

HIGH PERFORMANCE CMOS CAPACITIVE INTERFACE CIRCUITS
FOR MEMS GYROSCOPES

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Kanber Mithat SİLAY

ABSTRACT

HIGH PERFORMANCE CMOS CAPACITIVE INTERFACE CIRCUITS FOR MEMS GYROSCOPES

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This thesis reports the development and analysis of high-performance CMOS readout electronics for increasing the performance of MEMS gyroscopes developed at Middle East Technical University (METU). These readout electronics are based on unity-gain buffers implemented with source followers. High-impedance node biasing problem present in capacitive interfaces is solved with the implementation of a transistor operating in the subthreshold region.

A generalized fully-differential gyroscope model with force-feedback electrodes has been developed in order to simulate the capacitive interfaces with the model of the gyroscope. This model is simplified for the single-ended gyroscopes fabricated at METU, and simulations of resonance characteristics are done.

Three gyroscope interfaces are designed by considering the problems faced in previous interface architectures. The first design is implemented using a single-ended source follower biased with a subthreshold transistor. From the simulations, it is observed that biasing impedances up to several gigaohms can be achieved. The second design is the fully-differential version of the first design with the addition of a self-biasing scheme. In another interface, the second design is modified with an instrumentation amplifier which is used for fully-differential to single-ended conversion. All of these interfaces are fabricated in a standard 0.6 μm CMOS process.

Fabricated interfaces are characterized by measuring their ac responses, noise response and transient characteristics for a sinusoidal input. It is observed that, biasing impedances up to 60 gigaohms can be obtained with subthreshold transistors. Self-biasing architecture eliminates the need for biasing the source of the subthreshold transistor to set the output dc point to 0 V.

Single-ended SOG gyroscopes are characterized with the single-ended capacitive interfaces, and a 45 dB gain improvement is observed with the addition of capacitive interface to the drive-mode. Minimum resolvable capacitance change and displacement that can be measured are found to be 58.31 zF and 38.87 Fermi, respectively. The scale factor of the gyroscope is found to be 1.97 mV/(°/sec) with a nonlinearity of only 0.001% in ± 100 °/sec measurement range. The bias instability and angle random walk of the gyroscope are determined using Allan variance method as 2.158 °/ $\sqrt{\text{hr}}$ and 124.7 °/hr, respectively.

Keywords: Capacitive Interface Circuit, High-Impedance Node Biasing, Gyroscope, Microelectromechanical Systems (MEMS).

ÖZ

MEMS DÖNÜÖLÇERLER İÇİN YÜKSEK PERFORMANSLI CMOS SIĞASAL ARABİRİM DEVRELERİ

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Bu tez, Orta Doğu Teknik Üniversitesi'nde (ODTÜ) geliştirilmiş MEMS dönüölçerlerin performanslarını geliştirmek için tasarlanan yüksek performanslı CMOS okuma devrelerinin geliştirilmesini ve analizini anlatmaktadır. Bu okuma devreleri, kaynak takipçisi yapılarıyla oluşturulmuş kazancı 1 olan tampon devre tabanlıdır. Sığasal arabirim devrelerinde bulunan yüksek-empedanslı düğüm eğilimleme sorunu, eşikaltı bölgesinde çalışan bir tranzistör kullanımıyla çözülmüştür.

Sığasal arabirim devrelerinin dönüölçer modeli ile simüle edililebilmeleri için, geri-besleme elektrotlarını içeren genellenmiş tam-farksal bir dönüölçer modeli geliştirilmiştir. Bu model, ODTÜ'de üretilmiş tek-çıkışlı dönüölçerler için sadeleştirilmiş ve rezonans karakteristiği simülasyonları yapılmıştır.

Önceki arabirim devrelerindeki sorunlar göz önünde bulundurularak, üç türlü dönüölçer arabirimi tasarlanmıştır. İlk tasarım, eşikaltı tranzistörü ile eğilimlenmiş bir tek-çıkışlı kaynak takipçisi devresi ile oluşturulmuştur. Simülasyon sonuçlarına göre, eğilimleme empedansının gigaohm mertebelerine ulaşabileceği gözlemlenmiştir. İkinci tasarım ise oto-eğilimleme sistemi eklenmiş ilk tasarımın tam-farksal versiyonudur. Başka bir arabirime ise, ikinci tasarımın üzerine tam-farksal tek çıkış dönüşümü yapan bir aygıt kuvvetlendiricisi eklenmiştir. Tüm bu arabirim devreleri standart 0.6 μm CMOS sürecinde üretilmiştir.

Üretilen arabirim devrelerinin, ac, gürültü ve de sinüsoidal girişe geçiş tepkileri ölçülerek, devreler karakterize edilmiştir. Ölçüm sonuçlarından, 60 gigaohma kadar eğilimleme empedansı elde edilebileceği gözlemlenmiştir. Oto-eğilimleme yapısı, çıkışta 0 V dc noktası elde etmek için eşikaltı tranzistörünün kaynağının eğilimlenmesi gerekliliğini ortadan kaldırmıştır.

Tek-çıkışlı SOG dönüölçerleri, tek-çıkışlı sığasal arabirim devrelerine bağlanarak karakterize edilmiş ve sürüş-moduna takılan sığasal arabirimle 45 dB kazanç gelişimi gözlemlenmiştir. Arabirim tarafından ölçülebilecek asgari sığa ve mesafe değişiminin 58.31 zF ve 38.87 Fermi olduğu hesaplanmıştır. Dönüölçerin orantı katsayısı, ± 100 $^{\circ}/\text{sn}$ ölçüm aralığında %0.001 kadar doğrusallıktan sapma ile 1.97 mV/($^{\circ}/\text{sn}$) bulunmuştur. Dönüölçerin sabit kayma kararsızlığı ve açısal rastgele kayması Allan varyans metodu kullanılarak hesaplanmış ve sırasıyla 2.158 $^{\circ}/\sqrt{\text{saat}}$ ve 124.7 $^{\circ}/\text{saat}$ olarak bulunmuştur.

Anahtar kelimeler: Sığasal Arabirim Devresi, Yüksek-empedanslı düğüm eğilimleme, Dönüölçer, Mikroeletromekanik Sistemler (MEMS).

*To My Mother and Sister,
To the Memory of My Father,
And
To My Beloved Fiancée, Hüma*

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CHAPTER I

INTRODUCTION

A new era has been started in human history with the development of transistors and integrated circuit technology. This technology has enabled miniaturization of bulky components and enhanced operation capacity of electronic devices. Miniaturization of electronic components and development of integrated circuit technology decreased the costs paid for electronics, and hence, almost every aspect of life has been integrated with the cutting-edge technology for low costs.

Besides electronic circuits, development of miniaturization and silicon processing has enabled fabrication of miniaturized mechanical systems for sensing and actuation applications. These systems have minimum dimensions in the range of micrometers ($1\text{ }\mu\text{m} = 10^{-6}\text{ m}$) and combine micromechanics with microelectronics, and therefore, entitled as Micro-Electro-Mechanical Systems (MEMS). MEMS devices started entering many sensor and actuator applications, including military, biomedical, industrial and consumer applications. One of the most popular applications of MEMS technology is inertial navigation of objects. Inertial navigation is tracking the linear and angular movements of an object in order to determine its position and orientation, accurately. Common applications of inertial navigation are airplane, spacecraft and submarine navigation, platform stabilizations, guided-missile control, automotive safety, and personal navigation. In most of these applications, MEMS transducers replace bulky and expensive conventional

counterparts. Moreover, in some of the applications like automotive safety and personal navigation, MEMS devices are the only solution because of their low cost. There are two types of inertial measurement devices: 1) accelerometers that sense linear acceleration and 2) gyroscopes that sense angular rate applied on an object. Micromachined accelerometers have already been into mass production and been used in automotive market [1, 2]. On the other hand, structure and operation of gyroscopes are much more complex than accelerometers, and their performance has not achieved the desired level; and therefore, many applications like military navigation still use conventional gyroscopes. However, the advantages of miniaturization and the MEMS technology yield widespread research for MEMS gyroscopes.

Generally, MEMS gyroscopes are classified into three main groups according to their performance: rate-grade, tactical-grade, and inertial-grade. Table 1.1 shows the performance comparison of three different classes of gyroscopes for the most important performance parameters [3].

Table 1.1: Performance comparison of three different classes of gyroscopes for the most important performance parameters [3].

Performance parameter	Rate-Grade	Tactical-Grade	Inertial-Grade
Angle Random Walk, $^{\circ}/\sqrt{\text{hr}}$	> 0.5	$0.5 - 0.05$	< 0.001
Bias Drift, $^{\circ}/\text{hr}$	$10 - 1000$	$0.1 - 10$	< 0.01
Scale Factor Accuracy, %	$0.1 - 1$	$0.01 - 0.1$	< 0.001
Bandwidth, Hz	> 70	~ 100	~ 100

In order to replace the conventional gyroscopes with MEMS gyroscopes, performance of the MEMS gyroscopes should be improved. There are several factors limiting the performance of MEMS gyroscopes: Brownian

mechanical-thermal noise, mechanical and electrical cross-talk, temperature stability, and readout electronics noise. Therefore, to benefit from the advantages of MEMS gyroscopes, these limiting factors should be minimized. It has been seen that one of the most important design issues for high-performance gyroscopes is the design of readout electronics. This thesis reports development and analysis of high-performance CMOS readout electronics for increasing the performance of MEMS gyroscopes developed at Middle East Technical University (METU). It is verified that developed readout electronics solve challenging problems faced in high-performance MEMS gyroscopes.

The organization of this chapter is as follows; Section 1.1 includes a brief introduction to vibratory MEMS gyroscopes with basics of their operation and shows a few examples of MEMS gyroscopes in literature. Section 1.2 gives an overview of readout electronics for capacitive sensing. Then, Section 1.3 describes the gyroscope interfaces developed at METU, and Section 1.4 presents the research objectives and thesis organization.

1.1. Operation and Overview of Vibratory MEMS Gyroscopes

Figure 1.1 shows the basic operation principle of vibratory mechanical gyroscope. A vibratory mechanical gyroscope is composed of two main parts: drive frame and sense frame. These two frames are orthogonal to each other, and the plane enclosing these two frames is called the gyro frame. The third axis of the gyroscope is perpendicular to the gyro frame, and it is called the sensitive axis of the gyroscope. For operation of the gyroscope, the drive frame of the gyroscope continuously vibrates along a predefined axis. While the drive frame is moving, if the gyroscope is rotated along its sensitive axis, a portion of the energy stored in the drive frame is coupled to the sense frame. This creates a motion in the sense frame, and this effect is called the Coriolis effect. The motion in the sense frame is modeled by a fictitious

force or acceleration, and in literature, these are called Coriolis force and Coriolis acceleration, respectively.

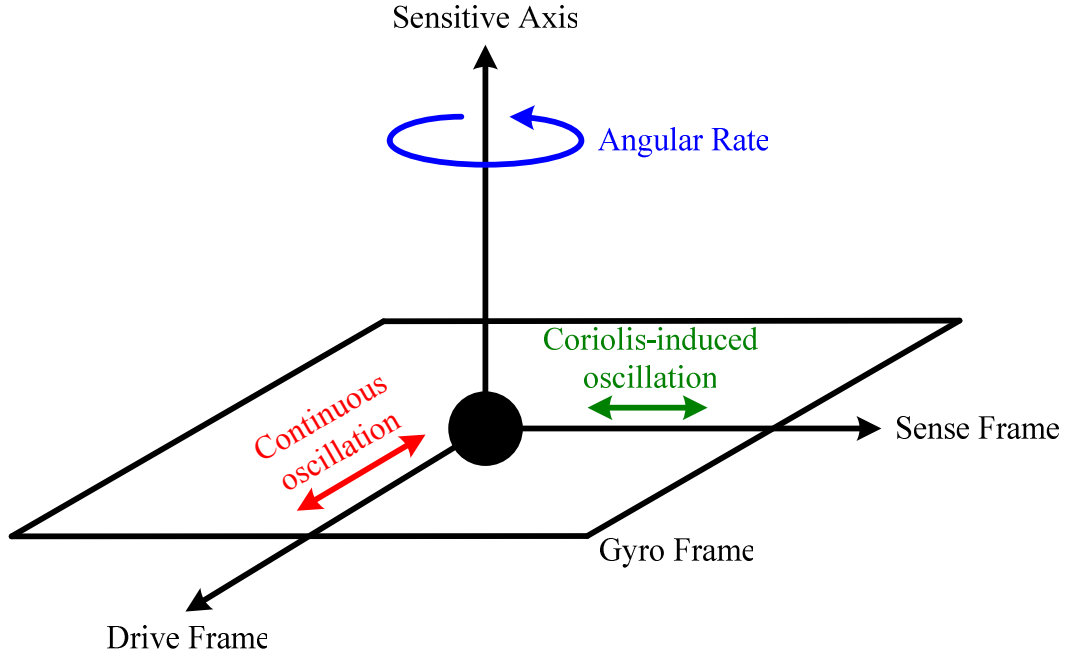


Figure 1.1: Basic operation principle of a vibratory mechanical gyroscope.

Mechanical structure of a MEMS gyroscope can be modeled as a second-order mass-damper-spring system. Assuming that two modes on the gyro frame are isolated from each other, there are two second-order mechanical structures present in the system. These mechanical structures demonstrate a bandpass mechanical filter characteristic, and hence, they have a resonance frequency, at which their response is maximum. Since the Coriolis-induced oscillation is proportional to the amplitude of oscillation in drive-mode and its frequency is same as the drive-mode oscillation frequency, the maximum response is obtained from the sense-mode if the resonance frequencies of two modes are set equal. There are special mechanisms which can tune the resonance frequency of a gyroscope by applying a dc voltage to generate

negative electrostatic spring constants. These structures can be found in literature, and are beyond the scope of this research.

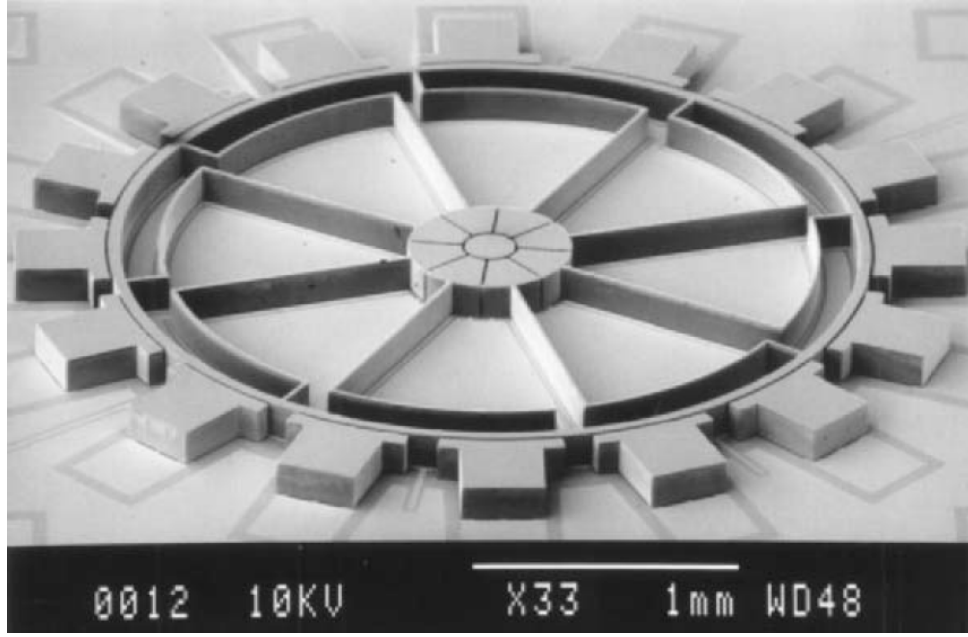
Actuation mechanisms for the drive frame of MEMS gyroscopes demonstrated in literature so far are primarily electrostatic, electromagnetic, or piezoelectric. On the other hand, the Coriolis-induced motion in the sense frame is generally detected by using capacitive, piezoelectric, piezoresistive, or optical detection mechanisms [3]. The most popular actuation/detection mechanism in MEMS gyroscopes is the electrostatic actuation/capacitive sensing scheme. Electrostatic actuation of MEMS structures is easy in terms of fabrication and control electronics. Similarly, capacitive sensing is advantageous, since its fabrication processes is straightforward, it has high sensitivity, and it is robust against temperature variations in the environment [4].

Currently, MEMS gyroscopes approach to $10^\circ/\text{hr}$ resolution, which is a quality of tactical-grade gyroscopes [3, 5, 6]. The University of Michigan reported a single-crystal silicon vibrating ring gyroscope with resolution of $10.4^\circ/\text{hr}$ and bias instability less than $\pm 1^\circ/\text{sec}$. Figure 1.2 shows the vibrating ring gyroscope developed at the University of Michigan [7].

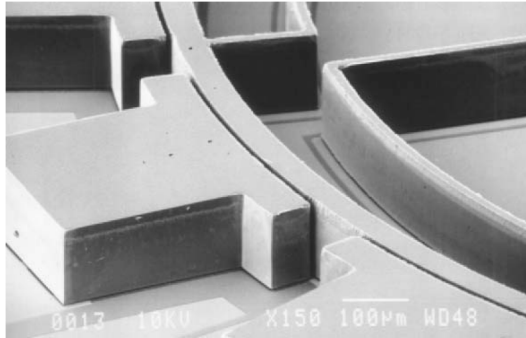
Moreover, micromachined gyroscopes have started entering commercial market for automotive applications and dead reckoning back up of global positioning satellite (GPS) navigation systems which satisfy $50^\circ/\text{hr}$ resolution floor. Figure 1.3 shows the die photograph of a mass-produced single-chip surface micromachined microgyroscope developed by Analog Devices [8].

There has been extensive research on the development of mechanical structures of MEMS gyroscopes at METU since 1998. During past 8 years, many different micromachined gyroscope designs are fabricated in several different processes. From these designs, advanced symmetric and decoupled gyroscope, namely single-ended SOG gyroscope, maintains robustness against rotary vibrations and has

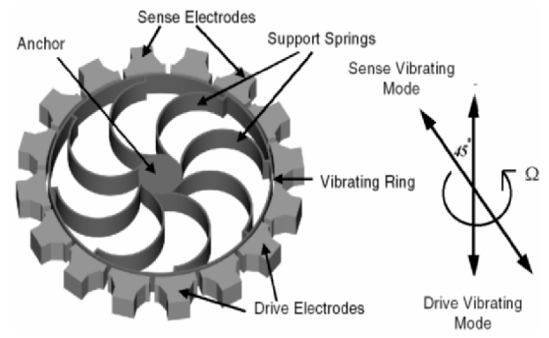
minimum mechanical cross-talk between the drive and sense modes, in addition to the common features of previous designs. Figure 1.4 shows the perspective view of the advanced symmetrical and decoupled gyroscope architecture fabricated in the silicon-on-glass (SOG) process developed at METU [9].



(a)



(b)



(c)

Figure 1.2: Vibrating ring gyroscope developed at the University of Michigan: (a) SEM picture of single-crystal silicon vibrating ring gyroscope, (b) close up view of ring structure, and (c) structure and operation of vibrating ring gyroscope [7].

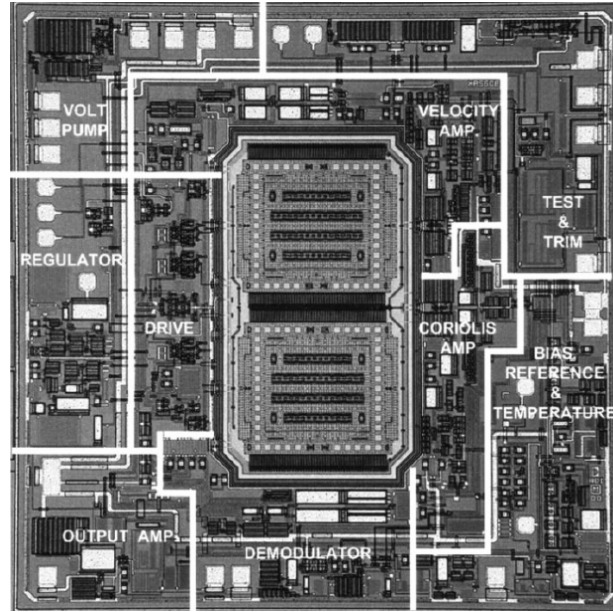


Figure 1.3: Die photograph of a mass-produced single-chip surface micromachined microgyroscope developed by Analog Devices [8].

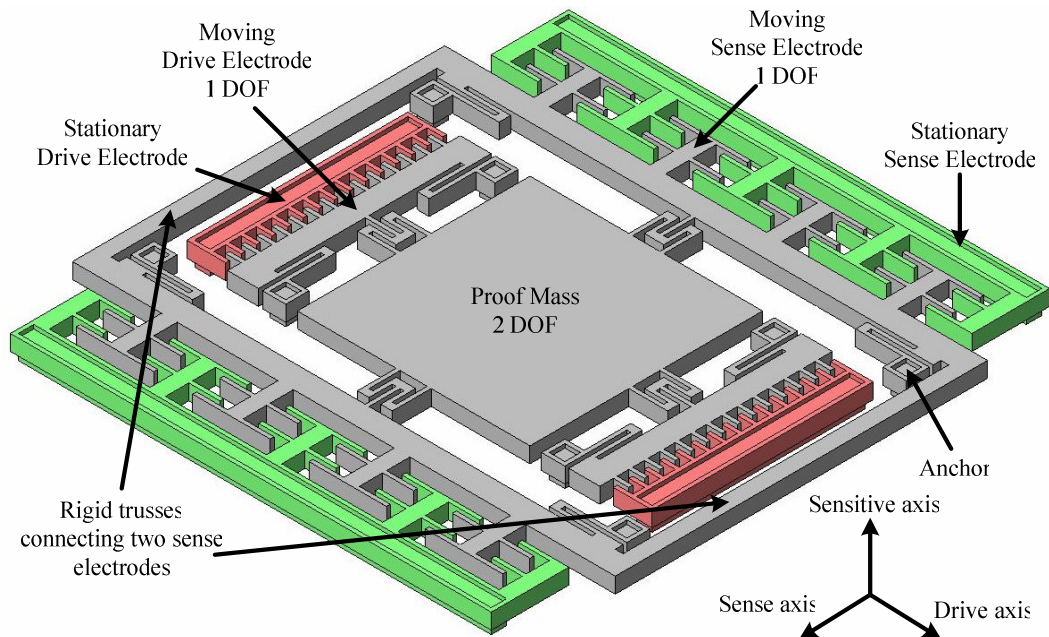


Figure 1.4: Perspective view of the advanced symmetrical and decoupled gyroscope architecture fabricated in SOG process developed at METU [9].

With the advanced design given in the figure above, an angular rate resolution of 36 °/hr is achieved at vacuum ambient. Moreover, fabricated gyroscope demonstrates an uncompensated quadrature signal of only 15 °/sec [10].

1.2. Overview of Readout Electronics for Capacitive Sensing

As mentioned before, besides ease of fabrication, capacitive sensors have high-sensitivity and are robust against ambient temperature changes. In the MEMS technology, capacitive sensing is widely used in many applications, like sensing position, pressure, and humidity.

In the case of micromachined capacitive gyroscopes, the capacitance of the sensor that is going to be measured is very small, being in the order of a few hundred femtofarads ($1 \text{ fF} = 10^{-15} \text{ F}$). Moreover, sense-mode of the gyroscope moves in the order of a few Angstroms ($1 \text{ Angstrom} = 10^{-10} \text{ m}$), which corresponds to even much smaller capacitance changes, in the order of sub-attofarads ($1 \text{ aF} = 10^{-18} \text{ F}$). This capacitance change must be detected by using proper circuitry in order to be used by external electronics. Therefore, highly sensitive readout electronics should be used to interface the sensor with the outside world.

Since the capacitance of the sensor is quite small, readout electronics is affected from the parasitic capacitances present in the system. These parasitic capacitances arise from the conductor plates around the sensor capacitance, connections between the sensor and readout chips, and pads and interconnect lines in the readout IC. Therefore, a careful design is necessary to minimize the parasitic capacitances in order to increase the sensitivity of the gyroscope.

Another limiting factor of the performance of gyroscopes is the electrical noise originating from the readout electronics. To have a highly sensitive gyroscope, the signal-to-noise ratio (SNR) at the output of the device must be maximized.

Therefore, the noise contributions present in the system should be minimum. This leads to have low-noise interface circuits for enhancing the operation of the gyroscope.

One of the most challenging problems in capacitive sensing is biasing the high-impedance node at the output of the sensor. Since the capacitance of the sensor is small, the input of the interface is a high-impedance node. This node cannot be connected to a low-impedance bias structure like a simple resistor, because connecting a low-impedance to the output of the sensor degrades the sensitivity. Therefore, a special design effort is essential to bias the input of a capacitive interface circuit.

In literature, there are many different capacitive sensor interfaces, but the most popular among these are: 1) ac bridge with voltage amplifiers, 2) transimpedance amplifiers, 3) switched-capacitor circuits, and 4) unity-gain buffers [11].

The ac bridge is one of the simplest capacitive readout techniques, and it requires a reference capacitor, a voltage amplifier, and demodulation circuitry. Figure 1.5 shows the ac bridge configuration for capacitive sensing. The operation principle of the ac bridge is quite simple. Anti-phase clock signals are applied to the sensor and reference capacitors, and change in the capacitance is simply detected by voltage division. It has been reported that a $0.02 \text{ aF}/\sqrt{\text{Hz}}$ capacitance noise floor is measured with this architecture [12]. The disadvantage of this architecture is that SNR may be affected from the parasitic capacitance at the input of the voltage amplifier, if this capacitance is large [11]. Therefore, this type of capacitive readout is most suitable for monolithic type capacitive sensors [1].

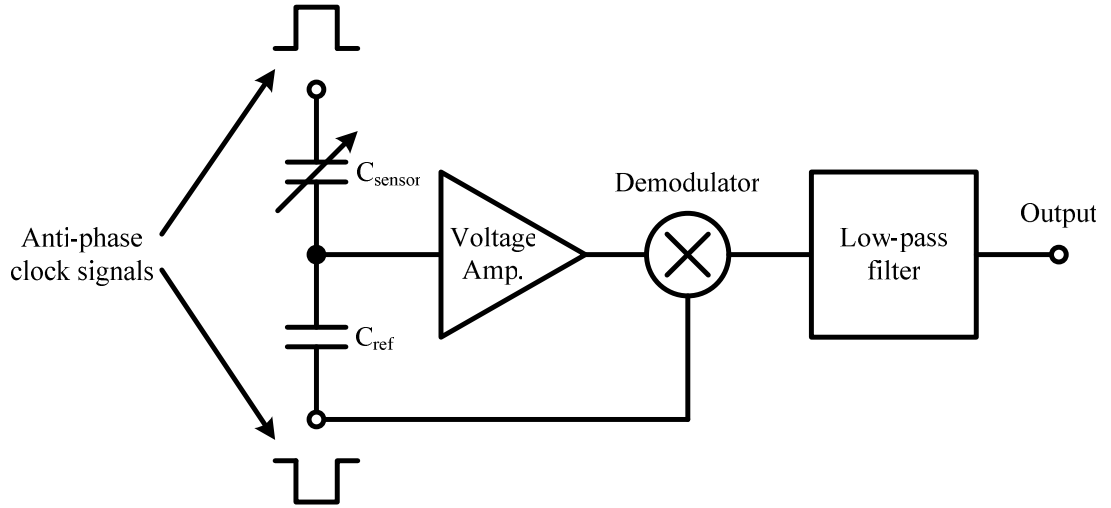


Figure 1.5: Ac bridge configuration for capacitive sensing.

Transimpedance amplifiers are also commonly used for capacitive position sensing. These types of amplifiers integrate the charge injected from the sensor over an impedance, implemented by either a capacitor or a resistor, or even both [13]. In fully-differential versions of these amplifiers, biasing the high-impedance node at the output of the sensor is a problem. Analog Devices reported a biasing circuit using controlled-impedance structure by matching of like-width FETs with significantly different lengths. Figure 1.6 shows the transimpedance amplifier with controlled-impedance structure by matching of like-width FETs with significantly different lengths proposed by Analog Devices [8]. With this biasing architecture, reported biasing impedance is $2.5 \text{ G}\Omega$ with 1/50 duty-cycle operation. The minimum detectable capacitance change with this architecture is given as 12 zF ($1 \text{ zF} = 10^{-21} \text{ F}$).

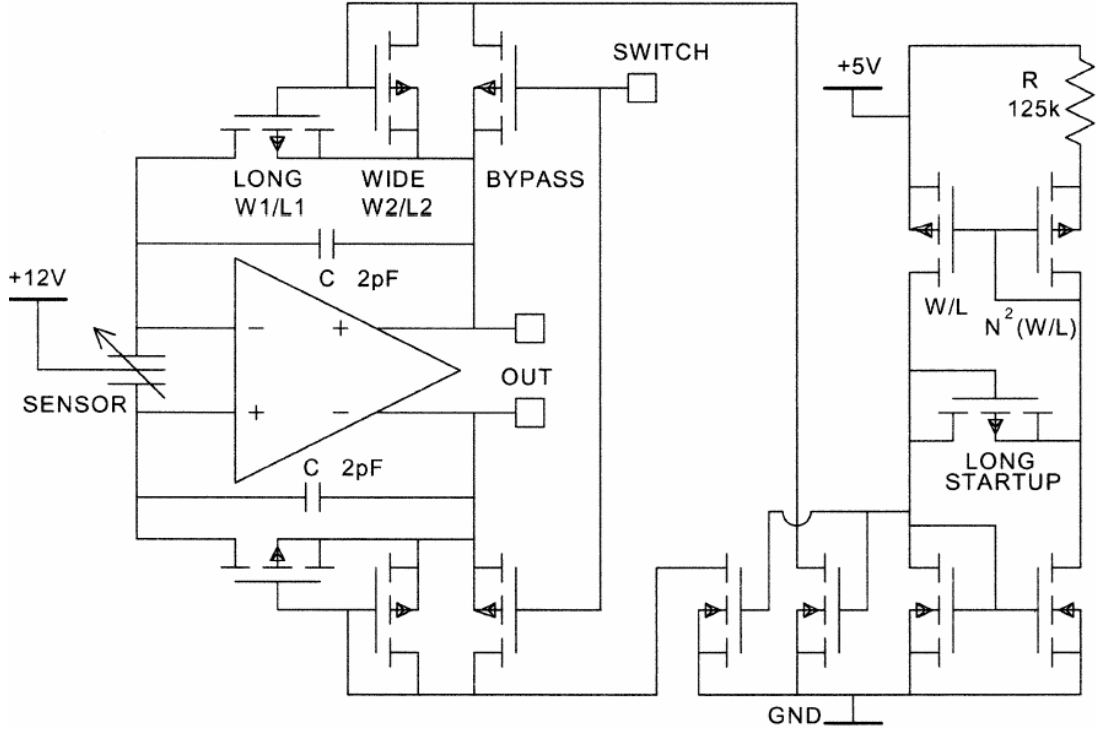


Figure 1.6: Transimpedance amplifier with controlled-impedance structure by matching of like-width FETs with significantly different lengths proposed by Analog Devices [8].

One other common capacitance readout technique is the use of switched-capacitor charge integrators. This type of readout is generally used for micromachined accelerometers where ac bridges are applicable [14-16]. Fully-differential versions of these readouts achieve sensitivity values of 10 aF although the structure is non-monolithic [17]. It is composed of a fully-differential charge-integrator with correlated double sampling. The charge integrator uses a fully-differential operational transconductance amplifier with a common-mode feedback. Charge-injected through the sensor is integrated over the integration capacitance, which is reset after the integration cycle.

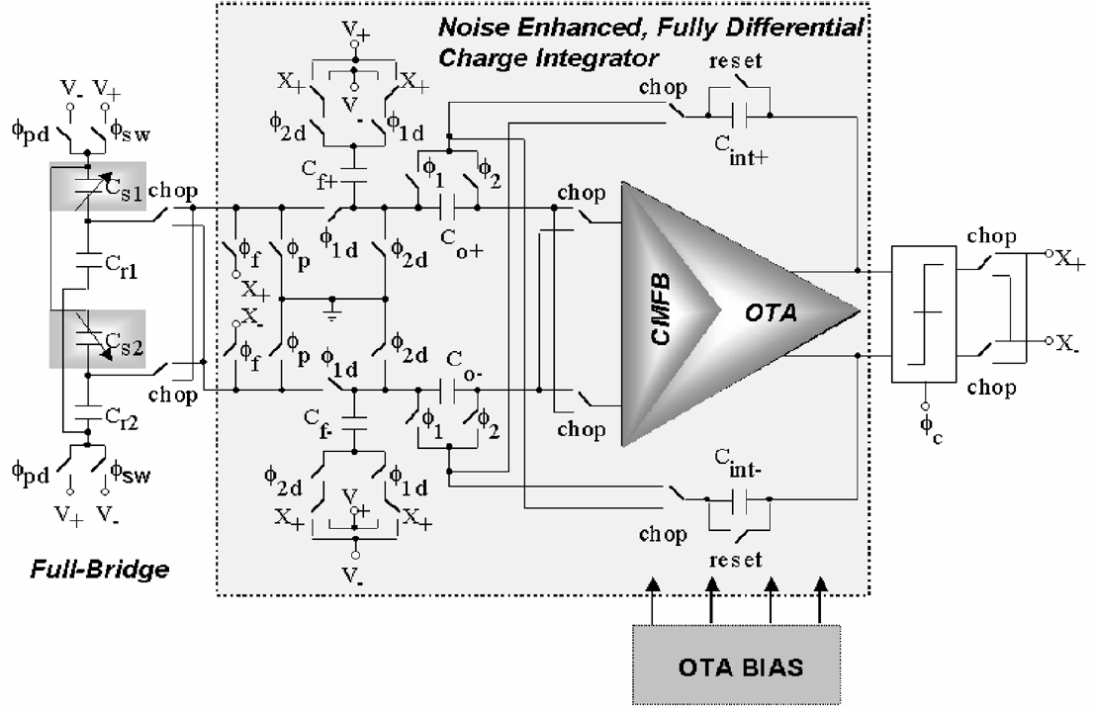


Figure 1.7: Schematic of switched-capacitor front-end circuit for capacitive sensing applications [17].

Unity-gain buffers are another common type of capacitive sensor interfaces. It is possible to obtain high-input impedance with this type of interfaces, on which the charge will be integrated. It has been reported that it is possible to obtain very small input capacitances around 4 fF at the input of the buffer [18]. It can be shown that smaller input capacitances yield larger signals at the input of the interface, which results in higher SNR. Moreover, unity-gain buffers use advantage of bootstrapping for minimization of input parasitic capacitances [18-20].

In conclusion, there are several types of capacitive sensor interfaces for different applications. Clearly, most of the research for capacitance measurement is focused on sensing micro-accelerometers using ac bridge structure driven with anti-phase clock signals. This type of readout may not be applicable in all cases, if there is not an ac bridge in the sensor or if it is not possible to use anti-phase signals. Therefore,

there is still a need for designing new capacitive interface circuits for high-performance capacitive gyroscopes. The next section describes the capacitive interface circuits developed at METU.

1.3. Gyroscope Interfaces Developed at METU

Although there has been extensive research on the mechanical structure and fabrication of MEMS gyroscopes at METU, there has not been sufficient research for development of gyroscope interfaces. The first prototype that was successfully used as a gyroscope interface was a unity-gain buffer biased with a MOSFET operated in its cut-off region. This design was inspired from the original unity-gain buffer biased with a reverse diode proposed in [18]. Figure 1.8 shows the schematic of the first prototype of gyroscope interface, which is a unity-gain buffer, developed at METU [9, 21].

The advantage of this structure is that, the gain from the input transistor's gate to its source and drain is very close to 1. This results in a very small input capacitance due to Miller effect. Simulated input capacitance at the input node is found to be only 13 fF. Biasing of the high-impedance node was done using a diode-connected PMOS transistor.

The major problem in this design was the biasing of the high-impedance input node. The T_{BIAS} transistor shown in Figure 1.8 is always in cut-off region, and hence, it is not possible to bias the input node to the desired level without an external excitation. Optical excitation was used in order to bias the input node at the beginning of operation. Unfortunately, when optical excitation is stopped, the biasing of the input node is lost.

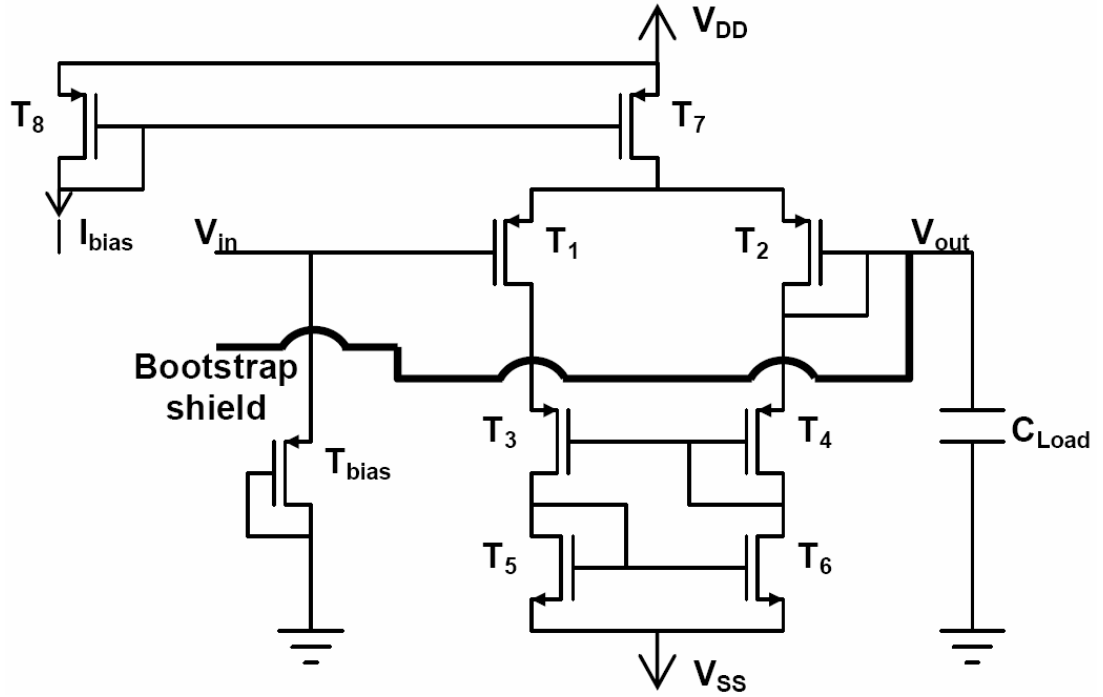


Figure 1.8: Schematic of the first prototype of gyroscope interface, which is a unity-gain buffer, developed at METU [9, 21].

The second problem in this architecture was the small linear range of the buffer. As there are four transistors in a single branch of the buffer, the circuit can operate up to supply voltages' difference minus four overdrive voltages. The maximum achievable linear range was 1.5 V for ± 2.5 V supplies.

Another designed and fabricated gyroscope interface was an improved version of previous unity-gain structure. This buffer circuit was using two more cascode transistors in the differential branches [22]. However, these circuits were suffering from static discharges, and therefore, their response were not stable.

These problems in the previous capacitive interfaces lead to the design of operational gyroscope interfaces with stable operation. The work presented in this thesis reports

the development of high-performance capacitive gyroscope interfaces with stable high-impedance node biasing architectures.

In the first phase of this research, the designed and fabricated gyroscopes were single-ended gyroscopes, i.e., the drive-mode of the gyroscope could be excited from only one electrode and its displacement could be measured from a single drive electrode. According to the needs, the first design aimed developing a simple preamplifier that would satisfy the requirements with a special biasing circuitry solving the biasing problem. For this design, a single-ended source follower biased with a subthreshold transistor is chosen. From the characterization of the interface with gyroscopes, it has been seen that the performance of the gyroscope can be improved by using a fully-differential readout architecture and special self-biasing technique.

The second phase of this research includes the development of fully-differential gyroscope interfaces using self-biasing architectures. The aim was to achieve a fully-differential gyroscope controlled with available single-ended external electronics. Therefore, a fully-differential to single-ended conversion circuit is also implemented for future needs. The operation of these circuits is verified with simulations and measurements; however, these circuits are not characterized with gyroscopes since there is no operational fully-differential gyroscope present in METU MEMS-VLSI Research Group during the writing of this thesis.

1.4. Research Objectives and Thesis Organization

The goal of this research is to develop and analyze high-performance gyroscope interfaces for MEMS gyroscopes developed at METU. These interfaces consist of three architectures according to their operation and purpose. Specific objectives of this study are as follows:

1. Analysis of MEMS gyroscopes and development of an accurate generalized gyroscope model for simulating gyroscope interfaces. This general model should cover the drive and sense modes of the gyroscope, and conversion of electrical and mechanical domains should be satisfied. Fully-differential gyroscopes should be considered for future designs. Advanced structures like force-feedback electrodes should also be involved for external control electronics.
2. Design of a single-ended gyroscope interface to obtain high-performance gyroscopes using a single-ended SOG gyroscope developed at METU MEMS-VLSI Research Group. This interface should solve the biasing problem faced in the previous interface designs and eliminate the need for optical excitation. It should also have small input capacitance to obtain high performance, therefore, bootstrapped interconnect lines should be implemented.
3. Design of fully-differential gyroscope interfaces for possible fully-differential gyroscope architectures. These interfaces should include self-biasing scheme in order to get rid of setting the high-impedance node bias voltage. Moreover, one of the designs should include a fully-differential to single-ended conversion circuitry for use of single-ended external electronics available at METU.
4. Characterization of gyroscope interfaces in terms of ac response, transient response, and noise response along with verification of high-impedance node biasing.
5. Characterization of the single-ended gyroscope interface with a single-ended SOG gyroscope in terms of angular rate sensitivity, linearity, bias instability, and minimum resolvable rate response.

Chapter 2 briefly explains the theory behind micromachined vibratory gyroscopes and gives the background for construction of a general model for these gyroscopes. Then, it analyzes the transduction mechanism with the analysis of governing equations behind gyroscope theory. After that, modeling of vibratory gyroscopes is introduced with electromechanical conversion, duality of electrical and mechanical domains, actuation and sensing in capacitive gyroscopes, a generalized model for fully-differential capacitive vibratory gyroscopes, and finally, a specific case of the general model developed for the SOG gyroscope. Lastly, requirements for high-performance gyroscope interfaces are described in this chapter.

Chapter 3 gives the theory and design procedure of capacitive interfaces for MEMS gyroscopes. It starts with the proposed solutions for challenges present in capacitive interfaces. Then, the design of single-ended capacitive interfaces is introduced with theoretical analysis and simulations. Following this, possible improvements for the single-ended capacitive interfaces are presented. After that, two architectures for fully-differential gyroscope interfaces are given with their simulations.

Chapter 4 presents the test setups used for the characterization and tests of the capacitive interface circuits developed in this study and gives the test results of these circuits. Three designs are tested in order to verify interfacing the high-impedance node to external electronics. These results are compared with the simulation results obtained in Chapter 3. Moreover, the single-ended interface is tested with the single-ended SOG gyroscope, and performance results with the gyroscope are obtained.

Finally, Chapter 5 summarizes this research and gives the conclusions from the results. Furthermore, suggestions for the following research route are given.

CHAPTER II

THEORY AND MODELING OF VIBRATORY GYROSCOPES

This chapter introduces the basic theory and modeling of vibratory gyroscopes. Section 2.1 briefly gives the basic theory behind vibratory gyroscopes and explains the operation of the gyroscopes with their basic modes. Then, Section 2.2 explains the transduction mechanism in these gyroscopes by clarifying the actuation and sensing mechanisms of a gyroscope. It also presents the governing equations which give the signal measured when a gyroscope interface is connected. Section 2.3 proposes the generalized fully-differential gyroscope model with details of each separate part present in the model. Then, a specific model is generated for the single-ended SOG gyroscope, and its resonance characteristics are simulated in the simulator. Section 2.4 gives the requirements necessary for obtaining high-performance gyroscope interfaces, and Section 2.5 provides a summary for this chapter.

2.1. Basic Theory of Vibratory Gyroscopes

In order to operate a vibratory gyroscope, the proof mass should be oscillated in drive-mode axis so that Coriolis acceleration creates a motion in the sense-mode axis when a rate input is applied about to the sensitive axis of the gyroscope. To generate the necessary electrostatic forces for oscillation of the proof mass, an excitation

voltage is applied to the driving electrode, while the proof mass is kept at a constant dc voltage. When the gyroscope is oscillating, if a rate input is applied to the system, Coriolis acceleration is converted to electrical signals at the sensing electrodes of the sense-mode.

Mechanical cross-talk from the drive-mode to the sense-mode is a major problem in vibratory gyroscopes. Oscillation of the drive-mode couples to the sense-mode due to imperfections in the fabrication and, hence, it results in a large output offset at the sensor output although there is no rate input. This error at the output of the sensor is called the quadrature error. For achieving maximum performance, drive and sense modes of the gyroscopes should be mechanically decoupled in order to minimize the mechanical cross-talk from the drive-mode to the sense-mode. Therefore, the gyroscopes used in this study have fully-decoupled architectures. A fully-decoupled architecture is achieved by limiting the motion of drive and sense modes to 1 degree-of-freedom (DOF) and proof mass to 2-DOF [9]. Figure 2.1 shows the block diagram of the fully-decoupled vibratory gyroscope architecture. The gyroscope is driven into resonance by applying an excitation signal to the actuation capacitance in order to vibrate the proof mass in the drive axis. When a rate input is applied to the system, the proof mass also vibrates in the sense-mode, and a Coriolis acceleration is sensed as a change in the sensing capacitance.

The gyroscope architecture described in Figure 2.1 is composed of mainly two parts: the first one is the drive-mode resonator which oscillates the proof mass, and the second part is the Coriolis accelerometer of the sense-mode which detects the rate input to the system from the motion of the sense-mode. These parts are explained in detail in the subsections below.

2.1.1. Drive-mode resonator

Figure 2.2 shows a second-order mass-spring-damper system. Both the drive-mode resonator and sense-mode accelerometer can be modeled with this system. Mass, m , in this model represents the proof mass of the system. Spring, k , is used for modeling the beams connecting the proof mass to the anchors suspending the micromachined gyroscope; while, damper, b , represents damping of the environment such as air damping at atmosphere.

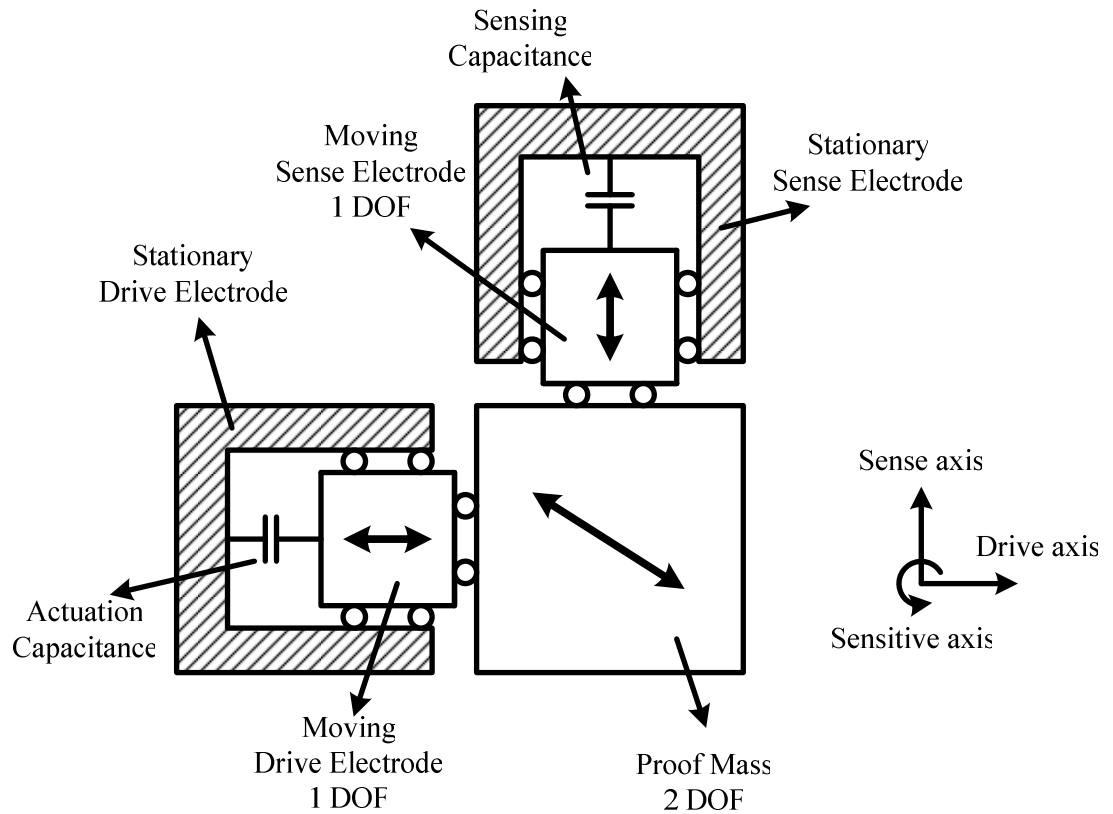


Figure 2.1: Block diagram of the fully-decoupled vibratory gyroscope architecture.

Force-deflection characteristic of the drive-mode resonator can be described with the second-order transfer function,

$$\frac{X_D(s)}{F_D(s)} = \frac{1}{s^2 \cdot m_D + s \cdot b_D + k_D} \quad (2.1)$$

where, $X_D(s)$ is the displacement of the drive mode, $F_D(s)$ is the force acting on the drive-mode, m_D is the total mass at the drive axis, b_D is the damping factor in the drive-mode and k_D is the total spring constant of the drive-mode.

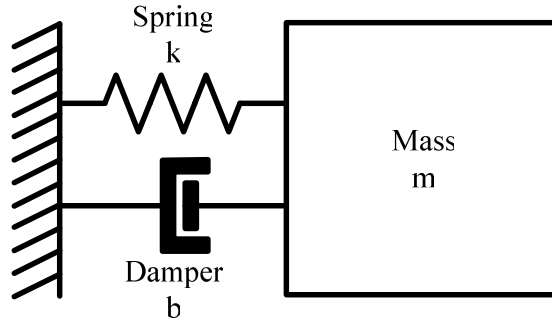


Figure 2.2: Second-order mass-damper-spring system.

The resonance frequency, ω_D , and quality factor, Q_D , of the second-order system described in Equation 2.1 are defined as

$$\omega_D = \sqrt{\frac{k_D}{m_D}} \quad (2.2)$$

$$Q_D = \frac{\sqrt{k_D \cdot m_D}}{b_D} \quad (2.3)$$

The maximum value of the transfer function occurs at the mechanical resonance frequency of the gyroscope; therefore, the gyroscope should be driven at the mechanical resonance frequency defined in Equation 2.2 to achieve the maximum oscillation by applying minimum force. If the gyroscope is driven at this frequency, Equation 2.1 simplifies to

$$\frac{X_D(s)}{F_D(s)} = -j \cdot \frac{Q_D}{k_D} \quad (2.4)$$

This equation implies that, if the gyroscope is driven into resonance, deflection of the resonator lags the applied force by 90°. Moreover, it can be concluded that, to maximize the gain of the system, quality factor should be increased, i.e., damping should be minimized. Damping of the system can be decreased by operating the gyroscope at vacuum, where damping is dominated by the material of the gyroscope.

2.1.2. Coriolis accelerometer of the sense-mode

The sense-mode also behaves like a second-order mass-damper-spring system as the drive-mode does. The difference between these modes is that the drive-mode is actuated by applying an electrical signal; whereas, the sense-mode starts oscillation when a rate input is applied to the system. The Coriolis force acting on the proof mass creates motion in the sense-mode, which is detected by the sensing capacitances. Equations 2.2 and 2.3 are also valid for the sense-mode, except the values of spring constant, damping and mass are replaced with the sense-mode mechanical parameters.

Detailed analyses are required for deriving the displacement of the sense-mode in response to the Coriolis force generated by the rate input [9]; however, these analyses are beyond the scope of this study. On the other hand, the parameter of

interest in this analyses is the sense-mode displacement when resonance frequencies of the drive and sense modes are matched and mismatched.

Assuming that the drive mode is resonated at its resonance frequency, ω_D , and a constant rate input of magnitude Ω_z is applied to the system, if the drive and sense modes of the gyroscope are matched, i.e., ω_D is very close to the resonance frequency of the sense mode, ω_S , then, displacement of the sense-mode at the frequency of the drive-mode will be [9]

$$Y(\omega_D) \approx \frac{2 \cdot \Omega_z \cdot x_D \cdot Q_S}{\omega_D} \cdot \frac{m_{PM}}{m_S} \quad (2.5)$$

where, x_D is the magnitude of the displacement of the drive-mode, Q_S is the quality factor of the sense-mode, m_{PM} is the mass of the 2-DOF proof mass, and m_S is the total mass present in sense-mode, i.e., the total of 2-DOF proof mass and sense electrodes.

When the modes are mismatched, i.e., difference between ω_D and ω_S is large, displacement of the sense-mode becomes [9]

$$Y(\omega_D) \approx j \cdot \frac{\Omega_z \cdot x_D}{(\omega_S - \omega_D)} \cdot \frac{m_{PM}}{m_S} \quad (2.6)$$

If the ratios of Equations 2.5 and 2.6 are taken

$$\frac{Y_{matched}}{Y_{mismatched}} = \frac{2 \cdot Q_S \cdot (f_S - f_D)}{j \cdot f_D} \quad (2.7)$$

If the difference between the frequencies of the two modes is around 1 kHz, the quality factor of the sense-mode is around 500, and the drive-mode resonance frequency is 5 kHz, then the ratio above will be 200 in magnitude. Therefore, matching the resonance frequencies of the two modes will improve the sensitivity significantly. However, matching the resonance frequencies may be problematic in terms of bias stability of the gyroscope.

2.2. Transduction Mechanism in Capacitive Vibratory Gyroscopes

Transduction mechanism in capacitive vibratory gyroscopes can be separated into two parts. The first one is the conversion of driving electrical signal to mechanical force. The second transduction is the conversion of displacement to the electrical signal in drive or sense modes. This section analyses the transduction mechanism in the following subsections.

2.2.1. Electrostatic force calculations for parallel-plate capacitors

Two charged and isolated plates try to pull each other if they have a potential difference. If one of the plates is fixed and the other one is movable, the electrostatic force generated between the plates tends to increase the amount stored energy in the capacitor, and therefore, the movable plate increases the capacitance. Figure 2.3 shows the parallel-plate capacitor configuration. The overlap area is defined by the overlap length, l_0 , and the overlap height, h_0 . The gap spacing between the parallel-plates is labeled as d_0 .

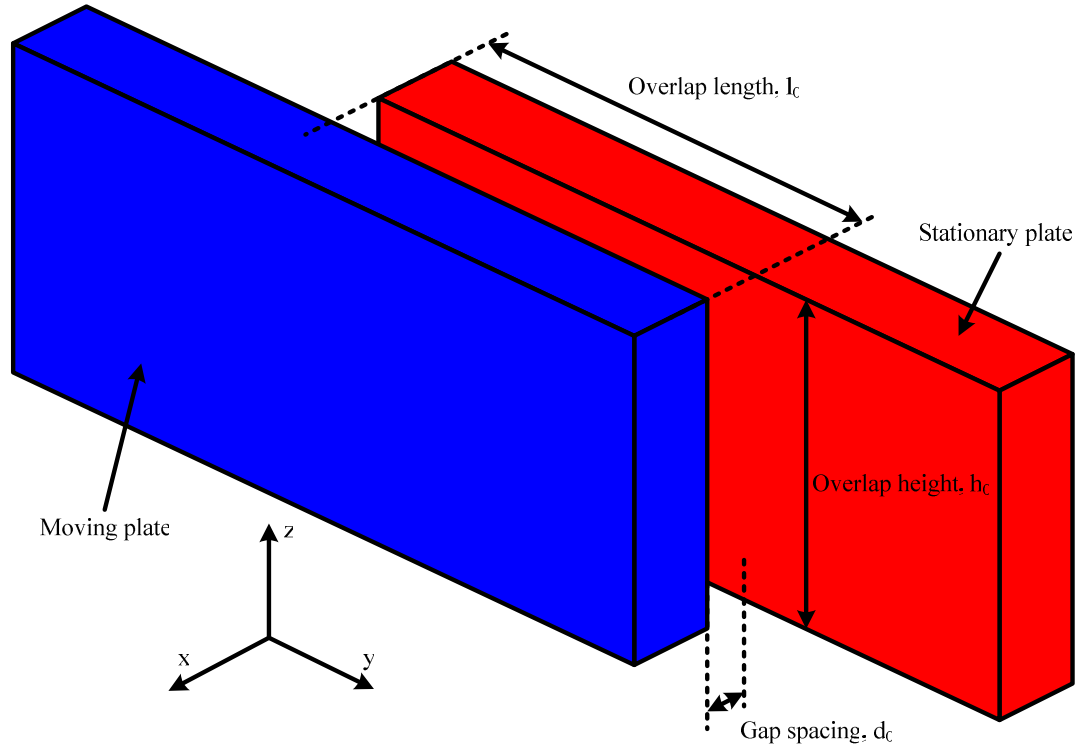


Figure 2.3: Parallel plate capacitor configuration.

The capacitance between these two plates is defined by the overlap area and can be calculated as

$$C = \alpha \cdot \epsilon_0 \cdot \frac{l(y) \cdot h(z)}{d(x)} \quad (2.8)$$

where, α is the fringing-field correction factor, and ϵ_0 is the permittivity of air. $d(x)$, $l(y)$, and $h(z)$ are position dependent parameters and can be defined using the equation set 2.9 – 2.11.

$$d(x) = d_0 + x \quad (2.9)$$

$$l(y) = l_0 + y \quad (2.10)$$

$$h(z) = h_0 + z \quad (2.11)$$

The force generated in a parallel-plate capacitor configuration described in Figure 2.3 can be found from differentiating the energy stored in the capacitor with respect to the displacement. The energy stored in a capacitor is

$$E = \frac{1}{2} \cdot C \cdot V^2 \quad (2.12)$$

where, V is the potential difference between the capacitor plates. Differentiating Equation 2.12 with respect to the displacement, one can obtain the electrostatic force generated as

$$F = \frac{\partial E}{\partial r} = \frac{1}{2} \cdot \frac{\partial C}{\partial r} \cdot V^2 \quad (2.13)$$

where, r is the position vector. This equation assumes that the voltage across the plates does not change with position, which is valid for actuation plates. Using Equation 2.8 in Equation 2.13 and taking the partial derivatives for each direction, x-y-z dependent electrostatic forces can be found as given equation set 2.14 – 2.16.

$$F_x = \frac{1}{2} \cdot \frac{\partial C}{\partial d} \cdot \frac{\partial d}{\partial x} \cdot V^2 = -\frac{1}{2} \cdot \alpha \cdot \epsilon_0 \cdot \frac{l_0 \cdot h_0}{(d_0 + x)^2} \cdot V^2 \quad (2.14)$$

$$F_y = \frac{1}{2} \cdot \frac{\partial C}{\partial l} \cdot \frac{\partial l}{\partial y} \cdot V^2 = \frac{1}{2} \cdot \alpha \cdot \epsilon_0 \cdot \frac{h_0}{d_0} \cdot V^2 \quad (2.15)$$

$$F_z = \frac{1}{2} \cdot \frac{\partial C}{\partial h} \cdot \frac{\partial h}{\partial z} \cdot V^2 = \frac{1}{2} \cdot \alpha \cdot \epsilon_0 \cdot \frac{l_0}{d_0} \cdot V^2 \quad (2.16)$$

In these equations, it is assumed that only one position parameter is varying at a time, e.g., when plate moves in x direction, it does not move in y or z directions. It should be noted that the force in y and z directions are position independent, whereas the force in x direction is position dependent, i.e., force applied on the plates increases if the gap spacing decreases. Moreover, it is a nonlinear function of displacement. Although F_x is position dependent and nonlinear with the change in displacement, it is larger than both F_y and F_z , since l_0 and h_0 are larger than d_0 in the gyroscopes under consideration.

2.2.2. Capacitive actuation

As mentioned before, in order to sense the Coriolis acceleration, the drive-mode must be oscillated. If the oscillation frequency is set to the mechanical resonance frequency of the drive-mode, the oscillation will be maximum for the applied force. Therefore, it will be better to excite the drive-mode of the gyroscope by a sinusoidal voltage at the resonance frequency. Assuming the amplitude of the applied sinusoid is V_{ac} and the its radial frequency is ω , the electrostatic force generated can be found as given in Equation 2.17.

$$F = \frac{1}{2} \cdot \frac{\partial C}{\partial r} \cdot (V_{ac} \cdot \sin(\omega t))^2 = \frac{1}{2} \cdot \frac{\partial C}{\partial r} \cdot \left\{ \underbrace{\frac{V_{ac}^2}{2}}_{\text{Dc force}} - \underbrace{V_{ac}^2 \cdot \cos(2\omega t)}_{\text{Sinusoidal force at } 2\omega} \right\} \quad (2.17)$$

Equation 2.17 shows that applying a pure sinusoidal voltage between the capacitor plates generates a constant force and a time-varying force at twice the frequency of the applied signal. In order to obtain the maximum deflection, applied force must be at the resonance frequency of the drive-mode, i.e., ω should be half the resonance frequency, ω_d . ω can be set to one half of ω_d by using external electronics like frequency dividers; however, this will complicate the drive-mode electronics. Instead of using more complex electronics, a simpler approach, which is adding a dc offset to the sinusoid, is preferred for obtaining maximum deflection for constant amplitude. When dc offset is added to the applied signal, Equation 2.17 becomes

$$F = \frac{1}{2} \cdot \frac{\partial C}{\partial r} \cdot (V_{dc} + V_{ac} \cdot \sin(\omega t))^2 = \frac{1}{2} \cdot \frac{\partial C}{\partial r} \cdot \left\{ \underbrace{V_{dc}^2 + \frac{V_{ac}^2}{2}}_{\text{Dc force}} + \underbrace{2 \cdot V_{dc} \cdot V_{ac} \cdot \sin(\omega t)}_{\text{Sinusoidal force at } \omega} - \underbrace{V_{ac}^2 \cdot \cos(2\omega t)}_{\text{Sinusoidal force at } 2\omega} \right\} \quad (2.18)$$

It can be seen that if a sinusoidal with a dc offset is applied between the stationary and movable electrodes of the drive-mode, sinusoidal forces at frequencies ω and 2ω can be obtained. Hence, if ω is set equal to ω_d , the external drive electronics will be much simpler as the only frequency at the output of the drive-mode will be its mechanical resonance frequency. The sinusoidal force at frequency 2ω will be filtered by the gyroscope, since the drive-mode is a high-quality bandpass filter with center frequency, ω_d .

In the literature, there are two basic finger configurations for actuating the drive-mode. The first method is using varying-overlap-area type fingers and the second one is the varying-gap type fingers. Figure 2.4(a) shows varying-overlap-

area type drive electrodes, whereas, Figure 2.4(b) shows varying-gap type drive electrodes. In both of these configurations, the moving and stationary electrodes are assumed to be aligned in z-direction, i.e., there is no net force applying on moving electrode in this direction. For the configuration given in Figure 2.4(a), since one finger of the moving electrode is same distance away from the neighboring fingers of the stationary electrode, F_{x1} is equal to F_{x2} . Therefore, there will be not net force in x-direction. However, if the oscillation magnitude in y direction increases, side instability may occur and fingers may also move in x-direction [23].

There will also be a net force in y-direction which can be derived from Equation 2.15 and found as

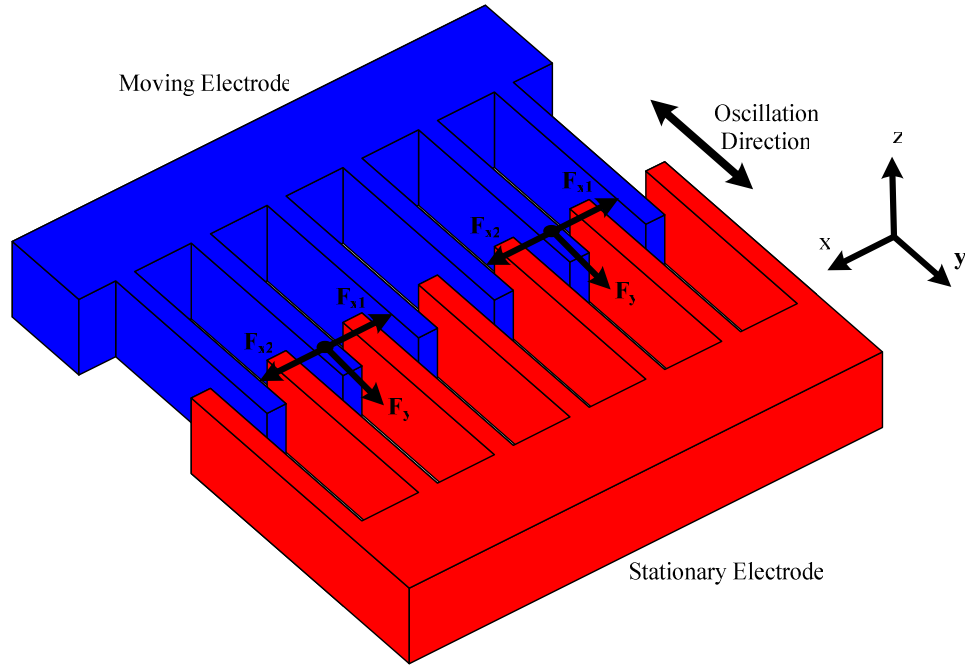
$$F_{total,y} = N \cdot F_y = N \cdot \alpha \cdot \epsilon_0 \cdot \frac{h_0}{d_0} \cdot V^2 \quad (2.19)$$

where, N is the number of comb finger pairs. Considering the configuration given in Figure 2.4(b), the net force along the oscillation direction is determined by the difference of F_{x0} , force applied by the pairing finger, and F_{xa} , force applied by the anti-gap finger. The net force can be found by the following equation:

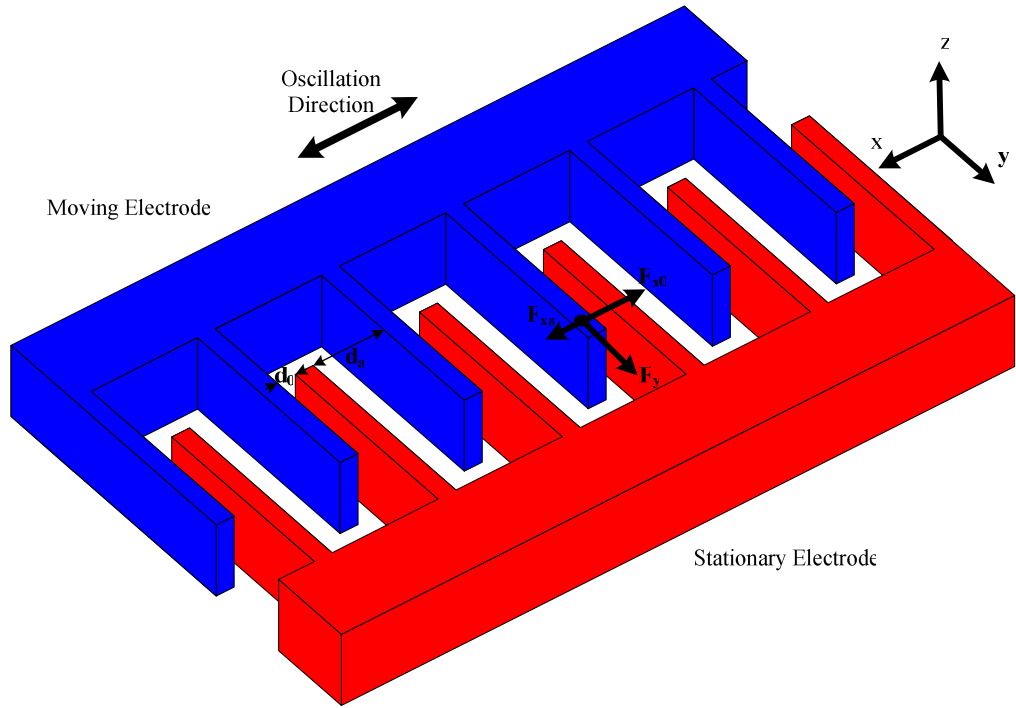
$$F_{total,x} = N \cdot (F_{x0} - F_{xa}) = \frac{1}{2} \cdot N \cdot \alpha \cdot \epsilon_0 \cdot h_0 \cdot l_0 \left(\frac{1}{(d_0 + x)^2} - \frac{1}{(d_a - x)^2} \right) \cdot V^2 \quad (2.20)$$

where, d_a is the spacing between two anti-gap fingers. As d_a is larger than d_0 and assuming displacement, x , is much smaller than d_0 , Equation 2.20 simplifies to

$$F_{total,x} = N \cdot (F_{x0} - F_{xa}) = \frac{1}{2} \cdot N \cdot \alpha \cdot \epsilon_0 \cdot h_0 \cdot l_0 \cdot \left(\frac{1}{d_0^2} - \frac{1}{d_a^2} \right) \cdot V^2 \quad (2.21)$$



(a)



(b)

Figure 2.4: (a) Varying-overlap-area type drive electrodes and (b) varying-gap type drive electrodes.

For the configuration in Figure 2.4(b), orthogonal forces, F_y , acting on the moving electrode will be canceled by the forces acting on the moving electrodes at the other end of the gyroscope. Therefore, there will be no net force in the y-direction.

2.2.3. Capacitive sensing

In the capacitive vibratory gyroscopes, capacitive sensing is required for two purposes: 1) sensing oscillation magnitude of the drive-mode, and 2) sensing Coriolis induced sense-mode displacements. The idea behind capacitive sensing is keeping the voltage across the variable gyroscope capacitor constant, so that a charge is injected from the capacitor when its capacitance changes. The injected current is converted to a voltage signal on the input impedance of the readout electronics connected in series to the capacitor of the gyroscope. This voltage is sampled by a voltage amplifier. Figure 2.5 shows a basic readout configuration for capacitive sensing in vibratory gyroscopes.

To maintain a constant voltage across the capacitor plates of the gyroscope, the proof mass is set to a constant dc voltage, V_{PM} , externally. The counter stationary electrode which the readout electronics are connected to is also set to a dc point by some biasing architecture which will be analyzed in detail in Chapter 3. $C_s(t)$ in Figure 2.5 models the sensor capacitance when the gyroscope is oscillating and can be defined with the function

$$C_s(t) = C_0 + \frac{\partial C}{\partial x} \cdot x(t) \quad (2.22)$$

where, C_0 is the stationary sensor capacitance and $x(t)$ is the time-varying displacement of the moving electrode of the sensor.

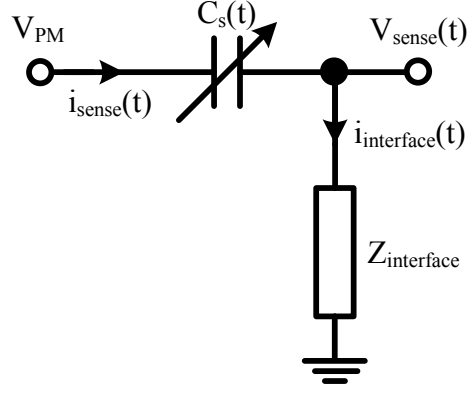


Figure 2.5: Basic readout configuration for capacitive sensing.

Voltage at the input of readout electronics, $V_{sense}(t)$, has two components. One of them is the dc voltage bias, and the other one is the sinusoidal voltage resulting from the charge induced from the oscillating gyroscope. $V_{sense}(t)$ is defined as

$$V_{sense}(t) = V_{BIAS} + v_{sense}(t) \quad (2.23)$$

To find the voltage induced at the input of the interface due to oscillation, first, the current injected from the capacitor should be calculated. The injected current can be found from the time derivative of the charge on the capacitor plates as

$$i_{sense}(t) = \frac{\partial Q_s}{\partial t} = \frac{\partial}{\partial t} (C_s(t) \cdot (V_{PM} - V_{sense}(t))) \quad (2.24)$$

Taking the derivative of multiplication of the capacitance and voltage yields

$$i_{sense}(t) = (V_{PM} - V_{sense}(t)) \cdot \frac{\partial C_s(t)}{\partial t} + C_s(t) \cdot \frac{\partial (V_{PM} - V_{sense}(t))}{\partial t} \quad (2.25)$$

As the $\partial C/\partial x$ of the sensor capacitor is a design parameter and constant for a gyroscope, the current equation can be analyzed with respect to displacement as

$$i_{sense}(t) = (V_{PM} - V_{BIAS} - v_{sense}(t)) \cdot \frac{\partial \left(C_0 + x(t) \cdot \frac{\partial C}{\partial x} \right)}{\partial t} + \left(C_0 + x(t) \cdot \frac{\partial C}{\partial x} \right) \cdot \frac{d(V_{PM} - V_{BIAS} - v_{sense}(t))}{dt} \quad (2.26)$$

Arranging the terms in Equation 2.26, the time-varying sense current can be found as

$$i_{sense}(t) = (V_{PM} - V_{BIAS}) \cdot \frac{\partial C}{\partial x} \cdot \frac{\partial x(t)}{\partial t} - \frac{\partial C}{\partial x} \cdot v_{sense}(t) \cdot \frac{\partial x(t)}{\partial t} - C_0 \cdot \frac{\partial v_{sense}(t)}{\partial t} - \frac{\partial C}{\partial x} \cdot x(t) \cdot \frac{\partial v_{sense}(t)}{\partial t} \quad (2.27)$$

To have a better insight on Equation 2.27, it is better to convert it to phasor domain as

$$\begin{aligned} I_{sense}(j\omega) = & (V_{PM} - V_{BIAS}) \cdot \frac{\partial C}{\partial x} \cdot j\omega \cdot X(j\omega) - \underbrace{\frac{\partial C}{\partial x} \cdot (V_{sense}(j\omega) * (j\omega \cdot X(j\omega)))}_{\text{signal component at } 2\omega} \\ & - C_0 \cdot j\omega \cdot V_{sense}(j\omega) - \underbrace{\frac{\partial C}{\partial x} \cdot (X(j\omega) * (j\omega \cdot V_{sense}(j\omega)))}_{\text{signal component at } 2\omega} \end{aligned} \quad (2.28)$$

The first term of Equation 2.28 represents the motional current, which is the transformation of the Coriolis acceleration to the electrical signal. Summation of the second and fourth term gives the modulation current, and its frequency is twice the frequency of the displacement signal. Therefore, when the closed loop control electronics are connected, this term will not actuate the gyroscope as the gyroscope is a high-quality mechanical filter. The third term of Equation 2.28 is called the feed-through current, and it flows through the stationary capacitance of the sensor.

Since the current injected from the sensor will only be drawn by the interface circuit, $i_{sense}(t)$ should be equal to $i_{interface}(t)$. Equating these two currents in phasor domain yields

$$I_{interface}(j\omega) = I_{sense}(j\omega) = (V_{PM} - V_{BIAS}) \cdot \frac{\partial C}{\partial x} \cdot j\omega \cdot X(j\omega) - C_0 \cdot j\omega \cdot V_{sense}(j\omega) \quad (2.29)$$

At this point, it should be noted that, the modulation current derived in Equation 2.28 is discarded for simplicity.

On the other hand, $i_{interface}(t)$ only includes an ac current since there is no path that dc current can flow through the gyroscope. Therefore, $i_{interface}(t)$ can be found as

$$I_{interface}(j\omega) = \frac{V_{sense}(j\omega)}{Z_{interface}(j\omega)} \quad (2.30)$$

Equating the results of Equations 2.29 and 2.30,

$$\frac{V_{sense}(j\omega)}{Z_{interface}(j\omega)} = (V_{PM} - V_{BIAS}) \cdot \frac{\partial C}{\partial x} \cdot j\omega \cdot X(j\omega) - C_0 \cdot j\omega \cdot V_{sense}(j\omega) \quad (2.31)$$

From this equation, $V_{sense}(j\omega)$ can be extracted as

$$V_{sense}(j\omega) = (V_{PM} - V_{BIAS}) \cdot \frac{\partial C}{\partial x} \cdot \left(Z_{interface}(j\omega) // \frac{1}{j\omega \cdot C_0} \right) \cdot j\omega \cdot X(j\omega) \quad (2.32)$$

Equation 2.32 shows that voltage at the input of the readout electronics can be related to the velocity of the oscillation in terms of some constants and the effective input-impedance which is defined by the equation

$$Z_{eff}(j\omega) = Z_{interface}(j\omega) // \frac{1}{j\omega \cdot C_0} \quad (2.33)$$

It can be seen that stationary sensor capacitance behaves like a capacitor added in parallel to the input impedance of the interface circuit.

2.2.4. Optimum capacitive electrodes for electrostatic actuation and capacitive sensing

For the drive-mode, the aim is to oscillate the gyroscope as large as possible so that the sensitivity is maximized. Additionally, it is preferred to have linear response at the drive-mode for the ease of controlling the amplitude of oscillation by external electronics. Although Equation 2.21 implies that varying-gap type fingers generate larger electrostatic forces compared to varying-overlap-area type fingers, the dependence of the force to the reciprocal of displacement creates the nonlinearity problem for large amplitude oscillations. As the fingers oscillate in the drive direction, the force applied by the stationary electrode changes for the varying-gap type fingers. When the displacement is small compared to the gap spacing, this effect is negligible. However, small oscillation magnitude results in small sensitivity to rate input. Therefore, it is not feasible to obtain large drive oscillation magnitude with varying-gap type fingers. With the current micromachining technologies like Deep Reactive-Ion-Etching (DRIE), the gap spacing is on the order of a few microns. Therefore, the maximum achievable oscillation magnitude with the varying-gap type fingers is around a few hundred angstroms. On the other hand, it is possible to obtain linear oscillation on the order of a few microns by using varying-overlap-area type fingers. However, to preserve the symmetry and obtain

large oscillation at the drive-mode, sensing-end of this mode should also be implemented by varying-overlap-area type fingers. Sensitivity of this type of fingers is small, but since the oscillation magnitude is large, the signal that will be generated at the sensing-end of the drive-mode will be large enough for detection by the interface electronics.

On the other hand, for the sense-mode, the movement due to Coriolis acceleration is very small, i.e., on the order of a few Angstroms. Therefore, a very-sensitive finger type should be used in the sense-mode, i.e., the change in the capacitance with respect to displacement, i.e., $\partial C/\partial x$, must be as large as possible. Considering $\partial C/\partial x$ of varying-overlap-area type fingers

$$\frac{\partial C}{\partial x} = \alpha \cdot \epsilon_0 \cdot \frac{h_0}{d_0} \quad (2.34)$$

On the other hand, $\partial C/\partial x$ of varying-gap type fingers is

$$\frac{\partial C}{\partial x} = -\alpha \cdot \epsilon_0 \cdot \frac{l_0 \cdot h_0}{d_0^2} \quad (2.35)$$

Since l_0 is much larger than d_0 in the current micromachining technologies, $\partial C/\partial x$ of varying-gap type fingers is much larger than $\partial C/\partial x$ of varying-overlap-area type fingers in magnitude. Moreover, the displacement in the sense-mode is so small that varying-gap does not create much nonlinearity. Therefore, it is better to use varying-gap type fingers for sensing the Coriolis acceleration in the sense-mode.

2.3. Modeling Capacitive Vibratory Gyroscopes

Development of an accurate model of a gyroscope plays an important role in simulating the readout circuits and external electronics, especially for closed-loop control electronics. To achieve a more realistic result, the model should include electrical and mechanical phenomena as much as possible.

In order to get a model in the electrical domain, an electromechanical conversion should be done for both actuation and sensing mechanisms. Figure 2.6 shows the block diagram of the electromechanical conversion algorithm used for the electrical model of the gyroscope. The model should have an electrical input for actuation, then, should convert electrical domain to mechanical domain by using the electrical equivalents of mechanical parameters. When the actuation is achieved, motions in the drive and sense modes of the gyroscope should be converted to electrical domain for sensing.

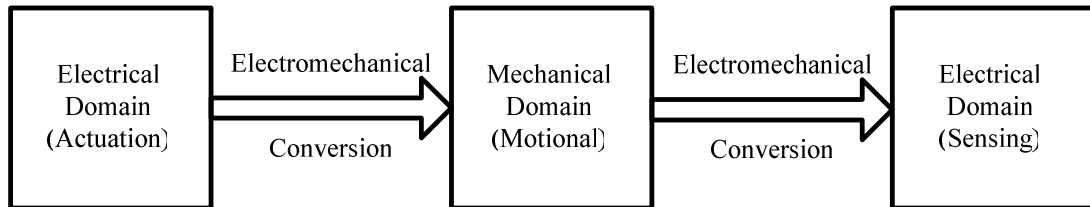


Figure 2.6: Block diagram of electromechanical conversion algorithm used for the electrical model of the gyroscope.

In order to make the electromechanical conversion, electrical equivalents of mechanical parameters should be known. Since the gyroscope is modeled as a second-order mass-damper-spring system, electrical equivalents of these parameters in addition to force, displacement, and velocity are needed. The electrical equivalent of the second-order mass-damper-spring system is the basic serially connected

resistor-inductor-capacitor (RLC) circuit. Table 2.1 gives the electrical equivalents of the mechanical parameters necessary for modeling the gyroscope [9]. This analogy can be easily extracted by equating the input voltage of the circuit to the total of voltages across each component. Figure 2.7(a) shows a 2-DOF mass-damper-spring system and Figure 2.7(b) shows the basic series RLC circuit modeling 1-DOF of this system. In the electrical domain model, mechanical analogies of electrical domain parameters are displayed for convenience.

The two different mechanisms in electromechanical conversion of the gyroscope are actuation and sensing. These parts are analyzed in detail in the following subsections.

Table 2.1: Electrical equivalents of the mechanical parameters necessary for modeling the gyroscope [9].

Mechanical Parameter	Electrical Equivalent
Mass (m)	Inductance (L)
Damping coefficient (b)	Resistance (R)
Spring (k)	Reciprocal of capacitance ($1/C$)
Force (F)	Voltage (V)
Displacement (x)	Electrical charge (Q)
Velocity ($\dot{x}$)	Current (I)

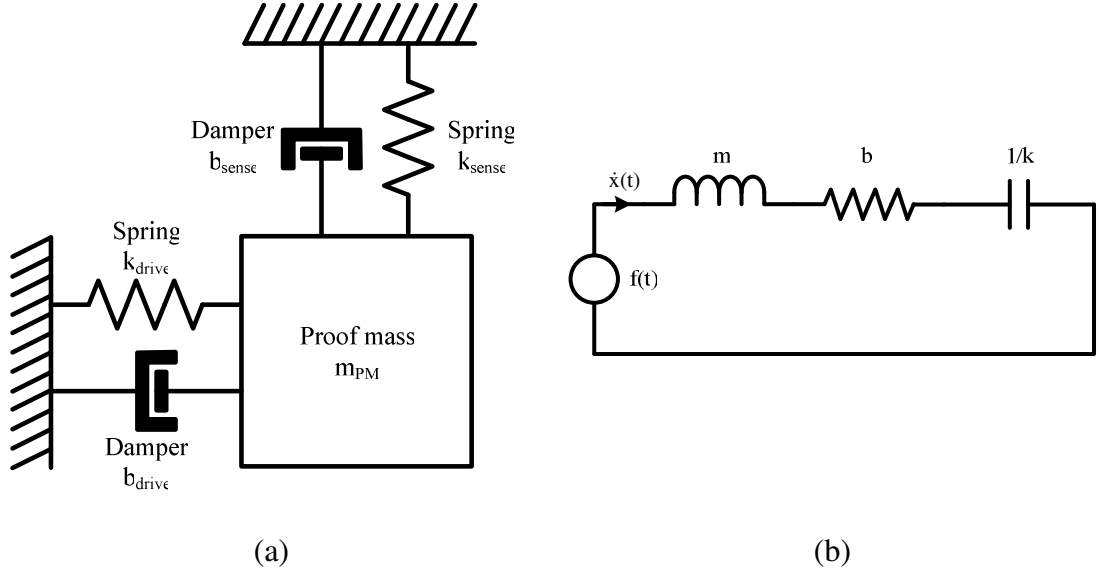


Figure 2.7: (a) 2-DOF mass-damper-spring system and (b) Basic series RLC circuit modeling 1-DOF of this system.

2.3.1. Modeling capacitive actuation in the vibratory gyroscope

The gyroscope is driven into resonance by the sinusoidal voltage applied to the stationary drive-motor electrode. The actuation voltage, $V_a(t)$, is defined as

$$V_a(t) = (V_{PM} - V_{DM}) + v_a(t) \quad (2.36)$$

where, V_{PM} is the proof mass voltage, V_{DM} is the dc point of the stationary drive-motor electrode, and $v_a(t)$ is the ac component of the actuation signal. Drive-motor capacitance, $C_{dm}(t)$, is

$$C_{dm}(t) = C_{0,dm} + x(t) \cdot \frac{\partial C_{dm}}{\partial x} \quad (2.37)$$

where, $C_{0,dm}$ is the stationary drive-motor capacitance. Using Equations 2.36 and 2.37, current through actuation source, $i_a(t)$, can be formulated as

$$i_a(t) = V_a(t) \cdot \frac{\partial C_{dm}(t)}{\partial t} + C_{dm}(t) \cdot \frac{\partial V_a(t)}{\partial t} \quad (2.38)$$

Using a similar analysis done in Section 2.2.3, $I_a(j\omega)$ is found as

$$\begin{aligned} I_a(j\omega) = & (V_{PM} - V_{DM}) \cdot \frac{\partial C_{dm}}{\partial x} \cdot j\omega \cdot X(j\omega) - \underbrace{\frac{\partial C_{dm}}{\partial x} \cdot (V_a(j\omega) * (j\omega \cdot X(j\omega)))}_{\text{signal component at } 2\omega} \\ & - C_{0,dm} \cdot j\omega \cdot V_a(j\omega) - \underbrace{\frac{\partial C_{dm}}{\partial x} \cdot (X(j\omega) * (j\omega \cdot V_a(j\omega)))}_{\text{signal component at } 2\omega} \end{aligned} \quad (2.39)$$

Similar to the result found in Equation 2.28, the current through the actuation source is composed of three parts. The first term in Equation 2.39 is the motional current, which generates the motion in the gyroscope. The summation of second and fourth terms is the modulation current and this component is filtered out in the mechanical filter. The third term in the above equation is the feed-through current and is not used for any purpose.

In order to convert the applied voltage signal to its mechanical analogue, which is force, a transformer can be used in the model. Then, voltage across the primary coil of the transformer is the actuation voltage, $V_a(t)$, whereas, the voltage across the secondary coil is the force applied on the moving end of the drive-mode electrode. Similarly, the current passing through the primary coil is the motional current,

whereas, the current passing through the secondary coil is the velocity of the drive-mode fingers. Figure 2.8 shows the model of the drive-mode of the gyroscope. $i_a(t)$ in the figure is the actuation current defined with Equation 2.39. Similarly, $i_f(t)$ is the feed-through current and $i_m(t)$ is the motional current. The transformer ratio given in Figure 2.8, n_{DM} , is used for transformation of the actuation voltage to the force applied on the drive-mode of the gyroscope and can be found by the current ratios in the transformer. Analyzing the model presented in Figure 2.8, the motional current in terms of the velocity drive-mode is given by

$$i_m(t) = n_{DM} \cdot \dot{x}_D(t) \quad (2.40)$$

Using the formula of the motional current given in Equation 2.39, Equation 2.40 in the phasor domain becomes

$$(V_{PM} - V_{DM}) \cdot \frac{\partial C_{dm}}{\partial x} \cdot j\omega \cdot X_D(j\omega) = n_{DM} \cdot j\omega \cdot X_D(j\omega) \quad (2.41)$$

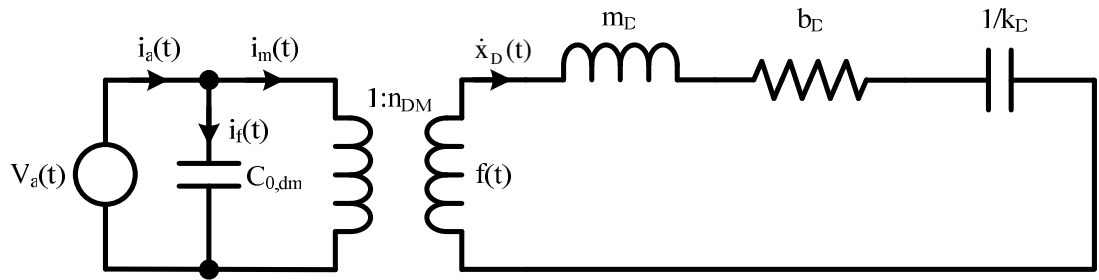


Figure 2.8: Model of the drive-mode of the vibratory gyroscope.

Therefore, transformer ratio, n_{DM} , is found as

$$n_{DM} = (V_{PM} - V_{DM}) \cdot \frac{\partial C_{dm}}{\partial x} \quad (2.42)$$

As the transformation ratio is obtained, one can derive the magnitude of oscillation. By using the transformer model, $F_D(j\omega)$ is found as

$$F_D(j\omega) = n_{DM} \cdot V_a(j\omega) \quad (2.43)$$

Since we need the ratio of the magnitude of oscillation to the actuation signal, using Equations 2.4 and 2.43 and assuming gyroscope is driven at its mechanical resonance frequency, one can obtain the ratio as

$$\frac{X_D(j\omega)}{V_a(j\omega)} = \frac{X_D(j\omega)}{F_D(j\omega)} \cdot \underbrace{\frac{F_D(j\omega)}{V_a(j\omega)}}_{n_{DM}} \quad (2.44)$$

$$\frac{X_D(j\omega)}{V_a(j\omega)} = -j \cdot \frac{n_{DM} \cdot Q_D}{k_D} \quad (2.45)$$

Converting the result found in Equation 2.45 into time domain and using Equation 2.42 yields

$$x_D(t) = \frac{(V_{PM} - V_{DM}) \cdot \frac{\partial C_{dm}}{\partial x} \cdot Q_D}{k_D} \cdot v_a \left(t - \frac{\pi}{2\omega} \right) \quad (2.46)$$

If we apply a sinusoidal signal to the input of the gyroscope, in the form of

$$v_a(t) = V_a \cdot \sin(\omega t) \quad (2.47)$$

then, drive-mode oscillation will be

$$x_D(t) = -\frac{(V_{PM} - V_{DM}) \cdot V_a \cdot \frac{\partial C_{dm}}{\partial x} \cdot Q_D}{k_D} \cdot \cos(\omega t) \quad (2.48)$$

which lags the input signal defined in Equation 2.47 by 90° .

2.3.2. Modeling Coriolis acceleration

The Coriolis force generated in the sense-mode due to a rate input is given as [9]

$$F_{Coriolis}(t) = 2 \cdot m_{PM} \cdot \Omega_z \cdot \dot{x}_D(t) \quad (2.49)$$

The current flowing in the secondary coil of the transformer in Figure 2.8 is the velocity of the drive-mode, and hence, we can model the Coriolis force by a current dependent voltage source. Figure 2.9 shows the model of the Coriolis force acting on the sense-mode of the gyroscope.

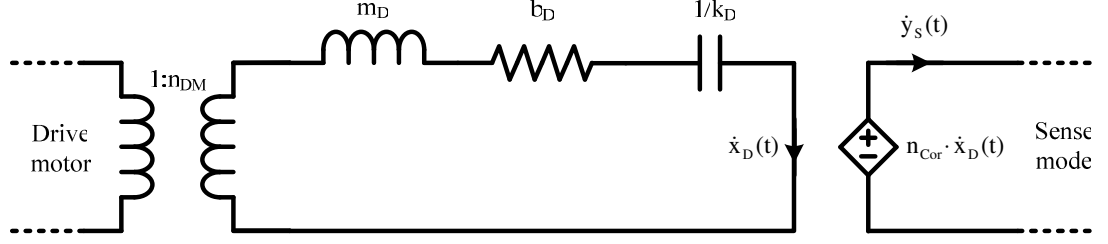


Figure 2.9: Model of Coriolis force acting on the sense-mode of the gyroscope.

n_{Cor} given in Figure 2.9 is the ratio of the Coriolis force to the velocity of the drive-mode and can be extracted from Equation 2.49 as

$$n_{Cor} = 2 \cdot m_{PM} \cdot \Omega_z \quad (2.50)$$

2.3.3. Modeling capacitive sensing in the vibratory gyroscope

As mentioned before, capacitive sensing is required for two purposes, one for sensing drive-mode displacements and the other for sensing Coriolis-induced sense-mode displacements. In this subsection, only drive-mode displacements will be modeled. Modeling of sensing the Coriolis-induced sense-mode displacements is very similar to sensing the drive-mode displacements.

In Section 2.2.3, the output voltage is expressed in terms of velocity in Equation 2.32. Therefore, it is possible to convert the mechanical parameter, $\dot{x}_D(t)$, to its electrical analogue, $i_{sense}(t)$, by using a current controlled current source. A transformer cannot be used in this case, since the impedance in electrical domain will affect the mechanical resonance frequency if a transformer is used. Figure 2.10 shows the model of the sensing-end of the drive-mode of the gyroscope. As seen from the figure, stationary capacitance of the drive-mode pick electrode is added in

parallel to the impedance of the capacitive interface due to the feed-through current generated between the plates of the pick electrode.

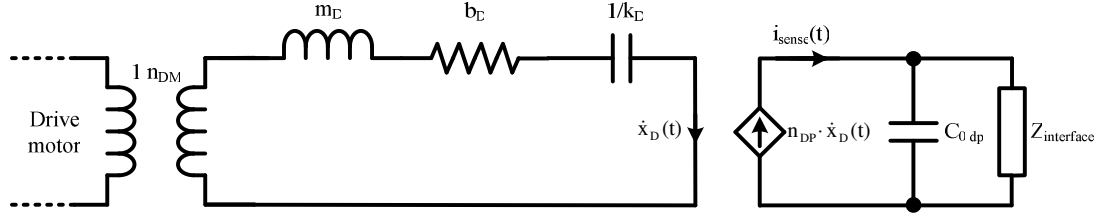


Figure 2.10: Model of sensing-end of the drive-mode of the vibratory gyroscope.

The current multiplier, n_{DP} , defined in Figure 2.10 is the ratio of the output current, $i_{sense}(t)$, to the velocity of the drive-mode, $\dot{x}_D(t)$, and is given by

$$i_{sense}(t) = n_{DP} \cdot \dot{x}_D(t) \quad (2.51)$$

Therefore, using Equations 2.32 and 2.51, the current multiplier is calculated as

$$n_{DP} = (V_{PM} - V_{BIAS}) \cdot \frac{\partial C_{dp}}{\partial x} \quad (2.52)$$

2.3.4. Modeling force-feedback in the vibratory gyroscope

The quality factor of the gyroscope is very high at vacuum, in the order of a few thousands, since air damping is negligible for vacuum operation. Therefore, the bandwidth of the gyroscope is limited to a few Hertz. In order to increase the bandwidth of the gyroscope and also to increase the linearity of the response, a

force-feedback control loop is implemented in high-performance gyroscopes. The idea behind the force-feedback is to stop the motion of the sense-mode due to the Coriolis force. This way, electronic damping is added to the system so that the quality factor of the sense-mode is decreased to a desired level.

Force-feedback electrodes are implemented similar to the varying-gap type fingers of the sense-mode. An actuation signal generated from an external control loop is applied to these electrodes so that the motion in the sense-mode is vanished during operation. The model of the force-feedback electrodes is the same as drive-motor electrodes. Figure 2.11 shows the model of the force-feedback electrodes in the sense-mode of the vibratory gyroscope. The aim of the force-feedback loop is to vanish the current, $\dot{y}_s(t)$, flowing in the mechanical domain given in Figure 2.11.

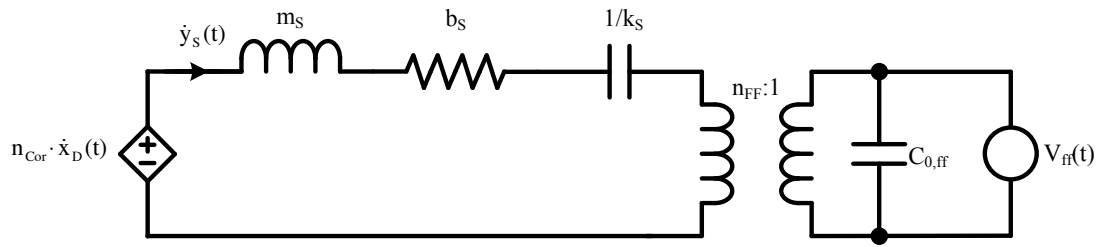


Figure 2.11: Model of the force-feedback electrodes in the sense-mode of the vibratory gyroscope.

The transformer ratio, n_{FF} , can be calculated as

$$n_{FF} = (V_{PM} - V_{FF}) \cdot \frac{\partial C_{ff}}{\partial x} \quad (2.53)$$

where, V_{FF} is the dc point of the force-feedback electrodes.

2.3.5. Generalized model for a fully-differential gyroscope with force-feedback electrodes

The generalized model for a fully-differential gyroscope can be generated from the circuits introduced in the previous four subsections. Figure 2.12 shows the schematic of the generalized model of a fully-differential gyroscope with force-feedback electrodes.

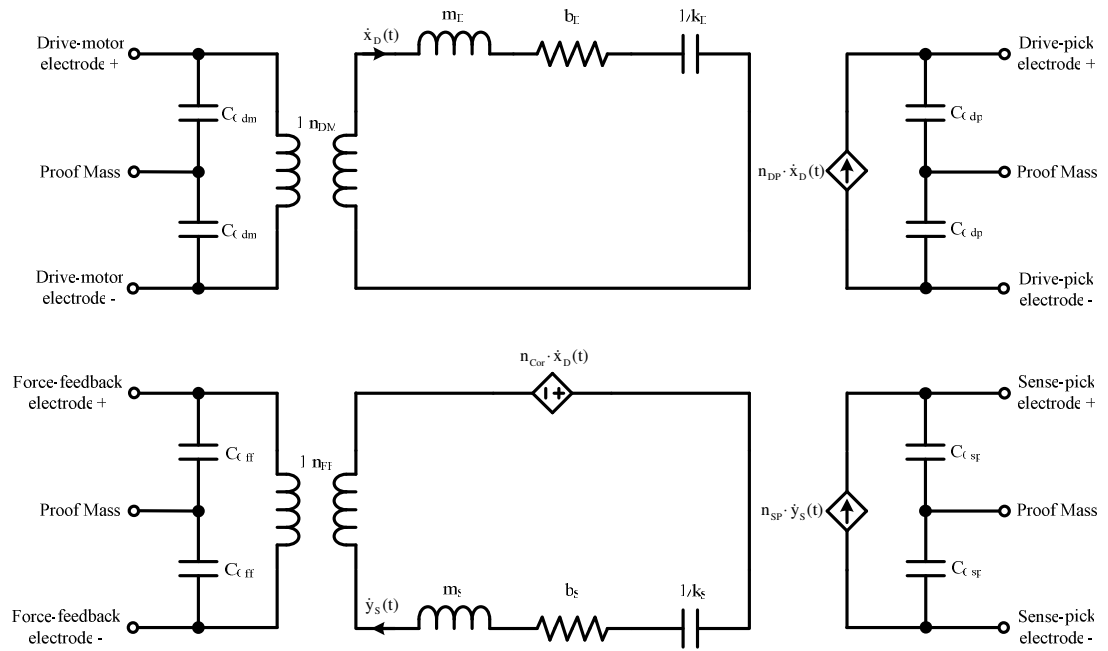


Figure 2.12: Schematic of the generalized model of a fully-differential gyroscope with force-feedback electrodes.

The parameters introduced in this figure correspond to both mechanical and electrical parameters. Some of these parameters are design parameters, and these are determined by either the designer of the mechanical structure or the designer of the interface circuitry. Another part of these parameters is the input to the system, where the input values depend on the application using the gyroscope. The final part

of these parameters includes some formulations, which are functions of either the design parameters, input parameters, or both. The equations given below summarize the formulations necessary to be implemented into the simulator software.

$$n_{DM} = (V_{PM} - V_{DM}) \cdot \left(\frac{\partial C_{dm}}{\partial x} - \frac{\partial C_{dm,a}}{\partial x} \right) \quad (2.54)$$

$$n_{DP} = (V_{PM} - V_{BIAS}) \cdot \left(\frac{\partial C_{dp}}{\partial x} - \frac{\partial C_{dp,a}}{\partial x} \right) \quad (2.55)$$

$$n_{Cor} = 2 \cdot m_{PM} \cdot \Omega_z \quad (2.56)$$

$$n_{SP} = (V_{PM} - V_{BIAS}) \cdot \left(\frac{\partial C_{sp}}{\partial x} - \frac{\partial C_{sp,a}}{\partial x} \right) \quad (2.57)$$

$$n_{FF} = (V_{PM} - V_{FF}) \cdot \left(\frac{\partial C_{ff}}{\partial x} - \frac{\partial C_{ff,a}}{\partial x} \right) \quad (2.58)$$

$$b_D = \frac{\sqrt{k_D \cdot m_D}}{Q_D} \quad (2.59)$$

$$b_S = \frac{\sqrt{k_S \cdot m_S}}{Q_S} \quad (2.60)$$

$$k_D = k_{D,mech} - \frac{\partial^2 C_d}{\partial x^2} \cdot V_{PM}^2 \quad (2.61)$$

$$k_S = k_{S,mech} - \frac{\partial^2 C_s}{\partial x^2} \cdot V_{PM}^2 \quad (2.62)$$

There are additional parameters added to the model due to the formulations given in Equations 2.54 – 2.62. Table 2.2 gives the explanations of the model parameters for

the general model of a fully-differential gyroscope with force-feedback electrodes given in Figure 2.12.

Table 2.2: Explanations of the model parameters for the general model of a fully-differential gyroscope with force-feedback electrodes given in Figure 2.12.

Parameter	Explanation	Parameter Type ¹
$C_{0,dm}$	Stationary drive-motor capacitance	D
$C_{0,dp}$	Stationary drive-pick capacitance	D
$C_{0,sp}$	Stationary sense-pick capacitance	D
$C_{0,ff}$	Stationary force-feedback electrode capacitance	D
m_D	Total mass in drive-mode	D
m_S	Total mass in sense-mode	D
m_{PM}	Effective proof mass	D
b_D	Drive-mode damping factor	F
b_S	Sense-mode damping factor	F
$1/k_D$	Reciprocal of total spring constant in drive-mode	F
$1/k_S$	Reciprocal of total spring constant in sense-mode	F
$k_{D,mech}$	Mechanical spring constant of the drive-mode	D
$k_{S,mech}$	Mechanical spring constant of the sense-mode	D
n_{DM}	Drive-motor transformation ratio	F
n_{DP}	Drive-pick current multiplier	F
n_{Cor}	Coriolis-induced rate transformation ratio	F
n_{SP}	Sense-pick current multiplier	F

¹ D : Design parameter, I : Input parameter, F : Formulation

Table 2.2 cont'd: Explanations of the model parameters for the general model of a fully-differential gyroscope with force-feedback electrodes given in Figure 2.12.

Parameter	Explanation	Parameter Type
n_{FF}	Force-feedback transformation ratio	F
V_{PM}	Proof mass dc voltage	I
V_{DM}	Drive-motor electrode dc voltage	I
V_{BIAS}	Bias voltage at the input of the interface circuit	I
V_{FF}	Force-feedback electrode dc voltage	I
$\partial C_{dm}/\partial x$	$\partial C/\partial x$ of drive-motor capacitance	D
$\partial C_{dm,a}/\partial x$	$\partial C/\partial x$ of drive-motor capacitance (anti-gap)	D
$\partial C_{dp}/\partial x$	$\partial C/\partial x$ of drive-pick capacitance	D
$\partial C_{dp,a}/\partial x$	$\partial C/\partial x$ of drive-pick capacitance (anti-gap)	D
$\partial C_{sp}/\partial x$	$\partial C/\partial x$ of sense-pick capacitance	D
$\partial C_{sp,a}/\partial x$	$\partial C/\partial x$ of sense-pick capacitance (anti-gap)	D
$\partial C_{ff}/\partial x$	$\partial C/\partial x$ of force-feedback capacitance	D
$\partial C_{ff,a}/\partial x$	$\partial C/\partial x$ of force-feedback capacitance (anti-gap)	D
$\partial^2 C_d/\partial x^2$	$\partial^2 C/\partial x^2$ of drive-mode capacitances	D
$\partial^2 C_s/\partial x^2$	$\partial^2 C/\partial x^2$ of sense-mode capacitances	D
Ω_z	Angular rate applied to the system	I
Q_D	Quality factor of the drive-mode	I
Q_S	Quality factor of the sense-mode	I

This model is modified for simulation of single-ended SOG gyroscope presented in Chapter 1. The details, parameters, and simulation results of the models of this gyroscope are given in the following subsection.

2.3.6. Electrical model of the single-ended SOG gyroscope

The single-ended SOG gyroscope presented in Figure 1.4 has one motor and one pick electrode for the drive-mode and two pick electrodes for the sense-mode. Therefore, the model proposed in Figure 2.12 should be adjusted for the single-ended SOG gyroscope. Figure 2.13 shows the modified model for the single-ended SOG gyroscope presented in Figure 1.4.

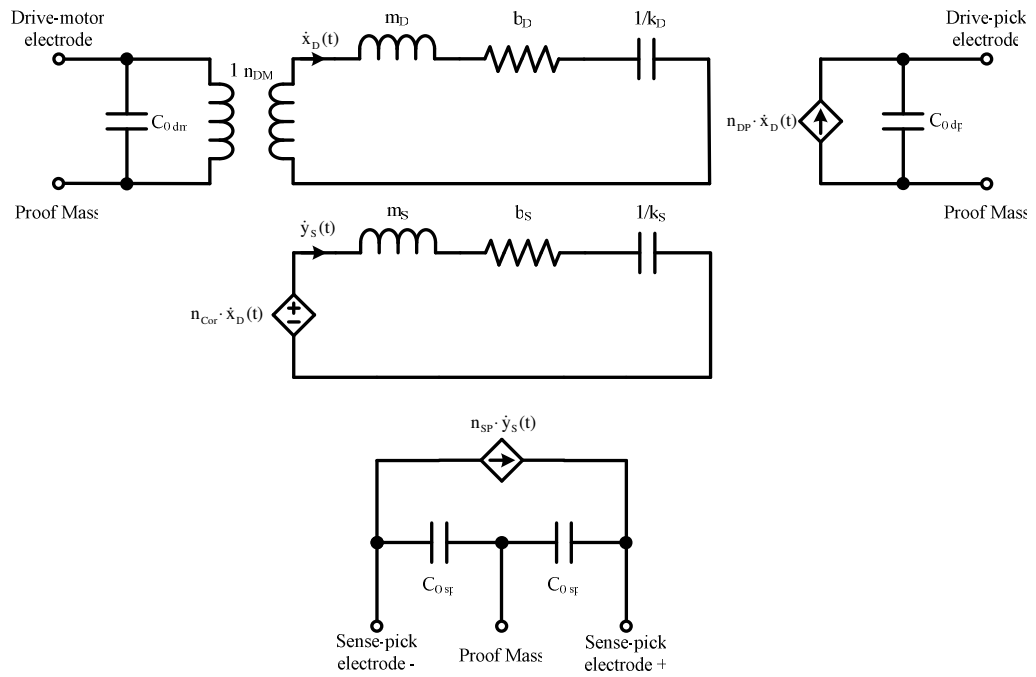


Figure 2.13: Modified model for the single-ended SOG gyroscope presented in Figure 1.4.

2.3.6.1. Model parameters for single-ended SOG gyroscope

This gyroscope is fabricated by using silicon-on-glass micromachining process described in [9]. During DRIE etching of the gyroscope, the structures get thinner

due to the undercut introduced at the beginning of etching and due to the lithography tolerances. Therefore, the parameters defined in Table 2.2 calculated from the layout of the gyroscope may differ after the fabrication of the gyroscope. Hence, it is better to simulate the model of the gyroscope using the parameters modified with a possible value of DRIE undercut. Table 2.3 shows the values of the model parameters for a single-ended SOG gyroscope for both the original design and expected undercut.

2.3.6.2. Simulations of the single-ended SOG gyroscope model

Resonance characteristics of the drive and sense modes of the single-ended SOG gyroscope model are simulated by using a load resistance of 100 k Ω and load capacitance of 3 pF connected in parallel. This load impedance models the behavior of the active probe that is going to be used in resonance response tests.

Figure 2.14 shows the simulation results for drive-mode resonance characteristics of single-ended SOG gyroscope. Resonance peak is observed at 3.644 kHz with magnitude -63.82 dB.

Figure 2.15 shows the simulation results for sense-mode resonance characteristics of the single-ended SOG gyroscope. From this simulation, the resonance frequency of the sense-mode is found as 3.643 kHz, and the gain of the sensor at this frequency is found as -32.52 dB.

Table 2.3: Values of the model parameters for the single-ended model of the SOG gyroscope for both the original design and expected undercut. The values given below are valid for the SOG gyroscope presented in [9]. Expected values after 2 μ m undercut also include the electrostatic fringe-field corrections¹.

Parameter	Unit	Designed Value	Expected Value with 2 μ m Undercut
$C_{0,dm}$	pF	1.753	1.368
$C_{0,dp}$	pF	1.753	1.368
$C_{0,sp}$	pF	15.537	12.119
m_D	kg	8.090×10^{-7}	8.030×10^{-7}
m_S	kg	1.330×10^{-6}	1.280×10^{-6}
b_D	sec/kg	3.389×10^{-4}	1.839×10^{-4}
b_S	sec/kg	2.221×10^{-2}	1.464×10^{-2}
$1/k_D$	1/(N/m)	7.042×10^{-4}	2.375×10^{-3}
$1/k_S$	1/(N/m)	6.821×10^{-4}	1.492×10^{-3}
n_{DM}	-	1.682×10^{-6}	1.063×10^{-6}
n_{DP}	-	1.764×10^{-6}	1.114×10^{-6}
n_{Cor}	-	2.087×10^{-8}	2.087×10^{-8}
n_{SP}	-	1.564×10^{-4}	5.889×10^{-5}

¹ These values are valid for matched-mode operation. The quality factors of the drive and sense modes are assumed to be 100 and 2, respectively. The input to the system is 1 °/sec. The drive-motor dc voltage is set to 0 V, and bias voltage at the input of the interface is -1.4V.

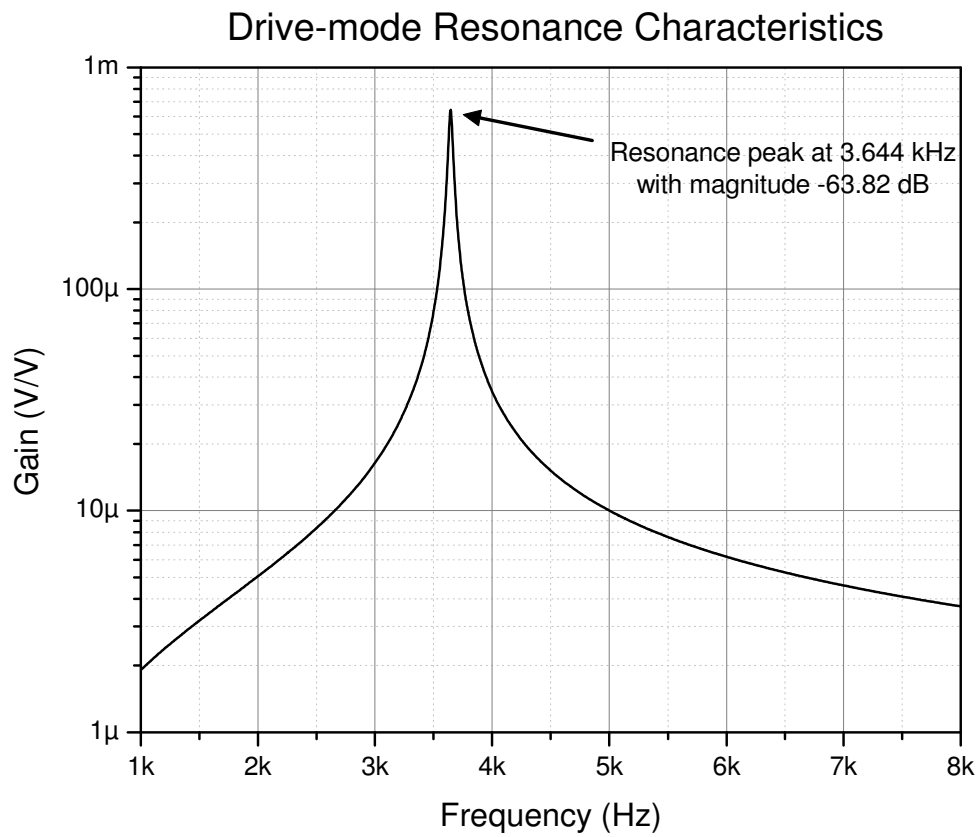


Figure 2.14: Simulation results for drive-mode resonance characteristics of the single-ended SOG gyroscope.

2.4. Requirements for High-Performance Gyroscope Interfaces

The aim of a gyroscope interface is to interface a high-impedance node with a low-impedance node, so that, the signal generated at the output of the mechanical sensor can be processed with conventional circuitry, like amplifiers, demodulators, phase-locked loops, and filters.

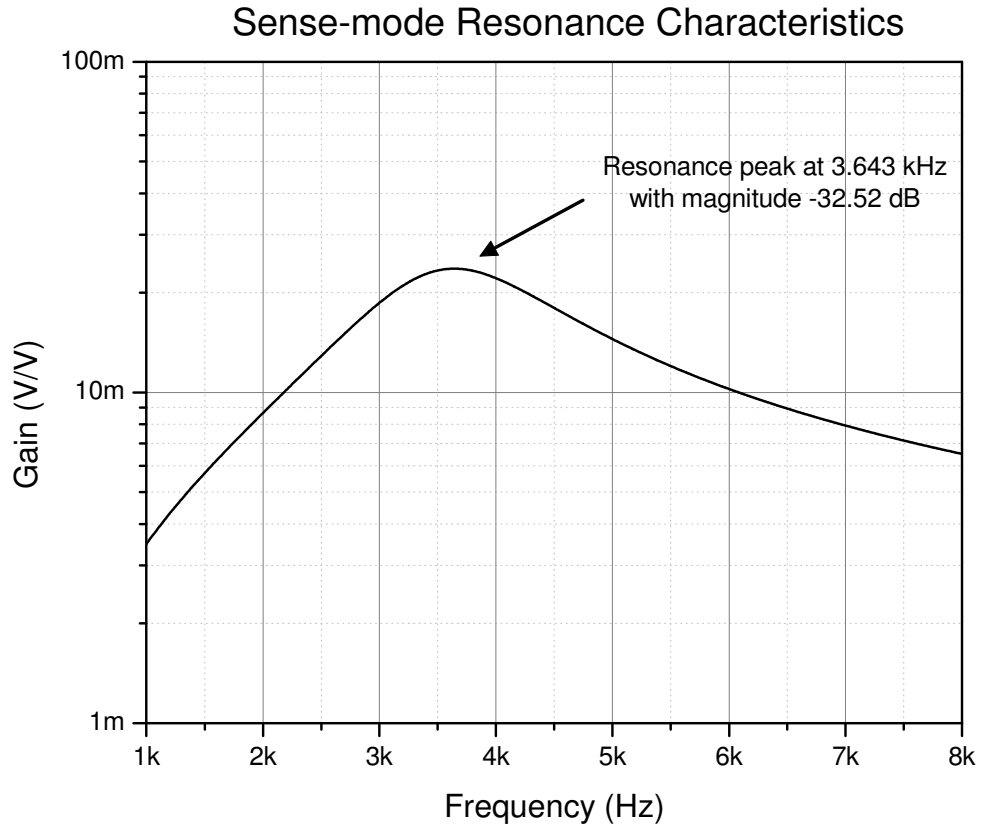


Figure 2.15: Simulation results for sense-mode resonance characteristics of single-ended SOG gyroscope.

To obtain the maximum performance from the capacitive vibratory gyroscope, the signal-to-noise ratio (SNR) of the output of the gyroscope should be maximized. The noise sources in the gyroscope are the mechanical-thermal noise [24] and the electrical noise present in the interface circuit. The mechanical-thermal noise, also known as Brownian noise, depends on several parameters of the mechanical structure. Therefore, this source of noise cannot be controlled by the circuit designer. On the other hand, it is possible to design low-noise gyroscope interfaces for high-performance applications.

The other possibility to increase the SNR of the gyroscope is to increase the signal induced from the motion due to the applied angular rate. The signal extracted from the gyroscope can be found from Equation 2.32, which depends on both the mechanical structure and interface circuit. Increasing the magnitudes of the multiplicands of the terms in this equation will generate larger signals. However, most of these terms are not quite controllable.

The first term of Equation 2.32 is the dc voltage applied across the sensing electrodes. The proof mass voltage in this term is fixed for matched-mode operation, and V_{BIAS} is limited by the supply voltages of the CMOS interface circuit. Therefore, it is not possible to effectively increase this parameter to obtain a high performance from the gyroscope.

The second term in Equation 2.32 is the gradient of capacitance with respect to position and completely depends on the mechanical structure of the sensor. Hence, this parameter is not controllable for the circuit designer.

The fourth term of the equation is Coriolis induced velocity of the sense-mode and is given in Equation 2.5 for the matched-mode operation. Displacement of the drive-mode is the only controllable parameter in this equation. However, there is a maximum limit for this value, and these are determined by the mechanical structure and desired linearity of the drive-mode.

The most important term for the interface designer is the third term of Equation 2.32, which is defined as the effective input-impedance of the interface circuit in Equation 2.33. This value not only limits the performance of the gyroscope, but also determines the type of the interface. There are two major interface types for capacitive gyroscopes. One of them is using resistive interfaces, whereas the other one is using capacitive interfaces.

The input-impedance of a gyroscope interface is generally composed of a parallel combination of a resistor and a capacitor. Figure 2.16 shows the parallel combination of the resistor and capacitor at the input of the gyroscope interface. According to this figure, the input-impedance of the gyroscope interface can be defined as

$$Z_{interface}(j\omega) = R_{interface} // \frac{1}{j\omega \cdot C_{interface}} \quad (2.63)$$

Inserting the above equation to Equation 2.33, the effective input-impedance of the interface becomes

$$Z_{eff}(j\omega) = R_{interface} // \frac{1}{j\omega \cdot C_{interface}} // \frac{1}{j\omega \cdot C_0} \quad (2.64)$$

$$Z_{eff}(j\omega) = R_{interface} // \frac{1}{j\omega \cdot (C_{interface} + C_0)} \quad (2.65)$$

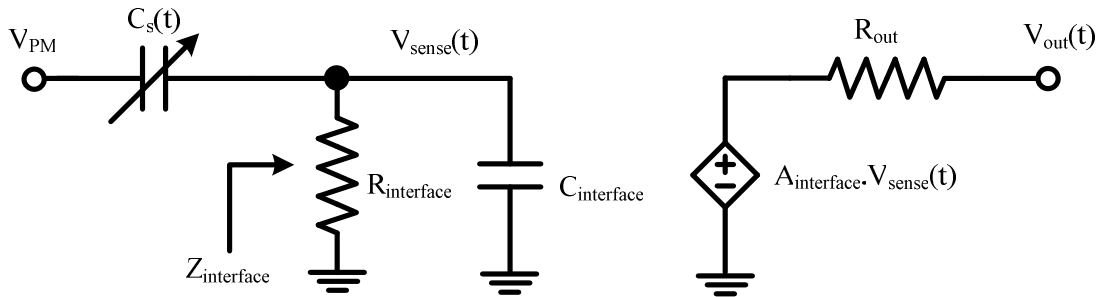


Figure 2.16: Parallel combination of the resistor and capacitor at the input of the gyroscope interface.

A gyroscope interface is called a resistive interface when its input-impedance given in Equation 2.63 is dominated by the resistive term, i.e., $R_{interface}$ is much smaller than $(\omega \cdot C_{interface})^{-1}$. On the contrary, if the impedance of the capacitance is much smaller than $R_{interface}$, then, the interface is called capacitive interface.

According to Equation 2.32, to obtain maximum signal at the input of the interface, $Z_{eff}(j\omega)$ should be maximized. The maximum value that $Z_{eff}(j\omega)$ can take is limited by the stationary capacitance of the gyroscope and can be written as

$$Z_{eff,max}(j\omega) = \frac{1}{j\omega \cdot C_0} \quad (2.66)$$

Therefore, the input-impedance of the interface should be as much as possible to pick larger signals at the output of the gyroscope.

It is also important to obtain a phase shift of either 0° or -90° from the input of the gyroscope to its output. If minimum phase shift from 0° or -90° with respect to the driving signal is obtained at the output of the gyroscope, the design of control and readout electronics after the output of the gyroscope will be much simpler. Therefore, the interface should add a minimum phase shift. To find the phase shift at the input of the interface, we need the transfer function from the input of the gyroscope to the input of the capacitive interface. Using Equations 2.32 and 2.45, the transfer function of the mechanical sensor connected to the input of the interface can be obtained. Transfer function for the drive-mode can be written as

$$H_{ms}(j\omega) = \frac{V_{sense}(j\omega)}{V_a(j\omega)} = n_{DM} \cdot n_{DP} \cdot \frac{Q_D}{k_D} \cdot \omega \cdot \left(Z_{interface}(j\omega) // \frac{1}{j\omega \cdot C_0} \right) \quad (2.67)$$

Using this equation, the phase shift can be found as

$$\angle H_{ms}(j\omega) = -\tan^{-1}(\omega \cdot (C_0 + C_{interface}) \cdot R_{interface}) \quad (2.68)$$

If the value of this equation is very close to either 0° or -90° , the phase shift from the interface will be minimum.

It is also possible to determine the type of the interface by examining Equation 2.68. If the multiplication inside the parenthesis is much larger than 1, then, the interface will be a capacitive interface, and its phase shift is -90° . If this term is much smaller than 1, then, the interface will be resistive, and the phase shift will be 0° .

Before choosing the type of the interface that is going to be used, it is important to get an intuition about the possible values of $Z_{eff,max}(j\omega)$. The gyroscopes used in this study have sense pick capacitances ranging from 2 pF to 15 pF. Moreover, their resonance frequencies are in the order of a few kilohertz, generally between 4 kHz to 15 kHz. Thus, $Z_{eff,max}(j\omega)$ ranges from 1 M Ω to 20 M Ω . In order to have the maximum signal at the output of the gyroscope, $Z_{interface}(j\omega)$ must be much larger than these values for maximum sensitivity. Since the performance is limited by the stationary capacitance of the mechanical sensor, it is not possible to use resistive interfaces to obtain maximum sensitivity. Therefore, capacitive interfaces are required for high performance gyroscopes.

When a capacitive interface without a biasing resistor is connected to the stationary sense electrode, there is no dc path at the input of the interface and therefore, the dc point of the stationary electrode cannot be set. This may cause improper operation of the interface. However, since the input of the interface is a high-impedance node, a very high resistance should be used to maintain minimum phase shift from -90° . For instance, if the impedance of the gyroscope stationary sense capacitance is

20 M Ω , the resistance that should be connected to the high impedance node should be at least 1.14 G Ω for 1° phase shift from -90°. The problem for the integrated circuit designer is that obtaining such high resistances on CMOS chips is not feasible in terms of functionality and die area. Therefore, special structures should be used for biasing high-impedance node at the input of the capacitive interface. These structures will be analyzed in detail in Chapter 3.

It is also important to keep the $C_{interface}$ value as small as possible so that the effective impedance at the input of the interface is maximized. This is another challenge for the designer to keep this value as small as possible so that assumption made in Equation 2.66 is valid. Using this assumption inside Equation 2.32, one can obtain

$$V_{sense}(j\omega) = (V_{PM} - V_{BIAS}) \cdot \frac{\partial C}{\partial x} \cdot \frac{1}{j\omega \cdot C_0} \cdot j\omega \cdot X(j\omega) \quad (2.69)$$

With this assumption, using Equation 2.35 for $\partial C / \partial x$ of a varying-gap type sense-mode fingers and assuming that $\partial C / \partial x$ of anti-gap capacitance and fringing-field effects are negligible, the signal at the input of the capacitive interface can be found as

$$V_{sense}(j\omega) = (V_{PM} - V_{BIAS}) \cdot \left(-\alpha \cdot \epsilon_0 \cdot \frac{l_0 \cdot h_0}{d_0^2} \right) \cdot \frac{1}{j\omega \cdot C_0} \cdot j\omega \cdot X(j\omega) \quad (2.70)$$

$$V_{sense}(j\omega) = (V_{PM} - V_{BIAS}) \cdot \left(\frac{C_0}{d_0} \right) \cdot \frac{1}{j\omega \cdot C_0} \cdot j\omega \cdot X(j\omega) \quad (2.71)$$

$$V_{sense}(j\omega) = (V_{PM} - V_{BIAS}) \cdot \frac{X(j\omega)}{d_0} \quad (2.72)$$

Equation 2.72 gives a hint about the design of the mechanical structure. If the stationary capacitance of the gyroscope is designed much larger than the input capacitance of the capacitive interface, the voltage induced at the input of the interface will be independent from the thickness and overlap of the varying-gap type fingers, and also, from the number of fingers. Therefore, the layout of the mechanical sensor can be designed with a minimum area.

In conclusion, high-performance gyroscope interfaces should provide low-noise, have low-input capacitances and high-impedance node biasing structures, and give minimum phase-shift to the system.

2.5. Summary

This chapter presents the basics of MEMS vibratory gyroscopes and their models. It describes the operation principles of a vibratory gyroscope with an introduction to drive and sense modes. Secondly, transduction mechanisms in capacitive MEMS gyroscopes are explained and equations for actuation and sensing of the gyroscope are given. Thirdly, a generalized fully-differential capacitive vibratory gyroscope model with force-feedback electrodes is proposed, and this model is specialized for a single-ended gyroscope fabricated at METU. Then, simulations of the single-ended model are done for verification of the model. Finally, the requirements for high-performance gyroscope interfaces are explained with their reasons.

CHAPTER III

CAPACITIVE INTERFACE DESIGN

This chapter presents the capacitive interfaces developed in this study and gives the simulations of these circuits. Section 3.1 proposes solutions for the common problems faced in the design of capacitive interface circuits. Then, Section 3.2 gives the design steps of single-ended capacitive interfaces. These interfaces are designed for a standard 0.6 μm CMOS process and simulated using Analog Artist simulator of Cadence Custom IC Design Tools. The simulation results are investigated around 5 kHz, which is the expected resonance frequency of the gyroscopes developed at METU. Then, layouts and photographs of the fabricated chip are given in this section. Section 3.3 introduces possible improvements for the single-ended interfaces, and Section 3.4 explains the design of improved gyroscope interfaces. This section also gives the simulations results, layouts, and photographs of the fabricated chips. Finally, Section 3.5 summarizes this chapter.

3.1. Proposed Solutions for Challenges in Capacitive Interfaces

There are four main challenges in the design of capacitive interfaces. These are: 1) high-impedance node biasing, 2) low input capacitance, 3) minimum phase shift, and 4) low input referred noise. Possible solutions for these challenges are introduced in the following subsections.

3.1.1. High-impedance node biasing

The challenge in the design of capacitive interfaces is biasing the high-impedance node at the input of the interface. This node should be biased with a very high resistance, in the order of gigaohms. There are several approaches [25] for biasing the high-impedance node like:

- Physical resistor
 - Low-doped polysilicon
 - Thin-film resistor
 - Diffusion resistor
- Back-to-back diodes
- Switched capacitors
- MOSFET in its subthreshold region

Implementation of a physical resistor is not quite feasible by using sheet resistances of layers available in CMOS processes. For instance, a 1 G Ω resistor implemented with high-poly resistance in XFAB XC0.6 μ m CMOS process [26] measures around 800 μ m x 800 μ m, which is the almost the size of an interface chip. Moreover, fabrication of such large resistances may be problematic. Therefore, physical resistors cannot be used for biasing the high-impedance node at the input of the interface.

Back-to-back diodes can also be used for biasing the high-impedance node at the input of the interface [18]. However, they add non-linear distortion to the system, which is not desired in high-performance gyroscopes.

Another common approach for biasing the high-impedance node is to use switched-capacitor circuits. Switched-capacitor circuits are commonly used in detection of very small capacitances. The most common approach for capacitive interfaces is to use fully-differential switched-capacitor charge integrator circuits

[14, 15, 17]. These circuits may reach high-performances, but they have many problems like common-mode feedback, charge injection, and large parasitic capacitances. Since the circuit is complex, the design of switched-capacitor circuits for high-impedance node biasing requires careful design. Additionally, since the circuit is not a continuous-time circuit, ac and noise simulations of the circuit become much more complex than a simple continuous-time circuit.

The final approach for biasing the high-impedance node is using a MOSFET operating in its subthreshold region. By using such a transistor, very high resistances in the order of gigaohms can be obtained [8, 27]. The resistance of the subthreshold transistor at low frequencies is controlled by its gate-to-source voltage. The controllability of this resistance is not so good, but since this resistance is much larger than the impedance of the capacitance seen at the high-impedance node, controllability is not a big issue. Moreover, the transistor is a continuous-time element, and therefore, the simulations of this transistor are much simpler when compared to a switched-capacitor type biasing circuit.

The impedance of a subthreshold transistor depends on several parameters like gate-to-source voltage, drain capacitances, and junction capacitances. This impedance can be defined with respect to these parameters as

$$Z_{sub} = R_{on} // \frac{1}{sC_d} // \frac{1}{sC_j} \quad (3.1)$$

where, R_{on} is the resistance of the subthreshold transistor, C_d is the total capacitance seen from the drain to the gate, source and bulk of the transistor, and C_j is the capacitance of the reverse-biased substrate junction at the drain of the transistor. Figure 3.1 shows the symbolic view of the input impedance of a transistor operating in subthreshold region. C_d represented in the figure corresponds to the total

capacitance of the drain with respect to ac ground excluding the capacitance of the junction.

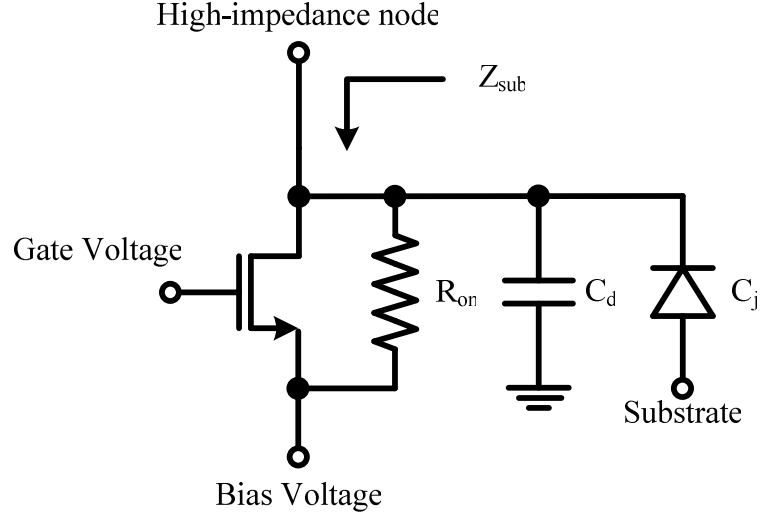


Figure 3.1: Symbolic view of the input impedance of a transistor operating in subthreshold region.

The resistance of a transistor operating in the subthreshold region can be derived from the drain current of the transistor in this region. The drain current of a subthreshold transistor is [28]

$$I_{DS} = I_0 \cdot \left[1 - \exp\left(-\frac{V_{DS}}{V_T}\right) \right] \cdot \exp\left(\frac{V_{GS} - V_{TH} - V'_{OFF}}{n \cdot V_T}\right) \quad (3.2)$$

where, V_T is the thermal voltage, V'_{OFF} is the offset voltage which determines the channel current when V_{GS} is zero, and n is the subthreshold swing parameter. By taking the derivative of the drain current with respect to drain-to-source voltage, R_{on} can be found as [27]

$$R_{on} = \frac{\partial I_{DS}}{\partial V_{DS}} = \frac{I_0}{V_T} \cdot \exp\left(\frac{V_{GS} - V_{TH} - V'_{OFF}}{n \cdot V_T}\right) \cdot \exp\left(-\frac{V_{DS}}{V_T}\right) \quad (3.3)$$

The drain capacitance of a subthreshold transistor is very small since the transistor is in weak inversion. The capacitance seen from the drain is the addition of capacitances with respect to source, gate and bulk of the transistor. Drain-to-source and drain-to-bulk capacitances are quite small compared to the drain-to-gate capacitance and therefore, they can be neglected. Drain-to-gate capacitance can be approximated as a parallel-plate capacitor by assuming that the capacitance is between the lateral diffused drain region and the gate. Hence, drain capacitance can be found as

$$C_d \cong \frac{\epsilon_{ox}}{t_{ox}} \cdot A_{ld} \quad (3.4)$$

where, t_{ox} is the thickness of the gate oxide and A_{ld} is the overlap area of the gate and laterally diffused drain of the transistor.

The junction capacitance of a MOSFET is composed of two parts: diffusion capacitance and depletion capacitance. The diffusion capacitance is the dominating term when the junction is forward-biased, whereas, the depletion capacitance dominates in reverse-biased mode. Since the p-n junctions in the drain and source of a MOSFET are reverse-biased under normal operating conditions, the depletion capacitance determines the junction capacitance. The depletion capacitance of a MOSFET can be found as [29]

$$C_{j,dep} = \frac{A}{2} \cdot \sqrt{\frac{2 \cdot q \cdot \epsilon}{V_0 - V}} \cdot N_d \quad (3.5)$$

where, A is the junction area, V_0 is the junction built-in potential, V is the applied voltage on the junction, and N_d is the donor concentration of a p⁺-n junction.

The terms in parallel given in Equation 3.1 dominates the impedance value at different situations. For instance, gate-to-source voltage determines the value of R_{on} and this value is the dominating term at low frequencies. At higher frequencies, parallel combination of the impedances of C_d and C_j determines the impedance of the subthreshold transistor. Figure 3.2 shows the dependence of the impedance of the subthreshold transistor with respect to frequency.

The values of C_d and C_j depends on the dimensions of the transistor and increases with increasing area of the transistor. Therefore, subthreshold transistors must be designed as small as possible to decrease these capacitance values. However, there is a practical limit for the sizes of these transistors. As a rule of thumb in analog layout design, width and length of the transistor should be at least 5 times the width and length of the minimum size transistor available in the process. The reason for such an adjustment is to minimize the possible mismatches in minimum size transistors.

Besides biasing the high-impedance node with a large resistance, the biasing transistor can be used to charge the high-impedance node with a small resistance by controlling the gate-to-source voltage. At the beginning of the operation, the resistance of the transistor can be decreased down to a few kilohms so that the high-impedance node is biased to the desired level very quickly. When biasing is achieved, the resistance can be increased by decreasing the gate-to-source voltage.

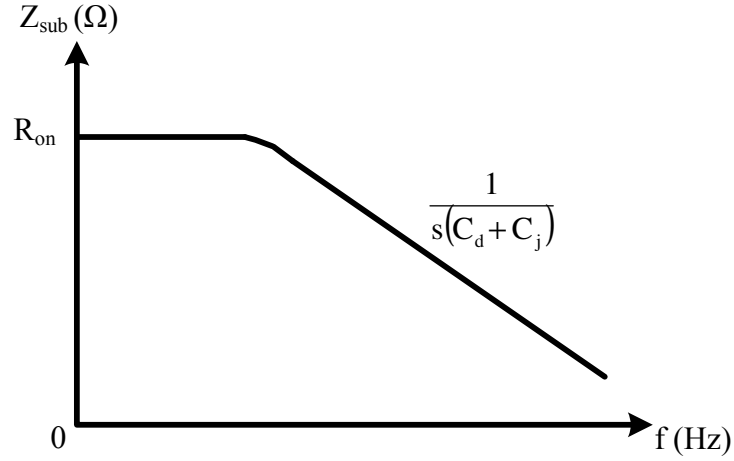


Figure 3.2: Dependence of the impedance of the subthreshold transistor with respect to frequency. At low frequencies, gate controlled resistance is the dominating impedance, whereas, at high frequencies, drain and junction capacitances dominate the impedance seen by high-impedance node.

The disadvantage of using subthreshold transistors as a biasing element for high-impedance nodes is that the resistance is not very controllable. The resistance value may decrease 15 times with a 0.1 Volt change in the gate-to-source voltage. However, the gate and source of the subthreshold transistor is connected to a dc supply and these supplies can be designed to bias these nodes accurately. The noise coming from the supplies should not have to be very low since the resistance of the subthreshold transistor is kept much larger than the impedance at the high-impedance node. Therefore, resistance change due to noise added on the gate voltage is not critical and neither the gain of the preamplifier nor SNR is affected by this source of noise.

3.1.2. Low input capacitance

As mentioned in Chapter 2, to obtain maximum sensitivity from the gyroscope, the input capacitance of the interface circuit should be kept at minimum. Since the electrode of the mechanical sensor is connected to the gate of input transistor of the preamplifier, the input capacitance seen by the sensor will be the gate capacitance of this transistor. The junction and drain capacitance of the biasing transistor described in previous subsection will also increase the input capacitance value; however, this capacitance is generally much smaller than the gate capacitance of the input transistor. Thus, the input capacitance is approximately equal to the gate capacitance of the input transistor of the preamplifier.

The gate capacitance of a MOSFET is composed of three parts: 1. Gate-to-drain capacitance, 2. Gate-to-source capacitance, 3. Gate-to-bulk capacitance. Figure 3.3 shows the gate capacitances of a MOSFET.

For a transistor operating in saturation region, the gate-to-bulk capacitance is very small and can be neglected in calculations. The gate-to-drain capacitance is simply the overlap capacitance of the gate and laterally-diffused drain region, and its value is given by Equation 3.4. The dominating term of the gate capacitance for a saturated MOSFET is its gate-to-source capacitance and can be found as

$$C_{gs} = \frac{2}{3} \cdot C_{ox} \cdot W \cdot L \quad (3.6)$$

where, C_{ox} is the gate-oxide capacitance, W is the width, and L is the length of the transistor. The value given in Equation 3.6 is generally much larger than gate-to-drain capacitance. Therefore, at first glance, gate-to-source capacitance should be decreased.

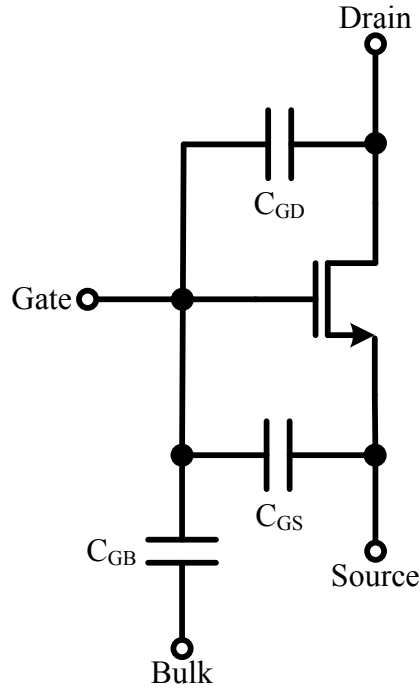


Figure 3.3: Gate capacitances of a MOSFET.

In standard capacitive readout circuits like fully-differential switched-capacitor charge integrators [14, 15, 17], the input capacitance of the fully-differential transconductance amplifier is in the order of a few picofarads, which is comparable to the stationary capacitance of the sense electrode of the mechanical sensor. Therefore, it is not possible to obtain low input capacitance by using transconductance amplifiers.

Since the aim of the capacitive interface is interfacing the high-impedance node with a low-impedance node, unity-gain buffer structures can be used as capacitive interfaces. The most common and basic unity-gain buffer is the common-drain amplifier, i.e., source follower. Figure 3.4 shows the basic source follower configuration with NMOS transistors.

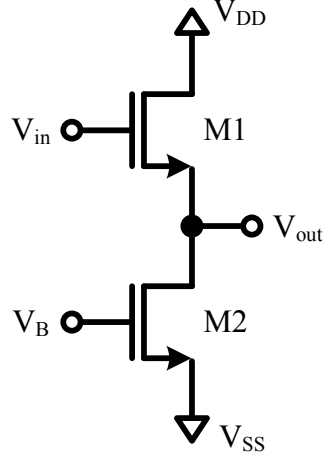


Figure 3.4: Basic source follower configuration with NMOS transistors.

Using the small-signal models of the transistors, the gain of source follower from input to output can be found as

$$A_{sf} = \frac{r_{o1} // r_{o2}}{r_{o1} // r_{o2} + \frac{1}{g_{m1} + g_{mb1}}} \quad (3.7)$$

where, r_o is the small-signal output resistance of a MOSFET, g_m is the transconductance, and g_{mb} is the body-effect transconductance of the transistor.

In general, r_o of a transistor is in the order of a few hundred kilohms, whereas, $1/g_m$ is around a few kilohms. Therefore, the second term in the denominator of Equation 3.7 is much smaller than the first term. Hence, the gain of the source follower can be set very close to 1.

Since there is a finite gain from the input node to the output node, i.e., from the gate of the input transistor to its source, the gate-to-source capacitance seen by the

mechanical sensor will be changed due to the Miller effect. According to the Miller's theorem, the impedance seen by the input will be affected by the voltage gain of the circuit and will be equal to [30]

$$Z_{in} = \frac{Z}{1 - A_v} \quad (3.8)$$

Hence, the effective input capacitance of the interface will be equal to

$$C_{in} = C_{gs} \cdot (1 - A_{sf}) + C_{gd} \quad (3.9)$$

where, A_{sf} is the gain of the source follower. As seen from Equation 3.9, the gate-to-source capacitance of the transistor is effectively decreased with the source follower architecture. It is possible to obtain A_{sf} values around 0.996, and therefore, gate-to-source capacitance decreases to its 0.4% value.

The input capacitance given in Equation 3.9 is not accurate when the devices are fabricated. The reason is that, there are additional parasitic capacitances from the bonding pads and interconnect lines on the chip. In addition, the wire bonding lines between the mechanical sensor and the interface will add more parasitic capacitances. Hence, Equation 3.9 must be manipulated for practical devices. A more realistic input capacitance can be written as

$$C_{in} = C_{gs} \cdot (1 - A_{sf}) + C_{gd} + C_{interconnect} + C_{bonding} \quad (3.10)$$

Interconnect capacitances present in the devices are due to the isolation oxide capacitance between the metal interconnect and silicon substrate. To decrease this capacitance, a shielding layer with a signal close to the signal at the interconnect line

is used to isolate the substrate from the metal interconnect [18, 19]. This method is called bootstrapping. For the shielding layer, polysilicon can be used. Figure 3.5a shows the layers that can be used for interconnections and shielding in a CMOS process.

In bootstrapping method, the shielding layer is connected to the buffered input signal. Therefore, $C_{met-poly}$ capacitance shown in Figure 3.5a will be multiplied by $(1 - A_{sf})$. If the gain is close to unity, parasitic capacitance that is added by bonding pads and interconnect lines will be minimized. Availability of bootstrapping is another advantage of using unity-gain source follower as capacitive interfaces in high performance gyroscopes. Figure 3.5b shows the bootstrapping method used for decreasing the capacitance from the bonding pad and interconnect lines.

Another advantage of using source follower as a capacitive interface is that the output dc point depends on the dc point of the input. Since gate-to-source voltage of M2 transistor is constant, this transistor behaves like a constant dc current source. Figure 3.6 shows the simplified schematic of the source follower with a constant dc current source. As constant current is drawn from M1 transistor, dc point of its gate-to-source voltage should be constant. On the other hand, the gate of the input transistor is biased to a fixed dc potential by the subthreshold transistor implemented. Hence, the dc point of the output voltage is

$$V_{OUT} = V_{IN} - V_{GS1} = V_{BIAS} - V_{GS1} \quad (3.11)$$

Therefore, the output dc value can be controlled by changing the bias potential of the subthreshold transistor. This eliminates the need for complex common-mode feedback circuitries to set the dc point of the output, and thus, simplifies the design of the interface.

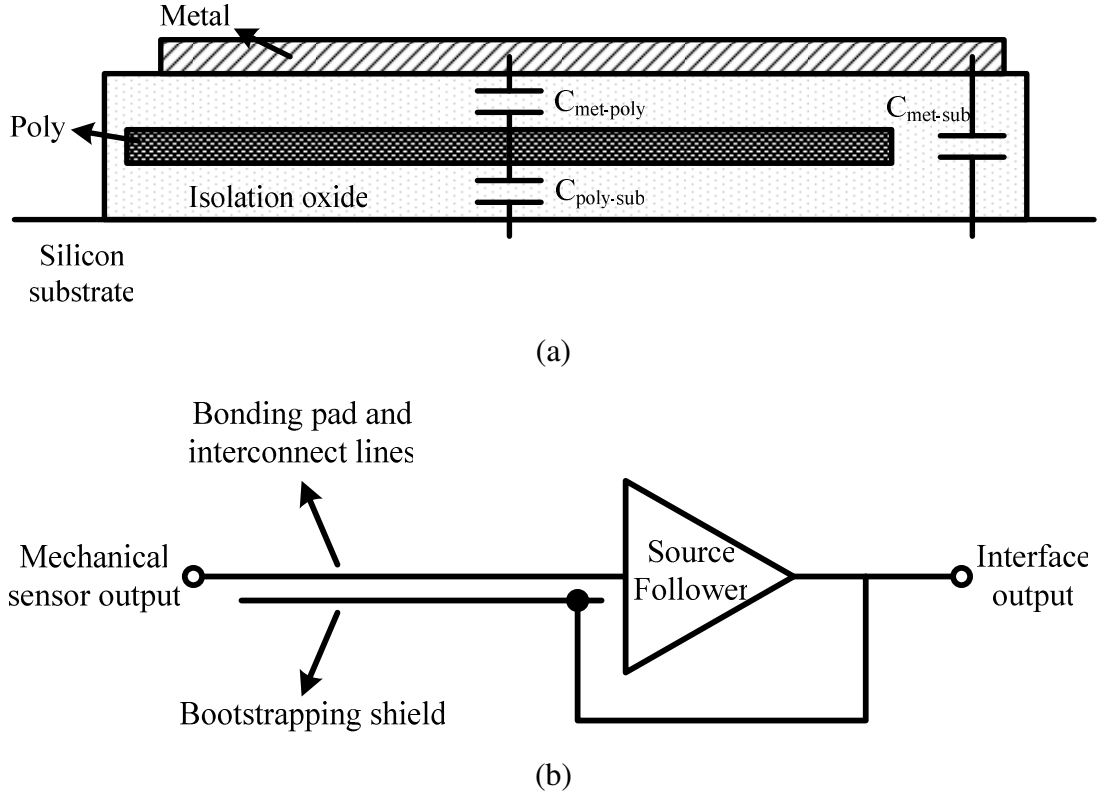


Figure 3.5: (a) Layers that can be used for interconnections and shielding in a CMOS process (b) Bootstrapping method used for decreasing the capacitance from the bonding pad and interconnect lines.

3.1.3. Minimum phase shift

To find the phase shift from the input of the gyroscope to its output, it is better to obtain the transfer function of the gyroscope. Modeling the interface as a voltage amplifier as in Figure 2.16 and using Equation 2.67, the transfer function of the gyroscope for the drive-mode can be found as

$$H_{\text{gyro}}(j\omega) = \frac{V_{\text{out}}(j\omega)}{V_a(j\omega)} = n_{DM} \cdot n_{DP} \cdot \frac{Q_D}{k_D} \cdot \omega \cdot \left(Z_{\text{interface}}(j\omega) // \frac{1}{j\omega \cdot C_0} \right) \cdot \frac{A_{sf}}{1 + j\omega \cdot R_{\text{out}} \cdot C_{\text{load}}} \quad (3.12)$$

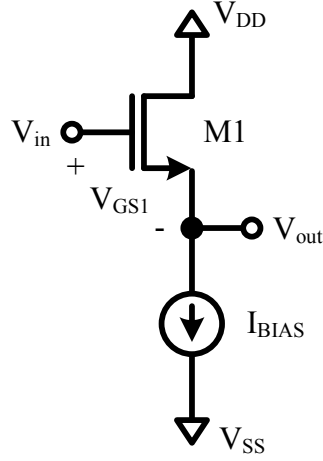


Figure 3.6: Simplified schematic of the source follower with a constant dc current source.

There are two poles present in Equation 3.12, one coming from the resistance and capacitance at the input of the interface, and the second due to the output resistance of the source follower and load capacitance at the interface output. Total phase shift due to these poles is

$$\angle H_{gyro}(j\omega) = -\tan^{-1}(\omega \cdot (C_0 + C_{interface}) \cdot R_{interface}) - \tan^{-1}(\omega \cdot C_{load} \cdot R_{out}) \quad (3.13)$$

As mentioned in Chapter 2, $R_{interface}$ should be very high to keep the first term as close to -90° in capacitive interfaces. For minimum phase shift, the second term of Equation 3.13 should be as small as possible. This term may become comparable with the phase shift due to the first pole, and, therefore, special attention must be paid on the design of the source follower for minimum phase shift.

3.1.4. Low input referred noise

For high performance gyroscope interfaces, SNR at the input of the interface should be as small as possible. Therefore, input referred noise should be minimum. In order to have low input referred noise, the common approach is using a high-gain low noise preamplifier as the input stage. The disadvantage of using source follower as the preamplifier of a capacitive interface is its unity-gain. Since the gain is unity, input referred noise will be higher when it is compared with a high-gain input stage. However, advantages of source follower overwhelm this disadvantage, and for this reason, source follower is a good choice for the preamplifier of a capacitive interface. To decrease the noise contribution coming from the stages following the input stage, a high-gain amplifier can be implemented right after the source follower. Noise analysis of the source follower biased with a subthreshold transistor will be discussed in detail in Section 3.2.3.

3.2. Single-Ended Capacitive Interfaces

In this study, capacitive interfaces for single-ended gyroscopes are implemented using source followers with subthreshold transistor biasing. As explained in the previous section, source followers and subthreshold transistors solve most of the problems present for high-performance capacitive interfaces. Therefore, it is advantageous to use simple source follower structures as the preamplifier part of the interface and bias the input of the preamplifier with transistors operating in subthreshold region.

For this design, XFAB 0.6 μ m XC0.6 process is used [26]. Design of the interface is mainly composed of two parts. The first part is the design of the biasing transistor operating in subthreshold mode, and the second one is the design of the source follower as the preamplifier of the interface. These parts are explained in detail in the subsections below.

3.2.1. Design of biasing transistor operating in subthreshold region

Biasing resistor can be implemented using a single NMOS transistor with its gate and source connected to the necessary dc biasing potentials. The resistance seen at the drain of the transistor can be controlled by changing the gate-to-source voltage. Figure 3.7 shows the single NMOS transistor operating in subthreshold region for biasing the interface input.

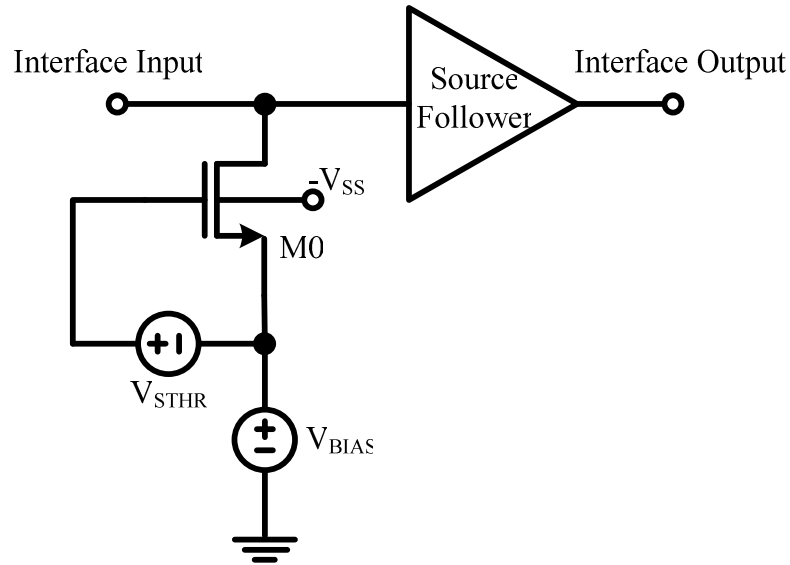


Figure 3.7: Single NMOS transistor operating in subthreshold region for biasing the interface input.

As mentioned in the previous section, the transistor should be implemented in minimum size to decrease the junction capacitances. Considering the fabrication tolerances, the limit for minimum size transistors is $4\text{ }\mu\text{m} / 3\text{ }\mu\text{m}$, 5 times the width and length of the minimum size transistor available in the process.

XFAB XC0.6 process is an n-well process and therefore, bulks of the NMOS transistors should be connected to the same potential. As the bulk of an NMOS transistor should be connected to the most negative potential present in the circuit, the body of the subthreshold transistor should be at the negative supply voltage, -2.5 V in this process. On the other hand, the source voltage of the transistor is at the biasing potential, V_{BIAS} . Therefore, the threshold voltage of the transistor is increased with the difference between V_{BIAS} and the negative supply voltage. For bulk-to-source voltage around 1.1 V, the threshold voltage of an NMOS transistor changes from 0.9 V to 1.4 V.

The impedance seen from the drain of the transistor is simulated by ac analysis and measuring the current drawn from the transistor. Figure 3.8 shows the simulation result of the output impedance seen from the drain of the biasing transistor for different values of V_{STHR} voltages. It can be seen that for low frequencies, the impedance is dominated by gate-controlled resistance, whereas, at high frequencies, it is dominated by depletion and junction capacitances.

Table 3.1 gives the output impedance values at the drain of the transistor for different values of gate-to-source voltage, V_{STHR} . As seen from the table below, the output impedance value at 0 Hz changes more than an order of magnitude with 0.1 V change in the gate-to-source voltage.

3.2.2. Design of single-ended source follower as a preamplifier for capacitive interfaces

In the basic source follower architecture shown in Figure 3.4, the transistors were chosen to be NMOS type. In order to use NMOS transistors in XFAB XC0.6 process, all of the bulks of the transistors should be connected to the most negative potential. Since the dc point of output of the source follower is set to 0 V, the threshold voltage of the input transistor will increase. Therefore, the dynamic range

of the source follower will decrease with the increase in the threshold voltage. On the other hand, PMOS transistors can have isolated bulk voltages, and therefore, the sources of the transistors can be connected to their bulks. Hence, using PMOS transistors in the source follower increases the dynamic range of the interface.

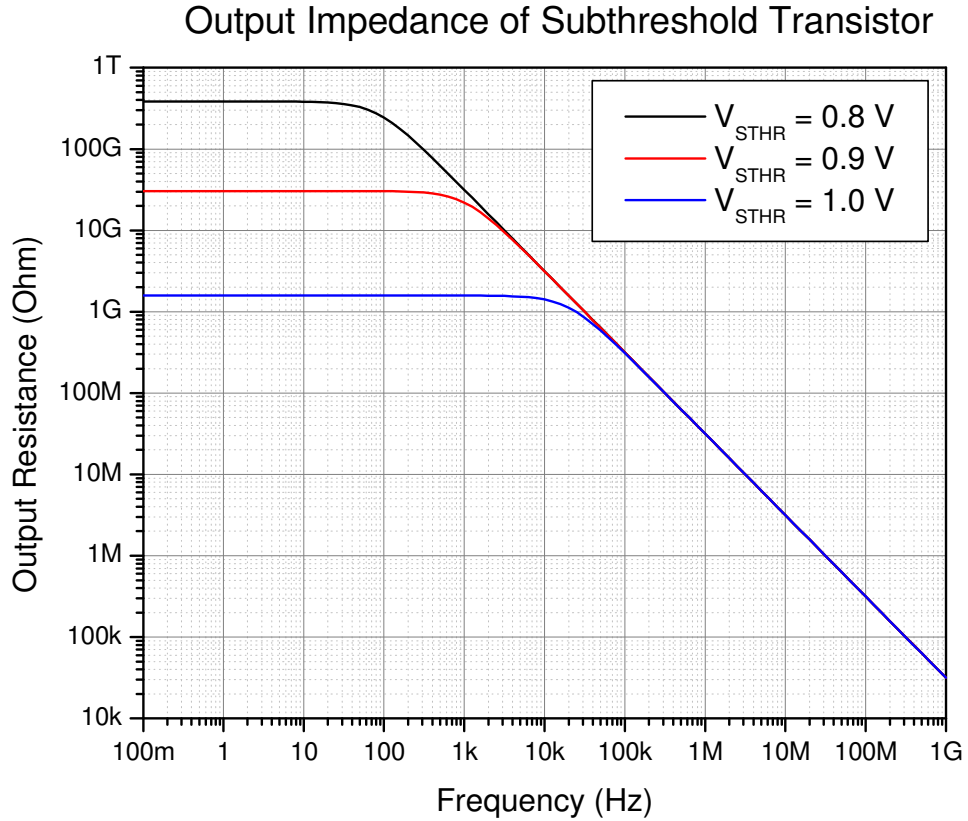


Figure 3.8: Simulation result of the output impedance seen from the drain of the biasing transistor for different values of V_{STHR} voltages. For this simulation, bias voltage of the biasing transistor is kept at -1.4 V. This value is chosen considering the dc point at the input of the source follower circuitry.

Table 3.1: Output impedance values at the drain of the transistor for different values of gate-to-source voltage, V_{STHR} .

Gate-to-source voltage, V_{STHR} (V)	Output Impedance at 0 Hz (Ω)	Output Impedance at 5 kHz (Ω)
0.8	384.8 G	6.319 G
0.9	30.46 G	6.187 G
1	1.533 G	1.533 G

Additionally, for an n-well process, PMOS transistors are less affected from the substrate currents compared to NMOS transistors, since their bulks are isolated from the substrate. Therefore, noise contribution from the substrate currents is decreased by using PMOS transistors.

Disadvantage of using PMOS transistors is that mobility of the PMOS transistors is small. Therefore, larger chip area is required for obtaining same bias current. However, area of the circuit is not a major problem in this study.

Figure 3.9 shows the basic source follower with PMOS transistors. The transistor dimensions for M1 and M2 transistors are chosen as $400 \mu\text{m} / 8 \mu\text{m}$ and $52 \mu\text{m} / 8 \mu\text{m}$, respectively. For $250 \mu\text{A}$ drain current, dc point of V_{in} is chosen to be -1.4 V while, V_B is kept at 0 V .

Figure 3.10 shows the simulation result of the ac analysis of the source follower at its dc operating point. At 5 kHz , the gain of the source follower is 0.9961 V/V and its phase shift is $-0.193 \text{ milidegrees}$.

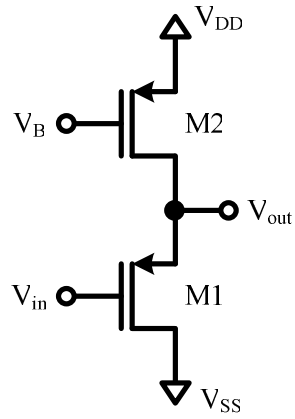


Figure 3.9: Basic source follower with PMOS transistors.

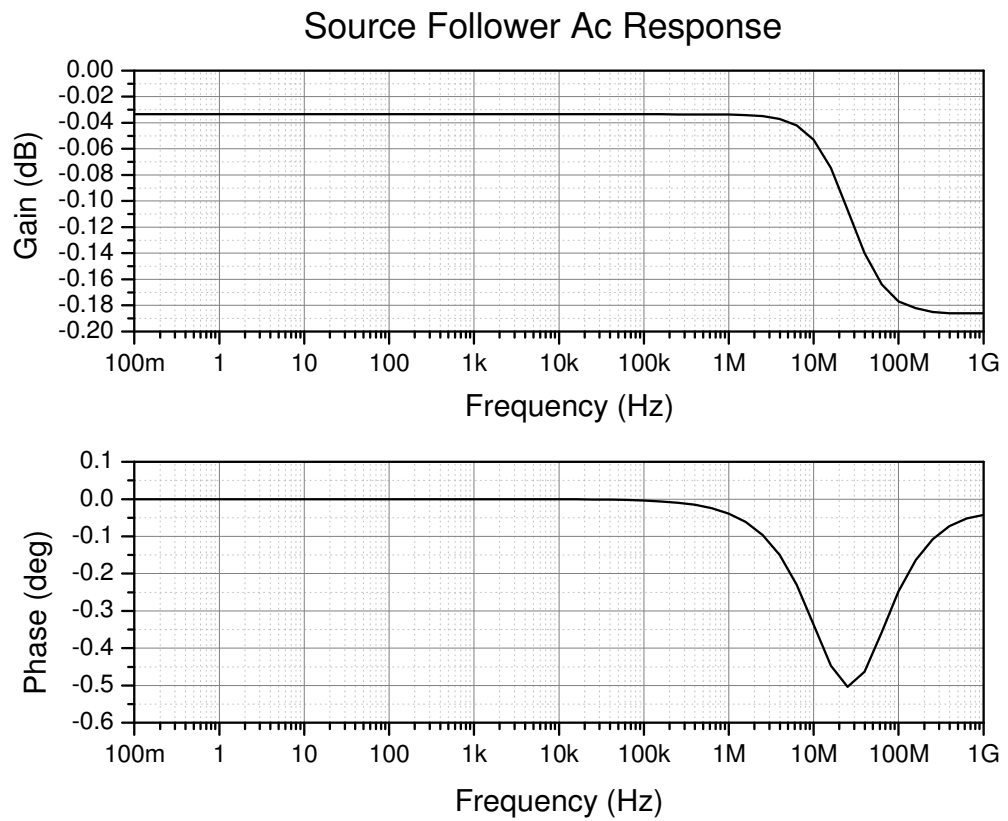


Figure 3.10: Simulation result of the ac analysis of the source follower at its dc operating point.

3.2.3. Single-ended source follower biased with subthreshold transistor

Single-ended capacitive interface is designed by connecting the PMOS source follower to biasing transistor operating in subthreshold region. Figure 3.11 shows the schematic of the single-ended source follower biased with subthreshold transistor. Table 3.2 shows the transistor dimensions of single-ended source follower biased with subthreshold transistor.

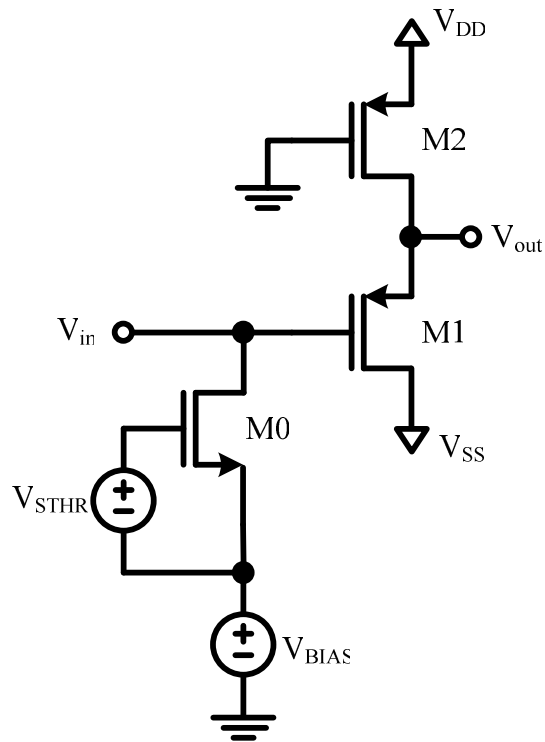


Figure 3.11: Schematic of the single-ended source follower biased with subthreshold transistor.

Before moving to the simulations of the design, it is important to estimate the noise of single-ended source follower biased with the subthreshold transistor. Figure 3.12 shows the noise sources present in the capacitive interface.

Table 3.2: Transistor dimensions of single-ended source follower biased with subthreshold transistor.

	M0	M1	M2
Width	4 μm	400 μm	52 μm
Length	3 μm	8 μm	8 μm

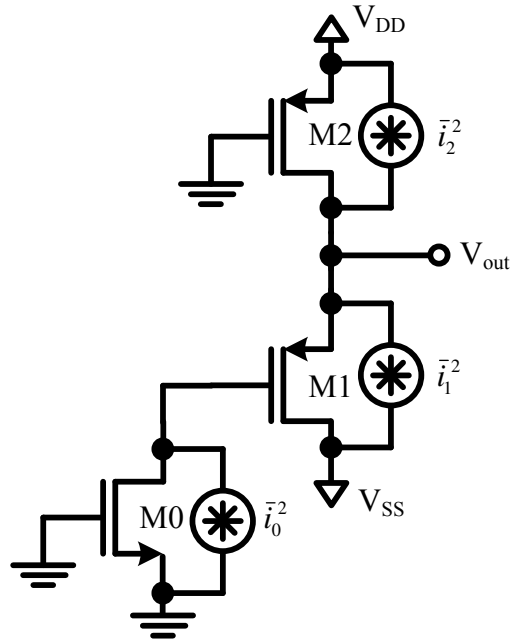


Figure 3.12: Noise sources present in the capacitive interface. This model excludes the flicker noise component and only shows the dominant noise sources.

The noise components described in the figure above are the current noise densities of the transistors. $\bar{i}_0^2$ is the current power spectral density due to the shot noise of the reverse biased junction diodes and it can be defined as given in Equation 3.14.

$$\bar{i}^2 = 2 \cdot q \cdot I_s \cdot \Delta f \quad (3.14)$$

where, q is the electron charge, Δf is the measurement bandwidth and I_s is the leakage current of the reverse biased junction transistors and is around 1 fA for M0 transistor. However, simulations have shown that the current noise power due to M0 transistor also depends on the gate-to-source voltage in the subthreshold region. Moreover, the noise power is inversely proportional to the resistance of the subthreshold transistor similar to the current noise power of a resistor. No useful information exists in the literature on the source of this noise.

$\bar{i}_1^2$ and $\bar{i}_2^2$ are the current power spectral densities for a transistor operating in saturation region. These spectral densities can be calculated using

$$\bar{i}^2 = \frac{8}{3} \cdot k \cdot T \cdot \frac{1}{g_m} \cdot \Delta f \quad (3.15)$$

where, k is the Boltzmann constant, T is the ambient temperature in Kelvin, and g_m is the transconductance of the transistor.

Making the noise analysis for the noise model given in Figure 3.12, the voltage noise at the output can be obtained as

$$\bar{v}_{out}^2 = Z_{out}^2(s) \cdot (\bar{i}_1^2 + \bar{i}_2^2) + A_{sf}^2(s) \cdot \left(\frac{1}{sC_{in}} \right)^2 \cdot \bar{i}_0^2 \quad (3.16)$$

where, Z_{out} is the output impedance of the capacitive interface. In a simple source follower, the output impedance is dominated by the reciprocal of the transconductance of the input transistor and is usually in the order of a few hundred

ohms. However, for the capacitive interface given in Figure 3.11, the gate of the input transistor is biased with a high impedance transistor, and therefore, the output impedance is not as small as a simple source follower.

To find the output impedance of the capacitive interface proposed in Figure 3.11, the small-signal model of the circuit is needed. Since gate-to-source voltage of M2 transistor is constant, this transistor behaves like a resistor of conductance g_{ds2} . In addition to this, subthreshold transistor biasing the high-impedance node can also be modeled as a load of impedance given in Equation 3.1. For M1 transistor, basic saturation mode small-signal model with gate-to-source and gate-to-drain capacitances can be used. Figure 3.13 shows the small-signal model of the capacitive interface for calculation of its output impedance.

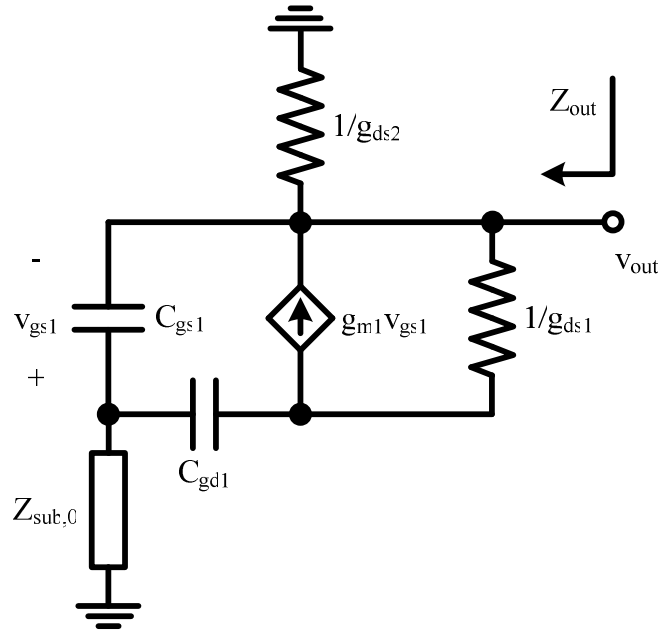


Figure 3.13: Small-signal model of the capacitive interface for calculation of its output impedance.

By analyzing the circuit given above, the output impedance of the capacitive interface can be calculated as

$$Z_{out}(s) = \left(g_{ds1} + g_{ds2} + \frac{(sC_{gs1} + g_{m1}) \cdot (1 + s \cdot R_{sub} \cdot (C_{gd1} + C_d + C_j))}{1 + s \cdot R_{sub} \cdot (C_{gs1} + C_{gd1} + C_d + C_j)} \right)^{-1} \quad (3.17)$$

Considering the values present in the above equation, some approximations can be done to simplify the equation. Assuming $sC_{gs1} \ll g_{m1}$, $C_{gd1} \gg C_d + C_j$, and $C_{gs1} \gg C_{gd1}$, Equation 3.17 can be approximated to

$$Z_{out}(s) \cong \left(g_{ds1} + g_{ds2} + \frac{g_{m1} \cdot (1 + s \cdot R_{sub} \cdot C_{gd1})}{1 + s \cdot R_{sub} \cdot C_{gs1}} \right)^{-1} \quad (3.18)$$

In a simple source follower without a subthreshold biasing transistor, R_{sub} is equal to zero, and thus, the output resistance of the circuit is approximately the reciprocal of the transconductance of the input transistor.

Table 3.3 shows the small-signal parameters of the capacitive interface circuit with transistor dimensions given in Table 3.2. Using the values given in the table below, the output impedance of the capacitive interface circuit is calculated as 57.56 k Ω , which is much larger than the output resistance of a simple source follower.

The output impedance of the capacitive interface circuit can also be simulated using s-parameter analysis of Analog Artist. Figure 3.14 shows the simulation result of output impedance of the capacitive interface circuit. For this simulation, gate-to-source and bias voltages of the subthreshold transistor are set to 0.8 V and -1.4 V, respectively. As seen from the figure, the output impedance is 56.34 k Ω at 5 kHz, which agrees well with the hand calculations.

Table 3.3: Small-signal parameters of the capacitive interface circuit with transistor dimensions given in Table 3.2.

Small-signal model parameter	Value
g_{m1}	960.3 μS
g_{ds1}	1.544 μS
g_{ds2}	2.171 μS
C_{gd1}	81.02 fF
C_{gs1}	5.708 pF
R_{sub}	384.8 G Ω

Since the output resistance increases when source follower is biased with a subthreshold transistor, the output voltage noise will also increase as given in Equation 3.16.

3.2.4. Simulations of the single-ended source follower biased with subthreshold transistor

Three different simulations are done for obtaining the response of the interface. These are discussed in detail in the following subsections. In these simulations, the gate-to-source and bias voltages of the subthreshold transistor are set to 0.8 V and -1.4 V, respectively.

3.2.4.1. Ac simulations

Figure 3.15 shows the schematic used for simulating the ac response of the capacitive interface. The input signal is applied through a large capacitor to isolate

the dc point of the capacitive interface from the ac voltage source. The gain from the source to the input of the interface will degrade due to the isolation capacitance; however, the impedance of this capacitor is much smaller than the impedance of input capacitance.

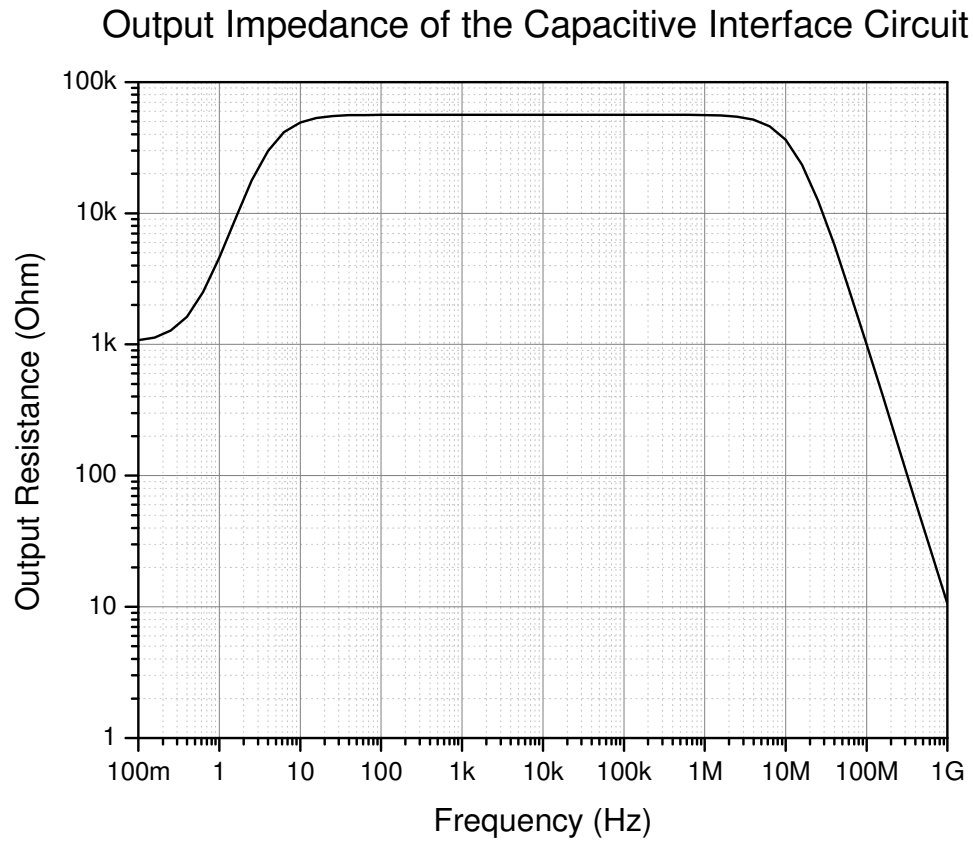


Figure 3.14: Simulation result of output impedance of the capacitive interface circuit.

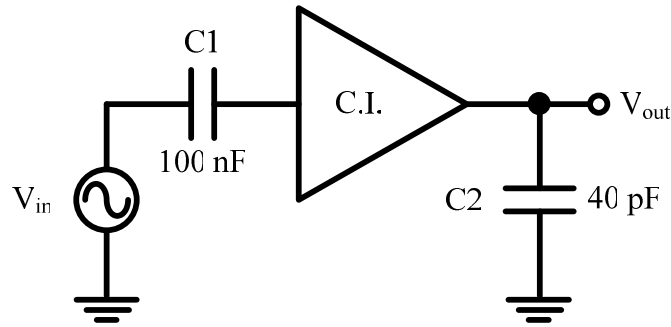


Figure 3.15: Schematic used for simulating the ac response of the capacitive interface.

Figure 3.16 shows the simulation result of the ac analysis of the capacitive interface. The gain of the interface is 0.9961 V/V at 5 kHz; while its phase shift is -74.92 milidegrees.

The effective input capacitance of the capacitive interface can be found using Equation 3.9. From dc analysis result, the gate-to-source capacitance and gate-to-drain capacitances are found as 5.708 pF and 81.02 fF, respectively. Hence, the effective input capacitance without interconnect parasitic capacitances can be found as 103.281 fF.

Moreover, power supply rejection ratio (PSRR) of the circuit can be simulated using ac analysis for characterization of this parameter. Figure 3.17 shows the simulation result of the ac analysis for PSRR calculation of the capacitive interface.

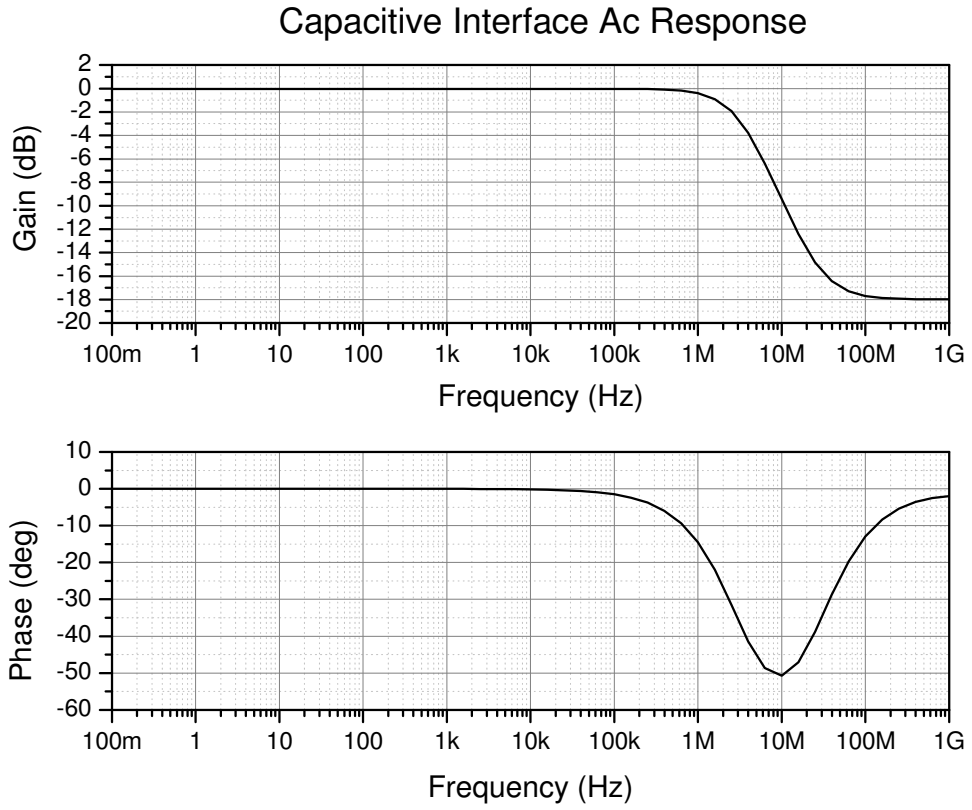


Figure 3.16: Simulation result of the ac analysis of the capacitive interface.

From the simulation result, at 5 kHz, PSRR of circuit are found as -24.55 dB, 1.277 dB, 38.3 dB, and 38.285 dB for the supplies V_{DD} , V_{SS} , V_{STHR} , and V_{BIAS} , respectively. It can be seen that, PSRR of the source follower is quite small especially for the positive supply. This is due to the gain of the source follower from the positive supply node to the output. This gain can be found small signal analysis as

$$A_{V_{DD}} = g_{m2} \cdot (r_{o1} // r_{o2}) \quad (3.19)$$

This gain can be high if the transconductance of M2 transistor is not small enough. Therefore, the PSRR of single-ended source follower can be very small.

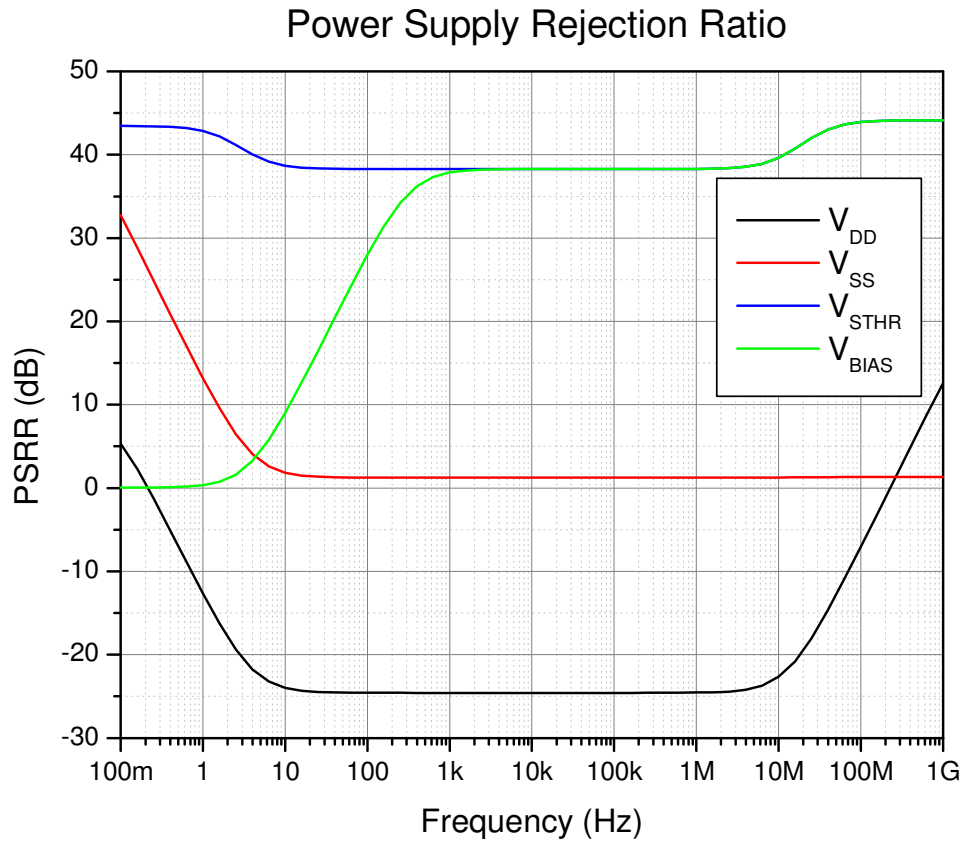


Figure 3.17: Simulation result of the ac analysis for PSRR calculation of the capacitive interface.

3.2.4.2. Transient simulations

For transient simulations, the voltage source given in Figure 3.15 is replaced with a current source of amplitude 100 pA and frequency 5 kHz. Transient analysis is done

for 2 ms and input of the interface, its output, current drawn from the drain of the subthreshold transistor and gate of the input transistor are monitored. Figure 3.18 shows the simulation result of the transient analysis of the capacitive interface.

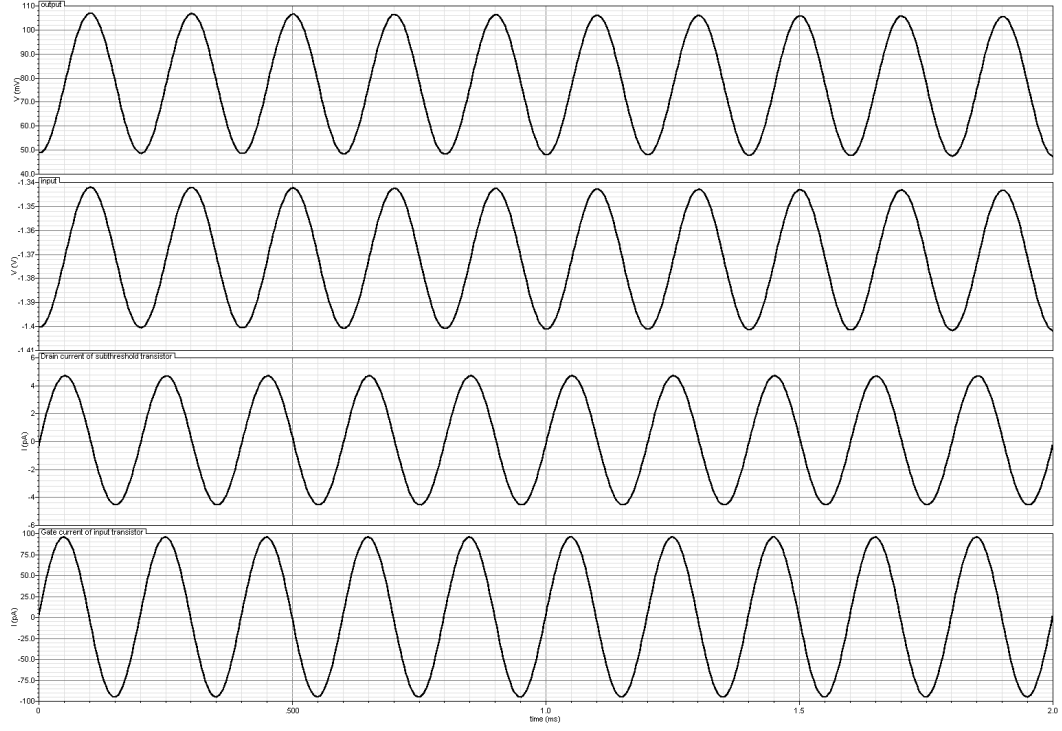


Figure 3.18: Simulation result of the transient analysis of the capacitive interface.

It is possible to extract the impedance of the biasing transistor from Figure 3.18. The ac current drawn from the gate of the input transistor is 95.168 pA; while the current through the drain of the biasing transistor is 4.655 pA. As the impedance of the input transistor is known, by taking the ratio of the currents, the impedance of the biasing transistor can be calculated as 6.3 G Ω , verifying the result given in Table 3.1.

3.2.4.3. Noise simulations

For the noise simulations of the capacitive interface, no external component like load capacitance is used in order to measure the response of circuit only. Figure 3.19 shows the simulation result of the output voltage noise of the capacitive interface circuit. Table 3.4 shows the noise contributions of the transistors for different gate-to-source voltages.

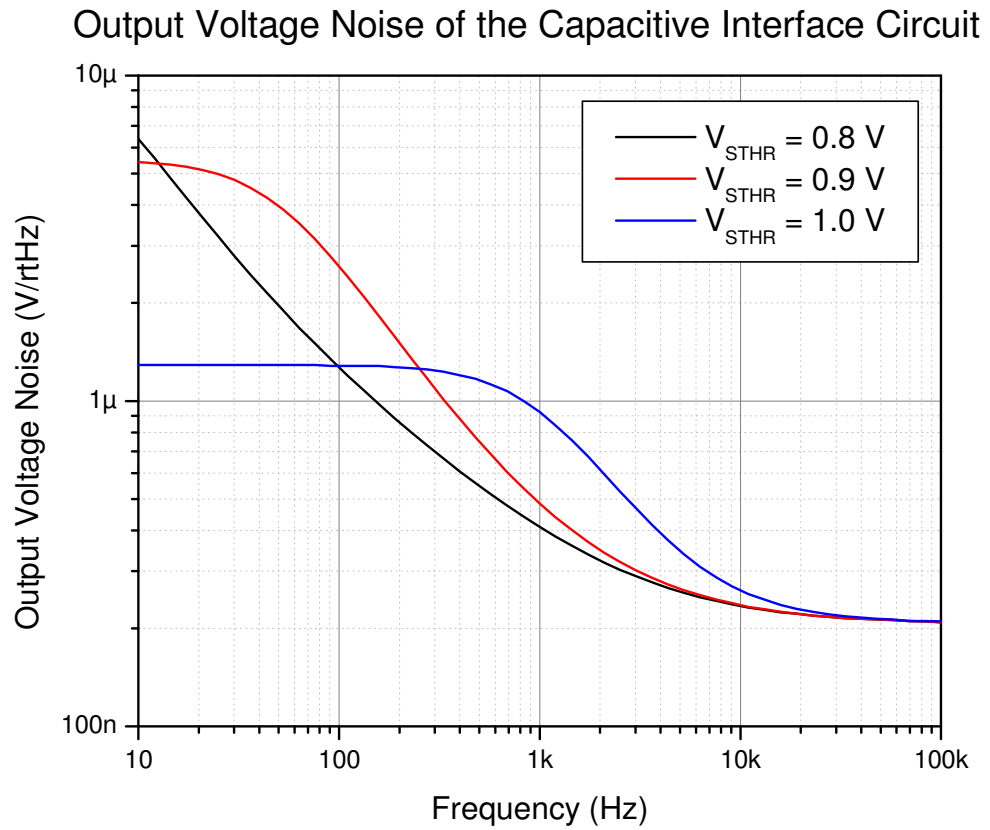


Figure 3.19: Simulation result of the output voltage noise of the capacitive interface circuit.

Table 3.4: Noise contributions of the transistors for different gate-to-source voltages. Most of the noise components that are less than 0.1% of the total noise are not included in this table.

V_{STHR} (V)	Total Output Voltage Noise at 5 kHz (V/ $\sqrt{\text{Hz}}$)	Transistor	Noise Type	Output Voltage Noise at 5 kHz (V/ $\sqrt{\text{Hz}}$)	Percentage of Total Output Voltage Noise at 5 kHz
0.8	259.548 n	M0	Current	11.883 n	0.21
			Flicker	36.578 f	0.00
		M1	Current	176.927 n	46.47
			Flicker	109.115 n	17.67
		M2	Current	106.838 n	16.80
			Flicker	112.625 n	18.83
0.9	264.722 n	M0	Current	53.484 n	4.08
			Flicker	57.809 f	0.00
		M1	Current	176.919 n	44.66
			Flicker	109.112 n	16.99
		M2	Current	106.378 n	16.15
			Flicker	112.614 n	18.10
1	347.964 n	M0	Current	236.963 n	46.38
			Flicker	58.453 f	0.00
		M1	Current	173.896 n	24.98
			Flicker	107.220 n	9.49
		M2	Current	104.560 n	9.03
			Flicker	110.661 n	10.11

It can be seen from Table 3.4 that for low V_{STHR} values, the dominant noise source is the current noise of the input transistor, M1. Current noise of this transistor decreases with increasing gate-to-source voltage, since the output resistance is

inversely proportional to the value of V_{STHR} . On the other hand, the current noise of the biasing transistor increases with increasing V_{STHR} since the resistance of this transistor decreases. Simulation result shows that it is better to operate the interface at V_{STHR} equal to 0.8 V.

3.2.4.4. Simulations with single-ended SOG gyroscope model

The simulations of single-ended source follower biased with subthreshold transistor are also done with the single-ended SOG gyroscope model proposed in Chapter 2. Resonance characteristics of the gyroscope are simulated with capacitive interfaces connected to the drive-mode and sense-mode outputs of the mechanical sensor.

Figure 3.20 shows the simulation results for drive-mode resonance characteristics of single-ended SOG gyroscope connected to single-ended source follower biased with subthreshold transistor. The gain of the gyroscope at resonance is increased by 47.96 dB when compared with the result obtained in Figure 2.14. This is due to the increase in the effective impedance at the high-impedance node.

Figure 3.21 shows the simulation results for sense-mode resonance characteristics of single-ended SOG gyroscope connected to single-ended source follower biased with subthreshold transistor. The gain of the resonance peak is increased by 16.66 dB with respect to simulation with active probe of the network analyzer. Increase in the gain is smaller with respect to increase in drive-mode because the effective impedance at the high-impedance node is smaller in sense-mode as its pick-up capacitance is much larger.

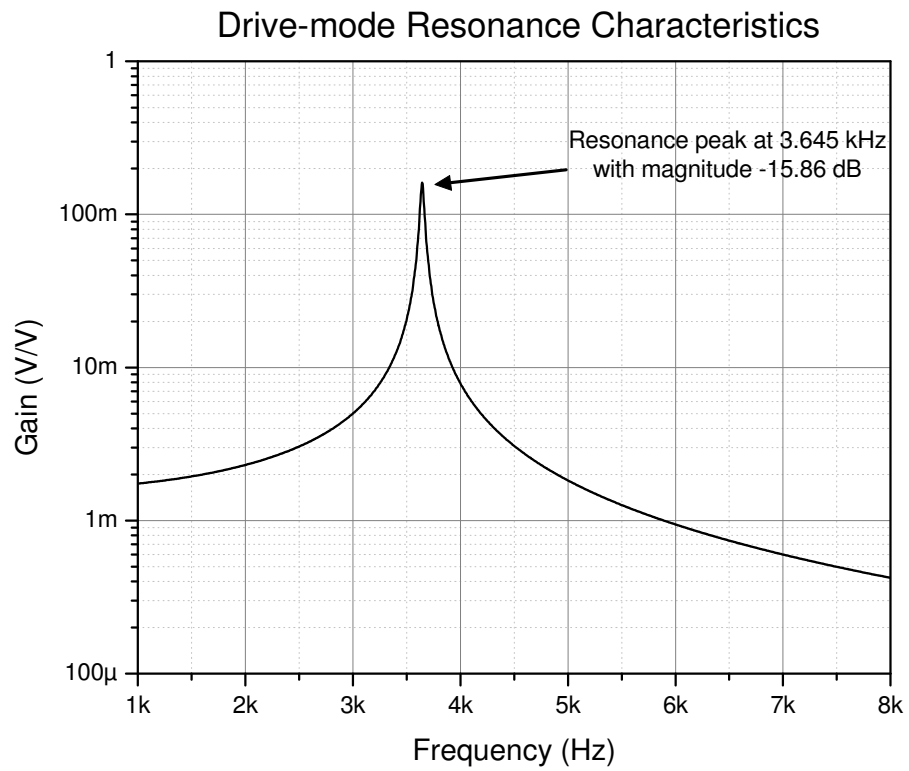


Figure 3.20: Simulation results for drive-mode resonance characteristics of single-ended SOG gyroscope connected to single-ended source follower biased with subthreshold transistor.

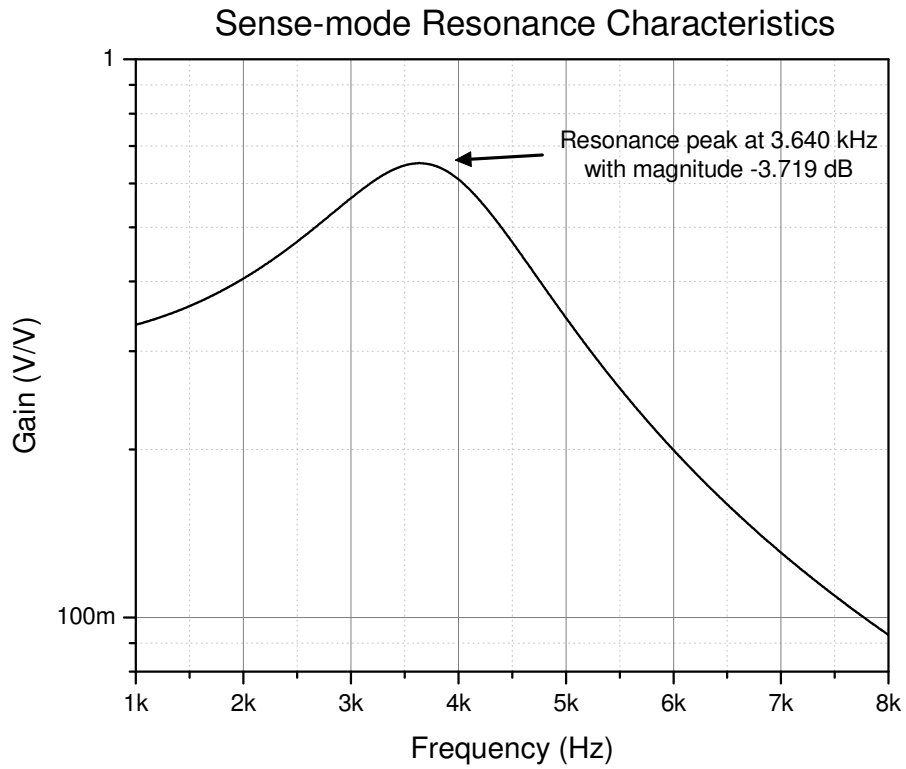


Figure 3.21: Simulation results for sense-mode resonance characteristics of single-ended SOG gyroscope connected to single-ended source follower biased with subthreshold transistor.

3.2.5. Fabrication of the single-ended source follower biased with subthreshold transistor

Figure 3.22 shows the layout of single-ended source follower biased with subthreshold transistor. This layout includes two source followers, two ac coupler capacitors, two op-amps, and op-amp bias cells. The chip measures only 815 μm x 994 μm . This chip is fabricated in XFAB XC0.6 process on November 2005. Figure 3.23 shows the photograph of the fabricated single-ended source follower biased with subthreshold transistor.

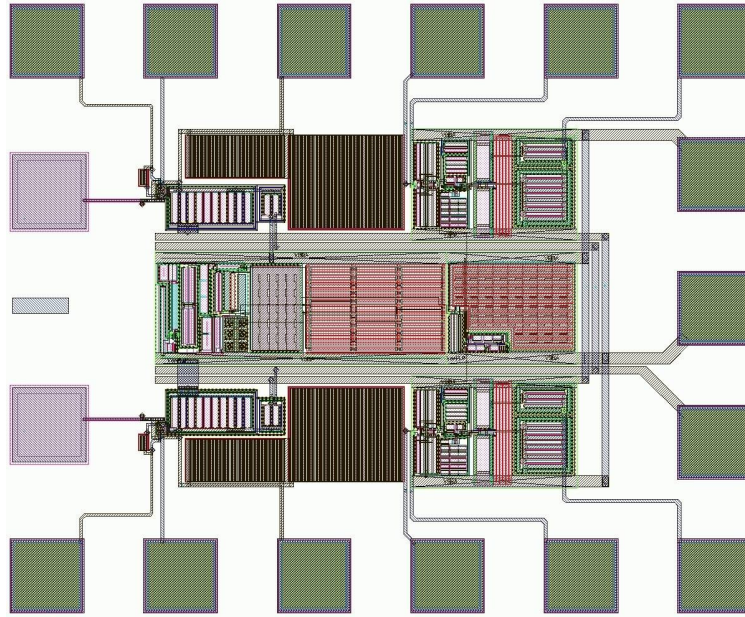


Figure 3.22: Layout of single-ended source follower biased with subthreshold transistor.

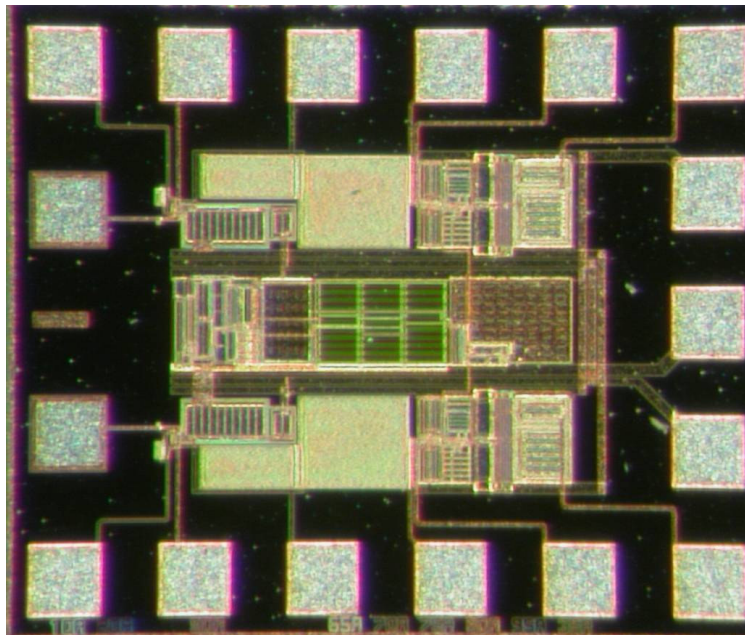


Figure 3.23: Photograph of the fabricated single-ended source follower biased with subthreshold transistor.

3.3. Improvements for Single-Ended Capacitive Interfaces

It is possible to improve the structure of the capacitive interface to obtain higher performances from the gyroscope. Three different solutions are proposed for improving the performance of the capacitive interface in this study:

1. Fully-differential architecture
2. Internal biasing with common-mode feedback
3. Fully-differential to single-ended conversion by instrumentation amplifier

These structures are analyzed in detail in the following subsections.

3.3.1. Fully-differential architecture

Fully-differential architectures have many advantages over single-ended structures. Some of these are [30, 31]:

1. Higher immunity to common-mode noise
2. Larger output voltage swing
3. Better linearity

The disadvantage of using fully-differential structures is that they generally require complex common-mode feedback circuits to set the dc point of the output nodes.

It is possible to implement the preamplifier architecture presented in Figure 3.4 in fully-differential architecture. Figure 3.24 shows the fully-differential source followers using PMOS transistors. This structure improves the performance of the capacitive interface with respect to the single-ended counterpart by increasing the immunity to common-mode noise and obtaining larger output voltage swing. The

linearity of the circuit is not a main design issue for these types of capacitive interfaces since the signal at the output of the gyroscope is quite small.

To generate the bias voltage at the gate of M2 and M4 transistors, basic current mirror configuration can be used. It is possible to increase the output swing by changing transistor dimensions and decreasing the overdrive voltages of transistors M2 and M4.

3.3.2. Internal biasing with common-mode feedback

The circuit given in Figure 3.24 does not require common-mode feedback circuitry as standard high-gain fully-differential architectures since the dc point of the output is determined directly with the common-mode of the input. In other fully-differential architectures, the mismatches may put the transistors into triode region; however, mismatches are not so critical in source followers compared to high-gain stages since gate-to-source voltage is less affected with the change in transistor parameters.

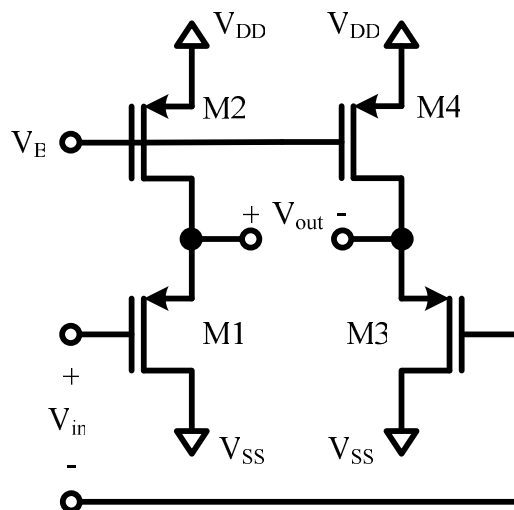


Figure 3.24: Fully-differential source followers using PMOS transistors.

On the other hand, although it is not essential, to simplify the biasing circuit, the output of the fully-differential source follower may be set to the desired point by use of a simple common-mode feedback circuit. For common-mode feedback circuits, general approach is to control the current passing through input transistor, in order to determine dc point of the outputs. On the other hand, in fully-differential source followers, it is possible to change dc point of the input directly, to control dc point of the output.

Common-mode feedback circuits are generally composed of two main parts. The first one is the common-mode level sense circuit and the second one is the feedback circuit. For the common-mode level sense circuit, the simplest approach is using two resistors between the output nodes and getting the common-mode level from the midpoint between the resistors. For the feedback circuit, a comparator can be used to compare the common-mode level with respect to a reference potential. The output of the comparator can be supplied to the input biasing circuit, in this case, subthreshold transistors. Figure 3.25 shows the common-mode feedback circuit implemented with fully-differential source follower. When common-mode voltage at the output, $V_{OUT,CM}$, is above the reference potential, V_{REF} , the feedback circuit will try to pull the bias voltage, V_{BIAS} , down. This will lead a decrease in $V_{OUT,CM}$ until the system is stabilized.

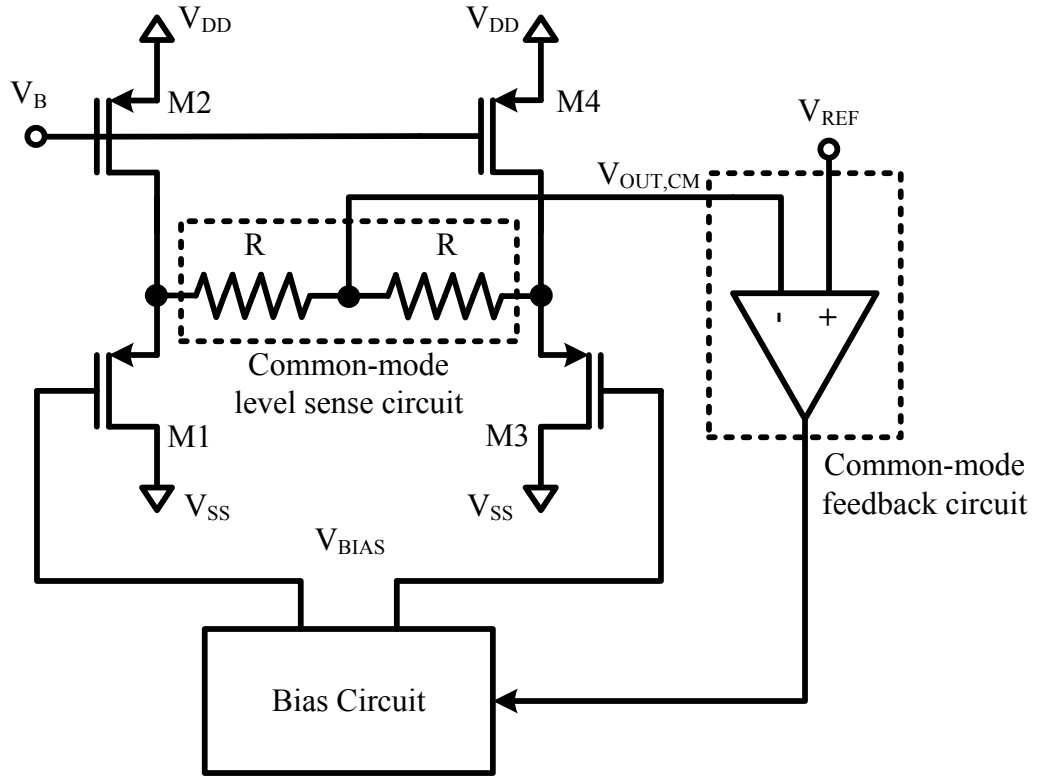


Figure 3.25: Common-mode feedback circuit implemented with fully-differential source follower.

Another advantage of using common-mode feedback is that, charging of the input node is accelerated. Assuming that at initial state, high-impedance node is around ground potential, the output of the comparator in Figure 3.25 will saturate to its low-output voltage, around -2 Volts. At this V_{BIAS} value, the threshold voltage of the subthreshold transistor is less than threshold voltage when V_{BIAS} is around -1.3 V. Therefore, the resistance of the subthreshold transistor is smaller and therefore, high-impedance node charges fast. When steady-state is achieved, the output of the comparator will return to its steady-state value, and hence, the resistance of the subthreshold transistor will increase. Because of this, fully-differential source follower with common-mode feedback charges faster than its counterpart without feedback.

3.3.3. Fully-differential to single-ended conversion by instrumentation amplifier

To reduce the complexity of the design, external control and sense electronics used in this study are designed to be single-ended. Therefore, the output of fully-differential capacitive interface must be converted to single-ended signal. It is wise to make the conversion right after the capacitive interface, because, if there are additional stages between the interface and converter, more mismatches will occur between the fully-differential branches. Most common approach for converting a fully-differential signal to single-ended version is the implementation of an instrumentation amplifier. Figure 3.26 shows the schematic of the instrumentation amplifier.

Choosing $R_1 = R_2 = R_3 = R_4$, the output voltage in terms of the input becomes

$$V_{out} = V_{OFF} + \left(1 + \frac{2 \cdot R_B}{R_G}\right) \cdot V_{in} \quad (3.20)$$

Therefore, fully-differential input can be converted to a single-ended signal by use of the instrumentation amplifier given in Figure 3.26. V_{OFF} voltage can be used to tune the dc offset at the output, e.g., if there is a dc offset at the input signal, by changing the value of V_{OFF} , one can obtain a signal with dc point at ground potential.

3.4. Fully-differential capacitive interfaces

In this study, two types of fully-differential capacitive interfaces are designed. The difference between the two is that, the second one also includes an instrumentation amplifier at the output stage. The designed circuits, their simulation results and layouts are given in the subsections below.

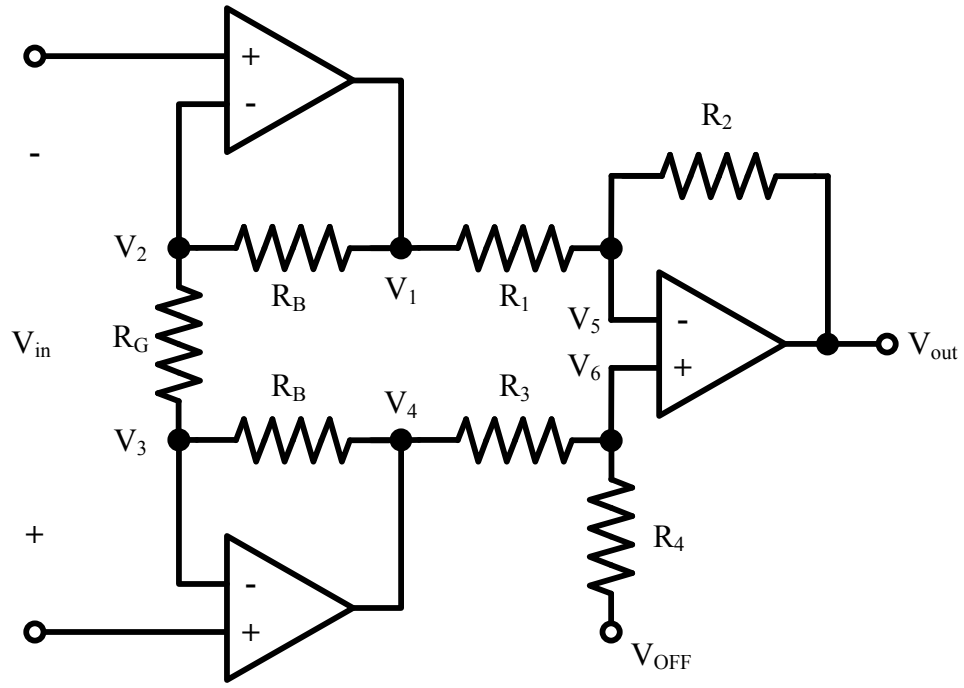


Figure 3.26: Schematic of the instrumentation amplifier.

3.4.1. Self-biased fully-differential source followers

Figure 3.27 shows the schematic of self-biased fully-differential source followers. Table 3.5 shows the transistor dimensions for self-biased fully-differential source followers.

Table 3.5: Transistor dimensions for self-biased fully-differential source followers.

	M0+ / M0-	M1 / M3	M2 / M4 / M6	M5
Width	4 μm	400 μm	200 μm	20 μm
Length	3 μm	5 μm	8 μm	20 μm

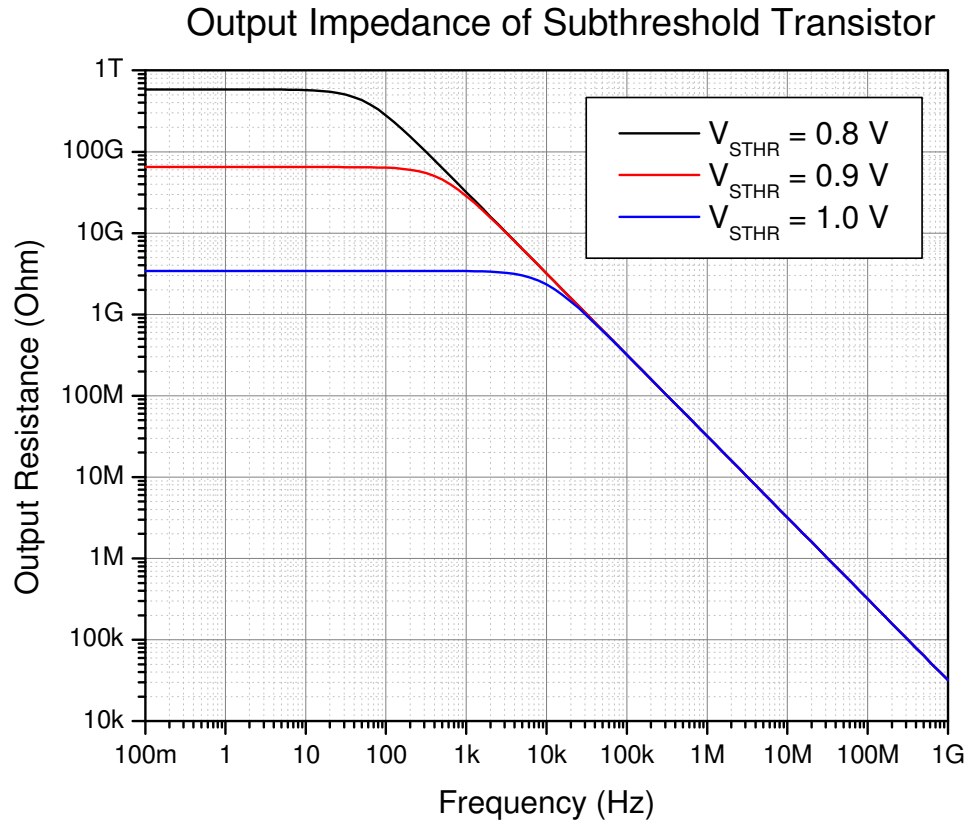


Figure 3.28: Simulation result of the output impedance seen from the drain of the biasing transistor for different values of V_{STHR} values. In this simulation, V_{BIAS} value is kept at -1.323 V considering the gate-to-source voltage at the input transistors.

Table 3.6: Output impedance values at the drain of the transistor for different values of gate-to-source voltage, V_{STHR} .

Gate-to-source voltage, V_{STHR} (V)	Output Impedance at 0 Hz (Ω)	Output Impedance at 5 kHz (Ω)
0.8	581.8 G	6.373 G
0.9	64.94 G	6.373 G
1	3.424 G	3.016 G

For source followers, the drain current is chosen to be 250 μA , which creates 1.323 V gate-to-source voltage at the input transistors. Figure 3.29 shows the simulation result of the ac analysis of fully-differential source follower at its dc operating point. At 5 kHz, the gain and phase shift of the source follower is 0.9970 V/V and -0.347 milidegrees.

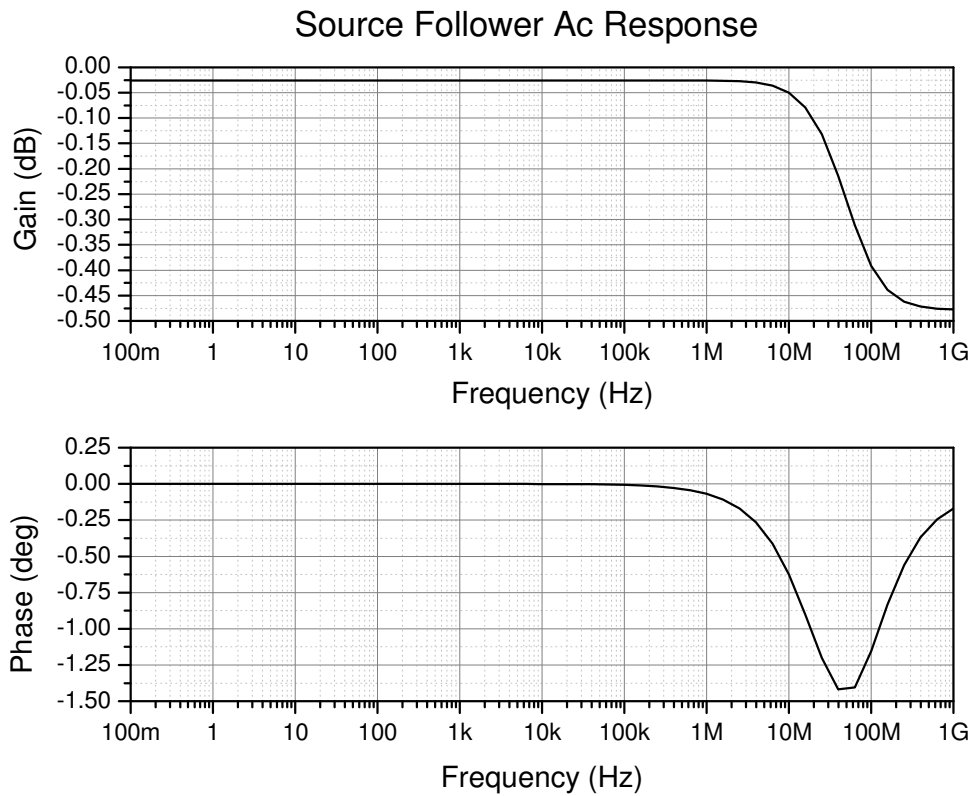


Figure 3.29: Simulation result of the ac analysis of fully-differential source follower at its dc operating point.

3.4.2. Simulations of self-biased fully-differential source followers

The simulations of self-biased fully-differential source followers are done for V_{STHR} value of 0.8 V.

3.4.2.1. Ac simulations

Figure 3.30 shows the schematic used for simulating the ac response of the capacitive interface. For the ac simulations of the fully-differential source followers, 40 pF load capacitor in Figure 3.15 is not used. The reason for this is that, there is no ac coupling capacitance implemented on-chip for the fully-differential source followers.

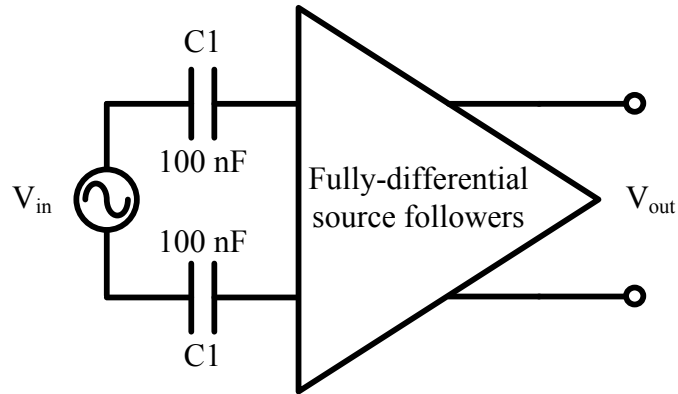


Figure 3.30: Schematic used for simulating the ac response of the capacitive interface.

Figure 3.31 shows the simulation result of the ac analysis of the self-biased fully-differential source followers. Gain and phase responses of the interface at 5 kHz are 0.988 V/V and -0.7011 milidegrees, respectively.

From the dc analysis result, the gate-to-source capacitance of the input transistor is 3.524 pF, while the gate-to-drain capacitance is 80.46 fF. Therefore, the effective input capacitance according to Equation 3.9 is 119.929 fF.

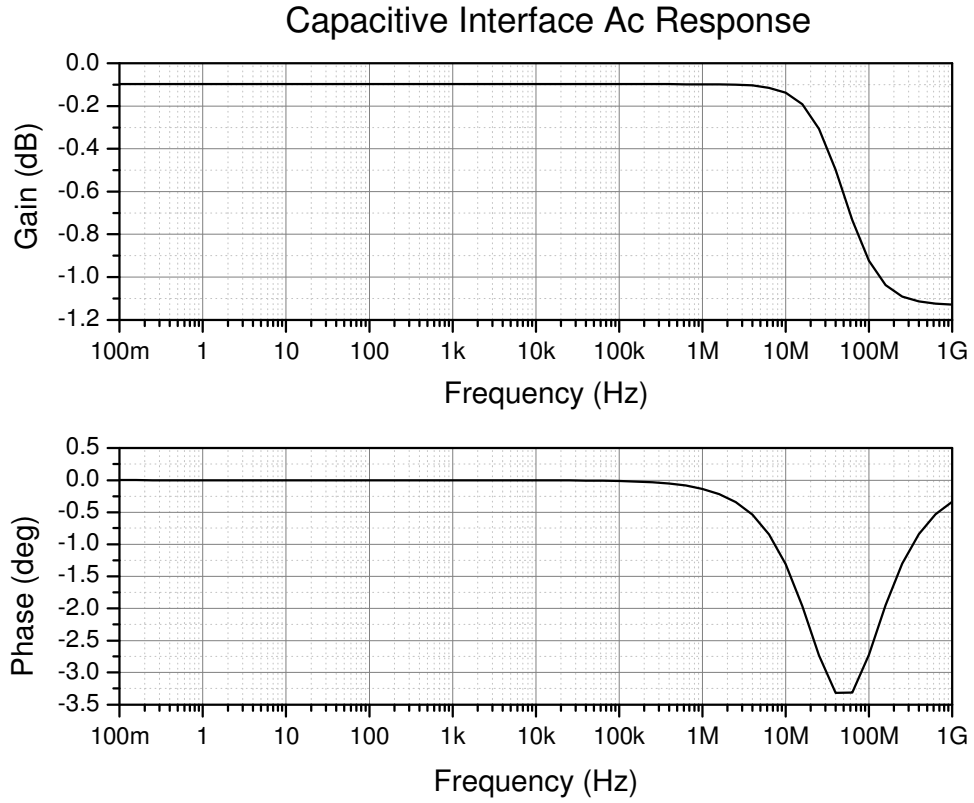


Figure 3.31: Simulation result of the ac analysis of the self-biased fully-differential source followers.

The PSRR simulation is also done for the self biased fully-differential source followers. Figure 3.32 shows the simulation result of the ac analysis for PSRR calculation of the self-biased fully-differential source followers.

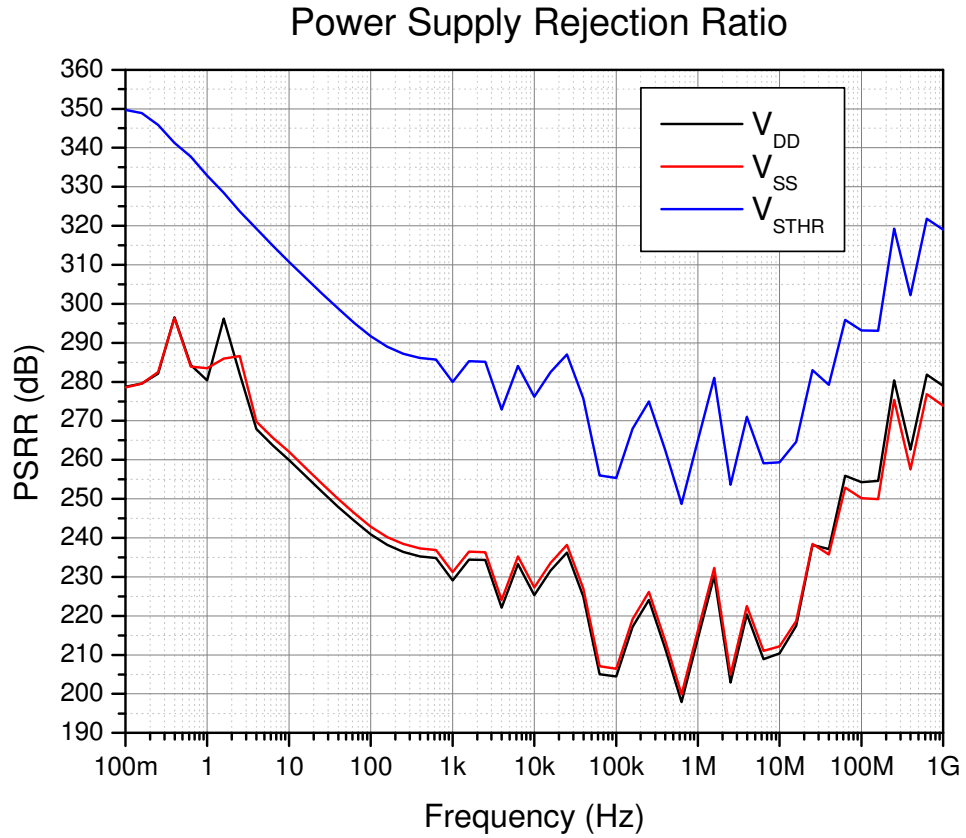


Figure 3.32: Simulation result of the ac analysis for PSRR calculation of the self-biased fully-differential source followers.

PSRR of the circuit is simulated to be 228 dB, 229.7 dB, and 278.7 dB for the supplies V_{DD} , V_{SS} , and V_{STHR} , respectively. Compared to the previous design, PSRR is highly improved since fully-differential architecture suppresses common-mode noise.

3.4.2.2. *Transient simulations*

For the transient simulation of the fully-differential interface, the voltage source in Figure 3.30 is replaced with a sinusoidal current source with amplitude 100 pA and

frequency 5 kHz. Transient analysis time is set to 2 ms and differential input, output, and current drawn from the drain of the subthreshold transistor and gate of the input transistor are monitored. Figure 3.33 shows the simulation result of the transient analysis of the self-biased fully-differential source followers.

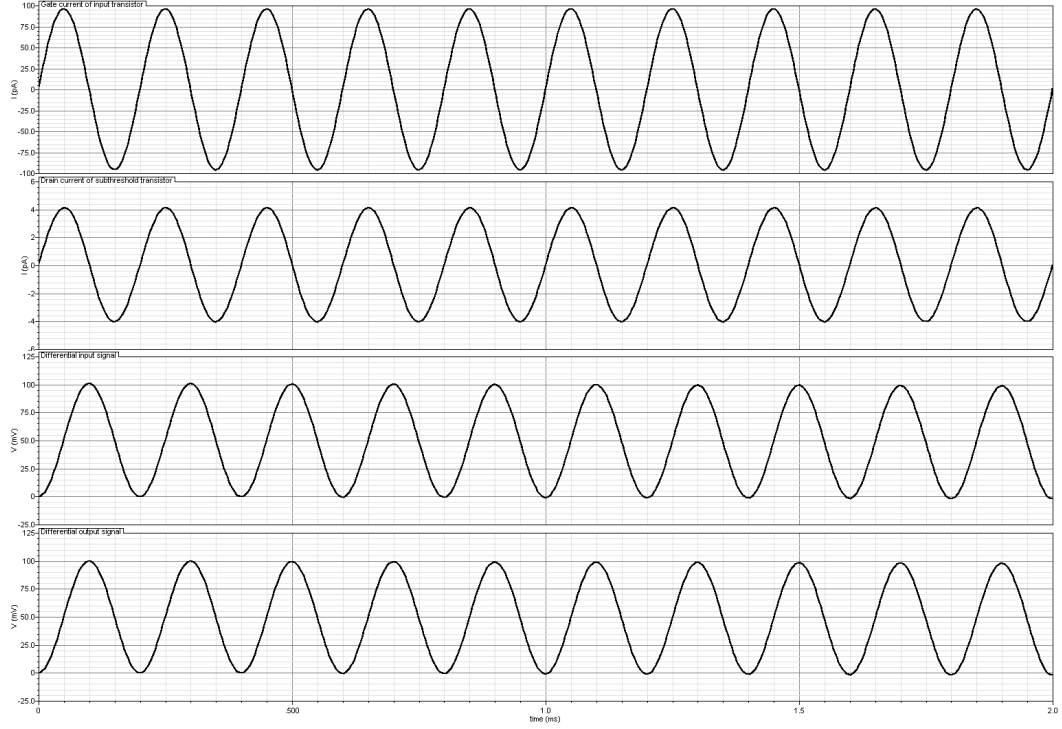


Figure 3.33: Simulation result of the transient analysis of the self-biased fully-differential source followers.

The ac current drawn from the gate of the input transistor is simulated to be 95.902 pA, while, the current from the drain of the subthreshold transistor is found to be 4.044 pA. Using the effective input capacitance of the source follower, the impedance of the biasing transistor can be calculated as 6.294 G Ω , which is close to the result found in Table 3.6.

Another transient simulation for understanding the self-biasing mechanism can be done as follows. The input of the interface is kept floating so that there is no loading at the high-impedance node. The time required for charging the input node can be measured using transient analysis. To make this analysis, the high-impedance node initial condition must be set to 0 V. Figure 3.34 shows the simulation result of the transient analysis showing charging of the high-impedance nodes. It can be seen from the figure that, up to 3.3 ms, V_{BIAS} voltage is at -2 V, which results in a lower biasing resistance in the subthreshold transistor. When the input is charged to the desired level, feedback forces V_{BIAS} to its steady-state value, -1.323 V.

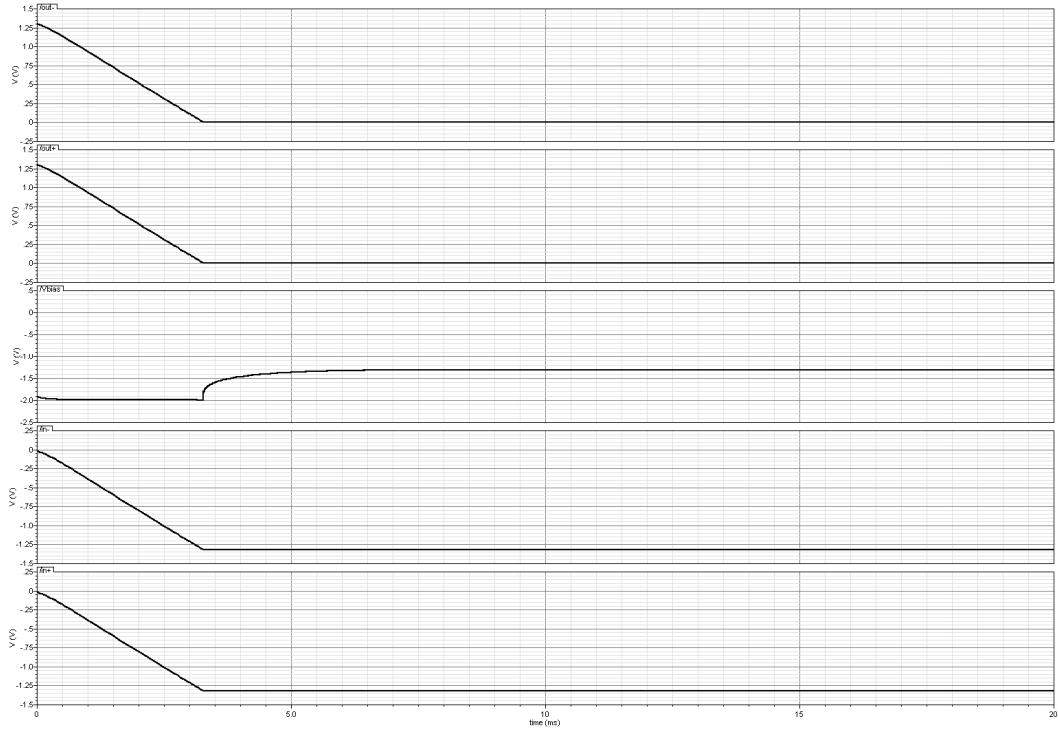


Figure 3.34: Simulation result of the transient analysis showing charging of the high-impedance nodes.

3.4.2.3. Noise simulations

Figure 3.35 shows the simulation result of the output voltage noise of self-biased fully-differential source followers.

From the simulation result, output voltage noise at 5 kHz is found to be 101.904, 104.149 and 139.936 nV/ $\sqrt{\text{Hz}}$ for V_{STHR} values of 0.8, 0.9 and 1 Volt, respectively.

There are many components present in the self-biased fully-differential source followers when compared to its single-ended counterpart. Therefore, the noise contributions from each component are not included in this thesis. However, when the contributions from the noise sources are analyzed, it can be seen that for low V_{STHR} values, the dominant source is the current and flicker noise of the input transistor as it was for the single-ended circuit. When V_{STHR} value is increased, the current noise of the subthreshold transistor increases exponentially and it dominates the noise at the output node. The components in the comparator do not affect the output voltage noise significantly since gain from the bias node to the output is quite small.

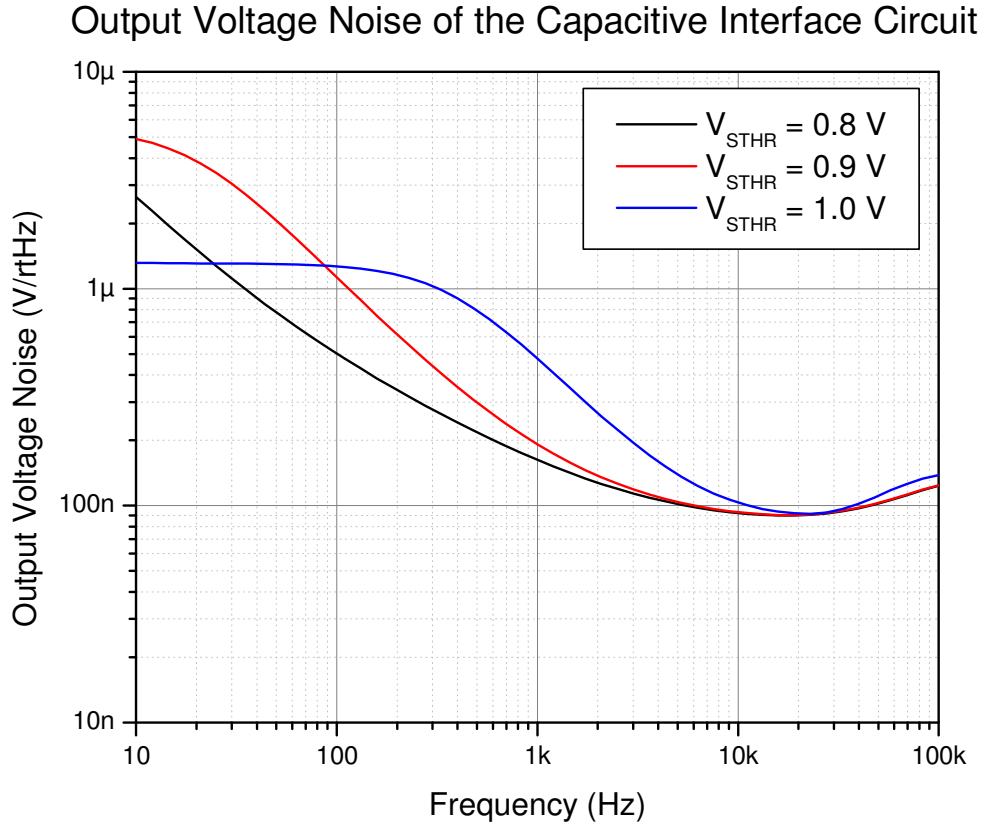


Figure 3.35: Simulation result of the output voltage noise of self-biased fully-differential source followers. Output voltage noise displayed in this figure is simulated from one of the output nodes with respect to ground.

3.4.3. Fabrication of the self-biased fully-differential source followers

Figure 3.36 shows the layout of self-biased fully-differential source followers. This layout includes two fully-symmetrical source followers, two symmetrical resistors for common-mode level sense circuit, one operational amplifier for common-mode feedback, and op-amp bias cells. Dimensions of the chip are $762\text{ }\mu\text{m} \times 783.5\text{ }\mu\text{m}$. This chip is fabricated in XFAB XC0.6 process on May 2006. Figure 3.37 shows the photograph of the fabricated self-biased fully-differential source followers.

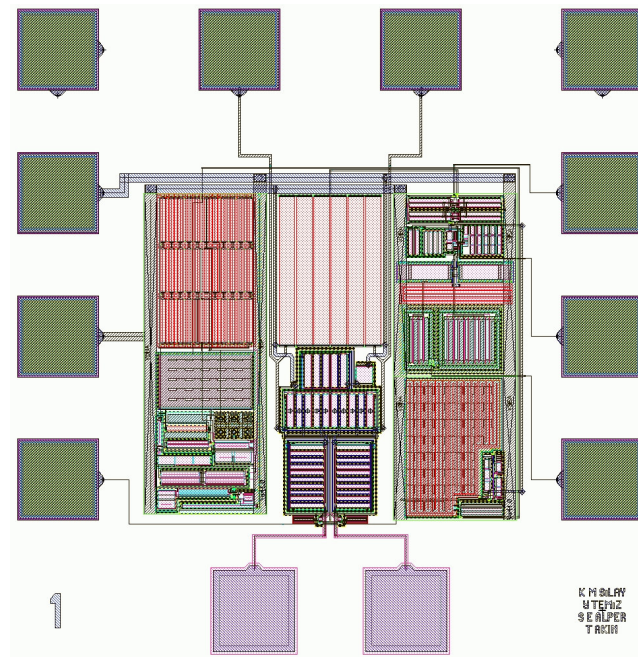


Figure 3.36: Layout of self-biased fully-differential source followers.

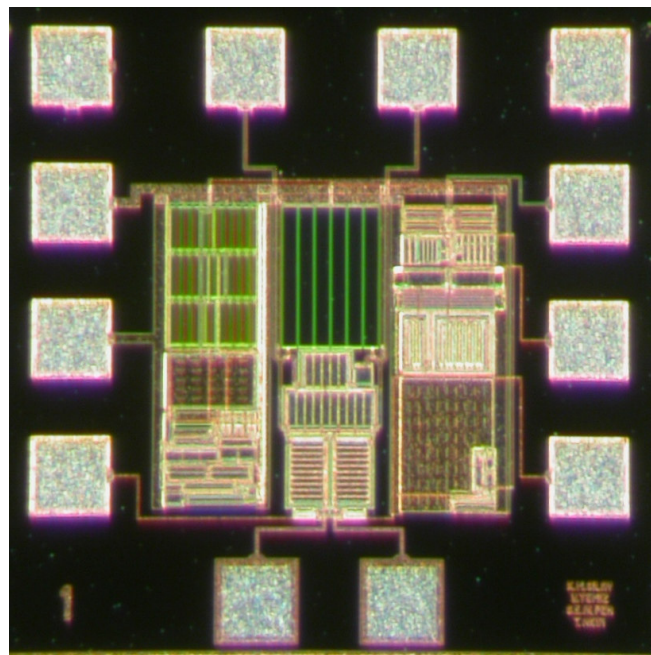


Figure 3.37: Photograph of the fabricated self-biased fully-differential source followers.

3.4.4. Self-biased fully-differential source followers with instrumentation amplifier

Input of the instrumentation amplifier presented in Figure 3.26 is directly connected to the self-biased fully-differential source follower, in order to convert fully-differential signal to single-ended signal. For the instrumentation amplifier, connections of resistor R_G are taken outside of the chip to have variable gain amplifier. All other resistor values are chosen to be 10 k Ω and these resistors are implemented on-chip.

3.4.5. Simulations of self-biased fully-differential source followers with instrumentation amplifier

V_{STHR} value is set to 0.8 V for simulations of this type of interface. Gain of the instrumentation amplifier is set to 3 by choosing R_G as 10 k Ω .

3.4.5.1. Ac simulations

Figure 3.38 shows the simulation result of the ac analysis of self-biased fully-differential source followers with instrumentation amplifier. At 5 kHz, the gain of the interface is simulated to be 2.9702 V/V, while, its phase shift is -170.574 milidegrees.

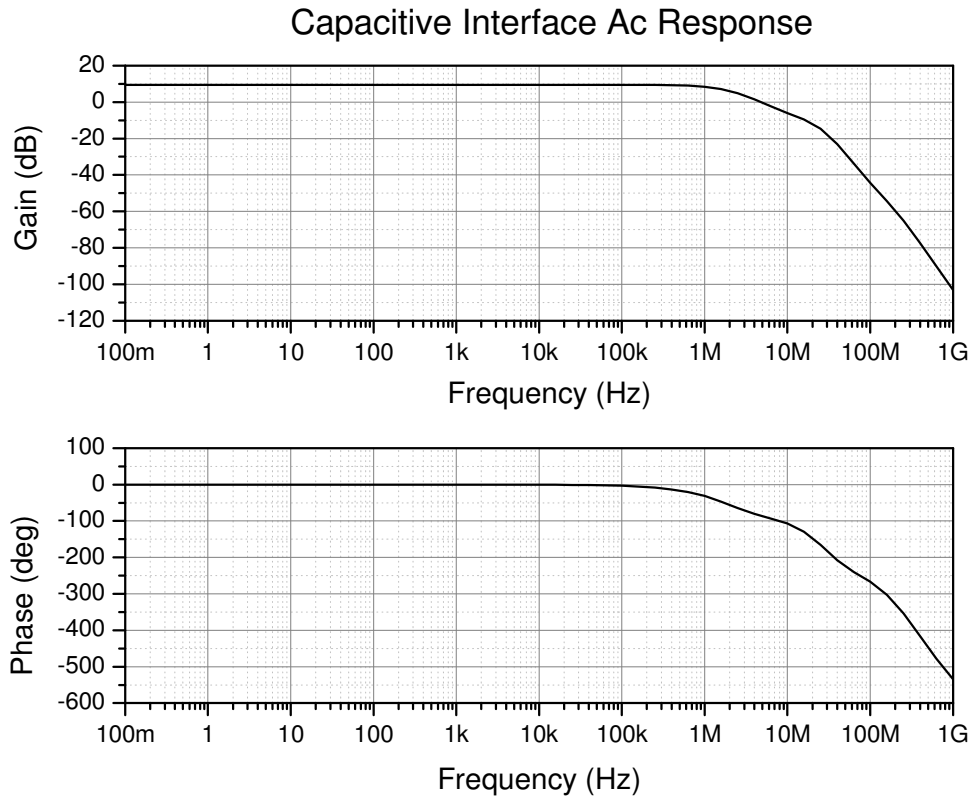


Figure 3.38: Simulation result of the ac analysis of self-biased fully-differential source followers with instrumentation amplifier.

Figure 3.39 shows the simulation result of the ac analysis for PSRR calculation of the self-biased fully-differential source followers with instrumentation amplifier.

From the simulation results, PSRR of the circuit is found as 58.76 dB, 104.1 dB, and 154.6 dB for the supplies V_{DD} , V_{SS} , and V_{STHR} , respectively. The reason for the degradation in PSRR is due to the addition of the power supply noise coming from the instrumentation amplifier.

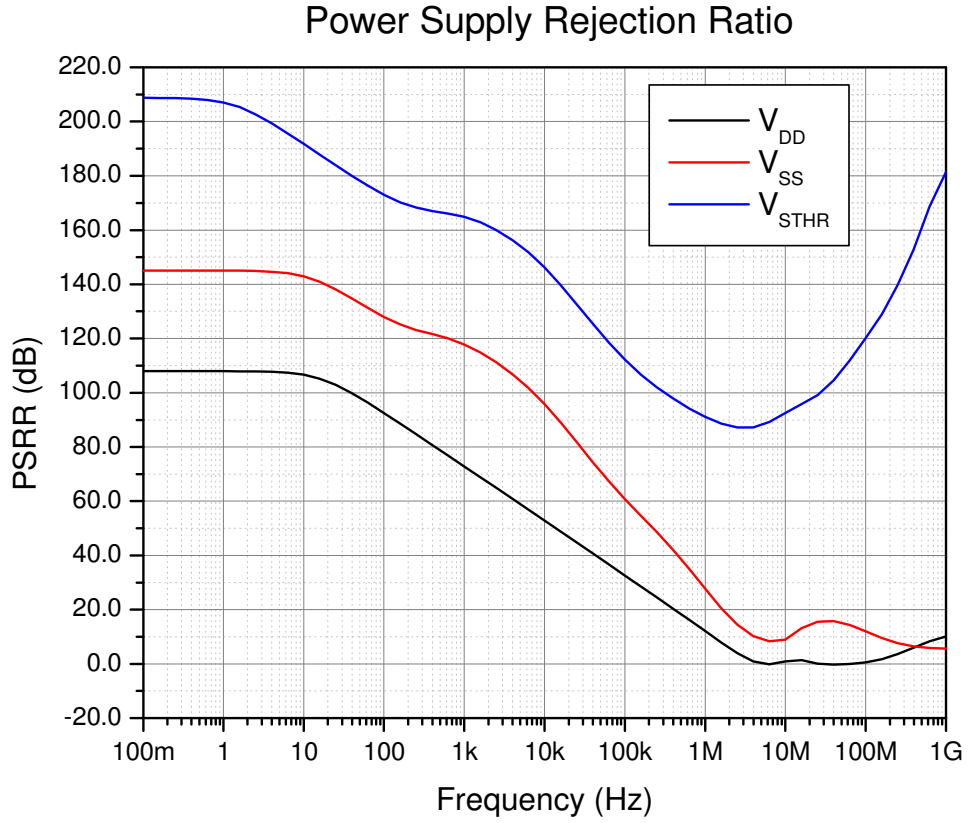


Figure 3.39: Simulation result of the ac analysis for PSRR calculation of the self-biased fully differential source followers with instrumentation amplifier.

3.4.5.2. Transient simulations

Transient analysis of the self-biased fully-differential source followers with instrumentation amplifier is the same analysis used in Section 3.4.2.2. Figure 3.40 shows the simulation result of the transient analysis of self-biased fully-differential source followers with instrumentation amplifier. The differential input signal for 100 pA amplitude current injected to differential nodes is 100.957 mV, while the output signal for this input is 300.121 mV. This implies that the gain of the capacitive interface is 2.973 V/V.

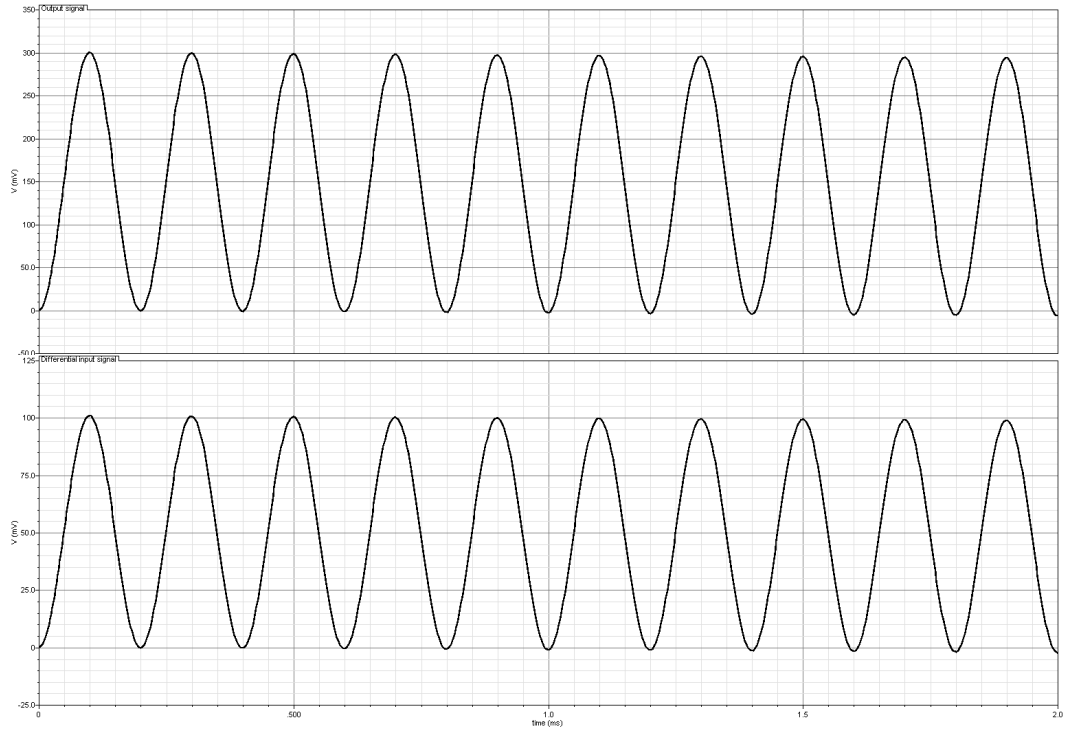


Figure 3.40: Simulation result of the transient analysis of self-biased fully-differential source followers with instrumentation amplifier.

Since the instrumentation amplifier does not affect charging of the high-impedance input node, the result found from Figure 3.34 is also valid for this capacitive interface. Charging time of the input is simulated to be 3.3 ms.

3.4.5.3. Noise simulations

Figure 3.41 shows the simulation result of the output voltage noise of self-biased fully-differential source followers with instrumentation amplifier.

At 5 kHz, voltage noise at the output of the capacitive interface is 581.287, 594.352 and 817.704 nV/ $\sqrt{\text{Hz}}$ for V_{STHR} values of 0.8, 0.9 and 1 Volt, respectively.

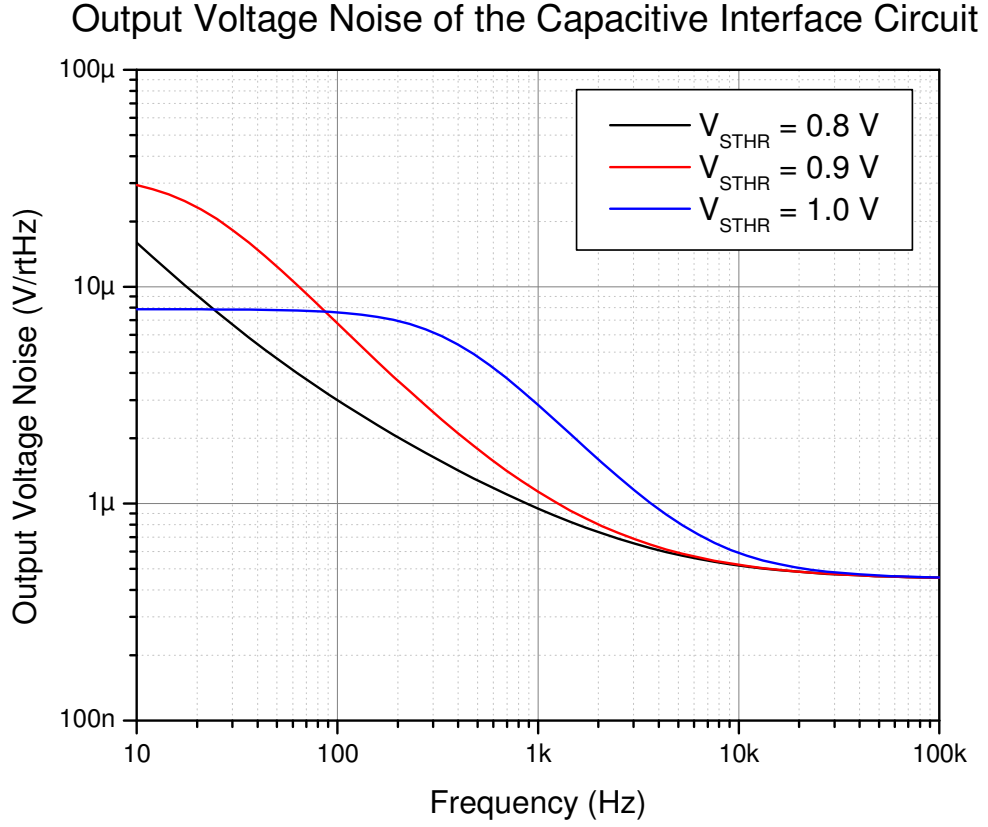


Figure 3.41: Simulation result of the output voltage noise of self-biased fully-differential source followers with instrumentation amplifier.

Output voltage noise of this interface can be represented as

$$\bar{v}_{out}^2 = A_{ins}^2 \cdot (2 \cdot \bar{v}_{dif}^2 + \bar{v}_{ins,ir}^2) \quad (3.21)$$

where, A_{ins} is the gain of the instrumentation amplifier, $\bar{v}_{dif}^2$ is the voltage noise power at the output of self-biased fully-differential source followers, and $\bar{v}_{ins,ir}^2$ is the voltage noise power of the instrumentation amplifier referred to its input. Detailed calculations are left to the reader's attention.

3.4.6. Fabrication of the self-biased fully-differential source followers with instrumentation amplifier

Figure 3.42 shows the layout of self-biased fully-differential source followers with instrumentation amplifier. This layout includes three op-amps and six resistors for the instrumentation amplifier, in addition to the self-biased fully-differential source followers given in 3.4.3. Dimensions of the chip are $767.7\ \mu\text{m} \times 882.8\ \mu\text{m}$. This chip is also fabricated in XFAB XC0.6 process on May 2006. Figure 3.43 shows the photograph of the fabricated self-biased fully-differential source followers with instrumentation amplifier.

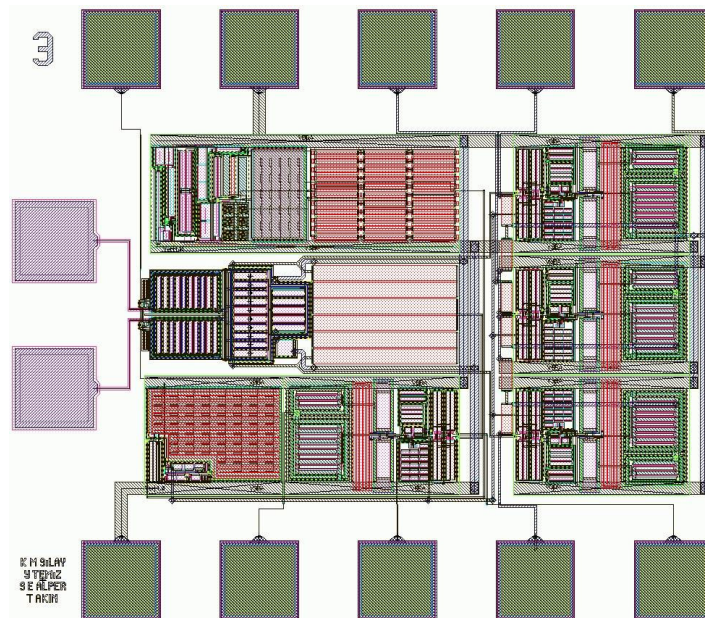


Figure 3.42: Layout of self-biased fully-differential source followers with instrumentation amplifier.

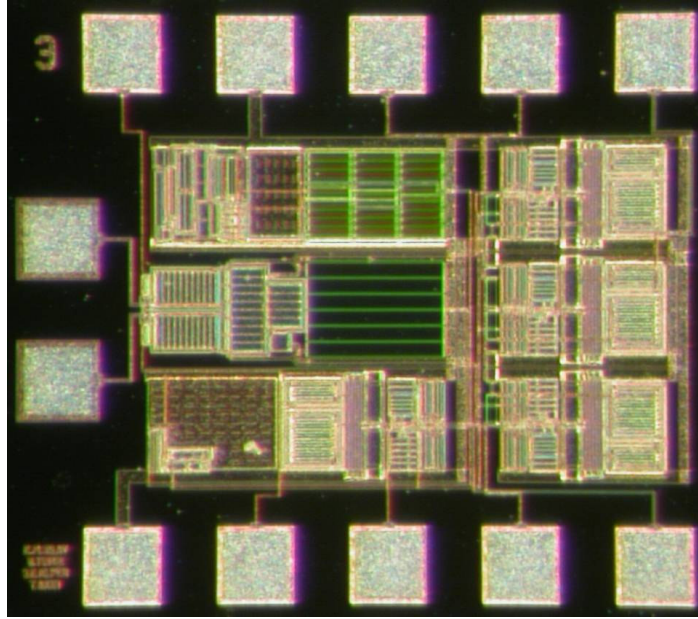


Figure 3.43: Photograph of the fabricated self-biased fully-differential source followers with instrumentation amplifier.

3.5. Summary

This chapter gave the details of the capacitive interfaces for MEMS vibratory gyroscopes developed in this study. It proposed solutions for the challenges faced in capacitive interfaces, especially for solving the high-impedance node biasing problem. Design of single-ended capacitive interfaces is explained and their simulations for verification of the theory are given. Moreover, the designed interface is simulated with the gyroscope model constructed for the single-ended SOG gyroscope and the effect of the readout is observed. Then, possible improvements for single-ended capacitive interfaces are proposed, and fully-differential gyroscope interfaces are designed according to the improvements given in this chapter. The simulations of these interfaces are also given verifying the expected improvements.

CHAPTER IV

CHARACTERIZATION AND TESTS OF CAPACITIVE INTERFACE CIRCUITS

This chapter gives the characterization and test results of the capacitive interface circuits designed in this research. Section 4.1 starts with explanation of the test setup used for characterization of interfaces. Then, it gives the test results of three designs developed in this study with each having ac characteristics, transient characteristics for sinusoidal inputs, characteristics of the biasing transistor, and noise characteristics. Section 4.2 gives the characterization of mechanical sensors of SOG gyroscopes used in this study, and Section 4.3 demonstrates the preliminary characterization of these gyroscopes with single-ended capacitive interfaces. Then, Section 4.4 explains the external readout and control electronics for MEMS gyroscopes, and Section 4.5 characterizes the single-ended SOG gyroscope with external electronics by giving its scale factor, linearity, bias instability and angle random walk. Finally, Section 4.6 summarizes this chapter.

4.1. Characterization of Capacitive Interface Circuits

Capacitive interface circuits designed in this study are tested in four different conditions to obtain the ac, transient and noise response of the circuit. The next subsections explain the test setup used for measuring the response of the circuit and give the test results for three interface architectures.

4.1.1. Test setup for characterization of capacitive interface circuits

Figure 4.1 shows the photograph of general purpose test setup used for characterization of the capacitive interfaces.

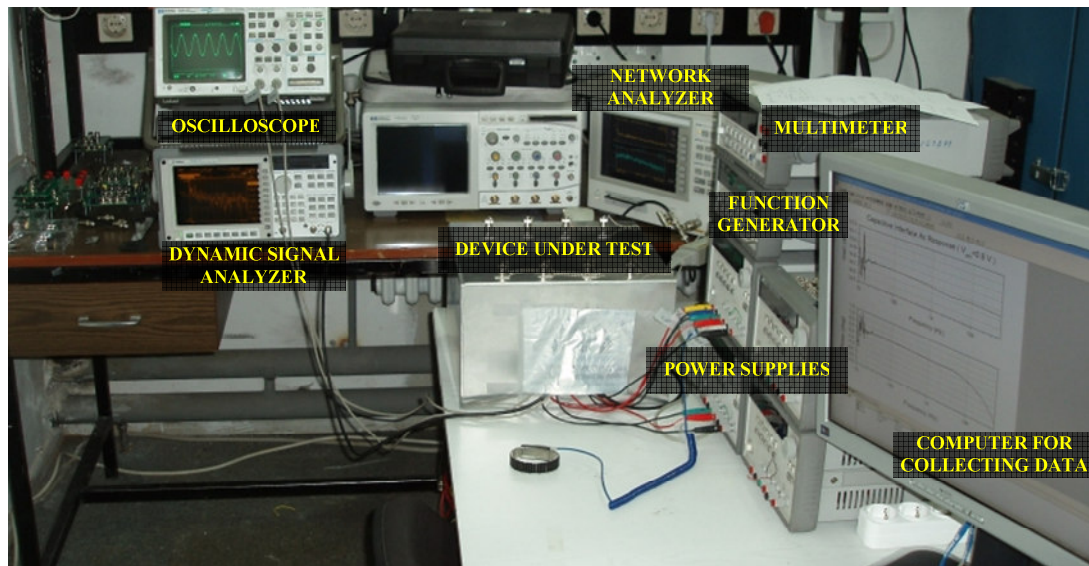


Figure 4.1: Photograph of general purpose test setup used for characterization of the capacitive interfaces.

4.1.1.1. Test setup for measuring ac response of the circuit

In order to measure the ac response of an interface, a sinusoidal input should be applied to the input of the interface and output signal should be measured. This measurement can be achieved by using a network analyzer (Agilent 4395A) and an active probe (Agilent 41800A). Ac source of the network analyzer should be connected to the high-impedance node and active probe to the output of the capacitive interface. The main problem for this measurement is that source of the network analyzer tries to bias the high-impedance node to its dc potential.

Therefore, dc point of the high-impedance node cannot be determined by the biasing transistor. Because of this, the effect of the biasing transistor cannot be characterized. To characterize the effect of this transistor, a series capacitance can be connected between the source of the network analyzer and input of the interface. The impedance of this coupling capacitor should be as small as possible to minimize its effect on the gain of measured ac response. For these designs, coupling capacitance is chosen as 100 nF, which corresponds to $318.31\ \Omega$ at 5 kHz.

To measure the ac response of fully-differential circuits, differential inputs should be applied to the inputs of the interface. However, it is not possible to get differential input from network analyzer. Therefore, anti-phase of the source of network analyzer must be generated. This can be achieved by implementing an inverting amplifier with gain -1. However, it is not possible to obtain 180° phase shift exactly as the amplifier will add a phase shift to the input.

Figure 4.2 shows the block diagram of the test setup for measuring ac response of the capacitive interfaces. For single-ended interfaces, the output of the inverting amplifier is not used. On the other hand, for fully-differential interfaces, one of the differential outputs is not measured since only one output can be measured at a time.

Figure 4.3 shows the photograph of the printed circuit board (PCB) used for characterization of capacitive interfaces.

In this measurement, network analyzer sweeps the frequency of constant amplitude signal at the source, measures the signal at the channel, compares these signals and extracts the gain and phase shift of the interface.

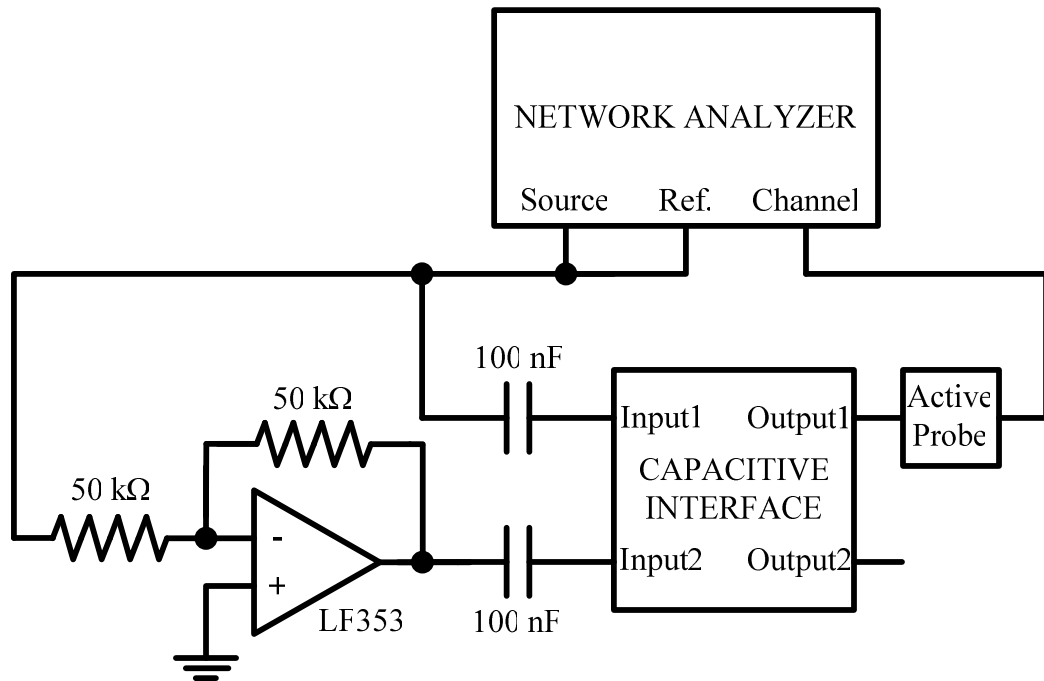


Figure 4.2: Test setup for measuring ac response of the capacitive interfaces. Source output of the network analyzer is connected to its reference input in order to compare the signal at source with the signal at the channel.

4.1.1.2. Test setup for measuring transient response of the circuit to a sinusoidal input

Setup for measurement of the transient response to a sinusoidal input is very similar to the setup given in Figure 4.2. The difference in this case is that, the sinusoidal signal is applied from a function generator (Agilent 33120A) set at frequency 5 kHz and the output is measured from the oscilloscope (Agilent 54645D).

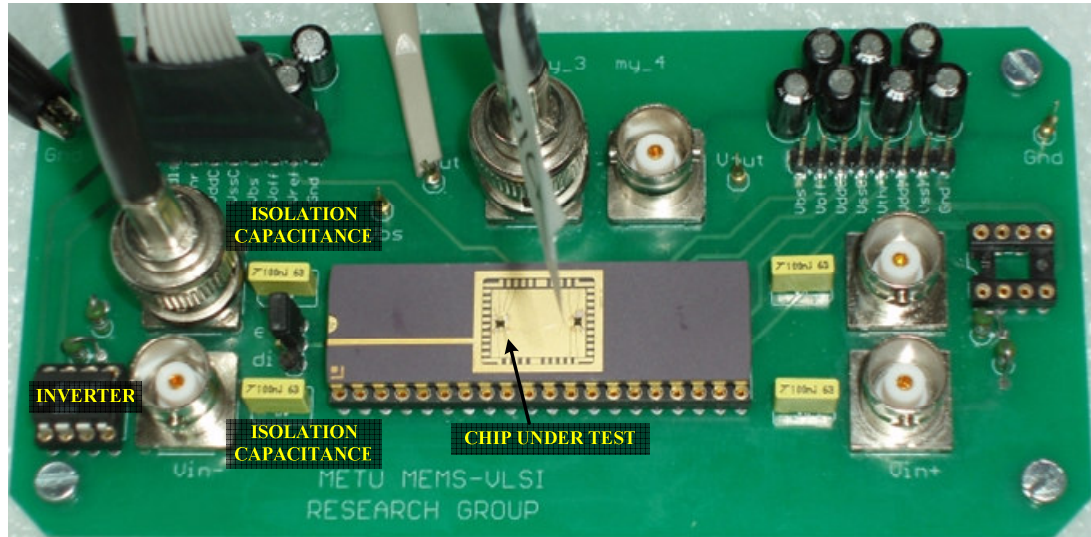


Figure 4.3: Photograph of the printed circuit board (PCB) used for characterization of capacitive interfaces.

4.1.1.3. Test setup for measuring the resistance of biasing transistor operating in subthreshold region

In order to measure the resistance of the biasing resistor, i.e., impedance of the subthreshold transistor at 0 Hz, one should find the time required for charging the high-impedance node. Unfortunately, the input capacitance of the capacitive interface is not known accurately, since parasitic capacitances coming from wire-bonding and package connections increases the capacitance at the high-impedance node. As it is necessary to know the input capacitance at the high-impedance node precisely, one should connect a large capacitor to that node, in order to dominate the capacitance seen by the subthreshold transistor. For this measurement, 100 nF capacitor is connected from the inputs of the capacitive interfaces to ground. Since the parasitic capacitances at the high-impedance node is in the order of a few picofarads, it can be said that the capacitance seen by the subthreshold transistor is 100 nF. As the capacitance seen by the subthreshold

transistor is known, one can calculate the resistance of this transistor from the simple charge/discharge equation given as

$$v(t) = v_{final} + (v_{initial} - v_{final}) \cdot \exp\left(-\frac{t - t_i}{R \cdot C}\right) \quad (4.1)$$

If initial and final values of the voltage at high-impedance node are known, one can extract the resistance of the transistor as

$$R_{sub} = \frac{t - t_i}{C} \cdot \ln\left(\frac{v_{initial} - v_{final}}{v(t) - v_{final}}\right) \quad (4.2)$$

As it is not possible to measure the voltages directly from the high-impedance node, one should measure the time constant from the output of the capacitive interface. Therefore, the initial and final values in Equation 4.1 are the initial and final values of the output voltage.

In some cases, time constant of the interface may be very high for small V_{STHR} values. If time constant is larger than 500 seconds, oscilloscope cannot measure the response of the circuit. In that case, the response of the circuit is first measured at a larger V_{STHR} value with 100 nF connected to the high-impedance node and resistance of the transistor is calculated for this value of V_{STHR} . After that, same measurement is done with no external capacitor connected and capacitance seen by the transistor is calculated from this measurement. Lastly, the response of the circuit is measured at the desired value of V_{STHR} and the resistance of the transistor is calculated using the capacitance value found from the second measurement.

4.1.1.4. *Test setup for measuring the noise characteristic of the circuit*

Measuring noise values below 1 $\mu\text{V}/\sqrt{\text{Hz}}$ accurately is not an easy task since the noise coming from the measurement setup will perturb the test results. For that reason, a high-gain noise amplifier is used in order to increase the signal levels measured so that the noise of the measurement setup is negligible.

Noise measured from the output of the noise amplifier will include both the noise of the amplifier and the device under test. Therefore, the measured data should be processed with the noise of the amplifier and its gain, in order to get the noise of the interface only. Noise measured at the output of the amplifier can be written as

$$\bar{v}_{int+amp}^2 = \bar{v}_{amp}^2 + A_{amp}^2 \cdot \bar{v}_{int}^2 \quad (4.3)$$

where, $\bar{v}_{int+amp}^2$ is the power of voltage noise at output of the noise amplifier, $\bar{v}_{amp}^2$ is the power of the noise amplifier only, and A_{amp} is the gain of the noise amplifier. Therefore, before measuring the noise of a capacitive interface, one should obtain the ac and noise response of the noise amplifier.

Figure 4.4 shows the schematic of the noise amplifier designed for measuring the noise of capacitive interfaces. Figure 4.5 shows the measured ac response of the noise amplifier given in Figure 4.4 using a dynamic signal analyzer (Agilent 35670A).

Noise measurements are performed in a metal cage and batteries are used as power supplies in order to eliminate the contribution of external noise sources. Figure 4.6 shows the photograph of the noise measurement setup in metal cage.

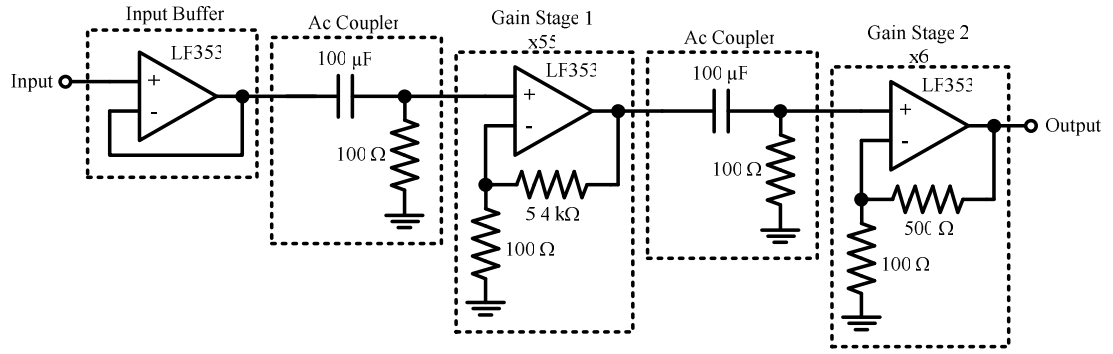


Figure 4.4: Schematic of the noise amplifier designed for measuring the noise of capacitive interfaces. The gain of the circuit is set to 330.

Noise spectrum of the noise amplifier is measured in the setup given in Figure 4.6 by connecting the input of the amplifier to ground and measuring the output with the dynamic signal analyzer. Figure 4.7 shows the measured noise spectrum of the noise amplifier given in Figure 4.4. From the test result, voltage noise at the output of the noise amplifier is calculated to be $9.47 \text{ nV}/\sqrt{\text{Hz}}$ in 50 kHz bandwidth.

4.1.2. Characterization of single-ended source follower biased with subthreshold transistor

During characterization of single-ended source follower biased with subthreshold transistor, bias voltage of the subthreshold transistor, V_{BIAS} , is kept at -1.4 V.

4.1.2.1. Ac characteristics

Figure 4.8 shows the ac characteristics of single-ended source follower biased with subthreshold transistor when V_{STHR} is set to 0.8 V.

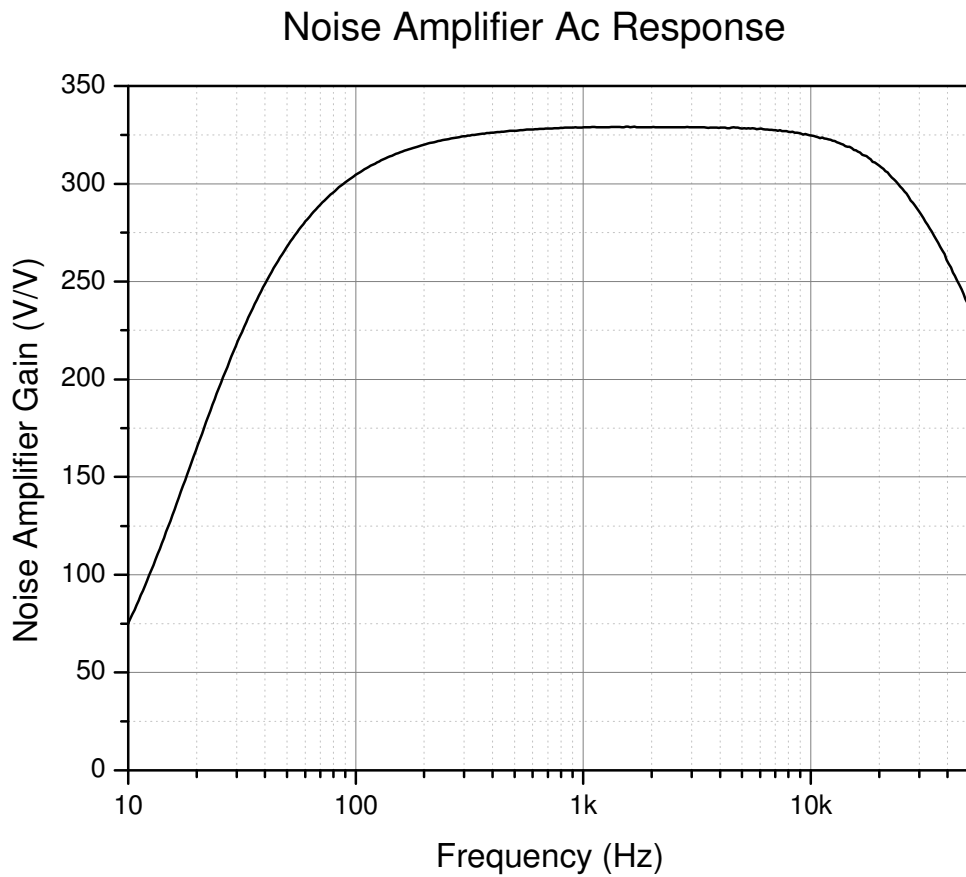


Figure 4.5: Measured ac response of the noise amplifier given in Figure 4.4 using a dynamic signal analyzer (Agilent 35670A).

For frequency range between 10 and 20 Hz, network analyzer cannot measure the output of the interface precisely. For that reason, there are glitches at the measured response. Around 5 kHz, the gain of the amplifier is 0.9843 V/V and its phase shift is -0.418° , which are consistent with the simulation results.

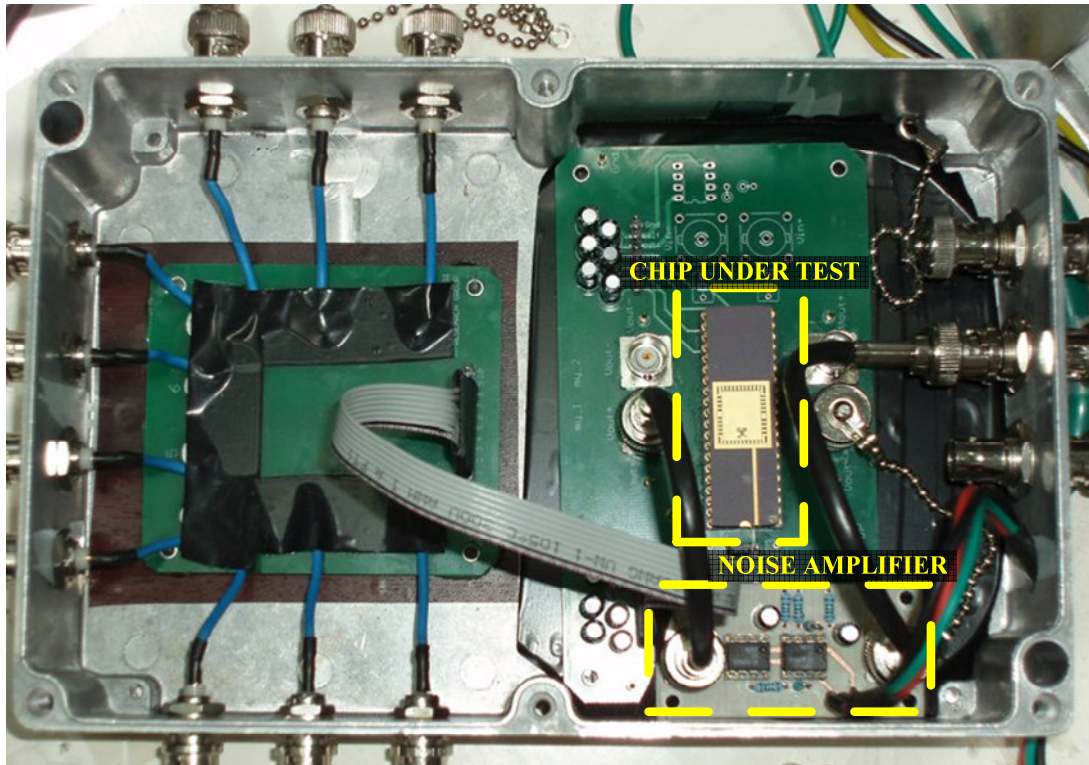


Figure 4.6: Photograph of the noise measurement setup in metal cage.

4.1.2.2. *Transient characteristics for a sinusoidal input*

Figure 4.9 shows the transient response of single-ended source follower biased with subthreshold transistor to a sinusoidal input with amplitude 250 mV and frequency 5 kHz.

The output signal measured at the output is very close to 0.5 V peak-to-peak, which means the gain of the circuit is very close to 1.

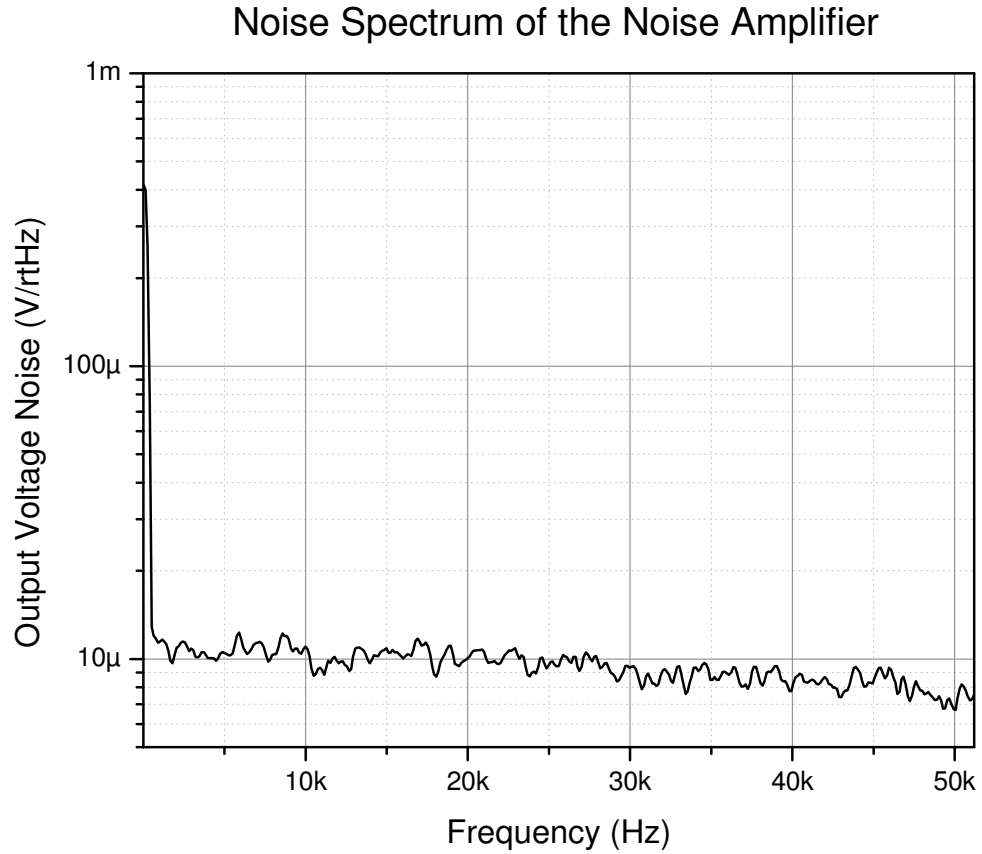


Figure 4.7: Measured noise spectrum of the noise amplifier given in Figure 4.4.

4.1.2.3. *Characterization of resistance of the biasing transistor*

During characterization of single-ended source follower, the resistance of the biasing transistor could not be measured for V_{STHR} value of 0.8 V, when 100 nF external capacitor is connected to the high-impedance node. Hence, the resistance is measured at 0.9 V firstly, and then, measured at 0.9 V when no external capacitance is connected. When time constants are scaled, it is possible to measure the resistance.

Firstly, measurement of the resistance of subthreshold transistor is done with 100 nF capacitor. Figure 4.10 shows the transient responses of the outputs of (a) capacitive interface 1 and (b) capacitive interface 2, when 0.9 V is applied between gate and source of the subthreshold transistor and 100 nF is connected to the high-impedance node. From the figure, using Equation 4.2, the resistances of the transistors biasing the high-impedance nodes are measured to be 524.1 M Ω and 1.75 G Ω for capacitive interfaces 1 and 2, respectively.

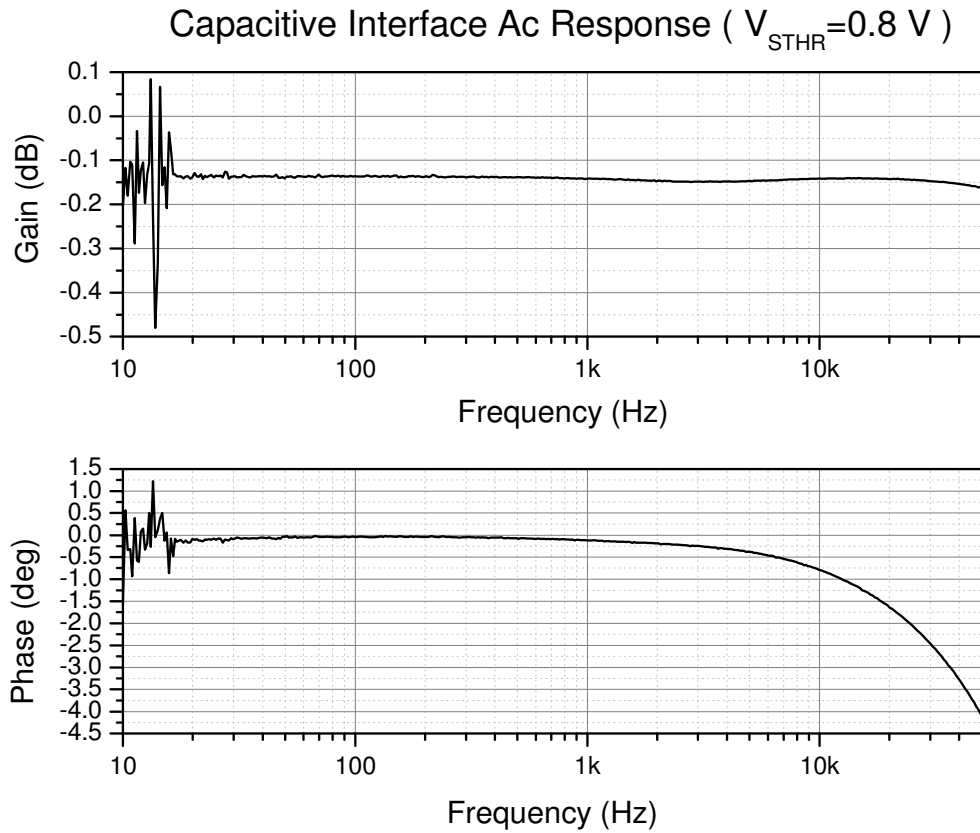


Figure 4.8: Ac characteristics of single-ended source follower biased with subthreshold transistor when V_{STHR} is set to 0.8 V.

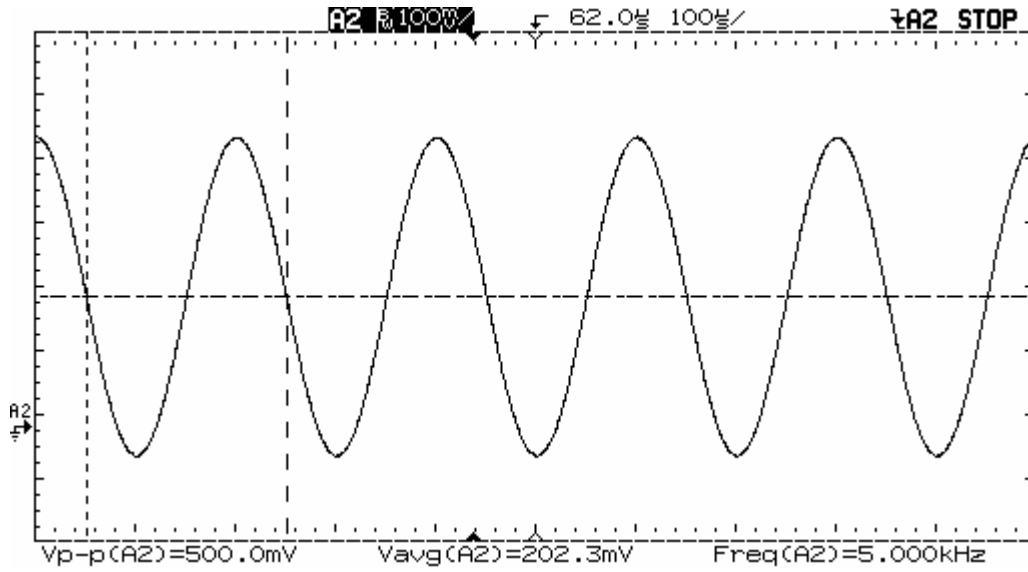
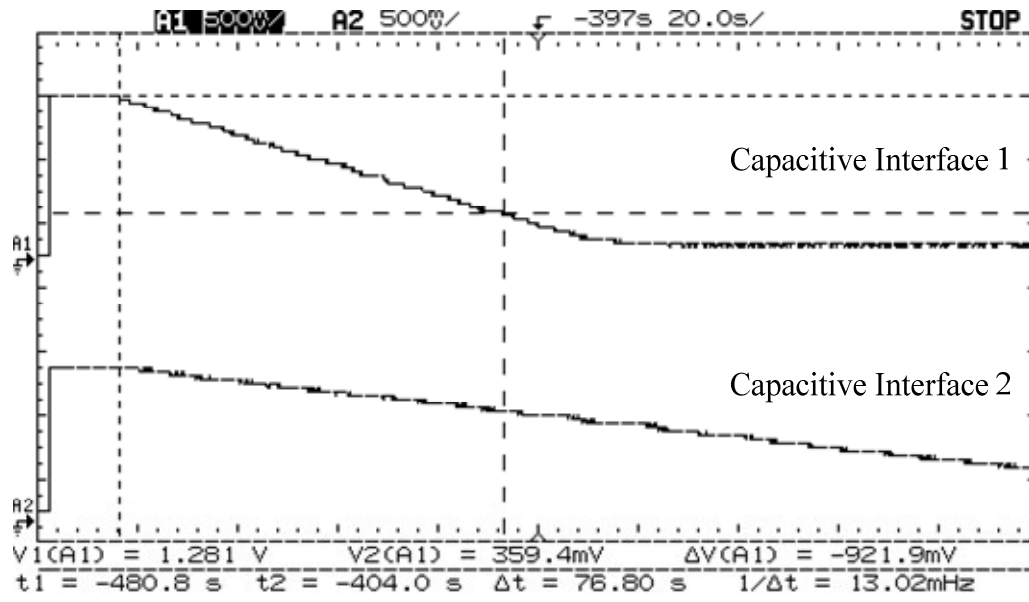


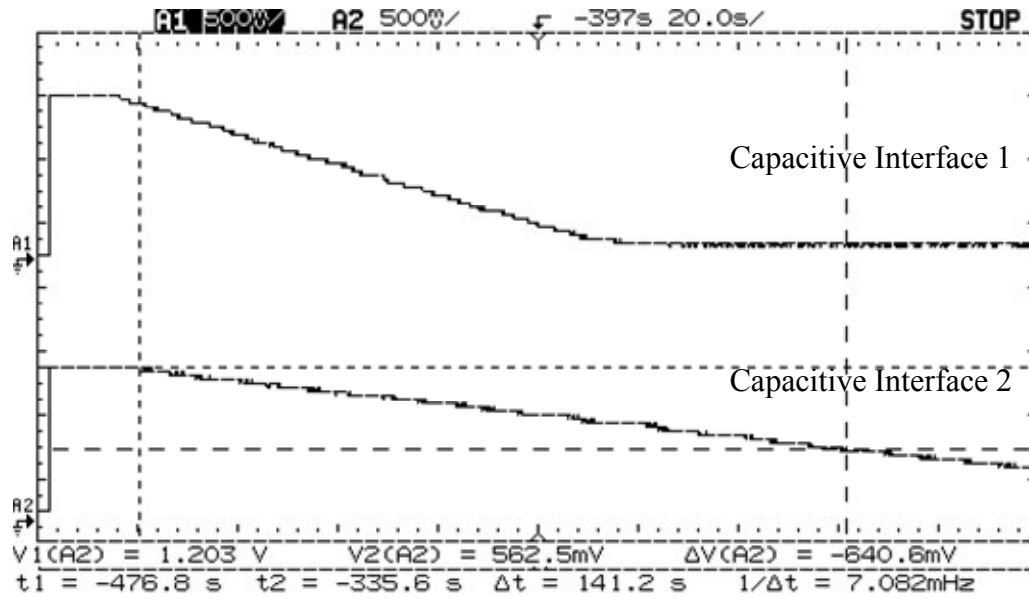
Figure 4.9: Transient response of single-ended source follower biased with subthreshold transistor to a sinusoidal input with amplitude 250 mV and frequency 5 kHz. For this measurement, V_{STHR} is set to 0.8 V.

Second measurement is done in the same conditions except that no external capacitor is connected to the high-impedance node. Figure 4.11 shows the transient responses of the outputs of (a) capacitive interface 1 and (b) capacitive interface 2, when 0.9 V is applied between gate and source of the subthreshold transistor and no external capacitor is connected to the high-impedance node.

Final measurement in order to determine the resistance of the transistor is measuring the discharge time constant when V_{STHR} is set to 0.8 V and there is no external capacitor in high-impedance node. Figure 4.12 shows the transient responses of the outputs of (a) capacitive interface 1 and (b) capacitive interface 2, when 0.8 V is applied between gate and source of the subthreshold transistor and no external capacitor is connected to the high-impedance node.

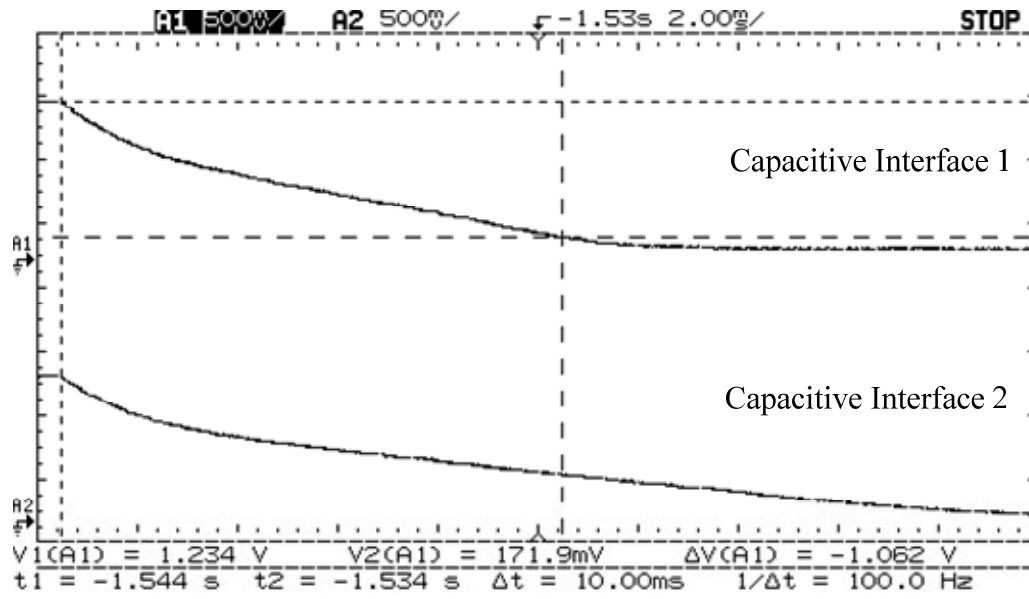


(a)

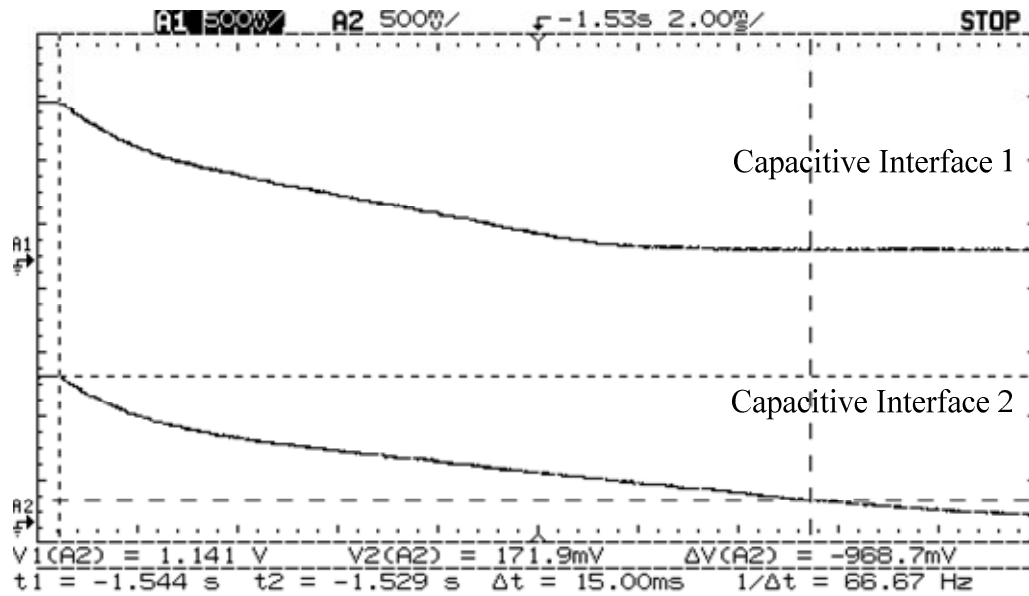


(b)

Figure 4.10: Transient responses of the outputs of (a) capacitive interface 1 and (b) capacitive interface 2, when 0.9 V is applied between gate and source of the subthreshold transistor and 100 nF is connected to the high-impedance node.

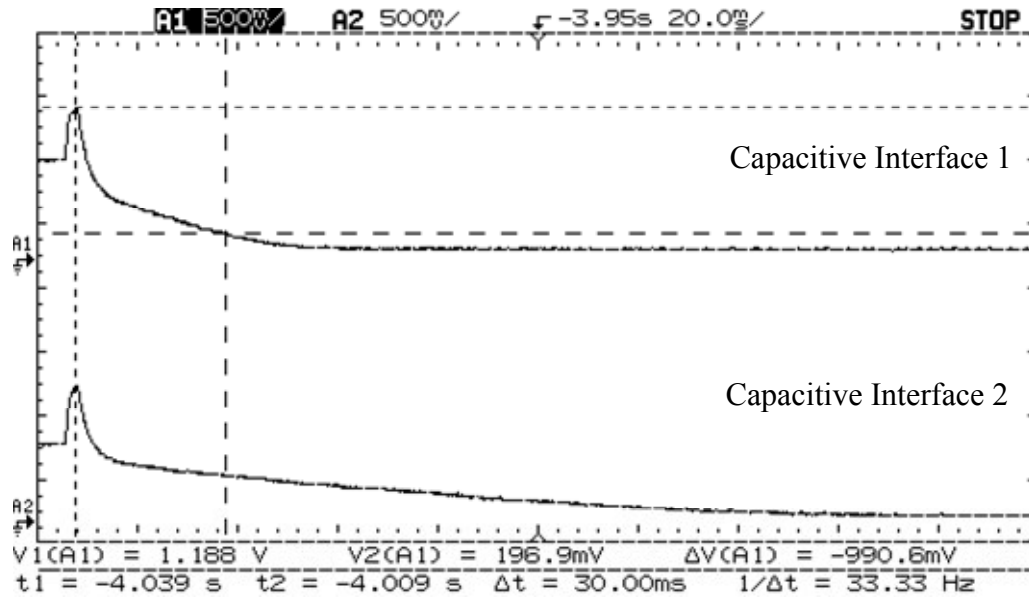


(a)

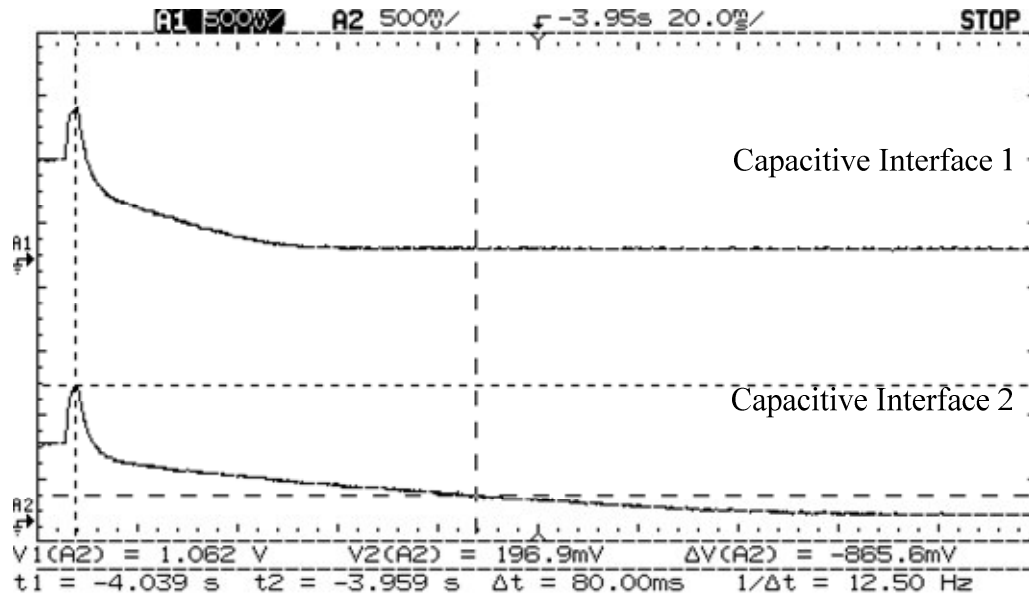


(b)

Figure 4.11: Transient responses of the outputs of (a) capacitive interface 1 and (b) capacitive interface 2, when 0.9 V is applied between gate and source of the subthreshold transistor and no external capacitor is connected to the high-impedance node.



(a)



(b)

Figure 4.12: Transient responses of the outputs of (a) capacitive interface 1 and (b) capacitive interface 2, when 0.8 V is applied between gate and source of the subthreshold transistor and no external capacitor is connected to the high-impedance node.

Taking the ratio of the time constants extracted from Figure 4.11 and Figure 4.12, and comparing this with the values found from Figure 4.10, we can find the resistance of the subthreshold transistor as 1.768 G Ω and 10.6 G Ω for capacitive interfaces 1 and 2, respectively.

In Chapter 3, the simulations of the subthreshold transistor imply that for V_{STHR} value of 0.8 V, the resistance of the transistor should be 384.8 G Ω . However, test results do not match the simulation values. The reason for this is the variation of the threshold voltage of the transistor in the process. NMOS transistors in XFAB XC0.6 process can have threshold voltages in the range of 0.85 V to 1.05 V for bulk-to-source voltage of 0 V [26]. Equation 3.3 implies that change in the threshold voltage of the transistor will affect the resistance of the transistor exponentially. Therefore, if the transistor tested is fabricated such that it has a lower threshold voltage than the simulated transistor, the resistance of the device will decrease significantly. It can be seen from Table 3.1 that 0.1 V change in the threshold voltage can decrease the resistance of the transistor by more than an order of magnitude.

Dependence of the resistance to the threshold voltage can be also understood from Figure 4.12. It can be seen that the resistance of the subthreshold transistor of capacitive interface 1 and 2 are quite different although they are located on the same die. The resistance of the biasing resistor in interface 2 is approximately 6 times larger than the biasing resistor value of interface 1. This implies that the resistance of the transistor can change significantly even on the same die.

Although the resistance of the transistor may change significantly, this change is not important if the impedance of the capacitance seen from the drain of the transistor is much smaller than the resistance of this transistor. If this condition is satisfied, dependence of the resistance on threshold voltage does not create a problem for the operation of the interface.

4.1.2.4. Noise characteristics

Figure 4.13 shows the measured noise characteristics of single-ended source follower biased with subthreshold transistor for V_{STHR} equal to 0.8 V. The measurement results are smaller than the expected results. The possible reason for this discrepancy is the simulation model of the flicker noise which heavily depends on the dimensions and operation region of the transistors. Silicon foundries cannot supply sufficient information about the flicker noise model of the transistors. Moreover, the noise model of the subthreshold transistor may also be insufficient for accurate results.

Noise measurement is repeated once more for a 400 Hz span around 5 kHz in order to characterize the response of the circuit around the possible operation frequency. Measured noise spectral density is 163.15 nV/ $\sqrt{\text{Hz}}$ for V_{STHR} value of 0.8 V, which was simulated to be 259.55 nV/ $\sqrt{\text{Hz}}$.

4.1.3. Characterization of self-biased fully-differential source followers

4.1.3.1. Ac characteristics

Figure 4.14 shows the ac characteristics of self-biased fully-differential source followers when V_{STHR} is set to 0.8 V.

In the vicinity of 5 kHz, gain of the amplifier is measured to be 0.9787 V/V, while its phase shift is -0.2305° . These values are in the acceptable range of desired operation.

Noise Characteristics of the Capacitive Interface Circuit

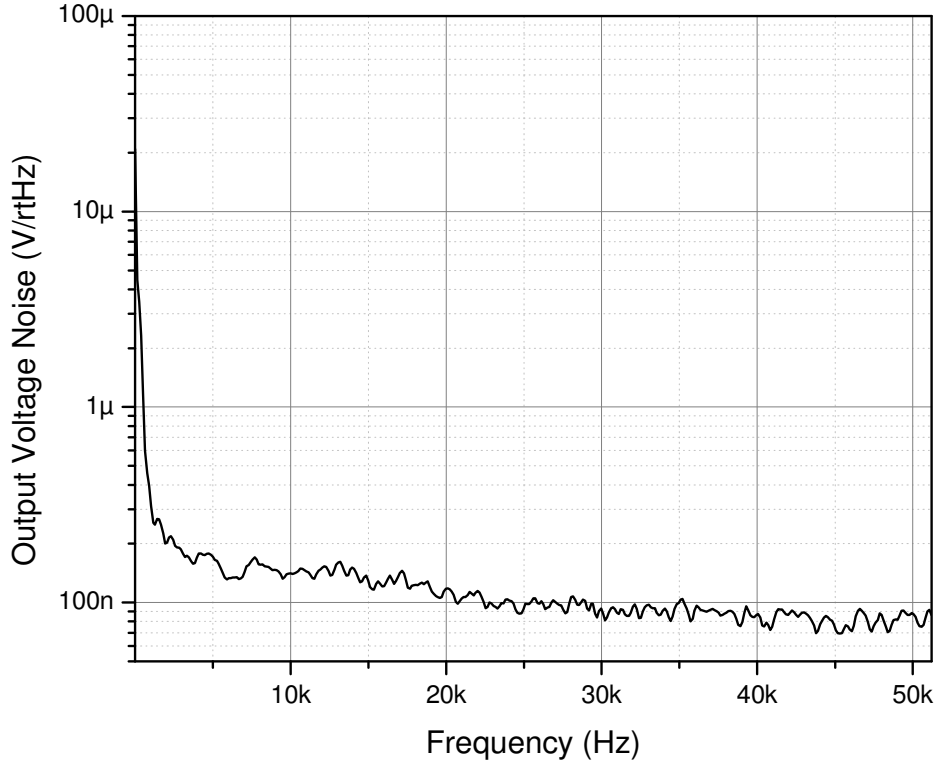


Figure 4.13: Measured noise characteristics of single-ended source follower biased with subthreshold transistor for V_{STHR} equal to 0.8 V.

4.1.3.2. *Transient characteristics for a sinusoidal input*

Figure 4.15 shows the transient response of self-biased fully-differential source followers to a sinusoidal input with amplitude 500 mV and frequency 5 kHz. The differential outputs of the interface are anti-phase with amplitude very close to 500 mV. Moreover, it can be seen that closed-loop for self-biasing the high-impedance node forced the dc point of the output to be very close to the reference potential, which is ground potential in this case.

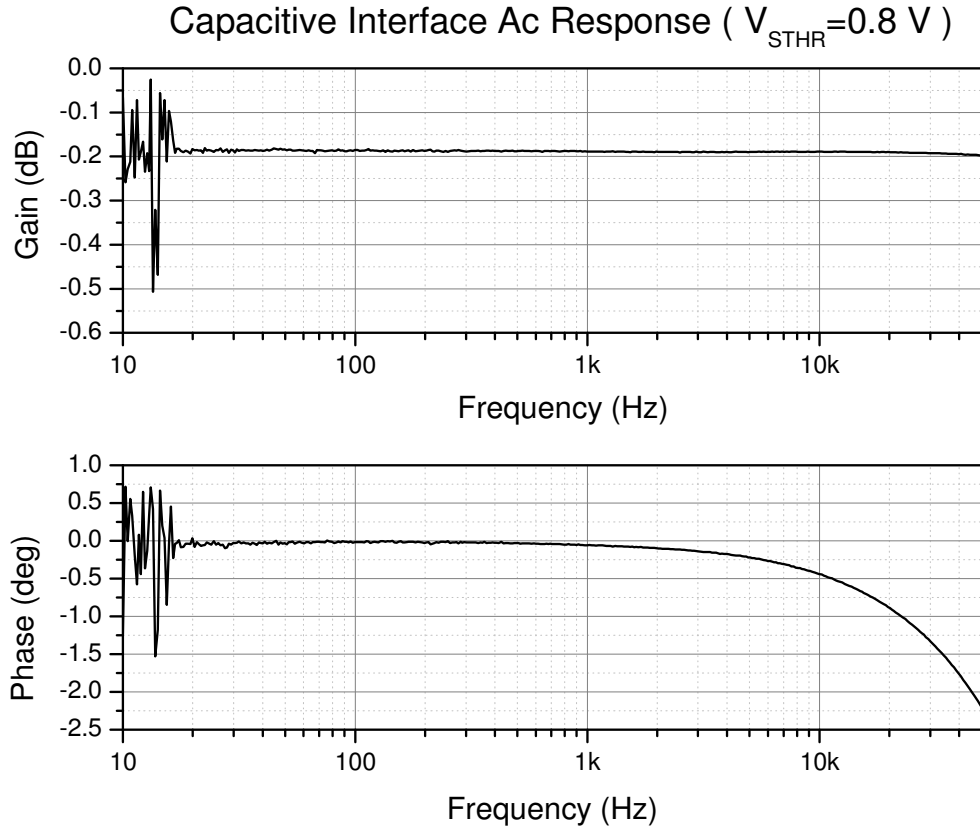


Figure 4.14: Ac characteristics of self-biased fully-differential source followers when V_{STHR} is set to 0.8 V.

4.1.3.3. *Characterization of resistance of the biasing transistor*

For measuring the resistance of the biasing resistor, a very similar test procedure explained in Section 4.1.2.3 is done. The resistance of the transistor is measured for V_{STHR} value of 1 V and 100 nF external capacitor is connected to the high-impedance node. After that, same measurement is repeated without the external capacitor. Finally, resistance of the transistor is extracted from the comparison of the time constants of measurements for V_{STHR} equal to 0.8 V and 1 V.

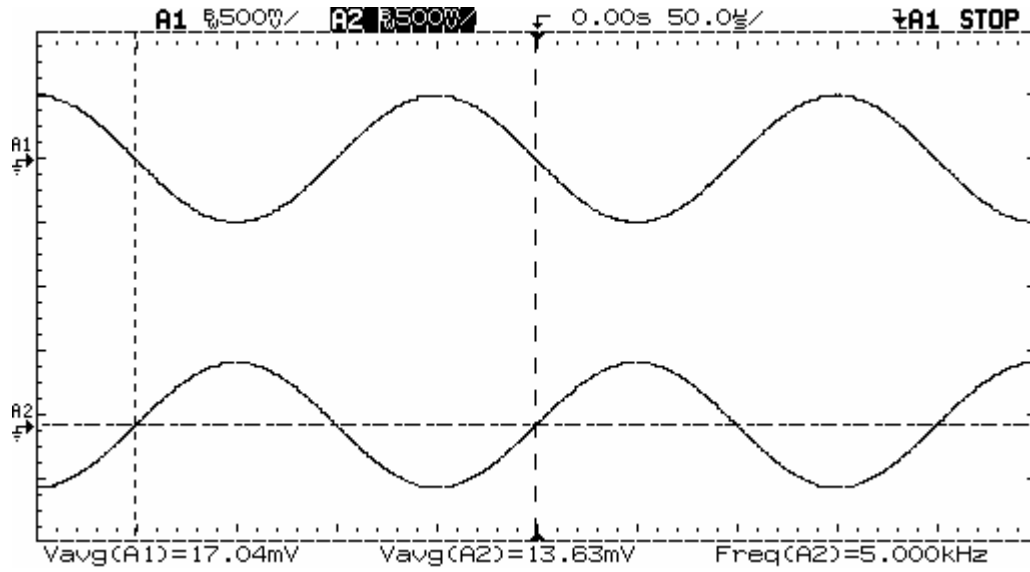


Figure 4.15: Transient response of self-biased fully-differential source followers to a sinusoidal input with amplitude 500 mV and frequency 5 kHz. For this measurement, gate-to-source voltage of the subthreshold transistors is set to 0.8 V.

Figure 4.16 shows the transient response of the output of self-biased fully-differential source followers when 1 V is applied between the gate and source of the subthreshold transistor and 100 nF is connected to the high-impedance node. From this figure, resistance of the biasing transistor can be calculated as 495 M Ω for V_{STHR} value of 1 V.

Figure 4.17 shows the transient response of the output of self-biased fully-differential source followers when 1 V is applied between the gate and source of the subthreshold transistor and no external capacitor is connected to the high-impedance node.

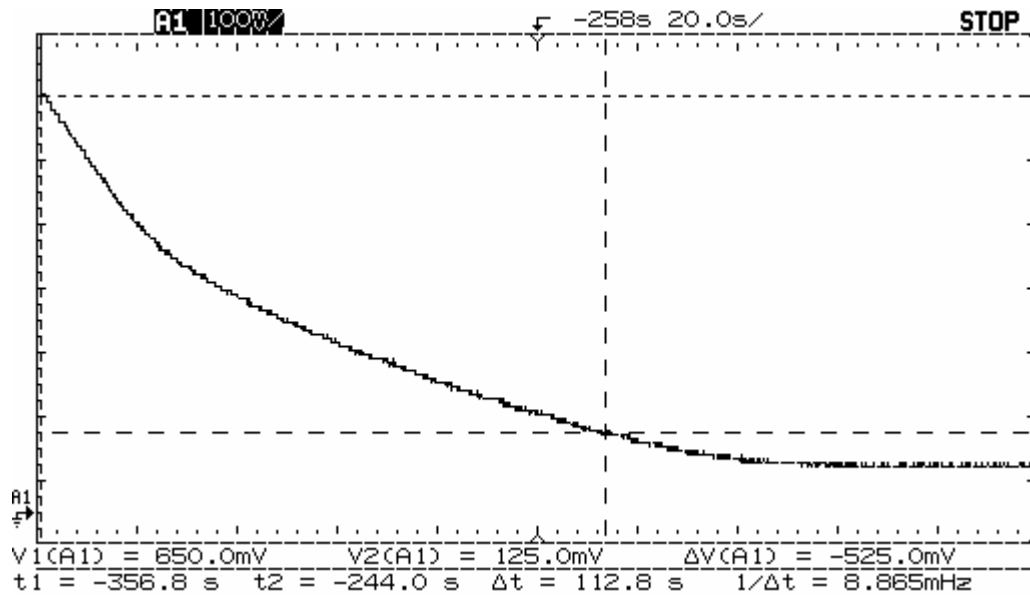


Figure 4.16: Transient response of the output of self-biased fully-differential source followers when 1 V is applied between the gate and source of the subthreshold transistor and 100 nF is connected to the high-impedance node.

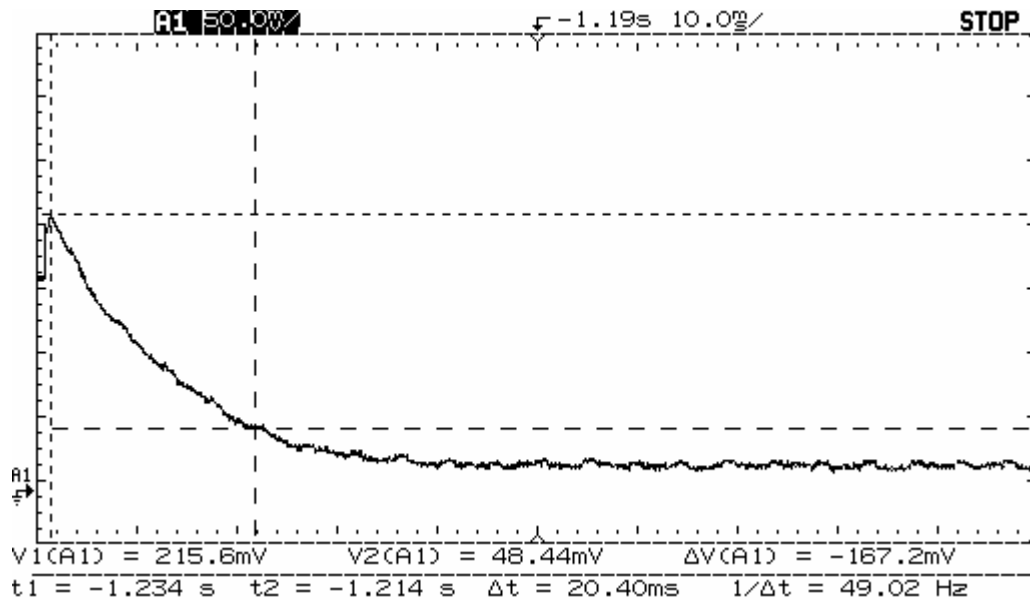


Figure 4.17: Transient response of the output of self-biased fully-differential source followers when 1 V is applied between the gate and source of the subthreshold transistor and no external capacitor is connected to the high-impedance node.

Figure 4.18 shows the transient response of the output of self-biased fully-differential source followers when 0.8 V is applied between the gate and source of the subthreshold transistor and no external capacitor is connected to the high-impedance node.

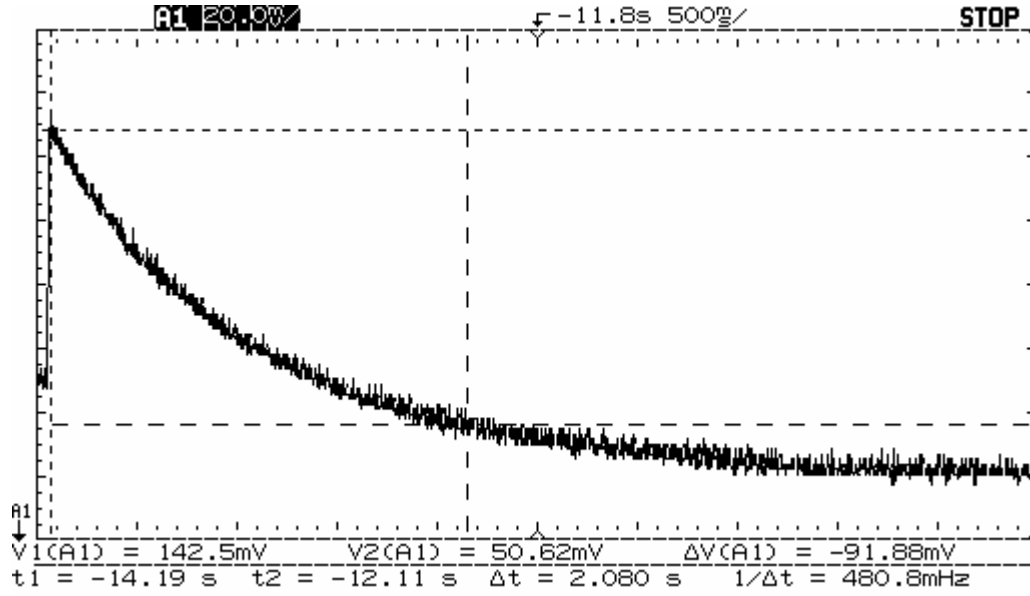


Figure 4.18: Transient response of the output of self-biased fully-differential source followers when 0.8 V is applied between the gate and source of the subthreshold transistor and no external capacitor is connected to the high-impedance node.

Using the measurements from Figures 4.16, 4.17 and 4.18, the resistance of the biasing transistor for V_{STHR} equal to 0.8 V can be found as 59.853 G Ω . It can be seen that the resistance of this transistor is much larger than the resistance of single-ended interface circuit. This is probably due to the threshold voltage variation in the process as explained in Section 4.1.2.3.

4.1.3.4. Noise characteristics

Figure 4.19 shows the measured noise characteristics of self-biased fully-differential source followers for V_{STHR} equal to 0.8 V. For the narrowed span around 5 kHz, output voltage spectral density is measured to be 70.182 nV/ $\sqrt{\text{Hz}}$ for V_{STHR} value of 0.8 V. From the simulation results, expected spectral density was found to be 101.904 nV/ $\sqrt{\text{Hz}}$. The discrepancy between the measurements and simulations are probably due to the reasons explained in Section 4.1.2.4.

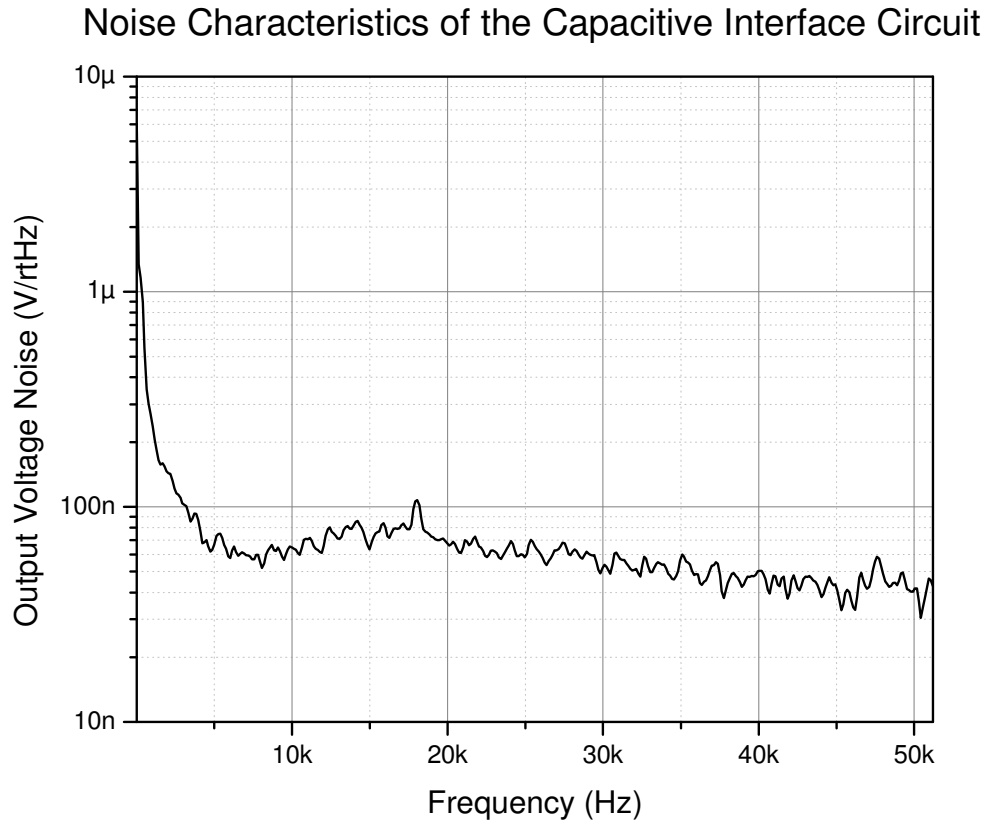


Figure 4.19: Measured noise characteristics of self-biased fully-differential source followers for V_{STHR} equal to 0.8 V.

4.1.4. Characterization of self-biased fully-differential source followers with instrumentation amplifier

For the characterization of this capacitive interface circuit, gain resistor R_G is chosen to be 10 k Ω for instrumentation amplifier gain of 3.

4.1.4.1. *Ac characteristics*

Figure 4.20 shows the ac characteristics of self-biased fully-differential source follower with instrumentation amplifier when V_{STHR} is set to 0.8 V.

At 5 kHz, gain of the circuit is measured to be 5.466 V/V. Normally, gain of the circuit is set to 3, but this gain is valid for differential signals. Since network analyzer cannot generate differential signals, the differential signal is generated externally. The amplitude of the signal generated is twice the amplitude of the source of the network analyzer. Therefore, the gain that is measured from the output of the interface should be twice the gain simulated. In conclusion, gain of the circuit is measured to be 2.733 V/V, whereas it was simulated to be 2.9702 V/V. Furthermore, from the measurements, phase shift at 5 kHz is -0.1349° , where it was -0.17057° from the simulations.

4.1.4.2. *Transient characteristics for a sinusoidal input*

Figure 4.21 shows the transient response of self-biased fully-differential source followers with instrumentation amplifier to a sinusoidal input with amplitude 50 mV and frequency 5 kHz. The signal at the output of the interface has 550 mV peak-to-peak voltage, which is 5.5 times the source signal. Therefore, the gain of the instrumentation amplifier to a differential signal is measured to be 2.75 V/V. This value is consistent with the ac characteristics.

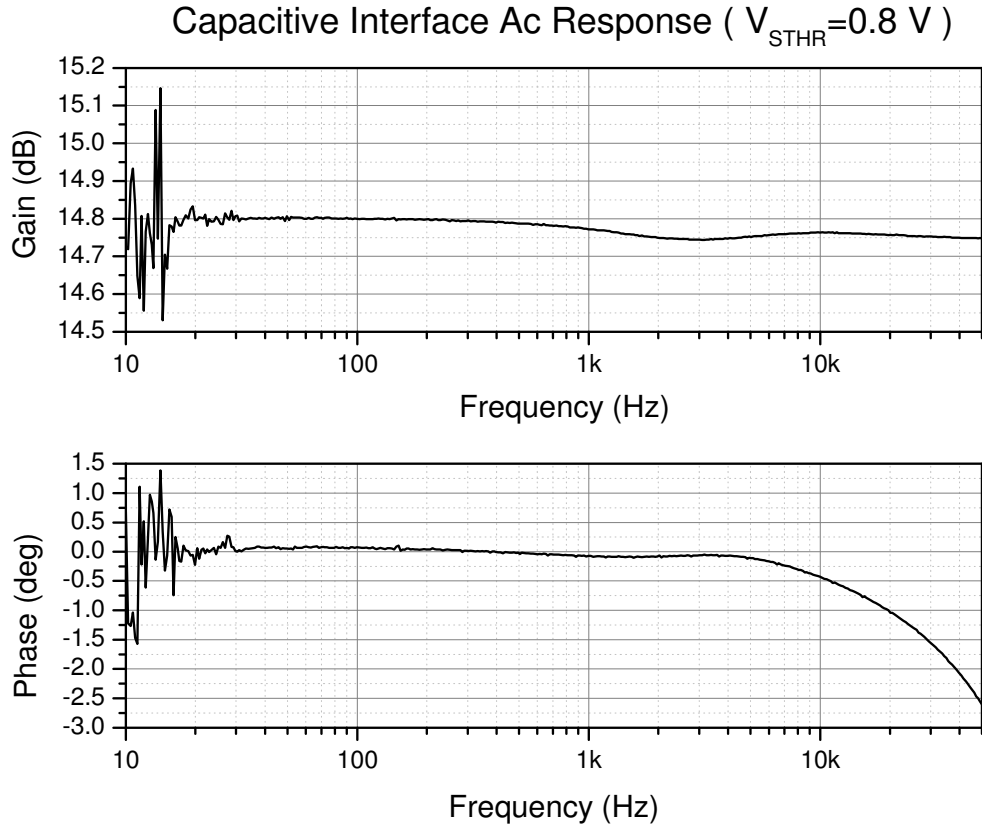


Figure 4.20: Ac characteristics of self-biased fully-differential source follower with instrumentation amplifier when V_{STHR} is set to 0.8 V.

4.1.4.3. Characterization of resistance of the biasing transistor

As it was done for the single-ended capacitive interface, the resistance of the biasing transistor is characterized by measuring the time constant of 100 nF capacitor for V_{STHR} equal to 0.9 V. After that, the time constants are measured for V_{STHR} equal to 0.8 V and 0.9 V, when there is no external capacitor at the high-impedance node. The resistance of the subthreshold transistor is extracted from these measurements.

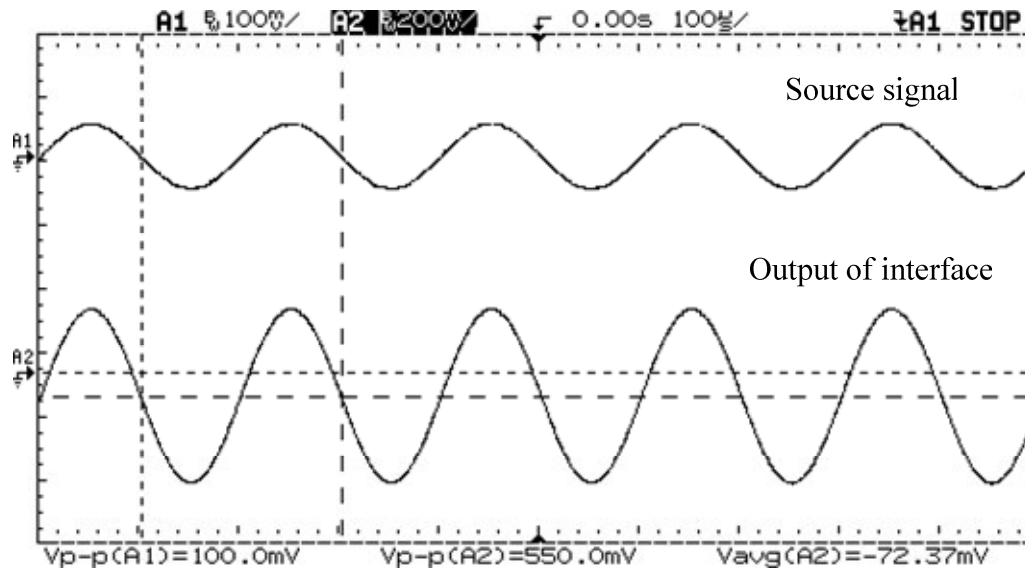


Figure 4.21: Transient response of self-biased fully-differential source followers with instrumentation amplifier to a sinusoidal input with amplitude 50 mV and frequency 5 kHz. For this measurement, V_{STHR} is set to 0.8 V.

Figure 4.22 shows the transient response of the output of self-biased fully-differential source followers with instrumentation amplifier when V_{STHR} is 0.9 V and 100 nF is connected to the high-impedance node. From this response, resistance of the biasing transistor is measured to be 1.093 G Ω .

Figure 4.23 shows the transient response of the output of self-biased fully-differential source followers with instrumentation amplifier when V_{STHR} is 0.9 V and no external capacitor is connected to the high-impedance node.

Figure 4.24 shows the transient response of the output of self-biased fully-differential source followers with instrumentation amplifier when V_{STHR} is 0.8 V and no external capacitor is connected to the high-impedance node.

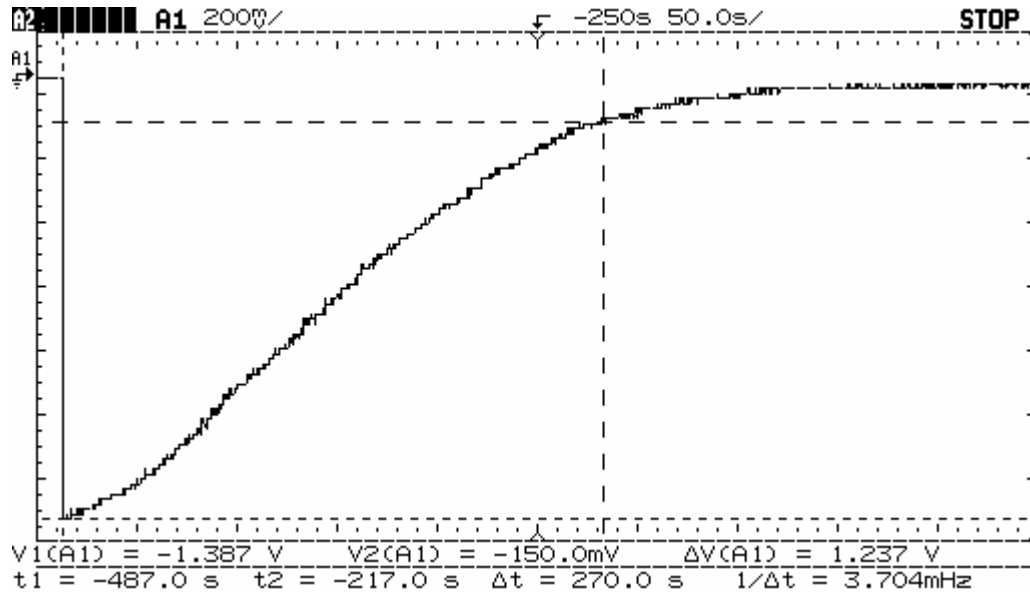


Figure 4.22: Transient response of the output of self-biased fully-differential source followers with instrumentation amplifier when V_{STHR} is 0.9 V and 100 nF is connected to the high-impedance node.

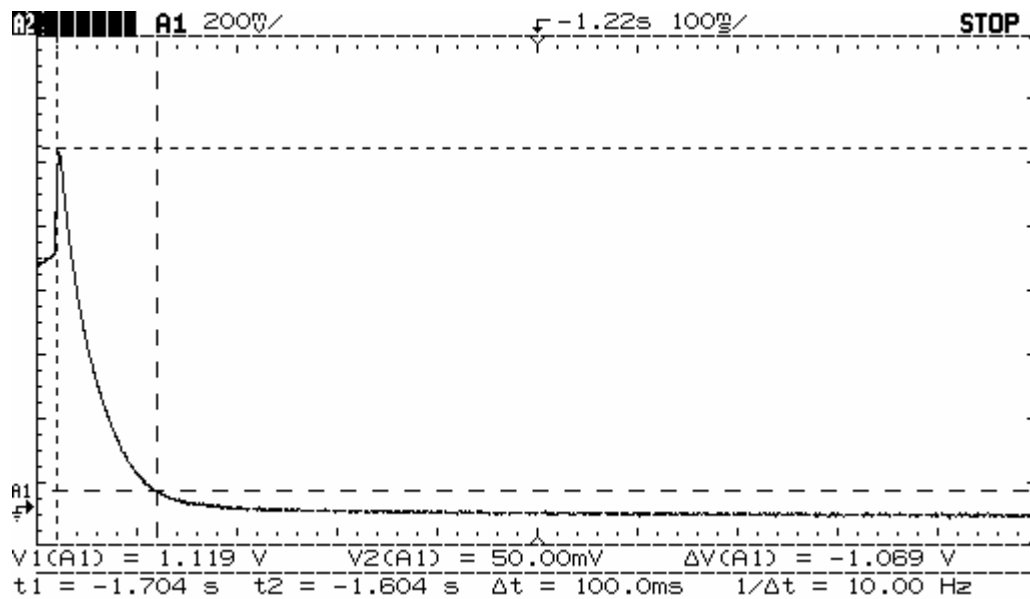


Figure 4.23: Transient response of the output of self-biased fully-differential source followers with instrumentation amplifier when V_{STHR} is 0.9 V and no external capacitor is connected to the high-impedance node.

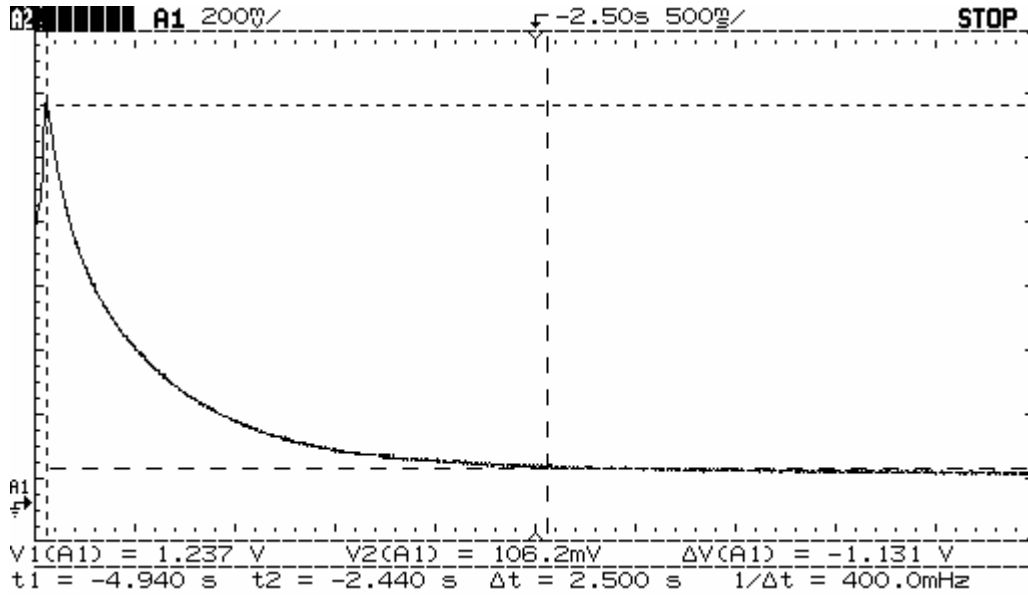


Figure 4.24: Transient response of the output of self-biased fully-differential source followers with instrumentation amplifier when V_{STHR} is 0.8 V and no external capacitor is connected to the high-impedance node.

Comparing the time constants extracted from Figures 1.23 and 1.24, the resistance of the biasing resistor can be calculated as 22.38 G Ω .

4.1.4.4. Noise characteristics

Figure 4.25 shows the measured noise characteristics of self-biased fully-differential source followers with instrumentation amplifier for V_{STHR} equal to 0.8 V. In the narrowed span around 5 kHz, the noise spectral density of the interface is 446.47 nV/ $\sqrt{\text{Hz}}$, which should be 581.287 nV/ $\sqrt{\text{Hz}}$ according to the simulations. The difference between the measured and simulated responses is most likely due to the errors present in the noise model of subthreshold transistor and also noise model of the flicker noise.

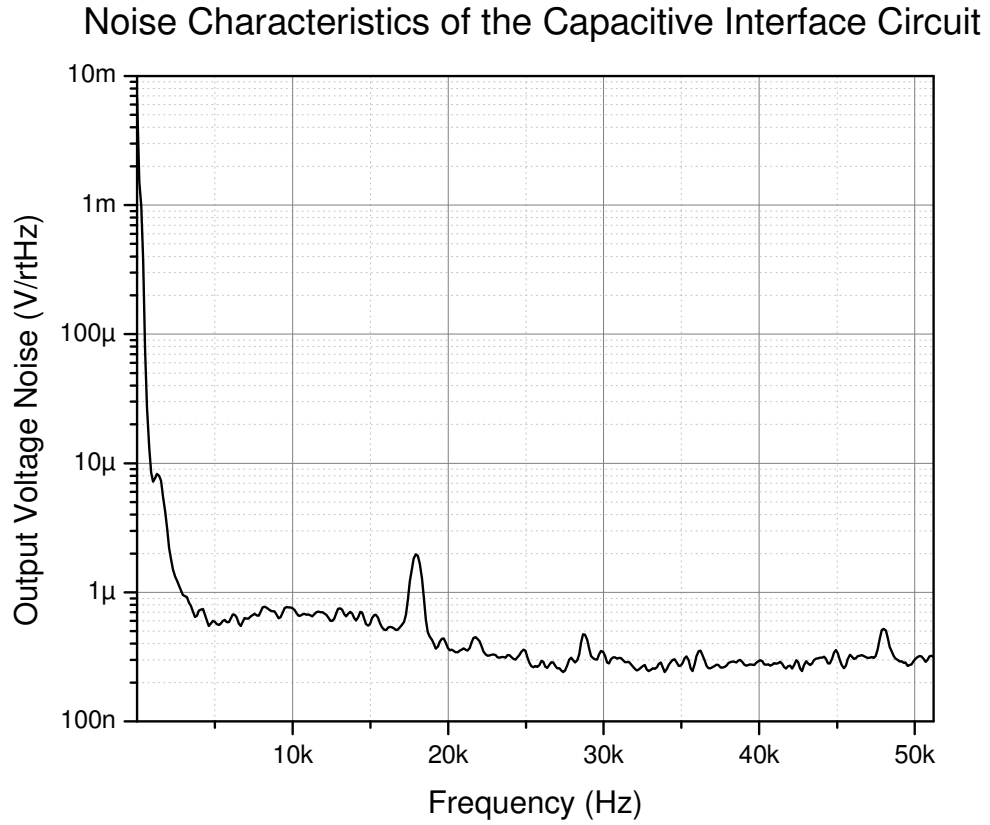


Figure 4.25: Measured noise characteristics of self-biased fully-differential source followers with instrumentation amplifier for V_{STHR} equal to 0.8 V.

4.2. Characterization of Mechanical Sensors of SOG Gyroscopes

Characterizations of the mechanical sensors used in this study are done by measuring the resonance characteristics of the gyroscopes using the network analyzer and a probe station. For these measurements, capacitive interface circuits are not connected to the output of the mechanical sensors, in order to get the response of the mechanical sensor only. The resonance characteristic measurements of these sensors

are done at atmospheric pressure; hence, the quality factors of the sensors are lower compared to measurements at vacuum due to air damping.

Figure 4.26 shows the resonance characteristics of the drive-mode of SOG gyroscope without capacitive interface circuit for proof mass voltage of 30 V. Resonance frequency of the drive-mode is at 3.64 kHz. The gain of the gyroscope at this frequency is measured to be -58.541 dB. This measurement is very close to the simulation result given Figure 2.14, which is obtained from the gyroscope model introduced in Chapter 2.

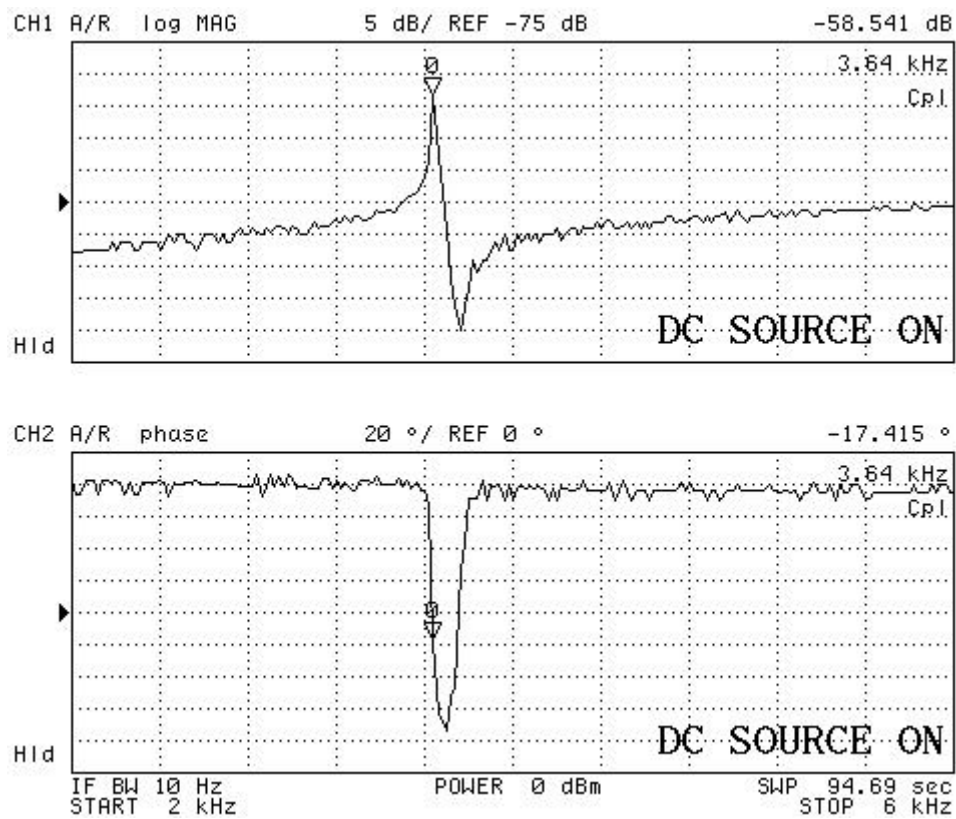


Figure 4.26: Resonance characteristics of the drive-mode of SOG gyroscope without capacitive interface circuit for proof mass voltage of 30 V.

In order to extract the displacement of the gyroscope for a given excitation, $\partial C/\partial x$ of the gyroscope must be known. Equation 2.67 gives the gain of the mechanical sensor. In this equation, the only unknown parameters are the $\partial C/\partial x$ value, input impedance of the active probe and the spring constant after fabrication. Spring constant of the sensor after fabrication can be extracted from the measured mechanical resonance frequency of this mode. The active probe used for measurement of the characteristics has an input impedance of 100 k Ω and 3 pF. 100 k Ω is much smaller than the impedance of 3 pF around the resonance frequency of the gyroscope, and therefore, it dominates the effective impedance. Hence, $\partial C/\partial x$ value can be extracted from these measurements.

When $\partial C/\partial x$ of the fabricated transistor is known, the oscillation amplitude of the gyroscope for a given ac excitation can be extracted from Equation 2.48.

Figure 4.27 shows the resonance characteristics of the sense-mode of SOG gyroscope without capacitive interface circuit for proof mass voltage of 30 V. The gain of the circuit is measured to be -38.119 dB when there is no capacitive interface at the output of the mechanical sensor. The measurement result, again, is close to the simulation result with the gyroscope model. The discrepancies between the measurement and simulation are due to the fabrication tolerances like DRIE undercut.

4.3. Preliminary Characterization of SOG Gyroscopes with Capacitive Interface Circuits

Single-ended SOG gyroscopes used in this study are characterized using the single-ended source followers biased with subthreshold transistors. Figure 4.28 shows the (a) top and (b) 3-D photographs of single-ended SOG gyroscope connected to single-ended source follower biased with subthreshold transistor. One

of the interfaces on the chip is connected to the pick-up capacitors of the drive-mode and the other one to the pick-up capacitors of the sense-mode.

For the preliminary tests of the gyroscope, resonance characteristics of the drive and sense modes are measured at atmospheric pressure and vacuum ambient in the presence of capacitive interface circuits.

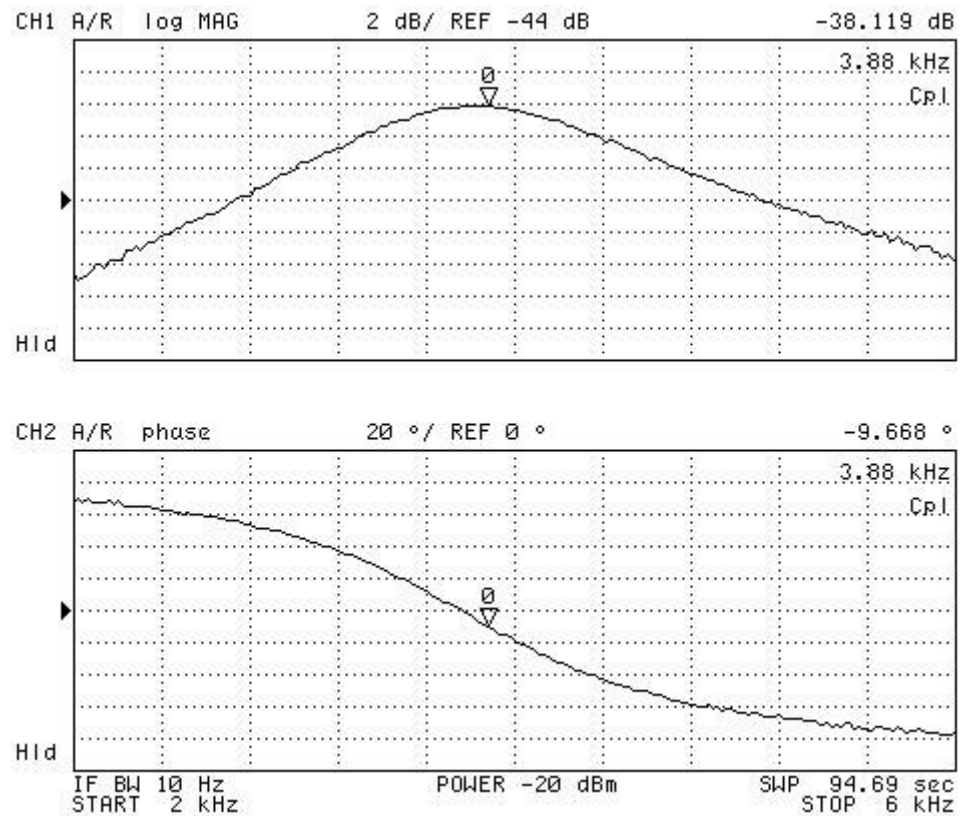
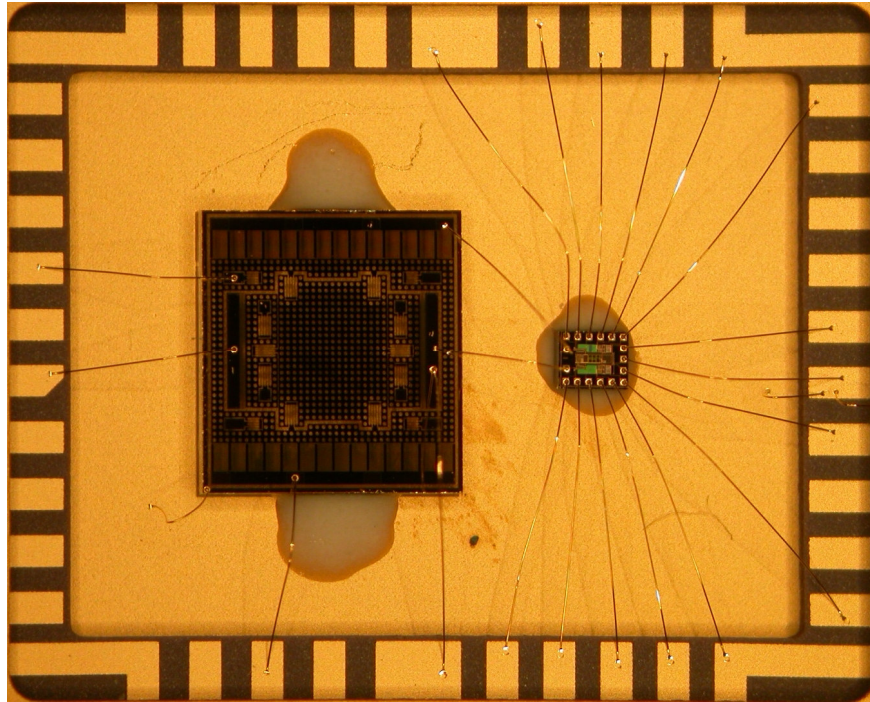
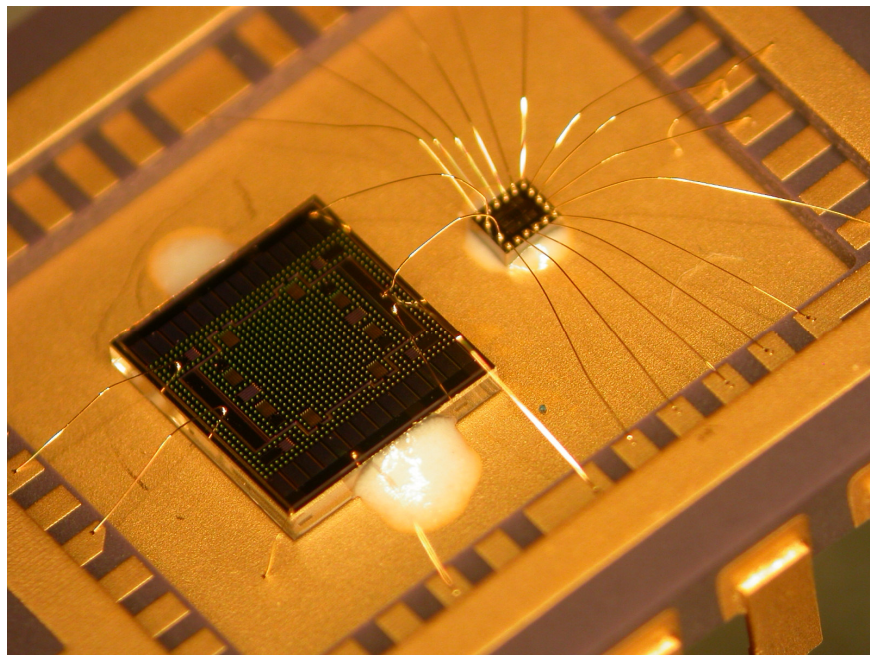


Figure 4.27: Resonance characteristics of the sense-mode of SOG gyroscope without capacitive interface circuit for proof mass voltage of 30 V.



(a)



(b)

Figure 4.28: (a) Top and (b) 3-D photographs of single-ended SOG gyroscope connected to single-ended source follower biased with subthreshold transistor.

Figure 4.29 shows the resonance characteristics of (a) drive-mode and (b) sense-mode of the single-ended SOG gyroscope connected to single-ended source follower biased with subthreshold transistor for proof mass voltage of -35 V and atmospheric pressure. As seen from the figure, the gain of the drive-mode is increased by more than 45 dB, when capacitive interface circuit is connected to its output. Measured responses are close to the simulated values when gyroscope model is connected to the capacitive interface.

At this point, it is possible to extract the minimum resolvable capacitance change that can be detected by the capacitive interface. Modifying Equation 2.32, signal measured at the output of the capacitive interface for capacitance change at the output can be found as

$$V_{out}(j\omega) = A_{sf} \cdot (V_{PM} - V_{BIAS}) \cdot j\omega \cdot \underbrace{\left(Z_{interface}(j\omega) // \frac{1}{j\omega \cdot C_0} \right)}_{Z_{eff}(j\omega)} \cdot \underbrace{\frac{\partial C}{\partial x}}_{\Delta C} \cdot X(j\omega) \quad (4.4)$$

If the voltage noise at the output of the capacitive interface is replaced with $V_{out}(j\omega)$ in Equation 4.4, minimum resolvable capacitance change by the capacitive interface can be found. However, $Z_{eff}(j\omega)$ should be calculated, first. Using Equation 2.67 and taking the ratio of measurements in Figure 4.27 and Figure 4.29b, effective impedance at the high-impedance node can be found as 3.427 M Ω . From this result, minimum resolvable capacitance change can be found as¹

¹ The gain and noise values used in Equation 4.5 are modified for the resonance frequency of the gyroscope, which is 3.62 kHz.

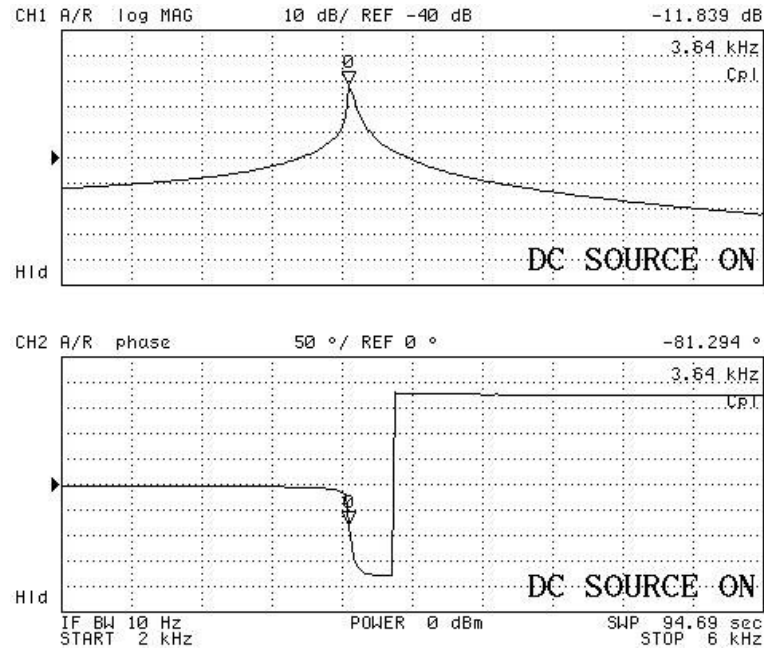
$$\Delta C = \left| \frac{162.6 \cdot 10^{-9}}{0.984 \cdot (-35 - 1.4) \cdot 2\pi \cdot 3.62 \cdot 10^3 \cdot 3.427 \cdot 10^6} \right| \frac{F}{\sqrt{Hz}} \quad (4.5)$$

$$\Delta C = 58.31 \text{ zF} / \sqrt{Hz} \quad (4.6)$$

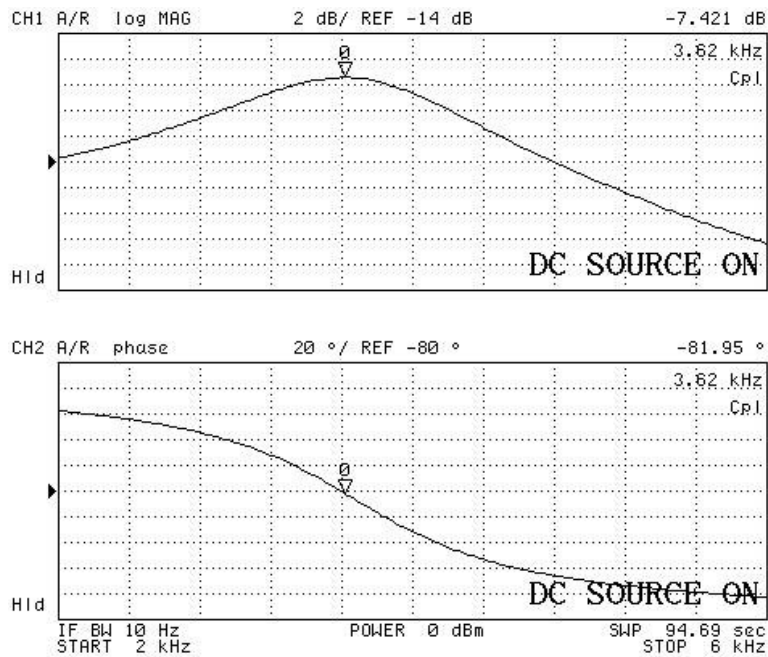
From this result, it can be concluded that, if the gyroscope is ideal, i.e., it does not generate Brownian noise, then, 58.31 zF (1 zF = 10^{-21} F) can be resolved with the capacitive interface in 1 Hz measurement bandwidth.

From Figure 4.27, $\partial C / \partial x$ of the sense-mode can be extracted as 1.5 $\mu\text{F/m}$. Using this value with minimum resolvable capacitance change, minimum resolvable displacement can be found as 38.87 Fermi (38.87×10^{-15} m).

Figure 4.30 shows the resonance characteristics of (a) drive-mode and (b) sense-mode of the single-ended SOG gyroscope connected to single-ended source follower biased with subthreshold transistor for proof mass voltage of -35 V and vacuum ambient.

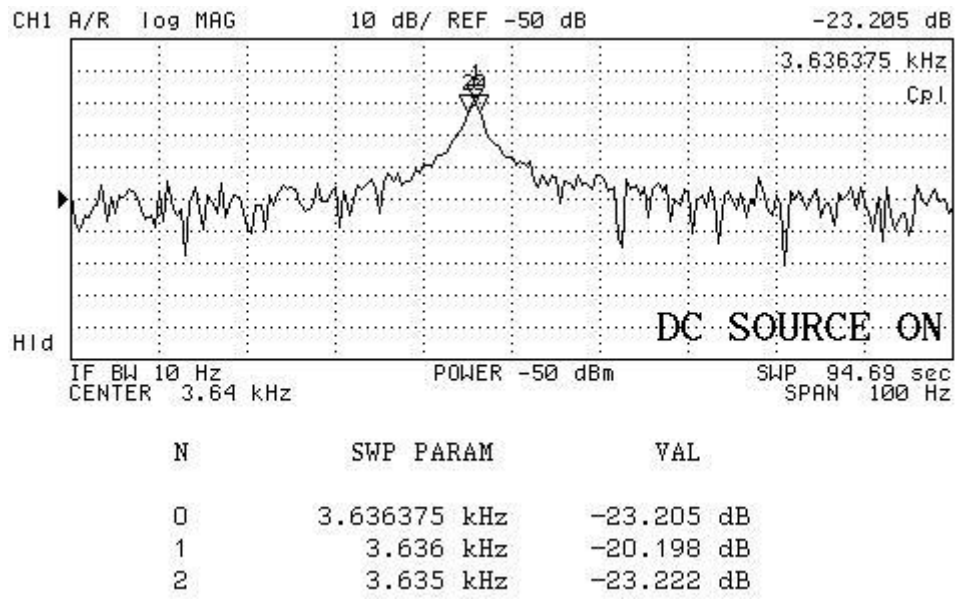


(a)

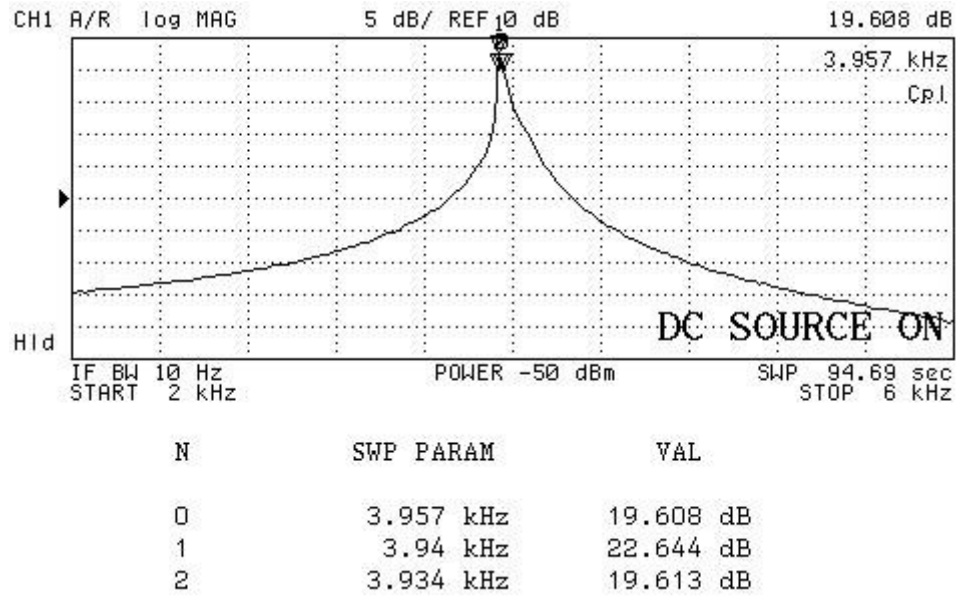


(b)

Figure 4.29: Resonance characteristics of (a) drive-mode and (b) sense-mode of the single-ended SOG gyroscope connected to single-ended source follower biased with subthreshold transistor for proof mass voltage of -35 V and atmospheric pressure.



(a)



(b)

Figure 4.30: Resonance characteristics of (a) drive-mode and (b) sense-mode of the single-ended SOG gyroscope connected to single-ended source follower biased with subthreshold transistor for proof mass voltage of -35 V and vacuum ambient.

4.4. External Readout and Control Electronics for MEMS Gyroscopes

4.4.1. Closed-loop drive-mode electronics

As mentioned in Chapter 1 and 2, in order to measure rate from the gyroscope, the drive-mode of the gyroscope should be kept in oscillation. Therefore, a closed-loop oscillator system should be implemented for proper operation. According to the Barkhausen's criteria for oscillation, the closed-loop transfer function should have gain equal to 1 and the total phase shift in the loop should be integer multiples of 360° at the frequency of oscillation [32]. The closed-loop that is going to keep the gyroscope in oscillation should meet these two criteria. Therefore, there should be two functional blocks in the closed-loop to satisfy these requirements, one will manipulate the gain of the system and the other one will set the phase of the system. Figure 4.31 shows the block diagram of closed-loop drive-mode oscillator satisfying Barkhausen's criteria.

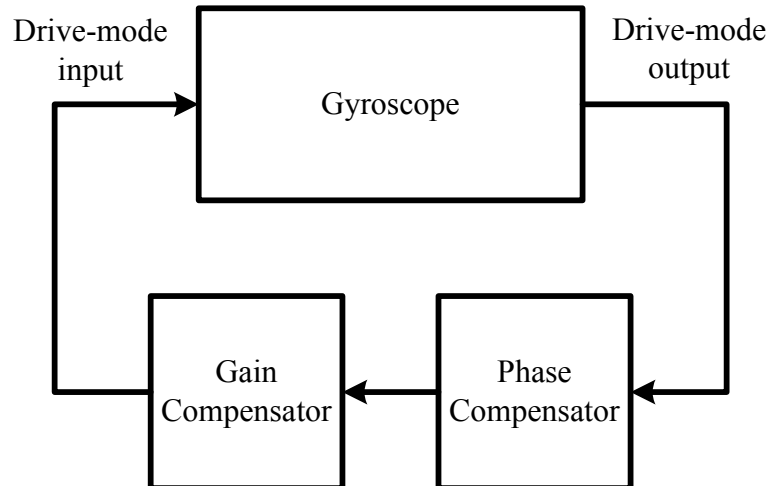


Figure 4.31: Block diagram of closed-loop drive-mode oscillator satisfying Barkhausen's criteria.

In order to sustain oscillation, the gain of the system should stay on the imaginary axis of the root-locus plot. If the gain of the compensator is small so that the poles stay in the left of the imaginary axis, oscillation will decay in time and will be lost. On the other hand, if the gain is large and poles remain in the right hand side of imaginary axis, oscillation will grow up to infinity and the system will blow up. Therefore, for proper operation, gain compensator should set the gain such that, the poles of the closed-loop should stay on the imaginary axis.

It is known that, in linear control systems, it is hard to keep the poles on the imaginary axis continuously, because a small shift in the gain will either decay or raise the oscillation. Therefore, a nonlinear control system is necessary to keep the robustness of the system, i.e., when the gain of the gyroscope decreases, gain compensator should increase its gain or vice versa. The description of the describing function of the nonlinear system may seem complicated; however, implementation of such a system in electronics is quite simple. A comparator or a high-gain stage with operational amplifiers will act as the describing function needed for oscillation. When the oscillation starts, high-gain stage will keep the gain of the closed-loop system more than 1, so that oscillation will start to increase. Positive-feedback present in the system will increase the amplitude of the signal generated by the gyroscope until the output of the gain stage saturates. The output of the gain stage will saturate at the supply limits and the input of the gyroscope will remain constant from then. Saturation of the gain stage compensates the gain of the closed-loop to be equal to 1. Hence, the first criterion of Barkhausen's criteria is met with a simple high-gain stage.

As the interfaces used in this study are capacitive interfaces, the voltage at the output of the capacitive interface lags the excitation voltage at the input of the gyroscope by -90° . Therefore, the phase compensator proposed in Figure 4.31 should shift its input's phase either by 90° or -270° . Implementation of this phase shifter is possible using two architectures, a differentiator or an integrator. Figure 4.32 shows (a) differentiator and (b) integrator implemented using an operational amplifier. The

advantages of using differentiator are that it cancels the offset present at its input and also its phase shift is very close to 90° . However, a differentiator is not a band-limited circuit and therefore, it increases the high-frequency noise in the system. This may result in glitches, if there is a square wave present in the loop. On the other hand, advantage of integrator is that its noise is band-limited, but an offset in the input may saturate the output. Moreover, a feedback resistor is required to keep the output stay away from saturation but this feedback resistor shift the phase away from 90° . Comparing these two circuits, advantages of differentiator is more dominating so that the phase compensator is implemented with a differentiator.

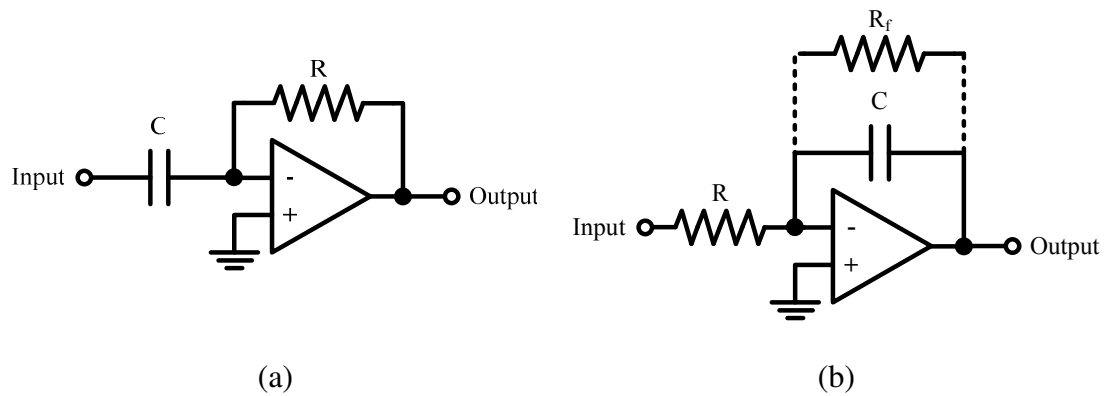


Figure 4.32: (a) Differentiator and (b) Integrator implemented using an operational amplifier.

Closed-loop drive-mode oscillator proposed in Figure 4.31 is implemented using a comparator as the gain compensator and differentiator as the phase compensator. Figure 4.33 shows the schematic of the closed-loop drive-mode electronics for sustaining oscillation in the drive-mode. The last stage is used for tuning the amplitude of the driving signal of the gyroscope. The gain of the last stage is less than or equal to 1 and is controlled with the use of a potentiometer.

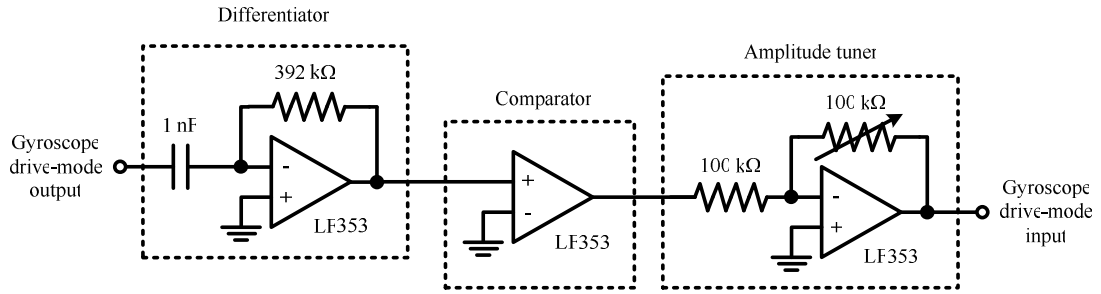


Figure 4.33: Schematic of the closed-loop drive-mode electronics for sustaining oscillation in the drive-mode.

4.4.2. Open-loop sense demodulation electronics

In many applications, it is preferable to have dc voltage at the output which is proportional to the rate measured. However, sense-mode of the gyroscope gives an amplitude modulated output at the resonance frequency of the drive-mode. Therefore, demodulation should be done to transfer the rate signal to the baseband. Moreover, the signal should be filtered from high-frequency components to give dc output. Figure 4.34 shows the block diagram of the open-loop sense demodulation electronics.

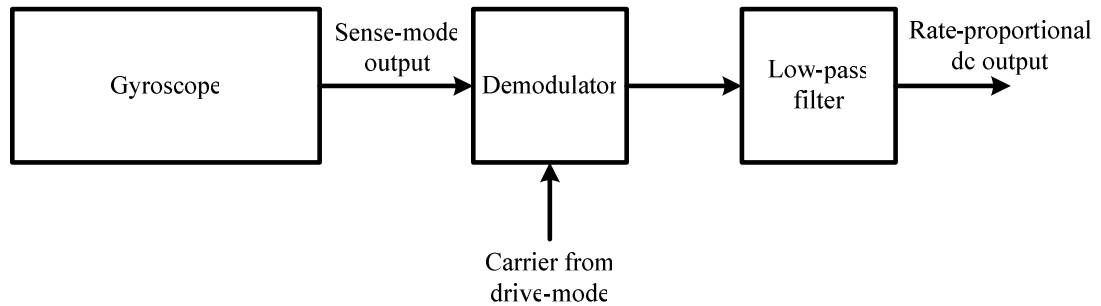


Figure 4.34: Block diagram of the open-loop sense demodulation electronics.

Before demodulation, the output of the gyroscope is buffered with a buffer connected operational amplifier to prevent loading. For demodulation of the buffered sense-mode output, a balanced modulator/demodulator integrated circuit is used [33]. The output of the demodulator is then filtered using a third-order Sallen-Key type low-pass filter. Then, a final gain stage is added in order to tune the scale factor to be measured at the output. Figure 4.35 shows the schematic of the open-loop sense demodulation electronics.

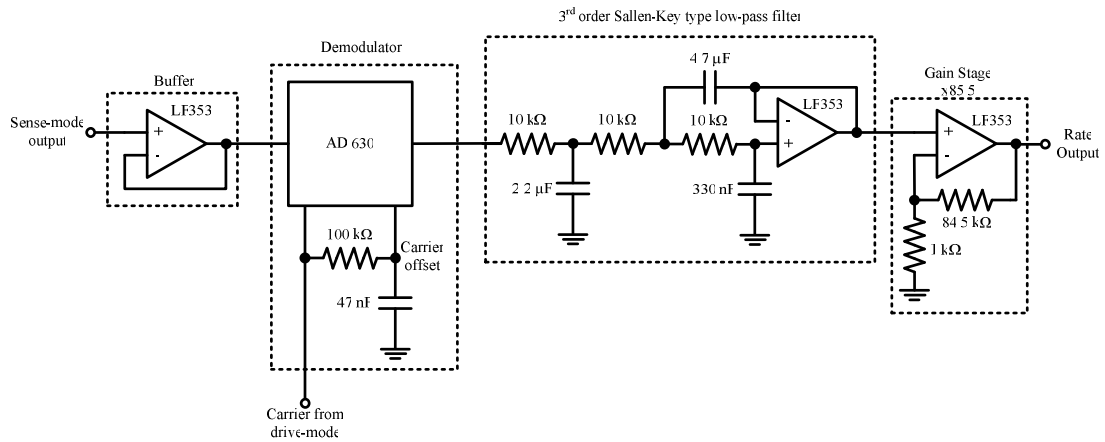


Figure 4.35: Schematic of the open-loop sense demodulation electronics.

4.4.3. Implementation of external readout and control electronics for MEMS gyroscopes

External electronics for the gyroscopes explained in Section 4.4.1 and 4.4.2 are implemented on a printed circuit board designed in METU MEMS-VLSI Research Group. Figure 4.36 shows the photograph of the printed circuit board for external readout and control electronics for MEMS gyroscopes.

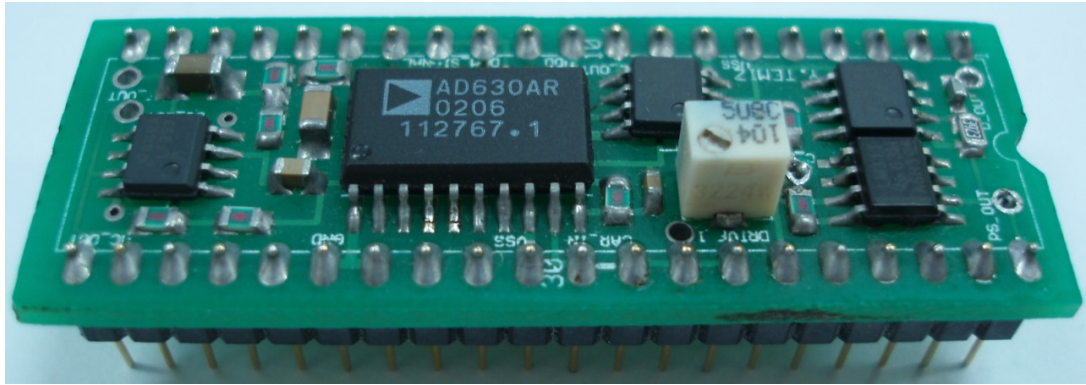


Figure 4.36: Photograph of the printed circuit board for external readout and control electronics for MEMS gyroscopes.

4.5. Characterization of SOG Gyroscopes with Capacitive Interface Circuits

There are four basic specifications of a MEMS gyroscope that must be measured for characterization. These are:

1. Scale factor
2. Linearity
3. Angle random walk
4. Bias instability

These parameters can be extracted from two different measurements. Test setups for these measurements and results of these tests are explained in the following subsections. For these measurements, gyroscope is operated at the mismatched mode. The reason is that, the bias instability of the gyroscope seriously increases, although the sensitivity is not increased that amount. Therefore, it has been seen that for low bias instability, mismatched operation is preferable.

4.5.1. Scale factor and linearity measurements

For characterization of the scale factor and linearity of the gyroscope, angular rate is applied to the system. Figure 4.37 shows the photograph of Ideal Aerosmith 1280 single-axis rate table used for scale factor and linearity measurements of MEMS gyroscopes.



Figure 4.37: Photograph of Ideal Aerosmith 1280 single-axis rate table used for scale factor and linearity measurements of MEMS gyroscopes.

For the scale factor and linearity tests, the gyroscope is driven into resonance using the closed-loop drive-mode and rate response is measured from the output of

open-loop sense demodulation electronics, which are explained in Section 4.4. Applied rate to the system is swept with 10 °/sec steps starting from 0 °/sec to 100 °/sec, then, to -100 °/sec and finally back to 0 °/sec. The output of sense demodulation electronics is continuously measured at sampling rate 22 Hz, using a multimeter (Agilent 34401A) and dedicated software (HPVVEE), while the rate is increased or decreased in 20 sec intervals. Figure 4.27 shows the scale factor and linearity test result for the single-ended SOG gyroscope.

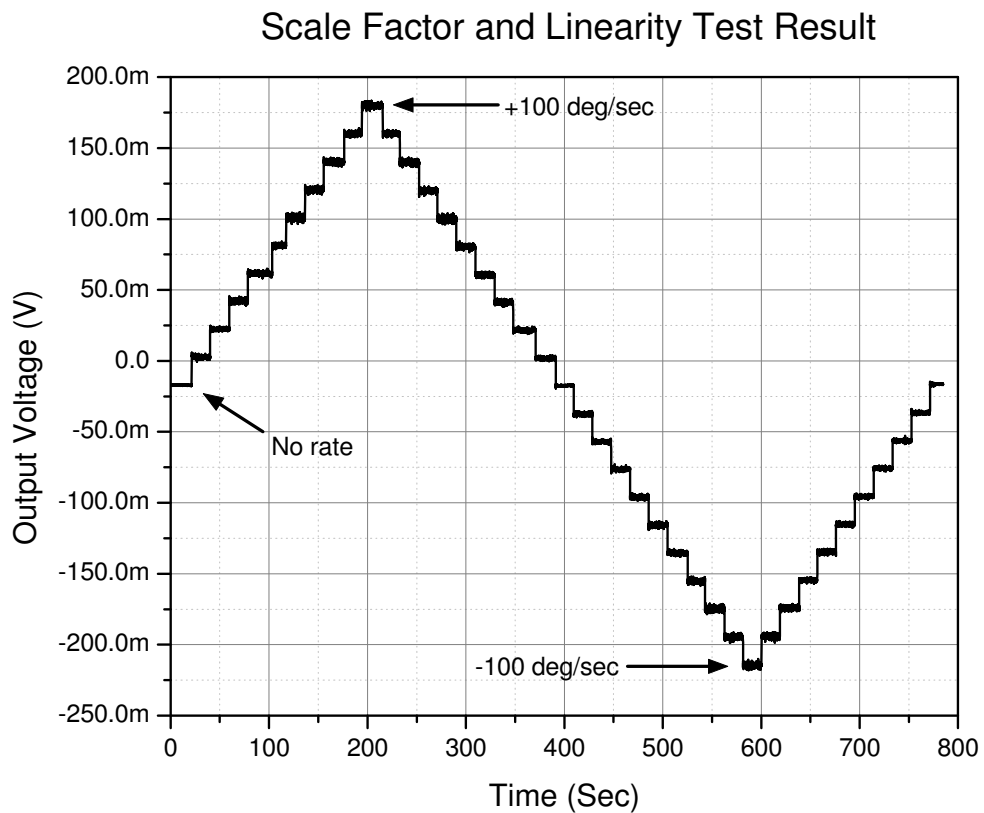


Figure 4.38: Scale factor and linearity test result for the single-ended SOG gyroscope.

From this result, output voltage vs. rate characteristics of the gyroscope can be extracted by taking the averages of the measured data for a given rate. Figure 4.39 shows the output voltage vs. rate characteristics of single-ended SOG gyroscope. From this graph, scale factor of the gyroscope is extracted as 1.97 mV/(°/sec), while its nonlinearity is 0.001% in ± 100 °/sec measurement range, which is a quite linear response. For zero rate, bias of the output voltage is measured to be -0.01721 V, which corresponds to -8.736 °/sec.

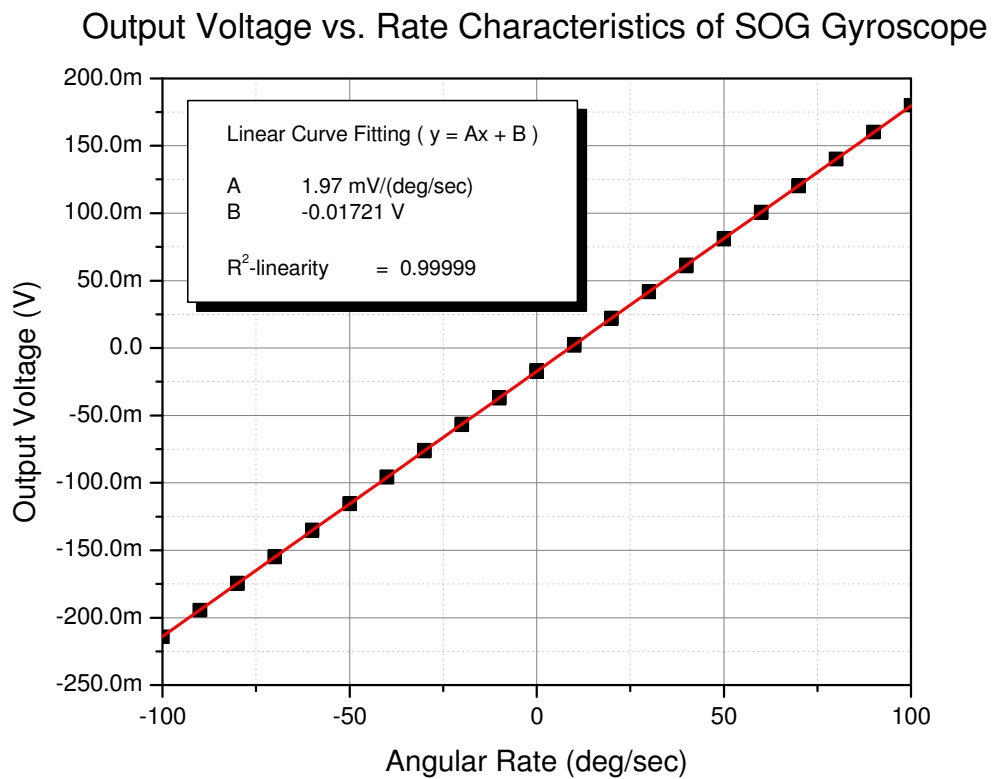


Figure 4.39: Output voltage vs. rate characteristics of single-ended SOG gyroscope.

4.5.2. Random walk and bias instability measurements

IEEE Standard 952-1997 [34] gives the details of required tests for characterizing the random walk and bias instability using Allan variance methodology. In these tests, rate output of the gyroscope is measured for a long time in the absence of rate input. Then, the collected data is processed using an Allan variance analysis tool and the Allan variance curve is obtained. Figure 4.40 shows a sample plot of Allan variance analysis results. The most important two parameters in this curve is the angle random walk, which gives the minimum resolvable rate and bias instability.

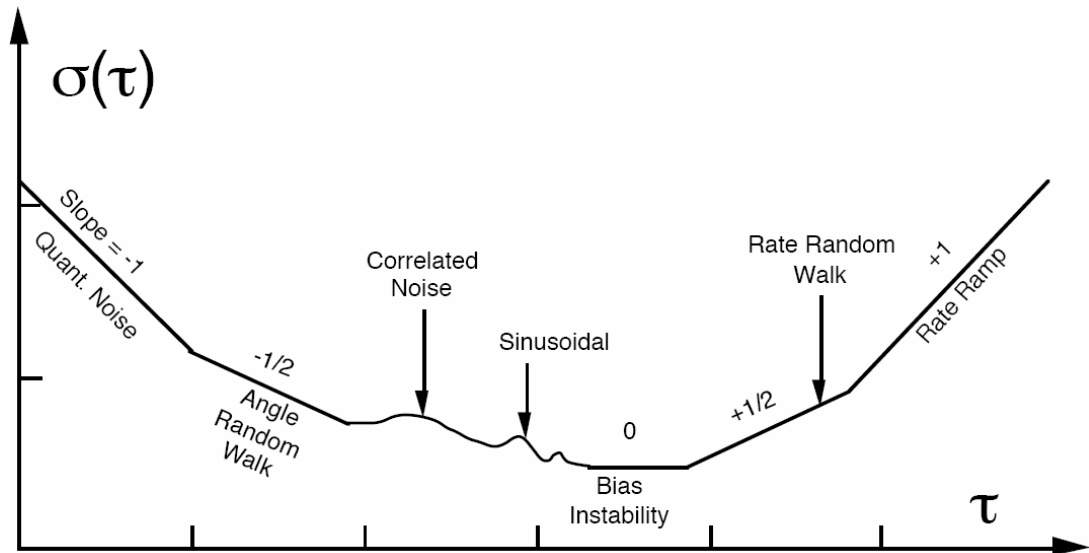


Figure 4.40: Sample plot of Allan variance analysis results [34].

Zero rate output voltage of the gyroscope is measured for a long time (approximately 2 hours sampled at 22 Hz) by using the same measurement setup used for scale factor and linearity tests. After that, the voltage measured is converted to rate by the scale factor of the output. Then, the data is processed with a free Allan variance

software, AlaVar 5.2 [35]. From the plot obtained with this software, angle random walk and bias instability specifications of the gyroscope can be extracted.

Figure 4.41 shows the Allan variance plot for SOG gyroscope using overlapping Allan variance method. From this plot, angle random walk and bias instability values are calculated to be $2.158\text{ }^{\circ}/\sqrt{\text{hr}}$ and $124.7\text{ }^{\circ}/\text{hr}$, respectively. Minimum resolvable rate is $2.158\text{ }^{\circ}/\sqrt{\text{hr}}$, and this value is limited by the noise of the external control and readout electronics. Noise of the capacitive interface is negligible when it is compared with the noise of external readout.

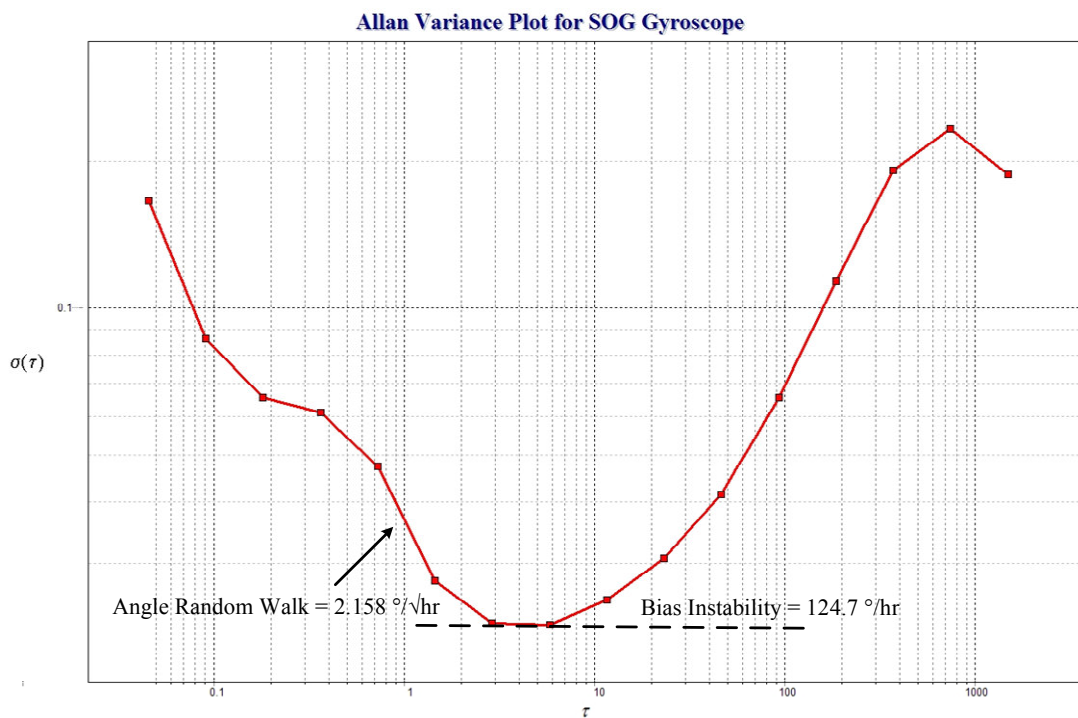


Figure 4.41: Allan variance plot for the SOG gyroscope using the overlapping Allan variance method.

4.6. Summary

This chapter presented the characterization and test results of the capacitive interface circuits developed in this study. Firstly, measurement setup used for characterization of the interfaces is presented. Then, each three designs are tested and their ac characteristics, transient characteristics for a sinusoidal input, and noise characteristics are presented. Moreover, the characteristics of the biasing transistor are obtained showing that high-impedance node can be biased with a resistance up to 60 G Ω . There have been discrepancies between the measurements and simulations in terms of the noise characteristics of the interface and resistance of the biasing transistor. The measured noise characteristics are found to be smaller than the simulated noise characteristics. The possible reason for this discrepancy is due to the insufficient noise model of the subthreshold transistor. The second discrepancy results from the variation of the threshold voltage of the transistor in the process. Measurement results show that the resistance of the transistor may change up to 6 times on a single die. However, measured resistances are in the safe region, and therefore, the operation is not affected much with the change in process.

Secondly, the characterizations of the mechanical sensors of SOG gyroscopes are done by obtaining resonance characteristics of the drive and sense modes using probe station and network analyzer.

Thirdly, preliminary characterizations of SOG gyroscopes with single-ended source followers biased with subthreshold transistors are obtained. The resonance characteristics of the drive and sense modes of the gyroscope are measured and the improvement is observed when the interface is connected to the sensor. Then, these results are compared with the values found from the simulations of the interface with gyroscope model. From resonance measurements with interface and noise measurements of the interface, minimum resolvable capacitance change for 1 Hz measurement bandwidth is found to be 58.31 zF. This value corresponds to 38.87 Fermi displacement in the SOG gyroscope.

Following this, external readout and control electronics for MEMS gyroscopes are presented with a basic introduction to closed-loop drive-mode electronics and open-loop sense demodulation electronics.

Finally, characterization of the SOG gyroscopes with single-ended capacitive interface circuits is done in order to obtain scale factor, linearity, angle random walk and bias instability of the gyroscope. The scale factor of the gyroscope is measured to be 1.97 mV/(°/sec) with a non-linearity of 0.001% in ± 100 °/sec range. Angle random walk and bias instability of the gyroscope are measured to be 2.158 °/√hr and 124.7 °/hr, respectively.

In conclusion, capacitive interfaces designed in this study are characterized and the obtained results are compared with the theoretical values. Most of the results obtained are consistent with the simulated values, although some of them have discrepancies due to modeling or fabrication.

CHAPTER V

CONCLUSIONS AND FUTURE WORK

The research reported in this dissertation includes development and analysis of high-performance CMOS readout electronics for increasing performance of MEMS gyroscopes developed at METU. A generalized gyroscope model is constructed in electrical domain for simulation of gyroscope interfaces with the model of the gyroscope. High-impedance node biasing problem present in previous interface architectures is solved by implementation of a MOSFET operating in subthreshold region. As the preamplifier part of the interface, PMOS source followers are used which gives unity-gain and decreases the input capacitance seen from high-impedance node. Moreover, fully-differential capacitive interfaces are implemented with self-biasing architectures and instrumentation amplifiers are added for fully-differential to single-ended conversion. The interfaces are characterized to verify successful operation. Furthermore, the responses of single-ended capacitive interfaces are measured with the single-ended SOG gyroscopes available.

Based on the results obtained from this study, following conclusion can be made:

1. Transduction mechanisms in MEMS vibratory gyroscopes are investigated and a generalized model for fully-differential gyroscopes with force-feedback electrodes is developed. Generalized model is modified for single-ended SOG gyroscope fabricated at METU and simulations of the resonance

characteristics are done using Cadence IC Design Tool. Results of the simulations match the theory of transduction.

2. Three gyroscope interfaces are designed according to the needs observed in the previous interface architectures. The first design is implemented using a single-ended source follower biased with subthreshold transistor. Simulations show that high-impedance node can be biased with a controlled-impedance in the order of gigaohms. Moreover, source follower structure satisfies the needs of a preamplifier although it has a simple structure. The second design is an improved version of the first design with fully-differential architecture and self-biasing scheme. Additionally, an instrumentation amplifier is implemented to the output of the fully-differential source followers to convert fully-differential signal to single-ended signal. All of these interfaces are fabricated in a $0.6\text{ }\mu\text{m}$ standard CMOS process by XFAB.
3. The designed interfaces are characterized after fabrication. Ac responses of the interfaces are measured for obtaining the gain and phase of the circuit with changing frequency. From these measurements, the gains of the interfaces are measured to be close to 1 as expected. Moreover, phase shift near the operational frequency of 5 kHz are measured to be around -0.4 degrees. Transient characteristics of the interfaces are measured in response to a sinusoidal input. With these measurements, the improvement in the linear range of the interface is observed. Linear ranges of the interfaces are found to be larger than 2.5 V. In addition to these measurements, characterization of the resistance of biasing transistor is done by measuring the charging time at the input node. With these measurements, resistances up to $60\text{ G}\Omega$ are achieved practically. From the results of these measurements, it is concluded that the resistance of the subthreshold transistor heavily depends on the threshold voltage of this transistor. As the threshold voltage of the transistor is process dependent and can vary 0.2 V, the resistance of the

transistor can change drastically from die to die. However, it is shown that the resistances measured are still much larger than the impedance of the stationary capacitance of the gyroscope, and therefore, they do not affect the performance effectively. Noise characteristics of the interfaces are also measured. The measurement results are compared to the simulation results, and it is seen that the measured values are smaller than simulated values. This is probably due to the errors in the model of flicker noise of a transistor and current noise of a transistor operating in subthreshold region.

4. Preliminary tests and characterization of single-ended SOG gyroscopes are done and the improvement of the performance when interface is connected is observed. It is seen that resonance peak in the drive-mode can be improved more than 45 dB. The results are compared with the simulated gyroscope model connected to single-ended interfaces, and it is seen that the simulations and measurements are consistent. From resonance frequency characteristics, minimum resolvable capacitance change of the interface is found to be 58.31 zF in 1 Hz measurement bandwidth. This capacitance change corresponds to 38.87 Fermi displacement for SOG gyroscopes.
5. External readout and control electronics are connected to the gyroscope and operation of the angular rate sensor is observed. Scale factor of 1.97 mV/(°/sec) is measured with a nonlinearity of only 0.001% in ± 100 °/sec measurement range. Zero rate output of the gyroscope is found to be -8.736 °/sec. The bias instability and angle random walk of the gyroscope are calculated using Allan variance method. Angle random walk of the device is found to be 2.158 °/√hr, while bias instability is 124.7 °/hr. Minimum resolvable rate value is limited by the noise of external readout electronics.

Although the major problems in the previous interface circuits are solved and fully-differential interfaces are realized, there is still need for further research for

improving the performance of MEMS gyroscopes developed at METU. Some of the issues that should be solved are explained below:

1. The main problem in the previous capacitive interfaces was biasing the high-impedance node. This problem is solved by implementing a subthreshold transistor. However, future research should consider implementing the biasing structure such that it is not affected profoundly from the process. Moreover, gate-to-source voltage of the subthreshold transistor can be generated on-chip.
2. Although the interfaces are designed to have small noise power density, the noise characteristics of the circuits can be improved. With proper analysis and modeling of the circuit, the noise response of the circuit can be optimized for maximum SNR.
3. Fully-differential gyroscopes should be fabricated and the fully-differential interfaces should be tested with these gyroscopes. Future research must evaluate fully-differential interfaces according to the test results and evince the drawbacks of these circuits, if there are any.
4. For maximizing the performance and eliminate the problems coming from parasitics, external electronics should be implemented on-chip, right after the capacitive interfaces. This will not only increase the performance of the gyroscope, but it will result in more compact devices with a single readout chip.
5. For further studies, monolithic gyroscopes should be considered, on which both the sensing element and readout are implemented. This will increase the performance of the gyroscope drastically since parasitic capacitances at the high-impedance node are eliminated.

In conclusion, the main goal of this research is the development and analysis of high-performance gyroscope interfaces for MEMS gyroscopes developed at METU. The main problem in previous interface designs are solved by implementation of source followers biased with subthreshold transistors. Moreover, fully-differential interfaces are designed for possible future gyroscope designs. In addition to these, a generalized gyroscope model is developed for simulation of interface circuits. Measurements show that the interface circuits are improved with new solutions. It is believed that, the results of this research will be helpful for future research and development of high-performance MEMS gyroscopes for industrial, commercial and military applications.

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