$ so that Eaton and Kortum (2002) assume that $\Theta > 1$. This Type-II extreme value distribution gives closed form solutions so that equations can be solved analytically in their model. As distinct from Eaton and Kortum (2002), we will use Weibull distribution which is Type-III extreme value distribution. There is also another extreme value distribution, Gumbel, which is Type-I extreme value distribution and all three extreme value distributions are related to each other. If the productivities follow a Weibull distribution, they will be distributed as follows:
Weibull distribution :

$$F_i(z) = 1 - e^{-\left(\frac{z}{\nu_i}\right)^{\alpha}} \quad where\ \nu_i > 0\ and\ z > 0\ and\ \alpha > 0. \tag{2.31}$$

Parameter α is the shape parameter which represents the comparative advantage and ν_i is the location parameter which represents country i's absolute advantage.Whereas if they follow Gumbel, they will be distributed as:

$$F_i(z) = e^{-e^{-\left(\frac{z}{\kappa_i}\right)}} \quad where\ \kappa_i > 0\ and\ z > 0 \tag{2.32}$$

κ_i represents the country i's absolute advantage

According to Head (2011), these extreme values are related to each other and their relations are:

- If Z has Fréchet distribution and Y=ln Z, then Y has Gumbel distribution with respective parameter arrangements.

- If Z has Weibull distribution and Y=1/Z, then Y has Fréchet distribution with respective parameter arrangements.

- If Z has Weibull distribution and Y=ln(1/Z), then Y has Gumbel distribution with respective parameter arrangements.

In order to see the relations of these distributions, we can look at Figure 2.1. At this figure we can see these three distributions with different comparative advantage and absolute advantage (related to the technology) parameters. Since Gumbel does not have comparative advantage parameter, its figure is shown only for different absolute advantage parameters. While Weibull and Gumbel distributions look similar for comparative advantage parameter values which are greater than one but there are differences when comparative advantage parameter is smaller than or equal to one which can be seen in Figure 2.2.

In this paper, we use Weibull distribution instead of Fréchet and assume comparative advantage parameter to be greater than zero. This means that there is no such restriction on comparative advantage parameter to be greater than one as in Eaton and Kortum (2002). This allow us to account for the possibility of having a comparative advantage parameter value smaller than one which might affect the welfare gains estimates significantly. Fréchet distribution provides great analytical convenience and we can get closed form solutions, however, as can be seen from the model with Weibull we cannot get those closed forms.

Figure 2.1: Extreme Value Distributions with Different Parameters

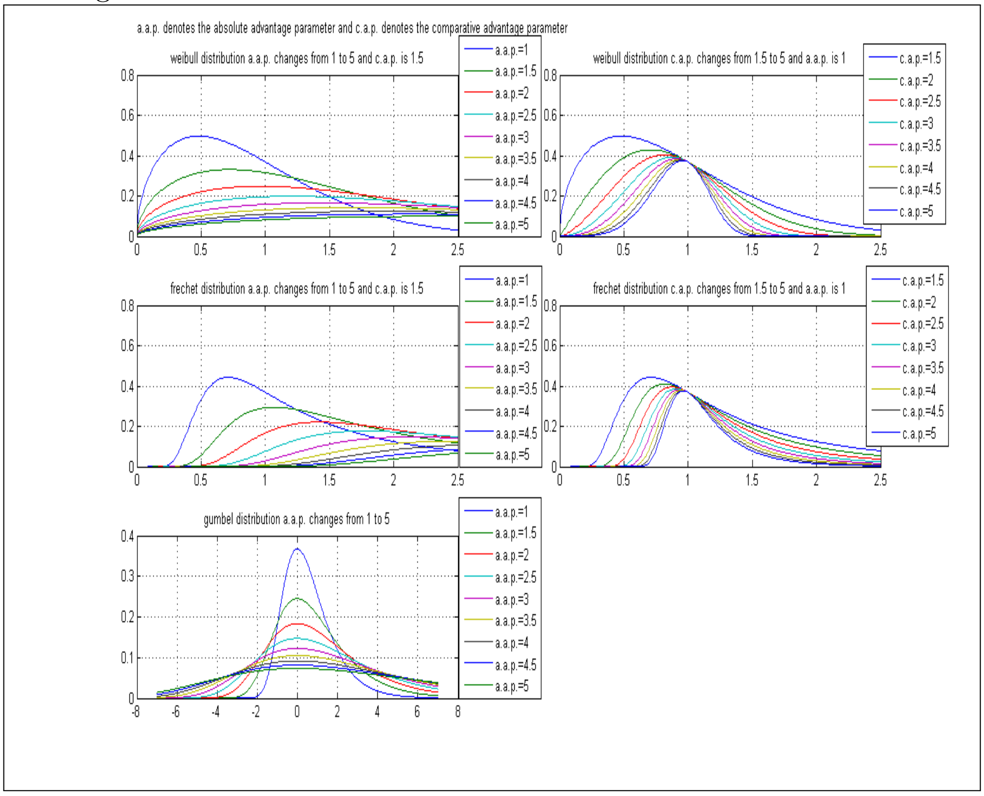
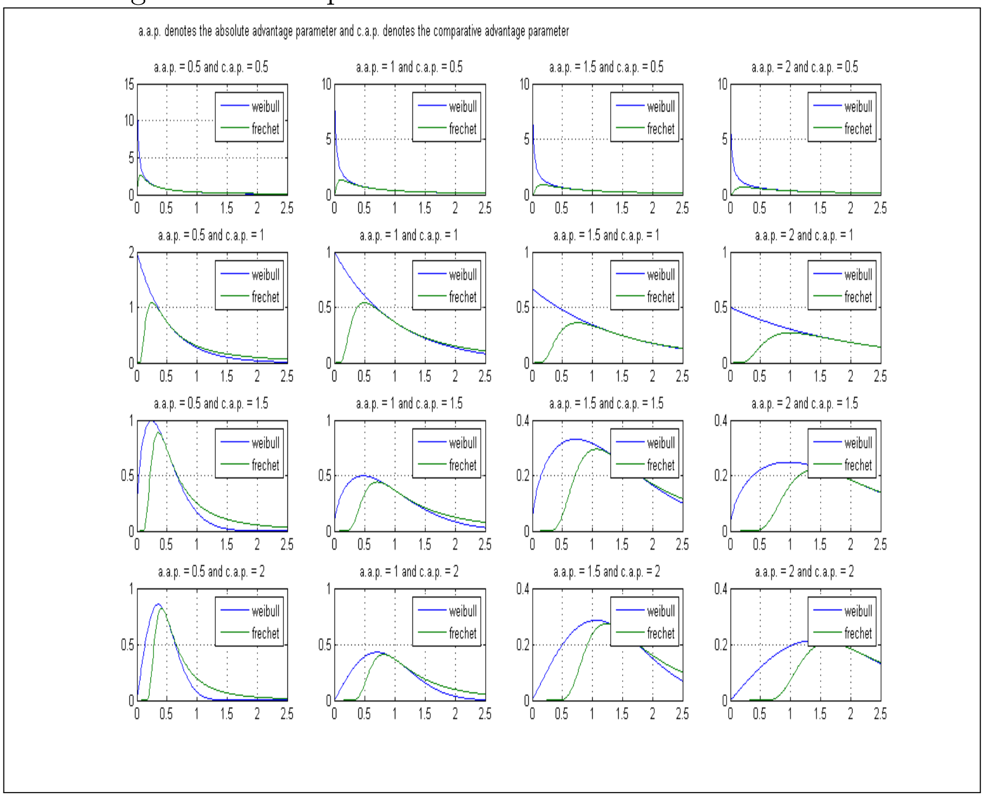


Figure 2.2: Comparisons of Weibull and Fréchet Distributions



CHAPTER 3

ESTIMATION

3.1 Data

All of the data we use is taken from the data sets of Eaton and Kortum (2002). In order to make the estimation, we need trade data. This means that we need for each country how much spend on manufactures from other countries (X_{ni}). In addition, we use total spending of a country on manufactures (X_n) which is the summation of the home purchases and imports from other 18 countries. Eaton and Kortum (2002) take these data from STAN database in local currencies (OECD (1995)). Our dependent variable are normalized, i.e. our dependent variable is $\frac{X_{ni}}{X_n}$. For this reason, there is no need for exchange rate translation.

We assume that labor is the only factor of production so that we use only the wage data from Eaton and Kortum (2002) for the cost of a bundle of inputs. Wages are normalized to the wages of the United States. This means that wage for the United States is equal to 1. For the price data, Eaton and Kortum (2002) use 50 manufacturing goods which are from The United Nations International Comparison Program 1990 benchmark study. This survey consists of 100 goods but they do not take some categories which are related to food and chemicals because they thought that these goods prices considerably change with proximity to natural resources and taxes on petroleum products. Again, prices are normalized to the prices in the United States for each good. Eaton and Kortum (2002) use distance, border, language, common trade area as proxies for geographic barriers. Distance, which is measured as the miles between central cities in each country, is divided to six areas. These are (in miles): [0,375); [375,750); [750,1500); [1500,3000); [3000,6000); [6000,maximum). English, French and German are the languages for 19 countries. English is spoken in the Australia, Canada, New Zealand, United Kingdom and United States. French is spoken in the Belgium and France. German is spoken in the Austria and Germany. The two trading areas are European Community (EC) and the European Free Trade Area (EFTA).

3.2 Estimation of Eaton-Kortum Model with Raw Data and Different β

In this part, we will estimate Eaton-Kortum model using different β. When labor is the only factor of production, this means $\beta = 1$. Also, if intermediate goods are considered, this means $\beta = 0.21$. We use the same data set as provided by Eaton and Kortum (2002). However, there is a slight discrepancy between the values of $(\frac{X_{ni}/X_n}{X_{ii}/X_i})$ calculated by Eaton and Kortum (2002) and the ones calculated from the raw bilateral trade data. [1] We will use the raw bilateral trade data, therefore we replicate all Eaton and Kortum (2002) estimation for these newly calculated $(\frac{X_{ni}X_n}{X_{ii}/X_i})$ values from the raw data. Eaton and Kortum(2002) uses method of moments as a first stage so

[1] When we calculate $(\frac{X_{ni}/X_n}{X_{ii}/X_i})$ values using the raw bilateral trade data provided by Samuel Kortum on his website (http://home.uchicago.edu/kortum/papers/tgt/maxdistx.prg), we get slightly different values than the ones we obtain when we run their code again provided by Kortum. They define $ln(\frac{X_{ni}/X_n}{X_{ii}/X_i})$ as $ln(X_{ni}/X_{nn})-(\beta/(1-\beta))((lnX'_{ni}-lnX'_{nn})-(lnX_{ni}-lnX_{nn}))$ in their codes and do not calculate it directly from raw data.

as to estimate the Θ. They use the below equation

$$\frac{X_{ni}/X_n}{X_{ii}/X_i} = \left(\frac{p_i d_{ni}}{p_n}\right)^{-\Theta} \tag{3.1}$$

They measure $ln(\frac{p_i d_{ni}}{p_n})$ with D_{ni} which is defined as

$$D_{ni} = \frac{\max 2_j\{r_{ni}(j)\}}{\frac{\sum_{j=1}^{50}(r_{ni}(j))}{50}} \tag{3.2}$$

where $r_{ni}(j) = lnp_n(j) - lnp_i(j)$ and $max2$ represents the second highest. Since anyone in country i which is able to sell to country n is also able to sell to country i, this gives $lnp_n(j) - lnp_i(j) < lnd_{ni}$ i.e., $r_{ni}(j) < lnd_{ni}$. This means that $r_{ni}(j)$ is bounded above by lnd_{ni}. They use second highest value of $r_{ni}(j)$ instead of the highest one in order to mitigate the effect of possible measurement error in the prices for particular commodities. The equation which is used for method of moments estimation becomes:

$$ln(\frac{X_{ni}/X_n}{X_{ii}/X_i}) = -\Theta D_{ni} \tag{3.3}$$

Eaton and Kortum (2002) obtain a method of moments estimate of $\Theta = 8.275930$. Notice that the estimated equation does not depend on cost structure so does not depend on β. As a result of this fact, this Θ value is same for any β. When we use raw data for method of moments estimation, we get the value 8.275827 for Θ. Since both the new $\frac{X_{ni}/X_n}{X_{ii}/X_i}$ values and the ones Eaton and Kortum (2002) calculate data are pretty close, the result of the estimation are very close as well.

In table 3.1, you can observe how the method of moments estimates change when we calculate trade shares from raw data.

Later, Eaton and Kortum (2002) estimate (2.26) with generalized least

Table 3.1: Method of Moments Estimation

	Method of Moments est. (Θ)
Eaton and Kortum (2002)	8.275930
Raw Data and $\beta = 0.21221$	8.275827
Raw Data and $\beta = 1$	8.275827

squares (GLS). As a proxy for geographic barriers, they use

$$lnd_{ni} = d_k + b + l + e_h + m_n + \delta_{ni} \tag{3.4}$$

for geographic barriers. Here $d_k(k = 1, ..., 6)$ represents the effect of the distance between two country, b represents the effect of sharing a border, l represents the effect of sharing a language, $e_h(h = 1, 2)$ represents the effect of belonging the same trading area and $m_n(n = 1, ..., 19)$ represents overall destination effect. In addition, to capture potential reciprocity, they assume that error term δ_{ni} consists of two components, δ^2_{ni}, δ^1_{ni}, the former affecting two-way trade while the latter affecting one-way trade. Then, (2.26) becomes

$$ln(\frac{X'_{ni}}{X'_{nn}}) = S_i - S_n - \Theta m_n - \Theta d_k - \Theta b - \Theta l - \Theta e_h + \Theta\delta^2_{ni} + \Theta\delta^1_{ni} \tag{3.5}$$

In their estimation, all of the variables on the right hand side of (3.5) are taken as dummy variables and some restrictions are imposed ($\sum_{i=1}^{19} S_i$=0 and $\sum_{n=1}^{19} m_n$=0). They do not mention in their text, however when we checked their codes, we have realized that they use a relative dummy which is basically source country dummy minus destination country dummy instead of the source country dummy only. Again, checking their codes we realized that they put the restriction on the relative dummy coefficients such that they will add up to zero and not on the source country coefficients. Table 3.2 represents the competitiveness of the countries for Eaton and Kortum (2002) case, raw data with $\beta = 0.21221$ case and raw data with $\beta = 1$ case.

Table 3.2: Source Country Competitiveness

	S_i					
Country	Eaton and Kortum (2002)		Raw Data and $\beta = 0.21221$		Raw Data and $\beta = 1$	
	est.	s.e.	est.	s.e.	est.	s.e.
Australia	0.19253	0.15	0.17194	0.15	-0.58808	0.15
Austria	-1.1615	0.12	-1.1576	0.12	-1.0345	0.12
Belgium	-3.3357	0.11	-3.1862	0.11	-0.010343	0.11
Canada	0.41181	0.14	0.31803	0.14	0.35176	0.14
Denmark	-1.7506	0.12	-1.7018	0.12	-0.91216	0.12
Finland	-0.52278	0.12	-0.55136	0.12	-0.92137	0.12
France	1.2818	0.11	1.2406	0.11	0.79137	0.11
Germany	2.3538	0.12	2.3307	0.12	1.6314	0.12
Greece	-2.8137	0.12	-2.7359	0.12	-2.5233	0.12
Italy	1.7823	0.11	1.7292	0.11	0.87870	0.11
Japan	4.1991	0.13	4.0797	0.13	2.6544	0.13
Netherlands	-2.1899	0.11	-1.9837	0.11	0.12134	0.11
New Zealand	-1.1977	0.15	-1.1717	0.15	-1.3168	0.15
Norway	-1.3465	0.12	-1.3393	0.12	-1.0110	0.12
Portugal	-1.5731	0.12	-1.5732	0.12	-1.3679	0.12
Spain	0.30356	0.12	0.23037	0.12	-0.44611	0.12
Sweden	0.010019	0.12	-0.036388	0.12	-0.045833	0.12
United Kingdom	1.3727	0.12	1.3830	0.12	0.97525	0.12
United States	3.9840	0.14	3.9535	1.4	2.7733	0.14

Note that the order of countries according to their competitiveness doesn't change when we use raw data for the same β value, $\beta = 0.21221$. However, as we can see from table 3.2, there is some change in the competitiveness of countries when we take $\beta = 1$. Again, as we can see from table 3.2, there is significant increase in the competitiveness of Belgium and Netherlands whereas there is significant decrease in the competitiveness of Spain, Finland and New Zealand for raw data with $\beta = 1$.

As in the Eaton and Kortum (2002), we estimate Θ using wage data but again using our raw data. We do this using our raw data for both $\beta = 1$ and $\beta = 0.21221$. Following Eaton and Kortum (2002), we relate technology to national stocks of research and development (R&D) and to human capital and estimate the following equation:

$$S_i = \alpha_0 + \alpha_R lnR_i - \alpha_H(1/H_i) - \Theta lnw_i + \tau_i \tag{3.6}$$

In Eaton and Kortum (2002), OLS estimate of Θ is equal to 2.84. For our raw data, OLS estimate of Θ is equal to 2.75 when we take $\beta = 0.21221$ and

Table 3.3: Estimates Using Wage Data

	OLS		2SLS	
	est. (Θ)	s.e.	est. (Θ)	s.e.
Eaton and Kortum (2002)	2.84	1.02	3.60	1.21
Raw Data and $\beta = 0.21221$	2.76	0.96	3.46	1.13
Raw Data and $\beta = 1$	0.81	0.31	0.76	0.36

Table 3.4: Estimates Using Price Data

	OLS		2SLS	
	est. (Θ)	s.e.	est. (Θ)	s.e.
Eaton and Kortum (2002)	2.44240	0.49404	12.862	1.64
Raw Data and $\beta = 0.21221$	2.44736	0.49311	12.86126	1.64
Raw Data and $\beta = 1$	2.44708	0.49309	12.86088	1.64

0.81 when we take $\beta = 1$. Eaton and Kortum (2002) conduct also a 2SLS estimation due to the endogeneity of wages and find the value of Θ to be 3.60. For our raw data, we find that 2SLS estimates of Θ is equal to 3.45 when $\beta = 0.21221$ and 0.75 when $\beta = 1$ as can be seen in table 3.3.

Eaton and Kortum (2002) also estimate (2.26) using D_{ni} instead of the geographic barrier proxies they used before where D_{ni} calculated according to (3.2) using price data. In Eaton and Kortum (2002), OLS estimate of Θ is equal to 2.44 and 2SLS estimate of Θ is equal to 12.86. Unlike the estimates using wage data, the estimates when price data is used are almost the same as can be seen in table 3.4.

3.3 Calculation of the Geographic Barriers Using Price Data

As defined in Eaton and Kortum (2002), another way of estimation the geographic barriers is to use the price data. According to Eaton and Kortum (2002), $r_{ni}(j) = lnp_n(j) - lnp_i(j)$ is bounded above by lnd_{ni} since anyone in i able to sell in n is also able to sell in i. This gives $r_{ni}(j) < lnd_{ni}$. In order to reduce the possible measurement error, they prefer to use the second maximum value of the $r_{ni}(j)$ for lnd_{ni}. Therefore, our geographic barriers

Table 3.5: Calculated Geographic Barriers (lnd_{ni}) Using Price Data

	ExporterCountries																	
Importer	Australia	Austria	Belgium	Canada	Denmark	Finland	France	Germany	Greece	Italy	Japan	Netherlands	N. Z.	Norway	Portugual	Spain	Sweden	U.K.	U.S.
Australia	0	0.2892	0.2573	0.4544	0.2885	0.2082	0.2628	0.3169	0.4221	0.2735	0.4305	0.2915	0.5709	0.1396	0.8641	0.4034	0.3285	0.4967	0.6858
Austria	0.5803	0	0.4139	0.5664	0.2569	0.1798	0.3769	0.5147	0.7211	0.3439	0.6159	0.5836	0.9828	0.2885	0.9299	0.4164	0.2588	0.5838	0.8939
Belgium	0.4883	0.4369	0	0.6386	0.2696	0.3258	0.2902	0.2669	0.6067	0.3640	0.7337	0.4683	0.8648	0.3035	0.8132	0.4758	0.4638	0.6180	0.9844
Canada	0.5635	0.5422	0.3547	0	0.4463	0.2055	0.4182	0.3973	0.6261	0.4374	0.7260	0.6369	1.0366	0.1893	0.8259	0.5665	0.3025	0.5442	0.6521
Denmark	0.7912	0.7371	0.6423	0.7505	0	0.2022	0.6542	0.6481	0.9309	0.6988	0.9214	0.5509	1.0619	0.2779	1.1438	0.6744	0.5560	0.7408	1.1879
Finland	0.8650	0.7282	0.6926	0.9223	0.7213	0	0.8743	0.8590	0.9775	0.7282	1.1820	0.7933	1.3877	0.4308	1.4141	0.7379	0.5168	0.8625	1.1629
France	0.7704	0.4882	0.3654	0.7733	0.3344	0.3946	0	0.3706	0.8003	0.4066	0.7878	0.6061	1.1470	0.3854	0.8643	0.5357	0.5107	0.5717	1.1642
Germany	0.6297	0.5097	0.2575	0.6838	0.2282	0.3714	0.3076	0	0.7803	0.3205	0.5688	0.4532	1.0062	0.2261	0.8694	0.4641	0.3887	0.5006	0.9431
Greece	0.6277	0.3987	0.4132	0.7655	0.3899	0.2873	0.3696	0.6008	0	0.5760	0.7774	0.5759	1.1272	0.2229	1.0456	0.3671	0.5029	0.6535	0.9805
Italy	0.6579	0.5687	0.4751	0.6423	0.3236	0.3175	0.3575	0.4643	0.7854	0	0.7095	0.6761	1.0344	0.3497	0.9234	0.4752	0.3897	0.5953	1.0394
Japan	0.7553	0.6557	0.4489	0.8243	0.3927	0.3734	0.7986	0.5801	0.8683	0.6829	0	0.6644	1.0964	0.5034	1.3421	1.0661	0.5449	0.6986	1.1064
Netherlands	0.6043	0.5273	0.3812	0.5635	0.1698	0.2775	0.5460	0.2401	0.4127	0.4019	0.5399	0	0.9289	0.1998	0.7299	0.4465	0.4036	0.5539	0.8555
New Zealand	0.5643	0.3847	0.3357	0.3744	0.2019	0.2310	0.3450	0.5332	0.3446	0.3689	0.3766	0.5443	0	0.2089	0.7067	0.3384	0.4139	0.6023	0.7002
Norway	1.0414	0.8185	0.7584	0.9741	0.5756	0.3225	0.9614	0.8669	0.9905	0.7258	1.2873	0.9219	1.2940	0	1.2301	0.6014	0.7198	0.8141	1.2693
Portugual	0.5066	0.3990	0.2053	0.4616	0.0994	0.1386	0.4292	0.4650	0.4414	0.2094	0.7758	0.3993	0.8831	0.1038	0	0.2299	0.3747	0.3538	0.8251
Spain	0.6841	0.5019	0.3458	0.6705	0.2962	0.3053	0.4885	0.5350	0.5410	0.4181	0.9554	0.4691	1.0606	0.3846	0.6785	0	0.4769	0.6515	1.1688
Sweden	0.8023	0.6752	0.6342	0.8636	0.4396	0.1979	0.6702	0.7587	0.8811	0.7288	0.7135	0.6916	1.1513	0.1555	1.0845	0.5416	0	0.8384	1.3741
U.K.	0.5555	0.2841	0.4262	0.5385	0.1926	0.3003	0.3991	0.3839	0.6443	0.2478	0.7832	0.3320	0.8325	0.2226	0.7673	0.4862	0.4102	0	1.0654
U.S.	0.5305	0.3277	0.2533	0.5697	0.4070	0.4799	0.0874	0.5202	0.5889	0.1953	0.8373	0.4349	0.7945	0.3069	0.6674	0.3244	0.3190	0.4113	0

becomes:

$$lnd_{ni} = \max 2_j \{r_{ni}(j)\} \tag{3.7}$$

Hence, we calculate geographic barriers using only price data. They do not report these values. However since we will use them in our estimation we calculate those geographic barriers and report them in table.

3.4 Estimation of Eaton-Kortum Model with Weibull

Our equation which will be estimated becomes:

$$\frac{X_{ni}}{X_n} = \frac{\int_0^\infty P_n(j)^{1-\sigma}\Big[\prod_{s\neq i} 1 - e^{-\left(\frac{c_s d_{ns}}{P_n(j)\nu_s}\right)^\alpha}\Big] e^{-\left(\frac{c_i d_{ni}}{P_n(j)\nu_i}\right)^\alpha} \left(\frac{c_i d_{ni}}{\nu_i}\right)^\alpha \alpha P_n(j)^{-\alpha-1} d(P_n(j))}{\int_0^\infty P_n(j)^{1-\sigma} d\left[1 - \prod_{i=1}^N (1 - e^{-\left(\frac{c_i d_{ni}}{P_n(j)\nu_i}\right)^\alpha})\right]} \tag{3.8}$$

Note that we assume that labor is the only factor of production i.e., we take $c_i = w_i$. While calculating these integrals we face the following problem:

In order to calculate these integrals, we have to apply integration by parts method about eighteen factorial times. Eventually, we need a lot of time to compute this integral manually. To overcome this, we try to compute these integrals using price data. First, we sort the prices of goods for each country in ascending order, then we evaluate the function in the integral for each goods. To have the approximate value of the integral, we simply apply the trapezoidal rule. The trapezoidal rule works by approximating the region under the graph of the function as a trapezoid and calculating its area. The lengths of the parallel edges of the trapezoid are the values of the function for the consecutive prices and the height of the trapezoid is the difference between the consecutive prices. Thus, we obtain the sum of area of forty nine trapezoids as an approximate value of the integral. In addition, even if we use this trapezoidal rule, calculating these integrals for 19 countries takes too much time. For this reason, we make this estimation for 10 countries which are:

Canada, France, Germany, Greece, Italy, Japan, Spain, Sweden, United Kingdom and United States. The general representation of our regression equations is:

$$Y_{ni} = f_i(X_{ni}, \beta) + \varepsilon_{ni} \tag{3.9}$$

where Y_{ni} is dependent variable and X_{ni} is vector of regressors. As in the Eaton and Kortum (2002) to deal with the problem which can be reciprocity, we assume the following:

$$\varepsilon_{ni} = \varepsilon_{ni}^2 + \varepsilon_{ni}^1 \tag{3.10}$$

where ε_{ni}^2 affects two way trade ($\varepsilon_{ni}^2 = \varepsilon_{in}^2$) and its variance σ_2^2. Besides, ε_{ni}^1 affects one way trade and its variance σ_1. Another crucial assumption is that ε_{ni}^2 is orthogonal to ε_{ni}^1. In the our equations, error terms u_n=$(\varepsilon_{n1}, \varepsilon_{n2}, \varepsilon_{n3}, \varepsilon_{n4}, \varepsilon_{n5}, \varepsilon_{n6}, \varepsilon_{n7}, \varepsilon_{n8}, \varepsilon_{n9}, \varepsilon_{n10})$ are independent because of the above assumption.

As a first stage, since we do not know the form of Ω, we calculate:

$$
\begin{aligned}
Var(\varepsilon_{ni}) &= E[\varepsilon_{ni}^2] - E[\varepsilon_{ni}]^2 \\
&= E[\varepsilon_{ni}^2] \qquad (by\ exogeneity\ assumption) \\
&= E[(\varepsilon_{ni}^2 + \varepsilon_{ni}^1)^2] \qquad (i) \\
&= E[(\varepsilon_{ni}^2)^2] + 2E[\varepsilon_{ni}^2\varepsilon_{ni}^1] + E[(\varepsilon_{ni}^1)^2] \\
&= \sigma_2^2 + \sigma_1^2 \quad (by\ orthogonolity\ assumption)
\end{aligned} \tag{3.11}
$$

$$
\begin{aligned}
Cov(\varepsilon_{ni}, \varepsilon_{in}) &= E[\varepsilon_{ni}\varepsilon_{in}] - E[\varepsilon_{ni}]E[\varepsilon_{in}] \\
&= E[(\varepsilon_{ni}^2 + \varepsilon_{ni}^1)(\varepsilon_{in}^2 + \varepsilon_{in}^1)] \\
&= E[(\varepsilon_{ni}^2)^2] + E[\varepsilon_{ni}^2\varepsilon_{in}^1] + E[\varepsilon_{ni}^1\varepsilon_{in}^2] + E[\varepsilon_{ni}^1\varepsilon_{in}^1] \\
&= \sigma_2^2
\end{aligned} \tag{3.12}
$$

$$
\begin{aligned}
Cov(\varepsilon_{ni}, \varepsilon_{ni'}) &= E[\varepsilon_{ni}\varepsilon_{ni'}] - E[\varepsilon_{ni}]E[\varepsilon_{ni'}] \\
&= E[(\varepsilon_{ni}^2 + \varepsilon_{ni}^1)(\varepsilon_{ni'}^2 + \varepsilon_{ni'}^1)] \\
&= E[\varepsilon_{ni}^2\varepsilon_{ni'}^2] + E[\varepsilon_{ni}^2\varepsilon_{ni'}^1] + E[\varepsilon_{ni}^1\varepsilon_{ni'}^2] + E[\varepsilon_{ni}^1\varepsilon_{ni'}^1] \\
&= 0
\end{aligned} \tag{3.13}
$$

Therefore we achieve the form of Ω. There remains to estimate the parameters in Ω.

In order to estimate the parameters in Ω, we first obtain the residuals from un-weighted regression model.[2] So, we minimize the following:

$$
\min_{\beta}\{(Y - f(X,\beta))'(Y - f(X,\beta))\} \tag{3.14}
$$

[2]For this estimation, we use the least squares curve fit (lsqcurvefit) function in the matlab.

From that minimisation we obtain $\widehat{\varepsilon_{ni}}$ and these are the parameters in Ω. To estimate the nonzero non-diagonal elements in Ω, we use $\Sigma_{n=1}^{10}\Sigma_{i=1}^{10}\frac{\widehat{\varepsilon_{ni}}\widehat{\varepsilon_{in}}}{100}$. For the diagonal elements, we use $\Sigma_{n=1}^{10}\Sigma_{i=1}^{10}\frac{\widehat{\varepsilon_{ni}}^2}{100}$. In the second step, to have the efficient estimator for β, we minimize the following:

$$\min_{\beta}\{(Y - f(X,\beta))'\Omega^{-1}(Y - f(X,\beta))\} \tag{3.15}$$

We estimate this equation using the least squares nonlinear estimation (lsqnonlin) function in matlab. This lsqnonlin function solves the problem which are of the form:

$$\min_{\beta}\{||f(X,\beta)||_2^2\} = \min_{\beta}\{(f_1(X,\beta)^2 + f_2(X,\beta)^2 + f_3(X,\beta)^2 + .. + f_n(X,\beta)^2)\} \tag{3.16}$$

Notice that, we need to separate Ω^{-1} into two. For this reason, we use the Cholesky Decomposition.[3] Thus, we get:

$$\min_{\beta}\{||(Y - f(X,\beta))'C'||_2^2\} \tag{3.17}$$

where $\Omega^{-1} = C'C$. This form is ready to use lsqnonlin function in matlab. As a result, we obtain the efficient and consistent estimation for β.

3.5 Estimation of Eaton-Kortum Model with Fréchet

In order to compare our results with Fréchet case, we need to estimate the model with our methods using Fréchet distribution. The main equation

[3]Cholesky decomposition is the decomposition of the square matrix with complex entries that is equal to its own conjugate transpose and positive definite matrix into the product of a lower triangular matrix and its conjugate transpose.

which we use:

$$\frac{X_{ni}}{X_n} = \frac{[\int_0^\infty Q_n(j)P_n(j)d(G_{n|i\ is\ winner}(p|i\ is\ winner))]\Pi_{ni}}{[\int_0^\infty Q_n(j)P_n(j)dG_n(p)]} \tag{3.18}$$

We put the associated demands for CES utility function and then we get:

$$\frac{X_{ni}}{X_n} = \frac{\left[\int_0^\infty \frac{P_n(j)^{1-\sigma}Y_n}{\int_0^1 P_n(s)^{1-\sigma}ds} d(G_{n|i\ is\ winner}(p|i\ is\ winner))\right]}{\left[\int_0^\infty \frac{P_n(j)^{1-\sigma}Y_n}{\int_0^1 P_n(s)^{1-\sigma}ds} d(G_n(p))\right]}\Pi_{ni} \tag{3.19}$$

In the below, there is the open form of (3.19):

$$\frac{X_{ni}}{X_n} = \frac{\int_0^\infty P_n(j)^{1-\sigma} d\left[\frac{\int_0^{P_n(j)} \Theta T_i(c_i d_{ni})^{-\Theta} q^{\Theta-1} e^{-q^\Theta \Phi_n} dq}{(T_i(c_i d_{ni})^{-\Theta})/\Phi_n}\right]}{\int_0^\infty P_n(j)^{1-\sigma} d(1 - e^{-P_n(j)^\Theta \Phi_n})}\Pi_{ni} \tag{3.20}$$

(3.20) simplifies to:

$$\frac{X_{ni}}{X_n} = \frac{\int_0^\infty P_n(j)^{1-\sigma} d\left[\int_0^{P_n(j)} \Theta T_i(c_i d_{ni})^{-\Theta} q^{\Theta-1} e^{-q^\Theta \Phi_n} dq\right]}{\int_0^\infty P_n(j)^{1-\sigma} d(1 - e^{-P_n(j)^\Theta \Phi_n})} \tag{3.21}$$

Applying the Leibniz Rule, (3.21) becomes:

$$\frac{X_{ni}}{X_n} = \frac{\int_0^\infty P_n(j)^{1-\sigma} \Theta T_i(c_i d_{ni})^{-\Theta} P_n(j)^{\Theta-1} e^{-P_n(j)^\Theta \Phi_n} dP_n(j)}{\int_0^\infty P_n(j)^{1-\sigma} e^{-P_n(j)^\Theta \Phi_n} \Theta \Phi_n P_n(j)^{\Theta-1} dP_n(j)} \tag{3.22}$$

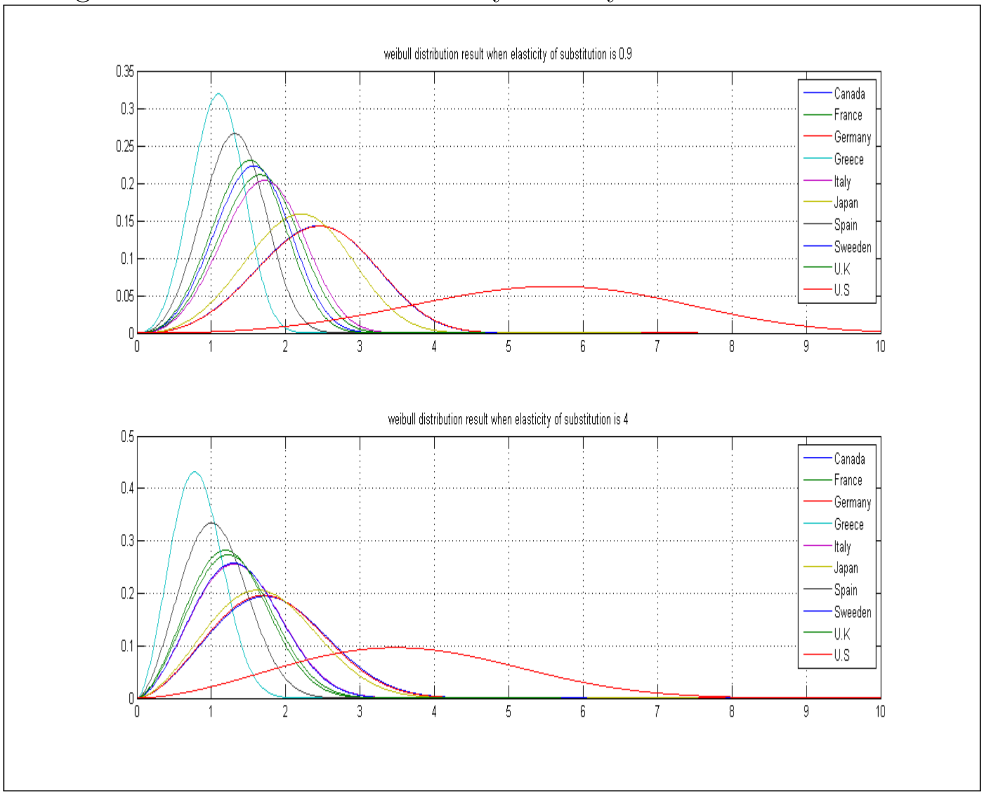
(3.22) forms the basis of estimation for the Fréchet case with $c_i = w_i$ assumption. We apply the same procedure with Weibull case in order to estimate (3.22).

CHAPTER 4

ESTIMATION RESULTS

In the table 4.1, we report the estimated values of the absolute advantage parameters (ν_i) and comparative advantage parameter (α) for the Weibull distribution. We can see that all estimated coefficients are highly statistically significant when we look at the t-values of the estimates. A closer look at the table shows that United States has the largest absolute advantage parameter which shows the highest state of technology among our countries. Then, second highest absolute advantage parameter belong to the Canada and Germany. On the other side, Greece has the lowest absolute advantage parameter and Spain has the second lowest absolute advantage parameter. Notice that the value of the absolute advantage parameter decreases as the elasticity of substitution increases. When elasticity of substitution is high, this means that substitutability among goods is high. If low elasticity of substitution is set, this means that we do not allow for that substitutability. So, that affect is probably captured by high state of technology (high T_i's, ν_i's) and high comparative advantage parameter (high Θ or high α) all of which indicates lower forces for trade. We can not estimate the elasticity of substitution for Fréchet case since it is simplified in the main equation. Besides, in the Weibull case, elasticity of substitution takes bigger values. For these reasons, we decide to put the values to elasticity of substitution as in

Figure 4.1: Estimated Probability Density Functions for Weibull Case



the Temple (2012) and Sancho (2009). However, the ordering of absolute advantage parameters of the countries does not change, except for France when elasticity of substitution changes. For the comparative advantage parameter, it can be seen that as the elasticity of substitution parameter σ increases, comparative advantage parameter α_i's decrease.

In the figure 4.1, we show the estimated productivity distributions of all our countries for the Weibull case. Since absolute advantage parameter is related with the location of the distribution, we can see that the graph for United States is located farthest to the right while the graph for Greece is located farthest to the left. Besides, for the bigger elasticity of substitution, graph is located farthest to the left since absolute advantage parameters decrease.

In the table 4.2, we can see the results for Fréchet case. We can see that the estimated coefficients of the United States, Germany, Sweden, France and Canada are statistically significant except for the Canada when elasticity of

Table 4.1: Estimation Results for Weibull Distribution

	σ=0.8	σ=0.9	σ=1.0	σ=4.0
$\nu_{UnitedStates}$	6.188	6.165	6.143	4.139
	(0.351)	(0.355)	(0.360)	(0.554)
$\nu_{Germany}$	2.704	2.694	2.684	2.030
	(0.165)	(0.167)	(0.168)	(0.141)
ν_{Canada}	2.696	2.686	2.675	2.052
	(0.479)	(0.475)	(0.471)	(0.303)
ν_{Japan}	2.426	2.418	2.409	1.934
	(0.174)	(0.175)	(0.176)	(0.233)
ν_{Italy}	1.893	1.886	1.878	1.558
	(0.121)	(0.121)	(0.121)	(0.116)
ν_{France}	1.825	1.819	1.813	1.414
	(0.093)	(0.094)	(0.095)	(0.075)
ν_{Sweden}	1.727	1.723	1.719	1.547
	(0.070)	(0.070)	(0.069)	(0.074)
$\nu_{UnitedKingdom}$	1.673	1.668	1.663	1.459
	(0.049)	(0.049)	(0.049)	(0.073)
ν_{Spain}	1.449	1.443	1.438	1.194
	(0.060)	(0.061)	(0.062)	(0.081)
ν_{Greece}	1.211	1.204	1.197	0.924
	(0.044)	(0.045)	(0.045)	(0.073)
α	3.585	3.565	3.545	2.676
	(0.468)	(0.468)	(0.469)	(0.319)
n	100	100	100	100
R-square	0.765	0.766	0.766	0.786

Notes: We see that all coefficients are highly statistically significant looking at the t-values of the estimates.

Notes: The numbers in parentheses are standard errors.

substitution is equal to 0.8. However, the estimated coefficients of Japan, Italy, United Kingdom, Spain and Greece are statistically insignificant. Besides, the value of the comparative advantage parameter is highly statistically significant. There is a difference from the Weibull case in terms of the value of the estimated parameters. In this case, absolute advantage parameters (T_i) are low while the comparative advantage parameter (Θ) is quite high which is approximately 22. However, there is no considerable difference for the sorting according to the absolute advantage compared to the Weibull case. Again, United States has the largest value for absolute advantage parameter (T_i), and Germany has the second largest value. Similarly, Greece has the lowest value and Spain follows it. Actually in estimation with Fréchet, these countries have very low absolute advantage parameters which are very close to zero. However, what we care about is the ranking of the state of technologies of these countries. While the ranking is quite similar to Weibull case an exception is Sweden. When we consider the Fréchet case, its rank is actually much higher than the Weibull case. In the Fréchet case, when we consider different elasticity of substitution levels like the Weibull case, the order of the countries do not change. This is not surprising as we know that in Eaton and Kortum (2002) the value of elasticity of substitution parameter has no effect on the results. Because in the Fréchet case, we know that elasticity of substitution parameter (σ) cancels out. Likewise, the value of Θ (comparative advantage parameter) does not change much.

According to the Eaton and Kortum (2002), absolute advantage parameters (T_i) determine the location of the distribution. In figure 4.2, we can observe that figures are close to the right when absolute advantage parameter increases. For example, United States has the largest state of technology value and its graph is located farthest to the right. Greece has the lowest state of technology parameter so its graph is located farthest to the left.

According to the Eaton and Kortum (2002) model, comparative advan-

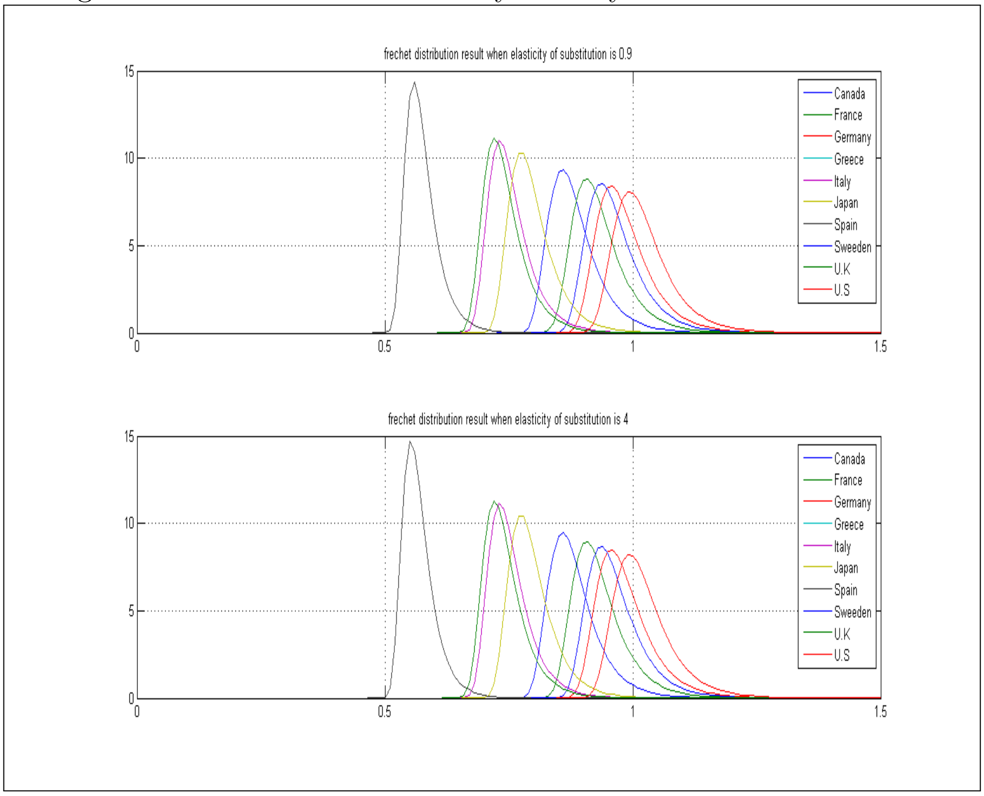
Table 4.2: Estimation Results for Fréchet Distribution

	σ=0.8	σ=0.9	σ=1.0	σ=4.0
$T_{UnitedStates}$	0.905***	0.905***	0.905***	0.905***
	(0.040)	(0.040)	(0.041)	(0.040)
$T_{Germany}$	0.373***	0.395***	0.381***	0.391***
	(0.060)	(0.054)	(0.058)	(0.055)
T_{Sweden}	0.227***	0.246***	0.233***	0.243***
	(0.050)	(0.047)	(0.049)	(0.047)
T_{France}	0.104***	0.121***	0.110***	0.118***
	(0.040)	(0.039)	(0.040)	(0.039)
T_{Canada}	0.028*	0.036**	0.031**	0.035**
	(0.018)	(0.019)	(0.018)	(0.019)
T_{Japan}	0.002*	0.004*	0.003*	0.004*
	(0.003)	(0.004)	(0.003)	(0.004)
T_{Italy}	0.001*	0.001*	0.001*	0.001*
	(0.001)	(0.001)	(0.001)	(0.001)
$T_{UnitedKingdom}$	0.000*	0.001*	0.001*	0.001*
	(0.001)	(0.001)	(0.001)	(0.001)
T_{Spain}	0.000*	0.000*	0.000*	0.000*
	(0.000)	(0.000)	(0.000)	(0.000)
T_{Greece}	0.000*	0.000*	0.000*	0.000*
	(0.000)	(0.000)	(0.000)	(0.000)
θ	23.708***	21.886***	23.042***	22.179***
	(4.742)	(4.001)	(4.501)	(4.122)
n	100	100	100	100
R-squared	0.973	0.973	0.972	0.973

Notes: ***p-value<0.01, **p-value<0.05, *p-value<0.1

Notes: The numbers in parentheses are standard errors.

Figure 4.2: Estimated Probability Density Functions for Fréchet Case



tage parameter denotes the amount of variation within the distribution. As expected, since for different values of elasticity of substitution comparative advantage parameters do not change much, in figure 4.2 we see the variation within the distributions is very similar for both graphs.

In figure 4.4 and 4.3, we show the estimated cumulative productivity distributions of countries for both Fréchet and Weibull. Figure 4.4 shows the cumulative productivity distribution when elasticity of substitution is equal to 0.9 and figure 4.3 shows when it is 4. In the Fréchet case, since comparative advantage parameter is higher than the Weibull case, the variation is low hence we see that Fréchet cumulative distribution functions reach 1 much faster. This result does not depend on the value of the elasticity of substitution parameter.

According to the Eaton and Kortum (2002), comparative advantage parameter demonstrates the amount of variation within the distribution. The comparative advantage sets heterogeneity in countries' relative efficiencies

Figure 4.3: Estimated Cumulative Distribution Functions When Elasticity of Substitution is Equal to 0.9

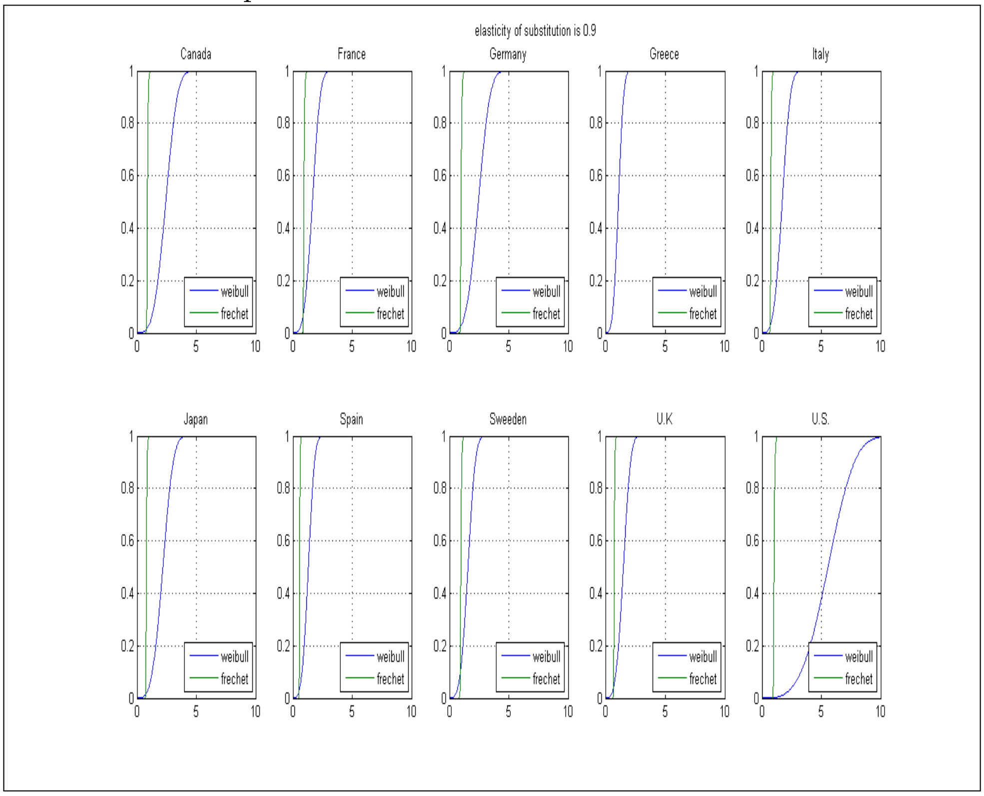
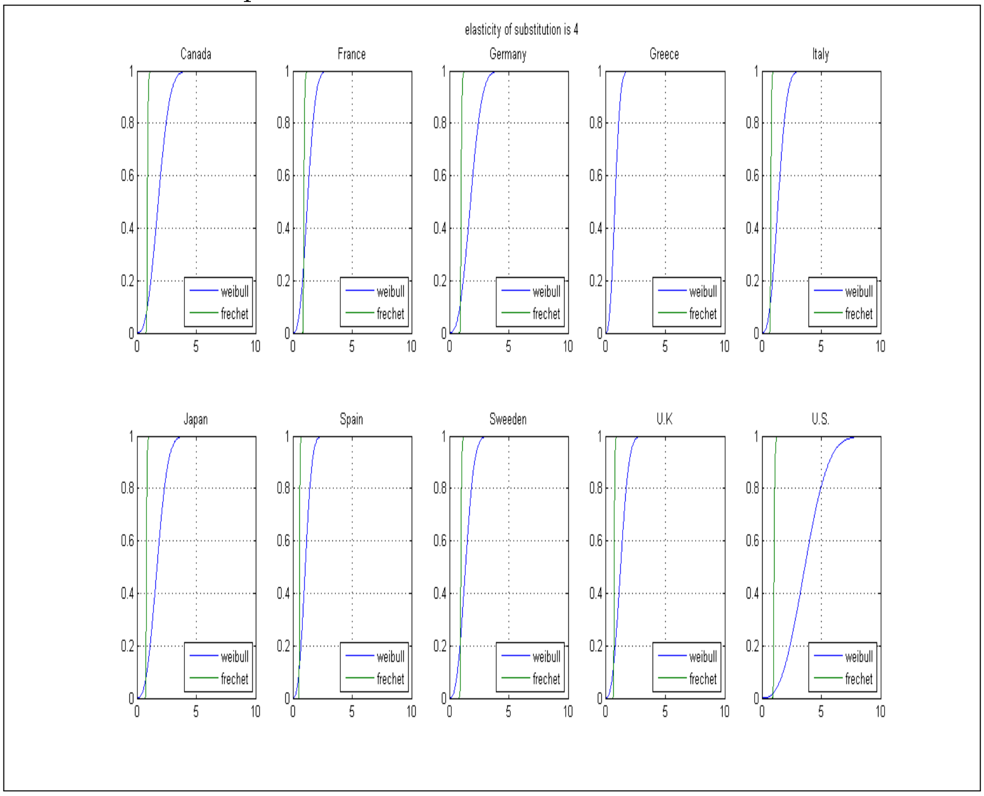


Figure 4.4: Estimated Cumulative Distribution Functions When Elasticity of Substitution is Equal to 4



across goods. Eaton and Kortum (2002) shows that if comparative advantage is low, then heterogeneity in efficiencies across goods increases. This implies that comparative advantage employs stronger force in favor of trade against the geographic barriers. We find that comparative advantage parameters for the Weibull case are approximately 3.6 (when elasticity of substitution is 0.9) and approximately 2.7 (when elasticity of substitution is 4). However, for the Fréchet case we find approximately 21.9 (when elasticity of substitution is 0.9) and 22.2 (when elasticity of substitution is 4). This means that if we take Weibull distribution instead of the Fréchet distribution for productivities, we will estimate even stronger forces in favor of trade against geographic barriers. Since welfare results depend heavily on the comparative advantage parameter, we believe considering different distributions is important since the estimated results of welfare will change significantly.

CHAPTER 5

CONCLUSION

From the recent studies, we know that estimates of the welfare gains depend on the productivity distributions. For this reason, in this paper, we study a Ricardian trade model where we use another extreme value distribution in an Eaton and Kortum (2002) framework. We use Weibull distribution for distribution of productivities instead of Fréchet. When we change the productivity distribution to Weibull, unfortunately we do not have closed form solutions unlike the Fréchet case. In order to make our computations simpler, we use only labor as the factor of production. Then, to compare our results to Eaton and Kortum (2002) we reestimate Eaton and Kortum (2002) model assuming labor is the only factor of production. According to Eaton and Kortum (2002), comparative advantage puts a force in favor of trade while geographic barriers put one against it. In this study, we find that comparative advantage parameter which is related with the Weibull distribution is lower than the comparative advantage parameter which is related with the Fréchet distribution. Therefore, using different distributions might provide further insights to gains from trade estimates.

Estimating the welfare gains is the most important future work and we will conduct the same counterfactuals with Eaton and Kortum (2002). Then, as a further research we will impose intermediate inputs to the model.

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APPENDICES

Data Sets

We work same data sets with Eaton and Kortum (2002) which are:

- United Nations- Statistics Canada bilateral merchandise trade data by 4-digit SITC as described in Feenstra, Lipsey and Bowen (1997) for 19 OECD countries in 1990.
- The United Nations International Comparison Program 1990 benchmark study gives prices of 100 products in each of 19 countries relative to the price in United States. (Eaton and Kortum uses 50 items identified by Hooper and Vrankovich (1995).)
- Average years of schooling in each of 19 countries in 1985 from Kyriacou (1991).
- Research stocks in 1990 from Coe and Helpman (1995).
- Labor forces in 1990 data from Summers and Heston (1991).
- Distances between countries from Eaton and Tamura (1994).

Codes

Weibull m.files

```
clear all;
close all;
clc;

n=10;
[trade1] = xlsread('trade1.xls');
wage=trade1(7202:7202+19-1,17);
c_old=exp(wage);
c(1,1)=c_old(4);
c(2,1)=c_old(7);
c(3,1)=c_old(8);
c(4,1)=c_old(9);
c(5,1)=c_old(10);
c(6,1)=c_old(11);
c(7,1)=c_old(16);
c(8,1)=c_old(17);
c(9,1)=c_old(18);
c(10,1)=c_old(19);

nu = sym(zeros(n,1));
for row = 1:n
nu(row, 1) = sym(sprintf('nu%d', row));
end

[D_ni]=xlsread('D_ni.xlsx');
dni=exp(D_ni);
syms el_sub alpha

[pricedat txt] = xlsread('pricedat.xlsx');
pricedata=ones(19,50);
```

```
for i=1:19
pricedata(i,:)=exp(pricedat(i*50-49:i*50,1));
end
pricedata=pricedata';

order_price_j_old=sort(pricedata,'ascend');
order_price_j_old=order_price_j_old';% Here we sort the prices
%of 50 goods of each 19 countries. So we have still 19x50 matrix.
order_price_j(1,:)=order_price_j_old(4,:);
order_price_j(2,:)=order_price_j_old(7,:);
order_price_j(3,:)=order_price_j_old(8,:);
order_price_j(4,:)=order_price_j_old(9,:);
order_price_j(5,:)=order_price_j_old(10,:);
order_price_j(6,:)=order_price_j_old(11,:);
order_price_j(7,:)=order_price_j_old(16,:);
order_price_j(8,:)=order_price_j_old(17,:);
order_price_j(9,:)=order_price_j_old(18,:);
order_price_j(10,:)=order_price_j_old(19,:);

order_price_j=round(order_price_j*100)/100;
global xdata
xdata= [c order_price_j dni];
[ydataa]=xlsread('xni_bl_Xn.xlsx');
global ydata
for i=1:10
   ydata((i*10)-9:i*10,1)=(ydataa(:,i));
end

x0 =1*[1.0000; 1.0000; 1.0000; 1.0000; 1.0000; 1.0000; 1.0000;
    1.0000;     1.0000; 1.0000; 1.0000; 1.0000];
lb=[0.0000; 0.0000; 0.0000; 0.0000; 0.0000; 0.0000; 0.0000;
    0.0000;    0.0000; 0.0000; 0.0000; 0.0000];
ub=1000*[1.0000; 1.0000; 1.0000; 1.0000; 1.0000; 1.0000; 1.0000;
    1.0000;    1.0000; 1.0000; 1.0000; 1.0000];
options = optimset('MaxFunEvals',1e10);
```

```
[X,RESNORM,RESIDUAL,EXITFLAG,OUTPUT,LAMBDA,JACOBIAN]=
lsqcurvefit(@modelfun,x0,xdata,ydata,lb,ub,options);

options = statset('nlinfit');
fdiffstep = options.DerivStep;
yfit=modelfun(X,xdata);
p = numel(X);
delta = zeros(size(X));
%This for loop computes the Jacobian matrix.
for k = 1:p
    if (X(k) == 0)
        nb = sqrt(norm(X));
        delta(k) = fdiffstep * (nb + (nb==0));
    else
        delta(k) = fdiffstep*X(k);
    end
    yplus = modelfun(X+delta,xdata);
    dy = yplus(:) - yfit(:);
    J(:,k) = dy/delta(k);
    delta(k) = 0;
end
r = ydata - yfit;
mse = (abs(r).^2)/(100-p);
tot=0;
for i=1:numel(mse)
tot=tot+mse(i);
end
mse=tot;

[Q,R] = qr(J,0);
Rinv = inv(R);
Sigma = Rinv*Rinv'*mse;
std_err=sqrt(diag(Sigma));%Standard error of estimates
```

```
y_mean=mean(ydata);
tot=0;
for i=1:numel(ydata)
tot=tot+(ydata(i)-y_mean)^2;
end
total_sum_of_squares=tot;%Total Sum of Squares

se = (abs(r).^2);
tot=0;
for i=1:numel(se)
tot=tot+se(i);
end
sum_of_squared_residuals=tot;%Sum of Squares of Residuals

R_square=(total_sum_of_squares-sum_of_squared_residuals)
/total_sum_of_squares;
%%%%%%%%%%%%%%%%%%%%%%%%%Weighted- LS%%%%%%%%%%%%%%%%%
R=reshape(RESIDUAL,10,10);
sum=0;
for i=1:10
    for j=1:10
    sum=sum+R(i,j)*R(j,i);
    end
end

non_diagonal_elements=sum/100;

sum=0;
for i=1:10
    for j=1:10
    sum=sum+R(i,j)*R(i,j);
    end
end

diagonal_elements=sum/100;
```

```
column1=ones(100,1);
column2=ones(100,1);

k=0;
for i=1:10
    for j=1:10
    k=k+1;
    column1(k)=j;
    column2(k)=i;
    end
end
%Weighting matrix,S is formulated.
S=eye(100,100)*diagonal_elements;
for i=1:100
    for j=1:100
       if column1(i)==column2(j) && column1(j)==column2(i)&&i~=j
       S(i,j)=non_diagonal_elements;
       end
    end
end
S_inv=inv(S);

global C
C=chol(S_inv);
options = optimset('MaxFunEvals',1e10);
[xw,resnorm,RESIDUAL] = lsqnonlin(@myfun,x0,lb,ub,options);

options = statset('nlinfit');
fdiffstep = options.DerivStep;
yfit=myfunfit(xw,xdata);
p = numel(xw);
delta = zeros(size(xw));
for k = 1:p
    if (xw(k) == 0)
        nb = sqrt(norm(xw));
```

```
        delta(k) = fdiffstep * (nb + (nb==0));
    else
        delta(k) = fdiffstep*xw(k);
    end
    yplus = myfunfit(xw+delta,xdata);
    dy = yplus(:) - yfit(:);

    J(:,k) = dy/delta(k);
    delta(k) = 0;
end
[Q,R] = qr(J,0);
ydata=(ydata'*C)';
r = ydata - yfit;
mse = (abs(r).^2)/(100-p);

tot=0;
for i=1:numel(mse)
   tot=tot+mse(i);
end
mse=tot;
Rinv = inv(R);
Sigma_w = Rinv*Rinv'*mse;
std_err_w=sqrt(diag(Sigma_w)); %Robust standard error of estimates

y_mean=mean(ydata);
tot=0;
for i=1:numel(ydata)
tot=tot+(ydata(i)-y_mean)^2;
end
total_sum_of_squares_w=tot;%Total Sum of Squares

se = (abs(r).^2);
tot=0;
for i=1:numel(se)
tot=tot+se(i);
```

```
end
sum_of_squared_residuals_w=tot;%Sum of Squares of Residuals
R_square_w=(total_sum_of_squares_w-sum_of_squared_residuals_w)
            /total_sum_of_squares_w;

estimates_result=[X std_err xw std_err_w];
tss_ssr_results=[total_sum_of_squares sum_of_squared_residuals
            total_sum_of_squares_w sum_of_squared_residuals_w];
R_square_results=[R_square R_square_w];
```

```
%This function evaluates the model that is estimated.
function F=modelfun(x,xdata)
%x=[nu el_sub alpha];
%xdata=[c order_price dni];

[n b]=size(xdata); %10x61
nu=x(1:n);
% el_sub=4.0;
% alpha=x(n+1);
el_sub=x(n+1);
alpha=x(n+2);
c=xdata(:,1);
order_price_j=xdata(:,2:51);
dni=xdata(:,52:b);

for i=1:n
   for j=1:n
       area=0;
       l=0;
       if i==10
            for jj=1:50
            crp=1;
            for k=1:n
                if j==k
                crp=crp*(order_price_j(i,jj)^(1-el_sub))
                *exp(-((round(10*((c(k)*dni(i,k))
                /order_price_j(i,jj))))/10)
                *(1/nu(k)))^alpha)*(((round(10*((c(k)*dni(i,k))))
                /10)/(nu(k)))^alpha)*alpha*(order_price_j(i,jj)^
                (-alpha-1));
                else
                crp=crp*(1-exp(-((round(10*((c(k)*dni(i,k))
                /order_price_j(i,jj))))/10)*(1/nu(k)))^alpha));
                end
            end
```

```
            l=l+1;
            y(l)=crp;
            end
Area_Xni_pricedat(i,j)=sum(y);
       else
            for jj=1:50
            crp=1;
            for k=1:n
                if j==k
                  crp=crp*(order_price_j(i,jj)^(1-el_sub))
                  *exp(-((round(10*((c(k)*dni(i,k))/
                  order_price_j(i,jj)))/10)*(1/nu(k)))^alpha)*
                  (((round(10*((c(k)*dni(i,k))))/10)
                  /(nu(k)))^alpha)*alpha*(order_price_j(i,jj)
                  ^(-alpha-1));
                else
                   crp=crp*(1-exp(-((round(10*((c(k)*dni(i,k))
                   /order_price_j(i,jj)))/10)*(1/nu(k)))^alpha));
                end
            end

                l=l+1;
                pp(l)=order_price_j(i,jj);
                y(l)=crp;
                if l>1
                small_area=((y(l)+y(l-1))/2)*(pp(l)-pp(l-1));
                area=area+small_area;
                end
            end
       Area_Xni_pricedat(i,j)=area;
       end
   end
end
Area_Xn_pricedat=Area_Xn(nu,alpha,dni,el_sub,order_price_j,c);
result_Weibull_Pricedat=ones(n,1);
```

```
k=0;
for i=1:n
    for j=1:n
        k=k+1;
        result_Weibull_Pricedat(k)=Area_Xni_pricedat(j,i)
        /Area_Xn_pricedat(j);
    end
end
F=result_Weibull_Pricedat;
```

```
%This function computes the Area_Xn.
function Area_Xn_pricedat=
                      Area_Xn(nu,alpha,dni,el_sub,order_price_j,c)

Area_Xn_pricedat=zeros(10,1);

arean=0;
for j=1:50
p=order_price_j(1,j);
derivative_Gn_1(j)=- 1/p^2*alpha*c(1)*
exp(-(1/p*c(1)/nu(1)*dni(1, 1))^alpha)
/nu(1)*dni(1,1)*(exp(-(1/p*c(2)/nu(2)*
dni(1, 2))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)*dni(1, 3))^alpha) - 1)
*(exp(-(1/p*c(4)/nu(4)*dni(1, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)
*dni(1, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*dni(1, 6))^alpha) - 1)
*(exp(-(1/p*c(7)/nu(7)*dni(1, 7))^alpha) - 1)*
(exp(-(1/p*c(8)/nu(8)*dni(1, 8))^alpha) - 1)*
(exp(-(1/p*c(9)/nu(9)*dni(1, 9))^alpha) - 1)*
(exp(-(1/p*c(10)/nu(10)*dni(1, 10))^alpha) - 1)*
(1/p*c(1)/nu(1)*dni(1, 1))^(alpha - 1) - 1/p^2*alpha*c(2)*
exp(-(1/p*c(2)/nu(2)*dni(1, 2))^alpha)/nu(2)*dni(1, 2)
*(exp(-(1/p*c(1)/nu(1)*dni(1, 1))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)
*dni(1, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)*dni(1, 4))^alpha) - 1)
*(exp(-(1/p*c(5)/nu(5)*dni(1, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)
*dni(1, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)*dni(1, 7))^alpha) - 1)
*(exp(-(1/p*c(8)/nu(8)*dni(1, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)
*dni(1, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)*
dni(1, 10))^alpha) - 1)*(1/p*c(2)/nu(2)*dni(1, 2))^(alpha - 1)
- 1/p^2*alpha*c(3)*exp(-(1/p*c(3)/nu(3)*dni(1, 3))^alpha)/nu(3)*
dni(1, 3)*(exp(-(1/p*c(1)/nu(1)*dni(1, 1))^alpha) - 1)*
(exp(-(1/p*c(2)/nu(2)*dni(1, 2))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(1, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(1, 5))^alpha) - 1)
*(exp(-(1/p*c(6)/nu(6)*dni(1, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)*
dni(1, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*dni(1, 8))^alpha) - 1)*
(exp(-(1/p*c(9)/nu(9)*dni(1, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)*
```

```
dni(1, 10))^alpha) − 1)*(1/p*c(3)/nu(3)*dni(1, 3))^(alpha − 1) −
1/p^2*alpha*c(4)*exp(−(1/p*c(4)/nu(4)*dni(1, 4))^alpha)/nu(4)*
dni(1, 4)*(exp(−(1/p*c(1)/nu(1)*dni(1, 1))^alpha)− 1)*
(exp(−(1/p*c(2)/nu(2)*dni(1, 2))^alpha) − 1)*
(exp(−(1/p*c(3)/nu(3)*dni(1, 3))^alpha) − 1)*
(exp(−(1/p*c(5)/nu(5)*dni(1, 5))^alpha) − 1)*(exp(−(1/p*c(6)/nu(6)*
dni(1, 6))^alpha) − 1)*(exp(−(1/p*c(7)/nu(7)*dni(1, 7))^alpha) − 1)*
(exp(−(1/p*c(8)/nu(8)*dni(1, 8))^alpha) − 1)*(exp(−(1/p*c(9)/nu(9)*
dni(1, 9))^alpha) − 1)*(exp(−(1/p*c(10)/nu(10)*
dni(1, 10))^alpha) − 1)*(1/p*c(4)/nu(4)*dni(1, 4))^(alpha − 1) −
1/p^2*alpha*c(5)*
exp(−(1/p*c(5)/nu(5)*dni(1, 5))^alpha)/nu(5)*dni(1, 5)
*(exp(−(1/p*c(1)/nu(1)*dni(1, 1))^alpha) − 1)*(exp(−(1/p*c(2)/nu(2)
*dni(1, 2))^alpha) − 1)*(exp(−(1/p*c(3)/nu(3)*dni(1, 3))^alpha) − 1)
*(exp(−(1/p*c(4)/nu(4)*dni(1, 4))^alpha) − 1)*(exp(−(1/p*c(6)/nu(6)
*dni(1, 6))^alpha) − 1)*(exp(−(1/p*c(7)/nu(7)*dni(1, 7))^alpha) − 1)
*(exp(−(1/p*c(8)/nu(8)*dni(1, 8))^alpha) − 1)*(exp(−(1/p*c(9)/nu(9)
*dni(1, 9))^alpha) − 1)*(exp(−(1/p*c(10)/nu(10)*
dni(1, 10))^alpha) − 1)*(1/p*c(5)/nu(5)*dni(1, 5))^(alpha − 1) −
1/p^2*alpha*c(6)*exp(−(1/p*c(6)/nu(6)*dni(1, 6))^alpha)/nu(6)*
dni(1, 6)*(exp(−(1/p*c(1)/nu(1)*dni(1, 1))^alpha) − 1)*
(exp(−(1/p*c(2)/nu(2)*dni(1, 2))^alpha) − 1)
*(exp(−(1/p*c(3)/nu(3)*dni(1, 3))^alpha) − 1)*(exp(−(1/p*c(4)/nu(4)
*dni(1, 4))^alpha) − 1)*(exp(−(1/p*c(5)/nu(5)*dni(1, 5))^alpha) − 1)
*(exp(−(1/p*c(7)/nu(7)*dni(1, 7))^alpha) − 1)*(exp(−(1/p*c(8)/nu(8)
*dni(1, 8))^alpha) − 1)*(exp(−(1/p*c(9)/nu(9)*dni(1, 9))^alpha) − 1)
*(exp(−(1/p*c(10)/nu(10)*dni(1, 10))^alpha) − 1)*(1/p*c(6)/nu(6)
*dni(1, 6))^(alpha − 1) − 1/p^2*alpha*c(7)*exp(−(1/p*c(7)/nu(7)
*dni(1, 7))^alpha)/nu(7)*dni(1, 7)*
(exp(−(1/p*c(1)/nu(1)*dni(1, 1))^alpha)− 1)*(exp(−(1/p*c(2)/nu(2)*
dni(1, 2))^alpha) − 1)*(exp(−(1/p*c(3)/nu(3)*dni(1, 3))^alpha) − 1)
*(exp(−(1/p*c(4)/nu(4)*dni(1, 4))^alpha) − 1)
*(exp(−(1/p*c(5)/nu(5)*dni(1, 5))^alpha) − 1)*(exp(−(1/p*c(6)/nu(6)
*dni(1, 6))^alpha) − 1)*(exp(−(1/p*c(8)/nu(8)*dni(1, 8))^alpha) − 1)
*(exp(−(1/p*c(9)/nu(9)*dni(1, 9))^alpha) − 1)*
```

```
(exp(-(1/p*c(10)/nu(10)*dni(1, 10))^alpha) - 1)*(1/p*c(7)/nu(7)*
dni(1, 7))^(alpha - 1) - 1/p^2*alpha*c(8)*exp(-(1/p*c(8)/nu(8)*
dni(1, 8))^alpha)/nu(8)*dni(1, 8)*(exp(-(1/p*c(1)/nu(1)
*dni(1, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*dni(1, 2))^alpha) - 1)
*(exp(-(1/p*c(3)/nu(3)*dni(1, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(1, 4))^alpha) - 1)
*(exp(-(1/p*c(5)/nu(5)*dni(1, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)
*dni(1, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)*dni(1, 7))^alpha) - 1)
*(exp(-(1/p*c(9)/nu(9)*dni(1, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)
*dni(1, 10))^alpha) - 1)*(1/p*c(8)/nu(8)*dni(1, 8))^(alpha - 1) -
1/p^2*alpha*c(9)*exp(-(1/p*c(9)/nu(9)*dni(1, 9))^alpha)/nu(9)*
dni(1, 9)*(exp(-(1/p*c(1)/nu(1)*dni(1, 1))^alpha) - 1)*
(exp(-(1/p*c(2)/nu(2)*dni(1, 2))^alpha) - 1)
*(exp(-(1/p*c(3)/nu(3)*dni(1, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(1, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(1, 5))^alpha) - 1)
*(exp(-(1/p*c(6)/nu(6)*dni(1, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)
*dni(1, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*dni(1, 8))^alpha) - 1)
*(exp(-(1/p*c(10)/nu(10)*dni(1, 10))^alpha) - 1)*(1/p*c(9)/nu(9)
*dni(1, 9))^(alpha - 1) - 1/p^2*alpha*c(10)*exp(-(1/p*c(10)/nu(10)
*dni(1, 10))^alpha)/nu(10)*dni(1, 10)*(exp(-(1/p*c(1)/nu(1)
*dni(1, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*dni(1, 2))^alpha) - 1)
*(exp(-(1/p*c(3)/nu(3)*dni(1, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(1, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(1, 5))^alpha) - 1)
*(exp(-(1/p*c(6)/nu(6)*dni(1, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)
*dni(1, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*dni(1, 8))^alpha) - 1)
*(exp(-(1/p*c(9)/nu(9)*dni(1, 9))^alpha) - 1)*(1/p*c(10)/nu(10)
*dni(1, 10))^(alpha - 1);derivative_Gn_1(j)
=derivative_Gn_1(j)*(p^(1-el_sub));
if j>1
small_arean=((derivative_Gn_1(j)+derivative_Gn_1(j-1))/2)
*(order_price_j(1,j)-order_price_j(1,j-1));
arean=arean+small_arean;
end
end
Area_Xn_pricedat(1)=arean;
```

```
arean=0;
for j=1:50
p=order_price_j(2,j);
derivative_Gn_2(j)=- 1/p^2*alpha*c(1)*exp(-(1/p*c(1)/nu(1)
*dni(2, 1))^alpha)/nu(1)*dni(2, 1)*(exp(-(1/p*c(2)/nu(2)*
dni(2, 2))^alpha)- 1)*(exp(-(1/p*c(3)/nu(3)*dni(2, 3))^alpha) - 1)
*(exp(-(1/p*c(4)/nu(4)*dni(2, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)
*dni(2, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*
dni(2, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)
*dni(2, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*dni(2, 8))^alpha) - 1)
*(exp(-(1/p*c(9)/nu(9)*dni(2, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)
*dni(2, 10))^alpha) - 1)*(1/p*c(1)/nu(1)*dni(2, 1))^
(alpha - 1) - 1/p^2*alpha*c(2)*exp(-(1/p*c(2)/nu(2)*
dni(2, 2))^alpha)/nu(2)*dni(2, 2)
*(exp(-(1/p*c(1)/nu(1)*dni(2, 1))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)
*dni(2, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)*dni(2, 4))^alpha) - 1)
*(exp(-(1/p*c(5)/nu(5)*dni(2, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)
*dni(2, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)*dni(2, 7))^alpha) - 1)
*(exp(-(1/p*c(8)/nu(8)*dni(2, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)
*dni(2, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)*dni(2, 10))^alpha)
- 1)*(1/p*c(2)/nu(2)*dni(2, 2))^(alpha - 1) - 1/p^2*alpha*c(3)*
exp(-(1/p*c(3)/nu(3)*dni(2, 3))^alpha)/nu(3)*dni(2, 3)*
(exp(-(1/p*c(1)/nu(1)*dni(2, 1))^alpha) - 1)
*(exp(-(1/p*c(2)/nu(2)*dni(2, 2))^alpha) - 1)
*(exp(-(1/p*c(4)/nu(4)*dni(2, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)
*dni(2, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*dni(2, 6))^alpha) - 1)
*(exp(-(1/p*c(7)/nu(7)*dni(2, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)
*dni(2, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*dni(2, 9))^alpha) - 1)
*(exp(-(1/p*c(10)/nu(10)*dni(2, 10))^alpha) - 1)*(1/p*c(3)/nu(3)
*dni(2, 3))^(alpha - 1) - 1/p^2*alpha*c(4)*exp(-(1/p*c(4)/nu(4)
*dni(2, 4))^alpha)/nu(4)*dni(2, 4)*(exp(-(1/p*c(1)/nu(1)
*dni(2, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*dni(2, 2))^alpha) - 1)
*(exp(-(1/p*c(3)/nu(3)*dni(2, 3))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)
*dni(2, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*dni(2, 6))^alpha) - 1)
```

```
*(exp(-(1/p*c(7)/nu(7)*dni(2, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)
*dni(2, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*dni(2, 9))^alpha) - 1)
*(exp(-(1/p*c(10)/nu(10)*dni(2, 10))^alpha) - 1)*(1/p*c(4)/nu(4)
*dni(2, 4))^(alpha - 1) - 1/p^2*alpha*c(5)*exp(-(1/p*c(5)/nu(5)
*dni(2, 5))^alpha)/nu(5)*dni(2, 5)*(exp(-(1/p*c(1)/nu(1)*
dni(2, 1))^alpha)- 1)*(exp(-(1/p*c(2)/nu(2)*dni(2, 2))^alpha) - 1
)*(exp(-(1/p*c(3)/nu(3)
*dni(2, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)*dni(2, 4))^alpha) - 1)
*(exp(-(1/p*c(6)/nu(6)*dni(2, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)
*dni(2, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*dni(2, 8))^alpha) - 1)
*(exp(-(1/p*c(9)/nu(9)*dni(2, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)
*dni(2, 10))^alpha) - 1)*(1/p*c(5)/nu(5)*dni(2, 5))^(alpha - 1)
- 1/p^2*alpha*c(6)*exp(-(1/p*c(6)/nu(6)*dni(2, 6))^alpha)/nu(6)*
dni(2, 6)*(exp(-(1/p*c(1)/nu(1)*dni(2, 1))^alpha) - 1)*
(exp(-(1/p*c(2)/nu(2)*dni(2, 2))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)
*dni(2, 3))^alpha) - 1)
*(exp(-(1/p*c(4)/nu(4)*dni(2, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)
*dni(2, 5))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)*dni(2, 7))^alpha) - 1)
*(exp(-(1/p*c(8)/nu(8)*dni(2, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)
*dni(2, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)*dni(2, 10))^alpha)
- 1)*(1/p*c(6)/nu(6)*dni(2, 6))^(alpha - 1) - 1/p^2*alpha*c(7)*
exp(-(1/p*c(7)/nu(7)*dni(2, 7))^alpha)/nu(7)*dni(2, 7)*
(exp(-(1/p*c(1)/nu(1)*dni(2, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)
*dni(2, 2))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)*dni(2, 3))^alpha) - 1)
*(exp(-(1/p*c(4)/nu(4)
*dni(2, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(2, 5))^alpha) - 1)
*(exp(-(1/p*c(6)/nu(6)*dni(2, 6))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)
*dni(2, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*dni(2, 9))^alpha) - 1)
*(exp(-(1/p*c(10)/nu(10)*dni(2, 10))^alpha) - 1)*(1/p*c(7)/nu(7)
*dni(2, 7))^(alpha - 1) - 1/p^2*alpha*c(8)*exp(-(1/p*c(8)/nu(8)
*dni(2, 8))^alpha)/nu(8)*dni(2, 8)*(exp(-(1/p*c(1)/nu(1)*
dni(2, 1))^alpha)- 1)*(exp(-(1/p*c(2)/nu(2)*dni(2, 2))^alpha) - 1)*
(exp(-(1/p*c(3)/nu(3)*dni(2, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(2, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(2, 5))^alpha) - 1)*
(exp(-(1/p*c(6)/nu(6)*dni(2, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)*
```

```
dni(2, 7))^alpha) − 1)*(exp(−(1/p*c(9)/nu(9)*dni(2, 9))^alpha) − 1)*
(exp(−(1/p*c(10)/nu(10)*dni(2, 10))^alpha) − 1)*(1/p*c(8)/nu(8)*
dni(2, 8))^(alpha − 1) − 1/p^2*alpha*c(9)*exp(−(1/p*c(9)/nu(9)
*dni(2, 9))^alpha)/nu(9)*dni(2, 9)
*(exp(−(1/p*c(1)/nu(1)*dni(2, 1))^alpha) − 1)*(exp(−(1/p*c(2)/nu(2)
*dni(2, 2))^alpha) − 1)*(exp(−(1/p*c(3)/nu(3)*dni(2, 3))^alpha) − 1)
*(exp(−(1/p*c(4)/nu(4)*dni(2, 4))^alpha) − 1)*(exp(−(1/p*c(5)/nu(5)
*dni(2, 5))^alpha) − 1)*(exp(−(1/p*c(6)/nu(6)*dni(2, 6))^alpha) − 1)
*(exp(−(1/p*c(7)/nu(7)*dni(2, 7))^alpha) − 1)*(exp(−(1/p*c(8)/nu(8)
*dni(2, 8))^alpha) − 1)*(exp(−(1/p*c(10)/nu(10)*dni(2, 10))^alpha)
− 1)*(1/p*c(9)/nu(9)*dni(2, 9))^(alpha − 1) − 1/p^2*alpha*c(10)*
exp(−(1/p*c(10)/nu(10)*dni(2, 10))^alpha)/nu(10)*dni(2, 10)*
(exp(−(1/p*c(1)/nu(1)*dni(2, 1))^alpha) − 1)*
(exp(−(1/p*c(2)/nu(2)*dni(2, 2))^alpha) − 1)
*(exp(−(1/p*c(3)/nu(3)*dni(2, 3))^alpha) − 1)*(exp(−(1/p*c(4)/nu(4)
*dni(2, 4))^alpha) − 1)*(exp(−(1/p*c(5)/nu(5)*dni(2, 5))^alpha) − 1)
*(exp(−(1/p*c(6)/nu(6)*dni(2, 6))^alpha) − 1)*(exp(−(1/p*c(7)/nu(7)
*dni(2, 7))^alpha) − 1)*(exp(−(1/p*c(8)/nu(8)*dni(2, 8))^alpha) − 1)
*(exp(−(1/p*c(9)/nu(9)*dni(2, 9))^alpha) − 1)*(1/p*c(10)/nu(10)
*dni(2, 10))^(alpha − 1);
derivative_Gn_2(j)=derivative_Gn_2(j)*(p^(1−el_sub));
if j>1
small_arean=((derivative_Gn_2(j)+derivative_Gn_2(j−1))/2)
*(order_price_j(2,j)−order_price_j(2,j−1));
arean=arean+small_arean;
end
end
Area_Xn_pricedat(2)=arean;

arean=0;
for j=1:50
p=order_price_j(3,j);
derivative_Gn_3(j)=− 1/p^2*alpha*c(1)*exp(−(1/p*c(1)/nu(1)
*dni(3, 1))^alpha)/nu(1)*dni(3, 1)*(exp(−(1/p*c(2)/nu(2)*
dni(3, 2))^alpha)− 1)*(exp(−(1/p*c(3)/nu(3)*dni(3, 3))^alpha) − 1
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)*(exp(-(1/p*c(4)/nu(4)*dni(3, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)
*dni(3, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*dni(3, 6))^alpha) - 1)*
(exp(-(1/p*c(7)/nu(7)*dni(3, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*
dni(3, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*dni(3, 9))^alpha) - 1)*
(exp(-(1/p*c(10)/nu(10)*dni(3, 10))^alpha) - 1)*(1/p*c(1)/nu(1)*
dni(3, 1))^(alpha - 1) - 1/p^2*alpha*c(2)*exp(-(1/p*c(2)/nu(2)*
dni(3, 2))^alpha)/nu(2)*dni(3, 2)*(exp(-(1/p*c(1)/nu(1)*dni(3, 1))
^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)*dni(3, 3))^alpha) - 1)*
(exp(-(1/p*c(4)/nu(4)*dni(3, 4))^alpha) - 1)
*(exp(-(1/p*c(5)/nu(5)*dni(3, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)
*dni(3, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)*dni(3, 7))^alpha) - 1)
*(exp(-(1/p*c(8)/nu(8)*dni(3, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)
*dni(3, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)*dni(3, 10))^alpha)
- 1)*(1/p*c(2)/nu(2)*dni(3, 2))^(alpha - 1) - 1/p^2*alpha*c(3)*
exp(-(1/p*c(3)/nu(3)*dni(3, 3))^alpha)/nu(3)*dni(3, 3)*
(exp(-(1/p*c(1)/nu(1)*dni(3, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*
dni(3, 2))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)*dni(3, 4))^alpha) - 1)
*(exp(-(1/p*c(5)/nu(5)*dni(3, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*
dni(3, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)*dni(3, 7))^alpha) - 1)*
(exp(-(1/p*c(8)/nu(8)*dni(3, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*
dni(3, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)*dni(3, 10))^alpha)
- 1)*(1/p*c(3)/nu(3)*dni(3, 3))^(alpha - 1) - 1/p^2*alpha*c(4)*
exp(-(1/p*c(4)/nu(4)*dni(3, 4))^alpha)/nu(4)*dni(3, 4)*
(exp(-(1/p*c(1)/nu(1)*dni(3, 1))^alpha)- 1)*(exp(-(1/p*c(2)/nu(2)
*dni(3, 2))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)
*dni(3, 3))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(3, 5))^alpha) - 1)
*(exp(-(1/p*c(6)/nu(6)*dni(3, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)
*dni(3, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*dni(3, 8))^alpha) - 1)
*(exp(-(1/p*c(9)/nu(9)*dni(3, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)
*dni(3, 10))^alpha) - 1)*(1/p*c(4)/nu(4)*dni(3, 4))^(alpha - 1) -
1/p^2*alpha*c(5)*exp(-(1/p*c(5)/nu(5)*dni(3, 5))^alpha)/nu(5)
*dni(3, 5)*(exp(-(1/p*c(1)/nu(1)*dni(3, 1))^alpha) - 1)*
(exp(-(1/p*c(2)/nu(2)*dni(3, 2))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)
*dni(3, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)*dni(3, 4))^alpha) - 1)
*(exp(-(1/p*c(6)/nu(6)*dni(3, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)
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*dni(3, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*dni(3, 8))^alpha) - 1)
*(exp(-(1/p*c(9)/nu(9)*dni(3, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)
*dni(3, 10))^alpha) - 1)*(1/p*c(5)/nu(5)*dni(3, 5))^(alpha - 1)
- 1/p^2*alpha*c(6)*exp(-(1/p*c(6)
/nu(6)*dni(3, 6))^alpha)/nu(6)*dni(3, 6)*(exp(-(1/p*c(1)/nu(1)
*dni(3, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*dni(3, 2))^alpha) - 1)
*(exp(-(1/p*c(3)/nu(3)*dni(3, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(3, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(3, 5))^alpha) - 1)
*(exp(-(1/p*c(7)/nu(7)*dni(3, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)
*dni(3, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*dni(3, 9))^alpha) - 1)
*(exp(-(1/p*c(10)/nu(10)*dni(3, 10))^alpha) - 1)*(1/p*c(6)/nu(6)
*dni(3, 6))^(alpha - 1) - 1/p^2*alpha*c(7)*exp(-(1/p*c(7)/nu(7)
*dni(3, 7))^alpha)/nu(7)*dni(3, 7)*(exp(-(1/p*c(1)/nu(1)*dni(3, 1))
^alpha)- 1)*(exp(-(1/p*c(2)/nu(2)*dni(3, 2))^alpha) - 1)*
(exp(-(1/p*c(3)/nu(3)*dni(3, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)*
dni(3, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(3, 5))^alpha) - 1)
*(exp(-(1/p*c(6)/nu(6)*dni(3, 6))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*
dni(3, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*dni(3, 9))^alpha) - 1)
*(exp(-(1/p*c(10)/nu(10)*dni(3, 10))^alpha) - 1)*(1/p*c(7)/nu(7)*
dni(3, 7))^(alpha - 1) - 1/p^2*alpha*c(8)*exp(-(1/p*c(8)/nu(8)*
dni(3, 8))^alpha)/nu(8)*dni(3, 8)
*(exp(-(1/p*c(1)/nu(1)*dni(3, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)
*dni(3, 2))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)*dni(3, 3))^alpha) - 1)
*(exp(-(1/p*c(4)/nu(4)*dni(3, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)
*dni(3, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*dni(3, 6))^alpha) - 1)
*(exp(-(1/p*c(7)/nu(7)*dni(3, 7))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)
*dni(3, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)*dni(3, 10))^alpha)
- 1)*(1/p*c(8)/nu(8)*dni(3, 8))^(alpha - 1) - 1/p^2*alpha*c(9)*
exp(-(1/p*c(9)/nu(9)*dni(3, 9))^alpha)/nu(9)*dni(3, 9)*
(exp(-(1/p*c(1)/nu(1)*dni(3, 1))^alpha) - 1)*
(exp(-(1/p*c(2)/nu(2)*dni(3, 2))^alpha) - 1)
*(exp(-(1/p*c(3)/nu(3)*dni(3, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(3, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(3, 5))^alpha) - 1)
*(exp(-(1/p*c(6)/nu(6)*dni(3, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)
*dni(3, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*dni(3, 8))^alpha) - 1)
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*(exp(-(1/p*c(10)/nu(10)*dni(3, 10))^alpha) - 1)*(1/p*c(9)/nu(9)
*dni(3, 9))^(alpha - 1) - 1/p^2*alpha*c(10)*exp(-(1/p*c(10)/nu(10)
*dni(3, 10))^alpha)/nu(10)*dni(3, 10)*(exp(-(1/p*c(1)/nu(1)
*dni(3, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*dni(3, 2))^alpha) - 1)
*(exp(-(1/p*c(3)/nu(3)*dni(3, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(3, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(3, 5))^alpha) - 1)
*(exp(-(1/p*c(6)/nu(6)*dni(3, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)
*dni(3, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*dni(3, 8))^alpha) - 1)
*(exp(-(1/p*c(9)/nu(9)*dni(3, 9))^alpha) - 1)*(1/p*c(10)/nu(10)
*dni(3, 10))^(alpha - 1);
derivative_Gn_3(j)=derivative_Gn_3(j)*(p^(1-el_sub));
if j>1
small_arean=((derivative_Gn_3(j)+derivative_Gn_3(j-1))/2)
*(order_price_j(3,j)-order_price_j(3,j-1));
arean=arean+small_arean;
end
end
Area_Xn_pricedat(3)=arean;

arean=0;
for j=1:50
p=order_price_j(4,j);
derivative_Gn_4(j)=- 1/p^2*alpha*c(1)*
exp(-(1/p*c(1)/nu(1)*dni(4, 1))^alpha)
/nu(1)*dni(4, 1)*(exp(-(1/p*c(2)/nu(2)*dni(4, 2))^alpha) - 1)
*(exp(-(1/p*c(3)/nu(3)*dni(4, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(4, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(4, 5))^alpha) - 1)
*(exp(-(1/p*c(6)/nu(6)*dni(4, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)
*dni(4, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*dni(4, 8))^alpha) - 1)
*(exp(-(1/p*c(9)/nu(9)*dni(4, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)
*dni(4, 10))^alpha) - 1)*(1/p*c(1)/nu(1)*dni(4, 1))^(alpha - 1) -
1/p^2*alpha*c(2)*exp(-(1/p*c(2)/nu(2)*dni(4, 2))^alpha)/nu(2)*
dni(4, 2)*(exp(-(1/p*c(1)/nu(1)*dni(4, 1))^alpha) - 1)*
(exp(-(1/p*c(3)/nu(3)*dni(4, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(4, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(4, 5))^alpha) - 1)
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*(exp(-(1/p*c(6)/nu(6)*dni(4, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)
*dni(4, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*dni(4, 8))^alpha) - 1)
*(exp(-(1/p*c(9)/nu(9)*dni(4, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)
*dni(4, 10))^alpha) - 1)*(1/p*c(2)/nu(2)*dni(4, 2))^(alpha - 1) -
1/p^2*alpha*c(3)*exp(-(1/p*c(3)/nu(3)*dni(4, 3))^alpha)/nu(3)*
dni(4, 3)*(exp(-(1/p*c(1)/nu(1)*dni(4, 1))^alpha) - 1)*
(exp(-(1/p*c(2)/nu(2)*dni(4, 2))^alpha) - 1)
*(exp(-(1/p*c(4)/nu(4)*dni(4, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)
*dni(4, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*dni(4, 6))^alpha) - 1)
*(exp(-(1/p*c(7)/nu(7)*dni(4, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)
*dni(4, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*dni(4, 9))^alpha) - 1)
*(exp(-(1/p*c(10)/nu(10)*dni(4, 10))^alpha) - 1)*(1/p*c(3)/nu(3)
*dni(4, 3))^(alpha - 1) - 1/p^2*alpha*c(4)*exp(-(1/p*c(4)/nu(4)
*dni(4, 4))^alpha)/nu(4)*dni(4, 4)*(exp(-(1/p*c(1)/nu(1)*dni(4, 1)
)^alpha)- 1)*(exp(-(1/p*c(2)/nu(2)*dni(4, 2))^alpha) - 1)*
(exp(-(1/p*c(3)/nu(3)*dni(4, 3))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5
)*dni(4, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*dni(4, 6))^alpha) - 1)
*(exp(-(1/p*c(7)/nu(7)*dni(4, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)
*dni(4, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*dni(4, 9))^alpha)
- 1)*(exp(-(1/p*c(10)/nu(10)*dni(4, 10))^alpha) - 1)*(1/p*c(4)/nu(4)
*dni(4, 4))^(alpha - 1) - 1/p^2*alpha*c(5)*exp(-(1/p*c(5)/nu(5)*
dni(4, 5))^alpha)/nu(5)*dni(4, 5)*
(exp(-(1/p*c(1)/nu(1)*dni(4, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)
*dni(4, 2))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)*dni(4, 3))^alpha) - 1)
*(exp(-(1/p*c(4)/nu(4)*dni(4, 4))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)
*dni(4, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)*dni(4, 7))^alpha) - 1)
*(exp(-(1/p*c(8)/nu(8)*dni(4, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)
*dni(4, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)*dni(4, 10))^alpha)
- 1)*(1/p*c(5)/nu(5)*dni(4, 5))^(alpha - 1) - 1/p^2*alpha*c(6)*
exp(-(1/p*c(6)/nu(6)*dni(4, 6))^alpha)/nu(6)*dni(4, 6)*
(exp(-(1/p*c(1)/nu(1)*dni(4, 1))^alpha) - 1)*
(exp(-(1/p*c(2)/nu(2)*dni(4, 2))^alpha) - 1)
*(exp(-(1/p*c(3)/nu(3)*dni(4, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(4, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(4, 5))^alpha) - 1)
*(exp(-(1/p*c(7)/nu(7)*dni(4, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)
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*dni(4, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*dni(4, 9))^alpha) - 1)
*(exp(-(1/p*c(10)/nu(10)*dni(4, 10))^alpha) - 1)*(1/p*c(6)/nu(6)
*dni(4, 6))^(alpha - 1) - 1/p^2*alpha*c(7)*exp(-(1/p*c(7)/nu(7)
*dni(4, 7))^alpha)/nu(7)*dni(4, 7)*
(exp(-(1/p*c(1)/nu(1)*dni(4, 1))^alpha)- 1)*
(exp(-(1/p*c(2)/nu(2)*dni(4, 2))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)
*dni(4, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)*dni(4, 4))^alpha) - 1)
*(exp(-(1/p*c(5)/nu(5)*dni(4, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)
*dni(4, 6))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*dni(4, 8))^alpha) - 1)
*(exp(-(1/p*c(9)/nu(9)*dni(4, 9))^alpha) - 1)*
(exp(-(1/p*c(10)/nu(10)*dni(4, 10))^alpha) - 1)*(1/p*c(7)/nu(7)
*dni(4, 7))^(alpha - 1) - 1/p^2
*alpha*c(8)*exp(-(1/p*c(8)/nu(8)*dni(4, 8))^alpha)/nu(8)*dni(4, 8)
*(exp(-(1/p*c(1)/nu(1)*dni(4, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)
*dni(4, 2))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)*dni(4, 3))^alpha) - 1)
*(exp(-(1/p*c(4)/nu(4)*dni(4, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)
*dni(4, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*dni(4, 6))^alpha) - 1)
*(exp(-(1/p*c(7)/nu(7)*dni(4, 7))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)
*dni(4, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)*dni(4, 10))^alpha)
- 1)*(1/p*c(8)/nu(8)*dni(4, 8))^(alpha - 1) - 1/p^2*alpha*c(9)
*exp(-(1/p*c(9)/nu(9)*dni(4, 9))^alpha)/nu(9)*dni(4, 9)*
(exp(-(1/p*c(1)/nu(1)*dni(4, 1))^alpha) - 1)*
(exp(-(1/p*c(2)/nu(2)*dni(4, 2))^alpha) - 1)
*(exp(-(1/p*c(3)/nu(3)*dni(4, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(4, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(4, 5))^alpha) - 1)
*(exp(-(1/p*c(6)/nu(6)*dni(4, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)
*dni(4, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*dni(4, 8))^alpha) - 1)
*(exp(-(1/p*c(10)/nu(10)*dni(4, 10))^alpha) - 1)*(1/p*c(9)/nu(9)
*dni(4, 9))^(alpha - 1) - 1/p^2*alpha*c(10)*exp(-(1/p*c(10)/nu(10)
*dni(4, 10))^alpha)/nu(10)*dni(4, 10)*(exp(-(1/p*c(1)/nu(1)
*dni(4, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*dni(4, 2))^alpha) - 1)
*(exp(-(1/p*c(3)/nu(3)*dni(4, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(4, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(4, 5))^alpha) - 1)
*(exp(-(1/p*c(6)/nu(6)*dni(4, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)
*dni(4, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*dni(4, 8))^alpha) - 1)
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*(exp(-(1/p*c(9)/nu(9)*dni(4, 9))^alpha) - 1)*(1/p*c(10)/nu(10)
*dni(4, 10))^(alpha - 1);
derivative_Gn_4(j)=derivative_Gn_4(j)*(p^(1-el_sub));
if j>1
small_arean=((derivative_Gn_4(j)+derivative_Gn_4(j-1))/2)
*(order_price_j(4,j)-order_price_j(4,j-1));
arean=arean+small_arean;
end
end
Area_Xn_pricedat(4)=arean;

arean=0;
for j=1:50
p=order_price_j(5,j);
derivative_Gn_5(j)=- 1/p^2*alpha*c(1)*
exp(-(1/p*c(1)/nu(1)*dni(5, 1))^alpha)
/nu(1)*dni(5, 1)*(exp(-(1/p*c(2)/nu(2)*dni(5, 2))^alpha) - 1)
*(exp(-(1/p*c(3)/nu(3)*dni(5, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(5, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(5, 5))^alpha) - 1)
*(exp(-(1/p*c(6)/nu(6)*dni(5, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)
*dni(5, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*dni(5, 8))^alpha) - 1)
*(exp(-(1/p*c(9)/nu(9)*dni(5, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)
*dni(5, 10))^alpha) - 1)*(1/p*c(1)/nu(1)*dni(5, 1))^(alpha - 1)
- 1/p^2*alpha*c(2)*exp(-(1/p*c(2)/nu(2)*dni(5, 2))^alpha)/nu(2)*
dni(5, 2)*(exp(-(1/p*c(1)/nu(1)*dni(5, 1))^alpha) - 1)*
(exp(-(1/p*c(3)/nu(3)*dni(5, 3))^alpha) - 1)*
(exp(-(1/p*c(4)/nu(4)*dni(5, 4))^alpha) - 1)
*(exp(-(1/p*c(5)/nu(5)*dni(5, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)
*dni(5, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)*dni(5, 7))^alpha) - 1)
*(exp(-(1/p*c(8)/nu(8)*dni(5, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)
*dni(5, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)*dni(5, 10))^alpha)-1)
*(1/p*c(2)/nu(2)*dni(5, 2))^(alpha - 1) - 1/p^2*alpha*c(3)*
exp(-(1/p*c(3)/nu(3)*dni(5, 3))^alpha)/nu(3)*dni(5, 3)*
(exp(-(1/p*c(1)/nu(1)*dni(5, 1))^alpha) - 1)*
(exp(-(1/p*c(2)/nu(2)*dni(5, 2))^alpha) - 1)
```

```
*(exp(-(1/p*c(4)/nu(4)*dni(5, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)
*dni(5, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*dni(5, 6))^alpha) - 1)
*(exp(-(1/p*c(7)/nu(7)*dni(5, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)
*dni(5, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*dni(5, 9))^alpha) - 1)
*(exp(-(1/p*c(10)/nu(10)*dni(5, 10))^alpha) - 1)*(1/p*c(3)/nu(3)
*dni(5, 3))^(alpha - 1) - 1/p^2*alpha*c(4)*exp(-(1/p*c(4)/nu(4)
*dni(5, 4))^alpha)/nu(4)*dni(5, 4)*(exp(-(1/p*c(1)/nu(1)*
dni(5, 1))^alpha)- 1)*(exp(-(1/p*c(2)/nu(2)*dni(5, 2))^alpha) - 1
)*(exp(-(1/p*c(3)/nu(3)*dni(5, 3))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)
*dni(5, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*dni(5, 6))^alpha) - 1)
*(exp(-(1/p*c(7)/nu(7)*dni(5, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)
*dni(5, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*dni(5, 9))^alpha) - 1)
*(exp(-(1/p*c(10)/nu(10)
*dni(5, 10))^alpha) - 1)*(1/p*c(4)/nu(4)*dni(5, 4))^(alpha - 1)
- 1/p^2*alpha*c(5)*exp(-(1/p*c(5)/nu(5)*dni(5, 5))^alpha)/nu(5)
*dni(5, 5)*(exp(-(1/p*c(1)/nu(1)*dni(5, 1))^alpha) - 1)*
(exp(-(1/p*c(2)/nu(2)*dni(5, 2))^alpha) - 1)*
(exp(-(1/p*c(3)/nu(3)*dni(5, 3))^alpha) - 1)
*(exp(-(1/p*c(4)/nu(4)*dni(5, 4))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)
*dni(5, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)*dni(5, 7))^alpha) - 1)
*(exp(-(1/p*c(8)/nu(8)*dni(5, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)
*dni(5, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)*dni(5, 10))^alpha)
- 1)*(1/p*c(5)/nu(5)*dni(5, 5))^(alpha - 1) - 1/p^2*alpha*c(6)*
exp(-(1/p*c(6)/nu(6)*dni(5, 6))^alpha)/nu(6)*dni(5, 6)*
(exp(-(1/p*c(1)/nu(1)
*dni(5, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*dni(5, 2))^alpha) - 1)
*(exp(-(1/p*c(3)/nu(3)*dni(5, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(5, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(5, 5))^alpha) - 1)
*(exp(-(1/p*c(7)/nu(7)*dni(5, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)
*dni(5, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*dni(5, 9))^alpha) - 1)
*(exp(-(1/p*c(10)/nu(10)*dni(5, 10))^alpha) - 1)*(1/p*c(6)/nu(6)
*dni(5, 6))^(alpha - 1) - 1/p^2*alpha*c(7)*exp(-(1/p*c(7)/nu(7)
*dni(5, 7))^alpha)/nu(7)*dni(5, 7)*(exp(-(1/p*c(1)/nu(1)*
dni(5, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*dni(5, 2))^alpha)
- 1)*(exp(-(1/p*c(3)/nu(3)*dni(5, 3))^alpha) - 1)*
```

```
(exp(-(1/p*c(4)/nu(4)*dni(5, 4))^alpha) - 1)
*(exp(-(1/p*c(5)/nu(5)*dni(5, 5))^alpha) - 1)*
(exp(-(1/p*c(6)/nu(6)*dni(5, 6))^alpha) - 1)*
(exp(-(1/p*c(8)/nu(8)*dni(5, 8))^alpha) - 1)
*(exp(-(1/p*c(9)/nu(9)*dni(5, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)
*dni(5, 10))^alpha) - 1)*(1/p*c(7)/nu(7)*dni(5, 7))^(alpha - 1)
- 1/p^2*alpha*c(8)*exp(-(1/p*c(8)/nu(8)*dni(5, 8))^alpha)/nu(8)
*dni(5, 8)*(exp(-(1/p*c(1)/nu(1)*dni(5, 1))^alpha) - 1)*
(exp(-(1/p*c(2)/nu(2)*dni(5, 2))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)
*dni(5, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)*dni(5, 4))^alpha)
- 1)*(exp(-(1/p*c(5)/nu(5)*dni(5, 5))^alpha) - 1)*
(exp(-(1/p*c(6)/nu(6)*dni(5, 6))^alpha) - 1)
*(exp(-(1/p*c(7)/nu(7)*dni(5, 7))^alpha) - 1)*
(exp(-(1/p*c(9)/nu(9)*dni(5, 9))^alpha) - 1)*
(exp(-(1/p*c(10)/nu(10)*dni(5, 10))^alpha) - 1)
*(1/p*c(8)/nu(8)*dni(5, 8))^(alpha - 1) - 1/p^2*alpha*
c(9)*exp(-(1/p*c(9)/nu(9)*dni(5, 9))^alpha)/nu(9)*dni(5, 9)
*(exp(-(1/p*c(1)/nu(1)
*dni(5, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*dni(5, 2))^alpha) - 1)
*(exp(-(1/p*c(3)/nu(3)*dni(5, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(5, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(5, 5))^alpha) - 1)
*(exp(-(1/p*c(6)/nu(6)*dni(5, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)
*dni(5, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*dni(5, 8))^alpha) - 1)
*(exp(-(1/p*c(10)/nu(10)*dni(5, 10))^alpha) - 1)*(1/p*c(9)/nu(9)
*dni(5, 9))^(alpha - 1) - 1/p^2*alpha*c(10)*exp(-(1/p*c(10)/nu(10)
*dni(5, 10))^alpha)/nu(10)*dni(5, 10)*(exp(-(1/p*c(1)/nu(1)
*dni(5, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*dni(5, 2))^alpha) - 1)
*(exp(-(1/p*c(3)/nu(3)*dni(5, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(5, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(5, 5))^alpha) - 1)
*(exp(-(1/p*c(6)/nu(6)*dni(5, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)
*dni(5, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*dni(5, 8))^alpha) - 1)
*(exp(-(1/p*c(9)/nu(9)*dni(5, 9))^alpha) - 1)*(1/p*c(10)/nu(10)
*dni(5, 10))^(alpha - 1);
derivative_Gn_5(j)=derivative_Gn_5(j)*(p^(1-el_sub));
if j>1
```

```
small_arean=((derivative_Gn_5(j)+derivative_Gn_5(j-1))/2)
*(order_price_j(5,j)-order_price_j(5,j-1));
arean=arean+small_arean;
end
end
Area_Xn_pricedat(5)=arean;

arean=0;
for j=1:50
p=order_price_j(6,j);
derivative_Gn_6(j)=- 1/p^2*alpha*c(1)*exp(-(1/p*c(1)/nu(1)
*dni(6, 1))^alpha)/nu(1)*dni(6, 1)*(exp(-(1/p*c(2)/nu(2)
*dni(6, 2))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)*dni(6, 3))^alpha) - 1)
*(exp(-(1/p*c(4)/nu(4)*dni(6, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)
*dni(6, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*dni(6, 6))^alpha) - 1)
*(exp(-(1/p*c(7)/nu(7)*dni(6, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)
*dni(6, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*dni(6, 9))^alpha) - 1)
*(exp(-(1/p*c(10)/nu(10)*dni(6, 10))^alpha) - 1)*(1/p*c(1)/nu(1)
*dni(6, 1))^(alpha - 1) - 1/p^2*alpha*c(2)*exp(-(1/p*c(2)/nu(2)
*dni(6, 2))^alpha)/nu(2)*dni(6, 2)*(exp(-(1/p*c(1)/nu(1)*dni(6, 1))
^alpha)- 1)*(exp(-(1/p*c(3)/nu(3)*dni(6, 3))^alpha) - 1)*
(exp(-(1/p*c(4)/nu(4)*dni(6, 4))^alpha) - 1)*
(exp(-(1/p*c(5)/nu(5)*dni(6, 5))^alpha) - 1)
*(exp(-(1/p*c(6)/nu(6)*dni(6, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)
*dni(6, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*dni(6, 8))^alpha) - 1)
*(exp(-(1/p*c(9)/nu(9)*dni(6, 9))^alpha)-1)*(exp(-(1/p*c(10)/nu(10)
*dni(6, 10))^alpha) - 1)*(1/p*c(2)/nu(2)*dni(6, 2))^(alpha-1)-1/p^2
*alpha*c(3)*exp(-(1/p*c(3)/nu(3)*dni(6, 3))^alpha)/nu(3)*dni(6, 3)
*(exp(-(1/p*c(1)/nu(1)*dni(6, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)
*dni(6, 2))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)*dni(6, 4))^alpha) - 1)
*(exp(-(1/p*c(5)/nu(5)*dni(6, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)
*dni(6, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)*dni(6, 7))^alpha) - 1)
*(exp(-(1/p*c(8)/nu(8)*dni(6, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)
*dni(6, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)*dni(6, 10))^alpha)-1)
*(1/p*c(3)/nu(3)*dni(6, 3))^(alpha - 1) - 1/p^2*alpha*c(4)*
```

```
exp(−(1/p*c(4)/nu(4)*dni(6, 4))^alpha)/nu(4)*dni(6, 4)*
(exp(−(1/p*c(1)/nu(1)*dni(6, 1))^alpha) − 1)*(exp(−(1/p*c(2)/nu(2)
*dni(6, 2))^alpha) − 1)*(exp(−(1/p*c(3)/nu(3)*dni(6, 3))^alpha) − 1)
*(exp(−(1/p*c(5)/nu(5)*dni(6, 5))^alpha) − 1)*(exp(−(1/p*c(6)/nu(6)*
dni(6, 6))^alpha) − 1)*(exp(−(1/p*c(7)/nu(7)*dni(6, 7))^alpha) − 1)
*(exp(−(1/p*c(8)/nu(8)*dni(6, 8))^alpha) − 1)*(exp(−(1/p*c(9)/nu(9)
*dni(6, 9))^alpha) − 1)*(exp(−(1/p*c(10)/nu(10)*dni(6, 10))^alpha)
− 1)*(1/p*c(4)/nu(4)*dni(6, 4))^(alpha − 1) − 1/p^2*alpha*c(5)*
exp(−(1/p*c(5)/nu(5)*dni(6, 5))^alpha)/nu(5)*dni(6, 5)*
(exp(−(1/p*c(1)/nu(1)*dni(6, 1))^alpha) − 1)*(exp(−(1/p*c(2)/nu(2)
*dni(6, 2))^alpha) − 1)*(exp(−(1/p*c(3)/nu(3)*dni(6, 3))^alpha) − 1)
*(exp(−(1/p*c(4)/nu(4)*dni(6, 4))^alpha) − 1)*(exp(−(1/p*c(6)/nu(6)
*dni(6, 6))^alpha) − 1)*(exp(−(1/p*c(7)/nu(7)*dni(6, 7))^alpha) − 1)
*(exp(−(1/p*c(8)/nu(8)*dni(6, 8))^alpha) − 1)
*(exp(−(1/p*c(9)/nu(9)*dni(6, 9))^alpha) −1)*(exp(−(1/p*c(10)/nu(10)
*dni(6, 10))^alpha) − 1)*(1/p*c(5)/nu(5)*dni(6, 5))^(alpha−1)−1/p^2
*alpha*c(6)*exp(−(1/p*c(6)/nu(6)*dni(6, 6))^alpha)/nu(6)*dni(6, 6)
*(exp(−(1/p*c(1)/nu(1)*dni(6, 1))^alpha) − 1)*(exp(−(1/p*c(2)/nu(2)
*dni(6, 2))^alpha) − 1)*(exp(−(1/p*c(3)/nu(3)*dni(6, 3))^alpha) − 1)
*(exp(−(1/p*c(4)/nu(4)*dni(6, 4))^alpha) − 1)*(exp(−(1/p*c(5)/nu(5)
*dni(6, 5))^alpha) − 1)*(exp(−(1/p*c(7)/nu(7)*dni(6, 7))^alpha) − 1)
*(exp(−(1/p*c(8)/nu(8)*dni(6, 8))^alpha) − 1)*(exp(−(1/p*c(9)/nu(9)
*dni(6, 9))^alpha) − 1)*(exp(−(1/p*c(10)/nu(10)*dni(6, 10))^alpha)−1)
*(1/p*c(6)/nu(6)*dni(6, 6))^(alpha − 1) − 1/p^2*alpha*c(7)*
exp(−(1/p*c(7)/nu(7)*dni(6, 7))^alpha)/nu(7)*dni(6, 7)*
(exp(−(1/p*c(1)/nu(1)*dni(6, 1))^alpha) − 1)*(exp(−(1/p*c(2)/nu(2)
*dni(6, 2))^alpha) − 1)*(exp(−(1/p*c(3)/nu(3)*dni(6, 3))^alpha) − 1)
*(exp(−(1/p*c(4)/nu(4)*dni(6, 4))^alpha) − 1)*(exp(−(1/p*c(5)/nu(5)*
dni(6, 5))^alpha) − 1)*(exp(−(1/p*c(6)/nu(6)*dni(6, 6))^alpha) − 1)*
(exp(−(1/p*c(8)/nu(8)*dni(6, 8))^alpha) − 1)*(exp(−(1/p*c(9)/nu(9)*
dni(6, 9))^alpha) − 1)*(exp(−(1/p*c(10)/nu(10)*dni(6, 10))^alpha)−1)
*(1/p*c(7)/nu(7)*dni(6, 7))^(alpha − 1) − 1/p^2*alpha*c(8)*
exp(−(1/p*c(8)/nu(8)*dni(6, 8))^alpha)/nu(8)*dni(6, 8)*
(exp(−(1/p*c(1)/nu(1)*dni(6, 1))^alpha)− 1)*(exp(−(1/p*c(2)/nu(2)*
dni(6, 2))^alpha) − 1)*(exp(−(1/p*c(3)/nu(3)
```

```
*dni(6, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)*dni(6, 4))^alpha) - 1)
*(exp(-(1/p*c(5)/nu(5)*dni(6, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)
*dni(6, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)*dni(6, 7))^alpha) - 1)
*(exp(-(1/p*c(9)/nu(9)*dni(6, 9))^alpha)-1)*(exp(-(1/p*c(10)/nu(10)
*dni(6, 10))^alpha) - 1)*(1/p*c(8)/nu(8)*dni(6, 8))^(alpha-1)-1/p^2
*alpha*c(9)*exp(-(1/p*c(9)/nu(9)*dni(6, 9))^alpha)/nu(9)*dni(6, 9)
*(exp(-(1/p*c(1)/nu(1)*dni(6, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)
*dni(6, 2))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)*dni(6, 3))^alpha) - 1)
*(exp(-(1/p*c(4)/nu(4)*dni(6, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)
*dni(6, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*dni(6, 6))^alpha) - 1)
*(exp(-(1/p*c(7)/nu(7)*dni(6, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)
*dni(6, 8))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)*dni(6, 10))^alpha)-1)
*(1/p*c(9)/nu(9)*dni(6, 9))^(alpha - 1) - 1/p^2*alpha*c(10)*
exp(-(1/p*c(10)/nu(10)*dni(6, 10))^alpha)/nu(10)*dni(6, 10)*
(exp(-(1/p*c(1)/nu(1)*dni(6, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*
dni(6, 2))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)*dni(6, 3))^alpha) - 1)*
(exp(-(1/p*c(4)/nu(4)*dni(6, 4))^alpha) - 1)*
(exp(-(1/p*c(5)/nu(5)*dni(6, 5))^alpha) - 1)
*(exp(-(1/p*c(6)/nu(6)*dni(6, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)
*dni(6, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*dni(6, 8))^alpha) - 1)
*(exp(-(1/p*c(9)/nu(9)*dni(6, 9))^alpha) - 1)*(1/p*c(10)/nu(10)
*dni(6, 10))^(alpha - 1);
derivative_Gn_6(j)=derivative_Gn_6(j)*(p^(1-el_sub));
if j>1
small_arean=((derivative_Gn_6(j)+derivative_Gn_6(j-1))/2)
*(order_price_j(6,j)-order_price_j(6,j-1));
arean=arean+small_arean;
end
end
Area_Xn_pricedat(6)=arean;


arean=0;
for j=1:50
p=order_price_j(7,j);
derivative_Gn_7(j)=- 1/p^2*alpha*c(1)*
```

```
exp(−(1/p*c(1)/nu(1)*dni(7, 1))^alpha)
/nu(1)*dni(7, 1)*(exp(−(1/p*c(2)/nu(2)*dni(7, 2))^alpha) − 1)
*(exp(−(1/p*c(3)/nu(3)*dni(7, 3))^alpha) − 1)*(exp(−(1/p*c(4)/nu(4)
*dni(7, 4))^alpha) − 1)*(exp(−(1/p*c(5)/nu(5)*dni(7, 5))^alpha) − 1)
*(exp(−(1/p*c(6)/nu(6)*dni(7, 6))^alpha) − 1)*(exp(−(1/p*c(7)/nu(7)
*dni(7, 7))^alpha) − 1)*(exp(−(1/p*c(8)/nu(8)*dni(7, 8))^alpha) − 1)
*(exp(−(1/p*c(9)/nu(9)*dni(7, 9))^alpha) − 1)*(exp(−(1/p*c(10)/nu(10)
*dni(7, 10))^alpha) − 1)*(1/p*c(1)/nu(1)*dni(7, 1))^(alpha−1)−1/p^2
*alpha*c(2)*exp(−(1/p*c(2)/nu(2)*dni(7, 2))^alpha)/nu(2)*dni(7, 2)
*(exp(−(1/p*c(1)/nu(1)*dni(7, 1))^alpha) − 1)*(exp(−(1/p*c(3)/nu(3)
*dni(7, 3))^alpha) − 1)*(exp(−(1/p*c(4)/nu(4)*dni(7, 4))^alpha) − 1)
*(exp(−(1/p*c(5)/nu(5)*dni(7, 5))^alpha) − 1)*(exp(−(1/p*c(6)/nu(6)
*dni(7, 6))^alpha) − 1)*(exp(−(1/p*c(7)/nu(7)*dni(7, 7))^alpha)−1)
*(exp(−(1/p*c(8)/nu(8)*dni(7, 8))^alpha) − 1)*(exp(−(1/p*c(9)/nu(9)
*dni(7, 9))^alpha) − 1)*(exp(−(1/p*c(10)/nu(10)*dni(7, 10))^alpha)−1)
*(1/p*c(2)/nu(2)*dni(7, 2))^(alpha − 1) − 1/p^2*alpha*c(3)*
exp(−(1/p*c(3)/nu(3)*dni(7, 3))^alpha)/nu(3)*dni(7, 3)*
(exp(−(1/p*c(1)/nu(1)*dni(7, 1))^alpha) − 1)*(exp(−(1/p*c(2)/nu(2)*
dni(7, 2))^alpha) − 1)*(exp(−(1/p*c(4)/nu(4)*dni(7, 4))^alpha) − 1)*
(exp(−(1/p*c(5)/nu(5)*dni(7, 5))^alpha) − 1)*(exp(−(1/p*c(6)/nu(6)*
dni(7, 6))^alpha) − 1)*(exp(−(1/p*c(7)/nu(7)*dni(7, 7))^alpha) − 1)*
(exp(−(1/p*c(8)/nu(8)*dni(7, 8))^alpha) − 1)*(exp(−(1/p*c(9)/nu(9)*
dni(7, 9))^alpha) − 1)*(exp(−(1/p*c(10)/nu(10)*dni(7, 10))^alpha)−1)
*(1/p*c(3)/nu(3)*dni(7, 3))^(alpha − 1) − 1/p^2*alpha*c(4)*
exp(−(1/p*c(4)/nu(4)*dni(7, 4))^alpha)/nu(4)*dni(7, 4)*
(exp(−(1/p*c(1)/nu(1)*dni(7, 1))^alpha) − 1)*(exp(−(1/p*c(2)/nu(2)
*dni(7, 2))^alpha) − 1)*(exp(−(1/p*c(3)/nu(3)*dni(7, 3))^alpha)−1)
*(exp(−(1/p*c(5)/nu(5)*dni(7, 5))^alpha) − 1)*(exp(−(1/p*c(6)/nu(6)
*dni(7, 6))^alpha) − 1)*(exp(−(1/p*c(7)/nu(7)*dni(7, 7))^alpha)−1)
*(exp(−(1/p*c(8)/nu(8)*dni(7, 8))^alpha) − 1)
*(exp(−(1/p*c(9)/nu(9)*dni(7, 9))^alpha) − 1)*(exp(−(1/p*c(10)/nu(10)
*dni(7, 10))^alpha) − 1)*(1/p*c(4)/nu(4)*dni(7, 4))^(alpha−1)−1/p^2
*alpha*c(5)*exp(−(1/p*c(5)/nu(5)*dni(7, 5))^alpha)/nu(5)*dni(7, 5)
*(exp(−(1/p*c(1)/nu(1)*dni(7, 1))^alpha) − 1)*(exp(−(1/p*c(2)/nu(2)
*dni(7, 2))^alpha) − 1)*(exp(−(1/p*c(3)/nu(3)*dni(7, 3))^alpha) − 1)
```

```
*(exp(-(1/p*c(4)/nu(4)*dni(7, 4))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)
*dni(7, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)*dni(7, 7))^alpha) - 1)
*(exp(-(1/p*c(8)/nu(8)*dni(7, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)
*dni(7, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)*dni(7, 10))^alpha)-1)
*(1/p*c(5)/nu(5)*dni(7, 5))^(alpha - 1) - 1/p^2*alpha*c(6)*
exp(-(1/p*c(6)/nu(6)*dni(7, 6))^alpha)/nu(6)*dni(7, 6)*
(exp(-(1/p*c(1)/nu(1)*dni(7, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*
dni(7, 2))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)*dni(7, 3))^alpha) - 1)*
(exp(-(1/p*c(4)/nu(4)*dni(7, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*
dni(7, 5))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)*dni(7, 7))^alpha) - 1)*
(exp(-(1/p*c(8)/nu(8)*dni(7, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*
dni(7, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)*dni(7, 10))^alpha)-1)
*(1/p*c(6)/nu(6)*dni(7, 6))^(alpha - 1) - 1/p^2*alpha*c(7)*
exp(-(1/p*c(7)/nu(7)*dni(7, 7))^alpha)/nu(7)*dni(7, 7)*
(exp(-(1/p*c(1)/nu(1)*dni(7, 1))^alpha)- 1)*(exp(-(1/p*c(2)/nu(2)
*dni(7, 2))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)*dni(7, 3))^alpha) - 1)
*(exp(-(1/p*c(4)/nu(4)*dni(7, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*
dni(7, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*dni(7, 6))^alpha) - 1)*
(exp(-(1/p*c(8)/nu(8)*dni(7, 8))^alpha) - 1)
*(exp(-(1/p*c(9)/nu(9)*dni(7, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)
*dni(7, 10))^alpha) - 1)*(1/p*c(7)/nu(7)*dni(7, 7))^(alpha - 1)-1/p^2
*alpha*c(8)*exp(-(1/p*c(8)/nu(8)*dni(7, 8))^alpha)/nu(8)*dni(7, 8)
*(exp(-(1/p*c(1)/nu(1)*dni(7, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)
*dni(7, 2))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)*dni(7, 3))^alpha) - 1)
*(exp(-(1/p*c(4)/nu(4)*dni(7, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)
*dni(7, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*dni(7, 6))^alpha) - 1)
*(exp(-(1/p*c(7)/nu(7)*dni(7, 7))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)
*dni(7, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)*dni(7, 10))^alpha)-1)
*(1/p*c(8)/nu(8)*dni(7, 8))^(alpha - 1) - 1/p^2*alpha*c(9)*
exp(-(1/p*c(9)/nu(9)*dni(7, 9))^alpha)/nu(9)*dni(7, 9)*
(exp(-(1/p*c(1)/nu(1)*dni(7, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*
dni(7, 2))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)*dni(7, 3))^alpha) - 1)*
(exp(-(1/p*c(4)/nu(4)*dni(7, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*
dni(7, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*dni(7, 6))^alpha) - 1)*
(exp(-(1/p*c(7)/nu(7)*dni(7, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*
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dni(7, 8))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)*dni(7, 10))^alpha)-1)
*(1/p*c(9)/nu(9)*dni(7, 9))^(alpha - 1) - 1/p^2*alpha*c(10)*
exp(-(1/p*c(10)/nu(10)*dni(7, 10))^alpha)/nu(10)*dni(7, 10)*
(exp(-(1/p*c(1)/nu(1)*dni(7, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*
dni(7, 2))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)*dni(7, 3))^alpha) - 1)*
(exp(-(1/p*c(4)/nu(4)*dni(7, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*
dni(7, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*dni(7, 6))^alpha) - 1)*
(exp(-(1/p*c(7)/nu(7)*dni(7, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*
dni(7, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*dni(7, 9))^alpha) - 1)*
(1/p*c(10)/nu(10)*dni(7, 10))^(alpha - 1);
derivative_Gn_7(j)=derivative_Gn_7(j)*(p^(1-el_sub));
if j>1
small_arean=((derivative_Gn_7(j)+derivative_Gn_7(j-1))/2)
*(order_price_j(7,j)-order_price_j(7,j-1));
arean=arean+small_arean;
end
end
Area_Xn_pricedat(7)=arean;

arean=0;
for j=1:50
p=order_price_j(8,j);
derivative_Gn_8(j)=- 1/p^2*alpha*c(1)*
exp(-(1/p*c(1)/nu(1)*dni(8, 1))^alpha)
/nu(1)*dni(8, 1)*(exp(-(1/p*c(2)/nu(2)*dni(8, 2))^alpha) - 1)
*(exp(-(1/p*c(3)/nu(3)*dni(8, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(8, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(8, 5))^alpha) -1)
*(exp(-(1/p*c(6)/nu(6)*dni(8, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)
*dni(8, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*dni(8, 8))^alpha) -1)
*(exp(-(1/p*c(9)/nu(9)*dni(8, 9))^alpha)-1)*(exp(-(1/p*c(10)/nu(10)
*dni(8, 10))^alpha) - 1)*(1/p*c(1)/nu(1)*dni(8, 1))^(alpha-1)-1/p^2
*alpha*c(2)*exp(-(1/p*c(2)/nu(2)*dni(8, 2))^alpha)/nu(2)*dni(8, 2)
*(exp(-(1/p*c(1)/nu(1)*dni(8, 1))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)
*dni(8, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)*dni(8, 4))^alpha) - 1)
*(exp(-(1/p*c(5)/nu(5)*dni(8, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)
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*dni(8, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)*dni(8, 7))^alpha-1)
*(exp(-(1/p*c(8)/nu(8)*dni(8, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)
*dni(8, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)*dni(8, 10))^alpha)-1)
*(1/p*c(2)/nu(2)*dni(8, 2))^(alpha - 1) - 1/p^2*alpha*c(3)*
exp(-(1/p*c(3)/nu(3)*dni(8, 3))^alpha)/nu(3)*dni(8, 3)*
(exp(-(1/p*c(1)/nu(1)*dni(8, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*
dni(8, 2))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)*dni(8, 4))^alpha) - 1)*
(exp(-(1/p*c(5)/nu(5)*dni(8, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*
dni(8, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)*dni(8, 7))^alpha) - 1)
(exp(-(1/p*c(8)/nu(8)*dni(8, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*
dni(8, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)*dni(8, 10))^alpha)-1)
*(1/p*c(3)/nu(3)*dni(8, 3))^(alpha - 1) - 1/p^2*alpha*c(4)*
exp(-(1/p*c(4)/nu(4)*dni(8, 4))^alpha)/nu(4)*dni(8, 4)*
(exp(-(1/p*c(1)/nu(1)*dni(8, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*
dni(8, 2))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)*dni(8, 3))^alpha) - 1)*
(exp(-(1/p*c(5)/nu(5)*dni(8, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*
dni(8, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)*dni(8, 7))^alpha) - 1)*
(exp(-(1/p*c(8)/nu(8)*dni(8, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*
dni(8, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)*dni(8, 10))^alpha)-1)
*(1/p*c(4)/nu(4)*dni(8, 4))^(alpha - 1) - 1/p^2*alpha*c(5)*
exp(-(1/p*c(5)/nu(5)*dni(8, 5))^alpha)/nu(5)*dni(8, 5)*
(exp(-(1/p*c(1)/nu(1)*dni(8, 1))^alpha)- 1)*(exp(-(1/p*c(2)/nu(2)*
dni(8, 2))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)*dni(8, 3))^alpha) - 1)*
(exp(-(1/p*c(4)/nu(4)*dni(8, 4))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*
dni(8, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)*dni(8, 7))^alpha) - 1)*
(exp(-(1/p*c(8)/nu(8)*dni(8, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*
dni(8, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)*dni(8, 10))^alpha)-1)
*(1/p*c(5)/nu(5)*dni(8, 5))^(alpha - 1) - 1/p^2*alpha*c(6)*
exp(-(1/p*c(6)/nu(6)*dni(8, 6))^alpha)/nu(6)*dni(8, 6)
*(exp(-(1/p*c(1)/nu(1)*dni(8, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)
*dni(8, 2))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)*dni(8, 3))^alpha) - 1)
*(exp(-(1/p*c(4)/nu(4)*dni(8, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)
*dni(8, 5))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)*dni(8, 7))^alpha) - 1)
*(exp(-(1/p*c(8)/nu(8)*dni(8, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)
*dni(8, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)*dni(8, 10))^alpha)-1)
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*(1/p*c(6)/nu(6)*dni(8, 6))^(alpha - 1) - 1/p^2*alpha*c(7)*
exp(-(1/p*c(7)/nu(7)*dni(8, 7))^alpha)/nu(7)*dni(8, 7)*
(exp(-(1/p*c(1)/nu(1)*dni(8, 1))^alpha) - 1)*
(exp(-(1/p*c(2)/nu(2)*dni(8, 2))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)*
dni(8, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)*dni(8, 4))^alpha) - 1)*
(exp(-(1/p*c(5)/nu(5)*dni(8, 5))^alpha) - 1)
*(exp(-(1/p*c(6)/nu(6)*dni(8, 6))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)
*dni(8, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*dni(8, 9))^alpha) - 1)
*(exp(-(1/p*c(10)/nu(10)*dni(8, 10))^alpha) - 1)*(1/p*c(7)/nu(7)
*dni(8, 7))^(alpha - 1) - 1/p^2*alpha*c(8)*exp(-(1/p*c(8)/nu(8)
*dni(8, 8))^alpha)/nu(8)*dni(8, 8)*(exp(-(1/p*c(1)/nu(1)*
dni(8, 1))^alpha)- 1)*(exp(-(1/p*c(2)/nu(2)*dni(8, 2))^alpha) - 1)
*(exp(-(1/p*c(3)/nu(3)*dni(8, 3))^alpha)-1)*(exp(-(1/p*c(4)/nu(4)*
dni(8, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(8, 5))^alpha)-1)*
(exp(-(1/p*c(6)/nu(6)*dni(8, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)*
dni(8, 7))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*dni(8, 9))^alpha)-1)
*(exp(-(1/p*c(10)/nu(10)*dni(8, 10))^alpha) - 1)*(1/p*c(8)/nu(8)*
dni(8, 8))^(alpha - 1) - 1/p^2*alpha*c(9)*exp(-(1/p*c(9)/nu(9)*
dni(8, 9))^alpha)/nu(9)*dni(8, 9)*(exp(-(1/p*c(1)/nu(1)*
dni(8, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*dni(8, 2))^alpha) - 1)
*(exp(-(1/p*c(3)/nu(3)*dni(8, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(8, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(8, 5))^alpha)-1)
*(exp(-(1/p*c(6)/nu(6)*dni(8, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)
*dni(8, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*dni(8, 8))^alpha)-1)
*(exp(-(1/p*c(10)/nu(10)*dni(8, 10))^alpha) - 1)*(1/p*c(9)/nu(9)
*dni(8, 9))^(alpha - 1) - 1/p^2*alpha*c(10)*exp(-(1/p*c(10)
/nu(10)*dni(8, 10))^alpha)/nu(10)*dni(8, 10)*(exp(-(1/p*c(1)/nu(1)
*dni(8, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*dni(8, 2))^alpha)-1)
*(exp(-(1/p*c(3)/nu(3)*dni(8, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(8, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(8, 5))^alpha)-1)
*(exp(-(1/p*c(6)/nu(6)*dni(8, 6))^alpha)-1)*(exp(-(1/p*c(7)/nu(7)
*dni(8, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*dni(8, 8))^alpha)-1)
*(exp(-(1/p*c(9)/nu(9)*dni(8, 9))^alpha) - 1)*(1/p*c(10)/nu(10)
*dni(8, 10))^(alpha - 1);
derivative_Gn_8(j)=derivative_Gn_8(j)*(p^(1-el_sub));
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if j>1
small_arean=((derivative_Gn_8(j)+derivative_Gn_8(j-1))/2)
*(order_price_j(8,j)-order_price_j(8,j-1));
arean=arean+small_arean;
end
end
Area_Xn_pricedat(8)=arean;

arean=0;
for j=1:50
p=order_price_j(9,j);
derivative_Gn_9(j)=- 1/p^2*alpha*c(1)*
exp(-(1/p*c(1)/nu(1)*dni(9, 1))^alpha)
/nu(1)*dni(9, 1)*(exp(-(1/p*c(2)/nu(2)*dni(9, 2))^alpha) - 1)
*(exp(-(1/p*c(3)/nu(3)*dni(9, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(9, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(9, 5))^alpha)-1)
*(exp(-(1/p*c(6)/nu(6)*dni(9, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)
*dni(9, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*dni(9, 8))^alpha) - 1)
*(exp(-(1/p*c(9)/nu(9)*dni(9, 9))^alpha) - 1)*
(exp(-(1/p*c(10)/nu(10)*dni(9, 10))^alpha) - 1)*(1/p*c(1)/nu(1)*
dni(9, 1))^(alpha-1)-1/p^2*alpha*c(2)*exp(-(1/p*c(2)/nu(2)*
dni(9, 2))^alpha)/nu(2)*dni(9, 2)*(exp(-(1/p*c(1)/nu(1)*
dni(9, 1))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)*dni(9, 3))^alpha)-1)
*(exp(-(1/p*c(4)/nu(4)*dni(9, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)
*dni(9, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*dni(9, 6))^alpha) - 1)
*(exp(-(1/p*c(7)/nu(7)*dni(9, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8
)*dni(9, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*dni(9, 9))^alpha)-1)
*(exp(-(1/p*c(10)/nu(10)*dni(9, 10))^alpha) - 1)*(1/p*c(2)/nu(2)
*dni(9, 2))^(alpha - 1) - 1/p^2*alpha*c(3)*exp(-(1/p*c(3)/nu(3)
*dni(9, 3))^alpha)/nu(3)*dni(9, 3)*(exp(-(1/p*c(1)/nu(1)*
dni(9, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*dni(9, 2))^alpha)-1)
*(exp(-(1/p*c(4)/nu(4)*dni(9, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)
*dni(9, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*dni(9, 6))^alpha)-1)
*(exp(-(1/p*c(7)/nu(7)*dni(9, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)
*dni(9, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*dni(9, 9))^alpha)-1)
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*(exp(-(1/p*c(10)/nu(10)*dni(9, 10))^alpha) - 1)*(1/p*c(3)/nu(3)*
dni(9, 3))^(alpha - 1) - 1/p^2*alpha*c(4)*exp(-(1/p*c(4)/nu(4)*
dni(9, 4))^alpha)/nu(4)*dni(9, 4)*(exp(-(1/p*c(1)/nu(1)*
dni(9, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*dni(9, 2))^alpha) - 1)
*(exp(-(1/p*c(3)/nu(3)*dni(9, 3))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)
*dni(9, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*dni(9, 6))^alpha) - 1)
*(exp(-(1/p*c(7)/nu(7)*dni(9, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)
*dni(9, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*dni(9, 9))^alpha)-1)
*(exp(-(1/p*c(10)/nu(10)*dni(9, 10))^alpha) - 1)*(1/p*c(4)/nu(4)
*dni(9, 4))^(alpha - 1) - 1/p^2*alpha*c(5)*exp(-(1/p*c(5)
/nu(5)*dni(9, 5))^alpha)/nu(5)*dni(9, 5)*(exp(-(1/p*c(1)/nu(1)
*dni(9, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*dni(9, 2))^alpha)-1)
*(exp(-(1/p*c(3)/nu(3)*dni(9, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(9, 4))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*dni(9, 6))^alpha)-1)
*(exp(-(1/p*c(7)/nu(7)*dni(9, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)
*dni(9, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*dni(9, 9))^alpha)-1)
*(exp(-(1/p*c(10)/nu(10)*dni(9, 10))^alpha) - 1)*(1/p*c(5)/nu(5)
*dni(9, 5))^(alpha - 1) - 1/p^2*alpha*c(6)*exp(-(1/p*c(6)/nu(6)
*dni(9, 6))^alpha)/nu(6)*dni(9, 6)*(exp(-(1/p*c(1)/nu(1)*
dni(9, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*dni(9, 2))^alpha) - 1)
*(exp(-(1/p*c(3)/nu(3)*dni(9, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(9, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(9, 5))^alpha) - 1)
*(exp(-(1/p*c(7)/nu(7)*dni(9, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)
*dni(9, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*dni(9, 9))^alpha) - 1)
*(exp(-(1/p*c(10)/nu(10)*dni(9, 10))^alpha) - 1)*(1/p*c(6)/nu(6)
*dni(9, 6))^(alpha - 1) - 1/p^2*alpha*c(7)*exp(-(1/p*c(7)/nu(7)
*dni(9, 7))^alpha)/nu(7)*dni(9, 7)*(exp(-(1/p*c(1)/nu(1)*
dni(9, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*dni(9, 2))^alpha) - 1)
*(exp(-(1/p*c(3)/nu(3)*dni(9, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(9, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(9, 5))^alpha) - 1)
*(exp(-(1/p*c(6)/nu(6)*dni(9, 6))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)
*dni(9, 8))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)*dni(9, 9))^alpha)-1)
*(exp(-(1/p*c(10)/nu(10)*dni(9, 10))^alpha) - 1)*(1/p*c(7)/nu(7)
*dni(9, 7))^(alpha - 1) - 1/p^2*alpha*c(8)*exp(-(1/p*c(8)
/nu(8)*dni(9, 8))^alpha)/nu(8)*dni(9, 8)*(exp(-(1/p*c(1)/nu(1)
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*dni(9, 1))^alpha) − 1)*(exp(−(1/p*c(2)/nu(2)*dni(9, 2))^alpha)−1)
*(exp(−(1/p*c(3)/nu(3)*dni(9, 3))^alpha) − 1)*(exp(−(1/p*c(4)/nu(4)
*dni(9, 4))^alpha) − 1)*(exp(−(1/p*c(5)/nu(5)*dni(9, 5))^alpha)−1)
*(exp(−(1/p*c(6)/nu(6)*dni(9, 6))^alpha) − 1)*(exp(−(1/p*c(7)/nu(7)
*dni(9, 7))^alpha) − 1)*(exp(−(1/p*c(9)/nu(9)*dni(9, 9))^alpha)−1)
*(exp(−(1/p*c(10)/nu(10)*dni(9, 10))^alpha) − 1)*(1/p*c(8)/nu(8)
*dni(9, 8))^(alpha − 1) − 1/p^2*alpha*c(9)*exp(−(1/p*c(9)/nu(9)
*dni(9, 9))^alpha)/nu(9)*dni(9, 9)*(exp(−(1/p*c(1)/nu(1)*
dni(9, 1))^alpha)− 1)*(exp(−(1/p*c(2)/nu(2)*dni(9, 2))^alpha)−1)
*(exp(−(1/p*c(3)/nu(3)*dni(9, 3))^alpha) − 1)*(exp(−(1/p*c(4)/nu(4)
*dni(9, 4))^alpha) − 1)*(exp(−(1/p*c(5)/nu(5)*dni(9, 5))^alpha)−1)
*(exp(−(1/p*c(6)/nu(6)*dni(9, 6))^alpha) − 1)*(exp(−(1/p*c(7)/nu(7)
*dni(9, 7))^alpha) − 1)*(exp(−(1/p*c(8)/nu(8)*dni(9, 8))^alpha)−1)
*(exp(−(1/p*c(10)/nu(10)*dni(9, 10))^alpha) − 1)*(1/p*c(9)/nu(9)*
dni(9, 9))^(alpha − 1) − 1/p^2*alpha*c(10)*exp(−(1/p*c(10)/nu(10)
*dni(9, 10))^alpha)/nu(10)*dni(9, 10)*(exp(−(1/p*c(1)/nu(1)*
dni(9, 1))^alpha) − 1)*(exp(−(1/p*c(2)/nu(2)*dni(9, 2))^alpha)−1)*
(exp(−(1/p*c(3)/nu(3)*dni(9, 3))^alpha) − 1)*(exp(−(1/p*c(4)/nu(4)
*dni(9, 4))^alpha) − 1)*(exp(−(1/p*c(5)/nu(5)*dni(9, 5))^alpha)−1)
*(exp(−(1/p*c(6)/nu(6)*dni(9, 6))^alpha)−1)*(exp(−(1/p*c(7)/nu(7)*
dni(9, 7))^alpha) − 1)*(exp(−(1/p*c(8)/nu(8)*dni(9, 8))^alpha)−1)*
(exp(−(1/p*c(9)/nu(9)*dni(9, 9))^alpha)−1)*(1/p*c(10)/nu(10)*
dni(9, 10))^(alpha − 1);
derivative_Gn_9(j)=derivative_Gn_9(j)*(p^(1−el_sub));
if j>1
small_arean=((derivative_Gn_9(j)+derivative_Gn_9(j−1))/2)
*(order_price_j(9,j)−order_price_j(9,j−1));
arean=arean+small_arean;
end
end
Area_Xn_pricedat(9)=arean;

arean=0;
for j=1:50
p=order_price_j(10,j);
```

```
derivative_Gn_10(j)=- 1/p^2*alpha*c(1)*exp(-(1/p*c(1)/nu(1)
*dni(10, 1))^alpha)/nu(1)*dni(10, 1)*(exp(-(1/p*c(2)/nu(2)
*dni(10, 2))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)*dni(10, 3))^alpha)
- 1)*(exp(-(1/p*c(4)/nu(4)*dni(10, 4))^alpha) - 1)*
(exp(-(1/p*c(5)/nu(5)*dni(10, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)
*dni(10, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)*dni(10, 7))^alpha)
- 1)*(exp(-(1/p*c(8)/nu(8)*dni(10, 8))^alpha) - 1)*
(exp(-(1/p*c(9)/nu(9)*dni(10, 9))^alpha)-1)*(exp(-(1/p*c(10)/nu(10)
*dni(10, 10))^alpha) - 1)*(1/p*c(1)/nu(1)*dni(10, 1))^(alpha - 1)
- 1/p^2*alpha*c(2)*exp(-(1/p*c(2)/nu(2)*dni(10, 2))^alpha)/nu(2)
*dni(10, 2)*(exp(-(1/p*c(1)/nu(1)*dni(10, 1))^alpha) - 1)*
(exp(-(1/p*c(3)/nu(3)*dni(10, 3))^alpha)-1)*(exp(-(1/p*c(4)/nu(4)*
dni(10, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*dni(10, 5))^alpha)-1)
*(exp(-(1/p*c(6)/nu(6)*dni(10, 6))^alpha)-1)*(exp(-(1/p*c(7)/nu(7)
*dni(10, 7))^alpha) - 1)*(exp(-(1/p*c(8)/nu(8)*dni(10, 8))^alpha)
- 1)*(exp(-(1/p*c(9)/nu(9)*dni(10, 9))^alpha)-1)*
(exp(-(1/p*c(10)/nu(10)*dni(10, 10))^alpha) - 1)*(1/p*c(2)/nu(2)
*dni(10, 2))^(alpha - 1) - 1/p^2*alpha*c(3)*exp(-(1/p*c(3)/nu(3)
*dni(10, 3))^alpha)/nu(3)*dni(10, 3)*(exp(-(1/p*c(1)/nu(1)
*dni(10, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*dni(10, 2))^alpha)
- 1)*(exp(-(1/p*c(4)/nu(4)*dni(10, 4))^alpha) - 1)*
(exp(-(1/p*c(5)/nu(5)*dni(10, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)
*dni(10, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)*dni(10, 7))^alpha)
- 1)*(exp(-(1/p*c(8)/nu(8)*dni(10, 8))^alpha) - 1)*
(exp(-(1/p*c(9)/nu(9)*dni(10, 9))^alpha) - 1)*
(exp(-(1/p*c(10)/nu(10)*dni(10, 10))^alpha) - 1)*(1/p*c(3)/nu(3)
*dni(10, 3))^(alpha - 1) - 1/p^2*alpha*c(4)*exp(-(1/p*c(4)/nu(4)
*dni(10, 4))^alpha)/nu(4)*dni(10, 4)*(exp(-(1/p*c(1)/nu(1)
*dni(10, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*dni(10, 2))^alpha)
- 1)*(exp(-(1/p*c(3)/nu(3)*dni(10, 3))^alpha) - 1)*
(exp(-(1/p*c(5)/nu(5)*dni(10, 5))^alpha) - 1)*
(exp(-(1/p*c(6)/nu(6)*dni(10, 6))^alpha) - 1)
*(exp(-(1/p*c(7)/nu(7)*dni(10, 7))^alpha) - 1)*
(exp(-(1/p*c(8)/nu(8)*dni(10, 8))^alpha) - 1)*
(exp(-(1/p*c(9)/nu(9)*dni(10, 9))^alpha) - 1)*
```

```
(exp(−(1/p∗c(10)/nu(10)∗dni(10, 10))^alpha) − 1)∗(1/p∗c(4)/nu(4)
∗dni(10, 4))^(alpha − 1) − 1/p^2∗alpha∗c(5)∗exp(−(1/p∗c(5)/nu(5)
∗dni(10, 5))^alpha)/nu(5)∗dni(10, 5)∗(exp(−(1/p∗c(1)/nu(1)
∗dni(10, 1))^alpha) − 1)∗(exp(−(1/p∗c(2)/nu(2)∗
dni(10, 2))^alpha) − 1)∗(exp(−(1/p∗c(3)/nu(3)∗dni(10, 3))^alpha)
− 1)∗(exp(−(1/p∗c(4)/nu(4)∗dni(10, 4))^alpha) − 1)∗
(exp(−(1/p∗c(6)/nu(6)∗dni(10, 6))^alpha) − 1)
∗(exp(−(1/p∗c(7)/nu(7)∗dni(10, 7))^alpha) − 1)∗
(exp(−(1/p∗c(8)/nu(8)∗dni(10, 8))^alpha) − 1)∗(exp(−(1/p∗c(9)/nu(9)
∗dni(10, 9))^alpha) − 1)∗(exp(−(1/p∗c(10)/nu(10)∗dni(10, 10))^alpha)
− 1)∗(1/p∗c(5)/nu(5)∗dni(10, 5))^(alpha − 1) − 1/p^2∗alpha∗c(6)∗
exp(−(1/p∗c(6)/nu(6)∗dni(10, 6))^alpha)/nu(6)∗dni(10, 6)∗
(exp(−(1/p∗c(1)/nu(1)∗dni(10, 1))^alpha) − 1)∗
(exp(−(1/p∗c(2)/nu(2)∗dni(10, 2))^alpha) − 1)
∗(exp(−(1/p∗c(3)/nu(3)∗dni(10, 3))^alpha) − 1)∗
(exp(−(1/p∗c(4)/nu(4)∗dni(10, 4))^alpha) − 1)∗(exp(−(1/p∗c(5)/nu(5
)∗dni(10, 5))^alpha) − 1)∗(exp(−(1/p∗c(7)/nu(7)∗dni(10, 7))^alpha)
− 1)∗(exp(−(1/p∗c(8)/nu(8)∗dni(10, 8))^alpha) − 1)∗
(exp(−(1/p∗c(9)/nu(9)∗dni(10, 9))^alpha)−1)∗(exp(−(1/p∗c(10)/nu(10)
∗dni(10, 10))^alpha) − 1)∗(1/p∗c(6)/nu(6)
∗dni(10, 6))^(alpha − 1) − 1/p^2∗alpha∗c(7)∗exp(−(1/p∗c(7)/nu(7)
∗dni(10, 7))^alpha)/nu(7)∗dni(10, 7)∗(exp(−(1/p∗c(1)/nu(1)
∗dni(10, 1))^alpha) − 1)∗(exp(−(1/p∗c(2)/nu(2)∗dni(10, 2))^alpha)
− 1)∗(exp(−(1/p∗c(3)/nu(3)∗dni(10, 3))^alpha) − 1)∗
(exp(−(1/p∗c(4)/nu(4)∗dni(10, 4))^alpha) − 1)∗
(exp(−(1/p∗c(5)/nu(5)∗dni(10, 5))^alpha) − 1)
∗(exp(−(1/p∗c(6)/nu(6)∗dni(10, 6))^alpha) − 1)∗(exp(−(1/p∗c(8)/nu(8)
∗dni(10, 8))^alpha) − 1)∗(exp(−(1/p∗c(9)/nu(9)∗dni(10, 9))^alpha)
− 1)∗(exp(−(1/p∗c(10)/nu(10)∗dni(10, 10))^alpha) − 1)∗
(1/p∗c(7)/nu(7)∗dni(10, 7))^(alpha − 1) − 1/p^2∗alpha∗c(8)∗
exp(−(1/p∗c(8)/nu(8)∗dni(10, 8))^alpha)/nu(8)∗dni(10, 8)∗
(exp(−(1/p∗c(1)/nu(1)∗dni(10, 1))^alpha) − 1)∗(exp(−(1/p∗c(2)/nu(2)∗
dni(10, 2))^alpha) − 1)∗(exp(−(1/p∗c(3)/nu(3)∗dni(10, 3))^alpha)−1)∗
(exp(−(1/p∗c(4)/nu(4)∗dni(10, 4))^alpha) − 1)∗(exp(−(1/p∗c(5)/nu(5)
∗dni(10, 5))^alpha) − 1)∗(exp(−(1/p∗c(6)/nu(6)∗dni(10, 6))^alpha)−1)
```

```
*(exp(-(1/p*c(7)/nu(7)*dni(10, 7))^alpha) - 1)*(exp(-(1/p*c(9)/nu(9)
*dni(10, 9))^alpha) - 1)*(exp(-(1/p*c(10)/nu(10)*dni(10, 10))^alpha)
- 1)*(1/p*c(8)/nu(8)*dni(10, 8))^(alpha - 1) - 1/p^2*alpha*c(9)*
exp(-(1/p*c(9)/nu(9)*dni(10, 9))^alpha)/nu(9)*dni(10, 9)*
(exp(-(1/p*c(1)/nu(1)*dni(10, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)
*dni(10, 2))^alpha) - 1)*(exp(-(1/p*c(3)/nu(3)*dni(10, 3))^alpha)
- 1)*(exp(-(1/p*c(4)/nu(4)*dni(10, 4))^alpha) - 1)*
(exp(-(1/p*c(5)/nu(5)*dni(10, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)
*dni(10, 6))^alpha) - 1)*(exp(-(1/p*c(7)/nu(7)*dni(10, 7))^alpha)-1)
*(exp(-(1/p*c(8)/nu(8)*dni(10, 8))^alpha)-1)*(exp(-(1/p*c(10)/nu(10)
*dni(10, 10))^alpha) - 1)*(1/p*c(9)/nu(9)*dni(10, 9))^(alpha - 1)
- 1/p^2*alpha*c(10)*exp(-(1/p*c(10)/nu(10)*dni(10, 10)
)^alpha)/nu(10)*dni(10, 10)*(exp(-(1/p*c(1)/nu(1)
*dni(10, 1))^alpha) - 1)*(exp(-(1/p*c(2)/nu(2)*dni(10, 2))^alpha)-1)
*(exp(-(1/p*c(3)/nu(3)*dni(10, 3))^alpha) - 1)*(exp(-(1/p*c(4)/nu(4)
*dni(10, 4))^alpha) - 1)*(exp(-(1/p*c(5)/nu(5)*
dni(10, 5))^alpha) - 1)*(exp(-(1/p*c(6)/nu(6)*dni(10, 6))^alpha)
- 1)*(exp(-(1/p*c(7)/nu(7)*dni(10, 7))^alpha) - 1)*
(exp(-(1/p*c(8)/nu(8)*dni(10, 8))^alpha) - 1)*
(exp(-(1/p*c(9)/nu(9)*dni(10, 9))^alpha) - 1)*(1/p*c(10)/nu(10)
*dni(10, 10))^(alpha - 1);
derivative_Gn_10(j)=derivative_Gn_10(j)*(p^(1-el_sub));
end
Area_Xn_pricedat(10)=sum(derivative_Gn_10);
```

```
%This function evaluates the weighted model that is estimated.
function F = myfun(x)

 global C
 global xdata
 global ydata
F1=modelfun(x,xdata);
F=((ydata-F1)'*C)';
```

```
%This function gives the weighted y_hat.
function yfit = myfunfit(x,xdata)

 global C
F1=modelfun(x,xdata);
yfit=(F1'*C)';
```

Frechet m.files

```
clear all;
close all;
clc;

n=10;
[trade1] = xlsread('trade1.xls');
wage=trade1(7202:7202+19-1,17);
c_old=exp(wage);
c(1,1)=c_old(4);
c(2,1)=c_old(7);
c(3,1)=c_old(8);
c(4,1)=c_old(9);
c(5,1)=c_old(10);
c(6,1)=c_old(11);
c(7,1)=c_old(16);
c(8,1)=c_old(17);
c(9,1)=c_old(18);
c(10,1)=c_old(19);

T = sym(zeros(n,1));
for row = 1:n
T(row, 1) = sym(sprintf('T%d', row));
end

[D_ni]=xlsread('D_ni.xlsx');
dni=exp(D_ni);
syms el_sub theta

[pricedat txt] = xlsread('pricedat.xlsx');
pricedata=ones(19,50);
for i=1:19
pricedata(i,:)=exp(pricedat(i*50-49:i*50,1));
```

```
end
pricedata=pricedata';

order_price_j_old=sort(pricedata,'ascend');
order_price_j_old=order_price_j_old';% Here we sort the prices of
% 50 goods of each 19 countries. So we have still 19x50 matrix.
order_price_j(1,:)=order_price_j_old(4,:);
order_price_j(2,:)=order_price_j_old(7,:);
order_price_j(3,:)=order_price_j_old(8,:);
order_price_j(4,:)=order_price_j_old(9,:);
order_price_j(5,:)=order_price_j_old(10,:);
order_price_j(6,:)=order_price_j_old(11,:);
order_price_j(7,:)=order_price_j_old(16,:);
order_price_j(8,:)=order_price_j_old(17,:);
order_price_j(9,:)=order_price_j_old(18,:);
order_price_j(10,:)=order_price_j_old(19,:);

order_price_j=round(order_price_j*100)/100;
global xdata
xdata= [c order_price_j dni];
[ydataa]=xlsread('xni_b1_Xn.xlsx');
global ydata
for i=1:10
   ydata((i*10)-9:i*10,1)=(ydataa(:,i));
end

x0 =1*[1.0000; 1.0000; 1.0000; 1.0000; 1.0000; 1.0000; 1.0000;
    1.0000; 1.0000; 1.0000; 1.5000];
lb=[0.0000; 0.0000; 0.0000; 0.0000; 0.0000; 0.0000; 0.0000; 0.0000;
    0.0000; 0.0000; 1.0000];
ub=1000*[1.0000; 1.0000; 1.0000; 1.0000; 1.0000; 1.0000; 1.0000;
    1.0000;    1.0000; 1.0000; 1.5000];
options = optimset('MaxFunEvals',1e10);

[X,RESNORM,RESIDUAL,EXITFLAG,OUTPUT,LAMBDA,JACOBIAN]=
```

```
lsqcurvefit(@modelfun,x0,xdata,ydata,lb,ub,options);

options = statset('nlinfit');
fdiffstep = options.DerivStep;
yfit=modelfun(X,xdata);
p = numel(X);
delta = zeros(size(X));
%This for loop computes the Jacobian matrix.
for k = 1:p
    if (X(k) == 0)
        nb = sqrt(norm(X));
        delta(k) = fdiffstep * (nb + (nb==0));
    else
        delta(k) = fdiffstep*X(k);
    end
    yplus = modelfun(X+delta,xdata);
    dy = yplus(:) - yfit(:);
    J(:,k) = dy/delta(k);
    delta(k) = 0;
end
r = ydata - yfit;
mse = (abs(r).^2)/(100-p);
tot=0;
for i=1:numel(mse)
tot=tot+mse(i);
end
mse=tot;

[Q,R] = qr(J,0);
Rinv = inv(R);
Sigma = Rinv*Rinv'*mse;
std_err=sqrt(diag(Sigma));%Standard error of estimates

y_mean=mean(ydata);
tot=0;
```

```
for i=1:numel(ydata)
tot=tot+(ydata(i)-y_mean)^2;
end
total_sum_of_squares=tot;%Total Sum of Squares

se = (abs(r).^2);
tot=0;
for i=1:numel(se)
tot=tot+se(i);
end
sum_of_squared_residuals=tot;%Sum of Squares of Residuals

R_square=(total_sum_of_squares-sum_of_squared_residuals)
/total_sum_of_squares;
%%%%%%%%%%%%%%%%%%%%%%%%%Weighted- LS%%%%%%%%%%%%%%
R=reshape(RESIDUAL,10,10);
sum=0;
for i=1:10
    for j=1:10
    sum=sum+R(i,j)*R(j,i);
    end
end

non_diagonal_elements=sum/100;

sum=0;
for i=1:10
    for j=1:10
    sum=sum+R(i,j)*R(i,j);
    end
end

diagonal_elements=sum/100;
column1=ones(100,1);
column2=ones(100,1);
```

```
k=0;
for i=1:10
    for j=1:10
    k=k+1;
    column1(k)=j;
    column2(k)=i;
    end
end
%Weighting matrix,S is formulated.
S=eye(100,100)*diagonal_elements;
for i=1:100
    for j=1:100
       if column1(i)==column2(j) && column1(j)==column2(i)&&i~=j
       S(i,j)=non_diagonal_elements;
       end
    end
end

S_inv=inv(S);
global C
C=chol(S_inv);
options = optimset('MaxFunEvals',1e10);
[xw,resnorm,RESIDUAL] = lsqnonlin(@myfun,x0,lb,ub,options);

options = statset('nlinfit');
fdiffstep = options.DerivStep;
yfit=myfunfit(xw,xdata);
p = numel(xw);
delta = zeros(size(xw));
for k = 1:p
    if (xw(k) == 0)
        nb = sqrt(norm(xw));
        delta(k) = fdiffstep * (nb + (nb==0));
    else
```

```
        delta(k) = fdiffstep*xw(k);
    end
    yplus = myfunfit(xw+delta,xdata);
    dy = yplus(:) - yfit(:);

    J(:,k) = dy/delta(k);
    delta(k) = 0;
end
[Q,R] = qr(J,0);
ydata=(ydata'*C)';
r = ydata - yfit;
mse = (abs(r).^2)/(100-p);

tot=0;
for i=1:numel(mse)
   tot=tot+mse(i);
end
mse=tot;
Rinv = inv(R);
Sigma_w = Rinv*Rinv'*mse;
std_err_w=sqrt(diag(Sigma_w)); %Robust standard error of estimates

y_mean=mean(ydata);
tot=0;
for i=1:numel(ydata)
tot=tot+(ydata(i)-y_mean)^2;
end
total_sum_of_squares_w=tot;%Total Sum of Squares

se = (abs(r).^2);
tot=0;
for i=1:numel(se)
tot=tot+se(i);
end
sum_of_squared_residuals_w=tot;%Sum of Squares of Residuals
```

```
R_square_w=(total_sum_of_squares_w-sum_of_squared_residuals_w)
                                  /total_sum_of_squares_w;

estimates_result=[X std_err xw std_err_w];
tss_ssr_results=[total_sum_of_squares sum_of_squared_residuals
               total_sum_of_squares_w sum_of_squared_residuals_w];
R_square_results=[R_square R_square_w];
```

```
%This function evaluates the model that is estimated.
function y_hat=modelfun(x,xdata)
%beta=[T el_sub theta];
%x=[c order_price dni];

[n b]=size(xdata); %10x61(n=10, b=61)
T=x(1:n);
el_sub=4.0;
theta=x(n+1);
c=xdata(:,1);
order_price_j=xdata(:,2:51);
dni=xdata(:,52:b);

phi_n=zeros(n,1);
for k=1:n
    phi_n(k)=0;
    for l=1:n
       phi_n(k)=phi_n(k)+(T(l)*(c(l)*dni(k,l))^-theta);
    end
    phi_n(k);
end
t_crp_frec=zeros(n,n);

AreaXnijFrechetPrcdta=zeros(n,n);
for i=1:n
    for j=1:n
        if i==10
            l=0;
            for jj=1:50
            t_crp_frec(i,j)=(order_price_j(i,jj)^(1-el_sub))
            *theta*(T(j)*(c(j)*dni(i,j))^-theta)*
            exp(-(order_price_j(i,jj)^theta)
            *phi_n(i))*(order_price_j(i,jj)^(theta-1));
            l=l+1;
            y(l)=t_crp_frec(i,j);
```

```
            end
        AreaXnijFrechetPrcdta(i,j)=sum(y);
        else
          l=0;
            area=0;
            for jj=1:50
            t_crp_frec(i,j)=(order_price_j(i,jj)^(1-el_sub))
            *theta*(T(j)*(c(j)*dni(i,j))^-theta)*
            exp(-(order_price_j(i,jj)^theta)
            *phi_n(i))*(order_price_j(i,jj)^(theta-1));
            l=l+1;
            pp(l)=order_price_j(i,jj);
            y(l)=t_crp_frec(i,j);
                if l>1
                small_area=((y(l)+y(l-1))/2)*(pp(l)-pp(l-1));
                area=area+small_area;
                end
            end
        AreaXnijFrechetPrcdta(i,j)=area;
        end
    end
end

t_crp_frec_Xn=zeros(n,1);
AreaXnjFrechetPrcdta=ones(n,1);
for j=1:n
    if j==10
        l=0;
        for jj=1:50
        l=l+1;
        t_crp_frec_Xn(j)=theta*exp(-(order_price_j(j,jj)^theta)
        *phi_n(j))         *(order_price_j(j,jj)^(theta-1))*
        (order_price_j(j,jj)^(1-el_sub));
        y(l)= t_crp_frec_Xn(j);
        end
```

```
    AreaXnjFrechetPrcdta(j)=sum(y);
    else
        area=0;
        l=0;
        for jj=1:50
        l=l+1;
        t_crp_frec_Xn(j)=theta*exp(-(order_price_j(j,jj)^theta)
        *phi_n(j))        *(order_price_j(j,jj)^(theta-1))*
        (order_price_j(j,jj)^(1-el_sub));
        y(l)= t_crp_frec_Xn(j);
        pp(l)=order_price_j(j,jj);
            if l>1
            small_area=((y(l)+y(l-1))/2)*(pp(l)-pp(l-1));
            area=area+small_area;
            end
        end
    AreaXnjFrechetPrcdta(j)=area;
    end
end

result_frechet_Pricedat=ones(n,1);
k=0;
for i=1:n
    for j=1:n
    k=k+1;
    result_frechet_Pricedat(k)=AreaXnijFrechetPrcdta(j,i)
    /AreaXnjFrechetPrcdta(j);
    end
end
y_hat=result_frechet_Pricedat;
```

```
%This function evaluates the weighted model that is estimated.
function F = myfun(x)

 global C
 global xdata
 global ydata
F1=modelfun(x,xdata);
F=((ydata-F1)'*C)';
```

```
%This function gives the weighted y_hat.
function yfit = myfunfit(x,xdata)

 global C
F1=modelfun(x,xdata);
yfit=(F1'*C)';
```