AN EXAMINATION OF FIFTH AND SIXTH GRADE STUDENTS’ PROPORTIONAL REASONING

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An Examination of Fifth and Sixth Grade Students' Proportional Reasoning

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Thesis Abstract

Şebnem Atabaş, "An Examination of Fifth and Sixth Grade Students' Proportional Reasoning"

The aim of the study was to investigate 5th and 6th grade students' understanding of proportional and non-proportional situations. The study also looks at reasons of erroneous solutions in proportional and non-proportional problems depending on the type of numbers used in the problems as integer ratio or non-integer ratio. Data collection instrument was a 12-item test that included four types of word problems, additive, constant, proportional comparison (PC), and proportional missing-value (PM), each with an integer and non-integer ratio. One hundred and twenty 5th and 101 6th grade students in a private school participated in the study.

5th and 6th grade students solved proportional and non-proportional situational problems with different success rates. In detail, constant problems were solved with the lowest success rate, while proportional missing-value problems with the highest success rate in both grades. When the use of erroneous strategies was calculated in percentages, the tendency to overuse proportional strategy in non-proportional situations was observed in both grade levels.

The study also examined number effect on students' success rate in proportional and non-proportional situations. The analysis showed that, fifth grade students' success rates in integer and non-integer numbered problems were significantly different in only additive problems. However, in the 6th grade the success rates differed significantly in additive and PC problems. Additionally, 5th grade students' choice of the methods significantly differed depending on the number change in only additive problems, while 6th grade students' choice of the strategies significantly differed in constant and PC problems.

In constant problems when problems included integer ratios 5th and 6th grade students tended to use proportional methods, and when problems included non-integer ratios they tended to prefer additive methods. Use of "other" method in the 5th grade also increased significantly by the number change in the problems. In additive problems, when numbers changed from integer ratio to non-integer ratio, there was a significant difference in the overuse of proportional methods. However, the expected difference in the overuse of additive strategies in proportional problems when numbers form non-integer ratios was not observed.

Qualitative findings revealed two important points. One of these points was students' understanding about "building-up" strategy. One another point was students' decision of the solution strategy without a full understanding of the problem. Frequently, students had a tendency to rely their solution strategy on the relation between numbers in the problems. Besides, students' difficulty in explaining their way of thinking, and students' beliefs about mathematical problems supported their insufficient understanding of the problem.

Tez Özeti

Şebnem Atabaş, "5 ve 6.Sınıf Öğrencilerinin Orantısal Düşünme Becerilerinin İncelenmesi"

Bu çalışmada 5 ve 6.sınıf öğrencilerinin orantısal ve orantısal olmayan durumları nasıl anlamlandırdıkları araştırılmaktadır. Araştırma ayrıca hatalı strateji kullanım sebeplerini sorularda yer alan sayıların birbirinin tam katı olma veya olmama durumlarına bağlı olarak incelemektedir. Veri toplama aracı olarak toplamsal, orantısal (bilinmeyen değer türünde), orantısal (niteliksel karşılaştırma soru türünde) ve sabit ilişki içeren dört farklı problem türü kullanılmıştır (sorular içerisinde kullanılan sayıların birbirinin tam katı olma ve birbirinin tam katı olmama durumlarına göre farklılaştırılmıştır). Veri toplama aracı, araştırmanın amacının öğrenci tarafından hissedilmemesi için dört adet araştırma amacından bağımsız soru olmak üzere toplam 12 problem içermektedir. Araştırmaya İstanbul'da bulunan bir özel okulda okumakta olan 120 beşinci sınıf ve 101 altıncı sınıf öğrencisi katılmıştır.

Beşinci ve altıncı sınıf öğrencileri orantısal ilişki içeren ve orantısal ilişki içermeyen problemleri farklı başarı yüzdeleri ile çözmüşlerdir. Sabit ilişki içeren problemler en düşük başarı yüzdesine sahipken, bilinmeyen değer türündeki orantısal ilişki içeren problemler en yüksek başarı oranı ile çözülmüşlerdir. Ayrıca, öğrencilerin hatalı strateji kullanma yüzdeleri incelendiğinde, orantısal olmayan durumlarda orantısal strateji kullanma eğilimi her iki sınıf seviyesinde de ortaya çıkmıştır.

Ayrıca problemde kullanılan sayıların, öğrencilerin başarı yüzdesine etkisi incelendiğinde, 5.sınıf öğrencilerinin sadece toplamsal ilişki içeren problemlerde, 6.sınıf öğrencilerinin ise hem toplamsal hem de niteliksel karşılaştırma sorularında başarı yüzdeleri anlamlı bir değişim göstermiştir. Strateji tercihleri incelendiğinde ise 5.sınıf öğrencilerinin toplamsal ilişki içeren problemlerde, 6.sınıf öğrencilerinin ise sabit ilişki ve niteliksel karşılaştırma gerektiren problemlerde tercih edilen çözüm stratejisi anlamlı bir değişim göstermiştir.

Sabit ilişki içeren problemde kullanılan sayıların birbirinin "tam katı olma" durumu öğrencilerin orantısal çözüm stratejisini kullanma eğilimlerini, "tam katı olmama" durumu ise toplamsal çözüm stratejisi kullanma eğilimlerini anlamlı bir şekilde arttırmıştır. 5.sınıf seviyesinde "diğer" başlıklı çözüm strateji kullanma eğilimleri de bu değişimden anlamlı bir şekilde etkilenmiştir. Toplamsal ilişki içeren problemlerde sayıların "tam katı olmama" durumundan "tam katı olma" durumuna geçişi de orantısal çözüm kullanımını arttırmıştır. Ancak orantısal olan durumlarda problem içerisinde kullanılan sayıların çözüm stratejisine anlamlı bir etkisi bulunmamıştır.

Araştırma devamında gerçekleşen nicel veri analizi araştırma sonuçları ile ilgili olarak iki önemli noktaya dikkat çekmiştir. İlk olarak, öğrenciler ile yapılan mülakatlarda öğrencilerin "toplamsal ilişki" ile "arttırma" stratejilerinin ayrımını yapamadıklarına dair ifadelere rastlanmıştır. Bir diğer önemli bulgu ise, öğrencilerin problemi tam olarak anlamadan çözüme karar vermeleridir. Öğrencilerin, problemlerdeki değişkenler arasındaki ilişkiden ziyade problemde yer alan sayılar arasındaki ilişkiyi göz önünde bulundurarak çözüm stratejisine karar vermeleri, çözüm stratejisine karar vermede nasıl bir akıl yürütme kullandıklarını açıklamada yaşanan zorluklar ve matematiksel problem ile ilgili kalıpsal beklentileri bu bulguyu desteklemektedir.

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CHAPTER I

INTRODUCTION

Some topics in mathematics education attract a special interest in learning and teaching process. The reason of this interest is usually multifaceted structure of these concepts and their relation with other concepts. To understand these concepts it is essential to make connections among some other concepts and learning one requires deep understanding of its related concepts. Researchers argue that these kinds of concepts may be considered as broad concepts. Some examples of these broad concepts include fractions, rational numbers, and multiplicative structures. Ratio and proportion stands as common concepts that are addressed as sub-concepts in all these broad concepts (Cramer, Post, & Delmas, 2002; Van Dooren, De Bock, & Verschaffel, 2010a; Thompson & Saldanha, 2003; Charalambos & Pitta-Pantazi, 2007; Pantziara & Philippou, 2011, Adjiage & Pluvinage, 2007; Lawton, 1993).

Proportional reasoning includes kinds of prerequisite abilities for high school mathematics and it has a wide range of applications in primary and secondary mathematics education and also for the following years of education (Modestaou & Gagatsis, 2007; Van Dooren et al., 2010a). The importance of the development of proportional reasoning is evident in National Council of Teaching Mathematics' (NCTM, 1989) documents, stating no matter how much time and effort is needed, anything required for the development of proportional reasoning must be provided. In addition to mathematics education, proportional reasoning has also wide usability in understanding basic scientific concepts and handling everyday problems and situations (Spinillo & Bryant, 1999).

Despite its importance, students have serious problems in understanding proportional relations, and most of the students graduate from high schools without acquiring fluency in proportional reasoning (Capon & Kuhn, 1979; Lawton, 1993; Modestou & Gagatsis, 2010). Research on proportional reasoning suggests that there is something missing in teaching the constructs of ratio and proportion since most students have difficulties in acquiring these concepts.

For a better understanding of the difficulty learners experience in the learning processes, what is expected from them in terms of “proportional reasoning” should be clarified. Although many studies state that proportional reasoning cannot be limited by the use of some algorithmic strategies (e.g. cross-multiplication), textbooks and instructions in schools mostly include these kinds of algorithms. Furthermore, understanding of proportional reasoning is oversimplified as an application of particular methods in particular problems (Cramer, Post, & Currier, 1993; Hiebert & Behr, 1988; Misailidou & Williams, 2003; Modestou & Gagatsis, 2008; 2010; Van Dooren, De Bock, Vluegels, & Verschaffel, 2010b; Van Dooren, De Bock, & Verschaffel, 2010a). However, overuse of proportional methods in non-proportional situations indicates a lack of understanding of proportional relations. Proportional reasoners need to differentiate proportional and non-proportional situations and to decide the appropriate solution method for each situation (Modestou & Gagatsis, 2008; Van Dooren et al., 2010a).

How students reason in proportionality tasks, what kinds of variables effect their way of reasoning, and how they give meaning to proportionality must be understood in detail by both researchers and educators. Understanding the situation may help arranging an effective learning process. Students’ success on proportional problems by using cross-multiplication is not enough for reasoning proportionally. Changes in numbers used in the problem, context, or problem structure may result in a difference on their performance. Independent from the superficial variables, students must be able to recognize and explain proportional and non-proportional relations, and use the appropriate solution method in each situation.

Schooling plays an important role in supporting the development of proportional reasoning, and expressing proportional relations in a mathematical way (Spinillo & Bryant, 1999). Oversimplification of proportional reasoning as a use of particular methods in particular problems in schools results in insufficient understanding of proportional relations and students experience difficulties in differentiating proportional and non-proportional situations (Singh, 2000). As a result, students may show a tendency to use incorrect methods when numbers in the problem, context of the problem, or structure of the problem changes. Although, some practices in schools may have an effect on the difficulty to differentiate proportional and non-proportional situations, schooling would play the role of supporting its development.

Statement of the Problem

Developing proportional reasoning skills may be accepted as one of the most important goal for middle school program. The reason why learning proportional reasoning is so important may be resulted from its role of being a prerequisite for understanding other subjects in high school and in the following years of education and for handling daily life problems and situations (Modestou & Gagatsis, 2007;

Modestou, Elia, Gagatsis, & Spanoudis, 2008; Spinillo & Bryant, 1999; Van Dooren et al., 2010a; 2010b; Vergnaud, 1988).

A current problem is that although students start to learn about proportional relations in the middle school, it remains to be problematic for many students who graduate from high school (Capon & Kuhn, 1979; Lawton, 1993, Fernández, Llinares, Van Dooren, De Bock, & Verschaffel, 2010; Modestou & Gagatsis, 2010). According to Lamon (1994), middle school children's efforts in making sense of proportional relations before and after the instruction give important clues about the methods children invented and used. Lo and Watanabe (1997) investigated development of the use of informal strategies on ratio and proportion problems in the 5^{th} grade. Additionally, Van Dooren et al. (2010a) stated almost half of the 5^{th} and 6^{th} grade students overused additive and proportional methods. Therefore, middle school years seem to be most prominent one, in which studies on proportional reasoning focus on revealing students' understanding about proportional relations and the overuse of particular strategies.

Proportional reasoning is a special form of multiplicative reasoning that requires considering the co-variation between variables, comparing the multiple variables at the same time, and using information as a whole (Lesh, Post, & Behr, 1988). Although in most cases and studies, proportional reasoning is considered as the skill to solve missing-value proportional problems, it is a more complex skill, which also requires the differentiation of the proportional and non-proportional situations (Cramer et al., 1993; Fernandez et al., 2010; Modestou & Gagatsis, 2008; 2010). When students' difficulties in developing proportional reasoning and the approach on proportionality in the studies and schools are considered, current situation on students’ understanding of the proportional and non-proportional relations deserves to be studied.

Literature points out that the transition from additive reasoning to proportional reasoning seems to be a quite slow and complicated process (Tourniaire & Pulos, 1985; Clark & Kamii, 1994). Therefore, it may not be possible to state a specific critical period for this transition. However, as students start learning about ratio and proportion mainly in the first two years of the middle school level, this time period provides a better research context to examine students’ difficulties in understanding proportional reasoning, the possible reasons of using erroneous methods, and their process of giving meaning to proportional relations when the instruction on proportions starts.

There are studies which investigate proportional reasoning in different grade levels starting from 4^{th} grade (Çeken & Ayas, 2010) to the university level (Duatepe & Akkuş-Çıkla, 2002) in Turkey. Studies focus on different situations about proportional reasoning, such as, interdisciplinary relation among Mathematics, Science and Technology, and Social Stuides curricular objective sequences in 4^{th}, 5^{th} and 6^{th} grade books (Çeken & Ayas, 2010), the relation between students’ proportional reasoning skills and problem posing skills in 7^{th} and 8^{th} grades (Çelik & Özdemir, 2011), misconceptions of 6^{th} grade students about proportional reasoning on velocity, perimeter and area relations, and mixture problems and classifications of the erroneous methods (Kaplan et al., 2011), solution strategy differences depending on the problem type in 6^{th}, 7^{th}, and 8^{th} grades (Duatepe et al, 2005), and a detailed analysis of the use of incorrect strategies on ratio and proportion tasks of pre-service middle school mathematics teachers (Duatepe & Akkuş-Çıkla, 2002) were investigated.

Although there are studies focusing on ratio and proportion in the Turkish context, this study differs from the previous studies conducted in Turkey in different ways. First, it is the first study that investigates 5th and 6th grade students' performance on proportional and non-proportional situations, in which the problems were written in the same context for the elimination of the effect of contextual variables. Besides this, most of the studies have been quantitative studies, so there is a need to hear the voices of the participants through a qualitative approach. Therefore, the current study uses interviews for a better understanding of students' making sense of proportional relations and problem solving process. Findings of the study will be important because understanding how students do reason in proportionality tasks, their performance in differentiating proportional and non-proportional relations, and variables affecting their way of reasoning in the problem solving process may give important clues for a better understanding of the current situation in Turkey. This understanding will enable the schools arrange learning process in a more effective way, and approach that proportional reasoning is more than solving proportion problems correctly with a consideration for the contextual variables, which have important effect on students' choice of the strategies.

Purpose of the Study

The purpose of this study is to examine 5th and 6th grade Turkish students' understanding of proportional and non-proportional situations by analyzing their performance on proportional and non-proportional problems. The study also looks at reasons of erroneous solutions in proportional and non-proportional problems. The purpose of this two-phase explanatory mixed-methods design is to obtain statistical quantitative results from the sample and then follow up with a few students to make sense of their understanding on proportional and non-proportional situations and their decision process in problem solving in more depth. In the first phase, quantitative data address students' performance on the proportional and non-proportional situations depending on the number characteristics of the problem, as the numbers in the problem involve integer ratio or non-integer ratios. In the second phase, qualitative semi-structured interviews were designed to explore the understanding of some students, who consistently used erroneous strategies or showed particular tendencies in quantitative data analysis, about proportional and non-proportional situations, problem solving process, and the effect of the number structure of the problem in their decision process. The reason for the qualitative follow-up data collection was to better understand the quantitative data that reveals students' performance in differentiating the proportional and non-proportional situations (Creswell, 2007).

Research Questions

The study is designed to answer the following research questions:

(1) Can 5^{th} grade students differentiate between the proportional problems (missing-value and comparison) and non-proportional situations (additive and constant situations)? In order to answer this question, the following questions will be examined: (a) Can 5^{th} grade students solve proportional and non-

proportional problems with the same success rate? (b) What is the distribution of students' erroneous strategies in each problem type?

(2) Can 6th grade students differentiate between the proportional problems (missing-value and comparison) and non-proportional situations (additive and constant situations)? In order to answer this question, the following questions will be examined: (a) Can 6th grade students solve proportional and non-proportional problems with the same success rate? (b) What is the distribution of students' erroneous strategies in each problem type?

(3) Does the use of integer and non-integer numbers in proportional problems (missing-value and comparison) and non-proportional problems (additive, and comparison) affect 5th grade students' solution strategies?

(4) Does the use of integer and non-integer numbers in proportional problems (missing-value and comparison) and non-proportional problems (additive, and comparison) affect 6th grade students' solution strategies?

(5) What are possible reasons for erroneous methods while solving proportional and non-proportion problems in the 5th and 6th grades?

Significance of the Study

Development of proportional reasoning skills is one of the most important goals for middle school programs across the world. The importance of proportional reasoning arise from its role of being a prerequisite for understanding related subjects in high school and in the following years of education, and different subjects such as basic scientific concepts, economics, and for handling daily life problems and situations, and so on. (Modestaou, 2007; Spinillo & Bryant, 1999; Van Dooren et al., 2010b).

Middle school year time can be accepted as the critical period for learning about ratio and proportional relations (Lamon, 1994; Lo & Watanabe 1997; Lobato, Ellis, & Zbiek, 2010; Van Dooren et al., 2010a; 2010b). Development of the use of informal strategies on ratio and proportion problems was investigated in 5th grade students (Lo & Watabe, 1997), and the over-application of additive and proportional methods were observed in almost half of the 5th and 6th grade students (Van Dooren et al., 2010a). There is a consensus in the literature about the development of proportional reasoning as a process of starting with qualitative reasoning (use of comparison words), use of build-up strategies (using addition and subtraction operations to maintain the proportional relationship), and finally understanding multiplicative reasoning (use of the properties in linear functions) (Fernandez et al., 2010). Literature suggests that understanding middle school students' way of reasoning in proportional and non-proportional situations are worthy of investigation. Identification of the variables, which effect students' performance on proportional and non-proportion problems give important clues about students' process of giving meaning to proportional relations. Findings of the current study may provide insight for a better organization of teaching process of proportional reasoning by considering possible reasons of the difficulties that students experience.

CHAPTER II

LITERATURE REVIEW

The purpose of this chapter is to provide a background on proportional reasoning. It is organized under six main titles: 1) Ratio and Proportion as a Sub-construct of Some Broad Concepts 2) Defining Proportional Reasoning, 3) Students' Difficulties in Proportionality, 4) Additive Reasoners and Proportional Reasoners, 5) The Variables Affecting the Performance on Proportional Situations, and 6) Studies on Proportional Reasoning in Turkey.

Ratio and Proportion as a Sub-construct of Some Broad Concepts

Some specific subjects in mathematics may attract a special interest of the researchers. The reason why researchers give a special importance to these subjects may be the importance, ubiquity of them, and students' difficulties in understanding these subjects. Some examples of the topics, which students have special difficulties with are fractions, rational numbers, multiplicative structures, or ratio and proportion (Cramer et al., 2002; Van Dooren et al., 2010b; Thompson & Saldanha, 2003; Charalambos & Pitta-Pantazi, 2005; Pantziara & Philippou, 2011, Adjiage & Pluvinage, 2007; Lawton, 1993). When ratio and proportion is considered, it stands for a common subconstruct in each of the following broad concepts as rational numbers, fractions and multiplicative structures.

The rational number concept comprises many subconstructs. Some examples of these comprises are as following; part-whole, quotient, and operator (Behr, Lesh, Post, & Silver, 1983), one another group is as following; quotient, measure, operator, and ratio (Kieren, 1993), and another one is; part-whole, quotient, measure, operator, and ratio (Marshall, 1993). Concepts, which are considered as subconstructs of rational numbers also include fraction order and equivalence and proportionality, measurement, probability, coordinates systems, graphing, part-whole model for fractions, flexible concept of unit, order and equivalence, estimation of addition and subtraction with fractions, and so on. (Behr et al.,1983; Behr, Harel, Post, & Lesh, 1992; RNP, 2001).

Kieren (1976) proposes the concept of fractions as the broader concept that consists of four interconnected concepts: ratio, operator, quotient and measure (including the part-whole subconstruct). When we consider 3/5 as a fraction, it can be conceived as a part-whole subconstruct (three out of five equal parts), as quotient subconstruct (three is divided by five), an operator subconstruct (three one fifths of a quantity), as a ratio subconstruct (three parts to five parts), and as a measure subconstruct (a point on the number) (Pantziara & Philippou, 2011).

Vergnaud (1994) conceptualize multiplicative structures as a special conceptual field, which is named as *Multiplicative Conceptual Field* (MCF). The collection of the concepts such as proportional relations, ratio and proportion, fractions, and the algorithms of division and multiplication constitute the MCF. In MCF the understandings of the concepts such as fraction, ratio and proportion, multiplication and division, or linear and/or bilinear functions must be considered in connection with each other. MCF forms broad concept in which all the other subconstructs stands in relation with each other.

What makes ratio and proportion special in the learning and teaching process may be its relation with these broad constructs. Ratio and proportion stands as a common concept in understanding fractions, rational numbers, and multiplicative structures. To understand ratio and proportion it is essential to make connections among these related broad concepts, and to make connections among the subconstructs for understanding broad concepts. For example, multiplication, ratio and proportion, division and rational numbers do not develop in isolation (Lo & Watanabe, 1997). Interconnection among these subconstructs and early exposure to the multiple perspectives of the broad concepts would support the full understanding of the domain (Charalambous & Pitta-Pantazi, 2007; Mamede, 2009; Moseley, 2005).

Vergnaud (2009) argues that connection between ratio, fraction, and rational numbers may support each other's meaning and deepen the understanding of each concept. However, poor understanding of one may result in difficulties to understand other interconnected concepts (Mulligan, 2002). For instance, if a learner has problems in understanding fractions, his or her understanding and interpretation about ratio may be negatively affected by the poor understanding of fractions. In general, future understandings of other related constructs may be negatively affected. As Thompson and Saldanha (2003) state how students understand a specific idea can have a strong effect on understanding other interrelated concepts. Therefore, since ratio and proportion stands as a common construct among some broad concepts, it is also essential to understand interconnected concepts for a better understanding of the concept.

Defining Proportional Reasoning

Ratio and proportion stands as a common subconstruct in all the aforementioned broad concepts. This may explain the reason why students have serious difficulties in understanding proportional relations and developing proportional reasoning. For understanding proportional relations, one needs to understand fractions, rational numbers, and/or multiplicative relations, and the other interconnected subconstructs.

Van Dooren et al. (2010a; 2010b) consider proportional situations as a special form of multiplicative situations. This shows the tight connection between multiplicative reasoning and proportional reasoning. Proportional reasoning is a form of multiplicative reasoning that includes the ability to reason by considering the co-variation between the variables, to reason on the second order relationships, to compare the multiple variables at the same time, and to use the pieces of information as a whole (Lesh, Post, & Behr, 1988). This ability requires seeing multiplicative relationships between different variables, expressing these relationships by using mathematical expressions, and reasoning quantitatively in addition to reasoning qualitatively.

Proportional reasoning requires an understanding of multiplicative relationships that exists among the variables representing proportional situation (Boyer, Levine, & Huttenlocher, 2008). Mathematical expression of the relation among proportional reasoning and multiplicative relations can be represented as: two interdependent variables, *x* and *y*, are corresponded in such a way that the correspondence existing between *x* and *y* also exists between all equal multiples of *x* and *y* (Karplus, Pulos, & Stage, 1983). It is possible to use linear and proportional relations interchangeably, both referring to the function of $f(x) = ax$ (with $a \neq 0$) and represented by a straight line passing through the origin on the coordinate plane (Van Dooren, De Bock, Hessels, Janssens, & Verschaffel, 2005).

Proportional reasoning is the milestone subject of middle graders mathematics. It includes kinds of prerequisite abilities for high school mathematics and also for the following years of education (Modestou & Gagatsis, 2007). The importance of the development of this concept is also evident in the NCTM documents (NCTM, 1989) stating no matter how much time and effort is needed anything required for the development of proportional reasoning must be provided. Also proportional reasoning has a wide range of applications in primary and secondary mathematics education and this wide applicability gives a special importance to proportional reasoning in education (Van Dooren et al., 2010a). Besides the importance of proportionality in learning mathematics, it has important role in understanding basic scientific concepts and handling everyday problems and situations (Spinillo & Bryant, 1999; Vergnaud, 1988).

Students' Difficulties with Proportionality

One of the main factors for the difficulties children face in understanding proportional relations is that it forms a multifaceted construct (Charalambous & Pitta-Pantazi, 2007) and it has a wide usability in daily life and everyday problems (Noss & Hoyles, 1996). For understanding proportional relations, one needs to understand fractions, rational numbers, and/or multiplicative relations, and the other interconnected subconstructs.

Limited understanding of multiplicative relations may increase the difficulty students experience in understanding proportional relations, therefore understanding proportional reasoning requires full understanding of multiplicative relations (Lo & Watanabe, 1997). The ability to think multiplicatively cannot be limited by an algorithm of multiplication (Vergnaud, 2009), or as the general way of introducing multiplication as repeated addition. Although considering multiplication as repeated addition is easy to teach and learn, connections among measurement, proportionality and fractions are conceptually separated (Thompson & Saldanha, 2003). Meaningful understanding of multiplicative relations is required for understanding proportional relations, otherwise making sense of ratio and proportion cannot go beyond a rule following process (Singh, 2000). Therefore, what students understand from multiplicative relations and the way they interpret these relations may have an effect on what they understand from proportional situations and to what extent they can interpret such situations.

Students may also have difficulties on abstraction of the concepts imparted from other contexts, as proportional situations exist in everyday problems and in areas other than mathematics. Context has an indicative role, sometimes more predominantly than the effect of the main construct (i.e., proportional reasoning). Prior knowledge about the problem may also affect the understanding of the children in a different way. As Noss and Hoyles (1996) state, it is clear that the expression of proportion has an extensity in daily life language, in which it may have a variety of meanings like "*roughly equivalent,*" or "*about the same shape.*" The use of these words in daily life may not have the exact mathematical meaning. What “similar” means in daily life is totally different than the concept of “similarity” in mathematics. Children need to be able to separate the daily life use and mathematical meanings. Since proportionality has a daily life aspect, its development starts with the experiences in life, and later these experiences affect students’ understanding (Alatorre, & Figueras, 2004).

Children have serious difficulties with transitioning from build-up strategies to multiplicative strategies when developing proportional reasoning (Fernández et al., 2010). The transition from the build-up strategies to multiplicative strategies requires the distinction between the multiplicative relations and additive relations. While students encounter many opportunities to deal with additive situations in primary school, students’ understanding about additive relations may be insufficient to deal with the multiplicative relations that are covered in middle school (Sowder et al., 1998).The difficulty in understanding multiplicative relations affect students’ understanding of proportional relations and their ability to differentiate proportional and non-proportional situations.

Non-Proportional Tasks

For the distinction between proportional and non-proportional situations, use of non-proportional tasks in instructions is important. Most common examples of non-proportional relations are additive relations, and constant relations (Modestou & Gagatsis, 2008). In constant problems, relation between the quantities is constant, as in this example: “Two shirts need 30 minutes to dry out in a sunny day. How much time do four shirts need to dry out under the same weather conditions? (Modestou & Gagatsis, 2007)". Relation between the quantities of time and number of shirts shows a constant one, independent from the number of shirts; time required for drying out is the same. Additive relations include a co-variation between the quantities as proportional tasks. However, the co-variation in additive situations is represented by an *invariant difference* between quantities (Van Dooren et al. 2010a). Example of an additive problem is "Tom and his sister Ana have the same birthday. Tom is 15 years old when Ana is 5 years old. They are wondering how old Ana will be when Tom is 75 (Van Dooren et al., 2010a)".

Proportionality Tasks

Understanding proportional reasoning and expressing multiplicative relations mathematically starts in elementary grades (Van Den Brink & Streefland, 1979). Students face with different types of proportionality tasks in school. Types of proportional reasoning may be classified according to various typical contexts in which proportional reasoning is required such as mixture problems, physical tasks, rate problems, and probability tasks (Tourniaire & Pulos, 1985).

In *mixture problems*, two or more substances bring out a different object, different than both of the substances. An example of a mixture problem can be making a jam, and the subject needs to know ingredients that form the mixture. *Physical tasks* are tasks that require physical knowledge and understanding in addition to proportional reasoning. An example for this type of a task can be speed comparisons. In these tasks knowing what speed means is an indispensable part for a successful performance on the proportional task. *Rate problems* require comparing dissimilar objects by using ratio, and an example can be a comparison of fuel consumptions of two cars. Rate problems do not require an additional knowledge related to the tasks itself. Hence this problem type can be easier to use proportional reasoning skills when compared to mixture or probability tasks. *Probability tasks* are much more similar to rate problems. In these two problems types, subjects need to know the meaning of probability to solve the task. An example of a probability task can be throwing a dice. Besides the understanding of proportional reasoning, knowledge of probability is necessary in the solution process. Therefore, in addition to one's ability to reason proportionally, contextual factors and related constructs also play a crucial role for a successful performance on the proportional tasks.

Hiebert and Behr (1988) classify seven different types of proportional tasks as: (a) missing-value problems, (b) comparison problems, (c) transformation problems, (d) mean value problems (e.g., geometric mean, harmonic mean, arithmetic mean), (e) proportions involving conversion from ratios to rates and fractions, (f) proportions involving unit labels as well as numbers, (g) between mode translation problems (writing ratio as a fraction, as a rate, or as a quotient). Although there are seven different problem types, Hiebert and Behr (1988) state that only comparison and missing value problem types dominates textbook-centered instructions and research. The rest of the problem types are neglected both in textbooks and research.

In literature, proportional reasoning tasks were also categorized as missing value problems (Cramer et al., 1993; Karplus et al., 1983), numerical comparison problems (Noelting, 1980; Karplus et al., 1983), and qualitative prediction and comparison problems (Behr et al., 1992; Modestou & Gagatsis, 2008). In missing value problems three values are given and fourth one is unknown (Kaput & West, 1994). Four values are given in comparison problems and ratio between two related values would be compared. No numerical information is given in qualitative problems, and proportional relation between variables is asked. Teaching of proportionality starts intensively in middle school where the most common problem type of proportional reasoning is missing value problems, which also have a dominant role compared to other proportionality problem types in textbooks.

Vergnaud (1983) classifies problem types according to mathematical structure of the problems. These problem types are *isomorphism of measures*, *product of measures*, and *multiple proportions*. Isomorphism of measures includes a simple direct proportion between two measure spaces. In product of measures, multiplication of two measure spaces gives a constant result. In multiple proportions, there are more than two measure spaces and relation among these measures results in multiple proportions.

Proportional reasoning is more than using algorithms in the solutions of proportionality problems, so people who are able to solve typical proportional problems are not always able to show proportional reasoning. Although many studies state that proportional reasoning cannot be limited by the use of some algorithmic strategies (e.g. cross-multiplication), textbooks and instructions in schools mostly include these kinds of algorithms (Van Dooren et al., 2010a; 2010b; Hiebert & Behr, 1988). Use of ratio and proportion and some specific strategies in solving missing-value-proportion problems have been perceived as the ability to reason proportionally (Cramer et al., 1993). Since proportional reasoning is a long-lasting and complicated process for both teaching and learning, there is a tendency to simplify the process. This simplification is using proportional reasoning as the ability to solve certain set of problems or use of some algorithmic strategies as an explanation of proportional reasoning. Consequently, some specific applications and tasks that are used in some studies started to define the term, especially how proportional reasoning is perceived in schools (Misailidou & Williams, 2003; Modestou & Gagatsis, 2008).

Strategies Used in Proportionality Tasks

There are different ways of dealing with proportional tasks. Researchers identified common successful and unsuccessful strategies that students use when solving these types of problems (Tourniaire & Pulos, 1985).

Successful Strategies Used in Proportionality Tasks

According to Adjiage and Pluvinage (2007), successful strategies can be *multiplicative* and *build-up strategies.* In multiplicative strategy, subjects can use whole numbers, decimals, and fractions to express a ratio, find a common reference to compare, or use cross-multiplication of two ratios. They also state that using fractions to express a ratio is a better indicator of proportional understanding. Although cross-multiplication strategy seems dominant in instruction, subjects rarely prefer to use this method; instead of this they establish a relationship between denominators and numerators within or between ratios. Although cross-multiplication occupies an important place in proportional reasoning models, it is not preferred as a solution strategy by most of the students (Tourniaire & Pulos, 1985). Instead of cross-multiplication method, students frequently use building-up strategy since it includes additive reasoning that students are more familiar with. Tourniaire and Pulos (1985) further differentiated six correct strategies under these two broad categories: unit-rate and building-up method.

Correct methods can also be subcategorized according to the construction of the variables as *functional* (comparing the ratio of two different variables using "external ratios" or between ratios, or a "functional method") and as *scalar* (comparing ratios of the same variable using "internal ratios" or within ratios, or a "scalar method") (Ben-Chaim, Fey, Fitzgerald, Benedetto, & Miller, 1998; Fernandez et al., 2010; Noss & Hoyles, 1996; Vergnaud, 1983). Suppose that 2 kg of sugar costs 1.7 TL and the price of 5 kg of sugar is asked. Working out the price per unit by dividing the price by 2 and multiplying the unit by 5 give the result, by using functional solution, the relation between amount of sugar and money is used. In the scalar solution, how to get 5 kg by using 2 kg is considered. Multiplying 2 by 2.5 gives the amount of 5 kg, then the same operation will be done on the price of 2 kg of sugar, multiplying the cost of 1.7 by 2.5 as well, and the result is found (Spinillo & Bryant, 1999). The situation is represented in the Figure 1.

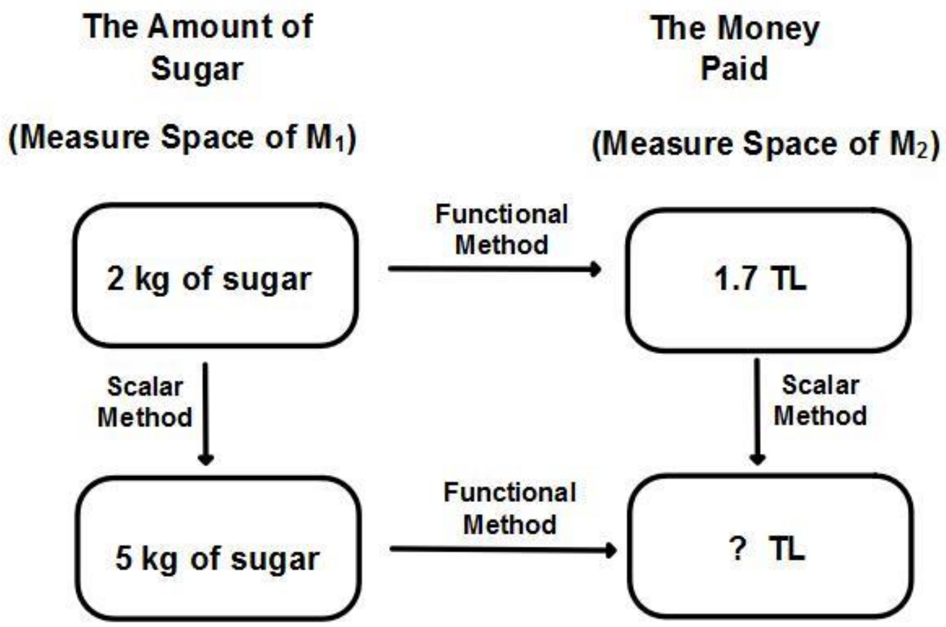

Figure 1. Scalar and functional methods in between and within measure spaces of M1 and M2.

Using *invariance of ratio* (Noelting, 1980) and *invariance of product* (Siegler, 1976) can be the other two general categories of correct solution methods. Children use comparison of two ratios as a correct solution method in solving proportional tasks such as comparing concentrations of two mixtures. Children also commonly use product of values of two variables in proportional tasks such as distance and weight problems in which product of the variables are kept constant.

Another successful method is *build-up strategy* or *unitizing* (Singh, 2000). In this method, subjects take ratio as a unit and iterate two number sequences (ratio) separately. This method can be accepted as a primitive version of a multiplicative strategy since the operation used is mainly addition. However key point in this strategy is the conceptualization of ratio as a unit. Therefore, although subjects use addition operation, repeated addition of variables preserves multiplicative relationship between the variables. Build-up strategy or additive strategies are taught and emphasized in schools. However, researchers argue that using only these kinds of methods show a deficient development of proportional reasoning (Lesh et al., 1988). The reason why this method is weak in providing the development of proportional reasoning ability is children's poor performance on proportional tasks that include non-integer numerical variables. Very small number of children can use building-up strategy successfully in non-integer tasks also (Tourniaire & Pulos, 1985).

Unsuccessful Strategies Used in Proportionality Tasks

One of the most common unsuccessful strategies is ignoring one part of data that is, solving problem just by relying one part of data. One another common unsuccessful strategy is *additive strategy* (Adjiage & Dluvinage, 2007). *Additive strategy* or *constant difference* is a method, in which ratio is calculated by adding the differences between two terms and then applying this difference to third term. Although this method has no sense of ratio, it is very frequent, especially in childhood. Main fault in this method is not using addition operation; it is also possible to maintain proportional relations by using addition (Tourniaire & Pulos, 1985). The main problem is being unable to maintain multiplicative relation between variables in the proportional situation by using an additive strategy. Usage of this method shows that subjects do not have the meaning of "ratio" and cannot identify multiplicative relation in proportional situations.

Additive and Proportional Reasoners

Although there are different types of proportionality tasks and solution methods to solve proportional problems, there may be more reference to some specific tasks and solution methods in schools (Hiebert & Behr, 1988; Modestou & Gagatsis, 2008). In most of the cases and studies, proportional reasoning is considered as the ability to solve missing-value proportional problems (Modestou & Gagatsis, 2008). In some situations, the ability to reason proportionally may be accepted as the ability to solve missing-value problems, even just application of cross-multiplication method to solve a problem. Having the focus on a specific problem type, or a specific solution method may limit what students understand from proportional reasoning. However, proportional reasoning is a complex process. It requires an understanding of multiplicative relationship that exists among the variables representing proportional situation and also ability to differentiate proportional and non-proportional situations (Cramer et al., 1993; Fernandez et al., 2010; Modestou & Gagatsis, 2008).

According to the study of Van Dooren et al. (2010a) there are four important profiles according to students' use of erroneous strategies depending on the problem type and number structure. These profiles are (a) additive reasoners, (b) proportional reasoners, (c) correct reasoners, and (d) number-sensitive reasoners. *Additive reasoners* overuse additive methods in proportional and constant situations, where it is inappropriate. Additive method relies on addition operation. *Proportional reasoners* overuse multiplicative (proportional) method in non-proportional situations for example, in additive situations or constant situations. Multiplicative method relies on multiplication operation. *Correct reasoners* are the ones, who can differentiate proportional and non-proportional situations and choose the appropriate solution strategy for each situation. *Number sensitive reasoners* are the ones, who solve problems including integer ratio proportionally, and problems including non-integer ratio additively.

According to the strategies students use in proportional situations and non-proportional situations, researchers identify two important types of reasoners: *additive reasoners* and *proportional reasoners.*

Additive Reasoners

Harel and Behr (1990) define additive reasoners who interpret changes in quantities as additively even the situations in which multiplicative relations are required. The ability to differentiate proportional and non-proportional situations is explored in research studies. It has been shown that people show a tendency toward overgeneralization of proportional reasoning, and they are unable to differentiate proportional and non-proportional situations (Fernandez et al., 2010).

The Reasons for Over-Generalization of Additive Strategies on Proportional Situations

Use of additive strategies in proportional situations is a frequently encountered problem in education, and students' tendency to use additive strategies in proportional situations is a heavily mentioned phenomenon (Van Dooren et al., 2010a). There are many possible reasons that may result in this situation. One of them may be students' expertise on addition and over-reliance on addition operation and counting routines (Boyer et al., 2008). Especially when students face with a problem in which there is a non-integer ratio between variables, they prefer to develop an additive method, in which calculation of numbers is much easier and familiar for the student (Clark & Kamii, 1996).

One other reason may be that additive reasoning develops intuitively through experiences in life, which are very primitive in nature. However, multiplicative reasoning does not develop naturally; it requires schooling and a deeper understanding of multiplicative relations and abstraction of the units in relation with each other (Sawder et al., 1998). Children have serious difficulties with transitioning from build-up strategies to multiplicative strategies when developing proportional reasoning (Fernandez et al., 2010).

Another reason may be students' insufficient understanding of multiplication and division, and multiplicative relations between variables. Multiplicative understanding supports the development of proportional reasoning independently from superficial effects resulted from contextual factors or overgeneralization of possible solution strategies. Especially focusing on particular solution strategies limits the understanding of proportional reasoning (Lo & Watanabe, 1997), and problem solving process becomes a rule-following procedure where students follow superficial clues (Van Dooren et al., 2010b).

Multiplicative thinking requires more than just doing repeated addition (Van Dooren et al., 2005; Thompson & Saldanha, 2003). So teaching multiplication just as repeated addition is not sufficient; students need to be given instructional opportunities to develop multiplicative and additive reasoning. However, in schools, teachers usually focus on multiplication as repeated addition and students confine multiplicative reasoning as repeated addition. As a result, students perform well on multiplicative problems that can be solved by repeated addition; however they fail on the problems in which multiplication should be conceived differently, for instance as a Cartesian product, which is also a multiplicative relationship (Van Dooren et al., 2010a).

Proportional Reasoners

Harel and Behr (1990) define multiplicative reasoners as the ones who can differentiate situations that include additive relations or multiplicative relations between quantities. It is important to make it clear that not only can proportional reasoners solve proportional problems correctly; they can also differentiate proportional and non-proportional situations and choose appropriate strategy for each situation.

The Overuse of Multiplicative Strategies on Additive Tasks

Overgeneralization does not result in a single direction; just like overuse of additive strategies in proportional situations, overuse of proportional strategies in additive situations are also documented in the studies. As Van Dooren et al. (2010a) state, students' tendency to use additive strategies in proportional situations changes in such a way that use of proportional strategies in additive situations also shows an increase in the following years of elementary school, especially in middle school years.

Overgeneralization of proportional strategies in non-proportional situations and vice versa is frequently observed in literature (Gagatsis & Modestou, 2007; Fernandez, Llinares, Van Dooren, De Bock, & Verschaffel, 2012; Fernandez et al., 2010). When asked directly, students indicate that they choose randomly between two strategies, but actually they respond systematically through artificial correlations that they develop based on their prior experiences (Perso, 1992; Lannin, Barker, & Townsend, 2007). Each choice of strategy reveals some relations and connections that are meaningful and useful for students according to their previous experiences (such as, considering linguistic structure of the problem, considering relation between given numbers as having integer ratio or non-integer ratio, making adjustments in the numbers when numbers are non-integer, considering that proportionality is applicable in all situations, etc.) This result is not just a reaction to proportionality tasks. In general, students have stereotyped formulation of mathematical problems that are disconnected with real life situations. In other words, students have a tendency to connect linguistic formulation of the problem and method they are supposed to use (Modestou & Gagatsis, 2007).

Another reason for overgeneralization of multiplicative methods may be "illusion of linearity" (Van Dooren, De Bock, Hessels, Janssens, & Verschaffel, 2007). People have a tendency to see linear relations in everywhere. This overgeneralization may be resulting from frequent use of mathematical tools for explaining different daily life situations. Everyday life and formal school experiences also result in overgeneralization of proportional reasoning (Van Dooren et al., 2005). Proportionality, ratio preservation, and linearity may be accepted as the universal models that are frequently used by people in everyday life. Therefore, linearity starts to seem as a panacea for nearly all problem situations even when linearity is not applicable (Modestou & Gagatsis, 2012).

The Variables Affecting the Performance on Proportional Situations

Studies on proportional reasoning suggest that students' performance on problems easily affected by the contextual properties (Lawton, 1993). These contextual factors may be familiarity of the context, presence of a mixture, presence of continuous quantities, and use of manipulatives (Tourniaire & Pulos, 1985; Lawton, 1993). Presence of an unfamiliar context, use of discrete or continuous quantities in different ways, presence of a mixture may increase difficulty of problems and presence of manipulative may have a supportive effect on students' performance (Tourniaire & Pulos, 1985).

Number Sensitive Reasoners

When students' tendencies to overuse some strategies are considered, Harel and Behr, (1990) define two common profiles as (a) additive reasoners, and (b) proportional reasoners. In addition to overuse of additive and proportional methods, Van Dooren et al. (2010a) added number-sensitivity as an additional reasoner profile criteria. Students' tendency to change their solution strategy was investigated in different studies. As the number in the task is changed in a way that unit ratio does not give a whole number, subjects show a tendency to change their reasoning completely and start to develop an additive strategy (Alatorre & Figueras, 2004; Singh, 2000).

Patterns of students' solution strategies depending on the number structure gave rise to another reasoner profile (Van Dooren et al., 2010a). Number sensitive reasoners overuse additive and proportional methods depending on the number structure of problems. If numbers in the problem form integer ratios, overuse of proportional method increases; while in non-integer numbered problems, the overuse of additive method increases.

Since context has an effect on students' choice of methods or strategies that they prefer to use, tasks presented in different contexts are perceived differently. Therefore, effect of number variable (use of integer and non-integer ratio), problem type (comparison problem or missing-value problem) and ability to differentiate proportional and non-proportional situations should be investigated separated from other variables, such as context that may affect choice of methods. In other words, context can be a confounding factor and there is a necessity to control this variable.

In order to examine whether students focus on superficial properties of problems, same context is used in all the problems in this study. Students' performance and strategies on these tasks were examined paying special attention to the effects of numbers included in tasks, and type of problem that are all presented in proportional linguistic structure. Problems that show similar linguistic structure of a proportional task, yet do not include multiplicative relations among the variables, are named as "pseudo-proportional" (Modestaou & Gagatsis, 2007). Additive problems and constant problems were presented as pseudo-proportional problems by making their linguistic structure very similar to proportional tasks. In other words, middle school students’ approach was assessed on multiplicative situations stated by word problems, additionally on the situations that are not multiplicative, but additive or constant.

In addition to the focus on students’ skill to differentiate proportional and non-proportional situations, use of only proportional missing-value problems would be insufficient to decide whether student can solve proportional problems or not. While studying students’ reasoning on proportional situations, use of only missing-value problems gives an insufficient picture of the performance since use of a single method provides correct answer of the problem (Modestou, & Gagatsis, 2010). In the light of literature, two different kinds of proportional tasks were used to measure students’ performance on reasoning proportionally, which were proportional missing-value and proportional comparison problems.

All these variables shape problems used in this study. More specifically, problems used in this study to collect data included: (a) two types of proportional situations (proportional missing value and proportional comparison), in which multiplicative relations (both missing value and constant types) among the variables should be considered with integer and non-integer ratios, (b) two types of non-proportional situations (additive and constant), that included integer and non-integer ratios.

In addition to the aim of understanding how students choose successful method, it is equally important to examine unsuccessful strategies for understanding students’ reasoning processes. Researchers must focus on understanding nature of students’ errors for supporting students’ conceptual and deeper understanding instead of their use of superficial clues for solving problems (Lannin et al., 2007). Actual processes and mechanisms students experience while deciding unsuccessful method can be revealed by carefully examining students' thinking processes as they solve problems.

Proportional Reasoning Studies in Turkey

In Turkish curriculum, children start to learn about ratio and proportion in 5th grade (Milli Egitim Bakanlıgı [MEB], 2009). Starting with 6th grade, students start to learn about algebraic expressions, equations and pronumerials. Instructional objectives about ratio and proportion also require students to solve proportional tasks by using their knowledge on algebraic expressions, equation and unknowns in this grade. 6th grade is the first time that students start to use cross-multiplication method to solve missing-value proportional problems.

In 2012-2013 school year, Ministry of Education in Turkey started a revision in Middle Grade Mathematics Curriculum. In the first year of this revision, middle school period included grade levels from 5th to 8th grades. First year of the revision was limited with a change in the grade levels of middle school. At the end of the first year of this revision, MEB declared that middle school mathematics program would incrementally be revised. The revision changed curricular objectives on ratio and proportion. In 5th grade, there was not any objective on ratio. Learning about ratio and proportion starts in 6th grade, and curricular objectives are as following: (1) use different representations of ratio for comparing quantities, (2) when a whole is divided into two groups, express ratio of one part to the other, or one part to the whole, and when ratio between the parts and a whole is given, find unknown, (3) express ratio of two quantities in same unit or in different units. Objectives on proportion placed for the first time in 7th grade year curriculum program. Starting with 2013-2014 school year, new mathematics curriculum started to be implemented in 5th grade level.

There exists a body of research in Turkey investigating proportional reasoning in Turkish schools. Çeken and Ayas (2010) investigated the inter-disciplinary relation among Mathematics, Science and Technology, and Social Studies curricula according to common curricular objectives that requires an understanding of proportional relations in elementary grade level. The study based on the supportive role of interdisciplinary relationship among these three subjects in the learning process. Curricular objectives of Science and Technology, and Social Studies lessons were examined by considering sequence of the objectives in 4th, 5th, and 6th grade curricula. The study revealed that these disciplines showed insufficient connection and correlation in the curricula considering timing of objectives, which require an understanding of proportional relations. This study indicated a wide usability of proportional reasoning in understanding other subjects that shows its importance.

Çelik and Özdemir (2011) examined relation between proportional reasoning skills and problem posing skills of the students in 7th and 8th grades. Researchers used proportional reasoning test that is developed by Akkuş and Duatepe (2006) and problem posing test that is developed by Çelik and Özdemir (2011). Analysis of these tests included written answers, and there were four levels of proportional reasoning skills, as Level 0, Level 1, Level 2, and Level 3 that was taken from Langrall and Swafford's study (2000). Çelik and Özdemir (2011) explained Level 3, most sophisticated level, as the use of algebraic expressions and different solution methods (e.g., equivalent fractions, cross-multiplication) while solving proportional problems. The study revealed a significant relation between proportional reasoning skills and problem posing skills. However, including only written answers in the analysis indicated that understanding proportional reasoning was limited by solving proportional problems by applying some algorithmic strategies in this study.

Kaplan, İşleyen, and Öztürk (2011) investigated 6^{th} grade students' misconceptions about proportional reasoning. Proportional tasks that were used in the study showed a wide range such as velocity problems, perimeter and area relations, and mixture problems. The study investigated erroneous strategies and misconceptions about proportional reasoning, and researchers classified incorrect strategies. Since tasks were different from each other, it was not possible to conclude common erroneous strategies that are used in proportional tasks. Hence, classification of students' common erroneous strategies did not go beyond categories such as "unable to focus on the problem", and "unable to use ratio." In other words, erroneous strategies that students used may have resulted from differences in proportional problem tasks since context, numbers, and type of the problem may affect students' choice of methods and performance (Tourniaire & Pulos, 1985; Lawton, 1993).

Duatepe, Akkuş-Çıkla, and Kayhan's (2005) investigated solution strategies of 6^{th}, 7^{th}, and 8^{th} grade students when solving proportional problems. Data collection instrument included three proportional missing-value problems, two proportional comparison problems, three qualitative comparison problems, one non-proportional problem, and one inverse relation problem. Study showed an unexpected result regarding choice of students' methods on missing-value type of problems. Although literature states cross-multiplication strategy as the least preferable solution method, this study shows that this strategy is the most preferable method among Turkish students. This result was presented as an expected result in the study since this method was the most referred solution method in instructions in Turkish schools. Furthermore, in the study, it was stated that in the comparison problems and additive problems students attempted to use cross-multiplication method, which implied that students have difficulties in differentiating proportional and non-proportional situations. However, this study did not investigate why students preferred specific solution strategies, how they explained their solution process, and variables that affected choice of solution strategies.

Detailed analysis of the incorrect strategies and participants' understanding about proportional relations is investigated in another study (Duatepe & Akkuş-Çıkla, 2002). Researchers studied pre-service teachers' reasoning on ratio and proportion tasks. Results of the study showed that although pre-service teachers can solve ratio and proportion tasks, they had problems on explaining concepts of ratio and proportion. As these concepts develop in middle grades, improving instruction in middle grades is necessary (Sowder et al., 2000). Understanding middle grade students' reasoning and detailed investigation on how participants decide their strategy, how they reason, and how they explain their thinking may give valuable information on understanding this process when solving proportional and non-proportional problems. Behind the successful performance on proportional problems, students may have insufficient understanding on proportional and non-proportional relations and how proportional relations differ from other types of relations. Therefore, in addition to written answers, students' explanations on their way of reasoning deserve to be investigated.

In most of the studies that were conducted in Turkey data collection was limited with written answers. However, detailed analysis on students' explanation of their choice of methods and their way of reasoning in primary school is needed (Çelik & Özdemir, 2011; Duatepe et al., 2005). There is a need to investigate the solution strategies that students use in detail. In addition to written answers, there is a need for a detailed analysis on how students choose specific strategies, when solving proportional and non-proportional problems and how they explain their choice of specific methods. Moreover, variables that affect choice of methods must be considered in data collection.

Summary of Literature

Proportional reasoning is the main subject of middle grade mathematics. It includes kinds of prerequisite abilities for high school mathematics and wide range of application in the following years of education, wide usability in understanding basic scientific concepts and handling everyday problems and situations (Modestaou & Gagatsis, 2007; Spinillo & Bryant, 1999; Van Dooren et al., 2010a). In contradiction with the importance of proportional reasoning, students have serious difficulties in developing proportional reasoning (Capon & Kuhn, 1979; Lawton, 1993).

For a better understanding of difficulties that students' experience, students' understanding of "proportional reasoning" must be investigated. Proportional reasoning is not a single concept. It requires an understanding of multiplicative relationships that exists among variables representing proportional situation, which is a slowly developing and complex process (Cramer et al., 1993). When problems that students encounter in school and instructions are considered, there is a tendency to limit proportional reasoning as the ability to solve certain set of problems or use of some algorithms (Hiebert & Behr, 1988; Modestou & Gagatsis, 2008). In addition to the ability to solve proportional problems, proportional reasoners need to differentiate proportional and non-proportional situations and to decide appropriate solution method for each situation (Harel & Behr, 1990).

In differentiating proportional and non-proportional situations, students' performances may easily be affected by contextual properties of the problems and students may focus on superficial properties of the problems (Gagatsis, 2007; Lawton, 1993; Tourniaire & Pulos, 1985; Van Dooren et al., 2005; Van Dooren et al., 2010a; 2010b). Use of numbers that gives an integer ratio or non-integer ratio, and context of the problem may easily affect students' choice of methods. Therefore, proportional reasoner must be able to differentiate proportional and non-proportional situations independent from these superficial and contextual variables.

Studies on proportional reasoning investigated current situation in Turkey starting from the 4th grade (Çeken & Ayas, 2010) to the university level (Duatepe & Akkuş-Çıkla, 2002). Interdisciplinary relation among Mathematics, Science and Technology, and Social Studies according to the sequence of the curricular objectives of 4th, 5th, and 6th grade books (Çeken & Ayas, 2010), relation between students' proportional reasoning skills and problem posing skills in 7th and 8th grades (Çelik & Özdemir, 2011), 6th grade students' misconceptions about proportional reasoning on velocity, perimeter and area relations, and mixture problems and classifications of erroneous methods (Kaplan et al., 2011), difference in 6th, 7th, and 8th grade students' solutions strategies depending on the problem type (Duatepe et al, 2005), and a detailed analysis of incorrect strategies pre-service middle school mathematics teachers' use in ratio and proportion tasks (Duatepe & Akkuş-Çıkla, 2002) were investigated.

In Turkey, students start to learn about ratio and proportion in 5th grade (MEB, 2009). In 6th grade, students learn about algebraic expressions, equation, and pronumerials, therefore instructional objectives require students to use this knowledge when solving proportional problems. While 5th grade students start to learn about ratio and proportion for the first time by using tables, students start to use cross-multiplication method in solving proportional problems in grade 6.

In 2012-2013 school year, MEB changed middle school level starting with 5th grade to 8th grade. Before the change, middle school involved 6th, 7th, and 8th grades. First year of the revision was limited with a change in grade levels of middle school. In the following year of the revision (school year of 2013-2014), MEB started to implement a new mathematics program in middle school. The revision changed curricular objectives on ratio and proportion. In 5th grade, there is not any objective on ratio. Learning about ratio and proportion starts in 6th grade, and curricular objectives require students to express ratio of given quantities, calculate part of a whole, or whole of a part in case of the ratio between two quantities were given. Solving proportion problems starts at 7th grade. Starting with 2013-2014 school year, new mathematics curriculum started to be implemented in 5th grade level. In 2014-2015 school years, new mathematics program will start to be implemented in 6th grade. Therefore, new curriculum will incrementally put in practice year by year.

In summary, research reveals that proportional reasoning requires the differentiation between proportional and non-proportional situations independent from contextual variables (Cramer et al., 1993; Fernandez et al., 2010; Modestou &

Gagatsis, 2008) and 5th and 6th grade levels are important periods in which instruction on ratio and proportion starts in Turkey. The purpose of this study is to investigate 5th and 6th grade students' proportional reasoning and identify type of limitations students have when solving proportionality problems. More specifically the study will be guided by the following research questions:

(1) Can 5th grade students differentiate between the proportional situations (missing-value and comparison problems) and non-proportional situations (additive and constant situations)? In order to answer this question, the following questions will be examined: (a) Can 5th grade students solve proportional and non-proportional problems with the same success rate? (b) What is the distribution of students' erroneous strategies in each problem type?
(2) Can 6th grade students differentiate between the proportional situations (missing-value and comparison problems) and non-proportional situations (additive and constant situations)? In order to answer this question, the following questions will be examined: (a) Can 6th grade students solve proportional and non-proportional problems with the same success rate? (b) What is the distribution of students' erroneous strategies in each problem type?
(3) Does the use of integer and non-integer numbers in proportional problems (missing-value and comparison) and non-proportional problems (additive, and comparison) affect 5th grade students' solution strategies?
(4) Does the use of integer and non-integer numbers in proportional problems (missing-value and comparison) and non-proportional problems (additive, and comparison) affect 6th grade students' solution strategies?

(5) What are possible reasons for erroneous methods while solving proportional and non-proportion problems in the 5th and 6th grades?

Research Hypotheses

In this section, research hypotheses based on literature review are stated.

(1) Can 5th grade students differentiate between the proportional situations (missing-value and comparison problems) and non-proportional situations (additive and constant situations)? In order to answer this question, we will examine the following questions: (a) Can 5th grade students solve proportional and non-proportional problems with the same success rate? (b) What is the distribution of students' erroneous strategies in each problem type?

Current study expected to find a higher success rate in proportional tasks compared with non-proportional tasks (additive and constant problems) (Fernández, Llinares, & Valls, 2008; Van Dooren et al., 2007). When two grade levels were compared, over-use of additive strategies was expected to be observed more frequently in the 5th grade (Van Dooren et al., 2010a). Besides this, comparing with 6th grade students, 5th grade students are expected to have a higher success rate in non-proportional situations and a lower success rate in proportional situations, as a result of students' tendency to apply additive method "anywhere" in the lower grade levels (Fernández et al., 2008; Van Dooren et al., 2010a).

Besides this, Fernandez et al. (2010) showed that from 5th to 6th grade, percentage of students' use of additive strategy in additive situations, and use of proportional strategy in proportional situations moderately increased. Van Dooren et al. (2010a) and Fernandez et al. (2010) indicated different age groups that show a trend in the development from additive reasoning to multiplicative reasoning. Fernandez et al. (2010) explains the reason why age groups differ in these two studies according to education traditions and mathematics curriculum.

(2) Can 6th grade students differentiate between the proportional situations (missing-value and comparison problems) and non-proportional situations (additive and constant situations)? In order to answer this question, we will examine the following questions: (a) Can 6th grade students solve proportional and non-proportional problems with the same success rate? (b) What is the distribution of students' erroneous strategies in each problem type?

Overuse of proportional methods in non-proportional tasks was observed more recently starting with the 6th grade (Van Dooren et al., 2010a). Current study expected to find over-use of proportional strategies more recently in the 6th grade comparing with the 5th grade and a higher success rate in proportional problems and a lower success rate in non-proportional problems as a result of the tendency to apply proportional strategy “anywhere” (Fernandez et al, 2010; Van Dooren et al, 2010a).

(3) Does the use of integer and non-integer numbers in proportional problems (missing-value and comparison) and non-proportional problems (additive, and comparison) 5th grade students' solution strategies?

As the number in the task is changed in a way that unit ratio does not give a whole number, subjects are expected to show a tendency to change their reasoning completely and start to develop an additive strategy (Fernandez et al., 2010; Singh, 2000) and when the number is changed from non-integer ratio to integer ratio subjects were expected to overuse proportional strategy (Van Dooren et al., 2010a). As Van Dooren et al. (2010a) and Fernandez et al. (2010) showed in their study, 5th grade students are expected to overuse additive strategy in proportional situations as the numbers were changed from integer ratio to non-integer ratio, overuse proportional strategy in non-proportional situations as the numbers were changed from non-integer ratio to integer ratio.

(4) Does the use of integer and non-integer numbers in proportional problems (missing-value and comparison) and non-proportional problems (additive, and comparison) 6th grade students' solution strategies?

As the numbers in the problem form integer ratios, overuse of proportional strategies is expected to increase (Van Dooren et al., 2010a), whereas when the numbers in the problem form non-integer ratios, the subjects are expected to show a tendency to develop an additive strategy (Singh, 2000). As Fernandez et al. (2010) indicated in their study, 6th grade students showed a higher success rate in non-integer numbered proportional problems when compared with 5th grade students' success rates.

CHAPTER III

METHOD

This chapter includes research design, research settings, research participants and sampling, data collection and data analysis sections.

Research Design

This study uses explanatory mixed method design (Creswell, 2008). The study started by collecting quantitative data first. Purpose of the quantitative data collection was to identify important points that deserve more focus for a better understanding of the situation. After quantitative data collection and analysis, qualitative data was collected for a deeper understanding of students' understanding of proportional and non-proportional situations, their choice of the strategies and reasons of these choices.

Research Setting

In each grade level, instruction of ratio and proportion was completed at least one month before conducting the study. Fifth and 6th grade mathematics curricula include ratio and proportion as a subject. Objectives of Turkish Ministry of Education (MEB) on ratio in grade 5 are "Expresses the relation between two variables as a ratio" and "Solves and creates ratio problems by organizing given data in tables" (MEB, 2011). An example on how a table is used while solving a ratio problem is as following. This example was taken from 5th Grade Mathematics Book (MEB, 2011) (Figure 2).

Problem:

One tin of yellow paint is mixed with two tins of blue paint to have green colour. If we use 5 tins of yellow paint, how many tins of blue paint do we need for having the same tone of green colour? Find the answer by presenting data on a table.

Answer:

Number of yellow colour tins	1	2	3	4	5
Number of blue colour tins	2	4	6	8	10

If we use 5 tins of yellow colour, we need to use 10

Figure 2. An example of a solution of a ratio problem by organizing data in a table.

MEB objectives on ratio and proportion in grade 6 are “Uses ratio to compare the quantities and expresses ratio in different ways” and “Explains ratio and the relation between quantities that have proportional relation” (MEB, 2011). Although the curricular objectives may differ according to the grade level, both 5th grade and 6th grade students should be able to solve proportional and non-proportional problems prepared for this study, according to MEB program.

In 2012-2013 school year, MEB started a revision in the Middle Grade Mathematics Curriculum. In the first year of this revision, middle school period included grade levels from 5th to 8th grades (it was from 6th to 8th grades in the previous program). First year of the revision was limited with a change in grade levels of middle school. Data collection of this study coincides with the first year of the revision, 2012-2013 school year. Although there was not any change in the curriculum and mathematics program, 5th grade was included in middle school years.

At the end of the first year of this revision, MEB declared that middle school mathematics program would incrementally be revised. The revision changed curricular objectives on ratio and proportion. In 5th grade, there is not any objective on ratio. Learning about ratio and proportion starts in 6th grade, and curricular objectives are as following: (1) use different representations of ratio for comparing quantities, (2) when a whole is divided into two groups, express the ratio of one part to the other, or one part to the whole, and when the ratio between the parts and the whole is given, find the unknown, (3) express ratio of two quantities in same unit or in different units. Objectives on proportion placed for the first time in 7th grade curriculum program. Starting with 2013-2014 school year, new mathematics curriculum started to be implemented in 5th grade level.

Research Participants and Sampling

Convenience sampling was used in order to choose the participants in this study. The researcher conducted this research in the school she worked in. Thus, research site was more accessible for the researcher. Administration and head of Mathematics Department were informed about the aim of this research and INAREK (Human Studies Evaluation Institutive Committee) form was shared with the administration (Appendix B). Students were informed that they were free to participate in the study.

Initially there were 252 participants 138 of students were in 5th grade (10-11 years old), and 114 of students were in 6th grade (11-12 years old). Students who had missing data were removed from sample (18 students from 5th grade and 13 students from 6th grade). Therefore, the final analysis included 120 students from 5th grade and 101 participants from 6th grade. Number of the boys and girls in the sample was approximately equal (113 boys and 108 girls). Most of the students were from high socioeconomic backgrounds since the school was a private one. For a better understanding of students' reasoning process and variables that affects students' reasoning, interviews were held with nine students (five students from 5th grade and four students from 6th grade).

Data Collection Procedures

Data collection took place in the participants' schools. Fifth grade and 6th grade participants were given the same data collection instrument during class time. The instrument (Appendix C) was a test that had 12 problems with four buffer items. Each set of problems had four proportional word problems (one integer numbered missing-value problem, one non-integer numbered missing-value problem, one integer numbered comparison problem and one non-integer numbered comparison problem) and four non-proportional word problems (one integer numbered constant problem, one non-integer numbered constant problem, one integer numbered additive problem and one non-integer numbered additive problem) (Table 1).

Table 1**.** Problem Types Used in the Test

Problem Types							
Constant		Additive		Proportional Missing-Value		Proportional Comparison	
I	NI	I	NI	I	NI	I	NI

Note. Integer (I), Non-integer (NI)

Four buffer items were chosen from the topics different than ratio and proportion from MEB textbook of 5th grade mathematics curriculum. Therefore, both 5th and 6th grade students were familiar with the buffer items. No time limit was set for working on the problems. Students were given as much time as they needed to respond to all problems.

Test was administrated by the researcher in the participating 5th and 6th grade classes in two weeks. Administration was held especially in morning sessions. Before starting the test, students were told that the test included items about two friends, Emre and Sıla. Students were asked to show their work on the problems. They were told that they could also write down sentences or make drawings in case they were not able to express their reasoning mathematically.

After analyzing written data, nine participants were chosen from the sample group for collecting qualitative data through semi-structured interviews. Changing solution strategy from an erroneous one to a correct one or changing solution strategy from a proportional to additive one when the numbers changed from integer ratio to non-integer ratio and use of proportional method in all problems were the most noticeable patterns among 5th and 6th grade students. Therefore, the participants were chosen according to these quantitative findings. Although interviews contained test questions that were shaped according to students' own written solutions in the test, participants could answer the questions during the interviews freely and in their own words.

Interviews aimed at revealing possible reasons (1) why students overused additive or proportional strategy, (2) why students changed their solution strategy when numbers in the problem changed from integer ratio to non-integer ratio or vice versa, and (3) for the use of erroneous solution strategies.

Semi-structured interviews were conducted with each participant individually. Interviews approximately took 15-30 minutes, depending on participants' use of time for thinking and expressing their way of reasoning. During these interviews, students were encouraged to explain their way of reasoning, process of deciding their solutions methods, and possible variables affecting students' decisions were investigated.

Data Analysis

First step in data analysis was to tabulate test results as shown in Table 2 for each grade level. In creating this table, students' answers in proportional and non-proportional situations were categorized according to the method they used in each problem. If the answer and strategy used to solve the problem is correct then this cell is named as "Correct." In case of an incorrect answer, erroneous strategy used to solve the problem is written in the cell.

Table 2. Classifications of Students' Answers to the Problems

	CONSTANT		ADDITIVE		PROPORTIONAL (MISSING-VALUE)		PROPORTIONAL (COMPARISON)	
STUDENTS	I (Q6)	NI (Q9)	I (Q8)	NI (Q4)	I (Q10)	NI (Q2)	I (Q11)	NI (Q3)
1	Incorrect (P)	Incorrect (P)	Correct	Correct	Correct	Correct	Correct	Incorrect (No)
2	Incorrect (P)	Incorrect (P)	Incorrect (P)	Incorrect (P)	Correct	Correct	Correct	Correct
3	Incorrect (A)	Incorrect (A)	Correct	Correct	Correct	Correct	Correct	Correct
…								
221								

Note. Proportional Method (P), Additive Method (A), No Calculation (No), Integer (I), Non-integer (NI)

Three main categories for an incorrect answer were identified in the data: (a) additive strategy (14% of constant situations were answered by using an additive strategy, 6% of proportional missing-value problems were answered by using an additive strategy), (b) proportional strategy (42% of constant problems and 41% of additive problems were answered by using a proportional strategy), and (c) other.

Sample incorrect responses to some questions given by students are as follows:

Question 10: Emre and Sıla go to a book store to buy books at discount. All the books are at discount and their prices are the same. Emre buys 3 books while Sıla buys 8 books. If Emre pays 15 TL for the books he buys, how much money does Sıla have to pay for the books she buys?

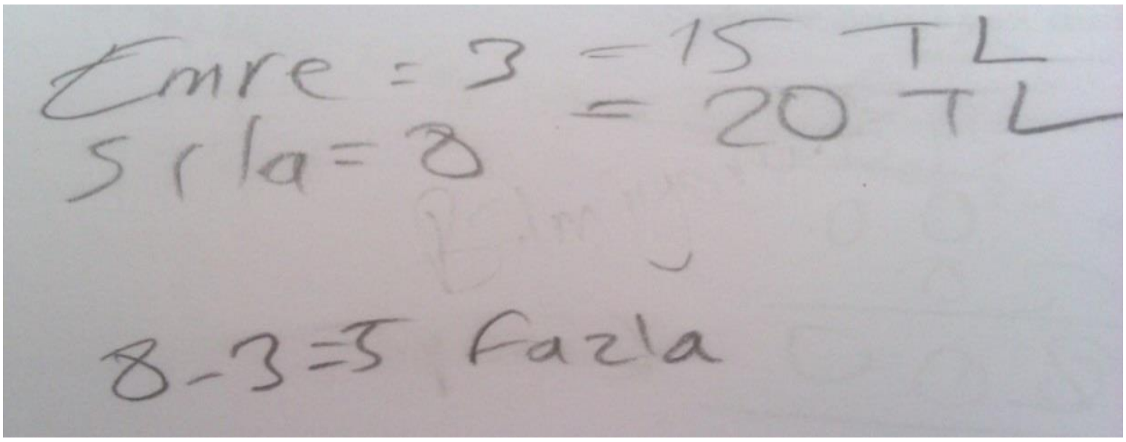

Figure 3. Incorrect use of additive strategy in a proportional problem

In Figure 3, the student calculates difference between number of books that Emre and Sıla buy. To find the price of Sıla's books, the student adds difference between numbers of books and price, which Emre pays for the books he buys. Students reason in a way that, Sıla has to pay 5 TL more than Emre since Sıla buys 5 more books than Emre, which means that one book was counted to cost 1 TL according to the student's explanation.

Question 6: Emre and Sıla go to a library to borrow some books. Emre borrows 8 books and Sıla borrows 16 books. The books must be returned to the library within 24 days. If Emre returns his books within 24 days, after how many days does Sıla have to return the books to the library?

An example of incorrect use of proportional method in a constant case is as follow:

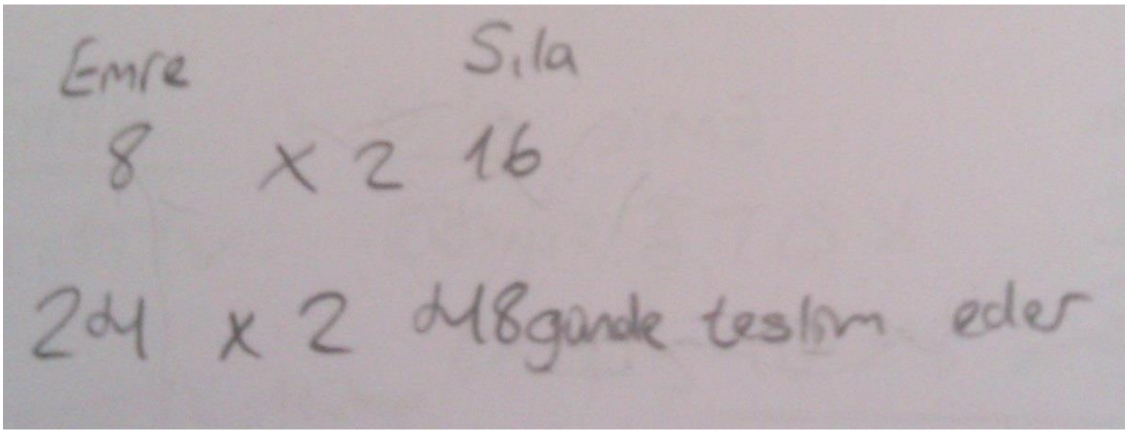

Figure 4. Incorrect use of proportional method in a constant problem.

In Figure 4, the student multiplies number of days by two, since number of the books is twice the other. Student reasons in this way; number of books that Sıla borrows from the library, is two times the number of books that Emre borrows. Since Emre has 24 days to return his books, Sıla has to return her books within 48 days, assuming that the quantities of number of books and number of days to return the books have a proportional relation.

After compiling data this way, analyses under each research question were conducted. Detailed explanations follow.

1. Can 5th grade students differentiate between the proportional situations (missing-value and comparison problems) and non-proportional situations (additive and constant situations)? In order to answer this question, the following questions will be examined: (a) Can 5th grade students solve proportional and non-proportional problems with the same success rate? (b) What is the distribution of students' erroneous strategies in each problem type?

In order to analyze how 5th grade students perform on proportional and non-proportional situations, one-way repeated measures ANOVA conducted. Students' scores were calculated by adding number of correct answers for integer and non-integer problem in each problem type. Repeated measures ANOVA allowed analyzing data when the same students take part in all situations of proportional and non-proportional situations. This method served the purpose of analyzing differences in students' success rates for each situation, and fulfilling the assumptions of sphericity (relationship between pairs of proportional and non-proportional problem scores must be similar), normality (success rate of students have a normal distribution), and randomness (scores between students are independent from each other) (Field, 2012). The assumption of sphericity could not be met and, therefore, the Greenhouse-Geisser correction has been applied.

In addition to this analysis that was based on the tabulation of students' answers as correct and incorrect, erroneous strategies were also categorized in four different groups, (a) additive, (b) constant, (c) proportional, and (d) other strategies. This tabulation provided a picture of overusing some strategies depending on problem type, which implied that students had difficulties on deciding appropriate solution strategy. Use of each erroneous method was calculated in percentages for each problem type and presented in a table. According to Fernandez et al. (2008), overuse of erroneous strategies revealed that students had problems in understanding the proportional relations, and difficulty in differentiating proportional and non-proportional situations.

2. Can 6th grade students differentiate between the proportional situations (missing-value and comparison problems) and non-proportional situations

(additive and constant situations)? In order to answer this question, we will examine the following questions: (a) Can 6th grade students solve proportional and non-proportional problems with the same success rate? (b) What is the distribution of students' erroneous strategies in each problem type?

Similarly, in order to analyze how 6th grade students perform on proportional and non-proportional situations, repeated measures of ANOVA was used. 6th grade students' scores were calculated by adding number of correct answers for integer and non-integer problem in each problem type. Since the assumption of sphericity could not be met, the Greenhouse-Geisser correction applied.

As in the above, erroneous strategies of the 6th graders were categorized in four different groups, (a) additive, (b) constant, (c) proportional, and (d) other. Use of each erroneous method was calculated as percentages for each problem type and presented in a table.

3. Does the use of integer and non-integer numbers in proportional problems (missing-value and comparison) and non-proportional problems (additive, and comparison) affect 5th grade students' solution strategies?

Cochran Q test was used to analyze whether there were significant differences in 5th grade students' answers while answering eight different problems. Cochran Q test is a repeated measure procedure that allows analysis of dichotomous data. However, Cochran Q test does not give information about problem pairs that caused the significant difference. To identify this, McNemar Test that enabled pairwise comparisons was used. In order to control Type I error rate, Bonferroni correction was applied when testing four pairwise comparisons (i.e., AI-ANI; CI-CNI; PMI-

PMNI; PCI-PCNI). In other words, significance level for pairwise comparisons was set to 0.013 (i.e., 0.05/4).

A further analysis regarding effect of numbers used in problems was also performed tabulating use of additive, proportional, constant, and other methods in each problem. Differences in the percentages of use of erroneous strategies were examined using Chi Square analyses.

4. Does the use of integer and non-integer numbers in proportional problems (missing-value and comparison) and non-proportional problems (additive, and comparison) affect 6^{th} grade students' solution strategies?

Cochran Q test was used to analyze whether there were significant differences in 6^{th} grade students’ answers while answering eight different problems. Cochran Q test is a repeated measures procedure that allows the analysis of dichotomous data. However, Cochran Q test analysis was followed by McNemar for identifying which problem pairs caused the significant difference. Significance level for pairwise comparisons was 0.013 (0.05/4) through Bonferroni correction.

A further analysis for analyzing effect of numbers used in the problems also performed by tabulating use of additive, proportional, constant, and other methods in each problem. Differences in percentages of use of erroneous strategies were examined by using Chi Square analyses.

5. What are possible reasons for erroneous methods while solving proportional and non-proportion problems in the 5^{th} and 6^{th} grades?

Related with quantitative phase of the study, qualitative data aimed to reveal possible reasons for 5^{th} and 6^{th} grade students’ use of erroneous strategies in proportional and non-proportional problems and their sensitivity to the number characteristics of problems. A widely used example of sequential quantitative and qualitative analysis is identification of group of individuals according to the quantitative data analysis. Qualitative data were collected after the identification of these groups for understanding possible factors, which led to their performance and strategy choice in proportional and non-proportionality problems, which varied according to numbers forming integer or non-integer ratios.

Quantitative analysis focused on students' success rate differences and choice of solution strategies depending on the number characteristics of problems and type of problem as additive, constant, PM, and PC. After determining general tendencies of the participants through analyzing quantitative data, qualitative data was collected for understanding possible reasons for the use of erroneous strategies thorough semi-structured interviews with some participants. After all 5th and 6th grade students' solution strategies were tabulated in an excel sheet, some patterns in their choice of strategies were searched. Since the study focuses on understanding reasons why students use erroneous strategies, students who consistently show the pattern of using erroneous strategies were chosen for interviews. Therefore, for understanding possible factors, which led to their performance and strategy choice in proportional and non-proportionality problems that varied in the number characteristics, five 5th grade and four 6th grade students were chosen from the sample group (Table 3). Students showed a tendency to change their strategy from an erroneous one to a correct one or change their strategy from a proportional to additive one when numbers changed from integer ratio to non-integer ratio and use of proportional method in all problems. These patterns were the most noticeable ones among 5th and 6th grade students.

Table 3. Interviewed Students' Solution Strategies in Each Problem Type

STUDENT	CONSTANT		ADDITIVE		PROPORTIONAL (MISSING-VALUE)		PROPORTIONAL (COMPARISON)	
	I (Q6)	NI (Q9)	I (Q8)	NI (Q4)	I (Q10)	NI (Q2)	I (Q11)	NI (Q3)
Ayça* (Grade 5)	Incorrect (P)	Incorrect (A)	Incorrect (P)	Correct	Correct	Correct	Correct	Correct
Arda (Grade 5)	Incorrect (P)	Incorrect (A)	Incorrect (P)	Correct	Incorrect (A)	Correct	Correct	Correct
Ali (Grade 5)	Incorrect (P)	Incorrect (A)	Incorrect (P)	Correct	Correct	Incorrect (O)	Incorrect (O)	Correct
Ata (Grade 5)	Incorrect (P)	Correct	Incorrect (P)	Correct	Incorrect (O)	Correct	Correct	Correct
Ayşe (Grade 5)	Incorrect (P)	Incorrect (P)	Incorrect (P)	Incorrect (P)	Correct	Correct	Correct	Correct
Ahmet (Grade 6)	Incorrect (P)	Incorrect (A)	Incorrect (P)	Correct	Correct	Correct	Correct	Correct
Akın (Grade 6)	Incorrect (P)	Correct	Incorrect (P)	Correct	Incorrect (O)	Incorrect (O)	Correct	Incorrect (O)
Atakan (Grade 6)	Incorrect (P)	Correct	Incorrect (P)	Incorrect (O)	Correct	Correct	Correct	Correct
Asım (Grade 6)	Incorrect (P)	Incorrect (A)	Incorrect (P)	Correct	Correct	Correct	Incorrect (O)	Correct

Note. Pseudo names were used in the study to indicate the participants in the interviews (*) Proportional Method (P), Additive Method (A), Other Method (O), Integer (I), Non-integer (NI)

Firstly, interviews focused on reasons why students changed their solution strategy when only numbers in the problems differed in non-proportional problems (additive and constant). Students' own written answers were given to each participant and they were kindly requested to answer following questions: (1) Do you agree on your solution method? (2) In what ways, these two problems (question 6 and question 9 were compared and question 4 and question 8 were compared) are different from each other? (3) In what ways, these two solution strategies are different from each other? (4) Why did you use two different solution strategies in these two problems, although you state that these problems are the same?

Interviews also focused on the pattern in which students use proportional strategy when it is inappropriate to use. Similarly, student's own written answers were given and the student was kindly requested to answer the following questions: (1) Do you agree on your solution method? (2) In what ways, these two problems (question 10 and question 2 were compared and question 11 and question 3 were compared) are similar to each other? (3) In what ways, these two problems are different from each other? (4) Why did you use the same strategy in these two problems, although you state that these problems are not the same? During these interviews, students were encouraged to explain their way of reasoning, process of deciding solution methods, and possible variables affecting students' decisions were investigated.

Although aim of the study was to reveal students' understanding about proportional and non-proportional situations, their understanding about "ratio and proportion" was not asked to students directly during the interviews. In case of a students' use of word "ratio" or "proportion" while explaining something, their aim of using this word was asked as an additional question.

Each interview was audiotaped, transcribed, and analyzed in order to provide detailed information about students' reasoning and problem solving processes on proportional and non-proportional situations. Qualitative analysis began with reading all transcripts, which were interview transcripts of nine students, several times with looking for details and a general view of the entire database. Then, text was divided into small units of phrases and sentences for the purpose of coding data. The researcher assigned code words to text segments, and looked for themes. Themes were meaningful combinations of codes, and they were assigned directly on printed transcripts.

CHAPTER IV

RESULTS

The following sections present findings under each research question.

Research Question 1: Can 5th grade students differentiate between the proportional situations and non-proportional situations (additive and constant situations)? In order to answer this question, the following questions will be examined: (a) Can 5th grade students solve proportional and non-proportional problems with the same success rate? (b) What is the distribution of students' erroneous strategies in each problem type?

In order to answer this question, following question will be examined first: Can 5th grade students solve proportional and non-proportional problems with the same success rate? One-way repeated measures ANOVA was used to analyze the data. 5th grade students' scores for each type of problem were calculated by adding students' scores in integer and non-integer problems.

From the Table 4, we can see that, on average, the 5th graders solved constant problems with the lowest success rate and proportional missing-value problems (PM) with the highest success rate.

Table 4. SPSS Output for Descriptive Statistics in the Fifth Grade

	Mean	Std. Deviation	N
Constant	,7833	,92748	120
Additive	1,2417	,86962	120
Proportional Missing	1,7167	,61060	120
Proportional Comparison	1,7000	,66862	120

Mauchly's test indicated that the assumption of sphericity had been violated, $X^2(5) = 57.870$, $p < .001$, therefore degrees of freedom were corrected using Greenhouse-Geisser estimatation of sphericity ($\varepsilon = .82$) (Table 5). Considering Greenhouse-Geisser correction, repeated measures ANOVA test showed that there was a significant effect of the problem type on the performance of the 5th grade students, $F = 41.761$, $df = 2.5$, $p < .01$ (Table 6).

Table 5. Mauchly's Test of Sphericity

Within Subjects Effect	Mauchly's W	Approx. Chi-Square	Df	Sig.	Epsilon		
					Greenhouse-Geisser	Huynh-Feldt	Lower-bound
Problem	,612	57,870	5	,000	,821	,840	,333

Table 6. Tests of Within-Subject Effects

Source		Type III Sum of Squares	Df	Mean Square	F	Sig.	Partial Eta Squared
Problem	Sphericity Assumed	70,723	3	23,574	41,761	,000	,260
	Greenhouse-Geisser	70,723	2,463	28,718	41,761	,000	,260
	Huynh-Feldt	70,723	2,519	28,079	41,761	,000	,260
	Lower-bound	70,723	1,000	70,723	41,761	,000	,260
Error (Problem)	Sphericity Assumed	201,527	357	,565			
	Greenhouse-Geisser	201,527	293,060	,688			
	Huynh-Feldt	201,527	299,725	,672			
	Lower-bound	201,527	119,000	1,694			

Since there are six different comparisons (Constant-Additive, Constant-PM, Constant-PC, Additive-PM, Additive-PC, PM-PC), Bonferroni correction was needed to control Type I error rate. Bonferroni correction changed significance level to 0.008 for testing comparisons (through dividing significance level by number of comparisons, which is six).

Examining pairwise comparisons (Table 7), following post-hoc contrasts have been found significant: success in constant problems was significantly different than additive ($p < .001$), proportional missing-value ($p < .001$), and proportional comparison problems ($p < .001$). Success on additive problems showed a significant difference when compared with proportional missing-value problems, and proportional comparison problems. A significant difference was not observed between proportional comparison problems and proportional missing-value problems. To sum up, these differences show that 5th grade students' scores between proportional and non-proportional problems and within non-proportional problems significantly differed.

Table 7. Pairwise Comparisons of Repeated Measures of ANOVA

(I) Problems	(J) Problems	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval for Difference	
					Lower Bound	Upper Bound
Constant	Additive	-,458*	,108	,000	-,747	-,169
	PM	-,933*	,097	,000	-1,193	-,674
	PC	-,917*	,103	,000	-1,193	-,641
Additive	Constant	,458*	,108	,000	,169	,747
	PM	-,475*	,102	,000	-,750	-,200
	PC	-,458*	,105	,000	-,740	-,176
PM	Constant	,933*	,097	,000	,674	1,193
	Additive	,475*	,102	,000	,200	,750
	PC	,017	,058	1,000	-,139	,172
PC	Constant	,917*	,103	,000	,641	1,193
	Additive	,458*	,105	,000	,176	,740
	PM	-,017	,058	1,000	-,172	,139

Note. Proportional Missing-value (PM), Proportional Comparison (PC)

Table 8. Effect Size between Proportional and Non-proportional Situations in Fifth Grade

	Proportional Situations	Non-proportional Situations
Mean	3,42	2,03
Standard Deviations	1,11	1,37

Effect size for this analysis (d = 1) was found. This value of d indicated a large effect size of types of situations as proportional and non-proportional.

In addition to studying students' performance in solving proportional and non-proportional problems, further analysis was performed at solution strategy level. 5^{th} grade students' solution methods were determined in each type of problem. Analysis of solutions methods allowed examining students' ability to use each solution strategy in appropriate situation, which indicated students' performance to differentiate proportional and non-proportional relations, and decide appropriate solution method for each type of problem. Table 8 was formed to represent students' choice of the solution strategies including percentages of the use of multiplicative, additive, constant, and other solution strategies in each problem type. As can be seen from the table, in constant problems, the 5^{th} graders had a tendency to overuse additive method (18% of the answers included additive method) and proportional method (34% of the answers included proportional method). Also, 31% of the answers included proportional method in additive problems. Although there appears a tendency to use proportional method in non-proportional situations (constant and additive problems), proportional situations elicited less overuse of additive method (8% of the answers included additive method in proportional missing-value problems, and 1% of the answers included additive method in proportional comparison problems) (Table 8).

Table 8. Fifth Grade Students' Use of Each Solution Method in Each Problem

	Constant				Additive				Proportional Missing-value				Proportional Comparison			
Method	P	A	C	O	P	A	C	O	P	A	C	O	P	A	C	O
Use in %	36.3	17.9	39.2	6.7	32.5	62.1	0	5.4	85.8	7.9	0	6.3	85.0	0.8	0	14.2

Note: Proportional Method (P), Additive Method (A), Constant Method (C), Other (O)

Research Question 2: Can 6th grade students differentiate between the proportional situations and non-proportional situations (additive and constant situations)? In order to answer this question, the following questions will be examined: (a) Can 6th grade students solve proportional and non-proportional problems with the same success rate? (b) What is the distribution of students' erroneous strategies in each problem type?

In order to answer this question, similar with the analysis of 5th grade students' data, following question is examined first: Can 6th students solve proportional and non-proportional problems with the same success rate? One-way repeated measures ANOVA was used to analyze data. 6th grade students' scores for each type of problem were calculated by adding students' scores in integer and non-integer problems.

From Table 9, we can see that, on average, 6th grade students solved constant problems with the lowest success rate and proportional missing-value problems (PM) with the highest success rate.

Table 9. SPSS Output for Descriptive Statistics

	Mean	Std. Deviation	N
Constant	,7426	,94502	101
Additive	,9109	,87291	101
ProportionalMissing	1,8515	,45577	101
ProportionalComparison	1,8317	,44876	101

Mauchly's test indicated that the assumption of sphericity had been violated, $X^2(5) = 65.391$, $p < .001$, therefore degrees of freedom were corrected using Greenhouse-Geisser estimation of sphericity ($\varepsilon = .79$) (Table 10). Considering Greenhouse-Geisser correction, repeated measures ANOVA test showed that there was a significant effect of problem type on the performance of 6th grade students, $F = 76.276$, $df = 2.4$, $p < .01$ (Table 11).

Table 10. Mauchly's Test of Sphericity

Within Subjects Effect	Mauchly's W	Approx. Chi-Square	Df	Sig.	Epsilon[b]		
					Greenhouse-Geisser	Huynh-Feldt	Lower-bound
Problem	,516	65,391	5	,000	,792	,813	,333

Table 11. Repeated Measures of ANOVA effect analysis

Source		Type III Sum of Squares	Df	Mean Square	F	Sig.	Partial Eta Squared
Problem	Sphericity Assumed	105,473	3	35,158	76,276	,000	,433
	Greenhouse-Geisser	105,473	2,377	44,366	76,276	,000	,433
	Huynh-Feldt	105,473	2,439	43,243	76,276	,000	,433
	Lower-bound	105,473	1,000	105,473	76,276	,000	,433
Error (Problem)	Sphericity Assumed	138,277	300	,461			
	Greenhouse-Geisser	138,277	237,731	,582			
	Huynh-Feldt	138,277	243,907	,567			
	Lower-bound	138,277	100,000	1,383			

Since we had six different comparisons, Bonferroni correction was applied to control Type I error. Bonferroni correction changed significance level to 0.008 for testing comparisons.

Examining pairwise comparisons (Table 12), following post-hoc contrasts have been found significant: success in constant problems was significantly different than proportional missing-value ($p < .001$), and proportional comparison problems ($p < .001$). Success rate on additive problems showed a significant difference when compared with proportional missing-value problems, and proportional comparison problems. A significant difference was not observed between non-proportional situations that are additive and constant problems and proportional situations that are proportional comparison and proportional missing-value problems. To sum up, these differences implied that 6^{th} grade students' scores were significantly different between proportional and non-proportional problems.

Table 12. Pairwise Comparisons of Repeated Measures of ANOVA

(I) Problem	(J) Problem	Mean Difference (I-J)	Std. Error	Sig.[b]	95% Confidence Interval for Difference[b]	
					Lower Bound	Upper Bound
Constant	Additive	-,168	,110	,774	-,464	,128
	PM	-1,109	,103	,000	-1,387	-,831
	PC	-1,089	,100	,000	-1,357	-,821
Additive	Constant	,168	,110	,774	-,128	,464
	PM	-,941	,100	,000	-1,211	-,671
	PC	-,921	,097	,000	-1,182	-,659
PM	Constant	1,109	,103	,000	,831	1,387
	Additive	,941	,100	,000	,671	1,211
	PC	,020	,051	1,000	-,117	,156
PC	Constant	1,089	,100	,000	,821	1,357
	Additive	,921	,097	,000	,659	1,182
	PM	-,020	,051	1,000	-,156	,117

b. Adjustment for multiple comparisons: Bonferroni.
Note. Proportional Missing-value (PM), Proportional Comparison (PC)

Analysis of solution methods enabled us to examine students' ability to differentiate additive, constant, and proportional relations and to use appropriate method for each individual relation in the problems, which has an implication of students' difficulties in differentiating proportional and non-proportional problems.

Table 8. Effect Size Sixth Graders

	Proportional Situations	Non-proportional Situations
Mean	3.68	1.62
Standard Deviations	0.77	1.45

Effect size for this analysis (d = 2.8) was found. This value of d indicated a large effect size of types of situations as proportional and non-proportional.

Table 13 was formed to represent students' choice of solution strategies including percentages of multiplicative, additive, constant, and other solution strategies in each problem type. As can be seen from the table, 6th graders had a tendency to use proportional method in inappropriate situations as 49.5% of the answers included the use of proportional method in constant problems and 53.0% of the answers included proportional method in additive problems. Although there was a tendency to use proportional method (higher than the 5th graders) in non-proportional situations (constant and additive problems), proportional situations elicited less overuse of additive method (3.0% of the answers included additive method in proportional missing-value problems, and 0.5% of the answers included additive method in proportional comparison problems) (Table 13).

Table 13. Sixth Grade Students' Use of Each Solution Method in Each Problem

	Constant				Additive				Proportional Missing-value				Proportional Comparison			
Method	P	A	C	O	P	A	C	O	P	A	C	O	P	A	C	O
Use in %	49.5	9.9	36.6	4.0	53.0	45.5	0.0	3.5	92.6	3.0	0.0	4.5	91.1	0.5	0.0	8.4

Note. Proportional Method (P), Additive Method (A), Constant Method (C), Other (O)

Research Question 3: Does the use of integer and non-integer numbers in proportional problems (missing-value and comparison) and non-proportional problems (additive, and comparison) affect 5th grade students' solution strategies?

Cochran Q test was used for examining students' success in eight problems. Null hypothesis for Cochran's Q test is that there are no differences in 5th graders' scores among eight types of problems (Table 14).

Table 14. Cochran Q Test for the Fifth Grade Students

Frequencies		
	Value	
	Incorrect	Correct
Constant Integer	74	46
Constant Non-integer	72	48
Additive Integer	52	68
Additive Non-integer	39	81
Proportional Missing Integer	18	102
Proportional Missing Non-integer	16	104
Proportional Comparison Integer	15	105
Proportional Comparison Non-integer	21	99

Test Statistics	
N	120
Cochran's Q	184,961
Df	7
Asymp. Sig.	,000

By using Cochran Q test, we found that there exists a significant difference (X_2 (7) = 184.961, p < .001) in 5th grade students' scores among eight different problems. As a reminder, these problems differed along two dimensions: four kinds of problems as constant, additive, proportional missing-value and proportional comparison and use of integer and non-integer numbers in each type of problem.

Cochran Q test indicated that there was a significant difference in students' answers to eight problems. However, Cochran Q test does not give information about which problem caused the difference. To identify this, McNemar Test, which enables pairwise comparisons, was used. In order to control Type I error rate, Bonferroni correction was applied when testing four pairwise comparisons (i.e., AI-ANI; CI-CNI; PMI-PMNI; PCI-PCNI). In other words, significance level for pairwise comparisons was 0.013 (0.05/4).

A pairwise comparison by using McNemar's tests revealed that students' success on additive problems were significantly affected by numbers in the problem, in grade 5 (p = .011 < .013). However, types of numbers did not result significant differences in 5^{th} grade students' correct solutions in constant problems, proportional missing-value problems, and proportional comparison problems.

Table 15. McNemar Pairwise Comparison of the Success Rates in Additive Problems

Additive Integer * Additive Non-Integer Crosstabulation

			Additive Non-integer		Total
			Incorrect	Correct	
Additive Integer	Incorrect	Count	34	18	52
		Expected Count	16,9	35,1	52,0
	Correct	Count	5	63	68
		Expected Count	22,1	45,9	68,0
Total		Count	39	81	120
		Expected Count	39,0	81,0	120,0

Chi-Square Tests

	Value	Exact Sig. (2-sided)
McNemar Test		,011
N of Valid Cases	120	

After investigating 5^{th} grade students' success rates depending on number characteristics of the problems, a further analysis on erroneous strategies was conducted. When percentages of use of each strategy were calculated depending on number characteristics of the problem, numbers showed an effect on 5^{th} grade students' choice of erroneous methods. Percentages of the use of each strategy in problems that differed in the use of an integer ratio number and non-integer ratio number were presented in Table 16.

Table 16. Overview of Solutions Strategies Used by the Fifth Grade Students (in %)

	Constant Problems				Additive Problems				Proportional Missing-Value Problems				Proportional Comparison Problems			
Method	P	A	C	O	P	A	C	O	P	A	C	O	P	A	C	O
I-P	51.67	8.33	38.33	1.67	41.67	56.67	0.00	1.67	85.00	9.17	0.00	5.83	87.50	0.83	0.00	11.67
NI-P	20.83	27.50	40.00	11.67	23.33	67.50	0.00	9.17	86.67	6.67	0.00	6.67	82.50	0.83	0.00	16.67

Note. Proportional Strategy (P), Additive Strategy (A), Constant Strategy (C), Other Strategies (O), Integer Problems (I-P), Non-integer Problems (NI-P)

When the Table 16 was examined, some points drew attention. First of all, there was a significant change in the overuse of additive methods in constant problems when numbers changed from integer ratio to non-integer ratio (X^2 (1) = 14.988, $p < .001$) (Table 17) (from 8.33% to 27.50%).

Table 17. Overuse of Additive Method in Constant Problems Depending on the Numbers

Numbers * AdditiveMETHODuse Crosstabulation

Count		AdditiveMETHODuse		Total
		USED	NOTUSED	
Numbers	INTEGER	10	110	120
	NON-INTEGER	33	87	120
Total		43	197	240

Chi-Square Tests

	Value	Df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	14,988	1	,000		
Continuity Correction	13,713	1	,000		
Likelihood Ratio	15,660	1	,000		
Fisher's Exact Test				,000	,000
Linear-by-Linear Association	14,925	1	,000		
N of Valid Cases	240				

Another point, which drew attention from Table 16 was that there was a decrease in the use of proportional methods in constant problems when the numbers in the problem changed from integer ratio to non-integer ratio (X^2 (1) = 24.683, $p < .001$) (Table 18) (from 51.67% to 20.83%).

Table 18. Overuse of Proportional Method in Constant Problems Depending on the Numbers

Numbers * proportionalMETHODuse Crosstabulation

Count

		proportionalMETHODuse		Total
		USED	NOTUSED	
Numbers	INTEGER	62	58	120
	NON-INTEGER	25	95	120
Total		87	153	240

Chi-Square Tests

	Value	Df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	24,683	1	,000		
Continuity Correction	23,367	1	,000		
Likelihood Ratio	25,285	1	,000		
Fisher's Exact Test				,000	,000
Linear-by-Linear Association	24,580	1	,000		
N of Valid Cases	240				

Therefore, in constant problems when problems included integer ratios 5th graders tended to use proportional methods, and when problems included non-integer ratios they tended to prefer additive methods.

In additive problems, when numbers changed from integer ratio to non-integer ratio, there was a significant decrease in the use of proportional methods (X^2 (1) = 9.729, $p < .01$) (Table 19) (from 41.67% to 23.33%).

Table 19. Overuse of Proportional Method in Additive Problems Depending on the Numbers

Numbers * proportionalMETHODuse Crosstabulation

Count

		proportionalMETHODuse		Total
		USED	NOTUSED	
Numbers	INTEGER	50	70	120
	NON-INTEGER	28	92	120
Total		78	162	240

Chi-Square Tests

	Value	Df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	9,193	1	,002		
Continuity Correction	8,376	1	,004		
Likelihood Ratio	9,287	1	,002		
Fisher's Exact Test				,004	,002
Linear-by-Linear Association	9,154	1	,002		
N of Valid Cases	240				

Although numbers used in the problems resulted in a significant difference in the overuse of erroneous methods in non-proportional situations, a similar statistically significant difference was not observed in proportional situations. Although an increase in the overuse of additive strategies as an erroneous strategy when numbers were changed from integer ratio to non-integer ratio was expected, data did not indicate a significant difference as expected.

In additive problems, when numbers changed from integer ratio to non-integer ratio, there was a significant decrease in the use of proportional methods (X^2 (1) = 9.729, $p < .01$) (Table 19) (from 41.67% to 23.33%).

Table 20. Other Method Use in Constant Problems Depending on the Numbers

Numbers * OtherMETHODuse Crosstabulation

Count

		otherMETHODuse		Total
		USED	NOTUSED	
Numbers	INTEGER	2	118	120
	NON-INTEGER	14	106	120
Total		16	224	240

Chi-Square Tests

	Value	Df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	9,643	1	,002		
Continuity Correction	8,103	1	,004		
Likelihood Ratio	10,767	1	,001		
Fisher's Exact Test				,003	,002
Linear-by-Linear Association	9,603	1	,002		
N of Valid Cases	240				

Research Question 4: Does the use of integer and non-integer numbers in proportional problems (missing-value and comparison) and non-proportional problems (additive, and comparison) affect 6th grade students' solution strategies?

Null hypothesis for Cochran's Q test is that there are no differences in 6th graders' performance in the eight types of problems (Table 20). According to Cochran Q test, there was a significant difference in 6th grade students' success among eight different problems (Cochran Q = 268.871, $p < .001$).

Table 21. Cochran Q Test for the Sixth Graders

Frequencies

	Value	
	Incorrect	Correct
Constant / Integer	65	36
Constant / Non-integer	62	39
Additive / Integer	65	36
Additive / Non-integer	45	56
Proportional Missing-value / Integer	5	96
Proportional Missing-value / Non-integer	10	91
Proportional Comparison / Integer	4	97
Proportional Comparison / Non-integer	13	88

Test Statistics

N	101
Cochran's Q	268,871[a]
Df	7
Asymp. Sig.	,000

a. 0 is treated as a success.

Cochran Q test indicated that there was a significant difference in students' answers to the eight problems. McNemar Tests were used with Bonferroni correction to test pairwise comparisons (i.e., AI-ANI; CI-CNI; PMI-PMNI; PCI-PCNI). Similarly, significance level for pairwise comparisons was 0.013 (i.e., 0.05/4).

A pairwise comparison between additive questions with integer and non-integer numbers using McNemar tests revealed that students' success on additive problems were significantly affected by numbers in the problems ($p < 0.001$).

Table 22. McNemar Test Comparing Additive Integer and Additive Non-integer Problems

Additive Integer * Additive Non-Integer Crosstabulation

			Additive Non-integer		Total
			Incorrect	Correct	
Additive Integer	Incorrect	Count	43	22	65
		Expected Count	29,0	36,0	65,0
	Correct	Count	2	34	36
		Expected Count	16,0	20,0	36,0
Total		Count	45	56	101
		Expected Count	45,0	56,0	101,0

Chi-Square Tests

	Value	Exact Sig. (2-sided)
McNemar Test		,000
N of Valid Cases	101	

a. Binomial distribution used.

In addition, pairwise comparison of proportional comparison problems with integer numbers and with non-integer numbers revealed a significant difference in students' performance in the 6th grade ($p < .013$).

Table 23. McNemar Test Proportional Comparison Problems Success Rate and Number Effect

Proportional Comparison Integer * Proportional Comparison Non-integer Crosstabulation

			Proportional Comparison Non-integer		Total
			Incorrect	Correct	
Proportional Comparison Integer	Incorrect	Count	3	1	4
		Expected Count	,5	3,5	4,0
	Correct	Count	10	87	97
		Expected Count	12,5	84,5	97,0
Total		Count	13	88	101
		Expected Count	13,0	88,0	101,0

Chi-Square Tests		
	Value	Exact Sig. (2-sided)
McNemar Test		,012
N of Valid Cases	101	

However, analysis also indicated that number variable did not have a significant effect on 6th grade students' performance in constant problems, and proportional missing-value problems.

In addition to success rate analysis of number effect, there was a need to analyze erroneous methods since analysis of erroneous methods enabled us to examine in what conditions students used proportional method in non-proportional situations and used additive strategy in the proportional or constant problems. To analyze students' use of the strategies, Table 23 was formed, representing percentages of the use of multiplicative, additive, constant, and other methods in constant, additive, proportional missing-value, and proportional comparison problems depending on number structure of the problem. The percentages of the use of each solution method in each problem type depending on the number characteristics of the problems were presented in Table 4.19.

Table 24. Overview of Solutions Given by Sixth Grade Students (in %)

	Constant Problems				Additive Problems				Proportional Missing-Value Problems				Proportional Comparison Problems			
Method	P	A	C	O	P	A	C	O	P	A	C	O	P	A	C	O
I-P	59.41	2.97	35.64	1.98	63.37	35.64	0.00	0.99	95.05	1.98	0.00	2.97	96.04	0.00	0.00	3.96
NI-P	39.60	16.83	37.62	5.94	38.61	55.45	0.00	5.94	90.10	3.96	0.00	5.94	86.14	0.99	0.00	12.87

Note. Proportional Solution (P), Additive Solution (A), Constant Solution (C), Other Solutions (O), Integer Problems (I-P), Non-integer Problems (NI-P)

When Table 23 was examined, some patterns were noticeable. First of all, there was a significant increase in the overuse of additive methods in constant problems when numbers in the problem changed from integer ratio to non-integer (X^2 (1) = 10.877, p = .001) (Table 24) (from 2.97% to 16.83%).

Table 25. Overuse of Additive Method in Constant Problems Depending on the Numbers

Numbers * AdditiveMETHODuse Crosstabulation

Count

		AdditiveMETHODuse		Total
		USED	NOTUSED	
Numbers	INTEGER	3	98	101
	NON-INTEGER	17	84	101
Total		20	182	202

Chi-Square Tests

	Value	Df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	10,877	1	,001		
Continuity Correction	9,379	1	,002		
Likelihood Ratio	11,896	1	,001		
Fisher's Exact Test				,002	,001
Linear-by-Linear Association	10,823	1	,001		
N of Valid Cases	202				

Another noticeable point is that there was a decrease in the use of proportional methods in constant problems when numbers in the problem changed from integer ratio to non-integer ratio (X^2 (1) = 7.922, p = .004) (Table 25) (from 59.41% to 39.60%).

Table 26. Overuse of Proportional Method in Constant Problems Depending on the Numbers

Numbers * ProportionalMETHODuse Crosstabulation

Count

		ProportionalMETHODuse		Total
		USED	NOTUSED	
Numbers	INTEGER	60	41	101
	NON-INTEGER	40	61	101
Total		100	102	202

Chi-Square Tests

	Value	Df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	7,922	1	,005		
Continuity Correction	7,149	1	,007		
Likelihood Ratio	7,974	1	,005		
Fisher's Exact Test				,007	,004
Linear-by-Linear Association	7,882	1	,005		
N of Valid Cases	202				

In additive problems, when numbers changed from integer ratio to non-integer ratio, there was a significant decrease in the overuse of proportional methods (X^2 (1) = 12.381, $p < .001$) (Table 26) (from 63.37% to 38.61%).

Table 27. Overuse of Proportional Method in Additive Problems Depending on the Numbers

Numbers * proportionalMETHODuse Crosstabulation

Count

		proportionalMETHODuse		Total
		USED	NOTUSED	
Numbers	INTEGER	64	37	101
	NON-INTEGER	39	62	101
Total		103	99	202

Chi-Square Tests

	Value	Df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	12,381	1	,000		
Continuity Correction	11,410	1	,001		
Likelihood Ratio	12,511	1	,000		
Fisher's Exact Test				,001	,000
Linear-by-Linear Association	12,320	1	,000		
N of Valid Cases	202				

Although type of numbers resulted in a significant difference in the overuse of erroneous methods in non-proportional situations, a similar difference was not observed in proportional situations. When numbers were changed from integer ratio to non-integer ratio, an increase in the overuse of additive strategies as an erroneous strategy in proportional cases was expected. However, data did not indicate this difference. These findings were in parallel with the findings of 5th grade students' data.

Research Question 5: *What are possible reasons for erroneous methods while solving proportional and non-proportion problems in the 5th and 6th grades?*

In accordance with quantitative results, 5th grade and 6th grade students showed similar tendencies in differentiating proportional and non-proportional situations and use of erroneous strategies depending to the number change in the problems. Therefore, last research question explained quantitative results and possible reasons for these results for 5th and 6th grade students together.

During the interviews, students' reflection on their written answers on proportional (missing-value and comparison problems) and non-proportional (additive and constant problems) problems, their explanation about their solution method, and understanding about proportional and non-proportional situations brought to light two themes.

First theme was students' lack of awareness of proportional relation in the building-up strategy and confusion between the use of additive strategy and building-up strategy. Three students out of nine stated that addition operation maintains proportional relation among quantities. However, use of addition operation was used as increasing or decreasing the quantities by the same number that cannot maintain the proportional relation. Ahmet's case is representative of this situation.

Ahmet, a 6th grade student, was reviewing his written answer on constant problem in which numbers form non-integer ratios. He solved the problem by using an additive strategy while he solved the same problem by using a proportional strategy when only the numbers in the problem differed as forming integer ratios. He was comparing his solution strategies and he stated that use of proportional strategy was more appropriate instead of additive strategy.

> R: Why do you think that you did solve the problem by using a proportional strategy when you first read the problem?
>
> Ahmet: Before I learnt about ratio and proportion this year, when I first learnt about them I used this. You take two from one quantity and two from the other one. Or you add two to one quantity and two to the other one. But, now since I learnt about ratio and proportion this year (the use of cross-multiplication), I used this method.
>
> R: What do you mean with "ratio and proportion"?
>
> Ahmet: Increasing or decreasing by the same ratio…
>
> R: How can you do this "increasing" or "decreasing"?
>
> Ahmet: You know… By making calculations.
>
> R: What kind of calculations are these?

> Ahmet: You multiply two numbers and divide the result by another number (see Appendix A.1.).

In this conversation, attention grabbing point is that Ahmet considers additive strategy and building-up strategy in the same way. He states that; “You take two from one quantity and two from the other one. Or you add two to one quantity and two to the other one” to explain increasing or decreasing the quantities by the same ratio. However, increasing quantities by the same number cannot maintain proportional relation between quantities. Since students use addition operation in building-up strategy, Ahmet showed an understanding that use of addition operation was enough for maintaining proportional relation.

Second theme was about students’ decision of the solution strategy without a full understanding of the problem. This theme was formed by three subthemes that were students’ decision of the solution strategy without a full understanding of the problem, students’ use of numbers in the problem as a superficial clue, and their belief about mathematic problems.

Seven students, out of nine, used numbers in the problem as superficial clues while explaining their way of thinking. Focus on the numerical characteristics of the problem affected students’ decision process and the choice of the solution strategy.

For example, a 5^{th} grade student, Ali, was trying to explain how he decided the solution steps in constant problem with integer numbers. Although he did not need to make any calculations to give the correct answer he used multiplicative method.

> B: Why did you think that it will be two times 24?

> Ali: Because… 12 is two times 6… So I found the answer as 48 (see Appendix A.2.).

Ali used the relation between the numbers as a reason for using particular strategy while solving the problem. In addition to this, Atakan, a sixth grader, had his written answer for the constant problem, in which numbers formed integer ratio. He solved the problem by using proportional method. When the reason why he solved the problem by using this method was asked, he explained:

> Atakan: If one has 24 days to return 16 books. 8 is …. No, not 8, 16 is two times 8. So, when I consider 24 days. I had to multiply by two, but it is not correct I guess. I had to divide instead of multiplying by two. The answer is 12, maybe (see Appendix A.3.).

Atakan had problems in explaining the reason of using some particular calculations. His focus was on the relation among the numbers, instead of the situation in the problem and relation among quantities. Therefore, he was confused about dividing or multiplying number of days by two, since both of them was possible when only multiplicative relation among the numbers were considered.

Another subtheme that supported students' insufficient understandings of the problem situation, was students' difficulty in explaining their way of thinking. Eight students, out of nine, showed difficulties in explaining their way of thinking in a convincing way. Two representative cases from each grade level are as following.

A 5th grade student, Arda, was reviewing his written answer on the additive problem in which numbers form non-integer ratios. He solved the problem by using correct solution strategy while he solved the same problem by using proportional strategy when only the numbers were changed from non-integer ratio to integer ratio. Arda was asked to compare these two additive problems and his solution strategies.

> Arda: My solution strategy in the 4th question seems to make more sense.

R: Can you explain, why?

Arda: Because… I do not know. It seems to be correct (see Appendix A.4.).

A 6th grade student, Akın was reviewing his written answer on constant problem in which numbers formed non-integer ratios. He solved the problem correctly. His explanation about the reason why he solved problem in this way was as following.

R: Why do you think you solved the problem in this written way?

Akın: When I solved the problem, I read the rest of the problem very quickly after I saw 25 days. I just wrote the answer as 25 days, then. Maybe… I do not know (see Appendix A.5.).

Although Akın solved the problem correctly, he could not give a convincing explanation about how he decided this solution strategy, and the reason why this method was correct. Instead of this, he was suspicious about the correctness of his way of thinking. When he solved the problem once more, he did not use correct strategy that was based on the constant relation among the quantities in the problem.

Lastly, there was another subtheme that supports a better understanding of students' use of erroneous strategies. This subtheme was students' beliefs about mathematical problems. During the interviews, two students (5th graders), out of nine, refused to state constant relation as a solution strategy, although they understood the relation among the quantities. This situation also pointed out students' views of about mathematics as a game that always required calculations.

A 5th grade student, Arda, was looking over his previous written answer in constant problem, in which numbers in the problem formed non-integer ratios. Arda solved problem by using an additive strategy. During the interview, he noticed constant relation and then he said:

Arda: Ok. I have an idea but it is not related with mathematics.

R: Just tell.

Arda: You know, at beginning of the question, it says that the books that were borrowed from the library must be turned back after 25 days. Maybe the answer is (giggling) 25…

R: Why do you think that this answer is not related with mathematics?

Arda: I don't know. It is much more a kind of logic. It sounds like incorrect.

R: Why, do not you think that mathematics includes logic and reasoning also?

Arda: Maybe includes a little, but it includes calculations in fact (see Appendix A.6.).

Arda noticed the constant relation among the quantities in the problem; however, he experienced a tension between his expectation from a mathematical problem (that mathematical problems always require calculations) and relation among the quantities, thus he changed his mind to use a constant relation among the quantities as a solution strategy.

CHAPTER V

DISCUSSION

In light of the current research on proportional and non-proportional reasoning, this section explores possible explanations for the participants' performances and use of erroneous strategies in proportional and non-proportional problems, and possible reasons for the difficulties students experience in problem solving process. As a reminder, the problems differed in terms of having an integer ratio or non-integer ratio and problem type (additive, constant, proportional missing-value, and proportional comparison problems). The study used an explanatory mixed method design to investigate students' understanding on proportional and non-proportional situations, variables affecting students' performance in differentiating proportional and non-proportional situations and possible reasons for the difficulty that students experience.

Review of Findings

First research question of the study was formed, considering the primary goal of the study that was to determine students' skills in differentiating the proportional and non-proportional situations and using the appropriate solution strategy in each situation. One-way repeated measures ANOVA revealed that there was a significant effect of the problem type on the performance of the 5^{th} and 6^{th} grade students. The lowest success rate was in constant problems and PM problems were solved with the highest success rate in both grade levels. This finding was in congruence with previous studies, which revealed that students were more successful in proportional tasks compared with the non-proportional tasks (additive and constant problems) (Fernandez et al., 2008; Van Dooren et al., 2007). In order to solve a constant problem, it is enough to report a number in the problem as an answer in case of realizing the constant relation among the quantities. However, constant problems elicited the least success rate when compared with the other problem types. For understanding the possible reasons for the low success in constant problems, the qualitative data analysis will be discussed in the following pages.

When percentages of the use of each solution strategy were analyzed, there was an overuse of proportional strategy in additive and constant problems. In both 5th and 6th grade, there was a tendency to overuse proportional method in non-proportional situations (in 5th grade, 34% of constant problems and 31% of additive problems and in 6th grade 47% of constant problems and 48% of additive problems were solved by using a proportional strategy). This finding was in parallel with previous studies, which stated that students had a tendency to apply proportional strategy to non-proportional (constant and additive) word problems (Fernandez et al., 2008; Fernandez et al., 2012; Gagatsis & Modestou, 2007; Van Dooren, Gillard, Verschaffel, Tzekaki, Kaldrimidou, & Sakonidis, 2009; Van Dooren et al., 2007; 2010a). Furthermore, these studies stated that there will be not only the use of proportional strategy in non-proportional problems students will tend to overuse additive strategy in proportional situations, also. However, in contrast with this finding, over-use of additive strategy in proportional situations was not observed in the current study, as expected. Summary of the findings on this issue is as following: proportional situations elicited less overuse of additive method in 5th grade, only 8% of PM problems and 1% of PC problems and in 6th grade, only 6% of PM problems and 1% of PC problems were solved by using an additive strategy. Additive errors in proportional problems were lower and use of proportional errors in non-proportional situations was higher than expected. This situation shows that independent from the number variable, students show a tendency to overuse proportional strategy. As Modestou and Gagatsis (2007) state, when students always deal with similar contextual or numbered problems, they generalize these characteristics to proportional solution methods. Therefore, typical proportional word problems may be resulted in seeing proportional relation in every situation dependent from the relation between quantities.

In the current study, 5th grade students' success rates were significantly affected by the numbers only for additive problems, while 6th grade students' success rates were significantly different in both additive and PC problems. These findings were not parallel with the previous research, which stated that students have a tendency to overuse additive strategy in proportional tasks when numbers in the problem form non-integer ratios and that would affect students' success rates in proportional and non-proportional situations (Fernandez et al., 2010; 2012; Harel & Behr, 1990; Singh, 2000; Van Dooren et al., 2007, 2010a). Both 5th and 6th grade students' success rates were significantly affected by number changes in the additive problems. As the numbers change from integer ratio to non-integer ratio, success rates were increased significantly. This situation may imply that non-integer numbers guide students to read the problem more carefully since numbers does not automatically evoke to use multiplication operation. An alternative solution strategy for proportional solution strategy seemed to be the additive one. Therefore, the change from proportional to additive strategy may result an increase in the success rates of additive problems.

In addition to success rate analysis, further analysis on erroneous strategies indicated that presence of integer or non-integer numbers resulted in significant differences in the overuse of additive and proportional strategies in some problem types. In constant problems, both 5^{th} grade and 6^{th} grade students started to overuse proportional strategy when numbers form integer ratios, while non-integer number use in constant problems increased the overuse of additive strategy. Besides this, in additive problems when numbers changed from integer ratio to non-integer ratios, there was a significant decrease in the use of proportional methods. Interestingly, presence of non-integer ratios in non-proportional problems (in additive and constant problems) decreased number of the use of proportional strategies in both grade levels. Present findings on the effect of number characteristics on students' erroneous strategies' choices were in parallel with Van Dooren et al (2007). In their study, they examined that non-proportional problems suggested more use of proportional errors when numbers in the problem form integer ratios and as numbers in the problem were changed from integer ratio to non-integer ratio, overuse of proportional strategy decreases.

In contrast with the findings of previous research, which stated that students show a tendency to overuse additive strategy in proportional tasks when numbers in the problem form non-integer ratios (Fernandez et al., 2010; 2012; Harel & Behr, 1990; Singh, 2000; Van Dooren et al., 2010a), students in the current study were not affected by number change in proportional problems, while they were significantly affected by number change in non-proportional situations. Numbers forming integer ratio or non-integer ratio did not result a significant difference in students' choice of the solution strategies in proportional problems, while the expected difference was observed in non-proportional situations depending on the numbers in the problems.

This unexpected result may be explained by students' high tendency to overuse of proportional strategy in inappropriate situations. Overuse of proportional strategy was so dominant that it was not affected by the change in numbers that were used in problems.

According to Van Dooren et al. (2010a), students experience a development from applying additive strategy in every situation, to applying proportional strategy in every situation. Between these two stages, numerous students change their solution between additive and proportional depending on the numbers, which form integer ratio or non-integer ratio. When current situation is discussed according to these findings, students in the current study showed some characteristics of between stage, in only constant and additive problems. However, in proportional problems, students' performances and solution strategy uses were not significantly affected by numerical characteristics of the problems. Students consistently used proportional strategy in proportional problems without a significant effect of the numerical changes in the problems.

Last research question aimed to investigate meanings, reasons, and interpretations that nine 5th and 6th grade students associated with their use of particular strategies depending on problem types including proportional or non-proportional relations and numbers forming integer or non-integer ratios. Aim of qualitative phase of the current study was to provide an in-depth understanding of students' own understandings about proportional and non-proportional relations. Analysis of the interviews suggested that there are two themes that help to better understand possible reasons why students had difficulties in differentiating proportional and non-proportional relations. These two themes were: students' lack of awareness of the proportional relation in the building-up strategy and students' decision of the solution strategy mainly based on superficial aspects of the problem without a full understanding of the problem.

Initially, some students seemed to be unaware that building-up strategy requires maintaining proportional relation among the quantities. These students showed an understanding that building-up strategy is only use of addition operation and increasing or decreasing two quantities by the same number maintains proportional relation. Since students think that use of addition operation is enough to maintain proportional relation, they used additive method and proportional method interchangeably. Therefore, some students may solve a problem by using additive strategy while they solved the same problem by using proportional strategy when only the numbers were changed from forming non-integer ratio to integer ratio. This finding was in congruence with research, which states that as students face with a problem in which there is a non-integer ratio between the variables, they prefer to develop an additive method, in which the calculation of numbers is much easier and familiar for the student (Clark & Kamii, 1996; Singh, 2000). When they were asked to compare these two problems and their solution strategies, students cannot recognize the difference between two solution strategies, and problematic use of building up strategy, their use of additive strategy, in which proportional relation could not be maintained. As Singh (2000) stated, multiplicative reasoning is bounded by teaching applications about proportional relations in school setting. Often times, without providing learning opportunities for making sense of proportional relations, children learn to use algorithms, which do not make any sense to them. As a result, although the use of algorithms provides getting the answer in proportionality problems, it does not provide a meaningful understanding of ratio and proportion for students. Therefore, students' differentiation between use of additive strategy and building-up strategy is difficult in case of instructional focus on algorithms.

Secondly, students' decision of the solution strategy without a full understanding of the problem was determined to be another theme that helped to understand why students used particular erroneous strategies. There are three sub-themes that formed this broad theme. First sub-theme was students' use of superficial clues in deciding the solution strategy of the problem. Second sub-theme was students' difficulty in explaining their way of thinking while solving the problem. The third one was students' beliefs about mathematical problems.

First sub-theme, which is students' use of superficial clues, especially numbers in the problem, explained reasons why students used erroneous strategies to some extent. Seven students out of nine stated that numbers in the problems was the reason to use particular strategies. Therefore, when numbers were changed from forming integer ratio to non-integer ratio or vice versa, since the relation among numbers was changed, students could change their solution strategies. Findings of the current study, about students' focus on numerical relations in the problems instead of the relation among the quantities in the problem, were parallel with previous research (Fernandez et al., 2008; 2010; 2012; Sowder, 1988; Van Dooren et al., 2007; 2010a; 2010b), which stated that numbers play an important role in deciding the solution strategy.

Second sub-theme was students' difficulties in explaining their way of thinking while solving problem and deciding solution strategy. Eight students, out of nine, showed difficulties in giving a convincing explanation about why they used a particular strategy. There is enough evidence for expecting students' difficulties in explaining their way of thinking, in Turkey. As Umay and Kaf (2005) stated, teachers focus on numerical answer of the problem instead of problem solving strategy and the way of thinking used, as a result of the highly focus on the multiple choice problems in national exams. Since only numerical answer is valued by teachers and system, students are not given enough opportunities to explain their way of thinking and reasons why they use particular solution strategies. In addition to this, Van Dooren et al. (2010b) criticize the focus on the correct and fluent implementation of the particular procedures that results in the negligence of questioning the applicability of the solution strategies.

Third sub-theme was students' beliefs about mathematical problems. Two students out of nine used some expressions, which implied that these students consider typical beliefs about the problems and shape their problem solving process in accordance with these beliefs. In addition to the need to consider the problem "mathematically", students feel a tension between doing calculations in a mathematical problem and the relation among the quantities. As the studies suggest (Reusser & Stebler, 1997; Verschaffel, 2000) while students solve problems in a classroom context, they experience a need to apply at least one arithmetic operation. This belief may explain the reason why students showed the lowest success rate in constant problems. Since students feel the need to make calculations in mathematics, they may insist on doing calculations although they realize the constant relation among the quantities in the problems.

Teaching Implications of the Study

Results of the current study have practical implications for a better understanding of teaching and learning experiences of students within the context of proportional and non-proportional relations. Students' use of superficial clues, especially numbers in the problem and their difficulty in explaining way of thinking shows that students are not aware of their decision process (Van Dooren et al., 2007). Teachers need to encourage students to explain their way of thinking, the reasons for the use of particular strategies (Umay & Kaf, 2005). When students do not feel the need to explain their way of reasoning, their reasoning becomes more fragile and easy to be effected from superficial clues.

As expected, numbers in the problems had an important effect on students' problem solving process and the choice of the strategies. As suggested by Greer (1987) solving the same problem by changing numbers can be an option for making students aware that numbers cannot be the indicative of the solution strategy. Therefore, students need to deal with the proportional problems dependent from the number variable and different numerical structural problems must be presented to the students in schools.

When students always deal with similar contextual or numbered problems, they generalize these characteristics to proportional solution methods (Modestou & Gagatsis, 2007). For the distinction between proportional and non-proportional situations, students should be offered to practice different relations (Fernandez et al., 2012). Therefore, in schools, the range of proportional and non-proportional situations must be diversified including constant, additive, and proportional relations.

Timing of the current study was important since data collection process of the study coincided with the revision on middle school program in Turkey, which started in 2012-2013 school year. When data collection was implemented, middle and elementary school periods were changed, within the revision. Elementary school comprised 1^{st} to 5^{th} grades and middle school from 6^{th} to 8^{th} grade in the previous program, and elementary school started to involve 1^{st} to 4^{th} grades and middle school 5^{th} to 8^{th} grades in the new program.

In the following year of the revision, 2013-2014 school year, the curricular objectives were revised. Some remarks on the curricular objectives of ratio and proportion in the new mathematics curriculum required to be placed for a better understanding of the revision. In the new program, students start to learn about ratio in 6^{th} grade for the first time. Curricular objectives require students to express ratio of given quantities, calculate the part of a whole, or the whole of a part in case of the ratio between two quantities were given. In 7^{th} grade, ratio and proportion and inverse relation are introduced to students. Students use equations or tables to express proportional relation among quantities, decide whether a given relation shows direct proportion or inverse proportion, solve direct proportion and inverse proportion problems, and students are encouraged to notice that graphs of linear relations pass through origin.

Information about the objectives in the new mathematics curriculum was obtained from MEB announcements (MEB, 2013). Details about the program and revised MEB book were available only for 5^{th} grade. MEB books, which are going to be used in the following years in 6^{th}, 7^{th}, and 8^{th} grades, were not published yet.

Therefore, the details about the revision on teaching ratio and proportion in middle school were limited.

When objectives on ratio and proportion and grade levels were considered some suggestions may come up in the light of the study. First of all, it would be meaningful to have some objectives aiming the differentiation of proportional and non-proportional situations in the revised mathematics curriculum in Turkey. As NCTM (2010) placed the first essential understanding for ratio, proportion, and proportional reasoning as understanding of how ratio differs from other types of reasoning. In addition to this, objectives on ratio and proportion start in 7^{th} grade. As the studies suggest, students have the potential to reason proportionally when they are younger (Van Den Brink & Streefland, 1979), although they are not able to express the proportional relation algebraically. Therefore, some objectives on explaining proportional (missing-value problems, comparison problems) and non-proportional situations (additive problems, constant problems), differentiating these situations in earlier grades would help students to form a basis for the development of proportional reasoning, and differentiation of proportional and non-proportional situations.

Limitations of the Study

The study has a number of limitations. Qualitative data were collected from only nine students. Students were not allowed to stay at school after the school day; therefore the interviews had to be conducted in the school time. When researchers' and students' schedules were considered, it was difficult to find appropriate time for the interviews. Although increasing number of participants in the interviews would enrich qualitative data of the study, number of the students in the interviews was limited with nine. Besides this, during the interviews, there was not any time limitation. Participant students had enough time for thinking, expressing their way of thinking and making sense of the situations.

Samples were selected from a private school because of the convenience factor. Private schools may not be a representative of the situation in public schools in Turkey since physical conditions and opportunities for students are not equal. Therefore, the study exemplifies the situation from a limited sample group and results may not be generalizable to public and all private schools in Turkey.

Recommendations and Suggestions for Further Research

In order to validate our results, additional investigations can be done according to the current studies' conclusions. In the current study, the same problem set was applied to both 6^{th} grade and 5^{th} grade students and that enabled us to control the context variable and problem structure variable, which were not the main interest of the study. In this study, data were collected from two grades. However, effect of grade level and curriculum can be investigated by collecting data from a larger sample of different grade levels. For instance, in order to analyze overuse of proportional and additive methods in a developmental way, data collection could include grades starting from elementary grades to high-school grades. Longitudinal studies can be very useful to have an understanding of the progress students make in their approach to additive and multiplicative relations. Besides this, change in middle school program and curriculum is worth to investigate students' understanding on proportional and non-proportional situations before and after the revision of the program.

In addition to students' understanding about proportional relations and reasoning about their use of erroneous methods, teachers' understanding about current difficulties students experience in differentiating proportional and non-proportional situations would be worthwhile to investigate. As Tourniaire and Pulos (1985) stated, proportional reasoning is a multifaceted construct and it must be presented to the students as such. Students' difficulties in understanding proportional relations may be a reflection of the learning and teaching process they experience. Therefore, it would be valuable to analyze teachers' understanding about proportional and non-proportional relations, and students' erroneous methods since teachers have the most essential role in students' understanding of these important concepts.

APPENDICES

APPENDIX A
Quotes in Turkish

1. R: Acaba neden soruyu ilk okuduğunda o şekilde çözmedin de bu şekilde çözdün (orantısal ilişki kullanarak)?

   Ahmet: Ya bu zaten oran-orantıyı ilk öğrendiğimde yani öğrenmeden önce bu vardı. Ondan iki tane, ondan iki tane…Ondan iki tane arttır, ondan iki tane arttır... Ama şimdi öğrendikten sonra bu aklıma geldi..

   R: Oran-Orantı ne demek?

   Ahmet: Birbirine oranlı bir şekilde arttırmak veya düşürmek…

   R: Birbiriyle oranlı bir şekilde arttırmak nasıl oluyor?

   Ahmet: Yani, ….. İşlemler yapmak…

   R: İşlemler yapmak… Nasıl işlemler yapıyorsun?

   Ahmet: Birşeyle birşeyi çarpıyorsun, bitanesine bölüyorsun.

2. R: Neden 24'ün iki katı dedin?
   Ali: Çünkü… 12 ile 6 yine birbirinin 2 katı… O yüzden 48 buldum…

3. Atakan: T: 16 kitap 24 günde teslim ediliyorsa, 16, 8 16'nın.. Ayy, yok, 16, 8'in 2 katı olduğu için… 24'ün de… evet bu da yanlış olmusş… 2'ye bölecektim, 12.. belki..

4. Arda: 4.sorudaki çözümüm (additive soruyu doğru çözdüğü çözüm), daha mantıklı.

   R: Neden?

   Arda:Çünkü, yani bilmiyorum. Bakınca bu daha doğruymuş gibi geliyor.

5. R: Yine çarpma işlemi yaparım diyorsun yani. Bu soruda neden çarpma işlemi yapmadın acaba?
   Akın: Ya işte orda, 25 günden sonrasını hızlı okudum. 25 gün yazdım. Herhalde. Bilmiyorum...

6. Arda: Tamam. Aklıma bişey geldi de alakası yok matematikle aslında.

   R: Söyle.

   Arda:Hani, başında da diyor, kütüphaneden alınan kitaplar, 25 gün sonra teslim edilmek zorundadır diyor ya. Belki 25 gün (gülüyor) olabilir.

   R: Neden matematiksel olmadığını düşünüyorsun bu çözümün?

Arda:Bilmem, mantık gibi. Yanlış oldu gibi.

R: Niye, matematiğin altında mantık yok mudur?

Arda:Vardır da, daha çok işlem vardır.

APPENDIX B

INAREK Form

BOĞAZİÇİ ÜNİVERSİTESİ
İnsan Araştırmaları Kurumsal Değerlendirme Kurulu (İNAREK) Toplantı Tutanağı
2012/7

17.12.2012

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Sayın Araştırmacı,

"5. Sınıf ve 6. Sınıf öğrencilerinin orantısal akıl yürütme becerilerinin belirlenmesi" başlıklı projeniz ile yaptığınız Boğaziçi Üniversitesi İnsan Araştırmaları Kurumsal Değerlendirme Kurulu (İNAREK) 2012/74 kayıt numaralı başvuru 17.12.2012 tarihli ve 2012/7 sayılı kurul toplantısında incelenerek etik onay verilmesi uygun bulunmuştur.

Saygılarımızla,

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APPENDIX C

Data Collection Instrument

Additive Problem with Integer Ratio Emre and Sıla read the same book. They read at the same speed. But, Sıla started to read the book before Emre. When Sıla read 12 pages, Emre read 6 pages. How many pages will Sıla have read when Emre reads 24 pages? (Emre ve Sıla aynı kitabı okumaya başlıyorlar. Her ikisi de kitabı aynı hızla okuyor. Ancak Sıla kitabı okumaya Emre'den daha önce başlıyor. Sıla 12 sayfa okuduğunda Emre 6 sayfa okumuş oluyor. Buna göre, Emre 24 sayfa okuduğunda Sıla kaç sayfa okumuş olur?)
Additive Problem with Non-integer Ratio Emre and Sıla read the same book. They read at the same speed. But, Sıla started to read the book before Emre. When Sıla read 10 pages, Emre read 4 pages. How many pages will Sıla have read when Emre reads 18 pages? (Emre ve Sıla aynı kitabı okumaya başlıyorlar. Her ikisi de kitabı aynı hızla okuyor. Ancak Sıla kitabı okumaya daha önce başlıyor. Sıla 10 sayfa okuduğunda Emre 4 sayfa okumuş oluyor. Buna göre, Emre 18 sayfa okuduğunda Sıla kaç sayfa okumuş olur?)
Proportional Problem (Missing-value structure) with Integer Ratio Emre and Sıla go to a book store to buy books at discount. All the books are at discount and their prices are the same. Emre buys 3 books while Sıla buys 8 books from the store. If Emre pays 15 TL for the books he buys, how much money does Sıla have to pay for the books she buys? (Emre ve Sıla kitapçıdan indirimde olan kitaplardan satın alıyorlar. İndirimdeki kitapların fiyatları aynıdır. Emre 3 kitap, Sıla ise 8 kitap satın alıyor. Emre kasaya 15 TL ödediğine göre, Sıla aldığı kitaplar için ne kadar ödeyecektir?)
Proportional Problem (Missing-value structure) with Non-integer Ratio Emre and Sıla go to a book store to buy books at discount. All the boosk are at discount and their prices are the same. Emre buys 4 books while Sıla buys 6 books from the store. If Emre pays 10 TL for the books he buys, how much money does Sıla have to pay for the books she buys? (Emre ve Sıla kitapçıdan indirimde olan kitaplardan satın alıyorlar. İndirimdeki kitapların fiyatları aynıdır. Emre 4 kitap, Sıla ise 6 kitap satın alıyor. Emre kasaya 10 TL ödediğine göre, Sıla aldığı kitaplar için ne kadar ödeyecektir?)

Proportional Problem (Comparison structure) with Integer Ratio Emre and Sıla go to a book store to buy some books, which are at discount, for their friends. Emre buys 8 short story books, and Sıla buys 4 fairy tale books. If Emre pays 32 TL for the books, and Sıla pays 20 TL, which book is more expensive? (Emre ve Sıla indirime giren kitapçıdan arkadaşları için kitap satın almaya giderler. Emre öykü kitabından 8 tane, Sıla ise masal kitabından 4 tane satın alır. Emre aldığı kitaplar için 32 TL, Sıla ise 20 TL ödediğine göre, kimin aldığı kitap daha pahalıdır?)
Proportional Problem (Comparison structure) with Non-integer Ratio Emre and Sıla go to a book store to buy some books, which are at discount, for their friends. Emre buys 8 math books, and Sıla buys 12 Turkish books. If Emre pays 20 TL for the books, and Sıla pays 45 TL, which book is more expensive? (Emre ve Sıla indirime giren kitapçıdan arkadaşları için kitap satın almaya giderler. Emre Matematik kitabından 8 tane, Sıla ise Türkçe kitabından 12 tane satın alır. Emre kasada 20 TL, Sıla ise 45 TL ödeme yaptığına göre, kimin aldığı kitap daha pahalıdır?)
Constant Problem with Integer Ratio Emre and Sıla go to the library to borrow some books. Emre borrows 8 books and Sıla borrows 16 books. The books must be returned to the library within 24 days. If Emre returns his books within 24 days, after how many days does Sıla have to return the books to the library? (Sıla ve Emre okul kütüphanesinden kitap ödünç alırlar. Aynı gün içerisinde Emre kütüphaneden 8 kitap, Sıla 16 tane kitap ödünç alır. Kütüphaneden alınan tüm kitaplar 24 gün sonunda geri teslim edilmek zorundadır. Emre kitaplarını 24 gün sonra teslim ettiğine göre, Sıla kitapları kaç gün sonra teslim etmelidir?)
Constant Problem with Non-integer Ratio Emre and Sıla go to the library to borrow some books. Emre borrows 10 books and Sıla borrows 12 books. The books must be returned to the library within 25 days. If Emre retuns his books within 24 days, after how many days does Sıla have to return the books to the library? (Sıla ve Emre okul kütüphanesinden kitap ödünç alırlar. Aynı gün içerisinde Emre kütüphaneden 10 kitap, Sıla 12 tane kitap ödünç alır. Kütüphaneden alınan tüm kitaplar 25 gün sonunda geri teslim edilmek zorundadır. Emre kitaplarını 25 gün sonra teslim ettiğine göre, Sıla kitapları kaç gün sonra teslim etmelidir?)

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