# THREE ESSAYS IN INDUSTRIAL ORGANIZATION

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BOĞAZİÇİ UNIVERSITY

2007

# THREE ESSAYS IN INDUSTRIAL ORGANIZATION

Dissertation submitted to the

Institute for Graduate Studies in the Social Sciences

in partial fulfillment of the requirements for the degree of

PhD

in

Economics

by

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2007

Three Essays in Industrial Organization

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December 2007

Thesis Abstract

Mehtap Işık, "Three Essays in Industrial Organization"

In the first chapter of the thesis, we consider the formation of networks over which a public good is produced (as opposed to considering given and fixed network structures). We identify that the multiplicity of equilibria of effort levels that are exerted in provision of public good on a given and fixed network structure differentiates the problem of formation of these networks from the existing literature. We suggest some stability definitions for these kinds of networks.

In the second chapter, we consider the problem of dealing with piracy in a monopolistic digital market in which piracy exists in the form of end-user copying and commercial reproduction. We find that the governmental protection against commercial piracy is not socially optimal due to end-user copying. Moreover, when the monopolist differentiates its product across quality, it can cover the most of its profit losses by deterring piracy.

In the third chapter, we formalize the discrete type space version of the corruption problem, which is identified by Koray and Saglam (2005a) in Baron and Myerson (1982) model of monopoly regulation. We propose a solution to the problem by using the framework driven by the Myerson (1979) for incentive compatible bargaining games. We also implement a simulation to gain a quantitative insight on the economic effects of the corruption bargaining game.

Tez Özeti

Mehtap Işık, "Sanayi İktisadı Üzerine Üç Çalışma"

Tezin birinci bölümünde, kamusal bir malın üretildiği ağların yapılanmasını (sabit ağ yapılarının ötesinde) incelemekteyiz. Kamusal malın sağlanması için gösterilen katkı miktarlarının çoklu dengeler oluşturabilmesi nedeniyle, bu sorunun literatürde çalışılan genel ağ yapılanmalarıyla ilgili problemlerden farklı olduğunu, farklı modellemeler gerektirdiğini ortaya koymaktayız. Bu doğrultuda bu tür ağ yapıları için kullanılabilecek bazı kararlılık / değişmezlik tanımları önermekteyiz.

İkinci bölümde, hem tüketici seviyesinde hem de ticari seviyede, usulsüz kopyalamanın varolduğu tekelci dijital ürünler piyasalarını modellemekteyiz. Kullanıcı seviyesindeki kopyalama nedeniyle, ticari korsanlığa karşı alınan idari / kurumsal tedbirlerin toplumsal faydayı eniyileyemediğini, diğer yandan tekelci firmanın, kalite üzerinden ürün çeşitlendirebildiğinde, korsan üretim ve tüketimi caydırarak, zararını telafi edebildiğini göstermekteyiz.

Üçüncü bölümde, Baron ve Myerson (1982) tarafından önerilen tekel düzenlemelerinde, Koray ve Sağlam (2005a) tarafından varlığı ispatlanan rüşvet pazarlığı oyununun ayrık tip uzayları için olan türünü, Myerson (1979) tarafından teşvik uyumlu pazarlık oyunları için çizilen çerçevede çözmekteyiz. Aynı zamanda bu oyunun nicel iktisadi etkilerini görebilmek amacıyla bir bilgisayar simulasyonu gerçekleştirmekteyiz.

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Acknowledgments

This thesis is the end of a very long journey in which I had many experiences and learned so much about life, not just academically but personally. There are many people that I would like to thank for different reasons. My gratitude is no less for those whose names I may have failed to mention.

In the first year of my thesis project, I worked with Murat R. Sertel, who inspired and encouraged me to study economics. He means a lot to me. I will always remember him as a mentor. I would like to thank to my supervisors Ünal Zenginobuz, İsmail Sağlam and Gökhan Özertan who have played a decisive role in saving me from leaving economics. The first chapter of this thesis was supervised by Ünal Zenginobuz. The second chapter was written under the supervision of Gökhan Özertan. İsmail Sağlam supervised the third chapter. Without their enthusiastic advice and suggestions this PhD thesis could not have been completed. I would also like to thank the other members of my PhD committee, Fikret Adaman, Murat Usman and Levent Yıldıran for monitoring my work and for their effort in reading and providing me with valuable comments. Members of the International Trade Department have supported me during the last three years. Intensive and stressful working period has been eased by their job offer, encouragements, and tolerance. Finally, I thank the Bogaziçi University Social Sciences Institute Editing Office for editing this thesis.

In the third year of my work, a surprising "rainbow" appeared in my life. Alkım has beautified and given a new meaning to every minute of my life, although she has exhausted my working time. I am deeply grateful to my dear parents Suna Özcanlı and Selahattin Özcanlı. They were always beside me whenever I needed them. Without them neither my life nor this thesis would be as it is. I thank to my mother in law Fatma Işık who has supported me during this work. My sisters and my best friends, Emel Akın and Sibel Işık have never left me alone.

In every step of my journey, I have been supported by all. Yet, without those hands that hold mine tightly, I could never walk in this way. My love, my very best friend, my husband Sinan Işık: This thesis is yours.

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# CHAPTER 1

# AN EXPLORATION ON MODELS OF NETWORK FORMATION WITH PUBLIC GOODS

## 1.1. Introduction

In today's world, the advances in information technologies and the impacts of globalization improves the communication possibilities for economic agents. As a result, agents now have more opportunities to increase their economic gains by carrying out costly activities in cooperating with other agents. Among these activities, those which are common to all geographical as well as social interactions, such as research and development activities or environmental spillovers, warrant special interest. Such activities can be regarded as involving the production of public goods. Economic agents expect to decrease the cost of public good production by locating themselves in a communication and/or cooperation structure. When R&D activities are undertaken by a firm, the improvements are followed by other firms that have been in cooperation with this firm. When anti-pollution programs are enacted in a country, nearby neighbors benefit. The public goods are typically non-excludable along links among agents. The public good nature of such activities by agents connected to each other makes the utilization of network models appropriate in studying these questions. The purpose of this chapter is to explore the formation of such communication structures over which a public good is produced. More specifically, the efficiency (the desirability) and the stability (whether the structure is immune to adding new links or breaking the existing links) of such structures are explored.

Bramoulle and Kranton (2007) build a network model of the public goods. They address such questions as how the social or geographical structure of a given network affect the level of public good provision, whether the agents rely on others or not, and how new links between the communities change the contribution levels and welfare. In their model, agents in a given network structure desire a public good. Whenever a link exist between two agents, the public good is common to both agents. If an agent has no link, he produces the public good himself and no one else can utilize the good. Agents should exert some effort to produce the public good. The effort exerted, which can be defined as the contribution to the provision of the public good, can be in terms of money, land, time, etc. Hence, the production of the public good is costly. The cost of effort exerted for an agent depends on the effort level exerted by that agent. The benefit that an agent gets from the public good accrues from the links that the agent has. Therefore, agents determine the optimal effort level that they should exert, depending on the network structure.

Bramoulle and Kranton (2007) work on the network structures which are fixed and given. This chapter of the thesis explores the *formation* of such structures in the model they study. For a given network, knowing which level of public good provision is exerted, we also address the following questions related to the stability and efficiency issues: Which network structures are the stable ones? How can we characterize the efficient network structures in which the welfare of the society is maximized? Is there a tension between the stable and efficient networks? Since the public good network models are somehow different from the existing network models in the literature, as we will discuss later on, the existing stability notions cannot be applied directly to our model. Hence, we suggest different alternative stability notions in identifying stable network structures.

The literature on the formation of networks originates from the work by Myerson (1977). He analyzes the different cooperation alternatives of a given coalition using graph theory. Myerson studies allocation rules which are defined as functions that assign a payoff value for every possible network structures in a given society. He defines fair allocation rules and introduces an allocation rule, namely Myerson's value, which is closely related to the Shapley value. He proves that Myerson's value is the unique fair allocation rule that is stable for a wide class of games.

Jackson and Wolinsky (1996) study the stability and efficiency of social and economic networks when self-interested agents form or violate links. They use the *pairwise stability* notion where no pair of agents wants to form a new link and no agent gains by severing an existing link. The efficiency notion used is a strong one, namely that the total sum of welfare over agents is maximized over the networks. They analyze the stability and efficiency of two network models, namely connections and co-author models. They show that the stable networks are not necessarily efficient in these two models. Then, the authors propose a general network model in which the payoffs of the agents in each network structure is exogenously determined. The value functions assign a certain payoff value to each network structure. This exogenously determined value of the network is redistributed to the agents according to an allocation rule. The links between the agents are costly. They show that in their model, the stable network is efficient only if the allocation rule assigns resources to free-riders. Moreover, there are some value functions for which under a broad class of allocation rules, the stable networks are not efficient. Therefore, how the total payoffs of the networks are determined and how this payoff is distributed among the agents plays a crucial role for the stability and efficiency of the networks.

The tension between stability and efficiency in the network structures is questioned in many more papers. Johnson and Gilles (2000) show that when the agents are located on a line and the costs of the links are related to the distance between the agents, the efficient and stable networks are shaped in different ways. The conflict between stability and efficiency still survive, in this geographical connection model.

Dutta and Mutuswami (1997) apply a mechanism design approach to overcome the struggle between stability and efficiency. The allocation rule is designed by a planner in this setting. Some properties of the allocation rule are applied only to the stable networks. The value function is again exogenously given. The design approach is used to define some allocation rules on the set of equilibrium networks. Dutta and Mutuswami construct allocation rules which satisfy some desirable properties on equilibrium networks. They use a strategic form game in which agents announce a set of agents with whom they want to form a link.

Jackson(2001) considers three different definitions of efficiency. The efficiency notion in which total welfare of the society is maximized over the network structures is a strong one and appropriate when the value is freely transferable among agents. A constrained efficiency notion or Pareto efficiency can be appropriate if the transfers between the agents are restricted. The characterization of the allocation rule may require revision of the efficiency notion. Jackson shows that Myerson's value leads to difficulties in supporting even Pareto efficiency.

Besides these non-directed networks, there are applications of directed links. The general non-directed networks in which two agents make invest in the link is referred as two sided links. When the cost of the link is provided by one agent, this agent can form a link to a second agent without the latter's consent. Bala and Goyal (2000) work on a non-cooperative non-directed network model. In this setting, each agent can benefit from the others via the formation of costly links. This model is introduced as one-sided links. Bala and Goyal work on one-sided links. Hence, the Nash equilibrium concept and the related refinements can be used in defining the equilibrium of the network formation games. They allow for different kinds of exogenous payoff functions. The payoff can be strictly increasing or decreasing with the number of people that are linked directly or indirectly. They study one-way and two-way flow of benefits. In the one-way flow case, the agent who forms the link benefits from the link. However, in the two-way flow case, both agents take the advantage of the link. Bala and Goyal work on both static and dynamic settings. In the static case, the result is that Nash networks are either connected or empty. They also use a stronger equilibrium concept to refine the many number of Nash networks that arise in the equilibrium. They conclude that the strict Nash networks are the empty networks and the wheel networks which is the structure where an agent is positioned at the center and connected to all others who are positioned in a circle structure with connections to two agents next to them. For the societies that composed of 4 agents or less, the wheel network is the complete network. In the dynamic setting, they use a version of the best-response dynamic game. The network formation game is played repeatedly, where agents form links in every period. While making the decision, an agent chooses a set of links that maximizes his payoffs. With some probability, agents choose the same strategy as in the previous period. Moreover, if more than one strategy are the best responses of the agent then he randomizes across these strategies. Bala and Goyal show that this dynamic process converges to a limit network.

Indeed, the pairwise stability notion in which no two agents can strictly gain by adding a new link and no agent can gain by severing an existing link is a weak notion. It might be thought of as a necessary condition for stability. When a network is not pairwise stable, it may not be formed, regardless of the actual process by which agents form links. Still, the actual process by which the agents form links is an issue of special interest in the literature and there exist many works which explicitly model the formation process as a game for different economic situations. Typically, the games that are used to explain the formation of a network have ad hoc features. Richer forms of deviations and threats to deviations can be considered and the far sighted approaches of the agents can be modeled in this line of the literature.

Aumann and Myerson (1988)'s game is the pioneer work in the literature on modeling the actual process of network formation. In that paper, Aumann and Myerson suggest a model for the endogenous formation of networks. The extensive form game that they model is not myopic. The game consists of exogenously ordered pairs of agents $(i_1 j_1, ...i_n j_n)$ where the link between $i_k j_k$ is formed according to a rule, namely the rule of order. The rule of order is defined as follows: A link is formed if both parties agree. Once it is formed, the link cannot be broken. The game is of perfect information: $i_k$ and $j_k$ decide on whether or not to form a link, knowing the decisions of all pairs coming before them and predicting which links will form as a result of given pairs' decisions. The rule of order is required to lead to a finite game. After the last link has formed, each of the pairs can form an additional link. In equilibrium, some network $g$ is determined. In the model, an exogenous value function determines the payoff of each network. The value of each graph is distributed to the agents according to the Myerson value. Aumann and Myerson are interested in the subgame perfect equilibrium of this network formation game. In the rule of order, the starting network may be an exogenously given graph or it may be the initial position with no links. If in any of the subgame perfect equilibria of the resulting game and for any choice of starting network, no new link is formed, then $g$ is called *stable*. Aumann and Myerson show that the resulting network of the game depends on the rule of order and allocations. Some outcomes may fail to be Pareto optimal and some outcomes will not be complete graphs. An important feature of the game form is that each pair is allowed to reconsider their decision when some other pair coming after them decides to form a link. It is this feature which allows the agents to make credible threats to other agents they want to form link.

Dutta, Nouweland, Tijs (1998) propose a version of the simultaneous game that is introduced by Myerson (1991). In this normal form game, each agent announces a set of agents with whom he wants to form a link. A link is formed between two agents if both agents want to form the link. Again an exogenously given value function assigns values to the possible networks. Once the network is constructed, the payoffs are distributed by some exogenously given allocation rule. Dutta et al. (1998) study the allocation rules that come from a specific class which includes the Myerson Value. They show that for any superadditive game, the complete network is both undominated Nash equilibrium outcome and coalition-proof Nash equilibrium outcome. Moreover, undominated Nash equilibria and coalition-proof Nash equilibria both involve networks in which payoffs are the equivalent to those of a complete network. Indeed, the results depend on the assumptions on the allocation rules. Allocation rules such as the Myerson value assume that agents gain from forming links. Therefore, it is not surprising to have the complete graph as the equilibrium outcome.

Slikker and Nouweland (2000) introduce an exogenous cost to the two game-theoretic models given by Aumann and Myerson (1988) and Dutta et al. (1998). The analysis becomes so complicated that they restrict the number of agents to 3 in the main part of the paper. They show that increasing costs may not lead agents to decrease the number of links they form. In the extensive-form game of link formation of Aumann and Myerson (1988), when the game is superadditive but not convex, the increasing costs lead to formation of initially, fewer links, subsequently more links, and consequently fewer links again. They give examples of games in which increasing costs lead to more links formed, when the number of agents are more than three. In contrast to the extensive-form games, in the strategic form games of link formation of Dutta et al. (1998), for all cases the increasing costs result in the formation of fewer links in the equilibrium.

Currarini and Morelli (2000) study the network formation process using a sequential game. The value of each network is exogenously given by a value function. The main difference of this model from the previous models is that they analyze situations in which the distribution of the value among agents is not exogenously determined. The authors construct a bargaining process in which endogenous allocation of the value and the formation of the network are realized at the same time. In this game, the agents are ordered exogenously. Each agent announces not just the agents with whom he wants to form a link, but a payoff demand. The game is of perfect information. At every decision node, agents know the history of the game. They announce a pair of an agents and a payoff with the perfect information of the history and an expectation about what kind of announcements will be made after them. Links are formed if two conditions are present: (i) Both agents involved in a link must want the link; (ii) The total payoff demands should not exceed the value of the resulting network when these links formed. Currarini and Morelli mainly show that, for value functions which satisfy a kind of weak superadditivity, the subgame perfect equilibria of the game are the efficient networks. They also show that the same results hold if the agents announce a specific payoff demands for each link, instead of demanding an aggregate payoff from the whole component.

The main characteristic of a network model of public goods is that the value of a network, on which a public good is demanded is determined endogenously not exogenously. This is the main feature that differentiates a network model with public goods from the general network models in the literature. Agents get benefits and incur costs from public goods that are produced by themselves or by their neighbors. Thus, the utilities of agents accrue from the links in a given network. However, different levels of effort can be chosen by the agents. Bramoulle and Kranton (2007) show that there may exist multiple Nash equilibria of public good provision in a given network. In a non-cooperative game form, the value of the network is defined as the sum of the utilities of the agents in the network, in a most natural way. Therefore, the value of the network is not exogenously determined, but also for some networks it is not unique.

Cornes, Hartly and Sandler (1999) derive a condition that solves the uniqueness problem of equilibrium in public good models. A group of people desire a public good and the public good produced is common to all society. Agents determine how much to contribute to the provision of the public good, depending on their utility functions. The best response function of agent $i$ gives the amount of the contribution level that agent $i$ should exert so that $i$ maximizes his utility given the level of contributions of the other agents. Simultaneous solution to the best response functions of all agents identifies the Nash equilibrium of the public good provision. The existence and the uniqueness of the solution to this system of equations depend on a condition. The normality condition that guarantees the existence and uniqueness of the Nash equilibrium of the public good provision is that for any agent $i$, there exist $\alpha_i < 1$ such that as the contribution of the neighbors increases by amount $e$, the best response contribution level of the agent $i$ decreases as $\alpha_i e$. In this model, all agents utilize the public good so that the society can be considered as a network structure in the form of a complete graph. Therefore, some restrictions on the utilities of the agents guarantee that the complete graph will have a unique value function. However, this is not the case for every network structure. In this chapter, we show that, even if the Nash equilibrium is unique in the complete graph under the suitable conditions on the utilities of the agents, multiple Nash equilibria exist in some other network structures of the same society under the same conditions.

The uniqueness issue appears as a difficulty that we have to overcome. We can consider assigning the efficient level of the public good in a given network as the value of this network. However, one of the main results given by Bramoulle and Kranton (2007) is that the efficient level of public good is not attained under any Nash equilibrium in any network. Therefore, in a non-cooperative game structure, if we want to construct the value function using the efficient value of the public good, we need a mechanism design approach. We will not consider such mechanisms in this work. Bramoulle and Kranton (2007) suggest a refinement of multiple Nash equilibria. Some Nash equilibria can be abandoned, because when an agent deviates from this equilibrium, the remaining agents respond in a way that creates a new Nash equilibrium. Whenever all agents stick to the given Nash equilibrium, this equilibrium is called stable. Unfortunately, for some given networks, there is no stable Nash equilibrium.

The presentation of the work is organized as follows: In the next section, we describe the model. In the third section, we characterize the Nash equilibria, the stable equilibria and the efficient effort profiles for a given network structure by demonstrating the results obtained by Bramoulle and Kranton (2007). In the subsequent section, we show that the uniqueness of the Nash equilibrium cannot be guaranteed for every network structures. In the fifth section, we suggest different definitions for pairwise stability and, we identify the strongly efficient network structures. In the last section, we conclude and present an agenda for future work.

## 1.2. The Model

In this section, we describe the model, and present the notations and definitions of the concepts that we use in the rest of this chapter. Agents from the set $N = \{1, 2, ..., n\}$ demand a public good. They can set links with each other. All agents benefit from the public good produced by agents to whom they are directly connected. $ij$ denotes the *link* between agents $i$ and $j$. *The network structure* that results from the links formed between the agents is represented by a graph. $G^N$ stands for all possible graphs for society $N$. The set of *connected agents* in a given graph $g$ is defined as $N(g) = \{i|\exists j, ij \in g\}$. When every agent has direct links with all other agents in a graph, we call this graph a *complete graph.* The graph $g' \subset g$ is a *component* of $g$ if $\forall i, j \in N(g')$, $i \neq j$, there exists a path that connects $i$ and $j$ and $\forall i \in N(g')$ and $\forall j \in N(g)$ if $ij \in g$ then $ij \in g'$. The set of *neighbors* of agent $i$ is defined as the agents who are directly linked to $i$, $N_i = \{j|ij \in g\}$. $N_i \cup \{i\}$ is the *neighborhood* of agent $i$. $|N_i|$ gives the number of neighbors of agent $i$ in a given graph. Whenever $|N_i| = k$ for all $i$ in the graph $g$, we call $g$ a *regular graph of degree k.*

Agents enjoy the public good according to a benefit function, $b(.)$, which is assumed to be strictly concave i.e. $b'(.) > 0$ and $b''(.) < 0$. Moreover, $b(0) = 0$. Each agent $i$ contributes to the provision of the public good at a level $e_i$, where $e_i \geq 0$. Agent $i$ benefits from the public good through the effort level $e_i$ he exerts, and the sum of effort levels exerted by his neighbors, namely, $e^g_{-i} = \sum\limits_{j \in N_i} e_j$. The effort level profile $e$ where $e = \{e_i\}^n_{i=1}$ shows how each agent $i$ contributes to the provision of public goods in a given graph $g$. The cost of contribution for any agent is assumed to be linear: $C(e_i) = ce_i$, where $c > 0$. Thus, the net utility of the agent $i$ from the consumption of the public good in a given graph $g$ is written as

$$U_i(g, e) = b(e_i + e^g_{-i}) - ce_i \tag{1.1}$$

How much public good is produced, who contributes to the provision of public good, the level of contribution for the contributors depends on the structure of the network. Hence, the total social welfare changes according to the network structure and the effort levels exerted by the agents in this structure. The welfare for a given graph $g$ and the effort level profile $e$ associated with $g$ is defined as

$$W(g, e) = \sum_{i \in N} U_i(g, e) = \sum_{i \in N} b(e_i + e^g_{-i}) - c \sum_{i \in N} e_i \tag{1.2}$$

### 1.2.1. Characterization of Nash Equilibria, and Stable and Efficient Effort Profiles

In this subsection, we demonstrate the results by Bramoulle and Kranton (2007). In the characterization of the Nash equilibria of the given graph $g$, each agent $i$

maximizes his utility, given the effort levels of the society:

$$\max_{e_i} \left\{ b(e_i + \sum_{j \in N_i} e_j) - ce_i \right\} \tag{1.3}$$

The aggregate effort level $e^*$ that gives the highest utility to any agent $i$ is given by the first order conditions for each agent:

$$b'(e^*) \leq c. \tag{1.4}$$

The benefit accrues from the neighbors in any network structure. Hence, given the effort levels of his neighbors, $e^g_{-i}$, agent $i$ maximizes his utility, $U_i$, over $e_i$ so that $e^* \leq e_i + e^g_{-i}$. Therefore, in a Nash equilibrium profile $e = (e_1, ..e_i, ..e_n)$, $e_i = 0$, if $e^* \leq e^g_{-i}$, and $e_i = e^* - e^g_{-i}$, if $e^* \geq e^g_{-i}$. A profile $e$ is called *specialized* if every agent either contributes in the maximum amount of $e^*$ or does not contribute at all. A profile $e$ is called as *distributed* if every agent is a contributor. A combination of these two extremes is called a *hybrid* profile. Using a graph theoretical approach, Bramoulle and Kranton show that for every graph there exists a specialized Nash equilibrium. (Bramoulle and Kranton, 2007, Theorem 1)

An equilibrium $e$ is defined as *stable effort level* if and only if there exist a positive number $\rho > 0$ such that for any vector $\varepsilon$ satisfying $\forall i, |\varepsilon_i| \leq \rho$ and $e_i + \varepsilon_i \rho \geq 0$ the sequence $e^{(n)}$ defined by $e^{(0)} = e + \varepsilon$ and $e^{(n+1)} = f(e^{(n)})$ converges to $e$ as n goes to infinity. $f = (f_1, ..., f_n)$ is defined as the best responses, where $f_i(e)$ is the best response of agent $i$ to a profile $e$. According to this definition, a profile $e$ is stable if when an agent $i$ has a tendency to deviate from the $e_i$, the rest of the society should stick to the profile $e^g_{-i}$. Whenever all the agents are contributors, if one agent $i$ decreases the amount of contribution, the other agents who are linked to $i$ increase their contributions accordingly. Thus, only specialized equilibria are stable. The process, indeed, characterizes the stable equilibria in detail. An equilibrium is stable if and only if it is specialized and every non-specialist is connected to at least two specialists. Therefore, there exist graphs on which no stable equilibrium can be found. (Bramoulle and Kranton, 2007, Theorem 2)

A profile $e$ is a *efficient effort level* for a given graph $g$ if and only if $e$ maximizes welfare in $g$. That is, $\forall e'$, $W(g,e) \geq W(g,e')$. Then, for any $i$, whenever $e_i > 0$, $\frac{\partial W(g,e)}{\partial e_i} = 0$. So,

$$b'(e_i + e_{-i}) + \sum_{j \in N_i} b'(e_j + e_{-j}) = c \tag{1.5}$$

and whenever $e_i = 0$, $\frac{\partial W(g,e)}{\partial e_i} \leq 0$.

An efficient profile for a given graph $g$ maximize the social welfare on $g$. However, it is obvious from the equations (1.3) and (1.4) that acting non-cooperatively, agents in structure $g$ never choose the efficient profile. In this set up, Nash equilibria are not efficient in a given graph. Bramoulle and Kranton (2007) show that in a given graph $g$, if the neighborhood of one agent is a subset of the neighborhood of another agent, then the one with the smaller neighborhood should exert no effort in the efficient effort profile. (Braumolle and Kranton, 2007, Section 4)

The model of Bramoulle and Kranton (2007) further provide characterization for the equilibrium effort levels and efficient effort levels in a given network structure. We will use their model and results in exploring the formation of such network structures.

### 1.2.2. On the Uniqueness of Nash Equilibrium for a Given Graph

In this section, we will show that the multiple Nash equilibria may exist, even under convex cost functions. The uniqueness of the Nash equilibrium in the public good provision game can be guaranteed under some conditions on the utility functions of the agents, when all agents are linked to each other in the network. Whenever the benefit function $b(.)$ is strictly increasing and concave, where $b(0) = 0$ and the cost function $c(.)$ is strictly increasing and convex, with that $b'(\infty) \leq c'(0)$, the equilibrium where all agents exert the same effort level is the unique Nash equilibrium for the complete graph as follows. (Cornes et al. (1999))

The optimization problem for each agent $i$ is $\max_{e_i} \left\{ b(e_i + \sum_{j \in N_i} e_j) - ce_i \right\}$. Since the network is complete, the problem turns out to be $\max_{e_i} \left\{ b(\sum_{j \in N} e_j) - ce_i \right\}$. By the above assumptions given above on b(.) and c(.), first order conditions for each $i$ identify the solution. $e_i = 0$, if $b'(\sum_{i \in N} e_i) \leq c'(e_i)$, and $e_i > 0$, if $b'(\sum_{i \in N} e_i) = c'(e_i)$. Therefore, given the assumptions on $b(.)$, the unique solution to this system is that $e_i = e > 0$ for all $i$. In other words, the Nash equilibrium is unique for the complete network.

However, the assumptions on the benefit and cost functions given above are not enough to generalize the uniqueness of the Nash equilibrium for all network structures. Suppose $N = 4$ and $b(e) = 2\sqrt{e}$ and $c(e) = \frac{1-\sqrt{2}}{\sqrt{2}} e^2 + \frac{1}{\sqrt{2}}$. $b(.)$ and $c(.)$ satisfy all the assumptions that guarantee the uniqueness for the complete graph. The unique Nash Equilibrium for the complete graph is that any of the four agents exerts the same effort level $e = 0.37$. If the network structure of the society is defined by a circle, as given in Figure 1.1 below, neighbors of the agents 1 and 4 are agents 2 and 3 and neighbors of the agents 2 and 3 are agents 1 and 4. Agents 2 and 3 and agents 1 and 4 are not linked to each other. The set of

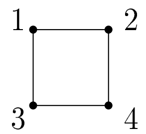


Figure 1.1. Circle

first order conditions are as follows:

$$b'(e_1 + e_2 + e_3) - c'(e_1) \leq 0$$
$$b'(e_4 + e_2 + e_3) - c'(e_4) \leq 0$$
$$b'(e_1 + e_2 + e_4) - c'(e_2) \leq 0$$
$$b'(e_1 + e_3 + e_4) - c'(e_2) \leq 0$$

The solutions to these problems are $e_1 = e_2 = e_3 = e_4 = 0.47$, $e_1 = e_4 = 0$, $e_2 = e_3 = 1$ and $e_2 = e_3 = 0$, $e_1 = e_4 = 1$. Therefore, three Nash equilibria on the circle are (0.47,0.47,0.47,0.47), (0,1,1,0), (1,0,0,1).

## 1.3. Formation of Network Structures with Public Goods: Some Stability Definitions

In this section, we consider the formation of network structures and study their stability and efficiency properties. We suggest some definitions for the stability of the network structures as a whole in the formation process. The question of whether agents in a given network $g$ wants to change the structure of this network by adding or severing links is related to the effort levels exerted in $g$, as the utility the agents derive come from their links in $g$. The value of a graph, for non-cooperative actions, can be defined most naturally as the sum of the utilities of the agents in this graph. An allocation rule would redistribute the value of the graph to the agents. The alterations of the payoffs of the agents under an allocation rule could then be considered as the transfers among the agents. The perfect information feature of the model makes such transfers possible under collateral agreements. However, for non-cooperative games, a mechanism should be implementable to make the transfers possible. We will not work on such mechanisms, instead we will use the allocation rule under which each agent obtains the utility that emerges at the equilibrium of the non-cooperative game.

We will define the following sets for a given graph $g$:

$NE(g) = \{e|$ e is a Nash equilibrium in g $\}$,

$S(g) = \{e|$ e is a stable equilibrium in g $\}$,

$E(g) = \{e|$ e is an efficient effort profile in g $\}$.

For a given graph structure $g$, the utilities of the agents will change as the effort profile $e$ on $g$ changes. We restrict ourselves to the Nash equilibria (of the effort levels) on $g$, and we define the value of $g$ depending on which $e$ is exerted in $g$.

The *value function* $v$ where $v : \{(g,e)|g \in G^N, e \in NE(g)\} \to \Re$ is defined as $v(g,e) = \sum_i U_i(g,e)$.

Suppose the set of values is $V$. A value of a graph is reallocated through an *allocation function*, $Y$. We define $Y$ as the collection of $Y_i$ where, $Y_i : \{(g,e)|g \in G^N, e \in NE(g)\} \times V \to \Re$ such that $Y_i((g,e),v)$ is the payoff of agent $i$ in graph $g$ when the effort profile $e$ is exerted and the aggregated value of $g$ is $v$. We will consider the reallocation of the value directly as the utilities of agents at equilbrium, i.e. $Y_i((g,e), \sum_i U_i(g,e)) = U_i(g,e)$

### 1.3.1. Stable Network Structures

Which network structure constitutes a stable structure for a society $N$ depends on the notion of stability adopted. The main definition in the literature is the pairwise stability where no pair of agents can strictly increase their payoffs by forming new links, and no one can gain by violating an existing link. However, the multiple equilibria prevent us from using this definition. We suggest four alternative stability definitions for a graph structure $g$. We will demonstrate how these definitions work for a society of four agents.

#### Alternative 1

The first stability notion we suggest is a strong one. In this version, given the graph $g$, the formation of the new links that do not exist in $g$ requires just the approval of two agents who are about to form the link. Also, an agent can freely violate his links that exist in $g$. In the decision process, agents consider any possible Nash equilibrium effort profile on the potential graph structure which is constituted after a link is formed or violated. If there exist a Nash equilibrium on the potential graph that gives higher utility to agent $i$, then $i$ can approve a new link or violate the existing link in $g$.

Formally, we call the graph $g$ with the effort profile $e$ *strongly pairwise stable* if and only if:

1. $\forall ij \in g$, $\forall e' \in NE(g - ij)$, $Y_i((g,e),v) \geq Y_i((g - ij, e'), v)$ and $Y_j((g,e),v) \geq Y_j((g - ij, e'), v)$, and
2. $\forall ij \notin g$, if $\exists e' \in NE(g + ij)$ such that $Y_i((g,e),v) < Y_i((g + ij, e'), v)$ then $Y_j((g,e),v) > Y_j((g + ij, e'), v)$.

According to this definition, the set of stable graphs with the associated effort levels can be expected to be a very small set. Any agent $i$ changes the given network structure easily, if he sees an opportunity for a gain. Indeed, the behavior is very myopic and risk loving in a sense. Agent $i$ does not care whether the Nash equilibrium which is immediately profitable for him is going to be a stable effort level or not. However, even if a profitable Nash equilibrium effort level in the potential graph exists, an agent other than $i$ can deviate from this equilibrium causing a loss for agent $i$, when this equilibrium of effort level is not a stable effort level.

Alternative 2

In this version, we assume a little more farsightedness on the part of the agents. When the agents decide whether to form or sever the links, they will compare their utility on graph $g$ with possible payoffs under the stable Nash equilibria effort profiles of the potential graphs. We call graph $g$ with effort profile $e$ as *weakly pairwise stable*, if the following hold:

1. $e$ is stable equilibrium profile in graph $g$ i.e. $e \in S(g)$,
2. $\forall ij \in g$, $\forall e' \in S(g - ij)$, $Y_i((g,e),v) \geq Y_i((g - ij, e'), v)$ and
   $Y_j((g,e),v) \geq Y_j((g - ij, e'), v)$, and
3. $\forall ij \notin g$, if $\exists e' \in S(g + ij)$ such that $Y_i((g,e),v) < Y_i((g + ij, e'), v)$ then
   $Y_j((g,e),v) > Y_j((g + ij, e'), v)$.

Note that checking weakly pairwise stability is very much like in the previous one. New links will be formed, if and only if two agents who are about to link directly to each other want to form this link. Moreover, any existing link can be violated freely (without the approval of the others) whenever it is beneficiary for an agent. In the next alternative, we require the approval of all agents for the formation of new links.

Alternative 3

The formation of a new link or violation of existing links will affect all agents in a graph structure. This is due to the fact that the effort levels of the agents are strategic substitutes. Even if an agent $i$ is not one of the agents who form the link, still his effort level/payoff can change in response to new links. Therefore, requiring the approval of all agents for the formation of new links is not an odd condition. We define the graph $g$ with effort profile $e$ as *approval stable* if the following hold:

1. $\forall ij \in g$, $\forall e' \in NE(g - ij)$, $Y_i((g,e),v) \geq Y_i((g - ij, e'), v)$ and
   $Y_j((g,e),v) \geq Y_j((g - ij, e'), v)$, and
2. $\forall ij \notin g$, if $\exists e' \in N(g + ij)$ such that $Y_i((g,e),v) < Y_i((g + ij, e'), v)$ then
   $\exists k \in N$ such that $Y_k((g,e),v) > Y_k((g + ij, e'), v)$.

In this version, we give veto power to agents that are directly or indirectly connected to each other for the formation of the new links. Note, however, the agents are allowed to violate their links freely. The violation of a link in a graph structure can be harmful for some agents, while it is beneficiary for the one who severs the link. Because the contribution to the provision of the public good is voluntary, when an agent wants to violate a link, it may not be easy to persuade the agent not to violate the link in some cases. In the next alternative, we consider a full approval framework that also requires the approval of all agents when an agent wants to break a link.

Alternative 4

Given graph $g$, agents need the consent of the society to sever or form links. We define $g$ with a profile $e$ as *full approval stable* if the following hold:

1. $\forall ij \in g$, if $\exists e' \in N(g - ij)$ such that $Y_i((g, e), v) < Y_i((g + ij, e'), v)$ then $\exists k \in N$ such that $Y_k((g, e), v) > Y_k((g - ij, e'), v)$, and
2. $\forall ij \notin g$, if $\exists e' \in N(g + ij)$ such that $Y_i((g, e), v) < Y_i((g + ij, e'), v)$ then $\exists k \in N$ such that $Y_k((g, e), v) > Y_k((g + ij, e'), v)$.

This definition will lead to a larger set of stable structures. Many structures with a stable equilibrium of effort levels become stable under this definition.

### 1.3.2. Efficient Network Structures

We call the network structure $g$ as *strongly efficient* if there exists $e \in E(g)$ such that for any $h \in G^N$ and for any $e' \in E(h)$, $W(e, g) \geq W(e', h)$.

We know from the work by Bramoulle and Kranton (2007) that the Nash equilibria profiles are not efficient. Therefore, the stability notions defined above will not involve strongly efficient graph structure in the model studied here. Note however that the stability definition offered in alternative 4 in the previous section carries some characteristics of Pareto efficiency. Given the graph structure $g$ which is stable in the sense of alternative 4, no one can improve himself by forming new links or by violating the existing links without hurting someone else in the society. Therefore, such a graph structure can be considered as a *pareto efficient* graph.

In the following analysis for a society of four agents, we demonstrate how the payoffs of agents depend on both the network structure and the effort profile exerted for that network. As mentioned before, the multiple equilibria create a difficulty in analyzing the decision process of the agents in forming or severing links.

### 1.4. An Analysis of Stability Notions Defined: The Case of Four Agents

We consider a society of four people. In this case, there exists 11 possible graph structures. First we characterize, for each graph $g$, the set of Nash equilibria effort profile on $g$, $NE(g)$, the set stable equilibria on g, $S(g)$, and the set of efficient effort profile e on $g$, $E(g)$. Then, we identify which graphs are stable with respect to the four different alternatives we suggest. In assessing the efficiency of the stable structures we adopt, for the sake of simplicity, the specific benefit function, $b(e) = 2\sqrt{e}$, and a specific cost function $C(e) = e$.

The utility of agent $i$ in a given graph $g$ is given by $U_i(g, e) = b(e_i + e_{-i}^g) - ce_i$. With the given assumptions on $b(.)$, the first order condition identify the Nash equilibria effort profiles. Let $e^*$ be the aggregate effort level such that $b'(e^*) = c$. Then, in a Nash equilibrium profile $e$, for all $i$, $e_i$ satisfies $e_i = e^* - e_{-i}^g$, if $e^* < e_{-i}^g$ and $e_i = 0$, otherwise. The network structure determines who the neighbors of agent $i$ are and what the value of $e_{-i}^g$ can be. Therefore, although the best response functions of all agents are the same for all graph structures, the best response effort level changes according to the graph structure as the aggregate effort levels exerted by the neighbors change.

Below, we classify the possible graph structures for four agents. The network structure can be classified according to the number of links each agent has. Let $\{l_1, ..l_i, ..l_n\}$ to denote the type of network structure so that agent $i$ has $l_i$ links. Since all agents are assumed to be symmetric, $\{l_1, ..l_i, ..l_n\}$ is not an ordered tuple but carries a set structure. For example, a network of type $\{1, 1, 0, 0\}$ in Figure

1.2, where link 12 (i.e. the link between agent 1 and agent 2) exists and a network of type $\{1,0,0,1\}$ in which link 14 exists characterize the same network structure in which only one link is formed between the agents.

The number of total links in a graph of type $\{l_1, ..l_i, ..l_n\}$, is given by $\sum_i l_i/2$. Hence, all possible network types for four agents are given in Figure 1.2 as a tree diagram. A line between two network types represents the possible construction of one network from the other by forming a new link or severing an existing link.

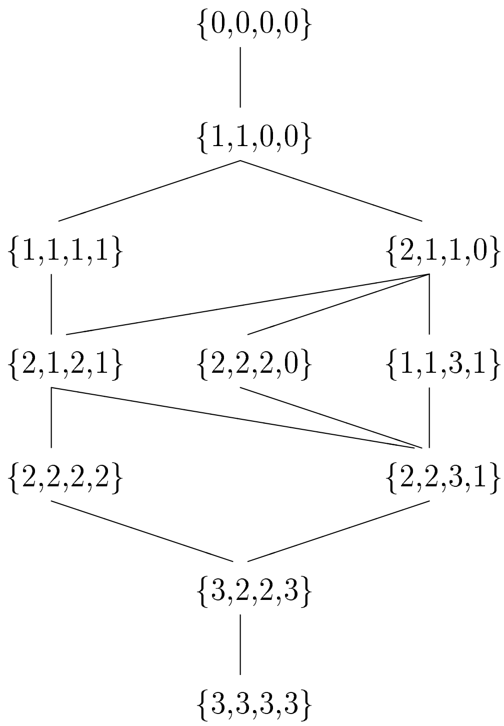



Figure 1.2. Network Tree for 4 agents

### 1.4.1. Type $\{0,0,0,0\}$

Let $g_1$ be a graph of this type on which no link is formed. The graph structure will look like as follows:

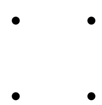


Figure 1.3. Graph of type $\{0,0,0,0\}$: $g_1$

The set of Nash equilibria, the set of stable equilibria and the set of efficient effort profiles on $g_1$ are listed in Table 1.1.

Table 1.1. Nash Equilibria, Stable Equilibria and Efficient Effort Profiles for $g_1$

| |
|---|
| $NE(g_1) = \{(e^*, e^*, e^*, e^*)\}$ |
| $S(g_1) = \{(e^*, e^*, e^*, e^*)\}$ |
| $v(g_1, (e^*, e^*, e^*, e^*)) = 4b(e^*) - 4c$ |
| $E(g_1) = \{(e^*, e^*, e^*, e^*)\}$ |
| $W(g_1, e) = 4b(e^*) - 4c$ |

This graph structure is the only strongly pairwise stable graph. As we can see from Figure 1.2, the only possible graph structure that can be obtained from $g_1$ is $g_2$. However, there is no stable effort level on $g_2$. (See Table 1.2)

### 1.4.2. Type $\{1, 1, 0, 0\}$

Let $g_2$ be a graph of this type on which one link is formed.

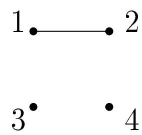



Figure 1.4. Graph of type $\{1, 1, 0, 0\}$: $g_2$

The set of Nash equilibria, the set of stable equilibria and the set of efficient effort profiles on $g_2$ are listed in Table 1.2. This graph structure is not stable according to any definition.

Table 1.2. Nash Equilibria, Stable Equilibria and Efficient Effort Profiles for $g_2$

| |
|---|
| $NE(g_2) = \{(e_1, e^* - e_1, e^*, e^*) \vert 0 \leq e_1 \leq e^*\}$ |
| $S(g_2) = \emptyset$ |
| $v(g_2, e) = 4b(e^*) - 3c$, $e \in NE(g_2)$ |
| $E(g_2) = \{(e_1, a_1 - e_1, e^*, e^*) \vert 0 \leq e_1 \leq a_1 \text{and }, b'(a_1) = c/2\}$ |
| $W(g_2, e) = 2.b(a_1) + 2b(e^*) - c(2 + a_1)$ |

1.4.3. Type $\{2, 1, 1, 0\}$

Let $g_3$ be a graph of this type on which two links are formed.

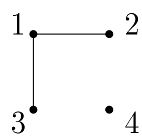



Figure 1.5. Graph of type $\{2, 1, 1, 0\}$: $g_3$

The set of Nash equilibria, the set of stable equilibria and the set of efficient effort profiles on $g_3$ are listed in Table 1.3. This graph structure is not stable under any definition.

Table 1.3. Nash Equilibria, Stable Equilibria and Efficient Effort Profiles for $g_3$

| |
|---|
| $NE(g_3) = \{(e^*, 0, 0, e^*), (0, e^*, e^*, e^*)\}$ |
| $S(g_3) = \{(0, e^*, e^*, e^*)\}$ |
| $v(g_3, (e^*, 0, 0, e^*)) = 4b(e^*) - 2c$, $v(g_3, (0, e^*, e^*, e^*)) = 4b(e^*) - 3c$ |
| $E(g_3) = \{(a_1, 0, 0, e^*) \vert b'(e_1) = c/3\}$ |
| $W(g_3, e) = 3b(a_1) + b(e^*) - c(a_1 + e^*)$ where $e \in E(g_3)$ |

1.4.4. Type $\{1, 1, 1, 1\}$

Let $g_4$ be a graph of this type on which two link is formed.

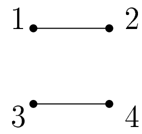



Figure 1.6. Graph of type $\{1,1,1,1\}$: $g_4$

The set of Nash equilibria, the set of stable equilibria and the set of efficient effort profiles on $g_4$ are listed in Table 1.4. With $e = (e^*, 0, 0, e^*)$, this graph structure is approval stable and full approval stable. As we can follow from Figure 1.2, the only possible graph structure which can be obtained from $g_4$ by adding a new link is $g_5$. The effort profile $e = (0, e^*, 0, e^*)$ on $g_5$ gives less utility to agents 2 and 4, so they will veto the link between agents 1 and 3. A similar idea works for the possible links between agents 1 and 4,and agents 2 and 3, and 2 and 4. The only possible graph structure which can be obtained from $g_4$ by violating an existing link is $g_2$. When an agent breaks a link, he cannot gain positive profit from this action on the new graph $g_2$.

Table 1.4. Nash Equilibria, Stable Equilibria and Efficient Effort Profiles for $g_4$

| |
|---|
| $NE(g_4) = \{(e_1, e^* - e_1, e_3, e^* - e_3) \mid 0 \leq e_1, e_3 \leq e^*\}$ |
| $S(g_4) = \emptyset$ |
| $v(g_4, e) = 4b(e^*) - 2c$, $e \in NE(g_4)$ |
| $E(g_4) = \{(e_1, a_1 - e_1, e_3, a_1 - e_3) \mid 0 \leq e_1, e_3 \leq a_1$ and $, b'(a_1) = c/2\}$ |
| $W(g_4, e) = 4b(a_1) - 2ca_1$ |

### 1.4.5. Type $\{2,1,2,1\}$

Let $g_5$ be a graph of this type on which three links are formed.

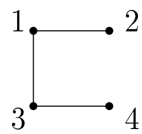



Figure 1.7. Graph of type $\{2,1,2,1\}$: $g_5$

The set of Nash equilibria, the set of stable equilibria and the set of efficient effort profiles on $g_5$ are listed in Table 1.5. With $e = (0, e^*, 0, e^*)$, this graph structure is full approval stable. As we can follow from Figure 1.2, the possible graph structures which can be obtained from $g_5$ by adding a new link are $g_8$ and $g_9$. When agents 2 and 4 add a new link, $g_9$ is obtained. However, with $e = (e^*, 0, 0, 0)$ on $g_9$, agent 1 will veto the new link. When agents 2 and 3 form a link to build $g_8$, with $e = (e^*, 0, 0, e^*)$ agent 1 will veto such an action. Graphs $g_3$ and $g_4$ can be obtained by violating links. However, with $e = (e^*, 0, 0, e^*)$ on $g_4$, and again with $e = (e^*, 0, 0, e^*)$ on $g_3$, agent 1 will veto the violation of links 13 or 34, respectively.

Table 1.5. Nash Equilibria, Stable Equilibria and Efficient Effort Profiles for $g_5$

| |
|---|
| $NE(g_5) = \{(e_1, e^* - e_1, 0, e^*), (0, e^*, e_3, e^* - e_3) \mid 0 \leq e_1, e_3 \leq e^*\}$ |
| $S(g_5) = \emptyset$ |
| $v(g_5, e) = b(e^* + e_1) + 3b(e^*) - 2c$, $e \in NE(g_5)$ and $0 \leq e_1 \leq e^*$,<br>$v(g_5, e) = b(e^* + e_3) + 3b(e^*) - 2c$, when $e \in NE(g_5)$ and $0 \leq e_3 \leq e^*$ |
| $E(g_5) = \{(e_1, 0, e_1, 0) \mid 0 \leq e_1 \leq e^*$ and $, 2b'(2e_1) + b'(e_1) = c\}$ |
| $W(g_5, e) = 2b(2e_1) + 2b(e_1) - 2ce_1$ |

### 1.4.6. Type {2, 2, 2, 0}

Let $g_6$ be a graph of this type on which three links are formed.

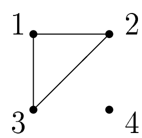


Figure 1.8. Graph of type $\{2, 2, 2, 0\}$: $g_6$

The set of Nash equilibria, the set of stable equilibria and the set of efficient effort profiles on $g_6$ are listed in Table 1.6. This graph structure is not stable under any definition.

Table 1.6. Nash Equilibria, Stable Equilibria and Efficient Effort Profiles for $g_6$

| |
|---|
| $NE(g_6) = \{(e_1, e_2, e_3, e^* \| e_1 + e_2 + e_3 = e^*\}$ |
| $S(g_6) = \{(e^*, 0, 0, e^*)\}$ |
| $v(g_6, e) = 4b(e^*) - 2c$, $e \in NE(g_6)$ |
| $E(g_6) = \{(e_1, e_2, e_3, e^*) \| b'(e_1 + e_2 + e_3) = c/3\}$ |
| $W(g_6, e) = 3b(e_1 + e_2 + e_3) + b(e^*) - 2c(e_1 + e_2 + e_3 + e^*)$, where $e \in E(g_6)$ |

### 1.4.7. Type $\{1, 1, 3, 1\}$

Let $g_7$ be a graph of this type on which three link are formed.

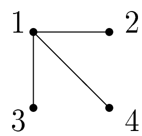



Figure 1.9. Graph of type $\{1, 1, 3, 1\}$: $g_7$

The set of Nash equilibria, the set of stable equilibria and the set of efficient effort profiles on $g_7$ are listed in Table 1.7. With $e = (0, e^*, e^*, e^*)$, this graph structure is weakly pairwise stable, approval stable and full approval stable. As we can follow from Figure 1.2, the only possible graph structure which can be obtained from $g_7$ by adding a new link is $g_8$. However, there is no stable effort level on $g_8$. Moreover, with $e = (e^*, 0, 0, e^*)$, agent 1 will veto the new link 24. The only possible graph obtained by violating a link is $g_3$. The only stable $e$ on $g_3$, $e = (0, e^*, e^*, e^*)$, does not bring any increase in utility to agent 4. Under the benefit function $b = 2\sqrt{e}$, this structure with $e \in E(g)$ is efficient, resulting in a total welfare of 16.

Table 1.7. Nash Equilibria, Stable Equilibria and Efficient Effort Profiles for $g_7$

| |
|---|
| $NE(g_7) = \{(0, e^*, e^*, e^*), (e^*, 0, 0, 0)\}$ |
| $S(g_7) = \{(0, e^*, e^*, e^*)\}$ |
| $v(g_7, (e^*, 0, 0, 0)) = 4b(e^*) - c$ and $v(g_7, (0, e^*, e^*, e^*)) = 4b(e^*) - 3c$ |
| $E(g_7) = \{(a_3, 0, 0, 0)\|b'(a_3) = c/4\}$ |
| $W(g_7, e) = 4b(a_3) - ca_3$ where $e \in E(g_7)$ |

### 1.4.8. Type $\{2, 2, 3, 1\}$

Let $g_8$ be a graph of this type on which four links are formed.

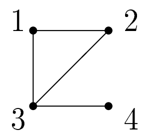



Figure 1.10. Graph of type $\{2, 2, 3, 1\}$: $g_8$

The set of Nash equilibria, the set of stable equilibria and the set of efficient effort profiles on $g_8$ are listed in Table 1.8. With $e = (0, 0, e^*, 0)$, this graph structure is full approval stable. As we can follow from Figure 1.2, the only possible graph structure which can be obtained from $g_8$ by adding a new link is $g_{10}$. No one can gain by adding link 24. Agent 3 can gain by breaking the link 34 with $e = (e^*, 0, 0, e^*)$ on $g_6$. However, agent 1 can veto such an action. Under the benefit function $b = 2\sqrt{e}$, this structure with $e \in E(g)$ is efficient, resulting in a total giving welfare of 16.

Table 1.8. Nash Equilibria, Stable Equilibria and Efficient Effort Profiles for $g_8$

| |
|---|
| $NE(g_8) = \{(e_1, e^* - e_1, 0, e^*), (0, 0, e^*, 0) \| 0 \leq e_1 \leq e^*\}$ |
| $S(g_8) = \emptyset$ |
| $v(g_8, e) = 3b(e^*) + b(2) - 2c$,when $e \neq (0, 0, e^*, 0)$ $v(g_8, (0, 0, e^*, 0)) = 4b(e^*) - c$ |
| $E(g_8) = \{(0, 0, a_3, 0) \| b'(a_3) = c/4\}$ |
| $W(g_8, e) = 4b(a_3) - ca_3$, where $e \in E(g_8)$ |

### 1.4.9. Type $\{2, 2, 2, 2\}$

Let $g_9$ be a graph of this type on which four links are formed. These networks are regular graphs of degree 2.

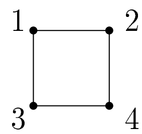



Figure 1.11. Graph of type $\{2, 2, 2, 2\}$: $g_9$

The set of Nash equilibria, the set of stable equilibria and the set of efficient effort profiles on $g_9$ are listed in Table 1.9. With $e = (0, e^*, e^*, e^*)$, this graph structure is weakly pairwise stable. As we can follow from Figure 1.2, the only possible graph structure which can be obtained from $g_9$ by adding a new link is $g_{10}$. The stable $e = (0, e^*, e^*, 0)$ on $g_{10}$, does not bring any increase in utility to those who may want to form a link. The only possible graph obtained by violating a link is $g_5$. There is no stable effort level on $g_5$. This graph structure is approval stable and full approval stable with $e = (0, 0, 0, e^*)$. Agent 4 may want to add a link with agent 1, but agent 1 will not accept.

Table 1.9. Nash Equilibria, Stable Equilibria and Efficient Effort Profiles for $g_9$

| |
|---|
| $NE(g_9) = \{(e^*, 0, 0, e^*), (0, e^*, e^*, 0)(e^*/3, e^*/3, e^*/3, e^*/3)\}$ |
| $S(g_9) = \{(e^*, 0, 0, e^*), (0, e^*, e^*, 0)\}$ |
| $v(g_9, e) = 2b(e^*) + 2b(2) - 2c$ if $e$ is a specialized equilibrium,<br>$v(g_9, e) = 2b(e^*) - c4/3$ |
| $E(g_9) = \{(e_1, e_2, e_3, e_4) \mid b'(e_1 + e_2 + e_3) + b'(e_1 + e_2 + e_4) +$<br>$b'(e_1 + e_3 + e_4) + b'(e_4 + e_2 + e_3) = c/4\}$,<br>for example $(a_1, a_1, a_1, a_1) \in E(g_9)$ where $b'(a_1) = c/4$ |
| $W(g_9, (a_1/3, a_1/3, a_1/3, a_1/3) = 4b(a_1) - ca_1 4/3$ |

### 1.4.10. Type $\{3, 2, 2, 3\}$

Let $g_{10}$ be a graph of this type on which five links are formed.

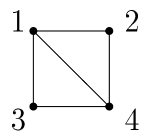



Figure 1.12. Graph of type $\{3, 2, 2, 3\}$: $g_{10}$

The set of Nash equilibria, the set of stable equilibria and the set of efficient effort profiles on $g_{10}$ are listed in Table 1.10. Under the benefit function $b = 2\sqrt{e}$, this structure with $e \in E(g)$, is efficient, resulting a total welfare of 16.

Table 1.10. Nash Equilibria, Stable Equilibria and Efficient Effort Profiles for $g_{10}$

| |
|---|
| $NE(g_{10}) = \{(e_1, 0, 0, e^* - e^*_e), (0, e^*, e^*, 0) \mid 0 \leq e_1 \leq e^*\}$ |
| $S(g_{10}) = \{(0, e^*, e^*, 0)\}$ |
| $v(g_{10}, e) = 4b(e^*) - c$ when $e \in NE(g_{10})$ and $e \neq (0, e^*, e^*, 0)$<br>$v(g_{10}, (0, e^*, e^*, 0)) = 2b(2) + 2b(e^*) - c$ |
| $E(g_{10}) = \{(e_1, 0, 0, a_3 - e_1) \mid 0 \leq e_1 \leq a_3$ and $b'(a_3) = c/4\}$ |
| $W(g_{10}, e) = 4b(a_3) - ca_3$ when $e \in E(g_{10})$ |

### 1.4.11. Type $\{3, 3, 3, 3\}$

This type represent the complete graph for four people as given in the Figure 1.13.

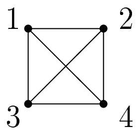



Figure 1.13. Graph of type $\{3, 3, 3, 3\}$: $g_{11}$

The set of Nash equilibria, the set of stable equilibria and the set of efficient effort profiles on $g_{11}$ are listed in Table 1.11. Under the benefit function $b = 2\sqrt{e}$, this structure with $e \in E(g)$ is efficient.

Table 1.11. Nash Equilibria, Stable Equilibria and Efficient Effort Profiles for $g_{11}$

| |
|---|
| $NE(g_{11}) = \{(e_1, e_2, e_3, e_4) \| e_1 + e_2 + e_3 + e_4 = e^*\}$ |
| $S(g_{11}) = \emptyset$ |
| $v(g_{11}, e) = 4b(e^*) - c$ |
| $E(g_{11}) = \{((e_1, e_2, e_3, e_4) \| b'(e_1 + e_2 + e_3 + e_4) = c/4\}$ |
| $W(g_{11}, e) = 4b(a_3) - ca_3$ where $e \in E(g_{11})$ and $e_1 + e_2 + e_3 + e_4 = a_3$ |

## 1.5. Conclusion

In this chapter of the thesis, our main aim was to explore the formation of network structures in a model with public goods. Our work contributes to the literature on network formation with public goods, by explicitly considering the formation of a network (as opposed to considering given and fixed network structures). We also deal with the question of multiplicity of equilibrium of effort levels exerted on a given network structure. We suggest stability definitions for such networks.

In our demonstrations of how the stability notions work, we use a linear cost structure and assume that the link formation is costless. We do not allow transfers among agents. The main result of the literature on the network formation continues to hold: the strong tension between efficiency and stability is still there. Any of the network structures that are defined as stable under different stability notions are not efficient.

As the first research question we can consider the implementation of efficient effort profiles on the networks by appropriately chosen allocation functions. The complete information of the model allows us to use many allocation functions and the binding agreements.

Another type of analysis can be considered involves costly links. With links that are costless to form for agents, they want to form as many links as they can. However, agents do not want the links created by their neighbors, as such links decrease the incentives of their neighbors to contribute to the public good. This substitutability nature of the efforts results in a conflict. When the links become costly, the opposing forces may be balanced in some proportion.

# CHAPTER 2

# DIGITAL GOODS: COPYING, PIRACY AND PROTECTION MECHANISMS

## 2.1. Introduction

Unauthorized reproduction of digital products has become easier with the advances in computer and information technologies. Whereas there are ethical issues concerning copying, more people are copying or using the copies of digital products as the copying techniques have become more user friendly, or the copies of the products are easily available for the users. Data related to the profit loss of the firms due to piracy is given frequently by statistics. It can be claimed that statistical data exaggerate the value of piracy or underestimate the current loss in the market due to copying. However, it is widely accepted that the firms in many digital markets incur losses from copying. Moreover, copying is of concern whether it causes losses or not.

Since the late 70s, the effects of the unauthorized reproduction of intellectual properties have been investigated in economic theory. Some of the discussions enlighten several aspects of copying concerning digital products. The conditions under which the firms lose due to copying, the conditions under which the firms can extract benefit from the copying, and even the conditions the firms can gain due to copying are analyzed in several studies. The markets of digital products such as software products, video and computer games, music and movies can be the application area for theoretical works. These markets change in their nature. Therefore, it is not surprising to consider these markets under different setups. The existence of the end-user copying or the commercial piracy on these markets cannot be rejected. However, piracy values excessively exaggerated relative to the market size is criticized by Peitz and Waelbroeck (2006). The authors claim that software products are complex. Hence, copying is not very attractive. Legal firms use protective devices in products, hence the copying of the products require some specialties. The support packs are important for some of the users. The cost of copying can be very high and it even may not be easy to correctly install the copied files. Some files might be missing or the copy can be of a very low quality. On the other hand, the benefit of the original is high, because software companies offer services, manuals, support-packs etc. When regulatory monitoring is applied for copyright protection, the probability of getting caught is high for companies when they make copies. Recently, the direct network effects are derived to be decreasing, because of the development of standards for file formats. Hence, the argument of using copies for the sale of originals is weakening. However, exceptions exist for the products where full compatibility is required. Moreover, software companies use the copies to give information about the product.

The market for the video and computer games is also interesting. We can observe that the protection policy has changed over the years. Because the quality difference between the copy and the original has increased with times, less precautions are taken by the firms in recent days. The copies of the games are not easily available from the internet and videogame producers sell their software with hardware devices. Thus, copying the software requires costly devices that cannot be used for other purposes. On the other hand, network effects are strong for the games of multi-players. Producers use this fact and sometimes give free versions of the games and then expect higher sales due to upgrading the product.

The music market is one of the most affected markets by end-user copying. The internet reduces the cost of copying. However, copying is still costly not just due to the time required, but copies may be so badly documented that users may download "wrong" files (spoof files). The network effects can be considered in this market, because the consumer may want to share experiences from downloaded gatherings. However, we cannot expect strong network effects. Lately, legal firms attempt to increase network effects in the market. Some official download sites create environments in which the users share their playlists. The internet can be seen as a substitute for marketing advertisement activities. Hence, the informational role played by the digital copies cannot be rejected especially for the music market.

Movies on DVDs are also subject to end-user copying. However, the effect of end-user copying is not as harsh on this market as it is on the music or games market. The cost of end-user copying of movies is very high compared to the music files. Storage of downloaded movie files requires a lot of space; sometimes a storage device is needed. Downloading a movie is also time consuming. For many consumers movies are watched only once or twice. Hence, consumers may prefer to rent movies. On the other hand, as the internet connections and the computer technologies are becoming faster, end-user copying has become more of a concern in recent days. The advances in the information technologies may lead the firms to appropriate the benefits of the digital area. Legal firms can offer digital versions of the movies from the internet. The real trouble for this market is commercial piracy. The cost of copying movies on DVDs is a fixed cost for the commercial pirates. Once the set-up for copying is constructed, then the marginal cost of copying is insignificant. A pirate firm can usually copy a movie on DVD in a quality very close to the original one. When the regulatory protection is high, the pirate firm may not reach the consumers. However, the pirate firm can easily reach the consumers on the streets and can charge very low prices if there is no protection. Therefore, we claim that an analysis on the market for movies on DVDs requires a model in which both a commercial pirate and end-user copying exists. To the best of our knowledge, there is no hybrid model where both commercial and end-user copying is considered.

The existing literature on the piracy of digital products investigates the issue from the viewpoint of copying by end-users. As opposed to general expectations, copying does not reduce the profit of the firms. Peitz and Waelbroeck (2006) survey the theoretical literature. They classify the research papers into four groups. We review the works in the literature according to this classification. The organization of this section is as follows: In the first subsection, we analyze the first group of articles in which there is no indirect appropriation of rents, no network externality, and consumers are perfectly informed about the quality of the products. In the second subsection, we demonstrate the second set in which the indirect appropriation of rent is possible. The appropriation can be across the consumers and across the products. In the third subsection, the third group of papers in which the network effects exist are given. In other words, the value that consumers give to the product depends on the decisions of other consumers. The fourth subsection is devoted to the fourth set of works which investigates asymmetric information. When there is no perfect information about the original product, copying can provide information to the consumers so that the demand for the original product increases. In the last subsection, we explain the similarities and differences of our model and the existing works in the literature.

The literature on end-user copying concentrates on two questions. Are the profits of the firms reduced by copying? What are the welfare implications of the copying? Peitz and Waelbroeck (2006) state that the answer to the first question is an obvious yes for the first group. The other groups expose conditions where the firms can gain from end-user copying. In most of the works, the second question is analyzed for given market structures and product decisions, in the short-run in particular. In the long-run, not only the economic environment but also the legal restrictions on the product decisions are important. In this sense, the socially optimal level of protection is not a deeply analyzed question in the existing works.

Copying alters the motivations of the firms to offer quality or to offer product variety. In the basic models, the original producer cannot extract a benefit from copying. Mainly, it is assumed that the copied product is of a lesser quality than the original one. The alternative of copying for the consumers limits the monopoly power of the original firm. The profit of the firm decreases. The firm can use different strategies against end-user copying. It can decrease the price so that all users prefer the original good, or it can allow some consumers to use the copies by charging higher prices. In order to fight against end-user copying, the firm or a regulator, such as the government, can design protection methods. Many works with different setups analyze these strategies. In some setups, the consumers' valuations for the product are the same, but the cost of copying changes from user to user. In other models, the heterogeneity of consumers with respect to copying costs is replaced by the heterogeneity of the valuations for the product. Mainly, two kinds of heterogeneity of consumers' valuation exist in the literature: (i) discrete types: high-valuation and low-valuation consumers. (ii) a continuum of types: the distribution is then typically assumed to be uniform in a specified interval. Some works model the consumers' utility with both heterogeneous valuations and heterogeneous copying costs.

### 2.1.1. Basic Models

Among many others, Novos and Waldman (1984), Johnson (1985), Yoon (2002), Belleflamme (2003), Bae and Choi (2006) analyze the social welfare implications of copying. Novos and Waldman (1984) consider a model in which the consumers are homogenous in their valuations for the product, but heterogeneous in the costs of copying. The monopolist produces the product in one quality and the copies are of the same quality of that value. The original gives the same utility to all consumers, but the copies give the same minus the cost of copying. Hence, if the price of the original good is not sufficiently low, then some consumers prefer copies to the originals. The monopolist chooses a lower quality than the socially optimal level, because the firm cannot extract all the benefits of quality improvements. Because the valuations of the consumers are the same, the increased copyright protection leads some consumers to use the original product instead of copying. Then, when the copying costs are taken into account, the increase in the protection level does not increase the social welfare loss due to underutilization. Bae and Choi (2006) confirm the same results. In their model, consumers vary in their valuations for the product. There exist two types of copying costs. The reproduction cost is fixed for all consumers, and the degradation costs change as the valuation of the consumer for the product changes. They show that the effects of the copying depend on the structure of the copying costs.

Chen and Png (2003) analyze social welfare when firms exert efforts to detect copying. Two groups of individuals exist in the setup. One group never copies and one group has copying costs. The consumers' expected utility depends on the detection probability. All consumers have heterogeneous valuations for the product where originals and the copies are identical. The firm can reduce the price or invest on detection technologies to increase the detection probability. They analyze the socially optimal government policy consisting of a tax, subsidy and penalties. They show that social costs arise when firms spend on detection. Chen and Png (2003)'s model is similar to Mussa and Rosen (1978)'s model in which consumers' heterogeneity comes from the utilities derived from the quality difference between the original and the copy. Consumers have unit demand and make a discrete choice between the original and the copy. Consumers differ in their willingness to pay for the products (the original and the copy). A consumer with a larger taste parameter has a higher willingness to pay. Belleflamme (2003) considers a similar model. He obtains the same results with Bae and Choi (2006). However, the long run analysis by Belleflamme is on extensive margin as the one by Johnson (1985). Johnson (1985) assumes that the valuations of the individuals vary horizontally. The cost of copying is heterogeneous for the consumers. There is no quality differentiation between the copied and the original products. Johnson (1985) focuses on the supplier response along extensive margin, so that the lower profits reduce the number of software products to be created.

Yoon (2002) considers a model in which protection adds costs to the consumers. Thus, the protection costs decreases consumer surplus directly and increases the profit of the monopolist. He shows that total surplus decreases at first and eventually increases as protection costs increase. The optimal protection level is the deterring level for copying. However, this also reduces the profits of the firms. On the other hand, Bae and Choi (2006), show that a higher level of protection decreases the demand for originals. This comes from the structure of the model. The cost of copying is type dependent due to copyright protection.

Alternative protection methods are suggested in different setups. Alvisi et al. (2002) analyze the pricing problem of the firm in a model very similar to Mussa-Rosen (1978) where there is no copying. Consumers' valuations come from a continuum type structure. The cost of copying for the consumers is of discrete type. For some consumers the copying costs are high, while for others the copying costs are low. They show that product differentiation can be the optimal response of the firm against copying. The firm can offer the product in high quality or low quality to discriminate between the consumers with high costs and low costs of copying. Optimal prices are characterized so that they are below copying costs for each product type and consumers with low copying costs do not have an incentive to copy high quality products. Cremer and Pestieau (2006) use a model in which consumers are of high valuation or low valuation. The type of the consumer is private information. The firm offers two sets of price-quality pairs. Individual copying costs increase on the quality of the product, q, and the level of copyright protection, e. e can be considered as a set determined by the firm corresponding to the technical protection or as a set determined by the government corresponding to public measures of copyright protection. The socially optimal level of protection and pricing is analyzed as well as profit maximizing pricing given that the protection level is determined by the government. Consumers have variable demands in the model by Sundararajan (2004), in which the profit-maximizing pricing decisions of firms are analyzed. A combination of the profit-maximizing and zero-copying pricing, where all consumers become indifferent between the original and its copies, is suggested. Sundararajan shows that if the firm uses digital rights management (DRM), it has to balance the value reduction with increased copying deterrence. Therefore, the author suggests an intermediate level of protection.

Takeyama (1997) considers a two-period model in which consumers are of two types, high valuation types have no propensity to copy and low valuation types have a strong propensity to copy. Without the copying problem, high type consumers expect a lower price in the second period, which makes the low types indifferent between buying and not buying. Then, the firm sets a price which is less than the valuation of the high type. The existence of copying causes greater loss in the profit. The firm should set the price in the second period as such that it makes the low types indifferent between buying and copying. Therefore, in the first period the price decreases. However, if the firm cannot deter in the second period then the originals are not sold in the second period so that the monopolist utilizes all profit from the high type consumers in the first period. In this case, due to end-user copying, the profit of the firm increases, even for a two period game.

Multi-product scenarios are considered in Johnson (1985), Belleflame and Picard (2004a, 2004b) When there is no copying, demands are independent. When copying is possible, if the cost of copying is a fixed cost, the demands of the products become interdependent. Then, a firm can increase its profits whenever the other firms set lower prices, because as the prices of the goods decrease, the consumer's incentive to buy the copying technology decreases. If the products are provided by a monopoly, whenever the cost of copying is sufficiently high, the firm sets the unconstrained monopoly price, otherwise it applies the deterring strategy. However, if each product is produced by a different firm, each firm applies the accommodate strategy.

The copying cost, which is an important determinant in differentiating whether the original and its copies are close substitutes or not, is affected by the copyright protection policy used by the firm that provides the product. Two types of protection policies are suggested in the literature. A broad-based enforcement policy affects all consumers in the same way. A targeted enforcement policy aims at high-value consumers. Harbaugh and Khemka (2001) compare the targeted and the broad-based copyright enforcements in a model of vertical differentiation with a continuum of consumer types. The former makes the copying more costly than the latter. Therefore, the group sizes of high and low valuation type consumers are determined by the scope of copyright enforcement. The optimal pricing under the targeted policy is greater than the monopoly pricing without end-user copying. Therefore, the surplus of the targeted consumers is reduced under optimal pricing when compared to the no copying situation. Thus, the incentives to enforce the copyright protection with the targeted policies may be insufficient from the social perspective.

Basic models, which assume consumer heterogeneity and product differentiation between the original and its copies, are considered as a variant of models in which a single product of high and low qualities exists. Consumers who are not willing to pay for the quality tend to use the copy. Therefore, the firms compete with end-user copying and have to lower their price. End-user copying expands the market and the consumer surplus increases in the short run. The cost of copying costs, which is compared to the marginal cost of production and the cost of protecting the product, plays an important role in the analysis of social welfare. When protection is costly, the society may prefer little or no protection. Most of the works deal with the analysis in the short term. The creativity issue should be taken into consideration in the long-term analysis.

### 2.1.2. Models with Indirect Appropriation

The second group of articles in the literature considers the situation where the firm extracts a benefit from copying. If the population is homogeneous, or the firm can discriminate the price effectively according to the consumers in the market, then they can appropriate from copying. Besen and Kirby (1989) consider a model of small-scale copying. The amount of direct appropriation rises if the marginal cost of copying is constant, and the amount of indirect appropriation rises if the marginal cost of copying is increasing with the number of copies. If the original and its copy are strong substitutes, and the marginal cost of copying increases with the number of copies, then the consumers create clubs to share the cost of the original. Then, the firm can set a higher price when the good is used by these clubs. Bakos et al. (1999) explain why the existence of clubs may increase the copyright owner's profits. The model highlights two contradictory facts: Aggregation effect and club diversity. The analysis presumes that clubs can extract all of consumer surplus. Varian (2000) analyzes the profitability of a monopolist selling to a club where the members pay the same price for the product. He shows that it may be more profitable to sell the product to the club instead of selling it to consumers directly, because of the cost effects.

Indirect appropriation may be possible due to the complementary goods. When the copy of a product is consumed together with a good which cannot be copied, copying can be seen as a smaller problem. For example, downloads can increase the popularity of an artist, which in the end, can increase the ticket sales for a concert. Gayer and Shy (2006), Curien et al. (2004), Krueger (2005) and Connolly and Krueger (2006) analyze such issues. The artists can be more tolerant of the Peer-to-Peer (P2P) networks than big record labels, because the concerts' profits go directly to the artists.

### 2.1.3. Models with Network Effects

The third group of models considers the network effect related with digital products. When network effects exist, the utility of the consumer depends on the consumption decisions of other users. Under some conditions, the network effects can imply that protection is not preferred by the society, or by the firm. Conner and Rumelt (1991) assume network effects in a model where the market demand is linear and consumers' valuations for the original good are heterogeneous. They show that end-user copying can increase or decrease the prices charged by the firm and the profits of the firm. Takeyama (1994) uses a model similar to the one of Conner and Rumelt (1991). However, Takeyama (1994)'s model considers full enforcement of copyright protection. The profit-maximizing strategy under no protection can create higher profits than the profit maximizing strategy with full protection. The author also analyzes the social welfare implications. When the firm sells to high valuation consumers only, and the end-user copying increases the profits of the firm, the Pareto improvement can occur with copying. Similarly, Gayer and Shy (2003b) consider a monopoly model in which the original and the copy are horizontally differentiated. The original may give higher fixed utility and result in stronger network effects. When network effects for the original are sufficiently strong, end-user copying can increase the profit of the firm.

Belleflamme (2003) extends his basic model, adding network effects. The difference from the model by Takeyama (1994), where the network size dependent utilities are type-dependent, is that only the fixed utilities depend on the network size. Moreover, unlike Takeyama, where only two types of consumers exist, he considers a continuum of the consumers. He concludes that with limited network effects, end-user copying always leads to profit losses. King and Lampe (2003) show that when a monopoly offers a downgraded version of the product with a lower but a positive price in order to benefit from the network effect, the end-user copying becomes difficult. If the cost of downgrading is high, then the monopoly cannot use this strategy. The copying can lead to higher profits for the firm who sells a single quality product. Gayer and Shy (2003a) analyze the effect of hardware taxation on the demand for software when network effects exist. They assume that all consumers buy the hardware. The copying decreases due to a tax on hardware. On the other hand, the demand for the software decreases as the prices of the pair consisting of the hardware and the software increase. They show that the profit-maximizing tax rate is below the tax rate that eliminates piracy.

Shy and Thisse (1999) extend the model of Conner and Rumelt (1991) to a duopoly framework. There exist two types of consumers: High value and low value types. Consumers are distributed in a Hotelling style between the two firms. Two types of equilibria appear under protection. If the network effects are sufficiently strong, the prices of both firms are low to sell some of the low types, and if the network effects are sufficiently weak, the firms set their prices so high that they sell only to high types.

#### 2.1.4. Models with Imperfect Information

When the consumers are not well informed about the digital product, copies can provide some information about the digital product. Therefore, the end-user copying may have some positive effects on the sale of digital products besides its negative effects on prices. Gopal et al. (2006) consider a model in which a single product is sold at a fixed price. Consumers do not have perfect information about the product. If downloading is not possible then some consumers have no idea about the product and do not buy it. Downloading provides information about the product so that intermediate consumers who like the product buy it. Therefore, due to the copies, the profit of the firm increases. Peitz and Waelbroeck (2006) assume a model in which the digital copies and P2P are the only way of gathering information about products. A firm sells different products which are horizontally differentiated. P2P allows sampling to users. Without digital copies, consumers choose among the products at random. The authors show that if the degree of product differentiation and the number of products is sufficiently large, then the firm can benefit from the informational role of digital copies that induce in the consumers higher willingness to pay. In another model, Peitz and Waelbroeck (2004) show that P2P reduce the cost of marketing and promotion. They conclude that because of digital copies, the revenues can decrease. However, the reduction in advertisement costs may actually increase the profit of the firms.

Duchene and Waelbroeck (2005) consider a model of P2P in which an original is more valuable than the copies, and the consumers are heterogeneous in the time they spend online searching for files. They assume that the only way of having information about the original product is digital copies. They show that increasing copyright protection directly decreases copying, but at the same time the demand for the product decreases as technological protection and prices increase. Zhang (2002) sees P2P technologies as a way for marginal artists to enter the market. Two horizontally differentiated goods are considered in the model. The star artist is supported by a big label and the marginal artist has no back up. The fixed prices are exogenously given, and are the same for each artist's product. Consumers are on unit interval as in Hotelling pricing. The products are asymmetric as if there is no digital copying, because the big label markets the star. In the model, it is assumed that only some consumers have access to digital copies and a fraction of these consumers purchase an original product after they download the product. The marginal artist distributes his product with digital copies. Some consumers who download the products of the marginal artist also purchase them legally. Therefore, the marginal artist can gain by P2P. Anderson (2004) exhibits an analysis on the informational role of the digital copies for niche artists, too.

If there is adverse selection in the market, copies can solve the problem.

Takeyama (2003) considers a monopoly that provides a good with one of two qualities, high or low. The information on the quality is private to the firm. The model is dynamic in two periods. Consumers are defined to be on the unit interval and of two types. One type never copies under some parameter restrictions, the other type can copy in the first period at price c. It is assumed that, due to the adverse selection problem, the firm cannot profit by high quality without copies. When copies are available, a consumer can see the quality of the product in the first period by copying the product. The copies are of less value than the originals. Therefore, consumers can purchase the product in the second period. A pooling equilibrium is indicated where the monopolist price-discriminates intertemporally, selling the consumers that purchase the low quality good in the first period and pricing other consumers in the second period. In a separating equilibrium, the existence of copies is suggested as a signal for high quality.

#### 2.1.5. The Differences and Similarities

The literature on the commercial piracy issue can be considered to start with the work by Banerjee (2003). He analyzes the government's role in restricting commercial piracy. A monopolist and a pirate firm who copies the product of the legal firm, are presented in the model. Banerjee examines the pricing strategies of firms when the pirate firm is a follower, or when both firms decide the pricing at the same time as in Hotelling's problem. They show that if it is profitable for a monopolist to prevent piracy by installing a protective device, then not monitoring is the equilibrium.

In our work, we model a market in which both end-user copying and commercial piracy exist. As the main example, we use the market of movies in DVD formats. In the model, we assume that a monopolistic firm M and a commercial pirate firm P play a sequential pricing game, where the firm M is the leader. Consumers can buy the copies from the commercial pirate, P. Moreover, consumers can copy the product by themselves using the internet or other sources. Our model is different from the literature in the sense that we consider commercial piracy and end-user copying together. On the other hand, we do not take into account indirect appropriation, network externalities and incomplete information. Therefore, our work belongs to the first group as classified by Peitz and Waelbroeck (2006). Table 2.1 on the next page summarizes the similarities and differences between our work and the existing studies.

In the first section, we will analyze the pricing strategy of a legal firm, firm M, when there is no copying choice for the consumers. The second section is devoted to the case when the firm M produces the good while consumers can copy the good by themselves. We assume that for now the pirate firm does not exist. In the third section, we show that the existence of the pirate firm P reduces the profit of the legal firm M even more. In the fourth section, we demonstrate the results when governmental protection is taken against the commercial pirate firm. In the fifth section, we analyze the market when firm M differentiates the product in quality and offers one more alternative to the users. We show that this action increases the firm's profit against piracy.

## 2.2. The Model

A firm, M, produces a digital product. A pirate firm, P, illegally produces the product and sells it to end-users. Each consumer uses only one unit of the product. There is no resale market for used goods.

In this set up, we define a sequential game in which first the firm M announces the market price of the good, $p_m$, then firm P copies the product and sells it at price $p_p$. The quality of the original product is $q_m$ and the quality of the product copied and sold by the pirate firm is $q_p$. Another alternative for consumers is that they can use the product by copying from other sources such as the internet. If so, the quality of the copied good is $q_c$. We assume that there are

Table 2.1. Properties of the Models in the Literature

| Chen & Png (2003) | Yoon (2002) | Banerjee (2003) | Alvisi et al. (2002) | Bae and Choi (2006) | Işık |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| Uniform | Uniform | Uniform | Uniform | Uniform | Discrete |
| No cost | Homogenous | No end-user copying | Discrete | Homogenous/ Heterogeneous | Discrete |
| No | No | Yes | No | No | Yes |
| No | No | No | No | No | No |
| Yes | No | Yes | No | Yes | Yes |
| Yes | No | Yes | No | Yes | Yes |
| No | Constant | No | No | No | No |
| No | No | Yes | No | Yes | No |

quality differences between the products that come from different sources. The quality of the product produced by the monopolist, $q_m$, has the highest quality. The pirate firm produces the good in quality $q_p$. We assume that qualities are exogenously given as $q_m \geq q_p \geq q_c$ and common knowledge to all.

The difference between quality is supposed to represent the fact that the copies are not supported by their sources so that they may turn out to be defective. When a consumer buys a product such as a movie DVD from a pirate, the images can be of very low quality. The situation is not different for the end-user copying case. After spending hours on the internet, it is likely that the downloaded file is of unsatisfactory quality or even a spoof.

There exist $N$ consumers in the model. Consumers are of two types, indexed by $\theta$, according to their valuations, where $\theta \in \{\theta_s, \theta_b\}$. Consumers of type $\theta_b$ have higher valuation for the product than consumers who are of type $\theta_s$, i.e. $\theta_s < \theta_b$. There exist $N_b$ people who are of type $\theta_b$ and $N_s$ people who are of type $\theta_s$. We denote these groups by high types and low types, correspondingly. The heterogeneity of the consumers in their valuations represents factors such as DVD usage frequency, degree of utilization, and so on.

We assume that buying from the firm M or from the pirate P is not the only way of using the product. Consumers can copy/reproduce the product by themselves. Internet (P2P networks), personal decoders and writers are possible tools of end-user copying. We do not take into account commercialized end-user copying actions or the commercial clubs in which the end-users share the product they have. We assume that the consumers are heterogeneous in the cost of end-user copying, too. Some consumers can copy the product by themselves more easily compared to the others. The cost of copying for these advanced users is smaller and denoted by $c_s$. The cost of copying for the rest is assumed to be higher and denoted by $c_b$. $c$ stands for the cost of copying where $c \in \{c_s, c_b\}$, and $c_s < c_b$. Some consumers cannot copy the product easily because of factors such as no easy access to the internet, no time to deal with such an activity, no information on the tools for copying, and so on. There exist $N_b^s$ consumers who have higher valuation for the product and smaller costs of copying. We call this group of users high types and small costs to copy. We name the consumers who are also of high type but have higher costs of copying as high types and big costs to copy as $N_b^b$. Similarly, consumers who have low valuations and larger costs of copying will be named as low types with small costs to copy and will be denoted by $N_s^b$. We call consumers who have smaller valuations and smaller costs of copying as low types with small costs to copy as $N_s^s$. At the end, $N_b^b + N_b^s = N_b$ and $N_s^b + N_s^s = N_s$.

We assume that the cost of developing or copying the product is sunk. Once the product is developed or copied, the marginal cost of production is negligible. When the demand for the original product is $D_m$ and the demand for the copied product by P is $D_p$, the profit of the firm M and illegal firm P becomes, $\pi_m = p_m D_m$ and $\pi_p = p_p D_p$, respectively.

Consumer surplus is defined as the total utility of society. We define social welfare as the sum of the profits of the firms and the consumer surplus.

### 2.3. No Copying in the Market

Following Tirole (1988), when any kind of copy is not available, the utility of a type $\theta$ consumer from using the good is,

$$u(\theta) = \begin{cases} \theta - p_m & \text{if the consumer buys the product} \\ 0 & \text{otherwise} \end{cases} \tag{2.1}$$

A consumer of type $\theta$ buys the product whenever $\theta - p_m \geq 0$. Then, when the monopolist is the only alternative for the consumers, there will be two different price choices of the monopolist. In the first one, in order to make all the consumers purchase the product, the monopolist sets the price as $p_m = \theta_s$. The demand for the good becomes, $D_m(p_m) = (N_s + N_b)$.

The profit of the monopolist is calculated for the first price option as,

$$\pi_m^1 = \theta_s(N_s + N_b). \tag{2.2}$$

The firm can set a higher price so that $p_m = \theta_b$. In this case, the low types do not buy the product. The demand for the good becomes, $D_m(p_m) = N_b$ and the profit of the firm is,

$$\pi_m^2 = \theta_b N_b. \tag{2.3}$$

The monopolist has a higher profit adopting the higher price strategy if

$$N_b(\theta_b - \theta_s) > \theta_s N_s. \tag{2.4}$$

Equation (2.4) shows that, the monopolist is more likely to prefer the high price equilibrium and sell only to high valuation types whenever the size of high types is larger and the difference between the high and small valuations is greater. Under this assumption, the monopoly equilibrium results are,

$$\begin{aligned} p_m^* &= \theta_b, \\ \pi_m^* &= N_b \theta_b, \\ CS &= 0 \end{aligned} \tag{2.5}$$

## 2.4. No Commercial Pirate in the Market

In this section, we analyze the market if there is no pirate firm P. After the firm M announces the price for the quality $q_m$, consumers decide to buy the product of quality $q_m$ at the price $p_m$ or to use the product of quality $q_c$ by copying the product at the cost $c_b$ or $c_s$. We assume that with such copying costs and quality of copies, copying is always better than not using the product for all groups of consumers in the market. Therefore, all consumers in the market are indeed users of the good, i.e. we assume a covered market.

The firm M can charge different prices in the subgame equilibrium. We assume that if the firm M sets $p_m$ as $p_m \leq (q_m - q_c)\theta_s + c_s$, then all users prefer $q_m$. In this case, the equilibrium price is $p_m = (q_m - q_c)\theta_s + c_s$, $D_m = N$. Therefore, the profit of the firm becomes $\pi_m = N[(q_m - q_c)\theta + c_s]$.

We assume that the high types with small costs to copy value the product $q_m$ are greater in number than the low types with big costs to copy. In other words, $(c_b - c_s) < (q_m - q_c)(\theta_b - \theta_s)$. Thus, if the firm M sets $p_m$ as $(q_m - q_c)\theta_s + c_s < p_m \leq (q_m - q_c)\theta_s + c_b$, then the whole market except the low types with small costs to copy prefer $q_m$. In this case, the equilibrium price is $p_m = (q_m - q_c)\theta_s + c_b$, $D_m = N_b + N_s^b$. Therefore, the profit of the firm becomes $\pi_m = N_b + N_s^b[(q_m - q_c)\theta_s + c_b]$.

If the firm M sets $p_m$ as $(q_m - q_c)\theta_s + c_b < p_m \leq (q_m - q_c)\theta_b + c_s$, then all high types prefer $q_m$. In this case the equilibrium price is $p_m = (q_m - q_c)\theta_b + c_s$, $D_m = N_b$. Therefore, $\pi_m = N_b[(q_m - q_c)\theta_b + c_s]$.

If the firm M sets $p_m$ as $(q_m - q_c)\theta_b + c_s < p_m \leq (q_m - q_c)\theta_b + c_b$, then all high types prefer $q_m$. In this case the equilibrium price is $p_m = (q_m - q_c)\theta_b + c_b$, $D_m = N_b^b$. Therefore, $\pi_m = N_b^b[(q_m - q_c)\theta_b + c_b]$.

Among these four equilibrium prices, the firm M prefers the $p_m$ which results in the highest profit. The highest profit depends on the conditions of the population distribution, the difference between the copying costs, $c_b - c_s$, the difference between the valuations of the consumers, $\theta_b - \theta_s$, and the difference between the qualities of the product, $q_m - q_c$, as follows: $p_m = (q_m - q_c)\theta_s + c_b$ is dominant to $p_m = (q_m - q_c)\theta_s + c_s$ if

$$\frac{N_b + N_s^b}{N_s^s} \geq \frac{(q_m - q_c)\theta_s + c_s}{c_b - c_s}.$$

$p_m = (q_m - q_c)\theta_b + c_s$ is dominant to $p_m = (q_m - q_c)\theta_s + c_b$ if

$$\frac{N_b}{N_s^b} \geq \frac{(q_m - q_c)\theta_s + c_b}{(q_m - q_c)(\theta_b - \theta_s) - (c_b - c_s)}.$$

$p_m = (q_m - q_c)\theta_b + c_b$ is dominant to $p_m = (q_m - q_c)\theta_b + c_s$ if

$$\frac{N_b^b}{N_b^s} \geq \frac{(q_m - q_c)\theta_b + c_s}{(c_b - c_s)}.$$

Whenever the number of people in each group, qualities of the product, costs of copying and the valuations of the people on the product satisfy the conditions above, we can conclude that the higher prices, which mean lower demands, return higher profits to the firms in the market. Then, firm M chooses the highest price $p_m = (q_m - q_c)\theta_b + c_b$. If the conditions do not hold in the market, then the firm M prefers the lowest price $p_m = (q_m - q_c)\theta_s + c_s$, and the whole market uses $q_m$. In any case, the existence of the end-user copying alternative causes further losses for monopoly. End-user copying reduces the monopoly power of the firm.

### 2.5. Presence of a Commercial Pirate in the Market

In this section, we assume that a commercial pirate enters the market. Besides, tools which make end-user copying possible for the users also exist in the market. We assume that $q_c\theta_s - c_b > 0$. Therefore, the low types with big costs to copy can still copy the good. Hence, the market is again a covered one.

Each consumer can use the good at price $p_m$ if he buys the original good from the firm M. He can use the good at price $p_p$ if he buys the copied good from firm P. He can use the good at no price if he copies the good by himself at copying cost $c$. We can set the utility of a type $\theta$ consumer who has copying costs $c$ as,

$$u(\theta, c, p) = \begin{cases} q_m\theta - p_m & \text{if } p = p_m \\ q_p\theta - p_p & \text{if } p = p_p \\ q_c\theta - c & \text{if } p = 0 \end{cases} \tag{2.6}$$

Population $N$ is diversified into four groups according to the valuations and their access to end-user copying tools. Then, the utilities of individuals when they use the product changes according to their types.As before, $\theta \in \{\theta_s, \theta_b\}$ with $c \in \{c_s, c_b\}$ . When an individual who is of high type with big costs and an individual who is of high type with small costs buy the product from firm M or pirate P, they will have the same utility $u(\theta_b, c_s, p_k) = u(\theta_b, c_s, p_k) = (q_k\theta_b - p_k)$. Similarly, when an individual who is of low type with big costs to copy and an individual who is of low type with small costs to copy buy the product from firm M or pirate P, they will have the same utility $u(\theta_s, c_s, p_k) = u(\theta_s, c_s, p_k) = (q_k\theta_s - p_k)$ where $k \in \{m, p\}$. Then, we will denote the utilities as

$$\begin{aligned} &u_i(p_k) = u(\theta_i, c_j, p_k) \text{ where } i \in \{s, b\} \text{ and } k \in \{m, p\} \\ &u_i^j(0) = u_i^j. \end{aligned} \tag{2.7}$$

We assume that $q_c(\theta_b - \theta_s) > c_b - c_s$. Therefore, when individuals copy by themselves, the utilities are ordered as:

$$u_s^b < u_s^s < u_b^b < u_b^s. \tag{2.8}$$

We assume that the firm M and the pirate P play a pricing game in which first the firm M announces the price $p_m$ expecting a pirate firm will follow it. Sequentially, the pirate P announces $p_p$. Then, consumers decide to buy the product from M, to buy the product from P, or to copy the product by themselves. Firm M sets the price, $p_m$, according to the best responses of the pirate firm P.

We assume that when an individual is indifferent about the products from two different sources, he prefers the one of the higher quality. Thus, firm M should set the price $p_m$ in such a way that the utility of the users of $q_m$ should be as large as the utility when they copy the product by themselves, or when they buy the product from pirate P.

When firm M announces $p_m$, the pirate P can announce $p_p$ as $p_p < p_m-(q_m-q_p)\theta_b$ so that all individuals in the market prefer $q_p$ to $q_m$ or $p_p < p_m-(q_m-q_p)\theta_s$ so that only low types prefer $q_p$ to $q_m$. Then, in any equilibrium, firm M should set the $p_m$ considering the least price $p_p$ that pirate P can charge, and firm M should set $p_m$ as

$$p_m \leq p_p + (q_m - q_p)\theta_b. \tag{2.9}$$

The different types of consumers have different utilities when they copy the product by themselves. Firm M should set different prices to persuade the different types of consumers to use the quality $q_m$. When firm M sets $p_m$ as $p_m^1 = (q_m - q_c)\theta_s + c_s$, then low types with small costs to copy prefer $q_m$ to $q_c$. If $p_m$ is set as $p_m^2 = (q_m - q_c)\theta_s + c_b$, low types with big costs to copy prefer $q_m$ to $q_c$. If $p_m$ is set as $p_m^3 = (q_m - q_c)\theta_b + c_s$, then high types with small costs to copy prefer $q_m$ to $q_c$. If $p_m = (q_m - q_c)\theta_b + c_b$, then high types with big costs to copy prefer $q_m$ to $q_c$. The order of the prices determine the possible demanders of the $q_m$. $p_m^1$ is the least price. Therefore, when $p_m = p_m^1$, all consumers prefer $q_m$ to $q_c$. Which one of the prices, $p_m^2$ or $p_m^3$ is lower, depends on the following condition:

$$c_b - c_s < (q_m - q_c)(\theta_b - \theta_s). \tag{2.10}$$

When the inequality in (2.10) holds, then $p_m^2 < p_m^3$. Hence, if $p_m = p_m^2$, all consumers but the low types with small costs prefer $q_m$ to $q_c$. When $p_m = p_m^3$, then high types prefer $q_m$ to $q_c$. If $p_m = p_m^4$, then high types with big costs to copy prefer $q_m$ to $q_c$. The assumption in (2.10) means that the high types with small costs to copy value $q_m$ more than the low types with big costs to copy do. When $p_m^2 > p_m^3$, if $p_m = p_m^3$ all consumers but the low types with small costs prefer $q_m$ to $q_c$ and, if $p_m = p_m^2$, then low types with big costs to copy and high types with big costs to copy prefer $q_m$ to $q_c$. We will assume that (2.10) is true in the rest of the work. However, this does not change the general form of the analysis. Similar results hold for $p_m^2 > p_m^3$.

The same pricing analysis holds for the pirate P. Under the assumption (2.10), we will analyze the equilibrium of the game for two cases

$$c_b - c_s > (q_p - q_c)(\theta_b - \theta_s), \tag{2.11}$$

$$c_b - c_s < (q_p - q_c)(\theta_b - \theta_s). \tag{2.12}$$

Depending on the population distribution, the qualities $q_m$, $q_p$, $q_c$, the difference of the valuations of the people, $\theta_b - \theta_s$, and the difference of the copying costs $c_b - c_s$, the firm M can target four different segments of consumers. The assumption in (2.10) identifies the four cases as follows: In the first case, $p_m$ is so low that the whole market demands $q_m$. In the second case, firm M targets all except the low types with small costs to copy. In the third case, all high types can demand $q_m$. In the fourth case, firm M sets $p_m$ so high that only high types with big costs to copy can demand $q_m$. If $c_b - c_s > (q_m - q_c)(\theta_b - \theta_s)$, in the third case, low types with big costs to copy and high types with big costs to copy demand $q_m$.

Some of the four cases do not constitute an equilibrium. We will show that under the assumption $c_b - c_s < (q_p - q_c)(\theta_b - \theta_s)$, the first and the third cases are subgame equilibria. If $c_b - c_s > (q_p - q_c)(\theta_b - \theta_s)$, then the first, the second and the third cases constitute subgame equilibria.

### 2.5.1. Case 1

When firm M targets the whole market, it should offer a price so that no one copies the product by himself and no one buys the product from the pirate P. $p_m \leq (q_m - q_c)\theta_s + c_s$ guarantees that $u_i^j < u_i^j(p_m)$ for all $i, j \in \{b, s\}$, so that all types in the society prefer to buy from M rather than to copy. Then, the pirate P can set the price $0 \leq p_p \leq p_m - (q_m - q_p)\theta_s$. Therefore, the firm M should set $p_m$ so that no one prefers $q_p$ to $q_m$. Thus, $p_m = (q_m - q_p)\theta_s$, $p_p = 0$ with $D_m = N$, $D_p = 0$ constitute a subgame equilibrium. We call this equilibrium equilibrium 1.

### 2.5.2. Case 2

If the firm M sets a higher price $(q_m - q_c)\theta_s + c_s < p_m \leq (q_m - q_c)\theta_s + c_b$, targeting low types with big costs, and all high types, then, $u_s^s > u_s^s(p_m)$, but for the rest of the society $q_m$ brings higher quality. The pirate P can set the price as $(q_p - q_c)\theta_s + c_s \leq p_p \leq p_m - (q_m - q_p)\theta_s$. Therefore, in order to derive the demand as $D_m = N_s^b + N_b$, the firm M should set the price $p_m = (q_m - q_c)\theta_s + c_s$, by the equation (2.9). However, this price results in the equilibrium in case 1.

### 2.5.3. Case 3

If the firm M wants to set a high price so that only high types demand $q_m$, then $(q_m - q_c)\theta_s + c_b < p_m \leq (q_m - q_c)\theta_b + c_s$ guarantees that $u_b^b < u_b^b(p_m)$ and $u_b^s = u_b^s(p_m)$. Now, all high types prefer $q_m$ to $q_c$. However, the pirate P can set the price so low that the whole market prefers $q_p$, namely, $p_p = (q_p - q_c)\theta_s + c_s$, or the pirate P can select a higher price than this price so that low types with small costs to copy do not prefer $q_p$.

When $c_b - c_s < (q_p - q_c)(\theta_b - \theta_s)$, if the pirate P sets $p_p = (q_p - q_c)\theta_s + c_b$, then all types except low types with small costs to copy prefer $q_p$. If the condition below holds, then pirate P can increase its profit by charging a higher price than the low types with big costs to copy want to pay.

$$\frac{N_s^b}{N_s^s} \geq \frac{(q_p - q_c)\theta_s + c_s}{c_b - c_s} \tag{2.13}$$

The numerator of the right hand side of the equation (2.13) is the value that the low types with small costs give to $q_p$. For different values of the variables in the equation (2.13), the inequality holds. However, we have assumed that $c_b - c_s < (q_p - q_c)(\theta_b - \theta_s)$. Therefore, the right hand side of the equation is greater than 1. Then, the condition in equation (2.13) requires that $N_s^b$ is greater than $N_b^s$ for sure. Hence, in this case, pirate P sets the price that $p_p = (q_p - q_c)\theta_s + c_b$. Therefore, equation (2.9) implies that $p_m = (q_m - q_c)\theta_b + c_b - (q_p - q_c)(\theta_b - \theta_s)$, $p_p = (q_p - q_c)\theta_s + c_b$, $D_m = N_b$, $D_p = N_s^b$ constitute an equilibrium. We call this equilibrium 2.

If the assumption on the population does not hold, then, $p_p = (q_p - q_c)\theta_s + c_s$ and $p_m = (q_m - q_c)\theta_b + c_s - (q_p - q_c)(\theta_b - \theta_s)$. Then, $D_m = N_b$ and $D_p = N_s$. No one in the society copies the product by himself. We call this equilibrium equilibrium 3. Low types with big costs to copy value $q_m$ as much as $p_m^2 = (q_m - q_c)\theta_s + c_b$. $p_m = (q_m - q_c)\theta_b + c_s - (q_p - q_c)(\theta_b - \theta_s)$ is greater than $p_m^2$ if

$$c_b - c_s < (q_m - q_p)(\theta_b - \theta_s). \tag{2.14}$$

Otherwise, $p_m = (q_m - q_c)\theta_b + c_s - (q_p - q_c)(\theta_b - \theta_s)$ is less than $p_m^2$. Thus, low types with big costs to copy value prefer $q_m$ to $q_c$. However, they prefer $q_p$ to $q_m$ in this equilibrium.

When $c_b - c_s > (q_p - q_c)(\theta_b - \theta_s)$, if the pirate P sets the price as $p_p =$

$(q_p - q_c)\theta_b + c_s$, then, all types except low types with small costs to copy prefer $q_p$. The other option of pirate P is to set $p_p$ as $p_p = (q_p - q_c)\theta_s + c_s$. Because of the condition $c_b - c_s > (q_p - q_c)(\theta_b - \theta_s)$, assumption (2.13) on the population implies that the pirate P prefers a higher price. In this case, $p_m = (q_m - q_c)\theta_b + c_s$, $p_p = (q_p - q_c)\theta_b + c_s$, $D_m = N_b$, $D_p = N_s^b$ constitute an equilibrium. We call this equilibrium as the equilibrium 4. When all variables in the equation (2.14) are the same but the small costs to copy are reduced so that (2.14) does not hold anymore, then, equilibrium 4 holds instead of equilibrium 2. This implies that $p_m$ decreases to hold the same demand in the market. So the profit of the firm decreases. In other words, we can expect less profits in the future as the copying costs decrease is satisfied by these equations.

When the assumption on the population does not hold, the pirate P gives the price as $p_p \geq (q_p - q_c)\theta_s + c_s$, and then the values of the prices, demands and profits are as in equilibrium 3. Because $c_b - c_s > (q_p - q_c)(\theta_b - \theta_s)$, we call this equilibrium as equilibrium 5.

### 2.5.4. Case 4

If M targets the high types with big costs to copy, then $(q_m - q_c)\theta_b + c_s < p_m \leq (q_m - q_c)\theta_b + c_b$. Therefore, the high types with big costs to copy prefer $q_m$ to $q_c$. If $c_b - c_s < (q_p - q_c)(\theta_b - \theta_s)$, then the follower firm, pirate P, can set the price as $p_p \geq (q_p - q_c)\theta_s + c_s$ (then the corresponding demand for $q_p$ is $D_p \leq N_s^s + N_s^b + N_b^s$) or $p_p \geq (q_p - q_c)\theta_s + c_b$ (the demand for $q_p$ becomes $D_p \leq N_s^b + N_b^s$) or $p_p \geq (q_p - q_c)\theta_b + c_s$ (the demand for $q_p$ is $D_p \leq N_b^s$). Whatever the choice of pirate P is, firm M should set its price as $p_m \leq (q_m - q_c)\theta_b + c_s$. With this pricing, the analysis follows the pattern in case 3.

If $c_b - c_s > (q_p - q_c)(\theta_b - \theta_s)$, then pirate P can set the price as $p_p \geq (q_p - q_c)\theta_s + c_s$(with the corresponding demand for $q_p$ as $D_p \leq N_s^s + N_s^b + N_b^s$) or $p_p \geq (q_p - q_c)\theta_b + c_s$ ( then the demand for $q_p$ is $D_p = N_s^b + N_b^s$) or $p_p \geq (q_p - q_c)\theta_s + c_b$

(so the demand for $q_p$ becomes $D_p = N_s^b$) If

$$\frac{N_b^s + N_s^b}{N_s^s} \geq \frac{(q_p - q_c)\,\theta_s + c_s}{(q_p - q_c)(\theta_b - \theta_s)} \tag{2.15}$$

then, pirate P prefers the second price $p_p \geq (q_p - q_c)\theta_b + c_s$ to the first one $p_p \geq (q_p - q_c)\theta_s + c_s$. Indeed, this assumption implies (2.13). With the assumption below, pirate P prefers the third price $p_p \geq (q_p - q_c)\theta_s + c_b$ to the second price $p_p \geq (q_p - q_c)\theta_b + c_s$:

$$\frac{N_b^s}{N_s^b} \geq \frac{(q_p - q_c)\,\theta_s + c_b}{c_b - c_s - (q_p - q_c)\,(\theta_b - \theta_s)}, \tag{2.16}$$

then, (2.15) and (2.16) imply that pirate P sets the price as $p_p \geq (q_p - q_c)\theta_s + c_b$. Therefore, it should be true that $p_m \leq (q_m - q_c)\theta_b + c_b - (q_p - q_c)(\theta_b - \theta_s)$. So, $p_m = (q_m - q_c)\theta_b + c_b - (q_p - q_c)(\theta_b - \theta_s)$ with $D_m = N_b^b$ and $p_p = (q_p - q_c)\theta_s + c_b$ with $D_m = N_s^b$ constitute an equilibrium. We call this equilibrium 6.

If the assumption (2.15) or (2.13) does not hold, then $p_p = (q_p - q_c)\theta_b + c_s$ or $p_p = (q_p - q_c)\theta_s + c_s$. Therefore, $p_m \leq (q_m - q_c)\theta_b + c_s$. With this pricing the analysis turns into the analysis in case 3.

We obtain the equilibria of the game as in the following proposition.

Proposition 1. Depending on the population distribution, the qualities $q_m$, $q_p$, $q_c$, the difference of the valuations of the people, $\theta_b - \theta_s$, and the difference of the copying costs $c_b - c_s$ we have shown in the case analysis above, firm M can set different prices so that 6 subgame equilibria occur. We can identify these equilibria as follows:

(i) In equilibrium 1, $p_m = (q_m - q_p)\theta_s$, $D_m = N$, $\pi_m^1 = N(q_m - q_p)\theta_s$ and $p_p = 0$ , $D_p = 0$, $\pi_p = 0$. The whole market uses $q_m$ in this equilibrium.

(ii) When $c_b - c_s < (q_p - q_c)(\theta_b - \theta_s)$, and (2.13) is satisfied, but (2.15) and (2.16) are not satisfied, equilibrium 2 holds where $p_m = (q_m - q_c)\theta_b + c_b - (q_p - q_c)(\theta_b - \theta_s)$, $D_m = N_b$, $\pi_m^2 = N_b[(q_m - q_c)\theta_b + c_b - (q_p - q_c)(\theta_b - \theta_s)]$, $p_p = (q_p - q_c)\theta_s + c_b$, $D_p = N_s^b$.

(iii) When $c_b - c_s < (q_p - q_c)(\theta_b - \theta_s)$, but (2.13) is not satisfied (which implies (2.15) and (2.16) are not satisfied) then, equilibrium 3 holds where $p_m = (q_m - q_c)\theta_b + c_s - (q_p - q_c)(\theta_b - \theta_s)$, $D_m = N_b$ and $\pi_m^3 = N_b(q_m - q_c)\theta_b + c_s - (q_p - q_c)(\theta_b - \theta_s)$ and $p_p = (q_p - q_c)\theta_s + c_s$, $D_p = N_s$.

(iv) When $c_b - c_s > (q_p - q_c)(\theta_b - \theta_s)$ and (2.13) is satisfied, but (2.15) and (2.16) are not satisfied then equilibrium 4 holds where $p_m = (q_m - q_c)\theta_b + c_s$,$D_m = N_b$ and $\pi_m^4 = N_b(q_m - q_c)\theta_b + c_s$,and $p_p = (q_p - q_c)\theta_b + c_s$,$D_p = N_s^b$.

(v) When $c_b - c_s > (q_p - q_c)(\theta_b - \theta_s)$, but (2.13) is not satisfied (so (2.15) and (2.16) are not satisfied, either ), then equilibrium 5 holds where $p_m = (q_m - q_c)\theta_b + c_s - (q_p - q_c)(\theta_b - \theta_s)$, $D_m = N_b$ and $\pi_m^5 = N_b(q_m - q_c)\theta_b + c_s - (q_p - q_c)(\theta_b - \theta_s)$ and $p_p = (q_p - q_c)\theta_s + c_s$, $D_p = N_s$.

(vi) When $c_b - c_s > (q_p - q_c)(\theta_b - \theta_s)$, and (2.15) and (2.16) are satisfied, then, equilibrium 6 holds in which $p_m = (q_m - q_c)\theta_b + c_b - (q_p - q_c)(\theta_b - \theta_s)$ with $D_m = N_b^b$, $\pi_m^6 = N_b^b[(q_m - q_c)\theta_b + c_b - (q_p - q_c)(\theta_b - \theta_s)]$ and $p_p = (q_p - q_c)\theta_s + c_b$ with $D_p = N_s^b$

Therefore, we can summarize the conditions under which the subgame equilibria of the game exist in Table 2.2.

Firm M sets $p_m$ so that the profit of the firm is maximized under the known conditions on population distribution, the qualities $q_m$, $q_p$, $q_c$, the difference of the valuations of the people, $\theta_b - \theta_s$, and the difference of the copying costs $c_b - c_s$. When we assume that the number of people in each of the four groups in the market distributed as higher prices ( although it means less customers ) bring higher profits to the firms, and when $c_b - c_s < (q_p - q_c)(\theta_b - \theta_s)$, then firm M chooses $p_m = (q_m - q_c)\theta_b + c_b - (q_p - q_c)(\theta_b - \theta_s)]$. When $c_b - c_s > (q_p - q_c)(\theta_b - \theta_s)$, then firm

Table 2.2. Subgame Equilibria in the Main Analysis

| | Subgame Equilibria | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| 2.11 | yes/no | no | no | yes | yes | yes |
| 2.12 | yes/no | yes | yes | no | no | no |
| 2.13 | yes/no | yes | no | yes | no | yes |
| 2.15 | yes/no | no | no | yes/no | no | yes |
| 2.16 | yes/no | no | no | yes/no | no | yes |
| $p_m$ | $p_m = (q_m - q_p)\theta_s$ | $(q_m - q_c)\theta_b + c_b - (q_p - q_c)(\theta_b - \theta_s)$ | $(q_m - q_c)\theta_b + c_s - (q_p - q_c)(\theta_b - \theta_s)$ | $(q_m - q_c)\theta_b + c_s$ | $(q_m - q_c)\theta_b + c_s - (q_p - q_c)(\theta_b - \theta_s)$ | $(q_m - q_c)\theta_b + c_b - (q_p - q_c)(\theta_b - \theta_s)$ |
| $p_p$ | 0 | $(q_p - q_c)\theta_s + c_b$ | $(q_p - q_c)\theta_s + c_s$ | $(q_p - q_c)\theta_s + c_s$ | $(q_p - q_c)\theta_s + c_s$ | $(q_p - q_c)\theta_s + c_b$ |
| $D_m$ | $N$ | $N_b$ | $N_b$ | $N_b$ | $N_b$ | $N_b^b$ |
| $D_p$ | 0 | $N_s^b$ | $N_s^b$ | $N_s^b$ | $N_s$ | $N_s^b$ |
| $\pi_m$ | $N(q_m - q_p)\theta_s$ | $N_b[(q_m - q_c)\theta_b + c_b - (q_p - q_c)(\theta_b - \theta_s)]$ | $N_b[(q_m - q_c)\theta_b + c_s - (q_p - q_c)(\theta_b - \theta_s)]$ | $N_b(q_m - q_c)\theta_b + c_s$ | $N_b(q_m - q_c)\theta_b + c_s - (q_p - q_c)(\theta_b - \theta_s)$ | $N_b^b[(q_m - q_c)\theta_b + c_b - (q_p - q_c)(\theta_b - \theta_s)]$ |
| $\pi_p$ | 0 | $N_s^b(q_p - q_c)\theta_s + c_b$ | $N_s^b(q_p - q_c)\theta_s + c_s$ | $N_s^b(q_p - q_c)\theta_s + c_s$ | $N_s(q_p - q_c)\theta_s + c_s$ | $N_s^b(q_p - q_c)\theta_s + c_b$ |

M chooses the same price function given by $p_m = (q_m - q_c)\theta_b + c_b - (q_p - q_c)(\theta_b - \theta_s)]$. These results imply that when the population distribution, the qualities $q_m$, $q_p$, $q_c$, the valuations of the people, $\theta_b$, $\theta_s$, and the copying cost $c_b$ do not change but $c_s$, which is the cost of copying for the consumers who do not have easy access to end-user copying does then the profit of firm M is reduced because the high types with small costs to copy prefer to use $q_c$ as $c_s$ decreases. Today, the improvements in communication technologies decrease the value of the cost of copying even for those who are not much interested in technology. Thus, it is not an unrealistic assumption that $c_b - c_s > (q_p - q_c)(\theta_b - \theta_s)$.

When the number of people in each of the four groups in the market is distributed in such a way that pirate P maximizes its profit with lower prices (so more customers), but firm M maximizes its profit with higher prices (correspondingly, less customers), the situation is similar. When $c_b - c_s < (q_p - q_c)(\theta_b - \theta_s)$ or on the contrary $c_b - c_s > (q_p - q_c)(\theta_b - \theta_s)$, firm M chooses the same pricing strategy as $p_m = (q_m - q_c)\theta_b + c_s - (q_p - q_c)(\theta_b - \theta_s)]$. These results imply that when the population distribution, the qualities $q_m$, $q_p$, $q_c$, the valuations of the people, $\theta_b$, $\theta_s$, and the copying costs $c_b$, do not change but $c_s$, which is the costs of copying for the consumers who have the easy access to end-user copying changes, then the profit of firm M is reduced because the price that firm M can charge decreases, however the demand for $q_m$ does not change.

In any of the equilibria, the profit of the firm decreases compared to no copying and/or no pirate firm markets. The activity of pirate P is illegal. The most natural way of forbidding the activity of pirate P is governmental monitoring. However, we will show that it is not always the optimal way to restrict the pirate.

We assume that the government only controls the supply side of the market. The government is responsible for monitoring and penalizing the pirate firm, P. Let $\mu$ and $F$ be the monitoring rate and the penalty level. Let $C(\mu)$ be the cost of monitoring. We assume that $C(\mu) = 0, C'(\mu) > 0, C'(0) = 0, C''(\mu) > 0$. The government chooses $\mu$ and F to maximize social welfare which is subject to a balanced budget constraint so that redistribution is not necessary. The social welfare which is the sum of the profits of the original producer, $\pi_m$ and the pirate firm $\pi_p$ and the consumer surplus, $CS$ is given as:

$$SW(\mu) = \pi_m(p_m, p_p) + \pi_p(p_m, p_p) + CS. \tag{2.17}$$

We denote the net expected revenue of the government from the regulation activities with $R$.

$$R = \mu F - C(\mu) \tag{2.18}$$

The balanced budget constraint requires $R = 0$. Therefore, the optimal penalty level is equal to the average cost of monitoring:

$$F = \frac{C(\mu)}{\mu}, \text{ where } \mu > 0 \tag{2.19}$$

No monitoring means no penalty. Thus, $F = 0$ if $\mu = 0$. Higher levels of monitoring mean higher levels of penalty. This is guaranteed by the assumption that as the monitoring level increases the marginal cost of monitoring also increases, which implies that the average cost of monitoring increases.

We assume that firms remain in the market if they are making nonzero profits. If the pirate firm is detected by the government with probability $\mu$, it has to pay the penalty F. Therefore, the profit of firm M and the expected profit of pirate P are,

$$\pi_m(p_m, p_p) = p_m D_m(p_m, p_p), \tag{2.20}$$

$$\pi_p(p_m, p_p) = (1-\mu) p_p D_p(p_m, p_p) - \mu F. \tag{2.21}$$

The game played between the government and the firms is as follows: At the first stage, the government announces $\mu$ and $F$ that maximize the social welfare subject to the balanced budget constraint. At the second stage, firm M sets a price, taking into consideration that a pirate may enter the market and there exist tools that make end-user copying possible. Pirate P acts accordingly.

Leader firm M and the follower firm P play a sequential game so that, given $\mu$ and $p_m$, pirate P maximizes its expected profit given by equation (2.21). Given $\mu$, firm M takes the best response pricing of pirate P and maximizes its profit given by (2.20). When the government cannot deter pirate P from entering the market, the pricing strategy of the pirate is analyzed in the previous section. In this section, we assume $c_b - c_s > (q_p - q_c)(\theta_b - \theta_s)$ and conditions given by (2.15) and (2.16) hold. Hence, in the equilibrium $p_m = (q_m - q_c)\theta_b + c_b - (q_p - q_c)(\theta_b - \theta_s)$ with $D_m = N_b^b$, $\pi_m^6 = N_b^b[(q_m - q_c)\theta_b + c_b - (q_p - q_c)(\theta_b - \theta_s)]$ and $p_p = (q_p - q_c)\theta_s + c_b$ with $D_p = N_s^b$, $\pi_p = N_s^b[(q_p - q_c)\theta_s + c_b]$ hold. The pricing of the pirate is set so that low type consumers with big costs to copy get the same utility when they copy or buy from the pirate and the high type consumers with big costs to copy get higher utility when they buy from firm M instead of copying the good or buying from the pirate. Hence, the profits of the firms and the consumer surplus

$CS$ become

$$\pi_m = N_b^b[(q_m - q_c)\theta_b + c_b - (q_p - q_c)(\theta_b - \theta_s)], \tag{2.22}$$

$$\pi_p = (1-\mu)N_s^b[(q_p - q_c)\theta_s + c_b] - \mu F, \tag{2.23}$$

$$CS_1 = N_s^s u_s^s + N_s^b u_s^b + N_b^s u_b^s + N_b^b u_b^b(p_m). \tag{2.24}$$

Let $\mu_1$ be the minimum monitoring rate under which pirate P does not enter the market. Then, $\mu_1$ satisfies that

$$(1-\mu_1)N_s^b[(q_p - q_c)\theta_s + c_b] - \mu_1 F = 0$$

or equivalently,

$$\frac{\mu_1 F}{(1-\mu_1)} = \frac{C(\mu_1)}{(1-\mu_1)} = N_s^b[(q_p - q_c)\theta_s + c_b]. \tag{2.25}$$

The right hand side of the equation (2.25) is the profit of the pirate when there is no governmental protection. Because we assumed that $C'(\mu) > 0$, for any $\mu$ such that $\mu < \mu_1$, the profit of the pirate is greater than 0, so that it enters the market. Whenever the government applies a protection policy in which $\mu \geq \mu_1$, then the pirate firm does not enter the market. Although pirate P cannot enter the market in this equilibrium, firm M cannot charge a higher price. Suppose, on the contrary, that firm M charges a price supposing that pirate P is not in the market. In this case, firm M charges the price as $p_m = (q_m - q_c)\theta_b + c_b$, so that the high types with big costs to copy prefer to buy from firm M instead of copying the product.

Following firm M, pirate P sets its price as $(q_p - q_c)\theta_s + c_b < p_p < (q_p - q_c)\theta_b + c_b$. Hence, the profit of pirate P becomes greater than 0 under the monitoring rate $\mu \geq \mu_1$. Therefore, when $\mu \geq \mu_1$, pirate P does not enter the market and $p_m = ((q_m - q_c)\theta_b + c_b - (q_p - q_c)(\theta_b - \theta_s)$, $D_m = N_b^b$. The rest of the population copy the product themselves, because P does not serve in the market. Hence, the profits of the firms and the consumer surplus $CS$ do not change where

$$\pi_m = N_b^b[(q_m - q_c)\theta_b + c_s], \tag{2.26}$$

and

$$CS_1 = N_s^s u_s^s + N_s^b u_s^b + N_b^s u_b^s + N_b^b u_b^b(p_m). \tag{2.27}$$

The price charged by firm M is equal to the one when the pirate firm P does not enter the market under the monitoring rate $\mu_1$. However, if the government sets the monitoring rate as $\mu = \mu_2$ where $\mu_2$ satisfies that

$$(1 - \mu_2)N_b^b[(q_p - q_c)\theta_b + c_b] - \mu_2 F = 0, \tag{2.28}$$

or equivalently,

$$\frac{\mu_2 F}{(1 - \mu_2)} = N_b^b[(q_p - q_c)\theta_b + c_b], \tag{2.29}$$

then, firm M can set the price as $p_m = (q_m - q_c)\theta_b + c_b$. Pirate firm P cannot offer such a price where consumers prefer to buy from pirate P. Therefore, firm M sets the price as $p_m = (q_m - q_c)\theta_b + c_b$, and

$$\pi_m = N_b^b[(q_m - q_c)\theta_b + c_b]. \tag{2.30}$$

The consumer surplus is less than the previous ones with,

$$CS_2 = N_s^s u_s^s + N_s^b u_s^b + N_b^s u_b^s + N_b^b u_b^b. \tag{2.31}$$

thus, we can write the social welfare function as

$$SW(\mu) = \begin{cases} N_b^b[(q_m - q_c)\theta_b + c_b - (q_p - q_c)(\theta_b - \theta_s)] + \\ (1-\mu)N_s^b[(q_p - q_c)\theta_s + c_b] - C(\mu) + CS_1 & \text{if } \mu < \mu_1 \\ N_b^b[(q_m - q_c)\theta_b + c_s] - C(\mu) + CS_1 & \text{if } \mu_1 \leq \mu < \mu_2 \\ N_b^b[(q_m - q_c)\theta_b + c_b] - C(\mu) + CS_2 & \text{if } \mu_2 \leq \mu \end{cases} \tag{2.32}$$

It is obvious that under any condition, the government does not prefer any monitoring rate $\mu$ as $\mu_1 \leq \mu < \mu_2$. Whenever $\mu < \mu_1$, the profits of the firms and the consumer surplus stay the same. However, $C(\mu)$ increases as the government increases the monitoring rate. Therefore, $\mu = 0$ is dominant to any $\mu$ that satisfy $0 < \mu < \mu_2$, and $\mu_2$ is dominant to any $\mu$ that satisfy $\mu < \mu_2$.

$$SW(0) - SW(\mu_2) = N_s^b[(q_p - q_c)\theta_s + c_b] + C(\mu_2). \tag{2.33}$$

The change in the social welfare given in equation (2.33) is always positive. When the government protection is applied against commercial piracy, the increase in the profit of the original firm is not enough to set off the decrease in the consumer surplus and the profit of the pirate firm. Therefore, the best strategy of the government is to choose no monitoring and thus, no penalty. The equilibrium strategies and profits are now as follows:

$$\begin{aligned}
&\mu = 0 \\
&p_m = (q_m - q_c)\theta_b + c_s \\
&p_p = (q_p - q_c)\theta_b + c_s \\
&\pi_m = N_b^b[(q_m - q_c)\theta_b + c_s] \\
&\pi_p = N_b^s[(q_p - q_c)\theta_b + c_s] \\
&CS = N_s^s u_s^s + N_s^b u_s^b + N_b^s u_b^s + N_b^b u_b^s.
\end{aligned} \tag{2.34}$$

The reason is mostly the existence of end-user copying as an alternative for the consumers. Whenever end-user copying is not available to the consumers and if the government applies monitoring rate $\mu_c$, where $\frac{C(\mu_c)}{(1-\mu)} = q_p\theta_s N_s$, then the pirate firm does not enter the market, and firm M gives the monopoly price as given in equation (2.5). In this case, only the high types use the product. Consumer surplus is 0. Social welfare is given as $SW(\mu_c) = q_m\theta_b N_b$. When there is no end-user copying, if the government does not follow a protection rule, then $p_m = q_m\theta_b$ and $p_p = q_p\theta_s$. The social welfare in this case is $SW_c(0) = q_m\theta_b N_b + q_p\theta_s N_s$.

$$SW(\mu_c) - SW_c(0) = q_m\theta_b N_b - q_m\theta_b N_b + q_p\theta_s N_s \tag{2.35}$$

The difference given above is always positive, assuming equation (2.4). Hence, the government chooses to protect the original firm M, when there is no end-user copying.

In conclusion, when there is end-user copying, no governmental protection is a socially optimal policy.

## 2.7. Product Differentiation

In this section, we analyze the product differentiation decision of firm M. Now, it can decide to offer the product in a lower quality, $q_l$, than $q_m$. For example, recently, legal firms have began to offer a downloadable version of movies in their official websites, after charging a price. Moreover, the consumers can rent the movies from the official offices of the legal firms. These versions of a movie are of a lower quality compared with the original DVD format of the movie.

Whenever firm M offers two qualities $q_m$ and $q_l$ with $q_l < q_m$, then $p_m$ and $p_l$, which is the price charged for $q_l$, should satisfy that

$$(q_m - q_l)\theta_s + p_l < p_m \leq (q_m - q_l)\theta_b + p_l. \tag{2.36}$$

Thus, high types prefer $q_m$ to $q_l$ and low types prefer $q_l$ to $q_m$. If the condition on the equation (2.36) does not hold, then there exists no demand for either $q_m$ or $q_l$.

In deciding the quality $q_l$, firm M should take into account that pirate P will set $p_p$ accordingly, because we assume a sequential pricing for the firms as follows: The qualities of the product $q_m$, $q_p$, $q_c$, the valuations of the consumers, $\theta_b$ and $\theta_s$, the costs of end-user copying $c_b$ and $c_s$ are of common knowledge. First, firm M announces the price $p_m$ and the quality $q_l$, at the price $p_l$. Then, pirate P announces $p_p$. Finally, consumers decide to buy the product from firm M at the quality $q_l$, or at the quality $q_m$, or to buy the product from pirate P at the quality $q_p$, or to copy the product by themselves at the quality $q_c$.

If $q_l < q_p$, then pirate P can set its price as $p_p \leq (q_p - q_l)\theta_s + p_l$, so the whole market prefers $q_p$ to $q_l$. This pricing implies $p_p < p_m - (q_m - q_p)\theta_b$ whenever $p_m = (q_m - q_l)\theta_b + p_l$, so the high types prefer $q_p$ to $q_m$. Therefore, in the subgame equilibrium, $p_m$ should satisfy $p_m < (q_m - q_c)\theta_b + p_p$. We observe that when $q_l < q_p$, as firm M decreases the price $p_l$, pirate P must decrease the price $p_p$ in order to create a demand for $q_p$. As a result, the low types demand $q_p$. Therefore, firm M decreases the price $p_m$ in order to persuade the high types to demand the product in quality $q_m$ instead of the product in quality $q_p$. Hence, the same number of people demand the product in quality $q_m$ with a smaller price. The profit of firm M decreases. Formally, if $p_l \leq c_s + (q_l - q_c)\theta_s$, then $p_p \leq (q_p - q_c)\theta_s + c_s$, so that firm M should set $p_m$ as $p_m \leq (q_m - q_p)\theta_b + (q_p - q_c)\theta_s + c_s$. If firm M does not decrease the $p_m$ more, then the high types demand $q_m$, low types demand $q_p$, If $p_l \leq c_b + (q_l - q_c)\theta_s$, then pirate P can set $p_p$ as $p_p = (q_p - q_c)\theta_s + c_s$ so that the low types with small costs to copy demand $q_p$. In this case, $p_m$ is given as the one in the previous case and the demand structure does not change. If the assumption (2.13) holds on the population, then pirate P can set $p_p$ as $p_p \leq (q_p - q_c)\theta_s + c_b$ so that the low types with small costs to copy prefer end-user copying, and firm M can set the price $p_m \leq (q_m - q_p)\theta_b + (q_p - q_c)\theta_s + c_b$. In this case, the demand for $q_l$ is 0. The demand for $q_m$ is the same as in the previous situation, but the profit of firm M is higher, since $p_m$ is higher. Therefore, when (2.13) holds, in the subgame equilibrium firm M announces $q_l < q_p$ with $p_l = c_b + (q_l - q_c)\theta_s$, and $p_m = (q_m - q_p)\theta_b + (q_p - q_c)\theta_s + c_b$, and then pirate P announces $p_p = c_b + (q_l - q_c)\theta_s$, so $D_m = N_b$,$D_p = N_s^b$, $D_l = 0$. If the assumption (2.13) does not hold, in the subgame equilibrium firm M announces $q_l < q_p$ with $p_l \leq c_s + (q_l - q_c)\theta_s$ , $p_m = (q_m - q_p)\theta_b + (q_p - q_c)\theta_s + c_b$, then pirate P announces $p_l = c_s + (q_l - q_c)\theta_s$, so $D_m = N_b$,$D_p = N_s$, $D_l = 0$. In any case, when the firm chooses the new quality $q_l$ as $q_l < q_p$, then firm M does not get any increase in its profit when we compare the results of the product differentiation and the main model.

When $q_l \geq q_p$, in a subgame equilibrium, $p_m$ and $p_l$ should satisfy that $(q_m - q_l)\theta_s + p_l < p_m \leq (q_m - q_l)\theta_b + p_l$ and $p_l \leq (q_l - q_p)\theta_s + p_p$. This pricing strategy of firm M implies that it targets the whole market where the high types prefer $q_m$ and low types prefer $q_l$. In any other pricing, pirate P can set a price so that there exists no demand for the quality $q_l$, or the quality $q_m$. If firm M sets $p_l$ as $p_l \geq (q_l - q_c)\theta_s + c_s$, then pirate P can set $p_p$ as $0 < p_p < (q_p - q_c)\theta_s + c_s$, so all low types prefer P. Therefore, $q_l \geq q_p$ with $p_l = (q_l - q_p)\theta_s$, $p_m = (q_m - q_l)\theta_b + (q_l - q_p)\theta_s$ and then, $p_p = 0$ with $D_m = N_b$, $D_l = N_s$, $D_p = 0$ is a subgame equilibrium.

In this equilibrium, $\pi_m = [(q_m - q_l)\theta_b + (q_l - q_p)\theta_s]N_b + (q_l - q_p)\theta_s N_s$. Table 2.3 summarizes all of the subgame equilibria in the market.

Therefore, firm M maximizes its profit by setting $q_l \geq_p$ with the corresponding equilibrium values. In this case, P cannot enter the market, and no individual copies the product by himself. Hence, by differentiating the quality of the product, the legal firm deters commercial and end-user copying in the market. We denote the profit of the firm as $\pi^d$ when firm M differentiates the quality of the product. Then, $\pi^d = [(q_m - q_l)\theta_b + (q_l - q_p)\theta_s]N_b + (q_l - q_p)\theta_s N_s$.

Whenever $q_l - q_p > q_p - q_c$, the profit of the firm $\pi^d$ is greater than $\pi^1$, $\pi^2$, $\pi^3$, $\pi^4$, $\pi^5$ and $\pi^5$. In other words, when firm M differentiates the quality of the product with $q_l - q_p > q_p - q_c$, its profit increases compared to the no product differentiation case.

Table 2.3. Subgame Analysis for Product Differentiation

| | Subgame Equilibria | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| 2.13 | yes | no | yes/no |
| $q_l$ | $q_l < q_p$ | $q_l < q_p$ | $q_l \geq q_p$ |
| $p_l$ | $c_b + (q_l - q_c)\theta_s$ | $c_s + (q_l - q_c)\theta_s$ | $(q_l - q_p)\theta_s$ |
| $p_m$ | $(q_m - q_p)\theta_b + (q_p - q_c)\theta_s + c_b$ | $(q_m - q_p)\theta_b + (q_p - q_c)\theta_s + c_b$ | $(q_m - q_l)\theta_b + (q_l - q_p)\theta_s$ |
| $p_p$ | $c_b + (q_l - q_c)\theta_s$ | $c_s + (q_l - q_c)\theta_s$ | 0 |
| $D_l$ | 0 | 0 | $N_s$ |
| $D_m$ | $N_b$ | $N_b$ | $N_b$ |
| $D_p$ | $N_s^b$ | $N_s$ | 0 |
| $\pi_m$ | $N_b[(q_m - q_p)\theta_b + (q_p - q_c)\theta_s + c_b]$ | $N_b[(q_m - q_p)\theta_b + (q_p - q_c)\theta_s + c_b]$ | $N_b[(q_m - q_l)\theta_b + (q_l - q_p)\theta_s] + N_s(q_l - q_p)\theta_s$ |
| $\pi_p$ | $N_s^b c_b + (q_l - q_c)\theta_s$ | $N_s[c_s + (q_l - q_c)\theta_s]$ | 0 |

## 2.8. Conclusion

In this work, we have analyzed the digital markets in which unauthorized reproduction exist. We partitioned the consumers into four groups. First, we classified the consumers into two groups with respect to their valuation of the product as the high type and the low type. Then, we further classified each group into two subgroups according to their costs in copying the product by themselves, as having small costs to copy and big costs to copy. In our setup, supporting the findings of the earlier works, (Novos and Waldman (1984) etc.) we observed that end-user copying reduces the monopoly power of the legal firm in the market when there is nothing that leads the firm to appropriate the rent of copying, such as network externality. We further observed that, in addition to end-user copying, the existence of commercial piracy causes the firm to experience more losses in the profit. Therefore, being different from existing literature on the piracy, we have used a hybrid model in which both commercial and end-user piracy is considered. We have shown that government's monitoring of piracy is not socially optimal. In other words, the governments who care about the society do not protect the legal firm against piracy in our model. Optimality of the no monitoring decision mainly results from the existence of the end-user copying. When the governmental protection deters the pirate firm to enter the market, some consumers tend to copy the product by themselves. Our result is supported by the earlier works that state that the increase in the protection level increases the social welfare loss due to underutilization. Moreover, our result does not contradict Novos and Waldman (1984)'s findings, since their model considers consumers who are homogenous in their valuations. On the other hand, Banerjee (2003) who models only commercial piracy in the digital markets, shows that governmental protection against piracy can be socially optimal under some circumstances. We have shown that when we exclude end-user copying from our setup, Banerjee (2003)'s result is supported. Additionally, we have found out that the differentiation of the quality of the product is an effective strategy to fight against piracy that is comparable to governmental protection.

As a further research agenda, one can consider modelling commercial piracy in a repeated game structure. The sequential pricing game we analyzed in our work can be played repeatedly so that the qualities of the product offered by the firm and by the pirate are determined endogenously. Furthermore, a real life simulation of the market can be implemented in order to support the theoretical evidence. Moreover, the research area on commercial piracy in digital markets is a fertile one. As computer technologies improve, commercial piracy in the digital area will expand. Besides the noncommercial clubs on the internet like P2P, commercialized illegal activities are now often seen in the web. The questions on the network effects, the social welfare implications of different protection methods, and the long-run effects of commercial piracy on digital markets can still be addressed.

# CHAPTER 3

# THE MANIPULABILITY OF THE INCENTIVE MECHANISMS AS A BARGAINING GAME

## 3.1. Introduction

In this chapter of the thesis, we formalize the discrete space version of the corruption problem which is defined to be a bargaining game by Koray and Saglam (2005a) in the Baron and Myerson (1982) model of monopoly regulation. Then, we characterize a solution to this problem using the framework of the incentive compatible bargaining games proposed by Myerson (1979).

From the early 90s, the debates on socialism and capitalism were replaced with those on how much governmental interference was required. In that sense, regulation, which analyzes the interference of governments in industrial activities, became one of the major areas of economics.

Vogelsang (2002) points out that by the end of the 70s, the economists were convinced about the failure of the traditional regulation of markets by the successful deregulation of some public utilities. Then, the economists tended to recommend deregulation or to search for new regulation methods. The introduction of the Bayesian incentive schemes to the field of regulation extended the real application areas.

In the Bayesian incentive regulatory mechanisms, the regulator announces a certain pricing for the firm that takes into account that the firm can misreport its costs. Under this pricing the firm can increase its profit by reducing the costs. The work by Baron and Myerson (1982) was the pioneer in the research. The acceleration in the improvement of the research was so encouraging that the use of the Bayesian approach in regulation was named "the new economics of regulation" by Laffont (1994). The incentive regulation was accompanied by privatization, liberalization and deregulation in many markets. In the U.K., many public utilities were privatized such as gas, water, electricity, and telecommunications. As noted by Crew and Kleindorfer (2002), in the U.S.A., the world's largest telecommunication company was liberalized and deregulated in the 80s. The changes in technology, demands, market structure and institutions required new regulation trends. In order to create a competitive environment, private ownership and deregulation were seen as necessities.

The incentive mechanisms fit in this picture since the regulator gives performance - based rewards with the incentive mechanisms in order to use the private information of the firms, instead of controlling their behavior. The main weakness of the traditional non-Bayesian regulatory mechanisms was that imperfect information had not been taken into account. Emphasizing the importance of informational asymmetries and using the incentive theory, the Bayesian mechanisms became a meaningful theoretical approach to the regulatory mechanisms. Indeed, Mougeot and Naegelen (2007) show that a monopoly that will be regulated by a Bayesian incentive scheme that is selected by an auction mechanism results in higher expected welfare than the duopoly competition does when the entry cost is low.

However, incentive mechanisms are criticized firstly, for not being applicable as much as the non-Bayesian regulation schemes. Furthermore, Crew and Kleindorfer (2002) criticize both deregulation and incentive mechanisms. They claim that the politicians, pressure groups and regulators do not apply rules to violate the features of regulation that distort the competitive markets.

Crew and Kleindorfer (2002) state that:

> The original rationale for regulation in the minds of economists was the desire to avoid monopoly inefficiency... However, the practice of regulation became more than this as elected representatives realized its considerable potential for providing them with opportunities of taxation and subsidization that had distinct advantages relative to the usual taxes and subsidies. Redistribution by regulation lacked the

> transparency and therefore, the accountability of traditional methods of taxation and subsidy. From the government's point of view, this was a huge advantage and provides a potentially convincing explanation of why deregulation is often a failure. Politicians preach deregulation while simultaneously retaining the redistribution mechanism that regulation provides.

Bayesian incentive schemes are reviewed in many more directions like the absence of common knowledge, lack of robustness, and manipulability of beliefs. Among many others Crew and Kleindorfer (1986, 2002), Vogelsgang (1988, 2002, 2004), Koray and Sertel (1990), Koray and Saglam (1999) point out the weakness of the Bayesian approach in regulation. Although many incentive mechanisms such as price caps, rate case moratoria, profit sharing and menus and options are in practical use in many markets, it is hard to say that they work efficiently. There exist works on the regulation of specific markets. Collette and Leitzinger (2002) on the gas industry, Hogan (2001) and Vogelsang (2004) on the electric industry analyze the regulation.

Koray and Saglam (2005a) examine the issue of manipulation in the Baron and Myerson (1982) model of monopoly regulation. In their following work, Koray and Saglam (2005b) present the lack of robustness even in cases where manipulability is not a point of question. Koray and Saglam (2005a and 2005b) show that corruption is a problem of Bayesian regulatory mechanisms under generalized principal agent framework even though the Bayesian approach is used. The manipulability of the optimal mechanism becomes possible, since the optimal mechanism in the Bayesian incentive schemes directly depend on the prior belief of the regulator.

Baron and Myerson (1982) model the regulation of a monopoly under incomplete information. A regulator designs a policy to regulate a monopolistic firm whose costs are unknown to the regulator. The regulator's objective is to maximize social welfare which is defined to be any linear combination of consumers' and producer's surplus where the consumers are weighted at least as much as the producer. The regulator does not know the exact cost of the firm. However, the regulator knows the cost structure and the interval on which the unknown cost type of the firm lies, and he has some prior belief in the unknown type of the firm over this interval. This prior belief, which is a probability distribution, and the demand function, is common knowledge to the consumers and the firm. The regulator designs a regulatory policy which consists of four components: Price, quantity, subsidy and probability of the firm being allowed to produce. In this process, the regulator has to take into account that the firm has an incentive to lie about the cost type. Therefore, the regulatory policy is designed under the Revelation Principle (Gibbard 1973 and Myerson 1979) so that the reported and real cost types are the same.

The optimal policy, in the Baron and Myerson (1982) model of monopoly, entirely depends on the belief of the regulator about the cost type of the firm. Hence, Koray and Saglam (2005a) conclude that the prior belief of the regulator about the cost of the firm becomes a strategic variable. They show that under a specific set of beliefs of the regulator, the producer maximizes its gain over the set of admissible beliefs and consumers maximize their ex-ante gain under some specific belief of the regulator. Moreover, under a certain set of beliefs of the regulator both parties gain almost nil. The existence of this threat belief gives rise to a situation in which a corrupt regulator may try to take advantage of the sensitivity of the agents to his beliefs.

The corruption problem defined by Koray and Saglam (2005a) includes asymmetric information. The optimal policy which is designed under incentive compatibility and individual rationality constraints in the maximization of expected social welfare makes the firm tell the truth. Then, the firm reports the cost parameter. When the regulator decides to have a belief in favor of one of the agents, even under the incentive compatible Bayesian regulatory mechanism, the firm can gain by misreporting its cost parameter, since the net utilities of the parties change in the bargaining. The party favored by the regulator shares some part of his gain with the regulator. A dishonest regulator has to design a solution to this bargaining in such a way that the firm has no incentive to lie about its type, expecting a gain as the outcome of the bargain. The regulator is not sure about the type of the regulated party. Hence, the use of Bayesian approach is needed in the analysis of the collective choice problem in addition to incentive compatibility.

In formulating the corruption problem, the work done by Myerson (1979) provides us with the framework we need. Myerson studies the problem of an arbitrator who is about to make a collective choice for a group of individuals when he has no complete information about their preferences and endowments. The arbitrator's main problem is not only to design a solution on which all agents agree but to make them tell the truth about their private information related to their preferences. In the Bayesian collective choice problem, the payoff of each individual depends on the types of the individuals and the alternative chosen by the arbitrator. A set of finite alternatives from which the arbitrator makes his choice is available to the group. Each individual has a private parameter that determines his type. Sets where the private parameters come from are common knowledge, while the type of each individual is his private information. The arbitrator asks every player to report his type and then assigns a probability to this type vector that these are the true types. The probability distribution over type sets are common knowledge. Moreover, each individual knows how much his payoff would be for a given type vector and for a choice of the arbitrator. The arbitrator's choice is allowed to be a probability distribution over the alternative set.

The choice mechanism is defined to be a probability distribution over the alternative set, given a type structure of the players. Since the players can misreport their types, the arbitrator restricts himself to the choice mechanisms which give incentive compatible payoffs to all individuals. Because no player knows the real types of the others, Myerson (1979) uses the Bayesian approach in the model. He shows that the set of expected utility allocations which are feasible with incentive compatible mechanisms include the equilibrium allocations. If conflict outcome is defined as the outcome of the game that results when there is no agreement among the players, the Bayesian collective choice problem becomes a bargaining problem on which Harsanyi and Selten's (1972) solution concept can be applied. Myerson (1979) shows that the arbitrator can find a unique solution to the problem, maximizing the generalized Nash product over the incentive compatible and individual rational choice mechanisms.

The work by Myerson (1979) is the pioneer work in the literature on bargaining under incomplete information. Informational asymmetries are observed in many economic problems where bargaining is involved. Contract theory, auction theory, social choice theory, cooperative and non-cooperative game theory, general equilibrium theory, mechanism design theory, public good theory and general principal-agent theory are all developed on the economic problems on which asymmetric information plays an important role. Following Myerson (1979), Chatterjee (1982) examines the question of designing efficient bargaining procedures in the context of a two-player model where each player has private information unavailable to the other. He obtains a class of incentive-compatible schemes in which the bargainers would wish to participate in the process by having the information on their reservation price. Weidner (1992) analyzes whether the generalized Nash solution which has been defined by Harsanyi and Selten for bargaining problems with incomplete information can be characterized in the mechanism framework introduced by Myerson (1979). He shows that the solution is uniquely determined by a set of axioms in the case of independently distributed types. It can be seen that the axioms given by Harsanyi and Selten cannot be used when the types are not independently distributed. Ichiishi and Idzik (1996) provide a formal framework within which to study cooperative behavior in the presence of incomplete information. They introduce and study the concepts of Bayesian society, Bayesian strong equilibrium and Bayesian incentive compatible strong equilibrium. The theoretical framework of the bargaining game under incomplete information is applied in many branches of economics. There exist hundreds of works which study bargaining on a 'cake' under these theories.

In our work, we formalize the discrete type space version of the corruption bargaining game which is proposed by Koray and Saglam (2005a) in Baron and Myerson's (1982) model of monopoly regulation for continuum. We characterize a solution to this problem by using the distinctive work of Myerson (1979). As Lovejoy (2006) suggests, the clear understanding and the extendibility of Myerson's (1982) work, together with the reality that the difference between finite and continuous-type spaces is just a mathematical detail, partially explain why the works on the continuous type sets are dominant in this literature. However, for the quantitative findings, the utilization of the discrete type space is almost necessary, which enables us to implement a simulation of the formalized solution in order to observe the economic effects of the corruption game, quantitatively.

The organization of the chapter is as follows: In section 2, we define a discrete-space version of the corruption game proposed by Koray and Saglam (2005a). In section 3, we solve the discrete version of the corruption problem defined by Koray and Saglam (2005a) by using the solution of incentive compatible bargaining games introduced by Myerson (1979). In section 4, we illustrate the solution characterized for the corruption bargaining game by implementing a simulation of the model. In section 5, we conclude.

## 3.2. The Corruption Problem for a Discrete Space Version of the Baron and Myerson Model Monopoly Regulation

In this section, we define a discrete-space version of the corruption game proposed by Koray and Saglam (2005a). They criticize the monopoly regulation proposed by Baron and Myerson (1982) by proving the existence of the corruption problem in this monopoly model.

Baron and Myerson (1982) regulate a monopoly whose marginal cost is unknown to the society by an incentive compatible mechanism. They assume that the linear cost structure of the firm is common knowledge, whereas the marginal cost, $\theta$, of the firm is a private information of the firm. However, $\theta$ is assumed to be bounded within a continuous space $\Theta = [\theta_0, \theta_1]$ with $\theta_0 < \theta_1$. The regulator has a belief about the type of cost $\theta$. A probability distribution, $f(.)$, over $\Theta$ which represents this belief is publicly known. A Borel field $\tau^{\Theta}$on $\Theta$ is assumed to be the set of all probability measures. Among all probability measures on $\tau^{\Theta}$, $\mathrm{A}^{\Theta}$ is defined with densities that are strictly positive at each $\theta$ on $\tau^{\Theta}$ as the admissable prior beliefs of the regulator. The regulator is restricted to the direct mechanisms which give the firm no incentive to lie about his private information under the generality of the Revelation Principle, following Gibbard (1973), Myerson (1979). The optimal mechanism consists of the probability that the regulator will allow the firm to produce, the regulatory price, $p$, the output quantity, and the subsidy. It maximizes the expected social welfare satisfying the incentive compatibility and individual rationality conditions for the consumers and the firm. The expected social welfare is the sum of the consumer surplus and the firm's profit weighted by $\alpha$. Baron and Myerson obtain the regulatory price, $p(.)$, which depends directly on the belief $f$ of the regulator as $p(\theta) = \theta + (1-\alpha)\frac{F(\theta)}{f(\theta)}$.

Koray and Saglam show that there exists no admissible belief that the consumers or the society strictly prefer, but consumers can expect to maximize their ex-ante gains. For the consumers, the most preferred social welfare function that is to be maximized by the regulator is the one that gives no weight to the producer i.e. $\alpha = 0$. The authors claim that for any admissible belief, $f$, of the regulator, we can find another admissible belief, $f_\alpha$, so that the expected social welfare under $f_\alpha$ with the weight $\alpha$ is equal to the expected consumers' surplus under $f$ that is the expected social welfare under $\alpha$ with zero weight given to the producer. $f_\alpha$ is given as

$$f_\alpha = (1-\alpha)\frac{f(\theta)}{F(\theta)^\alpha} \text{ for any } f \in \mathrm{A}^\Theta. \tag{3.1}$$

Now, we will obtain the results for the Baron and Myerson model of monopoly regulation, when the type space of the firm is a discrete one instead of a continuum.

As in the Baron and Myerson model, the marginal cost of production is the private information of the firm. Neither consumers nor the regulator knows the exact marginal cost of the firm. However, the regulator announces his belief about the marginal cost before he asks the producer for the true parameter. This belief is a probability distribution over a set of finite number of parameters $\Theta = \{\theta_1, \theta_2, ..., \theta_n\}$. The regulator designs a socially optimal regulatory policy that makes the producer tell the truth concerning the marginal cost.

The firm's cost function is in the form

$$C(q,\theta) = \begin{cases} k + \theta q & \text{if } q > 0 \\ 0 & \text{if } q = 0 \end{cases} \tag{3.2}$$

where the fixed cost $k$ and the form of the cost function is known by the whole society, but the marginal cost $\theta$ is the private information of the producer. The common knowledge is that $\theta$ comes from the set $\Theta = \{\theta_1, \theta_2, ..., \theta_n\}$, where $\theta_1 <$

$\theta_2 < ... < \theta_n$ and $n$ is finite. The regulator has a sequence of prior beliefs $\{\sigma(\theta_i)\}_{i=1}^n$, where $0 < \sigma(\theta_i) < 1$ for all $i$ and $\sum_{i=1}^{n} \sigma(\theta_i) = 1$.

The demand function is assumed to be common knowledge. The inverse demand function is denoted by $P(.)$ and assumed in the form

$$P(q) = a - bq \text{ where } a, b > 0. \tag{3.3}$$

The consumers' surplus is given by $V(q) - P(q)q$ where $V(q) = \int_0^q P(\tilde{q})d\tilde{q}$ is the total value of the produced quantity $q$. Given (3.3), the total value of $q$ is

$$V(q) = aq - \frac{b}{2}q^2. \tag{3.4}$$

The regulator designs a feasible regulatory policy $M = (r, p, q, s)$ which is defined as follows: For any reported parameter $\theta_i \in \Theta$, $r(\theta_i)$ is the probability that the regulator allows the producer to produce. Then,

$$0 < r(\theta_i) < 1. \tag{3.5}$$

Whenever the firm is permitted to produce, the regulatory price $p$ and the output quantity $q$ should be compatible with the inverse demand function. Therefore,

$$P(q(\theta_i)) = p(\theta_i). \tag{3.6}$$

The expected value of the subsidy is paid to the firm, whenever it is permitted to produce. When the firm is in production, and $s'$ is the amount of the transfer payment, then the expected amount will be $s(\theta_i) = s'(\theta_i)r(\theta_i)$.

We denote the utility of the producer with $u_p(\theta_j, \theta_i)$, that is the profit of the producer if it reports its marginal cost as $\theta_j$, when its real marginal cost is $\theta_i$.

$$u_p(\theta_j, \theta_i) = [p(\theta_j)q(\theta_j) - \theta_i q(\theta_j) - k]\, r(\theta_j) + s(\theta_j). \tag{3.7}$$

The producer has no incentive to lie about parameter $\theta_i$, when the policy satisfies incentive compatibility:

$$u_p(\theta_i) \equiv u_p(\theta_i, \theta_i) = \max_{\theta_j \in \Theta} u_p(\theta_j, \theta_i) \text{ for all } i. \tag{3.8}$$

A feasible policy must satisfy individual rationality:

$$u_p(\theta_i) \geq 0 \text{ for all } i. \tag{3.9}$$

A policy $M = (r, p, q, s)$ satisfying all conditions in (3.5)-(3.9) is a *feasible policy*, and it guarantees that the firm has no incentive to misreport its private information.

We denote the utility of the consumers' union as $u_c(\theta_i)$, that is the consumers' net gain, given the cost report $\theta_i$ of the producer:

$$u_c(\theta_i) = [V(q(\theta_i) - p(\theta_i)q(\theta_i)]\, r(\theta_i) - s(\theta_i) \tag{3.10}$$

We define the social welfare at the cost report $\theta_i$ as

$$\mathrm{SW}(\theta_\mathrm{i}) = u_c(\theta_i) + \alpha u_p(\theta_i), \text{ where } \alpha \in [0, 1]\,. \tag{3.11}$$

Following the Bayesian framework, the regulator's aim is to maximize the expected social welfare for the prior distribution $\sigma$. When the producer's net gain is $u_p^\sigma(\theta_i)$, the consumers' net gain is $u_c^\sigma(\theta_i)$ and the social welfare is $SW_\sigma(\theta_i)$, the regulator's objective is to find a feasible policy $M_\sigma = (r_\sigma, p_\sigma, q_\sigma, s_\sigma)$ that maximizes

$$w_\sigma = \sum_{i=1}^{n} SW_\sigma(\theta_i)\sigma(\theta_i), \tag{3.12}$$

for the given cost report $\theta_i$ and the belief $\sigma$. In the following lemma, we characterize the feasible policy in order to derive the optimal policy.

Lemma 1. *Regulatory policy $M = (r, p, q, s)$ is feasible if and only if the below conditions hold for all $i$:*

$$0 < r(\theta_i) < 1, \tag{3.13}$$

$$P(q(\theta_i)) = p(\theta_i), \tag{3.14}$$

$$u_p(\theta_i) = u_p(\theta_n) + \sum_{j=i}^{n-1} (\theta_{j+1} - \theta_j)q(\theta_{j+1})r(\theta_{j+1}), \tag{3.15}$$

$$u_p(\theta_n) \geq 0, \tag{3.16}$$

$$r(\theta_j)q(\theta_j) \geq r(\theta_i)q(\theta_i) \text{ for all } i \geq j. \tag{3.17}$$

Proof. First we show that feasibility conditions (3.5)-(3.9) imply the conditions above. Conditions (3.13), (3.14), (3.16) follow directly from the definition of feasibility. Then, we only need to show (3.15) and (3.17). From (3.8) we know that

$$u_p(\theta_j, \theta_{j+1}) = u_p(\theta_j) + (\theta_j - \theta_{j+1})\, q(\theta_j)\, r(\theta_j) \le u_p(\theta_{j+1}) \tag{3.18}$$

and

$$u_p(\theta_{j+1}, \theta_j) = u_p(\theta_{j+1}) + (\theta_{j+1} - \theta_j)\, q(\theta_{j+1})\, r(\theta_{j+1}) \le u_p(\theta_j)\,. \tag{3.19}$$

Since $\theta_j \le \theta_{j+1}$ for all $j$, then

$$(\theta_{j+1} - \theta_j)\, q(\theta_{j+1})\, r(\theta_{j+1}) \le u_p(\theta_j) - u_p(\theta_{j+1}) \le (\theta_{j+1} - \theta_j)\, q(\theta_j)\, r(\theta_j) \text{ for all } i. \tag{3.20}$$

So,

$$\sum_{j=i}^{n-1} (\theta_{j+1} - \theta_j) q(\theta_{j+1}) r(\theta_{j+1}) \le \sum_{j=i}^{n-1} (u_p(\theta_j) - u_p(\theta_{j+1})) \le \sum_{j=i}^{n-1} (\theta_{j+1} - \theta_j) q(\theta_j) r(\theta_j), \tag{3.21}$$

or equivalently

$$\sum_{j=i}^{n-1} (\theta_{j+1} - \theta_j) q(\theta_{j+1}) r(\theta_{j+1}) \le u_p(\theta_i) - u_p(\theta_n) \le \sum_{j=i}^{n-1} (\theta_{j+1} - \theta_j) q(\theta_j) r(\theta_j). \tag{3.22}$$

Therefore, for some $\varepsilon_j$, where $0 \le \varepsilon_j \le 1$ we can conclude that

$$u_p(\theta_i) = u_p(\theta_n) + \sum_{j=i}^{n-1} (\theta_{j+1} - \theta_j) q(\theta_{j+1}) r(\theta_{j+1}) + \sum_{j=i}^{n-1} (\theta_{j+1} - \theta_j) q(\theta_j) r(\theta_j) \varepsilon_j. \tag{3.23}$$

In the bargaining game, the value of $\varepsilon_j$ is very close to 1 so that we can neglect the last part of (3.23).

Lemma 2. *For any feasible regulatory policy,* $M_\sigma = (r_\sigma, p_\sigma, q_\sigma, s_\sigma)$ *at a given* *belief $\sigma$, the social welfare function, $w_\sigma$, in (3.12) is equal to*

$$w_\sigma = \left[V(q_\sigma(\theta_1) - \theta_1 q_\sigma(\theta_1) - k\right] r_\sigma(\theta_1)\sigma(\theta_1) + \sum_{i=1}^{n-1} (V(q_\sigma(\theta_{i+1}) - q_\sigma(\theta_{i+1})z_\sigma(\theta_{i+1}) - k)r_\sigma(\theta_{i+1})\sigma(\theta_{i+1}) - (1-\alpha)u_p^\sigma(\theta_n) \tag{3.24}$$

*where*

$$z_\sigma(\theta_{i+1}) = \theta_{i+1} + (1-\alpha)\frac{(\theta_{i+1} - \theta_i)\sum_{j=1}^{i}\sigma(\theta_j)}{\sigma(\theta_{i+1})} \tag{3.25}$$

*is assumed to be nondecreasing.*

Proof. From (3.7) and (3.8), we get

$$p(\theta_i)q(\theta_i)r(\theta_i) + s(\theta_i) = u_p(\theta_i) + (\theta_i q(\theta_i) - k)\, r(\theta_i). \tag{3.26}$$

Then,

$$w_\sigma = \sum_{i=1}^{n} \Big\{(V(q_\sigma(\theta_i) - \theta_i q_\sigma(\theta_{i+1}) - k)\, r_\sigma(\theta_i) - (1-\alpha)u_p^\sigma(\theta_i)\Big\}\sigma(\theta_i). \tag{3.27}$$

If we use (3.15) we get

$$\begin{aligned} \sum_{i=1}^{n} u_p(\theta_i)\sigma(\theta_i) &= \sum_{i=1}^{n}\left[u_p^\sigma(\theta_n) + \sum_{j=i}^{n-1}(\theta_{j+1} - \theta_j)q_\sigma(\theta_{j+1})r_\sigma(\theta_{j+1})\right]\sigma(\theta_i) \\ &= \sum_{i=1}^{n-1}\left(\sum_{j=1}^{i}\sigma(\theta_j)\right)(\theta_{j+1} - \theta_j)q_\sigma(\theta_{j+1})r(\theta_{j+1}) + u_p^\sigma(\theta_n). \end{aligned}$$

Substituting this into (3.26), we get

$$\begin{aligned} w_\sigma &= \sum_{i=1}^{n}\{(V(q_\sigma(\theta_i) - \theta_i q_\sigma(\theta_{i+1}) - k)\, r_\sigma(\theta_i)\} \\ &\quad -(1-\alpha)\left\{\sum_{i=1}^{n-1}\left(\sum_{j=1}^{i}\sigma(\theta_j)\right)(\theta_{j+1} - \theta_j)q_\sigma(\theta_{j+1})r_\sigma(\theta_{j+1}) + u_p^\sigma(\theta_n)\right\}. \end{aligned}$$

Then, by a straightforward simplification, we have

$$w_\sigma = [V(q_\sigma(\theta_1) - \theta_1 q_\sigma(\theta_1) - k]\, r_\sigma(\theta_1)\sigma(\theta_1) +$$
$$\sum_{i=1}^{n-1}\left(V(q_\sigma(\theta_{i+1}) - q_\sigma(\theta_{i+1})\left(\theta_{i+1} + (1-\alpha)\frac{(\theta_{i+1}-\theta_i)\sum_{j=1}^{i}\sigma(\theta_j)}{\sigma(\theta_{i+1})}\right) - k\right) r(\theta_{i+1})\sigma(\theta_{i+1})$$
$$-(1-\alpha)u_p^\sigma(\theta_n). \tag{3.28}$$

Now, assume that $z_\sigma(\theta_i)$ is nondecreasing over $\Theta$. Hence, the optimal policy $\bar{M}_\sigma = (\bar{r}_\sigma, \bar{p}_\sigma, \bar{q}_\sigma, \bar{s}_\sigma)$ satisfies the following conditions:

$$\bar{p}_\sigma(\theta_i) = z_\sigma \tag{3.29}$$

$$P(\bar{q}_\sigma) = \bar{p}_\sigma \tag{3.30}$$

$$\bar{r}_\sigma(\theta_i) = \begin{cases} 1 \text{ if } V(\bar{q}_\sigma(\theta_i)) - \bar{p}_\sigma(\theta_i)\bar{q}_\sigma(\theta_i) \geq k \\ 0 \text{ if } V(\bar{q}_\sigma(\theta_i)) - \bar{p}_\sigma(\theta_i)\bar{q}_\sigma(\theta_i) < k \end{cases} \tag{3.31}$$

$$\bar{s}_\sigma(\theta_i) = [\theta_i + k - \bar{p}_\sigma(\theta_i)]\, \bar{q}_\sigma(\theta_i)\bar{r}_\sigma(\theta_i) + \sum_{j=i}^{n-1}(\theta_{j+1} - \theta_j)q(\theta_{j+1})r(\theta_{j+1}) \tag{3.32}$$

The optimal policy depends on the belief of the regulator about the type of the monopolistic firm. The results obtained by Koray and Saglam (2005a), straightforwardly apply to the discrete type version of the Baron Myerson model that we have just described. There exists a probability distribution over $\Theta$ under which the monopolistic firm maximizes its profit. We define this belief as

$$\bar{\sigma}(\theta_i) = \begin{cases} 1 \text{ if } \theta_i = \theta_n \\ 0 \text{ otherwise} \end{cases} \tag{3.33}$$

The belief $\bar{\sigma}$ is not an admissible belief. However, there exists a sequence of admissible beliefs that converge to the belief $\bar{\sigma}(\theta_i)$, as it is shown by Koray and Saglam (2005a).

Moreover, even though there is no belief that maximizes the gains of the consumers', they mostly prefer the social welfare function that gives no weight to the firm. The belief $\sigma_\alpha$ gives social welfare equal to the one that does not give any weight to the producer for a belief $\sigma$. We obtain $\sigma_\alpha$ as follows:

$$\sigma_\alpha(\theta_i) = (1-\alpha)\sigma(\theta_i)\frac{\sum_{j=1}^{i-1}\sigma_\alpha(\theta_j)}{\sum_{j=1}^{i-1}\sigma(\theta_j)} \tag{3.34}$$

The threat belief of the regulator is $\underline{\sigma}$, which creates almost nil outcomes for both agents unless the firm is of the most efficient type.

$$\underline{\sigma}(\theta_i) = \begin{cases} 1 \text{ if } \theta_i = \theta_1 \\ 0 \text{ otherwise} \end{cases} \tag{3.35}$$

Again, even though the $\underline{\sigma}$ is not an admissible belief, there exist a sequence of admissible beliefs that converge to $\underline{\sigma}$ (Koray and Saglam, 2005a).

## 3.3. The Corruption as a Bargaining Game

In this section, we will solve the discrete version of the corruption problem defined by Koray and Saglam (2005a) by using the solution of incentive compatible bargaining games introduced by Myerson (1979).

Myerson (1979) works on a framework in which the net gains of individuals depend on the types of the individuals and the choice taken by an arbitrator. Myerson's arbitrator deals with settling the conflicting interests of individuals, when individuals can gain by lying about their private information and by manipulating the decision of the arbitrator. In that sense, we formulate the corruption problem in the same way as Myerson's incentive compatible bargaining game.

The *regulator's problem* is an object of the form $(A, \Theta, v_1, v_2, \sigma)$ whose components are as follows. $A = \{a_0, a_1, a_2\}$ is the set of alternatives available to the regulator. Here, $a_0$ stands for the conflict outcome which is applied when parties are not willing to give a bribe. $a_1$ stands for the case when the regulator takes the bribe from the firm, and $a_2$ stands for the case when the regulator takes the bribe from the consumers' union. When the regulator chooses $a_1$, the consumers' union maximize their ex-ante gain. In this case, they pay the bribe $b_1$. $u_i^{\sigma_\alpha}(\theta_i)$ stands for the utility of agent $i$, $i \in \{p, c\}$ when the regulator's belief is $\sigma_\alpha$. $\sigma_\alpha$ assigns zero probability to all parameters on $\Theta$ except $\theta_1$ as given in equation (3.34)

When the regulator chooses $a_2$, the firm gains nearly the most it can gain for any belief of the regulator. In this case, it pays the bribe $b_2$. $u_i^{\bar{\sigma}}(\theta_i)$ stands for the utility of agent $i$, $i \in \{p, c\}$ when the regulator's belief is $\bar{\sigma}$. $\bar{\sigma}$ assigns zero probability to all parameters on $\Theta$ except $\theta_n$ as given in equation (3.33).

When the regulator chooses $a_0$, he pretends to believe that the firm is of the least efficient type, $\theta_1$. $\underline{\sigma}$ denotes this belief that assigns zero probability to all parameters on $\Theta$ except $\theta_1$ as given in equation (3.35).

$\Theta$ is the *type space* of the firm which is regulated. Here, $\Theta = \{\theta_1, \theta_2, ..., \theta_n\}$, where $\theta_1 < \theta_2 < ... < \theta_n$ and $n$ is finite. Each $\theta_i$ in $\Theta$ represents the cost parameter of the firm. The regulator has a sequence of prior beliefs $\{\sigma(\theta_i)\}_{i=1}^n$, where $0 < \sigma(\theta_i) < 1$ for all $i$ and $\sum_{i=1}^{n} \sigma(\theta_i) = 1$. $v_i$ is a utility such that $v_i : A \times \Theta \rightarrow \Re$ with $v_i(a, \theta_i)$ being the net payoff that the player $i$ gains if $a \in A$ is chosen and $\theta_i$ is the true private cost of the firm.

The net utilities of the agents depend on the decision taken by the regulator. When the regulator chooses $a_1$, the consumers' union gets the best of the utilities that it can get under any admissible belief of the regulator. For this sake, it pays the bribe $b_1$. Let $u_i^{\sigma_\alpha}(\theta_i)$ stand for the utility of player $i$, $i \in \{p, c\}$ when the belief of the regulator is $\sigma$ and the firm has no weight in the social welfare function so that the firm gets the whole welfare. However, the regulator can be accused of giving the whole weight to the consumers' union. Therefore, instead of giving no weight to the firm that is $\alpha = 0$ and announcing any admissible belief $\sigma$, the regulator uses the belief $\sigma_\alpha$ as given in equation (3.34), for any given $\alpha$.

Then $v_1(a, \theta_i)$ and $v_2(a, \theta_i)$ for any reported parameter $\theta$ can be defined as follows:

$$v_1(a, \theta_i) = \begin{cases} u_1^{\underline{\sigma}}(\theta_i) \text{ if } a = a_0 \\ u_1^{\sigma_\alpha}(\theta_i) - b_1 \text{ if } a = a_1 \\ u_1^{\bar{\sigma}}(\theta_i) \text{ if } a = a_2 \end{cases} \tag{3.36}$$

$$v_2(a, \theta_i) = \begin{cases} u_2^{\underline{\sigma}}(\theta_i) \text{ if } a = a_0 \\ u_2^{\sigma_\alpha}(\theta_i) \text{ if } a = a_1 \\ u_2^{\bar{\sigma}}(\theta_i) - b_2 \text{ if } a = a_2 \end{cases} \tag{3.37}$$

Now, we find out the bribes $b_1$ and $b_2$. The regulator can take the bribe from the consumers' union which is very close to its gain when it is the favored agent. Then,

$$\eta\left(u_1^{\sigma_\alpha}(\theta_i) - u_1^{\underline{\sigma}}(\theta_i)\right) \text{ for some } \eta \in (0,1). \tag{3.38}$$

The same idea does not work for the firm. The firm has the private information $\theta_i$. Because the regulator is not sure about the cost type of the firm, if the regulator asks the firm for a bribe proportional to its gain, the firm may have an incentive to conceal its type so that it can increase the gain. Therefore, what the regulator should do is to extract a gain as much as he can in line with the common expectation. That is

$$\max_{b_2} \sum_{i=1}^{n} b_2 p(\theta_i)\sigma(\theta_i), \tag{3.39}$$

where $p$ is the probability that the firm can pay the bribe. The firm cannot pay more than its gain when the producer favors it instead of punishing it. Let us show this amount with $\Pi(\theta_i)$.

$$\Pi(\theta_i) = u_2^{\bar{\sigma}}(\theta_i) - u_2^{\underline{\sigma}}(\theta_i) \tag{3.40}$$

Therefore $p$ can be defined as

$$p(\theta_i) = \begin{cases} 1 \text{ if } \Pi(\theta_i) \geq b_2 \\ 0 \text{ otherwise.} \end{cases} \tag{3.41}$$

When $\theta_k$ is the type of the firm such that $\Pi(\theta_k) = b_2$, then the problem in (3.39) can be written as $\max_{\theta_k} \sum_{i=1}^{k} \Pi(\theta_k)\sigma(\theta_i)$ or equivalently, Then, $\theta_k$ satisfies

$$\Pi(\theta_k)\sigma(\theta_k) - \sum_{i=1}^{k} \Pi(\theta_i)(\theta_{i+1} - \theta_i) = 0. \tag{3.42}$$

Therefore, the regulator can ask the consumers' union and the firm for the bribes $b_1$ and $b_2$, satisfying,

$$b_1 = \eta \left(u_1^{\sigma\alpha}(\theta_k) - u_1^{\underline{\sigma}}(\theta_k)\right) \text{ for some } \eta \in (0,1) \tag{3.43}$$

$$b_2 = \mu \left(u_2^{\bar{\sigma}}(\theta_k) - u_2^{\underline{\sigma}}(\theta_k)\right) \text{ for some } \mu \in (0,1) \tag{3.44}$$

where $\theta_k$ satisfies (3.42).

A solution to the problem is given as the procedure in which the regulator first asks the firm for some information about its cost and then selects a choice in $A$ or a probability distribution over $A$, using the information given to him. However, when the firm announces its private parameter, there is no way to understand if the firm is telling the truth or not. The only way to make the firm reveal its true type is to design a mechanism in which the firm cannot gain by lying. Then, the regulator makes his choice in such a way that the incentive compatibility condition for the firm is satisfied. The regulator can make a mixed choice of alternatives as well as a pure one.

The choice mechanism $c$, $c : A \times \Theta \to \Re$, is a probability distribution over $A$ such that $\sum_{a \in A} c(a, \theta_i) = 1$ and $c(a, \theta_i) \geq 0$ for every $a \in A$ when the firm reveals its type as $\theta_i$. $X_1(c)$ is *the expected utility for the consumers' union*, given that $\sigma$ is the common prior belief about the type of the firm and the choice mechanism is $c$. Then,

$$X_1(c) = \sum_{i=1}^{n} \sum_{a \in A} c(a, \theta) v_1(a, \theta_i) \sigma(\theta_i)$$

$X_2(c, \theta_j, \theta_i)$ is *the expected utility of the firm*, where the firm is of type $\theta_i$ whereas it reveals its type as $\theta_j$, when the choice mechanism is $c$. Then,

$$X_2(c, \theta_j, \theta_i) = \sum_{a \in A} c(a, \theta_j) v_2(a, \theta_i)$$

If the firm reveals its true type for a given choice mechanism $c$, then the expected utility for the firm is $X_2(c, \theta_i) \equiv X_2(c, \theta_i, \theta_i)$. A choice mechanism c is said to be *incentive compatible* if

$$X_2(c, \theta_i) \geq X_2(c, \theta_j, \theta_i). \tag{3.45}$$

If the regulator uses a choice mechanism $c$ and expects true responses then the set of incentive-feasible expected allocations is defined to be

$$F = \{X(c) : c \text{ is incentive-compatible}\} \tag{3.46}$$

where $X(c) = \left(X_1(c), (X_2(c, \theta_i))_{\theta_i \in \Theta}\right)$ is a vector of $1+|\Theta|$ dimensions. The regulator does not want to use a mechanism $c$ that is *strictly dominated by another mechanism* $c'$ in the sense that

$$X_1(c') \geq X_1(c) \text{ and } X_2(c', \theta_i) \geq X_2(c, \theta_i) \text{ for all } \theta_i. \tag{3.47}$$

Therefore, the regulator seeks for the solution from the set of choice mechanisms that are incentive compatible and not strictly dominated by any other incentive compatible mechanism. This kind of choice mechanism is called *incentive-*

*efficient.*

When the regulator cannot force the agents to agree on the bribe, he can threaten the agents with the *conflict outcome.* The conflict outcome is the outcome that occurs when the agents do not make an agreement. In the market problems, it may be defined as no production position. In the non-cooperative game problems, when a game in which the players have to play when they cannot agree on the collective choice is defined, the equilibrium of this game becomes the conflict outcome. In the corruption problem, the conflict outcome occurs when no agent wants to give a bribe.

Given the conflict outcome $a_0$, the *conflict payoff vector* is defined as

$$t = \left(t_1, (t_2(\theta_i))_{\theta_i \in \Theta}\right) \tag{3.48}$$

where

$$t_1 = \sum_{i=1}^{n} v_1(a_0,\theta_i) f(\theta_i) \text{ and } t_2(\theta_i) = v_2(a_0, \theta_i).$$

Here, $t_1$ is the consumers' expected utility payoff in the case of the conflict outcome, because the consumers do not know the true value of the private information of the firm. $t_2$ is the firm's profit in the case of the conflict outcome, because it is the only one who has private information. The conflict payoff vector is incentive-feasible. In other words, the firm has no incentive to lie when the regulator implements $a_0$.

The set of incentive-compatible mechanisms offer many solutions to the regulator who wants to find a way to make the agents voluntarily bribe him. Given the conflict outcome and correspondent payoffs, the problem of the regulator becomes a *bargaining problem* which is a version of the problem studied by Nash. The solution to this problem is formalized by Harsanyi and Selten. Myerson

(1979) generalizes the problem and uses the same solution concept. We define the solution set for the problem of the regulator by following Myerson (1979). *The incentive-feasible bargaining solution* is the vector $X(c)$ which maximizes the product

$$(X_1(c) - t_1) \prod_{\theta_i \in \Theta_i} (X_2(c, \theta_i) - t_{\theta_i})^{f(\theta_i)} \tag{3.49}$$

over the set $F_+$ where $F_+$ is the set of *individually rational allocations* in which no one expects to do worse than in the conflict outcome:

$$F_+ = F \cap \{Y : Y_i \geq t_i \text{ for all } i \text{ and all } \theta \in \Theta\} \tag{3.50}$$

Myerson (1979) shows that there exists a unique incentive-feasible bargaining solution when the conflict outcome $a_0$ is not incentive-efficient. Reinterpreting this conclusion, the regulator can implement a mechanism in which parties give bribes with the probabilities identified by the mechanism. Moreover, the solution given by the mechanism is paretoptimal and incentive compatible for the firm and individually rational for both agents.

Finally, the unique incentive feasible bargaining solution $x(c)$ maximizes

$$(x_1(c) - t_1) \prod_{\theta_i \in \Theta} [(x_2(c, \theta_i) - t_{\theta_i}]^{\sigma(\theta_i)} \tag{3.51}$$

where $t_1 = \sum_{i=1}^{n} u_1^{\sigma}(\theta_i)\underline{\sigma}(\theta_i)$, and $t_{\theta_i} = u_2^{\sigma}(\theta_i)$.

Let us denote $c(a_i, \theta_i) = c_i(\theta_i)$. Then, $x_1(c)$ and $x_2(c_i(\theta_i)$ satisfy

$$x_1(c) = \sum_{i=1}^{n} [c_0(\theta_i)u_1^{\sigma}(\theta_i) + c_1(\theta_i)\,(u_1^{\sigma_\alpha}(\theta_i) - b_1) + c_2(\theta_i)u_1^{\sigma}(\theta_i)]\,\sigma(\theta_i) \tag{3.52}$$

and

$$x_2(c_i(\theta_i) = \sum_{i=1}^{n} \left[c_0(\theta_i)u_2^{\underline{\sigma}}(\theta_i) + c_1(\theta_i)u_2^{\sigma_\alpha}(\theta_i) + c_2(\theta_i)\left(u_2^{\underline{\sigma}}(\theta_i) - b_2\right)\right]\sigma(\theta_i). \quad (3.53)$$

## 3.4. Simulation Results

In this section, we illustrate the utilization of the incentive compatible bargaining game in solving the corruption problem in the Baron-Myerson model monopoly regulation for discrete type spaces by implementing a simulation. The simulation is implemented in Matlab R2007a. We aim to analyze how the profit of the firm and the consumer surplus behave as the demand in the market changes under the optimal regulatory mechanisms for the different beliefs of the regulator. In addition, we explore the question of which party will be favored as the demand increases in the market.

We have assumed a linear inverse demand function, $P(q) = a - bq$. The type of the firm is from the set $\{\theta_1, \theta_2, ..., \theta_n\}$ where $\theta_{i+1} - \theta_i > k$ for all $i$ with $k > 0$. The social welfare function gives weight $\alpha$ to the producer. When the producer is favored by the regulator, some percentage of its gain, which is represented by $\eta$, is taken by the regulator as a bribe. If the favored agent is the consumers' union, then the percentage $\mu$ of its gain is the bribe given to the regulator.

The parameter values are the variables $a, b, \theta_1, \theta_n, \alpha, \eta, \mu$. We assume that the fixed cost of the firm is 0 so that the firm is always allowed to produce. The output of the simulation are the optimal policy components, $p(\theta_i), q(\theta_i), s(\theta_i)$, the utilities of the consumers' union, $u_1^{\underline{\sigma}}(\theta_i), u_1^{\sigma_\alpha}(\theta_i), u_1^{\bar{\sigma}}(\theta_i)$, the utilities of the firm when it tells the truth, $u_2^{\underline{\sigma}}(\theta_i), u_2^{\sigma_\alpha}(\theta_i), u_2^{\bar{\sigma}}(\theta_i)$, the utilities of the firm when it misreports the parameter $\theta_i$ as $\theta_j$, $u_2^{\underline{\sigma}}(\theta_j, \theta_i), u_2^{\sigma_\alpha}(\theta_j, \theta_i), u_2^{\bar{\sigma}}(\theta_j, \theta_i)$, the bribe values $b_1$ and $b_2$, the choice mechanism $c = (c_0, c_1, c_2)$ that maximizes the generalized Nash product, and the maximum value of the Nash product $f$, for all values of

$\theta_i$.

Scenario 1. Consider that $\Theta_3 = \{1, 2, 3\}$, so that the marginal cost of the firm can be one of these three types. The inverse demand function is known to be $P(q) = 10 - q$. The regulator is supposed to maximize a social welfare function that gives weight 0.80 to the firm. The regulator wants to extract all the gain that he can get from this game and asks for 99 percent of the gain of the favored agent. The following table represents the net utilities of the agents. The first and the second values in each cell belong to the consumers' union and the firm, respectively.

Table 3.1. Gains of the Parties under Different Beliefs for $\Theta_3$

| $\theta_i$ | $\underline{\sigma}$ | $\sigma_\alpha$ | $\bar{\sigma}$ |
|---|---|---|---|
| 1 | (40.5,0) | (28.5,12) | (25.5,15) |
| 2 | (0,0) | (26.5,5) | (25,7) |
| 3 | (0,0) | (22.5,0) | (24.5,0) |

In the above table, we observe that if the conflict outcome occurs, when the firm is of the most efficient type, the firm gets no profit, but consumers get their maximum profit of 40.5. When the firm is not the most efficient type both get nil. If the regulator pretends to believe that the firm's type is $\theta_3$, while the real type of the firm is $\theta_1$, the firm gets its maximum gain of 25.5. The consumers gain their second best gain of 28.5, if the regulator pretends to believe $\sigma_\alpha$, and the firm is of the most efficient type. Whenever the regulator favors the consumers and the firm is not of the most efficient type, the firm gains higher utilities as we expect from the model.

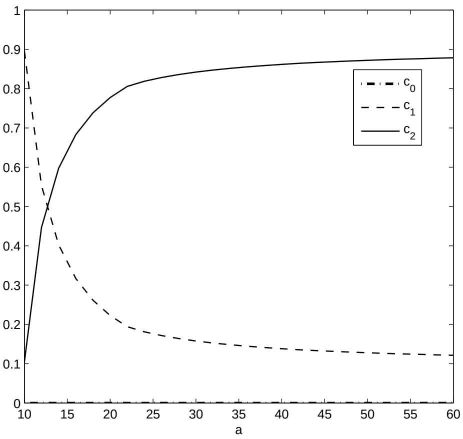
The choice mechanism that maximizes the generalized Nash product given by the above utility functions of the parties is as follows:

Table 3.2. Choice Mechanism for $\Theta_3$

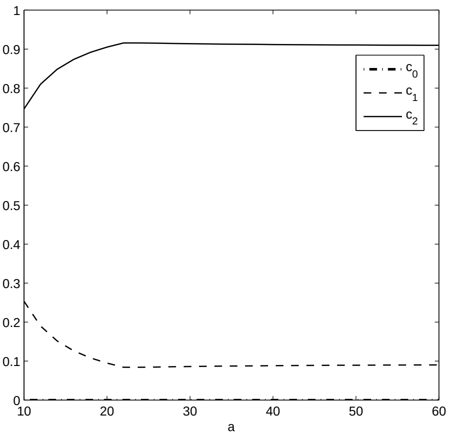
| $\theta_i$ | $c_0$ | $c_1$ | $c_2$ |
|---|---|---|---|
| 1 | 0 | 0.8958 | 0.1042 |
| 2 | 0 | 0.2532 | 0.7468 |
| 3 | 0 | 0 | 1 |

These results show that the regulator distributes his decision to take the bribe from consumers or the firm for efficient values of $\theta$. When $P(q) = 10 - q$ and the cost of the firm is the most efficient type, the regulator takes the bribe from consumers with probability 0.8958, and he takes the bribe from the firm with probability 0.1042. If the cost of the firm is of type $\theta_2$, the regulator takes the bribe from the producer with probability 0.7468. As the cost of the firm increases, the regulator tends to take the bribe from the firm, when the demand is fixed in the market.

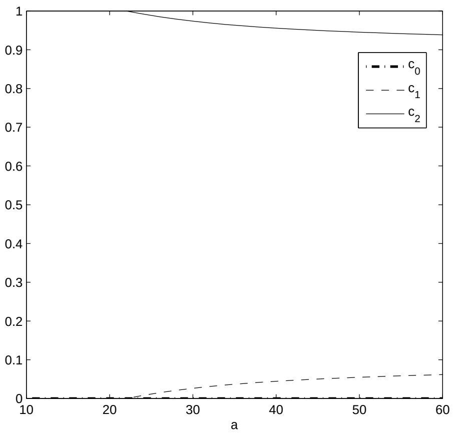
Next, we analyze the behavior of the regulator, when the cost type of the firm is fixed, but the demand increases in the market. Figure 3.1 shows the choice mechanisms applied by the regulator when the type set of the firm is $\Theta_3$. In Figure 3.1.a, the cost of the firm is 1 which is the most efficient type. As the price coefficient $a$ increases from 10 to 60, the demand for the product increases accordingly. For the low values of the demand, the regulator favors the consumers with higher probabilities. The bargaining game results on behalf of the firm, as the demand increases. In Figure 3.1.b, the cost of the firm is 2. In the choice mechanism, the probability of favoring the firm increases approximately from 0.75 to 0.9. Since the cost of the firm is higher with respect to the previous case, the probability of being the favored agent is higher for the lower demand values. In Figure 3.1.c, the cost of the firm is the least efficient type. The probability of favoring the firm starts from 1 for low demand values. It hardly deviates from these values for the high values of the demand.




a. $\theta = 1$



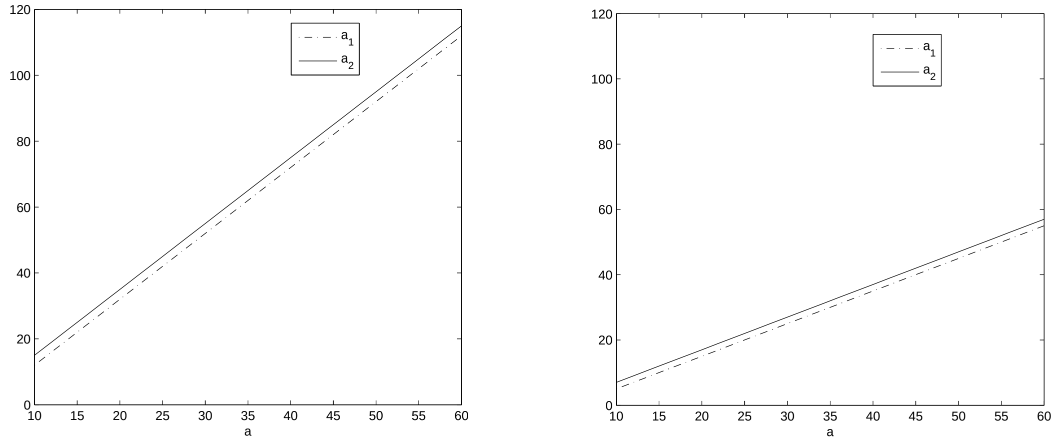

b. $\theta = 2$




c. $\theta = 3$

Figure 3.1. The choice mechanism under demand shocks for $\Theta_3$

Indeed, the choice mechanism converges approximately to 0.9 for any cost type of the firm. In Figure 3.2 given above, the behavior of the firm's profits under the beliefs $\bar{\sigma}$ and $\sigma_\alpha$ of the regulator with respect to the changes in demand is given, when the cost of the firm is 1 and 2 in Figure 3.1.a and in Figure 3.1.b, respectively. When the cost of the firm is the highest one, even though the regulator favors the firm in order to make him reveal his true cost, the firm gains no profit due to its high cost.




a. $\theta = 1$ b. $\theta = 2$

Figure 3.2. The producer's gain under demand shocks for $\Theta_3$

The analysis of the alteration of choice mechanism due to the changes in the demand for a fixed cost parameter is partially equivalent to the analysis of the variations of the choice mechanism for different cost types of the firm where the demand value is fixed. In both ways, the choice mechanism converges approximately to 0.9 probability for the firm and 0.1 probability for the consumers' union.

The size of the type space $\Theta_3$ of the firm is very small to make conclusions. Therefore, we test the same analysis for $\Theta_5 = \{1, 2, 3, 4, 5\}$ and $\Theta_8 = \{1, 2, 3, 4, 5, 6, 7, 8\}$.

Scenario 2. Now, consider that $\Theta_5 = \{1, 2, 3, 4, 5\}$, so that the marginal cost of the firm can be one of these five types. The inverse demand function $P(q) = 10-q$ is not an appropriate one for this case, since the demand turns out to be very small against the possible high costs of the firm. Then, we first assume that $P(q) = 50 - q$. The rest of the assumptions made in the previous scenario hold. The following table represents the gain of the consumers and the profit of the firm under different beliefs of the regulator as the cost type of the firm changes.

Table 3.3. Gains of the Parties under Different Beliefs for $\Theta_5$

| $\theta_i$ | $\underline{\sigma}$ | $\sigma_\alpha$ | $\bar{\sigma}$ |
|---|---|---|---|
| 1 | (1200.5,0) | (1024.5,176) | (1014.5,186) |
| 2 | (0,0) | (1022.5,129) | (1014,138) |
| 3 | (0,0) | (1018.5,84) | (1013.5,91) |
| 4 | (0,0) | (1012.5,41) | (1013,45) |
| 5 | (0,0) | (1004.5,0) | (1012.5,0) |

We see the difference between the gain of the consumers when the belief of the regulator is $\sigma_\alpha$ and $\bar{\sigma}$ is positive but small for each cost type of the firm. Economically, when the favored agent is the firm, the consumers do not lose much.

Figure 3.3 on the next page shows the choice mechanisms applied by the regulator for each cost type of the firm. The probability that the regulator assigns in taking the bribe from the firm is approximately 0.7, whereas 0.3 probability is assigned in taking the bribe from the consumers' union. Why this choice mechanism maximizes the generalized Nash product can be partially explained by the fact that the loss of the consumers is acceptable under this choice mechanism when the favored agent is the firm.

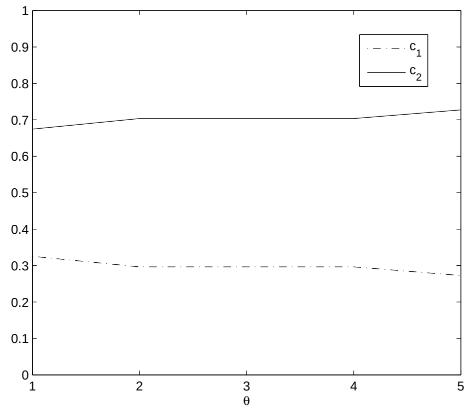
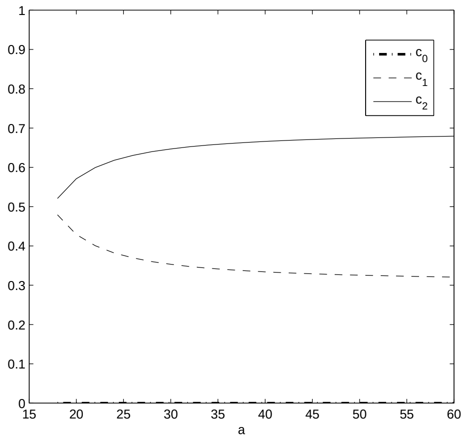
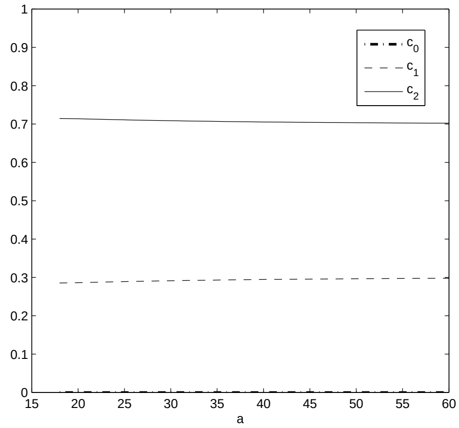



Figure 3.3. The choice mechanism for cost types in $\Theta_5$ when $a = 50$

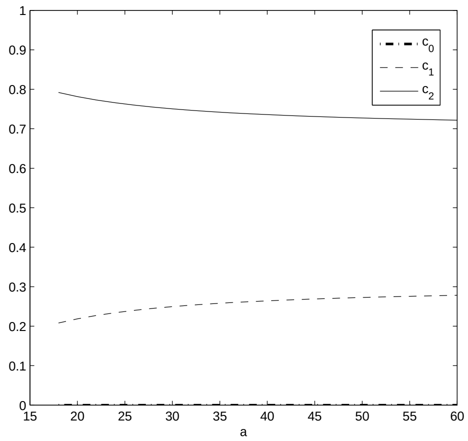
Now, in order to observe the effect of a demand raise in the choice mechanism, we present Figure 3.4 on the next page for the given costs of the firm. In Figure 3.4.a., the cost of the firm is of the most efficient type. The regulator's choice mechanism is almost identical when the cost of the firm is 2, 3 or 4. Figure 3.4.b involves $c_0$, $c_1$, $c_2$ when the cost is 2 as a representative. We observe that the choice mechanisms where $c_1$ and $c_2$ are around 0.3 and 0.7, respectively. In Figure 3.4.c, the cost of the firm is of the least efficient type. Although we notice an infinitesimal increase in $c_1$ for low values of the demand, in general, the choice mechanism does not deviate from the 0.7/0.3 level. The high cost of the firm can partially explain this incremental difference of the choice mechanism for the low demand values. Hence, although ignorable exceptions may arise due to the highest or smallest costs of the firm, the increase in demand does not really alter the choice mechanism for this scenario. The choice mechanism that maximizes the generalized Nash product is around 0.3 probability in favoring the consumers and 0.7 probability in favoring the producer.




a. $\theta = 1$



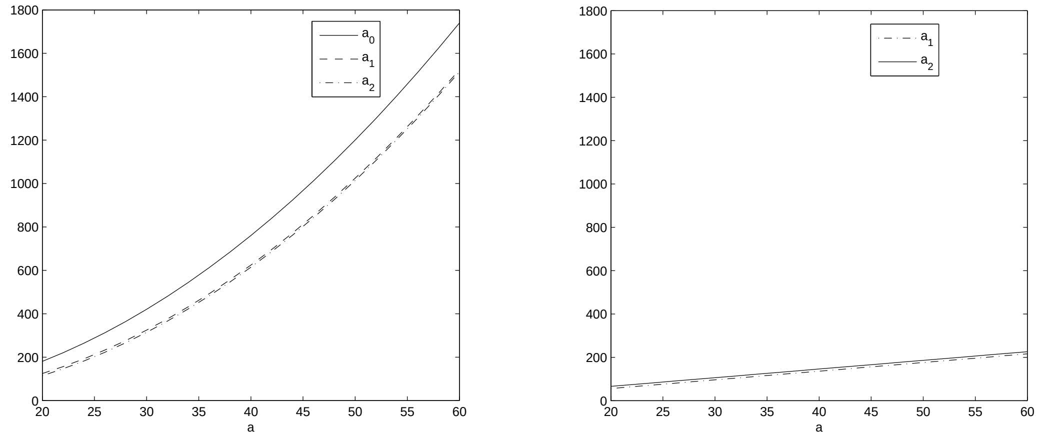

b. $\theta = 2$




c. $\theta = 5$

Figure 3.4. The choice mechanism under demand shocks for $\Theta_5$

Why the parties agree on this level, again, can be explained by the fact that the loss of the consumers remains to be acceptable under this choice mechanism, when the favored agent is the firm, even though the demand values increase. As a representative case, we set the cost type of the firm as the most efficient type and present the gain of the consumers in Figure 3.5.a and the profits of the firm in Figure 3.5.b as the demand changes.




a. Consumers' gain b. Producer's gain

Figure 3.5. Gains of the parties under demand shocks for $\Theta_5$

When we compare the two scenarios, the optimal choice mechanisms differ. We mostly explain this result with the fact that the type space $\Theta_3$ is considerably narrow. In order to verify the results in scenario two, we present the same analysis where the cost of the firm belongs to the set $\Theta_8 = \{1, 2, 3, 4, 5, 6, 7, 8\}$.

Scenario 3. Consider that $\Theta_8 = \{1, 2, 3, 4, 5, 6, 7, 8\}$, so that the marginal cost of the firm can be one of these eight types. First, we assume that the inverse demand function is $P(q) = 50 - q$. The rest of the assumptions on the input variables hold as in the previous scenarios.

The changes in the choice mechanism, as the cost type of the firm changes, are given in Figure 3.6. We observe that the optimal choice mechanism is around the same level as scenario 2, given in Figure 3.3. The probability in favoring the producer is around 0.7, while the probability in favoring the consumers' union is around 0.3.

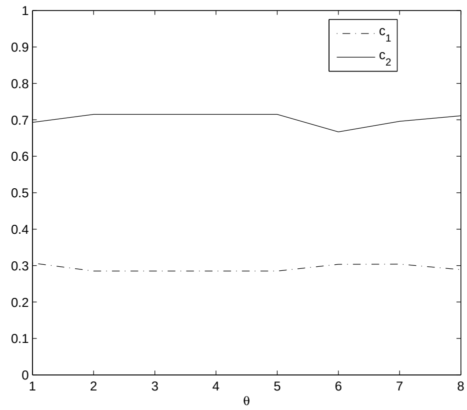
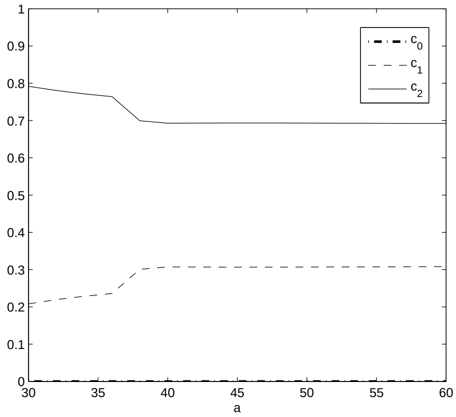
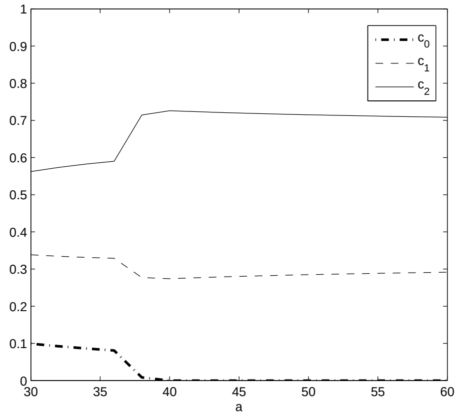



Figure 3.6. The choice mechanism for cost types in $\Theta_8$ when $a = 50$

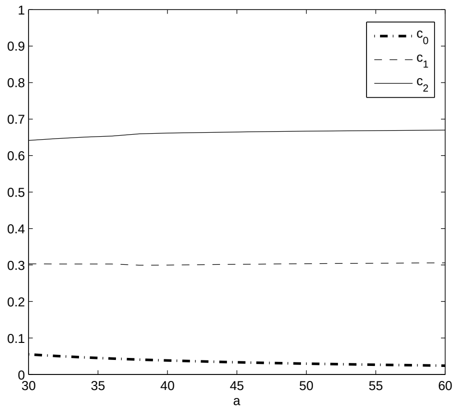
The changes in the choice mechanism with respect to the changes in the demand values for a given cost type of the firm are given in Figure 3.7. The behavioral and value change in the choice mechanisms are very similar to each other when the cost type of the firm is 1, 2, 3 or 4 and they are very similar when the cost type of the firm is 7 or 8. Therefore, we present the situation for the cost type 1 of the firm, in Figure 3.7.a, and for the cost type 7 of the firm, in Figure 3.7.d. When the demand is low in the market, and the cost of the firm is not the most or the least efficient type, there exists a room for conflict outcome in the choice mechanism. In these mechanisms, the probability of favoring the consumers does not deviate from the 0.3 level, however, the probability of favoring the producer is reduced, and a positive probability is given to the conflict outcome. Even though these exceptions arise, the general tendency of the choice mechanism in converging to the 0.7/0.3 level still holds.



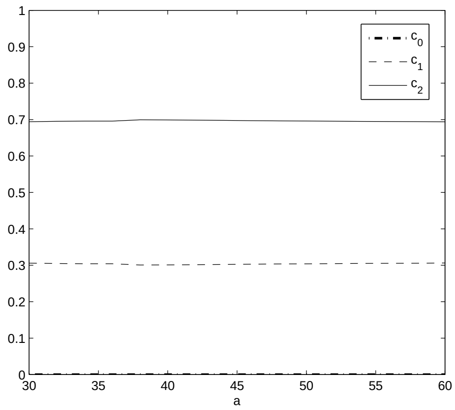

a. $\theta = 1$




b. $\theta = 5$



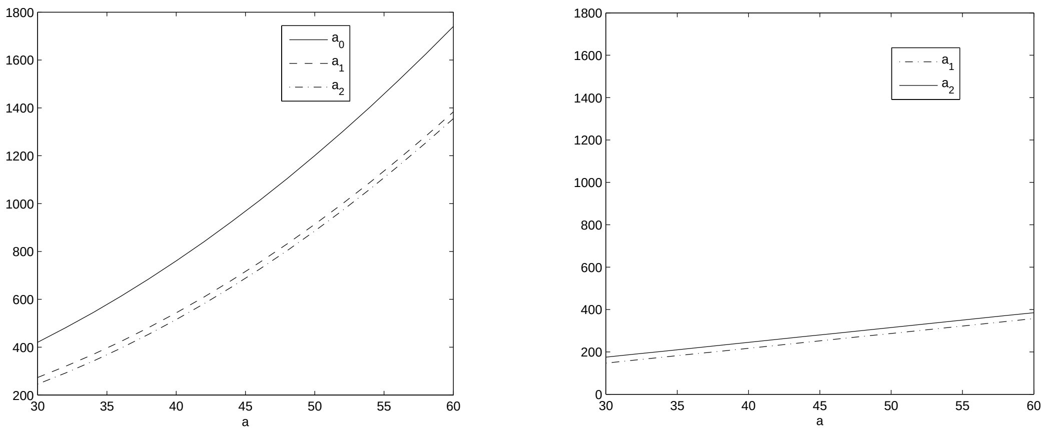

c. $\theta = 6$




d. $\theta = 7$

Figure 3.7. The choice mechanism under demand shocks for $\Theta_8$

In order to analyze the gains of the parties in the bargaining game under the beliefs $\bar{\sigma}$ and $\sigma_\alpha$ of the regulator, as the demand changes for a given cost type of the firm, we present Figure 3.8. Since we observe that the behavioral change in both the producer's and the consumers' gain does not differ as the cost type of the firm changes, we present only these two figures out of all to give the insight on the overall case.

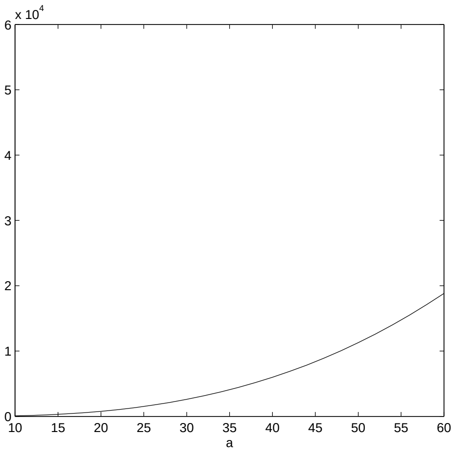



a. Consumers' gain

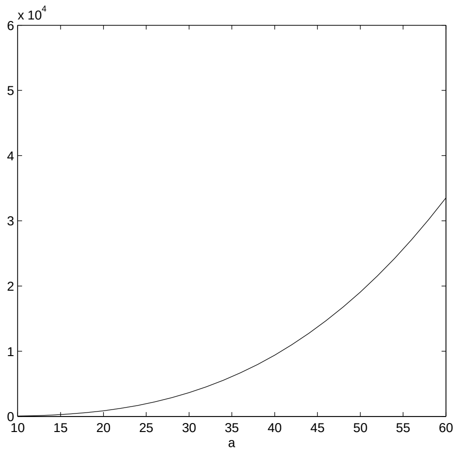
b. Producer's gain

Figure 3.8. Gains of the parties under demand shocks for $\Theta_8$

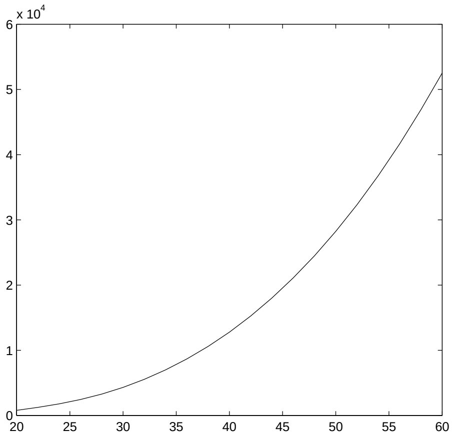
Overall Results. In each scenario, we observe that the gain of the producer increases linearly and the gain of the consumers increases quadratically, as the demand increases under each belief of the regulator. This result is also supported by the model. We observe from the simulation results that the cake to be shared becomes larger as the demand increases. Figure 3.9 shows that the generalized Nash product increases as the demand increases. We present the situation, when the firm's cost lies in $\Theta_3$, $\Theta_5$, $\Theta_8$, in Figure 3.9.a, Figure 3.9.b, Figure 3.9.c, respectively. Moreover, we observe from Figure 3.9, that for a fixed value of the price, the cake gets bigger as the type space of the firm enlarges. Figure 3.8 and Figure 3.9 show that for a fixed demand value the consumers' gain almost remains the same, whereas the profit of the firm increases. This is mostly because, as the type space of the firm enlarges, the ability of the regulator to guess the real cost of the firm weakens. Therefore, the regulator pays more informational rent to the producer. The generalized Nash product constitutes the expected value of the gains of the parties. Therefore, the enlargement of the cake, as the size and type space increases for a fixed demand shock, is supported by the model.




a. $\Theta = \Theta_3$




b. $\Theta = \Theta_5$




c. $\Theta = \Theta_8$

Figure 3.9. Genaralized Nash product under demand shocks

There is a general tendency in the optimal choice mechanism where the probability of favoring the firm is approximately 0.7 and the probability of favoring the consumers' union is approximately 0.3. Exceptions arise under some circumstances. In the first scenario, where the type set that includes the real cost of the firm is narrow, and the firm is the most or the least efficient type, the choice mechanism strongly deviates from the general level. In the other scenarios, when the firm is the most and the least efficient type, the choice mechanism slightly deviates from the general level favoring the consumers in the former case, and favoring the firm in the latter case. The reason behind that is supported by the incentive compatible bargaining game we used. The regulator should promise to favor the firm whose cost is the least efficient in order to persuade the firm to reveal its real cost. When the firm's cost is low, the regulator gives higher probabilities in favoring the consumers who gain their best if the firm is the least efficient type.

## 3.5. Conclusion

In this chapter of the thesis, we formalized the discrete type space version of the corruption problem which is proven to exist in the Baron Myerson(1982) model of monopoly regulation by Koray and Saglam (2005a)(K-S). Using the results of K-S obtained for the continuum, we identified the corruption problem as a bargaining game for the discrete case. We solved the difficulties that arise due to uncertainty about the private cost parameter of the firm, by applying Myerson (1979)'s solution for the incentive compatible bargaining games. We implemented a simulation of the model to quantitatively sense the economic effects of the corruption game that are expected under the theory.

We observed that the regulator persuades the consumers and the monopoly to agree on paying high percentages of their gains to him for being the favored party. The regulator's choice of whom will be favored does not deviate much from a general pattern in which the producer is highly favored, for both of the cases are identified by an increase in the demand under a given type of monopoly and an increase in the type of monopoly under a given demand. Infinitesimal deviations from this general pattern arise in favor of the consumers when the cost of the monopoly is too low (when the demand is too high) and in favor of the producer when the cost is too high (when the demand is too low). This result is supported by the model. In order to make the firm reveal its cost truthfully, the informational rent in the regulation and the favoring probability in the incentive compatible bargaining game given to the firm increase as the cost of the firm increases. The general choice of the regulator to favor the producer in higher probabilities is agreeable for the consumers, since the loss of the consumers is at an acceptable level under this choice. Moreover, we observed that the size of the cake to be bargained enlarges as the demand increases in the monopolistic markets, as we can expect from the model.

The incentive mechanisms are strongly criticized on their manipulability in the literature. (Crew and Kleindorfer 1986, Crew and Kleindorfer 2002, Vogelsang 1988, Vogelsang 2002, Vogelsang 2004, Koray and Sertel 1990, Koray and Saglam 1999, Koray and Saglam 2005a, Koray and Saglam 2005b). The regulator can manipulate the mechanism due to many reasons. A government which can strictly control the beliefs of the regulator, can redistribute the welfare of the society in the way it wants, while pretending to be fair and honest (Crew and Kleindorfer, 2002). Besides, the regulator can be captured by the politically dominant group (consumers' union or the producers) so that the policy applied is dictated according to the beliefs of this group (Posner, 1974). Overall, our contribution to this literature is the specification and verification of the well-known facts about the manipulability of the Baron and Myerson model of monopoly regulation in the discrete version. The utilization of the discrete type space made it possible to reach quantitative findings which lead us to sense the economic results further.

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