EGE UNIVERSITY GRADUATE SCHOOL OF APPLIED AND NATURAL SCIENCES

MASTER OF SCIENCE THESIS

SECOND LAW ANALYSIS OF THERMAL STORAGE SYSTEMS

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Presentation Date: 14.09.2007

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Sayın **Başak BARUTÇU** tarafından YÜKSEK LİSANS TEZİ olarak sunulan **"Second Law Analysis of Thermal Storage Systems"** başlıklı bu çalışma E.Ü. Lisansüstü Eğitim ve Öğretim Yönetmeliği ile E.Ü. Fen Bilimleri Enstitüsü Eğitim ve Öğretim Yönergesi'nin ilgili hükümleri uyarınca tarafımızdan değerlendirilerek savunmaya değer bulunmuş ve 14.09.2007 tarihinde yapılan tez savunma sınavında aday oybirliği/oyçokluğu ile başarılı bulunmuştur.

Jüri Üyeleri: **İmza**

Jüri Başkanı: **.........................**

Raportör Üye: **.........................**

Üye: **.........................**

ABSTRACT

SECOND LAW ANALYSIS OF THERMAL STORAGE SYSTEMS

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M.Sc. in Chemical Engineering

Supervisor: Prof. Dr. Firuz BALKAN

September 2007, 92 pages

In this thesis, the thermal storage systems are investigated from second law point of view. Some part of sensible energy of a hot waste gas is stored in the water as storage material in the tank. In storing energy, due heat transfer irreversibilities, some exergy is inevitably lost. To decrease exergy lost storage is done through cascade of n stages. Increasing the number of stages exergy lost decreased but this is offset by the increase in fixed cost of system. It is shown that optimum number of stages is five.

Keywords: Thermodynamic, Entropy, Exergy, TES

ÖZET

ISI DEPOLAMA SİSTEMLERİNİN İKİNCİ YASA ANALİZİ

BARUTÇU, Başak

Yüksek Lisans Tezi, Kimya Mühendisliği Bölümü
Tez Yöneticisi: Prof. Dr. Firuz BALKAN

Eylül 2007, 92 sayfa

Bu tezde ısı depolama sistemleri ikinci yasa bakımından incelenmiştir. Atık sıcak gazdaki hissedilir ısının bir kısmı tank içinde depolama maddesi olarak kullanılarak su içinde depolanmıştır. Enerji depolanırken, tersinmez ısı aktarımı nedeniyle bir miktar ekserji kaçınılmaz olarak kaybolur. Ekserji kaybını azaltmak için depolama seri olarak bağlanmış birden fazla kademede yapılır. Ancak, bu şekilde azaltılan ekserji kaybının yanında bu kez de artan tank maliyetleri gündeme gelir. Bu nedenle bir optimum tank sayısı olmalıdır. Bu çalışmada optimum tank sayısının beş olduğu bulunmuştur.

Anahtar Sözcükler: Termodinamik, Entropi, Ekserji, TES

ACKNOWLEDGEMENT

The author sincerely thanks to her supervisor Prof. Dr. Firuz BALKAN for his support, suggestions, critism and patience during the development of this thesis and for her entire career.

The author is also grateful to her family for their valuable support and patience in her entire life.

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NOMENCLATURE

A	Area, m^2
C	Cost, YTL/year
C_p	Specific heat, J / kgK
D	Diameter, m
e	Specific exergy, J/kg
E	Exergy, J
g	Gravity, m/s^2
h	Height of tank, m
k	Constant specific heat ratio, -
ke	Specific kinetic energy, J/kg
KE	Kinetic energy, J
L	Flow path length, m
$\dot{m}$	Flow rate of gas, kg/s
M	Mass of liquid bath, kg
N	Stage number
N_s	Dimensionless exergy loss number
Ntu	Number of transfer units
P	Pressure, atm, kPa
p	Perimeter, m
pe	Specific potential energy, J/kg
PE	Potential energy, J
Q	Heat Flowrate, W
S	Entropy, W/K
t	Time, s

NOMENCLATURE (Continued)

T	Temperature, ^{o}C
TES	Thermal energy storage
U	Overall heat transfer coefficient, W/m^{2}K
u	Specific internal energy, J
V	Volume of tank, m^3
W	Work, J
z	Height, m

Greek Letters

τ	Dimensionless temperature difference
θ	Dimensionless time
ρ	Density, kg/m^3
∞	Hot gas inlet

Subscripts

ad	Adiabatic system
amb	Ambient
c	Combined system
C	Coil
cv	Control volume
D	Destruction
e	Exit
g	Gas
gen	Generated
i	Inlet

NOMENCLATURE (Continued)

int	Internal
isol	Isolated system
L	Lateral
max	Maximum
min	Minimum
o	Ambient
opt	Optimum
out	Gas outlet
rev	Reversible
surr	Surrounding
T	Tank

Superscripts

CH	Chemical
e	Environment
KN	Kinetic
PH	Physical
PT	Potential

1. INTRODUCTION

The fact that the energy requirements increasing from day to day and increasing cost of energy impose both the invention of new energy resources and more efficient and better use of energy from limited amount of traditional sources. The studies on this area have been oriented especially to the minimization of availability losses (entropy generation). The real processes are always irreversible and they produce entropy.

Thermal energy storage (TES) is considered as one of the most crucial energy technologies and recently, increasing attention has been paid to the utilization of this essential technique. Economical aspects behind the design and operation of energy conversion systems has brought TES to the forefront. In conjunction with this, provisions must be included in an energy conversion system when the supply and demand for thermal energy do not coincide with time. TES appears to be the only solution to correcting the mismatch between the supply and demand of energy. TES is a key component of any successful thermal system and a good TES should allow minimum thermal energy losses, leading to energy savings, while permitting the highest possible extraction efficiency of the stored thermal energy (Dinçer, 2002).

Rosen (2003) investigates that exergy analysis is a method that uses the conservation of mass and conservation of energy principles together with the second law of thermodynamics for the design and analysis of energy systems. The exergy method can be suitable for furthering the goal of more efficient energy resource use, for it enables the location, type, and

true magnitude of wastes and losses to be determined. Therefore, exergy analysis can reveal whether or not and by how much it is possible to design more efficient energy systems by reducing the inefficiencies in existing systems. Therefore, exergy analysis appears to be a crucial tool in TES applications for energy conservation.

Exergy is defined as the maximum amount of work which can be produced by a stream of matter, heat or work as it comes to equilibrium with a reference environment. It is a measure of the potential of a stream to cause change, as a consequence of not being completely stable relative to the reference environment. Exergy is not subject to a conservation law; rather exergy is consumed or destroyed, due to irreversibilities in any process. Recently, exergy analysis has been recognized as the more powerful method for performance evaluation and design calculations of TES system rather than energy analysis (Rosen, 2003)

In the first part of this work we focus on the special class of heat exchangers intended for use in batch heating and batch cooling processes. The batch heating process appears frequently in the design of systems for thermal energy storage. One important characteristic of this class of heat exchangers is their transient operation; time becomes an important design parameter which plays a significant role in determining the thermodynamic irreversibility of the process. We study in detail the effect played by time-history on the loss of exergy during the exergy storage process.

In the second part, we focus on the economic analysis of the thermal energy storage systems.

2. BASIC LAWS OF THERMODYNAMICS

In the past 25 years engineering thermodynamics has undergone a revolution, both in terms of the presentation of fundamentals and in the manner that it is applied. In particular, the second law of thermodynamics has emerged as an effective tool for engineering analysis and design.

2.1 THE FIRST LAW OF THERMODYNAMICS, ENERGY

Energy is a fundamental concept of thermodynamics and one of the most significant aspects of engineering analysis. Energy can be stored within systems in various macroscopic forms: kinetic energy, gravitational potential energy, and internal energy. Energy can also be transformed from one form to another and transferred between systems. For closed systems, energy can be transferred by work and heat transfer. The total amount of energy is conserved in all transformations and transfers.

2.1.1 ENERGY

Supposing that a closed system undergoing a process that involves only work interactions with its surroundings, experiences an adiabatic process. On the basis of experimental evidence, it can be postulated that when a closed system is altered adiabatically, the amount of work is fixed by the end states of the system and is independent of the details of the process. This postulate, which is one way the first law of thermodynamics can be stated, can be made regardless of the type of work interaction involved, the type of process, or the nature of the system.

As the work in an adiabatic process of a closed system is fixed by the end states, an extensive property called energy can be defined for the system such that its change between two states is the work in an adiabatic process that has these as the end states. In engineering thermodynamics the change in the energy of a system is considered to be made up of three macroscopic contributions: the change in kinetic energy, KE, associated with the motion of the system as a whole relative to an external coordinate frame, the change in gravitational potential energy, PE, associated with the position of the system as a whole in the Earth's gravitational field, and the change in internal energy, U, which accounts for all other energy associated with the system. Like kinetic energy and gravitational potential energy, internal energy is an extensive property (Kreith, 2000).

In summary, the change in energy between two states of a closed system in terms of the work W_{ad} of an adiabatic process between these states is

$$(KE_2 - KE_1) + (PE_2 - PE_1) + (U_2 - U_1) = -W_{ad} \qquad (2.1)$$

where; the subscripts 1 and 2 denote the initial and final states, respectively, and the minus sign before the work term is in accordance with the previously stated sign convention for work. Since any arbitrary value can be assigned to the energy of a system at a given state 1, no particular significance can be attached to the value of the energy at state 1 or at any other state. Only changes in the energy of a system have significance.

The specific energy (energy per unit mass) is the sum of the specific internal energy, u, the specific kinetic energy, $v^2/2$, and the specific gravitational potential energy, gz, such that

$$specific\ energy = u + \frac{v^2}{2} + gz \qquad (2.2)$$

where the velocity v and the elevation z are each relative to specified data (often the Earth's surface) and g is the acceleration of gravity.

A property related to internal energy u, pressure P, and specific volume V is enthalpy, defined by

$$h = u + PV \qquad (2.3)$$

or on an extensive basis

$$H = U + PV \qquad (2.4)$$

2.2 THE SECOND LAW OF THERMODYNAMICS, ENTROPY

In a general sense, the second law says that temperature differences between systems in contact with each other tend to even out and that work can be obtained from these non-equilibrium differences, but that loss of heat occurs, in the form of entropy, when work is done. Pressure differences, density differences, and particularly temperature differences, all tend to equalize if given the opportunity. This means that an isolated system will eventually come to have a uniform temperature. A heat engine is a

mechanical device that provides useful work from the difference in temperature of two bodies:
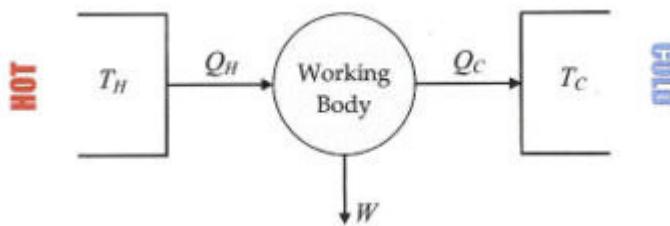
Figure 2.1. Heat Engine Diagram

Informal descriptions

The second law can be stated in various ways, including:

- It is impossible to produce work in the surroundings using a cyclic process connected to a single heat reservoir (Kelvin's Statement).
- It is impossible to carry out a cyclic process using an engine connected to two heat reservoirs that will have as its only effect the transfer of a quantity of heat from the low-temperature reservoir to the high-temperature reservoir (Clausius's Statement).
- If thermodynamic work is to be done at a finite rate, free energy must be expended.

Mathematical descriptions

In 1856, the German physicist Rudolf Clausius stated what he called the "second fundamental theorem in the mechanical theory of heat" in the following form:

$$\int \frac{\delta Q}{T} = -N$$

<div align="right">(2.5)</div>

where N is the "equivalence-value" of all uncompensated transformations involved in a cyclical process. Later, in 1865, Clausius would come to define "equivalence-value" as entropy. On the heels of this definition, that same year, the most infamous version of the second law was read in a presentation at the Philosophical Society of Zurich on April 24th, in which, in the end of his presentation, Clausius concludes:

"The entropy of the universe tends to a maximum."

This statement is the best-known phrasing of the second law. Moreover, owing to the general broadness of the terminology used here, e.g. universe, as well as lack of specific conditions, e.g. open, closed, or isolated, to which this statement applies, many people take this simple statement to mean that the second law of thermodynamics applies virtually to every subject imaginable. This, of course, is not true; this statement is only a simplified version of a more complex description.

In terms of time variation, the mathematical statement of the second law for a closed system undergoing an adiabatic transformation is:

$$\frac{dS}{dt} \geq 0$$

<div align="right">(2.6)</div>

where

S is the <u>entropy</u> and

t is <u>time</u>.

It should be noted that statistical mechanics gives an explanation for the second law by postulating that a material is composed of atoms and molecules which are in constant motion. A particular set of positions and velocities for each particle in the system is called a microstate of the system and because of the constant motion; the system is constantly changing its microstate. Statistical mechanics postulates that, in equilibrium, each microstate that the system might be in is equally likely to occur, and when this assumption is made, it leads directly to the conclusion that the second law must hold in a statistical sense. That is, the second law will hold on average, with a statistical variation on the order of $1/\sqrt{N}$ where N is the number of particles in the system. However, for systems with a small number of particles, thermodynamic parameters, including the entropy, may show significant statistical deviations from that predicted by the second law. Classical thermodynamic theory does not deal with these statistical variations. (Kreith, 2000)

2.2.1 ENTROPY AND ENTROPY GENERATION

Entropy

Consider two cycles executed by a closed system. One cycle consists of an internally reversible process A from state 1 to state 2, followed by an internally reversible process C from state 2 to state 1. The other cycle

consists of an internally reversible process B from state 1 to state 2, followed by the same process C from state 2 to state 1 as in the first cycle.

For these cycles,

$$\left(\int_1^2 \frac{\delta Q}{T} \right)_A + \left(\int_1^2 \frac{\delta Q}{T} \right)_C = -S_{gen} = 0 \tag{2.7}$$

$$\left(\int_1^2 \frac{\delta Q}{T} \right)_B + \left(\int_1^2 \frac{\delta Q}{T} \right)_C = -S_{gen} = 0 \tag{2.8}$$

where S_{gen} has been set to zero since the cycles are composed of internally reversible processes.

Subtracting these equations leaves

$$\left(\int_1^2 \frac{\delta Q}{T} \right)_A = \left(\int_1^2 \frac{\delta Q}{T} \right)_B \tag{2.9}$$

Since A and B are arbitrary, it follows that the integral of Q/T has the same value for any internally reversible process between the two states: the value of the integral depends on the end states only. It can be concluded, therefore, that the integral defines the change in some property of the system. Selecting the symbol S to denote this property, its change is given by

$$S_2 - S_1 = \left(\int_1^2 \frac{\delta Q}{T} \right)_{\substack{int \\ rev}} \tag{2.10}$$

where the subscript *int rev* indicates that the integration is carried out for any internally reversible process linking the two states. This extensive property is called entropy.

Since entropy is a property, the change in entropy of a system in going from one state to another is the same for all processes, both internally reversible and irreversible, between these two states. In other words, once the change in entropy between two states has been evaluated, this is the magnitude of the entropy change for any process of the system between these end states.

The definition of entropy change expressed on a differential basis is

$$dS = \left(\frac{\delta Q}{T} \right)_{\substack{\text{int} \\ rev}} \qquad (2.11)$$

Equation 2.11 indicates that when a closed system undergoing an internally reversible process receives energy by heat transfer, the system experiences an increase in entropy. Conversely, when energy is removed from the system by heat transfer, the entropy of the system decreases. This can be interpreted to mean that an entropy transfer is associated with (or accompanies) heat transfer. The direction of the entropy transfer is the same as that of the heat transfer. In an adiabatic internally reversible process of a closed system the entropy would remain constant. A constant entropy process is called an isentropic process.

On rearrangement, Equation 2.11 becomes

$$\left(\delta Q\right)_{\substack{\text{int} \\ rev}} = TdS \tag{2.12}$$

Then, for an internally reversible process of a closed system between state 1 and state 2,

$$Q_{\substack{\text{int} \\ rev}} = m\int_{1}^{2} TdS \tag{2.13}$$

When such a process is represented by a continuous curve on a plot of temperature vs. specific entropy, the area under the curve is the magnitude of the heat transfer per unit of system mass.

Entropy Balance

For a cycle consisting of an actual process from state 1 to state 2, during which internal irreversibilities are present, followed by an internally reversible process from state 2 to state 1,

$$\int_{1}^{2}\left(\frac{\delta Q}{T}\right)_{b} + \int_{1}^{2}\left(\frac{\delta Q}{T}\right)_{\substack{\text{int} \\ rev}} = -S_{gen} \tag{2.14}$$

where the first integral is for the actual process and the second integral is for the internally reversible process. Since no irreversibilities are associated with the internally reversible process, the term S_{gen} accounting for the effect

of irreversibilities during the cycle can be identified with the actual process only.

Applying the definition of entropy change, the second integral of the foregoing equation can be expressed as

$$S_1 - S_2 = \int_2^1 \left(\frac{\delta Q}{T} \right)_{\substack{int \\ rev}} \tag{2.15}$$

Introducing this and rearranging the equation, the closed system entropy balance results:

$$\underbrace{S_2 - S_1}_{\substack{entropy \\ change}} = \underbrace{\int_1^2 \left(\frac{\delta Q}{T} \right)_b}_{\substack{entropy \\ transfer}} + \underbrace{S_{gen}}_{\substack{entropy \\ generation}} \tag{2.16}$$

When the end states are fixed, the entropy change on the left side of Equation 2.16 can be evaluated independently of the details of the process from state 1 to state 2. However, the two terms on the right side depend explicitly on the nature of the process and cannot be determined solely from knowledge of the end states. The first term on the right side is associated with heat transfer to or from the system during the process. This term can be interpreted as the entropy transfer associated with (or accompanying) heat transfer. The direction of entropy transfer is the same as the direction of the heat transfer, and the same sign convention applies as for heat transfer: a

positive value means that entropy is transferred into the system, and a negative value means that entropy is transferred out (Kreith, 2000).

The entropy change of a system is not accounted for solely by entropy transfer, but is also due to the second term on the right side of Equation 2.16 denoted by S_{gen}. The term S_{gen} is positive when internal irreversibilities are present during the process and vanishes when internal irreversibilities are absent.

This can be described by saying that entropy is generated (or produced) within the system by the action of irreversibilities. The second law of thermodynamics can be interpreted as specifying that entropy is generated by irreversibilities and conserved only in the limit as irreversibilities are reduced to zero. Since S_{gen} measures the effect of irreversibilities present within a system during a process, its value depends on the nature of the process and not solely on the end states. Entropy generation is not a property.

When applying the entropy balance, the objective is often to evaluate the entropy generation term. However, the value of the entropy generation for a given process of a system usually does not have much significance by itself. The significance is normally determined through comparison. For example, the entropy generation within a given component might be compared to the entropy generation values of the other components included in an overall system formed by these components. By comparing entropy generation values, the components where appreciable irreversibilities occur can be identified and rank ordered. This allows attention to be focused on

the components that contribute most heavily to inefficient operation of the overall system.

To evaluate the entropy transfer term of the entropy balance requires information regarding both the heat transfer and the temperature on the boundary where the heat transfer occurs. The entropy transfer term is not always subject to direct evaluation, however, because the required information is either unknown or undefined, such as when the system passes through states sufficiently far from equilibrium. In practical applications, it is often convenient, therefore, to enlarge the system to include enough of the immediate surroundings that the temperature on the boundary of the enlarged system corresponds to the ambient temperature, T_{amb}. The entropy transfer term is then simply Q/T_{amb}. However, as the irreversibilities present would not be just those for the system of interest but those for the enlarged system, the entropy generation term would account for the effects of internal irreversibilities within the system and external irreversibilities present within that portion of the surroundings included within the enlarged system (Kreith, 2000).

A form of the entropy balance convenient for particular analyses is the rate form:

$$\frac{dS}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{S}_{gen} \qquad (2.17)$$

where dS/dt is the time rate of change of entropy of the system. The term $\dot{Q}_j / T_j$ represents the time rate of entropy transfer through the portion of the

boundary whose instantaneous temperature is T_j. The term $\dot{S}_{gen}$ accounts for the time rate of entropy generation due to irreversibilities within the system. For a system isolated from its surroundings, the entropy balance is

$$(S_2 - S_1)_{isol} = S_{gen} \qquad\qquad (2.18)$$

where S_{gen} is the total amount of entropy generated within the isolated system. Since entropy is generated in all actual processes, the only processes of an isolated system that actually can occur are those for which the entropy of the isolated system increases. This is known as the increase of entropy (Kreith, 2000).

3.0 EXERGY

Exergy is a term used in connection with the Second Law of Thermodynamics to describe the irreversible losses that occur within any thermal or power cycle. Inevitably, when heat is transferred from a high temperature source to a lower one, some exergy is destroyed.

Exergy is defined as the availability to perform useful work from a given energy source, and can be expressed in mathematical terms as the product of energy (or enthalpy) of the primary energy source and the ideal thermal (or power) conversion process.

3.1 EXERGY ANALYSIS

Exergy is defined as the maximum work potential of a material or of a form of energy in relation to its environment.

Traditional first-law analysis, based upon unit-performance characteristics coupled with energy balances, invariably leads to a correct final answer. However, such an analysis cannot locate and quantify the losses that lead to the obtained result. This is because the first law embodies no distinction between work and heat, no provision for quantifying the quality of energy. These limitations are not a serious drawback when dealing with familiar systems. For these, one can develop an intuitive understanding of the different parametric influences on system performance and a second-law qualitative appreciation of "grade-of-heat" and effect of

pressure loss. However, when analyzing novel and complex thermal systems, such an understanding should be complemented by a more rigorous quantitative method. Second-law analysis, or exergy analysis, provides such a tool. Second-law analysis is no substitute for first-law analysis, rather a supplement. (Bejan, 1996)

Exergy balance, including the loss (Irreversibility)

$$\sum_{in} \dot{m}_j \, e_j + \sum_l \dot{Q}_l \left(1 - \frac{T_0}{T_l}\right) = \sum_{out} \dot{m}_k \, e_k + \dot{W} + \dot{I} \qquad (3.1)$$

3.2 EXERGY COMPONENTS

The total exergy of a system E can be divided into four components: physical exergy E^{PH}, kinetic exergy E^{KN}, potential exergy E^{PT}, and chemical exergy E^{CH}.

$$E = E^{PH} + E^{KN} + E^{PT} + E^{CH} \qquad (3.2)$$

The sum of kinetic, potential and physical exergies is also referred to in the literature as the thermomechanical exergy.

Although exergy is an extensive property, it is often convenient to work with it on a unit-of-mass or molar basis. The total specific exergy on a mass basis e is given by

$$e = e^{PH} + e^{KN} + e^{PT} + e^{CH} \qquad (3.3)$$

when evaluated relative to the environment, the kinetic and potential energies of a system are in principle fully convertible to work as the system is brought to rest relative to the environment, and also they correspond to the kinetic and potential exergies, respectively. Accordingly,

$$e^{KN} = \frac{1}{2}V^2 \qquad\qquad (3.4)$$

$$e^{PT} = gz \qquad\qquad (3.5)$$

where V and z denote velocity and elevation relative to coordinates in the environment, respectively.

Considering a system at rest relative to the environment ($e^{KN} = e^{PT} = 0$), the physical exergy is the maximum theoretical useful work obtainable as the system passes from its initial state where the temperature is T and the pressure is p to the restricted dead state where the temperature is T_0 and the pressure is p_0. The chemical exergy is the maximum theoretical useful work obtainable as the system passes from the restricted dead state to the dead state where it is complete equilibrium with environment. In each instance heat transfer takes place with the environment only. (Bejan, 1996)

3.2.1 PHYSICAL EXERGY

The physical exergy of a closed system at a specified state is given by the expression

$$E^{PH} = (U - U_0) + p_0 (V - V_0) - T_0 (S - S_0) \qquad\qquad (3.6)$$

where U, V, and S denote, respectively, the internal energy, volume, and entropy of the system at the specified state, and U_0, V_0, and S_0 are the values of the same properties when the system is at the restricted dead state.
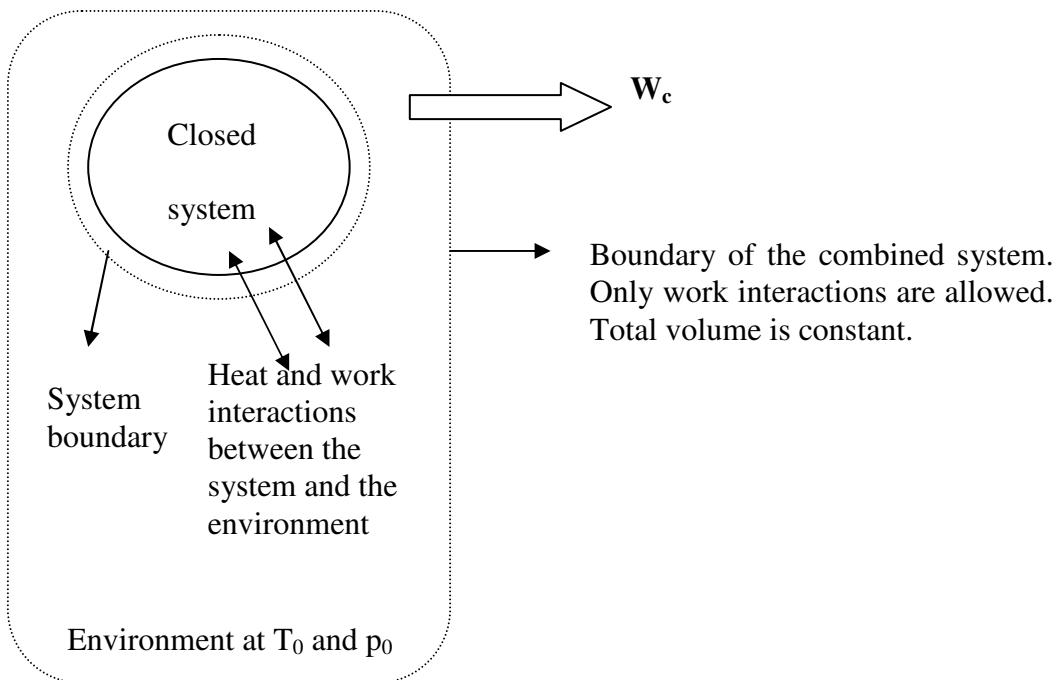
Figure 3.1. Combined System of Closed System and Environment

The system is at rest relative to the environment. As the objective is to evaluate the maximum work that could be developed by the combined system, the boundary of the combined system allows only energy transfers by work across it, ensuring that the work developed is not affected by heat transfers to or from the combined system. And although the volumes of the system and environment may vary, the boundary of the combined system is located so that the total volume remains constant. This ensures that the work

developed is useful: fully available for lifting a mass, say, and not expended in merely displacing the surroundings of the combined system.

An energy balance for the combined system reduces to

$$\Delta U_c = Q_c - W_c \qquad (3.7)$$

$$\text{or} \quad W_c = -\Delta U_c \qquad (3.8)$$

where W_c is the work developed by the combined system, and ΔU_c is the internal energy change of the combined system: the sum of the internal energy changes of the closed system and the environment. The internal energy of the system closed system initially is denoted by U_0. Accordingly, ΔU_c can be expressed as

$$\Delta U_c = (U_0 - U) + \Delta U^e \qquad (3.9)$$

where ΔU^e denotes the internal energy change of the environment. Since T_0, p_0, and the composition of the environment remain fixed, ΔU^e is related to changes in the entropy S^e and volume V^e of the environment through equation;

$$\Delta U^e = T_0 \Delta S^e - p_0 \Delta V^e \qquad (3.10)$$

Collecting the last three equations,

$$W_c = (U - U_0) - (T_0 \Delta S^e - p_0 \Delta V^e) \qquad (3.11)$$

As the total volume of combined system is constant, the change in volume of environment is equal in magnitude but opposite in sign to the volume change of the closed system:

$$\Delta V^e = - (V_0 - V).$$ (3.12)

The expression for work then becomes

$$W_c = (U - U_0) + p_0 (V-V_0) - T_0 \Delta S^e$$ (3.13)

This equation gives the work developed by the combined system as the closed system passes to the restricted dead state while interacting only with the environment. The maximum theoretical value for the work is determined using the entropy balance as follows: since no heat transfer occurs across its boundary, the entropy balance for the combined system reduces to give

$$\Delta S_c = S_{gen}$$ (3.14)

where S_{gen} accounts for entropy generation within the combined system as the closed system comes into equilibrium with the environment. The entropy change of the combined system, $\Delta S_{c,}$ is the sum of the entropy changes for the closed and environment, respectively,

$$\Delta S_c = (S_0 - S) + \Delta S^e$$ (3.15)

where S and S_0 denote the entropy of the closed system at the given state and restricted dead state, respectively. Combining the last two equations, solving for ΔS^e, and inserting the result into the expression for W_c gives

$$W_c = (U - U_0) + p_0 (V - V_0) - T_0 (S - S_0) - T_0 S_{gen} \qquad (3.16)$$

However, the values of S_{gen} depend on the nature of the process as the closed system passes to the restricted dead state. In accordance with the second law, this term is positive when irreversibilities are present and vanishes in the limiting case where there are no irreversibilities; it cannot be negative. Hence, the maximum theoretical value for the work of the combined system is obtained by setting S_{gen} to zero, leaving

$$W_{c,max} = (U - U_0) + p_0 (V - V_0) - T_0 (S - S_0) \qquad (3.17)$$

By definition, the physical exergy, E^{PH}, is the maximum value; and Equation 3.6 is obtained as the appropriate expression for calculating the physical exergy of a system. Various idealized devices can be invoked to visualize the development of work as a system passes from a specified state to the restricted dead state. (Bejan, 1996)

3.3 EXERGY BALANCE

As for the extensive properties mass, energy, and entropy, exergy balances can be written in alternative forms suitable for particular applications of practical interest. The objective of this section is to present such forms, beginning with the closed system case. The closed system

exergy balance is then used as a basis for extending the exergy balance concept to control volumes, which is the case of greater practical utility.

3.3.1 CLOSED SYSTEM EXERGY BALANCE

The exergy balance for a closed system shown in Figure 3.1 is developed by combining the energy and entropy balances:

$$(U_2 - U_1) + (KE_2 - KE_1) + (PE_2 - PE_1) = \int_1^2 \delta Q - W \qquad (3.18)$$

$$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + S_{gen} \qquad (3.19)$$

where W and Q represent, respectively, transfers of energy by work and heat between the system under study and its surroundings, T_b denotes the temperature on the boundary where energy transfer by heat occurs, and the term S_{gen} accounts for entropy generation owing to internal irreversibilities. Multiplying the entropy balance by the temperature T_0 and subtracting the resulting expression from the energy balance gives

$$(U_2 - U_1) + (KE_2 - KE_1) + (PE_2 - PE_1) - T_0(S_2 - S_1) = \int_1^2 \delta Q - T_0 \int_1^2 \left(\frac{\delta Q}{T}\right)_b - W - T_0 S_{gen}$$

$$(3.20)$$

Collecting the terms involving δQ and introducing Equation 3.17 on the left side, this expression can be rewritten as

$$\left(E_2 - E_1\right) - p_0\left(V_2 - V_1\right) = \int_1^2\left(1 - \frac{T_0}{T_b}\right)\delta Q - W - T_0 S_{gen} \qquad (3.21)$$

Rearranging, the closed system exergy balance results:

$$\left(E_2 - E_1\right) = \int_1^2\left(1 - \frac{T_0}{T_b}\right)\delta Q - \left[W - p_0\left(V_2 - V_1\right)\right] - T_0 S_{gen} \qquad (3.22)$$

For specified end states, the exergy change on the left side of Eq.3.22 can be evaluated from Eq.3.23 without regard for nature of the process. The terms on the right side of Eq.3.22 depend explicitly on the process, however. The first term on the right side is associated with heat transfer to or from the system during the process and can be interpreted as the exergy transfer associated with (or accompanying) the transfer of energy by heat:

$$E_q = \int_1^2\left(1 - \frac{T_0}{T_b}\right)\delta Q \qquad (3.23)$$

The second term on the right side is associated with the net useful work and can be interpreted as the exergy transfer associated with (or accompanying) the transfer of energy by work:

$$E_w = W - p_0(V_2 - V_1) \qquad (3.24)$$

The third term on the right side accounts for the destruction of exergy due to irreversibilities within the system. The exergy destruction E_D is related to the entropy generation by

$$E_D = T_0 S_{gen} \tag{3.25}$$

In the literature, the exergy destruction is also commonly referred to as the availability destruction, the irreversibility, and the lost work. Equation 3.25 is known also as the Gouy-Stodola theorem.

As for the exergy values at the states visited by the system, exergy transfers associated with heat and work are evaluated relative to the environment used to define exergy. Thus, on recognizing the term *$(1-T_0/T_b)$* as the Carnot efficiency, the quantity *$(1-T_0/T_b)$ δQ* appearing in Equation 3.22 can be interpreted as the work that could be generated by a reversible power cycle receiving energy by heat transfer δQ at temperature T_b and discharging energy by heat transfer to the environment at temperature *$T_0(<T_b)$*. It may also be noted that when T_b is less than T_0, the sign of the exergy transfer would be opposite to the sign of the heat transfer, so the heat transfer and the associated exergy transfer would be oppositely directed. Exergy transfers associated with work are also evaluated relative to the environment: The exergy transfer is the work of the system W less the work that would be required to displace the environment whose pressure is uniform at p_0, namely $p_0 \Delta V$, leaving *$W-p_0 \Delta V$* as shown in 3.24.

The exergy balance can be expressed in various forms that may be more appropriate for particular applications. A convenient form of the exergy balance for closed systems is the rate equation.

$$\frac{dE}{dt} = \sum_j (1 - \frac{T_0}{T_j})\, \dot{Q}_j - (\dot{W} - p_0\, \frac{dV}{dt}) - \dot{E}_D \qquad (3.26)$$

Where dE/dt is the time rate of change of exergy. The term $(1\text{-}T_0/T_j)$ $\dot{Q}_J$ represents the time rate of exergy transfer associated with heat transfer at the rate $\dot{Q}_J$ occurring at the location on the boundary where the instantaneous temperature is T_j. The term W represents the time rate of energy transfer by work, and the associated exergy transfer is given by $W\text{-}p_0$ dV/dt, where dV/dt is the time rate of change of system volume. Here, E_D accounts for the time rate of exergy destruction due to irreversibilities within the system and is related to the rate of entropy generation within the system by $\dot{E}_D = T_0\, \dot{S}_{gen}$.

3.3.2 CONTROL VOLUME EXERGY BALANCE

Building on the foregoing, we now introduce forms of the exergy balance applicable to control volumes. To allow for the chemical exergy to play a role, if necessary, when these balances are applied, we must be

prepared to specify the chemical make up of the environment in addition to the temperature T_0 and pressure p_0.
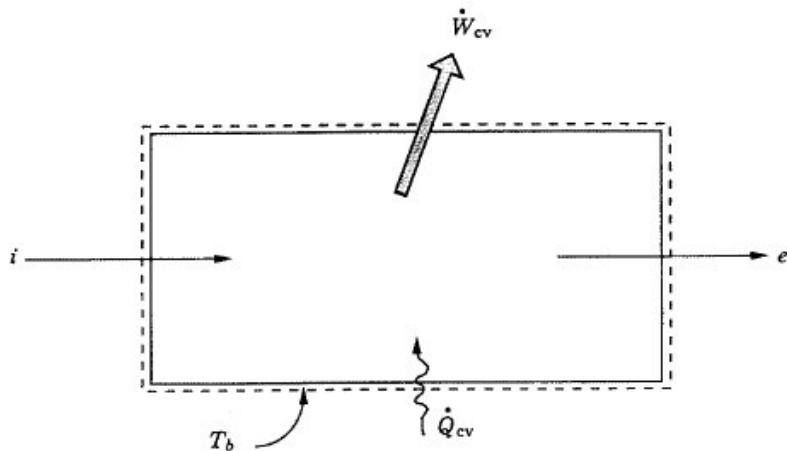

Figure 3.2. One Inlet, One Outlet Control Volume at Steady State

General Form: Like mass, energy, and entropy, exergy is an extensive property, so it can be transferred into or out of a control volume where streams of matter enter and exit. Accordingly, the counterpart of Eq. 3.26 applicable to control volumes requires the addition for such exergy transfers:

$$\frac{dE_{cv}}{dt} = \sum_j (1 - \frac{T_0}{T_j})\dot{Q}_j - (W_{cv} - p_0 \frac{dV_{cv}}{dt}) + \sum_i \dot{m}_i e_i - \sum_e \dot{m}_e e_e - \dot{E}_D \quad (3.27)$$

As for control volume rate balances, the subscripts i and e denote inlets and outlets, respectively.

In Equation 3.27, the term dE_{cv}/dt represents the time rate of change in the exergy in the control volume. The term $\dot{Q}_J$ represents the time rate of heat transfer at the location on the boundary of the control volume where the instantaneous temperature is T_j, and the associated exergy transfer is given by

$$\dot{E}_{q,j} = (1 - \frac{T_0}{T_j})\dot{Q}_j \qquad (3.28)$$

As in the control volume energy rate balance, W_{cv} represents the time rate of energy transfer by work other than flow work. The associated exergy transfer is given by

$$\dot{E}_w = \dot{W}_{cv} - p_0 \frac{dV_{cv}}{dt} \qquad (3.29)$$

Where dV_{cv}/dt is the time rate of change of volume of the control volume itself. The term $\dot{m}_i e_i$ accounts for the time rate of exergy transfer at the inlet i. Similarly, $\dot{m}_e e_e$ accounts for the time rate of exergy transfer at the outlet e. In subsequent discussions, the exergy transfer rates at control volume inlets and outlets are denoted, respectively, as $\dot{E}_i = m_i e_i$ and $\dot{E}_e = m_e e_e$. Finally, $\dot{E}_D$ accounts for the time rate of exergy destruction due to irreversibilities within the control volume, $\dot{E}_D = T_0 \dot{S}_{gen}$.

Steady-State Form: It is important to identify the steady-state form of the exergy rate balance. At steady state, $dE_{cv}/dt = 0$ and

$dV_{cv/dt} = 0$, so Equation 3.27 reduces to

$$0 = \sum_j (1 - \frac{T_0}{T_j})\dot{Q}_j - \dot{W}_{cv} + \sum_i \dot{m}_i \, e_i - \sum_e \dot{m}_e \, e_e - \dot{E}_D \qquad (3.30)$$

This equation states that the rate at which exergy is transferred into the control volume must exceed the rate at which exergy is transferred out; the difference is the rate at which exergy is destroyedwithin the control volume due to irreversibilities. Expressed in terms of the time rates of exergy transfer and destruction, Eq. 3.30 takes the form

$$0 = \sum_j \dot{E}_{q,j} - \dot{W}_{cv} + \sum_i \dot{m}_i \, e_i - \sum_e \dot{m}_e \, e_e - \dot{E}_D \qquad (3.31)$$

Where E_i and E_e are exergy transfer rates at inlets and outlets, respectively, and $E_{q,j}$ is given by Equation 3.28. For the thermodynamic analysis of control volumes at steady state, Equations 3.30 and 3.31 for exergy may be added to the steady-state forms of the mass, energy, and entropy balances. (Bejan, 1996)

Exergy Transfer at Inlets and Outlets: To complete the introduction of the control volume exergy balance, means are required to evaluate the exergy transfers at inlets and outlets represented by the terms e_i and e_e appearing in Equations 3.27 and 3.28. As for other exergy transfers, these terms must be evaluated relative to the environment used to define exergy.

Accordingly, the exergy associated with a stream of matter entering (or exiting) a control volume is the maximum theoretical work that could be obtained were the stream brought to the dead state, heat transfer occurring with the environment only. This work can be evaluated in two steps as follows:

In the first step the stream is brought to the restricted dead state, and in the second step from the restricted dead state to the dead state. The contribution of the second step to the work developed is evidently the chemical exergy e^{CH}. In the current application, however, the properties at the inlet of the device are those of the stream under consideration: h, s, V, and z, while at the outlet the corresponding properties are h_0, s_0, $V_0=0$, $Z_0=0$, where h_0 and s_0 denote, respectively, the specific enthalpy and specific entropy at the restricted dead state. Moreover, as heat transfer occurs with the environment only, the temperature T_b at which heat transfer occurs corresponds to T_0. Thus, the work developed in the first step, per unit of mass flowing, is $(h-h_0) - T_0(s-s_0) + \frac{1}{2}V^2 + gz$. In summary, for the two steps together we have on a unit-of-mass basis the following expression for the total exergy transfer associated with a stream of matter:

$$e = (h - h_0) - T_0(s - s_0) + \frac{1}{2}V^2 + gz + e^{CH} \qquad (3.32)$$

The underlined term in Equation 3.26 is conventionally identified as the physical component, e^{PH}, of the exergy transfer associated with a stream of matter:

$$e^{PH} = (h-h_0) - T_0(s - s_0) \qquad (3.33)$$

The physical exergy is associated with the temperature and pressure of a stream of matter.

For the special case of an ideal gas with constant specific heat ratio k, Eq. 3.33 can be expressed as

$$\frac{e^{PH}}{c_p T_0} = \left[\frac{T}{T_0} - 1 - \ln \frac{T}{T_0} \right] + \ln \left(\frac{p}{p_0} \right)^{(k-1)/k} \tag{3.34}$$

Finally, note that the two terms on the right hand side of Eq. 3.34 depend only on temperature and presssure, respectively. These terms may be referred to, respectively, as the thermal and mechanical components of the exergy associated with the ideal gas stream, each in dimensionless form. In general, however, the physical exergy cannot be represented in terms of these two components.

The character of the exergy transfer term given by Equation 3.32 can be seen from another perspective by expressing it alternatively as

$$e = e + (pv - p_0 v) \tag{3.35}$$

where e is the total specific exergy of the matter entering or exiting a control volume. The term pv is the specific flow work at the inlet (or outlet), and *(pv - p_0v)* accounts for the exergy transfer associated with the flow work. The exergy transfer term e is thus simply the sum of two contributions: the

exergy of the flowing matter and the exergy transfer associated with flow work. Although the first term of Equation 3.35, e, is never negative, the second term is negative when $p<p_0$, giving rise to negative values for e at certain states. (Bejan, 1996)

4.0 THERMAL STORAGE SYSTEM

Thermal energy storage (TES) is considered one of most important advanced energy technologies and recently, increasing attention has been paid to the utilization of this essential technique for thermal applications ranging from heating to cooling, particularly in buildings. Economies in the design and operation of energy conversion systems often result if some provision is made for the storage of thermal energy. Such provisions must be included in an energy conversion system when the supply of and demand for thermal energy do not coincide in time. The surveys conducted by several researchers have revealed that there is a wide range of practical opportunities for employing TES systems in industrial applications. In this regard, TES systems have a considerable high potential for more effective use of thermal energy equipment and for facilitating large-scale energy substitutions from the point of the economic perspective (Rosen, 1999)

In general, a coordinated set of actions has to be taken in several sectors of the energy system for the maximum potential benefits of storage to be realized. TES appears to be the only solution to correcting the mismatch between the supply and demand of energy. It can contribute significantly to meet society's needs for more efficient, environmentally benign energy use. TES is a key component of any successful thermal system in buildings and a good TES should allow minimum thermal energy losses, leading to energy savings, while permitting the highest possible extraction efficiency of the stored thermal energy. There are mainly three types of TES systems, i.e. sensible (e.g. water and rock), latent (e.g. water/ice and salt hydrates) and thermochemical (e.g. inorganic substances).

The selection of TES is mainly dependent on the storage period required, i.e. diurnal or seasonal, economic viability, operating conditions, etc. In practice, many research and development activities related to energy have been concentrated on the efficient energy use and energy savings, leading to energy-conservation. In this regard, TES appears to be one of the most attractive thermal applications and exergy as the best tool in analyzing their performances. In sensible TES system, energy is stored by changing the temperature of the storage medium (e.g. water, oil, bricks, sand, soil or rock beds) and a TES system consists of a storage medium, a container and input/output devices. The amount of energy stored by a TES device is proportional to the difference between the storage input and output temperatures, the mass of the storage medium, and the medium's heat capacity. Each medium has its own advantages and disadvantages. TES systems are available for both short and long-term storage (e.g. rock and earth beds, and water tanks for short time (daily) and rock and earth beds, large water or oil tanks, solar ponds and aquifers for long-term storage (monthly or annual). TES systems combined with solar heating, hot water and cooling applications in buildings have attracted interest in recent years (Dinçer, 1997).

4.1 CRUCIAL ASPECTS OF TES SYSTEMS

Dinçer (2002) investigated that there are numerous criteria to evaluate TES systems and applications such as technical, sizing, economic, energy saving and environmental. Each of these items should be considered carefully for a successful TES implementation.

4.1.1 Technical criteria for TES

Independent technical criteria for storage systems are difficult to establish since they usually are specific case and are closely related to and generally affected by the economics of the resultant systems. Nevertheless, certain technical criteria are desirable, although appropriate trade-offs must be made with such other criteria as;

- ❀ storage capacity,
- ❀ lifetime,
- ❀ size,
- ❀ cost,
- ❀ efficiency,
- ❀ safety,
- ❀ installation
- ❀ environmental standards.

Before proceeding with a project, a TES designer possess or obtain technical information on TES such as the types of storage available, the amount of storage required, the effect of storage on system performance, reliability and cost, and the storage systems or designs available. TES is difficult to employ at sites that have severe space restrictions. Also, TES tanks often have significant first capital costs. Financial analysis for TES-based projects can be complex, although most consulting energy engineers are now capable of performing financial calculations and evaluating TES benefits. (Dinçer, 2002)

4.1.2 Sizing criteria for TES

A need exists for improved TES-sizing techniques as analyses of projects reveal both undersized and oversized systems. Under-sizing can result in poor levels of indoor comfort, while over-sizing results not only in higher than necessary initial costs but also in the potential wasting of electricity if more energy is stored than is required. Another requirement for successful TES that affects sizing is proper installation and control. Using state-of-the-art equipment, properly designed and controlled storage systems often do not use more energy than conventional heating and cooling equipment. Performance data describing the use of TES for heating and cooling by shifting peak loads to off-peak-periods are limited, although the potential for such technologies is substantial. The initial costs of such systems can be lower than those for other systems. To yield the benefits, new construction techniques are required together with the use of more sophisticated thermal-design calculations that are as yet, unfamiliar to many builders and designers.

4.1.3 Economics of TES Systems

The economic justification for TES systems assumes that the annual income needed to cover capital and operating costs should be less than that required for primary generating equipment supplying the same service loads and periods. TES is mainly installed to lower initial costs and operating costs. Lower initial costs are usually obtained when the load to meet is of short-duration and there is a long time gap before the load returns. Secondary capital costs may also be lower for TES. For example, the

electrical service entrance size can sometimes be reduced because energy demand is lower. To complete the economic analysis, the initial costs must be determined. Equipment costs should be obtained from each of the manufacturers under the consideration and an estimate of installation cost should be made. The cost savings along with the net capital costs should be analyzed using the life cycle cost method or other suitable method to determine which system is best for the project.

4.1.4 Energy Savings by TES Systems

TES is an important element of energy saving programs in industry, in commercial buildings and in solar energy utilization. For many years TES systems have been investigated, which show that although many technically and economically successful TES systems have been in operation, no broadly valid basis for comparing the achieved performance of one storage with that of another operating under different conditions has found general acceptance. The development of such a basis for comparison has been received increasing attention, especially using exergy analysis technique, which is identified as one of the most powerful ways in evaluating the thermal performance of TES systems is based primarily on the second law of thermodynamics, as compared to energy analysis which is based on the first law and takes into account the quality of the energy transferred. Energy savings from TES's can be achieved in several ways;

❀ The consumption of purchased energy can be reduced by storing waste or surplus thermal energy available at certain times for use at other times. For example, solar energy can be stored during the day for heating at night.

❀ The demand of purchased electrical energy can be reduced by storing electrically produced thermal energy during off-peak-periods to meet the thermal loads that occur during high-demand periods. For example, an electric chiller can be used to charge a chilled water TES at night for reducing the electrical demand peaks usually experienced during the day.

❀ The purchase of additional equipment for heating, cooling or air-conditioning applications can be deferred and the equipment sizing in new facilities can be reduced. The equipment can be operated when thermal loads are low to charge TES systems. Energy can be withdrawn from storage to help meet the maximum thermal loads that exceed equipment capacity.

4.1.5 Environmental Impacts of TES Systems

TES systems can contribute significantly to meeting society's needs for more efficient, environmentally benign energy use in building heating and cooling, space power and utility applications. Suggested that in light of growing environmental concerns TES technology could play a vital role in the reduction of environmental pollution. The utilization of TES systems simply reduces the energy consumption and hence results in two most significant benefits:

(i)   conservation of fossil fuels

(ii)   reductions in CO_2, SO_2, NO_x and Chlorofluorocarbon (CFC) emissions.

4.1.6 General Evaluation of TES Systems

Dinçer (2002) also investigates that; since there are many factors that influence the selection, implementation, and operation of a TES system, it is necessary that comprehensive feasibility study should be developed. This study should take into consideration all variables which impact evaluation of the true cost benefits of a candidate TES implementation. However, sometimes it is practically impossible to conduct all evaluation tests. In practice, concerned people prefer a checklist on how to evaluate a TES system. In such cases, at least the following significant issues should be clarified and addressed before its implementation:

1. Management objectives (short- or long-term).
2. Environmental impact analysis.
3. Energy saving targets.
4. Economical aims.
5. Financial parameters of the project.
6. Available utility incentives.
7. New or existing TES system (of course, existing plant would reduce its implementation cost).
8. Net heating or cooling storage capacity (especially for peak-day requirements).
9. Utility rate schedules and associated energy charges.

10. Full or partial TES system.

11. TES system options best suited for the specific application.

12. Anticipated operating strategies for each of the TES options.

13. Space availability (e.g. tank).

14. Type of the TES (short- or long-term).

15. Type of the TES system (open or closed).

The implementation logistics for a TES being under consideration are as follows:

❀ New or existing TES system (if existing, the modifications to be required; if new, the system capacity).

❀ TES system location.

❀ Structural and environmental impact.

❀ Heat exchanger requirements.

❀ Piping arrangement.

❀ Automatic control requirements.

❀ Safety arrangement.

❀ New electrical service requirements.

For an existing facility, the TES feasibility study should also address the following checklist (for a facility under design, the requirements are generally the same, except that the database must first be developed, due to the nonexistence of the actual operating data).

Database:

❀ The utility's maximum incentive.

❀ The facility occupancy hours.

❀ The facility operating requirements.

❀ Existing physical constraints.

❀ The facility peak-day load or monthly average requirements.

❀ Historic energy consumption rates.

Analysis:

❀ The types of TES system.

❀ General implementation logistics for the TES system.

❀ The plant's yearly energy consumption with or without a TES system.

❀ The TES size utilization factor.

❀ The projected operating cost reduction for the TES system.

5. MODELLING OF THERMAL ENERGY STORAGE

5.1 TRANSIENT OPERATION OF STORAGE ELEMENTS

Consider the operation of thermal energy storage system shown schematically in Figure 5.1. The system consists of a large liquid bath of mass M and specific heat C placed in an insulated vessel. Hot gas, entering the system through one port, is cooled while flowing through a heat exchanger immersed in the bath, and is eventually discharged into the atmosphere. Gradually, the bath temperature T as well as the gas outlet temperature T_{out} rises, approaching the hot gas inlet temperature T_∞.

The bath is filled with an incompressible liquid such as water or oil. The stream m˙ carries an ideal gas, for example, high temperature steam or air. The bath liquid is thermally well mixed so that at any given time its temperature is uniform T(t). It is assumed that initially the bath temperature equals to the environment temperature T_0 (Bejan, 1982).

m, c_p
T_∞
T_{out}

T, M

$T_{surr} = T_o$

Temperature

T_∞

$T_{out}(t)$

$T(t)$

T_o

dx

T_g

stream

$T_g + dT_g$

U P dx $(T_g - T)$

T(t)

Liquid Bath

Time, t
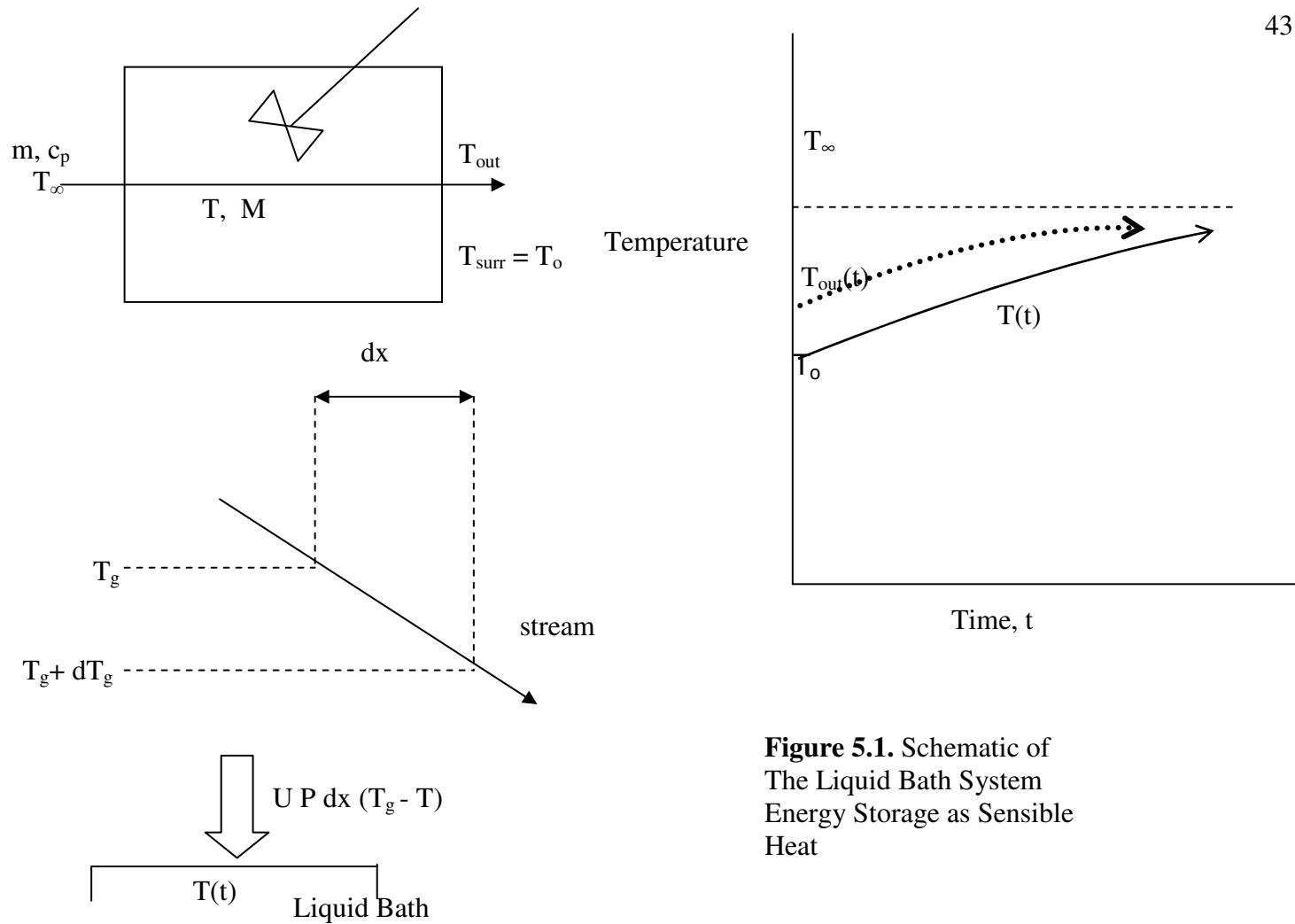
Figure 5.1. Schematic of
The Liquid Bath System
Energy Storage as Sensible
Heat

Bejan (1982) investigates the time dependence of the bath temperature T(t) and gas outlet temperature T_{out}(t) can be derived analytically using the drawing that appears in the left half of Figure 5.1. Locally, the gas liquid heat transfer is balanced by the enthalpy change in the gas stream,

In the gas stream

$$dq = UdA(T_g - T_{(t)}) = -\dot{m} C_p dT_g \tag{5.1}$$

$$dA = Pdx \tag{5.2}$$

$$-\frac{Up}{\dot{m}C_p} \int_0^L dx = \int_{T\infty}^{Tout} \frac{dT_g}{(T_g - T_{(t)})} \tag{5.3}$$

$$-\frac{UpL}{\dot{m}C_p} = \ln \frac{T_{out(t)} - T_{(t)}}{T_\infty - T_{(t)}} \tag{5.4}$$

$$\frac{UpL}{\dot{m}C_p} = N_{tu} \tag{5.5}$$

$$\frac{T_{out(t)} - T_{(t)}}{T_\infty - T_{(t)}} = e^{-Ntu} \tag{5.6}$$

Applying the 1st law of Thermodynamics to the Liquid Bath

Energy change rate of the tank for one second is $MC\dfrac{dT}{dt}$

Energy change of the gas is $\dot{m}C_p(T_\infty - T_{out})$

$$MC\frac{dT}{dt} = \dot{m} C_p(T_\infty - T_{out}) \tag{5.7}$$

From Eq. 5.6:

$$T_{out} = e^{-Ntu}(T_\infty - T_{(t)}) + T_{(t)} \tag{5.8}$$

Substituting this into Eq 5.8:

$$MC \frac{dT}{dt} = \dot{m} C_p (-y(T_{(t)} - T_\infty))$$ (5.9)

where $y = 1 - e^{-Ntu}$ (5.10)

$$\int_{T_o}^{T_{(t)}} \frac{dT}{-y(T_{(t)} - T_\infty)} = \int_0^t \frac{\dot{m} C_p}{MC} dt$$ (5.11)

$$\frac{1}{-y} \ln \frac{T_{(t)} - T_\infty}{T_o - T_\infty} = \frac{\dot{m} C_p}{MC} t$$ (5.12)

$$\ln \frac{T_{(t)} - T_\infty}{T_o - T_\infty} = -y\theta$$ (5.13)

$$\frac{T_{(t)} - T_\infty}{T_o - T_\infty} = e^{-y\theta}$$ (5.14)

$$\theta = \frac{\dot{m} C_p}{MC} t$$ (5.15)

As expected, both T(t) and T_{out} approach T_∞ asymptotically – the faster, the higher the Ntu. The ability to store thermal energy increases with increasing the charging time θ and the number of heat transfer units Ntu. It is shown later that there exists an optimum charging time for which the liquid bath of Figure 5.1 stores the most exergy per unit of exergy drawn from the high temperature gas source T_∞. (Bejan, 1982)

5.2 DESTRUCTION OF EXERGY DURING ENERGY STORAGE

As shown in Figure 5.2, the batch heating process is accompanied by two irreversibilities. First, the heat transfer between the hot stream and the cold bath always takes place across a finite ΔT. Second, the gas stream exhausted into the atmosphere is eventually cooled down to T_0 again by heat transfer across a finite ΔT. A third irreversibility source, neglected in the present analysis, is the frictional pressure drop on the gas side of the heat exchanger. The combined effect of these irreversibilities is a basic characteristic of all sensible heat storage systems, namely only a fraction of the exergy brought in by the hot stream is stored in the liquid bath. (Bejan, 1982)
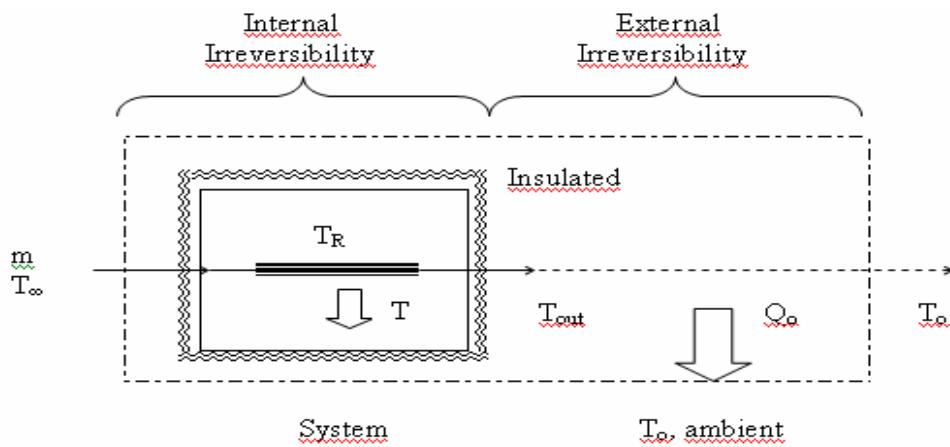

Figure 5.2. Sources of Thermodynamic Irreversibility in a Batch Heating Process

Multiplying Eq. 5.14 with minus sign

$$\frac{T_{(t)} - T_\infty}{T_\infty - T_o} = -e^{-y\theta} \qquad (5.16)$$

$$\frac{T_\infty - T_o}{T_\infty - T_o} + \frac{T_{(t)} - T_\infty}{T_\infty - T_o} = -e^{-y\theta} + 1 \qquad (5.17)$$

$$\frac{T_{(t)} - T_o}{T_\infty - T_o} = -e^{-y\theta} + 1 \qquad (5.18)$$

When we write the equation $T_{(t)}$ from Eq. 5.6:

$$T_{out(t)} - T_{(t)} = e^{-Ntu}(T_\infty - T_{(t)}) \qquad (5.19)$$

$$T_{(t)}(e^{-Ntu} - 1) = T_\infty - T_{out} \qquad (5.20)$$

$$T_{(t)} = \frac{T_\infty - T_{out}}{-y} \qquad (5.21)$$

Apply this result to Eq. 5.18

$$\frac{T_{out(t)} - T_o}{T_\infty - T_o} = 1 - ye^{-y\theta} \qquad (5.22)$$

The entropy generation rate in the system defined by the dashed boundary of Thermal Storage System:

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$$\dot{S}_{gen} = \Delta S_{hotgas} + \Delta S_{surr} + \Delta S_{bath} \tag{5.23}$$

The entropy change of ideal gas, ΔS_{hotgas}:

$$dS = \dot{m}\left[Cp\frac{dT}{T} - R\frac{dP}{P} \right] \tag{5.24}$$

at constant pressure

$$R\frac{dP}{P} = 0 \tag{5.25}$$

$$\therefore \int_{S_1}^{S_2} dS = \dot{m}\,Cp\int_{T_\infty}^{T_0} \frac{dT}{T} \tag{5.26}$$

$$S_2 - S_1 = \Delta S_{hotgas} = \dot{m}\,Cp\ln\frac{T_0}{T_\infty} \tag{5.27}$$

The entropy change of surrounding, ΔS_{surr}:

$$Q = \int_1^2 TdS \rightarrow \Delta S_{surr} = \frac{Q_o}{T_o} \tag{5.28}$$

$$Q_0 = \dot{m}\,Cp(T_{out} - T_o) \tag{5.29}$$

The entropy change of liquid bath, ΔS_{bath}:

$$dS = MC\frac{dT}{T} \rightarrow S = d(MC\ln T) \tag{5.30}$$

Dividing by dt we have,

$$\frac{dS}{dt} = \frac{d(MC\ln T)}{dt} = \Delta S_{bath} \tag{5.31}$$

So;

$$S_{gen} = \dot{m} Cp \ln \frac{T_o}{T} + \frac{Q_o}{T_o} + \frac{d}{dt}(MC \ln T) \qquad (5.32)$$

To calculate the entropy generated during the time interval 0-t; Eq. 5.32 is integrated additionally using Eq. 5.18:

$$\int_0^t \dot{S}_{gen}\, dt = t\,\dot{m}\,Cp\left(\ln \frac{T_o}{T_\infty} + \frac{(T_\infty - T_o)}{T_o} \right) + MC\left(\ln \frac{T_{(t)}}{T_o} - \frac{(T_{(t)} - T_o)}{T_o} \right)$$

$$(5.33)$$

From Eq. 5.18

$$T_{(t)} = (T_\infty - T_o)(1 - e^{-y\theta}) + T_o \qquad (5.34)$$

$$\frac{1}{MC}\int_0^t \dot{S}_{gen}\, dt = \theta\left(\ln \frac{T_o}{T_\infty} + \frac{(T_\infty - T_o)}{T_o} \right) + \ln\left[1 + \frac{(T_\infty - T_o)}{T_o}(1 - e^{-y\theta})\right] - \frac{(T_\infty - T_o)}{T_o}(1 - e^{-y\theta})$$

$$(5.35)$$

$$\text{Destroyed Exergy} = T_0 \int_0^t \dot{S}_{gen}\, dt \qquad (5.36)$$

Total Exergy, Ex: It is the exergy of gas flowing in the exchanger within t seconds. So; it is found by multiplying the specific exergy of the fluid by total mass.

$$Specific\, Exergy = (h - h_0) - T_0(s - s_0) \qquad (5.37)$$

$$Ex = \dot{m}tc_p \left(T_\infty - T_0 - T_0 \ln \frac{T_0}{T_\infty} \right) \tag{5.38}$$

Total exergy is the multiplication of specific exergy by mass so,

$$\text{Exergy-loss number} = N_s = \frac{Ex_{dest}}{Ex} \tag{5.39}$$

$$N_s = \frac{T_0 \int_0^t \dot{S}_{gen} dt}{Ex} \tag{5.40}$$

$$T_0 \int_0^t \dot{S}_{gen} dt = T_0 \dot{m}tc_p \left(\ln \frac{T_0}{T_\infty} + \frac{T_\infty - T_0}{T_0} \right) + MCT_0 \ln\left[1 + \frac{T_\infty - T_0}{T_0} \left(1 - e^{-y\theta}\right) \right]$$
$$- MCT_0 \left[\frac{T_\infty - T_0}{T_0} \left(1 - e^{-y\theta}\right) \right] \tag{5.41}$$

Eq. 5.38 can be written as,

$$Ex = T_0 \dot{m}tc_p \left(\ln \frac{T_0}{T_\infty} + \frac{T_\infty - T_0}{T_0} \right) \tag{5.42}$$

$$Ex = T_0 \dot{m}tc_p \left(\frac{T_\infty - T_0}{T_0} - \ln \frac{T_\infty}{T_0} \right) \tag{5.43}$$

$$Ex = T_0 \dot{m}tc_p \left(\frac{T_\infty - T_0}{T_0} - \ln\left(\frac{T_\infty - T_0 + T_0}{T_0} \right) \right) \tag{5.44}$$

Bejan (1982) defines dimensionless temperature as;

$$\tau = \frac{T_\infty - T_0}{T_0} \tag{5.45}$$

$$Ex = T_0 \dot{m} tc_p \left(\tau - \ln(1+\tau)\right) \tag{5.46}$$

$$N_s = \frac{T_0 \dot{m} tc_p \left(\ln \dfrac{T_0}{T_\infty} + \dfrac{T_\infty - T_0}{T_0}\right)}{T_0 \dot{m} tc_p \left(\ln \dfrac{T_0}{T_\infty} + \dfrac{T_\infty - T_0}{T_0}\right)} - \frac{MCT_0 \left\{\left[\tau\left(1 - e^{-y\theta}\right)\right] - \ln\left[1 + \tau\left(1 - e^{-y\theta}\right)\right]\right\}}{T_0 \dot{m} tc_p \left(\tau - \ln(1+\tau)\right)} \tag{5.47}$$

$$N_s = 1 - \frac{\left\{\left[\tau\left(1 - e^{-y\theta}\right)\right] - \ln\left[1 + \tau\left(1 - e^{-y\theta}\right)\right]\right\}}{\theta\left(\tau - \ln(1+\tau)\right)} \tag{5.48}$$

The optimum charging time can be calculated explicitly in the limit $\tau \to 0$ where the entropy generation number (Eq. 5.48) reduces to;

$$N_s = 1 - \frac{1}{\theta}(1 - e^{-y\theta})^2 \tag{5.49}$$

$$\theta_{opt} = \frac{1.256}{y} = \frac{1.256}{1 - e^{-Ntu}} \tag{5.50}$$

To find $N_{s,min}$;

$$N_{s,min} = 1 - \frac{1}{\dfrac{1.256}{1 - e^{-Ntu}}}(1 - e^{-y*\frac{1.256}{1 - e^{-Ntu}}})^2 \tag{5.51}$$

$$N_{s,\min} = 1 - \frac{1-e^{-Ntu}}{1.256}(1-e^{-(1-e^{-Ntu})*\frac{1.256}{1-e^{-Ntu}}})^2 \tag{5.52}$$

$$N_{s,\min} = 1 - \frac{1-e^{-Ntu}}{1.256}(1-e^{-(1-e^{-Ntu})*\frac{1.256}{1-e^{-Ntu}}})^2 \tag{5.53}$$

$$N_{s,\min} = 1 - \frac{1-e^{-Ntu}}{1.256}(1-e^{-1.256})^2 \tag{5.54}$$

$$N_{s,\min} = 1 - \frac{1-e^{-Ntu}}{1.256}(0.7152)^2 \tag{5.55}$$

$$N_{s,\min} = 1 - \frac{1-e^{-Ntu}}{1.256}(0.7152)^2 \tag{5.56}$$

$$N_{s,\min} = 1 - (0.407*(1-e^{-Ntu})) \tag{5.57}$$

$$N_{s,\min} = 1 - (0.407 - 0.407e^{-Ntu}) \tag{5.58}$$

$$N_{s,\min} = 1 - 0.407 + 0.407e^{-Ntu}) \tag{5.59}$$

$$N_{s,\min} = 0.593 + 0.407e^{-Ntu} \tag{5.60}$$

As shown in Figure 5.3, N_s depends on the charging time θ, the heat transfer area (y, Ntu), and the dimensionless temperature τ. It is clear that for any given τ and Ntu there exists an optimum time θ_{opt}. When the fraction of accumulated irreversibility N_s reaches its minimum. Away from this minimum, N_s approaches unity. In the $\theta \rightarrow 0$ limit, the entire energy content of the hot stream is destroyed by heat transfer to the liquid bath, which is initially at atmospheric temperature T_0. In the $\theta \rightarrow \infty$ limit, the irreversibility shifts outside the liquid bath. (Bejan, 1982)

$\tau \rightarrow 0$
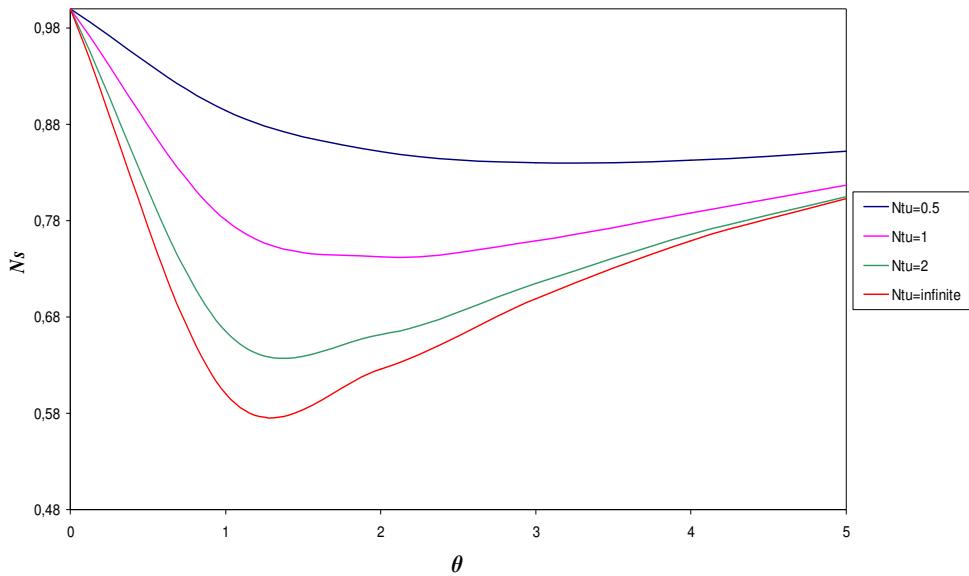
$\tau = 1$
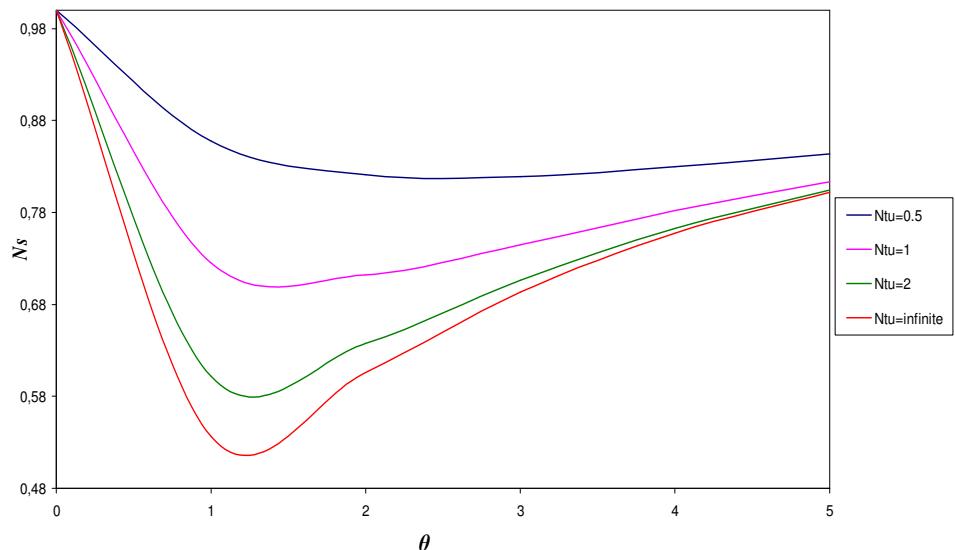
Figure 5.3. Fraction Exergy Destroyed During the Energy Storage Process

τ=2
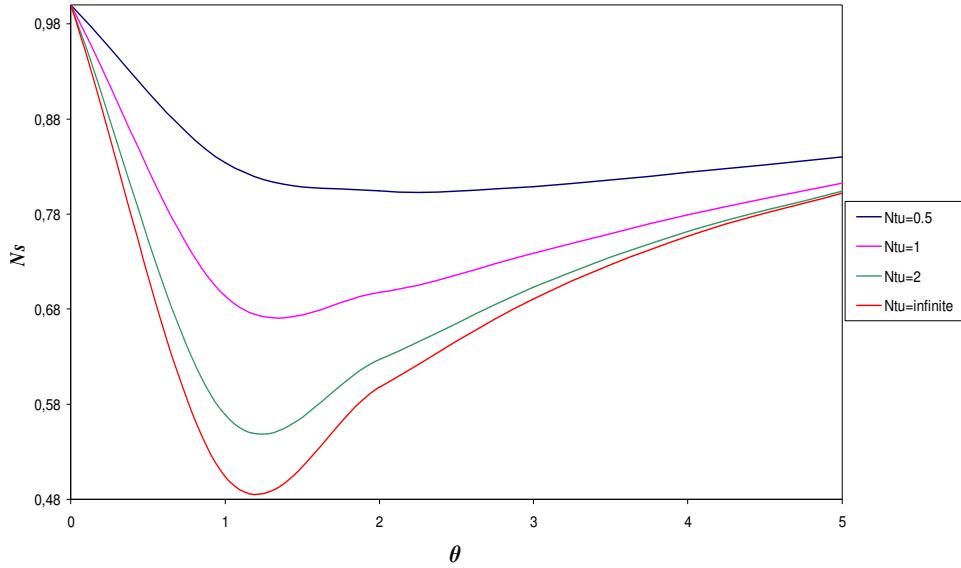
Figure 5.3. Fraction Exergy Destroyed during the Energy Storage Process (continued)

The gas stream exits from the heat exchanger as hot as it enters (T_∞). Hence, its exergy content is destroyed entirely by direct heat transfer to atmospheric temperature.

5.3 SERIES OF STORAGE UNITS

The engineering message of the proceeding section is that the energy storage process of Figure 5.1 is inefficient. Even under the best thermodynamic operating conditions, the process destroys approximately as much exergy as it stores. To make the storage process more efficient, we

must seek to reduce the internal and external heat transfer irreversibilities discussed earlier.

The reduction of these irreversibilities hinges on the ability to minimize the two temperature gaps, on the one hand the gap between hot inlet (T_∞) and storage unit (T), and on the other hand the temperature gap between exhaust (T_{out}) and ambient (T_0). The ambient temperature and at least in the initial phase of the process the temperature of the storage material are fixed (equal to T_0). Thus, we must concentrate on bringing the stream inlet temperature closer to T and also on bringing the exhaust temperature T_{out} closer to T_0. This task is accomplished in strikingly simple form by the use of a number of storage units positioned in series, as shown in Figure 5.4. The stream exhausted by unit (i-1) becomes the exergy source stream for unit (i). Next, the stream exhausted by unit (i) is not rejected to ambient temperature but to a higher temperature, the temperature of unit (i+1).

Storage Units in Series
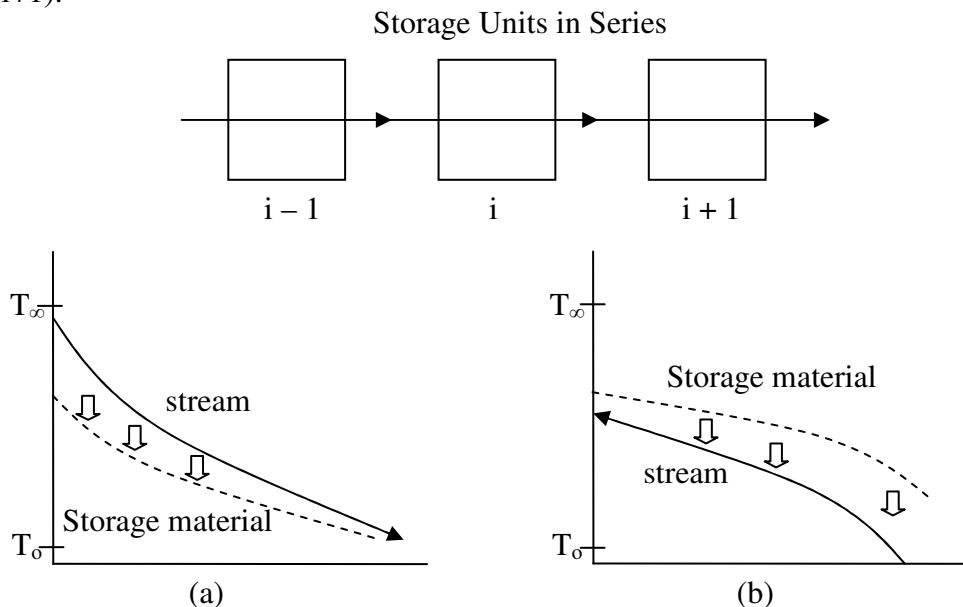
Figure 5.4. Series of Exergy Storage Units during Charging (a) and Discharging (b).

6. CASE STUDY
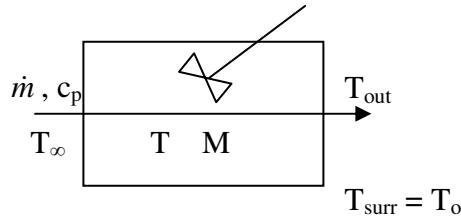
Figure 6.1. Case Study System

6.1 SYSTEM DESCRIPTIONS

The system consists of water bath of mass 10.000 kg placed in an insulated vessel. Hot waste gas enters the system at 1 kg/s and 300°C through one port, is cooled while flowing through a heat exchanger coil immersed in the bath, and is eventually discharged into the atmosphere.

The bath liquid is thermally well mixed so that at any given time its temperature is uniform T(t). It is assumed that initially the bath temperature equals the environment temperature 20°C.

Heat transfer area for this system and diameter of the stainless steel tank are chosen as 40 m². Heat exchanger tube diameter is chosen as 0,02 m.

6.1.1 Tank Dimension Analysis

Finding optimum h & D;

$$V = \frac{\pi}{4}D^2h \longrightarrow \text{Constraint} \tag{6.1}$$

$$h = \frac{4V}{\pi D^2} \tag{6.2}$$

$$A = \pi D h + 2\pi\frac{D^2}{4} = \pi D h + \frac{\pi D^2}{2} \tag{6.3}$$

$$A = f(h, D)$$

$$A = \pi D\left(\frac{4V}{\pi D^2}\right) + \frac{\pi D^2}{2} = \frac{4V}{D} + \frac{\pi D^2}{2} \tag{6.4}$$

$$\frac{dA}{dD} = 0 = -\frac{4V}{D^2} + \pi D \tag{6.5}$$

$$\frac{4V}{D^2} = \pi D \tag{6.6}$$

$$D_{opt} = \sqrt[3]{\frac{4V}{\pi}} \tag{6.7}$$

$$h_{opt} = \sqrt[3]{\frac{4V}{\pi}} \tag{6.8}$$

Hence at optimum $D = h$

58
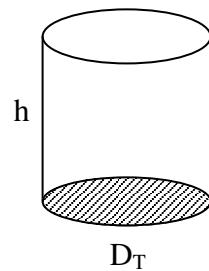
For a single tank;


Figure 6.2. Single Tank
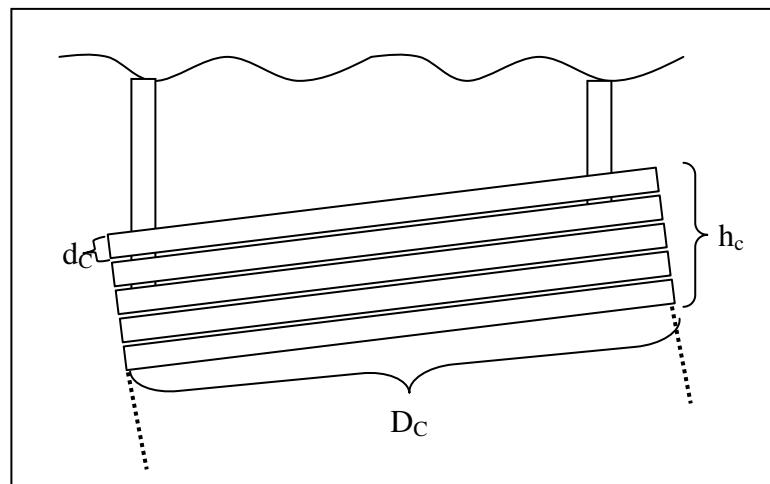
Figure 6.3. Coiled Tube Exchanger Submerged in the Tank

Volume of coil material:

$$A = 40 = \pi dL$$
$$d_C = 2\,cm$$
$$40 = \pi(0,02)L$$
$$L = 637\,m$$

dC

L

(6.9)

Volume of this coil;

$$V_{Coil} = \frac{\pi d_C^2}{4}(L) = \frac{\pi}{4}(0,02)^2(637)$$
$$V_{coil} = 0,2\,m^3$$

(6.10)

Taking $D_C = h_C$

N_c: Number of coil

$$A = N_c \pi^2 d_C D_C \xrightarrow{where} N_c d_C = h_C$$
$$= \pi^2 h_C D_C = \pi^2 D_C^2$$

(6.11)

$$40 = \pi^2 D_C^2$$
$$D_C = \frac{\sqrt{40}}{\pi} = 2,01m$$

(6.12)

D_T should be greater than 2,01 m as it is justified below:

$$V_T = \frac{M}{\rho} = \frac{10000}{1000} = 10\,m^3$$

(6.13)

$$V = \frac{\pi}{4}D_T^2 h = \frac{\pi}{4}D_T^3 \quad (\text{Since } D_{T\,opt} = h_{opt})$$

(6.14)

$$V_T = (V_{water} + V_{coil})1,1 \quad (with\ 10\%\ free\ space\ above\ water)$$
$$= (10 + 0,2)*1,1$$
$$V_T = 11,22\,m^3$$

(6.15)

$$V_T = \frac{\pi}{4} D_T^3$$

$$D_T = h = 2,43 \ m$$

$$(6.16)$$

As seen $D_T > 2,01 \ m$.

For N – stage;

    The same amount of storage material is filled in N tanks of equal volume. For example for 2 – stage system we have two tanks as shown below:
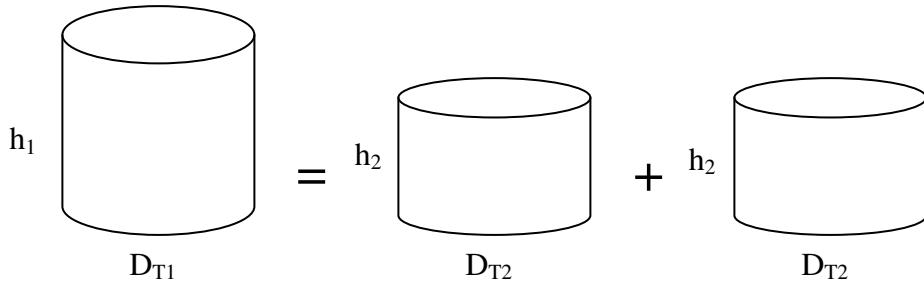

Figure 6.4. Multistage Tank Dimensions

$$M = \sum M_i = M_1 + M_2 = 2M_1 \qquad (6.17)$$

$$\rho V_{T1} = 2\rho V_{T2} \qquad (6.18)$$

$$V_{T2} = \frac{V_{T1}}{2} \qquad (6.19)$$

$$\frac{\pi D_{T2}^2 h_2}{4} = \frac{\pi D_{T1}^2 h_1 / 4}{2} \qquad (6.20)$$

Since $D_{T2} = h_2$ & $D_{T1} = h_1$

$$D_{T2} = \frac{D_{T1}}{(2)^{1/3}} \qquad (6.21)$$

Material for one tank (the function of lateral area)
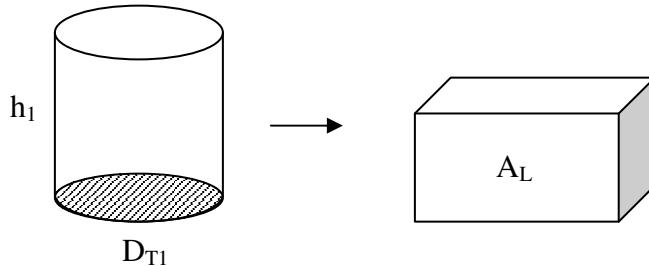
Let A_{L1} be lateral area of single tank and A_{L2} be that of 2 tank system.

These areas can be written as follows:

$$A_{L1} = \pi D_{T1} h_1 + 2\frac{\pi D_{T1}^2}{4} \qquad (6.22)$$

$$A_{L1} = \pi D_{T1}^2 + \frac{\pi D_{T1}^2}{2} \qquad (6.23)$$

$$A_{L1} = \frac{3}{2}\pi D_{T1}^2 = 1{,}5\,(\pi D_{T1}^2) \qquad (6.24)$$

$$A_{L2} = 2\left(\pi D_{T2} h_2 + \frac{\pi D_{T2}^2}{2} \right) \qquad (6.25)$$

$$A_{L2} = 2\left(\frac{3}{2}\pi D_{T2}^2 \right) \qquad (6.26)$$

$$A_{L2} = 3\pi D_{T2}^2 = 3\pi\frac{D_{T1}^2}{(2)^{2/3}} = 1{,}89(\pi D_{T1}^2) \qquad (6.27)$$

As seen $A_{L2} > A_{L1}$

Similarly, $A_{L\,n} > A_{L\,n-1} > ..... > A_{L2} > A_{L1}$
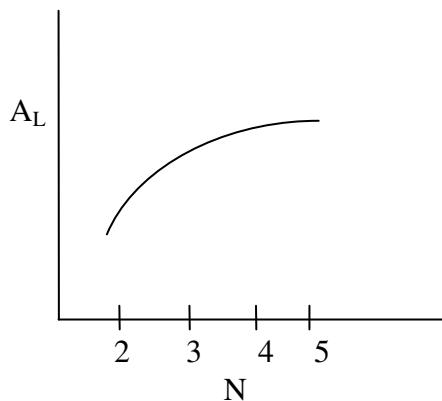
Figure 6.5. Increase in A_L with Stage Number N

As seen from Figure 6.5 as stage number increases A_L increases, too. Hence, fixed cost increases. But; as stage number increases, lost work decreases. When the total cost associated with work lost and fixed cost is drawn there must be an optimum stage number.
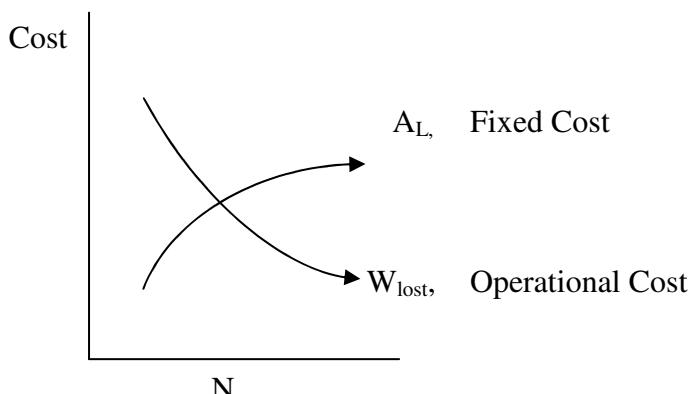

Figure 6.6. Fixed and Operational Cost for Different Number Stages

Lost work means lost electricity, so lost money.
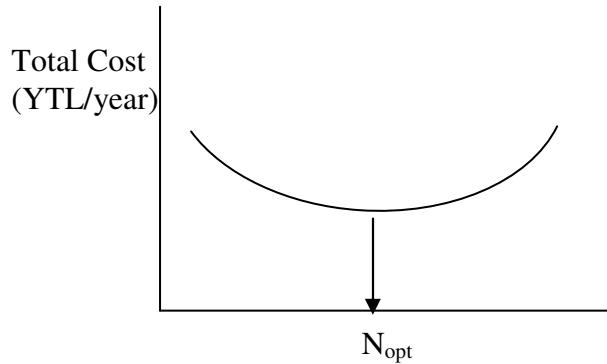
Figure 6.7. Optimum Stage Number for Economic Analysis

Total cost is the sum of fixed cost and operating cost annualized. It varies as in Figure 6.7 given above. It is expected an optimum stage number to exist when cost passes through a minimum.

6.2 VISUAL BASIC PROGRAMS

In the first part of the project two visual basic programs are written in order to calculate total entropy generation in single stage and N – stage system.

For single stage system the entropy generation is calculated with changing time and in N-stage entropy generation is calculated for different number of stages. The total entropy generated in N-stage system at all times is smaller than that of single stage system, so more exergy is stored.

THERMAL ENERGY STORAGE IN ONE STAGE

m_dot (kg/s) : 1,00   M : 10000, kg   T_gas (in) 300 C

time	teta	Ns	Sgen	T_out	T(Tank)
1	0,000	1,000	287	25	20
101	0,002	0,996	28927	26	21
201	0,005	0,993	57355	27	21
301	0,007	0,989	85574	27	22
401	0,010	0,985	113586	28	23
501	0,012	0,982	141394	29	23
601	0,015	0,978	169000	29	24
701	0,017	0,975	196406	30	25
801	0,019	0,971	223615	30	25
901	0,022	0,968	250629	31	26

RUN   END
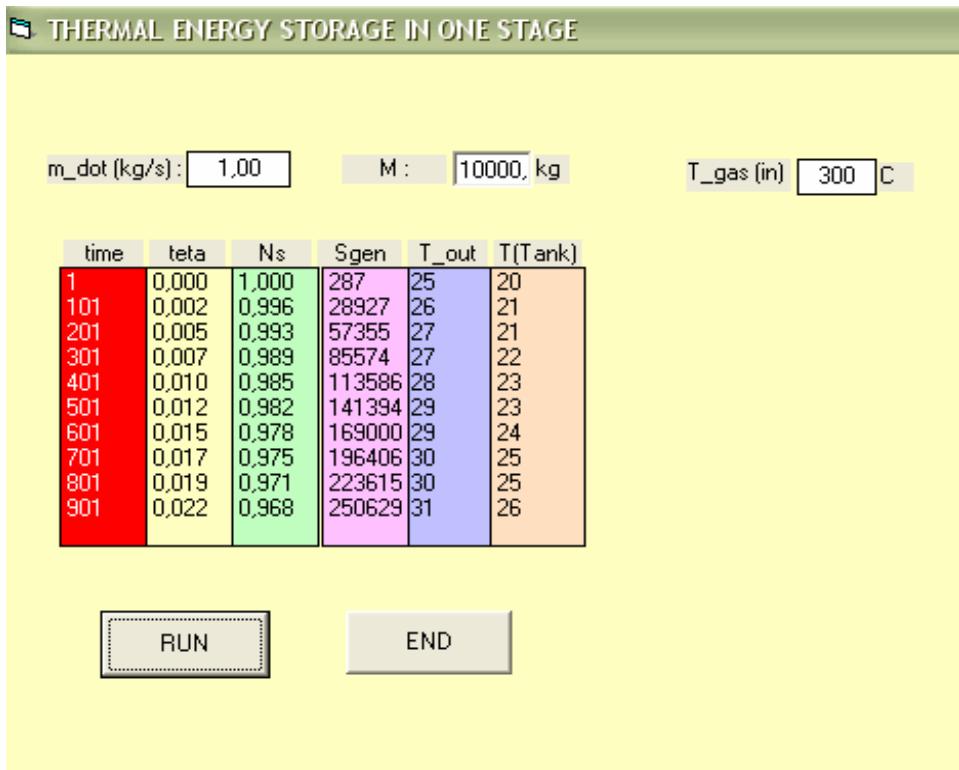
Figure 6.8. Thermal Energy Storage in a Single Stage Program

Storage

THERMAL ENERGY STORAGE IN N STAGES

M   10000,00 kg   RUN   CLEAR   END

m_dot   1,00 kg/s

T_gas_in   300 °C   time 10000 s   Stage Number 5
                          teta 0,241

N	Ns	Sgen	T_out	T(Tank)
1	0,906	520709	166	55
2	0,919	173696	96	38
3	0,927	53565	59	29
4	0,932	15645	41	25
5	0,934	4418	31	23

Sgen Single   2125383,206

Sgen Total   768033,125

$\dfrac{\text{Sgen Total}}{\text{Sgen Single}}$ = 0,361
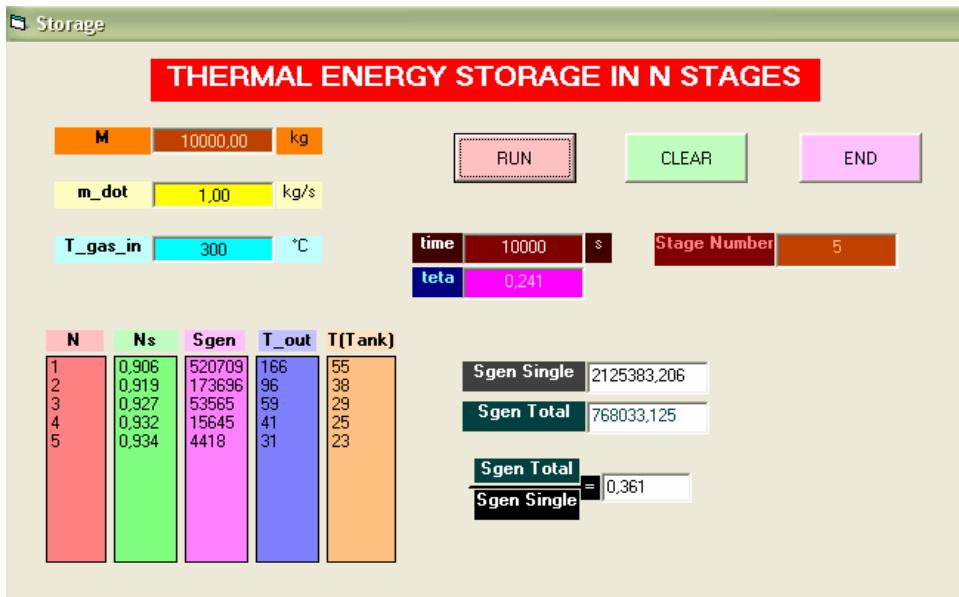
Figure 6.9. Thermal Energy Storage in N – Stage Program

In the second part of the project N – stage number system's program is upgraded from the economic analysis point of view. For different stage numbers and time, lost works and cost are calculated. These programs are provided in the Appendix A.

Using this program two different analyses are done for minimization of total cost. In the first, keeping storage time constant we calculated the total cost for various number of stages. Secondly, we calculated the total cost for various storage times, keeping stage number constant. For the analyses coil material is chosen as copper and tank wall thickness is chosen as 5 mm.
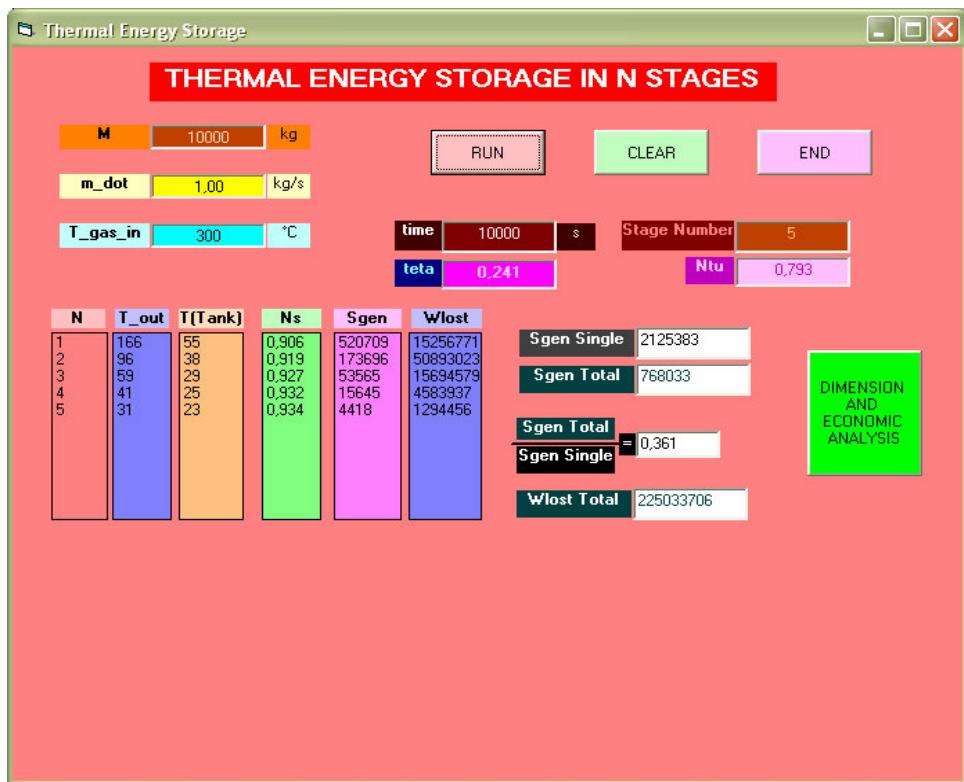

Figure 6.10. Program Output for N – Stage Thermal Energy Storage
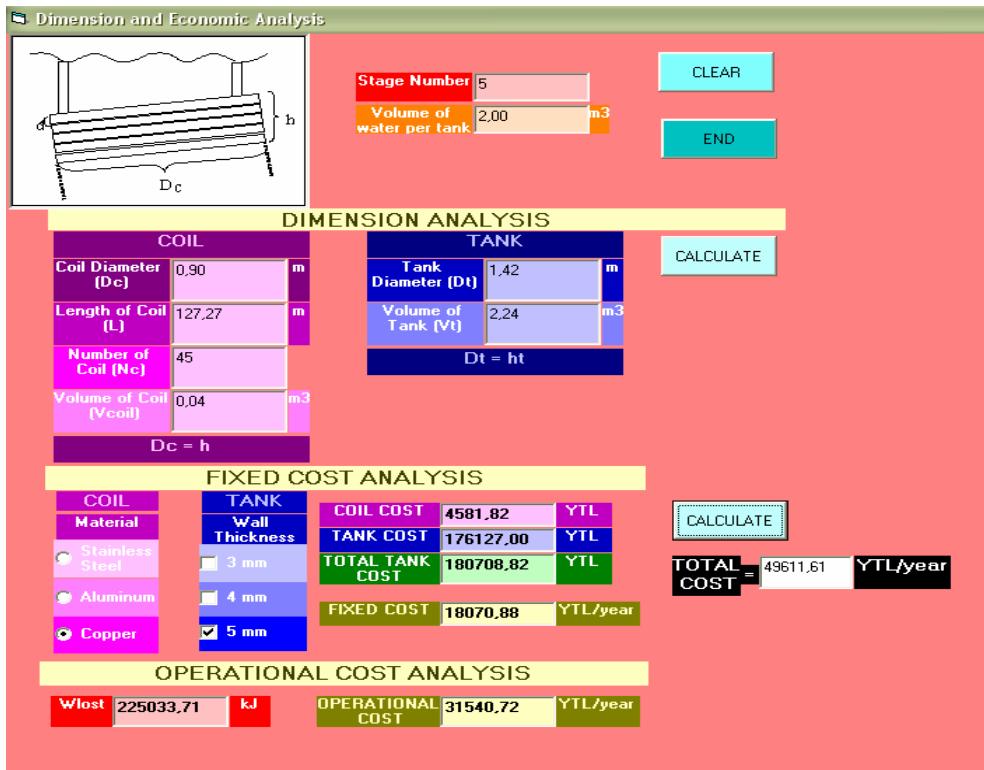
Figure 6.11. Dimension and Economic Analysis

7. NUMERICAL RESULTS OF THE CASE STUDY

The followings are the results of various cases of thermal energy storage. For each case the total annualized cost is calculated and the results are given in the following tables and figures. The unit costs for electricity, tank and coiled tube materials are given in Appendix B.

7.1 RESULTS FOR CONSTANT TIME

In this analysis time is held constant. For various stage numbers economic analysis are done and graphs are drawn.

Table 7.1. Results for t = 1 h

N	C_{total} (YTL/Year)
1	111.585
2	65.956
3	53.434
4	49.932
5	**49.421**
6	50.334
7	52.032
15	55.857
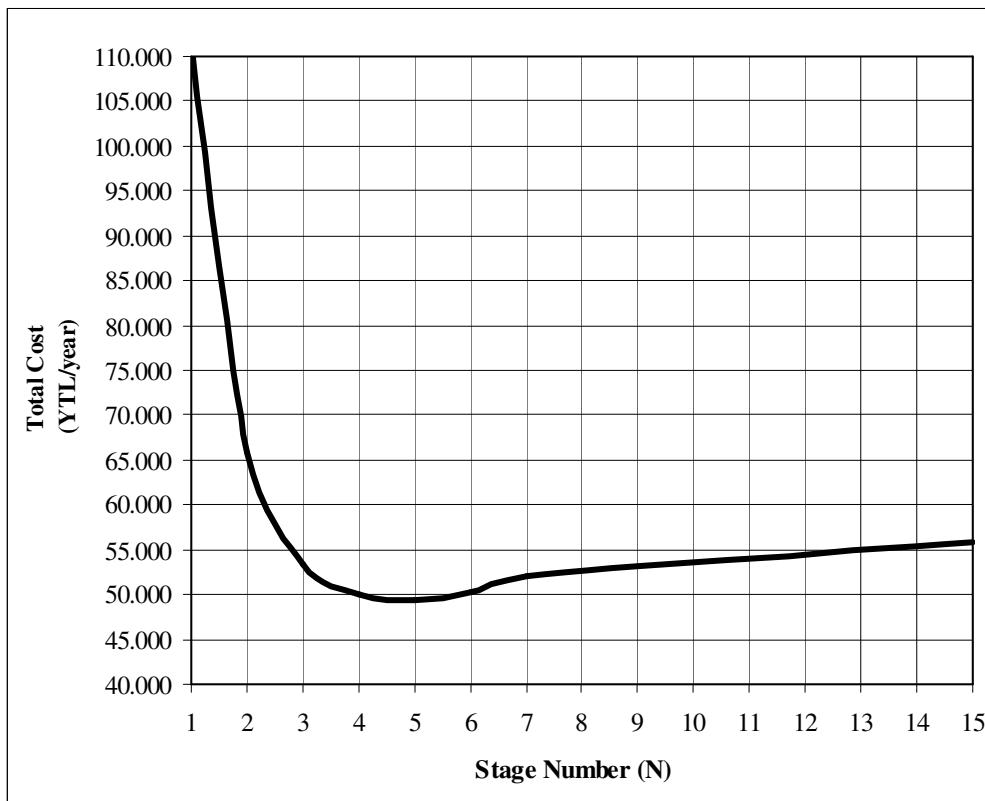
Figure 7.1. Variation of total cost with stage number (t = 1 h).

Table 7.2. Results for t = 2 h

N	C_{total} (YTL/Year)
1	101.061
2	63.280
3	52.554
4	49.706
5	**49.476**
6	50.520
7	52.282
15	56.118
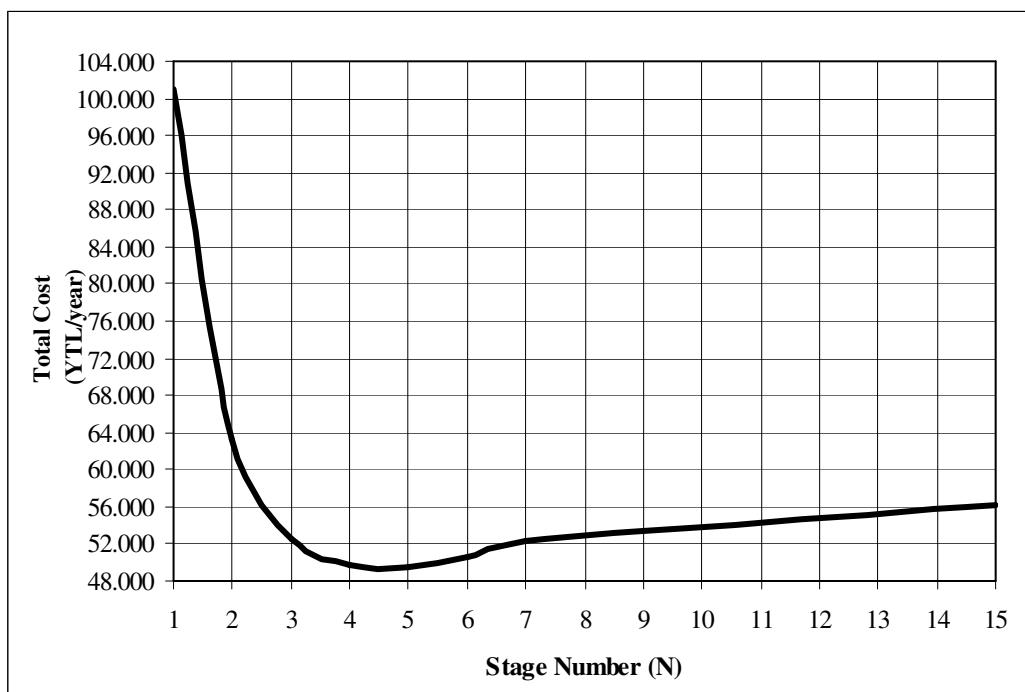
Figure 7.2. Variation of total cost with stage number (t = 2 h).

Table 7.3. Results for t = 3 h

N	C_{total} (YTL/Year)
1	93.029
2	61.497
3	52.086
4	49.700
5	**49.664**
6	50.795
7	52.594
15	56.389
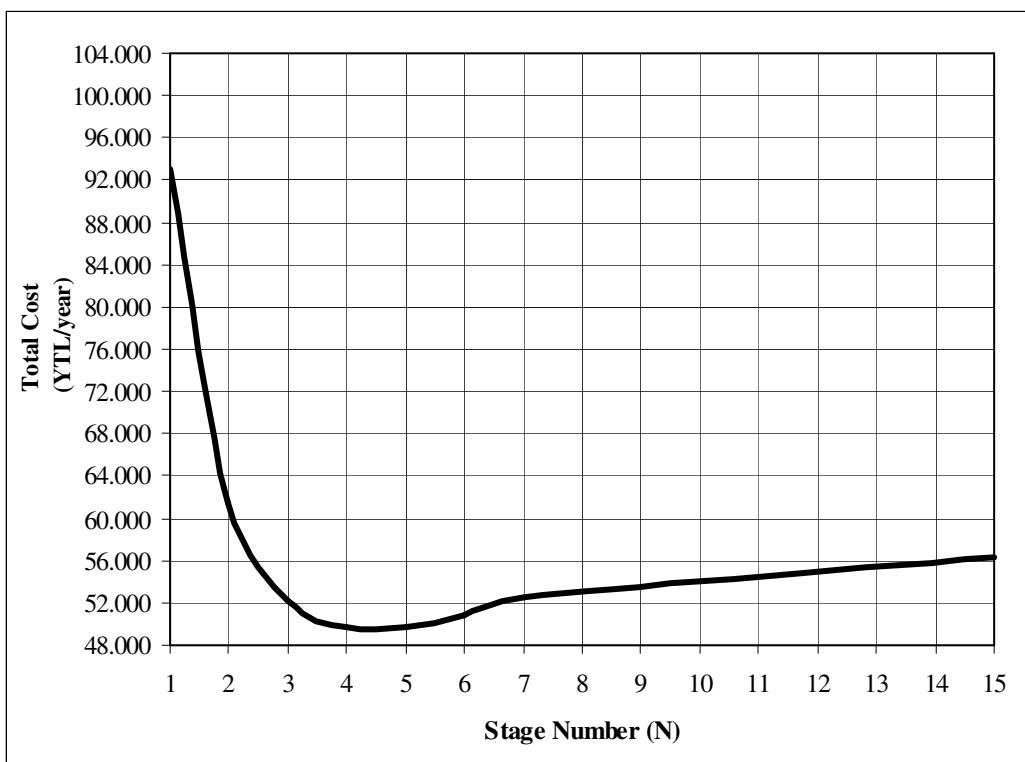
Figure 7.3. Variation of total cost with stage number (t = 3 h).

Table 7.4. Results for t = 10 h

N	C_{total} (YTL/Year)
1	72.429
2	60.852
3	55.424
4	53.807
5	**53.760**
6	54.678
7	56.221
15	58.582
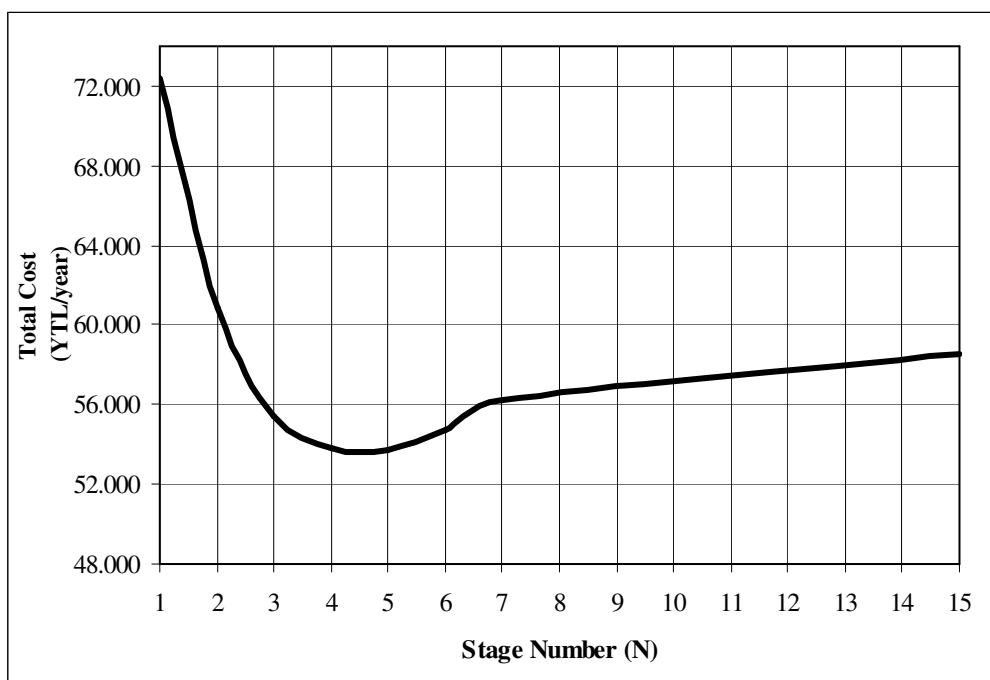
Figure 7.4. Variation of total cost with stage number (t = 10 h).

As a result, the optimum stage number for this analyses is determined to be five.

7.2 RESULTS FOR CONSTANT STAGE NUMBER

In this analysis stage number is held constant. For various times economic analysis are done and graphs are drawn.

Table 7.5. Results for N = 1

t (s)	θ	C_{total} (YTL/Year)
1	0,000	125.404
100	0,002	124.971
1.000	0,024	121.179
10.000	0,241	94.631
25.000	0,603	76.372
40.000	0,966	72.012
42.000	1,014	71.945
42.600	**1,028**	**71.941**
43.000	1,038	71.942
50.000	1,207	72.385
100.000	2,414	83.822
300.000	7,242	109.128
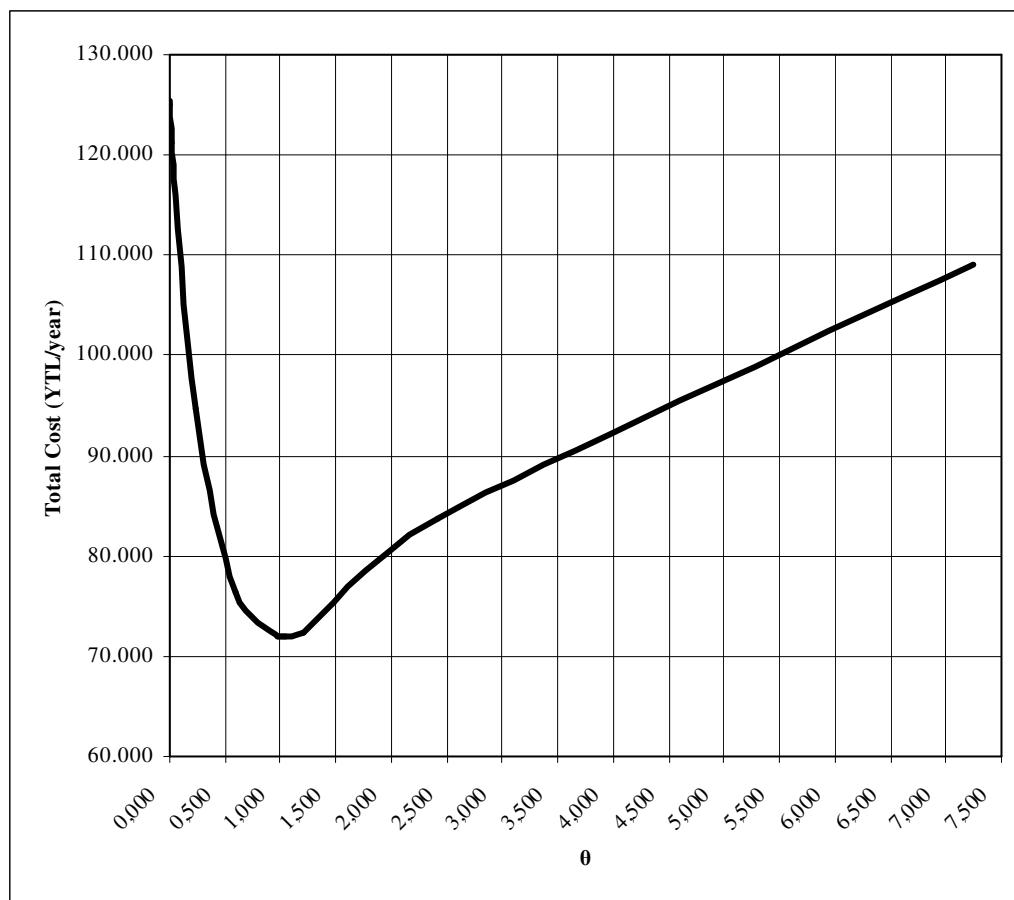
Figure 7.5. Variation of total cost with dimensionless time (N = 1).

Table 7.6. Results for N = 2

t (s)	θ	C_{total} (YTL/Year)
1	0,000	69.894,76
100	0,002	69.765,11
10.000	0,241	61.830,04
25.000	**0,603**	**59.460,94**
25.400	0,613	59.478,44
26.000	0,628	59.510,21
27.000	0,652	59.577,39
30.000	0,724	59.877,42
42.000	1,014	62.216,09
50.000	1,207	64.462,22
65.000	1,569	69.415,40
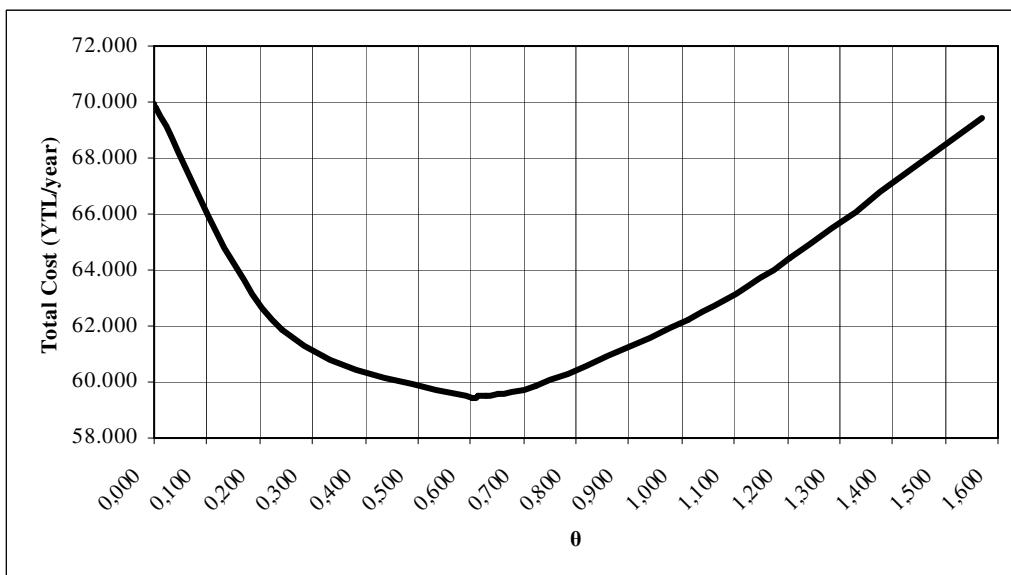
Figure 7.6. Variation of total cost with dimensionless time (N = 2).

Table 7.7. Results for N = 3

t (s)	θ	C_{total} (YTL/Year)
1	0,000	54.836
100	0,002	54.789
1.000	0,024	54.387
10.000	0,241	52.159
12.000	0,290	52.007
13.000	0,314	51.967
14.000	**0,338**	**51.950**
15.000	0,362	51.954
20.000	0,483	52.254
30.000	0,724	53.928
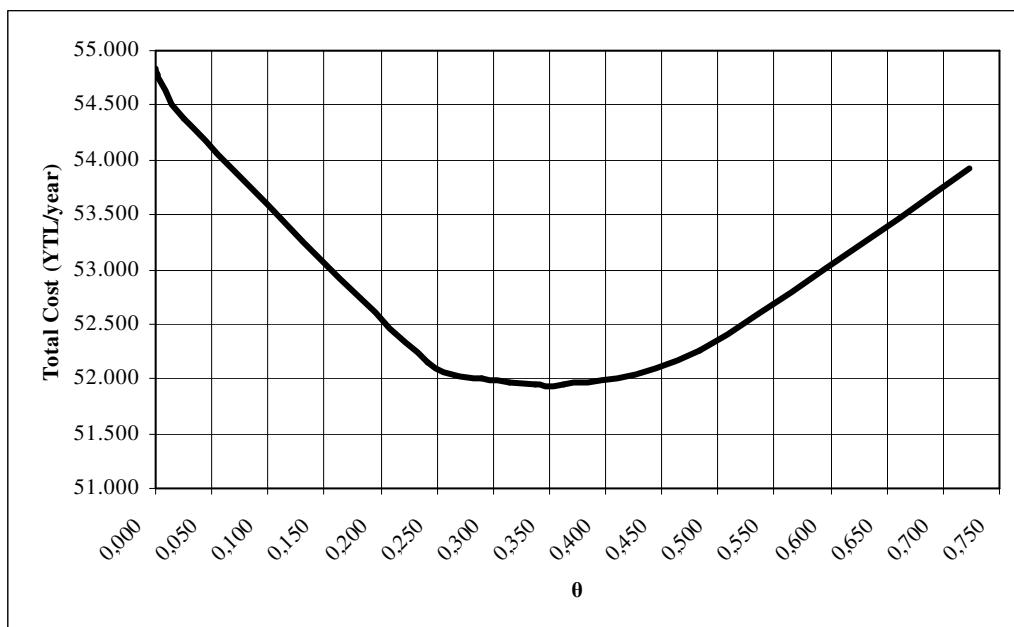
Figure 7.7. Variation of total cost with dimensionless time (N = 3).

Table 7.8. Results for N = 4

| t (s)	θ	C_total (YTL/Year)
1	0,000	50.419
100	0,002	50.402
1.000	0,024	50.255
3.000	0,072	49.994
4.000	0,097	49.895
6.000	0,145	49.755
7.000	0,169	49.712
8.000	0,193	49.687
9.000	**0,217**	**49.678**
9.500	0,229	49.679
12.000	0,290	49.742
13.000	0,314	49.792
15.000	0,362	49.932
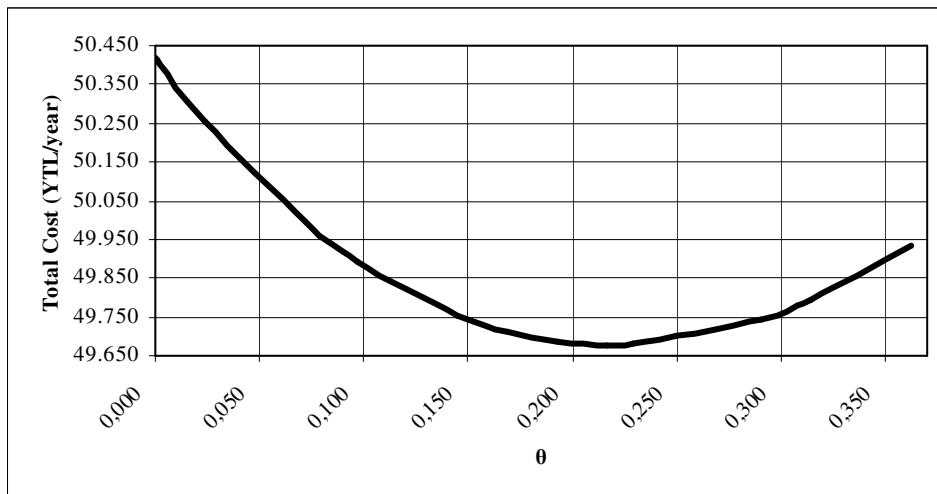
Figure 7.8. Variation of total cost with dimensionless time (N = 4).

Table 7.9. Results for N = 5

t (s)	θ	C_{total} (YTL/Year)
1	0,000	49.517
100	0,002	49.512
1.000	0,024	49.474
3.000	0,072	49.426
4.000	**0,097**	**49.419**
4.150	0,100	49.420
4.500	0,109	49.421
5.000	0,121	49.425
6.000	0,145	49.442
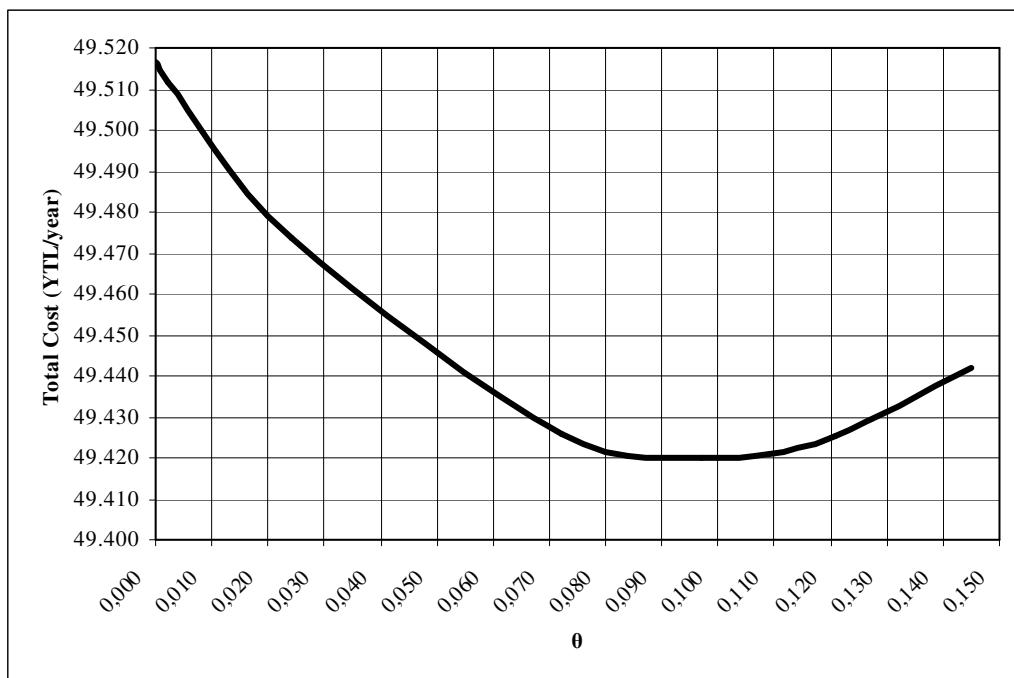
Figure 7.9. Variation of total cost with dimensionless time (N = 5).

Table 7.10. Results of Minimum Total Cost for Different Number of Stages

N	t (s)	θ	C_{total} (YTL/Year)
1	42.600	1,028	71.940,98
2	25.000	0,603	59.460,94
3	14.000	0,338	51.950,03
4	9.000	0,217	49.677,49
5	4.000	0,097	49.419,81

As seen from the figures 7.5 – 7.9 and the tables 7.5 – 7.9 optimum charging time decreases as N increases. So, for the system used optimum stage number and dimensionless time are determined to be 5 and 0,097 respectively.

8. CONCLUSIONS

In this study, the storage of sensible energy contained in a hot waste gas is considered, and water in the tank is selected as storage material. Due to inevitable heat transfer irreversibilities exergy is wasted, is destructed and consequently a thermal energy storage process causes considerable loss of exergy. Firstly, exergy losses are calculated for single stage at various storage times and it is shown that N_s (dimensionless exergy loss number) passes through a minimum at the time called "optimum charging time". It means that energy storage should be stopped at this time and discharging process should be started until water temperature decreases to its initial value.

Secondly, single stage storage system is compared with multistage systems and it is shown that total exergy loss for a multistage system is always lower than that of single stage system. As the number of stage is increased, the total exergy loss decreased. At this point one can conclude that a thermal energy storage system must have the stages as large number as possible. This is not true. Because the advantage of multistage storage with low exergy loss is offset by the increase in the fixed cost of the tank themselves. So, there must be an optimum number of stage. To determine this number, the operating and fixed costs are calculated for different number of stages using today's unit costs of electricity (for exergy loss), stainless steel (for tank material), and copper (for coiled tube heat exchanger). It is shown that optimum number of stage is five and optimum charging time is 0,097 (or t_{opt}=4000 s = 1,11 h). The values are obviously based on the unit costs used in the analysis.

It is clear that; as electricity prices increase, the optimum number shifts towards right while it decreases with increase in fixed and labour costs.

APPENDIX A

THERMAL ENERGY STORAGE IN SINGLE STAGE PROGRAM

Private Sub Command1_Click()

```
dteta = 0.00001
m_gas = Text5.Text
Cp = 1009
M = 10000          'kg
C = 4180
T0 = 20 + 273      'K
T_gas_in = 300 + 273  'K
U = 100
Area = 40          'm2
For t = 1 To 1000000 Step 10000
        Ntu = U * (Area) / (m_gas * Cp)
        y = 1 - Exp(-Ntu)
        thou = (T_gas_in - T0) / T0
        teta = (m_gas * Cp) * (t) / (M * c)
        a = 1 - Exp(-y * teta)
        numerator = thou * a - Log(1 + thou * a)
        denominator = teta * (thou - Log(1 + thou))
        Ns = 1 - numerator / denominator
        T_out = T0 + (T_gas_in - T0) * (1 - y * Exp(-y * teta))
        T_tank = T0 + (T_gas_in - T0) * a
        Ex_in = m_gas * (t) * Cp*(T_gas_in - T0 - T0 * Log(T_gas_in / T0))
        Sgen = Ns * Ex_in / T0
```

```
        List1(0).AddItem Format(t, "0")
        List1(1).AddItem Format(teta, "0.00000")
        List1(2).AddItem Format(Ns, "0.000000")
        List1(3).AddItem Format(Sgen, "0")
        List1(4).AddItem Format(T_out - 273, "0")
        List1(5).AddItem Format(T_tank - 273, "0")
Next t
Text5(0) = Format(m_gas, "0.00")
Text5(1) = Format(M, "0.00")
Text5(2) = Format(T_gas_in - 273, "0")


End Sub
```

THERMAL ENERGY STORAGE IN N – STAGES PROGRAM

FORM 1 – THERMODYNAMIC ANALYSIS

```
Private Sub Command1_Click()

dteta = 0.00001
m_gas = Text2.Text

Cp = 1009

M_water = Text1.Text

C = 4180

T_0 = 20 + 273

T_gas_in = Text3.Text

U = 100
Area = 40      'if it were single
```

$Ntu = U * Area / (m\_gas * Cp)$

$t = Text4(0).Text$

'Single Stage

$Ntu = U * Area / (m\_gas * Cp)$

$y = 1 - Exp(-Ntu)$

$thou_0 = (T\_gas\_in - T_0) / T_0$

$teta = (m\_gas * Cp) * t / (M\_water * c)$

$a = 1 - Exp(-y * teta)$

$Numerator_0 = thou_0 * a - Log(1 + thou_0 * a)$

$Denominator_0 = teta * (thou_0 - Log(1 + thou_0))$

$Ns_0 = 1 - Numerator_0 / Denominator_0$

$T\_out_0 = T_0 + (T\_gas\_in - T_0) * (1 - y * Exp(-y * teta))$

$T\_tank_0 = T_0 + (T\_gas\_in - T_0) * a$

$Ex\_in_0 = m\_gas * t * Cp * (T\_gas\_in - T_0 - T_0 * Log(T\_gas\_in / T_0))$

$Sgen_0 = N_{s0} * Ex\_in_0 / T_0$

$Text5.Text = Sgen_0$

$Text4(3).Text = Ntu$

'In N Stages

$N = Text4(1).Text$

$Wlost\_total = 0$

$Sgen\_total = 0$

For i = 1 To N

$Ntu = U * (Area / N) / (m\_gas * Cp)$

$y = 1 - Exp(-Ntu)$

$thou(i) = (T\_gas\_in - T_0) / T_0$

```
teta = (m_gas * Cp) * (t / N) / ((M_water / N) * C)
a = 1 - Exp(-y * teta)
Numerator(i) = thou(i) * a - Log(1 + thou(i) * a)
Denominator(i) = teta * (thou(i) - Log(1 + thou(i)))
Ns(i) = 1 - Numerator(i) / Denominator(i)
T_out(i) = T0 + (T_gas_in - T0) * (1 - y * Exp(-y * teta))
T_tank(i) = T0 + (T_gas_in - T0) * a
Ex_in = m_gas*(t / N)* Cp*(T_gas_in - T0 - T0 * Log(T_gas_in/T0))
Sgen(i) = Ns(i) * Ex_in / T0
Wlost(i) = Sgen(i) * T0


Sgen_total = Sgen_total + Sgen(i)
Wlost_total = Wlost_total + Wlost(i)
Ns_total = Ns_total + Ns(i)
T_gas_in = T_out(i)
List1(0).AddItem (i)
List1(1).AddItem Format(Wlost(i), "0")
List1(2).AddItem Format(Ns(i), "0.000")
List1(3).AddItem Format(Sgen(i), "0")
List1(4).AddItem Format(T_out(i) - 273, "0")
List1(5).AddItem Format(T_tank(i) - 273, "0")
Next i
Text1 = Format(M_water, "0")
Text2 = Format(m_gas, "0.00")
Text4(0) = Format(t, "0")
Text4(2) = Format(teta, "0.000")
Text4(3) = Format(Ntu, "0.000")
```

Text5 = Format($Sgen_0$, "0")

Text6(0) = Format(Sgen_total, "0")

Text6(1) = Format(Wlost_total, "0")

Text7 = Format(Sgen_total / $Sgen_0$, "0.000")

Text8 = Format(Ns_total, "0.000")

End Sub

FORM 2 – DIMENSION AND ECONOMIC ANALYSIS

Private Sub Command1_Click()

Text5(0).Text = N

pi = 22 / 7

density = 1000 'kg/m^3

Vwater = (M_water / N) / density 'm^3

Text5(1).Text = Format(Vwater, "0.00")

dcoil = 0.02 'm

For i = 1 To N

  Dc = ((Area / N) ^ 0.5) / pi

  Text1(0).Text = Format(Dc, "0.00")

  Nc = Dc / dcoil

  Text2(0).Text = Format(Nc, "0")

  L = (Area / N) / (dcoil * pi)

  Vcoil = pi * (dcoil ^ 2) * L / 4

  Text2(2).Text = Format(L, "0.00")

  Text2(1).Text = Format(Vcoil, "0.00")

  Vt = 1.1 * (Vwater + Vcoil)

  Text4.Text = Format(Vt, "0.00")

```
    Dt = (4 * Vt / pi) ^ (1 / 3)
    Text3.Text = Format(Dt, "0.00")
Next i
End Sub

Private Sub Command4_Click()

'For Coil Price
If Option3(2).Value = True Then
    price = 18.3 'YTL/m
    Price_coil = price * L
ElseIf Option2(2).Value = True Then
    weigth = 0.3 'kg/m
    price = 45 'YTL/kg
    W_total = weight * L 'kg
    Price_coil = price * W_total
ElseIf Option1(2).Value = True Then
    weigth = 0.6 'kg/m
    price = 60 'YTL/kg
    W_total = weight * L 'kg
    Price_coil = price * W_total * 3
End If
Text1(3).Text = Format(Price_coil, "0.00")

'For Tank Price

'Body 1x2 m plate AISI 316
'Cover 1.25 x 2.5 m AISI 316
```

```
If N = 1 Then
      PlateNumber = 12
ElseIf N <= 4 Then
      PlateNumber = (Dt * pi * Dt / 2) + 2
ElseIf N > 4 Then
      PlateNumber = (Dt * pi * Dt / 2) + 3
End If
price_plate = 31.98 'YTL/kg
If Dt > 1.25 Then
      CoverNumber = 4
Else
      CoverNumber = ((pi * (Dt ^ 2) / 4) / (1.25 * 2.5)) + 2
End If

price_cover = 39.75'YTL/kg

If Check1(0).Value Then
      weight_t = 48 'kg
      weight_c = 75 'kg
      price_tank=(price_plate*PlateNumber*weight_t + price_cover*
      CoverNumber *weight_c)*N
ElseIf Check1(1).Value Then
      weight_t = 64 'kg
      weight_c = 100 'kg
      price_tank=(price_plate*PlateNumber*weight_t + price_cover*
      CoverNumber *weight_c)*N
ElseIf Check1(2).Value Then
      weight_t = 80 'kg
```

```
        weight_c = 125 'kg
        price_tank=(price_plate*PlateNumber*weight_t + price_cover*
        CoverNumber *weight_c)*N
End If
Text1(1).Text = Format(price_tank, "0.00")
TotalPrice = price_tank + Price_coil
Text1(2).Text = Format(TotalPrice, "0.00")
TotalFixedCost = TotalPrice / 10
Text1(4).Text = Format(TotalFixedCost, "0.00")
t = Form1.Text4(0).Text
Wlost_total_kJ = Wlost_total / 1000
Text1(5).Text = Format(Wlost_total_kJ, "0.00")
Ope_cost = (Wlost_total_kJ * (0.16 / 3600) / t) * 3600 * 24 * 365
Text1(6).Text = Format(Ope_cost, "0.00")
total_cost = Ope_cost + TotalFixedCost
Text6.Text = Format(total_cost, "0.00")
End Sub
```

APPENDIX B

Table B.1. Physical Properties and Parameters Used in the Case Study

M	:	10.000 kg
$\dot{m}$	:	1 kg/s
$\mathbf{T_0}$	:	20°
$\mathbf{T_\infty}$	:	300°C
$\boldsymbol{\rho}_{\mathbf{water}}$	:	1.000 kg/m^3
$\mathbf{c_p}$	:	1.009 J/kgK
C	:	4.180 J/kgK
U	:	100 W/m^{2}K
A	:	40 m^2

Table B.2. Unit Costs for Electricity, Tank and Coiled Tube Materials*

Material Used		Unit Cost
ELECTRICITY		
1 kWh	:	0,16 YTL
TANK		
AISI 316 plate (1x2 m) (for body)	:	31,98 YTL/kg
AISI 316 (1.25 x 2.5 m) (for cover)	:	39,75 YTL/kg
COILED TUBE		
Stainless Steel	:	18,3 YTL/m
Aluminum	:	45 YTL/kg
Copper	:	60 YTL/kg

* Labour costs are included in unit costs of materials.

REFERENCES

Badescu, V., 2004, Optimal Operation of Thermal Energy Storage Units Based On Stratified And Fully Mixed Water Tanks, Applied Thermal Engineering, 24, 2101−2116.

Bejan, A., 1982, Entropy Generation through Heat and Fluid Flow, John Wiley and Sons Inc., 158 – 171.

Bejan, A., Tsatsaronis, G., Moran, M., 1996, Thermal Design and Optimization, John Wiley & Sons Inc., 113 – 156,

Dinçer, I., 2002, On Thermal Energy Storage Systems and Applications in Buildings, Energy and Buildings, 34, 377−388.

Dinçer, I., Dost, S., Li, X., 1997, Performance Analyses of Sensible Heat Storage Systems for Thermal Applications, IJER, 21, 1157−1171.

Kreith, F., 2000, Handbook of Thermal Engineering, CRC Press, 3-22.

Rosen, M., A., 1999, Second−Law Analysis of Aquifer Thermal Energy Storage Systems, Energy, 24, 167−182.

Rosen, M., A., Dinçer, I., 2003, Exergy Methods for Assessing and Comparing Thermal Storage Systems, IJER, 27, 415 – 430.

Rosen, M., A., Tang, R., Dinçer, I., 2004, Effect of Stratification on Energy and Exergy Capacities in Thermal Storage Systems, IJER, 28, 177−193.