T.C.
MARMARA UNIVERSITY
INSTITUTE FOR GRADUATE STUDIES IN PURE AND APPLIED SCIENCES

AXIOMATIC DESIGN APPROACH TO CONTROL SYSTEMS DESIGN

Ali ÖZYİĞİT
(Mechanical Engineering)
(141101820020147)

THESIS
FOR THE DEGREE OF MASTER OF SCIENCE
IN
MECHANICAL ENGINEERING PROGRAMME

SUPERVISOR
Prof. Dr. A.Kerim KAR

ISTANBUL 2006

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ACCEPTANCE AND APPROVAL DOCUMENT

THESIS TITLE

Established committee listed below, on 06.02.2006 and B.30.2.MAR.0.C1.00.00.sek./363 by the INSTITUTE FOR GRADUATE STUDIES IN PURE AND APPLIED SCIENCES' Executive Committee, have accepted Mr. Ali ÖZYİĞİT's Master of Science thesis, titled as "AXIOMATIC DESIGN APPROACH TO CONTROL SYSTEM DESIGN".

COMMITTEE

Advisor : Prof. Dr. Abdülkerim KAR

Member : Doç. Dr. Arif ADLI

Member : Yrd. Doç. Dr. Haluk KÜÇÜK

Date of Thesis' / dissertation's defense before the committee: 21.02.2006

APPROVAL

Mr. Ali ÖZYİĞİT has satisfactorily completed the requirements for the degree of Master of Science in Mechanical Engineering at Marmara University.

Mr. Ali ÖZYİĞİT is eligible to have the degree awarded at our convocation on 27.02.2006 Diploma and transcripts so noted will be available after that date.

İstanbul **DIRECTOR**

ACKNOWLEDGEMENT

I would like to express my sincere appreciation to my advisor, Professor A. Kerim KAR. His wise guidance and patience made this dissertation possible. I would like to thank him deeply for everything he has done for me. Professor A.Kerim KAR has provided the key technical insights, has critically read through several drafts of this work and has made himself readily accessible. He has made many direct contributions to this work and I have very much enjoyed and benefited from the intellectual discussions that we had during this time. I would like to thank him for his personal support, trust, and understanding during the difficult times (such as my business trips to Iran, Germany, Italy etc…) in my education journey.

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My thanks to the Mechanical Engineering department, for giving me chance to study here both during my BS and MS educations.

01.02.2006 **Ali ÖZYİĞİT**

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ÖZET

Otomatik Kontrol Sistemleri Dizaynında Aksiyomatik Dizayn Yaklaşımı

Doğadaki pek çok olayda çevredeki değişkenlerin istenilen bazı değerlere yaklaştırılması gerekir. Buna karşın unutulmamalıdır ki meydana gelen olay düzensiz ya da düzenli olsa bile ulaşılması istenen değer sabit olabilir.

Bu çalışmada, aksiyomatik tasarım yaklaşımı kullanılarak çeşitli klasik kontrol tasarımları incelenmiştir. Aksiyomatik tasarım yeni bir tasarım yaratılmasında ya da mevcut tasarımın geliştirilmesinde kullanılan bir metoddur. Bu metod, tasarım yapan kişilerin yapılan tasarımı daha iyi oluşturmasını ve anlayabilmesini sağlarken, en uygun tasarım için gereklilikleri de sorgular. Aksiyomatik tasarım bir tasarım ve yöntem olarak Massachusetts Teknoloji Enstitüsünde 20 yıl önce geliştirilmiş ve tasarımı yapan kişilerin kötü tasarımlarda meydana gelen problemlere odaklanmasını sağlamıştır. Teorinin sahibi Prof. Nam P. Suh, aksiyomatik tasarımın amacının tasarım yapan insanları daha yaratıcı yapmak, yapılan araştırmaları daha verimli hale getirmek, yapılan deneme-yanılma sayısını minimuma çekmek ve en iyi dizayna karar verebilmek olduğunu söylemiştir.

Bu çalışma boyunca aksiyomatik tasarım yaklaşımı P, I ve PI kontrol sistemleri üzerinde denenmiş ve sonuçlar belirtilmiştir. Özellikle 1., 2. ve 3. derece sistemler, P ve PI kontrol sistemleri temelinde araştırılmıştır. Bu tezin amacı istenilen değerlere uygun en iyi kontrol sisteminin aksiyomtik tasarım temelinde nasıl tasarlanabileceğini göstermektir.

Şubat, 2006 **Ali ÖZYİĞİT**

ABSTRACT

AXIOMATIC DESIGN APPROACH TO CONTROL SYSTEM DESIGN

In many situations in engineering, it is necessary that the value of a certain variable be kept near some target value. The target value may be constant, in which case the objective is regulation or it may vary.

In this thesis, axiomatic design has been used to investigate various classical control systems. Axiomatic design is a process for creating new designs and for diagnosing existing designs. It is a methodology that helps designers to structure and understand the design problems, thus facilitating the synthesis and analysis of suitable design requirements solution and processes. Axiomatic design is a theory and methodology developed at Massachusetts Institute of Technology (MIT) 20 years ago. According to Professor Nam P. Suh, the theory's creator, "The goal of axiomatic design is to make human designers more creative, reduce the random search process, minimize the iterative trial-end error process, and determine the best design among those proposed."

Axiomatic design approach will be applied to several P, I and PI classical control systems and solutions will be determined according to this new design concepts. Specifically first, second order and third order systems will be examined using classical controllers based on axiomatic design approach. Aim of the thesis is to determine which configuration is the best one according to required specifications.

February, 2006 **Ali ÖZYİĞİT**

ABBREVIATIONS

A/D – Analog-Digital Converter
B – Actual Damping Coefficient
b – Constant
$\mathbf{B_c}$ – Critical Damping Coefficient
C - Constraints
D – Derivative
D/A – Digital-Analog Converter
DP – Design Parameter
e – Tracking Error
$\mathbf{e_{ss}}$ – Steady State Error
F – Force
FR – Functional Requirement
I – Integral
k – Spring Constant
$\mathbf{K_d}$ – Derivative Gain
$\mathbf{K_i}$ – Integral Gain
$\mathbf{K_p}$ – Proportional Gain
m – Mass
$\mathbf{M_p}$ – Maximum Percent Overshoot
P – Proportional
PV – Process Variable
R – Desired input Value; Reangularity
s – Complex Variable
S - Semangularity
t – Time
T – Time Constant
$\mathbf{T_p}$ – Peak Time
$\mathbf{T_r}$ – Rise Time
$\mathbf{T_s}$ – Settling Time
x – Distance
Y – Actual Input
ζ – Damping Ratio
σ – Attenuation
ω_d – Damped Natural Frequency
ω_n – Undamped Natural Frequency
∞ - Infinity

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PART I

INTRODUCTION AND OBJECTIVES

I.1. INTRODUCTION TO PROBLEM

Product design requires the satisfaction of many objectives. Many multi objective approaches have been utilized for this purpose [20]. New methodologies have been developed to aid in decision making during product design. Axiomatic design of Suh [2,4] and Robust design of Taguchi [21] are the mostly known ones. Integration of robustness is in product at the design stage is the most desired activity in product design. Many attempts are made to increase the robustness of products, but in many cases, the issue of durability is left out for verification and testing stage of the product development.

In design of any component it is necessary to assign tolerances to all the dimensions and consider variability of all the inputs and outputs. The combination of all the tolerances with variability of inputs should guarantee, at least from a statistical basis, that the system will perform as expected. Usually there is a conflict in design and manufacturing engineers' interest of tolerances. This conflict of interest can be resolved if design engineer consider tolerances as a decision parameter in overall optimization problem. We need to understand how these tolerances affect the output of the system and tolerances which has significant impact may be kept tighter while other ones may be kept loose.

This thesis presents axiomatic design application in the field of control engineering as a first time. This is a new approach to control system design with the decomposition of functional requirements and design parameters.

Axiomatic design is a scientific approach to control system design that seeks to decouple design components and make direct links between functional requirements and design parameters.

Axiomatic Design is generally applied to new product, process and system design. In this thesis, axiomatic design approach has been used to investigate linear control systems to determine the best design configurations.

I.2. OBJECTIVE OF THE THESIS

Control system design requires the satisfaction of many objectives such as settling time, rise time, maximum percent overshoot etc… Many attempts have been done during this thesis to determine their behaviors. The purpose of the working paper is to make comparison between the control systems and to determine which one the best control system design is. The below methodology will be used during this thesis.

1. Detail investigation of Axiomatic Design research from literature.
2. Investigation of linear control systems.
3. Applications to first and second order control systems.

I.3. SCOPE OF THE THESIS

In this thesis, Part I presents introduction to the problem, objective and scope of the thesis. Part II presents general background of Axiomatic Design and Automatic Control Systems in the literature, Part III presents axiomatic design approach to the first and second order control systems, Part IV presents conclusion and finally Part V presents appendices.

PART II
REVIEW OF AXIOMATIC DESIGN PRINCIPLES AND AUTOMATIC CONTROL SYSTEMS

This part provides a literature review about Fundamentals of Axiomatic Design and Automatic Control Systems.

II.1. FUNDAMENTALS OF AXIOMATIC DESIGN

II.1.1. DESIGN PROCESS

Design has been defined in a variety of ways depending on the specific context or the field of interest. Whether the design solution is a tangible product, service, software, process, or something else, designers typically follow these steps [2]:

• Know their customers' needs.

• Define the problem they must solve to satisfy the needs.

• Create and select the solution through synthesis

• Perform analysis to optimize the proposed solution.

• Check the resulting design solution to see if it meets the original customer needs.

The design process is the development and selection of a means to satisfy objectives, subject to constraints. The questions that a designer has to think about in developing a design of an object, a process, an organization or a system may be stated as follows [2,4];

• What are the main goals that the design solution should achieve?

• What are the design parameters (DPs) that are needed to satisfy the functional requirements (FRs) of the design subject?

• Is this a good design? Why is this design better than others?

• Why didn't it work? In which phase of design has the problem occurred?

• Which parameters have to be changed? How should be the design re-organized in order to reach the optimum solution?

II.1.2. AXIOMATIC DESIGN

Axiomatic design provides a framework for describing design objects that is consistent for all types of design problems. Thus, different designers can quickly understand the relationships between the intended functions of an object and the means by which they are achieved. Design involves a continuous interplay between what we want to achieve and how we want to achieve it. Therefore a rigorous design approach must begin with an explicit statement of what we want to achieve and end with a clear description of how we will achieve them.

The ultimate goal of axiomatic design is to establish a science base for design and to improve design activities by providing the designer with a theoretical foundation based on logical and rational thought processes and tools. Axiomatic design is a process for creating new designs and for diagnosing existing designs. In the words of Professor Nam Suh of MIT, the inventor of this process, "The goal of axiomatic design is manifold: to make human designers more creative, reduce the random search process, minimize the iterative trial-and-error process, and determine the best design among those proposed [6, 11] ."

II.1.3. AXIOMATIC DESIGN FRAMEWORK

The underlying hypothesis of axiomatic design is that there exist fundamental principles that govern good design practice. It is a general theory of design, which provides a scientific basis for designers to make design decisions. Axiomatic design theory can be applied recursively throughout the design hierarchy. Design problems are stated; solutions are proposed and analyzed; and decisions are made. The components that distinguish axiomatic design from other design theories are domains, hierarchies, zigzagging, and the two design axioms: independence and information [6, 11].

a. The Concept of Domains

The world of design is made up of four domains: the customer domain, the functional domain, the physical domain, and the process domain. The domain on the left relative to the domain on the right represents "what we want to achieve", whereas the domain on the right represents the design solution of "how we propose to satisfy the requirements specified in the left domain". The definitions of the key expressions in the concept of domains are [4, 11]:

Functional requirements (FRs): FRs are a minimum set of independent requirements that completely characterizes the functional needs of the product (or software, organizations, systems, etc.) in the functional domain.

Constraints (Cs):

Cs are bounds on acceptable solutions. There are two kinds of constraints. Input constraints are imposed as part of the design specifications. System constraints are constraints imposed by the system in which the design solution must function.

Design parameters (DPs):

DPs are the key physical (or other equivalent terms in the case of other fields) variables in the physical domain that characterize the design that satisfies the specified FRs.

Process variables (PVs):

PVs are the key variables (or other equivalent terms in the case of other fields) in the process domain that characterizes the process that can generate the specified DPs.

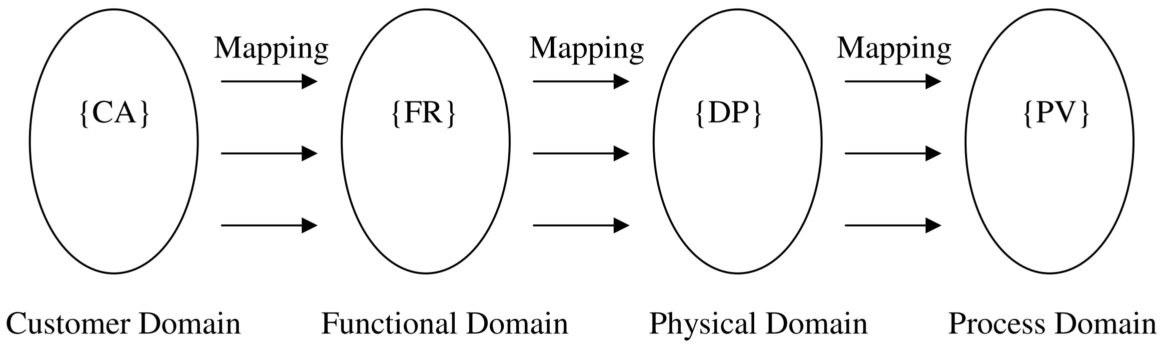


Figure 2.1: Domains of the Design World

The customer domain is characterized by customer needs (or the attributes) the customer is looking for in a product, process or system. In the functional domain, the customer needs are specified in terms of functional requirements (FRs) and constraints (Cs). In order to satisfy the specified FRs, we conceive a design described by design parameters (DPs) in the physical domain. Finally, to produce the design product specified in terms of DPs, we develop a process that is characterized by process variables (PVs) in the process domain.

b. Mapping from Customer Needs to Functional Requirements

The design of a product, system or an organization begins with understanding the customer needs and expectations. Once designers identify and define the perceived customer needs (or the attributes the customer is looking for in a product or system), they must be translated to functional requirements [2, 4].

c. Decomposition, Zigzagging and Hierarchy

The design process progresses from an abstract system level to levels of more detail. This may be presented in terms of a design hierarchy, and a design object is composed of hierarchies in each of the functional, physical and process domains. The decisions that are made at higher levels affect the statement of the problem at lower levels.

At a given level of the design object, there exists a set of functional requirements. Before these FRs can be decomposed, the corresponding design parameters must be selected. Once a functional requirement can be satisfied by a corresponding design parameter that FR can be decomposed into a set of sub-FRs, and the process is repeated. The designers go through a process whereby they zigzag between functional, physical and process domains, and process-in decomposing the design problem. The result of this zigzagging is the creation of hierarchical tree for both FRs and DPs [2, 4].

II.1.4. DESIGN AXIOMS

Designers follow a design process in which decisions are made about a design object with high level, system decisions and progressing to levels of increasing detail. In following this process to synthesize new designs at each level of detail, the steps through which the designer progresses can be described as a problem formulation, synthesis and analysis. The design axioms provide a tool for evaluating designs, particularly during conceptual design. The two axioms may be stated as follows [2]:

a. The Independence Axiom (Axiom 1)

Maintain the independence of the functional requirements (FRs).

Alternative statement: In an acceptable design, the DPs and the FRs are related in such a way that specific DP can be adjusted to satisfy its corresponding FR without affecting other functional requirements.

b. The Information Axiom (Axiom 2)

Minimize the information content of the design. Once a set of FRs has been formulated and possible sets of DPs have been generated, the two design axioms are used to evaluate the proposed designs (the design axioms can also be applied to analyze relationships between DPs and PVs).Independence axiom states that, during the mapping process from the FRs in the functional domain to the DPs in the physical domain, a change in a particular DP must affect only its referent FR. According to the information axiom, among all the feasible designs that satisfy the independence axiom, the one with the minimum information content is the best design.

The mapping process between the domains can be expressed mathematically in terms of the characteristic vectors that define the design goals and the design solutions. At a given level of the design hierarchy, the set of functional requirements that define the specific design goals constitutes a vector {FRs} in the functional domain.

Similarly, the set of design parameters constitutes a vector {DPs}. The relationship between these two vectors can be written as

$$\{FRs\}=[A]\,\{DPs\} \qquad (2.1)$$

Here [A] is defined as the design matrix that characterizes the design and shows the relationships between the FRs and DPs at a given level of the design hierarchy.

There are two special cases for the design matrix: the diagonal matrix where all Aij's except those i=j are equal to zero, and the triangular matrix where either upper or lower triangular elements are equal to zero as shown below.

$$\begin{Bmatrix} FR1 \\ FR2 \\ FR3 \end{Bmatrix} = \begin{bmatrix} X & X & X \\ X & X & X \\ X & X & X \end{bmatrix} X \begin{Bmatrix} DP1 \\ DP2 \\ DP3 \end{Bmatrix} \quad \text{Coupled Design} \qquad (2.2a)$$

$$\begin{Bmatrix} FR1 \\ FR2 \\ FR3 \end{Bmatrix} = \begin{bmatrix} X & 0 & 0 \\ X & X & 0 \\ X & X & X \end{bmatrix} X \begin{Bmatrix} DP1 \\ DP2 \\ DP3 \end{Bmatrix} \quad \text{Decoupled Design} \qquad (2.2b)$$

$$\begin{Bmatrix} FR1 \\ FR2 \\ FR3 \end{Bmatrix} = \begin{bmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{bmatrix} X \begin{Bmatrix} DP1 \\ DP2 \\ DP3 \end{Bmatrix} \quad \text{Uncoupled Design} \qquad (2.2c)$$

To satisfy the independence axiom, the design matrix must be either diagonal or triangular. When the design matrix [A] is diagonal, each of the FRs can be satisfied independently by means of one DP. Such a design is called uncoupled design (Equation 3.2c). When [A] is triangular, the independence of FRs can be guaranteed if and only if the DPs are changed in a proper sequence. Such a design is called decoupled design (Equation 3.2b). Any other matrix (Equation 3.2a) is known as a coupled design. In these equations, an X represents a strong effect by a DP on a FR, while a zero indicates a weak effect, relative to the tolerance associated with the FR.

II.1.5. USE AND BENEFITS OF AXIOMATIC DESIGN

Axiomatic design is a theoretical basis for rational design. It provides a framework for describing design objects that is consistent for all types of design problems and at all levels of detail.

Design axioms provide a rational means for evaluating the quality of proposed designs, and the design process which is used guides designers to consider alternatives at all levels of detail and to makes choices between these alternatives more explicit. Furthermore, axiomatic design theory encompasses a design process that has several benefits for the creation of designs. The design axioms provide a means for evaluating the quality of proposed designs so that design decisions may be made on a rational basis supported by easily understood analytical results.

The designer becomes more creative by understanding a clearly defined problem before design begins and identifying innovative ways to fulfill the functional requirements [2, 4, 11].

II.1.6. REANGULARITY, SEMANGULARITY AND IDEALITY

For any given design matrix, it is possible to find a quantitative measure of the independence. Suh gives two measures of independence, Reangularity, R, and Semangularity, S. Reangularity "measures the orthogonality between the DPs" and can be thought of as a measure of the interdependence among DPs. Semangularity measures the "angular relationship between the corresponding axes of DPs and FRs" and can be thought of as a measure of the correlation between one FR and any pair of DPs. Both have a maximum value of unity, which corresponds to an uncoupled (ideal) design. As the level of coupling increases, the Reangularity and Semangularity decrease [17].

Semangularity is a value that, when equal to unity, 1, then the design is uncoupled providing the Reangularity is also equal to unity, 1. Semangularity, S, is defined by:

$$S = \prod_{j=1}^{n} \frac{\left|A_{jj}\right|}{\left(\sum_{n}^{k=1} A_{kj}^{2}\right)^{1/2}} \tag{2.3}$$

Reangularity, R is a value that has an inverse relationship to coupling (i.e. as Reangularity decreases, coupling increases). If R is 0, the design is completely coupled. If R and S are unity, 1 then the design is uncoupled. Reangularity is defined by:

$$R = \prod_{\substack{i=1,n-1 \\ j=1,n}} \left(1 - \frac{\sum_{k=1}^{n} A_{ki} A_{kj}}{\left(\sum_{k=1}^{n} A_{ki}^2 \right) \left(\sum_{k=1}^{n} A_{kj}^2 \right)} \right)^{1/2} \tag{2.4}$$

These formulas can be rewritten according to 2x2 dimensional cases as

$$\mathrm{R} = \left[1 - \frac{(\mathrm{A}_{11}\mathrm{A}_{12} + \mathrm{A}_{21}\mathrm{A}_{22})^2}{(\mathrm{A}_{11}^2 + \mathrm{A}_{21}^2) \cdot (\mathrm{A}_{12}^2 + \mathrm{A}_{22}^2)} \right]^{1/2} \tag{2.5}$$

$$\mathrm{S} = \frac{|\mathrm{A}_{11}|}{(\mathrm{A}_{11}^2 + \mathrm{A}_{21}^2)^{1/2}} \cdot \frac{|\mathrm{A}_{22}|}{(\mathrm{A}_{12}^2 + \mathrm{A}_{22}^2)^{1/2}} \tag{2.6}$$

Using the above formulas, the independency of the design can be measured. As an example, the following system can be examined:

$$\begin{pmatrix} FR_1 \\ FR_2 \end{pmatrix} = \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix} * \begin{bmatrix} DP_1 \\ DP_2 \end{bmatrix} \tag{2.7}$$

Figure 2.2 shows the coupling relationship between FR_1 and FR_2. This is a fully coupled system. FRs are not parallel to DPs and FRs are not perpendicular to each other.

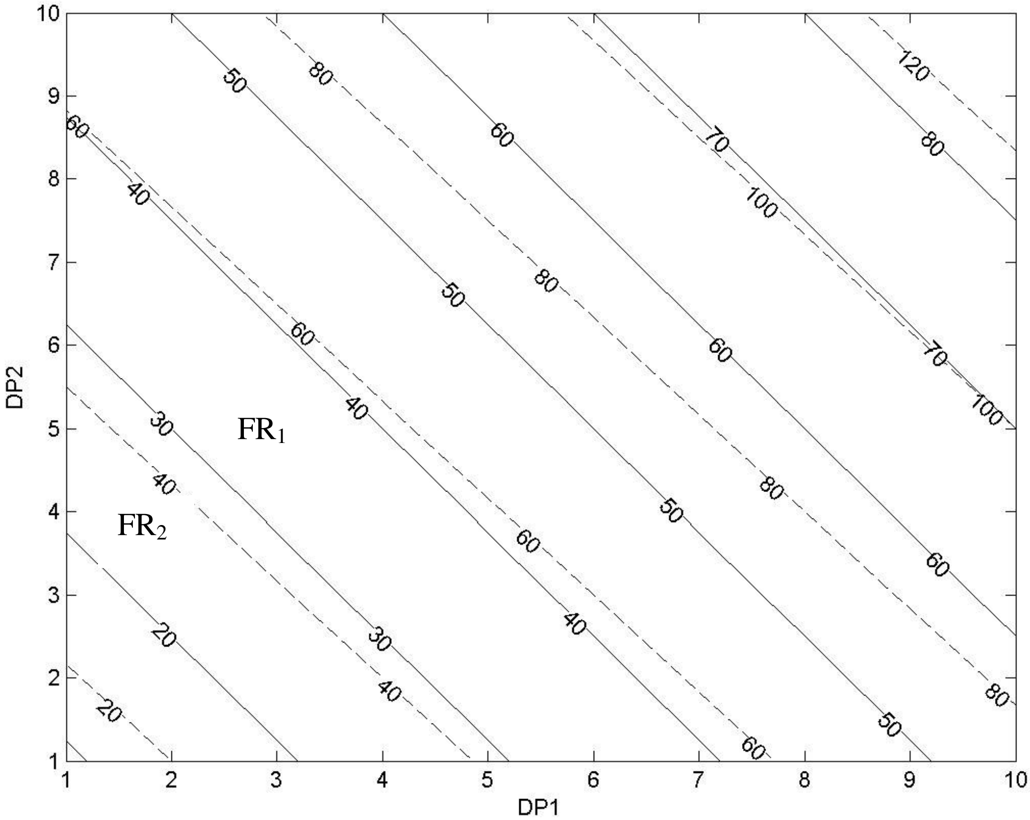


Figure 2.2: Graph of Equation 2.7

Reangularity "R" and Semangularity "S" can be calculated with the Equation 2.5 and 2.6 as S = 0.45138, R = 0.032241 which indicate coupled design.

System in Equation (2.7) was an example of a coupled design. Equation (2.8) is another example in which conditions are closer to the decoupled design. If we change A_{12} in the Equation (2.7) and make it closer to the zero as it is shown in the Equation (2.2b), we can see the behavior of the contour lines in a graph. Therefore our new design equation becomes;

$$\begin{pmatrix} FR_1 \\ FR_2 \end{pmatrix} = \begin{bmatrix} 4 & 0.5 \\ 6 & 7 \end{bmatrix} * \begin{bmatrix} DP_1 \\ DP_2 \end{bmatrix} \tag{2.8}$$

Figure (2.3) shows the independency of matrix in Equation (2.8).

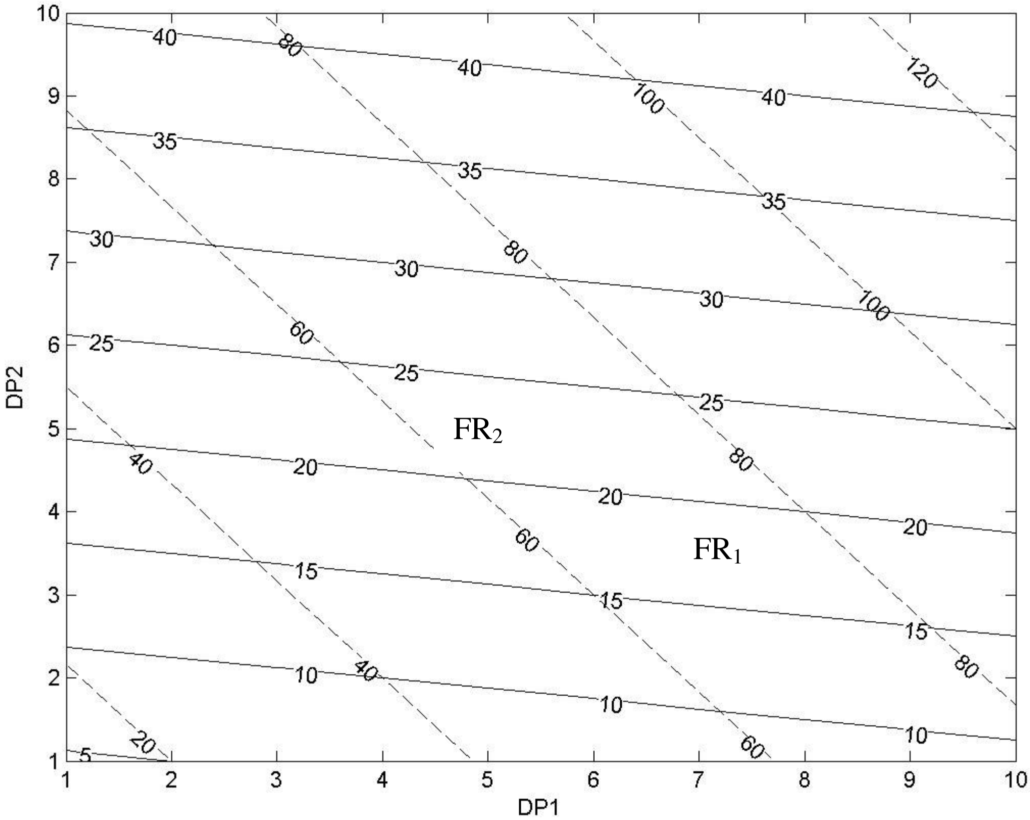


Figure 2.3: Graph of Equation 2.8

As it is seen at the Figure (2.3), the design is closer to decoupled design since FR_1 lines are almost parallel to DP_1 axis. Since the matrix is 2x2 dimensional, Reangularity "R" and Semangularity "S" can be calculated according to Equation (2.5) and (2.6), S = 0.55362, R = 0.50172. R and S are nearly the same and not equal to 1. Since R and S close to each other, this system can be assumed decoupled.

If we change A_{21} in the Equation (2.8) and make it closer to the zero as it is shown in the Equation (2.9), we can see the behavior of the contour lines in Figure (2.4). Therefore our new design equation becomes;

$$\begin{pmatrix} FR_1 \\ FR_2 \end{pmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.6 & 7 \end{bmatrix} * \begin{bmatrix} DP_1 \\ DP_2 \end{bmatrix} \qquad (2.9)$$

Figure (2.4) shows the independency of matrix in Equation (2.9).

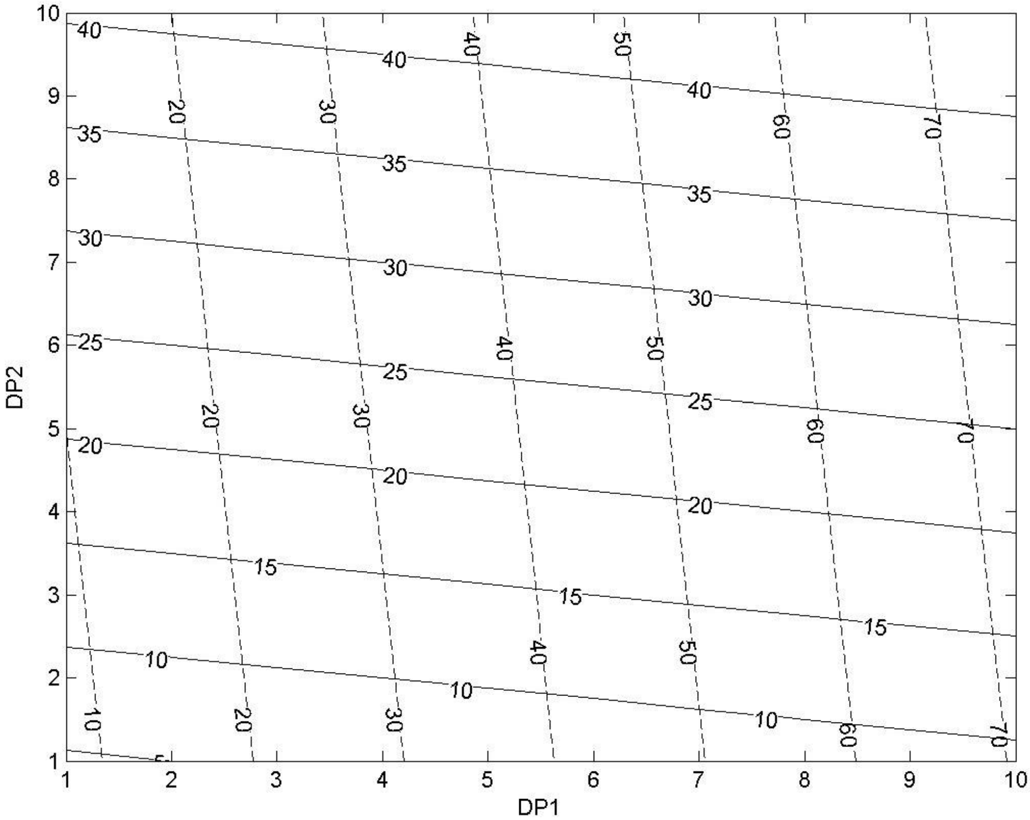


Figure 2.4: Graph of Equation 2.9

As it is seen at the Figure 2.4, the design is closer to the uncoupled design since FR_1 and FR_2 lines are almost parallel to $DP_1$and DP_2 axis respectively. Since the matrix is 2x2 dimensional, Reangularity “R” and Semangularity “S” can be calculated according to Equation (2.5) and (2.6), S=0.98701, R=0.9776. Since R and S almost equal to each other and closer to 1, this system can be assumed uncoupled design.

II.2. AUTOMATIC CONTROL SYSTEMS

II.2.1. INTRODUCTION

Automatic control has played a vital role in the advance of engineering and science. Since advances in the theory and practice of automatic control provide the means for attaining optimal performance of dynamic systems, improving productivity, relieving the drudgery of many routine repetitive manual operations, and more, most engineers and scientists must now have a good understanding of this field. There are many reasons why we need control systems, but the four most important ones are performance, economics, safety and reliability.

Many systems cannot achieve specified levels of performance without controls. A high-performance air-craft must have a control system to achieve good accuracy. A distillation column must be controlled if the chemical composition of its outlet stream is to be within specifications.

Economic considerations are also important, especially in process control, i.e. in the control of production systems. There are many so-called continuous processes, designed to operate in the steady state. Power plants, distillation columns, and paper machines are all examples of system that are designed to keep constant operating conditions except during start-up and shut-down. For example, the basis weight (weight per unit area) of paper produced by a papermaking machine is required to be above a certain lower limit. It is clear that, the lighter the paper, the smaller the quantity of pulp required, and hence the lower the cost. Another economic benefit is, one that is more difficult to predict, that a process can often be run at a higher capacity with improved control.

Safety is a third justification for control. An aircraft landing under poor visibility conditions must be controlled to follow a safe glide path. A nuclear reactor must operate in such a way that key variables are kept within safe limits. Many systems have “danger zones”; it is the task of control system to avoid them.

Finally, control is used to achieve better reliability. Physical systems are usually less prone to failure if they are operated smoothly and with low levels of fluctuation. Automobile manufacturers advise drivers to avoid quick starts and stops in order to lengthen the lives of their cars [1, 7].

II.2.2. CONTROL SYSTEM TERMINOLOGY

Before we discuss control systems, some basic terminologies must be defined in order to better explanation of the control systems.

Control Variable and Manipulated Variable: The controlled variable is the quantity or condition that is measured and controlled. The manipulated variable is the quantity or condition that is varied by the controller so as to affect the value of the controlled variable. Normally, the controlled variable is the output of the system. Control means measuring the value of the controlled variable of the system and applying the manipulated variable to the system to connect or limit deviation of the measured value from a desired value.

Plants: A plant may be a piece of equipment, perhaps just a set of machine parts functioning together, the purpose of which is to perform a particular operation.

Processes: It is described in Cambridge Advanced Dictionary as "a series of actions that you take in order to achieve a result"

Systems: A system is a combination of components that act together and perform a certain objective.

Disturbances: A disturbance (I), a signal tends to adversely affect the value of the output of a system. If a disturbance is generated within the system, it is called internal, while an external disturbance is generated outside the system and is an output.

Feedback Control: Feedback control refers to an operation that, in the presence of disturbances, tends to reduce the difference between the output of a system and some reference input and that does on the basis of this difference. Here only unpredictable disturbances are so specified, since predictable or known disturbances can always be compensated for within the system [1, 7].

II.2.3. PHYSICAL ELEMENTS OF CONTROL SYSTEMS

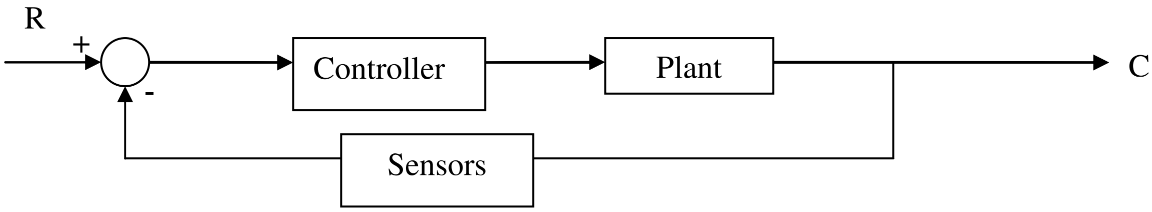


Figure 2.5: Physical Elements of Control Systems

Figure (2.5) depicts the elements of a control system. The word “plant” is used generally, to denote the object under control. A plant has output variables, some of which are the ones to be controlled. These variables are measured by sensors. A sensor is basically a transducer, i.e., a device that performs one type of physical quantity to another, usually electrical. Examples of sensors are tachometers, accelerometers, thermocouples, strain gages, and pH meters.

A plant must have input variables, which can be manipulated to affect the outputs. The elements that permit these manipulations are called actuators. Control valves, hydraulic actuators and variable voltage sources are examples of actuators. Actuators are often themselves complete system, with their own controls.

The role of the control elements is to carry out the control strategy, i.e. to derive command signals for the actuators in response to the sensor outputs. The control elements may be analog devices, but digital control has become prevalent. In the latter case, the control element includes the A/D and D/A converters and a portion of the computer software.

The operator interface is the window to the outside world that allows human monitoring and intervention. It receives information concerning the inputs and outputs, plus certain status variables from the control elements. It activates displays (dials, strip recorder charts, computer graphics) and triggers alarm indicators (red light, bells). It also serves as a means of altering the control strategy-for example, by changes in set points or control strategy parameters.

And also we should think that, not all systems are physical systems, and the above blocks not always correspond to physical devices. For example, the plant may be the economy of a nation, which have inputs such a monetary and fiscal policies, and outputs such as GNP growth rate, unemployment rate, and inflation rate, [7, 14].

II.2.4. PERFORMANCE REQUIREMENTS FOR CONTROL SYSTEMS

While the abilities to achieve tracking and reduce the effects of adverse conditions are some of the key features of feedback control systems, the behavior of the system, in terms of shapes of time responses of system signals, is usually used to specify how we want a closed-loop system to behave. Some of the most important ways to characterize the behavior of systems are given in the following:

- *Stability:* There are many ways to characterize stability, so that in other contexts "stability" may simply mean that all the system variables remain bounded by a fix constant. Or, an oscillating signal could be considered "unstable". We would normally want to design our control system so that the closed-loop system does not exhibit unstable behavior.
- *Rise time:* Rise time is a measure of how long it takes for the actual speed to get close to the desired speed when there is a step change in the set point speed. A short rise-time would normally be considered to be the best; however, physical characteristics of the vehicle (e.g., the size of the engine) will limit how short the rise time can be.
- *Overshoot:* Too much overshoot is not good, as you could get a speeding ticket if it is too large. Note that when we get a better (smaller) rise time, we get a worse (larger) overshoot. There is a "trade-off" between these two design objectives, and such trade-offs are often encountered in control system design. Creating a good control design often amounts to achieving an appropriate balance between competing objectives.
- *Settling time:* We would typically like to have a short settling time; however, if we get a short rise-time it can sometimes be more difficult to get a short settling time.
- *Steady state error:* We would typically like to have no steady state error, or at list very small one, in dependent of the desired speed. Special attention must be paid to this performance objective in design to ensure that this property holds.

Generally, when we begin the design process we may have in mind the many possible time responses that we might get for different controller choices. It is important to note that it is not sufficient to simply achieve a single adequate response for one set of conditions. The actual objective is to achieve this response, or something close to it, even if there are disturbances, plant parameter variations or sensor noise. In other words, we would like to performance, in term of time responses, to be 'robust' too many different conditions, including adverse conditions. We also want to have stability be robust under these same conditions. The problem of achieving such performance and stability robustness goals significantly complicates the control design problem; indeed, it is often the central issue to focus on in this design.

In control system design, there are below issues that must be considered that are often of equal or greater importance when you develop control systems. These include the following:

- *Cost:* How much money will it take to implement the controller, or how much time will it take to develop the controller? Costs of sensors, actuators, the controller, and other interfaces must be taken into consideration. Also, the number of person-years needed for development, implementation, commissioning, and maintenance must be considered.
- *Computational Complexity:* How much processor power and memory will it take to implement the controller? In some practical applications, controllers can quickly burden the available processors (e.g., in current automotive applications, relative simple inexpensive processors are used to achieve a variety of control functions in the engine, transmission, etc.). Clearly, in such situations, the concerns of computational complexity can significantly impact how you design your controller.
- *Manufacturability:* Does your controller have any extraordinary requirements with regard to manufacturing the hardware that is to implement it? It should be designed so that it is easy to manufacture.
- *Reliability:* Will the system always perform properly? What is its "mean time between failures"? What causes these failures? Sensors, actuators, communication links or controller? How can you design the system so that the number of failures is minimized? This can be particularly important in safety-critical applications such as aircraft control where redundant hardware is often used. Is your controller simply "too aggressive"? Does it try to achieve the best possible time responses, without giving enough attention to the need to be conservative to ensure that adverse conditions will be adequately dealt with, even ones that you cannot envision at this time? Sometimes experienced control engineers express such concerns when a new controller is developed and there are extraordinary performance claims.
- *Maintainability:* Will it be easy to perform maintenance and routine adjustments to the controller? As we discussed earlier, extra code is often added to the controller for a maintenance interface.

- *Adaptability:* Can the same design be adapted to similar applications so that the cost of later designs can be reduced? The controller engineer views her task as that of designing one controller that fits all the plants in a certain class. In practice, the design is sometimes provided with a set of instructions on how to tailor the design to each particular application.
- *Expandability:* Hoe much redesign work will we have to do to be able to add new hardware or functionality to the control system? Will we have to start over? Or, are the existing controller's functions and code established in a manner that makes it possible to easily add new functionality? Is the control system easy to interface to other system (is it open?)?
- *Understandability:* Will the right people be able to understand the approach to control? For example, will the people who implement, maintain, or test it be able to fully understand it? The importance of this characteristic cannot be overstated for some applications. You may have the best controller imaginable, but if it cannot be clearly explained to certain key personnel, it will not be chosen for implementation.
- *Politics:* Is your boss biased against your approach? Can you sell your approach to your colleagues? Is your approach too novel and does it thereby departs too much from standard company practice? Is your approach too risky?

Not only must a particular approach to control satisfy the basic performance objectives, but the above issues must also be taken into consideration; these can often force the control engineer to make some very practical decisions that can significantly affect. It is important, then, that the engineer has these issues in mind early in the design process to ensure that the best possible controller is delivered [15].

II.2.5. CONTROL SYSTEMS CHARACTERISTICS

Standard controllers have up to three "modes or terms in their control algorithm. (The word mode is used here to classify the output response of the controller to a set point and measurement and does not refer to whether the controller is in manual or automatic.) Figure 2.6 shows the mathematical definition and the two commonly used names for each mode.

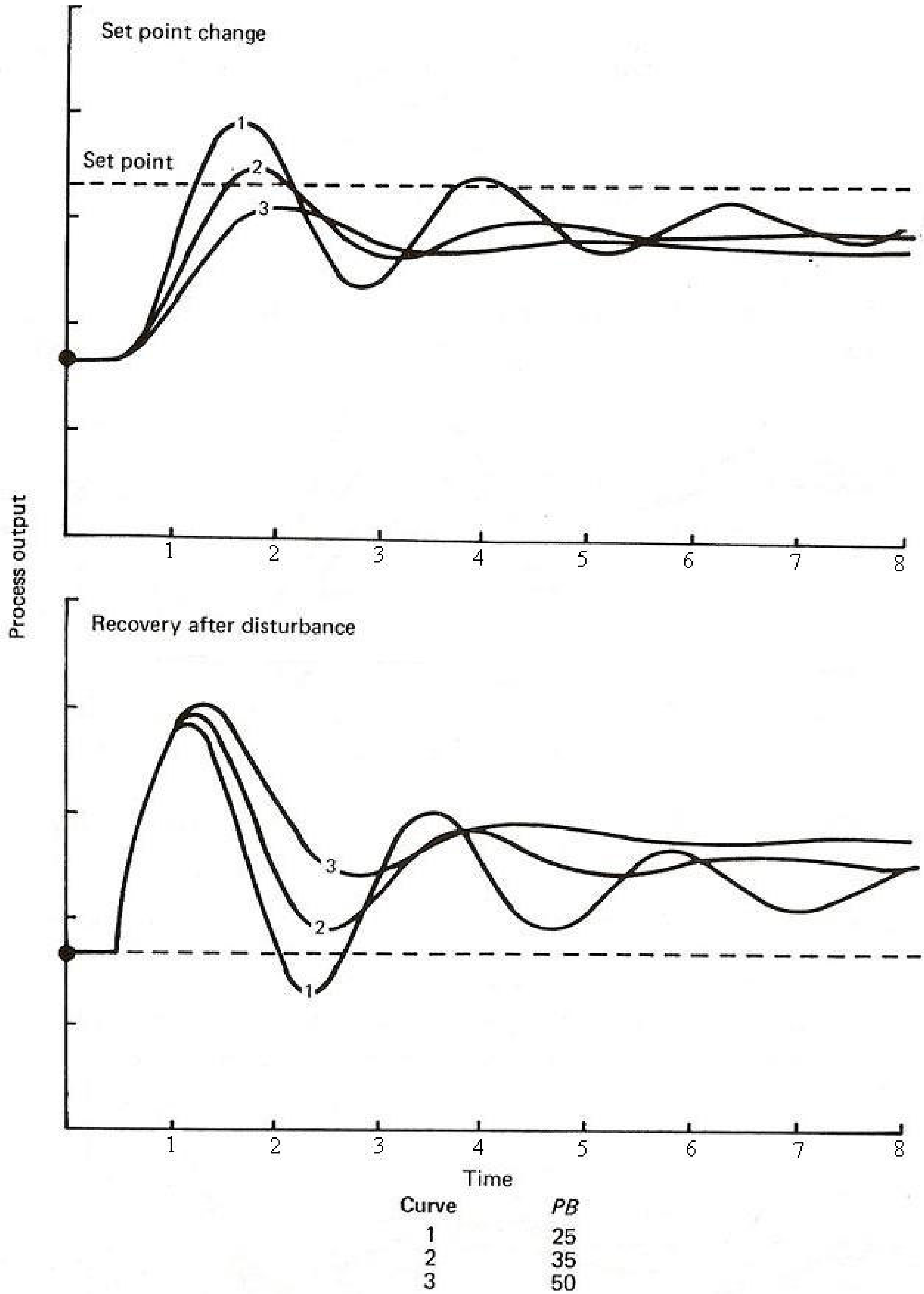


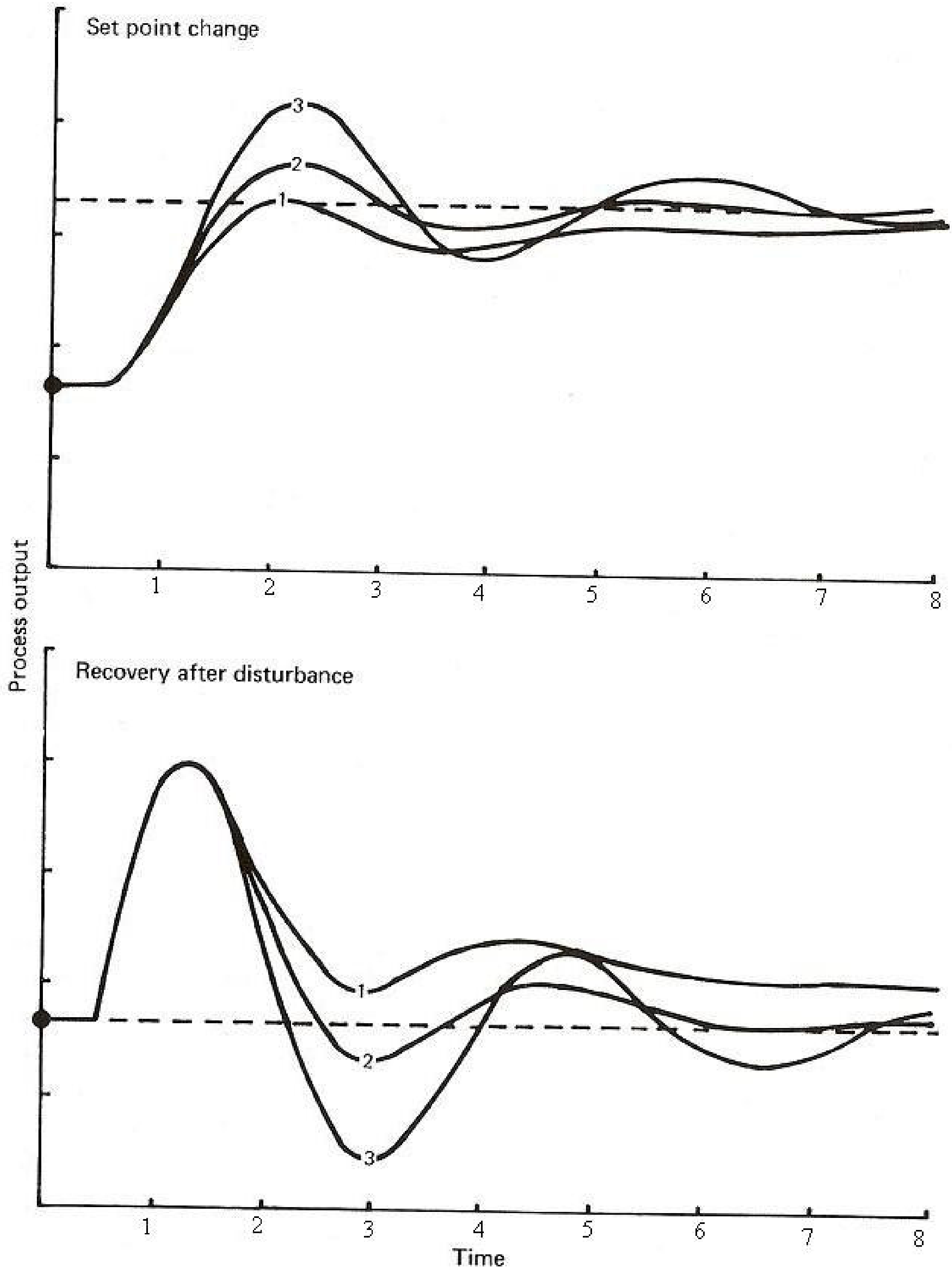
Figure 2.6: Effect of Proportional Mode on Response

a. Proportional Mode

Nearly all controllers have the proportional (gain) mode. This mode changes the controller output by an amount proportional to the change in error. The proportional band is the percent change in error necessary to cause a full-scale change in controller output. Proportional band is the inverse of controller gain multiplied by 100. Most of the analog controllers use proportional band whereas the majority of the new digital controllers use gain. Note that the proportional band setting also affects the integral and derivative modes. Figure 2.6 shows that as the proportional band is decreased (as the gain is increased), the offset (sustained error) decreases but the response becomes more oscillatory. If the proportional band is decreased until the oscillation are equal in amplitude and no other modes are used, the period of these oscillations is the natural period of the loop. This natural period is known as the ultimate period and depends on the dynamics of the process and the instrument components of the loop. If the oscillations are growing in amplitude, the measured period is shorter than the ultimate period. If the oscillations are decaying in amplitude, the measured period is longer than ultimate period.

b. Integral (reset) Mode

Most controllers also have an integral (reset) mode. This mode changes the controller output by an amount proportional to the integral of the error. The integral time is the time required for the integral (reset) mode contribution to equal (repeat) the proportional mode contribution for a constant error. The integral mode action lags the proportional mode action by this integral time. The use of the integral increases the permissible proportional band but eliminates offset. Most controllers use the inverse of the integral time so that the mode setting units are repeat per minute. Figure 2.7 shows that as the integral time is decreased (reset action in repeats per minute increased), the offset is eliminated faster but the response becomes more oscillatory. If the integral time is decreased too much, the oscillations develop into a reset cycle whose period is much longer than the ultimate period.

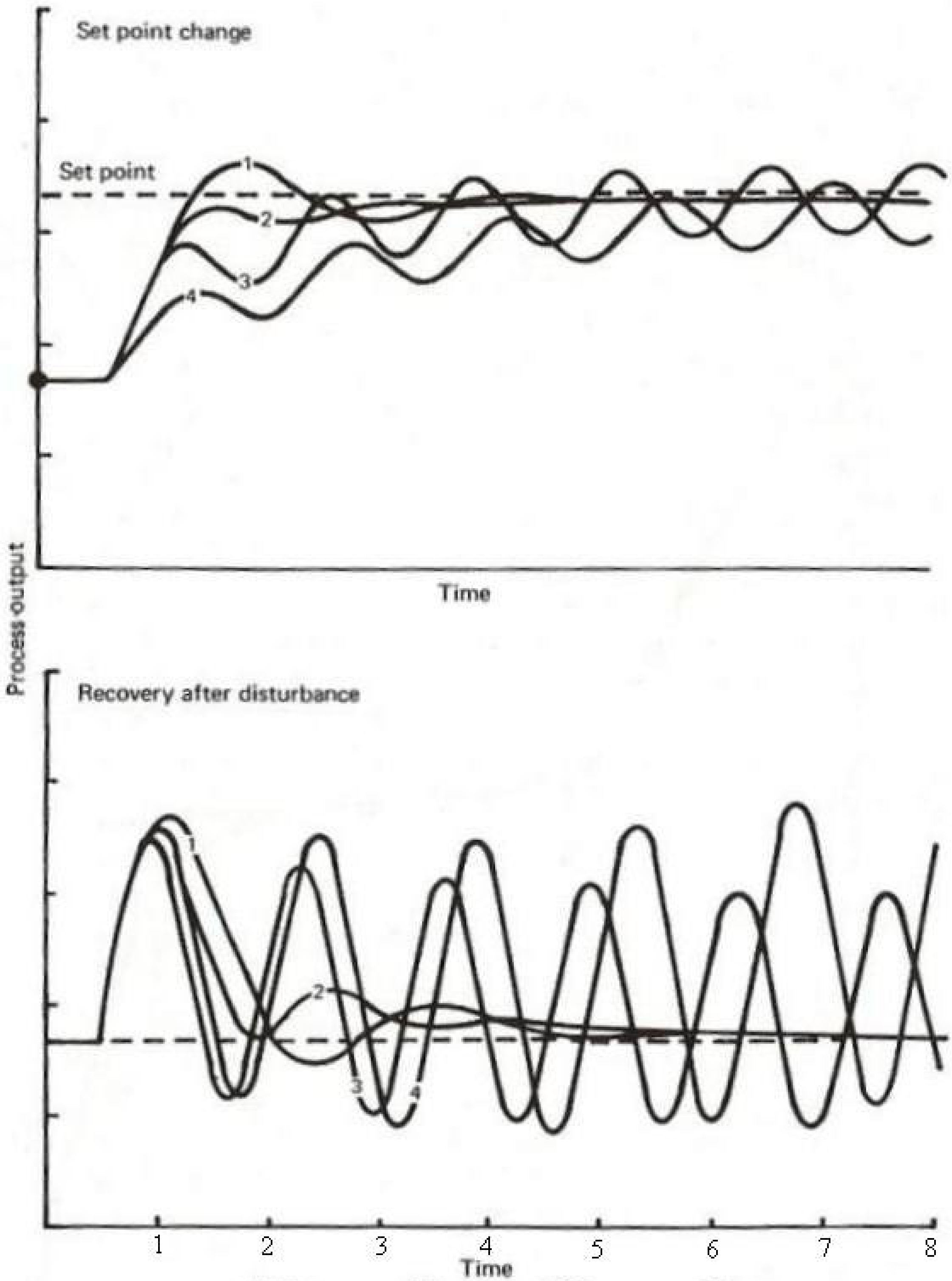


Curve	*PB*	1/*Ti*
1	50	0.02
2	50	0.05
3	50	0.10

Figure 2.7: Effect of Integral Mode on Response

c. Derivative (Rate) Mode

The derivative (rate) mode is used in only a few loops because the minimum setting available is too large for many loops, derivative action amplifies noise, and the tuning is more complicated as a result of interaction with the other modes in most industrial controllers. This mode changes the controller output by an amount proportional to the derivative of the error. It gives an anticipatory type of response that is useful for slow processes. The derivative time is the time required for the proportional mode contribution to equal the derivative (rate) mode contribution for a ramp error. The derivative action leads the proportional action by this derivative time. If there is no noise, the use of derivative action decreases the permissible proportional band. Figure 2.8 shows that the overshoot for set point changes and the peak time error for load disturbances are reduced but the response becomes more oscillatory. If the response turns back as the measurement approaches set point before it crosses set point, the derivative time is longer than normal. If the derivative time is increased too much, the oscillations develop into a rate cycle whose period is shorter than the ultimate period [19, 20].



Curve	PB	1/Ti	Td
1	35	0.05	2
2	35	0.05	5
3	35	0.05	10
4	60	0.05	20

Figure 2.8: Effect of Derivative Mode on Response

II.2.6. FIRST ORDER SYSTEMS

The simplified first-order system input-output relationship is given by

$$\frac{C(s)}{R(s)} = \frac{1}{Ts+1} \tag{2.10}$$

In the following, we shall analyze the system for the unit step response. The initial conditions are assumed to be zero.

Note that all systems having the same transfer function will exhibit the same output in response to the same input. For any given physical system, the mathematical response can be given a physical interpretation.

a. Unit Step Response of First Order Systems

Unit step response of system in Equation (2.10) for zero initial condition is

$$C(t) = 1 - e^{-t/T}, \text{ for } t \geq 0 \tag{2.11}$$

Equation (2.11) states that zero initial condition and finally it becomes unity. One important characteristic of such an exponential response curve C (t) is that at t=T the value of C(t) is 0.632, or the response C(t) has reached 63,2% of its total change.

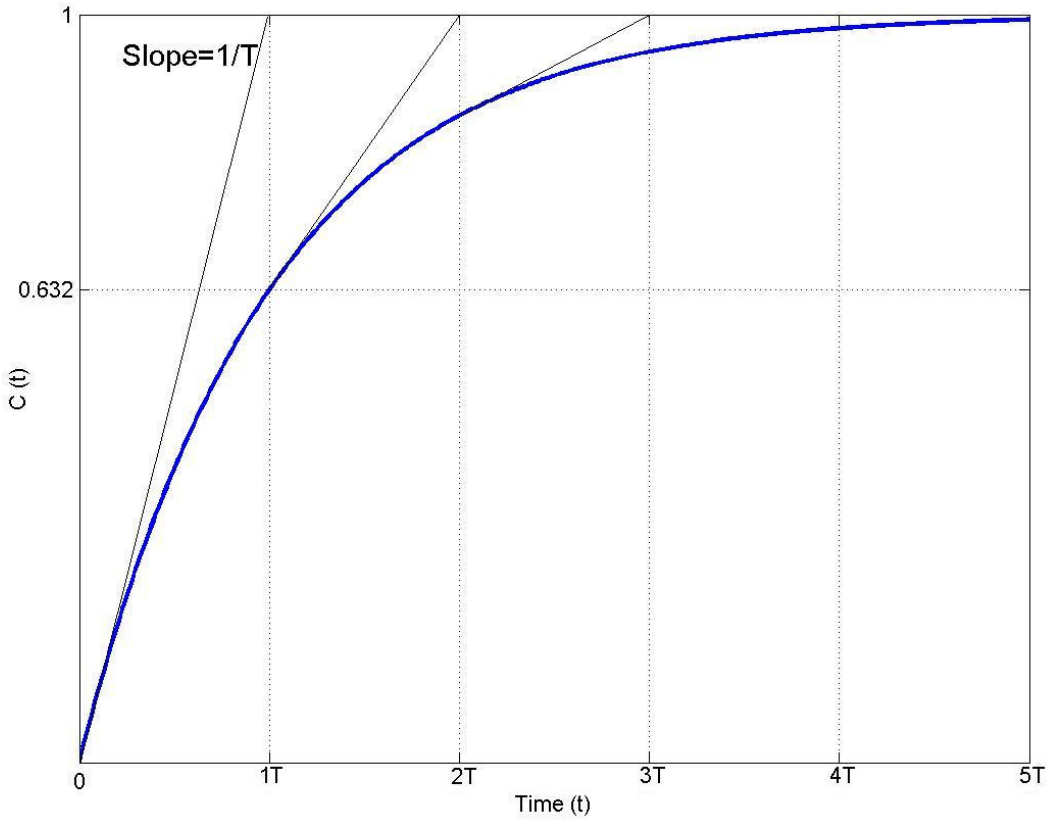


Figure 2.9: Exponential Response Curve

Note that the smaller the time constant T, the faster the system response. Another important characteristic of the exponential response curve is that the slope of the tangent line at t=0 is 1/T, since;

$$\left.\frac{dc}{dt}\right|_{t=0} = \left.\frac{1}{T}e^{-t/T}\right|_{t=0} = \frac{1}{T} \tag{2.12}$$

The output would reach the final value while t goes to infinity if it maintained its initial speed of response.

The exponential response curve C (t) given by Equation (2.11) is shown in Figure (2.9). In one time constant, the exponential response curve has gone from 0 to 63.2% of the final value. In two time constants, the response reaches 86.5% of the final value. T t=3T, 4T and 5T, the response reaches 95%, 98.2% and 99.3% respectively, of the final value. Thus, for t ≥ 4T, the response remains within 2% of the final value. As seen from Equation (2.11), the steady state is reached mathematically only after an infinite time. In practice, however, a reasonable estimate of the response time is the length of the time the response curve needs to reach and stay within the 2% line of the final value, or four time constants [1].

II.2.7. SECOND ORDER SYSTEMS

The standard form of the second order system is

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \tag{2.13}$$

Complex poles of this system can be defined in term of their real and imaginary parts, so we can refer to them as

$S_{poles} = -\sigma \pm j\ \omega_d$

The dynamic behavior of the second order system can than be described in terms of two parameters "ξ" and "w_n". If 0<ξ<1, the closed-loop poles are complex conjugates and lie in the left half s plane. The system is then called "under damped", and the transient response is oscillatory. If ξ=0, the transient response does not die out. If ξ=1, the system is called "critically damped". "Over damped" systems correspond to ξ>1.

Since solving the characteristic equation will give us the pole locations, we can equate the parameters between these equations.

$$\sigma = \zeta \cdot \omega_n \tag{2.14}$$

$$\omega_d = \omega_n * \sqrt{1-\zeta^2} \tag{2.15}$$

Where ζ is damping ratio, ω_n is undamped natural frequency and ω_d is damped natural frequency

a. Interpretations of Parameters

The damping ratio indicates how quickly the system will converge to its steady state value (high damping → fast convergence, but low damping → longer time for oscillations). The undamped natural frequency tells us the frequency of oscillations that would exists if the system had no damping.

In the presence of damping, the system response oscillates at the damped natural frequency. The location of the poles in the s-plane explicitly gives us this information, since the pole locations are defined as having a radius "ω_n" and an angle $\acute{\alpha}= \cos^{-1}\zeta$, as shown in Figure (2.10).

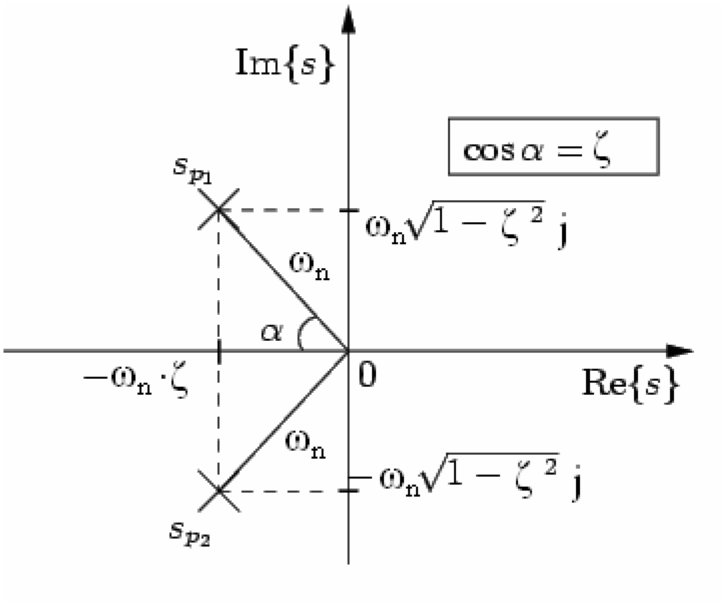


Figure 2.10: Positions of poles of second order systems

b. Time Response for Different Parameter Values

We can rewrite the second order transfer function Equation (2.13) in the form of

$$H(s) = \frac{\omega_n^2}{(s+\zeta*\omega_n)^2+\omega_n^2(1-\zeta^2)} \quad (2.16)$$

Then, after using the Laplace transform tables, we can convert this into the time domain, to give the impulse response

$$Y_{impulse}(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{\sigma t} * \sin(\omega_d t)1(t) \quad (2.17)$$

We could similarly find the step response as

$$Y_{step}(t) = 1(t) - e^{-\sigma t}\left[\cos(\omega_d t) + \frac{\sigma}{\omega_d} * \sin(\omega_d t)\right]1(t)$$ where 1(t) is the unit step function. (2.18)

c. Qualitative Effects of Damping Ratio on System Response

Looking at the responses we can observe the following effects of increasing the damping ratio

- Reduction in the maximum overshoot of the final value
- Shorter settling time
- Possibly longer rise time
- Longer time to peak of signal
- Slight decrease in actual (damped) frequency

With second order systems there is always a tradeoff between adjusting system parameters, e.g. you can't reduce both the overshoot and the rise time of a step response simultaneously. Sometimes it is preferable to have a large overshoot to obtain small settling times while other times we don't want any overshoot at all.

d. Quantitative Effects of System Parameters

First let's consider the maximum overshoot and peak time. The step response of the system is given by

$y(t) = 1 - e^{\zeta w_n t}(\cos \omega_{d.}t) + \frac{\zeta}{\sqrt{1-\zeta^2}} . \sin(\omega_{d.}t))$, for t>0

Peak time is obtained as

$t_p = \pi / \omega_d$ (2.19)

where $\omega_d = \sqrt{1-\zeta^2}$

Now that we have the peak time, we can easily calculate the percentage overshoot with

$Y(t_p) = 1 + M_p = 1 - e^{\frac{-\pi\sigma}{w_d}} (\cos(\pi) + \frac{\sigma}{\omega_d} . \sin(\pi)) = 1 + e^{\frac{-\pi\sigma}{w_d}}$

Hence the *maximum overshoot* is given by;

$M_p = e^{\frac{-\pi\sigma}{w_d}} = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$ (2.20)

This equation can be used to find the expected maximum overshoot for a given damping ratio, or inverted in a design problem to give the required damping ratio to provide a given overshoot specification.

We can quantitatively estimate the settling time by noting that this will be satisfied when the decaying exponential part of the response reaches 1%

Step response is $y(t) = 1 - e^{-\sigma t} (\cos(\omega_{d.}t) + \frac{\sigma}{\omega_d} . \sin(\omega_{d.}t))$ (2.21)

So settling time must satisfy $e^{-\zeta . wn . ts} = 0.01$

Hence the settling time is found explicitly by taking the natural logarithm of both sides,

$t_s = \frac{4}{\omega_n \zeta} = \frac{4}{\sigma}$ (2.22)

The rise time of each of the responses is a very complex relationship. Simplify – lets say it is about the same for different damping ratios.

However if the normalization of time w.r.t. natural frequency is considered, it is clear that a higher natural frequency will result in a shorter rise time.

If we consider the damping ratio = 0.5 to be indicative of a typical second order response, we get the following general rule of thumb for the *rise time*

$$t_r = \frac{1.8}{\omega_n} \qquad (2.23)$$

We have seen earlier that the natural (undamped) and actual (damped) frequencies are related by the equation,

$$\omega_d = \omega_n . \sqrt{1-\zeta^2} \qquad (2.24)$$

Hence we would expect the actual frequency observed to decrease with an increase in damping ratio.

II.2.8. CHARACTERISTICS OF THE P, I AND D CONTROLS

Let's examine the P, I and D controls with the following tutorial. This tutorial will show us the characteristics of the each of proportional (P), the integral (I), and the derivative (D) controls, and how to use them to obtain a desired response.

Proportional Control: Proportional control is a pure gain adjustment acting on the error signal to provide the driving input to the process. It is used to adjust the speed of the system.

Integral Control: Integral control is implemented through the introduction of an integrator. Integral control is used to provide the required accuracy for the control system.

Derivative Control: Derivative action is normally introduced to increase the damping in the system. The derivative term also amplifies the existing noise which can cause problems including instability.

In this tutorial, we will consider the following unity feedback system:

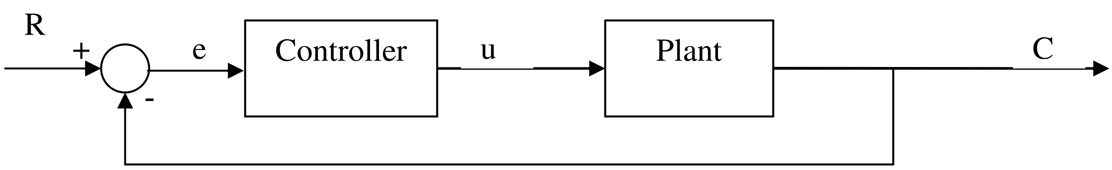


Figure 2.11: Closed-Loop System

Plant: A system to be controlled

Controller: Provides the excitation for the plant; Designed to control the overall system behavior

The transfer function of the PID controller is;

$$K_p + \frac{K_i}{s} + K_d s = \frac{K_d s^2 + K_p s + K_i}{s} \tag{2.25}$$

where K_P is proportional gain, K_i is integral gain, K_d is derivative gain.

First, let's take a look at how the PID controller works in a closed-loop system using the schematic shown above. The variable (e) represents the tracking error, the difference between the desired input value (R) and the actual output (Y). This error signal (e) will be sent to the PID controller, and the controller computes both the derivative and the integral of this error signal. The signal (u) just past the controller is now equal to the proportional gain (K_p) times the magnitude of the error plus the integral gain (K_i) times the integral of the error plus the derivative gain (K_d) times the derivative of the error.

$$u = K_p e + K_i \int e dt + K_d \frac{de}{dt} \tag{2.26}$$

This signal (u) will be sent to the plant, and the new output (Y) will be obtained. This new output (Y) will be sent back to the sensor again to find the new error signal (e). The controller takes this new error signal and computes its derivative and its integral again. This process goes on and on.

A proportional controller (Kp) will have the effect of reducing the rise time and will reduce, but never eliminate, the steady-state error. An integral control (Ki) will have the effect of eliminating the steady-state error, but it may make the transient response worse. A derivative control (Kd) will have the effect of increasing the stability of the system, reducing the overshoot, and improving the transient response. Effects of each of controllers K_p, K_d, and K_i on a closed-loop system are summarized in the table shown below.

Table 2.1: Effects of Kp, Kd and Ki on a closed-loop system

CL RESPONSE	RISE TIME	OVERSHOOT	SETTLING TIME	S-S ERROR
Kp	Decrease	Increase	Small Change	Decrease
Ki	Decrease	Increase	Increase	Eliminate
Kd	Small Change	Decrease	Decrease	Small Change

Note that these correlations may not be exactly accurate, because K_p, K_i, and K_d are dependent of each other. In fact, changing one of these variables can change the effect of the other two. For this reason, the table should only be used as a reference when we are determining the values for K_p, K_i, and K_d. During this thesis, this relationship will be determined by using Axiomatic Design.

II.2.9. GENERAL TIPS FOR DESIGNING A PID CONTROLLER

When we are designing a PID controller for a given system, the following steps help us to obtain a desired response.

Obtaining an open-loop response and determining what needs to be improved

Adding a proportional control to improve the rise time

Adding a derivative control to improve the overshoot

Adding an integral control to eliminate the steady-state error

Adjusting each of Kp, Ki, and Kd until we obtain a desired overall response.

Finally, it is important to keep the controller as simple as possible. For example, if a PI controller gives a good enough response, then we don't need to implement derivative controller to the system.

II.2.10. ZIEGLER-NICHOLS TUNING METHOD

Ziegler and Nichols proposed rules for determining values of the proportional gain Kp, integral constant Ki, derivative constant Kd, based on the transient response characteristics of a given plant. Such determination of the parameters of PID controllers or tuning of PID controllers can be made by engineers on-site by experiments on the plant. (Numerous tuning rules for PID controllers have been proposed since the Ziegler-Nichols proposal. They are available in the literature and from the manufacturers of such controllers.)

First Method: In the first method, we obtain experimentally the response of the plant to a unit-step input. If the plant involves neither integrator(s) nor dominant complex conjugate poles, then such a unit-step response curve may look S-shaped. This method applies if the response to a step input exhibits an S-shaped curve. Such step response curves may be generated experimentally or from a dynamic simulation of the plant.

Table 2.2: Ziegler-Nichols Tuning Rules Based on Step Response of Plant (First Method)

Type of Controller	Kp	Ki	Kd
P	T/L	0	0
PI	0.9T/L	0.3/L	0
PID	1.2T/L	1/2L	0.5L

The S-shaped curve may be characterized by two constants, delay time L and time constant T. The delay time and time constant are determined by drawing a tangent line at the inflection point of the S-shaped curve and determining the intersections of the tangent line with the time axis and line C(t)=K. The transfer function C(s)/U(s) may then be approximated by a first order system with a transport lag as follows:

$$\frac{C(s)}{U(s)} = \frac{Ke^{-Ls}}{Ts+1} \tag{2.27}$$

Ziegler and Nichols suggested setting the values of Kp, Ki and Kd according to the formula shown in Table 2.2.

Notice that the PID controller has a pole at the origin and double zeros as s=-1/L, [1].

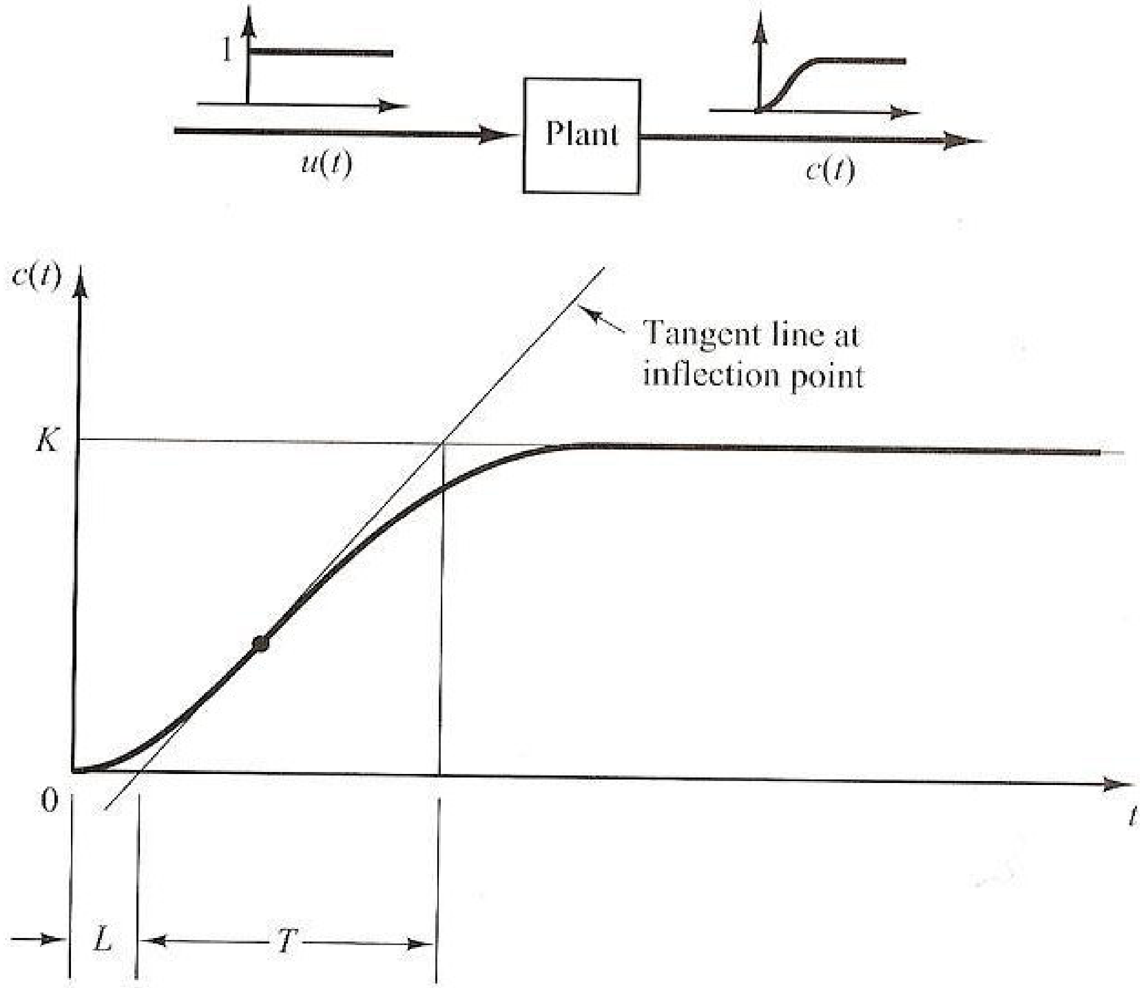


Figure 2.12: S-Shaped Response Curve

Second Method: In the second method, we first set Ki = 0 and $K_d = 0$. Using the proportional control action only, increase K_p from 0 to a critical value K_{cr} at which the output first exhibits sustained oscillations. (If the output does not exhibit sustained oscillations for whatever value K_p may take, then this method does not apply.) Thus, the critical gain K_{cr} and the corresponding period P_{cr} are experimentally determined. Ziegler and Nichols suggested that we set the values of the parameters K_p, K_i and K_d according to the formula shown in Table 2.2.

Table 2.3: Ziegler-Nichols Tuning Rules Based on Critical Gain K_{cr} and Critical Period P_{cr} (Second Method)

Type of Controller	Kp	Ki	Kd
P	0.5Kcr	0	0
PI	0.45Kcr	1.2Pcr	0
PID	0.6Kcr	2Pcr	0.125Pcr

Notice that the PID controller tuned by the second method of Ziegler-Nichols rules gives

$$G_c(s) = K_p(1+\frac{K_i}{s}+K_d s)$$
$$= 0.6K_{cr}(1+\frac{1}{0.5P_{cr}s}+0.125P_{cr}s)$$
$$= 0.075K_{cr}P_{cr}\frac{(s+\frac{4}{P_{cr}})^2}{s}$$

Thus, the PID controller has a pole at the origin and double zeros at s=-4/P_{cr}.
Note that if the system has a known mathematical model (such as the transfer function), then we can use the root-locus method to find the critical gain K_{cr} and the frequency of the sustained oscillation w_{cr}, where $2\pi/w_{cr} = P_{cr}$. These values can be found from the crossing points of the root locus branches with the jw axis, [1]. (Obviously, if the root-locus branches do not cross the jw axis, this method does not apply.)

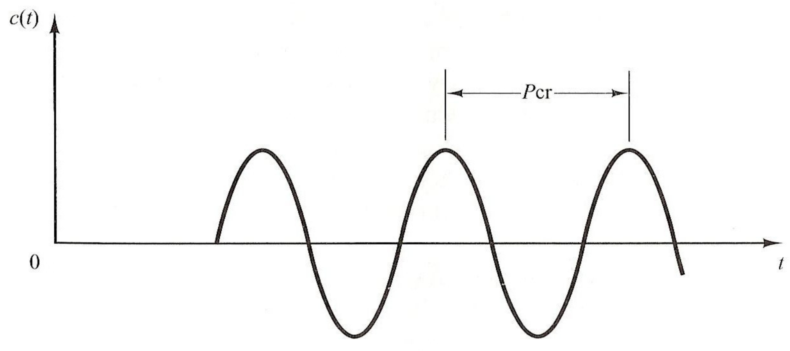


Figure 2.13: Sustained Oscillation with Period P_{cr}

PART III
INVESTIGATION OF CLASSICAL CONTROLLER USING AXIOMATIC DESIGN

In this part, axiomatic design approach will be applied to the first order systems and their responses will be investigated in the axiomatic design discipline.

III.1 INVESTIGATION OF PROPORTIONAL CONTROLLER ACTION ON FIRST ORDER SYSTEM

During this section, axiomatic design approach will be applied to the first order proportional controller system. In the following, a classical first order system as it is shown Figure (3.1) will be investigated and the results will be declared.

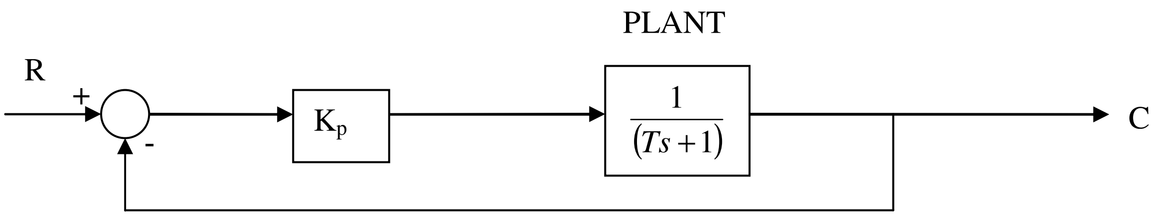


FIGURE 3.1: Closed-Loop First Order Proportional Controller System

For the ideal proportional controller system, below equations will be used during the investigation of the proportional controller design with the approach of the axiomatic design;

$DP_1 = K_p$ FR_1 = Steady State Error (e_{ss})

$DP_2 = T$ FR_2 = Settling Time (T_s)

The design equation that we are looking for should be of the form:

$$\begin{pmatrix} FR_1 \\ FR_2 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} DP_1 \\ DP_2 \end{pmatrix} \tag{3.1}$$

For the Equation (3.1), matrix parameters can be explained as below:

$$A_{11} = \frac{\partial FR_1}{\partial DP_1} = \frac{\partial e_{ss}}{\partial K_p} \tag{3.2a}$$

$$A_{12} = \frac{\partial FR_1}{\partial DP_2} = \frac{\partial e_{ss}}{\partial T} \tag{3.2b}$$

$$A_{21} = \frac{\partial FR_2}{\partial DP_1} = \frac{\partial T_s}{\partial K_p} \tag{3.2c}$$

$$A_{22} = \frac{\partial FR_2}{\partial DP_2} = \frac{\partial T_s}{\partial T} \tag{3.2d}$$

Since the system is first order, settling time:

$$T_s = \frac{4T}{\left(1 + K_p\right)} \tag{3.3}$$

We can write the input-output relationship for the Figure (3.1):

$$\frac{C(s)}{R(s)} = \frac{\dfrac{K_p}{(Ts+1)}}{1 + \dfrac{K_p}{Ts+1}} = \frac{K_p}{Ts + 1 + K_p} \tag{3.4}$$

Therefore E(s) / R(s) can be calculated as below:

$$\frac{E(s)}{R(s)} = 1 - \frac{C(s)}{R(s)} = \frac{Ts+1}{Ts + 1 + K_p} \tag{3.5}$$

By using Equation (3.5), steady state error for the unit step input:

$$e_{ss} = \lim_{s \to 0} s.E(s) = \lim_{s \to 0} s.\frac{(Ts+1)}{(Ts + K_p + 1)}.\frac{1}{s} = \frac{1}{(1 + K_p)} \tag{3.6}$$

Finally, by using Equations (3.2), (3.3) and (3.6), our design matrix Equation (3.1) becomes;

$$\begin{pmatrix} \Delta e_{ss} \\ \Delta T_s \end{pmatrix} = \begin{pmatrix} -1/(1+K_p)^2 & 0 \\ -4T/(1+K_p)^2 & 4/(1+K_p) \end{pmatrix} \begin{pmatrix} \Delta K_p \\ \Delta T \end{pmatrix} \tag{3.7}$$

So the functional requirements and the design parameters relationship can be shown as following:

$$\Delta FR_1 = f(K_p) \,.\, \Delta K_p \tag{3.8a}$$

$$\Delta FR_2 = f(K_p, \tau) \,.\, \Delta K_p + f(K_p) \,.\, \Delta\tau \tag{3.8b}$$

Hence Equation (3.7) can be drawn as below and shows the steady state error and the settling time behavior:

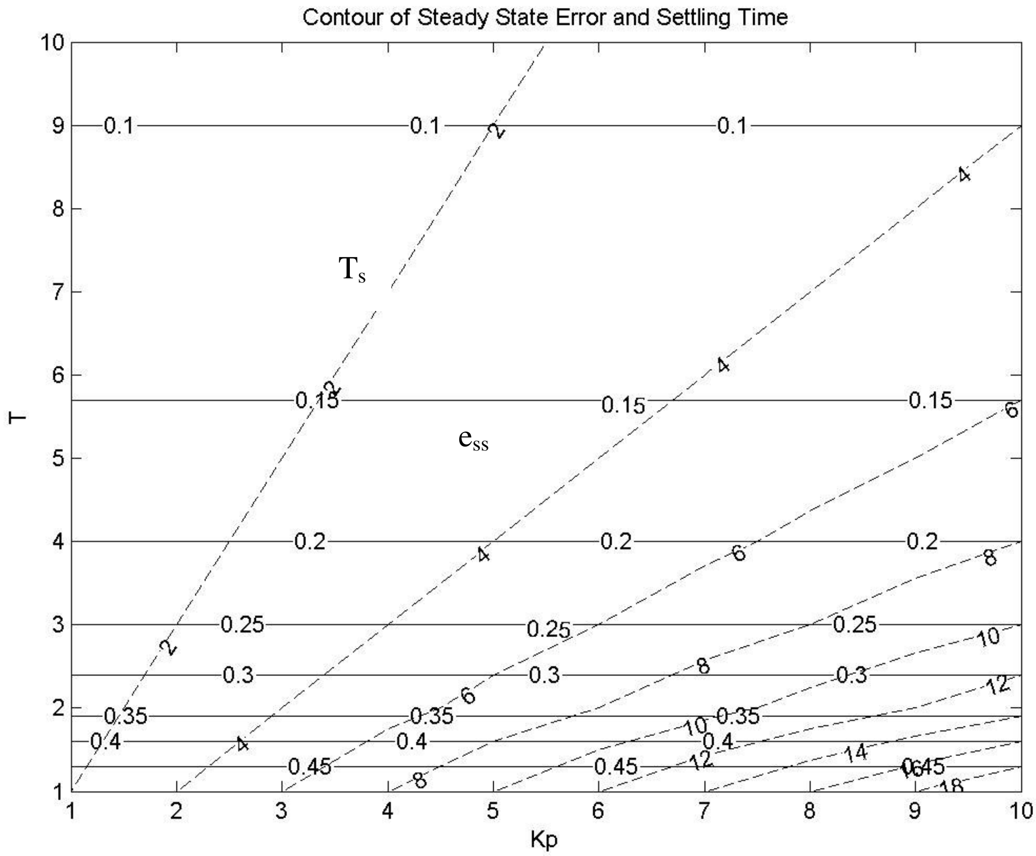


FIGURE 3.2: Graph of Steady State Error and Settling Time

As it is shown in the Figure (3.2), steady state error and the settling time are decoupled. As it was shown in the previous section, since design matrix is 2x2, for decoupled system, Reangularity and Semangularity always equal to each other. Figure (3.2) shows that the steady state error depends only on the change in the proportionality gain "K_p". It means that steady state error can be tuned by changing the "K_p". The change in the settling time caused by the change in "K_p" can be eliminated by changing the time constant "T". However, as it shown at the Figure (3.2), settling time is dependent both the "K_p" and the "T". But as it is seen, for the bigger value of "K_p" and the smaller value of "T", the effect of change in T can be neglected for the settling time.

III.2. INVESTIGATION OF INTEGRAL CONTROLLER ACTION ON FIRST ORDER SYSTEM

In this application, axiomatic design approach will be applied to the first order integral controller system. Figure (3.3) shows a classical first order system with the integral controller.

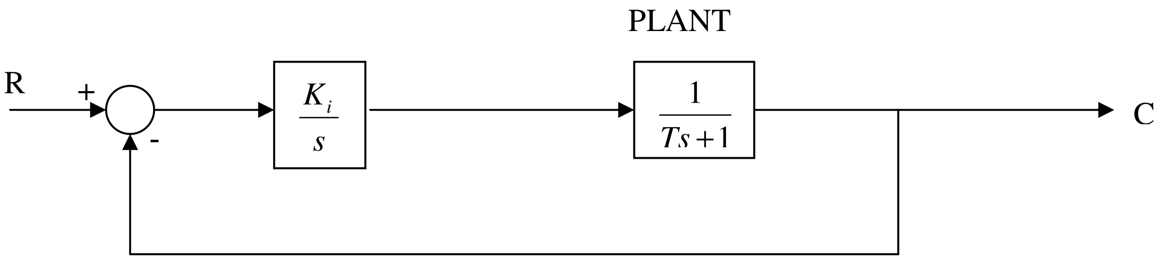


FIGURE 3.3: Closed-Loop First Order Integral Controller System

For an ideal integral controller system, below equations will be used during the investigation of the integral controller design with the approach of axiomatic design.

$DP_1 = K_i$ FR_1 = Steady State Error (e_{ss})

$DP_2 = T$ FR_2 = Settling Time (T_s)

We can write the input-output relationship for the Figure (3.5) as below:

$$\frac{C(s)}{R(s)} = \frac{\frac{K_i}{s(Ts+1)}}{1+\frac{K_i}{s(Ts+1)}} = \frac{\frac{K_i}{T}}{s^2+\frac{1}{T}s+\frac{K_i}{T}} \tag{3.9}$$

The steady state error for a unit step input can be calculated as below:

$$e_{ss} = \lim_{s\to 0} s.E(s) = \lim_{s\to 0} s.\frac{s(Ts+1)}{\left(s(Ts+1)+K_i\right)}.\frac{1}{s} = 0 \tag{3.10}$$

Since it is an integral controller there isn't any steady state error for this system.

The system given in the Equation (3.9) is a classical second order system as it was given in the Equation (2.13). So below transformations give us some information about the system behaviour.

$$\omega_n = \sqrt{\frac{K_i}{T}} \tag{3.11}$$

$$2\zeta\omega_n = \frac{1}{T} \tag{3.12}$$

If we investigate Equations (3.11) and (3.12), settling time can be calculated as it is stated in the previous section:

$$T_s = \frac{4}{\xi\omega_n} \cong 8T \tag{3.13}$$

As it is shown in the Equation (3.13), Settling time (T_s) is directly proportional with time constant (T) of the plant. Since the plant is constant, there is nothing we can do to design the system by changing settling time. But if we change the controller as it is in the next application, we can see in which condition the system can be designed. Therefore in the next application, proportional controller has been extended with the integral controller.

III.3. FIRST ORDER “PI CONTROLLER” APPLICATION

The following investigation will show us the first order proportional-integral controller system behavior with the approach of the axiomatic design. Figure (3.4) shows a classical first order PI controller system.

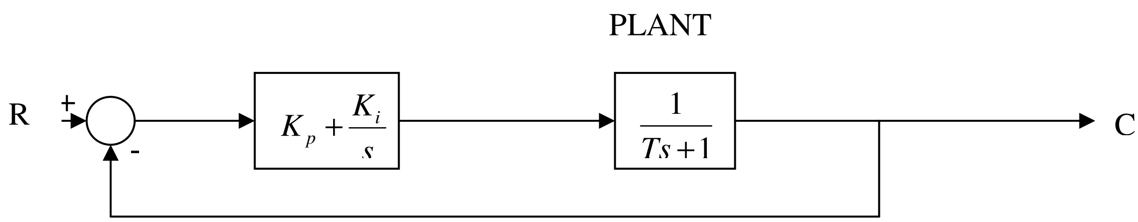


FIGURE 3.4: Closed-Loop First Order, PI Controller System

For an ideal PI controller system, below equations will be used during the investigation of the PI controller design with the approach of the axiomatic design.

$DP_1 = K_i$ FR_1 = Settling Time

$DP_2 = Kp$ FR_2 = Maximum Percent Overshoot (Mp)

We can write the input-output relationship for the Figure (3.4) as below:

$$\frac{C(s)}{R(s)} = \frac{\frac{K_p s + K_i}{s(Ts+1)}}{1+\frac{K_p s + K_i}{s(Ts+1)}} = \frac{\frac{K_p s + K_i}{T}}{s^2 + \frac{(1+K_p)}{T}s + \frac{K_i}{T}} \tag{3.14}$$

The steady state error for a unit step input can be calculated as below:

$$e_{ss} = \lim_{s\to 0} s.E(s) = \lim_{s\to 0} s.\frac{s(Ts+1)}{\left(s(Ts+1)+K_p s + K_i\right)}.\frac{1}{s} = 0 \tag{3.15}$$

Since it has an integral controller in the system, there is no steady state error. The system given in the Equation (3.14) is a classical second order system as it was given in the Equation (2.13). Hence below transformations give us some information about the system behaviour.

$$\omega_n = \sqrt{\frac{K_i}{T}} \tag{3.16}$$

$$\zeta = \frac{1+K_p}{2\omega_n T} \tag{3.17}$$

If we investigate Equation (3.16) and (3.17), settling time (T_s) and the maximum percent overshoot (M_p) can be found as below:

$$T_s = \frac{8T}{1+K_p} \tag{3.18}$$

$$M_p = e^{\frac{-\pi\sigma}{w_d}} = e^{\frac{-\pi(1+K_p)}{\sqrt{(4K_iT)-(1+K_p)^2}}} \tag{3.19}$$

Hence our design matrix becomes:

$$\begin{pmatrix} \Delta T_s \\ \Delta M \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \text{x} \begin{pmatrix} \Delta K_i \\ \Delta K_p \end{pmatrix} \tag{3.20}$$

$$A_{11} = \frac{\partial FR_1}{\partial DP_1} = \frac{\partial T_s}{\partial K_i} = 0$$

$$A_{12} = \frac{\partial FR_1}{\partial DP_2} = \frac{\partial T_s}{\partial K_p} = \frac{-8T}{(1+K_p)^2}$$

$$A_{21} = \frac{\partial FR_2}{\partial DP_1} = \frac{\partial M_p}{\partial K_i} = \frac{\pi(1+K_p)}{\sqrt{2T} * (4K_iT-(1+K_p)^2)} * e^{\frac{-\pi(1+K_p)}{\sqrt{(4K_iT)-(1+K_p)^2}}}$$

$$A_{22} = \frac{\partial FR_2}{\partial DP_2} = \frac{\partial M_p}{\partial K_p} = \frac{-\pi\sqrt{(4K_iT)-(1+K_p)^2} - \pi\sqrt{1+K_p}}{(4K_iT)-(1+K_p)^2} * e^{\frac{-\pi(1+K_p)}{\sqrt{(4K_iT)-(1+K_p)^2}}}$$

$$\begin{pmatrix} \Delta T_s \\ \Delta M_p \end{pmatrix} = \begin{pmatrix} 0 & X \\ X & X \end{pmatrix} x \begin{pmatrix} \Delta K_i \\ \Delta K_p \end{pmatrix} \tag{3.21}$$

As it can be seen from Equation (3.21), the relationship between settling time and the maximum percent overshoot is decoupled. This means that settling time and the maximum percent overshoot can be adjusted separately by changing K_p and K_i variables respectively while the time constant (T) is constant.

Equation (3.21) shows that the settling time depends only on the change in proportionality gain "K_p". Change in the "K_i" doesn't affect the settling time at all. However, maximum percent overshoot depends both the K_p and K_i.

III.4. SECOND ORDER "P CONTROLLER" APPLICATION

In this section, axiomatic design approach will be applied to the second order proportional controller system. Figure (3.5) shows a classical closed loop second order system and the P controller.

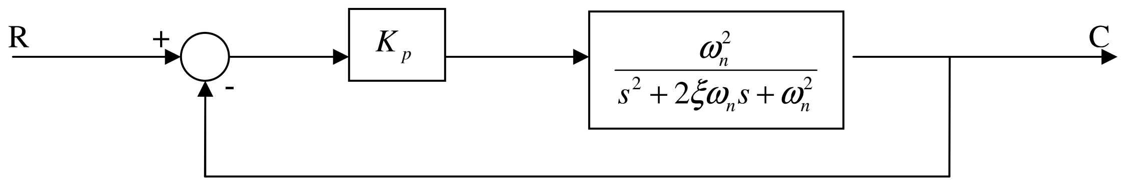


FIGURE 3.5: Closed-Loop Second Order Proportional Control System

We can write the input-output relationship for the above system as below:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{K_p\omega_n^2}{(s^2+2\xi\omega_n s+\omega_n^2+K_p\omega_n^2)} \tag{3.22}$$

For the ideal second order proportional controller system, below equations will be used during the investigation of the P controller design with the approach of the axiomatic design;

$DP_1 = Kp$ $\qquad$ FR_1 = Steady State Error (e_{ss})

$DP_2 = \omega_n$ $\qquad$ FR_2 = Settling Time (T_s)

$DP_3 = \zeta$ $\qquad$ FR_3 = Maximum Percent Overshoot (M_p)

The design equation that we are looking for should be of the form,

$$\begin{pmatrix} FR_1 \\ FR_2 \\ FR_3 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix} \begin{pmatrix} DP_1 \\ DP_2 \\ DP_3 \end{pmatrix} \tag{3.23}$$

For the Equation (3.23), matrix parameters can be explained as below:

$$A_{11} = \frac{\partial FR_1}{\partial DP_1} = \frac{\partial e_{ss}}{\partial K_p} \qquad A_{21} = \frac{\partial FR_2}{\partial DP_1} = \frac{\partial T_s}{\partial K_p} \qquad A_{31} = \frac{\partial FR_3}{\partial DP_1} = \frac{\partial M_p}{\partial K_p}$$

$$A_{12} = \frac{\partial FR_1}{\partial DP_2} = \frac{\partial e_{ss}}{\partial w_n} \qquad A_{22} = \frac{\partial FR_2}{\partial DP_2} = \frac{\partial T_s}{\partial w_n} \qquad A_{32} = \frac{\partial FR_3}{\partial DP_2} = \frac{\partial M_p}{\partial w_n}$$

$$A_{13} = \frac{\partial FR_1}{\partial DP_3} = \frac{\partial e_{ss}}{\partial \zeta} \qquad A_{23} = \frac{\partial FR_2}{\partial DP_3} = \frac{\partial T_s}{\partial \zeta} \qquad A_{33} = \frac{\partial FR_3}{\partial DP_3} = \frac{\partial M_p}{\partial \zeta}$$

When we make a comparison between the second order standard controller system design Equation (2.17) and the Equation (3.21) then the below equations can be found:

$$\omega_n^{'} = \omega_n \sqrt{(1+K_p)} \tag{3.24}$$

$$\omega_n^2 (1+K_p)\left(\zeta^{'}\right)^2 = \omega_n^2 \zeta^2 \tag{3.25}$$

$$2\zeta^{'} \omega_n^{'} = 2\zeta\omega_n \tag{3.26}$$

We obtain,

$$\zeta^{'} = \frac{\zeta}{\sqrt{1+K_p}} \tag{3.27}$$

For the new system, the settling time is;

$$T_s = \frac{4}{\omega_n^{'}\zeta^{'}} = \frac{4}{\omega_n\sqrt{1+K_p} * \frac{\zeta}{\sqrt{1+K_p}}} = \frac{4}{\omega_n\zeta} \tag{3.28}$$

As it is shown in the equation (3.28), settling time T_s is not depending on K_p. Maximum percent overshoot (M_p) with the same parameters can be written;

$$M_p = e^{\frac{-\zeta'\pi}{\sqrt{1-\zeta'^2}}} = e^{\frac{\frac{-\zeta\pi}{\sqrt{1+K_p}}}{\sqrt{1-\frac{\zeta^2}{1+K_p}}}} = e^{\frac{-\pi\zeta}{\sqrt{1+K_p-\zeta^2}}} \tag{3.29}$$

As it is shown in the equation, M_p is depending on K_p. In the second order control system E(s) / R(s) can be found

$$\frac{E(s)}{R(s)} = \frac{\left(R(s)-C(s)\right)}{R(s)} = \frac{s^2+2\zeta\omega_n s+\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2+K_p\omega_n^2} \tag{3.30}$$

Therefore by using Equation (3.30), steady state error for the unit step input can be calculated as following,

$$e_{ss} = \lim_{s\to 0} s.E(s) = \lim_{s\to 0} s.\ \frac{s^2+2\zeta\omega_n s+\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2+K_p\omega_n^2}\cdot\frac{1}{s} \tag{3.31}$$

And as a result steady state error is;

$$e_{ss} = \frac{1}{1+K_p} \tag{3.32}$$

Finally by combining Equations (3.28), (3.29) and (3.32), matrix Equation (3.23) becomes:

$$\begin{bmatrix} \Delta e_{ss} \\ \Delta T_s \\ \Delta M_p \end{bmatrix} = \begin{pmatrix} -1/(1+K_p)^2 & 0 & 0 \\ 0 & -4/\omega_n^2.\zeta & -4/\omega_n.\zeta^2 \\ \dfrac{\pi\zeta}{2(1+K_p-\zeta^2)^{3/2}} & 0 & e^{\frac{-\pi\zeta}{\sqrt{1+K_p-\zeta^2}}}\ \dfrac{-\pi(1+K_p)}{(1+K_p-\zeta^2)^{3/2}} \end{pmatrix} \mathrm{X} \begin{bmatrix} \Delta K_p \\ \Delta\omega_n \\ \Delta\zeta \end{bmatrix}$$

FIGURE 3.6: Second Order Proportional Controller Design Matrix

As it is shown above matrix equation, the functional requirements e_{ss}, T_s and M_p are decoupled in general. It seems that changing the steady state error is possible by changing K_p. If we change K_p, the parameters A_{11} and A_{31} automatically change. We need to change A_{11} to adjust steady state error but change in A_{31} is undesired since it changes maximum percent overshoot at the same time. Therefore to keep maximum percent overshoot (M_p) constant, the other M_p parameter A_{33} can be adjusted by changing "ζ" to eliminate the effect of change in A_{31}. Although keeping Maximum Percent Overshoot constant by changing "ζ", due to change in "ζ" A_{23} will be affected. Since A_{23} is changed this affects settling time. To keep settling time constant, only we can change A_{22} but if determine A_{22} and A_{23} carefully, we see that both parameters have the same constants "ω_n" and "ζ". Therefore we can not change A_{22} and A_{23} separately. This make impossible to keep settling time constant while adjusting steady state error by changing K_p.

However, there are certain 2x2 matrices in our 3x3 matrix where these FRs become decoupled or uncoupled. For this reason we will investigate each 2x2 matrices separately. First, we can investigate FR1 and FR2 which is shown also in the Figure (3.6) with the blue dashed lines.

$$\begin{bmatrix} \Delta e_{ss} \\ \\ \Delta T_s \end{bmatrix} = \begin{pmatrix} -1 / (1+K_p)^2 & 0 \\ \\ 0 & -4 / \omega_n^2.\zeta \end{pmatrix} \text{x} \begin{bmatrix} \Delta K_p \\ \\ \Delta \omega_n \end{bmatrix} \tag{3.33}$$

So the functional requirements and the design parameters relationship can be shown:

$$\Delta FR_1 = f\ (K_p)\ .\ \Delta K_p \tag{3.34a}$$

$$\Delta FR_2 = f\ (w_n, \zeta)\ .\ \Delta \omega_n \tag{3.34b}$$

By using the Equation (3.33), we can draw the steady state error and the settling time graph with the assumption of the ζ=0.5 as below:

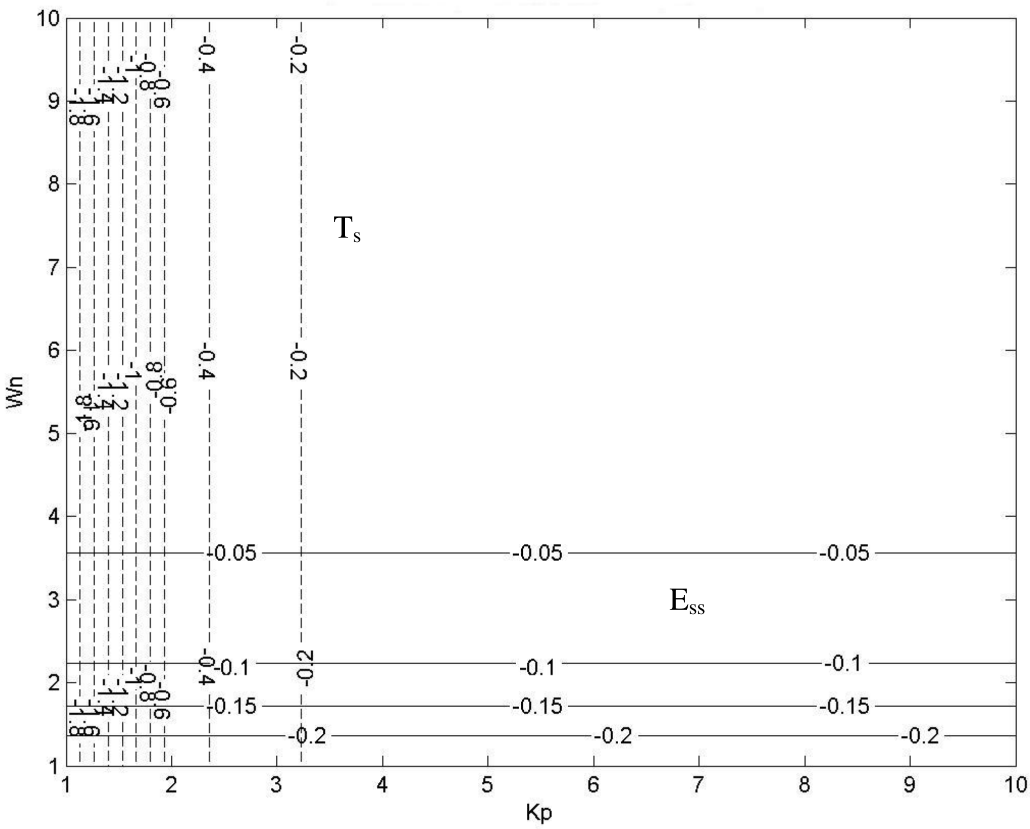


FIGURE 3.7: Contour Graph of Ess and Ts (ζ=0.5)

As it is shown in Figure (3.7), the relationship between steady state error and the settling time is uncoupled. This means that steady state error and the settling time can be adjusted separately by changing K_p and ω_n variables respectively while the damping ratio (ζ) is constant.

Secondly we can investigate FR2 and FR3 by using DP1 and DP2 which is shown also in the Figure (3.6) with the red dashed lines.

$$\begin{Bmatrix} \Delta T_s \\ \Delta M_p \end{Bmatrix} \begin{pmatrix} 0 & -4 / \omega_n^2 . \zeta \\ \dfrac{\pi\zeta}{2(1+K_p-\zeta^2)^{3/2}} & 0 \end{pmatrix} \begin{Bmatrix} \Delta K_p \\ \Delta \omega_n \end{Bmatrix} \qquad (3.35)$$

So the functional requirements and the design parameters relationship can be shown:

$\Delta FR_1 = f(\omega_n, \zeta) . \Delta \omega_n$ (3.36a)

$\Delta FR_2 = f(K_p, \zeta) . \Delta\omega_n$ (3.36b)

By using the Equation (3.33), we can draw the maximum percent overshoot and the settling time graph with the assumption of the ζ=0.5 as below:

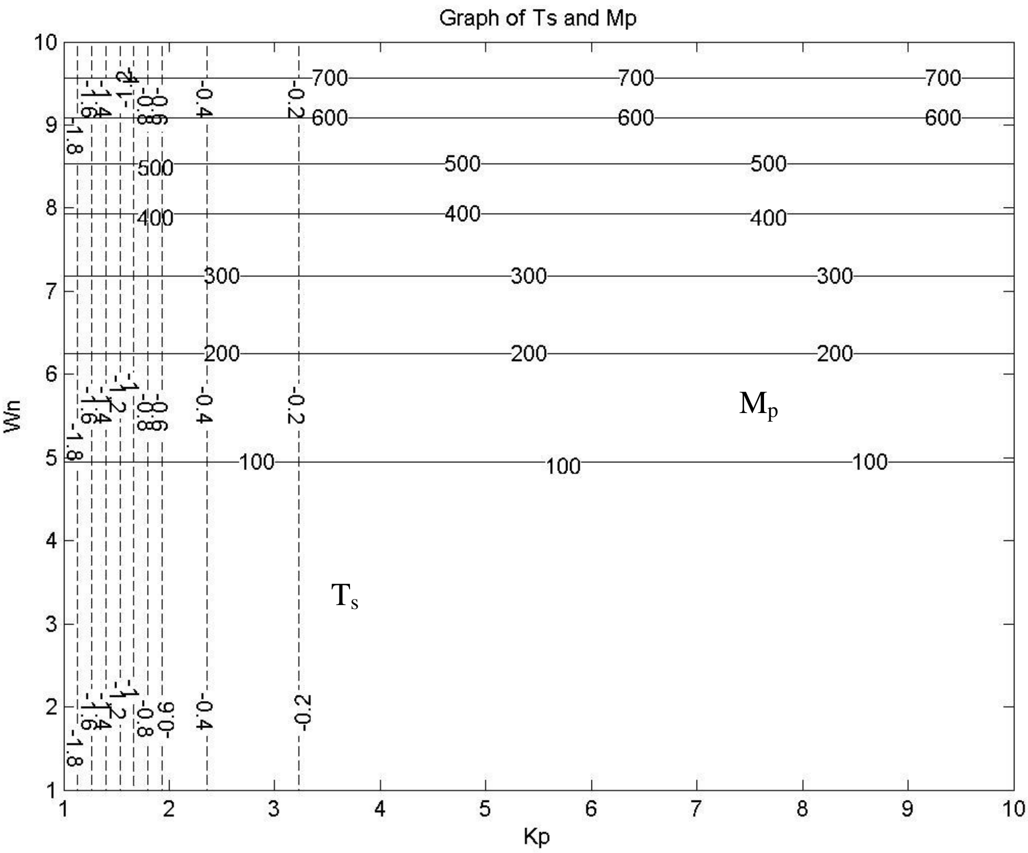


FIGURE 3.8: Graph of M_p and T_s (ζ=0.5)

As it is shown in Figure (3.8), the relationship between maximum percent overshoot and the settling time is also uncoupled. This means that maximum percent overshoot and the settling time can be adjusted separately by changing K_p and ω_n variables respectively while the damping ratio (ζ) is constant.

Thirdly we can investigate FR2 and FR3 by using DP2 and DP3 which is shown also in the Figure (3.6) with the green dashed lines.

$$\begin{pmatrix} \Delta T_s \\ \Delta M_p \end{pmatrix} \quad \begin{pmatrix} -4 / \omega_n^2.\zeta & -4 / \omega_n.\zeta^2 \\ 0 & e^{\frac{-\pi\zeta}{\sqrt{1+K_p-\zeta^2}}} \dfrac{-\pi(1+K_p)}{(1+K_p-\zeta^2)^{3/2}} \end{pmatrix} \quad \begin{pmatrix} \Delta\omega_n \\ \Delta\zeta \end{pmatrix} \qquad (3.37)$$

The functional requirements and the design parameters relationship can be shown:

$\Delta FR_1 = f(\omega_n, \zeta) . \Delta\omega_n$ (3.38a)

$\Delta FR_2 = f(K_p, \zeta) . \Delta\zeta$ (3.39b)

We can draw the maximum percent overshoot and the settling time graph by using the Equation (3.37) with the assumption of the K_p=1 as below:

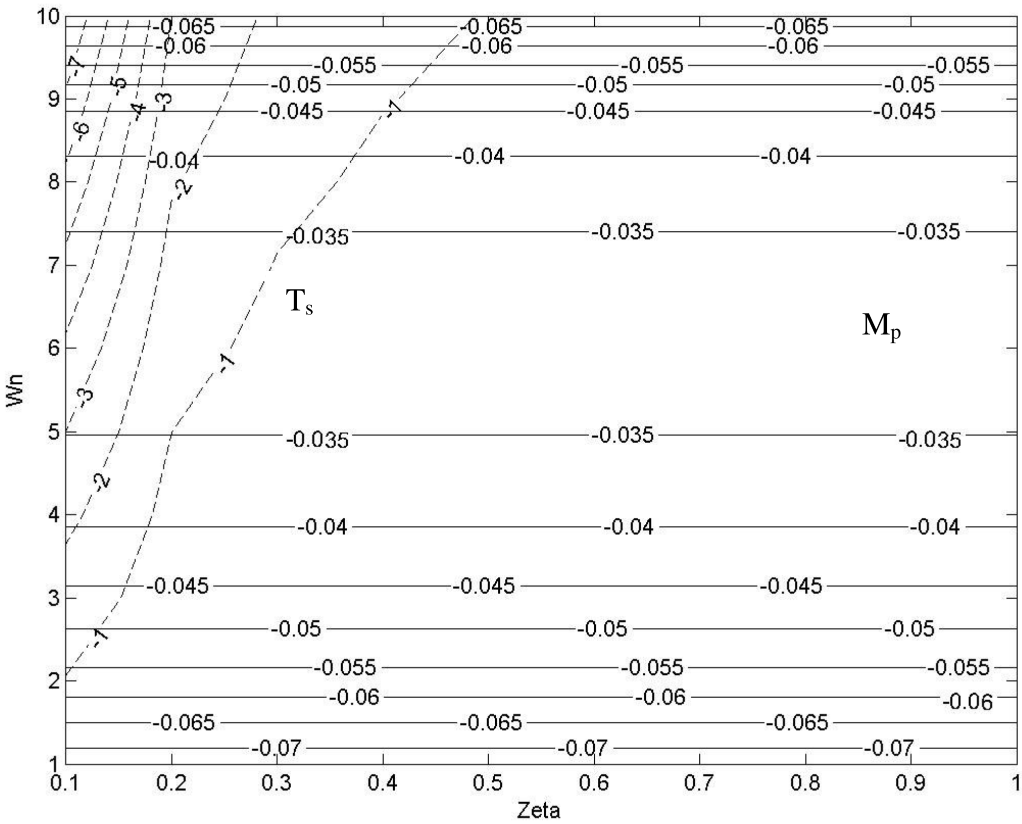


FIGURE 3.9: Graph of M_p and T_s (K_p=1)

As it is shown in Figure (3.9), the relationship between maximum percent overshoot and the settling time is decoupled while the proportional gain (K_p) is constant. This means that maximum percent overshoot can be adjusted separately by changing the damping ratio (ζ) however settling time can be adjusted by changing both the damping ratio (ζ) and undamped natural frequency (ω_n). Settling time (T_s) is dependent both of these variables while maximum percent overshoot dependent only “ζ”.

III.5. SECOND ORDER “PI CONTROLLER” APPLICATION

In the following, axiomatic design approach will be applied to the second order proportional and integral controller system. Figure (3.11) shows a classical closed loop second order system and the PI controller.

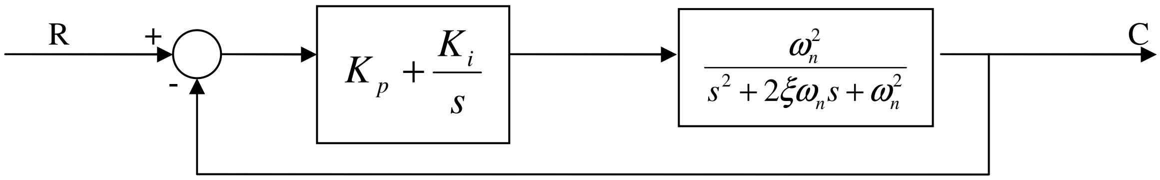


FIGURE 3.10: Closed-Loop Second Order Proportional & Integral Controller System

So the closed loop transfer function is;

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{(K_p s + K_i)\omega_n^2}{s^3 + 2\zeta\omega_n s^2 + (K_p s + s + K_i)\omega_n^2} \tag{3.40}$$

For the ideal second order PI Control System Design, below equations will be used during the investigation of the PI controller design with the approach of the axiomatic design;

DP1=Kp FR1= Settling Time (Ts)

DP2=Ki FR2= Maximum Percent Overshoot (Mp)

The design equation that we are looking for should be of the form,

$$\begin{pmatrix} FR_1 \\ FR_2 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} x \begin{pmatrix} DP_1 \\ DP_2 \end{pmatrix} \tag{3.41}$$

For the Equation (3.41), matrix parameters can be explained as below:

$$A_{11} = \frac{\partial FR_1}{\partial DP_1} = \frac{\partial T_s}{\partial K_p} \qquad A_{21} = \frac{\partial FR_2}{\partial DP_1} = \frac{\partial M_p}{\partial K_p}$$

$$A_{12} = \frac{\partial FR_1}{\partial DP_2} = \frac{\partial T_s}{\partial K_i} \qquad A_{22} = \frac{\partial FR_2}{\partial DP_2} = \frac{\partial M_p}{\partial K_i}$$

As it is shown in the Equation (3.38), our transfer function is third order. During our design, K_p and K_i will be determined between 1 and 100 interval and the damping ration (ζ) and undamped natural frequency (w_n) will be taken as below:

$\zeta=0.5$

$\omega_n=1$

So our transfer function become;

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{(K_p s + K_i)}{s^3 + s^2 + (K_p + 1)s + K_i} \tag{3.42}$$

Before we investigate above equation, we must use nyquist stability criterion to analyze stable and unstable interval of the equation according to K_p and K_i variables. Before we make an analysis we must show the routh table:

Table 3.1: The Routh Table.

s^3	1	Kp+1
s^2	1	Ki
s^1	(Kp+1)-Ki	
s^0	Ki	

Stability requires that there be no sign changes in the first column. Thus stability requires that $K_p+1-K_i>0$ and $K_i>0$.

Hence by using this conditions we can investigate K_p and K_i pairs for the PI controller. If we investigate K_p in the interval of [1,100] and K_i also in the same interval, we find totally 100x100=10,000 pairs. However, if we delete required pairs for the syability and calculate the 10 minimum value of M_p, we find the below table:

Table 3.2: Minimum "10" M_p values of the Appendix D.

	Kp	Ki	Ts	Mp
1	2	1	8	0.24614
2	3	1	7	0.25812
3	1	1	>10	0.26231
4	17	1	7.5	0.29276
5	17	3	7.5	0.31909
6	6	1	7.5	0.31916
7	17	4	7.5	0.33108
8	18	4	6.5	0.33572
9	17	5	7.5	0.3423
10	19	1	7	0.34348

For the minimum Maximum Percent Overshoot values, corresponding to settling time values can be seen in the Table (3.2). If we investigate Table (3.2) carefully, it will be seen that the minimum '10' maximum percent overshoot values are between the interval of [1,20] for K_p and K_i. Hence it is better to investigate our PI controller design in this interval instead of all values of K_p and K_i.

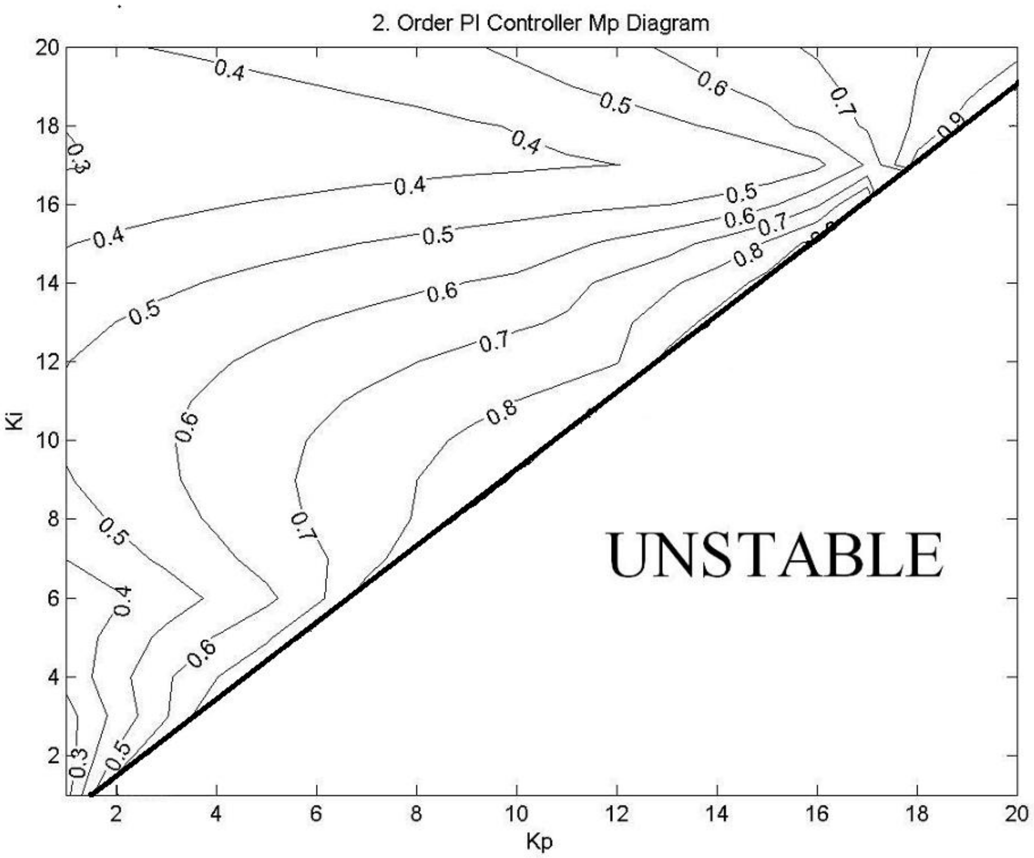


FIGURE 3.11: Graph of M_p corresponding to K_p and K_i Variables

As it is shown above Figure (3.11), proportional gain and the integral constant are coupled in general but there are some regions that it becomes decoupled for the maximum percent overshoot values. As it is shown in the Figure (3.11) that the interval of [14, 16.5] for K_p and the interval of [2, 16] for K_i, M_p becomes nearly parallel to the K_i-axes. Hence the affect of change in K_i for this interval close to the zero. In addition to this behavior, it is also observed that M_p becomes perpendicular to K_i-axes in the interval of [8, 12] for the K_p values. This means that in this interval change in M_p proportional only change in K_p, so also for this interval our design becomes decoupled. It should also be noticed that above graph shows only the stable interval for our PI Controller equation. Below Figure (3.12) shows the settling time behavior for this interval.

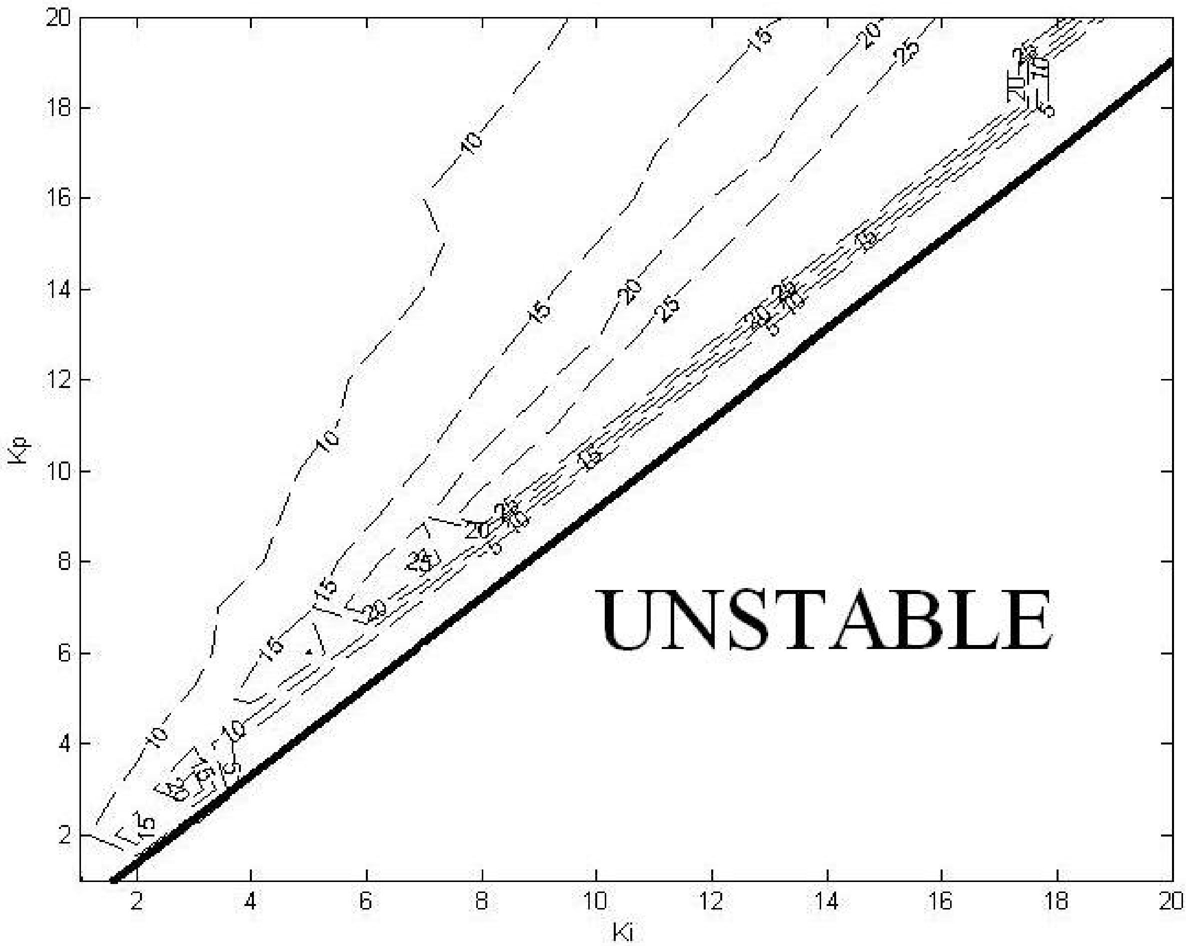


FIGURE 3.12: Graph of T_S corresponding to K_p and K_i Variables

As it is shown above Figure (3.12), proportional gain and the integral constant are coupled for the settling time values in general. It is also valid in the area which is shown by the dashed line. Thus, settling time depends both the change in K_p and K_i. It should also be noticed that above graph shows only the stable interval for our PI Controller equation. We can also investigate above two graphs by combining as it is shown below Figure (3.13).

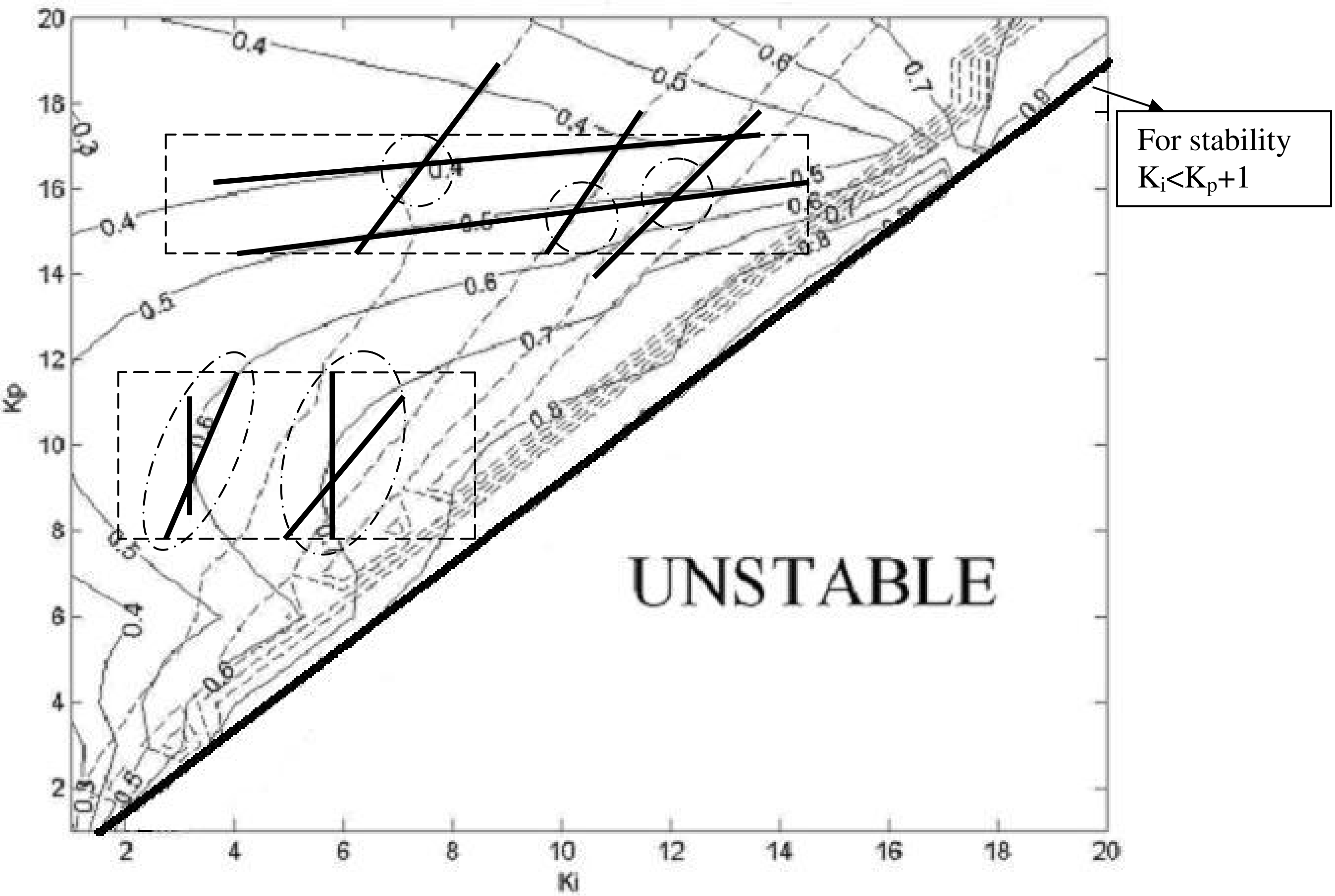


FIGURE 3.13: Graph of M_p and T_s corresponding to Kp and Ki Variables

As it is shown at the above figure, in the dashed lined area our design becomes decoupled. Small circled dashed lines in Figure 3.13 shows decoupled design. In this interval, while M_p depends only on the change in K_i, settling time depends on both change in K_p and K_i separately. We can explain this situation with the below matrix:

$$\begin{pmatrix} M_p \\ T_s \end{pmatrix} = \begin{pmatrix} 0 & X \\ X & X \end{pmatrix} \text{x} \begin{pmatrix} K_p \\ K_i \end{pmatrix} \qquad (3.43)$$

In addition to this decoupled area, also dashed ellipse lines show that there is another area where M_p becomes parallel to the K_p axis. In this interval, while M_p depends only on the change in K_p, settling time depends on both change in K_p and K_i separately. We can explain this situation with the below matrix:

$$\begin{pmatrix} M_p \\ T_s \end{pmatrix} = \begin{pmatrix} X & 0 \\ X & X \end{pmatrix} \text{x} \begin{pmatrix} K_p \\ K_i \end{pmatrix} \qquad (3.44)$$

The Equations (3.43) and (3.44) also shows that for the mentioned interval our controller is decoupled, but except this interval it is coupled as it is shown in the Figure (3.13).

PART IV
CONCLUSION

This thesis can show a methodology and example for the applications of axiomatic design to control system designing for the researchers who works in the field of control engineering. While this investigation of axiomatic design has achieved success in developing methods for axiomatic system design, the utility of the theorems presented would be greatly enhanced if they are integrated into design tools. Additional to methodologies, there is a great need for computing the results of non-linear equation. MATLAB is used to implement concepts and create documentation.

Axiomatic Design is an effective tool in control system design, because it logically identifies the essential system components and structures the once complex set of requirements and ideas into the coherent set of FR/DP architectures. Axiomatic Design leads to the betterment of the system design and ensures the effectiveness and the efficiency both in the product itself and its development.

The system level design requires a clear statement of FRs, with the conceptualizing DPs. The FR definition at this level serves the purpose of system organization and structuring.

The application level design starts with the clearly defined FR set, from the system level. In many cases, specific knowledge is required to select and 'design' the optimum DPs. Axiomatic Design is useful to break down the huge web of functionalities into an organic set of individual functions, which are easy to design.

A specific function of an application is designable, because of its clearly defined purpose and the well established input and output relationship. However, Axiomatic Design can still be used to design the sequential algorithm of a function, which is encountered frequently in control system design. The information contained in the design matrix is easily exported to construct the sequential functional diagram.

The Axiomatic Design system architecture and the accompanying design matrices and tables eases the tasks of system evaluation, maintenance and upgrade. Concerning the complexity of the modern machine control system, the information contained in the Axiomatic Design system architecture is essential to guide tests, troubleshooting and modifications in a fail-safe way, without passing 'bugs'.

The investigation of axiomatic design approach to control system design has been researched in this study.

It has been shown that for the first order systems with P controller, steady state error and the settling time are decoupled. The steady state error depends only on the change in the proportionality gain K_p. It means that steady state error can be tuned by changing the K_p. The change in the settling time caused by the change in K_p can be eliminated by changing the time constant T. However, settling time is dependent both the K_p and the T. But for the bigger value of K_p and the smaller value of T, the effect of change in T can be neglected for the settling time.

When we investigate first order control system with PI controller, the relationship between settling time and the maximum percent overshoot becomes decoupled. Hence the settling time depends only on the change in proportionality gain K_p. Change in the K_i doesn't affect the settling time at all. However, maximum percent overshoot depends both the K_p and K_i.

In second order systems, there isn't perfect decoupled or uncoupled relationship while there are three variables (K_p, ω_n, ζ). But when we make ζ constant, then our design becomes uncoupled. This means that, while steady state error or maximum percent overshoot can be tuned by changing proportionality gain, settling time can be tuned by changing the ω_n at the same time. Maximum percent overshoot also can be tuned by ζ when we take proportionality gain K_p constant.

In the second order systems with PI controller; there are some intervals for K_i and K_p that the relationship between M_p and T_s becomes decoupled.

This is brief study of how Axiomatic Design can be used to design a control system. Since this thesis has presented the design of a control system, the methodologies proposed here can be applied to the design of any type of control system, such as the automotive, the aircraft, the traffic and the factory management control system.

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APPENDICES

Appendix A. Matlab Programs

PROGRAM SEMANGULARITY AND REANGULARITY

```
% This program calculates Semangularity and Reangularity for the First Order Systems
% with P controller for the Section III.1.

% x = Proportionality gain (Kp), y = Time constant (T)

% This program plots Reangularity and Semangularity graphs.

x=1:1:10;
y=1:1:10;
[X,Y]=meshgrid(x,y);
i=1;
j=1;
for x=1:1:10,
for y=1:1:10,
A11=-1./(1.+x)^2;
A12=0;
A21=-4.*y./(1.+x).^2;
A22=4./(1+x);
S(i,j)=((abs(A11)).*(abs(A22)))./((sqrt((A11.^2)+(A21.^2))).*(sqrt((A12.^2)+(A22.^2))));
R(i,j)=sqrt(1-((((A11.*A12)+(A21.*A22))^2)/(((A11.^2)+(A21.^2))*((A12.^2)+(A22.^2)))));
COOR=[ X Y ]
R_VALUE=R(i,j)
S_VALUE=S(i,j)
A_MATRIX=[A11 A12;A21 A22]
j=j+1
end
```

```
i=i+1;
j=1;
end
X=1:1:10;
Y=1:1:10;
[c,h]=contour(X,Y,R,'-k');
clabel(c,h);
xlabel('Kp');
ylabel('T');
title('Contour Reangularity');
```

PROGRAM CONTOUR LINES

```
% This program calculates Steady State Error and Settling Time values for the
% proportionality gain and time constant variables on Section III.1.

% x = Proportionality gain (Kp), y = Time constant (T), Ess = Steady state error,
% Ts = Settling time.

% This program plots Steady State Error versus Settling Time for the
% proportionality gain and settling time variables.

x=1:1:10;
y=1:1:10;
[X,Y]=meshgrid(x,y);
i = 1;
j = 1;
for x = 1:1:10,
      for y = 1:1:10,
      Ess(i, j)= 1./(1.+x);
      Ts(i, j)= (4.*y)./(1.+x);
      j = j + 1;
   end
   i = i +1;
   j = 1;
```

```
end
[c,h] = contour(X,Y,Ess,'-k');
clabel(c,h);
hold;
[c,h] = contour(X,Y,Ts,'k--');
clabel(c,h);
xlabel('T');
ylabel('Kp');
```

SECOND ORDER SYSTEM DESIGN CODE

% This program calculates Rise Time, Maximum Percent Overshoot and Settling time

% for the maximum 5% Steady State Error

% T_s = Settling time, T_p = Peak time, T_r = Rise time, M_p = Maximum percent overshoot.

% K_p and K_i can be changed as variable and new system can be investigated.

```
t=0:0.5:10;
Kp=19;
Ki=1;
num=[0 0 Kp Ki];
den=[1 1 Kp+1 Ki];
 [y,x,t]=step(num,den,t);
plot(t,y)
grid
r=1;while y(r)<1.001, r=r+1;end;
Tr=(r-1)*0.5
[ymax,Tp]=max(y);
Mp=ymax-1;
s=21;while y(s)>0.95 & y(s)<1.05;s=s-1;end;
Ts=(s-1)*0.5;
[Mp’ Ts’ Tr’ Tp’]
```

PROGRAM M_P TABLE (PI Controller)

% This program calculates Maximum Percent Overshoot values when K_p and K_i
% variables between [1, 100] interval for the Section III.5.

% Totally 100*100=10,000 combination of K_p and K_i can be investigated, therefore
% 10,000 different values of M_p can be calculated.

```
t=0:0.5:10;
x=1:1:100;
y=1:1:100;
[X,Y]=meshgrid(x,y);
for x=1:1:100,
for y=1:1:100,
num=[0 0 x y];
den=[1 1 x+1 y];
[s,r,t]=step(num,den,t);
[smax,Tp]=max(s);
Mp(x,y)=smax-1;
y=y+1
end
x=x+1;
y=1;
end
```

PROGRAM SECOND ORDER PI CONTROLLER

% This program calculates Maximum Percent Overshoot and Settling Time values for
% the second order PI controller systems on Section III.5.

% x variable indicates K_i while y variable indicates K_p.

```
t=0:0.5:10;
x=1:20;
```

```
y=0.25:0.25:5;
[X,Y]=meshgrid(x,y);
for x=1:1:20;
        for y=0.25:0.25:5;
            num=[0 0 x y];
            den=[1 1 x+1 y];
            [s,r,t]=step(num,den,t);
           [smax,Tp]=max(s);
           Mp(i,j)=smax-1;
    m=21;while s(m)>0.95 & s(m)<1.05;m=m-1;end;
    Ts(i,j)=(m-1)*0.5;
           j=j+1;
      end
    i=i+1;
    j=1;
    end
[c,h] = contour(X,Y,Mp,'-k');
clabel(c,h);
hold;
[c,h] = contour(X,Y,Ts,'k--');
```

RESUME

Ali ÖZYIGIT was born in Istanbul in year 1980. He attended to Marmara University Mechanical Engineering Department (1997-2002). After his university education he started to pursue Master of Science Degree in the field of Mechanical Engineering. At the same time, at November 2002, he started to work at Arkas Holding as a Quality and System Engineer. In Arkas Holding he established quality systems in Arkas Ports according to ISO 9000 procedures. After he established all the quality system in Arkas Ports, one year later he started to work in Y&H Project Development and Management Ltd. as a Projects Engineer and Production Planning Responsible. During Y&H, he worked 6 months in Tehran/Iran as a project director to help Iranian people to increase their production efficiency with his Turkish team. He attended fairs and technological presentations related with his fields several times in all over the world.

After he has completed his military service in 2005, he has completed his M.Sc. Degree in Marmara University, Mechanical Engineering Department. Currently he is working at Bosch and Siemens Home Appliances as a Product Manager which is well known German Company as one of the world's leading domestic appliance manufacturers and the market leader in Western Europe.